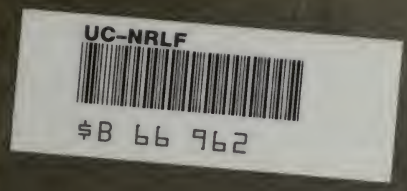

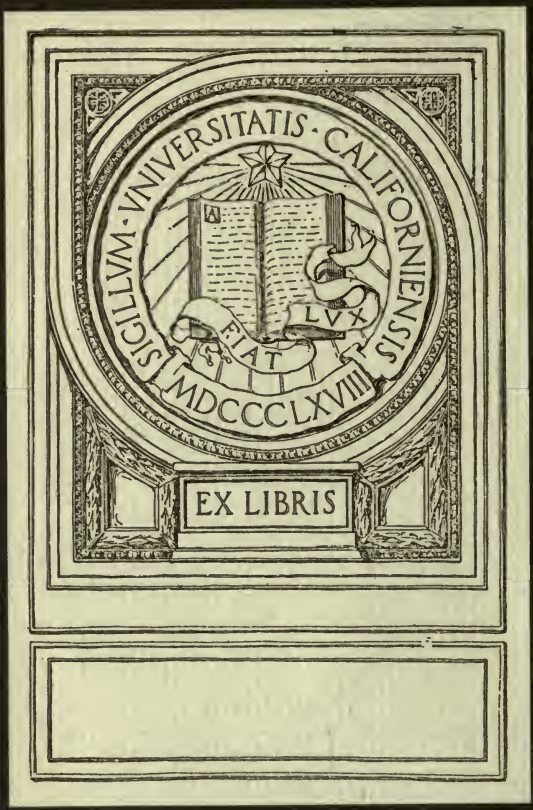
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Teacher's Manual

For

First Year Algebra Scales

Hotz

Published by
Teachers College, Columbia University
New York City
1922

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First Year Algebra Scales

By

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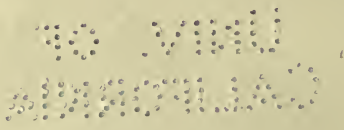

PREFACE

This manual is compiled for the purpose of assisting teachers of mathematics in the administration and practical use of my First Year Algebra Scales. There is a feeling that the original monograph, which appeared in the Teachers College Contributions to Education series, is too technical and, consequently, too difficult for most teachers to read intelligently and to determine from it with ease how to apply the scales most profitably. Suggestions concerning the purpose of these scales are incorporated in this manual in as simple and direct form as possible. Special training in statistical methods is not necessary for their comprehension.

Besides the Tentative Standards of Achievement proposed in the original monograph, scores more recently obtained in various cities and through school surveys have been included. Suggestions on presentation and diagnosis of results have also been added. In order that the progress of a class may be more scientifically determined, a revision of the tests, with the exercises arranged in duplicate or alternate scales of equal difficulty, will be published in the near future.

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CONTENTS

Teacher's Manual For First Year Algebra Scales

The First Year Algebra Scales were first published in 1918.[1] Since then they have been extensively used by teachers, school administrators, directors of educational research, and in various school surveys.

I. DESCRIPTION OF SCALES

The scales consist of five different sheets of algebraic exercises designed to measure the ability of pupils in elementary algebra. They are:

1. Addition and Subtraction
2. Multiplication and Division
3. Equation and Formula
4. Graphs
5. Problems

The first two scales, it will be seen, are designed to test the achievement of students in the fundamental operations, involving integral, fractional, and radical expressions; the second two, to test the ability of students in handling the instruments of quantitative thinking; while the last is composed of verbal problems of the type usually stressed in the first year of algebra.

The exercises in each scale are arranged in order of difficulty; that is, each scale begins with exercises so easy that they can be solved by practically every member of a class. Each succeeding exercise, however, becomes increasingly more difficult so that the last ones in each scale can be solved by only a relatively small number of students who try them.

Two series of each scale are offered—Series A and Series B. Series B is the longer and contains from eleven to twenty-five exercises in each scale. Series A is only about one half as long and contains from eight to twelve exercises in each scale. It covers just as wide a range of difficulty and has the added advantage of having the intervals between successive exercises and problems approximately equal: that is, Ex. 3 of a given scale is as much more difficult than Ex. 2, as Ex. 2 is more difficult than Ex. 1.

[1] Hotz, Henry G.: *First Year Algebra Scales.* Teachers College, Columbia University, Contributions to Education, No. 90.

In determining individual and class scores, the factor of primary importance is not so much how many exercises an individual can solve correctly in a given time, but rather how far along on the scale of exercises, arranged in order of increasing difficulty, he can perform satisfactorily. In other words, the pupil is measured almost entirely by the point which he reaches on the scale. For this reason the tests may very properly be technically called "scales," and are characterized as "difficulty tests" or "power tests" by specialists in educational measurements.

The scales were derived from data obtained from tests given to over 16,000 high-school students. The schools which coöperated in standardizing the scales varied all the way from the small rural high school to the large cosmopolitan high school. Classes were tested in eighty-four high schools located in the states of Massachusetts, Connecticut, Rhode Island, New York, New Jersey, Ohio, Wisconsin, Missouri, Oklahoma, Colorado, and Washington, and the results subjected to intricate statistical treatment. The difficulty of each exercise, or its position on the scale, was determined by the percentage of pupils solving each exercise correctly.[2]

2. SELECTION OF SCALES TO BE USED

Series A will be found on the whole to be more satisfactory than Series B, especially where the time available for testing purposes is limited. This is particularly true if the purpose of the test is primarily to determine degrees of attainment. If, however, the purpose of the test is mainly diagnostic, that is, to discover difficulties which the students are encountering, Series B should be used. It contains a richer variety of exercises and, consequently, a greater number of type processes. This makes it possible for teachers to make a more complete analysis of the mistakes made by pupils.

If only one scale can be used, it should be the Equation and Formula Scale, because it is more comprehensive and so tests a much wider range of functions. At least two scales should be used, however, and the scale which undoubtedly comes second in importance is the Problem Scale. If Series A is used there will be ample time to give both during a single class period of forty minutes.

[2] For a complete account of the method employed in locating each exercise on a linear scale, consult Hotz, Henry G.: *First Year Algebra Scales.*

Whenever it is possible to do so, all five of the tests of a given series should be used, since the achievement on all of the tests gives a much more reliable indication of a pupil's ability than the results from one or two tests would give.

Teachers have found it most practicable to use the tests in rotation somewhat as follows:

1. At the end of three months
   Addition and Subtraction Scale
   Equation and Formula Scale
2. At the end of six months
   Multiplication and Division Scale
   Problem Scale
3. At the end of nine months
   Equation and Formula Scale (repeated)
   Graph Scale

Whenever it is desired to use the same scale a second time, it is advisable to select it from a different series. It is feared that unless at least six months' time has elapsed since a given test was used some of the practice effect may survive.

3. WHEN TO GIVE THE TESTS

The scales may be used very profitably as early as the end of the third month of the school year.

Tentative Standards of Achievement,[3] based upon the 16,000 papers of the original study, were compiled for three-, six-, and nine-month intervals. It is, therefore, much more satisfactory for comparative purposes to submit these scales to algebra classes immediately after they have studied algebra for three, for six, or for nine months. However, data on the achievement at other intervals are being collected constantly, much of which is included in this pamphlet,[4] and as time passes more and more information with regard to the achievement that may reasonably be expected at other intervals will become available.

The scales are not intended to be used beyond the first year. For this reason very few results from classes having had algebra more than ten months have been reported.

[3] See p. 26.

[4] See pp. 27-34.

4. DIRECTIONS FOR ADMINISTERING THE TESTS

1. *Preliminary.* Before passing the papers see that the desks are cleared and pupils are provided with pencils. For the graph test rulers should also be provided. Then make the following statement:

> "I am going to give you a test to see how well you can solve exercises in algebra. Papers will be passed to you with the printed side down. Please leave them so until I tell you to turn them over."

2. Pass the papers, or have them distributed by the pupils in the front seats, with the printed side down (Series B, first page down) and the top end away from the pupil.

3. When all are ready say to the class:

> "Turn your papers. Write your name in the first blank space," etc. (The number of blank spaces to be filled out may be determined by the one giving the test. It is not necessary to have all filled in. Some teachers simply have pupils write their names in the upper right hand corner of the blank page so as to prevent the pupils from seeing any of the exercises before all of the directions have been given.)

4. Then repeat one of the following series of directions, depending upon the tests to be given:

Addition and Subtraction Scale

> "Attention! The exercises on these sheets are in addition and subtraction—collection of terms. Take the exercises in the order in which they are given. Work as many as you can and be sure you get them right. Work directly on these sheets and do not ask anybody any questions. When you have worked all the exercises you can, lay aside your pencils and remain quiet so as not to disturb those who are still working. You will have *twenty minutes* in which to work (Series B, *forty minutes*). Start."

Multiplication and Division Scale

> "Attention! The exercises on these sheets are in multiplication and division. Take the exercises in the order in which they are given. Work as many as you can and be

sure you get them right. All answers must be reduced to their simplest forms. Work directly on these sheets and do not ask anybody any questions. When you have worked all the exercises you can, lay aside your pencils and remain quiet so as not to disturb those who are still working. You will have *twenty minutes* in which to work (Series B, *forty minutes*). Start."

Equation and Formula Scale

"Attention! On these sheets you are given a number of equations and formulae to solve. Take the exercises in the order in which they are given. Solve as many as you can and be sure you get them right. Work directly on these sheets and do not ask anybody any questions. When you have worked all the exercises you can, lay aside your pencils and remain quiet so as to not disturb those who are still working. You will have *twenty minutes* in which to work (Series B, *forty minutes*). Start."

Graph Scale

"Attention! On these sheets you are given a number of graphs. Read each question carefully and then do as you are told to do. Take the exercises in the order in which they are given. Answer as many questions as you can and be sure you get them right. Work directly on these sheets and do not ask anybody any questions. When you have answered all the questions you can, lay aside your pencils and remain quiet so as not to disturb those who are still working. You will have *twenty-five minutes* in which to work. Start."

Problem Scale

"Attention! On these sheets you are given a number of questions to answer. Take the exercises in the order in which they are given. Answer as many questions as you can and be sure you get them right. In all the problems which call for the equation, for example No. 4, simply state the equation which will solve the problem. Take for example, this problem: A coat and hat cost \$30. The coat cost 5 times as much as the hat. Find the cost of each. The equation would be $x + 5x = \$30$.

(Write the equation on the board.) Work directly on these sheets and do not ask anybody any questions. When you have answered all the questions you can, lay aside your pencils and remain quiet so as not to disturb those who are still working. You will have *twenty-five minutes* in which to work (Series B, *forty minutes*). Start."

5. When the time is up, say "Stop" and collect the papers. Most of the pupils will have finished before this time. Those who have not, in all probability have done all they can.

A warning, stating the amount of time left, should be given three minutes before time is called for the tests of Series A and five minutes in advance for those of Series B.

With many classes which have had less than nine months of algebra, and especially those which have had only three months, it is perfectly safe to call time before the full time allowed for that particular test has elapsed.

Students may be provided with scratch paper for their own use. It has been found to be most satisfactory to pass quietly down the aisles a few minutes after the test is started and give each pupil a sheet of scratch paper. Pupils will find it more convenient, however, to work directly on the question sheets. For all but the problem test it is desirable to have as much of the work as possible on these sheets.

5. DIRECTIONS FOR SCORING THE PAPERS

In scoring the papers, answers are to be marked either right or wrong. All answers which may be accepted as correct are given on pages 13 to 19 of this manual. A few incorrect answers are also listed, to indicate more definitely the types of answers which must not be accepted.

No credit is given for answers that are partially right. This procedure, though somewhat arbitrary, greatly simplifies the task of scoring the papers. It saves time and offers less chance for variation in scoring the results. This last factor, uniformity in scoring, is an absolute essential in order that valid comparisons between different school systems may be made.

After each paper is scored, it has been found most convenient to record the total number of exercises solved correctly in the upper right-hand corner of the test sheet.

Answer Key to Addition and Subtraction Scale

Problem No.	Answers	Problem No.	Answers
1	$9r$	16	$\frac{2r^2}{r^2-z^2}$; $\frac{2r^2}{(r-z)\ (r+z)}$
2	$5x$		
3	$15b$	17	$\frac{7-4a}{6a}$
4	$2\frac{1}{2}c$; $\frac{5c}{2}$	18	$\frac{9-6x}{4}$; $\frac{9}{4}-\frac{6x}{4}$
5	$6x+2$		
6	$8a-6b$	19	$\frac{3y^2-6x^2}{2x^2y^2}$
7	m		
8	$5x$	20	$\frac{5-3a}{(a+1)\ (a^2-a+1)}$; $\frac{5-3a}{a^3+1}$
9	$r-5t+s$		
10	$5c+6$		
11	a^2-6b+4	21	$\frac{40-13x}{(x-3)\ (x-2)\ (x+5)}$; $\frac{40-13x}{x^3-19x+30}$
12	$4x-1$		
13	$\frac{3c}{8}$; $\frac{6c}{16}$	22	$\frac{1}{(x-1)^3}$; $\frac{1}{x^3-3x^2+3x-1}$
14	$\frac{7x}{6}$	23	$5\frac{1}{5}\sqrt{5}$; not $5\sqrt{5}+\frac{1}{5}\sqrt{5}$
15	$\frac{a-2x}{a^2-x^2}$	24	$\frac{6a-7}{a^2-4}$; $\frac{7-6a}{4-a^2}$

Answer Key to Multiplication and Division Scale

Problem No.	Answers	Problem No.	Answers
1	$21y$	14	$4x-8;\ 4(x-2)$
2	$3n$	15	$\frac{b}{a(m-n)};\ \frac{b}{am-an}$
3	$8a^2b^2$		
4	$3c$	16	$81x^4y^{12}$
5	$6m$	17	$c+d$; not $\frac{c^2-d^2}{c-d}$
6	$-2b$		
7	$-12x^2y^3$	18	$12x^2;\ 12x^{\frac{4}{2}}$
8	$6a^5$	19	$a-\frac{1}{2};\ \frac{2a-1}{2}$
9	$2m-3n$		
10	$\frac{2x^2}{5}$	20	$\frac{p-2}{3r(p-9)};\ \frac{p-2}{3pr-27}$
11	$10a^3+33a^2-52a+9$	21	$\frac{x^2-3x+9}{3(x-3)};\ \frac{x^2-3x+9}{3x-9}$
12	n^2-10	22	48
13	$\frac{4}{3a}$	23	3

Answer Key to Equation and Formula Scale

Problem No.	Answers	Problem No.	Answers
1	2	15	$\frac{1}{3}$
2	3		
3	2	16	154
4	7	17	$\frac{El}{R}$
5	2		
6	2; not $-z=-2$	18	$-\frac{1}{2}$; not $\frac{1}{-2}$; nor $-x=\frac{1}{2}$
7	9		
8	2	19	10, -5 (both roots)
9	4	20	-5, -1*; not $-x=5$
10	$\frac{15}{16}$	21	$x=2, y=4$*
11	40	22	$21\frac{1}{9}$; $\frac{190}{9}$
12	$4\frac{2}{7}$; $\frac{30}{7}$	23	1
13	15	24	$\sqrt{\frac{2s}{g}}$; $\sqrt{\frac{s}{\frac{1}{2}g}}$; $\frac{1}{g}\sqrt{2gs}$
14	$m=2, n=4$*	25	1

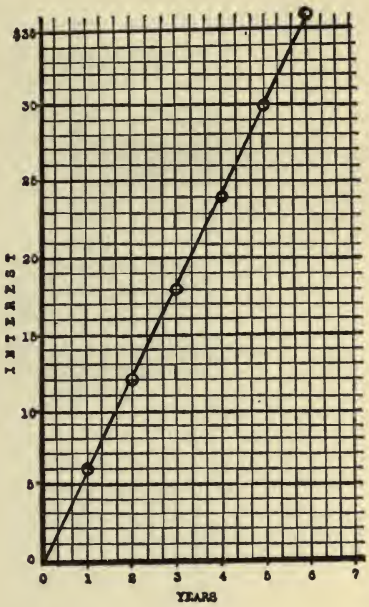
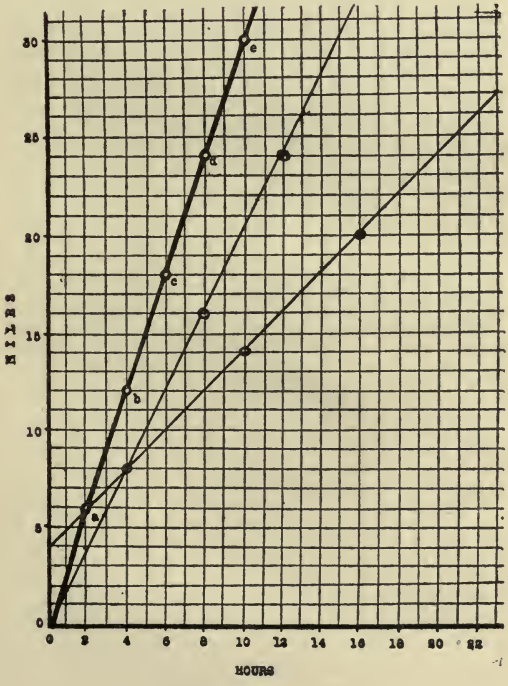
* Where two results are ordinarily required in an answer, the exercise is marked correct if the work is done correctly up to the point where only one value is obtained and stopped at that. If, however, the student makes an error in solving for the second value, the problem is scored incorrect.

Answer Key to Problem Scale

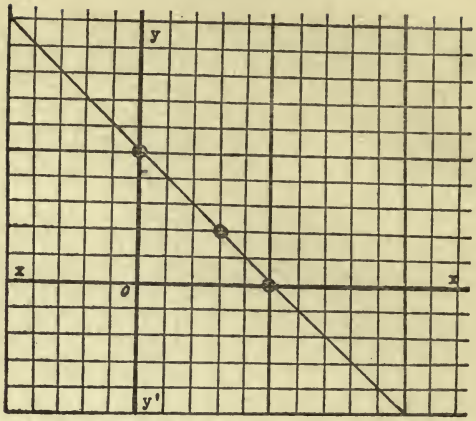
Problem No.	Answers
1	$3x$
2	$m - r$
3	$a + b$
4	$x + 10x = 132$. ($12 and $120)
5	$\frac{980}{v}$
6	$4x + 40 = 240$; $2x + 2(x - 20) = 240$; $4x - 40 = 240$. (50 ft. × 70 ft.)
7	$3x - 136 = 836$; $2x - 136 = 836 - x$; $x + y = 836$ and $x = 2y - 136$. (324 children and 512 adults)
8	$\frac{r}{dw}$; not $dwx = r$
9	$(x + 12)(x - 4) = x^2$. (6 ft. × 6 ft. and 2 ft. × 18 ft.)
10	$2\frac{1}{2} : 5\frac{3}{4} = 20 : x$; $x : 20 = 5$ ft. 9 in. : 2 ft. 6 in.; $x : 20 = 69 : 30$; eight times as high as the man; not $2.6 : 5.9 = 20 : x$. (46 ft.)
11	$x + y = 5000$ and $\frac{3x}{100} + \frac{4y}{100} = 172$; $\frac{3x}{100} + \frac{4(5000 - x)}{100} = 172$; $.03x + 200 - .04x = 172$. ($2800 and $2200)
12	$40x = 55(x - 2)$; $55x = 40(x + 2)$; $\frac{x}{40} - 2 = \frac{x}{55}$. (293⅓ miles)
13	$x + y = 20$ and $50x + 65y = 1200$; $50x + 65(20 - x) = 1200$; not $50x + 65(20 - x) = 12$. (6⅔ lbs. and 13⅓ lbs.)
14	$5x^2 = 180$; $5(x - 10)^2 = 180$. (16 in. × 16 in.)

The equations given above are those which are usually found. Modifications, which in the end equal the same, may be accepted. For example, $4x = 240 + 40$ is the same as $4x - 40 = 240$, and $\frac{69}{30} = \frac{x}{20}$ is the same as $69 : 30 = x : 20$. Where the problems have been worked out and the correct answers are given, they are to be scored as correct, though such a procedure on the part of students is to be discouraged.

Answer Key to Graph Scale

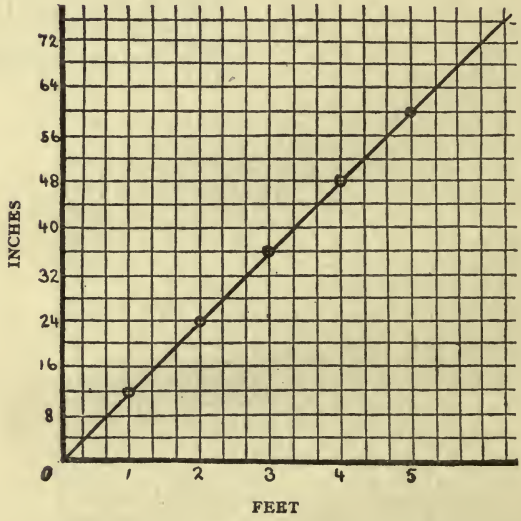
Problem No.	
1.	2000
2.	24
3.	3330 to 3350 (inclusive)
4.	
5.	−3 to −9 (inclusive)
6. and 9.	

7.

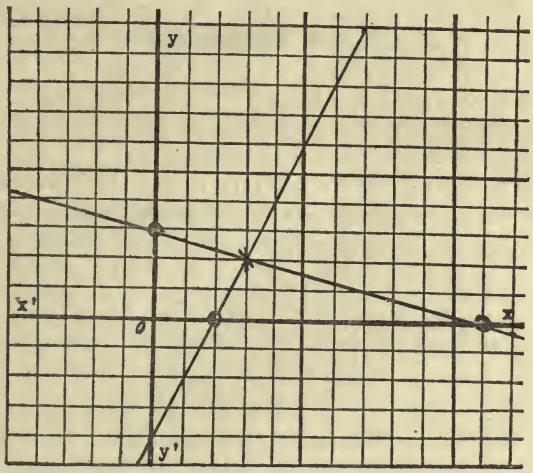


1 Space = 1 Unit

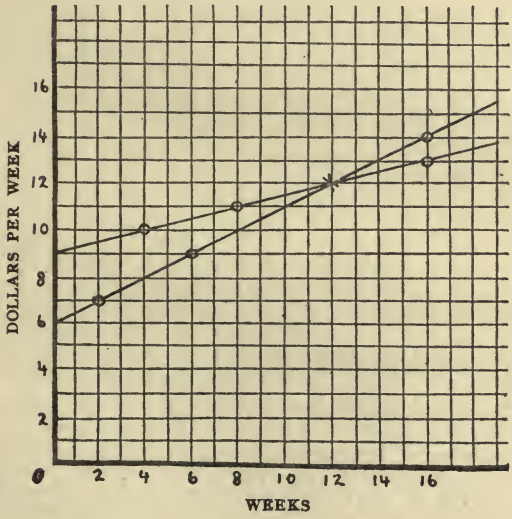
8.



10. $x = 3$
$y = 2.$



11. In twelve weeks; the thirteenth week.



The score assigned to each pupil for each test is the total number of "rights," that is, the total number of exercises solved correctly in the test. Individual scores made on the various tests by the different pupils of a class may be recorded on a sheet similar to Fig. 1.

City..... Date.....

School..... Teacher..... Grade.....

Names of Pupils	Scores on Each Test				
	I	II	III	IV	V
Median Score					
Standard Score					

FIG. 1

Individual Class Record Sheet Used in Recording the Results for Each Class

6. DIRECTIONS FOR DETERMINING THE MEDIAN OR CLASS SCORE

The median number of exercises correctly solved is used in connection with these scales as the class score. Though not entirely accurate scientifically, it is the most readily computed and is, therefore, for all practical purposes the most satisfactory measure of the achievement of a class. The median score represents the number of exercises solved correctly by just fifty per cent of a class. That is, there are just as many students in a class who solve a larger number as there are students who solve a smaller number of exercises.

In order to determine the median point of the achievement of a class, it is necessary to make a distribution table of the results of a test. Such a table shows the number of pupils who were unable to solve a single exercise correctly, the number who solved one exercise correctly, two exercises, three exercises, etc. Sample distributions for four of the tests, Addition and Subtraction, Multiplication and Division, Equation and Formula, and Problems, are shown in Table I, (page 22). Another distribution is given in Table II, (page 23).

TABLE I

DISTRIBUTION TABLE SHOWING SCORES ATTAINED FOR FOUR OF THE TESTS, OKMULGEE, OKLAHOMA [5]

City *Okmulgee* State *Oklahoma* Date *June 1921*

School *High* Teacher Grade *9th*

Remarks *9-month group*

No. of Examples Correct	Number of Pupils Making Score Indicated				
	Addition and Subtraction	Multiplication and Division	Equation and Formula	Problems	Graphs
25					
24					
23					
22					
21					
20					
19					
18					
17					
16					
15					
14					
13					
12	6	6	12		24
11	5	8	12		25
10	17	9	12	2	40
9	16	9	12	6	43
8	17	21	19	11	68
7	11	14	9	10	44
6	7	12	9	16	44
5	7	9	7	19	42
4	5	6	2	18	31
3	4	4	2	12	22
2	2			4	6
1					
0					
Total	97	98	96	98	
Median	8.74	8.19	9.0	5.79	
Standard	7.9	7.9	7.8	5.6	

[5] Form used by Bureau of Educational Research, University of Illinois, Urbana, Ill.

Table II also indicates in a clear and concise way the method of computing the median class score.[6] Since a thorough knowledge of the method of calculating the median is essential to the proper use of these scales, it is urged that teachers who are not familiar with educational statistics make a careful study of this table.

TABLE II

SAMPLE DISTRIBUTION OF SCORES MADE ON EQUATION AND FORMULA TEST, SERIES B

Score	Number of Pupils Making Each Score		Computation of Median
20			1. After checking the problems correctly solved, count the check marks in each paper, and indicate the number in the proper place in column 2.
19			
18			
17	||	2	
16	||||	4	2. Find the total number of scores (N).
15	|||	3	
14	|||||	5	3. Median equals middle score $= \frac{N}{2}$
13	||||| ||	7	
12	|||	3	
11	||||	4	
10	|||	3	Thus, $\frac{N}{2} = \frac{35}{2} = 17\frac{1}{2}$
9	||	2	
8			Beginning at 1 in the third column and counting up, it is necessary to count $3\frac{1}{2}$ of the 7 to make $17\frac{1}{2}$, thus:
7	|	1	
6	|	1	$17\frac{1}{2} = 1 + 1 + 2 + 3 + 4 + 3$ and $3\frac{1}{2}$ of the 7.
5			
4			Put $3\frac{1}{2}$ as a numerator over 7 and add to 13, the step on which the 7 occurs, thus:
3			
2			
1			
0			
Total Scores (N)		35	Median $= 13 + \frac{3.5}{7} = 13.5$
Median Score		13.5	

Errors in computing the median or class score are very common; and, when comparison is to be made with standard scores, an error of one-half point in its computation may do serious injustice to a class. As a further safeguard against error in this important detail, four additional class distributions are submitted in Table III, and a complete discussion of the method of determining the median or class score of these distributions follows.

[6] The method suggested here is an adaptation of the plan used by Clifford Woody in his *The Woody Arithmetic Scales and How To Use Them*, p. 19.

TABLE III

SAMPLE DISTRIBUTION OF SCORES MADE BY FOUR DIFFERENT CLASSES ON EQUATION AND FORMULA TEST

Number of Problems Solved	Class I	Class II	Class III	Class IV
0				
1	1		2	
2	2	1	1	1
3	2		2	3
4	1		1	2
5	4	1	2	3
6	6	5	3	2
7	3	4	1	
8	3	3	1	3
9	2	2	2	4
10	1	1	1	2
11	1	1		1
12				
13				1
14		1		
15				
Total	26	19	16	22
Median Score	6.5	7.6	6.0	7.5

According to this table, there were twenty-six students in Class I, nineteen students in Class II, sixteen students in Class III, and twenty-two students in Class IV. In Class I one student solved one problem correctly, two solved two problems correctly, two solved three correctly, etc. To find the median score of this class, it is necessary to find the point in the distribution of the class where there are just as many students who solved a greater number of problems as there are students who solved a smaller number. Since there are twenty-six students in the class, this point is obviously midway between the scores made by the thirteenth and fourteenth students, counting down in the distribution. That is, to include the thirteenth individual with the poorer group, it is necessary to count three of the six students who solved six problems; and, since it is assumed that the individuals are distributed

evenly through a step at equal distances from one another, the median point is just one half of the distance through this step, from six to seven. Therefore, the median score of this class is 6.5 problems solved correctly.

In Class II there are nineteen students. Thus, there are 9.5 individuals both above and below the exact median point in the distribution of this class. To include 9.5 individuals with the poorer group, it is necessary to count 2.5 of the 4 students who solved 7 problems. Hence, the median point is just $\frac{2.5}{4}$ of the distance through the 7th step, which makes the median score of this class 7.6 problems solved correctly.

Class III illustrates another situation still. There are 16 students in the class, and in counting out the 8 individuals for the poorer group, we exactly take up all the cases in the 5th step. The fact to be noticed here is that the median point is raised clear through the 5th step. The median score for this class, therefore, is 6.0 problems correctly solved.

Class IV is here included to assist in the solution of another difficulty which is often encountered. There are 22 individuals in the class. Counting from the top of the distribution, the 11 cases for the poorer group take us, as seen above, entirely through step 6. Likewise, counting from below, to include 11 cases, we have to go clear through step 8. From this it appears that the median point could be located all the way from 7 to 8 in the class distribution. Since, however, any given distance on a scale is best represented by its middle point, the median score of this class should be 7.5 problems solved correctly.

7. STANDARDS OF ACHIEVEMENT

Tentative standards of achievement were proposed in the original monograph.[7] When these were published, the scales had not been used very extensively, and some doubt was expressed with regard to the reliability of the tentative standards. There is as yet, however, no conclusive evidence that any of these standards should be materially revised.

There is some evidence that the tentative standards are on the whole a little too high. On the other hand, whenever the tests

[7] Hotz, Henry G: *First Year Algebra Scales*, p. 41.

have been submitted to classes in large high schools, where the teaching is generally more efficient and where there is more careful selection of the subject matter taught in elementary algebra, the results invariably surpass these standards.

TABLE IV

TENTATIVE MEDIAN STANDARDS OF ACHIEVEMENT, SERIES A

	Three-Month Group	Six-Month Group	Nine-Month Group
Addition and Subtraction	5.0	6.8	7.9
Multiplication and Division	5.3	6.3	7.9
Equation and Formula	4.9	7.1	7.8
Problem Test	4.3	4.9	5.6
Graph Test	2.8 (four and one-half months)		5.6

TABLE V

TENTATIVE MEDIAN STANDARDS OF ACHIEVEMENT, SERIES B

	Three-Month Group	Six-Month Group	Nine-Month Group
Addition and Subtraction	9.7	12.9	14.4
Multiplication and Division	9.6	14.0	16.3
Equation and Formula	7.8	14.3	16.0
Problem Test	5.4	6.5	7.5
Graph Test	3.7 (four and one-half months)		7.2

TABLE VI

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES A

ADDITION AND SUBTRACTION SCALE

City	No. of Months of Algebra Studied					
	3	6	8	9	10	14
Illinois Cities				6.9	7.3	
Virginia — Rural Schools		4.3	4.9			
Virginia — Small City Schools		5.2	5.9			
Virginia — Large City Schools	3.6	5.3	5.0			
North Carolina — Rural Schools	3.3	3.7		4.5		4.9
North Carolina — Small City Schools	2.9	5.4		3.9		4.3
North Carolina — Large City Schools	3.6	3.9				5.4
Atlantic City, N. J.		7.4				
Reading, Pa.		3.9				
Providence, R. I. (Moses Brown School)		6.7				10.5
Fayetteville, Ark. (University H. S.)	5.1	5.3		7.5		
Fayetteville, (City H. S.)	5.3					
Little Rock, Ark.		4.9		6.0		
Potsdam, N. Y.		6.3				
Saratoga, N. Y.		5.4				
Elmira, N. Y.		6.1				
Whitehall, N. Y.		6.9				
Cities of Original Study	5.0	6.8		7.9		

MULTIPLICATION AND DIVISION SCALE

City	No. of Months of Algebra Studied					
	3	6	8	9	10	14
Illinois Cities				7.2	7.4	
Athens, Ohio	3.8			4.8		5.2
Reading, Pa.		3.9				
Providence, R. I. (Moses Brown School)		7.7				10.4
Fayetteville, Ark. (University H. S.)	5.6	5.9		6.8		
Lockport, N. Y.		6.7				
Potsdam, N. Y.		5.8				
Saratoga, N. Y.		6.2				
Elmira, N. Y.		5.9				
Amsterdam, N. Y.				7.4		
Cities of Original Study	5.3	6.3		7.9		

TABLE VI

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES A

EQUATION AND FORMULA SCALE

City	No. of Months of Algebra Studied					
	3	6	8	9	10	14
Illinois Cities				7.7	7.9	
Virginia — Rural Schools		1.6	3.9			
Virginia — Small City Schools		4.6	5.8			
Virginia — Large City Schools	4.0	5.4	5.1			
North Carolina — Rural Schools	3.2	4.2		4.7		4.9
North Carolina — Small City Schools	3.1	2.8		1.5		5.1
North Carolina — Large City Schools	3.3	4.5				5.5
Reading, Pa.		1.8				
Atlantic City, N. J.		6.2				
Chicago, Ill. (Research Study by Eleanora Harris)					6.3	6.9
Providence, R. I. (Moses Brown School)		9.0				9.8
Fayetteville, Ark. (University H. S.)	5.9	6.3		8.6		
Philadelphia, Pa.				6.8		
Little Rock, Ark.		6.0		6.8		
Lockport, N. Y.		7.3				
Potsdam, N. Y.		6.4				
Saratoga, N. Y.		5.4				
Whitehall, N. Y.		7.8				
Amsterdam, N. Y.				8.1		
Cities of Original Study	4.9	7.1		7.8		

TABLE VI

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES A

GRAPH SCALE

City	No. of Months of Algebra Studied						
	4½	6	8	9	10	12	14
Chicago, Ill. (Research Study by Eleanora Harris)					4.7		
Fayetteville, Ark. (University H. S.)		4.5		6.0			
Providence, R. I. (Moses Brown School)		6.5					6.8
Reading, Pa.		1.8					
Illinois Cities				6.2	5.0		
Wellington, Kans. (1919)				6.6			
Wellington, Kans. (1920)				6.4			
Wellington, Kans. (1921)							
Rapid Group		7.8					
Average Group				4.8			
Slow Group				4.6			
Mississippi (Two Schools)							4.2
South West City, Mo.							
Hackensack, N. J.	2.3			5.6	5.2	7.3	7.4
Amsterdam, N. Y.				2.5			
Cities of Original Study	2.8			5.6			

TABLE VI

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES A

PROBLEM SCALE

City	No. of Months of Algebra Studied					
	3	6	8	9	10	14
Illinois Cities				6.4	5.0	
Athens, Ohio	1.9			2.5		3.8
Reading, Pa.		2.0				
Chicago, Ill. (Research Study by Eleanora Harris)					4.3	
Providence, R. I. (Moses Brown School)		5.8				7.2
Mount Holly, N. J.			3.6			
Fayetteville, Ark. (University H. S.)	4.3	4.3		6.2		
Saratoga, N. Y.		4.2				
Elmira, N. Y.		4.9				
Amsterdam, N. Y.				5.5		
Cities of Original Study	4.3	4.9		5.6		

TABLE VII

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES B

ADDITION AND SUBTRACTION SCALE

City	No. of Months of Algebra Studied						
	3	6	8	9	10	12	14
Cleveland, Ohio		14.7					
Fayetteville, Ark.							
(University H. S.)	9.4						
New Jersey Cities	12.0						
Racine, Wis. (1919)	13.0	17.3					
Racine, Wis. (1920)	13.6	18.8					
Wisconsin Cities (1918)	11.2			14.4			
Wisconsin, 22 Cities (1921)		14.8					
Wellington, Kans. (1919)				18.1			
Wellington, Kans. (1920)				15.9			
Wellington, Kans. (1921)							
Rapid Group		19.5					
Average Group				14.1			
Slow Group				10.8			
Elizabeth City, N. J.				12.8			
Andover, Mass.				10.0			
South West City, Mo.				17.0			
Hackensack, N. J.		12.0			14.5	18.0	20.0
Paragould, Ark.	10.8	12.6	12.1				
Cities of Original Study	9.7	12.9		14.4			

TABLE VII

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES B

MULTIPLICATION AND DIVISION SCALE

City	No. of Months of Algebra Studied						
	3	6	8	9	10	12	14
Cleveland, Ohio		14.8					
New Jersey Cities		15.4					
Racine, Wis. (1919)	13.6	18.1					
Racine, Wis. (1920)	14.7	18.6					
Waukesha, Wis.	14.0						
Wisconsin Cities (1918)	11.4			17.4			
Wisconsin, 23 Cities (1921)		16.2					
Wellington, Kans. (1919)				18.3			
Wellington, Kans. (1920)				17.4			
Wellington, Kans. (1921)							
Rapid Group		17.0					
Average Group				14.0			
Slow Group				13.1			
Andover, Mass.				14.2			
Ironwood, Mich.		11.0					
Hackensack, N. J.		10.4			15.9	16.5	19.5
Paragould, Ark.	11.3	13.1	14.9				
Cities of Original Study	9.6	14.0		16.3			

TABLE VII

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES B

EQUATION AND FORMULA SCALE

City	No. of Months of Algebra Studied						
	3	6	8	9	10	12	14
Cleveland, Ohio		14.9					
Fort Smith, Ark.	7.2	9.9					
New Jersey Cities		16.3					
Racine, Wis. (1919)		17.5					
Racine, Wis. (1920)		15.9					
Wisconsin Cities (1918)				17.2			
Wisconsin, 28 Cities (1921)		14.2					
Wellington, Kans. (1919)				18.1			
Wellington, Kans. (1920)				16.7			
Wellington, Kans. (1921)							
Rapid Group		20.6					
Average Group				14.4			
Slow Group				11.2			
Elizabeth City, N. C.				12.8			
Andover, Mass.				12.8			
Ironwood, Mich.		6.4					
Cold Springs, N. Y. (Holdane School)	13.5						
Hackensack, N. J.		9.0			14.5	20.0	20.3
Paragould, Ark.	11.0	12.5	15.8				
Cities of Original Study	7.8	14.3		16.0			

TABLE VII

SUMMARY OF MEDIAN SCORES ATTAINED IN VARIOUS CITIES WITH FIRST YEAR ALGEBRA SCALES, SERIES B

PROBLEM SCALE

City	No. of Months of Algebra Studied						
	3	6	8	9	10	12	14
Cleveland, Ohio		6.3					
Fort Smith, Ark.	4.7	5.0					
New Jersey Cities				7.0			
Waukesha, Wis.			7.9				
Wisconsin Cities (1918)				8.2			
Wisconsin, 31 Cities (1921)		7.1					
Wellington, Kans. (1919)				9.8			
Wellington, Kans. (1920)				9.4			
Wellington, Kans. (1921)							
Rapid Group		11.4					
Average Group				7.8			
Slow Group				5.8			
Elizabeth City, N. C.				8.1			
South West City, Mo.				8.3			
Mississippi (Two Schools)				4.7		6.3	
Cold Springs, N. Y. (Holdane School)	5.8						
Hackensack, N. J.		5.5			5.3	5.9	5.4
Paragould, Ark.	7.0	6.1	7.4				
Cities of Original Study	5.4	6.5		7.5			

8. GRAPHICAL REPRESENTATION AND STATISTICAL INTERPRETATION OF RESULTS

After the papers for one or more of the tests of a class have been scored, and the median has been computed, the results should be entered in tabular form on an Individual Class Record Sheet similar to the one shown in Fig. 1 (p. 20).

In large school systems it is usually necessary to combine the individual scores of various classes. For this purpose the tabulation shown in Table I (p. 22) has been devised. In order to save time, teachers very frequently use this form of tabulation for individual classes as well and compute the medians directly from these distributions.

TABLE VIII

SUMMARY OF RESULTS IN EQUATION AND FORMULA TEST, SERIES B, FORT SMITH, ARKANSAS

Score	Three-Month Group	Six-Month Group
20		1
19		
18		1
17		2
16		2
15		6
14	1	4
13	2	6
12	3	7
11	9	5
10	10	9
9	26	7
8	22	11
7	35	6
6	28	9
5	26	3
4	12	4
3	7	4
2	7	
1	9	
0	4	
Number of Pupils	201	87
Median Score	7.2	9.9

Table VIII represents, in somewhat greater detail, the results of one of the tests in a city high school having several algebra classes. It shows the distribution of the group that has studied algebra for three months, and the distribution of the group that has studied algebra for six months. Such a table indicates the extent of variation within a group and also the excessive amount of overlapping that exists between the two group distributions.

The facts in this table may be more strikingly portrayed if presented graphically as in Fig. 2. In this graph the results of two groups are drawn upon the same scale, one placed above the

other. In constructing these graphs, the X-axis represents the scores attained by the various pupils, and the Y-axis the percentage of pupils making each score.

Graphs of this type furnish a most efficient means for showing: (1) The wide range of abilities within a group. (2) The scores

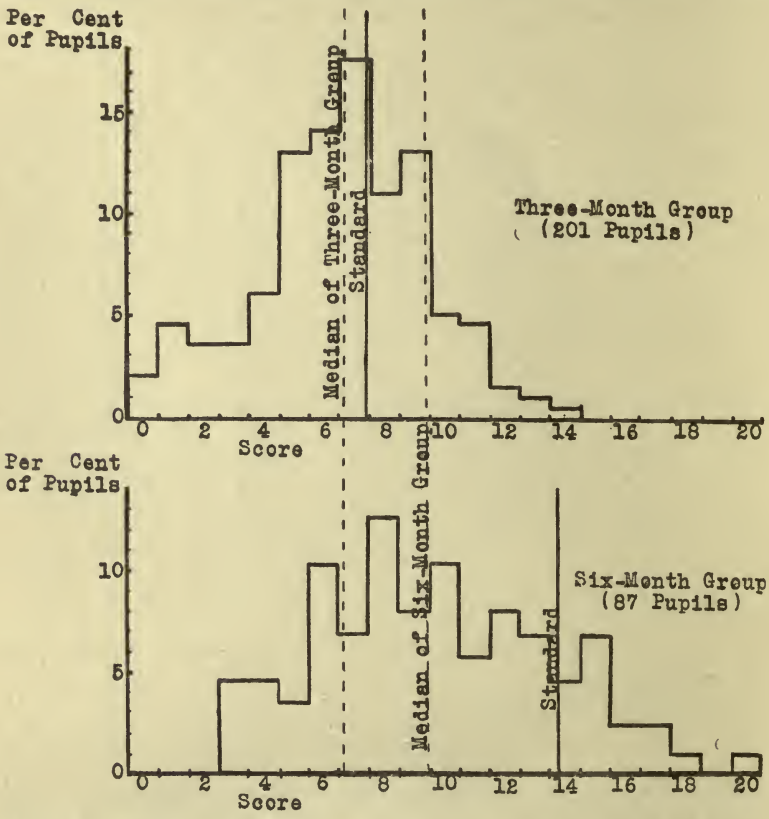


FIG. 2. Distribution and Median Scores, Equation and Formula Test, Series B, Fort Smith High School, Fort Smith, Ark.

most frequently made by pupils of a group. (3) The extent to which a group falls short or surpasses the median standard achievement, as indicated by the distance between the median for the group and the standard median. (4) The amount of progress made from group to group, as indicated by the distance between the group medians. (5) The amount of overlapping between the groups. In Fig. 2,

this last factor, the excessive amount of overlapping, is undoubtedly the most significant. It will be seen that 29 per cent of the pupils of the three-month group do as well as or better than, the median pupil in the six-month group, and that about 24 (24.4) per cent of the pupils of the six-month group fall below the achievement of the median pupil in the three-month group.

It is sometimes desirable to represent graphically the scores attained by each individual pupil in a given test. This is done in Fig. 3, where the results obtained from an entire class in the Addition and Subtraction Scale are exhibited. Here each pupil's score

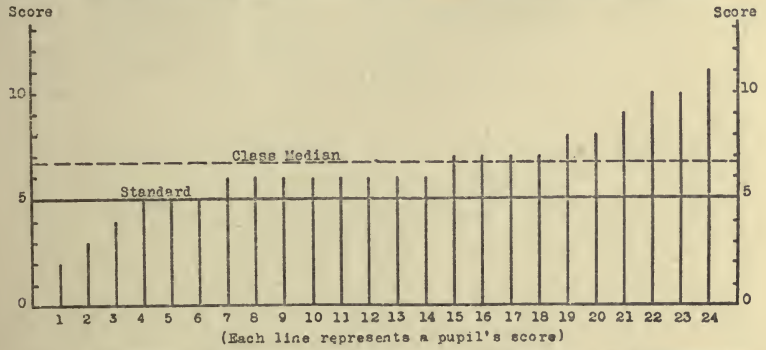


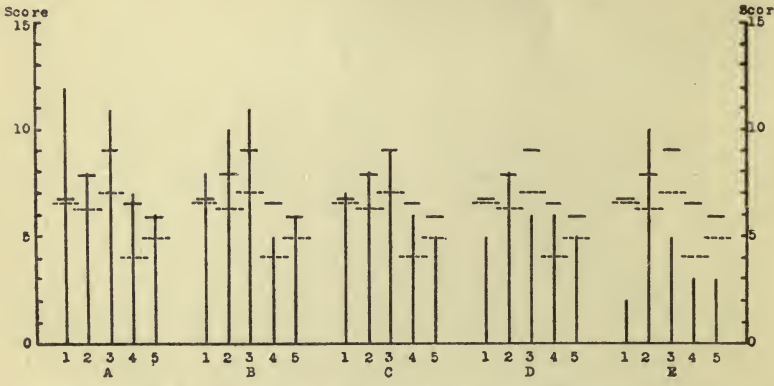
FIG. 3. Individual Scores of a Six-Month Group, Addition and Subtraction Test, Series A; Moses Brown School, Providence, R. I.

is represented by a vertical line, the height of the line indicating the size of the score. Thus the relative standing of each and every pupil is vividly portrayed. It will be seen that the best student in this class does more than five times as well on this test as the poorest student.

When a composite picture of the achievement of a pupil on all five tests is desired, a graph similar to that in Fig. 4 is suggested. The scores of five different pupils on each of the five different tests are here exhibited. The short horizontal lines indicate the median score of the class. Pupil A, it is evident, surpassed the median scores of the class in all of the tests; pupils B, C, and D fell below these medians in one or more tests; and pupil E did very poorly in all but the Multiplication and Division Test, in which he did exceptionally well. The relative standing of the members of a class is more readily determined from this graph if pupils are arranged roughly in order of excellence of achievement.

In Fig. 5 the median scores made in a city high school on all of the tests in 1920 are compared with the scores made in the same school in 1919 and with the standard scores. It is at once evident that the median scores made in 1920 are all well above the standard but not quite as high as the 1919 median scores.

A most interesting graph of results from the same school is reproduced in Fig. 6. These data were obtained during the school year of 1920–21, and clearly illustrate the advantage of homogeneous grouping of pupils. At the beginning of the year all first year algebra pupils were divided into three sections on the basis



Letters represent different pupils; perpendicular lines, pupil's individual scores in (1) Addition and Subtraction, (2) Multiplication and Division, (3) Equation and Formula, (4) Graph, (5) Problem, tests respectively; horizontal cross bars: — class median; - - - standard median.

FIG. 4. Individual Scores Attained by Pupils from a Six-Month Group on All Tests, Series A, Moses Brown School, Providence, R. I.

of results obtained with the Otis Intelligence Tests. The pupils making the highest grades in the intelligence test, 29 in all, were placed in one group and covered a year and a half of work in algebra in nine months. Twenty-two pupils were placed in the slow section and about 70 in the normal group. The bright group was tested at the end of six months and the other two groups at the end of nine months. The graph indicates that the normal group, covering the usual amount of ground, made a satisfactory showing on all the tests; and, moreover, that the bright group, at the end of six months, as shown by the results, possessed the ability to solve the algebra exercises that was far superior even to that represented by the nine-month standard.

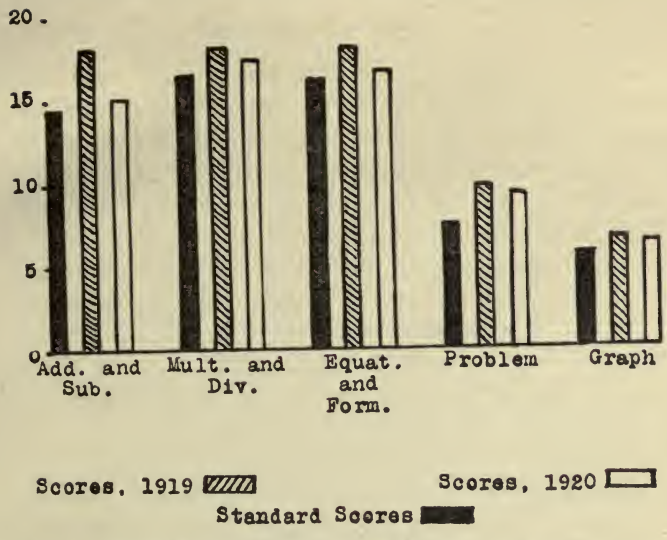


FIG. 5. Comparison of Median Scores Attained in 1919 and 1920, Series B, Wellington High School, Wellington, Kans.

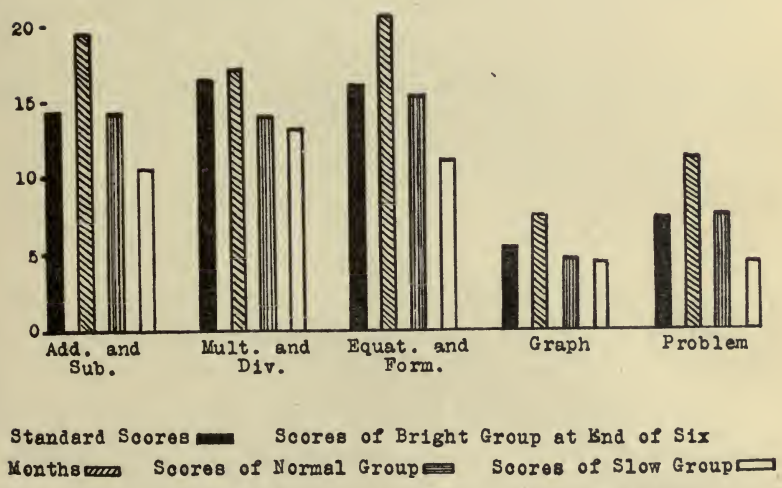


FIG. 6. Comparison of Median Scores Attained by Bright Group, Median Group, and Slow Group in All of the Tests, Series B, Wellington High School, Wellington, Kans.

9. ANALYSIS OF ERRORS

There is a tendency on the part of teachers and administrators to use standard tests merely for the purpose of determining degrees of attainment. Large numbers, the majority of the teachers perhaps, consider the net results, as shown by individual and class scores, as the all-important objective in the use of standardized tests. Such knowledge of total scores achieved is valuable, but it is merely the first step in the process of securing greater efficiency in instruction through the use of standard testing devices. Tests have other values which are much more significant to the classroom teacher. They should be used much more extensively to reveal weaknesses in teaching and to aid in the diagnosis of difficulties encountered by pupils.

In the analysis of the results of a test, the types of problems causing special difficulty are usually quite readily disclosed, but it is a much more intricate matter to determine with any degree of precision the mental processes that may be responsible for the various errors.

Three studies on the analysis of errors most frequently made in algebra are here reported in Tables IX, X, and XI.

TABLE IX

CLASSIFICATION OF 443 ERRORS[8] MOST FREQUENTLY MADE BY PUPILS OF THE FORT SMITH (ARK.) HIGH SCHOOL IN THE EQUATION AND FORMULA TEST, SERIES B

Results page 42

1. Performing wrong operation in solving for unknown:

Ex. 7. $\frac{2}{3}z = 6$ or $\frac{2}{3}z = 6$

$z = \frac{12}{3}$. $z = 6 - \frac{2}{3}$.

2. Error in sign in transposition:

Ex. 4. $5a + 5 = 61 - 3a$.
$5a - 3a = 61 + 5$.

3. Simple arithmetical errors:

Ex. 3. $3x = 9 - 3$
$3x = 9$.

[8] Fort Smith Survey: Classification of Errors in Algebra, made under the direction of A. M. Jordan, of the University of Arkansas.

4. Error in using the four fundamental operations of algebra:

Ex. 5. $7n - 3n = 12 - 4$
$10n = 8.$

Ex. 8. $c - 2(3 - 4c) = 12$
$c - 2 - 6 - 8c = 12.$

5. Adding denominators in addition of fractions:

Ex. 12. $\frac{y}{3} + \frac{y}{4} = \frac{5}{2}$

$\frac{y}{7} = \frac{5}{2}.$

6. Incomplete solution:

Ex. 12. $4y + 3y = 30$
$7y = 30.$

Ex. 18. $-4x = 2$
$-x = \frac{1}{2}.$

7. Error in sign in division:

Ex. 6. $-3z = -6$
$z = -2.$

8. Error in copying:

Ex. 14. $7m - 3n = 12$
$7n - 3n = 12.$

Ex. 12. $\frac{y}{3} = -\frac{y}{4} + \frac{5}{2}$

$\frac{y}{3} = \frac{5}{2}.$

9. Using exponent for coefficient:

Ex. 19. $p2 - 5p = 50$
$-3p = 50.$

10. Error in substituting the value of the unknown in a formula:

Ex. 16. Area of a triangle $= \frac{1}{2}bh.$
Find area when $b = 10$ ft.
and $h = 8$ ft.
Area of triangle $= 5 \times 4 = 20.$

11. Solving for wrong unknown in a formula:

Ex. 17. $RM = EL$, solve for M.

$R = \frac{EL}{M}.$

	No.	%
1. Performing the wrong operation in solving for unknown	126	28.4
2. Error in sign in transposition	86	19.4
3. Simple arithmetical errors	84	18.9
4. Error in using the four fundamental operations in algebra	57	12.8
5. Adding denominators in addition of fractions	37	8.5
6. Incomplete solution	15	3.4
7. Error in sign in division	12	2.8
8. Error in copying	11	2.5
9. Using exponent for coefficient	3	.6
10. Error in substituting the value of the unknown in a formula	2	.4
11. Solving for the wrong unknown in a formula	1	.2
12. Unclassified	9	2.1
	443	100%

FIG. 7.

Distribution of 443 Errors made by Three and Six-Month Groups on Equation and Formula Test, Series B

TABLE X

SUMMARY OF ERRORS MADE BY PUPILS IN 36 WISCONSIN HIGH SCHOOLS[1] ON FOUR TESTS, SERIES B, MARCH AND APRIL, 1921

	Per Cent of Total No. Errors
ADDITION AND SUBTRACTION TEST	
Failure to deal with parentheses correctly	38
Failure to write the denominator of a fractional answer	24
Wrong process (adding instead of subtracting, for example)	18
Writing the sum of the numerators for a new numerator and the sum of the denominators for a new denominator	4
Errors whose cause could not be discovered	8
Miscellaneous errors	8
Total	100
MULTIPLICATION AND DIVISION TEST	
Mistakes in dealing with exponents	37
Using the wrong process	14
Mistakes in the use of signs	13
Mistakes in factoring	11
Failure to deal with parentheses correctly	6
Errors whose cause could not be discovered	11
Miscellaneous errors	8
Total	100
EQUATION AND FORMULA TEST	
Failure to change signs when transposing	28
Multiplying by the coefficient of the unknown in order to solve	18
Use of wrong process	10
Mistakes in solving literal formulae	8
Errors in dealing with parentheses	7
Mistakes in substitution in formulae	7
Finding one root only in solving a quadratic	7
Dividing by the numerator of the coefficient of the unknown in order to solve	6
Errors which could not be explained	5
Miscellaneous errors	4
Total	100
PROBLEM TEST	
Errors due to ignorance of fundamental relationships (Those in length, breadth, thickness, and volume for example)	52
Errors due to misreading the problem	41
Errors which could not be explained	7
Total	100

[1]Osburn, W. J.: *Survey of Algebra Instruction in Wisconsin High Schools.* These pupils had studied algebra for six months. The total number of pupils tested varied from 1055 to 1635. Some of the schools did not find it possible to give all four tests.

TABLE XI

CLASSIFICATION OF ERRORS[10] MOST FREQUENTLY MADE BY PUPILS OF THE FORT SMITH (ARK.) HIGH SCHOOL IN THE PROBLEM TEST, SERIES B

1. Incorrect operation indicated, usually due to failure to comprehend the problem. Caused 55 per cent of the errors.

Prob. 1: If a coat costs x dollars, how much will 3 coats cost?

Answer: $x - 3$.

2. Conditions of the problem apparently understood, but the work left in incomplete form. Caused 15 per cent of the errors.

Prob. 7. The total number of circus tickets sold was 836. The number of tickets sold to adults was 136 less than twice the number sold to children. How many were sold of each?

Answer: x = No. of children's tickets sold
$2x - 136$ = No. of adults' tickets sold.

3. Failure to comprehend the problem, perhaps due to confusion and to use of technical terms. Caused 10 per cent of the errors.

Prob. 6. The width of a basket ball court is 20 feet less than its length. What is the length and width of the court if the perimeter (distance around) is 240 feet?

Equation: $x - x - 20 = 240$.

4. Inverting the order of terms in subtraction and in division. Caused 5 per cent of the errors.

Prob. 2: A man is m years old: how old was he r years ago?

Answer: $r - m$.

Prob. 5. The distance from Chicago to New York is 980 miles. If a train runs v miles an hour, what is the time required for the run?

Answer: $\frac{v}{980}$

5. Simple arithmetical errors. Caused 5 per cent of the errors.

6. Attempt to solve problems containing two unknowns with only one equation containing the two unknowns. Caused 2 per cent of the errors.

10. VALUE OF FIRST YEAR ALGEBRA SCALES

These scales may be used by teachers for three distinct and very useful purposes. They may be used (*a*) to indicate attainment, (*b*) to measure progress, and (*c*) to diagnose difficulties.

Scales which increase in difficulty by approximately equal steps furnish a most reliable objective means for determining the actual

[10] Fort Smith Survey: Classification of Errors in Algebra, made under the direction of A. M. Jordan, of the University of Arkansas.

achievement of a student or a group of students. Any one of the scales may be used for this purpose, though the Equation and Formula Scale is perhaps to be preferred, since, as previously stated, it is a more comprehensive test. It is well to keep in mind also, in this connection, that a low median class score is not always, nor even quite generally, due to poor instruction. Any one or a combination of several causes may be operating to keep a class score down. It is the duty of the teacher, however, to study these causes and to learn which ones are affecting the efficiency of the instruction, in order that proper remedial measures may be applied. This it is possible to accomplish with a much greater degree of certainty when the teacher knows the actual standard of achievement a class has attained. Such knowledge furnishes the teacher with a fact basis upon which to proceed and a motive with which to operate.

The extent of progress made by a class can be quite scientifically measured by submitting the same scale to a class at intervals of about three months. Teachers should be cautioned very specifically, however, not to do any drill work upon the exercises or problems appearing in the scales. If it is feared that some of the practice effect may survive, it is suggested that another scale in the same series, or the same scale in a different series, be used for the second test.[11] The most desirable method of measuring progress, very naturally, would be to have another parallel series of scales similar and equal in difficulty to those of Series A, and it is to be hoped that such a series will soon be constructed.

For diagnostic purposes the scales of Series B have been found to be more serviceable. They offer a richer variety of exercises and, therefore, a greater number of type processes. Hence, a more complete analysis of the mistakes made by students, and the difficulties they encounter, is made possible.

Finally, it must be stated emphatically that these are primarily power tests and as such should never be used for purposes of drill. Furthermore, with the time limits as now fixed, they are speed tests to a limited extent only. If a pure speed test is desired, the Standard Tests[12] devised by Dr. H. O. Rugg could be used to advantage. These would be particularly useful in determining whether a class has had sufficient drill upon the fundamentals.

[11] See suggestions for using the scales in rotation, p. 9.

[12] See *School Review*, October, 1917, 25:546.

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The scales may be secured from the publishers: Bureau of Publications, Teachers College, Columbia University, New York City, or from the following distributing centers: Public School Publishing Co., Bloomington, Illinois; Bureau of Educational Measurements and Standards, State Normal School, Emporia, Kansas.