TEXT BOOK OF MECHANICS

Designed for Colleges and Technical Schools

By LOUIS A. MARTIN, JR.

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TEXT-BOOK

OF

MECHANICS

BY

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Vol. VI.

THERMODYNAMICS

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FIRST THOUSAND

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PREFACE

THE kind reception of the first five volumes of this textbook by both teachers and students has led me to continue the same method of presentation.

I again seek to produce a text which will encourage the student to think and not to memorize, to do and not simply to accept something already done for him. At the same time care has been taken to furnish sufficient material in the way of explanation and example so that the student may not become discouraged.

In solving examples and exercises relating to steam the Steam Tables and Diagrams of Marks and Davis (Longmans, Green, and Co.) have been used.

I hereby gratefully acknowledge the assistance of my wife, Alwynne B. Martin, both in the preparation of the manuscript and in the reading of the proof. Mr. Gustav G. Freygang has also been kind enuf to read both manuscript and proof.

L. A. M., Jr.

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THERMODYNAMICS

INTRODUCTION

Thermodynamics is the science which deals with the relation between heat and other forms of energy. As most physical, chemical, and biological processes involve energy transformations involving heat, thermodynamics is a most fundamental science.

In this text-book the subject matter will be limited to the relation of heat to mechanical energy in both its potential and kinetic forms. It will include the mechanics of gases and vapors with special emphasis on technical or engineering applications.

Energy.—A body possesses energy when it can do work, when it can move itself or other bodies against resistances. The various forms in which energy is known to exist are

1. Kinetic energy
2. Potential energy
3. Heat energy
4. Radiant energy
5. Electric energy
6. Magnetic energy
7. Chemical energy.

Heat is a Form of Energy.—It has been experimentally demonstrated that other forms of energy may be converted into heat and that heat may be transformed into other forms of energy. Thus heat must be a form of energy.

It is useful to conceive a way in which this heat energy may reside in a body. As a working hypothesis it is usual to postulate the molecular constitution of matter and to point to the molecules as a storage place of heat energy. The Greek philosophers originated the idea that matter consists of ultimate particles or molecules separated by interspaces. By means of this hypothesis many physical properties of matter such as compressibility, solubility, etc. are readily explained; moreover, the laws of chemical combination are thereby explained with wonderful simplicity.

Internal Kinetic Energy.—The molecules of which matter is assumed to be composed are conceived to be in constant vibratory motion. Their mean velocity determines their kinetic energy and this mean velocity is assumed to increase with increasing temperature. The molecules of a body whose temperature is high may be conceived as producing the waves in the ether (which constitute radiant energy) owing to their rapid vibratory motion.

The higher the temperature of a body the greater will be its store of internal kinetic energy.

Internal Potential Energy.—The molecules of which matter is composed are conceived to exert an attraction for each other. This mutual attraction explains the cohesion which matter exhibits. If the mean distance between molecules is increased then work must be done in overcoming this attraction. Energy, equal in amount to the work done, is stored within the body as internal potential energy.

A change in the mean distance between the molecules may manifest itself as a change in volume or as a change in physical state. In either case a rearrangement of the molecules occurs.

External Work.—The heat supplied to a body from an external source does not reside wholly in the molecules of the body as internal kinetic and potential energies. Expansion usually accompanies the absorption of heat. Energy is necessary to do the work required in overcoming any external pressure exerted upon the body. This energy must be supplied to the body in the form of heat energy.

To compute the external work done in overcoming an external pressure, p, acting upon the expanding body note that the force on a differential element, dA, of the surface of the body is pdA. If this force is displaced normally to the surface thru a distance ds the work done is $pdAds$. As $dAds$ represents a differential volume of the increase in size of the body which may be denoted by dV we have as the external work performed, $\int pdV$.

The Conservation of Energy.—The heat absorbed by a body from some external source must be wholly accounted for, as energy has never been known to be either created or destroyed.

In order to establish an equation expressing the conservation of energy as applied to the above described forms of energy let

ΔQ be the heat absorbed by a body from external sources,

ΔP be that part of ΔQ which is stored in the body as an increase in internal potential energy,

ΔK be that part of ΔQ which is stored in the body as an increase in internal kinetic energy,

ΔW be that part of ΔQ which must be expended in overcoming the external pressure acting upon the body and which is stored as external potential energy,

then for our purpose the fundamental equation expressing the conservation of energy is

$$\Delta \mathbf{Q} = \Delta \mathbf{P} + \Delta \mathbf{K} + \Delta \mathbf{W}.$$

To illustrate the meaning of these symbols consider the heating of water. Here the increase in volume is very slight and therefore ΔW is very small. As no change in physical state occurs and as the mean distance between the molecules is not otherwise greatly increased by expansion ΔP is also small. The temperature does increase so that ΔK is relatively large and practically equals the whole of ΔQ. Thus the heat supplied is practically stored as increased internal kinetic energy due to an increase in the mean vibratory velocity of the molecules.

Consider now the conversion of boiling water into steam. Here the temperature remains constant so that ΔK is zero. ΔW and ΔP however are very large owing to the large increase in volume and to the change in physical state which calls for a rearrangement of the molecules, the mean distance between molecules having greatly increased.

The Units to be Used.—The units used by English-speaking engineers for measuring the energies above discussed are the **foot-pound** for the unit of work and energy and the **British thermal unit** for the unit of heat.

The British thermal unit (B.t.u.) is defined as $\frac{1}{180}$ of the heat which is absorbed by a mass of one pound of water while its temperature increases from 32 to 212° F, the pressure remaining constant and equal to the standard atmosphere. This unit is sometimes called the mean B.t.u. and it equals the heat required to change the temperature of one pound of water from 63 to 64° F under atmospheric pressure.

The First Law of Thermodynamics.—The first law of thermodynamics states that not only can heat be converted into other forms of energy (and vice versa) but that the complete transformation of a definite quantity of heat will always yield a definite quantity of mechanical energy.

Numerous experiments have led to the conclusion that 777.6 foot-pounds of mechanical energy always yield one B.t.u. of heat.

This constant, which for purposes of engineering calculations may be assumed to be 778, will for convenience be represented by the letter J in honor of Joule who in 1843 made the first experiments leading to a determination of this constant.

Thermal Capacity.—The heat required to raise the temperature of a body one degree F is called the thermal capacity of the body.

The thermal capacity of a body may vary with each degree of the thermometric scale and even for the same range of temperature the thermal capacity of a given body need not be constant under all conditions. It should be remembered that the heat supplied may be stored in three ways: (1) as internal kinetic energy manifested to our senses as a change in temperature, (2) as internal potential energy manifested as a change in volume or as a change in physical state, (3) as external work performed in overcoming external pressure. At least the third manner in which heat may be absorbed by a body must yield varying results with varying external conditions. These conditions have no connection with the nature of the body.

If ΔQ represents the heat absorbed under certain conditions with an increase in the temperature of the body from t_1 to t_2° F, then the **mean thermal capacity** of the body equals $\frac{\Delta Q}{t_2 - t_1}$ for this range of temperature and under the existing conditions.

The instantaneous value of **the thermal capacity** of a body at a temperature t° F would be represented by

$$\frac{dQ}{dt}.$$

If the thermal capacity be divided by the mass of the body in pounds the thermal capacity per unit of mass is obtained.

For water at 63.5° F and under standard atmospheric pressure, the thermal capacity per unit mass is by definition one B.t.u.

Specific Heat.—The specific heat of a substance at a given temperature and under given external conditions is the ratio of the thermal capacity per unit mass of this substance at the given temperature and under the given conditions to the thermal capacity per unit mass of water under some chosen standard temperature and external conditions.

If the chosen temperature and pressure for water are 63.5° F and atmospheric pressure then its thermal capacity per unit mass is unity and the specific heat of the substance becomes numerically equal to its thermal capacity per unit mass under the given conditions.

The heat absorbed by a given quantity of a substance would thus be computed from the continued product of its specific heat under the existing conditions, its mass, and the change in temperature.

GASES

CHAPTER I

THE LAWS OF IDEAL GASES

SECTION I

JOULE'S, BOYLE'S, AND CHARLES' LAW

THE laws about to be discussed are obeyed more or less exactly by all gases. The more the conditions under which the gases exist are removed from the conditions approaching liquefaction the more closely do the gases obey these laws. Thus at ordinary temperatures and pressures water vapor does not obey the laws even approximately, ammonia and carbon dioxide may as a first approximation be assumed to follow the laws of ideal gases, while hydrogen, oxygen, nitrogen, and air obey these laws with sufficient accuracy for all engineering purposes.

Any hypothetical gas which would obey absolutely the following laws is called an **ideal gas** (also a **perfect gas**). Actual gases approach this ideal condition more closely as their molecules become more and more separated and thus exert less and less influence upon each other. An ideal gas must thus be conceived to be so attenuated that intermolecular forces no longer exist. This conception leads to the first law of ideal gases.

Joule's Law.—In an ideal gas no change in internal potential energy occurs when the gas absorbs or rejects heat. Thus in an ideal gas any change in internal energy must be a change in internal kinetic energy which manifests itself as a change in temperature.

In our notation we have for ideal gases

$$\Delta \mathbf{P} = \mathbf{0}.$$

To test this law Joule devised the following experiment not knowing that Gay-Lussac had already performed it in a slightly different way. Joule connected two large receivers by means of a pipe closed with a valve. In one of these receivers he compressed air to 22 atmospheres; the other he exhausted. The whole apparatus was immersed in a well insulated water-bath whose temperature could be accurately determined. The experiment consisted in noting the change in temperature due to the flow of air from one receiver into the other after the valve was opened.

No change in temperature was detected.

In the notation of our fundamental energy equation

$$\Delta Q = \Delta P + \Delta K + \Delta W$$

this experiment was so devised that

$\Delta Q = 0$, for no heat was supplied to the air and

$\Delta W = 0$, for no external work was done during the flow of the air from one receiver into the other. Moreover the experiment showed that under the existing conditions

$\Delta K = 0$, for no change in temperature occurred. Thus ΔP must be equal to zero.

If the receivers in Joule's experiment are each immersed in a separate water bath, then considerable cooling would be noticed in the bath surrounding the compressed air receiver and the temperature of the bath surrounding the originally exhausted receiver would be correspondingly increased. This is explained by the fact that the air leaving the high pressure receiver does so with considerable velocity and therefore kinetic energy. This kinetic energy can be derived only from energy already in the system. Thus heat energy equal in amount to the kinetic energy disappears with a corresponding drop in temperature. Similarly the kinetic energy carried over into the other receiver is transformed by impact and friction into heat energy with a corresponding rise in temperature.

Boyle's Law.—At constant temperature the volumes of a given mass of an ideal gas are inversely proportional to the corresponding pressures.

Thus $$\frac{V_1}{V_2}=\frac{p_2}{p_1},$$

or $$p_1V_1=p_2V_2=\text{a constant},$$

when t is constant.

Law of Charles.—At constant pressure, each change of one degree Fahrenheit in temperature causes a change in the volume of an ideal gas at 32° F equal to $\frac{1}{492}$ of the volume at 32° F, and

At constant volume, each change of one degree Fahrenheit causes a change in the pressure at 32° F equal to $\frac{1}{492}$ of the pressure at 32° F.

To express the two forms of this law in symbols, let $\frac{1}{492}$ be represented by a for convenience, and let the zero subscript refer to the conditions existing at 32° F, so that p_0 and V_0 represent the pressure and the volume of the mass of ideal gas when the temperature is 32° F. Then the law of Charles may be written

$$V = V_0 + V_0 a(t - 32),$$

when p is constant and

$$p = p_0 + p_0 a(t - 32),$$

when V is constant, where p and V are the pressure and the volume when the temperature is t° F.

Absolute Zero.—The above symbolic expressions of the law of Charles may be simplified by using, instead of the Fahrenheit scale of temperature, a scale having the same degrees but whose zero is at the lowest conceivable temperature, the absolute zero.

The kinetic theory of gases shows that the pressure which a gas exerts may be explained as the result of the bombardment of the walls of the enclosing vessel by the molecules of the gas. The impact and therefore the pressure exerted depends upon the velocity of the impinging molecules. This velocity in turn depends upon the temperature of the gas. Thus no temperature means no vibratory velocity and no pressure. To find the absolute zero of temperature on the Fahrenheit scale we may thus place the pressure p in

$$p = p_0 + p_0 a(t - 32)$$

equal to zero and solve for t.

We find $t = -460°$ F. Therefore the absolute zero lies 492 Fahrenheit degrees below the freezing point of water.

Temperature referred to this absolute zero is called **absolute temperature** and will always be denoted by capital T.

Thus $$T=460+t$$

where t is the temperature on the Fahrenheit scale.

Placing $t=T-460$ in the equations expressing the law of Charles we obtain

$$V=V_0aT$$

and $$p=p_0aT.$$

The General Law for Ideal Gases.—A single law including Boyle's law and both forms of Charles' law is desirable.

To recapitulate we have

at constant temperature

$$pV=p_0V_0=\text{a constant},$$

at constant pressure

$$\frac{V}{T}=aV_0 \quad =\text{a constant},$$

at constant volume

$$\frac{p}{T}=ap_0 \quad =\text{a constant}.$$

A single equation combining these three equations may be obtained by equating the product of their left-hand members to the product of their right-hand members, as follows:

$$\frac{p^2V^2}{T^2}=a^2p_0{}^2V_0{}^2.$$

As $a=\frac{1}{492}$ and as $T=460+t$, whence $T_0=492$, we have

$$\frac{pV}{T}=\frac{p_0V_0}{T_0}.$$

Another method of obtaining this general relation between the pressure, the volume, and the temperature of any given mass of gas under any conditions and the corresponding values at 32° F consists in assuming an intermediate step in the transition as illustrated in Fig. 1. This intermediate step is necessary, because the application of either Boyle's or Charles' law requires in each case a constant condition. Thus we may maintain T constant during the change from A to B (Fig. 1) during which p becomes p_0 and V changes to V'. During the second stage of the transition p_0 must now be maintained constant, while V' changes to V_0 and T to T_0.

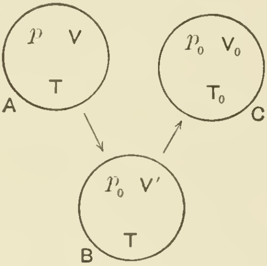

Fig. 1.

Applying Boyle's law during the change from A to B, we have

$$pV=p_0V'.$$

From Charles' law applied to the change from B to C we have

$$\frac{V'}{T}=\frac{V_0}{T_0}.$$

By the elimination of V' from these equations

$$\frac{pV}{T}=\frac{p_0V_0}{T_0}.$$

EXERCISE 1. Deduce the last equation assuming the pressure to remain constant during the first step of the transition and the temperature to remain constant during the second part of the transition.

In the above equations V represents the volume of a given mass of m pounds of gas. Let v represent the volume of one pound of the gas, then

$$V=mv, \text{ and of course } V_0=mv_0,$$

and we may write

$$\frac{pvm}{T}=\frac{p_0v_0}{T_0}m,$$

or

$$\mathbf{\frac{pV}{T}=mR}$$

where R is a constant for any given gas and

$$R=\frac{p_0v_0}{T_0} \quad \text{or} \quad \frac{pv}{T}.$$

R is called the **gas constant**. Its value depends not only upon the nature of the gas considered but also upon the units selected in the measurement of p, V, T, and m.

English-speaking engineers always measure p, the absolute pressure, in pounds per square foot, V, the volume, in cubic feet, T, the absolute temperature, in Fahrenheit degrees, m, the mass of the gas, in pounds.

Exercise 2. The weight of atmospheric air at normal temperature and pressure is 0.08071 pound per cubic foot. Find the gas constant for air.

Exercise 3. At what temperature will 50 pounds of air occupy 60 cubic feet when under an absolute pressure of 205 pounds per square inch? (R for air 53.3.)

Exercise 4. A cylindrical tank (capacity 3 cubic feet) contains 20 pounds of air at 60° F. To what internal fluid pressure is the tank subjected?

Section II

THE GAS CONSTANT OF ANY IDEAL GAS

In order to determine the gas constant for any ideal gas in terms of the molecular weight of the gas, Avogadro's law is used.

Avogadro's Law states that at the same temperature and pressure equal volumes of all ideal gases contain the same number of molecules.

Thus the weights of equal volumes of two ideal gases must be proportional to their molecular weights. As the weight of oxygen (molecular weight 32) is found to be 0.08922 pound per cubic foot at n.t.p. the specific weight of any other ideal gas whose molecular weight is μ is

$$(0.08922)\left(\frac{\mu}{32}\right) \text{ at n.t.p.}$$

The volume of one pound of any gas is thus $\frac{32}{(0.08922)(\mu)}$ cubic feet when under an absolute pressure of 14.7 pounds per square inch and at a temperature of 32° F.

The gas constant of any gas is therefore

$$\mathbf{R}=\frac{p_0v_0}{T_0}=\frac{14.7\times 144\times 32}{492\times 0.08922\times\mu}=\frac{\mathbf{1540}}{\mu},$$

where μ is the molecular weight of the gas.

Exercise 5. Show that the weight of a cubic foot of any gas at n.t.p. in pounds is

$$\frac{\mu}{358}.$$

Exercise 6. Compute R for nitrogen.

Exercise 7. A tank (capacity 5 cubic feet) contains 2 pounds of acetylene. At what temperature will the pressure in the tank reach 100 pounds per square inch gage?

Exercise 8. Two receivers, capacities 200 and 100 cubic feet, are maintained at temperatures of 150° F and 80° F respectively. These receivers are connected by a pipe and contain 120 pounds of air.

(*a*) Find the pressure in the system.

(*b*) What weight of air does the larger receiver contain?

Exercise 9. Two tanks, each having a capacity of 3 cubic feet, are filled at 1500 pounds per square inch gage, one with oxygen, the other with hydrogen, both at 70° F. If these tanks are of equal weight when empty, find the difference in their charged weights.

SECTION III

THE GAS CONSTANT OF GASEOUS MIXTURES

To compute the gas constant of a mixture of ideal gases Dalton's law is used.

Dalton's law states that the pressures exerted by a mixture of ideal gases upon the walls of the containing vessel equals the sum of the pressures due to each of the constituent gases, provided they each in turn alone occupied the containing vessel at the temperature of the mixture.

The law, $pV=mRT$, is applicable to the mixture as well as to each individual gas. It remains to find the value of R for the mixture in terms of the R's of the individual gases composing the mixture.

Let the mass m of the mixture be composed of the masses m_1, m_2, . . . of the constituent gases and let the pressure of the mixture be p and equal to $p_1+p_2+\ldots$, the sum of the pressures which each gas occupying the whole volume of the mixture would individually exert, this in accordance with Dalton's law.

Then if R_1, R_2, . . . are the gas constants of the constituent gases we have from the general equation of gases under the assumption that each gas alone occupies the original space of the mixture at the temperature of the mixture,

$$p_1V=m_1R_1T$$

$$p_2V=m_2R_2T, \text{ etc.}$$

Adding these equations we obtain

$$pV=(m_1R_1+m_2R_2+\ .\ .\ .)T.$$

Comparing this equation with the equation

$$pV=mRT$$

applied to the mixture and in which R is the gas constant of the mixture we see that

$$mR=m_1R_1+m_2R_2+. . .$$

or

$$R=\frac{m_1}{m}R_1+\frac{m_2}{m}R_2+. . .$$

Here $\frac{m_1}{m}$, $\frac{m_2}{m}$, . . . represent the parts by weight of the constituent gases forming the mixture.

EXERCISE 10. Show that the pressure due to any one gas of a mixture may be represented by

$$p_n=\frac{m_n}{m}\frac{R_n}{R}p.$$

EXERCISE 11. Air contains 23.6 per cent of oxygen and 76.4 per cent of nitrogen by weight.

(*a*) Compute the gas constant for air by means of the gas constants of oxygen and of nitrogen.

(*b*) What part of the 30 inches of mercury representing normal atmospheric pressure is due to the oxygen, and what part is due to the nitrogen of the atmosphere?

EXERCISE 12. Compute the apparent molecular weight of air by means of the gas constant computed in Exercise 11 (*a*).

EXERCISE 13. Compute the weight of a cubic foot of air at n.t.p. by means of the result obtained in Exercise 12.

EXERCISE 14. Analysis of a flue gas shows 12 parts of CO_2, 6 parts of O_2, and 82 parts of N_2 by volume.

Compute (*a*) the gas constant,

(*b*) the apparent molecular weight,

(*c*) the specific weight (pounds per cubic foot) at n.t.p. of this mixture.

SECTION IV

THE SPECIFIC HEAT OF GASES

It follows from the units and the standard conditions described in the Introduction that the specific heat of any substance is numerically equal to its thermal capacity per unit mass.

If c represents the specific heat under certain conditions of temperature and pressure then the heat ΔQ absorbed by a mass m of the given substance may be computed as follows

$$\Delta Q = \int mc\,dt.$$

If c is or is assumed constant then

$$\Delta Q = mc\Delta t.$$

The heat absorbed by a body may have other effects besides that indicated by a change in temperature. It may do external work. The heat absorbed computed by means of a specific heat must be interpreted as including not only the heat required to increase the temperature even tho it is computed by means of a change in temperature but also all the heat absorbed and stored or utilized in other ways as in doing external work.

Thus the specific heat of a body may have various values even for the same range of temperature depending upon the external conditions.

For solids and liquids the specific heats will vary only slightly, for a gas the specific heats may have any value from plus to minus infinity for the same range in temperature, under various external conditions.

The specific heat of a gas depends not only upon the nature of the gas but also upon the conditions under which the heat is supplied to the gas. These conditions determine how much of the heat energy must be used in doing external work during the heating of the gas.

The energy stored in an ideal gas must be stored wholly as internal kinetic energy (Joule's law), so that even with increasing volume and the accompanying increase in the mean distance between the molecules of an ideal gas no change in the internal potential energy occurs.

The energy equation for an ideal gas is therefore

$$\Delta Q = \Delta K + \Delta W.$$

The Specific Heat at Constant Volume.—If a gas is heated at constant volume no external work can be performed during the absorption of the heat.

Therefore $$\Delta W_v = 0,$$

and $$\Delta Q_v = \Delta K_v,$$

where the subscripts v are used to indicate change at constant volume. Under these conditions the whole heat energy absorbed is stored in the gas.

Let c_v represent the specific heat of the gas at constant volume then

$$\Delta Q_v = m\int_{T_1}^{T_2} c_v dT.$$

It can be shown mathematically (see Exercise 275) that the c_v for any gas obeying the law $pV = mRT$ must have the same value no matter at what pressure or at what volume the gas may be heated. But c_v may vary with the temperature and experiments have shown that the specific heats of all gases are linear functions of the temperature. Thus the changing value of c_v for any gas may be represented by

$$c_v = a + bT,$$

where a and b are empirical constants.

Moreover it has been shown experimentally that equal volumes of diatomic gases have the same thermal capacity at the same temperature. As the masses of equal volumes of diatomic gases are proportional to the molecular weights of these gases (Avogadro's law) it follows that the thermal capacities of masses of diatomic gases which are proportional to their molecular weights must be equal or

$$\mu_1(c_v)_1 = \mu_2(c_v)_2 = \mu_3(c_v)_3 = \ldots$$

where $(c_v)_1$, $(c_v)_2$ . . . are the specific heats at constant volume, and μ_1, μ_2, . . . are the corresponding molecular weights of the diatomic gases.

Langen has shown that for diatomic gases such as

H_2, molecular weight = 2.016
N_2, molecular weight = 28.08
O_2, molecular weight = 32.00
CO, molecular weight = 28.00
NO, molecular weight = 30.04

and for mixtures of these gases,

$$\mu c_v = 4.625 + 0.000588T,$$

where T is expressed in absolute Fahrenheit degrees.

For carbon dioxide ($\mu = 44$) Langen found

$$\mu c_v = 6.774 + 0.00210T$$

and for highly superheated steam ($\mu=18.016$)

$$\mu c_v = 4.72 + 0.00238T$$

where T is in absolute Fahrenheit degrees.

These experimental results show that the more a gas approaches the condition of the so-called permanent gases the less does the specific heat at constant volume vary with the temperature.

c_v for any ideal gas is assumed to remain constant for all temperatures.

The c_v of diatomic gases may be considered constant between 32 and 400° F for engineering computations, and may be taken as

0.171 for air
2.42 for H_2
0.174 for N_2
0.155 for O_2
0.172 for CO.

EXERCISE 15. (*a*) At what temperature is c_v for air equal to 0.171?

(*b*) What is the percentage increase in the c_v of any diatomic gas between 32 and 400° F?

EXERCISE 16. If $\mu c_v = a + bT$, where a and b are constants, compute the heat required to change the temperature of m pounds of the gas from T_1 to T_2 degrees at constant volume. What is the mean specific heat of the gas between T_1 and T_2 degrees?

The Specific Heat at Constant Pressure.—When a gas is heated under constant pressure its volume increases according to the law $pV=mRT$ and external work must be performed in overcoming the constant pressure p. This work equals

$$\Delta W_p = \frac{1}{J}\int_{V_1}^{V_2} p\,dV = \frac{1}{J}p(V_2 - V_1)$$

where the subscript p indicates constant pressure and the result is expressed in heat units.

During this change of volume the temperature of the gas has increased from T_1 to $T_2°$. Thus the internal energy of the gas has increased, and according to Joule's law its internal kinetic energy only has increased. As this increase in internal kinetic energy is independent of the change in volume and depends only on the change in temperature we see that ΔK_p is the same as any ΔK occurring under any other external conditions. We may put

$$\Delta K_p = \Delta K_v = m\int_{T_1}^{T_2} c_v\,dT.$$

If c_v is constant as it is for an ideal gas then

$$\Delta K_p = \Delta K_v = mc_v(T_2 - T_1).$$

The total heat energy added to an ideal gas changing its state under constant pressure is thus

$$\Delta Q_p = \Delta K_p + \Delta W_p = mc_v(T_2 - T_1) + \frac{1}{J}p(V_2 - V_1).$$

The heat absorbed by an ideal gas under constant pressure may also be computed by means of its specific heat at constant pressure, thus

$$\Delta Q_p = m\int_{T_1}^{T_2} c_p\,dT$$

and as c_p is constant

$$\Delta Q_p = mc_p(T_2 - T_1).$$

Note that the use of c_p leads directly to the value of ΔQ_p computed above in two steps by means of the energy equation.

The Relation between c_p **and** c_v.—As the ΔQ_p's just computed in two ways must be equal we may put

$$mc_v(T_2-T_1)+\frac{1}{J}p(V_2-V_1)=mc_p(T_2-T_1).$$

To simplify this equation eliminate the p and the V's by means of $pV=mRT$ from which we obtain

$$pV_1=mRT_1 \quad \text{and} \quad pV_2=mRT_2$$

for p by hypothesis remains constant.

Thus $mc_v(T_2-T_1)+\frac{1}{J}mR(T_2-T_1)=mc_p(T_2-T_1)$

or
$$c_p-c_v=\frac{R}{J}.$$

This proof of the fundamental relation between c_p and c_v is based upon the assumption that c_p and c_v are constant (as they would be for ideal gases) or that c_p and c_v are the mean specific heats.

By writing the energy equation in its differential form it is readily shown that the relation also holds for the instantaneous values of varying specific heats.

Thus
$$dQ_p=dK_p+dW_p$$
$$=dK_v+dW_p$$

or
$$mc_p dT=mc_v dT+\frac{1}{J}pdV.$$

From $pV=mRT$ with p constant

we have $pdV=mRdT$,

so that $$mc_pdT=mc_vdT+\frac{1}{J}mRdT,$$

whence $$\mathbf{c}_p-\mathbf{c}_v=\frac{\mathbf{R}}{\mathbf{J}}$$

as before.

The ratio $\frac{c_p}{c_v}$ occurs frequently in thermodynamics. This ratio will be denoted by k. Thus, by definition,

$$\frac{\mathbf{c}_p}{\mathbf{c}_v}=\mathbf{k}.$$

As c_p and c_v are constants, k is constant for ideal gases. Actually k varies with the temperature.

Specific Heats at Constant Pressure.—By means of the relation between c_p and c_v just established c_p may be computed when c_v is known.

As $$c_p-c_v=\frac{R}{J}$$

$$\mu c_p-\mu c_v=\frac{\mu R}{J}.$$

Substituting the value of R in terms of μ

$$R=\frac{1540}{\mu},$$

we find $$\mu c_p=\mu c_v+\frac{1540}{J}=\mu c_v+1.98.$$

Assuming Langen's values for μc_v we have for diatomic gases

$$\mu c_p=6.61+0.000588T.$$

In general $$c_p = c_v + \frac{1.98}{\mu}$$

and $$k = \frac{c_p}{c_v} = 1 + \frac{1.98}{\mu c_v}.$$

For purposes of engineering calculations c_p for diatomic gases between 32 and 400° F is usually considered constant. The following values may be used for c_p, c_v, and k.

	c_p	c_v	k
Air	0.240	0.171	1.40
H_2	3.40	2.42	1.40
O_2	0.217	0.155	1.40
N_2	0.244	0.174	1.40
CO	0.250	0.179	1.40
CO_2	0.21	0.16	1.31
NH_3	0.53	0.41	1.29

Exercise 17. Compute μc_v, μc_p, and k for diatomic gases (*a*) at 32° F, (*b*) at 212° F.

Exercise 18. Show that for diatomic gases

$$c_p - c_v = \frac{2}{\mu}, \quad c_v = \frac{5}{\mu}, \quad c_p = \frac{7}{\mu}, \quad \text{and} \quad k = \frac{7}{5} = 1.40$$

approximately.

Exercise 19. Show that

$$c_p = \frac{R}{J}\,\frac{k}{k-1}; \quad c_v = \frac{R}{J}\,\frac{1}{k-1}; \quad k = \frac{c_p}{c_p - \frac{R}{J}}.$$

Exercise 20. Show that the external work performed per degree change in temperature when an ideal gas expands at constant pressure is mR.

EXERCISE 21. Compute the mechanical equivalent of heat from the following experimental data. One pound of air at n.t.p. occupied 12.39 cubic feet, c_p for air at 32° F equals 0.2375 and c_v at 32° F. equals 0.1689.

EXERCISE 22. Ten pounds of air are heated at a constant pressure of 200 pounds per square inch absolute from 50 to 60° F.

(*a*) How much heat energy has been supplied?

(*b*) How much of this energy is stored in the air?

EXERCISE 23. One pound of air at 100° F expands at constant pressure while 20 B.t.u. are supplied.

(*a*) What is the final temperature of this air?

(*b*) How much external work has been done?

EXERCISE 24. How much heat is required to expand 20 cubic feet of air to 30 cubic feet under a constant pressure of 60 pounds per square inch absolute?

EXERCISE 25. Three pounds of air confined under constant pressure are heated from 50 to 100° F.

(*a*) How much heat is supplied?

(*b*) What increase in internal energy results?

(*c*) How much external work is done?

SECTION V

THE FUNDAMENTAL LAWS FOR IDEAL GASES

The fundamental laws for ideal gases may be summed up in mathematical language as follows.

$$\mathbf{pV = mRT}$$

where $$\mathbf{R = \frac{1540}{\mu}.}$$

$$\mathbf{dQ = dK + dW,}$$

and $$\mathbf{dK = mc_v dT}$$

under all conditions.

$$\mathbf{c_p - c_v = \frac{R}{J} = \frac{1.98}{\mu}}.$$

As $$dW = \frac{1}{J} p dV$$

the energy equation may be written

$$\mathbf{dQ = mc_v dT + \frac{1}{J} pdV}.$$

This equation may also be expressed in terms of dp, instead of dV by means of $pV = mRT$, from which we obtain

$$pdV + Vdp = mRdT$$

or $$pdV = mRdT - Vdp.$$

And as $$c_p - c_v = \frac{R}{J},$$

this equation reduces on elimination of R to

$$pdV = mJ(c_p - c_v)dT - Vdp,$$

so that $$\mathbf{dQ = mc_p dT - \frac{1}{J} Vdp}.$$

EXERCISE 26. Show that dQ may be expressed as follows:

$$dQ = mc_v dT + m(c_p - c_v) T \frac{dV}{V},$$

$$dQ = mc_p dT - m(c_p - c_v) T \frac{dp}{p},$$

$$dQ = mc_v T \frac{dp}{p} + mc_p T \frac{dV}{V}.$$

CHAPTER II

CHANGES OF STATE OF IDEAL GASES

SECTION VI

GENERAL DISCUSSION

IDEAL gases obey the law

$$pv=RT,$$

where R is a constant for any given gas.

The state of a gas is determined by its volume, its pressure, and its temperature.

In this chapter will be discussed some changes of state which a gas undergoes when any or all of its conditions of volume, pressure, and temperature vary.

Characteristic Equation and Surface.—As the state of a gas depends upon three conditions namely p, v, and T this state may be represented graphically by a point in space referred to three rectangular axes. The locus of all points representing possible states of the gas is a surface whose equation is

$$pv=RT,$$

the **characteristic equation** of the ideal gas.

Every gas has its own characteristic equation and corresponding surface because the parameter R of the equation depends upon the nature of the gas.

The **characteristic surface** of any ideal gas is a hyperbolic paraboloid, a portion of which is shown in Fig. 2. That this is so is evident from the following analysis of the equation $pv=RT$.

(1) The equations of all sections of the surface made by planes parallel to the pv-plane, for which $T=T_1$, a constant, are

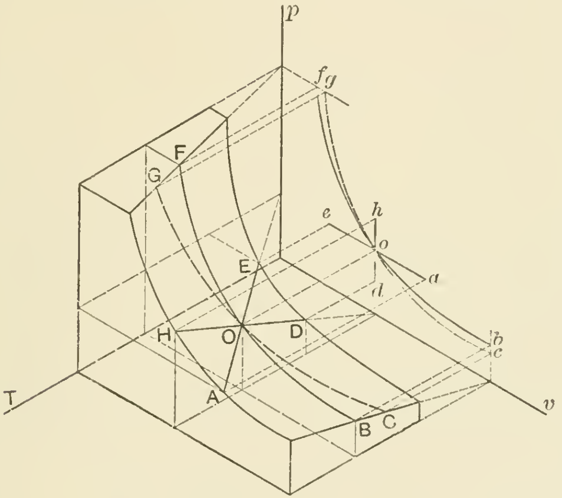

FIG. 2.

$$pv=RT_1=\text{a constant.}$$

These curves are thus equilateral hyperbolas.

(2) The equations of all sections of the surface made by planes parallel to the pT-plane, for which $v=v_1$, a constant, are

$$p=\left(\frac{R}{v_1}\right)T.$$

These sections are thus straight lines passing thru the v-axis.

(3) The equations of all sections of the surface made by planes parallel to the vT-plane, for which $p=p_1$, a constant, are

$$v=\left(\frac{R}{p_1}\right)T.$$

These sections are thus straight lines passing thru the p-axis.

The points on any line drawn upon the characteristic surface of an ideal gas between the points (p_1, v_1, T_1) and (p_2, v_2, T_2) represent a possible sequence of values thru which p, v, and T may pass while one pound of the gas changes its state from a condition determined by p_1, v_1, and T_1 to another condition determined by p_2, v_2, and T_2.

Exercise 27. What condition is imposed upon a change of state following the line (*a*) *AOE*, (*b*) *HOD*, (*c*) *FOB*, (*d*) *BC*, (*e*) *HA*, (*f*) *GF* traced upon the characteristic surface of an ideal gas, Fig. 2?

Exercise 28. How does (*a*) an increase in pressure at constant volume (*b*) a decrease in volume at constant pressure affect the temperature of an ideal gas?

Solve analytically, and also graphically by means of Fig. 2.

Instead of using lines traced upon the characteristic surface to represent changes of state of a gas it is customary to use only the projections of these lines upon the coordinate planes. As the external work done by a gas during a change of state always equals $\int pdV$ and as this external work is of great importance in engineering applications the projection upon the pv-plane is the most useful projection. Fig. 2 shows the pv-curves representing some of the more important changes of state. The projected lines naturally show only the changes which occur in the pressure and the volume of the gas; no changes in temperature can here be indicated.

Important Changes of State.—It is evident that changes of state may occur in innumerable ways. Any line drawn upon the characteristic surface would represent a possible way in which a change of state might occur. Of all possible changes of state the only ones of importance to the engineer are:

Isothermal changes, during which the temperature remains constant,

Isometric changes, during which the volume remains constant,

Isopiestic changes, during which the pressure remains constant,

Isodynamic changes, during which the internal energy remains constant,

Adiabatic changes, during which no heat is received from or rejected to external bodies,

Polytropic changes, during which the heat supplied to or withdrawn from the gas by external bodies is directly proportional to the change in temperature.

The discussion of any change of state should include:

(1) The equations showing the relations between p, v, and T during the change,

(2) The external work performed by or on the gas during the change,

(3) The quantity of heat supplied to or rejected by the gas during the change,

(4) The change in the intrinsic or internal energy of the gas during the change of state.

It should be remembered that the internal or intrinsic energy of an ideal gas changes only with the temperature and that this **change in internal energy during any change of state always equals**

$$\Delta K=\int_{T_1}^{T_2} mc_v dT=mc_v(T_2-T_1).$$

SECTION VII

ISOTHERMAL, ISOMETRIC, ISOPIESTIC, AND ISODYNAMIC CHANGES

Isothermal Change of State.—The equations for this change of state involve first

$$pv=RT$$

the fundamental equation which is always true. In addition we impose the condition of **constant temperature** so that

$$T=T_1, \text{ a constant.}$$

Thus $$pv=RT_1=C_1, \text{ a constant.}$$

This is Boyle's law and it is the equation of the pv-curve for isothermal change of state.

EXERCISE 29. What is the value of C_1 for the isothermal change of state of one pound of air at 90° F?

EXERCISE 30. Show that the construction illustrated in Fig. 3 yields a curve for which $pv=p_1v_1=$a constant.

EXERCISE 31. Draw to scale the isothermal pv-curve for one pound of air at (*a*) 90° F, (*b*) 200° F.

The external work performed is represented graphically by the area under the pv-curve, Fig. 3, for

$$\Delta W = \frac{1}{J}\int_{V_1}^{V_2} p\,dV.$$

The sign of ΔW will be assumed positive when external work is done by the gas during expansion and negative when external work is done on the gas during compression.

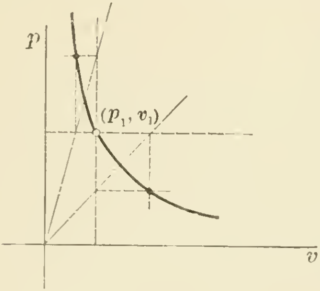

FIG. 3.

EXERCISE 32. Show that the external work performed during the isothermal change of state of one pound of ideal gas from p_1, v_1, T_1 to p, v, T_1 is

$$\Delta W = \frac{1}{J} p_1 v_1 \log_\epsilon \left(\frac{v}{v_1}\right),$$

when measured in B.t.u., and also equals

$$\frac{1}{J} p_1 v_1 \log_\epsilon \left(\frac{p_1}{p}\right) = \frac{1}{J} R T_1 \log_\epsilon \left(\frac{p_1}{p}\right),$$

where $$\log_\epsilon (x) = 2.303 \log_{10} (x).$$

The heat absorbed (or rejected) by a gas during isothermal change of state cannot be computed by means of a specific heat, for the change in temperature is zero.

We have, however, the equation

$$\Delta Q=\Delta K+\Delta W.$$

Here during isothermal change

$$\Delta K=0,$$

so that
$$\Delta Q_T=\Delta W_T=\frac{1}{J}p_1v_1\log_\epsilon\left(\frac{v}{v_1}\right).$$

Exercise 33. How much heat must be supplied to 3 pounds of air expanding at a constant temperature of 100° F from 200 to 30 pounds per square inch absolute?

Exercise 34. Gas expands, no heat is supplied to it. Where does the energy necessary for the performance of external work come from? Is this an isothermal change of state?

Exercise 35. During isothermal expansion all heat supplied is transformed into external work. Why is this not an ideal way of transforming heat into mechanical energy?

Isometric Change of State.—For all changes of state we have, $pv=RT$, if we now impose upon this general law change at **constant volume** then

$$v=v_1=\text{a constant}.$$

Combining these equations we find

$$\frac{p}{T}=\frac{R}{v_1}=C_2,\text{ a constant}.$$

This is one form of the law of Charles.

Exercise 36. Interpret the above equations graphically by means of Fig. 2.

Exercise 37. Describe the pv-curve for isometric change of state. How much external work is done during this change of state?

Exercise 38. How much heat must be supplied to a gas (specific heat at constant volume, c_v) in order to increase its pressure from p_1 to p_2 pounds per square inch absolute, the volume remaining v_1 during the change?

What becomes of this heat?

Exercise 39. Compute C_2 for air under a pressure of 200 pounds per square inch absolute and at 90° F. For what volume of air does this constant apply?

Isopiestic Change of State.—

Exercise 40. Write the equations governing isopiestic change of state. Interpret these equations on Fig. 2.

Exercise 41. Describe the pv-curve for isopiestic change of state and compute the external work done during a change from p_1, v_1, T_1 to p_1, v, T.

Exercise 42. Is the internal energy of a gas increased or diminished; is external work done by, or on; must heat be supplied to, or withdrawn from a gas during isopiestic (*a*) expansion, (*b*) compression?

Exercise 43. Ten pounds of air are kept under a constant pressure of 200 pounds per square inch absolute.

(*a*) What temperature has this air when it occupies 12 cubic feet?

(*b*) If its initial temperature was 300° F and its volume is then reduced to 12 cubic feet what external work was done upon the gas?

(*c*) How much intrinsic energy did it lose?

(*d*) How much heat was withdrawn from the gas?

Isodynamic Change of State.—By definition during an isodynamic change of state the internal energy remains constant.

The internal energy of an ideal gas is a function of the temperature only. Therefore constant internal energy implies constant temperature.

Thus isodynamic changes of state of ideal gases are isothermal changes of state.

Exercise 44. Express the change in internal energy of an ideal gas in terms of c_v and show from this expression that an isodynamic change is an isothermal change.

Section VIII

ADIABATIC CHANGE OF STATE OF IDEAL GASES

Definition and General Discussion.—During an adiabatic change of state no heat is received from or rejected to external bodies. Such a change of state would occur if the gas could be insulated, as far as heat is concerned, from all surrounding bodies. In practice adiabatic changes are approximated when expansion or compression takes place so rapidly that little time is allowed for the transfer of heat.

Gas expanding adiabatically does external work. The energy thus converted into external work is not derived from an external source of supply; the gas is insulated. This energy must therefore be drawn from the internal (intrinsic) energy of the gas. As the internal energy of an ideal gas depends only on its temperature, the temperature of an adiabatically expanding gas must fall. The internal energy which disappears during adiabatic expansion reappears as external work performed during the expansion.

In Fig. 2 the line *GOC* approximately represents an adiabatic change of state. Note the drop in temperature of the gas during expansion and the rise in temperature during compression.

EXERCISE 45. Explain the fact that the temperature remains constant during isothermal expansion and yet external work is done.

Is an increase in temperature possible during an expansion of a gas? Explain by means of Fig. 2.

EXERCISE 46. What terms in the equation

$$\Delta Q = \Delta K + \Delta P + \Delta W$$

disappear during an adiabatic change of state of an ideal gas? Why are these terms equal to zero?

Does a gas possess more or less internal energy after adiabatic expansion? To what is the change in internal energy equal?

The Equations Governing Adiabatic Change of State. The fundamental equation

$$pv = RT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

must always be satisfied.

During adiabatic change we have in addition the condition that

$$dQ = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This last equation must be expressed in terms of p, v, and T to make it useful.

As $$dQ = mc_v dT + \frac{1}{J} p dV$$

we have $$c_v dT + \frac{1}{J} p dv = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

a differential equation representing the conditions of adiabatic change of state in terms of p, v, and T.

In order to integrate this equation it must be expressed in terms of two variables by means of the fundamental relation $pv=RT$.

We may eliminate p from equation (3) by substituting for p its equal $\frac{RT}{v}$, thus

$$c_v dT+\frac{R}{J}\frac{T}{v}dv=0.$$

But $\frac{R}{J}$ always equals c_p-c_v, so that

$$c_v dT+(c_p-c_v)\frac{T}{v}dv=0,$$

or

$$\frac{dT}{T}=-\left(\frac{c_p}{c_v}-1\right)\frac{dv}{v}.$$

For ideal gases c_p and c_v are constant, so that

$$\frac{\mathbf{c}_p}{\mathbf{c}_v}=\mathbf{k},$$

a constant, by definition, and the differential equation may be written

$$\frac{dT}{T}=-(k-1)\frac{dv}{v}.$$

Integrating between the initial state p_1, v_1, T_1, and the final state, p, v, T we have

$$\log_\epsilon T\Big]_{T_1}^{T}=-(k-1)\log_\epsilon v\Big]_{v_1}^{v}$$

$$\log_\epsilon\frac{T}{T_1}=-(k-1)\log_\epsilon\frac{v}{v_1}$$

$$\log_e \frac{T}{T_1} = \log_\epsilon \left(\frac{v}{v_1}\right)^{1-k}$$

whence $$\frac{T}{T_1} = \left(\frac{v_1}{v}\right)^{k-1}.$$

This equation together with $pv=RT$ definitely fixes the relations between p, v, and T during an adiabatic change of state.

Exercise 47. (*a*) Deduce, in the manner outlined above, an equation in terms of p and v from $dQ=0$.

(*b*) Same in terms of p and T.

The equations governing the adiabatic change of state of any ideal gas are

$$\mathbf{pv=RT}$$

together with any one of the following equations

$$\frac{\mathbf{p}_1}{\mathbf{p}} = \left(\frac{\mathbf{v}}{\mathbf{v}_1}\right)^k, \quad \frac{\mathbf{T}_1}{\mathbf{T}} = \left(\frac{\mathbf{p}_1}{\mathbf{p}}\right)^{\frac{k-1}{k}}, \quad \frac{\mathbf{T}_1}{\mathbf{T}} = \left(\frac{\mathbf{v}}{\mathbf{v}_1}\right)^{k-1}.$$

Exercise 48. Interpret these equations geometrically by means of Fig. 2.

Exercise 49. Deduce the last three equations by means of the equations given in Exercise 26.

Starting with $pv=RT$ and $pv^k=p_1v_1{}^k$, the other relations between p, v, and T existing during adiabatic change of state may be deduced by eliminating either p or v. As an example let us eliminate p.

From $$pv^k=p_1v_1{}^k$$

we have $$\frac{p}{p_1} = \left(\frac{v_1}{v}\right)^k. \quad \ldots \ldots \quad (1)$$

Also $$pv=RT$$

gives $$\frac{pv}{T}=R=\frac{p_1v_1}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

To eliminate p we have from equation (2)

$$\frac{p}{p_1}=\frac{v_1T}{vT_1}$$

and from equation (1) $$\frac{p}{p_1}=\left(\frac{v_1}{v}\right)^k,$$

therefore $$\left(\frac{v_1}{v}\right)^k=\frac{v_1}{v}\,\frac{T}{T_1},$$

or $$\left(\frac{v_1}{v}\right)^{k-1}=\frac{T}{T_1}.$$

EXERCISE 50. Starting with $pv=RT$ and $pv^k=p_1v_1^k$ deduce the relation between p and T for an adiabatic change of state.

EXERCISE 51. Show that the final temperature of an ideal gas after an adiabatic change of state depends only upon the initial temperature and upon the ratio of expansion (or upon the ratio of the initial to the final pressure) but not upon the individual volumes (or pressures) involved.

Exponential and Logarithmic Computations.—As the equations just developed show, thermodynamic computations often require logarithmic computations. The following examples will serve as a review and they should be carefully studied.

1. What does $(0.035)^{0.21}$ equal?

Let $$N=(0.035)^{0.21}$$

then $$\log_{10} N=(.21)\log_{10}(.035)$$
$$=(.21)(8.5435-10)$$

$$= (.21)(-1.4565)$$

$$= -0.3059$$

$$= 10 - (0.3059) - 10$$

$$= 9.6941 - 10$$

therefore $N = 0.4944$

2. Find the value of $\log_\epsilon (0.085)$,

where $\epsilon = 2.718 \ldots$

Let $x = \log_\epsilon (0.085)$

then $\epsilon^x = 0.085.$

Taking logarithms to the base 10 for which complete tables are at hand we have

$$x \log_{10} \epsilon = \log_{10} (.085),$$

or

$$x = \frac{1}{\log_{10} \epsilon} \log_{10} (.085)$$

$$= \frac{1}{.434} \{8.9285 - 10\}$$

$$= 2.303 \{-1.0715\}$$

$$= -2.468.$$

Note that in general

$$\log_\epsilon N = \frac{1}{\log_{10} \epsilon} \log_{10} N = 2.303 \log_{10} N.$$

EXERCISE 52. One pound of air expands adiabatically from 300 pounds per square inch absolute and 200° F to 15 pounds per square inch absolute.

(*a*) What is its final temperature?
(*b*) What is its initial volume?
(*c*) What is its final volume?

Exercise 53. Four cubic feet of air at 60° F are compressed adiabatically until the volume is reduced to ½ cubic foot.

(*a*) What is its final temperature?

(*b*) What relation exists between the final and the initial pressures?

(*c*) If the mass of air had been one pound what would have been the final pressure?

The External Work Performed by or on an ideal gas during an adiabatic change of state may be computed in two ways,

(1) by means of the area under the *pv*-curve representing adiabatic change. That is, the area on the *pv*-plane bounded by the curve $pv^k = p_1v_1^k$, the *v*-axis, and the ordinates of the points representing the initial and the final states of the gas.

(2) by means of the change in internal energy which occurs during the adiabatic change of state, for all external work is done at the expense of the internal energy of the gas. Thus any loss in internal energy equals the external work performed, or

$$\Delta \mathbf{W} = -\Delta \mathbf{K} = -\mathbf{mc}_v(\mathbf{T} - \mathbf{T}_1),$$

where the result is expressed in B.t.u.

Exercise 54. Show that the external work performed during an adiabatic change of state when expressed in foot-pounds equals

$$Jmc_vT_1\left(1 - \frac{T}{T_1}\right) = Jmc_vT_1\left\{1 - \left(\frac{v_1}{v}\right)^{k-1}\right\} = Jmc_vT_1\left\{1 - \left(\frac{p}{p_1}\right)^{\frac{k-1}{k}}\right\}$$

where Jmc_vT_1 may be replaced by

$$\frac{Jc_vp_1V_1}{R} \quad \text{or} \quad \frac{p_1V_1}{k-1}.$$

Exercise 55. Show by means of the *pv*-curve that the external work done during an adiabatic change of state (expressed in foot-pounds) equals

$$\frac{p_1V_1-pV}{k-1}.$$

Exercise 56. Prove that the expression derived in Exercise 55 equals the expressions derived in Exercise 54.

Exercise 57. Compute the work done during the expansion of the air discussed in Exercise 52

(*a*) by means of the results of Exercise 54,

(*b*) by means of the results of Exercise 55.

Exercise 58. Compute the work for Exercise 53 assuming the mass of air to be one pound.

The Heat Absorbed from or rejected to external bodies during an adiabatic change of state is by definition equal to zero.

The Change in Internal Energy during an adiabatic change of state is

$$\Delta K=mc_v(T-T_1).$$

Section IX

POLYTROPIC CHANGE OF STATE OF IDEAL GASES

Definition and General Discussion.—During an isopiestic expansion from the state determined by p_1, v_1, T_1 to the state determined by p_1, v_2, T_2 the heat supplied to an ideal gas must be sufficient to increase the internal energy and to perform the external work done by the expanding gas. Thus the heat supplied must equal

$$\Delta Q_p=mc_v(T_2-T_1)+m\frac{1}{J}p_1(v_2-v_1)$$
$$=m\left(c_v+\frac{R}{J}\right)(T_2-T_1).$$

Here the factor $\left(c_v+\frac{R}{J}\right)$ is evidently a specific heat. It equals c_p as shown on page 25.

During any change of state of an ideal gas the specific heat c defined by the equation

$$dQ=mcdT$$

may similarly be regarded as composed of two terms, one always being c_v and the other term being analogous to $\frac{R}{J}$ but not equal to it. This second term depends upon the manner in which the gas changes its state and upon the external work done during this change. Therefore the specific heat of a given gas, c, may have as many different values as there are ways in which the gas may change its state. Even tho c_v be considered constant c need not remain constant during all possible changes of state.

By definition, a polytropic change of state is any change of state during which the specific heat c does remain constant, see page 33.

We must now determine the relations between p, v, and T during polytropic changes of state.

The Equations Governing Polytropic Change of State of Ideal Gases.—The equation governing all changes of state of ideal gases is

$$pv=RT.$$

In order to limit the change to a polytropic one the conditions expressed by the equation

$$dQ=mcdT,$$

where c is constant, must also be satisfied.

This second equation of condition must now be expressed in terms of p, v, and T. As

$$dQ = mc_v dT + \frac{1}{J} p dV$$

under all conditions of change of state we may write

$$cdT = c_v dT + \frac{1}{J} pdv,$$

or $$(c - c_v) dT = \frac{1}{J} pdv.$$

To establish a relation between the p's and the v's T must be eliminated by means of $pv = RT$, from which we obtain

$$dT = \frac{1}{R}(pdv + vdp)$$

so that $$(c - c_v)(pdv + vdp) = \frac{R}{J} pdv.$$

Replacing $\frac{R}{J}$ by $c_p - c_v$ and regrouping the terms we have

$$(c - c_p) pdv + (c - c_v) vdp = 0.$$

Integrating between the limits p_1, v_1, T_1 and p, v, T we have

$$(c - c_p) \log \frac{v}{v_1} + (c - c_v) \log \frac{p}{p_1} = 0,$$

or $$\left(\frac{v}{v_1}\right)^{c - c_p} = \left(\frac{p_1}{p}\right)^{c - c_v}.$$

Comparing this equation with the analogous equation for adiabatic change of state, namely,

$$\frac{p_1}{p} = \left(\frac{v}{v_1}\right)^k,$$

we note that the corresponding polytropic equation may be written

$$\frac{p_1}{p}=\left(\frac{v}{v_1}\right)^{\frac{c-c_p}{c-c_v}}.$$

Here $\frac{c-c_p}{c-c_v}$ is a constant, for c_p and c_v are constants for ideal gases and c is a constant by definition for polytropic changes of state. Let this constant exponent be denoted by n, then

$$\frac{p_1}{p}=\left(\frac{v}{v_1}\right)^n.$$

Thus during any polytropic change of state of ideal gases, we have

$$\mathrm{pv}=\mathrm{RT}$$

and

$$\mathrm{pv}^n=\mathrm{p}_1\mathrm{v}_1{}^n,$$

where the constant n has the value

$$\mathrm{n}=\frac{\mathrm{c}-\mathrm{c}_p}{\mathrm{c}-\mathrm{c}_v}.$$

Solving the last equation for c we may express the specific heat for any given polytropic change in terms of n as follows:

$$\mathrm{c}=\frac{\mathrm{n}-\mathrm{k}}{\mathrm{n}-1}\mathrm{c}_v.$$

Why do not the above equations hold for an actual gas?

EXERCISE 59. Deduce the relation between v and T for a polytropic change of state of an ideal gas starting with

(*a*) the fundamental equation $dQ=mcdT$,

(*b*) the equations $pv=RT$ and $pv^n=p_1v_1{}^n$.

EXERCISE 60. Deduce the relation between p and T for a polytropic change of state, starting with

(*a*) the fundamental equation $dQ = mc\,dT$,

(*b*) the equations $pv = RT$ and $pv^n = p_1 v_1^n$.

EXERCISE 61. Deduce the equations

$$\mathrm{pv}^n = \mathrm{p}_1\mathrm{v}_1^n; \quad \frac{\mathrm{T}}{\mathrm{T}_1} = \left(\frac{\mathrm{v}_1}{\mathrm{v}}\right)^{n-1}; \quad \frac{\mathrm{T}}{\mathrm{T}_1} = \left(\frac{\mathrm{p}}{\mathrm{p}_1}\right)^{\frac{n-1}{n}},$$

expressing the conditions involved in polytropic changes of state of ideal gases from the equations of Exercise 26.

Discussion of the Polytropic Equations.—The equations to be discussed are

$$pv^n = p_1 v_1^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where n may have any constant value, and

$$c = \frac{n-k}{n-1} c_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which the value of the constant specific heat may be computed for any given value of n.

(1) When $n = 0$, equation (1) becomes

$$p = p_1$$

and from equation (2)

$$c = kc_v = c_p.$$

Therefore a polytropic change during which $n = 0$ is an isopiestic change of state, represented in Fig. 4 by the line *aoc*.

(2) When $n = 1$, equation (1) becomes

$$pv = p_1 v_1$$

and from equation (2)

$$c = \frac{1-k}{0} c_v = \infty.$$

Therefore a polytropic change during which $n=1$ is an isothermal change, represented in Fig. 4 by the line *bof*.

Note that the specific heat of the gas is now infinite. The gas thus has an infinite capacity for heat; it can absorb or reject any quantity of heat without change in temperature provided it obeys the laws $pv=RT$ and $pv=p_1v_1$. During isothermal expansion this actually occurs. We must however bear in mind that the heat so absorbed without change in temperature does not remain in the gas but is at once transformed into external work performed by the gas, and the heat rejected without change in temperature is not derived from the gas but from the external work performed simultaneously on the gas, the internal energy of the gas remaining constant thruout the change.

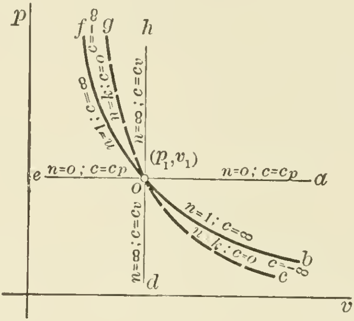

Fig. 4.

(3) When $n=k$, equation (1) becomes

$$pv^k=p_1v_1{}^k$$

and equation (2) shows that

$$c=0.$$

Therefore a polytropic change during which $n=k$ is an adiabatic change, represented in Fig. 4 by the line *cog*.

Note that when $pv^k=p_1v_1^k$ the gas has absolutely no capacity for heat ($c=0$). As $dQ=mcdT$ and $c=0$, an infinitesimal quantity of heat supplied to the gas under these conditions would cause an infinite rise in temperature. This must be so for under adiabatic conditions no heat should be added to or be extracted from the gas.

(4) When $n=\infty$, let $n=\frac{a}{b}$, where a and b are constants, then equation (1) becomes

$$p^bv^a=p_1{}^bv_1{}^a$$

and as b must approach 0 as n approaches ∞ we have for $n=\infty$

$$v=v_1.$$

From equation (2) we have

$$c=\frac{\frac{a}{b}-k}{\frac{a}{b}-1}c_v=\frac{a-bk}{a-b}c_v.$$

Thus when $n\doteq\infty$, $b\doteq 0$ and

$$c=c_v.$$

Therefore a polytropic change during which $n=\infty$ is an isometric change, represented in Fig. 4 by the line *doh*.

Polytropic changes of state thus include all the changes of state previously considered, together with other changes of state. It should however be noted that the polytropic conditions do **not include all** possible changes of state but only those during which the specific heat remains constant.

Of all the polytropic changes of state other than the isopiestic, isometric, isothermal, and adiabatic the only polytropics used in engineering are the ones for which n lies between 1 and k. The pv-curves of these polytropics lie between the lines fob and goc in Fig. 4. For these polytropics **the specific heat is negative.**

As $dQ = mcdT$ it follows that when c is negative and dQ is positive, dT must be negative therefore **when heat is supplied a drop in temperature occurs.** This apparently inconsistent condition will be understood when it is remembered that the external work performed plays an important part in the discussion. Thus under the condition $pv^n = p_1v_1^n$ when n lies between 1 and k work is performed so rapidly during the expansion of the gas that not only the whole heat supplied is converted into external work but this heat does not suffice to perform the work and the store of internal energy of the gas is called upon to aid with the evident result that a drop in temperature occurs.

During compression under these conditions the work done on a gas and converted into heat cannot be completely rejected by the gas and that which remains in the gas as an increase in internal energy causes a rise in temperature even tho heat is removed from the gas. Thus altho dQ is negative, dT is positive because c is negative.

Exercise 62. Trace the curve $pv^n = p_1v_1^n$, assuming $p_1 = 3$ and $v_1 = 2$, by means of several points, for (*a*) $n = -1$, (*b*) $n = -\frac{1}{2}$, (*c*) $n = -10$. What are the corresponding values of c?

Exercise 63. Describe the positions of the lines on the characteristic surface (Fig. 2) corresponding to $pv^n = p_1v_1^n$ for values of n between

(*a*) $+\infty$ and 1
(*b*) 1 and 0
(*c*) 0 and $-\infty$,

for both compression and expansion.

EXERCISE 64. Write the equations of the pv-curves for isothermal, adiabatic, isopiestic, and isometric changes of state.

(*a*) What values of a and b in $n = \frac{a}{b}$ and thus what values of n will change the polytropic law $pv^n = p_1v_1^n$ into each of the above mentioned changes of state?

(*b*) Discuss the value and the physical meaning of the specific heat for each of the above cases.

The Signs of dQ, dK, and dW.—In the equation

$$dQ = dK + dW$$

we have assumed

dQ positive when heat is supplied to the gas from some external source,
dK positive when the internal energy (and temperature) increases,
dW positive when external work is done by the gas.

Evidently during any polytropic expansion dW is positive; during any polytropic compression dW is negative.

From Fig. 2 it may be seen that an expansion does not necessarily entail a rise or a fall in temperature with corresponding increase or decrease in the internal energy. For all polytropic expansion lines lying in front of the isothermal OB (on the surface HOB) the temperature and the internal energy increase during expansion; thus

dT and dK are positive. For all polytropic expansion lines lying behind the isothermal OB (on the surface DOB) the temperature and the internal energy decrease during expansion; thus dT and dK are negative.

To determine the sign of dK for any polytropic change of state note that it depends only upon the sign of dT. To find the sign of dT analytically we may proceed as follows, starting with the equations governing polytropic change,

$$pv = RT, \quad pv^n = p_1 v_1^n.$$

As

$$T = \frac{pv}{R},$$

where p and v are both variable, eliminate one of these variables. If the change in T is to be expressed in terms of a change in volume eliminate p. Thus

$$T = \frac{p_1 v_1^n}{R} v^{1-n}$$

and

$$dT = \left(\frac{p_1 v_1^n}{R}\right)(1-n)(v^{-n})(dv).$$

The first factor is always positive, the sign of the second depends upon the value of n, the third factor is always positive, and the sign of dv is positive for an expansion and negative for a compression.

The sign of dT may also be found from the equation

$$\frac{T}{T_1} = \left(\frac{v_1}{v}\right)^{n-1}.$$

To illustrate, during an expansion $v > v_1$ so that $\frac{v_1}{v} < 1$.

If n is greater than unity $\left(\frac{v_1}{v}\right)^{n-1}$ is less than one and $T < T_1$, therefore dT is negative. If however $\frac{v_1}{v} < 1$ and n is less than unity, $\left(\frac{v_1}{v}\right)^{n-1}$ is greater than one and $T > T_1$, therefore dT is positive.

To determine the sign of dQ, remember that

$$dQ = mcdT,$$

so that the sign of dT does not alone determine the sign of dQ. The sign of c, the specific heat under the particular law $pv^n = p_1v_1{}^n$, must also be found. As an illustration, during any expansion for which n lies between 1 and k, c will be negative (see page 52) and dT will also be negative therefore dQ must be positive. Under these conditions heat must be supplied to the gas even tho the internal energy diminishes. The external work done absorbs not only the loss of internal energy but the heat supplied as well.

If n is greater than k, c is positive and dT is negative, therefore dQ must be negative. Under these conditions the expansion can only proceed according to the law $pv^n = p_1v_1{}^n$ provided heat is withdrawn from the gas. The loss in internal energy is so large that all of the energy lost by the gas cannot be utilized in the performance of external work.

EXERCISE 65. (*a*) Check the statements in the following table. (*b*) Sketch a figure similar to Fig. 4 and indicate the areas in which the polytropics lie for which (1) ΔW is $+$ and $-$, (2) ΔQ is $+$ and $-$, (3) ΔK is $+$ and $-$.

During **polytropic changes of state** we have

	when the values of n lie between	and the pv-curves, Fig. 4, lie between	then the values of c lie between	and the signs of		
				dW	dK	dQ
				are		
for expansion	$-\infty$ and 0	*oh* and *oa*	c_v and c_p; $c+$	$+$	$+$	$+$
	0 and 1	*oa* and *ob*	c_p and $+\infty c$; $+$	$+$	$+$	$+$
	1 and k	*ob* and *oc*	$-\infty$ and 0; $c-$	$+$	$-$	$+$
	k and $+\infty$	*oc* and *od*	0 and c_v; $c+$	$+$	$-$	$-$
for compression	$-\infty$ and 0	*od* and *oe*	c_v and c_p; $c+$	$-$	$-$	$-$
	0 and 1	*oe* and *of*	c_p and $+\infty$; $c+$	$-$	$-$	$-$
	1 and k	*of* and *og*	$-\infty$ and 0; $c-$	$-$	$+$	$-$
	k and $+\infty$	*og* and *oh*	0 and c_v; $c+$	$-$	$+$	$+$

The Experimental Determination of n.—In technical problems relating to heat engines the expansions and compressions are never isothermal or adiabatic altho these may be the ideal conditions whose attainment is sought. Thus the pv-curves representing the actual changes of state are not represented by the equations $pv=p_1v_1$ or $pv^k=p_1v_1^k$. It has been shown that the expansion and compression curves as drawn by an indicator may often be represented by the equation $pv^n=p_1v_1^n$, where n is a constant. Under these conditions the actual changes occurring in the engine cylinder are polytropic. Moreover the values of n are found to lie between 1 and k.

Three methods for finding the n of any polytropic traced by an indicator will be described.

(1) Take any two points on the curve and measure their p and v coordinates (always from the axes of absolute pressure and of absolute volume). Then assuming the

curve to be a polytropic n may be found by substituting in the formula

$$n = \frac{\log p_1 - \log p_2}{\log V_2 - \log V_1}.$$

Exercise 66. Develop the above formula.

(2) The areas $ABMN$ and $ABPQ$ (Fig. 5) may be found by means of a planimeter. Now if the curve AB is a polytropic whose equation is $pv^n = p_1 v_1^n$, then

$$n = \frac{\text{area } ABMN}{\text{area } ABPQ}.$$

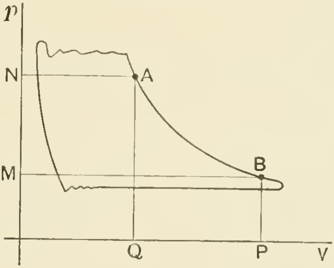

Fig. 5.

Exercise 67. Demonstrate the correctness of the above described method of finding n.

(3) The third and best method does not assume the curve AB (Fig. 5) to be a polytropic but shows whether it is or is not a polytropic. If it is not a polytropic this method shows how closely it approaches a polytropic and gives the value of n for the actual polytropic which best approximates the given curve. It often happens that a given curve may be very closely represented not by one but by two separate polytropics each for its own part

of the expansion or compression. This method of finding n will not only give the value of n for each polytropic but will also show the region in which each is applicable.

A curve whose equation is

$$pv^n = p_1 v_1{}^n = C, \text{ a constant}$$

becomes a straight line when plotted on axes of $\log_{10} p$ and $\log_{10} v$ instead of p and v. For the equation may be written

$$(\log_{10} p) = -n(\log_{10} v) + \log_{10} C,$$

which is the equation of a straight line when the variables are $(\log_{10} p)$ and $(\log_{10} v)$.

Thus if we replot the curve AB (Fig. 5) upon log-log paper (i.e., paper ruled according to the logarithmic scale instead of the usual decimal scale) the resulting curve will be a straight line if AB is a polytropic. In any case the best representative straight line thru the plotted points represents the log-log plot of a polytropic approximating the curve AB. The slope of this straight line with its sign changed gives the value of n in the equation $pv^n = p_1 v_1{}^n$ which approaches most closely to the given curve AB.

Exercise 68. Prove the last statement.

The External Work Performed during polytropic changes of state may be computed either by means of the pv-curve or by means of the fundamental energy equation.

Exercise 69. Show by means of $\int p\,dV$ that the external work performed during a polytropic change of state from the state p_1, V_1, T_1 to the state p, V, T is

$$J(\Delta W) = \frac{p_1 V_1 - pV}{n-1} = \frac{p_1 V_1}{n-1}\left[1 - \left(\frac{V_1}{V}\right)^{n-1}\right] = \frac{p_1 V_1}{n-1}\left[1 - \left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}\right].$$

Exercise 70. Starting with the equation $\Delta Q = \Delta K + \Delta W$, show that the external work performed during a polytropic change of state is

$$J(\Delta W) = mJ(c - c_v)(T - T_1) = \frac{mR}{n-1}(T_1 - T) = \frac{mRT}{n-1}\left(1 - \frac{T}{T_1}\right).$$

Exercise 71. Show that

$$\frac{\mathbf{p_1V_1 - pV}}{\mathbf{n-1}} = \mathbf{mJ(c - c_v)(T - T_1)}$$

and that the last expression always gives a positive result for expansion and always gives a negative result for compression.

The Heat Supplied during a polytropic change of state.

Exercise 72. Show that the heat supplied to an ideal gas during a polytropic change of state from p_1, V_1, T_1 to p, V, T is

$$\Delta Q = mc(T - T_1) = m\left\{c_v + \frac{R}{J(1-n)}\right\}(T - T_1).$$

Exercise 73. Show that the change in internal energy during a polytropic change is

$$\Delta K = mc_v(T - T_1) = \frac{1}{J}\,\frac{pV - p_1V_1}{k-1},$$

when expressed in B.t.u. (see also Exercise 86) and that therefore

$$\Delta Q = \frac{k-n}{k-1}(\Delta W),$$

whence

$$\mathbf{\Delta W : \Delta K : \Delta Q = k-1 : 1-n : k-n.}$$

Exercise 74. Show that the expression for ΔQ in Exercise 73 may be negative during certain polytropic expansions.

Exercise 75. Compute by means of Exercise 69 the external work done during an adiabatic change (*a*) from 10 pounds per square inch absolute and 17.16 cubic feet to 200 pounds per square inch absolute, and

(*b*) from 200 pounds per square inch absolute and 2 cubic feet to 10 pounds per square inch absolute.

Interpret the signs of the results.

EXERCISE 76. Air expands polytropically from 300 pounds per square inch absolute and 0.4 cubic foot to 15 pounds per square inch absolute with $n=3$.

Compute (*a*) the external work done, (*b*) the heat supplied, (*c*) the specific heat, the mass times the change in temperature from the result of (*b*), and thus the change in internal energy.

EXERCISE 77. Compute the external work done during an isothermal change of state from 20 pounds per square inch absolute and 25 cubic feet to 100 pounds per square inch absolute and 5 cubic feet.

CHAPTER III

GRAPHICS OF THE *pv*-PLANE

SECTION X

PLOTTING POLYTROPICS

Plotting the Polytropic Curve.—To construct the curve $pv^n = p_1 v_1{}^n$, assume any angle α, Fig. 6, and by means of

$$(1 + \tan\beta) = (1 + \tan\alpha)^n$$

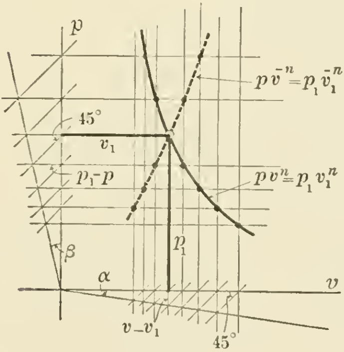

FIG. 6.

compute the corresponding angle β, using the assigned value of n. Set off these angles as shown in Fig. 6 and start the construction by drawing p_1 and v_1 from some known point on the required polytropic.

To demonstrate the correctness of this construction note that

$$\frac{v-v_1}{v_1}=\tan\alpha$$

and that

$$\frac{p_1-p}{p}=\tan\beta.$$

Hence

$$v=v_1(1+\tan\alpha)$$

and

$$p=\frac{p_1}{1+\tan\beta},$$

so that

$$pv^n=p_1v_1{}^n\frac{(1+\tan\alpha)^n}{1+\tan\beta}.$$

But

$$(1+\tan\beta)=(1+\tan\alpha)^n,$$

therefore

$$pv^n=p_1v_1{}^n.$$

This construction must be performed with great accuracy, for in it errors are cumulative.

Exercise 78. Starting with $(1+\tan\beta)=(1+\tan\alpha)^n$ deduce from it and the construction illustrated in Fig. 6 the fact that $pv^n=p_1v_1{}^n$.

Exercise 79. Show that the equation of the dotted curve in Fig. 6 is $pv^{-n}=p_1v_1{}^{-n}$.

Exercise 80. How could the points on the curve found by means of the construction illustrated in Fig. 6 be caused to lie closer together?

EXERCISE 81. Construct thru ($p_1=100$, $v_1=3$)

(*a*) an isothermal (see pages 35 and 63).

(*b*) an adiabatic

(*c*) a polytropic for $n=1.2$

for air, all on the same diagram.

EXERCISE 82. Plot a polytropic curve thru ($p_1=100$, $v_1=3$) or $n=1.2$ by direct substitution in the equation $pv^n=p_1v_1^n$. Assume values of v and compute p by means of

$$\log p=-n\log v+(\log p_1+n\log v_1).$$

Tabulate your computations as follows:

v	$\log v$	$n\log v$	$\log p$	p

A simpler and more rapid way of plotting a polytropic involves the use of log-log paper. On paper ruled both vertically and horizontally on the logarithmic scale locate the point (p_1, v_1), the given point thru which the polytropic should pass. Thru this point draw a straight line having a slope $-n$, where n is the exponent of the required polytropic. This line represents the polytropic on the log-log plane, see page 58. Now transfer as many points as may be desired from this plane to a plane ruled both vertically and horizontally on the decimal scale. These points will lie on the required polytropic in the pv-plane.

Construction of the Isothermal Curve.—The isothermal is readily constructed by the method given on page 35. Another method which does not involve the drawing of parallel lines is illustrated in Fig. 7. Here $OA = MN$, $OB = PQ$, etc.

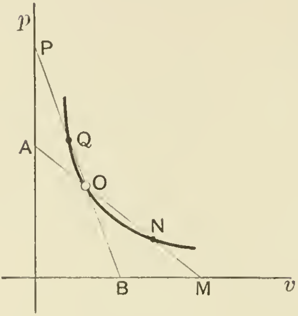

Fig. 7.

Exercise 83. Show that the construction illustrated in Fig. 7 yields a curve whose equation is $pv = p_1v_1$.

Section XI

GRAPHICAL REPRESENTATION OF ΔW, ΔK, AND ΔQ

Graphical Representation of External Work.—The external work performed during any change of state equals $\int_{v_1}^{v_2} pdv$. Therefore the area, Fig. 8, bounded by the curve representing the change of state, the end ordinates, and the v-axis represents the external work performed.

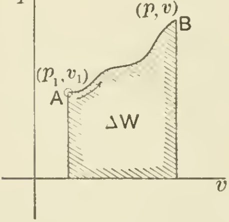

Fig. 8.

In order to determine the sign of this area the point tracing the boundary of the area (the stylus of a planimeter) should always start at the initial point of the line representing the change of state (A, Fig. 8) and then follow this line to the final point (p, v) and so on around the area. If the area is thus traced in the clockwise direction the area is to be considered positive, if in the reverse direction, negative. That this rule of signs agrees with the rules already established can be seen from Fig. 8. Here an expansion occurs from A to B and the area is traced in the clockwise direction and is thus positive.

EXERCISE 84. Under what conditions would the area bounded by the curve representing the change of state, the abscissas of the end points, and the portion of the p-axis intercepted between these abscissas represent the work done during the change of state considered?

Graphical Representation of Internal Energy.—The internal energy of an ideal gas is a function of the temperature only. When an ideal gas has a temperature of absolute zero it possesses no internal energy. An ideal gas expanding adiabatically does work. This work is done wholly at the expense of its internal energy. The work done thus equals the loss of internal energy. If the gas is assumed to expand adiabatically from any initial state until its volume is infinite and its pressure and temperature both zero the work done will equal the internal energy which the gas possessed at the initial state considered.

Graphically, the area under the adiabatic extending to infinity (Fig. 9) represents K_1. This internal energy must always be positive so that in tracing the area we must start at the point considered (p_1, v_1) and follow the adiabatic and so on around the area.

EXERCISE 85. Show that the internal energy of one pound of an ideal gas whose state is (p_1, v_1, T_1) is

$$\frac{p_1v_1}{k-1}, \text{ a finite quantity.}$$

Graphical Representation of the Change in Internal Energy.—The change in internal energy due to a change of state from p_1, v_1, T_1 to p, v, T (Fig. 10) would of course equal the difference between the areas K and K_1, or $K-K_1$.

EXERCISE 86. Show that $\Delta K = K - K_1 = Jmc_v(T-T_1)$.

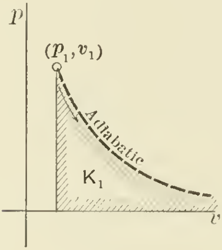

FIG. 9.

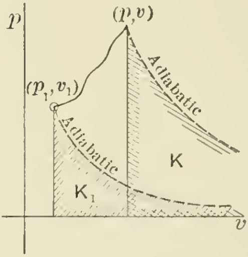

FIG. 10.

As the areas shown in Fig. 10 extend to infinity they cannot be used to find ΔK by means of a planimeter. To represent ΔK by an area finite in all its dimensions proceed as follows.

Draw an isothermal thru the initial point (p_1, v_1) and an adiabatic thru the final point, Fig. 11. These curves intersect in some point C. Note that the adiabatic approaches the v-axis more rapidly than the isothermal. The area under the adiabatic between the final point of the change of state considered and the intersection of the adiabatic and the isothermal equals ΔK for the change of state from (p_1, v_1, T_1) to (p, v, T).

To prove that this is so consider the cycle *ABCA* thru which the gas may be conceived to pass. It should be remembered that the gas actually changes its state only in accordance with the law represented by the line *AB*. The hypothetical cycle *ABCA* returns the gas to its initial state at *A*.

In passing thru the change from *C* to *A* the temperature (and therefore the internal energy) does not change. Thus any gain in internal energy during the change from *A* to *B* must be exactly neutralized by an equal loss of internal energy during the change from *B* to *C*. But the loss of internal energy from *B* to *C* is represented by the area ΔK. Therefore this area ΔK also represents the **gain** in internal energy during the change from *A* to *B*.

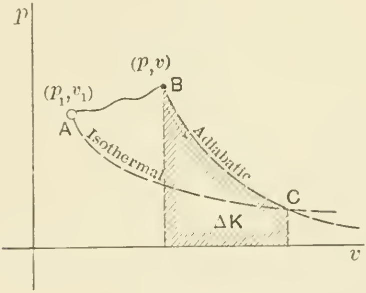

FIG. 11.

This area representing a gain must be positive so that we must start tracing the area at (p, v), then follow along the adiabatic to *C* and so on around the area. An area about which the tracing point would move in a counter-clockwise direction when following the above rule would

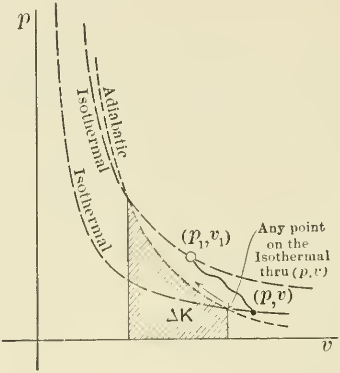

FIG. 12.

be negative and would indicate a loss of internal energy during the change of state from (p_1, v_1) to (p, v).

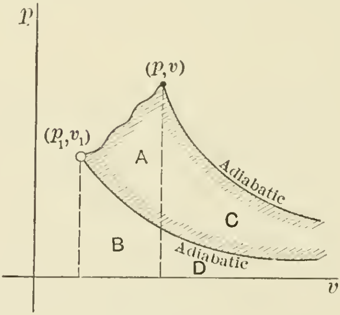

FIG. 13.

EXERCISE 87. Show that the construction illustrated in Fig. 12 gives ΔK for the change of state from (p_1, v_1) to (p, v).

Graphical Representation of the Heat Supplied.—The heat supplied during any change of state may be represented by the area included between the line representing the change of state and the adiabatics thru the points representing the initial and the final states of the gas, both adiabatics extending to infinity.

In Fig. 13 areas $A+B$ represent ΔW, $C+D$ represent K, $B+D$ represent K_1.

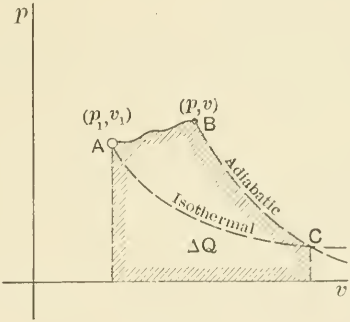

FIG. 14.

As $\Delta Q=\Delta K+\Delta W$, ΔQ would be represented by

$$\{(C+D)-(B+D)\}+\{A+B\} \quad \text{or} \quad A+C.$$

Another area, finite in all of its dimensions, which represents ΔQ is shown in Fig. 14. The sign of this area is determined by starting at the initial point then following the line representing the change of state, the adiabatic thru the final point, and so on around the area marked ΔQ. An area traced in a clockwise direction is plus as before.

Exercise 88. Show that the area shown in Fig. 14 represents the heat supplied during the change of state represented by AB.

Exercise 89. Indicate areas on the pv-plane finite in all their dimensions representing the external work, the change in internal energy, and the heat supplied during

(*a*) an isopiestic expansion,
(*b*) an isopiestic compression,
(*c*) an isometric decrease in pressure,
(*d*) an isothermal compression,
(*e*) an adiabatic expansion.

Note the signs of these areas. Do they agree with the signs given in the table on page 56?

CHAPTER IV

COMPRESSORS

Section XII

SINGLE-STAGE COMPRESSION

Piston compressors take in a charge of gas at a low pressure and after compression to a higher pressure deliver this gas into a receiver where it is stored at the higher pressure.

For an ideal compressor having no clearance these processes may be represented on the *pv*-plane as shown

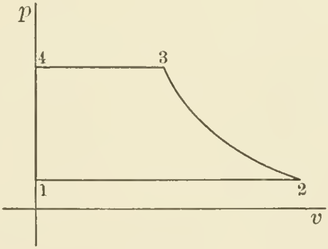

Fig. 15.

in Fig. 15. The line 1 2 does not represent an isopiestic change of state. During this process the mass of gas considered does not remain constant. This line represents the gradual filling of the cylinder as the piston moves toward the right. Thus during this process the temperature and the pressure of the gas remain constant but the volume and the mass of the gas in the cylinder continually increase as the piston moves toward the right.

When the cylinder is full the inlet valve closes and the piston moves toward the left, compressing the charge. During this process the mass of gas in the cylinder remains constant (assuming no leakage) and the change of state represented by the line 2 3 may be isothermal, adiabatic, or in general polytropic.

The compressed gas must now be expelled from the cylinder by a further motion of the piston to the left. The outlet valve opens and the gas is displaced as shown by the line 3 4 which does not represent an isopiestic change of state. During this process the temperature of the compressed gas is assumed to remain constant and equal to the temperature after compression at 3.

The work required to compress and deliver the gas may be computed from Fig. 15. The work done by and against the atmospheric pressure on the right-hand side of the piston during the forward and the backward stroke of the piston are equal and thus neutralize each other. On the left-hand side of the piston the gas entering the cylinder exerts a pressure p_1 (or p_2) upon the piston and thus does work represented by the area under the line 1 2. During the return stroke the gas exerts at first a variable pressure (represented by the ordinates of the line 2 3) upon the piston, and work represented by the area under the line 2 3 must be done by the machine upon the gas. Finally to deliver the gas the work done by the machine equals the area under the line 3 4.

The total work required for compression and delivery is thus represented by the area 1 2 3 4 1. This work must be done upon the gas and thus represents energy which must be supplied from some external source.

In developing formulas for the computation of this work care must be taken to note the signs of the expressions for the various areas concerned. Thus the area under the line 2 3 is

$$p_2v_2 \log_\epsilon \frac{v_3}{v_2},$$

if we assume isothermal compression. As v_3 is less than v_2 this expression is intrinsically negative. In passing from 3 to 4 the work done is

$$p_3(v_4 - v_3) = -p_3v_3,$$

along the line 1 2 we have

$$p_2(v_2 - v_1) = +p_2v_2.$$

Adding these expressions we find as the net work

$$p_2v_2 + (-p_3v_3) + p_2v_2 \log_\epsilon \frac{v_3}{v_2} = p_2v_2 \log_\epsilon \frac{v_3}{v_2},$$

a negative quantity, as it should be, for work is done *on* the gas.

Another method would be to write all areas in an intrinsically positive form, thus

$$_1W_2 = p_2v_2$$

$$_3W_2 = p_2v_2 \log \frac{v_2}{v_3}$$

$$_4W_3 = p_3v_3$$

and then add and subtract these expressions as indicated in Fig. 15, thus

$$p_3v_3+p_2v_2 \log \frac{v_2}{v_3}-p_2v_2.$$

This gives a positive answer which must of course be understood to represent the work necessary for compression and delivery and which must be supplied from outside sources of energy.

Exercise 90. Show that the work required to compress and deliver an ideal gas is

$$p_2V_2 \log_\epsilon \frac{p_3}{p_2}, \text{ for isothermal compression,}$$

$$\frac{k}{k-1}(p_3V_3-p_2V_2), \text{ for adiabatic compression,}$$

$$\frac{n}{n-1}p_2V_2\left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}-1\right], \text{ for polytropic compression,}$$

where p_2 and V_2, and p_3 and V_3 are the pressures and the volumes of the gas at the beginning and at the end of the compression, respectively, as indicated in Fig. 15.

If no heat is removed from the gas during compression the process represented by 2 3, Fig. 15, would be adiabatic. If it were possible to remove sufficient heat so that the temperature of the gas during compression would remain constant and equal to the temperature of the gas when compression starts then the process 2 3 would be isothermal.

As an adiabatic is a steeper curve than an isothermal more work must be done during adiabatic than during isothermal compression. Therefore in practice the cooling of the gas during compression is important. This cooling is effected by jacketing the cylinder or by injecting cold water into the cylinder. It is however impossible to remove sufficient heat during fairly rapid compression to effect even isothermal compression much less improve on this condition. Practically the compression curve is approximately polytropic with an exponent varying, with the design and operation of the compressor, from 1 to 1.4, with an average value of 1.25.

Exercise 91. A compressor is to take 100,000 cubic feet of air per hour at 15 pounds per square inch absolute and 70° F, and deliver it at 60 pounds per square inch absolute. As a first approximation compute the horse-power required to drive this compressor assuming

(*a*) adiabatic compression

(*b*) polytropic compression, $n=1.2$.

Exercise 92. Compute the heat to be removed from a gas during compression in order to produce a polytropic compression having an exponent n. Express the result in terms of the pressures and the volumes at the beginning and the end of compression and the gas constants. Solve by means of

(*a*) the equation $\Delta Q=\Delta K+\Delta W$,

(*b*) the specific heat of the gas.

Exercise 93. Assuming that the cooling water suffers an increase in temperature of 20° F during its passage thru the compressor described in Exercise 91 and that the cooling produced occurs only during compression, how much cooling water must be supplied?

What expenditure of power does this cooling water save?

The Effect of Clearance.—In all practical compressors the piston cannot come into contact with the cylinder-head at the end of its stroke. The volume between the valves and the piston when in its extreme position is called the clearance volume. The clearance is usually expressed as a percentage of the displacement volume of the piston which is the volume swept thru by the piston.

The effect of this clearance volume is shown on the *pv*-diagram in Fig. 16. After compression at 3 the volume of compressed gas in the cylinder is *a* 3. The piston can expel only the volume 3 4 from the cylinder for at 4 it reaches the end of its stroke. The volume *a* 4 of compressed gas thus remains in the cylinder when the piston starts its return stroke and expands during this return stroke of the piston as shown by the process 4 1. The inlet valves do not open until the point 1 is reached at which the pressure in the cylinder drops to the intake pressure.

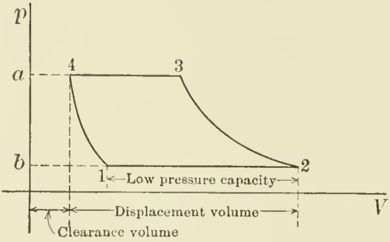

FIG. 16.

The volume of air admitted to the cylinder per stroke is thus $V_2 - V_1$. This volume is called the low-pressure capacity of the cylinder.

EXERCISE 94. The piston of a compressor has a displacement volume of 6 cubic feet. The clearance is 4 per cent. The compressor operates between 15 and 200 pounds per square inch absolute. Assuming $n = 1.2$, what volume of air does the compressor take in per stroke?

EXERCISE 95. Show that if

$$p_2 = p_1 \left(\frac{V + V_0}{V} \right)^n,$$

where p_2 is the delivery pressure,
p_1 is the intake pressure,
V is the displacement volume,
V_0 is the clearance volume,
the compressor delivers no gas.

What power is required to run the compressor under these conditions?

EXERCISE 96. At what pressure will delivery cease with isothermal compression if the intake pressure is 14 pounds per square inch absolute and the clearance is 5 per cent?

EXERCISE 97. Compute the volumetric efficiency of a compressor (i.e., the ratio of the low-pressure capacity to the displacement volume) in terms of the clearance c, the initial and the final pressures p_1 and p_2, and n.

EXERCISE 98. Compute the i.h.p. of a double-acting compressor with clearance in terms of its low-pressure capacity, the initial and the final pressures, and n.

SECTION XIII

COMPOUND COMPRESSION

As some of the above exercises show, the capacity of a cylinder is diminished by the necessary clearance. If the range of pressure is large this capacity may be very much diminished.

On account of this decrease in volumetric efficiency as well as on account of the saving of work that can be effected, compression to more than six times the intake pressure is never attempted in a single cylinder.

In two-stage compression the gas is thoroly cooled during its passage from the first cylinder to the second one. The ideal conditions (clearance neglected) are shown in Fig. 17. Thru 1 are drawn an adiabatic, a polytropic, and an isothermal. The area representing the saving of one-stage isothermal over one-stage adiabatic compression between the pressures p_1 and p_2 is at once evident.

Suppose now that the compression follows the polytropic law and that the compression in the first cylinder stops at *A* (pressure p'). As it is impossible to cool the gas thoroly in the cylinder let the gas be expelled from this cylinder (along *AC*) thoroly cooled in an intercooler and then returned to the second cylinder (along *CB*). This diminishes the volume of the gas from *AC* to *BC* but its mass is unchanged. Moreover by thoro cooling is to be understood that the gas has been cooled

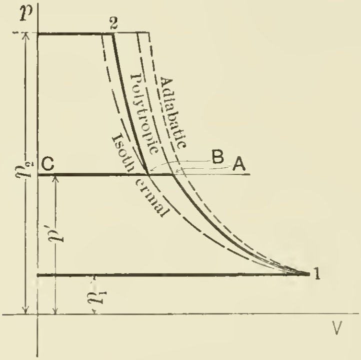

Fig. 17.

to its original temperature (the temperature at 1). Thus B lies on the isothermal thru 1 and

$$p_1 V_1 = p' V_B.$$

In the second cylinder the compression is continued from B to 2 and the gas finally delivered at the pressure p_2. Note carefully the area representing the work saved by two-stage polytropic over one-stage polytropic compression.

It is now important to determine the intermediate pressure p', which will yield the greatest saving of work.

The work done on the gas in the first cylinder is

$$W_1 = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p'}{p_1} \right)^{\frac{n-1}{n}} - 1 \right],$$

in the second cylinder it equals

$$W_2 = \frac{n}{n-1} p' V_B \left[\left(\frac{p_2}{p'} \right)^{\frac{n-1}{n}} - 1 \right],$$

if we assume the same exponent for the law of compression in both cylinders.

As the second cylinder must handle all the gas supplied by the first cylinder and as the temperature at B equals the temperature at 1 due to perfect intercooling we have by reason of $pV = mRT$

$$p_1 V_1 = p' V_B.$$

Thus the total work required to compress and deliver equals

$$W = W_1 + W_2 = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p'}{p} \right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p'} \right)^{\frac{n-1}{n}} - 2 \right].$$

Exercise 99. Show that if

$$p' = \sqrt{p_1 p_2}$$

a minimum of work is necessary for two-stage compression.

Exercise 100. Show that $W_1 = W_2$ when p' is adjusted for minimum work.

Exercise 101. When $p' = \sqrt{p_1 p_2}$ show that the temperature of the gas at the end of the first stage equals the temperature of the gas at the end of the second stage of compression.

Exercise 102. Show that the results obtained in Exercises 99 and 100 are not affected by whatever clearance may exist in either or both cylinders of a two-stage compressor.

Exercise 103. Compute the dimensions of the cylinders and the power required to drive a double-acting two-stage air compressor designed to deliver 3000 cubic feet of air per minute (measured at 15 pounds per square inch absolute) at 100 pounds per square inch absolute. Assume the intake pressure to be 15 pounds per square inch absolute, the revolutions 80 per minute, the piston speed 600 feet per minute, the clearance 4 per cent in each cylinder, and the exponents of the polytropic curves 1.3.

Three-stage Compression.—When compression is to be carried to very high pressures three stages may advantageously be employed. In order to determine the two intermediate pressures which will yield minimum work when three cylinders are used the calculus may be used, but this method leads to the investigation of a function of two independent variables.

Instead we may regard the first two cylinders as constituting a two-stage compressor and the intermediate and high-pressure cylinders as constituting another two-stage compressor.

Then if p_1 is the initial pressure,
p_2 the final pressure,
p' the pressure between the first and the second,
and p'' the pressure between the second and the third cylinders we have for minimum work

$$p' = \sqrt{p_1 p''} \quad \text{and} \quad p'' = \sqrt{p' p_2}.$$

Solving these equations for p' and p'' we find

$$p' = \sqrt[3]{p_1^2 p_2}$$

and

$$p'' = \sqrt[3]{p_1 p_2^2}.$$

EXERCISE 104. Find these values of p' and p'' by means of the calculus.

Turbo-compressors.—In turbo-compressors the gas is compressed by the action of rotating blades. The gas is not confined in cylinders by means of pistons. Under these conditions the indicator cannot be used to find the indicated horse-power.

The work required to compress and deliver a gas by means of a turbo-compressor can be computed from the following experimental data.

(1) The weight and the temperature rise of the cooling water used,

(2) the temperature rise of the compressed gas,

(3) the weight of the compressed gas.

Let p_1, V_1, T_1, represent the initial state, and
p_2, V_2, T_2, the final state of the gas,
W_c, the work of compression,
W, the work required to compress and deliver the gas, both in foot-pounds,

Q, the heat extracted during compression, in B.t.u.

Then as $$\Delta Q=\Delta K+\Delta W$$

we have $$-Q=mc_v(T_2-T_1)-\frac{1}{J}W_c,$$

or $$W_c=JQ+Jmc_v(T_2-T_1).$$

No matter what the law of compression may be

$$W=W_c+p_2V_2-p_1V_1,$$

thus $$W=JQ+Jmc_v(T_2-T_1)+p_2V_2-p_1V_1.$$

But $$pV=mRT$$

so that $$W=JQ+Jmc_v(T_2-T_1)+mR(T_2-T_1)$$

or $$W=JQ+mJc_p(T_2-T_1).$$

Exercise 105. A turbo-compressor delivers per minute at a temperature of 170° F, 16,000 cubic feet of air, measured at 14.7 pounds per square inch absolute. Assuming that no heat is extracted during the compression find the indicated horsepower of this compressor.

CHAPTER V

GAS CYCLES

SECTION XIV

INTRODUCTION

THE state of an ideal gas is determined by its pressure, volume, and temperature. The process thru which a gas passes when it changes its state may conveniently be represented by a line on the pV-plane. Thruout such a process the mass of gas considered must remain constant. The changes in temperature, not represented on the pV-plane, may be followed by means of the equation

$$pV = mRT.$$

A series of processes which after completion leave the gas in its initial state is called a cycle. Thus after the gas has passed thru a cycle the pressure, the volume, the temperature, and of course the internal energy have all returned to their initial values.

In Fig. 18, let the lines 1 2, 2 3, 3 1 represent the processes thru which the given mass of gas passes in order to complete the cycle 1 2 3 1. The work performed by the gas during the process 1 2 is represented by the area A 1 2 B. The work performed on the gas during the process 2 3 is represented by C 3 2 B and for the process 3 1 by A 1 3 C.

The net work obtained from the gas would thus be represented by

$$A\,1\,2\,B - C\,3\,2\,B - A\,1\,3\,C,$$

or as it may also be expressed,

$$A\,1\,2\,B + B\,2\,3\,C + C\,3\,1\,A.$$

The net work is thus the area enclosed between the lines representing the processes thru which the gas passes during the cycle, the shaded area in Fig. 18.

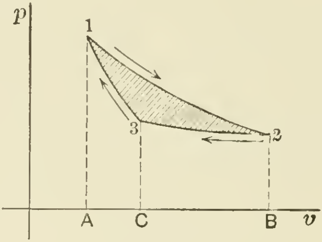

Fig. 18.

As will be shown, a gas cannot pass thru a cycle unless heat is supplied to it and is rejected by it during some of the processes of the cycle. Moreover when work is done by the gas passing thru a cycle more heat is invariably supplied to it than is rejected by it and the difference between the supplied and the rejected heats must equal the heat equivalent of the work done by the gas. This is of course a consequence of the law of conservation of energy.

A heat motor is any machine by means of which heat energy may be transformed into mechanical energy. In all known heat motors the following three prime elements appear in some form or other:

(1) **A hot body,** the source of heat,

(2) **A working substance,**

(3) **A cold body,** the receiver of heat.

During the operation of a heat motor heat energy leaves the hot body, passes to the working substance by means of which some of the heat energy is transformed into mechanical energy and the remaining heat energy is rejected by the working substance to the cold body in order that the working substance may return to its initial state ready to resume the cycle.

The working substance may be a solid, a liquid, or a gas. It is conceivable that a long bar of iron may act as the working substance of a heat motor. Assume one end of the bar to be firmly fixed while the other end acts as a pawl upon a ratchet-wheel with teeth of small pitch. The work performed may be stored as potential energy in a weight lifted by means of a cord wound about a drum fixed to the axle of the ratchet-wheel. If heat is supplied to the bar of iron (the working substance) by means of a flame (the hot body) the bar expands and work is done. In order to return the bar to its initial state so that the cycle may be repeated the bar must now be cooled and heat must be rejected to the cold body which may be the atmosphere. Heat motors operating on this principle have been used to wind freak clocks by means of the daily changes in the temperature of the atmosphere.

In the discussion of any cycle the following points should be considered.

(1) The relations between the states of the gas at the

beginning and at the end of the various processes involved in the cycle.

(2) The total heat supplied to the gas by the hot body and rejected by the gas to the cold body.

(3) The net external work performed by the gas during the cycle.

(4) The efficiency of the cycle.

It should be remembered that the algebraic sum of the heats supplied to the gas during the cycle (this to include the heats rejected by the gas which are negative) must equal the heat equivalent of the work performed by the gas.

The efficiency of a cycle is defined by the ratio

$$\frac{\text{output}}{\text{input}}=\frac{\text{heat converted into work}}{\text{heat supplied by the hot body}}.$$

The more important gas cycles which have been proposed or used will be discussed in this chapter. For convenience they have been grouped as cycles of hot-air engines and of internal-combustion engines.

SECTION XV

CYCLES OF HOT-AIR ENGINES

The Carnot Cycle.—No attempt has ever been made to build a heat motor operating on a Carnot cycle. Its mean effective pressure is very small so that a very large engine would yield but little power. Theoretically this cycle is however very important. Its efficiency, as will be shown, cannot be surpassed by that of any other cycle. It thus serves as a standard by means of which other cycles should be judged.

The Carnot cycle consists of two isothermal and of two adiabatic processes as shown in Fig. 19. The gas to be forced thru this cycle should be conceived as placed in a heat-insulated cylinder, Fig. 19, closed at one end by a non-conducting piston and at the other end by a conducting plate *A*. The mass of gas remains in the cylinder thruout the cycle. The end *A* of the cylinder may be covered at will by either the hot body, *H*, the cold body, *C*, or a non-conducting plate, *I*.

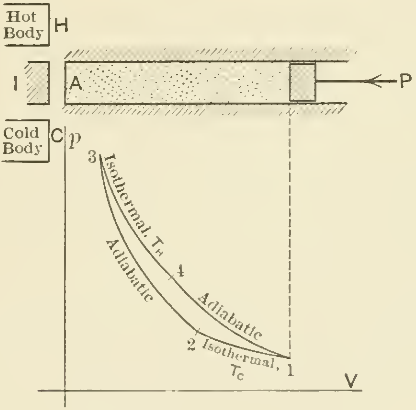

FIG. 19.

Assuming the cycle to start with the gas at its greatest volume and at its least pressure (1, Fig. 19) the gas must first be compressed along the isothermal 1 2 at the temperature of the cold body T_C. The work required for this compression must be furnished by the machine. It may be conceived to be derived from energy stored as kinetic energy in the fly-wheel of the machine. Moreover the force exerted on the piston *P* must be conceived as continually changing so as to remain only very slightly in excess of the force exerted on the piston by the confined gas. Thus the motion of the piston is very slow and the temperature of the gas rises but very slightly above T_C before the heat generated by the compression flows to the cold body *C* which must be placed at *A* during this process.

At 2 the law of compression should change to an adiabatic one. The cold body *C* must be removed from *A* and the non-conducting plate *I* substituted for it. The work required for compression is still derived from the machine but the temperature of the gas now rises from T_C to T_H, the temperature of the hot body. When the temperature T_H is reached (at 3) the isothermal expansion should begin.

Now the force *P* exerted by the mechanism upon the piston should remain always very slightly less than the force exerted by the gas. Thus *P* must vary. The gas now drives the piston and the work done must be partly stored in the machine for future use in compressing the gas and partly rejected by the machine. The isothermal expansion can occur only if *A* is now in contact with the hot body *H* so that the heat transformed into work may be supplied to the gas by conduction as soon as the temperature has fallen ever so slightly below T_H.

To complete the cycle *H* is removed and *I* substituted at *A* when the state 4 is reached. The gas continues to expand and do work at the expense of its internal energy and its temperature drops to T_C when state 1 is reached.

No engine reproducing exactly the processes above described could be built. The condition of practical equilibrium involved during isothermal transfers of heat make this impossible. However engines approximating these conditions may be conceived. So it is with all heat engines. The ideal conditions sought are never attained. The closer the actual cycle approaches the ideal the more nearly will the ideal efficiency be realized.

From the above description of the Carnot cycle it is evident that certain relations must exist between the pressures and between the volumes of the gas at 1, 2, 3, and 4 in order that the processes involved may form a closed cycle. For example, the isothermal expansion must cease at just the right point in order that the following adiabatic expansion may return the gas to its initial state at 1.

From the process 2 3 we have

$$\frac{T_2}{T_3}=\left(\frac{T_C}{T_H}\right)=\left(\frac{V_3}{V_2}\right)^{k-1}$$

and from the process 4 1

$$\frac{T_4}{T_1}=\left(\frac{T_H}{T_C}\right)=\left(\frac{V_1}{V_4}\right)^{k-1},$$

therefore

$$\frac{V_3}{V_2}=\frac{V_4}{V_1}.$$

EXERCISE 106. What relation must exist between the pressure at the beginning and at the end of the various processes of a Carnot cycle?

EXERCISE 107. Assuming the temperature of the source as 500° F and of the receiver as 70° F, the displacement volume of the piston as 10 cubic feet with a "clearance" volume of 2 cubic feet, find the highest pressure reached in a Carnot cycle operated under the above conditions provided the lowest pressure is 15 pounds per square inch absolute.

The heat supplied during the whole Carnot cycle illustrated in Fig. 19 consists of the sum of the heats supplied during the various processes. Thus

$$
\begin{aligned}
{}_1Q_2 &= \frac{1}{J} p_1 V_1 \log_\epsilon \frac{V_2}{V_1} \\
{}_2Q_3 &= 0 \\
{}_3Q_4 &= \frac{1}{J} p_3 V_3 \log_\epsilon \frac{V_4}{V_3}
\end{aligned}
$$

so that
$$
{}_1Q_1 = \frac{1}{J} p_1 V_1 \log_\epsilon \frac{V_2}{V_1} + \frac{1}{J} p_3 V_3 \log_\epsilon \frac{V_4}{V_3}.
$$

Note that $\frac{1}{J} p_1 V_1 \log_\epsilon \frac{V_2}{V_1}$ is intrinsically negative for $V_2 < V_1$. Thus ${}_1Q_1$ as written above represents the difference between the heat supplied by the hot body and the heat rejected to the cold body. It is the heat which disappears as heat and reappears as mechanical energy.

It is important to simplify the expression for ${}_1Q_1$ as follows:

$$
\begin{aligned}
{}_1Q_1 &= \frac{1}{J}\left(p_3 V_3 \log_\epsilon \frac{V_4}{V_3} - p_1 V_1 \log_\epsilon \frac{V_1}{V_2} \right) \\
&= \frac{1}{J}(p_3 V_3 - p_1 V_1) \log_\epsilon \frac{V_4}{V_3}.
\end{aligned}
$$

EXERCISE 108. Show that the heat converted into work during the operation of a Carnot cycle equals

$$
\frac{mR}{J}(T_H - T_C) \log_\epsilon r,
$$

where
$$
r = \frac{V_1}{V_2} = \frac{V_4}{V_3} = \frac{p_2}{p_1} = \frac{p_3}{p_4}.
$$

EXERCISE 109. Compute from Fig. 19 the area representing the work delivered during a Carnot cycle and show that the result is equivalent to the results expressed in Exercise 108.

EXERCISE 110. Show that **the efficiency of a Carnot cycle is**

$$\frac{T_H - T_C}{T_H}.$$

The Stirling Cycle.—The Stirling cycle (Fig. 20) consists of two isothermals combined with two isometric processes. This cycle affords larger mean effective pressures and thus more power for the same cylinder volume than the Carnot cycle.

Under ideal conditions the gas passing thru the processes of a Stirling cycle should be compressed while in thermal contact with the cold body (process 1 2, Fig. 20), then heated while its volume remains constant, 2 3, until its temperature is T_H, the temperature of the hot body, next expanded isothermally while in contact with the hot body, process 3 4, and finally cooled at constant volume to the temperature of the cold body, process 4 1.

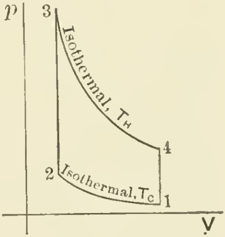

FIG. 20.

In this cycle the heat supplied during the process 2 3 is

$$_2Q_3 = mc_v(T_H - T_C),$$

and the heat supplied from 4 to 1 is

$$_4Q_1 = mc_v(T_C - T_H).$$

As the last expression is intrinsically negative $(T_C < T_H)$ it represents heat rejected. Also

$$_2Q_3 = {}_1Q_4.$$

Stirling proposed to store the heat rejected from 4 to 1 and utilize this heat during the process 2 3 in order that less heat need be drawn from the hot body, thus improving the efficiency. Stirling did this by means of a regenerator. The regenerator may be conceived as a cylinder filled with metal gauze. One end of this cylinder remains at the temperature of the hot body, the other at the temperature of the cold body. The hot gas passing thru this cylinder from the hot towards the cold end, heats the gauze so that the gas loses its heat gradually with gradually falling temperature and leaves the cylinder cold. Reversing the flow, the cold gas entering the cold end of the cylinder would (at least under ideal conditions) pick up the heat previously deposited in the gauze and would leave the cylinder hot without drawing upon the source of heat (the hot body).

Stirling built and operated engines based upon the above-described cycle. One of these engines indicating 50 horsepower is said to have realized a thermal efficiency of 30 per cent with 1.7 pounds of coal per i.h.p. hour.

EXERCISE 111. Show that the work performed by the gas during a Stirling cycle is

$$m(T_H - T_C) R \log_\epsilon r,$$

where

$$r = \frac{V_4}{V_3} = \frac{V_1}{V_2} = \frac{p_2}{p_1} = \frac{p_3}{p_4}.$$

EXERCISE 112. Show that with an ideal regenerator the efficiency of the Stirling cycle is

$$\frac{T_H - T_C}{T_H}.$$

EXERCISE 113. Compute the efficiency of the Stirling cycle without regeneration.

The Ericsson Cycle.—The Ericsson cycle consists of two isothermal and of two isopiestic processes. In Fig. 21 the cycle is 1 2 3 4 1.

The mass of gas passing thru the cycle may be conceived to remain in the cylinder during the whole cycle. It is first compressed while its temperature is maintained at the temperature of the cold body, process 1 2, Fig. 21.

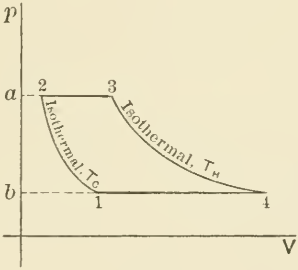

FIG. 21.

The gas is then heated while its volume increases at constant pressure until its temperature is T_H, the temperature of the hot body, process 2 3. Next isothermal expansion at a temperature T_H follows during which the gas must be heated from an external source, the hot body. Finally the gas is cooled at constant pressure to a temperature T_C the volume meanwhile reducing from V_4 to V_1.

Practically the gas cannot be made to pass thru the above-described processes in one cylinder for this would require that the cylinder be heated and cooled from a temperature T_C to T_H, and back to T_C during each cycle. This would entail a great waste of heat.

Ericsson invented, designed, and built engines operating on the above cycle in the following manner. The air was first compressed as nearly isothermally as possible in one cylinder of the engine. The action of this cylinder would be shown on a pV-diagram (Fig. 21) by the lines *b* 1, 1 2, 2 *a*. The compressed air is now heated while on its way to the second cylinder of the engine, the working cylinder. The compressed and heated air (equal in mass to the air discharged by the compressing cylinder) is now admitted to the working cylinder (process *a* 3, Fig. 21) expanded while heat is supplied to it by the hot body, process 3 4, and then expelled from the working cylinder by the piston, process 4 *b*. After leaving the working cylinder while on its way to the compressing cylinder preparatory to resuming the cycle the air is cooled at constant volume so that the mass of air having a volume of V_4 when it leaves the working cylinder enters the compressor with a volume V_1.

Exercise 114. Show that the heat which must be supplied to the gas during the process 2 3, Fig. 21, equals the heat which must be withdrawn during the process 4 1, and thus that a regenerator may be used to store the heat rejected along the process 4 1 for use during the process 2 3.

Exercise 115. Compute the efficiency of the Ericsson cycle (*a*) with, (*b*) without regeneration.

The Joule Cycle.—All engines operating on the cycle of Ericsson and of Stirling failed to give satisfaction in service on account of the failure of the metal plates, which are exposed on one side to the hot gases of the furnace and on the other to the air in the cylinder. The air in the working cylinder was also practically at rest so that the transfer of heat to it was very slow. The regenerators were also liable to rapid deterioration.

Joule proposed a cycle which overcame these difficulties. His cycle consisted of two adiabatics and two isopiestics, Fig. 22. The compression and the expansion being adiabatic, no attempt is made to heat or cool the air during these processes. Heat is supplied to the air only during the processes 2 3 and 4 1. That these transfers of heat may be effected under the best conditions Joule's engine consisted of two cylinders and the heating and cooling occurred during the passage of the air from one cylinder to the other.

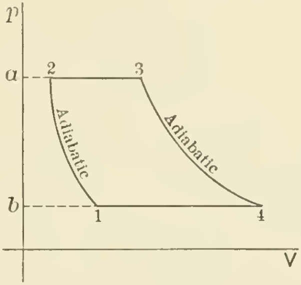

FIG. 22.

A diagrammatic sketch of the engine is shown in Fig. 23. The volume of the gas during compression is always less than the volume of the same mass of air during expansion (see Fig. 22) so that if the pistons have the same stroke the diameter of the working cylinder must be greater than the diameter of the compressing cylinder.

Assuming ideal conditions and cylinders without clearance the states of the gas during the performance of the cycle and the corresponding positions of the gas in the engine would be as follows. Intake into the compressor occurs along b 1 (Figs. 22 and 23), the temperature being T_1; delivery from the compressor occurs along 2 a, temperature T_2 where $T_2 > T_1$ owing to the adiabatic compression. The gas is now heated during its passage thru the "hot body" leaving it at a temperature T_3 and passing into the working cylinder a 3, $T_3 > T_2$. Adiabatic expansion in the working cylinder cools the gas to a temperature T_4 at this temperature it is delivered along 4 b to the "cold body" there to be further cooled to a temperature T_1 ($T_4 > T_1$), preparatory to reentering the compressor with a volume $V_1 = b$ 1.

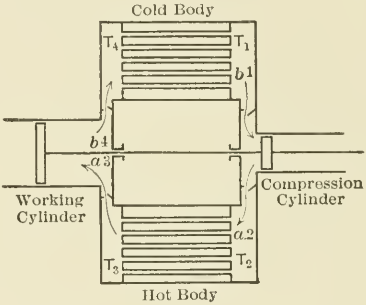

Fig. 23.

The temperature of the cold body under ideal conditions must be T_1 so that the gas coming in contact with it at a temperature T_4 may leave at a temperature T_1. Similarly the temperature of the hot body must be T_3, altho the gas reaches it at a temperature $T_2 (< T_3)$.

The difference between the conditions of heat transfer in this cycle and the cycles so far considered should be carefully noted. In the other cycles the gas was always at the same temperature as either the cold body or the hot body during the transfer of heat, a less practical but a theoretically more perfect condition and one which as we shall see leads to a higher efficiency. Whenever a transfer of heat from a body at higher to one of lower temperature occurs energy is invariably wasted.

For the analysis of the Joule cycle note that

$p_1 = p_4 = p_C$, the pressure in the " cold body",

$p_2 = p_3 = p_H$, the pressure in the " hot body".

We may obtain a relation between the volumes by means of either the processes 1 2 and 3 4 or 1 4 and 2 3. In the latter the temperatures are involved—about these we as. yet know nothing. The former gives the relations of the volumes to the pressures. Thus

$$\frac{p_C}{p_H} = \left(\frac{V_2}{V_1}\right)^k = \left(\frac{V_3}{V_4}\right)^k,$$

whence $$\frac{V_1}{V_2} = \frac{V_4}{V_3}.$$

Now using the processes 1 4 and 2 3 and remembering that in $pV = mRT$, p, m, and R are constant, we find

$$\frac{V_4}{V_1} = \frac{T_4}{T_1} \quad \text{and} \quad \frac{V_2}{V_3} = \frac{T_2}{T_3},$$

whence $$\frac{T_4}{T_1} = \frac{T_3}{T_2}.$$

The heats supplied during the four processes are

$$_1Q_2 = 0$$

$$_2Q_3 = mc_p(T_3 - T_2)$$

$$_3Q_4 = 0$$

$$_4Q_1 = mc_p(T_1 - T_4) = -mc_p(T_4 - T_1).$$

The work performed by the gas during each of the four processes is

$$_1W_2 = \frac{p_1V_1 - p_2V_2}{k-1} = -\frac{p_2V_2 - p_1V_1}{k-1}$$

$$_2W_3 = p_H(V_3 - V_2)$$

$$_3W_4 = \frac{p_3V_3 - p_4V_4}{k-1}$$

$$_4W_1 = p_C(V_1 - V_4) = -p_C(V_4 - V_1).$$

Exercise 116. Find the net work performed by the gas during a Joule cycle by means of two areas and show that the result equals the sum of the four areas given above.

Exercise 117. Find an expression for the net work performed during a Joule cycle in terms of the temperatures.

To find **the efficiency of the Joule cycle** the result of Exercise 117 is more useful than the result of Exercise 116. In general the efficiencies of a cycle can best be computed from the heat than from the standpoint of work directly. Thus the efficiency of the Joule cycle is

$$\frac{mc_p(T_3 - T_2) - mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}.$$

This expression may be simplified by means of the relation between the temperatures already deduced, namely,

$$\frac{T_4}{T_1}=\frac{T_3}{T_2}.$$

From this relation by subtracting unity from each member we obtain

$$\frac{T_4-T_1}{T_1}=\frac{T_3-T_2}{T_2},$$

or

$$\frac{T_4-T_1}{T_3-T_2}=\frac{T_1}{T_2}.$$

Therefore the efficiency reduces to

$$1-\frac{T_1}{T_2}=1-\frac{T_4}{T_3}=1-\left(\frac{p_C}{p_H}\right)^{\frac{k-1}{k}}.$$

It is important to note that this efficiency is less than the efficiency of a Carnot cycle operating between the same hot and the same cold bodies. The efficiency of the Carnot cycle is

$$\frac{T_H-T_C}{T_H}=1-\frac{T_C}{T_H},$$

or in the notation of this case

$$1-\frac{T_1}{T_3}.$$

As

$$T_3>T_2, \quad \frac{T_1}{T_3}<\frac{T_1}{T_2}$$

and

$$1-\frac{T_1}{T_3}>1-\frac{T_1}{T_2}.$$

EXERCISE 118. Show that $T_4>T_1$ and that therefore the efficiency of the Joule cycle is less than the efficiency of the Carnot cycle operating between the same extreme temperatures.

Condition Necessary for Greatest Efficiency.—It should be remembered that when heat is supplied to (and withdrawn from) the gas during isothermal processes and only during such processes that the efficiencies of the cycles (Carnot, Stirling, Ericsson, the last two with regeneration) are all the same and all greater than the efficiency of the Joule cycle. It will be shown that no cycle can exceed in efficiency the first-mentioned cycles. In these cycles thermal equilibrium always exists and this renders the exact fulfilment of the conditions involved impossible in practice.

Cycles Composed of Two Pairs of Polytropics.—All cycles so far considered have been composed of four polytropics, of which the opposite pairs have the same exponent. In general we may assume the

process 1 2 to be the polytropic $p_1v_1^n = p_2v_2^n = pv^n$,

process 2 3 to be the polytropic $p_2v_2^m = p_3v_3^m = pv^m$,

process 3 4 to be the polytropic $p_3v_3^n = p_4v_4^n = pv^n$,

process 4 1 to be the polytropic $p_4v_4^m = p_1v_1^m = pv^m$.

To establish **the general relation between the volumes** place the product of the first members of these equations equal to the product of the second members. Thus

$$p_1p_2p_3p_4\ v_1^nv_2^mv_3^nv_4^m = p_2p_3p_4p_1\ v_2^nv_3^mv_4^nv_1^m,$$

or $$(v_1v_3)^n\ (v_2v_4^m) = (v_2v_4)^n\ (v_3v_1)^m,$$

whence $$(v_1v_3)^{n-m} = (v_2v_4)^{n-m}$$

or $$\mathbf{v_1v_3 = v_2v_4}.$$

Similarly **the relation between the pressures** may be obtained as follows

$$p_1^{\frac{1}{n}}v_1 = p_2^{\frac{1}{n}}v_2$$

$$p_2^{\frac{1}{m}}v_2 = p_3^{\frac{1}{m}}v_3$$

$$p_3^{\frac{1}{n}}v_3 = p_4^{\frac{1}{n}}v_4$$

$$p_4^{\frac{1}{m}}v_4 = p_1^{\frac{1}{m}}v_1,$$

whence $$(p_1p_3)^{\frac{1}{n}}(p_2p_4)^{\frac{1}{m}} = (p_2p_4)^{\frac{1}{n}}(p_3p_1)^{\frac{1}{m}}$$

$$(p_1p_3)^{\frac{1}{n}-\frac{1}{m}} = (p_2p_4)^{\frac{1}{n}-\frac{1}{m}}$$

or $$\mathbf{p_1p_3 = p_2p_4}.$$

These relations are the same as those already established for each individual cycle discussed in this section. They do not depend upon the substance which passes thru the cycle but simply upon the laws of change of state followed.

The relation between the temperatures however depends upon the nature of the substance used. **For ideal gases the relation between the temperatures** may be developed as follows.

$$p_1v_1 = RT_1 \quad \text{and} \quad p_3v_3 = RT_3$$

therefore $$(p_1p_3)(v_1v_3) = R^2(T_1T_3).$$

Similarly $$(p_2p_4)(v_2v_4) = R^2(T_2T_4).$$

Thus by reason of the above-established relations between the volumes and the pressures

$$\mathbf{T_1T_3 = T_2T_4}.$$

Section XVI

HEAT PUMPS OR REFRIGERATING MACHINES

The cycles and engines considered in the last section were all cycles and engines utilized as heat motors, the object being the transformation of heat energy into mechanical energy. In all cases heat passed from the hot body to the cold body and during the transfer thru the working substance some of the heat leaving the hot body was transformed into mechanical energy thus never reaching the cold body.

If η represents the efficiency of the cycle considered and Q the heat units leaving the hot body then ηQ represents the heat units transformed into mechanical energy and $(1-\eta)Q$ the heat units rejected to the cold body.

Exercise 119. An ideal engine operating under a Joule cycle with air under extreme pressures of 40 and 100 pounds per square inch absolute will reject what fraction of each B.t.u. supplied by the hot body to the cold body, which is maintained at 70° F?

If the gas in any of the cycles already considered could be forced to change its state in such a manner that the point representing the state on the pV-plane would move around the cycle in a counter-clockwise direction then instead of yielding mechanical energy, mechanical energy would have to be supplied to the gas. Also instead of rejecting heat to the cold body the gas would withdraw heat from the cold body and the hot body would receive heat from the gas instead of supplying it with heat.

Moreover the heat reaching the hot body would equal the heat withdrawn from the cold body plus the heat equivalent of the net work done on the gas or the work required to run the engine under these conditions.

If the Carnot cycle (Fig. 19) is traced in the counter-clockwise direction the temperature of the hot body still persists along the process 4 3 and the temperature of the cold body along the process 2 1 is still the temperature at which heat is now transferred from the cold body to the gas.

If however the Joule cycle (Fig. 22) is traced in the counter-clockwise direction the temperatures cannot remain as they were during the motor cycle. During the motor cycle $T_2 < T_H$ and $T_3 = T_H$, heat passing from the hot body (temperature T_H) to the gas (whose temperature is less than T_H) during the process 2 3. But during the pump cycle heat must pass from the gas to the hot body during the process 3 2; this would be impossible if T_3 remained equal to T_H and $T_2 < T_H$. Thus during the pump cycle T_2 must equal T_H and T_3 must be less than T_H. During the motor cycle the gas reaches the temperature of the

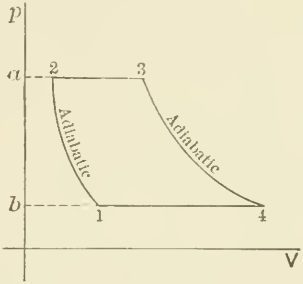

FIG. 22.

hot body at 3 while during the pump cycle the gas reaches the temperature of the hot body at 2.

Exercise 120. At what point of the Joule cycle, Fig. 22, does the gas attain the temperature of the cold body (*a*) during the motor cycle, (*b*) during the pump cycle? Explain the passage of the air thru the engine, Fig. 23, page 96, in connection with Fig. 22, when the engine operates as a heat pump.

Heat pumps are used as refrigerating machines. The cold body from which heat is withdrawn is the chamber to be maintained at low temperature and the hot body is the cooling water which carries away the heat rejected by the gas at the relatively higher temperature.

The lowest pressure in the cycle may be atmospheric. In this case the air of the refrigerated chamber may be taken directly into the compression cylinder and the working cylinder would discharge into the refrigerated chamber. This method is impractical on account of the large volume which the required mass of air occupies and also on account of the condensation and freezing of the moisture (carried by the air from the refrigerated chamber) in the ports and passages leading from the working cylinder. These objections may be overcome by working the air in a closed system of pipes at a pressure above the atmospheric pressure thus decreasing the volume of the required mass of air and preventing the absorption of moisture by the air.

Refrigerating machines working on the principle just described have been largely superseded by machines using liquids and their vapors instead of air. The air machines now are used only on shipboard or other confined places where accidents to machines using ammonia would be dangerous.

The action of refrigerating machines is judged by their coefficient of performance. The **thermodynamic coefficient of performance** of a refrigerating machine is defined as the heat extracted from the cold body divided by the heat equivalent of the work expended in driving the machine.

Referring to Fig. 22 the heat extracted from the cold body during the pump cycle is

$$mc_p(T_4-T_1).$$

The heat equivalent of the work done is the excess of the heat rejected to the hot body, $mc_p(T_3-T_2)$, over the heat extracted from the cold body $mc_p(T_4-T_1)$

thus $$mc_p\{(T_3-T_2)-(T_4-T_1)\}.$$

The coefficient of performance is thus under ideal conditions

$$\frac{T_4-T_1}{(T_3-T_2)-(T_4-T_1)}=\frac{1}{\dfrac{T_3-T_2}{T_4-T_1}-1}=\frac{1}{\dfrac{T_2}{T_1}-1}=\frac{T_1}{T_2-T_1},$$

by reason of the relation $T_1T_3=T_4T_2$.

From this result it should be noted that the smaller the range of temperature thru which the air is cooled by adiabatic expansion the larger will be the coefficient of performance and thus the more economical the action of the machine. Theoretically it is thus more advantageous to cool the whole mass of air to be refrigerated a few degrees than to cool a small portion thru many degrees and then mix this very cold air with uncooled air to obtain the proper temperature. Practically this would require very large cylinders with large friction losses and thus a reduction of the actual coefficient of performance. This points to one of the advantages of the liquid and vapor machines over the air machines for refrigeration.

Exercise 121. A refrigerating machine is to remove 1200 B.t.u. per minute from a cold-storage room. The temperature in this room is to be maintained at 34° F under atmospheric pressure. The machine is to draw air from the cold-storage room and compress it to 60 pounds gage while making 100 working strokes per minute. The cooling water maintains the coils at 80° F. What horse-power is required to drive this machine under ideal conditions and what must be the ideal displacement volumes of the compression and of the expansion pistons?

The Warming Engine.—A heat pump used to transfer heat from the cold outside air to the interior of a building may be called a warming engine. Lord Kelvin pointed out that the methods now used for warming buildings (by means of stoves or radiators) are very wasteful, at least theoretically. The heat supplied by a stove at high temperature is not used to the best advantage when it is simply transferred to the air of the room to be heated by convection and radiation. If this heat at high temperature be used in a heat motor and the energy so derived be utilized to drive a heat pump which transfers heat from the cold outer air to the building more heat could thus be delivered at the intermediate temperature of the building.

As a rough mechanical analogy consider two supplies of water one at a high level the other at a lower level and assume that water is required at an intermediate level. Three ways of obtaining this supply suggest themselves.

Firstly, the water from the higher level may simply be allowed to fall to the intermediate level. Secondly, the water falling to the intermediate level may be used in a turbine and the energy so recovered may be utilized to run a pump which raises water from the low to the intermediate level. Thirdly, the whole fall of the water from the high to the low level may be utilized in the turbine and the energy so recovered may be used to pump water from the low to the intermediate level.

Similarly to warm a building each B.t.u. liberated by the stove may be transferred directly to the air of the room but in this manner only one B.t.u. of heat will be available per B.t.u. supplied. Instead the B.t.u. supplied at high temperature may be partly transformed into mechanical energy by means of a heat motor before the remainder is rejected to the air of the room at room temperature. The mechanical energy obtained may be used to run a heat pump. Under ideal conditions this pump rejects as heat to the air at room temperature not only the mechanical energy supplied to run the pump but also such heat as it pumps from the outer air.

To take a concrete example assume the stove temperature as 500° F, the temperature of the room 60° F, and the outer air at 20° F. An ideal heat engine operating between 500 and 60° F would have an efficiency of $\frac{440}{960}=0.459$. Thus for each B.t.u. available at 500° F, 0.459 B.t.u. would be transformed into mechanical energy and 0.541 B.t.u. would be rejected to the air of the room at 60° F. An ideal heat motor operating on a reversible cycle between 60 and 20° F would have an efficiency of

$\frac{40}{520}=0.0769$. It would be very inefficient. If however this motor be reversed and used as a heat pump it would deliver one B.t.u. at 60° F for every 0.0769 B.t.u. supplied to it in the form of mechanical energy or it would transfer 0.9231 B.t.u. from the outer air to the room with an absorption of mechanical energy equivalent to 0.0769 B.t.u.; its ideal coefficient of performance would thus be $\frac{0.9231}{0.0769}=12.0$. As 0.459 B.t.u. are available in the form of mechanical energy from each B.t.u. furnished by the stove $\frac{0.0459}{0.0769}=5.97$ B.t.u. will be delivered by the heat pump to the room. Therefore each B.t.u. at 500° F may be made to furnish $5.97+0.541=6.51$ B.t.u. at 60° F, of course under ideal conditions.

Exercise 122. In the above example assume the ideal heat motor to operate between the stove and the outer air as hot and cold bodies respectively and the ideal heat pump to operate between the outside air and the building as cold and hot bodies respectively. Compute the B.t.u.'s supplied to the building under these conditions for each B.t.u. furnished by the stove.

Section XVII

CYCLES OF INTERNAL-COMBUSTION ENGINES

One of the difficulties met with in the operation of hot-air engines is the transfer of heat from the furnace to the air in the cylinder which passes thru the processes required to complete the cycle. To avoid this transfer of heat the engine may be designed so as to allow combustion to occur within the cylinder of the engine.

The Brayton Cycle.—Brayton designed an engine in which a mixture of gas and air is first compressed into a receiver, from this receiver it flows (thru wire gauze to prevent backfire) into the working cylinder where the mixture is ignited. During this process the pressure in the cylinder must be the same as the pressure in the receiver. After the flow of the mixture is cut off the heated products of combustion expand adiabatically, or nearly so, to atmospheric pressure provided the cylinder volume is sufficiently large. Finally the expanded products of combustion are expelled to the atmosphere at constant back pressure.

EXERCISE 123. Sketch the *pv*-diagram of the Brayton cycle and note that it is thermodynamically equivalent to the Joule cycle, see page 94.

The Otto Cycle.—An engine operating on the Otto cycle requires only one cylinder. The cycle is completed in four strokes of the piston. In Fig. 24 the processes are indicated as follows:

(1) The explosive mixture is drawn into the cylinder, process 1 2.

(2) This mixture is compressed, process 2 3.

(3) The compressed charge is ignited, the combustion assumed to be instantaneous causes an increase of pressure and temperature at constant volume, process 3 4.

(4) The products of combustion expand, process 4 5.

(5) The exhaust valve opens permitting the gases to escape to the atmosphere, process 5 2.

(6) The remaining products of combustion are now expelled into the atmosphere, process 2 1.

Thermodynamically the Otto cycle is represented by 2 3 4 5 2. The process 5 2 is regarded as a cooling at constant volume of the whole mass of gas taken in as the charge. This mass of gas is then conceived to be recompressed, 2 3, and reheated, 3 4, etc., altho actually a new charge must be taken in. Moreover in gas-engine design no account is usually taken of the fact that the contents of the cylinder is not pure air. Calculations made in this way give results which are known as the air standard.

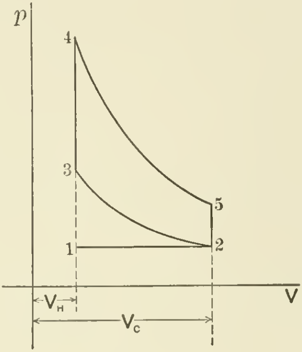

FIG. 24.

Analysis of the Otto Cycle.—Let $V_3=V_4=V_H$ and $V_2=V_5=V_C$ and assume the processes 2 3 and 4 5 (Fig. 24) to be polytropic.

Then
$$\frac{p_3}{p_2}=\left(\frac{V_C}{V_H}\right)^n=\frac{p_4}{p_5}.$$

As
$$\frac{pV}{T}=\text{a constant},$$

we have
$$\frac{p_3}{T_3}=\frac{p_4}{T_4} \quad \text{and} \quad \frac{p_5}{T_5}=\frac{p_2}{T_2}.$$

Thus $$\frac{p_3}{p_4}=\frac{p_2}{p_5}=\frac{T_2}{T_5}=\frac{T_3}{T_4}$$

see also page 101.

The amounts of heat supplied during the various processes are

$$\begin{aligned}
{}_2Q_3&=mc_n(T_3-T_2)\\
{}_3Q_4&=mc_v(T_4-T_3)\\
{}_4Q_5&=mc_n(T_5-T_4)=-mc_n(T_4-T_5)\\
{}_5Q_2&=mc_v(T_2-T_5)=-mc_v(T_5-T_2),
\end{aligned}$$

where $$c_n=\frac{n-k}{n-1}c_v.$$

The work done by the gas during the various processes is

$$\begin{aligned}
{}_2W_3&=\frac{p_2V_C-p_3V_H}{n-1}\\
{}_3W_4&=0\\
{}_4W_5&=\frac{p_4V_H-p_5V_C}{n-1}\\
{}_5W_2&=0
\end{aligned}$$

and the net work of the cycle is

$${}_2W_2=\frac{(p_4-p_3)V_H-(p_5-p_2)V_C}{n-1}.$$

As $$p_5=p_4\left(\frac{V_H}{V_C}\right)^n \quad\text{and}\quad p_2=p_3\left(\frac{V_H}{V_C}\right)^n$$

$$\begin{aligned}
{}_2W_2&=\frac{1}{n-1}(p_4-p_3)\left\{V_H-\left(\frac{V_H}{V_C}\right)^n V_C\right\}\\
&=\frac{p_3V_H}{n-1}\left(\frac{p_4}{p_3}-1\right)\left\{1-\left(\frac{V_H}{V_C}\right)^{n-1}\right\}.
\end{aligned}$$

EXERCISE 124. Show that the above result is equivalent to

$$
\begin{aligned}
{}_2W_2 &= \frac{p_2V_C}{n-1}\left(\frac{p_4}{p_3}-1\right)\left\{\left(\frac{V_C}{V_H}\right)^{n-1}-1\right\}\\
&= Jmc_vT_2\left(\frac{p_4}{p_3}-1\right)\left\{\left(\frac{V_C}{V_H}\right)^{n-1}-1\right\}.
\end{aligned}
$$

EXERCISE 125. Show that the mean effective pressure of the Otto cycle is

$$
\frac{{}_2W_2}{V_C-V_H}=p_2\left(\frac{p_4}{p_3}-1\right)\frac{\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}-1}{n-1}\cdot\frac{\left(\frac{p_3}{p_2}\right)^{\frac{1}{n}}}{\left(\frac{p_3}{p_2}\right)^{\frac{1}{n}}-1}.
$$

In gas-engine design the calculations are usually made under the assumption that the compression and the expansion are adiabatic. Under these conditions the net work of the cycle may be expressed as follows:

$$
{}_2W_2=J({}_3Q_4+{}_5Q_2)=Jmc_v(T_4-T_3+T_2-T_5).
$$

EXERCISE 126. Show that the efficiency of the Otto cycle is

$$
\eta=1-\frac{T_2}{T_3}=1-\frac{T_5}{T_4}=1-\left(\frac{V_H}{V_C}\right)^{k-1}=1-\left(\frac{p_2}{p_3}\right)^{\frac{k-1}{k}},
$$

when compression and expansion are adiabatic.

EXERCISE 127. (*a*) Show that the efficiency of the Otto cycle is less than the efficiency of the Carnot cycle both cycles operating between the same hot and the same cold bodies.

(*b*) Show that the efficiency of the Otto cycle increases with increasing compression.

The Diesel Cycle.—The Diesel cycle is particularly suited to the internal combustion of oil. In this cycle,

Fig. 25, the cylinder takes in a charge of air which is then compressed adiabatically, process 1 2, to about 500 pounds per square inch into the clearance space. The temperature of the compressed air is so high that the oil injected into the cylinder ignites spontaneously. The injection of oil and the heating of the contents of the cylinder continues during a portion of the working stroke of the piston. After the injection of oil ceases the contents of the cylinder expand adiabatically to the end of the stroke.

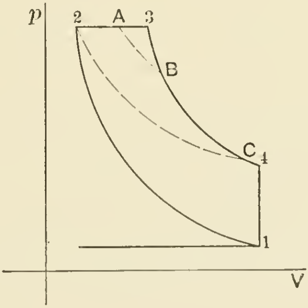

FIG. 25.

Theoretically at least the flow of oil may be regulated so as to maintain either

(*a*) constant pressure until the oil is cut off, process 2 3,

(*b*) constant pressure to *A* and then constant temperature to *B*, process 2 *A* *B*,

(*c*) constant temperature thruout the period of injection of oil, process 2 *C*.

Thermodynamically the cycle is completed after the adiabatic expansion 3 4, *B* 4, or *C* 4 by cooling at constant volume from 4 to 1, practically the products of combustion are expelled to the atmosphere.

EXERCISE 128. Show that the efficiency of the Diesel cycle under the conditions described above is

$$\frac{c_p(T_3-T_2)-c_v(T_4-T_1)}{c_p(T_3-T_2)}=1-\frac{1}{k}\left(\frac{T_4-T_1}{T_3-T_2}\right), \text{ for condition } (a),$$

$$1-\frac{T_4-T_1}{k(T_A-T_2)+(k-1)T_A \log\dfrac{V_B}{V_A}}, \text{ for condition } (b),$$

$$\frac{\dfrac{1}{J}p_2V_2\log\dfrac{V_C}{V_2}-mc_v(T_4-T_1)}{\dfrac{1}{J}p_2V_2\log\dfrac{V_C}{V_2}}=1-\frac{c_vT_1\left\{\left(\dfrac{V_C}{V_2}\right)^{k-1}-1\right\}}{\dfrac{R}{J}T_2\log\dfrac{V_C}{V_2}},$$

for condition (*c*).

EXERCISE 129. How many B.t.u. per i.h.p. per hour must be supplied to an ideal engine operating on an Otto cycle?

EXERCISE 130. Compute the ideal efficiency of the Otto cycle when compression is carried to 15 atmospheres.

EXERCISE 131. It is claimed that an oil engine uses 0.42 pound of fuel (18,000 B.t.u. per pound) per kilowatt-hour delivered by the generator to which it is directly connected, that 20 per cent of the indicated power is lost between the cylinder and the bus bars, that the highest temperature in the cylinder does not exceed 2500° F, and that the exhaust temperature is 800° F. Are the above claims probable?

CHAPTER VI

THE SECOND LAW OF THERMODYNAMICS

SECTION XVIII

THE FIRST LAW

Conservation of Energy.—Experience and experimentation have always led to the conclusion that the total energy in a given space cannot be increased or diminished unless energy passes thru the boundary of the space considered. This principle is known as the law of conservation of energy. In accordance with this law the total energy of the universe is a fixed unalterable quantity.

Transformation of Energy.—Altho the total energy within a given space cannot be increased or diminished without introducing energy into or withdrawing energy from the given space, energy of one form may be transformed into energy of another form within this space without producing changes in the amount of energy residing outside of the space considered. Thus mechanical energy may be converted into electrical energy and under ideal conditions it is possible to conceive that all of the mechanical energy which disappears would reappear as electrical energy. Practically this complete conversion is impossible. Unavoidable losses always result in the transformation of some of the mechanical energy into other forms of energy, principally heat, with a corresponding decrease in the amount of electrical energy produced.

The First Law of Thermodynamics states that whenever heat is transformed into mechanical energy or mechanical energy is transformed into heat a definite quantity of mechanical energy reappears for every definite quantity of heat which disappears and vice versa. It has been experimentally determined that the complete transformation of 777.64 foot-pounds of mechanical energy always results in the appearance of 1 B.t.u. of heat.

It should be noted that the first law of thermodynamics does not specify how much of a given quantity of heat energy can be transformed into mechanical energy. It simply states that if any portion of, or if the whole of any given quantity of heat is transformed each heat unit that disappears will reappear as a definite number of units of mechanical energy.

Section XIX

THE SECOND LAW OF THERMODYNAMICS

This law has been stated in many ways. As the proof of this second law must be based upon experience and experimentation the simplest form of the law is the following: **Heat has never been known to flow of its own accord from a cold to a relatively hotter body;** that is, an expenditure of energy is always necessary to cause heat to flow from a body to another body of higher temperature.

The consequences of this law are of the greatest importance. It will be shown by means of this second law that heat can never be completely converted into mechanical energy, not even under ideal conditions; that the fraction which is convertible depends upon the available extreme temperatures; that this fraction depends only upon these temperatures and not upon the working substance with which the heat is associated; and thus that every transformation (even under ideal conditions) of heat into mechanical energy is accompanied by an unavoidable waste of some of the heat energy involved.

In order to demonstrate these consequences of the second law we must first carefully consider the nature of reversible and irreversible processes and cycles.

Reversible Processes and Cycles.—A process is said to be reversible when the changes involved may be retraced in reverse order, thus returning the substance undergoing change to its initial state, and at the same time leave no change of any kind in any associated bodies.

As an example, during an isothermal expansion of ideal gases not only does the state of the gas change from p_1, V_1, T_1 to p_2, V_2, T_2 but heat is supplied to the gas during the change and the work performed by the gas must be stored in some way. Thus two bodies associated with the gas during the change of state were altered one by being deprived of some of its heat and the other by having its store of mechanical energy increased.

If this isothermal process is reversed and the gas is compressed to its initial state, work must be performed on the gas and heat must be withdrawn from the gas. The necessary work may be obtained from the body in which the work performed during expansion was stored. The heat withdrawn from the gas may be returned to the body which acted as the original source of heat. Then after the gas has been returned to its initial state no changes will remain in any bodies associated with the gas during its changes of state, because all work performed during expansion must be used for compression and all heat borrowed during expansion will be returned during compression.

It is at once evident that the processes just described are ideal. Practically it is impossible to realize storage and redelivery of the work involved without some loss due to friction. Nor can heat be transferred from the source to the gas while both are at the same temperature or while they differ in temperature by an infinitesimal amount as they must during a reversible change of state. Practical, actually realizable processes are never reversible.

Under ideal conditions processes may be conceived to be reversible and any cycle composed wholly of reversible processes must also be reversible in the thermodynamic sense. Thus the Carnot cycle is reversible. The Stirling and the Ericsson cycles with perfect regeneration are also reversible. Without regeneration the isopiestic processes of the Stirling cycle and the isometric processes of the Ericsson cycle are irreversible. During these processes the gas is not at the same temperature as the body supplying or receiving heat and a reversal of the process would require the flow of heat from a cold gas to a relatively hotter body.

It must not be supposed that the Stirling cycle without regeneration cannot be traced in a counter-clockwise direction and the cycle used in a heat pump. This can be done but only after the conditions of operation have been changed so that during the isometric processes the thermal contact between the gas and the hot and the cold bodies are interchanged.

EXERCISE 132. Sketch a Stirling cycle without regeneration and indicate during which processes the gas is in thermal contact with the hot body and with the cold body during (*a*) the motor cycle, (*b*) the pump cycle. How do these conditions differ from the conditions existing when a regenerator is used?

The Joule cycle is also irreversible, see page 103. Any process involving the transfer of heat from one body to another at an appreciably lower temperature is irreversible. As conduction of heat is always present in all actual machines reversibility can never be attained in actual practice. It is an ideal condition which may be imagined but not realized. The assumption of ideal reversible processes and cycles is very useful in arriving at important deductions of the second law of thermodynamics.

Irreversible Processes and Cycles.—As another example of an irreversible process consider the flow of air in Joule's experiment devised to show that the change in the internal potential energy of a gas is zero, see page 10. This change of state involves conditions which have not yet been studied. The internal energy of the compressed gas is partly changed to kinetic energy of the moving mass of gas during its passage from the high-pressure to the low-pressure receiver. This kinetic energy is then reconverted into heat energy in the low-pressure receiver thru impact and internal friction. The initial and the final states of the gas are readily shown on the pV-plane but the intermediate states cannot be represented by an adiabatic $pV^k =$ a constant even tho no heat is supplied to or withdrawn from the gas during the process because the whole mass of gas is not always under the same pressure and temperature.

That this process is not a reversible adiabatic process such as has been studied becomes evident when we attempt to restore the gas to its initial condition without leaving changes of any kind in any associated bodies. The gas will of course not flow back into the high-pressure receiver of its own accord. It must be pumped back. Assume the compression to be isothermal then work must be supplied and the body supplying this work will lose this energy. Assume the compression to be reversibly adiabatic then again must work be supplied and more work than during isothermal compression. Under the last assumption the compressed gas will be hotter than in its initial state. It may be argued that this extra heat may be withdrawn from the compressed gas, transformed into work, and this work used to repay the loan of work made for compressing the gas. It will be shown that the work so obtained will be insufficient to fully repay the loan of energy. Under any conceivable conditions return to the initial state is only possible when changes of some kind remain in associated bodies. If this is so then the process is irreversible.

Carnot's Principle.—This principle is a consequence of the second law of thermodynamics. It may be stated as follows:

No heat motor can have a greater efficiency than the efficiency of a heat motor operating on a reversible cycle provided all motors operate between the same extreme temperatures.

To prove this principle conceive two heat motors (marked 1 and 2, Fig. 26) operating between the same hot and the same cold body. Let motor 1 operate on a reversible cycle and assume that the efficiency of motor 2 is greater than the efficiency of motor 1, so that,

$$\eta_2 > \eta_1.$$

If the principle we are to prove is true this assumption should lead to an impossible condition.

As shown in Fig. 26 when both motors are running as motors we may assume conditions such that each motor draws the same quantity of heat from the source of heat in the same time. Under the assumption $\eta_2 > \eta_1$ the heat transformed into mechanical energy by motor 2 must be greater than the mechanical energy delivered by motor 1 or

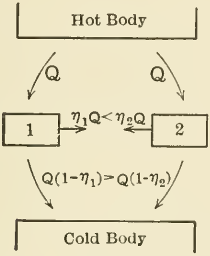

FIG. 26.

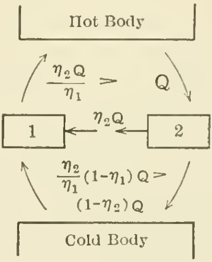

FIG. 27.

$$\eta_1 Q < \eta_2 Q.$$

Assume now that motor 1 is operated as a heat pump, Fig. 27, and that the whole mechanical energy obtained from motor 2 is used to drive the reversed motor 1. As this motor is able to transform into work η_1 B.t.u. per B.t.u. supplied when running as a motor it will deliver to the source of heat one B.t.u. for every η_1 B.t.u. supplied to it as work when it runs as a heat pump. Under our assumption motor 1 will receive $\eta_2 Q$ B.t.u. of mechanical energy and it must therefore reject to the hot body $\frac{\eta_2 Q}{\eta_1}$ B.t.u. But $\eta_2 > \eta_1$ by hypothesis. Therefore during the operation of the motors as shown in Fig. 27 more heat will be returned in a given time to the hot body by reversed motor 1 than motor 2 extracts from the hot body. Thus we have a self-contained system in which heat continuously flows from a cold to a relatively hotter body, an impossibility according to the second law of thermodynamics. We thus conclude that our hypothesis is impossible and that η_2 cannot be greater than η_1.

EXERCISE 133. Establish the inequalities indicated in Fig. 27.

The Consequences of the Second Law of Thermodynamics.—As has just been shown any heat motor operating on a reversible cycle has an efficiency at least as high as any other heat motor (reversible or not) utilizing the same hot and cold bodies as source and as receiver of heat.

Therefore all heat motors operating on reversible cycles must have the same efficiency.

Also, with a given hot body and a given cold body any heat motor operating on a reversible cycle is as efficient as any heat motor can possibly be, or no engine can convert a greater fraction of the heat energy supplied to it into mechanical energy than an engine operating on an ideal reversible cycle.

It should be noted that in the preceding discussion no mention is made of the working substance nor of the mode of action of the motor. Reversibility in the thermodynamic sense is the only stipulation. Thus any working substance passing thru any reversible cycle will transform into mechanical energy the greatest possible fraction of the heat supplied for any given conditions as regards the source and the receiver of heat.

We must now establish the efficiency of a reversible cycle in terms of the temperatures of the source and of the receiver of heat upon which alone this efficiency depends.

SECTION XX

KELVIN'S ABSOLUTE SCALE OF TEMPERATURE

Lord Kelvin established a scale of temperature which is independent of any particular substance and is in this sense absolute. That such a scale of temperature is necessary at least theoretically becomes evident when we remember that the readings of mercury thermometers differ with each other and with the readings of an air thermometer and that none of these agree with the reading of the standard hydrogen thermometer.

Kelvin arbitrarily defined temperature by means of the relation

$$\frac{\theta'}{\theta}=\frac{Q'}{Q},$$

where θ' and θ are the temperatures of a source and of a receiver of heat respectively and Q' and Q are the quantities of heat supplied by this source and rejected to this receiver by any heat motor operating on a reversible cycle between them.

As the efficiency of any heat motor is by definition

$$\eta = \frac{Q' - Q}{Q'},$$

and as $\frac{\theta'}{\theta} = \frac{Q'}{Q}$ or $\frac{Q}{\theta} = \frac{Q'}{\theta'} = r$, some constant,

we have $Q' = r\theta'$ and $Q = r\theta$,

so that $$\eta = \frac{\theta' - \theta}{\theta'}.$$

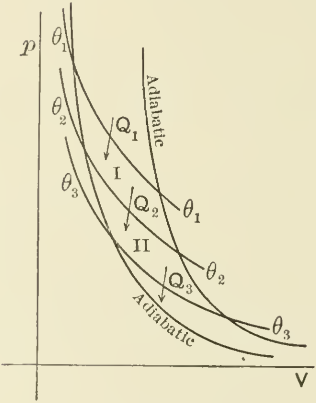

FIG. 28.

This is the efficiency of any and every ideal reversible cycle and it cannot be exceeded by any other cycle operating between the same source and the same receiver of heat. This expression is as yet meaningless. The significance of θ must be established. The scale of temperature must be determined.

The Carnot cycle is a typical reversible cycle. As shown in Fig. 28 this cycle may be represented by any two adiabatics and two isothermals. Let the cycle marked *I* be the cycle of a heat motor operating between the temperatures θ_1 and θ_2 and let Q_1 be the heat supplied at a temperature θ_1 and Q_2 be the heat rejected by the motor at a temperature θ_2. Further assume that the heat so rejected by motor *I* is at once supplied to another motor (whose cycle is marked *II*) at the temperature θ_2, and that this motor rejects Q_3 heat units at a temperature θ_3, and so on.

Under these conditions the efficiency of motor *I* is

$$\eta_1=\frac{Q_1-Q_2}{Q_1}=\frac{\theta_1-\theta_2}{\theta_1},$$

and for motor *II*

$$\eta_2=\frac{Q_2-Q_3}{Q_2}=\frac{\theta_2-\theta_3}{\theta_2}.$$

But, by definition,

$$\frac{\theta_1}{\theta_2}=\frac{Q_1}{Q_2} \quad \text{or} \quad \frac{Q_1}{\theta_1}=\frac{Q_2}{\theta_2},$$

and as θ_1, θ_2, θ_3 are to be consecutive points on a temperature scale the interval between θ_1 and θ_2 must equal the interval between θ_2 and θ_3 or

$$\theta_1-\theta_2=\theta_2-\theta_3.$$

Now from the expression for η_1 and η_2 we find

$$Q_1-Q_2=\frac{Q_1}{\theta_1}(\theta_1-\theta_2)$$

and

$$Q_2-Q_3=\frac{Q_2}{\theta_2}(\theta_2-\theta_3)$$

whence by reason of the above relations between the θ's it follows that

$$Q_1-Q_2=Q_2-Q_3.$$

Therefore the heat which disappears and which is converted into mechanical energy is the same in each of the ideal reversible motors when the temperatures of their sources and receivers of heat differ by the same number of degrees on Kelvin's absolute scale of temperature. Thus having assumed any two isothermals, Fig. 28, at say one degree apart the next isothermal another degree lower in temperature must be so located that the area of its cycle (marked *II*) must equal the area of the first cycle (marked *I*).

As each succeeding motor transforms a portion of the heat originally supplied to the first motor into mechanical energy and as the original supply of energy need not be infinite and as each motor transforms the same amount of heat as the preceding one, the heat will finally be wholly transformed. When this occurs the last motor must operate with a receiver at zero temperature and this will be an absolute zero. There is nothing beyond. The zero of Kelvin's scale is an absolute zero.

To put it in another way assume that in

$$\eta=\frac{\theta'-\theta}{\theta'}$$

θ becomes negative, a negative temperature on Kelvin's scale, then η becomes greater than one or the motor would deliver more mechanical energy than would be equivalent to the heat supplied to it. This would violate the law of conservation of energy and our assumption is impossible.

To establish a definite numerical value for Kelvin's scale of temperature assume θ_1 to correspond to 212° F and θ_2 to 32° F. Between θ_1 and θ_2 we have the following relations

$$\frac{\theta_1}{\theta_2}=\frac{Q_1}{Q_2}$$

and $$\theta_1-\theta_2=180.$$

Next assume a working substance which passes thru the ideal Carnot cycle. Any substance will do for all substances have the same efficiency. Let the substance be air. Now find by experimentation and calculation the value of $\frac{Q_1}{Q_2}$.* Its value is

$$\frac{Q_1}{Q_2}=1.3663.$$

Then as $$\frac{\theta_1}{\theta_2}=\frac{Q_1}{Q_2}=1.3663$$

and $$\theta_1-\theta_2=180,$$

we find that $$\theta_2=492^\circ \text{ F}.$$

Kelvin's scale for practical purposes is the same as the absolute scale already established on page 12.

The efficiency of all reversible cycles is therefore

$$\eta=\frac{T_H-T_C}{T_H}$$

where T_H is the absolute temperature of the source of heat and T_C is the absolute temperature of the receiver of heat.

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* Callendar: Phil. Mag. (6) 5, 48. 1903.

Section XXI

THE AVAILABILITY OF HEAT ENERGY

The availability of heat energy is measured by the fraction of the heat energy which can be transformed into mechanical energy under the best conceivable conditions. Thus the efficiency of any reversible cycle operating between the temperature of the given heat energy and the lowest obtainable temperature, i.e. the temperature of the coldest body which can be relied upon continuously to receive heat without increasing its temperature appreciably, is a measure of the availability of the heat supplied.

Thus never, even under the most ideal conditions which may be conceived altho not realized, can any given supply of heat be completely transformed into mechanical energy, for this would only become possible if a receiver of heat could be found whose temperature would remain permanently at absolute zero.

The degradation of heat energy is the term used to denote the idea that every transformation of heat energy into mechanical energy must be accompanied by an irretrievable loss, not of energy, but of transformable energy. All heat rejected to a receiver of heat whose temperature is the lowest obtainable is degraded; it can never be even partly transformed into mechanical energy. Such heat may be used for heating the receiver of heat but not for the production of mechanical energy.

The availability of heat energy for transformation into mechanical energy thus depends upon the fall in temperature between the source and the receiver of lowest obtainable temperature. It is measured by the efficiency of an ideal reversible cycle operating between the source and this receiver of heat.

Altho we can never attain this efficiency,

$$\eta=\frac{T_H-T_C}{T_H},$$

it is the standard by which all actual thermal efficiencies should be judged.

Exercise 134. What percentage of the heat supplied by a source at 600° F must necessarily be wasted if the lowest obtainable receiver temperature is 60° F?

Exercise 135. (*a*) The combustion of gases furnishes a source of heat with a temperature of 2500° F. Assuming that the lowest available temperature is 80° F and that the heat is supplied to a boiler and transmitted to steam at 380° F before being supplied to an engine, what is the best possible thermal efficiency of this engine?

(*b*) What would be the efficiency of an ideal engine capable of transforming the heat directly from the source?

Exercise 136. A steam engine requires 15,000 B.t.u. per i.h.p. hour. The temperature of the steam supplied to this engine is 300° F, and the temperature in the condenser is 130° F. What is the ratio of the thermal efficiency of this engine to the thermal efficiency of the best conceivable engine operating under the same conditions?

Three Types of "Perpetual Motion."—Perpetual motion of the first type would be realized by a machine which could create energy, a machine which gives us something for nothing. Any machine of this kind would violate the law of conservation of energy and is therefore impossible.

Perpetual motion of the second type is sought by inventors who attempt to design machines which are able to transform all of the heat energy supplied into mechanical energy without having a receiver at a temperature of absolute zero. In this case no attempt is made to obtain something for nothing and still the result is impossible of attainment for it would mean a violation of Carnot's principle and thus of the second law of thermodynamics.

Perpetual motion of the third type is embodied in an ideal mechanism which when once set in motion would continue in motion forever. A mechanism of this kind is not supposed to deliver energy and altho it is not theoretically impossible it could never actually be constructed.

CHAPTER VII

ENTROPY

SECTION XXII

INTRODUCTION

AT the outset it should be understood that no attempt is to be made in the following to explain what entropy is. Such matter would be out of place in a first course in thermodynamics. Nor is it necessary. We do not know what electricity or even gravitation is. It will be sufficient if we study the properties of and become familiar with entropy and above all learn to recognize its usefulness in our calculations.

The availability of Q heat units supplied at a temperature T_H when the lowest obtainable receiver temperature is T_C is

$$\eta=\frac{T_H-T_C}{T_H}=1-\frac{T_C}{T_H}.$$

During this ideal transformation of heat into mechanical energy $Q(1-\eta)$ heat units must be wasted or degraded,

and $$Q(1-\eta)=Q\frac{T_C}{T_H}=\left(\frac{Q}{T_H}\right)T_C.$$

The quantity $\frac{Q}{T_H}$ is called **the change in the entropy** of the source due to its loss of Q heat units at a constant temperature T_H.

The heat degraded during this ideal transformation of heat into mechanical energy may therefore be said to equal the entropy lost by the source $\left(\frac{Q}{T_H}\right)$ multiplied by the lowest obtainable receiver temperature T_C.

Whenever a body rejects heat its entropy is said to diminish, whenever a body receives heat its entropy is said to increase. The change in the entropy of a body is measured by the heat received (or rejected) divided by the absolute temperature of the body during this transfer of heat.

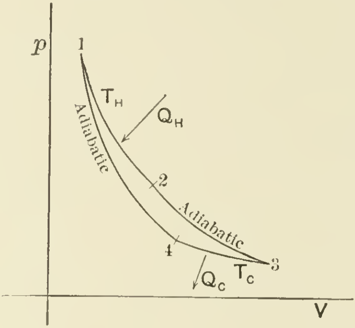

FIG. 29.

Let us study the changes of entropy which occur during the transfers and the transformation of heat during the operation of an ideal Carnot cycle, Fig. 29.

During the process 1 2 assume that the hot body rejects and the working substance receives Q_H heat units at an absolute temperature T_H. Then the entropy of the hot body is diminished by $\frac{Q_H}{T_H}$ and the entropy of the working substance is increased by $\frac{Q_H}{T_H}$. Similarly if the working substance rejects Q_C heat units to the cold body at a temperature T_C the entropy of the working substance is thereby diminished by $\frac{Q_C}{T_C}$ while the entropy of the cold body is increased by $\frac{Q_C}{T_C}$.

But $$\eta = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H},$$

so that $$\frac{Q_C}{T_C} = \frac{Q_H}{T_H},$$

see also page 123.

As no heat is transferred during the processes 2 3 and 4 1 no change in the entropy of either the working substance or of any associated body occurs. Finally during the operation of the cycle as a whole the entropy of the working substance returns periodically to its initial value and altho the entropy of the hot body continually diminishes and that of the cold body continually increases the entropy of the whole system remains constant.

Consider now a Carnot cycle in which owing to conduction the temperature of the working substance during the process 1 2 (Fig. 29) is T_1, a constant, where $T_1 < T_H$ and the temperature along the process 3 4 is T_2 where $T_2 > T_C$. Then if Q_H is the heat supplied by the hot body and Q_2 is the heat rejected to the cold body, we find that

the entropy lost by the hot body $= \frac{Q_H}{T_H}$,

the entropy gained by the working substance $= \frac{Q_H}{T_1}$, the entropy lost by the working substance $= \dfrac{Q_2}{T_2}$,

the entropy gained by the cold body $= \dfrac{Q_2}{T_C}$,

and that these are the only changes in entropy during one complete cycle.

As the efficiency of this cycle is less than the efficiency of the ideal cycle considered in Fig. 29

$$Q_H - Q_C > Q_H - Q_2,$$

or

$$Q_C < Q_2,$$

and as $T_1 < T_H$ and $T_2 > T_C$ by hypothesis,

$$\frac{Q_H}{T_H} < \frac{Q_H}{T_1} \quad \text{and} \quad \frac{Q_2}{T_2} < \frac{Q_2}{T_C}.$$

Also

$$\frac{Q_H}{T_1} = \frac{Q_2}{T_2}, \text{ see page 135.}$$

Therefore while the entropy of the working substance remains unchanged the entropy of the whole system now increases.

Moreover this cycle is not reversible. It is impossible, without aid from a source external to the system considered, to return the heat supplied by the hot body to it by means of the work obtained during the direct action of the cycle.

The above examples illustrate the fact that during the operation of reversible cycles the entropy of the system remains constant but that during the changes occurring in an irreversible cycle the entropy of the system increases.

As all actual transfers of heat involve conduction we may infer that the entropy of the universe is continually increasing.

The actual entropy of any body at any time is never computed nor need it be known. The changes in entropy of a substance due to a transfer of heat is however very useful. It is computed by means of the ratio

$$\frac{\text{heat received or rejected by the body}}{\text{the absolute temperature of the body during the transfer of heat}}.$$

Let S represent the entropy of a body receiving ΔQ heat units while the absolute temperature of the body remains T then

$$\Delta S=\frac{\Delta Q}{T}.$$

If the temperature of the body changes during the heat transfer then

$$dS=\frac{dQ}{T},$$

for during this transfer of dQ heat units the temperature of the body remains T.

A change in entropy will be denoted by dS if the mass of the substance is m pounds and by ds if the mass is one pound, so that

$$dS=mds.$$

The entropy of a body may be used as a coordinate to represent the state of the body. The change in the entropy of a body depends only upon the initial and the final states of the body and not upon the manner in which the change of state occurred.

To demonstrate this for an ideal gas note that

$$dQ = mc_v dT + \frac{1}{J} p dV,$$

and that by definition

$$dS = \frac{dQ}{T},$$

so that
$$dS = mc_v \frac{dT}{T} + \frac{1}{J} \frac{p dV}{T}.$$

From this equation it appears that the change in entropy dS depends upon T and pdV in addition to the c_v of the gas. As pdV represents the external work during the change of state considered and must vary with the processes involved it would seem that ds must also vary with these processes. But this is not so.

As
$$pV = mRT$$

replace p by $\frac{mRT}{V}$ and note that

$$dS = mc_v \frac{dT}{T} + m \frac{R}{J} \frac{dV}{V}.$$

This equation shows that the change in entropy depends upon the temperature and the volume of the gas as well as upon the change in temperature and the change in volume but as it can be integrated without knowing the relation between T and V the entropy does not depend upon the manner in which either T or V change.

Thus when the gas attains its new state its entropy reaches a definite value which is absolutely independent of what may have happened to the gas during its change of state.

The difference in this respect between a change in entropy and say the heat supplied during a change of state should be carefully noted. In changing from one state to another every particular process followed will require the addition of a definite amount of heat different in each case. Thus Q cannot be used as a coordinate to determine the final state of the gas. With entropy only one definite change in entropy will be found during any change of state provided the same final state is reached.

Thus instead of three coordinates to determine any particular state of an ideal gas as illustrated graphically in Fig. 2 we may now use four, i.e., p, V, T, and S. This does not mean that all four must be used simultaneously. In fact we have already explained that only two of the three coordinates p, V, and T are usually used; instead of the three-dimensional representation the two-dimensional representation of changes of state on the pV-plane is always preferred.

With the introduction of entropy we may use in addition to the planes of pV, pT, and TV, other planes such as pS, VS, TS. Of these new planes of projection it will be shown that the TS-plane is the most important.

SECTION XXIII

CHANGES IN ENTROPY OF IDEAL GASES DURING REVERSIBLE PROCESSES

Changes in Entropy during Reversible and Irreversible Processes.—It must be remembered that changes in the entropy of a substance may occur even when no heat is supplied to the substance from external sources.

All heat generated within the substance itself will cause a change in the entropy of the substance. To illustrate, consider a mass of gas flowing thru a pipe. Friction will cause some of the kinetic energy to be transformed into heat. Thus even tho the gas be thoroly insulated so that no heat may reach it from without its entropy will nevertheless increase. This is an example of an irreversible adiabatic change of state during which the entropy is not constant even tho dQ is zero. These cases will be considered more thoroly under the head of flow of fluids. It is at present sufficient to call attention to the fact that what follows applies only to reversible changes during which thermal equilibrium exists and all parts of the mass considered are always at the same temperature. Neglect of this precaution will lead to serious errors in the use of entropy.

Changes in the Entropy of an Ideal Gas during Reversible Processes.—Under these conditions as

$$dQ = mc_v dT + \frac{1}{J} p dV$$

we have
$$dS = \frac{dQ}{T} = mc_v \frac{dT}{T} + \frac{1}{J} \frac{pdV}{T}.$$

To integrate this equation either p or V must be eliminated by means of

$$pV = mRT.$$

Eliminating p we have

$$dS = mc_v \frac{dT}{T} + \frac{mR}{J} \frac{dV}{V},$$

whence
$$\begin{aligned} S &= mc_v \log_\epsilon T + m(c_p - c_v) \log_\epsilon V + C \\ &= mc_v \log_\epsilon (TV^{k-1}) + C. \end{aligned}$$

Assuming that the entropy changes from S_1 to S_2 while p, V, and T change from p_1, V_1, T_1 to p_2, V_2, T_2 we have

$$S_1 = mc_v \log T_1 V_1^{k-1} + C$$

and

$$S_2 = mc_v \log T_2 V_2^{k-1} + C$$

or the change in entropy

$$S_2 - S_1 = mc_v \log \left(\frac{T_2}{T_1}\right)\left(\frac{V_2}{V_1}\right)^{k-1}.$$

Exercise 137. Show that the entropy, S, of an ideal gas may be expressed in the following forms

$$S = mc_v \log (pV^k) + C_1$$

$$S = mc_v \log (TV^{k-1}) + C_2$$

$$S = mc_p \log (Tp^{\frac{1-k}{k}}) + C_3.$$

Exercise 138. Show by means of the equations of Exercise 26 that for reversible processes we may write the change in entropy due to a change of state from p_1, V_1, T_1, S_1 to p, V, T, S in the following forms

$$S - S_1 = mc_v \log \frac{T}{T_1} + m(c_p - c_v) \log \frac{V}{V_1},$$

$$S - S_1 = mc_p \log \frac{T}{T_1} - m(c_p - c_v) \log \frac{p}{p_1},$$

$$S - S_1 = mc_v \log \frac{p}{p_1} + mc_p \log \frac{V}{V_1}.$$

Exercise 139. Deduce the results of Exercise 138 from the results of Exercise 137 and transform them into the last form given in the illustrative example above.

The Temperature-Entropy Diagram.—During a reversible process

$$dQ = TdS.$$

Therefore the area under a line representing any reversible process on the TS-plane such as 1 2, Fig. 30, represents the heat supplied to the mass undergoing the change of state from 1 to 2 in accordance with the indicated process. If the process changes, the line 1 2 (Fig. 30) must change and the heat supplied must change but the change of entropy and position of point 2 remain unchanged.

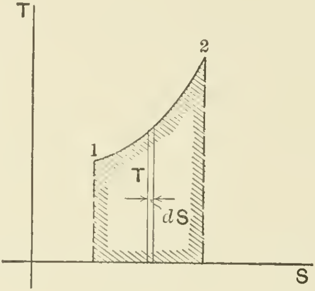

FIG. 30.

This graphical representation of heat supplied on the TS-plane is analogous to the graphical representation of work done on the pV-plane.

The area enclosed between the lines representing upon the TS-plane the processes of a reversible cycle represents the heat transformed into work during this cycle.

Reversible Isopiestic Processes on the TS-plane.—Let p_1, v_1, T_1, s_1 represent the initial conditions of one pound of gas and p_1, v, T, s any subsequent condition of this same gas after a reversible isopiestic process. This process is represented graphically on the pv-plane in Fig. 31. We must now transfer this representation to the Ts-plane, Fig. 32.

As
$$ds = \frac{dq}{T}$$
and as
$$dq = c_p dT$$
$$ds = \frac{c_p dT}{T}$$
and
$$s - s_1 = c_p \log_\epsilon \frac{T}{T_1}.$$

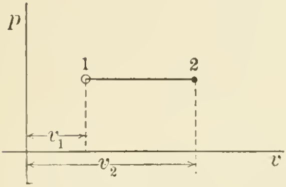

FIG. 31.

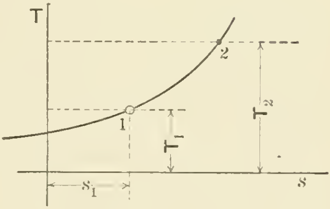

FIG. 32.

By means of $pv = RT$, T_1 may be computed and the horizontal line on which point 1 lies in Fig. 32 may be found. The value of s_1 may now be arbitrarily assumed for there is no absolute zero of entropy such as exists for pressure, volume, and temperature. In practice the entropy of the substance at the lowest temperature occurring in any calculation may conveniently be assumed to be zero. This would place point 1 in Fig. 32 on the axis of T.

T may also be computed by means of $pv = RT$. But s cannot be arbitrarily assumed after s_1 is fixed. As

$$s = s_1 + c_p \log_\epsilon \frac{T}{T_1},$$

the point 2 on the Ts-plane lies upon a logarithmic curve passing thru point 1.

Exercise 140. One pound of air expands at a constant pressure of 100 pounds per square inch absolute from a volume of 1.9 cubic feet to 3 cubic feet.

(*a*) Compute the change in entropy of this air.

(*b*) Compute the final entropy of this air.

The relation between any two isopiestic curves on the Ts-plane is important. The equation of any isopiestic curve is

$$s-s_1=c_p \log_\epsilon \frac{T}{T_1}.$$

By differentiation we obtain

$$\frac{dT}{ds}=\frac{T}{c_p}.$$

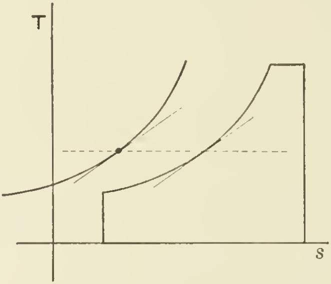

Fig. 33.

Therefore at any given temperature all isopiestic curves for a given gas have the same slope, $\frac{dT}{ds}$, Fig. 33.

As an application of this fact suppose a series of isopiestics on the Ts-plane are required. Then one such curve may be accurately plotted and a templet of the form indicated in Fig. 33 may be cut. This templet sliding along a straight edge coinciding with the s-axis will serve to draw all the required isopiestics.

EXERCISE 141. Compute the distance, measured on a line parallel to the s-axis, between two isopiestic curves for pressures p_1 and p_2 and show that this distance is independent of the isothermal along which it is measured.

EXERCISE 142. Isopiestics for 0, 10, 20, etc. pounds per square inch absolute are to be drawn on a Ts-plane. Are the distances between these isopiestics measured along any given isothermal equal?

Reversible Isometric Processes on the TS-plane.— It can be readily shown that the change in entropy during a reversible isometric process is

$$s-s_1=c_v \log \frac{T}{T_1}.$$

Therefore the isometrics on the Ts-plane are also logarithmic curves but their slopes are

$$\frac{dT}{ds}=\frac{T}{c_v}.$$

As $c_p>c_v$ for any ideal gas **isometrics are steeper than isopiestics** on the Ts-plane.

EXERCISE 143. Two pounds of air are confined in a receiver (capacity 10 cubic feet) at a temperature of 80° F. The pressure increases to 70 pounds per square inch absolute. Compute the increase in the entropy of this air during this change of state.

EXERCISE 144. Sketch a network of isopiestics and isometrics on both the pv- and the Ts-planes. Pay due attention to the slopes and the relative positions of the curves on the Ts-plane.

Reversible Isothermal, Adiabatic, and Polytropic Processes on the TS-plane.—Reversible isothermals are evidently represented by straight lines parallel to the s-axis. Reversible adiabatics for which $dQ=0$ and no heat is generated within the mass considered are processes for which $ds=0$ so that the entropy is constant. These are represented by straight lines parallel to the T-axis.

Exercise 145. Show that the equation of the Ts-curve representing any reversible polytropic process is

$$s-s_1=\left(\frac{n-k}{n-1}c_v\right)\log_\epsilon\frac{T}{T_1}.$$

From the last exercise it follows that polytropics are also logarithmic curves on the Ts-plane. Note that isometrics, isopiestics, and the general polytropic curves all have the equation

$$s-s_1=c\log_\epsilon\frac{T}{T_1},$$

the proper value of c to be introduced in each case.

The Temperature-Entropy Plane.—The preceding discussions and exercises lead to the results illustrated in Fig. 34. This figure should be compared with the corresponding diagram for the pv-plane, Fig. 4.

Note that isometrics and isopiestics are straight lines on the pv-plane but logarithmic lines on the Ts-plane. Isothermals and adiabatics are curves on the pv-plane but become straight lines on the Ts-plane.

Exercise 146. The state of an ideal gas changes from p_1, v_1, T_1, s_1 to p_2, v_2, T_2, s_2 in a reversible manner, assume that the change occurs in two steps, the first being

(*a*) adiabatic and the second isometric,
(*b*) isothermal and the second isometric,
(*c*) isopiestic and the second isometric,
(*d*) isometric and the second isopiestic.

Sketch these changes on the pv-plane and note the relative amounts of external work performed.

EXERCISE 147. Sketch the processes described in Exercise 146 on the Ts-plane and note the relative amounts of heat that must be supplied to produce these changes of state.

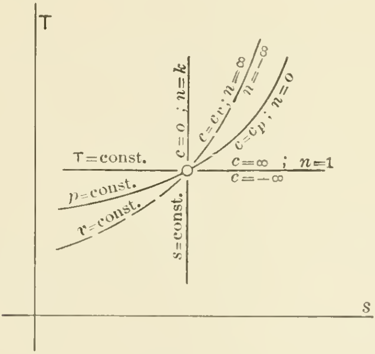

FIG. 34.

EXERCISE 148. Compute the changes in entropy during the processes described in Exercise 146 and show that the total change in entropy is the same in each case.

EXERCISE 149. Deduce the relation between p and v, v and T, and T and p for a reversible adiabatic process by means of the equations in Exercise 138.

The Logarithm Temperature-Entropy Diagram.—We have found that polytropic processes are represented upon the pv-plane by curves whose equations are $pv^n=C$. These curves are in general difficult to plot. Their use does not lead to simple graphical methods for the solution of problems.

On the Ts-plane polytropics are represented by logarithmic curves. Also all polytropic curves having the same exponent n may be drawn by means of the same templet (see page 142). This permits graphical calculations to be performed more easily on the Ts-plane than on the pv-plane. But it requires a templet for each polytropic.

For all graphical calculations it is very desirable to have to deal only with straight lines. All curves can be represented by straight lines by properly selecting the scales used on the coordinate axes.

The equation

$$s-s_1=c\log_\epsilon\frac{T}{T_1}$$

representing polytropic change on the Ts-plane is represented by the logarithmic curve if the decimal scales are used on both axes (Fig. 33). If however we rewrite the equation as follows

$$s=c\log_\epsilon T+(s_1-c\log_\epsilon T_1)$$

and put $$\log_\epsilon T=z,$$

we obtain $$s=cz+(s_1-c\log_\epsilon T_1),$$

the equation of a straight line when uniform decimal scales are used on the s and the z axes.

The scale on the z-axis would thus appear as shown in Fig. 35 (a). The reading of this scale, in terms of z is easy, in terms of T it is impossible for these intervals are non-uniform. As T is required and not z and in order to avoid reading z and then referring to a table of natural logarithms for the corresponding value of T as must be done when scale (*a*) is used, the scale shown in Fig. 35 (*b*) is used.

Paper ruled horizontally with the scale shown in Fig. 35 (*b*) and vertically with a uniform decimal scale can thus be used to plot any polytropic on the *Ts*-plane as a straight line.

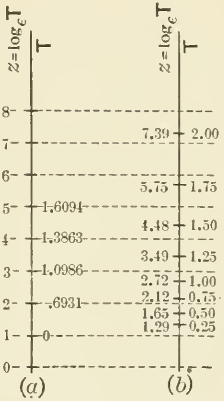

FIG. 35.

EXERCISE 150. If 100° F is represented by a point 1 inch above 0° F on the log T-axis how far below the point representing 0° F should $-200°$ F be located?

Where would the absolute zero be located?

For diagrams ruled as above described and which may be used for the graphical solution of problems relating to air see the diagrams accompanying Bulletin No. 63 of the Engineering Experiment Station at the University of Illinois.

SECTION XXIV

GAS CYCLES ON THE TS-PLANE

As all reversible processes can be represented on the *TS*-plane any cycle consisting of such processes can also be shown on this plane. The area included between the lines representing the processes represents the heat transformed into work during the cycle.

In sketching any cycle on the TS-plane care should be taken to note whether the temperature rises or falls, or whether heat is supplied to or rejected by the gas during each process as it occurs in the cycle.

EXERCISE 151. (*a*) Sketch a Carnot, a Stirling, an Ericsson, a Joule, an Otto, and a Diesel cycle on the pV-plane.

(*b*) Sketch these cycles on the TS-plane and number the points so as to correspond with the numbers on the pV-diagrams drawn in (*a*).

EXERCISE 152. Compute the efficiency of a Carnot cycle from the areas of its TS-diagram.

EXERCISE 153. Sketch superimposed TS-diagrams of a Carnot, a Stirling, and an Ericsson cycle (the last two when regenerators are used). Assume these cycles to operate between the same limiting temperatures and assume the gas to receive the same quantity of heat from the source in each case.

Show by means of these diagrams that the efficiencies of these cycles are the same.

EXERCISE 154. Compute the efficiency of the Otto cycle from the areas of its TS-diagram, assume adiabatic expansion and compression.

VAPORS

CHAPTER VIII

INTRODUCTION

SECTION XXV

PROPERTIES OF VAPORS

THE study of vapors can best be begun by an analysis of the behavior of water under varying conditions of pressure, temperature, and volume.

It has been determined experimentally that the pressures exerted by water-vapor at certain temperatures are those given in the following table. The last column gives the volume in cubic feet occupied by one pound of liquid water at the temperature given in the first column.

PROPERTIES OF WATER

Temperature, Deg. F.	Pressure, Pounds per Square Inch Absolute.	Specific Volume, Cubic Feet per Pound.
30	0.08	0.01602
120	1.69	0.01620
210	14.13	0.01670
300	67.00	0.01744
400	247.	0.0187
500	684.	0.0206
600	1574.	0.024

When the temperature of and the pressure on the water are those given in any line of this table and provision is made for expansion at this constant pressure then any addition of heat will cause vapor to form but no change in pressure or in temperature occurs.

To more clearly understand the nature of these changes it is useful to conceive the water to be confined as shown in Fig. 36. Here the only pressure exerted upon the water is that due to the heavy freely moving piston.

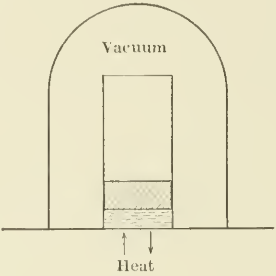

Fig. 36.

To fix our ideas let the mass of water be one pound and the weight of the piston be such as to exert a force of say 67 pounds per square inch upon this water. Then provided the temperature of the water is less than 300° F liquid water alone exists under the piston; no water-vapor is present. When the temperature reaches 300° F (due to the absorption of heat) vapor forms and more vapor forms as more heat is supplied but as long as liquid water is present the temperature remains 300° F and of course the pressure remains 67 pounds per square inch absolute.

Consider now the changes in volume involved in the above specific case. According to our table the volume of one pound of water at 300° F is 0.01744 cubic foot. Altho the pressure 67 is given in the same line of the table this does not mean that this must be the pressure exerted on the water when its specific volume is 0.01744. Experiment shows that water is practically incompressible (0.0000469 of one unit change in unit volume per atmosphere). Thus the volume of one pound of water at 300° F is 0.01744 cubic foot under any pressure likely to occur in engineering practice.

EXERCISE 155. (*a*) What is the specific volume of water at 32° F when under a pressure of 247 pounds per square inch absolute?

(*b*) What will be the specific volume of water under the same pressure when vapor forms?

Thus if in Fig. 36 we start with one pound of water at 32° F its volume will be 0.01602 cubic foot under a pressure of 67 pounds. As we heat this water under constant pressure its volume gradually increases to 0.01744 and its temperature to 300° F. Vapor now forms and the volume of the liquid and vapor rapidly increases until the liquid disappears and only (dry saturated) vapor at 300° F remains. The volume now is 6.47 cubic feet as set down in the steam tables. If more heat is supplied the temperature of the vapor rises and its volume increases. Some volumes and the corresponding temperatures of the now superheated vapor, always under a pressure of 67 pounds per square inch, are

6.99 cubic feet at 350° F

7.49 cubic feet at 400° F

8.45 cubic feet at 500° F

9.38 cubic feet at 600° F.

The above described changes of state may be represented graphically on the *pv*-plane as shown in Fig. 37. In this figure the names of the physical states of the substance have been added.

The point *A* at which the formation of vapor starts, the point separating the liquid from the liquid and vapor condition, lies on what is called the **liquid line.** The point *B* at which the liquid ceases to exist and at which the vapor is dry saturated lies on what is called the **saturation line.** The point *B* separates the saturated condition of the vapor from the superheated condition.

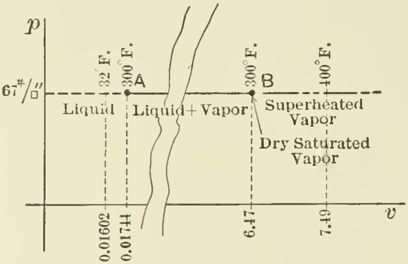

FIG. 37.

EXERCISE 156. Plot, by means of the steam tables, the saturation and the liquid lines for water on the *pv*-plane.

Plot the same lines on the *pt*-plane.

SECTION XXVI

THE CHARACTERISTIC SURFACE OF VAPORS

In order to illustrate the difference between the behavior of a vapor and an ideal gas the general nature of the characteristic surface of a vapor plotted in three dimensions on axes of p, v, and T is shown in Fig. 38. This figure does not represent any particular vapor nor is it drawn to scale.

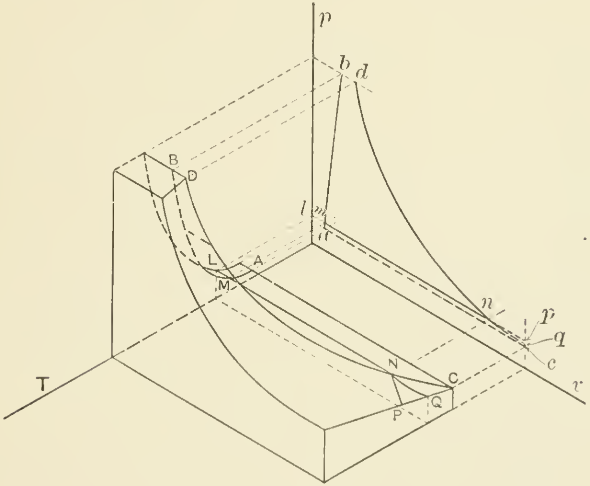

FIG. 38.

The portion of the characteristic surface between the liquid line AB and the saturation line CD is cylindrical. Its elements are parallel to the v-axis. This illustrates the condition noted above, that at constant pressure increase in volume occurs at constant temperature as long as some liquid is present.

To fix our ideas let us follow the point representing the state of say one pound of water while it is heated from 32° F under a constant pressure. At 32° F the volume of this water will be less than *LM* (or *lm*) Fig. 38, its state-point thus lies in the horizontal plane thru *LM* but nearer the *pv*-plane than *LM*. As the water is heated its volume and its temperature both increase until the temperature reaches the value corresponding to the assumed constant pressure as given in the steam tables, then the volume of the water equals *LM* and the state-point is at *M*. Vapor now begins to form. The volume of the water and its vapor rapidly increase at constant temperature and constant pressure and the state-point moves from *M* to *N*. At *N* the volume of the dry saturated vapor is that given in the steam tables for the given pressure and its saturation temperature. The supply of more heat now superheats the vapor and the state-point leaves the cylindrical surface upon which it moved during the formation of the vapor. It now follows some curve, such as *NP*, lying in a horizontal plane. Volume and temperature now both increase; corresponding values are given in the tables for superheated steam.

The portion of the characteristic surface upon which the state-point for the superheated condition of the vapor lies approximates a hyperbolic paraboloid (Fig. 2) more and more closely as the corresponding condition of the vapor recedes more and more from the saturated condition. This simply means that very highly superheated vapors may be assumed to obey the laws of ideal gases.

In fact actual gases under low temperatures and high pressures pass thru the state of superheated vapor to the wet vapor and the liquid states. The distinction between vapors and gases is thus one of degree, highly superheated vapors are gases and highly compressed and cooled gases are vapors.

The equation of the characteristic surface of superheated vapors is usually given in the form of a corrected equation of the characteristic surface of an ideal gas in the form

$$pv=RT-B,$$

or

$$p(v-b)=RT-B.$$

Here B is the correction term which indicates the deviation of the superheated vapor from the corresponding ideal gas, and b represents the least volume (or co-volume as it has been called) of the substance. This co-volume is practically the volume of the substance in the liquid state at low temperature; compared to the volume of the superheated vapor or gas it is very small.

As examples, Zeuner for steam put $B=Cp^{\frac{1}{4}}$, where C is a constant, in the first equation. Callendar proposed the equation

$$p(v-b)=RT-\frac{mp}{T^{3.5}},$$

where m is a constant for all vapors.

Linde proposed for superheated steam

$$pv=0.5962T-p(1+0.0014p)\left(\frac{150,300,000}{T^3}-0.0833\right),$$

where p is in pounds per square inch absolute, v is in cubic feet, and $T = t + 459.6$, the absolute Fahrenheit temperature, as an equation expressing experimentally determined facts.

Exercise 157. According to Linde's equation for superheated steam (*a*) what value of R for steam corresponds to 53.3 for air? (*b*) Does steam become an ideal gas when sufficiently superheated?

Section XXVII

ISOTHERMALS

If the temperature of a vapor is maintained constant while its volume is diminished the changes in volume and pressure are related as illustrated by the line *abcd* in Fig. 39.

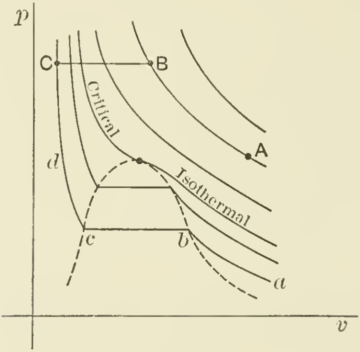

Fig. 39.

In the superheated region the pressure rises with decreasing volume, *ab*. Saturation is reached at *b*. Further decrease in volume does not cause increase in pressure but condensation of vapor. The vapor becomes wetter and wetter as compression continues until all vapor has been condensed to liquid, at *c*. During the change of state *bc* the vapor is continually saturated. After the vapor has wholly disappeared decrease in volume is accompanied by great increase in pressure always at constant temperature.

Isothermals for higher temperatures have the general form shown in Fig. 39. As already indicated in Fig. 38 the liquid and the saturation lines approach each other as the pressure increases. This means that some isothermal will exist for which the horizontal, straight portion is replaced by a point of inflection, the point at which the liquid and the saturation lines meet. This point is called the **critical point** and the corresponding pressure, temperature, and specific volume are the **critical pressure,** the **critical temperature,** and the **critical volume** of the particular gas considered.

Provided the temperature of any substance remains above its critical temperature, i.e., provided its state-point lies above and to the right of its critical isothermal (Fig. 39) the substance cannot be liquefied.

A gas may even be compressed into a liquid without ever condensing any part of it. This may be done as follows. Referring to Fig. 39, assume the gas heated well above its critical temperature (*A*), compress it at this temperature (*AB*). Under the conditions represented by *B* the substance is still a gas as its temperature is still above its critical temperature. Next cool the substance at constant pressure until the temperature falls below its critical temperature (*BC*). The gas has thus been reduced to the liquid state without passing thru the conditions of dry saturated and wet vapor.

Exercise 158. Indicate in Fig. 39 the region in which the state-point must lie if the substance is to

(*a*) be wholly liquid,
(*b*) be partly liquid,
(*c*) exist as a vapor,
(*d*) exist as a superheated vapor,
(*e*) exist in the gaseous state.

The Equation of Van der Waals.—The continuity of the properties of a substance in gaseous and liquid states indicated above led van der Waals to apply his equation

$$\left(p+\frac{a}{v^2}\right)(v-b)=RT,$$

to the liquid as well as the gaseous states of substances, altho this equation was originally designed to represent the deviation of actual gases from the ideal conditions represented by $pv=RT$.

In van der Waals' equation b represents the least volume of the gas and a is a measure of the attraction of the molecules. R, a, and b are usually considered constant altho a and b most likely vary with both temperature and pressure.

Van der Waals' equation is an attempt to represent in one equation the surfaces of the saturated and of the superheated vapors in Fig. 38 together with the extension of the latter surface into the gaseous region.

If in $\left(p+\frac{a}{v^2}\right)(v-b)=RT$, T is made constant the equation of the pv-projection of this isothermal is obtained. If in addition p is made constant the intersection of this isothermal on the pv-plane with the constant pressure line on the same plane is obtained. Under these conditions we have a cubic equation in v and the three roots of this equation give the volumes corresponding to the points A, B, and C, Fig. 40. Points A and C lie upon the liquid and the saturation lines respectively while B has only an indirect physical interpretation. When the three roots of the cubic equation are equal then the points A, B, and C coincide with each other and with the critical point of the substance. For any temperature higher than the temperature which causes A, B, and C to coincide the cubic equation will have only one real root.

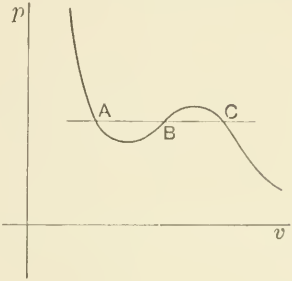

FIG. 40.

EXERCISE 159. Show that the critical temperature T_c, the critical pressure p_c, and the critical volume v_c as computed from van der Waals' equation are

$$T_c=\frac{8}{27}\frac{a}{bR}, \quad p_c=\frac{1}{27}\frac{a}{b^2}, \quad v_c=3b.$$

Van der Waals' equation does not completely represent the changes of state of a substance when a and b are considered constant not only on account of the curve between A and C, Fig. 40, but also because no second jog occurs in the isothermal at such points as would represent the change from the liquid to the solid states.

CHAPTER IX

FORMATION OF VAPORS AT CONSTANT PRESSURE

SECTION XXVIII

DRY SATURATED VAPORS

Heat Required to Warm the Liquid.—It is usual to assume the heat of the substance to be zero at 32° F. Referring to Fig. 36 assume the liquid under the piston to be at 32° F and under any pressure, p pounds per square foot. Then the heat required to warm one pound of liquid from 32° F to any temperature t° F, less than the temperature corresponding to the pressure as given in the vapor tables is

$$q=\int_{32^\circ}^{t} c'dt,$$

where c' is the specific heat of the liquid.

This specific heat is the specific heat at constant pressure and it varies with the temperature, but not appreciably with the pressure.

The heat of the liquid is defined as the heat which must be added to one pound of the liquid in order to change its temperature from 32° F to the temperature t' at which vapor forms under the constant pressure at which the liquid is heated.

Therefore
$$q'=\int_{32^\circ}^{t'} c'dt.$$

In what follows a prime (′) affixed to a letter always denotes a condition applying to liquids and especially to the conditions along the liquid line. A double prime (″) always denotes conditions along the saturation line.

In this case q' is the heat added to change the temperature of the liquid from 32° to the temperature at the liquid line, t', or which is the same thing the temperature for the same pressure at the saturation line, t''.

For approximate calculations the specific heat of water may be assumed constant and equal to unity. Under this assumption

$$q' = t'' - 32.$$

For accurate results the variation of the specific heat of water must be recognized.

At	40°	80°	120°	160°	200°	400°	600°
c' equals	1.0045	0.9970	0.9974	1.0002	1.0039	1.064	1.172

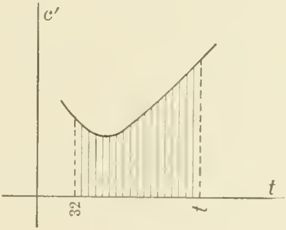

FIG. 41.

In order to integrate the equation

$$q' = \int_{32^\circ}^{t''} c' dt$$

either the variation of c' must be expressed as a function of t and the integral evaluated or c' must be plotted with respect to t, as shown in Fig. 41, and the area under the curve must be determined from the plot.

EXERCISE 160. By means of the steam tables, find the difference between $(t''-32)$ and $\int_{32}^{t''} c'dt$ for water heated at 0.505, 5, 14.7, 100, and 300 pounds per square inch absolute.

The fact that at 14.7 pounds or 212° F the values of $t''-32$ and $\int_{32}^{t''} c'dt$ are alike is due to the definition of the mean B.t.u. used. It is the $\frac{1}{180}$ part of the heat required to warm one pound of water from 32 to 212° F under a pressure of one standard atmosphere.

The Total Heat of Dry Saturated Vapor.—This is defined as the heat required to transform at constant pressure one pound of liquid, originally at 32° F and under the pressure at which the vapor is to form, into dry saturated vapor. It will be denoted by q''. Thus referring to Fig. 36, we start with one pound of liquid at 32° F under the piston which produces the given pressure at which the vapor is to be formed. When q'' heat units have been supplied then the space below the piston contains dry saturated vapor.

For water Regnault found as the result of his experiments that

$$q''=1091.7+0.305(t''-32),$$

see also page 164.

The Latent Heat of Vaporization.—This is defined as the heat required in order to change one pound of liquid at the saturation temperature corresponding to the constant pressure into dry saturated vapor at the same temperature. Denoting the latent heat of vaporization by r we have

$$r=q''-q'.$$

For water, according to Regnault,

$$r = 1091.7 - 0.695(t'' - 32).$$

External and Internal Latent Heat.—During the absorption of the latent heat of vaporization a large increase in volume occurs. Therefore external work must be done in overcoming the constant pressure. The volume of one pound of the liquid at saturation temperature is v', the volume of one pound of the dry saturated vapor at the same temperature is v'', the constant pressure is p, and the external work done equals $p(v''-v')$ foot-pounds which is equivalent to

$$\frac{p(v''-v')}{J} \text{ B.t.u.}$$

Thus of the r B.t.u. supplied during vaporization only

$$r - \frac{p(v''-v')}{J} = \rho \text{ B.t.u.}$$

remain as increase in internal energy within the vapor. This is the **internal latent heat.**

$\frac{p(v''-v')}{J}$ is called the **external latent heat**, this represents energy stored without the vapor.

The Internal Energy.—The internal energy of a substance, that is, the increase of the internal energy over its value at 32° F, will be denoted by u.

For the liquid at saturation temperature the heat supplied is q'. The increase in the specific volume is $v' - v_{32}$, where v_{32} is the specific volume of the liquid at 32° F which for water equals 0.01602. Therefore if q' is supplied at a constant pressure p the increase in the internal energy is

$$u'=q'-\frac{p(v'-v_{32})}{J}.$$

The last term is relatively small and is usually neglected. Under these assumptions

$$u'=q'$$

or it is assumed that the whole heat absorbed remains in the liquid as increased internal energy because v' is practically equal to v_{32}.

Exercise 161. Compute the external work done during the heating of one pound of water from 32° F to saturation temperature under a pressure of 300 pounds per square inch. Compare this with the heat which must be supplied.

For dry saturated vapor the internal energy is

$$u''=q''-\frac{p(v''-v_{32})}{J}$$

or

$$u''=u'+\rho,$$

where ρ is the internal latent heat.

The Total Heat of Dry Saturated Steam as Given in Marks and Davis' Steam Tables is denoted by H and is defined by the equation

$$H=u''+\frac{pv''}{J}-0.04,$$

where H is expressed in mean B.t.u.

To interpret this quantity physically it may be transformed so as to contain q''.

As

$$u''=q''-\frac{p(v''-v_{32})}{J}$$

$$H=q''+\frac{pv_{32}}{J}-0.04.$$

H therefore is not equal to the total heat as defined on page 162.

$\frac{pv_{32}}{J}$ represents the heat equivalent of the work done in introducing one pound of liquid (existing at 32° F under zero absolute pressure) under a piston (Fig. 36) producing a pressure of p pounds per square foot absolute.

0.04 equals the heat equivalent of the work done by the atmospheric pressure during the transfer of one pound of liquid at 32° F and under zero absolute pressure into the cylinder (Fig. 36) for

$$\frac{14.7\times 144\times 0.01602}{778}=0.0436.$$

Therefore H represents the total energy (expressed in heat units) that must be supplied in order to produce dry saturated steam at any pressure from water originally at 32° F and under a pressure of one atmosphere.

q'' represents the energy which must be supplied if the start is made with water at 32° F and under the pressure at which the dry saturated steam is to be produced.

$\frac{pv_{32}}{J}-0.04$, represents the heat equivalent of the work done by a feed-pump in introducing the water at 32° F into the boiler in which it is to be heated under constant pressure. H is greater than q'' by this amount.

Exercise 162. By how much does H for water exceed q'' at 300 pounds per square inch absolute?

What is the value of H under the above conditions?

The Heat Content of Dry Saturated Vapor.—The heat content of a dry saturated vapor is denoted by i'' and is defined by the equation

$$i''=u''+\frac{pv''}{J}.$$

Its physical significance becomes more apparent when we substitute for u'' its value

$$u''=q''-\frac{p(v''-v_{32})}{J},$$

thus $$i''=q''+\frac{pv_{32}}{J}.$$

From this equation it follows that the heat content represents the heat equivalent of the total energy which must be expended to produce the vapor from liquid initially at 32° F and under zero absolute pressure.

At low pressures $\frac{pv_{32}}{J}$ is very small so that under such conditions we may assume that

$$i''=q''.$$

The difference between H and i'' (i.e., -0.04) is always negligible when the degree of accuracy of the steam table is considered so that under all conditions

$$i''=H.$$

The Heat Content of the Liquid is denoted by i' and by definition

$$i'=u'+\frac{pv'}{J}.$$

Substituting the value of u' in terms of q' we have

$$i'=q'+\frac{pv_{32}}{J}.$$

Thus again at low pressures i' is practically equal to q'.

The total heat of the liquid, h, as given in Marks and Davis' Steam Tables is defined by the equation

$$h=u'+\frac{pv'}{J}-0.0436.$$

Therefore h is practically equal to i'.

EXERCISE 163. One pound of water at 32° F and under a pressure of 247 pounds per square inch absolute is to be transformed into dry saturated steam under a constant pressure of 247 pounds.

(*a*) How much heat is required?

(*b*) By how much is the internal energy of this water increased during the above change of state?

SECTION XXIX

WET VAPORS

The Quality of Wet Vapors.—During the transformation of the liquid into dry saturated vapor the substance is partly liquid and partly saturated vapor. Under these conditions the substance is called a wet vapor. That part of one pound of wet vapor which consists of saturated vapor is denoted by x and this fraction x is a measure of the **quality** of the wet vapor.

At the temperature of the wet vapor one pound of liquid occupies v' cubic feet and one pound of dry saturated vapor occupies v'' cubic feet. If the quality of the wet vapor is x its volume denoted by v equals

$$v=v''x+v'(1-x).$$

EXERCISE 164. Compute the volume of dry saturated vapor and of liquid in one pound of steam at 247 pounds per square inch absolute, the quality being 75 per cent.

The Total Heat, Internal Energy, and Heat Content of Wet Vapor.—It follows from the definitions already given that if x parts of one pound of liquid have been evaporated

$$q=q'+xr,$$

$$u=u'+x\rho,$$

and as $i=u+\frac{pv}{J}$, by definition,

we also have $i=u'+x\rho+\frac{1}{J}p\{v''x+v'(1-x)\}$

$$=\left(u'+\frac{pv'}{J}\right)+x\left\{\rho+\frac{p}{J}(v''-v')\right\}$$

or $i=i'+xr.$

Lines of Constant Quality.—In Fig. 42 are shown the liquid and the saturation lines of a vapor. Let the point A represent the state-point of the wet vapor when its quality is x and its pressure p. Then

$$aA'=v'$$

$$aA''=v''$$

$$aA=v=(v''-v')x+v',$$

and as $A'A=aA-aA'$

$$=(v''-v')x.$$

Therefore $A'A$ is the xth part of $A'A''$, and for any other pressure we would have

$$B'B=x(B'B'').$$

If x represents a constant quality then the locus of the points A or B as determined above is called a line of constant quality.

EXERCISE 165. On the plot of Exercise 156 (*a*) draw lines of constant quality for steam whose quality is (*a*) 50 per cent and (*b*) 75 per cent.

From the above discussion it should be noted that a given temperature and the corresponding pressure are not sufficient to determine the specific volume of a wet vapor (as was the case in an ideal gas). For wet vapors the quality must be known as well as the temperature or the pressure (these are not independent) before the volume can be computed.

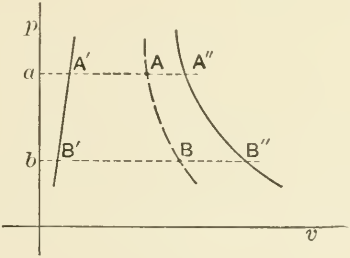

FIG. 42.

SECTION XXX

SUPERHEATED VAPORS

The Heat Required to Superheat Vapors at Constant Pressure.—After the dry saturated state has been passed the temperature of the vapor rises with added heat at constant pressure. The specific heat of superheated vapors varies with the pressure and with any given pressure it varies with the temperature.

Let c_p represent the specific heat of the vapor,
t'', the saturation temperature,
t_s, the temperature of the superheated vapor,
q, the total heat of the superheated vapor,

then $q-q''$ represents the heat that must be added to one pound of dry saturated vapor in order to increase its temperature to t_s under a constant pressure and

$$q-q''=\int_{t''}^{t_s} c_p dt,$$

$$=(c_p)_m(t_s-t''),$$

if $(c_p)_m$ represents the mean specific heat of the superheated vapor for the given pressure p and between the temperatures t'' and t_s. The range of temperature (t_s-t'') is called the **degrees of superheat.**

As both c_p and $(c_p)_m$ vary with pressure and with temperature, $q-q''$ is not easily calculated without the use of vapor tables.

The Internal Energy and the Heat Content of Superheated Vapor.—The internal energy of a superheated vapor equals

$$u=q-\frac{p(v-v_{32})}{J},$$

where v is the volume of the superheated vapor.

The heat content of superheated vapor is

$$i=u+\frac{pv}{J}$$

$$=q+\frac{pv_{32}}{J}.$$

The total heat of steam, h, as given in Marks and Davis' Steam Tables is practically equal to the heat content of the vapor.

As $$q=i-\frac{pv_{32}}{J}$$

and $$q''=i''-\frac{pv_{32}}{J},$$

$$q-q''=i-i''.$$

EXERCISE 166. (*a*) From the steam tables find the heat required to superheat one pound of dry saturated steam 200° F at 300 pounds per square inch absolute.

(*b*) Compute the mean specific heat of superheated steam for this pressure and this range of temperature.

EXERCISE 167. How much heat must be added per pound of steam, quality 0.80, in order to change it to steam at 427.8° F under a constant pressure of 100 pounds per square inch absolute?

RÉSUMÉ

The Heat Content or the total energy (expressed in heat units) required to change one pound of substance originally at 32° F and under zero absolute pressure to any other state under any given constant pressure is

$$i=u+\frac{pv}{J},$$

where u is the change in the internal energy (measured in heat units) of the substance due to the above change of state,

v is the volume of one pound of the substance in its final state, measured in cubic feet,

p is the constant pressure under which the change of state occurred, measured in pounds per square foot.

The heat content of the liquid, i', at saturation temperature is given in the vapor tables.

The heat content of the dry saturated vapor, i'', is given in the vapor tables.

The heat content of wet vapor, i, must be computed by means of the relation

$$i=i'+xr,$$

where x is the quality of the vapor, and r is the latent heat of evaporation which is given in the vapor tables and which also equals $i''-i'$.

The heat content of the superheated vapor, i, is given in the vapor tables.

The Internal Energy or more properly the change in the internal energy from its value for the substance at 32° F and under zero absolute pressure due to any change of state is

$$u=i-\frac{pv}{J}.$$

The internal energy of the liquid, u', at saturation temperature is

$$u'=i'-\frac{pv'}{J},$$

where i' is the heat content of the liquid at p the pressure corresponding to the temperature, and v' is the specific volume of the liquid at the same pressure (or temperature) as given in the vapor tables.

The internal energy of the dry saturated vapor, u'', is sometimes given in the vapor tables. It may be computed from

$$u''=i''-\frac{pv''}{J},$$

where i'' is the heat content of the dry saturated vapor at p, the pressure corresponding to the temperature, and v'' is the specific volume of the dry saturated vapor at the same pressure (or temperature) as given in the vapor tables.

The internal energy of wet vapor, u, must always be computed by means of the relation

$$u=u'+x\rho,$$

where x is the quality of the vapor and ρ is the internal latent heat which is sometimes given in the vapor tables but may always be computed by means of the relation

$$\rho=r-\frac{p(v''-v')}{J},$$

where r, v'', and v' are all taken from the vapor tables for the pressure (or temperature) for which ρ and u are sought.

The internal energy of superheated vapor must be computed from the relation

$$u=i-\frac{pv}{J},$$

where i and v are the heat content and the specific volume of the superheated vapor for the pressure p at which u is required, all given in the vapor tables.

CHAPTER X

ENTROPY OF VAPORS

Section XXXI

COMPUTATION OF THE CHANGE IN ENTROPY

The change in entropy of a substance depends only upon the initial and the final states of the substance and not upon the process followed in making the change.

The entropy tabulated in the vapor tables is the increase in the entropy over its value at 32° F.

By definition the change in entropy during any reversible process per pound of substance is

$$ds = \frac{dq}{T}.$$

The Entropy of the Liquid or the increase in the entropy of the liquid during the warming at a constant pressure from 32° F to the saturation temperature due to the absorption of the heat of the liquid q' is

$$s' = \int_{492}^{T'} \frac{c'dT}{T}.$$

If c' is constant and as s'_{32} is assumed to be zero, we have

$$s' = c' \log_\epsilon \frac{T}{492} = (2.303)c' \log_{10} \frac{T}{492}.$$

Actually c' is not constant. The computation of s' may be carried out by graphical integration of

$$s'=\int_{492}^{T'}\frac{c'}{T}dT.$$

This form of the integral suggests plotting $\frac{c'}{T}$ on an axis of T and finding the area under the curve.

The values of s' are given in the vapor tables.

EXERCISE 168. Plot the liquid line for water upon axes of absolute temperature and of entropy by means of the steam tables.

The Entropy of Evaporation.—The heat absorbed during evaporation is r, the latent heat of evaporation. During this absorption of heat the temperature remains constant. Let this absolute temperature be T, then the increase in entropy during evaporation equals

$$\frac{r}{T}.$$

It is given in the vapor tables.

EXERCISE 169. Check three values of the entropy of evaporation as given in the steam tables.

The Entropy of Dry Saturated Vapor or the increase in entropy of dry saturated vapor over the entropy of one pound of the substance at 32° F, denoted by s'', is given by

$$s''=s'+\frac{r}{T},$$

and may be found in the vapor tables.

EXERCISE 170. Plot the saturation line for steam by means of the steam tables upon the plot used for Exercise 168.

The Entropy of Wet Vapors, of quality x, is not given in the vapor tables. It must be computed from the relation

$$s = s' + x\frac{r}{T}.$$

EXERCISE 171. Plot lines of constant quality of 0.1, 0.5, and 0.8 upon the results of Exercises 168 and 170.

The Entropy of Superheated Vapor, s, may be computed from the equation

$$s - s'' = \int_{T''}^{T_s} \frac{d(q-q'')}{dT} = \int_{T''}^{T_s} \frac{c_p dT}{T},$$

where T'' and T_s are the absolute temperatures of saturation and of superheat respectively. s is given in the vapor tables for superheated vapors.

EXERCISE 172. Plot the path of the state-point representing the formation of superheated steam at a constant pressure of (*a*) 40, (*b*) 200 pounds per square inch absolute starting with water heated to the temperature of evaporation for the given pressure on the plot already used for Exercises 168, 170, and 171.

SECTION XXXII

THE TEMPERATURE-ENTROPY DIAGRAM FOR VAPORS

In this diagram the state of the substance is represented by its absolute temperature and by its entropy per pound. The liquid line and the saturation line have already been plotted in Exercises 168 and 170. These are shown in Fig. 43.

During the evaporation of a liquid the state-point representing the change of state on the *Ts*-plane moves from *B* to *C*, Fig. 43. The entropy increases with the addition of heat and the temperature remains constant. The area under *BC* represents the heat supplied during evaporation.

If more heat is supplied to the dry saturated vapor (state-point *C*) while the vapor remains under the same constant pressure which existed along *BC* the path of the state-point will resemble the line *CD* (see Exercise 172) and the area under *CD* represents the heat required to superheat the vapor under this pressure.

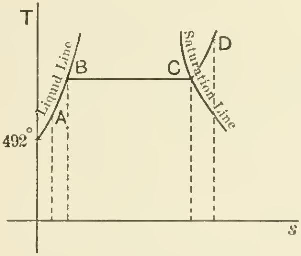

FIG. 43.

The liquid line as already plotted for water in Exercise 168 is not a line of constant pressure. Variation in pressure has however so little effect on the heat necessary to change the state of a liquid owing to the very small increase in volume and therefore the small amount of external work done that no account is taken of the variation in pressure. With this very close approximation in mind the liquid line plotted for variable pressure may be regarded as a line representing the change of state at constant pressure for any pressure during the heating of the liquid.

Thus the line representing the change at constant pressure starting with a liquid at any given temperature and ending with superheated vapor is *ABCD*, Fig. 43.

The areas under the lines *AB*, *BC*, and *CD* extending down to the axis of *s*, the line of absolute zero temperature, represent the heat required to produce the respective changes of state, or

$$\text{the area under } AB = q'_B - q'_A,$$

$$\text{the area under } BC = r,$$

$$\text{the area under } CD = q_D - q''_C.$$

Lines of Constant Quality.—These lines may be plotted on the *Ts*-diagram by means of the relation

$$s = s' + x\frac{r}{T},$$

s' and $\frac{r}{T}$ being given in the vapor tables s can be computed for various temperatures and any given quality.

Note also that the above equation in the form

$$s - s' = x\left(\frac{r}{T}\right)$$

suggests a convenient way of plotting lines of constant quality.

Lines of Constant Heat Content.—The heat content of a wet vapor equals

$$i = i' + xr.$$

Thus for any given heat content x may be computed for various temperatures after i' and r have been read from the steam tables. The corresponding entropy need not be computed, for x is evidently the ratio of the distance from B to the point sought to the distance BC, Fig. 43.

In the region of superheated vapor the temperature and the corresponding entropy of one pound of vapor for any given heat content must be found by interpolation in the vapor tables for superheated vapor.

EXERCISE 173. Plot on the result of Exercise 168 the lines of constant heat content for 1100 B.t.u. in the wet region and for 1150 B.t.u. in the superheated region.

Lines of Constant Volume.—For the wet region x the quality of the vapor for any given volume may be computed from

$$v = v''x + v'(1 - x)$$

or

$$x = \frac{v - v'}{v'' - v'}.$$

Thus for various temperatures find v'' and v' from the tables and compute the corresponding x. This value of x is sufficient to locate a point on the constant volume line corresponding to the temperature.

For the superheated region the entropy and the temperature corresponding to any given volume can be found from the tables for superheated vapors.

EXERCISE 174. Plot on the result of Exercise 168 the line of constant volume for 7.5 cubic feet per pound of wet and of superheated steam.

The following **graphical construction of lines of constant volume in the wet region** is interesting because it introduces the characteristic surface of the wet vapor referred to the coordinate axes of T, s, and v. It has already been pointed out that the state of a substance need not be given in terms of p, v, and T only.

Fig. 44 represents the elevation and the plan of the characteristic surface referred to the axes of T, s, and v. The line $A'A''$ represents the Ts-projection of the intersection of the surface and a plane parallel to the vs-plane.

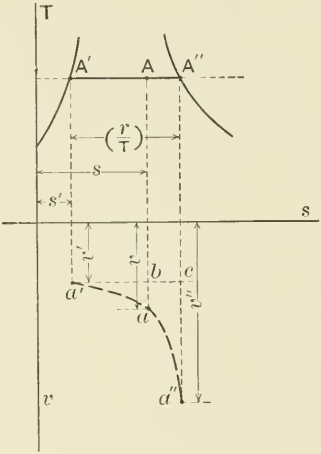

FIG. 44.

We must find the nature of the vs-projection of this same intersection represented on the figure by $a'aa''$.

As $$s=s'+x\left(\frac{r}{T}\right)$$

or $$s-s'=x\left(\frac{r}{T}\right)$$

it follows that $$(A'A)=x(A'A'')$$

or $$(a'b)=x(a'c).$$

But $$v=v''x+v'(1-x)$$

or $$(v-v')=x(v''-v').$$

Therefore $$x=\frac{(a'b)}{(a'c)}=\frac{v-v'}{v''-v'},$$

whence it follows that a (Fig. 44) must lie upon a straight line joining a' and a'', or the projection of the intersection

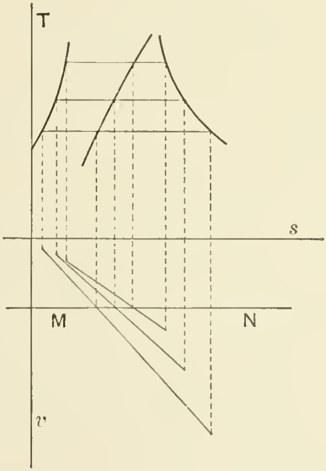

FIG. 45.

of the characteristic surface with a plane parallel to the vs-plane is a straight line.

As shown in Fig. 45 various rectilinear elements of the surface representing the wet region can easily be drawn. Then any plane parallel to the Ts-plane must contain the constant volume line. Three points on this line are shown projected on the *vs*-trace of the constant volume plane.

The *Ts*-projection of the required constant volume line is obtained as indicated in Fig. 45.

A Complete Temperature-Entropy Diagram for any given vapor contains, in addition to the vertical lines representing constant entropy and the horizontal lines representing constant temperature, lines of constant quality, constant degrees of superheat, constant volume, constant pressure, and constant heat content all drawn for constant intervals of the quantities represented.

Such a diagram more than replaces the vapor tables. When sufficient conditions of the vapor are known to determine the state-point on the diagram this diagram not only allows all other conditions to be read directly but it performs graphically all necessary interpolations in the superheated region and all the calculations involving quality in the wet region which must ordinarily be performed when the tables are used.

Moreover the diagram allows the direct reading of the final conditions due to various changes of state of the vapor for which the various lines are drawn as will shortly be explained.

Temperature-entropy diagrams were formerly much used but they have now been superseded by

The Mollier Diagram.—This diagram contains the same line as the *Ts*-diagram just described excepting the constant volume lines which are usually omitted. Instead of plotting these lines on axes of temperature and entropy Mollier devised the scheme of using axes of heat content and of entropy. The resulting diagram is more open and therefore more easily and accurately read than the *Ts*-diagram in the region near the saturation line, where such diagrams are of principal use to engineers.

A Mollier diagram for steam accompanies Marks and Davis' Steam Tables * and a Mollier diagram for ammonia has been drawn by Goodenough.†

EXERCISE 175. Plot the liquid and the saturation lines on axes of heat content and of entropy. Use the steam tables.

EXERCISE 176. Compute the heat content and the entropy of one pound of wet steam, quality 0.50, for pressures of (*a*) 200 and (*b*) one pound per square inch absolute.

EXERCISE 177. On the plot already used in Exercise 175 draw the constant pressure lines for (*a*) 200, and (*b*) one pound per square inch absolute, also the constant quality line for 0.50 and the lines of constant superheat for 100° and 300° of superheat.

Other Diagrams have been devised. One using axes of specific volume and "total heat" (in this case the total heat H as defined by Marks and Davis in their Steam Tables) has lately been drawn by Ellenwood and published by Wiley.§

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* Marks and Davis, Tables and Diagrams of Steam, Longmans, Green, & Co., New York, 1914.

† Goodenough, Properties of Steam and Ammonia, John Wiley & Sons, Inc., New York, 1915.

§ Ellenwood, Steam Charts, John Wiley and Sons, Inc., New York, 1914.

CHAPTER XI

CHANGES OF STATE OF VAPORS

SECTION XXXIII

WET VAPORS

IN this chapter are to be considered the energy changes occurring during various changes of state of vapors. Up to the present only changes of state occurring under constant pressure have been discussed.

The fundamental equation for these energy changes during which thermal equilibrium exists, i.e. during which the temperature thruout the vapor is uniform, altho not necessarily constant, is

$$\Delta Q = \Delta K + \Delta P + \Delta W,$$

where ΔQ represents the heat energy supplied to the substance by some external source,

ΔK, the change in internal (intrinsic) kinetic energy,

ΔP, the change in internal (intrinsic) potential energy,

ΔW, the external work performed by the substance during the change of state considered.

It is seldom necessary to separate the two kinds of intrinsic energy, so that we may put

$$\Delta U = \Delta K + \Delta P,$$

where ΔU represents the total internal energy of the substance, and the fundamental energy equation becomes

$$\Delta \mathbf{Q} = \Delta \mathbf{U} + \Delta \mathbf{W},$$

or when one pound of substance is considered

$$\Delta \mathbf{q} = \Delta \mathbf{u} + \Delta \mathbf{w}.$$

It is usual to represent all changes of state on either the pv- or the Ts-plane. These planes are selected because the area under the curve representing the process on the pv-plane represents external work done and the area under the Ts-curve of the process, provided the process is reversible, represents the heat supplied by an external source.

For **wet vapors** we have

$$i = i' + xr$$

$$u = u' + x\rho = i - \frac{pv}{J}$$

$$u' = i' - \frac{pv'}{J}$$

$$\rho = r - \frac{p(v'' - v')}{J}$$

$$v = v''x + v'(1 - x)$$

$$s = s' + x\left(\frac{r}{T}\right).$$

Pressure and Temperature Constant.—This change of state is represented on both the pv- and the Ts-planes in Fig. 46.

Let subscripts 1 denote initial and subscripts 2 final conditions, then

$$v_1 = v''x_1 + v'(1 - x_1)$$

$$v_2 = v''x_2 + v'(1 - x_2)$$

or $$v_1''=v_2''=v'',$$

and $$v_1'=v_2'=v'.$$

Thus the change in volume equals

$$v_2-v_1=(x_2-x_1)(v''-v').$$

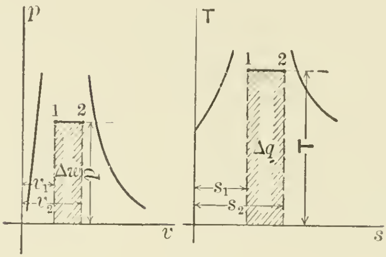

FIG. 46.

The heat received from (or rejected to, if negative) a external body per pound of substance is

$$\Delta q=q_2-q_1,$$

where $$q_2=i_2-\frac{pv_{32}}{J}$$

and $$q_1=i_1-\frac{pv_{32}}{J},$$

so that $$\Delta q=i_2-i_1.$$

But $$i_2=i'+x_1r$$

and $$i_1=i'+x_2r,$$

thus $$\mathbf{\Delta q=(x_2-x_1)r.}$$

The change in the intrinsic energy of one pound of substance is

$$\Delta u = u_2 - u_1,$$

where $u_2 = u' + x_2\rho$ and $u_1 = u' + x_1\rho$,

so that $\Delta \mathbf{u} = (\mathbf{x}_2 - \mathbf{x}_1)\rho.$

Finally the external work performed per pound equals

$$\Delta w = \frac{1}{J}p(v_2 - v_1) = \frac{1}{J}p(x_2 - x_1)(v'' - v') = \Delta q - \Delta u.$$

Exercise 178. (*a*) Show that $\Delta w = \Delta q - \Delta u$ by means of the above-developed expressions.

(*b*) Find Δq directly from Fig. 46.

Exercise 179. The volume of one pound of wet steam at 200 pounds per square inch absolute, quality 0.30, is increased by 0.50 of a cubic foot at constant pressure. Compute

(*a*) the final quality,
(*b*) the heat required,
(*c*) the change in intrinsic energy,
(*d*) the external work done.

Volume Constant.—The graphical representation of the change of state of wet vapor at constant volume is shown in Fig. 47.

Under these conditions we have

$$v_1 = x_1 v_1'' + (1 - x_1)v_1'$$

$$v_2 = x_2 v_2'' + (1 - x_2)v_2'$$

and as $v_1 = v_2$

$$x_1(v_1'' - v_1') + v_1' = x_2(v_2'' - v_2') + v_2'.$$

Thus the final quality can be computed when the initial quality and the initial and the final pressures or temperatures are known.

From the *pv*-plane, Fig. 47, it is evident that

$$\Delta \mathbf{w} = \mathbf{0}.$$

From the *Ts*-plane we see that Δq is now **not** equal to $q_2 - q_1$.

EXERCISE 180. Show in Fig. 47 an area representing (*a*) q_2, (*b*) q_1. Is $\Delta q = q_2 - q_1$?

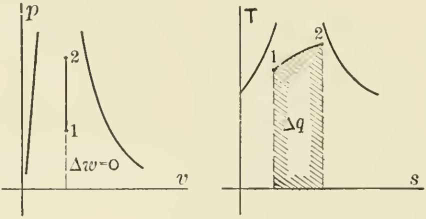

FIG. 47.

The change in intrinsic energy is always equal to

$$\Delta u = u_2 - u_1$$

and as

$$u = u' + x\rho$$

$$\Delta \mathbf{u} = \mathbf{u}_2' - \mathbf{u}_1' + \mathbf{x}_2\rho_2 - \mathbf{x}_1\rho_1.$$

As

$$u = i - \frac{pv}{J}$$

we may also write

$$\Delta \mathbf{u} = \mathbf{i}_2 - \mathbf{i}_1 - \frac{\mathbf{p}_2\mathbf{v} - \mathbf{p}_1\mathbf{v}}{\mathbf{J}},$$

where the *i*'s may be read from the Mollier diagram and the v from any diagram which contains constant volume curves, or v may be computed from the vapor tables.

Finally, as
$$\Delta w=0$$
$$\Delta q=\Delta u.$$

EXERCISE 181. Compute the quality of one pound of wet steam after the pressure drops at constant volume from 200 pounds per square inch absolute, when the quality was 0.90, to a vacuum of 28 inches of mercury.

EXERCISE 182. (*a*) What is the volume of the steam during the change of state described in Exercise 181?

(*b*) At what temperature and at what pressure would this steam become dry saturated, its volume remaining constant?

(*c*) Solve (*a*) and (*b*) by means of the steam diagrams.

EXERCISE 183. Compute the heat absorbed by the steam during the change of state indicated in Exercise 181.

EXERCISE 184. (*a*) Compute the entropy of one pound of steam at 90 pounds per square inch absolute, the volume being 4.5 cubic feet.

(*b*) Solve (*a*) by means of the steam diagrams.

Entropy Constant, Reversible Adiabatic Change.—

By definition
$$\Delta q=0,$$
thus
$$\Delta u=-\Delta w.$$

The process is represented in Fig. 48. To compute the final quality use is made of the fact that the entropy remains constant. Thus

$$s_1'+x_1\left(\frac{r}{T}\right)_1=s_2'+x_2\left(\frac{r}{T}\right)_2$$

from which x_2 may be found.

Knowing x_2 the final volume is found from

$$v_2 = v_2'' x_2 + v_2'(1 - x_2).$$

The change in intrinsic energy is always

$$\Delta u = u_2 - u_1 = (u_2' + x_2 \rho_2) - (u_1' + x_1 \rho_1)$$

and the external work done must equal

$$\Delta w = -\Delta u.$$

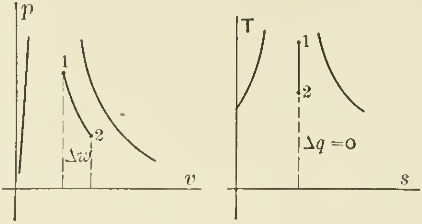

FIG. 48.

The Mollier diagram does not give the values of the intrinsic energy but as

$$u = i - \frac{pv}{J}$$

$$\Delta u = i_2 - i_1 - \frac{p_2 v_2 - p_1 v_1}{J},$$

in which all quantities are either given or may be read from the vapor diagrams.

EXERCISE 185. Compute the quality of steam after a reversible adiabatic expansion

(*a*) from dry saturated steam at 245 pounds per square inch absolute to 15 pounds per square inch absolute, (*b*) from 245 pounds per square inch absolute with a quality of 0.20 to 15 pounds per square inch absolute.

(*c*) Check the results by means of the Mollier diagram.

As the results of the above exercise show, the quality of low-quality steam is increased and the quality of high-quality steam is decreased by adiabatic expansion. The *Ts*-diagram for steam illustrates these facts. These conditions do not hold for all substances. For ether the liquid and the saturation lines both slope in the same direction, as shown in Fig. 49.

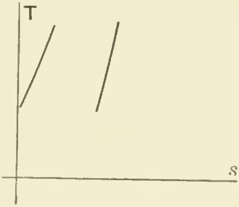

FIG. 49.

EXERCISE 186. Show that dry saturated steam condenses during adiabatic expansion and does not condense during adiabatic compression. How does ether behave under the same conditions?

EXERCISE 187. Compute the volume of one pound of wet steam before and after reversible adiabatic expansion from 350 pounds per square inch absolute and a quality of 0.90 to 50 pounds per square inch absolute.

Check by means of the steam diagrams.

EXERCISE 188. Compute the work performed during the reversible adiabatic expansion of 10 cubic feet of steam, quality 0.90, from a pressure of 80 pounds per square inch absolute to an absolute pressure of 10 inches of mercury.

EXERCISE 189. Solve Exercise 188 by means of the steam diagrams.

The *pv*-curve representing the reversible adiabatic of wet steam is of the approximate form shown in Fig. 48.

For an initial quality, x_1, lying between 0.70 and 1.00 the equation of this curve may be expressed by

$$pv^m = \text{a constant.}$$

$$\text{Rankine assumed } m = \frac{10}{9},$$

$$\text{Zeuner placed } \quad m = 1.035 + 0.1x_1,$$

and calculations based upon Marks and Davis' Steam Tables seem to show that

$$m = 1.059 - 0.000315p_1 + (0.0706 + 0.000376p_1)x_1.$$

Thus m may vary between 1.10 and 1.13 in problems usually arising in engineering.

As the area under any curve having the equation $pv^m = \text{a constant}$, where m is constant, equals

$$\frac{p_1v_1 - p_2v_2}{m - 1}$$

this expression affords another method of computing the work done during a reversible adiabatic change of state of wet steam. This method is not as accurate as the method based on the vapor tables on account of the questionable value of m and even of the equation $pv^m = \text{a constant}$.

Exercise 190. Using Zeuner's value of m, solve Exercise 188.

Quality Constant.—A change in state at constant quality is illustrated in Fig. 50. Under these conditions the final volume is evidently

$$v_2 = v_2''x + v_2'(1 - x),$$

where v_2'' and v_2' correspond to the final temperature or pressure and x is the constant quality.

The final entropy is

$$s_2=s_2'+x\left(\frac{r}{T}\right)_2.$$

In all changes of state heretofore discussed either one or all of the quantities Δq, Δu, or Δw could be computed by means of the vapor tables. This is not possible for all changes of state. The case under consideration is an illustration.

Here Δq is not equal to either q_2-q_1 or i_2-i_1.

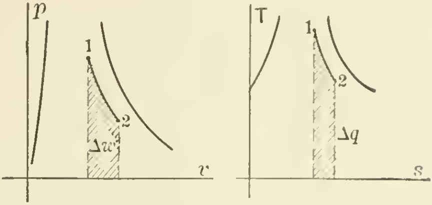

FIG. 50.

The change in intrinsic energy is however equal to

$$\begin{aligned}\Delta u&=u_2-u_1\\&=(u_2'-x\rho_2)-(u_1'-x\rho_1)\\&=\left(i_2-\frac{p_2v_2}{J}\right)-\left(i_1-\frac{p_1v_1}{J}\right).\end{aligned}$$

The external work done cannot be computed from the relation $\Delta q=\Delta u+\Delta w$ for Δq is not known.

In order to find Δw the equation of the pv-curve representing the process must be found. It has been found empirically that the equation of the saturation line on the pv-plane may be very closely represented by the equation

$$p(v'')^s = \text{a constant}$$

where $$s = 1.063.$$

As $$v = v''x + v'(1-x)$$

$$v'' = \frac{v - v'(1-x)}{x}$$

and the equation of a constant quality line is

$$p\left\{\frac{v - v'(1-x)}{x}\right\}^s = \text{a constant.}$$

If the pressures are relatively low and the quality high the term $v'(1-x)$ is small compared to v and this equation approximates

$$\frac{pv^s}{x^s} = \text{a constant.}$$

By means of this equation an approximate value of Δw may be computed from the area under the curve and thus a value of Δq may be found.

Exercise 191. One pound of steam expands with a constant quality from a volume of 2 cubic feet and a pressure of 200 pounds per square inch absolute to 18 pounds per square inch absolute.

(*a*) Find the equation of its pv-curve.
(*b*) Compute the external work performed.
(*c*) How much heat was supplied during this change of state?

SECTION XXXIV

SUPERHEATED VAPORS

Pressure Constant.—The change in specific volume and in entropy for a given change in temperature during a change of superheated vapor at constant pressure, illustrated in Fig. 51, may be obtained from the vapor tables by interpolation or may be read directly from the vapor diagrams.

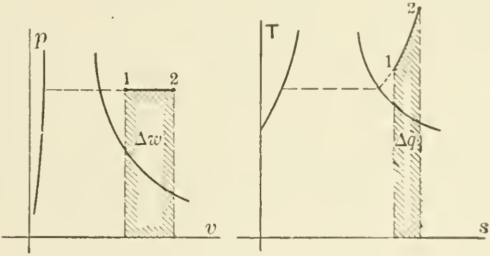

FIG. 51.

As may be seen from Fig. 51, the heat supplied during this change of state is

$$\Delta q = q_2 - q_1 = i_2 - i_1.$$

The change in intrinsic energy is always

$$\begin{aligned}\Delta u &= u_2 - u_1 \\ &= \left(i_2 - \frac{pv_2}{J}\right) - \left(i_1 - \frac{pv_1}{J}\right)\end{aligned}$$

and the external work done

$$\Delta w = \frac{p(v_2 - v_1)}{J}.$$

Exercise 192. (*a*) How much heat must be supplied to change one pound of dry saturated steam to superheated steam at a temperature of 600° F under a constant pressure of 200 pounds per square inch absolute?

(*b*) What is the mean specific heat of the steam under the above conditions?

Exercise 193. Compute the increase in the intrinsic energy during the change of state described in Exercise 192.

Temperature Constant.—If the temperature and the initial and the final pressures are known, the initial and the final volumes and entropies of one pound of vapor may be found by means of the vapor tables or the vapo diagrams.

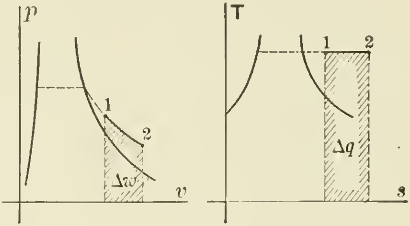

Fig. 52.

As Fig. 52 shows

$$\Delta q \neq q_2 - q_1.$$

Exercise 194. Show q_2 and q_1 on Fig. 52.

The heat supplied is, from Fig. 52,

$$\Delta q = T(s_2 - s_1).$$

The intrinsic energy is not constant even tho the temperature remains constant for the superheated vapor is not an ideal gas. Here as always

$$\begin{aligned}\Delta u &= u_2 - u_1\\ &= \left(i_2 - \frac{p_2v_2}{J}\right) - \left(i_1 - \frac{p_1v_1}{J}\right).\end{aligned}$$

To compute the external work done we have

$$\Delta w = \Delta q - \Delta u.$$

EXERCISE 195. One pound of steam expands at constant temperature from 300 pounds per square inch absolute and 50° superheat to 40 pounds per square inch absolute. Compute

(*a*) the final degrees of superheat.
(*b*) the increase in volume,
(*c*) the heat supplied,
(*d*) the change in intrinsic energy,
(*e*) the external work performed.

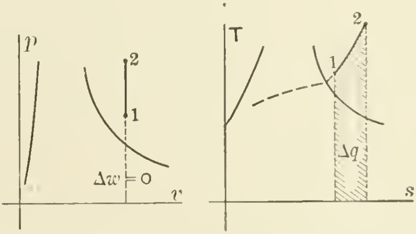

FIG. 53.

Volume Constant.—In this case $\Delta w = 0$ and $\Delta q = \Delta u$. But be careful to note that as Fig. 53 shows

$$\Delta q \neq q_2 - q_1 \neq i_2 - i_1.$$

Again $\Delta u = u_2 - u_1 = \left(i_2 - \frac{p_2v}{J}\right) - \left(i_1 - \frac{p_1v}{J}\right).$

EXERCISE 196. During a reduction in the pressure on one pound of steam from 290 pounds per square inch absolute and 140° superheat to 235 pounds per square inch absolute, the volume remained constant. Compute the heat supplied.

Entropy Constant, Reversible Adiabatic Change.—The changes in temperature, pressure, and volume of superheated vapor during a reversible adiabatic change of state, Fig. 54, can be found from the vapor tables by interpolation or can be read directly from the diagrams if it be remembered that the entropy remains constant.

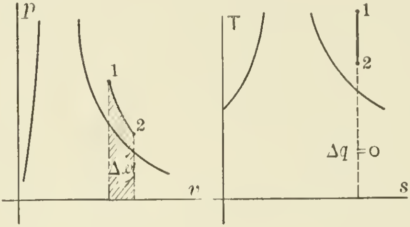

FIG. 54.

Under these conditions we have

$$\Delta q = 0,$$

$$\Delta w = -\Delta u,$$

and

$$\Delta u = u_2 - u_1$$

$$= \left(i_2 - \frac{p_2 v_2}{J}\right) - \left(i_1 - \frac{p_1 v_1}{J}\right).$$

EXERCISE 197. One pound of steam under 200 pounds per square inch absolute, superheated 160° F, expands adiabatically to 100 pounds per square inch absolute. Compute

(*a*) the final degrees of superheat,

(*b*) the work performed during expansion.

(*c*) Check the results by means of the steam diagrams.

The equation of the pv-curve representing reversible adiabatic change of state of superheated steam has been empirically determined to be

$$pv^{1.31} = \text{a constant.}$$

The external work performed may thus also be computed by means of

$$\Delta w = \frac{p_1v_1 - p_2v_2}{1.31 - 1},$$

altho this method is not as accurate as the method based upon the vapor tables.

Exercise 198. Solve Exercise 197 by means of the above formula.

Exercise 199. Steam initially under 160 pounds per square inch absolute and superheated 100° F expands with constant entropy to 0.50 pound per square inch absolute. Compute

(*a*) the final condition of the steam,

(*b*) the external work performed.

(*c*) Check these results by means of the steam diagrams.

Exercise 200. Assuming the equation of the pv-curve of the saturation line of steam to be

$$p(v'')^{1.063} = \text{a constant}$$

and that of the adiabatic expansion line of superheated steam to be

$$pv^{1.31} = \text{a constant},$$

find the pressure at which the steam in Exercise 199 becomes dry saturated.

Check this result by means of the steam diagrams.

CHAPTER XII

VAPOR CYCLES

SECTION XXXV

THE CARNOT CYCLE

THE Carnot cycle for ideal gases has been described on page 86. This cycle always consists of two isothermal and two adiabatic processes. For vapors this cycle can best be represented on the Ts-plane as shown in Fig. 55. Here the vapor thruout the various processes remains wet vapor.

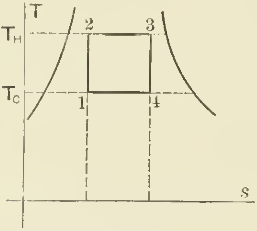

FIG. 55.

The whole cycle may be conceived to occur while the vapor remains in one cylinder. Starting with the state represented by the point 1 the first process consists of the reversible adiabatic compression, 1 2, during which the quality decreases with the volume while the pressure increases with the temperature. During this process no heat is supplied to or rejected by the vapor but work must be done on the vapor. During the next process 2 3 heat is supplied at the constant temperature of the hot body, T_H. If the vapor remains wet the pressure must remain constant while the volume increases. Then follows a reversible adiabatic expansion 3 4 during which the quality of the vapor diminishes as the temperature drops to the temperature of the cold body, T_C. During both the processes 2 3 and 3 4 external work is done by the vapor. Finally the wet vapor decreases in volume while in contact with the cold body at a constant temperature T_C (and necessarily constant pressure) until the quality returns to the initial quality at 1.

Actually the processes above described would not be performed in one cylinder for this would require the heating and the cooling of the cylinder between the extreme temperatures and this would entail a waste of heat. The evaporation 2 3 would be performed in a boiler, the condensation 4 1 in a condenser, and the adiabatic expansion 3 4 in one cylinder, the working cylinder, while the adiabatic compression 1 2 would be performed in another cylinder that of the feed-pump.

The *pv*-diagram of this cycle would appear as shown in Fig. 56. Here the wet vapor would leave the feed-pump with a volume *b* 2 and return from the boiler to the working cylinder with the volume *b* 3. After adiabatic expansion 3 4 the wet vapor would leave the engine cylinder with a volume 4 *a* and after passing thru the condenser would return to the feed-pump with a volume *a* 1.

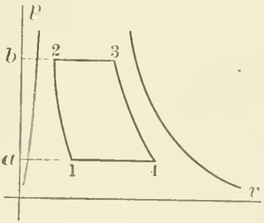

FIG. 56.

No heat engine has been built to operate upon this cycle for it would require a stopping of condensation at a definite quality while in practice complete condensation of the vapor is by far a simpler operation.

However as has already been shown the Carnot cycle gives an efficiency which cannot be surpassed by any other cycle and therefore sets a standard which it is desirable to approach as closely as possible, see page 122.

Exercise 201. Sketch on both the pv- and the Ts-planes a Carnot cycle in which the lowest quality of the vapor is zero and the vapor during its adiabatic expansion (*a*) changes from the superheated to the wet state, (*b*) remains superheated, for both water and ether.

Exercise 202. Show from the Ts-diagram that the efficiency of the Carnot cycle is under all conditions

$$\frac{T_H - T_C}{T_H},$$

where T_H is the absolute temperature of the hot body and T_C is the absolute temperature of the cold body.

Exercise 203. A Carnot cycle operates with steam between

(*a*) 150 and 15 pounds per square inch absolute,

(*b*) 150 and 1 pound per square inch absolute,

(*c*) 200 and 1 pound per square inch absolute.

Compute the efficiencies.

The Efficiency, the Mean Effective Pressure, the Heat Consumption, and the Mass of Substance, the last two per hour per indicated horse-power, may readily be computed for any cycle in the following manner.

The **thermal efficiency** of a cycle is

$$\eta = \frac{q_H - q_C}{q_H},$$

where q_H is the heat received from the hot body and q_C is the heat rejected to the cold body per pound of the working substance.

As the work performed per pound of the working substance passing thru the cycle is $J(q_H - q_C)$, the **mean effective pressure** may be found by dividing this work by the largest volume of one pound of the substance at the lowest pressure of the cycle.

One horse-power equals 33,000 foot-pounds per minute and this is equivalent to 2545 B.t.u. per hour. Even under ideal conditions a working substance passing thru a cycle can only transform a certain fraction η of the heat supplied to it by the source into mechanical energy (see page 128). Therefore the heat which must be supplied per hour in order to yield one indicated horse-power under the conditions under which η is computed or the **heat consumption** equals

$$\frac{2545}{\eta} \text{ B.t.u. per i.h.p. hour.}$$

The mass of the substance which must pass thru the cycle per hour per indicated horse-power, measured in pounds, is evidently equal to the heat supplied per i.h.p. hour divided by the heat supplied per pound of substance.

EXERCISE 204. Compute the work per pound of steam and the steam consumption in pounds per indicated horse-power hour when steam is used in a Carnot cycle between 150 and 3 pounds per square inch absolute with a quality varying between 0 and 1.

SECTION XXXVI

THE RANKINE CYCLE

It has not been found practicable to transform heat into mechanical energy by means of a vapor carried thru a Carnot cycle of changes of state.

The cycle which approximates most closely the cycle at present used in heat motors employing vapors as working substances is the Rankine cycle, illustrated in Fig. 57. During this cycle the heat is received and rejected by the substance while it is under constant pressure. This cycle further differs from the Carnot cycle in so far that the condensation of the vapor is continued until the vapor is completely liquefied, process 4 1, Fig. 57.

The liquid is then heated under a constant pressure corresponding to the highest pressure in the cycle, process 1 2, then vaporized in part, or wholly, or even possibly superheated, all under the highest constant pressure in the cycle, process 2 3, then expanded under reversibly adiabatic conditions 3 4, and finally cooled and condensed to the liquid state under the lowest pressure existing in the cycle.

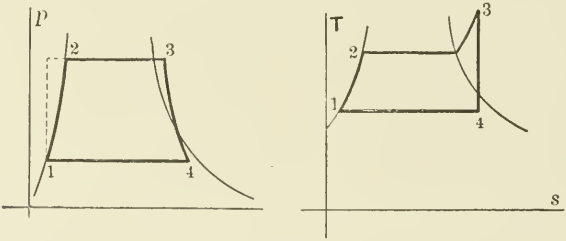

Fig. 57.

It will be noted that the process 1 2 as shown in Fig. 57 is not a constant pressure process. Actually the pressure suddenly increases (as shown by the dotted line) due to the action of the feed-pump and the heat is then supplied to the liquid under a constant pressure until vapor forms at 2. The line 1 2 is however so close to the p-axis and so nearly vertical (due to the small volume and the small change in the volume of liquid) that this distinction need not be made as it does not appreciably affect the calculations.

During the heating of the liquid, process 1 2, Fig. 57, the temperature of the liquid is below the temperature of the hot body, therefore the criterion for best efficiency, page 100, is not fulfilled and the efficiency of the Rankine cycle must be less than the efficiency of the Carnot cycle. For the same reason the Rankine cycle is irreversible in the thermodynamic sense.

The pv-diagram of the Rankine cycle must not be confused with the indicator diagram showing the conditions existing in the cylinder of an engine operating under this cycle. As actually carried out in practice the processes shown in Fig. 57 do not occur while the vapor is in the same cylinder. The processes 1 2 and 2 3 are performed in the feed-pump, the boiler, and the superheater, the process 3 4 occurs in the engine cylinder, and the process 4 1 in the condenser.

Fig. 58 shows the action of the engine cylinder. Here $b\ c$ is the ideal reversible adiabatic expansion corresponding to 3 4, Fig. 57. Only part of the expanded vapor is now displaced from the cylinder. The volume $g\ d$ remains in the cylinder and is compressed adiabatically to the volume $f\ a$. The quality of the vapor at d is the same as the quality at c (or 4, Fig. 57) while at a the state is the same as the state of the vapor about to be admitted to the cylinder, i.e., the state at b (or 3, Fig. 57). Thus vapor whose volume is $d\ c$ (Fig. 58) and whose quality is the same as the quality at 4 (Fig. 57) has been turned into the condenser, then as a liquid after condensation into the boiler, and finally is returned to the cylinder with a volume *a b* (Fig. 58) and superheated as indicated at 3 (Fig. 57).

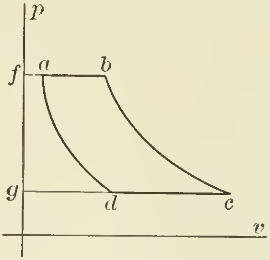

FIG. 58.

EXERCISE 205. Show from Fig. 57 that the net work performed by one pound of vapor during a Rankine cycle is very nearly $J(i_3 - i_4)$ foot-pounds, that the hot body supplies very nearly $i_3 - i_1$ B.t.u. during the cycle, and that therefore the efficiency of the cycle is

$$\eta = \frac{i_3 - i_4}{i_3 - i_1}.$$

EXERCISE 206. An ideal engine operates under a Rankine cycle between 160 and 1 pound per square inch absolute and the cylinder is supplied with dry saturated steam. Compute

(*a*) the thermodynamic efficiency,

(*b*) the work obtained per pound of steam,

(*c*) the steam consumption.

EXERCISE 207. Compare the results of Exercise 206 with the corresponding values for an engine operating under a Carnot cycle during which the steam is returned to the boiler in the liquid form and supplied to the cylinder as dry saturated steam.

EXERCISE 208. Compare the results of Exercise 206 with the results obtained when the steam in the Rankine cycle is supplied to the cylinder superheated 200° F instead of dry saturated.

EXERCISE 209. Sketch on both the *pv*- and the *Ts*-planes a Rankine cycle during whose adiabatic expansion the steam remains superheated.

The Rankine Cycle with Incomplete Expansion.—The large specific volume of steam at low pressures after expansion makes it impracticable to expand steam in the cylinders of a reciprocating engine down to a very low back pressure as shown in Figs. 57 and 58. The large cylinders required for complete expansion cause large friction and radiation losses which more than neutralize the gain even if the size, cost, and weight of the required engines were not prohibitive.

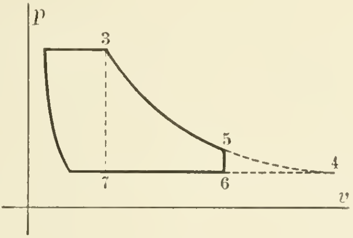

FIG. 59.

In reciprocating engines the steam is released at some pressure higher than the back pressure (5, Fig. 59) and the toe of the indicator card is cut off. The steam changes its state at constant volume during the added process 5 6.

In the steam turbine these disadvantages accompanying complete expansion do not exist, in fact the losses due to leakage past the blades and due to friction (windage) diminish with increasing specific volume of the steam so that the steam turbine is particularly fitted to utilize complete expansion to very low back pressures.

Steam can therefore be employed most advantageously in cylinders of reciprocating engines at high pressures and in turbines at low pressures. For this reason the exhaust of high-pressure reciprocating engines can be used in low-pressure turbines with higher efficiencies than could be obtained with either an all reciprocating or an all turbine installation.

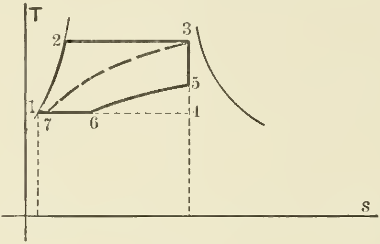

FIG. 60.

The *Ts*-diagram of the Rankine cycle with incomplete expansion (Fig. 60) shows the loss in efficiency due to the drop in pressure at constant volume. Note that the work performed with the same heat supply is diminished by the area 5 4 6 due to incomplete expansion. The extreme case in which the steam is used non-expansively is shown by the cycle 1 2 3 7 1.

EXERCISE 210. Compare the results of Exercise 206 with the results obtained under the same conditions with the exception that the expansion is continued to only 21 pounds per square inch absolute.

SECTION XXXVII

ENGINE EFFICIENCIES

The thermal efficiencies computed in the exercises of the previous sections do not give the complete facts concerning the operation of an actual engine. These efficiencies were computed under the assumptions that the engine parts were non-conducting, that the expansions were reversibly adiabatic (occurred with unchanging entropy), and that no losses due to throttling or wiredrawing occurred.

As the temperature of the steam varies during expansion the temperature of the cylinder must also vary. The high-pressure steam entering the cold cylinder must heat it. This causes a condensation of some of the incoming steam and not only reduces the quality of this steam below the quality in the steam-main but necessitates a greater supply of steam per stroke for the actual cylinder than would be the case in an ideal non-conducting cylinder. Some of this condensed steam is re-evaporated at the end of the stroke after the steam in the cylinder has been cooled by expansion.

Several methods may be used to overcome at least partly this cylinder condensation which greatly lowers the thermal efficiency of the engine. One of these is the use of superheated steam. As Exercise 208 shows, but little is gained in efficiency under ideal conditions by the use of superheated steam. Practically however a greater gain is produced due to the superheated steam losing its superheat in heating the cylinder and yet allowing expansion to begin with practically dry saturated steam.

Starting with the heat supplied by the hot body we have first the unavoidable losses even under ideal conditions which according to the second law of thermodynamics must accompany any transformation of heat into mechanical energy. Of the heat ideally transformable into work only a part is transmitted to the piston owing to unavoidable cylinder losses. Of the work actually reaching the piston only a part is delivered by the engine due to the mechanical friction in the mechanism.

These conditions may be represented as follows:

- Heat supplied
  - Ideally available work
    - Indicated work
      - Delivered work
      - Mechanical losses
    - Cylinder losses
  - Heat necessarily degraded
    - Cycle losses

The ideally available work may be based upon the work obtainable from a Carnot cycle operating under the conditions of the given engine. It is however more usual to use the Rankine cycle with complete expansion as the standard cycle for vapor engines.

If q represents the heat absorbed per pound of vapor used in the engine,

q_r, the heat transformed into work per pound of vapor by an ideal Rankine cycle under the conditions under which the engine runs,

q_a, the actual heat transformed into work per pound of steam supplied to the cylinder of the actual engine as determined by an indicator,

w_b, the work obtained at the brake per pound of steam supplied to the engine,

we have as defining equations the following,

ideal thermal efficiency $=$ cycle efficiency $=\eta_r=\dfrac{q_r}{q}$,

actual thermal efficiency $=$ indicated thermal efficiency $=\eta_a=\dfrac{q_a}{q}$,

cylinder efficiency $=\eta_c=\dfrac{q_a}{q_r}$,

mechanical efficiency $=\eta_m=\dfrac{w_b}{Jq_a}$.

Exercise 211. (*a*) Show that the actual thermal efficiency equals $\eta_c \cdot \eta_r$.

(*b*) Show that the total engine efficiency equals $\eta_r \cdot \eta_c \cdot \eta_m$.

If m_r represents the pounds of vapor required per indicated horse-power hour with the Rankine cycle, and

m_a represents the pounds of vapor required per indicated horse-power hour under the conditions existing in the actual engine,

the heat transformed into work per indicated horse-power hour $=q_r m_r=q_a m_a$.

As the cylinder efficiency is defined as

$$\eta_c=\frac{q_a}{q_r}$$

it follows that

$$\eta_c=\frac{m_r}{m_a}.$$

Exercise 212. An engine supplied with steam at 115 pounds per square inch absolute and containing one per cent of moisture was found to use 21 pounds of steam per indicated horse-power hour when the temperature in the condenser was 140° F. Compute

(*a*) the cylinder efficiency,

(*b*) the heat supplied per i.h.p. per minute,

(*c*) the actual thermal efficiency of the engine.

Exercise 213. An engine test showed the i.h.p. to be 500 and the heat supplied per i.h.p. per minute to be 290 B.t.u. with a boiler pressure of 170 pounds per square inch gage, quality of steam 0.98, vacuum 27 inches, and the barometer at 31 inches. Compute

(*a*) the ideal thermal efficiency,

(*b*) the ideal and

(*c*) the actual steam consumption,

(*d*) the cylinder efficiency,

(*e*) the actual thermal efficiency.

Exercise 214. An engine has a cylinder efficiency of 0.60, a mechanical efficiency of 0.85, and operates under conditions for which the Rankine cycle would transform 200 B.t.u. per pound of steam into work. What is the steam consumption of this engine per brake horse-power hour?

CHAPTER XIII

FLOW OF FLUIDS

SECTION XXXVIII

FUNDAMENTAL EQUATIONS

IN the following discussions the particles of the flowing fluid are assumed to move along stream lines. Any surface composed of stream lines and enclosing a portion of the flowing fluid may be regarded as a tube thru which the fluid flows. Note that no fluid passes thru the walls of this imaginary tube.

Steady flow is also assumed. This means that at any fixed point on any stream line the pressure, the specific volume, the temperature, the velocity, and in the case of wet vapors, the quality of the fluid remain constant.

As shown in Fig. 61 consider a portion of the moving fluid bounded by stream lines and by two sections (marked 1 and 2) normal to these stream lines. These sections may be assumed to be plane.

Let the following notation apply to any section, such as 1 or 2.

W is the weight of fluid passing any section per second in pounds per second,

v, the volume per unit weight of fluid at this section, in cubic feet per pound,

w, the velocity of the fluid, in feet per second at this same section for which

A, represents the area in square feet.

Then the volume of fluid passing this section per second equals both Aw and Wv so that

$$Aw = Wv.$$

As no fluid accumulates between sections the weight of the fluid passing any section must be constant, so that

$$W = \frac{Aw}{v} = \frac{A_1 w_1}{v_1} = \text{a constant.}$$

This is **the equation of continuity,** the first of the two fundamental equations governing the flow of fluids.

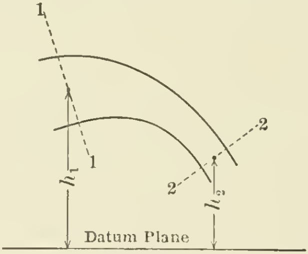

FIG. 61.

The second fundamental equation, a special form of the law of conservation of energy, will now be deduced.

The various forms of energy which may be present during a flow from section 1 to section 2 (Fig. 61) are kinetic energy, energy due to pressure, potential energy due to gravitation, internal energy of the fluid, and heat energy which may be supplied to the flowing fluid during its passing from section 1 to section 2.

Let p represent the absolute pressure at any section,
h, the position of that section with reference to any horizontal datum plane,
u, the internal energy of one pound of fluid at that section, in heat units,

then the kinetic energy of one pound of the fluid is

$$\frac{w^2}{2g}.$$

The energy of one pound of the fluid due to the existing pressure is

$$pv,$$

because as one pound of fluid occupies v cubic feet it could displace a piston (area A square feet) thru $\frac{v}{A}$ feet at the same time exerting a force of Ap pounds and therefore perform $(Ap)\left(\frac{v}{A}\right)=pv$ foot-pounds of work.

The potential energy due to gravitation of one pound of the fluid with reference to the datum plane would be h foot-pounds.

The changes in these various forms of energy in one pound of fluid due to its transfer from section 1 to section 2 (Fig. 61) are

$$\frac{w_2{}^2-w_1{}^2}{2g}, \quad p_2v_2-p_1v_1, \quad h_2-h_1$$

and at the same time the change in the internal energy of the fluid is

$$u_2-u_1.$$

The sum of the changes in all these forms of energy must equal the heat which has been supplied from some outside source to the flowing fluid during its motion from 1 to 2, let this heat per pound of fluid be denoted by ${}_1q_2$, then

$$\frac{w_2^2-w_1^2}{2g}+(p_2v_2-p_1v_1)+(h_2-h_1)+J(u_2-u_1)=J\,{}_1q_2$$

which is **the energy equation.**

Introducing the concept of the heat content i, defined by the equation

$$i=u+\frac{pv}{J},$$

the energy equation becomes

$$\frac{w_2^2-w_1^2}{2g}+(h_2-h_1)+J(i_2-i_1)=J\,{}_1q_2.$$

It will be of interest to note that if no heat is supplied $({}_1q_2=0)$ and if no change in the internal energy occurs $(u_2=u_1)$ the energy equation reduces to

$$\frac{w_2^2-w_1^2}{2g}+(p_2v_2-p_1v_1)+(h_2-h_1)=0.$$

This is Bernoulli's equation as used in hydraulics. For water flowing under these conditions $v_2=v_1=\frac{1}{62.4}$, and we have

$$\frac{w_2^2}{2g}+\frac{p_2}{62.4}+h_2=\frac{w_1^2}{2g}+\frac{p_1}{62.4}+h_1.$$

In the derivation of the above energy equation no mention is made of the effect of frictional resistances.

These would decrease the final kinetic energy owing to the transformation of mechanical energy into heat. This heat would either remain in the fluid or pass to some other body of the system. Thus the frictional resistances would not change the sum of the energies, but would effect a redistribution of energy among the various kinds present in the system.

Another form of the energy equation in which the effect of the frictional resistance appears is derived as follows.

In the fundamental equation

$$dq = du + \frac{1}{J} p dv,$$

in which we may replace u by i by means of the relation

$$i = u + \frac{pv}{J}$$

and obtain $$dq = di - \frac{vdp}{J},$$

q represents all the heat received by the substance either from an external source or generated by frictional resistance from mechanical energy within the substance itself.

Let ${}_1q_2$ be the heat energy received from external sources during the flow from 1 to 2, Fig. 61, and ${}_1f_2$ the mechanical energy which disappears during this flow to reappear as heat, then we have

$${}_1q_2 + \frac{1}{J}\, {}_1f_2 = (i_2 - i_1) - \frac{1}{J}\int_{p_1}^{p_2} vdp.$$

But from the energy equation already deduced

$$J(i_2-i_1)=-\frac{w_2^2-w_1^2}{2g}-(h_2-h_1)+J_1q_2,$$

therefore

$$_1q_2+\frac{1}{J}\,_1f_2=-\frac{1}{J}\frac{w_2^2-w_1^2}{2g}-\frac{1}{J}(h_2-h_1)+_1q_2-\frac{1}{J}\int_{p_1}^{p_2}vdp$$

or

$$\frac{w_2^2-w_1^2}{2g}+_1f_2+(h_2-h_1)=-\int_{p_1}^{p_2}vdp.$$

SECTION XXXIX

ADIABATIC, FRICTIONLESS FLOW

In most practical examples, at least as a first approximation, the flow may be considered frictionless and no heat may be assumed to be exchanged between the fluid and any external body.

Under these conditions $_1f_2=0$ and $_1q_2=0$ and the change of state becomes isentropic.

The equation of continuity is still

$$W=\frac{Aw}{v}=\text{a constant},$$

and the energy equations reduce to

$$\Delta\left(\frac{w^2}{2g}\right)=\frac{w_2^2-w_1^2}{2g}=J(i_1-i_2),$$

and

$$\Delta\left(\frac{w^2}{2g}\right)=\frac{w_2^2-w_1^2}{2g}=-\int_{p_1}^{p_2}vdp.$$

The term (h_2-h_1) is omitted as the difference in heads is usually small in practical applications to vapors and gases.

The second form of the energy equation is so important that it may be well to explain its derivation from the first form under the special conditions of reversible adiabatic change of state.

Under these conditions, by reason of

$$\Delta q = \Delta u + \Delta w$$

as $$\Delta q = 0$$

we have $$\Delta u = -\Delta w$$

or $$J(u_2 - u_1) = \int_{v_1}^{v_2} p\,dv.$$

Thus the energy equation

$$\Delta\left(\frac{w^2}{2g}\right) = J(i_1 - i_2)$$

or as it may be written

$$\Delta\left(\frac{w^2}{2g}\right) = J(u_1 - u_2) + p_1v_1 - p_2v_2$$

becomes $$\Delta\left(\frac{w^2}{2g}\right) = \int_{v_1}^{v_2} p\,dv + p_1v_1 - p_2v_2.$$

In Fig. 62, point 1 represents the state of the fluid existing at section 1, Fig. 61, and point 2 the state existing after flow to section 2, Fig. 61. Under the special conditions the process 1 2 is reversibly adiabatic and the area under this process equals $\int_{v_1}^{v_2} p\,dv$. Also the shaded area, Fig. 62, represents $\int_{v_1}^{v_2} p\,dv + p_1v_1 - p_2v_2$ which in turn equals $\int_{p_2}^{p_1} v\,dp$ or $-\int_{p_1}^{p_2} v\,dp$,

thus $$\Delta\frac{w^2}{2g} = -\int_{p_1}^{p_2} v\,dp.$$

Remember that during reversible adiabatic flow the change in kinetic energy of the flowing fluid equals the work represented by the area behind the curve representing the process on the pv-plane, the shaded area in Fig. 62.

EXERCISE 215. Show that

$$\frac{w_2^2 - w_1^2}{2g} = \frac{m}{m-1}(p_1v_1 - p_2v_2),$$

where $m = k = 1.40$ for ideal (diatomic) gases,

$m = 1.035 + 0.100x_1$, for wet steam with an initial quality x_1,

and $m = 1.31$ for superheated steam.

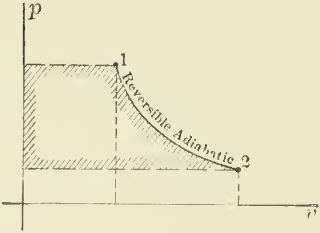

FIG. 62.

EXERCISE 216. Starting with $\Delta\left(\frac{w^2}{2g}\right) = J(i_1 - i_2)$, show that for adiabatic flow

$$\Delta\left(\frac{w^2}{2g}\right) = Jc_p(T_1 - T_2)$$

for ideal gases,

$$\Delta\left(\frac{w^2}{2g}\right) = J(q_1' - x_1r_1) - J(q_2' + x_2r_2) + (p_1 - p_2)v_{32}$$

for wet vapors,

$$\Delta\left(\frac{w^2}{2g}\right) = Jq_1 - J(q_2' - x_2r_2) + (p_1 - p_2)v_{32}$$

for superheated vapors changing to wet vapors during the flow.

EXERCISE 217. If the initial velocity of one pound of steam is zero and the conditions under which expansion occurs are such as to render 300 B.t.u. available for the production of velocity, what will be the velocity of the steam after expansion?

Solve this problem by means of the equations deduced above and check the result by means of the velocity scale on the Mollier diagram.

SECTION XL

FLOW THRU AN ORIFICE

A very short channel whose section continually decreases in the direction of the flow so that the last section has the smallest area is called an orifice. The area of the orifice is the least sectional area of the channel. The flow thru an orifice may be regarded as both adiabatic and frictionless, owing to the small surfaces of contact and to the short time the fluid requires to pass thru the orifice.

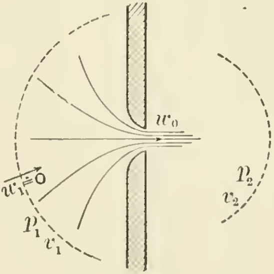

FIG. 63.

Let the state of the fluid on the up-stream side of the orifice be represented by p_1 and v_1 at a point where its velocity is practically zero (Fig. 63), also let A_0 be the area of the orifice, w_0, the velocity of flow at the section whose area is A_0, and p_2 and v_2, the state of the fluid on the downstream side of the orifice.

From the energy equation

$$\Delta\left(\frac{w^2}{2g}\right) = -\int_{p_1}^{p_2} v\,dp$$

we find
$$w_0^2 = 2g\frac{m}{m-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}\right\},$$

where m is the exponent of the equation of the adiabatic process $pv^m = p_1v_1{}^m$.

Exercise 218. Deduce the value of $w_0{}^2$ given above.

From the equation of continuity

$$W = \frac{Aw}{v} = \frac{A_0w_0}{v_2}$$

if we assume that the pressure and the specific volume of the fluid at the mouth of the orifice equal the pressure and the specific volume on the down-stream side of the orifice we find that the pounds of fluid passing the orifice per second equal

$$W = A_0\sqrt{2g\frac{m}{m-1}\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{m}} - \left(\frac{p_2}{p_1}\right)^{\frac{m+1}{m}}\right]}.$$

Exercise 219. Deduce this value of W.

As a special case of the above equations assume the pressure on the down-stream side of the orifice to be maintained at absolute zero, so that $p_2 = 0$. If we also assume, as the conditions seem to justify, that the pressure at the section whose area is A_0 is also $p_2 = 0$ we find the velocity at this section to be

$$w_0 = 2g\frac{m}{m-1}p_1v_1$$

and the discharge in pounds per second to be

$$W = 0.$$

This last conclusion can surely not agree with the facts especially as the velocity attained by the fluid is not zero.

Further investigation of the expressions for w_0 and W shows that as the pressure on the down-stream side of the orifice is made gradually less than p_1 the value of w_0 steadily increases, which is to be expected, but the value of W at first increases, reaches a maximum when the pressure on the down-stream side has been diminished to

$$p_c = p_1 \left(\frac{2}{m+1} \right)^{\frac{m}{m-1}}$$

and then decreases with diminishing down-stream pressure until it becomes zero when p_2 becomes zero. The particular pressure p_c which yields a maximum discharge is called the **critical pressure.**

EXERCISE 220. Deduce the value of the critical pressure given above.

In 1839, Saint Vénant and Wantzel advanced the hypothesis that the pressure at the mouth of an orifice is not always equal to the pressure on the down-stream side of the orifice, that is that p_0 is not always equal to p_2 and that for values of $p_2 < p_c$, p_0 no longer equals p_2 but remains constant and equal to p_c.

EXERCISE 221. The velocity of sound in any fluid equals

$$\sqrt{gmpv},$$

where g is the acceleration due to gravity,
m is the exponent of adiabatic expansion,
p is the pressure,
v is the specific volume,

the last three of the fluid under the conditions existing when the sound is transmitted. Show that the greatest possible velocity attainable by a fluid flowing thru an orifice equals the velocity of sound in this fluid when in the state existing at the mouth of the orifice.

The conclusion reached in Exercise 221 indicates a possible explanation for the existence of the critical pressure. Assume $p_2 < p_c$ then the jet issuing from the orifice at a pressure greater than p_2 and being surrounded by fluid under lesser pressure explodes and the pressure in the jet is suddenly reduced to p_2. This pressure p_2 is propagated toward the mouth of the orifice with the velocity of sound but the transmission of this pressure p_2 up to the mouth of the orifice is prevented by the outrush of the fluid which also moves with the velocity of sound.

Thus according to the hypothesis of Saint Vénant and Wantzel, which has been fully confirmed by experiment, two cases arise during the flow of fluids thru an orifice, namely

(1) when $p_2 > p_c$,

(2) when $p_2 \overset{=}{<} p_c$.

In the first case $(p_2 > p_c)$

$$w_0 = \sqrt{2g\frac{m}{m-1}p_1 v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}\right\}}$$

and
$$W = A_0\left(\frac{p_2}{p_1}\right)^{\frac{1}{m}}\sqrt{2g\frac{m}{m-1}\left(\frac{p_1}{v_1}\right)\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}\right]},$$

the values depending upon the back pressure.

In the second case ($p_2 \leqq p_c$)

$$w_0 = \sqrt{2g\frac{m}{m+1}p_1v_1}$$

and $$W = A_0\left(\frac{2}{m+1}\right)^{\frac{1}{m-1}}\sqrt{\frac{2gm}{m+1}}\sqrt{\frac{p_1}{v_1}},$$

which values are independent of the back pressure.

Exercise 222. Deduce the values of w_0 and W given above.

Exercise 223. Show that **the critical ratio** equals 0.528 for air and 0.577 for dry saturated steam.

Exercise 224. Dry saturated steam escapes to the atmosphere thru an orifice (area 0.3 square inch) from a boiler in which the pressure is maintained at 150 pounds per square inch absolute. Find the velocity of discharge

(*a*) by means of the above theory,
(*b*) by means of $w = \sqrt{2gJ(i_1 - i_2)}$,
(*c*) by means of the Mollier diagram.

Exercise 225. What would be the velocity of discharge in Exercise 224 if the whole fall in pressure could be utilized in producing velocity?

The Discharge thru an Orifice.—The discharge thru an orifice may be computed in any case for any fluid by means of the equations given above, provided the value of m is known. Two well-known formulas which are special cases of the more general formulas already given will now be deduced.

Fliegner's formula for air, for the discharge thru an orifice when $p_2 < p_c$, in terms of the initial pressure p_1 and the initial temperature T_1 of the air may be derived from

$$W = A_0\left(\frac{2}{m+1}\right)^{\frac{1}{m-1}}\sqrt{\frac{2gm}{m+1}}\sqrt{\frac{p_1}{v_1}}$$

by eliminating v_1 by means of the relation

$$p_1v_1=RT_1.$$

In this way we obtain

$$W=A_0\left(\frac{2}{m+1}\right)^{\frac{1}{m-1}}\sqrt{\frac{2gm}{m+1}}\sqrt{\frac{p_1^2}{RT_1}},$$

and as $m=k=1.40$, and $R=53.3$

$$W=0.53A_0\frac{p_1}{\sqrt{T_1}},$$

where W is the discharge of air in pounds per second,

A_0, the area of the orifice in square feet,

p_1, the pressure in pounds per square foot,

T_1, the absolute temperature in degrees F,

the last two on the up-stream side of the orifice.

As the critical ratio for air is 0.528, approximately 0.5, this formula for W can only be used for the discharge into the atmosphere when the reservoir pressure is at least twice the atmospheric pressure. Under these conditions the formula agrees with the results of Fliegner's experiments.

For the discharge of air from reservoirs in which the pressure is less than twice the atmospheric pressure into the atmosphere Fliegner proposed the empirical formula

$$W=1.06A_0\sqrt{\frac{p_a(p_1-p_a)}{T_1}},$$

where p_a is the atmospheric pressure in pounds per square foot. This formula may be used instead of the more cumbersome formula given on page 224.

Grashof's formula for wet steam when the discharge occurs under the condition $p_2 < p_c$,

where $$p_c = p_1 \left\{ \frac{2}{m+1} \right\}^{\frac{m}{m-1}}$$

and $$m = 1.035 + 0.100x_1$$

so that for $x_1 = 1.0$, $m = 1.135$, and $\dfrac{p_c}{p_1} = 0.577$

$x_1 = 0.9$, $m = 1.125$, and $\dfrac{p_c}{p_1} = 0.580$

$x_1 = 0.8$, $m = 1.115$, and $\dfrac{p_c}{p_1} = 0.582$

$x_1 = 0.7$, $m = 1.103$, and $\dfrac{p_c}{p_1} = 0.584$

is based upon the relation

$$p_1 (v_1'')^s = C = \text{a constant}$$

between the pressure and the specific volume of dry saturated steam for which s may be assumed to be 1.063 and for which

$$C = 14.7 \times 144 \times (26.79)^{1.063} = 69{,}600.$$

By means of this relation v_1 may be eliminated from the formula for W. As x_1 is the initial quality of the steam,

$$v_1 = x_1 v_1'' + (1 - x_1) v_1'$$

and as the quality x_1 is usually high and the pressure never excessively great we may assume as an approximation

$$v_1 = x_1 v_1''.$$

Thus $$p_1(v_1'')^s = p_1\left(\frac{v_1}{x_1}\right)^s = C$$

or $$v_1 = x_1\left(\frac{C}{p_1}\right)^{\frac{1}{s}}, \text{ approximately.}$$

Also from $m = 1.035 + 0.100x_1$, as x_1 remains near one we have approximately

$$m = 1.135.$$

Substituting these values of v_1 and m in the formula

$$W = A_0\left(\frac{2}{m+1}\right)^{\frac{1}{m-1}}\sqrt{\frac{2gm}{m+1}}\sqrt{\frac{p_1}{v_1}}$$

we obtain $$W = 0.019\frac{A_0 p_1^{0.97}}{\sqrt{x_1}},$$

where A_0 is measured in square feet, p_1 in pounds per square foot, and W in pounds per second.

If A_0 is expressed in square inches, p_1 in pounds per square inch and W in pounds per second then

$$W = 0.0165\frac{A_0 p_1^{0.97}}{\sqrt{x_1}}.$$

EXERCISE 226. Deduce Grashof's formula.

Napier's formula for dry saturated steam discharging under the conditions $p_2 < 0.58p_1$ is

$$W = \frac{A_0 p_1}{70},$$

where A_0 is to be measured in square inches, p_1 in pounds per square inch and W in pounds per second. It was deduced by Rankine from experiments by Napier. This formula altho not as accurate as Grashof's is still used on account of its simplicity.

The discharge of a vapor thru an orifice can most accurately be computed directly from the fundamental equations

$$w_0 = \sqrt{2gJ(i_1 - i_0)},$$

and

$$W = \frac{A_0 w_0}{v_0}.$$

When the back pressure is less than the critical pressure ($p_2 < p_c$) it must be remembered that p_c and not p_2 must be used in finding i_0 and v_0. Both i_0 and v_0 can most readily be found from the vapor diagrams.

EXERCISE 227. Compute the discharge in pounds per minute under the conditions described in Exercise 224, by means of

(*a*) Napier's formula,
(*b*) Grashof's formula,
(*c*) the steam diagrams.

The formulas for the flow of vapors deduced in this section are strictly applicable only to the flow of vapors initially slightly wet. The moisture must be present in very small drops distributed uniformly thruout the mass of vapor so as to approximate a homogeneous condition. These drops of liquid seem to serve as nuclei or centers of condensation and become larger as the quality of the flowing vapor decreases. If the vapor supplied is dry saturated or superheated it is found that the vapor does not change its quality with the expansion according to the theoretical indications but remains in a supersaturated condition owing to the lack of centers of condensation which seem necessary to start condensation.

If the liquid has separated from the vapor, if it is not uniformly distributed thruout the vapor as fog, it does not serve the purpose above described. It then simply flows along the walls of the orifice with much smaller velocity than the issuing vapor and the formulas deduced do not apply. The liquid under these conditions partially obstructs the orifice, lessens the discharge and if present in considerable amount, causes a sneezing action during discharge.

For further information on this subject the reader is referred to

Callendar: On the Steady Flow of Steam thru a Nozzle or Throttle, Journal I. Mech. E., p. 53, February, 1915; and

Leblanc: Machine Frigorifique a Vapeur D'Eau et a Éjecteur. Gauthier-Villars, Paris, 1911.

Section XLI

FLOW THRU A NOZZLE INCLUDING THE EFFECT OF FRICTION

As shown in the last section the velocity attained by a fluid flowing thru an orifice increases with decreasing back pressure until the back pressure attains a value equal to the critical pressure

$$p_1\left(\frac{2}{m+1}\right)^{\frac{m}{m-1}}$$

and then remains constant irrespective of any additional decrease in back pressure.

Under these conditions the energy $\int_2^{p_1} vdp$ which is liberated by the drop in pressure cannot be wholly converted into kinetic energy of the flowing fluid when the back pressure is less than the critical pressure.

A nozzle is a tube of such form that the fluid flowing thru it attains the full velocity and therefore kinetic energy which may be expected from a given drop in pressure.

The Form of a Nozzle.—Let us investigate the variation in the cross-section of a stream under frictionless, adiabatic flow during which the pressure gradually falls from an initial pressure p_1 to a final pressure p_2 and during which the velocity increases from zero to w_2 in accordance with the law

$$\frac{w^2 - w_1^2}{2g} = \int_p^{p_1} v\,dp$$

no matter how low p_2 may be made.

Under these conditions w continuously equals

$$w = \sqrt{\frac{2gm}{m-1} p_1 v_1 \left[1 - \left(\frac{p}{p_1}\right)^{\frac{m-1}{m}}\right]}$$

and the varying cross-section of the stream

$$A = \frac{vW}{w},$$

where W is a constant.

As $$pv^m = p_1 v_1^m$$

$$v = v_1 \left(\frac{p_1}{p}\right)^{\frac{1}{m}},$$

so that $$A = \frac{W v_1 \left(\frac{p_1}{p}\right)^{\frac{1}{m}}}{\sqrt{\frac{2gm}{m-1} p_1 v_1 \left\{1 - \left(\frac{p}{p_1}\right)^{\frac{m-1}{m}}\right\}}}.$$

To investigate the variation of A as the pressure p falls we first seek maximum and minimum values, by placing

$$\frac{dA}{dp}=0.$$

As $$A=\frac{Wv_1p_1^{\frac{1}{m}}}{\sqrt{\frac{2gm}{m-1}p_1v_1}}\times\frac{p^{-\frac{1}{m}}}{\sqrt{1-\left(\frac{p}{p_1}\right)^{\frac{m-1}{m}}}},$$

any value of p that makes A a maximum or a minimum will also make

$$A'=p^{-\frac{1}{m}}\left\{1-\left(\frac{p}{p_1}\right)^{\frac{m-1}{m}}\right\}^{-\frac{1}{2}}$$

$$=\left\{p^{\frac{2}{m}}-p_1^{\frac{1-m}{m}}p^{\frac{m+1}{m}}\right\}^{-\frac{1}{2}}$$

or even $$A''=p^{\frac{2}{m}}-p_1^{\frac{1-m}{m}}p^{\frac{m+1}{m}}$$

a maximum or a minimum.

Thus the equation

$$\frac{dA''}{dp}=\frac{2}{m}p^{\frac{2-m}{m}}-p_1^{\frac{1-m}{m}}\left(\frac{m+1}{m}\right)p^{\frac{1}{m}}=0$$

yields as a critical value

$$p=p_1\left(\frac{2}{m+1}\right)^{\frac{m}{m-1}}.$$

The cross-section of the stream will thus reach a maximum or a minimum value when the pressure in the fluid drops to the critical value just found. Let the critical pressure be denoted by p_c, the corresponding velocity and specific volume of the fluid by w_c and v_c respectively, and the corresponding cross-sectional area by A_c, then the velocity of the fluid at the critical section is

$$w_c = \sqrt{\frac{2gm}{m+1} p_1 v_1} = \sqrt{gmp_c v_c},$$

the velocity of sound in the fluid under the conditions existing at this section, see Exercise 221.

In order to demonstrate that A_c is a minimum cross-section of the stream it will be more convenient as well as more profitable to develop and study another form of $\frac{dA}{dp}$ instead of using $\frac{d^2A}{dp^2}$ which may be obtained from the value of $\frac{dA}{dp}$ already found.

The fundamental equations governing the flow under consideration are

$$d\left(\frac{w^2}{2g}\right) = -v\,dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$Aw = Wv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$pv^m = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

As we desire to find the change in area of the cross-section due to a change in pressure we must express dA in terms of dp. To obtain dA differentiate equation (2)

$$w\,dA + A\,dw = W\,dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Next eliminate dv and dw and introduce dp. From equation

(1)
$$dw = -\frac{gv\,dp}{w}$$

and from equation (3) or its equivalent

$$\log p + m \log v = \log \text{(a constant)}$$

$$\frac{dp}{p} + m\frac{dv}{v} = 0$$

or

$$dv = -\frac{1}{m}\,\frac{v}{p}\,dp.$$

Substituting these values in equation (4) we find

$$wdA - \frac{Agvdp}{w} = -\frac{W}{m}\,\frac{v}{p}dp$$

or

$$\frac{dA}{dp} = \frac{Agv}{w^2} - \frac{Wv}{wmp}.$$

Eliminating W by means of equation (2) we obtain

$$\frac{dA}{dp} = \frac{A\{gmpv - w^2\}}{mpw^2}.$$

Note that $gmpv$ is the square of the velocity of sound at any section A at which the velocity of the fluid is w in a medium whose pressure and specific volume are p and v.

All quantities in the right-hand member of this equation are necessarily positive; negative values of these quantities are excluded by our physical conception of the problem. However dp is always negative, for the pressure is assumed to decrease continually from the section A_1 to the section A_2.

Let us now study the signs of dA. This sign will determine whether the cross-section of the stream is increasing or decreasing at any point.

(1) As dp is never zero, **dA is zero** when

$$w = \sqrt{gmpv}.$$

That is, the cross-section of the stream is neither increasing nor decreasing, when the velocity of the fluid equals the velocity of sound in the fluid under the conditions existing at the section considered.

(2) As dp is negative, **dA is negative** when

$$w<\sqrt{gmpv}.$$

That is, the cross-section must decrease during that portion of the flow for which the velocity of the fluid is less than the velocity of sound in the fluid at the section considered.

(3) As dp is negative, **dA is positive,** when

$$w>\sqrt{gmpv}.$$

That is, the cross-section of the stream must increase during that portion of the flow for which the velocity of the fluid is greater than the velocity of sound in the fluid at the section considered.

Therefore a nozzle in which a continuous decrease in pressure with a continuous increase in velocity is to occur irrespective of any limiting pressures must at first diminish in section from a very large to a minimum section (called the throat of the nozzle) and then the section must continually increase with increasing velocity and decreasing pressure.

The length of the whole nozzle should be made as short as possible so as to keep down the losses due to friction. The portion of diminishing sectional area can be made very short, the section at first diminishing rapidly and then more slowly up to the throat of the nozzle. For practical reasons of manufacture an extremely short length of the nozzle at the throat is usually made cylindrical.

The portion of increasing sectional area cannot be made as short as the portion of diminishing sectional area for if the section of the nozzle enlarges too rapidly it is found that the stream no longer fills the nozzle, no longer follows the wall, and therefore ceases to expand in the desired manner. The diverging portion of the nozzle is commonly made conical in form and the angle of the cone should be about 6°.

Design of a Nozzle, Neglecting Friction.—The pressure at the throat of a nozzle is

$$p_c = p_1 \left(\frac{2}{m+1} \right)^{\frac{m}{m-1}},$$

the velocity at the throat may be computed by means of

$$w_c = \sqrt{2gJ(i_1 - i_c)}$$

or it may be read directly from the velocity scale on the Mollier diagram. Note that i_c is the heat content corresponding to the critical pressure p_c after a reversible adiabatic expansion from the initial state.

The sectional area at the throat is fixed by the number of pounds of fluid that must be discharged per second. From the equation of continuity we have

$$A_c = \frac{Wv_c}{w_c},$$

where v_c is the specific volume of the fluid under the conditions existing at the throat.

The exit area of the nozzle is found in the same manner by means of

$$w_2 = \sqrt{2gJ(i_1 - i_2)}$$

and $$Wv_2 = A_2 w_2,$$

where the subscripts 2 refer to the conditions existing at the end of the nozzle.

If the nozzle is to deliver energy at a certain rate in the form of kinetic energy the pounds of fluid passing thru the nozzle are computed as follows,

$$W=\frac{(\text{horse-power})(550)}{J(i_1-i_2)}.$$

EXERCISE 228. A nozzle is to deliver 5 horse-power in the form of kinetic energy of the moving fluid. It is to be supplied with steam at 125 pounds per square inch absolute, quality 0.93, and is to discharge against a pressure of 0.50 pound per square inch absolute. Compute the sectional area of the required nozzle at the throat and at exit, neglecting the effect of friction,

(*a*) by means of the Mollier diagram,

(*b*) by means of the steam tables.

The Ts-plane.—The solution of Exercise 228 by means of the steam tables involves the computation of the quality of the steam and the use of this quality in the computation of the drop in the heat content. By means of the graphical representation on the *Ts*-plane formulas may be developed in which the drop in the heat content is expressed in terms of quantities given in the vapor tables.

As $$i=q+\frac{pv_{32}}{J},$$

$$\Delta\left(\frac{w^2}{2g}\right)=J(i_1-i_2)=J(q_1-q_2)+(p_1-p_2)v_{32}.$$

If we assume that the liquid line during heating at the constant pressure p_2 coincides with the corresponding portion of the liquid line during heating at the constant pressure p_1, see page 204, then $q_1 - q_2$, the shaded area Fig. 64, may be expressed as follows when x_1 is the initial quality.

$$q_1 - q_2 = q_1'' - \left(\frac{r_1}{T_1}\right)(1 - x_1)(T_1 - T_2) - \{q_2'' - (s_2'' - s_2')T_2\}.$$

Thus

$$(i_1 - i_2) = i_1'' - i_2'' - \left(\frac{r_1}{T_1}\right)(1 - x_1)(T_1 - T_2) + (s_2'' - s_1'')T_2,$$

in which each expression is either given or can be read directly in the vapor tables.

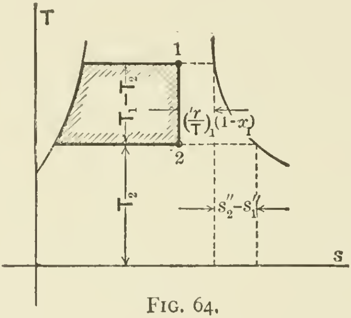

FIG. 64.

EXERCISE 229. Show by means of the Ts-plane that

$$\Delta\left(\frac{w^2}{2g}\right) = J\{i_1'' - i_2'' + (s_2'' - s_1'')T_2\}$$

when the vapor is initially dry saturated and expands at constant entropy.

EXERCISE 230. Compute the energy available for transformation into kinetic energy when one pound of steam expands adiabatically without friction from 200 to 10 pounds per square inch absolute, the initial quality being 0.95.

(*a*) by means of the formulas just developed,

(*b*) by means of the Mollier diagram.

EXERCISE 231. Deduce a formula similar to the one developed in Exercise 229, but for vapors initially superheated.

EXERCISE 232. Show on the Ts-plane an area representing the energy available for transformation into kinetic energy during an isentropic expansion of an ideal gas and from it compute this energy. Compare the result with the result obtained in Exercise 216.

Adiabatic Flow with Friction.—During frictionless, adiabatic flow the increase in kinetic energy due to a given drop in pressure may be represented on the pv-plane (Fig. 65) by the area behind the reversible adiabatic process 1 2, whose equation is

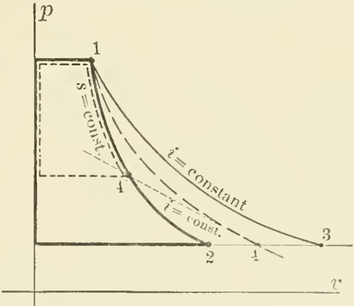

FIG. 65.

$$pv^m = p_1v_1^m.$$

Therefore
$$\frac{w_2^2 - w_1^2}{2g} = \int_{p_2}^{p_1} v\,dp \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Under these same conditions we have

$$\frac{w_2^2 - w_1^2}{2g} = J(i_1 - i_2) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

so that on the Ts-plane (Fig. 66) this increase in kinetic energy is represented by the heavily outlined area behind the line 1 2, plus a small quantity $\frac{(p_1-p_2)v_{32}}{J}$,

for by definition $$i=q+\frac{pv_{32}}{J}$$

and therefore $$i_1-i_2=q_1-q_2+\frac{(p_1-p_2)v_{32}}{J},$$

but q_1-q_2 is represented by the area just described.

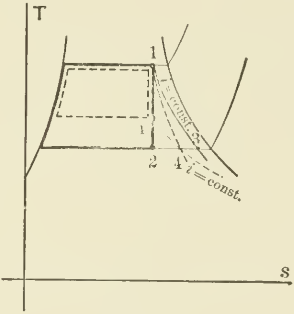

Fig. 66.

Now as an extreme case conceive sufficient frictional resistance during the adiabatic flow to destroy completely all velocity as it develops then $w_1=w=w_3$ and from equation (2)

$$i_1=i_3.$$

Let this final condition be represented by the points 3 in both Figs. 65 and 66 and the process altho adiabatic is now one of constant heat content represented by the line 1 3.

Note carefully that the areas behind the adiabatic, constant heat content lines no longer represent either the work or the heat transformed into kinetic energy. For surely more kinetic energy would not be obtained with the same drop in pressure when energy must be used in overcoming the frictional resistances.

The same reasoning applies to an actual nozzle for which the process lies between lines 1 2 and 1 3. Let the process be represented by 1 4. This also is an adiabatic but it is now neither an isentropic nor a line of constant heat content. The equation

$$\frac{w_4^2-w_1^2}{2g}=J(i_1-i_4)$$

still holds but now

$$\frac{w_4^2-w_1^2}{2g}+{}_1f_4=\int_{p_2}^{p_1} v\,dp,$$

where ${}_1f_4$ is the energy lost in friction between states 1 and 4 per pound of fluid, must be used instead of equation (1) given above, see page 217. Thus the area on the pv-plane behind the curve 1 4 not only represents the mechanical energy which appears as kinetic energy but also the mechanical energy re-transformed by friction into heat.

If thru the point 4 a line of constant heat content be drawn until it intersects the constant entropy line 1 2 at 4′, the areas behind the line 1 4′ represent the kinetic energy available after adiabatic frictional expansion.

The difference between the areas behind the lines 1 2 and 1 4′ does not represent a total loss. This energy is returned to the fluid as heat and causes the final state-point to lie at 4 instead of 4′ or 2, and therefore if a further transformation of heat into mechanical enregy is to occur the fluid contains more available heat energy than it would have contained if its state-point were at 2.

In Fig. 66 we have $i_4 = i_4'$ so that

$$i_2 - i_4' = i_2 - i_4,$$

therefore the area behind 2 4′ and up to the liquid line is practically equal to the area below the line 2 4 and down to the axis of absolute zero temperature.

EXERCISE 233. Deduce formulas for computing the quality of a wet vapor leaving a nozzle after an adiabatic expansion during which y per cent of the ideal heat drop is lost in friction.

EXERCISE 234. Sketch diagrams similar to Figs. 65 and 66 for a vapor expanding from a superheated to a wet condition.

On the is-plane, the Mollier diagram, the process described above on the *pv*- and the *Ts*-planes would appear as shown in Fig. 67. The numbering corresponds thruout with that on the preceding figures.

As it is impossible to locate the state-point 4 directly and as the process 1 4 is unknown it is customary to compute the difference between the heat contents at 1 and 2 assuming reversible adiabatic change of state (at constant entropy and therefore without frictional losses) and then subtract a certain fraction, y, of this heat drop, $i_1 - i_2$, so as to allow for the loss due to friction. On the *is*-plane the distance 1 2 represents $i_1 - i_2$ and the distance 4′ 2 represents $y(i_1 - i_2)$ so that the distance 1 4′ represents

$$(1 - y)(i_1 - i_2).$$

The final velocity of the vapor is computed from

$$\frac{w_4^2 - w_1^2}{2g} = J(1 - y)(i_1 - i_2).$$

The final quality of the vapor however must be read from the point 4 and not from either 4′ or 2.

Design of Nozzles Including Friction.— Computations involving the flow of vapors thru nozzles can most readily be performed by means of the Mollier diagram, Fig. 67. The flow is first assumed reversibly adiabatic and the state-points 1 and 2 are located, then the point 4′ is located so that the distance 2 4′ is the *y*th part of the distance 1 2 where *y* is the given fraction of the ideal heat drop that is lost in friction as determined by experiment. This fraction may vary from 0.08 to 0.20. The actual velocity can then be found from the velocity scale or by means of the fundamental formula.

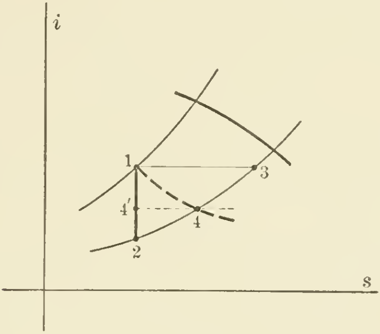

Fig. 67.

From the available heat drop and the given conditions the mass of fluid which must pass thru the nozzle can be determined.

To find the exit area of the nozzle the specific volume of the vapor at exit must be found and this in turn requires that the quality of the vapor leaving the nozzle be known. This quality can be read directly from the Mollier diagram at the state-point 4, Fig. 67, or it may be computed by means of the formulas developed in Exercise 233.

In computing the area at the throat of the nozzle it is customary to assume that the whole loss due to friction occurs between the throat and the exit section of the nozzle. That this assumption is reasonable appears from the facts that the length of the nozzle between entrance and throat is very short and that the velocity in this part of the nozzle is relatively low. Under these assumptions the velocity at the throat is due to the whole difference between the heat contents at the initial pressure and at the critical pressure existing at the throat.

As regards the length of the nozzle nothing can be determined by simple computations. In practice the rules laid down on page 236 are usually followed. For an interesting discussion on the determination of the form of nozzles see the pamphlet by Leblanc referred to on page 230.

Exercise 235. Design a nozzle for the conditions given in Exercise 228 assuming the ratio of the loss due to friction in the expanding portion of the nozzle to the total energy available under ideal frictionless conditions to be 10 per cent.

Section XLII

THROTTLING

A fluid is said to be throttled or wiredrawn when its pressure is diminished due to its flow thru a contracted passage. The flow thru pressure-reducing valves, expansion valves in ammonia refrigerating machinery, valves of steam and gas engines are cases in which throttling occurs.

The pressure on the down-stream side of the constriction depends upon the nature and the state of the fluid, the amount of constriction, and the velocity of approach. It is usually unnecessary to determine this pressure. The determination of the drop in temperature, of the change in specific volume, and of the loss in availability of the energy of the throttled fluid for a given drop in pressure is however of great importance.

During the throttling process illustrated in Fig. 68 some of the potential energy of the fluid is transformed into kinetic energy at the constriction. This kinetic energy is then re-transformed, wholly or in part, into heat by internal friction.

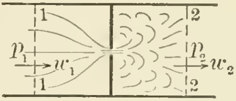

FIG. 68.

If the flow is adiabatic (altho not isentropic) we have

$$\frac{w_2^2 - w_1^2}{2g} = J(i_1 - i_2)$$

as the fundamental equation.

In most cases the kinetic energies per pound of fluid at sections 1 and 2 are relatively small and practically equal so that their difference is zero. This implies that $w_2 = w_1$ and then

$$\mathbf{i}_1 = \mathbf{i}_2.$$

Under these conditions the throttling process is a curve of constant heat content.

EXERCISE 236. Show that **for ideal gases**

$$u = c_v T, \quad i = c_p T,$$

also that the temperature remains unaltered, and that

$$p_1 v_1 = p_2 v_2$$

during throttling for which $w_2 = w_1$.

EXERCISE 237. Show that for ideal gases

$$w_2 = w_1 \frac{A_1 v_2}{A_2 v_1}$$

and that

$$t_1 - t_2 = \frac{1}{J c_p} \frac{w_1^2}{2g} \left[\left(\frac{A_1 v_2}{A_2 v_1} \right)^2 - 1 \right].$$

EXERCISE 238. Air flows from a pipe, in which $w_1 = 300$ feet per second, into a large receiver. Compute the change in temperature of the air.

EXERCISE 239. Show that for **superheated vapor**

$$u = \frac{1}{J} \frac{pv}{m-1}, \quad i = \frac{1}{J} \frac{mpv}{m-1}$$

and that for throttling in which the change in velocity may be neglected

$$p_1 v_1 = p_2 v_2.$$

EXERCISE 240. Assuming as the characteristic equation for superheated steam

$$pv = 85.85T - .256p,$$

show by means of this equation that the throttling of superheated steam for which $w_1 = w_2$ is always accompanied by a drop in temperature.

Von Linde's Process for the Liquefaction of Gases.— An ideal gas suffers no change in temperature when throttling occurs with $w_1 = w_2$. Thomson and Joule showed, in 1853, by means of their porous plug experiments that the throttling of actual gases such as air, nitrogen, oxygen, and carbon dioxide produces a slight drop in the temperature of these gases. This slight cooling of the gases due to throttling was utilized by von Linde for the production of liquid air as follows.

A compressor supplied compressed air at say 100 atmospheres to the inner tube of a long double worm thru which the air passes to a throttling valve. This valve reduces the pressure to say 20 atmospheres and the air suffers a drop of about 27° F in temperature. The air after throttling passes thru a vessel arranged to catch the liquid air produced and then returns thru the outer tube of the double worm to the compressor.

If the original temperature of the air after leaving the cooling coils placed between the compressor and the double worm is say 70° F, the temperature after throttling will be 43° F. This colder air returning thru the outer worm to the compressor cools the air on its way to the throttling valve. The air arriving at the valve soon has a temperature of 43° F, and the throttling cools it to 16° F. After the machine has been running for several hours the air finally reaches the state of a saturated vapor and liquid air is deposited after throttling. The whole machine must of course be thoroly lagged and a separate compressor must transfer air from the atmosphere to the high-pressure cycle.

The Throttling of Wet Vapors.—As already shown $i_1 = i_2$ whenever $w_1 = w_2$.

EXERCISE 241. Show that the quality of a wet vapor equals

$$x_2 = \frac{i_1' - i_2'}{r_2} + x_1 \frac{r_1}{r_2}$$

after throttling provided $w_1 = w_2$.

Exercise 242. What percentage of moisture may steam at 150 pounds per square inch absolute contain if throttling to 15 pounds per square inch absolute is to dry it?

Solve by means of (*a*) the steam tables, (*b*) the Mollier diagram.

Exercise 243. Steam initially at 150 pounds per square inch absolute is throttled to 20 pounds per square inch absolute. The observed temperature after throttling is 248° F. Find the initial quality by means of (*a*) the steam tables, (*b*) the Mollier diagram.

Exercise 244. Develop a formula for obtaining the initial quality of steam by means of the readings made on a throttling calorimeter and by means of the steam tables.

The Loss of Availability Due to Throttling.—On page 241, during the discussion of the effect of friction on the flow of fluids, it has already been shown that an adiabatic is not necessarily an isentropic curve.

During a reversible adiabatic process the fluid does work during the frictionless expansion either upon a piston or by imparting kinetic energy to its own mass. Under these conditions the entropy of the fluid remains constant and the adiabatic process is also an isentropic process.

During an irreversible adiabatic process the entropy increases even tho no heat reaches the fluid from an external source. Under these conditions some, or all, of the kinetic energy developed during the flow is transformed back into heat and this heat so returned to the fluid causes the increase in entropy.

During throttling for which $w_1 = w_2$ and during which adiabatic conditions exist no heat is lost and yet the ability of the fluid to do work has decreased. No energy has been lost or gained but the availability of this energy has been diminished.

To illustrate, consider an adiabatic flow of an ideal gas for which $w_1=w_2$ and therefore $i_1=i_2$, $T_1=T_2$, and $p_1v_1=p_2v_2$. Thus this process altho adiabatic is also isothermal as represented by the lines 1 2 in Figs. 69 and 70. The gas has passed thru the process 1 2 and yet the area under 1 2 in Fig. 69 does not represent the work done. That is even under ideal conditions no mechanical energy results from this change of state so that no energy can be stored so as to be available in assisting the return of the gas to its initial condition. It might be supposed that as the whole heat energy in the gas at 1 is still in the gas under the conditions represented by 2 the return of the gas to the condition 1 would require no external supply of energy.

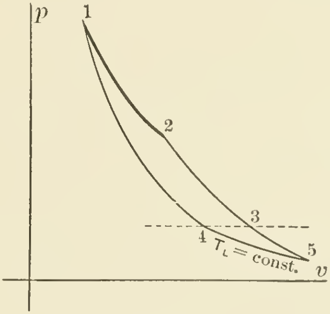

FIG. 69.

That this is not so is evident when we consider the effect of compressing the gas isothermally from 2 to 1. The external work required to do this is represented by the area under 2 1 in Fig. 69. When this work has been supplied the gas is exactly in its original state but some associated body must contain the heat rejected by the gas during this process. This heat, equivalent to the work done, cannot be completely transformed into work, thus availability has been lost. The work done by and the heat stored in bodies associated with the gas during the reversible isothermal compression equals

$$\frac{1}{J} p_1 v_1 \log \frac{v_2}{v_1}$$

heat units per pound of gas. If T_L is the lowest obtainable temperature then only

$$\left(\frac{T_1 - T_L}{T_1}\right) \frac{1}{J} p_1 v_1 \log \frac{v_2}{v_1}$$

can be again transformed into work under even the most ideal conditions.

As $$s_2 - s_1 = \frac{\Delta Q}{T_1} = \frac{1}{J} \frac{p_1 v_1}{T_1} \log \frac{v_2}{v_1}$$

the **loss of availability** due to the process 1 2 may be expressed as follows,

$$\frac{1}{J}(p_1 v_1) \log \frac{v_2}{v_1} - \left(1 - \frac{T_L}{T_1}\right) \frac{p_1 v_1}{J} \log \frac{v_2}{v_1}$$

$$= (s_2 - s_1) T_L,$$

the product of the change in the entropy of the gas due to the irreversible process and the lowest obtainable temperature expressed in absolute degrees.

It should be noted that the area under the process 1 2 in Fig. 70 does not represent heat supplied to the gas during the irreversible adiabatic process 1 2. It does represent the heat equivalent of the work that must be performed and of the heat that must be extracted from the gas during its change back to the initial state by reversible isothermal compression from 2 1 along the same process line.

The gas may be returned to its initial state 1 by a smaller expenditure of work than that required by the isothermal compression explained above. For instance let the horizontal dotted line in Fig. 69 represent the lowest obtainable pressure (the atmospheric pressure) then the gas may under ideal conditions be conceived to be expanded along a reversible adiabatic from 2 to 3, then cooled at constant pressure from 3 to 4, and finally compressed along a reversible adiabatic from 4 to 1. The energy expended during these processes is less than the work required for the isothermal compression discussed above.

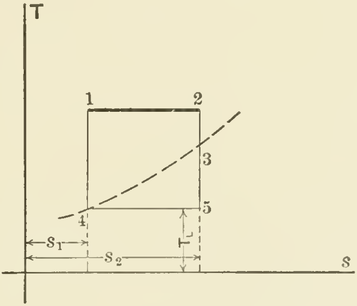

FIG. 70.

EXERCISE 245. Show that the energy required to produce the changes of state indicated by the processes 2 3 4 1, Fig. 69, amounts to $Jc_p(T_3-T_4)$ per pound of gas.

EXERCISE 246. Obtain the result of Exercise 245 from Fig. 70.

Even less energy than this would suffice to return the gas from state 2 to state 1 if instead of the lowest obtainable pressure use is made of the lowest obtainable temperature. As shown in Figs. 69 and 70, the gas at 2 may be expanded along a reversible adiabatic to 5, then compressed along a reversible isothermal at the lowest obtainable temperature to 4, and finally compressed adiabatically at constant entropy to 1. The work obtained from the gas during the process 2 5 is equal to the work required to compress the gas from 4 to 1. But during the process 5 4 work must be done on the gas by some associated body and heat equivalent to this work must be rejected by the gas to some associated body. This heat, as may be seen from Fig. 70, equals

$$T_L(s_5-s_4),$$

where T_L is the lowest obtainable temperature, and it is wholly unavailable for transformation into mechanical energy for its temperature is already the lowest obtainable.

The loss of availability due to the irreversible adiabatic process equals the increase in entropy during the irreversible process multiplied by the lowest obtainable temperature expressed in absolute units. It is still the same as the loss due to the direct reversible isothermal compression from 2 to 1.

During the throttling of a vapor similar conditions exist. As shown in Fig. 71 in which the points are numbered so as to agree with Figs. 69 and 70, the irreversible adiabatic 1 2 coincides with a constant heat content line, for now $i_1=i_2$. The series of processes by means of which the vapor at 2 can be returned to its initial condition 1 with the least expenditure of external energy is evidently represented by the line 2 5 4 1, provided the temperature along 5 4 is the lowest obtainable temperature T_L. Here also the loss of availability due to the irreversible process 1 2 is

$$T_L(s_2 - s_1).$$

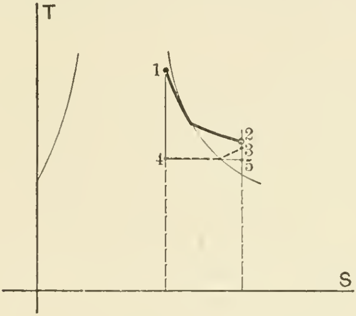

FIG. 71.

EXERCISE 247. Steam is supplied to an engine the back pressure in which is 3 pounds per square inch absolute. The steam is throttled from 150 pounds per square inch absolute and a quality of 0.99 to 120 pounds per square inch absolute before entering the engine. Compute the loss of available energy per pound of steam due to throttling.

SECTION XLIII

VENTURI METERS

Venturi meters are used for measuring the flow of fluids in pipes. These meters are especially useful when the quantity of fluid to be measured is so large that the displacement meters of the usual sort capable of handling the flow would be very large and correspondingly costly.

As already described in hydraulics the Venturi meter, Fig. 72, consists simply of a gradual reduction in the cross-section of the pipe line and a gradual enlargement of the section to its original diameter. Provision must be made for measuring the pressures at the up-stream section of the pipe before contraction occurs and at the throat of the meter.

By means of these pressures, the areas at the corresponding sections of the pipe, and the physical constants of the fluid, the velocity and the quantity of fluid passing any section can be computed.

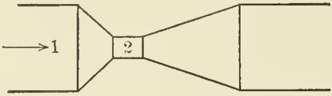

FIG. 72.

Let the subscripts 1 denote the state of the fluid in the up-stream section of the pipe and the subscripts 2 the state at the throat, then the fundamental equation is

$$\frac{w_2^2-w_1^2}{2g}=J(i_1-i_2)=\int_{p_2}^{p_1} v\,dp=\frac{m}{m-1}(p_1v_1-p_2v_2)$$

$$=\frac{m}{m-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}\right\},$$

where m is the exponent of the equation for reversible adiabatic change of state which is assumed to occur between 1 and 2, so that

$$\frac{p_1}{p_2}=\left(\frac{v_2}{v_1}\right)^m.$$

If W represents the pounds of fluid passing any section per second

$$W=\frac{A_1w_1}{v_1}=\frac{A_2w_2}{v_2}$$

and

$$w_2=w_1\frac{A_1v_2}{A_2v_1}=w_1\frac{A_1}{A_2}\left(\frac{p_1}{p_2}\right)^{\frac{1}{m}}.$$

EXERCISE 248. Show that in a Venturi meter

$$w_1{}^2=\frac{2gm}{m-1}p_1v_1\left\{\frac{1-\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}}{\left(\frac{A_1}{A_2}\right)^2\left(\frac{p_1}{p_2}\right)^{\frac{2}{m}}-1}\right\},$$

and that

$$W=A_1\sqrt{\frac{2gm}{m-1}\,\frac{p_1}{v_1}\left\{\frac{1-\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}}{\left(\frac{A_1}{A_2}\right)^2\left(\frac{p_1}{p_2}\right)^{\frac{2}{m}}-1}\right\}}.$$

For air $m=k=1.40$ and v_1 may be computed from $p_1v_1=RT_1$ by means of the temperature of the air taken in the up-stream section of the pipe.

For steam the value of W may be computed more readily and more accurately from the fundamental equations and the steam tables or diagrams than by means of the formulas developed in Exercise 248.

Thus we have

$$\frac{w_2{}^2-w_1{}^2}{2g}=J(i_1-i_2) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where (i_1-i_2) can be read from the Mollier diagram.

Also as
$$W=\frac{A_1 w_1}{v_1}=\frac{A_2 w_2}{v_2}$$

$$w_2=w_1\frac{A_1}{A_2}\frac{v_2}{v_1} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

from which, by means of the specific volumes v_1 and v_2, w_2 can be found in terms of w_1. Substituting this value in equation (1), we find w_1 and finally

$$W=\frac{A_1 w_1}{v_1}$$

yields the number of pounds of steam passing thru the pipe.

Experiments have shown that the Venturi meter furnishes a very reliable and accurate means of measuring the flow of fluids.

Exercise 249. A Venturi meter has a diameter of 6 inches at entrance and 3 inches at the throat. The steam entering this meter contains one per cent of moisture and is under a pressure of 120 pounds per square inch absolute. The pressure at the throat is 100 pounds per square inch absolute. How many pounds of steam pass thru the meter per second?

Section XLIV

FLOW THRU PIPES

The Hydraulic Formula.—The flow thru pipes of constant diameter will now be considered. As demonstrated in hydraulics the flow of liquids thru pipes is accompanied by a loss in pressure head and therefore of energy due to the frictional resistances encountered. This loss of head is

$$h_f=\frac{p_1-p_2}{w}=\frac{4fl}{d}\frac{v^2}{2g},$$

where f is the friction factor,
w the specific weight of the liquid in pounds per cubic foot,
v the velocity of flow in feet per second,
l the length of the pipe in feet,
d the diameter of the pipe in feet,
$p_1 - p_2$ the loss of pressure in pounds per square foot,
and $g = 32.2$ feet per second per second.

This formula expressed in the notation we have used in thermodynamics becomes

$$h_f = (p_1 - p_2)v = \frac{4fl}{d}\frac{w^2}{2g},$$

where v is the specific volume in cubic feet per pound
and w is the velocity of flow in feet per second.

When dealing with gases and vapors v and w cannot be regarded as constant because a decrease in pressure causes an appreciable change in the specific volume v.

The equation of continuity is now

$$W = \frac{Aw}{v} = \text{a constant},$$

and not $$Aw = \text{a constant},$$

as is the case when the flow of liquids is considered.

If however the drop in pressure is slight it may be assumed that the change in specific volume is small and as an approximation v may be regarded as constant.

The friction factor, f, is not constant. It depends upon the diameter of the pipe and the nature of and the specific volume of the fluid. Unwin deduced from experiments on the flow of gases

$$f = 0.0027\left(1 + \frac{3}{10d}\right), \text{ for air},$$

and $$f=0.0044\left(1+\frac{1}{7d}\right), \text{ for illuminating gas,}$$

where d is the diameter of the pipe in feet.

Assuming the specific volume of the fluid to remain constant we have

$$h_f=(p_1-p_2)v=f\frac{4l}{d}\frac{w^2}{2g},$$

so that $$p_1-p_2=f\frac{4l}{vd}\frac{w^2}{2g}.$$

It is customary to express this result in terms of V, the volume of fluid passing into the pipe per second.

As $$V=w\left(\frac{\pi d^2}{4}\right) \quad \text{or} \quad w=\frac{4V}{\pi d^2},$$

where the velocity w is assumed to remain constant thruout the length of the pipe,

we have $$p_1-p_2=\frac{32flV^2}{\pi^2gvd^5}$$

or $$V=\sqrt{\frac{\pi^2gvd^5(p_1-p_2)}{32fl}}=C\sqrt{\frac{vd^5(p_1-p_2)}{l}}.$$

This is known as **D'Arcy's formula.**

EXERCISE 250. The following formula based upon D'Arcy's formula is used by engineers for all gases and vapors. When p_1-p_2 is small it yields fairly accurate answers.

$$W=\frac{V_1}{v_1}=\sqrt{\frac{7000d^5(p_1-p_2)}{v_1l\left(1+\frac{3.6}{d}\right)}},$$

where W represents the pounds, and
V_1 the cubic feet of fluid passing into the pipe per minute,

d is the internal diameter of the pipe in inches,

l is the length of the pipe in feet,

v_1 is the specific volume of the fluid entering the pipe in cubic feet per pound,

$p_1 - p_2$ is the drop in pressure in pounds per square inch.

What is the value of f used in this formula?

EXERCISE 251. What weight of dry saturated steam under an initial pressure of 120 pounds per square inch absolute may be expected to pass thru a 4-inch pipe 700 feet long when the drop in pressure is 6 pounds per square inch?

EXERCISE 252. If the quality of steam flowing thru the pipe in Exercise 251 were doubled what would be the drop in pressure provided the initial pressure is maintained at its original value?

EXERCISE 253. Ten drills require 1100 cubic feet of free air (measured at 14.7 pounds and 60° F) per minute, what should be the diameter of a pipe 5000 feet long supplying these drills with compressed air at 55 pounds per square inch gage, the drop in pressure in the pipe line not to exceed 5 pounds and the air entering the pipe having a temperature of 60° F?

How fast does the air flow thru this pipe?

EXERCISE 254. It is stated that the approximate diameter of a pipe line may be obtained from the following formula

$$d = \sqrt[5]{\frac{0.0003 W^2 l v}{(p_1 - p_2)}},$$

in which the letters have the same significance as in Exercise 250.

(*a*) For what value of d is this formula correct?

(*b*) Solve Exercise 253 by means of this formula.

A More Accurate Formula for the flow of fluids thru pipes may be deduced by taking into consideration the variation of the specific volume and of the velocity of the flowing fluid.

The fundamental equations are the equations of continuity

$$W=\frac{Aw}{v}=\text{a constant},$$

and the energy equation deduced on page 217,

$$\frac{w_2{}^2-w_1{}^2}{2g}+{}_1f_2=-\int_{p_1}^{p_2}v\,dp,$$

where ${}_1f_2$ represents the energy lost thru friction per pound of fluid. This ${}_1f_2$ equals h_f so that

$${}_1f_2=h_f=f\frac{4l}{d}\frac{w^2}{2g}.$$

As w is a variable h_f varies. Assuming f constant for a given pipe and a given gas and placing

$$C=\frac{4f}{2gd}$$

the head lost per differential length of the pipe line may be expressed as follows,

$$d(h_f)=C(dl)w^2.$$

Substituting this value into the differential form of the energy equation which is

$$\frac{d(w^2)}{2g}+d({}_1f_2)+v\,dp=0$$

we have

$$\frac{d(w^2)}{2g}+C(dl)w^2+v\,dp=0$$

or

$$\frac{1}{2g}\frac{d(w^2)}{w^2}+Cdl+\frac{v\,dp}{w^2}=0.$$

The first two terms of this equation can be readily integrated. The third term must first be expressed in terms of a single variable, preferably p, before it can be integrated.

The equation of continuity affords a means of expressing w in terms of v

$$w=\frac{w_1}{v_1}v,$$

for the diameter of the pipe is assumed constant.

To express v in terms of p let us assume that the favorite polytropic relation of thermodynamics $pv^n=p_1v_1^n$, where n is a constant, holds during the flow thru the pipe. Then

$$\frac{vdp}{w^2}=\frac{v_1^2dp}{w_1^2v}=\frac{v_1}{w_1^2}\left(\frac{p}{p_1}\right)^{\frac{1}{n}}dp$$

and the integral of the energy equation is

$$\frac{1}{2g}\log_\epsilon w^2+Cl+\frac{n}{n+1}\frac{v_1}{w_1^2p_1^{\frac{1}{n}}}p^{\frac{n+1}{n}}=C_1.$$

To determine the constant of integration C_1, note the following conditions at the beginning and at the end of the pipe,

$$p=p_1,\qquad w=w_1,\qquad l=0$$

$$p=p_2,\qquad w=w_2,\qquad l=L,$$

where L is the length of the pipe. By means of the conditions at the beginning of the pipe we obtain

$$C_1=\frac{1}{2g}\log_\epsilon w_1^2+\frac{n}{n+1}\frac{v_1p_1}{w_1^2}.$$

The energy equation becomes

$$\frac{1}{2g}\log_\epsilon\frac{w^2}{w_1^2}+Cl+\frac{n}{n+1}\frac{v_1p_1}{w_1^2}\left\{\left(\frac{p}{p_1}\right)^{\frac{n+1}{n}}-1\right\}=0.$$

As we seek the velocity at entrance to the pipe note that

$$\frac{w}{w_1}=\frac{v}{v_1}=\left(\frac{p_1}{p}\right)^{\frac{1}{n}}$$

and solve

$$\frac{1}{n}\log_\epsilon\left(\frac{p_1}{p}\right)+gCl+g\frac{n}{n+1}\frac{v_1p_1}{w_1^2}\left\{\left(\frac{p}{p_1}\right)^{\frac{n+1}{n}}-1\right\}=0$$

for w_1, obtaining

$$w_1=\sqrt{\frac{g\frac{n}{n+1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right\}}{\frac{2fL}{d}+\frac{1}{n}\log_\epsilon\frac{p_1}{p_2}}}.$$

When L is large, the term $\frac{1}{n}\log_\epsilon\frac{p_1}{p_2}$ may be neglected. Also the temperature of the gas in the pipe may be assumed to remain constant, because any heat generated by friction would be dissipated by radiation and conduction. Therefore in $pv^n=$ a constant, n may be assumed to be unity. With these assumptions

$$w_1=\sqrt{\frac{gdp_1v_1}{4fL}\left\{\frac{p_1^2-p_2^2}{p_1^2}\right\}},$$

or

$$w_1=\sqrt{\frac{gdRT_1}{4fl}\left\{1-\left(\frac{p_2}{p_1}\right)^2\right\}}.$$

EXERCISE 255. Show that

$$W^2=\frac{\pi^2d^5g(p_1^2-p_2^2)}{64fLRT_1},$$

where W is in pounds per second.

EXERCISE 256. Show that the cubic feet of free air (measured at 14.7 pounds and 60° F) flowing per minute thru a pipe d inches in actual diameter and L feet long is

$$V=3.04\sqrt{\frac{d^5 p_1^2}{fL}\left\{1-\left(\frac{p_2}{p_1}\right)^2\right\}},$$

the pressures being in pounds per square inch absolute and the temperature of the air entering the pipe being 60° F.

EXERCISE 257. If the drop in pressure in the pipe line is small show that D'Arcy's formula may be derived as an approximation from the result of Exercise 255.

EXERCISE 258. A cast iron pipe 10.3 miles long and 0.98 foot in diameter is to deliver 135 cubic feet of free air per second. For this pipe $f=0.0023$. The initial pressure is 92 pounds per square inch gage. Compute the terminal pressure, the temperature thruout the pipe line being 60° F

(*a*) by means of the more accurate method,

(*b*) by means of the hydraulic method.

(*c*) Compute the initial and the final velocity of flow in this pipe.

CHAPTER XIV

GENERAL EQUATIONS OF THERMODYNAMICS

Section XLV

DIFFERENTIAL EXPRESSIONS FOR THE HEAT SUPPLIED

The State of a Body is not determined by its temperature alone. It has already been shown that for ideal gases and even for superheated vapors **three quantities are sufficient to determine the state** of the body. These may be the temperature, the pressure, and the specific volume and they are always so related by means of a characteristic equation that any two being arbitrarily assumed the third can be found.

It is by no means true that these or any other three quantities always determine the state of any body. Fortunately three variables suffice for many cases and to such cases the following discussions and equations are limited.

Mathematically this condition is expressed when the characteristic equation of the substance is

$$f(p, v, t)=0,$$

or

$$t=\phi(p, v).$$

The Heat Absorbed During any Change of State. By reason of the above assumption any two of the three variables p, v, and t may in general be assumed to be the independent variables during any change of state and the heat absorbed during this change of state may be computed in terms of these variables and of certain thermal capacities.

Assume p and v as the independent variables. Geometrically this means that the projection on the pv-plane of the actual change of state which occurs on the characteristic surface (Fig. 2) is to be considered.

The heat absorbed during any differential change of state when projected on the pv-plane may be conceived to occur in two steps one while v remains constant, the other while p remains constant (Fig. 73).

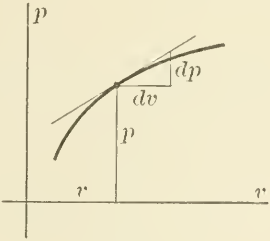

FIG. 73.

Let $\left(\frac{\partial q}{\partial v}\right)_p \equiv h_p$ be the heat absorbed by a unit mass of the substance **per unit change in volume while the pressure remains constant,**

and $\left(\frac{\partial q}{\partial p}\right)_v \equiv h_v$ be the heat absorbed by a unit mass of the substance **per unit change in pressure while the volume remains constant.**

Then if dq represents the whole heat absorbed per unit mass of the substance during the complete change of state

$$dq = \left(\frac{\partial q}{\partial v}\right)_p dv + \left(\frac{\partial q}{\partial p}\right)_v dp, \quad . \quad . \quad . \quad . \quad (1)$$

because the actual change in volume is dv and the actual change in pressure is dp.

The change in temperature does not appear explicitly in this equation. It is taken care of by the relation $f(p, v, t)=0$ which means that p and v cannot change without a corresponding definite change in t.

dq is not the heat absorbed along a differential element of a line on the pv-plane, but it is the heat absorbed during the corresponding actual change represented on the characteristic surface.

Two other expressions, similar to equation (1), for the same dq can be deduced from the projections of the actual change of state upon the other two coordinate planes.

If $\left(\frac{\partial q}{\partial t}\right)_p \equiv c_p$ represents the heat absorbed by a unit mass of the substance **per unit change in temperature while the pressure remains constant,**

and $\left(\frac{\partial q}{\partial p}\right)_t \equiv l_p$ represents the heat absorbed by a unit mass of the substance **per unit change in pressure while the temperature remains constant,**

then

$$dq=\left(\frac{\partial q}{\partial t}\right)_p dt+\left(\frac{\partial q}{\partial p}\right)_t dp. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also if $\left(\frac{\partial q}{\partial t}\right)_v \equiv c_v$ represents the heat absorbed by a unit mass of the substance **per unit change in temperature while the volume remains constant,**

and $\left(\frac{\partial q}{\partial v}\right)_t \equiv l_v$ represents the heat absorbed by a unit mass of the substance **per unit change in volume while the temperature remains constant,**

we have
$$dq=\left(\frac{\partial q}{\partial t}\right)_v dt+\left(\frac{\partial q}{\partial v}\right)_t dv. \quad . \quad . \quad . \quad . \quad (3)$$

It will be noticed that c_p and c_v are by definition the familiar specific heats at constant pressure and at constant volume respectively.

l_p and l_v are called latent heats because the temperature does not change during the changes of state to which they apply.

h_p and h_v have received no special names.

These six coefficients in the differential expressions for the heat absorbed are called **thermal capacities.**

In order to use equations (1), (2), and (3) the relations of the thermal capacities to each other and to the variables p, v, and t must be established.

Relations Between the Thermal Capacities.—Only four equations are available to determine the six thermal capacities. They are the characteristic equation of the substance, $f(p, v, t)=0$, and the three expressions for dq. Therefore two thermal capacities must be known before the others can be computed. Four independent relations between the thermal capacities can be established.

From equations (2) and (3) we obtain by elimination of dq

$$c_p dt+l_p dp=c_v dt+l_v dv. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This equation contains the differentials of the three variables p, v, and t. Any one of these can be eliminated by means of the characteristic equation

$$f(p, v, t)=0.$$

Let us eliminate dt. To do this put $f(p, v, t)=0$ in the form $t=\phi(p, v)$, from which we obtain

$$dt=\left(\frac{\partial t}{\partial v}\right)_p dv+\left(\frac{\partial t}{\partial p}\right)_v dp.$$

Exercise 259. Find dt for an ideal gas by means of this equation.

The above indicated elimination gives

$$(c_p-c_v)\left\{\left(\frac{\partial t}{\partial v}\right)_p dv+\left(\frac{\partial t}{\partial p}\right)_v dp\right\}+l_p dp=l_v dv$$

or $$\left\{(c_p-c_v)\left(\frac{\partial t}{\partial v}\right)_p-l_v\right\}dv+\left\{(c_p-c_v)\left(\frac{\partial t}{\partial p}\right)_v+l_p\right\}dp=0.$$

Altho the value of dt depends upon the values of both dp and dv, dp and dv are themselves independent of each other (no definite line on the characteristic surface having been assumed). dp and dv may thus vary in any manner with various changes of state. Therefore the only way in which the last equation can be true for all values of dp and dv is under the condition that the coefficients of dp and dv be both separately equal to zero.

Thus $$(c_p-c_p)=l_v\left(\frac{\partial v}{\partial t}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and $$(c_p-c_v)=-l_p\left(\frac{\partial p}{\partial t}\right)_v. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Other relations between the thermal capacities may be found by eliminating v from equation (4) by means of

$$dv=\left(\frac{\partial v}{\partial p}\right)_t dp+\left(\frac{\partial v}{\partial i}\right)_p dt,$$

obtained from $f(p, v, t)=0$.

In this way the relations

$$(c_p - c_v) = l_v\left(\frac{\partial v}{\partial t}\right)_p,$$

and

$$l_p = l_v\left(\frac{\partial v}{\partial p}\right)_t$$

are established.

The first one of these has already been found, equation (5). The second one altho apparently a new relation is really a result which may be obtained from equations (5) and (6) as follows.

From the calculus we have

when

$$z = f(x, y),$$

$$\frac{\partial y}{\partial x} = -\frac{\dfrac{\partial z}{\partial x}}{\dfrac{\partial z}{\partial y}}.$$

Therefore from the equation

$$t = \phi(p, v)$$

$$\frac{\partial p}{\partial v} = -\frac{\dfrac{\partial t}{\partial v}}{\dfrac{\partial t}{\partial p}}.$$

From equations (5) and (6)

$$l_v\left(\frac{\partial v}{\partial t}\right) = -l_p\left(\frac{\partial p}{\partial t}\right)$$

or

$$l_p = -\frac{\dfrac{\partial v}{\partial t}}{\dfrac{\partial p}{\partial t}} l_v$$

and

$$l_p = \frac{\partial v}{\partial p} l_v.$$

By the elimination of p from equation (4), we obtain the relations

$$(c_p-c_v)=-l_p\left(\frac{\partial p}{\partial t}\right)_v$$

and
$$l_v=l_p\left(\frac{\partial p}{\partial v}\right)_t,$$

both of which have already been obtained.

Thus so far we have found **two** relations for the determination of **four** thermal capacities.

EXERCISE 260. Assuming c_p and k to be known for a certain ideal gas, compute the latent heats of this gas.

Starting with equations (1) and (2), we have

$$h_p dv+h_v dp=c_p dt+l_p dp. \quad . \quad . \quad . \quad . \quad (7)$$

Eliminating dv the relations

$$l_p-h_v=h_p\left(\frac{\partial v}{\partial p}\right)_t$$

and
$$c_p=h_p\left(\frac{\partial v}{\partial t}\right)_p$$

are obtained.

EXERCISE 261. Deduce the relations

$$l_p-h_v=-c_p\left(\frac{\partial t}{\partial p}\right)_v,$$

$$l_v-h_p=-c_v\left(\frac{\partial t}{\partial v}\right)_p,$$

$$l_v-h_p=+h_v\left(\frac{\partial p}{\partial v}\right)_t,$$

and
$$c_v=h_v\left(\frac{\partial p}{\partial t}\right)_v.$$

The nine relations between the thermal capacities deduced above are not independent relations. As there exist only four equations which can be used to determine these relations only four independent relations can be found. Any four of the above nine relations involving the six thermal capacities will serve to compute the six thermal capacities when two of them are known.

SECTION XLVI

EXACT DIFFERENTIALS

Definition.—Many of the differential equations of thermodynamics are of the form

$$dz = M\,dx + N\,dy,$$

where M and N may be constants or functions of the variables x and y.

As an example we have

$$dt = \left(\frac{\partial t}{\partial v}\right)_p dv + \left(\frac{\partial t}{\partial p}\right)_v dp, \quad . \quad . \quad . \quad . \quad (a)$$

which is obtained by differentiating

$$t = \phi(p, v).$$

Here $$M = \left(\frac{\partial t}{\partial v}\right)_p \quad \text{and} \quad N = \left(\frac{\partial t}{\partial p}\right)_v.$$

As another example consider

$$dq = \left(\frac{\partial q}{\partial v}\right)_p dv + \left(\frac{\partial q}{\partial p}\right)_v dp, \quad . \quad . \quad . \quad . \quad (b)$$

where $$M = \left(\frac{\partial q}{\partial v}\right)_p = h_p \quad \text{and} \quad N = \left(\frac{\partial q}{\partial p}\right)_v = h_v.$$

Expressions of the form $Mdx+Ndy$ are divided into two classes. For every expression belonging to the first class, of which the right-hand member of equation (a) is an example, a definite relation exists between the variables, in the example the relation is $t=\phi(p, v)$. For all expressions belonging to the second class, of which the right-hand member of equation (b) is an example, no such functional relation exists.

Whenever a functional relation between the variables exists the value of z depends only upon the initial and the final values of the independent variables x and y and not in any way upon the manner in which the change is made from the initial to the final condition.

Referring to the first example, note that its integral $t=\phi(p, v)$ definitely fixes the change in the temperature when the values of p and v are known for both the initial and the final conditions without any reference to the path the state-point may have followed on the characteristic surface, as given by some relation between p and v, during its motion from the initial to the final point.

When these conditions are satisfied $Mdx+Ndy$ is called an **exact differential.**

Under these conditions the integration of

$$dt=\left(\frac{\partial t}{\partial v}\right)_p dv+\left(\frac{\partial t}{\partial p}\right)_v dp$$

can be performed without using a relation between p and v and

$$\int_{t_1}^{t_2} dt=t_2-t_1,$$

the change in temperature, is always the same when computed between the same initial and final points, (p_1, v_1)

and (p_2, v_2), irrespective of the line on the surface, $t=\phi(p, v)$, joining these points along which the integration may be performed.

EXERCISE 262. Write equation (*a*) so that it applies to an ideal gas and from it determine the change in temperature due to a change from (p_1, v_1) to (p_2, v_2).

If no functional relation exists between the variables x, y, and z in

$$dz=Mdx+Ndy$$

then the integration cannot be performed. No value of z can be found unless some assumption regarding the relation between x and y is made so that $Mdx+Ndy$ can be expressed in terms of one variable. If some relation between x and y is arbitrarily assumed each assumption will yield a different value of z.

Referring specifically to equation (*b*), the second example above, we know from physical considerations that the value of q_2-q_1 is not a definite constant quantity depending only upon the initial and the final values of p and v. The heat supplied between any two states of a substance depends not only upon these states but also upon the manner in which the heat is supplied. Therefore

$$\left(\frac{\partial q}{\partial v}\right)_p dv+\left(\frac{\partial q}{\partial p}\right)_v dp$$

is not an exact differential.

EXERCISE 263. Find z_2-z_1 between the limits $(x=0,\ y=0)$ and $(x=1,\ y=3)$ when

(1) $$dz=xdy-ydx,$$

(2) $$dz=xdy+ydx,$$

provided (*a*) $y=3x$, (*b*) $y^2=9x$.

Which of these expressions is an exact differential? Which can be integrated directly?

The Test for an Exact Differential.—The following criterion which is used to test expressions of the form $Mdx+Ndy$ for exactness may also be used in another way. Suppose an expression in this form is known to be exact, let us say from physical considerations, then the test about to be developed may be used to establish relations between the quantities involved in this expression.

If $Mdx+Ndy$ is exact then, by definition, some function $z=f(x, y)$ must exist which on differentiation gives

$$dz=Mdx+Ndy.$$

From $$z=f(x, y)$$

we have $$dz=\left(\frac{\partial z}{\partial x}\right)_y dx+\left(\frac{\partial z}{\partial y}\right)_x dy,$$

so that $$M=\frac{\partial z}{\partial x} \quad \text{and} \quad N=\frac{\partial z}{\partial y}.$$

Differentiating these identities the first with respect to y and the second with respect to x we have

$$\frac{\partial M}{\partial y}=\frac{\partial^2 z}{\partial x \partial y} \quad \text{and} \quad \frac{\partial N}{\partial x}=\frac{\partial^2 z}{\partial y \partial x}.$$

But as has been shown in the calculus

$$\frac{\partial^2 z}{\partial x \partial y}=\frac{\partial^2 z}{\partial y \partial x},$$

so that $$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}.$$

That is, if the change in a function z of two independent variables x and y, between fixed limits, for which

$$dz=Mdx+Ndy$$

is independent of the manner in which the change is produced, i.e., if $Mdx+Ndy$ is an exact differential, then

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}.$$

EXERCISE 264. The following differential must be exact,

$$(x^3+5xy+y)dx+(y^3+Ax^2+x)dy.$$

What is the value of A?

SECTION XLVII

THE DIFFERENTIAL EQUATIONS OF THERMODYNAMICS

Entropy As a Coordinate.—The coordinates we have already used in defining the state of a substance are the pressure, the specific volume, and the temperature (p, v, and t).

To these may be added the entropy, s, which for reversible processes is defined by

$$ds=\frac{dq}{T},$$

where T represents absolute temperature, so that

$$T=t+\text{a constant}.$$

It is important to show that the change in entropy depends only upon the change of state which occurs and not in any way upon the manner in which the change is made. In passing from one state to another the change in the entropy of the substance will be the same no matter what reversible path is followed in passing from the initial to the final state. That this is so may be shown as follows.

Any cycle may be divided into differential elementary Carnot cycles as shown in Fig. 74. The sum of these elementary cycles will be the same as the given cycle.

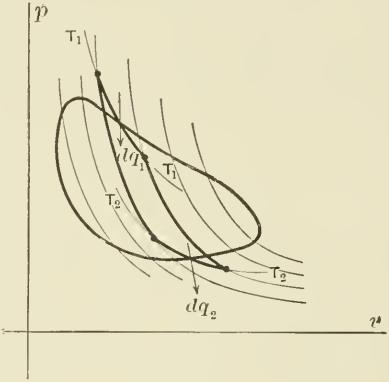

FIG. 74

For each of these elementary cycles we have

$$\frac{dq_1}{T_1}=\frac{dq_2}{T_2}$$

or

$$\frac{dq_1}{T_1}-\frac{dq_2}{T_2}=0,$$

where dq_1 and dq_2 are the quantities of heat absorbed and rejected by the working substance during the cycle at the absolute temperatures T_1 and T_2 respectively, see page 133.

Therefore in any reversible cycle for any substance

$$\int^{1}\frac{dq}{T}=0,$$

or the sum of the changes in entropy of any substance undergoing a reversible cycle of changes must be zero.

Now consider the change in entropy of a substance during any change of state represented by the line 1 *A* 2, Fig. 75. It would at first sight appear that as the heat supplied during the change of state varies with the nature of the path 1 *A* 2 the entropy $\int_1^2\frac{dq}{T}$ would also vary with the path.

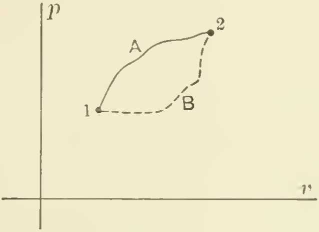

Fig. 75.

That this is not so becomes evident when we consider the return from state 2 to state 1 by any other path such as 2 *B* 1. As $\int_1^1\frac{dq}{T}=0$, the gain in entropy from 1 to 2 along the path 1 *A* 2 must equal the loss in entropy from 2 to 1 along the path 2 *B* 1, or

$$\int\frac{dq}{T}\text{ along path 1 }A\text{ 2}=\int\frac{dq}{T}\text{ along the path 1 }B\text{ 2.}$$

Therefore s depends only upon the state of the substance and not upon the manner in which the state was reached. Also

$$s = \int_1^2 \frac{dq}{T} = s_2 - s_1,$$

and **$\mathbf{ds} = \frac{\mathbf{dq}}{\mathbf{T}}$ must be an exact differential.**

Of the four coordinates p, v, t, and s which are functions of the state only and do not depend upon how the state may have been attained any two may be used as independent variables to determine the state of the substance. Two of these variables are always sufficient for this purpose; by means of these the other two can always be computed.

Thermodynamic Potentials. — The internal or intrinsic energy, u, of unit mass of a substance under certain conditions may be regarded as the energy in the substance due to its present state. It may also be regarded as the ability the substance possesses to perform work due to its thermal condition. From this point of view u may be called a thermal **potential;** u is a measure of the potency of the substance.

Evidently u depends only upon the state of the substance and not upon how the state may have been attained, therefore du is an exact differential.

$$dq = du + \frac{1}{J} p\,dv$$

$$du = dq - \frac{1}{J} p\,dv.$$

Thus $dq-\frac{1}{J}pdv$ is also an exact differential, altho neither dq nor pdv are exact.

The five quantities, p, v, T, s, and u are all functions of the state only therefore any combinations of these quantities must also be functions of the state only. Three such combinations have been found very useful in thermodynamic discussions. They are denoted by the letters i, f, and ϕ and are defined by the identities

$$\mathbf{i}=\mathbf{u}+\frac{1}{\mathbf{J}}\mathbf{pv},$$

$$\mathbf{f}=\mathbf{u}-\mathbf{Ts},$$

$$\phi=\mathbf{i}-\mathbf{Ts}.$$

u, i, f, and ϕ are called **thermodynamic potentials.** Of these u and i are the most useful to the engineer. They have received special names. u is the internal energy whose physical significance is well known, and **i is called the heat content** for the physical significance of which see page 166.

The other two potentials f and ϕ have received no special names; they are simply known as thermodynamic potentials. Applications of these potentials are to be found in physics and in chemistry.

The differentials of the four thermodynamic potentials u, i, f, and ϕ are

$$du=dq-\frac{1}{J}pdv,$$

$$di=du+\frac{1}{J}pdv+\frac{1}{J}vdp,$$

$$df = du - Tds - sdT,$$

and
$$d\phi = di - Tds - sdT.$$

These may all be expressed in terms of the four coordinates p, v, T, and s and each one in terms of two of them as independent variables.

For instance, as $dq = Tds$

$$\mathbf{du = Tds - \frac{1}{J} pdv.} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Using this relation we may transform di into

$$di = \left(Tds - \frac{1}{J} pdv\right) + \frac{1}{J} pdv + \frac{1}{J} vdp$$

or
$$\mathbf{di = Tds + \frac{1}{J} vdp.} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

EXERCISE 265. Show that

$$df = -sdT - \frac{1}{J} pdv, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and
$$d\phi = \frac{1}{J} vdp - sdT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Note that as u, i, f, and ϕ depend only upon the state of the substance their differentials are exact.

Maxwell's Relations.—The following four relations between the variables p, v, T, s, which are used as coordinates for the determination of the state of a substance in thermodynamics may be readily derived from the four differentials of the potentials. These relations, known as Maxwell's relations, are used in establishing relations between the thermal capacities and the variables p, v, and T.

From equation (8), if we remember that the differential is exact, we have

$$\left[\frac{\partial(T)}{\partial v}\right]_s = \left[\frac{\partial\left(-\frac{1}{J}p\right)}{\partial s}\right]_v$$

or

$$\left(\frac{\partial T}{\partial v}\right)_s = -\frac{1}{J}\left(\frac{\partial p}{\partial s}\right)_v. \quad . \quad . \quad . \quad . \quad (12)$$

Exercise 266. Show that

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{1}{J}\left(\frac{\partial v}{\partial s}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\left(\frac{\partial s}{\partial v}\right)_T = \frac{1}{J}\left(\frac{\partial p}{\partial T}\right)_v, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\left(\frac{\partial s}{\partial p}\right)_T = -\frac{1}{J}\left(\frac{\partial v}{\partial T}\right)_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

These are Maxwell's relations. They hold for all reversible changes of state and for all substances whose characteristic equation is $f(p, v, T)=0$. Note the physical meaning of these equations. For instance from equation (13) we see that the rate of change of the temperature with respect to the change in pressure along an isentropic path must always equal $\frac{1}{J}$ times the rate of change of the volume with respect to the change in entropy along an isopiestic path for any substance at any particular state.

Exercise 267. What geometrical interpretation may be attached to equation (15)?

Equation (12) may be written in terms of q instead of s. This will show its connection with the thermal capacities defined on pages 265–267. As $dq=Tds$ we have

$$\left(\frac{\partial T}{\partial v}\right)_q = -\frac{T}{J}\left(\frac{\partial p}{\partial q}\right)_v.$$

Note that in $\left(\frac{\partial T}{\partial v}\right)_q$ q now takes the place of s as the variable which remains constant during the change considered.

But as by definition $\left(\frac{\partial q}{\partial p}\right)_v=h_v$

we may write
$$h_v=-\frac{T}{J}\left(\frac{\partial v}{\partial T}\right)_q.$$

Also from equation (3),
$$dq=c_v dt+l_v dv;$$

now as q is constant, $dq=0$, and
$$\left(\frac{\partial v}{\partial T}\right)_q=-\frac{c_v}{l_v}$$

and
$$h_v=\frac{T}{J}\left(\frac{c_v}{l_v}\right).$$

EXERCISE 268. Show that
$$l_v=\left(\frac{\partial q}{\partial v}\right)_T=\frac{T}{J}\left(\frac{\partial p}{\partial T}\right)_v, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$
$$l_p=\left(\frac{\partial q}{\partial p}\right)_T=-\frac{T}{J}\left(\frac{\partial v}{\partial T}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

A General Expression for c_p-c_v.—As an application of Maxwell's relations let us find an expression for (c_p-c_v) in terms of p, v, and T.

From equation (5)
$$c_p-c_v=l_v\left(\frac{\partial v}{\partial t}\right)_p,$$

l_v may be eliminated by means of equation (16). Equation (16) may be written
$$l_v=\frac{T}{J}\left(\frac{\partial p}{\partial t}\right)_v,$$

for $T=t+$a constant.

Thus
$$(c_p-c_v)=\frac{T}{J}\left(\frac{\partial p}{\partial t}\right)_v\left(\frac{\partial v}{\partial t}\right)_p. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This relation is true for all substances having a characteristic equation of the form $f(p, v, t) = 0$.

EXERCISE 269. Show that altho c_p and c_v vary $c_p - c_v$ must be constant for all substances whose characteristic equation is $pv = RT$.

EXERCISE 270. Find $c_p - c_v$ for superheated steam assuming that Zeuner's characteristic equation

$$pv = RT - Cp^n,$$

where R, C, and n are constants, is correct.

The Heat Supplied, dq.—The heat supplied during any change of state to any substance whose characteristic equation is $f(p, v, t) = 0$ or $t = \phi(p, v)$ can now be expressed in terms of c_p, c_v, and the partial derivatives of p and of v with respect to t. To do this the thermal capacities l_v, l_p, h_v, and h_p must be eliminated from equations (1), (2), and (3) by means of the relations just deduced from Maxwell's relations.

From equation (3) by means of equation (16)

$$\mathrm{dq} = \mathrm{c}_v\mathrm{dt} + \frac{\mathrm{T}}{\mathrm{J}}\left(\frac{\partial \mathrm{p}}{\partial \mathrm{t}}\right)_v \mathrm{dv}, \quad . \quad . \quad . \quad . \quad (19)$$

and from equation (2) by means of equation (17)

$$\mathrm{dq} = \mathrm{c}_p\mathrm{dt} - \frac{\mathrm{T}}{\mathrm{J}}\left(\frac{\partial \mathrm{v}}{\partial \mathrm{t}}\right)_p \mathrm{dp}. \quad . \quad . \quad . \quad . \quad (20)$$

EXERCISE 271. From equations (19) and (20) derive expressions for dq for ideal gases.

To simplify equation (1), i.e.,

$$dq = \left(\frac{\partial q}{\partial v}\right)_p dv + \left(\frac{\partial q}{\partial p}\right)_v dp,$$

note that $\left(\frac{\partial q}{\partial v}\right)_p = h_p = l_v + c_v\left(\frac{\partial t}{\partial v}\right)_p$, Exercise 261,

that $l_v = \frac{T}{J}\left(\frac{\partial p}{\partial t}\right)_v$, equation (16),

and that $\left(\frac{\partial q}{\partial p}\right)_v = h_v = c_v\left(\frac{\partial t}{\partial p}\right)_v$, Exercise 261.

Substituting these values in equation (1)

$$dq = \left[\frac{T}{J}\left(\frac{\partial p}{\partial t}\right)_v + c_v\left(\frac{\partial t}{\partial v}\right)_p\right]dv + c_v\left(\frac{\partial t}{\partial p}\right)_v dp.$$

In order to eliminate $\left(\frac{\partial t}{\partial v}\right)_p$ and $\left(\frac{\partial t}{\partial p}\right)_v$ use equation (18) from which

$$\left(\frac{\partial t}{\partial v}\right)_p = \frac{\frac{T}{J}\left(\frac{\partial p}{\partial t}\right)_v}{(c_p - c_v)},$$

and

$$\left(\frac{\partial t}{\partial p}\right)_v = \frac{\frac{T}{J}\left(\frac{\partial v}{\partial t}\right)_p}{(c_p - c_v)}.$$

Finally equation (1) becomes

$$\mathbf{dq} = \frac{1}{\mathbf{J}}\,\frac{\mathbf{T}}{(\mathbf{c}_p - \mathbf{c}_v)}\left[\mathbf{c}_p\left(\frac{\partial \mathbf{p}}{\partial \mathbf{t}}\right)_v \mathbf{dv} + \mathbf{c}_v\left(\frac{\partial \mathbf{v}}{\partial \mathbf{t}}\right)_p \mathbf{dp}\right]. \quad . \quad (21)$$

Exercise 272. What form does equation (21) take for ideal gases? Compare the result with the third equation in Exercise 26.

Clapeyron's Formula.—Let us apply equation (19), i.e.,

$$dq = c_v dt + \frac{T}{J}\left(\frac{\partial p}{\partial t}\right)_v dv,$$

which is true for any substance and which has already been applied to an ideal gas in Exercise 271, to a vapor during the process representing the change from a liquid to a saturated vapor.

For a wet vapor we have

$$p=f(t)$$

and $$v=\phi(p, x),$$

where x is the quality of the vapor, see page 133.

Therefore $\left(\frac{\partial p}{\partial t}\right)_v$ is simply $\frac{dp}{dt}$ and as

$$\frac{dp}{dt}=f'(t)$$

$\frac{dp}{dt}$ is a constant for any temperature.

If the vapor forms at constant pressure the temperature is constant and dt is zero so that

$$q''-q'=\frac{T}{J}\left(\frac{dp}{dt}\right)\int_{v'}^{v''}dv=\frac{T}{J}\left(\frac{dp}{dt}\right)(v''-v').$$

As $q''-q'=r=$ the latent heat of vaporization

we have $$\mathbf{v''-v'=\frac{rJ}{T}\cdot\frac{1}{\left(\frac{dp}{dt}\right)}}. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This is **Clapeyron's formula** for the increase of volume during the evaporation of a liquid. From it when r and the relation between p and t have been determined experimentally $v''-v'$ can be computed.

EXERCISE 273. Deduce Clapeyron's formula directly from Maxwell's relation given in equation (14).

Other Important Relations.—The change in internal energy

$$du = dq - \frac{1}{J} p dv$$

may with the aid of equation (19) be expressed in terms of c_v, thus

$$du = c_v dt + \frac{1}{J}\left\{ T\left(\frac{\partial p}{\partial t}\right)_v - p \right\} dv$$

and as du is an exact differential

$$\left(\frac{\partial c_v}{\partial v}\right)_t = \frac{1}{J}\left[\frac{\partial \left\{ T\left(\frac{\partial p}{\partial t}\right)_v - p \right\}}{\partial t}\right]_v$$

$$= \frac{1}{J}\left[T\left(\frac{\partial^2 p}{\partial t^2}\right)_v + \left(\frac{\partial p}{\partial t}\right)_v - \left(\frac{\partial p}{\partial t}\right)_v \right]$$

or

$$\left(\frac{\partial c_v}{\partial v}\right)_t = \frac{T}{J}\left(\frac{\partial^2 p}{\partial t^2}\right)_v, \quad . \quad . \quad . \quad . \quad . \quad (23)$$

which shows the variation of c_v with respect to v while the temperature remains constant.

Similarly in

$$di = dq + \frac{1}{J} v dp, \text{ equation (9),}$$

c_p may be introduced by means of equation (20), and we obtain

$$di = c_p dt - \frac{1}{J}\left\{ T\left(\frac{\partial v}{\partial t}\right)_p - v \right\} dp.$$

EXERCISE 274. Show from this equation that

$$\left(\frac{\partial c_p}{\partial p}\right)_t = -\frac{T}{J}\left(\frac{\partial^2 v}{\partial t^2}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

which is called **the relation of Clausius.**

EXERCISE 275. Show by means of equations (23) and (24) that for ideal gases c_v does not depend upon the volume, that c_p does not depend upon the pressure, and that nevertheless both c_p and c_v probably vary with the temperature.

From the equation

$$di = c_p dt - \frac{1}{J}\left\{T\left(\frac{\partial v}{\partial t}\right)_p - v\right\}dp$$

we find that during an isothermal process for which dt must equal zero

$$\left(\frac{di}{dp}\right)_t = -\frac{1}{J}\left\{T\left(\frac{\partial v}{\partial t}\right)_p - v\right\}. \quad . \quad . \quad . \quad (25)$$

Also during throttling, for which i remains constant so that $di=0$, such as occurs in the Joule-Thomson porous plug experiment the same equation yields

$$\left(\frac{dt}{dp}\right)_i = \frac{1}{c_p J}\left\{T\left(\frac{\partial v}{\partial t}\right)_p - v\right\}. \quad . \quad . \quad . \quad (26)$$

$\left(\frac{dt}{dp}\right)_i$, the ratio of the drop in temperature to the drop in pressure during a process for which the heat content is constant, is called **the Joule-Thomson coefficient.**

EXERCISE 276. What drop in temperature occurs during the throttling of an ideal gas?

The equality of the absolute temperature used in the general equations of thermodynamics and the absolute temperature in the characteristic equation of an ideal gas may be shown by means of the equation

$$du = c_v dt + \frac{1}{J}\left\{T\left(\frac{\partial p}{\partial t}\right)_v - p\right\}dv.$$

Note that in this equation *du*, the change in internal energy, is independent of the change in volume when this equation is applied to an ideal gas, therefore the coefficient of *dv* must be zero, or

$$T\left(\frac{\partial p}{\partial t}\right)_v - p = 0$$

so that

$$\left(\frac{\partial p}{\partial t}\right)_v = \frac{p}{T}.$$

But from

$$pv = RT$$

$$\left(\frac{\partial p}{\partial t}\right)_v = \frac{R}{v} = \frac{p}{T},$$

therefore the two T's must be alike.

PROBLEMS FOR REVIEW

277. Determine the position of the absolute zero on the Fahrenheit scale from the following experimental result found by heating air at constant volume,

$$\frac{\text{pressure at } 100° \text{ C}}{\text{pressure at } 0° \text{ C}} = 1.3665.$$

278. Compute the internal energy of a mass m of an ideal gas whose state is p, V, T in terms of p, V, and k.

279. State four laws governing the behavior of an ideal gas.

280. Deduce the law $pV = mRT$ from the laws of Boyle and of Charles.

281. Fifty pounds of air enclosed in a receiver at a pressure of 200 pounds per square inch gage and at 210° F are cooled by radiation. What will be the final pressure if the temperature of the air becomes 50° F?

282. If one cubic foot of oxygen at n.t.p. weighs w pounds, compute the gas constant, R, for a gas whose molecular weight is μ.

283. At a pressure of 14.7 pounds per square inch absolute and a temperature of 32° F one pound of air occupies a volume of 12.39 cubic feet. The specific heats of air are said to be $c_p = 0.2375$ and $c_v = 0.1685$. From these data compute the gas constant for air by two methods.

284. Deduce a formula for the gas constant of a mixture of three ideal gases the gas constants of which are known.

285. A receiver (capacity 5 cubic feet) contains 9 pounds

of SO_2. At what temperature will the pressure in the tank reach 200 pounds per square inch gage? Assume that one pound of oxygen occupies 11.2 cubic feet at n.t.p. and that SO_2 behaves as an ideal gas under the above conditions.

286. Deduce the relation between c_p and c_v of an ideal gas and state the law (or laws) governing the behavior of ideal gases used in deducing this relation.

287. Develop from fundamental equations the relation between the pressures and the temperatures of two states of a given mass of gas during an adiabatic expansion.

288. Prove that

$$c_p=\frac{R}{J}\cdot\frac{k}{k-1}.$$

289. How much external work is performed by a gas during (*a*) an adiabatic, (*b*) an isothermal, (*c*) a polytropic change from the state determined by p_1 and V_1 to the state determined by p_2 and V_2.

290. During the compression of one pound of air 10 B.t.u. are removed by the cooling water and 30,000 foot-pounds of work are required. The initial temperature of this air was 80° F, compute the final temperature.

291. The temperature of air expanding in an air motor drops 120° F. The expansion follows the process $pv^{1.3}=c$. How much heat was transferred to or from the cylinder walls per pound of gas during this expansion?

292. Compute the least amount of work required per pound of dry air to draw it from the atmosphere at 70° F and deliver it to a receiver at 100 pounds per square inch gage under ideal conditions. The available cooling water has a temperature of 70° F ($R=53.3$).

293. Define polytropic change of state.

What is the value of the specific heat of an ideal gas during polytropic change of state?

Do the various polytropic changes include all possible changes of state?

294. How would you determine whether or not a given expansion or compression line on an indicator card represents a polytropic change of state?

295. The equation of the compression line of an air compressor card is found to be $pV^{1.35} = \text{a constant}$. During the compression the temperature of the air is raised 200° F. How much heat is transmitted to the cylinder walls during the compression of one pound of gas?

296. During a compression following the law $pV^{1.3} = \text{constant}$ the temperature of the air rises from 80 to 200° F. How much heat has been absorbed per pound of air?

297. How much external work was done by each pound of air during the change of state described in 296?

298. A balloon, capacity 9000 cubic feet, is filled with air at 300° F. The temperature of the surrounding air is 70° F. What total weight (including the weight of the balloon) is required to prevent its rising in still air?

299. Deduce a relation between T and v for an adiabatic change of state of a gas for which $c_v = a + bT$, the result to contain the constants a, b, R, and J.

300. The state of one pound of a certain gas for which $k = 1.4$ is changed from 15 pounds per square inch absolute, 80° F, and 192 cubic feet to 30 pounds per square inch absolute and 250° F. Compute the change in internal energy.

301. During the compression of an ideal gas 40,000 foot-pounds of work are expended and 8 B.t.u. are taken from it by conduction. What change in internal energy occurs during this process?

302. A tank held 40 cubic feet of air at 200 pounds per square inch gage. This air is cooled in the tank to 70° F and its pressure is now 150 pounds per square inch gage.

(*a*) How much heat has been extracted?

(*b*) How much has the entropy of this gas decreased?

303. Compute the change in the entropy of an ideal gas due to a change of state from (p_1, v_1, T_1) to (p_2, v_2, T_2) in terms of the temperatures and the volumes.

304. What is the temperature in the exhaust pipe of an air engine if the air is supplied at 100 pounds per square inch gage and 70° F and the expansion is assumed to be adiabatic to atmospheric pressure?

305. Three cubic feet of air at atmospheric pressure are to be compressed at constant temperature to 100 pounds per square inch gage.

(*a*) How much heat must be withdrawn from the gas to accomplish this?

(*b*) Where does this heat come from?

306. (*a*) Sketch the pv-diagram showing the operation of an air compressor without clearance in which the compression is completed in two adiabatic stages.

(*b*) Sketch upon the Ts-plane that portion of the diagram drawn in (*a*) which shows the saving effected by compounding.

307. (*a*) Indicate on the pv-plane an area, finite in all its dimensions, which is proportional to the heat supplied during a change of state from (p_1, v_1, T_1) to (p_2, v_2, T_2). Prove that this area represents the heat supplied.

(*b*) Indicate an area on the Ts-plane which represents the heat supplied during the above change of state.

308. In an air-compressor 10 cubic feet of air at a temperature of 100° F and under a pressure of 14 pounds per square inch absolute are compressed adiabatically. The final pressure is 60 pounds per square inch absolute. (*a*) Compute the volume of this air after compression. (*b*) Find the temperature of this air after compression.

309. Compute the most advantageous intermediate pressures for a three-stage compressor with perfect intercooling.

310. Compressed air is to be stored in cylinders 6 inches

in internal diameter and 6 feet long. These cylinders can withstand an internal pressure of 2000 pounds per square inch absolute. The greatest temperature to which they are liable to be exposed is 120° F. (*a*) What weight of air can be safely stored in each cylinder? (*b*) What should be the charging pressure at 60° F?

311. Air is compressed adiabatically from p_1 and T_1 and delivered at p_2. How much work must be supplied per pound of air? Assume ideal conditions and no clearance. Express the result in terms of the given pressures and temperature.

312. Compressed air is admitted to the cylinder of an air motor without clearance. Compare the work done during admission with the work it could perform if expanded indefinitely according to the law $pv^n = c$ after admission to the cylinder.

313. An air-compressor has a displacement volume of 10 cubic feet and a clearance of 5 per cent. What volume of air is taken in per stroke when the compressor operates adiabatically between 0 and 60 pounds per square inch gage?

314. Sketch on the pV-plane the cycle thru which the air passes during the operation of a dense-air refrigerating machine under ideal conditions. Indicate clearly on this diagram the points at which the air attains the temperature of (*a*) the cooling water, (*b*) the cold storage room.

315. In a dense-air refrigerating machine operating on a reversed Joule cycle the compressor takes in air at a pressure of 40 pounds per square inch gage and delivers it at a pressure of 120 pounds per square inch gage. At the beginning of compression the temperature is 34° F and the cooling water maintains the temperature of the cooling coils at 70° F. Determine the ratio of the heat extracted from the cold body to the heat equivalent of the work expended in driving the machine.

316. In a dense-air refrigerating machine operating on a reversed Joule cycle, the compression cylinder takes in air at 40 pounds per square inch gage and delivers it at 120 pounds

per square inch gage. At the beginning of compression the temperature is 34° F and the cooling water maintains the temperature of the cooling coils at 70° F. Compute the work expended per pound of air passed thru the cycle. ($R = 53.3$.)

317. Sketch the Ts-diagram corresponding to the pv-diagram shown in Fig. 76. Letter your diagram to correspond with Fig. 76.

318. Compute the change in entropy which occurs during the process $A\ B$, Fig. 76.

319. The clearance space of a Diesel engine is 0.1 of its piston displacement. If the cylinder is full of air at 160° F and 14 pounds per square inch absolute at the beginning of compression, determine the temperature at the end of compression provided no leakage occurs.

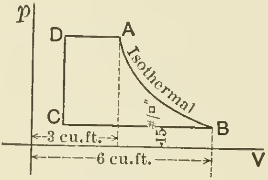

Fig. 76.

320. One pound of air is heated at constant pressure from 60 to 150° F. Compute

(a) the change in entropy,

(b) the heat supplied.

321. Prove that the efficiency of the Carnot cycle is $1 - \left(\frac{1}{r}\right)^{0.4}$, where $\frac{1}{r}$ is the ratio of the least volume to the volume before adiabatic compression.

322. Air is compressed isothermally from 70° F and 14.7 pounds per square inch absolute to 100 pounds per square

inch absolute, then expanded at constant pressure, and finally reduced to its initial state along an isometric process.

Compute (*a*) the efficiency of this cycle,

(*b*) the work performed per pound of air.

323. In Fig. 77 is shown a special case of the Atkinson cycle. This may be considered as an Otto cycle in which the expansion is carried to the atmospheric line. Compute the efficiency of this cycle in terms of T_1, T_4, v_1, v_2, v_4, and the physical constants of the gas.

324. Compute the efficiency of the Lenoir cycle (Fig. 78) in terms of the constant specific heats of the gas, the temperatures at 1 and 2, and the ratio of the volumes $\frac{v_4}{v_1}$.

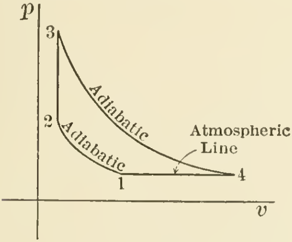

FIG. 77.

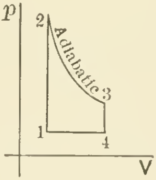

FIG. 78.

325. Sketch the Lenoir cycle (Fig. 78) on the *TS*-plane. Number the important points so as to correspond with Fig. 78.

326. One pound of air at 80° F is compressed at constant temperature from 0 to 150 pounds per square inch gage. It is then expanded at constant pressure and finally expanded adiabatically to its initial state. Compute the highest temperature reached during the cycle.

327. Compute the greatest and the least volume occupied by the air during the cycle described in 326.

328. The efficiency of an Otto cycle is $1-\frac{T_2}{T_3}$, where T_2 is

the absolute temperature at the beginning and T_3 is the absolute temperature at the end of compression.

The clearance space of a 6×12-inch Otto gas engine was found to hold 4 pounds of water. Compute the ideal efficiency of this engine assuming adiabatic expansion and compression.

329. The indicator card of an Otto engine shows that the pressure rises during compression from 15 to 200 pounds per square inch absolute. After ignition it is 400 pounds per square inch absolute.

Assuming the initial temperature of the gas to be 300° F and the compression to be polytropic ($n=1.3$) what is the probable temperature of the gas after ignition?

330. The inventor of an oil engine claims for it a consumption of 0.31 pound of fuel (yielding 19,000 B.t.u. per pound) per indicated horse-power hour. The edges of a nickel cube (melting point 2600° F) fuse in the cylinder of this engine and the exhaust temperature is 800° F. Test the probability of the inventor's claim by means of Carnot's principle. State this principle and your conclusion.

331. A room is heated by means of radiators maintained at 220° F. The temperature of the room is 70° F, that of the outside air 30° F. How many B.t.u. at 70° F could be transferred from the outside air to the room by means of an ideal heat-motor combined with an ideal heat-pump for every 100 B.t.u. furnished by the radiator?

332. (*a*) Explain the physical significance of a negative specific heat.

(*b*) For what values of n in the equation $pv^n=$ a constant is the specific heat of a diatomic gas negative during the change of state described by this equation?

333. State the second law of thermodynamics and prove Carnot's principle.

334. Determine analytically whether the temperature of an

ideal gas increases or decreases during a polytropic compression for which $n=0.90$.

335. A tank (capacity 10 cubic feet) contains air at atmospheric pressure and 70° F. Air is forced into this tank until the pressure rises to 200 pounds per square inch gage, and the temperature to 75° F. How many pounds of air have passed into the tank?

336. A certain producer gas is said to contain 0.6 per cent of H_2, 23.0 per cent of CO, 1.4 per cent of CH_4, 11.0 per cent of CO_2, and 64.0 per cent of N_2 by weight. Compute its gas constant.

337. Compute the mean specific heat of air at constant volume between 60 and 300° F.

338. What change in intrinsic energy occurs when 20 cubic feet of air expand to 30 cubic feet under a constant pressure of 60 pounds per square inch absolute?

339. (*a*) During an adiabatic change of state the temperature of 20 pounds of air changed from 80 to 150° F. How much work was done by the gas during this change?

(*b*) If the change had been polytropic with $n=1.2$, how much work would have been done by the gas?

340. Show without the aid of calculus, that the work required to compress and deliver a gas when the compression is isothermal equals the work done during compression.

341. Thru what per cent of the stroke must the piston of a compressor, having a displacement volume of 10 cubic feet and a clearance of 5 per cent, move before the outlet valves open when the compressor operates between 0.5 and 100 pounds per square inch gage? ($n=1.3$ and $p_a=14.7$.)

342. The displacement volume of the working piston of an engine operating on an Ericsson cycle is to be 100 cubic feet, the extreme pressures to be used 15 and 30 pounds per square inch absolute, the available extreme temperatures 70 and 500° F. (*a*) Compute the displacement volume of the compressor

piston neglecting all clearances. (*b*) What horse-power would be obtained under ideal conditions if the engine is single-acting and runs at 10 r.p.m.?

343. What is the most important difference between the conditions existing during the absorption of heat by the gas during a Carnot and a Joule cycle?

344. During an Otto cycle the heat supplied causes an increase in pressure from 60 to 250 pounds per square inch absolute. Compute the change in entropy per pound of air.

345. Air contained in a receiver is withdrawn until the gage shows 28.9 inches of vacuum. If the receiver originally contained air at atmospheric pressure and the barometer read 30.0 inches, what per cent by weight of the air originally present has been removed?

346. (*a*) Define total heat of vapors.

(*b*) Write the defining equation of the heat content of vapors.

(*c*) Develop the relation between the total heat and the heat content of a vapor. Explain each symbol.

(*d*) Which of these values are given in your steam tables?

347. (*a*) Compute the internal energy of steam, quality 0.90, at 100 pounds per square inch absolute.

(*b*) What does the result represent?

(*c*) What is the mean specific heat of superheated steam at a constant pressure of 150 pounds per square inch absolute between 100 and 300° F of superheat?

348. Compute by means of the steam tables the quality of steam after reversible adiabatic expansion from 200 pounds per square inch absolute and 100° F superheat to 10 pounds per square inch absolute.

349. Fourteen pounds of steam are confined in a closed receiver (capacity 20 cubic feet). The initial temperature is 400° F. The temperature then falls to 70° F. (*a*) What is the greatest and the least pressure to which the receiver is subjected if the

atmospheric pressure is 14.5 pounds per square inch? (*b*) Compute the heat lost by the steam.

350. One pound of steam occupies 1.5 cubic feet. Compute its entropy, the pressure being 250 pounds per square inch absolute?

351. One pound of dry saturated steam expands adiabatically from 250 to 15 pounds per square inch absolute. Find the external work performed measured in B.t.u. (Use the steam diagrams.)

352. Find the volume of 3 pounds of steam at 100 pounds per square inch absolute and 402° F.

353. The volume of an engine cylinder is 5 cubic feet. How many pounds of steam, quality 0.80 at 225 pounds per square inch absolute, are required to fill it?

354. How much heat must be extracted per pound of steam, initial quality 0.90, confined in a closed receiver in order to reduce the pressure from 120 to 15 pounds per square inch absolute?

355. Assuming that steam expands adiabatically in a frictionless nozzle from 125 pounds per square inch absolute, quality 0.95, to a temperature of 40° F, what would be the final quality?

356. A nozzle is to deliver 5 horse-power in the form of kinetic energy. It is supplied with steam at 125 pounds per square inch absolute, quality 0.93, and discharges at a pressure of 0.60 pounds per square inch absolute. Compute the area at the throat and at exit, assuming the critical ratio to be 0.579 and neglecting all frictional resistances.

357. Compute the mean specific heat of steam at a constant pressure of 103 pounds per square inch absolute between 440 and 500° F.

358. Steam is cooled at constant volume from 150° F superheat at 160 pounds per square inch absolute until the pressure drops to 15 pounds per square inch absolute. How much heat must be removed per pound of steam?

359. Express $i_1 - i_2$, for a reversible adiabatic flow of steam (initial quality x_1) during which the pressure drops from p_1 to p_2, in terms of x_1, p_1, p_2, and quantities which may be found in the steam tables. Use a *Ts*-diagram.

360. In a cycle steam changes its state at constant volume from 50 pounds per square inch absolute, quality 0.80, to one pound per square inch absolute. How much heat must be added per pound of steam during this process?

361. Sketch a Carnot and a Rankine cycle for steam on the *Ts*-plane. In both cases the expanding steam is to change its state from superheat to wet.

362. Develop, by means of the *Ts*-diagram, a formula from which the heat converted into work per pound of wet steam during a Rankine cycle with complete expansion may be computed by means of the steam tables.

363. Steam initially at 160 pounds per square inch gage and 500° F leaves a De Laval nozzle at one pound per square inch gage. What is the quality of the issuing steam if 10 per cent of the available energy is lost in friction?

364. (*a*) How much work can be obtained from one pound of steam passing thru a Rankine cycle between pressures of 140 and 0.50 pound per square inch absolute, provided the steam is superheated 250° F?

(*b*) How many pounds of steam must be circulated under the above conditions per i.h.p. hour?

365. Sketch the cycle described in 364 on the *Ts*- and on the *is*-planes.

366. An engine during a test developed an indicated horsepower of 170 with a boiler pressure of 130 pounds per square inch absolute, a condenser pressure of 2 pounds per square inch absolute, and a supply of 2500 pounds of dry saturated steam per hour. Compute the cylinder efficiency and the actual thermal efficiency of this engine.

367. A test on a compound steam engine shows that 386 B.t.u.

are supplied by the boiler per i.h.p. per minute when the total i.h.p. is 104.6, the boiler pressure 102 pounds gage, the vacuum 24.4 inches, the barometer 29.2 inches, and the priming 1.00 per cent. Find the ideal and the actual thermal efficiencies.

368. The test of a non-condensing, high-speed engine showed the i.h.p. to be 130, the b.h.p. 120, the steam pressure 115.3 pounds gage, the back pressure 0.30 pound gage, the atmospheric pressure 14.7 pounds, the quality of the steam 1.00, the steam per i.h.p. hour 30.5 pounds. Compute

(*a*) the ideal Carnot efficiency,
(*b*) the ideal Rankine efficiency,
(*c*) the actual thermal efficiency,
(*d*) the cylinder efficiency

under these conditions.

369. The test of an engine shows that 14 pounds of steam are consumed per indicated horse-power hour. The pressure and the temperature in the steam main are 150 pounds gage and 450° F. The vacuum maintained in the condenser was 28 inches, while the barometer read 30 inches. Compute (*a*) the actual thermal efficiency of this engine, (*b*) the thermal efficiency of a Rankine cycle with complete expansion operating under the same conditions.

370. Develop the two fundamental equations by means of which problems in the flow of fluids may be solved.

371. Compute the critical pressure for the flow of a fluid thru an orifice from

$$W = A_0\sqrt{2g\frac{m}{m-1}\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{m}} - \left(\frac{p_2}{p_1}\right)^{\frac{m+1}{m}}\right]}$$

where the letters have their usual significance.

372. Steam, quality 0.93, expands from a pressure of 100 pounds per square inch absolute to 0.40 pound per square inch absolute in a nozzle. If 10 per cent of the heat transformed

into kinetic energy under ideal conditions is required to overcome the frictional resistances, what would be (*a*) the velocity, (*b*) the quality of the issuing steam?

373. A De Laval turbine rated at 350 horse-power has seven nozzles. The steam supplied to this turbine at 180 pounds per square inch absolute is superheated 70° F. The pressure of the steam leaving the nozzles is one pound per square inch absolute. The loss of energy due to friction in the nozzles is 12 per cent and 70 per cent of the kinetic energy developed by the nozzles is available at the brake. Find

(*a*) the velocity of the steam leaving the nozzles,
(*b*) the quality of the steam leaving the nozzles,
(*c*) the steam required per brake horse-power per hour.

374. How many pounds of dry saturated steam pass per second thru a Venturi meter when pressures of 100 and 90 pounds per square inch absolute exist at the sections whose areas are 3 and 2 square inches respectively?

375. Steam before passing thru a throttling calorimeter was under a pressure of 150 pounds per square inch gage. On the down-stream side of the orifice the pressure is 10 pounds per square inch gage, and the temperature 260° F. What was the initial quality of the steam?

376. A throttling calorimeter shows a temperature of 287° F at 16 pounds per square inch gage. The initial temperature of the steam in the main was 382° F, the atmospheric pressure being 14.6. What was the quality of the steam in the main?

377. A superheat of at least 10° F must be recorded when a reliable determination of the quality of steam is to be made with a throttling calorimeter. What is the greatest percentage of moisture which can be measured in steam at 200 pounds per square inch absolute when the steam is throttled to 15 pounds per square inch absolute?

Indicate your method of solution on a sketch of a Mollier diagram.

378. How much dry saturated steam will escape thru an orifice (area $\frac{1}{2}$ square inch) into the atmosphere (14.7 pounds) from a receiver in which a pressure of 30 pounds per square inch absolute is maintained. Solve by means of the steam diagrams only.

379. Sketch a Rankine cycle with incomplete expansion on (*a*) the *pv*-plane, (*b*) the *Ts*-plane, and (*c*) the total heat entropy diagram.

380. Sketch on the *is*-plane three adiabatics between the same pressures and with the same initial point, one for frictionless flow thru a nozzle, another for flow thru a nozzle including friction, and the third for throttling without appreciable change of velocity.

381. Compute the heat content of one pound of steam containing 3 per cent of moisture at 150 pounds per square inch absolute in two different ways by means of the steam tables.

382. It is said that in the notation and according to the formula of Exercise 250 the velocity of the fluid entering the pipe is

$$w_1 = C\sqrt{\frac{(p_1 - p_2)dv_1}{l\left(1 + \frac{3.6}{d}\right)}}, \text{ feet per second.}$$

Show that this is so, and find the value of C.

ANSWERS

Marks and Davis Steam Tables and Diagrams have been used.

2. 53.3. **3.** 204° F.

4. 1280 lbs. per sq. in. abso. **6.** 54.9.

7. 238° F.

8. (*a*) 86.4 lbs. per sq. in. abso.; (*b*) 76.7 lbs.

9. 24 lbs.

11. (*a*) 53.2; (*b*) 6.4 and 23.6 ins. of Hg.

12. 28.9. **13.** 0.0806 lb. per cu. ft.

14. (*a*) 51.1; (*b*) 30.1; (*c*) 0.0842 lb. per cu. ft.

15. (*a*) 80° F.; (*b*) 4.4%.

16. $\frac{m}{\mu}(T_2-T_1)\left[a+\frac{b}{2}(T_1+T_2)\right]$; $\frac{1}{\mu}\left[a+\frac{b}{2}(T_1+T_2)\right]$.

17. (*a*) 4.914, 6.904, 1.406; (*b*) 5.020, 7.010, 1.397.

21. 777. **22.** (*a*) 24.0 B.t.u.; (*b*) 17.1 B.t.u.

23. (*a*) 183° F; (*b*) 5.75 B.t.u. **24.** 389 B.t.u.

25. (*a*) 36 B.t.u.; (*b*) 25.6 B.t.u.; (*c*) 10.4 B.t.u.

29. 29,300. **33.** 218 B.t.u.

38. $\frac{144\, mc_v v_1(p_2-p_1)}{R}$. **39.** 52.5; 1.016 cu. ft.

43. (*a*) 188° F; (*b*) +76.3 B.t.u.; (*c*) 191 B.t.u.; (*d*) 267 B.t.u.

47. (*a*) $pv^k=p_1v_1^k$; (*b*) $\frac{T_1}{T}=\left(\frac{p_1}{p}\right)^{\frac{k-1}{k}}$.

52. (*a*) −180° F; (*b*) 0.814 cu. ft.; (*c*) 6.94 cu. ft.

53. (*a*) 731° F; (*b*) $\frac{p}{p_1}=18.3$; (*c*) 883 lbs. per sq. in. abso.

57. 50,400 ft.-lbs. **58.** −89,500 ft.-lbs.

59. $\frac{T}{T_1}=\left(\frac{v_1}{v}\right)^{n-1}$. **60.** $\frac{T}{T_1}=\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}$.

75. (*a*) $-83{,}700$ ft.-lbs.; (*b*) $+83{,}700$ ft.-lbs.

76. (*a*) $+7460$ ft.-lbs.; (*b*) -38.4 B.t.u.; (*c*) -48.0 B.t.u.

77. $-116{,}000$ ft.-lbs.

91. (*a*) 187 horse-power; (*b*) 170 horse-power.

92. (*a*) $\frac{1}{J}\left[\frac{p_3V_3-p_2V_2}{n-1}+\frac{p_2V_2}{k-1}-\frac{p_3V_3}{k-1}\right]$; (*b*) $\frac{c_v}{R}\,\frac{n-k}{n-1}(p_3V_3-p_2V_2)$,

both of these expressions reduce to $\frac{m}{J(k-1)}\cdot\frac{n-k}{n-1}\cdot(p_3v_3-p_2v_2)$.

93. 2.5 lbs. per sec.; 17 h.p. **94.** 4.16 cu. ft.

96. 294 lbs. per sq. in. abso. **97** $1+c-c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$.

98. $\frac{2nN(\text{l.p.c.})p_1}{33{,}000(n-1)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right]$.

103. 30.9″×45″; 19.1″×45″; 417 horse-power.

105. 680 horse-power. **106.** $p_1p_3=p_2p_4$.

107. 163 lbs. per sq. in. abso.

113. $\frac{R(T_H-T_C)\log_\epsilon r}{RT_H\log_\epsilon r+Jc_v(T_H-T_C)}$, where $r=\frac{v_1}{v_2}=\frac{p_2}{p_1}$.

115. (*a*) $\frac{T_H-T_C}{T_H}$; (*b*) $\frac{R(T_H-T_C)\log_\epsilon r}{RT_H\log_\epsilon r+Jc_p(T_H-T_C)}$,

where $r=\frac{v_1}{v_2}=\frac{v_4}{v_3}=\frac{p_2}{p_1}=\frac{p_3}{p_4}$.

116. $\frac{k}{k-1}p_C(V_4-V_1)\left\{\left(\frac{p_H}{p_C}\right)^{\frac{k-1}{k}}-1\right\}$.

117. $Jmc_p(T_3-T_2-T_4+T_1)$. **119.** 0.77.

121. 16.8 h.p.; 4 cu. ft.; 2.74 cu. ft. **129.** $2545\ \frac{1}{\eta}$.

130. 54%.

131. Claimed effi., 56.4%; Carnot effi., 57.5%; No.

134. 49.5%. **135.** (*a*) 35.7%; (*b*) 81.8%.

136. 75.7%. **140.** (*a*) 0.109; (*b*) 0.109.

141. $\frac{R}{J} \log_\epsilon \frac{p_1}{p_2}$. **143.** 0.188.

150. 2.85 ins.; $-\infty$. **155.** (*a*) 0.01602 cu. ft.; (*b*) 0.0187 cu. ft.

157. (*a*) 85.9; (*b*) It becomes more than ideal.

160. −0.03; +0.18; 0.00; −2.5; −7.2 B.t.u.

161. 0.167 B.t.u.; 392.7 B.t.u. **162.** 0.844 B.t.u.; 1204.1 B.t.u.

163. (*a*) $1201.3 - .732 + .044 = 1200.6$ B.t.u.; (*b*) 1115.7 B.t.u.

164. $1.404 + 0.0047 = 1.409$ cu. ft.

166. (*a*) 115.2 B.t.u.; (*b*) 0.576. **167.** 231.0 B.t.u.

176. (*a*) 776.5 B.t.u., 1.045; (*b*) 587.1 B.t.u., 1.0541.

179. (*a*) 0.52; (*b*) 185.5 B.t.u.; (*c*) 167.0 B.t.u.; (*d*) 18.5 B.t.u.

181. 0.0058.

182. (*a*) 2.062 cu. ft.; (*b*) 223 lbs. per sq. in. abso., 391° F.

183. −965 B.t.u. **184.** (*a*) 1.515.

185. (*a*) 0.842; (*b*) 0.308. **187.** 1.20 cu. ft.; 6.83 cu. ft.

188. 317 B.t.u. or 246,000 ft.-lbs.

192. (*a*) 118.5 B.t.u.; (*b*) 0.542. **193.** 88.5 B.t.u.

195. (*a*) 200.2° F; (*b*) 12.01 cu. ft.; (*c*) 223.8 B.t.u.; (*d*) 18.5 B.t.u.; (*e*) 205.3 B.t.u.

196. −59.8 B.t.u. **197.** (*a*) 62.46° F; (*b*) 51.5 B.t.u.

198. 55 B.t.u. **199.** (*a*) 0.793; (*b*) 327 B.t.u.

200. 68 lbs. per sq. in. abso.; 71 lbs. per sq. in. abso.

203. (*a*) 0.178; (*b*) 0.314; (*c*) 0.334.

204. $\eta = 0.265$; 178,000 ft.-lbs.; 11.1 lbs. per h.p.-hr.

206. (*a*) 0.286; (*b*) 250,000 ft.-lbs.; (*c*) 7.9 lbs. per i.h.p.-hr.

207. (*a*) 0.318; (*b*) 212,500 ft.-lbs.; (*c*) 9.33 lbs. per i.h.p.-hr.

208. (*a*) 0.294; (*b*) 282,500 ft.-lbs.; (*c*) 7.01 lbs. per i.h.p.-hr.

219. (*a*) 0.189; (*b*) 166,000 ft.-lbs.; (*c*) 11.98 lbs. per i.h.p.-hr.

212. (*a*) 49.5%; (*b*) 376 B.t.u. per i.h.p. per min.; (*c*) 11.3%.

213. (*a*) 26.9%; (*b*) 8.72 and (*c*) 16.0 lbs. per i.h.p.-hr.; (*d*) 54.5%; (*e*) 14.7%.

214. 25 lbs. per b.h.p.-hr.
217. 3878 ft. per sec.
224. (*a*) 1491 ft. per sec.; (*b*) 1489 ft. per sec.
225. 2910 ft. per sec.
227. (*a*) 38.6; (*b*) 38.2; (*c*) 38.1 lbs. per min.
228. 0.00606 sq. in.; 0.188 sq. in.
230. 204 B.t.u.
231. $\Delta\left(\frac{w^2}{2g}\right)=J\left\{i_1-i_2+(s_2''-s_1)T_2\right\}$.
233. $s_1'+x_1s_1''=s_2'+x_2s_2''$; $x_4=x_2+\frac{y(i_1-i_2)}{100r_2}$
235. 0.0925 in.; 0.535 in. The corresponding values for Exercise 228 are 0.0879 in., 0.489 in.
238. +7.5° F.
242. 5%.
243. 0.967.
244. $x_1=\frac{i_2-i_1'}{r_1}$.
247. 14.4 B.t.u.
249. 10.1 lbs. per sec.
251. 93.2 lbs. per min.
252. 24 lbs. per sq. in.
253. 5.2 ins.; 24 ft. per sec.
254. (*a*) 3.3 ins.; (*b*) 5.6 ins.
258. (*a*) 71.8; (*b*) 73.5 lbs. per sq. in. gage; (*c*) 24.6, 30.3 ft. per sec.
260. $l_v=\frac{p}{R}\left(1-\frac{1}{k}\right)c_p$; $l_p=-\frac{v}{R}\left(1-\frac{1}{k}\right)c_p$.
263. (1), (*a*) 0, (*b*) −1; (2), (*a*) 3, (*b*) 3.
264. $\frac{5}{2}$.
270. $c_p-c_v=\frac{R}{J}\,\frac{RT}{(n-1)Cp^n+RT}$.
277. −459.6° F.
278. $\frac{pV}{k-1}$.
281. 149 lbs. per sq. in. gage.
282. $\frac{137.5}{\mu w}$.
283. 53.3; 53.7.
285. 255° F.
290. 247° F.
291. 6.8 B.t.u.
292. 58,000 ft.-lbs.
295. 4.86 B.t.u.
296. −6.84 B.t.u.
297. −21,300 ft.-lbs.
298. 203 lbs.
299. $a\log_\epsilon\left(\frac{T_2}{T_1}\right)+b(T_2-T_1)=-\frac{R}{J}\log_\epsilon\frac{v_2}{v_1}$.

300. 418 B.t.u.
301. 43.4 B.t.u.
302. 920 B.t.u.; 1.53.
304. $-165°$ F.
305. 16.8 B.t.u.
308. 3.54 cu. ft.; 390° F.
310. 11 lbs.; 1794 lbs. per sq. in.
311. $\frac{kRT_1}{k-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}-1\right\}.$
312. $(n-1)$.
313. 8.9 cu. ft.
315. 3.42.
316. 4660 ft.-lbs.
318. .0473.
319. 1160° F.
320. 0.0381; 21.6 B.t.u.
322. 0.193; 142.5 B.t.u.

323. $$1-\frac{c_p(T_4-T_1)}{c_v\left\{T_4\left(\frac{v_4}{v_2}\right)^{k-1}-T_1\left(\frac{v_1}{v_2}\right)^{k-1}\right\}}.$$

324. $$\frac{mc_v(T_2-T_1)-mc_v\left(\frac{v_4}{v_1}\right)\left\{T_2\left(\frac{v_1}{v_4}\right)^{k}-T_1\right\}-mc_pT_1\left\{\left(\frac{v_4}{v_1}\right)-1\right\}}{mc_v(T_2-T_1)}.$$

326. 620° F.
327. 1.212 cu. ft.; 13.6 cu. ft.
328. 42.9%.
329. 2306° F.
330. 43.2%; Carnot effi., 58.8%.
331. 371 B.t.u.
335. 10.1 lbs.
336. 57.7.
337. 0.173.
338. 277 B.t.u.
339. $-186{,}000$, $-373{,}000$ ft.-lbs.
341. 82.7%.
342. 55.2 cu. ft., 20.3.
344. 0.242.
345. 96.3%.
347. (*a*) 1023.7 B.t.u.; (*c*) 0.496.
348. 0.882.
349. 233, 14.1 lbs. per sq. in. gage; 12,600 B.t.u.
350. 1.3436.
351. 204 B.t.u.
352. 14.88 cu. ft.
353. 3.05 lbs.
354. 734 B.t.u.
355. 0.707.
356. 0.00606, 0.188 sq. in.
357. 0.4965.
358. 867 B.t.u.
360. -836.2 B.t.u.
363. 0.941.
364. 400 B.t.u., 6.36 lbs.

366. 63.5%; 15.8%.

367. 46.5%; 10.99%.

368. 16.7%; 15.8%; 8.28%; 52.0%.

369. 15.5%; 29.2%.

372. 3750 ft. per sec.; 0.762.

373. 3900 ft. per sec.; 0.837; 12 lbs.

374. 2.4 lbs. per sec.

375. 0.97.

376. 0.98.

377. 5%.

378. 0.218 lb. per sec.

381. 330.2+837.2; 1193.4−25.9.

382. 256.

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