TECHNIC

OF

SURVEYING INSTRUMENTS

AND

METHODS

INCLUDING

GENERAL AND DETAILED INSTRUCTIONS FOR FIELD AND OFFICE WORK OF EXTENDED STUDENTS' SURVEYS

BY

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PREFACE

This book is not intended to replace any general text-book on surveying, but to supplement the general directions of such a book by detailed directions for specific operations in field and office.

Part I is devoted to general directions and remarks.

Part II, on Technic of Surveying Instruments, is intended to so drill the student in the technic of instruments that he shall acquire working familiarity with the parts, motions, and readings in the shortest possible time; to the end that he may quickly grasp instruction on errors and methods of surveying, and enter upon his first surveys with a mind free to grapple with What, Where, and When to measure, letting habit take care of routine details of manipulation and reading. The drill is accomplished by a series of exercises, each of which, advancing the student by a single step, is designed to give him a maximum amount of practice in the routine sequence of manipulation and readings, in a given time. Incidentally each exercise is so designed as to bring sharply to notice the sources and range of error incident to the operations performed. Thus the exercises are also experiments, from which the student should gain such an intimate knowledge of errors as to enable him to form an intelligent judgment on the quality of the results of a given sequence of instrumental operations made under given conditions. The operations of each exercise are such that several pairs of students (each pair having exclusive use of one instrument) can conveniently work in a small area. The instructor may increase or diminish the number of repetitions named in any exercise whenever such change will best serve the student's needs; and he may omit the least important exercises if it is necessary on account of lack of time.

Part III contains detailed directions for performing a series of exercises in the use of surveying methods. In Part IV are given: detailed directions for field and office work of students' railroad surveys; general directions for students' topographic-hydrographic surveys; and general instructions to the advanced student who is required to plan as well as to perform a survey for a specified purpose.

In Part V will be found a variety of office exercises, including use of pantograph and polar planimeter; making of tables and diagrams to facilitate calculations; construction of slide-rule, vernier, slope scale, and quantity scales; graphical solution of topographic and mine-survey problems; and various devices and approximate formulas for expediting computations.

Part VI contains exercises in test and use of instruments of higher surveying, and some probable-error formulas.

The aim has been to make detailed directions so complete that the student can go through the routine of the exercises without verbal instructions; so that the instructor may be free to watch carefully the performance of each student, and give advice, explanation, suggestion, caution, or illustration at such time and in such quantity as best to serve the needs of that student.

For the best results the instructor must carefully select in advance the ground upon which each exercise shall be performed; so that the students, upon receiving equipment, may immediately begin work, and carry it forward without interruption. The work of many of the problems (*e.g.*, chaining, leveling, angle measurements, etc.) may be so systematized by means of permanent hubs on the surveying ground that the instructor can instantly judge of the character of the work done by mere inspection of the notes submitted.

This book combines the greater part of Webb's "Engineering Instruments," Fish's "Technic of Surveying Instruments" (all rewritten), and about a hundred pages of additional matter that has been used repeatedly in mimeographed form.

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TECHNIC OF SURVEYING INSTRUMENTS

PART I

GENERAL DIRECTIONS

"OTHERWISE DIRECTED"

THE student should at once become familiar with the fact that there are usually several excellent ways of performing and recording any of the operations of surveying. Practice is not uniform; it may vary from one locality to another, from one surveyor to another, and from survey to survey according to purpose or circumstance. At every turn in his work the surveyor selects that method of operation which he thinks best for the time, the place, and all the conditions under which the survey is to be prosecuted.

The methods prescribed in this book are in every-day use; but the instructor no doubt will find it necessary or expedient to change a stated requirement, here and there, in order more completely to harmonize the directions with local conditions. Of course the student is bound by the instructions of this book only to the extent that the instructor designates them as his own; hence, it is to be understood that the expression "unless otherwise directed," though inserted only occasionally and as a reminder, is applicable to every direction herein given.

INSTRUMENTAL MEASUREMENTS

One of the chief objects of this book is to give to the student an adequate idea of the degree of accuracy which is necessary and desirable in instrumental measurements, and also the degree of effort which is required to attain such accuracy. The story of the young man who paced off the diameter of a circle and then calculated its circumference to six places of decimals is an illustration of the kind of work which is constantly being done. The length of a line is measured and recorded to hundredths (or even thousandths) of a foot, when the method used involves a probable error which is greater than a hundredth of a foot on each tape- or chain-length. Levels are read with a target to thousandths of a foot with an instrument so handled or so out of adjustment that there is probably an error of one or two hundredths on each sight. The real trouble is generally caused by *unbalanced* accuracy. Some of the work is done with painstaking care and in the very next step some inaccuracy will be permitted which utterly wipes out the effect of the previous precision. By taking a large number of *independent* observations which should nominally give the same result (or which may be mathematically reduced to the same nominal result) and noting the discrepancies, an idea may be obtained of the accuracy of the individual results. But some ingenuity is required to insure that the observations are really independent. For example, if a line is measured several times, marking the positions of the ends of the successive tape-lengths on the tops of stakes, all of the intermediate stakes should be pulled between each set of measurements, or else the marks should be made on strips of paper or sheet metal, which should be changed before each new set of measurements, so that the observer is uninfluenced by any previous work. When a horizontal angle is measured with a transit, independent readings should be taken on different parts of the limb so that the readings as taken are actually different, the value of the angle being determined by differences. A similar method may be used for testing level readings, so that the various rod readings shall be different and yet may be reduced to the same nominal reading. One-thousandth of a foot in 300 feet subtends only 0.7 second. An ordinary wye-level bubble is graduated with about 20″ to 30″ of arc for each division. 0.001 foot at 300 feet therefore corresponds to about 1/36 of a division of a level bubble which moves 25″ of arc for each division. An inaccuracy of *half* a bubble division therefore corresponds to nearly 0.02 foot on the rod at 300 feet. Even the cross-hairs of the level may be so coarse that they will cover 0.01 foot at a distance of 300 feet; with a focal length of 12 inches, a cross-hair diameter of 0.0004 inch will do this.

It must not be understood that such elaborate methods of obtaining a large number of independent results are advocated for commercial work. But such drill-work gives the student an opportunity (which he seldom has later) of testing the character of his work, and teaches him what degree of effort is necessary and desirable to reach a certain degree of accuracy, so that on the one hand he need not waste his time uselessly when only approximate results are needed, and on the other hand may use all necessary precautions to insure precise work when it is needed.

ORGANIZATION AND EQUIPMENT

Unless otherwise directed, the class will be divided into parties of two men each. Each exercise of Parts

II and III will be performed twice. The one who acts as helper during the first performance will act as instrument man during the second.

Each student is held responsible for equipment issued to him by the school. In drawing and returning equipment, the student will follow prescribed procedure. He will leave no instrument unguarded in any hall, public room, street, or other thoroughfare.

STUDY

Study the text-book on every principle involved in each exercise soon after performing it, while all the details are fresh in mind. Better yet, do *some* of this studying *before* and thereby perform the exercise less haltingly. (For convenience of student who uses as a text Breed & Hosmer, Johnson & Smith, Raymond, or Tracy, a table of references has been inserted at the end of Part I, giving for each Exercise of this book the numbers of the pertinent articles of the foregoing texts.)

Many statements in the text-book will not be appreciated by the beginner on the first reading, because of his lack of experience. Furthermore, the beginner cannot remember all the statements which he can appreciate on first reading. It is therefore necessary for the beginner to *read over and over again*, in connection with the exercises, the pertinent portion of the text. The principles and rules of practice, given in the text-book, will be gradually fixed in mind, without special effort of memory, by applying them intelligently in field and drafting room.

Similarly, all of the present general instructions may not be thoroughly understood or fully appreciated at first reading. But the student will understand, at any time, such of the directions as apply to work then in hand. It is incumbent on him to run over all of Part I as often as may be necessary to make sure that he apply each direction at the times and at the places at which it is applicable.

Since this book is mainly concerned with ways of doing things, the student should, on encountering an unfamiliar term herein, refer immediately to his text-book for any needed definition, description, explanation or principle. He can find the matter in the text-book by means of its index, or in some cases by means of its table of contents.

Economy of time and effort requires that the student become familiar, by the actual use of the instruments, with their manipulation and reading, as speedily as possible, for such familiarity will enable him to read of principles, practices, and sources of error with quick understanding.

The student should scrutinize the computed errors in each exercise performed by him, and note particularly their range and variations. He should call to mind the entire procedure of the exercise together with all attendant circumstances, and try to form a judgment as to whether the errors do or do not appear to be within limits which might be reasonably expected under the circumstances. The beginner will not always be able to decide this question, but the conscientious effort to do so will lead him to give proper study to his work, trying to view it from all angles in order correctly to connect effect with cause. This serious attempt to appraise the degree of reasonableness of errors will profit the student vastly more than would judgments of a master surveyor; for, by acquiring the scrutinizing habit, the student will rapidly become a master himself. The student should therefore form his own judgment as to the reasonableness of the results obtained from an exercise, before asking for the judgment of his partner or instructor. The instructions in this paragraph will require exercise of will, but carrying them out conscientiously doubles the benefit derived from all other parts of the work.

FIELD NOTES

1. Use no pencil softer than 4H in making field notes. Keep the pencil sharp. Bear down hard enough to make figures show slightly on back of page.

2. Record in field book every reading of tape, rod, and instrument. Record every distance and angle, whether "measured" or "laid off."

3. Aim to confine page heading to space above vertical rulings. If heading must encroach on vertical rulings, begin the record of readings below that horizontal line which lies next below lowest part of heading.

4. The main words of general heading should be entered at top of left-hand page in upright capitals. Other parts of heading are entered in lower-case upright letters. The date and weather are placed on right-hand side of left page. In the heading space of right-hand page enter: (1) Inst., No., and make; (2) Kind of leveling rod (if any); (3) Name and position of each man in party; (4) Time spent by party on the exercise, *e.g.*, "Time $2\frac{3}{4}$ hr." Follow forms given in this book, except when otherwise directed by instructor.

5. Number right-hand pages of field book in upper right-hand corner. The left-hand page is not counted as a separate page. Each number refers to a double page.

6. Begin field notes on page 2, leaving page 1 for index (see 9 below).

7. Enter remarks on right-hand page, leaving sufficient room on left side of that page for entering all computed quantities which overrun left-hand page.

8. Having recorded one set of observations, record a succeeding set lower down on page (leaving at least two blank lines between sets), or on next double page.

9. The index (arranged in order of dates) will begin on left half of double-page 1 of field book. The heading of index page will be as follows:

INDEX

Date.	No. of exercise.	Name of survey or exercise.	Page no.

Index each piece of field work as soon as it is completed.

10. Make clear notes and well-formed letters and figures.

11. No notes need be copied from another's note book, except such figures as target readings of level rod, etc., taken by the helper, which will be necessary to complete record of the exercise.

12. Have text-book on hand for all drafting periods and, if the book is of pocket size, for all field work.

13. Before going into field, fill out page and column headings for exercise to be next undertaken.

14. In recording quantities, give each decimal point, degree mark, foot mark, etc., the same horizontal space as is given each digit. Do not record seconds in any case; but record anything less than one minute as a decimal of a minute. Thus, write 27° 35′.2 rather than 27° 35′ 12″. There is no advantage in recording

anything smaller than 0.1 minute in work with the ordinary transit. When recording a distance in feet and decimals it is well to place the foot mark above the decimal point thus, 20′.35. Write 28°03′ rather than 28°3′.

READINGS OF INSTRUMENTS

1. Each exercise is to be performed twice, the men changing places for the repetition.

2. At all times in the exercises, read the target to thousandths unless otherwise directed. Read without target to thousandths by counting tens and hundredths and estimating the thousandths. For short distances, the reading without target may be made with an uncertainty of one to three thousandths; as the distance increases the uncertainty increases, and at some distance (upwards of 250 ft. depending on the telescope) the wire covers so much of one-hundredth space as to make the uncertainty 0.005 ft.

3. All readings of the needle and of the transit verniers should be made with a magnifying glass.

4. When reading vernier of target or transit the observer should so place his eye as to eliminate parallax.

COMPUTING AND DRAFTING

1. The recorder may make computations in field if he can do so without unduly delaying the work. Field computing is always desirable when the computed quantities serve to check recorded readings at a station before the instrument is removed. Otherwise the computations for an exercise are to be made later in drafting room.

2. The true value of a distance or angle (other than a full circle) can never be known. Hence, true size of an error can never be known. In many exercises herein, the average of several values is *assumed* to be

the true value, and the error of each of the several values is calculated on this assumption.

3. All computing needful for each page of field notes will be carried out by the owner of the book, and at the bottom of the page he will write: "Computed by (*initials*) (*date*)," and within two weeks thereafter, the owner's partner will check these computations and enter below the owner's certificate, his certificate, thus: "Checked by (*initials*), (*date*)."

4. On each piece of office work, follow those directions and suggestions, given in the text-book, which apply to work in hand, unless they conflict with specific directions of instructor or of present book.

5. All computations in drafting room will be made in an orderly way on prescribed paper; and computations will be submitted as a whole at end of semester, along with drawings.

6. All drawings will be made on prescribed drafting paper. Use 6H pencil, sharpened to needle point, for all points and lines which have an influence on the result to be found from plotting. Make each point and each line as fine as practicable for the eyes, and at once surround each marked point with a small circle or half-circle to serve as finder. Draw no line into or through such circles. Do all lettering and dimensioning with 4H pencil in such manner as may be prescribed.

7. The student will write or letter his name and the date at the upper right-hand corner of each fresh sheet of computing paper or drawing paper, before putting any other marks on the sheet.

DEFINITIONS

1. To "set up" an instrument usually means to set the instrument up and make it ready in all ways for pointing and reading. Thus, to "set up" a level is to

set the level up, focus the eye-piece (if it is not already focused) and level the telescope; and to "set up" the transit is to set the transit up (over a mark usually), focus the eye-piece (if it is not already focused), and level the plate. If the transit is to be used for leveling, the "set-up" includes also leveling the telescope.

2. The "direct" (or "normal") position of the transit telescope is that in which the telescope bubble is underneath the telescope and the vertical circle is on the left. The "reversed" position of the transit telescope is "bubble up," or "circle right."

3. To "reverse," or "invert" the transit telescope is to turn it from direct to reversed position. To reverse the transit is to reverse the telescope and also turn it 180° about the vertical axis. To reverse the telescope of a level is to turn it end for end about its vertical axis.

4. To "return" the telescope is to turn it back from reversed to direct position.

5. To "plunge" the transit telescope is either to reverse it or to return it.

6. The "ocular" is the eye-piece. The "objective" is the object glass.

7. "Station" (point on a traverse or center line): stations are regularly 100 ft. apart; and are numbered consecutively 0, 1, 2, 3, . . . , beginning with the initial point of the traverse or center line. The word "station" is also applied to an intermediate point on traverse or center-line (see under "plus station" below).

"Plus station" }
"Plus point" } a point between two stations on a
"Substation" }
traverse or center-line; *e.g.*, the point 28.3 ft. beyond sta. 7 is a plus station, and is designated by the ex-

pression "7 + 28.3" which is read "seven-plus-twenty-eight-three." The 28.3 is referred to as "the plus." Often a plus station is called merely "station" when no ambiguity will result therefrom; *e.g.*, we say "What is the elevation of sta. 86 + 20?"

"Station" (a designating number): the number assigned to a station or plus station; *e.g.*, "the station of that point is 36, and the station of this point is 18 + 07.6 (read eighteen-plus-0-seven-six).

"Station" (unit of distance): 100 ft.; *e.g.*, "the curve is 6 stations long."

ABBREVIATIONS AND SYMBOLS

The following letters are used throughout the book with the meanings here given.

BM — bench mark (see text-book for description).
BS — backsight (verb or noun).
CC — point of change, on a compound or reversed curve, from one circular curve to another.
CT — point of change from circular curve to tangent.
CL — center line.
D — degree of circular curve.
FS — foresight (verb or noun).
H — horizontal.
HI — height of instrument.
I — intersection (*i.e.*, deflection) angle at PI.
L — lower.
LM — lower motion: the motion controlled by the clamp and tangent screw beneath the lower plate of the transit.
M — middle.
$\parallel$ — parallel.
$\perp$ — perpendicular.

PI — point of intersection of two consecutive tangents (produced).
T — tangent distance (= distance from TC to PI).
TC — point of change from tangent to circular curve.
TP — turning point (see text-book for description).
U — upper.
UM — upper motion: the motion controlled by the clamp and tangent screw at edge of the upper plate of the transit.
V — vertical.
VC — vertical circle of the transit.
VM — vertical motion: the motion controlled by the clamp and tangent screw of the vertical circle or vertical arc of the transit.

Table of References to Four Standard Surveying Texts

Exercise No. (This book.)	Breed and Hosmer. (Articles.)	Johnson and Smith. (Articles.)	Raymond. (Articles.)	Tracy. (Articles.)
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PART II

TECHNIC OF SURVEYING INSTRUMENTS

Exercises in manipulating and reading engineering instruments, and in measuring distances and angles; so designed as to bring the sources of errors prominently to the student's attention.

CHAIN AND TAPE

Link chains. — These have been almost entirely replaced by steel tapes, but since many are still in use, the student must be able to handle them properly. The chain should have been folded up by picking it up at the middle point (generally indicated by a *round* brass tag) and folding up four links at a time, placing

the links crosswise to the others so that they will have an "hour-glass" shape, as is shown in the illustration. A strap or heavy cord fastened around the center will then hold them together. To use the chain, select a clear stretch of ground of somewhat more than half a chain length. Then with the bulk of the chain in the right hand and holding the handles with the left hand, throw the chain out. If the chain has been properly folded up, a little practice will suffice to stretch the chain (doubled) along the ground. Picking up one handle, the chain can then be straightened out with little or no trouble from kinking. To guard against one source of inaccuracy as well as a possible injury to the chain, it should be examined link by link as it lies on the ground to make sure that there is no kink at any joint.

Never jerk a chain. It can be moved as desired without much effort unless it is caught. A jerk when it is caught frequently means a break. Even if you can see with certainty that it is not caught, a jerk is useless. A perfectly steady pull, with the rear end held rigidly at the pin or stake just set, is an absolute essential to good work. The two chainmen must learn by experience how much tension should be used; then when the rear chainman pulls back with a greater tension than the normal the front chainman should yield, for this should mean that the chain must be adjusted backward to bring the rear end to the proper point; if the tension is less than normal, this should mean that the front chainman should pull it forward until the sudden increase in the tension shows that the rear end has reached the proper point. With a little practice two chainmen can quickly become accustomed to each other so that the two essentials of proper position and proper tension are quickly obtained.

Do not stand facing in the direction of the chain and leaning backward, depending on the chain for support. Stand sidewise so that the line is not obstructed; stand squarely on both feet so that a perfect balance is obtained regardless of the tension on the chain; by leaning the right elbow on the right leg just above the knee, any proper tension can be easily given to the tape, while the alignment may be readily seen. One of the chief advantages of the attitude is its ease and steadiness. If the end of the tape must be held up high so as to be on a level with the other end, of course a plumb-bob must be used. Wind the plumb-bob string around the tape precisely on the zero mark — or over the end of the ring if that is the zero mark. Then the eyes may be steadily fixed on the plumb-bob, either to note its position or to adjust it to the position of the previously established point.

Steel tapes. — Much of the above instruction applies equally to the use of steel tapes. A tape is much more liable to be broken than a link chain, but the soft-steel tapes recently manufactured are capable of very rough usage. During wet weather or when a tape has become wet and muddy, it should be carefully wiped with an oily cloth before it is rolled up on the reel. Neglect to do this will quickly cause it to become rusty.

The student should learn to exercise special care that the tape is always held as truly horizontal as possible. Great care is often used to line in the tape with a transit, even using tacks on the tops of stakes to make sure that no error will arise from that cause, but at the same time the two ends of the tape have a difference of level of perhaps a foot or more. Precision in this case is simply wasted. A difference of level of one foot, which would frequently be ignored, will cause an error *one hundred times* as great as an error of one-tenth of a foot in horizontal alignment, an error which usually would be carefully avoided by transit alignment. The difficulty in holding the two ends at the same level is largely due to one's inability to correctly estimate the rate of slope, an optical illusion causing a truly level line on a hillside to appear sloping. Another inaccuracy is due to the desire to use full chain lengths of 100 feet rather than "break chain." A 6% or 7% grade does not appear very steep and so full 100-foot lengths will be attempted, but the impractibility of holding the down-hill end 7 feet above the ground, and the difficulty of holding it even 6 feet above the ground, generally causes an error of a foot or more in level, especially as the chainmen will honestly think that the tape is really level. The amount of error due to a difference of level of

Height of triangle.	Base.	Hypothenuse.	Error.
Feet.	Feet.	Feet.	Foot.
10	100	100.499	0.499
5	100	100.12499	0.12499
1	100	100.00500	0.00500
0.5	100	100.00125	0.00125
0.1	100	100.00005	0.00005

the two ends is shown in the tabular form. With the extreme difference of 10 feet the error is about 6 inches. The error varies about as the square of the difference of elevation. With one foot difference the error is 1/16 inch. Even with 0.1 foot difference of elevation (or of alignment) the error is less than six ten-thousandths of an inch. This shows the utter futility of great refinements in horizontal alignment under the ordinary methods of measurement.

To coil the tape. — (1) If the tape is wet, wipe it dry before beginning to coil. (2) Draw tape out full length. (3) Hold 100-foot end, flat up, in left hand, palm up; and stand so that tape passes across front of body. In the following steps make sure that tape does not turn over or on edge in right hand at any time, and that a firm grip is maintained on the coil with the left hand held palm up. (4) Grasp tape with right hand (palm down, thumb on top of tape) near left hand. (5) Slide right hand to 95-foot mark, without permitting tape to turn over or on edge as it slips through hand. (6) Grip tape at 95-foot mark, and swing right hand out around and to left of left hand, closing the swing by laying the 95-foot mark on top of the 100-foot mark. (7) Grip the two marks securely together in the left hand. Thus one loop is made, with a twist in it. (8) Next, in like manner, make a second loop of the next five feet of tape; and so continue until entire tape is looped and firmly held in left hand. (9) Tie coil firmly at zero mark with rawhide handle or twine. The coil has now the shape of figure 8. By proper manipulation the tape can now be sprung into a double coil of circular form, and so made fast by tying with the free rawhide handle or another twine.

To uncoil the tape. — Hold the tape firmly while untying, to prevent its springing into a tangle. Change the coil from circular to figure 8 form; then hold coil, at tape end, in left hand, as when coiling up the tape; and pay out tape with right hand, loop by loop, in the reverse order of coiling; at the same time walking along to keep uncoiled portion of tape from tangling.

1. LINK CHAIN

Equipment: Link-chain, set of marking-pins, two transit-poles, two plumb-bobs.

Location: When possible, these measurements should be taken between points (or permanent hubs) which are about 1000 feet apart and where the intervening ground has a steep and variable slope.

Method: Measure the distance *three* times, lining in by eye, taking precautions to have the two ends of the chain always at the same level, "breaking chain" if necessary, etc.

Always drag the chain (zero end forward) ahead its full length, even if it is necessary for the head chainman to walk back to "break chain." Always place a pin at each even hundred feet, and begin again at that

FORM OF NOTES

[LEFT-HAND PAGE OF NOTE-BOOK.]

Problem 1.

Line.	Meas.	Slope.			
Δ 15–Δ22	1087.4	down-hill			
Δ 22–Δ15	1086.9	up-hill			
Δ 15–Δ 22	1087.6	down-hill			

point for a new chain-length. Care should be taken that no greater length of chain is used than can be held truly level without holding the down-hill end inconveniently high. The student should learn that an optical illusion will cause a truly level line to appear as if it sloped down (opposite to the slope of the hill), and that when a chain is stretched down a hillside and held up so that it appears to an inexperienced eye to be level it is probably sloping down the hill. The pins may be aligned by eye, aided by the plumb-bobs.

Each student makes a *set* of three measurements acting as head chainman. The extreme range of the three values should not be more than $\frac{1}{1000}$ of the distance.

2. TAPE SUPPORTED AND UNSUPPORTED

Equipment: 4H or 6H pencil (use no pencil softer than 4H in the field book at any time); metallic tape (50 foot, of cloth containing longitudinal copper wires); steel tape (100 foot, with rawhides or handles); 2 bobs; 11 pins; 2 line rods.

The object of this exercise is to familiarize the student with two common ways of measuring distance with tape and to show him the range of error to be expected from measurements made under similar conditions.

(1) Set a line rod at each end of a comparatively level course, about 300 ft. long by pacing, or behind two permanent hubs on the surveying ground. (2) Set a pin on line, about 2 ft. inside of each rod, unless permanent hubs are used. Call the pins or the permanent hubs *A* and *B*. (3) With metallic tape measure distance between pins. (Zero point of metallic tape is usually at outer end of ring.) (4) Throw out steel tape. (5) With steel tape, measure distance *AB* three times; the tape being stretched straight and lying on ground at each application. Head tapeman takes zero end of tape. Find zero mark of steel tape by laying the 9–10 foot alongside the 0–1 foot. Read fractional foot at final pin *B*, to hundredths, counting tenths and estimating hundredths in following manner: (*a*) Head tapeman holds zero mark at *B*. Rear tapeman mentally notes distance in feet and tenths (counting feet and estimating tenths). (*b*) Rear tapeman, directing head tapeman to pull, lets tape slip through his hands till next foot-mark comes to his pin; and calls out new tape-reading (whole feet). (*c*) Head tapeman mentally notes fractional foot on rear side of *B* (counting tenths and estimating hundredths);

adds this fractional foot to rear tapeman's called distance less one foot; and calls out result. (NOTE. — If tape has an extra graduated foot beyond zero mark, the subtraction of one foot is not necessary.) (*d*) Rear tapeman compares this called result with distance which he mentally noted under (*a*) above, and if there is substantial agreement, calls out "check"; and then records distance in feet and hundredths. Obliterate each pin mark as soon as pin is pulled. (6) Men change places at end of three measurements. (7) Repeat the three measurements, holding tape shoulder high, using bobs and leveling by eye. (8) At close of this exercise do up tape in figure-8 form. (9) Computations: Find mean of each set of three measurements. Subtract mean from each measurement, and record difference with proper sign, in "Errors" column.

FORM OF NOTES

No. of obs.	Measured dist.	Errors.	Tape (supported or suspended).
1	306.13		Supported
2	306.10		Supported

3. TESTING CHAIN OR TAPE

Equipment: Chain (or tape) to be tested, standard tape, spring balance, two brass plumb-bobs, triangular scale, and a small strip of cardboard (*e.g.*, visiting-card).

Location: A passageway or hall is often the only available place, and serves very well, except for interference to and by others. A smooth level sidewalk will answer the purpose in fair weather. The vacant space under the seats of a large grand stand may sometimes be utilized as a permanent place for these as well as other long-tape tests.

1. Set one of the end marks at some well-defined mark on the floor (*e.g.*, a scratch in the top of a nail-head) and tack the strip of cardboard under the other end mark of the tape. Pull the tape to a tension of 16 lbs., and mark on the cardboard with a *sharp hard* pencil the position of the 100-foot mark. Make a similar measurement with the chain or tape to be tested, and measure the difference if any to hundredths of an inch, which is then reduced to thousandths of a foot. Make three independent trials.

2. Observe by three trials for each tension the changes in the length of the chain (or tape) by making the tension 8 lbs. and then 20 lbs. Compute the average change for one pound variation in pull.

3. Hold the chain (or tape) to be tested clear of the floor and note what tension is necessary to bring the end marks at the same distance apart as when supported throughout its length with a tension of 16 lbs. Use brass plumb-bobs in this test to bring the end marks over the points on the floor. Make three trials.

FORM OF NOTES

Test.	Standard Tape.	Tested Tape.	Diff.	Mean Error.		
No. 1	100.000	100.017 100.014 100.015	+0.017 +0.014 +0.015	+0.015		Tension 16 lbs.
No. 2		0″.16 0″.15 0″.16		0″.157	0″.013 per lb.	Stretch for change of tension from 8 lbs. to 20 lbs.
No. 3		17 lbs. 18 lbs. 16 lbs.			17 lbs.	Required tension, *i.e.*, " normal tension."

SETTING UP

Setting up a tripod. — Although it is possible to level up an instrument, and especially a compass which has a ball-and-socket joint, when the plate of the tripod head is considerably out of level, yet it is generally possible, even on very rough and steep ground, to so plant the three legs that the tripod plate will be practically level and very little adjustment will be necessary. The ability to quickly select positions for the tripod legs which will make the tripod plate practically level is largely a matter of experience, but there are certain principles which the student will do well to keep in mind and which will enable him to more readily acquire the skill to set up a tripod on uneven ground. The following discussion presupposes the use of the ordinary plain tripod with fixed legs. Extension-leg tripods look very attractive, but their positive disadvantages should not be forgotten. (1) There is always the danger that the clamps may not have been clamped so tightly but that they may slip. (2) Time may be wasted in adjusting the clamps when a little skill and ingenuity would set a plain tripod in place in far less time and with no danger of slipping. (3) Except in mines and tunnels, the occasions are rare when a plain tripod cannot be set up so that the tripod plate is practically horizontal. On a steep side hill, where the slope is approximately uniform, the best general method is to place two legs on the same contour, which is a little below the stake and equidistant from the stake. The third leg will be on the up-hill side on a line running almost directly up from the stake. If the side hill were actually plane, there is but one position for the third leg. Slight inequalities of the surface can then be easily allowed for by mere adjustment. When the side slope is very steep, it may even be necessary to point the third leg with an *upward* slope rather than a downward slope. If the two lower legs have been firmly planted, this position will be found to be sufficiently rigid. The student should learn by trial and experience that, when the tripod plate is very far from being level, it is often true that moving one leg-point in an arc of a circle about the stake, and without disturbing the centering of the instrument over the stake, will suffice to make the plate level. The student should test this by noting that when a tripod has been properly set, with the head level, the shifting of the point of one leg will throw the head far out of level and that it may easily be made level by the proper adjustment of that leg.

The above remarks apply to the setting up of all tripods. In the case of the level tripod it is all that is necessary, except when the elevation of the telescope is of importance. But with the compass tripod, and still more so with the transit tripod, the center of the tripod head must be vertically over the stake or hub. The student must train himself against the optical illusion on a side hill, which makes him think that the tripod head is directly over the stake when it is really a little too far down the hill.

When the tripod legs are provided with wing nuts where the legs are fastened to the head, as the best tripods are now made, the nuts should be tightened whenever the instrument is set up, and loosened again just before it is picked up for removal. The legs cannot be turned on the head unless the screw is somewhat loose and then the tripod is far less steady than when the wing nuts have been screwed up tightly. This is the feature that makes tripods having wing nuts better than the old-fashioned kind.

Leveling up. — Many a good instrument has had its leveling mechanism nearly spoiled by bad practice in leveling up. The large majority of transits and levels are provided with four leveling-screws, which oppose each other in pairs. It is quite possible for one who has strong fingers to turn a leveling-screw so hard that the leveling-plate becomes actually bent. When the screws of one pair have been turned against each other so that they hold the plate very tightly in the ball-and-socket joint, it becomes difficult to move the plate in the other direction by means of the other pair of leveling-screws. If at any time it becomes necessary to change the leveling of the instrument greatly, it is always better to loosen all four screws until the instrument turns readily in its ball-and-socket joint; then, by easy motion of the screws of each pair, the instrument can be quickly and easily leveled until the leveling is nearly accurate. The final step will be to screw the leveling-screws firmly against the tripod plate so that the instrument is properly leveled and yet there is no undue stress in the leveling-plate.

Focusing the telescope. — This not uncommonly gives much trouble to the beginner. All the trouble comes from the involuntary change that takes place in the lens of his eye as he transfers his thought from a near to a far object. He thinks of the cross-wires as very near; hence when focusing the eye-piece the lens of his eye is so shaped as to form a distinct image of the cross-wires on the retina. But when he begins to focus the object glass on the rod (and focusing object glass on rod is intended merely to shift rod image to the plane of cross-wires so that it can be distinctly seen along with them) he lets his mind turn to the distant rod (instead of keeping his attention firmly fixed on the rod *image* near the cross-wires), and the lens of his eye responds to his thoughts and reshapes itself to form on the retina a distinct image of the distant rod. When the eye lens is shaped for distinct distant vision, near objects (such as the cross-wires) become indistinct or invisible. The conclusion to be drawn from the foregoing is this: Do not transfer the thoughts from the near cross-wires to the distant rod; but keep the attention fixed on the cross-wires and the image of the rod, all the time thinking of both of them as being together and about 10 inches from the eye.

The effect of this involuntary change of the eye lens, as the attention is transferred from a near object to a far, is well shown by the following experiment. Look through a wire window screen in the direction of a distant object. If the attention is fixed entirely on the distant object, that will be distinctly seen while the wires of the screen will be invisible. If, next, the attention be fixed wholly on the wires of the screen, they will be distinctly seen, but the distant object will become invisible. Next, slowly transfer attention from screen to distant object and note that gradually the screen will become less and less distinct and the distant object will come into view and gradually grow more and more distinct; and that at some instant during this change, wires and distant objects will appear equally dim. At this instant the eye lens is shaped for distinctly seeing an object lying midway between screen and distant object.

4. SETTING UP AN INSTRUMENT

Equipment: Dumpy or wye-level, or transit.

The object of this exercise is to drill the beginner in setting up transit or level.

(1) One man will act as instrumentman while the other holds the watch and records the time (to seconds). (2) Set up the instrument. (CAUTION: Do not strain the leveling screws. If one pair works hard, loosen one screw of other pair. Do not let your hands, legs, or feet come in contact with tripod after setting it up. See that the set screws at top of tripod legs are snugly set but not too tight.) Read watch at beginning and at end of the operation. (3) Pick up instrument and place it on shoulder as for carrying. (4) Repeat (2) and (3) four or more times.

FORM OF NOTES

No. of set-up.	Time.		Time interval.		
	Beginning.	End.			

5. FOCUSING THE EYE-PIECE AND OBJECT GLASS

Equipment: Dumpy or wye-level, or transit; plain or extension leveling rod.

The object of this exercise is to give the student command of the telescope, and to bring his attention to the fact that parallax may be a source of considerable error in rod reading and in aligning.

(1) Set up the instrument (without leveling, if it be a level) so that, by simply turning the telescope about its vertical axis, the telescope can be pointed at sky near horizon. (2) Hold rod at distance of 100 ft.

(paced) from instrument, on a point suitable for a turning point (*e.g.*, on top of stake or other firm object having small top area). (3) Point telescope toward sky and focus eye-piece to make cross-wires as distinct as possible. It is well to keep both eyes open when looking into telescope, and hold a finger against barrel of telescope at "distance of distinct vision" (which is found by trial, and for most persons is about 10 in.); and while focusing the eye-piece, direct attention alternately on finger and cross-wire. The purpose is to keep eye lens in focus for "distinct vision." (4) Sight at rod, bringing rod into focus by turning focusing screw of object glass. While focusing on rod, keep the gaze riveted on cross-wires, and at same time think of them and the rod image as at about 10 in. from the eye. Make the V (vertical) wire cover the rod (*i.e.*, show against rod as background). (5) Move the eye vertically over opening in eye-piece, to extreme lower limit, and read rod to thousandths (the third decimal place by estimation. Do not use target in this exercise). (6) Move the eye to upper limit of opening, and while eye is in this position, read rod as before. (If the two readings are the same, there is no parallax; but if the two readings are different there is parallax.) (7) If there is parallax, move eye repeatedly up and down over opening, and at same time, gently turn focusing screw of object glass in one direction or other until there is no apparent motion of wire. (If screw is turned in wrong direction, apparent movement of wire increases. When apparent movement of wire is cut out, parallax is eliminated: the image has been brought precisely into the plane of the cross-wires.) In case the rod image or wire grows dim before parallax is entirely eliminated, the eye-piece should be refocused by looking toward sky, and all the foregoing repeated. Read the paragraph on "focusing the telescope," preceding Exercise 4. (8) Throw the eye-piece and object glass out of focus. (9) Repeat steps (3)–(8) four times.

FORM OF NOTES

No. of obs.	Reading.		Apparent movement.		
	Lower.	Upper.			

LEVEL

Reading level-bubble. — The refraction of the light through the glass of the level-bubble is apt to give deceptive results unless some care is taken to avoid parallax. The surest way to avoid parallax is to have the eye directly over the bubble, so that the ray of light from the bubble to the eye passes through the glass perpendicularly. But it is frequently impracticable to set up the level so that the eye can be placed over the bubble and the bubble must be viewed from the side. If care is taken that both ends of the bubble are read with the eye in a similar position regarding the bubble, the refraction of the light coming through the glass will be the same in both cases and its effect will be neutralized; but if the eye is so placed that it looks at one end of the bubble obliquely and at the other end perpendicularly, the resulting error may be considerable. In Part I, under "Instrumental Measurements," comment was made on the effect on the final result of certain small errors in the adjustment of the instrument or of inaccuracies in its use. The student should learn by actual test, as described in Exercise 9, what is the sensitiveness of his instrument. By this means he will know how closely the bubble should be adjusted to the precise center in order to obtain a given accuracy in the final result. He should also learn to constantly watch the bubble so that the effect of a microscopic settlement of the instrument or of its distortion by heating from the sun's rays shall not alter the line of collimation to an appreciable extent.

Cross-wires. — As a part of the general problem of studying the characteristics of his instrument the student should obtain a practical idea of the coarseness of the cross-wires, which practically means, in this case, how much space is covered on the rod by the cross-wire when the rod is at a distance of two or three hundred feet. When the cross-wire is very coarse, as is sometimes found with a cheaper grade of instrument, the space covered on the rod is very considerable and it makes the work correspondingly uncertain.

Use of target-rod or "self-reading" rod. — Many a student and many even of those who call themselves engineers waste considerable time in using a target when a self-reading rod would be sufficiently accurate for the purpose and also much more expeditious. The student should remember that much of the best geodetic leveling is done with self-reading rods, and it is even open to argument whether a self-reading rod is not the best to use under all circumstances. Of course, a target is almost an essential with a "New York" rod or "Boston" rod, since the marks on the plain rod are not visible for any considerable distance but with a "Philadelphia" rod or any other form of painted rod the graduations are plainly seen for as great a distance as it is practicable to do accurate work, and but little skill is required to estimate the readings to the nearest hundredth of a foot. This has as great a degree of accuracy as is of any practical value for the great bulk of ordinary leveling work. In Part I, under "Instrumental Measurements" there are given some figures on the accuracy necessary in order that target readings to the thousandth of a foot may have any practical value. The student should therefore rely on using a self-reading rod for the great bulk of all leveling work. When it becomes necessary to do the leveling with such accuracy that target readings to a thousandth of a foot are used, the student should take corresponding care that his instrument is in perfect adjustment, that the bubble is always precisely in the center when the reading is taken, that allowance is made, if necessary, for the thickness of the cross-hairs, and that every step of the work is taken with such careful accuracy that the readings to thousandths are justifiable. Unless such care is taken, it merely gives a false appearance of accuracy to the work to use a target and to read to thousandths of a foot.

6. TARGET READING

Equipment: Dumpy or wye-level; target rod.

The object of this exercise is to give the student practice in reading the target rod, and to show him by his own observations the range of error to be expected under the conditions of the work.

(1) Student will note whether target bears a vernier or merely a scale. He will set target at random and read; and repeat this until he is familiar with the operation. (2) Hold the rod on firm support (*i.e.*,

on a point suitable for a turning point) 100 ft. away from the instrument station. (3) Set up the level with two opposite leveling screws in line with rod. Focus eye-piece. (After instrument is placed on ground with lower plate set level by eye, loosen all four leveling screws, and rotate upper plate till two opposite leveling screws are brought into line with rod; then complete set-up by leveling the telescope.) (4) Sight at rod. Test for and cut out parallax, if there be any, as in preceding exercise. Set V wire on rod. (5) Unclamp target and move it up or down until its zero line is covered precisely by H wire (by middle H wire if there are three H wires). Clasp target in its position. Now look at bubble. If bubble stands at center, rodman reads target (to thousandths). If bubble is eccentric, bring it to center, and reset target. Look at bubble again, and when it is certain that bubble was in center at time of making the last target setting, read target. (Rodman reads target and keeps notes.) (6) Shift target. (7) Repeat four times this operation of setting, reading, and shifting target. (8) Repeat all the foregoing (except setting up) with rod at 200 ft. and at 300 ft. from instrument.

Remark: Whenever one levelman gives way to another in these exercises, it may be necessary for the other to refocus the eye-piece to obtain distinct vision of wires, because of difference in quality of vision in the two men.

(9) Computations: Add together the five readings of each set, and divide sum by 5, obtaining mean, or average reading. Subtract the mean from each of the 5 readings, and set remainder, with proper sign, in error column.

FORM OF NOTES

Dist.	No. of reading.	Target reading.	Error.		

7. DIRECT ROD READING

Equipment: Dumpy or wye-level; extension rod.

The object of this exercise is to give practice in reading the rod direct, *i.e.*, without using target, and in reading the rod extended. After performing this exercise, read the paragraph "Use of target, rod, etc.," preceding Exercise 6.

(1) Set up the level. Focus eye-piece. Hold rod 100 ft. away on firm point suitable for turning point. Throughout this exercise the BACK of rod is to face the level. (2) Bring two opposite screws in line with rod. (3) (The necessity of watching bubble cannot be overemphasized.) Just before making each of following readings of rod, levelman will see that bubble is precisely centered; and as soon as the reading is made he will again inspect bubble. If he finds bubble has moved during reading he will recenter it, and make new reading, and again examine bubble. This will be continued until a reading is obtained with bubble stationary in the center; and only this reading will be retained. (4) Levelman reads the rod *downward* (since back of rod is facing the level) direct, to thousandths (counting hundredths and estimating thousandths). (5) Rodman now reads and records vernier

at back of rod (graduations which are exposed to the levelman). (6) Rodman next unclamps extension set-screw, extends rod about an inch (something less than 0.1 ft.), and clamps extension screw. (7) Rodman reads vernier at back of rod as before. (8) Levelman reads rod as before. (9) This shifting and double reading of rod is repeated until 5 double readings are obtained. (10) Take a series of 5 double readings as directed above, with rod held at distance of 200 ft., and 300 ft. from the level. So long as distance is not too great to permit of subdividing a one-hundredth space by eye, levelman will read to thousandths. (11)

FORM OF NOTES

Dist.	No of reading.	Readings.		Differences.	
		Levelman's.	Rodman's.	Levelman's.	Rodman's.

Computations: In connection with levelman's readings: Subtract each reading from reading next below, and enter difference in appropriate column. In connection with the rodman's readings: Find and enter the corresponding differences for rodman's readings.

8. EXTENDED-ROD READING

Equipment: Dumpy or wye-level; extension rod; also 4 stakes and hatchet, unless firm points are furnished by concrete curb or sidewalk, in which case mark the points with keel.

The object of this exercise is to give practice in making direct and indirect readings of the extended rod.

(1) Choose positions for the level and rod 50 ft. (paced) apart, and of such difference in elevation that the telescope of the level will be 10 or 12 ft. above rod point. (The rod point must, each time, be the top of a stake, or a keeled point on concrete curb or sidewalk.) (2) Set up the level with two opposite leveling screws in line with rod. (3) Extend rod full length, and clamp. (4) Levelman reads rod (without use of target), to thousandths (by estimation). (5) Rodman unclamps rod, and, by direction from levelman, lets extended portion slide down till target index line is precisely covered by H wire; and clamps rod in this position. Rodman reads (and records) rod to thousandths by vernier. (6) Similarly take a pair of readings on rod held on firm points at 60, 70, and 90 ft. from the level, in line of sight already established.

FORM OF NOTES

	Dist.	Levelman's readings.	Rodman's readings.	Differences.	

(Estimate the distance.) (7) Computations: Subtract each of rodman's readings from corresponding levelman's reading, and enter remainder, with proper sign in "Difference" Column.

9. TESTING ADJUSTMENTS OF LEVEL

Equipment: Wye-level, level-rod, 100-foot tape, set of marking-pins.

The object of this problem is to make the student as familiar as possible with the adjustments, without subjecting the instrument to the injury that would result from excessive disturbance of the adjusting-screws, especially in unskilful hands. Therefore no adjusting-screws are to be disturbed, but the value of each error of adjustment is determined, assuming all previous adjustments as already made. Making three trials of each adjustment will demonstrate to the student that unless *extreme care* is taken in handling the instrument, the three trials will not exactly agree, and that he will sometimes find an apparent small error when the adjustment is practically perfect. This note applies equally to Exercise 21.

1. To make the line of sight parallel to the axis of the bubble.

a. Measure off a base 400 feet long on nearly level ground and set up the level at one end of the line. Sight at the rod at the other end. Have the vertical axis clamped, and the whole instrument as rigid as possible except the clips over the telescope, which should be loose so that the telescope can be rotated in the wyes without shaking the instrument. If the line of collimation is in adjustment the rod reading will be the same, when the telescope is rotated half-way, as at first. The difference, if any, is *twice* the error. *Make three tests.* Raise or lower the telescope before each new test so as to bring the readings on a different part of the rod.

b. Bring the telescope directly over a pair of leveling-screws and clamp the vertical axis. Bring the

FORM OF NOTES

Adjustment.		1st trial.	2d trial.	3d trial.	Mean.
1a	Tel. direct Tel. inverted ½ diff.	3.684 3.692 0.004	3.986 3.995 0.0045	3.902 3.909 0.0035	0.0040
1b	Eye end Object end ½ diff.	17.6 16.0 0.8	17.7 16.0 0.85	17.8 16.1 0.85	0.83 div.
2					
3	Eye end Object end ½ diff.	16.2 17.7 0.75	16.1 17.7 0.8	16.1 17.5 0.7	0.75 div.
Angular value of one division of level-tube	*E*	15.8 3.6 12.2	15.4 5.6 9.8	14.8 4.2 10.6	
	O	15.2 2.8 12.4	13.2 3.1 10.1	13.1 2.2 10.9	
	R	4.931 4.211 0.720	4.867 4.294 0.573	4.831 4.208 0.623	
	α''	30″.2	29″.7	29″.9	29″.93

bubble to the center with the leveling-screws. Reverse the telescope in the wyes. If the bubble again comes to the center, the wyes are level and the line of collimation is parallel to the axis of the bubble. If the bubble does not come to the center, move the leveling-screws until the bubble is moved *half-way* back. Reverse the telescope again, and the bubble will come very nearly if not quite to its position before reversal. Keep adjusting the leveling-screws until the bubble remains in one place in the tube for the telescope

Dist. = 400 feet. Rod-readings are *less* when telescope is direct; therefore line of collimation points *down* $\frac{.004}{400} = .00001 = \tan 0° \, 0' \, 02''$.

Eye end high; therefore telescope points *down* 0.83 div. of bubble.

Bubble moves forward about one division when swung to the right about 30°; moves backward when swung to the left.

Dist. = 400 feet.

in either position. Under those conditions the wyes are level and distance of the center of the bubble from the center of the tube is the error of adjustment. Record this distance measured in divisions of the bubble-tube. *Make three tests.* Make the determinations independent by purposely throwing the instrument out of level before each new test.

2. To bring the bubble axis into the vertical plane through the axis of the telescope. — Bring the bubble to the center of the tube, first observing that the bubble-tube hangs directly under the telescope. Rotate the telescope slightly in the wyes, and note whether the bubble remains in the center, or if it moves in one direction for a swing to the right and in the other direction for a swing to the left.

3. To make the axis of the wyes perpendicular to the vertical axis of the instrument. — Bring the bubble to the center over both pairs of leveling-screws and then rotate the instrument about its vertical axis 180°. Correct one-half the error, if any, with the leveling screws; make the same correction for the other leveling-screws. The bubble will then remain nearly, if not quite, in the same place for any position of the telescope. Keep adjusting until this is true. The distance of the center of the bubble from the center of the tube is a measure of the correction that should be made by the adjusting-screw under one of the wyes. Record the error, if any, in divisions of the bubble-tube. *Make three tests* by purposely throwing the instrument out of level after each test and making an independent determination.

Value in seconds of arc of one division of the level-tube. — Run the bubble to near one end of the tube, but still keeping both ends of the bubble within the graduated divisions on the tube. Take a reading on the rod held 400 feet away. Move the bubble to the other end of the tube and again read the rod.

Let E = the movement, in divisions, of the eye end of the bubble;

Let O = the movement, in divisions, of the object end of the bubble;

Let R = difference in rod readings;

Let $$\alpha'' = \frac{R}{400 \frac{E + O}{2} \sin 1''}.$$

Obtain three values.

COMPASS

Uncertainties of compass readings. — One of the principal objects of Exercises 10, 11, 12 is to demonstrate practically to the student the uncertainties and inaccuracies of compass work. For some kinds of surveying work where simplicity and rapidity are prime essentials and in which a considerable degree of inaccuracy, or at least uncertainty, may be tolerated, the compass is an exceedingly useful instrument. As an adjunct to transit work, to check the angle work, and to detect gross errors or the slipping of the plates, it is also useful, and its use in this way should not be neglected. But the student should be fully aware of the limitations of its use. The presence of large masses of iron and steel in buildings, as well as high-tension electric currents, limits its use in cities, and even barbed-wire fences and trolley lines along country highways introduce a local attraction when attempting to use a compass on such boundary lines. Even assuming that the diurnal, the lunar, and the annular variations, as well as the normal declination, have been duly allowed for, there still remain local attractions of an unknown and uncomputable amount, which are often so great in magnitude that they utterly overshadow all the others. These may be due to high-tension electric currents, masses of iron or iron ore (known or unknown), or an "electric storm." The only certain way of avoiding such errors is to take foresights and backsights on all lines. The closeness of the agreement of such pairs of readings will be a very fair test of the presence of local attraction.

Setting up the compass. — Since there is far less necessity for precision in the leveling of the compass plate, a compass is usually provided with a ball-and-socket joint. Although this is more troublesome to an inexperienced man than leveling-screws, it is possible to level up much more rapidly. The screw which clamps the ball should be screwed up until the friction is sufficient to hold the plate in any position and yet permit it to be moved and adjusted without great force. Only practice will show just how much this should be. After the plate is leveled the screw may be tightened still more, but it must be done with great care or the plate will be thrown out of level while doing it. Perhaps the better plan is to have the screw so tight, even during the process of adjustment, that it will not slip while the instrument is being turned about its axis.

A compass tripod is not usually provided with a plumb-bob or with a centered hook for a plumb-bob string. A compass ring is usually divided to half-degrees, and an estimation to sixths of a division (or 5′ of arc) is about as close as the needle can be read. Five minutes of arc at a distance of 500 feet means nine inches. Even if the compass is set up over a stake, it can easily be centered by eye to within an inch or so, which is far within the accuracy of the needle reading. When, as is so common, an offsetted line is run parallel to a fence, the offset can be measured directly from the instrument. A plumb-bob is therefore a useless refinement.

Compass readings. — Text-books are apt to emphasize, perhaps somewhat unduly, the magnetic attraction of small articles of steel which one is apt to carry with him, as, for example, keys, knives, key-chains, the steel band which may be in the rim of a derby hat, axes or hatchets which may be lying around, the steel aligning-rods, steel tape, etc. While it is always better to be on the safe side and to make sure that the needle is not affected by any of these things, it is quite possible to waste time and effort in removing such objects from the neighborhood of the instrument when it is wholly unnecessary. It should be remembered that the intensity of such attraction (or repulsion) varies as the *square* of the distance of the object from the needle. The steel in a hat-brim would be very likely to influence the needle, since the brim of the hat might approach very near to the compass-box while the needle was being read, while a bunch of keys whose magnetism and mass might be far greater would have no influence, since it would be two or three feet away. The student will do well to test the effect of any object by taking careful readings of the needle when the suspected object is placed alternately on diametrically opposite sides of the instrument. The difference of readings should be about twice the influence of the attraction. If no appreciable difference is observed, it shows that that particular object has no appreciable effect when it is at that distance from the instrument. To be of any value, this test should only be made in a way that might occur *naturally* while using the instrument.

Care should be taken that the eye which reads the needle is located in a vertical plane through the needle, and if possible very nearly above the needle-point in order to avoid "parallax." Even though the needle is only read by estimation to 5′ of arc, or 1/6 of a 30′ space, it is much easier to estimate the fraction of a space by observing the location of the point with a magnifying-glass.

10. ANGLES WITH COMPASS

Equipment: Surveyor's compass.

Set up the compass over some fixed hub designated by the instructor in charge and point at four well-defined points (*e.g.*, church steeples, corners of buildings, etc.), which will be also designated. After sighting at all four points in turn and taking the needle reading on each to the nearest 5′ of arc, move the declination arc slightly and again take the needle readings on all four points. Take four such sets of readings. Calling *O* the position of the instrument and *A*, *B*, *C*, and *D* the points sighted at, four values of the angle *AOB* may be obtained by taking the differences of the bearings. Similarly four values of each of the other angles may be obtained. The discrepancies will give some idea of the reliability of needle readings.

(Left-hand page) FORM OF NOTES

	1st reading.	2d reading.	3d reading.	4th reading.	Mean.
Point *A*	N 13° 20′ W	N 15° 10′ W	N 16° 25′ W	N 17° 10′ W	
" *B*	N 1° 35′ E	N 0° 20′ W	N 1° 30′ W	N 2° 15′ W	
" *C*	N 64° 30′ E	N 62° 35′ E	N 61° 15′ E	N 60° 45′ E	
" *D*	N 77° 45′ E	N 76° 0′ E	N 74° 55′ E	N 74° 15′ E	
Angle *AOB*	14° 55′	14° 50′	14° 55′	14° 55′	14° 54′
" *BOC*	62° 55′	62° 55′	62° 45′	63° 00′	62° 54′
" *COD*	13° 15′	13° 25′	13° 40′	13° 30′	13° 27′

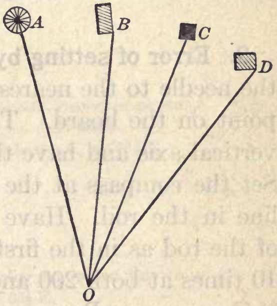
11. POINTING THE COMPASS

Equipment: Compass, transit-pole, foot-rule, board about 2 feet long and 6 or 8 inches wide.

1. Error of sighting. — Set up the compass where a clear view of 600 feet may be had. Have the rodman pace off 200 feet and lay the board on the ground with its length transverse to the line of sight, taking care to fix the board very firmly. If a bit of smooth clean pavement is at hand, it will be preferable to the board, being rigid. Sight at the rod held at some marked point near the center of the board. Clamp the vertical axis. Have the rodman measure the distance of this point from one end of the board and then move the rod away. Line in the rod again, and again have the rodman measure its position. Repeat at least 10 times. Compute the mean value; and subtract mean from each recorded value, and record the

(Right-hand page)

Tower Blind Asylum.
S. W. cor. 34th and Market Sts.
Tower of The Bartram.
N. W. cor. Chem. Lab.



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remainders as errors. Reduce the largest error to its angular value by dividing the error by the distance from the instrument. The quotient is the tangent of the angular error.

Repeat this test at a distance of 600 feet.

Form of Notes

Distance	Distance of rod from end of board	Error (d)			
	″	″			
200	12.0	0.18			
	12.8	0.62			
	12.6	0.42			
	10.8	1.38			
	etc.	etc.			
	....	....			
	....	....			
Sum =	121.8				
Mean =	12.18				
600	(simi	larly)			
200	(simi	larly)	Constant compass reading — N62° 25′ E		
600	(simi	larly)	Constant compass reading — N 67° 50′ E		

2. Error of setting by needle and lining in. — Read the needle to the nearest 5′ when pointing at some one point on the board. Turn the compass a little on its vertical axis and have the rodman move the rod away. Set the compass at the *same needle reading*, and again line in the rod. Have the rodman note the position of the rod as in the first part of the problem. Repeat 10 times at both 200 and 600 feet distance.

Caution. — Be sure that the *declination arc* is not disturbed during the progress of the work.

12. RUNNING LINE OF CONSTANT BEARING

Equipment: Compass, transit-pole, set of marking-pins.

Run out a line about ½ mile long, on uneven ground, setting pins at each station, using a *constant* bearing. Pace the distances between stations. Then run back by the reverse bearings and have the rodman note the discrepancy at each station. After noting the error the rodman transfers the marking-pin to the new position, which will make the entire return line continuous and independent. Be careful to select stations where there will be no *local attraction* (*e.g.*, from lamp-posts, iron fences, etc.). Where local attraction is suspected, take a backsight and determine its amount. The work will demonstrate the compensating character of the errors, the degree of accuracy that may be expected, and the frequency even in suburban localities of greater or less local attraction.

(Left-hand page) FORM OF NOTES (Right-hand page)

Sta.	Bearing	Distance	Errors on return	
0	S 65½° W	350	S 0.4	
1	S 65½° W	420	N 0.6	
2	S 60¼° W	370	N 1.8	Local attr. suspected at sta. 2. B. S. to sta. 1, N 60¼ E.
3	S 65½° W	640	N 0.4	B. S. to sta. 2, N 65½ E.
4	S 65½° W	260	S 1.0	
5	S 65½° W	480	S 0.3	
6	....	370	....	Bearing on return line from sta. 6, N 65½ E.

TRANSIT

Horizontal-angle work. — In cases where the horizontal angles are not required with minute accuracy, it may be justifiable to use only a single vernier (especially if the instrument is so well made that its eccentricity never exceeds one-half minute of arc) and to set the vernier at 0° as an initial reading; but when the greatest accuracy is required, not only should both verniers be read, but it is better to sight the instrument with the verniers reading *any* reading. The verniers on the horizontal plate of a well-made sharply graduated transit may be easily read or estimated by means of a microscope to the nearest half-minute of arc. By reading both verniers and taking their mean, in case the opposite readings should differ by even one half-minute, we have a mean value to the nearest 15 seconds.

A 6-inch circle has a circumference of 18.85 inches, but 15 seconds of arc is $\frac{1}{86400}$ of the circumference, which means but 0.00022 of an inch in this case. In order to set the vernier at precisely 0° to within 15 seconds of arc, the slow-motion screw must be turned so precisely that its position is located to within twenty-two hundred-thousandths of an inch. Although the tangent screw may be turned so that no variation from 0° is observable even with the microscope, there is always the chance for that psychological error of believing that the reading is some specially desired reading if the inaccuracy is really small. When the verniers are set at *any* reading, the mind is not biased by thinking that the reading must be any particular reading, but, in an unprejudiced way, can make the reading of each vernier to the nearest half-minute with the least probability of error.

Vertical-angle work. — When the transit is to be used only for measuring horizontal angles, any reasonable care in leveling the horizontal plate will suffice to avoid any measurable error owing to lack of horizontality of the horizontal plate, but many instrumentmen do not realize the care that must be taken to have the instrument truly level when a vertical angle is to be measured. The plate bubbles are always somewhat coarse and it is almost impossible to adjust them so that there is any feeling of certainty that the plate is level even to one minute of arc. No vertical-angle work should be done with a transit unless it is provided with a "long bubble" under the telescope, and even then it is advisable to check the adjustment, at least every day that it is used, as follows: After leveling the instrument as carefully as possible by means of the plate bubbles, clamp the telescope so that it is horizontal and with the slow-motion screw bring the long bubble exactly to the center. Then swing the whole instrument 180° about its vertical axis. The bubble should again be in the center, but if it is not, it means that the vertical axis of the instrument is not truly vertical. Correct half of the error by means of the slow-motion tangent-screw to the telescope and the other half by means of the leveling-screws. (Theoretically this should accomplish the desired result, but practically it may be found necessary to make repeated adjustments of the tangent-screw and the leveling-screws so that the bubble will remain in the center when the telescope is pointing in any direction.) Then note the reading of the vernier of the vertical arc. If it is adjustable, it should be adjusted to 0°. If it is not adjustable, the "index error" should be read, paying particular attention to its algebraic sign. For example, if it already reads +0° 5′, thus indicating that, according to the vernier, the telescope is pointing upward when we *know* it is horizontal, it would mean that every upward reading will be 5 minutes more than its true reading, and hence the index error is − 0° 5′, which means that we should *subtract* 0° 5′ from every upward or positive angle and *add* 0° 5′ to every downward or negative angle to get the true reading.

13. POINTING

Equipment: Transit.

The object of this exercise is to give additional practice with transit; and to draw the student's attention to the range of uncertainty in result of any single pointing made under the conditions governing the exercise.

(1) Set up transit. (Setting up transit includes focusing the ocular, if the ocular has not already been focused.) (2) At distance of 500 ft. from the transit, hold the open field book fast on the ground (or other fixed support) with the central H ruling of the right-hand page at right angles to line of sight. (3) Set line of sight on pencil held at the point, a_0, at middle of the said H ruling. Make all the dots on this one horizontal ruling. (4) Remove pencil. (5) Set pencil in line of sight, making small dot with an arrow running out to its letter, a_1. (6) Remove pencil. (7) Align pencil, marking the dot, a_2. (8) Remove pencil. (9) Align pencil, marking the dot a_3. (10) Repeat, obtaining dots a_4 and a_5.

(11) *Notes:* The plotted points with their letters should be accompanied by a brief statement of the exercise. (12) *Computations:* Make a dot a on the said H ruling, just beyond one of the end dots. Scale the distances as aa_1, aa_2 (using the 50-to-the-inch scale) and enter them in third column. Compute mean length. Subtract mean from each scale length, entering remainder, as an error, with proper sign, in fourth column. Compute angular error (as decimal of one minute) corresponding to largest error in length. (An error of one minute in angle would correspond to an error in length equal to 0.0003×500; since 0.0003 is the tangent of one minute.)

FORM OF NOTES

Dist. (ft.).	Line.	(Fiftieths of inch).	Error (fiftieths of inch).	Error (fraction of a minute).	
500	aa_1				
	aa_2				

14. SETTING VERNIER AND POINTING

Equipment: Transit; magnifying glass.

The object of this exercise is to continue practice with transit, and to show the student the range of uncertainty in combined result of vernier setting and pointing, when the work is done under the conditions governing this exercise.

(1) Set up transit. (2) At distance of 500 ft. from transit, hold open field book flat on ground (or other fixed support) with the central H ruling of right-hand page at right angles to line of sight, as in preceding exercise. (3) Hold pencil plumb on the point, a_0, at middle of said H ruling. Make all the dots on this one ruling. (4) Set line of sight on pencil, and clamp both UM and LM. (5) Read vernier under ocular. (Always use magnifying glass to read or set vernier.) (6) Unclamp UM, and swing away from pencil. (7) Remove pencil. (8) Set vernier at recorded reading by means of UM. (9) Align pencil, marking a fine dot a_1. (10) Loosen UM, swing away, and remove pencil. (11) Set vernier at recorded reading. (12) Align pencil, marking the dot a_2. (13) Continue till a_5 is obtained. (14) *Notes and computations:* Same as for preceding exercise. Notes should be kept in instrumentman's note-book. (15) Compare errors of this exercise with those of preceding exercise.

15. SINGLE ANGLE

Equipment: Transit; bob; 2 magnifying glasses; 3 pins.

The object of this exercise is to give practice in reading vernier and measuring horizontal angles, and to bring to the student's attention the range of error resulting from pointing and vernier reading with the transit used.

(1) Set up transit over marked point or permanent hub, *I*. (2) Set the three pins, each at about 200 ft. from *I*, and about 30 ft. apart. Call left-hand pin *A*, next pin *B*, and right-hand pin *C*. (Specified permanent marks, near or far, may be used instead of the pins.) (3) Clamp LM and with UM point at *A*. Read vernier under ocular. (While transitman is reading and recording vernier for each pointing, recorder will read and record opposite vernier in recorder's book.) (4) Loosen UM, and point at *B* by means of UM. (5) Read verniers. These two pairs of readings on *A* and *B* make Set 1. (6) Loosen UM and swing slightly — about 5°. (7) Clamp UM and loosen LM. (8) Point at *A* with LM. (9) Read vernier. (10) Loosen UM, and with UM point at *B*. Read vernier. These two pointings make Set 2.

(11) Repeat until you have 5 sets. (For each new set the initial pointing at *A* should be made with a new vernier reading containing odd minutes.)

(12) In like manner obtain 5 sets for angle *BIC*, and 5 sets for angle *AIC*.

(13) *Computations:* Subtract the mean of each five sets from the value for each set, and enter difference in "Errors" column. Express fractional part of a minute as a decimal (to the nearest tenth of a minute) — do not use seconds. (14) Find difference between

FORM OF NOTES

Set.	Pointing at pin.	Vernier *A* (or *B*).	Angle (name).	Angle (value).	Mean Angle (value).	Errors.
1	*A*		*AB*			
	B					

$AIB + BIC$ and AIC (using means), and apply one-third of it as correction to each mean in such manner as to make corrected angles satisfy the equation: $AIB + BIC = AIC$. This is the most reasonable adjustment of these angles to make them satisfy the axiom that the whole is equal to the sum of its parts.

16. ANGLE BY REPETITION

Equipment: Transit; bob; 2 magnifying glasses; 3 pins.

The object of this exercise is to continue practice with transit, and to exhibit a method often used to reduce the range of error in angle measurements.

(1) Set up transit and pins as in preceding exercise. (2) Unclamp needle. (3) While transitman reads vernier each time, recorder will read (north, *i.e.*, unweighted, end of) needle to nearest 5 minutes, by means of magnifying glass. (4) With telescope in normal position set vernier (under ocular) at 0° 00′; and point at pin *A* with LM. Record reading. (5) Loosen UM. With UM, point at *B*, and read vernier. (6) Loosen LM. With LM, point at *A*. (7) Loosen UM. With UM, point at *B*. (8) Loosen LM. With LM, point at *A*. (9) Loosen UM. With UM, point

at B, and read vernier. (10) Reverse telescope and repeat (4)–(9). (11) Repeat steps of (3)–(10) for angle BIC, and for angle AIC. (12) Clamp needle.

FORM OF NOTES

Pointing at	Tel. (N or R)	Vernier reading.	Needle reading.	$\frac{1}{3}$ of triple angle.	Mean of N & R	Corrected angle.	Angle by needle.

(13) *Computations:* Compute and record 1/3 of each triple angle. Take mean of each pair of normal and reverse angles, and record it. Adjust the three means, if necessary, to make $AIB + BIC = AIC$, as in preceding exercise; and enter the adjusted means under "Corrected angle." Calculate each angle from needle readings; and record. (14) Compare errors of this exercise with those of preceding.

17. ANGLE BY DIRECTION (OR SERIES)

Equipment: Transit; bob; 2 magnifying glasses; 3 pins.

The object of this exercise is to continue practice with the transit, to present a common method of measuring angles, and to bring out the range of error which may be expected under similar conditions.

(1) Set up transit and pins as in preceding exercise; except for making IA about 250 ft., IB about 300 ft., and IC about 350 ft., and angles AIB and BIC approximately equal by eye. (2) Unclamp needle. (3)

With telescope normal set vernier (under ocular) at 0° 00′, and point at pin *A* with LM. Record reading. (For each pointing, recorder will read the needle with magnifying glass, and record reading.) (4) Loosen UM. With UM, point at *B* and read vernier. (5) Loosen UM. With UM, point at *C*, and read vernier. (6) Loosen UM. With UM, point at *A*, and read vernier. (7) Reverse telescope, set vernier (under ocular) at 0° 00′, loosen LM, and point at *A*.

(8) Point with UM at *C*, *B*, and *A*, in order named, and read vernier for each pointing.

FORM OF NOTES

Pointing at	Tel. (*N* or *R*).	Vernier reading.	Needle reading.	Angle name.	Angle value.	Mean of *N* and *R*.	Angle by needle.

(9) *Computations:* Fill out columns "Angle name," "Angle value," etc., shown in form of notes. Adjust the angles, by adding $(360 - (AIB + BIC + CIA))/3$ to each of the three angles. Compare errors of this exercise with those of Exercises 15 and 16.

18. VERTICAL CIRCLE READING AND INDEX ERROR

Equipment: Transit with full vertical circle; magnifying glass; plain or extension level-rod; rod level.

The object of this exercise is to give practice in measuring vertical angles, and to introduce "index error" to the student, and to show the range of uncertainty in the value of a vertical angle found under the conditions of this exercise.

(1) Set up transit. (2) At distance of about 50 ft. from transit establish point on which to hold rod. (This point should be the equal of a turning point.) (3) Clamp UM. With LM, set V wire on rod. (4) With VM, set central H wire on each foot and half-foot mark of rod in turn, beginning with 2.5 ft. and ending with 7 ft.; and for each rod setting read VC. (In recording V angle, indicate by (+) or (−) whether slope is up or down.) Use rod-level on rod for this set of readings. (5) Reverse instrument, and repeat foregoing settings and readings.

FORM OF NOTES

Tel. (*N* or *R*).	Rod settings.	Vert. angle (+ or −).	Index error (+ or −).	Errors.	

(6) *Computations:* If there were no other error than index error, the difference between normal and reversed VC readings, taken on same rod point, would be twice the index error. Compute index error from the pair of circle readings for each rod setting; and enter it opposite normal rod setting. If all computed index errors have not same value, variations are due to errors of manipulation and reading. Since it may be nearly always assumed that true index error is constant throughout the exercise, the range of vari-

ation in values obtained for index error may be taken as the range of errors other than index error. Subtract average of index error values from each value, and set difference with proper sign in "Error" column. Read paragraph on "vertical-angle work" preceding Exercise 13.

19. STADIA READINGS

Equipment: Transit; bob; magnifying glass; metallic tape; plain or extension leveling rod; rod level.

The object of this exercise is to give further practice with the vertical circle, to give practice in stadia readings, and to show the range of uncertainty in results obtained under the conditions of this exercise.

(1) Set up transit. (2) Hold rod 20 ft. away on top of object suitable for a turning point. Use rod-level for plumbing rod. (3) Set V wire on rod, and clamp both LM and UM. (4) Tilt telescope until U wire cuts near top of rod; and clamp VM. (5) Read and record U, M, and L wires in order named; and read and record VC. (6) Subtract M from U, and L from M, and record differences in the "Half-intercept" column. (If the three wires are correctly set by instrument maker, the two computed half-intercepts should be equal. If they are not equal, the discrepancy indicates error or mistake in reading or mistake in recording. If discrepancy is unduly large, repeat readings until fair set is obtained or until it is certain that greater part of discrepancy is due to inequality in actual wire intervals in cross-wire ring. This test of readings is to be made for each of following sets of readings.) (7) Continue, taking readings with VC set, in turn, at about $\frac{3}{4}$, at about $\frac{1}{2}$, and at about $\frac{1}{4}$ of the first VC reading; and with VC set at 0° 00′.

FORM OF NOTES

VC.	U M } wire. L	Half-intercept.	Whole-intercept.	Computed distance.	Errors.
10° 35′	11.365 11.353 11.340	0.012 0.013	0.025		
					c = 0.6 ft. f = 0.95 ft.

(8) Measure c, the distance from center of H axis of telescope to object glass. Measure f, the distance from cross-wire ring to object glass. (Measure to nearest 0.1 ft. only.)

(9) *Computations:* Enter whole intercept in proper column. Compute from each set of readings, by one of methods shown under "Office work" in Exercise 36, the H distance from instrument to rod, and enter results in proper column. Subtract the mean of the five computed distances from each of them, and enter difference in "Errors" column. Note the amount by which average of computed distances differs from 20 ft.

20. STADIA FACTOR

Equipment: Transit; bob; magnifying glass; stadia rod; rod-level; 100 ft. steel tape; 11 pins; 5 stakes; hatchet.

Location: Choose an open line about 1000 ft. long on ground such that the entire rod at each rod point can be seen through the telescope.

The object of this exercise is to give further practice in stadia reading, to exhibit further the range of errors, and to show a method of ascertaining the value of the stadia factor (f/i) for a given transit.

(1) Drive stakes at *I*, *A*, *B*, *C*, and *D*, aligning them by eye, and making *IA* = 247 ft.; *AB* = 247 ft.; *BC* = 254 ft.; and *CD* = 254 ft. (2) Set up transit at *I*. (3) Hold rod on *A*, plumbing it with rod-level. (4) Clamp UM; and with LM bisect rod with V wire. Level the telescope with VM. (5) Read the three H wires. (Read to hundredths by estimation, at all times in this exercise. Compute half-intercepts for immediate check.) (6) Turn VC tangent screw about one-half revolution, and again read U, M, and L wires. (7) Again turn VC tangent screw a half-revolution, and again read the wires. (8) Repeat (5)–(7) without using rod-level. (Indicate under "Remarks" for which readings rod-level was used.) (9) Repeat (5)–(7) with rod at *B*, at *C*, and at *D*. (10) Measure *c* and *f*. (See (8) of Exercise 19.)

FORM OF NOTES

Dist. by tape.	No. of set.	U, M, L wire.	Half-intercept.	Whole intercept.	Errors (intercept).	Stadia factor.	Errors (Stadia factor).

(11) *Computation :* Enter whole intercept in proper column. Compute mean of each group of three intercepts, subtract mean from each intercept; and enter difference in "Errors (intercept)" column. Compute stadia factor from each mean intercept by use of stadia formula for H sights:

Dist. = $f + c +$ (stadia factor) $\times$ (rod intercept).

Find mean of stadia-factor values; subtract it from each stadia-factor value; and enter difference in column of "Errors (Stadia factor)."

21. TESTING ADJUSTMENTS OF TRANSIT *

Equipment: Transit, line rod, level-rod, 100 ft. steel tape, foot rule.

1. To make the plane of the plate bubbles perpendicular to the vertical axis. — Set up the transit and make the plate bubbles parallel to opposite leveling-screws. Bring both bubbles to the center; revolve exactly 180°. The distance (measured in divisions on the bubble-tube) of the *center* of the bubble from the *center* of the scale is *twice* the error. With the leveling-screws throw the transit out of level and begin over again so as to make an absolutely independent redetermination. *Obtain three independent values for the error of this adjustment.*

2. To make the line of sight perpendicular to the horizontal axis of the telescope. — Set the transit where a clear sight of about 300 feet each way in opposite directions may be obtained. Point the telescope at some well-defined point (*e.g.*, the corner of a building having a good background). Plunge the telescope and align the rodman at some point about 300 feet away. Revolve the alidade and point at the first point selected. Again plunge the telescope and align the rodman for a point beside the one previously set. The distance between these two points is *four* times the error. *Make three tests.*

3. To make the horizontal axis of the telescope perpendicular to the vertical axis of the transit. — Set up the transit about 20 feet from the vertical wall

* See first paragraph of Exercise 9.

of a building. Select a point about as high on the wall as the telescope can be pointed. Swing the telescope down and have the rodman make a mark on the wall down near the ground. Plunge the telescope, revolve the instrument 180°, and again point at the upper point. Swing the telescope down and have the rodman make another mark beside the first lower mark. A vertical plane through the instrument and the upper mark will pass midway between the two lower marks. Record the distance of the instrument from the wall, the vertical distance (estimated) between the upper and lower points and the distance between the two lower points. *Make three tests.*

4. To make the axis of the telescope bubble parallel to the line of sight. — Level the telescope and mark a point on the ground directly under the eye-piece (driving a peg if necessary). Measure the height of the center of the eye-piece above the point with the level-rod; call the reading a. Establish another point about 300 feet away and at about the same level. Read the level-rod placed on it; call the reading b. Transfer the instrument to the second point and set it up so that the eye-piece is directly over the point. Measure the height of the eye-piece above the point, calling it c, and also read the level-rod held on the first point, calling it d. Then we should have $d - a = c - b$ if the instrument is in adjustment. If not, then $(d - a) - (c - b) = 2m$. $\frac{m}{\text{distance}} = \tan \alpha$. If m is positive the line of collimation points upward. *Make three tests.*

5. To make the vernier of the vertical circle read zero when the line of sight is horizontal. — This test is only applicable to instruments having fixed vertical circles. As an adjustment it is only applicable to instruments having adjustable verniers. Make the telescope level by means of the level bubble and note the *amount* and *direction* of the index error. *Make three tests* by turning the telescope out of level between each test.

Best location for these tests. — Tests 2 and 4 require an unobstructed, nearly level stretch of 300 feet in one direction, and Test 2 a suitable point about 300 feet in the opposite direction. Test 3 requires a high wall about 20 feet from the instrument, and Tests 1 and 5 may be made anywhere. When an unobstructed line of sufficient length can be found, parallel to a wall and about 20 feet away, a single setting of the instrument will serve for all the tests.

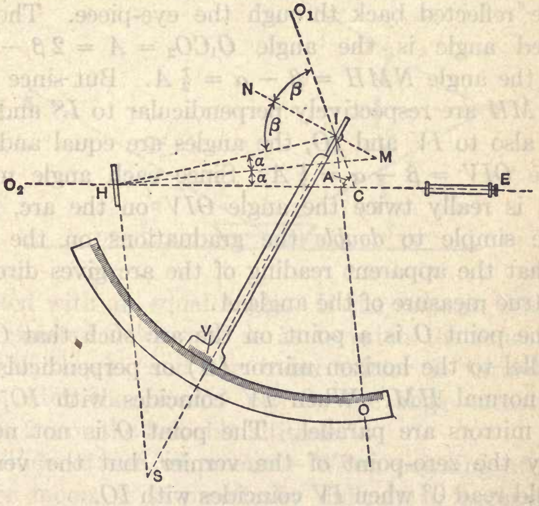
Form of Notes

Test.	1st Trial.	2d Trial.	3d Trial.	Mean.		
1	N 0.3 div.	N 0.5 div.	N 0.4 div.	N 0.40 div.		Bub. par. to tel.
	E 1.2 div.	E 1.0 div.	E 1.3 div.	E 1.17 div.		Bub. perp. to tel.
2	$1\frac{1}{2}''$	$1\frac{1}{4}''$	$1\frac{5}{8}''$	$1''.46$	$\frac{1.46}{4} = 0.365$	dist. about 350 ft.
3	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{5}{16} \div 2 = \frac{5}{32}$ dist. = 364	wall 24 ft. from transit; upper and lower points about 40 ft. apart.
4 *a*	4.26	4.51	4.51			
b	3.78	4.04	4.02			
c	4.61	4.61	4.47			
d	5.13	5.14	5.03			
m	0.02	0.03	0.035			
tan α	0.000 054	0.000 081	0.000 094			
α	$+0° 0' 11''$	$+0° 0' 16''$	$+0° 0' 19''$	$+0° 0' 15''$		
5	$+0° 3'$	$+0° 03\frac{1}{2}'$	$+0° 2\frac{1}{2}'$	$+0° 03'$		

SEXTANT

The sextant is in one respect the most remarkable instrument of precision ever invented — considering that with it angles may be measured to the nearest

10 seconds of arc, although the instrument is held with the hand by an operator who is perhaps on a vessel or even in a small unsteady boat where the use of a transit would be impracticable for even the roughest work. The sextant is used either for measuring the altitude of the sun or a star, or to measure the angle between two points. In the latter case the angle measured lies in the plane determined by the two points and the instrument, and does not necessarily equal the horizontal projection of that angle, as would be the case if the angle were measured with a transit. The sextant is the only instrument which will give directly the measurement of an angle lying in an oblique plane.



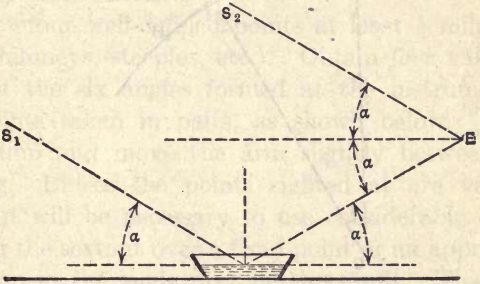
General principles of construction. — The ordinary sextant consists essentially of an arc of about 70° or 80°, having an arm, *VI*, revolving about the center (*I*) of the arc, having a mirror called the "index" mirror (*I*) attached to the arm, having a mirror called the "horizon" mirror (H) which is attached to the frame of the arc, and having also an eye-piece (E) which may be either a telescope or a small tube which merely directs the line of sight. The horizon mirror consists partly of clear glass, so that an object at O_2 may be seen through the glass and the eye-piece E. The sextant should be held in the right hand, the movable arm being adjusted by the left hand. The movable arm is provided with a clamp and slow-motion screw.

To obtain the angle between lines from the two points O_2 and O_1, the eye-piece E must be pointed at O_2 and the arm VI moved until a ray of light from O_1, falling on the index mirror (I), will be reflected to the silvered portion of the horizon mirror (H) and from there reflected back through the eye-piece. The required angle is the angle $O_1CO_2 = A = 2\beta - 2\alpha$. But the angle $NMH = \beta - \alpha = \frac{1}{2}A$. But since NM and MH are respectively perpendicular to IS and HS and also to IV and IO, the angles are equal and the angle $OIV = \beta - \alpha = \frac{1}{2}A$. Since each angle measured is really twice the angle OIV on the arc, it is more simple to *double* the graduations on the arc, so that the apparent reading of the arc gives directly the true measure of the angle A.

The point O is a point on the arc such that OI is parallel to the horizon mirror (H) or perpendicular to the normal HM. When IV coincides with IO, the two mirrors are parallel. The point O is not necessarily the zero-point of the vernier, but the vernier should read 0° when IV coincides with IO.

One theoretical defect of the sextant is the fact that the vertex of the angle (C) is a variable point which is somewhere within the instrument for all large angles, but which will be a long distance *behind* the operator for very small angles. If the uncertainty of either point sighted at is as great as the perpendicular distance from *I* to *HE* (two or three inches), as it is in all astronomical work and in terrestrial work where one point is at a distance of a mile or more, the error involved is too small for measurement.

Measurement of altitudes. — As a natural horizon is seldom obtainable, except at sea, the use of an "artificial horizon" becomes necessary. This is a basin containing a reflective liquid. Mercury is frequently used, but molasses is less disturbed by wind. As the free liquid assumes a level surface, a ray of light, S_1, striking the liquid at an angle α, will be

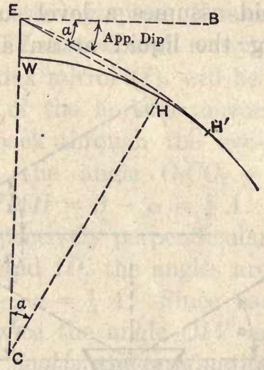


reflected with an equal angle. If the sextant, held at *E*, is pointed downward so as to catch the reflection from the liquid, and the index mirror is turned so as to catch the *parallel* ray S_2, the angle measured will be 2α or the "double altitude."

Altitude of sun or moon. — When pointing at the sun or moon, it is practically difficult to accurately superpose one image directly over the other, and it is much more accurate to move the arm until the discs are *tangent* to each other. The reflection (S_1) from the liquid is reflected *once* and is therefore inverted. The other image (S_2) is reflected *twice*, is therefore *re-*

inverted, and is seen erect. When the image from the liquid appears just *underneath* the other image, the two images of the *lower* limb are just in coincidence, and the altitude measured is therefore that of the *lower* limb, which is *less* than the altitude of the *center* by the angular value of the semi-diameter.

Dip of the horizon. — When observations are taken at sea, using the natural horizon, the instrument is always at some distance above the water. A line



to the real horizon, *EH*, would therefore dip below a horizontal line (*EB*) by an angle α which is equal to the angle *ECH*. But the refraction of the atmosphere complicates the solution by *apparently* raising up the horizon and also by rendering visible more of the sea than would be visible if there were no refraction, so that the *apparent dip* is less than α. Knowing the elevation of the sextant above the water, the *apparent dip* may be found by reference to the following table which gives the dip for all elevations from 6 feet to 30 feet.

APPARENT DIP OF THE HORIZON

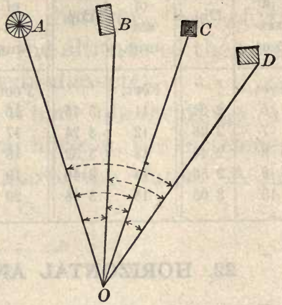
Height of sextant.	Dip.	Height of sextant.	Dip.	Height of sextant.	Dip.	Height of sextant.	Dip.	Height of sextant.	Dip.
Feet.	′ ″	Feet.	′ ″	Feet.	′ ″	Feet.	′ ″	Feet.	′ ″
6	2 24	11	3 15	16	3 55	21	4 30	26	5 0
7	2 36	12	3 24	17	4 03	22	4 36	27	5 06
8	2 46	13	3 32	18	4 10	23	4 42	28	5 11
9	2 56	14	3 40	19	4 16	24	4 48	29	5 17
10	3 06	15	3 48	20	4 23	25	4 54	30	5 22

22. HORIZONTAL ANGLES WITH SEXTANT

Equipment: Sextant.

Select four well-defined points at least $\frac{1}{2}$ mile away (*e.g.*, chimneys, steeples, etc.). Obtain *four* values for each of the six angles formed at the instrument by the points taken in pairs, as shown below. Loosen the clamp and move the arm slightly between each reading. Unless the points sighted at are very far away, it will be necessary to use considerable care in holding the sextant over a fixed point or an appreciable variation in the angle may be the result. This problem is useful as preliminary practice for the work of locating soundings by means of sextant-angles taken from a boat.

FORM OF NOTES

Angle.	Readings.	Mean.	
AOB	12° 16′ 20″		
	10		
	30		
	20	12° 16′ 20″	
BOC	14 22 40		
	etc.		
(similarly)			
COD			
AOC			
BOD			*A*, Church steeple, cor. E and F. sts.
			B, N. E. cor. G. and H. sts.
			C, Tower, City Hall,
AOD			*D*, S. W. cor. M. and N. sts.
			Sextant at N. E. cor. Chem. Lab.

[The student should place the sketch on a right-hand page of his note-book, the computations on left-hand page.]

23. TESTING ADJUSTMENTS OF SEXTANT *

Equipment: Sextant; also two blocks of wood about $\frac{7}{8}''$ in height.

1. The plane of the index-glass should be perpendicular to the plane of the arc. — The effect of an error in this adjustment is not readily capable of measurement. Set the index-arm at about 60° or 70°; hold the sextant nearly horizontal with the arc away from the eye; observe in the index-glass the reflection of the arc and note whether the arc as seen directly forms a continuation on both sides of the reflection seen in the index-glass.

2. The plane of the horizon-glass should be perpendicular to the plane of the arc. — An accurate test of this adjustment requires that the telescope be pointed

* See first paragraph of Exercise 9.

at a star, and it can be satisfactorily tested only when a *distant* and very sharply defined object may be sighted at. A finial, seen against a bright sky background, will often prove to be the best terrestrial mark obtainable. Use the highest power telescope that belongs to the sextant for this test. Set the index-arm at 0° and sight at the point; the direct and reflected images will nearly coincide. Move the index-arm slightly in either direction. If the sextant is held horizontally and the horizon-glass is not in adjustment, the reflected image of a point will be seen to pass the direct image either above or below it. A rough measure of the value of such a lack of adjustment may be obtained by reading the vernier when the direct and reflected images are directly above or below each other (the sextant being horizontal), and again reading it when the reflected image is as far (by estimation) to the right or left of the direct image as it is above or below it. The first of these readings should be 0° if there is no "index error." If more convenient, the sextant may be held vertical, in which case the reflected image will pass to the right or to the left of the direct image. The difference of the readings above will be four times the lack of perpendicularity of the glass. The effect on measured angles of a small error of this sort cannot be detected except with the closest work. Obtain *three* differences as above described on *each* side of the point where the images seem most nearly in coincidence.

3. The axis of the telescope should be parallel to the plane of the arc. — Set the sextant horizontal on a firm support which is 15 to 20 feet from a wall. Place two blocks on the arc so that a sighting may be taken over the blocks to the wall at the place at which the telescope points. The height of the blocks should be the same and equal to the height of the center of the telescope above the arc. The height of the telescope may be adjusted to the exact height of the blocks if the telescope is adjustable. Sight over the tops of the blocks and have an assistant make a mark on the wall. Then, without disturbing the instrument, note whether the telescope, previously focussed and directed to that place on the wall, is sighting above or below the mark. Make three trials.

4. To determine the index error. — Although all sextants are provided with adjusting-screws for correcting this error, it is considered best to be content with making it small and carefully computing its amount, which should then be applied to all readings. Sight at a well-defined point with the index arm at or

FORM OF NOTES

Adjustment.		1st Trial.	2d Trial.	3d Trial.	Mean.
1					
2	Refl. im. above .	−1′ 30″	−0′ 50″	−1′ 0″	
	" " oppos. .	+1 0	+1 50	+1 20	
	" " below. .	+3 10	+3 30	+3 0	
	Diff. above......	2′ 30″	24′ 0″	2′ 20″	2′ 30″
	" below......	2 10	14 0	1 40	1 50
3		+2½″	+2¾″	+2¼″	+2½″
		On arc.	Off arc.	½ diff.	
		33′ 40″	30′ 40″		
		50	15		
		60	20		
		40	40		
		60	30		
		75	50		
		60	50		
		70	15		
		80	10		
		75	35		
		610″	305″		
		34′ 01″	30′ 30″.5	−1′ 45″.2	

near 0°. Bring the two images into exact coincidence. The arc is graduated for a few degrees back of 0°. If the zero of the vernier is back of 0°, or "off the arc," all readings of angles will be too small and the correction is *positive.* If the zero of the vernier is "on the arc," the correction is *negative.* As shown in ¶ 2, a terrestrial object is not sufficiently well defined for accurate work. At night a star gives the best mark. In the daytime the sun may be sighted at, but, since it is very difficult to bring the two images of the sun into accurate coincidence, it is best to make the images tangent. Read the vernier. Then move the index-arm until the opposite limbs are tangent and again read the vernier. One reading will be off

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No perceptible error.

General mean 2′ 10″; error of glass 32″.5.

Wall 35′ from sextant. Telescope points *above* the arc.

Sighting on sun.

Mean reading is *on* the arc. Therefore the correction is *negative.*

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the arc and the other on the arc. One-half of the difference equals the index error, and its sign will be according as the reading on the arc or off the arc is greater. Obtain 10 readings on the sun off the arc and 10 readings on the arc.

OTHER INSTRUMENTS

24. PLANE TABLE *

Equipment: Plane table (including alidade, table-level, and compass-box), stadia-rod, steel tape, set of marking-pins. Students must each provide a sheet of drawing-paper about 16″ × 24″ (detail paper will answer the purpose), scale, *hard sharp* pencil, and rubber.

Measure a base-line (*AB*) about 300 feet long. Set up the plane table at *A*. Orient the table and draw in the base-line (30 feet to an inch) so that the proposed sketch will come properly on the paper. Draw a magnetic meridian through *A*, using the compass-box. Draw lines toward several points (trees, building corners, etc.), which are visible from both *A* and *B* and, determining their distances by stadia, plot them at once to scale.

Set up the table at *B*; orient by sighting to *A*; check the orientation with the compass; check the plotting of the other points by sighting to them. Sight to some station point *C*, visible from both *A* and *B*, drawing an indefinite line toward it.

Set up the table at *C*; orient by sighting to *B*; check the orientation with the compass; locate *C* on the drawing by sighting to *A*. Check the plotting of

* Complete directions for use of plane table are given in "A Plane Table Manual" (Appx. 7, U. S. C. & G. S. Report, 1905) by D. B. Wainright.

C by sighting at some of the other points already located.

Notes. — In the note-book record date, time of work, number of problem, and personnel of party. The drawing, properly lettered, should be handed in with the note-book. Each student of the party may plot *his own* sheet for each station while the instrument is set there — thus saving time. When another sheet is placed on the table at any station, it simply requires a readjustment of the orientation. The accuracy is not necessarily impaired. *Extreme accuracy* in drawing is essential for good results.

Alternative problem. — Since the amount of time which any one student may be permitted to devote to this problem is generally limited, it is sometimes preferable to make a somewhat extended survey involving frequent repetition of the above principles, and in turn assigning several parties of students to the problem. The base-line should then be increased to 500 feet or more and the scale reduced to 100 or even 200 feet per inch. A detailed topographical survey may thus be made of a considerable area, which will be a more interesting problem than the solution of abstract mathematical problems.

25. HAND-LEVEL

Equipment: Hand-level; target rod.

The object of this exercise is to give practice with the hand-level, and acquaint the student with the range of error to be expected under the conditions of the exercise.

(1) Instrumentman, standing beside rod, measures height of his eye above ground on which he is standing, and records height. (2) Instrumentman and rodman occupy points of equal elevation (as previously de-

termined by dumpy or wye-level) separated by 100 ft. (paced). The rod is held on the same point throughout the exercise. (3) Instrumentman takes sight on rod with hand-level, with bubble centered on cross-wire, and directs rodman to set target on line of sight. (4) Rodman then reads target to nearest 0.01 ft.; and records reading. (5) Repeat (3) and (4) four times. (6) Repeat (2)–(5) for distance of 200 ft.

FORM OF NOTES

Dist.	No. of obs.	Target reading.	Errors.		

(8) *Computations:* Find mean of target readings for each distance. Subtract mean from each reading, setting remainder down in column of "Errors." Give each error its proper sign.

NOTE. — The hand-level is sometimes used for differential and profile leveling; and in such cases the notes are kept just as for similar work with dumpy or wye-level.

NOTE. — Sometimes the hand-level is held on top of a 1 in. × 1 in. × 5 ft. light stick in order to keep the HI more nearly constant.

26. SLOPE-LEVEL (CLINOMETER)

Equipment: Slope-level (clinometer); target rod.

The object of this exercise is to give the student practice with the slope-level, and to show him the range of error that may be expected under conditions which obtain in this exercise.

(1) Measure height of instrumentman's eye above ground on which he stands. (2) Instrumentman and rodman take positions of equal elevation, 100 ft. (paced) apart. (3) Instrumentman sights at target set at 12 ft.; brings bubble to center by turning thumb screw; reads arc; and records both arc and rod reading. (4) Instrumentman repeats (3), sighting at target set at 10 ft., at 8 ft., at 6 ft., at 4 ft., and at 2 ft.

FORM OF NOTES

Dist.	Target setting.	Arc reading.	Computed slope.	Errors.	

(6) *Computation:* Compute slope angle corresponding to each target setting, entering result under heading "Computed slope." Subtract each computed slope from corresponding arc reading, and enter result with proper sign, under heading "Errors." Find index error of instrument.

27. POCKET OR PRISMATIC COMPASS

Equipment: Pocket compass, or prismatic compass.

The object of this exercise is to give practice with the pocket or prismatic compass, and to show the range of uncertainty to be expected in work done under the conditions of this exercise.

(1) Select three intervisible points, *A*, *B*, and *C*, to be the corners of a triangle having sides of from 100 to 300 ft. Select one of the points near a quantity of iron, if practicable. (2) At each of the three points take bearing of each o' the two other points; for example, at *A* take forward bearing *AB*, and back bearing *AC*.

(3) *Notes:* Make a sketch of the triangle; and record each bearing in proper position on sketch. Each bearing should be so placed as to read in direction in which sight is taken. For example, bearing *AB* will be written in direction *AB* of sketch; but bearing *BA* will be written in direction *BA*.

(4) *Computations:* (*a*) Compute interior angles of triangle, using forward bearings *AB*, *BC*, *CA*. (*b*) Compute interior angles, using back bearings *BA*, *CB*, *AC*. (*c*) Compute interior angle at each point by means of the two bearings (one forward, the other back) observed at that point. (*d*) Sum interior angles in each case, and note how much sum differs from 180°. In what way, if any, is the presence of iron at one of the stations indicated by the notes?

28. BAROMETER

Equipment: Mercurial and aneroid barometers, pocket thermometer. Each student must be provided with a watch, which should be compared before beginning work, and a small sheet of profile-paper (Plate A).

Use of the mercurial barometer. — The mercurial barometer is to be observed in its fixed position in the office. (*a*) Read the attached thermometer; this should be done first, as the heat of the observer's body will be sufficient to alter the reading in a short time. (*b*) Bring the surface of the mercury in the cistern to the tip of the ivory index. This is most accurately done by raising the mercury (by means of the large screw at the bottom) until the tip dips slightly into the mercury and forms a slight depression; lower the mercury until, as seen by the reflection of light on the surface of the mercury, the depression just disappears. A magnifier will be of assistance in doing this. Tap the barometer *very gently* to destroy the adherence of the mercury to the glass. (*c*) Raise or lower the slide near the top by means of the screw on the side until a horizontal plane through the lower edge of the slide is just tangent to the upper surface of the *meniscus* at the top of the column. (*d*) Read the vernier; the smallest reading is .002″, but odd thousandths may easily be estimated if necessary. (*e*) To eliminate the effect of temperature on the height of the column, reduce the readings to 32° F., by means of the accompanying Table I, interpolating when necessary. For example, assume readings of 29″.632 and 74°; interpolating between 29.5 and 30.0 for 74° we obtain a correction of .121, which is always *subtractive* for the range of values given in the table. 29.632 − .121 = 29.511. (*f*) It is preferable to hang the mercurial barometer in a place where, although sheltered from the sun, it may have the same temperature as the *external* air. If this is inconvenient, a thermometer should be placed immediately outside the office (sheltered from the sun) and readings taken when the barometer is read. The reduction to 32° should be determined from the *attached* thermometer, but the correction for temperature should be computed from the readings of the external thermometer and of the thermometer accompanying the aneroid. (*g*) Plot the readings immediately on profile-paper (Plate A), using time as abscissæ (one hour per inch) and reduced barometric readings as ordinates (.040″ of mercury

Form of Notes

Mercurial Barometer Readings.

Time.	Barom.	Att'd therm.	Red. to 32°.	Temp. of external air.	Corr. reading.
2:00 P.M.	29.846	72°	−0.117	63°	29.729
2:15	29.839	71	−0.114	63	29.725
etc.					

Aneroid Barometer Readings

Time.	Place.	Aneroid.	Therm.	Corr. aner.	Corr. merc.
2:10	Office	29″.582	74°	....	29″.726
2:25	Roof	.518	62	29.663	.723
2:40	Bridge	.616	65	.761	.720
2:50	Dock	.632	65	.777	.720
3:05	Office	.576	72	....	.722

per inch). If the curve of pressure is very irregular, it indicates an unstable condition of the atmosphere, or, possibly, very inaccurate readings of the barometer. In either case, the computed elevations will be very unreliable.

Use of the aneroid. — Handle the aneroid with *extreme care*, being especially careful that it is never violently jarred by concussion. Always allow it to lie flat when taking a reading. Tap it with the finger very gently before taking the reading. Ignore the thermometer inside of the aneroid and observe instead the pocket thermometer. If this has been carried in the pocket, allow it to remain in the air (shaded from the sun) long enough to indicate the true temperature of the air. Take readings * (1) beside the mercurial

* The directions are here purely local in their application, but were introduced as being suggestive of methods of obtaining a total difference of elevation of 100 ft. or over, which is sufficient to illustrate the use of these instruments.

Temp. of external air.	Approx. field read.	Approx. office read.	Diff.	Corr. for temp.	Diff. elev.
63°	308	253	+55	+2	+57
62	218	256	−38	−(+2)	−40
60	203	256	−53	−(+2)	−55

barometer; (2) on the roof of the College Building above the Society rooms; (3) on the floor of the South St. bridge; (4) on the edge of the dock immediately underneath the bridge; (5) finally, another reading beside the mercurial barometer. Correct the aneroid readings on the roof, bridge, and dock by the mean of the discrepancies between the aneroid readings in the office and the reduced mercurial readings. Draw a curve through the ordinates of pressures and scale off the amounts of the ordinates for the times when the aneroid readings were taken on the roof, bridge, and dock. Interpolate for temperature at these times. With these reduced and computed values for the pressures and temperatures, the differences of elevation may be computed from Tables II and III. For example, the calculation of the difference of elevation of "Office" and "Roof" is as follows:

From Table II, for 29.663, we have	366 − (6.3 × 9.2)	=	308
Similarly, " 29.723, " "	274 − (2.3 × 9.2)	=	253
Approx. diff. of elevation........................			55 ft.

62° + 63° = 125°. From Table III the coefficient for correction is 0.0262 + (5 × 0.00106) = 0.0315. For this and similar cases where the difference of elevation is small, it is only necessary to obtain from the table an approximate coefficient (*e.g.*, 0.03 in this case) by mere inspection. 0.03 × 55 = 2, the correction, obtained only to the nearest foot. The correction is *positive*, hence the difference of elevation is 57 feet.

TABLE I. — REDUCTION OF BAROMETER READING TO 32° F.

Temp.	Inches.										
	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0
45	−.039	−.039	−.040	−.041	−.042	−.042	−.043	−.044	−.045	−.045	−.046
46	.041	.042	.043	.043	.044	.045	.046	.046	.047	.048	.049
47	.043	.044	.045	.046	.047	.048	.048	.049	.050	.051	.052
48	.046	.047	.047	.048	.049	.050	.051	.052	.053	.053	.054
49	.048	.049	.050	.051	.052	.052	.054	.054	.055	.056	.057
50	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059	.060
51	.053	.054	.055	.056	.057	.058	.059	.060	.061	.062	.063
52	.055	.056	.057	.058	.059	.060	.061	.062	.064	.065	.066
53	.057	.058	.060	.061	.062	.063	.064	.065	.066	.067	.068
54	.060	061	.062	.063	.064	.065	.067	.068	.069	.070	.071
55	.062	.063	.064	.065	.066	.068	.069	.070	.071	.073	.074
56	.064	.065	.067	.068	.069	.070	.072	.073	.074	.075	.077
57	.067	.068	.069	.070	.072	.073	.075	.076	.077	.078	.080
58	.069	.070	.071	.073	.074	.076	.077	.078	.080	.081	.082
59	.072	.073	.074	.075	.077	.078	.080	.081	.083	.084	.085
60	.074	.076	.077	.078	.079	.081	.082	.084	.085	.086	088
61	.076	.077	.079	.080	.082	.083	.085	.086	.088	.089	091
62	.079	.080	.082	.083	.085	.086	.088	.089	.091	.092	.094
63	.081	.082	.084	.085	.087	.088	.090	.091	.093	.095	.096
64	.083	.085	.086	.088	.090	.091	.093	.094	.096	.097	.099
65	.086	.087	.089	.090	.092	.093	.095	.097	.099	.100	.102
66	.088	.089	.091	.093	.095	.096	.098	.099	.101	.103	.105
67	.090	.092	.094	.095	.097	.099	.101	.102	.104	.106	.108
68	.093	.094	.096	.098	.100	.101	.103	.105	.107	.108	.110
69	.095	.097	.099	.100	.102	.104	.106	.107	.110	.111	.113
70	.097	.099	.101	.103	.105	.106	.109	.110	.112	.114	.116
71	.100	.101	.103	.105	.107	.109	.111	.113	.115	.117	.119
72	.102	.104	.106	.108	.110	.112	.114	.116	.118	.120	.122
73	.104	.106	.108	.110	.112	.114	.116	.118	.120	.122	.124
74	.107	.109	.111	.113	.115	.117	.119	.121	.123	.125	.127
75	.109	.111	.113	.115	.117	.119	.122	.124	.126	.128	.130
76	.111	.113	.116	.118	.120	.122	.124	.126	.128	.130	.133
77	.114	.116	.118	.120	.122	.124	.127	.129	.131	.133	.136
78	.116	.118	.120	.122	.125	.127	.129	.131	.134	.136	.138
79	.118	.120	.123	.125	.127	.129	.132	.134	.137	.139	141
80	.121	.123	.125	.127	.130	.132	.135	.137	.139	.141	.144
81	.123	.125	.128	.130	.132	.134	.137	.139	.142	.144	.147
82	.125	.128	.130	.132	.135	.137	.140	.142	.145	.147	.149
83	.128	.130	.133	.135	.138	.140	.142	.145	.147	.149	.152
84	.130	.132	.135	.138	.140	.142	.145	.147	.150	.152	.155
85	.132	.134	.137	.140	.143	.145	.148	.150	.153	.155	.158
86	.135	.137	.140	.142	.145	.148	.150	.153	.155	.158	.161
87	.137	.139	.142	.144	.148	.150	.153	.155	.158	.161	.163
88	.139	.142	.145	.147	.150	.152	.155	.158	.161	.163	.166
89	.142	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169
90	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169	.172
91	−.146	−.149	−.152	−.155	−.158	−.160	−.163	−.166	−.169	−.172	−.175

Table II. — Barometric Elevations *

B	A	Diff. for .01.	B	A	Diff. for .01.	B	A	Diff. for .01
Ins.	Ft.	Ft.	Ins.	Ft.	Ft.	Ins.	Ft.	Ft.
20.0	11,047	−13.6	23.7	6,423	−11.5	27.4	2,470	−9.9
20.1	10,911	13.5	23.8	6,308	11.4	27.5	2,371	9.9
20.2	10,776	13.4	23.9	6,194	11.4	27.6	2,272	9.9
20.3	10,642	13.4	24.0	6,080	11.3	27.7	2,173	9.8
20.4	10,508	13.3	24.1	5,967	11.3	27.8	2,075	9.8
20.5	10,375	13.3	24.2	5,854	11.3	27.9	1,977	9.7
20.6	10,242	13.2	24.3	5,741	11.2	28.0	1,880	9.7
20.7	10,110	13.1	24.4	5,629	11.1	28.1	1,783	9.7
20.8	9,979	13.1	24.5	5,518	11.1	28.2	1,686	9.7
20.9	9,848	13.0	24.6	5,407	11.1	28.3	1,589	9.6
21.0	9,718	12.9	24.7	5,296	11.0	28.4	1,493	9.6
21.1	9,589	12.9	24.8	5,186	10.9	28.5	1,397	9.5
21.2	9,460	12.8	24.9	5,077	10.9	28.6	1,302	9.5
21.3	9,332	12.8	25.0	4,968	10.9	28.7	1,207	9.5
21.4	9,204	12.7	25.1	4,859	10.8	28.8	1,112	9.4
21.5	9,077	12.6	25.2	4,751	10.8	28.9	1,018	9.4
21.6	8,951	12.6	25.3	4,643	10.8	29.0	924	9.4
21.7	8,825	12.5	25.4	4,535	10.7	29.1	830	9.4
21.8	8,700	12.5	25.5	4,428	10.7	29.2	736	9.3
21.9	8,575	12.4	25.6	4,321	10.6	29.3	643	9.3
22.0	8,451	12.4	25.7	4,215	10.6	29.4	550	9.2
22.1	8,327	12.3	25.8	4,109	10.5	29.5	458	9.2
22.2	8,204	12.2	25.9	4,004	10.5	29.6	366	9.2
22.3	8,082	12.2	26.0	3,899	10.5	29.7	274	9.2
22.4	7,960	12.2	26.1	3,794	10.4	29.8	182	9.1
22.5	7,838	12.1	26.2	3,690	10.4	29.9	91	9.1
22.6	7,717	12.0	26.3	3,586	10.3	30.0	0	9.1
22.7	7,597	12.0	26.4	3,483	10.3	30.1	−91	9.0
22.8	7,477	11.9	26.5	3,380	10.3	30.2	181	9.0
22.9	7,358	11.9	26.6	3,277	10.2	30.3	271	9.0
23.0	7,239	11.8	26.7	3,175	10.2	30.4	361	9.0
23.1	7,121	11.7	26.8	3,073	10.1	30.5	451	8.9
23.2	7,004	11.7	26.9	2,972	10.1	30.6	540	8.9
23.3	6,887.	11.7	27.0	2,871	10.1	30.7	629	8.8
23.4	6,770	11.6	27.1	2,770	10.0	30.8	717	8.8
23.5	6,654	11.6	27.2	2,670	10.0	30.9	805	−8.8
23.6	6,538	−11.5	27.3	2,570	−10.0	31.0	−893	
23.7	6,423		27.4	2,470				

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Table XI.

Table III. Coefficients for Corrections for Temperature and Humidity.*

$t+t'$	C	Diff. for 1°.	$t+t'$	C	Diff. for 1°.	$t+t'$	C	Diff. for 1°.
0°	−0.1024	10.9	60°	−0.0380	10.7	120°	+0.0262	10.6
10	0.0915	10.9	70	0.0273	10.7	130	0.0368	10.4
20	0.0806	10.8	80	0.0166	10.8	140	0.0472	10.3
30	0.0698	10.6	90	−0.0058	10.7	150	0.0575	10.2
40	0.0592	10.6	100	+0.0049	10.7	160	0.0677	10.2
50	0.0486	10.6	110	0.0156	10.6	170	0.0779	10.0
60	−0.0380		120	+0.0262		180	+0.0879	

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Tables I, IV.

PART III

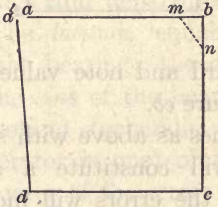
SURVEYING METHODS

Exercises devised to give the student familiarity with a number of methods commonly used in surveying, and, at the same time, additional practice in manipulating and reading instruments.

29. LAYING OUT RIGHT ANGLES WITH TAPE

Equipment: Steel tape, 11 pins, 2 bobs.

1. Select an area where a square about 250 ft. on a side can be laid out. Select *a* and *b* *about* 250 ft. apart. Lay out a right angle at *b*, using the "3, 4,

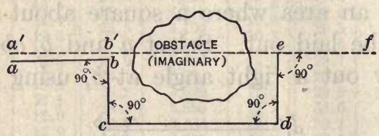


5 method." Measure mb = 30 ft. Fix the zero of the tape at *b* (fastening it with a marking-pin) and have one man hold the 100-foot mark at *m*. Make a loop by placing the 40-foot mark on the 50-foot mark, and swing both marks together until both sections of the tape are taut; and the angle *mbn* will then be a right angle.

Measure off $bc = ab$. Lay out a right angle at *c* and make $cd = ab$. Lay out a right angle at *d* and make $da' = ab$. Measure aa' and compute the "error of closure" (which equals aa' divided by the total perimeter).

The accuracy of the work will depend largely on having the point m exactly on the line ab and on establishing c exactly on the line bn produced, and on taking similar precautions at the other corners. This accuracy may be promoted, especially on rough ground, by sighting with the aid of a plumb-bob line. Similar precautions should be taken in the second part of this problem.

2. To chain a line through an obstacle (imaginary in this case). Select two points a and b as in figure. Lay off right angles at b, c, d, and e, making $bc = de$.



Produce ef backward and note value (if anything) of bb' and aa'. Measure eb.

NOTES. — Sketches as above with statements of the required results will constitute a sufficient report. The magnitude of the errors will indicate how much reliance can be placed on such methods.

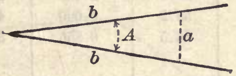
30. OBLIQUE ANGLES WITH TAPE

Equipment: Steel tape, 2 line rods, 11 pins.

Set five marking-pins as witnesses near five designated hubs, which should be about 300 ft. apart and form an irregular pentagon.

1. Measure all the angles by the method suggested in the figure, using the formula $\sin \frac{1}{2} A = \frac{a}{2b}$.

If the angle is much greater than 90°, it may be more convenient to produce one of the sides and measure the supplement of the angle (*e.g.*, angle 3 in the figure). Note whether the sum of all the interior angles is nearly 540°.



2. Measure the sides. Assume one side as meridian, and from the measured angles compute the corresponding bearings of the other sides. For example, the bearing of 3 . . . 4 is S 60° 06′ E; *i.e.*, the bearing of 4 . . . 3 is N 60° 06′ W. The angle at 4 is 87° 16′, which means that 4 . . . 5 runs in a direction 147° 22′ (the sum) from North or 32° 38′ from South. Therefore the bearing of 4 . . . 5 is S 32° 38′ W. Compute the latitudes and departures and then the error of closure. The *latitude* equals the distance times the *cosine* of the bearing; the *departure* equals the distance times the *sine* of the bearing. The *error of closure* is the quotient (generally expressed as a fraction whose numerator is one) of the square root of the sum of the squares of the errors of the latitudes and of the departures, divided by the total perimeter.

(For Form of Notes, see pp. 86 and 87.)

Stationed lines. — On some transit lines — for example, that of railroad preliminary survey — it is customary to establish "stations" at 100-foot intervals, and number them continuously forward from sta. 0 (station zero, the initial point).

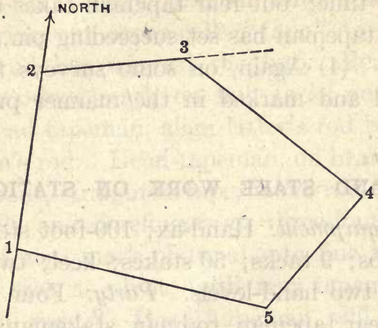
On some lines the transit points are marked temporarily only, by means of pins; but usually transit points and initial and final points are each marked by a "hub," which is a stake about two inches square

FORM OF NOTES

Angle.	b	a	$\frac{a}{2b}$	$\frac{1}{2}A$	A
1	40	58.7	0.734	47° 13′	94° 26′
2	40	62.8	0.785	51° 43′	103° 26′
3	40	29.5	0.369	21° 40′	(sup.) 136° 40′
4	40	55.2	0.690	43° 38′	87° 16′
5	40	68.5	0.856	58° 52′	117° 44′
					539° 32′
Course.	**Bearing. (meridian assumed.)**	**Distance.**	**Latitude.**	**Departure.**	
1......2	N	252.8	+252.8	....	
2......3	N 76° 34′ E	204.6	+ 47.5	+199.0	
3......4	S 60° 06′ E	320.7	−159.9	+278.0	
4......5	S 32° 38′ W	207.2	−174.5	−111.7	
5......1	N 85° 06′ W	363.8	+ 31.1	−362.5	
		1349.1	+331.4 −334.4	+477.0 −474.2	
			3.0	2.8	

and from one to two feet long driven plumb and till its top is within half-inch of ground. A tack is set in hub top to mark precise point. One foot (estimated) to the left (to the right, according to some surveyors) is driven a "guard stake" or "marker" on which station-and-plus of hub has been marked with keel from top downward, in such manner that the number faces hub, and stake leans about 15° toward the hub. The guard stake is like stakes used to mark stations between hubs: about one inch thick, two inches wide at the top, pointed, and two feet or less in length.

The method of marking station points, other than those at which hubs are set, is uniform throughout any one survey, though different methods are in use,



$$\text{Error of closure} = \frac{\sqrt{3.0^2 + 2.8^2}}{1349.1} = \frac{1}{329}$$

— among them the following: (1) On some surveys head tapeman sets pin at each station and rear tapeman pulls it, leaving no station mark. (2) On some surveys no pins are used; head tapeman marks station point by pressing rod into ground; stakeman at once drives numbered station-stake into hole made by rod; rear tapeman holds end of tape at center of stake (center determined by eye) in laying off next tape length. For more precise work the tape is again pulled taut after stake is driven, and precise station point is marked on top of stake by means of pencil, rod point, or ax blade; and rear tapeman holds end of tape on mark when laying off succeeding tape length. (3) On some surveys head tapeman sets pin at station point; stakeman then drives numbered station-stake six inches (estimated) in rear of pin. Thus true station point will lie a half-foot ahead of stake which marks it. Each hub, on the contrary, is driven precisely at station point. Stakeman does not disturb pin at any time; but rear tapeman takes up this pin, after head tapeman has set succeeding pin, and carries it forward. (4) Again, on some surveys the stations are located and marked in the manner prescribed in Exercise 31.

31. TAPE AND STAKE WORK ON STATIONED LINE

School Equipment: Hand-ax; 100-foot steel tape; 11 pins; 9 hubs; 9 tacks; 50 stakes; keel; two line rods; two bobs; two hand-levels. *Party:* Four men: head tapeman, rear tapeman, rodman, stakeman. The men will change positions at each eighth or tenth hub.

The *object* of this exercise is to give practice in the routine of locating and marking stations. *Location:* The line will be 4000 ft. long; consist of courses each approximately 250 ft. long; and follow favorable ground.

(1) At selected initial point, stakeman will drive hub plumb, and until top is within a half-inch of ground, and drive tack in top. He will then print, with keel, in large plain figures, on one face of a stake, the station number, "0 + 00"; beginning near top and writing in direction top to point. (Pass keel over each figure, pressing it into the wood, as many times as may be necessary to make a distinct and lasting record.) He will drive stake on left side of hub, and one foot (estimated) distant therefrom, in such manner that number will face hub and top will lean about 15° toward hub. (2) Stakeman will then mark another stake with figure "1," writing from top to point; and go forward with head tapeman, carrying stakes

and hand-ax. (3) Rodman will take position about 250 ft. in advance, and hold rod plumb on a chosen point until hub is driven at the point. (4) Head tapeman, carrying pins and rod, will take zero end of tape and walk toward rodman until rear tapeman, who is paying out tape, calls "tape" to give notice that rear end has reached the hub. Rear tapeman will hold 100-foot mark on hub tack, and, by hand signal to head tapeman, align latter's rod between hub and rodman's rod. Head tapeman, on hearing "tape," will face about, straighten tape, move rod (held plumb with thumb and forefinger at three-fourths height, and against zero mark of tape) into line by swinging about hub tack as pivot, until rear tapeman calls or signals "all right." Head tapeman will then carefully straighten and align tape, and shift rod, if necessary, to bring it against zero mark of tape. Thereupon, he will force point of rod into ground, and withdraw it, leaving mark at which stake 1 will be driven as soon as pin is set. (5) Head tapeman will then swing tape end aside about six inches, carefully pull again for distance and set pin at zero mark. (6) Stakeman will at once set stake 1 in hole left by rod, and with its number facing hub; and drive stake plumb and until it stands firm. (7) While stake is being driven, tapemen will move forward, each carrying his end of tape, until rear tapeman, on nearing station 1, calls "tape." Rear tapeman will note correctness of number on each driven stake as he comes to it, and call it out to stakeman who will then call back the number of stake he is about to drive.

(8) The work of stakeman and of head tapeman will be same at station 2 as at station 1; and rear tapeman meanwhile at station 1 will repeat his work of station 0, except that instead of holding at hub tack he will

hold at pin left by head tapeman; and on starting forward for station 2 he will pull this pin and carry it with him. Thus the work goes forward. (9) When head tapeman and stakeman come to rodman, stakeman will at once drive hub and tack at point on which rodman has been holding his rod; and drive guard stake, marked with station-and-plus of hub. (10) Tapemen will measure distance from this hub back to preceding station pin, in manner described in (5) (*a*) (*b*) of Exercise 2. (11) Rodman, on giving way to stakeman at second hub will go forward, along a chosen route, another 250 ft. (estimated), and select point for third hub; hold rod plumb on selected point to guide tapemen, as on preceding course. (12) The work from this point on will involve no new manipulations. Each succeeding course will be taken about 250 ft. long; and the line will continue to sta. 40.

(13) *Form of notes:* Notes should be taken all in one book, and later copied into other books of party. Run notes up the page. Enter station numbers in the first column; and immediately to left of station-and-plus of each hub, make small circle with a dot in its center — symbol for hub on stationed line.

(14) No computations are required for this exercise.

Remark. — The line staked in this exercise may be used for the supplementary work of the following exercise.

32. DIFFERENTIAL LEVELS

Equipment: Level, level-rod; and, if required, 2 hubs, 1 peg or TP spike, and hatchet. (Omit hubs if hubs or BM's have already been established. The peg or spike is to be driven to serve as TP and then pulled.)

The object of this exercise is to make the student acquainted with the customary method of using the engineer's level to find the difference in elevation between two given points.

(1) Set two hubs about $\frac{1}{4}$ to $\frac{1}{2}$ mile apart, at points differing in elevation by 50 ft. or more if this be practicable. The problem is to find difference of elevation of the two hubs, the elevations of intermediate points not being desired for themselves. Assume elevation of hub 1 to be 100.00 unless otherwise directed by the instructor. (2) Set up level so that a reading may be taken on level-rod when it is held on first hub. (To make a minimum number of "set-ups," the level should be set up as far away from hub as practicable, up to limit of about 200 ft., and yet permit reading rod set on hub. If natural slope is very steep, it may be difficult to make this distance much greater than 10 feet, which is about lower limit of distinct focusing with telescope.) (3) Levelman makes direct reading of rod to thousandths, and enters reading in (−) column opposite "Hub 1," in the "Sta." column. He then takes duplicate reading, using target; and if target reading checks direct reading within 0.004 ft., he calls direct reading O.K., and proceeds to read rod on first TP. (4) Rodman finds (or drives peg or TP spike to serve as) a suitable TP, which should preferably be as far from the level as the level is from hub, thus balancing horizontal distances of backsight and foresight. (TP may be any firm object which has flat or convex upper surface; *e.g.*, a protruding point of firm well-bedded stone.) (5) Levelman takes a direct reading (to thousandths) on rod held on turning point and enters it in (+) column, opposite "TP" of "Sta." column. He then checks reading by taking target reading as before. (Difference between (+) reading and (−) reading is, of course, the difference in elevation between initial hub and turning point.

N.B. — No mention of horizontal position of the instrument is to be made in tabulated notes.) (6) Remainder of work consists of a number of repetitions of operations described in (2)–(5). Levelman, after taking (+) reading on one point (initial hub, or TP) and (−) reading on succeeding TP, makes new set-up farther on toward final hub; and proceeds as before. At last set-up he takes a (−) reading on final hub. (7) He takes difference between sum of (+) sights and sum of (−) sights. This is difference in elevation between the two hubs. (8) Re-run the levels, starting at final hub. (9) Compare results of the two runs; and if possible form an opinion, based on your own experience in preceding exercise, as to whether or not the difference between the two results is unreasonably large.

FORM OF NOTES

Sta.	(+) Rod.	HI	(−) Rod.	Elev.
Hub 1	(1) 5.208	(*b*)		(*a*) 100.000
TP 1	(3) 6.755	(*d*)	(2) 3.246	(*c*)
TP 2	(5)	(*f*)	(4)	(*e*)

The numerals in parentheses in Form of Notes indicate order in which rod readings are entered in notes. Letters in parentheses indicate the order in which elevations are entered in notes. These numerals and letters are not used in field book, and are inserted here merely to make notes clear to beginner. The required computations are made in the following order:

$$(a) + (1) = (b)$$
$$(b) - (2) = (c).$$
$$(c) + (3) = (d)$$
$$(d) - (4) = (e)$$
$$(e) + (5) = (f)$$

Remark. — On some lines of differential leveling, levelman reads rod to nearest 0.01 ft. only, and without making check-reading with target; but the beginner should not omit check-reading with target even when reading only to hundredths.

33. PROFILE LEVELS

Equipment: Dumpy or wye-level; extension rod with target; hatchet; 15 stakes. (If stakes for this experiment have already been set, stakes may be omitted.)

Location: Instructor will select such a strip of ground, 1200 or more feet long, for this exercise, as to make it practicable to carry out the following directions to the letter.

(1) On chosen line set stakes 100 ft. apart, laying off the distances by pacing. (Stakes should be out of way of pedestrians and vehicles.) (2) Mark stakes in order, beginning at one end of line, 0, 1, 2, 3, ... (3) Establish BM near each end of line, if such BM have not already been established. (4) Set up the level where it will command sight of the rod (extended if necessary) held on BM near sta. 0, and stas. 0 and 1.

FORM OF NOTES

Sta.	(+) Rod (BM & TP).	HI	(−) Rod (BM & TP).	(−) Rod (Ground).	Elev.
BM	(1) 4.623	(*b*) 104.623			(*a*) 100.000
0 + 00				(2) 5.5	(*c*) 99.1
1 + 00				(3) 4.2	(*d*) 100.4
TP	(5) 5.375	(*f*) 106.667	(4) 3.331		(*e*) 101.292
2 + 00				(6) 2.6	(*g*)

[Numerals in parentheses in Form of Notes indicate the order in which rod readings are entered in notes. Similarly, letters indicate order of entering elevations. These parenthetical numerals and letters are not used in field book, and are inserted here merely to facilitate explanation. The computations required to reduce the readings to useful terms are indicated by the following equations:

$$(a) + (1) = (b)$$
$$(b) - (2) = (c)$$
$$(b) - (3) = (d)$$
$$(b) - (4) = (e)$$
$$(e) + (5) = (f)$$
$$(f) - (6) = (g)$$

Notice that the ground elevations, like the ground-rod readings, are written to the nearest 0.1 ft. only.]

(5) Write "BM" in column 1; and on opposite page and same line, write description and location of BM. Write elevation (given or assumed) of BM, in "Elev." column opposite "BM." (6) Read rod (to thousandths, without target) held on BM. Record reading in (+) column, opposite "BM." Take second reading, with target, and if this checks direct reading within 0.004 ft., call direct reading O.K. Add reading to elevation of BM, and write sum in "HI" column on line with "BM." (Until instrument is moved, each succeeding rod reading will be placed in one or other of (−) columns.) (7) Write "0 + 00" in "Sta." column on line next below "BM." Read rod (to nearest tenth without target) held on the ground at sta. 0 and record reading in fourth column opposite "0 + 00." Subtract this reading from HI; and enter remainder (to nearest tenth), in "Elev." column, opposite "0 + 00."

(8) Write "1 + 00" in "Sta." column, next below "0 + 00." Read rod (direct, to nearest tenth) held

on ground at sta. 1. Enter reading in fourth column opposite "1 + 00." Subtract this reading from HI, setting remainder (to nearest tenth) in "Elev." column, opposite "1 + 00."

(9) Write "TP" in "Sta." column, next below "1 + 00." Hold rod on top of stake at sta. 1. Read rod (to thousandths direct); and enter reading in fifth column, opposite "TP." Check direct reading by taking target reading, holding rod on top of stake as before. If target reading checks direct reading within 0.004 ft., call direct reading O.K. Subtract reading from HI and set remainder (retaining thousandths) in "Elev." column, opposite "TP."

(10) Set up the instrument, at or opposite, sta. 4 + 50. (11) Read rod (direct, to thousandths) held on TP (top of stake 1, as before). Enter reading in (+) column, opposite "TP." Take second reading, using target. If target reading checks direct reading within 0.004, call direct reading O.K. (11) Add reading in (+) column to elevation in "Elev." column (both opposite "TP"); and enter sum (retaining thousandths) in "HI" column, opposite "TP." (This sum is new HI, or elevation of line of sight, for present set-up. All succeeding ground rod-readings for this set-up are to be entered in fourth column, and subtracted from this HI.)

(12) Write "2 + 00" in "Sta." column, next below "TP." Read rod (to nearest tenth, direct) held on ground at sta. 2. Enter reading in fourth column, opposite "2 + 00." Subtract this reading from computed HI next above; and enter remainder (to nearest tenth), in fifth column, opposite "2 + 00."

(13) Write "2 + 50" in "Sta." column, on line below "2 + 00." Read rod (to nearest tenth) held on ground at sta. 2 + 50 (the plus being paced); and

enter reading in fourth column, opposite "2 + 50." (This reading at 2 + 50 is to illustrate the taking of an intermediate point where slope of ground changes.) From this point on, take rod readings at only such intermediate points as are required to fairly represent the profile.

(14) When elevation of final station has been taken, write "BM" in "Sta." column, on line below last station entered. Describe BM on right-hand page, opposite "BM" just written. Read rod (direct to thousandths) held on this BM, and enter reading in fifth column, opposite "BM" last written. (Check by target reading, as before on TP's.) Subtract reading from HI (next above) and enter remainder (retaining thousandths) in "Elev." column, opposite "BM" last written.

(15) Run line of check levels (differential levels) from terminal BM back to initial BM, paying no attention to stations. (See preceding exercise, 32.)

(16) Rodman will keep check notes, using form of Exercise 32 for all BM and TP readings.

(17) *Computations:* The routine computations have been carried out step by step as the readings have been taken. Further, compute the difference in elevation between the two BM's, from notes of check levels. Check the computations by method of Exercise 32.

(18) Make profile of the line or lines over which levels have been run. Use 10-in. Plate A profile paper, and, so far as they are applicable, follow the I.C.C. Specifications,* remembering that the model drawings

* "Specifications for Maps and Profiles prescribed by the Interstate Commerce Commission, in accordance with section 19A of the Act to Regulate Commerce." For sale at 15 cents by Superintendent of Public Documents, Washington, D. C.

therein are one-half size. Make all figures and capital letters (except in title) 3/20 in. high and lower-case letters, like a, c, e, . . . , 2/20 in. high on this profile.

Remark. — After performing this exercise, the student should be given a longer line of profile levels to run — from one-half mile upward, according to the time at disposal; and be required to plot profile. These levels may be run over staked line of Exercise 31. For this work, only general directions should be needed; the sequence of operations and the correct entries of readings have been learned in the work described above. For this work it is suggested that the following form of notes be used.

Alternative Form of Profile Notes

Sta.	Gr'd elev.	Gr'd rod (−)	HI	TP or BM rod. (+)	TP or BM rod. (−)	TP or BM elev.
BM			(*b*) 104.623	4.623		(*a*) 100.00
0 + 00	(*c*)	(2) 3.5				
1 + 00	(*d*)	(3)				
TP			(*f*) 106.667	(5) 5.375	(4) 3.331	(*e*) 101.292
2 + 00	(*g*)	(6) 2.6				

The figures and letters in parentheses are inserted in the form to indicate the order in which rod readings and elevations, respectively, are entered in the notes; and they are not to be inserted in the field book. The essential advantage of this form lies in the fact that "Sta." column and "Ground Elev." column lie adjacent to each other, and this is a distinct convenience for the profile plotter. The order of computing the quantities is indicated by the following equations:

$$(a) + (1) = (b)$$
$$(b) - (2) = (c)$$
$$(b) - (3) = (d)$$
$$(b) - (4) = (e)$$
$$(e) + (5) = (f)$$
$$(f) - (6) = (g)$$

Remark. — It is a common custom to read the rod on TP's and BM's only to the nearest 0.01 ft.; and to omit the check by target reading. The beginner, however, cannot profitably follow this custom, because of his liability to blunders.

34. TRAVERSE BY AZIMUTH

Equipment: Transit; bob; magnifying glass; 3 stakes; 3 tacks; hatchet.

The object of this exercise is to give practice in the use of the azimuth method of running a transit line, and to show the range of error to be expected from work done under the conditions given.

(1) Set stakes at *A*, *B*, and *C*, forming a triangle about 300 ft. on a side, and drive a tack in the top of each stake. (To give sight, the helper will hold a pencil plumb on each tack in turn.)

(2) Set up transit at *A*. (3) Unclamp needle. (4) Set vernier (under the ocular) at 0° 00′. (5) As soon as needle comes to rest, swing by LM until needle reads North (*i.e.*, until "N" scratch on compass ring is precisely under North end of needle). (6) Loosen UM; point at *B* with UM; read vernier; and read needle to nearest 5 minutes. (7) Clamp needle; unclamp LM.

(8) Set up at *B*. Unclamp needle. Reverse telescope. (Read vernier to make sure that reading is same as when pointing at *B* from *A*.) Unclamp LM.

(9) With LM, point at *A*. Unclamp UM. Return telescope. (10) With UM, point at *C*. Read vernier and needle. (11) Clamp needle. Unclamp LM.

(12) Set up at *C*. Unclamp needle. Reverse telescope. See that vernier has not slipped in moving from *B* to *C*. Unclamp LM. (13) With LM, point at *B*. Unclamp UM. Return telescope. (14) With UM, point at *A*. Read vernier and needle. (15) Clamp needle. Unclamp LM.

FORM OF NOTES

Transit at	Pointing at	Vernier reading.	Needle reading.	Calc'd. bearings.	
A	North	0° 00′	North		
	B	137 42	S 42° 15′ E		
B	*A*	137 42			Tel. rev.
	C	227 13	S 47 30 W		
C	*B*				Tel. rev.
	A				

(16) Set up at *A*. Unclamp needle. Reverse telescope. See that vernier has not slipped. Unclamp LM. (17) With LM, point at *C*. Unclamp UM. Return telescope. (18) With UM, point at *B*. Read vernier and needle. (19) Clamp needle. (20) *Computations:* Compute bearings from azimuths and enter in "Calc'd bearings" column. Compute interior angles from azimuths, and note whether their sum is 180°.

Remark. — Another way of orienting the transit at each set-up after the first, is the following: Instead of (8) and (9) as given above, substitute: (8′) Set up at *B*. Unclamp needle. Set vernier at azimuth $AB \pm 180°$. (9′) With LM point at *A*. Unclamp UM. (Telescope is not reversed when using this method of orienting.)

35. TRAVERSE BY DOUBLE DEFLECTION

Equipment: Transit; bob; magnifying glass; 3 stakes; 3 tacks. (If the stakes and tacks set for the preceding exercise are available, omit stakes and tacks from equipment here.)

The object of this exercise is to acquaint the student with the double-deflection method of running a transit line, which was devised to detect blunders and reduce the range of uncertainty in results.

(1) Set stakes at *A*, *B*, and *C*, forming a triangle about 300 ft. on a side. (Those set for the preceding exercise will serve if they are available.)

(2) Set up transit at *A*. Unclamp needle. (3) Set vernier *A* (under the ocular) at 0° 00′, with telescope normal. (4) Unclamp LM, and with LM point at *C*. (5) Read needle and reverse telescope. (6) Unclamp UM. Point at *B* with UM. Read vernier *A*. Record, adding R or L. (7) Unclamp LM. Point at *C* with LM. (8) Return telescope. Unclamp UM, and with UM point at *B*. (9) Read vernier *A*, obtaining the double deflection angle; and record. Read needle. (10) Clamp needle. Unclamp UM.

(11) Set up at *B*. Repeat (3)–(10), backsighting on *A* and foresighting on *C*.

(12) Set up at *C*. Repeat (3)–(10), backsighting on *B* and foresighting on *C*.

(13) Men change positions, and repeat (3)–(13).

(14) *Computations:* Fill in "Calc'd Bearing" column, as fast as readings are made, so as to obtain an immediate check. Find sum of interior angles of triangle, and note by how much it differs from 180° 00′.

FORM OF NOTES

Notes run up the page.

Sta.	Dist.	Calc'd bearing.	Defl. R or L	Needle FS	Needle BS	Double defl.
A						
C		S 59° 57′ W	109° 50′ R	S 60° 05′ E	N 49° 45′ W	219° 40′
B		S 49 53 E	100 07 R	S 50 00 E	S 30 15 W	200 14
A		N 30 00 E	150 00 R	N 30 00 E	N 60 00 E	300 00
C						

36. STADIA TRAVERSE AND LEVELS

Equipment: Transit with stadia wires; stadia rod; hand-ax; 4 stakes. (Plumb-bob and tacks are not needed. A set-up without bob is close enough when all distances are read to the nearest foot only.)

The object of this exercise is to give the student practice in the routine work (*a*) of running a traverse and finding elevation differences by the transit and stadia method; (*b*) of reducing the stadia notes; and (*c*) of plotting a traverse.

FIELD WORK

(1) Set the four stakes to mark corners *A*, *B*, *C*, *D*, of a quadrilateral such that (*a*) length of each side is between 200 and 500 feet, and (*b*) transit set at each corner will have a full sight of rod held at each adjacent corner.

(2) Set up transit at *A*. (It suffices in this survey to have transit center within 0.2 ft. or 0.3 ft. of vertical through station point. No bob is needed for this. Dropping a pebble from the plumb-bob hanger will test set-up.) (3) Measure HI (= V distance between top of stake and center of H axis of telescope) to nearest 0.1 ft., with stadia rod; and send rodman to *B*. (4) Unclamp needle, and set vernier *A* at 0° 00′. (5) Unclamp LM, and with LM bring needle to read N.

FORM OF NOTES

Transit point / HI	Rod pt.	Dist.	Bearing / Azim.	Vert. ang.	Elev. diff.	Elev.	Remarks and sketch.
A						1000.0	
4.8	*B*	625	S 20° 45′ E / 159–15	+ 3° 46′			
B							
5.1	*A*	626	N 20° 40′ W / 200–45	− 3° 48′			
	C	532	S 82° 35′ E / 107–21	+ 1° 34′			

(6) Unclamp UM.

(7) Sight on rod at *B* with UM. (8) Clamp M wire at reading = HI on rod at *B*. (9) With VM move L wire to nearest foot-mark on rod. (10) Read U wire to nearest 0.01 ft., subtract L wire reading, and record difference (omitting decimal point) under "Dist.," opposite "*B*." (11) With VM move M wire back to HI. (12) Read V circle to nearest minute; and record with proper sign under "Vert. ang.," opposite "*B*." (13) Read vernier A to nearest 5 minutes. (Read H limb to nearest half-degree and estimate minutes. It is not necessary to read vernier itself.) (14) Record reading under "Azim.," opposite "*B*." (15) Read needle to nearest 5 minutes; and record bearing immediately above azimuth just recorded. (16) Compare bearings and azimuth just recorded. If they do not fairly agree read azimuth and needle again, and make sure that both readings are correct before proceeding to next step. (17) Unclamp UM and VM. (18) Clamp needle.

(19) Set up transit at *B*. (20) Measure HI. (21) Send rodman to *A*. (22) Unclamp needle. (23) Set vernier A at azimuth of *AB* ± 180° 00′, and unclamp

LM. (24) Sight on rod at A with LM. (25) Read and record distance as before; and send rodman to C. (26) Read and record vertical angle and azimuth as before. (27) Compare readings from B to A with readings from A to B. Make sure that correct readings are obtained before taking next step. (28) Transit at C and at D: The transit work at each of these stations is the same as at B. After completing work at D move transit to A again. (29) Set up at A; measure HI; send rodman to D; sight on D with vernier A set at azimuth of $DA \pm 180° 00'$; and take and compare backsight readings as before, sending rodman to B when he is released from D. (30) Sight on D, and read and record azimuth and needle only.

OFFICE WORK

(31) Reduce stadia notes by one of the following methods:

Distance correction. — (32) Find to the nearest foot the subtractive correction, x, either by Eq. 1 or Eq. 2:

$$x = \frac{\text{recorded dist.}}{100} (100 - \text{table dist.}) \quad . \quad . \quad (1)$$

$$x = \frac{\text{recorded dist.}}{100} (0.03) (\text{slope, in degrees})^2 . \quad (2)$$

Take "recorded dist." from stadia notes and "table dist." from a stadia table for corresponding V angle. (100 – table dist. is obtained mentally; and the product may be obtained mentally, or by slide rule, or by abridged multiplication.) Enter x with minus sign immediately to the right of the corresponding recorded distance in the "Dist." column of the stadia notes. Make the indicated subtraction when plotting.

See example below. (Eq. 2 is approximate, but gives correction to nearest foot for distances up to 1000 ft. and V angles up to 16°; and for 1000 ft. and 4°, even when the nearest whole degree is used in the computation.) (33) Take $c + f$ as 1 ft.; and add it to each recorded distance when plotting.

Example. — Given from stadia notes:

Dist.	Vert. Ang.
625	3° 46′

Subtractive correction, x, is, by Eq. 1,

$$x = (625/100)\,(100 - 99.57) = 6.25 \times 0.43$$
$$= 3 \text{ (to nearest foot)}.$$

Or, by Eq. 2,

$$x = (625/100)\,(0.03)\,(3.75)^2 = 6.25 \times 0.03 \times 14$$
$$= 3 \text{ (to nearest foot)}.$$

Enter this 3 with minus sign immediately after recorded distance in field notes, making them appear thus:

Dist.	Vert. Ang.
625 − 3	3° 46′

Then, when plotting, subtract 3 from 625, and add 1 to the remainder, obtaining 623 as corrected horizontal distance to be plotted.

Elevation difference. — Three methods of computing difference in elevation between instrument point and rod point are given below.

(34) *First method.* — Multiply by abridged multiplication or slide-rule the "Diff. Elev." taken from a stadia table for given V angle, by the corresponding recorded distance /100; and add $(f + c)$ sin V angle.

Example. — Given the following stadia notes:

Dist.	Vert. Ang.	
625	3° 46′	$f + c = 1$ (to nearest foot)

For 3° 46′ we find from stadia table " Diff. Elev." =	6.56
Multiply this by 625/100, or	6.25
(Abridged multiplication)	39.4
	1.3
	.3
Obtaining the product	41.0
Add to this $(f + c)$ sin V angle, which we find near bottom of stadia table (opposite " $c + f = 1.00$ " and under " 3° " " Diff. Elev.") to be 0.06, or say	0.1
Obtaining required elevation difference	41.1

Which is now entered with proper sign in "Elev. Diff." column of stadia notes, opposite 625 and 3° 46′.

(35) *Second Method.* — Multiply corrected horizontal distance by natural tangent of corresponding vertical angle, using abridged multiplication or slide rule.

Example. — Given the following stadia notes:

Corrected dist.	Vert. Ang.
623	3° 46′

Multiply corrected distance	623
By nat. tan. 3° 46′	0.658
(Abridged multiplication)	37.4
	3.1
	.5
Obtaining required elevation difference	41.0

This is now entered in "Elev. Diff." column, opposite 3° 46′.

(36) *Third method.* — Use some one of the stadia slide-rules. Enter the computed elevation difference in "Elev. Diff." column, opposite corresponding vertical angle.

(37) Compute and record in field book *elevation* of each stadia station, assuming elevation of A to be 1000 ft. unless otherwise directed.

(38) *Plot traverse* by protractor and scale as follows: (39) Tack sheet of prescribed drawing paper on drawing board. (Do not roll this sheet at any time. File it flat.) Write or letter the draftman's name plainly in upper right-hand corner of sheet. (40) Plot traverse on scale of $1'' = 50'$ if map to this scale will lie wholly within sheet; otherwise use scale of $1'' = 100'$. (For latter scale use edge of engineers' scale divided 10 to the inch.) Usually the map, n, of terminal point, A, plotted from D, will not coincide with initial point A of plot. That is, owing to unavoidable errors of drafting, the plot usually does not close; and this is so even when field notes have been balanced before plotting. (41) *Close the plot* in the following manner: (42) Draw closing line nA, through plotted terminus, n, and initial point, A, aforesaid. (43) Through each intermediate corner — B, C, D — of plot draw line parallel to closing line nA. (44) Then move each corner in direction parallel to nA through a distance which bears to nA same relation which perimeter-distance-from-A-to-the-corner bears to perimeter-of-the-polygon. For example, to move corner C to its new position on map, lay off from plotted C, in direction parallel to nA, the distance $CC' = nA\ (AB + BC) / (AB + BC + CD + DA)$. Join points A and B', B' and C', and so on, and finally D' and A. New polygon $AB'C'D'A$ is corrected map of polygon. (45) In practice the error of closure is so small that the proportions for which formula is given above, need be only roughly approximated; and when computed shift for any corner is so small that adjacent new lines would lie practically on top of old, no shift is made at

that corner, which is permitted to stand as originally plotted. (46) It requires exercise of judgment to determine whether error of closure of plot is so large as to indicate some blunder in field work or in plotting. (47) Do such further work on plot as may be required by instructor.

37. STADIA TOPOGRAPHY

Equipment: Transit with stadia wires; stadia rod.

The object of this exercise is to give the student practice in the work (*a*) of obtaining, by the transit and stadia method, field data required for making a topographic map; (*b*) of reducing stadia notes; and (*c*) of plotting the field notes.

Form of notes. — Use the same headings as in the preceding exercise.

FIELD WORK

(1) Set up the transit at any point which commands entire area to be surveyed. Call this point *A*. (2) Measure HI to nearest 0.1 ft. (3) Send rodman to first rod point to be sighted on.

Selection of rod points in taking topography requires continual exercise of judgment. Choose rod points with an eye to determining positions of natural and artificial features of tract, including relief of ground, with required accuracy and with minimum total number of rod points. To survey straight line — straight fence, for example — select two rod points thereon as far apart as practicable. To survey curved line — for instance, a winding path — select rod points thereon, such that smooth curve drawn through their plotted positions on map will fairly represent curve. To determine the relief, select rod points at points of slope change along axes of valleys, crests of ridges, and lines crossing ridges and valleys approximately at right angles. In other words, one obtains H and V positions of such points as will enable one to plot profiles of ridges and valleys, and characteristic cross-sections thereof.

(4) Unclamp needle. (5) Set vernier A at 0° 00′. (6) Unclamp LM, and with LM bring needle to read North. (7) Unclamp UM. (8) Sight with V wire on rod at rod point 1 with UM. (9) Clamp M wire at reading HI on rod; and with VM move L wire to nearest foot mark on rod. (10) Read U wire to nearest 0.01 ft.; subtract L wire reading; and record difference under "Dist.," opposite "1." (11) With VM move M wire back to HI on rod. (12) Release rodman, who will go to hold his rod on point chosen for rod point 2. (13) Read V circle to nearest minute and record with proper sign under "Vert. Ang.," opposite "1." (14) Read vernier *A* to nearest 5 minutes; and record under "Azim.," opposite "1." (15) Read needle to nearest 5 minutes; and record bearing immediately above azimuth just recorded. (16) Compare bearing and azimuth, and make sure that both readings have been correctly made before proceeding to next step. (17) Unclamp VM and UM. (18) Sight on rod at rod point 2, and proceed as before. (19) Connect by dotted line on sketch each pair of points between which the ground is considered to be uniform in slope. Represent roughly to scale all the natural and artificial features on the sketch, such as water courses, fences, roads, buildings, etc., as soon as rod points are taken on them, or before. Each rod point number should appear on sketch.

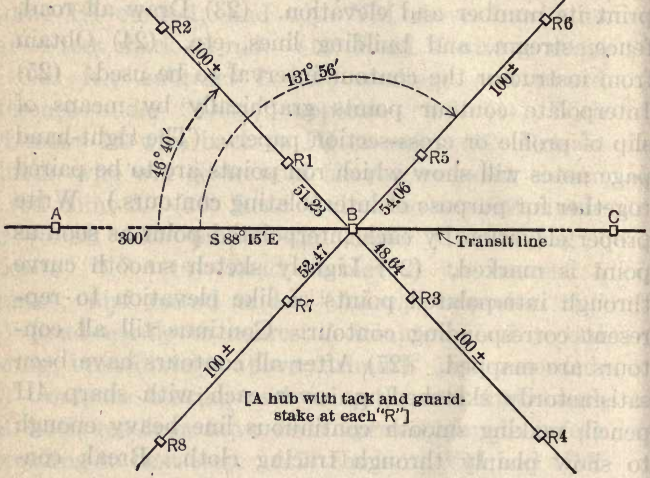
OFFICE WORK

(20) Reduce stadia notes as directed in preceding exercise (Stadia Traverse). (21) Compute elevations on the assumption that elevation of transit point is 1000 ft., unless otherwise directed by instructor. (22) Plot notes on prescribed paper to prescribed scale, laying off angles with protractor. Encircle each plotted point as soon as plotted, and by it write or print its number and elevation. (23) Draw all road, fence, stream, and building lines, etc. (24) Obtain from instructor the contour interval to be used. (25) Interpolate contour points graphically by means of slip of profile or cross-section paper. (The right-hand page notes will show which rod points are to be paired together for purpose of interpolating contours.) Write proper elevation by each interpolated point as soon as point is marked. (26) Lightly sketch smooth curve through interpolated points of like elevation to represent corresponding contour. Continue till all contours are mapped. (27) After all contours have been satisfactorily sketched, go over each with sharp 4H pencil, making smooth continuous line heavy enough to show plainly through tracing cloth. Break contours at buildings, double-lined roads, streams, etc. (28) Letter with pencil, in prescribed manner, the following title: "Stadia Topography. Scale 1″ = ——′. Drawn by ————, ——— 19—." (29) Letter freehand, with pencil, names of features represented on map. (30) If required, make tracing of map. For this, follow directions given under 32–34 of Students' Railroad Survey.

38. TO REFERENCE-OUT A POINT

Equipment: Transit; bob; magnifying glass; hatchet; 11 hubs; 11 stakes; 11 tacks; keel.

The object of this exercise is to give practice in referencing-out a point; *i.e.*, to establish witness points by means of which the original point can be re-established after it has been removed.



(1) Choose two points *A* and *B*, not more than 100 ft. apart. (2) Set a hub at *A*. Drive tack in the top of hub. Print "A" on a stake, with keel, to read from top toward bottom. Drive the stake 1 ft. from hub *A*, measured at right angles to *AB*, with top inclined toward *A*, and lettered side toward *A*. (3) Similarly set hub with tack, and witness stake at *B*. (4) Set up over *A*. Clamp UM. With LM sight on *B*. (5) Set a hub in line at *C*, about 50 ft. beyond *B*. Drive tack in hub *C* in line.

Assume that *ABC* is a transit line; and reference-out the transit station *B* as directed below.

(6) Set up at *B*. Unclamp needle. Set vernier at 0° 00′. (7) With LM sight on *A*. Read needle. (Record all readings on sketch drawn on the right-hand page with the central vertical line of that page representing transit line *AB*.) (8) Loosen UM. Swing to right approximately 45°, and clamp UM. Read vernier and needle. (9) Drive hub in line of sight and about 50 ft. from transit. Align tack in top of hub. Print "*R*1" (*i.e.*, reference point No. 1) on a stake (to read from top downward), and drive the stake as a guard (and witness) stake for hub. (10) Measure distance *B*-*R*1, and record it on sketch. (11) On same line of sight, set hub (with tack, and witness stake marked "*R*2") 100 ft. (estimated) beyond *R*1. Record the distance *R*1–*R*2 as 100 ± (the ± signifying that the distance was estimated, not taped). (12) Reverse telescope and set *R*3 and *R*4 similarly, and record taped distance *B*–*R*3 and estimated distance *R*3–*R*4.

(13) Set vernier at 0° 00′. Unclamp LM, and with LM sight on *R*1. (14) Unclamp UM. With UM swing to the right about 90°; and clamp UM. (15) Read vernier and needle. (16) Set *R*5, *R*6, *R*7, and *R*8; repeating steps (9)–(12).

(17) Unclamp UM, and set vernier at 0° 00′. Measure angle *A*–*B*–*R*5. (This gives a check on angle work.) (18) Clamp needle.

(19) Leave all hubs to serve for the following exercise.

39. TO REFERENCE-IN A POINT

Equipment: Transit; bob; magnifying glass; 10 ft. of string; hatchet; 2 nails; 1 tack; 1 hub; 2 stakes.

Location: Use reference points set in preceding exercise, and notes thereof.

The object of this exercise is to give practice in referencing-in a point, and to show something of the uncertainty attending the result of Exercise 38 and 39 under the given conditions.

(1) Remove hub *B*; fill the hole. (2) Set up at *R*2. (3) Set line of sight on *R*1. (4) Drive two stakes in line of sight, about 6 or 8 ft. apart, — one on one side of *AC* and the other on the other side. (5) Half drive a nail in line of sight in top of each stake. (6) Stretch string taut between the two nails. (7) Set up over *R*6. (8) Set line of sight on *R*5. Drive hub at point where line of sight cuts string (removing string to permit this). (9) Stretch string again from nail to nail. (10) Set tack in hub to mark intersection of string and line of sight.

(11) Test work by measuring carefully distance from *A* to *B* and comparing with recorded distance; and by setting up over *A*, setting line of sight on *C*, and measuring distance by which this line of sight misses tack at *B*. (12) Briefly describe work of referencing-in *B*, and record discrepancies found in (11), for line and distance.

Remarks. — It is evident that the record of referencing-out furnishes data for several other ways of referencing-in. With the greatest care in locating and guarding reference points some of them are liable to be knocked out; hence the need of putting in more reference points and measuring more angles and distances than are required for one way of referencing-in.

In some cases the outer reference point may be a tack on a blazed tree trunk or root, or the edge of a chimney or peak of a building, etc. In general, where the reference point is a hub (with a tack), it should be placed in a protected position, as by a fence or tree or building, etc. Reference points in construction should be so placed that the rising structure will not prevent the use of the reference points by interfering with intervisibility.

40. TOPOGRAPHIC CROSS-SECTIONS (CONTOUR POINTS LOCATED WITH HAND-LEVEL)

Equipment: Hand-level; plain leveling rod, or stadia rod; 50-foot metallic tape; hatchet; 20 stakes; keel.

Location: Select for this exercise a strip of ground 200 ft. or more wide and about 500 ft. long, that has some transverse slope. In the following it is assumed that the transverse fall is 10 ft. or more.

The object of this exercise is to give practice in one method of taking topography — a method in common use for highway, ditch, canal, and railway work.

FIELD WORK

(1) Along center of chosen ground set the 10 stakes (aligning them by eye), at uniform distance of 50 ft. apart. Let line of stakes be designated by CL (center line).

(2) Number the stakes in order, 0 + 00, 0 + 50, 1, 1 + 50, and so on, beginning at an end stake. (3) Assume elevation of ground at sta. 0 + 00 to be 98.3; and find the corresponding ground elevation of each succeeding stake by means of hand-level and rod. (4) Mark on each stake with soft pencil the ground elevation at the stake.

FORM OF NOTES

(Notes run up the page.)

Sta.	
0 + 80	
0 + 50	
0 + 00	

(5) Hand-levelman measures with rod the height of his eye above the ground. (Let us call this height *h*.) (6) Hand-levelman, with ring end of tape in one hand and hand-level in the other, backs down hill from 0 + 00 (in direction at right angles to CL) until he reads on rod (held on the ground at Sta. 0 + 00) r such that $98.3 + r - h = 95$. (Hand-levelman is then standing on contour point 95.) (7) Rodman, standing at 0 + 00, holding tape box, then reads "distance out" — say, for illustration, 37 ft. (8) Recorder enters in his notes the fraction $\frac{95}{37}$ (see form of notes below. Numerator = elevation; denominator = distance out. Recorder should face forward, and enter fraction just to right or left of central vertical line of right-hand page according as contour point is to right or left of station). (9) Rodman next goes out beyond hand-levelman until latter reads r' such that $r' = 5 + h$. Rod is then standing at contour point 90. (10) Rodman reads distance from hand-levelman to rod — let us say, 42 ft. (11) Recorder enters fraction, $\frac{90}{79}$, alongside preceding entry (denominator = total distance out, as always). (12) Succeeding contour points on low side are taken in similar manner. From one to five contour points on each side on line of stakes may be taken, according to slope of ground, or directions of instructor.

		$\frac{95}{0}$			
$\frac{105}{91}$	$\frac{100}{44}$	$\frac{96.5}{0}$	$\frac{95}{12}$	$\frac{90}{56}$	$\frac{85}{92}$
$\frac{105}{62}$	$\frac{100}{26}$	$\frac{98.3}{0}$	$\frac{95}{37}$	$\frac{90}{79}$	$\frac{85}{106}$

(13) In similar way find distance out to each contour point on up-hill side of station.

(14) Repeat foregoing operations at each succeeding stake, up to and including 4 + 50.

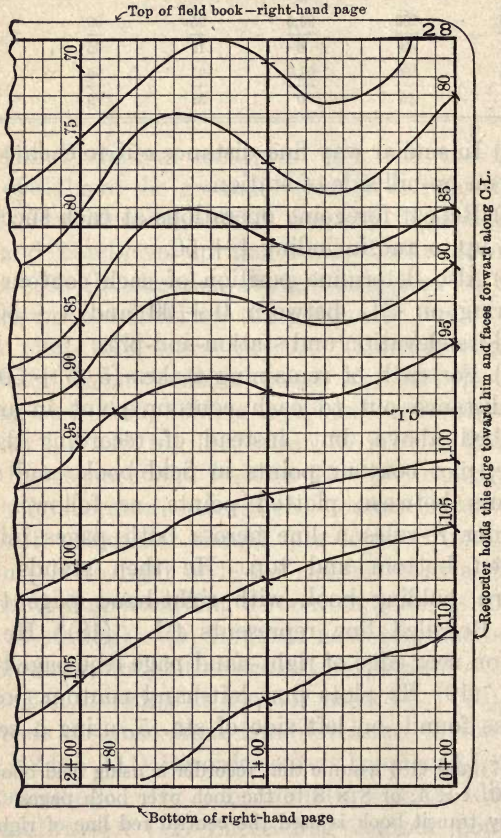
(15) Also determine position of each contour point occurring on CL, between 0 + 00 and 4 + 50, and record its elevation and station-and-plus.

(16) For each of remaining stakes (5, 5 + 50, etc.) find distance out to each contour point in manner described above, but, instead of recording data as before, plot contour points in field book, and sketch contours between plotted points, as follows. (17)* Recorder re-rules a line across both pages mid-way between bottom and top. He then stands facing forward, holding book with right-hand page toward him. Re-ruled line represents CL. (18)* He plots sta. 5 on near edge of right-hand page (the page toward him). (19) He plots first left-hand contour point as soon as found, on left side of sta. 5, using a scale of

* (17) and (18) assume that recorder is using field book ruled 10 × 10, 4 × 4, or 8 × 8 to the inch over both pages. If the ordinary transit book is used the central red line of right-hand page serves as CL, the plotting is carried upward on the right-hand page, and the station and elevation entries are made on the opposite page. In any case the plot should not be distorted by using one scale for the longitudinal and another for the transverse distances.

1 in. = 100 ft., and writes contour elevation by plotted point. (20) Each succeeding contour point is similarly plotted as soon as found. (21) In like manner

FORM OF NOTES



contour points are plotted for each succeeding station; the stations (50 ft. apart) being plotted at intervals of one-half inch along re-ruled line. (22) As soon as points are plotted for sta. 5 + 50, sketch contours for ground lying between the two rows of plotted points; and on completing plot of points at each succeeding station, continue contours up to row of plotted points.

OFFICE WORK

(23) From data make on prescribed paper a contour map as follows: (24) Rule hair-line, with a 6H pencil, to represent CL; plot stations on this, and label them with sharp 4H pencil. (25) Draw perpendicular hair-lines across CL at each station and substation. (A transparent triangle graduated from zero both ways on one edge, and with a perpendicular to the edge scratched from zero, facilitates this work.) (26) Plot each contour point with very sharp 6H pencil, and half-circle the point (the scale lies in the way of marking full circle.) (Zero of scale lies at station point; and all points on one side of station are plotted without shifting scale. Speed is increased and risk of blunder is lessened by having one's partner call off the notes.) (27) Write the elevation by at least one of the contour points in a cross-section as soon as points of that cross-section are plotted. (28) Sketch contours lightly with very sharp 4H pencil, leaving a little gap in contour at each plotted point, so as to preserve precise point for possible future use. (29) Retrace contours, 0, 20, 40, 80, 100, 120, etc. (after all have been sketched) with a fine, black-ink line, leaving a little gap at each pricked point. (30) Retrace all intermediate contours with sharp 4H pencil, using pressure enough to produce a line which will show clearly through tracing cloth; and leaving a little gap at each plotted point. (31) Letter the elevation figures of inked contours plainly, and with sufficient frequency for convenient interpretation of map. (32) As a title, print "Exercise 40," draftman's name, and the date — all freehand with 4H pencil.

41. TOPOGRAPHIC CROSS-SECTIONS (USING SLOPE-LEVEL)

Equipment: Slope-level; 50-foot metallic tape. (Also a level-rod, if partners differ much in height.)

Location: For this exercise use stakes set and marked (with station and elevation) in preceding exercise.

The object of this exercise is to give practice in another method of "taking topography," — a method not infrequently used on highway, water-way, and railway work.

FIELD WORK

(1) Levelman notes HI point on helper (*i.e.*, the point on helper at height of levelman's eye when the two men are standing on level ground. If rod is used, HI should be measured and rodman thereafter should hold one hand at HI point on rod whenever levelman is taking a sight). (2) Levelman stands at 0 + 00, holding ring end of tape in one hand and slope-level in the other. (3) Helper goes out to first break in slope on line at right angles to CL (center line of stakes), carrying tape box (and rod if rod is used). (4) Levelman sights slope-level at HI mark on helper (or on rod), brings bubble to center, and reads slope. (5) Helper reads distance out to break point. (If convenient, tape is held level for this measurement, but if slope is steep it is better to measure slope distance and indicate in notes that it is a slope distance by prefixing "*s*" to slope-distance record.) (6) Slope and distance out are written in the form of a fraction, slope as numerator and distance-out as denominator; *e.g.*, $\frac{+7° 30'}{s\,150}$. (7) Levelman then goes out to first break (where helper stood); helper goes out to second break; and slope and distance are taken as before.

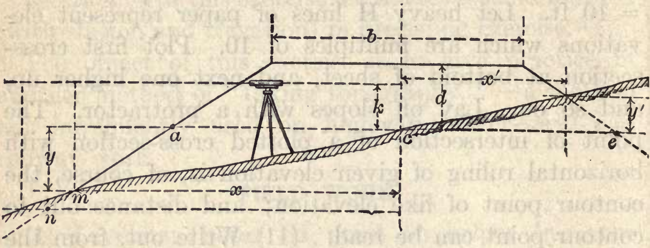
(8) Break points on other side of CL are taken in same way. (9) This work is repeated at each station and sub-station along CL. (10) Plot each cross-section on sheet (of prescribed size) of cross-section paper ruled 10 squares to the inch. Use H and V scales 1 in. = 10 ft. Let heavy H lines of paper represent elevations which are multiples of 10. Plot first cross-section at bottom of sheet, and next one higher up, and so on. Lay off slopes with a protractor. The point of intersection of a plotted cross-section with horizontal ruling of given elevation is, of course, the contour point of like elevation; and distance out to contour point can be read. (11) Write out, from the plotted cross-sections, the cross-section notes (in form used in first part of Exercise 40). (12) Plot these derived notes according to directions (23)–(32) of Exercise 40. (13) Compare plot with plot of Exercise 40. (14) Letter a title, freehand with 4H pencil, on plotted cross-sections.

42. EARTHWORK CROSS-SECTIONS AND SLOPE STAKES

Equipment: Level, level-rod (or light stadia-rod), 50-foot tape.

Location: Use same line of stakes that was used in Exercise 40 and 41, together with station and elevation markings. Instructor will prescribe grade elevation to be used at each stake. Assume all side slopes $1\frac{1}{2} : 1$, and roadbed widths to be 18 ft. in fill and 23 ft. in cut. *"F"* and *"C"* for fill and cut are less confusing than (−) and (+) which are often used.

Method a. Determine the location of the slope-stakes from the formula $x = \frac{1}{2}\, b + sd \pm sy$, in which s is the slope ratio. First, compute at any station $\frac{1}{2}\, b + sd$ — giving the point a. Then estimate by eye how much farther than this a point must be so that the added distance will be $1\frac{1}{2}$ times the added difference of elevation. Set up the rod at this total estimated distance. If the difference of elevation (y) satisfies the above formula, the required point has been



found. If not, a little study will show whether the point should be nearer to or farther from center and about how much. With practice, two trials will generally be sufficient to locate a point with sufficient accuracy. The upper side of the fill is determined similarly, except that the correction (sy) is negative.

FORM OF NOTES

Sta.	Surf. elev.	Grade elev.	Center cut (+) or fill (−).	Grade.	
24 + 60	90.8	99.2	− 8.4		
24	92.4	99.8	− 7.4	− 1.0%	

An easy check on the numerical calculation is found by subtracting $\frac{1}{2}b$ from the computed distance and noting whether the difference is $1\frac{1}{2}$ times the difference of elevation. For instance, take the result $\frac{-18.6}{36.9}$. $36.9 - 9.0 = 27.9$, which is $1\frac{1}{2}$ times 18.6; again, $15.3 - 9.0 = 6.3$, which is $1\frac{1}{2}$ times 4.2.

Determine the distance and elevation of all "breaks" $\left(e.g., \frac{-14.2}{20.5}\right)$ between the center and the slope-stakes.

Method b. Using an automatic slope-rod and a specially marked distance-tape.

The rod is ten feet long and has an endless graduated tape 20 feet long rolling over friction wheels at the ends, which are shod with metal shoes. The distance-tape is graduated on one side as usual; on the other side a zero-point is marked 9 feet (or more) from the ring. From the zero back to the ring the tape is graduated to feet and half-feet. Beyond the zero-point the tape is graduated to a scale s times the usual scale. Slope ratio, s, is usually $1\frac{1}{2}$.

(1) Set the rod-tape at zero — *i.e.*, so that the zero is at the bottom of the rod. (2) Hold the rod at the center stake (c) and note the reading k. (3) If k is $\left\{\begin{matrix}\text{less}\\\text{greater}\end{matrix}\right\}$ than d, $\left\{\begin{matrix}\text{raise}\\\text{lower}\end{matrix}\right\}$ the tape on the *back* side

(Read up the page.)

Left.		Right. $\left\{\begin{matrix} b = 18' \text{ in fill, } 23' \text{ in cut.}\\ s = 1\frac{1}{2} : 1.\end{matrix}\right.$	
$\frac{-18.6}{36.9}$	$\frac{-14.2}{20.5}$	$\frac{-2.2}{8.5}$	$\frac{-4.2}{15.3}$
$\frac{-16.2}{33.3}$	$\frac{-5.4}{18.6}$	$\frac{-1.6}{10.0}$	$\frac{-3.4}{14.1}$

of the rod by an amount equal to $\left\{\begin{matrix} d - k\\ k - d\end{matrix}\right\}$. (4) With the distance-tape, so held that its zero is $\frac{1}{2}\,b$ from the center, carry the rod out until the rod reading equals the reading indicated by the tape. The rod will then be at the required point. The proof is apparent from the figure.

OFFICE WORK

(1) Plot the earthwork cross-sections on sheet (or sheets), of prescribed size, of cross-section paper ruled 10 squares to the inch. Use 4H pencil with enough pressure to make all lines and figures easily followed by contrast with rulings. Plot on scale of 1 in. = 10 ft. both H and V. (2) Plot on each sheet one, two, or more vertical rows of cross-sections, according to total widths of sections, allowing for a clear margin of $1\frac{1}{2}$ inches at the left edge of the sheet for binding. (3) Choose a heavy vertical line of the paper to be line of centers for sections to be plotted in the left-hand vertical row; and re-rule this heavily in pencil to make it conspicuous. (4) Plot first section at bottom of left-hand row. Lay the roadbed, if in cut, on lowest heavy horizontal line of paper; if in fill, on lowest heavy horizontal that will give enough space below for whole section. As each point is plotted, write its "fraction," in very legible figures, near it. Place station number (written thus: "8 + 00" or "9 + 45," using bold heavy figures 0.2″ high) opposite and near to right-hand end of roadbed. (5) Plot next succeeding cross-section above the first, choosing a heavy horizontal line for roadbed as in (4) above. (6) Continue thus upward till left-hand row is plotted. Then begin at bottom of next vertical row to right of that plotted; and so on, until sheet is filled. (7) The sheet need not be inked; but it saves time, where cross-section sheets are to be frequently referred to, to ink the station numbers. (8) Letter bold title on lower right-hand corner of sheet: *e.g.*, "Exercise 42, John Doe, May 1, 1918." (9) Compute trapezoidal area between each segment of surface line and projection of segment on roadbed (produced if necessary).

Take sum of these areas. Compute triangular area lying between each side slope and roadbed produced; add the two; subtract their sum from trapezoidal sum, and enter difference (which is area of section) on plot of section, inside of outline. (10) Compute volume (cubic yards) of earthwork lying between each two contiguous sections by multiplying half the sum of their areas by 100/27; and enter volume on cross-section sheet between the plots of the two sections.

43. CIRCULAR CURVE (WITHOUT SUBCHORD)

Equipment: Transit; plumb-bob; 100-foot steel tape; 11 pins; hatchet; 3 hubs; 4 tacks; 8 stakes; keel.

Location: The ground should be free from obstructions to measurement, and of sufficient area to permit several parties to work side by side, on parallel curves.

The object of this exercise is to give drill in running circular curves; and to show the range of error which may be expected under the conditions of the exercise.

PRELIMINARY WORK

(1) Compute by formula the tangent distance, T, for a 4° curve of 5 full stations. Check result by use of table of tangent distances. (2) Compute deflections, and enter them in field book in manner indicated in Form of Notes, below. (3) Assume TC = sta. 1. Enter stations and TC and CT in field book.

FIELD WORK

N.B. — The PI, TC, and CT hubs of this exercise are to be left as driven, to serve in the following exercises.

(4) Set a hub (a tack is always set in an alignment hub) and guard stake marked "PI – 4° *R*"; and set up transit over PI hub. (5) Lay off T in a chosen direction and set hub at TC (sta. 1). Mark guard stake thus: "TC 1 + 00." Set another hub at *A* on same line, about 200 ft. beyond TC; and set a plain guard stake by it. (6) Turn off central angle, I, and by measurement set hub CT. Mark guard stake thus: "CT 6 + 00." (7) Set up at TC (sta. 1). (8) Set vernier at 0° 00′. Reverse telescope, and with LM sight on *A*. (9) Return telescope. (10) Loosen UM, and set vernier at recorded deflection for sta. 2. (11) Tapemen, working as directed in Exercise 30, set stake at sta. 2, in line of sight and 100 ft. from sta. 1. (Stake is first keeled with "2" reading from top downward, and then driven so that marked side faces sta. 1.) (12) Loosen UM, and set vernier at recorded deflection for sta. 3. (13) Tapemen set stakes for sta. 3, in line of sight, making distance 2–3 = 100 ft. (14) Continue this process; and finally obtain pin setting for sta. 6. (15) Measure and record distance by which pin 6 is "off line," and distance by which it is ahead of, or back of, CT hub. (16) Set up at CT. (17) Set vernier at total deflection angle, but on opposite side of zero of plate. Reverse telescope, and with LM set on TC. (18) Return telescope. Unclamp UM, and set vernier at 0° 00′. (19) Set stake at sta. 7 in line of sight, and 100 ft. from CT. Reverse telescope, and note by how much the line of sight misses PI. (20) Pull stakes at stas. 2, 3, 4, 5, and 7, and obliterate holes.

(21) Repeat operations (7)–(20) until each member of transit party has served in each position, unless otherwise directed. (22) Leave hubs undisturbed for use in next two exercises.

OFFICE WORK

(The following office work is to be done only in case the instructor does not supply substitute problems.)

(23) On sheet of paper of prescribed kind and size, go through field steps of laying out the curve, using protractor and scale instead of transit and tape. (24) Compute long chord (LC) by formula. Check by slide rule. Recheck by table of long chords. Scale the long chord. (25) Compute radius (R) by formula. Check by slide rule. Recheck by table of radii. Draw radius at TC. With compasses draw curve.

FORM OF NOTES

(Read up the page.)

Sta.	Alignment.	Vernier.	Tang. Defl.	Calc. Bearing.	Needle.
+02.3	CT	14° 23′	28° 46′	N 47° 10′ W	N 47° 15′ W
15	5° 42′ curve left for 28° 46′ Tan. dist. 257.88	14° 15′			
14		11° 24′			
13		8° 32′			
12		5° 42′			
11		5° 51′			
10	TC				
				N 18° 24′ W	N 18° 20′ W

N.B. — These notes are written for a curve having a subchord.

44. CIRCULAR CURVE (WITH SUBCHORD AND INTERMEDIATE SETUP)

Equipment: Transit; plumb-bob; 100-foot steel tape; 11 pins; 6 stakes; hatchet; keel; 3 tacks.

Location: The hubs set in the preceding exercise will be used for this exercise.

The object of this exercise is to give further practice in running circular curves, and in observing range of error in results.

PRELIMINARY WORK

(1) Data are same as in preceding exercise, except that here station of TC is assumed to be 3 + 25; so that in this exercise the first regular station of curve will lie at end of a 75-foot sub-chord. Mark guard stake for hub at TC thus: "TC 3 + 25," and guard stake for hub at CT thus: "CT 8 + 25." (2) Compute station of CT. (3) Compute deflections for CT and for stations on the curve between TC and CT. (4) Enter stations, deflections, etc., in field book (see form of notes given in preceding exercise).

FIELD WORK

(5) Set up at TC. With vernier set at 0° 00′ and with telescope reversed, backsight on *A*. Return telescope. (6) Set stas. 4 and 5. Assume that sta. 6 is not visible from TC. It is necessary then to set a tack in top of stake at sta. 5 (re-aligning and re-taping the tack carefully); and to "move up."

(7) Set up at sta. 5. (8) Set vernier at 0° 00′. Reverse telescope. Backsight on TC with LM. Return telescope. Unclamp UM. (9) Set stas. 6 and 7 by setting vernier at recorded deflections. Assume that sta. 8 is not visible from sta. 5. It is necessary then to "center" stake 7 (by setting tack as for sta. 5); and to move up.

(10) Set up at sta. 7. Assume that from sta. 7 no curve station back of sta. 5 can be seen. It is then proper to use sta. 5 for backsight. (11) Set vernier at deflection recorded for sta. 5. Reverse telescope. Backsight on sta. 5 with LM. Return telescope. (12) Set stake at sta. 8, and pin at sta. 8 + 25 by setting vernier at recorded deflections.

(13) Measure distance by which pin 8 + 25 is off line, and distance by which it is ahead of, or back of, hub at CT.

(14) Set up at CT. Set vernier at CT-deflection minus sta. 7-deflection, but on opposite side of zero. Reverse telescope. Backsight on sta. 7. Return telescope. Set vernier at 0° 00′. (15) Set pin at sta. 9, in line of sight and 75 ft. beyond CT. (16) Reverse telescope and note how far line of sight strikes to right or left of PI. (17) Pull up all stakes except guard stakes and hubs, and obliterate holes.

45. CIRCULAR CURVE BACKWARD

Equipment: Equipment, data, field notes and hubs, same as in preceding exercise.

The object of this exercise is to give practice in running a simple curve backward and in observing range of error in results.

(1) Set up at CT. Set vernier at recorded deflection for CT. By means of LM set line of sight on PI. (2) Unclamp UM, and set vernier at deflection recorded for station 8. Set stake at sta. 8 in line of sight and 25 ft. from CT. (3) Set stake at stas. 7 and 6 by setting vernier at their recorded deflections. Assume that sta. 5 cannot be seen from CT. Then it is necessary to center stake 6; and to move up. (4) Set up at sta. 6. Set vernier at deflection recorded for CT. Reverse telescope. Backsight on CT with LM. Return telescope. (5) Set stakes at stas. 5 and 4, and pin at sta. 3 + 25, by setting vernier at their recorded deflections.

(6) Note how far pin 3 + 25 is from hub TC, for line and distance. (7) Set up at TC. Set vernier at deflection recorded for sta. 6. Set line of sight on sta. 6 by LM. Loosen UM, and set vernier at 0° 00′.

(8) Reverse telescope, and note how far to right or left of point *A*; line of sight strikes. (9) Pull all stakes except guard stakes and hubs, and obliterate holes.

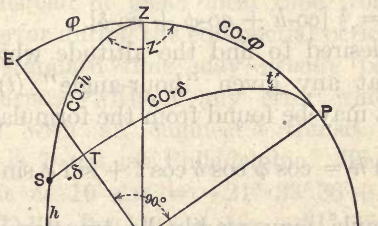
AZIMUTH

The azimuth of a line is the angle the line makes with a true meridian which intersects the line. It is usually reckoned from the south and measured to the right. Thus

an azimuth of	50°	means	S 50° W
" " "	160°	"	N 20° W
" " "	255°	"	N 75° E
" " "	350°	"	S 10° E

The ordinary magnetic needle is incapable of very close work even when it may be positively known that there is no local attraction and when the complicated effects of the various periodic changes in the declination are duly allowed for. The recent enormous extensions of trolley-lines, telegraph, telephone, and electric-light wires, and even barbed-wire fences, have rendered the employment of a magnetic needle worse than useless, in many places, as a means of determining true azimuth. Even when there is no apparent reason for local attraction a needle will often exhibit inexplicable vagaries, as is readily seen by making a traverse of a considerable area, measuring all angles exactly with the horizontal plates and taking the needle readings of all lines. The forward and backward readings of any one line will frequently show such discrepancies as to absolutely preclude any attempt at accuracy based on the needle readings. The United States Government requires that all surveys of the public lands shall be based on azimuths obtained by solar observations. The determination of true azimuth should be a part of all property surveys, topographical surveys, and it is a useful feature (as a check) on extended railroad surveys.

Solar azimuth may be obtained in two general ways: (*a*) by direct observation on the sun with the telescope of an ordinary "complete" transit, provided with a colored glass shade (as demonstrated in Exercise 47), and (*b*) by the use of a solar "attachment" or a "solar compass" (as demonstrated in Exercise 46). The first method requires very little in the way of special equipment, but necessitates the solution of a somewhat tedious calculation for each value obtained. The second method gives the true meridian directly, but requires the use of a somewhat expensive instrument. Accurate work requires that the numerous adjustments of both the transit and the attachment shall be perfect. An error of adjustment will frequently cause an error of azimuth which is several times the angular value of the error of adjustment. A proper appreciation of either method requires an understanding of certain astronomical relations.



The figure represents the orthographic projection of the celestial sphere, projected on the plane of the meridian of the observer.

HPZE represents the meridian of the observer.

Z is the zenith.

CP is the polar axis of the earth.
CE is the trace of the plane of the equator.
S is the position of the sun.
$EZ = \phi =$ the latitude of the observer.
$ZP = \text{co-}\phi = 90° - \phi$.
$SG = h =$ the *true* altitude of the sun.
$SZ = \text{co-}h = 90° - h$.
$Z =$ the zenith-angle *PZS*.
$ST = \delta =$ the declination of the sun north or south of the equator.
$SP = \text{co-}\delta = 90° - \delta$. The essential sign of δ must be considered.

If the sun is south of the equator (as it is from about Sept. 21 to March 21), δ is *negative;* and if the declination is (say) S. 20°, $\delta = -20°$. Then $\text{co-}\delta = 90° - \delta = 90° - (-20°) = 110°$.

From spherical trigonometry we have in the spherical triangle *SZP*:

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin (s - \text{co-}h) \sin (s - \text{co-}\phi)}{\sin \text{co-}h \sin \text{co-}\phi}}, \qquad (1)$$

in which $s = \frac{1}{2} [\text{co-}h + \text{co-}\phi + \text{co-}\delta]$.

If it is desired to find the altitude which the sun will have at any given "hour-angle" (t) from the meridian, it may be found from the formula

$$\sin h = \cos \phi \cos \delta \cos t + \sin \phi \sin \delta. \qquad (2)$$

This formula may readily be transformed to give the *time* when the sun will have a given altitude —

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

$$= \frac{\sin h}{\cos \phi \cos \delta} - \tan \phi \tan \delta. \qquad (3)$$

The sun describes each day a path which is approximately parallel to the equator, the change in declination being very small during June and December and fastest when the sun is crossing the equator in March and September. The declination of the sun must be known for the time of the observation; this is obtainable from the Nautical Almanac or Ephemeris as follows:

To find the declination (δ) *for any day and hour.* Suppose the time is 10 A.M. Jan. 12, 1897; the place, Philadelphia, Pa. The Ephemeris for 1897 gives the "apparent declination" for Greenwich at *noon* Jan. 12, 1897, S. 21° 33′ 21″.8 = S. 21° 33′.36; difference for 1 hour = +25″.14 = +0′.42. Greenwich is on the prime meridian — longitude is 0°. The longitude of Philadelphia (Univ. Pa.) is 75° 11′. The difference of time is, therefore, 5 hours (to a fraction of a minute), and therefore *in this case* and *for this purpose* the usual "standard 75th meridian time" may be used instead of "mean local time." In localities which are midway between the standard hour meridians the use of standard time instead of mean local time would cause a *maximum* error during the equinoctial periods of 29″.5. Therefore standard time may *usually* be employed for this purpose without any error which may be appreciable with an engineer's transit. 12 M. at Greenwich is 7 A.M. at Philadelphia. Therefore δ at Philadelphia at 10 A.M. = $-21° \ 33'.36 + (3 \times 0'.42)$ = $-21° \ 32'.10$; for 11 A.M. $\delta = -21° \ 32'.10 + 0'.42$ = $-21° \ 31'.68$. In applying these declinations in the above formula they need only be taken to the nearest tenth of a minute of arc.

Refraction. — Refraction causes the sun to appear higher than it actually is. Therefore when the *altitude* of the sun (or a star) is observed, the computed refraction should be *subtracted* from the *apparent altitude* to obtain the *true altitude.* The amount of the refraction is a function of the temperature and of the barometric pressure. For such work as may be done with an ordinary transit or sextant the values given in Table IV will suffice.

When working with an ordinary transit and at ordinary temperature and pressure, "mean refractions," taken directly from Table IV, may be used uncorrected, for it will usually be found that the correction is far within the lowest unit of angular measure. For example, at a temperature of 32° F. the refractions should be *increased* about 3%, but a barometric pressure of 28″.6 would *diminish* the refractions by about the same amount, and the two conditions together would give the same results as in the table. A low atmospheric pressure and high temperature will combine to reduce the refraction, but a correction of even 10% of the mean refractions can hardly be measured with an ordinary transit when the altitude is over 10°.

On account of the uncertainties in the refraction at low altitudes, it is generally undesirable to take observations at altitudes less than 10°.

Effect of refraction on declination. — The effect of refraction requires a modification of the declination-angle which is set off when using a solar attachment. When the sun is in the meridian, the declination is affected by the full value of the refraction; at other positions the effect on declination equals the true refraction for that altitude times the cosine of the angle at S (see figure on p. 129). The values of these effects of refraction for various latitudes, declinations, and hour-angles are given in Table V. As an illustration: latitude = 39° 57′; $\delta = +0° 10'$; hour angle = 3.5; take the value for latitude = 40°; by interpolation we obtain 1′ 23″ = 1′.4. The effect of refraction being to cause the sun to *appear* higher, the effect (in the northern hemisphere above lat. 23° 27′) is to make the apparent position farther north. Therefore this correction should be added *algebraically* to the declination. Therefore in the above case the modified declination set off is 0° 10′ + 1′.4 = 0° 11′.4.

All values in Table V which are *below* and to the *right* of the heavy lines indicate an altitude of less than 10°, in which case the refraction is somewhat uncertain and the observations correspondingly unreliable. For intermediate latitudes the true values may be found by interpolation. For all cases *above the heavy lines* (which means an altitude greater than 10°) the *maximum* error caused by such interpolation will not exceed 3″. When the latitude of the place of observation is within a degree or so of one of those given in the table, those values may generally be used without material error. Otherwise it will be advisable to construct a table (by interpolation) for the given latitude.

Inaccuracies in azimuth-work. — The azimuth obtained by these methods may be inaccurate for several reasons:

1. The instrument may be in poor adjustment. As it should be assumed that the instrument has been adjusted as perfectly as possible for this work, no discussion of this cause is necessary except that the error is determinable, as will be shown.

2. The latitude, declination, or altitude may have been inaccurately determined for use instrumentally or in the formula.

3. The latitude, declination, or altitude may have been inaccurately set off (or read), or there may be an index error of the vertical circle, or the level-bubbles

may be out of adjustment. The effect is substantially the same whatever the cause. The nominal angle is one thing, the real angle is something different and the resulting computed azimuth is more or less inaccurate.

In Table VI is given the effect on the azimuth in minutes of arc of an error of one minute in latitude, declination, or altitude. If it is discovered after the observations are made that there is an index error of the vertical circle, or that an incorrect declination or latitude was used, the observations need not be rejected — assuming that the error is only a few minutes of arc. By means of Table VI a correction may be applied to the azimuth obtained which will give the true azimuth. Great care should be taken that the correction is applied with its proper algebraic sign. The following rules should be observed in applying the corrections:

1. If the LATITUDE is used with a LARGER value than its *real* value, the angle *PZS* (see preceding figure) will be too LARGE.

2. If the DECLINATION is used with a LARGER (algebraic) value than its *real* value, the angle *PZS* will be too SMALL.

3. If the ALTITUDE is measured and used with a LARGER value than its *real* value, the computed value for *PZS* will be too LARGE. Of course these rules should also be used *vice versa*.

An inspection of Tables V and VI will show that accuracy depends largely on the time of day and that better work may be obtained in summer than in winter. Observations for azimuth cannot be taken at noon; errors will be extreme for observations near noon, and even one hour from the meridian is too close for accurate work. The best times are when the sun is as far from the meridian as possible and yet not so near the horizon that the refraction is uncertain. It will usually be possible to so plan work that observations for azimuth may be taken at favorable times without wasting time or interfering with other work.

TABLE IV. — MEAN REFRACTIONS. (Bessel.)

[True for barometer at 29″.6, temperature at 48° F.]

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
0° 0′	34′ 54″	3° 0′	14′ 15″	10°	5′ 16″	24°	2′ 09″
10	32 49	30	12 48	11	4 48	26	1 58
20	30 52	4 0	11 39	12	4 25	28	1 48
30	29 03	30	10 40	13	4 05	30	1 40
40	27 23	5 0	9 46	14	3 47	35	1 22
50	25 50	30	9 02	15	3 32	40	1 09
1 0	24 25	6 0	8 23	16	3 19	45	0 58
10	23 07	30	7 49	17	3 07	50	0 48
20	21 56	7 0	7 20	18	2 56	60	0 33
30	20 51	30	6 53	19	2 46	70	0 21
40	19 52	8 0	6 30	20	2 37	80	0 10
50	18 58	30	6 08	21	2 29	90	0 0
2 0	18 09	9 0	5 49	22	2 22		
30	16 01	30	5 32	23	2 15		

FACTOR *B*.

Barom.	*B*
28″.0	0.946
28 .5	0.963
29 .0	0.980
29 .5	0.997
30 .0	1.014
30 .5	1.031
31 .0	1.047

FACTOR *t*

Attached therm.	*t*
− 20° F.	1.005
0	1.003
+ 20	1.001
+ 40	0.999
+ 60	0.997
+ 80	0.996

FACTOR *T*

Temp. external air.	*T*	Temp. external air.	*T*
− 20° F.	1.156	40° F.	1.017
10	1.130	50	0.998
0	1.106	60	0.978
+ 10	1.082	70	0.960
20	1.060	80	0.942
30	1.038	90	0.925

True refraction = mean refraction $\times B \times t \times T$.

Table V. — Effect of Refraction on Declination

Latitude.	Hour-angle.	Declination.										
		+ 23°27′	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°	− 23°27′
		′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
+20°	0	−4	0	+5	+0 10	+0 15	+0 21	+0 27	0 33	0 40	0 48	0 55
	1	−3	0	6	11	16	22	28	34	41	49	0 56
	2	−1	+3	8	13	18	24	30	37	45	0 54	1 00
	3	+4	7	12	18	23	30	36	0 44	0 52	1 02	1 11
	4	13	17	22	28	0 35	0 42	0 50	1 00	1 11	1 25	1 36
	5	0 34	0 39	0 47	0 57	1 07	1 19	1 36	1 57	2 28	3 14	4 01
	6	2 06	2 27	3 12	4 24	6 43	11 56	......	......	......	......	......
+30°	0	0 07	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 58	1 09	1 18
	1	08	11	16	22	28	35	42	50	0 59	1 11	1 20
	2	10	14	19	25	32	39	46	0 55	1 06	1 18	1 27
	3	16	20	26	32	40	0 47	0 56	1 06	1 19	1 35	1 51
	4	28	32	0 39	0 47	0 56	1 06	1 19	1 36	1 58	2 28	2 59
	5	0 53	0 59	1 10	1 24	1 43	2 06	2 41	3 36	5 18	8 54	16 42
	6	2 11	2 33	3 23	4 51	8 00	17 27	......	......	......	......	......
+40°	0	0 17	0 21	0 27	0 33	0 40	0 48	0 58	1 09	1 22	1 40	1 55
	1	18	22	28	34	42	50	1 00	1 11	1 25	1 43	2 00
	2	22	26	32	39	47	0 56	1 07	1 19	1 36	1 58	2 18
	3	29	33	41	0 48	0 58	1 09	1 22	1 39	2 02	2 36	3 09
	4	0 41	0 47	0 56	1 07	1 20	1 36	1 58	2 30	3 19	4 47	6 45
	5	1 07	1 16	1 31	1 51	2 20	3 02	4 15	6 47	14 18	......	......
	6	2 11	2 35	3 27	5 03	8 42	22 26	......	......	......	......	......
+50°	0	0 29	0 33	0 40	0 48	0 58	1 09	1 22	1 40	2 03	2 37	3 12
	1	30	34	42	50	1 00	1 11	1 25	1 44	2 09	2 46	3 24
	2	34	39	47	0 56	1 07	1 19	1 36	1 58	2 29	3 18	4 13
	3	42	0 48	0 56	1 07	1 21	1 37	1 59	2 32	3 22	4 55	7 02
	4	0 55	1 02	1 15	1 29	1 49	2 17	2 58	4 07	6 32	13 37	......
	5	1 20	1 31	1 51	2 20	3 02	4 16	6 52	15 08	......	......	......
	6	2 11	2 37	3 30	5 09	9 12	26 44	......	......	......	......	......
+60°	0	0 43	0 48	0 58	1 09	1 22	1 40	2 03	2 37	3 32	5 10	7 46
	1	44	50	0 59	1 11	1 25	1 43	2 08	2 44	3 46	5 44	8 45
	2	48	0 54	1 05	1 18	1 34	1 55	2 26	3 11	4 34	7 44	13 44
	3	0 56	1 03	1 16	1 31	1 51	2 21	3 04	4 19	7 05	16 40	......
	4	1 10	1 19	1 35	1 57	2 28	3 17	4 44	8 08	22 10	......	......
	5	1 32	1 46	2 12	2 50	3 54	6 21	12 17	......	......	......	......
	6	2 12	2 37	3 31	5 12	9 30	30 13	......	......	......	......	......
Latitude.	Hour-angle.	+ 23°27′	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°	− 23°27′
		Declination.										

Table VI. — Errors in Azimuth*

— due to error of 1′ in	Hour-angle.	Latitude — φ. +20°	+30°	+40°	+50°	+60°
Declination	1	4′.11	4′.46	5′.04	6′.01	7′.73
	2	2 .13	2 .31	2 .61	3 .11	4 .00
	3	1 .50	1 .63	1 .85	2 .20	2 .83
	4	1 .23	1 .33	1 .51	1 .80	2 .31
	5	1 .10	1 .20	1 .35	1 .61	2 .07
	6	1 .06	1 .15	1 .31	1 .56	2 .00
Latitude	1	3′.97	4′.31	4′.87	5′.81	7′.46
	2	1 .84	2 .00	2 .26	2 .69	3 .46
	3	1 .06	1 .15	1 .31	1 .56	2 .00
	4	0 .61	0 .67	0 .75	0 .90	1 .15
	5	0 .29	0 .31	0 .35	0 .42	0 .54
	6	0 .00	0 .00	0 .00	0 .00	0 .00

	Latitude.	Hour-angle.	Declination — δ. +23° 27′	+10°	0°	−10°	−23° 27′
ALTITUDE	20°	1	0′.85	2′.44	3′.35	3′.69	3′.96
		2	0 .05	0 .82	1 .25	1 .52	1 .71
		3	0 .11	0 .48	0 .69	0 .85	0 .99
		4	0 .19	0 .37	0 .48	0 .56	0 .64
		5	0 .26	0 .34	0 .39	0 .42	0 .45
		6	0 .34	0 .36	0 .36	......	....
	30°	1	2 .19	3 .75	4 .07	4 .21	4 .29
		2	0 .80	1 .48	1 .75	1 .90	2 .00
		3	0 .54	0 .87	1 .03	1 .14	1 .23
		4	0 .48	0 .65	0 .74	0 .80	0 .86
		5	0 .48	0 .57	0 .61	0 .64	0 .65
		6	0 .54	0 .57	0 .58	......	....
	40°	1	4 .19	4 .69	4 .82	4 .88	4 .92
		2	1 .69	2 .10	2 .24	2 .32	2 .38
		3	1 .05	1 .28	1 .41	1 .48	1 .53
		4	0 .83	0 .98	1 .05	1 .09	1 .12
		5	0 .77	0 .85	0 .89	0 .91	....
		6	0 .80	0 .83	0 .84	......	....
	50°	1	5 .64	5 .82	5 .88	5 .90	5 .92
		2	2 .59	2 .80	2 .87	2 .91	2 .94
		3	1 .66	1 .83	1 .89	1 .93	1 .96
		4	1 .29	1 .40	1 .45	1 .48	....
		5	1 .15	1 .22	1 .25	1 .26	....
		6	1 .15	1 .18	1 .19	......	....
	60°	1	7 .56	7 .62	7 .64	7 .65	7 .67
		2	3 .71	3 .81	3 .84	3 .86	3 .87
		3	2 .49	2 .58	2 .62	2 .64	....
		4	1 .96	2 .04	2 .07	2 .08	....
		5	1 .74	1 .79	1 .81	......	....
		6	1 .69	1 .73	1 .73	......	....

* For method of using this table see under "Inaccuracies in Azimuth-work," above.

46. AZIMUTH, USING SOLAR ATTACHMENT

Equipment: Transit, with solar attachment; set of declination tables. The vertical circle of the transit should be *fixed* and have a vernier reading to half-minutes.

Location: This problem must be worked with the instrument at some point from which a clear view of the sun, unobstructed by trees, buildings, etc., may be obtained throughout the whole period of the work. The problem consists in finding the true azimuth of a line from the instrument (which should be set over a fixed hub) to some prominent mark (*e.g.*, a distant steeple) which is suitable as an azimuth-mark.

Time: From two to four hours before or after noon.

Method: Compute the declination-settings for each half-hour for the total period of the observations. Record these at once in the note-book, leaving an interval of about five lines for observations made during each half-hour. The ten desired observations will not generally require much more than one hour and with practice ten observations may be made in a few minutes.

Adjust the eye-pieces of both telescopes for parallax, focus the object-glass of the solar for observing the sun and focus the object-glass of the transit-telescope for observing the azimuth-mark.

Level up with *extreme care.* The plate-bubble parallel to the telescope is almost invariably too coarse for the accuracy required in this work. Clamp the telescope nearly horizontal. By repeated reversions of the whole instrument about its vertical axis, with corresponding adjustments of the leveling-screws and the tangent-screw to the vertical arc, the instrument may be so leveled that the telescope level-bubble will remain in the center of the tube for any position of the instrument. Under these conditions the vertical axis is truly vertical and the reading of the vertical arc will be its index error — assuming that the line of collimation is parallel to the axis of the bubble-tube. The exact error of the adjustment of the plate-bubbles is then apparent and should be noted, so that any accidental change of level that may occur during the time of taking observations may be at once detected.

Set vernier *A* of the horizontal plates at 0° and point at the azimuth-mark. Loosen the upper plate and swing the telescope approximately into the meridian, the telescope pointing *south*. If the declination is SOUTH, point the transit-telescope UPWARD with a vertical angle equal to the value of the declination as modified by refraction. If the declination is NORTH, point the telescope DOWNWARD with a vertical angle equal to the modified declination.* Then place the solar telescope in the *same vertical plane* as the transit-telescope and make it horizontal by means of its level-bubble, clamping it securely. Then make the transit-telescope parallel to the plane of the equator by pointing it UPWARD with a vertical angle equal to the COLATITUDE. The polar axis of the solar is thus made approximately parallel to the earth's axis; it will be exactly parallel when the transit-telescope is exactly in the meridian. Since the meridian is obtainable to within a small range by means of the magnetic needle or by an approximate solar observation, the most convenient method is to bring the transit-telescope into the meridian as nearly as possible and clamp the upper plate. Point the solar telescope at the sun by the

* To find the modified declination, see the discussion on AZIMUTH.

shadow of the sights on top and clamp the motion about the solar axis. Then, with one hand on the slow-motion screw of the polar axis and the other hand on the tangent-screw of the upper transit-plate, point the solar telescope exactly at the center of the sun. The reading of the horizontal plate gives one value of the angle between the azimuth-mark and the meridian. To obtain another value which shall be independent of the previous determination, loosen the upper transit-plate and the clamp-screws of the solar

FORM OF NOTES

Time.	Decl.	Refr. corr.	App. decl.	Angle between azimuth-mark and meridian.	Needle.
3:54	+0° 50′.4	+ 1′.5	+0° 51′.9	41° 21′	N 7° 30′ W
57			51 .9	19	
4:02	+0 50 .3	+1 .6	+0 51 .9	19	
07			51 .9	20	
14			51 .9	19	
17			51 .9	19	
19	+0 50 .0	+1 .9	+0 51 .9	19	
22			51 .9	18	
26			52 .0	18	
4:30	+0 49 .8	+2 .2	+0 52 .0	41 19	
				41° 19′.1	

and repeat the operation, making due allowance (if necessary) for any change of declination that may have taken place in the interval. With practice several observations may be taken in a very few minutes, during which time no *appreciable* change of declination will take place even when the motion of the sun in declination is most rapid. Watch the levels carefully for any indication of jarring of the instrument. The lower plate should be kept clamped throughout and the vernier should always read 0° when pointing at the azimuth-mark. Observe the needle reading when the instrument is in the meridian. It will show the declination of the needle for that time and place.

Take ten observations and compute the error of each observation assuming the mean to be the true value.

47. AZIMUTH, FROM ALTITUDE AND DECLINATION OF THE SUN, USING TRANSIT *

Equipment: Transit, provided with vertical circle, which should, if possible, be graduated to 30″ of arc. A colored glass shade is also necessary to protect the eye: this glass may be placed either over the eye-piece or over the object-glass, but if placed over the

d	d^2		
1.9	3.61	Philadelphia, Sept. 20, 1898	$\phi = 39°\ 57'$
0.1	0.01	δ (at noon at Greenwich)	$+0°\ 59'.0$
0.1	0.01	8 hours change at $-0'.97$	$-\ \ 7.76$
0.9	0.81	δ (3 P.M., Phila.)	$+0°\ 51'.24$
0.1	0.01	δ (4 P.M.)	$+0\ \ 50.27$
0.1	0.01	δ (5 P.M.)	$+0\ \ 49.30$
0.1	0.01		
1.1	1.21		
1.1	1.21		
0.1	0.01		
	6.90		

object-glass it must be optically correct, *i.e.*, the surfaces perfectly plane and parallel to each other so that there is no distortion or refraction of the light-rays passing through. When the transit has only a semicircular arc, or a movable arc which is set by clamping, METHOD A *must* be used. When the instrument is equipped with a full circular arc, METHOD B will be found more accurate.

Location: [See directions for location in Exercise 46.]

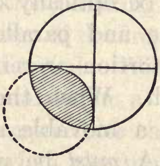
Time: From two to four hours before or after noon.

* See discussion on AZIMUTH.

METHOD A

Method: The problem consists in obtaining the altitude of the center of the sun above the horizon and also the horizontal angle between the sun's center and some definite mark at the same instant of time — which must be observed. Then, knowing the altitude and declination of the sun at that instant and also the latitude of the place of observation, the angle between the sun's center and the meridian is computable, and then by mere addition or subtraction the azimuth of the mark from the place of observation is obtained.

Set up the transit over the designated hub and level up with especial care (using precautions suggested in Exercise 46), since the accurate determination of vertical angles is necessary. Point at the azimuth-mark with the horizontal plates reading 0°. With the *upper* plate loose, point at the sun, observing the time, altitude, and the horizontal angle from the azimuth-mark. Take SIX observations, which will, of course, be different, as the altitude and azimuth are constantly changing. Finally, point at the azimuth-mark to test whether the lower plate has slipped. The reading on the azimuth-mark should be 0°.



Pointing at the sun. The sun's angular diameter is about 0° 32′. With the comparatively high power telescopes now generally used on transits, this fills a large part of the field of view and it is impossible to accurately bisect such a large angular width, especially as the apparent motion of the sun across the field of view is very rapid. It therefore becomes advisable to sight the cross-wires on the *edges* of the sun, as shown in the figure, and make due allowance for the semi-diameter of the sun. The effect of this is to obtain an altitude which differs from the true altitude (h) by the angular value of the semi-diameter. The observed azimuth differs from the true azimuth by the $\frac{\text{semi-diameter}}{\cos h}$. When the sun is at the horizon, $\cos h = 1$, and the allowance equals the semi-diameter both for altitude and azimuth. For high altitudes the allowance for azimuth is much larger than the semi-diameter, since the divisor ($\cos h$) is small. If several observations are taken within a short interval, the *change* in this allowance for azimuth may be too small for notice and one value may be sufficiently accurate for all the observations. It will probably be found easier to obtain simultaneous contact of both wires by using the lower left-hand (or upper right-hand) *corner* of the field of view for morning work and the upper left-hand corner (or lower right-hand corner) for afternoon work. There is a slight variation in the semi-

Time of year.	Semi-diam. of the sun in min. of arc.
Jan. 1	16′.31 (max.)
Apr. 1	16 .03
July 1	15 .77 (min.)
Oct. 1	16 .03

diameter, as is shown by the accompanying tabular form giving average values, which may be used by interpolation if closer values are desired.

Form of Notes*

Time.	Apparent altitude.	α	h	δ	Z
4:50 P.M.	22° 48′.5	237° 41′	22° 30′.3	14° 45′.6	89° 16′.6
4:53	22 12 .5	238 11	21 54 .3	45 .6	88 46 .6
4:55	21 44 .5	238 34	21 26 .2	45 .6	88 23 .3
4:58	21 19 .0	238 55	21 0 .7	45 .7	88 02 .4
5:00	20 49 .5	239 19.5	20 31 .1	45 .7	87 38 .0
5:03	20 28 .0	239 38	20 9 .5	14° 45 .7	87 19 .9

* These figures were obtained by one of the author's students and represent his first attempt at this kind of work.

In the Form of Notes the column-headings signify as follows:

α = horizontal angle from the azimuth-mark, the angle being measured to the *right.*

h = apparent altitude − refraction ± semi-diameter of sun, in which

semi-diam. is + when sun is *above* the horizontal cross-wire,
semi-diam. is − when sun is *below* the horizontal cross-wire.

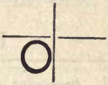
δ = declination.

Z = computed angle (as illustrated below).

Latitude: The latitude of the place of observation to the nearest half-minute of arc is necessary. This is generally known or may be obtained as shown in Exercises 48 and 49.

Reducing the observations. Compute the declinations for the given times of observation. If several observations are taken, it is generally sufficiently accurate to compute the declinations for the times of the first and last observations and interpolate for the others. The observations may be most readily reduced by using a regular form, as given below.

semi-diam. / cos app. alt.	True azim. of mark.	
17′.2	213° 19′.6	April 29, 1897
17 .2	19 .6	ϕ = 39° 58′
17 .1	19 .8	semi-diam. sun = 15′.9
17 .1	19 .7	
17 .0	19 .5	
17 .0	213 19 .1	



213° 19′.55 = mean value.

True azimuth of mark $= 540° \pm \frac{\text{semi-diam.}}{\text{cos alt.}} \pm Z - \alpha$, in which Z is + for A.M. and − for P.M., and the

$\frac{\text{semi-diam.}}{\text{cos alt.}}$ is + when the sun is on the *left* of the middle wire (as above).

$\frac{\text{semi-diam.}}{\text{cos alt.}}$ is − when the sun is on the *right* of the middle wire.

If " true azimuth of mark " is computed to be more than 360°, subtract 360° from it.

As a numerical example of the reduction: App. decl. Greenwich mean noon April 29, 1897, +14° 38′.0; hourly change +0′.77; difference of time between Greenwich and Philadelphia 5.0 hours; 5 P.M. at Philadelphia is 10 P.M. at Greenwich; therefore δ for 5 P.M. at Philadelphia = + 14° 38′.0 + (10 × 0′.77) = +14° 45′.7. From Eq. 1 (see article on Azimuth, preceding):

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin (s - \text{co.}\ h) \sin (s - \text{co.}\ \phi)}{\sin \text{co.}\ h \sin \text{co.}\ \phi}}.$$

co. h = 67° 29′.7	s − co. h = 28° 53′.3,	sin = 9.684041
co. ϕ = 50 02′.0	s − co. ϕ = 46 21 .0,	sin = 9.859480
co. δ = 75 14 .4		9.543521
192° 46′.1	sin co. h = 9.965599	
s = 96 23 .0	sin co. ϕ = 9.884466	
	9.850065	9.850065
		2) 9.693456
		9.846728 = sin 44° 38′.3

$\tfrac{1}{2} Z = 44° 38'.3; \quad \therefore \ Z = 89° 16'.6.$

$$\frac{\text{semi-diam. sun}}{\text{cos app. alt.}} = \frac{15.9}{\cos 22° 48'} = 17'.2.$$

$$540° + 17'.2 = \quad 540° \ 17'.2$$
$$-Z - \alpha = -89° \ 16'.6 - 237° \ 41' = -326 \ 57'.6$$
$$\text{True azimuth of mark} = \quad 213° \ 19'.6$$

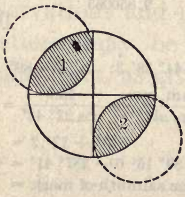
Method B

Method: The method differs from the preceding in that the observations are made in pairs, the telescope being plunged and the upper plate swung 180° in

Form of Notes for Method B

Time.	Apparent altitude.	α	Refraction.	h	δ
3:27	19° 39′	99° 52′			
3:29	19 52	99 49			
3:28	19° 45′ 30″	99° 50′ 30″	2′ 40″	19° 42′ 50″	−9° 30′ 00″
3:32	18° 46′	100° 55′ 30″			
3:34	19 03	100 49			
3:33	18° 54′ 30″	100° 52′ 15″	2 48	17 51 42	−9 30 00
3:36	18° 04′ 30″	101° 46′			
3:38	18 23 30	101 35			
3:37	18° 14′	101°40′ 30″	2 54	18 11 06	−9 30 00
3:40	17° 26′ 30″	102° 29′ 30″			
3:42	17 43	102 21			
3:41	17° 34′ 45″	102° 25′ 15″	3 0	17 31 45	−9 30 15

azimuth between the two observations of each pair. This has the effect of eliminating all index error of the



vertical circle, error of collimation, and error due to difference of height of the telescope-standards. The instrument *must* be equipped with a complete vertical circle. Then, if the observations of a pair are taken in diagonal *corners* of the field of view (as shown in the figure), even the correction for semi-diameter of the sun is eliminated. Observe as before the time, altitude, and azimuth. Correct the altitude for refraction. Add

Z	True azim. of mark.	
		October 17, 1898 $\phi = 39° 57'$
121° 55′ 30″	318° 14′ 00″	
120 54 30	318 13 15	
120 05 30	318 14 00	
119 22 00	318 12 45	
	318° 13′ 30″	

(or subtract) 180° to the reading of vernier A for the readings when the telescope is inverted.

Compute a mean Z for each pair of observations as before except that

α = the mean horizontal angle,
h = the mean altitude − refraction,
δ = the declination for the mean time, and
True azimuth of mark = $540° \pm Z - \alpha$.

Take *four* pairs of observations.

METHOD C

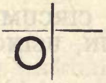
The two preceding methods have the disadvantage that the results of the field-work require considerable calculation before they have any value or before any idea of their accuracy may be obtained. The following method gives the meridian directly, but has the disadvantage that the observation *must* be taken at some previously calculated instant of time, at which time the sun may be obscured by clouds or the observation is unobtainable for any one of many reasons. But when the weather may be depended on and sufficient precautions are taken so that nothing interferes with taking an observation at the required instant, the method is perhaps the best of any of the solar methods.

FORM OF NOTES FOR METHOD C

Apparent time.	h	Refraction.	h + semi-diam. + refraction.	Z	$\frac{\text{semi-diam.}}{\text{cos alt.}}$
9:00	26° 20′ 54″	1′ 55″	26° 38′ 54″	128° 40′ 00″	18′ 00″
05			[27 22 36]		
10			[28 06 18]		
9:15	28 32 10	1 45	28 50 00	132 02 30	18 21
20			[29 31 22]		
25			[30 12 44]		
9:30	30 36 24	1 36	30 54 05	135 34 50	18 41

Method: Calculate from formula (2) (see under "Azimuth") the altitude (h) of the center of the sun for a given time and date. Then calculate from formula (1) the angle Z corresponding to that altitude. By this method the horizontal angle from the sun to the meridian at any chosen instant becomes known. Setting off this horizontal angle and altitude a few minutes before the given time, move the whole instrument in azimuth until the telescope is pointing directly at the sun. A 0° reading of the horizontal circle should then give the true meridian. Then sight on the azimuth-mark and read vernier *A*, reading the row of degree numbers that runs to 360°. Since the azimuth readings and the altitudes are different for each observation, the readings on the azimuth-mark are independent observations on the value of the required angle.

As explained in Method A, the cross-wires of the telescope should preferably be pointed at the *edges*

$180° - Z - \frac{\text{semi-diam.}}{\text{cos alt.}}$	Azimuth of mark.	
51° 02′ 00″	318° 11′	October 14, 1898
[49 54 23	318 13	$\phi = 39° 57'$
48 46 46]	318 12	Semi-diam. of sun 16′ 05″
47 39 09	318 14	
[46 28 16	318 12	
45 17 23]	318 11	
44 06 29	318 14	

Mean = 318° 12′ 26″

of the sun, which requires that the altitude should be corrected by the semi-diameter of the sun (as well as by the refraction) and that the azimuth should be corrected by the $\frac{\text{semi-diam.}}{\text{cosine app. alt.}}$. Since more accurate work may be done when the sun is apparently moving *toward* the wires, the telescope should be so pointed that the sun is in the lower left-hand corner of the field of view in the morning and in the upper left-hand corner in the afternoon. The altitude and azimuth set off should be correspondingly altered.

The *hour-angle* (*t*) is the angle from the true meridian and differs from *standard* time and even from *mean local* time. The exact time is unimportant in this problem, it being only necessary to know *apparent* time with such accuracy that the instrument may be properly set a few seconds before the time for taking the observation.

Make calculations for three observations 15 minutes apart and interpolate so as to take four intermediate observations.

Since errors of altitude produce magnified errors in the resulting azimuth, the quantities are computed to seconds of arc. In setting off the angles the closest possible setting to the real value should be made. The angles [180° − Z − (semi-diam. ÷ cos alt.)] should be set off on the *left* side of the 0° in the forenoon and on the *right* side in the afternoon, noting carefully, however, the algebraic sign of (semi-diam. ÷ cos alt.).

48. LATITUDE, FROM CIRCUMMERIDIAN ALTITUDES OF THE SUN, USING TRANSIT *

Equipment: Transit, provided with a vertical arc and a colored glass shade to protect the eye. [Note the required equipment for Exercise 47.]

Location: [See directions for LOCATION in Exercise 46.]

Time: The transit should be set up so that observations can be commenced at about 15 minutes before "apparent" noon, which is *not* "standard time" noon nor even "mean local time" noon, but is the time that the sun actually crosses the meridian, which differs from "mean local noon" by the "equation of time." If a solar ephemeris is available and if true mean local time is accurately known, the precise

* The method here developed is based on that used in refined astronomical calculations, but is very greatly simplified by approximations, which, however, do not cause errors as great as $\frac{1}{10}$ of a minute of arc, which is far within the limit of accuracy of an ordinary engineer's transit.

instant of apparent noon is readily obtainable and the reductions are simplified — as shown below. A rough approximation of the time of apparent noon may be found by interpolating in the tabular form, true for 1916 and within a few seconds for any year.

Date.		Mean local time of apparent noon.			
		h.	m.	s.	
Jan. 1		12	3	11	
Feb. 12	(approx.)	12	14	24	(max.)
Mar. 15	"	12	09	6	
Apr. 15	"	12	0	0	
May 14	"	11	56	12	(min.)
June 14	"	12	0	0	
July 26	"	12	06	20	(max.)
Sept. 1	"	12	0	0	
Oct. 1		11	49	44	
Nov. 3	"	11	43	21	(min.)
Dec. 1		11	49	6	
Dec. 25	"	12	0	0	

Method: Level up the instrument with extreme care, using precautions suggested in Exercise 46. At about 15 minutes before apparent noon sight the horizontal cross-wire on the upper edge of the sun, noting the time of exact contact and then reading the vertical angle. Repeat these observations as rapidly as is consistent with accurate work and continue the observations until about 15 minutes after noon.

Reduction of observations. An observation at apparent noon will have a greater altitude than an observation before or after noon — disregarding the effect of change of declination, which will hardly be appreciable with an ordinary engineer's transit. Observations taken before and after noon can therefore be compared with one at noon by *adding* to them a computed correction, which consists of a series of terms of which only the first need be taken. This correction

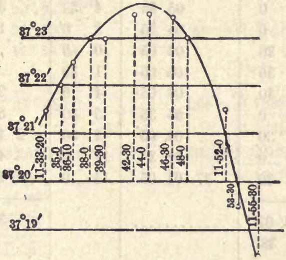
TABLE VII. — VALUES OF $\frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$, USED IN THE REDUCTION TO THE MERIDIAN FOR CIRCUMMERIDIAN ALTITUDES

Min.	0 sec.	10 sec.	20 sec.	30 sec.	40 sec.	50 sec.
0	0″.0	0″.1	0″.2	0″.5	0″.9	1″.4
1	2	3	3	4	5	7
2	8	9	11	12	14	16
3	18	20	22	24	26	29
4	31	34	37	40	43	46
5	49	52	56	59	63	67
6	71	75	79	83	87	92
7	96	101	106	110	115	120
8	126	131	136	142	147	153
9	159	165	171	177	183	190
10	196	203	210	216	223	230
11	238	245	252	260	267	275
12	283	291	299	307	315	323
13	332	340	349	358	367	376
14	385	394	403	413	422	432
15	442	451	461	472	482	492
16	502	513	524	534	545	556
17	567	578	590	601	612	624
18	636	648	660	672	684	696
19	708	721	733	746	759	772
20	785	798	811	825	838	852

$x = \frac{2 \sin^2 \frac{1}{2} t}{\sin 1''} \frac{\cos \phi \cos \delta}{\sin (\phi - \delta)}$, in which t = the "hour-angle," ϕ = latitude, and δ = declination. The value of t is found as shown below; ϕ may be found with sufficient accuracy for the purpose from the equation $\phi = 90° - (A - \text{semi-diam.} - \delta)$, in which A is the maximum altitude obtained during the observations. $\frac{\cos \phi \cos \delta}{\sin (\phi - \delta)}$ is sensibly a constant for any one set of observations, while $\frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$ depends only on the hour-angle from the meridian, and the values for it are tabulated for each 10 seconds of time up to 20 minutes from the meridian in Table VII. A value for ϕ is obtainable from each observation by the formula

$$\phi = 90° - (A - \text{refrac.} - \text{semi-diam.} - \delta + x).$$

Hour-angle (t). This is the angular value of the difference of time between the time of the observation and the time of apparent noon. When the observer is provided with accurate mean local time, he may obtain the time of apparent noon as illustrated numerically below; otherwise the time of apparent noon may be obtained with sufficient accuracy for this purpose by plotting the altitudes on profile-paper, plotting differences of time at the scale of 8 minutes of time per inch horizontally, and differences of altitude at the scale of 2 minutes of arc per inch vertically. The points should plot into a parabolic curve with vertical axis, which axis will denote the time of apparent noon.



If an observation was taken exactly at apparent noon or at culmination, it would be plotted at the vertex of the parabola and this observation would have no correction (x). The time need only be taken to the nearest even 10 seconds. As an illustration, the observations recorded in the FORM OF NOTES have been plotted as shown. The vertical circle used was graduated to single minutes and estimated to half-minutes. The coarseness of these readings will partially account for the variations between the plotted points and the curve. The reliability of this method (in the absence of an Ephemeris) is shown by the fact that although the ordinate at 11 h. 39 m. 30 s. is evidently very far from the vertex of the curve, yet if the "x" corrections were computed on the basis of apparent noon occurring at that time instead of at 11 h. 42 m. 30 s. the final result would be altered only 3″ of arc, which is far within the lowest unit of these observations.

FORM OF NOTES

Time.	Altitude.	Altitude corrected for refr. and semi-diam.	Hour-angle (t).	Correction (x).	Altitude reduced to the meridian.
h. m. s.			m. s.		
11 33 20	37° 21′ 30″	37° 04′ 05″	9 10	2′ 33″	37° 06′ 38″
35 0	22 0	04 35	7 30	1 42	06 17
36 10	22 30	05 05	6 20	1 13	06 18
38 0	23 0	05 35	4 30	37	06 12
39 30	23 0	05 35	3 0	17	05 52
42 30	23 30	06 05	0 0	0	06 05
44 0	23 30	06 05	1 30	4	06 09
46 30	23 30	06 05	4 0	29	06 34
48 0	23 0	05 35	5 30	55	06 30
52 0	21 30	04 05	9 30	2 44	06 49
53 30	20 0	02 35	11 0	3 41	06 16
11 55 30	18 30	37 01 05	13 0	5 18	37 06 23
Mean.........	37° 22′ 07″	....	......	1′ 38″	37° 06′ 20″
Mean x.......	+ 1 38			δ =	12 58 0
	37° 23′ 45″			co-ϕ =	50° 04′ 20″
Refr. +				ϕ =	39 55 40
semi-diam.	17 25				
Mean alt......	37° 06′ 20″				

It should be noticed that the final result (the mean value of ϕ) may be obtained from columns 1, 2, 4, and 5, without reducing each separate observation, by employing the *mean altitude* and *mean x*. The work is thereby much shortened, but it has the disadvantage of giving but little idea of the probable error of the result. A mere inspection of the independent results given in column 6 shows the accuracy of the work *except as it may be affected by constant errors,* such as errors of adjustment.

49. LATITUDE FROM CIRCUMMERIDIAN ALTITUDES OF THE SUN, USING SEXTANT

Equipment: Sextant, artificial horizon, chronometer or watch.

[For *Location, Time,* and *Reduction of Observations* see Exercise 48.]

October 22, 1898. Upper limb of sun.

Max. alt.	=	37° 06′ 05″	
− (− δ)	=	+12 58 0	
		50° 04′ 05″	
Approx. φ	=	39° 56′............	cos = 9.8847
δ for Gr. noon	=	12° 53′.8	
5 × 0.84	=	4 .2	
δ	=	12° 58′.0...........	cos = 9.9888
			9.8735
φ − (− δ)	=	52° 54′...............	sin = 9.9018
Coeff. for (x)	=	0.937......................	9.9717
Refr.	=	1′ 16″	
Semi-diam. sun	=	16 09	
		17′ 25″	

Eq. of time,	−16 m. 03 s.
Mean local time of app. noon,	11 h. 43 m. 57 s.
Watch *slow* (mean local time),	1 m. 20 s.
Watch time of app. noon,	11 h. 42 m. 27 s.

Method: The method is nearly identical with that of Exercise 48 except that the double altitude is observed, since an artificial horizon is used. Also, as shown in Exercise 48, the observations need not be reduced separately, but the means may be used after the corrections (x) have been computed.

To obtain the altitude of the *upper* limb, bring the two images into contact so that the direct reflection (from the artificial horizon) is *above* the index-image.

FORM OF NOTES

Time.	Double altitude.	Hour-angle (P).	Correc. to alt. (x).		
		m. s.			
11:39	89° 05′ 10″	15 30	504″		
42	10 40	12 30	328		
44:30	13 10	10	210		
47	16 00	7 30	118		
48:40	17 50	5 50	71		
50:30	19 40	4	34		
53:30	20 40	1	2		
55:30	20 40	1	2		
57:20	19 30	2 50	17		
59	19 50	4 30	43		
12:01:10	16 50	7 20	113		
03:20	13 20	8 50	164		
05:30	12 30	11	254		
07	89 08 00	12 30	328		
	89° 15′ 16″		156″		
Index error	− 1 45		2′ 36″		
2)	89° 13′ 31″				
	44 36 45				
Corr. (x)	+ 2 36				
Refr.	− 59				
	44° 38′ 22″				
Semi-diam.	− 16 03				
	44° 22′ 19″				
$-(-\delta) =$	+ 5 41 50				
co ϕ	50° 04′ 09″				
ϕ	39 55 51				

Of course the inversion of an inverting telescope, if used, must be allowed for. Obtain the altitude of the *lower* limb by bringing the direct reflection *below* the index-image.

Oct. 7, 1898. Upper limb of sun.

Approx. long. = 75° 16′ 30″. 16′ 30″ = 1 m. 06 s.

Eq. of time = 12 m. 12.06 s.

5 × 0.71 = 3.55 s.

Corr. eq. of time = 12 m. 15.61 s.

Apparent noon occurred at 11 h. 47 m. 44 s. mean local time

or at 11 h. 54 m. 30 s. watch time,

the watch being 5 m. 40 s. fast of 75° time

and 6 m. 46 s. fast of 75° 16′ 30″ time.

Max. double alt.	= 89° 20′ 40″		
Corr. for index error	= 89 18 10		
Altitude	= 44 39 05		
Corr. for semi-diam.	= 44 23		
(Approx.) δ	= 5 42	$\cos\delta$ =	9.9979
co ϕ	50° 05′	$\cos\phi$ =	9.8848
(Approx.) ϕ	39 55		9.8827
$\phi - (-\delta)$	= 45 37	$\sin(\phi - \delta)$	9.8540
		Factor for (x) = 1.07 ...	0.0287

PART IV

SURVEYS

STUDENTS' TOPOGRAPHIC-HYDROGRAPHIC SURVEY
STUDENTS' RAILROAD SURVEYS
OTHER STUDENTS' SURVEYS

STUDENTS' TOPOGRAPHIC-HYDROGRAPHIC SURVEY

The following directions were originally written for the purpose of supplementing the general treatment of the subject, as given in a text-book on surveying, by specific instructions applied to the locality of the survey — a section of Fairmount Park, including a stretch of the Schuylkill River and a considerable area along the banks. They have been incorporated, with some revision, into this book because it is believed that they are sufficiently applicable to any similar survey to warrant their publication.

The triangulation methods are those for tertiary work, and the directions for "precise" base-line work, and latitude, were inserted as exercises for the upper classmen, and not because they are essential to the proper conduct of the survey.

A. Reconnoissance; Location of Stations

1. Two parties are necessary — one on each side of the river. Each party should be supplied with stakes, a heavy hatchet or axe, a pocket sextant (or prismatic compass), three transit-rods with flags tacked on, and a light crowbar.

2. In general four lines will run from each station — two to stations above and below on the same shore and two to stations on the opposite shore. The distance between stations along one shore should be, when possible, a little more than the width of the river so as to obtain nearly equilateral triangles.

3. The *forward* rodman on each side is controlled by the party on the *other* side. When the forward rodman reaches a suitable location that gives satisfactory angles (as near 60° as possible) to *both* parties he remains there until dismissed by *both* parties. The rear flagman on each side occupies the station last occupied by the main party.

4. The angles between stations should be measured with a pocket sextant to the nearest degree. These should be plotted at night so that any necessary modifications can be promptly made, and then the stations can be properly numbered as a basis for future field orders.

5. All triangulation stations should be marked by stakes 2″ × 2″ × 18″, centered with tacks, driven to within 4″ of the ground, using a crowbar to make a hole if necessary. The stakes for the ends of the base-line should be longer; they should project at least 12″.

B. Base-line. Precise Method; accuracy 1 : 1 000 000

1. The site for the base-line should be nearly level, and on as uniform a surface as possible. The roadways on the river-banks generally afford the best sites.

2. A hub should be set every 300 feet, measuring from one end; also, a hub should be placed within 10 feet of the other end and at an even 10-foot distance from the last 300-foot hub placed. The hubs should be long enough to project 12″. At every 50 feet between the hubs 3-foot stakes should be driven so

that the base-line would pass about $\frac{1}{4}''$ from the face of the stake. Screw-eyes with $3''$ hooks should be screwed into the faces of these stakes so that the bottom of each hook is on the grade-line between the tops of the adjacent hubs. The hooks should have the points bent upward, and the screws should be so set that the hooks will point outward and swing freely in both directions. Strips of zinc should be nailed to the tops of the hubs. While obtaining the grade-line between hubs, the differences of elevation of all adjacent hubs should be noted.

3. The tape stretcher should be used with a tension of 16 lbs. The rear end of the tape is placed exactly on the station mark, the forward end screwed up until the tension of 16 lbs. is obtained, every hook support critically examined to see that the tape rests with perfect freedom in the hooks, and then a fine mark is scratched in the zinc strip on the hub with a fine steel point. Then the tape is moved ahead. While measuring, two thermometers should be tied to the tape near the 100-ft. and 200-ft. points (marked 90 and 190 on the U. Pa. 300-foot tape), and the readings taken to the nearest tenth of a degree at every measurement. These thermometers should be carefully standardized and their exact errors known. The last sub-distance (less than 10 feet) is measured with a fine 10-foot steel tape, graduated to $0.01'$ and estimated to $0.001'$.

4. Measurements should not be made, unless for mere practice, while the sun is shining on the tape. A calm, cloudy day should be chosen, and even better results are obtained during a drizzling rain. In default of cloudy days, measurements should be taken immediately after sundown or before sunrise. For the best results the earth and atmosphere should have the

same temperature. This probably occurs during the evening, but then lanterns must be used, endangering the accuracy.

C. Base-line. Ordinary Method; probable accuracy, 1 : 50000

1. Measure the base-line *three* times with an ordinary 100-foot steel tape, using fine brass plumb-bobs to plumb down the tape ends to the tops of stakes set every 100 feet. The mean temperature of the air should be noted with a pocket thermometer, and corrections made accordingly. Care should be used to have a uniform tension of as near 16 lbs. as possible. A spring-balance should be used for this.

2. The mean of these measurements may be used as the final value for the triangulation should anything prevent the "precise" measurement (B).

3. A check-base should be measured at or near the other end of the system of triangulation.

D. Azimuth

1. Azimuth may be determined by a set of observations with a solar attachment. At least twenty observations should be obtained having an extreme range not greater than 5′ of arc (see Exercise 46). The observations should be taken at a triangulation station, and each observation consists in finding the angle between the meridian, as determined by that observation, and a line to another triangulation station.

2. Azimuth may also be determined by one of the methods given in Exercise 47. Some such method is necessary when the equipment does not include a solar attachment.

3. A valuable check on the work may be obtained by taking azimuth at stations near each end of the

triangulation, making corrections, if necessary, for the convergence of meridians.

4. True latitude (as close as the reading limit of the vertical arc of the transit) should be known. If it is not obtainable from an accurate map of the section of the country, it must be observed as provided below.

E. Latitude

1. Observations for latitude may be made by observing the culmination of the sum at noon by one of the methods described in Exercise 48 or in Exercise 49.

2. Observations for latitude should be taken at two triangulation stations which differ considerably in latitude. These observations may be checked by allowing 101 feet for each second of arc in the difference of the latitudes obtained and comparing this result with that computed directly from the triangulation.

F. Triangulation

1. Sightings may be taken (1) at a transit-rod, held by a rodman pointing as near the bottom of the rod as possible, noting by the vertical cross-hair the verticality of the rod, or (2) at a transit that may be occupying the station, pointing at the plumb-bob or at the vertical spindle underneath the plates. When it is evident that a station occupied by a transit party is being sighted at by another party, care must be taken to facilitate their work by not obstructing the line of view and always giving them a good mark to sight at. If a transit which is being sighted at is removed from the station, it should be promptly replaced by a rod.

2. Angles should be measured by the method of Exercise 16. Measure by repetition until six *consecu-*

tive values are obtained having an extreme range not exceeding 2′ of arc. Compute the mean value.

3. All triangles must "close" with an error not exceeding 1′ of arc.

4. As fast as the angles of the triangles are *satisfactorily* measured, the sides should be computed from the measured base, the true azimuth of each triangulation line determined, and the whole triangulation plotted.

G. LEVELING.

1. A line of levels should be run along each bank of the river, noting the elevation of each triangulation station and also of the water surface near each station.

2. A tide-gauge should be set up in a suitable place, the elevation of its zero determined, and readings to the nearest tenth of a foot taken as often as proves necessary, while the line of levels is run and while stadia topography is being taken.

H. SOUNDINGS

1. Soundings to a depth of 18 to 20 feet may be made with a sounding-pole. A sounding-line with weight will be needed for a large part of the soundings above the Girard Ave. Bridge (max. depth about 36 feet).

2. Lines of soundings should be run nearly perpendicular to the course of the river and about 300 to 500 feet apart; the individual soundings about 50 feet apart.

3. The soundings should be located by two transits on shore pointing at each other and at the boat. As a check, a sextant in the boat points at the two transits. Each transit is set at 0° on the other transit, the lower plate always clamped, the upper plate free, the movement of the boat constantly followed with the telescope.

4. At the instant of taking a sounding, a signal will be given from the boat — a waving flag or a whistle. The shore parties will note *first* the *time* of the signal, and *second* the angle to the boat. Since it will not increase the accuracy of the plotting, the angles need not be read closer than the nearest 10′ or 15′, and so it will be easier and better to take the reading directly from the index, not even reading the vernier. The transitmen should point at the sounding-pole as nearly as possible.

5. The shore parties and the boat party must be supplied with watches, previously compared to a close fraction of a minute. It will save trouble in identifying the shore and boat records if the soundings (and signals) are made always at an even minute of time, and never oftener than once per minute. An occasional signal of *two* whistles (marked as such in all the note-books) will also aid in identifying the soundings, especially when mistakes have been made.

6. The shore parties should be located at triangulation stations when they are suitable. Otherwise, points which are especially desirable for this purpose may be selected and their locations accurately determined from the triangulation stations.

7. The following form of notes should be used for the note-book in the sounding-boat:

Line.	Time.	Sextant angle.	Angle.	Depth.	Bottom.
4-D_2	2:34	*E*-boat-*D*	62° 20′	15.7	Sandy
	2:36		68° 40′	18.2	Sandy

The right-hand page should record the personnel of the party and any other desired information. D_2 means the second point on the shore below Sta. *D* toward (or away from) which a line of soundings has been run. The note-books of the shore parties should observe the following form:

Inst. at	Line.	Time.	Shore angle.	Angle.	
Δ *D*	4–D_2	2:34 2:36	boat–*D*–*E*	45° 10′ 42° 30′.	

8. During the evening following the sounding work, the soundings in the sounding-book should be numbered consecutively and the corresponding shore angles numbered with the same numbers, the correspondence being determined by the identity of time, verified by double signals, etc. These checks of double signals, etc., are often proved valuable.

I. Topography

1. Topography is taken by stadia lines, which must always begin and end on triangulation stations or other points equally well determined. Beginning at some station, the telescope should be pointed at some other known station, with the plates set at the known azimuth of that line.

2. Distance and vertical angle are read both for foresight and backsight between stadia stations. Side shots are taken to all desired points within a radius of 500 to 600 feet.

3. One line should be run near enough to the shore line to take in all the configurations of the shore and also a belt of land from 200 to 600 feet wide — varying with the interference of trees and bluffs. The vertical angles to shore points should be noted with especial care, as they furnish a valuable check on the levels.

4. The course of the other meander lines should depend largely on the topography. In general, they should follow roads, etc., so as to pass as near as possible to the important points that must be observed, and yet must pass so near to the other lines that no feature of topography will be omitted.

5. If the transits contain fixed stadia wires, a careful test of them should be made at the beginning of the season's work, and if the interval does not correspond to the graduation of the rod, a reduction coefficient should be computed and applied to each reading. If the wires are adjustable, tests should be made frequently, even daily, considering how simple it is. Measure off a base-line 600 feet long on level ground near headquarters. Each morning, before beginning stadia work, set up the transit at a distance "$f + c$" behind one end of the base-line, and read the rod held at the other end. An error of more than 1 : 600 should be immediately adjusted, after consultation with an instructor. If the first "set-up" of the transit is to be at a triangulation station, it may be more convenient to begin there at once and point to another station whose distance is known, and take the reading, making due allowance for the "$f + c$."

6. Further instructions on this subject, together with a form of notes, may be found in Exercises 36 and 37.

J. Computations; Plotting

1. The closure of triangles within the required limits must be determined as soon as possible, and the angles remeasured if necessary.

2. The azimuth and length of all triangulation lines must be computed as soon as sufficient data are obtained.

3. To check topographic work, the coordinates of all triangulation stations from some chosen meridian and parallel should be computed before topographic work commences.

4. The latitudes and departures of the lines between stations of the topographic work should be computed and checked with the previously computed coordinates of the triangulation stations to which the stadia lines run.

5. For such lines as are run on this survey — probably less than ½ mile between such stations as are used to check on — the errors should be within 10 feet horizontal distance and 1 foot in vertical height.* Greater errors than these call for a thorough revision of notes to discover a possible error, and perhaps a resurvey.

6. The triangulation should be plotted to a scale of 200 feet to the inch as soon as the computations are complete.

7. All soundings should be plotted during the progress of the survey, or at least sufficient of them to show the general accuracy and reliability of the notes.

8. All topographical notes should be plotted during the survey (at least in pencil) — plotting first the transit stations until the line checks with sufficient accuracy on some known station and then plotting the side shots, locating all drives, walks, buildings, railroads, bridges, etc.

* These errors are very large considering what accuracy is possible with the stadia, but it is doubtful if much closer work than this can be expected of inexperienced students.

STUDENTS' RAILROAD SURVEY

A. INTRODUCTION

1. Student surveys necessarily differ from commercial surveys in many ways.

A prime requisite in commercial field work is to obtain the desired data at minimum cost, and therefore at maximum speed; in the class field work, to give each student a maximum of practice in each position. Therefore each member of the commercial corps is kept in one position throughout the survey, and the corps need be broken in only once; while each member of the class corps takes a new position each field day, and the corps must be broken in anew each time it enters the field. Hence, speed in breaking in is of far greater relative importance on class surveys than on commercial surveys. The time required for each daily breaking in is materially shorter when each member of the corps has learned in advance something of the duties which usually pertain to the position assigned to him for the day; and the less the time taken for breaking in, the greater the time remaining for carrying the survey forward.

The following pages have been prepared expressly for the purpose of enabling the beginner to learn something of the duties assigned to him a little in advance of his undertaking their performance.

Again, chance of making mistakes in any part of the survey decreases as the duties of each position come to be performed more or less mechanically. Hence, chances for making mistakes are greater on class surveys where, every field day, each position is filled by a beginner, than on commercial surveys on which, after the beginning, each position is filled by a skilled man. Furthermore, in class work every member of the corps performs all the steps of the office work, so that a mistake discovered late may render useless, so far as advancing the solution of the main problem is concerned, the work of the entire corps for an hour or even for a whole afternoon. Hence it is decidedly to each student's interest to use every means to discover his own mistakes in each step before using the results or turning them over to the corps for use in the succeeding step.

Another difference between class surveys and commercial surveys is this: many factors which influence the locating engineer's decisions are here ignored altogether, and many decisions are arbitrarily stated for the student. This is necessary for two reasons. In the first place, the student lacks the knowledge of railroad location, construction, maintenance, and operation, necessary for the solution of a complete problem in railroad location; and in the second place, he could not take the time, in a course in railroad surveying, to apply that knowledge if he had it.

The following directions for making railroad surveys are written strictly for the limiting conditions under which the class works, and would require amplification and revision in several parts to meet the requirements of any given commercial Preliminary Survey. See Beahan, Lavis, and Wellington.

B. Location and Schedule of Work.

2. Location. — The instructor will make a reconnoissance for a line about one mile * long, of constant

* One mile of Preliminary Survey is about the maximum for a party of seven or eight students limited to eighty hours in the field and sixty in the drafting room. There is a tendency to undertake too great length of line, with the result that students are unable to carry the office work through the later stages.

gradient, running diagonally on side-hill ground of such character as to require several curves; and narrow the choice of route to a certain strip of territory. Within this strip, the choice of position for the line will be based upon a more detailed survey of the strip, which survey is a topographic survey and is called a Preliminary Survey. A topographic survey can be made by any one of several methods, or by any one of several combinations of these methods. Thus, for example, the Preliminary Survey may be made by plane-table method, or by transit-and-stadia method, or by method of traverse and cross-section (wherein cross-sections are taken as in Exercise 40 or as in Exercise 41). Unless otherwise specified by the instructor, contours of students' Preliminary Survey will be obtained by cross-section method of Exercise 40.

3. Schedule. — The work of making the students' railroad survey may be outlined as follows:

PRELIMINARY SURVEY

A. Obtaining field data —

1. Transit party stakes traverse lengthwise through the strip.
2. Level party takes elevations of stations.
3. Topography party determines positions of contours, and other features of strip.

B. Distributing field data to all members of class —

4. Chief of corps reports on field work.
5. Recorders transcribe field notes into office books.
6. Recorders transcribe reduced field notes on blackboard.
7. Each student transcribes reduced field notes into his own field book.

C. Computing and plotting —

8. Each student computes rectangular coordinates of traverse.

9. Each student plots assigned part of topography.

10. Each student plots entire profile.

11. Each student traces complete map of Preliminary Survey.

LOCATION SURVEY

D. Planning location —

12. Each student plans and draws grade line and alignment for a proposed railroad on ground represented by Preliminary Survey.

13. Each student prepares complete field notes from his own paper location.

E. Making field location —

14. Transit party stakes out paper location, using field notes prepared therefor.

15. Level party takes elevations of stations of staked location.

16. Earthwork cross-section parties determine positions for slope stakes and of break-points on cross-section of each station of staked location.

17. Land-line party obtains necessary data for right of way map showing ownership.

18. Drainage survey party makes rough, rapid survey of each drainage area on up-stream side of located line.

19. Each student adds to map of Preliminary Survey, data obtained on Location Survey.

F. Office work following staking out of location —

20. Each student makes profile of staked location.

21. Each student plots earthwork cross-sections of assigned stations.
22. Each student computes yardage for certain assigned stations.
23. Each student plots mass curve; and decides on distribution of material.
24. Each student makes detailed estimate of quantities involved in construction of located line.

C. Organization and Equipment

4. Sections and Corps. — The class in railroad surveying may be divided into sections, each section having a separate time schedule. A small section will constitute a single surveying corps. If a section is large enough it is divided into two or more corps. All members of a corps should have same drafting periods, as well as same field periods.

5. Corps letter. — In order to avoid confusion in field, drafting room, and office, each corps will be designated by a letter (as A, B, or C, etc.); and members of a corps will mark all stakes, books, maps, profiles, and sheets of computations conspicuously with the corps' designating letter.

6. Rank and file. — In the field each member of a corps will occupy in turn all positions (with possible minor exceptions) named below, as directed by instructor. In the following outline relative ranks of positions are indicated by arrangement.

- Chief of Corps
  - *Transit Party*
    - Transitman
      - Recorder (sometimes omitted)
      - Head Tapeman
        - Rear Tapeman
        - Stakeman
        - Axman
        - Front Flagman
        - Rear Flagman
  - *Level Party*
    - Levelman
      - Rodman
  - *Topography Party*
    - Topographer
      - Handlevel Man
      - Rodman

Duties of each position in field are described under corresponding head, below.

7. Personal equipment. — Equipment to be furnished by student for each position in field is stated in paragraph devoted to that position in following pages.

8. School equipment. — The following school equipment is ordinarily provided by the school for use in field.

Issued to	Issued for	Equipment.
Transitman	Transit party	Transit and shade Hood (in rainy weather) Bob for transit Bobs for tapemen (2 or 3) 100-foot steel tape (with 2 handles or rawhides) 11 pins Line rods (2 or more) Sack (or basket) Hand-axes (2 or more) Keel (1 or more pieces) Tacks and 10d nails Stakes, hubs, pegs
Levelman	Level party	Level (dumpy or wye) and shade Hood (in rainy weather) Hand level Target rod Hatchet Keel BM nails or tacks TP iron spike or wood pegs
Topographer	Topography party	Hand-level 50-foot metallic tape Plain leveling rod, or stadia rod Hand-ax Pocket compass Slope-level Aneroid

9. Issue and return of equipment. — Unless otherwise directed by instructor, the transitman, the levelman, and the topographer, each in his turn will: (1) Stand outside of issuing door or window of instrument room and receive, and distribute to his men standing near, all equipment prescribed for the day for his party. (2) Sign receipt with his regular signature. (3) Retain carbon copy of receipt. (4) Be responsible for proper use and care of all equipment receipted for.

(5) Receive equipment from his men, on return from field, and return it through receiving door or window of instrument room. (6) Take up his receipt.

D. Duties and Equipment of Members of Corps

10. Chief of corps. — Equipment and duties of chief of corps are as follows: (1) Personal equipment: text-book, field book, 4H pencil, pocket knife, list of men and their positions. (2) School equipment: pocket compass, hand-level (or slope-level), line rod with attached flag, aneroid barometer. (3) Become conversant in advance, as far as practicable, with what has been done to date on the survey; and with object and general methods of field work about to be undertaken. (4) Supervise corps equipping at beginning of field period, and see that each recorder has all needful data for starting work. (5) Direct entire work in field — taking counsel from instructor as need arises — directing each party to its starting station, passing from party to party, giving such suggestions, advice, or directions as may seem required to make work proceed smoothly. Inspect notes of recorders from time to time, to see that ample information is being obtained and intelligently recorded. Pick out line ahead of transit party; and keep informed of elevations so as to know whether to advance on higher or lower ground. (6) Record in field book such data as recorders have not been instructed to record — for example: names of property owners; required areas of waterways; and estimate of the material underlying the surface, as having a bearing on excavating cuts and foundation pits. (7) Obtain from each of recorders, on stopping work for the period, starting station and stopping station for the period. (8) Obtain report blank from instructor at close of period, fill it out, and return it to instructor. (If no report blank is provided by instructor, chief will follow Form D, of Appendix.) (9) See that recorders thoroughly check their notes, original and reduced, and copy their checked notes into the office books before following drafting period. (This does not apply if field notes have been taken directly in office books.) (10) See that recorders copy reduced notes on blackboard at beginning of succeeding drafting period.

11. Transitman. — Transitman's duties and equipment will be as follows: (1) Personal equipment: magnifying glass, 4H pencil, scratch pad, and pocket knife. (2) School equipment: transit, hood (in rainy weather), bob. (3) Under supervision of chief, direct transit party. (4) Run traverse by double deflection angles, turning the angles as directed in Exercise 35. (5) Set hubs so that first calculated bearing and each deflection angle will read to even ten minutes, if this be practicable, in order to minimize work of computing latitudes and departures. (6) Read the needle on each foresight and compare reading immediately with calculated bearing. (7) Take rough stadia reading on back flag rod to check tapemen. (8) Read needle on each backsight, when there appears to be local attraction. (9) Align tapemen and axmen when necessary. It is not necessary to set intermediate station stakes with precision on the line: they can be aligned by eye well enough whenever the eye has in view two marks already on line. (10) Establish transit ahead of tapemen whenever it is advantageous to do so, letting tapemen work toward transit. (11) Read and follow (4) and (6) of Art. 21 following. (12) Determine, by stadia or otherwise, positions of such topographic details on each side of traverse as may be specified by instructor. (Angle at which traverse crosses a fence can be plotted, by orienting sketch page of field book on fence, and sighting along fence.)

(13) If acting as recorder as well as transitman, follow also directions given in following article.

12. Recorder. — Equipment and duties of recorder will be as follows: (1) Personal equipment: text-book; field book; 4H pencil; pocket triangle; protractor; pocket knife; emery paper; and two or more sheets of blank paper (for use in case computing becomes necessary in field); and rubber bands or paper clips to hold pages open. (2) Prepare field book, some time before field period, by inserting main heading and column headings for notes he is going to take. Use form of notes given herewith unless otherwise directed by instructor. (3) Compute length of each course

FORM OF NOTES

Preliminary traverse. Line *C*.

Sta.	Dist.	Calc'd bearing.	Defl'n angle (*R* or *L*).	Needle FS.	Needle BS.	Double Defl'n angle.

N.B. — The notes run up the page.

N.B. — Calculated bearing of first course (starting from sta. 0 + 00) to be taken = needle reading on that course.

N.B. — Indicate each hub by placing encircled dot ⊙ immediately after station and plus of hub.

N.B. — Central V line of right-hand page represents traverse line in all sketching. Enter in proper columns, at bottom of page, station of last hub set during preceding field period, and calculated bearing of preceding course.

as soon as taped, and enter in "Dist." column. Check length of course with back stadia reading. (4) Copy into recorder's book at convenient times pluses taken by rear tapeman, making certain that all are copied before corps is disbanded for the day. Check transcript with help of rear tapeman. (5) Sketch on right-hand page all topographic details taken by transitman, and enter on sketch all measurements made to determine magnitude and position of objects sketched. Central V line of right-hand page represents traverse whether latter is straight or broken. (6) Compute calculated bearing without delay, so that any discrepancy in reading deflection angle and needle can be investigated before transit is moved ahead. (7) Check all computations made on notes in field, after field period and before copying notes. (8) Copy notes into office book (if notes were not taken in office book), prior to succeeding drafting period. (9) Copy on the designated blackboard the following notes (whether taken in recorder's book or office book), at beginning of succeeding drafting period:

Sta. of hub.	Defl'n angle (*R* or *L*).	Calculated bearing.

(In case these data are manifolded and distributed to all corps members, blackboard copy will not be required.) (10) Check transcript.

13. Head tapeman. — Equipment and duties of head tapeman will be as follows: (1) Equipment; 100-foot steel tape with handles; 11 tape pins with white or red cloth flags attached; plumb bob for taping; line rod with white cloth flag attached. (2) Carry zero end of tape and line rod and pins. Locate and mark station points and hubs as prescribed in Exercise 31. (3) Inspect stake marking and driving. (4) In taking a plus, hold zero-end of tape at the point of which plus is desired; and direct rear tapeman to come forward to station within tape-length and take plus by reading whole feet and estimating fractional foot. Then shift tape one foot forward, and measure fractional foot at zero-end; and call out result to rear tapeman for record. (5) Pick out new transit points (setting hub and tack to mark each), when this is not done by chief, and let hubs come at full stations, or, at least, at pluses which are multiples of 10; and, where practicable, so as to make deflection angle a multiple of 10 minutes. (6) Direct axmen in work of clearing.

14. Rear tapeman. — Equipment and duties of rear tapeman will be as follows: (1) Personal equipment: field book; 4H pencil; pocket knife. (2) School equipment: plumb bob for taping. (3) Prepare field book in advance, by inserting page heading and column headings. (4) Hold 100-foot mark of tape at each station to fix position of next succeeding station. (5) Follow directions given in Exercise 31 for rear tapeman. (6) Read all pluses to nearest 0.1 ft. (First, note whole feet, and estimate fractional foot; and thus be able to pass judgment on correctness of head tapeman's measuring of fractional foot. (7) Record station wherever plus is taken, together with name or description of object to which plus is taken, whether latter be hub, fence, road, land line, stream, or other thing. (8) Let recorder copy record of pluses into transit book every time the two come together, and make sure that all pluses have been copied by time corps leaves field. Help recorder check transcript.

15. Stakeman. — Equipment and duties of stakeman will be as follows: (1) Equipment: sack (or basket, to carry stakes, etc.); stakes; hubs; tacks; pegs; hand-ax (or ax); keel. (2) Mark and drive station stakes, hubs and hub guards, as prescribed in Exercise 31. Prefix corps letter to each stake number.

16. Axman. — Equipment and duties of axman are as follows: (1) Equipment: ax (or hand-ax); tacks; keel; and, if specially ordered, sack of stakes. (2) Work under direction of chief or head tapeman. (3) When not otherwise employed, assist stakeman. (4) Avoid damaging property, including fences and crops.

17. Front flagman. — (1) Front flagman's equipment is as follows: Line rod with white cloth flag attached; and, if specially ordered, sack; stakes; hubs; tacks; hand-ax; keel. (2) Clear line and also work under chief, giving foresights to transit party.

18. Rear flagman. — Equipment and duties of this position will be as follows: (1) Personal equipment: pocket knife and two or more sheets of rather stiff white note-paper. (2) School equipment: line rod with white cloth flag attached; plumb bob; dozen 10d. nails; hatchet. (3) Remain at hub next back of transit until transitman signals "forward"; then proceed to hub which transitman is leaving. (4) Keep eye on transitman and anticipate his need of backsight. When transitman wants backsight, hold rod (or suspend bob) plumb, over tack in hub. If rod is warped, hold it so that it will appear straight to transitman. (5) Leave "butter-fly" where directed to by transitman. (A "butter-fly" is a piece of white paper doubly impaled on a nail partially driven in vertical position in top rail or board of a fence or in top of a hub, just behind the tack, in such manner as to serve as a backsight to transitman.) (6) He will carry all lunches, spare coats, and other baggage of party.

19. Levelman. — Equipment and duties of levelman are as follows: (1) Personal equipment: field book; 4H pencil; pocket knife; emery paper; two or three sheets of blank paper (for field computations); and copy of descriptions and elevations of all BM's established by the corps. (2) School equipment: engineers' level; hood (if weather is rainy); hand-level. (3) Prepare field book in advance. Insert page heading and column headings as shown in first Form of Notes given in Exercise 33, unless directed by instructor to use second form given in Exercise 33. Enter station, elevation, and description, of BM, TP, or hub from which leveling will start. (4) Take two readings on each BM, TP, and hub — direct reading and target reading — thus: (*i*) Take direct reading to nearest 0.01 ft. and record reading. (*ii*) Take target reading to nearest 0.001 ft. (rodman records reading and calls or signals it to levelman.) (*iii*) If target reading checks direct reading (to nearest 0.01 ft.), make a check mark at record of direct reading; and send rodman to next point. (*iv*) If target reading does not check direct reading, take additional direct and target readings till sure of correct reading. (Cross out — do not erase — an incorrect entry; and see that rodman does likewise.) (5) When rod is held on ground, take one direct reading, and this only to nearest 0.1 ft. (6) Establish substantial BM's at intervals of 2000 ft., or less if practicable. Under (4) above, every transit hub becomes a temporary BM. (7) Check on all previously established BM's in passing. (8) Describe each BM on right-hand page on same horizontal line with record of its elevation. Examples of definite descriptions: "BM, top of hub, 21 +

30." "BM, nail, root of Live Oak 18″ diam., 56 + 10, 45′ R." (9) For TP's, use top of station stake, or hub guard, or hub, when convenient, or pointed top of boulder. If none of these be at hand, have rodman drive TP spike or peg, to serve as TP. (10) Describe each TP on right-hand page, on same horizontal line with elevation of TP. Examples: "TP, top of hub, 12 + 60." "TP, top of guard stake, 10″ L 12 + 60." "TP, top of boulder, 42 + 30, 10′ R." "TP, spike." (11) Keep BS and FS distances for TP's and BM's approximately balanced. (12) Take ground elevation at every station. In case the top of station stake or hub guard is used as TP, take elevation of ground either before taking FS on TP, or after taking BS on TP. (13) Take ground elevation at pluses, such as CL of highway, top of stream bank, bed of stream, and decided humps and hollows (consult instructor). Describe plus on right-hand page. For ditch or small stream, take elevation of top of near bank, and note width and depth on right-hand page. (14) To save time, use hand-level, instead of engineer's level, to obtain elevations of ground at and near bottoms of deep and narrow depressions, and make remark to this effect on right-hand page. Carry regular leveling over depression. Also, use hand-level, if convenient, to try for extreme-position set-ups. (15) Compute elevations while rodman is passing from station to station. Record ground elevations to nearest 0.1 ft., only. Have rodman compute his elevations while the level is moving forward to new set-up. (16) Compare elevations with rodman's while passing him. (17) Report to chief, before separating from him at end of field period, initial and final stations of leveling work for period. (18) After field period, check notes and all computations thereon (using adding machine, if it be at hand). (19) Copy field notes into office book (unless field notes were taken directly in office book) before the following drafting period of corps. Check copying. (20) Copy stations and ground elevations (whether taken in levelman's or office book), at designated blackboard at beginning of next drafting period. Check copying. (If notes be mimeographed and copies passed around, it will not be necessary to copy notes on the blackboard.)

20. Level-rodman. — Equipment and duties of level-rodman are as follows: (1) Personal equipment: field book; 4H pencil; pocket knife. (2) School equipment: target rod; wooden TP (or TP spike); nails (or tacks, for BM's); hand-ax; keel. (3) Prepare field book in advance. (Level-rodman will keep "peg level" or check notes.) Insert page heading. Insert column headings as given in FORM OF NOTES of Exercise 32. Enter station, elevation, and description of BM, TP, or hub from which work of level party will start. (4) Follow instructions to levelman under (4) to (13), inclusive, of Art. 19 in so far as they apply to rodman. (5) Select TP's with care, especially on rough ground, so that line of sight may not be obstructed by stumps, rocks, etc. (6) Keel tops of all stakes and hubs which serve as TP's or temporary BM's. (7) Call out station and character of each rod point on approaching it; *e.g.*, say whether it is to be "TP, top of stake," or "BM, hub," or "Ditch, top of near bank"; and so on. (8) Help levelman to check notes, to copy into office book, and to copy on blackboard.

21. Topographer. — Equipment and duties of topographer will be as follows: (1) Personal equipment: field book; 4H pencil; pocket knife; emery paper. (2) Prepare field book in advance. Insert page heading, "Topography, Line ——." (Put corps letter in blank space.) See (3) below. Copy into field book, from office book, the stations and elevations of that part of line on which topography is to be taken. (3) Form of notes: In first two columns of left-hand page, record "station" and "ground elevations." (Run notes up the page.) On left-hand page record contour elevations and distances out in form of fractions, as shown in Exercise 40; and on right-hand page plot contour points to scale, and sketch contours through plotted points. Put on each contour its elevation. Sketch on right-hand page such other topographic details as are located by topography party; and enter on sketch of such, all measurements made to determine size and position of each detail.

(4) The following fundamental principles should control taking of topography. (*i*) Preliminary map must show every feature and topographic detail which may affect choice of location of center line of proposed road. Thus, map must show configuration of ground surface, and all fences, land lines, ownerships, ditches, streams, lakes, roads, railroads, buildings, etc., etc.; boundaries of orchards, gardens, vineyards, and other cultivated lands, and of forests, waste lands, etc.; which lie within the strip mapped. (*ii*) The maximum permissible error in measuring distance in connection with topographic details depends on two things: First: on scale of map. For example, if scale is 1 in. = 100 ft., any field error of less than 2 ft. will scarcely affect map; and we may estimate distances under 10± ft., and pace distances under 60± ft., without detriment to map. Second: maximum permissible error depends on probability that detail to which distance pertains will affect location of CL of road. This means that positions of topographic details remote from traverse need not be determined with precision required for near-by

details; provided the traverse lies approximately in central portion of strip to be mapped. (Sometimes obstacles force traverse close to one edge, or even entirely outside, of strip.) Consequently, sketching of details along the outer fringe of strip is based usually on estimated distances, controlled when convenient by "intersecting angles" taken from traverse by transit or compass.

(5) Direct and record work of locating contours, which is to be performed in accordance with Exercise 40. (6) Learn from instructor or chief what other topographic details are to be taken by topographic party and corresponding measurements to be made. (7) Report initial and final stations of topography taken during field period, to chief before corps disbands at end of period. (8) Copy field notes into office book (unless notes were taken in office book) before following drafting period of corps. Check copying. (9) Copy stations and "fractions" on designated blackboard at beginning of corps' next drafting period. Check copying. (Copying on blackboard will not be required if office supplies each member of corps with mimeographed copy of notes.)

22. Hand-levelman. — Equipment and duties of hand-levelman will be as follows: (1) School equipment: hand-level; and 50 foot metallic tape. Also, if so ordered, pocket compass or prismatic compass; slope level; and aneroid. (2) Run the hand-level and hold the 50 foot end of tape for location of contour points on both sides of traverse, at each station, according to method of Exercise 40. (3) Measure such bearings, elevations, slopes, and distances as topographer may require for determinations of positions of other topographic details.

23. Topography rodman. — Work with hand-levelman under direction of topographer; and carry the following equipment: hand-ax, and plain leveling rod (or stadia rod).

E. Distribution of Field Data

24. Importance of timely distribution. — In order that the succeeding steps in field and office work of the Preliminary Survey may be planned and executed intelligently and without loss of time on the part of individual students, a clear record of past field steps must be kept up to date in the office; and results of field work must be placed before corps members in drafting room as soon as they are ready to use them in computing and plotting. Therefore the following directions are of the first importance. (1) Chief will fill out, in prescribed form, a report blank immediately on returning from field work, and file report at once with instructor. See under (8) of Art. 10, above. (2) Recorder, levelman, and topographer will each check his field notes and computed entries, and copy both into office book (if notes have not been taken in office book) before next drafting period of his corps, and check transcript. See (8) and (9) of Art. 12, above; (19) and (20) of Art. 19; and (8) and (9) of Art. 21. (3) Recorder, levelman, and topographer will each copy his field notes (or the essential portions thereof) on designated blackboard at beginning of next succeeding drafting period of his corps; and check copy. (4) Each member of corps will at once copy notes into his own book from blackboard, or, in some cases, from office book or original field notes; and check copy.

F. Computation of Coordinates

25. Identification marks. — Each member of corps will place his name and corps letter and the date conspicuously in upper right-hand corner of every sheet of paper — computing paper, detail paper, profile paper, and cross-section paper — before he begins work on it.

26. Sequence of steps. — The following parts of office work connected with Preliminary Survey are to be begun as soon as possible and carried forward as continuously as rate of field work will permit; but the steps need not, in every case, be taken in the same order as given here. For example, if traverse notes are not at hand and level notes are, proceed with profile (Art. 42, following).

27. Departures and latitudes. — Compute departure and latitude of each course of traverse to nearest 0.1 ft.; using logarithms, or machine or traverse table or both, to obtain products. Details of logarithmic method are given in Art. 28, following; of machine method, in Art. 29.

28. Log computation. — Form A (see Appendix) is convenient for log computation. Rule this form on as many sheets of prescribed paper as there are data for. (1) Column 1. — The first entry of column 1 of sheet 1 is "0 + 00," as shown on form A. Next, enter station of first angle-point beyond 0 + 00, leaving 5 blank horizontal lines between the two entries. (See Form A.) So continue, entering stations of angle-points until bottom of the sheet is reached. At top of sheet 2 repeat last entry of sheet 1. Continue with column 1 from sheet to sheet, always repeating last entry on each sheet at top of following sheet, until supply of data is exhausted. (2) Check copying, by trading notes with partner and calling back. (3)

Column 2. Bearings. — Enter bearings in column 2 of all sheets, on lines shown on Form A. Check copying. (4) Column 2. Distances. — Enter distance in column 2 on line immediately above bearing, as indicated in Form A. (If distances were not computed in field, obtain each distance by subtracting each station from following station.) Continue this work till end of recorded data is reached. (5) Check computed distances by taking their sum (on adding machine, if at hand) which should equal last station entered on last sheet. (6) Column 3. Log Dist. — Enter log of each distance in column 3, opposite the distance. Continue this work till it is completed for all sheets. (7) Column 3. Log sin and log cos. — Enter log sin of each bearing in column 3, just above log distance; and enter log cos in column 3 just below log distance. So continue, to end of data. (8) Add logs. — (Use adding machine, if it is at hand.) In each group of three logs, add middle log to upper, setting sum just above upper; and add middle log to lower, setting sum just below lower; continue till the two sums have been written for each of the groups. (9) Anti-logs. — Look up anti-log of upper log of each group of five logs, and set it, opposite the log, in column 4 if bearing is E, or in column 5 if bearing is W. Enter anti-log of lower log opposite the log, in column 6 if bearing is N, or in column 7 if bearing is S. (Anti-logs are to be entered to nearest 0.1 ft.) So continue till anti-logs of all groups have been entered. (10) Total departures and latitudes. — Total departure of sta. 0 + 00 is taken as zero. Find by algebraic addition, total departure of each succeeding angle-point, and set result opposite station of angle-point, in column 8 if total departure is (+), or in column 9 if total departure is (−). Total lat. of sta. 0 + 00 is taken as zero. Find, by algebraic addition, total latitude of each succeeding angle-point, and set result opposite station of the angle-point, in column 10 if total latitude is (+), or in column 11 if total latitude is (−).

Check this work, using following equations: Sum col. 4 + sum col. 5 = last entry in col. 8 or 9. Sum col. 6 + sum col. 7 = last entry in col. 10 or 11. (11) The coordinates (total departures and latitudes) found in columns 8, 9, 10, and 11 can be used for plotting, but close watch must be kept on signs. (12) Coordinates all positive. — When total departures and latitudes have been computed for entire Preliminary Survey, make total departures and latitudes all positive in following manner: (*i*) If there are any negative departures in column 9, note the greatest. Add algebraically to each departure in column 8 and to each departure in column 9, that power of 10 next greater than the noted greatest departure in column 9, and set sum directly opposite departure which enters into sum, in column 12. For example, if greatest departure in column 9 is (−) 763.4, add 1000 algebraically to every total departure, and set results in column 12 opposite that total departure; or, if your greatest negative departure in column 9 is (−) 1000.4, add 10,000 to every total departure. (*ii*) Similarly, make all total latitudes positive, and set results in column 13.

29. Machine computation. — Form B (see Appendix) is convenient for machine computation. Rule this form on as many sheets of computing paper as data will fill. (1) Column 1. — First entry at top of column 1 of sheet 1 is 0 + 00, as shown on Form B. Next, enter station of first angle-point beyond, leaving two blank H lines between the two entries. So continue, entering station of each angle-point in turn, till bottom of sheet is reached. Repeat bottom entry of sheet 1 at top of sheet 2. Continue with column 2 from sheet to sheet, always repeating bottom entry of each sheet at top of following sheet, until data are exhausted. (2) Check copying. (3) Column 2. Bearings. — Follow directions under (3) of Art. 28. (4) Column 2. Distances. — Follow directions under (4) of Art. 28. (5) Column 3. — Look up natural sine and natural cosine of each bearing, and enter them in column 3 in positions indicated on Form B. Continue till natural sines and cosines have been entered for all bearings entered on sheets. (6) Find both departure and latitude of each course before going on to next course.

$$\text{Dep.} = \text{dist.} \times \text{nat. sine}$$
$$\text{Lat.} = \text{dist.} \times \text{nat. cosine}$$

By machine, find departure and enter it (to nearest 0.1 ft.) in column 4 (if bearing is E) or in column 5 (if bearing is W), opposite natural sine. Next, by machine, find corresponding latitude and enter it (to nearest 0.1 ft.) in column 6 (if bearing is N) or column 7 (if bearing is S), opposite natural cosine. Repeat foregoing for each course. (7) Total departures and latitudes. — Follow directions under (10) of Art. 28. (8) Coordinates now found in columns 8, 9, 10, and 11 can be used for plotting; but must be used with due regard to signs. (9) Coordinates all positive. — Follow directions under (12) of Art. 28. As soon as total departures and total latitudes have been computed for entire Preliminary Survey, coordinates should be made all positive, and if any plotting has been done with negative total departures and latitudes the figures on the lines of the five-inch squares should be changed to give their readings in terms of coordinates all positive.

G. Plotting the Data

30. Plotting traverse. — If the student has calculated "coordinates all positive" (columns 12 and 13 of Forms A and B), he will use those coordinates for plotting traverse. If he has not filled out columns 12 and 13, he will use for plotting the coordinates of columns 8, 9, 10, and 11 — and observe carefully the sign of each departure and each latitude. (1) Plot traverse to scale of 1 in. = 100 ft. on 15 in. × 22 in. sheets of first quality "detail paper," unless otherwise directed by instructor. (2) Sheet 1. — Rule one sheet with hair-lines into five-inch squares; giving the lines such directions across the sheet that the traverse when plotted will lie lengthwise (as nearly as may be) left to right, and not run so near either lower or upper edge of sheet as to leave insufficient room for plotting topography on both sides of traverse. (3) Station 0 + 00. — This initial point of the traverse should lie not less than three or four inches inside left edge of the sheet. (4) Mark each hair line with its abscissa or ordinate; placing the figure (distinct, a little large, and made with soft pencil) along margins of sheet. (5) Plot initial point, 0 + 00, then each succeeding angle-point of traverse. Encircle each point, freehand, as soon as plotted, and write its station by it. (6) Check by scaling length of each plotted course and comparing with recorded distance. Lettering on sheets may be executed without using guide lines, but it should be clearly legible even when seen through tracing cloth. (7) Join plotted points in proper order, forming map of traverse. (8) Mark each station on traverse by very short cross mark. (9) Figure stations 0, 5, 10, 15, 20, . . . , — every fifth station. (10) Write calculated bearing on each course. (11) It is not necessary to ink sheets;

though circles around angle-points, traverse, station marks, station numbers, and squares are sometimes inked before topography is plotted so as to permit of erasing mistakes in latter.

(12) Sheet 2. — (*i*) Remove tacks from sheet 1. Arrange sheets 1 and 2 on board. (*ii*) Slip left end of sheet 2 under right end of sheet 1 far enough to provide continuous paper for plotting contours on both sides of traverse even after next step. (*iii*) Swing right end of sheet 2, about right end of traverse on sheet 1, into such position that the advancing traverse will fall approximately lengthwise of sheet 2, and not near enough its upper or lower edge to crowd any of strip of topography (yet to be plotted) off sheet. (*iv*) Pin both sheets to board in this position.

(13) Continue system of hair-line squares over sheet 2. (14) Write on each hair-line its total departure or latitude. (15) Plot traverse across sheet 2; and complete work on sheet 2 as on sheet 1. (16) In same way, adjust sheet 3 to sheet 2; and so on, with remaining sheets. (17) Do not roll sheets. File them flat.

31. Plotting topography. — Each student will not plot all topography taken by his corps, but only for such stations as are assigned to him by instructor. (1) Each student will plot topography assigned to him, in manner described in Exercise 40. (2) Plot all other topographic data: fences, roads, land lines, buildings, ditches, streams, ownership, culture; and soft-pencil each with its station-and-plus, its bearing, or whatever descriptive matter may be necessary to make pertinent information complete. (3) Draw double arrow on sheet, in style prescribed by instructor. (4) Letter title of sheet as prescribed by instructor. Include corps letter in title.

(5) Check the map. — When the sheet is apparently

completed, look it over to see whether all the information is presented, whether it is all legible, whether it is bold enough to show through tracing cloth, and whether it is all correct. The Check List for Map Lines and Check List for Map Lettering may help in this work. (See Forms F and G of Appendix.) (6) After the sheet has been checked and all deficiencies have been made good, write "OK," date, and signature of draftsman, in middle of upper margin of sheet. (7) File sheet flat.

32. Tracing traverse. — Sheets of tracing cloth will be 15 in. × 22 in. unless otherwise prescribed by instructor. Use dull side. Do not roll up sheets, but lay them away flat. The following is suggested as convenient order of steps in making tracings. (1) Rub pumice powder over dull side of sheet; and afterward carefully remove all of powder. (2) Lay out on drawing board all sheets of plotted traverse in such a way as to show entire traverse precisely in its true dimensions and shape; and tack them down in this position. (The system of squares will now be continuous over all of the sheets.) (3) Arrange sheets of tracing cloth over plotted sheets. It is not necessary to make edges of tracing sheets flush with edges of plotted sheets. On the contrary, it may be possible so to lay tracing sheets that strip of topography will lie upon them more satisfactorily than it does on plotted sheets; and if this be possible it should be done. Tack the tracing sheets, with sheet 1 overlapping 2, sheet 2 overlapping 3, and so on. (4) If ink-lettering of all tracing sheets is done at one time, better lettering will be produced than if ink-lettering is alternated with ink-line work. For this reason it is proposed that in the following steps all figures, letters, and words be put on each tracing with soft pencil and with very free hand; and that any ink-lettering be deferred till all ink-line work has been completed for all tracing sheets. (5) Trace the squares, using black-ink hair-lines. (6) Soft-pencil the coordinates of sides of squares. (7) With bow compass make small black-ink hair-line circle centered at each angle-point of traverse, at initial station and at final station. (8) Soft-pencil the station of each circle. (9) Trace courses of traverse from circle to circle, with black-ink hair-line; but do not let lines extend into circles. (10) Pencil the bearing of each course of traverse. (11) Pick out each station with very short hair-line drawn across traverse, with black ink. (12) Pencil station numbers 0, 5, 10, 15, 20, and so on. (13) Trace double arrow on each sheet. (14) Pencil draftman's name, corps letter, and sheet number on each tracing sheet.

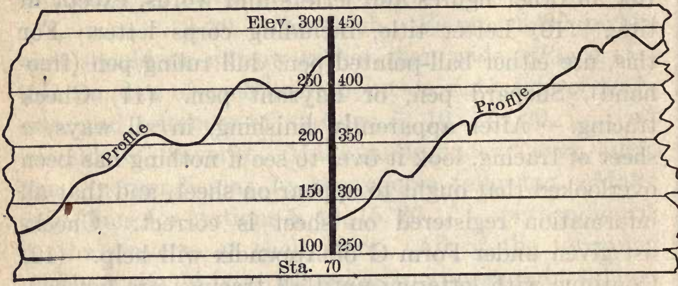
33. Tracing topography. — The tracing sheets are now ready for topography. Each student will have plotted the assigned stations of topography on one or more of his own detail-paper sheets. (1) Lay on the board a detail-paper sheet on which topography has been plotted, and lay over it the corresponding tracing sheet. Adjust the two sheets till traced traverse coincides with underlying traverse; and tack them together to the board. (2) Before tracing contours on tracing sheet, practice using contour pen and Shepard pen as follows: On spare strip of tracing cloth draw rapidly a lot of freehand curves — some flat curves and some sharp — with soft pencil. Practice following the curves with contour pen loaded with orange ink, until the penciled curves can be followed accurately, even around sharpest bends, and at same time an ink line of uniform thickness can be produced. Handle of pen must be maintained perpendicular to drawing surface; otherwise poor work will result. Do this with pen opened for fine lines, for medium lines, and for heavy lines. Note that the amount of ink in pen and the speed affect the result. (3) With contour pen, or Shepard pen, trace contours 0, 20, 40, 60, 80, 100, 120, and so on — multiples of 20; using brown ink, or, lacking that, orange ink; and making the lines of medium thickness. (4) Soft-pencil contour elevations 0, 20, 40, . . . (5) Trace intermediate contours in orange with fine lines. (Beginners nearly always make intermediate contours too heavy.) (6) Trace all stream lines in black. (7) Trace all fences, roads, land lines, buildings, etc., in black. (8) Soft-pencil all remaining figures, letters and words. (9) Look tracing over to make sure that all data are recorded on it and that the record is correct and legible. The check list given in Forms F and G of Appendix will help in this work. (10) Student will trace any other sheet or sheets of his own topography in same way. (11) After he has traced his own topography, he will borrow topographic sheets plotted by other members of his corps and trace them in same way. (12) Carry topography out to extreme edge of each tracing sheet, so that where two sheets overlap there will be duplication of topography. (13) Continue till all topography of Preliminary Survey is traced. Check the work by the aid of Form F of Appendix.

34. Ink-lettering tracings. — Practice lettering with black ink on spare strip of tracing cloth, until a satisfactory pen is found and uniformly satisfactory lettering can be done. If tracing has tendency to move, under draftman's pen, it should be held down with pencil. Submit practice work to instructor before passing to next step. (Pen strokes of students are commonly without sufficient body, especially at extreme ends of stroke. Student should have at hand his text-book on lettering while doing this work.) (1)

Rule guide lines with sharp 4H pencil for all lettering and figuring on tracings. Do not erase guide lines, even after lettering has been completed. Make all figures (except contour elevations which are to be 2/20 inches high, and figures and letters in title) and capitals 3/20 inches high, and lower-case letters a, c, e, i, m, . . . 2/20 inches high. Figures 3/20 inches high should be narrow rather than wide. Place station number of each angle point on right side of traverse, and numbers of stations 5, 10, 15, 20, . . . on left side of traverse. Make guide lines at each angle point perpendicular to the course following. Make guide lines for stations 5, 10, 15, 20, . . . perpendicular to the traverse at each point. All of the foregoing numbers should be so placed as to appear right side up when the user of the map is looking forward along the traverse. Place the bearing of each course of the traverse on the left side thereof, and so that it shall be read in forward direction. The bottom guide line of bearing should be parallel to, and about 1/20 inch distant from, its course. For all other lettering on Preliminary Survey tracings, follow the usual rule for working drawings, as exemplified in I.C.C. Specifications: On all lines having a component parallel to the bottom edge of the sheet, make the lettering read from left to right; and on vertical lines, make the lettering read from bottom toward top of the sheet. (2) If student is in doubt as to his ability to letter in ink directly, with only guide lines to guide him, he will first put lettering in place between guide lines, very carefully, with sharp, soft pencil; and submit the work to instructor before inking. (3) All lettering, including figuring of contour elevations, is to be done with black ink. (4) Figure coordinates of lines forming system of five-inch squares. (5) Figure station of each angle-point, of initial station, and of final station of traverse. (6) Figure stations 0, 5, 10, 15, 20, and so on. (7) Figure station and bearing of each straight line that crosses traverse. (8) Letter names of owners, roads, streams, buildings, culture, etc. (9) Ink all other figures and letters and words, except in title. (10) Letter title, including corps letter. For this, use either ball-pointed pen, dull ruling pen (free-hand), Shepard pen, or Payzant pen. (11) Check tracing. — After apparently finishing, in all ways, a sheet of tracing, look it over to see if nothing has been overlooked that ought to appear on sheet, and that all information registered on sheet is correct. Check-list given under Form G of Appendix will help. (12) Continue with lettering until all tracings are finished. (13) File tracings flat.

35. Plotting preliminary profile. — Plot profile from level notes of Preliminary Survey on 10-inch, Plate A profile paper. Do not roll profile paper, but fold it to 22 inches and lay it away flat. All work on this profile should be done rapidly. No ink need be used except when called for herein. This preliminary profile is merely a temporary working sheet, and is not to be a "finished" drawing. (1) If height of profile paper is not sufficient for plotting profile as one continuous line (because of great range of elevation), break profile line as indicated below. (2) Suggested order of work: Follow, as nearly as may be by eye, the I.C.C. specifications (see (18) of Exercise 33) for size, style, weight, and position of lettering. Letter legibly and boldly, but rapidly, with pencil only. (3) Figure rapidly with sharp, soft pencil, the elevations of heaviest H lines of profile paper. These lines are 50 ft. apart, and elevation of each is to be some multiple of 50, as is indicated in diagram below.

Bottom line should be given that multiple of 50 just below lowest elevation of profile levels. For example, if lowest elevation is 128.6, bottom line of profile paper should be marked 100. (4) Figure rapidly with soft, sharp pencil, heavy V lines of profile paper with



SHOWING BREAK IN PROFILE

station numbers: 0, 10, 20, and so on (multiples of 10). Put 0 not less than 6 inches from left end of paper to allow room for title. (5) Plot points of profile with sharp 6H pencil. Join each point, after plotting it, to preceding point, with light line. Do not encircle points. (6) With 4H pencil put in pluses of ditches, streams, roads, etc.; and enter names of these in neat freehand lettering in position indicated on I.C.C. model profile. (7) Pencil rapidly, but in bold legible letters, at left end of paper:

> "Line —————" (putting corps letter in blank space).
> "PRELIMINARY PROFILE,"
> Name of draftsman.
> Date of completing profile.

(8) Redraw surface line carefully with smooth, continuous black-ink hair-line. For this, use 404 pen, or other medium pen, without pressure. (This same

sheet of profile paper is to be used later for penciling profiles of trial locations made on tracings.) (9) If Preliminary was run on side-hill ground, plot on same profile paper, using old station and elevation numbers, profile of higher edge of mapped strip of topography, and also profile of lower edge of this strip. Redraw these profiles with light, black-ink dotted line. (These two auxiliary profiles show in a general way the limits within which grade line must lie in order that the corresponding location may fall within mapped strip of land throughout its length, and on supporting ground.)

H. Introduction to Paper Location

36. Field data. — The tracings and profile now show all data obtained by Preliminary Survey. The next work is to make the "paper location"; in other words, to "project" the location on the topographic map and the profile.

37. Arbitrary data. — In making paper location the student will be governed by the following arbitrary data, except as otherwise specified by the instructor:

(1) The located center-line must lie entirely within mapped strip of topography.
(2) Minimum tangent = 100 ft.
(3) Maximum degree of curve = 20°.
(4) Grade line must pass approximately through extremities of preliminary profile.
(5) Make cuts and fills approximately balance; *i.e.*, make total volume of excavation approximately equal to total volume of fills.
(6) Make volume of earthwork a minimum.
(7) Cross-section in cut: roadbed = 18′; side-slopes 1 : 1.

(8) Cross-section in fill: roadbed = 14′; side-slopes $1\frac{1}{2}$: 1.

(9) Do not compensate grade on curves.

(10) Do not allow for spirals.

(The object of many of the data above is to limit the problem to a small number of simple elements, so that the beginner may be able to work out complete logical solution in available time.)

38. Slope scale. — Transverse slope of ground at stations is one of data necessary for rapid determination of volume of cut and fill; and by far the quickest way of ascertaining slopes, when Preliminary map shows contours, is by reading them from map by means of a "slope scale." Student will construct a slope scale in the manner described in Art. 11, Sec. E, Part VI, and by means of it obtain transverse slopes from his map as occasion arises.

39. Set of curves. — Selection of curve for connecting two tangents already drawn on map, or for averaging given stretch of grade contour, is greatly facilitated by using a "set of curves" of various radii, drawn on a piece of tracing cloth or other transparent material. Student will provide set of curves for himself, either by constructing or tracing, or both, in following manner: (1) If he can borrow set of curves, drawn on detail paper or on tracing cloth, he will do so; and make tracing of set. Otherwise, he will construct a set, on either detail paper or tracing cloth, in pencil, thus: (2) Draw an arc for each of the following curves: 20°, 16°, 12°, 8°, 4°; using scale of Preliminary map; placing all centers on same straight line, and at such distances apart that arcs will cut the straight line, produced, in order named and at $\frac{3}{4}$ inch intervals. (3) Draw a tangent to each curve where it cuts line of centers. (4) Mark each curve, and also its center, with its degree; and put scale and draftsman's name on sheet. (5) If pencil work has been done directly on tracing cloth, ink it with black-ink hair-lines; and if it has been done on detail paper, trace with black-ink hair-lines. (6) File tracing flat. For use of the set of curves, see Arts. 50 and 51.

40. Study of grade-line problem for hillside locations. — The following discussion — continued in Arts. 41, 42, 43, 44 — assumes that finally located CL must lie within strip of land plotted on map; and that grade line must pass (approximately) through extremities, *A* and *Z*, of profile of ground surface along Preliminary traverse. From this discussion it is hoped that the student will be able to obtain a few simple ideas which will be of direct service to him in beginning his work of planning location. In Figs. 1 to 3, there

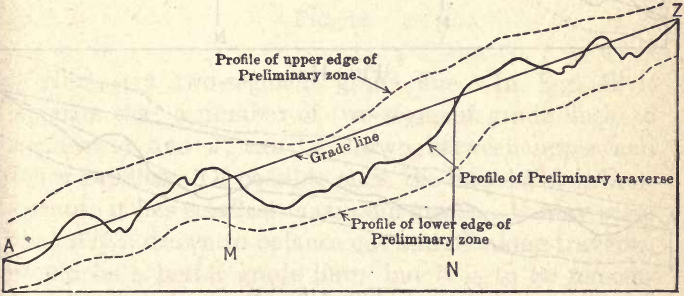


FIG. 41.

are three profiles: the central one is profile of traverse; and upper and lower are respectively the profiles of upper and lower edges of strip of topography shown on Preliminary map.

41. Straight grade line. — Fig. 41. The straight grade line, *AZ*, gives fill (volume) greatly in excess of cut (volume). This is apparent on mere inspection. However, between *M* and *N* (where lies greatest fill) the ground within the strip rises higher than grade line; and by throwing location away from traverse, toward up-hill side of mapped strip, it can be made to rest on ground which follows straight grade line more closely. In technical terms, we have in this case "supporting ground" throughout and within the mapped strip, for straight grade line.

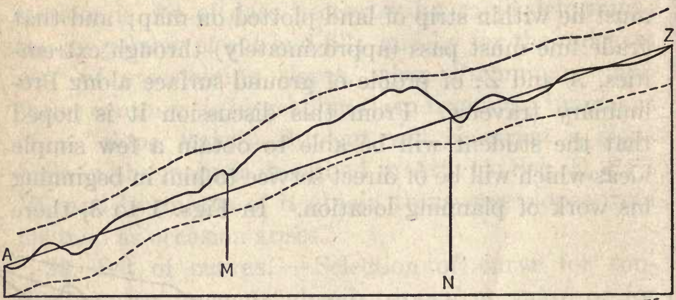


FIG. 41*a*.

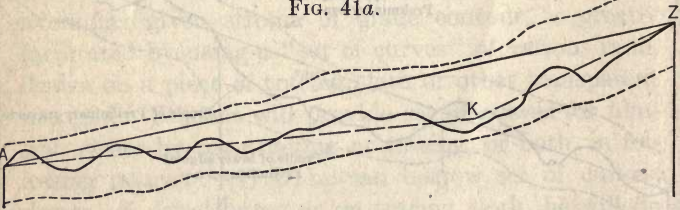


FIG. 41*b*.

In Figs 41*a* and 41*b* we have same condition as in Fig. 41: straight grade line, *AZ*, lies, throughout strip, between profiles of upper and lower edges of mapped topography; and this means that supporting ground can be found within mapped strip for entire length of straight grade line, *AZ*.

42. Two-segment grade line. — Fig. 42. Volume of fill will greatly exceed volume of cut if we use straight grade line and keep location close to traverse. Fill can be decreased by throwing location up-hill from traverse, especially in region of *K*. It is plain, however, that fill cannot be reduced sufficiently by this expedient; because upper edge of mapped strip is not as elevated in region of *K* as is grade line, *AZ*, and this means that there is not supporting ground within and throughout the strip for straight grade line, *AZ*.

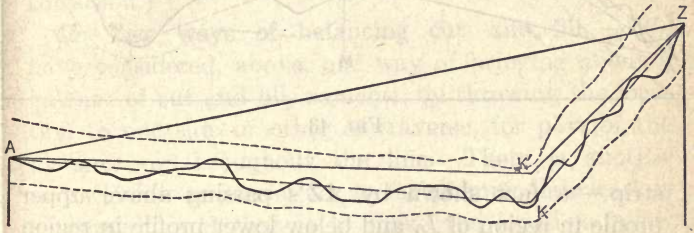


FIG. 42.

Next, try two-segment grade line. In Fig. 42 it appears that a number of two-segment grade lines, to connect *A* and *Z*, can be drawn between upper and lower profiles. Of possible ones, the line *AK'L* is best, because it has smallest maximum grade. It may seem that *AKZ*, drawn to balance cut and fill along traverse, would be a better grade line; but it is to be remembered that it is possible to throw location up-hill from traverse far enough to make its profile follow as closely the line *AK'Z* as traverse profile follows *AKZ*.

Conditions shown on Fig. 42 may be taken as typical for a two-segment grade line, under limitations of student surveys.

43. Three-segment grade line. — With profile of Fig. 43, the straight grade line, *AZ*, gives approximately balanced cut and fill. The cuts and fills are, however, very heavy and should be greatly diminished. They could be diminished by throwing location down-hill from traverse between *A* and *M*, and up-hill from traverse between *M* and *Z*; but there is not supporting ground for grade line, *AZ*, within limits of mapped

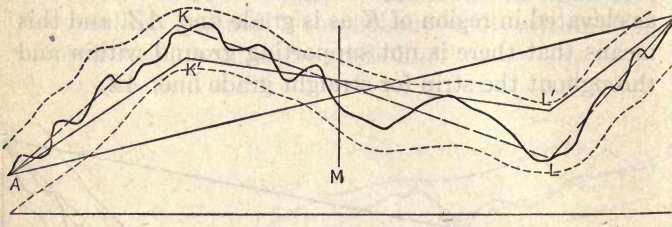


FIG. 43.

strip — a fact shown by *AZ*'s passing above upper profile in region of *L*, and below lower profile in region of *K*. Therefore the straight grade line is not feasible under the imposed conditions.

Trying, next, for a two-segment grade line, we find that *A* and *Z* cannot be connected by two-segment line lying wholly between upper and lower profiles. In other words, there is not continuous supporting ground for two-segment grade line.

Next, try for three-segment grade line. It is seen to be possible to connect *A* and *Z* by several three-segment lines lying wholly between upper and lower profiles. Thus *AKLZ* is a supported grade line. This line will apparently balance cut and fill, provided the location is made to follow traverse very closely. *AK′L′Z* is, however, the best continuously supported grade line which can be laid, because it has lowest maximum grade of all.

Fig. 43, showing two great bends in strip lying between upper and lower profiles, may be taken as showing conditions calling for three-segment grade line.

44. Rule for choosing grade line. — From Arts. 40, 41, 42, 43, it would appear that a tentative rule for obtaining the best grade line is to make the grade line lie between upper and lower profiles, and make it as nearly straight as possible, and with fewest possible breaks. (There are, in practice, many exceptions to this rule; but the student's attempt at this stage to consider the exceptions would lead only to his certain confusion.)

45. Two ways of balancing cut and fill. — We have considered, above, one way of bringing about a balance of cut and fill, namely: by throwing the location to one side or other of traverse, for part of the distance or throughout the line. There is another way which is much simpler, so far as immediate work is concerned; and that is to make the location follow traverse very closely, and lay the grade line so as to balance approximately cut and fill on traverse profile. For example, in Fig. 42, we can lay grade line *AKZ*, for we have supporting ground for this along the traverse. The second method will not be used in class work.

J. Making the Paper Location

(Use only pencil on the profile and tracing cloth, unless otherwise directed.)

46. Choosing grade line. — Let initial point of surface line of Preliminary Profile be called *A*; and final point, *Z*. (1) Draw straight line from *A* to *Z*. Does this line throughout lie between upper and lower profiles? If so, proceed with Art. 47. (2) If *AZ* passes, at any point, above upper profile or below lower profile, try for a two-segment grade line (Art. 42

and Fig. 42); and if successful, proceed with Art. 48. (3) If necessary, resort to a three-(or more-) segment grade line (Art 43 and Fig. 43); and proceed with Art. 48.

47. Grade contour for straight grade line. — (Read Part V, E, 16.) If a straight grade line (Art. 46) has been chosen, construct corresponding grade contour on map (tracings) in following manner: (1) Let g be gradient of AZ. (g = rise per 100 horizontal feet.) Take g to nearest 0.1 ft. (2) If contour interval is 5 ft. (and it will be so assumed in the following) it requires 500 horizontal feet to rise from one contour to the next higher, on a 1% grade. Therefore, to rise from one contour to the next on a $g\%$ grade will require an H distance of $500/g, = r$, say. (3) If A is not on a contour, but n ft. below a contour, open dividers to radius $r' = (n/5)(500/g)$. From A as center, describe short arc cutting next higher (or lower) contour at x, say. Mark x with encircled dot. (If A is on a contour, the work starts off in manner similar to that described below for starting from x.) (4) Open dividers to radius $r = 500/g$, and keep to this radius in subsequent steps. (5) From x as center, describe short arc cutting next higher (or lower) contour, at y, say. Mark y with encircled dot. (6) Continue, stepping from contour to contour and marking each point in turn. *Exception:* When the bend in line of points, fixed by this method, would be much sharper than a 20° curve (look at 20° curve on the set of curves), double or treble the step-radius and cut second or third contour above (or below). In other words, do not carry the grade contour up ravines or around ridges which are too narrow to turn with specified maximum curve. For example, in Fig. 47, if grade contour were carried from C to D' and D'', and so on into and out of the ravine, plainly it could not be followed by a 20° curve; so, in view of this fact, the grade contour is carried by the two steps, *CD* and *DE*, made in a straight line, to *E*, which is a point on the second contour above *C*. (7) When grade contour has been

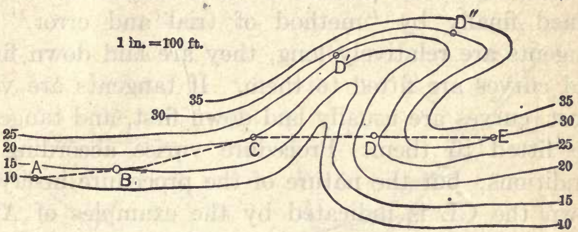


FIG. 47.

carried to end of traverse, submit the work to instructor. (The grade contour will not come out precisely at map of *Z*. Why will it not?) (8) After grade contour points have been marked and work has been approved by instructor, join them two-and-two, with a black-ink, dotted hair-line.

48. Grade contour for broken grade line. — Work is same for this case as for straight grade line (see foregoing article — 47), except that, with broken grade line, we change opening of dividers on reaching each break in grade, re-setting them to new radius figured from succeeding rate of grade.

49. Laying the center line. — (For all work on tracing cloth, referred to below, use soft pencil, kept sharp, and make penciled lines dark enough to be followed easily among the inked lines. Likewise, make all penciled lettering easily legible. Too much graphite is better than too little. After paper location has been completed and approved by instructor, do some further work on tracings with ink.)

The next work is to lay out on map the CL of proposed railroad, making it follow grade contour as closely as may be. This CL consists of a series of tangents and circular curves, and is built up tangent by tangent, and curve by curve. Position and degree of each curve and position of each tangent are determined finally by "method of trial and error." If tangents are relatively long, they are laid down first, and curves are fitted to them. If tangents are very short, curves are usually laid down first, and tangents are fitted to them. Procedure varies according to conditions; but the nature of the procedure in laying down the CL is indicated by the examples of Arts. 50 and 51.

Caution. — Do not spend much time laying out first two or three tangents and curves. Work rapidly. Decide quickly each question, and without fear of consequences. The only difference between effect of good judgment and that of bad judgment, in laying down first trial location, lies in amount of revision subsequently needed; because even with best judgment, considerable revision will be required. From nature of case — the method being that of "trial and error" — it cannot be otherwise.

50. Example of laying tangents first. — Given the grade contour (dotted, broken line) of Fig. 50. (1) Draw tangents *AM*, *MP*, and *PR*, of indefinite length, each in such position as to "average," by eye, the corresponding stretch of grade contour; remembering that curves will cut across angles formed by tangents. (2) Applying "set of curves" to map (Art. 39), move one curve after another up against tangents *AM* and *MP*, and observe where points of tangency, *L* and *N*, fall for each curve. (3) Similarly, apply the set of curves to tangents *MP* and *PR*, and observe where *O* and Q fall for each curve. (4) NO must not be less than 100 ft. (Art. 37). Of course, the sharper the curves, the greater NO. (5) If NO is less than 100 ft., even with two 20° curves, ignore the middle tangent, MP, and consider AM and PR as consecutive tangents;

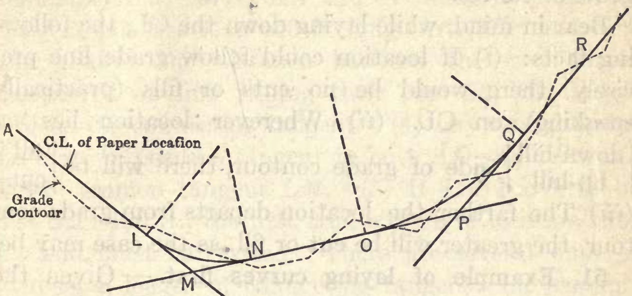


FIG. 50. LAYING TANGENTS FIRST

and find by trial that even-degree curve which will most nearly average the intervening stretch of grade contour. As soon as the chosen curve has been placed in its chosen position, prick its center through into the map; encircle the prick mark on the map, and by it write "$D = \cdots °$." Draw the curve from tangent point to tangent point on the map, and draw the two radii running to points of tangency. (6) If NO is not less than 100 ft. with two 20° curves, select for each apex the flattest even-degree curve which can be used without making NO less than 100 ft. Placing the selected curve in position for one apex, prick its center through on the map; encircle the prick mark on the map, and write by it "$D = \cdots °$"; and draw the two radii running to points of tangency. Repeat the operations for curve at other apex. (7) The foregoing operations are repeated for next two or three stretches of grade contour and so on. Work on any one stretch may develop need of revising, more or less, the CL laid down on preceding stretch. Each tangent must be adjusted to the ground and to preceding and following curve; and each curve must be adjusted to preceding and following segments of CL, whether they be tangents or curves.

Bear in mind, while laying down the CL, the following facts: (*i*) If location could follow grade line precisely, there would be no cuts or fills (practically speaking) on CL. (*ii*) Wherever location lies on $\left|\begin{array}{c}\text{down-hill}\\ \text{up-hill}\end{array}\right|$ side of grade contour, there will be $\left|\begin{array}{c}\text{fill}\\ \text{cut}\end{array}\right|$. (*iii*) The farther the location departs from grade contour, the greater will be cut or fill, as the case may be.

51. Example of laying curves first. — Given the grade contour (dotted, broken line) of Fig. 51. (1) Slide the set of curves (Art. 39) over map, to bring one

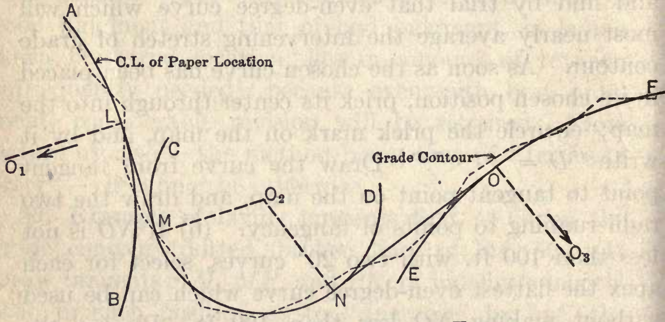


FIG. 51. LAYING CURVES FIRST

curve after another into position to "average" first stretch of grade contour. (2) On finding curve which seems best to satisfy all requirements, bring it to position *AB*, averaging (by eye) the first stretch of grade contour. Prick its center and one of its points through onto map; encircle these prick marks on map;

and by the encircled center write "$D = \cdots °$." Draw the curve, giving it generous length on map; using sharp, soft lead in the compasses. (3) In same way, select and draw curve *CD* for second stretch of grade contour. In selecting and placing curve *CD*, remember that curves *AB* and *CD* must be joined by tangent not less than 100 ft. long (Art. 37); and that the greater the distance between the two curves (measured on line joining their centers), the longer will be the connecting tangent. (4) Lay straight-edge on map, in position tangent to both *AD* and *CD*; and draw common tangent *LM*. (5) If *LM* prove to be less than 100 ft., the first trial stretch of location thus far laid must be revised. There are several ways of increasing length of connecting tangent; for example (*i*) shorten radius of curve *AL*, or of curve *MN*, or do both. (*ii*) Move the two curves away from each other, or one away from the other. (6) In choosing degree and position for curve, *CD*, the requirements of connecting with following curve, *EF*, as well as with preceding curve, *AB*, must be kept in mind. (7) In same manner, the succeeding pairs of curves are dealt with.

Any revision of degree or position of a curve will, of course, make necessary more or less revision of line immediately preceding and immediately following.

52. Marking the stations. — After entire CL has been projected (*i.e.*, laid), on map and approved by instructor, mark its every station, beginning with 0 + 00 at initial point, which is sta. 0 + 00 of traverse. Determine positions of stations by stepping along the line, both straight and curved parts, with dividers opened to 100 ft. Write station number by each station whose number is a multiple of five.

53. Profile of first paper location. — Plot profile of ground surface along projected CL, on Preliminary Profile, using old station-numbers and elevation-numbers. Label this plainly, "First Paper Location."

54. Studying profile of first paper location. — Study this profile in connection with adopted grade line. Observe cuts and fills. (Do cut and fill apparently balance? Are there any particularly heavy cuts or fills? If so, can they be decreased?) (2) If it is plain to the eye that cut exceeds fill, or fill exceeds cut, decide, from study of profile, along what stretches to shift location; which way to shift it (up-hill or down-hill); and about how much to shift at each place. In this study for revision of location, keep in mind the double requirement: (*i*) Cut and fill are to be made to balance. (*ii*) Total earthwork is to be made as small as possible. (3) Make the shift, or shifts, on the map. (This will probably introduce one or more "long" or "short" stations, but the effect of such may be ignored at this stage of work.) (4) Revise profile of shifted portion of location; and note whether or not revised profile shows balanced cut and fill. (5) Repeat revising process, if necessary, until cut and fill approximately balance, and total yardage is a minimum — as far as can be determined by mere inspection of profile.

55. Estimating volume from profile of paper location. — When work of projecting location has been brought to such stage that, by inspection of the profile and map, cut and fill balance and total volume is a minimum, the next step is to apply a more precise method for testing for balance of cut and fill, which method consists of finding the volume of earthwork for each station and comparing total fill volume with total cut volume. While estimating volumes from profile, the student should not forget that his volume factors are but roughly approximate, and avoid undue refinement in calculating volume at this stage. Comparatively large errors (not mistakes or blunders) which are as likely to be positive as negative may be tolerated, because they tend to neutralize one another; but the eye should be kept on any cumulative error to see that it does not exceed reasonable limit.

(1) Student will systematically record all earthwork data and results, at this stage, on specified computing paper ruled into columns to be headed as follows:

VOLUMES FROM PROJECTED PROFILE

Sta.	Center heights.		Transverse slope.	Volume for level sections.		Correction.		Corrected volumes.	
	Cut.	Fill.		Cut.	Fill.	%	Cu. yd.	Cut.	Fill.
1	2	3	4	5	6	7	8	9	10

(2) Find volume of each station by method of Art. 56 if ground is level or nearly level transversely, and by method of Art. 57 or 58 if ground has considerable transverse slope.

(3) If estimated cut and fill volumes do not balance within specified limit, study profile and map to determine where, and in what manner, revision should be made in location as laid on map. Make revision on map. Revise profile correspondingly. Recompute volumes for those cuts and fills affected by revision; and again see if cut and fill balance within specified limit.

(The student might go on revising the paper location repeatedly, bringing the result nearer to perfection with each succeeding revision; but, since available time is limited, he is expected to repeat the revising process only so far as may be necessary to enable him thoroughly to understand it. In practice, also, the work of revision is not carried to a fine point, because of inexactness in data.)

(4) Having made final revision of location on map, label resulting CL, "Final Paper Location." Having made profile correspond to final CL on map, label profile, "Final Paper Location."

(5) Ink final paper location. Use black ink. Make CL heavy. Rule guide lines for all the lettering, and do not erase them. Make all capitals and figures 3/20 inches high. Make lower-case letters like a, c, e, i, m, . . . 2/20 inches high. For position of letters and figures, follow the usual rule for working drawings, as exemplified by I.C.C. Specifications: On all lines having a component parallel to the bottom edge of the sheet, make the lettering read from left to right; and on vertical lines make the lettering read from bottom toward top of sheet. Observe that the Location CL, unlike the Preliminary traverse, has each regular station number, 5, 10, 15, 20, . . . , written along (instead of perpendicular to) the CL.

56. Estimating level-section volume from profile. — In nearly all railroad surveying texts there are tables which give by inspection the volume per station for given roadbed width, side slopes and center height, for those cases in which the cross-section of the ground is level. (1) Scale center height of cut (or fill) from profile at each mid-station (*i.e.*, at each +50), using engineers' scale of 20 divisions to the inch. Enter station in column 1 (of ruled sheet described in Art. 55);

enter cut heights in column 2 and fill heights in column 3. (2) Find volume for each station by means of a table of level sections; or by means of a volume diagram (See Part V, B, Art. 2), or by means of a volume scale (See Part V, C, Art. 1). Enter cut volumes in column 5 and fill volumes in column 6. (3) Find total volume for each cut and each fill; and total cut and total fill for entire line.

57. Estimating side-hill volume from profile. — (1) Enter stations in column 1 and center heights in columns 2 and 3, as directed under (1) of Art. 56. (2) Scale from map, at each +50, transverse slope of ground, using slope scale of Art. 38. Enter slope in column 4 of ruled sheet. (3) Find volume for each station from a table giving volumes directly from center height and transverse slope;* or from a corresponding volume diagram,† or by means of a corresponding volume scale.‡ Enter cut volumes in column 9 and fill volumes in column 10. (4) Find total volume of each cut and each fill; and total cut and total fill for entire line.

58. Level-section volume corrected for side-hill. — Station volumes for ground having more or less transverse slope can be found in the following manner: (1)

* Tables giving station volumes for given center heights and transverse slopes will be found in Crandall's "Earthwork," Hauch and Rice's "Tables of Quantities for Preliminary Estimates," and C. P. Howard's "Tables and Data for Railway Locating Engineers."

† Volume diagrams for sloping sections are given in James's "Earthwork Diagrams" and in Wellington's "Computations from Diagrams of Railway Earthwork."

‡ Volume scales for sloping sections are given by Crandall in "Cornell Civil Engineer" of March, 1907; in Crandall and Barnes's "Railroad Construction"; in C. P. Howard's "Tables and Data for Railway Locating Engineers"; and in James's "Earthwork Diagrams."

First, find and record the station volumes as if ground were level transversely, as directed under Art. 56. (2) Scale from map, and record the transverse slope at each + 50, as directed under (2) of Art. 57. (3) From diagram (Fig. 58), find correction (per cent) corresponding to each recorded slope, and enter correction

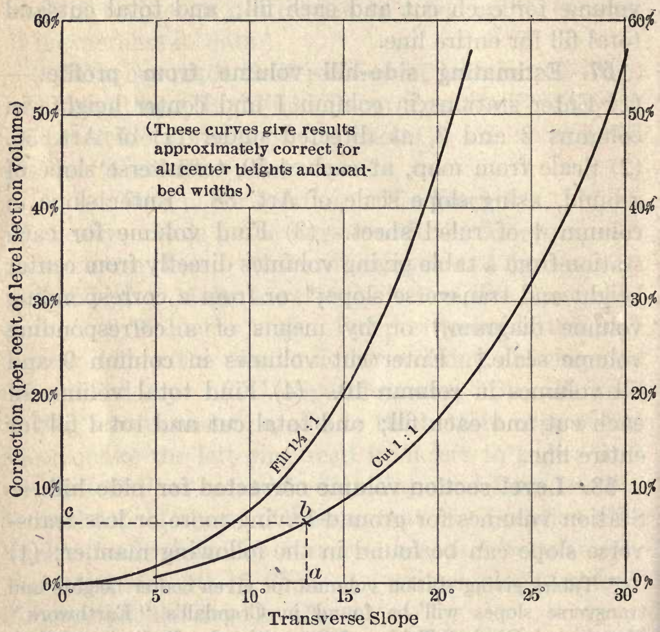


FIG. 58. DIAGRAM GIVING SIDE-HILL CORRECTION TO LEVEL-SECTION VOLUME

in column 7 of ruled sheet. (4) Multiply each volume in columns 5 and 6 by its correction ratio, and enter product in column 8. (5) Add each volume in column 8 to volume standing opposite in column 5 or 6, and place sum in column 9 if cut, in column 10 if fill. (6) Find corrected volume of each cut and each fill; and total cut and fill for entire line.

K. Transfer of Paper Location to Field

59. Two methods of transferring paper location to field. — As the Preliminary traverse was laid out on paper from data taken from the field, so the planned Location CL is to be staked out on the ground from data taken from the map. There are two general methods of transferring the paper location to the ground, one of which is described in Art. 60 and the other in Art. 61. The instructor will choose one of the two methods.

60. First method of transferring paper location to field. — In this method a complete set of alignment notes — including curve deflections — is written out from the CL planned and penciled on Preliminary map; and the CL is then staked in the field in accordance with these notes. For this method there are two different ways of computing the alignment notes from the paper location; one of which, employing rectangular coordinates, is described in Art. 63; and the other employing polar coordinates, in Art. 64. Each student will prepare a set of alignment notes from his paper location according to the directions of Art. 63 or of Art. 64.

61. Second method of transferring paper location to field. — The second method may be presented best by example. Refer to Fig. 61, which shows first portion of traverse and paper location on Preliminary map. (1) Scale and record the following from map: offset 3–*B*, at sta. 3; offset 4–*C*, at sta. 4; offset 8–*D*, at sta. 8; and so on. (See (13) below, for variations.) (2) Record degree of each curve of paper location. (3) In the field, lay off recorded offsets, marking points *A*, *B*, *C*, *D*, and so on, on ground, with pins, stakes, or line rods. (4) Produce *A–B* and *D–C* to intersect at

PI. Set hub at PI. (For directions for marking stakes, see under (3) of Exercise 31.) (5) Tape the line *A*–PI, setting station stakes the whole distance, in same manner as on traverse. (This may be done while transitman is computing tangent distance and curve deflections.) (6) Measure deflection angle *I*, at PI

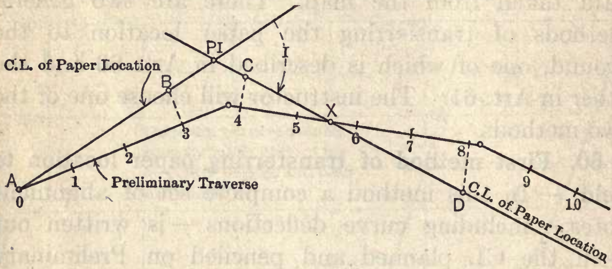


FIG. 61.

with transit. (7) Compute the following: tangent distance, *T*, from recorded *D* and measured *I*; length of curve, *L*; station of TC and CT; and deflections for curve stations. (8) Lay off forward tangent distance, and set hub and guard stake at CT. (9) Set hub and guard stake at TC. (10) Pull stakes driven on line between TC and PI. (11) Set up transit at TC and run in curve, carrying station numbering continuously from 0 + 00. (12) Set up transit at CT and run toward D. And so on. This process is repeated with variations according to conditions, for each curve. (13) In some cases an angle may well be used, instead of an offset, to help determine position of a Location tangent with respect to Preliminary traverse. For example, angle *B*–*A*–3 might be used instead of offset *B*–3; and tangent *C*–*D* might be located in field by means of station of *X* and angle 4–6–*C*.

62. Comments on the two methods of transfer. — Some of the disadvantages of method of Art. 60, compared with method of 61, are; (1) Any deviation from the alignment notes, at any part of the line, may render useless notes previously written for the alignment from that part forward; and a deviation may be caused not only by discovery of a blunder in Preliminary Survey or in office work, but also by recognition, in field, of chances for improvement which could not be seen on the map, owing to small scale. (2) The work of preparing field data from map is very much greater in the first method than in the second. (3) The computations must be made by a person previously trained in the method, because the engineer on location has not time to teach the method. Advantages: (1) Assuming that there is no need to revise paper location as staked in field the first time, the total cost by the first method is much less than that by the second, because extra office cost (of one man) is only a small part of corresponding saving (of time of several men) in field. (2) To the engineer who must direct an unskilled surveying corps from the office, this is the only method that gives complete control at every stage.

63. Preparation of alignment notes by rectangular coordinates. — It is assumed that tangents and curves have been drawn on Preliminary map. (1) Produce tangents to intersect, thus marking position of PI of every curve. (2) Scale the coordinates of initial point of paper location, of terminal point, and of each PI; writing by each point the two scaled coordinates; first x (or departure); then y (or latitude). (3) Compute bearing and length of each segment of broken line determined by initial point, final point, and PI's using Form E of Appendix. Write on each line its length and bearing, thus: PI–PI = 1262.7, S. 86° 30′ E. (4) Compute central angle, I, of each curve, and write each computed I within the curve, thus: I = 35° 14′. (5) Compute tangent distance, T, for each curve, and write it inside the curve, thus: T = 241.0. (6) Compute length, L, of each curve, and write it inside the curve, thus: L = 509.6. (7) Compute length of each CL tangent; *i.e.*, distance from 0 + 00 to TC of first curve, distance from CT of first curve to TC of second curve, and so on. Write each tangent length on its line on Location CL, thus: CLT = 342.8. (8) Compute station of each TC and CT. Write these results on map, at right angles to CL, thus: TC 12 + 41.7; CT = 16 + 09.0. (9) Compute deflections for each curve. (10) Write alignment notes in field book, using the following FORM OF NOTES:

FORM OF NOTES

Sta.	Alignment.	Vernier.	Calc'd bearing.	Needle.	Curve data.
34					
33			N 79° 02′ E	N 79° 10′ E	
32 + 00.2	⊙ CT	16° 15′	N 62° 47′ E	N 62° 45′ E	
32		16–14.6			I = 32° 30′
31		13–14.6			L = 541.67
30	⊙	10–14.6			R = 955.37
29		7–14.6			T = 278.47
28		4–14.6			PI = 29 + 37.0
27		1–14.6			
26 + 58.53	⊙ TC	0° 00′	N 46° 32′ E	N 46° 40′ E	
26					
25					

(11) Check points: At any point at which Location CL cuts Preliminary CL there is opportunity in field to observe agreement between the two surveys — Location and Preliminary — provided the Preliminary station and the Location station of intersection point have been computed. To prepare check data proceed as follows: (*i*) Write equation of each of two lines whose inter-

section is to be noted in field as a check point. (*ii*) Compute coordinates of intersection point. (*iii*) Find Preliminary station and Location station of intersection point. Enter check data in alignment notes in field book. In case Location CL does not cross Preliminary at intervals frequent enough to furnish desired number of check points, a check line may be introduced between one station on Preliminary and another on Location; length and direction of check line may be computed; and check line run in field.

64. Preparation of alignment notes by polar coordinates. — In this method of computing alignment notes from paper location no rectangular coordinates are used. By the present method, the steps leading from projected CL to alignment notes ready to be run in field, are as follows: (1) Scale the following distances and write them on map, each on its line: 0 + 00 to first PI; first PI to second PI; and so on; and, finally, last PI to final station. (2) With protractor scale the bearing of each of these lines. Write on each line its scaled bearing. (3) Compute central angle, I, for each curve, from scaled bearings. Write the angle inside curve, thus: I = 27° 36′. (4) Compute length, L, of each curve; and write it within curve, thus: L = 373.6. (5) Compute tangent distance, T, for each curve, and write it within the curve, thus: T = 786.5 (6) Compute length of each CL tangent: *i.e.*, distance from 0 + 00 to TC of first curve, distance from CT of first curve to TC of second curve, and so on. Write length on each CL tangent, thus: CLT = 262.1. (7) Compute station of each TC and CT, and of final point, taking them in order, from 0 + 00. Write by each of these points its computed station-and-plus, thus: CT 6 + 37.3; TC 12 + 70.8. (8) Check points: Compute station of selected points of intersection of Location CL and Preliminary CL. Enter these on map, at the check points. (9) Check lines: if no intersection point occurs where a check is desired, draw check line from one station on Preliminary to another station on Location, and compute length and bearing of this, so that it can be run in field. (10) Compute deflections for each curve. (11) Write alignment notes in field book, using FORM OF NOTES given in Art. 63.

65. Staking the center line. — (1) Run CL by first method (Art. 60) or by second (Art. 61), as directed by instructor. (2) Run curves of less than 8° with 100 foot chords; curves of 8°–15° with 50 foot chords; and curves of 16° and upward with 25 foot chords. (3) Set hub (with guard stake) at each point occupied by transit. (4) Set stake at each station, and at each plus where occurs a conspicuous break in profile. (5) Letter stakes as indicated in the following examples, where "L" stands for "Location" and "B" is corps letter. "R" and "L," following the degree of curve, stand for "Right" and "Left," respectively.

Station stake	*LB* 28
Plus stake	*LB* 28 + 40.7
TC guard stake	*LBTC* 35 + 16.2 (facing hub); 8° *R* (on back face)
CT guard stake	*LBCT* 38 + 62.0
PI guard stake	*LBPI* – 86° 42′ *R*

(6) Reference out the PI's, TC's, CT's, and initial and final points. (See Exercise 38.)

66. Location levels. — (1) Run profile levels over staked location, according to directions given in Arts. 19 and 20 for running profile levels of Preliminary survey. Note the following additional directions: (2) Copy all BM elevations and descriptions of Preliminary Survey into field book before going into field to run location levels. (3) Read rod on all convenient Preliminary Survey BM's, and immediately compare computed elevation with elevation as given in BM list.

67. Earthwork cross-sections. — The corps will be divided into cross-section parties; and to each party will be assigned a certain series of stations for cross-sectioning. Take cross-section at each station and at each plus at which ground changes. Follow method of Exercise 42.

L. Office Work from Staked Location

68. Profile of staked location. — All preceding profiles have been made without particular regard to appearance. This profile is to be inked in first-class manner to conform in arrangement, dimensions, and style of lettering to I.C.C. specifications, (see (18) of Exercise 33). As a general rule, figures and capital letters outside the title should be $\frac{3}{20}$ in. high, and all lower-case letters $\frac{2}{20}$ in. high. (The model drawings in I.C.C. Specifications are reduced one-half from the originals; hence the student should use $\frac{3}{20}$ in. as height of letter on his profile if height of corresponding letter on model is $\frac{3}{40}$ in.) The Check List for Profile (Form H of Appendix) should be used in reviewing the profile. (1) Plot profile of staked location on a fresh sheet of Plate A, ten-inch profile paper; placing sta. 0 at least 6 in. from left edge of sheet. (2) Place pluses and names of roads, streams, etc., on the paper. (3) Draw grade line. (4) Write grade (per cent) on each segment of grade line. (5) Write grade elevations for each end and each break point of grade line. (6) Record alignment on profile. (7) Pencil the title at left end of sheet, making it truly symmetrical about a V axis. (Title need not be repeated at right end of profile, as this profile is very short.) The name of the railroad may be taken as "Line ———," the corps letter being put in blank space. (8) Pencil guide lines for all lettering and figuring which run on diagonal or V lines, making letters and figures conform to I.C.C. specifications. (9) Ink the profile, taking special pains to do good work. The following order of work is suggested. (10) Encircle, with bow-pen and red ink, extreme points and break points of grade line. (11) Rule grade line with red ink, making a line narrower than medium. (12) From each encircled point on the grade line, rule a line, about 2 in. long, downward, with red ink. (13) Go over surface line with smooth, even, fine, black-ink line. (No. 1 or No. 2 Shepard pen may give better results than common lettering pen.) (14) Rule alignment lines with black ink, making heavy, — full dash for tangents and dashed for curves. (See I.C.C. model profile, and remember that all dimensions on it are half-size.) (15) Letter elevation numbers and grade-rate numbers with red ink. (16) Letter all else with black ink. (17) Check the lettering by means of Form J of Appendix.

69. Plotting earthwork cross-sections. — Certain cross-sections will be assigned to each student for plotting according to directions given in Exercise 42.

70. Computing volumes from cross-sections. — Certain stations will be assigned to each student for earthwork computation. He will compute yardage for each station and substation, using a method specified by instructor. (1) Make the computations on prescribed computing paper, and maintain an orderly arrangement of computations and results. (2) Pencil the computed volumes, station by station, on field-location profile; obtaining from other students the volumes from sections assigned to them. (Instructor will explain details of placing volumes on profile.) (3) Assuming that material neither swells nor shrinks, do cut and fill balance?

71. Prismoidal correction. — The prismoidal correction to "end-area" volume is usually so small as to be insignificant in comparison with those uncertainties, in end-area volume, which arise from practically unavoidable approximations made in taking earthwork cross-sections. The prismoidal correction should be ignored, therefore, except where ground surface is

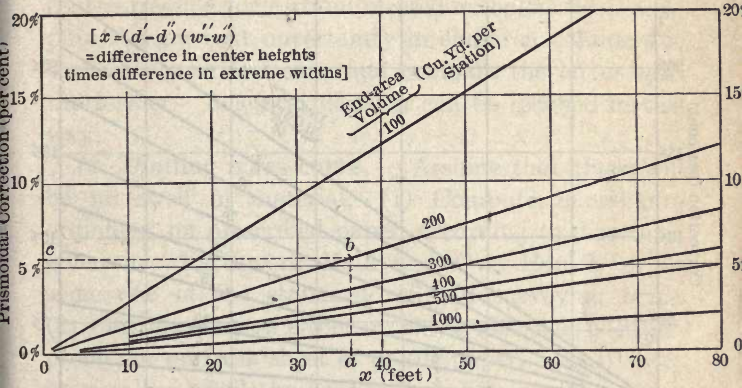


FIG. 71. DIAGRAM GIVING PRISMOIDAL CORRECTION IN PER CENT OF AVERAGE-END-AREA VOLUME

smooth and of prismoidal form between consecutive cross-sections and it can be shown that computation of the correction is justified. The accompanying diagram, Fig. 71, gives prismoidal correction in per cent of end-area volume for various values of product obtained by multiplying difference between distances out-to-out by difference between center-heights. For example, if $(w''-w')\ (d'-d'') = 36$, and end-area volume is 200 cu. yd., the prismoidal correction is found by starting at *a* on horizontal axis, running up to *b* on diagonal marked "200," thence to left to c, where read "5 + %," which is correction sought.

72. Correction for curvature. — (*a*) The correction for curvature, in per cent of average-end-area volume, can be obtained from the accompanying diagram, Fig. 72, thus: (1) Place pencil point at center of gravity (position determined by eye) of plotted cross-section. (2) Read distance (to nearest foot) from CL to pencil point; and call this excentricity, e. (3) In

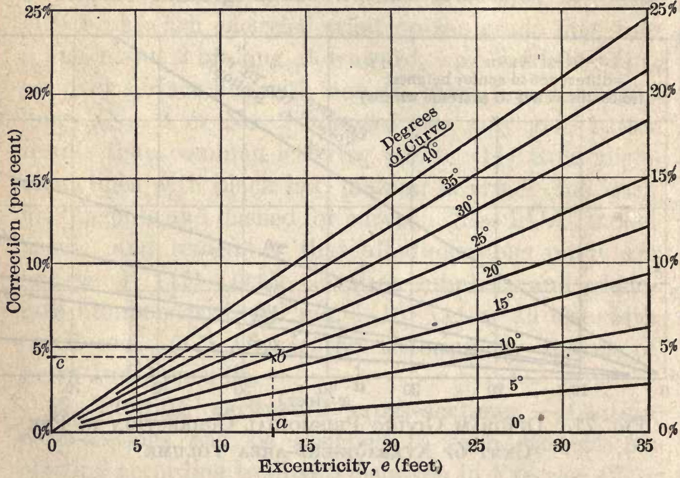


FIG. 72. DIAGRAM GIVING VOLUME CORRECTION FOR CURVATURE IN PER CENT OF AVERAGE-END-AREA VOLUME

Fig. 72, find value of e on horizontal axis, follow up corresponding vertical to its intersection with diagonal of given degree of curve, and thence move horizontally to right or left and read required per cent correction on per cent axis. For example, for excentricity of 13 ft. on a 20° curve, start at a; run up to b, on diagonal marked "20°"; thence to left to c; and read "4 + %." We then add (or subtract, as case may require) 4% of itself to average-end-area station volume, and obtain the corrected volume. (*b*) Correction is added if center of gravity lies on outside of curve; subtracted if it lies on inside. (*c*) Correction for curvature should be applied only when this is required for substantial accuracy. In many cases the correction is insignificant in comparison with uncertainties, in end-area volume, arising from approximations made in cross-sectioning. The only way to tell what range of excentricity can well be ignored is to find the per cent correction for certain selected excentricities; compute the per cent uncertainty in end-area volume due to field approximations; and compare the latter with the former. A logical decision can be reached in this way.

73. Plotting mass curve. — Assume that there will be no swell of material. (1) Compute mass-curve ordinates, on prescribed paper, according to directions given in "Earthwork Haul and Over-Haul"* or in some one of the standard railroad surveying texts. (2) Plot mass curve either on sheet with field location profile, or on fresh sheet of profile paper. (3) If sheet bearing the profile be used, use station numbers there lettered. If fresh sheet be used, place station numbers exactly as they are placed on field location profile. (4) Choose largest convenient V scale which can be used without running mass curve off paper at top or bottom when base line is laid on one of the five very heavy H lines of paper. Submit choice of V scale to instructor before beginning to plot mass curve. (5) After points are plotted, connect them by lines ruled with straight-edge. Go over mass curve with some colored ink (not red), ruling fine lines to connect points in order. This work should be done with precision

* By J. C. L. Fish, New York: John Wiley & Sons, $1.50.

because results are later to be computed from mass-curve scalings. (Do not ink the circles penciled around plotted points on mass curve.) (6) If mass curve is drawn on fresh sheet, place title at left end. If mass curve is drawn on sheet with field location profile, label mass curve. In either case, letter V scale numbers with color used for mass curve.

74. Distribution of material. — (If the student has the wish and the time to include overhaul computations in the work yet to be done, he will obtain from instructor the assumptions on which to base overhaul calculations.) (1) Plan distribution of material by means of mass curve, so as to make total haul a minimum. (2) On field location profile, mark off by V lines the several resulting bodies of cut and fill. Draw an arrow to indicate proposed destination of each body of cut, and write above the arrow the volume of the body. (3) Determine, by inspection of profile, the center of gravity of each body of material in cut and in fill. Compute haul for each body, and write haul below arrow. Compute total haul, and enter it near right end of grade line. (4) Find total haul represented by each mass-curve area, by means of planimeter; and record the haul on area. (5) Any further work on distribution will be assigned by instructor.

75. Additional work will be outlined for members of the class who complete the foregoing before the end of the course. Examples of additional work: Making surveys of drainage areas. (See p. 41 of Am. Ry. Eng. Assoc. "Manual" 1915.) Making an estimate of cost of grading, culverts, ballast, ties, rails, etc. Assigned reading in Wellington, in Beahan, in Lavis, and in Raymond, on the topics of reconnoissance, preliminary, and location surveys, and spirals.

M. Final Inspection of Student's Work

76. Work to be submitted. — Each student will deliver to instructor for final inspection, on or before a specified date, the following: (*i*) Field book, prepared as directed in Art. 77 below; (*ii*) Computations, prepared as directed in Art. 78 below; and (*iii*) Drawings, prepared as directed in Art. 79 below.

77. Preparation of field book. — The student will letter his name plainly on "back-bone" of his field book, and on the outside of front cover, with black drawing ink; and make headings, cross-references, and index complete.

78. Preparation of computations. — (*i*) The student will reletter with black drawing ink, all side-heads and center-heads of his computations made on prescribed computing paper, *e.g.*, "Prob. A," "Exercise 4," "Lats. & Deps. of Prelim. Traverse." (*ii*) Make an index of computations, on one or more sheets of computation paper; and place the index before pages of computations when binding. (*iii*) Take out of curl any pages which have been inadvertently filed in a roll, so that they will lie flat. Place all pages in proper sequence. (*iv*) Bind whole set of pages firmly together, with edges flush, in manila cover. (*v*) On outside of front cover, letter with black drawing ink, a title similar to the following:

C.E.6a
COMPUTATIONS
John Doe
1916

Keep the book flat; and deliver it flat.

79. Preparation of drawings. — Bind all detail-paper sheets, tracing, profile, cross-section sheets, and mass-curve sheet, together into one book of plates fastened firmly at left end with McGill fasteners or otherwise; thus: (*i*) Before binding, take the curl out of any sheets which may have been inadvertently filed in a roll. (*ii*) Arrange sheets in pile, with lower and right-hand edges flush, and in such sequence that when bound into a book the sheets will appear in following order from front to back of book: (*a*) Detail-sheets on which earlier exercises were drawn, in proper sequence. (*b*) Preliminary profile (with profiles of trial paper locations and final paper location) so folded that it does not project beyond right end of detail-paper sheets. (*c*) Field Location profile, folded as above. (*d*) Mass-curve sheet (if mass-curve is on separate sheet), folded as above. (*e*) Cross-section sheets in proper sequence; trimmed so as not to project beyond detail-paper sheets. (*f*) Tracings, in sequence, trimmed so as not to project beyond detail-paper sheets. (*g*) Detail-paper sheets of Preliminary map, in sequence. (*h*) Fasten sheets firmly together at left end. (*i*) File and deliver the book flat.

OTHER STUDENT SURVEYS

1. On planning surveys. — The chief methods and the order of steps for an extended survey requiring the coordinate working of several field parties, are planned in advance by the man in charge of the survey. In many cases the man in charge issues to the heads of such parties written or printed specifications or directions for the field work. Written instructions often specify permissible errors and prescribe forms for field notes for various parts of the survey. The man in charge of one party, also, plans in advance the chief methods and the order of steps for the survey, although his subordinates are not always aware that his verbal directions all follow a plan which he has thought out in advance and carries in his head. The student should understand, then, that to become a successful head of even a small surveying party, he must be able to do something more than merely handle his instruments with skill: he must be able to plan surveys in advance. The work of planning a survey is acknowledged to be of higher order than the work of performing the details of the field work. Because of the relatively high importance of correctly planning surveys, practice in survey planning is given a prominent place in the following General Directions. A student will derive valuable experience from conscientiously planning a survey, and discussing the plan with the instructor, even if it be not feasible to carry out the plan because of lack of time or facilities.

The student should fix well in his mind this all-important fact: A full appreciation of the purpose of the survey is absolutely necessary to one who would successfully plan the survey.

2. General directions for any student's survey. — The student in charge of survey will report progress to instructor at close of each office period and of each field period on prescribed form. (See forms C and D, in Appendix.) (1) Student will submit to instructor (or instructor to student, in case latter does not come to former with definite survey in mind) a written description of (*i*) purpose (real or assumed) of proposed survey, and (*ii*) location of proposed survey. (2) After conferring with student, instructor will assign such reading and study as he thinks the student needs by way of preparation for planning the survey. (3) Instructor, by questioning student on subject matter of assigned reading, will ascertain when student is prepared to plan survey. (4) Student will specify, in writing: (*i*) classes of data to be obtained in field, if object of survey is to obtain information; or classes of results to be accomplished in field, if purpose of survey is to stake out a structure or set of lines; and (*ii*) quality of data to be obtained in field, or quality of results to be obtained in staking out given structure or set of lines. (5) Student will submit foregoing specifications to instructor for criticism and approval, before taking following step. (6) Student will next make a reconnoissance of area to be surveyed, to obtain field data which will have bearing on planning of survey. While in field he will estimate distances and angles and make sketch, approximately to scale, showing all features of area which may influence choice of surveying methods and general planning of survey. (7) Student will plan survey in detail, and specify, in writing, accompanied by a one-sheet copy of his reconnoissance sketch, the following: (*i*) Proposed methods of surveying to be used, and order in which they will be used. (*ii*) Proposed checks on field work. (*iii*) Men required for work; their positions and equipment. (*iv*) Days and hours on which it is proposed to do field work; to do office work. (*v*) Order of steps in office work. (8) Student will submit foregoing plans and specifications, including sketch, to instructor for criticism and approval before proceeding with next step. (9) Student will devise, and enter in back part of his field book, a form of notes for each part of survey as planned; and submit same to instructor, before entering upon next step. (10) Student will next proceed with field work in accordance with approved plan and specifications. (11) Student will perform office work following field work, under supervision of instructor. (12) Student will make, on prescribed paper, a final report to instructor on entire work, describing concisely the following: (*i*) Location of survey. (*ii*) Real or assumed object of survey. (*iii*) Methods used. (*iv*) Checks applied. (*v*) Results of survey. (*vi*) Personnel of party. (13) Student will submit his field notes, computations, and drawings, — all arranged and bound in an orderly style — for final inspection on date specified by instructor.

3. List of surveys. — The following list is given to suggest different kinds of surveys from which choice may be made by student or instructor. (1) Preliminary surveys (with transit or plane table, or with pocket instruments): (*a*) reservoir site and dam site. (*b*) Ball field or tennis court on side-hill. (*c*) Drive-way ascending side-hill diagonally. (*d*) Highway diversion. (*e*) Stream diversion. (*f*) Canal or ditch. (*g*) Sewer system. (*h*) Water supply distribution. (*i*) Bridge site. (*j*) Building site. (*k*) Tunnel site. (*l*) Drainage area. (2) Location surveys: (*a*) Dam. (*b*) Ball field or tennis court. (*c*) Driveway ascending on side-hill ground. (*d*) Highway diversion. (*e*) Stream diversion. (*f*) Canal or ditch. (*g*) Sewer or drain. (*h*) Water pipe. (*i*) Bridge. (*j*) Building. (*k*) Electric power line. (*l*) Pipe line. (3) Other surveys: (*a*) Hydrographic. (*b*) Mine. (*c*) Parting off land. (*d*) Subdivisions. (*e*) Re-surveys and deed descriptions. (*f*) Simple methods of finding true meridian.

PART V

OFFICE EXERCISES

OFFICE INSTRUMENTS — MAKING TABLES AND DIAGRAMS — CONSTRUCTION OF SCALES — COMPUTATIONS — TOPOGRAPHIC PROJECTION

A. Office Instruments

1. Pantograph. — *Equipment:* Pantograph, sheet of drawing-paper (detail paper will suffice) about 18″ × 24″, two triangles, scale, *hard* pencil. All equipment except the pantograph should be furnished by the student.

Use *extreme care* not to strain the arms or clamps while altering the ratios of the arm lengths. Record in each case by a sketch the arrangement of the fixed point, tracing point and pencil, and also the distances (measured to the nearest .01″) of these points from the pivots.

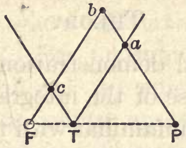
1. Set the instrument so that it will *magnify* 3.5 diameters and prove the work by making an enlarged copy of a 2″ square, and also of a 2″ circle.

2. Set the instrument so that it will *reduce* 2.5 diameters and prove the work by making a reduced copy of a 5″ square and also of a 5″ circle.

3. Set the instrument to retrace *at the same scale* any given figure and prove by applying it to some irregular figure sketched on the sheet.

Notes. — The drawing, with all results suitably recorded, will constitute the report. In the field-book record simply date, number of problem, etc.

Principles involved. Let F represent the fixed point, T the tracer, and P the pencil or marker. F, T, and P must *always* be in a straight line. FcT and FbP must always be *similar* triangles; in this instrument they are also *isosceles* triangles. $abcT$ is always a



parallelogram. The ratio $PF : TF$ is the ratio of enlargement. To reduce, transpose P and T. To retrace at same scale, make the parallelogram equilateral and transpose F and T. The retraced figure will then be inverted.

2. Polar Planimeter. — The Polar Planimeter is no longer considered as being merely an ingenious mathematical curiosity. The instrument is capable of such marvellous precision, and the practical uses to which it can be put are so varied, that it must now form a part of every complete engineering equipment. The essential work of the instrument is to find the area of any plane figure, no matter how irregular its perimeter may be. This permits the solution of many related problems. The planimeter may be used as follows:

1. To obtain the area of plotted figures which are bounded by irregular lines, such as drainage areas, ponds, property bounded by water lines, etc.

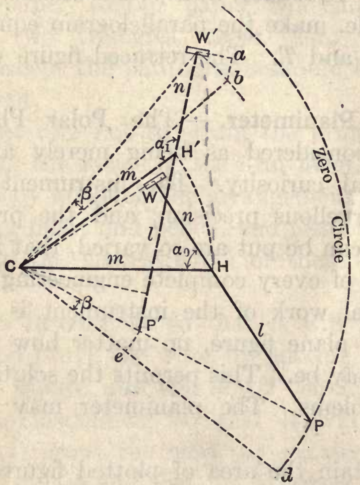
2. To obtain the area of plotted earthwork sections, especially when very irregular; also profiles, indicator diagrams, etc.

3. To obtain the average of observations taken at either regular or irregular intervals. This is done by

plotting the observations as vertical ordinates at horizontal intervals proportionate to the intervals of the observations. The tops of the ordinates are then joined by an irregular line. The area of the figure divided by the length of the base gives the average ordinate.

Theory

The mathematical demonstration of this problem is best made by the use of the integral calculus; but, for the benefit of those unfamiliar with the calculus, the following demonstration has been given — a demonstration involving nothing higher than elementary algebra,



geometry, and trigonometry, although the proof is rigidly accurate.

1. When the planimeter is in the position *PHWC* (see Fig.), the plane of the wheel, which is perpendicular to the axis *PH*, passes through *C*. If the instrument is revolved about *C*, with the angle *WHC*

($= \alpha_0$) always *constant*, the motion of the wheel over the paper will have no component in the direction of its plane, and the wheel will not revolve. The pointer in this position describes the "zero circle."

2. When the planimeter is in the position $P'H'W'C$ and is revolved about C, with the angle $W'H'C$ ($= \alpha_1$) always *constant*, the wheel will have a combined sliding and rolling motion. For an infinitesimal movement ($W'b$) the wheel will roll an amount $W'a$ and slide perpendicular to its plane an amount ab. When rolling in this direction, the movement is called *negative*.

3. When the point P is moved from P to P', the wheel W will both slide and roll, but its rolling will all be in a *negative* direction.

4. If the pointer were to move back from P' to P, the wheel would again slide and roll in precisely the same amounts but in contrary directions, and when it reached P it would have identically the same position, and the reading of the index would be identical with the previous reading at P.

5. If the pointer were to move from e to d, the amount and direction of both the slipping and rolling would be the same as when it moved from P' to P.

6. If the pointer were to start from P, move to P', thence to e, thence to d, and thence back to P, the resultant rolling of the wheel is the same as that for the line $P'e$ alone; for the rolling for dP is zero (¶ 1), and the rolling for PP' will be just neutralized by that for ed (¶¶ 3, 4, and 5).

7. Therefore when the pointer is moved to the right on the arc of a circle within the zero circle, about C as a center, the indication is *negative* and is the same as if the pointer moved around the area included between the arc, the corresponding arc of the zero circle, and the including radii.

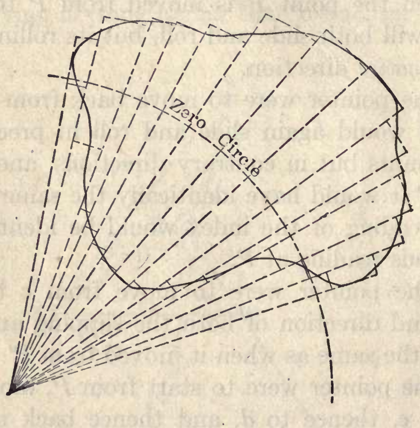
8. If the pointer moved in the opposite direction, the indication would be the same in amount but *positive.*

9. By similar demonstrations, similar facts may be shown for any other elementry area, except that

(*a*) When the pointer is *outside* the zero circle and moving to the *right*, the indication is *positive.*

(*b*) When outside and moving to the *left*, the indication is *negative.*

10. The perimeter of any area may be considered as made up of a combination of *infinitesimal* arcs and



radial lines having the fixed point of the planimeter as center. Its total area is the *algebraic* sum of all the infinitesimal areas lying between *each* arc and the zero circle.

11. If the pointer of the planimeter moves around each infinitesimal area in turn in such a manner that when moving on the perimeter it moves in the same direction as though moving continuously around the perimeter only, the pointer will move over all interior lines an *even* number of times in *opposite* directions.

Therefore the *accumulated* registration of the wheel will be the same as though it moved on the perimeter only, for all registration on interior lines will be neutralized by the equal motion on them in opposite directions (¶¶ 4 and 6).

12. Referring to the first Figure under "Polar Planimeter,"

$$P'e = CP' \times \beta = \sqrt{m^2 + l^2 + 2\,ml \cos \alpha} \times \beta.$$

$W'b = CW' \times \beta$. The rolling of the wheel $= W'a$ (§ 2). $W'a = (W'h \times W'b) \div CW' = (n - m \cos \alpha_1)\,\beta$, since $W'b \div CW' = \beta$ and $W'h = n - m \cos \alpha_1$.

Area $PP'ed$

$$= \tfrac{1}{2}(PC \times \beta)\,PC - \tfrac{1}{2}(P'C \times \beta)\,P'C$$
$$= \tfrac{1}{2}\beta\,(PC^2 - P'C^2)$$
$$= \tfrac{1}{2}\beta\,[(l^2+n^2+2nl)+(m^2-n^2)-(m^2+l^2+2ml \cos \alpha_1)]$$
$$= \beta l\,(n - m \cos \alpha)$$
$$= l \times W'a \text{ (i.e., } l \text{ times the rolling of the wheel).}$$

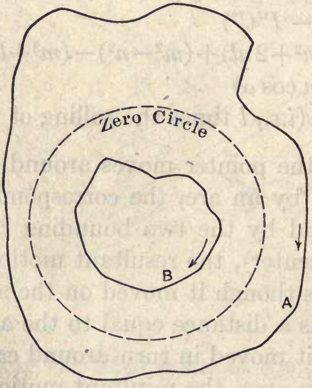
13. When the pointer moves around an elementary area bounded by an arc, the corresponding arc of the zero circle, and by the two bounding radial lines (all having C as center), the resultant motion of the wheel is the same as though it moved on the arc alone (§ 6); the wheel rolls a distance equal to the area divided by l (¶ 12). If it moved in turn around each elementary area of a large area, the resultant motion of the wheel would be the same as though it moved continuously around the perimeter of the large area (¶¶ 10 and 11), and therefore the total resultant motion of the wheel will equal the area of the figure divided by l.

14. Therefore if c = the circumference of the wheel, n the number of turns recorded by the index, and l the length of the arm from H to P, then

$$\text{Area} = lnc.$$

15. *Fixed center inside the figure.* If the pointer is moved around the perimeter A (see Figure below) to the *right*, the indication will be positive (¶ 9), but will indicate only the area between A and the zero circle. Therefore the total area will equal the indicated area (lnc) *plus* the area of the zero circle [$\pi (m^2 + l^2 + 2\,nl)$]. If the pointer is moved to the *right* around perimeter B, the record will be *negative* (¶ 9) and will correspond to the area between B and the zero circle. Therefore the *algebraic* sum (the numerical difference) will give the true area.

16. *To find the area of the zero circle.* The accurate measurement of l, m, and n, directly from the instru-



ment, is impracticable, but the area of the zero circle may be obtained from the consideration that it is equal to the algebraic difference (the numerical sum) of the readings when an area is measured (1) with the fixed center *outside* the figure and (2) with the fixed center *inside* the figure. Therefore draw some figure large enough so that the fixed center may be placed *inside* and the pointer may travel all around the perimeter, and yet not so large but that the fixed center may be placed *outside* and the pointer may reach all parts of the perimeter. If the pointer is always moved around the figure to the *right*, the reading when the fixed center is *inside* will be *negative*, and when the fixed center is *outside* will be *positive*. Their algebraic difference, which is their numerical sum, will be the area of the zero circle. A 6-inch square will be the best figure for this purpose, for reasons given below.

Practical Use

These instruments are generally constructed so that the arm *PH* (the length *l*) may be made variable, and the arm is graduated so that, by setting it at given marks, the wheel will give the area directly in almost any desired unit. If the desired unit is not marked on the arm, draw a square having 1, 2, or 3 of the desired *linear* units on a side and take its reading. The reading for this square will be 1, 4, or 9 times the reading for *one* of the desired square units. Then the reading for the given irregular figure divided by the reading for one square unit will give the number of such square units.

When running the pointer along straight lines it will increase the accuracy to guide the pointer by running it along a straight edge. Therefore in obtaining the area of trial unit figures, or that of the zero circle (¶ 16), it is best to use straight-lined figures.

If there is any doubt of the accuracy of the markings on the arm *PH*, they may be tested by using a test-bar that generally accompanies the instruments. This bar has a needle center and drilled points at even inches or centimeters from the needle. By placing the pointer in one of these drilled points, an accurate circle may be described by the pointer about the fixed needle center. This circle has a known area, and the wheel reading should correspond.

The previous work has assumed that the *initial* reading is *zero*, and then the final reading is the true "reading" for the area. The practical difficulty in having the initial reading *precisely* zero, when the pointer is at the desired starting-point, justifies the practice of taking both initial and final readings (whatever they may be) and then taking the *difference* as the "reading" or measure of the area.

Care must be taken, especially in work involving the zero circle, to note the *direction* of motion of the index wheel and also whether it runs past the zero of that wheel and how often. When the fixed center is *inside* the figure, the index may turn completely around two or three times — turning negatively or backward. Then if the initial reading is (say) 4.642, call it 34.642. Then watch the wheel run back through zero, and it is in the *twenties*; again through zero, and it is in the "*teens*"; again through zero, and it is in the single units, and the reading is (say) 8.796. Then (34.642 − 8.796) is the true difference or *negative reading*.

See below for practical exercises illustrating these principles.

Exercise with Planimeter

Equipment: Planimeter, sheet of drawing-paper about 15″ × 22″, two triangles, scale, drawing-compasses, *hard* pencil. All equipment except the planimeter should be furnished by the student.

Draw on the paper, with *extreme accuracy*, a circle 2″ in diameter, a circle 6″ in diameter, a 2″ square and a 4″ square. The 6″ circle should be nearly in the center of the sheet, so that when the fixed point is placed inside the circle and the pointer traced around the circumference, the rolling wheel shall not run off the paper. Set the index of the arm of the planimeter at 295.5. With the pointer on some definite point of the perimeter of one of the figures read an initial reading (*not* necessarily zero) on the wheel and vernier. Move the pointer around the perimeter, turning to the right. When the starting-point is again reached, take the final reading. The difference in the readings divided by the known area in square inches gives the reading per square inch. Compute the quotient to three significant figures. Trace each figure similarly *twice.* The readings per square inch on all the figures should agree to a small fraction of one per cent.

Obtain the area of the "zero-circle" by taking the mean of two readings of the six-inch circle with the fixed point *inside* the circle, and then taking the *algebraic difference* of the mean readings (with the fixed point first inside, then outside), which difference, divided by the average reading per square inch, gives the area of the zero-circle in square inches. In taking the reading with the fixed point inside the circle, and the pointer turning as usual to the right, the index wheel will run *backward*, giving a *negative* quantity. The number obtained when the fixed point is outside the circle is *positive.* Therefore their algebraic difference is their numerical sum.

To compute the circumference of the rolling wheel. — Use the formula $A = lnc$, in which c = the required circumference, n = the mean record of revolutions per square inch, $l = \left(\frac{295.5}{2}\right)^{mm} = 147.75\,mm = 5.817$ inches, $A = 1$ square inch.

Check on area of irregular figure. — Draw a "free-hand" figure enclosing an area, the figure being purposely made very irregular. Obtain its area by tracing the pointer around the perimeter, and dividing the difference of the initial and final readings by the previously obtained average reading for one square inch. The quotient is the area in square inches. Check this result by carefully cutting out the irregular figure and also one of the squares and weighing the pieces on a chemical balance. With a drawing-paper of uniform thickness and texture the weights should be proportional to the areas.

NOTES. — The drawing, with all results suitably recorded on it, will constitute the report. In the field-book record simply the date, number of problem, etc.

B. MAKING TABLES AND DIAGRAMS

1. Calculating a table of quantities. — (*a*) The method of calculating a table of quantities can be most readily understood from an example. Suppose it is required to make a table which shall give the areas of level sections for different center-heights. Assume road-bed to be 18 ft. wide, side slopes 1 : 1. Then area of level section is given by the formula $A = 18\,h + h^2$ where h is center-height.

TABULATION: $A = 18\,h - h^2$

When h is (ft.).	Then A is (sq. ft.)	First differences.	Second differences.
1	19		
		21	
2	40		2
		23	
3	63		2
		25	
4	88		2
		27	
5			2
		29	
6			2
		31	

If the values of A, corresponding to four or more consecutive values of h, be calculated and set in parallel columns, and the differences between consecutive values of A be set in a third column, it will be seen that the differences increase by the constant quantity 2, as shown in the fourth column. The remainder of the table to any desired value of h, can, therefore, be built up thus: Continue down column of first differences, making each number 2 greater than the preceding. Next, fill in second column numbers: the value of A for $h = 5$ is $88 + 27 = 115$; of A for $h = 6$ is $115 + 29 = 144$; and so on. By computing A directly from the formula for $h = 10$, $h = 20$, etc., the whole table can be well checked.

(*b*) The second difference is always constant for an equation of second degree; the third difference is always constant for an equation of third degree; and so on. (*c*) Exercises: Construct a table giving for each of the depths, 1 ft., 2 ft., 3 ft., . . . , and 10 ft., the quantity (cubic feet) of water in an inverted conical tank, 10 ft. high and 8 ft. across the base.

2. Construction of computing diagram. — (*a*) If we plot the equation $y = x^2$ we obtain a curve which, with the two graduated axes, forms a computing diagram. The value of y, for any given value of x, can be read directly from the diagram. (*b*) The case is not so simple when there are three variables, as, for example, with the equation $A = hb/2$ which gives the area of a triangle in terms of altitude h and base b. In such a case the equation is arbitrarily transformed into an equation between two variables (as A and h) by giving b a fixed value during the time of once plotting the equation. Another value is then given to b while plotting the equation a second time; and so on. For example, plot on one set of axes the following equations formed by giving different fixed values to b in $A = hb/2$:

$$A = h/2 \quad \text{(for } b = 1\text{)}$$
$$A = 2\,h/2 = h \quad \text{(for } b = 2\text{)}$$
$$A = 3\,h/2 = 1.5\,h \quad \text{(for } b = 3\text{)}$$
$$A = 4\,h/2 = 2\,h \quad \text{(for } b = 4\text{)}$$

On the plot of $A = h/2$, write $b = 1$; on the plot of $A = h$, write $b = 2$; and so on. (*c*) Exercise: Make a diagram for computing station volume (cu. yd.) for various center heights and roadbed widths, assuming level sections and side slopes of $1\frac{1}{2}$ to 1. (*d*) Exercise: Make a diagram giving corrections to be applied to stadia readings to reduce them to horizontal distance.

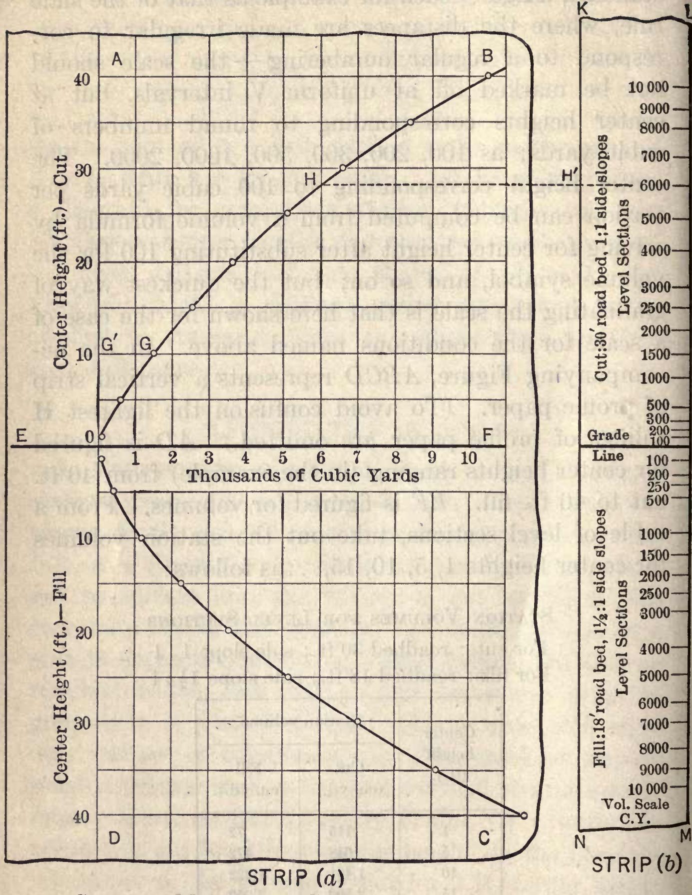
C. Construction of Scales

1. Construction of volume scale. — (*a*) This is a scale so graduated and figured that, when its zero is placed on the grade line of a profile at a mid-station, (*i.e.*, a +50 point) one reads, at point where scale is cut by surface line, the volume of cut (or fill, as the case may be) in that station. (*b*) Since volume of a station depends not only on center height, but also on roadbed width, side slopes, and transverse slope of ground, it is plain that one volume scale will serve only one set of conditions. (*c*) The quickest way to make a volume scale is to cut a vertical strip of profile paper; mark its central heavy H line "0"; write the volume of cut for 1 foot center height on the one-foot line above 0, the volume of cut for 2 ft. center height on the two-foot line above 0, and so on; and similarly write fill volumes for center heights of 1 ft., 2 ft., etc., on corresponding lines below 0. The volumes may be copied from a table. (*d*) The scale just described is inconvenient to use because of irregularity in scale numbers. To make the scale numbers conform to standard usage — such for example as that of the slide rule, where the distances are made irregular to correspond to a regular numbering — the scale should not be marked off at uniform V intervals, but at center heights corresponding to round numbers of cubic yards; as 100, 200, 300, 500, 1000, 2000. The center height corresponding to 100 cubic yards per station can be computed from a volume formula by solving for center height after substituting 100 for the volume symbol, and so on; but the quickest way of graduating the scale is that here shown for the case of a scale for the conditions named above. In the accompanying Figure, *ABCD* represents a vertical strip of profile paper. (To avoid confusion the lightest H rulings of profile paper are omitted.) *AD* is figured for center heights ranging (in the example) from 40 ft. cut to 40 ft. fill. *EF* is figured for volumes. From a table of level sections, take out the station volumes for center heights 1, 5, 10, 15,. . .as follows:

STATION VOLUMES FOR LEVEL SECTIONS

For cuts: roadbed 30 ft.; side slope 1 : 1
For fills: roadbed 18 ft.; side slope $1\frac{1}{2}$: 1

Center height (ft.).	Station volume.	
	Cut (cu. yd.).	Fill (cu. yd.).
1	115	72
5	648	472
10	1,481	1,222
15	2,500	2,250
20	3,704	3,556
25	5,093	5,139
30	6,667	7,000
35	8,426	9,139
40	10,370	11,556



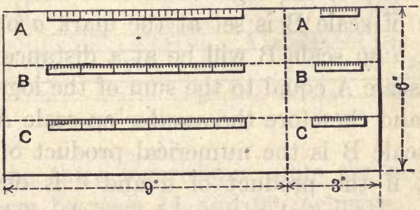
SHOWING METHOD OF CONSTRUCTING VOLUME SCALE

Now, with *EF* as an axis of volumes and *EA* as an axis of center heights, plot a volume curve, *GHB*, thus: Volume of cut for 10 ft. center height is 1481 cu. yds. — taken from foregoing table. On the horizontal of 10 ft. fill, lay off, to right of *EA*, distance *G'G* equal to 1481 to scale *EF*, and mark point *G*. *G* is on required volume curve. In like manner other points are plotted until all data above are represented by plotted points. Draw smooth curve *GHB* through plotted points above *EF*. The curve of fill volumes below *EF* is constructed in the same way. In the figure, above, *KLMN* represents a strip of stiff paper, originally blank, set alongside strip *ABCD* and fastened in position with its long edges parallel to V rulings of strip *ABCD*. Graduate the edge *LM*, thus: From point *H*, where volume curve *GHB* crossed vertical marked "6," draw to the right a horizontal *HH'* to mark edge *LM* at *H'*. Write "6000" on the mark at *H'*. By repeating this process for each of the points in which volume curve is crossed by a vertical, the entire edge *LM* is graduated and figured. Write on the slip the roadbed width and side-slope ratio. The strip *KLMN* is now ready for use. (*e*) The scale is placed, with left hand, in V position on profile so that zero point lies on grade line of profile at a mid-station, (*i.e.*, at a +50 point). Station volume is then read off at point where edge *LM* crosses grade line, and, with the free right hand, recorded for use. A volume scale for different fixed conditions, including any other given transverse slope, can be made in the manner described above. (A set of "Level Section Transparent Scales for Plate A Profiles, for Graduation, Arch Masonry, and Iron Culverts" is sold by F. W. Steber, 2101 McKinney Ave., Dallas, Texas, for $2.50.)

2. Construction of quantity scale for culverts, etc. — (*a*) The reauired length of culvert for a given fill depends on (*i*) depth of flow-line of culvert at center line of fill, (*ii*) side-slope ratio of fill, (*iii*) slope of culvert, (*iv*) diameter of culvert, (*v*) on the presence or absence of headwalls, and (*vi*) on length of shortest practicable piece of pipe (the regular length of cast-iron pipe is 12 ft., but half-lengths can be had). (*b*) On preliminary and location surveys for railroads, estimates of quantities of pipe, or stone, etc., for culverts, as well as of other quantities, must be made very rapidly. Where there are many pipe culverts along the profile, time will be saved by making a quantity scale from which, when its zero point is applied to the grade line above the pipe site, one may read, at the point where scale is cut by surface line of profile, the required quantity of pipe. The construction and use of this scale are the same as for volume scale of preceding exercise. (*c*) Similarly, scales may be made for finding required quantities of other culvert materials. If desired, the scale can be made so that cost can be read, instead of lineal feet, cubic yards, etc. (*d*) Exercise: Make a scale by which to read required lineal feet of vitrified clay culvert-pipe under fills, from the following data: top width of fills is 20 ft., side slopes are $1\frac{1}{2} : 1$. Pipe may be assumed to be horizontal. The pipe is 2 ft. in diameter, and comes in two-foot lengths. There are no headwalls. (See last sentence of preceding paragraph for notice of ready-made scales.)

3. Vernier construction. — *Equipment:* Sheet of drawing-paper 15″ × 22″, a decimal scale, a pair of hair-spring (or bow-spring) dividers, a *hard* pencil, triangles, etc., all of which should be furnished by the student.

Principles involved. A vernier is a device which determines the position of an index-point, which is movable along the face of a scale, with a greater exactitude than the finest division of the scale. A *direct* vernier has n divisions which are made equal to $(n - 1)$ divisions on the scale. The finest reading (except by estimation) is $\frac{1}{n}$ of the smallest division of the scale. On a direct vernier the vernier readings increase in the *same* direction as the scale readings. A *retrograde* vernier has n divisions which are made equal to $(n + 1)$ divisions on the scale. The finest reading is likewise $\frac{1}{n}$ of the smallest division. The vernier readings increase in a *contrary* direction to the scale readings. A *double* vernier, which may be either direct or retrograde, has a full set of vernier graduations on both sides of the index. It facilitates the use of a scale which is read in both directions, as is the case with the horizontal limb of a transit.



Construction of scales. Construct the required scales on a single sheet, and cut them out afterward. Lay off the verniers by setting the dividers (by trial) at such a space that n repetitions of the spacing shall equal $n - 1$ (or $n + 1$) divisions on the scale.

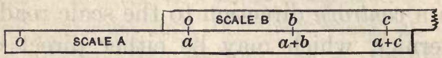
a. Draw a scale 6 inches long, divided into fifths of an inch; also a single direct vernier by which the scale may be read to $\frac{1}{25}$ inch.

b. Draw a scale 6 inches long, divided into tenths of an inch; also a double direct vernier by which the scale may be read to $\frac{1}{100}$ inch.

c. Draw a scale 6 inches long, divided into inches, tenths, and half-tenths of an inch; also a single retrograde vernier by which the scale may be read to half-hundredths of an inch.

4. Slide-rule construction. — *Equipment:* Sheet of drawing-paper 15″ × 22″, decimal scale, *hard* pencil, triangles, etc., all of which should be furnished by the student.

Principles involved. An ordinary slide-rule is a combination of two logarithmic scales. A logarithmic

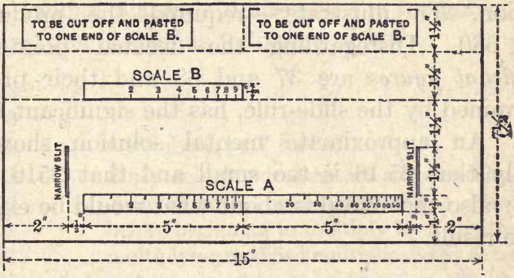


scale is one on which the distance of each numbered mark from the origin is proportional to the logarithm of that number. Since the logarithm of the product *ab* is the sum of the logarithms of *a* and of *b*, if the origin (*o*) of scale B is set at the mark *a* on scale A, the mark *b* on scale B will be at a distance from the origin of scale A equal to the sum of the logarithms of *a* and *b*, and therefore the *number* on scale A opposite to *b* on scale B is the numerical product of *a* and *b*. Similarly, if the product of *a* and *c* is desired, the relative position of the two scales need not be altered. It is only necessary to observe the position of *c* on scale B and note the corresponding number on scale A, which will be the required product of *a* and *c*.

Since the logarithm of 1 is zero, the number at the origin is always 1 (10, 100, 1000, etc.). The number 2 (20, 200, etc.) is always placed at .301030 . . . of a *logarithmic unit* from 1. The number 8 comes at .903090 of the logarithmic unit from the origin, and 9 and all the subdivisions between 8 and 10 must come between .903090 and 1.000000.

Since it frequently happens that when the origin of scale B is set at the desired point on scale A the other factor on scale B would come beyond the range of one logarithmic unit on scale A, it is necessary to give scale A a length of two logarithmic units. Two logarithmic units for A and one for B are all that is necessary for simple operations, but two logarithmic units for B are sometimes more convenient for the more complicated operations which are possible with a slide-rule.

Construction of scales. Lay off the scales as shown in the sketch. Set off between the extremities of



each of the three logarithmic units the positions for all numbers between 11 and 99 inclusive. Take the quantities from any table of logarithms. Use the "20" decimal scale. $\frac{1}{20}''$ will then equal .01 of the logarithmic unit. The finest space (that between 99 and 100) will then equal .0044, or slightly less than $\frac{1}{40}''$. Estimate fractions of the smallest scale unit, $\frac{1}{20}''$.

Use *extraordinary care* in laying off the scales, marking the positions with a needle-point. Ink the scales, using very fine lines.

Cut out the narrow slits. Cut off scale B and lengthen it by pasting on the two other pieces. Slide scale B through the slits so that it passes close to scale A.

Test. Test the work by the solution of problems, which are verified by ordinary numerical processes. When made with sufficient care, even such a hand-made scale will have practical value when used for roughly checking numerical computations whose accurate solution requires more tedious methods.

Position of decimal point. Rules may be formulated for determining the location of the decimal point in the final result, but their application is sometimes troublesome and liable to lead to error. A simpler and surer method is to make an approximate mental solution. To illustrate — required the product of $.37 \times 680$. Disregarding the decimal points, the *significant figures* are 37 and 68 and their product, determined by the slide-rule, has the significant figures 2516. An approximate mental solution shows instantly that 25.16 is too small and that 2516 is too large; also that 251.6 is about what would be expected as the result.

D. Computations

1. Abridged multiplication. — The method of abridged multiplication will be presented by means of examples. *Example* 1. — What is the elevation difference corresponding to a stadia-reading 7.53 and slope-angle 8° 16′? From a stadia reduction table, for stadia-reading 1.00, and slope 8° 16′, we have, neglecting the effect of $f + c$,

$$\text{elev. diff.} = 14.23;$$

therefore the required elev. diff. = 14.23 × 7.53. The indicated multiplication will now be performed in the ordinary way and in the ordinary way reversed.

Ordinary process	*Ordinary process reversed*
14.23	14.23
7.53	7.53
4269	99.61
7115	7.115
9961	.4269
107.1519	107.1519

In the abridged method the ordinary process is reversed and certain digits are disregarded in the multiplicand, according to the number of decimal places to be retained in the product. Suppose the retention of two decimal places is desired in the foregoing case. The operation is then as follows:

14.23	
7.53	
99.61	........multiplying by 7 (units).
14.23	Since only hundredths are to be retained the
7.53	3 (hundredths) in multiplicand is crossed off
99.61	before beginning to multiply by 5 (tenths).
7.12	........multiplying modified multiplicand by 5 (tenths), and carrying 2 from product .5 × .03.
14.23	The 2 of multiplicand is next cancelled and
7.53	modified multiplicand is multiplied by 3
99.61	(hundredths).
7.12	
.42	........multiplying by 3 (hundredths), and carrying
107.15	nothing from product of .03 × .2.

If it were desired to retain only one decimal place in the product, the steps would be as follows: Note that 7 (units) times the right-hand figure, 3 (hundredths), of the multiplicand gives hundredths. Therefore at once cancel the right-hand figure of multiplicand.

Since 7 (units) times 2 (tenths) gives tenths, and we wish to retain tenths, we now multiply the modified multiplicand by 7:

14.23	
7.53	
99.6	multiplying 14.2 by 7, and carrying the 2 from product 7 × .03.
14.23	Next, cancel the 2 of 14.2, and multiply
7.53	modified multiplicand by 5 (tenths):
99.6	
7.1	multiplying 14 by 5 (tenths), and carrying the 1 from product .5 × .2.
14.23	Next, cancel the 4 of 14, and multiply modi-
7.53	fied multiplicand by 3 (hundredths):
99.6	
7.1	
.4	Multiplying 1 by 3 (hundredths), and carry-
107.1	ing the 1 from product .03 × 4.

Example 2. — What is area of circle of which the squared radius is 39.47 inches? Area = 3.1416 × 39.47.

Area to 2 decimal places:

		3.1416	(cancel the 6 at once)
		39.47	
3 (tens) × 3.141	=	94.25	carrying 2 from 3 × 6
9 (units) × 3.14	=	28.27	carrying 1 from 9 × 1
4 (tenths) × 3.1	=	1.26	carrying 2 from 4 × 4
7 (hundredths) × 3	=	.22	carrying 1 from 7 × 1
		124.00	

Area to 4 decimal places:

		3.14160	(annex "0" at once)
		39.47	
3 (tens) × 3.14160	=	94.2480	
9 (units) × 3.1416	=	28.2744	
4 (tenths) × 3.141	=	1.2566	(carrying 2)
7 (hundredths) × 3.14	=	.2199	(carrying 1)
		123.9989	

2. Notation by powers of ten. — Small whole numbers are more quickly sensed than very large numbers or very small decimal fractions. For example, we sense the approximate value of 2.1408×4.653 instantly, and without doubt or hesitation place the decimal point correctly in the product obtained by slide-rule or by calculating machine; whereas we do not instantly sense the approximate value of 214.08×0.004653, and considerable conscious effort is required to place correctly the decimal point in the product whether the multiplication be performed by slide-rule or calculating machine. In either case the use of logarithms would give us mechanically, through the combining of characteristics, the correct position of decimal point. The advantages of using small whole numbers (between one and ten units), and of using the logarithmic characteristic in fixing the decimal point, are combined in the method of notation by powers of ten, which is exhibited in the following examples.

Example 1.— Find the product of 214.08×0.004653.

Now,
$$214.08 = 2.1408 \times 10^2,$$
$$0.004653 = 4.653 \times 10^{-3},$$

and

$$\begin{aligned} 214.08 \times 0.004653 &= 2.1408 \times 10^2 \times 4.653 \times 10^{-3} \\ &= 2.1408 \times 4.653 \times 10^{-1} \\ &= 9.96 \times 10^{-1} \text{ (by slide-rule)} \\ &= 0.996. \quad \text{Ans.} \end{aligned}$$

In practice, the only necessary work on paper would be as follows: Write the numbers as given; assume the decimal points moved to make each number lie between 1 and 10; and set above each number the exponent of the corresponding power of 10; so that the work would appear thus:

$$\begin{matrix} +2 & & -3 & & -1 \\ 214.08 & \times & 0.004653 & = & 9.96 = 0.996. \quad \text{Ans.} \end{matrix}$$

Example 2. Reduce $\dfrac{3147.6 \times 83.12 \times 0.096}{42.8 \times 0.00523 \times 594}$.

Assume decimal points shifted to make each factor read in units (less than 10) and decimals, and write the exponent of the power of ten corresponding to the shift in decimal point, above (or below) the number, thus:

$$\begin{matrix} +3 \quad\quad +1 \quad\quad\quad -2 = +2 \\ \dfrac{3147.6 \times 83.12 \times 0.096}{42.8 \times 0.00523 \times 594} = 1.895 = 189.5. \quad \text{Ans.} \\ +1 \quad\quad -3 \quad\quad\quad +2 = 0 \end{matrix}$$

Find mentally the approximate product

$$\frac{3 \times 8 \times 9}{4 \times 5 \times 5} = 2 \pm;$$

then find the result 1895 of the given expression on the slide-rule, and set it down as 1.895 as indicated by the approximate mental result 2 ±. Add the upper row of exponents algebraically and set the sum, +2, over 1.895. Add the lower row of exponents, and set the sum, 0, below 1.895. Subtract, algebraically, the lower sum from the upper, and multiply 1.895 by that power of 10 whose exponent is the difference just found. In this case the difference is +2, and 10 to the +2th power is 100. The answer is therefore 189.5. (See Holman: "Computation Rules and Logarithms," Macmillan and Co., 1896.)

3. Computing and applying corrections. — In many cases it is far easier to find a desired value by computing and applying a correction than by the direct computation of the quantity. The correction is usually a relatively small quantity and often may be computed mentally or with slide-rule with all needful precision even when the slide-rule would not be precise enough for computing the value sought. Thus it is easier to reduce a stadia reading to the horizontal by calculating and applying a correction, than by direct computation. (See under "Office work" in Exercise 36. Other examples follow.)

4. Versed sine instead of cosine. — Since $\cos A = 1 - \text{vers}\, A$, it is often advantageous, when the angle is small (from zero to 20°) to find latitude by the formula: lat. = dist. − dist. × vers bearing. *Example.* — What is the latitude of the course 257.3, N 8° 07′ E? By the usual formula we have 257.3 × 0.98998, whereas by the foregoing formula we have 257.3 − 257.3 × 0.01002 and the latter product can be computed mentally. In like manner, when a bearing is near 90°, it saves time to compute the departure by the formula: dep. = dist. − dist. × vers co-bearing. *Example.* To find the departure of course 358.6, N 80° 15′ E. By the usual formula we have 358.6 × 0.98556, whereas by the substitute formula we have (since 90° 00′ − 80° 15′ = 9° 45′) 358.6 − 358.6 × 0.01444; and the latter product can be found with slide-rule with sufficient accuracy.

5. Difference between base and hypotenuse of right triangle. — If h be the hypotenuse, b the base, and a the altitude of a right triangle, and x the difference between base and hypotenuse, then

$$x = A + B + C + D + \cdots, \qquad (1)$$

where

$$A = a^2/2\,h; \quad B = A^2/2\,h; \quad C = AB/h; \quad D = 2.5\,B^2/h.$$

Also,

$$x = A' - B' + C' - D' + \cdots, \tag{2}$$

where

$$A' = a^2/2\,b;\ B' = A'^2/2\,b;$$
$$C' = A'B'/b;\ D' = 2.5\,B'^2/b.$$

Eqs. 1 and 2 are of more interest than value except as a means of finding the error involved by the use of eqs. 1*a* and 2*a* in any given case. Eqs. 1*a* and 2*a*, on the contrary, are of such wide application in practice and so easily applied mentally that the student would do well to memorize them.

Example 1. — A 100 foot tape was stretched on a slope, and upper end of tape was 30 ft. higher than lower end. What is corresponding horizontal distance? Here h and a are given and b is required. $b = h - x$ and x is computed by eq. 1 thus:

$$\begin{aligned} A &= 900/200 &&= 4.50 \\ B &= (4.50)^2/200 &&= 0.1013 \\ C &= 4.5 \times 0.1013/100 &&= 0.0046 \\ D &= 2.5 \times 0.1013^2/100 &&= \underline{0.0003} \\ & & x &= 4.6062 \\ & & h &= \underline{100} \\ & & b &= 95.3938. \text{ Ans.} \end{aligned}$$

It will be observed that the terms in eqs. 1 and 2 rapidly diminish. For the great majority of cases it suffices to retain only the first term in each case. That is, for most cases the value of x is found with sufficient precision by

$$x = a^2/2\,h \text{ (approx.)}, \tag{1a}$$

or by

$$x = a^2/2\,b \text{ (approx.)}. \tag{2a}$$

Example 2. — If a 100 foot tape is held with one end 2 ft. higher than the other, what is the correction for horizontal distance? Here $a = 2$ and $h = 100$. $x = 2^2/100 = 0.04$. The horizontal distance is therefore $100 - 0.04 = 99.96$ ft. To test the precision of this value of x, compute B of eq. 1: $B = (a^2/2\,h) = 0.04^2/200 = 0.000008$ — certainly a negligible distance in ordinary surveying. (See "Engineering News," 1906 — ii — 173.)

6. Convenient values of trigonometric functions. — For many cases, sines of small angles may be assumed to vary directly with the angle; and tangents may be assumed to be the same as the sines. The following values are convenient for mental or slide-rule calculations in office or field.

$$\sin 1° = \tan 1° = 0.0175 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

$$\sin 1' = \tan 1' = 0.0003 \text{ ("three ciphers three")} \quad (2)$$

$$\sin 1'' = \tan 1'' = 0.000005 \text{ ("five ciphers five")} \quad (3)$$

$$\sin A \text{ (in degrees)} = 0.0175\,A \text{ (in degrees)} \quad . \; . \quad (4)$$

$$1/\sin A \text{ (in degrees)} = 57.1/A \text{ (in degrees)} \quad . \; . \; . \quad (5)$$

$$\sin A \text{ (in minutes)} = 0.0003\,A \text{ (in minutes)} \quad . \; . \quad (6)$$

$$1/\sin A \text{ (in minutes)} = 3437/A \text{ (in minutes)} \quad . \; . \quad (7)$$

Example. — What is the degree of curve for radius 800 ft.? From formula $\sin D/2 = 50/R$ we have, approximately, $D \times 60/2 = 50/(R \sin 1') = (50/800) \times 3437 = 215$ minutes; or $D = 7° \, 10'$. (See "Engineering News," 1902–i–147.) (*a*) By the use of eq. 2, above, the usual formula for subchord deflection,

$$\sin (d/2) = (c/100) \sin (D/2),$$

becomes

$$(d/2) \sin 1' = (c/100)\,(60\,D/2) \sin 1' \text{ (approx.)},$$

from which we find the subchord deflection in minutes to be

$$d/2 = 0.3\,cD. \qquad (8)$$

7. Corrections to approximate values of curve elements. — The following equations give practically true values of radius R, tangent distance T, and external distance E, of a circular curve:

$$R = 5730/D + (0.073\,D - 0.425/D) \qquad (1)$$

$$T = (T \text{ for } 1^\circ \text{ curve})/D + (0.073\,D - 0.073/D) \tan (I/2) \qquad (2)$$

$$E = (E \text{ for } 1^\circ \text{ curve})/D + (0.073\,D - 0.073/D) \operatorname{exsec} (I/2). \qquad (3)$$

The second term in each equation is small — often negligible — and may be calculated mentally or with slide-rule with sufficient precision. *Examples.* — For $D = 10^\circ$ and $I = 90^\circ$, find R, T, and E.

Approx. $R = 5730/10 =$	573.0
Correction $= +(0.73 - 0.04) =$	+ .69
True $R =$	573.69
Approx. $T = 5729.7/10 =$	572.97
Correction $= +(0.73 - 0.01) \tan 45^\circ =$	+ .72
True $T =$	573.69
Approx. $E = 2373.3/10 =$	237.33
Correction $= +(0.73 - 0.01)\ 0.4 =$	+ .29
True $E =$	237.62

8. Difference between arc length and chord length. — For circular curve of n stations, arc exceeds long chord by $0.00127\,n^3D^2$ (approximately), where D is degree of curve. The error by the use of this approximate formula is only 0.1 ft. for 5 stations of a 10° curve.

9. Tangent offset. — The tangent offset, O, for the nth station on a D° curve is, roughly, $O = (7/8)\,n^2D$. From this equation we have, also,

$$D = (8/7)\,O/n^2;$$

and

$$O' - O = (7/8)\, n^2\, (D' - D);$$

and

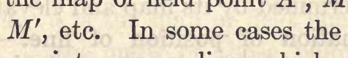
$$(D' - D) = (8/7)\, (O' - O)/n^2;$$

where D'-degree curve and D-degree curve have common point of tangency and turn in same direction. These formulas are useful in location work.

E. Topographic Projection

1. Introduction. — Many topographic problems and some problems of geology and mine surveying are solved by the general method of descriptive geometry, in which two planes of projection are used: one to show the relative horizontal, the other the relative vertical, positions of points; so that problem data are shown by "plan" and "elevation." But data of topographic problems are shown on one plane, and this condition leads to a modified manner of applying descriptive geometry methods. Topographic projection treats of the application of descriptive geometry methods to topographic problems.

2. Notation. — (1) The map of a field point will be designated by one of the capital letters $A, B, \ldots O$, and the field point itself by the same letter primed. Thus A is the map of field point A', M is the map of field point M', etc. In some cases the maps of those equidistant points on a line which correspond to contours of the ground surface will be designated by their respective elevation numbers, thus:



The points 2, 3, 4, 5, and 6 are the maps of points on the line d', having the elevations of 2, 3, 4, 5, and 6 ft., respectively. (2) The map of a field line will often be designated by one of the lower-case letters, *a*, *b*, . . . , *o*, and the field line itself by the same letter primed. For example, the line *b* here shown, ___*b*___, is the map of the field line *b*′. (*a*) However, maps of contours (horizontals) of a plane or other surface will most frequently be designated by the numbers which express their respective elevations; the contours themselves being designated by the same numbers primed. Thus 6 is the name given to the map of the contour of 6 ft. elevation. When it is desired particularly to distinguish contours of one surface from those of another, the letter of the surface and the number of the contour may be combined; thus, *Q* 7 may be used to designate the map of contour 7′ of surface *Q*. (*b*) Also a slope line (line of greatest declivity) of a surface, as *P*, will be designated by *p*′, and its map by *p*. (3) A plane or other surface will be designated by one of the letters *P*, *Q*, . . . , *Z*.

3. Determination of position of point. — The H position of a field point, as *A*′, is given by its map *A*. The V position is given by writing the elevation of *A*′ alongside point *A* on the map. Thus, in Fig. 1, *A* 25 is the map of field point *A*′ of which the elevation is 25 (feet being understood in these pages except where otherwise stated). The position of the field point is completely determined by its map and elevation.

4. Determination of position of line. — (*a*) The H position of a field line, as *b*′, may be given by its map, *b*. The V position of the line may be given in any one of several ways. We know that if field line *b*′ passes through two points *D*′ and *E*′, *b* (map of *b*′) passes through *D* and *E* (maps of *D*′ and *E*′, respectively). Thus *b* drawn through *D* and *E* in Fig. 1, is the map of *b*′, the straight line which passes through *D*′ and *E*′. (*b*) If we have map and elevation of each of two points of a straight field line, the field line is determined in V as well as H position. Thus, in Fig. 1, if b' contains D' and E', the position of b' is known from $D2$ and $E5$. (*c*) Other ways of deter-

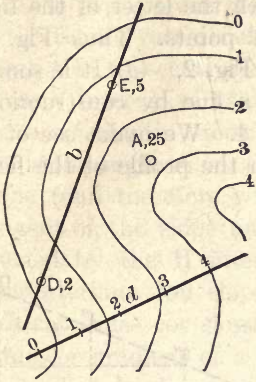


FIG. 1.

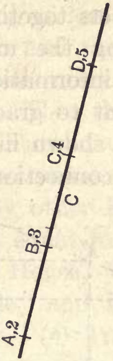


FIG. 2.

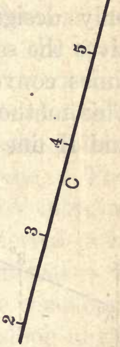


FIG. 3.

mining the position of a field line will be given in problems below.

5. To graduate a line. — (*a*) To graduate a line is to mark upon the map of it the maps of those equidistant points of the field line which correspond to contours of the ground surface. Fig. 1 shows a graduated line, *d*. *d* is the map of field line d', and points 0, 1, 2, . . . are maps of those points of d' which have elevations 0, 1, 2, . . . respectively. (*b*) In Fig. 2, line *c*, drawn through *A* and *D*, is the map of a field line c' which passes through field points A' and D'. The map shows that the elevation of A' is 2; of D' 5 ft. To graduate AD we mark points $B3$ and $C4$, the positions of which are found by proportion thus:

$$AB/AD = (3-2)/(5-2). \quad AC/AD = (4-2)/(5-2);$$

hence $AB = AD/3$, and $AC = 2\,AD/3$. We lay off AB from A toward D and mark $B3$, and lay off AC from A toward D and mark $C4$. The equidistant marks $A, B, C, \ldots$ graduate the line. (*c*) Ordinarily the letters of the graduation marks are omitted, the elevation numbers together with the letter of the line fully designating the marked points. Thus Fig. 3 gives the same information as Fig. 2. (*d*) It is sometimes convenient to graduate a line by construction. This method is shown in Fig. 4. We make use of V and H lines in connection with the profile of the field

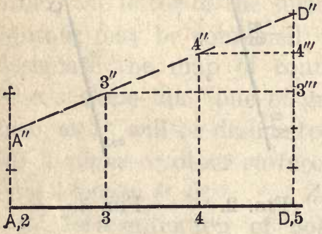


Fig. 4.

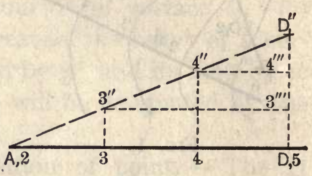


Fig. 5.

line. Erect at A and D, respectively, the $\perp$s $AA'' = 2$ and $DD'' = 5$, and mark points A'' and D''. $A''D''$ is the profile of field line $A'D'$. Also mark points $3'''$ and $4'''$, making $D3''' = 3$ and $D4''' = 4$. Through $3'''$ and $4'''$ draw $\|$s to AD cutting $A''D''$ at $3''$ and $4''$ respectively. From $3''$ and $4''$ drop $\perp$s to strike AD at points 3 and 4. The marks $2, 3, 4, \ldots$ graduate the line. (*e*) It is evident that any one scale may be used to lay off the V distances $AA'', DD'' \ldots$; however, it should be remembered that, unless V and H scales are the same, the profile will be distorted and and show distorted slope angle and slope distance. (*f*) Fig. 5 shows the same construction as Fig. 4, except that in the former AD is taken as the H of elevation 2 for the profile.

6. Scale of slope of field line. — (*a*) The graduated map of a field line is sometimes called the scale of slope of the line. For example in Fig. 3, line *c* with the marks 2, 3, 4 . . . , is the scale of slope of field line *c*′. The field distance represented by the space between two consecutive marks, as 2 and 3 of the scale, is the H interval of field line; and the difference between two consecutive elevations marked on the scale is the V interval of field line. Thus the V interval of the field line *c*′ is 1 foot. (*b*) The slope angle of a field line is the angle which that line makes with the datum plane (and therefore with any other H plane). The tangent of the slope angle is equal to the V interval divided by the H interval. Hence, V interval = H interval times tan slope angle; and H interval = V interval times cot slope angle. (*c*) To give the slope, grade, or gradient of a line is to give the slope angle of the line in direct or indirect terms. Some terms used in practice to indicate the steepness of a slope are given in the following quotations: (*i*) "The steepest gradient on the A. B. & C. Ry. is 52.8 feet per mile." (*ii*) "The steepest grade on the A. B. & C. Ry. is 1%." (*iii*) "By the laws of one State, the maximum permissible grade of highways at railroad crossings is 5°." (*iv*) "The slope of that stream is 1 : 5000." (1 : 5000 is read "one to five thousand," meaning 1 V to 5000 H.) (*v*) "The slope of the upper face of this dam is 3 : 1, and of the lower face, 5 : 1." (3 : 1 is read "three to one," meaning three H to one V.) (*vi*) "The side slopes of this rock cut are $\frac{1}{2}$: 1, but the side slopes of the fill are $1\frac{1}{4}$: 1." ($\frac{1}{2}$: 1 is read "one-half to one," and means one-half H to one V.) (Note. — The expressions in (*iv*), (*v*), and (*vi*) have this in common: the V component is taken as unity; but while with very flat slopes the V component is mentioned before the H, with very steep slopes it is mentioned after the H.) (*vii*) "The house drain has a drop of $\frac{1}{4}$ inch to the foot."

7. Problems on point and line. — Use H scale of 1 in. = 1 ft., in solving the following problems. (1) Given $A0$ and $B6$, on the map. (Mark two points some distance apart, at random, on a sheet of paper. By one point write $A0$; and by the other, $B6$.) (*a*) Compute distance $A'B'$. (*b*) Compute slope angle of $A'B'$. (*c*) Compute H interval corresponding to a V interval of 1 foot. Graduate the line. (*d*) Check preceding results by graphical process. (2) Line b' contains A' and has slope angle of 30°. Given $A2$ and b on map. (*a*) Compute H interval corresponding to a V interval of 1 foot. Graduate the line. (*b*) Check the foregoing results by construction. (3) Given two coincident lines B and D on map, passing through three points $E2$, $F8.5$, and $G4$; b' contains E' and F', d' contains G' and is $\perp$ to b'. (*a*) Graduate both lines by construction. (*b*) What is slope angle of each line? (*c*) What is relation between H interval of b' and H interval of d'? (4) Given line b, on map, passing through $A1.2$ and $C4.6$; d passing through $E3$ and crossing b; b' contains A' and C'; d' contains E' and crosses 3 ft. vertically above b'. (*a*) Construct scale of slope of d. (*b*) What is slope of b'? Of d'? (5) Given, on map, $A2$, $B5.2$, and $C3.5$. Graduate the line which passes through C' parallel to line $A'B'$. (6) What is slope per cent corresponding to each of following slopes (or gradients)? 0°01'; 1°00'; 5°00'; 26.4 ft. per mile; 1 : 300; 1 : 4000; 1.5 : 1. (7) Given the scale of slope of a line. Mark the map of that point of the line of which the elevation is 1.2 ft.; of which the elevation is 4.7 ft. (8) Given two graduated lines, b and d, on map. Given also line f crossing b

and d. Graduate f on the supposition that f' intersects both b' and d'.

8. Representation of a plane. — (a) Any three points not in the same (straight) line determine a plane; and any two || or intersecting lines determine a plane. The irregular surface of the ground is commonly represented on the map by contours, or, more strictly, by the maps of contours, of the surface. It is often convenient to represent a plane surface in the same way, that is, by the maps of two or more of its contours. (b) The contours of a plane are straight || lines, and the maps of such contours are straight and ||. Thus in Fig. 6 the portion $ABCD$ of the map exhibits contours straight and ||, and this indicates that

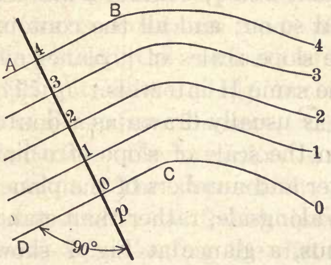


FIG. 6.

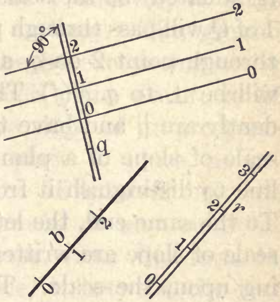


FIG. 7.

the corresponding portion of ground surface is plane. (The direction of a contour of a stratum of rock is called the "strike" of the stratum.) (c) Line of greatest slope of a plane. — Of all the lines of a plane the contours have the least slope (the slope angle is zero), and those which are ⊥ to the contours have the greatest slope; and the maps of the latter are ⊥ to the maps of the former. Thus, in Fig. 6, if p is drawn ⊥ to the contours and p' is a line of the plane represented by the contours, p' is a line of greatest slope of the plane. Further, the slope of p' is the slope of the plane. (The slope angle of a stratum of rock is called the "dip" of the stratum.) (*d*) Scale of slope of a plane. — In Fig. 6 the contours graduate the line p; and the points 0, 1, 2, . . . , of p's scale of slope are points at which contours 0, 1, 2, . . . , respectively, cross p. The scale of slope of p' may be considered the scale of slope of the plane. Thus to construct the scale of slope of a plane is to graduate any one of the lines of greatest slope of the plane. (*e*) A plane is determined by its scale of slope, because, given the scale of slope p, we can draw at once the map of any designated contour of the plane. Thus in Fig. 7 let q (graduated) be the scale of slope of plane Q. Contour 1 of Q will pass through point 1 on q; contour 2 will pass through point 2 on q, and so on; and all the contours will be $\perp$ to q. (*f*) The slope scales of $\|$ planes evidently are $\|$ and have the same H intervals. (*g*) The scale of slope of a plane is usually drawn as a double line to distinguish it from the scale of slope of a line. To the same end, the letter and numbers of the plane's scale of slope are written alongside, rather than standing upon, the scale. Thus, a glance at Fig. 7 shows that b represents a line while r represents a plane. (*h*) From 6 (*b*) it will be evident that the H interval between the contours of a plane is equal to V interval multiplied by cotan slope; and that the scale of slope or the contours of a plane being given on the map, we can scale the H interval and compute the slope of the plane by the formula, $\cot s = h/v$; where s is the slope, h is the H and v is the V interval.

9. Representation of irregular surfaces. — The student is supposed to be familiar with contour maps. He is therefore assumed to know that the surface of the ground is often represented by the maps of its contours. In many cases the irregular surface of the ground is represented on the map by the maps and elevations of a number of surface points. The student will recognize the fact that, after he has plotted stadia points and written by each its elevation, and before he has interpolated the contours, the map represents the ground surface; and that the only reason for adding the contours is that the system of contours is more convenient than the system of isolated points for many of the uses to which a topographic map is put.

10. Scale of slope of ground surface. — (1) It was pointed out in 8(h) that the slope of a plane that is represented by its contours can be found by the formula cot slope = h/v (v is the V interval; h is the H interval and is found by scaling). The ground surface is sometimes made plane by excavating or by filling; and the contours of such plane areas are of course straight and ||; but the slope of an irregular ground surface changes from place to place, as indicated by the variation in the H interval between contours. The slope of the ground at any point on the map can be found, however, from the formula above, after scaling the distance, h, between the two immediately adjacent contours taken on a line passing through the point. (2) When it is necessary to find the slope of the ground at many points, on a contour map, the method just stated is slow. It is a short job to make a "slope scale" by means of which the ground slope can be instantly ascertained at any point on a contour map. The construction of such a scale is described below.

11. Scale for reading slope from contour map. — As we have seen, the H interval at any point between two adjacent contours is equal to the V interval (or equidistance) multiplied by cotan of slope. That is, in symbols, $h = v$ cot slope. (1) By this formula, compute the H interval between contours for each of the following slopes, using the V interval of the map to which the scale is to be applied: 30°, 20°, 10°, and 5°. (2) Cut a strip of stiff paper, about one by six inches. (3) Graduate one edge of the strip thus: Draw a line ⊥ to the edge of the strip, near one end, and write "90°" on the line (see Fig. 7*a*). To the scale of the map on which the slope scale is to be used, lay off along the edge of strip, starting at the 90° line, the

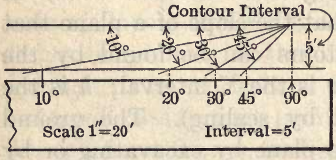


Fig. 7*a*.

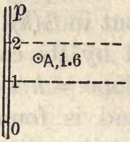


Fig. 8.

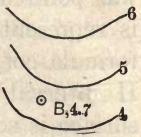


Fig. 8*a*.

computed H interval for 30° slope; draw a line at point thus located, ⊥ to the edge; and label the line "30°." In like manner locate and label the 20° graduation; the 10° graduation; and the 5° graduation; in each case taking the 90° line as the initial point of measure. (4) Write on the strip, "Slope Scale," the scale of the map on which the slope scale is to be used; and the name of the draftsman. Fig. 7*a* shows graphical construction of slope scale for use on map of which scale is 1 in. = 20 ft. and contour interval is 5 ft.

12. To assume point in surface. — Given *A* and the slope scale *p* (Fig. 8) of plane *P*. *A′* lies in *P*. To find the elevation of *A′*. (*a*) Drawing the contours of *P* in the vicinity of *A*, we find that *A* lies between contours 1 and 2. Scaling the distance from *A* to contour 1, and the distance from contour 1 to contour 2, we find that the former distance is 6/10 of the latter.

Therefore A' is higher than contour 1 by 6/10 of the V interval between contours 1 and 2; that is, the elevation of A' is $(0.6 \times 1) + 1 = 1.6$ ft. (*b*) The same method is followed if the contours represent a surface other than plane. For example, given *B* in Fig. 8*a*, along with the contours of an irregular ground surface. B' lies in the surface represented by the contours. What is the elevation of B'? By inspection (or scaling) we find elevation of B' to be 4.6 ft. (*c*) *Problem.* — Given a contour map with *A*5 marked at random on the map. Find the V distance between A' and the surface.

13. To assume line in surface. — Given the scale of slope *q* (Fig. 9), of plane *Q*, and any line *b*, — *b* being the map of some line b' of *Q*. To graduate *b*. (*a*)

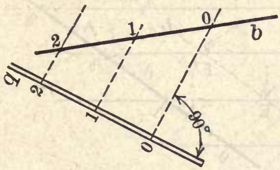


FIG. 9.

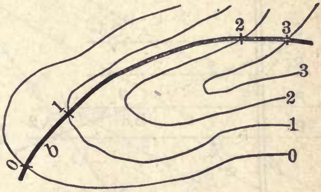


FIG. 9*a*.

Draw the contours 0, 1, 2, . . . , of plane *Q*. These contours graduate *b*. The point 1 of *b*'s scale is the point at which *b* cuts contour 1; point 2 of *b*'s scale is the point which *b* cuts from contour 2; and so on. (*b*) The same method is followed if the surface is not plane, or if the line is not straight. In any case the crossing of the map of the line and the map of a contour is the map of that point of the line of which the elevation is that of the contour. In Fig. 9*a*, *b* is a curved line drawn at random. If b' is a line of the surface represented by the contours, points 1, 2, 3, . . . , of *b* fall on contours 1, 2, 3, . . . , respectively.

14. To pass plane through three points. — Given on the map (Fig. 10), *A*2, *B*3, and *C*5. Plane *P* contains field points *A′* and *B′*, and *C′*. To construct scale of slope, *p*. (*a*) Graduate the line drawn through two of the points as *A* and *B*. Draw a line through *C* and that point of *AB*'s slope scale having the same elevation as *C*; that is, through *C* and point 5 on line *AB*. This line, just drawn, is the map of contour 5 of *P*, and may be designated as *P*5. Contour *P*4 is now drawn through point 4 of *AB* and || to *P*5; *P*3 is drawn through point 3 of *AB* || to *P*5; and so on. In a convenient place we draw the double line *p* ⊥ to the contours. It remains only to write the numbers on *p* at the scale marks made by the contours.

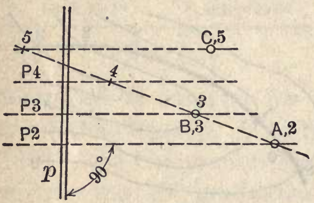


FIG. 10.

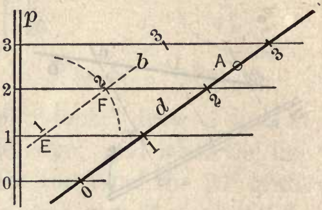


FIG. 11.

15. To locate line of given slope in plane. — Given the plane *P* by its slope scale *p*, Fig. 11. The line *b′* lies in *P* and has a slope of 1 (V) to 5 (H). To construct the slope scale *b*. (*a*) Draw contours of *P*. We know that points 1, 2, . . . , of scale *b* must fall on contours 1, 2, . . . , respectively. Also we know that the H interval of scale *b* is 5 times the V interval; that is, the H interval of *b* is $5 \times 1 = 5$ ft. Therefore from any point (as *E*) on some contour (as 1) and with a radius of 5 ft., draw an arc cutting contour 2 at some point *F*. Draw line *b* through *E* and *F*. *E* is point 1 and *F* is point 2 on the scale *b* required. (*b*) If a line d' has slope 1 : 5, lies in P and also passes through A', then d passes through A || to b; and the contours graduate d.

16. To locate surface line of given slope on contour map. — The same method serves for locating a line of given uniform slope in a surface which is not plane. Of course if the contours are curved the located line cannot be straight, but in most cases it may be considered, for all practical purposes, to be straight from contour to contour, changing direction only at points at which it crosses the contours. In proceeding along the surface line of uniform slope the H interval must of course be constant. In Fig. 11*a*,

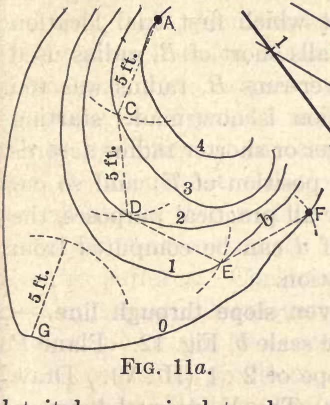


FIG. 11*a*.

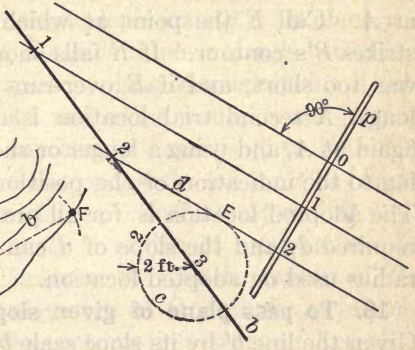


FIG. 12.

let it be required to draw the map of surface line b' which starts from point A' on contour 4 and has uniform slope of 1 : 5. Open the dividers to 5 ft. (that is, 5 times the V interval between adjacent contours). From A as center, strike arc cutting contour 3 at C. From C as center, strike arc cutting contour 2 at D. From D as center strike arc cutting contour 1 at E; and so on. The broken line $ACDE$ . . . is the map b of b'. Points A, C, D, E, . . . , are respectively

4, 3, 2, 1, . . . , of b's slope scale. (*Remark.* — The segment $A'C'$ passes a little below the surface. How do we know this? Is there more than one solution to the problem above? What if we had tried to start the line uphill from G?)

17. To draw surface line of uniform slope between two surface points. — (Figure is to be drawn by student.) Given the contours of the surface, also $A7$ on contour 7 and $B2$ on contour 2. d' is a surface line which has uniform slope and runs from A' to B'. Construct the map d of d' and find the uniform slope. (d and the slope must be found by trial.) For the first trial location use slope a little flatter than slope of air line from A' to B'; and start the work of striking arcs at A. Call E the point at which first trial location strikes B's contour. If E falls short of B, radius used was too short; and if E over-runs B, radius was too long. A second trial location is now made, starting again at A, and using a longer or shorter radius according to the indication of the position of E; and so on. The adopted location is, for all practical purposes, the required d; and the slope of d can be computed from radius used on adopted location.

18. To pass plane of given slope through line. — Given the line b' by its slope scale b, Fig. 12. Plane P contains line b' and has slope of 2 : 1 (H : V). Draw one-foot contours of P. (a) The V interval between contours is 1 foot; therefore the H interval between contours is $2 \times 1 = 2$ ft. From any scale-mark of b, as 3, as center, with radius = 2 ft., draw circumference c. Through adjacent scale-mark, 2, draw line d tangent to c. Call point of tangency E. d is the map of contour 2 of plane P. Remaining contours 0, 1, 3, . . . , of P are drawn || to d and through points 0, 1, 3, . . . , respectively, of slope scale b. (*Remark.*

— The construction will be more accurate if we take a longer radius, as 4 ft., or 6 ft., etc. Solve the problem using a radius of 6 ft. Is there more than one solution to the foregoing problem?)

19. Problems on line and surface. — The student will represent on paper the data of each of the following problems; and assume that the scale of the map is, in each case, 1 in. to 10 ft. unless otherwise instructed. (*a*) To draw plane through point || to another plane. — Given the slope scale *p* and *A*3. Plane *Q* is || to plane *P* and contains point *A*′. Construct the slope scale *q*. (|| planes have || slope scales and the same interval.) (*b*) To draw plane through line || to another line. — Given the slope scales *b* and *d*. Plane *Q* contains *b*′ and is || to *d*′. Construct slope scale *q*. (A plane is || to a given line if it contains a line which is || to the given line.) (*c*) To pass a plane through point || to two lines. — Given *A*2 and slope scales *b* and *d* on the map. Plane *P* contains *A*′ and is || to both *b*′ and *d*′. Construct the slope scale *p*. (*d*) To find intersection of two surfaces. — Surfaces *P* and *Q* are given by their contours. *b*′ is line of intersection of the two surfaces. Construct *b*. (Each contour of *P* cuts contour of like elevation of *Q*, and the point of intersection of the two contours is a point of line *b*′.) (*e*) To construct profile of surface line shown on contoured map. — Given the surface *R* by its contours. Draw line *b* across the map. *b*′ is a line of the given surface. Construct the profile of *b*′. (The map *b* is graduated by the contours.) (*f*) To find piercing point of line with surface. — Given the surface *R* by its contours. Given the slope scale *b* of line *b*′. *b*′ pierces *R* at point *A*′. Find *A* and elevation of *A*′. (Any plane, as *Q*, which contains *b*′ will cut from *R* some line *x*′ which will cut *b*′ at *A*′. It is convenient sometimes to let *b* be the slope scale of the auxiliary plane *Q*. At other times it may be convenient to make *Q* vertical, so that it will cut a profile from plane *P*. Less frequently it is better to draw *Q*'s contours oblique to *b*.) (*g*) To draw a line through point ⊥ to plane. — Given *A*4 and *p*. *A′* is not necessarily a point of *P*. Line *b′* contains *A′* and is ⊥ to *P*. Construct slope scale *b*. (*b* will be || to *p*, and the H interval of *b* will be the reciprocal of the H interval of *p*, and the numbers on the two scales will increase in opposite directions.) (*h*) To pass plane through line ⊥ to plane. — Given the slope scales *p* and *b*. Plane *Q* contains *b′* and is ⊥ to *P*. Construct the slope scale *q*. (Pass a line *x′* through some point of *b′* ⊥ to *P*. *Q* is the plane of *b′* and *x′*.) (*i*) To pass line through point ⊥ to line. — Given *A*6 and the slope scale *b*, *A* not being on *b*. Line *c′* contains *A′* and is ⊥ to *b′*. Construct slope scale *c* and find ⊥ distance from *A′* to *b*. (Pass an auxiliary plane *Q* through *A′* ⊥ to *b′*. *b′* pierces *Q* at *E′*. *c′* contains *A′* and *E′*; and the required distance is *A′E′*.) (*j*) To find line which cuts at right angles two non-intersecting lines. — Given the slope scales *b* and *c*. Line *d′* cuts both *b′* and *c′* and is ⊥ to both. Required slope scale *d* and ⊥ distance between *b′* and *c′*. (Pass a plane *P* through *b′* || to *c′*. Through two points of *c′* draw lines *f′* and *g′* ⊥ to *P*. *f′* pierces *P* at *F′*; and *g′* pierces *P* at *G′*. Line *F′G′* cuts *b′* at *J′*. Line *d′* passes through *J′* and is ⊥ to *P*.)

20. Problems in location, earthwork, geology and mining. — Some of the following problems have been suggested by Joseph W. Roe's "Application of Descriptive Geometry to Mining Problems," a paper presented before a meeting of the American Institute of Mining Engineers in March, 1910; published in the Transactions of the Institute; and reprinted as Chapter IX in "Practical Field Geology."* This paper gives clear, detailed solutions of a variety of mining problems.

(1) On a contour map mark three points, A, B, C, at random. Let them be the maps of points at which contacts have been made with a certain stratum of rock, and which are described as follows: A' is 43 ft. below surface in drilled hole; B' is at breast of tunnel; and C' is on an outcrop. Find the strike (see 8(b)) and dip (see 8(c)) of stratum.

(2) Find depth of V shaft necessary to reach the stratum of rock of (1), from a given surface point D', the map of which is given.

(3) Given, on a contour map, three points of outcrop of a rock stratum, and the position of a V shaft of given depth. Find position and length of shortest H tunnel which can be driven from bottom of shaft to the stratum.

(4) Given, on a contour map, the maps and elevations of three points of the hanging wall of a vein, and the map and elevation of one point of the foot wall. Find the thickness of the vein.

(5) On contour map of narrow valley, draw center line of top of a proposed earth dam, of which top width is 20 ft., upstream slope is 2 : 1, and downstream slope is 3 : 1. Let top of dam run from a contour on one side of the valley to the contour of same elevation on the other side. Draw the two edges of the dam crest, and find the lower edges of the two faces of the dam.

(6) A railroad cut is to be made through a hill, and following the cut is a fill running out to a pier abut-

* By J. H. Farrel and Alfred J. Moses, New York: McGraw-Hill Book Co., Inc. $3.

ment (wingless) supporting one end of a bridge. Given all necessary data on a contour map of the site, find top of side slopes of cut, and toe of slope of fill on both sides of fill and at the end where it flows around abutment.

(7) Given a contour map of an irregular side-hill. Draw a polygon of five or more sides on the map, to represent the boundary line of a property. On the higher side of the property mark a point to represent a tunnel entrance. Find the limits of the area within which a level dumping-trestle can be located without causing toe of dump to overrun any of the property lines. The stable slope of the dump may be taken as 1.25 : 1.

(8) Given the data of (1). Find the line of outcrop of the stratum of rock.

(9) Given a contour map of a piece of ground having a slope of five per cent or more; scale 1″ = 20′; contour interval one foot. Draw on the map, to scale, the outline of a tennis court area, 60 ft. by 120 ft., giving the long dimension the same general direction as the contours near it. Find the elevation which the court must have in order that the cut may just make the fill. Assume the side slopes of cut and of fill to be 1.5 : 1, and that there will be neither swell nor shrinkage of the excavated material.

(10) Given a contour map of a side-hill, and other necessary data. To find the location of road between two given points on side-hill, such that the resulting excavation shall just make fills.

(11) Given the necessary data, find the elevation of bottom of a proposed reservoir on a side-hill, assuming that cut and fill shall balance.

(12) Find volume of dam of (5). Find the capacity of resulting reservoir.

(13) Find on a contour map the length and H position of tunnel which is to be driven with given upward slope — say 7° — from bottom of a shaft to a rock stratum given by three of its points. The tunnel is to be of minimum length possible considering the fact that it must lie entirely within given property lines.

PART VI

HIGHER SURVEYING EXERCISES
PROBABLE ERROR

MODULUS OF ELASTICITY OF STEEL TAPE

Equipment: 300-foot (or 100-foot) steel tape, provided at one end with a fine scale (preferably with a vernier) so that minute changes of length may be detected; a spring balance or other apparatus for measuring tension, which should be determined with great accuracy. If a screw and nut is so arranged at each end that the tape may be held at any required tension without any unsteadiness, the accuracy will be greatly increased.

FORM OF NOTES

Tension.	Vernier scale.	Mean.	P_2 and P_1	$v_2 - v_1$	C_s
12 lbs........	0.0292 0.0291 0.0291	0.02913	16, 12	0.02087	0.00128
14 lbs........	0.0395 0.0393 0.0394	0.03940	18, 14 20, 16	0.02057 0.02007	0.00108 0.00075
16 lbs........	0.0500 0.0500 0.0500	0.05000	18, 12	0.03084	0.00206
18 lbs........	0.0599 0.0600 0.0600	0.05997	20, 14 20, 12	0.03067 0.04094	0.00139 0.00237
20 lbs........	0.0700 0.0701 0.0701	0.07007			

Location: These tests can best be made between fixed hubs or monuments at some place which is permanently arranged for them. The space under the seats of a grand stand will often prove a favorable place for such work.

Method: If it is readily possible to support the tape throughout its length, the effect of sag may be eliminated; otherwise the tape must be supported by freely swinging hooks set at regular intervals. Set the tape in place so that it swings freely in the hooks, with the zero at one end exactly on the index-mark on the monument at that end. Stretch the tape by means of the screws until the pull is 12 lbs. and take the exact reading on the fine scale at the other end of the tape. Slacken the tape slightly and again stretch it to 12 lbs. and again take the reading. Take *three* such readings with tensions of 12, 14, 16, 18, and 20 lbs.

Computations: Since the tape was swung on hooks

a	*E*	
0.01959	30 165 000	
0.01949	30 330 000	
0.01932	30 597 000	
0.02878	30 810 000	
0.02928	30 283 500	
0.03857	30 652 500	

spaced 30 feet apart, there is a *difference* in the sag for the various tensions, which equals

$$C_s = \frac{L}{24}(wd^2)\left(\frac{1}{P_1^2} - \frac{1}{P_2^2}\right),$$

in which L = length of tape in feet (300), w = weight* of one linear unit of the tape in pounds (0.000574), d = the uniform distance between supports in the *same* unit as w, and P_1 and P_2 are the various tensions. If l_1 and l_2 are the true lengths, v_1 and v_2 the mean

* It is difficult to obtain *with accuracy* the weight of a linear unit and also the cross-section. The weight of a linear unit is sometimes obtained by weighing the whole tape and dividing by the length, and then the cross-section is obtained by dividing the weight of a linear unit by the assumed weight of a cubic unit of steel of that quality. These results are vitiated by the fact that handles, rivets, brass sleeves, etc., which are not readily removable add to the weight of the tape and also because the weight of the cubic unit is rather uncertain. To obtain the cross-section accurately by direct measurement is impracticable because the cross-section is not truly rectangular, the sides of the cross-section being more nearly circular, and again because the thickness (generally about 0.015″) cannot be readily measured, even with micrometer calipers, with a sufficient *percentage* of accuracy. When it is possible to obtain from the manufacturer a sample of the same kind of tape, the following method is more accurate: A piece of the tape 2.52 inches long was weighed on a chemical balance as 656.2 m.g. (= 0.0014467 lb.). It was also weighed in distilled water having a temperature of 16°.4 C., the weight being 572.4 m.g. From these figures and the known weight of distilled water the weight of the steel per cubic inch was determined to be 0.2825 lb. Dividing the weight of the piece by the weight per cubic inch gives the volume, which divided by the length gives the area of the cross-section (0.002032 sq. in.). Dividing the weight of the piece by its length gives the weight per linear inch (0.000574 lb.). Multiplying the area of cross-section by 40 000 shows that a pull of over 80 lbs. will be required to strain the tape 40 000 lbs. per square inch, which is probably within the elastic limit.

vernier readings, and s_1 and s_2 the sags for two tensions, $l_1 = v_1 + s_1$ and $l_2 = v_2 + s_2$. The stretch a for the difference in tension $= l_2 - l_1 = v_2 + s_2 - v_1 - s_1 = (v_2 - v_1) - (s_1 - s_2) = (v_2 - v_1) - C_s$.

The Modulus of Elasticity $E = \dfrac{(P_2 - P_1)\,L}{Sa}$, in which S is the cross-section of the tape in square inches (0.00203).

As a sample computation,

$$C_s = \frac{300}{24}(0.000574 \times 360)^2\left(\frac{1}{12^2} - \frac{1}{16^2}\right) = 0.00128.$$

Then

$$(v_2 - v_1) - C_s = 0.02087 - 0.00128 = 0.01959$$
$$= a \text{ (in feet)}.$$

$$E = \frac{(16 - 12)\,3600}{0.00203 \times (0.01959 \times 12)} = 30\,165\,000.$$

LEVEL-TRIER

Equipment: Level-trier; level-tube to be tested, which may be an unattached tube or a tube in an instrument which may be placed bodily on the level-trier.

Location: For accurate work the level-trier must be placed where it is perfectly steady and not subject to vibrations; as, for instance, on a shelf fastened to a masonry wall, or on a solid masonry pier.

Constants of the trier. It is necessary to know the pitch of the screw and the length of the arm. The determination of these, with such accuracy as is necessary, requires that the screw be taken completely out, involving excessive wear if done frequently and by inexperienced hands. These values should therefore be determined definitely for the particular instrument used, by the instructor in charge, and the values given to the student.

Value of one division of the level-tube. Place the level-tube (or instrument) on the trier. If the large telescope level-tube of a transit is to be tested, place the whole instrument (including the leveling-head) on the flat plate of the trier provided for that purpose. Level up the transit by means of its leveling-screws, being very particular to orient the instrument so that the tube to be tested is exactly parallel to the axis of the trier. Use the transit leveling-screws to set the bubble at any desired position in the tube and thus save wear on the fine micrometer-screw. Bring the bubble near one end of the tube, yet so that the micrometer-index points to some even tenth or twentieth division or preferably at zero. Note the position on the scale of *both* ends of the bubble — estimating tenths of a division. A microscope will be of assistance in estimating tenths. Turn the micrometer to the next even fifth (tenth or twentieth) division. Again read and record the position of the ends of the bubble. Continue these operations until the bubble is near the other end of the tube. The bubble always drags somewhat behind its true position; therefore, when moving the bubble through the tube, the screw should always be turned in the *same* direction and never reversed. When the bubble has been moved as far in one direction as desired, turn the screw a few divisions farther in that direction and then turn backward to the same micrometer reading it had before. On account of lagging, the bubble readings will be somewhat different. Move the screw backward to each reading in turn as before. The bubble readings will differ from those before obtained for the same micrometer readings, and the differences will be practically constant and will equal *twice* the effect of the lagging. The *position of the center of the bubble* is found by taking one-half of the difference of the two end readings, which gives the distance of the center of the bubble from the center of the tube. The difference in the positions of the center of the bubble for corresponding settings of the micrometer-screw gives twice the effect of the lagging. This quantity should be determined in order to obtain a measure of the uncertainty due to this cause in any random reading. The motion of the center of the bubble for two consecutive positions of the bubble is a measure of the length of bubble-tube corresponding to a definite angular motion. Since the micrometer intervals are equal, the average difference is the mean value corresponding to that micrometer interval. The sum of each pair of end bubble readings is a measure of the length of the bubble. The differences in the bubble lengths may be caused by (*a*) variation in temperature, (*b*) variation in calibre of tube, (*c*) errors of reading. If the temperature is fairly constant and the tube presumably of good workmanship, any large variation will probably be due to the third cause. When the bubble is sluggish or excessively sensitive, considerable time (three to five minutes) *may* be required before the bubble becomes stationary.

If α'' = angular value in seconds of arc of one division of the index-head of the screw,
n = number of micrometer divisions turned before each new position of bubble,
d = mean movement in divisions of the bubble,

then angular value of one division of bubble $= \dfrac{n\alpha''}{d} = D.$

Radius of the tube. Let m = number of divisions per inch on the bubble-scale; then $\dfrac{mn\alpha''}{d}$ $(= mD)$ is the angular value of one inch of the tube. Since there are 206 265 seconds in the arc which equals its radius, *the radius of the tube in inches* $= \dfrac{206\,265\,d}{mn\alpha''}\left(=\dfrac{206\,265}{mD}\right)$.

FORM OF NOTES

No.	Disk reading.	Bubble readings. Left.	Bubble readings. Right.	Position of center of bubble.	Motion for consecutive positions.	Change of position — same screw reading.
1	40	6.1	23.8	− 8.85		
2	60	7.9	22.0	− 7.05	1.8	
3	80	10.0	20.0	− 5.00	2.05	
4	100	11.5	18.4	− 3.45	1.55	
5	120	13.3	16.8	− 1.75	1.70	
6	140	15.0	15.1	− 0.05	1.70	
7	160	16.8	13.6	+ 1.6	1.65	
8	180	18.6	11.8	+ 3.4	1.80	
9	200	20.0	10.2	+ 4.9	1.5	
10	220	21.8	8.6	+ 6.6	1.7	
11	240	24.1	6.3	+ 8.9	2.3	
11	240	24.6	5.9	+ 9.35		+ 0.45
10	220	22.4	8.0	+ 7.2	2.15	0.60
9	200	20.5	10.0	+ 5.25	1.95	0.35
8	180	18.7	11.7	+ 3.5	1.75	0.10
7	160	17.2	13.2	+ 2.0	1.5	0.40
6	140	15.5	14.9	+ 0.3	1.7	0.35
5	120	13.9	16.6	− 1.35	1.65	0.40
4	100	12.1	18.3	− 3.1	1.85	0.35
3	80	10.2	20.1	− 4.95	1.75	0.05
2	60	8.6	21.9	− 6.65	1.7	0.40
1	40	6.9	23.7	− 8.4	1.75	+ 0.45
					1.775	0.354

The plate-bubbles on a transit are too coarse to require such refined methods of work. It is advisable, however, to obtain some idea of their sensitiveness. This may be done by observing how much motion of the micrometer-screw is required to move the bubble some definite amount — say $\frac{1}{20}''$. The screw motion, reduced to angular measure, gives the angular value of such a motion of the bubble, and therefore of such an error in the adjustment of the bubble.

When testing a transit, test both plate-bubbles as just given, and also test the large telescope-bubble by the accurate method.

Length of bubble.	
	Length of arm — 17.92 inches
29.9	Pitch of screw — 60 per inch
29.9	One div. of disk raises arm $\frac{1}{100} \times \frac{1}{60} = \frac{1}{6000}$ inch
30.0	$\alpha : 206\,265 :: \frac{1}{6000} : 17.92$
29.9	$\therefore \alpha = 1''.918$
30.1	$n = 20$; $d = 1.775$ div.
30.1	
30.4	$D = \frac{n\alpha}{d} = \frac{20 \times 1.918}{1.775} = 21.6$
30.4	
30.2	$m = 15$ div. per inch
30.4	Rad. of tube in inches $= \frac{206\,265}{mD}$
30.4	
30.5	$= \frac{206\,265}{15 \times 21.6} = 637$
30.4	637 inches = 53.1 feet
30.5	
30.4	Mean lagging of bubble $= \frac{0.354}{2} = 0.177$ div.
30.4	
30.4	0.177 div. $= 3''.8$.
30.5	
30.4	
30.3	
30.5	
30.6	

The uniform interval for the screw readings should be varied according to the sensitiveness of the bubble; a very sensitive bubble may move about one bubble division for one or two divisions of the screw, while a coarser bubble may require a movement of 5, 10, or 20 divisions of the screw to move the bubble one division.

INEQUALITY OF TELESCOPE COLLARS OR OF TELESCOPE AXIS PIVOTS

Equipment: Telescope of a wye-level or the telescope of a theodolite having open wyes, and a suitable striding-level. When testing the collars of a theodolite telescope, dummy wyes may be provided, which will leave the horizontal plates free and available for other tests — such as those for eccentricity, errors of runs, etc.

Form of Notes

Telescope.	Striding-level	Bubble.		Center of bubble.	Motion of bubble for alt. readings.	d	d^2
Direct	Direct	22.7	22.3	+ 0.2	...4.90		
	Reversed	24.5	20.3	+ 2.1	...5.25	0.395	0.156025
Inverted	Direct	27.3	17.1	+ 5.1	...6.05	0.045	0.002025
	Reversed	29.5	14.8	+ 7.35	...5.45	0.755	0.570025
Direct	Direct	21.0	22.9	− 0.95	...5.25	0.155	0.024025
	Reversed	23.8	20.0	+ 1.9	...4.85	0.045	0.002025
Inverted	Direct	26.0	17.4	+ 4.3	...5.75	0.445	0.198025
	Reversed	28.4	14.9	+ 6.75	...5.05	0.455	0.207025
Direct	Direct	20.1	23.0	− 1.45	...5.30	0.245	0.060025
	Reversed	23.2	19.8	+ 1.7	...5.10	0.005	0.000025
Inverted	Direct	25.8	18.1	+ 3.85		0.145	0.038025
	Reversed	28.8	15.2	+ 6.8			
					52.95		
					5.295		1.257250

Location: [See "Location" under "Level-trier.]"

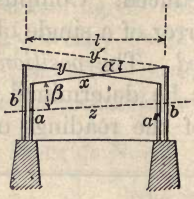
Method: Set the telescope in its wyes, place the striding-level on the pivots (or collars), and level up until the bubble is nearly in the center. Record the readings of both ends of the bubble. Reverse the striding-level and record the bubble readings. Reverse the telescope in the wyes and apply the striding-level both in direct and reversed position. Again reverse the telescope in the wyes and proceed as before until *three* sets of observations have been taken with the telescope in both positions in each set. From the figures obtained *ten* values of the inequality of the collars may be deduced. Compute the mean value and the probable error of a single observation.

Principles involved. The *position of the center* of the bubble can always be determined by taking one-half of the difference of the readings of the bubble ends,

½ motion of bubble for reversal.	d	d^2	
0.950	0.346	0.119716	$E_1 = 0.6745\sqrt{\frac{1.25725}{9}} = .252$
1.125	0.171	0.029241	$E_0 = \frac{.252}{\sqrt{10}} = .080$
1.425	0.129	0.016641	Error due to collars $= \frac{5.295}{4} \pm \frac{.080}{4}$
1.225	0.071	0.005041	$= 1.324 \pm 0.020$
1.575	0.279	0.077841	$E_1' = 0.6745\sqrt{\frac{.280521}{5}} = .160$
1.475	0.179	0.032041	$E_0' = \frac{0.160}{\sqrt{6}} = .065$
7.775		0.280521	Error of level adj. $= 1.296 \pm .065$
1.296			

The *right-hand* pivot (telescope direct) is *too large*. The *right leg* of the striding-level (when direct) is too high.

and this is true regardless of whether the graduations actually begin in the center or at some distance each side of the center. The *movement* of a bubble is then found by determining the motion of the *center*, and its angular value by multiplying the movement in divisions by the angular value of one division. If a represents the diameter of one collar and b that of the other (the inequality is grossly exaggerated in the figure), then $b - a$ is the difference of diameter. If we consider the telescope changed in the wyes but the level *unchanged* (relative to the wyes), the level will be



inclined by the angle α, which is the angle subtended by $2(b - a)$ at a distance of l. This is *four* times the angle between the axis of the collars and the line joining the bases of the striding-level, which is the required angle; *i.e.*, β, the required angle, is equal to $\frac{1}{4}\alpha$. Therefore the required angle is one-fourth of the angular value corresponding to the motion of the center of the bubble due to a change of the telescope in the wyes but no change of the striding-level relative to the wyes. This is found by comparing *alternate* readings of the striding-level when used as described above ("Method").

Error of adjustment of the bubble-tube. As this is determinable from the observations just described, without additional instrumental work, the method of computation is here given. One-half of the algebraic sum of the distances of the center of the bubble from the center of the tube, for readings of the bubble when the level is reversed but the telescope is unchanged in the wyes (readings toward one end of the tube being considered positive and toward the other end negative), gives the value in bubble divisions of the error of adjustment of the bubble-tube. The above-described observations will give six values of this error. Compute the probable error of one determination.

CONSTANTS OF A LEVEL OF PRECISION

Equipment: Level of precision, leveling-rod (or paper scale — see next paragraph), and steel tape.

Location: Ordinarily such tests are made in the field, the only requirement being a fairly smooth stretch of 300 to 400 feet; but in unfavorable weather a very good test may be made indoors by sighting at an illuminated scale graduated to 50ths of an inch (or to millimeters) from a distance of about 35 feet. $\frac{1}{50}''$ at 35′ subtends about 10 seconds of arc, and the 50ths may be easily subdivided by estimation. The following NOTES are taken from an actual in-door test.

FIXED CONSTANTS

1. *Angular value of one division of bubble-tube.* Sight at rod (or scale) when the bubble is as near one end of the tube as possible — so that the bubble-reading at each end may be taken. By means of the micrometer-screw under one wye, run the bubble to the other end and again read the scale (reading the middle cross-wire only) and both ends of the bubble. Take SIX such readings: five differences may be obtained from them. Note for each observation the position of the *center* of the bubble; then for each pair the motion of the bubble-center and the difference of scale-readings; then for each pair the difference of scale per bubble-division. Call r the mean difference of scale-readings per bubble-division; d = distance from instrument to scale; and v the value of one bubble-division in seconds of arc; then

$$v = \frac{r}{d \sin 1''}.$$

2. *Inequality of the pivot-rings.* [The theory and method have been developed under "Inequality of Telescope Collars" and are not repeated here, especially as

FORM OF NOTES.

Test.	Scale.	Bubble.		Center of bubble.	Motion of bubble-cen.	Diff. of scale-readings.	Diff. of scale per bubble-division.
		E	*O*				
1	640.5	+ 1.5	−34.5	−16.5	34.7	15.3	0.441
	655.8	+36.3	0.0	+18.2	38.2	16.8	0.440
	639.0	− 2.0	−38.0	−20.0	43.5	17.1	0.393
	656.1	+37.0	+10.0	+23.5	41.8	16.1	0.385
	640.0	0.0	−36.5	−18.3	38.6	16.5	0.428
	656.5	+38.5	+ 2.0	+20.3			
2							

Test.	Wire read.	Wire interv'l.
3	331.0, 525.0	194.0
	342.5, 536.0	193.5
	335.9, 530.0	194.1
	346.8, 540.5	193.7
	380.3, 574.2	193.9
	394.0, 587.5	193.5
	421.8, 615.5	193.7
	453.0, 646.8	193.8
	471.5, 665.5	194.0
	491.5, 685.0	193.5

Test.	Wire read., tel. dir.	Wire read., tel. rev.	Mean wire, tel. dir.	Mean wire, tel. inv.	½ Diff. for inversion.
4	753.8, 656.0, 558.7	752.0, 654.5, 556.5	656.17	654.33	0.92
	771.2, 673.8, 576.2	769.0, 671.1, 574.5	673.73	671.53	1.10
	786.8, 689.5, 591.8	785.2, 688.0, 589.0	689.37	687.40	0.985

Test.	Level dir.		Level rev.		Cen. of bub., level dir.	Cen. of bub., level rev.	½ Motion for reversion.
	E	*O*	*E*	*O*			
5	18.0	18.0	18.5	17.5	0.0	+0.5	+0.25
	22.0	14.0	22.5	13.5	+4.0	+4.5	+0.25
	10.5	25.0	10.5	25.0	−7.25	−7.25	0.0

$d = 33.86$ feet $= 406.32$ inches $r = \dfrac{0.4174}{50}$ inch $= 0.008348$ inch	$v = \dfrac{r}{d \sin 1''}$ $= \dfrac{0.008348}{406.32 \times 0.00000485} = 4''.24$

[Exercise, Inequality of Collars. Test not repeated here.]

$d = 33.50$ feet $= 402.0$ inches
$f = 14.4$ inches
$c = 7.2$ inches
$f + c = 21.6$ inches

$$s = \frac{193.77}{50} = 3.8754$$

$$r = \frac{d - f - c}{s}$$

$$= \frac{380.4}{3.8754} = 98.1$$

Mean error in collimation $= \dfrac{1.00}{50} = 0.0200$ inch at distance of 406 inches
$= 0.0049$ feet per 100 feet,
or 4.9 mm. per 100 meters.

∴ with telescope direct, the line of collimation points *downward* (the scale was inverted) by 4.9 mm. per 100 m.

Mean inclination of bubble = 0.17 div. = 0.72 seconds of arc.

∴ when level is direct and in center, telescope points *downward* an angle of 0.72″ or 0.00035 foot per 100 feet.

a careful determination of the value is sufficient for one problem.]

3. *Angular value of the wire interval.* Read the extreme cross-wires on the rod (or scale). Move the telescope by means of the micrometer-screw just enough to give different (and independent) scale-readings. Take ten such readings. If d = distance from center of instrument to rod; s = mean difference of wire-readings; $(f + c)$ = the distance from the center of the instrument to a point as far in front of the object-glass as the cross-wires are behind the object-glass; then r, the required quantity, is the fixed ratio of $(d - f - c)$ to s. When this test is made indoors the distances should be measured with corresponding accuracy.

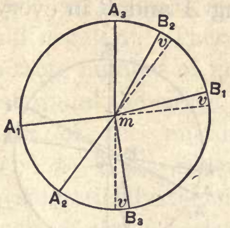
Daily Tests

4. *Error of the line of collimation.* Read the three wires on the rod or scale. Rotate the telescope 180° in the wyes and again read the three wires. Take three pairs of readings, moving the telescope slightly with the micrometer-screw before each new pair. Raise the clips (if any) from the collars, and remove the striding-level so that the telescope may be rotated with as little jar as possible. The difference in the mean wire-readings is *twice* the collimation error *for that distance.* Reduce the collimation error to its value in decimals of a foot (or meter) per 100 feet (or 100 meters).

5. *Error of striding-level.* Observe the bubble-readings at both ends when the striding-level is direct and when it is reversed. Take three readings with level direct and three with level reversed. One-half the motion of the bubble-center during reversal indicates the bubble inclination.

ECCENTRICITY AND ERRORS OF GRADUATION OF A GRADUATED CIRCLE

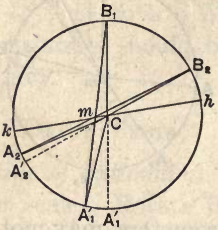
The readings of two opposite verniers (or micrometers) will usually differ slightly, owing to one or more of three causes. These differences are frequently less than the least unit of measurement in a *well-made* engineer's transit, but micrometer microscopes permit the reading of such a small unit of measurement that the absolutely unavoidable errors of the finest mechanical construction become apparent. The three causes



are: (*a*) the micrometers may not be 180° apart — measured on a circle whose center is the center of the vertical axis carrying the telescope and the micrometers; (*b*) the center of this axis may not coincide with the center of the graduated plate; (*c*) there is more or less error in the graduations. There is still a fourth cause, *viz.*: inaccuracies of reading due to inexperience or "personal equation," but in the investigation of the errors of graduation of a circle such errors are necessarily bound up with the errors of graduation. Call A the reading of micrometer A in any position and B the corresponding reading of micrometer B ± 180°; then $B - A$ = the difference, which is usually a variable for different parts of the circle. Then $B - A$ for any position of the micrometers includes the combined effect of these three causes.

Cause **a.** Let A_1B_1, A_2B_2, etc., represent the various positions of the micrometer, m representing the vertical axis carrying the telescope and micrometers. As may be readily seen from the figure, v is a constant for all parts of the circle. Then $(B - A)$ includes a *constant* v, which may be large or small and possibly absolute zero. Micrometers are usually adjustable, so that v may be reduced to a very small quantity.

Cause **b.** Since v is a constant in $(B - A)$, we may assume for simplicity that it has been eliminated, or that m, the center of the axis supporting the telescope, is on a line joining A and B in every position. Let C



be the center of the graduated circle. Then ACA' measures that part of $(B - A)$ due to cause b. The effect of cause b is plainly seen to be variable for various positions of the micrometers, and when B reaches h (or k) the effect is zero, *i.e.*, mh is the "line of no eccentricity." Let h be the reading at h, and t the reading for *any* position of B; then $(t - h) = BCh$; $mC = e$, the *linear* eccentricity; $BC = R$; $mC \div BC = e \div R$, which may be called $e'' \sin 1''$. From the triangle mCB (in *any* position of B) we have

$$\sin mBC = \frac{mC}{Bm} \sin BCh.$$

Let $x = ACA' = 2\,mBC$. Since mC is exceedingly small, Bm is practically equal to BC; $\therefore\ mC \div Bm$

$= e \div R = e'' \sin 1''$. $\therefore \sin \frac{1}{2} x = \frac{1}{2} x \sin 1'' = e'' \sin 1'' \sin (t - h)$. $x = 2 e'' \sin (t - h)$. But note that if the telescope is swung around 180° from any given position the algebraic sign of x is changed, although x is numerically the same as before. Therefore $2 e'' \sin (t - h)$ is numerically the same but of opposite sign for all *pairs* of positions 180° apart.

Cause c. Errors of graduation and reading are variable and wholly irregular. Call them g. Evidently the error of any graduation is its variation from a position in an ideal system whose differences from the actual marks are as small as possible and are equally positive and negative. Therefore the average of all the g's for a large number of graduations, taken at regular intervals around the circle, is zero.

For any position of the micrometers $(B - A) = v + 2 e'' \sin (t - h) + g$. Suppose that readings for $(B - A)$ are systematically taken at points of the circle 10° or 20° apart, a set of variable values will be found. Since g is assumed to average zero, and since for every value of $2 e'' \sin (t - h)$ there is an equal value of opposite sign, the *mean* of all the values of $(B - A)$ is evidently v. Then if v is subtracted from each value of $(B - A)$ we have $(B - A) - v = x + g$. But if $(B - A - v)_{d°} = x + g$, then $(B - A - v)_{d° + 180°} = - x - g$. Let $[(B - A - v)_{d°} - (B - A - v)_{d° + 180°}] = 2 D = 2 x + 2 g$. Then $D = x + g = 2 e'' \sin (t - h) + g$. It seems impracticable to wholly eliminate g from the values of D, but it may be seen from the following method of solution that if we call $D = 2 e'' \sin (t - h)$ and obtain values for e'' and h, the values obtained will be very nearly correct, since the *mean* value of g is zero and in the summations will almost cancel out. We thus have a series of variable numerical values for

D, each of which equals $2\,e'' \sin(t - h)$, in which e'' is a required quantity, t has certain known values varying by 10° or 20°, and h is unknown. The only practicable method of solution is one involving the principles of the Method of Least Squares. The numerical work is simple and may be accepted by the student until he has mastered the theory. Only the numerical process will be here given.

For each numerical case we have (assuming that the readings are taken at each even 20° around the circumference)

$$D_1 = 2\,e'' \sin(0° - h), \quad D_2 = 2\,e'' \sin(20° - h), \text{ etc.,}$$

which may be written

$$D_1 = 2\,e'' (\sin 0° \cos h - \cos 0° \sin h),$$
$$D_2 = 2\,e'' (\sin 20° \cos h - \cos 20° \sin h), \text{ etc.}$$

Multiply D_1 by sin 0°, D_2 by sin 20°, etc., and add the results; then multiply D_1 by cos 0°, D_2 by cos 20°, etc., and add the results. We then have

$$\Sigma(D \sin t) = 2\,e'' \cos h\,(\sin^2 0° + \sin^2 20° + \cdots + \sin^2 160°) - 2\,e'' \sin h\,(\sin 0° \cos 0° + \sin 20° \cos 20° + \cdots + \sin 160° \cos 160°). \quad (a)$$

$$\Sigma(D \cos t) = 2\,e'' \cos h\,(\sin 0° \cos 0° + \sin 20° \cos 20° + \cdots + \sin 160° \cos 160°) - 2\,e'' \sin h\,(\cos^2 0° + \cos^2 20° + \cdots + \cos^2 160°) \quad (b)$$

In (a), sin 0° cos 0° = 0, sin 20° = sin 160°, and cos 20° = − cos 160°; therefore sin 20° cos 20° + sin 160° cos 160° = 0; and similarly all the quantities in the second term of (a) and also the first term of (b) will cancel out. It may be readily shown that ($\sin^2 0° + \sin^2 20° + \cdots + \sin^2 160°$) = 4.5 precisely, *i.e.*, $\frac{1}{2}\frac{180°}{20°}$. If the uniform interval were 10°, the sum would be $\frac{1}{2}\frac{180°}{10°} = 9$. Likewise it may be shown that ($\cos^2 0° + \cos^2 20° + \cdots + \cos^2 160°$) = 4.5.

Then

$$\Sigma(D \sin t) = ne'' \cos h, \text{ in which } n = \frac{180°}{20°} = 9;$$

$$\Sigma(D \cos t) = - ne'' \sin h;$$

$$\frac{\Sigma(D \cos t)}{\Sigma(D \sin t)} = - \tan h.$$

h becoming known, we may solve for e'' from the equation

$$e'' = \frac{- \Sigma(D \cos t)}{n \sin h}.$$

Knowing the reading h for the line of no eccentricity and the value in seconds of the eccentricity (e''), the variable values of x are obtainable. By subtracting each value of x from the corresponding value of $B - A - v$ the error of graduation (g) for that angle may be found. We know that the values of x *must* vary according to a definite system, depending on the position of the line of no eccentricity, and the above method gives the *most probable* system. The method gives the most probable constant value for v, the most probable *system* of values for x, the most probable position of the line of no eccentricity and the angular value of the eccentricity, and throws the remainder into errors of graduation and reading. The *linear* eccentricity (e) equals $e'' R \sin 1''$.

For a numerical illustration see the following Exercise.

THEODOLITE WITH MICROMETERS — EXERCISES AND TESTS

Equipment: Only the lower part of the instrument is needed for this test. The telescope, striding-level, etc., may be removed and used in other problems if desired. The instrument may be set up in any convenient place where a good light may be obtained for illuminating the micrometers. The instrument need not be leveled. If the light comes from only one direction (a window), it is best to keep the lower plate loose so that it may be turned on the leveling-screws

FORM OF NOTES.

Micr. A.	Micr. B.	Error of runs.		B—A	B—A—v	2 D	D
		A	B				
° ′ ″	′ ″	″	″	″	″		
0 02 (25 26	02 (21 22	+ 1	+ 1	− 4.0	−3.6	−6.5	−3.2
20 02 (26 26	02 (17 20	0	+ 3	− 7.5	−7.1	−8.0	−4.0
40 02 (31 30	02 (22 28	− 1	+ 6	− 5.5	−5.1	−4.0	−2.0
60 02 (18 18	02 (10 14	0	+ 4	− 6.0	−5.6	−9.0	−4.5
80 02 (38 37	02 (34 38	− 1	+ 4	− 1.5	−1.1	−3.5	−1.7
100 02 (36 36	02 (36 40	0	+ 4	+ 2.0	+2.4	−1.0	−0.5
120 02 (32 30	02 (37 40	− 2	+ 3	+ 7.5	+7.9	+6.5	+3.2
140 02 (36 35	02 (33 35	− 1	+ 2	− 1.5	−1.1	+1.0	+0.5
160 02 (37 42	02 (42 40	+ 5	− 2	+ 1.5	+1.9	+2.5	+1.2
180 02 (25 22	02 (28 24	− 3	− 4	+ 2.5	+2.9		
200 02 (36 34	02 (36 35	− 2	− 1	+ 0.5	+0.9		
220 02 (29 29	02 (26 29	0	+ 3	− 1.5	−1.1		
240 02 (19 17	02 (21 21	− 2	0	+ 3.0	+3.4		
260 02 (31 31	02 (34 32	0	− 2	+ 2.0	+2.4		
280 02 (19 21	02 (21 25	+ 2	+ 4	+ 3.0	+3.4		
300 02 (24 24	02 (24 26	0	+ 2	+ 1.0	+1.4		
320 02 (17 21	02 (14 19	+ 4	+ 5	− 2.5	−2.1		
340 02 (25 21	02 (22 22	− 4	0	− 1.0	−0.6		
		+12 −16	+41 − 9	+23.0 −31.0			
		− 4 − 0.2	+32 + 1.8	− 8.0 $v = -0.4$			

and allow the micrometers to always have the same illumination.

Method: Set the micrometers so that the center of the comb of micrometer "A" is approximately at 0° 02′ + ; the center of the comb of micrometer "B" (assuming that the eccentricity is not excessive) will read 180° 02′ +. Disregard the degrees of micrometer B, only noting the minutes and seconds. Turn the

sin t	cos t	D sin t	D cos t	$t-h$	$2e''$ sin $(t-h)$	g
				°	″	″
0.00	1.00	0	−3.20	−107	−3.7	+ 0.1
0.34	0.94	−1.36	−3.76	− 87	−3.9	− 3.2
0.64	0.77	−1.28	−1.54	− 67	−3.6	− 1.5
0.87	0.50	−3.91	−2.25	− 47	−2.8	− 2.8
0.98	0.17	−1.67	−2.89	− 27	−1.7	+ 0.6
0.98	−0.17	−0.49	+0.08	− 7	−0.5	+ 2.9
0.87	−0.50	+2.78	−1.60	+ 13	+0.9	+ 7.0
0.64	−0.77	+0.32	−0.38	+ 33	+2.1	− 3.2
0.34	−0.94	+0.41	−1.13	+ 53	+3.1	− 1.2
				+ 73	+3.7	− 0.8
				+ 93	+3.9	− 3.0
				+113	+3.6	− 4.7
				+133	+2.8	+ 0.6
				+153	+1.7	+ 0.7
				+173	+0.5	+ 3.9
				+193	−0.9	+ 2.3
				+213	−2.1	0.0
				+233	−3.1	+ 2.5
		−8.71	−16.75			+20.6
		+3.51	+ 0.08			−20.4
		−5.20	−16.67			

$\frac{\Sigma(D\cos t)}{\Sigma(D\sin t)} = -\tan h = \frac{-16.67}{-5.20} = 3.20$; $\tan h = -3.20$; $h = 107°\ 21'$ (call it 107°).

$e = \frac{-\Sigma(D\cos t)}{9\sin 107°\ 21'} = \frac{+16.67}{8.64} = +1''.93$. ∴ the centre is *toward* the 107° mark.

$R = 4.5$ inches; $e = 1.93 \times 4.5 \times 0.0000048 = 0.000042$ of an inch.

Note that the *mean* value of g is almost exactly zero, as the theory requires.

micrometer-screw of micrometer A until the movable parallel wires are directly over (or equidistant from) the 0° mark. Note from the comb the minutes and from the wheel the seconds. Turn the screw so as to bring the parallel wires equidistant from the 0° 05′ mark. When the micrometer is properly adjusted, the reading on the wheel in this position should *nearly* agree with the reading when the wires were over the 0° mark. Obtain two readings similarly with micrometer B. Record the readings of seconds on the micrometer-wheels, enclosing corresponding readings in braces (as shown in the preceding Notes. It is desirable (especially for a novice) that after a reading of the micrometers is taken the parallel wires be moved out of position slightly and again set and read. If this is done several times, various values, having a range of, perhaps several seconds, will probably be found. The extreme range of several such readings will give an idea of the closeness obtainable. In such cases a mean value should be chosen. Turn the upper part of the instrument until micrometer A is at 20° 02′ + and clamp it. Then turn the *whole instrument* until the micrometers are substantially in the same position as before and have the same illumination. Again read micrometers A and B as before. Repeat this operation for the 18 different positions around the circumference.

Subtracting the first micrometer reading from the second gives the "error of runs." This should be substantially the same for all positions and should average not more than 5″ and as much less as is possible.

Find the eccentricity of the micrometers and the eccentricity of the center of the vertical axis.

PROBABLE ERROR — FORMULÆ AND USE

[Nothing but a mere statement of working formulæ, with their use, is here given, which the student must take on trust if he has not studied the theory. The proof of these formulæ is taken up in Geodesy.]

1. When a number of separate observations of any kind have been made with equal care, their *arithmetical mean* is the *most probable value* of the quantity observed.

2. The *probable error* is such a quantity that it would be an even wager that it is greater or that it is less than the real error. For example, if the measurement of a base-line is given as 622.456 ± 0.096, it means that the arithmetical mean of several equally good measurements was 622.456; that the probable error (computed as below) is 0.096; that according to the mathematical laws of the theory of probability there is *as much chance* that the real error of 622.456 is *less* than 0.096 as that it is *more.*

3. Let n = the number of observations;
d = the difference between any one observation and the arithmetical mean (or the "weighted mean");
E_1 = the probable error of a single observation;
E_m = the probable error of the mean;
c = 0.6745, a constant determined by computation according to the theory;
Σ = a symbol signifying that a summation should be made of all the quantities represented by the following letter or letters.

Then

$$E_1 = c\sqrt{\frac{\Sigma d^2}{n-1}},$$

$$E_m = \frac{E_1}{\sqrt{n}} = c\sqrt{\frac{\Sigma d^2}{n(n-1)}}.$$

For example:

Angle.	d	d^2
66° 54′ 12″.5	+1.1	1.21
13 .5	+0.1	0.01
11 .3	+2.3	5.29
16 .5	−2.9	8.41
12 .3	+1.3	1.69
15 .5	−1.9	3.61
Mean, 66° 54′ 13″.6		20.22

$$E_1 = 0.6745\sqrt{\frac{20.22}{6-1}} = \pm 1''.33.$$

$$E_m = 0.6745\sqrt{\frac{20.22}{6(6-1)}} = \pm 0''.54.$$

$$\text{Angle} = 66^\circ\ 54'\ 13''.6 \pm 0''.54.$$

4. Weighted observations. — When observations have not been made with equal care or are not equally reliable they are "weighted" according to their reliability. If M represents any observation and w its weight, then the "weighted mean" $= \dfrac{\Sigma\,(wM)}{\Sigma\,(w)}$. Subtract each observation from the weighted mean to obtain d. Then, with the other symbols as before,

$$E_1 = c\sqrt{\frac{\Sigma\,(wd^2)}{n-1}}$$ = the probable error of an observation of weight unity;

$$E_m = c\sqrt{\frac{\Sigma\,(wd^2)}{(\Sigma w)(n-1)}}$$ = the probable error of the weighted mean.

For example:

Angle.	w	Seconds (weighted)	d	d^2	wd^2
116° 43′ 48″.81	5	244.05	+0.83	0.69	3.45
48 .76	4	195.04	+0.88	0.77	3.08
49 .53	5	247.65	+0.11	0.01	0.05
51 .56	3	154.68	−1.92	3.69	11.07
50 .38	2	100.76	−0.74	0.55	1.10
49 .84	5	249.20	−0.20	0.04	0.20
116° 43′ 49″.64	24	1191.38			18.59
		49.64			

$$E_1 = 0.6745\sqrt{\frac{18.95}{6-1}} = 1''.32.$$

$$E_m = 0.6745\sqrt{\frac{18.95}{24(6-1)}} = 0''.27.$$

$$\text{Angle} = 116° \, 43' \, 49''.64 \pm 0''.27.$$

APPENDIX

Form A — Computation of coordinates of traverse (logs).
B — Computation of coordinates of traverse (machine).
C — Student's office-work report.
D — Student's field-work report.
E — Computation of distance and bearing from rectangular coordinates.
F — Check list for map lines.
G — Check list for map lettering.
H — Check list for profile.

FORM A. COMPUTATION OF COORDINATES OF TRAVERSE (LOGS)

Sta. and + of ∠ pt.	Dist. / Brg.	Logs.	Dep. E +	Dep. W −	Lat. N +	Lat. S −	Total D. E +	Total D. W −	Total L. N +	Total L. S −	Totals all positive. D	Totals all positive. L
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0+00							0	0	0	0	....	....
		log dep.	......	......								
		log sin brg.										
	dist.	log dist.										
	brg.	log cos brg.										
		log. lat.			....	....						
...+..											....	....
		log dep.	......	......								
		log sin brg.										
	dist.	log dist.										
	brg.	log cos brg.										
		log lat.			....	....						
...+..											....	....

(And so on.)

Form B. Computation of Coordinates of Traverse (Machine)

Sta. and + of ∠ pt.	Dist. / Brg.	Nat. sin / Nat. cos	Dep. E +	Dep. W −	Lat. N +	Lat. S −	Total D. E +	Total D. W −	Total L. N +	Total L. S −	Totals all positive. D.	Totals all positive. L.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0+00							0	0	0	0	....	....
	dist.	nat. sin	......	......								
	brg.	nat. cos			....	....						
...+..											....	....
	dist.	nat. sin	......	......								
	brg.	nat. cos			....	....						
...+..											....	....

(And so on.)

FORM C. *STUDENT'S OFFICE-WORK REPORT* Course: C. E..........

(Letter size sheet)

Section......... Corps........... Day of week.....................Date......... 19...................

Exercise or Problem. No....... and Title...

Mr.................................... (Instructor)

Dear Sir: I submit the following report on my office work for the period of the date above.

Time (hrs.)	Topics read on.	Author.	Book title.	Chapters or pages.
	....	....	....	....
	....	....	....	....

	Quantities computed.	How computed.	How checked.
	....	....	....
	....	....	....

	Data on.	Plotted.	Inked.	Traced.	Lettered.
	....	....	....	....	....
	....	....	....	....	....

REMARKS:

Yours truly,

....

(Use other side if necessary)

FORM D. *STUDENT'S FIELD REPORT* Course: C. E. ....

(Letter size sheet)

Section............ Corps............ Day of week.................. Date.........19.................

Name of assistant in charge...

Exercise or Problem. No....... and Title..

Mr...(Instructor)

Dear Sir: I submit the following report on the work done under my direction during the field period of the date above.

	Position.	Student.	Remarks.
Transit party. Ran from Sta........................... to Sta...........................	Transitman Recorder Head tape Rear tape Stakeman Rear flag	....	Transverse? Triangulation? Preliminary? Location? No. of setups:.......... Time (hrs.):........
Stadia party. Ran from Sta........................... to Sta...........................	Transitman Recorder Rodman Rodman	....	Traverse or details? Total No. of shots:....... No. of setups:............ Time (hrs.):........

	Position.	Student.	Remarks.
Level party. Ran from Sta........................ to Sta........................	Levelman Recorder Rodman	....	Differential or profile? No. of setups:............ Time (hrs.):........
Cross-section party. Ran from Sta........................ to Sta........................	Recorder Levelman Rodman	....	Topo. or earthwork? No. of cross-sections:........... Time (hrs.):........
.... *Party.*	....	....	No. of........................ Time (hrs.):........

REMARKS:

Yours truly,

....

(Chief of Party).

(Use other side if necessary)

Form E

Computation of Distance and Bearing from Rectangular Coordinates (Logs)

		$0 + 00$ to PI_1	PI_1 to PI_2	PI_2 to
1	x			
2	y			
3	x'			
4	y'			
5	$x - x'$			
6	$y - y'$			
7	$\log (x - x')$			
8	$\log (y - y')$			
9	log tan bearing			
10	log sin bearing			
11	log distance			
12	distance			
13	bearing			
Check:				
14	$(x - x')^2$			
15	$(y - y')^2$			
16	$(x - x')^2 + (y - y')^2$			
17	$[(x - x')^2 + (y - y')^2]^{\frac{1}{2}}$			

Form F. **Check List for Map Lines**

	Complete.	Legible.	Bold.	Correct.
Squares (coordinates)				
Preliminary traverse				
Angle points encircled				
Traverse stations picked out by short cross lines				
CL of staked location				
0 + 00, TC's, CT's, etc., encircled				
CL stations picked out by short cross lines				
Contours 0, 20, 40, 60				
Contours, intermediate				
Right-of-way lines				
Fences				
Roads and streets				
Railroads and electric roads				
Pipe lines				
Telegraph and telephone lines				
Streams, ditches and canals				
Flow arrows				
"To" arrows on roads, etc.				
North arrow				
Property lines				
Lot, district, township, etc., lines				
Culture				
Buildings				
All other lines				

Form G. Check List for Map Lettering

	Complete.	Legible.	Bold.	Correct.
Figuring of squares....................				
Preliminary angle-point stations.......				
Stations 0, 5, 10, 15, .. of preliminary..				
Preliminary CL bearings..............				
Symbol and sta. of each TC, CT, etc..				
Curve data of each curve..............				
Bearings of location tangents..........				
Stations 0, 5, 10, 15, .. of location CL..				
Contour elevations 0, 20, 40, 60.... ...				
Right-of-way widths............... ..				
"To .. .. " at each end of each				
road..............................				
railroad or electric road............				
pipe line..........................				
telegraph and electric line.........				
Name of each road and street..........				
railroad and electric road..........				
pipe line..........................				
telegraph and telephone line.......				
stream, ditch and canal...........				
building...........................				
property owner....................				
district, township, county, etc.....				
Lot numbers.........................				
Dimensions of ditches and canals......				
Culture..............................				
All other descriptive matter...........				
Title................................				

FORM H. CHECK LIST FOR PROFILE

Lines.	Complete.	Legible.	Bold.	Correct.
Profile line of traverse..................				
Profiles of trial paper locations.........				
Profile of staked location..............				
Mass curve............................				
Alignment............................				
Subgrade.............................				
Culverts, bridges, and trestles.........				
High- and low-water lines.............				
Highway and other crossings..........				
Mile posts...........................				
Arrows indicating distribution.........				
Other lines..........................				
LETTERING				
Stations 0, 10, 20, 30..................				
Elevations 0, 50, 100, 150..............				
Plus and name or number of structures				
roads, streets.......................				
streams, etc.........................				
Gradients on subgrade line............				
Plus and elev. of each grade-change point...............................				
Estimated quantities in each structure				
Station volumes......................				
High- and low-water elevations........				
Descriptive notes.....................				
Alignment notes......................				
Title................................				
Mile-post numbers....................				
BM locations and elevations...........				
Mass curve results:				
stations............................				
distances...........................				
volumes.............................				
haul................................				
overhaul............................				