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Faculty Working Papers

TAX AND TURNOVER

Peter F. Colwell, Associate Professor,
Department of Finance
Wei-long W. Lee, Graduate Student,
University of Illinois

#591

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

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August 7, 1979

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Summary:

The premise of this paper is that real estate investment decisions must, at least, be based on the entire life cycle of a single investment ...from acquisition through disposition. When to sell the property is a key decision in this life cycle. A useful rule of thumb indicating when to sell cannot involve only the tax shelter aspects but must also relate to the reversion. The rule or condition developed in this paper is that the sale should take place when the present value of after tax cash flow equals the loss in the present value of after tax equity reversion.

TAX AND TURNOVER

The tax consequences of real estate investment are of primary importance. In fact, investors may appear to be motivated by little else. Many investigators have felt that ownership periods (i.e., turnover rates) are determined primarily by tax policy. It is a widely held notion that rapid turnover rates (i.e., short ownership periods) are the result of rapid accelerated depreciation methods.

> Present income tax arrangements operate strongly to inhibit long-term ownership of income-producing real estate. Important tax advantages can in most instances be obtained by a sale after a rather brief interval of holding ... because the tax saving depreciation allowances are highest in the first few years.[1]

Thus the trend toward lower tax depreciation rates has been a response intended to reduce turnover rates.

This paper traces the major features of the income tax as they affect the equity investor over the ownership period.[2] Significant events in the life of the investment are identified. Of these, the loss of the tax shelter and the distinctly different optimal time to sell are the most important. The identification of the optimal time to sell requires that a model of owner behavior be suggested. Finally, some public policy implications of the model are discussed.

THE FLOWS

In Figures 1 and 2, the rate at which the various dollar flows occur is plotted against the points in time when the specific rates occur. Thus in these two Figures, the area under a curve between two points in time gives the magnitude of the flow during the period defined by the two points in time. For example, the net operating income (NOI) during the third year is shown as the shaded area in Figure 1.

Before Tax Considerations

Net operating income (NOI) is shown growing through time in Figure 1. The annual rate of growth in NOI is assumed to be 5% for this graphical illustration. Also in Figure 1, debt service (DS) is shown as being constant over the life of the loan. The life of the loan is 25 years. Debt service payments are, of course, composed of interest payments and payments on the principal (PRIN). A 9% rate of interest is implicit in the shape of the PRIN curve in Figure 1.

Note that in Figure 1 the debt coverage ratio (i.e., NOI/DS) is always greater than unity. In today's world, this is not likely to be the case for two reasons: (1) mortgage payments are constant rather than graduated with most mortgages, and (2) high rates of inflation and resulting high nominal interest rates require that the real value of debt service be much higher in the early than the later years of a loan. So properties are often sold today on the basis that debt service equals or even exceeds net operating income initially.

Figure 2 is of necessity much more complicated than Figure 1, since Figure 2 incorporates the most important features of the tax on income flowing to real estate investments. Before tax cash flow is the difference between net operating income and debt service.

$$BTCF = NOI - DS.$$

Before tax cash flow (BTCF) grows through time as shown in Figure 2, because NOI grows while DS remains constant.

Tax and Its Consequences

The path of taxable income (TI) through time is shown in Figure 2. Taxable income equals before tax cash flow plus payment on the principal

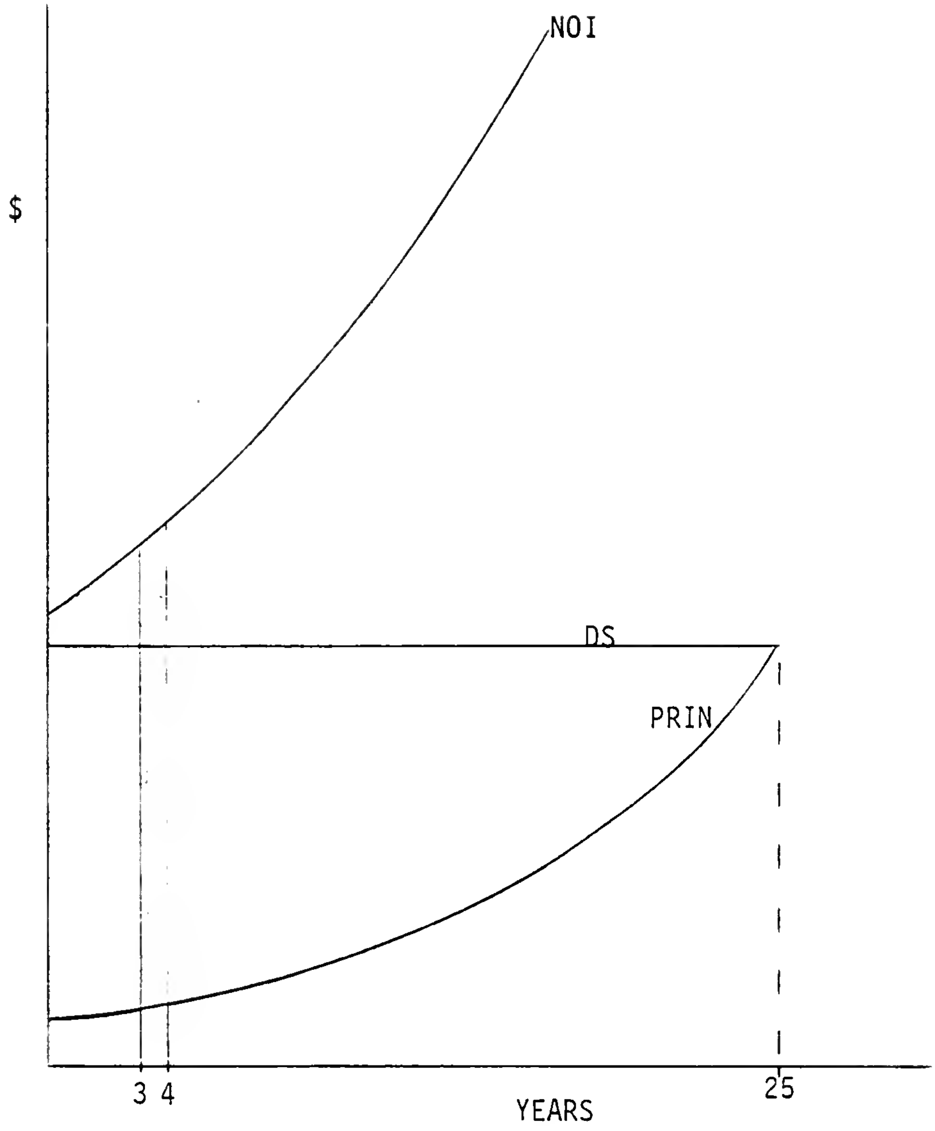

FIGURE 1

and minus depreciation deductions.

$$TI = BTCF + PRIN - DEPR$$

The BTCF curve is found in Figure 2, the PRIN curve is found in Figure 1, and the depreciation deductions (DEPR) curve is developed in Figure 2. The development of the DEPR curve is based on the depreciable basis (which will be discussed in connection with Figure 3), the depreciation method, and the economic life of the property. Having assumed that the subject property consists of new apartments, double declining balance is the depreciation method selected.[3] For simplicity, it is further assumed that component depreciation is not selected. These assumptions give the property an economic life of 40 years for tax purposes.[4] Depreciation deductions (DEPR) are shown in Figure 2 to the end of the economic life. Notice that depreciation deductions decline through the 21st year of ownership and are constant thereafter. This occurs because it is optimal to switch from accelerated to straight line depreciation in the 22nd year.[5] Finally, subtracting depreciation deductions at each point in time from the sum of before tax cash flow and principal payments yields the taxable income (TI) curve as shown in Figure 2. Note that taxable income is initially negative but increases with time becoming zero after 4 years of ownership. After this point in time, taxable income becomes positive and continues to grow.

For simplicity, tax is assumed to be a proportion of taxable income. This tax (TX) is equal to taxable income multiplied by the equity investor's marginal tax rate.[6] Assuming that the marginal tax rate is .5, the TX curve is always halfway between the TI curve and the horizontal axis as shown in Figure 2. That is,

$$TX = .5(TI).$$

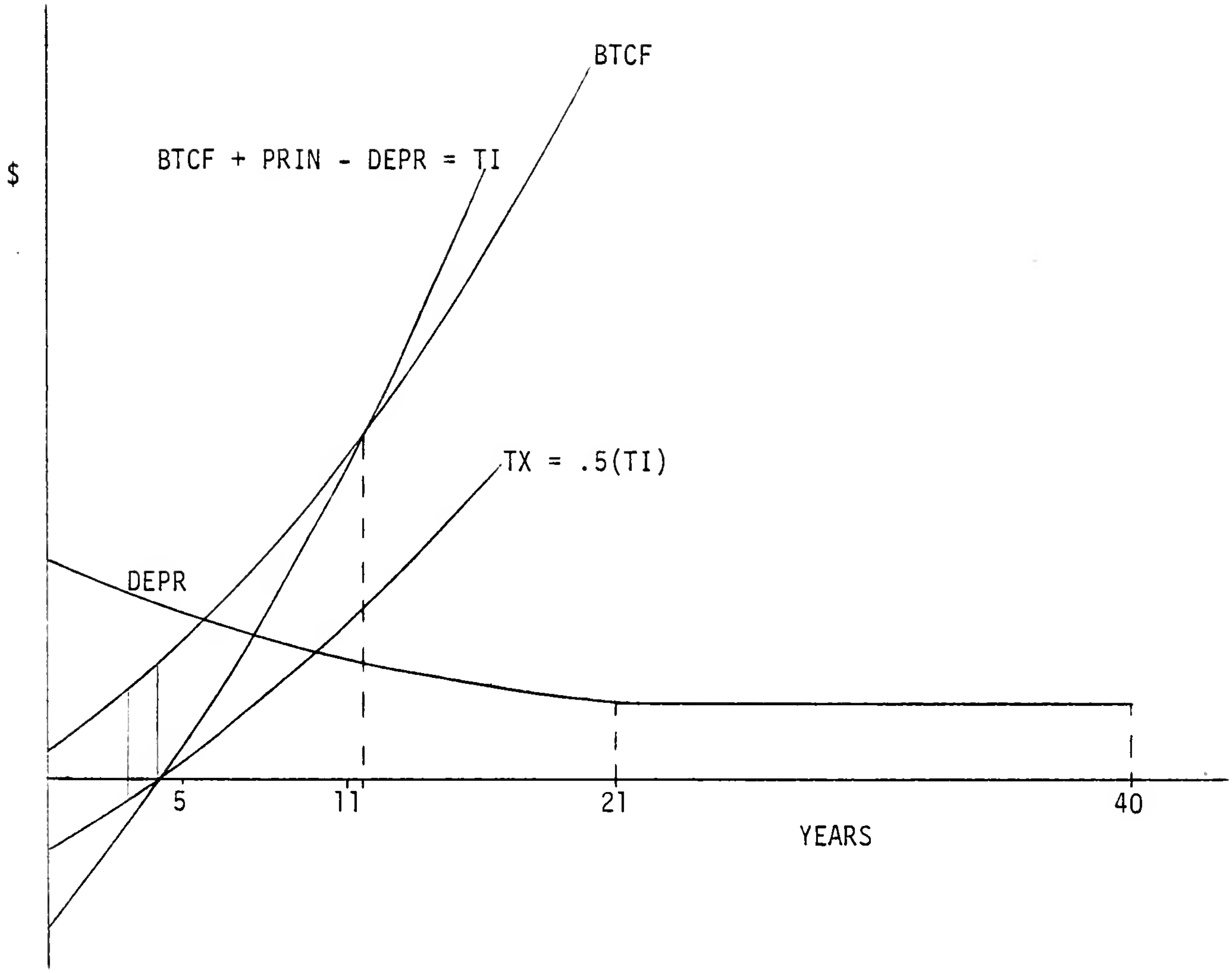

FIGURE 2

The last problem which relates to the impact of taxes on the income which flows to real estate is to identify after tax cash flow (ATCF), the amount that the equity investor gets to keep. After tax cash flow is simply the difference between before tax cash flow and tax.

$$ATCF = BTCF - TX.$$

Thus the shaded area in Figure 2 is the after tax cash flow during the third year of ownership.

Three Significant Dates

One date stands out as being significant. Just about 4 years into the ownership period, taxable income and tax are zero. Graphically, the TI and TX curves cross the horizontal axis at this point in time. It is at this point in time when the tax shelter on other income is lost. To many equity investors, this is a signal to sell.

It is clear from Figure 2 that the loss of the tax shelter on other income occurs when before and after tax cash flows are equal. Before this point in time, after tax cash flow exceeds before tax cash flow because of the tax savings (i.e., negative taxes). After this point in time, after tax cash flow is less than before tax cash flow.

Another study has incorrectly suggested that the point in time when taxable income equals before cash flow indicates the earliest reasonable time to sell.[7] This date is referred to as the "first turning point." Of course, this date is important in that it indicates that the tax shelter on the subject property's income is at an end. This occurs in the 11th year of ownership in the graphical example. (See Figure 2)

As will be clear, this point in time is no more significant for determining the optimal time to sell than the point at which the shelter on other income is eliminated.

Still another significant date is when after tax cash flow becomes negative (not shown on Figure 2). It has been incorrectly suggested that holding property beyond this date is irrational.[8] But again, it is impossible to identify the optimal time to sell without reference to both cash flow and reversionary magnitudes.

THE REVERSION

Figures 3, 4, and 5 deal strictly with the reversion or sale of the property. In these Figures, various consequences of a sale such as the expected selling price are plotted against the time at which a sale might occur. The initial selling price, the amount paid for the property by the equity investor, may be decomposed two ways. First, the initial selling price equals the sum of the mortgage loan and the cost of equity. A loan to value ratio of 80% is assumed for Figure 3. Second, the initial selling price equals the sum of depreciable and non-depreciable basis assuming that buying expenses are zero. The ratio of depreciable to total basis is assumed to be 87.5% for Figure 3. Of course, this also has an impact on the DEPR curve in Figure 2. Each of these decompositions is clear from Figure 3. The scale on the vertical axes of Figures 3, 4, and 5 is ten times that of Figures 1 and 2.

From Selling Price to Before Tax Reversion

At the top of Figure 3, selling price (SP) is shown to be growing through time at a 5% rate. Just below the SP curve, the amount realized

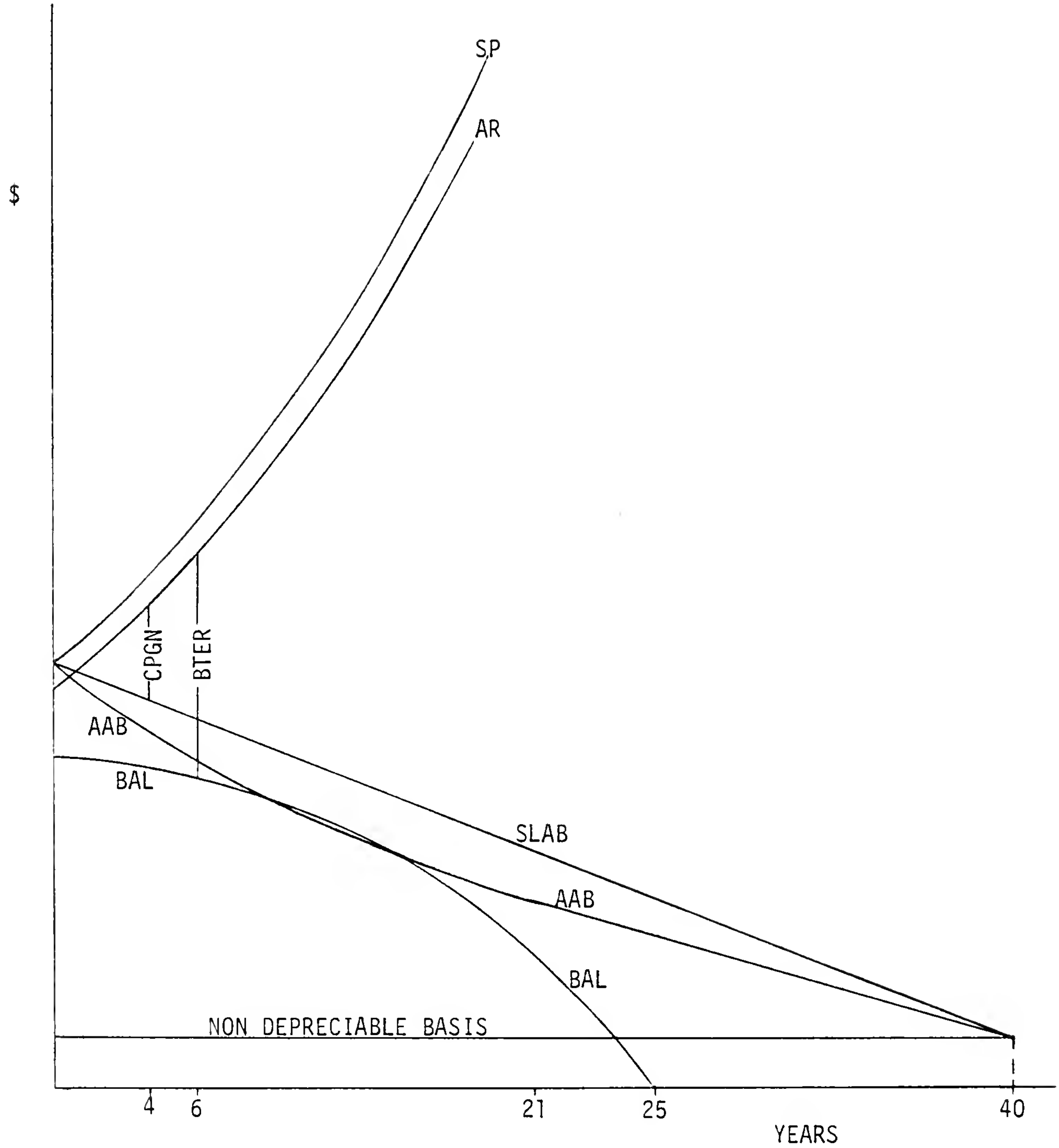
SP
AR
$
CPGN
BTER
AAB
BAL
SLAB
AAB
BAL
NON DEPRECIABLE BASIS
4
6
21
25
40
YEARS

FIGURE 3

(AR) from a sale is shown. The difference between the selling price and the amount realized is the selling expense.

AR = SP - (selling expense).

Selling expense is assumed to run 6% of the selling price.

The straight line adjusted basis (SLAB) curve is found by connecting the initial selling price with the non-depreciable basis at the end of the economic life as shown in Figure 3. The vertical distance between the amount realized and the straight line adjusted basis is the capital gain (CPGN). The capital gain is labeled in Figure 3 as if a sale were to occur after 4 years of ownership. The actual adjusted basis (AAB) falls below the straight line adjusted basis during the economic life. But at the end and the beginning of the economic life, SLAB and AAB are equal as shown in Figure 3. It is clear in Figure 3 that using the accelerated depreciation method causes the adjusted basis to decline at a decreasing rate through the 21st year and to decline at a constant rate thereafter. As indicated previously, this difference is caused by the switch to straight line depreciation in the 22nd year. Had sum of the year's digits depreciation been selected, it would never be reasonable to switch to straight line depreciation.

The balance due (BAL) on the mortgage loan is illustrated in Figure 3. The initial balance due is, of course, the amount of the loan. The vertical distance between the amount realized (AR) and the balance due (BAL) curves is the before tax equity reversion (BTER).

BTER = AR - BAL.

Before tax equity reversion is labeled in Figure 3 as if there is a sale after 6 years of ownership.

Figure 3 deals with the income tax consequences of the reversion. The before tax equity reversion is found in Figure 3 and plotted in Figure 4 as the BTER curve. After tax equity reversion (ATER) is ultimately found by subtracting the capital gains tax and the recapture of excess depreciation from BTER.

Recapture

The recapture of excess depreciation is relatively straightforward. Excess depreciation (XDEPR) is the distance between the straight line adjusted basis curve (SLAB) and the actual adjusted basis (AAB) curve in Figure 3.

$$XDEPR = SLAB - ABB.$$

It can be seen in Figure 3 that excess depreciation is at its peak sometime around 14 years into the ownership period. Of course, excess depreciation is zero at both the beginning and the end of the economic life. The tax on excess depreciation, or the recapture of excess depreciation, is the product of the investor's marginal tax rate and excess depreciation. Again assuming that the marginal tax rate is .5, the tax is half the excess depreciation. This recapture is labeled in Figure 4 as if a sale occurs after 15 years of ownership.

Capital Gains Tax

Capital gain (CPGN) is the vertical distance between AR and SLAB curves in Figure 3.

$$CPGN = AR - SLAB.$$

The capital gains tax is 20% of this distance. The amount of the capital gains tax is subtracted from the BTER curve in Figure 4. The

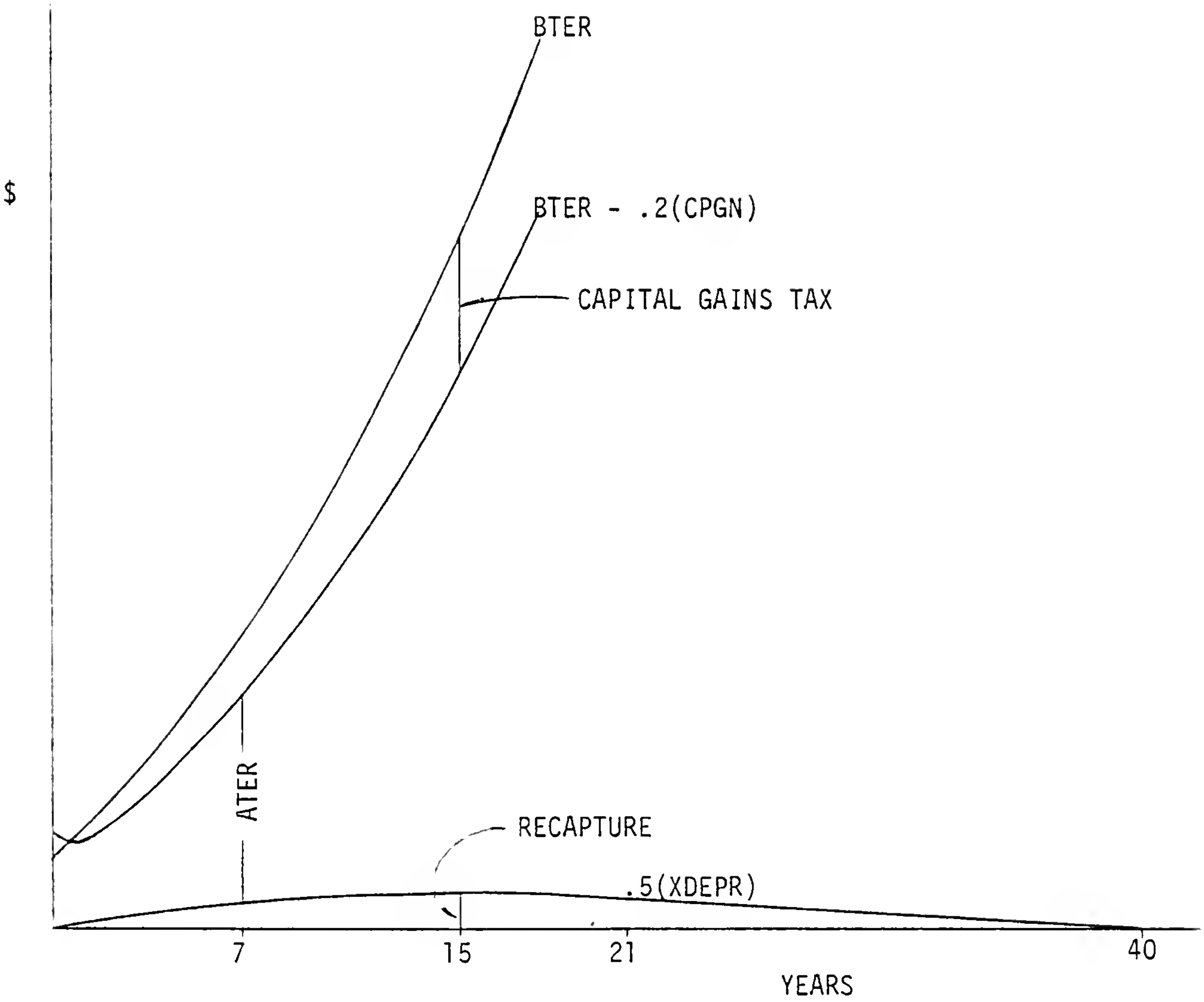
BTER
$
BTER - .2(CPGN)
CAPITAL GAINS TAX
ATER
RECAPTURE
.5(XDEPR)
7
15
21
40
YEARS

FIGURE 4

curve which results from this subtraction is labeled BTER-.2(CPGN) or before tax equity reversion minus the capital gains tax. The 20% capital gains tax rate comes from the product of .4, the proportion of capital gains taxed, and .5 the investor's marginal tax rate on ordinary income. The capital gains tax is labeled in Figure 4 as if a sale occurs after 15 years of ownership.

After Tax Reversion

Subtracting the capital gains tax and the recapture from before tax equity reversion yields the after tax equity reversion.

$$ATER = BTER - .5(XDEPR) - .2(CPGN).$$

The after tax equity reversion is labeled in Figure 4 as if a sale occurs after 7 years of ownership. Thus it is possible to derive the ATER curve as shown in Figure 5.

TURNOVER

Some limited partners, intent on sheltering their other income from taxation tend to think of the loss of the shelter as a signal to sell. The shelter is gone when after and before tax cash flows are equal. Even given the appeal of this rule, it still seems that a condition which indicates the optimal time to sell must relate in some way to the reversion. As so it must. Considering only a single real estate investment, and not the too little known rotation problem that relates to a series of investments, and maximizing the present value of the investor's equity, the optimal condition is quite simple. (See Appendix I.) The optimal time to sell is when the amount gained in terms of cash flow as a result of holding the property an additional unit of time is exactly offset by what is lost due to a lower equity reversion. In practical terms, this means that the change in the present value of the after tax equity reversion multiplied by -1 would have to equal the present value of after tax cash flow.

The optimal condition says that when, in present value terms, the amount gained through cash flow equals the amount lost to a decline in the reversion, it is time to sell. For this condition to be relevant, it is necessary that prior to this time the reversionary loss is less than the cash flow gain and afterward the loss is greater than the gain. At a time short of the optimal time to sell, the present value of after tax cash flow (atcf) which would be received if the property were held a bit longer will more than offset the reduction in the present value of after tax equity reversion (ater) which results from the delayed sale.

Of course it is possible for ATCF and atcf to be negative. In this situation, property may be held because of growth in ater. It would be optimal to sell such a property when the loss from atcf is just equal to the gain in ater. This assumes that prior to this time the loss in atcf is less than the gain in ater. Thus by delaying the sale until the optimal time, the negative cash flows are more than compensated by the growth in equity reversion.

Turnover Graphically

It is helpful to view the optimal condition graphically. The first step is to determine the present value of after tax equity reversion (ater). For the purpose of this illustration, a discount rate of 18% is used. The present value of after tax equity reversion falls below after tax equity reversion itself if the discount rate is positive.

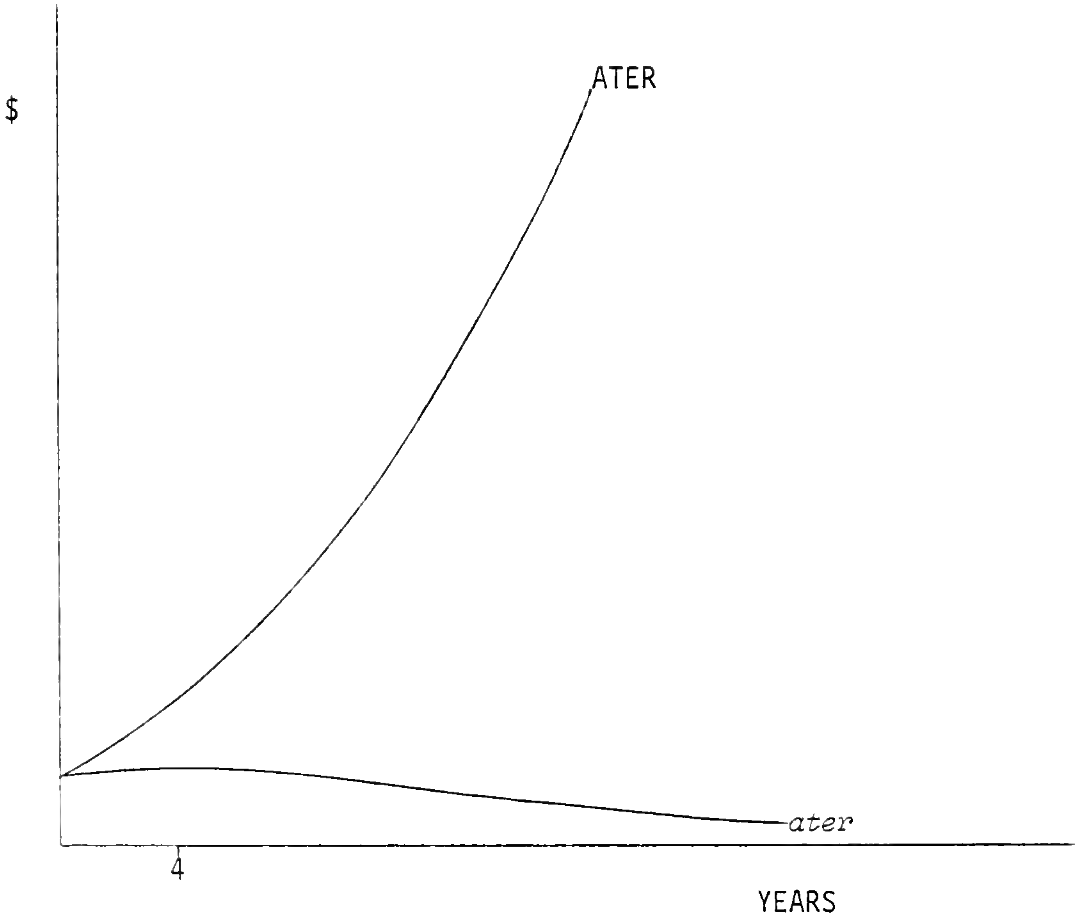

FIGURE 5

As shown in Figure 5, the present value may actually decline. The slope of the ater curve, is multiplied by -1 to obtain the loss in terms of after tax equity reversion. The scale of the vertical axis in Figure 6 is one fifth the scale in Figures 1 and 2.

The loss in terms of after tax equity reversion is plotted in Figure 6. Next, the after tax cash flow (ATCF) found in Figure 2 is plotted in Figure 6. Again using a discount rate of 18%, the present value of after tax cash flow (atcf) is determined and plotted in Figure 6.

The optimal time to sell is found in Figure 6 where the atcf curve crosses the loss of ater curve. Thus it is optimal to sell the property in the graphical example after an ownership period of 10 years. Taking the BTCF from Figure 2 and reproducing it in Figure 6 allows a comparison between the optimal time to sell and the time suggested by the elimination of the tax shelter. The shelter is gone when the BTCF and ATCF curves cross in Figure 6. Thus the shelter is gone after an ownership period of about 4 years. It is true that a higher discount rate would cause the atcf curve to shift downward, the loss of ater curve upward, and thus the optimal ownership period to decrease. Yet there is no natural reason for the optimal time to sell to correspond to the time at which the shelter disappears.

PUBLIC POLICY

Utilizing the model developed in this paper, it is possible to determine the direct effects of tax policy changes on the optimal ownership period. Direct effects refer to holding other things constant.

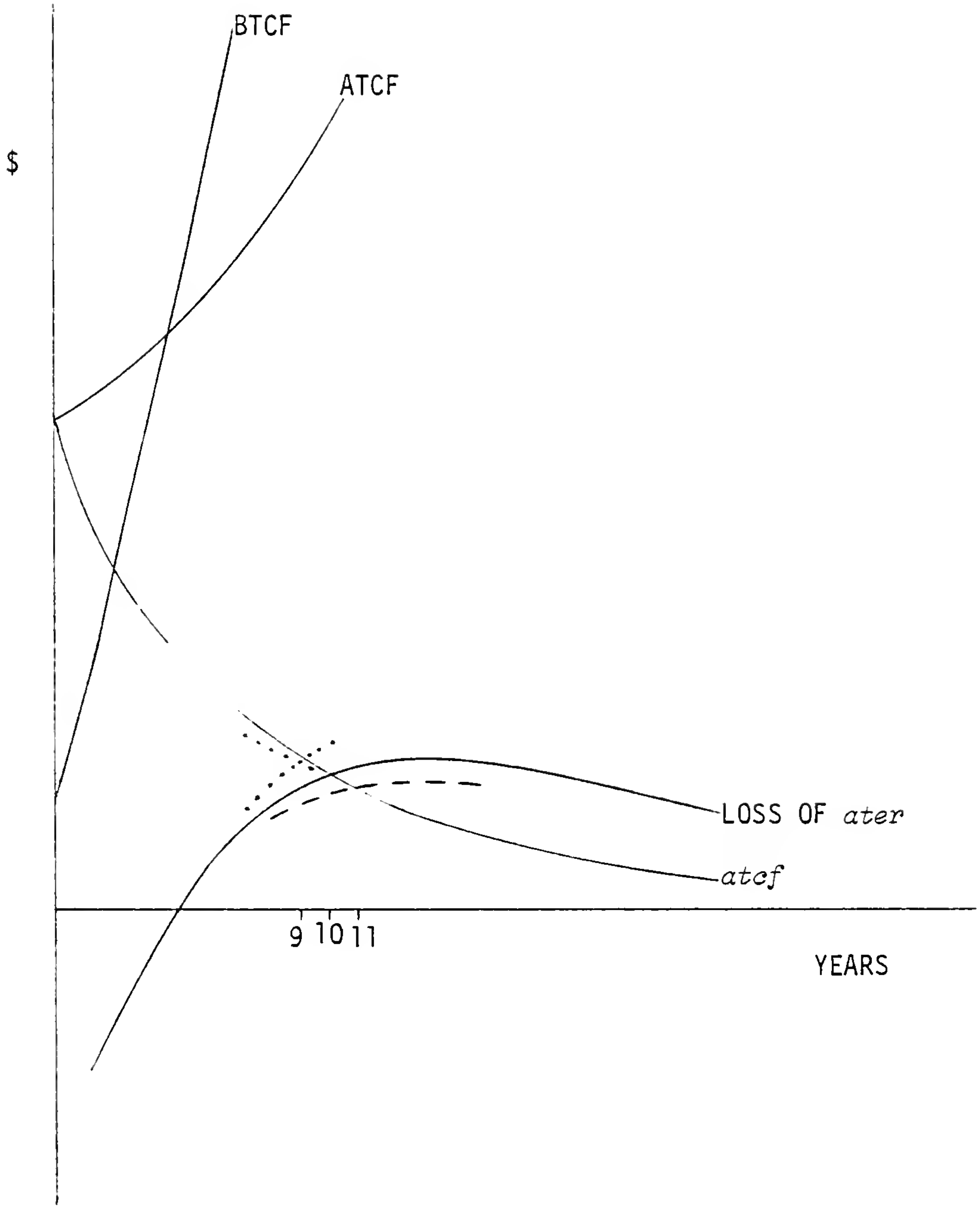

FIGURE 6

While there certainly are indirect effects of tax policy, effects which relate to protracted market adjustment processes and new long-run equilibrium levels of rents, costs, and selling prices, they are beyond the scope of this paper and model. Yet the model is useful for judging the direct or ceteris paribus effects.

The Capital Gains Tax Rate

Suppose that the percentage of capital gains taxed increases from the present 40% to the recent 50% or even beyond. What would be the impact on the optimal holding period? The change in the capital gains tax causes the loss of _ater_ curve to shift. The direction of the shift depends on whether the present value of capital gains is increasing or decreasing through time. (See Appendix II.) If it is decreasing, the loss of _ater_ curve shifts downward, the _atcf_ curve is unaffected, of course, and the optimal ownership period is extended. Alternatively, if it is increasing, the loss of _ater_ curve shifts upward, and the optimal ownership period is reduced. Finally, no change in the present value of capital gains would mean that the increase in the capital gains tax rate would have no effect on the optimal ownership period. Since the direction of change in the present value of capital gains is itself likely to change through time (e.g., first increasing then decreasing), the "shift" in the loss of _ater_ curve is likely to be more a rotation than a shift in the same direction all along its length.

In the example illustrated in the Figures, the present value of capital gains peaks at around 6 years and decreases slightly with time in the neighborhood of the initial optimum. Therefore, an increase in the capital gains tax rate would cause the loss of _ater_ curve to rotate clockwise around a point on the curve 6 years into the ownership period. This causes a slight increase in the optimal ownership period. For example, an increase in percent taxed from 40% to 60% would cause an increase in the optimal ownership period of about 1 year. This is shown by the intersection of the new loss of ater curve, the dashed line in Figure 6, intersects the atcf curve. On the other hand, the time at which the tax shelter ends is unchanged because neither the ATCF curve nor the BTCF curve are affected by a change in the capital gains tax rate. It is important to note that with a lower atcf curve the impact of the change in the capital gains tax on the optimal ownership period could be reversed.

The Tax Depreciation Rate

The tax depreciation rate (i.e., the percent declining balance) is one of the more important features of the tax on income from real estate which has been subject to substantial policy manipulation. It is conventional to imagine that by reducing the tax depreciation rate the ownership period can be lengthened. While this notion has intuitive appeal, it is just not so.

Changing the tax depreciation rate causes the atcf curve to shift as shown in equation (6A) of Appendix III. The amount of the shift is the product of the investor's marginal tax rate and the present value of the change in the depreciation deduction. When the depreciation deduction is the same before and after the change in the tax depreciation rate, the old and new atcf curves intersect. Given a reduction in the rate from 200% to 125%, the intersection occurs after about 15 years of ownership, well after having switched from the 125% declining balance to straight line depreciation. Thus, a change in the tax depreciation rate causes the atcf curve to rotate rather than shift in a single direction. Specifically, a reduction in the tax depreciation rate causes the atcf curve to rotate counter-clockwise.

The tax depreciation rate has a similar impact on the loss of ater curve. The shift resulting from a change in the tax depreciation rate has two terms as shown in equation (7A) of Appendix III. The first of these is equal to the shift in the atcf curve. If this were the only term causing the loss of ater curve to shift, then both curves would shift in the same direction and by the same amount. There would be no change in the optimum ownership period. However, there is an additional term in the shift of the loss of ater curve. This term is the product of the marginal tax rate, the discount rate, and the present value of the difference in excess depreciation. It causes an additional upward shift in the event of a reduction in the tax depreciation rate. This is as if the policy change has two effects. The first results in no change in the optimal ownership period. The second results in an upward shift in only the loss of ater curve. This second effect guarantees that a reduction in the depreciation rate would result in a decrease in the optimal ownership period. In the graphical example, the decrease in the tax depreciation rate from 2.0 to 1.25 causes the atcf and loss of ater curves in Figure 6 to shift to the dotted curves and the optimal ownership period to decline from 10 to 8+ years.

SUMMARY AND CONCLUSIONS

The premise of this paper is that real estate investment decisions must, at least, be based on the entire life cycle of a single investment ...from acquisition through disposition. When to sell the property is a key decision in this life cycle. A useful rule of thumb indicating when to sell cannot involve only the tax shelter aspects but must also relate to the reversion. The rule or condition developed in this paper is that the sale should take place when the present value of after tax cash flow equals the loss in the present value of after tax equity reversion.

While a more theoretically satisfying optimal condition would relate to a series of investments (i.e., the rotation problem), much of its intuitive appeal would have been lost. In addition, not much would have been gained in terms of different and more accurate estimates of optimal ownership periods. In terms of the numerical example embodied in the Figures, preliminary calculations indicate that the optimal ownership periods would probably be reduced by no more than 2 years even considering the most extreme case of an infinite series of investments.

The graphical representation of the optimal condition was made possible by projecting the paths of a number of important magnitudes through time. First graphed were the annual rates of the dollar flows from net operating income to after tax cash flow. Next, the paths of reversionary magnitudes, from selling price to after tax equity reversion, were projected. Finally, the present value of after tax cash flow was brought together with the loss in present value of after tax equity reversion. The intersection of these last two curves was shown to indicate the optimal time to sell.

The implications for private investment policy are obvious: to use the loss of the tax shelter as an index of when to sell is suboptimal. Some of the public policy implications are less obvious. An increase in the tax depreciation rate, for example, results in longer optimal ownership periods and slower turnover. Since a convincing case can be made for less frequent turnovers causing higher levels of maintenance, public policy makers may be encouraged to increase the tax depreciation rate in order to check urban decay. Of course, this runs against conventional wisdom.

FOOTNOTES

[1] National Commission on Urban Problems, Building the American City, Praeger, New York, 1969, p. 403.

[2] Among the more important features of the tax system excluded from the analysis are the minimum tax on preference items and the impact on gains and loss of holding the property for less than one year.

[3] No pretensions are made here for having selected the optimal depreciation method. Sum of the years' digits might be preferred. But this is another story.

[4] Alvin L. Arnold, Tax Shelter in Real Estate Under the Tax Reform Act of 1976, Warren, Gorham & Lamont, 1977, pp. 21-22.

[5] The declining balance depreciation deduction in the year of the switch to straight line depreciation is less than or equal to the straight line deduction.

$$\frac{a}{L}\left(1 - \frac{a}{L}\right)^{m-1} \leq \frac{\left(1 - \frac{a}{L}\right)^{m-1}}{L - m + 1}$$

where a = the tax depreciation rate or the percent declining balance divided by 100,
L = the economic life for tax purposes,
m = the year of the switch,
$\frac{a}{L}\left(1 - \frac{a}{L}\right)^{m-1}$ = the declining balance depreciation deduction in m^{th} year per dollar of depreciable basis,
$\left(1 - \frac{a}{L}\right)^{m-1}$ = the adjusted basis as a proportion of the depreciable basis after the $(m-1)^{th}$ year,
$L - m + 1$ = the remaining economic life after the $(m-1)^{th}$ year, and
$\frac{\left(1 - \frac{a}{L}\right)^{m-1}}{L - m + 1}$ = the straight line deduction in the m^{th} year per dollar of depreciable basis.

It is a simple matter to show that

$$L - \frac{L}{a} + 1 \leq m.$$

Substituting the magnitudes used in this paper,

$$40 - \frac{40}{2} + 1 = 21 \leq m.$$

[6] It is assumed throughout that the investor's marginal tax rate is unchanged at the relatively low amount of .5. This kind of assumption is common in real estate investment analysis.

[7] Friedman, Jack P., Real Estate Issues, Summer 1978, p. 72.

[8] Ibid.

Appendix

I. The Optimal Ownership Period

The mathematical justification of the optimal condition is as follows:

(1A) $$V = \int_0^{\theta} \underline{atcf}\, dt + \underline{ater}$$

where V = the present value of the equity investment,

$\underline{atcf}$ = the present value of after tax cash flow, and

$\underline{ater}$ = the present value of after tax equity reversion.

Maximizing V,

(2A) $$\frac{dV}{d\theta} = \underline{atcf} + \frac{d}{d\theta}\,(\underline{ater}) = 0 \; .$$

Thus a condition for the optimal time to sell is

(3A) $$\underline{atcf} = -\frac{d}{d\theta}\,(\underline{ater}) \; .$$

That is, the present value of after tax cash flow must equal the negative time derivative of the present value of after tax equity reversion.

II. Change in Capital Gains Tax Rate

In order to determine the direction in which the loss of ater curve shifts as a result of changing the capital gains tax rate, it is necessary to begin by specifying the ATER function.

$$\text{(4A)} \qquad \text{ATER} = \text{BTER} - \tau\alpha(\text{AR}-\text{SLAB}) - \tau(\text{XDEPR})$$

where τ = the investor's marginal tax rate, and

α = the proportion of capital gains taxed at ordinary income tax rates.

The shift in the loss of ater curve resulting from a change in α can be described as follows:

$$\Delta\{-\frac{d}{dT}[(\text{ATER})e^{-\phi T}]\} = \Delta\frac{d}{dT}[\tau\alpha(\text{AR}-\text{SLAB})e^{-\phi T}]$$

$$\text{(5A)} \qquad = \tau\Delta\alpha\frac{d}{dT}[(\text{AR}-\text{SLAB})e^{-\phi T}] \ .$$

Thus the direction of the shift depends on the direction of the change in α and whether the time derivative of the present value of capital gains (i.e., AR-SLAB) is positive or negative.

III. Change in Tax Depreciation Rate

In order to describe the shift in the atcf curve, the ATCF function must be specified.

$$\text{ATCF} = \text{BTCF} - \tau(\text{BTCF} + \text{PRIN} - \text{DEPR})$$

$$= (1 - \tau)\text{BTCF} - \tau(\text{PRIN}) + \tau(\text{DEPR})$$

The shift is as follows:

$$\Delta[(\text{ATCF})e^{-\phi T}] = \tau e^{-\phi T}\Delta(\text{DEPR}) \tag{6A}$$

Using equation (4A), the shift in the loss of ater curve can be described as follows:

$$\Delta\{-\frac{d}{dT}[(\text{ATER})e^{-\phi T}]\} = \Delta\frac{d}{dT}\tau[(\text{XDEPR})e^{-\phi T}]$$

$$= \tau e^{-\phi T}\Delta\frac{d}{dT}(\text{XDEPR}) - \tau\phi e^{-\phi T}\Delta(\text{XDEPR})$$

$\frac{d}{dT}(\text{XDEPR}) = \frac{d}{dT}(\text{SLAB}) - \frac{d}{dT}(\text{AAB})$, $-\frac{d}{dT}(\text{AAB}) = \text{DEPR}$, and

$\Delta\frac{d}{dT}(\text{XDEPR}) = \Delta\text{DEPR}$.

Thus, the shift is as follows:

$$\Delta\{-\frac{d}{dT}[(\text{ATER})e^{-\phi T}]\} = \tau e^{-\phi T}\Delta(\text{DEPR}) - \tau\phi e^{-\phi T}\Delta(\text{XDEPR}). \tag{7A}$$

Note that equation (7A) contains a term which does not appear in equation (6A).

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