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TABLES
FOR THE FORMATION OF
LOGARITHMS & ANTI-LOGARITHMS
TO
TWENTY-FOUR
OR
ANY LESS NUMBER
OF
PLACES.

ETER GRAY, F. R. A. S.

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T A B L E S

FOR THE FORMATION OF

LOGARITHMS & ANTI-LOGARITHMS

TO

T W E N T Y - F O U R

OR

ANY LESS NUMBER

OF

P L A C E S ;

WITH

EXPLANATORY INTRODUCTION

AND

HISTORICAL PREFACE.

BY

PETER ^aGRAY, F.R.A.S.,

HONORARY MEMBER OF THE INSTITUTE OF ACTUARIES;
AND AUTHOR OF "TABLES AND FORMULE FOR THE COMPUTATION OF
LIFE CONTINGENCIES," "TABLES FOR THE FORMATION OF LOGARITHMS AND
ANTI-LOGARITHMS TO TWELVE PLACES," ETC. ETC.

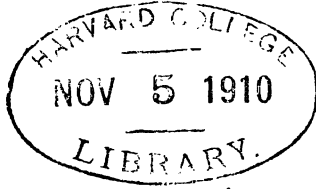
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PREFACE.

I PROPOSE to give here a brief account of the origination of the methods for the formation of logarithms and anti-logarithms which are developed in the succeeding pages.

A systematic and practical method for the formation of logarithms will, probably of necessity, consist in the resolution of the number whose logarithm is required into factors, and the addition of the logarithms of these factors, taken from a table previously prepared; the sum being the logarithm required. Also, the formation of an anti-logarithm will be effected by the decomposition of a given logarithm into a series of logarithms taken from a table, and the multiplication of the corresponding numbers, the product being the required anti-logarithm. The extent of the data is minimized if, which is not necessarily the case, the same table is used in both the direct and the inverse processes; and both processes will depend more or less on the form of the factors whose logarithms are tabulated.

The earliest systematic logarithmic method with which I am acquainted is Mr. Manning's, of which an account is given in the *Philosophical Transactions* for 1806. Mr. Manning's paper is reprinted, ("nearly as it stands," Mr. Young says,) in Young's "*Elementary Essay on the Computation of Logarithms.*"* Mr. Manning first reduces the number to be dealt with, in the manner described on pp. [3] and [32] of the following Introduction, to the form $1 + N_1$, where N_1 is a decimal fraction; and he then multiplies the number thus modified down to unity, by the employment of a succession of factors, $1 - \cdot 1$, $1 - \cdot 01$, $1 - \cdot 001$, &c., (the general form being $1 - \cdot 1^r$, where r takes the values 1, 2, 3, &c., successively,) till the decimal portion of $1 + N_1$ is exhausted. It is obvious that the product of the factors by which this exhaustion is effected will be the reciprocal of $1 + N_1$; and hence that the sum of their co-logarithms will be the logarithm of $1 + N_1$. It is the co-

* Second Edition, London, 1835, pp. 67 to 79.

logarithms, with the first nine multiples of each, which form the data in this method.

Mr. Manning's method is remarkably simple, but it is also exceedingly tedious. Multiplication by a factor of the form $1 - \cdot 1^r$, is effected by subtracting from the multiplicand the multiplicand itself, removed r places to the right; and the resolving process thus consists of series of subtractions of the simplest character, which moreover do not need to be formally continued after the exhaustion of the first half of the decimal places in N . Still, with this abatement, the number of subtractions requisite remains so numerous as to render the process exceedingly irksome. In an example of it now before me, the formation of $\log \pi$ to sixteen places, the number of subtractions is thirty-eight, occupying seventy-six lines, and the number of tabular entries is twenty-one.

The next step, constituting a great advance, was made by Mr. Thomas Weddle, subsequently Professor in the Royal Military College, Sandhurst. In *The Mathematician* for November, 1845, Mr. Weddle gives an account of both a logarithmic and an anti-logarithmic method, "discovered," he says, "nearly seven years ago." The first of these methods he describes as a modification of Mr. Manning's. Mr. Manning's factors are of the form $1 - \cdot 1^n$, when n is always unity; Mr. Weddle's are of the same form, but in them n takes all values from 1 to 9; the effect of which is, that at the cost of a few multipliers by factors of a single digit, the number of subtractions is reduced by about four-fifths. In the example already referred to, involving by Mr. Manning's process thirty-eight subtractions, six only are needed when it is worked by Mr. Weddle's. The number of tabular entries is about the same in both processes.

Mr. Manning did not attempt to apply his data to the converse problem. Mr. Weddle, as intimated, made this application of his data. His tabulated logarithms, (extending to sixteen places,) being those of the reciprocals of the factors, he decomposes the complement of the given logarithm, by subtraction, into a series of values taken from his table; and multiplication of the corresponding factors gives the required number.

The next step, as I suppose, was taken by myself. It consisted in the construction of a new table, to twelve places, in which the factors were of the form $1 - (\cdot 01)^n$, permitting n consequently to take any value of two figures, that is, from 1 to 99. The effect of this was, in both the direct and the inverse processes, a reduction in the number of tabular entries of nearly one-half, and in the number of figures of about two-fifths.

In my table, which, with the requisite description and examples, appeared in the *Mechanics' Magazine* for October and November, 1846, the data were for the first time arranged in columns, corresponding to successive values of r ; while the values of n occupy the argument column.

It will be convenient, in indicating the extent of any specified table, to designate it as a one-figure, a two-figure, a three-figure, &c. table, according as its argument consists of one, two, three, &c. figures, respectively. My table, just referred to, was a two-figure table, and Mr. Weddle's was a one-figure table.

The next publication was a paper by Mr. Hearn, of the Royal Military College, Sandhurst, which appeared in the *Mathematician* for March, 1847. It was entitled "*Practical Method of forming Logarithms and Anti-Logarithms, independently of Extensive Tables;*" and it contained two one-figure tables, to ten places, of which the first was intended for the formation of logarithms, and the second for the formation of anti-logarithms. The first of these was similar to Mr. Weddle's table, and the manner of using it was the same. It therefore needs no further remark here. In the second table the factors, instead of $1 - (\cdot 1)^r n$, were of the form $1 + (\cdot 1)^r n$; so that in its application to the purpose for which it was intended, the subtractions previously requisite were replaced by additions, thus materially improving the anti-logarithmic operation.

I saw that this would be still further improved by the adaptation to it of a two-figure table. I accordingly constructed such a table to twelve places, and I found that in its use my expectations were fully realized. The number of tabular entries was reduced one-half, and the number of additions in a still higher ratio.

My paper, descriptive of this anti-logarithmic method, and accompanied by the new table, appeared in the *Mechanics' Magazine* for February 12, 1848. Soon after the preparation of this paper, I discovered a method of applying a table of Mr. Hearn's form to the construction of logarithms; and I described this method, with an example, in a paper which appeared in the same journal a fortnight later, in the number for February 26.

The result of my extension of the two tables had proved so satisfactory, as regards the abbreviation and improvement in other respects of the processes, that I now resolved on the construction of a three-figure table, in which the data should extend to twenty-four places, and which would therefore be available for formations to the number of places named. I adopted for the extended table the form proposed

by Mr. Hearn, as possessing on the whole the greatest facilities for both the direct and the inverse operations.

It is the table resulting from this extension that forms the basis of the present Work. The methods employed in the construction and verification of it are briefly described in Section IV. of the Introduction. The labour attending these operations, (in which I had no assistance,) was no doubt very considerable; but, from the evidence already acquired of the power and utility of the table, I am satisfied that the labour was not ill bestowed.

The table was completed upwards of twenty years ago. Knowing it was not of a character likely soon to repay the cost of printing, I suffered it to lie by me in manuscript, (using it occasionally for my own purposes,) for a number of years. At length I made an abridgment of it, adapted for formations of twelve places, which, with the requisite explanatory matter, I communicated to the *Assurance Magazine*, in the twelfth volume of which my papers appeared.* They were subsequently collected and issued in a separate form, in 1865, by Messrs. Layton. This tract, after some years, came under the notice of a gentleman with whom I had no previous acquaintance, Thomas Warner, Esq., F.R.A.S., of 47, Sussex Square, Brighton. Mr. Warner opened a correspondence with me, in the course of which he was pleased to speak very favourably of the probable utility of the large table, and he offered me a most handsome contribution towards the expense of putting it in print. Having mentioned this circumstance to two gentlemen interested in such matters, they each offered, quite spontaneously, a liberal contribution, in supplement of Mr. Warner's, towards the same object. I have pleasure in naming the gentlemen who gave this gratifying proof of their appreciation of the value of the table. They were, J. W. L. Glaisher, Esq., F.R.S., of Trinity College, Cambridge, and H. D. Hoskold, Esq., Civil and Mining Engineer, of Dean Forest, Gloucestershire.

Encouraged by the approval of such competent judges, I put the table to press. During the preparation of the introductory matter, I have had many suggestions, by which I have profited, from Mr. Glaisher, Mr. Warner, and my friend Major-Gen. Hannynghton, to all of whom I desire here to record my thanks. To Mr. Glaisher, in particular, for the kindly interest he has taken in the progress of my Work, and the encouragement thus afforded me, my special thanks are due.

It may be fitting that, ere I close, I should advert briefly to other

* I think it needful to mention that, in the papers referred to, I explained and exemplified the use of both Weddle's and Hearn's one-figure tables, and of my own two-figure tables.

methods that have been proposed for the formation of logarithms, akin to those I have described, and to other publications of the same or similar methods.

The first in order of date with which I am acquainted are two methods suggested by the late Mr. William Orchard, in a letter which appeared in the *Mechanics' Magazine* for Feb. 26, 1848, the same number which, as already intimated, contained the first example of my present logarithmic method. In his first method, Mr. Orchard, availing himself of my extended table of Hearn's form, dealing with the number whose logarithm is required as a decimal fraction, multiplies it *up* to unity by the use of the tabular factors. The sum of the logarithms of the factors employed is the co-logarithm of the given number. The operation in this method is a very compact one.

In Mr. Orchard's second method he proposed to multiply *up* the number as before, by a series of factors of the form $(1.1)^n$, $(1.01)^n$, $(1.001)^n$, &c., n having in each case a *suitable* value, never exceeding 9; and he used the binomial factors in the multiplications. As the method was only suggested, Mr. Orchard gave no table for its application.

The next in order is Mr. Oliver Byrne, whose method is described in a work published in 1849, entitled, "*Practical, Short and Direct Method of Calculating the Logarithm of any Given Number, and the Number corresponding to any Given Logarithm.*" Mr. Byrne informs us in his Introduction that he discovered his method "about twenty years ago."

Mr. Byrne having, by an ingenious application of Lagrange's Theorem, provided himself with a series of ten numbers, ranging from 1 to 10, the *figures* of which are the same as those of their logarithms, respectively, generally multiplies the given number *up* to the next greater of these tabulated numbers, by the requisite factors of the form employed in Orchard's second method; effecting the multiplications, also as in Orchard's method, by aid of the binomial co-efficients. The sum of the logarithms of the factors, subtracted from the number employed (or its logarithm) is the logarithm required. The method, it thus appears, is virtually Orchard's, the main distinction being that, in the last named, the final subtraction is the formation of an arithmetical complement.

I refer next to Gen. Shortrede's *Logarithmic Tables*, published in 1849. Gen. Shortrede being home on furlough, and occupied with the issue of his tables, my papers in the *Mechanics' Magazine* came under his notice, as also Weddle's and Hearn's papers in the *Mathematician*. He computed extended tables of the forms used by those gentlemen, and introduced them into his volume as follows:—first, a two-figure

table of each form, to 16 places; and, secondly, a one-figure table, also of each form, to 25 places. General Shortrede makes no claim to originality in the matter, but duly indicates the sources of his information.*

I have next to notice "*Tables de Logarithmes à 27 Décimales pour les Calculs de Précision*. Par Fédor Thoman; Paris, 1867." In this work the author gives one-figure tables of both Weddle's and Hearn's forms, and his methods are *essentially* those of the writers named. With a view to abbreviation of the work, he introduces certain modifications, involving the use of several auxiliary tables; but it may fairly be doubted whether his success in this respect is more than compensation for the added complexity in the operations.

M. Thoman does not state how far he lays claim to originality in the methods he presents. It is, I think, not unlikely that he was acquainted with what had been done by Weddle and Hearn;† and if so, it will generally be considered that he ought to have named them.

I have now to refer to, first, a tract of 16 pages, entitled "*A Simple Method of calculating Logarithms to any number of Figures*," London, Pickering, 1873; and, secondly, a paper headed, "*On the Calculation of Logarithms*," in the *Messenger of Mathematics*, vol. iii., pp. 66 to 92. The author of the tract and paper just mentioned is the Rev. Henry Wace, Brasenose College, Oxford, Chaplain of Lincoln's Inn. The method, (which term, as used by Mr. Wace, means both a logarithmic and an anti-logarithmic method,) developed in the tract, Mr. Wace states was "suggested by a study of Mr. Oliver Byrne's *Dual Arithmetic*, or rather of his *Young Dual Arithmetician*;" and he informs us in the paper, (which is an amplification of the tract,) that he "worked it out in its present form before he knew that anything of the kind had been attempted before." He tells us further that, his tract having come under the notice of Mr. Glaisher, that gentleman "at once recognized in it the substance of a method already known," which was "discovered in 1845 ‡ by Mr. Weddle," &c.

Substantially, then, Mr. Wace's logarithmic method is that of Mr. Weddle, and his anti-logarithmic method is that of Mr. Hearn, of whose claim however he does not appear to have been aware. The paper

* It is only recently that I have noticed that, in General Shortrede's two-figure table of Hearn's form, the second half of Col. I. is wanting. The omission is legitimate in a table of Weddle's form, but not so in one of Hearn's.

† My papers in the *Assurance Magazine*, (of which Magazine, as a writer on Interest and Annuities, M. Thoman is probably a reader,) in which the methods in question are described, may have come under his notice. My Example 15 in those papers is Example 19 in M. Thoman's tract. The example was original with me, and it is hardly one likely to have suggested itself to independent writers. It is Example 21, p. [22], in the present Work.

‡ Seven years previous to 1845. See p. iv. *ante*.

comprises a one-figure table, to twenty places, of both forms. The manner in which his methods are worked out by Mr. Wace is deserving of high commendation for its elegance and perspicuity.

I should have liked to give an example of each of the methods to which reference has been made. Want of space has prevented this; but I have in all cases indicated where the examples may be found. I have now a method to explain which has not hitherto appeared in print, and of which therefore it is necessary to exhibit an example. This method has been devised by Mr. Warner, to whom I have already referred in this Preface. It consists in an application of my three-figure table, an early copy of which I had communicated to him.

Mr. Warner's example follows:—

$\pi = \begin{array}{r} \cdot 314\ 159\ 265\ 358\ 979\ 323\ 846\ 264 \\ \hline \cdot 998\ 52 \\ \quad 505\ 62 \\ \quad \quad 842\ 70 \\ \quad \quad \quad 1\ 138\ 44 \\ \quad \quad \quad \quad 3\ 113\ 22 \\ \quad \quad \quad \quad \quad 1\ 027\ 14 \\ \quad \quad \quad \quad \quad \quad 2\ 690\ 28 \\ \quad \quad \quad \quad \quad \quad \quad 840 \\ \hline \cdot 999\ 026\ 463\ 841\ 554\ 249\ 831\ 120 \\ \quad 973\ 026 \\ \quad \quad 25\ 324 \\ \quad \quad \quad 450\ 962 \\ \quad \quad \quad \quad 819\ 134 \\ \quad \quad \quad \quad \quad 539\ 596 \\ \quad \quad \quad \quad \quad \quad 242\ 526 \\ \quad \quad \quad \quad \quad \quad \quad 809 \\ \hline \cdot 999\ 999\ 515\ 617\ 335\ 923\ 670\ 455 \\ \quad \quad 483\ 516 \\ \quad \quad \quad 483\ 516 \\ \quad \quad \quad \quad 249\ 260 \\ \quad \quad \quad \quad \quad 298\ 628 \\ \quad \quad \quad \quad \quad \quad 162\ 140 \\ \quad \quad \quad \quad \quad \quad \quad 447 \\ \hline \cdot 999\ 999\ 999\ 617\ 101\ 482\ 461\ 042 \\ \quad \quad \quad 381\ 618 \\ \quad \quad \quad \quad 381\ 618 \\ \quad \quad \quad \quad \quad 381\ 618 \\ \quad \quad \quad \quad \quad \quad 235\ 694 \\ \quad \quad \quad \quad \quad \quad \quad 39 \\ \hline \cdot 999\ 999\ 999\ 999\ 101\ 482\ 314\ 775 \\ \quad \quad \quad \quad 898\ 517\ 685\ 225 \\ \quad \quad \quad \quad \quad 5 \quad 6 \quad 7 \quad 8 \end{array}$	$318 = 1 \cdot 272 \div 4$ $974 \quad 2$ $484 \quad 3$ $382 \quad 4$ $382 \quad 4$ $272 \quad 1$ $974 \quad 2$ $484 \quad 3$ $382 \quad 4$ $898 \quad 5$ $517 \quad 6$ $685 \quad 7$ $225 \quad 8$ $-\log \pi$
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Here, as in Orchard's method, the given number is multiplied up to unit in the next higher denomination, the sum of the logarithms of the

a*

factors employed being obviously the co-logarithm of the given number; but the requisite multiplications are performed by the aid of Crelle's Table, which gives (twice over) the product of every pair of numbers consisting of not more than three places.* There is, as will be seen, a remarkable adaptation in Crelle's three-figure factors to the three-figure periods of the logarithmic table employed. The first step is a preparatory one. The number is multiplied by that factor of three figures which will give the greatest product not exceeding unity, and which factor consequently must not exceed the reciprocal of the first period of the number. Turning to Barlow or Oakes, we find for the reciprocal of 314, the first period, 3184713. The number therefore is to be multiplied by 318, and this is done period by period. Opening Crelle at 318, we find on the same page all the products we want. Thus, opposite 314 we have 99852, opposite 159 we have 50562, and so on; and each product, as it is taken out, is arranged, under the multiplicand, three places in advance of the preceding. The sum of these partial products is the entire product of 314159 by the factor 318.

It should be noted that in this preparatory operation, the partial products are all set down one place to the left of what would ordinarily be their position, the object being to fill up the first period preparatory to the subsequent operations. This causes no derangement, since the multiplicand has not here, as in the succeeding steps, to be included in the addition.

The factor here used is to be broken up, if necessary, by multiplication or division, (as shown in the margin,) into two factors, of which one has unit for its leading figure; and so the logarithms of both can be obtained from the tables.†

The factors employed in the multiplying up being of the form $1 + (.001)^n$, a little consideration will show that, generally, n , the multiplier at any point, will be the complement to 999, of the first period of the multiplicand that differs from 999. This will be observed to hold in the example in the case of all the multipliers except that which comes into use in the step immediately following the preparatory step. The cause of the exception is, that in the second period there is a large defect from 999, and the multiplicand is consequently not sufficiently near to unity to allow the rule to hold. This causes but little inconvenience. For, opening Crelle at 973, (the complement to 999 of 026,) we see that the product ($973 \times 999 =$) 972027 is insufficient to bring the second period up to 999, even when allowance is made for

* It is, in fact, a multiplication table, whose limit is 1000×1000 .

† Instead of $1.272 \div 4$, 318 might be broken up into $1.908 \div 6$, 1.590×2 , or 1.060×3 .

the effect of the succeeding partial products. We therefore use 974, the product of which by 999, namely, 973026, is found on the same line as that of 973; and this product answers.

In the next steps we see that 515 and 484 are complementary, and so also are 617 and 382. When this last multiplier has been used, the first four periods have been each brought up to 999; and the remaining multipliers, (with which it is of course unnecessary to operate,) are the complements of the last four periods.

The logarithms of the factors taken from the tables are used as shown; and the arithmetical complement of their sum is the required logarithm, which is here true in the last place.

It is worthy of being noted, that while the entries in Crelle, for the third and fourth steps, are apparently six and five respectively, in consequence of repetitions they are in reality only five and three.

We shall not, however, always succeed in bringing up the first period to 999 by the preparatory step as above prescribed. This is shown in the following example:—

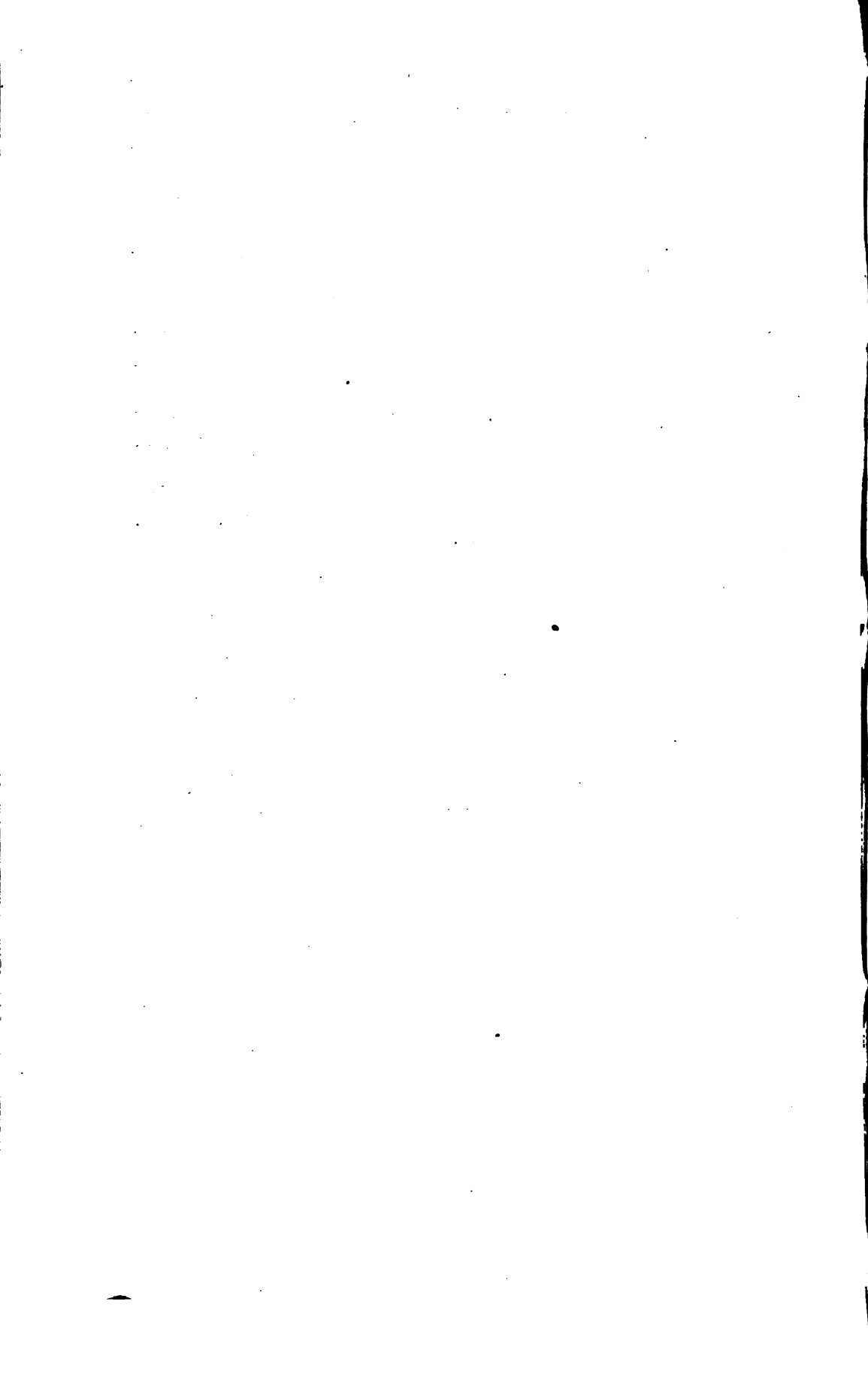
$N = 320\ 908\ 469\ 721$ $\underline{995\ 20}$ $2\ 823\ 88$ $\quad 1\ 458\ 59$ $\quad \quad 2\ 242$ $\underline{998\ 025\ 340\ 832}$ $\quad 998\ 025\ 341$ $\underline{999\ 023\ 366\ 173}$ $\quad 976\ 023$ $\quad \quad 22\ 471$ $\quad \quad \quad 358$ $\underline{999\ 999\ 412\ 002}$ $\quad \quad 587\ 998$	$\times 311 = 1.244 \div 4$ $001\ 1$ $977\ 2$	$\cdot 397\ 940\ 008\ 672$ $\quad 94\ 820\ 380\ 355$ $\quad \quad 434\ 077\ 479$ $\quad \quad \quad 424\ 098\ 570$ $\quad \quad \quad \quad 254\ 931$ $\quad \quad \quad \quad \quad 433$ $\underline{\cdot 493\ 618\ 820\ 440}$ $\cdot 506\ 331\ 179\ 560$	$\left. \begin{array}{l} \text{colog } 4 \\ 244 \\ 001 \\ 977 \\ 587 \\ 998 \end{array} \right\} 1$ $977\ 2$ $587\ 3$ $998\ 4$ $-\log N$
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Proceeding as directed, we find the reciprocal of 320, the first period, 3125. But if we use 312 as multiplier, we shall overshoot the mark, the first period of the product being 1001. We must therefore use 311. This, however, we see, gives for first period only 998. But a remedy at once presents itself. Multiplication by 001 effects the desired result, giving rise, in the second part of the operation, to an additional entry in Col. I.

The advantages gained by the employment of Crelle's table in the multiplications are, greater facility, with at least equal assurance of accuracy in the results; and, generally, a saving in the number of figures written. In the first of the foregoing examples there is a saving of eighty figures; but in consequence of the additional entry in Col. I., there is no saving in the second.

In the compounding process, for finding the anti-logarithm, the use of Crelle's Table is sufficiently obvious.

LONDON, 25 Nov. 1876.



INTRODUCTION.

IN the present work computers are furnished with a method of forming logarithms and anti-logarithms to a greater number of places than are afforded by the common tables; which method, it is believed, will be found to be easier and more efficient than any heretofore available. The principle of the present method, which is also that of all other *systematic* methods that have been proposed for a like purpose, is the resolution of the number whose logarithm is required into factors whose logarithms are tabulated; and the logarithm is obtained by summation of the logarithms of those factors, taken from the table. The converse operation—the formation of anti-logarithms—adapts itself to the provision made for the performance of the other.

The specialty of the method now proposed, being that on which its claim to attention chiefly rests, consists in the peculiar form and the extent of the factors into which the number is resolved, which, while admitting of a lucid and systematical arrangement of the logarithms of the factors, also lead to easy and elegant arithmetical processes for the *resolution* required in the formation of a logarithm, and the *composition* required in the formation of an anti-logarithm.

I.—DESCRIPTION OF THE TABLES.

The tables are two, the Auxiliary Table, and the Principal Table.

The Auxiliary Table is on page 1. It contains simply the logarithms and the co-logarithms, that is, the logarithms of the reciprocals, of the natural numbers, 1 to 9.

The Principal Table occupies pages 2 to 41. It consists of five columns, headed I., II., III., IV., V., respectively. Three more columns, VI., VII., and VIII., are wanted when the table is used to its full extent; but these are implicitly contained in Col. V., and therefore do not need to be separately exhibited. For Col. VI., the last period is dropped from Col. V.; for Col. VII., the last two periods; and for Col. VIII., the last three.

The arguments for the use of the table, which extend from 000 to 999, occupy the small side column, headed n ; and the values in the several main columns are related to the corresponding argument, as follows:—If n denote any argument, then we have corresponding,

$$\begin{aligned} &\text{in Col. I., } \log(1 + \cdot 001 n), \\ &'' \text{ II., } \quad \quad \quad (1 + \cdot 001^2 n), \\ &'' \text{ III., } \quad \quad \quad (1 + \cdot 001^3 n), \\ &'' \text{ IV., } \quad \quad \quad (1 + \cdot 001^4 n), \\ &'' \text{ V., } \quad \quad \quad (1 + \cdot 001^5 n); \end{aligned}$$

and so on. Thus, corresponding to argument 475 we have,

$$\begin{aligned} &\text{in Col. I., } \log 1 \cdot 475, \\ &'' \text{ II., } \quad \quad \quad 1 \cdot 000 \ 475, \\ &'' \text{ III., } \quad \quad \quad 1 \cdot 000 \ 000 \ 475; \end{aligned}$$

and so on, up to Col. VIII., the logarithm in which is that of $1 \cdot (000)^7 475$, the

symbol (000)⁷ indicating that seven periods of the form shewn intervene between the initial and the terminal figures of the number represented.

Anterior periods of the logarithms in the columns after the first, consist of ciphers, and are not inserted. Col. II. has one such period, Col. III. has two, and so on; and from this it will appear that in all the columns the last figure occupies the twenty-fourth decimal place.

It will be observed that numbers are placed in the margin of the several pages of the table. On pages 20 and 21, for instance, we find 450, and underneath, in different type, ¹⁶¹195. The first, 450, is the leading argument on the opening presented; and the others are the first periods of the leading values in Cols. I. and II. respectively. The purpose of the insertion of those numbers is to facilitate the use of the table; and they will be found to answer this purpose very effectually.* The upper number guides to the proper opening when the table is used *directly*, that is, when the operation in hand is the formation of a logarithm; and the lower numbers serve a like purpose when the table is used *inversely*, that is, when the operation is the formation of an anti-logarithm.

II.—ON THE USE OF THE TABLES.

The tables are adapted to the formation of logarithms and anti-logarithms of any number of places not exceeding twenty-four; and the two problems thus arising will be dealt with in order.

It might seem at first sight that the simpler and more natural order of proceeding would be, to commence with cases in which the number of places required in the logarithm or anti-logarithm to be formed is comparatively small, say, not exceeding twelve, and to ascend thence until the higher limit, twenty-four, is reached. I have found it otherwise. In the suggested mode of proceeding, a separate set of rules for each extent of the *quæsitum* would be requisite, whereby we should be overloaded with rules; whereas, when the alternative mode of proceeding is adopted, and commencement is made with the case in which the tables are called into use to their full extent, the rules for that case are found to suffice for all. In descending from the more to the less extended cases the manner of operating in each follows so easily and naturally from that in the preceding, that nothing more is needed than a remark calling attention to the point or points of divergence.

PROBLEM I.—To find the logarithm of a given number to twenty-four places.

Case First.—When the given number is one whose leading figure is unit.

The resolving process—the resolution of the given number into factors of the requisite form—constitutes the principal part of the work in the solution of this problem; and it is thought that the rules to be immediately given will be more readily apprehended if a summary description of the process in question is premised.

The process consists of a division, which, so far as the dividend—the given number—is concerned, is strictly continuous. But the divisor varies: it receives accessions from point to point. These points are well marked, and the formation of the successive divisors is an operation of extreme simplicity. The dividend, as just hinted, is the given number, and the successive divisors constantly approximate to the same number. The fourth divisor, so far as it needs to be formed, is practically identical with the given number to the same extent; and upon its formation the process merges into pure *contracted* division.

I now give the rules for the systematic conduct of the operation.

1. Place the decimal point after the first figure of the given number, and separate the portion of this which follows the point into eight periods of three

* The idea of the arrangement above described was obtained from the well-known English reprint of Lalanda.

figures each, supplying the place of deficient figures by ciphers, or cutting off such as would extend beyond the eighth period. It is convenient to distinguish the periods by placing the proper numeral over each.*

2. Separate, for the first divisor, in the usual way, the leading figure and the first period of the number, and by it divide the remaining portion till three quotient figures have been obtained; observing that one such figure must be got for each figure taken in or taken down from the third period.

3. For the second divisor take, besides the leading figure, the first three periods of the given number diminished by the remainder of the previous division, the figures of which remainder will be found to stand directly under those of the number from which they have to be subtracted. This admits of the subtraction being most readily performed, and the result—the new divisor—can be at once set down in its place. Annex now to the previous remainder the fourth and fifth periods of the given number, and proceed with the division till another triad of quotient figures has been obtained. This will use up the sixth period.

4. The formation of the third divisor is in strict analogy with that of the second, namely, by subtraction of the remainder of the second division from the portion of the number over it. The divisor would thus comprise six periods. But, contraction commencing at this point, we want and can make use of only five periods. The divisor is, therefore, formed only to this extent; and the previous remainder being extended by the annexation to it of the seventh and eighth periods of the number, the division goes on, now in the contracted form, one quotient figure being got for each figure struck off from the divisor, till another triad has been obtained.

5. The fourth divisor is formed in the same way as the others, but so as to comprise only four periods. It will generally coincide with the given number to the same extent, never differing from it by more than a unit in the last place. The division now proceeds till the number is exhausted, four more quotient triads being obtained in the process.

6. Enter now the columns of the principal table, in order, with the eight triads, of which the first is that which appears in the first divisor, and the others are the seven formed by division. The sum of the results, with the proper index prefixed, which will be determined by reference to the given number, will be the required logarithm.

Case Second.—When the number whose logarithm is required is one whose leading figure is other than unit.

1. Multiply or divide the given number by any number, (which will be called the preparing number,) consisting of a single digit, that will give a result having unit for its first figure; and proceed with this result, as regards the resolving process, as in the first case with the given number.

2. With the preparing number enter the auxiliary table, of logarithms if the preparation was made by division, or of co-logarithms if it was made by multiplication; and the successive columns of the principal table, as before, with the eight triads formed in the operation. Summation of the results will, as in the former case, give the mantissa of the required logarithm, to which the proper index must be prefixed.

Every number admits of four or five modes of preparation, as above prescribed, by the employment of different preparing numbers. Two of them may be used when verification is desired.

The rules laid down will now be illustrated by examples.

Example 1.—Find, to twenty-four places, the logarithm of the number 1.2599 as below.

* The entire process is much facilitated by the employment of paper ruled in squares, having every third vertical line deepened either by pencil or red ink.

unused; contraction therefore here commences, and the third divisor is formed one period short, and so as to contain five in all. Division, now in the contracted form, gives the quotient triad 174, and remainder 1218714

The fourth divisor, which takes up and continues the work of the third, is of the same extent to which this has been reduced in the formation of the fourth triad; and it is usually, as here, identical with the given number to a like extent. Division by it gives, without further interruption to continuity, the remaining four triads 967, 294, &c.* It will be observed that, in the present example, the divisor last formed is pushed one place to the left by an encroachment of the same extent by the remainder. This occasionally, but rarely, happens in the case of either the third or the fourth divisor.†

By the process above described, the given number is resolved into the following factors:—

$$\begin{array}{r}
 1\cdot259 \\
 1\cdot000,731 \\
 1\cdot000,000,572 \\
 \vdots \\
 1\cdot(000)^7182
 \end{array}$$

The remaining portion of the work, which is the direct formation of the logarithm, needs no remark. The eight triads which form the arguments are set down at the side, and, in line with them, their results. The sum of these, ·100343 . . . with the index 0 prefixed, is the logarithm required.

The given number in the present example is the cube root of 2; and, as it happens, the logarithm formed is true in the last place, as appears by comparison of it with the logarithm of 2. I say, “as it happens,” because obviously the degree of accuracy here attained is not such as can be expected always to subsist. Considering that a large part of the resolving process consists of *contracted* division, and that the logarithms summed are only *conventionally*, not *absolutely*, true,‡ we must be prepared for occasional departures from exactitude in our results. It is not easy to assign a limit to the amount of error that is possible in the formation of a logarithm by this method; but I am able to say, that in the course of my experience I have in only one instance, so far as I am aware, found an error exceeding a unit in the last place; and, what may seem singular, it was in the formation of a logarithm of twelve places that the error referred to, amounting to 2, was found.

It will be convenient to have the means of indicating in a compact form the results of the resolving process when applied to a specified number, the method above used being cumbersome, and occupying too much space. For this purpose I shall enclose in parentheses the triads given by the process, and which are the tabular arguments, in order; and the results may then be exhibited in the form of equations, which will be called the analyses of the numbers represented.

* The numbers attached to the several quotient triads, while indicating their order, also direct to the columns of the table to be entered with them respectively.

† It may be well to show here the actual composition of the several divisors.

1st Divisor.	<u>1·259</u>
	1·259 921 049
	720
2nd ”	<u>1·259 920 329</u>
	1·259 921 049 894 873
	220 444
3rd ”	<u>1·259 921 049 674 429</u>
	1·259 921 049 894
	1
4th ”	<u>1·259 921 049 893</u>

It will be noted that the last figures of the third and fourth divisors, which are here 9 and 3, appear in the example as 8 and 4, respectively. The reason is that these last are seen to be nearest to the truth if the divisors were extended. But attention to this matter is a refinement, neglect of which will rarely have an appreciable effect on the final result. The first step in the employment of those divisors is to cut off their last figure.

‡ That is, although “true to the nearest figure in the last place,” yet, being interminate numbers, they all err either in excess or defect.

Thus, the analysis of the number given in the present example will be denoted as follows:—

$$\sqrt[4]{2} = (259:731:572:174:967:294:041:132).$$

The n th factor can be immediately written out at length by prefixing to the figures of the n th triad, $1 \cdot (000)^{n-1}$. As an instance, the fourth factor here is $1 \cdot (000)^3 174$, that is, 1·000 000 000 174.

Example 2.—Find the logarithm, to twenty-four places, of e , the Napierian base, = 2·71828 as below.

	1	2	3	4	5	6	7	8	
	271	828	182	845	904	523	536	028	7) × 4
	1·087	312	781	383	618	094	144	115	287 2
		217	4						
		95	38						
		86	96						
		8	871						
		7	609						
	1 087	311	969	762	883	618	0		701 3
		761	118	378	3				
		1	265	239	794				
		1	087	311	969				
	1 087	312	731	205	699	177	927	825	144 115 163 4
		108	731	273	120	569			
		69	196	552	023	546			
		65	238	763	872	341			
		3	957	788	151	205			
		3	261	938	193	617			
	1 087	312	781	383	695	849	957	588	639 5
					652	887	638	830	
					43	462	318	758	
					82	619	381	941	
					10	842	936	817	
					9	785	814	582	
					1	057	122	235	972 6
					978	581	458		
					78	540	777		
					76	111	891		
					2	428	886		
					2	174	625		
					254	261			233 7
					217	463			
					36	798			
					32	619			
					4	179			
					8	262			
					917				844 8
					870				
					47				
					43				
					4				
	397	940	008	672	087	609	572	522	colog 4
	86	229	544	086	294	539	926	257	087 1
	124	624	683	526	633	340	112		287 2
	304	440	325	107	858	048			701 3
	70	790	000	544	461				168 4
	277	514	173	936					639 5
	422	184	236						972 6
	101	191							233 7
	367								844 8
	0·434	294	481	903	251	827	651	130	—M

The number here proposed falls under the second case: its first figure is greater than unit. It needs preparation, and for this purpose multiplication by 4 is employed. Multiplication by 5, 6, or 7 would have also answered; but in

using 4 we have the smallest first divisor we should obtain, by which the work is somewhat facilitated.

The following is the analysis of the number:—

$$\epsilon = 10 \times 4^{-1}(087:287:701:163:639:972:238:844),$$

in which 4 is the preparing number.

The logarithm formed is the modulus of the common logarithms, and as such is usually denoted by M. The last period, which is here 180, should be 129.

Example 3.—Required, to twenty-four places, the logarithm of π , the semi-circumference to radius unity.

	1	2	3	4	5	6	7	8	
$\pi = 3.141592653589793238462643$									÷ 2
1.570796326794896619281322									507 2
785 0									
11 326									
10 990									
1 570 795 990)336 794 896 6									214 3
314 159 198 0									
22 635 698 61									
15 707 959 90									
6 927 738 719									
6 233 183 960									
1 570 796 326 150 34)644 554 759 231 322									410 4
628 318 530 460 137									
16 236 228 771 185									
15 707 963 261 503									
1 570 796 326 794)528 265 509 682									396 5
471 238 898 038									
57 026 611 644									
47 123 889 804									
9 902 721 840									
9 424 777 960									
477 943 880									304 6
471 238 898									
6 704 982									
6 233 185									
421 797									268 7
314 159									
107 638									
94 248									
13 390									
12 566									
824									525 8
785									
39									
31									
8									
.301 029 995 663 981 195 213 739									log 2
.195 899 652 409 233 736 761 481									570 1
220 131 503 702 946 921 894									507 2
92 939 009 182 822 263									214 3
178 060 737 543 831									410 4
145 922 945 919									336 5
132 025 522									304 6
116 391									268 7
228									525 8
0.497 149 872 694 133 854 351 268									-log π

The logarithm here formed is true in its last place.

It sometimes happens that, by the employment of different modes of preparation, we obtain for the same number series of factors which, after the

first, are identical. Thus, for the number π , if we prepare with 4 and 6 as multipliers, we have the following analyses, to each of which I append the last period of the logarithm it yields:—

$$\pi = \begin{matrix} (10 \times 4^{-1}(256:507:214:410:336:304:268:525) \dots 268 \\ (10 \times 6^{-1}(884:507:214:410:336:304:268:523) \dots 267 \end{matrix}$$

In these two analyses, as well as in that which arises as above from division by 2, the second and following factors are identical. The identity arises from the *casual* circumstance that in each of the three cases the significant figures in the product of the preparing number and the first factor are the same. Thus, $1570 \times 2 = 3140$, and $1256 \times 4^{-1} = 1884 \times 6^{-1} = 314$. The figures that arise in the three processes are altogether different.

I subjoin another analysis of the same number, which, with those just given, will serve as further examples if any choose to work them out:—

$$\pi = 3(047:188:682:962:908:664:150:368) \dots 269$$

I shall now give some examples of formations to fewer than twenty-four places. The number π , used in last example, will be retained, curtailing it of one period in each succeeding example. The degree of accuracy attained in each case will thus be plainly visible.

Example 4.—Find the logarithm of π , to twenty-one places.

$\pi = 3$	1	2	3	4	5	6	7	÷ 2
1570	796	326	794	896	619	232	507	2
		785 0						
		11 326						
		10 990						
$1\ 570\ 795\ 990$	336	794	896	6			214	3.
		314	159	198	0			
		22	685	698	61			
		15	707	959	90			
		6	927	798	719			
		6	283	183	960			
$1\ 570\ 796\ 826\ 15$	644	554	759	232			410	4
		628	318	590	460			
		16	236	228	772			
		15	707	963	262			
$1\ 570\ 796\ 826$	327	528	265	510			336	5
		471	238	898				
		57	026	612				
		47	123	890				
		9	902	722				
		9	424	778				
		477	944				304	6
		471	239					
		6	705					
		6	283					
		422					269	7
		314						
		108						
		94						
		14						
		14						
$301\ 029\ 995\ 663\ 981\ 195\ 214$							log 2	
$195\ 899\ 652\ 409\ 233\ 736\ 761$							570	1
$220\ 131\ 503\ 702\ 946\ 922$							507	2
$92\ 939\ 009\ 182\ 822$							214	3
$178\ 060\ 737\ 544$							410	4
$145\ 922\ 946$							336	5
$132\ 026$							304	6
117							269	7
$0.497\ 149\ 872\ 694\ 188\ 854\ 852$							= log π	

The changes here are, that the third and fourth divisors have each one period fewer than in the twenty-four-figure process. The formation of the fourth divisor might, indeed, be dispensed with entirely, by merely altering, when necessary, the last figure of the third period in the previous divisor. The alteration is, however, seldom necessary. It is made here, and it produces no effect. The logarithm formed errs in its last place by a unit in excess.

Example 5.—Required the logarithm of π , to eighteen places.

$\pi = 3 \cdot 141\ 592\ 653\ 589\ 793\ 238$	1 2 3 4 5 6	$\div 2$
$1 \cdot 570)796\ 326\ 794\ 896\ 619$	785 0	507 2
	11 326	
	10 990	
$1\ 570\ 795\ 990)336\ 794\ 896\ 6$	314 159 198 0	214 3
	22 635 698 61	
	15 707 959 90	
	6 927 738 719	
	6 283 188 960	
$1\ 570\ 796\ 32\cancel{0})644\ 554\ 759$	628 318 530	410 4
	16 236 229	
	15 707 963	
	528 266	336 5
	471 239	
	57 027	
	47 124	
	9 908	
	9 425	
	478	304 6
	471	
	7	
	6	
$301\ 029\ 995\ 668\ 981\ 195$		log 2
$195\ 899\ 652\ 409\ 233\ 737$		570 1
$220\ 181\ 503\ 702\ 947$		507 2
$92\ 989\ 009\ 188$		214 3
$178\ 060\ 738$		410 4
$145\ 923$		336 5
132		304 6
$0 \cdot 497\ 149\ 872\ 694\ 133\ 855$		= log π

The third divisor here loses one period, and it is identical with the dividend as far as the third period of the latter, since its next figure would be 1. There is consequently no need for forming a fourth divisor. The logarithm formed errs by a unit in excess.

Example 6.—Required the logarithm of π , to fifteen places,

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 1 & 2 & 3 & 4 & 5 \\
 \pi = 3 \cdot 141 & 592 & 653 & 589 & 793 & \\
 \hline
 1 \cdot 570 & 796 & 326 & 794 & 897 & \\
 \hline
 & 785 & 0 & & & \\
 & \underline{11} & 326 & & & \\
 & & 10 & 990 & & \\
 1 & 570 & 795 & 99 & 336 & 794 & 897 & \\
 \hline
 & & & & 314 & 159 & 198 & \\
 & & & & \underline{22} & 635 & 699 & \\
 & & & & & 15 & 707 & 960 & \\
 & & & & & \underline{6} & 927 & 739 & \\
 & & & & & & 6 & 283 & 184 & \\
 & & & & & & \underline{644} & 555 & & \\
 & & & & & & & 628 & 318 & \\
 & & & & & & & \underline{16} & 237 & \\
 & & & & & & & & 15 & 708 & \\
 & & & & & & & & \underline{529} & & \\
 & & & & & & & & & 471 & \\
 & & & & & & & & & 58 & \\
 & & & & & & & & & \underline{47} & \\
 & & & & & & & & & 11 & \\
 & & & & & & & & & \underline{11} & \\
 & & & & & & & & & & 11 &
 \end{array}
 \end{array}
 \begin{array}{l}
 \div 2 \\
 507 \ 2 \\
 \\
 214 \ 3 \\
 \\
 410 \ 4 \\
 \\
 337 \ 5
 \end{array}$$

$$\begin{array}{r}
 \cdot 301 \ 029 \ 995 \ 663 \ 981 \quad \log 2 \\
 \cdot 195 \ 899 \ 652 \ 409 \ 234 \quad 570 \ 1 \\
 \quad 220 \ 131 \ 503 \ 703 \quad 507 \ 2 \\
 \quad \quad 92 \ 939 \ 009 \quad 214 \ 3 \\
 \quad \quad \quad 178 \ 061 \quad 410 \ 4 \\
 \quad \quad \quad \quad 146 \quad 337 \ 5 \\
 \hline
 0 \cdot 497 \ 149 \ 872 \ 694 \ 134 \quad = \log \pi
 \end{array}$$

There is here no necessity for the formation of a third divisor. It would comprise two periods, and as the second divisor is to that extent identical with the dividend, it is sufficient to continue the division till the close with the second divisor. The logarithm formed is true in its last figure.

Example 7.—Find the same logarithm, to twelve places.

$$\begin{array}{r}
 \begin{array}{cccccc}
 & 1 & 2 & 3 & 4 & \\
 \pi = 3 \cdot 141 & 592 & 653 & 590 & & \\
 \hline
 1 \cdot 570 & 796 & 326 & 795 & & \\
 \hline
 & 785 & 0 & & & \\
 & \underline{113} & 26 & & & \\
 & & 109 & 90 & & \\
 1 & 570 & 795 & 336 & 795 & \\
 \hline
 & & & & 314 & 159 & \\
 & & & & \underline{22} & 636 & \\
 & & & & & 15 & 708 & \\
 & & & & & \underline{6} & 928 & \\
 & & & & & & 6 & 283 & \\
 & & & & & & \underline{645} & & \\
 & & & & & & & 628 & \\
 & & & & & & & \underline{17} & \\
 & & & & & & & & 16 & \\
 & & & & & & & & \underline{1} & \\
 & & & & & & & & & & 1 &
 \end{array}
 \end{array}
 \begin{array}{l}
 \div 2 \\
 507 \ 2 \\
 \\
 214 \ 3 \\
 \\
 410 \ 4
 \end{array}$$

$$\begin{array}{r}
 \cdot 301 \ 029 \ 995 \ 664 \quad \log 2 \\
 \cdot 195 \ 899 \ 652 \ 409 \quad 570 \ 1 \\
 \quad 220 \ 131 \ 504 \quad 507 \ 2 \\
 \quad \quad 92 \ 939 \quad 214 \ 3 \\
 \quad \quad \quad 178 \quad 410 \ 4 \\
 \hline
 0 \cdot 497 \ 149 \ 872 \ 694 \quad = \log \pi
 \end{array}$$

Here the second divisor comprises only two periods, and it is to the same extent identical with the dividend, subject occasionally (not in the present

instance) to an abatement of a unit in the last place; and with this divisor the division goes on to its close. The logarithm formed is true in its last figure.

Example 8.—Required the same logarithm, to nine places.

$$\begin{array}{r|l}
 \pi = 3 \cdot 141\ 592\ 654 & \div 2 \\
 \hline
 1\ 570\ 796\ 327 & 507\ 2 \\
 \hline
 785\ 0 & \\
 \hline
 11\ 327 & \\
 \hline
 10\ 990 & \\
 \hline
 1\ 57\ 837 & 214\ 3 \\
 \hline
 814 & \\
 \hline
 23 & \\
 \hline
 16 & \\
 \hline
 7 & \\
 \hline
 6 & \\
 \\
 \hline
 301\ 029\ 996 & \log 2 \\
 195\ 899\ 652 & 570\ 1 \\
 220\ 132 & 507\ 2 \\
 93 & 214\ 3 \\
 \hline
 0\ 497\ 149\ 873 & = \log \pi
 \end{array}$$

Here we may avoid the trouble of writing down a second divisor by altering, when necessary, (as it is in this instance,) the last figure of the first divisor. Again the logarithm formed is correct in its last place.

It must not be concluded that in each of the last five examples (4 to 8) the figures of the resolving processes are derived from those of the preceding by merely cutting off a period from the point where contraction begins. Each example has been conscientiously and legitimately worked out as if it were a primary example; and hence, with those that have gone before, (as well as those that will follow,) they serve to indicate, by comparison of their results with that of Example 3, the degree of accuracy that may generally be expected in the employment of the method now set out.

Example 9.—Find the logarithm, to twelve places, of 52943.

$$\begin{array}{r|l}
 \begin{array}{r}
 1\ 2\ 3\ 4 \\
 529\ 43 \\
 \hline
 1\ 058\ 860\ 000\ 000 \\
 846\ 4 \\
 \hline
 13\ 60 \\
 \hline
 10\ 58 \\
 \hline
 3\ 020 \\
 2\ 116 \\
 \hline
 1\ 058\ 857\ 904\ 000 \\
 847\ 087 \\
 \hline
 56\ 913 \\
 \hline
 52\ 943 \\
 \hline
 3\ 970 \\
 \hline
 3\ 177 \\
 \hline
 793 \\
 \hline
 741 \\
 \hline
 52 \\
 \hline
 53
 \end{array} & \begin{array}{l}
 \times 2 \\
 812\ 2 \\
 \\
 853\ 3 \\
 \\
 750\ 4
 \end{array}
 \end{array}$$

$$\begin{array}{r|l}
 698\ 970\ 004\ 336 & \text{colog } 2 \\
 24\ 485\ 667\ 699 & 058\ 1 \\
 352\ 504\ 022 & 812\ 2 \\
 870\ 453 & 853\ 3 \\
 326 & 750\ 4 \\
 \hline
 4\ 723\ 808\ 546\ 886 & = \log \text{ req.}
 \end{array}$$

The logarithm here formed is true in its last place. This logarithm, as is pointed out by Bremiker, in the Preface to his Seven-figure Vega, has been incorrectly given in all the older seven-figure tables, as Gardiner, Sherwin, Taylor, Callet, (1795,) Borda (Delambre), &c., and also in others of more recent date, as Hutton, Babbage, Sang, &c. It appears in those tables as .7238086, its true value to seven places being as above, .7238085. In the following recent tables the error has been corrected:—Bremiker's Vega (above cited), Schrön, Callet, (1862, by Dupuis,) and Bruhns, all of which appear to have been very carefully edited.

The logarithms proposed for formation in the next four examples are logarithms for which we shall have occasion hereafter. They all present specialties in the resolving process, which it is advantageous to have brought under notice.

Example 10.—Find the logarithm of 1.0471, to eighteen places.

1	2	3	4	5	6			
1.0471	000	000	000	000	000	095	2	
	94	23						
	5	770						
	5	235						
1 047	099	465	535	000	000	510	3	
			523	549	732			
			11	450	267			
			10	470	994			
1.047	099	999	979	272	850	985	4	
			942	389	999			
			36	882	851			
			31	413	000			
			5	469	851			
			5	235	500			
			234	351	223		5	
			209	420				
			24	931				
			20	942				
			3	989				
			3	141				
			848	810		6		
			838					
			10					
			10					
			0					
.019	946	681	678	842	334	047	1	
	41	256	016	151	068	095	2	
		221	490	129	291	510	3	
			406	065	840	985	4	
			96	848	223		5	
			352	810		6		
0.019	988	159	591	285	233	=log req.		

The only peculiarity here is that all the periods, after the first, are made up by the annexation of ciphers.

Example 11.—Find the logarithm of 1.000 093 160, to eighteen places.

tions of Problem I., exhibit abnormal features in the resolving process, some of which are very interesting.

Example 14.—Required the logarithm, to twenty-four places, of 1.123034 . . . as below.

	1	2	3	4	5	6	7	8		
1.123)034 202 171 630 867 322 959										030 2
			33 69							
1 123 033 690)512 171 630 8										456 3
			449 213 476 0							
			62 958 154 86							
			56 151 684 50							
			6 806 470 367							
			6 738 202 140							
1 123 034 202 103 362) 68 268 227 322 959										060 4
			67 382 052 126 202							
1 123 034 202 17)886 175 196 757										789 5
			786 123 941 520							
			100 051 255 237							
			89 842 736 174							
			10 208 519 063							
			10 107 307 819							
			101 211 244							090 6
			101 073 078							
			138 166							123 7
			112 303							
			25 863							
			22 461							
			3 402							
			3 369							
			33							030 8
			34							
050 379 756 261 457 784 687 079										123 1
13 028 639 028 489 260 761										030 2
198 038 238 595 167 865										456 3
26 057 668 913 413										060 4
342 658 346 222										789 5
39 086 503										090 6
53 418										123 7
13										030 8
0.050 392 982 965 125 235 515 274										=log req.

This example demands notice on account of the symmetrical character of the analysis. This is,

$$(123:030:456:060:789:090:123:030).$$

But there is nothing in this, the number having been constructed to yield the above result. It shows, however, the mode of dealing with ciphers, when they occur, in various circumstances.

Example 15.—Find, to twenty-four places, the logarithm of .1428571 . . . as below.

	1	2	3	4	5	6	7	8		
1.428)571 428 571 428 571 428 571										400 2
			571 2							
1 428 571 200)228 571 428 5										160 3
			142 857 120 0							
			85 714 308 57							
			85 714 272 00							
1 428 571 428 571 392) 36 571 428 571										000 4
			28 571 428 571							025 5
			8 000 000 000							
			7 142 857 143							
			857 142 857							600 6
			857 142 857							
			0							

154 728 207 440 155 586 974 892	428	1
173 683 058 464 918 822 638	400	2
69 487 111 545 551 517	160	3
10 857 362 048	025	5
260 576 689	600	6
1.154 901 959 985 743 169 287 784	= colog 7	

This is a much more remarkable resolution than that in the preceding example. The number here resolved, which, being the reciprocal of 7, is not, like the last, a fancifully-constructed number, gives on analysis the following singular result:—

$$7^{-1} = 10^{-1}(428:400:160:000:025:600:000:000).$$

We have not in previous examples met with an instance of a triad composed entirely of ciphers, while here there are no fewer than three such, besides two which contain each two ciphers, and one which contains one cipher.

We have thus, out of twenty-four figures, no fewer than fifteen ciphers. This singularity is no doubt connected, *in some way*, with the circumstance that the given number is a circulate of *six* figures, and so comprising two of our periods. But the present is not the place to enquire into the connexion.

In the operation we see that when the triads 2 and 3 have been formed, the *complete* remainder consists of only four periods: it was therefore unnecessary to form the third divisor to more than the same number. Had this been attended to, then, on supplying the triad of ciphers (No. 4), on account of the missing period, the division would have gone on without interruption. Oddly enough, the next example presents the same phenomenon, and at the same point. It is dealt with in the manner here suggested.

The logarithm formed in the present example is true in the last place.

Example 16.—Required the logarithm, to twenty-four places, of 1.999 999 . . . as below.

1 2 3 4 5 6 7 8		
1.999)999 999 999 999 999 999 999 999	500	2
999 5		
1 999 999 500)	499 999 999 9	250 3
	399 999 900 0	
	100 000 099 99	
	99 999 975 00	
2 000 000 000 00)	125 000 000 000	062 5
	120 000 000 000	
	5 000 000 000	
	4 000 000 000	
	1 000 000 000	500 6
	1 000 000 000	
	0	
.800 812 794 118 116 939 414 048	999	1
217 092 972 230 208 281 913	500	2
108 573 606 904 112 659	250	3
26 926 257 878	062	5
217 147 241	500	6
0.301 029 995 663 981 195 213 739	= log 2	

The number here proposed is practically 2, and the logarithm formed is the logarithm of 2, true in the last place. The analysis is about as remarkable as that in the preceding example. It is—

$$2 = (999:500:250:000:062:500:000:000).$$

Here, as already intimated, the leading period of the remainder following the formation of the second and third triads is missing, and the treatment is that above suggested. The third divisor becomes 2, followed by four periods of ciphers, and to simplify the work, the dividend, instead of 12499 . . . is written 12500 . . . which amounts to the restoration of the unit in the twenty-fourth decimal place, by which the number as given is short of 2. The absence of

1 079 080	950	7
1 021 566		
57 464		
56 754		
710	625	8
681		
29		
23		
6		
6		

064 995 861 529 141 524 889 810	185	1
28 228 223 916 372 450 869	065	2
1 737 177 924 188 651	004	3
97 716 258 417 239	225	4
6 948 712	016	6
412 580	950	7
271	625	8

3.055 024 091 587 952 087 257 632	= log 881 ⁻¹	

The number here proposed is the reciprocal of 881, and we have,

$$881^{-1} = 1000^{-1}(135:065:004:225:000:016:950:626).$$

Here one third of the whole number of figures are ciphers, and, as in the preceding example, the fifth quotient triad consists entirely of ciphers. And so, also as in the last example, the third divisor is the last that needs be formed. The logarithm is true in its last figure.

How far the singularity of the analyses of the numbers proposed in Examples 15, 17, and 18, is owing to those numbers being reciprocals of simple *prime* numbers, and in what respects they would be modified if the quotients were restricted to two figures or extended to four, are enquiries on which I have not entered.

In the foregoing examples I have employed the ordinary form of division in the resolving process; but it is strongly recommended in practice to use the *short* form, in which the remainders only of the several partial divisions are set down. This manner of conducting the operation, while it saves the writing of many figures, and imparts great compactness to the work, will also, after a little practice, be found to be easier than the usual and much more lengthy one. It is however, it must be admitted, subject to one disadvantage: it does not possess the same facility for revision as the other. But then repetition of the work is so easy, that this is not found to be a serious drawback.

For illustration, the resolving process of Example 1, worked as here suggested, is subjoined.

1	2	3	4	5	6	7	8		
1-259	921	049	894	878	164	767	211	781	2
	39	74							
	1	979							
1-259	920	329	720	894	878	1		572	3
	90	934	708	66					
	2	740	285	634					
1 259	921	049	674	433	220	444	976	767	211
	94	452	871	799	768				
	6	258	398	322	558				
1 259	921	049	894	1	218	714	123	860	967
	84	785	178	955					
	9	189	915	961					
	370	468	612					294	6
	118	484	402						
	5	091	508						
	51	824						041	7
	1	427							
	167							132	8
	41								
	3								

PROBLEM II.—To find the number, to twenty-four places, corresponding to any given logarithm.

This problem is the converse of Problem I. The operation here consists of, first, the decomposition of the given logarithm into a series of tabulated logarithms; and secondly, the *compounding*, or multiplication, of the corresponding numbers, which are the factors of the required number.

Case First.—When the given logarithm, index apart, is less than $\cdot 301\ 029\ 995\ 663\ 981\ 195\ 213\ 739$, which is the logarithm of 2.

1. Neglecting the index, cut down, or make up by the annexation of ciphers, the given logarithm to twenty-four places, separating them into periods of three figures each.*

2. Decompose the logarithm thus modified, by successive subtraction of values taken from the several columns of the principal table, in order: observing that the values thus taken are to be, in each case, the greatest contained in the column in use which does not exceed that from which it has to be subtracted; and place the arguments corresponding to them in the margin, attaching to each the number of the column to which it belongs. By this process, in no case requiring or admitting of more than one entry in the same column, the given logarithm will be exhausted; and unless, as will occasionally happen, one, or perhaps two,—rarely more,—columns have been passed over, as not containing suitable values, eight triads will be found in the margin. The fact of one or more columns having been passed over, and the situation of the missing triad or triads, will be indicated by the consequent break or breaks in the sequence of the triad numbers. Missing triads are to be considered as consisting of ciphers, and, when needed, their places are to be supplied accordingly.

3. The first multiplicand will consist of unity followed by eight periods of three figures each, of which periods the first four are composed of ciphers, and the remaining four of the figures of the last four triads, Nos. 5 to 8, found as above, written in order.

4. The rest of the process consists, practically, of contracted multiplication, with interruptions to the continuity as follows:—The multiplying digits, which follow each other in regular succession from the decimal point, are the figures of the triads 1 to 4 in order. As each triad is used up, the partial products are added to the multiplicand, and the sum, in each case as it arises except the last, is dealt with as a new multiplicand. The sum last formed is the required number.

The multiplicand is to be curtailed of one figure for each digit as it comes into use; and as the curtailment commences in the last period of the multiplicand, preparatory to the use of the first multiplying digit, it follows that the number of the period, reckoning from right to left, in which the contraction commences in each multiplicand, is the same as the number of the corresponding triad.† Attention to this remark will obviate any difficulty that might be felt, especially in the case of missing triads, as to the proper placing of the partial products.

Case Second.—When the mantissa of the given logarithm is not less than the logarithm of 2.

1. Commence by subtracting from the given logarithm any logarithm in the Auxiliary Table, (which logarithm we may call the *preparing logarithm*,) that

* This is most conveniently done by the use of properly-ruled paper. See note, p. [3].

† The relation here pointed out may be stated in a somewhat different manner, as follows:—The sum of the number of any multiplying triad and that of the multiplicand period in which the former commences its operation, is always *one more than the number of periods in the multiplicand*. Thus when, as here, we have eight periods in the multiplicand, we find $1+8=9$, $2+7=9$, $3+6=9$, and so on, the former of the two numbers on the left-hand side of each of these equations being the number of a multiplying triad, and the latter the number of the corresponding multiplicand period.

This relation holds universally, whatever be the number of periods in the multiplicand.

the illustration the example affords of a point adverted to above. It will be observed that, corresponding to the *first* triad, the contraction of the multiplicand commences in the last period—that is, the *first* when reckoned in reverse order; corresponding to the *second* triad, the contraction commences in the *second* period, reckoning in the same order, and so on. And it may be added that the same thing holds, whatever be the extent of the required formation, as will be hereafter pointed out.

The compounding process, as above exhibited, admits of very considerable abbreviation. How this is to be obtained, will now be shown.

1	$\begin{array}{r} 967\ 294\ 041\ 13\cancel{x} \\ 193\ 458\ 808\ 226 \\ 48\ 364\ 702\ 057 \\ 8\ 705\ 646\ 370 \end{array}$	259 1
1 259 000 000 001	$\begin{array}{r} 217\ 828\ 19\cancel{x}\ 784 \\ 881\ 3 \\ 87\ 77 \\ 1\ 259 \end{array}$	731 2
1 259 920 329 001	$\begin{array}{r} 218\ 71\cancel{x}\ 426\ 541 \\ 629\ 960\ 164\ 500\ 609\ 357 \\ 88\ 194\ 423\ 030\ 085\ 310 \\ 2\ 519\ 840\ 658\ 002\ 437 \end{array}$	572 3
1 259 921 049	$\begin{array}{r} 675\ 64\cancel{x}\ 902\ 123\ 645 \\ 125\ 992\ 104\ 967\ 565 \\ 88\ 194\ 473\ 477\ 295 \\ 5\ 039\ 684\ 198\ 703 \end{array}$	174 4
	$\hline 1\ 259\ 921\ 049\ 894\ 873\ 164\ 767\ 208$	= $\frac{1}{2}$

The above is a repetition of the process, in its abbreviated form. The abbreviation consists, first, in the direct multiplication of the leading unit of the first multiplicand by the complete first factor, 1259, entering the product in its place as the leading portion of the second multiplicand; and secondly, in the omission of the *runs* of ciphers. When properly-ruled paper is used,* and due care is exercised, the omission of the ciphers is found to be perfectly safe. This form of the process in the present example effects a saving of seventy figures.

There is another form of the compounding process, which I give here as applied to the same example.

1	$\begin{array}{r} 1\ 259 \\ 881\ 3 \\ 87\ 77 \\ 1\ 259 \end{array}$	731 2
1 259 920 329	$\begin{array}{r} 629\ 960\ 164\ 5 \\ 88\ 194\ 423\ 03 \\ 2\ 519\ 840\ 658 \end{array}$	572 3
1 259 921 049	$\begin{array}{r} 674\ 42\cancel{x}\ 188 \\ 125\ 992\ 104\ 967\ 443 \\ 88\ 194\ 473\ 477\ 210 \\ 5\ 039\ 684\ 198\ 698 \end{array}$	174 4
1 259 921 049 89	$\begin{array}{r} 654\ 450\ 643\ 351 \\ 1\ 133\ 928\ 944\ 904 \\ 75\ 595\ 262\ 994 \\ 8\ 819\ 447\ 349 \\ 251\ 984\ 210 \\ 113\ 392\ 894 \\ 5\ 039\ 684 \\ 50\ 397 \\ 1\ 260 \\ 126 \\ 38 \\ 1 \end{array}$	967 5 294 6 041 7 181 8
	$\hline 1\ 259\ 921\ 049\ 894\ 873\ 164\ 767\ 208$	= $\frac{1}{2}$

* See note, p. [3].

The distinction between this form and that which precedes it is, that here the multiplying triads are used in direct and unbroken order from the first to the last; and a key to the placing of the partial products will be obtained if the blank spaces in the first nine lines be filled up with ciphers, and the contraction indicators be inserted in their places. Nine more lines are required, but twenty-six figures are saved.

The remark already made, with reference to the identity of the numbers (in reverse order) of the periods in the successive multiplicands in which the contraction commences, with the numbers of the corresponding triads, is equally applicable in this form of the process as in the other.

In succeeding examples I shall confine myself to the first form of the compounding process.

The last period of the number formed in Ex. 19 should be 211, (*see* Ex. 1.) so that, using either form of the compounding process, we have an error of 3 in the last—the twenty-fifth—place: greater departures from exactitude than this will occasionally show themselves. We shall see that, generally, n places in a logarithm do not usually give back, accurately, the $n+1$ places in the number from which it was deduced. The cause of this is to be sought in the relation that subsists between a number and its logarithm.

Example 20.—Find the anti-logarithm, to twenty-four places, of M , the modulus of the common system of logarithms.

	1	2	3	4	5	6	7	8		
$M =$	434	294	481	903	251	827	651	129	log 2	
	301	029	995	663	981	195	213	739		
	133	264	486	239	270	632	437	390	359	1
	133	219	456	732	494	311	459	129		
	45	029	506	776	320	978	261	103	2	
	44	730	028	079	131	898	841			
	299	478	697	189	079	420	689	3		
	299	228	794	947	032	488				
	249	902	242	046	932	575	4			
	249	719	327	022	575					
	182	915	024	357	421	5				
	182	837	976	881						
	77	047	476	177	6					
	76	870	123							
	177	353	408	7						
	177	192								
	161	371	8							
	161									
		5	6	7	8					
1	421	177	408	371	359	1				
	126	353	222	511						
	21	058	870	419						
	3	790	596	675						
1	359	000	000	000	572	380	097	976	103	2
	135	9	57	238	010					
	4	077	1	717	140					
1	359	189	977	000	572	487	053	126	689	3
	815	483	986	200	343	463				
	108	731	198	160	045	795				
	12	232	259	793	006	152				
1	359	140	913	448	017	592	447	536	575	4
	679	570	456	724	008					
	95	189	868	941	361					
	6	795	704	567	240					
1	359	140	914	229	522	617	680	145	$\times 2$	
2	718	281	828	459	045	235	360	290	$= e$	

This example, which is the converse of Ex. 2, belongs to the second case, the mantissa of the given logarithm being greater than log 2. The number formed is e , the Napierian base. The last period should be 287.

Log 2 is here used as the preparing logarithm; but any one of the following might have been employed:—Colog 4, colog 5, (the same as log 2,)

colog 6, colog 7. The final multiplication by 2 is in accordance with the use of log 2 as above.

Example 21.—The greatest number that can be expressed by three figures in the ordinary notation is 9^9 ; required the first twenty-four figures of the number thus denoted.

The number here represented is the power of 9 whose exponent is $9^9 = 729^9 = 387\ 420\ 489$. Its logarithm consequently is $(\log 9) \times 387\ 420\ 489$, or,

369 693 099 631 570 358 743 543 095 095 483,

log 9 to thirty-six places, as obtained from Callet, being used in the formation.

The operation is as follows:—

369 693 099 631 570 358 743 543 095 095 483	log 4
602 059 991 327 962 390 427 478	
<u>29 510 367 415 580 704 668 005</u>	070 1
29 383 777 685 209 640 834 541	
126 589 780 371 063 833 464	291 2
<u>126 361 309 554 876 500 049</u>	
228 420 816 187 333 415	525 3
<u>228 004 543 148 019 870</u>	
416 273 039 313 545	958 4
<u>416 054 113 464 025</u>	
218 925 849 520	504 5
<u>218 884 418 879</u>	
41 430 641	095 6
<u>41 257 976</u>	
172 665	397 7
<u>172 415</u>	
250	576 8
<u>250</u>	

1	504 095 897 576	070 1
	<u>35 286 677 830</u>	
1 070 000 000 000 539 382 076 406		291 2
214 0	107 876 415	
96 80	48 544 387	
<u>1 070</u>	<u>539 382</u>	
1 070 311 370 000 539 536 035 590		525 3
535 155 685 000 269 770		
21 406 227 400 010 791		
<u>5 351 556 850 002 698</u>		
1 070 311 931 914 006 789 818 849		958 4
963 280 738 722 608		
53 515 596 595 700		
<u>8 562 495 455 312</u>		
1 070 311 932 939 367 620 092 469		× 4
<u>4 281 247 731 757 470 480 369 876</u>		= 9 ⁹

In this example log 4 is used as the preparing logarithm, and it gives the first of the two following analyses: the second is the analysis we should have obtained by the use of colog 3:—

$$4(070:291:525:958:504:095:397:576)$$

$$3^{-1}(284:291:525:958:504:095:397:576).$$

It will be noticed that, in the two analyses the second and following factors are identical. This is in analogy with what we found in the case of Ex. 3; and it arises from a similar cause:—the figures of the two products, 4×1070 and $3^{-1} \times 1284$, are the same.

The number, a small portion of which is here determined, is an integer consisting, as appears by the index of its logarithm, of 369 693 100 figures; and it may excite surprise when it is stated, that to write it down, working day and night, at the rate of two figures per second, would occupy nearly six years.

Example 22.—Given $\log \pi = 0.49714\dots$, as below; required π , to twenty-four places.

0.497 149 872 694 183 854 351 268	log 3
.477 121 254 719 662 437 295 028	
<u>20 028 617 974 471 417 056 240</u>	047 1
19 946 681 678 842 333 835 200	
81 936 295 629 083 221 040	188 2
81 639 688 707 506 334 294	
296 606 921 576 886 746	682 3
296 188 735 657 670 368	
418 185 919 216 378	962 4
417 791 291 389 971	
394 627 826 407	908 5
394 339 389 568	
288 436 839	664 6
288 371 536	
65 303	150 7
65 144	
159	367 8
159	
1	908 664 150 367
	36 346 566 015
	6 860 649 052
<u>1 047 000 000 000 951 371 367 434</u>	188 2
104 7	95 137 187
83 76	76 109 709
8 376	7 610 971
<u>1 047 196 836 000 951 556 228 251</u>	682 3
628 318 101 600 570 930	
83 775 746 880 076 124	
2 094 393 672 001 903	
<u>1 047 197 550 189 197 702 872 208</u>	962 4
942 477 795 170 274	
62 831 853 011 351	
2 094 395 100 378	
<u>1 047 197 551 196 597 746 154 211</u>	x 3
3.141 592 653 589 793 238 462 633	= π

The last period, here 633, should be 643; so that there is in this case an error of a unit in the twenty-fourth place.

I now give some examples of formations to fewer than twenty-four places.

Example 23.—Find π , to twenty-one places, its logarithm to the same number of places being given.

0.497 149 872 694 183 854 351	log 3
.477 121 254 719 662 437 295	
<u>20 028 617 974 471 417 056</u>	047 1
19 946 681 678 842 333 835	
81 936 295 629 083 221	188 2
81 639 688 707 506 334	
296 606 921 576 887	682 3
296 188 735 657 670	
418 185 919 217	962 4
417 791 291 390	
394 627 827	908 5
394 339 390	
288 437	664 6
288 372	
65	150 7
65	

	4	5	6	7		
1	962	908	664	15 6	047	1
	38	516	346	566		
	6	740	360	649		
1	047	000	001	008	188	2
	104	7	100	816		
	88	76	80	653		
	8	376	8	065		
1	047	196	837	008	682	3
		628	318	102		
		83	775	746		
		2	094	393		
1	047	197	551	196	x 3	
		597	746	153		
		3	141	592		-π

Although there are here only seven triads, we can still, without sensible error, as will be hereafter shown, assume 1·(000)³ followed by the figures of the triads 4 to 7, for the product of the last four factors; and the rest of the work is as shown. It will be noticed, moreover, and the same thing will be seen in succeeding examples, that the identity, *in reverse order*, of the numbers of the periods in which, in successive multiplicands, the contraction commences, with the numbers of the corresponding triads, holds here as in the twenty-four-figure process. See also note, p. [18].

The last period should be 463, so that there is in the present example an error of 4 in the twenty-second place.

Example 24.—Required π, to eighteen places, its logarithm to the same number of places being given.

	1	2	3	4	5	6	
	0·497	149	872	694	133	854	log 3
	·477	121	254	719	662	487	
		20	028	617	974	471	047
		19	946	631	678	842	
		81	936	295	629	083	188
		81	639	688	707	506	
		296	606	921	577		682
		296	188	735	658		
		418	185	919			962
		417	791	291			
			394	628			908
			394	339			
			289				665
			289				
			4	5	6		
1	962	908	66 4	15	6	047	1
	38	516	346	566	6		
	6	740	361	649	6		
1	047	000	001	008	16 8	372	188
	104	700	000	100	817	2	
	83	760	000	080	653	3	
	8	376	000	008	065	3	
1	047	196	837	00 8	854	907	682
		628	318	102	205	3	
		83	775	746	960	3	
		2	094	393	674	3	
1	047	197	551	196	597	746	x 3
		597	746	153	3	-π	
		3	141	592	653	589	

The above, after what precedes, seems to require no remark. The number formed is true in the nineteenth place.

Example 25.—Required π , to fifteen places, its logarithm to the same extent being given.

1	2	3	4	5		
0·497	149	872	694	134		log 3
·477	121	254	719	662		
20	028	617	974	472		047 1
19	946	681	678	842		
	81	936	295	630		188 2
	81	639	688	708		
	296	606	922			682 3
	296	188	736			
		418	186			962 4
		417	791			
		395				910 5
		395				
1		3	4	5		047 1
		682	962	916		
		27	818	516		
		4	780	740		
1	047	000	715	062		188 2
		104	700	071		
		83	760	057		
		8	376	005		
1	047	197	551	196		× 3
		8·141	592	653		= π

This, too, needs no remark. The last period, here 794, should be 793.

Example 26.—Required π , to twelve places, its logarithm to the same extent being given.

1	2	3	4			
0·497	149	872	694		log 3	
·477	121	254	720			
20	028	617	974		047 1	
19	946	681	679			
	81	986	295		188 2	
	81	689	689			
	296	606			682 3	
	296	189				
		417			960 4	
		417				
1		3	4		047 1	
		682	966			
		27	818			
		4	781			
1	047	000	715	059		188 2
		104	700	072		
		83	760	057		
		8	376	006		
1	047	197	551	194		× 3
		8·141	592	653		= π

There is here an error of 8 in the thirteenth place, as the last period should be 590. As it happens, all the logarithms that come into use in the first part of the operation have been increased in the last place, which will help to account for the deviation.

Example 27.—Required π , to nine places, its logarithm to the same extent being given.

0·497 149 873	log 3
<u>477 121 255</u>	
20 028 618	047 1
<u>19 946 682</u>	
81 936	188 2
<u>81 640</u>	
296	682 3
<u>296</u>	
1 188 682	047 1
<u>7 547</u>	
<u>1 321</u>	
1 047 197 550	× 3
<u>3 141 592 650</u>	= π

The last period should be 654, so that there is an error here of 4 in the tenth place.

I now give some miscellaneous examples.

Example 28.—Required the anti-logarithm of 0·708, to eighteen places.

0·708 000 000 000 000 000	log 5
<u>698 970 004 836 018 805</u>	
9 029 995 663 981 195	021 1
<u>9 025 742 086 910 247</u>	
4 258 577 070 948	009 2
<u>8 908 632 748 308</u>	
344 944 822 640	794 3
<u>844 829 681 734</u>	
114 640 906	263 4
<u>114 219 449</u>	
421 457	970 5
<u>421 266</u>	
191	440 6
<u>191</u>	
1 268 970 44	021 1
<u>5 279 409</u>	
268 970	009 2
<u>1 021 269 517 819</u>	
9 189 000 002 426	794 3
<u>1 021 009 189 267 516 245</u>	
714 706 432 489	009 2
<u>91 890 827 084</u>	
4 084 036 757	794 3
<u>1 021 009 999 950 812 525</u>	
5·105 049 999 754 062 625	× 5
	= log ⁻¹ 0·708

This example is noticeable on account of the succession of nines in the number formed. If the result were required to only six figures, to determine the last of them correctly it would be necessary to carry the computation as far as the eleventh. The number here obtained is true in the last—the nineteenth—place.

Example 29.—Find the value, to eighteen places of significant figures, of

$$\Delta^2 \log^{-1} x = (\log^{-1} \Delta x - 1)^2 \log^{-1} x,$$

when $x = .1376$, and $\Delta x = .0001$.

On substitution, the above becomes,

This example is the converse of Example 15; and it is given here in further illustration of the compounding process, in its application to exceptional cases. The triads 4, 7, and 8, are wanting. See p. [15].

Hermite* has shown that for a certain class of integer values of Δ , the value of $e^{\pi\sqrt{\Delta}}$ is very nearly equal to an integer; and it will afford a good example of the use of the tables to verify Hermite's statement in the cases of $\Delta=43$ and $\Delta=67, 43$ and 67 being numbers which belong to the class in question.

Example 31.—Find the number denoted by $e^{\pi\sqrt{43}}$.

$\log 43$	1.633 468 455 579 586 526	÷ 2	
$\sqrt{43}$	0.816 784 227 789 793 263		
$M\pi$	0.134 934 183 994 670 643		
	0.951 668 411 784 463 906	log 8	
	.903 089 986 991 943 586		
	48 578 424 792 520 820	118 1	
	48 441 803 550 404 491		
	136 621 242 115 829	814 2	
	136 347 061 948 992		
	274 180 166 837	631 3	
	274 039 781 621		
	140 435 216	323 4	
	140 277 118		
	158 098	364 5	
	158 083		
	15	035 6	
	15		
	4 5 6		
1	323 364 037	118 1	
	32 336 404		
	3 233 640		
	2 586 912		
1 118	361 526 991	814 2	
335 4	108 456		
11 18	3 615		
4 472	1 446		
1 118 351	052 367 634 508	631 3	
	671 010 631 417		
	38 550 581 571		
	1 118 351 052		
log $e^{\pi\sqrt{43}}$	1.118 351 758 041 148 548	× 8	
	8.946 814 064 329 188 384	log 8	
	.903 089 986 991 943 586		
	43 724 077 337 244 798	105 1	
	43 362 278 021 129 503		
	361 799 316 115 295	833 2	
	361 616 710 966 741		
	182 605 148 554	420 3	
	182 403 644 095		
	201 504 459	463 4	
	201 078 345		
	426 114	981 5	
	426 043		
	71	164 6	
	71		
	4 5 6		
1	463 981 167	105 1	
	46 393 116		
	2 319 906		
1 105	512 697 186		

* *Théorie des équations modulaires*, p. 48; see also *Comptes Rendus*, t. xlix., pp. 18, 118.

1 105	512 69 186	833
884 0	410 159	
88 15	15 381	
8 315	1 538	
1 105 920 465 51 126 264	420 3	
442 368 186 205		
22 118 409 810		
1 105 920 929 999 721 779	x 8	
8 847 367 439 997 774 232		
$e^{\pi\sqrt{48}} =$	884 736 743 999 777 423	

Example 32.—Find the number denoted by $e^{\pi\sqrt{67}}$.

log 67	1 826 074 802 700 826 434 149 132	÷ 2
" $\sqrt{67}$	0 913 037 401 350 413 217 074 566	
" $M\pi$	0 134 934 133 994 670 643 474 237	
	1 047 971 585 345 083 860 548 803	116 1
	047 664 194 601 559 944 684 238	
	307 390 743 523 915 864 565	708 2
	307 371 696 441 949 190 978	
	19 047 081 966 673 537	043 3
	18 674 662 320 334 592	
	372 419 646 338 995	857 4
	372 190 370 831 608	
	229 275 507 392	527 5
	228 873 191 963	
	402 315 429	926 6
	402 156 690	
	158 739	365 7
	158 517	
	222	511 8
	222	

1	5 27 926 365 51 1	116 1
	52 792 636 551	
	5 279 263 655	
	3 167 558 198	
1 116	589 165 82 910	708 2
	781 2 412 416 077	
	8 928 4 713 326	
1 116 790 128 000 589 58 953 813	043 3	
	44 671 605 120 023 533	
	3 350 370 384 001 769	
1 116 790 176 022 56 086 978 665	857 4	
	893 432 140 818 052	
	55 839 508 801 123	
	7 817 531 232 158	
1 116 790 176 979 654 267 830 003		
log $e^{\pi\sqrt{67}}$	11 167 901 769 796 542 678 300 03	471 1
	167 612 672 727 530 111 220 716	
	289 097 069 012 567 079 814	666 2
	288 709 845 078 197 070 618	
	337 223 934 370 008 696	891 3
	336 956 210 986 830 934	
	267 723 333 177 712	616 4
	267 525 400 770 005	
	197 982 407 707	455 5
	197 603 989 266	
	378 418 441	871 6
	378 270 494	
	147 947	340 7
	147 660	
	237	661 8
	237	

1	$\begin{array}{r} 455\ 871\ 340\ 66\cancel{x} \\ 182\ 348\ 536\ 264 \\ 81\ 910\ 998\ 846 \\ \hline 455\ 871\ 341 \end{array}$	471	1
1 471	$\begin{array}{r} 670\ 586\ 74\cancel{x}\ 112 \\ 882\ 6 \qquad 402\ 352\ 045 \\ 88\ 26 \qquad 40\ 235\ 205 \\ 7\ 355 \qquad 8\ 352\ 934 \end{array}$	665	2
1 471 978 215 000 671 032 682 296	$\begin{array}{r} 1\ 177\ 582\ 572\ 000\ 536\ 826 \\ 132\ 478\ 039\ 350\ 060\ 393 \\ \hline 1\ 471\ 978\ 215\ 000\ 671 \end{array}$	891	3
1 471 979 526 533 268 598 280 186	$\begin{array}{r} 883\ 187\ 715\ 919\ 956 \\ 14\ 719\ 795\ 265\ 333 \\ 8\ 881\ 877\ 159\ 200 \end{array}$	616	4
$\epsilon^{\pi\sqrt{67}}$	$\begin{array}{r} 1\ 471\ 979\ 527\ 439\ 999\ 986\ 624\ 675 \\ \hline 147\ 197\ 952\ 743\ 999\ 998\ 662\ 468 \end{array}$		

To explain the method of solution, referring to Ex. 31. The number required is that whose Napierian logarithm is $\pi\sqrt{43}$, (ϵ being the Napierian base,) and consequently whose common logarithm is $M\pi\sqrt{43}$, M and π being as on pp. 6, 7. The number $M\pi\sqrt{43}$ is formed by logarithms, and found equal to 8.946 . . . ; and the anti-logarithm of this number, namely, 88473 . . . is the number required. Analogous remarks apply to the solution of Ex. 32.*

Hermite himself has calculated the value of $\epsilon^{\pi\sqrt{43}}$ to 16 figures, and his result agrees with that obtained above to this extent. I believe the actual value of $\epsilon^{\pi\sqrt{67}}$ has not been exhibited before.

Besides the interest that always attaches to the direct verification of a numerical theorem, which has been obtained as the result of an elaborate and complicated process, I think the above examples are noteworthy on account of the illustrations they afford of the occasional necessity that is felt for logarithms to a great many places: it is clear that no verification of Hermite's remarkable theorem could be so satisfactory as that derived from direct calculation; and for such a calculation the ordinary tables are inadequate. Generally, the verification of any theorem relating to the decimal part of a quantity would, if the integral part were large, require many decimal places in the logarithms employed. Another case in point is afforded by the curious approximate representations of quadratic surds by exponentials, which were given by Kronecker in the Berlin

Monatsberichte (Jan. 22, 1863, p. 49), such as, *ex. gr.*, that $4 + \sqrt{17} = \frac{2}{9} \epsilon^{\frac{2}{9}\pi\sqrt{17}}$,

$882 + 145\sqrt{37} = \frac{1}{8} \epsilon^{\frac{1}{8}\pi\sqrt{37}}$, very nearly. None of the results given by Kronecker are beyond the reach of ten-figure tables, but in the more interesting examples, in which the numbers under the radical sign are larger, and the approximations more exact, the use of tables giving more places would be requisite.†

* The quantity $\log M\pi$, (which occurs in the above examples,) $= \log \log \epsilon^{\pi} = \log \log \frac{1}{q}$, (mod. angle 45°) is one that is of importance in the theory of the elliptic and theta functions; and its value to fifteen places is given by Verhulst, (*Traité des fonctions elliptiques*, 1841, pp. 253, 309). It is worth while to place it on record here, along with $\log M$. The formation to twenty-four places, believed to be true in the last place, is as follows:—

log M	1.637 784 311 800 536 789 122 969
„ π	0.497 149 872 694 133 854 351 268
„ $M\pi$	0.184 934 183 994 670 643 474 237

† It is to Mr. J. W. L. Glaisher that I am indebted for the foregoing references to Hermite's and Kronecker's papers; and for a number of valuable suggestions besides.

III.—INVESTIGATION OF THE RULES.

The rules exemplified in the preceding section, as applied to the formation of logarithms and anti-logarithms, have now to be investigated.

Let N be any given number whose logarithm to t places is required. If N be divided or multiplied by a , any number that will give a result having unit for its first figure, and if the decimal point be placed after the first figure, the result so modified will be of the form $1+N_1$, where N_1 is a decimal fraction. Then if s denote the number of places through which the decimal point was moved—*plus* or *minus*, according as the movement was to the left or to the right—we should have,

$$N = 10^{\pm s} \cdot a^{\pm 1} (1 + N_1);$$

or, in logarithms,

$$\log N = \pm s \pm \log a + \log (1 + N_1).$$

Now a is a small number, which needs not, and in practice does not, exceed 9: its logarithm therefore can be obtained from a small auxiliary table, such as that on p. 1. Moreover s is an integer, and consequently affects only the characteristic of $\log N$. But this characteristic is determinable by reference to the position of the decimal point in N ; and hence we need here attend only to the formation of logarithms of numbers of the form $1+N_1$, in which N_1 is, as stated, a decimal fraction.

Let $1+n_1$ denote the leading unit and the first m figures (m being any number*) of the decimal portion of $1+N_1$; subtract $1+n_1$ from $1+N_1$, and let the remainder be r_1 ; divide r_1 by $1+n_1$, carrying the quotient to m places, and let the quotient and the remainder be n_2 and r_2 , respectively; divide r_2 by $(1+n_1)(1+n_2)$, carrying the quotient to m places as before, and let the quotient and the remainder of this division be n_3 and r_3 , respectively. Proceeding thus, always forming quotients of m figures, we find that, generally, division of r_{p-1} by $(1+n_1)(1+n_2) \dots (1+n_{p-1})$ gives for quotient and remainder n_p and r_p respectively.

The operation just described may be typically represented as follows:—

$$\begin{array}{r} 1+N_1 \\ 1+n_1 \\ 1+n_1 \overline{) r_1} \quad [n_2 \\ \quad (1+n_1)n_2 \\ (1+n_1)(1+n_2) \overline{) r_2} \quad [n_3 \\ \quad \quad (1+n_1)(1+n_2)n_3 \\ \quad \quad \quad r_3 \\ \quad \quad \quad \vdots \\ (1+n_1)(1+n_2) \dots (1+n_{p-1}) \overline{) r_{p-1}} \quad [n_p \\ \quad \quad \quad (1+n_1)(1+n_2) \dots (1+n_{p-1})n_p \\ \quad \quad \quad \quad r_p \end{array}$$

From this we obviously have,

$$\begin{array}{rcl} r_1 & = & (1+N_1) - (1+n_1), \\ r_2 & = & r_1 - (1+n_1)n_2, \\ r_3 & = & r_2 - (1+n_1)(1+n_2)n_3, \\ & \vdots & \vdots \\ r_{p-1} & = & r_{p-2} - (1+n_1)(1+n_2) \dots (1+n_{p-2})n_{p-1}, \\ r_p & = & r_{p-1} - (1+n_1)(1+n_2) \dots (1+n_{p-1})n_p; \end{array}$$

or, by transposition,

* But it is convenient that m should be an aliquot part or sub-multiple of t , the number of places in the logarithm to be formed.

$$\begin{aligned}
 (1+N_1) - r_1 &= (1+n_1), \\
 r_1 - r_2 &= (1+n_1)n_2, \\
 r_2 - r_3 &= (1+n_1)(1+n_2)n_3, \\
 &\vdots \\
 r_{p-2} - r_{p-1} &= (1+n_1)(1+n_2) \dots (1+n_{p-2})n_{p-1}, \\
 r_{p-1} - r_p &= (1+n_1)(1+n_2) \dots (1+n_{p-1})n_p.
 \end{aligned}$$

The foregoing equations may be written as follows:—

$$\begin{aligned}
 (1+N_1) - r_1 &= (1+n_1), \\
 r_1 - r_2 &= (1+n_1)(1+n_2) - (1+n_1), \\
 r_2 - r_3 &= (1+n_1)(1+n_2)(1+n_3) - (1+n_1)(1+n_2), \\
 &\vdots \\
 r_{p-2} - r_{p-1} &= (1+n_1)(1+n_2) \dots (1+n_{p-1}) - (1+n_1)(1+n_2) \dots (1+n_{p-2}), \\
 r_{p-1} - r_p &= (1+n_1)(1+n_2) \dots (1+n_p) - (1+n_1)(1+n_2) \dots (1+n_{p-1}).
 \end{aligned}$$

By addition of these last equations we obtain,

$$\begin{aligned}
 (1+N_1) - r_p &= (1+n_1)(1+n_2)(1+n_3) \dots (1+n_p), \\
 \text{or, } 1+N_1 &= (1+n_1)(1+n_2)(1+n_3) \dots (1+n_p) + r_p \dots (1)
 \end{aligned}$$

The remainders $r_2, r_3, \&c.$, successively recede from the decimal point, and hence the product $(1+n_1)(1+n_2) \dots$ constantly approximates to $1+N_1$; from which it follows that, if the division shall have been continued till r_p has no significant figure in the first t decimal places, the equation

$$1+N_1 = (1+n_1)(1+n_2)(1+n_3) \dots (1+n_p) \dots (2)$$

will be true to t places of decimals, or to $t+1$ places in all. Now the first t places of a logarithm are not affected by any figure beyond the $(t+1)$ th place of the corresponding number*; and therefore, on taking the logarithms on both sides of the last equation, we shall have, true to t places,

$$\log(1+N_1) = \log(1+n_1) + \log(1+n_2) + \dots + \log(1+n_p) \dots (3)$$

From this it appears that by the process described $1+N_1$ is resolved into a series of factors of the same form; and hence that, if provided with a suitable table, we shall be able to assign the logarithm of $1+N_1$ to the required extent.

There are t places in N_1 †, and, the division being continued till these are exhausted, there will be the same number in the aggregate of the quotients $n_1, n_2, \&c.$ The number of the quotients, consequently, (m places in each) is $t \div m$. This is also the number of factors, and therefore of tabular entries requisite.

The quotients, as regards their position in the scale, descend regularly from the decimal point. This is implied in the expressions for the factors $1+n_1, 1+n_2, \&c.$; but it will be accurately represented if, denoting by $n_1, n_2, \&c.$, the figures of the several quotients, we write the factors as follows:—

$$1 + (.1^m)n_1, 1 + (.1^m)^2n_2, \dots 1 + (.1^m)^{t \div m}n_{t \div m}.$$

From this, it might at first sight be inferred that a table adapted for the application of (3) to the formation of logarithms to t places, when arranged in columns, would require one column for each position in the scale occupied by a quotient, being $t \div m$ in all. But a much less number suffices.

* This is easily shown by aid of the expression,

$$\log(x+h) - \log x = M \left(\frac{h}{x} - \frac{h^2}{2x^2} + \dots \right),$$

where M is the modulus, = .43429

† We have just seen that for the formation of a logarithm to t places, we need use no more than t places in N_1 ; but it is also convenient, if the number of places in N_1 is less than t , to make it up to that number by the annexation of ciphers.

From the expression

$$\log(1+x) = M^*(x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots)$$

we learn that when x is a fraction commencing with k ciphers, if we use the first term of the series only, the equation $\log(1+x) = Mx$ will be true to $2k$ places. Now, the $(t \div 2m + 1)$ th quotient commences with $t \div 2m$ periods of ciphers, being $t \div 2$ ciphers in all; hence the $(t \div 2m + 1)$ th and following columns of the table, true to t places, will consist of a series of multiples of M by the successive arguments. And no column beyond the $(t \div 2m + 1)$ th needs be exhibited, since the values belonging to such are to be obtained by simply dropping the requisite number of periods of m places from the corresponding values in the last column exhibited.

The number m may have any value; but it is obvious that the value given to it, and the extent of the table, are mutually dependent. The number of the factors into which $1 + N_1$ is resolved, which is also the number of tabular entries, varies inversely as m ; hence the greater the value given to m the more will the labour involved in the formation of a logarithm be diminished. In the same circumstances the extent of the table requisite is also largely increased. When a table suited to an extended value of m has once been formed, however, all objection on this ground loses its force: and we have the advantage of the saving of labour such extended value affords, without the corresponding drawback.

In the present work m is taken equal to 3, the quotients being extended to three places; and the factors into which $1 + N_1$ is resolved are,

$$1 + (.1^3)n_1, \quad 1 + (.1^3)^2n_2 \dots 1 + (.1^3)^{t \div 3}n_{t \div 3},$$

or, $1 + (.001)n_1, \quad 1 + (.001)^2n_2 \dots 1 + (.001)^{t \div 3}n_{t \div 3}.$

Also, the subjoined tables being adapted, when used to their full extent, to the formation of logarithms of twenty-four places, t is in such case 24, and the number of the factors, in the same circumstances, is $24 \div 3 = 8$. Moreover, the number of columns in the principal table, in accordance with what has just been shown, is $(24 \div 6) + 1 = 5$; the last of them, in which the fifth and following entries are to be made, being in effect little else than a series of multiples of the modulus by the corresponding arguments.

In the conduct of the operation indicated in the foregoing investigation, various modifications present themselves by which the resolving process is much facilitated, and which give rise to the form of the process prescribed in the rules on pp. [2], [3].

The following is an example of the resolving process, as indicated by the investigation, and applied to Ex. 1, p. [4].

	$\frac{1}{2} = 1.259\ 921\ 049\ 894\ 873\ 164\ 767\ 211$ $\quad\quad\quad 1\ 259$	
$1 + n_1$	$-1\ 259$ $\quad\quad 881\ 3$ $\quad\quad\quad 37\ 77$ $\quad\quad\quad\quad 1\ 259$	$)921\ 049\ 894\ 873\ 164\ 767\ 211$ $\quad\quad 881\ 3$ $\quad\quad\quad 89\ 74$ $\quad\quad\quad 37\ 77$ $\quad\quad\quad\quad 1\ 979$ $\quad\quad\quad\quad 1\ 259$
$(1 + n_1)(1 + n_2)$	$= 1\ 259\ 920\ 329$ $\quad\quad 629\ 960\ 164\ 629\ 960\ 164\ 5$ $\quad\quad\quad 88\ 194\ 423$ $\quad\quad\quad\quad 2\ 519\ 841$	$)720\ 894\ 873\ 164\ 767\ 211$ $\quad\quad 90\ 934\ 708\ 66$ $\quad\quad\quad 88\ 194\ 423\ 03$ $\quad\quad\quad\quad 2\ 740\ 285\ 634$ $\quad\quad\quad\quad 2\ 519\ 840\ 658$
$(1 + n_1)(1 + n_2)(1 + n_3)$	$= 1\ 259\ 921\ 049\ 674\ 423$ $\quad\quad 125\ 992$ $\quad\quad\quad 88\ 194$ $\quad\quad\quad\quad 5\ 040$	$)220\ 444\ 976\ 767\ 211$ $\quad\quad 125\ 992\ 104\ 967\ 443$ $\quad\quad\quad 94\ 452\ 871\ 799\ 768$ $\quad\quad\quad 88\ 194\ 473\ 477\ 210$ $\quad\quad\quad\quad 6\ 258\ 398\ 322\ 558$ $\quad\quad\quad\quad 5\ 039\ 634\ 198\ 698$
$(1 + n_1) \dots (1 + n_4)$	$= 1\ 259\ 921\ 049\ 892\ 654$	$)1\ 218\ 714\ 123\ 860$

* M is here, as usual, the modulus, .43429

Here we commence, as directed, by subtracting $1+n_1$ from $1+N_1$; but the figures of $1+n_1$ being identical with the first four figures of $1+N_1$, this formality may obviously be omitted, and the object in view will be attained, as on p. [4], by simply separating from $1+N_1$, by the usual indicator, its first four figures for a divisor. And so two lines are saved.

The division by $1+n_1$ then proceeds till the quotient $731 (=n_2)$ is formed. The next divisor is $(1+n_1)(1+n_2)$, and it is formed by adding to $1+n_1$ its product by n_2 . Now the products of $1+n_1$ by the digits of n_2 already appear in the process (8813, 3777, &c.), and require merely to be transferred to their places under $1+n_1$, when addition of the four lines gives the complete divisor. Division by this divisor gives $572 (=n_3)$, after which a third divisor is formed, in like manner as the second, as shown. Another division is made, and finally a fourth divisor is formed as before. The effective portion of this divisor is identical with the first half of $1+N_1$, and it serves without alteration till the close of the process, as will be seen on reference to p. [4].

It is unnecessary, as is done here, to bring down to the dividend the entire remainder on the completion of each division. The rule directs (p. [3]), to bring down only the number of periods by which at that point the divisor is extended; and the division then goes on as usual.

The method of forming the successive divisors here employed, although sufficiently simple, is cumbrous: fortunately it admits of improvement.

By equation (1),

$$(1+n_1)(1+n_2) \dots (1+n_p) = (1+N_1) - r_p.$$

Now the left hand member of this equation is, by the type, p. [32], the p th divisor, and r_p is the remainder at the same point; hence each new divisor may be formed by subtracting the remainder, at the point of the operation attained, from the portion of $1+N_1$ directly over it. It is this method which is prescribed in the rules, and which is followed in all the examples given. Its application to the formation of the divisors in Ex. 1 is shown in the second note on p. [5].*

It will be interesting, as further illustrative of the resolving process, and as showing the changes that take place in its form corresponding to changes in the value of m , to have presented here an example worked out for the values 1, 2, 3, 4, of m , successively. The example is the formation of $\log \pi$ to 16 places.

$m=1.$		$m=2.$	
$\pi = 3.1415926535897932$	$\div 3$		
<u>10)471975511965977</u>	4 2	$\pi = 3.1415926535897932$	$\div 3$
40		1 04)71 97 55 11 96 59 77	69 2
1040)7197	6 3	<u>62 4</u>	
6240		<u>9 57</u>	
1046240)9575511	9 4	9 36	
9416160		1 04 71 76)21 55 11 9	20 3
10471816160)15935196597	1 5	<u>20 94 35 2</u>	
10171816160		1 04 71 96 94 3)60 76 76 59 77	58 4
10471920878)54633804377	5 6	<u>52 35 98 47 18</u>	
52359604391		<u>8 40 78 12 59</u>	
1047197323)2274199986	2 7	<u>8 37 75 75 55</u>	
2094394648		1 04 71 97 5)8 02 37 04	02 5
104719753)179805338	1 8	<u>2 09 43 95</u>	
104719753		<u>92 93 09</u>	88 6
10471975)75085585	7 9	<u>83 77 58</u>	
73303828		<u>9 15 51</u>	
1781757	1 10	<u>8 37 76</u>	
1047198		<u>77 75</u>	74 7
734559	7 11	<u>73 30</u>	
733038		<u>4 45</u>	
1521	1 13	<u>4 19</u>	
1047		<u>26</u>	
474			

* It is hardly necessary to remark that the subtractions are most commodiously performed by inspection, the results being at once set down in their places.

474	4	14	
419			
<u>55</u>	5	15	
52			
<u>3</u>	3	16	
·4771212547196624	log 3.		
170333892987804	4	2	
25979807199086	6	3	
3906892499101	9	4	
43429231045	1	5	
21714669809	5	6	
868588877	2	7	
43429448	1	8	
30400614	7	9	
434294	1	10	
304006	7	11	
434	1	13	
174	4	14	
22	5	15	
1	3	16	
<u>0·4971498726941339</u>			= log π

26	25	8	
21			
<u>5</u>			
·47 71 21 25 47 19 66 24	log 3		
1 70 33 33 92 98 78 04	04	1	
29 86 34 08 56 78 50	69	2	
8 68 58 02 78 03	20	3	
25 18 90 72 65	58	4	
86 85 89	02	5	
38 21 79	88	6	
32 14	74	7	
11	25	8	
<u>0·49 71 49 87 26 94 13 39</u>			= log π

m=3.				
π* = 3·141 592 653 589 793	÷ 3			
<u>1·047</u> 197 551 196 598	188			
104 7				
<u>92 85</u>				
83 76				
9 091				
8 876				
1 047 196 837)715 196 598	682	3		
<u>628 318 102</u>				
86 878 496				
83 775 747				
<u>3 102 749</u>				
2 094 394				
1 008 355	962	4		
<u>942 477</u>				
65 878				
62 832				
<u>3 046</u>				
2 094				
952	910	5		
942				
<u>10</u>				
·477 121 254 719 662	log 3			
19 946 681 678 842	047	1		
81 639 688 708	188	2		
296 188 786	682	3		
417 791	962	4		
395	910	5		
<u>0·497 149 872 694 134</u>				= log π

m=4.				
π = 3·1415 9265 3589 7932	÷ 3			
<u>1·0471</u> 9755 1196 5977	9816	2		
9423 9				
<u>831 21</u>				
314 13				
17 089				
10 471				
6 6186				
6 2826				
104719757)3360 5977	3209	3		
<u>3141 5927</u>				
219 0050				
209 4395				
9 5655				
9 4247				
1408	1345	4		
1047				
<u>361</u>				
314				
47				
42				
<u>5</u>				
·4771 2125 4719 6624	log 3			
199 8815 9591 2852	0471	1		
4045 6989 4768	9316	2		
1393 6510	3209	3		
584	1345	4		
910	910	5		
<u>0·4971 4987 2694 1338</u>				= log π

The gain by the increase of m is here apparent. The last formation has little more than half the number of figures required in the first, while the work, and consequently the liability to error, is diminished in a still greater ratio. In the first formation eight divisors are needed, all except the first having to be formed by subtraction, while in the last only two are requisite, both of which are given by inspection. In the first also there are no fewer than fifteen tabular entries, while the last has only five.

The gain is progressive: it is less in the second and third formations than in the fourth. The third, however, to which the table that follows is adapted, is not much behind the fourth in this respect. To realize the advantage of the fourth, as far as formations of sixteen places are concerned, we should require a table of three columns, with 10,000 values in each.

* See Ex. 6, p. [10].

The tabular values requisite in the foregoing formations, for $m=1$ are got from the appended table. Thus, the 2nd, 3rd, 4th, &c., values are those corresponding to 040 and 006 in col. I., 900 in col. II., and so on. The values for $m=2$ are got from Shortrede's Table V., which is a table of the same form as the present, in which $m=2$. The values for $m=3$ are of course taken from the subjoined table; and those for $m=4$ are got from Examples 10 to 13, in the present Introduction, which were proposed with a view to the employment of their results in this place.

The rules delivered for the formation of anti-logarithms have now to be investigated. The given logarithm is decomposed into a series of tabular logarithms, the numbers corresponding to which are known, and by the multiplication of these the required number is produced.

The decomposition of the given logarithm, as directed by the rules, and shown in the examples, is effected by successive subtraction from it of values taken from the columns of the principal table in order, preceded when necessary by the subtraction of one value from the auxiliary table.

The necessity for an entry in the auxiliary table arises when the given logarithm, apart from the index, (which in the present connexion is disregarded,) is greater than $\log 2$; and such a value is taken from the table as will, when subtracted, leave a remainder less than $\log 2$. This remainder is always exhausted by a single entry in each column of the principal table. Sometimes indeed a column may have to be passed over, but in no case is there more than one entry in the same column.

The exhaustion of the given logarithm in the manner described is a consequence of the structure of the table. In each column the difference between any two successive terms, *including the last in this and the first* in the preceding column*, is less than the least term in the column in question. Hence the remainder of a subtraction, made as directed in the rules, will be less than the least term in the column from which the subtrahend has been taken; and the next entry will of necessity have to be made in a subsequent column. And when the eighth column is reached (which is the fifth curtailed of its last three periods,) exhaustion necessarily takes place, since the column named contains all values from the greatest that the remainder at that point can have down to nothing.

It is here supposed that the table is being used to its full extent; but that the same things will happen, *mutatis mutandis*, when it is used to any less extent, is too obvious to need further illustration.

I close the present section with a few remarks illustrative of the *compounding* process. The factors to be compounded or multiplied together are,

$$1 + (\cdot 001)^{n_1}, 1 + (\cdot 001)^{2n_2} \dots 1 + (\cdot 001)^{8n_8},$$

in which the numbers $n_1, n_2, \&c.$, are the tabular arguments of the logarithms taken from the successive columns. Multiplication by a factor of the form shown is effected with great facility: it is performed by adding to the multiplicand its product by the fractional part of the factor. But we have a further facility here, arising from the form of the factors, and their orderly succession in powers of $\cdot 001$: we can assign at once, by inspection, the product of half the factors, at least. This is shown as follows:—

Let the number of factors be eight. The latter half of them are,

$$1 + (\cdot 001)^{4n_5}, 1 + (\cdot 001)^{6n_6}, 1 + (\cdot 001)^{7n_7}, 1 + (\cdot 001)^{8n_8};$$

and if these be multiplied together we shall obtain for product,

$$1 + (\cdot 001)^{4n_5} + (\cdot 001)^{6n_6} + (\cdot 001)^{7n_7} + (\cdot 001)^{8n_8} + (\cdot 001)^{11n_5n_6} + (\cdot 001)^{12n_5n_7} + \dots$$

* By the *first* term in a column is to be understood the term corresponding to argument 001. Also, in the case of Col. I., the "first term in the preceding column" must be taken to mean $\log 2$, this being the logarithm that would follow in natural order the last term in Col. I., if this column were extended by the addition of another term.

By a first interpolation, nine terms were inserted in each interval of the series thus formed; and by a second, nine terms in each interval of the new series. Thus the columns were completed.

In the case of these columns, further verification would hardly be considered necessary. In series formed by interpolation, if the operation has been properly conducted, we have strong assurance of the accuracy of the work in the periodical coincidence of terms of the series being formed with terms of the parent series. Nevertheless, for greater security against error, the columns II. to V., were subjected to a further test, as will presently appear.

Like care to that bestowed on the construction of the table was exercised to ensure that, as printed, it should be a correct transcript of the original. The proofs were read* with both the printer's copy and the original computations, four times in all, and all corrections made, after which the pages were stereotyped, a final verification being reserved for an impression of the whole *from the plates*. On this impression the terms occupying the several columns were summed in groups of ten. The sums of the groups in Col. I. were then very carefully compared with the corresponding sums which had been verified as above described. In this column no error was discovered: the correspondence of the two sets of values was complete.

The sums of the groups in the remaining columns were examined by differencing them out, as far as was necessary. In this way two errors were brought to light. The first, in Col. III., was a printer's error, which, notwithstanding the care bestowed in the examination of the proofs, had up to this point escaped detection. The other, in Col. V., was an error of transcription, which arose in the preparation of the copy of the table used by the printer.

Both these errors have been corrected; and the author believes he has good grounds for cherishing a hope that there remain now no more errors to be discovered.

* This part of the work was performed by a valued friend of the author, Lieut.-Col. W. H. Oakes. Being himself, while the table was in the press, temporarily incapacitated, he most gratefully accepted Col. Oakes's kind and considerate offer to relieve him of the task.

TABLE.

LOGARITHMS.	
1	'000 000 000 000 000 000 000 000
2	'301 029 995 663 981 195 213 739
3	'477 121 254 719 662 437 295 028
4	'602 059 991 327 962 390 427 478
5	'698 970 004 336 018 804 786 261
6	'778 151 250 383 643 632 508 767
7	'845 098 040 014 256 830 712 216
8	'903 089 986 991 943 585 641 217
9	'954 242 509 439 324 874 590 056
CO-LOGARITHMS.	
1	'000 000 000 000 000 000 000 000
2	'698 970 004 336 018 804 786 261
3	'522 878 745 280 337 562 704 972
4	'397 940 008 672 037 609 572 522
5	'301 029 995 663 981 195 213 739
6	'221 848 749 616 356 367 491 233
7	'154 901 959 985 743 169 287 784
8	'096 910 013 008 056 414 358 783
9	'045 757 490 560 675 125 409 944

	#	I.	II.
.000 000	000	'000 000 000 000 000 000 000 000	000 000 000 000 000 000 000
	1	'000 434 077 479 318 640 668 921	000 434 294 264 756 155 641
	2	'000 867 721 531 226 912 492 843	000 868 588 095 218 697 966
	3	'001 300 933 020 418 118 800 826	001 302 881 491 388 495 560
	4	'001 733 712 809 000 529 768 027	001 737 174 453 266 417 006
	5	'002 166 061 756 507 676 230 421	002 171 466 980 853 330 883
	6	'002 597 980 719 908 592 311 963	002 605 759 074 150 105 769
	7	'003 029 470 553 618 007 169 326	003 040 050 733 157 610 239
	8	'003 460 532 109 506 486 157 228	003 474 341 957 876 712 864
	9	'003 891 166 236 910 521 715 281	003 908 632 748 308 282 214
	010	'004 321 373 782 642 574 275 188	004 342 923 104 453 186 855
	1	'004 751 155 591 001 063 485 043	004 777 213 026 312 295 353
	2	'005 180 512 503 780 310 045 455	005 211 502 513 886 476 268
	3	'005 609 445 360 280 428 450 162	005 645 791 567 176 598 159
	4	'006 037 954 997 317 170 921 777	006 080 080 186 183 529 584
	5	'006 466 042 249 231 722 831 324	006 514 368 370 908 139 095
	6	'006 893 707 947 900 449 888 204	006 948 656 121 351 295 243
	7	'007 320 952 922 744 597 385 283	007 382 943 437 513 866 579
	8	'007 747 778 000 739 941 781 816	007 817 230 319 396 721 646
	9	'008 174 184 006 426 394 904 990	008 251 516 767 000 728 990
	020	'008 600 171 761 917 561 048 937	008 685 802 780 326 757 150
	1	'009 025 742 086 910 247 248 148	009 120 088 359 375 674 664
	2	'009 450 895 798 693 927 000 342	009 554 373 504 148 350 069
	3	'009 875 633 712 160 157 711 932	009 988 658 214 645 651 897
	4	'010 299 956 639 811 952 137 389	010 422 942 490 868 448 678
	5	'010 723 865 391 773 104 081 934	010 857 226 332 817 608 941
	6	'011 147 360 775 797 468 635 156	011 291 509 740 494 001 210
	7	'011 570 443 597 278 197 201 327	011 725 792 713 898 494 008
	8	'011 993 114 659 256 927 590 373	012 160 075 253 031 955 854
	9	'012 415 374 762 432 929 431 677	012 594 357 357 895 255 267
	030	'012 837 224 705 172 205 171 071	013 028 639 028 489 260 761
	1	'013 258 665 283 516 546 909 664	013 462 920 264 814 840 847
	2	'013 679 697 291 192 549 341 333	013 897 201 066 872 864 036
	3	'014 100 321 519 620 579 044 010	014 331 481 434 664 198 834
	4	'014 520 538 757 923 700 378 158	014 765 761 368 189 713 746
	5	'014 940 349 792 936 558 244 094	015 200 040 867 450 277 273
	6	'015 359 755 409 214 217 948 145	015 634 319 932 446 757 914
	7	'015 778 756 389 040 962 425 918	016 068 598 563 180 024 166
	8	'016 197 353 512 439 047 069 297	016 502 876 759 650 944 523
	9	'016 615 547 557 177 412 402 101	016 937 154 521 860 387 475
	040	'017 033 339 298 780 354 847 722	017 371 431 849 809 221 512
	1	'017 450 729 510 536 155 830 385	017 805 708 743 498 315 120
	2	'017 867 718 963 505 669 450 080	018 239 985 202 928 536 782
	3	'018 284 308 426 530 868 969 580	018 674 261 228 100 754 978
	4	'018 700 498 666 243 352 350 380	019 108 536 819 015 838 188
	5	'019 116 290 447 072 807 072 795	019 542 811 975 674 654 886
	6	'019 531 684 531 255 434 473 865	019 977 086 698 078 073 546
	7	'019 946 681 678 842 333 835 200	020 411 360 986 226 962 638
	8	'020 361 282 647 707 846 451 271	020 845 634 840 122 190 630
	9	'020 775 488 193 557 859 907 201	021 279 908 259 764 625 987

n	III.	IV.	V.
000 1 2 3 4 5 6 7 8 9	000 000 000 000 000 000 000 434 294 481 686 105 000 868 588 962 937 915 001 302 883 443 755 430 001 737 177 924 138 651 002 171 472 404 087 578 002 605 766 883 602 210 003 040 061 362 682 548 003 474 355 841 328 591 003 908 650 319 540 340	000 000 000 000 000 000 434 294 481 903 000 868 588 963 806 001 302 883 445 708 001 737 177 927 610 002 171 472 409 511 002 605 766 891 412 003 040 061 373 312 003 474 355 855 212 003 908 650 337 112	000 000 000 000 000 434 294 482 000 868 588 964 001 302 883 446 001 737 177 928 002 171 472 410 002 605 766 891 003 040 061 373 003 474 355 855 003 908 650 337
010 1 2 3 4 5 6 7 8 9	004 342 944 797 317 794 004 777 239 274 660 954 005 211 533 751 569 819 005 645 828 228 044 390 006 080 122 704 084 667 006 514 417 179 690 649 006 948 711 654 862 336 007 383 006 129 599 729 007 817 300 603 902 828 008 251 595 077 771 632	004 342 944 819 011 004 777 239 300 909 005 211 533 782 808 005 645 828 264 706 006 080 122 746 603 006 514 417 228 500 006 948 711 710 396 007 383 006 192 293 007 817 300 674 188 008 251 595 156 083	004 342 944 819 004 777 239 301 005 211 533 783 005 645 828 265 006 080 122 747 006 514 417 229 006 948 711 710 007 383 006 192 007 817 300 674 008 251 595 156
020 1 2 3 4 5 6 7 8 9	008 685 889 551 206 141 009 120 184 024 206 356 009 554 478 496 772 277 009 988 772 968 903 903 010 423 067 440 601 235 010 857 361 911 864 272 011 291 656 382 693 015 011 725 950 853 087 464 012 160 245 323 047 617 012 594 539 792 573 477	008 685 889 637 978 009 120 184 119 873 009 554 478 601 766 009 988 773 083 660 010 423 067 565 553 010 857 362 047 446 011 291 656 529 338 011 725 951 011 229 012 160 245 493 121 012 594 539 975 012	008 685 889 638 009 120 184 120 009 554 478 602 009 988 773 084 010 423 067 566 010 857 362 048 011 291 656 529 011 725 951 011 012 160 245 493 012 594 539 975
030 1 2 3 4 5 6 7 8 9	013 028 834 261 665 042 013 463 128 730 322 312 013 897 423 198 545 288 014 331 717 666 333 970 014 766 012 133 688 357 015 200 306 600 608 450 015 634 601 067 094 248 016 068 895 533 145 752 016 503 189 998 762 961 016 937 484 463 945 876	013 028 834 456 902 013 463 128 938 792 013 897 423 420 682 014 331 717 902 571 014 766 012 384 460 015 200 306 866 348 015 634 601 348 236 016 068 895 830 123 016 503 190 312 010 016 937 484 793 897	013 028 834 457 013 463 128 939 013 897 423 421 014 331 717 903 014 766 012 385 015 200 306 867 015 634 601 349 016 068 895 830 016 503 190 312 016 937 484 794
040 1 2 3 4 5 6 7 8 9	017 371 778 928 694 497 017 806 073 393 008 823 018 240 367 856 888 854 018 674 662 320 334 592 019 108 956 783 346 034 019 543 251 245 923 183 019 977 545 708 066 036 020 411 840 169 774 596 020 846 134 631 048 861 021 280 429 091 888 831	017 371 779 275 783 017 806 073 757 668 018 240 368 239 554 018 674 662 721 438 019 108 957 203 323 019 543 251 685 207 019 977 546 167 090 020 411 840 648 973 020 846 135 130 856 021 280 429 612 738	017 371 779 276 017 806 073 758 018 240 368 240 018 674 662 722 019 108 957 204 019 543 251 686 019 977 546 168 020 411 840 649 020 846 135 131 021 280 429 613

000
000

n	I.	II.
021 021	050 '021 189 299 069 938 072 793 505 1 '021 602 716 028 242 220 083 769 2 '022 015 739 817 720 259 399 711 8 '022 428 371 185 486 518 386 617 4 '022 840 610 876 527 803 420 613 5 '023 252 459 633 711 469 867 819 6 '023 663 918 197 793 454 113 922 7 '024 074 987 307 426 267 581 289 8 '024 485 667 699 166 952 949 293 9 '024 895 960 107 485 002 792 097	021 714 181 245 155 137 172 022 148 453 796 294 592 645 022 582 725 913 183 860 863 023 016 997 595 823 810 281 023 451 268 844 215 309 350 023 885 539 658 359 226 522 024 319 810 038 256 430 242 024 754 079 983 907 788 955 025 188 349 495 314 171 103 025 622 618 572 476 445 124
060 1 2 8 4 5 6 7 8 9	'025 305 865 264 770 240 846 731 '025 715 383 901 340 666 122 884 '026 124 516 745 450 260 064 451 '026 533 264 523 296 756 971 474 '026 941 627 959 029 377 889 766 '027 349 607 774 756 528 174 118 '027 757 204 690 553 458 929 656 '028 164 419 424 469 892 534 562 '028 571 252 692 537 612 446 049 '028 977 705 208 778 017 490 146	026 056 887 215 395 479 456 026 491 155 424 072 142 533 026 925 423 198 507 302 785 027 359 690 538 701 828 641 027 793 957 444 656 588 528 028 228 223 916 372 450 869 028 662 489 953 850 284 085 029 096 755 557 090 956 595 029 531 020 726 095 336 813 029 965 285 460 864 293 153
070 1 2 8 4 5 6 7 8 9	'029 383 777 685 209 640 834 541 '029 789 470 831 855 633 842 442 '030 194 785 356 751 215 004 087 '030 599 721 965 951 084 141 297 '031 004 281 363 536 802 079 149 '031 408 464 251 624 135 977 610 '031 812 271 330 370 370 514 706 '032 215 703 297 981 585 111 557 '032 618 760 850 719 897 388 371 '033 021 444 682 910 673 039 266	030 399 549 761 398 694 026 030 833 813 627 699 407 840 031 268 077 059 767 302 999 031 702 340 057 603 247 906 032 136 602 621 208 110 963 032 570 864 750 582 760 565 033 005 126 445 728 065 108 033 439 387 706 644 892 984 033 873 648 533 334 112 582 034 307 908 925 796 592 291
080 1 2 8 4 5 6 7 8 9	'033 423 755 486 949 702 312 561 '033 825 693 953 310 343 281 997 '034 227 260 770 550 632 093 087 '034 628 456 625 320 360 367 695 '035 029 282 202 368 119 948 658 '035 429 738 184 548 315 165 181 '035 829 825 252 828 142 798 510 '036 229 544 086 294 539 926 257 '036 628 895 362 161 099 822 603 '037 027 879 755 774 956 090 456	034 742 168 884 033 200 494 035 176 428 408 044 805 572 035 610 687 497 832 275 906 036 044 946 153 396 479 873 036 479 204 374 738 285 845 036 913 462 161 858 562 194 037 347 719 514 758 177 289 037 781 976 433 437 999 497 038 216 232 917 898 897 181 038 650 488 968 141 738 702
090 1 2 8 4 5 6 7 8 9	'037 426 497 940 623 635 200 513 '037 824 750 588 341 877 611 063 '038 222 638 368 718 427 641 227 '038 620 161 949 702 792 269 256 '039 017 321 997 411 969 026 386 '039 414 119 176 137 143 155 676 '039 810 554 148 350 354 204 143 '040 206 627 574 711 132 215 483 '040 602 340 114 073 103 689 543 '040 997 692 423 490 567 473 697	039 084 744 584 167 392 419 039 518 999 765 976 726 686 039 953 254 513 570 609 858 040 387 508 826 949 910 285 040 821 762 706 115 496 315 041 256 016 151 068 236 293 041 690 269 161 808 998 562 042 124 521 738 338 651 463 042 558 773 880 658 063 332 042 993 025 588 768 102 505

n	III.	IV.	V.
050	021 714 723 552 294 507	021 714 724 094 620	021 714 724 095
1	022 149 018 012 265 889	022 149 018 576 501	022 149 018 577
2	022 583 312 471 802 976	022 583 313 058 382	022 583 313 059
3	023 017 606 930 905 769	023 017 607 540 262	023 017 607 541
4	023 451 901 389 574 267	023 451 902 022 142	023 451 902 023
5	023 886 195 847 808 471	023 886 196 504 022	023 886 196 505
6	024 320 490 305 608 380	024 320 490 985 901	024 320 490 987
7	024 754 784 762 973 995	024 754 785 467 780	024 754 785 468
8	025 189 079 219 905 316	025 189 079 949 658	025 189 079 950
9	025 623 373 676 402 342	025 623 374 431 536	025 623 374 432
060	026 057 668 132 465 074	026 057 668 913 413	026 057 668 914
1	026 491 962 588 093 511	026 491 963 395 290	026 491 963 396
2	026 926 257 043 287 654	026 926 257 877 167	026 926 257 878
3	027 360 551 498 047 502	027 360 552 359 043	027 360 552 360
4	027 794 845 952 373 056	027 794 846 840 919	027 794 846 842
5	028 229 140 406 264 316	028 229 141 322 794	028 229 141 324
6	028 663 434 859 721 281	028 663 435 804 669	028 663 435 806
7	029 097 729 312 743 951	029 097 730 286 543	029 097 730 288
8	029 532 023 765 332 328	029 532 024 768 417	029 532 024 769
9	029 966 318 217 486 409	029 966 319 250 291	029 966 319 251
070	030 400 612 669 206 197	030 400 613 732 164	030 400 613 733
1	030 834 907 120 491 690	030 834 908 214 036	030 834 908 215
2	031 269 201 571 342 889	031 269 202 695 908	031 269 202 697
3	031 703 496 021 759 793	031 703 497 177 780	031 703 497 179
4	032 137 790 471 742 402	032 137 791 659 652	032 137 791 661
5	032 572 084 921 290 718	032 572 086 141 522	032 572 086 143
6	033 006 379 370 404 739	033 006 380 623 393	033 006 380 625
7	033 440 673 819 084 465	033 440 675 105 263	033 440 675 107
8	033 874 968 267 329 897	033 874 969 587 133	033 874 969 588
9	034 309 262 715 141 035	034 309 264 069 002	034 309 264 070
080	034 743 557 162 517 878	034 743 558 550 870	034 743 558 552
1	035 177 851 609 460 427	035 177 853 032 739	035 177 853 034
2	035 612 146 055 968 682	035 612 147 514 607	035 612 147 516
3	036 046 440 502 042 642	036 046 441 996 474	036 046 441 998
4	036 480 734 947 682 307	036 480 736 478 341	036 480 736 480
5	036 915 029 392 887 678	036 915 030 960 208	036 915 030 962
6	037 349 323 837 658 755	037 349 325 442 074	037 349 325 444
7	037 783 618 281 995 538	037 783 619 923 939	037 783 619 926
8	038 217 912 725 898 026	038 217 914 405 805	038 217 914 407
9	038 652 207 169 366 219	038 652 208 887 669	038 652 208 889
090	039 086 501 612 400 118	039 086 503 369 534	039 086 503 371
1	039 520 796 054 999 723	039 520 797 851 398	039 520 797 853
2	039 955 090 497 165 033	039 955 092 333 261	039 955 092 335
3	040 389 384 938 896 049	040 389 386 815 124	040 389 386 817
4	040 823 679 380 192 771	040 823 681 296 987	040 823 681 299
5	041 257 973 821 055 198	041 257 975 778 849	041 257 975 781
6	041 692 268 261 483 331	041 692 270 260 711	041 692 270 263
7	042 126 562 701 477 169	042 126 564 742 572	042 126 564 745
8	042 560 857 141 036 713	042 560 859 224 433	042 560 859 227
9	042 995 151 580 161 963	042 995 153 706 294	042 995 153 708

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n	I.	II.
041 043	100 '041 392 685 158 225 040 750 200 1 '041 787 318 971 751 775 282 557 2 '042 181 594 515 766 244 082 919 3 '042 575 512 440 190 598 661 470 4 '042 969 073 393 180 097 017 761 5 '043 362 278 021 129 502 532 936 6 '043 755 126 968 679 453 920 777 7 '044 147 620 878 722 806 394 496 8 '044 539 760 392 410 944 205 234 9 '044 931 546 149 160 064 707 206	043 427 276 862 669 637 314 043 861 527 702 363 536 088 044 295 778 107 850 667 156 044 730 028 079 131 898 841 045 164 277 616 208 099 466 045 598 526 719 080 137 349 046 032 775 387 748 880 808 046 467 023 622 215 198 156 046 901 271 422 479 957 706 047 335 518 788 544 027 765
110	1 '045 322 978 786 657 434 103 479 1 '045 714 058 940 867 615 025 388 2 '046 104 787 246 038 666 097 650 3 '046 495 164 334 708 313 640 236 4 '046 885 190 837 710 095 657 172 5 '047 274 867 384 179 478 261 437 6 '047 664 194 601 559 944 684 238 7 '048 053 173 115 609 057 015 977 8 '048 441 803 550 404 490 825 332 9 '048 830 086 528 350 042 801 925	047 769 765 720 408 276 641 048 204 012 218 073 572 636 048 638 258 281 540 784 053 049 072 503 910 810 779 189 049 506 749 105 884 426 341 049 940 993 866 762 593 802 050 375 238 193 446 149 861 050 809 482 085 935 962 808 051 243 725 544 232 900 929 051 677 968 568 337 832 505
120	1 '049 218 022 670 181 611 567 172 1 '049 605 612 594 973 151 796 990 2 '049 992 856 920 142 601 799 137 3 '050 379 756 261 457 784 687 079 4 '050 766 311 233 042 283 291 396 5 '051 152 522 447 381 288 948 839 6 '051 538 390 515 327 424 308 322 7 '051 923 916 046 106 540 292 210 8 '052 309 099 647 323 487 350 464 9 '052 693 941 924 967 861 144 304	052 112 211 158 251 625 816 052 546 453 313 975 149 142 052 980 695 035 509 270 756 053 414 936 322 854 858 932 053 849 177 176 012 781 938 054 283 417 594 983 908 043 054 717 657 579 769 105 510 055 151 897 130 369 242 603 055 586 136 246 785 187 579 056 020 374 929 017 808 697
180	1 '053 078 443 483 419 722 795 227 1 '053 462 604 925 455 293 834 380 2 '053 846 426 852 252 625 986 430 3 '054 229 909 863 397 245 921 271 4 '054 613 054 556 887 775 106 067 5 '054 995 861 529 141 524 889 310 6 '055 378 331 375 000 066 947 785 7 '055 760 464 687 734 779 226 490 8 '056 142 262 059 052 367 500 794 9 '056 523 724 079 100 362 689 302	056 454 613 177 067 974 210 056 888 850 990 936 552 370 057 323 088 370 624 411 426 057 757 325 316 132 419 624 058 191 561 827 461 445 208 058 625 797 904 612 356 420 059 060 033 547 586 021 497 059 494 268 756 383 308 677 059 928 503 531 005 086 191 060 362 737 871 452 222 272
140	1 '056 904 851 336 472 594 045 100 1 '057 285 644 418 214 638 352 308 2 '057 666 103 909 829 245 254 023 3 '058 046 230 395 281 738 837 043 4 '058 426 024 457 005 395 597 922 5 '058 805 486 675 906 798 914 191 6 '059 184 617 631 371 170 143 810 7 '059 563 417 901 267 676 475 155 8 '059 941 888 061 954 715 649 106 9 '060 320 028 688 285 177 674 074	060 796 971 777 725 585 146 061 231 205 249 826 043 041 061 665 438 287 754 464 178 062 099 670 891 511 716 778 062 533 903 061 098 669 059 062 968 134 796 516 189 235 063 402 366 097 765 145 520 063 836 596 964 846 406 124 064 270 827 397 760 839 253 064 705 057 396 509 313 113

n	III.	IV.	V.
100	043 429 446 018 852 918	043 429 448 188 154	043 429 448 190
1	043 863 740 457 109 579	043 863 742 670 013	043 863 742 672
2	044 298 034 894 931 945	044 298 037 151 872	044 298 037 154
3	044 732 329 332 320 017	044 732 331 633 731	044 732 331 636
4	045 166 623 769 273 795	045 166 626 115 590	045 166 626 118
5	045 600 918 205 793 278	045 600 920 597 447	045 600 920 600
6	046 035 212 641 878 467	046 035 215 079 305	046 035 215 082
7	046 469 507 077 529 361	046 469 509 561 162	046 469 509 564
8	046 903 801 512 745 961	046 903 804 043 018	046 903 804 046
9	047 338 095 947 528 267	047 338 098 524 875	047 338 098 527
110	047 772 390 381 876 278	047 772 393 006 730	047 772 393 009
1	048 206 684 815 789 995	048 206 687 488 585	048 206 687 491
2	048 640 979 249 269 418	048 640 981 970 440	048 640 981 973
3	049 075 273 682 314 546	049 075 276 452 295	049 075 276 455
4	049 509 568 114 925 379	049 509 570 934 149	049 509 570 937
5	049 943 862 547 101 919	049 943 865 416 002	049 943 865 419
6	050 378 156 978 844 164	050 378 159 897 855	050 378 159 901
7	050 812 451 410 152 114	050 812 454 379 708	050 812 454 383
8	051 246 745 841 025 771	051 246 748 861 560	051 246 748 865
9	051 681 040 271 465 132	051 681 043 343 412	051 681 043 346
120	052 115 334 701 470 200	052 115 337 825 263	052 115 337 828
1	052 549 629 131 040 973	052 549 632 307 114	052 549 632 310
2	052 983 923 560 177 452	052 983 926 788 965	052 983 926 792
3	053 418 217 988 879 636	053 418 221 270 815	053 418 221 274
4	053 852 512 417 147 526	053 852 515 752 664	053 852 515 756
5	054 286 806 844 981 121	054 286 810 234 514	054 286 810 238
6	054 721 101 272 380 423	054 721 104 716 362	054 721 104 720
7	055 155 395 699 345 429	055 155 399 198 211	055 155 399 202
8	055 589 690 125 876 142	055 589 693 680 058	055 589 693 684
9	056 023 984 551 972 560	056 023 988 161 906	056 023 988 166
130	056 458 278 977 634 684	056 458 282 643 753	056 458 282 647
1	056 892 573 402 862 513	056 892 577 125 600	056 892 577 129
2	057 326 867 827 656 048	057 326 871 607 446	057 326 871 611
3	057 761 162 252 015 288	057 761 166 089 291	057 761 166 093
4	058 195 456 675 940 235	058 195 460 571 137	058 195 460 575
5	058 629 751 099 430 887	058 629 755 052 981	058 629 755 057
6	059 064 045 522 487 244	059 064 049 534 826	059 064 049 539
7	059 498 339 945 109 307	059 498 344 016 670	059 498 344 021
8	059 932 634 367 297 076	059 932 638 498 513	059 932 638 503
9	060 366 928 789 050 550	060 366 932 980 357	060 366 932 985
140	060 801 223 210 369 730	060 801 227 462 199	060 801 227 466
1	061 235 517 631 254 616	061 235 521 944 041	061 235 521 948
2	061 669 812 051 705 207	061 669 816 425 883	061 669 816 430
3	062 104 106 471 721 504	062 104 110 907 725	062 104 110 912
4	062 538 400 891 303 507	062 538 405 389 565	062 538 405 394
5	062 972 695 310 451 215	062 972 699 871 406	062 972 699 876
6	063 406 989 729 164 629	063 406 994 353 246	063 406 994 358
7	063 841 284 147 443 749	063 841 288 835 086	063 841 288 840
8	064 275 578 565 288 574	064 275 583 316 925	064 275 583 322
9	064 709 872 982 699 105	064 709 877 798 764	064 709 877 804

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n	I.	II.
.060 .065	150 '060 697 840 353 611 683 654 038 1 '061 075 323 629 791 801 848 963 2 '061 452 479 087 193 241 086 228 3 '061 829 307 294 699 021 640 973 4 '062 205 808 819 712 623 702 573 5 '062 581 984 228 163 113 543 705 6 '062 957 834 084 510 247 507 818 7 '063 333 358 951 749 553 930 049 8 '063 708 559 391 417 393 106 005 9 '064 083 435 963 595 995 422 082	065 139 286 961 092 695 905 065 573 516 091 511 855 830 066 007 744 787 767 661 083 066 441 973 049 860 979 860 066 876 200 877 792 680 352 067 310 428 271 563 630 747 067 744 655 231 174 699 234 068 178 881 756 626 753 995 068 613 107 847 920 663 212 069 047 333 505 057 295 063
160	1 '064 457 989 226 918 477 760 325 1 '064 832 219 738 573 838 290 172 2 '065 206 128 054 311 929 758 716 3 '065 579 714 728 448 411 390 457 4 '065 952 980 313 869 679 506 868 5 '066 325 925 362 037 776 975 400 6 '066 698 550 422 995 281 596 931 7 '067 070 856 045 370 173 539 975 8 '067 442 842 776 380 681 929 342 9 '067 814 511 161 840 110 696 292	069 481 558 728 037 517 725 069 915 783 516 862 199 371 070 350 007 871 532 208 171 070 784 231 792 048 412 295 071 218 455 278 411 679 908 071 652 678 330 622 879 173 072 086 900 948 682 878 250 072 521 123 132 592 545 297 072 955 344 882 352 748 470 073 389 566 197 964 355 920
170	1 '068 185 861 746 161 643 796 561 1 '068 556 895 072 363 129 902 042 2 '068 927 611 682 071 846 671 236 3 '069 298 012 115 529 244 702 975 4 '069 668 096 911 595 671 277 298 5 '070 037 866 607 755 073 986 742 6 '070 407 321 740 119 684 360 677 7 '070 776 462 843 434 681 584 741 8 '071 145 290 451 082 836 416 776 9 '071 513 805 095 089 135 400 110	073 823 787 079 428 235 799 074 258 007 526 745 256 254 074 692 227 539 916 285 429 075 126 447 118 942 191 466 075 560 666 263 823 842 506 075 994 884 974 562 106 685 076 429 103 251 157 852 138 076 863 321 093 611 946 997 077 297 538 501 925 259 390 077 731 755 476 098 657 444
180	1 '071 882 007 306 125 385 474 395 1 '072 249 897 613 514 799 083 647 2 '072 617 476 545 236 559 880 511 3 '072 984 744 627 930 369 125 227 4 '073 351 702 386 900 972 877 145 5 '073 718 350 346 122 670 076 110 6 '074 084 689 028 243 801 610 410 7 '074 450 718 954 591 220 467 456 8 '074 816 440 645 174 743 062 761 9 '075 181 854 618 691 581 842 259	078 165 972 016 133 009 284 078 600 188 122 029 183 030 079 034 403 793 788 046 802 079 468 619 031 410 468 716 079 902 833 834 897 316 885 080 337 048 204 249 459 420 080 771 262 139 467 764 430 081 205 475 640 553 100 020 081 639 688 707 506 334 294 082 073 901 340 328 335 352
190	1 '075 546 961 392 530 759 252 386 1 '075 911 761 482 777 503 171 876 2 '076 276 255 404 217 623 898 580 3 '076 640 443 670 341 872 784 144 4 '077 004 326 793 350 282 608 789 5 '077 367 905 284 156 489 787 922 6 '077 731 179 652 392 038 301 760 7 '078 094 150 406 410 666 838 606 8 '078 456 818 053 292 575 041 911 9 '078 819 183 098 848 675 950 713	082 508 113 539 019 971 292 082 942 325 303 582 110 210 083 376 536 634 015 620 198 083 810 747 530 321 369 346 084 244 957 992 500 225 742 084 679 168 020 553 057 471 085 113 377 614 480 732 615 085 547 586 774 284 119 254 085 981 795 499 964 085 466 086 416 003 791 521 499 323

<i>n</i>	III.	IV.	V.
150	065 144 167 399 675 341	065 144 172 280 602	065 144 172 285
1	065 578 461 816 217 283	065 578 466 762 440	065 578 466 767
2	066 012 756 232 324 931	066 012 761 244 277	066 012 761 249
3	066 447 050 647 998 285	066 447 055 726 114	066 447 055 731
4	066 881 345 063 237 344	066 881 350 207 951	066 881 350 213
5	067 315 639 478 042 109	067 315 644 689 787	067 315 644 695
6	067 749 933 892 412 579	067 749 939 171 623	067 749 939 177
7	068 184 228 306 348 755	068 184 233 653 458	068 184 233 659
8	068 618 522 719 850 637	068 618 528 135 293	068 618 528 141
9	069 052 817 132 918 224	069 052 822 617 127	069 052 822 623
160	069 487 111 545 551 517	069 487 117 098 961	069 487 117 105
1	069 921 405 957 750 516	069 921 411 580 795	069 921 411 586
2	070 355 700 369 515 220	070 355 706 062 628	070 355 706 068
3	070 789 994 780 845 630	070 790 000 544 461	070 790 000 550
4	071 224 289 191 741 746	071 224 295 026 293	071 224 295 032
5	071 658 583 602 203 567	071 658 589 508 125	071 658 589 514
6	072 092 878 012 231 094	072 092 883 989 956	072 092 883 996
7	072 527 172 421 824 327	072 527 178 471 787	072 527 178 478
8	072 961 466 830 983 265	072 961 472 953 618	072 961 472 960
9	073 395 761 239 707 909	073 395 767 435 448	073 395 767 442
170	073 830 055 647 998 258	073 830 061 917 277	073 830 061 924
1	074 264 350 055 854 314	074 264 356 399 106	074 264 356 405
2	074 698 644 463 276 075	074 698 650 880 935	074 698 650 887
3	075 132 938 870 263 541	075 132 945 362 764	075 132 945 369
4	075 567 233 276 816 714	075 567 239 844 591	075 567 239 851
5	076 001 527 682 935 592	076 001 534 326 419	076 001 534 333
6	076 435 822 088 620 175	076 435 828 808 246	076 435 828 815
7	076 870 116 493 870 464	076 870 123 290 073	076 870 123 297
8	077 304 410 898 686 459	077 304 417 771 899	077 304 417 779
9	077 738 705 303 068 160	077 738 712 253 724	077 738 712 261
180	078 172 999 707 015 566	078 173 006 735 550	078 173 006 743
1	078 607 294 110 528 678	078 607 301 217 375	078 607 301 224
2	079 041 588 513 607 496	079 041 595 699 199	079 041 595 706
3	079 475 882 916 252 019	079 475 890 181 023	079 475 890 188
4	079 910 177 318 462 248	079 910 184 662 847	079 910 184 670
5	080 344 471 720 238 183	080 344 479 144 670	080 344 479 152
6	080 778 766 121 579 824	080 778 773 626 492	080 778 773 634
7	081 213 060 522 487 170	081 213 068 108 315	081 213 068 116
8	081 647 354 922 960 221	081 647 362 590 136	081 647 362 598
9	082 081 649 322 998 979	082 081 657 071 958	082 081 657 080
190	082 515 943 722 603 442	082 515 951 553 779	082 515 951 562
1	082 950 238 121 773 611	082 950 246 035 599	082 950 246 044
2	083 384 532 520 509 485	083 384 540 517 419	083 384 540 525
3	083 818 826 918 811 065	083 818 834 999 239	083 818 835 007
4	084 253 129 316 678 351	084 253 129 481 058	084 253 129 489
5	084 687 415 714 111 343	084 687 423 962 877	084 687 423 971
6	085 121 710 111 110 040	085 121 718 444 695	085 121 718 453
7	085 556 004 507 674 443	085 556 012 926 513	085 556 012 935
8	085 990 298 903 804 551	085 990 307 408 331	085 990 307 417
9	086 424 593 299 500 366	086 424 601 890 148	086 424 601 899

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065

	n	I.	II.
°079 °086	200	°079 181 246 047 624 827 722 506	086 850 211 648 957 228 900
	1	°079 543 007 402 906 048 927 113	087 284 419 072 272 142 264
	2	°079 904 467 666 720 716 099 590	087 718 626 061 467 107 483
	3	°080 265 627 339 844 743 839 669	088 152 832 616 542 992 620
	4	°080 626 486 921 805 747 544 782	088 587 038 737 500 665 738
	5	°080 987 046 910 887 188 863 145	089 021 244 424 340 994 895
	6	°081 347 307 804 132 503 952 926	089 455 449 677 064 848 147
	7	°081 707 270 097 349 214 632 999	089 889 654 495 673 093 548
	8	°082 066 934 285 113 022 510 290	090 323 858 880 166 599 149
9	°082 426 300 860 771 886 168 237	090 758 062 830 546 233 000	
210		°082 785 370 316 450 081 500 400	091 192 266 346 812 863 144
	1	°083 144 143 143 052 245 272 747	091 626 469 428 967 357 627
	2	°083 502 619 830 267 401 997 694	092 060 672 077 010 584 488
	3	°083 860 800 866 572 974 202 474	092 494 874 290 943 411 766
	4	°084 218 686 739 238 776 173 935	092 929 076 070 766 707 495
	5	°084 576 277 934 330 991 261 401	093 363 277 416 481 339 710
	6	°084 933 574 936 716 132 818 767	093 797 478 328 088 176 440
	7	°085 290 578 230 064 988 866 506	094 231 678 805 588 085 713
	8	°085 647 288 296 856 550 553 830	094 665 878 848 981 935 554
9	°086 003 705 618 381 924 500 769	095 100 078 458 270 593 985	
220		°086 359 830 674 748 229 099 487	095 534 277 633 454 929 026
	1	°086 715 663 944 882 474 853 679	095 968 476 374 535 808 695
	2	°087 071 205 906 535 428 834 463	096 402 674 681 514 101 006
	3	°087 426 457 036 285 463 330 731	096 836 872 554 390 673 972
	4	°087 781 417 809 542 388 771 442	097 271 069 993 166 395 600
	5	°088 136 088 700 551 270 996 955	097 705 266 997 842 133 900
	6	°088 490 470 182 396 232 956 005	098 139 463 568 418 756 874
	7	°088 844 562 727 004 240 904 531	098 573 659 704 897 132 524
	8	°089 198 366 805 148 875 182 086	099 007 855 407 278 128 850
9	°089 551 882 886 454 085 641 183	099 442 050 675 562 613 847	
230		°089 905 111 439 397 931 804 440	099 876 245 509 751 455 511
	1	°090 258 052 931 316 307 824 016	100 310 439 909 845 521 830
	2	°090 610 707 828 406 652 317 372	100 744 633 875 845 680 796
	3	°090 963 076 595 731 643 152 982	101 178 827 407 752 800 393
	4	°091 315 159 697 222 877 259 205	101 613 020 505 567 748 604
	5	°091 666 957 595 684 535 529 100	102 047 213 169 291 393 411
	6	°092 018 470 752 797 032 893 577	102 481 405 398 924 602 792
	7	°092 369 699 629 120 653 634 843	102 915 597 194 468 244 723
	8	°092 720 644 684 099 172 011 706	103 349 788 555 923 187 176
9	°093 071 306 376 063 458 267 901	103 783 979 483 290 298 121	
240		°093 421 685 162 235 070 094 182	104 218 169 976 570 445 527
	1	°093 771 781 498 729 829 614 557	104 652 360 035 764 497 359
	2	°094 121 595 840 561 385 966 600	105 086 549 660 873 321 580
	3	°094 471 128 641 644 763 545 427	105 520 738 851 897 786 148
	4	°094 820 380 354 799 895 980 501	105 954 927 608 838 759 023
	5	°095 169 351 431 755 145 914 049	106 389 115 931 697 108 158
	6	°095 518 042 323 150 810 649 499	106 823 303 820 473 701 505
	7	°095 866 453 478 542 613 737 935	107 257 491 275 169 407 015
	8	°096 214 585 346 405 182 570 228	107 691 678 295 785 092 633
9	°096 562 438 374 135 512 042 065	108 125 864 882 321 626 306	

n	III.	IV.	V.
200	086 858 887 694 761 886	086 858 896 371 964	086 858 896 381
1	087 293 182 089 589 111	087 293 190 853 781	087 293 190 863
2	087 727 476 483 982 043	087 727 485 335 596	087 727 485 344
3	088 161 770 877 940 680	088 161 779 817 412	088 161 779 826
4	088 596 065 271 465 022	088 596 074 299 227	088 596 074 308
5	089 030 359 664 555 071	089 030 368 781 041	089 030 368 790
6	089 464 654 057 210 825	089 464 663 262 855	089 464 663 272
7	089 898 948 449 432 285	089 898 957 744 669	089 898 957 754
8	090 333 242 841 219 450	090 333 252 226 482	090 333 252 236
9	090 767 537 232 572 322	090 767 546 708 294	090 767 546 718
210	091 201 831 623 490 899	091 201 841 190 107	091 201 841 200
1	091 636 126 013 975 181	091 636 135 671 919	091 636 135 682
2	092 070 420 404 025 169	092 070 430 153 730	092 070 430 163
3	092 504 714 793 640 864	092 504 724 635 541	092 504 724 645
4	092 939 009 182 822 263	092 939 019 117 351	092 939 019 127
5	093 373 303 571 569 369	093 373 313 599 162	093 373 313 609
6	093 807 597 959 882 180	093 807 608 080 971	093 807 608 091
7	094 241 892 347 760 697	094 241 902 562 780	094 241 902 573
8	094 676 186 735 204 919	094 676 197 044 589	094 676 197 055
9	095 110 481 122 214 848	095 110 491 526 398	095 110 491 537
220	095 544 775 508 790 481	095 544 786 008 205	095 544 786 019
1	095 979 069 894 931 821	095 979 080 490 013	095 979 080 501
2	096 413 364 280 638 867	096 413 374 971 820	096 413 374 983
3	096 847 658 665 911 618	096 847 669 453 627	096 847 669 464
4	097 281 953 050 750 074	097 281 963 935 433	097 281 963 946
5	097 716 247 435 154 237	097 716 258 417 239	097 716 258 428
6	098 150 541 819 124 105	098 150 552 899 044	098 150 552 910
7	098 584 836 202 659 679	098 584 847 380 849	098 584 847 392
8	099 019 130 585 760 959	099 019 141 862 653	099 019 141 874
9	099 453 424 968 427 944	099 453 436 344 457	099 453 436 356
230	099 887 719 350 660 635	099 887 730 826 261	099 887 730 838
1	100 322 013 732 459 032	100 322 025 308 064	100 322 025 320
2	100 756 308 113 823 135	100 756 319 789 867	100 756 319 802
3	101 190 602 494 752 943	101 190 614 271 669	101 190 614 283
4	101 624 896 875 248 457	101 624 908 753 471	101 624 908 765
5	102 059 191 255 309 677	102 059 203 235 272	102 059 203 247
6	102 493 485 634 936 602	102 493 497 717 073	102 493 497 729
7	102 927 780 014 129 233	102 927 792 198 874	102 927 792 211
8	103 362 074 392 887 570	103 362 086 680 674	103 362 086 693
9	103 796 368 771 211 613	103 796 381 162 474	103 796 381 175
240	104 230 663 149 101 361	104 230 675 644 273	104 230 675 657
1	104 664 957 526 556 815	104 664 970 126 072	104 664 970 139
2	105 099 251 903 577 975	105 099 264 607 870	105 099 264 621
3	105 533 546 280 164 840	105 533 559 089 668	105 533 559 102
4	105 967 840 656 317 412	105 967 853 571 465	105 967 853 584
5	106 402 135 032 035 689	106 402 148 053 262	106 402 148 066
6	106 836 429 407 319 671	106 836 442 535 059	106 836 442 548
7	107 270 723 782 169 360	107 270 737 016 855	107 270 737 030
8	107 705 018 156 584 754	107 705 031 498 651	107 705 031 512
9	108 139 312 530 565 854	108 139 325 980 446	108 139 325 994

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n	I.	II.
250	'096 910 013 008 056 414 358 783	108 560 051 034 779 875 973
1	'097 257 309 693 419 955 046 489	108 994 236 753 160 709 575
2	'097 604 328 874 410 875 235 604	109 428 422 037 464 995 048
3	'097 951 070 994 150 000 282 599	109 862 606 887 693 600 326
4	'098 297 536 494 697 634 795 300	110 296 791 303 847 393 340
5	'098 643 725 817 056 944 126 810	110 730 975 285 927 242 019
6	'098 989 639 401 177 322 402 698	111 165 158 833 934 014 289
7	'099 335 277 685 957 747 145 768	111 599 341 947 868 578 073
8	'099 680 641 109 250 120 562 359	112 033 524 627 731 801 294
9	'100 025 730 107 862 597 553 767	112 467 706 873 524 551 868
260	'100 370 545 117 562 900 516 011	112 901 888 685 247 697 711
1	'100 715 086 573 081 620 990 867	113 336 070 062 902 106 738
2	'101 059 354 908 115 508 230 669	113 770 251 006 488 646 858
3	'101 403 350 555 330 744 739 095	114 204 431 516 008 185 978
4	'101 747 073 946 366 208 849 777	114 638 611 591 461 592 006
5	'102 090 525 511 836 724 404 238	115 072 791 232 849 732 842
6	'102 433 705 681 336 297 590 325	115 506 970 440 173 476 388
7	'102 776 614 883 441 341 001 960	115 941 149 213 433 690 540
8	'103 119 253 545 713 884 980 707	116 375 327 552 631 243 195
9	'103 461 622 094 704 776 299 303	116 809 505 457 767 002 243
270	'103 803 720 955 956 864 246 987	117 243 682 928 841 835 575
1	'104 145 550 554 008 174 176 116	117 677 859 965 856 611 077
2	'104 487 111 312 395 068 569 237	118 112 036 568 812 196 635
3	'104 828 403 653 655 395 685 465	118 546 212 737 709 460 130
4	'105 169 427 999 331 625 844 677	118 980 388 472 549 269 441
5	'105 510 184 769 973 975 407 720	119 414 563 773 332 492 445
6	'105 850 674 385 143 518 510 525	119 848 738 640 059 997 017
7	'106 190 897 263 415 286 609 664	120 282 913 072 732 651 027
8	'106 530 853 822 381 355 896 624	120 717 087 071 351 322 344
9	'106 870 544 478 653 922 637 704	121 151 260 635 916 878 835
280	'107 209 969 647 868 366 496 172	121 585 433 766 430 188 364
1	'107 549 129 744 686 301 892 993	122 019 606 462 892 118 791
2	'107 888 025 182 798 617 462 116	122 453 778 725 393 537 975
3	'108 226 656 374 928 503 656 042	122 887 950 553 665 313 772
4	'108 565 023 732 834 468 557 047	123 322 121 947 978 314 034
5	'108 903 127 667 313 341 949 157	123 756 292 908 243 406 614
6	'109 240 968 588 203 267 705 677	124 190 463 434 461 459 358
7	'109 578 546 904 386 684 546 761	124 624 633 526 633 340 112
8	'109 915 863 023 793 295 221 210	125 058 803 184 759 916 720
9	'110 252 917 353 493 024 166 427	125 492 972 408 842 057 020
290	'110 589 710 299 248 963 700 116	125 927 141 198 880 628 852
1	'110 926 242 266 420 308 797 056	126 361 309 554 876 500 049
2	'111 262 513 659 065 280 503 981	126 795 477 476 830 538 444
3	'111 598 524 880 394 038 045 308	127 229 644 964 743 611 868
4	'111 934 276 332 681 579 672 169	127 663 812 018 616 588 147
5	'112 269 768 417 270 632 306 928	128 097 978 638 450 335 106
6	'112 605 001 534 574 530 035 067	128 532 144 824 245 720 567
7	'112 939 976 084 080 081 496 066	128 966 310 576 003 612 350
8	'113 274 692 464 350 426 224 595	129 400 475 893 724 878 271
9	'113 609 151 073 027 879 993 094	129 834 640 777 410 386 144

n	III.	IV.	V.
250	108 573 606 904 112 659	108 573 620 462 241	108 573 620 476
1	109 007 901 277 225 171	109 007 914 944 036	109 007 914 958
2	109 442 195 649 903 388	109 442 209 425 830	109 442 209 440
3	109 876 490 022 147 311	109 876 503 907 623	109 876 503 922
4	110 310 784 393 956 939	110 310 798 389 416	110 310 798 403
5	110 745 078 765 332 274	110 745 092 871 209	110 745 092 885
6	111 179 373 136 273 314	111 179 387 353 002	111 179 387 367
7	111 613 667 506 780 059	111 613 681 834 793	111 613 681 849
8	112 047 961 876 852 511	112 047 976 316 585	112 047 976 331
9	112 482 256 246 490 668	112 482 270 798 376	112 482 270 813
260	112 916 550 615 694 531	112 916 565 280 166	112 916 565 295
1	113 350 844 984 464 100	113 350 859 761 956	113 350 859 777
2	113 785 139 352 799 375	113 785 154 243 746	113 785 154 259
3	114 219 433 720 700 355	114 219 448 725 535	114 219 448 741
4	114 653 728 088 167 041	114 653 743 207 324	114 653 743 222
5	115 088 022 455 199 433	115 088 037 689 113	115 088 037 704
6	115 522 316 821 797 530	115 522 332 170 901	115 522 332 186
7	115 956 611 187 961 333	115 956 626 652 688	115 956 626 668
8	116 390 905 553 690 842	116 390 921 134 475	116 390 921 150
9	116 825 199 918 986 057	116 825 215 616 262	116 825 215 632
270	117 259 494 283 846 977	117 259 510 098 948	117 259 510 114
1	117 693 788 648 273 604	117 693 804 579 834	117 693 804 596
2	118 128 083 012 265 936	118 128 099 061 619	118 128 099 078
3	118 562 377 375 823 974	118 562 393 543 404	118 562 393 560
4	118 996 671 738 947 717	118 996 688 025 188	118 996 688 041
5	119 430 966 101 637 166	119 430 982 506 972	119 430 982 523
6	119 865 260 463 892 321	119 865 276 988 756	119 865 277 005
7	120 299 554 825 713 182	120 299 571 470 539	120 299 571 487
8	120 733 849 187 099 749	120 733 865 952 322	120 733 865 969
9	121 168 143 548 052 021	121 168 160 434 104	121 168 160 451
280	121 602 437 908 569 999	121 602 454 915 886	121 602 454 933
1	122 036 732 268 653 683	122 036 749 397 668	122 036 749 415
2	122 471 026 628 303 072	122 471 043 879 449	122 471 043 897
3	122 905 320 987 518 168	122 905 338 361 229	122 905 338 379
4	123 339 615 346 298 969	123 339 632 843 009	123 339 632 861
5	123 773 909 704 645 476	123 773 927 324 789	123 773 927 342
6	124 208 204 062 557 688	124 208 221 806 568	124 208 221 824
7	124 642 498 420 035 607	124 642 516 288 347	124 642 516 306
8	125 076 792 777 079 231	125 076 810 770 125	125 076 810 788
9	125 511 087 133 688 561	125 511 105 251 903	125 511 105 270
290	125 945 381 489 863 597	125 945 399 733 681	125 945 399 752
1	126 379 675 845 604 338	126 379 694 215 458	126 379 694 234
2	126 813 970 200 910 785	126 813 988 697 235	126 813 988 716
3	127 248 264 555 782 938	127 248 283 179 011	127 248 283 198
4	127 682 558 910 220 797	127 682 577 660 787	127 682 577 680
5	128 116 853 264 224 362	128 116 872 142 562	128 116 872 161
6	128 551 147 617 793 632	128 551 166 624 337	128 551 166 643
7	128 985 441 970 928 608	128 985 461 106 111	128 985 461 125
8	129 419 736 323 629 290	129 419 755 587 886	129 419 755 607
9	129 854 030 675 895 678	129 854 050 069 659	129 854 050 089

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	<i>n</i>	I.	II.
113 130	300	113 943 352 306 836 769 206 505	130 268 805 227 061 003 781
	1	114 277 296 561 586 254 399 679	130 702 969 242 677 598 991
	2	114 610 984 232 173 142 887 687	131 137 132 824 261 039 581
	3	114 944 415 712 584 690 619 013	131 571 295 971 812 193 354
	4	115 277 591 395 901 393 281 308	132 005 458 685 331 928 111
	5	115 610 511 674 299 766 709 164	132 439 620 964 821 111 652
	6	115 943 176 939 055 116 643 049	132 873 782 810 280 611 771
	7	116 275 587 580 544 297 888 333	133 307 944 221 711 296 263
	8	116 607 743 988 248 462 923 019	133 742 105 199 114 032 918
	9	116 939 646 550 755 800 002 586	134 176 265 742 489 689 525
810		117 271 295 655 764 260 810 054	134 610 425 851 839 133 868
	1	117 602 691 690 084 277 699 138	135 044 585 527 163 233 731
	2	117 933 835 039 641 470 578 106	135 478 744 768 462 856 895
	3	118 264 726 089 479 343 481 693	135 912 903 575 738 871 137
	4	118 595 365 223 761 970 878 182	136 347 061 948 992 144 232
	5	118 925 752 825 776 673 758 495	136 781 219 888 223 543 953
	6	119 255 889 277 936 685 553 913	137 215 377 393 433 938 070
	7	119 585 774 961 783 807 928 769	137 649 534 464 624 194 349
	8	119 915 410 257 991 056 494 224	138 083 691 101 795 180 557
	9	120 244 795 546 365 296 489 003	138 517 847 304 947 764 455
820		120 573 931 205 849 868 472 706	138 952 003 074 082 813 802
	1	120 902 817 614 527 204 077 066	139 386 158 409 201 196 357
	2	121 231 455 149 621 431 860 311	139 820 313 310 303 779 872
	3	121 559 844 187 500 973 309 516	140 254 467 777 391 432 099
	4	121 887 985 103 681 129 035 618	140 688 621 810 465 020 789
	5	122 215 878 272 826 655 205 515	141 122 775 409 525 413 687
	6	122 543 524 068 754 330 255 442	141 556 928 574 573 478 537
	7	122 870 922 864 435 511 929 596	141 991 081 305 610 083 081
	8	123 198 075 031 998 684 687 716	142 425 233 602 636 095 057
	9	123 524 980 942 731 997 525 132	142 859 385 465 652 382 202
830		123 851 640 967 085 792 248 550	143 293 536 894 659 812 248
	1	124 178 055 474 675 122 250 600	143 727 687 889 659 252 928
	2	124 504 224 834 282 261 825 984	144 161 838 450 651 571 969
	3	124 830 149 413 859 206 071 792	144 595 988 577 637 637 098
	4	125 155 829 580 530 161 414 363	145 030 138 270 618 316 036
	5	125 481 265 700 594 026 804 832	145 464 287 529 594 476 505
	6	125 806 458 139 526 865 625 293	145 898 436 354 566 986 222
	7	126 131 407 261 984 368 347 260	146 332 584 745 536 712 904
	8	126 456 113 431 804 305 983 943	146 766 732 702 504 524 262
	9	126 780 577 012 008 974 377 576	147 200 880 225 471 288 007
840		127 104 798 364 807 629 362 871	147 635 027 314 437 871 846
	1	127 428 777 851 598 912 847 419	148 069 173 969 405 143 484
	2	127 752 515 832 973 269 849 687	148 503 320 190 373 970 624
	3	128 076 012 668 715 356 534 991	148 937 465 977 345 220 965
	4	128 399 268 717 806 439 289 678	149 371 611 330 319 762 204
	5	128 722 284 338 426 784 873 490	149 805 756 249 298 462 036
	6	129 045 059 887 958 041 689 910	150 239 900 734 282 188 152
	7	129 367 595 722 985 612 214 054	150 674 044 785 271 808 243
	8	129 689 892 199 301 016 617 493	151 108 188 402 268 189 994
	9	130 011 949 671 904 247 629 163	151 542 331 585 272 201 089

<i>n</i>	III.	IV.	V.
300	130 288 325 027 727 771	130 288 344 551 432	130 288 344 571
1	130 722 619 379 125 571	130 722 639 033 205	130 722 639 053
2	131 156 913 730 089 076	131 156 933 514 977	131 156 933 535
3	131 591 208 080 618 286	131 591 227 996 749	131 591 228 017
4	132 025 502 430 713 203	132 025 522 478 521	132 025 522 499
5	132 459 796 780 373 825	132 459 816 960 292	132 459 816 980
6	132 894 091 129 600 153	132 894 111 442 062	132 894 111 462
7	133 328 385 478 392 187	133 328 405 923 832	133 328 405 944
8	133 762 679 826 749 927	133 762 700 405 602	133 762 700 426
9	134 196 974 174 673 373	134 196 994 887 371	134 196 994 908
310	134 631 268 522 162 524	134 631 289 369 140	134 631 289 390
1	135 065 562 869 217 381	135 065 583 850 909	135 065 583 872
2	135 499 857 215 837 944	135 499 878 332 677	135 499 878 354
3	135 934 151 562 024 212	135 934 172 814 444	135 934 172 836
4	136 368 445 907 776 187	136 368 467 296 211	136 368 467 318
5	136 802 740 253 093 867	136 802 761 777 978	136 802 761 800
6	137 237 034 597 977 253	137 237 056 259 744	137 237 056 281
7	137 671 328 942 426 345	137 671 350 741 510	137 671 350 763
8	138 105 623 286 441 142	138 105 645 223 275	138 105 645 245
9	138 539 917 630 021 646	138 539 939 705 040	138 539 939 727
320	138 974 211 973 167 855	138 974 234 186 805	138 974 234 209
1	139 408 506 315 879 770	139 408 528 668 569	139 408 528 691
2	139 842 800 658 157 391	139 842 823 150 332	139 842 823 173
3	140 277 095 000 000 717	140 277 117 632 096	140 277 117 655
4	140 711 389 341 409 750	140 711 412 113 858	140 711 412 137
5	141 145 683 682 384 488	141 145 706 595 621	141 145 706 619
6	141 579 978 022 924 932	141 580 001 077 383	141 580 001 100
7	142 014 272 363 031 082	142 014 295 559 144	142 014 295 582
8	142 448 566 702 702 937	142 448 590 040 905	142 448 590 064
9	142 882 861 041 940 499	142 882 884 522 666	142 882 884 546
330	143 317 155 380 743 766	143 317 179 004 426	143 317 179 028
1	143 751 449 719 112 739	143 751 473 486 185	143 751 473 510
2	144 185 744 057 047 418	144 185 767 967 945	144 185 767 992
3	144 620 038 394 547 802	144 620 062 449 704	144 620 062 474
4	145 054 332 731 613 893	145 054 356 931 462	145 054 356 956
5	145 488 627 068 245 689	145 488 651 413 220	145 488 651 438
6	145 922 921 404 443 191	145 922 945 894 978	145 922 945 919
7	146 357 215 740 206 399	146 357 240 376 735	146 357 240 401
8	146 791 510 075 535 312	146 791 534 858 491	146 791 534 883
9	147 225 804 410 429 932	147 225 829 340 248	147 225 829 365
340	147 660 098 744 890 257	147 660 123 822 003	147 660 123 847
1	148 094 393 078 916 288	148 094 418 303 759	148 094 418 329
2	148 528 687 412 508 025	148 528 712 785 514	148 528 712 811
3	148 962 981 745 665 468	148 963 007 267 268	148 963 007 293
4	149 397 276 078 388 616	149 397 301 749 022	149 397 301 775
5	149 831 570 410 677 471	149 831 596 230 776	149 831 596 257
6	150 265 864 742 532 031	150 265 890 712 529	150 265 890 739
7	150 700 159 073 952 297	150 700 185 194 282	150 700 185 220
8	151 134 453 404 938 269	151 134 479 676 034	151 134 479 702
9	151 568 747 735 489 946	151 568 774 157 786	151 568 774 184

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	n	I.	II.
130 151	350	'130 333 768 495 006 116 671 345	151 976 474 334 284 709 211
	1	'130 655 349 022 030 591 309 455	152 410 616 649 306 582 038
	2	'130 976 691 605 617 124 054 227	152 844 758 530 338 687 247
	3	'131 297 796 597 622 972 554 640	153 278 899 977 381 892 510
	4	'131 618 664 349 125 511 219 756	153 713 040 990 437 065 500
	5	'131 939 295 210 424 534 307 441	154 147 181 569 505 073 884
	6	'132 259 689 531 044 550 517 733	154 581 321 714 586 785 329
	7	'132 579 847 659 737 069 128 433	155 015 461 425 683 067 497
	8	'132 899 769 944 482 877 710 317	155 449 600 702 794 788 050
9	'133 219 456 732 494 311 459 129	155 883 739 545 922 814 646	
	360	'133 538 908 370 217 514 181 387	156 317 877 955 068 014 939
	1	'133 858 125 203 334 690 970 783	156 752 015 930 231 256 583
	2	'134 177 107 576 766 352 611 816	157 186 153 471 413 407 228
	3	'134 495 855 834 673 551 747 066	157 620 290 578 615 334 522
	4	'134 814 370 320 460 110 844 382	158 054 427 251 837 906 110
	5	'135 132 651 376 774 842 000 010	158 488 563 491 081 989 634
	6	'135 450 699 345 513 758 613 563	158 922 699 296 348 452 734
	7	'135 768 514 567 822 278 970 494	159 356 834 667 638 163 048
	8	'136 086 097 384 097 421 767 606	159 790 969 604 951 988 210
9	'136 403 448 133 989 993 616 901	160 225 104 108 290 795 851	
	370	'136 720 567 156 406 768 562 927	160 659 238 177 655 453 602
	1	'137 037 454 789 512 659 648 574	161 093 371 813 046 829 090
	2	'137 354 111 370 732 882 564 127	161 527 505 014 465 789 938
	3	'137 670 537 236 755 111 414 154	161 961 637 781 913 203 768
	4	'137 986 732 723 531 626 636 697	162 395 770 115 389 938 199
	5	'138 302 698 166 281 455 108 983	162 829 902 014 896 860 849
	6	'138 618 433 899 492 502 473 783	163 264 033 480 434 859 329
	7	'138 933 940 256 923 677 720 282	163 698 164 512 004 741 253
	8	'139 249 217 571 607 010 053 236	164 132 295 109 607 434 228
9	'139 564 266 175 849 758 083 961	164 566 425 273 243 785 860	
	380	'139 879 086 401 236 511 376 544	165 000 555 002 914 663 754
	1	'140 193 678 578 631 284 382 529	165 434 684 298 620 935 508
	2	'140 508 043 038 179 602 797 107	165 868 813 160 363 468 723
	3	'140 822 180 109 310 582 369 710	166 302 941 588 143 130 994
	4	'141 136 090 120 739 000 201 747	166 737 069 581 960 789 913
	5	'141 449 773 400 467 358 564 017	167 171 197 141 817 313 071
	6	'141 763 230 275 787 941 266 211	167 605 324 267 713 568 055
	7	'142 076 461 073 284 862 610 720	168 039 450 959 650 422 452
	8	'142 389 466 118 836 108 962 835	168 473 577 217 628 743 843
9	'142 702 245 737 615 572 969 221	168 907 703 041 649 399 809	
	390	'143 014 800 254 095 080 456 433	169 341 828 431 713 257 927
	1	'143 327 129 992 046 410 041 046	169 775 953 387 821 185 772
	2	'143 639 235 274 543 305 482 830	170 210 077 909 974 050 916
	3	'143 951 116 423 963 480 812 238	170 644 201 998 172 720 928
	4	'144 262 773 761 990 618 263 321	171 078 325 652 418 063 376
	5	'144 574 207 609 616 359 043 021	171 512 448 872 710 945 824
	6	'144 885 418 287 142 286 967 650	171 946 571 659 052 235 834
	7	'145 196 406 114 181 904 997 187	172 380 694 011 442 800 966
	8	'145 507 171 409 662 604 697 906	172 814 815 929 883 508 774
9	'145 817 714 491 827 628 663 648	173 248 937 414 375 226 815	

n	III.	IV.	V.
350	152 003 042 065 607 330	152 003 068 639 538	152 003 068 666
1	152 437 336 395 290 419	152 437 363 121 289	152 437 363 148
2	152 871 630 724 539 214	152 871 657 603 039	152 871 657 630
3	153 305 925 053 353 715	153 305 952 084 789	153 305 952 112
4	153 740 219 381 733 922	153 740 246 566 539	153 740 246 594
5	154 174 513 709 679 835	154 174 541 048 288	154 174 541 076
6	154 608 808 037 191 453	154 608 835 530 037	154 608 835 558
7	155 043 102 364 268 777	155 043 130 011 786	155 043 130 039
8	155 477 396 690 911 807	155 477 424 493 534	155 477 424 521
9	155 911 691 017 120 543	155 911 718 975 281	155 911 719 003
360	156 345 985 342 894 985	156 346 013 457 028	156 346 013 485
1	156 780 279 668 235 132	156 780 307 938 775	156 780 307 967
2	157 214 573 993 140 986	157 214 602 420 521	157 214 602 449
3	157 648 868 317 612 545	157 648 896 902 267	157 648 896 931
4	158 083 162 641 649 810	158 083 191 324 013	158 083 191 413
5	158 517 456 965 252 781	158 517 485 865 757	158 517 485 895
6	158 951 751 288 421 458	158 951 780 347 502	158 951 780 377
7	159 386 045 611 155 840	159 386 074 829 246	159 386 074 858
8	159 820 339 933 455 928	159 820 369 310 990	159 820 369 340
9	160 254 634 255 321 723	160 254 663 792 733	160 254 663 822
370	160 688 928 576 753 223	160 688 958 274 476	160 688 958 304
1	161 123 222 897 750 429	161 123 252 756 218	161 123 252 785
2	161 557 517 218 313 340	161 557 547 237 960	161 557 547 268
3	161 991 811 538 441 958	161 991 841 719 701	161 991 841 750
4	162 426 105 858 136 281	162 426 136 201 442	162 426 136 232
5	162 860 400 177 396 311	162 860 430 683 183	162 860 430 714
6	163 294 694 496 222 046	163 294 725 164 923	163 294 725 196
7	163 728 988 814 613 487	163 729 019 646 663	163 729 019 677
8	164 163 283 132 570 633	164 163 314 128 402	164 163 314 159
9	164 597 577 450 093 486	164 597 608 610 141	164 597 608 641
380	165 031 871 767 182 045	165 031 903 091 880	165 031 903 123
1	165 466 166 083 836 309	165 466 197 573 618	165 466 197 605
2	165 900 460 400 056 279	165 900 492 055 355	165 900 492 087
3	166 334 754 715 841 955	166 334 786 537 092	166 334 786 569
4	166 769 049 031 193 337	166 769 081 018 829	166 769 081 051
5	167 203 343 346 110 425	167 203 375 500 565	167 203 375 533
6	167 637 637 660 593 218	167 637 669 982 301	167 637 670 015
7	168 071 931 974 641 718	168 071 964 464 037	168 071 964 497
8	168 506 226 288 255 923	168 506 258 945 771	168 506 258 978
9	168 940 520 601 435 834	168 940 553 427 506	168 940 553 460
390	169 374 814 914 181 451	169 374 847 909 240	169 374 847 912
1	169 809 109 226 492 774	169 809 142 390 974	169 809 142 424
2	170 243 403 538 369 803	170 243 436 872 707	170 243 436 906
3	170 677 697 849 812 538	170 677 731 354 440	170 677 731 388
4	171 111 992 160 820 978	171 112 025 836 172	171 112 025 870
5	171 546 286 471 395 124	171 546 320 317 904	171 546 320 352
6	171 980 580 781 534 976	171 980 614 799 636	171 980 614 834
7	172 414 875 091 240 534	172 414 909 281 367	172 414 909 316
8	172 849 169 400 511 798	172 849 203 763 097	172 849 203 797
9	173 283 463 709 348 768	173 283 498 244 827	173 283 498 279

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<i>n</i>	I.	II.
400	146 128 035 678 238 025 925 955	173 683 058 464 918 822 638
173 1	146 438 135 285 774 600 383 079	174 117 179 081 515 163 793
2	146 748 013 630 639 852 277 761	174 551 299 264 165 117 827
3	147 057 671 028 359 912 753 526	174 985 419 012 869 552 282
4	147 367 107 793 786 471 519 067	175 419 538 327 629 334 699
5	147 676 324 241 098 697 650 179	175 853 657 208 445 332 617
6	147 985 320 683 805 153 558 523	176 287 775 655 318 413 573
7	148 294 097 434 745 702 156 376	176 721 893 668 249 445 098
8	148 602 654 806 093 407 246 372	177 156 011 247 239 294 723
9	148 910 993 109 356 427 165 098	177 590 128 392 288 829 978
410	149 219 112 655 379 901 709 247	178 024 245 103 398 918 386
1	149 527 013 754 347 832 372 930	178 458 361 380 570 427 470
2	149 834 696 715 784 955 924 547	178 892 477 223 804 224 752
3	150 142 161 848 558 611 351 537	179 326 592 633 101 177 748
4	150 449 409 460 880 600 201 143	179 760 707 608 462 153 973
5	150 756 439 860 309 040 345 213	180 194 822 149 888 020 940
6	151 063 253 353 750 213 196 901	180 628 936 257 379 646 159
7	151 369 850 247 460 404 407 018	181 063 049 930 937 897 136
8	151 676 230 847 047 738 067 623	181 497 163 170 563 641 376
9	151 982 395 457 474 004 450 316	181 931 275 976 257 746 382
420	152 288 344 383 056 481 306 568	182 365 388 348 021 079 652
1	152 594 077 927 469 748 757 279	182 799 500 285 854 508 684
2	152 899 596 393 747 497 798 616	183 233 611 789 758 900 971
3	153 204 900 084 284 332 451 067	183 667 722 859 735 124 005
4	153 509 989 300 837 565 578 499	184 101 833 495 784 045 275
5	153 814 864 344 529 008 403 884	184 535 943 697 906 532 267
6	154 119 525 515 846 753 748 220	184 970 053 466 103 452 466
7	154 423 973 114 646 953 019 058	185 404 162 800 375 673 351
8	154 728 207 440 155 586 974 892	185 838 271 700 724 062 403
9	155 032 228 790 970 230 291 565	186 272 380 167 149 487 096
430	155 336 037 465 061 809 956 705	186 706 488 199 652 814 905
1	155 639 633 759 776 357 518 076	187 140 595 798 234 913 299
2	155 943 017 971 836 755 211 599	187 574 702 962 896 649 748
3	156 246 190 397 344 475 994 693	188 008 809 693 638 891 716
4	156 549 151 331 781 317 510 428	188 442 915 990 462 506 666
5	156 851 901 070 011 130 007 889	188 877 021 853 368 362 059
6	157 154 439 906 281 538 244 007	189 311 127 282 357 325 353
7	157 456 768 134 225 657 391 992	189 745 232 277 430 264 003
8	157 758 886 046 863 802 981 390	190 179 336 838 588 045 461
9	158 060 793 936 605 194 894 655	190 613 440 965 831 537 177
440	158 362 492 095 249 655 445 011	191 047 544 659 161 606 598
1	158 663 980 813 989 301 560 254	191 481 647 918 579 121 170
2	158 965 260 383 410 231 097 026	191 915 750 744 084 948 334
3	159 266 331 093 494 203 309 984	192 349 853 135 679 955 530
4	159 567 193 233 620 313 500 145	192 783 955 093 365 010 195
5	159 867 847 092 566 661 866 601	193 218 056 617 140 979 764
6	160 168 292 958 512 016 585 651	193 652 157 707 008 731 667
7	160 468 531 119 037 471 141 301	194 086 258 362 969 133 334
8	160 768 561 861 128 095 930 960	194 520 358 585 023 052 192
9	161 068 385 471 174 584 170 049	194 954 458 373 171 355 664

n	III.	IV.	V.
400	173 717 758 017 751 444	173 717 792 726 557	173 717 792 761
1	174 152 052 325 719 825	174 152 087 208 286	174 152 087 243
2	174 586 346 633 253 913	174 586 381 690 015	174 586 381 725
3	175 020 640 940 353 706	175 020 676 171 744	175 020 676 207
4	175 454 935 247 019 205	175 454 970 653 472	175 454 970 689
5	175 889 229 553 250 410	175 889 265 135 199	175 889 265 171
6	176 323 523 859 047 321	176 323 559 616 927	176 323 559 653
7	176 757 818 164 409 937	176 757 854 098 653	176 757 854 135
8	177 192 112 469 338 260	177 192 148 580 380	177 192 148 616
9	177 626 406 773 832 288	177 626 443 062 105	177 626 443 098
410	178 060 701 077 892 023	178 060 737 543 831	178 060 737 580
1	178 494 995 381 517 463	178 495 032 025 556	178 495 032 062
2	178 929 289 684 708 609	178 929 326 507 280	178 929 326 544
3	179 363 583 987 465 461	179 363 620 989 004	179 363 621 026
4	179 797 878 289 788 019	179 797 915 470 728	179 797 915 508
5	180 232 172 591 676 282	180 232 209 952 451	180 232 209 990
6	180 666 466 893 130 252	180 666 504 434 174	180 666 504 472
7	181 100 761 194 149 927	181 100 798 915 896	181 100 798 954
8	181 535 055 494 735 309	181 535 093 397 618	181 535 093 436
9	181 969 349 794 886 396	181 969 387 879 340	181 969 387 917
420	182 403 644 094 603 189	182 403 682 361 061	182 403 682 399
1	182 837 938 393 885 688	182 837 976 842 782	182 837 976 881
2	183 272 232 692 733 893	183 272 271 324 502	183 272 271 363
3	183 706 526 991 147 804	183 706 565 806 222	183 706 565 845
4	184 140 821 289 127 420	184 140 860 287 941	184 140 860 327
5	184 575 115 586 672 743	184 575 154 769 660	184 575 154 809
6	185 009 409 883 783 771	185 009 449 251 378	185 009 449 291
7	185 443 704 180 460 506	185 443 743 733 096	185 443 743 773
8	185 877 998 476 702 946	185 878 038 214 814	185 878 038 255
9	186 312 292 772 511 092	186 312 332 696 531	186 312 332 736
430	186 746 587 067 884 944	186 746 627 178 248	186 746 627 218
1	187 180 881 362 824 502	187 180 921 659 964	187 180 921 700
2	187 615 175 657 329 765	187 615 216 141 680	187 615 216 182
3	188 049 469 951 400 735	188 049 510 623 395	188 049 510 664
4	188 483 764 245 037 411	188 483 805 105 110	188 483 805 146
5	188 918 058 538 239 792	188 918 099 586 825	188 918 099 628
6	189 352 352 831 007 879	189 352 394 068 539	189 352 394 110
7	189 786 647 123 341 673	189 786 688 550 253	189 786 688 592
8	190 220 941 415 241 172	190 220 983 031 966	190 220 983 074
9	190 655 235 706 706 377	190 655 277 513 679	190 655 277 555
440	191 089 529 997 737 288	191 089 571 995 391	191 089 572 037
1	191 523 824 288 333 904	191 523 866 477 103	191 523 866 519
2	191 958 118 578 496 227	191 958 160 958 815	191 958 161 001
3	192 392 412 868 224 256	192 392 455 440 526	192 392 455 483
4	192 826 707 157 517 990	192 826 749 922 236	192 826 749 965
5	193 261 001 446 377 431	193 261 044 403 946	193 261 044 447
6	193 695 295 734 802 577	193 695 338 885 656	193 695 338 929
7	194 129 590 022 793 429	194 129 633 367 366	194 129 633 411
8	194 563 884 310 349 987	194 563 927 849 074	194 563 927 893
9	194 998 178 597 472 251	194 998 222 330 783	194 998 222 375

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	<i>n</i>	I	II
161 195	450	'161 368 002 234 974 892 119 108	195 388 557 727 414 911 172
	1	'161 667 412 437 735 873 656 905	195 822 656 647 754 586 135
	2	'161 966 616 364 074 909 222 906	196 256 755 134 191 247 968
	3	'162 265 614 298 021 529 152 368	196 690 853 186 725 764 086
	4	'162 564 406 523 019 031 427 203	197 124 950 805 359 001 899
	5	'162 862 993 321 926 093 865 651	197 559 047 990 091 828 815
	6	'163 161 374 977 018 380 773 677	197 993 144 740 925 112 240
	7	'163 459 551 769 990 144 080 923	198 427 241 057 859 719 578
	8	'163 757 523 081 955 818 983 906	198 861 336 940 896 518 228
	9	'164 055 291 893 451 614 119 061	199 295 432 390 036 375 589
	460	'164 352 855 784 437 096 288 126	199 729 527 405 280 159 056
	1	'164 650 215 934 296 769 758 233	200 163 621 986 628 736 022
	2	'164 947 372 621 841 650 158 997	200 597 716 134 082 973 877
	3	'165 244 326 125 310 832 998 750	201 031 809 847 643 740 008
	4	'165 541 076 722 373 056 821 993	201 465 903 127 311 901 800
	5	'165 837 624 690 128 261 030 020	201 899 995 973 088 326 636
	6	'166 133 970 305 109 138 386 552	202 334 088 384 973 881 895
	7	'166 430 113 843 282 682 230 147	202 768 180 362 969 434 955
	8	'166 726 055 580 051 728 415 007	203 202 271 907 075 853 189
	9	'167 021 795 790 256 492 001 732	203 636 363 017 294 003 971
	470	'167 317 334 748 176 098 719 460	204 070 453 693 624 754 668
	1	'167 612 672 727 530 111 220 716	204 504 543 936 068 972 649
	2	'167 907 810 001 480 050 150 210	204 938 633 744 627 525 276
	3	'168 202 746 842 630 910 048 719	205 372 723 119 301 279 911
	4	'168 497 483 523 032 670 113 070	205 806 812 060 091 103 914
	5	'168 792 020 314 181 799 833 179	206 240 900 566 997 864 641
	6	'169 086 357 487 022 759 526 945	206 674 988 640 022 429 445
	7	'169 380 495 311 949 495 793 774	207 109 076 279 165 665 677
	8	'169 674 434 058 806 931 907 324	207 543 163 484 428 440 686
	9	'169 968 173 996 892 453 168 045	207 977 250 255 811 621 818
	480	'170 261 715 394 957 387 235 928	208 411 336 593 316 076 417
	1	'170 555 058 521 208 479 463 833	208 845 422 496 942 671 822
	2	'170 848 203 643 309 363 251 605	209 279 507 966 692 275 372
	3	'171 141 151 028 382 025 441 180	209 713 593 002 505 754 402
	4	'171 433 900 943 008 266 772 686	210 147 677 604 563 976 245
	5	'171 726 453 653 231 157 421 545	210 581 761 772 687 808 233
	6	'172 018 809 424 556 487 636 411	211 015 845 506 938 117 691
	7	'172 310 968 521 954 213 497 746	211 449 928 807 315 771 946
	8	'172 602 931 209 859 897 816 688	211 884 011 673 821 638 320
	9	'172 894 697 752 176 146 193 826	212 318 094 106 456 584 132
	490	'173 186 268 412 274 038 257 364	212 752 176 105 221 476 701
	1	'173 477 643 452 994 554 100 074	213 186 257 670 117 183 340
	2	'173 768 823 136 649 995 934 375	213 620 338 801 144 571 362
	3	'174 059 807 725 025 404 984 736	214 054 419 498 304 508 077
	4	'174 350 597 479 379 973 636 555	214 488 499 761 597 860 790
	5	'174 641 192 660 448 452 860 543	214 922 579 591 025 496 807
	6	'174 931 593 528 442 554 931 587	215 356 658 986 588 283 430
	7	'175 221 800 343 052 351 460 933	215 790 737 948 287 087 956
	8	'175 511 813 363 447 666 760 496	216 224 816 476 122 777 683
	9	'175 801 632 848 279 466 557 961	216 658 894 570 096 219 905

n	III.	IV.	V.
450 1 2 3 4 5 6 7 8 9	195 432 472 884 160 221 195 866 767 170 413 897 196 301 061 456 233 279 196 735 355 741 618 367 197 169 650 026 569 160 197 603 944 311 085 660 198 038 238 595 167 865 198 472 532 878 815 777 198 906 827 162 029 394 199 341 121 444 808 717	195 432 516 812 491 195 866 811 294 199 196 301 105 775 906 196 735 400 257 613 197 169 694 739 319 197 603 989 221 025 198 038 283 702 730 198 472 578 184 435 198 906 872 666 140 199 341 167 147 844	195 432 516 856 195 866 811 338 196 301 105 820 196 735 400 302 197 169 694 784 197 603 989 266 198 038 283 748 198 472 578 230 198 906 872 712 199 341 167 194
460 1 2 3 4 5 6 7 8 9	199 775 415 727 153 746 200 209 710 009 064 481 200 644 004 290 540 922 201 078 298 571 583 069 201 512 592 852 190 922 201 946 887 132 364 480 202 381 181 412 103 745 202 815 475 691 408 716 203 249 769 970 279 392 203 684 064 248 715 774	199 775 461 629 547 200 209 756 111 251 200 644 050 592 954 201 078 345 074 656 201 512 639 556 358 201 946 934 038 059 202 381 228 519 761 202 815 523 001 461 203 249 817 483 161 203 684 111 964 861	199 775 461 675 200 209 756 157 200 644 050 639 201 078 345 121 201 512 639 603 201 946 934 085 202 381 228 567 202 815 523 049 203 249 817 531 203 684 112 013
470 1 2 3 4 5 6 7 8 9	204 118 358 526 717 863 204 552 652 804 285 657 204 986 947 081 419 157 205 421 241 358 118 363 205 855 535 634 383 275 206 289 829 910 213 893 206 724 124 185 610 217 207 158 418 460 572 247 207 592 712 735 099 983 208 027 007 009 193 424	204 118 406 446 561 204 552 700 928 259 204 986 995 409 958 205 421 289 891 656 205 855 584 373 354 206 289 878 855 051 206 724 173 336 748 207 158 467 818 444 207 592 762 300 140 208 027 056 781 835	204 118 406 494 204 552 700 976 204 986 995 458 205 421 289 940 205 855 584 422 206 289 878 904 206 724 173 386 207 158 467 868 207 592 762 350 208 027 056 832
480 1 2 3 4 5 6 7 8 9	208 461 301 282 852 572 208 895 595 556 077 425 209 329 889 828 867 985 209 764 184 101 224 250 210 198 478 373 146 222 210 632 772 644 633 899 211 067 066 915 687 282 211 501 361 186 306 371 211 935 655 456 491 166 212 369 949 726 241 667	208 461 351 263 530 208 895 645 745 225 209 329 940 226 919 209 764 234 708 613 210 198 529 190 306 210 632 823 671 999 211 067 118 153 691 211 501 412 635 383 211 935 707 117 075 212 370 001 598 766	208 461 351 314 208 895 645 795 209 329 940 277 209 764 234 759 210 198 529 241 210 632 823 723 211 067 118 205 211 501 412 687 211 935 707 169 212 370 001 651
490 1 2 3 4 5 6 7 8 9	212 804 243 995 557 874 213 238 538 264 439 787 213 672 832 532 887 406 214 107 126 800 900 731 214 541 421 068 479 762 214 975 715 335 624 499 215 410 009 602 334 941 215 844 303 868 611 090 216 278 598 134 452 945 216 712 892 399 860 505	212 804 296 080 456 213 238 590 562 147 213 672 885 043 836 214 107 179 525 526 214 541 474 007 215 214 975 768 488 903 215 410 062 970 591 215 844 357 452 279 216 278 651 933 966 216 712 946 415 653	212 804 296 133 213 238 590 614 213 672 885 096 214 107 179 578 214 541 474 060 214 975 768 542 215 410 063 024 215 844 357 506 216 278 651 988 216 712 946 470

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	n	I.	II.
176	500	*176 091 259 055 681 242 081 289	217 092 972 230 208 281 913
217	1	*176 380 692 243 270 389 531 155	217 527 049 456 459 830 995
	2	*176 669 932 668 149 584 959 724	217 961 126 248 851 734 438
	3	*176 958 980 586 908 154 574 132	218 395 202 607 384 859 525
	4	*177 247 836 255 623 440 482 914	218 829 278 532 060 073 538
	5	*177 536 499 929 862 161 903 566	219 263 354 022 878 243 753
	6	*177 824 971 864 681 771 849 316	219 697 429 079 840 237 448
	7	*178 113 252 314 631 809 313 127	220 131 593 702 946 921 894
	8	*178 401 341 533 755 246 966 830	220 565 577 892 199 164 363
	9	*178 689 239 775 589 834 393 252	220 999 651 647 597 832 123
	510	*178 976 947 293 169 436 869 073	221 433 724 969 143 792 438
	1	*179 264 464 339 025 369 716 095	221 867 797 856 837 912 572
	2	*179 551 791 165 187 728 238 517	222 301 870 310 681 059 784
	3	*179 838 928 023 186 713 263 713	222 735 942 330 674 101 332
	4	*180 125 875 164 053 952 303 966	223 170 013 916 817 904 470
	5	*180 412 632 838 323 816 356 477	223 604 085 069 113 336 452
	6	*180 699 201 296 034 732 358 940	224 038 155 787 561 264 527
	7	*180 985 580 786 730 491 317 863	224 472 226 072 162 555 941
	8	*181 271 771 559 461 552 126 744	224 906 295 922 918 077 940
	9	*181 557 773 862 786 341 091 137	225 340 365 339 828 697 765
	520	*181 843 587 944 772 547 177 550	225 774 434 322 895 282 656
	1	*182 129 214 052 998 413 003 066	226 208 502 872 118 699 848
	2	*182 414 652 434 554 021 582 463	226 642 570 987 499 816 577
	3	*182 699 903 336 042 578 849 571	227 076 638 669 039 500 073
	4	*182 984 967 003 581 691 969 493	227 510 705 916 738 617 566
	5	*183 269 843 682 804 643 458 271	227 944 772 730 598 036 282
	6	*183 554 533 618 861 661 126 468	228 378 839 110 618 623 445
	7	*183 839 037 056 421 183 863 105	228 812 905 056 801 246 274
	8	*184 123 354 239 671 123 276 260	229 246 970 569 146 771 990
	9	*184 407 485 412 320 121 206 633	229 681 035 647 656 067 808
	530	*184 691 430 817 598 803 130 226	230 115 100 292 330 000 941
	1	*184 975 190 698 261 027 466 283	230 549 164 503 169 438 600
	2	*185 258 765 296 585 130 806 524	230 983 228 280 175 247 993
	3	*185 542 154 854 375 169 081 631	231 417 291 623 348 296 325
	4	*185 825 359 612 962 154 680 900	231 851 354 532 689 450 799
	5	*186 108 379 813 205 289 540 870	232 285 417 008 199 578 616
	6	*186 391 215 695 493 194 218 678	232 719 479 049 879 546 973
	7	*186 673 867 499 745 132 965 839	233 153 540 657 730 223 065
	8	*186 956 335 465 412 234 818 031	233 587 601 831 752 474 085
	9	*187 238 619 831 478 710 716 445	234 021 662 571 947 167 223
	540	*187 520 720 836 463 066 676 155	234 455 722 878 315 169 666
	1	*187 802 638 718 419 313 016 909	234 889 782 750 857 348 598
	2	*188 084 373 714 938 169 671 662	235 323 842 189 574 571 201
	3	*188 365 926 063 148 267 588 119	235 757 901 194 467 704 656
	4	*188 647 295 999 717 346 238 455	236 191 959 765 537 616 139
	5	*188 928 483 760 853 447 252 360	236 626 017 902 785 172 824
	6	*189 209 489 582 306 104 188 427	237 060 075 606 211 241 882
	7	*189 490 313 699 367 528 458 891	237 494 132 875 816 690 484
	8	*189 770 956 346 873 791 422 622	237 928 189 711 602 385 795
	9	*190 051 417 759 206 002 661 214	238 362 246 113 569 194 979

n	III.	IV.	V.
500	217 147 186 664 833 772	217 147 240 897 339	217 147 240 952
1	217 581 480 929 372 744	217 581 535 379 025	217 581 535 433
2	218 015 775 193 477 422	218 015 829 860 710	218 015 829 915
3	218 450 069 457 147 807	218 450 124 342 395	218 450 124 397
4	218 884 363 720 383 897	218 884 418 824 080	218 884 418 879
5	219 318 657 983 185 693	219 318 713 305 764	219 318 713 361
6	219 752 952 245 553 195	219 753 007 787 448	219 753 007 843
7	220 187 246 507 486 404	220 187 302 269 131	220 187 302 325
8	220 621 540 768 985 318	220 621 596 750 814	220 621 596 807
9	221 055 835 030 049 938	221 055 891 232 496	221 055 891 289
510	221 490 129 290 680 264	221 490 185 714 178	221 490 185 771
1	221 924 423 550 876 296	221 924 480 195 860	221 924 480 253
2	222 358 717 810 638 034	222 358 774 677 541	222 358 774 734
3	222 793 012 069 965 478	222 793 069 159 222	222 793 069 216
4	223 227 306 328 858 628	223 227 363 640 902	223 227 363 698
5	223 661 600 587 317 483	223 661 658 122 582	223 661 658 180
6	224 095 894 845 342 045	224 095 952 604 261	224 095 952 662
7	224 530 189 102 932 313	224 530 247 085 940	224 530 247 144
8	224 964 483 360 088 287	224 964 541 567 619	224 964 541 626
9	225 398 777 616 809 966	225 398 836 049 297	225 398 836 108
520	225 833 071 873 097 352	225 833 130 530 974	225 833 130 590
1	226 267 366 128 950 444	226 267 425 012 652	226 267 425 072
2	226 701 660 384 369 241	226 701 719 494 328	226 701 719 553
3	227 135 954 639 353 745	227 136 013 976 005	227 136 014 035
4	227 570 248 893 903 955	227 570 308 457 681	227 570 308 517
5	228 004 543 148 019 870	228 004 602 939 356	228 004 602 999
6	228 438 837 401 701 492	228 438 897 421 031	228 438 897 481
7	228 873 131 654 948 819	228 873 191 902 706	228 873 191 963
8	229 307 425 907 761 853	229 307 486 384 380	229 307 486 445
9	229 741 720 160 140 592	229 741 780 866 054	229 741 780 927
530	230 176 014 412 085 037	230 176 075 347 727	230 176 075 409
1	230 610 308 663 595 189	230 610 369 829 400	230 610 369 891
2	231 044 602 914 671 046	231 044 664 311 072	231 044 664 372
3	231 478 897 165 312 610	231 478 958 792 744	231 478 958 854
4	231 913 191 415 519 879	231 913 253 274 416	231 913 253 336
5	232 347 485 665 292 854	232 347 547 756 087	232 347 547 818
6	232 781 779 914 631 536	232 781 842 237 757	232 781 842 300
7	233 216 074 163 535 923	233 216 136 719 428	233 216 136 782
8	233 650 368 412 006 016	233 650 431 201 098	233 650 431 264
9	234 084 662 660 041 815	234 084 725 682 767	234 084 725 746
540	234 518 956 907 643 321	234 519 020 164 436	234 519 020 228
1	234 953 251 154 810 532	234 953 314 646 104	234 953 314 710
2	235 387 545 401 543 449	235 387 609 127 772	235 387 609 191
3	235 821 839 647 842 072	235 821 903 609 440	235 821 903 673
4	236 256 133 893 706 402	236 256 198 091 107	236 256 198 155
5	236 690 428 139 136 437	236 690 492 572 774	236 690 492 637
6	237 124 722 384 132 178	237 124 787 054 440	237 124 787 119
7	237 559 016 628 693 625	237 559 081 536 106	237 559 081 601
8	237 993 310 872 820 778	237 993 376 017 772	237 993 376 083
9	238 427 605 116 513 637	238 427 670 499 437	238 427 670 565

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	n	I.	II.
190 238	550	190 331 698 170 291 484 452 965	238 796 302 081 717 985 198
	1	190 611 797 813 604 942 459 447	239 230 357 616 049 623 610
	2	190 891 716 922 169 632 639 317	239 664 412 716 564 977 372
	3	191 171 455 728 558 524 403 955	240 098 467 383 264 913 636
	4	191 451 014 464 895 460 029 434	240 532 521 616 150 299 554
	5	191 730 393 362 856 310 339 284	240 966 575 415 222 002 273
	6	192 009 592 653 670 126 672 425	241 400 628 780 480 888 940
	7	192 288 612 568 120 289 150 586	241 834 681 711 927 826 698
	8	192 567 453 336 545 651 259 484	242 268 734 209 563 682 687
	9	192 846 115 188 841 680 757 931	242 702 786 273 389 324 045
	560	193 124 598 354 461 596 929 011	243 136 837 903 405 617 907
	1	193 402 903 062 417 504 187 392	243 570 889 099 613 431 405
	2	193 681 029 541 281 522 056 768	244 004 939 862 013 631 671
	3	193 958 978 019 186 911 531 369	244 438 990 190 607 085 831
	4	194 236 748 723 829 197 835 425	244 873 040 085 394 661 011
	5	194 514 341 882 467 289 594 388	245 307 089 546 377 224 332
	6	194 791 757 721 924 594 431 669	245 741 138 573 555 642 914
	7	195 068 996 468 590 131 004 584	246 175 187 166 930 783 875
	8	195 346 058 348 419 637 493 127	246 609 235 326 503 514 328
	9	195 622 943 586 936 676 555 154	247 043 283 052 274 701 386
	570	195 899 652 409 233 736 761 481	247 477 330 344 245 212 158
	1	196 176 185 039 973 330 524 334	247 911 377 202 415 913 750
	2	196 452 541 703 389 088 532 560	248 345 423 626 787 673 267
	3	196 728 722 623 286 850 706 905	248 779 469 617 361 357 810
	4	197 004 728 023 045 753 688 649	249 213 515 174 137 834 478
	5	197 280 558 125 619 314 874 794	249 647 560 297 117 970 367
	6	197 556 213 153 536 513 012 961	250 081 604 986 302 632 572
	7	197 831 693 328 902 865 369 094	250 515 649 241 692 688 182
	8	198 106 998 873 401 501 481 000	250 949 693 063 289 004 286
	9	198 382 130 008 294 233 510 709	251 383 736 451 092 447 971
	580	198 657 086 954 422 623 208 560	251 817 779 405 103 886 319
	1	198 931 869 932 209 045 501 902	252 251 821 925 324 186 412
	2	199 206 479 161 657 748 721 182	252 685 864 011 754 215 327
	3	199 480 914 862 355 911 476 202	253 119 905 664 394 840 140
	4	199 755 177 253 474 696 195 211	253 553 946 883 246 927 923
	5	200 029 266 553 770 299 339 490	253 987 987 668 311 345 748
	6	200 303 182 981 584 998 305 993	254 422 028 019 588 960 681
	7	200 576 926 754 848 195 030 582	254 856 067 937 080 639 787
	8	200 850 498 091 077 456 304 326	255 290 107 420 787 250 130
	9	201 123 897 207 379 550 815 265	255 724 146 470 709 658 769
	590	201 397 124 320 451 482 928 020	256 158 185 086 848 732 761
	1	201 670 179 646 581 523 213 539	256 592 223 269 205 339 161
	2	201 943 063 401 650 235 741 238	257 026 261 017 780 345 021
	3	202 215 775 801 131 502 145 740	257 460 298 332 574 617 391
	4	202 488 317 060 093 542 480 340	257 894 335 213 589 023 317
	5	202 760 687 393 199 932 869 308	258 328 371 660 824 429 844
	6	203 032 887 014 710 619 971 055	258 762 407 674 281 704 013
	7	203 304 916 138 482 932 264 152	259 196 443 253 961 712 863
	8	203 576 774 977 972 588 168 128	259 630 478 399 865 323 431
	9	203 848 463 746 234 701 010 945	260 064 513 111 993 402 751

n	III.	IV.	V.
550 1 2 3 4 5 6 7 8 9	238 861 899 359 772 203 239 296 193 602 596 474 239 730 487 844 986 451 240 164 782 086 942 134 240 599 076 328 463 523 241 033 370 569 550 618 241 467 664 810 203 419 241 901 959 050 421 927 242 336 253 290 206 140 242 770 547 529 556 059	238 861 964 981 101 239 296 259 462 766 239 730 553 944 429 240 164 848 426 093 240 599 142 907 756 241 033 437 389 418 241 467 731 871 080 241 902 026 352 742 242 336 320 834 403 242 770 615 316 063	238 861 965 047 239 296 259 529 239 730 554 011 240 164 848 492 240 599 142 974 241 033 437 456 241 467 731 938 241 902 026 420 242 336 320 902 242 770 615 384
560 1 2 3 4 5 6 7 8 9	243 204 841 768 471 684 243 639 136 006 953 015 244 073 430 245 000 052 244 507 724 482 612 796 244 942 018 719 791 245 245 376 312 956 535 400 245 810 607 192 845 261 246 244 901 428 720 828 246 679 195 664 162 102 247 113 489 899 169 081	243 204 909 797 724 243 639 204 279 383 244 073 498 761 043 244 507 793 242 702 244 942 087 724 360 245 376 382 206 018 245 810 676 687 676 246 244 971 169 333 246 679 265 650 990 247 113 560 132 646	243 204 909 866 243 639 204 348 244 073 498 830 244 507 793 311 244 942 087 793 245 376 382 275 245 810 676 757 246 244 971 239 246 679 265 721 247 113 560 203
570 1 2 3 4 5 6 7 8 9	247 547 784 133 741 766 247 982 078 367 880 157 248 416 372 601 584 255 248 850 666 834 854 058 249 284 961 067 689 567 249 719 255 300 090 782 250 153 549 532 057 704 250 587 843 763 590 331 251 022 137 994 688 664 251 456 432 225 352 704	247 547 854 614 302 247 982 149 095 958 248 416 443 577 613 248 850 738 059 268 249 285 032 540 922 249 719 327 022 575 250 153 621 504 229 250 587 915 985 882 251 022 210 467 534 251 456 504 949 186	247 547 854 685 247 982 149 167 248 416 443 649 248 850 738 130 249 285 032 612 249 719 327 094 250 153 621 576 250 587 916 058 251 022 210 540 251 456 505 022
580 1 2 3 4 5 6 7 8 9	251 890 726 455 582 449 252 325 020 685 377 901 252 759 314 914 739 058 253 193 609 143 665 922 253 627 903 372 158 491 254 062 197 600 216 767 254 496 491 827 840 748 254 930 786 055 030 436 255 365 080 281 785 829 255 799 374 508 106 929	251 890 799 430 838 252 325 093 912 489 252 759 388 394 140 253 193 682 875 790 253 627 977 357 440 254 062 271 839 089 254 496 566 320 738 254 930 860 802 387 255 365 155 284 035 255 799 449 765 682	251 890 799 504 252 325 093 986 252 759 388 468 253 193 682 950 253 627 977 431 254 062 271 913 254 496 566 395 254 930 860 877 255 365 155 359 255 799 449 841
590 1 2 3 4 5 6 7 8 9	256 233 668 733 993 735 256 657 962 959 446 246 257 102 257 184 464 464 257 536 551 409 048 388 257 970 845 633 198 018 258 405 139 856 913 354 258 839 434 080 194 395 259 273 728 303 041 143 259 708 022 525 453 597 260 142 316 747 431 757	256 233 744 247 330 256 658 038 728 976 257 102 333 210 623 257 536 627 692 269 257 970 922 173 914 258 405 216 655 559 258 839 511 137 204 259 273 805 618 848 259 708 100 100 492 260 142 394 582 135	256 233 744 323 256 668 038 805 257 102 333 287 257 536 627 769 257 970 922 250 258 405 216 732 258 839 511 214 259 273 805 696 259 708 100 178 260 142 394 660

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	n	I.	II.
*204 260	600	*204 119 982 655 924 780 854 956	260 498 547 390 346 817 855
	1	*204 391 331 919 299 733 193 141	260 932 581 234 926 435 769
	2	*204 662 511 748 218 854 527 338	261 366 614 645 733 123 522
	3	*204 933 522 354 144 824 840 146	261 800 647 622 767 748 136
	4	*205 204 363 948 144 696 972 119	262 234 680 166 031 176 632
	5	*205 475 036 740 890 882 915 830	262 668 712 275 524 276 029
	6	*205 745 540 942 662 137 038 326	263 102 743 951 247 913 341
	7	*206 015 876 763 344 536 243 437	263 536 775 193 202 955 583
	8	*206 286 044 412 432 457 085 376	263 970 806 001 390 269 765
9	*206 556 044 099 029 549 844 991	264 404 836 375 810 722 894	
610	1	*206 825 876 031 849 709 579 993	264 838 866 316 465 181 976
	2	*207 095 540 419 218 044 160 438	265 272 895 823 354 514 014
	1	*207 365 037 469 071 839 300 687	265 706 924 896 479 586 007
	3	*207 634 367 388 961 520 599 013	266 140 953 535 841 264 953
	4	*207 903 530 386 051 612 595 995	266 574 981 741 440 417 847
	5	*208 172 526 667 121 694 862 764	267 009 009 513 277 911 681
	6	*208 441 356 438 567 355 130 144	267 443 036 851 354 613 446
	7	*208 710 019 906 401 139 469 660	267 877 063 755 671 390 127
	8	*208 978 517 276 253 499 537 367	268 311 090 226 229 108 710
9	*209 246 848 753 373 736 891 363	268 745 116 263 028 636 177	
620	1	*209 515 014 542 630 944 393 851	269 179 141 866 070 839 506
	2	*209 783 014 848 514 944 708 529	269 613 167 035 356 585 674
	3	*210 050 849 875 137 225 904 070	270 047 191 770 886 741 656
	4	*210 318 519 826 231 874 174 376	270 481 216 072 662 174 423
	5	*210 586 024 905 156 503 686 280	270 915 239 940 683 750 944
	6	*210 853 365 314 893 183 565 288	271 349 263 374 952 338 185
	7	*211 120 541 258 049 362 029 947	271 783 286 375 468 803 110
	8	*211 387 552 936 858 787 685 328	272 217 308 942 234 012 680
	9	*211 654 400 553 182 427 986 128	272 651 331 075 248 833 853
1	*211 921 084 308 509 384 879 799	273 085 352 774 514 133 585	
630	1	*212 187 604 403 957 807 640 091	273 519 374 040 030 778 830
	2	*212 453 961 040 275 802 901 355	273 953 394 871 799 636 539
	3	*212 720 154 417 842 341 903 892	274 387 415 269 821 573 659
	4	*212 986 184 736 668 164 960 607	274 821 435 234 097 457 136
	5	*213 252 052 196 396 683 155 161	275 255 454 764 628 153 912
	6	*213 517 756 996 304 877 281 802	275 689 473 861 414 530 929
	7	*213 783 299 335 304 194 036 981	276 123 492 524 457 455 124
	8	*214 048 679 411 941 439 472 823	276 557 510 753 757 793 431
	9	*214 313 897 424 399 669 722 516	276 991 528 549 316 412 784
1	*214 578 953 570 499 079 007 564	277 425 545 911 134 180 112	
640	1	*214 843 848 047 697 884 936 890	277 859 562 839 211 962 343
	2	*215 108 581 053 093 211 107 675	278 293 579 333 550 626 401
	3	*215 373 152 783 421 967 017 798	278 727 595 394 151 039 209
	4	*215 637 563 435 061 725 299 696	279 161 611 021 014 067 686
	5	*215 901 813 204 031 596 285 432	279 595 626 214 140 578 748
	6	*216 165 902 285 993 099 912 697	280 029 640 973 531 439 311
	7	*216 429 830 876 251 034 981 447	280 463 655 299 187 516 285
	8	*216 693 599 169 754 345 770 832	280 897 669 191 109 676 581
	9	*216 957 207 361 096 986 026 027	281 331 682 649 298 787 104
1	*217 220 655 644 518 780 324 532	281 765 695 673 755 714 758	

<i>n</i>	III.	IV.	V.
600	260 576 610 968 975 623	260 576 689 063 778	260 576 689 142
1	261 010 905 190 085 195	261 010 983 545 421	261 010 983 624
2	261 445 199 410 760 473	261 445 278 027 063	261 445 278 106
3	261 879 493 631 001 457	261 879 572 508 704	261 879 572 588
4	262 313 787 850 808 148	262 313 866 990 345	262 313 867 069
5	262 748 082 070 180 544	262 748 161 471 986	262 748 161 551
6	263 182 376 289 118 646	263 182 455 953 626	263 182 456 033
7	263 616 670 507 622 454	263 616 750 435 266	263 616 750 515
8	264 050 964 725 691 969	264 051 044 916 906	264 051 044 997
9	264 485 258 943 327 189	264 485 339 398 545	264 485 339 479
610	264 919 553 160 528 116	264 919 633 880 183	264 919 633 961
1	265 353 847 377 294 748	265 353 928 361 821	265 353 928 443
2	265 788 141 593 627 087	265 788 222 843 459	265 788 222 925
3	266 222 435 809 525 131	266 222 517 325 096	266 222 517 407
4	266 656 730 024 988 882	266 656 811 806 733	266 656 811 889
5	267 091 024 240 018 339	267 091 106 288 369	267 091 106 370
6	267 525 318 454 613 501	267 525 400 770 005	267 525 400 852
7	267 959 612 668 774 370	267 959 695 251 641	267 959 695 334
8	268 393 906 882 500 945	268 393 989 733 276	268 393 989 816
9	268 828 201 095 793 226	268 828 284 214 911	268 828 284 298
620	269 262 495 308 651 213	269 262 578 696 545	269 262 578 780
1	269 696 789 521 074 906	269 696 873 178 179	269 696 873 262
2	270 131 083 733 064 305	270 131 167 659 812	270 131 167 744
3	270 565 377 944 619 410	270 565 462 141 445	270 565 462 226
4	270 999 672 155 740 221	270 999 756 623 077	270 999 756 708
5	271 433 966 366 426 739	271 434 051 104 709	271 434 051 189
6	271 868 260 576 678 962	271 868 345 586 341	271 868 345 671
7	272 302 554 786 496 891	272 302 640 067 972	272 302 640 153
8	272 736 848 995 880 527	272 736 934 549 603	272 736 934 635
9	273 171 143 204 829 868	273 171 229 031 233	273 171 229 117
630	273 605 437 413 344 916	273 605 523 512 863	273 605 523 599
1	274 039 731 621 425 669	274 039 817 994 492	274 039 818 081
2	274 474 025 829 072 129	274 474 112 476 121	274 474 112 563
3	274 908 320 036 284 295	274 908 406 957 750	274 908 407 045
4	275 342 614 243 062 167	275 342 701 439 378	275 342 701 527
5	275 776 908 449 405 745	275 776 995 921 006	275 776 996 008
6	276 211 202 655 315 029	276 211 290 402 633	276 211 290 490
7	276 645 496 860 790 019	276 645 584 884 260	276 645 584 972
8	277 079 791 065 830 715	277 079 879 365 886	277 079 879 454
9	277 514 085 270 437 117	277 514 173 847 512	277 514 173 936
640	277 948 379 474 609 225	277 948 468 329 138	277 948 468 418
1	278 382 673 678 347 039	278 382 762 810 763	278 382 762 900
2	278 816 967 881 650 560	278 817 057 292 387	278 817 057 382
3	279 251 262 084 519 786	279 251 351 774 012	279 251 351 864
4	279 685 556 286 954 719	279 685 646 255 635	279 685 646 346
5	280 119 850 488 955 358	280 119 940 737 259	280 119 940 828
6	280 554 144 690 521 702	280 554 235 218 882	280 554 235 309
7	280 988 438 891 653 753	280 988 529 700 504	280 988 529 791
8	281 422 733 092 351 510	281 422 824 182 126	281 422 824 273
9	281 857 027 292 614 973	281 857 118 663 748	281 857 118 755

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	n	I.	II.
217 282	650	*217 483 944 213 906 282 831 489	282 199 708 264 481 326 445
	1	*217 747 073 262 793 633 453 493	282 633 720 421 476 489 064
	2	*218 010 042 984 363 411 400 350	283 067 732 144 742 069 510
	3	*218 272 853 571 447 486 164 208	283 501 743 434 278 934 677
	4	*218 535 505 216 527 865 925 406	283 935 754 290 087 951 456
	5	*218 797 998 111 737 543 394 402	284 369 764 712 169 986 735
	6	*219 060 332 448 861 339 099 050	284 803 774 700 525 907 401
	7	*219 322 508 419 336 742 126 492	285 237 784 255 156 580 336
	8	*219 584 526 214 254 748 328 876	285 671 793 376 062 872 420
	9	*219 846 386 024 360 696 002 066	286 105 802 063 245 650 532
	660	*220 108 088 040 055 099 046 499	286 539 810 316 705 781 546
	1	*220 369 632 451 394 477 619 273	286 973 818 136 444 132 337
	2	*220 631 019 448 092 186 286 523	287 407 825 522 461 569 773
	3	*220 892 249 219 519 239 685 123	287 841 832 474 758 960 722
	4	*221 153 321 954 705 135 702 677	288 275 838 993 337 172 050
	5	*221 414 237 842 338 676 184 768	288 709 845 078 197 070 618
	6	*221 674 997 070 768 785 178 341	289 143 850 729 339 523 287
	7	*221 935 599 828 005 324 720 133	289 577 855 946 765 396 913
	8	*222 196 046 301 719 908 178 939	290 011 860 730 475 558 351
	9	*222 456 336 679 246 711 160 546	290 445 865 080 470 874 452
	670	*222 716 471 147 583 279 984 076	290 879 868 996 752 212 067
	1	*222 976 449 893 391 337 738 461	291 313 872 479 300 438 042
	2	*223 236 273 102 997 587 927 750	291 747 875 528 176 419 221
	3	*223 495 940 962 394 515 713 877	292 181 878 143 321 022 445
	4	*223 755 453 657 241 186 765 527	292 615 880 324 755 114 555
	5	*224 014 811 372 864 043 721 654	293 049 882 072 479 562 385
	6	*224 274 014 294 257 700 278 217	293 483 883 386 495 232 769
	7	*224 533 062 606 085 732 906 621	293 917 884 266 802 992 540
	8	*224 791 956 492 681 470 212 340	294 351 884 713 403 708 525
	9	*225 050 696 138 048 779 942 164	294 785 884 726 298 247 550
	680	*225 309 281 725 862 853 648 461	295 219 884 305 487 476 438
	1	*225 567 713 439 470 989 018 824	295 653 883 450 972 262 011
	2	*225 825 991 461 893 309 879 432	296 087 882 162 753 471 087
	3	*226 084 115 975 823 843 880 426	296 521 880 440 831 970 481
	4	*226 342 087 163 630 607 871 545	296 955 878 285 208 627 005
	5	*226 599 905 207 357 430 976 276	297 389 875 695 884 307 471
	6	*226 857 570 288 723 525 372 685	297 823 872 672 859 878 687
	7	*227 115 082 589 125 214 789 100	298 257 869 216 136 207 456
	8	*227 372 442 289 636 250 722 775	298 691 865 325 714 160 583
	9	*227 629 649 571 008 666 389 611	299 125 861 001 594 604 866
	690	*227 886 704 613 673 538 413 010	299 559 856 243 778 407 103
	1	*228 143 607 597 741 746 259 877	299 993 851 052 266 434 089
	2	*228 400 358 703 004 729 431 753	300 427 845 427 059 552 617
	3	*228 656 958 108 935 242 419 059	300 861 839 368 158 629 475
	4	*228 913 405 994 688 107 426 355	301 295 832 875 564 531 451
	5	*229 169 702 539 100 964 876 516	301 729 825 949 278 125 328
	6	*229 425 847 920 695 021 701 687	302 163 818 589 300 277 890
	7	*229 681 842 317 675 797 428 837	302 597 810 795 631 855 914
	8	*229 937 685 907 933 868 067 719	303 031 802 568 273 726 178
	9	*230 193 378 869 045 607 808 978	303 465 793 907 226 755 455

n	III.	IV.	V.
650	282 291 321 492 444 142	282 291 413 145 369	282 291 413 237
1	282 725 615 691 839 017	282 725 707 626 990	282 725 707 719
2	283 159 909 890 799 598	283 160 002 108 610	283 160 002 201
3	283 594 204 089 325 886	283 594 296 590 230	283 594 296 683
4	284 028 498 287 417 879	284 028 591 071 849	284 028 591 165
5	284 462 792 485 075 578	284 462 885 553 468	284 462 885 647
6	284 897 086 682 298 984	284 897 180 035 087	284 897 180 128
7	285 331 380 879 088 096	285 331 474 516 705	285 331 474 610
8	285 765 675 075 442 913	285 765 768 998 323	285 765 769 092
9	286 199 969 271 363 437	286 200 063 479 940	286 200 063 574
660	286 634 263 466 849 667	286 634 357 961 557	286 634 358 056
1	287 068 557 661 901 603	287 068 652 443 173	287 068 652 538
2	287 502 851 856 519 245	287 502 946 924 789	287 502 947 020
3	287 937 146 050 702 593	287 937 241 406 405	287 937 241 502
4	288 371 440 244 451 648	288 371 535 888 020	288 371 535 984
5	288 805 734 437 766 408	288 805 830 369 635	288 805 830 466
6	289 240 028 630 646 874	289 240 124 851 249	289 240 124 947
7	289 674 322 823 093 047	289 674 419 332 863	289 674 419 429
8	290 108 617 015 104 926	290 108 713 814 476	290 108 713 911
9	290 542 911 206 682 510	290 543 008 296 089	290 543 008 393
670	290 977 205 397 825 801	290 977 302 777 701	290 977 302 875
1	291 411 499 588 534 798	291 411 597 259 313	291 411 597 357
2	291 845 793 778 809 501	291 845 891 740 925	291 845 891 839
3	292 280 087 968 649 910	292 280 186 222 536	292 280 186 321
4	292 714 382 158 056 026	292 714 480 704 147	292 714 480 803
5	293 148 676 347 027 847	293 148 775 185 757	293 148 775 285
6	293 582 970 535 565 374	293 583 069 667 367	293 583 069 766
7	294 017 264 723 668 608	294 017 364 148 977	294 017 364 248
8	294 451 558 911 337 548	294 451 658 630 586	294 451 658 730
9	294 885 853 098 572 194	294 885 953 112 194	294 885 953 212
680	295 320 147 285 372 545	295 320 247 593 802	295 320 247 694
1	295 754 441 471 738 603	295 754 542 075 410	295 754 542 176
2	296 188 735 657 670 368	296 188 836 557 017	296 188 836 658
3	296 623 029 843 167 838	296 623 131 038 624	296 623 131 140
4	297 057 324 028 231 014	297 057 425 520 231	297 057 425 622
5	297 491 618 212 859 897	297 491 720 001 837	297 491 720 104
6	297 925 912 397 054 485	297 926 014 483 442	297 926 014 586
7	298 360 206 580 814 780	298 360 308 965 047	298 360 309 067
8	298 794 500 764 140 781	298 794 603 446 652	298 794 603 549
9	299 228 794 947 032 488	299 228 897 928 256	299 228 898 031
690	299 663 089 129 489 901	299 663 192 409 860	299 663 192 513
1	300 097 383 311 513 020	300 097 486 891 463	300 097 486 995
2	300 531 677 493 101 845	300 531 781 373 066	300 531 781 477
3	300 965 971 674 256 376	300 966 075 854 669	300 966 075 959
4	301 400 265 854 976 614	301 400 370 336 271	301 400 370 441
5	301 834 560 035 262 557	301 834 664 817 872	301 834 664 923
6	302 268 854 215 114 207	302 268 959 299 474	302 268 959 405
7	302 703 148 394 531 563	302 703 253 781 074	302 703 253 886
8	303 137 442 573 514 625	303 137 548 262 675	303 137 548 368
9	303 571 736 752 063 393	303 571 842 744 275	303 571 842 850

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n	I.	II.
230 303	700 ·230 448 921 378 273 928 540 170 1 ·230 704 313 612 569 017 187 356 2 ·230 959 555 748 569 070 889 967 3 ·231 214 647 962 601 030 016 559 4 ·231 469 590 430 681 309 029 074 5 ·231 724 383 328 516 525 203 165 6 ·231 979 026 831 504 225 212 150 7 ·232 233 521 114 733 609 582 083 8 ·232 487 866 352 986 255 025 443 9 ·232 742 062 720 736 834 660 869	303 899 784 812 491 810 518 304 333 775 284 069 758 134 304 767 765 321 961 465 069 305 201 754 926 167 798 088 305 635 744 096 689 623 952 306 069 732 833 527 809 418 306 503 721 136 683 221 243 306 937 709 006 156 726 179 307 371 696 441 949 190 978 307 805 683 444 061 482 388
710	1 ·232 996 110 392 153 836 126 389 2 ·233 250 009 541 100 277 593 503 3 ·233 503 760 341 134 421 689 497 4 ·233 757 362 965 510 487 335 312 5 ·234 010 817 587 179 359 506 269 6 ·234 264 124 378 789 296 922 910 7 ·234 517 283 512 686 637 679 211 8 ·234 770 295 160 916 502 815 358 9 ·235 023 159 495 223 497 842 283 ·235 275 876 687 052 412 225 099	308 239 670 012 494 467 152 308 673 656 147 249 012 016 309 107 641 848 325 983 718 309 541 627 115 726 248 996 309 975 611 949 450 674 586 310 409 596 349 500 127 219 310 843 580 315 875 473 626 311 277 563 848 577 580 533 311 711 546 947 607 314 666 312 145 529 612 965 542 746
720	1 ·235 528 446 907 548 916 832 566 2 ·235 780 870 327 560 259 359 670 3 ·236 033 147 117 635 957 730 395 4 ·236 285 277 448 028 491 487 705 5 ·236 537 261 488 693 991 177 758 6 ·236 789 099 409 292 925 735 327 7 ·237 040 791 379 190 787 877 370 8 ·237 292 337 567 458 777 511 681 9 ·237 543 738 142 874 483 167 517 ·237 794 993 273 922 561 455 055	312 579 511 844 653 131 493 313 013 493 642 670 947 623 313 447 475 007 019 857 851 313 881 455 937 700 728 888 314 315 436 434 714 427 444 314 749 416 498 061 820 224 315 183 396 127 743 773 933 315 617 375 323 761 155 272 316 051 354 086 114 830 939 316 485 332 414 805 667 631
730	1 ·238 046 103 128 795 414 560 530 2 ·238 297 067 875 393 865 783 862 3 ·238 547 887 681 327 833 125 544 4 ·238 798 562 713 917 000 929 571 5 ·239 049 093 140 191 489 589 107 6 ·239 299 479 126 892 523 321 618 7 ·239 549 720 840 473 096 020 137 8 ·239 799 818 447 098 635 187 294 9 ·240 049 772 112 647 663 958 760 ·240 299 582 002 712 461 222 670	316 919 310 309 834 532 040 317 353 287 771 202 290 859 317 787 264 798 909 810 774 318 221 241 392 957 958 472 318 655 217 553 347 600 635 319 089 193 280 079 603 944 319 523 168 573 154 835 077 319 957 143 432 574 160 710 320 391 117 858 338 447 514 320 825 091 850 448 562 159
740	1 ·240 549 248 282 599 719 841 614 2 ·240 798 771 117 331 202 983 723 3 ·241 048 150 671 644 398 569 376 4 ·241 297 387 109 993 171 840 005 5 ·241 546 480 596 548 416 055 469 6 ·241 795 431 295 198 701 326 433 7 ·242 044 239 369 550 921 588 157 8 ·242 292 904 982 930 939 722 083 9 ·242 541 428 298 384 230 831 588 ·242 789 809 478 676 523 678 220	321 259 065 408 905 371 314 321 693 038 533 709 741 644 322 127 011 224 862 539 809 322 560 983 482 364 632 470 322 994 955 306 216 886 285 323 428 926 696 420 167 907 323 862 897 652 975 343 988 324 296 868 175 883 281 178 324 730 838 265 144 846 123 325 164 807 920 760 905 467

n	III.	IV.	V.
700	304 006 030 930 177 867	304 006 137 225 874	304 006 137 332
1	304 440 325 107 858 048	304 440 431 707 473	304 440 431 814
2	304 874 619 285 103 934	304 874 726 189 072	304 874 726 296
3	305 308 913 461 915 527	305 309 020 670 670	305 309 020 778
4	305 743 207 638 292 826	305 743 315 152 268	305 743 315 260
5	306 177 501 814 235 830	306 177 609 633 865	306 177 609 742
6	306 611 795 989 744 541	306 611 904 115 462	306 611 904 224
7	307 046 090 164 818 959	307 046 198 597 058	307 046 198 705
8	307 480 384 339 459 082	307 480 493 078 654	307 480 493 187
9	307 914 678 513 664 911	307 914 787 560 250	307 914 787 669
710	308 348 972 687 436 447	308 349 082 041 845	308 349 082 151
1	308 783 266 860 773 688	308 783 376 523 440	308 783 376 633
2	309 217 561 033 676 636	309 217 671 005 034	309 217 671 115
3	309 651 855 206 145 290	309 651 965 486 628	309 651 965 597
4	310 086 149 378 179 650	310 086 259 968 221	310 086 260 079
5	310 520 443 549 779 717	310 520 554 449 814	310 520 554 561
6	310 954 737 720 945 489	310 954 848 931 406	310 954 849 043
7	311 389 031 891 676 967	311 389 143 412 999	311 389 143 525
8	311 823 326 061 974 152	311 823 437 894 590	311 823 438 006
9	312 257 620 231 837 043	312 257 732 376 181	312 257 732 488
720	312 691 914 401 265 640	312 692 026 857 772	312 692 026 970
1	313 126 208 570 259 943	313 126 321 339 363	313 126 321 452
2	313 560 502 738 819 952	313 560 615 820 952	313 560 615 934
3	313 994 796 906 945 667	313 994 910 302 542	313 994 910 416
4	314 429 091 074 637 089	314 429 204 784 131	314 429 204 898
5	314 863 385 241 894 217	314 863 499 265 720	314 863 499 380
6	315 297 679 408 717 050	315 297 793 747 308	315 297 793 862
7	315 731 973 575 105 590	315 732 088 228 895	315 732 088 344
8	316 166 267 741 059 836	316 166 382 710 483	316 166 382 825
9	316 600 561 906 579 789	316 600 677 192 070	316 600 677 307
730	317 034 856 071 665 447	317 034 971 673 656	317 034 971 789
1	317 469 150 236 316 812	317 469 266 155 242	317 469 266 271
2	317 903 444 400 533 882	317 903 560 636 828	317 903 560 753
3	318 337 738 564 316 659	318 337 855 118 413	318 337 855 235
4	318 772 032 727 665 142	318 772 149 599 997	318 772 149 717
5	319 206 326 890 579 331	319 206 444 081 582	319 206 444 199
6	319 640 621 053 059 227	319 640 738 563 166	319 640 738 681
7	320 074 915 215 104 828	320 075 033 044 749	320 075 033 163
8	320 509 209 376 716 136	320 509 327 526 332	320 509 327 644
9	320 943 503 537 893 150	320 943 622 007 914	320 943 622 126
740	321 377 797 698 635 870	321 377 916 489 497	321 377 916 608
1	321 812 091 858 944 296	321 812 210 971 078	321 812 211 090
2	322 246 386 018 818 428	322 246 505 452 659	322 246 505 572
3	322 680 680 178 258 266	322 680 799 934 240	322 680 800 054
4	323 114 974 337 263 811	323 115 094 415 821	323 115 094 536
5	323 549 268 495 835 062	323 549 388 897 400	323 549 389 018
6	323 983 562 653 972 019	323 983 683 378 980	323 983 683 500
7	324 417 856 811 674 682	324 417 977 860 559	324 417 977 982
8	324 852 150 968 943 051	324 852 272 342 138	324 852 272 464
9	325 286 445 125 777 126	325 286 566 823 716	325 286 566 945

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n	I.	II.
243	750	243 038 048 686 294 440 284 738
325	1	325 598 777 142 732 325 852
	2	326 032 745 931 059 973 917
	3	326 466 714 285 744 716 297
	4	326 900 682 206 787 419 628
	5	327 334 649 694 188 950 538
	6	327 768 616 747 950 175 658
	7	328 202 583 368 071 961 613
	8	328 636 549 554 555 175 026
	9	329 070 515 307 400 682 519
	9	329 504 480 626 609 350 708
	760	245 512 667 814 149 821 605 156
	1	329 938 445 512 182 046 210
	2	330 372 409 964 119 635 637
	3	330 806 373 982 422 985 599
	4	331 240 337 567 092 962 706
	5	331 674 300 718 130 433 560
	6	332 108 263 435 536 264 766
	7	332 542 225 719 311 322 922
	8	332 976 187 569 456 474 627
	9	333 410 148 985 972 586 475
	9	333 844 109 968 860 525 057
	770	247 973 266 361 806 627 555 684
	1	334 278 070 518 121 156 964
	2	334 712 030 633 755 348 782
	3	335 145 990 315 763 967 096
	4	335 579 949 564 147 878 488
	5	336 013 908 378 907 949 536
	6	336 447 866 760 045 046 817
	7	336 881 824 707 560 036 905
	8	337 315 782 221 453 786 371
	9	337 749 739 301 727 161 784
	9	338 183 695 948 381 029 711
	780	250 420 002 308 893 979 937 282
	1	338 617 652 161 416 256 714
	2	339 051 607 940 833 709 355
	3	339 485 563 286 634 254 191
	4	339 919 518 198 818 757 779
	5	340 353 472 677 388 086 672
	6	340 787 426 722 343 107 421
	7	341 221 380 333 684 686 572
	8	341 655 333 511 413 690 673
	9	342 089 286 255 530 986 264
	9	342 523 238 566 037 439 887
	790	252 853 030 979 893 169 570 383
	1	342 957 190 442 933 918 080
	2	343 391 141 886 221 287 376
	3	343 825 092 895 900 414 309
	4	344 259 043 471 972 165 407
	5	344 692 993 614 437 407 200
	6	345 126 943 323 297 006 210
	7	345 560 892 598 551 828 959
	8	345 994 841 440 202 741 968
	9	346 428 789 848 250 611 754
	9	346 862 737 822 696 304 829

n	III.	IV.	V.
750	325 720 739 282 176 908	325 720 861 305 294	325 720 861 427
1	326 155 033 438 142 396	326 155 155 786 871	326 155 155 909
2	326 589 327 593 673 590	326 589 450 268 448	326 589 450 391
3	327 023 621 748 770 490	327 023 744 750 024	327 023 744 873
4	327 457 915 903 433 096	327 458 039 231 600	327 458 039 355
5	327 892 210 057 661 409	327 892 333 713 176	327 892 333 837
6	328 326 504 211 455 427	328 326 628 194 751	328 326 628 319
7	328 760 798 364 815 152	328 760 922 676 326	328 760 922 801
8	329 195 092 517 740 583	329 195 217 157 900	329 195 217 283
9	329 629 386 670 231 720	329 629 511 639 474	329 629 511 764
760	330 063 680 822 288 564	330 063 806 121 047	330 063 806 246
1	330 497 974 973 911 113	330 498 100 602 620	330 498 100 728
2	330 932 269 125 099 369	330 932 395 084 193	330 932 395 210
3	331 366 563 275 853 331	331 366 689 565 765	331 366 689 692
4	331 800 857 426 172 999	331 800 984 047 336	331 800 984 174
5	332 235 151 576 058 373	332 235 278 528 908	332 235 278 656
6	332 669 445 725 509 453	332 669 573 010 478	332 669 573 138
7	333 103 739 874 526 240	333 103 867 492 049	333 103 867 620
8	333 538 034 023 108 733	333 538 161 973 619	333 538 162 102
9	333 972 328 171 256 932	333 972 456 455 188	333 972 456 583
770	334 406 622 318 970 837	334 406 750 936 757	334 406 751 065
1	334 840 916 466 250 448	334 841 045 418 326	334 841 045 547
2	335 275 210 613 095 766	335 275 339 899 894	335 275 340 029
3	335 709 504 759 506 790	335 709 634 381 462	335 709 634 511
4	336 143 798 905 483 519	336 143 928 863 029	336 143 928 993
5	336 578 093 051 025 956	336 578 223 344 596	336 578 223 475
6	337 012 387 196 134 098	337 012 517 826 163	337 012 517 957
7	337 446 681 340 807 946	337 446 812 307 729	337 446 812 439
8	337 880 975 485 047 501	337 881 106 789 294	337 881 106 921
9	338 315 269 628 852 762	338 315 401 270 859	338 315 401 403
780	338 749 563 772 223 729	338 749 695 752 424	338 749 695 884
1	339 183 857 915 160 402	339 183 990 233 988	339 183 990 366
2	339 618 152 057 662 782	339 618 284 715 552	339 618 284 848
3	340 052 446 199 730 867	340 052 579 197 116	340 052 579 330
4	340 486 740 341 364 659	340 486 873 678 679	340 486 873 812
5	340 921 034 482 564 157	340 921 168 160 241	340 921 168 294
6	341 355 328 623 329 362	341 355 462 641 803	341 355 462 776
7	341 789 622 763 660 272	341 789 757 123 365	341 789 757 258
8	342 223 916 903 556 889	342 224 051 604 926	342 224 051 740
9	342 658 211 043 019 212	342 658 346 086 487	342 658 346 222
790	343 092 505 182 047 241	343 092 640 568 047	343 092 640 703
1	343 526 799 320 640 976	343 526 935 049 607	343 526 935 185
2	343 961 093 458 800 417	343 961 229 531 167	343 961 229 667
3	344 395 387 596 525 565	344 395 524 012 726	344 395 524 149
4	344 829 681 733 816 419	344 829 818 494 285	344 829 818 631
5	345 263 975 870 672 979	345 264 112 975 843	345 264 113 113
6	345 698 270 007 095 245	345 698 407 457 400	345 698 407 595
7	346 132 564 143 083 218	346 132 701 938 958	346 132 702 077
8	346 566 858 278 636 897	346 566 996 420 515	346 566 996 559
9	347 001 152 413 756 281	347 001 290 902 071	347 001 291 041

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n	I.	II.
800	255 272 505 103 306 069 803 795	347 296 685 363 540 687 706
1	255 513 712 819 533 326 047 602	347 730 632 470 784 626 893
2	255 754 786 643 044 169 386 901	348 164 579 144 428 988 898
3	255 995 726 722 401 958 180 879	348 598 525 384 474 640 224
4	256 236 533 205 922 925 687 090	349 032 471 190 922 447 372
5	256 477 206 241 676 727 858 928	349 466 416 563 773 276 840
6	256 717 745 977 486 989 626 071	349 900 361 503 027 995 125
7	256 958 152 560 931 849 662 938	350 334 306 008 687 468 720
8	257 198 426 139 344 503 650 183	350 768 250 080 752 564 116
9	257 438 566 859 813 746 034 215	351 202 193 719 224 147 801
810	257 678 574 869 184 510 289 744	351 636 136 924 103 086 261
1	257 918 450 314 058 407 690 282	352 070 079 695 390 245 978
2	258 158 193 340 794 264 591 579	352 504 022 033 086 493 433
3	258 397 804 095 508 658 232 883	352 937 963 937 192 695 105
4	258 637 282 724 076 451 060 953	353 371 905 407 709 717 467
5	258 876 629 372 131 323 581 689	353 805 846 444 638 426 993
6	259 115 844 185 066 305 744 266	354 239 787 047 979 690 153
7	259 354 927 308 034 306 862 598	354 673 727 217 734 373 414
8	259 593 878 885 948 644 078 983	355 107 666 953 903 343 241
9	259 832 699 063 483 569 374 711	355 541 606 256 487 466 096
820	260 071 387 985 074 795 132 460	355 975 545 125 487 608 439
1	260 309 945 794 920 018 255 224	356 409 483 560 904 636 727
2	260 548 372 636 979 442 846 540	356 843 421 562 739 417 414
3	260 786 668 654 976 301 456 758	357 277 359 130 992 816 952
4	261 024 833 992 397 374 900 056	357 711 296 265 665 701 791
5	261 262 868 792 493 510 646 909	358 145 232 966 758 938 376
6	261 500 773 198 280 139 796 699	358 579 169 234 273 393 153
7	261 738 547 352 537 792 635 119	359 013 105 068 209 932 562
8	261 976 191 397 812 612 781 024	359 447 040 468 569 423 043
9	262 213 705 476 416 869 927 360	359 880 975 435 352 731 031
880	262 451 089 730 429 471 180 776	360 314 909 968 560 722 961
1	262 688 344 301 696 471 004 512	360 748 844 068 194 265 263
2	262 925 469 331 831 579 769 147	361 182 777 734 254 224 366
3	263 162 464 962 216 670 915 752	361 616 710 966 741 466 695
4	263 399 331 334 002 286 736 009	362 050 643 765 656 858 674
5	263 636 068 588 108 142 773 790	362 484 576 131 001 266 724
6	263 872 676 865 223 630 852 731	362 918 508 062 775 557 263
7	264 109 156 305 808 320 734 276	363 352 439 560 980 596 706
8	264 345 507 050 092 460 410 659	363 786 370 625 617 251 466
9	264 581 729 238 077 475 037 294	364 220 301 256 686 387 953
840	264 817 823 009 536 464 508 994	364 654 231 454 188 872 576
1	265 053 788 504 014 699 684 450	365 088 161 218 125 571 738
2	265 289 625 860 830 117 263 375	365 522 090 548 497 351 843
3	265 525 335 219 073 813 320 695	365 956 019 445 305 079 291
4	265 760 916 717 610 535 502 160	366 389 947 908 549 620 478
5	265 996 370 495 079 173 885 729	366 823 875 938 231 841 800
6	266 231 696 689 893 250 513 073	367 257 803 534 352 609 649
7	266 466 895 440 241 407 595 508	367 691 730 696 912 790 413
8	266 701 966 884 087 894 398 661	368 125 657 425 913 250 481
9	266 936 911 159 173 052 810 176	368 559 583 721 354 856 236

n	III.	IV.	V.
800	347 435 446 548 441 373	347 435 585 383 627	347 435 585 522
1	347 869 740 682 692 170	347 869 879 865 183	347 869 880 004
2	348 304 034 816 508 674	348 304 174 346 738	348 304 174 486
3	348 738 328 949 890 883	348 738 468 828 293	348 738 468 968
4	349 172 623 082 838 799	349 172 763 309 847	349 172 763 450
5	349 606 917 215 352 422	349 607 057 791 401	349 607 057 932
6	350 041 211 347 431 750	350 041 352 272 954	350 041 352 414
7	350 475 505 479 076 785	350 475 646 754 507	350 475 646 896
8	350 909 799 610 287 526	350 909 941 236 060	350 909 941 378
9	351 344 093 741 063 973	351 344 235 717 612	351 344 235 860
810	351 778 387 871 406 126	351 778 530 199 164	351 778 530 341
1	352 212 682 001 313 985	352 212 824 680 715	352 212 824 823
2	352 646 976 130 787 551	352 647 119 162 266	352 647 119 305
3	353 081 270 259 826 823	353 081 413 643 816	353 081 413 787
4	353 515 564 388 431 801	353 515 708 125 366	353 515 708 269
5	353 949 858 516 602 486	353 950 002 606 916	353 950 002 751
6	354 384 152 644 338 877	354 384 297 088 465	354 384 297 233
7	354 818 446 771 640 973	354 818 591 570 013	354 818 591 715
8	355 252 740 898 508 777	355 252 886 051 562	355 252 886 197
9	355 687 035 024 942 286	355 687 180 533 109	355 687 180 679
820	356 121 329 150 941 501	356 121 475 014 657	356 121 475 161
1	356 555 623 276 506 423	356 555 769 496 204	356 555 769 642
2	356 989 917 401 637 051	356 990 063 977 750	356 990 064 124
3	357 424 211 526 333 386	357 424 358 459 296	357 424 358 606
4	357 858 505 650 595 426	357 858 652 940 842	357 858 653 088
5	358 292 799 774 423 173	358 292 947 422 387	358 292 947 570
6	358 727 093 897 816 626	358 727 241 903 932	358 727 242 052
7	359 161 388 020 775 785	359 161 536 385 476	359 161 536 534
8	359 595 682 143 300 650	359 595 830 867 020	359 595 831 016
9	360 029 976 265 391 222	360 030 125 348 563	360 030 125 498
830	360 464 270 387 047 500	360 464 419 830 106	360 464 419 980
1	360 898 564 508 269 484	360 898 714 311 649	360 898 714 461
2	361 332 858 629 057 175	361 333 008 793 191	361 333 008 943
3	361 767 152 749 410 571	361 767 303 274 733	361 767 303 425
4	362 201 446 869 329 674	362 201 597 756 274	362 201 597 907
5	362 635 740 988 814 483	362 635 892 237 815	362 635 892 389
6	363 070 035 107 864 999	363 070 186 719 355	363 070 186 871
7	363 504 329 226 481 220	363 504 481 200 895	363 504 481 353
8	363 938 623 344 663 148	363 938 775 682 435	363 938 775 835
9	364 372 917 462 410 782	364 373 070 163 974	364 373 070 317
840	364 807 211 579 724 122	364 807 364 645 512	364 807 364 799
1	365 241 505 696 603 169	365 241 659 127 051	365 241 659 280
2	365 675 799 813 047 922	365 675 953 608 588	365 675 953 762
3	366 110 093 929 058 381	366 110 248 090 126	366 110 248 244
4	366 544 388 044 634 546	366 544 542 571 663	366 544 542 726
5	366 978 682 159 776 418	366 978 837 053 199	366 978 837 208
6	367 412 976 274 483 996	367 413 131 534 735	367 413 131 690
7	367 847 270 388 757 280	367 847 426 016 271	367 847 426 172
8	368 281 564 502 596 270	368 281 720 497 806	368 281 720 654
9	368 715 858 616 000 967	368 716 014 979 341	368 716 015 136

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	n	I.	II.
267 368	850	267 171 728 403 013 801 594 712	368 993 509 583 238 474 060
	1	267 406 418 752 904 119 340 494	369 427 435 011 564 970 332
	2	267 640 982 345 915 526 101 671	369 861 360 006 335 211 429
	3	267 875 419 318 897 563 740 683	370 295 284 567 550 063 725
	4	268 109 729 808 478 274 974 866	370 729 208 695 210 393 591
	5	268 343 913 951 064 681 131 470	371 163 132 389 317 067 395
	6	268 577 971 882 843 258 615 280	371 597 055 649 870 951 504
	7	268 811 903 739 780 414 092 995	372 030 978 476 872 912 282
	8	269 045 709 657 622 958 398 506	372 464 900 870 323 816 088
9	269 279 389 771 898 579 163 210	372 898 822 830 224 529 283	
860	1	269 512 944 217 916 312 175 471	373 332 744 356 575 918 221
	2	269 746 373 130 767 011 473 323	373 766 665 449 378 849 255
	3	269 979 676 645 323 818 174 505	374 200 586 108 634 188 737
	4	270 212 854 896 242 628 047 889	374 634 506 334 342 803 014
	5	270 445 908 017 962 557 830 356	375 068 426 126 505 558 431
	6	270 678 836 144 706 410 293 158	375 502 345 485 123 321 331
	7	270 911 639 410 481 138 061 790	375 936 264 410 196 958 055
	8	271 144 317 949 078 306 193 369	376 370 182 901 727 334 940
	9	271 376 871 894 074 553 515 529	376 804 100 959 715 318 320
		271 609 301 378 832 052 730 788	377 238 018 584 161 774 529
870	1	271 841 606 536 498 969 290 370	377 671 935 775 067 569 896
	2	272 073 787 500 009 919 041 416	378 105 852 532 433 570 748
	3	272 305 844 402 086 424 651 517	378 539 768 856 260 643 410
	4	272 537 777 375 237 370 814 492	378 973 684 746 549 654 204
	5	272 769 586 551 759 458 241 316	379 407 600 203 301 469 449
	6	273 001 272 063 737 656 440 072	379 841 515 226 516 955 462
	7	273 232 834 043 045 655 288 826	380 275 429 816 196 978 558
	8	273 464 272 621 346 315 405 253	380 709 343 972 342 405 047
	9	273 695 587 930 092 117 316 893	381 143 257 694 954 101 239
		273 926 780 100 525 609 435 841	381 577 170 984 032 933 440
880	1	274 157 849 263 679 854 841 697	382 011 083 839 579 767 954
	2	274 388 795 550 378 876 876 589	382 444 996 261 595 471 083
	3	274 619 619 091 238 103 556 036	382 878 908 250 080 909 124
	4	274 850 320 016 664 810 799 445	383 312 819 805 036 948 375
	5	275 080 898 456 858 564 483 987	383 746 730 926 464 455 128
	6	275 311 354 541 811 661 325 613	384 180 641 614 364 295 674
	7	275 541 688 401 309 568 590 928	384 614 551 868 737 336 301
	8	275 771 900 164 931 362 643 648	385 048 461 689 584 443 295
	9	276 001 989 962 050 166 329 351	385 482 371 076 906 482 940
		276 231 957 921 833 585 202 194	385 916 280 030 704 321 515
890	1	276 461 804 173 244 142 597 300	386 350 188 550 978 825 298
	2	276 691 528 845 039 713 552 453	386 784 096 637 730 860 566
	3	276 921 132 065 773 957 582 766	387 218 004 290 961 293 589
	4	277 150 613 963 796 750 311 958	387 651 911 510 670 990 639
	5	277 379 974 667 254 613 963 853	388 085 818 296 860 817 982
	6	277 609 214 304 091 146 717 723	388 519 724 649 531 641 884
	7	277 838 333 002 047 450 931 066	388 953 630 568 684 328 608
	8	278 067 330 888 662 560 233 396	389 387 536 054 319 744 411
	9	278 296 208 091 273 865 494 631	389 821 441 106 438 755 553
		278 524 964 737 017 539 671 614	390 255 345 725 042 228 287

n	III.	IV.	V.
850	369 150 152 728 971 370	369 150 309 460 875	369 150 309 618
1	369 584 446 841 507 479	369 584 603 942 409	369 584 604 100
2	370 018 740 953 609 294	370 018 898 423 943	370 018 898 581
3	370 453 035 065 276 816	370 453 192 905 476	370 453 193 063
4	370 887 329 176 510 044	370 887 487 387 008	370 887 487 545
5	371 321 623 287 308 978	371 321 781 868 540	371 321 782 027
6	371 755 917 397 673 618	371 756 076 350 072	371 756 076 509
7	372 190 211 507 603 965	372 190 370 831 603	372 190 370 991
8	372 624 505 617 100 018	372 624 665 313 134	372 624 665 473
9	373 058 799 726 161 777	373 058 959 794 664	373 058 959 955
860	373 493 093 834 789 242	373 493 254 276 194	373 493 254 437
1	373 927 387 942 982 414	373 927 548 757 724	373 927 548 919
2	374 361 682 050 741 292	374 361 843 239 253	374 361 843 400
3	374 795 976 158 065 876	374 796 137 720 782	374 796 137 882
4	375 230 270 264 956 167	375 230 432 202 310	375 230 432 364
5	375 664 564 371 412 164	375 664 726 683 838	375 664 726 846
6	376 098 858 477 433 867	376 099 021 165 365	376 099 021 328
7	376 533 152 583 021 276	376 533 315 646 892	376 533 315 810
8	376 967 446 688 174 392	376 967 610 128 419	376 967 610 292
9	377 401 740 792 893 214	377 401 904 609 945	377 401 904 774
870	377 836 034 897 177 742	377 836 199 091 470	377 836 199 256
1	378 270 329 001 027 976	378 270 493 572 996	378 270 493 738
2	378 704 623 104 443 917	378 704 788 054 520	378 704 788 219
3	379 138 917 207 425 564	379 139 082 536 045	379 139 082 701
4	379 573 211 309 972 917	379 573 377 017 569	379 573 377 183
5	380 007 505 412 085 977	380 007 671 499 092	380 007 671 665
6	380 441 799 513 764 742	380 441 965 980 615	380 441 966 147
7	380 876 093 615 009 215	380 876 260 462 138	380 876 260 629
8	381 310 387 715 819 393	381 310 554 943 660	381 310 555 111
9	381 744 681 816 195 278	381 744 849 425 181	381 744 849 593
880	382 178 975 916 136 869	382 179 143 906 703	382 179 144 075
1	382 613 270 015 644 166	382 613 438 388 224	382 613 438 557
2	383 047 564 114 717 169	383 047 732 869 744	383 047 733 038
3	383 481 858 213 355 879	383 482 027 351 264	383 482 027 520
4	383 916 152 311 560 295	383 916 321 832 784	383 916 322 002
5	384 350 446 409 330 417	384 350 616 314 303	384 350 616 484
6	384 784 740 506 666 246	384 784 910 795 821	384 784 910 966
7	385 219 034 603 567 781	385 219 205 277 340	385 219 205 448
8	385 653 328 700 035 022	385 653 499 758 857	385 653 499 930
9	386 087 622 796 067 970	386 087 794 240 375	386 087 794 412
890	386 521 916 891 666 623	386 522 088 721 892	386 522 088 894
1	386 956 210 986 830 984	386 956 383 203 408	386 956 383 376
2	387 390 505 081 561 050	387 390 677 684 924	387 390 677 858
3	387 824 799 175 856 823	387 824 972 166 440	387 824 972 339
4	388 259 093 269 718 302	388 259 266 647 955	388 259 266 821
5	388 693 387 363 145 487	388 693 561 129 470	388 693 561 303
6	389 127 681 456 138 378	389 127 855 610 984	389 127 855 785
7	389 561 975 548 696 976	389 562 150 092 498	389 562 150 267
8	389 996 269 640 821 280	389 996 444 574 012	389 996 444 749
9	390 430 563 732 511 291	390 430 739 055 525	390 430 739 231

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'278 390 900 1 2 3 4 5 6 7 8 9	'278 753 600 952 828 961 536 333 '278 982 116 865 443 138 289 356 '279 210 512 601 395 127 061 996 '279 438 788 287 020 455 310 730 '279 666 944 048 455 540 107 342 '279 894 980 011 638 106 328 276 '280 122 896 302 307 603 746 679 '280 350 693 046 005 623 030 558 '280 578 370 368 076 310 650 526 '280 805 928 393 666 782 700 537	390 689 249 910 131 028 864 391 123 153 661 706 023 535 391 557 056 979 768 078 546 391 990 959 864 318 060 142 392 424 862 315 356 834 562 392 858 764 332 885 268 048 393 292 665 916 904 226 834 393 726 567 067 414 577 155 394 160 467 784 417 185 241 394 594 368 067 912 917 322
910 1 2 3 4 5 6 7 8 9	'281 033 367 247 727 537 635 044 '281 260 687 055 012 867 925 966 '281 487 887 940 081 270 642 878 '281 714 970 027 295 856 959 779 '281 941 933 440 824 760 591 814 '282 168 778 304 641 545 165 307 '282 395 504 742 525 610 524 442 '282 622 112 878 062 597 977 913 '282 848 602 834 644 794 488 882 '283 074 974 735 471 535 811 522	395 028 267 917 902 639 623 395 462 167 334 387 218 367 395 896 066 317 367 519 775 396 329 964 866 844 410 065 396 763 862 982 818 755 454 397 197 760 665 291 422 153 397 631 657 914 263 276 373 398 065 554 729 735 184 322 398 499 451 111 708 012 205 398 933 347 060 182 626 224
920 1 2 3 4 5 6 7 8 9	'283 301 228 703 549 608 577 461 '283 527 364 861 693 651 335 397 '283 753 383 332 526 554 547 147 '283 979 284 238 479 859 543 405 '284 205 067 701 794 156 442 434 '284 430 733 844 519 481 034 938 '284 656 282 788 515 710 638 336 '284 881 714 655 452 958 923 631 '285 107 029 566 811 969 718 101 '285 332 227 643 884 509 786 966	399 367 242 575 159 892 580 399 801 137 656 640 677 470 400 235 032 304 625 847 089 400 668 926 519 116 267 628 401 102 820 300 112 805 277 401 536 713 647 616 326 224 401 970 606 561 627 696 653 402 404 499 042 147 782 745 402 838 391 089 177 450 680 403 272 282 702 717 566 634
930 1 2 3 4 5 6 7 8 9	'285 557 309 007 773 760 597 239 '285 782 273 779 394 709 066 888 '286 007 122 079 474 537 302 499 '286 231 854 028 553 011 328 551 '286 456 469 746 982 868 811 445 '286 680 969 354 930 205 781 405 '286 905 352 972 374 862 355 356 '287 129 620 719 110 807 463 869 '287 353 772 714 746 522 585 270 '287 577 809 078 705 384 489 973	403 706 173 882 768 996 781 404 140 064 629 332 607 293 404 573 954 942 409 264 339 405 007 844 821 999 834 084 405 441 734 268 105 182 692 405 875 623 280 726 176 325 406 309 511 859 863 681 141 406 743 400 005 518 563 295 407 177 287 717 691 688 941 407 611 174 996 383 924 229
940 1 2 3 4 5 6 7 8 9	'287 801 729 930 226 046 998 101 '288 025 535 388 362 821 753 458 '288 249 225 571 986 058 016 878 '288 472 800 599 782 521 481 978 '288 696 260 590 255 772 116 356 '288 919 605 661 726 541 031 208 '289 142 835 932 333 106 382 388 '289 365 951 520 031 668 305 884 '289 588 952 542 596 722 890 683 '289 811 839 117 621 435 191 996	408 045 061 841 596 135 307 408 478 948 253 329 188 322 408 912 834 231 583 949 415 409 346 719 776 361 284 727 409 780 604 887 662 060 396 410 214 489 565 487 142 557 410 648 373 809 837 397 341 411 082 257 620 713 690 880 411 516 140 998 116 889 300 411 950 023 942 047 858 726

n	III.	IV.	V.
900	390 864 857 823 767 008	390 865 033 537 037	390 865 033 713
1	391 299 151 914 588 431	391 299 328 018 550	391 299 328 195
2	391 733 446 004 975 560	391 733 622 500 061	391 733 622 677
3	392 167 740 094 928 396	392 167 916 981 573	392 167 917 158
4	392 602 034 184 446 938	392 602 211 463 083	392 602 211 640
5	393 036 328 273 531 186	393 036 505 944 594	393 036 506 122
6	393 470 622 362 181 140	393 470 800 426 104	393 470 800 604
7	393 904 916 450 396 801	393 905 094 907 613	393 905 095 086
8	394 339 210 538 178 168	394 339 389 389 123	394 339 389 568
9	394 773 504 625 525 242	394 773 683 870 631	394 773 684 050
910	395 207 798 712 438 022	395 207 978 352 140	395 207 978 532
1	395 642 092 798 916 508	395 642 272 833 647	395 642 273 014
2	396 076 386 884 960 700	396 076 567 315 155	396 076 567 496
3	396 510 680 970 570 599	396 510 861 796 662	396 510 861 977
4	396 944 975 055 746 204	396 945 156 278 168	396 945 156 459
5	397 379 269 140 487 515	397 379 450 759 674	397 379 450 941
6	397 813 563 224 794 533	397 813 745 241 180	397 813 745 423
7	398 247 857 308 667 257	398 248 039 722 685	398 248 039 905
8	398 682 151 392 105 687	398 682 334 204 190	398 682 334 387
9	399 116 445 475 109 824	399 116 628 685 694	399 116 628 869
920	399 550 739 557 679 667	399 550 923 167 198	399 550 923 351
1	399 985 033 639 815 216	399 985 217 648 702	399 985 217 833
2	400 419 327 721 516 471	400 419 512 130 205	400 419 512 315
3	400 853 621 802 783 433	400 853 806 611 707	400 853 806 797
4	401 287 915 883 616 101	401 288 101 093 210	401 288 101 278
5	401 722 209 964 014 476	401 722 395 574 711	401 722 395 760
6	402 156 504 043 978 557	402 156 690 056 213	402 156 690 242
7	402 590 798 123 508 344	402 590 984 537 714	402 590 984 724
8	403 025 092 202 603 837	403 025 279 019 214	403 025 279 206
9	403 459 386 281 265 037	403 459 573 500 714	403 459 573 688
930	403 893 680 359 491 943	403 893 867 982 214	403 893 868 170
1	404 327 974 437 284 556	404 328 162 463 713	404 328 162 652
2	404 762 268 514 642 874	404 762 456 945 211	404 762 457 134
3	405 196 562 591 566 899	405 196 751 426 710	405 196 751 616
4	405 630 856 668 056 631	405 631 045 908 208	405 631 046 097
5	406 065 150 744 112 069	406 065 340 389 705	406 065 340 579
6	406 499 444 819 733 213	406 499 634 871 202	406 499 635 061
7	406 933 738 894 920 063	406 933 929 352 698	406 933 929 543
8	407 368 032 969 672 620	407 368 223 834 195	407 368 224 025
9	407 802 327 043 990 883	407 802 518 315 690	407 802 518 507
940	408 236 621 117 874 852	408 236 812 797 185	408 236 812 989
1	408 670 915 191 324 528	408 671 107 278 680	408 671 107 471
2	409 105 209 264 339 910	409 105 401 760 175	409 105 401 953
3	409 539 503 336 920 999	409 539 696 241 669	409 539 696 435
4	409 973 797 409 067 793	409 973 990 723 162	409 973 990 916
5	410 408 091 480 780 295	410 408 285 204 655	410 408 285 398
6	410 842 385 552 058 502	410 842 579 686 148	410 842 579 880
7	411 276 679 622 902 416	411 276 874 167 640	411 276 874 362
8	411 710 973 693 312 036	411 711 168 649 132	411 711 168 844
9	412 145 267 763 287 362	412 145 463 130 623	412 145 463 326

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290 412	950	290 034 611 362 518 011 287 794	412 383 906 452 507 465 279
	1	290 257 269 394 518 069 381 597	412 817 788 529 496 575 081
	2	290 479 813 330 673 009 954 443	413 251 670 173 016 054 246
	3	290 702 243 287 854 384 968 976	413 685 551 383 066 768 891
	4	290 924 559 382 754 266 128 542	414 119 432 159 649 585 125
	5	291 146 761 731 885 612 194 208	414 553 312 502 765 369 059
	6	291 368 850 451 582 635 302 597	414 987 192 412 414 986 799
	7	291 590 825 658 001 166 707 405	415 421 071 888 599 304 449
	8	291 812 687 467 119 020 687 482	415 854 950 931 319 188 110
	9	292 034 435 994 736 358 724 338	416 288 829 540 575 503 881
960	1	292 256 071 356 476 051 851 910	416 722 707 716 369 117 858
	2	292 477 593 667 784 042 441 443	417 156 585 458 700 896 135
	3	292 699 003 043 929 705 004 308	417 590 462 767 571 704 801
	4	292 920 299 600 006 206 075 578	418 024 339 642 982 409 947
	5	293 141 483 450 930 863 181 169	418 458 216 084 933 877 657
	6	293 362 554 711 445 502 891 343	418 892 092 093 426 974 015
	7	293 583 513 496 116 817 963 374	419 325 967 668 462 565 101
	8	293 804 359 919 336 723 576 134	419 759 842 810 041 516 993
	9	294 025 094 095 322 712 659 395	420 193 717 518 164 695 766
			294 245 716 138 118 210 320 583
970	1	294 466 226 161 592 927 371 744	421 061 465 634 047 198 246
	2	294 686 624 279 443 212 959 471	421 495 339 041 808 254 090
	3	294 906 910 605 192 406 300 494	421 929 212 016 117 001 091
	4	295 127 085 252 191 187 525 693	422 363 084 556 974 305 311
	5	295 347 148 333 617 927 635 202	422 796 956 664 381 032 810
	6	295 567 099 962 479 037 567 344	423 230 828 338 338 049 646
	7	295 786 940 251 609 316 384 055	423 664 699 578 846 221 872
	8	296 006 669 313 672 298 575 513	424 098 570 385 906 415 541
	9	296 226 287 261 160 600 486 604	424 532 440 759 519 496 703
			296 445 794 206 396 265 867 934
980	1	296 665 190 261 531 110 553 995	425 400 180 206 407 785 687
	2	296 884 475 538 547 066 271 168	425 834 049 279 684 725 595
	3	297 103 650 149 256 523 578 173	426 267 917 919 518 017 167
	4	297 322 714 205 302 673 941 600	426 701 786 125 908 526 440
	5	297 541 667 818 159 850 949 137	427 135 653 898 857 119 446
	6	297 760 511 099 133 870 663 109	427 569 521 238 364 662 218
	7	297 979 244 159 362 371 116 907	428 003 388 144 432 020 784
	8	298 197 867 109 815 150 956 919	428 437 254 617 060 061 170
	9	298 416 380 061 294 507 232 523	428 871 120 656 249 649 399
			298 634 783 124 435 572 336 731
990	1	298 853 076 409 706 650 100 022	429 738 851 434 316 933 468
	2	299 071 260 027 409 551 039 944	430 172 716 173 196 361 342
	3	299 289 334 087 679 926 769 005	430 606 580 478 640 801 127
	4	299 507 298 700 487 603 563 403	431 040 444 350 651 118 834
	5	299 725 153 975 636 915 095 111	431 474 307 789 228 180 470
	6	299 942 900 022 767 034 329 839	431 908 170 794 372 852 041
	7	300 160 536 951 352 304 593 383	432 342 033 366 085 999 550
	8	300 378 064 870 702 569 808 852	432 775 895 504 368 488 996
	9	300 595 483 889 963 503 907 273	433 209 757 209 221 186 377
			300 812 794 118 116 939 414 048

n	III.	IV.	V.
950	412 579 561 832 828 395	412 579 757 612 114	412 579 757 808
1	413 013 855 901 935 134	413 014 052 093 604	413 014 052 290
2	413 448 149 970 607 580	413 448 346 575 094	413 448 346 772
3	413 882 444 038 845 732	413 882 641 056 584	413 882 641 254
4	414 316 738 106 649 590	414 316 935 538 073	414 316 935 736
5	414 751 032 174 019 154	414 751 230 019 562	414 751 230 217
6	415 185 326 240 954 425	415 185 524 501 050	415 185 524 699
7	415 619 620 307 455 402	415 619 818 982 538	415 619 819 181
8	416 053 914 373 522 086	416 054 113 464 025	416 054 113 663
9	416 488 208 439 154 476	416 488 407 945 512	416 488 408 145
960	416 922 502 504 352 572	416 922 702 426 999	416 922 702 627
1	417 356 796 569 116 375	417 356 996 908 485	417 356 997 109
2	417 791 090 633 445 884	417 791 291 389 971	417 791 291 591
3	418 225 384 697 341 099	418 225 585 871 456	418 225 586 073
4	418 659 678 760 802 021	418 659 880 352 941	418 659 880 555
5	419 093 972 823 828 649	419 094 174 834 425	419 094 175 036
6	419 528 266 886 420 983	419 528 469 315 909	419 528 469 518
7	419 962 560 948 579 024	419 962 763 797 393	419 962 764 000
8	420 396 855 010 302 771	420 397 058 278 876	420 397 058 482
9	420 831 149 071 592 224	420 831 352 760 358	420 831 352 964
970	421 265 443 132 447 384	421 265 647 241 840	421 265 647 446
1	421 699 737 192 868 250	421 699 941 723 322	421 699 941 928
2	422 134 031 252 854 823	422 134 236 204 804	422 134 236 410
3	422 568 325 312 407 102	422 568 530 686 284	422 568 530 892
4	423 002 619 371 525 087	423 002 825 167 765	423 002 825 374
5	423 436 913 430 208 779	423 437 119 649 245	423 437 119 855
6	423 871 207 488 458 177	423 871 414 130 725	423 871 414 337
7	424 305 501 546 273 281	424 305 708 612 204	424 305 708 819
8	424 739 795 603 654 092	424 740 003 093 682	424 740 003 301
9	425 174 089 660 600 609	425 174 297 575 161	425 174 297 783
980	425 608 383 717 112 833	425 608 592 056 639	425 608 592 265
1	426 042 677 773 190 762	426 042 886 538 116	426 042 886 747
2	426 476 971 828 834 399	426 477 181 019 593	426 477 181 229
3	426 911 265 884 043 741	426 911 475 501 070	426 911 475 711
4	427 345 559 938 818 790	427 345 769 982 546	427 345 770 193
5	427 779 853 993 159 545	427 780 064 464 021	427 780 064 674
6	428 214 148 047 066 007	428 214 358 945 597	428 214 359 156
7	428 648 442 100 538 175	428 648 653 426 971	428 648 653 638
8	429 082 736 153 576 050	429 082 947 908 446	429 082 948 120
9	429 517 030 206 179 631	429 517 242 389 920	429 517 242 602
990	429 951 324 258 348 918	429 951 536 871 393	429 951 537 084
1	430 385 618 310 083 911	430 385 831 352 866	430 385 831 566
2	430 819 912 361 384 611	430 820 125 834 339	430 820 126 048
3	431 254 206 412 251 018	431 254 420 315 811	431 254 420 530
4	431 688 500 462 683 130	431 688 714 797 283	431 688 715 012
5	432 122 794 512 680 949	432 123 009 278 754	432 123 009 494
6	432 557 088 562 244 475	432 557 303 760 225	432 557 303 975
7	432 991 382 611 373 707	432 991 598 241 696	432 991 598 457
8	433 425 676 660 068 645	433 425 892 723 166	433 425 892 939
9	433 859 970 708 329 290	433 860 187 204 635	433 860 187 421

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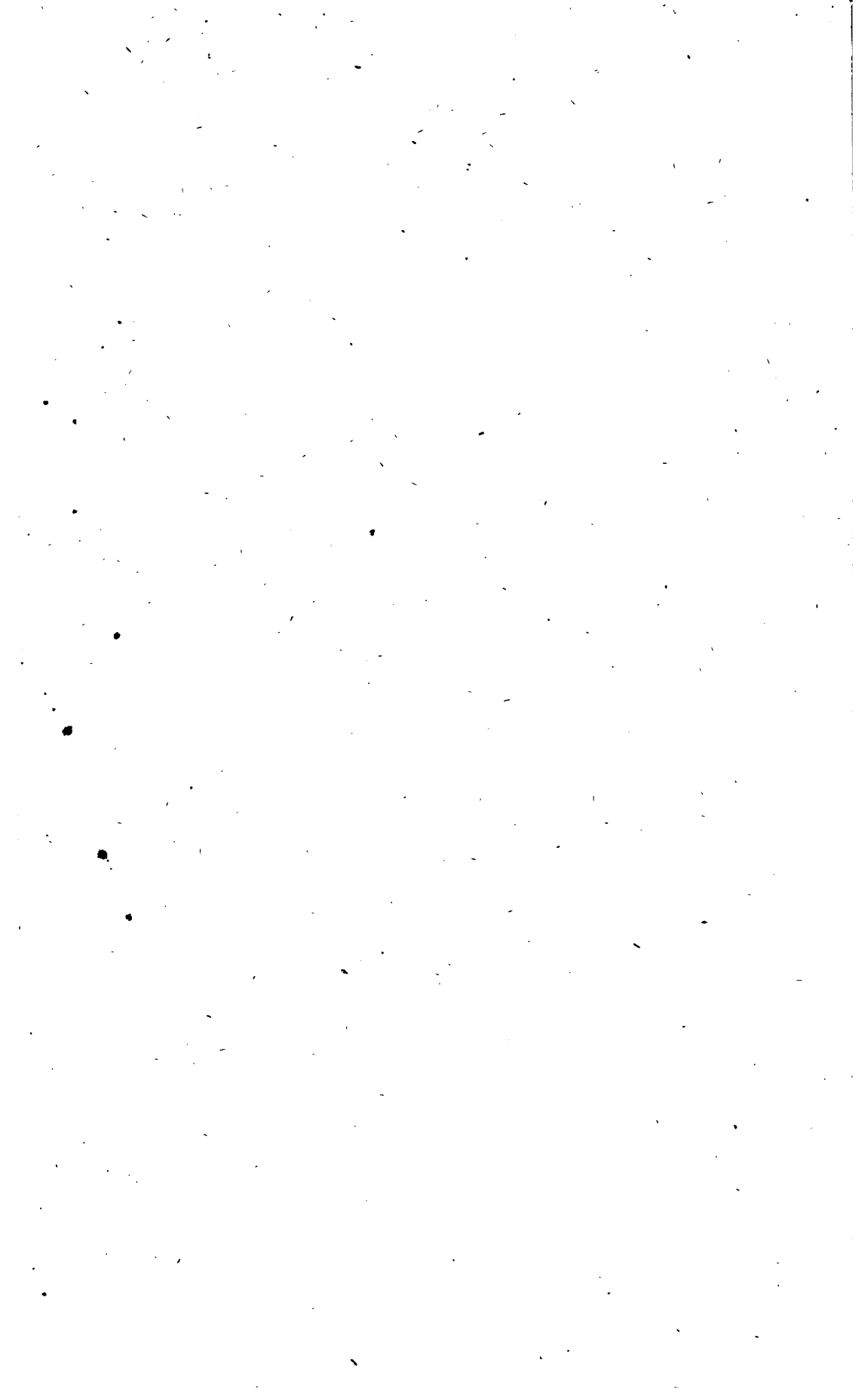
COMPRISING A LARGE EXTENSION OF THE PRINCIPAL TABLE.

* * * Possessors of the First Issue will be supplied with the Additional Matter, free of charge, on application to the Publishers.

"The distinct characteristic of this work is the introduction of *Gauss's Table* into the calculation of life contingencies; and by this one introduction the name of the author will live in the history of the subject, while the book itself will long be of use to those who practise it. Many of our readers may know what a logarithm is, and yet not know Gauss's table. For their information we state that it enables any one to find the logarithm of a *sum* or *difference* by knowing the logarithms *only* of the components. An addition or subtraction and one direct entry of the tables is used, instead of two inverse entries and one direct one. Mr. Peter Gray has recomputed both the subdivisions of Gauss's table; with arguments of four figures, and tabular results of six. This recomputation, accompanied by comparison with Matthiessen's table, renders the present work a new and valuable authority, and the most easily accessible form of Gauss's table to Englishmen. In a long introduction, very explicit discussion is given of the mode of calculating tables of life contingencies and of using Gauss's table in detached questions. In a valuable historical chapter, Barrett's method of forming tables,—one of the greatest improvements ever introduced into the subject,—is most successfully fixed to its right owner's name. This method was refused by the Royal Society, and would probably have been lost had it not been for Francis Baily; who thereupon published it, under Barrett's name, as an appendix to his own work. Some have since attributed it to Mr. Griffith Davies,—its improver, but not its inventor; and one writer has given the credit of it to the late William Morgan, on the strength of his having given a table which *might* have been used in Barrett's way if the giver had only seen how. But so far from seeing how, Mr. Morgan did not see the merit of the method when Barrett presented it to the Royal Society; and as he was on the Council of that body at the time, and was then the most eminent writer on life contingencies, the rejection of the method must be attributed to him,—not the invention. All this is well put by Mr. Gray; who has done himself honour in this as in every other part of the book,—and who will certainly take his place as a standard writer."—*Athenæum*.

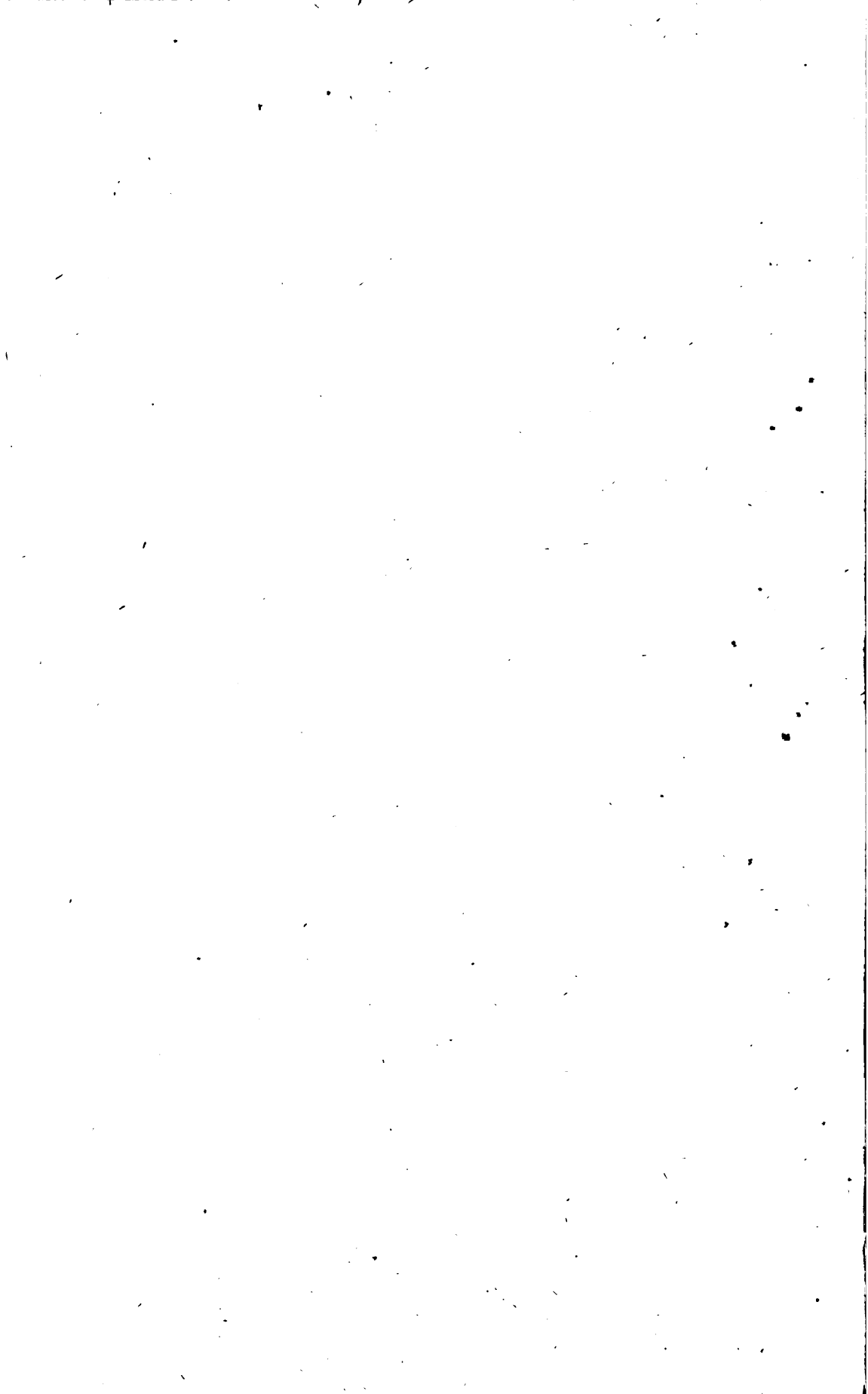
"If this work were the production of one 'to the manner born,' and the business of whose daily life led to such investigations, the reader, however much he might appreciate its merits, would have no reason for surprise; but when we state, as we believe we may correctly do, that it has been a labour of love, a thing quite apart from the author's ordinary occupation, we feel that it must be looked upon with considerable admiration were its intrinsic worth less than it really is. The work might have been, as it appears to us, more accurately described as a treatise on the art of constructing tables, particularly those required in determining the value of Life Contingencies, with Logarithmic Tables appended; for such it really is. The ground, moreover, is almost entirely new; no writer has hitherto touched upon it to any extent, so far as regards Life Tables, although many of the methods described were known; but the introduction of the use of Gauss's Tables, we believe, originates with Mr. Gray, as well as several very important and ingenious contrivances, which, to use the author's own words, afford greater facilities for the formation of tables than have hitherto been at the command of the computer.

"Such then is a brief and imperfect outline of the work before us. Of the order and precision which characterize it throughout, the neatness and simplicity of the demonstrations, and the minute and laborious accuracy of its details, we fail to give any adequate idea. The book, however, is nothing short of a standard one, and might, we would venture to suggest, be classed with great propriety amongst those selected as the text books in the future Examinations of the Institute."—*Assurance Magazine*.





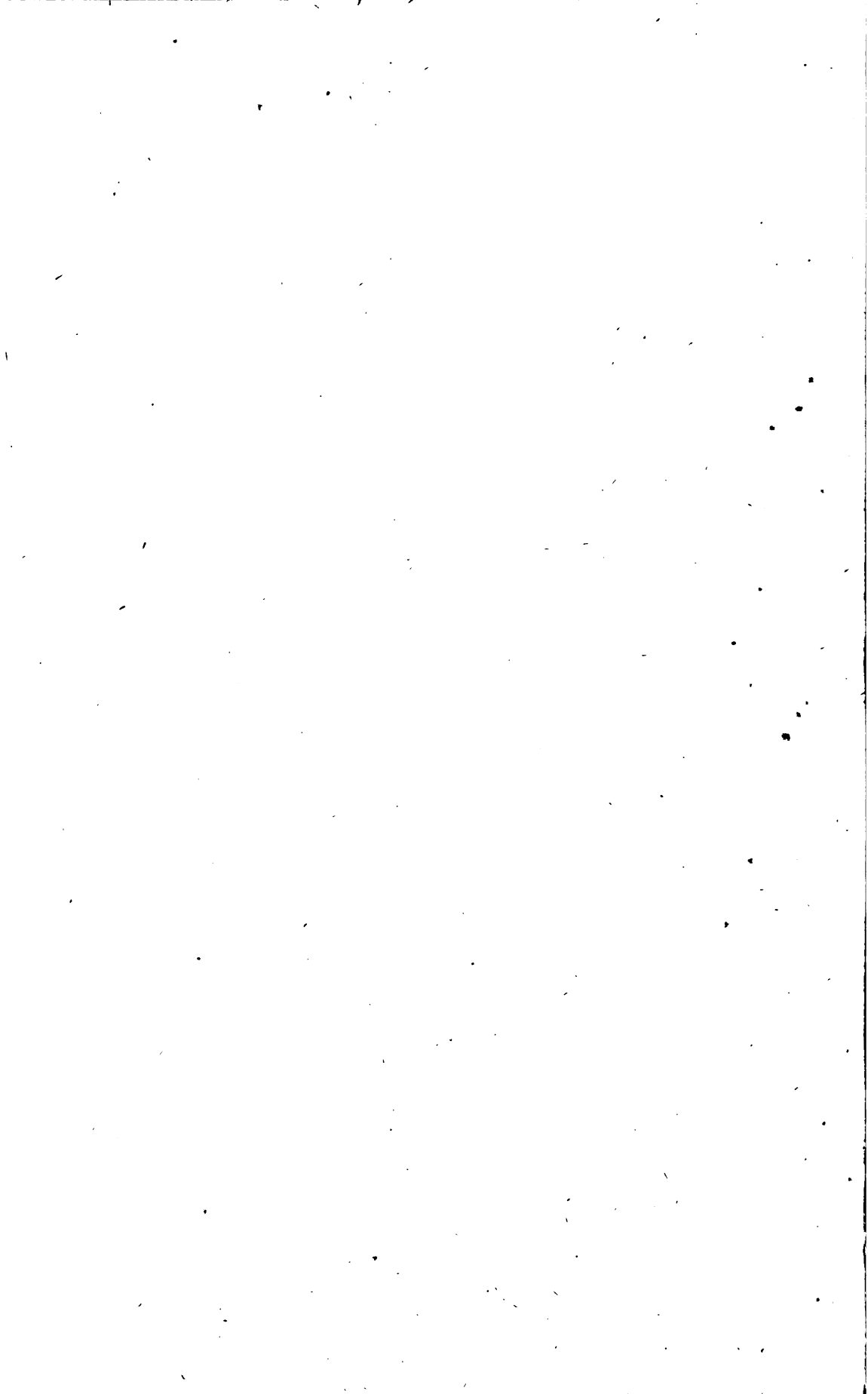


















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