IN PREPARATION

TEXT-BOOK OF MECHANICS

BY

LOUIS A. MARTIN, Jr.

Vol. II

KINEMATICS AND KINETICS

TEXT-BOOK

OF

MECHANICS

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VOL. I.

STATICS

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PREFACE

Answers to the examples given in this book will be supplied to students by the publishers, free of charge, on the receipt of the written order of the instructor.

of Statics, the study of which can advantageously be begun with such knowledge of mathematics as is required for admission to most Colleges and all Technical Schools. The elements of Analytical Geometry are introduced as the treatment of the subject requires, but no Calculus is used in the first volume.

The second volume, the MS. of which is nearly ready, will treat of Kinematics and Kinetics. In this volume extensive use will be made of the Calculus.

Thus, while studying Mechanics, the student becomes acquainted with a use to which pure mathematics, as now taught in our schools, may be put. Starting with a knowledge of Algebra, Geometry, and Trigonometry,

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PREFACE

THE Text-book of Mechanics, of which this is the first volume, is designed as an introductory course to Applied or Technical Mechanics for Technical Schools and Colleges. It is based upon notes prepared for the use of the Freshman and Sophomore classes at Stevens Institute of Technology.

At the same time an attempt has been made to produce a graded course in the application of mathematics. With this end in view the first volume treats of Statics, the study of which can advantageously be begun with such knowledge of mathematics as is required for admission to most Colleges and all Technical Schools. The elements of Analytical Geometry are introduced as the treatment of the subject requires, but no Calculus is used in the first volume.

The second volume, the MS. of which is nearly ready, will treat of Kinematics and Kinetics. In this volume extensive use will be made of the Calculus.

Thus, while studying Mechanics, the student becomes acquainted with a use to which pure mathematics, as now taught in our schools, may be put. Starting with a knowledge of Algebra, Geometry, and Trigonometry, he at once puts them to practical use, and it is designed to have this application keep pace with his advancing knowledge of pure mathematics.

Throughout it is aimed to make the book a *teachable* one. As a course in Mechanics should primarily fit the student to solve its problems, *numerous examples and many exercises* are introduced to *illustrate each principle as developed.*

Of the two hundred and fifty-five problems contained in this volume many are to be found in most books on Mechanics; some have been especially prepared, while others have been selected from examinations set at Stevens.

As regards the subject-matter in a book of this nature nothing new can be expected. My only claim to originality lies in the presentation. Should any errors be found in the work, I shall esteem it a favor to be informed of them.

In concluding, I would here express my thanks to my wife, Alwynne B. Martin, for her aid in preparing the MS. and in reading the proof.

LOUIS A. MARTIN, Jr.

HOBOKEN, N. J., March, 1906.

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TEXT-BOOK OF MECHANICS

INTRODUCTION

THAT branch of physics which is the simplest and also the oldest and which is usually treated as introductory to the other branches of this science is called Mechanics.

Mechanics treats of the motions of bodies and the equilibrium of forces.

It is divided into Kinematics and Dynamics.

Kinematics treats of the motions of bodies without reference to the forces producing these motions or to the masses moved.

Dynamics is that branch of Mechanics which treats of the equilibrium of forces and the motions of bodies under the action of forces. It is accordingly subdivided into two parts,—Statics and Kinetics.

Statics treats of the equilibrium of forces.

Kinetics treats of the motions of bodies as related to the forces producing these motions and to the masses moved.

Motion is change of position.

Velocity is the rate of change of position.

Acceleration is the rate of change of velocity.

By **rate of change** is meant the total change divided by the time occupied in making the change, provided the change progresses uniformly.

Thus, the population of a town in 1890 was 1500, in 1900 it was 4000; to find the rate at which the population changed we have:

$$\text{Rate of change} = \frac{\text{Total change}}{\text{Time}} = \frac{4000-1500}{10}$$
$$= 250 \text{ persons per year.}$$

A body which moves uniformly over a space of 60 feet in 5 seconds changes its position at the rate of $\frac{60}{5} = 12$ feet per second.

The velocity of a uniformly moving body is therefore calculated thus: velocity $= \frac{\text{space}}{\text{time}}$. In algebraic language this is expressed, $v = \frac{s}{t}$, where velocity, space, and time are denoted by their initial letters.

Exercise 1. If a body moves 144 feet in 3 seconds, what is its velocity?

Exercise 2. A body moves with a uniform velocity of 40 miles and 1600 yards per hour; what is its velocity in feet per second, and how many feet will it traverse in 10 seconds?

Exercise 3. A railway-train explodes two torpedoes which are placed on the rails 176 feet apart. Two seconds of time elapse between the explosions. Compare the velocity during that interval with the mean velocity, which is indicated by the statement that the train takes an hour and a half to perform the journey between two stations 45 miles apart.

If the velocity of a moving body changes, the *rate of change of velocity* is called its "acceleration." Thus, if a moving body has a velocity of 20 feet per second at one point of its path, and this velocity increases uniformly so that 3 seconds later its velocity is 50 feet per second, its acceleration is $\frac{50-20}{3}=10$ feet-per-second per second.

This shows that

$$\text{Acceleration}=\frac{\text{Change in velocity}}{\text{Time required to produce change}},$$

or $a=\frac{v_2-v_1}{t_2-t_1}$, where v_2 and v_1 are the velocities of the body at the times t_2 and t_1 respectively.

Acceleration, being measured by the velocities imparted per second, is stated in terms of feet-per-second *per second.*

EXERCISE 4. A body starts from rest with an acceleration of 2 feet-per-second per second. When will it have a velocity of 1000 feet per second?

EXERCISE 5. The velocity of a body decreases during 10 seconds from 10 feet per second to 7 feet per second. What is its acceleration?

EXERCISE 6. A body is made to record its own velocity, which shows that at a certain instant it is moving at the rate of 112 feet per second; but after an interval of 1/20 second its velocity is 114 feet per second. What is its acceleration?

From the above it will be noticed that the unit of velocity is obtained from the formula $v=\frac{s}{t}$, by placing $s=1$ foot and $t=1$ second; thus, $v=\frac{1}{1}=1$ foot per second.

This may be stated thus: A body has Unit Velocity when it traverses uniformly a unit of distance in a unit of time.

Similarly, the unit of acceleration is derived from $a=\frac{v_2-v_1}{t_2-t_1}$ by placing $v_2-v_1=1$ foot per second and $t_2-t_1=1$ second; thus, $a=\frac{1}{1}=1$ foot-per-second per second; or, A body has Unit Acceleration when its velocity increases uniformly by a unit of velocity in a unit of time.

This method of deriving units will be found of universal application.

Matter is the material of which bodies are composed.

A Body is a limited portion of matter.

Mass is the scientific name for a quantity of matter.

We can recognize in bodies a property which depends partly on their size and partly on the substance of which they are composed. Thus, if we take two balls of iron of considerably different sizes and hang them up by long strings of the same length, a very slight effort is sufficient to give considerable motion to the small ball, while a strong push is necessary to displace appreciably the large ball. These balls are said to *differ in mass*.

A heavy fly-wheel, properly mounted on ball-bearings, continues to rotate for a long time when set in motion; much effort, however, is required to either stop it or to start it when at rest. The fly-wheel is said to have *mass*, and the greater the mass the greater the effort required to stop it in a given time.

These and similar observations lead us to recognize that property of bodies to which the name of "Mass" has been given.

The Unit of Mass is the mass of a certain lump of platinum marked "P. S., 1844, 1 lb." (P. S. being the abbreviation of Parliamentary Standard).

Momentum.—Observation leads us to recognize a quantity which depends upon both the mass of a body and its velocity. Two bodies of equal mass moving with equal and opposite velocities will on impact (collision) come to rest. If, however, the masses are unequal or their velocities different, the result of impact is not rest. The motion which a given body can communicate by impact to another body depends upon this quantity, which we call "Momentum."

The momentum of a body is the product of its mass and its velocity, or

$$\text{Momentum} = m \times v.$$

The Unit of Momentum is the momentum of a mass of 1 pound moving with a velocity of 1 foot per second.

EXERCISE 7. A baseball of 5 oz. mass has a horizontal velocity of 66 feet per second. A blow causes it to travel back on the same line with a velocity of 62 feet per second. Find the change of momentum.

Momentum is a fundamental property of a moving body. If we observe the motions of bodies, we will find but few cases in nature in which the velocity is uniform: bodies generally have acceleration. This leads us to consider change in momentum. That which changes the momentum of a body is called "Force."

Force is measured by the rate of change of momentum, or

$$\text{Force} = \frac{\text{Change of momentum}}{\text{Time required for change}}.$$

$$F = \frac{m \times \text{change of velocity}}{\text{Time required for change}} = \frac{m(v_2 - v_1)}{t_2 - t_1};$$

but $$\frac{v_2 - v_1}{t_2 - t_1} = \text{acceleration} = a.$$

Therefore $$F = m \times a.$$

The Force acting on any mass is equal to the product of the mass by the acceleration produced by the force.

The **Unit of Force** is called the "Poundal," and is that force which when acting upon a mass of 1 pound produces an acceleration of 1 foot-per-second per second for $F = m \times a = 1 \times 1 = 1$ poundal.

From the formula $F = \dfrac{m(v_2 - v_1)}{t_2 - t_1}$ it will be noticed that $F(t_2 - t_1) = m(v_2 - v_1)$, or Force multiplied by the time during which it acts equals the change of momentum produced.

The product $F(t_2 - t_1)$ is called Impulse.

$\therefore$ Impulse = change of momentum.

It has been experimentally demonstrated that any body near the earth's surface falls with a sensibly constant acceleration, usually denoted by g. This means that the velocity of the body, and therefore its momentum, is changing. In the light of what has been said we should say that a force is acting on the body.

This force is known as "Weight" and is the result of the attraction which the earth exercises on all bodies in accordance with the law of gravitation.

Thus, if w represents the weight of a body whose mass is m and as

$$\text{Force} = \text{mass} \times \text{acceleration},$$

we have
$$w = m \times g.$$

As g is approximately equal to 32 feet-per-second per second, the weight of a body whose mass is, say, 10 lb. equals

$$10 \times 32 = 320 \text{ poundals.}$$

In engineering practice it is not usual to use the absolute unit of force,—the poundal. Instead, forces are measured in terms of the weight of some given body, usually the weight of the standard of mass,—1 pound. Thus, when in practice we speak of a force p we do not mean a force of p poundals, but a force p times as great as the weight of 1 pound. Such a unit is known as a *gravitational unit of force.*

If we wish to use as the unit of force the weight of a mass of 1 pound, as before defined, and still retain the equation $w = mg$, which is a necessary deduction from the laws of mechanics, we must change the unit of mass.

The necessity of this change becomes evident from the equation $m = \frac{w}{g}$, which gives $m = 1$ only upon making $w = g$. The new unit of mass is then the mass of a body possessing g times the weight of the original unit of mass.

If, then, we speak of a mass of $5g$ lb., we mean a mass possessing $5g$ times the weight of the original unit of mass, or it contains $\frac{5g}{g}$ $(=5)$ times the new unit of mass, and is therefore a *mass of 5 units.*

As, by definition, a poundal is that force which acting on a mass of 1 lb. produces an acceleration of 1 foot-per-second per second, and as we observe in the case of falling bodies that a force equal to the weight of 1 pound of mass acting upon a mass of 1 pound produces an acceleration of g feet-per-second per second, the force equal to the weight of 1 pound of mass must equal g poundals. This is expressed more compactly as follows: 1 pound $=$ g poundals.

In the Absolute System of Units the unit of mass is fundamental, and the unit of force is derived.

In the Gravitational System of Units we assume the unit of force and then derive the unit of mass from it.

Example.—A force of 10 lb. acts upon a weight of 20 lb. for 5 seconds. Find the change of momentum produced and the mass of the moving body in gravitational units.

Solution.—We have the law $Ft = mv$.

$\therefore$ The change of momentum $= mv = Ft = (10)\ (5) =$ 50 units of momentum.

The moving body has a weight of 20 lb.; one unit of mass has a weight of g lb. $\therefore$ The mass of the moving body is $\frac{20}{g}$ units of mass.

It should be observed that no specific name has been assigned to either the unit of momentum or the unit of mass.

Example.—What velocity would a mass of 64 pounds acquire in 10 seconds if acted on by a force of 4 pounds? ($g = 32$ ft.-per-sec. per sec.)

Solution.—Find first the acceleration produced by the force by means of the formula $F = ma$, wherein $F = 4$ pounds and $m = \frac{64}{32} = 2$ units of mass.

Thus, $a = \frac{4}{2} = 2$ ft.-per-sec. per sec.

As the acquired velocity will be the change in velocity and as the acceleration is the rate of change of velocity, we have

$$2 = \frac{v}{10}. \quad \therefore\ v = 20 \text{ ft. per sec.}$$

The following exercises should be solved in gravitational units.

EXERCISE 8. A mass of 100 pounds possesses an acceleration of 10 feet-per-sec. per sec. What force must be acting upon it?

EXERCISE 9. What is the weight of a mass on which a force of 10 pounds produces an acceleration of 3 feet-per-sec. per sec.?

EXERCISE 10. What force would increase the velocity of a 30-pound mass from -10 feet per sec. to 30 feet per sec. in 5 minutes?

EXERCISE 11. What acceleration would a force of 6 pounds produce in a mass of 32 units of mass?

STATICS

ANALYTICAL STATICS

CHAPTER I

FORCES ACTING AT A SINGLE POINT

SECTION I

TRIANGLE OF FORCES

Statics treats of the equilibrium of forces.

When a body is acted on by forces which are in equilibrium the body is at rest or moves with a constant velocity.

Representation of a Force.—A force can conveniently be represented by a straight line; one end of the line will represent the point of application of the force, the direction of the line gives the direction of the line of action of the force, the number of units of length in the line represents the number of units of force, while the arrowhead shows the "sense" of the force.

It can be experimentally demonstrated that if OA and OB (Fig. 1) represent two forces acting upon a particle O, the diagonal OC of the parallelogram constructed upon OA and OB represents in direc-

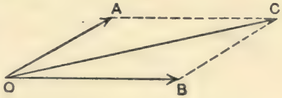


FIG. 1

tion and magnitude the resultant of the forces OA and OB, or the single force OC may replace the forces OA and OB without affecting the state of the particle O.

If we assume a force OC' (Fig. 2) equal in magnitude, but opposite in direction to OC, acting upon O together with OA and OB, it is evident that equilibrium of the forces would result.

Let us now start with any one of the forces OC', OA, OB, as OA (Fig. 2), and in any order put the remaining

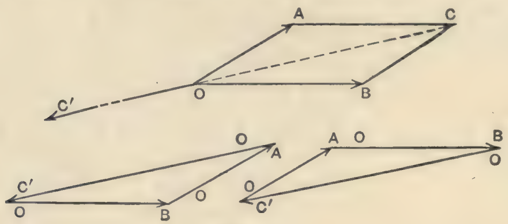


FIG. 2

forces end to end, being careful to preserve their lengths and direction, and it will be found that a closed triangle results. (Prove this by Geometry.) Therefore,

If three forces impressed on a particle are in equilibrium, they can be represented in direction and magnitude by the sides of any triangle drawn so as to have its sides parallel to the forces.

Before applying this principle to some examples we will call attention to the manner in which the actions of cords, planes, and supports in general may be represented by their equivalent forces.

A flexible cord or string is always assumed to transmit any force along its fibres in the direction of its length. If two equal and opposite forces keep a string in equilibrium, the string is said to be in tension and the tension is constant throughout its length. This tension is measured by the force applied at *either* end. The tension in a cord passing over a smooth (frictionless) peg or surface is unaltered. If a string is knotted to another string at any point of its length, the tension will no longer be the same for both portions, in fact we may consider the two portions as two separate strings.

The action of a string can thus always be represented by a force having for direction the direction of the string, and this force will measure the tension to which the string is subjected.

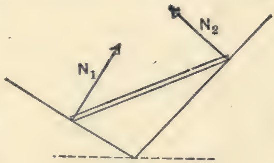


FIG. 3

A smooth (frictionless) plane always exerts a force normal to its surface. Thus in Fig. 3, representing a beam resting upon two smooth surfaces, the arrows N_1 and N_2 represent the reactions of the planes.

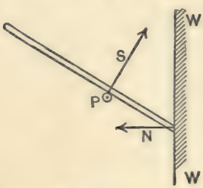


FIG. 4

If a body rests upon a smooth peg, the peg exerts a force normal to the surface of the body at the point of contact. This is illustrated in Fig. 4, representing a beam resting upon a peg, P, and against a smooth wall, WW; here S represents the reaction of the peg, and N the reaction of the wall.

A body which has been separated from all supports and upon which the reactions of the supports are all represented by their equivalent forces is called a *free body*. Of course the action of gravity must also be shown as a force, representing the weight of the body, acting upon the free body.

In statics it is of the utmost importance always to represent the body to which the principles of equilibrium are to be applied as a free body. The greatest care must be taken to represent *all* the forces acting upon the body by their respective arrows.

In solving problems involving the above principles the method to be pursued may be outlined as follows:

(*a*) Draw a diagrammatic sketch to aid in obtaining a clear conception of the problem.

(*b*) Find a point at which *three* forces act in equilibrium.

(*c*) Locate this point with its impressed forces, making it a "free body."

(*d*) Construct the triangle of forces.

(*e*) Apply your knowledge of mathematics (Geometry, Algebra, and Trigonometry) so as to calculate the required forces.

(*f*) Interpret your answer so as to obtain its mechanical significance.

Example.—A picture weighing 3 lb. hangs vertically from a nail by a cord passing through two rings in the frame. The parts of the string form an equilateral triangle. Find the tension of the string.

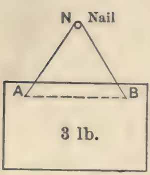


FIG. 5

Solution.—(*a*) Fig. 5 illustrates the problem.

(*b*) At N (Fig. 5) we have three forces acting, the tensions of the two strings and the reaction of the nail, which is equal to the weight of the picture and acts upward.

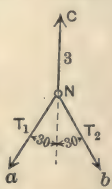


FIG. 6

(*c*) In Fig. 6 NC represents the reaction of the nail, and Na $(=T_1)$ and Nb $(=T_2)$ the tensions of the strings NA and NB respectively. These lines represent *forces* and are in no way connected with the lines in Fig. 5, except by reason of their parallelism.

(*d*) Fig. 7 represents the corresponding triangle of forces.

(*e*) It now remains to calculate the values T_1 and T_2 from the triangle of forces. This may be done in many ways; for instance, from Q drop $\perp QS$ to PR; then as $PQ=QR$, $PS=SR=3/2$ (Geometry). From $\triangle PSQ$,

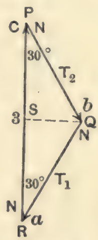


FIG. 7

$$\frac{PS}{PQ}=\frac{3/2}{T_2}=\cos 30^\circ=\frac{\sqrt{3}}{2}.$$

$$\therefore\ T_2=\frac{3}{2}\cdot\frac{2}{\sqrt{3}}=\sqrt{3}\text{ lb.}=1.7+\text{ lb.}$$

and $\qquad T_1=T_2.$ (Geometry.)

(*f*) We thus arrive at the conclusion that the tensions in the strings are equal, and that in each it equals 1.7 + lb. This means that if the string is strong enough to support a weight of 1.7 + lb., as indicated in Fig. 8, it will safely support the picture as shown in Fig. 5.

FIG. 8

Example.—A piece of machinery weighing 4000 pounds is suspended by two ropes making angles of 30° and 45° with the vertical. Find the tensions in the ropes.

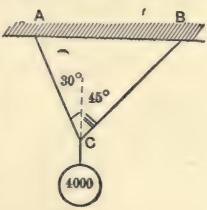


FIG. 9

Solution.—(*a*) Fig. 9 illustrates the problem.

(*b*) At C we have three forces acting.

FIG. 10

(*c*) Fig. 10 shows these forces, the unknown forces due to the action of the ropes being denoted by R and S.

(*d*) Fig. 11 represents the triangle of forces.

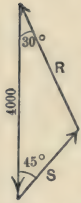


FIG. 11

(*e*) To calculate the magnitude of the forces R and S note that the angles between the forces 4000 and R and between 4000 and S (Fig. 11) are respectively equal to 30° and 45°. (Why?) Therefore the angle between the forces R and S becomes 105°. By the law of sines (Trigonometry) we have

$$\frac{S}{\sin 30^\circ}=\frac{R}{\sin 45^\circ}=\frac{4000}{\sin 105^\circ}\left(=\frac{4000}{\sin 75^\circ}\right).$$

$$\therefore\ S=\frac{4000 \sin 30^\circ}{\sin 75^\circ}=2070 \text{ pounds,}$$

and $$R=\frac{4000 \sin 45^\circ}{\sin 75^\circ}=2928 \text{ pounds.}$$

(*f*) These results to be interpreted as in the previous example.

Example.—A heavy particle whose weight is W pounds is placed on a smooth (frictionless) inclined plane, AB. The height of the plane is a and its length is c (Fig. 12). Find the force acting parallel to AB required to sustain the particle upon the plane. Also find the pressure exerted by the particle upon the plane.

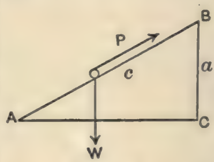


FIG. 12

Solution.—(*a*) Fig. 12 illustrates the problem.

(*b*) Whenever a surface enters into a problem it should be remembered that *there always exists a normal reaction perpendicular to the surface*, which in the case of a frictionless plane is the only reaction. The particle then becomes the point at which three forces act in equilibrium.

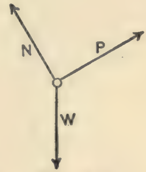


FIG. 13

(*c*) The three forces are (Fig. 13):

1st. The weight, W.

2d. The sustaining force, P.

3d. The normal reaction of the plane, N.

(*d*) The triangle of forces is shown in Fig. 14.

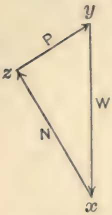


FIG. 14

(*e*) In this example no angle is given. This precludes the use of Trigonometry. On due consideration it will be noticed that $\triangle xyz$ is similar to $\triangle ABC$, (why?) and therefore

$$\frac{yz}{xy}=\frac{BC}{AB},$$

or more simply $\dfrac{P}{W}=\dfrac{a}{c}$; $\therefore\ P=W\dfrac{a}{c}$;

also $$\frac{N}{W}=\frac{\sqrt{c^2-a^2}}{c};\quad \therefore\ N=W\frac{\sqrt{c^2-a^2}}{c}.$$

(*f*) This shows us that the force P necessary to sustain the particle upon the plane is less than W, the weight of the particle, for $a<c$.

As the reaction of the plane upon the particle is N, it follows that an equal but opposite force must be the pressure exerted by the particle upon the plane.

EXERCISE 12. A picture weighing 7 lb. hangs by a cord passing through two hooks and over a small nail. The hooks are 18 inches apart and the cord is 4 feet long. Find the tension of the cord.

Exercise 13. If the cord in Exercise 12 be lengthened to 6 feet, how will the tension in it be altered?

Exercise 14. A cord is stretched horizontally between two posts 6 feet apart. What is the tension in it, and how much longer does it become when a 7-lb. weight suspended from its middle makes it droop 3 inches?

Exercise 15. A particle weighing 10 lb. is sustained upon a smooth inclined plane by a force whose line of action makes an angle of 60° with the horizontal. If the angle of the plane is 30°, find the sustaining force.

Exercise 16. Same as preceding exercise with angle of plane 45° and angle between line of action of the force and horizontal 30°.

Exercise 17. An inclined plane has a base of 4 feet and a length of 5 feet. What will be the reaction of the plane upon a particle whose weight is 100 pounds, if the sustaining force be applied parallel to the base of the plane?

Exercise 18. A weight w is supported on a smooth plane inclined at an angle α to the horizon, by means of a force inclined at an angle θ to the plane. Find the magnitude of the force, and the pressure on the plane. If there is no pressure on the plane, in what direction does the force act?

Exercise 19. A man pushes a garden-roller weighing 80 pounds up a plank 10 feet 3 inches long and with one end 2 feet 3 inches above the ground. If the handle is horizontal, find the force applied and the pressure of the roller on the plank.

Exercise 20. Two sticks, AB and BC, loosely jointed at A, rest upon horizontal ground at B and C, making angles of 65° and 40° respectively with the ground. If a weight of 200 pounds be suspended from A, find the forces transmitted by the sticks to the ground.

Exercise 21. Draw a triangle ABC with base BC horizontal and its vertex A under BC. Let AB and AC be threads fastened to two fixed points B and C, and to a third thread at A. If the third thread supports a weight W and the angles of the triangle are A, B, and C respectively, find the tensions in the threads.

Section II

COMPONENTS. COMPOSITION OF FORCES

We have just noticed that the triangle of forces gives us an easy method for calculating the relations existing between *three* forces in equilibrium.

For the consideration of the equilibrium of more than three forces another method must be used. To introduce this, we will first consider the

Resolution of Forces

In Fig. 15 assume the force R and the lines of action of two other, unknown forces OA and OB. Through C draw $CD \parallel$ to OA, and $CE \parallel$ to OB. Let OD and OE then represent the forces F_y and F_x respectively. By the parallelogram of forces, F_x and F_y would have R as resultant.

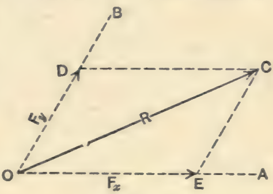


FIG. 15

F_x and F_y are called the *Components* of R; and R is said to be *resolved* into the component forces F_x and F_y.

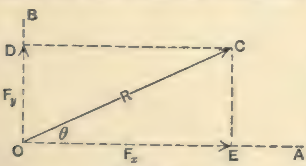


FIG. 16

In case the lines of action of the components OA and OB are oblique, the analytical calculation of F_x and F_y is not easily accomplished. To simplify the calculation we shall, whenever possible, assume OA and OB at right angles as in Fig. 16. Then if $\angle COE = \theta$,

$$F_x = R \cos \theta$$

and
$$F_y = R \sin \theta.$$

Exercise 22. A force of 100 pounds acts at an angle of 30° to the horizontal. Resolve same into vertical and horizontal components.

Exercise 23. Same as Exercise 22 if force is inclined at 45° to the horizontal.

For convenience we shall designate all forces acting *upward* and *to the right* as plus, and therefore all forces acting *downward* and *to the left* as minus.

Exercise 24. Resolve a force of 2 tons, acting at an angle of 160° to the horizon, into vertical and horizontal components.

Exercise 25. Same as Exercise 24, if the line of action of the force makes an angle of (*a*) 270°; (*b*) 210°; (*c*) 315° with the horizontal.

Exercise 26. The pull on the rope of a canal-boat is 100 pounds, and the direction of the rope makes an angle of 60° with the parallel banks. Find the force urging the boat forward.

In the propulsion of a canal-boat, is a long or a short rope more advantageous? Why?

Example.—Explain the action of the wind in propelling a ship.

Solution.—Let AB, Fig. 17, represent the direction of the ship's keel; CD the position of the sail, which we assume flat. Let the pressure of the wind be equivalent to a force P acting on the sail in the direction indicated. Resolve the force P into two components T and N, one parallel to the sail and the other at right angles to the sail. The first component, T, produces little or no effect, and we will neglect it. The component N acts on the ship through the mast. We may resolve N into the components X and Y, parallel and at right angles to the keel. The resistance offered by the water to a motion in a direction at right angles to the keel is so great that the component Y produces but little motion. The ship is so built that the water may offer only a small

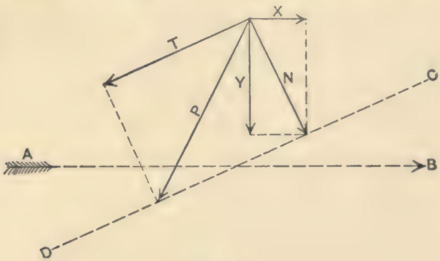


FIG. 17

resistance to motion parallel to the keel, and the ship moves in this direction under the action of the force X.

EXERCISE 27. Show that a vessel may sail due east against a southeast wind.

EXERCISE 28. If F be the force of the wind, α and β the inclinations of the wind and sail to the keel of a boat, find the headway force and the leeway force.

EXERCISE 29. A boat is sailing due west; the wind is from the northeast. The sail is set at an angle of 75° with the keel of the boat. Of the two possible positions of the sail select the most advantageous and calculate the headway force.

EXERCISE 30. A boat is towed along the centre of a canal, 25 feet wide, by mules on each bank; the length of each rope is 36 feet. Find the force exerted by each mule when the force urging the boat forward is 200 pounds.

Composition of Forces

The method of resolving a force into its components leads us to a method of combining forces which is, for purposes of calculation, much more convenient than the principle of the parallelogram of forces. Consider three forces F', F'', F''' all acting upon the point O and inclined to the horizontal at angles θ', θ'', θ''', respectively (Fig. 18). From what has preceded it follows that each of the three forces may be replaced by a horizontal and vertical component, i.e.,

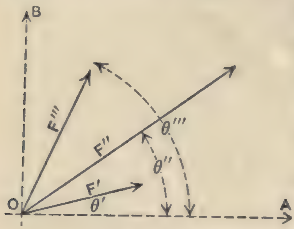


FIG. 18

F'	by	$F' \cos \theta'$	and	$F' \sin \theta'$;
F''	by	$F'' \cos \theta''$	and	$F'' \sin \theta''$;
F'''	by	$F''' \cos \theta'''$	and	$F''' \sin \theta'''$.

To simplify the notation, we shall designate $F' \cos \theta'$ by F_x' and $F' \sin \theta'$ by F_y', etc., and therefore call the horizontal line OA the axis of X, and OB the axis of Y.

As F_x', F_x'', F_x''' all have a common line of action (the axis of X), we may add them algebraically to obtain their resultant,

$$R_x = F_x' + F_x'' + F_x''';$$

similarly,

$$R_y = F_y' + F_y'' + F_y''',$$

where R_x and R_y are the x and y components of the resultant, R, of the three forces F', F'', F'''.

Fig. 19 shows how R_x and R_y may be utilized to obtain the resultant

$$R=\sqrt{R_x^2+R_y^2},$$

and the direction of its line of action

$$\alpha=\tan^{-1}\frac{R_y}{R_x}.$$

It should be noticed that the argument just applied would be in nowise affected if the X axis and the Y axis

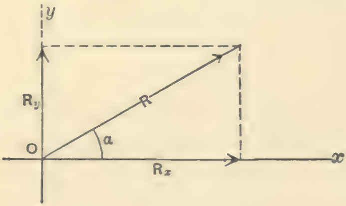


FIG. 19

were not horizontal and vertical so long as they remained mutually perpendicular.

EXERCISE 31. Three forces of 6, 8, and 10 pounds act on a particle at angles of 120° to each other. Find the resultant in magnitude and direction:

1st. By assuming axis of x coincident with force 6;
2d. " " " " x " " " 8;
3d. " " " " x " " " 10.

EXERCISE 32. $ABCDEF$ is a regular hexagon. Find the magnitude of the resultant of the forces represented by AB, AC, AD, AE, and AF. (Assume $AB=a$).

EXERCISE 33. Three smooth pegs are driven into a vertical wall and form an equilateral triangle whose base is horizontal.

Find the pressure on each peg, if a thread, having equal weights of 10 pounds attached to its ends, be hung over the pegs.

EXERCISE 34. Three men pull an iron ring. The first pulls with a force of 100 pounds in a southeasterly direction, the second pulls northeast with a force of 70 pounds, and the third pulls towards the north with a force of 50 pounds. In which direction will the ring move?

EXERCISE 35. Two forces of 20 pounds each and one of 21 pounds act at a point. The angle between the first and second is 120°, and between the second and third 30°. Find the resultant in direction and magnitude.

SECTION III

CONDITIONS FOR EQUILIBRIUM

We know that the resultant of any number of forces may be expressed by $R=\sqrt{R_x^2+R_y^2}$.

If a force equal in magnitude but opposite in direction to R is applied to the body, it will balance the original forces.

If the original forces are themselves in equilibrium, no other force is necessary to produce rest or motion with constant velocity, and their resultant is therefore zero.

Whenever forces are in equilibrium we may set

$$R=0,$$

and as $R^2=R_x^2+R_y^2$, both R_x and R_y must be 0. (Why?)

Therefore $\qquad F_x'+F_x''+F_x'''+\ldots=0,$

and $\qquad F_y'+F_y''+F_y'''+\ldots=0;$

whence follows the theorem:

If any number of forces acting on a body keep it in equilibrium, the respective sums of the components of the forces along any two straight lines at right angles to each other are equal to zero.

In the solution of problems based upon this theorem we may proceed as follows:

(*a*) Draw the sketch illustrating the problem.

(*b*) Find a point at which the forces act in equilibrium.

(*c*) In a separate sketch show this point with *all* the forces acting upon it and draw "any two straight lines at right angles to each other." These lines are usually drawn so as to coïncide with as many forces as possible.

(*d*) Write the equation expressing the fact that the respective sums of the components of the forces along these lines or axes are equal to zero.

(*e*) Solve the equations for the unknown quantities they may contain.

(*f*) Interpret the results so obtained.

Example.—A rod AB, whose weight may be neglected, is hinged at A and supports a weight W at B. It is held up by a wire BM fastened to a fixed point M vertically above A. If AB is horizontal and the angle $ABM = 30°$, find the tension, T, in the wire and the compression, C, in the rod.

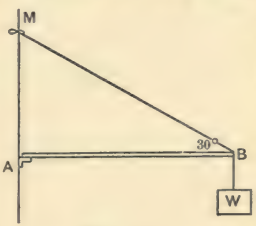


FIG. 20

Solution. — (*a*) Fig. 20 illustrates the problem.

(*b*) The wire and the rod both act upon the point B, and by the nature of the problem this point is in equilibrium.

(*c*) In Fig. 21 the forces acting at the point B are shown. BX and BY are the lines along which the components will be taken.

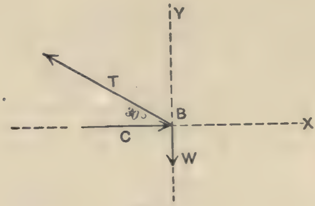


FIG. 21

(*d*) The components of the forces in the X and Y directions are:

Forces	X Components	Y Components
C	C	0
W	0	$-W$
T	$-T\cos 30^\circ$	$T\sin 30^\circ$

Therefore $C+0-T\cos 30^\circ=0$ and $0-W+T\sin 30^\circ=0$.

$$(e)\quad \left.\begin{aligned} &C-T\cos 30^\circ=0,\\ &-W+T\sin 30^\circ=0,\end{aligned}\right\} \text{ or } \left\{\begin{aligned} &C-T\frac{\sqrt{3}}{2}=0,\\ &-W+T\frac{1}{2}=0.\end{aligned}\right.$$

$$\therefore\ C=\frac{T\sqrt{3}}{2},\quad T=+2W,\quad \text{and}\quad C=W\sqrt{3}.$$

(*f*) In Fig. 21 the arrow T represents the action of the wire upon the point B. The reaction upon the wire at B is necessarily an equal but opposite one and may be represented as in Fig. 22. As the wire is in equilibrium, an equal force must act upon the wire at M, this force being supplied by the wall. These equal and opposite forces $(T=2W)$ at M and B, Fig. 22, produce a tension of $T=2W$ lb. in the wire.

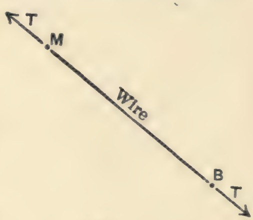


FIG. 22

Explain the effect of the forces acting upon the rod AB.

EXERCISE 36. A man weighing 160 lb. rests in a hammock suspended by ropes which are inclined at 30° and 45° to two vertical posts. Find the pull in each rope.

EXERCISE 37. A man weighing W lb. is seated in a loop at the end of a rope l feet long, the other end being fastened to a point above. What horizontal force will pull him m feet from the vertical, and what will be the pull on the rope? If the rope is just strong enough to support the man when the rope is vertical, will it support the man when displaced from the vertical?

EXERCISE 38. A thread whose length is $2l$ is fastened at two points, A and B, in the same horizontal, and distant l from each other. The thread carries a smooth ring of weight W. Find the tension in the thread.

EXERCISE 39. What weight can be sustained on a smooth inclined plane, the ratio of whose height to base is 5 : 12, by a horizontal force of 10 pounds and a force of 50 pounds parallel to the plane. What is the pressure on the plane?

EXERCISE 40. Five men pull upon five ropes knotted together towards the south, northeast, east, northwest, and 30° south of west respectively. If the first three exert forces of 70, 30, and 40 pounds respectively and the knot does not move, find the forces exerted by the remaining two men.

SECTION IV

STATICAL FRICTION

Thus far the surfaces of bodies in contact have been assumed perfectly smooth, that is, they offered no resistance to the motion of the bodies parallel to their surfaces. In reality no body can be made perfectly smooth, and thus if a body weighing W lb. is to be moved along a horizontal plane, a certain force P is necessary to start the body, Fig. 23.

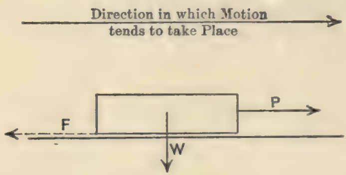


FIG. 23

This force P overcomes an equal but opposite force F, arising from the irregularities in the surfaces of contact which fit more or less closely into one another. This is shown in Fig. 24, which represents a very highly magnified view of the contact between the weight and plane. *This force, F, is called Friction.* From an inspection of Fig. 24 it is evident that the irregularities will always grip in such a way as to resist any motion of the weight W. So that: *The direction in which friction acts is always opposite to the direction in which motion would take place if there were no friction.*

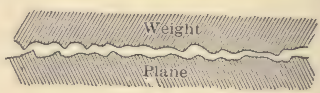


FIG. 24.

The amount of friction up to a certain limit is always just sufficient to prevent motion. But only a limiting amount of friction can be called into play. We may suppose that this limiting amount of friction is reached when either the irregularities give way and break, or when the body lifts sufficiently to allow the irregularities to clear one another.

Thus, if a body rests upon a horizontal table, the pressure of the table balances the weight; and as these forces are both vertical, there is no tendency to move in the horizontal direction and no friction is called into play. Apply a small force parallel to the surface; the body does not move; sufficient friction is exerted to just balance the applied force; increase the applied force and still the friction increases so as to hold the force in equilibrium until the applied force reaches a certain magnitude, which the friction cannot reach, and the body moves. (Try this experiment with a book.)

Experiment shows *that the amount of this limiting friction varies as the normal pressure between surfaces in contact*, or is directly proportional to the force with which they are pressed together. Therefore, if

$$F = \text{limiting friction}$$

and $$N = \text{normal pressure},$$

$$F \propto N,$$

and $$\therefore\ F = \mu N,\text{*}$$

where μ is a constant depending upon the material of the surfaces in contact and the state of their polish, but not on their area or shape.

This constant μ, the *Coefficient of Friction*, is deter-

* The sign $\propto$ is read "varies as," and μ is called a proportionality factor.

mined by experiment and for some substances is approximately as follows:

Wood on wood or metal, surface dry........	0.4 to 0.6
" " " " ' " lubricated..	0.1 to 0.2
Metal on metal, surface dry.	0.2
" " " " lubricated	0.075
Steel on ice.	0.02

EXERCISE 41. Find the greatest horizontal force which can be applied to a sled shod with steel and resting on a horizontal sheet of ice without causing motion, if the weight of the sled is 50 lb. and the load is 200 lb.

EXERCISE 42. An iron mass of 1000 lb. rests upon a horizontal wooden floor. What will be the least horizontal force necessary to move the same if the coefficient of friction is 0.5? Between what limits may the applied force vary without producing motion?

Consider a weight W placed upon a rough inclined plane (Fig. 25). The tendency to move is in the down-

FIG. 25

FIG. 26

ward direction and we thus have the friction acting upward and parallel to the plane.

This force with the other forces acting upon W are shown in Fig. 26, and as the weight is assumed at rest we have:

Forces	X Components	Y Components
W	$-W \sin \theta$	$-W \cos \theta$
f	f	0
n	0	n

$$\Sigma X = -W \sin \theta + f = 0,$$
$$\Sigma Y = -W \cos \theta + n = 0,$$

where Σ (sigma, the Greek S) denotes summation. Thus ΣX is read, *the sum of the X components.* Therefore $n = W \cos \theta, \quad f = W \sin \theta.$

Thus the friction $f = W \sin \theta$, where θ is the inclination of the plane.

Assume θ to vary from 0 to $\frac{\pi}{2}$, then $\sin \theta$ varies from 0 to 1 and the friction, to preserve equilibrium, would have to vary from 0 to W.

It is evident that f cannot possibly equal W or even approach this value, and therefore equilibrium cannot be preserved throughout the whole change of θ. As θ increases from 0, some limiting angle must be reached beyond which the plane cannot be inclined without causing the body to slide. If we call this limiting angle α, then in Fig. 27 the weight will be on the point of sliding and is retained in equilibrium by the limiting friction F, and the values of n and f above deduced become

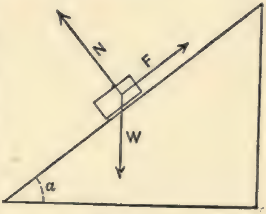


FIG. 27

$$N = W \cos \alpha, \qquad F = W \sin \alpha,$$

from which $\mu = \frac{F}{N} = \frac{W \sin \alpha}{W \cos \alpha} = \tan \alpha$ can be obtained

The limiting angle, α, is known as the *Angle of Friction* or the *Angle of Repose;* therefore: *The tangent of the angle of friction is equal to the coefficient of friction.*

As has already been pointed out (page 15), a surface always exerts a normal pressure or reaction upon a body in contact with it. If the plane is rough we will have, under certain conditions, the friction as an additional reaction (Fig. 28). The resultant of these forces is called the *Total Reaction* of the surface. If ϕ be the angle between the normal to the surface and the total reaction, we have

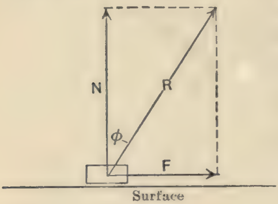


FIG. 28

$$\tan\phi = \frac{F}{N},$$

and as

$$\frac{F}{N} = \mu = \tan\alpha,$$

$$\tan\phi = \tan\alpha,$$

$$\therefore\ \phi = \alpha.$$

Therefore:

The greatest angle that the total reaction of a rough surface can make with the normal is equal to the angle of friction.

Example.—A heavy body rests on a rough plane inclined at an angle 30° to the horizontal, the coefficient of friction being $\frac{2}{\sqrt{3}}$. A horizontal force is applied to the body, and is gradually increased until the body begins to move up the plane; find the magnitude of the horizontal force.

Solution.—As $\mu=\frac{2}{\sqrt{3}}$, the tangent of the limiting angle (α) to which the plane may be inclined without causing the weight to slide is $\frac{2}{\sqrt{3}}$, or

$$\tan\alpha=\frac{2}{\sqrt{3}}=1.15+;$$

$$\therefore\ \alpha=49^\circ+.$$

As the plane is only inclined at 30°, the body will remain at rest upon the plane with only part of the total amount of friction (acting upward) brought into play (Fig. 29).

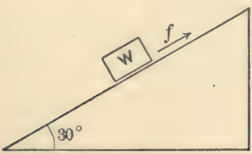


FIG. 29

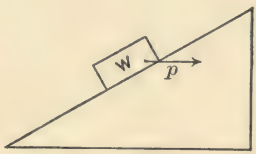


FIG. 30

If the force p, Fig. 30, be now applied, this force will tend to cause motion up the plane, thus relieving f, and as p increases it causes f to act down the plane, finally overcoming f, thus producing motion *up* the plane.

The value of p which we are to consider is the limiting one just sufficient to balance the limiting value of f acting downward. We will call these values of p and f, P and F.

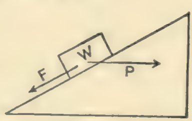


FIG. 31

Fig. 31 shows this condition of equilibrium.

Fig. 32 shows us *all* the forces acting upon the body. We have:

Forces	X Components	Y Components
P	P	0
N	$-N\sin 30^\circ$	$N\cos 30^\circ$
F	$-F\cos 30^\circ$	$-F\sin 30^\circ$
W	0	$-W$

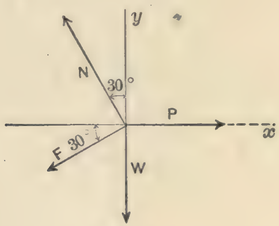


FIG. 32

and the conditions of equilibrium are

$$\Sigma X = P - N\sin 30^\circ - F\cos 30^\circ = 0,$$

$$\Sigma Y = N\cos 30^\circ - F\sin 30^\circ - W = 0,$$

or

$$P = \frac{1}{2}N + \frac{\sqrt{3}}{2}F, \quad \dots\dots \quad (1)$$

$$W = \frac{\sqrt{3}}{2}N - \frac{1}{2}F. \quad \dots\dots \quad (2)$$

These equations contain three unknown quantities, N, P, and F. So that we must find another equation before we can effect a solution. This third equation is obtained through the law of friction. We know the coefficient of friction, and as F is the *limiting value of the friction* and N the *normal pressure* between the surfaces,

we have

$$F = \mu N = \frac{2}{\sqrt{3}}N = \frac{2\sqrt{3}}{3}N. \quad \dots \quad (3)$$

We can now find P in terms of W from equations (1), (2), and (3). Substituting F from (3) in (1) and (2),

$$P=\frac{1}{2}N+N=\frac{3}{2}N, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$W=\frac{\sqrt{3}}{2}N-\frac{\sqrt{3}}{3}N=\frac{\sqrt{3}}{6}N. \quad . \quad . \quad . \quad (5)$$

Substitute N from (4) in (5),

$$W=\frac{\sqrt{3}}{9}P;$$

$$\therefore\ P=3\sqrt{3}W.$$

EXERCISE 43. A weight W rests in equilibrium on a rough inclined plane, being just on the point of slipping down. On applying a force W parallel to the plane, the weight is just on the point of moving up. Find the angle of the plane and the coefficient of friction.

EXERCISE 44. What is the coefficient of friction when a body weighing 50 lb. just rests on a plane inclined at 30° to the horizontal? If the plane were horizontal, what horizontal force would be required to move the body?

EXERCISE 45. What horizontal force will be required to support a weight of 300 lb. upon a smooth inclined plane whose height is 3/5 of its length?

EXERCISE 46. A block of iron weighing 10 lb. rests on a level plate. A string attached to the block passes over a pulley so placed above the plate that the string makes an angle of 45° with the vertical. After passing over this pulley the string supports a weight. Find the least value of this weight which will make the block slip, the coefficient of friction being 0.2.

EXERCISE 47. Explain with diagrams two methods for experimentally determining the coefficient of friction.

Exercise 48. A weight of 60 pounds rests on a rough level floor. Find the least horizontal force that will move it ($\mu=0.5$). Find the total reaction of the floor. What is its direction?

Exercise 49. Find the least angle of inclination of a wooden incline that stone blocks may slide down under the action of gravity. $\left(\mu=\frac{1}{\sqrt{3}}.\right)$

Exercise 50. On a hill sloping 1 in 50 a loaded sled weighing one ton is kept from sliding down. Show that the pull of the horses may vary from 360 to 440 pounds, the coefficient of friction between sled and snow being 0.2.

Section V

MOMENTS

We will now consider another method of obtaining the relations existing between forces acting on a particle.

In Fig. 33 consider the force F acting on a particle at A and conceive a fixed point O to be rigidly connected to A.

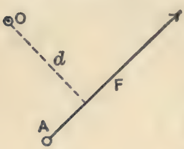


Fig. 33

The force F will then tend to move the particle A about O as centre. The effect of the force in producing rotation about O depends not only upon the magnitude of the force F, but also upon the distance of the line of action of F from O. This *perpendicular distance, d, is called the arm of the force.* We may then state as a definition that the

Moment of a force with reference to a point is the product of the force and its arm,

wherein the word *moment* is used in its old-fashioned sense of *importance* or *influence;* so that the moment of a force with reference to a point means its influence in producing rotation about this point.

The point O, from which the perpendicular is drawn, is called the *origin of moments* and may be chosen arbitrarily.

The algebraic sign of a moment is considered positive if it tends to turn the system in a direction opposite to that of the hands of a watch, and negative if in the other direction. This assumption is purely arbitrary, but it will be noticed that it conforms with the positive and negative directions assumed in the measurement of angles in Trigonometry. Thus in Fig. 34 the moments of the forces with reference to the origin of moments O are $-F_1d_1$; $+F_2d_2$; $+F_3d_3$; $F_4(0)=0$; and $-F_5d_5$.

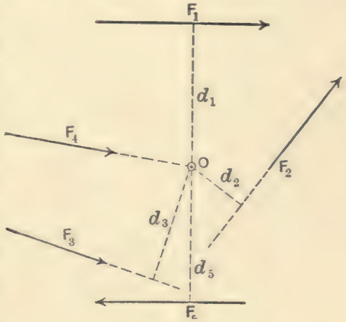


FIG. 34

As the moment of a force $=Fd$, the unit moment is the moment due to a force of 1 pound whose arm is 1 foot. No name has been assigned to this unit.

EXERCISE 51. A force of 5 pounds acts along one side of an equilateral triangle whose side is 2 feet long. Find the moment about the vertex of the opposite angle.

EXERCISE 52. A force of P pounds acts along the diagonal of a square whose side is $2n$ feet. Find the moments of P about each of the four vertices.

EXERCISE 53. If P is the thrust along the connecting-rod of an engine, r the crank-radius, and the connecting-rod is inclined to the crank at 150°, show that the moment of the thrust about the crank-axis is one-half the greatest moment possible.

EXERCISE 54. At what height from the foot of a tree must one end of a rope, whose length is l feet, be fastened so that a given force acting at the other end may have the greatest tendency to overturn it, assuming the force applied at the ground.

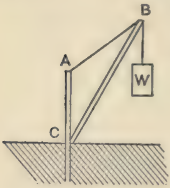


FIG. 35

EXERCISE 55. The post AC, Fig. 35, of a jib-crane is 10 feet long; the jib CB is inclined at 30°, and the tie AB at 60°, to the vertical. If the weight lifted is 10 tons, find the moment about C tending to upset the crane.

The moment of a force may be geometrically represented by twice the area of a triangle whose base is the force and whose altitude is the arm of the force. Thus in Fig. 36, 2(area of $\triangle OAB$) $= Fd =$ moment of F about O.

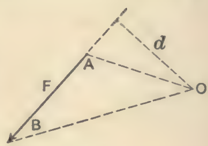


FIG. 36

This geometrical representation will be used to prove *Varignon's Theorem of Moments:*

The sum of the moments of two forces, F_1 and F_2, about any point O in their plane is equal to the moment of their resultant, R, about the same point.

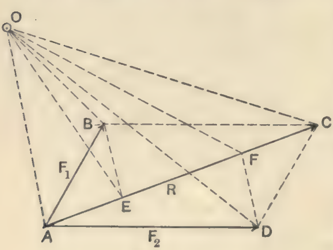


FIG. 37

In Fig. 37 (moment of F_1 about O) $= 2\triangle OAB$, (moment of F_2 about O) $= 2\triangle OAD$, (moment of R about O) $= 2\triangle OAC$. We must prove that (moment of F_1) + (moment of F_2) = (moment of R),

or $$2\triangle OAB + 2\triangle OAD = 2\triangle OAC.$$

Draw BE and $DF \parallel OA$. Then

$\triangle OAB \backsimeq \triangle OAE$ and $\triangle OAD \backsimeq \triangle OAF$. (Why?)

Prove $AE = CF$, by means of $\triangle ABE$ and $\triangle DCF$. Then

$$\triangle OAE \backsimeq \triangle OFC. \quad \text{(Why?)}$$

From this the remainder of the proof may be easily deduced.

Theorem: If any number of forces acting upon a particle are in equilibrium, the algebraic sum of their moments about any point will be zero.

To prove this proposition consider the forces F_1, F_2, . . ., Fig. 38, to be in equilibrium, and O any origin of moments. We must now prove

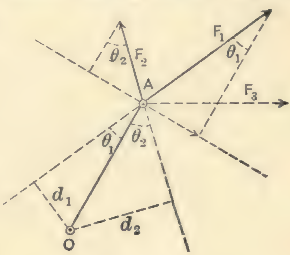


FIG. 38

$$F_1 d_1 + F_2 d_2 + \ldots = 0.$$

Join O and A and find the component forces $\perp$ to OA; then

$$F_1 \sin\theta_1 + F_2 \sin\theta_2 + \ldots = 0. \quad \text{(Why?)}$$

But $$\sin\theta_1 = \frac{d_1}{OA}, \quad \sin\theta_2 = \frac{d_2}{OA}, \quad \text{etc.}$$

Therefore $$F_1 \frac{d_1}{OA} + F_2 \frac{d_2}{OA} + \ldots = 0$$

and $$F_1 d_1 + F_2 d_2 + \ldots = 0;$$

or $$\Sigma \text{ moment} = \Sigma M = 0.$$

Example.—A brace AB rests against a smooth vertical wall and upon a rough horizontal plane, and supports a weight W at its upper end. Find the compression in the brace if the angle CAB is θ.

Solution.—The forces acting at A are W, N, and C, Fig. 39.

As these forces are in equilibrium we may select *any* point as our origin of moments and equate the sum of the moments to zero. But as we seek a relation between C and W, we should attempt to exclude N from our equation of moments. This can only be done by choosing the origin of moments upon the line of action of N, thus reducing its arm to zero. Besides, the origin of moments should be chosen conveniently for calculation. Vertically above B at O is a good position. Thus,

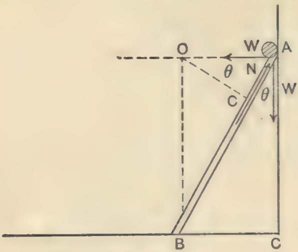


FIG. 39

$$\Sigma M = -W(\overline{OA}) + C(\overline{OA}\cos\theta) = 0; \quad \therefore\ C = W\sec\theta.$$

EXERCISE 56. Find the normal reaction of the wall in the preceding example.

EXERCISE 57. A weight W is attached to a string which is secured at A to a vertical wall and pushed from the vertical by a strut BC perpendicular to the wall; find the pressure on BC when the angle CAB is θ.

EXERCISE 58. A rod whose length is $BC = l$ is secured at a point B in a horizontal plane, and the end C held up by a cord AC so that $\angle ABC$ is θ and the distance $AB = a$; required the compression in BC due to a weight W applied at C,

CHAPTER II

FORCES ACTING ON A RIGID BODY

SECTION VI

RESULTANT OF TWO FORCES. COUPLES

To the present our work has been confined to the investigation of forces applied to a single point or particle. We shall now consider the action of forces applied at different points of a rigid body.

A body is said to be rigid when the particles of which it is composed retain their relative positions no matter what external forces may be applied to the body. Practically no such thing as a rigid body is found in nature, but if the body considered "gives" under the action of the applied forces, the methods now to be described may still be employed, provided the positions of the points of application considered are assumed after all changes in the body have ceased.

FIG. 40

If any force F (Fig. 40) be applied to a body at A, the effect of this force will remain unchanged if its point of application be *moved to any other point in the line of action of the force*, such as A'. This principle is known as the *transmissibility of a force*.

Let the question be proposed to find the *resultant of two non-parallel forces* F_1 and F_2, Fig. 41, applied to a body, B, at the points A_1 and A_2.

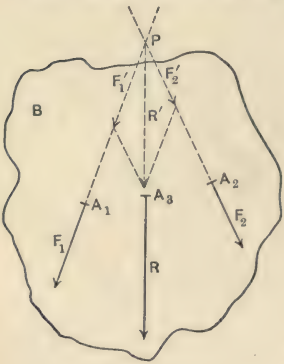


FIG. 41

Then by the above principle the force F_1 may have its point of application transferred to *any* point in its line of action; the same holds for the force F_2. Let us select the point P at the intersection of their lines of action as the new point of application. The transference of the forces F_1 and F_2 to the new point P reduces the problem to the simple case of the parallelogram of forces, and the resultant R may now be applied to any point in its line of action PA_3.

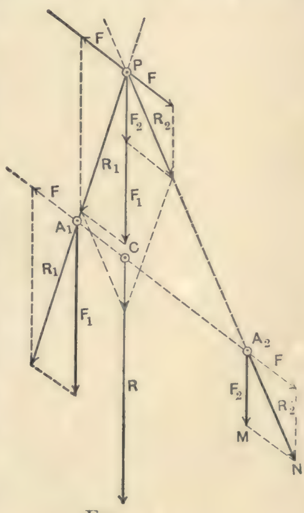


FIG. 42

Consider now the computation of *the resultant of two parallel forces* F_1 and F_2, Fig. 42. It is evident that the method of transmissibility of forces will in this case lead to no result. (Why?) So a special artifice must be used.

Two equal and oppositely directed forces F having a common line of action A_1A_2 are introduced. As these forces neutralize each other their introduction will not affect the original problem. Combine these forces with F_1 and F_2 and obtain the forces R_1 and R_2. Now, instead of considering F_1 and F_2, find the resultant of R_1 and R_2, which we will denote by R, whose point of application is anywhere in the line PC, at C if we so choose.

By resolving R_1 and R_2 at P into components along PC and parallel to A_1A_2 the *magnitude of R will be found to be F_1+F_2*, and its direction will be parallel to F_1 and F_2. (Prove this.) To completely determine the resultant it is sufficient to locate *any* convenient point, say C on A_1A_2, on its line of action. From the similar triangles A_2MN and PCA_2 we find that

$$\frac{F_2}{F}=\frac{PC}{CA_2}; \quad \text{similarly} \quad \frac{F_1}{F}=\frac{PC}{A_1C};$$

$$\therefore \frac{F_2}{F_1}=\frac{CA_1}{CA_2}, \quad \text{or} \quad F_1(CA_1)=F_2(CA_2).$$

As the angle between the forces F_1, F_2 and the line A_1A_2 does not enter into this discussion, it should be noticed that the resultant R always passes through C irrespective of the direction of the parallel forces.

This point C is therefore called the *Center of the Parallel Forces.*

EXERCISE 59. Show how to find the resultant of two unequal parallel forces acting in opposite directions but not in the same straight line.

Through Ex. 59 we arrive at the result shown in Fig. 43. The resultant of the two unequal parallel forces, F_1 and F_2, is parallel to them, but divides the line A_1A_2

externally at C. The resultant has the same direction as the greater of the two forces, F_2, and equals F_2-F_1. Also $F_2(CA_2)=F_1(CA_1)$.

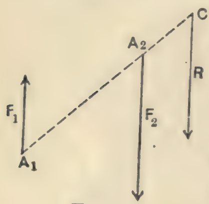


FIG. 43

EXERCISE 60. Three forces are represented by the lines AD, BC, and DB in a parallelogram $ABCD$. Show that AC is their resultant.

EXERCISE 61. Two forces acting upon a body are located with reference to a set of rectangular axes as follows: F_1, point of application $(-4, -4)$, with arrow representing force extending to $(2, 6)$; F_2, point of application $(1, 7)$, arrow extending to $(14, 6)$. Find the resultant.

EXERCISE 62. Two parallel forces of 3 and 5 pounds act in the same direction at points 2 feet apart. Find their resultant in magnitude and position.

EXERCISE 63. Two parallel forces of 3 and 5 pounds act in opposite directions at points 2 feet apart. Find their resultant in magnitude and position.

EXERCISE 64. "We have a set of hay-scales and sometimes we have to weigh wagons that are too long to go on them. Can we get the correct weight by weighing one end at a time and then adding the two weights?"

Consider the *case of two equal parallel forces acting in opposite directions* (Fig. 44). If we here apply the artifice used when the forces are unequal, we find that the lines of action of R_1 and R_1 are still parallel and the method fails.

But from the results obtained in Ex. 59 we have $R=F_1-F_1=0$ and $F_1(CA_1)=F_1(CA_2)$; $\therefore CA_1=CA_2$.

But as C must divide A_1A_2 externally, CA_1 can only equal CA_2, as $CA_2 \doteq \infty$. (The sign $\doteq$ is read "approaches," $\therefore$ CA_2 approaches infinity.) In other words, *the two forces cannot be reduced to a single force in a definite position.*

This combination of two equal parallel forces acting in opposite directions, but not in the same straight line, which cannot be replaced by a single force, is called a couple.

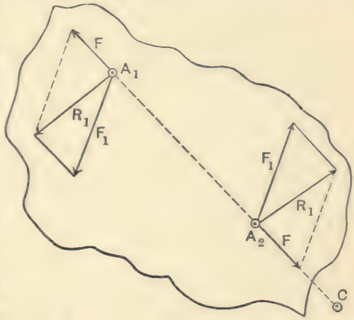


FIG. 44

As shown a couple cannot be reduced to a single force, and its action therefore is not that of a force. It thus becomes imperative to study more fully the properties of couples.

The tendency of a couple is to cause rotation. This is illustrated in the operation of a screw copying-press, or in the winding of a watch or clock. This tendency to produce rotation is measured, as explained on page 38, by the moments of the forces.

Select any point O, Fig. 45, in the plane of the couple and draw the line Oab perpendicular to the lines of action of the forces F. The sum of the moments of the forces about O is $-F(aO)+F(bO)=F(bO-aO)=F(ab)$. The *distance* ab *is called the arm of the couple, and we have shown that the moment of the couple* $[F(ab)]$ *about any point in its plane is the prod-*

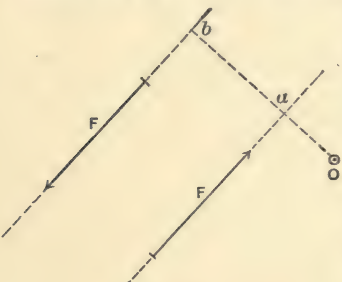


FIG. 45

uct of one of the forces and the arm; the sign to be determined, as on page 39, by the direction in which the body acted on would tend to turn.

Theorems Concerning Couples

In the proofs of the following theorems we shall always consider the forces of the couple applied at the ends of their arm. That a couple can always be reduced to this form is evident from Fig. 45. Here we may move the forces F along their lines of action until the tails of their arrows are respectively at a and b.

We will first prove that the action of a couple remains unaltered if it is moved anywhere in its plane in such a position that the forces remain parallel to their original directions, or

The translation of a couple does not alter its effect upon the body.

In Fig. 46 consider the couple $F(A_1A_2)F$. We wish to show that it is equivalent to the couple $F(A_1''A_2'')F$, in which the F's are the unmarked ones.

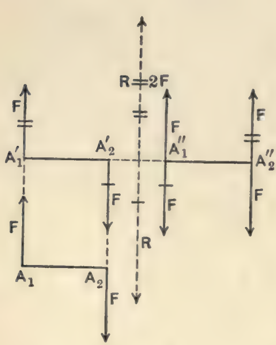


FIG. 46

To do this first move its forces by the principle of transmissibility to the position $A_1'A_2'$. Then anywhere on $A_1'A_2'$ prolonged take $A_1''A_2''$ equal to $A_1'A_2' = A_1A_2$. Introduce at A_1'' and A_2'' two pairs of equal and oppositely directed forces F. The body originally acted on by the couple $F(A_1A_2)F$ will now be acted upon by six forces, but its state of rest or motion will not be altered.

Now the singly marked F's may be replaced by the singly marked $R=2F$, their resultant. Similarly for the doubly marked forces. The R's balance each other and their action may be neglected. Thus all forces but the unmarked ones at A_1'' and A_2'' are accounted for. These evidently form the couple $F(A_1''A_2'')F$, which is therefore equivalent to the couple $F(A_1A_2)F$.

We will now show that *a couple may be rotated about any point without altering its action.*

In Fig. 47 consider the couple $F(A_1A_2)F$. On any line MA_2' lay off $MA_1'=MA_1$ and $A_1'A_2'=A_1A_2$. At A_1' and A_2' introduce the pairs of forces $F \perp$ to $A_1'A_2'$. Now the singly marked F's reduce to their singly marked resultant R. Similarly for the doubly marked F's.

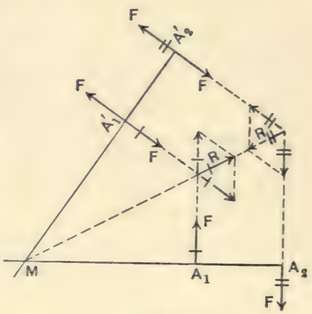


FIG. 47

The resultants R have a common line of action (which bisects $\angle A_1MA_1'$) and are of equal magnitude. (Prove this.) They thus neutralize each other and there remains only the couple $F(A_1'A_2')F$, which proves the proposition.

As a corollary to the above theorems we can state that *A couple can be shifted into any position in its plane without altering its action.*

Another important theorem concerning couples is the following:

Any couple may be replaced by another couple of equal moment.

This may be demonstrated thus: In Fig. 48 consider the couple $F(A_1A_2)F$. Let A_3 be any point in A_1A_2 prolonged. Introduce two pairs of equal and opposite forces $P \perp$ to A_1A_3 at A_3 and A_2. Let us, however, make P of such magnitude that $P(A_2A_3) = F(A_1A_2)$. The resultant of the singly marked P and F will be $R_1 = F + P$ acting at A_2 (see page 44), and the resultant of the doubly marked P and F is $R_2 = F + P$ acting at A_2. These resultants neutralize each other and there remains only the couple $P(A_2A_3)P$; but as $P(A_2A_3)$ was made equal to $F(A_1A_2)$, this new couple has the same moment as the original couple. Combining this result with the previous theorem we arrive at our present theorem.

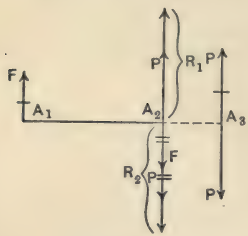


FIG. 48

We can now state that *the resultant of any number of couples is a couple whose moment is the sum of the moments of the given couples.*

For the given couples may all be reduced to equivalent couples having a given arm. They may then be shifted until the lines of action of their forces coincide in pairs. Their forces may then be added and there results a couple whose moment is equal to the sum of the moments of the given couples.

EXERCISE 65. Prove the last theorem for the case of three couples whose moments are Pa, Qb, and Rc and whose arms are a, b, and c respectively.

EXERCISE 66. Along the sides AD and CB of a rectangle

$ABCD$, whose side AD is s feet long and whose side AB is s yards long, forces of F pounds act, and along AB and CD forces of $5F$ pounds act. Find the moment of the equivalent couple. In what position should this equivalent couple be placed with reference to the rectangle $ABCD$?

It is important to notice that *a single force F acting at C*, Fig. 49, *and a couple $P(a)P$ acting on a body in the same plane cannot be in equilibrium.* For, the couple $P(a)P$ may be replaced by a couple $F(b)F$, if $Pa = Fb$, and this new couple may be placed anywhere in the plane. If placed as shown in Fig. 49, the forces at C are in equilibrium, leaving the force F at D unbalanced.

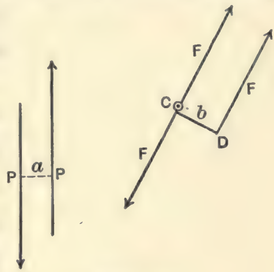


FIG. 49

EXERCISE 67. Assume a force of 100 pounds and a couple whose forces are 50 pounds and whose arm is 4 feet. Find their resultant.

SECTION VII

RESULTANT OF ANY NUMBER OF FORCES

We are now in position to find the resultant of any number of forces acting upon a rigid body.

Let the forces $F_1, F_2, F_3, \ldots$ act at the points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3, y_3) \ldots$ of a body referred to rectangular axes. Consider the force F_1 resolved into two components X_1 and Y_1, Fig. 50. At the origin O apply two opposing forces each equal and parallel to X_1, and similarly for Y_1. This will not alter the given problem. X_1 acting at P_1 may now be replaced by X_1 acting at O *plus* a couple whose moment is $-X_1y_1$, and Y_1 acting at P may be replaced by Y_1 acting at O *plus* a couple whose moment is Y_1x_1. Similarly for all the other forces F_2, F_3, . . .

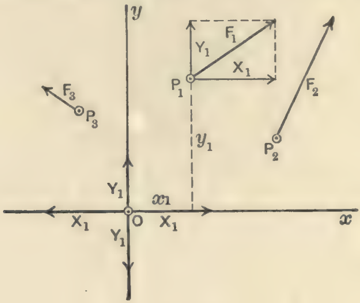


FIG. 50

The resultant of all the component forces acting at O may now be found as already explained on pages 24 and 25. We have

$$R_x = X_1 + X_2 + X_3 \ . \ . \ . = \Sigma X,$$

$$R_y = Y_1 + Y_2 + Y_3 \ . \ . \ . = \Sigma Y,$$

and $\therefore$ $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$ and $\tan\alpha = \dfrac{\Sigma Y}{\Sigma X}$.

We have thus found the magnitude and direction of the resultant, but it is evident that the line of action of the resultant need not pass through the origin.

So far we have neglected the couples introduced above. By adding their moments we may obtain the moment of the resultant couple:

$$M = (Y_1x_1 - X_1y_1) + (Y_2x_2 - X_2y_2) + (Y_3x_3 - X_3y_3) + \ . \ . \ .$$
$$= \Sigma(Yx - Xy).$$

We shall assume this to be the moment of a couple whose forces are R, R, and whose arm is a. R having been previously determined, the equation $M = Ra = \Sigma(Yx - Xy)$ determines a. These results are illustrated in Fig. 51. R acting at O shows the magnitude and direction of the forces acting at O, Fig. 50, and RaR the couple whose moment is M, placed in the convenient position for finding their resultant (see page 51).

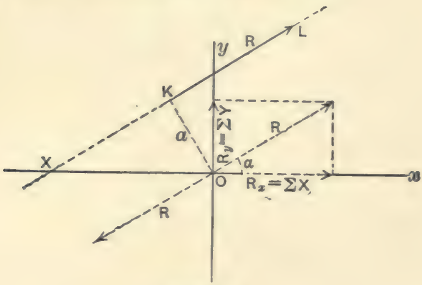


FIG. 51

The forces R, R acting at O neutralize each other, and this leaves KL as the final resultant, which may be applied at any point in its line of action KL.

The intercept of the line of action of the resultant KL, Fig. 51, upon the X-axis may readily be found to be $(OX) = \frac{a}{\sin \alpha}$, and as $\sin \alpha = \frac{\Sigma Y}{R}$, we have

$$(OX) = \frac{aR}{\Sigma Y}.$$

To condense this theory for actual computation we need simply fill out the following table:

Forces	x	y	X	Y	Yx	Xy	$Yx - Xy$
F_1	x_1	y_1	X_1	Y_1	Y_1x_1	X_1y_1	$Y_1x_1 - X_1y_1$
F_2	x_2	y_2	X_2	Y_2	Y_2x_2	X_2y_2	$Y_2x_2 - X_2y_2$
....	....	....	....	....	....	....	....

Then obtain the ΣX, ΣY, $\Sigma(Yx-Xy)=Ra$. Finally,

$$R=\sqrt{(\Sigma X)^2+(\Sigma Y)^2}, \quad \alpha=\tan^{-1}\frac{\Sigma Y}{\Sigma X},$$

and the intercept of the line of action of the resultant upon the X-axis is

$$(OX)=\frac{Ra}{\Sigma Y},$$

which gives us all the data necessary for the location of the resultant.

EXERCISE 68. Three forces F_1, F_2, F_3 have their points of application at (2, 4), (4, −1), (−2, −2), and the arrows representing their ends at (6, 7), (−2, −6), and (−7, 10), respectively. Find their resultant.

EXERCISE 69. Find the resultant of the following forces, given a point on each line of action, the angle each line of action makes with the X-axis, and their magnitudes:

1st force: (3, 2); 100°; 50 pounds.
2d force: (−1, 3); 200°; 100 pounds.
3d force: (−2, −4); 30°; 60 pounds.

EXERCISE 70. Find the resultant of the following forces: $P_1=40$ pounds, $P_2=30$ pounds, and $P_3=90$ pounds, whose lines of action make angles of 60°, 280°, and 140° with the X-axis and whose intercepts on the X-axis are 9 feet, 5 feet, and 0 feet, respectively.

SECTION VIII

CONDITIONS FOR EQUILIBRIUM

The motion of a rigid body may always be considered as either a *translation*, a *rotation*, or a *combination of the two.*

In the *motion of translation* the spaces described simultaneously by the different parts of the body are parallel and equal to each other. Thus if, in Fig. 52, A_1, B_1, C_1 represent the positions of the particles of a body at a time t_1, and A_2, B_2, C_2 represent their positions at some subsequent time t_2, the body is said to possess a motion of translation.

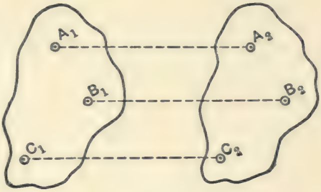


FIG. 52

In the *motion of rotation* the parts of the body describe concentric arcs of circles about a certain line, called the *axis of rotation*. Thus in Fig. 53 the two positions shown are those of a body possessed of a motion of rotation, and a perpendicular to the plane of the paper passing through O would be the axis of rotation.

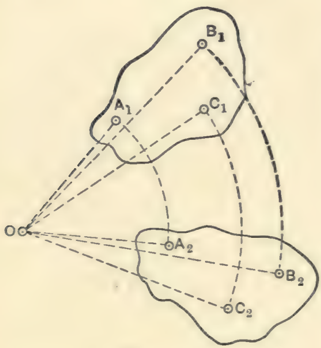


FIG. 53

Every more complex motion can be considered as the combination of a translation and of a rotation. Thus, in Fig. 54, if the body $A_1B_1C_1$ moves to the position $A_2B_2C_2$, we may *first* conceive it to move by translation to $A'B'C'$ and *then* by rotation about O as an axis from $A'B'C'$ to $A_2B_2C_2$.

Thus, only *two kinds of motion, Translation and Rotation, need be considered.*

Translation is always caused by *the resultant of the* *forces*, and *Rotation* by *the resultant of the couples* acting on a body.

The forces shown in Fig. 50, page 52, acting on the body have been reduced, as explained on pages 52 and 53, to the force R and a couple whose moment is $M = Ra$. This body, if acted on only by the forces F_1, F_2, F_3, . . . , would possess a compound motion; the translation of which would be due to the force R, and the rotation of which would be due to a couple whose moment is Ra.

FIG. 54

Conceive now that the body is at rest, possessing neither translation nor rotation. This would imply that R (the cause of the translation) is zero and $M = Ra$ (the cause of the rotation) is zero.

Therefore, if *any rigid body is in equilibrium under the action of forces*,

$$\Sigma X = 0, \quad \Sigma Y = 0, \quad \text{and} \quad \Sigma(Yx - Xy) = 0,$$

the X and Y axes to be assumed at pleasure.

In the application of this principle proceed as follows:

(a) Draw a sketch illustrating the problem.

(b) Draw a diagram showing the body considered as a *free body*.

(*c*) Into the last sketch introduce the coördinate axes and fill out a table similar to that shown on page 53, thus obtaining the ΣX, the ΣY, and the $\Sigma(Yx-Xy)$.

The choice of the position of the axes must be left largely to the ingenuity of the student.

(*d*) Solve the equations obtained by putting the three sums just found each equal to zero for whatever unknown quantities are sought.

(*e*) Interpret the results.

Example.—A beam AB rests on smooth horizontal ground at A and on a smooth inclined plane at B; a string is fastened at B and, passing over a smooth peg at the top of the plane, supports a weight P. If W, the weight of the beam, acts at the centre of the beam and α be the inclination of the plane, find P and the reactions on the rod.

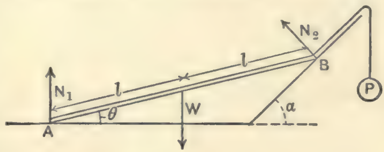


FIG. 55

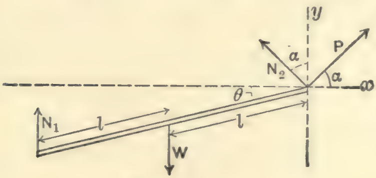


FIG. 56

Solution.—(*a*) Draw a diagram illustrating the problem (Fig. 55).

(*b*) Draw the rigid body under consideration (beam) alone (Fig. 56), showing *all* the reactions and other forces acting upon it.

(*c*) Introduce the coördinate axes and complete the table:

F	x	y	X	Y	Yx	Xy	$Yx-Xy$
N_1	$-2l\cos\theta$	$-2l\sin\theta$	0	N_1	$-2N_1l\cos\theta$	0	$-2N_1l\cos\theta$
W	$-l\cos\theta$	$-l\sin\theta$	0	$-W$	$Wl\cos\theta$	0	$Wl\cos\theta$
N_2	0	0	$-N_2\sin\alpha$	$N_2\cos\alpha$	0	0	0
P	0	0	$+P\cos\alpha$	$P\sin\alpha$	0	0	0

Then, as the beam is in equilibrium,

$$\Sigma X = 0 = -N_2 \sin\alpha + P\cos\alpha = 0,$$

$$\Sigma Y = 0 = N_1 - W + N_2\cos\alpha + P\sin\alpha = 0,$$

and $$\Sigma(Yx - Xy) = -2N_1 l\cos\theta + Wl\cos\theta = 0.$$

(*d*) Solving the equations, we obtain

$$N_1 = \frac{W}{2};\quad P = \frac{W}{2}\sin\alpha;\quad N_2 = \frac{W}{2}\cos\alpha.$$

Exercise 71. A beam 20 feet long and weighing 1000 pounds rests at an angle of 60° with one end against a smooth vertical wall and the other end on smooth horizontal ground. It is held from slipping by a rope extending horizontally from the foot of the beam to the foot of the wall. Find the tension in the string and the reaction at the ground and wall.

Exercise 72. Same as Exercise 71, but assuming the wall rough, with $\mu = 0.2$.

Exercise 73. Same as Exercise 71, but assuming the ground rough, $\mu = 0.3$.

Exercise 74. Same as Exercise 71, but assuming the wall rough, $\mu = 0.2$, and the ground rough, $\mu = 0.3$.

Exercise 75. A beam 20 ft. long rests with its upper end against a smooth vertical wall. Its lower end rests on a smooth horizontal plane and is prevented from slipping by a rope 16 feet long fastened to it and the base of the wall. The weight of the beam is W and acts at a point 5 feet from its upper end. Find the reactions of the wall and the plane and the tension in the rope.

Exercise 76. A cellar-door AB, hinged at its upper edge, A, rests at an angle of 45° with the horizontal when a horizontal force F is applied to its lower edge, B. If the weight of the door, W, be assumed to act at its midpoint, find the force F and the reaction at the hinge A.

Exercise 77. A uniform rod, length $2a$ and weight W, rests with one end against the inner surface of a smooth hemispherical bowl whose edge is horizontal and whose radius is r. The rod is also supported at some point of its length by the edge of the bowl. Find its position of equilibrium.

Another method of obtaining the equation $\Sigma(Yx-Xy)=0$, used in the solution of the above exercises, which is perhaps less laborious than that described on page 57, is the following:

From Fig. 57 it follows, by Varignon's theorem of moments, page 40, that

$$Yx-Xy=-Fd;$$

$$\therefore\ \Sigma(Yx-Xy)=\Sigma(Fd).$$

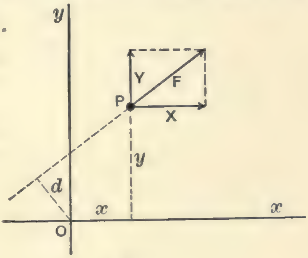


FIG. 57

But $\Sigma(Fd)$ represents the sum of the moments of all the forces about the origin O, and as the origin O may be taken in any position we have as the *conditions of equilibrium* the three equations:

Sum of the components of all the forces along any line $=0$;
Sum of the components of all the forces along a line $\perp$ *to the first line* $=0$;
Sum of the moments of all the forces about any point $=0$.

Example.—A uniform cylindrical shell of radius r stands upon a horizontal plane; two smooth spheres of radii a and b, such that $a+b>r$, are placed within it. Show that the cylinder will not upset if the ratio of its weight to the weight of the upper sphere exceeds the ratio $2r-a-b:r$.

Solution.—Fig. 58 illustrates the problem. Fig. 59

shows the cylindrical shell as a free body, W being its weight. From this figure we obtain

$$\Sigma X = R_1 - R_2 = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\Sigma Y = N - W = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\Sigma(\text{moments about } O) = 2rN - rW - zR_1 = 0. \quad . \quad (3)$$

(We select O as origin, as this eliminates R_2 from equation(3)).

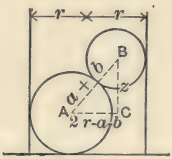


FIG. 58

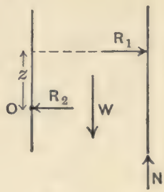


FIG. 59

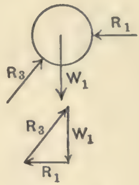


FIG. 60

Consider now the equilibrium of the upper sphere, W_1 being its weight, and the corresponding triangle of forces (Fig. 60).

This triangle is similar to $\triangle ABC$ (Fig. 58).

$$\therefore \frac{R_1}{W_1} = \frac{2r-a-b}{z}, \quad \text{or} \quad R_1 = \frac{2r-a-b}{z} W_1. \quad . \quad (4)$$

From (2), (3), and (4) we have $\dfrac{W}{W_1} = \dfrac{2r-a-b}{r}$.

EXERCISE 78. Write the equations for equilibrium for Exercises 75, 76, and 77 by the method just described.

An important principle which in many cases simplifies the solution of problems is the following:

If three forces in the same plane keep a body in equilibrium, they must be parallel or meet in a point.

From pages 44 and 45 it is evident that parallel forces acting on a body may be in equilibrium. If the three forces are not parallel, two of them at least intersect and their resultant acting at the point of intersection must balance the third force. Therefore the line of action of the third force must pass through the point of intersection of the first two; otherwise a couple would result and the body would not be in equilibrium.

Example.—A rod AB whose weight may be neglected and which is 35 inches long carries a weight, W, at C, 20 inches from A. A thread 49 inches long is tied to the ends of the rod and slung over a smooth peg D. Find the tension in the thread and the inclination of the rod to the horizontal when it comes to rest.

Solution.—In Fig. 61, as the thread ADB passes over a *smooth* peg at D, the tensions in AD and DB must be equal to, say, T pounds.

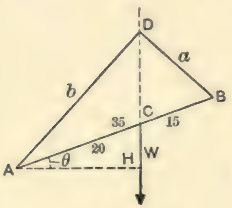


FIG. 61

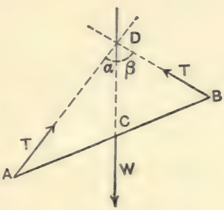


FIG. 62

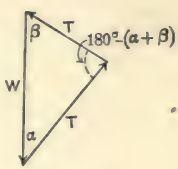


FIG. 63

In Fig. 62 the lines of action of the forces T, T, W, as there are only three, must pass through D, and we may apply the method of the triangle of forces, Fig. 63.

As $T=T$, $\alpha=\beta$, and

$$\frac{T}{\sin\alpha}=\frac{W}{\sin\{180-(\alpha+\beta)\}}=\frac{W}{\sin(\alpha+\beta)}.$$

From Fig. 61,

$$AC:CB=b:a=20:15; \quad \text{(Why?)}$$

$$\therefore\ b+a:a=35:15; \quad \text{(Why?)}$$

$$49:a=35:15;$$

$$a=\frac{49\times 15}{35}=21;$$

$$b=28.$$

As $35^2=28^2+21^2$, $\angle ADB=90°$; $\therefore\ \alpha=\beta=45°$;

then $$\cos\theta=\frac{AH}{AC}=\frac{28(\frac{1}{2}\sqrt{2})}{20}=\frac{7\sqrt{2}}{10}.$$

Also $$T=W\tfrac{1}{2}\sqrt{2}.$$

Exercise 79. A uniform beam weighing W pounds rests on two smooth planes inclined at 30° and 60° to the horizontal. Find the angle which the beam makes with the horizontal in the position of equilibrium, and also the pressure on the planes.

Exercise 80. A spherical shot weighing 100 pounds lies between two smooth planes inclined respectively at 30° and 60° to the horizontal. Find the pressure on each plane.

Exercise 81. A straight rod 4 inches long is placed in a smooth hemispherical cup and when in equilibrium one inch projects over the edge. Find the radius of the cup.

Exercise 82. A rod 3 feet long is in equilibrium resting upon a smooth pin and with one end against a smooth vertical wall. If the pin is one foot from the wall, show that the inclination θ to the horizontal is given by $3\cos^3\theta=2$.

Exercise 83. A balanced window-sash, whose height is 3 ft. and width 4 ft., weighs 100 pounds. If one of the cords is broken, what must be the least coefficient of friction in order that the remaining cord may sustain the sash?

EXERCISE 84. A hemisphere, whose center of gravity is at a distance of $\frac{3}{8}$ of its radius from the centre and is on the radius perpendicular to its plane surface at its centre, is sustained by friction against a vertical wall and a horizontal plane of equal roughness. Find the greatest inclination of the plane of the base to the horizon.

SECTION IX

PARALLEL FORCES. CENTROIDS OR CENTRES OF GRAVITY

In the consideration of the conditions for the equilibrium of forces acting on a body we meet a difficulty not involved in the consideration of the forces acting on a single particle. While considering a particle the force of gravity (the weight of the particle) was assumed to act at the point occupied by the particle itself, but in the consideration of a body assumed to consist of many particles, each having its own weight, it becomes imperative to find *the resultant force of gravity* before we can apply the conditions of equilibrium.

To this end we will first find the

Resultant of any Number of Parallel Forces

In Fig. 64 conceive the forces F_1, F_2, F_3, . . . to be parallel and to have their points of application at $P_1(x_1y_1)$, $P_2(x_2y_2)$, $P_3(x_3y_3)$, ... The forces do not necessarily lie in the plane xy, but may have any direction whatsoever so long as they remain parallel.

Consider first the forces F_1 and F_2; their resultant, by the principle described on page 45, is $F' = F_1 + F_2$, and the line of action of the resultant divides P_1P_2 in the ratio $F_2:F_1$. The coördinates of P' may now be determined for

$$\frac{l_1}{l_2}\left(=\frac{F_2}{F_1}\right)=\frac{x'-x_1}{x_2-x_1}=\frac{y'-y_1}{y_2-y_1} \text{ (by similar triangles),}$$

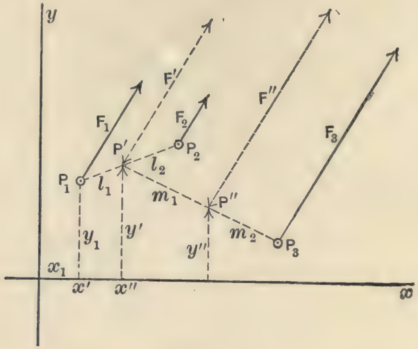


FIG. 64

from which we obtain

$$x'=\frac{F_2x_2+F_1x_1}{F_1+F_2} \quad \text{and} \quad y'=\frac{F_2y_2+F_1y_1}{F_1+F_2}.$$

Find now the resultant F'' of F' and F_3. Here

$$F''=F'+F_3=F_1+F_2+F_3 \quad \text{and} \quad \frac{m_1}{m_2}=\frac{F_3}{F'}.$$

$$\therefore \; x''=\frac{F_3x_3+F'x'}{F_3+F'}=\frac{(F_1+F_2)\dfrac{F_2x_2+F_1x_1}{F_1+F_2}+F_3x_3}{F_1+F_2+F_3}$$

$$=\frac{F_1x_1+F_2x_2+F_3x_3}{F_1+F_2+F_3};$$

similarly $$y''=\frac{F_1y_1+F_2y_2+F_3y_3}{F_1+F_2+F_3}.$$

The resultant of F'' and F_4 may now be found, etc. The final resultant will be $R = F_1 + F_2 + F_3 + \ldots$, and the *Centre of the Parallel Forces* (see page 45) will be at the point $(\bar{x}, \bar{y})$* where

$$\bar{x} = \frac{F_1x_1 + F_2x_2 + F_3x_3 + \ldots}{F_1 + F_2 + F_3 + \ldots}, \quad \bar{y} = \frac{F_1y_1 + F_2y_2 + F_3y_3 + \ldots}{F_1 + F_2 + F_3 + \ldots},$$

which we will abbreviate to

$$\bar{x} = \frac{\Sigma Fx}{\Sigma F} \quad \text{and} \quad \bar{y} = \frac{\Sigma Fy}{\Sigma F}.$$

Note particularly that the above operations are wholly independent of the direction of the parallel forces, and the position of the point $(\bar{x}, \bar{y})$ would remain unchanged even if the direction of the forces were altered.

EXERCISE 85. The legs of a right triangle are 3 and 4 feet long; a force of one pound acts at the right angle, a negative force of 2 pounds at the greater acute angle, and a force of 3 pounds at the smaller, all forces being parallel. Find the centre of the forces.

EXERCISE 86. Four forces of 2, 3, 1, and -4 pounds act perpendicularly to the plane xy at $(1, -1)$, $(0, 4)$, $(-1, 3)$, and $(1, 2)$ respectively. Find the resultant and one point on its line of action.

It can be shown that the attraction which the earth exercises on a particle is directed toward the centre of the earth.

In Fig. 65 let O represent the centre of the earth, and P_1 and P_2 two particles near its surface; then the arrows

---

* $\bar{x}$ is read "x bar."

W_1 and W_2 would represent the forces of attraction. If instead of the particles P_1 and P_2 we consider a body AB, then each particle of the body is acted on by a force as represented. These forces are certainly not parallel, but bodies such as we ordinarily deal with in mechanics are very small in comparison to the earth, under which circumstances the lines of action of the weights of the particles are very nearly parallel and for practical purposes can be assumed perfectly parallel.

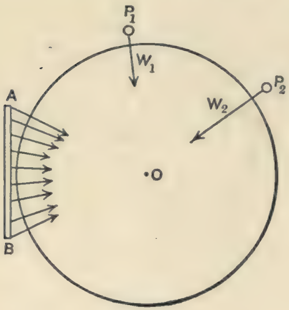


FIG. 65

Under these conditions the *centre of the parallel forces representing the weights of the particles of a body is called its centre of gravity, or centroid.*

As this point is a point in the line of action of the resultant weight, it follows that a force equal and opposite to this resultant acting through the centroid will support or balance the body in any position.

To find the position of the centroid we may use the formulæ above derived for parallel forces, if we remember that the forces F_1, F_2, F_3, . . . under consideration are now the weights W_1, W_2, W_3, ... Thus the *coördinates of the centroid* are

$$\bar{x}=\frac{\Sigma Wx}{\Sigma W} \quad \text{and} \quad \bar{y}=\frac{\Sigma Wy}{\Sigma W}.$$

Example.—Find the centre of gravity of three particles each weighing W pounds placed upon the circumference of a circle, radius r, so that the first and third are 90° from the second.

Solution.—Draw diagram illustrating problem (Fig. 66). Assume two rectangular axes. Then the position of the C. of G. is found as follows:

$$\bar{x} = \frac{\Sigma Wx}{\Sigma W} = \frac{W(r) + W(0) + W(-r)}{W + W + W},$$

$$\bar{x} = 0;$$

$$\bar{y} = \frac{\Sigma Wy}{\Sigma W} = \frac{W(0) + W(r) + W(0)}{W + W + W},$$

$$\bar{y} = \frac{r}{3}.$$

$\therefore$ the C. of G. is really at C, Fig. 66.

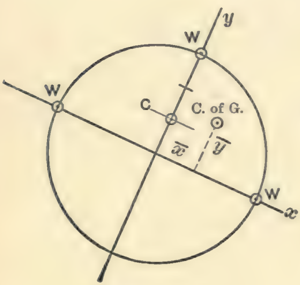


FIG. 66

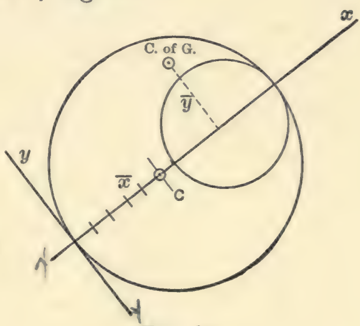


FIG. 67

Example.—Find the centroid of a circular plate or lamina, radius r inches, in which a circular hole tangent to the circumference and of radius $\frac{r}{2}$ has been drilled (Fig. 67).

Solution.—In this case consider by reason of symmetry that the C. of G. of the whole plate is at its centre, at which point its weight may be considered as concentrated. Similarly for the hole, with the exception that the weight is taken with the *negative sign*, signifying that this weight has been *removed from the whole plate.*

Assume the density of the plate to be w pounds per square inch; then the weight of the whole disc is $\pi r^2 w$, and of the removed portion $-\pi \frac{r^2}{4} w$.

$$\therefore\ \bar{x} = \frac{\Sigma Wx}{\Sigma W} = \frac{(\pi r^2 w)(r) + \left(-\pi \frac{r^2}{4} w\right)\left(\frac{3r}{2}\right)}{\pi r^2 w + \left(-\pi \frac{r^2}{4} w\right)} = \frac{r - \frac{3r}{8}}{1 - \frac{1}{4}} = \frac{5r}{6} \text{ in.}$$

and

$$\bar{y} = \frac{\Sigma Wy}{\Sigma W} = \frac{(\pi r^2 w)(0) + \left(-\pi \frac{r^2}{4} w\right)(0)}{(\pi r^2 w) + \left(-\pi \frac{r^2}{4} w\right)} = 0,$$

which places the centroid at C, Fig. 67.

EXERCISE 87. Weights of 1, 2, and 3 ounces are placed at the vertices of an equilateral triangle whose sides are 6 inches long. Find the distance of their C. of G. from the vertices.

EXERCISE 88. A common form of a cross-section of a reservoir wall or embankment wall is a trapezoid whose top and bottom sides are parallel (Fig. 68). If the top side equals 4 feet, bottom side 8 feet, and height 9 feet, find the centre of gravity of the section with reference to the left and lower sides. (Divide the figure into a rectangle and a triangle, or into two triangles. C. of G. of rectangle is at intersection of diagonals and of triangle at intersection of medians.)

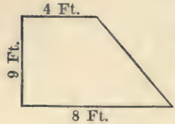


FIG. 68

EXERCISE 89. A lamina has the form of a square with an isosceles triangle attached to one side. The side of the square is a, the height of the triangle is h. Find the position of the centre of gravity of the figure, and determine the value of h, if the C. of G. lies in the base of the triangle.

EXERCISE 90. Where must a circular hole of one foot radius be punched out of a circular disc of 3 foot radius, so that the centre of gravity of the remainder may be 2 inches from the centre of the disc?

EXERCISE 91. A circular disc, 8 inches in diameter, has a hole 2 inches in diameter punched out of it; the centre of the hole is 3 inches from the circumference of the disc. Find the C. of G. of the remaining portion.

EXERCISE 92. Find the centroids of the plates shown in Fig. 69.

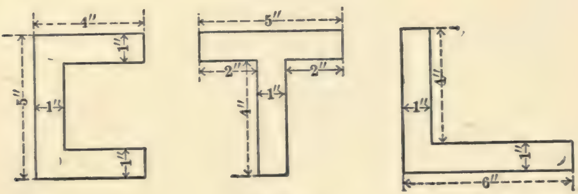


FIG. 69

SECTION X

SUMMARY OF THE METHODS OF STATICS

As may be observed from the preceding sections, we may divide the methods employed in statics into two large classes, based upon the treatment of forces acting—

I. At a single point.

II. On a rigid body.

If the *forces acting at a single point* are only *three in number*, the "Triangle of Forces" may be used advantageously.

If *more than three forces act at a single point*, we may best proceed by putting

$$\Sigma(X\text{-components of the forces}) = 0$$

and $$\Sigma(Y\text{-components of the forces}) = 0.$$

If *the forces* considered *are applied to a rigid body*, then they will be in equilibrium if

$$\Sigma(X\text{-components of the forces}) = 0,$$

$$\Sigma(Y\text{-components of the forces}) = 0,$$

and $$\Sigma(\text{Moments about any point}) = \Sigma(Yx - Xy) = 0.$$

In dealing with rigid bodies the weight of the body considered must always be assumed as concentrated at the centre of gravity, or centroid, of the body.

The method of finding the centroid as developed in Section IX, although universally true, can be applied only to a system of concrete particles or to a system of bodies the positions of whose centroids are known until the student has familiarized himself with the methods of the calculus.

It is not always necessary, in dealing with the equilibrium of the forces acting on a rigid body, to write out all the equations for equilibrium as explained in Section VIII. It lies with the ingenuity of the student to select that one of the equations $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma(\text{Moment}) = 0$ which will involve only the one unknown force sought. This is illustrated in the following

Example.—A straight rod 10 feet long, when unweighted, balances about a point 4 feet from one end;

but when loaded with 20 pounds at this end and 4 pounds at the other it balances about a point 3 feet from the end. Find the weight of the rod.

Solution.—Draw two diagrams representing the conditions of the problem; and also a diagram showing all the forces acting on the rod (Fig. 70).

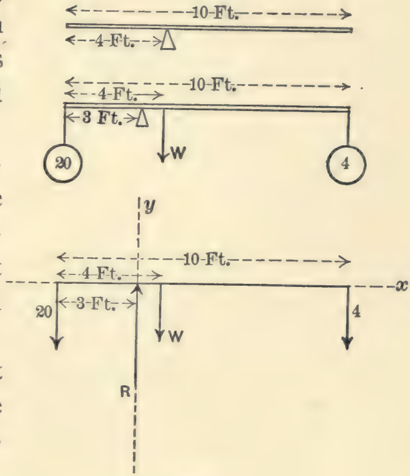


FIG. 70

From the first statement, and from the definition of the centroid, we can say that the centroid of the rod is 4 feet from the left-hand end, and that the weight W may be assumed there concentrated.

For the solution of this example it will be most convenient to write the equation for the moments about the point of support when loaded, for the reaction of the support (an unknown force) will then not enter our equation of moments.

Thus $\Sigma(M)=(20)(3)-(W)(1)-(4)(7)=0;$

$$\therefore\ W=32 \text{ pounds.}$$

If the reaction (R) of the support were required, the moments should have been taken about the centroid, as then the unknown weight would not have entered our equation. Thus

$$\Sigma(M)=(20)(4)-R(1)-(4)(6)=0;$$
$$\therefore\ R=56 \text{ pounds.}$$

If the weight were known and the reaction of the support were required,

$$\Sigma Y=-20+R-32-4=0$$

would give the result

$$R=56 \text{ pounds.}$$

Solve each of the following exercises by the use of a single equation.

EXERCISE 93. A weightless rod ten feet long is supported at its centre. 100 pounds is hung at one end and 200 pounds 3 feet from this end. Find the pressure on the support if enough force is applied at the other end to hold the rod in equilibrium.

EXERCISE 94. A heavy rod 5 feet long has a weight of 200 pounds attached to one end and is supported 2 feet from that end. What force must be applied at the other end to produce a pressure of 150 pounds on the support?

EXERCISE 95.—A rod weighing 25 pounds and loaded at each end with 200 pounds is supported so as to be in equilibrium. If the unloaded rod balances on a knife-edge placed 1 foot from one end and is 6 feet long, find the pressure on the support.

EXERCISE 96. A man carries a load of 40 pounds attached to the end of a stick resting on his shoulder. If the man exerts a force of 20 pounds at the other end of the stick, what is the pressure on his shoulder

(*a*) if the stick is horizontal?

(*b*) if the forward end of the stick dips 30°?

CHAPTER III

APPLICATIONS OF THE PRINCIPLES OF STATICS TO THE SIMPLE MACHINES

SECTION XI

THE LEVER AND THE WHEEL AND AXLE

THE principles of statics as discussed in Chapters I and II will now be applied to what are usually termed simple machines.

Any contrivance for making a force applied to a body at a given point and in a given direction available at some other point or in some other direction is called a *machine*.

The machines considered will all be supposed in equilibrium under the applied force, the available force, and the reactions of the supports of the machine. If motion is desired, the applied force must be slightly increased.

The ratio of the available force to the applied force is called the *mechanical advantage of the machine.*

The *simple machines* usually include the lever, the wheel and axle, the pulley, the inclined plane, the wedge, and the screw.

In the solution of problems involving simple machines no special formulæ will be deduced or required; each case is to be analyzed, all the forces acting are to be shown, and the required forces calculated in terms of the known forces and the dimensions of the machines by the *direct* application of the principles of equilibrium already studied.

The Lever

The lever is a rod or bar, either straight or curved, supported at one point. This point is called the fulcrum.

Example.—A straight rod is loaded so that its centroid is one third of its length from one end. When weights of 5 and 10 pounds are supported from the ends, the rod balances about the middle point. Find the weight of the rod.

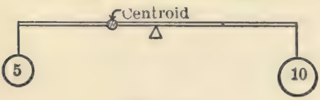


FIG. 71

Solution.—(1) Draw a diagram illustrating the problem (Fig. 71).

(2) Draw a sketch showing all forces acting on the rod (Fig. 72); these include the reaction of the fulcrum and the weight.

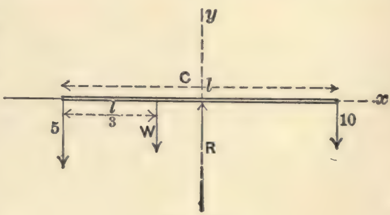


FIG. 72

Assume the origin of moments so as to exclude from the moment equation ($\Sigma M=0$) the reaction R, which is not required. Then

$$\Sigma M = 5\left(\frac{l}{2}\right) + W\left(\frac{l}{6}\right) - 10\left(\frac{l}{2}\right) = 0;$$

$$\therefore\ W = 15 \text{ pounds.}$$

EXERCISE 97. A lever is to be cut from a bar weighing 3 pounds per foot. What must be its length that it may balance about a point 2 feet from one end, when weighted at this end with 50 pounds?

EXERCISE 98. A straight lever 6 feet long weighs 10 pounds, and its centre of gravity is 4 feet from one end. What weight at this end will support 20 pounds at the other when the lever is supported at one foot from the latter, and what is the pressure on the fulcrum?

EXERCISE 99. A lever is supported at its centroid, which is nearer to one end than the other. A weight P at the end of the shorter arm is balanced by 2 pounds at the end of the longer; and the same weight P at the longer arm is balanced by 18 pounds at the shorter. Find P.

EXERCISE 100. In a pair of nut-crackers the nut is placed one inch from the hinge; the hand is applied at a distance of 6 inches from the hinge. How much pressure must be applied by the hand if the nut requires a pressure of 20 pounds to break it, and what will be the force acting on the hinge?

EXERCISE 101. The oar of a boat is $10\frac{1}{2}$ feet long; the rowlock is $2\frac{1}{2}$ feet from the end. A man applies a force of 70 pounds at 2 feet from the rowlock; the average pressure of the water is exerted at 6 inches from the other end of the oar. Find the force urging the boat forward, and the total pressure of the water on the blade of the oar.

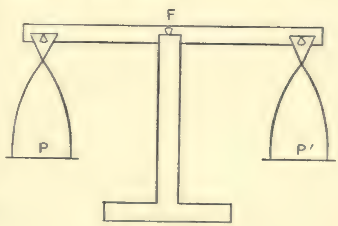


FIG. 73

The balance (Fig. 73) in its simplest form is a lever which turns very freely about a fulcrum F. It is used for comparing the weights of two bodies. A balance is true if the beam is horizontal whenever equal masses are placed in the scale-pans PP'.

EXERCISE 102. Find the condition that a balance may be true.

EXERCISE 103. A false balance rests with the beam horizontal when unloaded, but the arms are not of equal lengths. A weight W when hung at the end of the shorter arm, b, balances a weight P, and when hung at the end of the longer arm, a, it balances a third weight, Q. Find the correct weight of W.

Can you suggest another way of ascertaining correctly the weight of W?

EXERCISE 104. The arms of a false balance are in the ratio of 20 to 21. What will be the loss to a tradesman who places articles to be weighed at the end of the shorter arm if he is asked for 4 pounds of goods priced at $1.50 per pound?

The Common, or Roman, Steelyard (Fig. 74) consists of a lever supported on knife-edges at D. The object to be weighed is placed at W, and the distance of Q from the fulcrum is adjusted until the lever is horizontal.

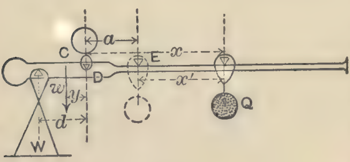


FIG. 74

The centroid, C, of the lever and scale-pan does not usually coincide with the point of suspension D.

Example.—To graduate the steelyard.

Assume the weight of the lever to be w. With no load at W adjust Q until the lever is horizontal; then $x=a$.

Thus $$\Sigma(M)=wy-Qa=0.$$

Now assume a load W in the pan; then

$$\Sigma(M)=Wd+wy-Qx=0;$$

$$\therefore\ Wd=Qx-wy=Qx-Qa.$$

In this equation d, Q, and a are known quantities to be determined by experiment; the variable quantities are W and x.

Assume $$d = 3 \text{ inches},$$
$$Q = 6 \text{ pounds},$$
$$a = 2 \text{ inches};$$
then $$3W = 6x - 12,$$
$$W = 2x - 4.$$

This is the equation of the straight line plotted in Fig. 75. Assuming any value of W we obtain a certain value of x.

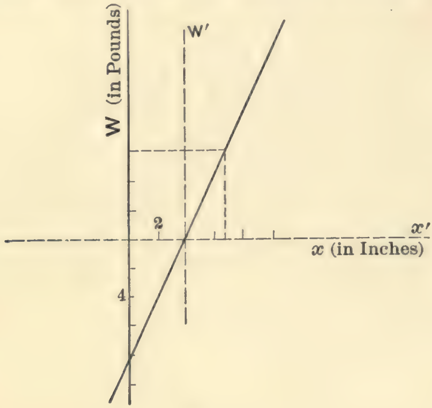


FIG. 75

If $W = 0$, then $x = 2$, so that when the steelyard is unloaded Q must be placed 2 inches from D.

If $W = 1$, then $x = 5/2$ inches. If $W = 2$, then $x = 3$ inches, etc.

The zero mark would then be placed at 2 inches to the right of D, the one-pound mark 1/2 inch farther to the right, the two-pound mark 1/2 inch still farther, etc.

If now, instead of measuring x from D, x be measured from E, the zero mark, the equation $W=2x-4$ would have to be transformed to a new set of axes W', x' (Fig. 75), so that $W=W'$ and $x=x'+2$; whence

$$W'=2(x'+2)-4, \text{ or } W'=2x'.$$

If now $W=W'=0$, then $x'=0$;

$W=1$, " $x'=1/2$;

$W=2$, " $x'=1$, etc.

If, in the equation $W'=2x'$, x' is given a minus value, that is, is measured to the left of the zero mark, the value $W'=W$ becomes minus (Fig. 75).

The physical interpretation of this is that instead of a weight an upward push must be supplied at W to produce equilibrium.

EXERCISE 105. In a common steelyard the pan is supported 3 inches to one side of the fulcrum, and the center of gravity is 1 inch to the other side. The weight of the lever is 3 pounds, that of the movable constant weight is 2 pounds. What is the smallest load that can be weighed?

The Danish Steelyard (Fig. 76) consists of a bar

FIG. 76

terminating in a ball. The load is placed at W, and the fulcrum F is moved until equilibrium is established.

EXERCISE 106. Graduate a Danish steelyard, assuming the distance from the load to the centroid to be 12 inches and the weight of bar and scale-pan 20 pounds.

The Wheel and Axle

consists of a wheel or drum of considerable diameter to which is rigidly attached a drum or axle of smaller diameter. Both drums turn freely upon the same axis (Fig. 77).

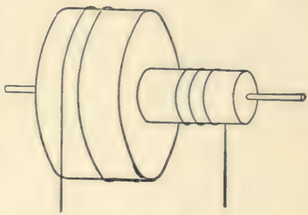


FIG. 77

A rope is coiled around each drum, one clockwise, and the other counter-clockwise.

The forces are applied to these ropes. An end view of this machine is shown in Fig. 78, from which it may be seen that the wheel and axle is simply a lever with fulcrum F at the axis.

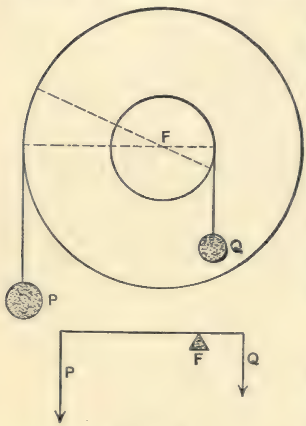


FIG. 78

The wheel and axle may be called a continuous lever; for, if motion results, new radii take the place of the ones previously in use and the machine operates as before.

EXERCISE 107. In a wheel and axle the diameter of the wheel is 10 feet and the radius of the axle is 18 inches. A force of 100 pounds is applied to the rope coiled about the axle. Find the available force at the end of the rope coiled about the wheel if this rope leaves on a horizontal tangent. Find, also, the mechanical advantage. What pressure do the bearings sustain?

EXERCISE 108. In a wheel and axle the radius of the wheel is 3 feet. The axle is of square section, the side of the square being 6 inches long. Find (*a*) the greatest and (*b*) the least vertical power that must be exerted to slowly lift a weight of 252 pounds attached to the rope coiled around the wheel. If the wheel and axle weighs 150 pounds, what is the pressure on the bearings?

EXERCISE 109. The circumference of a wheel is 60 inches, the diameter of the axle is 5 inches. What force must be applied to the circumference of the axle to support 100 pounds at the circumference of the wheel?

SECTION XII

THE PULLEY

The pulley is a small wheel with a groove cut in its outer edge. The pulley can turn on an axis through its centre; the ends of this axis are carried by a block within which the pulley turns. If the block is so fastened as to be immovable, the pulley is said to be "fixed"; otherwise the pulley is designated as "movable."

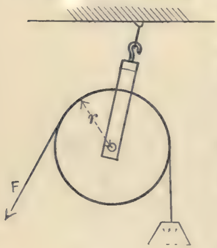


FIG. 79

Fig. 79 shows a fixed pulley; the applied force is F pounds, the available force is W pounds.

Example. — Calculate F and the reaction of the staple supporting the block.

Solution.—Fig. 80 shows the forces acting upon the pulley. R is the force transmitted by the block from the staple to the axis of the pulley.

Since three forces acting in equilibrium upon a rigid body (pulley) have their lines of action meeting in a point (page 60), Fig. 81 may be used to represent the forces.

Using C, Fig. 80, as the origin of moments, we have

$$\Sigma M = Fr - Wr = 0; \quad \therefore\ F = W.$$

In Fig. 81, assuming the x and y axes as shown,

$$\Sigma Y = R - W\cos\theta - W\cos\theta = 0\cdot$$

$$\therefore\ R = 2W\cos\theta,$$

as, by Geometry,

$$\angle mOC = \angle nOC.$$

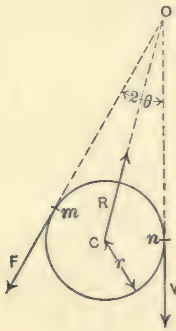


FIG. 80

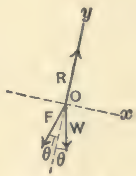


FIG. 81

EXERCISE 110. Find the force necessary to sustain a weight of 100 pounds by means of a fixed pulley; find also the magnitude and direction of the force on the staple to which the pulley is fastened, if the applied force makes an angle of (*a*) 0°, (*b*) 30°, (*c*) 90°, (*d*) 150° with the vertical.

From the above it follows that a fixed pulley has a mechanical advantage of unity (it does not increase the available over the applied force): it simply changes the direction of the applied force.

EXERCISE 111. Design a system of fixed pulleys by means of which a horse walking on horizontal ground may raise a bale of hay weighing 200 pounds from the ground to a vertical height of 20 feet. Show by diagrams all the forces acting on the various pulleys.

In Fig. 82 a movable pulley is shown.

Example. — Assuming the pulley, Fig. 82, to weigh w pounds, find the force F necessary to sustain the weight W.

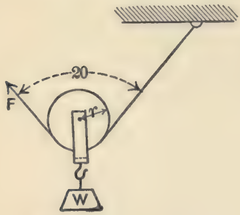


FIG. 82

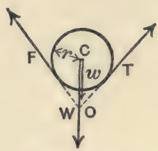


FIG. 83

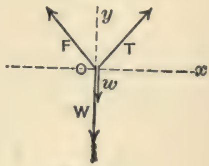


FIG. 84

Solution.—Fig. 83 shows all forces acting on the pulley, T representing the tension in the rope.

Taking moments about C,

$$\Sigma M = -Fr + Tr = 0; \quad \therefore F = T.$$

From Fig. 84,

$$\Sigma Y = F \cos\theta + F \cos\theta - W - w = 0;$$

$$\therefore F = \frac{W + w}{2 \cos\theta}.$$

EXERCISE 112. Find the force necessary to sustain a weight of 100 pounds by means of a single movable pulley; also, find direction and magnitude of the force acting on the staple supporting one end of the rope if the force makes an angle of (*a*) 90°, (*b*) 35°, (*c*) 0° with the horizon.

System of Pulleys

Various combinations of pulleys are in use. For convenience we shall designate three important systems as the first, second, and third; these systems are illustrated in Figs. 85, 86, and 87, respectively.

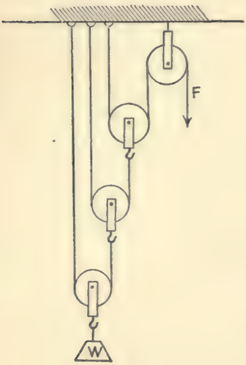


FIG. 85

FIG. 86

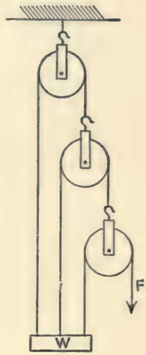


FIG. 87

In finding the various forces acting in a system of pulleys, it should be remembered that *the tension in any one string is the same throughout its length*, and that each portion of the system may be considered in equilibrium by itself.

Example.—Find the force necessary to sustain a weight W by means of the third system if one movable pulley (weighing w pounds) is used. Also find the pull on the staple if the fixed pulley weighs w_1 pounds.

Solution.—The machine is shown in Fig. 88.

Consider first the equilibrium of the movable pulley, Fig. 89, where t, t_1 represent the tensions in the several strings.

$$\Sigma Y = t_1 - F - w - F = 0; \quad \therefore\ t_1 = 2F + w.$$

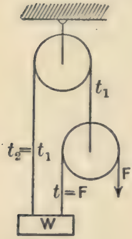


FIG. 88

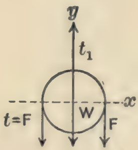


FIG. 89

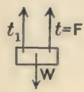


FIG. 90

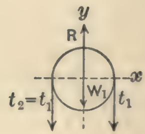


FIG. 91

The equilibrium of the weight gives (Fig. 90)

$$\Sigma Y = -W + t_1 + t = 0;$$

$$\therefore\ W = t + t_1 = F + 2F + w, \quad \text{and} \quad F = \frac{W - w}{3}.$$

Consider now the equilibrium of the fixed pulley, Fig. 91.

$$\Sigma Y = R - t_1 - w_1 - t_1 = 0; \quad \therefore\ R = 2t_1 + w_1,$$

or

$$\begin{aligned} R &= 4F + 2w + w_1 \\ &= \frac{4W - 4w}{3} + 2w + w_1 \\ &= \frac{4W + 2w + 3w_1}{3}. \end{aligned}$$

EXERCISE 113. What force is necessary to sustain a weight of 1000 pounds by means of the first system of pulleys, using two movable pulleys and regarding them as weightless? What is the pull upon each point of support?

EXERCISE 114. Same as Exercise 113, considering each pulley to weigh 20 pounds.

EXERCISE 115. What weight will a force of 10 pounds applied to the first system, containing five weightless pulleys, support?

EXERCISE 116. A force of 20 pounds applied to the second system, containing two pulleys in the movable block with the rope fastened to the fixed block, will sustain what weight? What is the pull on the point of support?

EXERCISE 117. Same as Exercise 116, but having the rope fastened to the movable block.

EXERCISE 118. A system of the third class has three weightless movable pulleys. What weight will a force of 10 pounds sustain, and what is the tension in the staple?

EXERCISE 119. Same as Exercise 118, if each pulley weighs 2 pounds.

EXERCISE 120. In the system illustrated in Fig. 92 find the force, necessary to sustain 100 pounds, and the pull on the staples, if each pulley weighs 4 pounds.

FIG. 92

SECTION XIII

THE INCLINED PLANE AND THE WEDGE

Any plane inclined to the horizon is an inclined plane.

In solving problems involving inclined planes, it is simply necessary to consider the equilibrium of the weight, considered as a particle, upon the plane. Care should be taken not to neglect the *normal reaction* of the plane, and, if friction is involved, the *friction force.*

The inclination of a plane is usually denoted by the angle it makes with the horizon. When referring to roads or railways the inclination is generally expressed as the *grade* of the incline, which means the ratio of the height to the length. Thus on a "five per cent grade" a rise of 5 feet would be obtained on walking 100 feet up the incline.

Less frequently the term *pitch* is used to denote the ratio of the height to the base.

EXERCISE 121. What force acting parallel to the plane is needed to support two tons on a smooth incline, if the grade is 8 per cent?

EXERCISE 122. Same as Exercise 121, if the coefficient of friction is 0.3.

EXERCISE 123. The pitch of a plane is 0.25, and the coefficient of friction between a certain body and the plane is 0.35. Will the body, if placed on the plane, slide down or remain at rest? What will be the friction force exerted?

EXERCISE 124. A weight rests on a smooth inclined plane. Show that the least force which will keep it in equilibrium must act along the plane. (Assume a force acting at an angle θ with the plane and show that θ is zero for a minimum force.)

EXERCISE 125. Two unequal weights W_1 and W_2 on a rough inclined plane are connected by a string which passes over a smooth pulley in the plane. Find the greatest inclination of the plane consistent with equilibrium, in terms of W_1, W_2, and the coefficients of friction μ_1 and μ_2.

EXERCISE 126. Two rough bodies W_1 and W_2 rest upon an inclined plane and are connected by a string parallel to the plane. If the coefficient of friction is not the same for both, determine the greatest inclination and the tension of the string, consistent with equilibrium, in terms of W_1, W_2, and the coefficients of friction μ_1 and μ_2.

The Wedge

This machine consists of a double inclined plane made of some hard material, such as iron or steel. It is used for splitting wood, or overcoming great resistances over short distances. In Fig. 93, ABC represents a wedge. The applied force, F, acts normally to BC; the available force will be a force equal and opposite to R, normal to AB.

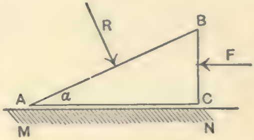


FIG. 93

Example.—Assuming the angle shown in Fig. 93, and all surfaces to be frictionless, find the relation existing between R and F.

Solution.—Fig. 94 shows all forces acting on the wedge, N being the normal resultant of the reactions of the plane MN on AC.

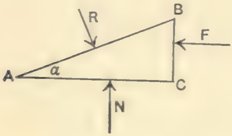


FIG. 94

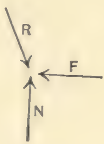


FIG. 95

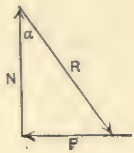


FIG. 96

Three forces to be in equilibrium must always act through one point. Therefore Fig. 95 truly represents the forces, and by the principle of the triangle of forces Fig. 96 is obtained.

From which we have

$$\frac{F}{R} = \sin \alpha,$$

or

$$F = R \sin \alpha.$$

EXERCISE 127. In the wedge shown in Fig. 97 find the relation between F and P in terms of the angles α, β, and θ. All surfaces to be considered smooth, and GG being guides preventing lateral motion of the rod transmitting the force P.

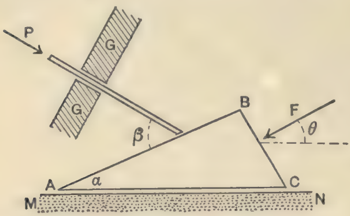


FIG. 97

Hint.—Find a force R normal to AB balancing F and then consider the rod as a free body with a force equal and opposite to R acting upon it together with the reactions of the guides, etc.

EXERCISE 128. A wedge in the form of a right triangle, whose sides are 5, 4, and 3, is driven by a horizontal force of 300 pounds, applied normally to side 3, along a horizontal plane. What weight, supported so as to prevent its moving horizontally, may be lifted if applied to the side 5, friction neglected?

EXERCISE 129. In Fig. 97, assuming $\alpha = 45°$, $\theta = 30°$, $\beta = 120°$, and $F = 500$ pounds, find P.

Action of the Wedge, including Friction

In Fig. 98 let $ABCD$ be the wedge driven by a force P along the plane MN, HE a block sliding upon AB, and GG a fixed guide preventing the horizontal motion of HE.

Let θ be the inclination of AB, and μ, μ_1, μ_2 the coefficients of friction at the surfaces DC, AB, and GG, respectively.

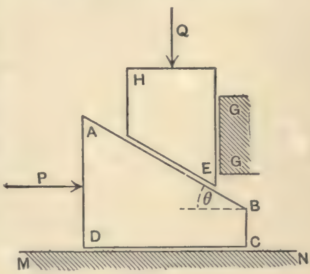


FIG. 98

The problem consists in finding the relation between P and Q, assuming the wedge and block HE as weightless.

In Figs. 99 and 100 are shown the wedge and block respectively, with all forces acting on the same.

From Fig. 99,

$$\Sigma X = P - \mu R - N_1 \sin\theta - \mu_1 N_1 \cos\theta = 0, \quad . \quad (1)$$

$$\Sigma Y = R - N_1 \cos\theta + \mu_1 N_1 \sin\theta = 0. \quad . \quad . \quad . \quad (2)$$

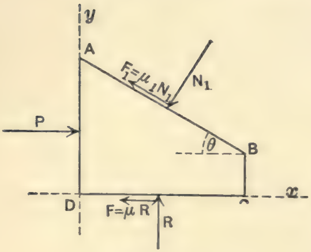


FIG. 99

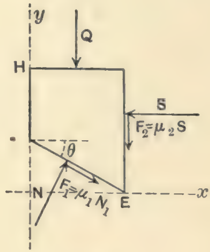


FIG. 100

From Fig. 100,

$$\Sigma X = N_1 \sin\theta + \mu_1 N_1 \cos\theta - S = 0, \quad . \quad . \quad . \quad (3)$$

$$\Sigma Y = N_1 \cos\theta - \mu_1 N_1 \sin\theta - Q - \mu_2 S = 0. \quad . \quad (4)$$

From (1) and (2), by elimination of R,

$$P = [(1 - \mu\mu_1)\sin\theta + (\mu + \mu_1)\cos\theta]\, N_1.$$

From (3) and (4), by elimination of S,

$$Q = [(1 - \mu_1\mu_2)\cos\theta - (\mu_1 + \mu_2)\sin\theta] N_1;$$

$$\therefore \frac{P}{Q} = \frac{(1 - \mu\mu_1)\sin\theta + (\mu + \mu_1)\cos\theta}{(1 - \mu_1\mu_2)\cos\theta - (\mu_1 + \mu_2)\sin\theta},$$

which is the required solution.

If the coefficients of friction are equal, or $\mu = \mu_1 = \mu_2 = \tan\alpha$, where α is the angle of friction (see page 34),

$$\frac{P}{Q} = \frac{(1-\mu^2)\sin\theta + 2\mu\cos\theta}{(1-\mu^2)\cos\theta - 2\mu\sin\theta} = \frac{\dfrac{\sin\theta}{\cos\theta} + \dfrac{2\mu}{1-\mu^2}}{1 - \left(\dfrac{2\mu}{1-\mu^2}\right)\dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{\tan\theta + \tan 2\alpha}{1 - \tan 2\alpha\tan\theta} = \tan(\theta + 2\alpha),$$

by the trigonometric relations

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x},$$

and $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$

EXERCISE 130. Find the relation of P to Q in the machine shown in Fig. 98, if Q is applied normally to AB by means of a block restrained from moving parallel to AB.

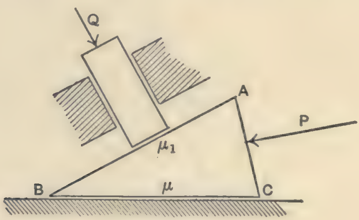


FIG. 101

EXERCISE 131. Find P, if $Q = 1000$ pounds, and the coefficient of friction $\mu = 0.2$, and $\mu_1 = 0.4$, from the wedge shown in Fig. 101, where $\triangle ABC$ is isosceles and $\angle ABC = 60°$.

SECTION XIV

MISCELLANEOUS MACHINES

The Bent-lever Balance.—This balance is shown in Fig. 102, where ABC is the bent lever turning about a pivot at B. D is the index which points to some division on the graduated arc MDN. C is the centroid of the lever CBA.

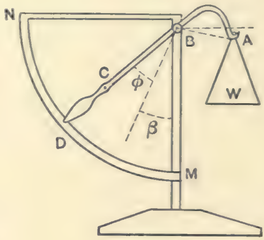


FIG. 102

EXERCISE 132. Find the relation existing between ϕ and W, if W is the load in the pan and ϕ is the angle through which BC is displaced from its position, when $W=0$, by the load W. Assume as constants of the balance $BC=a$, $BA=b$, w=weight of pan, W_1=weight of lever CBA, and that when $W=0$ the end A is on a horizontal through B while BC makes an angle β to the vertical.

EXERCISE 133. Find the relation between F and W in the combination of levers shown in Fig. 103.

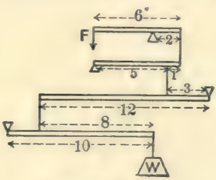


FIG. 103

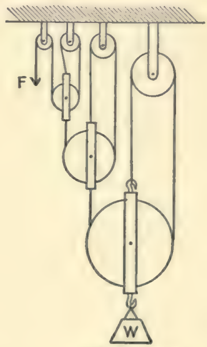


FIG. 104

EXERCISE 134. Find the weight, W, that can be sustained by a force of 100 pounds and the pull on each point of support in the combination of pulleys shown in Fig. 104, if the movable pulleys weigh 5 pounds each and the fixed pulleys 3 pounds each.

The Differential Wheel and Axle.—This machine is shown in Fig. 105. It is similar to the wheel and axle, but instead of one axle it has two of different diameters with the same rope coiled in opposite directions around them. In the loop of this rope hangs a single movable pulley to which the weight is attached.

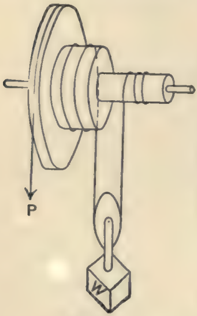


FIG. 105

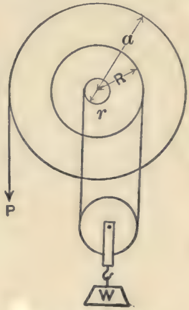


FIG. 106

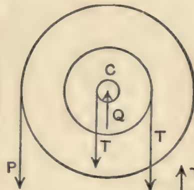


FIG. 107

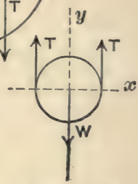


FIG. 108

An end view of the machine is shown in Fig. 106, and the forces acting on the separate parts in Figs. 107 and 108, where T represents the tension in the rope.

Taking moments about C, Fig. 107, we have

$$\Sigma M = Pa + Tr - TR = 0.$$

From Fig. 108,

$$\Sigma Y = 2T - W = 0; \quad \therefore\ T = \frac{W}{2}.$$

Thus,
$$Pa = T(R-r) = \frac{W}{2}(R-r),$$

$$P = \frac{W(R-r)}{2a}.$$

From this equation it should be noted that the nearer $r=R$ the smaller does P become until $R=r$ when $P=0$.

EXERCISE 135. In a differential wheel and axle the radius of the wheel is 3 feet, and the radii of the axles 20 inches and 18 inches respectively; find the weight sustained, and the pressure on the bearings, if applied force is 200 pounds.

A form of **Platform Scales** is shown in Fig. 109. The levers are so arranged that x pounds at A balance the

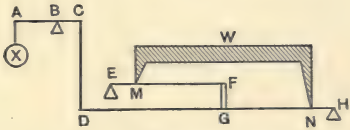


FIG. 109

same load W wherever it may be put on the platform. This result is secured by making the horizontal arms EM and NH equal, and also EF equal to GH.

In this machine there are three fulcrums at B, E, and H, while the platform rests on the levers EF and DH at M and N respectively.

EXERCISE 136. Assume the distances (Fig. 109) as follows: $EF=GH=10$ ft., $EM=NH=2$ ft., $HD=25$ ft., $BC=1$ inch, $AB=30$ inches. Find x, if $W=3$ tons and is placed (*a*) at the centre of the platform, (*b*) one foot from the left edge of the platform.

The combination of levers shown in Fig. 110 is sometimes called **a knee.** This machine can be used to advantage where very great pressure is required to act through very small space only, as in coining money, in the printing-press, etc.

At A and D are fixed pivots, and the levers AB and DC are joined by pins to BC at B and C.

As BC transmits force only in the direction BC, the forces acting upon the levers AB and DC can be represented as in Fig. 111.

EXERCISE 137. In Fig. 110, assuming $\angle CAB = \angle BCA = 30°$, $AH = 10$ inches, $AC = 3$ feet, $DC = 2$ feet, and $DK = 1\frac{1}{2}$ feet, find pressure at K produced by a horizontal force of 100 pounds acting at H.

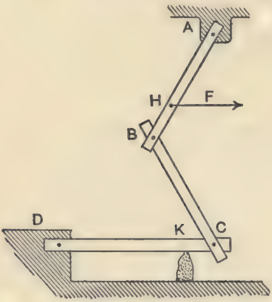


FIG. 110

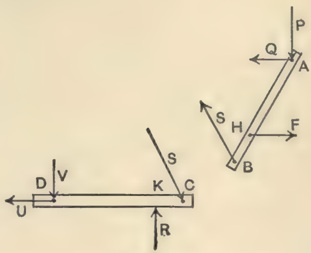


FIG. 111

EXERCISE 138. Draw diagrams showing the forces acting on the fixed points A and D and on the bar BC.

EXERCISE 139. Calculate the direction and magnitude of the forces acting at A and D due to the force given in Exercise 137.

GRAPHICAL STATICS

CHAPTER IV

GRAPHICAL ARITHMETIC

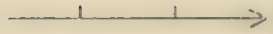
Section XV

SUMMATION, DIVISION, AND MULTIPLICATION

The theory of statics and the solution of statical problems has already been considered. The results, however, were obtained by the use of analytical methods. Methods for solving statical problems graphically will now be developed.

As an introduction, some problems in *Graphical Arithmetic* will first be considered.

Numbers may be represented by the lengths of lines drawn to some scale. Thus,  represents 5.

If plus and minus numbers are to be used, some assumption as to signs becomes necessary. It will be assumed that *plus numbers* are to be shown by lines drawn to the *right*, and *minus numbers* by lines drawn to the *left.* Thus,  represents $+3$, and  represents -3, the *arrow-head* being used to show *direction.*

Summation (addition and subtraction) is performed graphically by placing the arrows representing the numbers to be summed tail to head, and measuring the distance between the tail of the first and the head of the last arrow, care being taken to denote the direction of this line by the appropriate sign. Thus $3+2-4$ is represented graphically by 

and equals , $+1$.

EXERCISE 140. Sum graphically:

(*a*) $5+3-2-6-3$.

(*b*) $-6+4-5+7-2$.

(*c*) $12-3-4+2-10+5$.

EXERCISE 141. Rewrite each sum in Exercise 140, changing the order of the terms, and then again sum graphically.

Multiplication, when performed graphically, must always be preceded by **Division.**

In order to perform division, it is necessary to use lines drawn vertically, and another assumption as to signs must be made. Assume lines drawn *upward* as *plus* and *downward* as *minus.*

Then $2 \div 3$ is shown graphically in Fig. 112. The dividend is placed vertically, the divisor horizontally, always tail to head. The tail of the dividend is then joined to the head of the divisor. The quotient, although not drawn in Fig. 112, is the tangent of the angle θ, for $\tan \theta = \frac{2}{3}$. This quotient may be obtained graphically by multiplying by

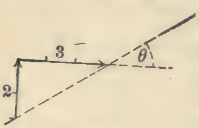


FIG. 112

one. In Fig. 113 *this is done by making the multiplier a continuation of the divisor.* Then x, the required quotient, is drawn vertically to meet the sloping line.

EXERCISE 142. Prove geometrically that in Fig. 113 $x=\frac{2}{3}$.

Example.—Find graphically $x=\frac{-4}{3}$.

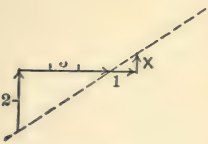


FIG. 113

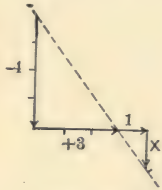


FIG. 114

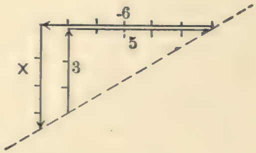


FIG. 115

Solution in Fig. 114 shows $-4\div 3$ to obtain the dotted line, then the multiplication by unity to obtain x.

EXERCISE 143. Find graphically:

(*a*) $x=\frac{6}{3}$; (*b*) $x=\frac{-4}{-2}$; (*c*) $x=\frac{-2}{2}$.

Example.—Find graphically $x=\frac{(3)(-6)}{5}$.

The solution in Fig. 115 is obtained by dividing 3 by 5, thus obtaining the direction of the sloping line, then multiplying by (-6), as above explained, to find x.

EXERCISE 144. Find x graphically:

(*a*) $x=\frac{(-2)(4)}{3}$; (*b*) $x=\frac{(2)(-4)}{3}$; (*c*) $x=\frac{(2)(4)}{-3}$;
(*d*) $x=\frac{(-2)(-4)}{3}$; (*e*) $x=\frac{(-2)(4)}{-3}$; (*f*) $x=\frac{(-2)(-4)}{-3}$.

It is important to note that the triangles used in multiplication and division need not be right-angled. Thus, $x=\frac{(5)(3)}{2}$ may be found from each of the Figs. 116 to 118.

EXERCISE 145. Give a geometric proof that $x=\frac{(5)(3)}{(2)}$ for the construction shown in each of the figures from 116 to 118.

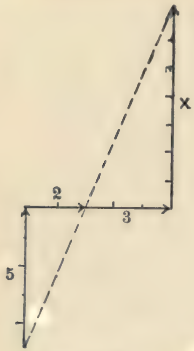


FIG. 116

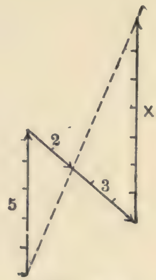


FIG. 117

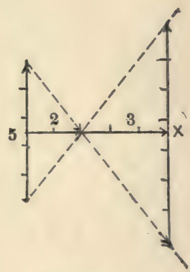


FIG. 118

Example.—Find graphically $x=\frac{(3)(2)(7)}{(-5)(-3)}$. To find x the form of the expression should first be changed to

$$x=\frac{\left\{\frac{(3)(2)}{-5}\right\}(+7)}{(-3)}.$$

Then find $y=\frac{(3)(2)}{(-5)}$, as shown in Fig. 119, and then $x=\frac{y(7)}{-3}$ as shown in Fig. 120. Or, better, we may solve the entire problem in one diagram by combining Figs. 119 and 120, as in Fig. 121.

From the preceding work it becomes evident that, in performing multiplication and division graphically, the problem must be put in a fractional form, in which the numerator always contains one more factor than the denominator. This may be accomplished by the introduction of unit factors. Thus, $x=\frac{(2)(3)(4)}{(5)}$ should be put in the form $x=\frac{(2)(3)(4)}{(5)(1)}$.

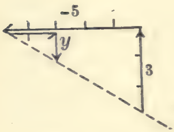


FIG. 119

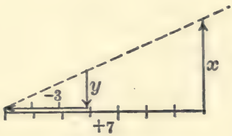


FIG. 120

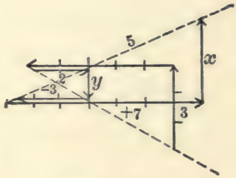


FIG. 121

Again, in $x=\frac{90}{14}$ an inconveniently large diagram would result if the form be changed to $x=\frac{(90)(1)}{(14)}$. This may be avoided by factoring; thus $x=\frac{(3)(6)(5)}{(2)(7)}$.

EXERCISE 146. Find x graphically:

(*a*) $x=\frac{(3)(4)(5)}{(7)(11)}$; (*b*) $x=\frac{(2)(3)}{(5)(7)}$; (*c*) $x=\frac{35}{4}$;

(*d*) $x=\frac{(5)}{(3)(2)}$; (*e*) $x=\frac{75}{7}$; (*f*) $x=\frac{(4)(5)(6)(11)}{(7)(13)}$.

EXERCISE 147. Find x graphically:

(*a*) $x=\frac{(-5)(-5)}{(-7)}$; (*b*) $x=\frac{(2)(-4)(-7)}{(-3)(-5)}$;

(*c*) $x=\frac{-75}{-7}$; (*d*) $x=\frac{(-35)(-7)}{(-9)}$.

EXERCISE 148. By means of cross-section paper find x, y, and z. In each case select an appropriate scale.

$$x=\frac{(-101)(-113)(9)(59)}{(76)(78)(-31)};\quad y=\frac{(13750)(-4725)(1379)}{(6375)(3957)};$$

$$z=\frac{(.087)(.0035)(.01975)}{(-.029)(.00995)}.$$

SECTION XVI

COMBINED MULTIPLICATION AND SUMMATION

Consider now a problem such as $u=\frac{2\cdot 3+4\cdot 5+6\cdot 7}{8}$. It is found most convenient to change the given expression into one composed of parts similar to those already solved; thus, $u=\frac{2\cdot 3}{8}+\frac{4\cdot 5}{8}+\frac{6\cdot 7}{8}$. Instead of solving each term separately and then summing the results, it is more convenient to perform all the divisions in one diagram (Fig. 122).

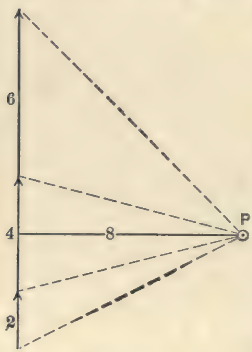


FIG. 122

Here P (called the Pole) is selected anywhere at a distance 8 from the line along which 2, 4, and 6 are set off, and the rays drawn from P to the extremities of 2, 4, 6. The division is then performed as in the left-hand portion of Fig. 118.

To perform the multiplication the rays in Fig. 122 could be prolonged (Fig. 123) and the distance 3, 5, 7 laid down horizontally and lines x,

y, z located between their proper rays and at their proper distance from P, as in Fig. 118.

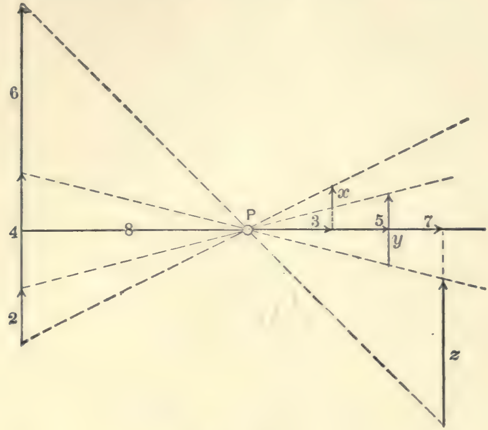


FIG. 123

By this construction the lines x, y, z will not fall in the same straight line, and therefore their sum is not found directly. A more convenient construction is shown in Fig. 124. Here at any point, O, in a vertical line OM, construct a triangle similar to the one with base 2 in Fig. 123, but with altitude 3, thus obtaining x. This is most conveniently done by drawing the sides of the triangle parallel to the corresponding rays of Fig. 122. Next construct a triangle similar to the one with base 4, but with altitude 5, etc. Thus we obtain x, y, z all on the

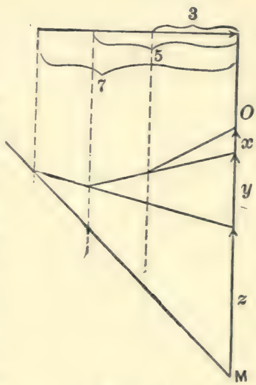


FIG. 124

line OM and following one another head to tail. Then $OM = u$ is the required result.

The diagram in Fig. 124 can appropriately be called a *Summation Polygon* or *Diagram.*

EXERCISE 149. Find graphically:

$$(a)\ x=\frac{5\cdot 3+2\cdot 4+30}{7}; \qquad (b)\ x=\frac{7\cdot 8+5\cdot 6+3\cdot 4+1\cdot 2}{5}.$$

EXERCISE 150. Find x graphically:

$$(a)\ x=\frac{2\cdot 3+(3)(-4)+(5)(6)}{7};$$

$$(b)\ x=\frac{(-2)(3)+(3)(-5)-(6)(-7)}{8}.$$

EXERCISE 151. A horizontal weightless rod 10 feet long has weights of 2, 3, and 4 pounds attached at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of its length. Write down the value of the $\bar{x}$ of its centroid according to the formula $\bar{x}=\frac{\Sigma Wx}{\Sigma W}$, and find $\bar{x}$ graphically.

EXERCISE 152. Same as Exercise 151, with additional weights of 1 and 5 pounds at the ends of the rod.

The construction just studied lends itself well to finding the centroids of plates.

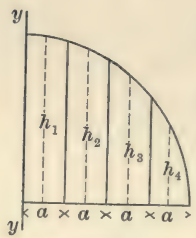


FIG. 125

Example. — Find the centroid of a quadrant of a circle, Fig. 125.

Solution. — Consider the quadrant divided into sections of equal width as shown. Let the plate weigh w pounds per square foot and let the width of each section be a feet. Each section will have a mean altitude, as shown by the dotted lines; denote these by h_1, h_2, ...

Assume each section to be a trapezoid, and assume their centroids to be at the midpoints of their medians. Neither of these assumptions is strictly true, but by decreasing the width of the sections, and thus increasing their number, the assumptions approach nearer and nearer the truth. It follows that

the area of any section is ah,
" weight " " " " wah,

which weight is assumed to act at the midpoint of the median.

Placing the y-axis as shown in Fig. 125 and putting $\bar{x}$ = the abscissa of the centroid, we have

$$\bar{x}=\frac{\Sigma Wx}{\Sigma W}=\frac{wah_1\left(\frac{a}{2}\right)+wah_2\left(\frac{3a}{2}\right)+wah_3\left(\frac{5a}{2}\right)+\ \ldots}{wah_1+wah_2+wah_3+\ \ldots}$$

$$=\frac{h_1\left(\frac{a}{2}\right)+h_2\left(\frac{3a}{2}\right)+h_3\left(\frac{5a}{2}\right)+\ \ldots}{h_1+h_2+h_3+\ \ldots},$$

which is an expression precisely suiting our graphical methods.

The graphical construction for $\bar{x}$ is shown in Fig. 126. At (a) is shown the quadrant with the mean altitudes (the sections are not shown, as these are unnecessary for purposes of construction). At (b) the divisions are performed. Here, instead of using the h's as indicated in the formula, half h's are used throughout. Why does this not affect the result? At (c) we have the summation polygon, and $\bar{x}$, the required abscissa, is here found. $\bar{x}$ is then plotted as shown in (a); the centroid must lie on the dotted line.

Where is the final position of the centroid?

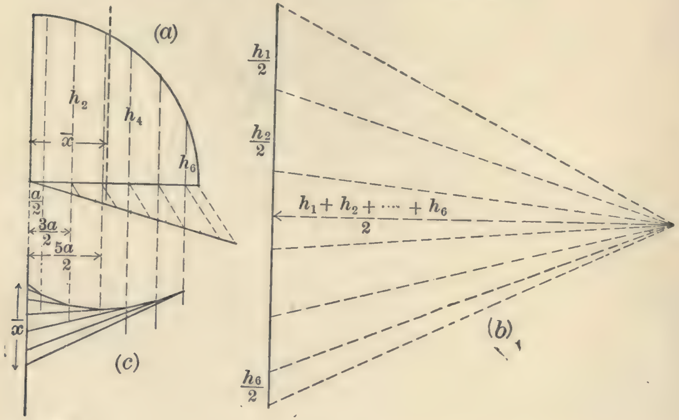


FIG. 126

EXERCISE 153. Construct a parabola whose parameter is 8 and find the centroid of a plate of this shape between the vertex and the latus rectum. Divide the plate into four sections.

EXERCISE 154. Same as Ex. 153, but divide the plate into 8 sections and compare the result with that of Ex. 153.

CHAPTER V

FORCES ACTING AT A SINGLE POINT

SECTION XVII

COMPONENTS. RESULTANTS

WE already know that the diagonal of a parallelogram constructed upon two forces as sides, correctly represents the resultant if the tails of all forces are at the one vertex of the parallelogram. Thus, in Fig. 127, if AB and AC represent forces acting on a particle at A, AD represents their resultant. If now the arrows representing the forces AB and AC be drawn to a scale, i.e., each unit of their length being accurately made to represent a unit of their force, and the parallelogram be accurately drawn, then, by applying the same scale of length to AD, the number of units of length in AD would correctly indicate the number of units of force in the resultant of AB and AC. This is, then, a graphical method of obtaining the resultant of two forces acting at a point.

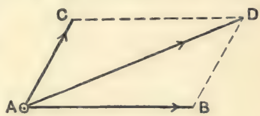


FIG. 127

In many constructions it is inconvenient to complete the parallelogram as above, so the triangle of forces is used. This is illustrated in Fig. 128. It is here required to find the resultant of the forces represented by AB and

AC. In a detached diagram, ab is drawn $\parallel$ and $=$ to AB and bc $\parallel$ and $=$ to AC, and the triangle then completed by the line ac. It will be remembered that the force ca represents the force producing equilibrium with AB and AC and thus ac, acting in the opposite direc-

FIG. 128

tion to the other forces about the triangle, represents the resultant of AB and AC in magnitude.

This resultant is now, however, not in its proper position. A force parallel and equal to ac acting at A is the resultant in both magnitude and position.

Notice the difference between the parallelogram and the triangle of forces, the first gives the complete resultant in position, magnitude, and direction, while the second gives *only* the magnitude and direction.

Instead of the notation employed in Fig. 128 it will be found more convenient to use the notation of Fig. 129.

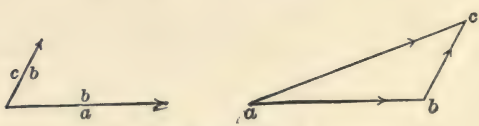


FIG. 129

Here the forces are denoted by naming the letters on each side of the forces; thus, the horizontal force would be ab. In the triangle of forces these letters are placed at the ends of the arrow representing the force.

EXERCISE 155. Find graphically the resultant of

(*a*) 3 lb. and 5 lb., included angle 90°;
(*b*) 5 " " 7 " " " 60°;
(*c*) 4 " " 9 " " " 120°;

both by the parallelogram and by the triangle of forces.

Components

To find the components of a force P, Fig. 130 (*a*), in the direction of the lines 1 and 2, proceed as indicated in Fig. 130 (*b*). Draw $p \parallel$ and $=$ to P, and through its ends

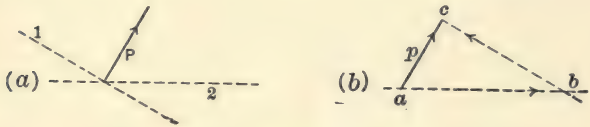


FIG. 130

draw lines $\parallel$ to 1 and 2 respectively; then *ab* and *bc* represent the required components of P, in magnitude and direction.

EXERCISE 156. Demonstrate the correctness of the above construction by means of the triangle of forces.

EXERCISE 157. Find the components of $P = 10$ lb. inclined at 45° to the horizon along lines inclined at

(*a*) 0° and 90°;
(*b*) 30° " 60°;
(*c*) −30° " 120° to the horizon.

EXERCISE 158. Find graphically the components of a force whose magnitude, direction, and point of application are 20 lb., 210°, and (2, 6) respectively, along lines inclined at angles of 45° and 60° to the axis of X.

Resultants

If it is desired to find the resultant of more than two forces acting upon a particle, the principle of the parallelogram of forces may still be applied, provided the resultant of any two forces be found, and this resultant be then combined with another of the given forces, etc.

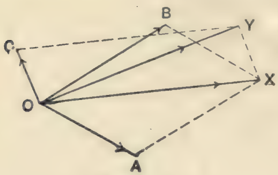


FIG. 131

This is illustrated in Fig. 131, where the given forces are OA, OB, OC. The resultant of OA and OB is OX, and the resultant cf OX and OC is OY. Therefore the resultant of OA, OB, OC is OY.

As the number of forces increases it is evident that this construction becomes more and more complicated.

In its place a construction based upon the *polygon of forces* is used. This is shown in Fig. 132. The forces whose resultant is required are shown in Fig. 132 (*a*). In

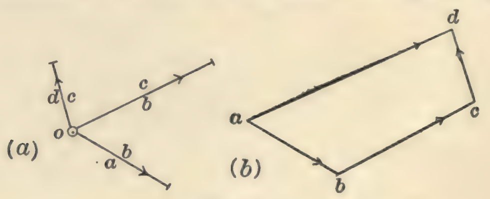


FIG. 132

the detached diagram, Fig. 132 (*b*), an open polygon is drawn having for its sides lines parallel and equal to the forces *ab*, *bc*, and *cd* arranged tail to head. The arrow *ad* required to close the polygon and taken in a direction around the polygon opposed to that of the given forces represents the required resultant in magnitude and direction but not in position. A force parallel and equal to *ad* acting at *o* completely represents the resultant.

EXERCISE 159. Find the resultant of the forces shown in Fig. 133, by means of the polygon of forces, and demonstrate the dependence of this construction upon that of the triangle of forces.

EXERCISE 160. Find the resultant of the forces shown in Fig. 133 by means of the polygon of forces, and demonstrate the correctness of the construction by resolving each force into vertical and horizontal components and showing that

$$\text{Resultant} = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}.$$

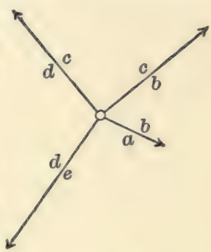


FIG. 133

EXERCISE 161. Assume three sets of forces of 3, 4, and 5 forces respectively, and find the resultant of each set.

EXERCISE 162. Six men pull upon six cords knotted together. They exert forces of 50, 60, 40, 100, 80, and 125 pounds and pull towards the N., N. 30° E., E. 60° S., W., NW., and S. 30° W., respectively. Find the direction in which the knot will move.

EXERCISE 163. If O is any point in the plane of the triangle ABC, and D, E, F are the midpoints of the sides of the triangle, show that the system of forces OA, OB, OC is equivalent to the system OD, OE, OF.

SECTION XVIII

CONDITIONS FOR EQUILIBRIUM

For the equilibrium of forces acting on a particle it is evident that the resultant must be zero. Therefore from the above discussion of the polygon of forces it follows that if the *forces acting upon a particle are known to produce equilibrium, the polygon formed of them must close,* *Example.*—A particle weighing 5 pounds is supported upon a rough inclined plane by forces of 3 and 2 pounds acting horizontally and vertically respectively. Find the normal reaction of the plane and the friction force, if the inclination of the plane is 30°.

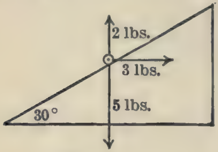


FIG. 134

Solution.—Fig. 134 illustrates the problem. To solve graphically *all* forces acting on the particle should be shown, as in Fig. 135. As the reaction of the plane, *de*, and the friction force, *ea*, are unknown, they are represented by their *lines of action only.*

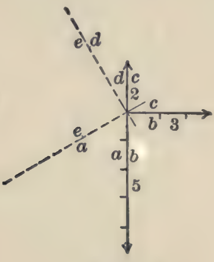


FIG. 135

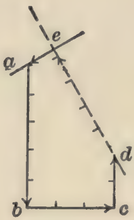


FIG. 136

In Fig. 136 the polygon of forces is shown. To construct this start with the known forces and then close the polygon by lines parallel to the lines of action of the unknown forces.

The arrows *de* and *ea* represent the normal reaction of the plane and the friction force respectively. If measured with the same scale as was used in plotting the known forces, their magnitudes are directly obtained. The total reaction of the plane, being defined as the sum of the normal reaction and friction, is represented by *da*. (Why?)

In the following exercises diagrams illustrating the frames should first be drawn to scale so as to obtain the relative inclination of the members.

Exercise 164. Fig. 137 shows a derrick, ABC, and a chain, DCE, supporting a load of 10 tons. Assume $AC = 25$ feet, $AB = 9$ feet, $BC = 20$ feet, and D the midpoint of AB.

Find graphically the stresses in AC and BC if the chain is fastened at C. (This releases DC of stress.)

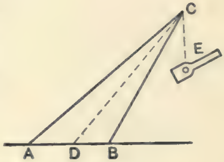


Fig. 137

Exercise 165. Find stresses in AC and BC, Fig. 137, if the chain passes freely over a pin at C. (This causes a stress of 10 tons in DC.)

Exercise 166. Find the stress in AB and BC of the triangular frame, Fig. 138, if 2 tons are suspended from B.

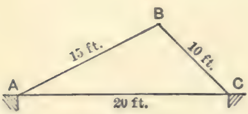


Fig. 138

Exercise 167. What will be the stress in AC, Fig. 138, if the frame is loaded as in Exercise 166?

Exercise 168. If in Fig. 138 two cables inclined to the horizon at 45° and 90° and stretched with forces of 2 and 5 tons respectively be attached to B, find the stress in AB and BC.

Exercise 169. Find stresses in AB and BC, Fig. 139, if the dotted line represents a rope passing without friction over B and fastened at E.

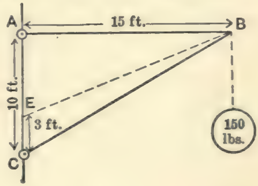


Fig. 139

Exercise 170. If a kite in equilibrium in the air makes an angle of 45° with a horizontal plane, and if the pressure of the air on the kite is equal to three times the weight of the kite, find the direction of the string at its point of attachment to the kite, and the magnitude of the tension in the string in terms of the weight of the kite.

EXERCISE 171. A string $ABCD$ is fastened to supports at A and D which are on the same level. Two weights of 10 pounds and x pounds are tied to the string at B and C. If AD is 30 feet and BC is horizontal and $AB=BC=CD=12$ feet, find x and the tensions in AB, BC, and CD.

When considering the relation between *forces acting on a body* the important principle concerning the equilibrium of three forces stated on page 60 should be carefully remembered; for by it the problem can sometimes be reduced to the case considered in this section.

EXERCISE 172. Find the reactions at the points of support A and B when the crane, shown in Fig. 140, is loaded at C with 4000 pounds. (The support at A consists of a collar, and at B of a footstep.)

EXERCISE 173. Make the rod DC (Fig. 140) a "free body" and thus find the stress in the rod EF.

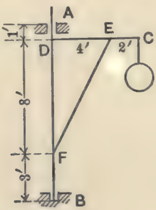


FIG. 140

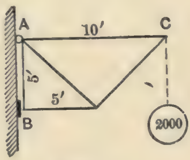


FIG. 141

EXERCISE 174. If in Fig. 141 the frame ABC is hinged to the wall at A and rests against the wall at B, find the supporting forces at A and B.

CHAPTER VI

FORCES ACTING ON A RIGID BODY

SECTION XIX

RESULTANTS

THE finding of the resultant of forces acting on a rigid body is next to be considered. Assume the forces shown in Fig. 142 and let their resultant be required. It will be remembered (see p. 43) that the point of application of a force may be placed anywhere upon its line of action and thus the forces 1 and 2 may be applied at the point of intersection of their lines of action and the parallelogram completed to obtain the resultant *X*. *X*, considered as replacing 1 and 2, may now be similarly combined with 3, and *Y*, the resultant of 1, 2, and 3, obtained in both magnitude and position. This resultant, *Y*, may have its point of application anywhere along its line of action.

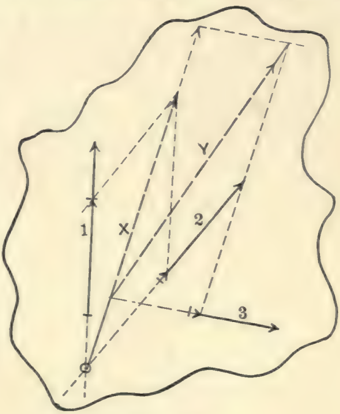
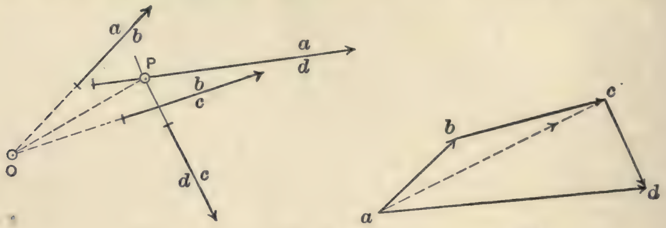


FIG. 142

This use of the *parallelogram of forces* is cumbersome when a larger number of forces is to be dealt with. The triangle or polygon of forces is again, as in Section XVII, used to simplify the construction.

In Fig. 143 (*a*) the forces are shown, and in Fig. 143 (*b*) the forces are combined to obtain the resultant. First *ab* and *bc* give *ac*, then *ac* and *cd* give the final resultant



(*a*) FIG. 143 (*b*)

a l, Fig. 143 (*b*), in *magnitude and direction* but *not in position.*

Notice that from no portion of the work already completed can we infer at what point of the body this resultant should act.

To find the position of the resultant, return to Fig. 143 (*a*) and find the intersection of the lines of action *ab* and *bc* at *O*. Through this point we know that their resultant must act; therefore through *O* draw a line parallel to *ac* of Fig. 143 (*b*). This partial resultant is now combined with *cd*, and the intersection of their lines of action is found at *P*. Finally through this point the resultant of *ac* and *cd* (or of *ab*, *bc*, and *cd*), that is the force *ad*, must pass. The problem is thus completely solved.

EXERCISE 175. Find the resultant of the forces shown in Fig. 144

(*a*) by means of the parallelogram of forces;
(*b*) by means of the triangle of forces.

If it should happen that the given forces are parallel or even nearly so, it is evident that the method for finding their resultant outlined above is inapplicable, as the intersection of the lines of action of any two such forces cannot be found at all, or is inconveniently distant. It is therefore necessary to develop some modification of the preceding method.

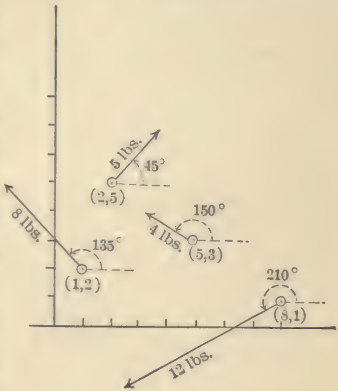


FIG. 144

Consider the forces *ab*, *bc*, *cd*, and *de*, shown in Fig. 145. Instead of finding the resultant of the given forces directly, introduce into the problem a force *oa*, taken entirely at random or to suit convenience, and then proceed to find the resultant of this new set of forces, as already explained on page 114.

This is done in Figs. 145 and 146, and *oe* is found to be the resultant of *oa*, *ab*, *bc*, *cd*, and *de*. Thus *oe* equals the sum of the given forces *plus oa*. But as *oa* was introduced for convenience of solution only, it should now be removed, i.e., *subtracted from oe*, to obtain the sum of the given forces or the required resultant. This is done by drawing the line *ae*; for, if the direction of the arrows be noted, we see that

$$oa + ae = oe;$$

$$\therefore\ ae = oe - oa.$$

This is equivalent to reversing the direction of the force *oa* and adding it to *oe*.

Thus a force parallel and equal to *ae*, having its line of action passing through *P*, the intersection of the lines of action of *oa* and *oe*, is the required resultant.

The polygon formed of the forces *ab*, *bc*, *cd*, *de*, and *ae*, Fig. 146, is called the *polygon of forces*, or more dis-

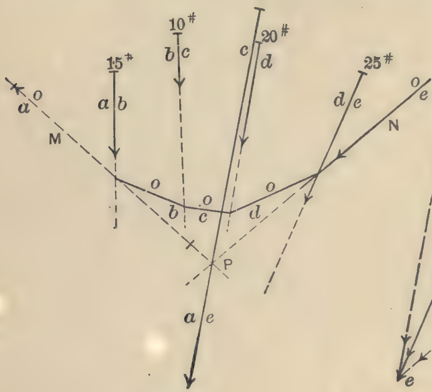


FIG. 145

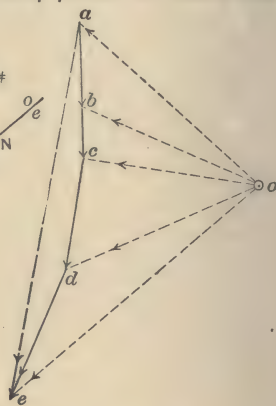


FIG. 146

tinctively the *magnitude polygon*. It depends only upon the forces irrespective of their positions. The point *o* is called the *pole*, and the lines *oa*, *ob*, . . . , *oe* are *rays*.

The polygon formed by the lines of action of the forces *oa*, *ob*, *oc*, *od*, and *oe*, Fig. 145, is called the *funicular polygon*. This is the Latin equivalent of string polygon, and it is so named because it gives the form assumed by a weightless string fastened at the points *M* and *N* and subjected to the given forces *ab*, *bc*, . . . , *de*.

In applying this method to a set of given forces, it will be found convenient to first letter all the forces, then draw the magnitude polygon, thus determining the magnitude and the direction of the resultant, as in Fig. 146. Next, instead of assuming the force *oa* as in the above demonstration, place the pole in any convenient position and draw the rays. Then start the funicular polygon by drawing the line of action of *oa* anywhere in the diagram, showing the positions of the given forces, parallel to the ray *oa*. Through the intersection of the first and last sides of the funicular polygon draw the resultant parallel and equal to the arrow representing the resultant in the magnitude polygon.

Notice particularly the system of lettering employed and follow it closely in the exercises.

EXERCISE 176. Determine graphically the resultant of the following forces: (20 lb., 45°, 0, 0), (15 lb., 90°, 13, 0), (30 lb., 60°, 7, 0), and (25 lb., 315°, 30, 0). Each force is given by its magnitude, the direction of its line of action with reference to the X-axis, and one point on its line of action.

EXERCISE 177. Find the resultant of (20 lb., 105°, 5, 0), (25 lb., 263°, 11, 0), (30 lb., 98°, 15, 0), (10 lb., 80°, 17, 0), (15 lb., 280°, 2, 0).

(*Note.*—The angles in this exercise can be conveniently plotted by the use of a table of natural tangents.)

EXERCISE 178. Find the resultant of forces of 10, 20, 30, 40, 50 lb. acting vertically upward, with successive intervals between their lines of action of 3, 2, 6, and 4 feet.

EXERCISE 179. Same as Ex. 178, but with 10- and 30-lb. forces acting vertically downward.

SECTION XX

CONDITIONS FOR EQUILIBRIUM

The magnitude polygon shown in Fig. 146 is said to be "open" because, in addition to the given forces, *ab*, . . . , *de*, an extra force, *ae*, the resultant, is required to close the polygon. If the given forces placed tail to head do of themselves form a "closed" magnitude polygon, no closing line is required and the given forces have no resultant.

This, however, does not mean that the body on which these forces act is in equilibrium, for the *absence of a resultant simply precludes translation.*

It yet remains to be shown that no couple acts upon the body. To do this we turn to the funicular polygon. From Fig. 145 it will be noted that the funicular polygon has all but one of its vertices on the lines of action of the given forces, and this one vertex P lies on the line of action of the resultant. The polygon is said to be "open" because *all* of its vertices do not lie on the lines of action of the *given* forces. If, however, the lines of action of *ao* and *oe* should coincide, then the funicular polygon would be "closed," i.e., have each of its vertices upon one of the lines of action of the given forces, and the forces *ao* and *oe* would neutralize each other and produce equilibrium. This can only happen if the magnitude polygon is also closed. Study carefully Fig. 147.

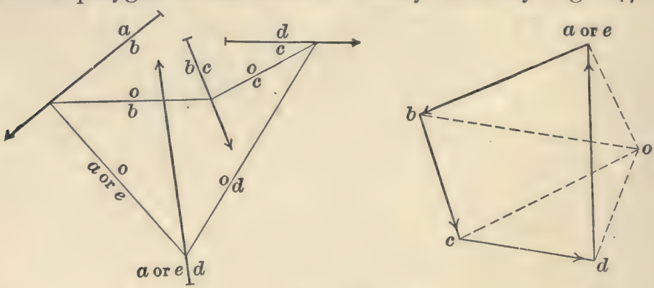


FIG. 147

It is, however, possible that the magnitude polygon be closed, and that the first and last sides of the funicular polygon do not coincide but are parallel. Under these conditions the funicular polygon is open and a couple, whose forces are *ao* and *oe*, results. Study carefully Fig. 148.

Thus it is seen that for *equilibrium of translation the graphical requirement is a closed magnitude polygon, and for equilibrium of rotation a closed funicular polygon is*

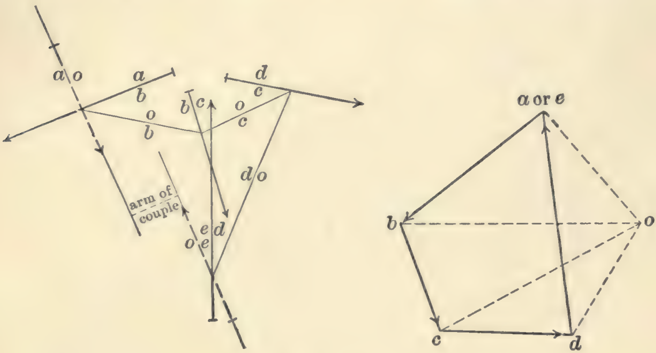


FIG. 148

necessary. These, then, are the graphical conditions of equilibrium.

The following examples illustrate the method of procedure in the application of the above principles.

Example.—Upon a uniform beam, *AB*, 10 feet long, weighing 150 pounds, a force of 100 pounds acts at an angle of 315° at a point 6 feet from *A*: find the reactions of the supports if the beam is horizontal, and its end *A* is hinged to a vertical wall while the beam rests upon a smooth knife-edge one foot from *B*.

Solution.—Fig. 149 (*a*) illustrates the problem. Fig. 149 (*b*) shows the beam as a "free body." There is an unknown vertical reaction P at C, and a reaction Q, unknown in magnitude and direction, at A.

Letter the forces, known and unknown, and start the magnitude polygon, Fig. 149 (c), using any convenient scale. Proceed to the line of action of cd, the point d being undetermined.

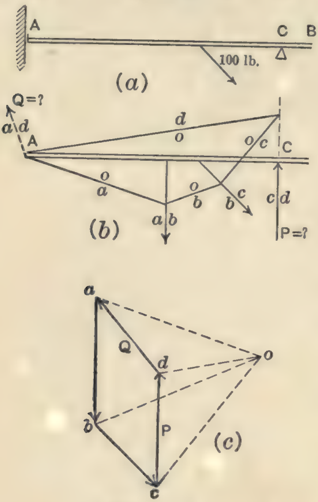


FIG. 149

Assume a pole, o, and draw the rays, ao, bo, and co. Now *start the funicular polygon*, Fig. 149 (b), *at* A, *the only known point in the line of action of* Q. *This is essential, as otherwise the funicular polygon cannot be closed.* Complete the funicular polygon, closing it by the line od, and through o, Fig. 149 (c), draw od parallel to od, Fig. 149 (b), and its intersection with cd determines the point d.

Close the magnitude polygon by drawing da. Then $cd = P = 140$ pounds is the vertical reaction at C and $da = Q = 110$ pounds is the reaction at A in both direction and magnitude.

These are the forces which represent the actions of the knife-edge and hinge upon the beam. What forces do the wall and knife-edge have to resist?

Example.—The weightless bar, Fig. 150 (a), supported by smooth pegs at A and B and by the cord CD, is in equilibrium. Find the reactions of the supports.

Solution.—Fig. 150 (b) shows the bar as a "free body."

Letter the forces (known ones first), and start the magnitude polygon, Fig. 150 (c). Having carried the magnitude polygon as far as possible, *start the funicular polygon at E, the intersection of the lines of action of the two last-named unknown forces;* this, again, is essential for the closing of the funicular polygon.

Close first the funicular polygon and then the magnitude polygon by drawing de and ea in Fig. 150 (c) through d and a parallel to the corresponding lines of action in Fig. 150 (b).

The reactions are found to be $cd = 730$ lb., $de = 725$ lb., and $ea = 200$ lb.

The application of the graphical principles of equilibrium offers no difficulties with the exception of the two illustrated in the above examples. These should therefore be carefully noted.

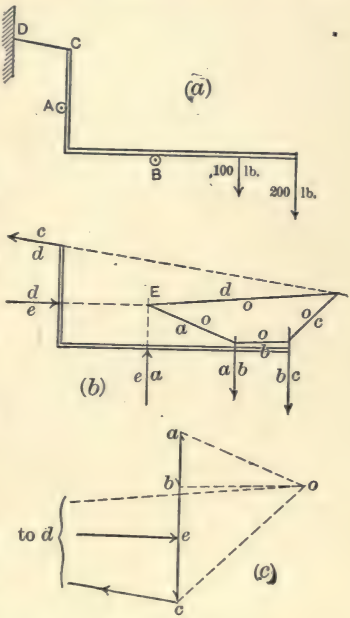


FIG. 150

Notice, also, as a check upon the correctness of a funicular polygon, that any two of its adjacent sides must meet upon the line of action of that force designated by the same letters as the sides after omitting the o's. Thus the sides oa and ob meet on the line of action of the force ab, etc.

Exercise 180. Assume four forces and find a fifth wholly unknown force necessary to produce equilibrium.

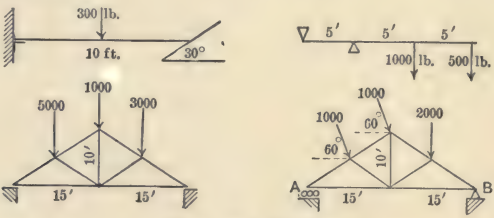
Exercise 181. Assume four non-parallel forces and the lines of action of two other forces. Find the magnitudes of these forces for equilibrium.

Note.—This system of forces requires the lines of action of *all* forces involved to pass through one point. This may be shown by replacing the known forces by their resultant and noting that this resultant and the two unknown forces can only be in equilibrium under the above conditions. (Why?)

Exercise 182. Same as Exercise 181, but all forces parallel.

Exercise 183. Assume two forces and the line of action of another. Find a fourth wholly unknown force and the magnitude of the third force for equilibrium.

Exercise 184. Assume two forces and the lines of action of three other forces. Find the magnitudes of these forces for equilibrium.



Figs. 151 to 154

Exercise 185. Find the reactions of the supports of the beam shown in Fig. 151.

Exercise 186. Find the reactions of the supports of the beam shown in Fig. 152.

Exercise 187. Find the reactions of the abutments of the frame shown in Fig. 153.

Exercise 188. Find the reactions of the abutments of the frame shown in Fig. 154. The end *A* rests on frictionless rollers, and *B* is hinged to the foundations.

CHAPTER VII

APPLICATIONS TO STRUCTURES

Section XXI

STRESSES IN MEMBERS OF FRAMED STRUCTURES

Up to the present only the forces acting on a rigid body have been considered. These forces are called the *applied forces* or *external forces*. The external forces acting upon a body cause a strained condition of the body due to the interaction of the various parts of the body in withstanding the action of the applied forces. This interaction of the parts of the body allows a transmission of forces from one part of the body to another. The forces so transmitted are known as *internal forces*.

Structures are contrivances for resisting forces. They may be divided into two types, *framed* and *non-framed*. A *framed structure*, or *frame*, is one composed of a system of straight bars fastened together, at their ends only, by pins so as to allow a hinge-like motion. Since the triangle is the only geometric figure in which a change of shape is impossible without a change in the length of its sides, the triangle is necessarily the basis of the arrangement of the bars in a frame.

Non-framed structures consist of one continuous member, or a number of members so fastened together throughout their lengths as to make one solid piece.

Only the forces acting in framed structures will be considered.

Consider the simple frame shown in Fig. 155. A load of 1000 pounds acts at the apex M, and the frame rests upon two abutments N and P.

The first step in finding the forces acting on and transmitted by the frame is to show the frame as a "free body," Fig. 156. In this figure, in order to readily refer

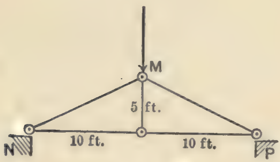


FIG. 155

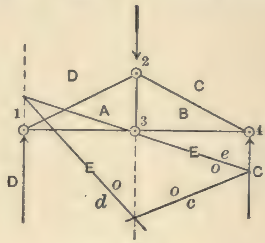


FIG. 156

to the various members of the frame and the applied forces, designate each portion into which the plane is divided by a letter as shown. The load of 1000 pounds would then be referred to by naming the areas on each side, as CD; similarly the members of the frame are EA, AD, AB, etc.

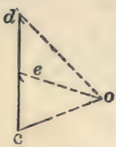


FIG. 157

Find now the reactions of the abutments DE and EC by means of the magnitude and funicular polygons as in Figs. 157 and 156. The reactions are found in Fig. 157 to be ce and ed. Thus a complete knowledge of the external forces is obtained.

To find the internal forces, consider the pin 1 joining the bars AD and EA. The forces acting on this pin are shown in Fig. 158 (*a*). *DE* is the known reaction, and *AD* and *EA* are the unknown forces transmitted by the members of the same name. These forces must be in equilibrium, therefore the triangle of forces *must close*, Fig. 158 (*b*), and *ae* and *da* are the required internal forces transmitted by *AE* and *AD* respectively.

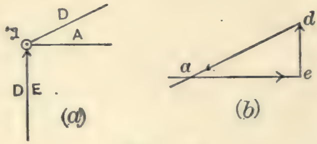


FIG. 158

Consider now pin 2. The forces acting on it are shown in Fig. 159 (*a*). Here, in addition to *DC*, the given load, *AD*, is known. For, as the member *AD* itself is in equilibrium and presses upon pin 1 in the direction *da*, Fig. 158 (*b*), it must press with an equal and opposite force on pin 2.

The polygon of forces for pin 2 is shown in Fig. 159 (*b*).

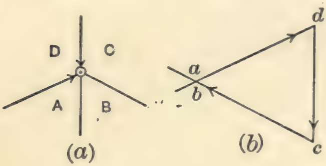


FIG. 159

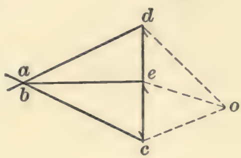


FIG. 160

This polygon shows that the member *AB* transmits no force and is therefore useless *for this particular loading of the frame.*

The process begun above may be continued until as many force polygons as there are pins have been constructed. Instead, however, it is much shorter to combine all these polygons into one diagram based upon the magnitude polygon, Fig. 157, as shown in Fig. 160. The reason for this is evident if we consider that each polygon has for its sides one or more sides already belonging to preceding polygons.

In order to facilitate the construction of the polygons of internal forces, the names of all the forces acting at each pin, *always taken in one direction* (clockwise) and always starting with known forces, are set down thus:

For pin 1, *EDAE*; pin 3, *EABE*;
pin 2, *ADCBA*; pin 4, *CEBC*.

Then, as in Fig. 160, draw lines parallel to the corresponding members transmitting the forces, through the proper points, until all the schemes have been completed.

Consider now very carefully the information conveyed by these polygons of internal forces (Fig. 160). If we consider the action of the member *AD* upon pin 1 we need the scheme *EDAE*. Follow this in Fig. 160 and note that *DA* presses pin 1 downward and to the left. For equilibrium it is necessary for pin 1 to react with an equal and opposite force, *pushing member AD upward and to the right*. Now regard pin 2; here the scheme is *ADCBA*. This, with the assistance of Fig. 160, shows that *AD* pushes pin 2 upward and to the right. Thus *pin* 2 reacts on *AD* by *pushing it downward and to the left*. Thus *AD* is pushed upward and to the right at 1, and downward and to the left at 2, by equal forces. These equal and opposite forces produce in *AD* a *stress*, and in this case the stress is known as a *compression*, and its magnitude is given by the length *ad*, Fig. 160. The member *AD* must be made of sufficient strength to withstand this compression.

EXERCISE 189. Explain the stress in member AE, Fig. 156.

EXERCISE 190. Explain the stress in member BC, Fig. 156.

EXERCISE 191. Find the stresses in the various members of the frame shown in Fig. 155 if P is rigidly fastened to the abutment and N rests on frictionless rollers, and the load of 1000 lbs. at the apex is inclined at 45° to the horizon.

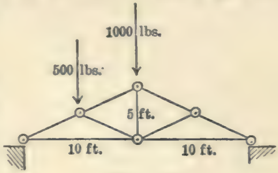


FIG. 161

EXERCISE 192. Find the stresses in the members of the frame shown in Fig. 161.

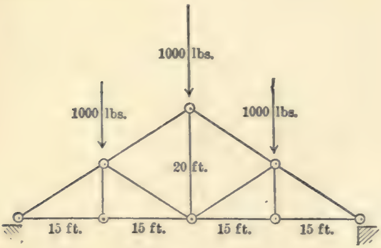


FIG. 162

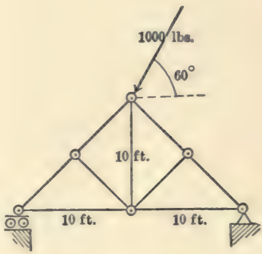


FIG. 163

EXERCISE 193. Same as Ex. 192 for Fig. 162.

EXERCISE 194. Same as Ex. 192 for Fig. 163.

SECTION XXII

THE FUNICULAR POLYGON FOR PARALLEL FORCES CONSIDERED AS A MOMENT DIAGRAM

Fig. 164 shows a horizontal bar supported at the ends and supporting the weights P and Q. The magnitude polygon is shown at the right, and the funicular polygon directly under the bar. From these polygons we derive the magnitudes of the reactions cd and da.

As this bar is in equilibrium the sum of the moments of all the forces acting on the bar about any point, A, must be zero. Thus, the sum of the moments of the forces to the left of A must be equal and opposite in sign to the sum of the moments of the forces to the right of A. At A, then, there is a tendency to bend the bar by rotating the left-hand end clockwise and the right-

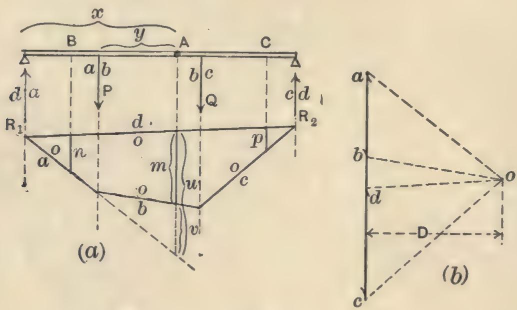


FIG. 164

hand end counter-clockwise about A. The measure of this tendency to bend the beam at A is called the *bending moment*, or simply the *moment*, at A, and it is equal to the sum of the moments of the forces, *either* to the left *or* to the right of A, about A.

In Fig. 164 this moment would be

$$M = -R_1x + Py.$$

To obtain M graphically we must remember that in graphics *division must always precede multiplication.* From Fig. 164 (*b*) we see that $R_1 = da$ and $P = ab$ are both divided by D (see page 100). It only remains to multiply by x and y respectively. This is done in Fig. 164 (a), where $u=\frac{R_1}{D}x$ and $v=\frac{P}{D}y$ (see page 101);

$$\therefore\ m=u-v=\frac{R_1}{D}x-\frac{P}{D}y,$$

or

$$mD=R_1x-Py,$$

$$\therefore\ mD=-M.$$

As the sign is immaterial, depending only on which side of A we consider the moment as acting, we see that

$m \cdot D$ is the bending moment at A.

m being a vertical distance bounded by the funicular polygon, we see that the funicular polygon can be considered as a moment diagram.

EXERCISE 195. Construct a diagram similar to Fig. 164 and show that (a) nD is the bending moment at B; (b) pD is the bending moment at C.

In the application of the above principle care must be taken to measure the various distances in the proper units. A good rule to follow is to *measure* the forces and other *vertical distances with the scale of force* (in pounds) and all *horizontal distances with the scale of space* (in feet). Thus, m, p, ab, cd, etc. (Fig. 164) should be measured in pounds and x, y, D, etc. in feet.

EXERCISE 196. Find graphically and check by calculation the bending moments at B, C, D, E, and F in the following horizontal beam: The beam, AG, is 30 feet long, $AB=BC=$ etc. $=5$ feet; abutments are at A and G; loads of 3000, 1000, 2000 lbs. are hung at B, C, and E respectively.

EXERCISE 197. A horizontal beam AE, 20 feet long, is divided into four equal parts by B, C, and D. The beam is supported at A and B and loaded at C, D, and E with 1000, 3000, and 2000 lbs. respectively. Find graphically the bending moments at B, C, D, and E.

EXERCISE 198. Check the results of Ex. 197 analytically.

EXERCISE 199. A beam similar to that of Ex. 197 is supported at A and C and loaded with 3000, 2000, and 3000 lbs. at B, D, and E respectively. Find the moments at B, C, and D.

EXERCISE 200. A horizontal beam AF, 25 feet long, supports loads of 1000, 500, and 300 pounds at one end and at 10 and 25 feet from this end respectively. The supports are 5 and 15 feet from this same end. Find graphically the moments at 5-foot intervals along the beam.

SECTION XXIII

GRAPHICAL METHOD FOR FINDING CENTROIDS

To find the centroid of a lamina, divide the lamina into portions the position of whose centroids are known, and consider the weights of these portions to act at their respective centroids in any convenient direction. By means of the funicular polygon find the resultant of these weights. The centroid must lie somewhere on the line of action of this resultant. Now assume the weights to act in any other direction, always through their respective centroids, and again find their resultant. The centroid must also lie upon the line of action of this resultant. The centroid is thus located at the intersection of the lines of action of these resultants.

This is illustrated in the following

Example.—Find the centroid of the plate shown in Fig. 165.

Divide the plate into rectangles. The weight of each rectangle is proportional to its area; thus the weights of the rectangles can be represented by 20×6, 20×4, 8×4 respectively.

Consider first the weights *ab*, *bc*, and *dc* as acting vertically downward; their resultant is found to be *ad*.

Next assume the weights to act horizontally. Repre-

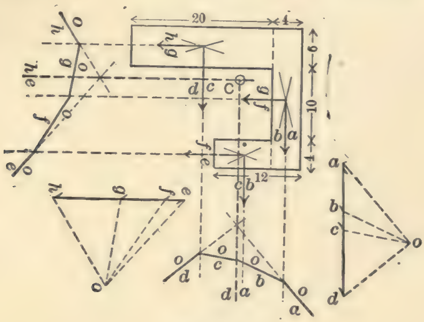


Fig. 165

sent them by *ef*, *fg*, and *gh*; their resultant is now *eh*. Therefore the centroid of the plate is at *C*, the intersection of the lines of action of the resultants.

Exercise 201. Find graphically the centroid of the plates shown in Fig. 69.

Exercise 202. Find graphically the centroid of the lamina described in Exercise 153.

APPENDIX

APPLICATION OF TWO-DIMENSIONAL METHODS TO FINDING STRESSES IN THREE-DIMENSIONAL STRUCTURES

In this volume only two-dimensional statics has been considered. A few examples will illustrate how the principles already studied may be applied to three-dimensional structures.

Example.—Find the stresses in the members of the structure represented pictorially in Fig. 166 (*a*). Here *AD*,

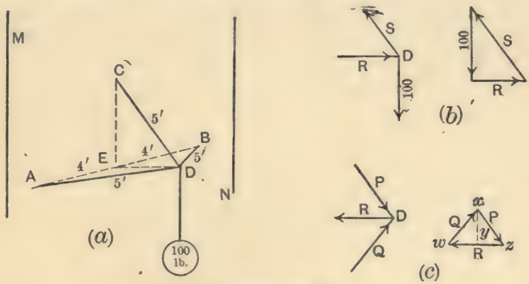


FIG. 166

BD, and *CD* represent three rods fastened at *A*, *B*, and *C* to a vertical wall *MN*. *A* and *B* are on the same level and 8 feet apart; *EC* is perpendicular to *AB* at its midpoint. The rods are each 5 feet long.

Solution.—Pass a plane through C, D, and E and find the stresses in the member CD, and in an imaginary member ED, by considering the equilibrium of the point D under the action of the forces transmitted by these members and the weight of 100 pounds, Fig. 166 (b).

By the similarity of triangles, we have

$$\frac{R}{100}=\frac{3}{4}, \quad \therefore R=75 \text{ pounds};$$

also
$$\frac{S}{100}=\frac{5}{4}, \quad \therefore S=125 \text{ pounds}.$$

The tension in CD is thus found to be $S=125$ pounds.

Now pass a plane through A, B, and D. This plane contains the rods AD and BD and the imaginary rod DE. As AD and BD are to replace ED, a force equal and opposite to R, Fig. 166 (b), together with the forces transmitted by AD and BD, must be in equilibrium. These forces are shown in their true relative positions, together with their triangle of forces, in Fig. 166 (c).

By comparing $\triangle xzw$ with $\triangle ABD$ we can show that $P=Q$, $\therefore wy=yz=\frac{75}{2}$. As $\triangle xyz$ is similar to $\triangle EBD$ we have

$$\frac{P}{\frac{75}{2}}=\frac{5}{3}, \quad \text{or} \quad P=62.5 \text{ pounds}.$$

Thus the compressions in AD and BD are equal each to 62.5 pounds.

Exercise 203. A pair of "sheer-legs" is formed of two equal spars fastened together at the top so as to form an inverted V. The spars are inclined at 60° to each other, and their plane is inclined at 60° to the ground. A rope attached to the top of the sheers and inclined at 30° to the horizon lies in a vertical plane bisecting the sheers and holds them in position. Find the stresses in the spars and in the rope when a weight of 10 tons is lifted.

EXERCISE 204. A pair of sheers, such as described in Ex. 203 but of the following dimensions, supports a load of 122 tons. The legs of the sheers are 116 feet long; they are 45 feet apart at the bottom; the supporting guy is 146 feet long, and the sheers afford a horizontal reach of 35 feet. Find the stresses in the legs.

Example.—The vertical post of a crane is 10 feet long. The jib is 30 feet long and the stay is 24 feet long. There

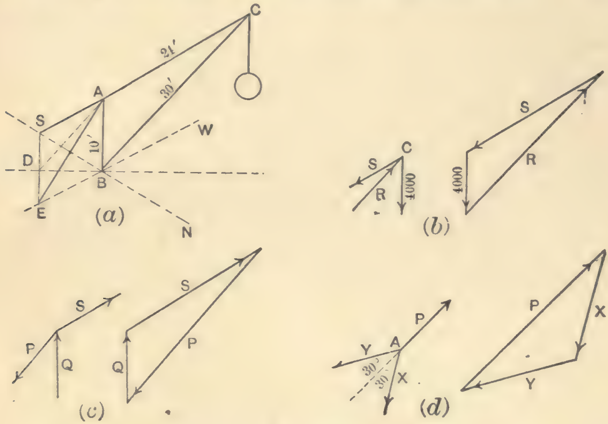


FIG. 167

are two back-stays, making angles of 45° with the horizontal; these lie in planes due south and due east of the post. A weight of 4000 pounds is sustained by the crane. Find the forces transmitted by the jib- and back-stays when the jib lies to the northwest of the post.

Solution.—Fig. 167 (a) illustrates the problem. Consider the equilibrium of the forces acting at C and lying in the plane ABC. If their triangle of forces, Fig. 167 (b), is drawn to scale, we obtain the forces transmitted by AC and BC graphically. Represent these by S and R.

Now consider the point A in equilibrium under the action of AC, AB, and an imaginary stay AD. These forces all lie in the plane $ACBD$. From the triangle shown in Fig. 167 (c) find the forces transmitted by AB and AD, let these be Q and P.

Then consider A in equilibrium under the action of AS, AE, and a force equal and opposite to P, Fig. 167 (c). These forces are shown in their true relative positions in Fig. 167 (d). From this figure we obtain $Y = X$, the stresses in the stays AS and AE.

EXERCISE 205. Calculate the stresses found graphically in the preceding example.

EXERCISE 206. Find the stresses in the structure described in the preceding example when the jib lies to the north of the post.

EXERCISE 207. A weight of 100 pounds is sustained by a tripod having equal legs so arranged that the distance between each pair of feet is equal to the legs. Find the compression in each leg.

EXERCISE 208. Find the compression in each leg of a tripod the feet of whose legs rest at the vertices of an equilateral triangle whose sides are $\sqrt{3}$ feet long, if the legs are 5 feet long and the tripod sustains a weight of 1000 pounds.

EXERCISE 209. A, B, and C are the vertices of an isosceles triangle drawn upon a horizontal ceiling. AB, the base of the triangle, is 6 feet long; the altitude CD is 5 feet long. To A and B two strings, each 5 feet long, are attached; at C a string 3 feet long is fastened. The strings are joined at their lower ends and support a weight of 100 pounds. Find the tension in each string.

PROBLEMS FOR REVIEW

210. Two weights of P and Q pounds are attached to a string 21 inches long at points 8 and 6 inches respectively from the ends. If the ends of the string are fastened to two points on the same level 14 inches apart, and the central portion of the string is horizontal when equilibrium results, find the ratio of P to Q.

211. Draw an equilateral triangle ABC with the base AB horizontal and C downward. Let a weight at C be tied by threads AC and BC to the fixed points A and B: if the thread BC is cut, by how much does the tension in AC increase?

212. A load, W, of 2000 pounds is hung from a pin, P, at which pieces AP and BP meet like the tie-rod and jib of a crane. The angles WPB and WPA are respectively 30° and 60°. Find the forces transmitted by AP and BP, and state which piece acts as a strut and which as a tie.

213. Draw an equilateral triangle ABC. Let BC represent a weightless lever acted on at B by a force of 13 pounds, acting from A to B, and at C by a force of 9 pounds, acting from A to C. Find graphically the pressure on, and the position of, the fulcrum.

214. An incline, the ratio of the height to the base being 1 to 10, supports a body weighing 100 pounds. If the coefficient of friction is .2, what force inclined at 30° to the incline would move the body up the plane?

215. A body rests upon a smooth plane inclined at 35° to the horizon. If the body weighs 10 pounds, find the tension in the supporting string when the string is inclined to the plane at an angle of (*a*) 25°, (*b*) $-10°$.

216. A dead-weight safety-valve is 4 inches in diameter. The weight of the valve is 20 pounds. What additional weight must be added so that steam should blow off when the pressure reaches 80 pounds per square inch?

217. A square lamina is divided into four equal squares by lines parallel to the sides; a circle is inscribed in one of these squares, and the portion of the lamina within the circle is removed. Find the centroid of the remainder.

218. A uniform beam, whose length is 4 feet and whose weight is 20 pounds, has weights of 4 and 8 pounds suspended from its extremities. Where must a single support be placed to produce equilibrium?

219. A, B, and C are three smooth pegs in a vertical wall, A being the highest. AB and AC make angles of 30° and 60° respectively with the vertical through A on opposite sides. A string, carrying two weights of 12 pounds each, passes over the pegs and the weights hang freely. Find the pressure on each peg.

220. A sphere, diameter 1 foot, hangs against a smooth vertical wall by a string 6 inches long fastened to its surface and to the wall; find the tension of the string and the pressure of the sphere against the wall.

221. A uniform wire AD, 15 inches long, is bent upwards at right angles 4 inches from A and 6 inches from D. Prove that if it be suspended from A it will rest with the second bend vertically below A.

222. A step-ladder has the form of the letter A. The semi-angle at the vertex is θ. If it rests on a smooth horizontal plane, and its legs are kept from slipping by a cord connecting them together half-way up, how much greater does the tension in the cord become when a weight W is placed on top of the ladder?

223. The post BA of a crane is 10 feet high; the jib BC is 24 feet long and is movable about B. The tie is shortened so as to drop the load of 6000 pounds at a point, D, 20 feet from B. Find graphically the stresses in the tie and jib.

224. A rod of length b is supported horizontally, and to its extremities are attached the ends of a string of length s. If a heavy ring of weight W is slung on the string, find the compression in the rod.

225. Draw a square $ABCD$; a force of 8 pounds acts from A to D, and two forces of 12 pounds each act from A to B and from C to D; find their resultant.

226. A beam balances about the midpoint of its axis when weights of 20 and 40 pounds are suspended one from each end, and it balances about a point one-third of the length from one end when the weights are interchanged. Find the weight of the beam and the distance of its centroid from one end.

227. A round table is supported on three legs A, B, and C. $AB = AC = 3$ feet, $BC = 4$ feet. A weight of 100 pounds is placed at P; the distances of P from AB and AC are respectively $\frac{1}{3}$ and 1 foot. What is the compression in each leg?

228. A sheer-leg is formed by two sheer-poles, BC and DC, each 25 feet in length and secured to a base-plate in the ground at B and D. The wire guy, AC, is attached to the ground at a point A which is 60 feet from BD. The vertical from the top, C, of the poles meets the ground at a distance of 10 feet from the center of BD, which is 15 feet long. Find the stresses in the guy (and the compression in the poles) when a weight of 20 tons is suspended from C.

229. A uniform beam 12 feet long and weighing 56 pounds rests on and is fastened to two props 5 feet apart, one of which is 3 feet from one end of the beam. A load of 35 pounds is placed (*a*) at the end farthest from a prop, (*b*) at the middle of the beam, (*c*) at the end nearest a prop. Calculate the pressure on each prop in each case.

230. A crane, whose post, tie-rod, and jib measure 15, 20, and 30 feet respectively, supports a load of 10 tons suspended by a chain passing over a pulley at the jib-head. Find the stresses in each member (1) when the lifting-chain passes

from the pulley to the drum parallel to the jib, (2) when the drum is placed so that the chain passes from the jib-head parallel to the tie-rod.

231. In a pair of pincers the jaws meet one inch from the pin forming the joint. If the handles are grasped with a force of 30 pounds on each handle at a distance 6 inches from the pin, find the compression exerted on an object held between the jaws, and also the force resisted by the pin.

232. A painter's scaffold 20 feet long and weighing 150 pounds is supported by vertical ropes attached 2 feet from each end; if two painters weighing 125 and 175 pounds are at 4 feet and 9 feet from one end of the scaffold respectively, and pots of paint weighing 30 pounds are at 6 feet from the same end, find the tensions in the ropes.

233. A uniform bar projects 6 inches beyond the edge of a table, and when 2 ounces is placed one inch from the projecting end the bar topples over; when it is pushed out so as to project 8 inches beyond the edge, one ounce at the end makes it topple over. Find the weight of the bar and its length.

234. In a common steelyard the weight of the beam is 10 pounds, and acts at a distance of 2 inches from the fulcrum. Where must a weight of 4 pounds be applied to balance it?

235. The weight of a window-sash 3 feet wide is 5 pounds; each of the weights acting on the cords is 2 pounds. If one of the cords be broken, find at what distance from the middle of the sash the hand must be placed to raise it with the least effort. What pressure must the hand exert?

236. A piece of lead placed in one pan, A, of a balance is balanced by 10 pounds in the other pan, B. When the same piece of lead is placed in the pan B it required 11 pounds in the pan A to balance it. Find the ratio of the lengths of the arms of the balance.

237. Two equal uniform spheres of weight W and radius a rest in a smooth spherical cup of radius r. Find the pressure between either sphere and cup and the pressure between the spheres.

238. A uniform beam weight W is hinged to a horizontal plane and rests against a vertical wall. Find the reaction of the hinge and the pressure on the wall if the inclination of the beam is α.

239. A smooth uniform beam rests against two smooth horizontal rods, and its lower end rests against a smooth horizontal plane. The beam is $2l$ feet long, and the rods touch it at points a and b feet from its lower end. If the inclination of the beam is α, find the reactions of the supports.

240. A beam rests between two rough horizontal rods. The beam lies in a vertical plane. Assume all necessary data and write the equations for equilibrium.

241. ABC is a rigid equilateral triangle; the weight is not considered; the vertex B is fastened by a hinge to a vertical wall, while the vertex C rests against the wall under B. If 100 pounds is hung from A, find the reactions at B and C graphically.

242. Three forces of 10, 15, and 50 pounds, making angles of $30°$, $90°$, and $-135°$ with the horizon, act upon a particle. Find the resultant force acting upon the particle. In what direction will the particle move?

243. A uniform beam weighing 10 pounds is supported at its ends by two props. If the length of the beam is 5 feet, find where a weight of 30 pounds must be attached so that the pressures on the props may be 15 and 25 pounds respectively.

244. A carriage-wheel, whose weight is W and whose radius is r, rests upon a level road. Find the least horizontal force applied at the axle necessary to draw the wheel over an obstacle whose height is h.

245. A mass whose weight is 750 pounds rests on a horizontal plane and is pulled by a force, P, inclined at $15°$ to the horizon. Find the value of P, which will just start the mass if the coefficient of friction is .62.

246. Find the total resistance of the plane in Ex. 245.

247. A body whose weight is 10 pounds is supported on a

smooth inclined plane by a force of two pounds acting along the plane and a horizontal force of 5 pounds. Find the inclination of the plane.

248. Two equal rafters l feet long support a weight, W, at their upper ends. Find the stress in the tie-rod a feet long connecting their lower ends.

249. A davit is supported by a foot-step at A and by a collar at B placed 6 feet apart. A boat weighing 2 tons is supported by two such davits and is about to be lowered. Assuming that the boat hangs 5 feet from the vertical through the foot-step and collar, and that each davit supports one-half the weight of the boat, determine the forces at A and B.

250. A man, sitting upon a board suspended from a single movable pulley, pulls downward at one end of the rope which passes under the movable pulley and over a pulley fixed to a beam overhead, the other end of the rope being fixed to the same beam. If the man weighs 180 pounds, what force must he exert so as to maintain equilibrium?

251. Make sketches of (*a*) a system of weightless pulleys in which one pound balances 32 pounds, (*b*) a system of weightless pulleys in which one pound balances 15 pounds.

252. Given four weightless pulleys, three movable and one fixed, around each pulley passes a separate rope; the load is a man weighing 160 pounds. Find the pull exerted by the man on the free end of the last rope in order to maintain equilibrium.

253. A rod 10 inches long can turn freely about one of its ends; a body weighing 4 pounds is hung at a point 3 inches from this end. If the free end of the rod is supported by a string inclined to it at an angle of 120°, find the tension in the string and the reaction at the fixed end of the rod.

254. Find the height of a cylinder which can just rest on an inclined plane the angle of which is 60°, the radius of the cylinder being r.

255. What is the minimum coefficient of friction necessary to prevent the sliding of the cylinder in Ex. 254?

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