A TEXT-BOOK

OF

PLANE SURVEYING

BY

WILLIAM G. RAYMOND, C.E.

MEMBER AMERICAN SOCIETY OF CIVIL ENGINEERS; PROFESSOR OF GEODESY,
ROAD ENGINEERING, AND TOPOGRAPHICAL DRAWING, IN THE
RENSSELAER POLYTECHNIC INSTITUTE

NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

RAYMOND'S PL. SURV.

W. P. I

PREFACE.

THIS book has been prepared to meet the needs of those beginning the study of surveying. The subject treated is a simple one, and an effort has been made to make its presentation clear. The book is a text-book, not a treatise, and it is hoped that the teachers who use it will find it possible to devote their lecture work to amplification, rather than to explanation, of the matter it embraces.

So far as seemed necessary the plan of giving first the general method and then the details has been adopted, at the risk of some repetition, because I believe this to be the clearest method of presentation. A special effort has been made to render clear and comprehensible those points which an experience of fourteen years of practice and teaching has indicated to be the ones presenting the greatest difficulties to students. Simpler matters have been left to the student to work out from suggestions. The book can be read understandingly by any one who has completed Trigonometry, two formulas only being given whose derivation requires anything beyond. These may be derived by the teacher for such students as are sufficiently advanced.

Particular attention is called to the systematic arrangement of computations in Chapter VI.; to the article on the slide rule; to the discussion of practical surveying methods in Book II.; to the full treatment of coördinates; to the large number of examples; and to the use of the terms "latitude difference" and "longitude difference" for the old terms "latitude" and "departure."

The whole general scheme of terms is thought to be much more logical than that heretofore in use; and in this I have the support of Professors Merriman and Brooks, who have adopted practically the same nomenclature in their "Handbook for Surveyors," recently issued.

The logarithmic tables are from Professor C. W. Crockett's "Trigonometry," and are particularly suitable for surveyors' use.

I am indebted to many persons and books for valuable assistance. Especial acknowledgment is due to Professor H. I. Randall of the University of California, who drew Plate IV.; to Mr. J. J. Ormsbee, Mining Engineer, who drew Plate V.; to Mr. John H. Myers, Jr., A.B., C.E., for the problems on coördinates and for many suggestions; and to Professors R. S. Woodward of Columbia, and Frank O. Marvin of the University of Kansas, for valuable suggestions. Mr. E. R. Cary, C.E., Instructor in Geodesy, Rensselaer Polytechnic Institute, has given much help in the preparation of examples.

I also acknowledge my indebtedness to the following instrument makers for the use of cuts: Messrs. Buff & Berger, Boston, Mass.; W. & L. E. Gurley, Troy, N. Y.; Keuffel & Esser Company, New York; G. N. Saegmuller, Washington, D. C.; L. Beckman, Toledo, O.; Mahn & Co., St. Louis, Mo.; F. E. Brandis, Sons & Co., Brooklyn, N. Y. The principal instrument cuts, furnished by the Messrs. Gurley, Keuffel & Esser, and Mahn & Co., will be known by the firm name on the cut. Those of Buff & Berger are Figs. 19, 20, 48, 107, 148, 151, and 153. G. N. Saegmuller furnished Fig. 54. All of the cuts used are covered by copyright.

The book is submitted to my fellow teachers and students of surveying in the hope that it may prove useful to them in their work.

TROY, N. Y., AUGUST. 1896. WILLIAM G. RAYMOND.

CONTENTS.

BOOK I. PRINCIPAL INSTRUMENTS AND ELEMENTARY OPERATIONS.

BOOK II. GENERAL SURVEYING METHODS.

APPENDIX.

BOOK I.

PRINCIPAL INSTRUMENTS AND ELEMENTARY OPERATIONS.

INTRODUCTION.

1. Preliminary conceptions. An ellipse of axes AB and CD (Fig. 1), being revolved around its shorter axis CD, will generate the surface of an oblate spheroid of revolution.

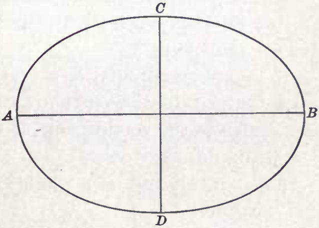


FIG. 1.

If we imagine the sea to extend underneath the surface of the earth so that the visible solid portions of the earth will be, as it were, floating on a ball of water, the shape of that ball will be, approximately, that of an oblate spheroid of revolution. The surface of this ball is called the mean surface of the earth.

The shorter axis is that connecting the poles; the longer is the diameter of the circle called the equator. In the case of the earth these two axes do not differ much in length, and hence the earth is usually spoken of as a "sphere slightly flattened at the poles." It may seem strange to the student that a difference of twenty-seven miles should be spoken of as a slight difference. But when it is said that this difference is about one third of one per cent of the length of the longer axis, the meaning is clearer.

The lengths of the two axes according to the latest determinations[1] are:

Shorter or polar axis	41,709,790 feet.
Longer or equatorial axis . .	41,852,404 feet.

[1] Clarke's spheroid of 1880. The values as found for Clarke's spheroid of 1866 are those generally used by geodesists. They are: shorter, 41,710,242 feet; longer, 41,852,124 feet.

If a plane is passed through an oblate spheroid of revolution, parallel to its shorter axis, it will cut from the spheroid an ellipse. If passed parallel to the longer axis, it will cut a circle. So with the earth: a plane passed parallel to the polar axis cuts from the mean surface of the earth an ellipse, while one passed parallel to the equator cuts a circle. Hence meridians of longitude are ellipses, and parallels of latitude are circles.

The surface of the sea, or that surface extended as before mentioned, forms what is called a level surface, and a line lying in this surface is a level line.

A line perpendicular to this surface at any point is a vertical line *for that point.* (A plumb line at any point is a vertical line for that point.)

A line perpendicular to a vertical line is a horizontal line.

A tangent to the earth's mean surface at any point is perpendicular to the vertical line at that point, and hence is a horizontal line for that point.

An inclined line is a straight line that is neither vertical nor horizontal.

A vertical plane at any point is a plane including the vertical line at that point.

A horizontal plane at any point is a plane perpendicular to the vertical line at that point.

A vertical angle is an angle formed by lines in a vertical plane.

A horizontal angle is an angle formed by lines in a horizontal plane.

At any point on the earth's surface there can be but one vertical line, but there may be an indefinite number of horizontal lines; there can be but one horizontal plane, but there may be an indefinite number of vertical planes.

If water collects upon the earth's surface in some depression above the mean surface, as in a lake or pond, or even as in a small glass, and if the water is still, its surface will be nearly parallel to that portion of the mean surface of the earth that is vertically below it; hence it will be a level surface, and a line drawn on it will be a level line. Such a line will be longer than the corresponding line drawn on the mean surface of the earth between the verticals through the extremities of the upper line.

The visible solid parts of the earth above the mean surface and the invisible solid parts below, make up a very irregular body. It is customary to speak of the visible parts of the earth's surface, both fluid and solid, as the "surface of the earth." In the definition in Art. 2, however, this term must be understood to mean not only the visible parts of the earth's crust, but also those parts that must be reached in connection with the operations of mining, bridge building, or other engineering works that extend below the visible surface.

2. Surveying defined. Surveying is the art of finding the contour, dimensions, position, etc., of any part of the earth's surface, and of representing on paper the information found.

The operations involved are the measurement of distances, — level, horizontal, vertical, and inclined, — and of angles, — horizontal, vertical, and inclined; and the necessary drawing and computing to represent properly on paper the information obtained by the field work.

The drawn representation is called a map. It may be a map showing by conventional signs the shape of that part of the earth's surface that has been measured; or it may be simply an outline showing the linear dimensions of the bounding lines, together with the angles that they make with the meridian, or with each other, and sometimes the position within the tract of structures, roads, or streams.

A map of the former kind is called a topographical map, and the operations necessary to its production constitute a topographical survey.

A map of the latter kind is a land map, and the operations necessary to produce it constitute a land survey.

Either one of these surveys is a geodetic survey, if the tract is so large that the curvature of the earth's surface must be taken into account. This limit is supposed to be reached when the tract is greater than one hundred square miles, but many surveys of tracts of much greater area than this are made without considering the mean surface to be other than plane.

Such surveys are of course inaccurate, but may be sufficiently correct for the purpose they are to serve.

A plane survey is one made on the assumption that the mean surface of the earth is a plane, above which is the irregular visible surface broken by hills and valleys. Almost all land surveys are plane surveys. Only plane surveys will be considered in this book.

In plane surveying all measurements are referred to a plane.

In geodetic surveying all measurements are referred to a sphere, or spheroid, according to the area covered and the accuracy desired.

It must be borne in mind that no physical measurements are exact. The art of surveying makes it possible to determine that a field of land contains a certain area, *more or less*, that a mountain is so many feet high, *more or less*, that a mine is so many feet deep, *more or less*, etc. That is to say, it is physically impossible to measure *exactly* either distance or angles. The precision attainable or desirable in any surveying operations will be discussed elsewhere in this book.

CHAPTER I.

MEASUREMENT OF LEVEL AND HORIZONTAL LINES.

3. The line to be measured. The distance between two points on the surface of the earth is the length of the level line joining the verticals through the points. If one of these points is much higher than the other (further from the mean surface), there may arise confusion as to which of several lines is meant by the above definition. In geodetic surveying it is customary to reduce the distance, when measured, to the length of the level line lying in the mean surface, and contained between the verticals through the points. The distance as measured will always be approximately the length of the level line lying midway as to altitude between the two points; and this length is that used in plane surveying. The length of this line is obtained by measuring a series of short horizontal lines; the sum of these lines approximates to the length of the required level line, just as the regular polygon of an infinite number of sides approximates to the circle.

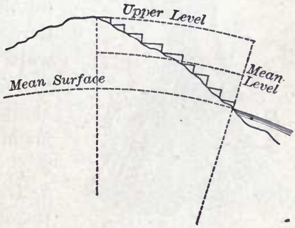


FIG. 2.

Fig. 2 will serve to make the above statements clearer.

INSTRUMENTS USED.

4. Chains. The instruments used are chains, tapes, and wooden or metallic rods. Chains are of two kinds — Gunter's chain and the engineer's chain. These chains are alike in form, but vary in the length of the links and the length of the entire chain. In Gunter's chain the links are 7.92 inches long, and in the engineer's chain they are 12.00 inches, or one foot, long. With this exception, one description will apply to both.

A chain consists of one hundred "links" made of iron or steel wire. Number 12 steel wire is best. Fig. 3 shows the form of the links. A link includes one of the long pieces and two or three rings, according as there are two or three rings used to connect the long pieces. The rings are introduced to enable one to handle the chain more readily. Brass tags with the proper number of points mark the ten-link divisions from each end toward the center, and a round tag marks the center or fifty-link division. The handles are of brass, and are usually made adjustable, so that slight changes in the length of the chain may be corrected. A special form of handle is sometimes used, having a knife edge on which is filed a notch indicating the zero of the chain for the day, the chain being compared daily with a standard kept for the purpose.

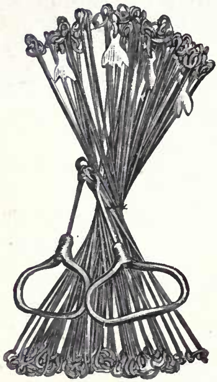


FIG. 3.

The Gunter's chain, having 100 links of 7.92 inches each, is 66.00 feet long, and the engineer's chain is 100.00 feet long. The former was devised by Mr. Edward Gunter, for the purpose of facilitating the computations of areas that have been measured. Its length was so taken that 10 square chains make one acre. It is the chain referred to in the table of surveyor's square measure, which should be carefully memorized. This table may be found in almost any arithmetic.

In all surveys of the public lands of the United States the Gunter's chain is used, and all descriptions of land, found in deeds or elsewhere, in which the word "chain" is used, are based on this chain. It is not convenient for use in connection with engineering works, such as railroad construction, canal building, bridge building, etc., where the unit of measure is the foot, and hence in such work the engineer's chain is used.

5. Tapes. Steel tapes are better than any sort of chain for most engineering work and for all fine surveying. These tapes are made in various forms, from thin ribbons half an inch wide to flat wires about one eighth of an inch wide and one fiftieth of an inch thick. The ribbon tapes are graduated on the front to feet, tenths, and hundredths of a foot, or to feet, inches, and eighths, and on the back to links of 7.92 inches. They usually come in box reels and are from twenty-five feet to one hundred feet long. They are suitable for very nice work of limited extent, and particularly for measurements for structures, such as bridges and buildings, both in the shop and in the field. They are not suitable for ordinary field operations of surveying, because they are easily broken. For such work the narrower, thicker tapes are preferable. These may be obtained in any lengths up to a thousand feet or more; but the lengths usually kept in stock are fifty feet and one hundred feet. They are graduated, usually to ten feet and sometimes to fifty feet only, but may be graduated to suit the purchaser. For surveying work the following graduation is recommended: Graduate to feet, numbering every tenth foot from one end of tape to the other, and not from each end to the middle, as in the chain. Have the tape one foot longer next the zero end than its nominal length and divide the extra foot into tenths. If it is required to make measurements closer than to tenths of a foot, carry a pocket steel tâpe from three to five feet long and graduated in feet, tenths, and hundredths. Hundredths can usually be estimated with sufficient precision.

6. Reels. For the narrow tapes there are a number of patterns of reels, most of them aiming to furnish an open reel, of form convenient to go in the pocket when not in use, and so constructed as to enable the surveyor readily to reel the tape. There is but one reel that has come to the author's attention that combines all three of these requisites. This is shown

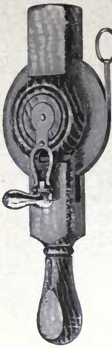


FIG. 4.

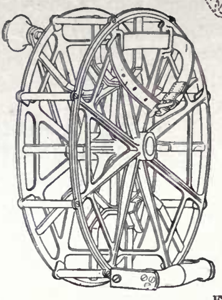


FIG. 5.

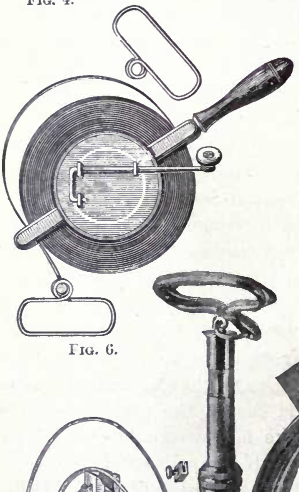


FIG. 6.

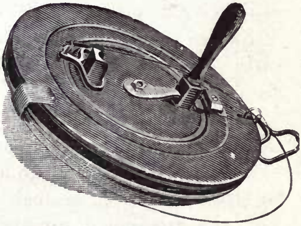


FIG. 7.

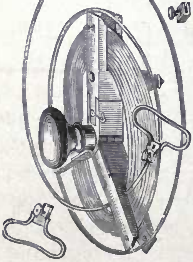


FIG. 8.

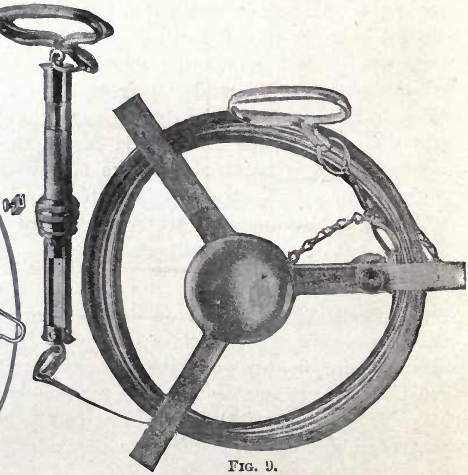


FIG. 9.

in Fig. 6, and a modified form in Fig. 4. Other forms of reels are shown in Figs. 5, 7, 8, and 9. Fig. 5 is a reel for a tape from 300 feet to 1000 feet in length. Fig. 9 shows a tape fitted with a spring balance for measuring the pull on the tape when in use, a level to show when the tape is held horizontal, and a thermometer to give the temperature. The necessity for these attachments will appear hereafter. Such a tape is used for land surveys in the city of New York.

Tapes should always be kept dry, and if wet by use, should be wiped dry and rubbed with a cloth or leather that has the smallest possible quantity of mineral oil on it.

7. Linen tapes. In addition to the steel tapes, linen and "metallic" tapes are used for rough work. The ordinary linen tape is well known to everyone. The metallic tape is a linen tape with a few strands of fine brass wire woven through it. The linen tape is subject to great change in length with changes of moisture in the atmosphere, is soon stretched, and is easily worn out. The metallic tape is not so subject to change in length with change of atmospheric conditions; it is soon stretched, but is not nearly so soon worn out as is the linen tape. Both these tapes, being easily stretched, soon become quite inaccurate for any but the commonest kinds of work, where the measurements are short and need not be closer than to the nearest tenth of a foot. They are graduated in feet, tenths, and half-tenths, and on the reverse side in links of 7.92 inches. Sometimes they are graduated in inches. They are sold in paper or leather box reels.

8. Rods. While some rough measurements are made with the ordinary ten-foot pole or a similar arrangement, no other surveying measurements are now made with wooden or metallic rods, except measurements of base lines in connection with important geodetic surveys; in these the rods, usually metallic, are arranged with other devices into a very elaborate piece of apparatus. It is believed that the narrow steel and brass tapes will entirely supersede the elaborate base apparatus now in use.

9. Pins. These are used with the chain for the purpose of marking chain lengths. They are about fourteen inches long, made of steel or iron wire somewhat less than a quarter of an inch thick (No. 4 to No. 6 wire gauge) with a ring at one end into which is fastened a strip of cloth to insure ready finding of the pin when stuck in tall grass or brush. The other end is pointed. Eleven of these pins constitute a set. They are usually carried on a ring like a large key ring, or loose in the hand.

10. Range poles. Poles are used to range out the line to be measured. They are usually of wood, round or hexagonal, six to eight feet long, tapering from the bottom to the top, shod with a pointed iron shoe, and painted red and white in alternate strips one foot long. Gas pipe is sometimes used, but is not recommended, because, while it does not break, and while from its weight such a pole is easily balanced on its point, it is also very easily bent, and very difficult to straighten, and is too heavy to be handled with ease. A good pole for nice work in cities or on railway surveys is made of hexagonal steel about three eighths to five eighths of an inch thick, painted like the wooden poles, and pointed at one end.

METHODS.

11. Preliminary statement. The accurate measurement of a line on a comparatively level piece of ground is a task difficult for a beginner and not simple for an expert chainman, however easy it may seem. The method of doing this work on ordinary farm surveys, where the smallest unit of measure is the link (7.92 inches), and where an error of one in three hundred to one in five hundred may be tolerated, will be described; and the errors incident to this method with the necessary corrections, as well as the more precise methods applied to city work, will then be discussed.

12. Chaining. It will be noticed when the chain is received from the maker that it is so folded together as to be compact in the center of the bundle and somewhat bulky at the ends, in shape not unlike an hour glass. This results from doing up the chain as follows: Take the two links at the center of the chain in the left hand, with the fifty-link tag on the left. Take the right hand ends of the pair of links next but one to those in the left hand, in the right hand, and lay the right hand pair and the intermediate pair in the left hand diagonally across the pair already there. In like manner proceed to the ends of the chain, being careful always to place the new links diagonally across the links already in the left hand and always diagonally the same way. It is better, however, to do up the chain from one end instead of from the middle. The method is the same except that the two end links are first taken in the left hand, the handle end to the right. A little more time is required, but the chain is more readily loosened for service.

It will be assumed that the ground on which the line is to be measured is comparatively level, and that the ends of the line are visible, one from the other. If there is no visible object to mark the further end of the line, a range pole is placed there, toward which the measurement is to be made. If the rear end of the line is also marked, the head chainman will be able, without difficulty, to put himself in approximate line, thus saving time. The strap with which the chain is fastened is removed, and, if the chain has been done up from the middle, the two handles are taken in the left hand of the forward chainman and the chain bundle in the right hand, allowing a few links next the handles to fall off. The chain bundle is then thrown out in a direction opposite to that in which the measurement is to be made, the chainman retaining the handles in his left hand. The chain should be thrown with sufficient force to straighten it out. The forward man, usually called the "head chainman," then takes the forward end of the chain and the pins, and starts toward the further end of the line, while the rear chainman allows the chain to slip through his hands to see that it is not kinked or bent. If he finds any bends he straightens them. If the chain has been done up from one end, it should be laid down near the starting point with one handle uppermost, the latter to be taken by the head chainman, who moves off toward the further end of the line. The rear chainman allows the chain to slip through his hands as before. The chain gets enough rough service that can not be avoided, without subjecting it to the additional unnecessary wear arising from throwing it out forcibly, to be kinked or caught in the brush or other obstruction.

One pin is left with the rear chainman. As the head chainman walks out, he places one pin in the hand that carries the chain, the remaining pins being in the other hand. When the chain is almost out, the rear chainman calls "Chain." The head chainman then stops, turns, and straightens the chain while being put into approximate line by the rear chainman. The chain being taut and approximately "lined," the head chainman assumes the position shown in Fig. 10, and the rear chainman, by motions or the words "right" and "left," accurately aligns the pin held by the head chainman and cries "Stick." The head chainman then forces the pin into the ground, taking care that it marks exactly the end of the chain, and cries "Stuck."

FIG. 10.

The rear chainman then, and not till then, draws his pin, keeping hold of the chain, and follows the head chainman, who moves on toward the forward end of the line, and the whole operation is repeated. After one pin has been placed, the head chainman, on being stopped by the call of the rear chainman, can quickly put himself in approximate line by sighting back over the pin last set to the flag left at the starting point. The work thus proceeds till the further end of the line is reached, when the head chainman walks right on past the point till the chain is all drawn out. He then returns to the point and notes the fraction of a chain between the last pin and the point. This added to the number of chains gives the distance required. If the distance is more than ten chains, the head chainman, when he sticks his last or tenth pin, calls "Stuck out." He then waits by the pin till the rear chainman comes up with the pins he has collected, which should, with the pin he started with, be ten. He counts them, as does the head chainman, as a check, and they note one "tally." At any time the number of tallies plus the number of pins in the rear chainman's hands gives, in tens of chains and chains, the distance that has been measured.

13. Hints. The following hints may be of service to beginners:

The rear chainman should not use the pin to brace himself.

He should hold the outside edge of the handle flush with the rear side of the pin, without moving the pin.

He should not stop the head chainman with a jerk.

He should not sit down on the ground while holding the pin.

Motions and words should be sharp and distinct.

Motions and instructions should be proportionate to the distance that the pin is to be moved; for example, the arms should not be swung wildly when the pin is to be moved an inch.

The head chainman should see that the rear chainman is looking when he tries to straighten the chain.

The chain should not be jerked in straightening it; it should be straightened by an undulatory motion.

In straining the chain, the head chainman should pull steadily.

Attention to these matters will greatly facilitate the work.

14. Chaining on slopes. In chaining up or down hill, one end of the chain is raised till both ends are as nearly as possible in a horizontal line.

If the slope is so steep that one end of a full chain cannot be raised enough to bring both ends in a horizontal line, the chain is "broken," that is, the distance is measured by using a part of the chain at each measurement. To do this, the chain should be carried out as if a full chain were to be used, the head chainman returning to such a point on the chain (preferably a ten-link point) that the portion of chain between himself and the rear chainman may be properly leveled. A measurement is made with this portion, then with the next succeeding portion, and so on till the whole chain has been used. Care must be taken not to get the pin numbering confused. The rear chainman should have but one pin for the whole chain.

The high end of the chain is transferred to the ground in one of several ways, according to the precision desired. If the work is to be done with care, a plumb line is used. If an error of a tenth of a link in each chain is not important, a pin may be dropped from the high end, and stuck in the ground where it is seen to fall. The pin should be dropped ring down. A small pebble will serve the purpose for rough work. In careful work the plumb bob should not be dropped and the pin placed in the hole made; but it should be noticed where the bob will drop, and the ground should be made smooth with the foot, and the bob swung down till it is still and just clearing the ground; then it should be carefully lowered till it touches. The chainman should then lower his grasp on the string, hand over hand, keeping the bob steadily in its place, and place a pin in the ground at the point of the bob. The pin should be put in the ground in an inclined position across the line, so that the point where it enters the ground is that covered by the bob. The position should then be checked. In place of a pin a small wire brad may be used and left in the ground.

In chaining up hill, the rear chainman must hold the bob directly over the pin which has been set in an inclined position, and must at the same time align the head chainman and see that he sticks at a moment when the bob is directly over the point. It will be at once inferred that it is easier to measure down hill correctly than up hill. Therefore, where close work is required on inclined ground, the measurements should always, if possible, be made down hill.

ERRORS INVOLVED.

15. Classes. The errors involved in the method of chaining just described, whether the work is done with a chain or a tape, are of two classes: (*a*) constant or cumulative errors, and (*b*) accidental or compensating errors.

(*a*) Cumulative errors are such as occur each time in the same direction. They are not necessarily equal, but may be so. Thus a line so long as to require that a chain one inch too short shall be applied to it ten times, will be recorded ten inches too long, the error of the chain being added each time the chain is applied. In this case the errors are equal.

(*b*) Compensating errors are such as tend to balance; that is, they are as likely to be in one direction as in another. Thus the error that may be made in setting the pin, if it is attempted to set it just right, will be a compensating error, for it will be set ahead of the true point about as often as it will be set behind it. Error in plumbing is of the same character.

16. Causes. Cumulative errors arise from five causes: (*a*) erroneous length of chain, (*b*) errors in judgment in making the chain horizontal in chaining up or down hill, (*c*) erroneous alignment of the chain, (*d*) failure to straighten the chain for each measurement, (*e*) sag of the chain when not supported throughout its length.

Compensating errors arise from accidental inaccuracies in setting the pin, and from irregularities in the pull exerted on the chain or tape. They are remedied by care, and, in fine work, by measuring the pull on the tape by a spring balance.

Erroneous length of chain may arise from any one of six causes.

(1) One or more links may be bent, making the chain too short. The remedy is to see that the links are straight or to use a tape.

(2) Mud or grass may get in the links and rings with the same effect. The remedy is obvious.

(3) A bent link that has been straightened has been permanently lengthened, thus making the chain too long. The remedy is to compare the chain or tape frequently with a standard tape kept for this purpose. If the chain is found to be slightly too long, it may be adjusted by the nuts at the handle, or if such a handle as is described in Art. 4 is used, the standard length of the chain for the day may be marked on the handle.

(4) The links and rings wear, thus making the chain too long. While the wear is slight, it may be adjusted at the handle. When it becomes excessive, it must be known and allowed for as hereafter described.

(5) The chain may be lengthened by too hard pulling, but this does not often occur. The remedy is the same as in (3).

(6) The chain may be too long or too short according as the temperature is higher or lower than that for which the chain is standard. The remedy is to know the temperature at which the chain is standard and that at which the work is done and make the necessary correction to the recorded measurements.

In general it may be said that erroneous length of chain may be corrected by adjusting the handles, or by comparing the tape or chain with a standard and correcting the records taken according to the errors found. It should be carefully noted that, in measuring the distance between two points, a long chain gives the distance too short and a short chain gives the distance too long, while in laying out a line of given length the errors are just reversed. Failure to appreciate this difference often causes confusion and error, and hence the student should thoroughly fix it in mind. Since similar figures are in area as the squares of their homologous sides, the *erroneous area* of a field determined from measurements with an erroneous chain, will be to the *true area* as the square of the *nominal length* of the chain is to the square of its *true length.*

17. Temperature. The coefficient of expansion of steel is about 0.0000065. (Tapes and chains being alike subject to this error, this discussion will do for both.) A tape or chain will expand or contract sixty-five ten-millionths of its length for each Fahrenheit degree change of temperature. Thus a line about ten chains long, if measured in the summer with the chain at a temperature of, say, 80° F., the chain being standard at a temperature of 62° F., will be recorded 0.117 links too short; while the same line measured with the same chain in midwinter with the chain at a temperature of 0° F., will be recorded 0.403 links too long, making a total difference of 0.52 links between the two measurements. This is an error of one in two thousand for the extreme difference in temperature of 80° F.

It is thus seen that for all ordinary work the temperature correction may be neglected; but in city work where an inch in frontage may be worth several thousand dollars, it is very necessary that the temperature be determined and the standard temperature of the tape known. The tape shown in Fig. 9 is adjustable for the effect of temperature. A scale numbered to correspond to the thermometer readings indicates the proper setting of the adjusting screw. The spring balance insures a constant pull.

18. Sag. The effect of sag in shortening a tape that is unsupported except at the ends is given by the following formula in which l is the unsupported length of the tape, w the weight of a unit of length, and P the pull in pounds.

$$x = \frac{l}{24}\left(\frac{wl}{P}\right)^2 \text{ [1].}$$

This formula the student will have to accept until he has studied the elements of Mechanics and Calculus. It assumes that the tape is supported only at the ends, and that it is standard for no pull when supported its entire length. If the tape is standard for a pull of P_0 pounds, substitute in the formula for P the difference $P - P_0$.

If the tape is applied n times in measuring a line and each time is supported only at the ends, and the pull is always the same, the correction for the whole line is n times the above expression.

The formula gives the difference between the length of the curve of the unsupported tape and its chord. The real distance measured is the chord, while the distance read is the length of the curve or of the whole tape. It is evident, therefore, that the distance is read too long, and hence the formula is a negative correction.

19. Pull. If the chain were of constant cross section as is a tape, the amount that the chain would stretch for a pull of P pounds would be given by the following formula in which l is the length of the chain in inches, S is the area of its cross section in square inches, and E is the modulus of elasticity of the metal of which it is made:

$$y = \frac{Pl}{SE}.$$

[1] This formula has been developed by Prof. J. B. Johnson.

E for steel is variable, but may be taken at 28,000,000. There is no such thing as a perfectly elastic material. If there were, the amount that a given length of the material would be stretched by varying pulls would be proportional to the pulls, and supposing the piece to be of unit cross section, as one square inch, the pull that would stretch it by its own length is known as E, the modulus of elasticity. For any other than a unit cross section the stretch for a given pull will be inversely proportional to the cross-sectional area. Hence the formula. The lengthening effect of a given pull on a tape would be as in the formula. In the case of a chain, the effect would be somewhat greater, owing to the elongation of the rings.

20. Elimination of sag and pull. To find the pull that will just balance the effect of sag, equate the values of x and y and solve for P. Since the units are inches in the y formula, they must be inches in the x formula, and l must be the length of the tape in inches, and w the weight of an inch of the tape. The solution gives

$$\frac{l}{24}\left(\frac{wl}{P}\right)^2 = \frac{Pl}{SE};$$

whence

$$P = \sqrt[3]{\frac{w^2l^2SE}{24}}.$$

A good practical way to determine this value is as follows: Mark on a smooth level floor a standard tape or chain length, with the tape supported its entire length, and with only enough pull to straighten it. Raise the tape, and supporting it only at the ends, measure with a spring balance the pull necessary to bring the ends over the marks on the floor. It will be best to have one end fastened in a firm hook in the wall for the test, and afterward to have both ends held by the chainmen, that they may see just the difficulties involved. The test should be made for the whole chain, the half chain, and the quarter chain. The only way in which this work can be done with extreme nicety is by employing mechanical means to pull the chain steadily, and a telescopic line of sight to transfer the floor marks upward to the tape ends. As in all work, except the measurement of base lines for geodetic surveys, or elaborate triangulation surveys of cities, the chain or tape is to be held in the hands of the chainmen, it will be unnecessary, except for comparisons, to resort to the nicer methods. Experiments of the kind noted above will demonstrate that the whole chain should never be used unsupported, and that the tape is by far the most satisfactory measuring instrument. In rough work, where a precision of one in five hundred, or even one in five thousand, as a maximum limit, is sufficient, the chain *may* be used. In close work requiring a precision of one in five thousand and upward, the tape should invariably be used.

21. Alignment. Errors due to inaccuracy of alignment of the chain are usually not great. In ordinary work no great pains need be taken to align the chain within an inch or two, except where stakes are to be driven on the line. In close work, of course, the chain should be correctly aligned.[1]

22. Slope. In chaining on slopes, errors of judgment in making the chain horizontal are eliminated by the use of a level tube fastened to one end of the chain, which tube, if properly adjusted, will indicate when the chain is horizontal. This is rarely used with a chain, but frequently with a tape. Much can be done without such a level by having a third man stand on one side of the chain and compare the parallelism of the chain and the horizon, or the horizontal lines of some building. If there is no horizontal line visible, he can still judge better from the side as to the horizontality of the tape, than can the chainmen at the ends. It is almost always true that the lower end of the chain is not raised high enough, because a horizontal line on a hillside extending in the direction of the slope, always appears to dip into the hill. Hand levels (see Art. 52) carried by the chainmen are of great service in hilly country. The effect of neglecting the slope entirely, which is also the correction to be applied if the line has been measured on the slope instead of in horizontal lines, is given in Appendix, Table I., page 361.

[1] Let the student compute the error arising in a ten-chain line from placing the end of the chain first six inches on one side of the line and then six inches on the other side, throughout the measurement.

It will be seen that the error caused by neglecting a slope of five in one hundred is about one in one thousand, while a slope of ten in one hundred, which is not unusual in hilly country, causes an error of one in two hundred. Fifty feet in one hundred is about the steepest slope met with in nature, aside from rock cliffs, and the error here is more than one in ten.

On a slope where close work is required, it is considered best to measure along the slope, keeping the tape or chain supported throughout its entire length, and making the necessary reductions when the line has been measured. The reduction can be made exactly by the use of a table of versed sines if the angle of slope is known. It may be approximately obtained from Table I., page 361, by interpolating for the small angles, or it may also be approximately obtained by the use of the following formula when the rise in a tape length or in the entire line, if it is of uniform slope, is known:

The square of the rise divided by twice the known side, be it base or hypotenuse, gives the difference between the base and hypotenuse.

Demonstration: Let B be the base, H the hypotenuse, and R the rise; C being the difference between B and H. Then $B = H - C$ and $H = B + C$. Assuming H known, there is written

$$H^2 - (H - C)^2 = R^2;$$

whence

$$C = \frac{R^2 + C^2}{2H}.$$

Neglecting C^2 as a very small quantity, there results

$$C = \frac{R^2}{2H}.$$

Similarly if B is known, there may be written

$$(B + C)^2 - B^2 = R^2,$$

$$C = \frac{R^2 - C^2}{2B};$$

and as before, neglecting C^2, there results

$$C = \frac{R^2}{2B}.$$

Hence the rule already given

23. Precision to be obtained. In measuring lines the degree of precision obtainable should be known by the surveyor. The author suggests the degrees of precision mentioned below as those that should be attained ordinarily before the surveyor can say he is doing good work. The figures given do not refer to the absolute lengths of the lines, involving a knowledge of the absolute length of the chain or tape, but merely to the probable error of the mean of two measurements of the same line made with the same tape under different conditions. Not all conditions of work are covered; but only such as usually exist. The surveyor will be able to judge as to how closely the conditions under which he is working at any time correspond to those given.

In good, fairly level ground, good work will be represented by differences between two measurements of one in twenty-five hundred, and excellent work by differences of one in five thousand, assuming the work to be done with a chain. These differences give the probable error of the mean value as one in seventy-five hundred and one in fifteen thousand, and the probable error of a single determination rather better than $\frac{1}{5000}$ and $\frac{1}{10000}$. On hilly ground, rough and covered with brush, one in one thousand might be considered good and one in five hundred passable, where the land is not of great value. These differences give the probable errors of mean and single measurement as $\frac{1}{3000}$ to $\frac{1}{1500}$ and $\frac{1}{2000}$ to $\frac{1}{1000}$ respectively. It should be remembered that the value of the land measured, or the object of the survey, is a better basis for judgment as to passable work than the conditions under which the work is done.

In work in large cities the author thinks that a precision of one in fifty thousand should be obtained. That is, it is thought that the probable error of the mean of two measurements should not be greater than one in fifty thousand. This will require that the same line measured under totally different conditions as to weather should be recorded, after the necessary corrections for pull, grade, and temperature have been made, both times alike, within about one in seventeen thousand, or, in round numbers, three tenths of a foot in a mile.

When but two observations of a quantity have been taken, the probable error of the mean is $\frac{1}{3} D$, where D is the difference of values determined. The probable error of either of the observations is 0.47 D or, roughly, $\frac{1}{2} D$. (See any treatise on Least Squares.) This supposes that all cumulative errors and mistakes have been eliminated by correction and that only accidental errors remain.

The following are the requirements for securing a precision of one in five thousand and one in fifty thousand. For intermediate standards, the requirements will lie between those mentioned:

For a precision of one in five thousand, using a tape, no corrections for sag, grade, pull, or small changes of temperature need be made. The tape may be stretched by hand, the pull and horizontality being estimated by the tapemen. The plumb line will be used on uneven ground as in close chaining. The temperature of the air may be compared with that for which the tape is standard, and a corresponding correction deduced.

For a precision of one in fifty thousand, the temperature of the tape should be known within a degree or two Fahrenheit; the slope should be determined by measuring over stakes whose elevations have been determined by a level, or by measuring on ground whose slope is known. The pull should be known to the nearest pound, and hence should be measured with spring balances. If the tape is held on stakes, the sag correction must be considered. The work may be done in any ordinary weather, but is best done on cloudy days, so that the temperature of the tape may be more constant. In sunny weather the mercurial thermometers attached to the tape may indicate a very different temperature from that of the tape. If the absolute length of the tape is not known, of course the absolute length of the line is not determined.

CHAPTER II.

VERNIER AND LEVEL BUBBLE.

24. Before proceeding with a description of surveying instruments, it is necessary to describe two important attachments common to almost all such instruments. These are the vernier and the level bubble.

VERNIER.

25. Vernier. This is a device for reading scales to a greater degree of precision than is possible with the finest convenient division of the scale. Thus a scale graduated to read tenths of an inch, may be read to hundredths of an inch by the aid of a vernier. This is done by making an auxiliary scale called a vernier, with divisions one one-hundredth of an inch smaller or larger than those of the main scale. If the divisions are larger than the main scale, the vernier is called a retrograde vernier ; and if the divisions are smaller, it is called a direct vernier. The reason for this distinction will appear hereafter. In Fig. 11 S is a scale divided into inches and tenths. V is the vernier made by dividing a space equal to nine of the small divisions of the main scale into ten equal parts, thus making each division on the vernier one one-hundredth of an inch shorter than a division of the main scale. The first division line of the vernier falls one one-hundredth of an inch toward zero from the first division line of the main scale. If then the first division line of the vernier is made to coincide with the first line of the main scale

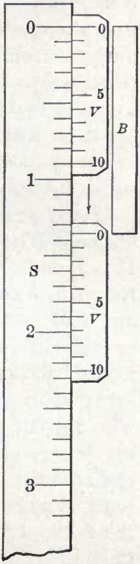


Fig. 11.

the vernier will have been moved one one-hundredth of an inch. Similarly the second division of the vernier is two one-hundredths of an inch toward zero from the second line of the main scale, and hence if the vernier is moved along till the second line of the vernier coincides with the second division of the scale, the movement has been two one-hundredths of an inch, and so on. If the vernier is moved till the zero is opposite some other division of the scale than the zero division, the first line of the vernier will be one one-hundredth short of the line of the main scale next ahead of the zero of the vernier; the second line of the vernier will be two one-hundredths short of the second line of the main scale, and so on. If the vernier is moved along a little further till, say, the fourth line of the vernier has been brought into coincidence with the fourth line of the main scale ahead, the vernier has been moved a further distance of four one-hundredths of an inch. Hence to tell how far the zero of the vernier has moved from the zero of the main scale, note the inches and tenths on the scale from zero to the zero of the vernier, and get the fractional tenth expressed in hundredths by looking along the vernier and finding the division that coincides with a division of the main scale. This vernier is called direct, because in reading it one looks forward along the vernier in the direction in which the vernier has moved.

Let it be required to read the length of the bar *B*. Place one end of it opposite the zero of the main scale and vernier. It will be noticed that the other end is opposite a point on the main scale between one and three tenths inches, and one and four tenths inches. Move the vernier till the zero is opposite this end of the bar. To read the length of the bar, read on the main scale one and three tenths inches and look along the vernier and find that the sixth division coincides with a division of the scale and that therefore the length of the bar is one and thirty-six one-hundredths inches. It will be observed that the divisions of the vernier are one tenth of one tenth of an inch smaller than the divisions of the scale. That is, the value of the smallest division on the main scale divided by the number of divisions of the vernier gives the smallest reading that may be had with the vernier. This is called the least count.

In the retrograde vernier a space equal to a given number of divisions of the main scale is divided into a number one less on the vernier. Thus for a vernier reading to hundredths of an inch with a scale graduated to tenths, eleven divisions of the scale will be divided on the vernier into ten spaces, making each division one tenth of one tenth of an inch longer than those of the scale. The vernier is therefore placed as shown in Fig. 12, back of the zero of the scale instead of ahead of it, as in the direct vernier. "Back" and "ahead" are used with reference to the direction in which measurements are to be made. From the portion of the scale extended above the zero in the figure, it will be seen that the first line of the vernier is back of the first line of the scale by one one-hundredth of an inch, the second line by two one-hundredths, and so on. The principle of operation is the same as in the direct vernier, except that one must look backward along the vernier to find the coinciding line.

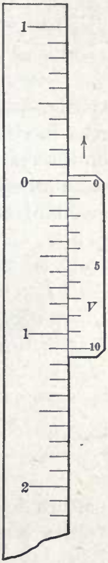


FIG. 12.

A vernier to read angles is generally used when the angles are to be read to the nearest minute or less. The principle of construction is the same as for linear verniers. A vernier to read minutes will usually occur with a circle graduated to read half degrees. If a space equal to twenty-nine of such divisions is divided on a vernier into thirty equal parts, each division of the vernier will be one thirtieth of thirty minutes, or one minute, less than a division of the main scale, and the instrument is said to read to minutes. If a circle is to be read to the nearest twenty seconds, it is usually divided into twenty minute spaces, and a vernier must then have sixty divisions, since

$$\frac{20'}{n} = \frac{1'}{3};$$
$$n = 60 \text{ divisions.}$$

That is, fifty-nine parts of the scale must be divided on the vernier into sixty parts, making each part of the vernier one sixtieth of twenty minutes, or one third of a minute, less than a division of the main scale. Figures 13, 14, and 15 show three double verniers. They are called double, because there are really two verniers in each figure, one on each side of the vernier zero. They are thus arranged so that angles may be read in either direction, the circle graduations being numbered both ways for the same purpose. The student should determine whether the first two are direct or retrograde, the least count of each, and their readings. The third is a peculiar

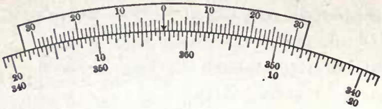


FIG. 13.

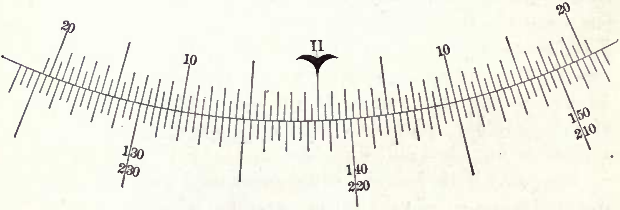


FIG. 14.

pattern found ordinarily only on compasses. It is a double vernier, direct as to division (though it is sometimes made retrograde), and the lower left-hand and upper right-hand portions form one vernier. It is used where there is lack of space to make the ordinary form. To read an angle measured to the right, read on the scale to the last division before reaching the zero of the vernier, follow to the right along the vernier, noting the lower line of figures for a coinciding line, and if none is found, pass to the extreme left end of the vernier and look along toward the right, noting the upper line of figures till a coinciding line is found. Thus the reading of the vernier in the figure is 355° 20′, or 4° 40′.

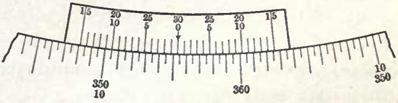


FIG. 15.

LEVEL BUBBLE.

26. Description. The spirit level consists of a glass tube almost filled with ether, the remaining space being filled with the vapor of ether. The bubble of vapor will always seek the highest point in the tube. If the tube were perfectly cylindrical, the bubble would occupy the entire length of the tube when the tube is horizontal, and the same thing would be true if the tube were but slightly inclined to the horizon, thus making it impossible to tell when the tube is in a truly horizontal position. The tube is, therefore, ground on the inside so that a longitudinal section will show a circular arc. A line tangent to this circle at its middle point, or a line parallel to this tangent, is called the axis of the bubble tube. This axis will be horizontal when the bubble is in the center of its tube. Should the axis be slightly inclined to the horizon, the bubble will move toward the higher end of the tube, and if the tube is ground to the arc of a circle, the movement of the bubble will be proportional to the angle made by the axis with the horizon. Therefore, if the tube is graduated into divisions, being a portion of the circumference of a very large circle (so large in fact that the arc of a few seconds is quite an appreciable length), it will be possible to determine, within the limits of the tube, the angle that the axis may make at any time with the horizon, provided the angular value of one of the divisions of the tube is known. This is done by simply noting how many divisions the center of the bubble has moved from the center of the tube.

It will be evident that divisions of uniform length will cover arcs of less angular value as the radius of the tube increases, and also that the bubble with a given bubble space will become more elongated as the radius is increased. Therefore the bubble is said to be sensitive in proportion to the radius of curvature of the tube, and this is also indicated by the length of the bubble. The length of the bubble, however, will change with change of temperature, becoming longer in cold weather and shorter in warm weather. In the best class of tubes there is a partition near one end, with a small hole in it at the bottom, so that the amount of liquid in the main tube may be regulated, thus regulating the size of the bubble. This is necessary, because independently of the effect of long radius a longer bubble is more sensitive than a shorter one. A bubble should settle quickly, but should also move quickly and easily.

27. Determining the angular value of one division. There are several methods of determining the angular value of one division of the bubble tube, all essentially the same in principle.

The axis is moved through a small angle, and the movement of the bubble is recorded in divisions; then the angular value of one division is at once found by dividing the angle by the number of divisions through which the bubble has moved.

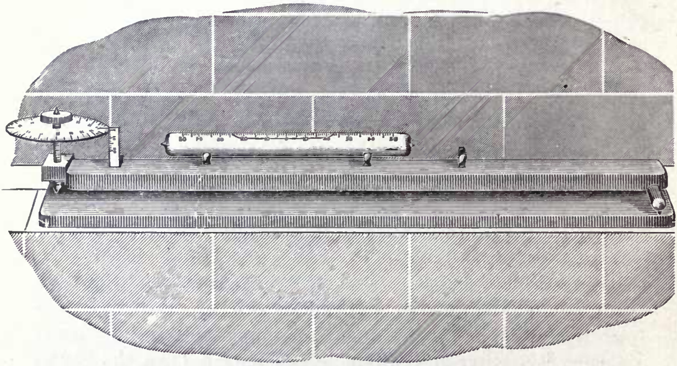


Fig. 16.

It is not easy to measure the small angle exactly but it is not very difficult if closely approximate results are sufficient.

In Fig. 16 is shown a level vial, as it is sometimes called, resting on a level trier. The construction of the level trier is perhaps sufficiently clear from the cut. It consists simply of a board resting on a knife edge at one end, and capable of being raised or lowered at the other by means of a screw so divided as to tell the angle of inclination of the board. The screw is called a micrometer screw, because it will measure a very small movement. Suppose the pitch of the micrometer screw is one sixtieth of an inch. Then the divisions on the vertical scale attached to the movable board will be one sixtieth of an inch ; so that a single revolution of the screw will move the scale past the edge of the screw head by one division. If the circumference of the disk head of the screw is divided into one hundred parts, and the screw is turned only so much as will cause one division on the disk to pass the scale, the board has been moved vertically through one one-hundredth of one sixtieth of an inch. If now the length of the bar from pivot to screw is known, the angular movement of the bar may be computed. Thus, if the length of the bar is eighteen inches, and the bar is raised so that one division of the scale passes the micrometer head, and so that in addition ten divisions of the micrometer head pass the scale, the linear elevation of the end of the bar is

$$\tfrac{11}{10} \times \tfrac{1}{60} \text{ inch} = 0.018+ \text{ inch}.$$

Since there are 206,265 seconds in an arc equal in length to radius, there results the proportion, in which x is the angle in seconds,

$$\frac{x}{206265} = \frac{0.018+}{18}$$

Whence $$x = 206.265 \text{ seconds}.$$

If now a bubble tube were resting on the bar, and the run of the bubble were observed, for the above movement, to be ten divisions, the value of one division would be 20.6 seconds.

Example. In the above example it is found that the run of the bubble is one inch. Find the radius of curvature of the bubble tube.

Other methods of finding the angular value of a division of the tube will be suggested in the problems on Chapters III. and IV.

Many of the level vials found on compasses, and on the lower plates of many other instruments, are not graduated, are ground to short radii, and not uniformly, and hence are not fit for accurately leveling the instruments to which they are attached ; but such bubbles are cheaper than others, and when placed on a compass or other instrument not intended for high-class work, they are sufficiently precise for the purpose for which they are used.

28. Principles. The proper adjustment of a bubble on an instrument so that one can determine when the instrument is level, depends on the following principles :

I. *If a frame carrying a bubble tube, and resting on two supports that lie in a level line, is reversed end for end on the supports, the bubble will occupy the same position in the tube for both positions of the frame.*

In Fig. 17 it is seen that the axis of the tube makes the same angle with the horizon in both positions, and the same end is higher.

Conversely, if a frame to which a bubble tube is rigidly attached is reversed on two supports, and the bubble occupies the same position in its tube for both positions of the frame, the supports lie in a level line, or, as is usually said, are level. It should be noted in the above that the bubble is not necessarily in the center of the tube, but merely retains the same position in the tube for the direct and reversed positions of the frame.

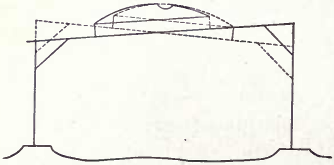


FIG. 17.

If the axis of the tube in the foregoing cases is parallel to the line joining the supports, the bubble will lie in the center of the tube, and if not parallel, the deviation of the bubble from the center of the tube will be that due to the angle between the line of support and the axis of the tube. If in the latter case the bubble is brought to the center of the tube, the line of supports will make an angle with the horizon (be out of level) equal to that between the axis of the tube and the line of supports. If now the frame carrying the level is reversed, the movement of the bubble will be twice that due to the angle between the axis and the line of supports.[1] If the tube is now raised at one end, or lowered at the other, till the bubble has moved halfway back to its former position, the axis of the tube is made parallel to the line of support. The line of support may now be made level by raising the lower, or low-

[1] Let the student make a diagram showing this.

ering the higher, support till the bubble stands in the center of its tube.

If two levels are attached to a plate at right angles to each other and parallel to the plate, the plate will be level when both bubbles are centered. If the tubes are not parallel to the plate, it will be difficult to determine when the plate is level, as the position of each bubble for level plate must be determined by trial. If the tubes are so fastened to the plate as to permit of being adjusted, their parallelism may be tested and, if necessary, corrected by the method of this article.

II. *If a frame carrying a level tube is revolved about a vertical axis, the bubble will maintain a constant position in its tube.*

For the axis of the tube maintains a constant angle with the horizon.

Conversely, if a frame carrying a bubble tube is revolved about an axis, and the bubble remains in one position in the tube, the axis of revolution is vertical. If the constant position occupied by the bubble is the center of its tube, the tube is horizontal, and consequently perpendicular to the axis of revolution.

CHAPTER III.

MEASURING DIFFERENCES OF ALTITUDE, OR LEVELING.

29. General principle. It will be evident that if by any means a line of sight may be made to revolve about a vertical axis to which it is perpendicular, it will describe a horizontal plane. Omitting consideration of the curvature of the earth, a rod graduated from the bottom up, and held at any point on the ground, will be cut by this horizontal plane at a distance above the ground equal to the height of the line of sight above the ground at the point where the rod is held. The distance above the bottom of the rod, as indicated by the graduations on the rod, is called the reading of the rod. If the elevation of the line of sight above some assumed base or datum, as sea level, is known, the rod reading subtracted from that elevation will give the elevation of the point where the rod is held, referred to the same base. Conversely, if the elevation of the point where the rod is held is known, and it is required to find the elevation of the line of sight, it is done by adding the rod reading to the elevation of the point. While there are many details to be considered, such as the curvature of the earth, the adjustment of instruments, atmospheric conditions, etc., the above contains the essential principle of leveling.

INSTRUMENTS.

30. General description of level. Any instrument used for the purpose of securing a horizontal line of sight may be called a leveling instrument; or, as is more usual, simply a level. There are three comparatively common forms of levels, shown in Figs. 18–20.

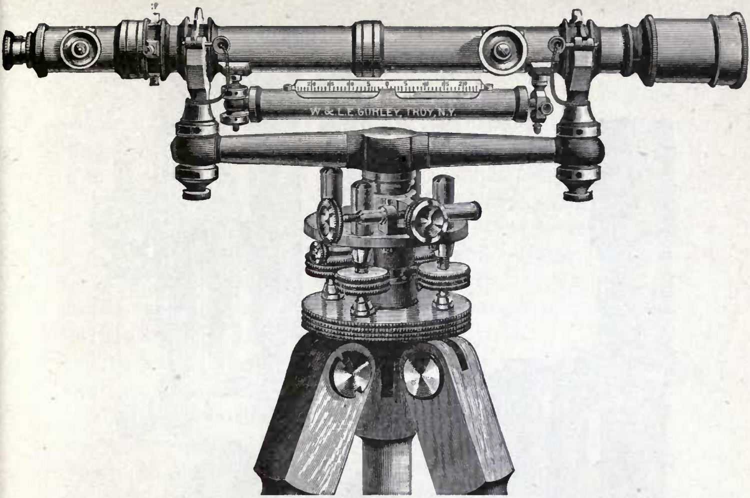


FIG. 18.

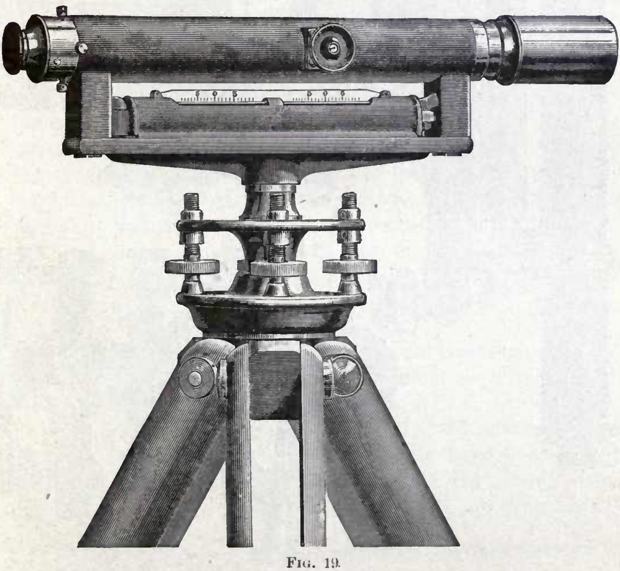


FIG. 19.

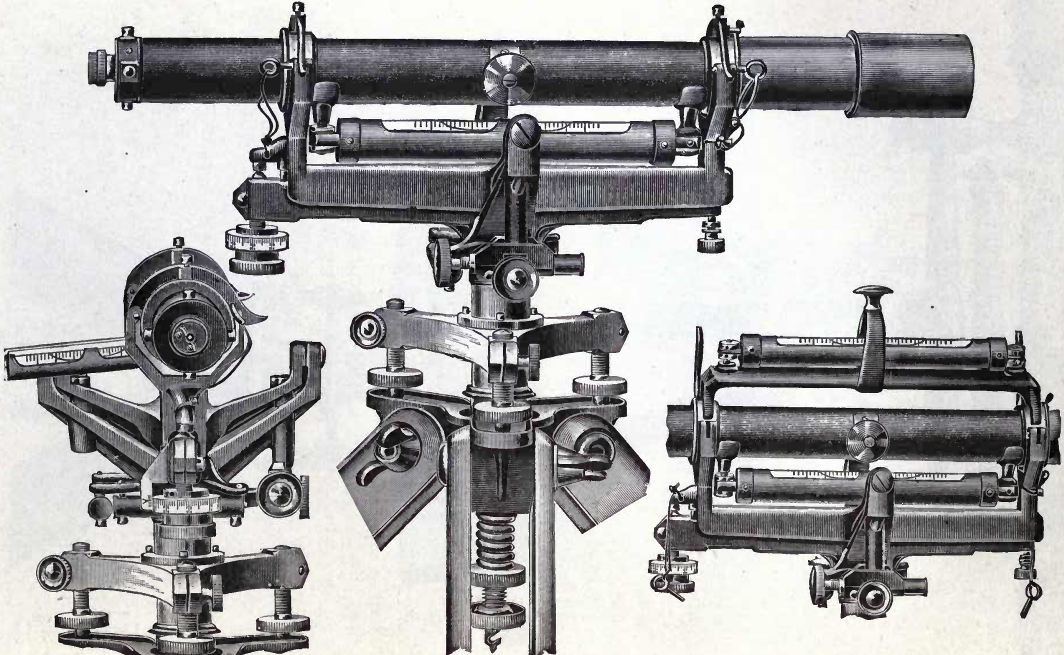


FIG. 20.

Fig. 20 is a precise level, or level of precision. Fig. 18 is known as a Y level. Fig. 19 is a dumpy level. The most common of these is the Y level, so called because the telescope rests in Y-shaped supports. The instrument consists essentially of a telescopic line of sight and an attached bubble tube, whose axis may, by adjustment, be made parallel to the line of sight, so that when the bubble is in the center of its tube it will be known that the line of sight is horizontal. These are combined with a leveling head which contains the vertical axis, and screws on to a tripod. A sectional view of a Y level is shown in Fig. 21.

The dumpy level is so called because of its short telescope with large aperture.

The precise level is simply a modification of the Y level, so improved as to make it capable of doing work to a greater degree of precision than can be obtained by the use of either the Y or the dumpy level. The dumpy level is sufficiently precise for all work that does not require the precise level, and it is considerably cheaper than a Y level of the same make. From the standpoint of the optician, the Y level is the more perfect instrument, because of its many easy adjustments; but this very feature is to some extent an undesirable one from the standpoint of the engineer, who wants, for all ordinary work, an instrument with few parts to get out of adjustment. The dumpy level can not be so easily and exactly adjusted for collimation as the Y level, but, as has been before stated, it is sufficiently precise for all work not requiring a precise level. It is used almost altogether by English engineers, having been invented by an Englishman named Gravatt, whence the level is frequently called the Gravatt level.

31. Telescope.[1] The telescope of the level consists of a barrel in which slide two tubes. One of these tubes is the eyepiece tube carrying the eyepiece lenses *LLLL*, Fig. 21, and the other is the objective tube carrying the objective, or object glass *O*. The objective tube is moved in and out by means of a pinion, which works in a rack attached to the sliding tube. The tube is made to move in the axis of the barrel by having

[1] For a discussion of the principles of telescopes see any good book on Physics.

it pass through the ring at C, which is accurately centered in the barrel. The eyepiece tube moves in and out of the barrel at the other end in a similar way, and is centered by the ring shown at AA. Instead of a rack and pinion movement, the eyepiece should be moved by turning it around a small screw extending through the barrel and into a helical slot in the eyepiece tube. This is a better plan than the rack and pinion arrangement, because the eyepiece is but seldom changed, and when once set should not be easily disturbed. Some instruments, however, have the rack and pinion movement, and it is preferred by some surveyors. In addition to these two tubes there is at R a ring (shown separately in perspective in Fig. 22) which carries two fine wires, one vertical and one horizontal. These wires are either spider lines or fine platinum wires. The spider lines are more common. This ring is centered in the barrel by means of the screws at BB. There are four of these screws, called capstan-headed screws, arranged as seen at W in Fig. 18. If the ring is to be moved to the right, the screw on the left is first loosened, and then the screw on the right is tightened, thus drawing the ring over to the right. Similarly for the vertical movement of the ring—if the ring is to be moved up, the lower screw is first loosened and the upper screw is then tightened, and *vice versa*. The

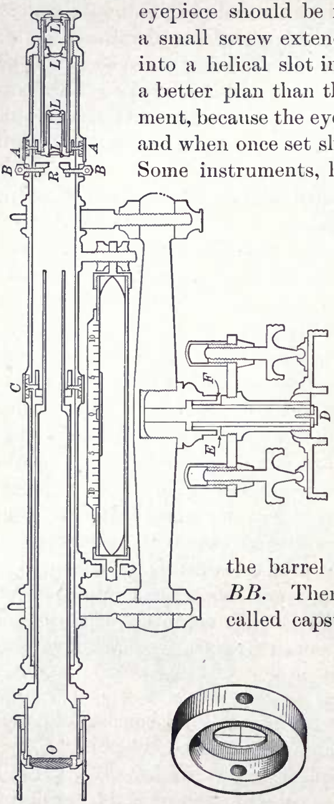


FIG. 21. FIG. 22.

difference between this telescope and the ordinary or Galilean telescope, is in the introduction of these wires and a difference in eyepiece necessitated by them. In the ordinary telescope, such as is found in field glasses, there are no wires and it is impossible to say to what particular point in an object looked at the axis of the telescope is directed. Moreover, it is useless to put such wires in a field glass, since no image of the object looked at is formed in the tube of the telescope. With an angle-measuring instrument, or any instrument that must be pointed to a definite point, it is indispensable that the exact point to which the axis of the telescope is directed be known. It is perhaps inaccurate to say that the direction of the axis must be known, for any other fixed line in the telescope would do as well, provided the adjustments hereafter to be described could be made with that fixed line. It is more convenient to have the fixed line at least very close to the axis of the telescope, for reasons that will appear.

The imaginary line joining the optical center of the object glass and the intersection of the cross wires is known as the *line of collimation*, and this is the line that is directed to the precise spot toward which it is desired to point the telescope; or rather, in the level, it is the line that indicates the point towards which the telescope is directed.

The area seen at one time through the telescope, or rather the angle between the rays of light from the extreme edges of this area, measured at the instrument, is known as the field of view. This field of view is larger as the magnifying power of the telescope is smaller, and varies from about one and one half degrees to about fifty minutes. The former is for the commoner kinds of surveyor's transits, and the latter corresponds to a magnifying power of about thirty-five diameters, or about what is found in the better leveling instruments.

An image of an object within the field of view is formed at a point back of the object glass, and the glass is moved in or out till this image falls in the plane of the wires. This is called focusing the objective. The point in the image covered by the intersection of the wires is that toward which the telescope is directed.

It would be practically impossible to tell when the focusing had been done, were it not for the eyepiece, which is nothing more nor less than a microscope with which to obtain a magnified view of the wires and the image formed by the objective. This is done by first focusing the microscope eyepiece till the wires are plainly seen. Then, any other objects in the same plane with the wires will be seen at the same time, and objects not in that plane can not be distinctly seen, and hence it may be determined when the image formed by the objective is in the plane of the wires, and also what point in the field of view is covered by the intersection of the wires.

The objective will need to be focused anew for each different object looked at, unless, as will rarely occur, all the objects viewed are at the same distance. The eyepiece, on the other hand, since it has to be focused for only one distance, need be changed only for different individuals; and hence, if only one person is to use the instrument for a long time, the eyepiece may be focused once, and not again disturbed during the time it is used by this person.

Telescopes should be corrected for spherical aberration and should be achromatic.[1]

The eyepiece shown in Fig. 21 has four lenses, and is commonly known as an erecting eyepiece. The image formed by the objective is inverted, and the eyepiece inverts the image so that the object appears right side up or erect. This eyepiece has been generally used in American surveying instruments because of a supposed difficulty in the use of one that shows the object inverted. Such an eyepiece, shown in Fig. 23, has two lenses less than the erecting eyepiece, and consequently absorbs less light and secures better definition of the object viewed. It is to be preferred for all surveying instruments, and is well-nigh indispensable for some kinds of work; for instance, for stadia measurements, to be hereafter described. The inconvenience

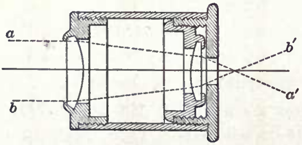


FIG. 23.

[1] Students unfamiliar with these terms can find their meaning in any good dictionary or encyclopædia, in any text-book on Physics, or in Baker's "Engineer's Surveying Instruments."

attending the use of a telescope that shows objects inverted is largely imaginary. A few hours with an inverting glass makes its use as natural as the use of an erecting glass. Dumpy levels and precise levels are almost always inverting instruments; while the Y level, the most commonly used in this country, is almost always erecting. As has already been stated, there now seems to be little reason for the existence of the Y level.

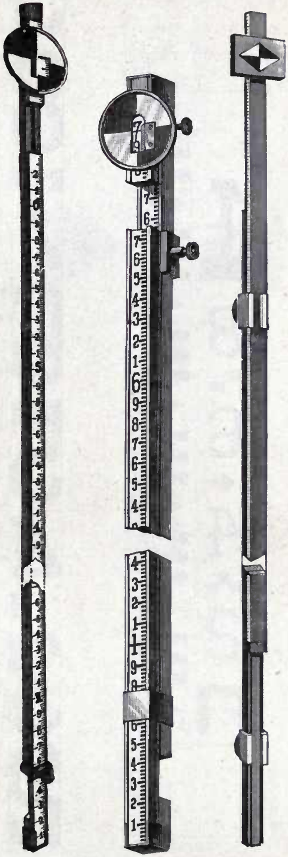


Fig. 24. Fig. 25. Fig. 26.

32. Leveling Rods. There are three common patterns of leveling rods and an innumerable number of uncommon ones. The three common patterns are shown in Figs. 24, 25, and 26. Each of these is made in two pieces about seven or less feet long. The New York rod (Fig. 24) and the Philadelphia rod (Fig. 25) differ in that the Philadelphia rod is so graduated as to be easily read at ordinary distances by the leveler, while with the New York rod the target must be set and the reading taken and called out by the rodman. The target of the New York rod is provided with a direct vernier, usually placed below the center of the target. This causes some confusion to the beginner, who has been taught to read the scale at the zero of the vernier and the fractional reading by looking along the vernier in the direction of motion for the coinciding line. There need be no confusion if the rodman remembers that it is the center of the target that is set by the leveler, and not the zero of the vernier. A little study of the vernier will show him that the main scale is read opposite the ten of the vernier and the fractional reading by noting the number of the vernier line coinciding with a division of the scale. This rod is graduated to hundredths of a foot, and the vernier permits readings to thousandths. When a reading greater than 6.5 feet is required, the target is clamped at 6.5 and the rod extended. There will be found on the side a second vernier, which, when the rod is closed, reads 6.5 feet and which gives the readings when the rod is extended. This vernier is of the usual construction.

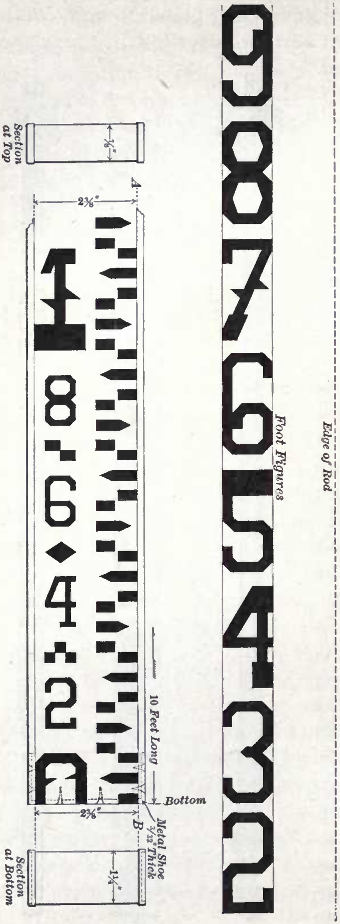


FIG. 27.

The Philadelphia rod target has no vernier, but a tenth of a foot is divided into half-hundredths, permitting a direct reading of 0.005 foot and by estimation to 0.001 foot. The target is set at seven feet for greater readings than seven feet, and, if the rod is to be read by the leveler, it is extended to its full length, the graduation then being continuous from bottom to top. If the target is to be set, the rod is extended until the target is in the line of sight; it is then clamped, and the rod is read by the rodman by means of a scale on the back.

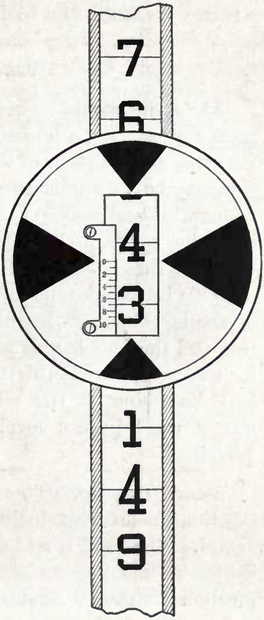


Fig. 28.

The target of the Boston rod (Fig. 26) is fixed to the rod. Readings are all obtained by extending the rod. It is held with the target end down, for readings less than 5.5 feet, and is inverted for readings greater than this. It is read altogether by vernier, the scales and vernier being on the sides. It is read to 0.001 foot. It is the lightest, neatest rod of the three, and the least used. The Philadelphia rod, which is the heaviest of the three, is the most used because of the fact that it may be quickly read by the leveler. In the great majority of sights, readings are taken to 0.1 foot only; and, on turning points (hereafter described), it is usual to read to 0.01 foot only. Readings to 0.001 foot are required in but a very small percentage of work done with the level, even on turning points or bench marks. For this reason many engineers prefer a "self-reading" rod without target, and made in one piece. Such a rod, the standard of the Lake Shore and Michigan Southern Railroad, is shown in Fig. 27.

It will be noticed that the figures are so made as to mark the divisions into hundredths.

With a target rod, much better work may be done if the target is painted with diamond-shaped figures instead of with quadrants, as is customary. The target may be set more precisely if the wire has a sharp angle to bisect or sharp point to

H. I. Cory

cover, than if it is to be made coincident with the edge of a dark surface. Professor Baker finds that at 300 feet the error of setting a quadrant target may be about 0.002 of a foot or more while that of setting a diamond-shaped target may be a little over 0.001 of a foot. Such a target is shown in Fig. 28.

USE OF THE LEVEL.

33. Adjustment. The level has been said to be an instrument for securing a horizontal line of sight. It will be evident from the construction of the ordinary leveling instruments that this may be accomplished with those instruments if the line of collimation and bubble axis are made parallel; because, if this condition exists, and the bubble is brought to the center of the tube, the line of sight will be horizontal.

This introduces the idea of adjustment, and the adjustment of the level consists essentially in making the line of collimation and the bubble axis parallel. There are other adjustments for convenience, but this is the only necessary one. The general discussion of the adjustments will be deferred till the use of the adjusted level in doing simple leveling has been explained.

34. Setting up. To "set up" the level is to place it in position for leveling, including making the line of sight horizontal. The level is an instrument that is rarely set "on line," except in making certain adjustments. It is placed in that position that will command the greatest possible number of points whose elevations are to be determined. To set up, plant the legs firmly in the ground with the leveling plates approximately horizontal. Focus the eyepiece on the wires. Bring the telescope and attached level over one set of diagonally opposite leveling screws and, by the screws, bring the bubble to the center of its tube. Perform the same operation over the other set of screws. This will to some extent disturb the former work. Therefore turn the telescope again over the first set of screws and relevel; again over the second set, etc., till the instrument is level over both sets. If the instrument is in adjustment, the line of sight will now be horizontal in whatever direction it is pointed.

35. Differential leveling. To determine the difference in elevation between two points, both of which are visible from a possible position of the level, set up the level in a position such that a rod held on either point will be visible. Turn the telescope toward one point and read a rod held there by a rodman. The rodman will then carry the rod to the other point, and the telescope will be directed toward that point and the rod read. The difference in readings will evidently be the difference of elevation required. Care must be taken to see that the bubble is in the center of its tube when each reading is taken.

If the elevation above some base surface of one of the points is known, the difference of elevation applied to the known elevation gives the elevation of the second point. This operation is capable of further analysis, thus: The rod reading on the point of known elevation, added to the known elevation, will give the elevation of the line of sight, and is therefore called a plus sight. The rod reading on the second point subtracted from the elevation of the line of sight will give the elevation of the second point. This reading is, therefore, called a minus sight. From these considerations the following definitions are formulated:

A plus sight, or reading, is any reading taken on a point of known or assumed elevation for the purpose of determining the elevation of the line of sight.

A minus sight, or reading, is any reading taken for the purpose of determining the elevation of the point on which the rod is read.

A very bad custom of calling plus sights, "backsights," and minus sights, "foresights," has prevailed in the past. It has been a source of confusion to the beginner and is illogical. It probably arose from the fact that the work in leveling is considered to proceed from the point of known elevation toward the point of unknown elevation, and that, therefore, plus sights are taken in a backward direction, and minus sights in a forward direction. This is not always true, as will appear later, and hence the nomenclature, "backsight" and "foresight" is ill-chosen. It is, moreover, true that when a minus sight is taken in what may be considered a backward direction, the beginner becomes confused and applies the wrong sign. Hence the terms should be abandoned.

If consideration of curvature is neglected, the instrument should be set midway between the two points, in order to do correct work. It will be apparent that if this is done, the amount of the curvature of the earth, for half the distance between the two points will be added in the plus sight, and subtracted in the minus sight; that is, each reading will be too great by the curvature. The amount of this curvature is about 8 inches in one mile and varies with the square of the distance. The student may determine the effect on a rod reading when the rod is held 528 feet from the instrument and when held 279 feet away. Three hundred feet is about as great a distance as will permit a definite reading of the rod with the average level; though in work requiring no great exactness, much longer sights may be taken.

If the points whose difference in altitude is required are so located that rod readings can not be had on both from one position of the instrument, an intermediate point is chosen that may be used with the first one, and the readings are taken on the first and on the intermediate point. The difference in altitude between the first and intermediate point is thus obtained. The level is then moved to a position between the intermediate and final point, and their difference of altitude is determined. The two differences added or subtracted, as the case may be, will give the required difference. It may be necessary to introduce several intermediate points. The work is simply a succession of operations like those of the first case.

It is unnecessary to determine the differences of altitude of each set of intermediate points, as the difference between the sum of all the plus sights and the sum of all the minus sights will give the difference of altitude of the first and final point. The student should show this.

The intermediate points that are chosen should, if the work is to be well done, be firm, definite points, as the projecting part of a firm rock, the top of a peg firmly driven in the ground, etc.

When extensive differential leveling operations are to be carried on, requiring close work (as the careful determination of the altitude of an observatory or other point involving the carrying of levels over many miles and the introduction of many intermediate "turning points"), it is well for the rodman to carry a "point" with him. A very convenient form is shown in Fig. 29. This is made of a triangular piece of boiler plate about three sixteenths of an inch to one quarter of an inch thick and about five or six inches on a side. The three corners are bent down to form, as it were, a three-legged stool, and a round-headed rivet is set in the center. A small hole is drilled in one side, in which to fasten a string or chain. When used, the points are firmly pressed into the ground with the foot, and the rod is held on the rivet. In some work, notably railroad leveling, this kind of point is not suitable, because it is well to leave every turning point so that it can be again found.

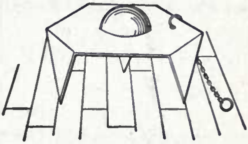


FIG. 29.

In leveling down or up a steep hill, the distance from the instrument to the rod, called the length of the sight, may be greater or less for minus sights than for plus sights. This may be avoided by zig-zagging. Distances on each side of the instrument are made nearly enough equal by pacing.

The form of notes that is kept in differential leveling is as simple as the work. There are three vertical columns, one for the name or number of the point observed, one for the plus sights, and one for the minus sights. *The readings, both plus and minus,* taken on any point, should appear in their proper columns *opposite the number of that point.*

36. Profile leveling. A profile of a line laid out on the ground is the bounding line of a vertical section that includes the line whose profile is desired. It shows the elevations and depressions along the line. Profile leveling differs from differential leveling in that the elevations of a number of points along the line whose profile is required are obtained from a single setting of the instrument. The principle is the same as that of differential leveling, but the method of keeping the notes and of doing the work is a little different.

The line whose profile is required is first marked out on the ground by stakes or other marks placed at such intervals as may be necessary. These intervals are usually regular, and in railroad surveys are generally one hundred feet. In city streets the interval may be fifty feet. In other surveys the interval may be less or more, according to the nature of the survey. The object is usually to reproduce to scale on paper, the profile desired. For this purpose profile paper is generally used, on which the notes are plotted, as will be described later.

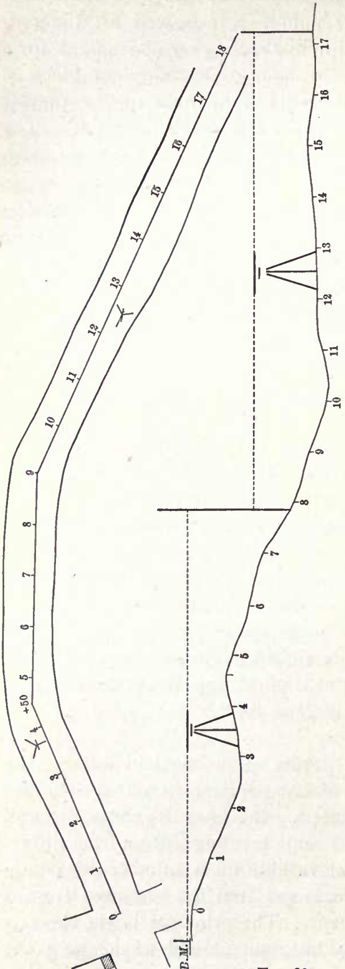


Fig. 30.

Fig. 30 shows a map of a line whose profile is desired, which may be assumed to be the center line of a road. It also shows in exaggerated form, and to no scale, the position of the level along the road, both in plan and elevation.

The process of leveling is as follows:

Some convenient point is chosen as a "bench mark," either because its elevation above some accepted datum is known, or because it is a convenient permanent point whose elevation may be arbitrarily assumed. A bench mark in leveling is a permanent point of known or assumed elevation from which leveling operations may proceed. In the example given, the B. M. is the corner of the water table of a building, and its elevation is assumed to be 1000.000 feet above some arbitrary datum surface. The elevation of the B. M. should be so assumed as to avoid any minus elevations; that is, the datum surface should

Levels along Grafton Road, Quackenkill to Cropseyville.

Leveler . . . . Touceda.
Rod. . . . . Higgins.

March 24, 1895.

Sta.	+S.	H. I.	−S.	Elev.	
B. M. No. 1.	0.462	1000.462		1000.000	On the top of the water table at the
0			0.6	999.8	S.E. cor. of the brick store of P. Gosling.
1			2.2	98.3	Elev. assumed.
2			4.3	96.2	
+50			5.5	95.0	
3			5.2	95.3	
4			4.5	96.0	
+60			3.8	96.7	
5			4.3	96.2	
6			6.0	94.5	
7			7.2	93.3	
T. P.	3.602	994.041	10.023	990.439	On stone marked +, ten feet right of
8			4.0	990.0	Sta., 7 + 80.
9			5.3	88.7	
10			7.0	87.0	
+20			7.2	86.8	
11			6.6	87.4	
12			6.2	87.8	
13			6.0	88.0	
14			5.8	88.2	
15			5.2	88.8	
16			5.7	88.3	

Fig. 31.

be assumed so far below the B. M. that it will be below any point reached in the leveling.

Having selected the B. M., write in the notebook, Fig. 31, under the column marked "Elev.," the elevation assumed, and in the column of stations mark the name of the station whose elevation is given, in this case B. M. No. 1. On the right-hand page write a description of the B. M. so clear and full that another, coming to the place long after, may determine with certainty the exact spot used.

Set up the level at a point convenient to the B. M. and to as much of the line as possible, remembering that it is better to make sights to turning points approximately equal. Having the level set up, read the rod held on the B. M., and record the reading in the column marked "+S.," and add it to the elevation of the B. M. to get the height of the line of collimation, which record in the column headed "H. I." (height of instrument). The rod is now held on station 0, and is read, and the reading is recorded in the column of minus sights, opposite station 0. It should be noted that it is usually the elevation of the ground that is wanted, and not that of the top of the stake that marks the station. Therefore, the rod is held on the ground, and not on the stake. The rod is usually read at station points only to the nearest tenth, as it would be folly to try to determine the elevation of rough ground to hundredths. Points used as B. M.'s are usually read to hundredths or thousandths. Hundredths are generally close enough.

The rod is now carried to station 1 and read, and the reading is recorded in the column headed "−S.," opposite station 1. The leveler, if he is quick, may make the subtraction of the minus sight on one station while the rodman is passing to the next station, and he should practice doing this. He may check his work at noon or night, or at any other time when the survey is, for any reason, temporarily stopped. The rod is now read on station 2, and recorded as before. Between station 2 and station 3 there is a decided change in slope, and the rod is held at the point of change, the rodman either measuring the distance from station 2 to the point or estimating it by pacing, according to the importance of exactness in the particular survey. He calls out the "plus" as (in this case) "plus fifty," and the leveler records the distance as shown in the notes, and then reads the rod and records the reading opposite the recorded distance in the −S. column.

It will be noticed that a number of minus sights have been taken with the instrument pointing backward with reference to the direction of progress. It is these sights that are sometimes confused with "backsights" when the terms "backsight" and "foresight" are used.

Thus the work proceeds till the rod is about as far beyond the instrument as it was in the beginning back of it. Not more than three hundred to five hundred feet on either side of the instrument should be used, if fairly good work is to be done. A point is then selected that is tolerably permanent, as a solid stone or a peg driven into the ground. This point need not be on the line that is being leveled, and, indeed, should not be there, if the line is the center of a traveled road.

The rod is held on the peg, which is called a turning point, or simply "peg," and is read to the same unit as on the B. M., and the reading is recorded in the column of −S., opposite the station T. P. The elevation is then worked out at once, and the level is carried beyond the peg a convenient distance and again set up. A reading is then taken on the rod held on the peg, is recorded in the column +S., and is added to the elevation of the peg to get a new height of instrument, which is noted in the proper column opposite the station from which it was determined. A description of the peg is written on the right-hand page, so that it may be again found, if wanted within a reasonable length of time. The work then proceeds as before. After an interval of, say, half a mile or a mile, a permanent B. M. is established. It may be used as a turning point, or merely determined as is any station, except that the reading will be taken to the unit used for B. M.'s. All points that are likely to be wanted again should be very fully described. Sketches are valuable helps to words in making descriptions.

There are many hints that could be given to facilitate the work of leveling; but they would occupy too much space, and the beginner will have to learn details from experience. One point may be mentioned: Never clamp the spindle. It is unnecessary to do this except in adjusting.

Other forms of notes are sometimes used by engineers. Some of them have station numbers and elevation in adjacent columns; others have a separate column for minus sights on turning points, and others a separate column for elevations of turning points, etc.

37. Making the profile. The profile is made by plotting to scale the elevations of the points, at the proper horizontal distance apart, and connecting the plotted points by a line. Profile paper is generally used for this purpose. One form of ruling for such paper, known as Plate A, is shown in Fig. 32. There are three common forms, known as Plate A, Plate B, and Plate C. Plate A has four horizontal divisions and twenty vertical divisions to an inch. Plate B has four by thirty divisions per inch; and Plate C five by twenty-five divisions. Profile paper comes in rolls twenty inches wide, or in sheets.

If profile paper is used, a convenient elevation is assumed for the bottom horizontal line, and a value is assigned to the space between successive lines. For railroad profiles it is usual to assume one foot as the value of the smallest interval; the paper is so ruled that the five-feet and twenty-five-feet, or fifty-feet lines are accentuated. A smaller value may be assumed for the smallest interval when it is necessary to work to a large scale and to show minute irregularities. It is usual, also, in railroad work to assume the value of the interval between successive vertical lines to be one hundred feet or "*one station.*" Each tenth station is then accentuated by the ruling. This scale may also be increased, if necessary. The scales, both horizontal and vertical, having been assumed, it is only necessary to find the position, horizontally and vertically, that will correctly represent any point whose elevation and distance or station from the beginning are known, and to make a small dot at that place. So with the other points. These dots may then be connected with a smooth line without angles. It may be necessary, occasionally, to make angles, as at the bottom of a steep-sided, narrow ravine; but ordinarily such angles do not occur in nature.

The following method of making the profile, assuming the scale as small as described for railroad work, is rapid. The elevation of the first station is called, and a point is made on the paper; the elevation of the second station is called, the pencil is put down on the paper in the proper place, and a line is drawn back to the first point without making a decided dot at the second station. The line is drawn freehand. The work then proceeds in this way. It seems unnecessary to say that turning points are not plotted, unless they happen to be critical points on line at the ground level — a rare occurrence. The notes in Fig. 31 are shown plotted in Fig. 32.

If profile paper is not used, a base line of assumed elevation is drawn on a piece of paper, the horizontal distances to the points whose elevations have been determined are laid out to scale, and perpendiculars are erected on which are laid off, to vertical scale, the elevations of the points above that assumed for the base line. The points thus formed are connected by a line.

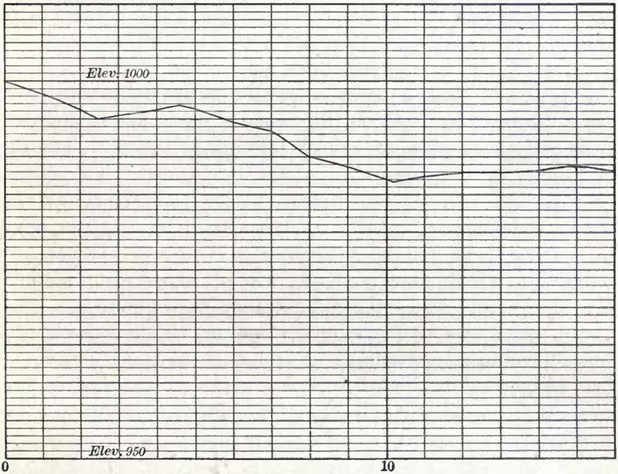


Fig. 32.

A convenient scale for city streets is :

Horizontal, 1 inch to 100 feet.
Vertical, 1 inch to 10 feet.

Where such a scale is used, it will generally be better to connect the points found by ruled lines.

The vertical scale is usually exaggerated, except in profiles made for detail plans of buildings or other works. This is necessary to make the irregularities appeal properly to the eye, since the vertical distances are so small, compared with the horizontal ones, that when a long horizontal distance is laid out to small scale, what would be considered appreciable variations in altitude on the ground, will, if drawn to the same scale as the horizontal distances, appear insignificant on the drawing.

38. Leveling over an area. Sometimes it is required to determine the elevations of a number of points in a comparatively small area, as when a city block is to be graded and the quantity of earth to be removed or supplied is required. In this case, it is customary to divide the block into rectangles or squares, of, say, twenty feet on a side. The elevations of all the corners of these squares or rectangles are determined, and after the grading is done, the same squares are reproduced from reference stakes, previously set outside the work, and the elevations of the corners are again determined. The difference of elevation at any corner, before and after grading, shows the depth of material that has been cut away or filled in. Thus there will be a number of truncated prisms, the lengths of whose edges and the area of whose right sections are known, and hence the volume may be computed.

In such a case as this, after determining the elevation of the line of collimation, readings are taken to as many corners as may be seen before the instrument is moved. The work is precisely the same as profile leveling, except that consecutive readings are not necessarily along any one line.

39. Errors. In engineering work the level is more used than any other surveying instrument, and to describe the details of its use in the various surveys in which it finds a place would be beyond our purpose. These properly belong to a description of the methods of laying out and executing such engineering works. Some hints as to errors arising through carelessness will be given. A discussion of minute errors involved is necessary only in a treatment of precise leveling.

To avoid errors of adjustment, the effect of the earth's curvature and refraction, see that plus and minus sights are of equal length. See that the turning points are firm so that there may be no settlement between sights. They should also be such that the rod may be sure to rest on the same point each time.

The bubble should be in the middle of the tube when the reading is taken; therefore, observe just before and after the

FIG. 33.

reading to see that the bubble has not moved. On unstable ground, throwing the weight of the body from one foot to the other may throw the bubble out. Fine work can not be done on soft ground. A mirror similar to that on the precise level may be attached to a dumpy or Y level, so that the bubble may be observed as the reading is taken. Unequal heating of different parts of the bubble tube will cause the bubble to move toward the warmer part. This may be shown by resting the finger lightly on the tube, to one side of the bubble.

Ordinary work must be done in all kinds of weather, and no great precaution can be taken to avoid the effect of the sun on the instrument.

The rod should be held vertical. To accomplish this on nice work a "rod level" is used. Fig. 33 shows one form of such a level. It is held against one corner of the rod. Waving the rod back and forth, from and toward the instrument, is recommended in the absence of the rod level. When this is done, the lowest reading observed is used. This serves very well when the reading is several feet; but if it is near the bottom of the rod, and the rod is resting on the middle of its base, a considerable error may be introduced.[1] If the rod is rested only on its front edge, the method is correct.

The resultant error in leveling a long line will be proportional to some function of the distance or the number of set-

[1] Let the student show why.

tings of the instrument, or both. It is common to say it is proportional to the square root of the distance. With readings taken carefully to thousandths of a foot, excellent work with ordinary levels would be indicated by a difference of 0.01 foot to twenty-four settings of the instrument; that is, if a line of levels is run for twelve settings of average length, and rerun in the opposite direction, the difference in elevation of the starting point, as assumed at the start and as found at the finish, should not be greater than 0.01 foot. If it is as much as 0.04 foot, the work, if errors of this amount are important, should be rerun. The object of the leveling is a better guide to a decision as to what is good work than any arbitrary limit.

The author's students ran a line of levels from San José, California, to Mount Hamilton, a distance of twenty-seven miles, and were required to repeat the work if the error of any day's run proved to be more than 0.01 foot for twelve settings of the instrument in advance. The difference in altitude was about 4200 feet; and at one or two places along the line, differences of as much as 0.1 foot were observed, while the difference between the elevation of the mountain, as determined by the advance work and that determined by the backward work, was 0.023 foot. This indicates the compensating character of the errors of observation. The compensation was probably partly due to the fact that the different stretches were run by different men, since it seemed that one man had a tendency to make errors in one direction and another to make them in the opposite direction. The work was done by running a half day ahead, and then rerunning the same stretch backward.

40. Curvature and refraction. The effect of curvature and refraction is shown as follows: In the exaggerated figure let A be a point in the line of sight of a level telescope. Let FD be a normal to the earth's surface, at some distance K, — approximately equal to AB, GF, or AD, — from the instrument. The level line through A cuts the normal at B, while the horizontal line of sight cuts it at D. DB is

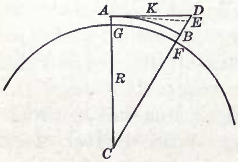


FIG. 34.

then the *correction for curvature* for the distance K. Radius CA being taken equal to GC, we have by geometry and a small approximation that the student may discover,

$$DB = \frac{K^2}{2R}.$$

The effect of curvature is to make the rod read high and hence to make the point F seem low.

Refraction has the opposite effect, that is, the point E is seen in the line AD. The ray of light from any distant object is bent to a curve that is irregular because of the irregularities in the atmosphere, but which is almost always concave toward the earth's surface, and which is considered for average conditions to be the arc of a circle of diameter about seven times that of the earth. Therefore the effect of refraction is to reduce the correction for curvature by about one seventh.

It is not likely that this value is even closely approximate so near the ground as leveling work is done. The combined effect of curvature and refraction is about 0.001 foot in 220 feet, and varies as the square of the distance.

41. Reciprocal leveling. So long as plus and minus sights are at equal distances, no error arises in the work because of curvature and refraction or lack of adjustment of the instrument. It not infrequently happens that sights must be made of unequal lengths, as in crossing a wide river. The following method avoids the errors of curvature, refraction, and adjustment. Establish a peg near each end of the long sight. From a point near one peg read a rod on both pegs. Move the instrument near the other peg and again read on both. The mean of the two differences of elevation obtained will be the correct difference.[1]

ADJUSTMENTS OF THE LEVEL.

42. Focusing the eyepiece. The first adjustment (so called) in any instrument having a telescopic line of sight is the careful focusing of the eyepiece on the wires. If this is not done, there will appear to be a movement of the wire over the image

[1] The student should show this with a diagram.

as the eye is moved a little up or down or to the right or left. This is due to the fact that the wire and image are not in the same plane. The eyepiece is best set by turning the telescope toward the sky and moving the eyepiece till the wires appear sharp and black.

43. Adjustments named. In addition to this focusing of the eyepiece, which can hardly be called an adjustment, the adjustments of the level that the surveyor is usually called upon to make are three in number:

(1) The collimation adjustment, which consists in making the line of collimation parallel to the axis of the telescope tube.

(2) The bubble adjustment, which consists in making the bubble axis parallel to the line of collimation.

(3) The Y adjustment, which consists in making the axis of the Y supports that carry the telescope and attached bubble, at right angles to the vertical axis of the instrument, so that the level when leveled in one position may be turned on the spindle 180° and still be level.

The first two are essential to correct work. The third is merely for convenience.

44. Collimation adjustment. To make the first adjustment, set up the instrument and note whether the vertical wire will coincide with some vertical line, as a plumb line or the edge of a building. If not, loosen all four adjusting screws that hold the wire ring, and turn the ring till the condition described is fulfilled. Set the screws and unfasten the clips that hold the telescope in the Y's, and, by means of the leveling screws and the horizontal motion of the instrument about its vertical axis, bring the intersection of the wires to cover a minute distant point. Clamp the horizontal motion and make the coincidence of the wires and point more perfect by the slow-motion screw. Carefully turn the telescope over in the Y's, not end for end, and note whether the intersection of the wires still covers the point. If not, move the ring that carries the wires, by the capstan-headed screws, until the error seems to be one half corrected. This is done by correcting first one wire, then the other. Reset the wires on the point, by the leveling screws and the slow-motion screw, and carefully turn the telescope in the Y's to its former position. If the intersection of the wires now remains on the point, the adjustment is correct for the distance at which the point is, and this is all that is usually given for this adjustment.

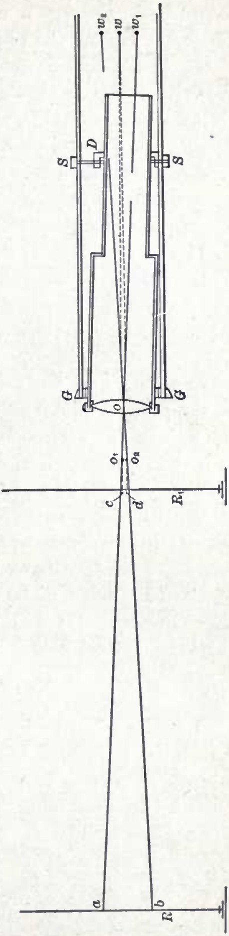


FIG. 35.

45. Adjustment of objective slide. It is also correct for all distances if the object slide is in adjustment; that is, if its ring is so centered that the optical axis of the objective moves in the axis of the telescope tube. To determine whether this is so, test for the collimation error as just described, but on a very near point. If there appears to be an error, it must be corrected by adjusting the object slide. To do this, remove the ring that covers the screws controlling the slide ring, and adjust with these screws so that one half the error appears to be corrected. Test to see whether the adjustment is correct, and again test on the distant point, and, if necessary, adjust the wire, and repeat till the adjustments are both correct. The correctness of this method is shown by Fig. 35. The figure is much distorted.

Let o be the optical center of the objective. It is assumed that the maker has so placed it in its ring that it is practically in the axis of the telescope when drawn clear back as shown. This is probably not exactly true, but the error is less than can be determined by the surveyor. Let D be the ring that carries the object slide. It is adjusted by the screws *SS*. It is shown much out of the center of the tube. Let w_1 be a wire supposed not to be in the center of its tube, that is, out of adjustment. The outer end of the objective tube slides in the fixed ring *G*. If the instrument is perfectly made, it is evident that, if the objective could be brought to the plane of this ring *G*, the optical center would be in the axis of the telescope tube. When the glass is focused on a very distant object, the objective approximates to the plane of the ring, and hence the optical center is nearly in the axis of the tube, but not necessarily exactly so. In the case presented, it could not be just in the axis.

If the telescope is directed to a distant object, as a rod *R*, the point *a* of the rod, that will appear to be covered by the intersection of the wires, will be found in the line joining the intersection of the wires with the optical center of the objective. If the telescope is turned upside down, the wire will move to w_2, and the point that will seem to be covered will be the point *b*, found as was *a*. If now the optical center is in the axis of the telescope, and the wire is moved to the position *w* in the prolongation of the line joining the point midway between *a* and *b* and the optical center, it will be in the center of the tube and in adjustment. If the optical center is not exactly in the center of the tube, the position of the wire when moved to read halfway between *a* and *b* will not be the center of the tube but will be very near it, depending on the error of the objective. It will be off center about the same amount that the optical center is off. Let it be assumed that the optical center is right, and that therefore the wire is now right. Make the telescope again normal.

Let the glass be focused on a near object, as a rod at R_1. Let it be supposed that in throwing the glass out for this purpose *o* moves to o_1. The reading on the rod will be *c*, obtained by producing the line wo_1. Invert the telescope, and the reading will be *d*. If the ring *D* is now moved, the object slide will swing about *GG* as a pivot, and o_2 will be moved toward o_1. It is evident that, if this movement is enough to make the wire read halfway between *c* and *d*, the objective is centered. If it was right for the distant rod, it may now be assumed to be right for all intermediate distances, since it is supposed that the maker has made the tube true in form. If this is doubted, it may be tested in the same manner for any distance. Since the objective was not exactly centered for the distant rod, the wire was not exactly adjusted, and hence if the test is not made again on the distant rod, a slight error will be discovered which is to be corrected by moving the wire, and the test must again be made on the near rod, which will also probably show some error of objective which must be corrected. By continual approximations the work is completed.

Because the wire is not quite centered the first time, the work may be hastened by adjusting the *slide* for about or quite the whole error instead of half the error. Each wire is adjusted separately, and the objective slide first in one direction and then in the other at right angles thereto. It is well to note that this adjustment of the objective is seldom made because it is seldom necessary. It may be considered to be necessary only when the instrument has been taken apart or has had extraordinarily severe usage. When it is not necessary, the adjustment of the line of collimation is a very simple matter. In some makes of instruments the object slide is permanently adjusted, and hence no provision is made for its adjustment by the engineer.

46. Bubble adjustment. The second adjustment is made in either of two ways. The first depends on the equality of diameter of the rings of the telescope tube that rests in the Y's. Since by the first adjustment the line of collimation has been made to coincide with the axis of the tube, it is parallel to the axis of the Y's, if the rings mentioned are equal in diameter. If now the attached bubble tube is made parallel to the axis of the Y's, it will be parallel to the line of collimation. To test it, set up the instrument, and, having leveled over both sets of screws, unfasten the clips and level more carefully over one set. Remove the telescope very carefully from the Y's and turn it end for end and replace. If the bubble returns to the center of its tube, the tube is parallel to the axis of the Y's. If it does not, bring it halfway to the center by the adjusting screws at the end of the bubble tube. The adjustment is now complete if the instrument has not been moved in making it.

Test it by releveling and turning end for end in the Y's, as before.[1]

47. The "peg" method. This method is considered better by almost all engineers, and is as follows: Set the instrument in a fairly level plot of ground midway between two stakes that have been driven firmly into the ground, from two hundred to four hundred feet apart. Read a rod on each stake. (Remember to have the bubble always in the center of its tube when a reading is taken.) Since the rods are at equal distances from the instrument, errors of reading due to errors of adjustment will disappear in subtracting, and hence the difference in readings will give the true difference in elevation of the two stakes. The instrument is now removed to a point in line with the two stakes, but beyond one of them. Some engineers take the instrument beyond one stake a distance equal to one tenth of the distance between the stakes. Others set the instrument at one of the stakes so that the eye end of the instrument in swinging around will just clear a rod held on the stake. The procedure, adopting the first method, will be explained, and the student may work out the process for the second method. It will be supposed that the instrument has been set up beyond one stake a distance equal to one tenth of that between the stakes, and with one diagonally opposite pair of leveling screws in line with the stakes. Any other proportion would do as well.

After leveling, with the telescope pointing in the line of the stakes, a reading is taken on the near stake. If there were no curvature of the earth, the difference in level of the two stakes applied to the reading on the first would give the reading that should be obtained on the second, provided the line of collimation is horizontal, that is, parallel to the axis of the bubble tube. Since there is the earth's curvature to consider, even if the instrument is in adjustment, the rod at the distant stake will read more than above noted, by the amount of the earth's curvature, less the effect of refraction.[2] Therefore, to obtain the proper reading for the distant rod there must be added

[1] The principle on which this adjustment depends is stated in Art. 28.
[2] See Art. 40.

to the reading on the near rod, the difference of elevation plus the correction for curvature for the given distance.

The target is then set at this reading and the rod is held on the distant stake. If the level, when sighted to the rod, reads above the target, the line of collimation is not parallel to the bubble tube, but points up. If it reads below the target, it points down. The amount of error in reading is the amount that the line of collimation points up or down in the *distance between the stakes*. In the distance from the level to the distant stake, it points up or down eleven tenths of this apparent error. The proportion will vary with the varying proportions of the distances from stake to stake and stake to instrument. Hence to get a horizontal line from the instrument to the far rod, set the target in the line of collimation and then raise or lower it, as the case may be, by an amount equal to eleven tenths of the apparent error, and point the telescope to it by manipulating the leveling screws.

The line of collimation now being horizontal, the bubble tube will be parallel to it if the tube is adjusted with reference to the telescope so that the bubble comes to the center. This is done, as explained in the first method, by the screws at one end of the tube. The test should be again made to see that no movement has taken place during the operation. It is sometimes stated that the adjustment may be made by moving the bubble *or* the wire. This is not correct, as will be evident from the preceding discussion of collimation adjustment. If the objective is permanently adjusted by the maker, the second adjustment by reversing in the Y's may be made first, and the peg method may follow, adjusting the *wire*.

48. Lateral adjustment. In many old levels, the telescope is free to turn in the Y's. If this is the case, and the axis of the bubble tube is not in the vertical plane through the line of collimation, and the telescope tube becomes turned a little, the line of collimation will not be horizontal when the bubble is in the center of its tube. To test this, level the instrument and turn the telescope slightly in the Y's. If the bubble remains in the center of its tube, the adjustment is perfect, and nothing need be done. If not, adjust the tube laterally by the lateral adjusting screws at one end of the tube. Most levels have a little pin in one of the clips that fits in a notch cut in a ring on the telescope and prevents the telescope from turning in the Y's. Nevertheless, it is well to know that the adjustment is not much out.

49. Y Adjustment. The third adjustment depends on Principle II. stated in Art. 28. It is performed as follows: The test consists in carefully leveling the instrument over both sets of screws and more carefully over one set, and then swinging the telescope around 180° on the vertical axis and noting whether the bubble remains in the center of the tube. If not, the axis of the instrument is not vertical, and the axis of the Y's is not perpendicular to it. To correct the error, that is, to make the axis of the Y's perpendicular to the vertical axis of the instrument, move one of the Y's up or down (as may appear necessary by the position assumed by the bubble) by manipulating the large capstan nuts that hold the Y to the bar, till the bubble has moved *halfway* back to the center of its tube. The axis of the Y's is then perpendicular to the vertical axis of the instrument, provided no movement has taken place. To test, relevel and reverse the telescope and make any further correction necessary, and test again until the work is complete.

In making any of the adjustments it is practically impossible to manipulate the adjusting screws without to some extent disturbing the instrument. Therefore, an adjustment is rarely made perfectly the first time, and must always be tested and repeated till complete. If an instrument is badly out of adjustment, it will be advisable first to adjust it approximately all round, and then readjust more carefully.

50. To adjust the dumpy level. Reference to the cut of the dumpy level will show that the axis of the telescope is made by the maker perpendicular to the vertical axis of revolution. It is not supposed that this condition will change. It will also be noticed in the cut that there are no adjusting screws for the object slide. It is permanently adjusted. This is true of all the engineering instruments made by the maker of this level. Therefore, since the axis of the telescope is perpendicular to the axis of revolution and the optical axis of the objective is permanently adjusted, if the axis of the bubble tube is made perpendicular to the axis of revolution, and the line of collimation is made parallel to the bubble axis, the instrument will be correctly adjusted. In this instrument, then, the order of adjustment is reversed.

First, the bubble tube is adjusted to be perpendicular to the axis of revolution. The test for this condition is made as in the test for the Y adjustment of the Y level. If adjustment is needed, it is made with the screws at the end of the bubble tube. One half the apparent error is corrected.

Second, the line of collimation is made parallel to the axis of the bubble tube. The test for this condition is made by the "peg" method, as already described. The adjustment is made by moving the wires instead of the bubble.

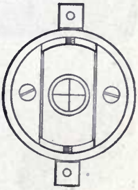


FIG. 36.

The dumpy level, and in fact all levels, should be so made that the vertical wire is not adjustable. A form of ring meeting this requirement is shown in Fig. 36, taken from a valuable English work on Engineers' Surveying Instruments.[1]

51. To adjust any level on a metal base. To adjust in this way, say, a carpenter's level, or a striding level for engineering instruments, place the level on a plane surface and mark its position, and note the position of the bubble in its tube. Reverse the level, end for end, and place it in the same position on the plane surface. If the bubble is as far from the center in the reversed position as it was in the first, the axis is parallel to its base, which means that it is in adjustment. If not, correct one half the difference in readings by whatever screws may be provided for the adjustment of the bubble. If convenient, the bubble may be brought to the center in the first position by tilting the plane surface; then when the level is reversed, the movement of the bubble from the center will correspond to twice the error of adjustment, which may then be corrected.

[1] Stanley's "Surveying and Leveling Instruments."

MINOR INSTRUMENTS.

52. Locke hand level. This is a small instrument for quick approximations where no great degree of precision is required. It is shown in Fig. 37.

It consists of a small bubble mounted on a brass tube over a hole in the tube, beneath which, and within the tube, is a prism occupying one half of the tube. Across the hole is stretched a wire. There is usually an eyepiece of low magnifying power. The eye, in looking through the tube, will see the wire and the bubble reflected by the prism; and when, if the instrument is in adjustment, the bubble appears to be bisected by the wire, the line of vision past the wire is horizontal. In use, the leveler and hand level constitute the leveling instrument, and between plus and minus readings the observer must remain stationary. There are other forms of the Locke level than that shown in the figure. The principle is the same in all.

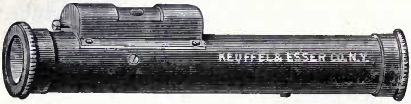


Fig. 37.

53. Abney level and clinometer. This is a combination of a hand level and a slope-measuring instrument. Such an instrument is shown in Fig. 38.

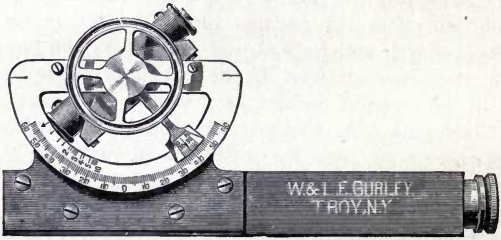


Fig. 38.

It is essentially a hand level, in which the bubble is attached to an arm that carries a vernier, and may be moved over a small arc. To determine a slope or vertical angle, bring the line of sight to the slope, and turn the vernier arm till the bubble is seen on the wire. The bubble axis will then be horizontal, and the angle between it and the line of collimation will be indicated by the vernier reading.

54. Gurley's monocular and binocular hand levels. These are telescopic hand levels by which readings are more definitely determined on a rod at some distance than is possible

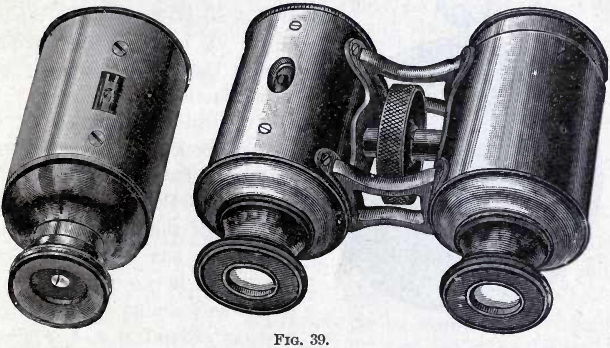


FIG. 39.

with the ordinary hand level which is not a telescope. The Gurley levels are shown in Fig. 39. The author thinks the monocular hand level the best yet devised.

55. Adjustment of the Locke and Abney levels. For the Locke level, get by some means a horizontal line, as a sheet of water of limited extent; if the latter is not at hand use a leveling instrument, or indeed the hand level itself, held midway between two rods or trees some distance apart. Having obtained the horizontal line, stand with the hand level at one end of it, and, with the eye and wire in the horizontal line, note whether the bubble appears to be on the wire. If it does not, adjust by moving the frame carrying the wire by means of the little screws at the end of the bubble mounting.

To adjust the Abney level and clinometer, set the vernier to read zero, and test as for the hand level; adjust the bubble by screws provided for that purpose, or adjust the wire by a slide arrangement provided in some instruments.

LEVELING WITH THE BAROMETER.

56. Barometer described. Very rough leveling work is sometimes done with the aneroid barometer. The barometer is an instrument for measuring the pressure of the atmosphere. The only reliable form is the mercurial barometer. This is not a convenient portable instrument. The aneroid barometer is an attempt at portableness. It is made in various forms, the more common of which is shown in Fig. 40.

Fig. 40.

It consists essentially of a metallic box, with a thin corrugated top, from which the air is exhausted. With the varying pressure of the atmosphere, the top of the box rises and falls. Its motion is communicated by levers and chains to a pointer moving around a dial, graduated by comparing the movement of the pointer with that of a mercurial barometer. Hence the "inches" on the dial are not true inches nor of uniform length.

57. Theory. The use of the barometer to determine differences of altitude depends on the supposition that the atmosphere is composed of a series of layers of air of uniformly upward decreasing density. If this were true, the pressure indicated by the barometer would grow uniformly less as the barometer were carried to higher altitudes, and it would be easy to establish by experiment a relation between the altitude and the barometric reading. The supposition is not entirely correct. The pressure of the atmosphere at any one place is affected by humidity, and very considerably by temperature. Moreover, owing to the movement of the atmosphere, two distant places of equal altitude, having at a given time equal temperatures, will not have at the same time equal barometer readings.

It is therefore difficult to establish a relation between the altitude and the barometer readings. It has been attempted by many persons, with the result that there are many formulas for determining the differences in altitude from the readings of the barometer. These formulas do not differ so much in method of deduction as in the experimental constants used. Perhaps one of the best is that developed by Mr. William Ferrall.[1]

Modified by omitting the terms for correcting for humidity and latitude, which are small in effect, and one or two other still smaller terms, this formula is

$$H = 60521.5 \log \frac{B}{B_1}\Big(1 + 0.001017\,(t + t' - 64°)\Big),$$

in which H is the difference in altitude of two points where observations are taken, B and B_1 are the barometer readings at the lower and higher points respectively, and t and t' are the Fahrenheit temperatures at the two points.

58. Barometric tables. It is convenient in tabulating this formula for practical use to consider that $t + t'$ is 100°, and that H is the difference in altitude between two points whose altitudes above a given datum are to be directly determined from the tables. Thus if $t + t' = 100°$, and the given datum is such that the barometer reading is 30 inches, the formula is

$$H = 60521.5\,(1 + .001017 \times 36)\left(\log \frac{30}{B_1} - \log \frac{30}{B}\right)$$
$$= 62737 \log \frac{30}{B_1} - 62737 \log \frac{30}{B}.$$

Tables may be constructed for the quantity $62737 \log \frac{30}{B}$, with B as the argument, and such a table would be entered twice, once for each barometer reading, and the difference of the tabular values found would be the value of H if $t + t' = 100°$ and there is no correction for humidity. Table III., page 362, is such a table. For other than a mean temperature of 50°, and for average hygrometric conditions, a correction is to be applied to the result. This correction is a linear function of the value of H already found such that

$$H = (A_1 - A_2)(1 + C),$$

[1] See Appendix, 10, "United States Coast and Geodetic Survey Report" for 1881.

in which A_1 and A_2 are the values from Table III., and C is the temperature and humidity correction coefficient. Values of C are given in Table II., page 361.

59. Practical suggestions. The aneroid barometer should be frequently compared with a mercurial barometer, and the errors of reading at different parts of the scale should be noted. This can be well done only under the bell of an air pump.

The scale of altitudes on some barometers is useless.

Some barometers are marked "compensated," meaning that no temperature corrections need be applied. This is a mistake. The temperature corrections should always be applied.

Always read the barometer when it is horizontal. Carry it in a strong case, and do not let the warmth of the body affect it. Do not use it for altitudes just before or after, or during a storm.

In determining the difference of altitude of two widely separated points, stop once or twice for a half hour or so on the journey between the points, note the reading of the barometer at the beginning and end of the stop, and thus try to determine whether the atmospheric conditions are changing and, if so, at what rate. A correction can thus be determined for the first reading, on the supposition that the observed change has taken place also at the starting point. Tap the box gently before reading.

60. Accuracy of the method. If barometer readings are taken simultaneously daily at two widely separated stations for from one to six years, the utmost that can be expected is an error of about one half of one per cent, though occasional results are much closer. Single observations taken on the same day with the same or compared barometers and at distances of a few hours' walk will give results with errors of from one per cent to three per cent and more. An error in temperature of five degrees will cause an error in result of one per cent or more. Single observations taken a day or more apart and at stations a day's journey or more apart can not be depended on to give results much closer than the nearest ten per cent, though they will frequently do better than this and sometimes worse.

CHAPTER IV.

DETERMINATION OF DIRECTION AND MEASUREMENT OF ANGLES.

61. Instruments mentioned. In almost all surveys it is necessary to measure distances and angles.

Angles are measured by means of one of several instruments, according to the character of the work. The most commonly used are the compass and the transit.

The compass is an instrument for determining directions and, indirectly, angles; while the transit is an instrument for determining angles and, indirectly, directions.

Almost all of the old land surveys of this country have been made with the compass as the angle-measuring instrument; and while this instrument has been, to a great extent, supplanted by the transit, there are still a great many compasses sold annually. As a discussion of the compass will make clearer some surveying methods, it will be here described. Mention must be made of the sextant, which is an angle-measuring instrument whose use in surveying is confined to certain particular operations, as in exploratory surveying or in the location of soundings taken off shore. A description of this instrument is given on pages 294–298.

Mention should also be made of the solar compass and transit, instruments for determining the true meridian by an observation on the sun. The solar transit is described on pages 116–126.

THE COMPASS.

62. Description. The compass consists of a line of sight attached to a graduated circular box, in the center of which is hung, on a pivot, a magnetic needle.

At any place on the earth's surface, the needle, if allowed to swing freely, will assume a position in what is called the magnetic meridian of the place. If, then, the direction of any line is required, the compass may be placed at one end of the line and the line of sight may be made to coincide with the line. The needle lying in the magnetic meridian, and the zero of the graduations of the circular needle box being in the line of sight, the angle that the line on the ground makes with the magnetic meridian is read on the graduated circle. If the magnetic meridian coincided everywhere with the true meridian, or even if the angle between them were constant, the compass would be a far more valuable instrument than it is. The compass, however, has its field, and in that field is a very valuable tool. A very good form of compass is shown in Fig. 41.

It is sometimes set on a tripod head and sometimes on a jacob staff, which is a single-pointed staff with a

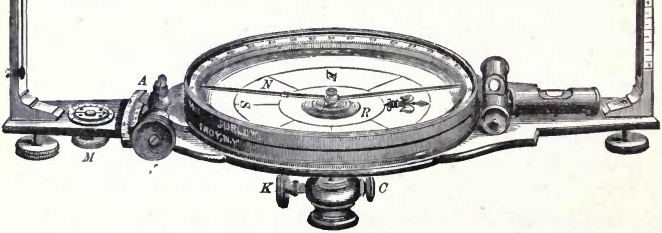


Fig. 41.

head fitted to receive the compass. The sights SS are attached to the main plate of the instrument by the screws seen below the plate. The level vials encased in brass tubes are fastened to the plate by screws passing through the plate from below, by which screws the bubbles may be adjusted parallel to the plate. The needle N rests on a steel pivot that is screwed into the plate, and the needle is lifted from the pivot, when not in use, by the ring R operated by a lever, which is in turn moved by the screw whose head is seen just below the plate directly in front. On the left is a marker M used to keep track of the chaining. Next to this is an arc A and a vernier, moved by the tangent screw V. The arc is attached to the plate, and the vernier to an arm that is fastened to the compass box, which box may be thus turned through a small angle by the tangent screw. This vernier is called a declination vernier. Its use will appear later. The compass box is graduated from the north and south points in each direction 90°. When the zero of the vernier coincides with the zero of the arc A, the line of zeros of the compass box is coincident with the line of sight. The spring catch C holds the compass on the tripod head, and the clamp K, acting on a concealed spring that encircles the tripod head, serves to clamp the instrument so that it will not revolve about the vertical axis. There is a small coil of wire seen on one end of the needle. This counteracts the magnetic force that would otherwise cause the needle to *dip*. This means that the direction of magnetic force or magnetic pull is not horizontal, but is inclined, and if the needle is free to dip, it will lie in the direction of magnetic pull. This *vertical component* of magnetic force that causes the dip, varies in different places, and the little coil of wire may be moved nearer to, or further from, the pivot to allow for this variation.

Since only horizontal angles are measured with the compass and used in land surveying, it is necessary to have the plates horizontal, and the sights vertical when working. This is accomplished by means of a ball and socket joint attached to the tripod head, and the bubbles on the plate. The plate having been leveled, the joint is clamped by simply screwing tight the milled head by which the spindle is attached to the head of the tripod.

COMPASS ADJUSTMENTS.

63. Requirements mentioned. To determine properly the direction of lines with the compass, it is necessary (1) that the plate be level, the sights vertical with their line passing through the zeros of the graduated box when the zeros of the vernier and arc A coincide; (2) that the needle be straight; (3) that the pivot be in the center of the circle.

64. Plate bubbles. The instrument is readily leveled if the bubbles are parallel to the plate and the plate perpendicular to the vertical axis. Test the parallelism of the bubbles and plate by the method of Art. 28, and correct, if necessary, by the screws already mentioned. In doing this the plate is reversed on the vertical axis. It is assumed that the plate is perpendicular to the vertical axis. If not, it must be made so by the maker.

65. Sights. To test the perpendicularity of the sights, turn the leveled instrument on a suspended plumb line and see whether the line traverses the slits in the standards, turning each end separately to the plumb line. If the sights are found to be *out of plumb*, they may be adjusted by removing them and filing the bottoms till they are found to be correct. With proper care they should not get out, and this is not a common error. Strips of paper may be inserted under one edge, if the surveyor prefers this to filing.

To determine whether the plane of the sights includes the line of the zeros when the vernier is set to read zero, stretch two fine threads through the slits in the sights and, looking down, see whether the plane of the threads includes the line of zeros of the graduated box. If not, the error should be corrected by the maker. The error may be allowed for, however, by moving the vernier till the zeros are in the plane of the threads, or parallel to it, noting the angle that has been turned, and seeing that thereafter the vernier is set at this angle when working. If the line of sights fails to pass through the center by one tenth of an inch, the error in the direction of a line ten feet long will be about three minutes, and will be less as the line is longer.

66. Needle and pivot. If the needle is straight and the pivot is in the center, the two ends will read 180° apart. If both of these conditions are not fulfilled, the two ends will read other than 180° apart. If it is observed that one of these faults exists, the instrument should first be tested to determine which one. This is done by turning the plates on the vertical axis and noting whether the difference in the two end readings remains constant: if so, the needle is bent; if not, the pivot is not in the center, and the needle may or may not be bent. If the needle is found to be bent, remove the glass cover from the box by unscrewing it, take the needle from the pivot, and carefully bend it till straight. If the pivot is found to be out of center, remove the needle and carefully bend the top of the pivot over until it is in the center. To determine which way to bend it, turn the box till that position of the box is found that gives the greatest error in end readings of the needle, and bend the pivot at right angles to the position of the needle.[1]

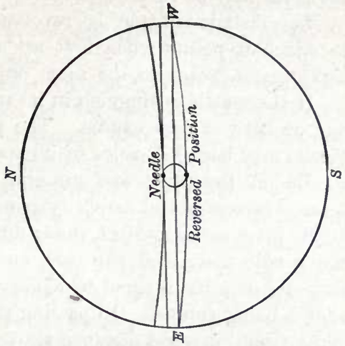


FIG. 42.

If there is reason to think that both errors are involved, turn the instrument as before to get the maximum error of end readings, and read the angle greater than 180° between the ends, calling this R. Now revolve the instrument 180°, in the way described below, and read the angle on the same side of the needle, calling this reading R'. Let p be half of the error due to eccentricity of pivot, and n be half of the error due to bent needle; then, from Fig. 42, it is seen that

$$180° + (2p + 2n) = R; \qquad (1)$$
$$180° - (2p - 2n) = R'. \qquad (2)$$

Solving for p by subtraction, and for n by addition, there results

$$p = \frac{R - R'}{4};$$
$$n = \frac{R + R' - 360}{4}.$$

Bend the needle so that each end moves over n degrees. Then bend the pivot at right angles to the needle, toward the center, till the needle ends read 180° apart.

Otherwise, turn the box till the needle ends read 180° apart. Turn the box 180°, and if the two ends do not read opposite

[1] The student should show the correctness of these statements by a diagram.

divisions, bend the ends through half the difference between the first and second positions. The needle is now straight, and the pivot may be tested and corrected as above described.

The instrument may be revolved 180° by noting an object to which it points when first set and then revolving till the sights again point to the same object.

If the needle is sluggish in its movement, it may be due to one or both of two causes. The pivot may be blunt, or the needle may have lost some of its magnetic strength. Take the needle off the pivot and examine the pivot. If it is found blunt, unscrew it and carefully grind it down on a fine oilstone. If the pivot seems perfect, the trouble is with the needle. Lay the needle down and rub each end from the center out with that end of a bar magnet which attracts the end of the needle that is being rubbed. In passing the magnet back from end to center, raise it some distance above the needle; otherwise the backward movement will tend to counteract the rubbing. The needle may also be remagnetized by placing it in the magnetic field of a strong electro-magnet, as a dynamo. If placed on the magnet in a wrong direction, the south end will, when replaced, point north. To correct this, replace the needle on the magnet in the opposite direction.

Two needles at the same place may not point in the same direction. This will probably be due to the fact that their magnetic axes do not coincide with their geometric axes. This seems to be unavoidable. For this reason the needle should be narrow and deep rather than flat and wide. There is a difference of opinion about this, based on difference of manufacturing processes.

The angle between the true meridian and the magnetic meridian should be determined by the instrument that is to be used in a given piece of work.

The glass cover of the compass box may become electrified by rubbing, and attract the needle. This is remedied by touching the glass with the moistened finger.

When the compass is being moved, the needle should always be lifted from the pivot. When put away, the needle should be allowed to swing to the magnetic meridian and then lifted. It retains its magnetic strength longer in the meridian.

USE OF THE COMPASS.

67. Bearing. The use of the compass is to determine the directions, or, technically, the *bearings* of lines; or to mark out lines whose bearings are given.

The bearing of a line is the horizontal angle between a vertical plane including the line, and the meridian plane through one end of the line, and is measured from the north or south points 90° each way. Thus the bearing of a line extending in a direction midway between the north and east, would be N. 45° E. or S. 45° W., according to the end from which the bearing is read. This gives rise to two terms, *forward* bearing and *back* bearing, the forward bearing being the bearing in the direction in which the survey is being run, and the back bearing being that read in the reverse direction. These two bearings (omitting the effect of convergence of the meridians of the two ends of the line, which is an inappreciable amount in an ordinary compass survey) should be numerically equal, but with opposite letters to express the direction. The bearings are never read, east so many degrees south or north, or west so many degrees north or south; but always from the north or south points.

68. To determine the bearing of a line. Set the compass over one end, level the plates, lower the needle, turn the north end of the compass box toward the distant end of the line, and bring the line of sights carefully in the direction of the line. If the declination vernier reads zero, the north end of the needle will show the bearing of the line.

69. To lay out a line of given bearing. Set the compass over a point on the line, level the plates, drop the needle, turn the compass on its vertical axis till the north end of the needle reads the given bearing. The line of the sights is now in the required line, which may be ranged out.

70. To run a traverse. This is to determine the lengths and bearings of a series of connected lines. These may form the sides of a farm, or inclose a pond, or be the center line of a crooked road. If there are no natural objects to mark the angle points, flags are set at those points by a flagman, the compass is set at each point in succession, and the back bearing of the preceding line and the forward bearing of the line ahead are determined while the lines are being measured by the chainmen. The back bearings are read as a check on the forward bearings and to detect local attraction, as will be hereafter explained. In occasional hasty work the compass will be set only at every second corner, and the back bearing of one line (afterward changed to forward bearing) and the forward bearing of the other line will be determined. This should not ordinarily be done. If the chainmen have no point ahead to chain toward, they may be kept in line by the man at the compass.

It is frequently necessary to produce a line for a considerable distance through woods or over hills, one end not being visible from the other. If the bearing of the line is known, turn the compass, placed on one end of the line, in the direction of the line, and direct the chainmen as far as they can be well seen. Establish a point by driving a stake in the line at the end of the last measurement, or by merely setting the flag in the ground, to be pulled out when the compass is brought up, and set the compass over the established point and continue the line.

If a tree or small building obstructs the line, and a few inches on one side or the other are of no consequence, as is the case in most surveys made with the compass, set the compass beyond the obstruction, as nearly on the line as can be estimated, and, turning the sights in the direction of the known bearing, proceed with the line. Other obstacles are discussed on pages 201–204.

71. Notes. The notes that are taken may be kept in the following form, if nothing more than a record of the traverse is required:

Station.	Bearing.	Dist. Chains.
etc.	etc.	etc.
D	S. 72° 45′ E.	10.40
C	S. 52° 00′ E.	7.30
B	N. 23° 30′ E.	6.74
A	N. 36° 15′ W.	4.63

The notes read up the page so that the line may appear to be ahead as one walks along with the open notebook. Other notes are usually required, as distances to points passed, fences crossed, roads and streams crossed, distances right or left to objects near the line, as will be hereafter described. In such cases, a sketch of the work with all the necessary measurements marked on it is the best possible form of notes to take in the field. This may afterwards be converted into the form shown on page 229. The name and date of survey should always be recorded on every page of the notebook used and the names of those engaged on the work should be entered at least once.

72. Angles. To determine the angle formed by two lines at their point of intersection, set the compass over the point of intersection and read the bearing of each line. From these bearings determine the angle. There will always be two supplementary angles. If the lines are conceived to be run from the end of one to the point of intersection, to the end of the second, and so on, as in running a traverse, the angle between the prolongation of line one, and line two, measured in the direction in which the line is conceived to bend, is called the deflection angle, or, in a closed survey, the exterior angle. Some examples in converting bearings to angles, and the reverse, will be found in the problems in the Appendix, pages 324, 325.

73. Cautions. It is better to keep the north end of the compass box ahead, and to read the north end of the needle. The north end of the needle is distinguished either by its peculiar shape or by the fact that the coil of wire is on the other end. The north end of the compass box is distinguished by a peculiar figure, usually a conventional fleur de lis. If the south end of the box is ahead, read the south end of the needle. Care should be exercised in the use of the compass, both in directing the sights and in reading the needle; and the beginner should be particularly careful to read from the north or south point according to which is nearer the north end of the needle. Greater differences in results may be expected from different instruments used by the same man, than from the same instrument used by different men, if the men use care in their work.

MAGNETIC DECLINATION.

74. Declination defined. Thus far all bearings have been assumed to be taken with reference to the magnetic meridian. If the magnetic meridian and the true meridian were coincident at all places on the earth's surface, or if at one place the angle between them were constant, the determination of the true bearing of a line by the compass would be comparatively simple. The needle, however, points to the true north at very few places on the earth's surface, and the angle that the magnetic meridional plane makes at any point with the true meridional plane is called the magnetic declination.

75. Variations of declination. The declination is subject at every place to changes, called variations of the declination. These are: secular variation, annual variation, lunar inequalities, and diurnal variation. There are also irregular variations due to magnetic storms. The annual variation and the lunar variation are both very small and may be neglected entirely. The diurnal variation is nothing at about 10.30 A.M. and 8.00 P.M., and varies between these hours, the north end being furthest east at about 8 o'clock in the morning and furthest west at about 1.30 in the afternoon. The total movement is from five to ten minutes, and the corrections to be applied are as in the Appendix, Table V., page 364. It will be noticed that the variation changes with the seasons. It also varies slightly with the latitude, and the corrections in the table are about a mean for the United States for the latitudes given.

The secular variation is by far the most important. It is supposed to be periodic in its character, requiring from two hundred to four hundred years to make a complete cycle.[1]

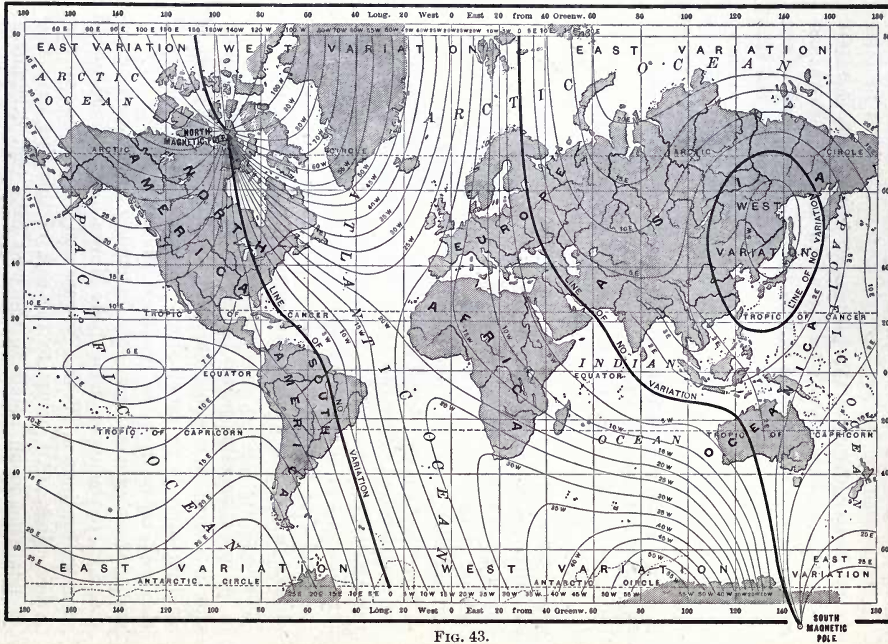
The needle in Paris in 1580 pointed about 9° or 10° east of north, while in 1800 it pointed a little more than 22° west of north. The change seems to have been fairly but not absolutely uniform in rate. The rate of change seems to differ with difference of place, and also with time at the same place. It is almost nothing for a few years when the needle is near its extreme position in either direction; and again, while at one

[1] For an extended article on this subject see "United States Coast and Geodetic Survey Report" for 1882.

place the declination may vary from one side of the meridian to the other, at another place the entire movement is on one side.

76. Determination of the declination. Since it is the magnetic meridian that is given by the needle, and since this is not a fixed direction, it should always be required to reduce the magnetic bearings of lines run with the compass to the true bearings. This was not done in the old land surveys made in this country, and the declination was not noted. The result is that it is difficult to rerun the old lines, as will hereafter appear. From what has been said about secular variation, it will readily be seen that the declination must be known for the place of the survey for the time of the survey. In this country it is sufficient to determine the declination each year for a given place. It is done by the surveyor in the manner to be hereafter described. From observations conducted over a number of years in many of the cities of this country, Mr. Charles Schott, of the Coast Survey, has deduced empirical formulas for different places for determining the declination at those places, which formulas will give approximately correct results for a number of years to come. These formulas are given in the Appendix, Table VIII., pages 368–370. The use of such formulas is not advisable except to check, in a general way, the results of the observations of the surveyor, who should always, in going to a new place, determine the declination for himself.

A line connecting points having the same magnetic declination is called an isogonic line, and the line joining points of no declination is an agonic line. In Plate VI., at the end of the book, taken from the report of the Coast Survey for 1888 and 1889, an agonic line is seen extending from near Charleston, S.C., to the upper end of Lake Huron, and passing through or near Huntington, W.Va., Newark and Toledo, O., and Ann Arbor, Mich. All points to the east of this line have west declination, and all points to the west have east declination. Moreover, this line is moving westward, and hence west declinations are increasing and east declinations diminishing. The declination for a given place may be determined approximately



Long. West East from Greenw.
EAST VARIATION
WEST VARIATION
EAST VARIATION
ARCTIC OCEAN
NORTH MAGNETIC POLE
NORTH AMERICA
SOUTH AMERICA
EUROPE
AFRICA
ASIA
OCEANICA
ATLANTIC OCEAN
PACIFIC OCEAN
INDIAN OCEAN
LINE OF NO VARIATION
TROPIC OF CANCER
EQUATOR
TROPIC OF CAPRICORN
ARCTIC CIRCLE
ANTARCTIC CIRCLE
SOUTH MAGNETIC POLE

FIG. 43.

from this chart, though, from the extreme irregularity of the lines in those portions of the country where observations have been most frequent, as, for instance, in Missouri, it will be seen that too much reliance can not be placed on the correctness of the lines. Figure 43 is a chart on a very small scale, showing in a general way the declination at all points on the earth.

The declination is determined by the surveyor by first determining a true meridian and then setting the compass over the southern end of a line in this meridian and reading the bearing of the line. This is, in amount, the declination. The direction is, however, reversed, since there is read the bearing of the true meridian referred to the magnetic meridian, instead of that of the magnetic meridian referred to the true meridian. The zeros of the declination vernier and its arc should coincide, or the reading will be erroneous. To read the declination more precisely than can be done on the circle, which is graduated only to half degrees, bring the plane of the sights carefully into the meridian, and then move the vernier by the tangent screw till the needle reads zero, or north. The reading of the vernier will be the declination. If now the vernier is left set at this reading, a survey may be conducted by the compass and referred to the true meridian, without mental reduction, as the compass will give the true bearing of the lines at once.

77. Determination of the true meridian. The only real difficulty in finding the declination lies in determining the true meridian. This is done in several ways, one of which, very old and only roughly approximate, is to mark on the ground or on a flat surface, the extremities of the shadows cast by a vertical rod or other object when the sun is at equal altitudes above the eastern and western horizons. This is done by noting the shadow at about 9 or 10 o'clock in the morning and drawing a circle through the extremity of the shadow, taking the foot of the rod as a center, and then marking the point on this circle that is just touched by the shadow in the afternoon. The point midway between the two positions of the extremity of the shadow and the center of the circle lies in the meridian.

The better way to determine the meridian is by an observation on the star Polaris, or, as it is usually called, the North Star. This star is not directly at the north pole, but is at present about one and one quarter degrees from the pole, about which it seems to revolve in 23 hours and 56 minutes, thus bringing it to the meridian twice every day. It is approximately on the meridian when a plumb line traverses the pole star and the star next the end of the handle of the *dipper*, or Zeta Ursæ Majoris, that is, the star Zeta in the constellation of the Great Bear, which is the name of the constellation including the dipper. The stars in every constellation are lettered or numbered. Polaris is known as Alpha Ursæ Minoris, or Alpha of the constellation of the Lesser Bear.

Two thirds of a minute, 1895, after both of these stars are covered by the plumb line, Polaris is on the meridian. For each year after 1895 add 0.35 minutes. It is not best to observe the star at culmination, that is, on the meridian, because at that time its motion is all horizontal, giving a considerable error for a small error in time of observation. It is better to observe the star at the time when it is furthest east or west, or, as is said, when it is at eastern or western elongation. The method of observation is essentially the same in both cases. Suspend a plumb line from a beam or overhanging limb which is high enough so that a line of sight to the star, from a point twenty feet or more to the south, will pass below the point of suspension. This will usually require a plumb line at least twenty feet long, the length depending on the latitude ; the elevation of the pole above the horizon being equal to the latitude of the place. The top of the plumb line should be illuminated with a light screened from the observer. Select a point south of the plumb line, approximately in the meridian and such that the star may be seen just below the top of the string ; and drive two stout stakes in an east and west line, making sure that the meridian will fall between the stakes. Drive the stakes till their tops are in a level line and then nail to their tops a stiff, smooth board. Take off one of the sights of the compass and rest it on this board. It may be necessary to set it in a block because of the pins on the bottom. At a little before the time of culmination or elongation, whichever is to be observed, bring the sight "into line" with the plumb line and the star.

Keep it in line by moving the sight till the time of culmination or elongation, and then mark on the board the position of the sight. The work may now be left for the night. The next morning at a little after 10 o'clock, set up the compass over the point occupied the previous night by the sight, and turn the sights on the point over which the plumb line was suspended. If culmination has been observed, the compass is now pointing in the meridian, and the needle reading will give, in amount, the magnetic declination for that place.

78. Azimuth at elongation. If elongation has been observed, the true bearing of the star at elongation, called the "azimuth" at elongation, must be determined, and a bearing equal to this must be set off to get the true meridian. The explanation of how this is done is as follows: The angle that is required is the angle between the vertical plane including the pole and the position of the observer, and the vertical plane through the latter point and the star at elongation. This latter plane is tangent to the circle of apparent revolution of the star. If, then, the observer were on the equator, the required angle would be equal to the pole distance of the star. As the position of the observer is moved to the north, the plane becomes tangent to the circle of revolution at a point above a horizontal line through the pole. Therefore, as the latitude of the observer is increased, the angle between the two planes is also

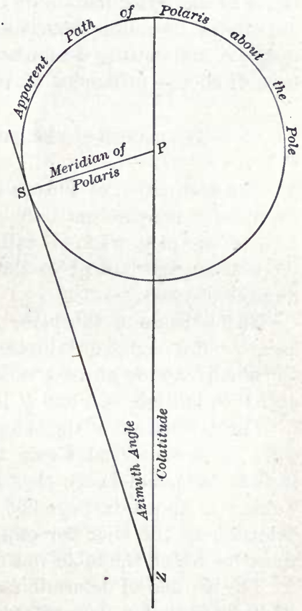


FIG. 44.

increased. These two planes, extended to the celestial sphere, will cut from that sphere two arcs which, with the arc of the meridian of the star from pole to star, will form a spherical triangle. The arcs are the arcs zenith-pole, zenith-star, and pole-star; the triangle formed being zenith-pole-star, right-angled at star. In Fig. 44 the points are lettered Z, P, and S. The plane of the terrestrial equator extended to the celestial sphere cuts from that sphere the celestial equator, and the zenith of the place of observation is north of that equator by an arc equal to the latitude of the place. Therefore, the pole is distant from the zenith by an arc equal to the colatitude. On the celestial sphere the term "declination" is used instead of "latitude," and hence the star is distant from the pole by an arc equal to the codeclination of the star. In the figure the arcs are shown. Calling codeclination of the pole star the pole-distance, and letting ϕ represent the latitude, there results at once from the principles of trigonometry the relation

$$\text{sine azimuth at elongation} = \frac{\text{sine pole distance}}{\text{cosine } \phi}.$$

The declination of Polaris is constantly increasing, and will continue to increase until the star is within about thirty minutes of the pole, when it will begin to decrease. Table IV. Appendix, page 364, gives the polar distance of Polaris for a number of years.

The latitude of the place may be taken from a good map, an error of a degree of latitude involving in the territory below 50° north latitude an error of $2\frac{1}{2}'$ in latitude 50°, $1\frac{1}{2}'$ in latitude 40°, $1'$ in latitude 30°, and $\frac{1}{2}'$ in latitude 20°.

The calculation of the time of elongation involves a knowledge of astronomical terms that it is not thought wise to include here, and hence the times of elongation are given in Table VI. Appendix, page 365, for the year 1897, with rules for determining the time for other years and other latitudes than those for which the table was computed.

The method of determining the meridian just given is sufficiently precise for compass work. A more precise method will be given in the discussion of the transit. As has been said, almost all of the original surveys made in the eastern part of this country were made with the magnetic compass. Many of them were made and recorded with reference to the magnetic meridian, without recording the magnetic declination at the place at the time of the survey; and hence, if it were required to retrace one of the old surveys, all but one of whose corners had been lost, it would be found to be a difficult thing to do. The method of procedure will be discussed further on.

79. Local attraction. The needle may be drawn from its mean position for a given locality by the attraction of large or small masses of iron or other magnetic substances. These may be either minerals hidden in the ground or manufactured articles, as agricultural implements in a barn near by, railroad rails, nails in an adjacent house, etc. The chain used in the survey may influence the needle if brought too near, while keys in the pocket of the observer, or steel wire in the rim of a stiff hat, and steel button molds on a coat have been known to give trouble.

Local attraction may be very troublesome, and must be carefully looked out for. It may be discovered and allowed for by simply occupying every corner of a survey, and reading the bearing of each line back as well as forward. It will be evident that at any one station where the compass is set, the local attraction, if any exists, will affect by the same amount all readings taken from that point; and hence, if the bearings of two lines meeting at a point are both determined at that point, the angle of intersection is correctly deduced, even though there is local attraction and neither bearing is correctly determined. If the bearing of a line is determined from each end, and it is found that the two readings are not numerically the same, and it is certain that the bearings are correctly read, there is probably local attraction at one end or the other, or possibly at both ends.

If now another line is set off from one end of the first line, and the bearing of the new line is read at both ends, and the readings are found to agree, the local attraction affected the reading of the first line at the other end. If trouble is encountered on the second line, a third line may be set off from the other end of the first line, the same observations being made as on the second line. If the auxiliary lines are properly chosen, one will usually be sufficient. If it is feared that local attraction exists, the best method is to read the bearings of all lines at each end, determining the angles between adjacent lines from the bearings taken at their points of intersection. One line may then be assumed to be correctly read, or the true bearing may be determined, and the bearings of the others may be computed from the series of angles.

80. Special forms of compasses. Some compasses are made with an additional full circle of 360° and a vernier reading to minutes, with suitable clamps and tangent screws; but it

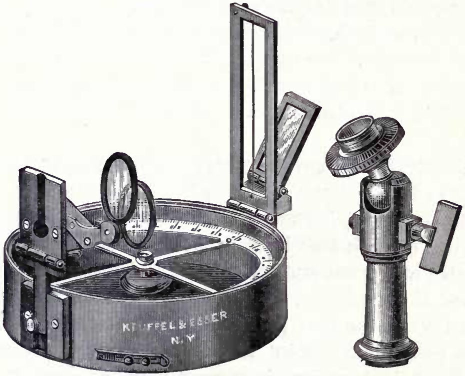


FIG. 45.

may be said that the precision obtained in pointing with the open sights is hardly sufficient to warrant this. If angles are required to be read to single minutes, it is better to use a transit.

The prismatic compass is a very convenient instrument to use on exploratory work. It has folding sights, and, by means of the prism, may be read while being pointed, which is a convenience when the instrument is held in the hand instead of being mounted on a jacob staff or tripod. It is usually held in the hand, though it may be used either way. The instrument is shown in Fig. 45.

THE TRANSIT.

81. Description. The instrument most used by surveyors and engineers for measuring horizontal angles, and, with certain attachments, for measuring vertical angles and distance, and for leveling, is called a transit.

The transit consists of a telescope attached to a pointer which may be moved around a graduated circle. There are suitable attachments for controlling the motion of the telescope and the pointer, and for enabling the graduated circle to be made horizontal. The pointer may be clamped to the graduated circle, and they may be revolved together. If this is done, and the telescope is pointed to a given object, and the graduated circle is clamped so that it will not revolve, and if the pointer is then unclamped from the circle, the pointer and telescope may be turned together till the telescope points to a second object. The number of degrees of the circle over which the pointer has passed will be the angle subtended, at the point where the transit is placed, by the two objects seen through the telescope.

The pointer is the zero mark of a vernier. There are usually two double verniers placed 180° apart.

In the transit shown in Fig. 46, T is the telescope which may, by means of the horizontal axis A, be revolved in a vertical plane.

The horizontal axis rests in bearings at the top of the standards X, which are rigidly attached to the circular plate that carries the vernier V.

On this plate are set two level tubes which, when adjusted to be parallel to the plate, will show whether the plate is level. This plate is made by the maker perpendicular to the axis on which it revolves, and hence, when the plate is level, the axis of revolution is vertical. This axis is conical, and fits inside a conical socket which is the inside of the conical axis of the plate that carries the graduated circle. This latter conical axis revolves in a socket that connects the top and bottom plates of the leveling head. The upper plate is leveled by the leveling screws L. The lower plate of this leveling head screws on to the tripod. The clamp C fastens together the vernier plate (sometimes called the *alidade*) and the plate carrying the graduated circle. The latter plate is called the *limb*. When the two plates are clamped together, the vernier plate may still

FIG. 46.

be moved a small amount, relatively to the limb, by means of the tangent screw *S*. This is for the purpose of setting the vernier at a given reading, or pointing the telescope at a point, more precisely than would be ordinarily possible by simply turning the alidade by the hand. The collar K surrounds the spindle of the limb, and by the clamp screw C' may be fastened to that spindle. The lug M, which is attached to the collar, being held by the spring in the barrel P and the opposing screw S', in turn fastened to the upper leveling plate, prevents the revolution of the limb, when the collar is clamped to its axis. The limb may, however, be moved a small amount by the tangent screw S' working against the pressure of the spring in the barrel P.

Some instruments when put away in the box in which they are kept when not in use, are separated into two parts. The upper part of the instrument is separated from the leveling head, and the two parts are placed separately in the box. Others are so made as to be put away in one piece, being merely unscrewed from the tripod and screwed to a board that slides into the box.

There are various forms of transits and various patterns for the different parts, according to the ideas of the different makers. A careful examination of an instrument should be made before putting it to use, that the user may become perfectly familiar with its construction.

The circles are usually so graduated that angles may be read to minutes. Many instruments are graduated to read 30 seconds, some to read 20 seconds, and a few to read 10 seconds. For ordinary land surveying, single minutes are sufficiently precise, while for fine city surveying 20 seconds, or even 10 seconds, is not too fine.

The telescope is essentially the same as that used in the level, but is shorter and usually of lower magnifying power. It should be inverting, but is usually an erecting glass. The use of the inverting glass is growing. There is a proper relation between the magnifying power of the telescope and the least count of the vernier used; and it is not necessarily the best instrument that has the smallest count vernier, for the reason that the magnifying power of the telescope may be such that a movement of several of the smallest units that can be read by the vernier may give no perceptible lateral motion to the line of collimation. The power of the telescope and the least count of the vernier should be so adjusted to each other that a barely perceptible movement of the vernier will cause a barely perceptible movement of the line of collimation. The same is true of the magnifying power and the level under the telescope. Methods of testing these relations will suggest themselves.

The verniers are usually read with small magnifying glasses known as reading glasses.

In order that the transit may be set precisely over a point on the ground, there is fastened to the center of the lower part of the leveling head a ring or hook, from which may be suspended a plumb line. A sectional view of the lower part of the transit in Fig. 46 is shown in Fig. 47. It will be seen that the

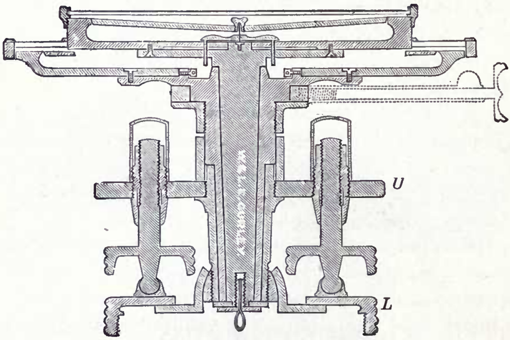


FIG. 47.

upper leveling plate U—in some forms four arms instead of a plate—is attached to the socket in which the vertical axis revolves, and that to this upper plate are attached the nuts in which the leveling screws work. These leveling screws rest in little cups, that in turn rest on the lower leveling plate L. The ball and socket joint that permits the leveling of the instrument is separate from the lower leveling plate, and is not quite so large as the hole in the center of that plate. There is an extension plate to this ball and socket joint that extends under the lower leveling plate. When the leveling screws are tightened, this extension plate binds against the leveling plate, and the instrument can not be shifted on the lower plate. If the leveling screws are loosened, the upper part of the instrument may be shifted laterally on the lower plate by an amount depending on the relative diameters of the ball joint and the hole in the center of the lower plate. The device is called a shifting center and is found in all modern transits.

82. The tachymeter. Such a transit as has been described, while still in very general use, is being rapidly displaced by the complete engineer's transit, or tachymeter. This instrument consists of such a transit as has been described, with the following attachments:

(1) A level under the telescope, making the transit a good leveling instrument.

(2) A vertical circle or arc, by which vertical angles may be read.

(3) A gradienter attachment by which very small vertical angles may be set off, as so many feet rise or fall in one hundred.

(4) A pair of horizontal wires in the telescope, known as stadia wires.

There are various other attachments for special purposes that will be found described in instrument makers' catalogues, but which it is not necessary to mention here. There are also special forms of transits for use underground. One form of the complete instrument is shown in Fig. 48.

The level under the telescope and the vertical circle are readily recognized. The gradienter and slow-motion screw, for vertical motion of telescope, is shown at *G*. The telescope is clamped in vertical motion by the clamp *C*. It may then be moved slowly a small amount by the slow-motion screw. The head of this screw is graduated with reference to the pitch of the screw, so that a single revolution of the head corresponds to an angular motion of the telescope of one foot in one hundred, or half a foot in one hundred. In the figure, the head is graduated in the latter way, as is indicated by the double row of figures. The use of the gradienter is chiefly in setting out grades.

Many transits are fitted with the slow-motion screw without the gradienter head. All makers make both the plain and complete transits.

USE OF THE TRANSIT.

83. Carrying the transit. While in use, the transit is not removed from the tripod when it is carried from one point to another; but is carried with the tripod on the shoulder of the

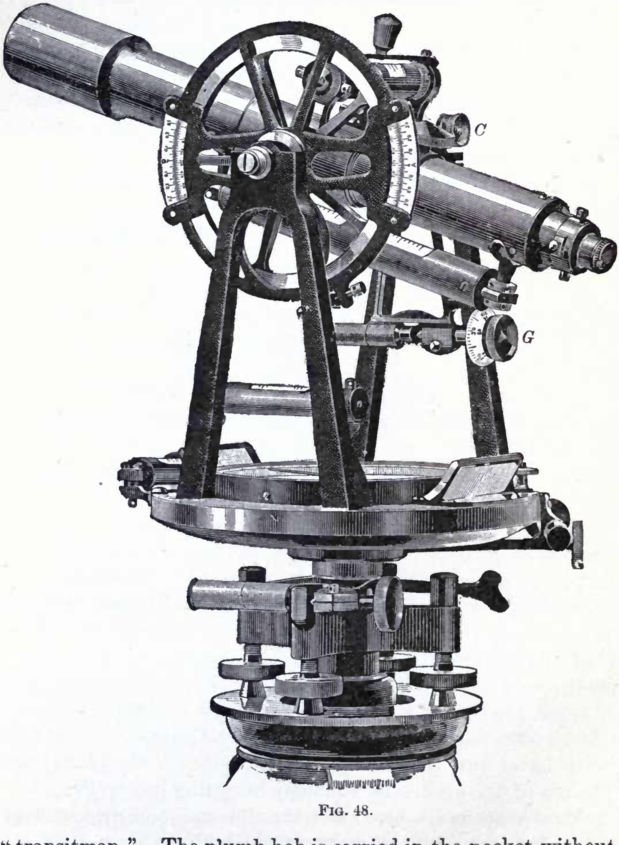


FIG. 48.

"transitman." The plumb bob is carried in the pocket without removing the string from the instrument. Before the transit can be properly set up and the angles measured, the various parts must be in proper adjustment. The discussion of the adjustments will, however, be deferred till the use of the adjusted transit has been described.

84. "Setting up" the transit. To "set up" the transit over a point, let the plummet swing free, grasp in one hand the uppermost leg of the tripod, and plant it firmly in the ground. Next grasp the other two legs, one in each hand, and place them symmetrically about the point so that the plumb bob covers the point as nearly as may be. The plates should be at the same time observed and made approximately level. It should be noticed that a sidewise motion of a leg changes the level of the plates without much disturbing the plumb bob, and that a radial motion changes the level of the plates to a less degree, but does disturb the plumb bob. It is necessary to become expert in setting the transit, as much time may be lost by awkward work in setting up. After the transit is set approximately over the point, press the legs firmly into the ground so as to bring the bob more nearly over the point and to make the instrument firmer. Finish centering by the adjustable or shifting center, by loosening all the leveling screws and moving the whole upper part of the instrument till it is centered. If there is no adjustable center, the instrument must be centered by manipulating the legs. When the instrument is finally centered, level the limb by the leveling screws. To do this, place one plate bubble parallel to one set of opposite leveling screws; then the other bubble will be parallel to the other set. Level one bubble at a time and note that leveling one will, to a slight extent, disturb the other, which must then be leveled again. If the lower plate is not fairly level, "leveling up" will disturb the centering of the plumb bob, which must then be corrected. Continue till both bubbles are in the centers of the tubes. Set the vernier that is to be used at the proper reading (this will often be zero), and the instrument is set up.

85. To produce a straight line with the transit. Set the instrument over one extremity of the line, and with the limb free to turn, turn the telescope on the other extremity or on a point in the line. Clamp all motions except the vertical motion of the telescope, and, with either of the horizontal tangent screws, but preferably with the lower screw, bring the line of collimation to bisect exactly the distant point. Transit the telescope and set a point beyond it at any desired place, in the line of collimation. This point will, if the instrument is in adjustment, be in the straight line produced. If it is feared that the transit is slightly out of adjustment, unclamp the limb, turn the instrument till the line of collimation again cuts the first point, clamp and set precisely, and then transit and set a new point beside the first one set; the point midway between the two set will be a point in the continuation of the straight line. This is called "double centering," and is the same operation as the test of the adjustment of the vertical wire, to be hereafter explained. The method of ranging out a number of points in a straight line in the same direction will be apparent without explanation.

86. To measure an angle with the transit. Set the instrument over the vertex of the angle, with the vernier at zero. With the lower motion bring the line of collimation to bear approximately on one of the distant points, clamp, and with the lower tangent screw make an exact bisection. Loosen the alidade and bring the line of collimation approximately to the second point, clamp, and complete the bisection with the upper tangent screw. Read the vernier. The reading will be the angle sought. This assumes the numbering to extend both ways from 0° to 360°. It is, of course, unnecessary to set the vernier at zero. When the instrument has been set on the first point, a reading may be taken and noted and subtracted from the reading found on pointing the telescope to the second point. In this case the vernier may pass the 360° point, and it will then be necessary to add 360° to the final reading before subtracting the first.

87. Azimuth. The azimuth of a line is the horizontal angle the line makes with a reference line, as a meridian. It differs from bearing, in that it is measured continuously from 0° to 360°. If the azimuth of a point is mentioned, there is implied another point and a meridian through it from which the azimuth is measured.

In making surveys with the transit, except possibly city surveys and others where angles are to be repeated several times, it is better to use azimuths than bearings. It is necessary, however, in writing descriptions of property, to reduce the azimuths to bearings.

It is customary to reckon azimuths from the south point around by way of the west, north, east, south, 360°. This is the practice of astronomers and geodesists. It is believed to be more convenient for the surveyor to begin with zero at the north point and read to the right 360°. The reason for this will appear in Chapter VI.

88. Traversing. The method of traversing with the transit is as follows: In Fig. 49, let it be required to determine the

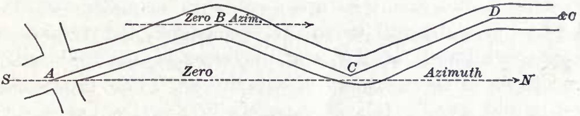


FIG. 49.

lengths and azimuths of the courses of the crooked road *A*, *B*, *C*, *D*, etc. It will be assumed that the magnetic meridian *SN* is the meridian of the survey. Set the transit over a tack in a stake driven at *A*, and, with the vernier set at zero, turn the telescope by the lower motion in the direction *AN* as defined by the needle, and clamp the lower motion. For the purpose of again using this line, if necessary, set a stake in the line *AN*, some distance toward *N*, and put a tack in it in the exact line. This is necessary because the needle would not give the direction twice alike to the nearest minute, which is desired in transit work. Unclamp the alidade and turn the telescope toward a point in the first turn of the road, as *B*. Clamp the alidade and set a stake at *B*. In most transit work all stakes marking stations to be occupied by the transit are "centered" by setting a tack in the precise line. Read and record the vernier and the needle. They should agree within the limit of precision of the needle reading. The angle read is that which has been turned to the right, and in the figure is greater than 270°. (The mistake of reading the wrong circle must be avoided. The needle reading may be recorded as bearing or may be mentally reduced to azimuth and so recorded. If the compass box is graduated continuously 360°, the needle will give azimuths at once.)

Measure *AB*, and, while this is being done, take the transit to *B* and set up over the tack that has been put in the stake. With the same vernier used at *A* (let it be called vernier *A*) bring the instrument to read the azimuth *AB* plus 180°, which is the "back azimuth" of *AB*. Point the telescope to *A* by using the lower motion, clamp, and make the exact pointing with the lower tangent screw. By so doing the instrument is *oriented;* that is, the zeros of the limb are made parallel to their positions at *A*. (The telescope has not been transited, and, if the alidade is now unclamped and the telescope turned in azimuth just 180°, the line of sight will be in *AB* prolonged, the vernier will read the azimuth of *AB*, and the zeros of the limb will be seen to lie in the meridian of the survey. This intermediate step is not usually taken separately in practice, but is introduced here for the clearer understanding of the method.)

With the alidade unclamped, turn the telescope in the direction of the next point, as *C*. Clamp the alidade, set a stake and tack at *C*, and read and record the vernier and needle. The reading of the vernier will be the azimuth of *BC* referred to *AN*, and the needle should read the same or the corresponding bearing. Measure *BC*, take the transit to *C*, set the vernier at the back azimuth of *BC*, and proceed as at *B*. Thus continue the work till the traverse is complete.

89. Transit vs. compass. The form for the field notes of the survey just outlined is the same as for a compass traverse, except that azimuths instead of bearings are recorded and another column is used in which to record the needle readings as a future evidence of the correctness of the work. Any land survey may be made with the transit, the object being, of course, the same as if made with the compass. The essential difference is that the work is done with a greater degree of precision with the transit and azimuths are used instead of bearings. One difference that should also be noted is this:

the use of the compass makes every line independent of every other line, so far as direction is concerned, since the direction of each line is determined independently by the needle. With the transit this is not so. An error in one line is carried through the remainder of the survey; and, being an angular error, the error of position of the final point of the survey is much greater than it would be if there were an error in the direction of but one line of the survey. It may be said, however, that with the needle check always applied, there is very little probability of an error of this kind, and the use of the transit is advised in all work of importance. Where speed and roughly approximate results are the chief requisites, the compass is the better instrument to use, unless the measurements may be made with the stadia, in which case the transit still remains the better tool. The two may be combined. The directions may be determined with the compass attached to the transit, and the distances read with the stadia, thus securing a maximum economy of time. In this case the stakes would not be centered with tacks, one or two inches making little error in such work, since the compass may not be read to less than five minutes, and this by estimation. Five minutes of arc means 0.15 of a foot in one hundred feet. Moreover, the distances, if read by the stadia, may not be determined closer than the nearest half foot or possibly the nearest foot.

90. Determining the meridian. In many surveys it is not necessary or common to determine the meridian before the survey is made. One line of the survey, usually the first run, is assumed as a meridian, and the direction in which the survey proceeds along this line is assumed as zero azimuth. The azimuths of all lines of the survey are then determined with reference to this one line. If, for any reason, it is desired to know the true bearing or azimuth of the lines of the field or survey, an observation for the meridian is made, and, when the meridian is determined, the angle that it makes with one side of the survey is found with the transit, and all the azimuths are corrected by this angle to make them read from the true meridian. The method of determining the meridian with the transit is the same as with the compass, except for the difference in pointing.

The time of the elongation of Polaris is determined, and a few minutes before that time the transit is set up over a stake in an open space where the star may be observed. As this observation is usually made at night, it is necessary to make some provision for illuminating the wires in the transit so that they may be seen. This is usually done by throwing a faint light into the object end of the telescope by holding a piece of paper a little to one side and before it, and a lamp — a bull's eye is best — behind it so as to get a reflection of light from the paper in the telescope. It is troublesome to do this, and it takes two or three observers. A better way is to have a reflector like one of those in Fig. 50, which fits on to the object end of the telescope and is illuminated by a lamp held back of the observer. A still better way, when the instrument is made for it, is to have the horizontal axis of the transit hollow, with a small mirror near the center of the telescope. A small bull's-eye lamp is placed on a stand that is fastened to the standards and throws a beam of light upon the mirror, by which it is reflected to the wires. The light must not be too strong or the star will be indistinct. This is particularly true when, as may be the case, another and less bright star than Polaris is used.

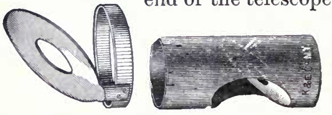


Fig. 50.

The transit being set up and provision being made for illuminating the wires, the telescope is turned on the star and both plates are clamped. The vertical wire is then made to cover the star as nearly as possible and is made by the tangent screw of the alidade to follow the star as it seems to move to the right or left, that is, in azimuth, until it seems to be stationary in azimuth and to be moving only vertically. The telescope is then plunged and a point set in the ground some distance away and left till morning.

The setting of the point requires some patience. Perhaps the best way to accomplish this is to provide a box open on two sides. Cover one side with thin tissue paper, and place a candle in the box. This improvised lantern may be approximately set from the transit in the proper line, and the point to drive the stake may be determined by holding a plumb line in front of the box, the papered side being turned toward the transit. This line will be put in position from the instrument and the stake driven. The stake will then be centered by the use of the line, and a tack driven. On the next day at about 10 o'clock the transit is set over the point occupied the previous evening; and the azimuth of Polaris at elongation, which has been previously computed as explained on page 364, is turned off from the line of stakes to the right, if western elongation has been observed, and to the left, if eastern elongation has been observed. Another stake is now set in the line thus determined and the line defined by the point occupied by the transit, and the last stake will be the true meridian.

With the instrument set on this line the reading of the needle will give the magnetic declination in amount, but with the opposite sign. That is to say, the needle will read the magnetic bearing of the true meridian; and if the magnetic meridian lies west of the true meridian, thus making the declination west, the needle will read the bearing of the true meridian as east.

91. Needle checks on azimuths. In making surveys, if the compass box is not provided with a declination plate or vernier, on which the declination may be set off so that the needle will read north when pointing in the true meridian, it is often convenient to make the meridian of the survey, the magnetic meridian of the place, to facilitate checking by the needle the angles measured with the transit. It is inconvenient to do this even then, because the compass is graduated from two points 90° each way, while for azimuths, the transit is graduated continuously through 360°. This difficulty is obviated by making a similar graduation on the compass box. If one has a transit that is not graduated in this way, he may graduate a paper ring and paste it on the glass cover of the box. The graduations need be only to degrees. It will be found that such a check on angle measurements is very valuable. If the ring has been properly set, and if the magnetic meridian has been chosen as the meridian of the survey, the reading of the compass should always agree practically with that of the transit. If, instead of the magnetic meridian some arbitrary meridian has been assumed as the meridian of the survey, the paper circle should be placed so that the needle will read zero when the telescope is pointed in the direction of the assumed zero azimuth, the vernier of the transit reading zero. The needle check is good to the nearest five minutes except where there is local attraction.

Some transits have the graduations of the limb numbered as are those of the compass box. This is an old method. It is better to provide continuous numbering on the compass box.

ADJUSTMENT OF THE TRANSIT.

92. Requirements. The transit is used for measuring horizontal angles, that is, angles subtended at a given point by the vertical planes through the two other observed points. If these two other points are not in the same horizontal plane, the vertical motion of the telescope must be used to bring the line of sight down from the higher point to the horizontal plane of the instrument and the line of sight of the lower point up to the same plane. Now, the line of collimation is the line in the instrument that is directed to the distant point, and the line that is revolved down or up, as the case may be, to the horizontal plane of the instrument, which is the horizontal plane through the center of the horizontal axis. It is evident that this line must revolve in a vertical plane, to properly project the distant points into the horizontal plane of the instrument. It will also be evident that the vertical axis of the transit must be truly vertical in order that the line of sight when projected into the horizontal plane of the instrument and then turned in azimuth, may move in a horizontal plane. If the axis of revolution is not vertical, the lateral motion of the instrument will be in an inclined plane. In order that these necessary conditions may obtain, certain adjustments of the instruments must be properly made. Every adjustment consists of two parts: the test to determine the error, and the rectification of the error found.

93. The plate bubbles. If the plate bubbles are perpendicular to the axis of revolution, that axis will be vertical when both bubbles are in the centers of their respective tubes. The test is made and the adjustment is performed as described for the compass. If one bubble gets broken, the other may be used for both, by first leveling with the bubble in one position and then in another, 90° from the first, exactly as in setting up the level, and by repeating the operation till the axis is vertical. But two operations would be necessary were it not for the fact that the second leveling will, to some extent, disturb the first.

The adjustment being made, all points of the instrument will revolve in horizontal planes if the bubbles are both in the centers of their tubes. This adjustment should be tested every day, although it will probably be found correct for a considerable number of days in succession. If the transit is out of adjustment all around, it is better to make each adjustment approximately in order and then carefully repeat them all.

94. To make the line of collimation revolve in a vertical plane when the telescope is turned on its horizontal axis. This adjustment consists of several parts. One of these is to make the axis of revolution of the telescope horizontal. If the instrument is provided with a striding level, this part may be performed first. If not, as is usually the case, it must be done last, and possibly at the expense of repeating some of those adjustments that have preceded it. It will be assumed that no striding level is used.

If the line of collimation is not perpendicular to the axis of revolution of the telescope, it will describe, in revolving, the surface of a cone, whose axis is the horizontal axis of revolution of the telescope; and if the line of collimation is perpendicular to the axis and the axis is not horizontal, the line of collimation will describe, on being revolved, an inclined instead of a vertical plane. The student should make a mental picture of these conditions.

The intersection of the wires should be in the line of motion of the optical center of the objective; for, if not, then the line of collimation will not be fixed in its position in the telescope tube, and, if made perpendicular to the axis of revolution for one distance, it would not be so for some other distance that would require a shifting of the object slide for new focusing. The object slide should move parallel to, if not absolutely coincident with, the axis of the telescope tube, in order that the line of collimation may be made perpendicular to the horizontal axis of revolution. Theoretically, its line of motion should pass through the horizontal axis. In the instrument shown in Fig. 48 the objective is permanently adjusted in motion by the maker. In some other instruments, however, this is not the case.

The two wires are adjusted separately, and it is not unusual to omit the adjustment of the horizontal wire. This should not be omitted if the instrument is to be used for leveling or for measuring vertical angles. The first adjustment of the vertical wire is to make it vertical. This can be done only approximately, if the horizontal axis has not been previously adjusted.

I. *To make the vertical wire vertical.* Carefully focus the eyepiece, level the instrument, turn the telescope on a suspended plumb line, and observe whether the vertical wire coincides or is parallel with it. The corner of a plumb building will sometimes answer the purpose if the wind interferes too much with the swinging plumb line. If the wire is found to be out of plumb, it should be made plumb by loosening all four of the capstan-headed screws that hold the ring carrying the wire, and by moving the ring around by the screws till the wire is vertical. The holes by which the adjusting screws pass through the telescope tube are slotted for this purpose.

The remaining part of the adjustments will be described as it is considered best to make them in the complete transit or tachymeter. (See Art. 114, page 127.) The adjustments that are to be made when only the plain transit is to be adjusted are those of Art. 93 and numbers I., III., and IV., of this article.

II. *To make the line of collimation parallel to or coincident with the geometric axis of the telescope.* Construct of wood a pair of Y standards fastened to a firm block of wood and with such a distance between them that the telescope tube may, after being removed from its standards, rest in the Y's near its ends. Having made the vertical wire vertical as described in the last adjustment, remove the telescope with its horizontal axis from the standards, and rest it in the Y supports. Now adjust the wires (and the object slide if it is adjustable and needs it) in precisely the same manner as described in the first adjustment of the level. If the geometric axis of the telescope has been made exactly perpendicular to the horizontal axis, and the two are coincident where they cross, the line of collimation is now perpendicular to the horizontal axis. The foregoing adjustment is, in all good instruments, sufficient for the horizontal wire. After replacing the telescope in its standards, the vertical wire is tested and, if necessary, corrected as explained in III.

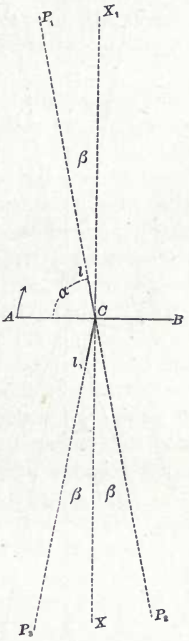


FIG. 51.

III. If the line of collimation as defined by the optical center of the objective and the vertical wire (which is the wire most used in the transit) is not perpendicular to the horizontal axis of revolution, and the instrument is set over the middle one of three points that are in a straight line, the conditions will be as shown in Fig. 51. AB is the axis of revolution, Cl the line of collimation, showing only the object end to avoid confusion; the latter makes with the axis the angle α. P_1, C, and P_2, are the three points in a straight line, the instrument being centered over C. XX_1 is a line drawn perpendicular to the axis of revolution. The error of perpendicularity is the angle $\beta = 90° - \alpha$. The instrument is shown with the line of collimation directed toward the point P_1.

If now the telescope is revolved, the axis of revolution remaining in the same straight line, the line of collimation will take the position CP_3, making with the line CP_2, an angle equal to 2β. Since the lens can not be moved, the wire must be moved so that the line of collimation will fall in the line CX found by taking a point halfway between P_3 and P_2. Since the wire is nearer the observer than the lens, the wire must be moved in a direction opposite to that in which the forward end of the line of collimation is to be moved. With erecting instruments, since points that are really on the left or right appear so, this rule would be followed. With inverting instruments, in which an object on the right of the line of collimation appears to be on the left, and *vice versa*, if the line should apparently be moved to the left, it should really be moved to the right; and hence the wire should be moved in the direction in which it appears that the correction should be made.

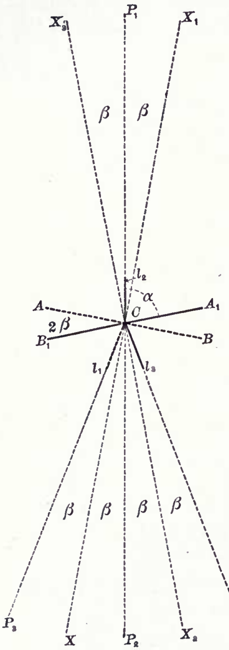


FIG. 52.

If the position of P_2, and hence also that of X, is not known, the explanation is as follows: If, while the line of collimation is pointing to P_3, a point is established there, and the instrument is then turned on its vertical axis in the direction indicated by the arrow till the line of collimation again points to P_1, the line of collimation and its axis of revolution have been turned in azimuth through an angle equal to $180° - 2\beta$, and would be in the position shown at $A_1 B_1$, in Fig. 52. The angle between the new and former positions of the axis of revolution will be 2β. If now the line of collimation is revolved on its axis, it will fall in the line CP_4 as much to the right of CP_2 as it was before to the left; and the angle between its two positions pointing to P_3 and P_4, will be 4β; and hence, if P_2 had not been established, but only P_1, C, P_3, and P_4, the instrument would be corrected by shifting the wire till the line of collimation should fall on X_2 one fourth the distance from P_4 to P_3. It should be noted that unless the axis of revolution has been made horizontal, the points P_1, P_3, and P_4 should all be in about the same horizontal plane; that is, comparatively level ground should be used for this adjustment.

From the foregoing explanation there results the following rule for making this adjustment of the vertical wire, or, as it is usually called, the adjustment of the line of collimation:

Set the instrument over a point in a comparatively level stretch of ground. After leveling, establish a point about two hundred feet in one direction and turn the line of collimation on this point, clamping all motions except the vertical motion of the telescope. Transit the telescope and set a point in the opposite direction, directly in the line of collimation. Loosen one clamp for motion in azimuth and turn the instrument in azimuth till the line of collimation cuts the first point, and clamp all motions except the vertical motion of the telescope. Transit the telescope, and set a point by the side of the second point. If the adjustment is perfect, the second and third points will coincide. If they do not, move the vertical wire to one side, loosening first one screw and then tightening the opposite, till the line of collimation cuts a point one fourth the distance from the third point toward the second. Repeat the test for a check.

It will usually be found necessary to perform the operation more than once. All bisections should be made with the greatest possible precision, using the clamps and slow-motion screws of all motions except the vertical motion of the telescope.

The horizontal wire will probably not be appreciably disturbed by this adjustment, but may be tested as before. If it is found necessary to correct it, the vertical wire must again be tested as above. If the instrument is to be used for leveling or for reading vertical angles, it is just as necessary as in the level that the horizontal wire be properly adjusted so that the line of collimation shall be correct for all distances.

IV. The adjustment of the axis of revolution of the telescope is made in any one of several ways. Two will be given.

(1) Hang a long plumb line from some tall and firm support, as a second-story window sill. Having set up the transit a short distance, say twenty feet, from the line, turn the line of collimation, as defined by the intersection of the wires, on a point in the line near its upper end, and clamp azimuth motion. Swing the telescope vertically, noting whether the intersection of the wires remains on the plumb line. If not, raise or lower one end of the axis of revolution of the telescope till the intersection of the wires will follow the plumb line.

(2) Set up the instrument near some tall building or other high object, setting the intersection of the wires on some well-defined point near the top, and clamp the azimuth motion. Plunge the telescope downward and set a point on the ground near the building or object. Reverse in azimuth, transit the telescope, and set again on the point near the top and clamp in azimuth. Drop the telescope, and see whether the intersection of the wires falls on the same point as before near the bottom. If it does, the axis is horizontal and needs no adjustment. If not, set a point midway between the two points at the bottom and adjust the axis by raising or lowering one end till the wire will cut that point when plunged from the upper point. The student will be able to make a diagram showing the correctness of these methods and will be able to tell whether the axis is moved in the second adjustment, so as to make the line of collimation cut a point one fourth the distance from the second to the first lower point or one half that distance. Unless this adjustment is very badly out, the vertical wire will not be again disturbed. If thought necessary, the screws may again be loosened and the vertical wire may be made truly vertical, after which the adjustments for collimation must again be tested. The instrument as a transit is now adjusted.

95. Level under the telescope. To use the transit as a level, the level under the telescope must be adjusted by the peg method as described for the leveling instrument in Chapter III., adjusting the bubble, not the wire.

96. Vertical circle. If the transit has a vertical circle, that should be adjusted so as to read 0° when the bubble under the telescope is in the center of its tube, after the last-mentioned adjustment for leveling has been made. For this purpose the vernier of the vertical circle is adjustable. To make the adjustment, loosen the small screws that fasten the vernier to the standard and slide the vernier along as is indicated by the position of the zero of the circle till the zeros of scale and vernier coincide, the bubble being in the center of its tube.

97. Eyepiece. After the wires have been adjusted they may not appear in the center of the field of view. This is because the eyepiece is not properly centered. There need be no inaccuracy in work done with the transit if this is not corrected, but better seeing will result if it is corrected. This may be done by moving the ring through which the eyepiece slides, just as the wire ring was moved, there being a set of screws for this purpose next back of the wire screws. In the instrument shown in Fig. 48, this adjustment is permanently made. In such an instrument, the horizontal wire may usually be adjusted with sufficient precision for ordinary work by merely bringing it by eye to the center of the field of view. The vertical wire is then adjusted as described in Art. 94, III., and it is unnecessary to remove the telescope from the standards for the adjustment of the line of collimation. In inverting instruments, in which the field of view is limited by the eyepiece itself, this may not be true unless the eyepiece is in adjustment. It usually is.

98. Eccentricity. There may exist errors of graduation; but such as are likely to occur in modern machine-graduated instruments cannot be detected by ordinary means. The center of the graduated circle may not lie in the axis of rotation, and the line joining the zeros of the verniers may not pass through the center of the graduated circle. If the latter condition exists, the verniers will not read 180° apart, except possibly at some one point in case the first condition also obtains. The second condition simply means that the verniers are not 180° apart, and no error will result from this cause if the same vernier is read for both pointings for the measurement of an angle. If the second condition exists and the first does not, the angular distance between verniers will be the same for all parts of the circle ; while, if the first condition exists, the angular distance for different parts of the circle will not be constant. These two facts furnish methods of testing for eccentricity that will be evident to the student who is familiar with the discussion of eccentricity in the compass.

THE SOLAR TRANSIT.

99. What it is. The solar transit consists of a transit with an attachment for determining the true meridian by an observation on the sun. The solar transit and the solar compass, essentially the same as the solar part of the solar transit, have been extensively used in laying out the public lands of the United States. The solar compass was invented by William A. Burt of Michigan, and has become known as Burt's solar compass. The United States Land Office has specified that the work of subdividing the public lands must be done with solar instruments or transits. The solar compass is not now much used. When a solar instrument is used, it is usually the solar transit.

100. Fundamental conception. Before describing the solar transit, it will be necessary to explain the conceptions on which its action is based. For this purpose let the student imagine a celestial sphere, concentric with the earth and of infinite radius. This is not quite true to fact, but will assist the understanding of the following statements :

Let it be imagined that the equatorial plane and the axis of the earth are extended till one cuts from the celestial sphere a circle, called the celestial equator, and the other cuts the celestial sphere in two points that may be known as the north and south poles. Imagine further the meridian plane of the place of the reader extended to the celestial sphere. It will cut from that sphere a meridian circle. Let the earth be conceived to be very small as compared with the celestial sphere, so that points on its surface are practically at the center of the sphere. If the reader imagines himself to be at the equator, the zenith will be the intersection of the celestial meridian and equator. If he now imagines that he moves north, his zenith point will move north by an angular amount equal to the latitude he covers. Moreover, his horizon, which at the equator included the poles, will be depressed below the north pole by an equal angular amount. Hence the altitude of the north pole will at any place indicate the latitude of the place, the angular distance from the pole to the zenith will be the colatitude; the angular distance zenith-equator will be the latitude, and the angular distance equator-south horizon will be the colatitude.

It is well known that, because of the inclination of the ecliptic to the earth's axis, the sun is below the celestial equator for six months of the year and above it for six months. The amount that it is above or below it is constantly changing, and the angular distance of the sun from the celestial equator at any moment is known as the *declination* of the sun for that moment. It is the same as terrestrial latitude. Let it be forgotten for a time that the sun is fixed and that it is the revolution of the earth on its axis that causes the sun to appear to rise in the east and set in the west, and let it be imagined that the sun does the moving just as it appears to do. Then, if the sun were to maintain a constant declination for a whole day, its path in the heavens would correspond to a parallel of latitude above or below the equator by the amount of the sun's declination for the day. On the 21st of June it would correspond to the extension of the Tropic of Cancer, and on the 21st of December to the extension of the Tropic of Capricorn.

If a pointer of any kind should be directed from the center of the celestial sphere toward the sun at any time during the day under consideration, it would make with the equatorial plane an angle equal to the declination, and with the polar axis an angle equal to the codeclination. If now this pointer is revolved about the polar axis, keeping the angle between them constant, the pointer will at all times point to some point in the path of the sun for the day; and if it is revolved just as fast as the sun moves, it will all day point to the sun.

101. Description. The solar attachment consists essentially of an axis that is made to be parallel with the earth's axis, and a line of sight (pointer) that is set at an angle to the instrumental polar axis equal to the codeclination of the sun for the time of observation.

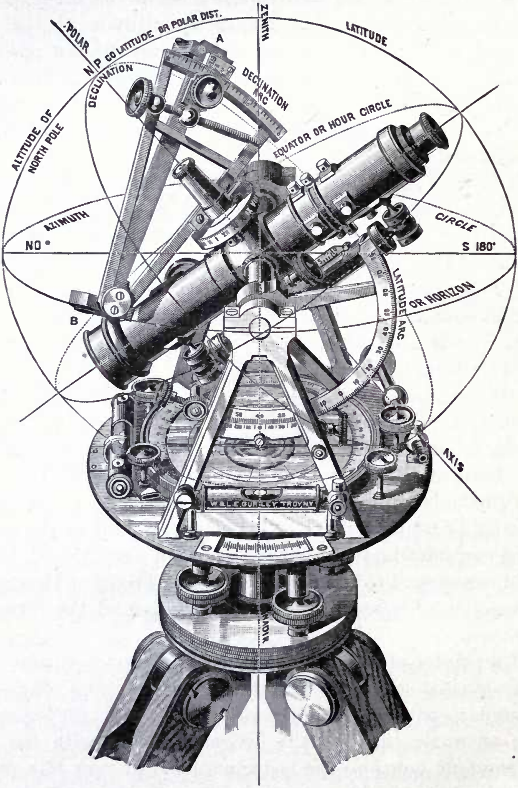


FIG. 53.

In Fig. 53 the polar axis is marked. It is made at right angles to the telescope tube, and hence if the telescope tube is brought into the plane of the equator as shown, and is also in the meridian plane, the polar axis must be parallel to the terrestrial or celestial polar axis, or, as is commonly said, it must be pointing to the pole. If now the arm marked AB, which carries a line of sight, is brought to a zero reading on the declination arc, it will be perpendicular to the polar axis and practically coincident with the equatorial plane. If the sun is for the day on the equator, the line of sight, on being revolved about the polar axis, will cut from the celestial sphere the path of the sun, or it can be at any time turned on the sun. If the sun is a few degrees above the equator and its declination is set off on the declination arc, with the arc in the position shown in the figure, and the line of sight is then revolved about the polar axis, it will cut from the celestial sphere a circle parallel to the equator which will be for the day the path of the sun. So, as before, the line of sight may be at any time turned on the sun by simply revolving it about the polar axis. If now the whole transit is revolved in azimuth so that the polar axis no longer points to the pole, the line of sight will not, on being revolved, cut from the heavens the path of the sun and can not be set on the sun at any time by merely revolving it about the polar axis. There may be one instant at which it can be thus set.

102. Method of use. This, then, furnishes the key to the method of use of the instrument, which is as follows: The direction of the meridian plane being unknown and desired, set off on the vertical circle of the instrument the colatitude of the place, so that the polar axis, when in the meridian, may point to the pole. Set off on the declination arc the declination for the time of the observation. Now with the plates level, so that revolution about the vertical axis may be only in azimuth, revolve the instrument in azimuth and the line of sight about the polar axis till the sun is found to be in the line of sight. When this occurs, the polar axis and telescope lie in the meridian, and the instrument may be clamped and the line ranged out. There is a small circle which will then give the time of day, which was of course known beforehand, in order to compute the declination.

The declination is found in the "Nautical Almanac"[1] for the longitude of Greenwich and for noon of each day in the year, and with the hourly change. The longitude of the place being known, the declination for the time and place may be readily computed. It must be remembered that in most places standard time, which probably is not the same as local time, is used and, if very different from local time, allowance must be made. A difference of fifteen minutes will not ordinarily make any appreciable error in the resulting work. The latitude and longitude may be taken from any good map, or the latitude may be observed by measuring on a previous night the altitude of Polaris at culmination, and adding or subtracting from the result the pole distance of the star. See Table IV., page 364. It may also be observed with the solar transit, as will be explained later.

The line of sight AB consists of a lens at A and a small silver disk at B. The line of sight is directed toward the sun by bringing the image of the sun formed by the lens into the center of a square ruled on the opposite silver plate. In order that the line of sight may be used when the sun is below the equator, there is a lens in each end, and a silver plate opposite each lens. If the declination were south, the declination arc would be reversed from the position shown in the cut; and the lens in B and the plate in A would be used.

103. Limitations. Since there is a horizontal, that is, azimuth, and a vertical component to the sun's motion, except just at noon, when the motion appears to be all in azimuth, the image of the sun will appear to move off the square on the disk, if it is left stationary for a time, in a diagonal direction, and can be kept in the square only by revolving the line of sight about the polar axis and shifting the arm on the declination arc, the polar axis being in the meridian. Just at noon, however, the motion of the sun is apparently all horizontal, and since at noon the line of sight will be in the same vertical plane as the polar axis and telescope, the sun's image will move out of the square horizontally or between two of the lines, so that

[1] The "Nautical Almanac" is published by the Navy Department at Washington. The portion relating to the sun's declination is published separately by W. & L. E. Gurley and by G. N. Saegmuller, and perhaps by other instrument makers.

it could be kept in the square for a little time by moving the polar axis a little in azimuth. At this time, therefore, the meridian cannot be correctly determined. It can not be well done within one or two hours either side of noon.

104. Latitude. For the same reason, since at noon the exact meridian is not needed to get the sun in the line of sight, this is the time to observe for latitude, as follows:

Set up the instrument a little before noon. Set off the co-latitude (as nearly as known) on the vertical circle, and the declination for noon on the declination arc. Now bring the sun's image upon the silver disk between the horizontal lines by using the azimuth motion of the transit and the vertical circle tangent screw. The image will appear to get lower as the sun goes higher. Keep the image in the square on the disk till it appears to begin to move upward. The sun is then at its highest point; and if the declination has been properly set off and the plates carefully leveled, the vertical circle will read the colatitude of the place.

The solar transit may then be used at noon for finding latitude, and between 8 o'clock and 10.30 o'clock A.M. and 1.30 o'clock and 5 o'clock P.M. for determining a meridian. It could be used earlier and later but for refraction, which is of unknown and very irregular amount near the horizon.

105. Refraction. Thus far nothing has been said of refraction. It must always be considered in setting off the declination. The effect of refraction has been discussed in the chapter on leveling. It there appears that the object is always seen higher than it really is. Hence, if the sun's declination is north, and the exact amount is set off on the declination arc, and the polar axis is brought into the meridian, and the line of sight pointed toward the sun, the sun's image would not be formed exactly in the little square, because the sun's rays would seem to come from a higher point than its true place. It is true that for small differences, like that of refraction or small errors in setting the latitude, the sun's image may be brought on the square by slightly turning the instrument in azimuth, thereby destroying the correctness of the determination of meridian. Hence it is necessary to know and correctly set off the latitude and the declination *corrected for refraction.* The correction for refraction is greatest near the horizon, and is nothing at the zenith. Since it always makes a luminous body appear higher than it is, the correction must be added to north declinations and subtracted from south declinations so as to result in setting the line of sight to point higher than if the correction were not applied. The corrections to be used are as given in Appendix, Table VII., page 366.

ADJUSTMENTS OF THE SOLAR TRANSIT.

106. Named. The adjustments of the solar apparatus are simple. They consist in making the lines of collimation parallel to each other and at right angles to the polar axis when the declination arc reads zero ; and in making the polar axis perpendicular to the telescope. The transit is supposed to be adjusted.

107. Lines of Collimation. These are made parallel by making each line parallel to the edges of the blocks containing them. To do this, remove the bar carrying the line of collimation, and replace it with a bar called an adjuster, which is simply a table upon which to rest the lines of collimation while adjusting. Rest the bar containing the lines of collimation on the adjuster, and, by any means, bring the sun into one line of collimation. Quickly turn the bar over (not end for end). If the image still falls in the square, the line of collimation is parallel to the two edges of the blocks. If not, move the silver disk through one half the apparent error of position of the sun's image and try again till complete. Turn the bar end for end, and adjust the other line of collimation. The two now being parallel to the blocks are parallel to each other. Remove the adjuster, and replace the bar on the instrument.

108. Declination vernier. Bring the declination vernier to read zero, and, by any means, bring the sun into one line of collimation. Quickly and carefully revolve the bar on the polar axis, and note whether the sun is in the other line of collimation. If so, the vernier is in adjustment. If not, move the arm till the image is centered, and note the reading of the vernier. Adjust the vernier by loosening it and moving it one half the apparent error. Test again.

109. Polar axis. To make the polar axis perpendicular to the telescope axis, first carefully level the plates and the telescope, and then level the solar apparatus by the capstan-headed screws shown underneath the attachment. This is precisely the same as leveling up an ordinary instrument, and is performed by the aid of an auxiliary level that rests on the blocks of the collimation bar. This bar is set so that the declination is zero, and is brought into the plane of the main telescope and leveled, then at right angles to this position and leveled again. This is an important adjustment, and should not be omitted.

SAEGMULLER SOLAR ATTACHMENT.

110. Description and use. Fig. 54 shows another form of solar attachment, which consists simply of a small theodolite (so called because the telescope will not transit) attached to the top of the telescope of an engineer's transit. This is the invention of Mr. George N. Saegmuller, of Washington, D.C., and is made by him and furnished by other makers as well.

In operation it is similar to the last-described attachment. The difference is that a telescopic line of sight is substituted for the lens and disk, and the small level is used in conjunction with the vertical circle for a declination arc.

Suppose the transit to be turned with the object end of the telescope to the south, and the telescope level. Assume north declination. Turn the object end of the telescope down an amount equal to the corrected declination and bring the small bubble to the center of its tube. The angle between the main and small telescopes then equals the declination. If the object end is now pointed to the equator by setting off the colatitude upward from zero (not from the declination reading), the instrument is set ready for use.

Turn the instrument in azimuth and the small instrument about its polar axis till the sun's image is seen in the small telescope; then the large telescope and polar axis lie in the meridian. There is a diagonal eyepiece to the smaller telescope to facilitate observations. The instrument is approximately pointed by the small disk sights above the level tube. The objective is turned up in setting off south declination.

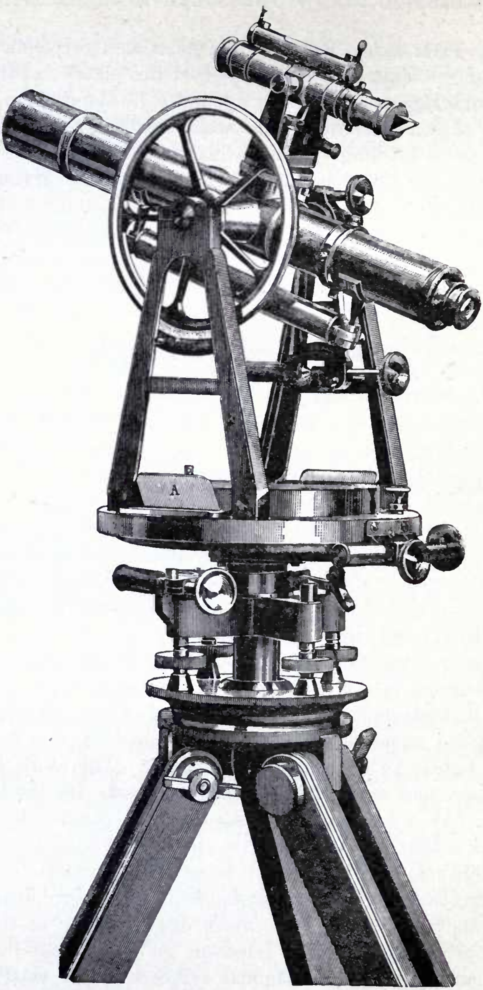


Fig. 54.

111. Adjustments. The adjustments of this instrument are two: the adjustment of the polar axis, and that of the line of collimation and small bubble. The first is performed, after the transit has been carefully adjusted, by making the main telescope level, then leveling the small telescope over one set of capstan-headed screws at the base of the attachment and adjusting, and then over the other precisely as in adjusting the plate bubbles of the transit, correcting first by screws and next by tipping the small telescope. The second adjustment is performed indirectly by making the two lines of collimation parallel. Measure the distance between the centers of the two horizontal axes and draw on a piece of paper two parallel lines the same distance apart; tack this up at some distance from the instrument and about on the same level, with the lines horizontal. Make the two bubbles parallel by making both level, with the telescopes pointing toward the paper. Set the line of collimation in the large telescope on the lower line on the paper and adjust the wires of the small telescope until the line of collimation of the small telescope cuts the upper line.

MERIDIAN AND TIME BY TRANSIT AND SUN.

112. Meridian. If the sun's altitude at any moment is measured with the transit, its azimuth at the same moment may be computed if its declination and the latitude of the place of observation are known.[1] For in the spherical triangle pole-zenith-sun, the three sides will be known and the angle Z (the azimuth) may be computed.

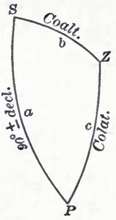


FIG. 55.

By Trigonometry if $s = \frac{1}{2}(a+b+c)$,

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}},$$

from which, if $\delta =$ declination, $\phi =$ latitude, and $h =$ altitude, and if $s' = \frac{1}{2}(\text{pole dist.}\ (= 90° \pm \delta) + \phi + h)$,

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin (s' - \phi) \sin (s' - h)}{\cos \phi \cos h}}. \qquad (1)$$

[1] The same is true of any known star.

Set up the transit at a convenient hour, over a tack in a stake and, with vernier at zero, set a stake some distance away, approximately north, and set the line of collimation on this with the lower motion. Unclamp the alidade and turn the line of collimation toward the sun and measure its altitude, at the same moment clamping the alidade. A small piece of red glass placed inside the cap of the eyepiece will enable the observer to look at the sun; or, with the eyepiece drawn fully out, the image of sun and wires may be received on a card held just back of the eyepiece. To observe most accurately, bring the horizontal and vertical wires tangent to the sun's image, correct the altitude by the sun's semi-diameter, $=0° 16'$, and the observed azimuth by $16' \times \sec h$. The observed altitude must be corrected for refraction. The corrections given in Table IV., page 364, for Polaris, will answer if the column of latitude is taken as altitude. Substitute the corrected altitude, the latitude, and the declination, in Equation 1, and solve for Z. The difference between the observed azimuth of the sun and Z is the azimuth of the line of stakes, from which the meridian may be laid off.[1]

113. Time. The angle at P in Fig. 55 is called t, the hour angle. It is given by

$$\sin t = \frac{\sin Z \cos h}{\cos \delta}. \tag{2}$$

Reduced to time, it is the true sun time, before or after noon, of the observation. This must be corrected by the equation of time (difference between true sun time and mean time) found in the "Nautical Almanac" to give mean local time. This must again be corrected by the difference between mean local and standard time. The result compared with the observed time of observation will give the error of the watch used.

[1] Let the student show the error in azimuth resulting from an error of 0° 01′ in either latitude or altitude by computing Z for values of ϕ differing by 0° 01′ and the same for values of h. This should be done for latitudes near 30° and 50° and altitudes of 10° and 60° to show the effect of variations in these quantities. Practically the same errors arise with the solar transit for like errors of latitude and declination.

CHAPTER V.

STADIA MEASUREMENTS.

114. Defined. The stadia is a device for reading distances by means of a graduated rod and auxiliary wires in a telescope. In many farm surveys where the ground is very rough and not very valuable, the measurements may be made with the stadia. When so made, they will probably be somewhat more accurate than if made with a chain or tape with the care that would ordinarily be taken in the kind of work mentioned. In fact, with the use of care and judgment, the stadia will serve for almost all farm surveys.

The stadia is the best distance measurer for extensive topographical surveys. The discussion of the stadia that follows will perhaps indicate its limitations. The term "stadia" has been very loosely used in this country. The word is the plural of the Latin word "stadium," which means a standard of measure. "Stadia" was the word adopted by the Italian engineer Porro (who invented the method to be described) to indicate the rod used by him in the application of his method. The English have retained this use of the word and apply the word "tacheometer" to the transit equipped with the additional wires used in the method. "Tacheometer" means quick measurer, and is applied by at least one American manufacturing firm to their high-class instruments that are equipped with vertical circle, level to telescope, and stadia wires. The word as used by them is "tachymeter." As generally used in this country the word "stadia" means the combination of instrument and rod, but it is thought that it should be applied to the rod only, and the word "tacheometer" or "tachymeter" is a very suitable word to apply to the complete transit instrument equipped for stadia work. Such a transit should have an inverting telescope of comparatively high power for the best work. The term "stadia wires" is properly used to designate the wires used with the stadia.

115. Method explained. Stadia measurements depend simply on the proportionality of the corresponding sides of similar triangles.

The telescope is fitted with two wires in addition to those already described, one above and one below the horizontal wire, and both parallel to it. When a rod is held at some distance from the instrument, and the telescope is properly focused on the rod, an image of the rod will be formed in the plane of the wires, and a certain definite portion of this image will be seen between the two stadia wires. The rod is usually held vertical. If the telescope is horizontal, the conditions will be as shown in Fig. 56. The two wires are shown at U and L. By drawing lines from U and L through the optical center of the objective O, to the rod R, the space on the rod whose image is included between the wires is seen to be lu.

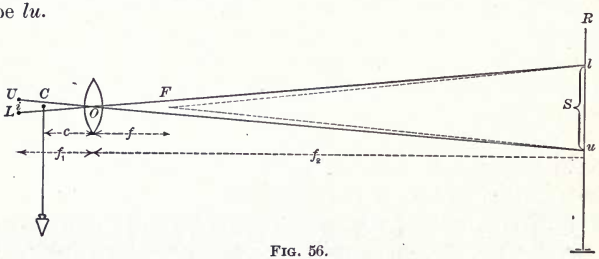


FIG. 56.

From the similar triangles OLU and Olu

$$\frac{f_2}{f_1} = \frac{S}{i}. \qquad (1)$$

A law of optics is that the sum of the reciprocals of the conjugate focal distances of a convex lens is equal to the reciprocal of the focal length of the lens. From this law, if f is the focal length of the objective,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}.$$

Whence
$$\frac{1}{f_1}=\frac{f_2-f}{ff_2}. \tag{2}$$

Equating this value of $\frac{1}{f_1}$ with that obtained from (1), there results
$$f_2=\frac{f}{i}S+f. \tag{3}$$

This is the distance from the objective to the rod. It is usual to require the distance from the point over which the transit is set, to the rod. If the distance from the objective to the center of the horizontal axis, which is vertically over the plumb line. is represented by c, the total distance required is
$$D=\frac{f}{i}S+(f+c). \tag{4}$$

If, then, a graduated rod is held at an unknown distance from the instrument, the distance will become known by multiplying the space intercepted on the rod by the ratio of f to i and adding to the product the constant quantity $(f+c)$.

116. Spacing of the wires. The value of $\frac{f}{i}$ must be known. Sometimes the wires are made so that the space between them is adjustable. This is not considered by the author as good practice. The wires should be fixed. They are attached to the same ring that carries the cross wires, and should be spaced at equal distances from the horizontal wire. This is not necessary, but is very convenient in reading, because it will occasionally happen that but one stadia wire and the horizontal wire can be read. This is likely to occur in brushy country, but should not be permitted when it is possible, at an expense commensurate with the importance of the work, to avoid it. In case it should occur, it is convenient to have the wires equally spaced, because the reading of one wire multiplied by two will give a sufficiently accurate determination of the distance. In case the wires are not equally spaced, separate values of the intervals may be determined for the wires.

It is thought best to have the ratio $\frac{f}{i}=100$ for convenience. If one is introducing stadia wires in his transit, he will find that it is practically impossible to accomplish this result accurately himself. It can rarely be done outside of the maker's shop. However, it is not absolutely necessary to make this ratio 100, as the rod may be graduated to suit the instrument.

117. To approximate the value of *f*. Focus the telescope on a distant point (theoretically the point should be infinitely distant, as a star) and measure the distance from the center of the objective to the plane of the wires. This is the value of f.

118. The value of *c*. This is not quite constant in most instruments, since the objective is moved in and out in focusing for different distances. For almost all work done by the stadia the error from this cause will not exceed one tenth of an inch, which is inappreciable compared with the smallest unit readings taken by this method. Single distances may not be read in careful work much closer than the nearest half foot; and, in a very large percentage of work done with the stadia, the distances are read only to the nearest yard or meter.

The value of c may be determined by focusing on an object at, say, two hundred feet distance, and measuring the distance from the objective to the center of the horizontal axis. In some inverting instruments the focusing of the image on the wires is accomplished by moving the eyepiece and wires together, instead of the objective. In such an instrument c is a constant.

119. The value of $\frac{f}{i}$ and $(f+c)$. If the value of $\frac{f}{i}$ is not known, it may be determined as follows: Select a strip of level ground, drive a stake and center it with a tack. Set the transit over this point. From this point measure two distances, say, one hundred feet and two hundred feet. Hold a rod at each of the two distant points, and note the space intercepted on the rod at each point.

$$D = \frac{f}{i}S + (f+c), \tag{5}$$

$$D' = \frac{f}{i}S' + (f+c). \tag{6}$$

In each of these equations, D and S are known, and hence $\left(\frac{f}{i}\right)$ and $(f+c)$ may be found.

120. The rod. While a rod graduated by lines to feet, tenths and hundredths, as the ordinary leveling rod (see page 47), may be used as a stadia, it is more convenient to use a rod that is graduated so as to give the distance intercepted by the wires directly by inspection, without reading the lower and the upper wire and making a mental subtraction, as would be necessary in the use of the level rod. Various patterns of rods have been devised by different surveyors. A rod suitable for a topographic survey covering a large area and mapped to a very small scale would not be suitable for a similar survey of a small tract to be mapped on a large scale. In the former case the units on the rod should be yards or meters, while in the latter case the units should be feet. The reason for this difference will be apparent. For any work connected with land surveys the rod should be graduated to feet, tenths, and hundredths.

FIG. 57.

An excellent form of rod is that designed by Mr. T. D. Allin of Pasadena, Cal.[1] Fig. 57 is a drawing of a rod designed on the same principle as that of Mr. Allin. The form of the figures is somewhat modified, to the improvement of the rod. The essential features of a stadia rod are, that it shall be well graduated in such designs as will render it easily and quickly read. There should always be some of the white rod showing at every graduation, to give definiteness to the position of the wire. It will be noticed that this has been accomplished in the rod shown in Fig. 57. The peculiarity of this rod is that the different divisions are very distinctly marked so as to make the reading of the distance very rapid and easy. It is assumed that the value of $\frac{f}{i}$ is 100, though this is not necessary, as a rod could be graduated with these same diagrams to fit any instrument. In the rod shown, the distance from point to point of the finest divisions is two one-hundredths of a foot, corresponding to two feet of distance. The distance from

[1] See "Engineering News," August 2, 1894.

point to point of the larger divisions shown on the right of the rod, is one tenth of a foot, corresponding to ten feet distance. On the left side of the rod the half-foot points stand out plainly, corresponding to fifty feet distance, and each whole foot is also plainly seen, giving one hundred feet distance. These whole feet are again marked in sets of three, giving three hundred feet distance. To read the rod, set the upper wire on some fifty-foot point and then run down the rod to the lower wire, taking in at once all the three-hundred-foot spaces, adding the hundreds and the final fifty, then the tens, and lastly the single units.

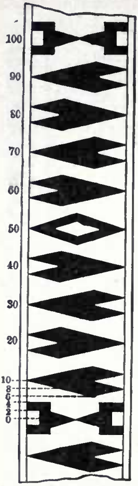


FIG. 58.

Another good rod for long-distance work, is shown in Fig. 58. The manner of reading this rod is seen from the figures at the side of the rod. This rod is better suited for yard or meter units than the Allin rod, and the author has used it with feet units with success.

121. Inclined readings. It has been assumed thus far that all readings taken with the stadia are horizontal. It will be evident that there will probably be more readings that are inclined than horizontal. When the readings are inclined, the formulas already derived must be modified.

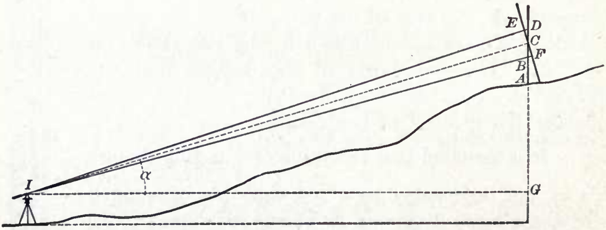


FIG. 59.

In Fig. 59 let the distance from I to A be required. If the rod were held perpendicular to the line of sight, the reading EF would at once give the inclined distance IC, which then multiplied by the cosine of the angle α would give the required horizontal distance. The angle α is determined by setting the horizontal cross wire on a point on the rod as much above A as the center of the telescope is above the ground at I, and then reading the vertical circle of the transit. The rod is usually held vertical. Therefore, the reading obtained will be DB. If the angles at E and F are considered right angles, an approximation sufficiently exact for the purpose, EF is given by

$$EF = DB \cos \alpha. \tag{7}$$

If, then, R is the reading DB, the distance IC is given by

$$IC = \frac{f}{i} R \cos \alpha + (f + c). \tag{8}$$

But it is the distance IG that is required.

$$\begin{aligned} IG &= IC \cos \alpha, \\ &= \frac{f}{i} R \cos^2 \alpha + (f + c) \cos \alpha. \end{aligned} \tag{9}$$

From this equation the distance is obtained. If it were necessary to perform this multiplication for every sight taken, very much of the usefulness of the stadia would be offset by this great labor. The value of $(f+c)$ varies from, say, nine inches to fifteen inches or more, according to the construction of the instrument. Where such an error is unimportant, the final term may be neglected. For vertical angles between five degrees and six degrees, the exact angle depending on the value of $(f+c)$, this term is just balanced by the difference between $\frac{f}{i} R$ and $\frac{f}{i} R \cos^2 \alpha$; hence for angles near this value no correction need be applied, and the distance may be recorded as read.

For the more ready computation of the results, tables have been computed giving the value of $\frac{f}{i} R \cos^2 \alpha$, for $R = 1$, and angles from 0° to 30°. Such a table is Table XIV., Appendix, pages 380–382. To explain the use of the table, suppose a reading of 1.52 feet, usually read at once 152 feet, is had with a vertical angle of 5° 30′. Under 5° 30′ Hor. Dist. find 99.08, which multiply by 1.52, and add the correction found at the bottom of the page for c, which is $(f + c)$. This multiplication may be hastened by the use of a slide rule,[1] which is an instrument that every surveyor should own and use. The column in the table headed Diff. Elev. is used when it is desired to know the elevation of the position of the rod above the instrument.

122. Difference of elevation. In Fig. 59, since the instrument is directed to a point on the rod as much above A as the center of the instrument is above the ground at I (found by standing a rod alongside the instrument), the difference in elevation between the ground at I and at A will be CG, and

$$\begin{aligned} CG &= IC \sin \alpha \\ &= \frac{f}{i} R \cos \alpha \sin \alpha + (f + c) \sin \alpha \\ &= \frac{f}{i} R \frac{\sin 2\alpha}{2} + (f + c) \sin \alpha. \qquad (10) \end{aligned}$$

For any reading and vertical angle the value of CG is found from the table in the same way as was the horizontal distance.

123. Diagram. A still more rapid method of reducing the observations is by means of diagrams. Omitting the second term in equation (9), the distance for any given vertical angle is proportional to the reading. To make a diagram to give the correction to apply to the readings to get the required distance, considering only the first term, a distance equal to the longest probable reading is laid off along one side of a sheet of cross-section paper ruled to tenths of inches or other small units. Suppose the distance to be 1000 feet. A space of ten inches may then be laid off, making the scale of distance 100 feet to an inch. Select an angle, as, say, 10°, and find from Table XIV. the true distance for a reading of 1000 feet and a vertical angle of 10°. The difference between this value and 1000 feet will be the correction to apply to the reading to get the true distance.

At the extremity of the distance line AB, Fig. 60, lay off at right angles to AB, and to a scale of, say 10 feet to an inch, the correction just found for the distance and angle. Suppose the point found is C. Draw from C to A a straight line.

[1] See Chapter VI.

Since the correction for this angle of 10° will be proportional to the reading, the correction for any other reading than 1000 feet will be given by the ordinate to the line just drawn at the proper distance from A. This will be evident from the similarity of the triangles involved. Thus the correction for any reading, as say 450 feet, is found by looking along AB to the 450-foot point, and taking to the scale of 10 feet to an inch

Fig. 60.

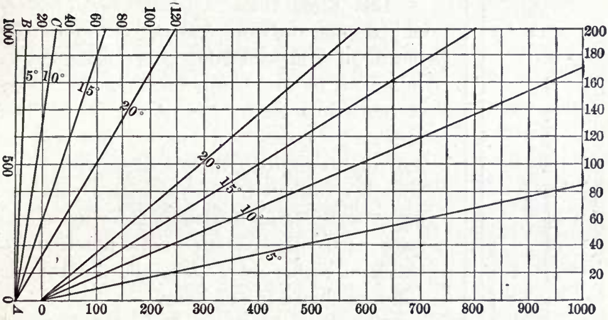


Fig. 61.

the ordinate at the 450-foot point to the 10° line. A similar line should be drawn in a similar manner for all other angles. It will be sufficiently exact to draw the lines for degrees only, estimating the minutes by eye.

A diagram for differences in elevation may be made in the same manner (see Fig. 61), except that it will be necessary to draw the ten-minute lines, as the elevations may not be estimated with sufficient exactness without these lines.

It should be noticed that the lines marked 1°, 2°, 3°, etc., do not make with the horizontal line angles of 1°, 2°, 3°, etc., but are drawn to the points found by laying off on the vertical the correction to the reading for the given angle, as found in the tables or as worked out from the equations. For the differences in elevation, the differences as found in the table for the given distance and angles are laid off on the extreme vertical line. The ten-minute lines may ordinarily be drawn with sufficient exactness by dividing the space between two adjacent

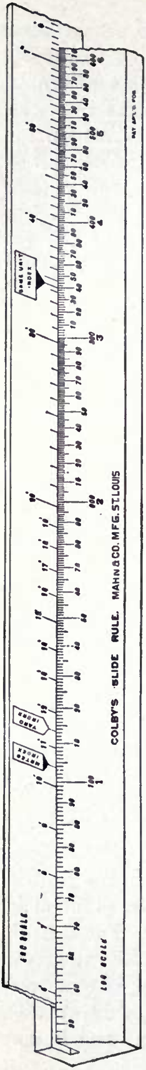


Fig. 62.

degree lines into six equal parts. These two diagrams may be drawn on one sheet of paper.[1] There are other forms of diagrams made for the same work.[2]

The second term of the equations must be applied separately when it may be necessary to apply it at all.

124. Slide rule. The most rapid method of reducing the differences of elevation is by means of a special slide rule[3] designed for this work by Mr. Colby of St. Louis. A cut of this rule is shown in Fig. 62. It is about four and one half feet long by one and one half inches high by three inches wide. The slide is a thin strip of wood with a varnished paper scale. Full instructions for use accompany each rule. With a little practice with this rule, rather more than 300 observations can be reduced in one hour.

125. Graduating a stadia. The stadia is not usually kept in stock by instrument makers, probably because there is no one design that is generally adopted by surveyors, each surveyor having his own idea as to the best form. Any instrument maker will make these rods after a model pattern furnished by the surveyor, but the surveyor may make his own rods very much cheaper than he can get them made.

Select a straight-grained piece of any light wood that is not too soft, have it dressed to the required length, width, and thickness. These dimensions are usually twelve or fifteen

[1] Such diagrams are printed and published by John Wiley and Sons, New York.

[2] See "Engineers' Surveying Instruments" by Professor Ira O. Baker.

[3] The principle of this rule will be understood after reading the article on the Slide Rule in Chapter VI.

feet by three to four inches by seven eighths of an inch. The wood is generally made uniform in dimensions throughout, but may be made thinner toward the top and may be shod with a strip of iron or brass at the bottom. Some prefer to have no top or bottom, but to have the rod graduated symmetrically from the center out to the ends, so that whichever end is turned up the reading is made equally well. There seems to be no great advantage in this. The rod shown in Fig. 57 is three inches wide inside of the side strips.

Give the rod three light coats of white paint, letting each coat become thoroughly dry before the next is put on. After the final coat is dry, graduate the rod. To do this, set up the transit that is to be used with the rod, on a level strip of ground, and, having determined the value $(f + c)$, lay this distance off in front of the point of the plumb bob. From the point thus obtained, measure carefully with a steel tape distances of one hundred feet, two hundred feet, three hundred feet, and more if desired. (If the rod is to be graduated to yards or meters, the distances should be one hundred, two hundred, and three hundred, of the units to be used.) Hold the blank rod at these distances and carefully mark with a hard pencil the points cut by the two wires at the different distances.

The spaces cut should, of course, be strictly proportional to the distances; but owing to inaccuracies of observation they may not be exactly so. Measure the several spaces intercepted and divide the three-hundred space by three, the two-hundred space by two, and average the quotients with the one-hundred space to determine the average space intercepted at one hundred feet. These spaces should be measured with great care, using either a good steel tape or a standard draughtsman's scale. This value of the space intercepted at one hundred feet having been determined, it is divided into as many equal parts as may be required, and the rod is spaced off for the diagrams that have been selected for use. The diagrams are then laid out in pencil and are painted on the rod, with black paint. A good quality of paint should be used, one that will dry with a dead rather than a glossy, surface, that is, the oil should be "killed" with turpentine.

The rod should then be allowed to dry. When it is thoroughly dry, strips of hard wood, about one eighth to one fourth of an inch wider than the rod is thick, are screwed to the edges of the rod. These strips, which are about a quarter of an inch thick, are to protect the graduated surface of the rod. They are very necessary to a rod that is to be much used. The rod should be shod with brass or iron about one eighth of an inch thick, at both ends if it is to be used either end up, and at the bottom if it is to have a bottom.

For some kinds of work rods are graduated to read directly from the instrument without the introduction of the $(f + c)$ correction. This is done by measuring the distances one hundred, two hundred, three hundred, or more, units from the plumb bob, noting the spaces intercepted at these distances, taking a mean value for one hundred units, and graduating the rod by this. The readings will be correct on such a rod for but one distance. If the rod is graduated by measuring from the instrument a distance which is assumed to be about a mean of those that will be read with the rod, and dividing the space intercepted into the required number of units, the readings taken thereafter will be correct only for the distance for which the rod was graduated. The space intercepted will always be that between the dotted lines radiating from *F*, Fig. 56.

It will be observed that, with a rod graduated as first described, it is not necessary that $\frac{f}{i}$ should be one hundred, or any other definite quantity, since the actual space intercepted at one hundred feet may be divided into one hundred units, and the rod may then be read as easily as if it had been graduated to hundredths of a foot and $\frac{f}{i}$ were equal to one hundred. $\frac{f}{i}$ is equal to one hundred of the units used. It is frequently convenient to use a level rod for single measurements (as the crossing of a stream on a survey in which the stadia is not being regularly used), or to be able to graduate rods directly in feet and hundredths instead of taking the trouble to graduate them as has been described, it being often practical to use such rods for a time as level rods. For these reasons, and others, it is thought best to make i equal $\frac{f}{100}$.

126. Smith's observations. It is possible for the maker to do this for the conditions of atmosphere obtaining at the time of the setting.

Mr. L. S. Smith, C. E., has recently shown[1] that systematic errors are introduced into work when it is assumed that a wire interval determined at any one time is correct for all future work. This is due to the fact that the effect of refraction is a variable quantity depending on the relative temperatures of air and ground. The effect of refraction is very much greater near the ground than a few feet above. It is much greater at noon than before or after. The effect varies for different distances. It is also shown that observers have different "personal equations."

It follows from the above, that to secure the best results with the stadia, and indeed to avoid cumulative errors, the rod, if graduated by the first method, should be graduated with a unit obtained as the average result of a series of observations at varying distances, at all hours of the day, for several days.

Also if a wide difference occurs in climatic or temperature conditions between those obtaining when the rod is graduated and those at the time of any survey, the rod should be regraduated for the particular survey, or a factor applied to the results. This factor should be determined as noted below.

It is also shown by these conclusions and the above result of them that it is better to graduate the rod to standard units and depend on the maker to place the wires at the interval $\frac{f}{100}$. It will be sufficient for many surveys to consider this as correct and to use the reduction tables or diagrams at once.

But if close work is to be done, it will be better to determine the wire interval by observations at all hours of the working day, and on several days, so that the average result shall conform to the average to be expected from the average conditions to prevail during the survey.

Thus it will be found that wires set $\frac{f}{100}$ apart will intercept on the rod held at distances of $100 + (f + c)$, $200 + (f + c)$, etc., from the instrument, varying lengths, from possibly a

[1] Bulletin of the University of Wisconsin, Engineering Series, Vol. I. No. 5.

little more than one unit to a little less than one unit for each 100 feet. The average result of all the observations would give a factor by which all observed distances on the survey should be affected before entering the reduction tables or diagrams. Thus suppose the average intercept for 100 + $(f + c)$ distance as determined from all observations should be 1.0039, then all distances read would be read too long, and should be multiplied by $\frac{1}{1.0039}$ before entering the reduction tables. The multiplication is very quickly performed by the aid of an auxiliary diagram, or table, or with the slide rule. It is not strictly correct to say that all distances will be read too long. Some will be too long and some too short, but the average result will be too long. By supplying this factor perhaps no one of the distances is made correct, but the errors introduced are no longer cumulative, as they would be were the factor not used. They are now compensating. The factor should be determined by the person who is to make the observations on the survey.

By adopting these precautions and determining a factor for each of several average conditions, — say for each of the four seasons, — a degree of precision of one in two thousand to one in five thousand may be obtained with the stadia.

127. Notes. Owing to the fact that vertical angles must be observed in traversing with the stadia, the form of notes to be kept in such work differs from that used with the transit and tape or chain. The form shown below is a good one.

Traverse of Mud Turnpike, June 21, 1895.					Right page for remarks and descriptive notes and sketches.
Station.	Azimuth.	Distance, Feet.	Vert. Angle.	Needle.	
A	37° 42′	476	+ 4° 00′	37° 45′	Describe A.
B	63° 26′	324	− 0° 26′	63° 25′	Describe B.
etc.	etc.	etc.	etc.	etc.	etc.

When elevations are to be kept, the form of notes shown on page 252 is better. If it is necessary to correct the distances, the corrected distances may be written in red in the distance column, leaving the original distances for possible future checks.

CHAPTER VI.

LAND SURVEY COMPUTATIONS.

128. General considerations and definitions. It will be evident that, if the lengths and bearings of all the sides of a closed field have been determined, these lengths may be reproduced to scale on paper, with the proper bearings referred to a meridian line drawn on the paper. It will be further evident that, if these lines and bearings have been correctly determined and have been laid off consecutively, joining end to end, they should form a closed figure similar to the field on the ground. A survey of a closed field rarely "closes" on the drawing board, owing to the facts that distances can not be exactly measured, that the various sides, offering different obstacles will not be measured with the same proportionate error, that if a compass is used, the bearings are ordinarily determined only to the nearest quarter degree, and that even if a transit is used, these bearings are not exactly determined. If the drawing is done with the aid of a protractor, the survey may close on the drawing, even though there is some error, because the errors are too small to show with the scale used. It is therefore necessary to resort to computation to determine just what the error of closure is.

To balance a survey is to determine the error of closure and, if it is not greater than is allowable, to distribute a quantity equal to the error, but of opposite sign, among the several sides, changing the notes accordingly, so that the survey shall "balance" or close. This is necessary because a description of a piece of property appearing in a deed ought to be such that it will indicate a possible closed tract, and such descriptions are, or should be, made from surveyors' notes. Before describing the method of doing this, it will be necessary to explain a few terms.

The latitude of a point is its perpendicular distance from an assumed base parallel.

The longitude of a point is its perpendicular distance from an assumed base meridian.

The latitude of a line is the latitude of its middle point.

The longitude of a line is the longitude of its middle point.

The perpendicular distance of the final end of a given line from a latitude parallel through its initial end is the latitude difference of the line.

The perpendicular distance of the final end of a given line from the meridian through its initial end is the longitude difference.

In Fig. 63 the latitude difference of the line AB is BD; the longitude difference is CB. If α is the bearing of the line,

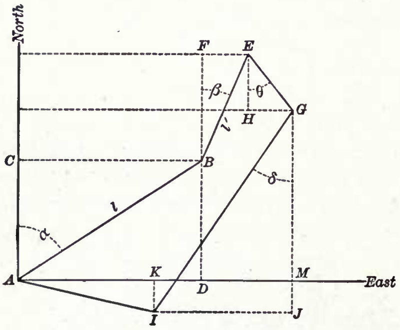


Fig. 63.

$$BD = CA = l \cos \alpha$$
$$CB = \qquad l \sin \alpha$$

Therefore the latitude difference is equal to the length of the line multiplied by the cosine of the bearing; and the longitude difference equals the length of the line multiplied by the sine of the bearing.

The latitude and longitude differences of the line BE are respectively:

$$BF = l' \cos \beta,$$

$$FE = l' \sin \beta.$$

The latitude and longitude of the point B, referred to the meridian and parallel through A, are respectively AC and BC.

The latitude and longitude of the point E, referred to the same meridian and parallel, are respectively $AC + BF$ and $CB + FE$.

Latitude differences measured north are considered positive, and those measured south negative. Longitude differences measured east are considered positive, and those measured west negative.

North latitudes are likewise positive, and south latitudes negative. East longitudes are positive, and west longitudes negative.

The latitude and longitude differences of the line EG are respectively $-EH$ and $+HG$. The latitude and longitude of the point G are respectively:

$$\begin{aligned} \text{Lat.} &= AC + BF - EH = GM, \\ \text{Long.} &= CB + FE + HG = AM. \end{aligned}$$

Similarly the latitude and longitude of the point I are respectively:

$$\begin{aligned} \text{Lat.} &= AC + BF - EH - GJ = -MJ = -KI, \\ \text{Long.} &= CB + FE + HG - JI \ = +AK. \end{aligned}$$

From the above discussion the method of determining the latitude and longitude of any point in a traverse will readily be seen.

The latitude of any point in a traverse is the latitude of the initial point of the traverse plus the algebraic sum of the latitude differences of all the courses of the traverse to the given point.

The student may formulate a similar statement for the longitude of a point in a traverse.

In the figure the latitude and longitude of the initial point are zero. In some cases this may not be so.

Since in a closed traverse the initial and final points are coincident, their latitudes and longitudes must be the same, and hence the algebraic sum of the latitude differences and the algebraic sum of the longitude differences should each equal zero.

129. Error of closure. If the latitude and longitude differences of the courses are computed, and the algebraic sum of each equals zero, the field closes; otherwise it does not.

If the field does not close, and if it is drawn on paper by laying off from a meridian and parallel the latitudes and longitudes of the several corners, in order, it will be found that the last point, which should be coincident with the first, is some distance from it. This distance is known as the error of closure, and is the hypotenuse of a right triangle, whose two sides are the error in latitude and the error in longitude. Its length, therefore, is the square root of the sum of the squares of the errors of the latitude and longitude. If this error is *very* small, or nothing at all, it indicates that the angles of the field have been correctly determined (though the bearings may none of them be right), and that if one is wrong, all are wrong by an equal amount. It also indicates that the lengths of the sides have been measured correctly so far as their relative lengths are concerned, though, perhaps, their true lengths have not been determined. The correctness of the bearings depends, if a compass has been used, on the correctness with which the declination has been determined; and the determination of the true length of the lines depends on the length of the chain or tape and the precision with which it has been handled.

It is usually assumed, however, that the constants of the compass and chain have been correctly determined, and that, therefore, the error of closure is a true measure of the precision of the survey. If a transit has been used to measure angles, from which the bearings of the sides have been computed, the error will be almost wholly due to erroneous measurement. The surveyor should know whether the error of closure is such as to be the result of justifiable or unjustifiable lack of precision in the work, or gross error. It may be said in this connection that, in ordinary farm surveying, an error of one in five hundred, obtained by dividing the error of closure by the total perimeter of the field, is tolerable, and that the precision should be, for good farm work, as high as one in two thousand, or better.

BALANCING THE SURVEY.

130. Fundamental hypotheses. If a compass is used, the method of correcting the notes is based on the supposition that any errors are due as much to inaccuracy of angle measurement as to a lack of precision in measuring lines. This does not mean that the error of closure due to any one line is as much owing to erroneous linear measurement as to erroneous determination of bearing; but that it is probable that in a line in which an error of bearing would tend to produce the observed error of closure, such an error has been made, and little or no error in measurement of length; while in a line in which an error of length would tend to produce the observed error of closure, such an error in length has been made with little or no error in bearing. This means that a line whose bearing is about normal to the line which represents the error of closure, would be corrected in bearing and not in length; while a line that is parallel to the direction of the line representing the error of closure, would be corrected in length and not in bearing. Lines whose bearings are intermediate between those mentioned would be corrected in both bearing and length, the greater portion of the error being given to bearing or length in proportion to the effect that each would have on the error of closure; thus, if the bearing of the line were nearly normal to the line representing the error of closure, it would be corrected more for bearing than for length, and if nearly parallel to that line it would be corrected mostly in length.

The plausibility of the supposition that errors of closure are due as much to errors in bearing as to errors in length, is clear when it is remembered that in using the compass, bearings are not read with greater precision than to the nearest quarter degree, thus making possible an error of from nothing to seven and one half minutes. The error resulting from an error in bearing is proportional to the length of the line on which the error is made, and an error of seven minutes gives an erroneous position for the end of a line of one foot in five hundred, or as much as would be allowed in measurement of lines.

It is assumed that errors in length will be proportional to the lengths of the lines measured, although it is more probable that they are proportional to the difficulties involved in the measurement, and to the lengths.

131. The method explained. If the assumptions are all true, it will follow that the error of closure should be distributed among the several sides of the field in proportion to their several lengths. The demonstration of this is as follows:

In Fig. 64 let $ABCDE$ be the plot of the notes, EA being the error of closure, known as the closing line. The line AF is the error in latitude, and EF is the error in longi-

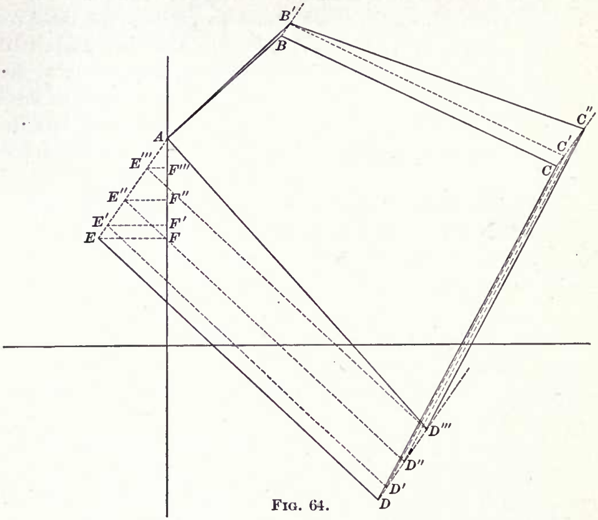


FIG. 64.

tude. Let the length of $AB = a$, $BC = b$, $CD = c$, $DE = d$, and let the sum of these lengths be s. Divide EA into four parts,

$$EE' = \frac{a}{s}\overline{EA},\quad E'E'' = \frac{b}{s}\overline{EA},\quad E''E''' = \frac{c}{s}\overline{EA},$$

$$E'''A = \frac{d}{s}\overline{EA}.$$

Lay off the lines BB', CC', DD', equal and parallel to EE'. Lay off also $C'C''$ and $D'D''$, equal and parallel to $E'E''$. Lay off also $D''D'''$ equal and parallel to $E''E'''$. Connect $AB'C''D'''A$. This will be the correctly closed field, and to each side will have been given such a portion of the entire error of closure, as that side is of the whole perimeter. From the similarity of the triangles AEF, $AE'F'$,

$AE''F''$, etc., it will be seen that the same proportion of the error of latitude and the error of longitude has been given to each side, whence the following rule:

RULE: *Correct the latitude difference and longitude difference of each course by an amount determined by the proportion; the required correction in latitude (or longitude) is to the total error in latitude (or longitude) as the length of the course in question is to the entire perimeter of the field.*[1]

It will be observed that the courses nearly parallel to EA have been corrected mostly in length, while those nearly normal to EA have been corrected mostly in bearing.

From the corrected latitude and longitude differences new bearings and lengths are computed.

132. The practice. This method is not strictly followed in practice. It is, as has been seen, based on certain assumptions as to the probable mode of occurrence of the errors. In a given case it may be that the surveyor knows, or is reasonably certain, that a greater portion of the error is due to the difficulty encountered in measuring one side, and in such a case he would give the greater portion of the error to that side. In determining that this is true, he must first ascertain the direction of the closing line to see whether the line supposed to be difficult to measure is parallel or nearly parallel to the closing line, so that an error in measurement would be responsible for the resulting closing line. It may be that it is believed that the error is not confined to one course but is distributed over all the courses, though not in the proportion of their lengths, as some may have offered greater difficulties to measurement than others. If this is so, the error is distributed in the following manner:

RULE: *Determine by judgment which line has offered the least difficulties to measurement and number this line* 1. *Give to each of the other sides a number that shall represent its relative difficulty of measurement, as* 2, 3, $2\frac{1}{2}$, *etc. Multiply each length by its number and find the sum of the multiplied lengths. Distribute the total errors of latitude and longitude by the follow-*

[1] Least square demonstration of this rule will be found in Wright's "Adjustment of Observations."

ing proportion: The correction to the latitude (or longitude) difference of any course is to the total error in latitude (or longitude) as the multiplied length of the course is to the sum of the multiplied lengths.

This is termed weighting the courses in proportion to their probable error, and results in distributing the whole error in proportion to the difficulties encountered and the lengths of the lines.

When the lines are all of equal difficulty of measurement, or when one who knows nothing of the field work balances the survey, the first method should be used. In case the person making the survey also balances it (as should always be the case, when possible), the second method should be followed.

The original notes of the survey should not be destroyed. The new latitude and longitude differences and lengths and bearings of courses may be written in the notebook in red ink over the originals. The originals may be useful in the future in determining how the corrections were made. It is not common to correct the observed bearings and lengths to correspond to the corrected latitude and longitude differences; but it is well to do this.

133. When a transit is used. In case there is no reason to believe there is any error in bearings, as when a transit is used for determining angles, and therefore the entire error of closure is due to lack of uniformity in measurement, the survey should be balanced in such a manner that only the lengths shall be corrected. No satisfactory rule has yet been devised to accomplish this, and it must usually be done by trial.

If the field is rectangular and one side may be taken parallel to the meridian of the survey, or assumed so for the purpose, the following rule accomplishes the desired result:

RULE: *The correction in latitude (or longitude) difference for any course is to the total error in latitude (or longitude) as the latitude (or longitude) difference of the course is to the arithmetical sum of the latitude (or longitude) differences.*

It is believed that the student can demonstrate the correctness of this rule for the assumed conditions, if he remembers that the sides must be corrected in length only, their directions remaining unchanged.

SUPPLYING OMISSIONS.

134. Necessity for. At times it is impossible to measure the length of a line; at other times it is impracticable to determine directly its bearing; and sometimes it is impossible to do either. Moreover, it may happen that the length of one line and the bearing of another have not been determined. If any one of these conditions prevails, it becomes necessary to supply the omission in the notes. To do this leaves the work without a check, as all the errors are thrown into the quantity that is supplied; hence no omission that can in any way be supplied in the field, should be permitted.

The cases that may occur are the following:

I. The length of one side may be wanting.
II. The bearing of one side may be wanting.
III. The bearing and length of one side may be wanting.
IV. The bearing of one side and the length of another may be wanting.
V. The lengths of two sides may be wanting.
VI. The bearings of two sides may be wanting.

135. General discussion. It should be clear from what has preceded, that the algebraic sums of the latitude and longitude differences of the known sides are respectively equal to the latitude and longitude differences necessary to close the field (but are of opposite signs), or to the latitude and longitude differences of the wanting sides plus the errors in latitude and longitude of the known sides. Since there is no means of determining these errors, the sums are taken as the latitude and longitude differences of the defective sides.

It is therefore possible to write two independent equations as follows:

I. The algebraic sum of the products of the lengths of the defective courses by the cosines of their respective bearings equals the algebraic sum of the latitude differences of the known sides, but has the opposite sign.

II. The algebraic sum of the products of the lengths of the defective courses by the sines of their respective bearings equals the algebraic sum of the longitude differences of the known sides, but has the opposite sign.

Having but two equations, there can be but two unknown quantities, and these may be as in Art. 134.

If l_1 and l_2 and θ_1 and θ_2 are the lengths and bearings of the defective sides, and if Σ is taken as a sign indicating the algebraic sum of any series of quantities, as in this case the latitude and longitude differences, and L and M represent latitude and longitude differences respectively, the above two equations may be written as follows :

$$l_1 \cos\theta_1 + l_2 \cos\theta_2 = -\Sigma L, \tag{1}$$

$$l_1 \sin\theta_1 + l_2 \sin\theta_2 = -\Sigma M. \tag{2}$$

These are, of course, equivalent to the following general equations :

$$\Sigma L = 0,$$
$$\Sigma M = 0.$$

136. Cases I., II., and III. These are most readily solved by applying equations (1) and (2). If but one quantity is wanting, as the length or bearing of a side, either equation (1) or equation (2) will solve, one of the left-hand terms becoming part of the right member.

If both the length and bearing of one side are wanting, one of the left-hand terms in each equation becomes part of the right member, and equation (2) may be divided by equation (1), giving

$$\tan\theta = \frac{-\Sigma M}{-\Sigma L}.$$

Having θ_1, the length is found by substituting in either equation.

The signs of the sums of the known differences will indicate the direction of the defective side.

137. Cases IV., V., and VI. The author thinks the remaining cases are better solved by special methods as follows :

Case IV. 1. *The two imperfect sides are adjacent.* With the algebraic sums of the known latitude and longitude differences, compute the length and bearing of a closing side. This side will form with the two defective sides a triangle in which two sides and one angle are known.[1] The triangle may then be solved, giving the required quantities.

[1] The student should draw a diagram showing this.

2. *The defective sides are not adjacent.* Imagine some of the sides shifted till the defective sides are adjacent and proceed as before. Thus, in Fig. 65, the defective sides are DE and AB. If the side AE is imagined shifted parallel to itself to the position BF, a closing line DF may be computed, and this will form with ED and EF $(= AB)$ a triangle.

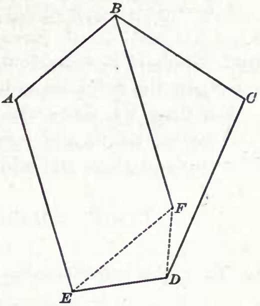


FIG. 65.

Case V. Treat in the same way as Case IV. In the triangle that results there will be known the three sides. This case is indeterminate if the defective sides are parallel, unless the area is known.

Case VI. is solved in the same way, and in this case there is known in the triangle one side and all the angles.

Cases IV. and VI. are not determinate unless enough is known of the field to show which of two angles that correspond to a given sine or cosine is the correct one to use. This will be evident from the following algebraic discussion of Cases IV., V., VI.

138. Algebraic solution. Let the sum of the known latitude differences be denoted by L and the sum of the longitude differences by M. Then for these cases may be written,

$$l_1 \cos \theta_1 + l_2 \cos \theta_2 = - L, \tag{3}$$

$$l_1 \sin \theta_1 + l_2 \sin \theta_2 = - M. \tag{4}$$

In Case IV. let there be wanting l_1 and θ_2. Solving (3) for cosine θ_2 and (4) for sine θ_2, squaring and adding and then solving for l_1, there results

$$l_1 = \pm \sqrt{l_2^2 - M^2 - L^2 + (L \cos \theta_1 + M \sin \theta_1)^2} - (L \cos \theta_1 + M \sin \theta_1).$$

It is evident that there are here two values for l_1. If the proper value may be determined by a knowledge of the field, θ_2 may be found by substitution in either (3) or (4).

For Case V. there will be wanting in (3) and (4) l_1 and l_2.

Solve each for l_1, equate the resulting expressions, solve for l_2, and derive

$$l_2 = \frac{M\cos\theta_1 - L\sin\theta_1}{\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2} = \frac{M\cos\theta_1 - L\sin\theta_1}{\sin(\theta_1 - \theta_2)},$$

from which it is seen that the value is indeterminate when $\theta_1 = \theta_2$, or the defective sides are parallel.

For Case VI. there will be wanting θ_1 and θ_2. Solve equation (3) for $\cos\theta_1$ and equation (2) for $\sin\theta_1$. Square both resulting equations and add, getting

$$L\cos\theta_2 + M\sin\theta_2 = \frac{l_1^2 - L^2 - M^2 - l_2^2}{2l_2}.$$

Let the right-hand member be represented by P. Then

$$M\sqrt{1-\cos^2\theta_2} = P - L\cos\theta_2;$$

square, and solve for $\cos\theta_2$, deriving

$$\cos\theta_2 = \frac{PL \pm M\sqrt{M^2 + L^2 - P^2}}{M^2 + L^2},$$

from which it is evident that there are two values, the correct one to be determined by a knowledge of the survey. In the algebraic solutions, careful attention must be paid to the signs of the trigonometric functions. Since no bearing is greater than 90°, the signs, according to usual trigonometric conceptions, would all be positive; but in equations (3) and (4) those signs must be used that will produce the proper sign for the latitude and longitude differences; thus, for a S.E. bearing, the cosine would be negative and the sine positive.

AREAS.

139. Double longitudes. When the survey has been balanced, the area may be computed. This is usually done by the use of a formula involving the products of the latitude difference and double longitude of each side of the survey. The double longitude of a line is simply twice its longitude, or the sum of the longitudes of its ends. The latitude differences are taken from the table of corrected latitude differences that has been prepared in balancing the survey. There must be found a convenient method for determining the double longitudes. In Fig. 66 let NS be the reference meridian, the longitude differences of the various sides being as shown, with the signs prefixed to each. It is evident that the double longitudes of the first and last courses will be numerically equal to their respective longitude differences, but the sign of the D.L. of the final course will be opposite to that of its longitude difference. The D.L. of any other course, as BC, is the sum of the longitudes of its extremities.

$$\text{D.L. of } BC = bB + cC = d_1 + d_1 + (-d_2).$$

In words, the D.L. of BC is the D.L. of AB plus the longitude difference of AB, plus the longitude difference of BC.

Again, the D.L. of CD is

$$\begin{aligned} cC + (-dD) &= [d_1 + (-d_2)] + [d_1 + (-d_2) + (-d_3)] \\ &= [d_1 + d_1 + (-d_2)] + (-d_2) + (-d_3), \end{aligned}$$

or the D.L. of CD equals the D.L. of BC plus the longitude difference of BC plus the longitude difference of CD.

From the result of this investigation may be formulated the

RULE: *The double longitude of any course is equal to the double longitude of the preceding course, plus the longitude difference of that course, plus the longitude difference of the course itself.*

In applying this rule, due attention must be paid to the signs of the quantities. This rule applies to the first course as well as to the others, if the preceding course is considered to be zero.

To determine the D.L. of the various courses, select one course as the first. Its longitude difference is its D.L. To this add its longitude difference and the longitude difference of the second course, and obtain the D.L. of the second course; to this add the longitude difference of the second course and the longitude difference of the third course, and obtain the D.L. of the third course, etc. The correctness of the work is proved if the D.L. of the final course is found numerically equal to its longitude difference.

With some it is customary to select the most westerly station of the survey as the point through which to pass the reference meridian. This practice makes all of the D.L.'s

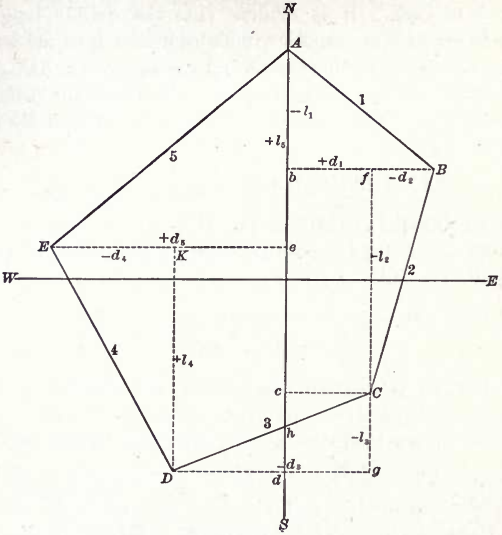


FIG. 66.

positive, and consequently obviates the necessity of considering their signs in the subsequent work of finding areas, and to this extent simplifies the work.

140. To find the area of the field. The area in Fig. 66 evidently equals,

1. The triangles *ABb*, *Cch*, and *EeA*, plus
2. The trapezoids *bBCc* and *EeDd*, minus
3. The triangle *Ddh*.

Beginning with the triangle *ABb* and taking the triangles and trapezoids in order around the figure to the right, there results the following equation :

Area $ABCDE =$

$$\left\{ \tfrac{1}{2}\, l_1 bB + l_2 (bB + cC) + l_3 (cC - Dd) + l_4 (Dd + Ee) + l_5 Ee \right\}:$$

The third term is twice the difference of the triangles cCh and Ddh.[1] This equation may be written, paying attention to the signs of the latitude and longitude differences and remembering that distances to the right of the meridian are positive and to the left negative,

$$\text{Area } ABCDE = \tfrac{1}{2}\{ -l_1d_1 - l_2(d_1 + d_1 - d_2) - l_3(2d_1 - d_2 - d_2 - d_3) + l_4(2(d_1 - d_2) - d_3 - d_3 - d_4) - l_5d_5 \}.$$

It is seen that the coefficient of each of the latitude differences is the double longitude of the corresponding course, whence the

RULE: *To find the area of a closed survey: Multiply the latitude difference of each course by the double longitude of the course, and note the signs of the products. Divide the algebraic sum of these products by* 2.

The sign of the result is of no consequence and depends on the position assumed for the reference meridian and the direction of survey around the field, whether clockwise or counter clockwise.

As stated, the work is simpler if the reference meridian is chosen through the most westerly corner.

141. Irregular areas by offsets. It frequently occurs that one side of a field that is to be surveyed is bounded by a

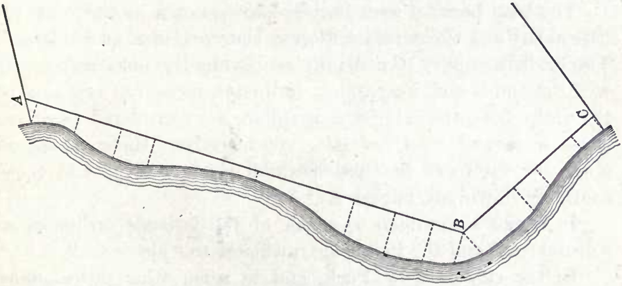


FIG. 67.

stream or the shore of a lake, and that the bounding line is quite irregular and not easily run out.

In Fig. 67 the irregular shore line is a boundary. To

[1] The student should prove this.

determine the area, two auxiliary courses, AB and BC, are run and used with the remaining sides to compute the area on their left. Left and right refer to the direction in which the lines are run. The additional area between these lines and the shore is obtained by measuring offsets normal to these lines. These offsets are measured to the points of change of direction of the shore, and their lengths and distances from A, B, or C, are noted. The area is then computed by considering the portions between the shore and base line and two adjacent offsets to be trapezoids or triangles. When the curve of the shore is such that the offsets may be taken at regular intervals along a base line, the area is found by applying the following rule :

RULE : *To the half sum of the initial and final offsets add the sum of all the intermediate offsets, and multiply the result by the common distance between offsets.*[1]

If the offsets are taken at irregular distances, the area may be found as described in Art. 146.

COÖRDINATES.

142. Definitions. It is becoming common to call the latitude and longitude of a point the "coördinates" of the point.

The base parallel and meridian are known as the "coördinate axes," and their intersection as the "origin of coördinates." The coördinates of the origin are evidently both zero. As with latitudes and longitudes, ordinates measured east, or to the right from the reference meridian, are considered positive; those measured west, or left, are negative; those measured north, or up from the base parallel, are positive; and those south, or down, are negative.

It is not uncommon to speak of the latitude ordinates as "ordinates," and the longitude ordinates as "abscissas."

In the city of New York, and in some other cities, many corners to public property, and many or all private property corners, are located by coördinates, the well-established line of some important thoroughfare being taken as a reference meridian, the origin and meridian being marked by monuments.

[1] The student will be able to show the truth of this rule.

The true meridian through a well-defined point would perhaps do as well, except that, if the subdivision of the city is on the rectangular plan, and not exactly "with" the cardinal points, a great deal of computation would be avoided by making the axes parallel to the street lines.

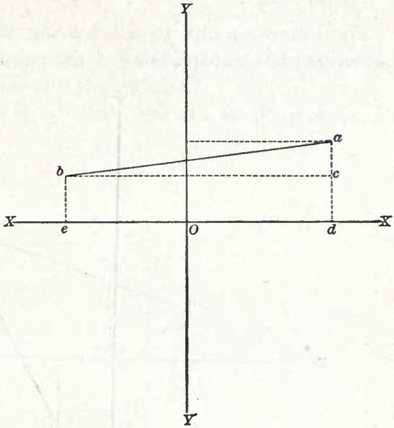


FIG. 68.

143. Elementary problems. A large amount of surveying work is much facilitated by the use of coördinates, particularly by the use of the methods of the two following problems:

I. Given the coördinates of two points, to find the bearing and length of the line joining them.[1]

In Fig. 68 the coördinates of a and b are given. The angle cab is the bearing of ab, but the angle cba will be first found because the smaller angles can be found with greater precision.

$$\tan cba = \frac{ac}{bc}.$$

$$ac = ad - cd = \text{difference of ordinates.}$$
$$bc = Od - (-Oe) = \text{difference of abscissas.}$$

$$\text{Length of } ab = \frac{bc}{\cos cba}.$$

To solve this problem, therefore,

log tan smaller angle = log smaller difference of ordinates *minus* log larger difference of ordinates.

(The angle thus obtained is the bearing when the latitude dif-

[1] The same as supplying a wanting course, Art. 136.

ference is the greater, and is the complement of the bearing when the longitude difference is the greater.)

log length = log greater difference of ordinates *minus* log cos of angle found.

The letters to affix to the bearing may be determined by inspection of the coördinates of the points.

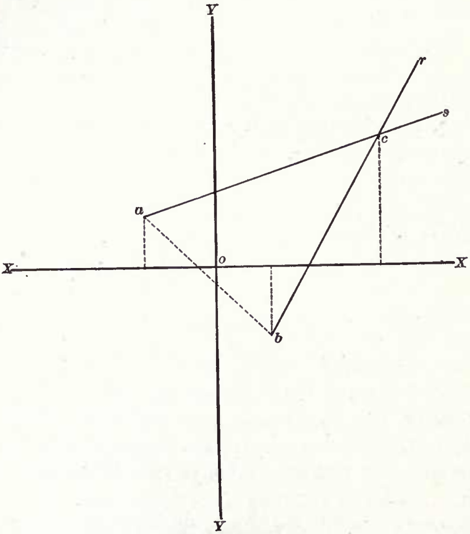


FIG. 69.

II. Given the bearings of two lines and the coördinates of a point on each, to find the location, *i.e.* the coördinates, of the point of intersection of the lines.[1]

The coördinates of *a* and *b* are given, and the bearings of the lines *as* and *br*. By Problem I. find the bearing of *ab* and the *logarithm* of its length. From the known bearings of the three sides of the triangle *abc*, the three angles may be found and the triangle solved for either *ac* or *bc*. From the determined length of either *bc* or *ac* and its bearing, its latitude and

[1] This problem is the same as supplying two wanting lengths.

longitude differences may be found and applied to the coördinates of a or b, according to the line chosen.

It is not necessary to find the length of either side unless the conditions of the particular case require it; only the logarithm need be found.

144. To find the area. Areas are frequently more readily computed by the method of "coördinates."

It will be seen in Fig. 70 that the area of the closed field

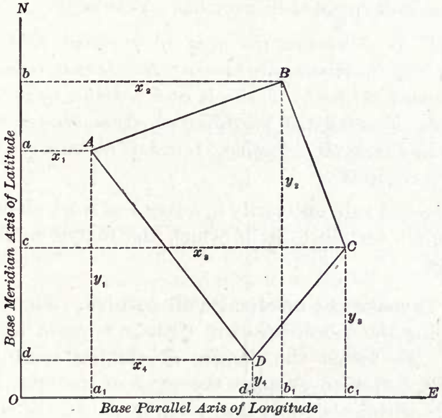


Fig. 70.

$ABCD$ is the sum of the two areas, $bBCc$ and $cCDd$, less the sum of the two areas, $bBAa$ and $aADd$. Expressed in an equation this becomes

$$A = (y_2 - y_3)\frac{x_2 + x_3}{2} + (y_3 - y_4)\frac{x_3 + x_4}{2} - (y_2 - y_1)\frac{x_1 + x_2}{2}$$
$$- (y_1 - y_4)\frac{x_1 + x_4}{2}$$
$$= \tfrac{1}{2}\{y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1 - (y_1x_4 + y_2x_1 + y_3x_2 + y_4x_3)\} \quad (1)$$

This equation expressed in words as a rule is:

RULE: *To determine the area of a closed field when the coördinates of its corners are known: Number the corners consecutively around the field. Multiply each ordinate by the following abscissa and sum the products. Multiply each ordinate by the preceding abscissa and sum the products. One half the difference of the two sums, subtracting the second from the first, is the area of the field.*

Equation (1) may be written,

$$A = \tfrac{1}{2}\{y_1(x_2 - x_4) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_1 - x_3)\}. \quad (2)$$

This equation expressed in words as a rule is:

RULE: *To determine the area of a closed field when the coördinates of its corners are known: Number the corners consecutively around the field. Multiply each ordinate by the difference between the following and preceding abscissas, always subtracting the preceding from the following. One half the sum of the product is the area required.*

The second rule ordinarily involves less work than the first, but there are certain cases in which the first is used to better advantage.

145. To make the coördinates all positive. The method of determining the coördinates will perhaps suggest itself to the student. To lessen the danger of making errors in signs it will be better to arrange the axes so that the field shall lie wholly within the northeast quadrant. All the coördinates will then be positive. This may be done as follows: Determine in the usual way the latitudes and longitudes of the corners of the field with reference to the meridian and parallel through one corner, preferably the most westerly corner. If the most westerly corner is chosen, the longitudes will all be positive without further arrangement. To make the latitude ordinates all positive, add to each latitude a quantity equal to the greatest southern latitude. This will move the reference parallel to the most southern point. If considered more convenient, any round number, as 100, greater than the most southern latitude, may be added. An inspection will ordinarily be sufficient to enable the computer to assume beforehand proper coördinates for the first corner.

146. Elongated areas by offsets. A special case, in which the application of this method of coördinates is advantageously used, is the determination of elongated, irregular areas, the measurements for which consist of offsets at unequal intervals along a straight line. In this case, the line from which the offsets are measured is assumed as the reference parallel. Thus, in Fig. 71, it is required to determine the area between the line *AH*, the irregular line *abcdefgh*, and the two end offsets.

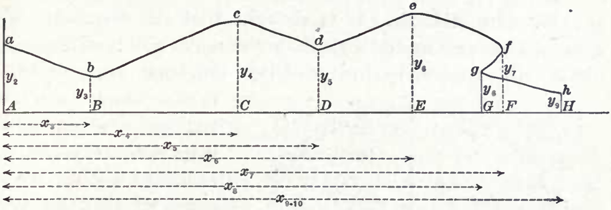


FIG. 71.

sets. The corners of the closed field are *AabcdefghH* and their coördinates are as shown in the figure. x_1 and y_1 and x_2 and y_{10} are zero. The equation that would be written, following the second rule of Art. 144, would be as follows:

$$A = \tfrac{1}{2}\{y_1(x_2 - x_{10}) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_5 - x_3) + y_5(x_6 - x_4) + y_6(x_7 - x_5) + y_7(x_8 - x_6) + y_8(x_9 - x_7) + y_9(x_{10} - x_8) + y_{10}(x_1 - x_9)\}.$$

y_1 and y_{10} being zero, the first and last terms will disappear; x_1 and x_2 being zero, the second term becomes y_2x_3, and the third term y_3x_4. What is, perhaps, a less confusing system of writing these quantities is to write each ordinate and its corresponding abscissa in the form of a fraction, connecting each ordinate with the abscissas whose difference is to be taken as a multiplier. Thus, calling the x's ordinates and the y's abscissas,

$$\frac{-}{y_{10}}\ \frac{x_1}{y_1}\ \frac{x_2}{y_2}\ \frac{x_3}{y_3}\ \frac{x_4}{y_4}\ \frac{x_5}{y_5}\ \frac{x_6}{y_6}\ \frac{x_7}{y_7}\ \frac{x_8}{y_8}\ \frac{x_9}{y_9}\ \frac{x_{10}}{y_{10}}\ \frac{-}{y_1}$$

The downward lines to the right show the following or positive abscissas, and the downward lines to the left show the preceding or negative abscissas. The lines may be omitted as soon as the student becomes thoroughly familiar with the work. This arrangement applies to any closed field and not alone to the elongated strip last described, though it is particularly applicable to that method since the quantities may be arranged for computation as they are taken in the field. Applications of the coördinate method of surveying will be found in the problems, pages 328–335.

147. Zero azimuth. It is thought that the reason for suggesting that zero azimuth shall be the north point, will now be clear. Since north latitude and east longitude are considered positive, and south latitude and west longitude, negative, a system of azimuths should be so arranged that the signs of the trigonometric functions of any azimuth shall agree with the signs of the corresponding latitude and longitude differences. Fig. 72 shows the arrangement of signs of coördinates, and the correspondence of the signs of the trigonometric functions.

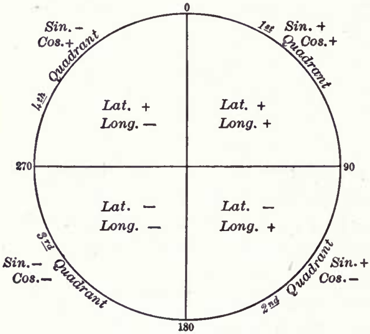


FIG. 72.

There is thus no necessity actually to convert azimuth into bearing in order to determine the signs of the latitude and longitude differences, nor to carry in mind any other than the ordinary scheme of signs given in any work on Trigonometry. The signs of the coördinates could, of course, be changed to suit a south zero azimuth, but the custom among all people to look to the north as the orienting point, and the long use of the signs given for coördinates, seems to make it better for the surveyor to use the north as zero azimuth. Were it merely a matter of changing technical terms used only by the surveyor, such as changing "departure" to "longitude difference," the case would be different.

DIVIDING LAND.

148. Occurrence of the problem. It sometimes becomes necessary to divide a field into two or more parts of equal or known areas. This occurs when one man, as John Jones, sells to another, as Paul Smith, x acres to be laid off in the northeast corner of Jones's field. It also occurs in the division of inherited lands among the heirs, and in the determination of lands sold for taxes. When the taxes are not paid on a given piece of land, the land is sold to the lowest bidder. This means that the land is put up at auction for the taxes and expenses of sale, and that the person who agrees to take the least part of the whole piece, and pay therefor the taxes and expense of sale, is given a title to that portion of the land that he agrees to take. This title is redeemable by the original owner within a certain time, specified by law, after the expiration of which time, if the title has not been redeemed, it becomes vested in the purchaser forever. In any case that may arise the original tract will be fully known, either by previous surveys or by surveys made at the time and for the purpose of the subdivision. It will also be known in what way the land is to be divided, and the problem then becomes simply one of Geometry or Trigonometry.

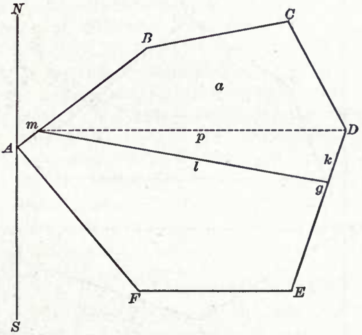


FIG. 73.

149. Solution of the problem. The method of solving two common problems will be given, and others may be readily devised, on which the student may test his ingenuity.

I. It is required to lay off from a given field A acres by a line beginning at a given point in a given side. Plot the field to scale. Let Fig. 73 represent the field so plotted. Let m be the given point. Imagine the line mD to have been run to the corner nearest the probable ending point of the required line mg. The point g on the line DE will first be determined as follows: Consider $mBCD$ as a closed field with length and bearing, or azimuth, of one side, mD, wanting. Find bearing and length of mD, and area of $mBCD$, which call a. Then

$$\text{Area } mDg = A - a.$$

In the triangle mDg, the angle at D is known, and the side mD. The area of the triangle is

$$A - a = \tfrac{1}{2}\, pk \sin D.$$

Whence
$$k = \frac{A - a}{\tfrac{1}{2}\, p \sin D}.$$

The triangle may then be further solved, giving the bearing and length of mg.

II. It is required to lay off from a given field A acres by a line extending in a given direction.

Let Fig. 74 represent the given plotted field. Select a corner of the field, as A, such that from this corner a line may be run in the given direction, cutting off, as nearly as may be, the required area A acres. Let mg be the required line, the area $AFEgm$ being equal to A. If Am were known, the line mg could be run. Since the length l should be computed for a check, and since it is somewhat simpler to determine l first rather than Am, l will be first determined. Imagine the line Ah run in the given direction. Consider $AFEh$ a closed field with two wanting lengths — viz., Ah and Eh. Determine these and the area $AFEh$, which call a. The area $Amgh = A - a$. Then

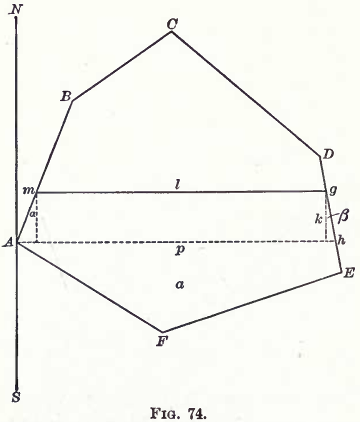


FIG. 74.

$$\frac{l+p}{2}k = A - a, \quad (1) \qquad l = p - k(\tan\alpha + \tan\beta). \quad (2)$$

α and β are known from the bearings.

Whence $$k = \frac{p-l}{\tan\alpha + \tan\beta}. \quad (3)$$

Substituting in (1), $$\frac{l+p}{2} \times \frac{p-l}{\tan\alpha + \tan\beta} = A - a.$$

Whence $$l = \pm\sqrt{p^2 - 2(A-a)(\tan\alpha + \tan\beta)}. \quad (4)$$

l being known, k is determined from (1), or (3), and Am, or hg, from the small right triangles. In the field, find the point m, and run l on the given bearing to its intersection with DE at g. See that the length agrees with the computed length, and that gE as measured agrees with gE as computed.

MODEL EXAMPLES.

150. Logarithms. When formerly, in land surveying, bearings were read only to quarter degrees, there were published for convenience what were known as traverse tables. These were nothing more nor less than tables of natural sines and cosines of the angles from 0° to 90° for every quarter-degree multiplied by 1, 2, 3, 4, etc., to 10, and in some tables to 100. The use of such a table made it unnecessary to multiply the sines and cosines of the bearings to get the latitude and longitude differences, since for each digit in the number expressing the length of the course the differences could be read from the table and brought to the right amount by moving the decimal point. The several quantities were then added.

With modern methods of work the compass is read to the nearest five minutes, and when a transit is used the angles are determined to minutes. Such a table as has been described then becomes useless. The proper table to use is one of logarithmic sines and cosines. A single computation involving not more than two or three figures can perhaps be more quickly performed without the use of logarithms, but any series of computations or a single computation involving five or more figures can be more quickly performed by logarithms. This does not mean that one unaccustomed to the use of logarithms can work with them so fast as without, but a very little practice with them will in any case substantiate the above claim. The student should familiarize himself with a good set of logarithmic tables.[1] The question will arise whether four-place, five-place, six-place, or seven-place tables should be used. The decision must be based on the size of the

[1] The tables of Bremicker or Vega are recommended for work requiring great precision. Gauss or Crockett are recommended for five-place tables.

quantities involved in the computation and the precision required in the result. A four-place table will give results correct to three significant figures and almost correct to four significant figures, probably within one or two units in the extreme end of the table. A five-place table will give results correct to four significant figures and within one or two units in the end of the table, to five significant figures, and so on. For very many surveying computations four-place tables are good enough, but for the general use of surveyors five-place tables are considered better, and for use in connection with very accurate city surveys, six-place tables will not be too extensive, though almost all cases may be solved properly by the use of five-place tables. For general field use five-place tables are ample. Logarithmic tables should have auxiliary tables of proportional parts for quickly getting the logarithm of a number greater than any given in the table and for getting the number corresponding to a logarithm not in the table. Such tables, with proportional parts in the trigonometric functions for tenths of minutes instead of for seconds, will be found on pages 420–464, taken from Crockett's Trigonometry.

151. Example stated. The notes of the courses of a survey are as follows:

N. 69° E.	437.0 ft.
S. 19° E.	236.0 ft.
S. 27° W.	244.0 ft.
N. 71° W.	324.0 ft.
N. 19° W.	183.5 ft.
	1424.5

It is required to balance the survey and determine the area of the field. This example will be worked out in detail as a model for the student. He is advised to note carefully the systematic arrangement of the work, as by such system much time is saved. It is a case from practice.

152. Balancing. Letting L represent latitude differences and M longitude differences, the computation is arranged as shown on page 167.

Explanation. — We first write the logarithm of the length of the course, and above it the logarithmic cosine of the bearing, and below it the logarithmic sine. The logarithm of L is then obtained by adding up, and the logarithm of M by adding down. The L's and M's are then taken out and placed in their respective columns with their signs, and each column added algebraically, giving the result -4.9 in L and $+7.9$ in M. The error of closure is then found. In the example given it is entirely too large, and the field should be rerun. The errors in L and M are now distributed among the courses in proportion to the length of the sides. It is not necessary to be exact about this, and it is done by inspection. Thus $\frac{437}{1424.5}$ is a little less than one third, hence the first corrections are 1.5 in L and 2.5 in M. The other fractions are treated in the same way, the second being somewhat less than one sixth, etc. If the corrections thus determined do not sum up exactly to

COMPUTATIONS FOR THE HARRINGTON SURVEY.

QUANTITY.	LOGS.	L	M	dL's AND dM's.	QUANTITY.	LOGS.
L	2.19481	+ 156.6		$+\frac{43.}{1424.5} \times 4.9 = 1.5$	435.2	2.63872
cos 69°	9.55433	+ 158.1			sin 68° 42′	9.96927
437.0	2.64048				405.5	2.60799
sin 69°	9.97015		+ 405.5		158.1	2.19893
M	2.61063		+ 408.0	$- \quad \times 7.9 = 2.5$	tan 68° 42′	0.40906
L	2.34858	− 223.1		$+\frac{236}{1424.5} \times 4.9 = 0.8$		
cos 19°	9.97567	− 222.3				
236.0	2.37291					
sin 19°	9.51264		+ 75.5			
M	1.88555		+ 76.8	$- \quad \times 7.9 = 1.3$		
L	2.33727	− 217.4		$+\frac{244}{1424.5} \times 4.9 = 0.8$		
cos 27°	9.94988	− 216.6				
244.0	2.38739					
sin 27°	9.65705		− 112.1			
M	2.04444		− 110.8	$- \quad \times 7.9 = 1.3$		
L	2.02319	+ 105.5		$+\frac{324}{1424.5} \times 4.9 = 1.1$		
cos 71°	9.51264	+ 106.6				
324.0	2.51055					
sin 71°	9.97567		− 308.2			
M	2.48622		− 306.4	$- \quad \times 7.9 = 1.8$		
L	2.23931	+ 173.5		$+\frac{183.5}{1424.5} \times 4.9 = 0.7$		
cos 19°	9.97567	+ 174.2				
183.5	2.26364					
sin 19°	9.51264		− 60.7			
M	1.77628		− 59.7	$- \quad \times 7.9 = 1.0$		
		− 440.5	+ 484.8			
		+ 435.6	− 476.9			
		− 4.9	+ 7.9			

$$\text{Error of closure} = \sqrt{4.9^2 + 7.9^2} = 9.4 \pm = \frac{9.4}{1424.5} = \frac{1}{151} - .$$

the respective total corrections, some one or more of them is slightly altered to make the sum correct; thus, in the above example the corrections first written for the fourth L and the fifth M were 1.2 and 1.1 respectively, and these were changed to 1.1 and 1.0 to make the sums equal 4.9 and 7.9 respectively. The balanced L's and M's are now written in the column of L's and M's under the old L's and over the old M's. The lengths and bearings are now inconsistent with the balanced L's and M's, and should be corrected to be consistent. This is done in this example for the first course only. The tangent of the bearing is $\frac{M}{L}$, hence write the log M and subtract the log L and get log tan. Above log M write log sin and, subtracting up, get log length.

This completes the balancing.

153. Areas by latitude differences and double longitudes. From the balanced L's and M's the double longitudes are computed, and from the L's and D's, as we may call the double longitudes, the double areas are computed. The work is systematized as in the following table:

			Logs.	Double Areas.
M of the First course is its $D =$	405.5		2.60799	
$+ M =$	$+ 405.5$	$+ L$	2.19893	
$+ M =$	$+ 75.5$		4.80692	$+ 64110$
Second course $D =$	$+ 886.5$		2.94768	
$+ M =$	$+ 75.5$	$- L$	2.34694	
$+ M =$	$- 112.1$		5.29462	$- 197069$
Third course $D =$	$+ 849.9$		2.92937	
$+ M =$	$- 112.1$	$- L$	2.33566	
$+ M =$	$- 308.2$		5.26503	$- 184089$
Fourth course $D =$	$+ 429.6$		2.63306	
$+ M =$	$- 308.2$	$+ L$	2.02776	
$+ M =$	$- 60.7$		4.66082	$+ 45795$
Fifth course $D =$	$+ 60.7$		1.78319	
		$+ L$	2.24105	
			4.02424	$+ 10574$
				$- 381158$
				$+ 120479$
				2)260679
			43560)	130339.5
				2.992+ acres.

It is believed that the work is self-explanatory.

The results of the foregoing computations are usually tabulated in the following form:

Station.	Course.	Distance, Feet.	Lat. Diff.		Long. Diff.		Balanced.		Double Longitudes.	Double Areas.	
			+	−	+	−	Lat. Diff.	Long. Diff.		+	−
A	N. 69° E.	437.0	156.6		408.0		+ 158.1	+ 405.5	+ 405.5	64110	
B	S. 19° E.	236.0		223.1	76.8		− 222.3	+ 75.5	+ 886.5		197069
C	S. 27° W.	244.0		217.4		110.8	− 216.6	− 112.1	+ 849.9		184089
D	N. 71° W.	324.0	105.5			306.4	+ 106.6	− 308.2	+ 429.6	45795	
E	N. 19° W.	183.5	173.5			59.7	+ 174.2	− 60.7	+ 60.7	10574	
		1424.5	435.6	440.5	484.8	476.9	0.0	0.0		120479	381158
				4.9	7.9						

$$2\overline{)260679}$$
$$43560\overline{)130339.5}$$
$$2.992+ \text{ acres.}$$

$$\text{Error of closure} = \sqrt{\overline{4.9}^2 + \overline{7.9}^2} = 9.4 \pm = \frac{9.4}{1424.5} = \frac{1}{151} -$$

If the field is balanced by weighting the sides, a column of weights and one of weighted lengths are inserted between the third and fourth columns of the table. If the distances are in chains the result is square chains, and is divided by 10 instead of 43560 to reduce to acres.

154. Areas by coördinates. We shall next work out by coördinates the area of the field just determined. The work is the same up to and including the determination of the balanced L's and M's. An inspection of these demonstrates that the first corner is the most westerly corner, and that the fourth corner is the most southerly, and that it is 283.9 feet south of the first. Therefore if it is desired to make all coördinates positive, the reference meridian will be passed through the first corner, and the origin of coördinates will be taken on this meridian 300 feet south of the first corner. The coördinates of the corners and the area of the field are then found as in the table, in which the y's are the latitude ordinates and the x's are the longitude ordinates.

Corner No.	Y	Logs.	Diff. X's	X	Double Areas. Sq. Feet.
1	300.0	2.47712		0.0	
		2.53757	−344.8=2−5		
		5.01469			+103440
	+158.2			405.5	
2	458.1	2.66096		405.5	
		2.68214	+481.0=3−1		
		5.34310			+220341
	−222.3			+ 75.5	
3	235.8	2.37254		481.0	
		1.56348	− 36.6=4−2		
		3.93602			− 8630
	−216.6			−112.1	
4	19.2	1.28330		368.9	
		2.62356	−420.3=5−3		
		3.90686			− 8070
	+106.6			308.2	
5	125.8	2.09968		60.7	
		2.56691	−368.9=1−4		
		4.66659			− 46408
	+174.2			−60.7	
	300.0			0.0	+323781
					− 63108
					2)260673
					43560)130336.5
					2.992+ acres.

Rather more work is required by this method than by the method of double longitudes. This is not the case when the corners have been determined by random lines rather than by a continuous traverse around the field.

When the coördinates of the corners are alone known, the method just given is by far the quickest, since it would be necessary to compute the L's and M's for each side before the D's could be obtained or the areas, just as it was here necessary to compute the coördinates from the L's and M's.

155. Supplying an omission. I. In the example already used, let the bearing and length of the second course be wanting. Adding the original L's and M's, we get

Course.	L	M
1	+ 156.6	+ 408.0
2		
3	− 217.4	− 110.8
4	+ 105.5	− 306.4
5	+ 173.5	− 59.7
	+ 435.6	− 476.9
	− 217.4	+ 408.0
	+ 218.2	− 67.9

There is then to be added south latitude and east longitude in order to close the field, and hence the line to be supplied runs southeast.

The tangent of its bearing is

	Logs.	
$\tan\theta = \dfrac{67.9}{218.2}$	1.83187 + 2.33885	
Log tan θ	9.49302	$\theta = 17° \ 17' +$.
Log sin θ	9.47300	
Log length	2.35887	length = 228.5.

The whole error of closure being thrown into this course, its bearing and length have been materially altered.

II. Let the lengths of sides one and two be wanting. Adding the original L's and M's, we get

Course.	L	M
1	+ 156.6	+ 408.0
2		
3	− 217.4	− 110.8
4		
5	+ 173.5	− 59.7
	+ 330.1	+ 408.0
	−217.4	− 170.5
	+ 112.7	+ 237.5

It is seen that southwest bearing must be that of the closing line. Its bearing and length are obtained as in the last example.

	Logs.	
$\tan\theta = \dfrac{237.5}{112.7}$	2.37566 2.05192	
Log tan θ	0.32374	$\theta = 64° \ 37'$ S.W.
Log sin θ	9.95591	
Log length	2.41975	length = 262.9.

We now have a triangle composed of this closing side and the two wanting sides. This triangle[1] is formed by shifting one of the wanting sides parallel to itself. In this triangle there are known the bearing and length of the closing side just found and the bearings of the two other sides, and hence there are known all the angles and one side.

If the apexes are lettered A, B, C, and the sides opposite the apexes a, b, and c, respectively, and if a is the closing side just found, b side 2, and c side 4, we have from the known bearings

Angle $A = 52° 00'$	Whence	$b = \dfrac{a \sin B}{\sin A} = \dfrac{262.9 \sin 44° 24'}{\sin 52° 00'}$		
" $B = 44° 23'$				
" $C = 83° 37'$		Log 262.9	2.41975	
$180° 00'$		Log sin 44° 23′	9.84476	
			2.26451	
		Log sin 52° 00′	9.89653	
		Log b	2.36798	$b = 233.3.$

Solve for course 4. The other problems are similarly solved.

THE PLANIMETER.

156. Description. The most elegant and rapid method to obtain the area of an irregular figure is to draw the figure to scale and measure the area with a planimeter. There are three kinds of planimeters, shown in Figs. 75, 76, and 77.

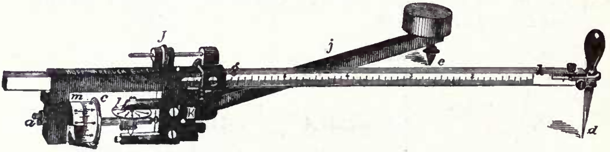


FIG. 75.

Fig. 75 shows the polar planimeter, the most commonly used. Fig. 76 is a suspended planimeter, which is a polar planimeter so arranged that the wheel c, Fig. 75, will roll on a polished surface instead of on the drawing. In Fig. 76 the *axle* of the wheel is turned by contact with a surface. This instrument is more accurate in its results than the polar planimeter. Fig. 77 is a rolling planimeter. This is the most costly and the most accurate of the three forms. Its principle of action is somewhat different from that of the polar planimeter.

[1] The student should draw the triangle.

The polar planimeter, the one most used, consists of two arms, h and j, one of which, j, is of fixed length, and the other is adjustable through the frame shown on the left. There is a

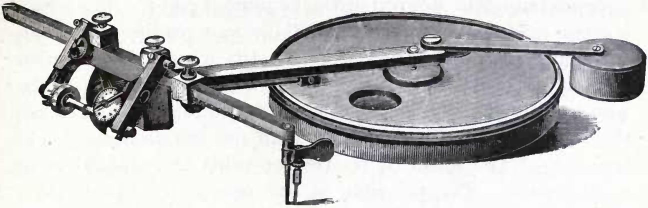


Fig. 76.

clamp back of the point g and a slow-motion screw f for setting the adjustable arm to the required length. The arm j is pivoted at k, and directly over k, on the frame, is a single graduation which indicates the length of the adjustable arm. There

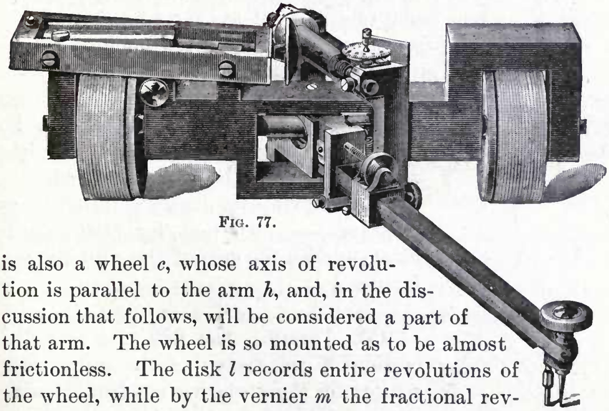


Fig. 77.

is also a wheel c, whose axis of revolution is parallel to the arm h, and, in the discussion that follows, will be considered a part of that arm. The wheel is so mounted as to be almost frictionless. The disk l records entire revolutions of the wheel, while by the vernier m the fractional revolutions are read.

157. Use. The point e is fixed in the drawing board preferably outside of the figure to be measured. The tracer d is then placed on a point in the circumference of the figure, and the wheel *c* is read. The wheel could be set to zero, but this is not easy to do, and hence it is read at whatever it happens to be. The tracer *d* is now moved carefully around the circumference and stopped at the beginning point. The wheel is again read, and the difference of the two readings indicates the number of revolutions made by the wheel. The instrument is so made that if the tracer is moved clockwise, the wheel will roll in the direction of its graduation, and hence the final reading will be greater than the initial reading, and *vice versa*. The wheel by its vernier reads to thousandths of a revolution. The principle of the instrument is such that the distance rolled by the wheel, or the number of revolutions times the circumference in inches, multiplied by the length of the arm in inches, is the area in square inches bounded by the path of the tracer *d*. The arm *h* is generally so set that ten times the number of revolutions is the area. If other units, as tenths of a foot, are used for the above-named linear units, the area will be given in square units of the same kind. In any event, it is the area of the drawing that is measured. If this drawing has been made to a scale of say forty feet to an inch, a square inch of paper is equivalent to sixteen hundred square feet, and the area given by the instrument must be multiplied by this quantity to get the area in square feet. If the fixed point is placed inside of the area to be measured so that the tracer *d* in circumscribing the required area makes a complete revolution about *e*, there must be added to the result in square inches a certain constant area — a constant for each instrument — called the area of the zero circumference.

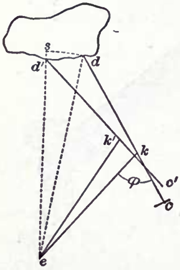


FIG. 78.

158. Theory. The following is a discussion of the theory of the instrument, written for those who are not familiar with the principles of the Calculus.

In the figures that follow, the essential parts of the instrument are lettered as in Fig. 75. The instrument is so constructed that neither *d* nor

c can cross j. In circumscribing an area, the curved path of d may be conceived as divided into an infinite number of infinitesimal portions, each a straight line.

Each of the small portions may be conceived as made up of two parts or component motions, one radial with reference to e, and the other circumferential.

With the radial motion the value of the angle ϕ is constantly changing, while with the circumferential motion the value of ϕ and the length of the line ed remains fixed. In Fig. 78 the component motions sd and sd', and their resultant motion dd', are shown greatly enlarged. It will be evident, that, in circumscribing a closed figure, each minute movement of d *toward* e will have its corresponding movement *from* e with the *same value of* ϕ. Each element of right-hand circumferential motion will have its corresponding element of left-hand circumferential motion; but, since d will be farther from e for one than for the other, these corresponding elements will not be made with equal values of ϕ.

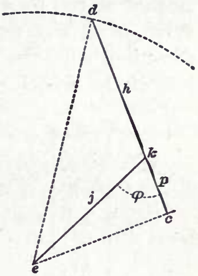


FIG. 79.

When the plane of the wheel c, Fig. 79, passes through e, the angle dce is a right angle, and, if d is revolved about e with ϕ constant, there will be no rolling of the wheel c, because the direction of motion of all points of the instrument about e is circumferential, and the radius of motion of the point c is ec, and this is normal to the axis of the wheel, and hence the motion of the wheel is parallel to its axis, and the wheel simply slips and makes no record. The path described by d for the particular value of ϕ that brings about the above result, is known as the zero circumference. Its radius, ed, may be easily shown to be

$$R=\sqrt{j^2+h^2+2ph}, \tag{1}$$

$$\phi=\cos^{-1}\frac{p}{j}. \tag{2}$$

It will be evident that if d could be moved *outward* till c should fall in the line ek, and then rotated clockwise, the motion of *c* would be all rolling motion, and would be, looking from *c* to *d*, clockwise. The wheel is graduated so as to record positively for this kind of motion. Between these two positions the motion of the wheel will be partly slip and partly roll, the amount of each depending on the value of ϕ; and the roll will all be clockwise. It will be further evident that, if *d* were moved *in* till *c* should fall in the line *ke* produced, and then rotated clockwise, the motion of *c* would be all roll and would be counter-clockwise or left-handed, when looking as before from *c* to *d*. For *d* between the zero circumference and the last-named position, clockwise motion will produce a motion of *c* partly roll and partly slip. The amount of each will depend on the value of ϕ, and the roll will be counter-clockwise. Hence, clockwise motion of *c* will be caused by positive motion of *d* *outside* the zero circumference, and by negative motion of *d* *inside* the zero circumference, and *vice versa;* and, since the amount of roll for a given motion of the tracer depends on the value of ϕ, any two equal infinitesimal motions in opposite directions with the same value of ϕ will produce no resultant roll of the wheel, while, if made with unequal values of ϕ, there will be a resultant roll of the wheel that can be read. For these reasons, the radial components in circumscribing a closed figure cause no resultant rolling of the wheel and may be neglected, while the circumferential components do cause a resultant roll and must be considered. It will be shown that the roll of the wheel for a given circumferential motion of *d* is proportional to the area included between the path of *d*, the radial lines from *e* to the initial and final points of *d*'s path, and the arc of the zero circumference included between those lines.

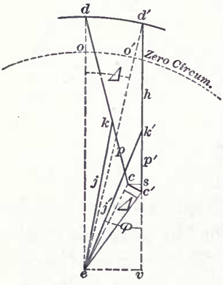


FIG. 80.

Let *dd'*, Fig. 80, be a minute circumferential component of *d*'s motion due to the movement of the instrument about *e* as a center, through the angle Δ, ϕ remaining constant. The wheel will move through the arc *cc'*, and will partly roll and partly slip. The rolling component of its motion will, of course, be normal to its axis, and may be represented by the line cs, csc' being considered an infinitesimal right-angled triangle. Let A be the length in linear units of an arc of radius unity and central angle Δ. (For any other radius X the length of the arc for a central angle of Δ would be X times A.) The angle cec' is Δ, and the arc cc' is given by

$$cc' = \overline{c'e} \cdot A.$$

The roll of the wheel is

$$cs = \overline{c'e} \cdot A \cdot \cos c'cs.$$

The angle Δ being very small, cc' may be considered perpendicular to $c'e$, whence

$$c'cs = ec'v,$$

ev being drawn perpendicular to $d'c'$ produced. Then

$$c'v = \overline{c'e} \cdot \cos c'cs.$$

Whence $$cs = A \cdot \overline{c'v}.$$

Now $$c'v = j \cos \phi - p.$$

Therefore $$cs = A (j \cos \phi - p), \tag{3}$$

which is the roll of the wheel for the motion of d through the arc dd'. To show that this is proportional to the area $dd'o'o$, there must be deduced an expression for that area.

From Trigonometry

$$ed = \sqrt{j^2 + h^2 + 2jh \cos \phi},$$
$$dd' = ed \cdot A.$$

The area of a sector of a circle is the product of one half its arc by its radius, whence

$$\text{Area } edd' = \tfrac{1}{2} A (j^2 + h^2 + 2jh \cos \phi). \tag{4}$$

Using the value of the radius of the zero circumference given in equation (1), there results for the value of the area eoo',

$$\text{Area } eoo' = \tfrac{1}{2} A (j^2 + h^2 + 2\,ph). \tag{5}$$

Subtracting (5) from (4), there results

$$\text{Area } dd'o'o = Ah (j \cos \phi - p). \tag{6}$$

This area is equation (3), the roll of the wheel, multiplied by the length of the adjustable arm, and hence is proportional to the roll of the wheel. Q. E. D.

It is now to be shown that, in tracing a closed area, the record of the wheel is correctly summed. In Fig. 81 let the tracing point move about the area $dd_1d_2d_3d$ clockwise. Motion from d to d_1 will cause a clockwise roll of the wheel proportional to the area $dd_1o'o$. The motion from d_1 to d_2 will be neutralized by motion from d_3 to d. Motion from d_2 to d_3 will cause counter-clockwise roll of the wheel proportional to the area d_2d_3oo', and the resulting roll will therefore be proportional to the

area $dd_1d_2d_3$.

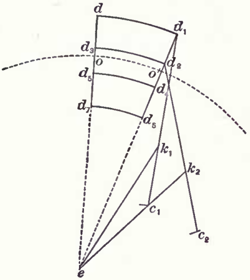


FIG. 81.

The student may reason similarly for the other areas. Since the roll of the wheel is proportional to the area lying between d's path and the zero circumference and is positive, or clockwise, when d is outside the zero circumference and moves to the right, and negative when d is inside and moves to the right, it follows that if an area is traced with e inside that area, so that d must complete a revolution about e, there must be added to the area obtained by multiplying the roll of the wheel by h, the area of the zero circumference. This area is

$$Z = \pi (j^2 + h^2 + 2\,ph).$$

159. To find the zero circumference. This area is usually furnished with the instrument when it comes from the maker, but may be found thus :

Measure a known area with the point e within it and compare the result by the instrument with what is known to be the correct area. The difference is Z. This should be done a number of times, and a mean value of the several determinations used.

160. To find the circumference of the wheel. If n is the number of revolutions, and c is the circumference,

$$\text{Roll of wheel} = nc.$$

In a given circumscribed area with e outside,

$$A = hnc.$$

If c is not known, measure a known area with any convenient length of arm and note the reading of the wheel, which is n. From the known quantities compute c. This should likewise be done a number of times.

161. Length of arm. It is very convenient to make h such a length as will reduce the work of multiplying hnc to a minimum. Most instruments are so made that the length of the arm may be such that

$$A = 10n.$$

To find what this length is for a given instrument in which c is known, let it be assumed that *one* revolution of the wheel shall correspond to ten square inches;

then $$10 = hc,$$

and $$h = \frac{10}{c}.$$

If the arm h is not graduated, it may be set by trial so that $A = 10n$ and the value c will not be required. Some cheap forms of the instrument are made with the arm h fixed in length. When so made they are usually proportioned so that $A = 10n$.

The drawing on which the instrument is to be used should be perfectly smooth

THE SLIDE RULE.[1]

162. Described. The slide rule is an instrument for mechanically performing multiplication, division, involution, and evolution. It is merely a series of scales, which are the logarithms of numbers laid off to scale, so arranged that by sliding one scale on the other the logarithms may be mechanically added or subtracted. The divisions are numbered with the numbers to which the plotted logarithms correspond.

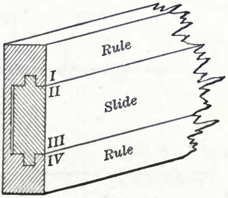


FIG. 82.

The rule is constructed in many forms, but the principles involved are

[1] Written by C. W. Crockett, C.E., A.M., Professor of Mathematics in the Rensselaer Polytechnic Institute.

the same in all. The ordinary rule, about ten inches long, consists of a framework called the *rule* and a movable part called the *slide*, arranged as shown in Fig. 82. On their surfaces, which should be in the same plane, are scales at I and IV on the rule, and at II and III on the slide. The initial points of these scales are in a line perpendicular to the upper edge of the rule. A *runner*, acting on the principle of a T-square, assists in finding points on the scales that are at a common distance from the initial points of the scales.

The slide may be *inverted* — turned end for end — so that II is adjacent to IV, and III to I, or *reversed* so that the other side of the slide becomes visible. One form of the slide rule is shown in Fig. 83.

FIG. 83.

163. Historical. In 1624 Gunter proposed the use of the logarithmic scale, as shown in Art. 165. In 1630 Oughtred suggested that two scales, sliding by each other, could be used. In 1685 Partridge fastened two scales together by bits of brass, another scale sliding between them. In 1851 Mannheim introduced the runner.

164. Construction of the scales. A *logarithmic scale* is one on which the distance from the initial point to any division is proportional to the *mantissa* of the logarithm of the number corresponding to that division. The slide rule usually bears two of these scales, constructed as follows:

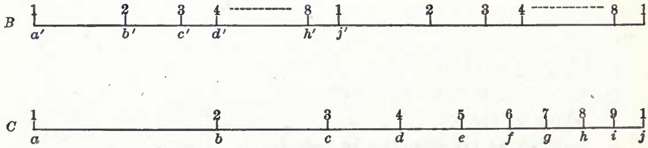


FIG. 84.

On scale B, Fig. 84, let a distance of 5 inches represent unity in the logarithm, so that, if $a'j' = 5''$, we have

log $1 = 0.00$; and the beginning of the scale is marked 1.
log $2 = 0.30$; and at b', so that $a'b' = 1.5''$, we mark 2.
log $3 = 0.48$; " " c', " " $a'c' = 2.4''$, " " 3.
log $4 = 0.60$; " " d', " " $a'd' = 3.0''$, " " 4.
. . . .
. . . .
log $8 = 0.90$; and at h', so that $a'h' = 4.5''$, we mark 8.
log $10 = 1.00$; " " j', " " $a'j' = 5.0''$, " " 1.

This is duplicated on the right of j', so that the total length of scale B is 10 inches.

On scale C, let a distance of 10 inches represent unity in the logarithm, so that, if $aj = 10''$, we have

log $1 = 0.00$; and the beginning of the scale is marked 1.
log $2 = 0.30$; and at b, so that $ab = 3.0''$, we mark 2.
log $3 = 0.48$; " " c, " " $ac = 4.8''$, " " 3.
log $4 = 0.60$; " " d, " " $ad = 6.0''$, " " 4.
. . . .
. . . .
log $8 = 0.90$; and at h, so that $ah = 9.0''$, we mark 8.
log $9 = 0.95$; " " i, " " $ai = 9.5''$, " " 9.
log $10 = 1.00$; " " j, " " $aj = 10.0''$, " " 1.

The initial points, a and a', of these scales are in the same vertical line, and aj is exactly twice the length of $a'j'$.

165. Use of the scales. The following examples will show the use of the scales:

Suppose we wish to find the product of 2 and 3, using scale C. The logarithm of the product is equal to the sum of the logarithms of the two

FIG. 85.

numbers. Then, if with a pair of dividers we lay off $rs = ab = \log 2$ and $st = ac = \log 3$, rt will represent the logarithm of the product, and we find by comparison with scale C that $rt = af = \log 6$.

To divide 6 by 2 we must subtract log 2 from log 6. Lay off with the dividers $rt = af = \log 6$ and $rs = ab = \log 2$. Then st will represent the logarithm of the quotient, and by comparison with scale C we find that $st = be = \log 3$.

166. The Mannheim rule. This is a straight rule, in which the scales I and II are constructed in the way described for scale B, and scales III and IV in that described for C.

The *carpenter's rule* is also a straight rule, in which the scales I, II, and III are similar to B, while IV is similar to C.

The scales on the slide will be denoted by A, so that the arrangement is shown in Fig. 86.

The *Thacher rule*, described in Art. 182, is equivalent to a straight carpenter's rule 720 inches long, the slide bearing only one scale A, similar to B.

This explanation of the slide rule will be confined to the carpenter's rule for two reasons: first, the principles are the same for all the rules, and, second, the explanation will also apply to the Thacher rule, which is extensively used on account of its size and consequent accuracy.

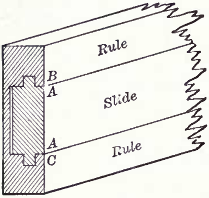


Fig. 86.

A rule may be easily constructed that will be of great assistance in following the explanations. Cut out two pieces of stiff paper or cardboard about twelve inches long, one being two inches wide, to form the rule, and the other one half inch wide, to form the slide. Along the middle of the wider piece draw two lines one half inch apart and lay off a scale B above the upper line, and a scale C below the lower. On both the upper and the lower margins of the narrower piece lay off scales similar to scale B. It will be sufficient to lay off the distances corresponding to the whole numbers from one to ten, the logarithms being taken from any table.

167. Use of the rule. The following illustrations will show the use of the rule:

1. *Multiplication.* To find the product of two numbers, say 2×3, with the rule, set 1 of scale A opposite 2 of scale B, and opposite 3 of scale A

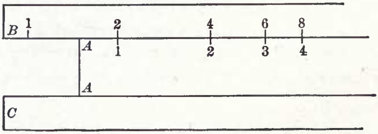


Fig. 87.

find the product 6 on scale B. For in this way to log 2 on scale B we add log 3, found on scale A, giving log 6 on scale B.

If 2 is to be multiplied by any other number, as 4, the same setting is made, and opposite 4 of A we find the product on B.

If any constant number b is to be multiplied by a series of numbers, so that we wish to find the value of bx for different values of x—opposite b of B, set 1 of A and opposite x of A read the product bx on B. This operation may be stated as follows, reading vertically, the lines B and A representing the two scales used:

B.	Opp. b.	Read bx.
A.	Set 1.	Opp. x.

2. *Division.* To divide a number by another, as $8 \div 2$, set 2 of A opposite 1 of B, and opposite 8 of A read the quotient 4 on B. For in this way

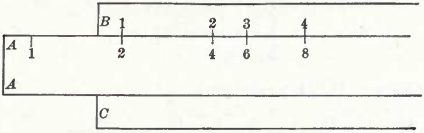


FIG. 88.

we take away log 2 from log 8 on scale A and then find the number on B that corresponds to the remainder.

To divide 6 by 2 we could use the same setting, reading the quotient on B opposite 6 of A.

To divide a series of numbers x by a constant number a, opposite 1 of B set a of A, and opposite x of A read the quotient $x \div a$ on B.

B.	Opp. 1.	Read $x \div a$.
A.	Set a.	Opp. x.

3. *Proportion.* An expression in the form $\frac{bx}{a}$, as $\frac{4 \times 3}{2}$, may be written $\frac{4}{2} \times 3$. Opposite 4 of B set 2 of A, and the index (1) of A will be opposite

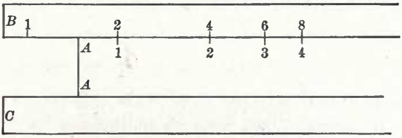


FIG. 89.

the quotient $4 \div 2$ on B; then opposite 3 of A read the result 6 on B, for in this way we add log 3 to the logarithm of the quotient.

If a and b are constants and x a variable, we may use the following setting for $\frac{bx}{a}$, reading vertically:

B.	Opp. b.	Read $bx \div a$.
A.	Set a.	Opp. x.

It will be seen that $\frac{bx}{a}$ becomes bx when $a = 1$, and $\frac{x}{a}$ when $b = 1$. Compare the settings for multiplication and division with that for proportion.

168. Infinite extent of the logarithmic scale. In the common system of logarithms a single mantissa will correspond to a given sequence of figures, the position of the decimal point affecting only the characteristic; and to a given mantissa a single sequence of figures will correspond, the position of the decimal point being fixed by the characteristic. Thus,

$$\log 3 = 0.48;$$
$$\log 30 = 1.48;$$
$$\log 300 = 2.48.$$

If we make $aj = jk = kl = nm = ma$, and if $aj = \log 10$, we have,

FIG. 90.

$$ak = 2 \log 10 = \log 10^2 = \log 100;$$
$$al = 3 \log 10 = \log 10^3 = \log 1000;$$
$$am = - \log 10 = \log 10^{-1} = \log \frac{1}{10} = \log 0.1;$$
$$an = - 2 \log 10 = \log 10^{-2} = \log \frac{1}{10^2} = \log 0.01.$$

Hence, the logarithm of a number between 10 and 100 will be represented by a distance greater than aj and less than ak. The logarithmic scale, therefore, is not merely the distance aj, but it is composed of a series of such parts, extending without limit on both sides of a. Any one of the points n, m, a, j, k, l, and so on, may be considered as the starting point ($\log 1 = 0$); thus, if the scale is supposed to start at n, we would mark 1 at

FIG. 91.

n, 10 at m, 100 at a, 1000 at j, and so on. For this reason the indices of scales B and C are marked 1, and not with multiples of 10.

To illustrate this, suppose we wish to multiply 5 by 8. In Fig. 85, using scale C, make $rs = ae = \log 5$, and $st = ah = \log 8$. Then comparing the distance rt with scale C, we find that rt is greater than aj, the excess being equal to $ad = \log 4$. Hence,

$$\log (5 \times 8) = \log 5 + \log 8 = aj + ad = \log 10 + \log 4 = \log 40.$$

Again, suppose we wish to divide 4 by 8. In Fig. 91, make $st = \log 4$, and $tr = \log 8$. Then, sr is a *negative* mantissa, and

$$-rs = - ms + mr = - 1 + mr.$$

By comparison with scale C we find that mr is the mantissa of log 5, so that the quotient is 0.5.

169. Sequence of figures. In operations with the slide rule, we use the mantissas of the logarithms of the quantities involved, and the distances found represent the mantissas of the results. We find, therefore, the sequence of figures in the result, the position of the decimal point being determined either by special rules or by rough computation.

170. Shifting the slide. As we use the mantissas only, and as the addition or subtraction of the distance between the indices of the scales would affect the characteristic alone, we can change the position of the slide, by bringing one index into the position previously occupied by another index, without affecting the sequence of figures in the result.

171. Use of the runner.

Let us find the value of $2 \times 3 \times 4$. Opposite 2 of B set 1 of A, and 3 of A will be opposite $6 \, (= 2 \times 3)$ on B. Instead of reading this number, set the runner over 3 of A. Bring 1 of A to the runner, and now read the final result 24 on B opposite 4 of A.

In order that the runner may be used, without intermediate reading, it is necessary that the result should be found on the *rule*.

172. Squares and square roots.

The distance on C representing $\log 2 \, (= 3.0'')$ is just twice that representing $\log 2 \, (= 1.5'')$ on B. Hence, if x is any number, $\log x$ on C is twice $\log x$ on B. If the initial points of the two scales are in the same vertical line, any number x on C will correspond to its square, x^2, on B; for

$$\log x \text{ on } C = 2 \log x \text{ on } B = \log x^2 \text{ on } B.$$

Conversely, any number on B will correspond to its square root on C.

1. *Squares.* To find the square of any number, find the number on C, and the corresponding number on B will be its square.

2. *Square roots.* To find the square root of any number, find the number on B, and the corresponding number on C will be its square root. To avoid artificial rules it is necessary to know approximately the first figure of the root. Divide the numbers into sections of two digits each, commencing at the decimal point, as in arithmetic, and call the first section at the left that contains a significant figure the *left section.* If this left section is 10 or greater, the first figure of the root will be 3 or greater, and if it is less than 10, the first figure of the root will be 3 or less. In the carpenter's rule the middle index of B corresponds to 3.16+ on C since

$\sqrt{10} = 3.1622+$. Therefore, if the left section is less than 10, the number whose square root is required is found on the left scale of B, and if greater than 10, it is found on the right scale of B.[1]

173. General statement of problems. If a and b are constants, and x is a variable number, the following expressions may be solved, each with one setting, with the carpenter's or with the Thacher rule, the slide being in the position indicated and the result being read on the scale named:

SLIDE DIRECT.		SLIDE INVERTED.	
EXPRESSION.	READ ON	EXPRESSION.	READ ON
1. $y = \frac{bx}{a}$.	Rule (B).	8. $y = \frac{ba}{x}$.	Rule (B).
2. $y = \sqrt{\frac{bx}{a}}$.	Rule (C).	9. $y = \sqrt{\frac{ba}{x}}$.	Rule (C).
3. $y = \frac{b^2x}{a}$.	Rule (B).	10. $y = \frac{b^2a}{x}$.	Rule (B).
4. $y = \sqrt{\frac{b^2x}{a}}$	Rule (C).	11. $y = \sqrt{\frac{b^2a}{x}}$.	Rule (C).
5. $y = \frac{ax}{b^2}$.	Slide (A).	12. $y = \frac{ba}{x^2}$.	Slide (A).
6. $y = \frac{ax^2}{b}$.	Slide (A).	13. $y = \frac{b^2a}{x^2}$.	Slide (A).
7. $y = \frac{ax^2}{b^2}$.	Slide (A).		

In these expressions either a or b may be equal to unity.

174. Slide direct.

1.
$$y = \frac{bx}{a}.$$
$$\therefore \log y = (\log b - \log a) + \log x.*$$

From $\log b$ we must take away $\log a$, and then add $\log x$ to the difference. Hence $\log x$ must be found on the same scale as $\log a$, the result being read on the scale on which $\log b$ was found (see Fig. 89). Reading vertically, we have

B.	Opp. b.	Read y.
A.	Set a.	Opp. x.
A.		
C.		

[1] Or the number is found on the left scale of B when there is an odd number of digits to the left or an odd number of ciphers to the right of the decimal point; otherwise it is found on the right scale of B.

* The quantity in the parentheses will, in every case, indicate the setting.

Ex. $b=3,\ a=2,\quad x=4,\ y=6;$
$x=6,\ y=9;$
$x=8,\ y=12.$

2. $y=\sqrt{\dfrac{bx}{a}}.$

$$\therefore\ \log y=\tfrac{1}{2}\{(\log b-\log a)+\log x\}.$$

Use the same setting as in the preceding case, but read the result on C instead of on B (see Art. 172).

B.	Opp. b.	
A.	Set a.	Opp. x.
A.		
C.		Read y.

Note that the square roots are always *read* on C.

When a number on C is to be compared with a number on A, it will be more convenient, in the carpenter's rule, to use the scale A that is adjacent to C. Thus, in this case we would find x on the lower scale of the slide. With this caution, the upper scale of the slide will be used in all the settings.

3. $y=\dfrac{b^2x}{a}.$

$$\therefore\ \log y=(\log b^2-\log a)+\log x.$$

Since b is squared, we find b on C, so that the number opposite to it on B is b^2.

B.		Read y.
A.	Set a.	Opp. x.
A.		
C.	Opp. b.	

Ex. $b=3,\ a=2;\quad x=4,\ y=18;$
$x=6,\ y=27;$
$x=8,\ y=36.$

4. $y=\sqrt{\dfrac{b^2x}{a}}.$

$$\therefore\ \log y=\tfrac{1}{2}\{(\log b^2-\log a)+\log x\}.$$

B.		
A.	Set a.	Opp. x.
A.		
C.	Opp. b.	Read y.

5. $y = \dfrac{ax}{b^2}$.

$$\therefore \log y = (\log a - \log b^2) + \log x.$$

B.		Opp. x.
A.	Set a.	Read y.
A.		
C.	Opp. b.	

In this case we must read on the slide; for b^2 and x must both be found on the rule or both on the slide, the latter case being impossible, since the slide does not contain a scale similar to *C*.

Ex. $a = 6$, $b = 3$; $x = 2$, $y = 1.33$;
$x = 4$, $y = 2.67$;
$x = 8$, $y = 5.33$.

Note that $\sqrt{y}$ can not be determined without first reading the value of y, since the slide does not contain a scale corresponding to *C*.

6. $y = \dfrac{ax^2}{b}$.

$$\therefore \log y = (\log a - \log b) + \log x^2.$$

B.	Opp. b.	
A.	Set a.	Read y.
A.		
C.		Opp. x.

Ex. $a = 3$, $b = 2$; $x = 4$, $y = 24$;
$x = 6$, $y = 54$;
$x = 8$, $y = 96$.

7. $y = \dfrac{ax^2}{b^2}$.

$$\therefore \log y = (\log a - \log b^2) + \log x^2.$$

B.		
A.	Set a.	Read y.
A.		
C.	Opp. b.	Opp. x.

Ex. $a = 3$, $b = 2$; $x = 3$, $y = 6.75$;
$x = 6$, $y = 27.00$;
$x = 8$, $y = 48$.

175. Slide inverted. The arithmetical complement of the logarithm (written *colog*) of a number is the logarithm of its reciprocal. The mantissa of the cologarithm of a number is found by subtracting the mantissa of the logarithm from unity. Thus

$$\begin{aligned}\text{mantissa of } \log 2 &= .301030\\ \text{mantissa of } \log 0.5 &= \underline{.698970}\\ \therefore \text{ sum of mantissas} &= 1.000000\end{aligned}$$

Hence, if in Fig. 84 ab represents $\log 2$ on C, bj will represent colog 2, since $bj = aj - ab = \log 10 - \log 2 = 1 - \log 2$.

With the slide inverted. the numbers on the slide increase from right to left, so that the distance from the left index to a number represents the mantissa of the *cologarithm* of that number.

8. $y = \dfrac{ba}{x}$.

$$\begin{aligned}\therefore \log y &= \log b + \log a - \log x\\ &= (\log b - \operatorname{colog} a) + \operatorname{colog} x.\end{aligned}$$

Opposite b of B set a of A, thus subtracting colog a from log b and placing the index (1) of A opposite the product ba on B. Then opposite x of A read y on B.

	B.	Opp. b.	Read y.
Inv.[1]	A.	Set a.	Opp. x.
	A.		
	C.		

Ex. $a = 5$, $b = 12$; $x = 2$, $y = 30.0$;
$x = 4$, $y = 15.0$;
$x = 8$, $y = 7.5$.

9. $y = \sqrt{\dfrac{ba}{x}}$.

$$\therefore \log y = \tfrac{1}{2}\{(\log b - \operatorname{colog} a) + \operatorname{colog} x\}.$$

	B.	Opp. b.	
Inv.	A.	Set a.	Opp. x.
	A.		
	C.		Read y.

[1] Inverted.

10. $y = \frac{b^2a}{x}$.

$$\therefore \log y = (\log b^2 - \text{colog } a) + \text{colog } x.$$

	B.		Read y.
Inv. {	*A.*	Set a.	Opp. x.
	A.		
	C.	Opp. b.	

Ex. $a = 15$, $b = 2$; $x = 3$, $y = 20.0$;
$x = 5$, $y = 12.0$;
$x = 8$, $y = 7.5$;

11. $y = \sqrt{\frac{b^2a}{x}}$.

$$\therefore \log y = \tfrac{1}{2}\{(\log b^2 - \text{colog } a) + \text{colog } x\}.$$

	B.		
Inv. {	*A.*	Set a.	Opp. x.
	A.		
	C.	Opp. b.	Read y.

In the other two cases the result must be read on the slide, so that the distances from the left index (1) will be the mantissas of the *cologarithms* of the corresponding numbers, while the distances on the rule will correspond to the mantissas of the *logarithms*.

12. $y = \frac{ba}{x^2}$.

$$\therefore \text{colog } y = \text{colog } a + \text{colog } b - \text{colog } x$$
$$= (\text{colog } a - \log b) + \log x^2.$$

	B.	Opp. b.	
Inv. {	*A.*	Set a.	Read y.
	A.		
	C.		Opp. x.

Ex. $a = 5$, $b = 12$; $x = 2$, $y = 15.00$;
$x = 4$, $y = 3.75$;
$x = 5$, $y = 2.4$.

13. $y = \frac{b^2a}{x^2}$.

$$\therefore \text{colog } y = \text{colog } a + \text{colog } b^2 - \text{colog } x^2$$
$$= (\text{colog } a - \log b^2) + \log x^2.$$

	$B.$		
Inv.	$A.$ $A.$	Set $a.$	Read $y.$
	$C.$	Opp. $b.$	Opp. $x.$

Ex. $a = 15,\ b = 2;\ x = 3,\ y = 6.67;$
$x = 4,\ y = 3.75;$
$x = 5,\ y = 2.4.$

NOTE. — Expressions similar to (1), (8), and (10), Art. 173, may be solved in such a way that the result may be read on the slide A.

176. Use of the runner in complicated expressions.

If we have found the value of an expression, say $\frac{ab}{c}$, and marked its place on the rule by placing the runner there, then to multiply it by a fraction $\frac{d}{e}$ we bring the denominator e on the slide to the runner and read the result on the rule opposite d on the slide. For this subtracts $\log e$ from $\log \frac{ab}{c}$ and then adds $\log d$ to the remainder.

$$y = \frac{abcde}{ghk} = \frac{ab}{g} \cdot \frac{c}{h} \cdot \frac{d}{k} \cdot \frac{e}{1}.$$

Denoting the runner by R, we have

$B.$	Opp. $a.$							Read $y.$
$A.$	Set $g.$	R to $b.$	h to $R.$	R to $c.$	k to $R.$	R to $d.$	1 to $R.$	Opp. $e.$

EXAMPLES.

$$\frac{4 \cdot 5 \cdot 6 \cdot 7}{2 \cdot 3 \cdot 4} = \frac{4 \cdot 5}{2} \cdot \frac{6}{3} \cdot \frac{7}{4} = 35.$$

$$\frac{5 \cdot 6 \cdot 7 \cdot 8}{2 \cdot 4 \cdot 5 \cdot 6} = \frac{5 \cdot 6}{2} \cdot \frac{7}{4} \cdot \frac{8}{5} \cdot \frac{1}{6} = 7.$$

$$\frac{8 \cdot 9}{2 \cdot 3 \cdot 4 \cdot 6} = \frac{8 \cdot 9}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{6} = 0.5.$$

An expression of the form

$$y = \frac{a(m^2 + n^2 + r + sp + q^3)}{b}$$

may be written

$$y = \frac{a}{b}m^2 + \frac{a}{b}n^2 + \frac{a}{b}r + \frac{a}{b}sp + \frac{a}{b}q^3,$$

and the values of the several terms found separately. Notice that one setting will answer for the first three terms, if the results are read on the slide.

177. Gage points. When a formula is often used, as that for computing the horse power of an engine, it will be found convenient to combine the constant factors, sometimes using the resulting constant, and sometimes its reciprocal. Such a constant is called a *gage point.*

Thus the formula for the horse power of an engine is

$$\text{H.P.} = \frac{p \times 0.7854\, d^2 \times 2s \times r}{33000},$$

where p = mean effective steam pressure in the cylinder in lb. per sq. in.,
d = diameter of piston in inches;
s = stroke in feet,
r = number of revolutions per minute.

But $$\frac{0.7854 \times 2}{33000} = \frac{1}{21008}; \qquad \therefore \text{H.P.} = \frac{d^2psr}{21008},$$

where 21008 is the gage point for the formula

$$\text{H.P.} = \frac{d^2p}{21008} \times \frac{s}{1} \times \frac{r}{1}.$$

B.						Read H.P.
A. *A.*	Set 21008.	*R* to *p*.	1 to *R*.	*R* to *s*.	1 to *R*.	Opp. *r*.
C.	Opp. *d*.					

178. Extraction of cube roots.

$y = \sqrt[3]{b}$. Since $\sqrt[3]{8.000} = 2$, $\sqrt[3]{80.000} = 4+$, $\sqrt[3]{800.000} = 9+$, the cube root of any sequence of figures will depend upon the position of the decimal point, so that the first figure of the root should be known approximately in order that the wrong number may not be taken. This first figure may be found by dividing the given number into sections containing three digits each, commencing at the decimal point, and extracting the cube root of the left section. Then, with the slide *inverted*, opposite b of B, set one of the extreme indices (1) of A, and find the number on C that is opposite the same number on A. This will be the cube root required. Sometimes the right index of A must be used, at others the left index.

	B.	Opp. *b*.	
Inv.	*A.* *A.*	Set 1.	Opp. *y*. or Opp. *y*.
	C.		Read *y*.

179. Slide reversed. The reverse of the slide sometimes has along one edge a scale of logarithmic sines and along the other a scale of logarithmic tangents, the scales being so arranged that the numbers (degrees) increase from left to right when, the slide being reversed, the corresponding edge of the slide is adjacent to B.

180. Scale of logarithmic sines, S.

If we reverse the slide and place S adjacent to B so that their initial points coincide, we find that the beginning of B corresponds to $0° 34' +$ on S, the middle index of B to $5° 44' +$, and the right index of B to $90°$; for

$$\begin{aligned} \log\sin\ 0° 34' + &= 8.00\ (=-2), \\ \log\sin\ 5° 44' + &= 9.00\ (=-1), \\ \log\sin 90° \quad &= 0.00, \end{aligned}$$

unity in the characteristic being represented by the same distance on S and on B.

In this position the natural sine of an angle may be found by reading the number on B corresponding to the angle on S.

The distance from the beginning of S to any division, in the direction of increasing numbers, represents the mantissa, or the mantissa + 1, of the logarithmic sine of the angle corresponding to that division; and the distance from any division to the other end (90°) of S represents the mantissa, or the mantissa + 1, of the cologarithm of the sine.

In expressions containing $\cos x$, since $\cos x = \sin(90° - x)$, we may subtract the angle x from 90° and then use the sine of the remainder, finding it on S.

1. $a = c \sin x$.

$$\therefore\ \log a = \log c + \log\sin x = \log c - \operatorname{colog}\sin x.$$

Opposite c of B set the beginning of S, and opposite x of S read a on B, for this adds $\log\sin x$ to $\log c$. If x should fall outside of the rule, we would set the *right* end of S opposite c of B, and then opposite x of S read a on B, for this subtracts $\operatorname{colog}\sin x$ from $\log c$.

2. $b = c\cos x$.

$$\therefore\ \log b = \log c + \log\sin(90° - x) = \log c - \operatorname{colog}\sin(90° - x),$$

and the methods given for the multiplication of a sine by a number ($a = c\sin x$) are applicable, using $90° - x$ instead of x.

3. $c = a \div \sin x$.

$$\therefore\ \log c = \log a - \log\sin x = \log a + \operatorname{colog}\sin x.$$

Opposite a of B set x of S, and opposite the end of S read c on B; for this subtracts $\log\sin x$ from $\log a$ when we read opposite the left end, and adds $\operatorname{colog}\sin x$ to $\log a$ when we read opposite the right end.

4. $c = b \div \cos x$.

$$\therefore \log c = \log b - \log \sin(90° - x) = \log b + \text{colog} \sin(90° - x),$$

and the methods for $c = a \div \sin x$ are applicable, using $90° - x$ instead of x.

5. $a = c \sin x \div \sin z$.

$$\therefore \log a = \log c - \log \sin z + \log \sin x.$$

Opposite c of B set z of S, and opposite x of S read a on B; for this first subtracts $\log \sin z$ from $\log c$, and then adds $\log \sin x$ to the remainder. If x falls outside the rule, set the runner over the end of S that is in the rule, shift the slide until the other end of S comes to the runner, and then opposite x of S read a on B.[1]

The expression for a becomes $c \sin x$ when $z = 90°$, and $c \div \sin z$ when $x = 90°$. Compare the setting just given with those for $c \sin x$ and $a \div \sin x$.

6. $\sin x = a \sin z \div c$.

$$\therefore \log \sin x = \log \sin z - \log c + \log a.$$

Opposite c of B set z of S, and opposite a of B read x on S; for this subtracts $\log c$ from $\log \sin z$, and adds $\log a$ to the remainder.

This becomes $\sin x = a \div c$ when $z = 90°$.

7. $\cos x = b \div c$.

$$\begin{aligned}\therefore \log \sin(90° - x) &= \log b - \log c \\ &= \log \sin 90° - \log c + \log b.\end{aligned}$$

Opposite c of B set $90°$ of S, and opposite b of B read $90° - x$ on S.

181. Scale of logarithmic tangents, T.

If we reverse the slide and place T adjacent to B so that their initial points coincide, we find that the beginning of B corresponds to $0° 34' +$ on T, the middle index of B to $5° 42' +$, and the right index to $45°$; for

$$\begin{aligned}\log \tan\ 0° 34' + &= 8.00\ (= -2), \\ \log \tan\ 5° 42' + &= 9.00\ (= -1), \\ \log \tan 45° &= 0.00.\end{aligned}$$

Unity in the characteristic, therefore, corresponds to the same distance on T, A, and S.

In this position the natural tangent of an angle less than $45°$ may be found by reading the number on B opposite the given angle on T. To find the tangent of an angle greater than $45°$, invert the slide so that T is adjacent to C, their ends coinciding, and read the number on B corresponding to the complement of the angle on T.

The distance from the beginning of T to any division, in the direction of increasing numbers, represents the mantissa, or the mantissa + 1, of the logarithmic tangent of the angle corresponding to that division; while the

[1] In all cases when the result cannot be read, the slide is shifted over the length of its scales, since this merely changes the characteristic.

distance from that division to the end (45°) of the scale represents the mantissa, or the mantissa + 1, of the cologarithm of the tangent.

Since $\tan x = 1 \div \cot x$, we have

$$\log \tan x = \operatorname{colog} \cot x$$

and

$$\operatorname{colog} \tan x = \log \cot x.$$

Hence the distance that represents $\log \tan x$ also represents $\operatorname{colog} \cot x$, and that representing $\operatorname{colog} \tan x$ also represents $\log \cot x$.

When x is greater than 45°, subtract x from 90° and use the cotangent of the remainder instead of $\tan x$, and the tangent of the remainder instead of $\cot x$.

1. $a = b \tan x$.

$$x < 45°. \quad \therefore \log a = \log b + \log \tan x = \log b - \operatorname{colog} \tan x.$$

Opposite b of B set the end of T, and opposite x of T read a on B; for this adds $\log \tan x$ to $\log b$ when the left end is used, and subtracts colog tan x from $\log b$ when the right end is used.

$$\begin{aligned} x > 45°. \quad \therefore \log a &= \log b + \log \cot\ (90° - x) \\ &= \log b - \log \tan\ (90° - x) \\ &= \log b + \operatorname{colog} \tan\ (90° - x). \end{aligned}$$

Opposite b on B set $90° - x$ of T, and opposite the end of T read a on B; for this subtracts $\log \tan\ (90° - x)$ from $\log b$ when the left end is used, and adds $\operatorname{colog} \tan\ (90° - x)$ to $\log b$ when the right end is used.

2. $b = a \div \tan x$.

$$x < 45°. \quad \therefore \log b = \log a - \log \tan x = \log a + \operatorname{colog} \tan x.$$

Opposite a of B set x of T and opposite the end of T read b on B; for this subtracts $\log \tan x$ from, or adds colog tan x to, $\log a$ when we read opposite the left or the right end respectively.

$$\begin{aligned} x > 45°. \quad \therefore \log b &= \log a - \log \cot\ (90° - x) \\ &= \log a + \log \tan\ (90° - x) \\ &= \log a - \operatorname{colog} \tan\ (90° - x). \end{aligned}$$

Opposite a of B set the end of T and opposite $90° - x$ of T read b on B; for this adds $\log \tan\ (90° - x)$ to, or subtracts $\operatorname{colog} \tan\ (90° - x)$ from, $\log a$ when we use the left or the right end respectively.

3. $b = a \cot x$.

$$\therefore b = a \div \tan x,$$

and the settings given under (2) are used.

4. $b = a \div \cot x$.

$$\therefore b = a \tan x,$$

and the settings given under (1) are used.

182. The Thacher rule. This rule (Fig. 92) consists of a wooden base bearing two upright metallic standards with a large circular opening in each, the line joining the centers of the openings being perpendicular to the planes of the standards. Attached to the standards are two circular plates of metal,

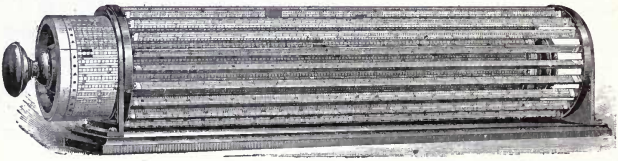


FIG. 92.

each with a large circular opening concentric with those in the standards, so arranged that they can revolve around the line joining the centers of the openings as an axis. These plates are united by twenty bars a little more than 18 inches long, triangular in section, and perpendicular to the plates.

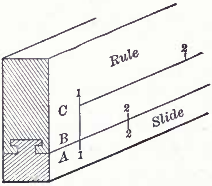


FIG. 93.

These bars are arranged at equal distances around the circular openings, with their vertices outward so that their bases form a cylindrical envelope, the distances between the bars being approximately equal to the width of their bases. A cylindrical slide fits in this cylindrical envelope, moving with either a rotary or a longitudinal motion. The system of bars and plates may be rotated about the axis of the cylindrical envelope without disturbing the relative position of the bars and the slide.

The system of bars is the rule, and the cylinder is the slide. The latter bears only one scale A, while the former contains the scales B and C. The Thacher rule is equivalent to a carpenter's rule in which the slide bears only one of the two A scales. The arrangement of the scales is practically the same as that in a straight rule of the form shown in Fig. 93.

The scale A is laid down on the cylinder as follows:

Let M and N be two logarithmic scales 360 inches long, and let each be divided into forty equal parts, $a, b, \cdots, a', b', \cdots$. Draw forty equidistant elements on a cylinder and lay off on them the segments of the scales as shown in Fig. 95, which represents the development of the cylinder. Then the scales will read continuously from left to right.

The twenty bars are graduated on each side and bear two different

FIG. 94.

scales. The one nearer the cylindrical slide B is constructed in the same way as that on the slide. The outer one C is formed by dividing a logarithmic scale 720 inches long into eighty equal parts and placing them in order above $a, b, \cdots, k, l$; $a', b', \cdots, k', l'$.

The Thacher rule gives results that should never be in error by more than one unit in the fourth significant figure, while the fifth figure can often be found with only a small error. The error should not exceed one part in ten thousand, so that this rule is intermediate in accuracy between a four- and a five-place logarithmic table.

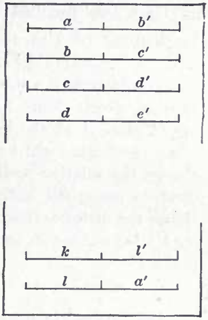


FIG. 95.

183. Settings for the Thacher rule. The settings given for the carpenter's rule will answer for the Thacher rule.[1] All the cases that can be solved with a single setting of the instrument have been mentioned. Problems that can not be solved with one setting may sometimes be easily computed by the use of the runner, and sometimes by computing the different parts of the expression, reading the results, and then combining them with the aid of the rule.

184. Fuller's slide rule. This rule is shown in Fig. 96. The hollow sleeve C, which bears the graduations, is capable of sliding along and revolving about the continuous cylinder HH, the latter being held by the handle attached to it. F is

[1] Mr. Thacher has devised two simple statements that give the settings for all the expressions in Art. 173 with the exception of the third.

a fixed index fastened to H, and by moving the sleeve, any division of the scale may be made to coincide with F. The cylinder H is hollow, forming a guide for the motion of a third cylinder that is attached to the flange G, its axis being coincident with the common axis of C and H. A and B are two indices fixed to and moving with G, the distance AB being

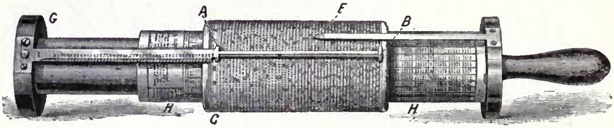


FIG. 96.

equal to the axial length of the scale. This scale, 500 inches in length, is wrapped on the sleeve C in the form of a helix, its beginning being at the end towards G. Stops are provided, so that the indices may be readily made to coincide with the beginning of the scale.

To illustrate the use of the indices,[1] suppose that we wish to find the value of $a \times b$. Move the sleeve C until a is opposite F, and then by moving G, place A at the beginning of the scale; the distance from A to F, along the helix, will be $\log a$. Move the sleeve C, being careful not to change the relative positions of G and H, until b is opposite A; then the distance along the helix from the beginning of the scale to A is $\log b$. Hence the distance from the beginning of the scale to F, along the helix, is $\log b + \log a = \log ab$, so that the product will be found on the scale opposite F.

185. The Mannheim rule. With the Mannheim rule (Art. 166) we can find with one setting the value of any expression in a fractional form with two factors in the numerator and one in the denominator, one of the numbers being variable and one or all of the numbers being squared, and also the value of the square root of such an expression, the result being always read on the rule.

The carpenter's and the Thacher rules do not possess this power, since the slide does not bear a scale similar to C.

[1] This is intended only to illustrate the principle of the instrument.

BOOK II.

GENERAL SURVEYING METHODS.

CHAPTER VII.

LAND SURVEYS.

186. Obstacles and problems. In the field work of the surveyor, various obstacles arise which must be overcome. A few of the more common difficulties, with the methods of surmounting them, will be given.

I. In Fig. 97 it is required to produce the line *XA* in distance and direction.

(1) At *A* erect the perpendicular *AC*, to which erect the perpendicular *CD*, which make long enough to pass the obstacle. At the point *D* erect the perpendicular *DB* equal to *AC*, and at *B* erect the perpendicular *BY*, which is the line produced. The distance *AB* equals *CD*.

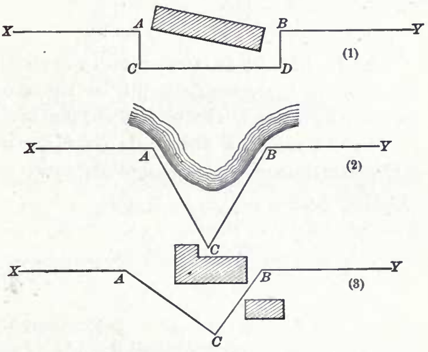


FIG. 97.

(2) At *A* set off an angle of 60° and measure *AC* of such length that a line making an angle of 60° with *AC* will pass the obstacle. At *C* set off from *AC* an angle of 60° and measure *CB* equal to *AC*. At *B* set off the angle *ABC* 60°. The instrument will then be in the line *XA* produced, pointing toward *A*, and the distance *AB* equals *AC* or *CB*.

(3) At *A* set off any convenient angle *BAC* and measure any distance *AC*. At *C* set off any angle *ACB*, making it

90° if convenient, and measure CB, a distance determined by solving the triangle ABC, and at B set off an angle ABC determined in the same way.

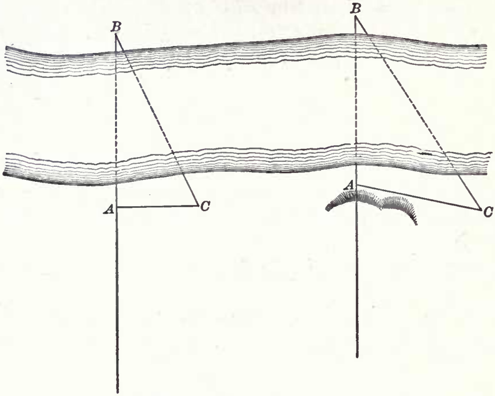


FIG. 98.

II. In Fig. 98 it is required to determine the distance AB, B being an inaccessible point, or the stream too wide to be spanned by a single chain or tape length.

(1) At A set off the angle BAC equal to 90° and measure AC a convenient distance, not so short as to make the angle

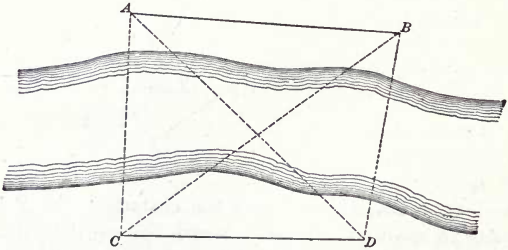


FIG. 99.

ABC less than 20°, unless this is unavoidable, and at C measure the angle BCA. Solve the right-angled triangle.

(2) At A set off any convenient angle BAC and measure any convenient distance AC, observing the previous caution

with regard to the angle at B. Measure the angle at C and solve the triangle ACB.

III. In Fig. 99, it is required to determine the length and bearing of the inaccessible line AB.

(1) Measure the line CD, from each end of which both A and B may be seen, and determine its bearing. With the instrument first at C and then at D measure three angles at each point. Solve the triangle ACD for AC, BCD for CB, and ABC for AB, and an angle at A and B. Another combination of triangles could be used.

IV. In Fig. 100, it is required to determine the length and bearing of the line AB.

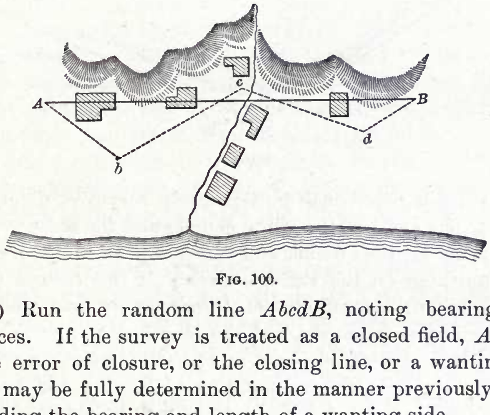


FIG. 100.

(1) Run the random line $AbcdB$, noting bearings and distances. If the survey is treated as a closed field, AB will be the error of closure, or the closing line, or a wanting side which may be fully determined in the manner previously given for finding the bearing and length of a wanting side.

187. Two common problems. I. In Fig. 101, it is required to determine the lengths and bearings of the sides of the field $ABCDEF$, the lines being through woods and no two corners visible, the one from the other. The positions of the corners are, however, known.

Run the random line $AabBcdCefDgEhikFlA$, and compute the latitudes and longitudes, or the coördinates, of the points $ABCDEF$, referred to the true or any convenient meridian, preferably the true meridian through the most westerly point of the survey. From these coördinates determine the latitude and longitude differences and the bearing and length of each course.

II. If a corporation, as a mining company, proposes to purchase a considerable tract of ground containing many small parcels owned by different individuals, and a description of each parcel with its area is wanted, there are two ways of obtaining the information.

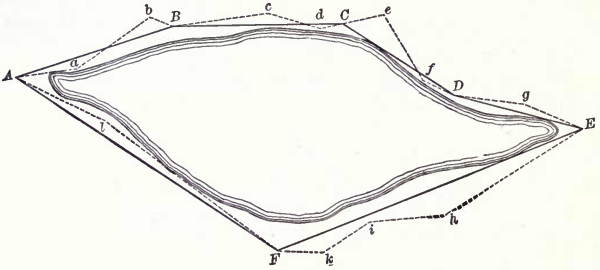


FIG. 101.

(1) Each parcel may be separately surveyed, determining the bearings and lengths of the courses and the areas.

(2) By far the quicker and better way is to run a random field, touching on the various corners of the various parcels, to compute the coördinates of these corners, and from these the bearings, or azimuths, and the length of the sides and the areas. If a separate description of each piece is not required, but only the area, the computation of bearing and length of course may be omitted and the areas may be obtained at once from the coördinates of the corners.

SURVEYING WITH THE CHAIN ALONE.

188. In most surveys the chain or tape is used in connection with some instrument for measuring angles, since, when the sides and angles of a polygon (a field) are known, the polygon may be drawn and its area computed. But it is frequently convenient in approximating to make an entire survey, usually of a very small tract, with the chain alone. This entire survey may be a part of a larger survey, but is, nevertheless, being a closed survey, complete in itself. A method of making such a survey will therefore be described briefly, introducing some methods that are employed as well when the compass or transit is used with the chain.

189. Preliminary examination. Let Fig. 102 represent the map of a farm, a survey of which is desired, and let it be supposed that there is no instrument available except a chain or tape. It will of course be impossible to determine bearings. It is assumed that it is the area that is desired.

The first thing to do in any land survey is to make a rough sketch of the tract to be surveyed, drawing it as nearly as possible in correct proportion, from an inspection made by walking over the field, or from a description of the field taken from the deed, if one can be obtained. From the deed will be obtained only the description of the boundary; and the other features that may be desired must be sketched in the field. On inspection it is found that Mr. Miller owns a farm bounded on one side by the center of the road, on another by the Green River, and on three remaining sides by fences, broken on one side by a pond, which is owned partly by Mr. Miller and partly by his neighbor.

But one convenient way to get the area of the field is known to the surveyor, and that is to divide the field into triangles, measure the sides of the triangles or two sides and an included angle of each, whereupon the area of each may be computed and the whole summed. It is evident in the case of the Miller farm that what might be considered as two sides, those formed by the river, are not straight, and therefore can not be taken as sides of a triangle. Two auxiliary sides, DE and EF, are chosen, lying as nearly as may be parallel to the two sides. It is found impossible in the field to choose the point E so that A may be seen from it, hence it is so chosen that it shall lie on a perpendicular to the line AF drawn through F. This makes the triangle AFE a right-angled triangle, and hence AE need not be measured. If the house were so located that the line BF could be conveniently laid out, it might be measured and the two triangles ABF and BFE might be used instead of AFE and AEB.

190. Survey. To begin the survey, select a point, F, in the center of the road, and, placing a flag in the fence at B, or merely sighting along the fence, if it is straight, locate a point in the center of the road at A, in Ba prolonged. Measure FA. Measure AB. If the fence aB is not straight, or is a

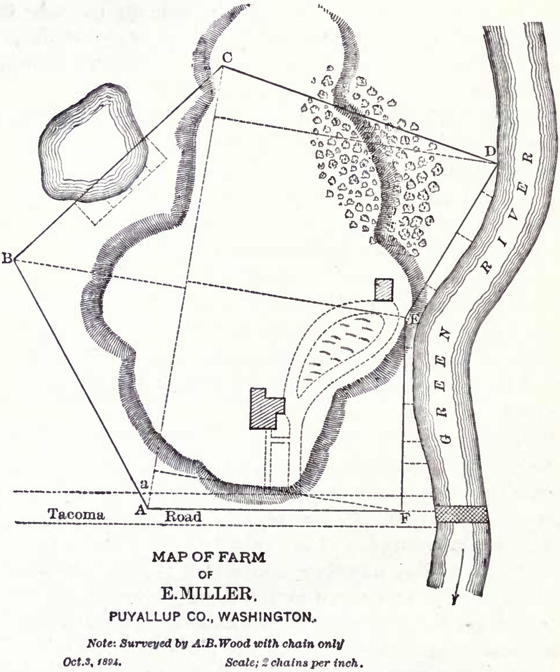


FIG. 102.

rough rail fence, so that it is not convenient to measure on the line AB, measure along a parallel line offsetting a short distance from the fence, making no note of the offset, but recording merely the length AB and the distance from A to a. Measure along the line BC, or on a small offset parallel to it toward C, until the pond is reached. Note the distance, and erect a perpendicular offset long enough to permit a line parallel to BC to pass the pond. At the extremity of this offset erect a perpendicular which will be parallel to BC, and measure along the perpendicular far enough to clear the pond, and then, by a process like that just used, get back on the line CB. When measuring along the parallel line, take offsets to the pond as often as may be necessary, noting the distance to each offset and the length of the offset. Having reached C, measure along the line CD to D, then along DE to E, noting the distance to and the length of each offset that it appears necessary to take to correctly locate the river; and having reached E, measure EF, noting similarly the offsets and also the distance to the fence at the roadside.

Perhaps the ordinary method of procedure now would be to measure the diagonals DB and BE; then the field would be divided into four triangles, ABE, BCD, BED, and AEF, the areas of which could be computed, and to their sum, the area between the lines DE and EF and the river (computed as described in Chapter VI.) could be added, and the area of the entire field would thus be obtained. The line DB would be a difficult one to run, because both D and B being lower than the ground between them, the line must be ranged out, and, moreover, the intervening woods make it even more difficult to determine. If the woods were thick, it could be obtained only by running a trial line and correcting that to the true line. Moreover, the method above described gives no check on the work; and the constant thought of the surveyor should be, "Where can I find a check on my work?" Not more than one check on one piece of work is, however, necessary; though, if more may be obtained without waste of time, they may sometimes prove advantageous.

Having measured the boundary of the field, it is probably best now to measure the line AC, finding by trial the points at which perpendiculars to this line will pass through F, B, E, and D. Measure these perpendiculars, and now four sides of the boundary are the hypotenuses of right-angled triangles, of which the other two sides are known. The other two sides are sides of trapezoids, or may be considered the hypotenuses of right triangles. The computed value of each of the sides of the boundary should agree with its measured length. If it does not, there is an error in the measurement of either the line AC, one or more of the perpendiculars, or one or more of the sides of the boundary. If all is right except one side, the error is in that side. The work should then be remeasured in so far as it may be found wrong. If this latter method is not adopted, a rough check may be had on the former method by measuring the angles made by the sides. These should measure on a drawing the same as found in the field.

The work necessary to obtain the area of the farm is now completed. If it is desired to locate the drives, buildings, and other objects within the inclosure, it may be done by running auxiliary lines at known angles and from known points on the various sides. For instance, a line could be run from E, at right angles to EF, and at stated points on this auxiliary line offsets could be measured to the corners of the objects it is desired to locate; or if it were the driveway, the offsets could be taken at frequent noted intervals to the sides of the drive. As the point D can not be seen from the line AC, a point may be chosen at random, as near as possible to the proper position, and a perpendicular run out to a point opposite D, whereupon the length of the perpendicular and the distance that the point on AC must be shifted to be in its proper place become known.

191. Notes. The measurements should be written on the sketch, which must be made large enough to permit this to be done without confusion. It is believed that no form of notes for such work is so good as a sketch on which all information is written. For the auxiliary lines locating drives, etc., the sketch would consist of a straight line, with distances to offsets marked along it, and offsets and objects offsetted to sketched in with dimensions. These need not be to any scale.

FARM SURVEYS.

192. Classes. All land surveys may be divided into three classes, — original surveys, resurveys, and location surveys. These surveys are made with a compass or transit. The surveys of former years, in the older settled portions of the United

States, were all made with the compass, and almost always with reference to the magnetic meridian.

193. Original surveys. Original surveys are those made for the purpose of mapping a field whose boundaries are marked in some way, for determining its area, and for making a description from which it could be again laid out if the boundaries should be destroyed. Thus, Mr. Brown may own a farm, a portion of which is timber land, which Mr. Black wishes to buy. The boundaries are sufficiently marked by the edges of the growth of timber. After the timber is gone, however, there will be nothing to mark the boundaries, unless the tract is fenced. There is no way in which Mr. Black may know how many acres he is to buy, unless the tract is surveyed. Neither can Mr. Brown give Mr. Black a deed to the property that would contain a definite description from which the tract could be laid out on the ground. A surveyor is called in and shown the tract and asked to make a survey of it, to compute the acreage, and to write a description of the plot.

194. Making an original survey. At the corners that are shown him he sets monuments, preferably of stone. Too many surveyors set merely small stakes, which soon rot or are pulled out. These monuments are "witnessed" by trees, or other natural objects near by, whose positions relative to the corner are observed and noted. The witness points are marked as indicated in Art. 196. The surveyor then determines the bearing and length of each side, usually beginning at one corner and working round the field in one direction till he closes on the corner from which he started. He keeps notes of the work in a notebook, preferably in the form shown on page 229, or on a sketch, which is afterwards "written up" in the form just mentioned, and from the notes he "tables" the survey and computes the area as explained in Chapter VI.

195. Making the map. He may, and usually does, make a map of the survey. This map should conform to the requirements stated in the Appendix, page 355. The easiest and most rapid method of plotting the map is by "latitudes and longitudes." Two reference lines at right angles are drawn, and one is assumed as the meridian and the other as a line of zero latitude. The total latitudes and longitudes of the corners are determined from the tabling work, and are laid off from these base lines. The latitudes are measured from the zero parallel, and the longitudes from the meridian. It will be convenient to make the reference meridian pass through the most westerly point of the survey. This is not necessary. To avoid negative signs for latitude, compute all latitudes as if the reference parallel were through the same point as the meridian; then add to all latitudes a sum equal to the greatest negative latitude; or assume the latitude of the most westerly point to be so large that there shall be no negative latitudes.

When the corners are all plotted they are connected by right lines, and the outline is complete. It remains to number the corners, to write the bearings and lengths along the sides, and to put on the necessary descriptive matter. The work should all be done with the utmost neatness, the lettering being preferably the simple Roman, the most effective and most difficult letter that is made.

196. Description. Having made the map, the surveyor writes a description, somewhat in the following form:

Beginning at a post marked B.1., at the S.E. corner of the land of Joseph Brown, from which post a hard maple tree, 8 inches in diameter, bears S. 10° W. 10 links, and a white ash, 12 inches in diameter, bears N. 70° W. 50 links, both of which trees are blazed and marked B.1.B.T. (Black 1 Bearing Tree), and running thence N. 10° 30′ W. along the easterly line of said Joseph Brown, six and forty-two one-hundredths ($6\frac{42}{100}$) chains to a post marked B.2., from which post a hickory tree, 10 inches in diameter and marked B.2.B.T. bears S. 68° W. 30 links; thence N. 84° W. seven and fifty one-hundredths ($7\frac{50}{100}$) chains to a stone about 12 inches long and 6 inches square, set flush with the ground and marked B.3., from which stone a beech tree, 15 inches in diameter and marked B.3.B.T. bears, etc.,.. thence, etc., to the point of beginning, containing .... acres, more or less.

The surveyor usually does his tabling and writes his descriptions in a book which he keeps for the purpose, for future reference. Copies are made, and, with a tracing of the map, are furnished the person for whom the survey is made. This is the simplest kind of land surveying.

197. Resurveys. These are far more difficult than original surveys. A resurvey consists in tracing on the ground an original survey, from a description similar to that just given. The difficulties arise from the destruction of monuments and from errors in original work, from change in declination of the needle (if the surveys were run by the magnetic compass and referred to the magnetic meridian), from insufficient data in the original description, such as failure to state whether the survey is referred to the true or magnetic meridian, failure to state the declination on which the survey was made, lack of bearing trees or other reference points, etc., and from conflicting testimony of interested owners as to where the corners were. It is impossible to point out all the difficulties that will be encountered by a surveyor in his attempt to reëstablish the monuments of an old survey. The more of this work that he does, the more firmly will it be impressed upon him that the only kind of corners to establish are those that will be as nearly as possible permanent, and that minuteness of detail and accuracy in all descriptions are well worth the time they take.

198. Reasons for a resurvey. A resurvey becomes necessary for various reasons. Among others might be the following, referring to the original survey previously described: After Mr. Black has owned the wood lot for a number of years, and perhaps has cleared it, and it has descended to his son and his son's son, and the corners are mostly obliterated, it is sold to Mr. Johnson who, desiring to fence it off, and, moreover, to see whether the land he is paying for is all there, employs a surveyor to "run it out," giving him the description written by the original surveyor as he finds it in his deed, with possibly some errors in copying.

199. Procedure. If the surveyor can recover a single line of the original survey, and the notes are correct, his work will be comparatively easy. (He simply has to establish a series of lines of given bearing and length.) He therefore endeavors to do this. If this can not be done, the next best thing is to recover any two corners and determine in the field the bearing and length of the line joining them. From the original notes he will then compute the bearing and length of this line, and the angle made with it by one of the sides of the field joining it, and he then can lay off this angle from his field-determined line and locate that side. As there will be four sides joining the line between the two corners, it will be seen that the surveyor has a good beginning for his work.

200. Change of declination. It might seem that if he could find but one corner, he could run out the field, knowing the bearings. This would be true if he knew also the declination on which the original survey was made and the declination at the time of his resurvey.

The following consideration will explain this: Let it be assumed that the original survey was made with reference to the magnetic meridian and that the declination at that time was east 10°. Then, any line that is recorded due north will be 10° east of north. Any line recorded due south will be 10° west of south, etc. That is, all the points of the compass are turned 10° to the right. Let it be assumed that the declination at the time of the resurvey is 8° east. A line recorded as due north will be 8° east of north, and a line recorded as due south will be 8° west of south. The points of the compass have been turned back two degrees to the left. And hence, if a line were run out with the original bearing, it would lie two degrees to the left of the true place. It would be necessary to run the line with a bearing two degrees to the right of its originally recorded bearing. Thus, if its original bearing were north, it must be rerun with the needle reading N. 2° E. Hence, the following rule is serviceable:

RULE: *To rerun lines recorded by their magnetic bearings, change the bearings by an amount equal to the change in declination and in a direction opposite to that change* — that is, *left or right.*

If the compass has a declination vernier, that vernier may be set so that the compass, when the line of sight is pointing in a known bearing, as magnetic north, shall read a bearing as many degrees to the right or left of the known magnetic bearing as the declination has changed to the right or left. It must be remembered that, in using the declination vernier, the compass box moves with reference to the line of sight. In the above case it should read N. 2° W., when the sights are pointing to magnetic north. Hence, to be able to retrace the survey with a compass, using the original bearings, the following rule is serviceable :

RULE : *Set the declination vernier so that the compass shall read magnetic bearings erroneously by an amount equal to and in the direction (left or right) of the change in declination.*

To find the change in declination, if it is not known, determine the bearing of a line of the original survey and compare this with the bearing recorded. The difference is the change in declination in amount and in opposite direction (left or right).

If one line can not be found, but two corners may be, connect the two corners by a random line as described in Art. 186. Find the bearing of the closing line and compare it with the bearing of the same line computed from the original notes.

201. Transit or compass. When a transit is used, or indeed a compass, the angles at the corners of the field may be determined from the recorded bearings and, when one side has been recovered, the angles and distances may be measured, to recover the other sides. This does away with all consideration of, or change in, declination. It is by far the better method to pursue with lands of any considerable value. The angles determined from compass readings can not be depended on to minutes, and may be found to vary five or ten minutes. It would be well if, when a transit is first used on a resurvey, the corrected bearings and distances could be introduced in a correction deed which could be filed with the proper authority. If angles are to be used, and one side can not be directly recovered, but a closing line joining two corners is determined, proceed as follows: Over one known corner, set the line of sight in the closing line mentioned, and turn off an angle determined, by computation from the description, to be the angle between this line and a side adjacent to the corner occupied.

This should give the direction of that side. Measure the recorded length of that side, and look about over a considerable area for evidences of the corner. Look for recorded witness trees, or their stumps. Carefully shovel off the top of the soil — do not dig it up by spadefuls. A careful shoveling will frequently reveal the hole formerly occupied by a stake, now rotted entirely away, with evidences of the decayed wood. Continue the work till all corners are found or satisfactorily relocated. *The only thing that can absolutely insure the correctness of the resurvey is the finding of the old corners.*

202. Report. Upon the completion of the resurvey, the surveyor should report to his employer just what he finds. It is not his business to decide controversies. He may advise, just as an attorney would do; but he has no authority to correct errors or to establish corrected corners as *the* corners.[1] His business is to make an examination, to reset lost corners in their original positions, if he can find them, and to report his method of procedure and the reasons for his action to his employer, who may then take such action as he chooses. A neat and explicit map should accompany the report. The surveyor may possibly be assisted in his work by an understanding of the principles of Art. 204, deduced from many court decisions.[2]

203. Application of coördinates. In all of this work the coördinate system is very helpful. Usually the true corners of a tract of land can not be occupied by the instrument, nor can the lines be seen throughout their length. If it is known where the corners are, and it is desired to make a survey for a map or a description, a random survey is made with corners as near as practicable to the true corners. . This survey is balanced and the coördinates of its corners are determined. From such of the corners as are near true corners, angles or azimuths,

[1] See "Judicial Functions of Surveyors," by Judge Cooley, Appendix, pages 341–350.

[2] These and many other decisions may be found in a valuable "Manual of Land Surveying," by Hodgman and Bellows.

and distances to the true corners are noted, and from these the coördinates of the true corners are determined. From these coördinates, lengths and bearings of true lines may be found.

If the survey is for the purpose of relocating lost corners, and the bearings of the lines and at least one point in each are known, the corners may be located even though the lines can not be occupied by the instrument, provided the corners and known points are accessible. This would be done by running a random survey as before with pointings to the known points (and such corners as are known) then finding the coördinates of the known points, and by Problem II., Art. 143, the coördinates of the wanting corners. Knowing now the coördinates of the points in the random survey and those of the true corners, by Problem I., Art. 143, find the bearing and distance from a corner of the random to the nearest wanting true corner, and locate the corner.[1]

204. Principles for guidance in resurveys. I. *Construing descriptions.* The following principles have been applied to the construing of descriptions that are inconsistent, obscure in meaning, or imperfect.

(1) The description is to be construed favorably to the purchaser, unless the intent of both parties can be certainly ascertained. If that intent can be ascertained, the description will be construed accordingly.

(2) The deed must be construed according to the conditions existing, and in the light of the facts known and in the minds of the parties, at the time the instrument was drawn.

(3) Every requirement of a description must be met, if possible. Nothing is to be rejected if all requirements are mutually consistent.

(4) If some parts are evidently impossible and, by rejecting such parts, the remainder forms a perfect description, such impossible parts may be rejected.

(5) A deed is to be construed so as to make it effectual rather than void.

[1] For model example, applied to city property, see Appendix, page 328. The student may tell how to proceed if the bearings and lengths are all given, with but two corners known, and lines so grown over or occupied by structures that they can not be run out directly without great labor,

(6) If land is described as that owned and occupied by an individual, the actual line of occupation is a requirement or call to be met in the location.

(7) A line described as running a definite distance to a definite known line or object, will be construed as running to that object, whatever distance is required. If the known object is uncertain as to position, the written distance may be used.

(8) The terms "northerly," "southerly," "easterly," and "westerly," are to be construed, in the absence of other information, as meaning due north, south, east, and west.

(9) When a definite quantity of land is sold and nothing appears to indicate its form — as, for instance, ten acres in the northeast corner of B's land, the land will be laid out as a square, unless this is manifestly impossible.

(10) A description by "metes and bounds" will convey all the land within the boundaries, be it more or less than the area mentioned in the deed.

(11) Property described as bounded by a highway extends to the center of the highway, unless specifically noted otherwise.

(12) A description by metes and bounds, followed by a statement that the land described is a particular well-known parcel, will be construed to convey the well-known parcel, though the metes and bounds do not fulfill the necessary conditions.

II. *Water boundaries.* (1) Local laws of different states give different constructions to the word "navigable" and the surveyor must examine the laws of the state in which he works. The United States statutes provide as follows for the streams within the area known as the public lands :

"All navigable rivers, within the territory occupied by the public lands, shall remain and be deemed public highways; and, in all cases where the opposite banks of any streams not navigable belong to different persons, the stream and the bed thereof shall become common to both."

(2) Grants of land bordering on navigable streams carry only to high-water mark, while on non-navigable streams they carry to the center, or "filum aquæ."

(3) The common law holds those streams only to be navigable in which the tide ebbs and flows. The civil law considers a stream navigable that is capable of being used as a commercial highway.

The courts of Pennsylvania, North Carolina, South Carolina, and Alabama follow the civil law, while those of Maine, New Hampshire, Massachusetts, Connecticut, New York, Maryland, Virginia, Ohio, Illinois, Indiana, and Michigan follow the common law.

(4) The bank is the continuous margin where vegetation ceases. The shore is the sandy space between it and low-water mark.

(5) A description reading "to the bank," or "along the bank," is construed to mean "the bank," and to include no portion of the stream.

(6) Islands in rivers fall under the same rule as the land under water, and belong to one adjoining proprietor or the other — unless previously lawfully appropriated — according as they are on one side of the center or the other. The filum aquæ is midway between lines of ordinary low-water mark, without regard to the position of the main channel.

(7) Riparian rights, unless expressly limited, extend to the middle of the navigable channel.

(8) In some states the tide lands are held to belong inalienably to the people of the state and may not be sold to individuals. In others a different policy has been pursued.

(9) A boundary by the shore of a millpond carries to low-water mark.

(10) Boundary lines of lots fronting on a river extend into the river at right angles to the thread of the stream, without regard to the form of the bank.

(11) Land made by the drying up of a lake or the deposit of alluvium along a river accrues to the adjacent owners and should be so distributed among them that each will receive such a portion of the made area as his former frontage on the water was of the entire former frontage. If, however, the water front is the valuable item, as it would be along a navigable river or lake in a city, the new frontage is to be distributed according to the old frontages.

III. *Special field rules.* (1) Monuments control courses and distances. That is, if the location of an original monument can be certainly ascertained and the recorded distance does not reach that monument, the line must, nevertheless, be run to the monument. In the absence of sufficient evidence to determine the monument, the description will govern. (The surveyor should use every effort to find evidence as to the location of the lost corner.)

(2) Adverse possession of land for a definite period of time (varying in different states), even without color of title, constitutes title in fee; but the possession must be adverse; that is, the true line must be known to the parties or the line of occupation must be acquiesced in by them. If the true boundary is unknown, and each claims to own only to the true line, no adverse possession can arise.

(3) Boundaries and monuments may be proved by any evidence that is admissible in establishing any other facts.

(4) A resurvey after original monuments have been lost is for the purpose of finding where they *were*, and not where they *should have been.*

(5) Purchasers of town lots have a right to locate them according to the stakes which they find planted and recognized, and no subsequent survey can be allowed to unsettle them. The question afterwards is not where they should have been, but where they were planted with authority, and where lots were purchased and taken possession of in reliance on them.

(6) Of two surveys that disagree, made many years apart — the monuments being lost — the original survey will be preferred, particularly if the line of the first survey has remained unquestioned for many years.

(7) When streets have been opened and long acquiesced in, in supposed conformity to a plot, they should be accepted as fixed monuments in locating lots or blocks contiguous thereto.

(8) A beginning corner is of no greater dignity or importance than any other corner.

(9) A call for a lot by a name or number that it bears on a mentioned plot will prevail over courses and distances, and sometimes over monuments.

205. Location surveys. These consist in laying out on the ground lines previously determined by computation or drawing. Such surveys are not infrequently connected with either original surveys or resurveys. Surveys for the partition and division of land are of this class. If Joseph Brown sells to John Black five acres of land in the shape of a square, one corner and the direction of one side being fixed, the location of the sides and corners would be a location survey. Such surveys are comparatively simple, involving sometimes the running of one or more trial lines for data to compute the location. Many complex problems, however, arise.

UNITED STATES PUBLIC LAND SURVEYS.

206. Value and character of work. The original surveys for the subdivision of the public lands of the United States are location surveys. The method adopted, imperfectly as the work has been done, has been of incalculable value in definitely describing each separate tract or parcel of land sold to individuals, and in providing that lines once established by the United States deputy surveyors shall remain as the lines they purport to be, even though found to be improperly placed. The latter provision has been of particular value in making the land lines permanent. The work of subdividing the public lands is almost completed, and hence the work of the surveyor of the future will be largely resurveying, that is, relocating corners that have been or are supposed to be lost, and dividing into smaller parcels the areas already located.

If the original surveys had been properly executed, the work would not be difficult. In some instances the work was most wretchedly done, either willfully or through ignorance; and the corners, when established, were placed far from their proper positions. The notes have generally been returned in a form indicating correct work; and hence has arisen more or less difficulty in relocating the corners.

207. General scheme of subdivision. Certain points have been selected in different parts of the country through which true north and south lines, called "principal meridians," have been run and marked out on the ground. Intersecting these principal meridians at the initial point are run parallels of latitude, known as "base lines." On either side of these meridians the land is laid out in approximately square parcels, six miles on a side, called "townships." A tier of these townships running north and south is called a "range." The townships are described as being "Township No. —— south or north of a named base line and range No. —— east or west of a named principal meridian." This definitely locates every township.

The lines that divide the ranges are called "range lines," and those that divide a southern from a northern township are called "township lines." Each township is further divided into thirty-six "sections," each approximately one mile square. These sections are numbered from one to thirty-six. Each section, then, is definitely located by its number, township, and range. The sections have been further divided into quarter sections as, the northeast quarter, the southwest quarter, etc. Sometimes the sections have been divided into halves, described as the north half or the east half, etc. The Government does not divide the land into smaller divisions than quarter sections, but it sells less areas than this, and, in such cases, and when original purchasers sell a portion of their purchase, it is usual to sell a quarter of a quarter, or half of a quarter, or even a quarter of a quarter of a quarter section; and the method of describing these fractional portions is the same as that used to describe the quarter sections. The description would be written as follows for the piece described: The N. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$ of Sec. 26, Tp. 8 N., R. 4 E., Mt. Diablo Meridian.

The positions of the meridians being chosen more or less at random and at different times, as the necessity for surveys in different localities developed, it is to be expected that the surveys extending east from one meridian will not close with regular full sections or townships on the surveys extended west from the next easterly meridian. The same is true of tiers of townships extended north and south from adjacent base lines. The result is fractional townships and sections. Sometimes these are larger than the standard division, sometimes smaller. If very much larger, the surplus in a township is divided into lots which are numbered. These lots are made to contain as nearly as possible 160 acres. There are other circumstances that will appear, that cause a departure from the ordinary method of subdividing and describing.

The work in a given state is under the direction of a United States surveyor general. The work has all been done by contract, a surveyor taking a contract to perform a definite portion of work for a specified sum per mile. This has been the principal cause of much bad work.

The lands classified as public lands and subdivided according to the method outlined include all land north of the Ohio River and west of the Mississippi River, except Texas, and including Mississippi, Alabama, and Florida, except in all the above-named territory such lands as belonged to individuals at the time the territory became a part of the United States.

The public lands of Texas are, by a provision of the laws admitting Texas into the Union, the property of the state. No general scheme for their subdivision has been developed. They have been sold in parcels as nearly square as may be. The Spanish vara is the unit that has been adopted for measurement. The vara is, in Texas, $33\frac{1}{3}$ inches. There are 5645 square varas in an acre.

N

	36	30	24	18	12	6	
	35	29	23	17	11	5	
	34	28	22	16	10	4	
W	33	27	21	15	9	3	*E*
	32	26	20	14	8	2	
	31	25	19	13	7	1	

S

FIG. 103.

The following is very much condensed from the "Manual of Surveying Instructions" issued by the General Land Office in Washington.

208. Historical note. The first surveying of the public lands was done in Ohio under an act passed by Congress in 1785. The territory included in this early survey is now known as "The Seven Ranges." The townships were divided into thirty-six sections one mile square, numbered consecutively, as in Fig. 103.

In 1796 the method of numbering sections was changed to that shown in Fig. 104, and this method is still in use.

An act of 1805 directs the subdivision of public lands into quarter sections, and provides that all the corners *marked* in the public surveys shall be established as the proper corners of sections which they were intended to designate, and that corners of half and quarter sections *not marked* shall be placed, as nearly as possible, "equidistant from those two corners which stand on the same line." This act further provides that "the boundary lines *actually run* and *marked* . . . *shall be established* as the *proper boundary lines* of the sections or subdivisions for which they were intended; and the length of such lines *as returned by* . . . *the surveyors shall be held and considered as the true length* thereof. . . ."

N

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

W *E*

S

FIG. 104.

An act of 1824 provides "that whenever, in the opinion of the President of the United States, a departure from the ordinary mode of surveying land on any river, lake, bayou, or water course would promote the public interest, he may direct the surveyor general in whose district such land is situated . . . to cause the lands thus situated to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or water course, and running back the depth of forty acres."[1]

An act of 1820 provided for the sale of public lands in half-quarter sections, the quarters to be divided by lines running north and south; and an act of 1832 provided for the sale of the public lands in quarter-quarter sections, and that the half sections should be divided by lines running east and west. The latter act also provided that the secretary of the treasury should establish rules for the subdivision of fractional sections.

209. Legal requirements inconsistent. Existing law requires that the public lands be laid out in townships *six miles square* by

[1] Let the student determine the width and depth in chains of such a strip. This provision is carried out where the water front rather than area is the valuable item.

lines running due north and south, and others east and west, also that the township shall be divided into thirty-six sections, by two sets of parallel lines, one governed by true meridians and the other by parallels of latitude, the latter intersecting the former at right angles, at intervals of one mile; and each of these sections must contain, as nearly as possible, six hundred and forty acres. These requirements are manifestly impossible because of the convergency of the meridians, and the discrepancies will be the greater as the land divided is farther north. The law also provides that the work of subdivision shall be so performed as to throw all shortages or surplus into the northern and western tiers of sections in each township. To harmonize these various requirements as much as possible, the following methods have been adopted by the general land office.

210. Principal reference lines. Initial points are established astronomically under special instructions. From the initial point a "principal meridian" is run north and south. Through the initial point a "base line" is run as a parallel of latitude east and west.[1] On the principal meridian and base line the section and quarter corners, and meander corners at the intersection of the line with all streams, lakes, or bayous, prescribed to be meandered, will be established. These lines may be run by solar instruments, but methods involving the use of the transit with observations on Polaris at elongation are now preferred.[2]

211. Standard parallels. — Such parallels, called also correction lines, are run east and west from the principal meridian as parallels of latitude at intervals of twenty-four miles north and south of the base line. "Where standard parallels have been placed at intervals of thirty or thirty-six miles, regardless of existing instructions, and where gross irregularities require additional standard lines, from which to initiate new, or upon which to close old, surveys, an intermediate correction line should be established to which a *local* name may be given, *e.g.*, 'Cedar Creek Correction Line'; and the same will be run, in all respects, like the regular standard parallels."

[1] A list of base lines and principal meridians will be found in the Appendix, pages 357–360.

[2] See the "Manual of Surveying Instructions" for detail of methods.

212. Guide meridians. Guide meridians are extended *north* from the base line, or standard parallels, at intervals of twenty-four miles east and west from the principal meridian.

When existing conditions require the guide meridians to be run *south* from a standard parallel or a correction line, they are initiated at properly established closing corners on the given parallel. This means that they are begun from the point on the parallel at which they would have met it if they had been run north from the next southern parallel. The point is obtained by computation, and is less than twenty-four miles from the next eastern or western meridian by the convergence of the meridians in twenty-four miles.

In case guide meridians have been improperly located too far apart, auxiliary meridians may be run from standard corners, and these may be designated by a local name, *e.g.*, "Grass Valley Guide Meridian."

213. Angular convergence of two meridians. This is given by the equation

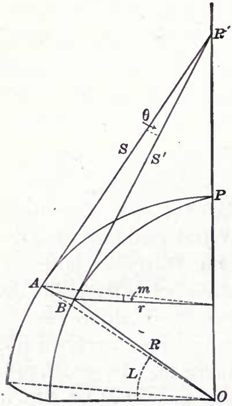


FIG. 105.

$$\theta = m \sin L, \qquad (1)$$

where m is the angular difference in longitude of the meridians, and L is the mean latitude of the north and south length under consideration. The linear convergence in a given length l is

$$c = l \sin \theta. \qquad (2)$$

The derivation of equation (1) is as follows, assuming the earth to be a sphere, which will introduce no error of consequence in this work. In the figure, R is the mean radius, r is the radius of a parallel, S and S' are tangents to the two meridians. The angle θ between these is the angular convergence of the meridians in the latitude L. m is the difference in longitude.

$$r = R \cos L. \tag{3}$$

$$S = R \cot L. \tag{4}$$

S and S' may be considered as radii of the arc AB, which has also the radius r. Since a given length of arc subtends angles inversely proportional to the radii with which it may be drawn,

$$\frac{\theta}{m} = \frac{r}{S} = \frac{R \cos L}{R \cot L}, \tag{5}$$

whence $\theta = m \sin L$, which was to be found.

Since the distance between meridians is usually given in miles, this must be reduced to degrees. This is done by first finding the linear value of one degree for the mean latitude, using the value of r given in equation (3). To make (3) and (4) strictly correct, the normal at A should be used instead of R; but the mean radius will give results sufficiently close for land surveying. See Tables IX. and X., pages 371, 372, for values of θ and length of $1'$ of longitude in various latitudes.

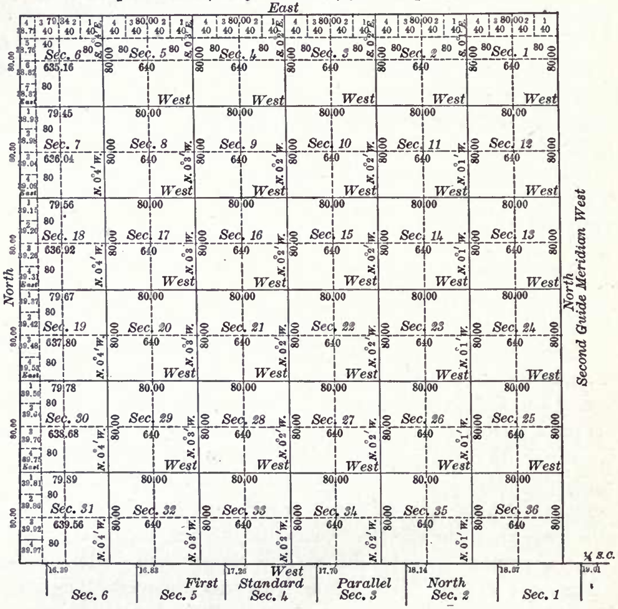
214. Township exteriors. Each twenty-four mile "square" block is, when practicable, subdivided into townships at one time, the work being done as follows :

Beginning with the southwestern township, the meridional boundaries or range lines are first run, and on these are established the section and quarter corners. These are run as true meridians from south to north. Next the east and west lines, or township lines, are run from east to west between corresponding corners of the range lines. On each such township line the section and quarter corners are established at full distances from the eastern range line of each range, the shortage being thrown into the most westerly half mile in each range. A random line is run from east to west, temporary corners being set at correct distances. The distance north or south by which the random line fails to reach the proper corner is observed, and from this and the known length and bearing of the random line a correct bearing is determined and the correct line run eastward, on which are placed the permanent corners. If a random line fails to meet the required corner by more than three chains, it must be rerun. Deviations from the foregoing methods are sometimes made necessary by the topographical features of the territory.

215. Subdivision of townships. Each township is next subdivided into sections as follows:

Beginning on the south line of the township, at the corner common to sections 35 and 36, run northerly *parallel* to the eastern *range line* of the range in question, one mile, setting a quarter corner at the half mile. Establish the corner common

Township No. 5 North, Range No. 9 West, of a Principal Meridian



The above plot represents a theoretical township with perfect subdivisions, contiguous to the north side of a Standard Parallel; in assumed Latitude 42 15′ N., and Longitude 100°00′ W. of Gr. Area 23024.16 A.

FIG. 106.

to sections 25, 26, 35, and 36, and run east on a random line to the corner common to sections 25 and 36 on the range line, setting at the half mile a temporary quarter corner. From the observed failure to meet the corner on the range line, compute the bearing of a true line, and run this from the corner west, setting the permanent quarter corner at the *middle* point of the line.

Proceed thus with each succeeding section to the north till section 1 is reached. From the corner common to sections 1, 2, 11, and 12, the meridional line is run to the section corner on the next township line, by a random line corrected back; but the quarter corner is set at forty chains from the south end so as to throw whatever discrepancy there may be into the north half mile. In case the township is the most northerly of a twenty-four mile block, the line between sections 1 and 2 is *not run to the corner on the correction line*, but is run *parallel to the range line*, to an intersection with the correction line where a closing corner is established and its distance from the section corner noted.

This process is repeated for each tier of sections until the fifth tier is completed. The east and west lines of the sixth tier are run from east to west as random lines and corrected back, but the quarter corner is set just forty chains from the eastern boundary so as to throw the discrepancies into the most westerly half miles. Fig. 106 represents a township with perfect subdivisions. The directions written along the various lines indicate the directions in which they are run. Probably no townships that have been surveyed are like that in the figure, all of them being more or less distorted by inaccuracies, and some of them being very much distorted. When very bad work is discovered in time to correct it, before such alteration will interfere with acquired rights of individuals, the corrections may be made under rules prescribed by the General Land Office.

When individuals have bought the land, the corners as actually set must remain the corners, however erroneously placed.

216. Meandering a stream. This consists in running lines along its bank to determine its direction and length. The left and right banks of a stream refer to the banks as they would be to one passing down stream.

The "Instructions" provide for the meandering of navigable streams and those whose width is three chains and upward.

They are to be meandered on both banks. Lakes, deep ponds, bayous, etc., of twenty-five acres or more are also meandered. Corners called "meander corners" are established wherever a meander crosses standard base lines, township lines, or section lines.

A meander of tidal waters follows the high water line.

217. Corners. Stones, posts, trees, and earth mounds are used to mark the corners. The kind of corner post depends on the material afforded by the country. The "Instructions" designate the kinds of corners to be established and the method of marking them.

When it is impossible to establish a corner in its proper place, auxiliary corners are established within twenty chains of the true place, on each of the lines approaching the corner. These corners are called "witness corners." The center quarter corner of a section is located by connecting the opposite north and south quarter corners by a straight line, and placing the center corner at the middle of this line. Exceptions that will suggest themselves are necessary in the northern and western tiers of sections.

Any person having to do with surveys of lands that have been subdivided as part of the public lands, should procure the "Instructions" and study them carefully. These instructions have varied from time to time and it is well to consult those in force at the time the original surveys in hand were made.[1]

218. Notes. The following sample page of the field notes prescribed in the instructions is given as an excellent form of notes for any land survey. For field purposes the author prefers a sketch on which all notes are made, and from which each night the field book may be written up in the form given. Variations in items noted and in nomenclature will suggest themselves to the maker of other kinds of surveys.

[1] He will also be much helped by a book entitled "Manual of Land Surveying" by Hodgman and Bellows. This work contains the gist of many court decisions and instructions, resulting from many years' experience in this work. Dorr's "Surveyor's Guide" will also be found to contain many practical suggestions.

6TH GUIDE MERIDIAN EAST, THROUGH TPS. 13 N., BETWEEN RS. 24 AND 25 E.

Chains.	
	Survey commenced August 29, 1890, and executed with a W. & L. E. Gurley light mountain transit, No. —, the horizontal limb being provided with two opposite verniers reading to $30''$ of arc.
	I begin at the Standard Corner of Township 13 North, Ranges 24 and 25 East, which I established August 29, 1890. Latitude 45° 34′.5 N., longitude 107° 24′ W.
	At this corner, at $8^h\ 54^m$ P.M., by my watch, which is $3^m\ 49^s$ fast of local mean time, I observe Polaris at *eastern elongation* in accordance with instructions in the manual, and mark the point in the line thus determined by a tack driven in a wooden plug set in the ground, 5.00 chs. north of my station.
	August 29, 1890.
	August 30: At $6^h\ 30^m$ A.M., I lay off the azimuth of Polaris, 1° 49′.5 to the *west*, and mark the TRUE MERIDIAN thus determined by a cross on a stone firmly set in the ground, west of the point established last night.
	The *magnetic bearing* of the true meridian is N. 18° 05′ W., which reduced by the table on page 100 of the Manual gives the *mean mag. decl. 18° 02′ E.*
	From the standard cor. I run
	North, bet. Secs. 31 and 36.
	Descend over ground sloping N. W.
2.60	Creek 10 lks. wide in ravine, 45 ft. below the Tp. cor., course N. 32° W.
7.50	To edge of table land, bears N. E. and S. W.; thence over level land.
17.40	Bluff bank, bears N. 58° W. and S. 58° E.; descend abruptly 40 ft.
19.00	Bottom of ravine, course S. 58° E.; ascend 50 feet to
22.00	Edge of table land, bears S. 58° E. and N. 58° W.; thence over level land.
	Difference between measurements of 40.00 chs., by two sets of chainmen, is 18 lks.; position of middle point
	By 1st set, 40.09 chs.
	By 2d set, 39.91 chs.; the mean of which is
40.00	Set a limestone, 16 × 7 × 5 ins., 11 ins. in the ground, for ¼ sec. cor., marked ¼ on W. face, and raise a mound of stone, 2 ft. base, 1½ ft. high, W. of cor.
42.60	Stream, 6 lks. wide, in ravine 15 ft. deep, course N. 60° W.
47.00	Enter heavy oak timber, bears E. and W.
53.00	An oak, 30 ins. diam., on line, I mark with 2 notches on E. and W. sides.
55.20	Creek, 20 lks. wide, 1 ft. deep, course N. 83° W.
55.40	Right bank of creek, begin very steep rocky ascent.
60.00	Top of ridge, 250 ft. above creek, bears N. 80° W. and S. 80° E.
64.00	Begin descent.
	Difference bet. measurements of 80.00 chs., by two chainmen, is 22 lks.; position of middle point
	By 1st set, 79.89 chs.
	By 2d set, 80.11 chs.; the mean of which is
80.00	The point for sec. cor., 150 ft. below top of ridge, falls on a flat rock *in place*, 10 ft. E. and W. by 6 ft. N. and S., on which I
	Cut a cross (×) at the exact cor. point, for cor. of secs. 25, 30, 31, and 36, marked with 5 grooves on N. and 1 groove on S. sides; from which
	An oak, 10 ins. diam., bears N. 22° E., 54 lks. dist., marked T. 13 N., R. 25 E., S. 30, B. T.
	A dogwood, 5 ins. diam., bears S. 64½° E., 40 lks. dist., marked T. 13 N., R. 25 E., S. 31, B. T.
	An ash, 13 ins. diam., bears S. 51° W., 37 lks. dist., marked T. 13 N., R. 34 E., S. 36, B. T.
	An oak, 9 ins. in diam., bears N. 34° W., 42 lks. dist., marked T. 13 N., R. 24 E., S. 25, B. T.
	Land, level and mountainous.
	Soil, gravel and rock; 4th rate.
	Timber, oak.
	Mountainous or heavily timbered land, 33.00 chs.

CITY SURVEYING.

219. Precision required. Surveying to determine land lines in a town or city does not differ in principle from surveying for similar purposes in the country. The difference lies in the degree of precision required. In the country a square foot of land is worth from one cent to, perhaps, in extreme cases, five or ten cents. In the heart of a great city it may be worth many dollars, and a single inch frontage of a lot one hundred feet deep may be worth several hundred dollars. The brick wall of a modern office building of ten or more stories, built an inch over the line of the owner's land, would be a costly wall to move, and an error so locating it would virtually place the owner at the mercy of the possessor of the adjoining property. It is therefore readily seen that an error in closure of one in three hundred, or one in five hundred, which may be tolerated in farm surveying, would be altogether out of the question in surveys in the heart of a great city.

In such situations a precision of one in fifty thousand should be secured. In villages and small towns a precision of one in five thousand will frequently be sufficiently close, though, if the place has "prospects," it may be well to make closer surveys. The methods of making measurements to the various degrees of precision required are noted in Chapter I.

To lay off an angle so that the position of a line may not depart from its true position by more than $\frac{1}{50000}$ of its length, requires that the angle have no greater error than about four seconds. A transit reading to thirty seconds will nearly accomplish this if the angle is measured three times before reading, the reading being divided by three.

An instrument reading to twenty seconds will ordinarily do a little better than the requirement, and an instrument reading to ten seconds will usually accomplish the required result with a single measurement of the angle, though it should always be measured more than once, if for no other reason than to secure a check on the work. Of course the magnifying power of the telescope must correspond to the fineness of the graduations.

It has been stated that angles are read. Bearings are not used, nor are regular traverses run out in city surveys. Bearings are usually worked up from the angles for all the lines of a survey, in order to compute the latitudes and longitudes for determining the error of the survey. Azimuth would do as well.

220. Extent of survey. City land surveys are usually, except where "additions" are being laid out, of small extent, covering perhaps a single lot of 25 feet by 100 feet, more or less. In a well-monumented city the entire survey may be confined to the block in which the lot lies. In more cases, it will be necessary to go several blocks away to obtain monuments for determining the necessary lines. In cities that have been laid out with irregular lines, it may often be necessary to carry the lines and measurements over the roofs of the solid blocks of buildings in order to determine the angle points in the side lines of the lots. Each case involves new difficulties that the surveyor must meet by exercising his ingenuity. Many difficulties are solved by the proper use of the coördinate system, following the general methods outlined in Art. 203.

221. Instruments. A surveyor who undertakes to do city work should be supplied with the best instruments for measuring lines and angles. The tape mentioned in Chapter I., as used in New York City, is a very good tool. It should not be used, however, until it has been tested by the surveyor himself to verify the "pull scale" for various temperatures. The city surveyor should have, in his office, a standard length marked out on the floor, or some other place not subject to change, by which standard he may test his tapes. The United States Coast and Geodetic Survey Department in Washington, D.C., will test tapes sent there for that purpose, and will report the temperature, pull, etc., for which they are standard, and their constants. For this service a very small fee is required.

The city surveyor's transit should be of high magnifying power, and should read to thirty seconds and preferably to twenty seconds or even ten seconds. It should be well made, and of a pattern to insure stiffness and permanency of adjustment. A good form is shown in Fig. 107.

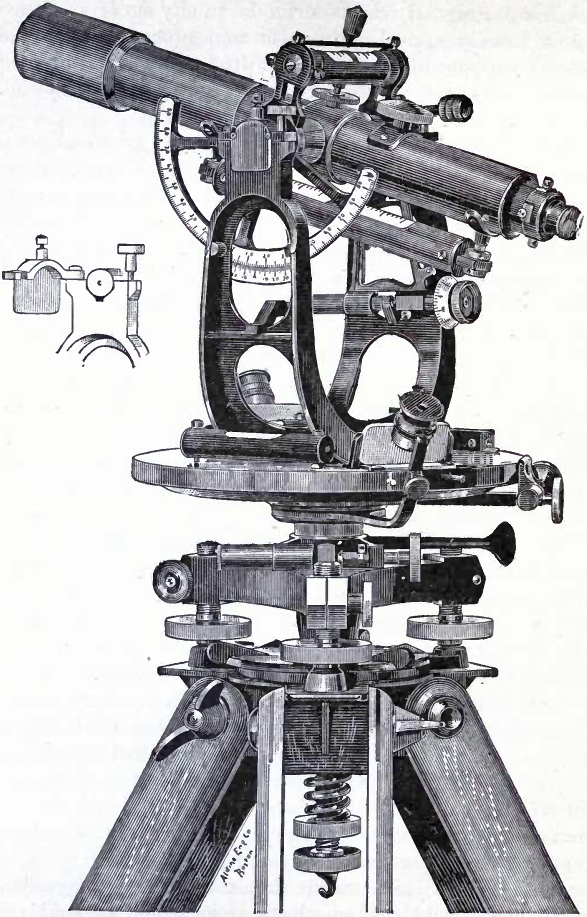


Fig. 107.

The compass being of little use in city surveying, the space usually occupied by the compass box is used to form the base of the horseshoe-shaped standards. The transit is made with either three or four leveling screws. The advantage of three screws is wider leveling base, and therefore, with given pitch of thread, a finer adjustment of the horizontal plate for a given turn of the screw. The author prefers four screws, perhaps because he has never become accustomed to the use of three.

222. Description of a city lot. The description of a city lot as found in a deed is either by "lot and block" or by "metes and bounds."

By the former method it will be described as "Lot No.______of Block No.______of the original survey of the city of______________." Or, "Lot No.________of Block No._______of______________________'s addition to the city of_____________________as the same is shown and delineated on a certain map entitled (Then follows the title of the map) filed in the office of the recorder of_______________ county, ___(month)___ ___(day)___ ___(year)___"

By the method of metes and bounds it will be described as:

"Beginning at a point in the_______________(N., S., E., or W.) line of________________street distant thereon__________________ (Direction)_________________________(Distance) feet from the corner formed by the intersection of the said_______________line of ____________street, and running thence along the said____________ line of__________________street_______________feet, thence at an angle of__________degrees________________________(Direction written easterly, northerly, etc.)______________feet; thence at an angle of___________degrees_________________(Direction)_____________feet; thence at an angle of___________degrees___________________(Direction)____________feet to the point of beginning." To this will sometimes be added: "being Lot No.________of block No.________," etc., as was written above.

223. Finding a city lot. If the description is by lot and block, it will be necessary to refer to the map of the survey mentioned for the data as to the widths of streets, angles in the lines, and positions of lots, before the survey can be begun. The map should also show the character and position of monuments located at the time of the original subdivision of the tract. This it will rarely do if the survey is an old one, and it may not do so even if the survey is a late one.[1]

It will often happen that even though the map shows the positions of the monuments that originally marked the lines, those monuments are not now to be found. In such cases, the best that can be done is to find any monuments having any bearing on the survey in question, and further to note existing lines of permanent improvements that have been long in place, and to endeavor to reconcile these with the figures shown on the map. This work requires the best judgment of the surveyor, for it will almost always be found that the discrepancies are considerable. This is not so true in the centers of very large cities, for here surveys have been gone over and over by most careful workmen, and the lines have become pretty definitely fixed ; but in smaller and newer cities there is an endless amount of difficulty.

224. Marking corners. When the boundaries of the required lot are finally fixed upon, each corner is marked. If the ground is open, a stake with a small tack may mark each corner, if the marking is for temporary use.

The marking may be a mark made on a building or in the stone flagging of the sidewalk, or otherwise. A sketch of the lot with a note of the marks made and a certificate of survey should be furnished the person for whom the survey is undertaken.

225. Discrepancies. If any discrepancies are discovered, they should not be "fudged in" and hidden, but mention should be made of just what is found. If discrepancies are found, the surveyor's work is doubled, for he must be sure the apparent errors are not in his own work. If the discrepancies found are small, no greater than the surveyor would expect to find in repeating his own work — that is, if they are within the limit of precision required — no note need be made of them. When, for instance, a block that is recorded 500 feet long is found to be 500.04 feet long, and it is required to lay out a lot in any portion of the block, the measurement from one end of the block to locate the lot should be to the recorded

[1] Consult Appendix, page 354.

measurement as 500.04 is to 500. Such discrepancies may be thus distributed.

If the street lines on both sides of the block are well defined, and the block is found to overrun or fall short of the recorded length, and if the description of the lots sold has been only by lot and block, the error should ordinarily be divided proportionally among the various lots. If the description is by metes and bounds, it is more difficult to say what to do, and the best the surveyor can do is to report what he finds and ask his employer for further instructions. He must, in no case, locate the lot in what he thinks should be its proper place and attempt to defend such an action as the only proper course. Such procedure has proved a source of much litigation for the owner and loss of reputation to the surveyor.

If the description is by metes and bounds only, the surveyor must determine the starting point by finding the street line on which it lies and the other street line which, with the former, forms the intersection from which the beginning point is located. This is frequently difficult to do because of lack of monuments or marks. When it is accomplished, the surveyor's work is clear: he follows the description. If he finds discrepancies, he reports what he finds. If the description is both by metes and bounds and by block and lot, it is more difficult properly to locate discrepancies, and such as are found must be reported. The interpretation of the deed would perhaps usually be the intent of the buyer and seller, if that could be discovered. The surveyor must remember that he has no right to interpret the deed. That is the business of a court. Descriptions in deeds are often inconsistent in themselves and indicate impossible plots.

226. Planning additions. Whenever possible, additions should be so laid out that the streets may be continuations of those in the adjacent subdivided portion of the city. Not enough consideration has been given in the past to this important feature. Unless the ground is very irregular and the addition is a remote suburb, the subdivision should be rectangular. If the ground is irregular and on the extreme outskirts of the city, in a part that will probably never be thickly built up, it may be laid out in curved lines to conform more nearly to the surface of the ground, involving less work in grading streets and lots, and making the tract more beautiful. In such cases the tract will not be cut up into very small lots. No lot should have less than one hundred feet front, and it may be said that such subdivision should not ordinarily be undertaken if the lots are to be less than one acre in area.

In planning a curved subdivision, the streets should usually be located so as to give good drainage to the lots. This will generally mean that the streets will occupy the lower ground. To do this work most satisfactorily, a contour map of the tract is desirable. This is made as described in Chapter IX.

227. Making the survey and map for an addition. The first step in subdividing an addition is to make a complete survey of the entire tract, using the same care as for city work. Many discrepancies of greater or less amounts will be found. The descriptions of the tract are probably from old farm surveys. If no greater discrepancies are found than might be expected from such work, and the corners are well established, no trouble need arise. The new distances and angles are the ones that will be recorded on the map that is to be made. The surveyor will carefully note the intersection of the boundary with the street lines of the adjoining tract, and the directions of those lines. The survey will be very carefully balanced and mapped.

Probably the best method of making the original map is to divide the sheet into squares of, say, one hundred to one thousand units on a side — the number will depend on the size and scale of the map. One of the lines of division will be assumed as the reference meridian, and another, at right angles, as the base parallel. Their intersection may be taken as the origin or may be given any arbitrary number of hundreds as its coördinates. Each of the corners of the squares will then have for its ordinates quantities expressed in whole hundreds. The coördinates of any point in the survey being determined, it is at once known what square the point falls in, and the point may be measured in from a corner of that square, and be certainly within reach of a single rule length. If the map is small, this method has no advantage over a single meridian on which latitude ordinates are measured and perpendicular to which longitude ordinates are measured.

A system of subdivision will then be planned and drawn on the map,[1] and then located on the ground. The data for placing the street lines on the ground will not be scaled from the map, but will always be computed. The coördinate system is recommended for this work.

The location on the ground consists in placing monuments at all intersections of street lines. These are sometimes placed at the intersection of center lines and sometimes at the block corners. Neither practice is good. Probably the best place to locate them is in the sidewalk area, at a given offset, say five feet, from the property line. They are less likely to be disturbed here than elsewhere. If the addition is to be laid out on curved lines, the monuments should mark the intersections of street lines and the points on the arcs where the radii change. For methods of locating curves, see Chapter VIII.

The monuments should be of stone, durable in quality and about two feet long, with the top dressed to about six inches square and the bottom left rough. The top face should be smooth, with a hole drilled in its center, in which is set with lead a copper bolt with a cross marked in its head. The stone should be set below the disturbing action of frost. A piece of iron pipe may reach to the surface of the ground and terminate with a cast cover. Where stone is costly, concrete may be used, formed of sand and cement, inside of a piece of stove pipe. The copper bolt should be set as before. Other forms may suggest themselves to the surveyor.

A carefully constructed map finishes the work. This map should show the position of each monument, preferably by giving its coördinates referred to some origin, the relation of the monuments to the property lines, and such further information as is indicated in the Appendix, page 355. If curves are used, the coördinates of the centers and beginning and ending points of the curves should be noted.

[1] See an excellent paper by J. Stübben, of Cologne, Germany, in Vol. XXIX., "Transactions American Society Civil Engineers."

CHAPTER VIII.

CURVES.

228. Use of curves. When a road, railroad, or canal is built, the center line is laid out on the ground. The center line is, in the case of a railroad, always a series of straight lines connected by arcs of circles to which the straight lines are tangent. In the case of a canal or wagon road, this is not always true, as the line is sometimes irregular, following more closely the conformation of the ground than does a railroad. It would be better if canals and wagon roads were always laid out as railroads are. In the case of wagon roads, the radii of connecting curves may, of course, be short. Many park roads and suburban streets are located as arcs of circles and straight line tangents. Park drive curves are often irregular and sometimes are successive arcs of varying radii, known as compound curves. Railroad curves are frequently compound curves.

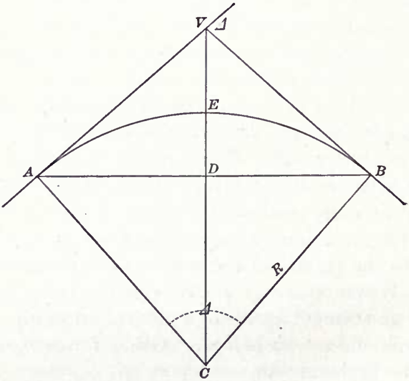


FIG. 108.

229. Principles. Only the fundamental principles of the laying out of these curves will be here given. The straight lines are called technically "tangents." The curves are known by the angle subtended at the center by a chord of one hundred feet. Thus, if such a chord subtends 4°, the curve is known as a 4° curve. Any two tangents not parallel intersect at some point, and the angle at the point, between one tangent produced and the other tangent, measured in the direction in which the line is to bend, is known as the intersection angle. Curves are measured in chords of one hundred feet. This distance is a "station," whether used on curve or tangent.

A curve four hundred feet long means a curve of four stations. It does not mean that the arc is just four hundred feet long, but that there are four chords, each of one hundred feet. In Fig. 108, two tangents, AV and VB, are connected with a curve of radius R. AB is known as the long chord, C. DE is the middle ordinate, M. $AV = VB$ is the tangent distance, T. VE is the external distance, E. From the figure it is evident that

$$T = R \tan \tfrac{1}{2}\Delta, \quad (1) \qquad M = R \operatorname{vers} \tfrac{1}{2}\Delta, \quad (3)$$

$$C = 2R \sin \tfrac{1}{2}\Delta, \quad (2) \qquad E = R \operatorname{ex\,sec} \tfrac{1}{2}\Delta. \quad (4)$$

External secant is a trigonometrical function not commonly mentioned in Trigonometry, but very useful in handling curves. It is the "secant minus one." In the above equations it will be seen that the radius R is used, while it has been said that curves are known by their "degrees." If AB is one hundred feet, Δ becomes D, the degree of the curve, and from (2)

$$R = \frac{50}{\sin \frac{1}{2} D}. \qquad (5)$$

For any other curve of degree D',

$$R' = \frac{50}{\sin \frac{1}{2} D'}. \qquad (6)$$

$$\therefore \frac{R}{R'} = \frac{\sin \frac{1}{2} D'}{\sin \frac{1}{2} D}.$$

For small angles it may be assumed that the sines vary as the angles; whence

$$\frac{R}{R'} = \frac{\frac{1}{2} D' \sin 1^\circ}{\frac{1}{2} D \sin 1^\circ} = \frac{D'}{D}. \qquad (7)$$

The radius of a 1° curve is 5729.65 feet. Except in close land surveys or surveys of city streets, the radius may be taken as 5730.00 feet. In using curves in land surveys, the term "degree" should be abandoned and all circles designated by their radii. Since the values of all the functions given in equations (1), (2), (3), (4), vary directly as R, Δ remaining constant, these functions will all, assuming the correctness of equation (7), vary inversely as D. They would vary exactly in this way if the one hundred feet that measures D were measured on the arc instead of as a chord. This will be clear from the principles of Geometry. The length of a curve for a given value of Δ and D is in stations,

$$N = \frac{\Delta}{D}, \tag{8}$$

and in feet measured in chords of one hundred feet,

$$L = 100\frac{\Delta}{D}. \tag{9}$$

It is frequently required in practice, to know the values T and E for given or assumed values of Δ and D. If a table is made of values of these quantities for a great number of values of Δ and for $D = 1°$, the value of T or E for any other value of D would be at once obtained by dividing the value found in the table opposite the given Δ, by the given value of D. In practical railroad work this method may be used for curves of less than 8° or 10° without serious error. For sharper curves than these the quantities should be computed by equations (1), (2), (3), and (4). If, however, the sharper curves are measured in fifty-foot chords, up to say 14° to 16° curves, and in twenty-five-foot chords thereafter, the approximate method may be used up to at least 20° curves or 24° curves without serious error. This means that a 14° curve will be defined as one in which a chord of fifty feet subtends at the center an angle of 7°. A 20° curve is one in which a twenty-five-foot chord subtends an angle of 5°; etc.

230. Laying out the curve. The point at which the curve is to begin is known when the point V and the tangent distance T are known. For the latter, Δ and D must be known. Let the point of beginning, called the P. C. (point of curve), be A. A transit is set at A, Fig. 109, an angle $= \frac{1}{2} D$ turned from the tangent AV to C. The chain or tape is stretched from A and the farther end made to coincide with the line of sight at C.[1]

From the line CA an angle equal to $\frac{1}{2}D$ is laid off, and the chain stretched from C and the farther end put in the line of sight at D. Thus the points one hundred feet apart are located. The final deflection angle VAB is $\frac{1}{2}\Delta$. If the curve is not of a length expressed by a whole number of stations, there must be turned off for the "subchord" a deflection usually made proportional to the length of the subchord; thus in the figure, if EB is fifty feet, the deflection EAB is one half of $\frac{1}{2}D$, etc. This again assumes that the curve is measured on the arc, and the method is practically correct for curves of less than 10°. For sharper curves than this a computation should be made. Thus, since Δ is known, there remains to turn off from AE an angle equal to $\frac{1}{2}\Delta - VAE$. This being half the angle subtended by the chord EB, that chord may be computed. If the curves of higher degree are laid out with short chords as before described, the proportionality of chord to deflection angle may still be assumed.

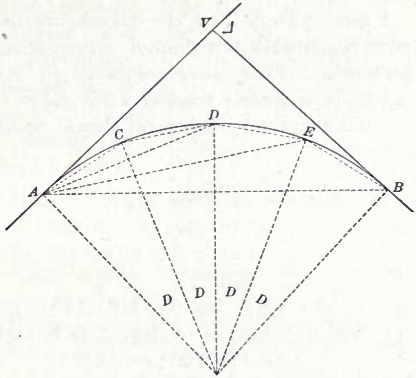


FIG. 109.

If the whole of the curve can not be seen from the beginning, the transit may be moved to a point on the curve already located and the work continued as follows: Let it be assumed in Fig. 109 that there is an obstruction in the line AE so that E can not be seen from A. Move the transit to D, and with the verniers set at zero, with the lower motion, turn the line of collimation on A. Now lay off to the right an angle equal to D (the deflection from A to D) and the line of collimation will

[1] Let the student show that this procedure locates the point C on the curve.

lie in the tangent to the curve at D. Transit the telescope and deflect to the right for the station E, etc., as usual. Not much more than 90° of an arc should be located from one point.[1]

Points occupied by the transit are usually solid "hubs" driven flush with the ground with a small tack to mark the exact point. The other points are, in railroad and common road surveys, merely marked with stakes. In close land surveys and city work, each point should be marked with hub and tack.

231. Location by chain alone. A method for locating arcs approximately by the use of the chain or tape is as follows, and is known as the method by "chord offsets":

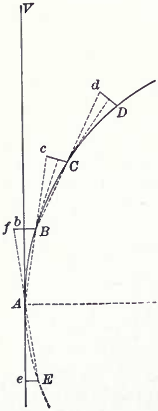


FIG. 110.

In Fig. 110, AV is a tangent to which a curve of degree D is to be joined at A. If AB is one station,

$$Bb = 100 \sin \tfrac{1}{2} D.$$

Imagine the curve continued back one station to E. $eE = Bb$, and f, in EA produced, is distant from b by an equal amount, therefore

$$fB = 2\,Bb.$$

Bb is known as the tangent offset t, and fB is the chord offset $= 2\,t$. To locate the curve, swing the chain from A and place the farther end of it a distance t from the tangent. Stretch the chain from B in line with AB and make a mark at c. Swing the chain about B till cC equals $2\,t$. Proceed in the same manner for the remainder of the curve. If the curve begins with a fractional station, the tangent offset for the fractional station may be computed and laid off; and for the remainder of a full station back of A, the same thing may be done. The direction of the chord of a full station on the curve is now determined, and the work may proceed as usual.

[1] Let the student draw a figure of a curve carrying it to the half circle and show why the 180° point could not be definitely located from the beginning.

232. Compound curves. These are used in railroad and other locations to bring the line of the work to conform more nearly to the shape of the surface along which it lies. Fig. 111 shows such a curve of two arcs only. Compound curves have frequently several arcs of different radii. In compound curves the tangent distances are always unequal, and in the case of two arcs the shorter tangent will be adjacent to the arc of shorter radius. The two arcs must have a common tangent at their junction P. The sum of the central angles will, of course, be equal to the angle Δ. The arcs are laid out as are simple curves, one at a time. The arc AP is laid out from A to P, and the transit is then moved to P, and the arc PB is laid out. The problems in simple and compound curves are merely exercises in the Geometry of the circle, often solved by Trigonometry.

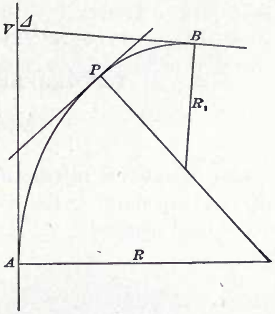


FIG. 111.

One element of curves is of much assistance in solving the problems that arise in laying them out and changing them to better fit the ground where they are located. This element is:

PROPOSITION: *If two circular arcs begin at a common tangent point and end in parallel tangents, their long chords are coincident in direction and vary only in length.*

In Fig. 112 let AV and VB be the tangents to the curve AB; and AV' and $V'C$, the tangents to the curve AC. A is a common tangent point. Since AV and AV' are coincident, and VB and $V'C$ are parallel, $\Delta = \Delta'$. The deflection angles from the common tangent to the points B and C will each equal $\frac{1}{2}\Delta$. Therefore, AB is coincident with AC.

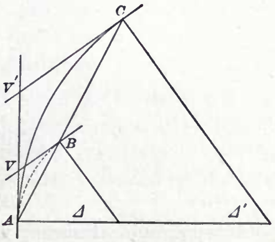


FIG. 112.

CHAPTER IX.

TOPOGRAPHICAL SURVEYING.

TOPOGRAPHY.

233. Methods of representing surface form. A topographical map is a map showing the conformation of the surface of the area of land mapped.

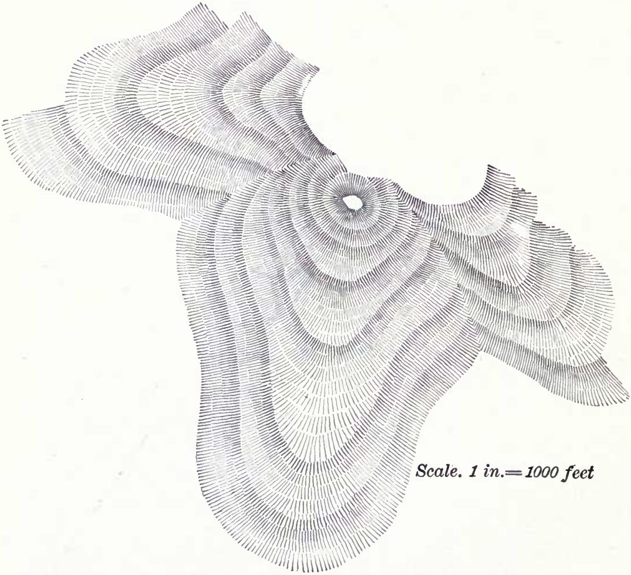


FIG. 113.

A topographical survey is a survey conducted for the purpose of obtaining information for the production of a topographical map of the area surveyed.

There are two common ways of representing the conformation of any small portion of the earth's surface.

(1) By means of hachures or, as more commonly called, hill shading or hatching. Fig. 113 is an example of this method. It is evident that it is little better than sketching, since it gives no definite notion of the height of the hills or the rate of slope of the surface. It is now employed only for the

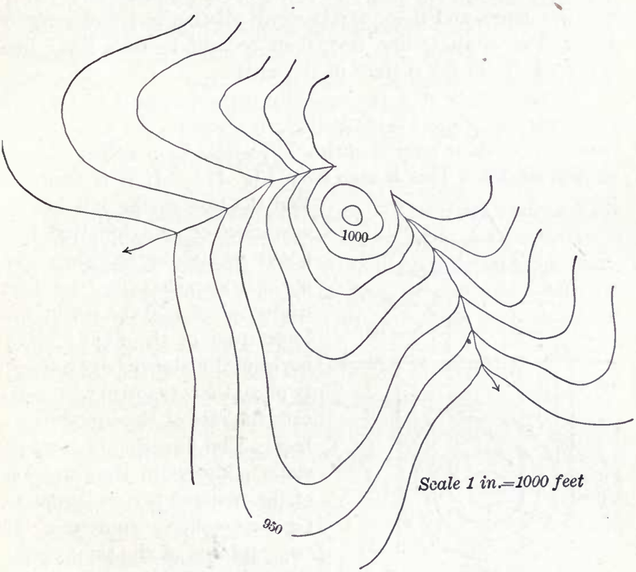


FIG. 114.

purpose of making pictures to be intelligible to those not acquainted with technical methods for representing surfaces, and to convey to them as well as to trained surveyors, a general notion of the conformation of the surface where no definite information is required.

(2) By means of contours, or lines of equal elevation. Fig. 114 is an example of this method.

The word "contour" as used in surveying means a line all points of which have the same elevation above or below some assumed level surface, as sea level. Let the student imagine some section of country with which he is familiar covered with water so deep that the tops of the highest hills are just in the surface of the water. Let it be imagined that the water flows away till the surface has dropped five feet vertically. The intersection of the water surface and the land surface would be a five-foot contour line. Let the surface drop five feet more, and there would result another five-foot contour line. The contour line may then be said to be a level line lying wholly in the surface of the earth.

The contours of a theoretically perfect conical hill would be a series of circles having their centers in the axis of the cone. A contour map of such a hill would be a series of concentric circles. This is seen from Fig. 115. If it is assumed that the base of the hill has an elevation of one hundred feet above some datum, then the contour map showing "ten-foot contours" would be as in the lower part of the figure. The horizontal distance between any two contours shown will indicate the rate of slope of the surface. The direction of steepest slope is always in the direction of the common perpendicular to two consecutive contours. If two contours of the same elevation were to be found on a map running together and continuing as a single line and then separating again, there would be indicated a perfect knife-edge ridge or hollow. Since such a thing does not exist in nature, no such representation can be found on a correct contour map. Moreover, except in the case of an overhanging bluff or a cave, contours of the same or of different elevations could not cross, and hence a correct contour map will not show contour lines crossing. A contour line that closes upon itself indicates a peak higher than the surrounding

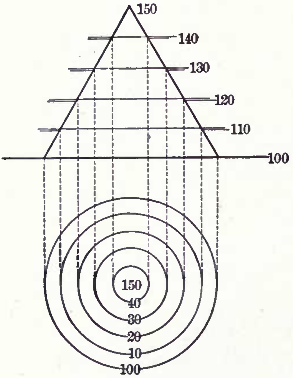


Fig. 115.

ground or a hole without an outlet. Since any contour line, except one that closes on itself as above described, must continue indefinitely, closing on itself somewhere, to be sure, no contour line can begin or end within the border of a map unless it closes on itself. With the exception noted, all contours must appear first in the border of the map and disappear in the same or another edge. When contours cross a stream, it is customary in mapping to run them up to one bank and stop them, beginning again on the opposite bank. This merely means that the portions that run up the banks of the stream and unite in the bed, are omitted. In some cases, however, the contours are carried across the stream.

A contour map is thus seen to show with definiteness the conformation of the surface.

234. Field methods for small area. There are various ways of making topographical surveys, according to the use to which the information obtained is to be put, or the extent of territory to be mapped. A more or less precise and extensive topographical survey is required when a large and important building is to be erected, when a park is to be laid out, or any landscape gardening done, when a railroad, canal, or road is to be built, or when a system of irrigation or drainage is to be planned.

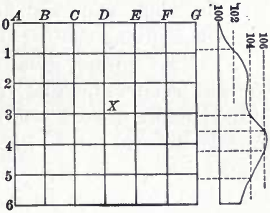


FIG. 116.

When the area to be mapped is small, as a city lot, or block, or a very small farm, the following method is used:

The area is divided into squares or rectangles of from twenty feet to one hundred feet on a side, stakes being driven at each corner. The stakes along one edge will usually be lettered and along another edge they will be numbered. Any interior stake would then be known by its number in its lettered row: thus, stake X (Fig. 116) would be known as stake D_3. Levels are now taken at all corners and any points of abrupt change of slope not at a corner. From the level notes the contours are obtained, as described below. Contours are usually named with their elevations above the assumed datum. Thus, there would be the 110-foot contour, the 112-foot contour, etc. In work covering small areas it will usually be necessary to locate contours with vertical intervals of from one foot to five feet. In one map the interval between contours will be uniform.

235. Contour map. To draw the contour map from the notes that have been taken, lay out in pencil, on paper, to scale, a map of the squares that have been used in the field. Note lightly in pencil the elevations of the points where levels have been taken. Assume that the slope along any line is uniform from one known elevation to the next. If the contours are being drawn with an interval of two feet, for instance, and if either of the known elevations is an even two feet, as 112 feet or 114 feet, then a contour line passes through the known elevation point. If not, and this will be the usual case, the point where the contour crosses the line between the two points of known elevation must be found by interpolation. When found, the point is numbered with the elevation of the line passing through it, and other points are found. This is done over the whole map, and finally points of equal elevation are connected, giving contour lines.

The lines will not usually be drawn straight between points, but will be curved, either from a knowledge of the ground or, in the absence of such information, so as to form no angles, but to make smooth, curved lines. Sometimes a profile of each line is drawn in pencil, as indicated for line *G*, and on this profile are noted the points having elevations of required contours. These are projected down on to the line, giving the points on the line intersected by the given contours. The contour lines are inked in in black or brown or red, according to the purpose of the map. The outside lines of the square are inked in black. The contour lines are numbered with their elevations usually at every fifth contour or at the even tens.

236. Field methods for large area. When the area is large, as a farm of twenty or more acres, or even less, or a city or county, or a portion of a given valley, the following method is adopted:

Points are located, more or less at random, over the area to be mapped, and their elevations are obtained. The points are then mapped, and it is assumed that the slope is uniform between those that are adjacent; and the positions of contours between points are determined by interpolation, as was done with the squares. Equal elevations are then connected by curved lines, which are the contour lines. Sketches are made in the field to assist in shaping the contours.

237. Transit and stadia method. In the field the points are located by "polar coördinates"; that is, by direction and distance from a known point. This work, as well as the determination of elevations, is best done with a transit and stadia.

The field work is as follows: A starting point, as the intersection of two streets, the corner of a farm, or simply an arbitrary spot, is established by driving a firm stake that will not be easily disturbed. The transit is set over this point with vernier reading zero, and the instrument is pointed, by the lower motion, in the direction of the meridian. This may be the true meridian previously determined, the magnetic meridian as shown by the needle, or an arbitrary meridian assumed for the purpose of the survey. If either of the latter, a stake is driven a distance away, so that the direction may be again accurately found. It is assumed that the true meridian will have been so marked as to be found again. The elevation of the point over which the instrument is set, if not known, is assumed and written in the notebook. A traverse line is now run, noting the azimuths of all courses, as described in Chapter V.

The distances are measured with the stadia and recorded in the book. The vertical angles are also noted. Vertical angles and distances are read from both ends of any course as a check. The traverse line is chosen with a view to obtaining from each station the largest possible number of pointings to salient features of the area to be surveyed, and, while the instrument is set at any station and before the traverse line is completed, these are taken. These additional pointings are usually called "side shots." For each pointing the distance, azimuth, and vertical angle are read. These readings will locate the point and determine its elevation.

After the instrument has been oriented at any station, a rod is held alongside and the height of the axis of the telescope is noted in the book as a heading to the notes taken from that station. While this is being done, another rodman (there are usually two or more) selects, by the direction of the person in charge of the survey or by his own judgment, a second point to be occupied by the transit. He drives a stake and marks it $\boxdot_1$ (Stadia Station 1). The rodmen are provided with hatchets for the purpose of making and driving stakes and clearing away trifling obstructions to sight. If there is much clearing to do, axmen are employed.

The rodman holds his rod vertical and just behind the stake from the instrument, with its edge turned toward the observer. The observer directs, by the upper azimuth motion, the line of collimation to the edge of the rod, making the exact bisection by the tangent screw. He then reads the azimuth and calls it to the recorder, who records it. He also reads the needle, and this is likewise recorded, to serve as a witness should an error be discovered in mapping the work. Having read the azimuth, the observer signals the rodman, who turns the graduated side of the rod toward the observer. The latter turns the middle horizontal wire to a point on the rod corresponding to the height of the instrument, using the clamp and slow motion screw of the vertical motion. He then notes the lower wire to see whether it is near a fifty or hundred graduation, makes it coincide with such graduation, and reads the distance. He again brings the horizontal wire to the proper height and signals that he is through. He then reads the vertical angle, which is recorded.

While this is being done, the rodman who was at the instrument selects by direction or judgment a point for a side shot, and stands there with the graduated side of the rod toward the observer. The observer now unclamps the upper motion, and does not, as a rule, clamp it again till he leaves the station. He turns the line of sight on the rod, and after reading the distance he reads the azimuth — the needle is neglected on side shots — and vertical angle as already described for the stadia station, and signals that he is through. The rodman who set the station has now found a point, and his rod is observed as last described, and the work continues in this way, observing first on one rod and then on another, according to which is first ready.

The points that are selected for side shots are such as will enable contours to be well drawn on a map on which those points are plotted. They should be therefore along ridges and hollows at all changes of the slope. They should be taken frequently along a stream to locate its course. It is usually required to obtain information for the mapping of artificial structures, as roads, fences, houses, etc. Therefore pointings should be taken to all fence corners and angles, to enough corners of all buildings to enable the building to be mapped, and to a sufficient number of points in all roads. If it is desired to indicate wooded lands, then pointings must be taken around the edges of timber. While all this is being done, the recorder is recording the notes and endeavoring to keep a sketch on the right-hand page of the book to assist the memory in mapping.

After all the side shots from $\boxdot_0$ have been taken, the instrument is moved to $\boxdot_1$ and a rodman goes to $\boxdot_0$. The instrument is oriented on $\boxdot_0$, the edge of the rod being first sighted. The needle is read and recorded, as well as the distance and vertical angle. After one of the rodmen, who has come up to the station, has noted the height of the instrument, he goes ahead to select a new stadia station, which is established and located as was $\boxdot_1$, and the work proceeds. Instead of requiring a rodman to be at the instrument each time a new point is occupied, the recorder or observer may carry a light graduated rod with which to determine the height of the transit. It is convenient to have the five-foot point of the stadia painted red to expedite the setting of the wire on the proper point for the determination of the vertical angle.

The form of notes used in stadia surveying is given on page 252. The left page of the notebook is represented. The right page is used for sketching and for additional notes descriptive of the stations occupied or observed. The sketch should be as full as possible and neatly drawn.

RENS. POLY. INST. PRACTICE SURVEY. JUNE, 1894.

INST. ON $\boxdot_0$. ELEV. 830.0. H. I. 4.90. SATURDAY, 14. ADEY, OBSERVER. DAVIS, RECORDER.

STATION.	AZIMUTH.	DISTANCE.	VERT. ANGLE.	DIFF. ELEV.	ELEV.
$\boxdot_1$	331° 39′	556	– 1° 41′	– 16.3	
W. side road	34 30	61	– 0 50	– 0.9	829.1
Beginning slope . . .	243 08	80	– 0 22	– 0.5	829.5
C. P. (Contour point) .	31 21	119	+ 1 05	+ 2.2	832.2
C. P.	286 48	110	– 2 00	– 3.8	826.2
C. P.	19 39	141	+ 1 10	+ 2.9	832.9
Top slope	313 26	45	– 1 20	– 1.0	829.0
C. P. west road . . .	356 20	123	– 3 00	– 6.4	823.6
C. P. on hill	347 32	94	– 3 47	– 6.2	823.8
C. P.	331 24	103	– 5 20	– 9.4	820.6
C. P.	334 05	120	– 5 40	– 11.7	818.3

INST. ON $\boxdot_1$. ELEV. 813.7. H. I. 5.0.

$\boxdot_0$	151 39	556	+ 1 41	+ 16.3	
$\boxdot_2$ (described on rt. p.)	352 05	69	+ 2 55	+ 3.5	
C. P. on hill	152 45	436	+ 0 31	+ 3.9	817.6
C. P.	151 56	289	– 1 00	– 5.1	808.6
C. P., W. side road . .	141 11	191	+ 0 35	+ 2.0	815.7
C. P.	151 16	198	– 1 26	– 4.9	808.8
C. P., E. side road . .	141 33	305	+ 0 35	+ 3.6	817.3
Beginning of cross rd. W.	145 47	97	– 0 55	– 1.6	812.1

INST. ON $\boxdot_2$. ELEV. 817.2. H. I. 4.9.

$\boxdot_1$	172 05	69	– 2 54	– 3.5	
$\boxdot_3$	348 09	156	– 2 11	– 6.1	
etc.			etc.		

The elevations of the stadia stations are not written in the sixth column, but in each heading. It sometimes happens that the vertical angles do not check so well as in the example given. When this is the case, the mean value of the difference of elevation is used for determining the elevation.

An instrument that has poor plate bubbles is not suitable for stadia work. To secure good work with such an instrument it is necessary to use the telescope bubble for leveling it.

If it is necessary to correct the horizontal distance, it is done by the use of the table in Appendix, page 380. The differences in elevation are obtained from the same table, a diagram, or Colby's slide rule.

SIMPLE TRIANGULATION.

238. When used. If the area to be surveyed covers several square miles, or is an elongated strip, as a narrow valley, it is best to hang the topography on a system of triangles. Unless this area is extensive, as a large county or a state, these triangles may be considered plane triangles, the curvature of the earth being neglected. In case the triangles are to be considered as plane, it will be best to assume for the meridian of the survey a true meridian near the middle of the area, east and west. The process of laying out such a system of triangles is as follows:

239. Measuring the base line. A fairly level stretch of ground is selected, if possible, near the middle of the area to be mapped, and a line of from one thousand feet to a half mile long, or longer, is measured very carefully. This line is called the base line. Its ends are marked with substantial monuments of stone or with solid stakes. If the survey is of sufficient importance to warrant the permanent preservation of the system of triangles for future reference, the monuments that mark the base line, and those that mark the apexes of triangles, should be stones not less than six inches square in cross section and two feet long. In the top face, which should be dressed, there is drilled a hole large enough to take a half-inch copper bolt, which is leaded in. The head of the bolt is marked with a cross to determine the exact point, and the stone is set deep enough to be beyond the disturbing action of frost. The point may be carried to the surface by "plumbing," and a temporary point set for use. The location of each monument should be fully described with reference to surrounding objects of a permanent character, so that it may be readily found at a future date. If no permanent objects that can be used are to be found, then constructions of stone, brick, or timber are built nearly on the surface of the ground, and the point is described with reference to these. Such precautions are usually necessary only for extensive geodetic surveys.

The measurement of the base line for the surveys of smaller areas, which are the only surveys to be described here, should be made with a precision of from one in five thousand to one in fifty thousand, according to the scale of the map that is to be made, the area surveyed, and the importance of the work.

The two ends of the base line being determined and marked, the transit is set over one end, and a line of stakes ranged out between the two ends. The alignment is made as perfect as possible, and the stakes are set as nearly as practicable at single tape lengths apart, center to center, or at half or quarter tape lengths. These stakes are made quite firm in the ground, and should be not less than two inches square in section. For high-class work the stakes should have on their tops tin or zinc strips on which to mark the measurements with fine scratches.

The measurements are then made, the temperature, pull, and grade elements being noted, and the distance between supports. The requisites for securing the required degree of precision are mentioned in Chapter I. If the absolute length of the tape at some standard temperature and pull is unknown, of course the absolute length of the line remains undetermined. For a precision of one in one million, which is attempted in the measurement of base lines for extensive geodetic surveys, as of a state or continent, much more refined methods than those described are necessary. These methods properly belong to the subject of geodetic surveying.

240. Measuring the angles. After the base line is established and measured, or even while it is being measured, prominent points in the area are chosen as "triangulation stations," that is, apexes of triangles. These are so chosen that no angle shall be less than 30 degrees, nor more than 120 degrees. The first point is chosen, as nearly as possible, in a line at right angles to the base line at its middle point. A second point may be chosen opposite the first one, and the others are then located with reference to these and to one another. When these stations are established, signals are erected over them. For work of the magnitude here considered, these may be simply wooden poles, about one and one half inches in diameter, to which a flag of some white and dark cloth is tacked to make the signal more easily found. Such signals are usually called flags. A transit is then set over one extremity of the base line, and the angles formed at that point by lines to the adjacent stations are measured. When all of these angles have been measured, the transit is moved to the other extremity of the base line, and similar measurements are made there, and subsequently at each triangulation station. There are two methods for measuring angles, called respectively the method by repetition and the method by continuous reading.

In the first method, each angle is measured separately a number of times, and the mean result is taken. The process is as follows: With vernier at 0° 00′,[1] set by lower motion on the left-hand station.

Unclamp above, set on right station.
Unclamp below, set on left station.
Unclamp above, set on right station.

Continue as many times as may be necessary to use practically the whole circle of 360°, and finally read. Divide the reading by the number of repetitions for the value of the angle. Reverse the telescope and repeat the above operation from right to left. The readings are taken in both directions to eliminate errors due to clamping and unclamping and personal mistakes in setting. They are taken with the telescope both direct and reverse, to eliminate errors of adjustment. They are made to include the whole circle to eliminate errors of graduation. Both verniers are usually read to eliminate the effect of eccentricity of verniers.

When such an instrument as the engineer's transit is used, this method is the simplest and probably the best.

The second method, by continuous reading, differs from the first in that each angle is not measured independently of

[1] Any other setting could be used, but it is considered better in this method to set at zero.

the others at the point occupied. The telescope is pointed at each of the distant stations consecutively, and the vernier is read for each pointing, — the difference between the consecutive readings being the angle between the corresponding points. Thus, in Fig. 117, the instrument being at I, the telescope is first pointed to A and the vernier is read, then to B and the vernier is read, then to C, D, E, F, and A in succession. The reading on A subtracted from the reading on B gives the angle AIB. In this method it is necessary to read both to the right and left, and with the telescope both direct and inverted. Moreover, since each angle is measured on only one part of the limb, it is necessary, after completing the readings once around and back, to shift the vernier to another part of the limb and repeat the readings both forward and back, with telescope both direct and inverted. This is done as many times as there are sets of readings. Each set of complete readings to right and left, with telescope direct and inverted, gives one value for each angle. If it is desired to measure each angle three or five times, then three or five sets of readings must be taken. The amount that the vernier is shifted between each two sets is $\frac{180°}{n}$, where n is the number of sets.[1]

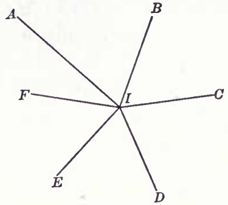


FIG. 117.

241. Notes. A good form for keeping the record of the notes of the triangulation, assuming that it is done by the repeating method and with an engineer's transit reading to thirty seconds, is shown on page 257.

The notes of one set of readings are given. As many sets as may be desired are taken. For the purposes of a small topographical survey, one, or at most two, sets will be sufficient. The check marks indicate the number of times the angle is measured, a check being made after each measurement. Three hundred and sixty degrees must be added to the reading to the right each time the vernier passes the zero of the limb, or must

[1] The student should make a programme for observations by this method.

SATURDAY, JUNE 24, 1894. A.M.

INSTRUMENT ON △[1] A.

STATION OBSERVED.	VERNIER A	B	MEAN READING.	COMPUTATION AND MEAN ANGLE.
△ B	0° 00′ 00″	– 30″	359° 59′ 45″	15 01 45
				360
	✓ ✓ ✓ ✓ ✓ ✓			360
Inst. direct . .	360			735 01 45
				359 59 45
				6)375 02 00
△ C	15° 02′ 00″	01′ 30″	15° 01′ 45″	62° 30′ 20″
△ C	15° 02′ 00″	01′ 30″	15° 01′ 45″	15 01 45
				360
	✓ ✓ ✓ ✓ ✓ ✓			360
Inst. reversed .	360			735 01 45
	360			359 58 15
				6)375 03 30
△ B	359° 58′ 30″	58′ 00″	359° 58′ 15″	62° 30′ 35″

be subtracted from the readings to the left. The latter process brings the same result as adding to the initial reading.

If the method of continuous reading is used, the form of record shown on page 258 is considered good.

The readings are begun on the line *AB* of the triangulation system, and carried to the right and then back, the readings taken to the right being recorded down the page, and the reverse readings up the page. Such a series, when complete, forms a set from which a value of each angle is determined. Provision is made in the form given, for three sets of readings, while but one complete set is shown. The mean of the three determinations of each angle would be taken. The errors are purposely shown large. With a transit reading to thirty seconds, there will usually be no appreciable difference in vernier readings, and it is always possible with a little practice to read the vernier by estimation to one half of its least count.

A third method of measuring angles, which may be called a combination of the first two, saves some time, and gives results as good as the class of work warrants. It consists in

[1] Triangulation station.

SATURDAY, JUNE 24, 1894. A.M.

INSTRUMENT ON △ A.

TELESCOPE.	STATION OBSERVED.	VERNIER A	B	MEAN READING FROM AB.	MEAN ANGLE WITH AB.	MEAN ANGLE.
Direct . . Inverted .	B 1 2 3	000° 00′ 00″ 359 59 30	— 00′ 30″ 59 00	— 000° 00′ 15″ — 45″	— 000° 00′ 30″	
Direct . . Inverted .	C 1 2 3	62 30 30 62 30 00	30 00 29 30	62 30 15 62 29 45	62 30 30	62° 30′ 30″
	D 1 2 3	111 06 30 111 06 30	06 00 06 00	111 06 15 111 06 15	111 06 45	48 36 15
	E 1 2 3	183 48 00 183 48 00	47 30 47 30	183 47 45 183 47 45	183 48 15	72 41 30
	F 1 2 3	278 12 00 278 12 30	11 30 12 00	278 11 45 278 12 15	278 12 30	94 24 15
	B 1 2 3	359 59 30 359 59 30	59 00 59 00	359 59 15 359 59 15	359 59 15	81 46 45 359 59 15

measuring each angle a stated number of times, reading, measuring the succeeding angle the same number of times, reading, and continuing. After all the angles are read successively to the right, the telescope is inverted and pointed on the right-hand station, and the operation is repeated to the left. Three repetitions of each angle are considered sufficient. This provides for all errors except those of graduation, which are usually too small to affect work done to the degree of precision required.

The form of notes to be taken with this third method is:

SATURDAY, JUNE 24, 1894.

INSTRUMENT ON △ A.

STATION.	NO. MEAS.	READINGS.			MEAN ANGLE WITH *AB*.	MEAN ANGLE.
		VERNIER *A*.	*B*.	MEAN.		
	✓✓✓	000° 00′ 00″	+00′ 30″	15″		
B				000 00 30	000° 00′ 30″	
	✓✓✓	000 00 30	01 00	45		
	✓✓✓	187 31 00	31 30	15		
C				187 31 30	187 31 30	3)187° 31′ 00″
						62 30 20
	✓✓✓	187 31 30	32 00	45		
	✓✓✓	398 51 00	51 30	15		
D				398 51 30	398 51 30	3)211 20 00
						70 26 40
	✓✓✓	398 51 30	52 00	45		
	✓✓✓	567 59 30	00 00	45		
E				567 59 45	567 59 45	3)169 08 15
						56 22 45
	✓✓✓	567 59 30	00 00	45		

242. Adjusting the triangles. All the angles of a given triangle are measured. If but two were measured and the third computed, the entire error of measurement of the two angles would be thrown into the third angle. It will be found almost invariably, on adding the measured angles of a triangle, that the sum of the three angles is less or more than 180°. The error should be less than one minute, using the engineer's transit. If there is no reason to suppose that one angle is measured more carefully than another, this error is divided equally among the three angles of the triangle, and it is the corrected angles that are used in computing the azimuths and the lengths of the sides. This distribution of the error is called "adjusting" the triangle. With the large systems of extensive geodetic surveys, very much more elaborate methods are employed, since a large number of triangles must be adjusted simultaneously so that they will all be geometrically consistent, not only each by itself, but one with another.

243. Computing the triangles. When all the angles are measured and adjusted, and the azimuth of the base line, or any other line of the system, is determined, the azimuth of each of the sides may be found by computation. It is to be remembered that the azimuth of any line xy is the azimuth of the line $yx + 180°$. From the measured base line and angles, the lengths of each side of the system may be computed by the ordinary sine formula.

The computations can be much facilitated if systematically arranged. The following arrangement of the computations of the system of triangles shown in Fig. 118 is suggested. AB is supposed known. The adjusted angles are used. AC is determined before BC, because BC is to be used in the triangle BCD.

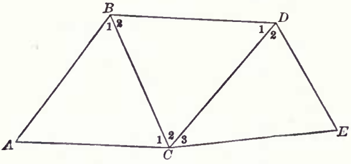


Fig. 118.

$\log AB$	————
$\log \sin C_1$ . . .	————
$\log \frac{AB}{\sin C_1}$ . . .	————
$\log \sin B_1$ . . .	————
$\log \sin A$	————
$\log AC$ (value) . .	————
$\log BC$ ( ) . .	————
$\log \sin D_1$ . . .	————
$\log \frac{BC}{\sin D_1}$ . . .	————
$\log \sin C_2$ . . .	————
$\log \sin B_2$	————
Etc.	

$\log AB - \log \sin C_1 = \log \frac{AB}{\sin C_1}$.

$\log \frac{AB}{\sin C_1} + \log \sin B_1 = \log AC$.

$\log \frac{AB}{\sin C_1} + \log \sin A = \log BC$, etc.

Cologs can be used if preferred. Write the values found for AC, BC, etc., in the brackets.

$\log \frac{AB}{\sin C_1}$ may be written between $\log \sin B_1$ and $\log \sin A$ if preferred.

244. Use of triangles. The elevation of each triangulation station is determined by leveling from some bench mark. Any station may now serve as a beginning point for the topographical survey. The transit being set over Station X is oriented on Station Y by bringing the vernier to read the azimuth of the line XY, and setting the line of collimation on Y by the lower motion. The stadia traverse and work then begin. The azimuth work may always be checked in the field by occupying each triangulation station as it is reached, and determining the azimuth of one of the triangle sides radiating from the station occupied. If this agrees with the known azimuth, the work is correct. If it does not agree within one or two minutes, the work should be rerun.

The elevations also serve to check the vertical angles.

MAPPING.

245. The triangles. If the topographical survey has been based on a system of triangles, these triangles are first drawn. To draw them, assume one station as the origin of the survey, and, knowing the lengths and azimuths of all the sides, compute the latitude and longitude of each station with reference to the assumed origin. Proceed then to plot the stations by latitudes and longitudes. It will be usual to assume one end of the base line as the origin, and the direction of the base line as that of the reference meridian. If, however, the work is referred to another meridian, that should be chosen as the meridian of reference for the computing and plotting.

The lines may also be drawn by means of a protractor; but this, while quicker and sufficiently precise, if a large protractor is used, for much work, is of course not so accurate as latitudes and longitudes.

Another method is to use the dividers and the lengths of the triangle sides, describing arcs whose intersections locate the triangulation stations.

246. Outline of method for topography. A stadia line is first plotted to see that it closes properly. When this has been satisfactorily done, the side shots are plotted. Then the contours are drawn and all other objects that it is desired to map. If preferred, of course the detail work can begin before the completion of the mapping of the side shots, and it is perhaps better to draw in buildings and other structures as soon as the points locating them are plotted.

247. The stadia line. The best method for plotting the stadia lines is by latitudes and longitudes, the computations for which are made with sufficient exactness by a diagram or by the trigonometer shown in Fig. 119. The arm is set to the

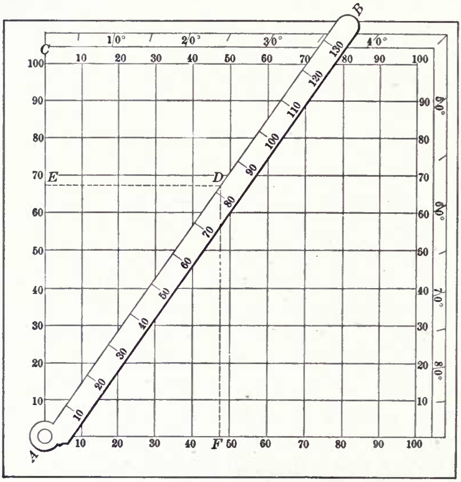


Fig. 119.

proper azimuth and the length of sight is noted on the arm. The latitude and longitude differences are then read on the cross-section scale.[1]

If the latitudes and longitudes of the triangulation stations have been determined, it will be known on the completion of computations for any one stadia line whether or not that line is correct. If it does not close on the proper triangulation station, and the error is small, that error may be distributed

[1] The degree numbering should be continued on this figure to 360°, and the signs for latitude and longitude differences, marked on each quadrant of numbers. Let the student show how this numbering should be placed, and write the proper signs.

among the various courses, as in a land survey. If the error is great, it must be found. If the azimuths were checked in the field, the error must be in distance. Note the direction of the closing line and see whether it corresponds to any course. If it does, the error is probably in that course, and it should be remeasured. If it does not correspond to any one course, there is probably more than one error. In case of error of geographical position due to mistakes in measurement, the elevations will not check unless the error is made on practically level ground; hence if no field check of azimuths has been possible and the elevations check while geographical positions do not, the error is in azimuth.

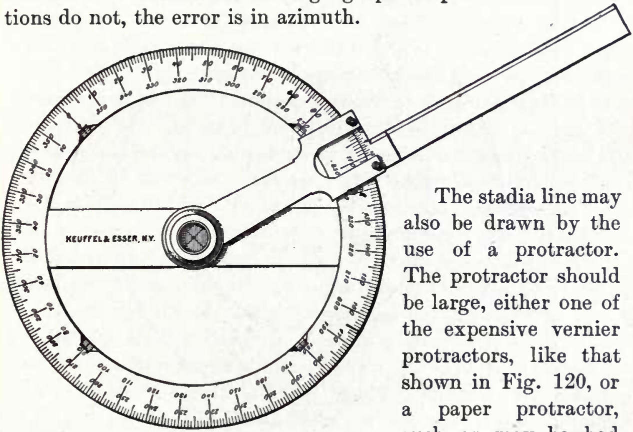


FIG. 120.

The stadia line may also be drawn by the use of a protractor. The protractor should be large, either one of the expensive vernier protractors, like that shown in Fig. 120, or a paper protractor, such as may be had from any dealer in surveyors' supplies. The paper protractor should be about fourteen inches in diameter and graduated to quarter degrees.

248. Side shots. It may be prepared for use in mapping side shots as follows, see Fig. 121 :[1]

Number the graduations from zero to the right 360°. Draw through the zero and the 180° points and the center a straight line, and continue the line to the end of the sheet. The edges are trimmed true and square, and not less than an inch of paper is left outside the printed circle. With a pair of dividers

[1] Modification of a form suggested by Prof. J. B. Johnson.

set to the length of the longest average shots, when measured to the scale of the map, describe an arc *acb* with the center of the protractor as a center. With a radius equal to the radius of the inner circle of the graduations, and with a center on the zero–180° line a little nearer the center than the inner circle of the graduations, describe an arc *adb*. With a sharp knife cut out the crescent thus formed. Cut out also a small triangle, the middle of one side of which shall be the center of the cir-

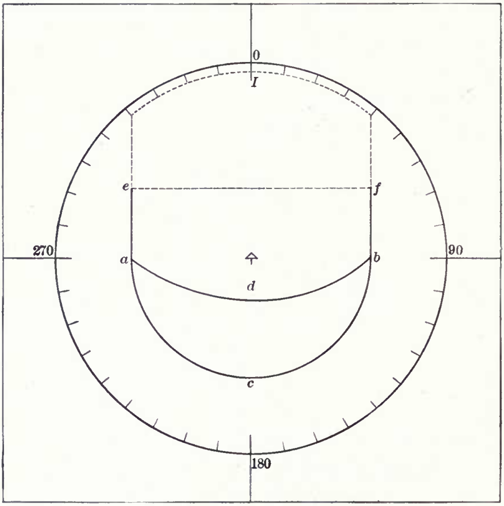


FIG. 121.

cle. Midway between *d* and *I* erect a perpendicular to *dI*, as *ef*. With the knife, cut along the lines *ea* and *fb*. Fold the flap *eabf* thus made on the line *ef*. A protractor with so much material taken from its center is good only for mapping side shots.

Perhaps a better way to prepare the protractor for side shots is to cut out about three quarters of the circle *acb*, Fig. 121, leaving a trifle more than a quadrant which will contain the center and which may be folded back after the protractor is oriented on a point. This form will give a somewhat larger opening for plotting the shots. If a field map is to be made from which a finished map is to be traced, it may be made on the protractor sheet, or larger sheets may be had with a protractor printed on them. The azimuths may be transferred over the sheet with a parallel ruler.

To use the protractor, draw a meridian through each stadia station. With the flap unfolded, center and orient the protractor, weight it down, and fold back the flap. Paste under the zero of the scale that is to be used, a small piece of drawing paper, or other firm paper, through which, at the zero graduation, a fine needle is passed. The needle is stuck into the drawing at the station, and the rule may then be revolved about it as a center. The edge of the scale is brought to each azimuth in succession, and the distance is scaled at once and marked on the paper. If the point is merely taken for the purpose of obtaining elevations, its elevation is noted on the map when it is plotted. Otherwise, the name of the object is marked or the object is sketched in. In case the distance is so great that the point falls under the paper of the protractor, a mark is made with the name or elevation of the point and, when all the other points are plotted, the protractor is lifted and this point put in. With the aid of the sketches and the assumption that the slope is regular between adjacent plotted points, the contours and other objects are now drawn in. The triangulation stations, if there are any, should be inked in before any stadia plotting is done. The stadia lines are not drawn nor are the stadia stations inked. Buildings, fence lines, etc., should be inked before the contours are drawn, to prevent confusion. It is not customary, except in maps of small scale, to run contours across a road, or stream, or through a building.

249. Colby's protractor. A very convenient form of protractor for this work is Colby's, shown in Fig. 122. It is made in three pieces. The largest piece is the limb *L*, *L*, *L*, graduated from 0° to 360°, with fifteen-minute divisions ; *E*, *E*, are projections, upon which weights are placed. The limb has four indexes *B*, *B*, *B*, *B*, 90° apart, by which the protractor is oriented. *A*, *A*, *A*, is the alidade, fitted to revolve inside the limb. *C*, *C*, are indexes carried by the alidade, 180° apart. To the bar *D*, of the alidade is attached the scale *S*, by the small screws and nuts *N*, *N*. The scale has its zero mark in

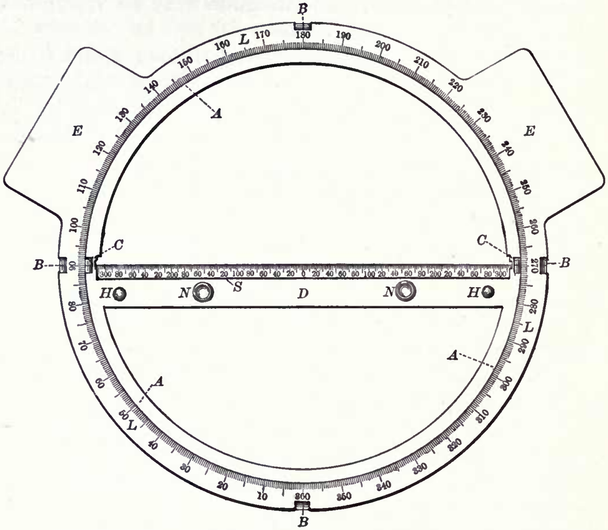


Fig. 122.

the middle, and is graduated both ways. It can be taken off by unscrewing the nuts *N*, *N*, and a scale of different denomination may be substituted when desired.

250. Ockerson's protractor. Another cheaper form is shown in Fig. 123. It is centered with a needle point, about which it will revolve. A meridian is drawn through the point over which the protractor is to be set, and the graduations are such that, if a given azimuth is brought to the meridian line, the diameter, on which is a scale for plotting the points, is in the given azimuth. The protractor was designed by Mr. J. A. Ockerson for use in the Mississippi River surveys.

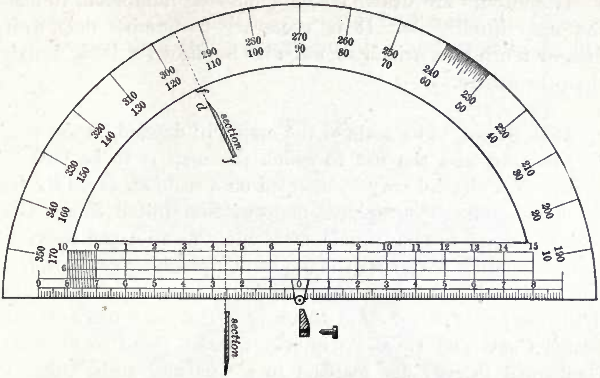


FIG. 123.

251. Finishing the map. If the map is to be finished in black, it may be finished as shown on Plate I., at the end of the book. If it is to be finished in color, a very effective result will be obtained by following the example given in Plate II., which shows the scheme in use by the United States Geological Survey. Plate III. shows a more effective and more elaborate scheme. The contours are brown, being a mixture of crimson lake and burnt sienna. The red is vermilion. Streams and all water will be blue. Roads may be black and dotted or full lines.[1]

252. Requirements for maps. It must be remembered that no map is complete without the following items :

(1) A neat, explicit title, preferably in Roman letters.
(2) Scale, both drawn on the map and given by figures.
(3) The date of the survey.
(4) Name of surveyor and draughtsman.
(5) Direction of meridian.
(6) A key to any topographic symbols that are used.
(7) A neat line border.[2]

[1] An excellent work on "Topographical Drawing and Surveying" is that by Lieutenant Henry S. Reed, U.S.A.

[2] Other requirements for land maps will be found in Appendix, page 355.

If contours are drawn, these should be numbered, to make the map intelligible. It is necessary to number only every fifth or tenth line, which should also be drawn a little heavier than the others.

253. Scale. The scale of the map will depend on the territory covered and the use to which the map is to be put. A survey of a city lot may be mapped on a scale of, say, fifty feet per inch. The topographical maps of the United States Geological Survey are on a scale of about one mile per inch. A railroad preliminary survey may be mapped on a scale of from one thousand feet per inch, for ordinary purposes, to one hundred feet per inch, for close detailing. The maps of the United States Coast and Geodetic Survey and of the United States Geological Survey are mapped to a "natural scale," that is, a given distance on the map is some round fraction of the distance it represents, as $\frac{1}{60000}$ or $\frac{1}{10000}$, etc. Maps of land surveys, or others made to show to non-technical persons, should be made at so many feet, or chains, or miles, to the inch; and the top of the map should be north. Maps of large territories, mainly for the use of those who are conversant with surveying methods, may be made with advantage to the natural scale. The original map may be copied by tracing on cloth or on paper, and transferring to other drawing paper.

THE PLANE TABLE.

254. Description. Detail topographical surveys are sometimes made with the plane table, indeed, this is the standard instrument of the United States Coast Survey for such work, and is also largely used by the United States Geological Survey.

The plane table consists essentially of a drawing board with a suitable leveling device, mounted on a tripod, and a ruler for drawing. The ruler is attached to a line of sight, usually telescopic, but sometimes merely open sights, like those of the compass. The combined ruler and telescope is called the alidade. Fig. 124 is a complete plane table. Clips for holding the paper are shown, as well as the alidade, compass box, and levels, and a device for setting a point on the paper over a point on the ground. The plane table is, in its ordinary form, a very awkward instrument, and is used very little outside the two surveys mentioned. A much simpler form of leveling head than that generally used is one invented by Mr.

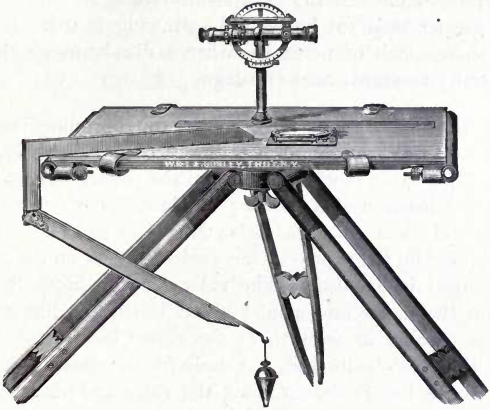


FIG. 124.

W. D. Johnson, and this device has been approved by the topographers of the United States Geological Survey. It is shown in Fig. 125. By loosening the wing nut *d*, the table may be leveled, and when leveled, the nut *d* is tightened.

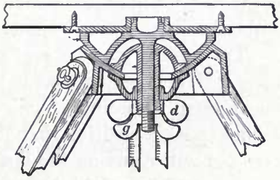


FIG. 125.

When the nut *g* is loosened, the table may be turned in azimuth without disturbing its horizontality. The whole arrangement is very light. It does not permit of as close leveling as does the ordinary form with leveling screws, and should not be used where contours are to be carefully determined by vertical angles. In making a map of a park showing the location of each tree, bush, etc., for planning new work, and when work may be, without loss, performed on pleasant days and omitted on wet or damp days, the plane table may be used with advantage, and, aside from the United States Coast and Geological Surveys, it is for this class of work that it is most used. The author prefers for contour work to use the transit and stadia. If the advantage of mapping at once in the field is desired, it may be obtained by having an assistant and a light drawing board. It is believed that a lack of notes is rather a disadvantage than, as is generally assumed, an advantage.

255. Use. The plane table is used for the immediate mapping of a survey made with it, no notes of angles being taken, but the lines being plotted at once on the paper. The simplest case is the location of a number of places from one point by azimuth and distance. The table is set up so that some convenient point on the paper is over a selected spot on the ground, and clamped in azimuth. The ruler is then brought to the point on the paper and swung about it till the line of sight which is parallel in azimuth to the ruler[1] is directed toward a point that is to be located. A scale for the drawing is determined, and a line is drawn along the ruler and made to scale, equal to the distance to the desired point, which distance is found by measurement or by the stadia. The point is then located. A similar procedure locates other points. This is called the method of radiation. If a plane table is set up in the interior of a field all of whose corners are visible from the position of the table, the corners may be thus located and connected and a map of the field is at once obtained. There is evidently nothing by which to determine the area except to scale the map for additional data or to use a planimeter.

Traversing may be performed with the plane table as follows: If it is a field that is to be "run out," set the table over one corner, choosing a point on the paper to represent that corner in such position that the drawing of the field to the scale selected will come on the board. In the four-sided field shown in Fig. 126, it would be necessary to occupy but two corners to map the field, though it would be better for a check to occupy the third. The figure shows the field and, to a very exaggerated scale, the plane table.

[1] It is only necessary that the two shall have a fixed angle in azimuth.

The table is first set over B and a point b marked on the paper directly over B. The instrument being clamped, the alidade is brought to bear on A, and a line is drawn to scale equal to BA, which is found by measurement or by stadia. The alidade is then directed to C, and the line bc is drawn to scale. The table is then removed to C, and so set up that the point c shall be over the point C and the line cb in the direction CB. This is hard to do with the plane table. If

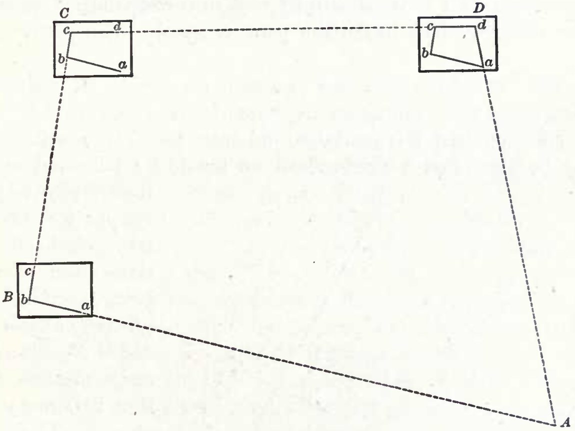


FIG. 126.

the scale is large, it must be done; but if the scale is very small, the following is a sufficiently close approximation: Set the table level, with c over C and with cb approximately in line with CB. Loosen the azimuth motion and, with the alidade on the line cb, bring the line of sight on B. Clamp in azimuth, and the table is set. The point c is not exactly over C, but the error will be inappreciable.

The table being set over C and oriented on CB, turn the alidade toward D and draw cd. da may now be connected, giving the entire field. It will be better to occupy D and see whether the line DA will pass through a on the paper, and whether the length da equals to scale the length DA.

Another method very much used with the plane table is known as the method of intersections. Let it be required to locate the points *A*, *B*, *C*, Fig. 127, from the points *D* and E. Measure *DE*, and lay off to scale on the paper a line equal to *DE*, in proper position so that the points desired will fall on the paper. Set the table over *D* and orient on *E*. Swinging the alidade about *d*, draw lines toward *C*, *B*, and *A*. Set the table over *E* and orient on *D*. Swinging the alidade about *e*, draw lines toward *C*, *B*, and *A*, and note the intersections of these lines with those drawn from *d* to corresponding points. These intersections locate the points.

256. The three-point and two-point problems. In filling in topography that is hung on a system of triangles, it is common to complete the triangulation and map it. There will usually be drawn on a single sheet on the plane table only the topography adjacent to one or two triangulation stations, the work being carried on precisely as in the use of the transit and stadia, except that it is mapped at once. The first sheet will ordinarily have plotted on it at least one triangulation station and a line toward another. These, indeed, might be drawn at random, so that they be located conveniently on the sheet for the work to be done. Usually there will be two or more triangulation stations, or previously occupied plane table stations, mapped on a new sheet. The table may be set over one and oriented on another. It not unfrequently happens that the table can not conveniently be set over any one of the stations mapped on the sheet. Then, if the table is set over any point at random and

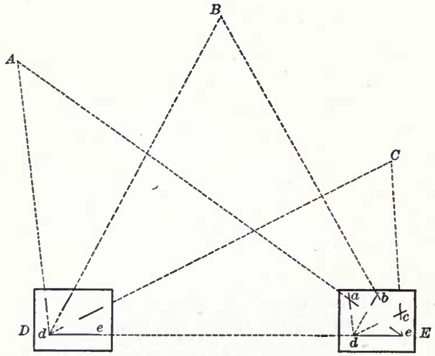


FIG. 127.

properly oriented (assuming that the latter could be done by the needle or otherwise) and the alidade is then revolved separately about each plotted point and directed toward the place in the field thereby represented, a line being drawn on the paper in this direction, the lines so drawn from the various points should all intersect at one point, which would be the properly mapped point over which the instrument is set. In Fig. 128, if points A, B, C, have been properly mapped in a, b, and c, and if the table is set up so that ab is parallel to AB, and consequently bc is parallel to BC, and lines are drawn through a, b, and c in the directions Aa, Bb, and Cc, they should intersect in the point d which is over D over which the table is set. The difficulty is to orient the table. This is done (when three points are mapped) by what is called the "three-point problem" and, when two points are mapped, by the "two-point problem," or "location by resection on the known points." There are several solutions of these problems. The following are perhaps the simplest and most satisfactory.

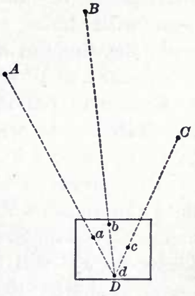


FIG. 128.

(1) *Three-point problem.* If three points are mapped and may be seen from the position of the table, place a piece of tracing paper on the table, and, assuming any convenient point on the tracing paper, draw lines from this toward the three field points. Remove the alidade and shift the tracing paper till the three lines pass through the three mapped points. Prick through the intersection of the three lines. This is the point required. Now orient the table on any one of the points.

(2) *Two-point problem.* In this problem two points are mapped on the paper, and a third is occupied on the ground; the position of this third point is to be correctly mapped, and the table oriented so that work may proceed from the third point. To do this, a fourth point is occupied temporarily. In Fig. 129 the field points have been mapped and are shown on the table in a and b. The point C is to be occupied. Place a piece of tracing paper on the table and set the table over a fourth point D, conveniently chosen. Through the point on

the tracing paper that is over D draw lines toward A, B, and C. Estimate the distance DC, and lay it off to scale to c'. Set c' over C and orient on D. Draw lines toward A and B, and note the intersection of these lines with those already drawn from d'. The figure $a'b'c'd'$ will be similar to the figure $ABCD$. It will, however, not be drawn to the proper scale since the distance DC was only estimated. Therefore, $a'b'$ will be longer or shorter than it should be to represent properly the line AB to the scale that it is proposed to use. Let it be supposed to be shorter. Remove the alidade and shift the tracing paper till a' is over a, and the line $a'b'$ is coincident with ab. Construct a quadrilateral on ab similar to $a'b'c'd'$. This will give the quadrilateral $abcd$, and c is properly located relatively to a and b. The point is pricked through into the drawing, and the table is set up with c over C and oriented on any one of the other points, and the work proceeds. The tracing paper may be dispensed with, the drawing work being thereby somewhat increased. This method is seen to be merely a modification of the three-point problem.

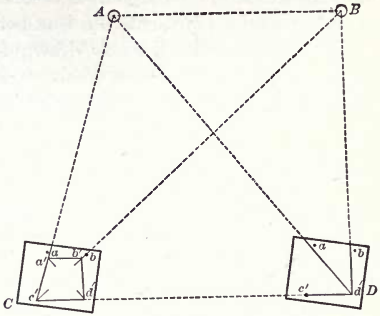


FIG. 129.

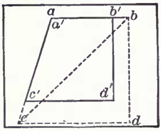


FIG. 130.

257. Adjustments. The bubbles for leveling the table are adjusted to be parallel to the base of the alidade. This is done by the method already given for adjusting a level having a plane metallic base. See Art. 51.

The line of collimation and the axis of the telescope bubble are made parallel by the "peg" adjustments as applied to the transit.

The horizontal axis is adjusted like that of the transit.

CHAPTER X.

EARTHWORK COMPUTATIONS.

ORDINARY METHODS.

258. Occurrence of problem. The surveyor is frequently called upon to measure the volume of earth moved or to be moved in connection with various grading operations. These may consist of the grading of a city block to conform to the bounding street grades, the simple excavation of a foundation pit or cellar, the grading of a street, the building of a reservoir, the dredging of material from the bottom of a river, lake, or bay, etc.

Payment for earthwork is usually by the cubic yard; sometimes, for small quantities, by the cubic foot. The measurement of earthwork is based on the geometrical solids, the prism, wedge, pyramid, and prismoid.

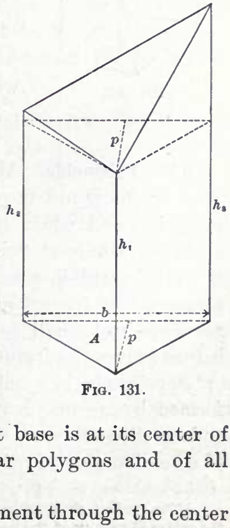


FIG. 131.

259. Prisms. The volume of any doubly truncated prism is $V=Ah$, in which A is the area of a right section, and h is the element of length through the center of gravity of the right section or is the length of the line joining the centers of gravity of the two ends.

In any prism having a symmetrical right base, the center of gravity of that base is at its center of form. This is true of all the regular polygons and of all parallelograms.

In such prisms the length of the element through the center of gravity is the mean of all the edges. This is further true of a triangular prism.

This may be shown as follows:

In the truncated triangular prism shown in Fig. 131, the area of the right base is $A = \frac{1}{2}bp$, and $p = \frac{2A}{b}$. The volume is $V = \frac{(h_2 - h_1) + (h_3 - h_1)}{2} b \frac{p}{3} + \frac{3h_1 A}{3}$. Substituting for p its value $\frac{2A}{b}$, $V = \frac{h_1 + h_2 + h_3}{3} A$.[1]

A truncated parallelopiped may be divided into two truncated triangular prisms in two ways, as shown in Fig. 132, and each set may be treated as above and the resulting volumes added. This gives twice the volume of the solid, which divided by two gives

$$V = \frac{h_1 + h_2 + h_3 + h_4}{4} A.$$

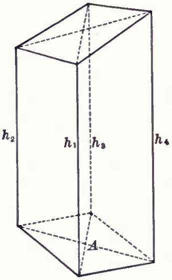


FIG. 132.

A prism having for its right base a regular pentagon may be divided into three triangular truncated prisms in five ways and treated as the parallelopiped, when the result $V = \frac{h_1 + h_2 + h_3 + h_4 + h_5}{5} A$ is obtained.

260. Prismoids. Many of the forms dealt with in earthwork are more nearly prismoids than either prisms, wedges, or pyramids, and when this is the case the prismoidal formula gives more correct volumes.

A prismoid is a solid having two parallel polygonal bases connected by triangular faces. This is equivalent, if the faces are conceived small enough, to the following, which somewhat better defines the forms dealt with in earthwork:

A prismoid is a solid having parallel plane ends with sides formed by moving a ruled line around the perimeters of the ends as directrices. Such a solid may be conceived as made up of a combination of prisms, pyramids, and wedges.

[1] The student should show that $\frac{h_1+h_2+h_3}{3}$ is the length of the element through the center of gravity of the bases.

261. Prismoidal formula. If a single expression can be found giving the volume of a prism, pyramid, and wedge, it will give the volume of a prismoid, since such a solid is made up of the above three elementary solids, all having, in any given case, equal heights.

In the right

$$\left.\begin{array}{l}\text{prism}\\[2ex] \text{pyramid}\\[2ex] \text{wedge}\end{array}\right\} V=\left\{\begin{array}{c}Ah\\[2ex] \dfrac{Ah}{3}\\[2ex] \dfrac{Ah}{2}\end{array}\right\} \begin{array}{c}\text{which may be}\\ \text{written}\end{array} \left\{\begin{array}{l}\dfrac{h}{6}\,(A_1 + A_2 + 4\,A_m)\\[2ex] \dfrac{h}{6}\,(A_1 + A_2 + 4\,A_m)\\[2ex] \dfrac{h}{6}\,(A_1 + A_2 + 4\,A_m)\end{array}\right.$$

in which A_1 is one base, A_2 is the other base, and A_m is a section midway between the two end bases and parallel to them.

The single expression thus found is called the prismoidal formula, and is the correct formula for finding the volumes of prismoids.

A_m is not a mean of A_1 and A_2, but each of its linear dimensions is a mean of the corresponding dimensions of A_1 and A_2.

This formula is applicable to the sphere and all the regular solids of revolution.[1]

262. Approximations. To avoid the labor involved in computing the volume of prismoids by the prismoidal formula, one of two assumptions is frequently made. These are:

(1) The volume of a prismoid is $\dfrac{A_1 + A_2}{2}h$, called the method of average end areas.

(2) The volume of a prismoid is hA_m, called the method of mean areas.

The error of the first assumption is twice that of the second and of the opposite sign. To show this, let a_1, a_2, and a_m be homologous sides of the similar end and middle sections of a given prismoid.[2] Then since the areas of similar figures are as

[1] Let the student show the truth of this statement for the sphere and both spheroids of revolution.

[2] Prismoid end areas are not necessarily similar. They are here assumed to be so for simplicity.

the squares of their homologous sides, there may be written the following expressions for the volume of the prismoid, based on the true and approximate formulas. In these expressions K is any constant ratio by which a^2 is to be multiplied to give area. Remembering that $a_m = \frac{a_1 + a_2}{2}$,

By prismoidal formula

$$V_p = 2K\frac{h}{6}(a_1^2 + a_1a_2 + a_2^2) = \frac{4}{12}Kh(a_1^2 + a_1a_2 + a_2^2). \quad (1)$$

By average end areas

$$V_{ea} = \frac{Kh}{2}(a_1^2 + a_2^2) = \frac{6}{12}Kh(a_1^2 + a_2^2). \quad (2)$$

By mean areas

$$V_m = K\frac{h}{4}(a_1^2 + 2a_1a_2 + a_2^2) = \frac{3}{12}Kh(a_1^2 + 2a_1a_2 + a_2^2). \quad (3)$$

Subtracting (1) from (2),

$$V_{ea} - V_p = \frac{2Kh}{12}(a_1 - a_2)^2. \quad (4)$$

Subtracting (1) from (3),

$$V_m - V_p = -\frac{Kh}{12}(a_1 - a_2)^2. \quad (5)$$

It is seen that equation (5) is half of equation (4) and of opposite sign. From equations (4) and (5) it is also seen that the average end area method gives results too great, while the mean area method gives results too small. There are certain peculiar cases when the end areas are not similar figures in which the average end area method gives results too small.

By the prismoidal formula, or one of the approximations mentioned, the volume of all masonry work or earth work may be computed.

It is seen from equations (4) and (5) that the errors of the assumptions given vary with the square of the difference in dimensions of the two end areas, and therefore it may be concluded that in cheap work, such as earthwork, if successive end areas are nearly alike, the simpler method of average end areas may be used, while if the areas are quite different, the prismoidal formula should be used; and in costly work, such as masonry, the prismoidal formula should always be used.

263. Area grading. When a large area, as a city block, is to be filled or excavated, the area may be divided into rectangles of such size that the four corners of each may be assumed to be in one plane. The lines of division will be so referenced that their intersections can be again found after the grading is done. Elevations of the corners of the rectangles will be found before grading and after the grading is done. The difference in elevation at any one corner is the depth of cut or fill at that corner.

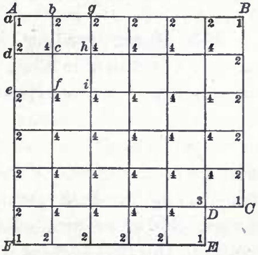


FIG. 133.

The depth of cut or fill is marked on the stakes set at the corners to guide the workmen. The total volume is thus divided into a number of prisms of equal bases and known altitudes. The volume in cubic yards of one prism is, letting A be its base and a, b, c, d, its four corner heights,

$$V = \frac{A}{27} \cdot \frac{a + b + c + d}{4}.$$

The student may show that the volume of all the prisms is given by

$$V = \frac{A}{4 \times 27} (\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4),$$

in which Σ is the sign for "sum of" and h_1, h_2, etc., are the corner heights of the several prisms, the subscript numeral indicating the number of prisms of which the h to which it is affixed is one corner. Thus c would be an h_4, b an h_2, etc.

The rectangles may be made larger by conceiving a diagonal as drawn in each rectangle. This would be drawn in the correct direction by inspection in the field, so that the assumed plane tops of the triangular prisms thus formed would most nearly correspond with the ground surface.

Assuming that A is still the area of a rectangle, the student should show that the volume of the series of prisms is given by

$$V = \frac{A}{6 \times 27} (\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4 + 5\Sigma h_5, \text{ etc.}),$$

and that the highest subscript may be 8.

The sides of such rectangles as have been mentioned may vary from twenty feet to one hundred feet, according to the character of the ground surface.

264. Street grading. In grading streets the sides are usually cut down or filled up vertical where possible, the principle being that the adjacent property must look out for itself. The best way to get the volume in street grading is to make cross profiles of the section to be graded, at such intervals as may be necessitated by the character of the ground, say from fifty to one hundred feet apart. On these cross profiles will be drawn the cross profile of the finished street, and the area between the two profiles will be the area in cut or fill at that section. The volume between any two adjacent sections is computed as a prismoid. The areas may be drawn to scale on cross-section paper, and measured with a planimeter, or they may be computed.

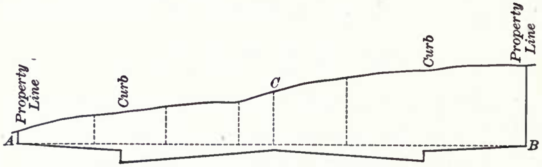


FIG. 134.

Some engineers work with the grade of the center of the street, some with the grade of the curb lines, and some with the grade of the property lines. Stakes are usually set at the center and sides on which are marked the depths of cut or fill as a guide to the workmen. Where this is not possible, a list of cuttings may be furnished the contractor. Assuming the grade of the property line to be used in Fig. 134, the depths of cut to the line AB will be determined at such points in the cross section as may be necessary to give a correct area. The area above the line AB is computed from the field notes and used in getting the volume. The volume of the cutting below the line AB is a constant per unit of length of street, and may be readily computed for any given stretch. The slide rule is very useful in saving time in all volumetric computations.

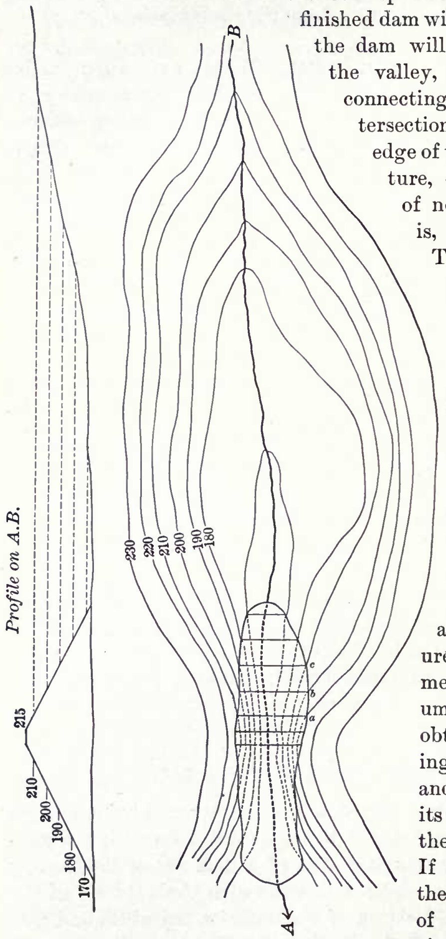
265. Excavation under water. When excavation under water is to be measured, it may be done by sounding on known lines, both before and after the excavating is done, and the volume is computed as in surface grading. This is not always the best method. The material is usually measured after removal. If it is placed in scows, the displacement of the scow for each load is obtained, and this, divided by the specific gravity of the material, gives the volume in the scow. It is necessary to observe the displacement of the scow for no load, and subtract this from the observed displacement for a given load.

The displacement is obtained by measuring the model lines of the vessel at varying depths, and computing the volume for these depths by the prismoidal formula. These volumes may be plotted on cross-section paper by laying off the depths to scale on one side of the paper, and at these points laying off the computed volumes to scale perpendicular to this axis of depths. A curve may now be drawn through the points thus plotted. Ordinates to this curve, at any scaled depth, will be to scale the volume for that depth.

To observe the displacement correctly, the water in the hold must be at the level obtaining when the displacement of the empty vessel was noted. The depth of the vessel in the water is best noted at four symmetrically situated points, and the mean is taken.

ESTIMATING VOLUMES FROM A MAP.

266. A reservoir. Fig. 135 is a contour map of a portion of a valley or ravine that it is proposed to convert into a storage reservoir by the construction of a dam across the narrow part shown. It is required to determine the capacity of the reservoir. At the location of the dam, its top width and side slopes being determined, the lines representing the top, and what will be contour lines of the finished surface, may be drawn on the map as indicated. The intersection of any contour with the corresponding contour of the finished dam will be a point where the dam will join the side of the valley, and a line, *abc*, connecting such points of intersection will indicate the edge of the proposed structure, or will be a line of no cut or fill, that is, a "grade line." The storage space may be conceived to be divided into a series of horizontal layers whose top and bottom areas are the closed figures formed by corresponding contours of surface and dam. These areas may be measured with a planimeter. The volume of any layer is obtained by averaging its two end areas and multiplying by its height, which is the contour interval. If h is that interval, the volume of a series of layers would be given by



Plan FIG. 135.

$$V_{cy} = \frac{h}{27}\left(\frac{A_1}{2} + A_2 + A_3 + \text{etc. } \frac{A_n}{2}\right).$$

If it is preferred to use the prismoidal formula, the height of each prismoid will be twice the contour interval, and every other area will be a middle area, and the volume of the whole, assuming an even number of layers, will be

$$V_{cy} = \frac{h}{6 \times 27}(A_1 + 4\,A_2 + 2\,A_3 + 4\,A_4 + 2\,A_5 + 4\,A_6 + \text{etc.} \cdots A_n).$$

If there is an odd number of layers, the final layer may be computed separately, either by average end areas or by interpolating a middle area on the map and measuring it with the planimeter.

The volume of material in the dam may be obtained in the same way.[1]

267. Application to surface grading. This method is applicable also to the measurement of irregular earthwork in general.

Let Fig. 136 represent a rectangular area that it is proposed to surface as indicated. The full irregular lines represent contour lines of the original surface. The more regular curved full lines represent contours of the surface as it is proposed to have it. The figures represent elevations above some datum surface. Conceive the original and graded surface to exist at the same time, and imagine the contour planes passed into the hill at their respective elevations, and consider particularly the 82-foot plane. The areas *a*, *c*, *b* and *n*, *o*, *p* will be in excavation, while the areas *b*, *s*, *n* and *p*, *q*, *r* will be in embankment. The points *a*, *b*, *n*, *p*, and *r* are "at grade," being neither in cut nor fill.

Connecting corresponding grade points, there result the dotted grade lines 84, *f*, *b*, 81; *t*, *n*, *w*; *u*, *p*, *v*, which are bounding lines of bodies of cut and fill.

Thus, the whole central portion of the figure is in excavation, while the upper left corner is in embankment.

The horizontal area in excavation or embankment at any level is the area included between original and finished surface contours at that level.

[1] The student should show how to do this, using Fig. 135.

To get the volume of any single body of cut or fill, measure successive areas with the planimeter, and assume these to be end areas of figures that are as nearly prismoids as anything else, with altitudes equal to the contour interval. If the prismoidal formula is to be used for computing the volume, the

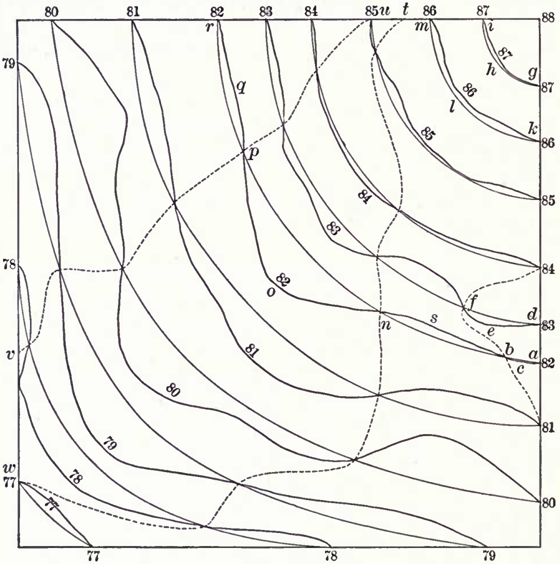


Fig. 136.

altitude of a single prismoid is taken as twice the contour interval, that is, two layers are taken to make one prismoid. The volumes may be computed by the formulas of the last article. Assuming average end areas, the small body of cut on the right is found to have areas :

At elevation 81, 00.
At elevation 82, *acb*.
At elevation 83, *def*.
At elevation 84, 00.

$$\text{Volume} = \frac{1}{27}\left(\frac{00}{2} + acb + def + \frac{00}{2}\right) = \frac{1}{27}(acb + def).$$

The portion of fill between this cut and the large central body of cut has areas:

At 88-foot level, 00.
At 87-foot level, *ghi*.
At 86-foot level, *klm*.
etc., etc.

268. Application to structures. One example of the application of this method of estimating volumes to regular structures built on irregular ground will be given. The form of the

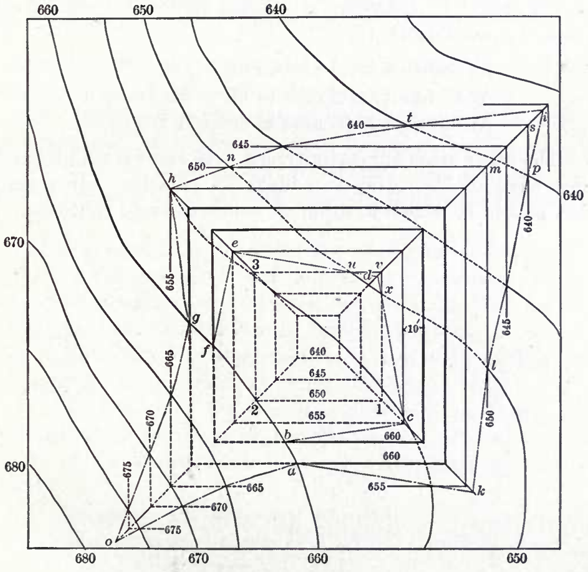


Fig. 137.

structure is not presented as an example of good engineering, but merely to show the method of estimating volumes.

A small reservoir is to be built on a hillside, and will be partly in excavation and partly in embankment. Fig. 137 shows such a case. The contours, for the sake of simplicity, are spaced five feet apart. The top of the reservoir (shown by the heavy lines making the square) is 10 feet wide, and at an elevation of 660 feet. The reservoir is 20 feet deep, with side slopes — both inside and outside — of two to one, making the bottom elevation 640 feet, and 20 feet square, the top being 100 feet square on the inside. The dotted lines are contours that would be invisible if both original surface and completed reservoir were supposed to exist at the same time. The areas of fill all fall within the broken line marked *abcdefghik*, and the cut areas all fall within the broken line marked *abcdefgo*. These broken lines are grade lines. The areas of fill and cut are readily traced by following the closed figures formed by contours of equal elevation:

At 640-foot level area in fill is *pst*.
At 650-foot level area in fill is *lmnuvxl*.
At 650-foot level area in cut is 1 2 3 *ux* 1.

The other areas are easily traced. In the figures given, the lines have all been drawn in black for printing. In practice they should be drawn in different colors to avoid confusion.

CHAPTER XI.

HYDROGRAPHIC SURVEYING.

269. Definition. A hydrographic survey is a survey having to do with any body of water. A topographic survey may be, and frequently is, partly a hydrographic survey.

270. Objects. A hydrographic survey may be undertaken for any one of the following purposes :

(1) To determine the topography of a portion of the bed of the sea, a bay, or harbor, or river, in order that it may be mapped for the information of seamen. In this case it is necessary merely to locate the channels, dangerous rocks, and shoals.

(2) To determine definitely the configuration of a small portion of the bed of the sea, a bay, harbor, or river, for the purpose of planning works to rest on or in the bed, such as lighthouses, sea walls or docks, bridge piers, etc., or to estimate the quantities of materials to be moved by dredging or blasting, for the improvement of the channels or harbor.

(3) To determine the flow of a given river or estuary and the direction of the currents, for the purpose of studying the physics of the stream or for planning waterworks or drainage systems.

(4) To secure the necessary information for planning the improvement of marshy shores of a lake or stream by lowering the water level or otherwise.

271. Work of the surveyor. The survey ends with the determination of the required information. The planning of the works and their execution is hydraulic engineering. The laying out of the plan on the ground requires the methods of the surveyor. The work of the surveyor, then, is to make a topographical map of the area to be covered, to compute material moved, to determine cross sections of streams, their velocities, their discharge, the direction of their currents, and the character of their beds, and to lay out projected improvements.

272. General statement of methods. The configuration or topography of the bed of a body of water is determined by sounding, that is, measuring the depth of water. If many points are observed, a contour map of the bottom may be drawn, the water surface being the plane of reference.

The cross section of a stream is determined by taking soundings along a fixed transverse line.

The velocity of flow of a stream is determined by noting how long a float requires to drift a known distance, or by the use of a current meter.

Since the velocity is not the same at all parts of a given cross section, many determinations for velocity must be made before an average for the whole cross section can be obtained.

The discharge of a large stream like the Mississippi, Missouri, Ohio, or Hudson River is determined by measuring a cross section and finding the velocity of flow past it. The discharge of a small stream, like any one of the thousands of creeks, is obtained by weir measurements. A weir is a notch cut in a dam. A dam is constructed across the stream, and a notch is built in it, through which the water flows. From the known size of the notch and the depth of water flowing over it, the flow may be calculated by methods that are explained in full in any work on hydraulics. Insignificant streams may be measured by any simple means that may occur to the surveyor.

The direction of surface currents may be determined by means of floats. Subcurrents are best determined by means of the direction meter.

The character of the bed of a body of water may be ascertained by securing samples in a cup attached to the sounding lead, or by putting tallow in a cavity in the bottom of the lead. Some of the bottom will adhere to the tallow. If the character of the bed for some distance below the bottom is required, as when bridge piers are to be founded on firm bottom which is at an unknown depth, various methods are resorted to, according to the depth of water, the soil below, and the difficulties to be overcome. Simple cases may be handled with gaspipe rods driven through the bottom by hammers to a hard layer of material. In other cases drills driven by pile drivers are used, or diamond drills if rock is encountered, and it is necessary to determine its character and depth.

The method of making weir measurements, being properly a part of the study of hydraulics, and requiring considerable space to treat adequately, will not be given here; but such methods as require the use of surveying instruments and meters will be described in sufficient detail to enable the student to undertake such work.

SOUNDINGS.

273. Making soundings. Deep-sea soundings are made with special elaborate apparatus which will not be described.

Soundings in moderately deep water are made with a weight, known as a lead, attached to a suitable line.

For depths of less than fifteen or twenty feet a pole is best.

The lead may be any heavy weight. It is preferably of lead, molded about an iron rod. The form is shown in Fig. 138. The cup at the bottom is for collecting samples of the bottom. There is a leather washer that slides up and down the rod between the bottom of the lead and the top of the cup. This keeps the soil that is collected, in the cup, while the lead is being raised. Very often the cup is omitted, and if samples of the bottom are required, tallow is used as already described.

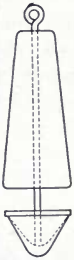


Fig. 138.

The line for very deep work may be of wire, but must be used with a reel. For ordinary work, a hemp line or a chain is best. A line must be stretched, but not too much, as it may shrink. It must be frequently tested.

For soundings for navigation charts, the depths are taken in feet to four fathoms, and thereafter in fathoms. The party required to make soundings consists of a leadsman, a recorder, one or more oarsmen, and, when soundings are located from a boat, one or two observers to read the angles. If the boat is located by angles read on shore, there must be one or more observers on shore.

The leadsman casts the lead and announces the depths. For this purpose the line is graduated by suitable tags, so that the depths can be readily determined. In still, deep water the boat may be stopped while the sounding is being taken and recorded, but more often it is not stopped. The leadsman stands in the bow of the boat and throws his lead ahead, so that, as he judges, it will be on the bottom and the line will be vertical when the boat is over it. He will become expert at this after a time. The recorder notes the number of the sounding and the depth that the leadsman gives him.

The method of using rods will be obvious.

The reference plane for depths is the surface of the water, and as this is continually changing, except in still ponds or lakes, it is necessary to know the stage of the water at the time the soundings are taken. This is done by establishing a gage in the vicinity of the work. The soundings on all navigation charts are referred to mean low tide. The zero of the gage should be set, if possible, at this elevation. The gage may be a graduated board nailed to a pile in a pier in the vicinity of the work and read at hourly intervals during the progress of the sounding. Gages for rivers are frequently inclined, being laid along the shore at right angles to the stream, and the points of equal differences of altitude are determined by leveling. A tide gage that is to remain permanently in place should be self-registering. Such a gage may consist essentially of a float protected by a surrounding house or tube, and attached by suitable mechanism to a pencil that has a motion proportional to the rise and fall of the float. The pencil bears against a piece of graduated paper fastened to a drum that is revolved by clockwork. There will thus be drawn on the paper a profile in which the horizontal units are time, and the vertical units are feet, rise, and fall. The stage of the tide for any instant can be read from the profile.

274. Locating the soundings. (1) The soundings may be located by two angles read simultaneously from the opposite ends of a line on shore, the recorder in the boat signaling the observers on shore when a sounding is about to be taken, and again when it is taken.

(2) The soundings may be located by establishing two flags on shore in a line nearly normal to the shore line and taking the

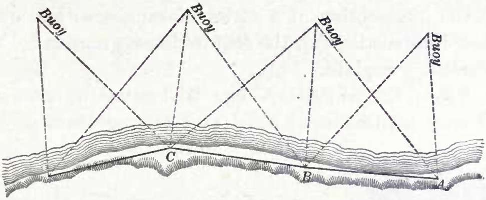


FIG. 139.

soundings in line with the range thus formed, the position of the boat being determined by an angle read on shore, or by time intervals. A series of such ranges is laid out, and soundings are taken along each range. The point selected for occupation by the angle-measuring instrument on shore should be so chosen that the angles of intersection with the range lines will be large enough to give good locations. If the shore is narrow and precipitous, the range may be established by placing a flag on shore and a buoy in the

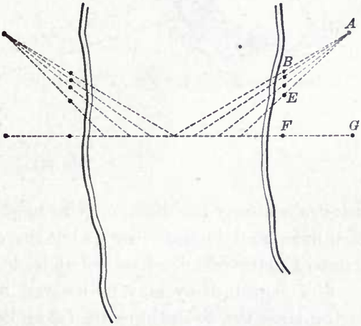


FIG. 140.

water. The position of the buoy is determined as was described in the first method for locating soundings. The distances between the shore points will be measured for base lines, and the angles will be measured from these base lines. Fig. 139 will explain the method. The buoys may be any visible float, anchored in place by any heavy weight attached to the buoy by a rope. The rope must be sufficiently long to permit the buoy to be seen at high tide. At low tide the position of the buoy will not be certain, for obvious reasons. A good kind of buoy is a tapering spar of wood. If it is not readily visible, a flag may be set in the end.

(3) If many soundings are to be taken at different times on one cross section of a river, they may be located by taking them at the intersection of a series of ranges so laid out as to have their intersections on the required cross section. Fig. 140 will sufficiently explain.

(4) Again, the soundings may be located by stretching a rope or wire across the channel and taking them at marked

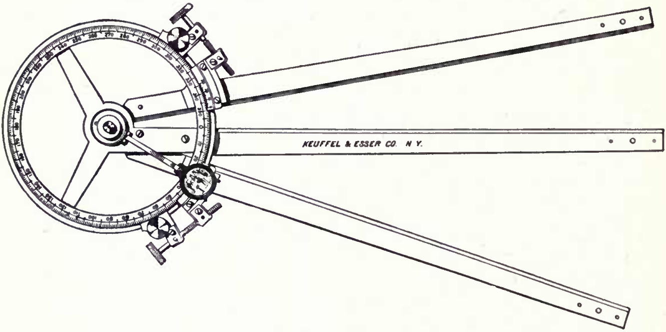


Fig. 141.

intervals along the line. This method is adapted to narrow channels, and is used somewhat in connection with measurement of materials dredged in such channels.

(5) Again, they may be located by measuring in the boat, at the time the soundings are taken, two angles to three known points on shore. The angles are measured with a sextant. There are several methods of obtaining from the data thus secured the location of the point where the sounding is taken. The problem to be solved is known as the "three-point problem."

The most convenient way to plot the point on a map on which have been already plotted the known shore points, is by the use of a three-arm protractor, shown in Fig. 141. The two measured angles are set on the protractor and the arms are made to coincide with the plotted points. The center then gives the required position of the sounding.

If no three-arm protractor is available, draw three lines on a piece of tracing paper so that the two measured angles are included between the lines. Shift the paper over the map till the three lines pass through the three points. The point from which they radiate is now over the position of the sounding, and may be pricked through.

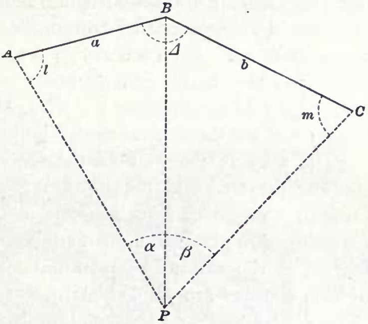


FIG. 142.

The position of the point may be found algebraically as follows: In Fig. 142, the known points are A, B, C, determined by the angle Δ, and the sides a and b. The measured angles are α and β, and the required point is P. From the triangles APB and CPB there is obtained

$$PB = \frac{a \sin l}{\sin \alpha} = \frac{b \sin m}{\sin \beta}. \tag{1}$$

Also, $$l + m = 360° - (\Delta + \alpha + \beta) = S, \tag{2}$$

whence, $$m = S - l,$$

and $$\sin m = \sin S \cos l - \cos S \sin l. \tag{3}$$

From (1) and (3) determine l, which being found, the triangle APB may be solved and P located. To solve for l, substitute in (1) the value of $\sin m$ found in (3), reduce to common denominator, divide by $\sin l$, transpose, and get

$$\cot l = \frac{a \sin \beta + b \sin \alpha \cos S}{b \sin \alpha \sin S} = \cot S\left(\frac{a \sin \beta}{b \sin \alpha \cos S} + 1\right). \tag{4}$$

275. Occurrence of methods. The fifth method is used almost exclusively in locating soundings in bays, harbors, and off the seacoast. The others are used in connection with river and small lake surveys.

276. Survey of a harbor, river, etc. In making a complete map of a harbor, coast, or river, a system of triangles of greater or less extent is laid out. The triangulation stations thus locate the prominent points. These points are then used as a basis for locating the soundings. If too far apart, other points are located along shore by running a traverse between triangulation stations. Such a traverse is usually run in any event to determine the detail configuration of the shore line and the topography adjacent thereto. This work is best done with the transit and stadia or with the plane table.

The map is finished like any topographical map. The contour lines of the bottom are frequently drawn as dotted lines to a depth of four fathoms, and beyond that depth the soundings are given in figures expressing fathoms and quarters. Charts of the various harbors and many portions of the coast of this country are published by the United States Coast and Geodetic Survey, and may be purchased for a small price. On these charts are located buoys and beacons. These are established to denote shoals and rocks, and are so arranged that in entering a harbor a red buoy with even number is to be passed on the right; a black buoy with odd number is to be passed on the left, and buoys with red and black stripes may be passed on either the right or the left. Buoys in channels are painted with black and white vertical stripes.

Beacons and buoys are different things. A beacon is a permanent fixed signal, usually on a shoal or dangerous rock; while a buoy is a float of some kind, anchored by a chain. It is used to denote either danger or channel.

THE SEXTANT.

277. Description. It has been said that when the angles are read in the boat, they are measured with a sextant. The sextant is shown in Fig. 143. It consists of an arc of sixty degrees, but, from the principle of the instrument, reading angles up to one hundred and twenty degrees. *I* is a mirror attached to the movable arm that carries the vernier *V*. The arm is centered under the mirror. This mirror is called the index glass. The vernier reads to ten seconds. *H* is another glass, called the horizon glass. Its lower portion is a mirror, and its upper portion is unsilvered. *GG* are colored glasses to protect the eye when making observations on the sun. They may be turned back out of the way when not needed.

278. Use. To use the sextant to measure an angle between two terrestrial objects, hold the plane of the arc in the plane of

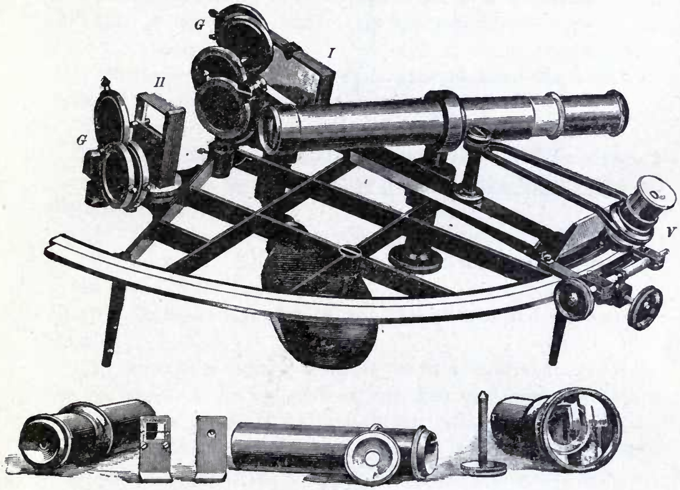


Fig. 143.

the observer's eye and the two points. The telescope should be directed toward the fainter object. It may be necessary to hold the sextant upside down to do this. Swing the vernier arm till the image of the second point reflected from *I* to *H* to the telescope is seen superimposed on the fainter object seen directly. Clamp the vernier, bring the images into exact coincidence, using the tangent screws, and read the vernier. The reading is the angle sought. It will be observed that the angle read is not horizontal, unless the distant points are in the same level with the observer. It requires some practice to become expert in the use of the sextant. The eyepieces shown in the figure are of different kinds. There is usually one astronomical eyepiece, one for terrestrial work, and one without magnifying power.

The instrument is that used by seamen for observing for latitude and longitude.

279. Theory. The principle on which the sextant is constructed is as follows: If a ray of light is reflected successively

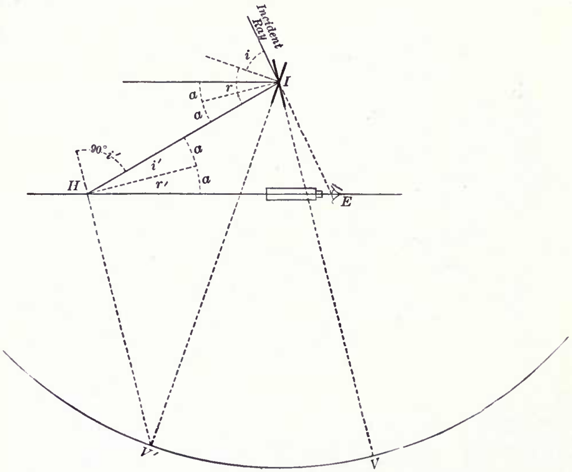


FIG. 144.

from two plane mirrors, the angle between the incident and finally reflected ray is twice the angle of the mirrors.

Referring to Fig. 144, since the angles of incidence and reflection are equal, $i = r$ and $i' = r'$, and from Geometry

$$E = (i + r) - (i' + r') = 2(r - r'),$$

and $$V' = (90° - i') - (90° - r) = (r - r').$$

Therefore $E = 2V'$, which was to be shown.

To show that the angle read by the vernier on the arc or limb is the angle V', suppose the two mirrors to be parallel and the telescope to be directed to an object infinitely distant, so that the rays from it are parallel. An image of the object will be seen in the same line as the object, or will appear superimposed on the object. This will be evident from the equality of the angles a. The vernier will then be in the position V and this point of the arc is graduated *zero*. If now the vernier is moved to another point V' for the purpose of getting the reflection of a second point to cover the direct image of the first point, the angle VIV' will equal that at V', the angle of the mirrors. It is, however, the angle at E that is required, and this is twice the angle through which the vernier has moved. Hence the arc VV', instead of being numbered to give the angle V', is numbered to give at once the angle E. That is, each degree is marked two degrees, etc.

280. Adjustments. There are four adjustments of the sextant:

(1) To make the index glass perpendicular to the plane of the limb.

(2) To make the horizon glass perpendicular to the limb.

(3) To make the line of collimation of the telescope parallel to the plane of the limb.

(4) To make the vernier read zero when the mirrors are parallel.

(1) Bring the vernier to about the middle of the arc. Hold the eye at about the upper G of Fig. 143, and observe the arc near the zero point directly, and the reflected image of the other end of the arc in the index glass. If the glass is perpendicular to the plane of the limb, the reflected and direct portions will seem to form one continuous arc. Adjust the glass, if necessary, by means of the screws at its base. It may be necessary to place slips of thin paper under the base.

(2) Direct the telescope toward a star or other very distant object and note whether the direct and reflected images seem, when the vernier is moved, to separate or overlap laterally. If they do not, the horizon glass is in adjustment. The adjustment is made, when necessary, by the screws of the glass.

(3) Place the sextant on a plane surface and direct the telescope to a point not far away. Place the two peep sights shown in Fig. 143 on the extreme ends of the limb and note whether the line of sight through them coincides in elevation

with the line of sight of the telescope. If not, adjust the telescope by the screws in its collar. Any other objects of equal altitude will serve as well as the peep sights.

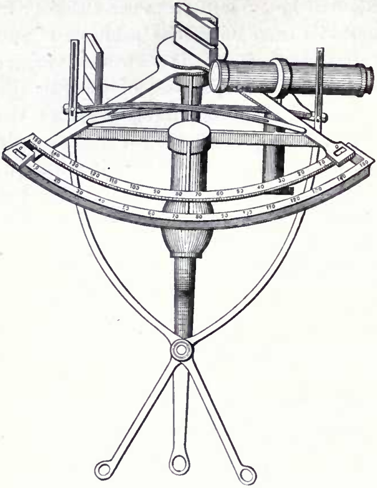


FIG. 145.

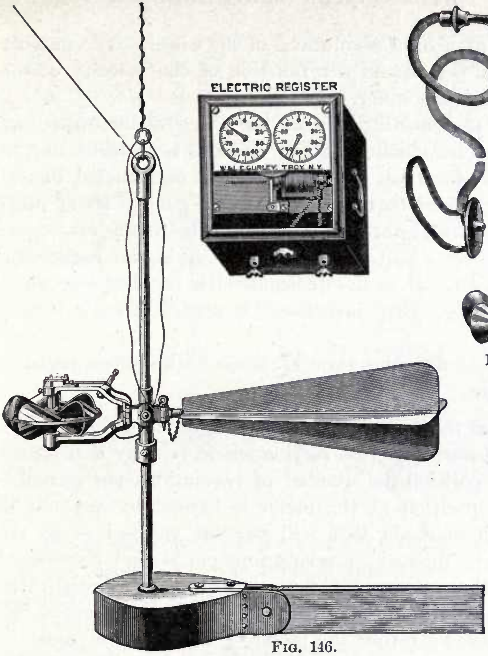
(4) Bring the direct and reflected images of a very distant point to coincide, and read the vernier. It should read zero. If it does not, note the error as an index correction to be applied to all angle readings. The error may be corrected by adjusting the horizon glass, but this is not usually done.

281. Other forms. By a double sextant, Fig. 145, two angles can be measured quickly by one observer, one sextant measuring one angle and the other sextant, the other.

MEASURING VELOCITY AND DISCHARGE.

282. Position of maximum velocity. The velocity of a stream varies in different portions of a cross section and in different cross sections. The maximum in a cross section is, when there is no wind, at about one third the depth in the middle of the channel. The surface velocity may be greater if the wind is favorable. To determine the mean velocity in a given cross section, the velocity in many parts of the section must be found. This is best done with a current meter.

283. Current meters. These are of various patterns. Those shown in Figs. 146–148 are considered good forms. Fig. 146 is a meter devised by W. G. Price, United States assistant engineer. There is an electric connection with a register that



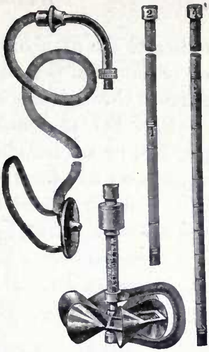


FIG. 147.

FIG. 146.

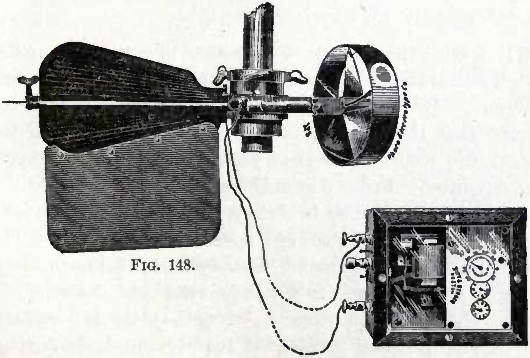


FIG. 148.

indicates the number of revolutions of the wheel. The number of revolutions per second is a function of the velocity of the current through the meter.

Fig. 147 is a modification of the Price meter, called an audible, or acoustic, meter, and for streams is probably as convenient as any form yet devised. It is so constructed that at each tenth or twentieth revolution a small gong is struck, and the sound is carried through a rubber tube to the ear of the observer above in a boat. This meter requires no registering apparatus at all. It is not recommended for deep-sea work, in which a strong, heavy instrument is needed, since it is very light.

Fig. 148 is a different form of meter with electric registering attachment.

284. Use of the meter. In use the meter is simply held in different positions in a cross section whose velocity is required, and in each position the number of revolutions per second is noted. The position of the meter is located by any one of many possible methods that will suggest themselves to the surveyor. The number of revolutions per second bears some relation to the velocity of the current going through the propeller-like wheel of the meter, and the determination of this relation is called "rating the meter." Every meter must be rated before the velocity of current corresponding to an observed velocity of wheel can be told.

285. Rating the meter. The meter is rated by moving it through still water at a known rate and noting the revolutions per second. By moving it at different velocities it will become apparent that the velocity is not strictly proportional to the speed of the wheel but bears a relation that must be expressed by an equation. From a number of observations at different velocities, a diagram may be drawn that will give the velocity corresponding to any observed speed of the wheel. This is done as follows: The meter is moved over a known distance and a stop watch is used to determine the time occupied. The distance divided by the time in seconds gives the velocity in feet per second. The number of revolutions made during the time is recorded, and this divided by the time gives the number of revolutions per second. The following observations were made by Mr. Price, all on a distance of two hundred feet.

In the table, R is the whole number of revolutions in the two hundred feet, T the whole time, r the revolutions per second, and v the velocity per second.

No.	R	T	r	v
1	100	53	1.886	3.774
2	101	44	2.295	4.544
3	101	41	2.464	4.878
4	96	124	0.774	1.613
5	94	152	0.618	1.316
6	90	193	0.466	1.036
7	91	181	0.503	1.105
8	103	28	3.678	7.142
9	100	53	1.886	3.774
10	98	73	1.342	2.740
11	193	29	3.552	6.896
			11)19.464	38.818
			1.769	3.529

The average speed of the wheel is seen to be 1.769 revolutions per second, and the average velocity of flow, or current, is 3.529 feet per second.

Two axes at right angles are now drawn. On one of these axes are laid off the several values of r, and parallel to the other are laid off the corresponding values of v. In Fig. 149, the horizontal axis is that of r, and the vertical axis is the velocity axis. In the first observation, r is 1.886 and v is 3.774. Assuming suitable scales the observations are plotted in the same way as a point by latitudes and longitudes. Thus, the latitude of the first observation is 3.774 and its longitude is 1.886. If cross-section paper is used the work will be facilitated. Each observation being plotted, it is observed that they all fall nearly on a straight line. It may be assumed that they should all fall on the straight line and that they do not because of small errors in the observations. If the mean values of r and v are plotted it may be assumed that the line should pass through the point thus found and that it must then be swung to average the other points as nearly as may be. A piece of fine thread is stretched through the point of average r and v, and swung around till it seems to the eye to average the other points. This line is that on which it is most probable that the observa-

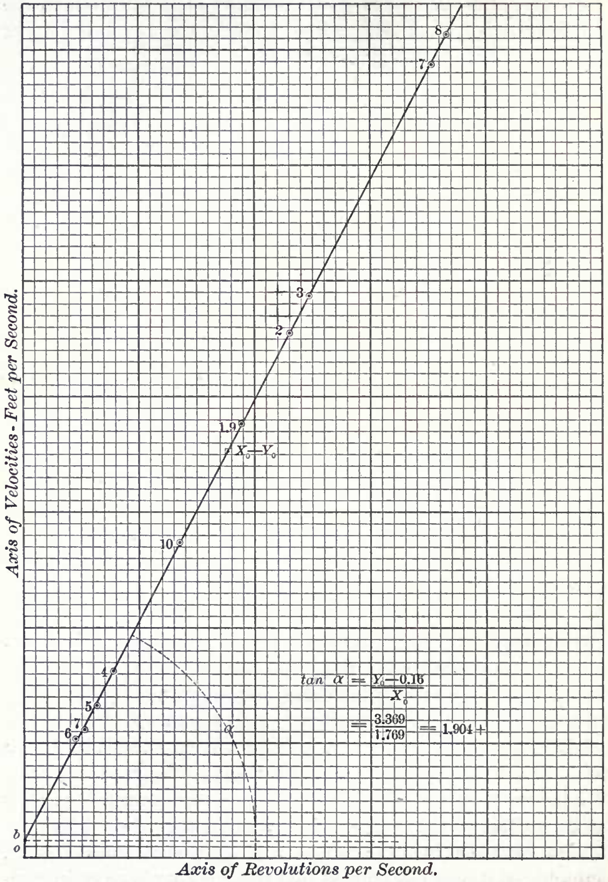


FIG. 149.

tions should all fall, and hence if it is drawn and the meter is held in a running stream and the revolutions per second are noted, the corresponding velocity may be determined by laying off the observed r on its axis and measuring the corresponding v up to the line. If a horizontal line is drawn on the diagram through b, where the line cuts the axis of v, it will be seen that for any value of r the corresponding value of v is $r \tan \alpha + bo$. If b represents bo, we may write

$$v = r \tan \alpha + b.$$

From the values of the average r and v in the example given

$$v = 1.904\, r + 0.16.$$

In its general form this equation is

$$v = ar + b,$$

and this is known in Analytical Geometry as an equation of a straight line. This equation is the general equation of the relation between v and r. The rating of the meter then consists in finding values for a and b from observations that give v and r. Since there are two unknown quantities, two independent observations should be sufficient; but since no observation can be perfect, many are taken, and mean values are determined for a and b. This may be done analytically by the method of Least Squares.

286. Rod floats. A very good method of obtaining velocities, when a current meter is not available, is to observe the velocities of rod floats in various parts of the stream. For this purpose two ranges are established on shore, one above and one below the section in which it is desired to measure the velocity. The ranges are laid out normal to the stream. A transit is placed on each range, and the rod is started just above the upper range. The transit man on the upper range signals the lower observer when the rod is about to cross the upper range, and just as it crosses; and the lower observer reads an angle to the rod at the instant it crosses the upper range. The operation is repeated when the rod crosses the lower range, the lower observer making the signals, and the upper observer reading the angle to the rod. Each notes the time of crossing the range on which he is stationed. In this way the path of the rod and the time it takes to travel that path are known. The rod may be of wood, or it may be a long cylindrical tin can. In either case it should be weighted so as to remain vertical and just clear the bottom. The velocity of the float will then be approximately that of the vertical filament of the stream in which it floated. The immersion of the rod should be about nine tenths of the depth of the stream. If this rule is observed, the mean velocity in the vertical filament will be given by $V_m = V_o[1 - 0.116(\sqrt{D} - 0.1)]$[1] in which V_o is the observed velocity of the rod, and D is the ratio of the depth of the water below the bottom of the rod to the total depth of the water.

In small streams, wires or ropes graduated with tags may be stretched across the stream at the upper and lower ends of the stretch of river to be observed, and the time of passing of the float under each wire or rope may be observed.

By observing the velocities in many vertical filaments, the mean velocity of the section may be determined. It would not be a mean of the several observed velocities unless all "filaments" had equal areas. The mean velocity is that which multiplied by the area will give the discharge, hence it is $\frac{\text{Discharge}}{\text{Area}}$.

287. The discharge. To get the discharge, simply multiply the mean velocity of each filament by its cross-sectional area, and add the products. If the velocity is determined by meter, the work need be done in but one cross section; if by rod floats, it will usually be best to measure two or more sections, preferably at the quarter points of the stretch, from which to get a mean section.

DIRECTION OF CURRENT.

288. How determined. It is sometimes necessary to know the direction of both surface and subsurface currents. The necessity may arise in the determination of the proper place to discharge sewage, or in surveys for the improvement of harbors and the approaches thereto. The surface currents may be determined by watching floats. The subsurface currents are best found by a direction meter. This is a form of meter that not only gives velocities, but also shows the direction of the current in which it is placed. It was devised by Messrs. Ritchie and Haskell. It is said to be one of the best forms of meters for measuring currents alone.

[1] Francis's "Lowell Hydraulics."

CHAPTER XII.

MINE SURVEYING.

SURFACE SURVEYS.

289. Coal mines. The surface surveys in connection with coal mining operations consist in making land surveys of the property that may be owned by the mining company, and locating all shafts, tunnel openings, buildings, railways, roads, and other structures. In addition to this it is well, if the area owned is extensive, to make a complete topographical survey of the property. Frequently a mining company mines coal under the lands of many small holders and pays a royalty for the coal so mined. It is therefore necessary to know the positions of the land lines of all owners under whose land it is probable that operations may extend. The location of these lines is best performed by careful stadia surveys — reading the stadia to the smallest possible unit. (There may be cases of sufficient importance to warrant most careful transit and tape work.) The topographical survey may be carried on at the same time. Indeed, the location of the land lines becomes merely an incident in the topographical survey. The positions of the corners are determined by computing their latitudes and longitudes.

Among other points that will be located in the topographical survey or land survey will be the positions of exploration drill holes that have been put down in advance of mining operations for the purpose of locating veins, or beds, as they are frequently called. The entire survey may or may not be hung on a system of triangles. If it is of any considerable extent, it is best to reference it thus for the sake of the checks it gives on the field work, even though not more than two or three triangles are used. The whole work, except the side shots, is best plotted by latitudes and longitudes, these being computed with sufficient accuracy for this purpose by the aid of a trigonometer. The side shots, except those to land corners, may be plotted with a protractor. The surface map should show all outcroppings of mineral.

290. Metal mines. Surface surveys for metal mines may be of the same character as those for coal mines, but in the United States there are certain laws governing the size and form of mining claims that are located on public land, and the method of surveying them.

291. Form of mining claim. By the provisions of these laws, a mining claim may not exceed fifteen hundred feet in length along the vein nor six hundred feet in width (three hundred feet each side of the vein). It may be of any length or width less than these limits, according to local custom; but no rule may be made that will compel the claim to be less than twenty-five feet on each side of the vein. The side lines need not be straight or parallel, but the end lines must be both. The miner has the right to follow the vein he has discovered into the ground to any depth, even should it depart so much from the vertical as to pass beyond the vertical planes through the side lines; but he may not mine beyond the vertical planes through the end lines.

When claims intersect, the subsequently located claim is said to conflict with the previous one and carries with it only the area that is not in conflict. The foregoing rules apply to what are called lode claims.

A "lode" is the name sometimes given to a vein of ore.

A "placer" claim is a claim taken for the purpose of obtaining gold or silver or other metal from the surface materials, as in washing gold from a river bed or from gravel in the mines worked by hydraulic process in California. Such a claim may not exceed twenty acres, and, if located on surveyed lands, its lines must conform to the legal subdivisions. A placer claim carries no right to mine beyond the vertical planes through its bounding lines. It carries the right to mine a vein discovered within its boundaries after the claim is located, but not if the vein is known to exist when the claim is located, unless specially mentioned.

292. Surveying the claim. When a miner has found a vein of ore, he has a claim staked out on the ground, conforming to the legal requirements, using a width and length that he thinks best. He also posts the notices required by law. This survey may be made by any surveyor. After the miner has expended five hundred dollars in working his claim he is entitled to have it deeded to him by the government. This deed is called a "patent," and the claim is said to be "patented." For the purpose of this patent, a survey is made, from which to write a description. This final survey, on which the deed to the miner is based, must be made by a United States officer, known as a United States deputy mineral surveyor. Such officers are under bonds of ten thousand dollars, and are required to pass an examination before receiving their licenses. They make the final survey and tie it to a corner of the public surveys or to a special mineral survey monument, and return to the surveyor general of the state the notes, with an affidavit that the required five hundred dollars have been expended on the mine.

The form of field notes and the instructions issued to United States deputy mineral surveyors by the surveyor generals of the various mining states may be had on application to the surveyor general's office.

293. Surface monuments. The surface monuments of a mine that is to be worked for any length of time should be of a permanent character. At least two of them should be located near the entrance to the mine, and these two should be particularly permanent, for use in connecting the underground and surface surveys. These may be auxiliary monuments located for the express purpose of connecting the two surveys. They may be built of stone or brick, with a properly centered copper bolt to mark the exact spot. The bolt may be removable for the purpose of substituting a signal. These monuments must be constructed with a view to being proof against disturbance by frost, or travel, or by the operations about the mine. A true north and south line permanently monumented is perhaps the best base for all surveys.

UNDERGROUND SURVEYS.

294. General statement. Underground surveys consist of traversing, leveling, and sometimes measuring volumes. The necessary traverses are run to determine the location of the various portions of the mine, so that a map or a plan of the mine can be made. Leveling is done to determine the relative elevations of the different parts of the mine, the grade of the tunnels or drifts, the direction and amount of dip of the vein, etc. The quantity of ore mined is sometimes determined by measuring the volume excavated in the mine, and the measurement of the quantity of ore "in sight" is usually by the volumetric method. The ore mined is usually measured by weight after it is brought out of the mine.

The only differences between underground and surface surveys are due to the difficulties encountered in working in dark, cramped passages. These necessitate certain modifications in surface methods to make them applicable to underground work.

295. Definitions. The following are a few definitions of technical terms used in mining.[1] There are many others that the mining surveyor must know.

Shaft. A pit sunk from the surface.

Adit. A nearly horizontal passage from the surface, by which a mine is entered and through which the water that collects in it is removed. In the United States an adit is usually called a tunnel, though the latter, strictly speaking, passes entirely through a hill, and is open at both ends.

Drift. A horizontal passage underground. A drift follows the vein, as distinguished from a cross-cut, which intersects it, or a level or gallery, which may do either.

Level. A horizontal passage or drift into or in a mine. It

[1] Taken from "A Glossary of Mining and Metallurgical Terms" by R. W. Raymond, mining engineer.

is customary to work mines by levels at regular intervals in depth, numbered in their order below the adit or drainage level.

Cross-cut. A level driven across the course of a vein.

Winze. An interior shaft, usually connecting two levels.

Strike. The direction[1] of a horizontal line, drawn in the middle plane of a vein or stratum, not horizontal.

Dip. The inclination of a vein or stratum below the horizontal. The dip is necessarily at right angles[2] with the strike or course, and its inclination is steeper than that of any other line drawn in the plane of the vein or stratum.

Pitch. The inclination of a vein, or of the longer axis of an ore body.

Incline or *Slope.* A shaft not vertical; usually on the dip of a vein.

Stope. To excavate ore in a vein by driving horizontally upon it a series of workings, one immediately over the other, or *vice versa.* Each horizontal working is called a stope (probably a corruption of step), because when a number of them are in progress, each working face being a little in advance of the next above or below, the whole face under attack assumes the shape of a flight of steps. The process is called *overhand* stoping, when the first stope is begun at a lower corner of the body of ore to be moved, and, after it has advanced a convenient distance, the next is commenced above it, and so on. When the first stope begins at an upper corner, and the succeeding ones are below it, it is *underhand* stoping. The term "stoping" is loosely applied to any subterranean extraction of ore except that which is incidentally performed in sinking shafts, driving levels, etc., for the opening of the mine.

296. Location and form of station marks. In underground work, stations can not be stakes driven in the floor of the mine. The reasons are obvious. Where the roof is solid rock, holes may be drilled in it and wooden plugs inserted, in which may be driven a nail or tack. A good form is a horseshoe nail with flattened head, in which is cut a triangular hole with an apex of the triangle toward the head end of the nail. Such a nail

[1] Azimuth.

[2] That is, the direction of the vertical plane in which the amount of the dip is measured is at right angles.

driven in the roof insures that a plumb line suspended from it will always hang from the same point. A small round opening about one eighth inch in diameter is preferred by some. When the roof is soft and the drift timbered, temporary stations may consist of nails driven in the timbers. It is probably better in such cases to reference the station by two points on the side of the drift. If the two points are properly located, and the distance from the vertical through the station to each of them is recorded, the station may be recovered at any time.

When about to occupy a station, transfer it to the floor with a plumb line and set there a temporary mark consisting of a piece of slate with a cross scratched on it, or a small metallic cone carried for the purpose. Stations underground should be plainly marked. White paint is good if kept bright. The stations in a mine should be consecutively numbered, and the same number should not be used twice. They should be numbered at the station points as well as in the notebook. Probably as good a way to do this as any is to drive round-headed nails or tacks in the timbering near the station, locating them all in the same position relative to the station. The tacks may be driven in a small board or plug fastened to the side of the drift near the station. The nails are arranged to give the number of the station, thus ⋮:⁞ for 324 and ⋮0: for 302, a washer being used for the 0. Practically this method was followed in the New Almaden mine in California. The nail heads can be felt and read, even though not clearly visible.

As the mine is extended and new stations are added, they should receive their proper numbers and be permanently located. There is no branch of surveying where method and system count for so much as in mine surveys.

297. Instruments used. For all angle work in mine surveys, except the location of short tortuous passages, the transit is the only suitable instrument. The work must be done with great precision. Results need not be so precise, perhaps, as in high class city work, but a precision of one in five thousand should certainly be secured. It is best to use azimuths and perform the traverses as already described under traversing with the transit.

The transit should be a complete transit or tachymeter, so that leveling may be done either directly or by vertical angles. The vertical angle method is the better ordinarily, as it saves much time in the mine. The tripod should be either short or adjustable in length. A diagonal eyepiece is desirable for pointings at high inclinations. The wires are made visible by means of a reflector described in Art. 90, or by a hollow axis and reflector within the telescope. Often enough light is secured by having an assistant flash a candle near the objective. It is sometimes necessary to point upward or downward at a considerable inclination, as when an incline is to be traversed. Pointing up is accomplished by means of an auxiliary diagonal eyepiece, but pointing down can not be done when the angle is so great that the line of sight cuts the plates of the instrument.

Various devices have been invented to overcome this difficulty. Sometimes an auxiliary telescope is mounted above, and parallel with, the main telescope of the transit, as in Fig. 150. In some transits, the auxiliary telescope is mounted on the end of the horizontal axis, as in Fig. 151. Some transits are made with the standards leaning forward far enough to permit the line of sight to clear the

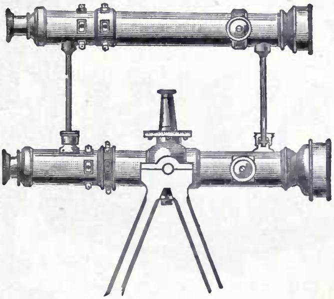


FIG. 150.

plates when turned vertical. Others are made with double standards, one as usual, and one bracketed to the ordinary standards. The telescope is made so as to be readily changed from one bearing to the other. If the side telescope is used, the transit should be set over a point to one side of the station occupied, or the line of collimation of the auxiliary telescope should be directed to a point as much to one side of the true station as the auxiliary telescope is to one side of the main telescope. This is conveniently done by having the object sighted to, as a candle, fastened to a flag at the proper distance from it. The flag is held on the true station.

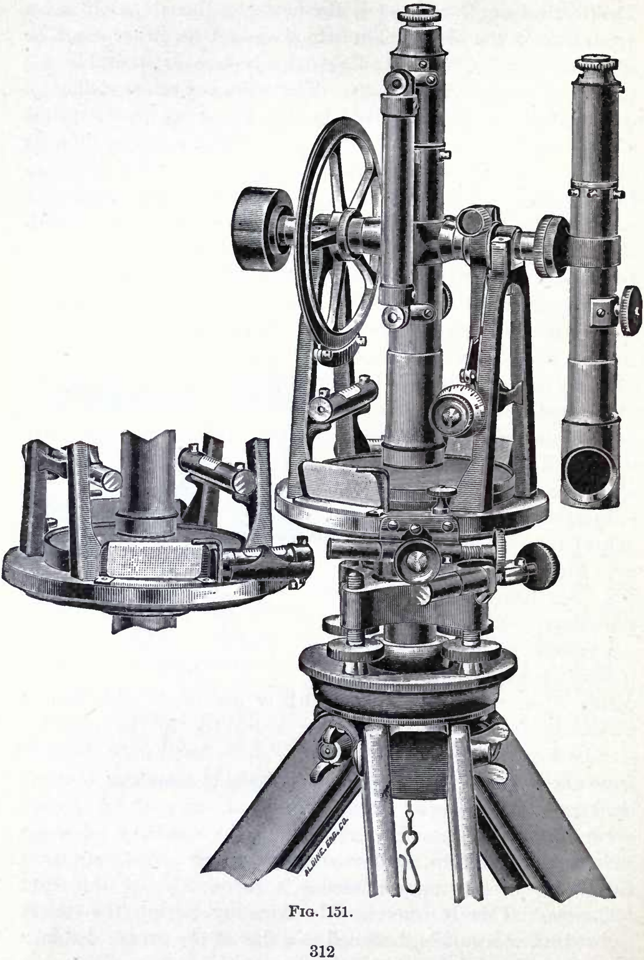


Fig. 151.

For tortuous passages in which a transit can not be set up, so crooked that one end can not be seen from the other, resort is had to what is sometimes called a German dial, and a hanging clinometer. One of the best forms of the dial is shown in Fig. 152.[1]

The compass is hung on a wire or line stretched from one station to the next, and as it levels itself like a mariner's compass, it may be properly read. The bearings are not depended on, but angles are read. Passages so tortuous as to require the use of such methods are not usually of great length, and the resulting errors are small. They are not likely to occur in coal mines, but occur frequently in quicksilver mines, and sometimes in gold and silver mines. Coal mines usually involve fewer irregularities of alignment than any other class of mines.

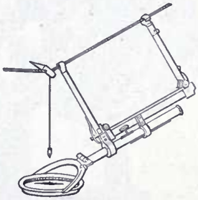


FIG. 152.

The linear measurements are made with a steel tape. In some cases of long, straight drifts, a long tape, two hundred feet, or three hundred feet, may be used to advantage; but usually a tape either fifty feet or one hundred feet long is best. The tape should be as long as possible, and no more stations should be used than seem necessary. It is very necessary to expedite work in a mine, because mining work in a drift must be largely suspended while the survey is in progress. The flat wire tapes are best for mine work, since they are not so easily broken as the ribbon tapes. A pocket tape, graduated to hundredths of feet, may be carried for the necessary plus measurements at stations.

If a level rod is used, it must be a short rod. Bench marks are best placed on the side of the drifts where they will not be disturbed.

298. Devices for making stations visible. The object observed at a station is usually a plumb line made visible by holding a piece of oiled paper or milk glass behind it and a candle behind the transparency. The plumb line may be illu-

[1] From Brough's "Treatise on Mine Surveying."

minated by holding a light in front of it, shielded from the observer.

Another method is to sight the flame of a candle or lamp, properly centered over the station. Another way is to use a plummet lamp, which is a lamp made in the form of a plummet. Perhaps the neatest method, at least in coal mines, is to use a second and third tripod on which can be mounted a target lamp, such as is shown in Fig. 153. The lamp is behind the target from the observer, and the target may be accurately centered over the station point. The target and transit are interchangeable on the tripods, the particular form shown being used with a special transit that is interchangeable above the leveling screws. The method of use is somewhat as follows: The back station is occupied by a target, as is also the forward station, and that at which the observation is to be made is occupied by the transit. When the observation is complete, the tripod at the back station is carried to the next forward station, the others remaining in place. The back lamp is placed on the tripod that was occupied by the transit while the latter is being carried to the tripod occupied by the previous forward lamp, which in turn is carried to the next forward station. The lamps should neither of them be moved till the observations at the station occupied by the

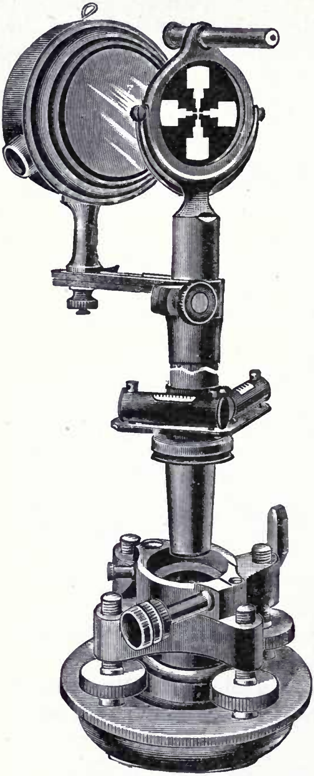


FIG. 153.

transit are complete. This is a very expeditious method of work.[1]

The entire angular work may be done first, and the linear measurements made afterward, or *vice versa*, or the two may be carried on at the same time. The latter method will probably consume more of the miner's time, since the linear work would retard the angle work. The linear work may be carried on without seriously interfering with the work of the mine. The extent or character of the work will guide the surveyor in choosing his methods. When lateral drifts depart at frequent intervals from one long tunnel or drift, the stations at the angle points of the main drift should be first located and the substations to be used in running the lateral drifts should be set in the line of the main drift. This may be done at the time the angles of the main drift are measured. In this case, the angle work should be done first.

When an extensive survey of a mine is to be made, the surveyor should go through the mine and lay out his programme of work as closely as may be. It will almost always be possible to mark out the main stations before taking the transit into the mine, or at least before setting it up.

299. Notes. In an underground traverse in which notes are to be taken at intervals along each course to objects on the sides, a good form is as follows: Prepare the page of the notebook by ruling a column down the center, in which to write distances and azimuths, and leave the sides to note the objects measured to, and such other information as need be recorded. The form would be similar to that shown on page 229, except that the column for distances would also contain the azimuths and would be in the middle of the page instead of at one side. Notes taken on the right of a line that is run out should be noted on the right of the column, and those taken on the left should be recorded on the left.

[1] An important point in favor of the three-tripod system, when long tapes are used for measuring, is that all ordinary distances may be measured from center of the telescope to center of back-sight or fore-sight flame, the vertical angle being read and noted. Having then the length of the line of sight and its angle, the true horizontal distance between stations is easily calculated. Oftentimes this is a much safer course than to rely on presumably horizontal measurements along the floor of the gangways.

J. J. Ormsbee, *Mining Engineer.*

A line should be drawn across the page between each two courses.

For the purpose of computation of latitudes and longitudes, the azimuths and distances may be copied in more compact form in the office notebook.

If elevations are carried by vertical angles, a good form is that given in Art. 127 for traversing with the transit and stadia, with sufficient vertical space given each course to write the details of the line along the side or right-hand page under remarks. The center line up the middle of the right-hand page may be used to represent the transit line, and the objects may be sketched to scale on this page. There should always be two sets of notes in existence, kept in different places, and when not in use they should be stored in fire-proof vaults.

CONNECTING SURFACE AND UNDERGROUND WORK.

300. When the mine is entered by a tunnel. In order that the underground workings may be plotted with reference to the surface lines, or even so that they may be properly oriented, it is necessary to connect the underground surveys with those on the surface. The simplest case occurs when the mine is entered by an adit or drift. From a point in the surface survey, located conveniently for the purpose, run a traverse into the drift.

301. When there are two shafts. If the mine is entered by shafts, and there are two or more of these some distance apart, the surface and underground surveys are connected by running a traverse on the surface and another underground between the shafts. The azimuth of the line joining the shafts is computed from the surface survey; and that of the same line underground, from the underground survey, a zero azimuth being assumed for this purpose. The two computed azimuths will differ by the angle between the true zero and the assumed zero, and the assumed underground azimuths may be corrected accordingly. The precise points at the shafts are made the same by using plumb lines. In deep shafts the lines are best made of piano wire, and the plummet must weigh from ten to twenty-five pounds.

The line is first lowered with a light bob, and the heavy one is attached at the bottom. The line may be lowered by a reel. Signals may be given for lowering or raising by tapping on the guides or pipes in the shaft. Wherever possible, this is an excellent method, since small errors of position of the bobs are not multiplied.

Another method, that may be applied in shallow mines, is to use the transit and auxiliary telescope, either setting the points in the bottom in line with a given line on top, or setting the transit in the bottom by trial, so that the line of collimation may be in line with a surface line. Perhaps a better way than the latter is to set the transit on the bottom and, with the horizontal motion clamped, set two points in a line on opposite sides of the shaft at the surface, which line is produced into the mine as the first course of the traverse. The points located on the surface are afterward connected with the surface survey. This method of using the transit and auxiliary telescope is adapted only to shallow shafts that are tolerably free from smoke. The better method is that using the plummets. Work may be carried from one level to another through winzes and air shafts by the same method. As high vertical angles occur in this work, double centering is an absolute essential to correct work.

Levels are carried down the shafts by measuring with a steel tape, or, what is the same thing, by lowering a weighted wire and then measuring the wire. If the tape is used, and is shorter than the shaft, it is first hung on a nail near the surface, the elevation of the nail being determined by leveling. The reel and observers are then slowly lowered in the cage until the tape is all paid out, when a second nail is driven at a measured distance from the first. A piece of white cloth is fastened to this second nail, to make it visible; and the cage is raised, the tape unhooked, the cage again lowered to the second nail, and the operation repeated from there down. A nail is left for a bench mark opposite each level.

302. When the mine is entered by one shaft. To connect a surface survey with an underground survey through a single deep shaft is the most difficult task the mining surveyor will encounter. If the mine is of small extent and the shaft shallow, it may be done with the transit and auxiliary telescope, as already described under the preceding article. But if the shaft is deep, it can not be done this way. The method is to use two plumb lines. The utmost care is necessary in their use, for from a base line from six to twelve feet long must be produced a survey one or two or more miles long. It may be necessary to drive a second shaft a mile or two distant, and it must be driven to meet an underground drift. An error of one tenth of an inch in a base line ten feet long will cause a shaft two miles distant to be driven nearly nine feet out of place. Two methods of using two plumb lines have given satisfactory results.

(1) In Fig. 154, M and M' are surface monuments, and AB the two plumb lines. The distance AB is carefully measured, and the distance MA and MB and the angles at M. The azimuth of AB is then determined. T is the transit set in the mine below. The angles at T, and the distances AT and BT are measured. The triangle TAB is then solved, from which the azimuth of TD becomes known.

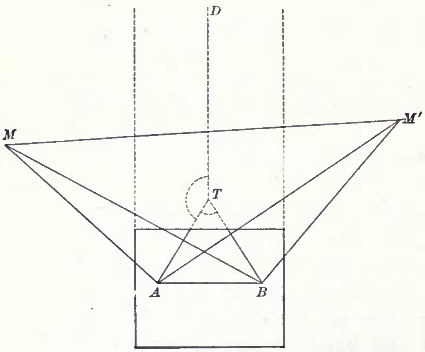


FIG. 154.

(2) This method is probably not so good as the second, which consists in suspending the plumb lines so that the transit may be set up in line with them below. The difficulty in all this work is that the plumb lines are never still, but continually oscillate. The mean position of the line is best determined by placing a fine scale behind it and noting the amplitude of the vibrations. The transit is pointed to the

mean position. The oscillations are not in a straight line, but are more or less elliptical; hence the scale must be placed a little behind the wire, but should be as close as possible to it, in order to avoid parallax. The transit is set up in the mine as close to the nearest line as possible (this will be from ten to fifteen feet distant), and each line is separately observed and the transit gradually brought by trial into line with the mean positions of both wires. The two plummets should hang in water, or, better, in some more viscous liquid, as oil, or even molasses. The oscillations will be slow, a line three hundred and twenty-five feet long requiring about ten seconds for one complete vibration in air. Of course the cages can not be in use during the operation.

The points from which the bobs are suspended must be as firm as possible and as well defined, so that the line joining them may be connected with the surface surveys. Two transits are really required to insure good work. One is left on the surface with an observer to see that there is no movement of the points of suspension during the operation, and to note what change, if any, takes place. It must be known that the lines do not hang against any projections on the sides of the shaft. This may be told, in some instances, by looking up the shaft; in others, by passing a candle or other light slowly around the wire at the bottom, and observing from the top that it is visible throughout the motion. In case neither method is practicable, a system of movements of the wire may be arranged. When the observer below is ready, he will signal by rapping on the pipes or guides, and the observer above will, at agreed intervals of time, move the wire out one inch, two inches, etc. The observer below notes whether corresponding movements take place there.

On account of the stoppage of machinery this connection of surface and underground surveys is an expensive operation. It should nevertheless, in important cases, be performed several times, and a mean of the results taken for the first course of the underground traverse. When the bottom of the shaft can be seen from the top, a method used at the Severn tunnel will give good results. A wire is stretched from one side of the shaft, about one hundred or more feet into the drift, and accurately aligned for the few feet visible by a transit on the surface. In the case of the Severn tunnel the wire was stretched over horizontal screws, the wire lying in the groove between the threads. The wire was then moved laterally for aligning by turning the screws.

MAPPING THE SURVEY.

303. Metal mines. Three maps are necessary for a complete representation of a mine with many levels running in different directions — (1) the plan; (2) a longitudinal section; (3) a transverse section. Such a set of maps is shown in Plate IV. at the back of the book. The notes taken by the surveyor should be such as to enable him to make these three maps. These maps may be in addition to one of the surface survey, and, if so, there would be four maps necessary for a complete representation of a mining property. It is usual to tint the portions shown on the plans as worked, and the different levels are sometimes tinted with different colors to distinguish them. If several lodes are shown on one map, it will be better to color each lode, working with a single color to distinguish it from the other lodes.

304. Coal mines. The representation of a coal mine is usually a simpler matter than the mapping of a metal mine. The coal ordinarily lies in beds nearly horizontal, and the workings may often be represented by a plan alone, the different levels being distinguished by colors, and the elevations being written in. It is customary to show on the map the direction and amount of dip of the seams. There are a number of symbols used to represent various objects; but the practice is not uniform. Arrows may be used to show the direction of air currents, blue for inlets, and red for outlets, etc. The plan should show all the details of the mine, all stop walls, all doors, all sumps or reservoirs, all pumps or machinery, the location of faults, etc.

It is extremely necessary that complete, correct maps should be kept up, so that if any connections are to be made, they may be correctly directed. Very serious accidents have occurred from incorrect maps of mines.

305. Scale. The state of Pennsylvania provides that there shall be kept at each mine a plan on a scale of one to twelve hundred, or one hundred feet per inch, for the use of the state inspector. The English law requires the plan to be on a scale of twenty-five inches per mile, or about one half that required in this country. Other countries require different scales. The maps of the mining claims made by the deputy mineral surveyors are to a scale of two hundred feet to the inch. This would seem to be large enough for working drawings, except for details of structures.

As in the case of notes, two maps should be in existence and kept in different places. A good example of a coal mine map is shown in Plate V. at the back of the book.

306. Problems. The problems that arise in mining surveying call for some ingenuity in handling Trigonometry and Descriptive Geometry, although they may all be solved without a knowledge of the latter subject by name.

Some problems, such as seeking a lost vein, require more the principles of Geology than those of Mathematics.

The commonest problems are the following:

(1) The determination of the course and distance from a point in one compartment of a mine to a point in another compartment. By course is meant azimuth and vertical angle. This includes such problems as the determination of the distance and direction to run a tunnel to tap a shaft.

(2) Knowing the strike and dip of a vein, to determine the length of a shaft or drift started at a given point and run in a given direction to meet the vein, or to determine the direction to run a tunnel from a given point so as to cut the vein in the shortest possible distance, and to find the distance required.

It is believed that the student can solve such problems as are given in the Appendix, page 340, if he remembers that the pitch or dip is always measured in a vertical plane normal to a horizontal element of the vein, and that the strike is the direction of such horizontal element.

APPENDIX.

I. PROBLEMS AND EXAMPLES.

CHAPTER I.

The problems in the use of the chain may be solved on the blackboard with string and chalk.

1. To range out a line between two points on opposite sides of a hill: Mark the points so that they will be visible from the top of the hill. Two men with range poles then place themselves near the top of the hill, facing each other, so that each can see one end of the line. They then alternately align each other with the visible ends of the line until both are in the line. The same principle is used in "fudging" one's self into line between two distant visible points.

2. To range a line across a deep ravine or valley: The inability to do this by eye alone arises from the fact that the eye can not carry a vertical line on a hillside, nor can it be sure of transferring a point vertically downward, even in level country. The observer stands on one side of the valley, swinging a plummet over a point in the line to be ranged. He places his eye in line with the plumb line and a distant point in the line to be ranged, and, casting his eye down the plumb line, directs the placing of a flag held by an assistant in the valley.

3. To erect a perpendicular to a line: A triangle whose sides are in the ratio 4, 5, 6, is a right triangle. Hence, fasten one end and the thirty-link division at one point in the line and the ten-link division at another point. Carry the eighteen-link division out till the three portions of the chain used form a triangle. If the line is to be erected at a given point, the method will suggest itself to the student.

4. Using the chain and pins in the way a string and pencil or a pair of dividers would be used on the drawing board, perform the above problem in a different way.

5. To drop a perpendicular on a line from a given point: Run an inclined line from the point to the given line, erect a perpendicular to the latter, and produce it till it intersects the inclined line. The required

perpendicular is parallel to the perpendicular erected, and the consideration of the similar figures will enable the student to solve.

6. To run a line parallel to a given line: Erect two equal perpendiculars to the given line. If the parallel is to pass through a given point, drop a perpendicular from the point to the line and erect a perpendicular to this at the point. Otherwise: The diagonals of a parallelogram bisect each other.

7. To measure an angle with a chain: In an isosceles triangle of legs a, included angle A, and base b, $2\,a \sin \frac{1}{2} A = b$.

8. To lay out a given angle: Reverse the process just suggested.

9. A line is measured with a chain that is afterward found to be one link too long, and is found to be 10.36 chains long. What is its true length?

10. A line is measured with a 100-foot tape and found to be 723.36 feet long. The tape is afterward found to be 0.02 foot short. What is the true length of the line?

11. A triangular field is measured with a chain that is afterward found to be one link too long. The sides as measured are 6 chains, 4 chains, and 3 chains, respectively. What is the resulting area and what is the true area?

12. An irregular field is measured with a chain three links short. The area is found to be 36.472 acres. What is the true area?

13. A 100-foot tape is standard length for a pull of 10 pounds, when supported its entire length, and at a temperature of 62° F. A line is measured and found to be 1000.00 feet long. The tape is of steel, has a cross section of 0.002 square inch, and weighs 0.00052 lb. per inch. In the measurement it is at a temperature of 80° F., and is unsupported except at the ends. The pull is 25 pounds. What is the true length of the line?

14. A steel tape weighs 0.0061 pounds per lineal foot. It is 100 feet long and is standard for no pull when entirely supported, at 62° F. It has a cross section of 0.002 square inch. What pull must be exerted to keep the tape standard length if it is supported only at its ends, the temperature remaining unchanged?

15. A line is measured along the surface on a hillside. The first 300 feet has a vertical angle of 3°, the next 250 feet has a vertical angle of 4° 30′, and the last 700 feet has a vertical angle of 1°. What is the true length of the line?

16. A similar line, of equal surface lengths, is measured, and the rise per hundred feet of each stretch is found to be respectively 5 feet, 8 feet, and $1\frac{3}{4}$ feet. What is the true length of the line?

17. Measure a line of about 1000 feet or more in length, at least twice with a chain and twice with a tape, and determine the difference of measurement. Do this, if possible, over two lines of about equal length, but offering very unequal difficulties to measurement, and note the differences.

CHAPTER III.

1. Determine the angular value of one division of each bubble available in the school collection of levels. The method is in brief as follows: Hold a rod at a known distance; read the rod and bubble with the bubble near one end of its tube, and then read with the bubble near the other end. By the differences of readings and the known distances, determine the value of one division. If there are any striding levels, use these as described for the level with metal base.

2. Other exercises with the level will suggest themselves to the student or teacher. A considerable amount of differential and profile leveling should be done, and profiles drawn. The leveling should always be checked by rerunning in the opposite direction.

CHAPTER IV.

1. Radiating from a point A, are eight lines of bearings as follows:

AB, N. 53° 45′ W.	AF, S. 86° 45′ E.
AC, N. 36° 42′ W.	AG, S. 24° 36′ E.
AD, N. 18° 34′ E.	AH, S. 20° 30′ W.
AE, N. 34° 28′ E.	AI, S. 36° 20′ W.

Required the angles between the following bracketed lines — always considering the smaller angle:

$\left\{\begin{matrix}AB\\AC\end{matrix}\right.$ $\left\{\begin{matrix}AB\\AD\end{matrix}\right.$ $\left\{\begin{matrix}AB\\AE\end{matrix}\right.$ $\left\{\begin{matrix}AB\\AF\end{matrix}\right.$ $\left\{\begin{matrix}AB\\AG\end{matrix}\right.$ $\left\{\begin{matrix}AB\\AH\end{matrix}\right.$ $\left\{\begin{matrix}AB\\AI\end{matrix}\right.$

$\left\{\begin{matrix}AC\\AD\end{matrix}\right.$ $\left\{\begin{matrix}AC\\AE\end{matrix}\right.$ $\left\{\begin{matrix}AC\\AF\end{matrix}\right.$ $\left\{\begin{matrix}AC\\AG\end{matrix}\right.$ $\left\{\begin{matrix}AC\\AH\end{matrix}\right.$ $\left\{\begin{matrix}AC\\AI\end{matrix}\right.$

$\left\{\begin{matrix}AD\\AE\end{matrix}\right.$ $\left\{\begin{matrix}AD\\AF\end{matrix}\right.$ $\left\{\begin{matrix}AD\\AG\end{matrix}\right.$ $\left\{\begin{matrix}AD\\AH\end{matrix}\right.$ $\left\{\begin{matrix}AD\\AI\end{matrix}\right.$

$\left\{\begin{matrix}AE\\AF\end{matrix}\right.$ $\left\{\begin{matrix}AE\\AG\end{matrix}\right.$ $\left\{\begin{matrix}AE\\AH\end{matrix}\right.$ $\left\{\begin{matrix}AE\\AI\end{matrix}\right.$

$\left\{\begin{matrix}AF\\AG\end{matrix}\right.$ $\left\{\begin{matrix}AF\\AH\end{matrix}\right.$ $\left\{\begin{matrix}AF\\AI\end{matrix}\right.$

$\left\{\begin{matrix}AG\\AH\end{matrix}\right.$ $\left\{\begin{matrix}AG\\AI\end{matrix}\right.$

$\left\{\begin{matrix}AH\\AI\end{matrix}\right.$

2. A traverse is run with the following bearings: N. 42° E., N. 36° E., S. 1° W., N. 50° W. Determine the interior angles. If the traverse is of a closed field, what should be the sum of the interior angles?

3. A traverse is run with the following azimuths, zero azimuth being north: 42°, 144°, 181°, 230°. What are the bearings?

4. Another is run with the following azimuths: 306° 15′, 18° 34′, 93° 15′, 200° 30′. What are the bearings?

5. What are the azimuths of the following lines of Example 1: *AC*, *AE*, *AG*, *AI*?

6. Determine the exterior or deflection angles in Example 4.

7. Let the eight lines of Example 1 be the first eight consecutive courses of a traverse. Determine the deflection angles.

8. The magnetic declination at a given place is found to be 10° 30′ E. What will be the bearing of the true north? the true south? the true east? the true west?

9. The magnetic declination is 7° 10′ W. What will be the bearings of the four true cardinal points?

10. A line of an old survey is recorded as N. 18° E. mag. bear. It now reads N. 16° 30′ E. What has been the change in declination in direction and amount?

11. A line of an old survey is recorded as N. 36° 15′ E. mag. bear. It now reads N. 34° 30′ E., and the magnetic declination is now 10° 30′ W. What was the declination at the time of the original survey?

12. A line of an old survey is recorded as N. 36° 15′ E. mag. bear., and the declination is recorded as having been 10° 30′ W. at the time of the survey. The declination is now 12° 00′ W. What magnetic bearing should the line now show?

13. A line of an old survey is recorded as S. 26° 00′ E. mag. bear., declination 10° 30′ W. The declination is now 9° W. What magnetic bearing should be used to retrace the line?

14. The magnetic declination is 10° 30′ E. If the declination vernier is attached to the south side of the compass box, in what direction and by what amount should it be moved so that true bearings may be read by the needle? With sights pointed to the magnetic north, what would the needle read after moving the vernier?

15. A line of an old survey is recorded as N. 30° 15′ E. mag. bear. The declination is now 4° W., and the same line reads N. 30° E. It is desired to set the declination vernier so that the remainder of the survey may be retraced by the recorded bearings. The vernier being attached to the south side of the compass box, what is its movement in amount and direction?

16. Suppose, in the above example, the former bearing had been S. 26° W., and the present bearing S. 26° 30′ W. What would be the movement of the vernier?

17. Is it necessary in the foregoing examples to know the present declination?

18. Determine the angular value of the plate bubbles and the telescope bubble of each transit in the school collection. Do this work as suggested

for the level, and also for the telescope bubble, by the use of the vertical circle.

19. Determine the meridian by an observation on Polaris and by the solar transit, and compare the results.

20. Measure all the angles of a polygon that has been laid out on the ground and note whether they sum up properly.

21. Set over each station in succession and run the polygon as a traverse, using azimuths, and, reoccupying the first station, redetermine the azimuth of the first course from that of the last, and note whether the azimuth found agrees with that first used.

In fairly good transit work, the error in a perimeter of a mile or more should not exceed one minute.

22. The adjustments of the transit should be made by the student.

CHAPTER V.

1. Determine for the transits in the school collection the value of $\frac{f}{i}$ and $(f + c)$.

2. Reduce the horizontal distances and differences of elevation in the set of notes shown on pages 337–338.

CHAPTER VI.

Find the error of closure, balance the survey, plot, and compute the areas of the following fields:

	Sta.	Bearing.	Chains.	
1.	A	S. 51° 10′ E.	5.05	
	B	S. 58° 10′ W.	4.63	
	C	N. 29° 15′ W.	4.16	
	D	N. 45° 30′ E.	2.87	
				Area, 1.676 + A.
2.	A	N. 84° 00′ W.	9.04	
	B	S. 21° 15′ W.	12.34	
	C	N. 72° 15′ E.	12.92	
	D	N. 9° 30′ E.	6.68	
				Area, 9.264 + A.
3.	1	S. 7° 25′ W.	4.35	
	2	S. 62° 05′ W	6.94	
	3	N. 2° 35′ E.	4.01	
	4	N. 34° 25′ E.	3.64	
	5	N. 83° 00′ E.	4.51	
				Area, 3.124 + A.

	Sta.	Bearing.	Chains.
4.	1	S. 62° 15′ E.	5.12
	2	S. 71° 15′ E.	4.66
	3	S. 5° 30′ W.	12.00
	4	S. 80° 15′ W.	12.46
	5	N. 2° 15′ E.	9.46
	6	N. 25° 45′ E.	9.40

Area, 17.683 + A.

	Sta.	Azimuth.	Chains.
5.	1	107° 15′	16.40
	2	196° 15′	24.10
	3	246° 05′	19.60
	4	13° 55′	37.00

Area, 48.36 + A.

6. The latitudes and longitudes of a series of points are as follows, measured in chains:

$$A\begin{cases}\text{Latitude} & +\ 13.63\\ \text{Longitude} & +\ 10.20\end{cases}$$

$$B\begin{cases}\text{Latitude} & +\ 1.26\\ \text{Longitude} & +\ 5.10\end{cases}$$

$$C\begin{cases}\text{Latitude} & -\ 3.30\\ \text{Longitude} & +\ 7.20\end{cases}$$

$$D\begin{cases}\text{Latitude} & -\ 3.30\\ \text{Longitude} & -\ 6.60\end{cases}$$

$$E\begin{cases}\text{Latitude} & +\ 7.10\\ \text{Longitude} & -\ 10.20\end{cases}$$

Find the area included by the broken line joining the points. (Shift the coördinate axes so that the coördinates are all positive.)

7. The following offsets were taken on the sides of a line indicated, at points 100 feet apart. Required the area between extreme boundaries.

L.	Dist.	R.
137.4	0	55.1
93.5	100	76.5
49.3	200	83.3
78.2	300	79.9
102.0	400	69.7
100.5	500	59.5
96.6	600	83.3
144.2	700	70.5
111.9	800	54.4
71.4	900	101.2

Area, 153,085 square feet.

8. The following offsets were taken on one side of a base line, to determine the area between it and an irregular boundary line. The distances are all from zero:

Distance. Feet.	Offset. Feet.	
0	20.7	
202	31.5	
358	42.6	
825	53.2	
984	36.1	
1223	40.7	Area, 49,698.25 sq. ft.

9. In Example 1, let it be supposed that the second course is wanting. Supply it.

10. In Example 2, let it be supposed that the bearing of the second course and the length of the third course are wanting. Supply them.

11. In the third example, let it be supposed that the bearings of the second and fourth courses are wanting. Supply them.

12. In the fifth example, let it be assumed that the lengths of the second and fourth courses are wanting. Supply them.

13. The wheel of a planimeter has a circumference of 2.26 inches. What should be the length of tracer that the number of revolutions multiplied by 10 shall give in square inches the area circumscribed?

14. An area is circumscribed by a planimeter with the fixed point inside, and the resulting reading is found to be 11.26 square inches. The area is known to be 120 square inches. What is the area of the zero circumference?

COÖRDINATES.[1]

Model Example.

1. To determine the coördinates of the corners *Abcdef* (Fig. 155) and plot the survey, the corner *e* and bearing *fe* being unknown, but point *e′* in prolongation of *fe* and bearing of *de*, as well as other corners, known:

Run the random traverse *ABCDEF*. Make a rough sketch of the traverse and place on it lengths of lines and angles measured. (Note that angles and not azimuths or bearings are read. This is careful work where angles are repeated several times, and hence it is impracticable to read azimuths.) Balance the angles so that their sum will be "twice as many right angles less four as the figure has sides," distributing any error equally among the various angles. In the example the angles "close."

[1] The problems here given were furnished the author by Mr. John H. Myers, Jr., A.B., C.E., Assistant Engineer Department of Public Works, Brooklyn, N.Y. The examples are from practice in New York City and were treated as here indicated.

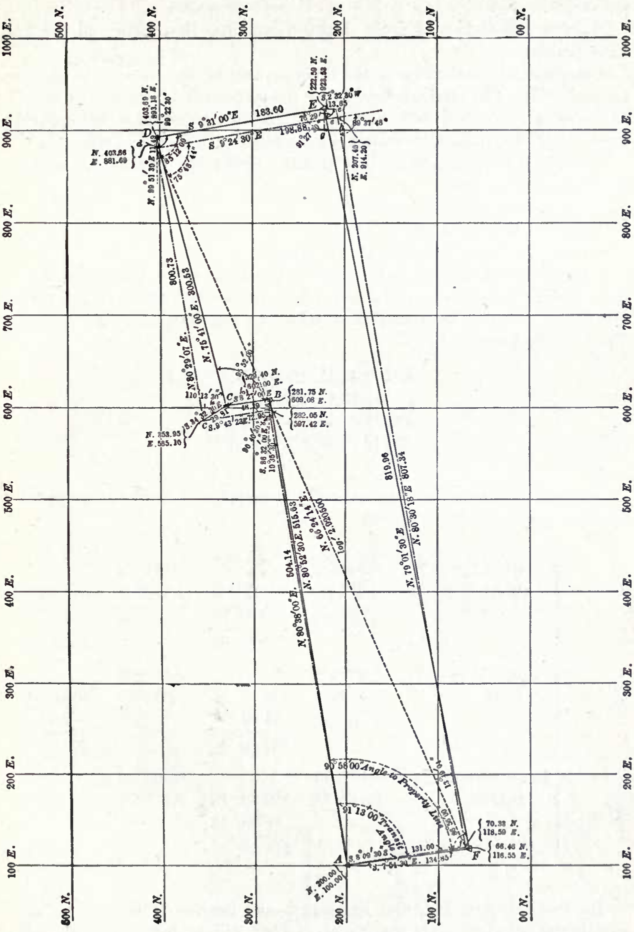

1000 E.
900 E.
800 E.
700 E.
600 E.
500 E.
400 E.
300 E.
200 E.
100 E.
500 N.
400 N.
300 N.
200 N.
100 N.
00 N.
S 9°31′00″E
183.60
S 9°24′30″E
198.88
N. 75°41′00″E. 300.53
300.73
N. 80°29′07″E.
504.14
N. 80°38′00″E.
N. 80°52′30″E. 515.63
819.96
807.34
N. 79°01′30″E
N. 80°30′18″E
90°58′00″ Angle to Property Line
91°13′00″ Transit Angle
131.00
134.85
N. 403.86
E. 881.69
281.75 N.
609.06 E.
282.05 N.
597.42 E.
N. 253.95
E. 585.10
N. 207.46
E. 914.20
222.59 N.
923.53 E.
70.33 N.
118.59 E.
66.46 N.
116.55 E.
N. 200.00
E. 100.00
A
B
C
D
E
F

Fig. 155.

Following a previous survey in this district, assume bearing of *DE* to be S. 9° 31′ E., and compute bearings of remaining lines and place them on the sketch.

Assume coördinates of point *A* 200 N. and 100 E. to bring all coördinates positive.

Compute the coördinates of the other corners of the traverse as shown on page 331. The student should note the systematic layout of the work. The error of closure, 0.05 E. and 0.14 N., is distributed as in balancing a survey, and the balanced coördinates are used in all subsequent work.

Plot the traverse as shown in Fig. 155. Check the plotting by scaling the lengths of the lines.

To locate the property corners;

Corner *A* is a corner of the random.

Course *Af* was actually run on the ground.

Calculate from the recorded angles the bearings of the lines joining random corners and true corners, and compute the latitude and longitude differences of these lines, and from these the coördinates of the corners. They will be found to be:

b, 597.42 E., 282.05 N.
c, 585.10 E., 353.95 N.
d, 881.69 E., 403.66 N.
f, 118.59 E., 70.33 N.

The computations are systematically arranged as follows, the computations for *c*, *d*, and *f*, being shown:

Cc . . .	N. 34° 32′ 30″ W.	9.753587		9.915776	
	29.81	1.474362	602.00 E.	1.474362	329.40 N.
		1.227949	16.90 W.	1.390138	24.55 N.
	c		585.10 E.	. . . .	353.95 N.
Dd . . .	S. 89° 51′ 30″ W.	9.999990		7.417968	
	11.49	1.060320	893.18 E.	1.060320	403.69 N.
		1.060319	11.49 W.		0.03 S.
	d		881.69 E.	. . . .	403.66 N.
Af . . .	S. 8° 09′ 30″ E.	9.152010		9.995582	
	131.00	2.117271	100.00 E.	2.117271	200.00 N.
			18.59 E.		129.67 S.
	f		118.59 E.	. . . .	70.33 N.

By Prob. 1, Art. 143, find the length and bearing of the courses the coördinates of whose ends are known. They will be found to be as indicated on the figure. The arrangement of the work for a single course is shown on page 332.

Coördinates of *A*			+ 100.00 E.			+ 200.00 N.
N. 80° 52′ 30″ E.	log sin 80° 52′ 30″	9.994469		log cos 80° 52′ 30″	9.200273	
515.63	log 515.63	2.712338	08	log 515.63	2.712338	75
		2.706807	+ 509.10 E.		1.912611	+ 81.77 N.
Coördinates of *B*			609.08 E.			+ 281.75 N.
N. 8° 27′ 00″ W.	log sin 8° 27′ 00″	9.167159	+ 609.10 E.	log cos 8° 27′ 00″	9.995260	281.77 N.
48.19	log 48.19	1.682957		log 48.19	1.682957	65
		0.850116	− 7.08 W.		1.678217	+ 47.66 N.
Coördinates of *C*			602.00 E.			+ 329.40 N.
N. 75° 41′ 00″ E.		9.986299	602.02 E.		9.393191	329.43
300.53		2.477888	18		2.477888	29
		2.464187	291.19 E.		1.871079	74.31 N.
Coördinates of *D*			893.18 E.			403.69
S. 9° 31′ 00″ E.		9.218363	893.21		9.218363	403.74
183.60		2.263873			2.263873	10
		1.482236	30.35 E.		2.257855	181.07 S.
Coördinates of *E*			923.53 E.			222.59 N.
S. 79° 01′ 30″ W.		9.991983	923.56 E.		9.279623	222.67 N.
819.96		2.913793	98		2.913793	13
		2.905776	804.96 W.		2.193416	156.10 S.
Coördinates of *F*			118.55 E.			66.46 N.
N. 7° 54′ 30″ W.		9.138582	118.60 E.		9.995850	66.57 N.
134.85		2.129851			2.129851	
		1.268433	18.55 W.		2.125701	133.57 N.
Coördinates of *A*			100.00			200.00
			100.05			200.14
Error			0.05 E.			0.14 N.

cd	
353.95	585.10
403.66	881.69
49.71	296.59
1.696444	
2.472156	N. 80° 29′ 07″ E.
9.224288	tan 9° 30′ 53″
9.993984	cos do.
2.478172	300.73

To find the corner *e* find the coördinates of *e*′, then the bearing of *e*′*f* by Prob. I., Art. 143. The bearing of *de* is known to be S. 9° 24′ 30″ E. By Prob. II., Art. 143, find the coördinates of *e*, the intersection of *de* and *fe*′. The quantities found appear on the sketch. Prob. II. is arranged thus:

	403.66 N.	881.69 E.	
	70.33 N.	118.59 E.	
	333.33	763.10	
	2.522874		
	2.882581	N. 66° 24′ 14″ E.	
	9.640293	23° 35′ 46″	
	9.962081		
	2.920500		2.920500
75° 48′ 44″	9.986547	13° 49′ 04″	9.378097
89° 37′ 48″	0.000009		0.000009
	2.907056		2.398606
	807.34 *ef*		198.88 *de*

From the length and bearing of *de* and coördinates of *d* find coördinates of *e*, and from coördinates of *e* and *f* find bearing of *fe*.

Plot the three corners as the randoms. In close work it is best to use seconds, but on the final map the nearest half minute may be recorded. Minutes and tenths will serve as well as minutes and seconds.

If tne area is required, it is obtained as follows, making the computations by logaritnms. The work is here carried to a needless number of decimals.

$$\frac{200.00}{100.00} \times \frac{282.05}{597.42} \times \frac{353.95}{585.10} \times \frac{403.66}{881.69} \times \frac{207.46}{914.20} \times \frac{70.33}{118.59} \times \frac{200.00}{100.00}$$

200.00 × 597.42 = 119484.0000	100.00 × 282.05 = 28205.0000
282.05 × 585.10 = 165027.4550	597.42 × 353.95 = 211456.8090
353.95 × 881.69 = 312074.1755	585.10 × 403.66 = 236181.4660
403.66 × 914.20 = 369025.9720	881.69 × 207.46 = 182915.4074
207.46 × 118.59 = 24602.6814	914.20 × 70.33 = 64295.6860
70.33 × 100.00 = 7033.0000	118.59 × 200.00 = 23718.0000
997247.2839	746772.3684
746772.3684	

250474.9155 ÷ 2 = 125237.4577 square feet = 2.875 acres.

EXAMPLES.

1. In Fig. 155 determine whether the line *fd* passes to the north or south of the corner *b*; and how far *b* is from *fd*, measured on a line at right angles to *fd*.

2. In Fig. 156 the line *ABCDEF* represents a portion of a survey which has been made to locate a street or railroad. It has been decided to connect the lines *AB* and *EF* by a circular curve of radius 716.779 feet, and it is required to find how far from *B* the curve will start and how far from *E* it will end. The method in outline is as follows: Assume coördinates for

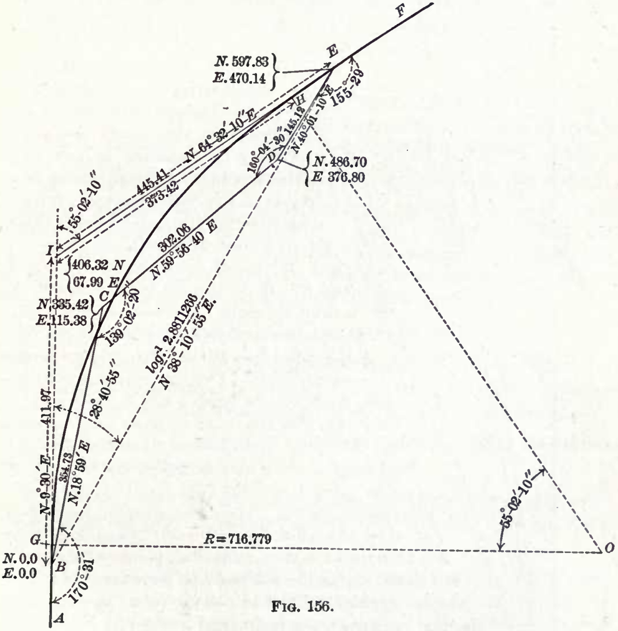


FIG. 156.

the point *B*, and by means of the connecting courses work up coördinates for the point *E*. By Problem I., Art. 143, find the bearing of *BE* and the logarithm of its length. Then by Problem II. find the distances from *B* and from *E* to the intersection *I* of *AB* and *EF* produced. Then the radius multiplied by the tangent of ½ the angle *GOH*, which angle is equal to the angle at *I*, gives the "tangent distance" *HI*, and this subtracted from *IB* and from *IE* gives the required distances *GB* and *HE*.

3. Fig. 157 illustrates the location of two piers, B and C, for the approach to a bridge. The piers were on a curve and were located by the usual method of running railway curves, the necessary deflections and distances being shown in the figure. As the points B and C would have to be removed during the construction of the piers, it became necessary to so "reference" them that they could easily and quickly be recovered at any time. This was done by placing two or more points on the radial lines through the centers of the piers, since the intersections of these lines with the lines AB and AC would determine the points B and C. If the ground had been solid, the instrument could have been set at B and at C, and, by turning off the proper angles from A, the radial lines BO and CO could easily have been established. This, however, it was impossible to do, as the ground was so swampy as to render the instrument too unsteady for good work. The following method was therefore adopted: Beginning at A, a random traverse $Abcf$ was run on firm ground, and the angles and distances were measured as shown. The distances from b and from c at which the radial lines BO and CO would intersect this traverse were then calculated, points were set at e and d, and the angles beB and cdC were measured to see whether their actual and calculated values agreed. This having been done, the instrument was set at e and at d, and by backsighting on B and C and reversing, any number of points could be established on the radial lines BO and CO. The points B and C are thus referenced. The problem for the student is therefore to calculate the distances be and cd. It will be well to use the tangent AX as a meridian and to make the coördinates of A 00 N and 00 E. (Problem II., Art. 143.)

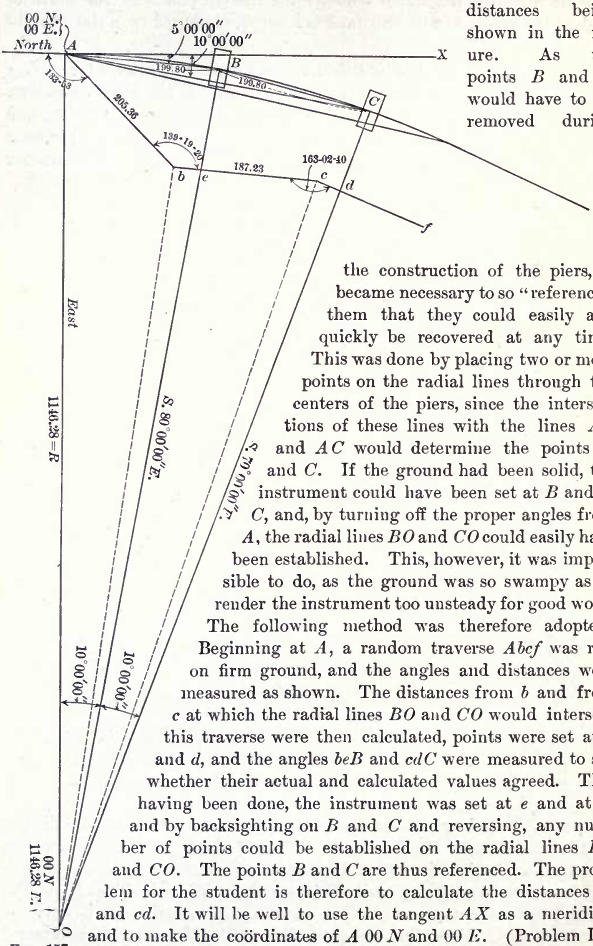


Fig. 157.

4. From the data given on Fig. 158 calculate the depths *ac* and *bd* of lot number 4.

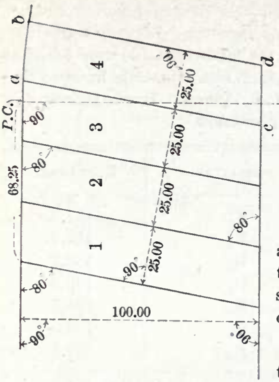


Fig. 158.

CHAPTER VIII.

1. A 4° curve begins at station 36 + 50, and the central angle is 16° 00′. Tabulate the deflection angles from the P. C. to the several stations and determine the station of the P. T.

2. In the above example what is the tangent distance? the external distance? the long chord? the middle ordinate?

3. In the above example what is the true length of the circular arc?

4. Two tangents intersect at station 182 + 75, with a deflection angle of 26° 30′. It is required to connect them with a 3° 30′ curve. What will be the station of the P. C. and, if the station numbering is changed to read around the curve instead of along the tangents, what will be the station of the P. T.? (The stationing usually reads along the curve.)

5. Tabulate the deflections to each station on the curve.

6. It is required to connect the tangents named in Example 4 by a curve that shall pass approximately 39 feet inside the vertex. What degree curve should be used, and what will be the station of P. C. and P. T.?

7. A 24° curve is to be located by 25-foot chords. What is the correct deflection for each chord?

8. If the 24° curve is to be so located that four chord lengths shall cover the arc subtended by a 100-foot chord, what is the length of each chord and what the deflection?

9. A 4° curve begins at station 376 + 50, and at station 381 is compounded to a 6° curve which ends at station 387 + 25. It is required to find the central angle of each arc, the total central angle, and the two tangent distances P. C. to intersection, and intersection to P. T.

10. Find all the offsets necessary to lay out a 3° curve, with the chain alone, by offsets from the chord produced; the curve beginning at station 367 + 30 and ending at station 372 + 60.

11. What is the degree of the curve shown in Fig. 156? That shown in Fig. 157?

CHAPTER IX.

1. By leveling, the elevations given in the following table were found at the points indicated. The tract is in the form of a square six hundred feet on a side and divided into one-hundred-foot squares. Draw the map to a scale of 100 feet per inch, locating five-foot contours.

Station.	Elevation.	Station.	Elevation.
A_0	153.0	B_0	154.5
A_1	155.5	B_1	159.6
A_2	158.0	B_2	163.7
A_3	158.5	B_3	166.5
A_4	156.4	B_4	165.5
A_5	152.8	B_5	156.0
A_6	149.5	B_6	155.5
C_0	155.4	D_0	155.3
C_1	162.1	$D_0 + 80$	160.5
C_2	169.0	D_1	160.0
C_3	173.7	D_2	170.2
$C_3 + 50$	175.5	D_3	180.1
C_4	170.4	$D_3 + 50$	192.2
$C_4 + 20$	166.0	D_4	178.5
$C_4 + 60$	167.0	D_5	168.7
C_5	164.8	D_6	161.5
C_6	159.4		
E_0	158.2	F_0	157.5
E_1	164.2	F_1	162.3
E_2	168.8	F_2	165.5
E_3	175.0	$F_2 + 50$	166.0
$E_3 + 20$	176.0	F_3	165.5
$E_3 + 60$	172.5	$F_3 + 50$	164.0
E_4	172.5	F_4	164.9
E_5	166.0	F_5	163.1
E_6	161.5	F_6	158.5
G_0	154.5	$G_3 + 40$	155.5
G_1	159.6	G_4	159.0
G_2	161.0	G_5	157.5
G_3	159.5	G_6	155.0
$0_C + 50$	154.0	$3_D + 15$	181.8
$2_D + 40$	167.5	$4_B + 75$	170.8
$3_C + 80$	182.0	$4_C + 10$	170.0

2. Plot a contour map with five-foot contours from the following notes of a stadia survey, all pointings being taken from one position.

INST. AT ⊡$_0$ ON SUMMIT OF BROWN'S HILL.

Zero Az. = Mag. North. Elev. ⊡$_0$ assumed 100.00.

STATION.	AZIMUTH.	DIST.	VERT. ∠	DIFF. ELEV.	ELEV.
1.	357°00′	1025	– 2°12′	40.0	60.0
2. In ravine running northerly.	355°10′	885	– 2°28′	38.0	62.0
3.	3°15′	860	– 2°20′	34.8	65.2
4. In ravine running northerly.	1°25′	720	– 2°47′	35.0	65.0
5.	15°40′	635	– 2°42′	30.0	70.0
6.	26°50′	985	– 1°58′	33.8	66.2
7.	30°00′	1175	– 1°48′	37.0	63.0
8.	42°00′	1370	– 1°42′	40.5	59.5
9.	48°30′	1120	– 1°45′	34.2	65.7
10.	35°20′	730	– 2°15′	28.7	71.3
11.	52°50′	825	– 2°08′	30.7	69.3
12. In ravine running northerly.	17°10′	460	– 3°26′	27.5	72.5
13.	26°55′	420	– 3°25′	25.0	75.0
14.	50°05′	470	– 2°54′	23.7	76.3
15.	71°55′	960	– 2°03′	34.2	65.8
16.	72°30′	715	– 2°24′	30.0	70.0
17.	59°35′	340	– 3°22	20.0	80.0
18.	30°35′	230	– 4°47	19.0	81.0
19.	5°00′	135	– 6°26′	15.0	85.0
20.	48°15′	140	– 4°52′	11.8	88.2
21.	86°35′	85	– 6°48′	10.0	90.0
22. Head of ravine running easterly.	94°30′	235	– 4°56′	20.0	80.0
23.	82°25′	290	– 3°45′	19.0	81.0
24.	81°15′	545	– 2°49′	26.8	73.2
25. In ravine.	95°05′	490	– 3°16′	28.0	72.0
26. " "	90°35′	655	– 2°50′	32.5	67.5
27. " "	89°05′	835	– 2°32′	37.0	63.0
28. " "	91°10′	905	– 2°27′	38.8	61.2
29.	102°55′	860	– 2°18′	34.5	65.5
30.	118°10′	1020	– 2°07′	37.5	62.5
31.	130°50′	1185	– 1°57′	40.0	60.0
32.	147°15′	860	– 2°12′	33.0	67.0
33.	147°15′	580	– 2°43′	27.5	72.5
34.	124°15′	690	– 2°32′	30.5	69.5
35.	143°30′	365	– 3°33′	22.6	77.4

Station.	Azimuth.	Dist.	Vert. ∠	Diff. Elev.	Elev.
36.	118°45′	420	– 3°14′	23.7	76.3
37.	130°30′	125	– 5°33′	12.0	88.0
38.	194°00′	140	– 7°30	18.0	82.0
39.	168°50′	755	– 2°28′	32.5	67.5
40.	191°55′	615	– 2°49′	30.4	69.6
41.	191°55′	315	– 3°58′	21.8	78.2
42.	219°45′	340	– 4°00′	23.6	76.4
43. In ravine running southwesterly.	229°15′	425	– 4°05′	30.0	70.0
44. In ravine running southwesterly.	232°30′	710	– 3°01′	37.5	62.5
45.	223°25′	640	– 3°00′	33.7	66.3
46.	203°35′	835	– 2°25′	35.2	64.8
47. In ravine.	229°35′	945	– 2°31′	41.5	58.5
48.	224°15′	1025	– 2°21′	42.0	58.0
49. In ravine.	229°15′	1185	– 2°13′	46.0	54.0
50.	343°00′	960	– 2°12′	37.0	63.0
51.	331°00′	1180	– 1°59′	40.5	59.5
52.	320°05′	1350	– 1°52′	44.0	56.0
53.	315°30′	1115	– 2°01′	38.9	61.1
54.	326°30′	780	– 2°22′	32.2	67.8
55.	343°20′	440	– 3°16′	25.2	74.8
56.	300°05′	795	– 2°20′	32.5	67.5
57.	287°15′	930	– 1°58′	31.8	68.2
58.	2°30′	315	– 4°01′	22.0	78.0
59.	300°30′	300	– 3°55′	20.4	79.6
60.	274°45′	770	– 2°27′	33.0	67.0
61.	276°55′	470	– 2°59′	24.5	75.5
62.	340°45′	140	– 5°43′	13.8	86.2
63.	285°30′	175	– 4°37′	14.0	86.0
64.	262°55′	275	– 3°51′	18.5	81.5
65. Head of ravine	240°20′	225	– 5°10′	20.0	80.0
66.	253°45′	570	– 2°49′	28.0	72.0
67.	243°00′	715	– 2°42′	33.8	66.2
68.	255°55′	900	– 2°25′	38.0	62.0

CHAPTER X.

1. Assume in Example 1, Chapter IX., that the area shown is to be graded to a plane surface of uniform elevation of 160 feet. Determine the depth of cut or fill at each corner of the small squares and from these the volumes.

2. Determine the volumes by the approximate graphical method, using the planimeter for the areas.

CHAPTER XI.

The following observations[1] for rating a Price acoustic current meter were made by Mr. W. G. Price, June 21, 1895, by the method described by him in "Engineering News," Jan. 10, 1895. The student will use the first, fourth, fifth, and sixth columns. The second and third columns will be understood by a reference to the article in "Engineering News." Get the equation for rating the meter.

RATING OF PRICE ACOUSTIC CURRENT METER NO. 4, JUNE 21, 1895.

No.	1st Wire. Feet.	2d Wire. Feet.	Distance. Feet.	Rev.	Time. Seconds.
1	31.09	79.72	48.63	20	24.4
2	24.84	74.28	48.44	20	22.4
3	37.50	86.32	48.82	20	35.0
4	32.76	82.50	49.74	20	56.0
5	26.21	76.54	50.33	20	56.0
6	13.61	64.21	50.60	20	99.0
7	27.55	77.78	58.23	20	94.6
8	13.45	62.31	48.86	20	25.0
9	23.28	71.27	48.09	20	15.8
10	32.22	80.35	48.13	20	10.2
11	27.84	76.01	48.17	20	7.4
12	32.08	103.95	71.87	30	20.4
13	39.99	87.98	47.99	20	6.0
14	35.08	82.68	47.60	20	5.8
15	24.20	72.84	48.64?	20	6.0
16	38.86	86.39	47.53	20	5.8
17	24.67	72.47	47.80	20	5.2
18	39.13	86.85	47.72	20	5.2
19	35.74	83.51	47.77+	20	11.2
20	20.15	67.97	47.82+	20	11.2
21	30.92	79.17	48.25	20	13.4
22	21.86	70.30	48.44	20	11.6
23	23.67	72.09	48.42+	20	19.4
24	14.47	62.71	48.24+	20	20.0
25	39.27	88.65	49.38	20	32.4
26	46.18	85.82	49.64	20	31.4
27	40.82	90.57	49.75	20	37.4
28	26.14	101.15	75.01	30	64.4

The center of the meter wheel was 2.1 feet below the water surface. In Nos. 19, 20, 23, and 24 I felt sure the time interval used to start the watch and stop the first wire from passing out, after I had heard the click of the

[1] Kindly furnished the author by Mr. Price.

meter, was longer than the time consumed for the same purpose at the end of the run. I therefore put a + mark to indicate that the distance was too small. No. 15 was doubtful, as the skiff rocked during the trip. The six columns are an exact copy of the field notes.

CHAPTER XII.

1. From the following data of the survey of a tunnel and shaft for a connection, determine the azimuth, length, and grade of the connecting drift. First draw a map of the survey and indicate the connecting drift.

From a monument at the mouth of the tunnel run in the tunnel, azimuth 36° 30′, 436 feet, vert. ∠ + 1° 00′; thence azimuth 52° 10′, 200 feet, vert. ∠ + 1° 10′, to point near breast of tunnel.

From the monument at the mouth of the tunnel run on the surface, azimuth 86° 30′, 232 feet, vert. ∠ − 3° 30′; thence azimuth 40° 20′, 636 feet, vert. + 13° 50′, to center of shaft 110 feet deep.

2. The strike of a vein of ore observed on the point of an outcrop is N. 36° W. or S. 36° E. The dip of the vein is found to be 13° from the vertical and to the northeast (right angle to the strike). From a point which bears from the before-mentioned point of outcrop N. 36° E. 300 feet, vert. ∠ − 15° a cross-cut tunnel is to be driven level and in a direction S. 55° W. How long must it be to reach the vein? What will be the difference if the tunnel is run on a one-per-cent grade?

3. If, in the above example, the tunnel is to run S. 60° W., how long must it be?

4. How long must it be, if the dip is to the southwest, the tunnel being run S. 54° W.?

5. How long must it be, if the dip is to the southwest and the tunnel is run S. 50° W.?

6. In Example 4 what would be the azimuth, grade, and length of the shortest tunnel that could be run to reach the vein?

II. THE JUDICIAL FUNCTIONS OF SURVEYORS.[1]

By Justice Cooley of the Michigan Supreme Court.

When a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that, under some circumstances, he must recognize inaccuracies, and govern his action by facts which lead him away from the results which theoretically he ought to reach. Observation warrants us in saying that this remark may frequently be made of surveyors.

In the state of Michigan all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes; and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is that the lines were very carelessly run, the monuments inaccurately placed; and, as the recorded witnesses to these were many times wanting in permanency, it is often the case that when the monument was not correctly placed it is impossible to determine by the record, with the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plots were more accurately surveyed, as indeed they should have been, for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them, however, were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate the importance of having correct lines to determine boundaries when land should become dear. The fact probably is that town surveys are quite as inaccurate as those made under authority of the general government.

It is now upwards of fifty years since a major part of the public surveys in what is now the state of Michigan were made under authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold and the work of improvement begun. A generation has passed away since they were converted into cultivated farms, and few, if any, of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were handy, but often they were not, and the witness trees must be relied upon after the stake was gone. Too often the first settlers were careless in fixing

[1] A paper prepared for the Michigan Society of Surveyors and Engineers.

their lines with accuracy while monuments remained, and an irregular brush fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing were substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary.

If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determined the boundary line between the proprietors. However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though the effect be to make one half-quarter section ninety acres and the one adjoining but seventy; for parties buy or are supposed to buy in reference to those monuments, and are entitled to what is within their lines, and no more, be it more or less. *McIver* v. *Walker*, 4 Wheaton's Reports, 444; *Land Co.* v. *Saunders*, 103 U. S. Reports, 316; *Cottingham* v. *Parr*, 93 Ill. Reports, 233; *Bunton* v. *Cardwell*, 53 Texas Reports, 408; *Watson* v. *Jones*, 85 Penn. Reports, 117.

While the witness trees remain there can generally be no difficulty in determining the locality of the stakes. When the witness trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men whose theoretical education is supposed to make them experts, who think that when the monuments are gone the only thing to be done is to place new monuments where the old ones should have been, and where they would have been if placed correctly. This is a serious mistake. The problem is now the same that it was before, — to ascertain, by the best lights of which the case admits, where the original lines were. The mistake above alluded to is supposed to have found expression in our legislation; though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed.

An act passed in 1869, Compiled Laws, § 593, amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows:

"*Second.* Extinct interior section-corners must be reëstablished at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

"*Third.* Any extinct quarter-section corner, except on fractional lines, must be reëstablished equidistant and in a right line between the section corners; in all other cases at its proportionate distance between the nearest original corners on the same line."

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near.

To estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct. If the monument was set for an interior-section corner, but did not happen to be "at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that, notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between the government subdivisions.

There are two senses in which the word "extinct" may be used in this connection: one the sense of physical disappearance; the other the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man lose his deed he shall lose his land altogether.

But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

(1) There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

(2) But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

(3) No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between state and federal law; it is a question of property right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them; and any legislation, whether state or federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

(4) In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hearing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged.

The general duty of a surveyor in such a case is plain enough. He is not to assume that a monument is lost until after he has thoroughly sifted the evidence and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary when no other is attainable; and the surveyor should inquire when it originated, how, and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the state statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation can not harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark, which may or may not have been trustworthy; but to bring these lines into discredit when the people concerned do not question them not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance and perhaps discredit, since in a legal controversy the law as well as common sense must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have disappeared. *Stewart* v. *Carleton*, 31 Mich. Reports, 270; *Diehl* v. *Zanger*, 39 Mich. Reports, 601; *Dupont* v. *Starring*, 42 Mich. Reports, 492. And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession must often appear in cities and villages. In towns the block and lot stakes soon disappear; there are no witness trees and no monuments to govern except such as have been put in their places, or where their places were supposed to be. The streets are likely to be soon marked off by fences, and the lots in a block will be measured off from these, without looking farther. Now it may perhaps be known in a particular case that a certain monument still remaining was the starting point in the original survey of the town plot; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plot to be accurate, he will then undertake to find all streets and all lots by course and distance according to the plot, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done; the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and use for many years, and the proprietors in a certain block have laid off their lots in reference to this practical location. Two lot owners quarrel, and one of them calls in a surveyor that he may be sure that his neighbor shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was, or not, and the first result is, he notifies the lot owners that there is error in the street line, and that all fences should be moved, say, one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course, if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot owners generally will allow the new survey to unsettle their possessions, but there is always a probability of finding some one disposed to do so. We shall then have a lawsuit; and with what result?

It is a common error that lines do not become fixed by acquiescence in a less time than twenty years. In fact, by statute, road lines may become conclusively fixed in ten years; and there is no particular time that shall be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. *McNamara* v. *Seaton*, 82 Ill. Reports, 498; *Bunce* v. *Bidwell*, 43 Mich. Reports, 542. Public policy requires that such lines be not lightly disturbed, or disturbed at all after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment, or to report the facts one way when he believes them to be another. He has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the rights of parties as if it were at another. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go, as an officer of the law, is to express his opinion where the monument should be, at the same time that he imparts the information to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer and entrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument, for any considerable period, as strong, if not conclusive, evidence of such settlement. The peace of the community absolutely requires this rule. *Joyce* v. *Williams*, 26 Mich. Reports, 332. It is not long since that, in one of the leading cities of the state, an attempt was made to move houses two or three rods into a street, on the ground that a survey under which the street had been located for many years had been found on more recent survey to be erroneous.

From the foregoing it will appear that the duty of the surveyor where boundaries are in dispute must be varied by the circumstances. (1) He is to search for original monuments, or for the places where they were originally located, and allow these to control if he finds them, unless he has reason to believe that agreements of the parties, express or implied, have rendered them unimportant. By monuments in the case of government surveys, we mean, of course, the corner and quarter stakes: blazed lines or marked trees on the lines are not monuments; they are merely guides or finger posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. (2) If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any state statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts; giving due prominence to the acts of parties concerned, and always keeping in mind, *first*, that neither his opinion nor his survey can be conclusive upon parties concerned; *second*, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and rules that will govern theirs. On town plots if a surplus or deficiency appears in a block, when the actual boundaries are compared with the original figures, and there is no evidence to fix the exact location of the stakes which marked the division into lots, the rule of common sense and of law is that the surplus or deficiency is to be apportioned between the lots, on an assumption that the error extended alike to all parts of the block. *O'Brien* v. *McGrane*, 29 Wis. Reports, 446; *Quinnin* v. *Reixers*, 46 Mich. Reports, 605.

It is always possible when corners are extinct that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, can not, on the basis of mere consent, be compelled to do so; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course it is desirable that all such agreements be reduced to writing; but this is not absolutely indispensable if they are carried into effect without.

Meander Lines.— The subject to which allusion will now be made is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless it is often found that surveyors misapprehend them, or err in their application; and as other interesting topics are somewhat connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that the meander lines are boundary lines, and all in front of them remains unsold. Of course this is erroneous. There was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the middle line of the river; and while some courts have held that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the state, while conceding to the owners of the banks all riparian rights. The practical difference is not very important. In this state the rule that the center line is the boundary line is applied to all our great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation in some cases with the stream in the main shallow, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander line to the center line of the river. Of course the original survey supposes that each purchaser of land on the stream has a water front of the length shown by the field notes; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore by the gradual change of course of the stream or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely; and in some cases, if it were continued in the same direction to the center line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation, or other valuable use of the water, while the other might have a water front much greater than the length of a line crossing it at right angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule: Extend the line of division between the two parcels from the meander line to the center line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and it is not limited to government surveys, but applies as well to water lots which appear as such on town plots. *Bay City Gas Light Co.* v. *The Industrial Works*, 28 Mich. Reports, 182. It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out.

When the stream is very crooked, and especially if there are short bends, so that the foregoing rule is incapable of strict application, it is sometimes very difficult to determine what shall be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action can not be conclusive; but if he adopts one that follows, as nearly as the circumstances will admit, the general rule above indicated, so as to divide, as near as may be, the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties and may constitute a sufficient guide when peculiar cases arise. Each riparian lot owner ought to have a line on the legal boundary, namely, the center line of the stream, proportioned to the length of his line on the shore; and the problem in each case is, how this is to be given him. Alluvion, when a river imperceptibly changes its course, will be apportioned by the same rules.

The existence of islands in a stream, when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the original admeasurement. Wherever that was the case, the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purposes of governmental sale afterwards, for the reason that the rights of the bank owners are fixed by their purchase: when making that, they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale; and having this right, anything which their purchase would include under it can not afterward be taken from them. It is believed, however, that the federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the Great Lakes.

On all the little lakes of the state which are mere expansions near their mouths of the rivers passing through them — such as the Muskegon, Père Marquette, and Manistee — the same rule of bed ownership has been judicially applied that is applied to the rivers themselves; and the division lines are extended under the water in the same way. *Rice* v. *Ruddiman*, 10 Mich. 125. If such a lake were circular, the lines would converge to the center; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as such. It is sometimes important, however, to run the lines out for some considerable distance, in order to determine where one may lawfully moor vessels or rafts, for the winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. *Lorman* v. *Benson*, 8 Mich. 18; *People's Ice Co.* v. *Steamer Excelsior*, recently decided.

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them.

As regards the lakes and ponds of the state, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not:

(1) To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them?

(2) What public rights exist in them?

(3) If there are islands in them which were not surveyed out and sold by the United States, can this be done now?

Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers can not be applied to the Great Lakes. Perhaps it should be held that the boundary is at low-water mark, but improvements beyond this would only become unlawful when they became nuisances. Islands in the Great Lakes would belong to the United States until sold, and might be surveyed and measured for sale at any time. The right to take fish in the lakes, or to cut ice, is public like the right of navigation, but is to be exercised in such manner as not to interfere with the rights of shore owners. But so far as these public rights can be the subject of ownership, they belong to the state, not to the United States; and, so it is believed, does the bed of a lake also. *Pollard* v. *Hagan*, 3 Howard's U. S. Reports. But such rights are not generally considered proper subjects of sale, but, like the right to make use of the public highways, they are held by the state in trust for all the people.

What is said of the large lakes may perhaps be said also of many of the interior lakes of the state; such, for example, as Houghton, Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size — say, even a mile across — there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon before. But the general rules for the extension of shore lines, which have already been laid down, should govern such cases, or at least should serve as guides in their settlement.[1]

[1] Since this address was delivered, some of these questions have received the attention of the Supreme Court of Michigan in the cases of *Richardson* v. *Prentiss*, 48 Mich. Reports, 88, and *Backus* v. *Detroit*, Albany Law Journal, vol. 26, p. 428.

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have any rights in it whatever. Where it is not so included, it is believed they have rights of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the states may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation.

If the state should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as a trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but the state could not drain, even for this purpose, against the will of the shore owners, unless their rights were appropriated and paid for.

Upon many questions that might arise between the state as owner of the bed of a little lake and the shore owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not and can not be judicial officers, but in a great many cases they act in a *quasi* judicial capacity with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said can not contribute much to their enlightenment, but I trust will not be wholly without value.

III. THE OWNERSHIP OF SURVEYS, AND WHAT CONSTITUTES A SURVEY AND MAP.[1]

There seems to be a difference of opinion among surveyors as to how much of the information obtained, and how much of the work done in making a survey shall be furnished to the individual for whom the survey is made. Many surveyors keep what are called "private notes." All men doing business as surveyors keep notes of all surveys in a convenient form for ready reference. The extent to which these notes are private has not been rightly comprehended by all surveyors, and hence has resulted the difference of opinion mentioned.

This article is an attempt to present a side of this question that has not heretofore been fully considered. An endeavor has also been made to point out to the young surveyor a line of action expedient for him to follow, which will at the same time be found advantageous to the community in which he works.

In this discussion the question arises at once, "What constitutes a survey?" and the answer obviously depends on the object of the survey. This discussion will be confined to land surveys; that is, to surveys made for the purpose (1) of subdividing a large tract of land into smaller parcels to be sold; (2) of determining the boundary of a tract the description of which is known; (3) of determining the description when the boundaries are known.

The principle to be enunciated applies to any other survey as well, be it railroad, canal, bridge, or topographical survey. Indeed, it is well understood in all such surveys, but seems to be ignored by many engineers having to do with land surveys.

A survey is the operation of finding the contour, dimensions, position, or *other particulars* of any part of the earth's surface, . . . tract of land, etc., and *representing the same on paper*.

In making a survey it is necessary to set certain points, called monuments or corners, and to determine a description of these points. These items therefore become a part of the survey. Then a map must be drawn. This map, to be a faithful representation of the ground and the work done, should, together with the notes, show all of the items mentioned.

The object of establishing monuments or corners and describing them is twofold: (1) to mark on the ground the boundaries of the tract, and (2) to secure definite information as to its location with reference to other points or tracts, so that from this information the land may at a future time be found. For a complete survey, therefore, the corners must be fixed, information that will preserve their location must be obtained, and the facts must be delineated on a map with accompanying notes.

To whom belongs this survey? It would appear to be evident that it belongs to the individual who pays to have it made. It is not readily seen in

[1] A paper prepared by the author for "The Polytechnic," the student journal of the Rensselaer Polytechnic Institute, from which it is taken.

what way the survey, or any part of it, becomes the sole property of the surveyor. He may keep a copy of his notes to facilitate his future work; but he has not the shadow of a claim to a single note, the time for taking which has been paid for by his employer. If his charge for his work is on a time basis, there can be no question as to the correctness of the above statements. If he contracts to do the work for a definite sum for the entire job, he may take as much time as he likes, and may keep as many private notes as he desires; but he is bound in honor to return to his employer the complete survey; and, if he does so, the private notes would thereafter be of no great assistance to him in securing further employment, particularly when it is remembered that men of repute do not bid against each other for professional work. His reputation for accuracy and honesty will be a far more potent factor in securing employment than any set of private notes fairly obtained.

It is true that a great many surveyors hold a different opinion, and purposely return their maps and notes in such condition, that, while they may serve the purpose for which they are primarily made, they do not tell the whole story, nor enough to make it easy for another surveyor to relocate the tract surveyed. When this is done, the person ordering the survey does not receive what he pays for. Something is withheld. It seems to need no argument to show that this is radically wrong.

But there is another reason for condemning this practice. The correct and permanent location of all public land lines, as streets, alleys, etc., as well as the permanent location of party lines between private owners, is a matter of the gravest importance, and no information that will at all serve to fix such lines in their correct positions for all time, should be withheld from the owner who pays for the survey, be it private citizen, municipality, county, or state.

The records of monuments and street lines made by a city engineer are no more his private property than are the records in the offices of the clerk, auditor, or treasurer, the property of the individuals who held office at the time the records were made. The correctness of the position assumed has been indicated by court decisions.

A great deal of laxity is shown in the conduct of offices of city engineers and county surveyors. The methods of regulating the pay of these officers has doubtless had much to do with this. It is frequently the case that the surveyor receives no salary, but is allowed to collect certain specified fees for work performed, and this gives color to his claim that his work is private and belongs to him. That this is not true concerning the public work he does is evident from what has preceded. That the records of work done for private citizens are not the property of the public needs no demonstration; but such work belongs to those citizens for whom it was done.

A different policy should be pursued with regard to these offices. In every case such an office should be a salaried one, with such salaried assistants as may be necessary. Certain fees should be prescribed for performing the various kinds of work that the surveyor may be called upon to do within the limits of the territory of the political division whose servant he is. These fees should cover all work connected with public construction and public or private land lines, and should be returned to the public treasury.

Their amount may be regulated, from time to time, so that they shall aggregate a sum sufficient to pay the expenses of the office. They should, of course, not cover work of a private character not having to do with land lines. But the entire public is interested in the permanency of land lines, and all records concerning them made by a public official should become public property. The permanency of land lines is too important a matter to be subject to avaricious and jealous rivalry; and all the surveyors in a given district should coöperate to preserve, in their correct places, all lines within the district.

To this end, the returns of every surveyor made to the owner should be thoroughly complete. Maps made for filing as public records should be so finished as to enable any surveyor to relocate the land without the least uncertainty as to the correctness of his work. That this is done in very few instances is well known to every surveyor who has had occasion to examine public records for data for surveys which he has been called upon to make.

Because of the fact that in most cases neither owners nor attorneys have been fully posted as to what constitutes a complete description, sufficient for relocation, and because surveyors have been willing to let matters stand as they were, great carelessness has arisen in the practice of making and filing maps for record.

While in some states good laws exist prescribing what shall appear on a map before it will be received as a public record, in more states there is nothing whatever to guide either owner, surveyor, attorney, or recorder in the matter. In the county records in such states, anything that is made up of lines and figures and labeled "this is a map," is considered a sufficient basis for a correct description and location of the property it purports to represent — no matter whether it is drawn by hand, photo-lithographed, or simply printed with "rule" and type. The records are full of auctioneers' circulars, manufactured in a printing office from information coming from nobody knows where, filed at the request of the auctioneer's clerk, with no name of owner or other interested party attached, except as the name of the auctioneer appears in the accompanying advertisement. Further than this, these maps are frequently purposely distorted to create a favorable impression of the property to be sold. Wide streets are shown where only narrow ones exist, streets appear opened for the full width where they have been opened for but half their width, subdivisions are indicated as rectangles that really may not be even parallelograms, etc. Such maps as these frequently form the only basis for the description and location of the property they are supposed to represent. Such misrepresentations are bad, *very* bad for those who buy; but is the information given by these circulars much worse than that furnished by many of the maps made by surveyors and filed at the request of the owners?

On these plots, if of "additions," we find lines indicating the boundaries of blocks and lots, all of which blocks and lots are numbered; the names of streets appear in neat letters; a few dimensions, possibly all linear dimensions, will be given; the streets or blocks may be delicately tinted, and the whole set off with a fine border and title. As an exhibition of the draughtsman's skill, these maps are perhaps valuable. As a source of information as to the location of the lines they purport to show, they are worth little more than the auctioneer's circular. Perhaps they have a few more figures, and the presumption may be a little stronger that the figures are correct.

Examine one of these maps closely. There will be found no evidence that a monument has been set in the field; not an angle is recorded, though the lines may cross at all sorts of angles; and dimensions are given that do not agree among themselves, so that the angles can not be calculated. There will be found no name signed except, possibly, that of the surveyor, who thus advertises what we shall charitably call his stupidity.

Frequently no monuments are set except small stakes at the corners of the blocks; but even the fact that such stakes have been set is not recorded on the plot. One who is acquainted with the practice of surveyors in a given district knows at what points to look for such stakes, and if they have been set and not pulled out to make room for a fence post or building, he may succeed in finding them. Some surveyors are accustomed to set stakes a certain distance away from the point the stake is supposed to mark, but no mention of this fact appears on the map. In fact, the map is so drawn that no one but the surveyor who made it can write a description of any one of the parcels of land shown, or correctly locate it on the ground. Furthermore, the surveyor himself finds it impossible, after the lapse of a few years and the destruction of his "private marks," to rerun any one of the lines exactly as originally laid out.

It is easy to see to what this leads — impossible descriptions of property, giving opportunity for differences in judgment as to interpretation of what was intended; disputes as to position of party lines; costly litigation and expensive movement of structures begun or completed; and the actual shifting of lines back and forth by different surveyors, or even by the same surveyor, honestly trying to locate the lines properly.

The writer has seen enough trouble of this sort to indicate to him that a radical change is needed in the field work and mapping of cities, towns, and additions, not to mention farms and other tracts of land that it may be necessary to lay out and describe. So long as fallible man is responsible for the accuracy of surveys, maps, and descriptions of properties, so long will there be errors; but that it is possible greatly to reduce their number by proper regulation the writer is fully persuaded. What we have been describing are not maps at all, or at most they are very imperfect maps, and "what constitutes a map?" thus seems to be a very pertinent question.

A map of a city, town, or addition, or other tract of land, serving as a basis for the description of property, should furnish all the information necessary for the proper description and location of the various parcels shown, and also of the whole piece. It should further show the exact location of the whole tract relatively to the lands immediately adjoining; particularly should this be done when an offset or angle in a street line occurs. To accomplish these things, there should appear on the map the following items:

(1) The lengths of all lines shown.

(2) The exact angle made by all intersecting lines.

(3) The exact position and character of all monuments set, with notes of reference points.

(4) The number of each block and lot.

(5) The names of all streets, streams or bodies of water, and recognized land marks.

(6) The scale.

(7) The direction of the meridian and a note as to whether the true or magnetic meridian is shown. (It should be the true meridian.)

(8) The angles of intersection made by the lines of adjoining property with the boundaries of the tract mapped.

(9) The exact amount of offset in lines that may extend from the outside through the tract mapped.

(10) A simple, complete, and explicit title, including the date and the name of the surveyor.

All this is necessary to make the map valuable for *description and location* of the property it represents.

Of course monuments will not be shown if none have been set, and very frequently none are set, either from carelessness on the part of the surveyor, or an unwillingness on the part of the owner to pay their cost. Monuments of a permanent character should be set at each corner of a tract surveyed, and at least two, visible the one from the other, should be on the line of each street. If these monuments are not placed on the center lines of the streets, they should be at *uniform* distances from the center or property lines. If placed with reference to the center line they should all be on the same side of the center. In streets extending east and west the monuments should all be on the north of the center, or they should all be on the south, and at uniform distances. In streets extending north and south the monuments should all be on the east of the center or all on the west. Uniformity in such practice saves a vast amount of time.

Monuments may be set at uniform distances from the block lines, in the sidewalk area, and this is an excellent practice. The stakes or monuments set at the corners of the blocks in additions, or town sites, should never be the *only* stakes or monuments set in the tract.

That the map may be reliable there should appear on it the following:

(1) The certificate of the surveyor that he has carefully surveyed the

land, that the map is a correct representation of the tract, and that he has set monuments (to be described) at the points indicated on the map.

(2) The acknowledged signature of all persons possessing title to any of the land shown in the tract, and, if possible, signatures of adjoining owners.

(3) If of an addition, the acknowledged dedication to public use forever of all areas shown as streets or roads.

(4) If a street of full width, whose center line is a boundary of the tract, is shown, the acknowledged signature of the owner of the adjoining property, unless his half of the street has been previously dedicated.

It has been already stated that, in some states, a map may be filed at the request of any person, and without signature. This practice frequently leads to trouble. The writer knows of cases in which owners of large tracts of land have had those tracts subdivided and have taken land of adjoining non-resident owners for street purposes without the consent or knowledge of those owners. When, at a later day, the owners of the land so taken have objected and attempted to close half of the street, trouble of a serious character has arisen. The same trouble has occurred where streets have been run through narrow gores of land and have subsequently been completely closed, leaving houses built on the mapped property without outlet. Time and again have cases of this sort come to the knowledge of the writer.

Having pointed out certain evils, it remains to suggest a remedy. It lies in the enactment of laws governing these matters. There should be included in the statutes of every state a law explicitly defining what shall appear on every map filed for reference, and making it a misdemeanor to file a map that does not strictly conform to the requirements. In the absence of such laws it is believed that the young surveyor can assist greatly in a much-needed reform, by following the principles suggested in this paper as the correct ones, and avoiding the errors here indicated.

It is hoped that those graduates of our engineering schools who drift into this class of work will be guided by a higher principle than that which actuates the surveyor who covers up his tracks, at the expense of his employer, in order to secure a monopoly of the business of the locality. The young surveyor can spend his energies to greater advantage in devising new and better methods of work, than in inventing ways for hiding information that belongs to his employer. Certainly a thorough education should so broaden the young man's views as to make it impossible for him to be controlled by those meaner instincts which, if indulged, lead ever to narrow the vision and prevent one from perceiving the greater problems that continually present themselves for solution.

IV. GEOGRAPHICAL POSITIONS OF BASE LINES AND PRINCIPAL MERIDIANS GOVERNING THE PUBLIC SURVEYS.

The system of rectangular surveying, authorized by law May 20, 1785, was first employed in the survey of United States public lands in the state of Ohio.

The boundary line between the states of Pennsylvania and Ohio, known as "Ellicott's line," in longitude 80° 32′ 20″ west from Greenwich, is the meridian to which the first surveys are referred. The *townships* east of the Scioto River, in the state of Ohio, are numbered from south to north, commencing with No. 1 on the Ohio River, while the *ranges* are numbered from east to west, beginning with No. 1 on the east boundary of the state, *except* in the tract designated "U. S. military land," in which the townships and ranges are numbered, respectively, from the south and east boundaries of said tract.

During the period of one hundred and nine years since the organization of the system of rectangular surveying, numbered and locally named principal meridians and base lines have been established, as follows:

The first principal meridian begins at the junction of the Ohio and Big Miami rivers, extends north on the boundary line between the states of Ohio and Indiana, and roughly approximates to the meridian of longitude 84° 48′ 50″ west from Greenwich. The ranges of the public surveys in the state of Ohio, west of the Scioto River, are, in part, numbered from this meridian. For further information in regard to numbering of townships and ranges of the early surveys in Ohio, the reader is referred to the state map prepared in the General Land Office.

The second principal meridian coincides with 86° 28′ of longitude west from Greenwich, starts from a point two and one half miles west of the confluence of the Little Blue and Ohio rivers, runs north to the northern boundary of Indiana, and, with the base line in latitude 38° 28′ 20″, governs the surveys in Indiana and part of those in Illinois.

The third principal meridian begins at the mouth of the Ohio River and extends north to the northern boundary of the state of Illinois, and with the base line in latitude 38° 28′ 20″, governs the surveys in the state *east* of the third principal meridian, with the exception of those projected from the *second principal meridian*, and the surveys on the *west*, to the Illinois River. This meridian is nearly coincident with 89° 10′ 15″ of west longitude from Greenwich.

The fourth principal meridian begins at a point on the right bank of the Illinois River, in latitude 40° 00′ 30″ north, and longitude 90° 28′ 45″ west from Greenwich, and with the base line running west from the initial point, governs the surveys in Illinois west of the Illinois River and west of that part of the *third* principal meridian which lies north of the river.

The fourth principal meridian also extends north through Wisconsin and northeastern Minnesota, and, with the south boundary of Wisconsin as its base line, governs all the surveys in the former and those in the latter state lying east of the Mississippi River, and the third guide meridian west (of the fifth principal meridian system), north of the river.

The fifth principal meridian starts from the old mouth of the Arkansas River, and with the base line running west from the old mouth of the St. Francis River, governs the surveys in Arkansas, Missouri, Iowa, North Dakota; those in Minnesota, west of the Mississippi River and west of the third guide meridian north of the river; and in South Dakota all east of the Missouri River, and the surveys on the west side of the river to a limiting line following the third guide meridian (of the sixth principal meridian system), White River, and the west and north boundaries of the Lower Brulé Indian Reservation. This meridian is nearly coincident with 91° 03′ 42″ longitude west from Greenwich.

The sixth principal meridian, which is approximately the meridian of 97° 23′ west longitude from Greenwich, extends from the base line coincident with the north boundary of Kansas in latitude 40° north, *south* through the state to its south boundary, in latitude 37° north, and *north* through Nebraska to the Missouri River; and governs the surveys in Kansas and Nebraska; the surveys in Wyoming, except those referred to the *Wind River meridian* and base line, which intersect in latitude 43° 01′ 20″ north, and longitude 108° 48′ 40″ west from Greenwich; the surveys in Colorado, except those projected from the *New Mexico* and *Ute meridians*, the latter intersecting its base line in latitude 39° 06′ 40″ north and longitude 108° 33′ 20″ west from Greenwich; and the surveys in South Dakota extended, or to be extended, over the tract embracing the Pine Ridge and Rosebud Indian reservations.

In addition to the above-mentioned *numbered* principal meridians, other principal meridians with *local names* have been established as follows:

The Michigan meridian, in longitude 84° 22′ 24″ west from Greenwich, with a base line in latitude 42° 26′ 30″ north (eight miles north of Detroit), governs the surveys in Michigan.

The Tallahassee meridian, in longitude 84° 16′ 42″ west from Greenwich, runs north and south from the initial point on the base line at Tallahassee, in latitude 30° 28′ north, and governs the surveys in Florida.

The Saint Stephen's meridian, in longitude 88° 02′ west from Greenwich, begins at the initial point (Ellicott's corner), on the base line, in latitude 31° north, extends south to Mobile Bay and north to latitude 33° 06′ 20″, and governs the surveys in the southern district of Alabama, and in Pearl River district lying east of the river and south of the Choctaw base line, in latitude 31° 52′ 40″ north, in the state of Mississippi.

The Huntsville meridian begins on the northern boundary of Alabama, in latitude 34° 59′ north, longitude 86° 34′ 45″ west from Greenwich, extends south to latitude 33° 06′ 20″ north, and governs the surveys in the northern district of Alabama.

The Choctaw meridian begins on the Choctaw base line, latitude 31°

54′ 40″ north, longitude 90° 14′ 45″ west from Greenwich, runs north to the south boundary of the Chickasaw cession, in latitude 34° 19′ 40″ north, and governs the surveys east and west of the meridian, and north of the base line.

The Chickasaw meridian begins on the north boundary of Mississippi in latitude 34° 59′ north, longitude 89° 15′ west from Greenwich, extends south to latitude 33° 48′ 45″ north, and governs the surveys in north Mississippi.

The Washington meridian begins on the base line in latitude 31° north, longitude 91° 9′ 15″ west from Greenwich, extends *north* to the Mississippi River, and governs the surveys in the southwestern angle of the state of Mississippi.

The Saint Helena meridian begins at the initial point of the *Washington meridian*, in latitude 31° north, and longitude 91° 09′ 15″ west of Greenwich, extends *south* to the Mississippi River, and governs the surveys in the Greensburg and southeastern districts of Louisiana, east of the Mississippi River.

The Louisiana meridian, in longitude 92° 24′ 15″ west of Greenwich, extends from the Gulf of Mexico to the north boundary of Louisiana, and with the base line through the initial point, conforming to the parallel of 31° north latitude, governs all the surveys in the state west of the Mississippi River.

The New Mexico meridian, in longitude 106° 53′ 40″ west from Greenwich, extends through the territory, and with the base line, in latitude 34° 15′ 25″ north, governs the surveys in New Mexico, *except* those in the northwest corner of the territory, referred to *Navajo meridian* and base line, which have their initial point in latitude 35° 45′ north, longitude 108° 32′ 45″ west from Greenwich.

The Salt Lake meridian, in longitude 111° 54′ 00″ west from Greenwich, has its initial point at the corner of Temple Block, in Salt Lake City, Utah, extends north and south through the territory, and, with the base line, through the initial, and coincident with the parallel of 40° 46′ 04″ north latitude, governs the surveys in the territory, except those referred to the *Uintah meridian* and base line projected from an initial point in latitude 40° 26′ 20″ north, longitude 109° 57′ 30″ west from Greenwich.

The Boisé meridian, longitude 116° 24′ 15″ west from Greenwich, passes through the initial point established south 29° 30′ west, nineteen miles distant from Boisé City, extends north and south through the state, and, with the base line in latitude 43° 46′ north, governs the surveys in the state of Idaho.

The Mount Diablo meridian, California, coincides with the meridian of 121° 54′ 48″ west from Greenwich, intersects the base line on the summit of the mountain from which it takes its name, in latitude 37° 51′ 30″ north, and governs the surveys in the state of Nevada, and the surveys of all central and northern California, *except* those belonging to the Humboldt meridian system.

The Humboldt meridian, longitude 124° 08′ west from Greenwich, intersects the base line on the summit of Mount Pierce, in latitude 40° 25′ 12″ north, and governs the surveys in the northwestern corner of California, lying west of the Coast range of mountains, and north of township 5 south, of the *Humboldt meridian system.*

The San Bernardino meridian, California, longitude 116° 56′ 15″ west from Greenwich, intersects the base line on Mount San Bernardino, latitude 34° 07′ 10″ north, and governs the surveys in southern California, lying *east* of the meridian, and that part of the surveys situated *west* of it which is *south* of the eighth standard parallel south, of the *Mount Diablo meridian system.*

The Willamette meridian, which is coincident with the meridian of 122° 44′ 20″ west from Greenwich, extends *south* from the base line, in latitude 45° 31′ north, to the north boundary of California, and *north* to the international boundary, and governs all the public surveys in the states of Oregon and Washington.

The Black Hills meridian, longitude 104° 03′ west from Greenwich, with the base line in latitude 44° north, governs the surveys in the state of South Dakota, north and west of White River, and west of the Missouri River (between latitudes 45° 55′ 20″ and 44° 17′ 30″), the north and west boundaries of the Lower Brulé Indian Reservation, and the west boundary of range 79 west, of the fifth principal meridian system.

The Montana meridian extends north and south from the initial monument on the summit of a limestone hill, eight hundred feet high, longitude 111° 38′ 50″ west from Greenwich, and with the base line on the parallel of 45° 46′ 48″ north latitude, governs the surveys in the state of Montana.

The Gila and Salt River meridian intersects the base line on the south side of Gila River, opposite the mouth of Salt River, in latitude 33° 22′ 40″ north, longitude 112° 17′ 25″ west from Greenwich, and governs the surveys in the territory of Arizona.

The Indian meridian, in longitude 97° 14′ 30″ west from Greenwich, extends from Red River to the south boundary of Kansas, and with the base line in latitude 34° 30′ north, governs the surveys in the Indian Territory, and in Oklahoma Territory all surveys *east* of 100° west longitude from Greenwich.

The Cimarron meridian, in longitude 103° west from Greenwich, extends from latitude 36° 30′ to 37° north, and with the base line in latitude 36° 30′ north, governs the surveys in Oklahoma Territory *west* of 100° west longitude from Greenwich.

V. TABLES.

TABLE I.

Correction to One Hundred Units measured along the Slopes Given.

Units rise in 100.	Corresponding Vertical Angle.	Correction.
1.02	0° 35′	0.005
2.01	1° 09′	0.020
3.03	1° 44′	0.046
4.02	2° 18′	0.081
5.01	2° 52′	0.125
6.00	3° 26′	0.179
7.00	4° 00′	0.244
8.02	4° 35′	0.320
9.01	5° 09′	0.404
10.01	5° 43′	0.497
20.01	11° 19′	1.617
30.00	16° 42′	4.218
40.00	21° 48	7.151
50.00	26° 34′	10.559

TABLE II.[1]

Correction Coefficient for Temperature and Hygrometric Conditions.

This correction is used when no hygrometric observations have been made. To the difference in altitude found in Table III. for the given barometer readings is added algebraically the product of that difference and the correction below given, according to the formula, Diff. Alt. = (Diff. by Table III.) $(1 + c)$.

Sum O. T.[2]	Corr. Coeff.[3]	Sum O. T.	Corr. Coeff.	Sum O. T.	Corr. Coeff.
0°	0.1024	70°	0.0273	140°	0.0471
10	0.0915	80	0.0166	150	0.0575
20	0.0806	90	0.0058	160	0.0677
30	0.0698	100	0.0049	170	0.0779
40	0.0592	110	0.0156	180	0.0879
50	0.0486	120	0.0262		
60	0.0380	130	0.0368		

[1] Computed from Tables I. and IV., Appendix 10, "U. S. Coast Survey Report" for 1881.
[2] Sum of Observed Temperatures. [3] Correction Coefficient.

TABLE III.[1]

Barometric Elevations.

Giving altitudes above arbitrary sea level (barometer reading 30 inches) for various barometer readings *B*.

To determine difference of elevation of two points having barometer readings B and B_1, take from the table the altitudes corresponding to B and B_1, and correct *their difference* by Table II. The corrected difference is the quantity required.

B.	*A.*	Diff. for .01.	*B.*	*A.*	Diff. for .01.	*B.*	*A.*	Diff. for .01.
Inches.	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*
11.0	27,336	−24.6	14.0	20,765	−19.5	17.0	15,476	−16.0
11.1	27,090	24.4	14.1	20,570	19.3	17.1	15,316	15.9
11.2	26,846	24.2	14.2	20,377	19.1	17.2	15,157	15.8
11.3	26,604	24.0	14.3	20,186	18.9	17.3	14,999	15.7
11.4	26,364	23.8	14.4	19,997	18.8	17.4	14,842	15.6
11.5	26,126	23.6	14.5	19,809	18.6	17.5	14,686	15.5
11.6	25,890	23.4	14.6	19,623	18.6	17.6	14,531	15.4
11.7	25,656	23.2	14.7	19,437	18.5	17.7	14,377	15.4
11.8	25,424	23.0	14.8	19,252	18.4	17.8	14,223	15.3
11.9	25,194	22.8	14.9	19,068	18.2	17.9	14,070	15.2
12.0	24,966	22.6	15.0	18,886	18.1	18.0	13,918	15.1
12.1	24,740	22.4	15.1	18,705	18.0	18.1	13,767	15.0
12.2	24,516	22.2	15.2	18,525	17.9	18.2	13,617	14.9
12.3	24,294	22.1	15.3	18,346	17.8	18.3	13,468	14.9
12.4	24,073	21.9	15.4	18,168	17.6	18.4	13,319	14.7
12.5	23,854	21.7	15.5	17,992	17.5	18.5	13,172	14.7
12.6	23,637	21.6	15.6	17,817	17.4	18.6	13,025	14.6
12.7	23,421	21.4	15.7	17,643	17.3	18.7	12,879	14.6
12.8	23,207	21.2	15.8	17,470	17.2	18.8	12,733	14.4
12.9	22,995	21.0	15.9	17,298	17.1	18.9	12,589	14.4
13.0	22,785	20.9	16.0	17,127	16.9	19.0	12,445	14.3
13.1	22,576	20.8	16.1	16,958	16.9	19.1	12,302	14.2
13.2	22,368	20.6	16.2	16,789	16.8	19.2	12,160	14.2
13.3	22,162	20.4	16.3	16,621	16.7	19.3	12,018	14.1
13.4	21,958	20.1	16.4	16,454	16.6	16.4	11,877	14.0
13.5	21,757	20.0	16.5	16,288	16.4	19.5	11,737	13.9
13.6	21,557	19.9	16.6	16,124	16.3	19.6	11,598	13.9
13.7	21,358	19.8	16.7	15,961	16.3	19.7	11,459	13.8
13.8	21,160	19.8	16.8	15,798	16.2	19.8	11,321	13.7
13.9	20,962	−19.7	16.9	15,636	−16.0	19.9	11,184	−13.7
14.0	20,765		17.0	15,476		20.0	11,047	

[1] Taken from Appendix 10, "U.S. Coast and Geodetic Survey Report" for 1881.

TABLE III. (*continued*).

B.	*A.*	DIFF. FOR .01.	*B.*	*A.*	DIFF. FOR .01.	*B.*	*A.*	DIFF. FOR .01.
Inches.	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*
20.0	11,047	−13.6	23.7	6,423	−11.5	27.4	2,470	− 9.9
20.1	10,911	13.5	23.8	6,308	11.4	27.5	2,371	9.9
20.2	10,776	13.4	23.9	6,194	11.4	27.6	2,272	9.9
20.3	10,642	13.4	24.0	6,080	11.3	27.7	2,173	9.8
20.4	10,508	13.3	24.1	5,967	11.3	27.8	2,075	9.8
20.5	10,375	13.3	24.2	5,854	11.3	27.9	1,977	9.7
20.6	10,242	13.2	24.3	5,741	11.2	28.0	1,880	9.7
20.7	10,110	13.1	24.4	5,629	11.1	28.1	1,783	9.7
20.8	9,979	13.1	24.5	5,518	11.1	28.2	1,686	9.7
20.9	9,848	13.0	24.6	5,407	11.1	28.3	1,589	9.6
21.0	9,718	12.9	24.7	5,296	11.0	28.4	1,493	9.6
21.1	9,589	12.9	24.8	5,186	10.9	28.5	1,397	9.5
21.2	9,460	12.8	24.9	5,077	10.9	28.6	1,302	9.5
21.3	9,332	12.8	25.0	4,968	10.9	28.7	1,207	9.5
21.4	9,204	12.7	25.1	4,859	10.8	28.8	1,112	9.4
21.5	9,077	12.6	25.2	4,751	10.8	28.9	1,018	9.4
21.6	8,951	12.6	25.3	4,643	10.8	29.0	924	9.4
21.7	8,825	12.5	25.4	4,535	10.7	29.1	830	9.4
21.8	8,700	12.5	25.5	4,428	10.7	29.2	736	9.3
21.9	8,575	12.4	25.6	4,321	10.6	29.3	643	9.3
22.0	8,451	12.4	25.7	4,215	10.6	29.4	550	9.2
22.1	8,327	12.3	25.8	4,109	10.5	29.5	458	9.2
22.2	8,204	12.2	25.9	4,004	10.5	29.6	366	9.2
22.3	8,082	12.2	26.0	3,899	10.5	29.7	274	9.2
22.4	7,960	12.2	26.1	3,794	10.4	29.8	182	9.1
22.5	7,838	12.1	26.2	3,690	10.4	29.9	91	9.1
22.6	7,717	12.0	26.3	3,586	10.3	30.0	00	9.1
22.7	7,597	12.0	26.4	3,483	10.3	30.1	−91	9.0
22.8	7,477	11.9	26.5	3,380	10.3	30.2	181	9.0
22.9	7,358	11.9	26.6	3,277	10.2	30.3	271	9.0
23.0	7,239	11.8	26.7	3,175	10.2	30.4	361	9.0
23.1	7,121	11.7	26.8	3,073	10.1	30.5	451	8.9
23.2	7,004	11.7	26.9	2,972	10.1	30.6	540	8.9
23.3	6,887	11.7	27.0	2,871	10.1	30.7	629	8.8
23.4	6,770	11.6	27.1	2,770	10.0	30.8	717	8.8
23.5	6,654	11.6	27.2	2,670	10.0	30.9	805	−8.8
23.6	6,538	−11.5	27.3	2,570	−10.0	31.0	−893	
23.7	6,423		27.4	2,470				

TABLE IV.

POLAR DISTANCE OF POLARIS. For January 1 of years named.

1894	1897	1900	1903	1906	1909	1912	1915	1918	1921
1° 15.43′	1° 14.50′	1° 13.55′	1° 12.62′	1° 11.68′	1° 10.75′	1° 09.82′	1° 08.88′	1° 07.97′	1° 07.03′

$$\text{Sin of azimuth at elongation} = \frac{\text{sin polar distance}}{\text{cosine latitude}}.$$

Latitude = altitude of Polaris at culmination ± polar distance — refraction correction given below.

LATITUDE.	CORRECTION, MINUTES.	LATITUDE.	CORRECTION, MINUTES.
20°	2.60	50°	0.80
30	1.65	60	0.55
40	1.13		

TABLE V.[1]

AMOUNT AND VARIATION OF THE MAGNETIC NEEDLE FROM ITS MEAN DAILY POSITION.

The letters E and W indicate which side of the mean position the needle points.

SEASON AND POSITION IN LATITUDE.	LOCAL MEAN TIME; MORNING HOURS.						
	6^h	7^h	8^h	9^h	10^h	11^h	12^h
	′	′	′	′	′	′	′
December, January, February:							
Latitude 37° to 49°	0.7 E	1.1 E	1.9 E	2.2 E	1.5 E	0.1 W	1.8 W
Latitude 25° to 37°	0.1 W	0.1 E	1.0 E	2.0 E	2.2 E	1.1 E	0.5 W
March, April, May:							
Latitude 37° to 49°	2.6 E	3.8 E	4.4 E	3.5 E	1.2 E	1.6 E	3.8 W
Latitude 25° to 37°	1.6 E	2.8 E	3.3 E	2.6 E	1.1 E	0.6 W	1.9 W
June, July, August:							
Latitude 37° to 49°	4.0 E	5.6 E	5.7 E	4.5 E	1.7 E	1.6 E	4.1 W
Latitude 25° to 37°	2.4 E	4.0 E	4.2 E	2.9 E	0.5 E	1.6 W	2.8 W
September, October, November:							
Latitude 37° to 49°	1.8 E	2.6 E	3.1 E	2.5 E	1.0 E	1.5 E	3.3 W
Latitude 25° to 37°	0.9 E	2.1 E	2.6 E	2.1 E	0.6 E	0.9 W	2.1 W

SEASON AND POSITION IN LATITUDE.	LOCAL MEAN TIME; AFTERNOON HOURS.						
	0^h	1^h	2^h	3^h	4^h	5^h	6^h
	′	′	′	′	′	′	′
December, January, February:							
Latitude 37° to 49°	1.8 W	2.9 W	2.8 W	2.1 W	1.3 W	0.7 W	0.2 W
Latitude 25° to 37°	0.5 W	1.5 W	1.8 W	1.6 W	1.0 W	0.4 W	0.1 W
March, April, May:							
Latitude 37° to 49°	3.8 W	4.8 W	4.6 W	3.8 W	2.5 W	1.4 W	0.7 W
Latitude 25° to 37°	1.9 W	2.6 W	2.8 W	2.4 W	1.6 W	0.9 W	0.5 W
June, July, August:							
Latitude 37° to 49°	4.1 W	5.6 W	5.6 W	4.6 W	3.0 W	1.4 W	0.6 W
Latitude 25° to 37°	2.8 W	3.2 W	3.1 W	2.4 W	1.5 W	0.8 W	0.4 W
September, October, November:							
Latitude 37° to 49°	3.3 W	4.0 W	3.4 W	2.3 W	1.2 W	0.6 W	0.1 W
Latitude 25° to 37°	2.1 W	2.3 W	1.9 W	1.2 W	0.7 W	0.4 W	0.2 W

[1] From "Manual of Instructions" issued by the U. S. Land Office to Surveyors General.

TABLE VI.[1]

APPROXIMATE LOCAL MEAN TIMES (COUNTING FROM NOON 24 HOURS) OF THE ELONGATIONS AND CULMINATIONS OF POLARIS IN THE YEAR 1897 FOR LATITUDE 40° N.; LONGITUDE 6^h W. FROM GREENWICH.

DATE.		EAST ELONGATION.		WEST ELONGATION.		UPPER CULMINATION.		LOWER CULMINATION.	
		h.	m.	h.	m.				
Jan.	1	0	38.2	12	27.8	6	33.6	18	31.6
	15	23	39.0	11	32.5	5	38.6	17	36.3
Feb.	1	22	31.8	10	25.4	4	31.2	16	29.2
	15	21	36.6	9	30.2	3	35.9	15	33.9
Mar.	1	20	41.4	8	34.9	2	40.7	14	38.7
	15	19	46.3	7	39.8	1	45.7	13	43.7
Apr.	1	18	39.3	6	32.8	0	38.6	12	36.7
	15	17	44.3	5	37.8	23	39.7	11	41.7
May	1	16	41.5	4	35.0	22	36.8	10	38.8
	15	15	46.6	3	40.1	21	41.9	9	43.9
June	1	14	39.9	2	33.4	20	35.3	8	37.3
	15	13	45.0	1	38.5	19	40.4	7	42.4
July	1	12	42.4	0	35.9	18	37.8	6	39.8
	15	11	47.5	23	37.1	17	42.9	5	44.9
Aug.	1	10	41.0	22	30.6	16	36.4	4	38.4
	15	9	46.1	21	35.7	15	41.5	3	43.5
Sept.	1	8	39.5	20	29.1	14	34.9	2	36.9
	15	7	44.6	19	34.2	13	40.0	1	42.0
Oct.	1	6	41.8	18	31.4	12	37.2	0	39.2
	15	5	46.8	17	36.4	11	42.2	23	40.3
Nov.	1	4	40.0	16	29.6	10	35.4	22	33.4
	15	3	44.8	15	34.4	9	40.2	21	38.2
Dec.	1	2	41.8	14	31.4	8	37.2	20	35.2
	15	1	46.5	13	36.1	7	41.9	19	39.9

To refer to any calendar day other than the first and fifteenth of each month, subtract 3.94^m for every day between it and the preceding tabular day, or add 3.94^m for every day between it and the succeeding tabular day.

To refer the tabular times to any year subsequent to the year 1897, add 0.25^m (nearly) for every additional year (after 1900, 0.2^m).

Also, For the second year after a leap year, add, 0.9^m.
For the third year after a leap year, add, 1.7^m.
For leap year *before* March 1, add, 2.6^m.
For leap year *on* and *after* March 1, subtract, 1.2^m.

For the first year after a leap year the table is correct, except for the regular annual change.

To refer the tabular times to other longitudes than six hours, add when east, and subtract when west of six hours, 0.16^m for each hour.

To refer to any other than the tabular latitude between the limits of 25° and 50° north, *add* to the time of west elongation 0.13^m for every degree south of latitude 40°, and *subtract* from the time of west elongation 0.18^m for every degree north of 40°. Reverse these signs for corrections to the times of east elongation. For latitudes as high as 60°, diminish the times of *west* elongation and increase the times of *east* elongation by 0.23^m for every degree north of latitude 40°.

[1] Computed from information contained in the "Manual of Instructions" issued by the General Land Office. The information was furnished by the U. S. Coast and Geodetic Survey.

TABLE VII.[1]

Refraction Corrections to Declination of the Sun.

The hour angle is the time either side of noon.

Latitude.	Hour Angle.	Declinations. +20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
° ′	h.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
25 00	0	0 05	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 57
	2	0 08	0 14	0 19	0 25	0 31	0 38	0 46	0 54	1 05
	3	0 12	0 18	0 24	0 30	0 37	0 44	0 53	1 04	1 18
	4	0 23	0 29	0 35	0 45	0 53	1 03	1 16	1 31	1 52
	5	0 49	0 59	1 10	1 24	1 52	2 07	2 44	3 46	5 43
27 30	0	0 08	0 13	0 18	0 24	0 30	0 36	0 44	0 52	1 02
	2	0 11	0 16	0 22	0 28	0 34	0 41	0 49	1 00	1 10
	3	0 17	0 22	0 28	0 35	0 42	0 50	1 00	1 11	1 26
	4	0 28	0 35	0 42	0 50	1 00	1 11	1 26	1 43	2 09
	5	0 54	1 05	1 18	1 34	1 54	2 24	3 11	4 38	8 15
30 00	0	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 57	1 08
	2	0 14	0 19	0 25	0 31	0 38	0 46	0 54	1 05	1 18
	3	0 20	0 26	0 32	0 39	0 47	0 55	1 06	1 19	1 36
	4	0 32	0 39	0 46	0 52	1 06	1 19	1 35	1 57	2 29
	5	1 00	1 10	1 24	1 52	2 07	2 44	3 46	5 43	13 06
32 30	0	0 13	0 18	0 24	0 30	0 36	0 44	0 52	1 02	1 14
	2	0 17	0 22	0 28	0 35	0 42	0 50	1 00	1 11	1 26
	3	0 23	0 29	0 35	0 43	0 51	1 01	1 13	1 28	1 47
	4	0 35	0 43	0 51	1 01	1 13	1 27	1 46	2 13	2 54
	5	1 03	1 15	1 31	1 53	2 20	3 05	4 25	7 36	—
35 00	0	0 15	0 21	0 27	0 33	0 40	0 48	0 57	1 08	1 21
	2	0 20	0 25	0 32	0 38	0 46	0 55	1 05	1 18	1 35
	3	0 26	0 33	0 39	0 47	0 56	1 07	1 21	1 38	2 00
	4	0 39	0 47	0 56	1 07	1 20	1 36	1 59	2 32	3 25
	5	1 07	1 20	1 38	2 00	2 34	3 29	5 14	10 16	—

[1] Computed from formula

$$C = 57'' \cot(\delta + N),$$

in which δ is the declination, plus when north, and minus when south; and N an auxiliary angle found by

$$\tan N = \cot \phi \cos t,$$

in which ϕ is the latitude of the place, and t the angle between the meridian of the place and the meridian through the sun at the given time, — called the "hour angle." The formulæ are from Chauvenet's "Spherical and Practical Astronomy," vol. I., p. 171. The table was computed by Mr. Edward W. Arms, C.E., for Messrs. W. & L. E. Gurley, of Troy, N.Y., and is here used by their permission.

TABLE VII. (*continued*).

Latitude.	Hour Angle.	Declinations.								
		+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
° ′	h.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
37 30	0	0 18	0 24	0 30	0 36	0 44	0 52	1 02	1 14	1 29
	2	0 22	0 28	0 35	0 42	0 50	1 00	1 12	1 26	1 45
	3	0 29	0 36	0 43	0 52	1 02	1 14	1 29	1 49	2 16
	4	0 43	0 51	1 01	1 13	1 27	1 49	2 14	2 54	4 05
	5	1 11	1 26	1 44	2 10	2 49	3 55	6 15	14 58	—
40 00	0	0 21	0 27	0 33	0 40	0 48	0 57	1 08	1 21	1 03
	2	0 25	0 32	0 39	0 46	0 52	1 06	1 19	1 35	1 57
	3	0 33	0 40	0 48	0 57	1 08	1 21	1 38	2 02	2 36
	4	0 47	0 55	1 06	1 19	1 36	1 58	2 30	3 21	4 59
	5	1 15	1 31	1 51	2 20	3 05	4 25	7 34	25 18	—
42 30	0	0 24	0 30	0 36	0 44	0 52	1 02	1 14	1 29	1 49
	2	0 28	0 35	0 39	0 50	1 00	1 12	1 26	1 45	2 11
	3	0 36	0 43	0 52	1 02	1 13	1 29	1 49	2 17	2 59
	4	0 50	1 00	1 11	1 26	1 44	2 10	2 49	3 55	6 16
	5	1 16	1 36	1 58	2 30	3 22	5 00	9 24	—	—
45 00	0	0 27	0 33	0 40	0 48	0 57	1 08	1 21	1 39	2 02
	2	0 32	0 39	0 46	0 52	1 06	1 19	1 35	1 57	2 29
	3	0 40	0 47	0 56	1 07	1 21	1 38	2 00	2 34	3 29
	4	0 54	1 04	1 16	1 33	1 54	2 24	3 11	4 38	8 15
	5	1 23	1 41	2 05	2 41	3 40	5 40	12 02	—	—
47 30	0	0 30	0 36	0 44	0 52	1 02	1 14	1 29	1 49	2 18
	2	0 35	0 42	0 50	1 00	1 12	1 26	1 45	2 01	2 51
	3	0 43	0 51	1 01	1 13	1 28	1 47	2 15	2 56	4 08
	4	0 56	1 09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
	5	1 27	1 46	2 12	2 52	4 01	6 30	16 19	—	—
50 00	0	0 33	0 40	0 48	0 57	1 08	1 21	1 39	2 02	2 36
	2	0 38	0 46	0 55	1 06	1 18	1 35	1 57	2 28	3 19
	3	0 47	0 56	1 06	1 19	1 36	2 29	2 31	3 23	5 02
	4	1 02	1 14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
	5	1 30	1 51	2 19	3 04	4 22	7 28	24 10	—	—
52 30	0	0 36	0 44	0 52	1 02	1 14	1 29	1 49	2 18	3 05
	2	0 43	0 50	0 59	1 11	1 26	1 42	2 23	2 49	3 55
	3	0 50	1 00	1 11	1 26	1 45	2 11	2 51	2 58	6 22
	4	1 05	1 18	1 35	2 10	2 28	3 19	4 53	8 42	—
	5	1 34	1 56	2 27	3 16	4 47	8 52	—	—	—
55 00	0	0 40	0 48	0 57	1 08	1 21	1 39	2 02	2 36	3 33
	2	0 46	0 55	1 05	1 18	1 34	1 56	2 30	3 15	4 47
	3	0 55	1 06	1 19	1 35	1 58	2 30	3 21	4 58	9 19
	4	1 10	1 23	1 42	2 06	2 43	3 44	5 49	12 41	—
	5	1 37	2 01	2 34	3 28	5 15	10 18	—	—	—

TABLE VIII.[1]

Magnetic Declination.

Formulas giving approximately the magnetic declination at the places named and for any time within the limits of the period of observation. The places are divided into three groups, as follows:

Group I. — Magnetic stations on the eastern coast of the United States and inclusive of the region of the Appalachian range, with some additional stations in Newfoundland and other foreign localities.

Group II. — Magnetic stations mainly in the central part of the United States between the Appalachian and Rocky Mountain ranges, with additions in British North America, Canada, the West Indies, and Central America.

Group III. — Magnetic stations on the Pacific coast and Rocky Mountain region; also in Mexico and Alaska and in some foreign countries.

D stands for declination, + indicating west, and — east declination; m stands for $t - 1850.0$ or for the difference in time, expressed in years and fraction of a year, for any time t and the middle of the century; a * indicates uncertainty.

No.	Name of Station and State.	Latitude.	West Longitude.	The Magnetic Declination expressed as a Function of Time.
	Group I.	° ′	° ′	° ° °
1	Saint Johns, Newfoundland.	47 34.4	52 41.9	$D = +21.94 + 8.89 \sin(1.05\,m + 63.4)$*
2	Quebec, Canada.	46 48.4	71 14.5	$D = +14.66 + 3.03 \sin(1.4\,m + 4.6)$
				$+ 0.61 \sin(4.0\,m + 0.3)$
3	Charlottetown, P.E.I.	46 14	63 27	$D = +15.95 + 7.78 \sin(1.2\,m + 49.8)$
4	Montreal, Canada.	45 30.5	73 34.6	$D = +11.88 + 4.17 \sin(1.5\,m - 18.5)$
				$+ 0.36 \sin(4.9\,m + 19)$
5	Eastport, Me.	44 54.4	66 59.2	$D = +15.18 + 3.79 \sin(1.25\,m + 31.1)$*
6	Bangor, Me.	44 48.2	68 46.9	$D = +13.86 + 3.55 \sin(1.30\,m + 8.6)$
7	Halifax, Nova Scotia.	44 39.6	63 35.3	$D = +16.18 + 4.53 \sin(1.0\,m + 46.1)$*
8	Burlington, Vt.	44 28.5	73 12.0	$D = +10.81 + 3.65 \sin(1.30\,m - 20.5)$
				$+ 0.18 \sin(7.0\,m + 132)$
9	Hanover, N. H.	43 42.3	72 17.1	$D = +9.80 + 4.02 \sin(1.4\,m - 14.1)$*
10	Portland, Me.	43 38.8	70 16.6	$D = +11.40 + 3.28 \sin(1.30\,m + 2.7)$
11	Rutland, Vt.	43 36.5	72 55.5	$D = +10.03 + 3.82 \sin(1.5\,m - 24.3)$
12	Portsmouth, N. H.	43 04.3	70 42.5	$D = +10.71 + 3.36 \sin(1.44\,m - 7.4)$
13	Chesterfield, N. H.	42 53.5	72 24	$D = +9.60 + 3.84 \sin(1.35\,m - 16.1)$*
14	Newburyport, Mass.	42 48.9	70 49.2	$D = +10.07 + 3.02 \sin(1.35\,m - 1.0)$
15	Williamstown, Mass.	42 42.8	73 13.4	$D = +8.84 + 3.13 \sin(1.4\,m - 14.0)$*
16	Albany, N. Y.	42 39.2	73 45.8	$D = +8.17 + 3.02 \sin(1.44\,m - 8.3)$
17	Salem, Mass.	42 31.9	70 52.5	$D = +9.98 + 3.85 \sin(1.4\,m - 5.1)$*
18	Oxford, N. Y.	42 26.5	75 40.5	$D = +6.19 + 3.24 \sin(1.35\,m - 18.9)$
19	Cambridge, Mass.	42 22.9	71 07.7	$D = +9.54 + 2.69 \sin(1.30\,m + 7.0)$
				$+ 0.18 \sin(3.2\,m + 44)$
20	Boston, Mass.	42 21.5	71 03.9	$D = +9.48 + 2.94 \sin(1.3\,m + 3.7)$
21	Provincetown, Mass.	42 03.1	70 11.3	$D = +9.67 + 3.04 \sin(1.3\,m + 11.0)$*
22	Providence, R. I.	41 50.2	71 23.8	$D = +9.10 + 2.99 \sin(1.45\,m - 3.4)$
				$+ 0.26 \sin(7\,m + 84)$
23	Hartford, Conn.	41 45.9	72 40.4	$D = +8.06 + 2.90 \sin(1.25\,m - 26.4)$
24	New Haven, Conn.	41 18.5	72 55.7	$D = +7.78 + 3.11 \sin(1.40\,m - 22.1)$
25	Nantucket, Mass.	41 17.0	70 06.0	$D = +8.61 + 2.83 \sin(1.35\,m + 19.7)$
26	Cold Spring Harbor, Long Island, N. Y.	40 52	73 28	$D = +7.19 + 2.52 \sin(1.35\,m - 11.4)$
27	New York City, N. Y.	40 42.7	74 00.4	$D = +7.04 + 2.77 \sin(1.30\,m - 18.1)$
				$+ 0.14 \sin(6.3\,m + 64)$

[1] From Appendix 7, "U. S. Coast and Geodetic Survey Report" for 1888.

No.	Name of Station and State.	Latitude.	West Longitude.	The Magnetic Declination expressed as a Function of Time.
		° ′	° ′	° ° °
28	Bethlehem, Pa.	40 36.4	75 22.9	$D=+\ 5.40+\ 3.13\sin(1.55\,m-38.3)$
29	Huntingdon, Pa.	40 31	78 02	$D=+\ 3.76+\ 2.93\sin(1.48\,m-35.2)$
30	New Brunswick, N.J.	40 29.9	74 26.8	$D=+\ 5.11+\ 2.94\sin(1.30\,m+\ 4.2)$
31	Jamesburg, N. J.	40 21	74 27	$D=+\ 6.03+\ 2.94\sin(1.40\,m-22.4)$
32	Harrisburg, Pa.	40 15.9	76 52.9	$D=+\ 2.93+\ 2.98\sin(1.50\,m+\ 0.2)$
33	Hatboro, Pa.	40 12	75 07	$D=+\ 5.17+\ 3.16\sin(1.54\,m-16.7)$ $+\ 0.22\sin(4.1\ m+157)$
34	Philadelphia, Pa.	39 56.9	75 09.0	$D=+\ 5.36+\ 3.17\sin(1.50\,m-26.1)$ $+\ 0.19\sin(4.0\ m+146)$
35	Chambersburg, Pa.	39 55	77 40	$D=+\ 2.79+\ 3.10\sin(1.55\,m-30.6)$ $+\ 0.20\sin(4.6\ m+124)$
36	Baltimore, Md.	39 17.8	76 37.0	$D=+\ 3.20+\ 2.57\sin(1.45\,m-21.2)$
37	Washington, D. C.	38 53.3	77 00.6	$D=+\ 2.73+\ 2.57\sin(1.45\,m-21.6)$ $+\ 0.14\sin(12\ m+27)$
38	Cape Henlopen, Del.	38 46.7	75 05.0	$D=+\ 4.01+\ 3.22\sin(1.35\,m-25.2)$
39	Williamsburg, Va.	37 16.2	76 42.4	$D=+\ 2.33+\ 2.56\sin(1.5\ m-38.1)$
40	Cape Henry, Va.	36 55.6	76 00.4	$D=+\ 2.42+\ 2.25\sin(1.47\,m-30.6)$
41	Newbern, N. C.	35 06	77 02	$D=+\ 0.63+\ 2.56\sin(1.45\,m-18.2)$*
42	Milledgeville, Ga.	33 04.2	83 12	$D=-\ 3.10+\ 2.53\sin(1.40\,m-61.9)$*
43	Charleston, S. C.	32 46.6	79 55.8	$D=-\ 1.82+\ 2.75\sin(1.40\,m-12.1)$*
44	Savannah, Ga.	32 04.9	81 05.5	$D=-\ 2.13+\ 2.55\sin(1.40\,m-40.5)$*
45	Paris, France.	48 50.2	2 20.2E	$D=+6^{\circ}.479+16^{\circ}.002\sin(0.765\,m+118^{\circ}\,46'.5+[0.85-0.35\sin(0.69\,n)]\sin[(4.04+0.0054\,n+.000035\,n^2)n]$
46	St. George's, Bermuda	32 23	64 42	$D=+\ 6.95+0.0145\,m+0.00056\,m^2$*
47	Rio de Janeiro, Brazil	−22 54.8	43 09.5	$D=+\ 2.19+9.91\sin(0.80\,m-10.4)$*
	Group II.			
1	York Factory, British North America	56 59.9	92 26	$D=+\ 7.34+16.03\sin(1.10\,m-97.9)$
2	Fort Albany, British North America.	52 22	82 38	$D=+15.78+\ 6.95\sin(1.20\,m-99.6)$*
3	Duluth, Minn., and Superior City, Wis.	46 45.5 / 46 39.9	92 04.5 / 92 04.2	$D=-\ 7.70+\ 2.41\sin(1.4\ m-120.0)$*
4	Sault Ste. Marie, Mich.	46 29.9	84 20.1	$D=+\ 1.54+\ 2.70\sin(1.45\,m-58.5)$
5	Pierrepont Manor, N. Y.	43 44.5	76 03.0	$D=+\ 5.95+\ 3.78\sin(1.4\ m-22.2)$
6	Toronto, Canada.	43 39.4	79 23.5	$D=+\ 3.60+\ 2.82\sin(1.4\ m-44.7)$ $+\ 0.09\sin(9.3\ m+136)$ $+\ 0.08\sin(19\ m+247)$
7	Grand Haven, Mich.	43 05.2	86 12.6	$D=-\ 4.95+\ 0.0380\,m+0.00120\,m^2$
8	Milwaukee, Wis.	43 02.5	87 54.2	$D=-\ 4.12+\ 3.60\sin(1.45\,m-64.5)$*
9	Buffalo, N. Y.	42 52.8	78 53.5	$D=+\ 3.66+\ 3.47\sin(1.4\ m-27.8)$
10	Detroit, Mich.	42 20.0	83 03.0	$D=-\ 0.97+\ 2.21\sin(1.5\ m-15.3)$
11	Ypsilanti, Mich.	42 14	83 38	$D=-\ 1.20+\ 3.40\sin(1.40\,m-4.1)$
12	Erie, Pa.	42 07.8	80 05.4	$D=+\ 2.17+\ 2.69\sin(1.5\ m-27.3)$
13	Chicago, Ill.	41 50.0	87 36.8	$D=-\ 3.77+\ 2.48\sin(1.45\,m-62.5)$
14	Michigan City, Ind.	41 43.4	86 54.4	$D=-\ 3.23+\ 2.42\sin(1.4\ m-48.0)$
15	Cleveland, O.	41 30.4	81 41.5	$D=+\ 0.47+\ 2.39\sin(1.30\,m-14.8)$
16	Omaha, Neb.	41 15.7	95 56.5	$D=-\ 9.30+\ 3.34\sin(1.30\,m-54.7)$
17	Beaver, Pa.	40 44	80 20	$D=+\ 1.41+\ 2.72\sin(1.40\,m-39.6)$

No.	Name of Station and State.	Latitude.	West Longitude.	The Magnetic Declination expressed as a Function of Time.
		° ′	° ′	° ° °
18	Pittsburg, Pa.	40 27.6	80 00.8	$D=+\ 1.85+\ 2.45\sin(1.45\,m-28.4)$
19	Denver, Col.	39 45.3	104 59.5	$D=-15.30+\ 0.011\,m+0.0005\,m^2$
20	Marietta, O.	39 25	81 28	$D=+\ 0.02+\ 2.89\sin(1.4\ \ m-40.5)$
21	Athens, O.	39 19	82 02	$D=-\ 1.51+\ 2.63\sin(1.4\ \ m-24.7)$
22	Cincinnati, O.	39 08.4	84 25.3	$D=-\ 2.59+\ 2.43\sin(1.42\,m-37.9)$
23	Saint Louis, Mo.	38 38.0	90 12.2	$D=-\ 5.91+\ 3.00\sin(1.40\,m-51.1)$*
24	Nashville, Tenn.	36 08.9	86 48.2	$D=-\ 3.57+\ 3.33\sin(1.35\,m-68.5)$*
25	Florence, Ala.	34 47.2	87 41.5	$D=-\ 4.25+\ 2.33\sin(1.3\ \ m-52.8)$
26	Mobile, Ala.	30 41.4	88 02.5	$D=-\ 4.38+\ 2.69\sin(1.45\,m-76.4)$
—	Pensacola, Fla.	30 20.8	87 18.3	$D=-\ 4.40+\ 3.16\sin(1.4\ \ m-59.4)$
27	New Orleans, La.	29 57.2	90 03.9	$D=-\ 5.20+\ 2.98\sin(1.40\,m-69.8)$
28	San Antonio, Tex.	29 25.4	98 29.3	$D=-\ 7.40+\ 2.88\sin(1.35\,m-81.8)$*
29	Key West, Fla.	24 33.5	81 48.5	$D=-\ 4.31+\ 2.86\sin(1.30\,m-23.9)$
30	Havana, Cuba.	23 09.3	82 21.5	$D=-\ 4.25+\ 2.74\sin(1.25\,m-23.3)$*
31	Kingston, Port Royal, Jamaica.	17 55.9	76 50.6	$D=-\ 3.81+\ 2.39\sin(1.10\,m-10.6)$
32	Barbados, Caribbee Islands.	13 05.7	59 37.3	$D=-\ 1.38+\ 2.84\sin(1.10\,m+09.4)$
33	Panama, Colombia.	8 57.1	79 32.2	$D=-\ 5.66+\ 2.22\sin(1.10\,m-27.8)$
	Group III.			
1	Acapulco, Mexico.	16 50.5	99 52.3	$D=-\ 4.48+\ 4.41\sin(1.0\ \ m-85.7)$*
2	Vera Cruz, Mexico.	19 11.9	96 08.8	$D=-\ 5.09+\ 4.22\sin(1.2\ \ m-63.4)$*
3	City of Mexico, Mex.	19 26.0	99 11.6	$D=-\ 5.34+\ 3.28\sin(1.0\ \ m-87.9)$*
4	San Blas, Mex.	21 32.5	105 18.4	$D=-\ 5.21+\ 4.26\sin(1.15\,m-96.5)$
5	San Lucas, Lower Cal.	22 53.3	109 54.7	$D=-\ 5.94+\ 3.68\sin(1.20\,m-116.8)$*
6	Magdalena Bay, Lower Cal.	24 38.4	112 08.9	$D=-\ 6.33+\ 4.17\sin(1.15\,m-119.2)$*
7	Cerros Island, Lower Cal.	28 04	115 12	$D=-\ 7.40+\ 4.61\sin(1.05\,m-107.0)$
8	El Paso, Tex.	31 45.5	106 27.0	$D=-\ 9.08+\ 3.40\sin(1.3\ \ m-108.4)$
9	San Diego, Cal.	32 42.1	117 14.3	$D=-10.32+\ 3.00\sin(1.10\,m-126.5)$
10	Santa Barbara, Cal.	34 24.2	119 43.0	$D=-11.52+\ 3.32\sin(1.10\,m-123.1)$
11	Monterey, Cal.	36 36.1	121 53.6	$D=-13.25+\ 2.83\sin(1.10\,m-144.0)$
12	San Francisco, Cal.	37 47.5	122 27.3	$D=-13.94+\ 2.65\sin(1.05\,m-135.5)$
13	Cape Mendocino, Cal.	40 26.3	124 24.3	$D=-15.25+\ 2.45\sin(1.10\,m-128.0)$*
14	Salt Lake City, Utah.	40 46.1	111 53.8	$D=-12.40+\ 4.25\sin(1.4\ \ m-121.6)$*
15	Vancouver, Wash.	45 37.5	122 39.7	$D=-17.93+\ 3.12\sin(1.35\,m-134.1)$*
16	Walla Walla, Wash.	46 04	118 22	$D=-17.80+\ 3.30\sin(1.3\ \ m-129.0)$*
17	Cape Disappointment, Wash.	46 16.7	124 02.8	$D=-19.39+\ 2.54\sin(1.25\,m-158.7)$
18	Seattle, Wash.	47 35.9	122 20.0	$D=-19.19+\ 3.14\sin(1.4\ \ m-136.1)$*
19	Port Townsend, Wash.	48 07.0	122 44.9	$D=-18.84+\ 3.00\sin(1.45\,m-122.1)$
20	Neah Bay, Wash.	48 21.8	124 38.0	$D=-19.83+\ 2.91\sin(1.40\,m-141.6)$
21	Nootka, Vancouver Isl.	49 35.5	126 37.5	$D=-21.25+\ 2.74\sin(1.30\,m-152.0)$*
22	Captain's and Iliuliuk Harbors.	53 52.6	166 31.5	$D=-18.01+\ 1.82\sin(1.3\ \ m-69.6)$*
23	Sitka, Alaska.	57 02.9	135 19.7	$D=-25.79+\ 3.30\sin(1.30\,m-104.2)$
24	St. Paul, Kadiak Island.	57 48.0	152 21.3	$D=-22.21+\ 5.18\sin(1.35\,m-72.5)$
25	Port Mulgrave, Alaska.	59 33.7	139 45.9	$D=-24.03+\ 7.77\sin(1.30\,m-85.8)$
26	Port Etches, Alaska.	60 20.7	146 37.6	$D=-23.71+\ 7.89\sin(1.35\,m-80.9)$
27	Port Clarence, Alaska.	65 16	166 50	$D=-18.98+\ 7.99\sin(1.3\ \ m-68.4)$*
28	Chamisso Isl., Alaska.	66 13	161 49	$D=-23.62+\ 7.64\sin(1.3\ \ m-64.0)$*
29	Petropaulovsk, Siberia.	53 01	201 17	$D=-\ 3.35+\ 2.97\sin(1.3\ \ m+12.2)$

TABLE IX.

Angular Convergences and Distances between Meridians.

1. Angular convergence of meridians per mile of easting or westing.
2. Distance between meridians converging by one minute.

Latitude.	Angular Convergence per Mile. Minutes.	Distance for Convergence of 1'. Feet.	Latitude.	Angular Convergence per Mile. Minutes.	Distance for Convergence of 1'. Feet.
°			°		
1	0.015	348733	31	0.521	10140
2	.030	174314	32	.542	9751
3	.045	116150	33	.563	9382
4	.061	87052	34	.585	9034
5	.076	69578	35	.607	8703
6	.091	57917	36	.630	8387
7	.107	49578	37	.653	8087
8	.122	43337	38	.677	7801
9	.137	38436	39	.702	7527
10	.153	34525	40	.727	7265
11	.169	31320	41	.753	7013
12	.184	28642	42	.780	6770
13	.200	26371	43	.808	6537
14	.216	24419	44	.836	6313
15	.232	22723	45	.866	6097
16	.249	21234	46	.897	5888
17	.265	19916	47	.929	5686
18	.282	18740	48	.962	5491
19	.299	17685	49	.998	5301
20	.316	16731	50	1.032	5118
21	.333	15864	51	1.069	4940
22	.350	15073	52	1.108	4766
23	.368	14348	53	1.149	4597
24	.386	13680	54	1.191	4433
25	.404	13062	55	1.236	4271
26	.423	12488	56	1.283	4115
27	.442	11955	57	1.333	3962
28	.461	11457	58	1.385	3813
29	.480	10990	59	1.440	3666
30	.500	10552	60	1.499	3523

TABLE X.[1]

LENGTH OF ONE MINUTE OF LATITUDE AND ONE MINUTE OF LONGITUDE TO THE NEAREST WHOLE FOOT.

LATITUDE.	1′ LATITUDE. FEET.	1′ LONGITUDE. FEET.	LATITUDE.	1′ LATITUDE. FEET.	1′ LONGITUDE. FEET.
°			°		
1	6046	6086	31	6062	5222
2	6046	6083	32	6063	5167
3	6046	6079	33	6064	5110
4	6046	6072	34	6065	5052
5	6046	6064	35	6066	4992
6	6047	6054	36	6067	4930
7	6047	6042	37	6068	4867
8	6047	6028	38	6069	4803
9	6047	6013	39	6070	4737
10	6048	5995	40	6071	4670
11	6048	5976	41	6072	4601
12	6049	5955	42	6074	4531
13	6049	5932	43	6075	4459
14	6050	5908	44	6076	4386
15	6050	5881	45	6077	4312
16	6051	5853	46	6078	4236
17	6051	5823	47	6079	4159
18	6052	5791	48	6080	4081
19	6052	5758	49	6081	4001
20	6053	5722	50	6082	3921
21	6054	5685	51	6083	3839
22	6055	5647	52	6084	3756
23	6055	5606	53	6085	3671
24	6056	5564	54	6086	3586
25	6057	5520	55	6087	3499
26	6058	5475	56	6088	3412
27	6059	5427	57	6089	3323
28	6059	5379	58	6090	3234
29	6060	5328	59	6091	3143
30	6061	5276	60	6092	3051

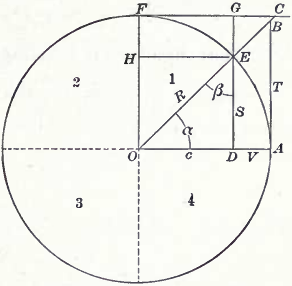
[1] Abbreviated from the Smithsonian Geographical Tables.

TABLE XI.

Trigonometric Functions and Formulas. Solution of Triangles.

By definition, if $R = 1$,

$ED =$ sine α.
$OD =$ cosine α.
$DA =$ versed sine α.
$HF =$ coversed sine α.
$BA =$ tangent α.
$FC =$ cotangent α.
$OB =$ secant α.
$OC =$ cosecant α.



If R is other than 1, it follows from the above definitions and the proportionality of similar figures, that

1. $ED = R \sin \alpha$.
2. $OD = R \cos \alpha$.
3. $DA = R \operatorname{versin} \alpha$.
4. $HF = R \operatorname{coversin} \alpha$.
5. $BA = R \tan \alpha$.
6. $FC = R \cot \alpha$.
7. $OB = R \sec \alpha$.
8. $OC = R \operatorname{cosec} \alpha$.

from which also in *any right triangle* of angles α and β, if o be the side *opposite* the angle α, a the side *adjacent* thereto, and h the hypotenuse,

9. $\sin \alpha = \frac{o}{h} = \cos \beta$.

10. $\cos \alpha = \frac{a}{h} = \sin \beta$.

11. $\tan \alpha = \frac{o}{a} = \cot \beta$.

12. $\cot \alpha = \frac{a}{o} = \tan \beta$.

13. $\sec \alpha = \frac{h}{a} = \operatorname{cosec} \beta$.

14. $\operatorname{cosec} \alpha = \frac{h}{o} = \sec \beta$.

15. $\operatorname{vers} \alpha = \frac{h-a}{h} = \operatorname{covers} \beta$.

16. $\operatorname{covers} \alpha = \frac{h-o}{h} = \operatorname{vers} \beta$.

Hence,

17. $\begin{cases} o = h \sin \alpha = h \cos \beta. \\ h = \dfrac{o}{\sin \alpha} = \dfrac{o}{\cos \beta}. \end{cases}$

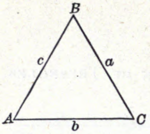
18. $\begin{cases} a = h \cos \alpha = h \sin \beta. \\ h = \dfrac{a}{\cos \alpha} = \dfrac{a}{\sin \beta}. \end{cases}$

19. $\begin{cases} o = a \tan \alpha = a \cot \beta. \\ a = \dfrac{o}{\tan \alpha} = \dfrac{o}{\cot \beta}. \end{cases}$

20. $\begin{cases} a = o \cot \alpha = o \tan \beta. \\ o = \dfrac{a}{\cot \alpha} = \dfrac{a}{\tan \beta}. \end{cases}$

21. $\begin{cases} h = a \sec \alpha = a \operatorname{cosec} \beta. \\ a = \dfrac{h}{\sec \alpha} = \dfrac{h}{\operatorname{cosec} \beta}. \end{cases}$

22. $\begin{cases} h = o \operatorname{cosec} \alpha = o \sec \beta \\ o = \dfrac{h}{\operatorname{cosec} \alpha} = \dfrac{h}{\sec \beta}. \end{cases}$



23. $o = \sqrt{h^2 - a^2} = \sqrt{(h + a)(h - a)}$.

24. $a = \sqrt{h^2 - o^2} = \sqrt{(h + o)(h - o)}$.

25. $h = \sqrt{o^2 + a^2}$.

26. Area $= \dfrac{oa}{2}$.

Oblique triangles may be solved by some one of the following formulas:

	Given.	Sought.	Formulas.
27.	$A, B, a,$	$C, b, c,$	$C = 180° - (A + B),\ b = \dfrac{a}{\sin A} \sin B,$ $c = \dfrac{a}{\sin A} \sin (A + B).$
28.	$A, a, b,$	$B, C, c,$	$\sin B = \dfrac{\sin A}{a} b,\ \ C = 180° - (A + B),$ $c = \dfrac{a}{\sin A} \sin C.$
29.	$C, a, b,$	$\frac{1}{2}(A + B),$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C.$
30.		$\frac{1}{2}(A - B),$	$\tan \frac{1}{2}(A - B) = \dfrac{a - b}{a + b} \tan \frac{1}{2}(A + B).$
31.		$A, B,$	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B);$ $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B).$
32.		$c,$	$c = (a + b) \dfrac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b) \dfrac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}.$
33.		Area,	Area $= \frac{1}{2} ab \sin C.$
34.	$a, b, c,$	$A,$	If $s = \frac{1}{2}(a + b + c),$ $\sin \frac{1}{2} A = \sqrt{\dfrac{(s - b)(s - c)}{bc}},$ $\cos \frac{1}{2} A = \sqrt{\dfrac{s(s - a)}{bc}},$ $\tan \frac{1}{2} A = \sqrt{\dfrac{(s - b)(s - c)}{s(s - a)}}.$ $\sin\ A = \dfrac{2\sqrt{s(s - a)(s - b)(s - c)}}{bc},$ $\text{vers}\ A = \dfrac{2(s - b)(s - c)}{bc}.$
35.		Area,	Area $= \sqrt{s(s - a)(s - b)(s - c)}.$
36.	$A, B, C, a,$	Area,	Area $= \dfrac{a^2 \sin B \sin C}{2 \sin A}.$

From the definitions of the trigonometric functions, the geometrical properties of right triangles and in some cases algebraic transformations, it may be shown that if A is any angle and B any other angle,

37. $\sin^2 A + \cos^2 A = 1.$

38. $\sin A = \dfrac{1}{\operatorname{cosec} A} = \sqrt{1 - \cos^2 A} = \tan A \cos A$

$= 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \operatorname{vers} A \cot \tfrac{1}{2} A$

$= \sqrt{\tfrac{1}{2} \operatorname{vers} 2A} = \sqrt{\tfrac{1}{2}(1 - \cos 2A)}.$

39. $\cos A = \dfrac{1}{\sec A} = \sqrt{1 - \sin^2 A} = \cot A \sin A$

$= 1 - \operatorname{vers} A = 2\cos^2 \tfrac{1}{2} A - 1 = 1 - 2 \sin^2 \tfrac{1}{2} A$

$= \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A = \sqrt{\tfrac{1}{2} + \tfrac{1}{2} \cos 2A}.$

40. $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{1}{\cot A} = \sqrt{\sec^2 A - 1}$

$= \sqrt{\dfrac{1}{\cos^2 A} - 1} = \dfrac{\sqrt{1 - \cos^2 A}}{\cos A} = \dfrac{\sin 2A}{1 + \cos 2A}$

$= \dfrac{1 - \cos 2A}{\sin 2A} = \dfrac{\operatorname{vers} 2A}{\sin 2A} = \cot \tfrac{1}{2} A (\sec A - 1).$

41. $\cot A = \dfrac{\cos A}{\sin A} = \dfrac{1}{\tan A} = \sqrt{\operatorname{cosec}^2 A - 1}$

$= \dfrac{\sin 2A}{1 - \cos 2A} = \dfrac{\sin 2A}{\operatorname{vers} 2A} = \dfrac{1 + \cos 2A}{\sin 2A} = \dfrac{\tan \frac{1}{2} A}{\sec A - 1}.$

42. $\operatorname{vers} A = 1 - \cos A = \sin A \tan \tfrac{1}{2} A = 2 \sin^2 \tfrac{1}{2} A = \cos A (\sec A - 1).$

43. $\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A.$

44. $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B.$

45. $\sin \tfrac{1}{2} A = \sqrt{\dfrac{1 - \cos A}{2}} = \sqrt{\dfrac{\operatorname{vers} A}{2}}.$

46. $\sin 2A = 2 \sin A \cos A.$

47. $\cos \tfrac{1}{2} A = \sqrt{\dfrac{1 + \cos A}{2}}.$

48. $\cos 2A = 2 \cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A.$

49. $\tan \tfrac{1}{2} A = \dfrac{\tan A}{1 + \sec A} = \operatorname{cosec} A - \cot A = \dfrac{1 - \cos A}{\sin A} = \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}.$

50. $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 \frac{1}{2} A}.$

51. $\cot \tfrac{1}{2} A = \dfrac{\sin A}{\operatorname{vers} A} = \dfrac{1 + \cos A}{\sin A} = \dfrac{1}{\operatorname{cosec} A - \cot A}.$

52. $\cot\ 2\,A = \dfrac{\cot^2 A - 1}{2\cot A}$.

53. $\text{vers}\ \frac{1}{2}\,A = \dfrac{\frac{1}{2}\,\text{vers}\,A}{1+\sqrt{1-\frac{1}{2}\,\text{vers}\,A}} = \dfrac{1-\cos A}{2+\sqrt{2(1+\cos A)}}$.

54. $\text{vers}\ 2\,A = 2\sin^2 A$.

55. $\sin\ A + \sin\ B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$.

56. $\sin\ A - \sin\ B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$.

57. $\cos\ A + \cos\ B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$.

58. $\cos\ B - \cos\ A = 2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$.

59. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$.

60. $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$.

61. $\tan\ A + \tan\ B = \dfrac{\sin(A+B)}{\cos A\cos B}$.

62. $\tan\ A - \tan\ B = \dfrac{\sin(A-B)}{\cos A\cos B}$.

TABLE XII.

Lengths of Circular Arcs of Radius 1, and Various Circular Measures.

No.	Degrees.	Minutes.	Seconds.	No.	Degrees.	Minutes.	Seconds.
1	.0174533	.0002909	.0000048	6	.1047198	.0017453	.0000291
2	.0349066	.0005818	.0000097	7	.1221730	.0020362	.0000339
3	.0523599	.0008727	.0000145	8	.1396263	.0023271	.0000388
4	.0698132	.0011636	.0000194	9	.1570796	.0026180	.0000436
5	.0872665	.0014544	.0000242	10	.1745329	.0029089	.0000485

Degrees in arc of length equal to radius, 57.° 295 780.
Degrees in arc of length equal to π, 180.° 000 000.
Circumference $= 2\,\pi r =$ 360.° 000 000.
Area $= \pi r^2$.

If l = length of circular arc
d = number of degrees in same
r = radius of same
c = chord of same
m = middle ordinate

$d = \dfrac{l}{r}\cdot\dfrac{180^\circ}{\pi}$.

$r = \dfrac{l}{d}\cdot\dfrac{180^\circ}{\pi}$.

$l = \dfrac{d}{180}\,\pi r$.

Area of sector $= \frac{1}{2}\,lr$.

Area of sector $= \dfrac{d}{360}\,\pi r^2$.

Approximate area of segment $= \frac{2}{3}\,cm$.

TABLE XIII.

Linear Transformations.

1. *Gunter's Chains to Feet.*

Chains.	0.0	0.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0		.66	1.32	1.98	2.64	3.30	3.96	4.62	5.28	5.94
.1	6.60	7.26	7.92	8.58	9.24	9.90	10.56	11.22	11.88	12.54
.2	13.20	13.86	14.52	15.18	15.84	16.50	17.16	17.82	18.48	19.14
.3	19.80	20.46	21.12	21.78	22.44	23.10	23.76	24.42	25.08	25.74
.4	26.40	27.06	27.72	28.38	29.04	29.70	30.36	31.02	31.68	32.34
.5	33.00	33.66	34.32	34.98	35.64	36.30	36.96	37.62	38.28	38.94
.6	39.60	40.26	40.92	41.58	42.24	42.90	43.56	44.22	44.88	45.54
.7	46.20	46.86	47.52	48.18	48.84	49.50	50.16	50.82	51.48	52.14
.8	52.80	53.46	54.12	54.78	55.44	56.10	56.76	57.42	58.08	58.74
.9	59.40	60.06	60.72	61.38	62.04	62.70	63.36	64.02	64.68	65.34

	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
0.0		66	132	198	264	330	396	462	528	594
10.0	660	726	792	858	924	990	1056	1122	1188	1254
20.0	1320	1386	1452	1518	1584	1650	1716	1782	1848	1914
30.0	1980	2046	2112	2178	2244	2310	2376	2442	2508	2574
40.0	2640	2706	2772	2838	2904	2970	3036	3102	3168	3234
50.0	3300	3366	3432	3498	3564	3630	3696	3762	3828	3894
60.0	3960	4026	4092	4158	4224	4290	4356	4422	4488	4554
70.0	4620	4686	4752	4818	4884	4950	5016	5082	5148	5214
80.0	5280	5346	5412	5478	5544	5610	5676	5742	5808	5874

2. *Gunter's Chains to Meters.*

Chains.	0.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
0.0	—	.2012	.4023	.6035	.8047	1.0058	1.2070	1.4082	1.6093	1.8105
.1	2.0117	2.2128	2.4140	2.6152	2.8163	3.0175	3.2187	3.4198	3.6210	3.8222
.2	4.0233	4.2245	4.4257	4.6268	4.8280	5.0292	5.2303	5.4315	5.6326	5.8338
.3	6.0350	6.2361	6.4373	6.6385	6.8396	7.0408	7.2420	7.4431	7.6443	7.8455
.4	8.0466	8.2478	8.4490	8.6501	8.8513	9.0525	9.2536	9.4548	9.6560	9.8571
.5	10.0583	10.2595	10.4606	10.6618	10.8630	11.0641	11.2653	11.4665	11.6676	11.8688
.6	12.0700	12.2711	12.4723	12.6735	12.8746	13.0758	13.2770	13.4781	13.6793	13.8805
.7	14.0816	14.2828	14.4840	14.6851	14.8863	15.0875	15.2886	15.4898	15.6910	15.8921
.8	16.0933	16.2945	16.4956	16.6968	16.8980	17.0991	17.3003	17.5015	17.7026	17.9038
.9	18.1050	18.3061	18.5073	18.7084	18.9096	19.1108	19.3119	19.5131	19.7143	19.9154

	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
0.0	—	20.1166	40.2332	60.3499	80.4665	100.5831	120.6996	140.8163	160.9329	181.0495
10.	201.1662	221.2828	241.3994	261.5160	281.6326	301.7493	321.8659	341.9825	362.0991	382.2157
20.	402.3323	422.4489	442.5656	462.6822	482.7988	502.9154	523.0320	543.1487	563.2653	583.3819
30.	603.4985	623.6151	643.7317	663.8483	683.9649	704.0816	724.1982	744.3148	764.4314	784.5480
40.	804.6646	824.7812	844.8979	865.0145	885.1311	905.2477	925.3643	945.4810	965.5976	985.7142
50.	1005.8308	1025.9474	1046.0640	1066.1806	1086.2973	1106.4139	1126.5305	1146.6471	1166.7637	1186.8803
60.	1206.9969	1227.1135	1247.2302	1267.3468	1287.4634	1307.5800	1327.6966	1347.8133	1367.9299	1388.0465
70.	1408.1631	1428.2797	1448.3963	1468.5129	1488.6296	1508.7462	1528.8628	1548.9794	1569.0960	1589.2126
80.	1609.3292	1629.4459	1649.5625	1669.6791	1689.7957	1709.9123	1730.0290	1750.1456	1770.2622	1790.3788

3. *Feet to Meters.*

Feet.	0	1	2	3	4	5	6	7	8	9
	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.
0	0.000	0.305	0.610	0.914	1.219	1.524	1.829	2.134	2.438	2.743
10	3.048	3.353	3.658	3.962	4.267	4.572	4.877	5.182	5.486	5.791
20	6.036	6.401	6.706	7.010	7.315	7.620	7.925	8.229	8.534	8.839
30	9.144	9.449	9.753	10.058	10.363	10.668	10.972	11.277	11.582	11.887
40	12.192	12.496	12.801	13.106	13.411	13.716	14.020	14.325	14.630	14.935
50	15.239	15.544	15.849	16.154	16.459	16.763	17.068	17.373	17.678	17.983
60	18.287	18.592	18.897	19.202	19.507	19.811	20.116	20.421	20.726	21.031
70	21.335	21.640	21.945	22.250	22.555	22.859	23.164	23.469	23.774	24.079
80	24.383	24.688	24.993	25.298	25.602	25.907	26.212	26.517	26.822	27.126
90	27.431	27.736	28.041	28.346	28.651	28.955	29.260	29.565	29.870	30.174
100	30.479	30.784	31.089	31.394	31.698	32.003	32.308	32.613	32.918	33.222

4. *Meters to Feet.*

Meters.	0	1	2	3	4	5	6	7	8	9
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
0	0.00	3.28	6.56	9.84	13.12	16.40	19.69	22.97	26.25	29.53
10	32.81	36.09	39.37	42.65	45.93	49.21	52.49	55.78	59.06	62.34
20	65.62	68.90	72.18	75.46	78.74	82.02	85.30	88.58	91.87	95.15
30	98.43	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.96
40	131.24	134.52	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164.04	167.33	170.61	173.89	177.17	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.42	206.70	209.98	213.26	216.54	219.82	223.10	226.38
70	229.66	232.94	236.22	239.51	242.79	246.07	249.35	252.63	255.91	259.19
80	262.47	265.75	269.03	272.31	275.60	278.88	282.16	285.44	288.72	292.00
90	295.28	298.56	391.84	305.12	308.40	311.69	314.97	318.25	321.53	324.81
100	328.09	331.37	334.65	337.93	341.21	344.49	347.78	351.06	354.34	357.62

1 statute mile = 1.6093 kilometers
1 kilometer = 0.6214 statute miles

TABLE XIV.

Horizontal Distances and Differences of Elevation.

For stadia readings of 100 units at various vertical angles. The values for other readings are obtained by multiplying the quantities under the proper vertical angle by the stadia reading in *hundreds of units*; thus, if the reading is 204, multiply by 2.04. Use a slide rule. Computed by Mr. Arthur Winslow while assistant geologist, second Geological Survey of Pennsylvania.

M.	0°		1°		2°		3°		4°		5°		6°		7°		8°		9°		10°	
′	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78	97.55	15.45	96.98	17.10
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15	98.05	13.84	97.53	15.51	96.96	17.16
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21	98.03	13.89	97.52	15.56	96.94	17.21
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26	98.01	13.95	97.50	15.62	96.92	17.26
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32	98.00	14.01	97.48	15.67	96.90	17.32
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38	97.98	14.06	97.46	15.73	96.88	17.37
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43	97.97	14.12	97.44	15.78	96.86	17.43
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49	97.95	14.17	97.43	15.84	96.84	17.48
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55	97.93	14.23	97.41	15.89	96.82	17.54
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60	97.92	14.28	97.39	15.95	96.80	17.59
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66	97.90	14.34	97.37	16.00	96.78	17.65
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72	97.88	14.40	97.35	16.06	96.76	17.70
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77	97.87	14.45	97.33	16.11	96.74	17.76
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83	97.85	14.51	97.31	16.17	96.72	17.81
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88	97.83	14.56	97.29	16.22	96.70	17.86
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94	97.82	14.62	97.28	16.28	96.68	17.92
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00	97.80	14.67	97.26	16.33	96.66	17.97
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05	97.78	14.73	97.24	16.39	96.64	18.03
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11	97.76	14.79	97.22	16.44	96.62	18.08
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17	97.75	14.84	97.20	16.50	96.60	18.14
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22	97.73	14.90	97.18	16.55	96.57	18.19
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28	97.71	14.59	97.16	16.61	96.55	18.24
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33	97.69	15.01	97.14	16.66	96.53	18.30
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39	97.68	15.06	97.12	16.72	96.51	18.35
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45	97.66	15.12	97.10	16.77	96.49	18.41
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50	97.64	15.17	97.08	16.83	96.47	18.46
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56	97.62	15.23	97.06	16.88	96.45	18.51
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61	97.61	15.28	97.04	16.94	96.42	18.57
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67	97.59	15.34	97.02	16.99	96.40	18.62
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73	97.57	15.40	97.00	17.05	96.38	18.68
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
$c = 0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10	0.74	0.11	0.74	0.12	0.74	0.14
$c = 1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13	0.99	0.15	0.99	0.16	0.98	0.18
$c = 1.25$	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16	1.23	0.18	1.23	0.21	1.23	0.23

TABLE XIV. (*continued*).

M.	11°		12°		13°		14°		15°		16°		17°		18°		19°		20°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	96.36	18.73	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
2	96.34	18.78	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83	88.26	32.18
4	96.32	18.84	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87	88.23	32.23
6	96.29	18.89	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92	88.19	32.27
8	96.27	18.95	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97	88.15	32.32
10	96.25	19.00	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01	88.11	32.36
12	96.23	19.05	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06	88.08	32.41
14	96.21	19.11	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10	88.04	32.45
16	96.18	19.16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15	88.00	32.49
18	96.16	19.21	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19	87.96	32.54
20	96.14	19.27	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24	87.93	32.58
22	96.12	19.32	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28	87.89	32.63
24	96.09	19.38	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33	87.85	32.67
26	96.07	19.43	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38	87.81	32.72
28	96.05	19.48	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42	87.77	32.76
30	96.03	19.54	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47	87.74	32.80
32	96.00	19.59	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51	87.70	32.85
34	95.98	19.64	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56	87.66	32.89
36	95.96	19.70	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60	87.62	32.93
38	95.93	19.75	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65	87.58	32.98
40	95.91	19.80	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69	87.54	33.02
42	95.89	19.86	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74	87.51	33.07
44	95.86	19.91	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78	87.47	33.11
46	95.84	19.96	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83	87.43	33.15
48	95.82	20.02	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87	87.39	33.20
50	95.79	20.07	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92	87.35	33.24
52	95.77	20.12	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96	87.31	33.28
54	95.75	20.18	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01	87.27	33.33
56	95.72	20.23	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05	87.24	33.37
58	95.70	20.28	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09	87.20	33.41
60	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14	87.16	33.46
$c=0.75$	0.73	0.15	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25	0.70	0.26
$c=1.00$	0.98	0.20	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33	0.94	0.35
$c=1.25$	1.22	0.25	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34	1.20	0.36	1.19	0.38	1.19	0.40	1.18	0.42	1.17	0.44

TABLE XIV. (*concluded*).

M.	21°		22°		23°		24°		25°		26°		27°		28°		29°		30°	
′	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30
2	87.12	33.50	85.93	34.77	84.69	36.01	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49	77.91	41.48	76.45	42.43	74.95	43.33
4	87.08	33.54	85.89	34.82	84.65	36.05	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52	77.86	41.52	76.40	42.46	74.90	43.36
6	87.04	33.59	85.85	34.86	84.61	36.09	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55	77.81	41.55	76.35	42.49	74.85	43.39
8	87.00	33.63	85.80	34.90	84.57	36.13	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59	77.77	41.58	76.30	42.53	74.80	43.42
10	86.96	33.67	85.76	34.94	84.52	36.17	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62	77.72	41.61	76.25	42.56	74.75	43.45
12	86.92	33.72	85.72	34.98	84.48	36.21	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66	77.67	41.65	76.20	42.59	74.70	43.47
14	86.88	33.76	85.68	35.02	84.44	36.25	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69	77.62	41.68	76.15	42.62	74.65	43.50
16	86.84	33.80	85.64	35.07	84.40	36.29	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72	77.57	41.71	76.10	42.65	74.60	43.53
18	86.80	33.84	85.60	35.11	84.35	36.33	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76	77.52	41.74	76.05	42.68	74.55	43.56
20	86.77	33.89	85.56	35.15	84.31	36.37	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79	77.48	41.77	76.00	42.71	74.49	43.59
22	86.73	33.93	85.52	35.19	84.27	36.41	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82	77.42	41.81	75.95	42.74	74.44	43.62
24	86.69	33.97	85.48	35.23	84.23	36.45	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86	77.38	41.84	75.90	42.77	74.39	43.65
26	86.65	34.01	85.44	35.27	84.18	36.49	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89	77.33	41.87	75.85	42.80	74.34	43.67
28	86.61	34.06	85.40	35.31	84.14	36.53	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92	77.28	41.90	75.80	42.83	74.29	43.70
30	86.57	34.10	85.36	35.36	84.10	36.57	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96	77.23	41.93	75.75	42.86	74.24	43.73
32	86.53	34.14	85.31	35.40	84.06	36.61	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99	77.18	41.97	75.70	42.89	74.19	43.76
34	86.49	34.18	85.27	35.44	84.01	36.65	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02	77.13	42.00	75.65	42.92	74.14	43.79
36	86.45	34.23	85.23	35.48	83.97	36.69	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06	77.09	42.03	75.60	42.95	74.09	43.82
38	86.41	34.27	85.19	35.52	83.93	36.73	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09	77.04	42.06	75.55	42.98	74.04	43.84
40	86.37	34.31	85.15	35.56	83.89	36.77	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12	76.99	42.09	75.50	43.01	73.99	43.87
42	86.33	34.35	85.11	35.60	83.84	36.80	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16	76.94	42.12	75.45	43.04	73.93	43.90
44	86.29	34.40	85.07	35.64	83.80	36.84	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19	76.89	42.15	75.40	43.07	73.88	43.93
46	86.25	34.44	85.02	35.68	83.76	36.88	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22	76.84	42.19	75.35	43.10	73.83	43.95
48	86.21	34.48	84.98	35.72	83.72	36.92	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26	76.79	42.22	75.30	43.13	73.78	43.98
50	86.17	34.52	84.94	35.76	83.67	36.96	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29	76.74	42.25	75.25	43.16	73.73	44.01
52	86.13	34.57	84.90	35.80	83.63	37.00	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32	76.69	42.28	75.20	43.18	73.68	44.04
54	86.09	34.61	84.86	35.85	83.59	37.04	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35	76.64	42.31	75.15	43.21	73.63	44.07
56	86.05	34.65	84.82	35.89	83.54	37.08	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39	76.59	42.34	75.10	43.24	73.58	44.09
58	86.01	34.69	84.77	35.93	83.50	37.12	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42	76.55	42.37	75.05	43.27	73.52	44.12
60	85.97	34.73	84.73	35.97	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30	73.47	44.15
$c = 0.75$	0.70	0.27	0.69	0.29	0.69	0.30	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35	0.66	0.36	0.65	0.37	0.65	0.38
$c = 1.00$	0.93	0.37	0.92	0.38	0.92	0.40	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46	0.88	0.48	0.87	0.49	0.86	0.51
$c = 1.25$	1.16	0.46	1.15	0.48	1.15	0.50	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58	1.10	0.60	1.09	0.62	1.08	0.64

TABLES XV., XVI.

COMMON LOGARITHMS OF NUMBERS.
LOGARITHMS OF TRIGONOMETRIC FUNCTIONS.

EDITED BY C. W. CROCKETT.

NOTE.

THE well-known tables of Gauss, Hoüel, Becker, and Albrecht have been taken as the standards, and the figures compared with the more extensive tables, the doubtful cases being recomputed.

EXPLANATION OF THE TABLES

INTRODUCTORY.

1. When we have a number with six or more decimal places, and we wish to use only five:

(*a*) If the sixth and following figures of the decimal are less than 5 in the sixth place, they are dropped; thus, 0.46437 4999 is called 0.46437.

(*b*) If the sixth and following figures of the decimal are greater than 5 in the sixth place, the fifth place is increased by unity and the sixth and following places are dropped; thus, 0.46437 5001 is called 0.46438.

(*c*) If the sixth figure of the decimal is 5, and if it is followed only by zeros, make the fifth figure the nearest *even* figure; thus, 0.46437 500 is called 0.46438, while 0.46438 500 is also called 0.46438. The number is thus increased when the fifth figure is odd, and decreased when it is even, the two operations tending to neutralize each other in a series of computations, and hence to diminish the resultant error.

2. Hence any number obtained according to Art. 1 may be in error by half a unit in the fifth decimal place.

3. When the last figure of a number in these tables is 5, the number printed is too large, the 5 having been obtained according to Art. 1 (*b*); if the 5 is without the minus sign, the number printed is too small, the figures following the 5 having been dropped according to Art. 1 (*a*).

4. The marginal tables contain the products of the numbers at the top of the columns by 1, 2, 3, ⋯ 9 *tenths*, and may be used in multiplying and dividing in interpolation.

(*a*) To multiply 38 by .746:

$$
\begin{aligned}
38 \times .7 &= &&= 26.6\\
38 \times .4 &= 15.2\,; \therefore\ 38 \times .04 &&= 1.52\\
38 \times .6 &= 22.8\,; \therefore\ 38 \times .006 &&= \underline{.228}\\
&\therefore\ 38 \times .746 &&= 28.348
\end{aligned}
$$

	38
1	3.8
2	7.6
3	11.4
4	15.2
5	19.0
6	22.8
7	26.6
8	30.4
9	34.2

In multiplying by the second figure (hundredths), the decimal point in the table is moved *one* place to the left; in multiplying by the third (thousandths), *two* to the left; and so on.

(*b*) To divide 28 by 38:

Dividend,	28			
Next less,	26.6	corresponding to	.7	
Remainder,	1 4			
Next less,	1 1.4	corresponding to	.03	
Remainder,	2 6			
Nearest,	2 6.6	corresponding to	.007	
∴ Quotient,			·737	

	38
1	3.8
2	7.6
3	11.4
4	15.2
5	19.0
6	22.8
7	26.6
8	30.4
9	34.2

to the nearest third decimal place. The decimal point is moved one place to the right in each remainder, since the next figure in the quotient will be one place farther to the right.

To divide 23 by 38:

Dividend,	23		
	22.8	corresponding to	.6
	2		
	0.0	corresponding to	.00
	2 0.		
Nearest,	1 9.0	corresponding to	.005
∴ Quotient,			.605

The computer should use the marginal tables mentally.

LOGARITHMS.

5. The logarithm of a number is the exponent of the power to which a given number called the *base* must be raised to produce the first number. If $A = e^a$, a is called the logarithm of the number A to the base e, written $\log_e A = a$.

6. If $A = e^a$, and $B = e^b$, or (omitting subscripts) $\log A = a$, and $\log B = b$, we have

$$A \times B = e^{a+b};\ \therefore \log(A \times B) = a + b;\ \therefore \log(A \times B) = \log A + \log B.$$

$$A \div B = e^{a-b};\ \therefore \log(A \div B) = a - b;\ \therefore \log(A \div B) = \log A - \log B.$$

$$A^n = e^{na};\ \therefore \log(A^n) = na;\ \therefore \log(A^n) = n \log A.$$

$$\sqrt[n]{A} = e^{\frac{1}{n}a};\ \therefore \log \sqrt[n]{A} = \frac{1}{n} a;\ \therefore \log \sqrt[n]{A} = \frac{1}{n} \log A.$$

7. When the base is not specified, it is generally understood that logarithms to the base 10, or *common logarithms*, are meant. In this system, since

$$
\begin{array}{llll}
0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}, & \log 0.001 & = & -3; \\
0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}, & \log 0.01 & = & -2; \\
0.1 = \frac{1}{10} = \frac{1}{10} = 10^{-1}, & \log 0.1 & = & -1; \\
1. = 10^0, & \log 1 & = & 0; \\
10. = 10^1, & \log 10 & = & +1; \\
100. = 10^2, & \log 100 & = & +2; \\
1000. = 10^3, & \log 1000 & = & +3.
\end{array}
$$

8. The logarithm of a number between 100 and 1000 will be a number between 2 and 3, or $2 + m$ where m will be a decimal called the *mantissa*, the integral portion of the logarithm being the *characteristic*. The mantissa is always considered *positive;* thus log 0.002 will be a number between -2 and -3, that is, either $-3 + m$ or $-2 - m'$, the first form being used. We write $\log 0.002 = \bar{3}.30103$, the negative sign being placed over the characteristic to show that the characteristic alone is negative.

9. Since

$$\log (A \times 10^n) = \log A + \log 10^n = \log A + n \log 10 = \log A + n,$$

and $\log (A \div 10^n) = \log A - \log 10^n = \log A - n \log 10 = \log A - n,$

we have, if $\log 37.3 = 1.57171$,

$$
\begin{array}{lll}
\log 373. = 2.57171, & \text{and} & \log 3.73 = 0.57171; \\
\log 3730 = 3.57171, & \text{and} & \log 0.373 = \bar{1}.57171; \\
\log 37300 = 4.57171, & \text{and} & \log 0.0373 = \bar{2}.57171.
\end{array}
$$

Hence the position of the decimal point affects the characteristic alone, the mantissa being always the same for the same sequence of figures. For this reason the common system of logarithms is used in practice.

10. The characteristic is found as follows: *When the number is greater than* 1, *the characteristic is positive, and is one* less *than the number of digits to the left of the decimal point; when the number is less than* 1, *the characteristic is negative, and is one* more *than the number of zeros between the decimal point and the first significant figure.*

11. To avoid the use of negative characteristics we add 10 to the characteristic and write -10 after the mantissa, *i.e.* adding and subtracting the same quantity, 10. Thus $\log 0.2 = \bar{1}.30103$ would be written

$9.30103 - 10$. The -10 is often omitted for brevity when there is no danger of confusion, but its existence must not be forgotten. Such logarithms are called *augmented* logarithms.

In this case the characteristic of the logarithm of a pure decimal is 9 *diminished by the number of ciphers to the left of the first significant figure.* Thus the characteristic of $\log 0.004$ is $9 - 2$, or 7, and that of $\log 0.94$ is $9 - 0$, or 9.

12. The arithmetical complement of the logarithm (written *colog*) of a number is the logarithm of its reciprocal, and is found by subtracting each figure of the logarithm from 9, commencing at the left, except the last significant figure on the right, which is subtracted from 10.

For $\qquad \log\frac{1}{x} = -\log x = 10 - \log x - 10;$

thus, if $\qquad \log x = 2.46403, \text{colog}\, x = 7.53597 - 10;$

if $\qquad \log x = 8.43000 - 10, \text{colog}\, x = 1.57000.$

TABLE XV.

13. Page 397 contains the logarithms of numbers from 1 to 100, to five decimal places.

Pages 398–415 contain the mantissas of the logarithms of numbers from 1000 to 10009, to five decimal places.

Pages 416, 417, contain the mantissas of the logarithms of numbers from 10000 to 11009, to seven decimal places.

NOTE. — The mantissas of the logarithms of numbers, except those of the integral powers of 10, are incommensurable, the mantissas in the tables being found as shown in Art. 1.

To find the Logarithm of a Number.

14. The *characteristic* is found by the rules in Arts. 10 and 11, and the *mantissa* from the tables, as shown in Arts. 15, 16, 17, 18.

15. *When the number has four figures.* — Find on pages 398–415 the first three figures in the column marked N, and the fourth at the top of one of the other columns. The last three figures of the mantissa are found in this column on the horizontal line through the first three figures of the given number in column N. The first two figures of the mantissa are those under L in the same line with the number, or else those nearest above it, unless the last three figures of the mantissa as given in the tables are preceded by a *, when the first two figures are found under L in the first line *below* the number. Thus (page 398),

$$\log 1136 = 3.05538;\ \log 1137 = 3.05576;\ \log 1138 = 3.05614;$$
$$\log 1370 = 3.13672;\ \log 1371 = 3.13704;\ \log 1372 = 3.13735;$$
$$\log 1380 = 3.13988;\ \log 1381 = 3.14019;\ \log 1382 = 3.14051.$$

16. *When the number has less than four figures,* annex ciphers on the right and proceed as in Art. 15. Thus,

$$\log 1.13 = 0.05308;\ \log 12.8 = 1.10721;\ \log 130 = 2.11394;$$
$$\log 15 = 1.17609;\ \log 16 = 1.20412;\ \log 17 = 1.2304\bar{5}.$$

17. *When the number has more than four figures,* as 11.4672. — Since the mantissa is independent of the position of the decimal point, point off the first four figures and find the mantissa of log 1146.72. This will be between the mantissas of log 1146 and log 1147. Hence find from the tables the mantissas corresponding to 1146 and 1147; multiply the difference between them (called the tabular difference) by .72, and add the product (called the correction) to log 11.46; the result will be the logarithm required.

Mantissa of log 1146 = 05918	log 11.46 = 1.05918
Mantissa of log 1147 = 05956	correction = 38 × .72 = 27.36
Tabular difference = 38	∴ log 11.4672 = 1.05945 36
	or = 1.05945

NOTE. — Since any mantissa in the tables may be in error by half a unit in the fifth decimal place (Art. 2), no advantage is gained by using the sixth place in the interpolated logarithm. Thus, according to Art. 1, we drop the .36, and call log 11.4672 = 1.05945.

NOTE. — The marginal tables should be used in multiplying the tabular difference to find the correction (Art. 4).

NOTE. — It is assumed that the change in the mantissa is proportional to that in the number, as the latter increases from 1146 to 1147. An increase of 1 in the number causes an increase of 38 in the mantissa; hence an increase of .72 in the number will cause an increase of 38 × .72 in the mantissa.

NOTE. — We could also find the mantissa of log 11.4672 by subtracting the product of the tabular difference by .28 (or 1.00 − .72) from the mantissa corresponding to 1147; that is, the required mantissa is 05956 − (38 × .28) = 05956 − 10.64 = 05945 as before.

18. *The general rule is: Find the mantissa corresponding to the first four figures of the number; multiply the tabular difference by the fifth and following figures treated as a decimal; and add the product to the mantissa just found.*

The tabular difference is the difference between the mantissas corresponding to the two numbers in the tables, between which the given number lies.

$$\log 1.62163 = 0.20995;\ \log 0.38024 = \bar{1}.58006;\ \log 0.0852763 = \bar{2}.93083;$$
$$\log 189.524 = 2.27767;\ \log 0.38602 = \bar{1}.58661;\ \log 0.0085938 = \bar{3}.93419;$$
$$\log 19983.4 = 4.30067;\ \log 3.98743 = 0.60070;\ \log 0.090046 = \bar{2}.95446.$$

NOTE. — Page 397 is used when the number contains less than three figures, the number being found in the column *N*, and the logarithm in the column headed *Log.* The characteristic is given for whole numbers, and must be changed for decimals.

NOTE. — When a number is composed of three figures, find on pages 398–415 the number in the column *N*, and the mantissa corresponding in the column *L*. 0.

To find the Number corresponding to a Given Logarithm.

19. From the tables we find the sequence of figures corresponding to the given mantissa, as shown in Arts. 20, 21, and 22, the position of the decimal point being determined by the characteristic (Arts. 10, 11).

20. *When the given mantissa can be found in the tables.* — Find on pages 398–415 the first two figures of the mantissa under *L* in the column headed *L*. 0. The last three figures of the mantissa are then sought for in the columns headed 0, 1, 2, ⋯ 9, in the same line with the first two figures, or in one of the lines just below, or in the line next above (where they would be preceded by a *). The first three figures of the required number will be found in the column headed *N*, in the same horizontal line with the last three figures of the mantissa, and the fourth figure of the number at the top of the column in which the last three figures of the mantissa are found. Thus (page 398),

$0.06221 = \log 1.154$; $0.06558 = \log 1.163$; $0.06893 = \log 1.172$;
$0.07004 = \log 1.175$; $0.07188 = \log 1.180$; $0.08063 = \log 1.204$.

21. *When the given mantissa can not be found in the tables.* — If we wish to find the number whose logarithm is 2.16531, we enter the tables with 16531, and find that it lies between 16524 and 16554, so that the given mantissa corresponds to a number between 1463 and 1464. Also 16531 exceeds 16524 by 7, and this difference, divided by the tabular difference 30, gives .23 ⋯ as the amount by which the required number exceeds 1463. Hence $2.16531 = \log 146.323 \cdots$, which we call 146.32, according to Art. 1, the incompleteness of the tables making the sixth figure uncertain.

NOTE. — The marginal tables should be used in dividing the difference between the given mantissa and the one next less in the tables by the tabular difference.

22. *The general rule is: Find the number corresponding to the mantissa in the tables next less than the given mantissa; divide the excess of the given mantissa over the one found in the tables by the tabular difference; and annex the quotient to the first four figures already found.*

The tabular difference is the difference between the two mantissas in the tables, between which the given mantissa lies.

$\bar{1}.16600 = \log 0.14656$; $0.18002 = \log 1.5136$; $2.18200 = \log 152.06$;
$1.19000 = \log 15.488$; $4.19680 = \log 15773$; $1.20020 = \log 15.856$.

23. For the use of the numbers S', T', S'', T'', see Arts. 35–38.

TABLE XVI.

24. This table (pages 420–464) contains the logarithms, to five decimal places, of the trigonometric sines, cosines, tangents, and cotangents of angles from 0° to 90°, for each minute. The logarithms in the columns headed *L. Sin*, *L. Tan*, and *L. Cos*, are augmented, and should be diminished by 10 (Art. 11), while those in the columns headed *L. Cot* are correctly given.

25. Since $\sec x = \frac{1}{\cos x}$, and $\operatorname{cosec} x = \frac{1}{\sin x}$, the logarithms of the secant and cosecant of an angle are the arithmetical complements of those of the cosine and sine respectively (Art. 12).

To find the Logarithmic Functions of an Angle Less than 90°.

26. *When the angle is less than* 45°, the degrees are found at the *top* of the page, and the minutes on the *left.* The numbers in the same horizontal line with the minutes of the angle are the logarithmic functions indicated by the notation at the *top* of the columns. Thus (page 428),

$$\log \sin 8° 4' = 9.14714 - 10, \qquad \log \tan 8° 4' = 9.15145 - 10,$$
$$\log \cot 8° 4' = 0.84855, \qquad \log \cos 8° 4' = 9.99568 - 10.$$

27. *When the angle is greater than* 45°, the degrees are found at the *bottom* of the page, and the minutes on the *right.* The numbers in the same horizontal line with the minutes of the angle are the logarithmic functions indicated by the notation at the *bottom* of the columns. Thus (page 428),

$$\log \sin 81° 25' = 9.99511 - 10, \qquad \log \tan 81° 25' = 0.82120,$$
$$\log \cot 81° 25' = 9.17880 - 10, \qquad \log \cos 81° 25' = 9.17391 - 10.$$

28. *When the angle is given to decimals of a minute.* — In finding $\log \sin 30° 8'.48$, for example, we see that it will lie between the logarithmic sines of $30° 8'$ and $30° 9'$, that is, between 9.70072 and 9.70093, their difference 21 being the change in the logarithmic sine caused by a change of $1'$ in the angle. Hence, to find the correction to $\log \sin 30° 8'$ that would give $\log \sin 30° 8'.48$ we multiply 21 by .48 (Art. 4). The product 10.08 added to $\log \sin 30° 8'$, since $\log \sin 30° 9'$ is greater than $\log \sin 30° 8'$, gives $\log \sin 30° 8'.48 = 9.70082$ (Art. 1). Similarly, $\log \tan 30° 8'.48 = 9.76391$, $\log \cot 30° 8'.48 = 0.23609$, $\log \cos 30° 8'.48 = 9.93691$, the correction being subtracted in the last two cases, since the cotangent and the cosine decrease as the angle increases.

29. *The general rule is: Find the function corresponding to the given degrees and minutes; multiply the tabular difference by the decimals of a minute; add the product to the function corresponding to the given degrees and minutes when finding the logarithmic sine or tangent, and subtract it when finding the logarithmic cosine or cotangent.*

The tabular differences are given in the columns headed *d.* and *c. d.*, the latter containing the common difference for the *L. Tan* and *L. Cot* columns. *The difference to be used is that between the functions corresponding to the two angles between which the given angle lies.*

For $30° 39'.38$: $\log \sin = 9.70747$; $\log \cos = 9.93462$;
$\log \tan = 9.77285$; $\log \cot = 0.22715$.

For $59° 43'.46$: $\log \sin = 9.93632$; $\log \cos = 9.70257$;
$\log \tan = 0.23375$; $\log \cot = 9.76625$.

30. *When the angle is given to seconds,* the correction may be found by multiplying the tabular difference by the number of seconds, and dividing the product by 60.

To find the Acute Angle corresponding to a Given Logarithmic Function.

31. The column headed *L. Sin* is marked *L. Cos* at the bottom, while that headed *L. Cos* is marked *L. Sin* at the bottom; hence, if a logarithmic sine or cosine were given, we should expect to find it in one of these two columns. Similarly, we should expect to find a given logarithmic tangent or cotangent in one of the two columns headed *L. Tan* and *L. Cot.*

32. *When the function can be found in the tables.*— If a logarithmic sine is given, find it in one of the two columns marked *L. Sin* and *L. Cos;* if found in the column headed *L. Sin,* the degrees are taken from the top, and the minutes from the left of the page; if in the column headed *L. Cos* but marked *L. Sin* at the bottom, the degrees are taken from the bottom, and the minutes from the right of the page. Thus,

$9.70115 = \log \sin 30° 10'$; $9.93457 = \log \sin 59° 20'$;
$9.93724 = \log \cos 30° \ 4'$; $9.70590 = \log \cos 59° 28'$;
$9.76406 = \log \tan 30° \ 9'$; $0.23130 = \log \tan 59° 35'$;
$0.23420 = \log \cot 30° 15'$; $9.76870 = \log \cot 59° 35'$.

33. *When the function can not be found in the tables.* — If we wish to find the angle whose logarithmic sine is 9.70170, we see on page 450 that the given logarithmic sine lies between 9.70159 and 9.70180, and hence the angle is between $30° 12'$ and $30° 13'$. The given logarithmic sine differs from $\log\sin 30° 12'$ by 11, and this difference, divided by the tabular difference 21, gives .52 + as the decimal of a minute by which the angle exceeds $30° 12'$. Hence $9.70170 = \log\sin 30° 12'.52$, which we call $30° 12'.5$, since the incompleteness of the tables (Art. 1) makes the hundredths of a minute uncertain.

34. *The rule is: For a logarithmic sine or tangent find the degrees and minutes corresponding to the function in the tables next* less *than the given function; divide the difference between the given function and the one next less by the tabular difference; and the quotient will be the decimal of a minute to be added to the degrees and minutes already found. For a logarithmic cosine or cotangent find the degrees and minutes corresponding to the function next* greater *than the given function, since the cosine and cotangent decrease as the angle increases, and divide the difference between the given function and the one next greater by the tabular difference, to find the decimal of a minute.*

The tabular difference is the difference between the two functions in the tables, between which the given function lies.

$$9.70000 = \log\sin 30°\ 4'.7;\quad 9.93500 = \log\sin 59°\ 25'.7;$$
$$9.93400 = \log\cos 30°\ 47'.6;\quad 9.70500 = \log\cos 59°\ 32'.2;$$
$$9.77000 = \log\tan 30°\ 29'.5;\quad 0.23200 = \log\tan 59°\ 37'.4;$$
$$0.23300 = \log\cot 30°\ 19'.1;\quad 9.76400 = \log\cot 59°\ 51'.2.$$

Angles Near 0° *or* 90°.

35. The assumption that the variations in the functions are proportional to the variations in the angles if the latter are less than 1′ fails when the angle is small, shown by the rapid changes in the tabular differences on pages 420, 421, and 422.

36. The quantities S' and T' which are used in this case are defined by the equations

$$S' = \log\frac{\sin\alpha}{\alpha'},$$

$$T' = \log\frac{\tan\alpha}{\alpha'},$$

where α' is the number of minutes in the angle. Their values from 0° to 1° 40′ (= 100′) are given at the bottom of pages 397–415; from 1° 40′ to 3° 20′ at the left margin of pages 398 and 399, the first three figures being found at the top; and from 3° to 5° on page 418. Thus,

for $1' = 1'$ (page 399), $S' = 6.46\,373$, $T' = 6.46\,373$;
for $15' = 15'$ (page 399), $S' = 6.46\,372$, $T' = 6.46\,373$;
for $2° 40' = 160'$ (page 399), $S' = 6.46\,357$, $T' = 6.46\,404$;
for $4° 20' = 260'$ (page 418), $S' = 6.46\,331$, $T' = 6.46\,456$.

Each of these numbers should have -10 written after it (Art. 11).

NOTE. — The logarithmic cosine of a small angle is found by the ordinary method. The cotangent of an angle is the reciprocal of the tangent, and hence the logarithmic cotangent is the arithmetical complement of the logarithmic tangent. The formulas for finding the logarithmic cosine, tangent, and cotangent of angles near 90° are given on page 419.

37. *To find the logarithmic sine or tangent of a small angle.* — From Art. 36, we have

$$\log \sin \alpha = S' + \log \alpha',$$
$$\log \tan \alpha = T' + \log \alpha'.$$

Hence, to find the logarithmic sine or tangent of an angle less than 5°, find the value of the S' or T' corresponding to the angle, interpolating if necessary, and add it to the logarithm of the number of minutes in the angle.

Find $\log \sin 0° 42'.6$. Since the angle is nearer $43'$ than $42'$, we take

$$\begin{aligned} S' &= 6.46\,371 \\ \log 42.6 &= 1.62\,941 \\ \therefore \log \sin 0° 42'.6 &= 8.09\,312 \end{aligned}$$

Find $\log \tan 1° 53'.2$. Since the angle is nearer $1° 53'$ ($= 113'$) than $114'$, we take

$$\begin{aligned} T' &= 6.46\,388 \\ \log 113.2 &= 2.05\,385 \\ \therefore \log \tan 1° 53'.2 &= 8.51\,773 \end{aligned}$$

NOTE. — When the angle is given in seconds, either reduce the seconds to decimals of a minute, or use the values of S'' and T'' given at the bottom of pages 397–417 and on page 418. They are defined by the equations

$$S'' = \log \frac{\sin \alpha}{\alpha''}, \text{ and } T'' = \log \frac{\tan \alpha}{\alpha''},$$

where α'' is the number of seconds in the angle. Hence

$$\log \sin \alpha = S'' + \log \alpha'', \text{ and } \log \tan \alpha = T'' + \log \alpha''.$$

38. *To find the small angle corresponding to a given logarithmic sine or tangent.* — From Art. 36,

$$\left.\begin{aligned} \log \alpha' &= \log \sin \alpha - S', \\ \log \alpha' &= \log \tan \alpha - T', \end{aligned}\right\}$$

or

$$\left.\begin{aligned} \log \alpha' &= \log \sin \alpha + \text{cpl } S', \\ \log \alpha' &= \log \tan \alpha + \text{cpl } T'. \end{aligned}\right\}$$

When the angle is less than 3°, find on pages 420–422 the value of cpl S' (or cpl T') corresponding to the function, interpolating if necessary, and add it to $\log \sin \alpha$ (or $\log \tan \alpha$); the sum will be the logarithm of the number of minutes in the angle.

In finding the angle whose logarithmic sine is 8.09006, we see from the *L. Sin* column (page 420) that the angle is between 0° 42′ and 0° 43′, and that the value of cpl S' must be either 3.53628 or 3.53629. The given logarithmic sine is nearer that of 42′ than that of 43′; hence we take

$$\begin{aligned} \text{cpl } S' &= 3.53628 \\ \log \sin \alpha &= \underline{8.09006} \\ \log \alpha' &= 1.62634 \quad \therefore\ \alpha' = 42'.300. \end{aligned}$$

When the angle is between 3° and 5°, we may find S' and T' from page 418 after finding the angle approximately from pages 423 and 424. Thus in finding the angle whose logarithmic tangent is 8.77237 we find from page 423 that the angle is between 3° 23′ and 3° 24′, being nearer 3° 23′. Then on page 418 we have

$$\begin{aligned} T' &= 6.46423 \\ \log \tan \alpha &= \underline{8.77237} \\ \therefore\ \log \tan \alpha - T' = \log \alpha' &= 2.30814 \quad \therefore\ \alpha' = 203'.30 = 3^\circ\ 23'.30. \end{aligned}$$

Angles Greater than 90°.

39. To find the logarithmic sine, cosine, tangent, or cotangent of an angle greater than 90°, subtract from the given angle the largest multiple of 90° contained therein. If this multiple is even, find from the tables the logarithmic sine, cosine, tangent, or cotangent of the remaining acute angle. If the multiple is odd, the logarithmic *cosine*, *sine*, *cotangent*, or *tangent*, respectively, of the remaining acute angle will be the function required; thus, sin 120° = sin (90° + 30°) = cos 30°.

$x =$	I. QUADRANT. α	II. QUADRANT. $90^\circ +$	III. QUADRANT. $180^\circ + \alpha$	IV. QUADRANT. $270^\circ + \alpha$
$\sin x =$	$+\sin\alpha$	$+\cos\alpha$	$-\sin\alpha$	$-\cos\alpha$
$\cos x =$	$+\cos\alpha$	$-\sin\alpha$	$-\cos\alpha$	$+\sin\alpha$
$\tan x =$	$+\tan\alpha$	$-\cot\alpha$	$+\tan\alpha$	$-\cot\alpha$
$\cot x =$	$+\cot\alpha$	$-\tan\alpha$	$+\cot\alpha$	$-\tan\alpha$

Or we could find the difference between the angle and 180° or 360°, and find from the tables the same function of the remaining acute angle; thus, cos 300° = cos (360° − 60°) = cos 60°, etc.

$x =$	I. QUADRANT. α	II. QUADRANT. $180^\circ - \alpha$	III. QUADRANT. $180^\circ + \alpha$	IV. QUADRANT. $360^\circ - \alpha$ or $-\alpha$
$\sin x =$	$+\sin\alpha$	$+\sin\alpha$	$-\sin\alpha$	$-\sin\alpha$
$\cos x =$	$+\cos\alpha$	$-\cos\alpha$	$-\cos\alpha$	$+\cos\alpha$
$\tan x =$	$+\tan\alpha$	$-\tan\alpha$	$+\tan\alpha$	$-\tan\alpha$
$\cot x =$	$+\cot\alpha$	$-\cot\alpha$	$+\cot\alpha$	$-\cot\alpha$

To indicate that the trigonometric function is negative, *n* is written after its logarithm.

40. To find the angle corresponding to a given function, find the acute angle α corresponding thereto, and the required angle will be α, $180° \pm \alpha$, or $360° - \alpha$, according to the quadrant in which the angle should be placed.

41. There are always two angles less than 360° corresponding to any given function. Hence there will be ambiguity in the result unless some condition is known that will fix the angle; thus, if the sine is positive, the angle may be in either of the first two quadrants, but if we also know that the cosine is negative, the angle must be in the second quadrant.

Given One Function of an Angle, to find Another without finding the Angle.

42. Suppose $\log \tan \alpha = 9.79361$, and $\log \cos \alpha$ is sought. On page 451 the tabular difference for $\log \tan \alpha$ is 28, and that for $\log \cos \alpha$ is 8, the given logarithmic tangent exceeding 9.79354 by 7. Hence $28 : 7 = 8 : x$; $\therefore x = 2 =$ correction to 9.92905, giving $\log \cos \alpha = 9.92903$.

In the margin are tables to facilitate the process. In the column headed $\frac{8}{28}$, the numerator is the tabular difference for the logarithmic cosines, and the denominator that for the logarithmic tangents[1]. The correction for the logarithmic cosine will be 0 when the given logarithmic tangent exceeds the next smaller logarithmic tangent, found in the tables, by less than 1.8, 1 for an excess between 1.8 and 5.2, 5 for an excess between 15.8 and 19.2, etc. In the example above, the excess was 7, which is between 5.2 and 8.8, so that the correction is 2.

For example, if we have given the logarithms of the sides of a right-angled triangle, $\log a = 2.98227$ and $\log b = 2.90255$, to find the hypotenuse, we use the formulas

$$\tan \alpha = \frac{a}{b}, \text{ and } c = \frac{a}{\sin \alpha} = \frac{b}{\cos \alpha}.$$

$$\begin{aligned} \log a &= 2.98227 \ (1) \\ \therefore \log \sin \alpha &= 9.88571 \ (4) \\ \log b &= 2.90255 \ (2) \\ \therefore \log \tan \alpha &= 0.07972 \ (3) \\ \therefore \log c &= 3.09656 \ (5) \end{aligned}$$

The value of $\log \tan \alpha$ being found in the column marked *L. Tan* at the bottom, the right column will contain the logarithmic sine of the corresponding angle. Also, the correction to 9.88563 is $20 \times \frac{10}{26}$, which we find to be 8 from the table headed $\frac{10}{26}$.

[1] For angles $< 45°$.

TABLE XV.

COMMON

LOGARITHMS OF NUMBERS

FROM 1 TO 11000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	—	**20**	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 32$\bar{5}$	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 45$\bar{0}$
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 88$\bar{5}$	89	1.94 939
10	1.00 000	**30**	1.47 712	**50**	1.69 897	**70**	1.84 510	**90**	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 51$\bar{5}$	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 04$\bar{5}$	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
20	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309	**100**	2.00 000

	S′.	T′.
0′	6.46 373	373
1	373	373

	S″.	T″.
0° 0′ = 0″	4.68 557	557
0 1 = 60	557	557
0 2 = 120	557	557

S'.	T'. 6.46	N.	L. 0	1	2	3	4	5	6	7	8	9
366	385	**100**	00 000	043	087	130	173	217	260	303	346	389
366	385	101	432	475	518	561	604	647	689	732	775	817
366	385	102	860	903	945	988	*030	*072	*115	*157	*199	*242
366	386	103	01 284	326	368	410	452	494	536	578	620	662
366	386	104	703	745	787	828	870	912	953	995	*036	*078
366	386	105	02 119	160	202	243	284	325	366	407	449	490
366	386	106	531	572	612	653	694	735	776	816	857	898
366	387	107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
365	387	108	03 342	383	423	463	503	543	583	623	663	703
365	387	109	743	782	822	862	902	941	981	*021	*060	*100
365	387	**110**	04 139	179	218	258	297	336	376	415	454	493
365	388	111	532	571	610	650	689	727	766	805	844	883
365	388	112	922	961	999	*038	*077	*115	*154	*192	*231	*269
365	388	113	05 308	346	385	423	461	500	538	576	614	652
365	389	114	690	729	767	805	843	881	918	956	994	*032
365	389	115	06 070	108	145	183	221	258	296	333	371	408
364	389	116	446	483	521	558	595	633	670	707	744	781
364	389	117	819	856	893	930	967	*004	*041	*078	*115	*151
364	390	118	07 188	225	262	298	335	372	408	445	482	518
364	390	119	555	591	628	664	700	737	773	809	846	882
364	390	**120**	918	954	990	*027	*063	*099	*135	*171	*207	*243
364	391	121	08 279	314	350	386	422	458	493	529	565	600
363	391	122	636	672	707	743	778	814	849	884	920	955
363	391	123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
363	391	124	09 342	377	412	447	482	517	552	587	621	656
363	392	125	691	726	760	795	830	864	899	934	968	*003
363	392	126	10 037	072	106	140	175	209	243	278	312	346
363	392	127	380	415	449	483	517	551	585	619	653	687
363	393	128	721	755	789	823	857	890	924	958	992	*025
362	393	129	11 059	093	126	160	193	227	261	294	327	361
362	393	**130**	394	428	461	494	528	561	594	628	661	694
362	394	131	727	760	793	826	860	893	926	959	992	*024
362	394	132	12 057	090	123	156	189	222	254	287	320	352
362	394	133	385	418	450	483	516	548	581	613	646	678
362	395	134	710	743	775	808	840	872	905	937	969	*001
361	395	135	13 033	066	098	130	162	194	226	258	290	322
361	395	136	354	386	418	450	481	513	545	577	609	640
361	396	137	672	704	735	767	799	830	862	893	925	956
361	396	138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
361	396	139	14 301	333	364	395	426	457	489	520	551	582
361	397	**140**	613	644	675	706	737	768	799	829	860	891
360	397	141	922	953	983	*014	*045	*076	*106	*137	*168	*198
360	397	142	15 229	259	290	320	351	381	412	442	473	503
360	398	143	534	564	594	625	655	685	715	746	776	806
360	398	144	836	866	897	927	957	987	*017	*047	*077	*107
360	398	145	16 137	167	197	227	256	286	316	346	376	406
360	399	146	435	465	495	524	554	584	613	643	673	702
359	399	147	732	761	791	820	850	879	909	938	967	997
359	399	148	17 026	056	085	114	143	173	202	231	260	289
359	400	149	319	348	377	406	435	464	493	522	551	580
359	400	**150**	609	638	667	696	725	754	782	811	840	869
		N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

		S.'	T.'
1'	6.46	373	373
2		373	373
10		373	373
13		373	373
14		372	373
15		372	373

			S.''	T.''
0°	1' = 60''	4.68	557	557
0	2 = 120		557	557
0	3 = 180		557	557
0	16 = 960		557	558
0	17 = 1020		557	558
0	18 = 1080		557	558
0	19 = 1140		557	558

			S.''	T.''
0°	19' = 1140''	4.68	557	558
0	20 = 1200		557	558
0	21 = 1260		557	558
0	22 = 1320		557	558
0	23 = 1380		557	558
0	24 = 1440		557	558
0	25 = 1500		557	558

S′.	T′. 6.46	N.	L. 0	1	2	3	4	5	6	7	8	9
359	400	**150**	17 609	638	667	696	725	754	782	811	840	869
359	401	151	898	926	955	984	*013	*041	*070	*099	*127	*156
358	401	152	18 184	213	241	270	298	327	355	384	412	441
358	401	153	469	498	526	554	583	611	639	667	696	724
358	402	154	752	780	808	837	865	893	921	949	977	*005
358	402	155	19 033	061	089	117	145	173	201	229	257	285
358	402	156	312	340	368	396	424	451	479	507	535	562
358	403	157	590	618	645	673	700	728	756	783	811	838
357	403	158	866	893	921	948	976	*003	*030	*058	*085	*112
357	404	159	20 140	167	194	222	249	276	303	330	358	385
357	404	**160**	412	439	466	493	520	548	575	602	629	656
357	404	161	683	710	737	763	790	817	844	871	898	925
357	405	162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
356	405	163	21 219	245	272	299	325	352	378	405	431	458
356	406	164	484	511	537	564	590	617	643	669	696	722
356	406	165	748	775	801	827	854	880	906	932	958	985
356	406	166	22 011	037	063	089	115	141	167	194	220	246
356	407	167	272	298	324	350	376	401	427	453	479	505
355	407	168	531	557	583	608	634	660	686	712	737	763
355	408	169	789	814	840	866	891	917	943	968	994	*019
355	408	**170**	23 045	070	096	121	147	172	198	223	249	274
355	408	171	300	325	350	376	401	426	452	477	502	528
354	409	172	553	578	603	629	654	679	704	729	754	779
354	409	173	805	830	855	880	905	930	955	980	*005	*030
354	410	174	24 055	080	105	130	155	180	204	229	254	279
354	410	175	304	329	353	378	403	428	452	477	502	527
354	411	176	551	576	601	625	650	674	699	724	748	773
353	411	177	797	822	846	871	895	920	944	969	993	*018
353	411	178	25 042	066	091	115	139	164	188	212	237	261
353	412	179	285	310	334	358	382	406	431	455	479	503
353	412	**180**	527	551	575	600	624	648	672	696	720	744
353	413	181	768	792	816	840	864	888	912	935	959	983
352	413	182	26 007	031	055	079	102	126	150	174	198	221
352	414	183	245	269	293	316	340	364	387	411	435	458
352	414	184	482	505	529	553	576	600	623	647	670	694
352	415	185	717	741	764	788	811	834	858	881	905	928
351	415	186	951	975	998	*021	*045	*068	*091	*114	*138	*161
351	415	187	27 184	207	231	254	277	300	323	346	370	393
351	416	188	416	439	462	485	508	531	554	577	600	623
351	416	189	646	669	692	715	738	761	784	807	830	852
350	417	**190**	875	898	921	944	967	989	*012	*035	*058	*081
350	417	191	28 103	126	149	171	194	217	240	262	285	307
350	418	192	330	353	375	398	421	443	466	488	511	533
350	418	193	556	578	601	623	646	668	691	713	735	758
350	419	194	780	803	825	847	870	892	914	937	959	981
349	419	195	29 003	026	048	070	092	115	137	159	181	203
349	420	196	226	248	270	292	314	336	358	380	403	425
349	420	197	447	469	491	513	535	557	579	601	623	645
349	421	198	667	688	710	732	754	776	798	820	842	863
348	421	199	885	907	929	951	973	994	*016	*038	*060	*081
348	422	**200**	30 103	125	146	168	190	211	233	255	276	298
		N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	29	28	27	26	25	24	23	22	21
1	2.9	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1
2	5.8	5.6	5.4	5.2	5.0	4.8	4.6	4.4	4.2
3	8.7	8.4	8.1	7.8	7.5	7.2	6.9	6.6	6.3
4	11.6	11.2	10.8	10.4	10.0	9.6	9.2	8.8	8.4
5	14.5	14.0	13.5	13.0	12.5	12.0	11.5	11.0	10.5
6	17.4	16.8	16.2	15.6	15.0	14.4	13.8	13.2	12.6
7	20.3	19.6	18.9	18.2	17.5	16.8	16.1	15.4	14.7
8	23.2	22.4	21.6	20.8	20.0	19.2	18.4	17.6	16.8
9	26.1	25.2	24.3	23.4	22.5	21.6	20.7	19.8	18.9

		S.′	T.′
1′	6.46	373	373
2		373	373
15		372	373
20		372	373

		S.″	T.″
0° 2′ = 120″	4.68	557	557
0 3 = 180		557	557
0 4 = 240		557	558
0 25 = 1500		557	558
0 26 = 1560		557	558
0 27 = 1620		557	558
0 28 = 1680		557	558
0° 28′ = 1680″	4.68	557	558
0 29 = 1740		557	559
0 30 = 1800		557	559
0 31 = 1860		557	559
0 32 = 1920		557	559
0 33 = 1980		557	559
0 34 = 2040		557	559

N.	L. 0	1	2	3	4	5	6	7	8	9
200	30 103	125̄	146	168	190	211	233	255̄	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	75̄0	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345̄	366
206	387	408	429	45̄0	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765̄	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015̄	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305̄	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675̄	695̄	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325̄	345̄	365̄	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985̄	*005̄	*025̄
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655̄	674	694	713	733	753	772	792	811
223	830	85̄0	869	889	908	928	947	967	986	*005
224	35 025̄	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315̄	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965̄
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305̄	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605̄	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365̄	383	401	420	438	457
237	475̄	493	511	530	548	566	585̄	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985̄	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525̄	543
243	561	578	596	614	632	65̄0	668	686	703	721
244	739	757	775̄	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235̄	252
247	270	287	305̄	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655̄	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

		S.′	T.′
2′	6.46	373	373
3		373	373
20		372	373
25		372	373

		S.″	T.″
0° 3′ = 180″	4.68	557	557
0 4 = 240		557	558
0 5 = 300		557	558
0 33 = 1980		557	559
0 34 = 2040		557	559
0 35 = 2100		557	559
0 36 = 2160		557	559

		S.″	T.″
0° 36′ = 2160″	4.68	557	559
0 37 = 2220		557	559
0 38 = 2280		557	559
0 39 = 2340		557	559
0 40 = 2400		557	559
0 41 = 2460		556	560
0 42 = 2520		556	560

N.	L. 0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	967	98$\bar{5}$	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	17$\bar{5}$	192	209	226	243	261	278	29$\bar{5}$
253	312	329	346	364	381	398	41$\bar{5}$	432	449	466
254	483	500	518	53$\bar{5}$	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	87$\bar{5}$	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*09$\bar{5}$	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*09$\bar{5}$	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	32$\bar{5}$	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	63$\bar{5}$
267	651	667	684	700	716	732	749	76$\bar{5}$	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	18$\bar{5}$	201	217	233	249	26$\bar{5}$	281
271	297	313	329	34$\bar{5}$	361	377	393	409	42$\bar{5}$	441
272	457	473	489	50$\bar{5}$	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	96$\bar{5}$	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	29$\bar{5}$	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	54$\bar{5}$
279	560	576	592	607	623	638	654	669	68$\bar{5}$	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 02$\bar{5}$	040	056	071	086	102	117	133	148	163
283	179	194	209	22$\bar{5}$	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	$\bar{5}$00	51$\bar{5}$	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*01$\bar{5}$	*030	*04$\bar{5}$	*060	*07$\bar{5}$
289	46 090	10$\bar{5}$	120	13$\bar{5}$	1$\bar{5}$0	16$\bar{5}$	180	19$\bar{5}$	210	22$\bar{5}$
290	240	25$\bar{5}$	270	28$\bar{5}$	300	31$\bar{5}$	330	34$\bar{5}$	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	83$\bar{5}$	8$\bar{5}$0	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	30$\bar{5}$	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	18	17	16	15	14
1	1.8	1.7	1.6	1.5	1.4
2	3.6	3.4	3.2	3.0	2.8
3	5.4	5.1	4.8	4.5	4.2
4	7.2	6.8	6.4	6.0	5.6
5	9.0	8.5	8.0	7.5	7.0
6	10.8	10.2	9.6	9.0	8.4
7	12.6	11.9	11.2	10.5	9.8
8	14.4	13.6	12.8	12.0	11.2
9	16.2	15.3	14.4	13.5	12.6

	S.′	T.′
2′	6.46 373	373
3	373	373
25	372	373
26	372	373
27	372	374
30	372	374

	S.″	T.″
0° 4′ = 240″	4.68 557	558
0 5 = 300	557	558
0 41 = 2460	556	560
0 42 = 2520	556	560
0 43 = 2580	556	560
0 44 = 2640	556	560
0 45 = 2700	556	560

	S.″	T.″
0° 45′ = 2700″	4.68 556	560
0 46 = 2760	556	560
0 47 = 2820	556	560
0 48 = 2880	556	560
0 49 = 2940	556	560
0 50 = 3000	556	561

N.	L. 0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	430	444	458	473	487	501	515	530	544	558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	831	845	859	872	886	900	914	927	941	955
316	969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	188	202	215	228	242	255	268	282	295	308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375	388	401	414	427	440	453	466	479	492
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815	827	840	853	866	879
338	892	905	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	782	794	807	820	832	845	857	870	882	895
346	908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	15	14	13	12
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

		S.′	T.′
3′	6.46	373	373
4		373	373
30		372	374
35		372	374

		S.″	T.″
0° 5′ = 300″	4.68	557	558
0 6 = 360		557	558
0 50 = 3000		556	561
0 51 = 3060		556	561
0 52 = 3120		556	561
0 53 = 3180		556	561
0 54 = 3240		556	561

		S.″	T.″
0° 54′ = 3240″	4.68	556	561
0 55 = 3300		556	561
0 56 = 3360		556	561
0 57 = 3420		555	561
0 58 = 3480		555	562
0 59 = 3540		555	562

N.	L. 0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
351	531	543	555	568	580	593	605	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	145	157	169	182	194	206	218	230	242	255
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715	727	739
361	751	763	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	967	979
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	229	241	253	265	277	289	301	312	324	336
366	348	360	372	384	396	407	419	431	443	455
367	467	478	490	502	514	526	538	549	561	573
368	585	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
371	937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	403	415	426	438	449	461	473	484	496	507
376	519	530	542	553	565	576	588	600	611	623
377	634	646	657	669	680	692	703	715	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	195
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	535
385	546	557	569	580	591	602	614	625	636	647
386	659	670	681	692	704	715	726	737	749	760
387	771	782	794	805	816	827	838	850	861	872
388	883	894	906	917	928	939	950	961	973	984
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	550	561	572	583	594	605	616	627	638	649
395	660	671	682	693	704	715	726	737	748	759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	945	956	966	977
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	13	12	11	10
1	1.3	1.2	1.1	1.0
2	2.6	2.4	2.2	2.0
3	3.9	3.6	3.3	3.0
4	5.2	4.8	4.4	4.0
5	6.5	6.0	5.5	5.0
6	7.8	7.2	6.6	6.0
7	9.1	8.4	7.7	7.0
8	10.4	9.6	8.8	8.0
9	11.7	10.8	9.9	9.0

	S.′	T.′
3′	6.46 373	373
4	373	373
35	372	374
39	372	374
40	372	375

	S.″	T.″
0° 5′ = 300″	4.68 557	558
0 6 = 360	557	558
0 7 = 420	557	558
0 58 = 3480	555	562
0 59 = 3540	555	562
1 0 = 3600	555	562
1 1 = 3660	555	562

	S.″	T.″
1° 1′ = 3660″	4.68 555	562
1 2 = 3720	555	562
1 3 = 3780	555	562
1 4 = 3840	555	563
1 5 = 3900	555	563
1 6 = 3960	555	563
1 7 = 4020	555	563

N.	L. 0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	73$\bar{5}$
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	88$\bar{5}$	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*04$\bar{5}$	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	21$\bar{5}$	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	39$\bar{5}$	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	80$\bar{5}$	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	04$\bar{5}$	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	31$\bar{5}$
420	32$\bar{5}$	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	65$\bar{5}$	66$\bar{5}$	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	15$\bar{5}$	16$\bar{5}$	17$\bar{5}$	18$\bar{5}$	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	37$\bar{5}$	38$\bar{5}$	39$\bar{5}$	404	414	424	434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	88$\bar{5}$	89$\bar{5}$	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	21$\bar{5}$
449	22$\bar{5}$	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

	S.′	T.′
4′	6.46 373	373
5	373	373
40	372	37$\bar{5}$
42	372	37$\bar{5}$
43	371	37$\bar{5}$
44	371	37$\bar{5}$
45	371	375

	S.″	T.″
0° 6′ = 360″	4.68 557	558
0 7 = 420	557	558
0 8 = 480	557	558
1 6 = 3960	55$\bar{5}$	563
1 7 = 4020	55$\bar{5}$	563
1 8 = 4080	55$\bar{5}$	563
1 9 = 4140	55$\bar{5}$	563

	S.″	T.″
1° 9′ = 4140″	4.68 55$\bar{5}$	563
1 10 = 4200	554	563
1 11 = 4260	554	564
1 12 = 4320	554	564
1 13 = 4380	554	564
1 14 = 4440	554	564
1 15 = 4500	554	564

N.	L. 0	1	2	3	4	5	6	7	8	9
450	65 321	331	341	350	360	369	379	389	398	408
451	418	427	437	447	456	466	475	485	495	504
452	514	523	533	543	552	562	571	581	591	600
453	610	619	629	639	648	658	667	677	686	696
454	706	715	725	734	744	753	763	772	782	792
455	801	811	820	830	839	849	858	868	877	887
456	896	906	916	925	935	944	954	963	973	982
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66 087	096	106	115	124	134	143	153	162	172
459	181	191	200	210	219	229	238	247	257	266
460	276	285	295	304	314	323	332	342	351	361
461	370	380	389	398	408	417	427	436	445	455
462	464	474	483	492	502	511	521	530	539	549
463	558	567	577	586	596	605	614	624	633	642
464	652	661	671	680	689	699	708	717	727	736
465	745	755	764	773	783	792	801	811	820	829
466	839	848	857	867	876	885	894	904	913	922
467	932	941	950	960	969	978	987	997	*006	*015
468	67 025	034	043	052	062	071	080	089	099	108
469	117	127	136	145	154	164	173	182	191	201
470	210	219	228	237	247	256	265	274	284	293
471	302	311	321	330	339	348	357	367	376	385
472	394	403	413	422	431	440	449	459	468	477
473	486	495	504	514	523	532	541	550	560	569
474	578	587	596	605	614	624	633	642	651	660
475	669	679	688	697	706	715	724	733	742	752
476	761	770	779	788	797	806	815	825	834	843
477	852	861	870	879	888	897	906	916	925	934
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	215	224	233	242	251	260	269	278	287	296
482	305	314	323	332	341	350	359	368	377	386
483	395	404	413	422	431	440	449	458	467	476
484	485	494	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	655
486	664	673	681	690	699	708	717	726	735	744
487	753	762	771	780	789	797	806	815	824	833
488	842	851	860	869	878	886	895	904	913	922
489	931	940	949	958	966	975	984	993	*002	*011
490	69 020	028	037	046	055	064	073	082	090	099
491	108	117	126	135	144	152	161	170	179	188
492	197	205	214	223	232	241	249	258	267	276
493	285	294	302	311	320	329	338	346	355	364
494	373	381	390	399	408	417	425	434	443	452
495	461	469	478	487	496	504	513	522	531	539
496	548	557	566	574	583	592	601	609	618	627
497	636	644	653	662	671	679	688	697	705	714
498	723	732	740	749	758	767	775	784	793	801
499	810	819	827	836	845	854	862	871	880	888
500	897	906	914	923	932	940	949	958	966	975
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	10	9	8
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

	S.′	T.′
4′	6.46 373	373
5	373	373
45	371	375
48	371	375
49	371	376
50	371	376

	S.″	T.″
0° 7′ = 420″	4.68 557	558
0 8 = 480	557	558
0 9 = 540	557	558
1 15 = 4500	554	564
1 16 = 4560	554	565
1 17 = 4620	554	565
1 18 = 4680	554	565

	S.″	T.″
1° 18′ = 4680″	4.68 554	565
1 19 = 4740	554	565
1 20 = 4800	554	565
1 21 = 4860	553	566
1 22 = 4920	553	566
1 23 = 4980	553	566
1 24 = 5040	553	566

N.	L. 0	1	2	3	4	5	6	7	8	9
500	69 897	906	914	923	932	940	949	958	966	975
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70 070	079	088	096	105̄	114	122	131	140	148
503	157	165	174	183	191	200	209	217	226	234
504	243	252	260	269	278	286	295̄	303	312	321
505	329	338	346	355̄	364	372	381	389	398	406
506	415	424	432	441	449	458	467	475	484	492
507	501	509	518	526	535	544	552	561	569	578
508	586	595̄	603	612	621	629	638	646	655̄	663
509	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
511	842	851	859	868	876	885̄	893	902	910	919
512	927	935	944	952	961	969	978	986	995̄	*003
513	71 012	020	029	037	046	054	063	071	079	088
514	096	105̄	113	122	130	139	147	155	164	172
515	181	189	198	206	214	223	231	240	248	257
516	265̄	273	282	290	299	307	315	324	332	341
517	349	357	366	374	383	391	399	408	416	425̄
518	433	441	45̄0	458	466	475̄	483	492	5̄00	508
519	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
521	684	692	700	709	717	725	734	742	750	759
522	767	775	784	792	800	809	817	825	834	842
523	850	858	867	875	883	892	900	908	917	925̄
524	933	941	95̄0	958	966	975̄	983	991	999	*008
525	72 016	024	032	041	049	057	066	074	082	090
526	099	107	115	123	132	140	148	156	165̄	173
527	181	189	198	206	214	222	230	239	247	255
528	263	272	280	288	296	304	313	321	329	337
529	346	354	362	370	378	387	395̄	403	411	419
530	428	436	444	452	460	469	477	485̄	493	501
531	509	518	526	534	542	550	558	567	575̄	583
532	591	599	607	616	624	632	640	648	656	665̄
533	673	681	689	697	705	713	722	730	738	746
534	754	762	770	779	787	795̄	803	811	819	827
535	835	843	852	860	868	876	884	892	900	908
536	916	925̄	933	941	949	957	965	973	981	989
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
538	73 078	086	094	102	111	119	127	135̄	143	151
539	159	167	175̄	183	191	199	207	215	223	231
540	239	247	255	263	272	280	288	296	304	312
541	320	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	616	624	632
545	640	648	656	664	672	679	687	695	703	711
546	719	727	735	743	751	759	767	775̄	783	791
547	799	807	815̄	823	830	838	846	854	862	870
548	878	886	894	902	910	918	926	933	941	949
549	957	965	973	981	989	997	*005̄	*013	*020	*028
550	74 036	044	052	060	068	076	084	092	099	107
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

		S.′	T.′
5′	6.46	373	373
6		373	373
50		371	376
55		371	376

		S.″	T.″
0° 8′ = 480″	4.68	557	558
0 9 = 540		557	558
0 10 = 600		557	558
1 23 = 4980		553	566
1 24 = 5040		553	566
·1 25 = 5100		553	566
1 26 = 5160		553	567

		S.″	T.″
1° 26′ = 5160″	4.68	553	567
1 27 = 5220		553	567
1 28 = 5280		553	567
1 29 = 5340		553	567
1 30 = 5400		553	567
1 31 = 5460		552	568
1 32 = 5520		552	568

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	8
561	896	904	912	920	927	935	943	950	958	966	1 0.8
562	974	981	989	997	*005	*012	*020	*028	*035	*043	2 1.6
563	75 051	059	066	074	082	089	097	105	113	120	3 2.4
564	128	136	143	151	159	166	174	182	189	197	4 3.2
565	205	213	220	228	236	243	251	259	266	274	5 4.0
566	282	289	297	305	312	320	328	335	343	351	6 4.8
567	358	366	374	381	389	397	404	412	420	427	7 5.6
568	435	442	450	458	465	473	481	488	496	504	8 6.4
569	511	519	526	534	542	549	557	565	572	580	9 7.2
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	7
581	418	425	433	440	448	455	462	470	477	485	1 0.7
582	492	500	507	515	522	530	537	545	552	559	2 1.4
583	567	574	582	589	597	604	612	619	626	634	3 2.1
584	641	649	656	664	671	678	686	693	701	708	4 2.8
585	716	723	730	738	745	753	760	768	775	782	5 3.5
586	790	797	805	812	819	827	834	842	849	856	6 4.2
587	864	871	879	886	893	901	908	916	923	930	7 4.9
588	938	945	953	960	967	975	982	989	997	*004	8 5.6
589	77 012	019	026	034	041	048	056	063	070	078	9 6.3
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

	S.′	T.′
6′	6.46 373	373
55	371	376
56	371	376
57	371	377
58	371	377
59	370	377
60	370	377

	S.″	T.″
0° 9′ = 540″	4.68 557	558
0 10 = 600	557	558
1 31 = 5460	552	568
1 32 = 5520	552	568
1 33 = 5580	552	568
1 34 = 5640	552	568
1 35 = 5700	552	569

	S.″	T.″
1° 35′ = 5700″	4.68 552	569
1 36 = 5760	552	569
1 37 = 5820	552	569
1 38 = 5880	552	569
1 39 = 5940	551	569
1 40 = 6000	551	570

N.	L. 0	1	2	3	4	5	6	7	8	9
600	77 815	822	830	837	844	851	859	866	873	880
601	887	895̄	902	909	916	924	931	938	945	952
602	960	967	974	981	988	996	*003	*010	*017	*025̄
603	78 032	039	046	053	061	068	075̄	082	089	097
604	104	111	118	125	132	140	147	154	161	168
605	176	183	190	197	204	211	219	226	233	240
606	247	254	262	269	276	283	290	297	305̄	312
607	319	326	333	340	347	355̄	362	369	376	383
608	390	398	405̄	412	419	426	433	440	447	455̄
609	462	469	476	483	490	497	504	512	519	526
610	533	540	547	554	561	569	576	583	590	597
611	604	611	618	625	633	640	647	654	661	668
612	675	682	689	696	704	711	718	725̄	732	739
613	746	753	760	767	774	781	789	796	803	810
614	817	824	831	838	845	852	859	866	873	880
615	888	895̄	902	909	916	923	930	937	944	951
616	958	965	972	979	986	993	*000	*007	*014	*021
617	79 029	036	043	050̄	057	064	071	078	085̄	092
618	099	106	113	120	127	134	141	148	155	162
619	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
621	309	316	323	330	337	344	351	358	365	372
622	379	386	393	400	407	414	421	428	435̄	442
623	449	456	463	470	477	484	491	498	505̄	511
624	518	525	532	539	546	553	560	567	574	581
625	588	595̄	602	609	616	623	630	637	644	650
626	657	664	671	678	685	692	699	706	713	720
627	727	734	741	748	754	761	768	775	782	789
628	796	803	810	817	824	831	837	844	851	858
629	865	872	879	886	893	900	906	913	920	927
630	934	941	948	955̄	962	969	975	982	989	996
631	80 003	010	017	024	030	037	044	051	058	065̄
632	072	079	085	092	099	106	113	120	127	134
633	140	147	154	161	168	175̄	182	188	195	202
634	209	216	223	229	236	243	250	257	264	271
635	277	284	291	298	305̄	312	318	325	332	339
636	346	353	359	366	373	380	387	393	400	407
637	414	421	428	434	441	448	455̄	462	468	475
638	482	489	496	502	509	516	523	530	536	543
639	550	557	564	570	577	584	591	598	604	611
640	618	625̄	632	638	645	652	659	665	672	679
641	686	693	699	706	713	720	726	733	740	747
642	754	760	767	774	781	787	794	801	808	814
643	821	828	835̄	841	848	855̄	862	868	875	882
644	889	895	902	909	916	922	929	936	943	949
645	956	963	969	976	983	990	996	*003	*010	*017
646	81 023	030	037	043	050	057	064	070	077	084
647	090	097	104	111	117	124	131	137	144	151
648	158	164	171	178	184	191	198	204	211	218
649	224	231	238	245̄	251	258	265̄	271	278	285̄
650	291	298	305̄	311	318	325̄	331	338	345̄	351
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

		S.′	T.′
6	6.46	373	373
7		373	373
60		370	377
63		370	377
64		370	378
65		370	378

		S.″	T.″
0° 10′ = 600″	4.68	557	558
0 11 = 660		557	558
1 40 = 6000		551	570
1 41 = 6060		551	570
1 42 = 6120		551	570
1 43 = 6180		551	570
1 44 = 6240		551	571

		S.″	T.″
1° 44′ = 6240″	4.68	551	571
1 45 = 6300		551	571
1 46 = 6360		551	571
1 47 = 6420		550	572
1 48 = 6480		550	572
1 49 = 6540		550	572

N.	L. 0	1	2	3	4	5	6	7	8	9
650	81 291	298	305	311	318	325	331	338	345	351
651	358	365	371	378	385	391	398	405	411	418
652	425	431	438	445	451	458	465	471	478	485
653	491	498	505	511	518	525	531	538	544	551
654	558	564	571	578	584	591	598	604	611	617
655	624	631	637	644	651	657	664	671	677	684
656	690	697	704	710	717	723	730	737	743	750
657	757	763	770	776	783	790	796	803	809	816
658	823	829	836	842	849	856	862	869	875	882
659	889	895	902	908	915	921	928	935	941	948
660	954	961	968	974	981	987	994	*000	*007	*014
661	82 020	027	033	040	046	053	060	066	073	079
662	086	092	099	105	112	119	125	132	138	145
663	151	158	164	171	178	184	191	197	204	210
664	217	223	230	236	243	249	256	263	269	276
665	282	289	295	302	308	315	321	328	334	341
666	347	354	360	367	373	380	387	393	400	406
667	413	419	426	432	439	445	452	458	465	471
668	478	484	491	497	504	510	517	523	530	536
669	543	549	556	562	569	575	582	588	595	601
670	607	614	620	627	633	640	646	653	659	666
671	672	679	685	692	698	705	711	718	724	730
672	737	743	750	756	763	769	776	782	789	795
673	802	808	814	821	827	834	840	847	853	860
674	866	872	879	885	892	898	905	911	918	924
675	930	937	943	950	956	963	969	975	982	988
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052
677	83 059	065	072	078	085	091	097	104	110	117
678	123	129	136	142	149	155	161	168	174	181
679	187	193	200	206	213	219	225	232	238	245
680	251	257	264	270	276	283	289	296	302	308
681	315	321	327	334	340	347	353	359	366	372
682	378	385	391	398	404	410	417	423	429	436
683	442	448	455	461	467	474	480	487	493	499
684	506	512	518	525	531	537	544	550	556	563
685	569	575	582	588	594	601	607	613	620	626
686	632	639	645	651	658	664	670	677	683	689
687	696	702	708	715	721	727	734	740	746	753
688	759	765	771	778	784	790	797	803	809	816
689	822	828	835	841	847	853	860	866	872	879
690	885	891	897	904	910	916	923	929	935	942
691	948	954	960	967	973	979	985	992	998	*004
692	84 011	017	023	029	036	042	048	055	061	067
693	073	080	086	092	098	105	111	117	123	130
694	136	142	148	155	161	167	173	180	186	192
695	198	205	211	217	223	230	236	242	248	255
696	261	267	273	280	286	292	298	305	311	317
697	323	330	336	342	348	354	361	367	373	379
698	386	392	398	404	410	417	423	429	435	442
699	448	454	460	466	473	479	485	491	497	504
700	510	516	522	528	535	541	547	553	559	566
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

		S.′	T.′
6′	6.46	373	373
7		373	373
65		370	378
69		370	378
70		370	379

		S.″	T.″
0° 10′ = 600″	4.68	557	558
0 11 = 660		557	558
0 12 = 720		557	558
1 48 = 6480		550	572
1 49 = 6540		550	572
1 50 = 6600		550	572
1 51 = 6660		550	573

		S.″	T.″
1° 51′ = 6660″	4.68	550	573
1 52 = 6720		550	573
1 53 = 6780		550	573
1 54 = 6840		550	573
1 55 = 6900		549	574
1 56 = 6960		549	574
1 57 = 7020		549	574

N.	L. 0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528	53$\bar{5}$	541	547	553	559	566
701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	66$\bar{5}$	671	677	683	689
703	696	702	708	714	720	726	733	739	74$\bar{5}$	751
704	757	763	770	776	782	788	794	800	807	813
705	819	825	831	837	844	8$\bar{5}$0	856	862	868	874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	98$\bar{5}$	991	997
708	85 003	009	016	022	028	034	040	046	052	058
709	06$\bar{5}$	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	17$\bar{5}$	181
711	187	193	199	205	211	217	224	230	236	242
712	248	254	260	266	272	278	28$\bar{5}$	291	297	303
713	309	315	321	327	333	339	345	352	358	364
714	370	376	382	388	394	400	406	412	418	42$\bar{5}$
715	431	437	443	449	45$\bar{5}$	461	467	473	479	485
716	491	497	503	509	516	522	528	534	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	62$\bar{5}$	631	637	643	649	65$\bar{5}$	661	667
719	673	679	68$\bar{5}$	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
721	794	800	806	812	818	824	830	836	842	848
722	854	860	866	872	878	884	890	896	902	908
723	914	920	926	932	938	944	9$\bar{5}$0	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	064	070	076	082	088
726	094	100	106	112	118	124	130	136	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	25$\bar{5}$	261	267
729	273	279	28$\bar{5}$	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	47$\bar{5}$	481	487	493	499	504
733	510	516	522	528	534	540	546	552	558	564
734	570	576	581	587	593	599	605	611	617	623
735	629	63$\bar{5}$	641	646	652	658	664	670	676	682
736	688	694	700	705	711	717	723	729	73$\bar{5}$	741
737	747	753	759	764	770	776	782	788	794	800
738	806	812	817	823	829	835	841	847	853	859
739	864	870	876	882	888	894	900	906	911	917
740	923	929	93$\bar{5}$	941	947	953	958	964	970	976
741	982	988	994	999	*005	*011	*017	*023	*029	*03$\bar{5}$
742	87 040	046	052	058	064	070	075	081	087	093
743	099	10$\bar{5}$	111	116	122	128	134	140	146	151
744	157	163	169	17$\bar{5}$	181	186	192	198	204	210
745	216	221	227	233	239	24$\bar{5}$	251	256	262	268
746	274	280	286	291	297	303	309	31$\bar{5}$	320	326
747	332	338	344	349	355	361	367	373	379	384
748	390	396	402	408	413	419	42$\bar{5}$	431	437	442
749	448	454	460	466	471	477	483	489	49$\bar{5}$	500
750	506	512	518	523	529	535	541	547	552	558
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	7		6		5
1	0.7	1	0.6	1	0.5
2	1.4	2	1.2	2	1.0
3	2.1	3	1.8	3	1.5
4	2.8	4	2.4	4	2.0
5	3.5	5	3.0	5	2.5
6	4.2	6	3.6	6	3.0
7	4.9	7	4.2	7	3.5
8	5.6	8	4.8	8	4.0
9	6.3	9	5.4	9	4.5

		S.′	T.′
7′	6.46	373	373
8		373	373
70		370	379
71		370	379
72		369	379
74		369	379
75		369	380

		S.″	T.″
0° 11′ = 660″	4.68	557	558
0 12 = 720		557	558
0 13 = 780		557	558
1 56 = 6960		549	574
1 57 = 7020		549	574
1 58 = 7080		549	57$\bar{5}$
1 59 = 7140		549	57$\bar{5}$

		S.″	T.″
1° 59′ = 7140″	4.68	549	57$\bar{5}$
2 0 = 7200		549	575
2 1 = 7260		549	575
2 2 = 7320		548	576
2 3 = 7380		548	576
2 4 = 7440		548	576
2 5 = 7500		548	577

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.	
750	87 506	512	518	523	529	535	541	547	552	558		
751	564	570	576	581	587	593	599	604	610	616		
752	622	628	633	639	645	651	656	662	668	674		
753	679	685	691	697	703	708	714	720	726	731		
754	737	743	749	754	760	766	772	777	783	789		
755	795	800	806	812	818	823	829	835	841	846		
756	852	858	864	869	875	881	887	892	898	904		
757	910	915	921	927	933	938	944	950	955	961		
758	967	973	978	984	990	996	*001	*007	*013	*018		
759	88 024	030	036	041	047	053	058	064	070	076		
760	081	087	093	098	104	110	116	121	127	133		**6**
761	138	144	150	156	161	167	173	178	184	190	1	0.6
762	195	201	207	213	218	224	230	235	241	247	2	1.2
763	252	258	264	270	275	281	287	292	298	304	3	1.8
764	309	315	321	326	332	338	343	349	355	360	4	2.4
765	366	372	377	383	389	395	400	406	412	417	5	3.0
766	423	429	434	440	446	451	457	463	468	474	6	3.6
767	480	485	491	497	502	508	513	519	525	530	7	4.2
768	536	542	547	553	559	564	570	576	581	587	8	4.8
769	593	598	604	610	615	621	627	632	638	643	9	5.4
770	649	655	660	666	672	677	683	689	694	700		
771	705	711	717	722	728	734	739	745	750	756		
772	762	767	773	779	784	790	795	801	807	812		
773	818	824	829	835	840	846	852	857	863	868		
774	874	880	885	891	897	902	908	913	919	925		
775	930	936	941	947	953	958	964	969	975	981		
776	986	992	997	*003	*009	*014	*020	*025	*031	*037		
777	89 042	048	053	059	064	070	076	081	087	092		
778	098	104	109	115	120	126	131	137	143	148		
779	154	159	165	170	176	182	187	193	198	204		
780	209	215	221	226	232	237	243	248	254	260		**5**
781	265	271	276	282	287	293	298	304	310	315	1	0.5
782	321	326	332	337	343	348	354	360	365	371	2	1.0
783	376	382	387	393	398	404	409	415	421	426	3	1.5
784	432	437	443	448	454	459	465	470	476	481	4	2.0
785	487	492	498	504	509	515	520	526	531	537	5	2.5
786	542	548	553	559	564	570	575	581	586	592	6	3.0
787	597	603	609	614	620	625	631	636	642	647	7	3.5
788	653	658	664	669	675	680	686	691	697	702	8	4.0
789	708	713	719	724	730	735	741	746	752	757	9	4.5
790	763	768	774	779	785	790	796	801	807	812		
791	818	823	829	834	840	845	851	856	862	867		
792	873	878	883	889	894	900	905	911	916	922		
793	927	933	938	944	949	955	960	966	971	977		
794	982	988	993	998	*004	*009	*015	*020	*026	*031		
795	90 037	042	048	053	059	064	069	075	080	086		
796	091	097	102	108	113	119	124	129	135	140		
797	146	151	157	162	168	173	179	184	189	195		
798	200	206	211	217	222	227	233	238	244	249		
799	255	260	266	271	276	282	287	293	298	304		
800	309	314	320	325	331	336	342	347	352	358		
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.	

		S.′	T.′
7′	6.46	373	373
8		373	373
75		369	380
80		369	380

			S.″	T.″
0° 12′ = 720″	4.68		557	558
0 13 = 780			557	558
0 14 = 840			557	558
2 5 = 7500			548	577
2 6 = 7560			548	577
2 7 = 7620			548	577
2 8 = 7680			547	578

		S.″	T.″
2° 8′ = 7680″	4.68	547	578
2 9 = 7740		547	578
2 10 = 7800		547	578
2 11 = 7860		547	579
2 12 = 7920		547	579
2 13 = 7980		547	579
2 14 = 8040		546	579

N.	L. 0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
801	363	369	374	380	385̄	390	396	401	407	412
802	417	423	428	434	439	445̄	450	455	461	466
803	472	477	482	488	493	499	504	509	515̄	520
804	526	531	536	542	547	553	558	563	569	574
805	580	585̄	590	596	601	607	612	617	623	628
806	634	639	644	650	655	660	666	671	677	682
807	687	693	698	703	709	714	720	725	730	736
808	741	747	752	757	763	768	773	779	784	789
809	795̄	800	806	811	816	822	827	832	838	843
810	849	854	859	865̄	870	875	881	886	891	897
811	902	907	913	918	924	929	934	940	945̄	950
812	956	961	966	972	977	982	988	993	998	*004
813	91 009	014	020	025	030	036	041	046	052	057
814	062	068	073	078	084	089	094	100	105	110
815	116	121	126	132	137	142	148	153	158	164
816	169	174	180	185̄	190	196	201	206	212	217
817	222	228	233	238	243	249	254	259	265̄	270
818	275	281	286	291	297	302	307	312	318	323
819	328	334	339	344	350	355̄	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
821	434	440	445̄	450	455	461	466	471	477	482
822	487	492	498	503	508	514	519	524	529	535̄
823	540	545	551	556	561	566	572	577	582	587
824	593	598	603	609	614	619	624	630	635̄	640
825	645	651	656	661	666	672	677	682	687	693
826	698	703	709	714	719	724	730	735̄	740	745
827	751	756	761	766	772	777	782	787	793	798
828	803	808	814	819	824	829	834	840	845̄	850
829	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	950	955̄
831	960	965	971	976	981	986	991	997	*002	*007
832	92 012	018	023	028	033	038	044	049	054	059
833	065̄	070	075̄	080	085	091	096	101	106	111
834	117	122	127	132	137	143	148	153	158	163
835	169	174	179	184	189	195̄	200	205	210	215
836	221	226	231	236	241	247	252	257	262	267
837	273	278	283	288	293	298	304	309	314	319
838	324	330	335̄	340	345	350	355	361	366	371
839	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
841	480	485̄	490	495	500	505	511	516	521	526
842	531	536	542	547	552	557	562	567	572	578
843	583	588	593	598	603	609	614	619	624	629
844	634	639	645̄	650	655̄	660	665	670	675	681
845	686	691	696	701	706	711	716	722	727	732
846	737	742	747	752	758	763	768	773	778	783
847	788	793	799	804	809	814	819	824	829	834
848	840	845̄	850	855̄	860	865	870	875	881	886
849	891	896	901	906	911	916	921	927	932	937
850	942	947	952	957	962	967	973	978	983	988
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	S.′	T.′
8′	6.46 373	373
9	373	373
80	369	380
81	369	381
82	368	381
85	368	381

	S.″	T.″
0° 13′ = 780″	4.68 557	558
0 14 = 840	557	558
0 15 = 900	557	558
2 13 = 7980	547	579
2 14 = 8040	546	579
2 15 = 8100	546	580
2 16 = 8160	546	580

	S.″	T.″
2° 16′ = 8160″	4.68 546	580
2 17 = 8220	546	580
2 18 = 8280	546	581
2 19 = 8340	546	581
2 20 = 8400	545	582
2 21 = 8460	545	582
2 22 = 8520	545	582

N.	L. 0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962	967	973	978	983	988
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039
852	93 044	049	054	059	064	069	075̄	080	085̄	090
853	095̄	100	105	110	115	120	125	131	136	141
854	146	151	156	161	166	171	176	181	186	192
855	197	202	207	212	217	222	227	232	237	242
856	247	252	258	263	268	273	278	283	288	293
857	298	303	308	313	318	323	328	334	339	344
858	349	354	359	364	369	374	379	384	389	394
859	399	404	409	414	420	425̄	430	435̄	440	445̄
860	45̄0	455̄	460	465̄	470	475	480	485	490	495
861	500	505	510	515	520	526	531	536	541	546
862	551	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	682	687	692	697
865	702	707	712	717	722	727	732	737	742	747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
871	94 002	007	012	017	022	027	032	037	042	047
872	052	057	062	067	072	077	082	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	211	216	221	226	231	236	240	245
876	250	255	260	265	270	275	280	285	290	295
877	300	305̄	310	315̄	320	325̄	330	335̄	340	345̄
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
881	498	503	507	512	517	522	527	532	537	542
882	547	552	557	562	567	571	576	581	586	591
883	596	601	606	611	616	621	626	630	635	640
884	645	650	655	660	665̄	670	675̄	680	685̄	689
885	694	699	704	709	714	719	724	729	734	738
886	743	748	753	758	763	768	773	778	783	787
887	792	797	802	807	812	817	822	827	832	836
888	841	846	851	856	861	866	871	876	880	885
889	890	895	900	905̄	910	915̄	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
891	988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95 036	041	046	051	056	061	066	071	075	080
893	085	090	095̄	100	105̄	109	114	119	124	129
894	134	139	143	148	153	158	163	168	173	177
895	182	187	192	197	202	207	211	216	221	226
896	231	236	240	245	250	255	260	265̄	270	274
897	279	284	289	294	299	303	308	313	318	323
898	328	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405̄	410	415̄	419
900	424	429	434	439	444	448	453	458	463	468
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

	S.′	T.′
8′	6.46 373	373
9	373	373
85	368	381
86	368	382
89	368	382
90	368	383

	S.″	T.″
0° 14′ = 840″	4.68 557	558
0 15 = 900	557	558
2 21 = 8460	545	582
2 22 = 8520	545	582
2 23 = 8580	545̄	583
2 24 = 8640	545̄	583
2 25 = 8700	545̄	583

	S.″	T.″
2° 25′ = 8700″	4.68 545̄	583
2 26 = 8760	544	584
2 27 = 8820	544	584
2 28 = 8880	544	584
2 29 = 8940	544	585̄
2 30 = 9000	544	585

N.	L. 0	1	2	3	4	5	6	7	8	9
900	95 424	429	434	439	444	448	453	458	463	468
901	472	477	482	487	492	497	501	506	511	516
902	521	525	530	535	540	54$\bar{5}$	5$\bar{5}$0	554	559	564
903	569	574	578	583	588	593	598	602	607	612
904	617	622	626	631	636	641	646	650	655	660
905	66$\bar{5}$	670	674	679	684	689	694	698	703	708
906	713	718	722	727	732	737	742	746	751	756
907	761	766	770	775	780	78$\bar{5}$	789	794	799	804
908	809	813	818	823	828	832	837	842	847	852
909	856	861	866	871	875	880	885	890	89$\bar{5}$	899
910	904	909	914	918	923	928	933	938	942	947
911	952	957	961	966	971	976	980	985	990	99$\bar{5}$
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042
913	96 047	052	057	061	066	071	076	080	085	090
914	09$\bar{5}$	099	104	109	114	118	123	128	133	137
915	142	147	152	156	161	166	171	175	180	18$\bar{5}$
916	190	194	199	204	209	213	218	223	227	232
917	237	242	246	251	256	261	265	270	27$\bar{5}$	280
918	284	289	294	298	303	308	313	317	322	327
919	332	336	341	346	350	355	360	36$\bar{5}$	369	374
920	379	384	388	393	398	402	407	412	417	421
921	426	431	435	440	44$\bar{5}$	4$\bar{5}$0	454	459	464	468
922	473	478	483	487	492	497	501	506	511	515
923	520	52$\bar{5}$	530	534	539	544	548	553	558	562
924	567	572	577	581	586	591	595	600	60$\bar{5}$	609
925	614	619	624	628	633	638	642	647	652	656
926	661	666	670	675	680	68$\bar{5}$	689	694	699	703
927	708	713	717	722	727	731	736	741	745	750
928	75$\bar{5}$	759	764	769	774	778	783	788	792	797
929	802	806	811	816	820	82$\bar{5}$	830	834	839	844
930	848	453	858	862	867	872	876	881	886	890
931	89$\bar{5}$	900	904	909	914	918	923	928	932	937
932	942	946	951	956	960	96$\bar{5}$	970	974	979	984
933	988	993	997	*002	*007	*011	*016	*021	*025	*030
934	97 03$\bar{5}$	039	044	049	053	058	063	067	072	077
935	081	086	090	095	100	104	109	114	118	123
936	128	132	137	142	146	151	155	160	16$\bar{5}$	169
937	174	179	183	188	192	197	202	206	211	216
938	220	22$\bar{5}$	230	234	239	243	248	253	257	262
939	267	271	276	280	285	290	294	299	304	308
940	313	317	322	327	331	336	340	345	3$\bar{5}$0	354
941	359	364	368	373	377	382	387	391	396	400
942	405	410	414	419	424	428	433	437	442	447
943	451	456	460	46$\bar{5}$	470	474	479	483	488	493
944	497	502	506	511	516	520	52$\bar{5}$	529	534	539
945	543	548	552	557	562	566	571	575	580	58$\bar{5}$
946	589	594	598	603	607	612	617	621	626	630
947	63$\bar{5}$	640	644	649	653	658	663	667	672	676
948	681	685	690	69$\bar{5}$	699	704	708	713	717	722
949	727	731	736	740	74$\bar{5}$	749	754	759	763	768
950	772	777	782	786	791	795	800	804	809	813
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

		S.′	T.′
9′	6.46	373	373
10		373	373
90		368	383
91		368	383
92		367	383
94		367	383
95		367	384

		S.″	T.″
0° 15′ = 900″	4.68	557	558
0 16 = 960		557	558
2 30 = 9000		544	585
2 31 = 9060		544	585
2 32 = 9120		543	586
2 33 = 9180		543	586
2 34 = 9240		543	587

		S.″	T.″
2° 34′ = 9240″	4.68	543	587
2 35 = 9300		543	587
2 36 = 9360		543	587
2 37 = 9420		542	588
2 38 = 9480		542	588
2 39 = 9540		542	588

N.	L. 0	1	2	3	4	5	6	7	8	9
950	97 772	777	782	786	791	795	800	804	809	813
951	818	823	827	832	836	841	845	850	85$\bar{5}$	859
952	864	868	873	877	882	886	891	896	900	90$\bar{5}$
953	909	914	918	923	928	932	937	941	946	950
954	95$\bar{5}$	959	964	968	973	978	982	987	991	996
955	98 000	00$\bar{5}$	009	014	019	023	028	032	037	041
956	046	050	05$\bar{5}$	059	064	068	073	078	082	087
957	091	096	100	10$\bar{5}$	109	114	118	123	127	132
958	137	141	146	150	15$\bar{5}$	159	164	168	173	177
959	182	186	191	195	200	204	209	214	218	223
960	227	232	236	241	245	250	254	259	263	268
961	272	277	281	286	290	29$\bar{5}$	299	304	308	313
962	318	322	327	331	336	340	34$\bar{5}$	349	354	358
963	363	367	372	376	381	385	390	394	399	403
964	408	412	417	421	426	430	43$\bar{5}$	439	444	448
965	453	457	462	466	471	475	480	484	489	493
966	498	502	507	511	516	520	52$\bar{5}$	529	534	538
967	543	547	552	556	561	565	570	574	579	583
968	588	592	597	601	605	610	614	619	623	628
969	632	637	641	646	650	65$\bar{5}$	659	664	668	673
970	677	682	686	691	695	700	704	709	713	717
971	722	726	731	735	740	744	749	753	758	762
972	767	771	776	780	784	789	793	798	802	807
973	811	816	820	82$\bar{5}$	829	834	838	843	847	851
974	856	860	86$\bar{5}$	869	874	878	883	887	892	896
975	900	90$\bar{5}$	909	914	918	923	927	932	936	941
976	94$\bar{5}$	949	954	958	963	967	972	976	981	985
977	989	994	998	*003	*007	*012	*016	*021	*025	*029
978	99 034	038	043	047	052	056	061	06$\bar{5}$	069	074
979	078	083	087	092	096	100	10$\bar{5}$	109	114	118
980	123	127	131	136	140	14$\bar{5}$	149	154	158	162
981	167	171	176	180	18$\bar{5}$	189	193	198	202	207
982	211	216	220	224	229	233	238	242	247	251
983	255	260	264	269	273	277	282	286	291	295
984	300	304	308	313	317	322	326	330	33$\bar{5}$	339
985	344	348	352	357	361	366	370	374	379	383
986	388	392	396	401	405	410	414	419	423	427
987	432	436	441	44$\bar{5}$	449	454	458	463	467	471
988	476	480	484	489	493	498	502	506	511	515
989	520	524	528	533	537	542	546	550	55$\bar{5}$	559
990	564	568	572	577	581	585	590	594	599	603
991	607	612	616	621	62$\bar{5}$	629	634	638	642	647
992	651	656	660	664	669	673	677	682	686	691
993	69$\bar{5}$	699	704	708	712	717	721	726	730	734
994	739	743	747	752	756	760	76$\bar{5}$	769	774	778
995	782	787	791	795	800	804	808	813	817	822
996	826	830	83$\bar{5}$	839	843	848	852	856	861	865
997	870	874	878	883	887	891	896	900	904	909
998	913	917	922	926	930	93$\bar{5}$	939	944	948	952
999	957	961	965	970	974	978	983	987	991	996
1000	00 000	004	009	013	017	022	026	030	03$\bar{5}$	039
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

	S.′	T.′
9′	6.46 373	373
10	373	373
95	367	384
98	367	384
99	367	38$\bar{5}$
100	366	38$\bar{5}$

	S.″	T.″
0° 15′ = 900″	4.68 557	558
0 16 = 960	557	558
0 17 = 1020	557	558
2 38 = 9480	542	588
2 39 = 9540	542	588
2 40 = 9600	542	589
2 41 = 9660	542	589

	S.″	T.″
2° 41′ = 9660″	4.68 542	589
2 42 = 9720	541	590
2 43 = 9780	541	590
2 44 = 9840	541	590
2 45 = 9900	541	591
2 46 = 9960	541	591
2 47 = 10020	540	592

N.	L. 0	1	2	3	4	5	6	7	8	9
1000	000 0000	0434	0869	1303	1737	2171	2605	3039	3473	3907
1001	4341	4775	5208	5642	6076	6510	6943	7377	7810	8244
1002	8677	9111	9544	9977	*0411	*0844	*1277	*1710	*2143	*2576
1003	001 3009	3442	3875	4308	4741	5174	5607	6039	6472	6905
1004	7337	7770	8202	8635	9067	9499	9932	*0364	*0796	*1228
1005	002 1661	2093	2525	2957	3389	3821	4253	4685	5116	5548
1006	5980	6411	6843	7275	7706	8138	8569	9001	9432	9863
1007	003 0295	0726	1157	1588	2019	2451	2882	3313	3744	4174
1008	4605	5036	5467	5898	6328	6759	7190	7620	8051	8481
1009	8912	9342	9772	*0203	*0633	*1063	*1493	*1924	*2354	*2784
1010	004 3214	3644	4074	4504	4933	5363	5793	6223	6652	7082
1011	7512	7941	8371	8800	9229	9659	*0088	*0517	*0947	*1376
1012	005 1805	2234	2663	3092	3521	3950	4379	4808	5237	5666
1013	6094	6523	6952	7380	7809	8238	8666	9094	9523	9951
1014	006 0380	0808	1236	1664	2092	2521	2949	3377	3805	4233
1015	4660	5088	5516	5944	6372	6799	7227	7655	8082	8510
1016	8937	9365	9792	*0219	*0647	*1074	*1501	*1928	*2355	*2782
1017	007 3210	3637	4064	4490	4917	5344	5771	6198	6624	7051
1018	7478	7904	8331	8757	9184	9610	*0037	*0463	*0889	*1316
1019	008 1742	2168	2594	3020	3446	3872	4298	4724	5150	5576
1020	6002	6427	6853	7279	7704	8130	8556	8981	9407	9832
1021	009 0257	0683	1108	1533	1959	2384	2809	3234	3659	4084
1022	4509	4934	5359	5784	6208	6633	7058	7483	7907	8332
1023	8756	9181	9605	*0030	*0454	*0878	*1303	*1727	*2151	*2575
1024	010 3000	3424	3848	4272	4696	5120	5544	5967	6391	6815
1025	7239	7662	8086	8510	8933	9357	9780	*0204	*0627	*1050
1026	011 1474	1897	2320	2743	3166	3590	4013	4436	4859	5282
1027	5704	6127	6550	6973	7396	7818	8241	8664	9086	9509
1028	9931	*0354	*0776	*1198	*1621	*2043	*2465	*2887	*3310	*3732
1029	012 4154	4576	4998	5420	5842	6264	6685	7107	7529	7951
1030	8372	8794	9215	9637	*0059	*0480	*0901	*1323	*1744	*2165
1031	013 2587	3008	3429	3850	4271	4692	5113	5534	5955	6376
1032	6797	7218	7639	8059	8480	8901	9321	9742	*0162	*0583
1033	014 1003	1424	1844	2264	2685	3105	3525	3945	4365	4785
1034	5205	5625	6045	6465	6885	7305	7725	8144	8564	8984
1035	9403	9823	*0243	*0662	*1082	*1501	*1920	*2340	*2759	*3178
1036	015 3598	4017	4436	4855	5274	5693	6112	6531	6950	7369
1037	7788	8206	8625	9044	9462	9881	*0300	*0718	*1137	*1555
1038	016 1974	2392	2810	3229	3647	4065	4483	4901	5319	5737
1039	6155	6573	6991	7409	7827	8245	8663	9080	9498	9916
1040	017 0333	0751	1168	1586	2003	2421	2838	3256	3673	4090
1041	4507	4924	5342	5759	6176	6593	7010	7427	7844	8260
1042	8677	9094	9511	9927	*0344	*0761	*1177	*1594	*2010	*2427
1043	018 2843	3259	3676	4092	4508	4925	5341	5757	6173	6589
1044	7005	7421	7837	8253	8669	9084	9500	9916	*0332	*0747
1045	019 1163	1578	1994	2410	2825	3240	3656	4071	4486	4902
1046	5317	5732	6147	6562	6977	7392	7807	8222	8637	9052
1047	9467	9882	*0296	*0711	*1126	*1540	*1955	*2369	*2784	*3198
1048	020 3613	4027	4442	4856	5270	5684	6099	6513	6927	7341
1049	7755	8169	8583	8997	9411	9824	*0238	*0652	*1066	*1479
1050	021 1893	2307	2720	3134	3547	3961	4374	4787	5201	5614
N.	L. 0	1	2	3	4	5	6	7	8	9

	S.″	T.″		S.″	T.″
2° 46′ = 9960″	4.68 541	591	2° 51′ = 10260″	4.68 540	593
2 47 = 10020	540	592	2 52 = 10320	539	594
2 48 = 10080	540	592	2 53 = 10380	539	594
2 49 = 10140	540	592	2 54 = 10440	539	595
2 50 = 10200	540	593	2 55 = 10500	539	595

N.	L. 0	1	2	3	4	5	6	7	8	9
1050	021 1893	2307	2720	3134	3547	3961	4374	4787	5201	5614
1051	6027	6440	6854	7267	7680	8093	8506	8919	9332	9745
1052	022 0157	0570	0983	1396	1808	2221	2634	3046	3459	3871
1053	4284	4696	5109	5521	5933	6345	6758	7170	7582	7994
1054	8406	8818	9230	9642	*0054	*0466	*0878	*1289	*1701	*2113
1055	023 2525	2936	3348	3759	4171	4582	4994	5405	5817	6228
1056	6639	7050	7462	7873	8284	8695	9106	9517	9928	*0339
1057	024 0750	1161	1572	1982	2393	2804	3214	3625	4036	4446
1058	4857	5267	5678	6088	6498	6909	7319	7729	8139	8549
1059	8960	9370	9780	*0190	*0600	*1010	*1419	*1829	*2239	*2649
1060	025 3059	3468	3878	4288	4697	5107	5516	5926	6335	6744
1061	7154	7563	7972	8382	8791	9200	9609	*0018	*0427	*0836
1062	026 1245	1654	2063	2472	2881	3289	3698	4107	4515	4924
1063	5333	5741	6150	6558	6967	7375	7783	8192	8600	9008
1064	9416	9824	*0233	*0641	*1049	*1457	*1865	*2273	*2680	*3088
1065	027 3496	3904	4312	4719	5127	5535	5942	6350	6757	7165
1066	7572	7979	8387	8794	9201	9609	*0016	*0423	*0830	*1237
1067	028 1644	2051	2458	2865	3272	3679	4086	4492	4899	5306
1068	5713	6119	6526	6932	7339	7745	8152	8558	8964	9371
1069	9777	*0183	*0590	*0996	*1402	*1808	*2214	*2620	*3026	*3432
1070	029 3838	4244	4649	5055	5461	5867	6272	6678	7084	7489
1071	7895	8300	8706	9111	9516	9922	*0327	*0732	*1138	*1543
1072	030 1948	2353	2758	3163	3568	3973	4378	4783	5188	5592
1073	5997	6402	6807	7211	7616	8020	8425	8830	9234	9638
1074	031 0043	0447	0851	1256	1660	2064	2468	2872	3277	3681
1075	4085	4489	4893	5296	5700	6104	6508	6912	7315	7719
1076	8123	8526	8930	9333	9737	*0140	*0544	*0947	*1350	*1754
1077	032 2157	2560	2963	3367	3770	4173	4576	4979	5382	5785
1078	6188	6590	6993	7396	7799	8201	8604	9007	9409	9812
1079	033 0214	0617	1019	1422	1824	2226	2629	3031	3433	3835
1080	4238	4640	5042	5444	5846	6248	6650	7052	7453	7855
1081	8257	8659	9060	9462	9864	*0265	*0667	*1068	*1470	*1871
1082	034 2273	2674	3075	3477	3878	4279	4680	5081	5482	5884
1083	6285	6686	7087	7487	7888	8289	8690	9091	9491	9892
1084	035 0293	0693	1094	1495	1895	2296	2696	3096	3497	3897
1085	4297	4698	5098	5498	5898	6298	6698	7098	7498	7898
1086	8298	8698	9098	9498	9898	*0297	*0697	*1097	*1496	*1896
1087	036 2295	2695	3094	3494	3893	4293	4692	5091	5491	5890
1088	6289	6688	7087	7486	7885	8284	8683	9082	9481	9880
1089	037 0279	0678	1076	1475	1874	2272	2671	3070	3468	3867
1090	4265	4663	5062	5460	5858	6257	6655	7053	7451	7849
1091	8248	8646	9044	9442	9839	*0237	*0635	*1033	*1431	*1829
1092	038 2226	2624	3022	3419	3817	4214	4612	5009	5407	5804
1093	6202	6599	6996	7393	7791	8188	8585	8982	9379	9776
1094	039 0173	0570	0967	1364	1761	2158	2554	2951	3348	3745
1095	4141	4538	4934	5331	5727	6124	6520	6917	7313	7709
1096	8106	8502	8898	9294	9690	*0086	*0482	*0878	*1274	*1670
1097	040 2066	2462	2858	3254	3650	4045	4441	4837	5232	5628
1098	6023	6419	6814	7210	7605	8001	8396	8791	9187	9582
1099	9977	*0372	*0767	*1162	*1557	*1952	*2347	*2742	*3137	*3532
1100	041 3927	4322	4716	5111	5506	5900	6295	6690	7084	7479
N.	L. 0	1	2	3	4	5	6	7	8	9

	S.″	T.″		S.″	T.″
2° 55′ = 10500″	4.68 539	595	3° 0′ = 10800″	4.68 538	597
2 56 = 10560	539	595	3 1 = 10860	537	598
2 57 = 10620	538	596	3 2 = 10920	537	598
2 58 = 10680	538	596	3 3 = 10980	537	599
2 59 = 10740	538	597	3 4 = 11040	537	599

3°

′	M.	S′.	T′.	Sec.	S″.	T″.
		6.46			4.68	
0	180	353	412	10800	538	597
1	181	353	413	10860	537	598
2	182	352	413	10920	537	598
3	183	352	414	10980	537	599
4	184	352	414	11040	537	599
5	185	352	415	11100	537	599
6	186	351	415	11160	536	600
7	187	351	415	11220	536	600
8	188	351	416	11280	536	601
9	189	351	416	11340	536	601
10	190	350	417	11400	535	602
11	191	350	417	11460	535	602
12	192	350	418	11520	535	603
13	193	350	418	11580	535	603
14	194	350	419	11640	534	604
15	195	349	419	11700	534	604
16	196	349	420	11760	534	605
17	197	349	420	11820	534	605
18	198	349	421	11880	533	606
19	199	348	421	11940	533	606
20	200	348	422	12000	533	607
21	201	348	422	12060	533	607
22	202	348	423	12120	532	608
23	203	347	423	12180	532	608
24	204	347	424	12240	532	609
25	205	347	424	12300	532	609
26	206	347	425	12360	531	610
27	207	346	425	12420	531	610
28	208	346	426	12480	531	611
29	209	346	426	12540	531	611
30	210	346	427	12600	530	612
31	211	345	427	12660	530	612
32	212	345	428	12720	530	613
33	213	345	428	12780	530	613
34	214	345	429	12840	529	614
35	215	344	429	12900	529	614
36	216	344	430	12960	529	615
37	217	344	430	13020	529	615
38	218	344	431	13080	528	616
39	219	343	431	13140	528	616
40	220	343	432	13200	528	617
41	221	343	432	13260	528	617
42	222	342	433	13320	527	618
43	223	342	434	13380	527	618
44	224	342	434	13440	527	619
45	225	342	435	13500	526	620
46	226	341	435	13560	526	620
47	227	341	436	13620	526	621
48	228	341	436	13680	526	621
49	229	340	437	13740	525	622
50	230	340	437	13800	525	622
51	231	340	438	13860	525	623
52	232	340	439	13920	525	623
53	233	339	439	13980	524	624
54	234	339	440	14040	524	625
55	235	339	440	14100	524	625
56	236	338	441	14160	523	626
57	237	338	441	14220	523	626
58	238	338	442	14280	523	627
59	239	338	443	14340	522	628
60	240	337	443	14400	522	628

4°

′	M.	S′.	T′.	Sec.	S″.	T″.
		6.46			4.68	
0	240	337	443	14400	522	628
1	241	337	444	14460	522	629
2	242	337	444	14520	522	629
3	243	336	445	14580	521	630
4	244	336	446	14640	521	631
5	245	336	446	14700	521	631
6	246	336	447	14760	520	632
7	247	335	447	14820	520	632
8	248	335	448	14880	520	633
9	249	335	449	14940	520	634
10	250	334	449	15000	519	634
11	251	334	450	15060	519	635
12	252	334	450	15120	519	635
13	253	333	451	15180	518	636
14	254	333	452	15240	518	637
15	255	333	452	15300	518	637
16	256	332	453	15360	517	638
17	257	332	454	15420	517	638
18	258	332	454	15480	517	639
19	259	332	455	15540	516	640
20	260	331	456	15600	516	640
21	261	331	456	15660	516	641
22	262	331	457	15720	515	642
23	263	330	457	15780	515	642
24	264	330	458	15840	515	643
25	265	330	459	15900	514	644
26	266	329	459	15960	514	644
27	267	329	460	16020	514	645
28	268	329	461	16080	513	646
29	269	328	461	16140	513	646
30	270	328	462	16200	513	647
31	271	328	463	16260	512	648
32	272	327	463	16320	512	648
33	273	327	464	16380	512	649
34	274	327	465	16440	511	650
35	275	326	465	16500	511	650
36	276	326	466	16560	511	651
37	277	326	467	16620	510	652
38	278	325	467	16680	510	652
39	279	325	468	16740	510	653
40	280	325	469	16800	509	654
41	281	324	469	16860	509	654
42	282	324	470	16920	509	655
43	283	324	471	16980	508	656
44	284	323	472	17040	508	656
45	285	323	472	17100	508	657
46	286	323	473	17160	507	658
47	287	322	474	17220	507	659
48	288	322	474	17280	507	659
49	289	321	475	17340	506	660
50	290	321	476	17400	506	661
51	291	321	477	17460	506	661
52	292	320	477	17520	505	662
53	293	320	478	17580	505	663
54	294	320	479	17640	505	664
55	295	319	479	17700	504	664
56	296	319	480	17760	504	665
57	297	319	481	17820	503	666
58	298	318	482	17880	503	666
59	299	318	482	17940	503	667
60	300	317	483	18000	502	668

TABLE XVI.

THE LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FOR EACH MINUTE.

Formulas for the Use of the Auxiliaries *S* and *T*.

1. When α is in the first five degrees of the quadrant:

$\log\sin\alpha = \log\alpha' + S.'$
$\log\tan\alpha = \log\alpha' + T.'$
$\log\cot\alpha = \text{cpl}\log\tan\alpha.$

$\log\alpha' = \log\sin\alpha + \text{cpl } S.'$
$\quad = \log\tan\alpha + \text{cpl } T.'$
$\quad = \text{cpl}\log\cot\alpha + \text{cpl } T.'$

$\log\sin\alpha = \log\alpha'' + S.''$
$\log\tan\alpha = \log\alpha'' + T.''$
$\log\cot\alpha = \text{cpl}\log\tan\alpha.$

$\log\alpha'' = \log\sin\alpha + \text{cpl } S.''$
$\quad = \log\tan\alpha + \text{cpl } T.''$
$\quad = \text{cpl}\log\cot\alpha + \text{cpl } T.''$

2. When α is in the last five degrees of the quadrant:

$\log\cos\alpha = \log(90^\circ - \alpha)' + S.'$
$\log\cot\alpha = \log(90^\circ - \alpha)' + T.'$
$\log\tan\alpha = \text{cpl}\log\cot\alpha.$

$\log(90^\circ - \alpha)' = \log\cos\alpha + \text{cpl } S.'$
$\quad = \log\cot\alpha + \text{cpl } T.'$
$\quad = \text{cpl}\log\tan\alpha + \text{cpl } T.'$

$\log\cos\alpha = \log(90^\circ - \alpha)'' + S.''$
$\log\cot\alpha = \log(90^\circ - \alpha)'' + T.''$
$\log\tan\alpha = \text{cpl}\log\cot\alpha.$

$\log(90^\circ - \alpha)'' = \log\cos\alpha + \text{cpl } S.''$
$\quad = \log\cot\alpha + \text{cpl } T.''$
$\quad = \text{cpl}\log\tan\alpha + \text{cpl } T.''$

$$\alpha = 90^\circ - (90^\circ - \alpha).$$

0°

″	′	L. Sin.	d.	Cpl. S′.	Cpl. T′.	L. Tan.	c. d.	L. Cot.	L. Cos.	
0	0	—		—	—	—		—	0.00 000	60
60	1	6.46 373	30103	3.53 627	3.53 627	6.46 373	30103	3.53 627	0.00 000	59
120	2	6.76 476	17609	3.53 627	3.53 627	6.76 476	17609	3.23 524	0.00 000	58
180	3	6.94 085	12494	3.53 627	3.53 627	6.94 085	12494	3.05 915	0.00 000	57
240	4	7.06 579	9691	3.53 627	3.53 627	7.06 579	9691	2.93 421	0.00 000	56
300	5	7.16 270	7918	3.53 627	3.53 627	7.16 270	7918	2.83 730	0.00 000	55
360	6	7.24 188	6694	3.53 627	3.53 627	7.24 188	6694	2.75 812	0.00 000	54
420	7	7.30 882	5800	3.53 627	3.53 627	7.30 882	5800	2.69 118	0.00 000	53
480	8	7.36 682	5115	3.53 627	3.53 627	7.36 682	5115	2.63 318	0.00 000	52
540	9	7.41 797	4576	3.53 627	3.53 627	7.41 797	4576	2.58 203	0.00 000	51
600	10	7.46 373	4139	3.53 627	3.53 627	7.46 373	4139	2.53 627	0.00 000	50
660	11	7.50 512	3779	3.53 627	3.53 627	7.50 512	3779	2.49 488	0.00 000	49
720	12	7.54 291	3476	3.53 627	3.53 627	7.54 291	3476	2.45 709	0.00 000	48
780	13	7.57 767	3218	3.53 627	3.53 627	7.57 767	3219	2.42 233	0.00 000	47
840	14	7.60 985	2997	3.53 628	3.53 627	7.60 986	2996	2.39 014	0.00 000	46
900	15	7.63 982	2802	3.53 628	3.53 627	7.63 982	2803	2.36 018	0.00 000	45
960	16	7.66 784	2633	3.53 628	3.53 627	7.66 785	2633	2.33 215	0.00 000	44
1020	17	7.69 417	2483	3.53 628	3.53 627	7.69 418	2482	2.30 582	9.99 999	43
1080	18	7.71 900	2348	3.53 628	3.53 627	7.71 900	2348	2.28 100	9.99 999	42
1140	19	7.74 248	2227	3.53 628	3.53 627	7.74 248	2228	2.25 752	9.99 999	41
1200	20	7.76 475	2119	3.53 628	3.53 627	7.76 476	2119	2.23 524	9.99 999	40
1260	21	7.78 594	2021	3.53 628	3.53 627	7.78 595	2020	2.21 405	9.99 999	39
1320	22	7.80 615	1930	3.53 628	3.53 627	7.80 615	1931	2.19 385	9.99 999	38
1380	23	7.82 545	1848	3.53 628	3.53 627	7.82 546	1848	2.17 454	9.99 999	37
1440	24	7.84 393	1773	3.53 628	3.53 627	7.84 394	1773	2.15 606	9.99 999	36
1500	25	7.86 166	1704	3.53 628	3.53 627	7.86 167	1704	2.13 833	9.99 999	35
1560	26	7.87 870	1639	3.53 628	3.53 627	7.87 871	1639	2.12 129	9.99 999	34
1620	27	7.89 509	1579	3.53 628	3.53 626	7.89 510	1579	2.10 490	9.99 999	33
1680	28	7.91 088	1524	3.53 628	3.53 626	7.91 089	1524	2.08 911	9.99 999	32
1740	29	7.92 612	1472	3.53 628	3.53 626	7.92 613	1473	2.07 387	9.99 998	31
1800	30	7.94 084	1424	3.53 628	3.53 626	7.94 086	1424	2.05 914	9.99 998	30
1860	31	7.95 508	1379	3.53 628	3.53 626	7.95 510	1379	2.04 490	9.99 998	29
1920	32	7.96 887	1336	3.53 628	3.53 626	7.96 889	1336	2.03 111	9.99 998	28
1980	33	7.98 223	1297	3.53 628	3.53 626	7.98 225	1297	2.01 775	9.99 998	27
2040	34	7.99 520	1259	3.53 628	3.53 626	7.99 522	1259	2.00 478	9.99 998	26
2100	35	8.00 779	1223	3.53 628	3.53 626	8.00 781	1223	1.99 219	9.99 998	25
2160	36	8.02 002	1190	3.53 628	3.53 626	8.02 004	1190	1.97 996	9.99 998	24
2220	37	8.03 192	1158	3.53 628	3.53 626	8.03 194	1159	1.96 806	9.99 997	23
2280	38	8.04 350	1128	3.53 628	3.53 626	8.04 353	1128	1.95 647	9.99 997	22
2340	39	8.05 478	1100	3.53 628	3.53 626	8.05 481	1100	1.94 519	9.99 997	21
2400	40	8.06 578	1072	3.53 628	3.53 625	8.06 581	1072	1.93 419	9.99 997	20
2460	41	8.07 650	1046	3.53 628	3.53 625	8.07 653	1047	1.92 347	9.99 997	19
2520	42	8.08 696	1022	3.53 628	3.53 625	8.08 700	1022	1.91 300	9.99 997	18
2580	43	8.09 718	999	3.53 629	3.53 625	8.09 722	998	1.90 278	9.99 997	17
2640	44	8.10 717	976	3.53 629	3.53 625	8.10 720	976	1.89 280	9.99 996	16
2700	45	8.11 693	954	3.53 629	3.53 625	8.11 696	955	1.88 304	9.99 996	15
2760	46	8.12 647	934	3.53 629	3.53 625	8.12 651	934	1.87 349	9.99 996	14
2820	47	8.13 581	914	3.53 629	3.53 625	8.13 585	915	1.86 415	9.99 996	13
2880	48	8.14 495	896	3.53 629	3.53 625	8.14 500	895	1.85 500	9.99 996	12
2940	49	8.15 391	877	3.53 629	3.53 624	8.15 395	878	1.84 605	9.99 996	11
3000	50	8.16 268	860	3.53 629	3.53 624	8.16 273	860	1.83 727	9.99 995	10
3060	51	8.17 128	843	3.53 629	3.53 624	8.17 133	843	1.82 867	9.99 995	9
3120	52	8.17 971	827	3.53 629	3.53 624	8.17 976	828	1.82 024	9.99 995	8
3180	53	8.18 798	812	3.53 629	3.53 624	8.18 804	812	1.81 196	9.99 995	7
3240	54	8.19 610	797	3.53 629	3.53 624	8.19 616	797	1.80 384	9.99 995	6
3300	55	8.20 407	782	3.53 629	3.53 624	8.20 413	782	1.79 587	9.99 994	5
3360	56	8.21 189	769	3.53 629	3.53 624	8.21 195	769	1.78 805	9.99 994	4
3420	57	8.21 958	755	3.53 629	3.53 623	8.21 964	756	1.78 036	9.99 994	3
3480	58	8.22 713	743	3.53 629	3.53 623	8.22 720	742	1.77 280	9.99 994	2
3540	59	8.23 456	730	3.53 630	3.53 623	8.23 462	730	1.76 538	9.99 994	1
3600	60	8.24 186		3.53 630	3.53 623	8.24 192		1.75 808	9.99 993	0
.		L. Cos.	d.			L. Cot.	c. d.	L. Tan.	L. Sin.	′

89°

″	′	L. Sin.	d.	Cpl. S′.	Cpl. T′.	L. Tan.	c. d.	L. Cot.	L. Cos.	
3600	0	8.24 186	717	3.53 630	3.53 623	8.24 192	718	1.75 808	9.99 993	60
3660	1	8.24 903	706	3.53 630	3.53 623	8.24 910	706	1.75 090	9.99 993	59
3720	2	8.25 609	695	3.53 630	3.53 623	8.25 616	696	1.74 384	9.99 993	58
3780	3	8.26 304	684	3.53 630	3.53 623	8.26 312	684	1.73 688	9.99 993	57
3840	4	8.26 988	673	3.53 630	3.53 622	8.26 996	673	1.73 004	9.99 992	56
3900	5	8.27 661	663	3.53 630	3.53 622	8.27 669	663	1.72 331	9.99 992	55
3960	6	8.28 324	653	3.53 630	3.53 622	8.28 332	654	1.71 668	9.99 992	54
4020	7	8.28 977	644	3.53 630	3.53 622	8.28 986	643	1.71 014	9.99 992	53
4080	8	8.29 621	634	3.53 630	3.53 622	8.29 629	634	1.70 371	9.99 992	52
4140	9	8.30 25$\bar{5}$	624	3.53 630	3.53 622	8.30 263	625	1.69 737	9.99 991	51
4200	**10**	8.30 879	616	3.53 630	3.53 621	8.30 888	617	1.69 112	9.99 991	**50**
4260	11	8.31 495	608	3.53 630	3.53 621	8.31 50$\bar{5}$	607	1.68 495	9.99 991	49
4320	12	8.32 103	599	3.53 631	3.53 621	8.32 112	599	1.67 888	9.99 990	48
4380	13	8.32 702	590	3.53 631	3.53 621	8.32 711	591	1.67 289	9.99 990	47
4440	14	8.33 292	583	3.53 631	3.53 621	8.33 302	584	1.66 698	9.99 990	46
4500	15	8.33 875	575	3.53 631	3.53 620	8.33 886	575	1.66 114	9.99 990	45
4560	16	8.34 450	568	3.53 631	3.53 620	8.34 461	568	1.65 539	9.99 989	44
4620	17	8.35 018	560	3.53 631	3.53 620	8.35 029	561	1.64 971	9.99 989	43
4680	18	8.35 578	553	3.53 631	3.53 620	8.35 590	553	1.64 410	9.99 989	42
4740	19	8.36 131	547	3.53 631	3.53 620	8.36 143	546	1.63 857	9.99 989	41
4800	**20**	8.36 678	539	3.53 631	3.53 620	8.36 689	540	1.63 311	9.99 988	**40**
4860	21	8.37 217	533	3.53 631	3.53 619	8.37 229	533	1.62 771	9.99 988	39
4920	22	8.37 7$\bar{5}$0	526	3.53 632	3.53 619	8.37 762	527	1.62 238	9.99 988	38
4980	23	8.38 276	520	3.53 632	3.53 619	8.38 289	520	1.61 711	9.99 987	37
5040	24	8.38 796	514	3.53 632	3.53 619	8.38 809	514	1.61 191	9.99 987	36
5100	25	8.39 310	508	3.53 632	3.53 619	8.39 323	509	1.60 677	9.99 987	35
5160	26	8.39 818	502	3.53 632	3.53 618	8.39 832	502	1.60 168	9.99 986	34
5220	27	8.40 320	496	3.53 632	3.53 618	8.40 334	496	1.59 666	9.99 986	33
5280	28	8.40 816	491	3.53 632	3.53 618	8.40 830	491	1.59 170	9.99 986	32
5340	29	8.41 307	485	3.53 632	3.53 618	8.41 321	486	1.58 679	9.99 985	31
5400	**30**	8.41 792	480	3.53 632	3.53 617	8.41 807	480	1.58 193	9.99 985	**30**
5460	31	8.42 272	474	3.53 632	3.53 617	8.42 287	475	1.57 713	9.99 98$\bar{5}$	29
5520	32	8.42 746	470	3.53 633	3.53 617	8.42 762	470	1.57 238	9.99 984	28
5580	33	8.43 216	464	3.53 633	3.53 617	8.43 232	464	1.56 768	9.99 984	27
5640	34	8.43 680	459	3.53 633	3.53 617	8.43 696	460	1.56 304	9.99 984	26
5700	35	8.44 139	455	3.53 633	3.53 616	8.44 156	455	1.55 844	9.99 983	25
5760	36	8.44 594	450	3.53 633	3.53 616	8.44 611	450	1.55 389	9.99 983	24
5820	37	8.45 044	445	3.53 633	3.53 616	8.45 061	446	1.54 939	9.99 983	23
5880	38	8.45 489	441	3.53 633	3.53 616	8.45 507	441	1.54 493	9.99 982	22
5940	39	8.45 930	436	3.53 633	3.53 615	8.45 948	437	1.54 052	9.99 982	21
6000	**40**	8.46 366	433	3.53 634	3.53 615	8.46 38$\bar{5}$	432	1.53 615	9.99 982	**20**
6060	41	8.46 799	427	3.53 634	3.53 61$\bar{5}$	8.46 817	428	1.53 183	9.99 981	19
6120	42	8.47 226	424	3.53 634	3.53 61$\bar{5}$	8.47 245	424	1.52 75$\bar{5}$	9.99 981	18
6180	43	8.47 6$\bar{5}$0	419	3.53 634	3.53 614	8.47 669	420	1.52 331	9.99 981	17
6240	44	8.48 069	416	3.53 634	3.53 614	8.48 089	416	1.51 911	9.99 980	16
6300	45	8.48 48$\bar{5}$	411	3.53 634	3.53 614	8.48 505	412	1.51 49$\bar{5}$	9.99 980	15
6360	46	8.48 896	408	3.53 634	3.53 614	8.48 917	408	1.51 083	9.99 979	14
6420	47	8.49 304	404	3.53 634	3.53 613	8.49 325	404	1.50 67$\bar{5}$	9.99 979	13
6480	48	8.49 708	400	3.53 63$\bar{5}$	3.53 613	8.49 729	401	1.50 271	9.99 979	12
6540	49	8.50 108	396	3.53 63$\bar{5}$	3.53 613	8.50 130	397	1.49 870	9.99 978	11
6600	**50**	8.50 504	393	3.53 63$\bar{5}$	3.53 613	8.50 527	393	1.49 473	9.99 978	**10**
6660	51	8.50 897	390	3.53 63$\bar{5}$	3.53 612	8.50 920	390	1.49 080	9.99 977	9
6720	52	8.51 287	386	3.53 635	3.53 612	8.51 310	386	1.48 690	9.99 977	8
6780	53	8.51 673	382	3.53 635	3.53 612	8.51 696	383	1.48 304	9.99 977	7
6840	54	8.52 055	379	3.53 635	3.53 611	8.52 079	380	1.47 921	9.99 976	6
6900	55	8.52 434	376	3.53 635	3.53 611	8.52 459	376	1.47 541	9.99 976	5
6960	56	8.52 810	373	3.53 636	3.53 611	8.52 83$\bar{5}$	373	1.47 165	9.99 975	4
7020	57	8.53 183	369	3.53 636	3.53 611	8.53 208	370	1.46 792	9.99 97$\bar{5}$	3
7080	58	8.53 552	367	3.53 636	3.53 610	8.53 578	367	1.46 422	9.99 974	2
7140	59	8.53 919	363	3.53 636	3.53 610	8.53 94$\bar{5}$	363	1.46 055	9.99 974	1
7200	**60**	8.54 282		3.53 636	3.53 610	8.54 308		1.45 692	9.99 974	**0**
		L. Cos.	d.			L. Cot.	c. d.	L. Tan.	L. Sin.	′

2°

"	'	L. Sin.	d.	Cpl. S'.	Cpl. T'.	L. Tan.	c. d.	L. Cot.	L. Cos.	
7200	**0**	8.54 282	360	3.53 636	3.53 610	8.54 308	361	1.45 692	9.99 974	**60**
7260	1	8.54 642	357	3.53 636	3.53 609	8.54 669	358	1.45 331	9.99 973	59
7320	2	8.54 999	355	3.53 637	3.53 609	8.55 027	355	1.44 973	9.99 973	58
7380	3	8.55 354	351	3.53 637	3.53 609	8.55 382	352	1.44 618	9.99 972	57
7440	4	8.55 705	349	3.53 637	3.53 609	8.55 734	349	1.44 266	9.99 972	56
7500	5	8.56 054	346	3.53 637	3.53 608	8.56 083	346	1.43 917	9.99 971	55
7560	6	8.56 400	343	3.53 637	3.53 608	8.56 429	344	1.43 571	9.99 971	54
7620	7	8.56 743	341	3.53 637	3.53 608	8.56 773	341	1.43 227	9.99 970	53
7680	8	8.57 084	337	3.53 637	3.53 607	8.57 114	338	1.42 886	9.99 970	52
7740	9	8.57 421	336	3.53 638	3.53 607	8.57 452	336	1.42 548	9.99 969	51
7800	**10**	8.57 757	332	3.53 638	3.53 607	8.57 788	333	1.42 212	9.99 969	**50**
7860	11	8.58 089	330	3.53 638	3.53 606	8.58 121	330	1.41 879	9.99 968	49
7920	12	8.58 419	328	3.53 638	3.53 606	8.58 451	328	1.41 549	9.99 968	48
7980	13	8.58 747	325	3.53 638	3.53 606	8.58 779	326	1.41 221	9.99 967	47
8040	14	8.59 072	323	3.53 638	3.53 605	8.59 105	323	1.40 895	9.99 967	46
8100	15	8.59 395	320	3.53 639	3.53 605	8.59 428	321	1.40 572	9.99 967	45
8160	16	8.59 715	318	3.53 639	3.53 605	8.59 749	319	1.40 251	9.99 966	44
8220	17	8.60 033	316	3.53 639	3.53 604	8.60 068	316	1.39 932	9.99 966	43
8280	18	8.60 349	313	3.53 639	3.53 604	8.60 384	314	1.39 616	9.99 965	42
8340	19	8.60 662	311	3.53 639	3.53 604	8.60 698	311	1.39 302	9.99 964	41
8400	**20**	8.60 973	309	3.53 639	3.53 603	8.61 009	310	1.38 991	9.99 964	**40**
8460	21	8.61 282	307	3.53 640	3.53 603	8.61 319	307	1.38 681	9.99 963	39
8520	22	8.61 589	305	3.53 640	3.53 603	8.61 626	305	1.38 374	9.99 963	38
8580	23	8.61 894	302	3.53 640	3.53 602	8.61 931	303	1.38 069	9.99 962	37
8640	24	8.62 196	301	3.53 640	3.53 602	8.62 234	301	1.37 766	9.99 962	36
8700	25	8.62 497	298	3.53 640	3.53 602	8.62 535	299	1.37 465	9.99 961	35
8760	26	8.62 795	296	3.53 640	3.53 601	8.62 834	297	1.37 166	9.99 961	34
8820	27	8.63 091	294	3.53 641	3.53 601	8.63 131	295	1.36 869	9.99 960	33
8880	28	8.63 385	293	3.53 641	3.53 601	8.63 426	292	1.36 574	9.99 960	32
8940	29	8.63 678	290	3.53 641	3.53 600	8.63 718	291	1.36 282	9.99 959	31
9000	**30**	8.63 968	288	3.53 641	3.53 600	8.64 009	289	1.35 991	9.99 959	**30**
9060	31	8.64 256	287	3.53 641	3.53 599	8.64 298	287	1.35 702	9.99 958	29
9120	32	8.64 543	284	3.53 642	3.53 599	8.64 585	285	1.35 415	9.99 958	28
9180	33	8.64 827	283	3.53 642	3.53 599	8.64 870	284	1.35 130	9.99 957	27
9240	34	8.65 110	281	3.53 642	3.53 598	8.65 154	281	1.34 846	9.99 956	26
9300	35	8.65 391	279	3.53 642	3.53 598	8.65 435	280	1.34 565	9.99 956	25
9360	36	8.65 670	277	3.53 642	3.53 598	8.65 715	278	1.34 285	9.99 955	24
9420	37	8.65 947	276	3.53 642	3.53 597	8.65 993	276	1.34 007	9.99 955	23
9480	38	8.66 223	274	3.53 643	3.53 597	8.66 269	274	1.33 731	9.99 954	22
9540	39	8.66 497	272	3.53 643	3.53 596	8.66 543	273	1.33 457	9.99 954	21
9600	**40**	8.66 769	270	3.53 643	3.53 596	8.66 816	271	1.33 184	9.99 953	**20**
9660	41	8.67 039	269	3.53 643	3.53 596	8.67 087	269	1.32 913	9.99 952	19
9720	42	8.67 308	267	3.53 643	3.53 595	8.67 356	268	1.32 644	9.99 952	18
9780	43	8.67 575	266	3.53 644	3.53 595	8.67 624	266	1.32 376	9.99 951	17
9840	44	8.67 841	263	3.53 644	3.53 594	8.67 890	264	1.32 110	9.99 951	16
9900	45	8.68 104	263	3.53 644	3.53 594	8.68 154	263	1.31 846	9.99 950	15
9960	46	8.68 367	260	3.53 644	3.53 594	8.68 417	261	1.31 583	9.99 949	14
10020	47	8.68 627	259	3.53 644	3.53 593	8.68 678	260	1.31 322	9.99 949	13
10080	48	8.68 886	258	3.53 645	3.53 593	8.68 938	258	1.31 062	9.99 948	12
10140	49	8.69 144	256	3.53 645	3.53 592	8.69 196	257	1.30 804	9.99 948	11
10200	**50**	8.69 400	254	3.53 645	3.53 592	8.69 453	255	1.30 547	9.99 947	**10**
10260	51	8.69 654	253	3.53 645	3.53 592	8.69 708	254	1.30 292	9.99 946	9
10320	52	8.69 907	252	3.53 646	3.53 591	8.69 962	252	1.30 038	9.99 946	8
10380	53	8.70 159	250	3.53 646	3.53 591	8.70 214	251	1.29 786	9.99 945	7
10440	54	8.70 409	249	3.53 646	3.53 590	8.70 465	249	1.29 535	9.99 944	6
10500	55	8.70 658	247	3.53 646	3.53 590	8.70 714	248	1.29 286	9.99 944	5
10560	56	8.70 905	246	3.53 646	3.53 589	8.70 962	246	1.29 038	9.99 943	4
10620	57	8.71 151	244	3.53 647	3.53 589	8.71 208	245	1.28 792	9.99 942	3
10680	58	8.71 395	243	3.53 647	3.53 589	8.71 453	244	1.28 547	9.99 942	2
10740	59	8.71 638	242	3.53 647	3.53 588	8.71 697	243	1.28 303	9.99 941	1
10800	**60**	8.71 880		3.53 647	3.53 588	8.71 940		1.28 060	9.99 940	**0**
		L. Cos.	d.			L. Cot.	c. d.	L. Tan.	L. Sin.	'

87°

′	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	
0	8.71 880	240	8.71 940	241	1.28 060	9.99 940	60
1	8.72 120	239	8.72 181	239	1.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	1.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	1.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	1.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	1.26 868	9.99 937	55
6	8.73 303	232	8.73 366	234	1.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	1.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	1.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	1.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	1.25 708	9.99 934	50
11	8.74 454	226	8.74 521	227	1.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	1.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	1.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	1.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	1.24 577	9.99 930	45
16	8.75 575	220	8.75 645	222	1.24 355	9.99 929	44
17	8.75 795	220	8.75 867	220	1.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	1.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	1.23 694	9.99 927	41
20	8.76 451	216	8.76 525	217	1.23 475	9.99 926	40
21	8.76 667	216	8.76 742	216	1.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	1.23 042	9.99 925	38
23	8.77 097	213	8.77 173	214	1.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	1.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	1.22 400	9.99 923	35
26	8.77 733	210	8.77 811	211	1.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	1.21 559	9.99 920	31
30	8.78 568	206	8.78 649	206	1.21 351	9.99 919	30
31	8.78 774	205	8.78 855	206	1.21 145	9.99 918	29
32	8.78 979	204	8.79 061	205	1.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	1.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	1.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	1.20 327	9.99 915	25
36	8.79 789	201	8.79 875	201	1.20 125	9.99 914	24
37	8.79 990	199	8.80 076	201	1.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	1.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	1.19 326	9.99 911	20
41	8.80 782	196	8.80 872	196	1.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	1.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	1.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	1.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	1.18 347	9.99 907	15
46	8.81 752	192	8.81 846	192	1.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	1.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	1.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	1.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	1.17 390	9.99 903	10
51	8.82 701	187	8.82 799	188	1.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	1.17 013	9.99 901	8
53	8.83 075	186	8.83 175	186	1.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	1.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	1.16 453	9.99 898	5
56	8.83 630	183	8.83 732	184	1.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	1.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 718	9.99 895	1
60	8.84 358		8.84 464		1.15 536	9.99 894	0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	′

P. P.

	241	239	237	236	234
1	24.1	23.9	23.7	23.6	23.4
2	48.2	47.8	47.4	47.2	46.8
3	72.3	71.7	71.1	70.8	70.2
4	96.4	95.6	94.8	94.4	93.6
5	120.5	119.5	118.5	118.0	117.0
6	144.6	143.4	142.2	141.6	140.4
7	168.7	167.3	165.9	165.2	163.8
8	192.8	191.2	189.6	188.8	187.2
9	216.9	215.1	213.3	212.4	210.6

	232	231	229	227	226
1	23.2	23.1	22.9	22.7	22.6
2	46.4	46.2	45.8	45.4	45.2
3	69.6	69.3	68.7	68.1	67.8
4	92.8	92.4	91.6	90.8	90.4
5	116.0	115.5	114.5	113.5	113.0
6	139.2	138.6	137.4	136.2	135.6
7	162.4	161.7	160.3	158.9	158.2
8	185.6	184.8	183.2	181.6	180.8
9	208.8	207.9	206.1	204.3	203.4

	224	222	220	219	217
1	22.4	22.2	22.0	21.9	21.7
2	44.8	44.4	44.0	43.8	43.4
3	67.2	66.6	66.0	65.7	65.1
4	89.6	88.8	88.0	87.6	86.8
5	112.0	111.0	110.0	109.5	108.5
6	134.4	133.2	132.0	131.4	130.2
7	156.8	155.4	154.0	153.3	151.9
8	179.2	177.6	176.0	175.2	173.6
9	201.6	199.8	198.0	197.1	195.3

	216	214	213	211	209
1	21.6	21.4	21.3	21.1	20.9
2	43.2	42.8	42.6	42.2	41.8
3	64.8	64.2	63.9	63.3	62.7
4	86.4	85.6	85.2	84.4	83.6
5	108.0	107.0	106.5	105.5	104.5
6	129.6	128.4	127.8	126.6	125.4
7	151.2	149.8	149.1	147.7	146.3
8	172.8	171.2	170.4	168.8	167.2
9	194.4	192.6	191.7	189.9	188.1

	208	206	203	201	199
1	20.8	20.6	20.3	20.1	19.9
2	41.6	41.2	40.6	40.2	39.8
3	62.4	61.8	60.9	60.3	59.7
4	83.2	82.4	81.2	80.4	79.6
5	104.0	103.0	101.5	100.5	99.5
6	124.8	123.6	121.8	120.6	119.4
7	145.6	144.2	142.1	140.7	139.3
8	166.4	164.8	162.4	160.8	159.2
9	187.2	185.4	182.7	180.9	179.1

	198	196	194	192	190
1	19.8	19.6	19.4	19.2	19.0
2	39.6	39.2	38.8	38.4	38.0
3	59.4	58.8	58.2	57.6	57.0
4	79.2	78.4	77.6	76.8	76.0
5	99.0	98.0	97.0	96.0	95.0
6	118.8	117.6	116.4	115.2	114.0
7	138.6	137.2	135.8	134.4	133.0
8	158.4	156.8	155.2	153.6	152.0
9	178.2	176.4	174.6	172.8	171.0

	188	186	184	182	181
1	18.8	18.6	18.4	18.2	18.1
2	37.6	37.2	36.8	36.4	36.2
3	56.4	55.8	55.2	54.6	54.3
4	75.2	74.4	73.6	72.8	72.4
5	94.0	93.0	92.0	91.0	90.5
6	112.8	111.6	110.4	109.2	108.6
7	131.6	130.2	128.8	127.4	126.7
8	150.4	148.8	147.2	145.6	144.8
9	169.2	167.4	165.6	163.8	162.9

P. P.

4°

′	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	′
0	8.84 358	181	8.84 464	182	1.15 536	9.99 894	60
1	8.84 539	179	8.84 646	180	1.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	1.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	1.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	1.14 637	9.99 890	55
6	8.85 429	176	8.85 540	177	1.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	1.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	50
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	1.12 894	9.99 880	45
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	1.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	1.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	1.12 047	9.99 876	40
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	1.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	1.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	1.11 217	9.99 871	35
26	8.88 817	163	8.88 948	163	1.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	1.10 563	9.99 867	31
30	8.89 464	161	8.89 598	162	1.10 402	9.99 866	30
31	8.89 625	159	8.89 760	160	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	1.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	1.09 601	9.99 861	25
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24
37	8.90 574	156	8.90 715	157	1.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	1.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	1.08 971	9.99 857	21
40	8.91 040	155	8.91 185	155	1.08 815	9.99 856	20
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	1.08 505	9.99 854	18
43	8.91 502	153	8.91 650	153	1.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	1.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	1.08 043	9.99 851	15
46	8.91 959	151	8.92 110	152	1.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	1.07 284	9.99 845	10
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	1.06 984	9.99 843	8
53	8.93 007	147	8.93 165	148	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	1.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	1.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	1.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	1.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	1.05 951	9.99 836	1
60	8.94 030		8.94 195		1.05 805	9.99 834	0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	′

P. P.

	182	181	180	179	178
1	18.2	18.1	18.0	17.9	17.8
2	36.4	36.2	36.0	35.8	35.6
3	54.6	54.3	54.0	53.7	53.4
4	72.8	72.4	72.0	71.6	71.2
5	91.0	90.5	90.0	89.5	89.0
6	109.2	108.6	108.0	107.4	106.8
7	127.4	126.7	126.0	125.3	124.6
8	145.6	144.8	144.0	143.2	142.4
9	163.8	162.9	162.0	161.1	160.2

	177	176	175	174	173
1	17.7	17.6	17.5	17.4	17.3
2	35.4	35.2	35.0	34.8	34.6
3	53.1	52.8	52.5	52.2	51.9
4	70.8	70.4	70.0	69.6	69.2
5	88.5	88.0	87.5	87.0	86.5
6	106.2	105.6	105.0	104.4	103.8
7	123.9	123.2	122.5	121.8	121.1
8	141.6	140.8	140.0	139.2	138.4
9	159.3	158.4	157.5	156.6	155.7

	172	171	170	169	168
1	17.2	17.1	17.0	16.9	16.8
2	34.4	34.2	34.0	33.8	33.6
3	51.6	51.3	51.0	50.7	50.4
4	68.8	68.4	68.0	67.6	67.2
5	86.0	85.5	85.0	84.5	84.0
6	103.2	102.6	102.0	101.4	100.8
7	120.4	119.7	119.0	118.3	117.6
8	137.6	136.8	136.0	135.2	134.4
9	154.8	153.9	153.0	152.1	151.2

	167	166	165	164	163
1	16.7	16.6	16.5	16.4	16.3
2	33.4	33.2	33.0	32.8	32.6
3	50.1	49.8	49.5	49.2	48.9
4	66.8	66.4	66.0	65.6	65.2
5	83.5	83.0	82.5	82.0	81.5
6	100.2	99.6	99.0	98.4	97.8
7	116.9	116.2	115.5	114.8	114.1
8	133.6	132.8	132.0	131.2	130.4
9	150.3	149.4	148.5	147.6	146.7

	162	161	160	159	158
1	16.2	16.1	16.0	15.9	15.8
2	32.4	32.2	32.0	31.8	31.6
3	48.6	48.3	48.0	47.7	47.4
4	64.8	64.4	64.0	63.6	63.2
5	81.0	80.5	80.0	79.5	79.0
6	97.2	96.6	96.0	95.4	94.8
7	113.4	112.7	112.0	111.3	110.6
8	129.6	128.8	128.0	127.2	126.4
9	145.8	144.9	144.0	143.1	142.2

	157	156	155	154	153
1	15.7	15.6	15.5	15.4	15.3
2	31.4	31.2	31.0	30.8	30.6
3	47.1	46.8	46.5	46.2	45.9
4	62.8	62.4	62.0	61.6	61.2
5	78.5	78.0	77.5	77.0	76.5
6	94.2	93.6	93.0	92.4	91.8
7	109.9	109.2	108.5	107.8	107.1
8	125.6	124.8	124.0	123.2	122.4
9	141.3	140.4	139.5	138.6	137.7

	152	151	150	149	148
1	15.2	15.1	15.0	14.9	14.8
2	30.4	30.2	30.0	29.8	29.6
3	45.6	45.3	45.0	44.7	44.4
4	60.8	60.4	60.0	59.6	59.2
5	76.0	75.5	75.0	74.5	74.0
6	91.2	90.6	90.0	89.4	88.8
7	106.4	105.7	105.0	104.3	103.6
8	121.6	120.8	120.0	119.2	118.4
9	136.8	135.9	135.0	134.1	133.2

P. P.

85°

′	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	
0	8.94 030	144	8.94 195	145	1.05 805	9.99 834	**60**
1	8.94 174	143	8.94 340	145	1.05 660	9.99 833	59
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52
9	8.95 310	140	8.95 486	141	1.04 514	9.99 824	51
10	8.95 450	139	8.95 627	140	1.04 373	9.99 823	**50**
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49
12	8.95 728	139	8.95 908	139	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	1.03 953	9.99 820	47
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	**40**
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	**30**
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795	26
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23
38	8.99 194	128	8.99 405	129	1.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21
40	8.99 450	127	8.99 662	129	1.00 338	9.99 787	**20**
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785	18
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	**10**
51	9.00 828	123	9.01 055	124	0.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1
60	9.01 923		9.02 162		0.97 838	9.99 761	**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	′

P. P.

	147	146	145	144
1	14.7	14.6	14.5	14.4
2	29.4	29.2	29.0	28.8
3	44.1	43.8	43.5	43.2
4	58.8	58.4	58.0	57.6
5	73.5	73.0	72.5	72.0
6	88.2	87.6	87.0	86.4
7	102.9	102.2	101.5	100.8
8	117.6	116.8	116.0	115.2
9	132.3	131.4	130.5	129.6

	143	142	141	140
1	14.3	14.2	14.1	14.0
2	28.6	28.4	28.2	28.0
3	42.9	42.6	42.3	42.0
4	57.2	56.8	56.4	56.0
5	71.5	71.0	70.5	70.0
6	85.8	85.2	84.6	84.0
7	100.1	99.4	98.7	98.0
8	114.4	113.6	112.8	112.0
9	128.7	127.8	126.9	126.0

	139	138	137	136
1	13.9	13.8	13.7	13.6
2	27.8	27.6	27.4	27.2
3	41.7	41.4	41.1	40.8
4	55.6	55.2	54.8	54.4
5	69.5	69.0	68.5	68.0
6	83.4	82.8	82.2	81.6
7	97.3	96.6	95.9	95.2
8	111.2	110.4	109.6	108.8
9	125.1	124.2	123.3	122.4

	135	134	133	132
1	13.5	13.4	13.3	13.2
2	27.0	26.8	26.6	26.4
3	40.5	40.2	39.9	39.6
4	54.0	53.6	53.2	52.8
5	67.5	67.0	66.5	66.0
6	81.0	80.4	79.8	79.2
7	94.5	93.8	93.1	92.4
8	108.0	107.2	106.4	105.6
9	121.5	120.6	119.7	118.8

	131	130	129	128
1	13.1	13.0	12.9	12.8
2	26.2	26.0	25.8	25.6
3	39.3	39.0	38.7	38.4
4	52.4	52.0	51.6	51.2
5	65.5	65.0	64.5	64.0
6	78.6	78.0	77.4	76.8
7	91.7	91.0	90.3	89.6
8	104.8	104.0	103.2	102.4
9	117.9	117.0	116.1	115.2

	127	126	125	124
1	12.7	12.6	12.5	12.4
2	25.4	25.2	25.0	24.8
3	38.1	37.8	37.5	37.2
4	50.8	50.4	50.0	49.6
5	63.5	63.0	62.5	62.0
6	76.2	75.6	75.0	74.4
7	88.9	88.2	87.5	86.8
8	101.6	100.8	100.0	99.2
9	114.3	113.4	112.5	111.6

	123	122	121	120
1	12.3	12.2	12.1	12.0
2	24.6	24.4	24.2	24.0
3	36.9	36.6	36.3	36.0
4	49.2	48.8	48.4	48.0
5	61.5	61.0	60.5	60.0
6	73.8	73.2	72.6	72.0
7	86.1	85.4	84.7	84.0
8	98.4	97.6	96.8	96.0
9	110.7	109.8	108.9	108.0

P. P.

′	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	60
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	0.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755	55
6	9.02 639	118	9.02 885	120	0.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	0.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	50
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	0.96 052	9.99 741	45
16	9.03 805	115	9.04 065	116	0.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	0.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	0.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	0.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	40
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	0.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	30
31	9.05 497	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	0.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	0.93 776	9.99 713	25
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	0.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	0.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	0.93 334	9.99 707	21
40	9.06 481	108	9.06 775	110	0.93 225	9.99 705	20
41	9.06 589	107	9.06 885	109	0.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	0.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	0.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	0.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	0.92 680	9.99 698	15
46	9.07 124	107	9.07 428	108	0.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	0.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	0.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	0.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	0.92 142	9.99 690	10
51	9.07 653	105	9.07 964	107	0.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	0.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	5
56	9.08 176	104	9.08 495	105	0.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	0.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	0.91 190	9.99 677	1
60	9.08 589		9.08 914		0.91 086	9.99 675	0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	′

P. P.

	121	120	119	118
1	12.1	12.0	11.9	11.8
2	24.2	24.0	23.8	23.6
3	36.3	36.0	35.7	35.4
4	48.4	48.0	47.6	47.2
5	60.5	60.0	59.5	59.0
6	72.6	72.0	71.4	70.8
7	84.7	84.0	83.3	82.6
8	96.8	96.0	95.2	94.4
9	108.9	108.0	107.1	106.2

	117	116	115	114
1	11.7	11.6	11.5	11.4
2	23.4	23.2	23.0	22.8
3	35.1	34.8	34.5	34.2
4	46.8	46.4	46.0	45.6
5	58.5	58.0	57.5	57.0
6	70.2	69.6	69.0	68.4
7	81.9	81.2	80.5	79.8
8	93.6	92.8	92.0	91.2
9	105.3	104.4	103.5	102.6

	113	112	111	110
1	11.3	11.2	11.1	11.0
2	22.6	22.4	22.2	22.0
3	33.9	33.6	33.3	33.0
4	45.2	44.8	44.4	44.0
5	56.5	56.0	55.5	55.0
6	67.8	67.2	66.6	66.0
7	79.1	78.4	77.7	77.0
8	90.4	89.6	88.8	88.0
9	101.7	100.8	99.9	99.0

	109	108	107	106
1	10.9	10.8	10.7	10.6
2	21.8	21.6	21.4	21.2
3	32.7	32.4	32.1	31.8
4	43.6	43.2	42.8	42.4
5	54.5	54.0	53.5	53.0
6	65.4	64.8	64.2	63.6
7	76.3	75.6	74.9	74.2
8	87.2	86.4	85.6	84.8
9	98.1	97.2	96.3	95.4

	105	104	103
1	10.5	10.4	10.3
2	21.0	20.8	20.6
3	31.5	31.2	30.9
4	42.0	41.6	41.2
5	52.5	52.0	51.5
6	63.0	62.4	61.8
7	73.5	72.8	72.1
8	84.0	83.2	82.4
9	94.5	93.6	92.7

P. P.

′	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	
0	9.08 589	103	9.08 914	105	0.91 086	9.99 675	**60**
1	9.08 692	103	9.09 019	104	0.90 981	9.99 674	59
2	9.08 795	102	9.09 123	104	0.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	0.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	0.90 566	9.99 667	55
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	0.90 360	9.99 664	53
8	9.09 405	101	9.09 742	103	0.90 258	9.99 663	52
9	9.09 506	100	9.09 845	102	0.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	**50**
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	0.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	0.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	45
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	**40**
21	9.10 697	98	9.11 056	99	0.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38
23	9.10 893	97	9.11 254	99	0.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	35
26	9.11 184	97	9.11 551	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	0.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	0.88 155	9.99 629	31
30	9.11 570	96	9.11 943	97	0.88 057	9.99 627	**30**
31	9.11 666	95	9.12 040	98	0.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	0.87 862	9.99 624	28
33	9.11 857	95	9.12 235	97	0.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	0.87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	0.87 572	9.99 618	25
36	9.12 142	94	9.12 525	96	0.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	0.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	0.87 091	9.99 610	**20**
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	15
46	9.13 078	93	9.13 478	95	0.86 522	9.99 600	14
47	9.13 171	92	9.13 573	94	0.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	0.86 239	9.99 595	11
50	9.13 447	92	9.13 854	94	0.86 146	9.99 593	**10**
51	9.13 539	91	9.13 948	93	0.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	0.85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	0.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	0.85 680	9.99 584	5
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	0.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	0.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	0.85 312	9.99 577	1
60	9.14 356		9.14 780		0.85 220	9.99 575	**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	′

P. P.

	105	104	103
1	10.5	10.4	10.3
2	21.0	20.8	20.6
3	31.5	31.2	30.9
4	42.0	41.6	41.2
5	52.5	52.0	51.5
6	63.0	62.4	61.8
7	73.5	72.8	72.1
8	84.0	83.2	82.4
9	94.5	93.6	92.7

	102	101	99
1	10.2	10.1	9.9
2	20.4	20.2	19.8
3	30.6	30.3	29.7
4	40.8	40.4	39.6
5	51.0	50.5	49.5
6	61.2	60.6	59.4
7	71.4	70.7	69.3
8	81.6	80.8	79.2
9	91.8	90.9	89.1

	98	97	96
1	9.8	9.7	9.6
2	19.6	19.4	19.2
3	29.4	29.1	28.8
4	39.2	38.8	38.4
5	49.0	48.5	48.0
6	58.8	58.2	57.6
7	68.6	67.9	67.2
8	78.4	77.6	76.8
9	88.2	87.3	86.4

	95	94	93
1	9.5	9.4	9.3
2	19.0	18.8	18.6
3	28.5	28.2	27.9
4	38.0	37.6	37.2
5	47.5	47.0	46.5
6	57.0	56.4	55.8
7	66.5	65.8	65.1
8	76.0	75.2	74.4
9	85.5	84.6	83.7

	92	91	90
1	9.2	9.1	9.0
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

P. P.

′	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	
0	9.14 356	89	9.14 780	92	0.85 220	9.99 575	60
1	9.14 445	90	9.14 872	91	0.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	0.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	0.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	0.84 764	9.99 566	55
6	9.14 891	89	9.15 327	90	0.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	0.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	0.84 312	9.99 557	50
11	9.15 333	88	9.15 777	90	0.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	40
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	0.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	0.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	0.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	0.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	0.82 550	9.99 520	30
31	9.17 055	84	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	25
36	9.17 474	84	9.17 965	86	0.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	20
41	9.17 890	83	9.18 391	84	0.81 609	9.99 499	19
42	9.17 973	82	9.18 475	85	0.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	0.81 440	9.99 495	17
44	9.18 137	83	9.18 644	84	0.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	0.81 272	9.99 492	15
46	9.18 302	81	9.18 812	84	0.81 188	9.99 490	14
47	9.18 383	82	9.18 896	83	0.81 104	9.99 488	13
48	9.18 465	82	9.18 979	84	0.81 021	9.99 486	12
49	9.18 547	81	9.19 063	83	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	10
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	0.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1
60	9.19 433		9.19 971		0.80 029	9.99 462	0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	′

P. P.

	92	91	90
1	9.2	9.1	9.0
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

	89	88
1	8.9	8.8
2	17.8	17.6
3	26.7	26.4
4	35.6	35.2
5	44.5	44.0
6	53.4	52.8
7	62.3	61.6
8	71.2	70.4
9	80.1	79.2

	87	86	85
1	8.7	8.6	8.5
2	17.4	17.2	17.0
3	26.1	25.8	25.5
4	34.8	34.4	34.0
5	43.5	43.0	42.5
6	52.2	51.6	51.0
7	60.9	60.2	59.5
8	69.6	68.8	68.0
9	78.3	77.4	76.5

	84	83
1	8.4	8.3
2	16.8	16.6
3	25.2	24.9
4	33.6	33.2
5	42.0	41.5
6	50.4	49.8
7	58.8	58.1
8	67.2	66.4
9	75.6	74.7

	82	81	80
1	8.2	8.1	8.0
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0

P. P.

′	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	60
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	0.79 218	9.99 442	50
11	9.20 302	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	45
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	30
31	9.21 836	76	9.22 438	78	0.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	25
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	0.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	0.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	20
41	9.22 583	74	9.23 206	77	0.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	0.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	0.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	0.76 490	9.99 368	15
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	0.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	0.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	10
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	0.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	0.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	5
56	9.23 679	73	9.24 335	75	0.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	0.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967		9.24 632		0.75 368	9.99 335	0
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	′

P. P.

	82	81	80
1	8.2	8.1	8.0
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0

	79	78	77
1	7.9	7.8	7.7
2	15.8	15.6	15.4
3	23.7	23.4	23.1
4	31.6	31.2	30.8
5	39.5	39.0	38.5
6	47.4	46.8	46.2
7	55.3	54.6	53.9
8	63.2	62.4	61.6
9	71.1	70.2	69.3

	76	75	74
1	7.6	7.5	7.4
2	15.2	15.0	14.8
3	22.8	22.5	22.2
4	30.4	30.0	29.6
5	38.0	37.5	37.0
6	45.6	45.0	44.4
7	53.2	52.5	51.8
8	60.8	60.0	59.2
9	68.4	67.5	66.6

	73	72	71
1	7.3	7.2	7.1
2	14.6	14.4	14.2
3	21.9	21.6	21.3
4	29.2	28.8	28.4
5	36.5	36.0	35.5
6	43.8	43.2	42.6
7	51.1	50.4	49.7
8	58.4	57.6	56.8
9	65.7	64.8	63.9

	$\frac{3}{79}$	$\frac{3}{78}$	$\frac{3}{77}$
0	13.2	13.0	12.8
1	39.5	39.0	38.5
2	65.8	65.0	64.2
3			

	$\frac{3}{76}$	$\frac{3}{75}$	$\frac{3}{74}$
0	12.7	12.5	12.3
1	38.0	37.5	37.0
2	63.3	62.5	61.7
3			

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	2	60
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	0.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	2	55
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	3	50
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	2	45
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	0.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	2	40
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	2	35
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	3	30
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	3	25
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	0.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	2	20
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	2	18
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	2	15
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	0.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	2	10
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	3	5
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	3	2
59	9.27 995	65	9.28 798	67	0.71 202	9.99 197	2	1
60	9.28 060		9.28 865		0.71 135	9.99 195		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	74	73	72
1	7.4	7.3	7.2
2	14.8	14.6	14.4
3	22.2	21.9	21.6
4	29.6	29.2	28.8
5	37.0	36.5	36.0
6	44.4	43.8	43.2
7	51.8	51.1	50.4
8	59.2	58.4	57.6
9	66.6	65.7	64.8

	71	70	69
1	7.1	7.0	6.9
2	14.2	14.0	13.8
3	21.3	21.0	20.7
4	28.4	28.0	27.6
5	35.5	35.0	34.5
6	42.6	42.0	41.4
7	49.7	49.0	48.3
8	56.8	56.0	55.2
9	63.9	63.0	62.1

	68	67	66
1	6.8	6.7	6.6
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	3
1	6.5	0.3
2	13.0	0.6
3	19.5	0.9
4	26.0	1.2
5	32.5	1.5
6	39.0	1.8
7	45.5	2.1
8	52.0	2.4
9	58.5	2.7

	$\frac{3}{74}$	$\frac{3}{73}$	$\frac{3}{72}$
0			
1	12.3	12.2	12.0
2	37.0	36.5	36.0
3	61.7	60.8	60.0

	$\frac{3}{71}$	$\frac{3}{70}$	$\frac{3}{69}$	$\frac{3}{68}$
0				
1	11.8	11.7	11.5	11.3
2	35.5	35.0	34.5	34.0
3	59.2	58.3	57.5	56.7

P. P.

'	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.28 060	65	9.28 865	68	0.71 135	9.99 195	3	60
1	9.28 125	65	9.28 933	67	0.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	0.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	0.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	3	56
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	2	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335	67	0.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175	3	52
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	2	51
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	3	50
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165	3	48
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	0.70 134	9.99 157	2	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	3	44
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	0.69 805	9.99 145	3	40
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135	3	36
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	2	35
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	3	31
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	2	30
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	2	25
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	2	20
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	0.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	2	15
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	3	10
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	2	9
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	3	8
53	9.31 370	60	9.32 311	62	0.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	63	0.67 627	9.99 056	2	6
55	9.31 490	59	9.32 436	62	0.67 564	9.99 054	3	5
56	9.31 549	60	9.32 498	63	0.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	2	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	3	2
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	3	1
60	9.31 788		9.32 747		0.67 253	9.99 040		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'

P. P.

	68	67	66
1	6.8	6.7	6.6
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	64	63
1	6.5	6.4	6.3
2	13.0	12.8	12.6
3	19.5	19.2	18.9
4	26.0	25.6	25.2
5	32.5	32.0	31.5
6	39.0	38.4	37.8
7	45.5	44.8	44.1
8	52.0	51.2	50.4
9	58.5	57.6	56.7

	62	61	60
1	6.2	6.1	6.0
2	12.4	12.2	12.0
3	18.6	18.3	18.0
4	24.8	24.4	24.0
5	31.0	30.5	30.0
6	37.2	36.6	36.0
7	43.4	42.7	42.0
8	49.6	48.8	48.0
9	55.8	54.9	54.0

	59	3
1	5.9	0.3
2	11.8	0.6
3	17.7	0.9
4	23.6	1.2
5	29.5	1.5
6	35.4	1.8
7	41.3	2.1
8	47.2	2.4
9	53.1	2.7

	3/67	3/66	3/65
0			
	11.2	11.0	10.8
1			
	33.5	33.0	32.5
2			
	55.8	55.0	54.2
3			

	3/64	3/63	3/62
0			
	10.7	10.5	10.3
1			
	32.0	31.5	31.0
2			
	53.3	52.5	51.7
3			

	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'	P. P.

'	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.31 788	59	9.32 747	63	0.67 253	9.99 040	2	60
1	9.31 847	60	9.32 810	62	0.67 190	9.99 038	3	59
2	9.31 907	59	9.32 872	61	0.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	62	0.67 067	9.99 032	2	57
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	3	56
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	3	55
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	2	54
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	3	53
8	9.32 261	58	9.33 242	61	0.66 758	9.99 019	3	52
9	9.32 319	59	9.33 303	62	0.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	2	50
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	3	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	3	47
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	2	46
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	3	45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	2	42
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	3	41
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	3	40
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	3	39
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	2	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	3	37
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	3	36
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	3	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	2	34
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	3	33
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	3	30
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	2	29
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	3	28
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	3	26
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	3	25
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	3	24
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	2	23
38	9.33 987	56	9.35 051	60	0.64 949	9.98 936	3	22
39	9.34 043	57	9.35 111	59	0.64 889	9.98 933	3	21
40	9.34 100	56	9.35 170	59	0.64 830	9.98 930	3	20
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	2	17
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	3	16
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	3	15
46	9.34 436	55	9.35 523	58	0.64 477	9.98 913	3	14
47	9.34 491	56	9.35 581	59	0.64 419	9.98 910	3	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	3	12
49	9.34 602	56	9.35 698	59	0.64 302	9.98 904	3	11
50	9.34 658	55	9.35 757	58	0.64 243	9.98 901	3	10
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	2	9
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	3	8
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	3	5
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	57	0.63 721	9.98 875	3	1
60	9.35 209		9.36 336		0.63 664	9.98 872		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'

P. P.

	63	62	61
1	6.3	6.2	6.1
2	12.6	12.4	12.2
3	18.9	18.6	18.3
4	25.2	24.8	24.4
5	31.5	31.0	30.5
6	37.8	37.2	36.6
7	44.1	43.4	42.7
8	50.4	49.6	48.8
9	56.7	55.8	54.9

	60	59
1	6.0	5.9
2	12.0	11.8
3	18.0	17.7
4	24.0	23.6
5	30.0	29.5
6	36.0	35.4
7	42.0	41.3
8	48.0	47.2
9	54.0	53.1

	58	57
1	5.8	5.7
2	11.6	11.4
3	17.4	17.1
4	23.2	22.8
5	29.0	28.5
6	34.8	34.2
7	40.6	39.9
8	46.4	45.6
9	52.2	51.3

	56	55	3
1	5.6	5.5	0.3
2	11.2	11.0	0.6
3	16.8	16.5	0.9
4	22.4	22.0	1.2
5	28.0	27.5	1.5
6	33.6	33.0	1.8
7	39.2	38.5	2.1
8	44.8	44.0	2.4
9	50.4	49.5	2.7

	$\frac{3}{62}$	$\frac{3}{61}$	$\frac{3}{60}$
0	10.3	10.2	10.0
1	31.0	30.5	30.0
2	51.7	50.8	50.0
3			

	$\frac{3}{59}$	$\frac{3}{58}$	$\frac{3}{57}$
0	9.8	9.7	9.5
1	29.5	29.0	28.5
2	49.2	48.3	47.5
3			

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	′
0	9.35 209	54	9.36 336	58	0.63 664	9.98 872	3	60
1	9.35 263	55	9.36 394	58	0.63 606	9.98 869	2	59
2	9.35 318	55	9.36 452	57	0.63 548	9.98 867	3	58
3	9.35 373	54	9.36 509	57	0.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	58	0.63 434	9.98 861	3	56
5	9.35 481	55	9.36 624	57	0.63 376	9.98 858	3	55
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	3	50
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	3	46
15	9.36 022	53	9.37 193	57	0.62 807	9.98 828	3	45
16	9.36 075	54	9.37 250	56	0.62 750	9.98 825	3	44
17	9.36 129	53	9.37 306	57	0.62 694	9.98 822	3	43
18	9.36 182	54	9.37 363	56	0.62 637	9.98 819	3	42
19	9.36 236	53	9.37 419	57	0.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	3	40
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39
22	9.36 395	54	9.37 588	56	0.62 412	9.98 807	3	38
23	9.36 449	53	9.37 644	56	0.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	3	35
26	9.36 608	52	9.37 812	56	0.62 188	9.98 795	3	34
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	55	0.62 020	9.98 786	3	31
30	9.36 819	52	9.38 035	56	0.61 965	9.98 783	3	30
31	9.36 871	53	9.38 091	56	0.61 909	9.98 780	3	29
32	9.36 924	52	9.38 147	55	0.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27
34	9.37 028	53	9.38 257	56	0.61 743	9.98 771	3	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	3	25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	56	0.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	55	0.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	3	20
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	4	19
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	3	18
43	9.37 497	52	9.38 754	54	0.61 246	9.98 743	3	17
44	9.37 549	51	9.38 808	55	0.61 192	9.98 740	3	16
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	3	15
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	3	14
47	9.37 703	52	9.38 972	55	0.61 028	9.98 731	3	13
48	9.37 755	51	9.39 027	55	0.60 973	9.98 728	3	12
49	9.37 806	52	9.39 082	54	0.60 918	9.98 725	3	11
50	9.37 858	51	9.39 136	54	0.60 864	9.98 722	3	10
51	9.37 909	51	9.39 190	55	0.60 810	9.98 719	4	9
52	9.37 960	51	9.39 245	54	0.60 755	9.98 715	3	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	3	5
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	4	1
60	9.38 368		9.39 677		0.60 323	9.98 690		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	58	57	56
1	5.8	5.7	5.6
2	11.6	11.4	11.2
3	17.4	17.1	16.8
4	23.2	22.8	22.4
5	29.0	28.5	28.0
6	34.8	34.2	33.6
7	40.6	39.9	39.2
8	46.4	45.6	44.8
9	52.2	51.3	50.4

	55	54	53
1	5.5	5.4	5.3
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	51
1	5.2	5.1
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	$\frac{4}{55}$	$\frac{4}{54}$	$\frac{3}{58}$	$\frac{3}{57}$
0	6.9	6.8	9.7	9.5
1	20.6	20.2	29.0	28.5
2	34.4	33.8	48.3	47.5
3	48.1	47.2	—	—
4				

	$\frac{3}{56}$	$\frac{3}{55}$	$\frac{3}{54}$
0	9.3	9.2	9.0
1	28.0	27.5	27.0
2	46.7	45.8	45.0
3			

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.38 368	50	9.39 677	54	0.60 323	9.98 690	3	**60**
1	9.38 418	51	9.39 731	54	0.60 269	9.98 687	3	59
2	9.38 469	50	9.39 785	53	0.60 215	9.98 684	3	58
3	9.38 519	51	9.39 838	54	0.60 162	9.98 681	3	57
4	9.38 570	50	9.39 892	53	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	54	0.60 055	9.98 675	4	55
6	9.38 670	51	9.39 999	53	0.60 001	9.98 671	3	54
7	9.38 721	50	9.40 052	54	0.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	54	0.59 788	9.98 659	3	**50**
11	9.38 921	50	9.40 266	53	0.59 734	9.98 656	4	49
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 121	49	9.40 478	53	0.59 522	9.98 643	3	45
16	9.39 170	50	9.40 531	53	0.59 469	9.98 640	4	44
17	9.39 220	50	9.40 584	52	0.59 416	9.98 636	3	43
18	9.39 270	49	9.40 636	53	0.59 364	9.98 633	3	42
19	9.39 319	50	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 369	49	9.40 742	53	0.59 258	9.98 627	4	**40**
21	9.39 418	49	9.40 795	52	0.59 205	9.98 623	3	39
22	9.39 467	50	9.40 847	53	0.59 153	9.98 620	3	38
23	9.39 517	49	9.40 900	52	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	53	0.59 048	9.98 614	4	36
25	9.39 615	49	9.41 005	52	0.58 995	9.98 610	3	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	53	0.58 839	9.98 601	4	32
29	9.39 811	49	9.41 214	52	0.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	**30**
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 958	48	9.41 370	52	0.58 630	9.98 588	4	28
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27
34	9.40 055	48	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	49	9.41 526	52	0.58 474	9.98 578	4	25
36	9.40 152	48	9.41 578	51	0.58 422	9.98 574	3	24
37	9.40 200	49	9.41 629	52	0.58 371	9.98 571	3	23
38	9.40 249	48	9.41 681	52	0.58 319	9.98 568	3	22
39	9.40 297	49	9.41 733	51	0.58 267	9.98 565	4	21
40	9.40 346	48	9.41 784	52	0.58 216	9.98 561	3	**20**
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	52	0.58 113	9.98 555	4	18
43	9.40 490	48	9.41 939	51	0.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	52	0.57 959	9.98 545	4	15
46	9.40 634	48	9.42 093	51	0.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	4	12
49	9.40 778	47	9.42 246	51	0.57 754	9.98 531	3	11
50	9.40 825	48	9.42 297	51	0.57 703	9.98 528	3	**10**
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	4	9
52	9.40 921	47	9.42 399	51	0.57 601	9.98 521	3	8
53	9.40 968	48	9.42 450	51	0.57 550	9.98 518	3	7
54	9.41 016	47	9.42 501	51	0.57 499	9.98 515	4	6
55	9.41 063	48	9.42 552	51	0.57 448	9.98 511	3	5
56	9.41 111	47	9.42 603	50	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	0.57 347	9.98 505	4	3
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	3	2
59	9.41 252	48	9.42 755	50	0.57 245	9.98 498	4	1
60	9.41 300		9.42 805		0.57 195	9.98 494		**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	54	53
1	5.4	5.3
2	10.8	10.6
3	16.2	15.9
4	21.6	21.2
5	27.0	26.5
6	32.4	31.8
7	37.8	37.1
8	43.2	42.4
9	48.6	47.7

	52	51	50
1	5.2	5.1	5.0
2	10.4	10.2	10.0
3	15.6	15.3	15.0
4	20.8	20.4	20.0
5	26.0	25.5	25.0
6	31.2	30.6	30.0
7	36.4	35.7	35.0
8	41.6	40.8	40.0
9	46.8	45.9	45.0

	49	48	47
1	4.9	4.8	4.7
2	9.8	9.6	9.4
3	14.7	14.4	14.1
4	19.6	19.2	18.8
5	24.5	24.0	23.5
6	29.4	28.8	28.2
7	34.3	33.6	32.9
8	39.2	38.4	37.6
9	44.1	43.2	42.3

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	4/54	4/53	4/52	4/51
0	6.8	6.6	6.5	6.4
1	20.2	19.9	19.5	19.1
2	33.8	33.1	32.5	31.9
3	47.2	46.4	45.5	44.6
4				

	3/54	3/53	3/52	3/51
0	9.0	8.8	8.7	8.5
1	27.0	26.5	26.0	25.5
2	45.0	44.2	43.3	42.5
3				

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.41 300	47	9.42 805	51	0.57 195	9.98 494	3	60
1	9.41 347	47	9.42 856	50	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	51	0.57 094	9.98 488	4	58
3	9.41 441	47	9.42 957	50	0.57 043	9.98 484	3	57
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	4	56
5	9.41 535	47	9.43 057	51	0.56 943	9.98 477	3	55
6	9.41 582	46	9.43 108	50	0.56 892	9.98 474	3	54
7	9.41 628	47	9.43 158	50	0.56 842	9.98 471	4	53
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	3	52
9	9.41 722	46	9.43 258	50	0.56 742	9.98 464	4	51
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460	3	50
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	4	49
12	9.41 861	47	9.43 408	50	0.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	47	9.43 508	50	0.56 492	9.98 447	4	46
15	9.42 001	46	9.43 558	49	0.56 442	9.98 443	3	45
16	9.42 047	46	9.43 607	50	0.56 393	9.98 440	4	44
17	9.42 093	47	9.43 657	50	0.56 343	9.98 436	3	43
18	9.42 140	46	9.43 707	49	0.56 293	9.98 433	4	42
19	9.42 186	46	9.43 756	50	0.56 244	9.98 429	3	41
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	4	40
21	9.42 278	46	9.43 855	50	0.56 145	9.98 422	3	39
22	9.42 324	46	9.43 905	49	0.56 095	9.98 419	4	38
23	9.42 370	46	9.43 954	50	0.56 046	9.98 415	3	37
24	9.42 416	45	9.44 004	49	0.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	0.55 947	9.98 409	4	35
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	3	34
27	9.42 553	46	9.44 151	50	0.55 849	9.98 402	4	33
28	9.42 599	45	9.44 201	49	0.55 799	9.98 398	3	32
29	9.42 644	46	9.44 250	49	0.55 750	9.98 395	4	31
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	30
31	9.42 735	46	9.44 348	49	0.55 652	9.98 388	4	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826	46	9.44 446	49	0.55 554	9.98 381	4	27
34	9.42 872	45	9.44 495	49	0.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	48	0.55 456	9.98 373	3	25
36	9.42 962	46	9.44 592	49	0.55 408	9.98 370	4	24
37	9.43 008	45	9.44 641	49	0.55 359	9.98 366	3	23
38	9.43 053	45	9.44 690	48	0.55 310	9.98 363	4	22
39	9.43 098	45	9.44 738	49	0.55 262	9.98 359	3	21
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	4	20
41	9.43 188	45	9.44 836	48	0.55 164	9.98 352	3	19
42	9.43 233	45	9.44 884	49	0.55 116	9.98 349	4	18
43	9.43 278	45	9.44 933	48	0.55 067	9.98 345	3	17
44	9.43 323	44	9.44 981	48	0.55 019	9.98 342	4	16
45	9.43 367	45	9.45 029	49	0.54 971	9.98 338	4	15
46	9.43 412	45	9.45 078	48	0.54 922	9.98 334	3	14
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	4	13
48	9.43 502	44	9.45 174	48	0.54 826	9.98 327	3	12
49	9.43 546	45	9.45 222	49	0.54 778	9.98 324	4	11
50	9.43 591	44	9.45 271	48	0.54 729	9.98 320	3	10
51	9.43 635	45	9.45 319	48	0.54 681	9.98 317	4	9
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	4	8
53	9.43 724	45	9.45 415	48	0.54 585	9.98 309	3	7
54	9.43 769	44	9.45 463	48	0.54 537	9.98 306	4	6
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	3	5
56	9.43 857	44	9.45 559	47	0.54 441	9.98 299	4	4
57	9.43 901	45	9.45 606	48	0.54 394	9.98 295	4	3
58	9.43 946	44	9.45 654	48	0.54 346	9.98 291	3	2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	4	1
60	9.44 034		9.45 750		0.54 250	9.98 284		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	51	50	49
1	5.1	5.0	4.9
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47	46
1	4.8	4.7	4.6
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44
1	4.5	4.4
2	9.0	8.8
3	13.5	13.2
4	18.0	17.6
5	22.5	22.0
6	27.0	26.4
7	31.5	30.8
8	36.0	35.2
9	40.5	39.6

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	$\frac{4}{50}$	$\frac{4}{49}$	$\frac{4}{48}$	$\frac{4}{47}$
0	6.2	6.1	6.0	5.9
1	18.8	18.4	18.0	17.6
2	31.2	30.6	30.0	29.4
3	43.8	42.9	42.0	41.1
4				

	$\frac{3}{51}$	$\frac{3}{50}$	$\frac{3}{49}$	$\frac{3}{48}$
0	8.5	8.3	8.2	8.0
1	25.5	25.0	24.5	24.0
2	42.5	41.7	40.8	40.0
3				

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.44 034	44	9.45 750	47	0.54 250	9.98 284	3	60
1	9.44 078	44	9.45 797	48	0.54 203	9.98 281	4	59
2	9.44 122	44	9.45 845	47	0.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	48	0.54 108	9.98 273	3	57
4	9.44 210	43	9.45 940	47	0.54 060	9.98 270	4	56
5	9.44 253	44	9.45 987	48	0.54 013	9.98 266	4	55
6	9.44 297	44	9.46 035	47	0.53 965	9.98 262	3	54
7	9.44 341	44	9.46 082	48	0.53 918	9.98 259	4	53
8	9.44 385	43	9.46 130	47	0.53 870	9.98 255	4	52
9	9.44 428	44	9.46 177	47	0.53 823	9.98 251	3	51
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	4	50
11	9.44 516	43	9.46 271	48	0.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	47	0.53 681	9.98 240	3	48
13	9.44 602	44	9.46 366	47	0.53 634	9.98 237	4	47
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	4	46
15	9.44 689	44	9.46 460	47	0.53 540	9.98 229	3	45
16	9.44 733	43	9.46 507	47	0.53 493	9.98 226	4	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	3	42
19	9.44 862	43	9.46 648	46	0.53 352	9.98 215	4	41
20	9.44 905	43	9.46 694	47	0.53 306	9.98 211	4	40
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	3	39
22	9.44 992	43	9.46 788	47	0.53 212	9.98 204	4	38
23	9.45 035	42	9.46 835	46	0.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	0.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	3	35
26	9.45 163	43	9.46 975	46	0.53 025	9.98 189	4	34
27	9.45 206	43	9.47 021	47	0.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	46	0.52 932	9.98 181	4	32
29	9.45 292	42	9.47 114	46	0.52 886	9.98 177	3	31
30	9.45 334	43	9.47 160	47	0.52 840	9.98 174	4	30
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	0.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	0.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	25
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	0.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	20
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	0.52 148	9.98 117	4	15
46	9.46 011	42	9.47 897	46	0.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	0.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	0.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	0.51 920	9.98 098	4	10
51	9.46 220	42	9.48 126	45	0.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	0.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	0.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	0.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	4	5
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	0.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	0.51 511	9.98 063	3	1
60	9.46 594		9.48 534		0.51 466	9.98 060		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	48	47	46
1	4.8	4.7	4.6
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44	43
1	4.5	4.4	4.3
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41
1	4.2	4.1
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	$\frac{4}{48}$	$\frac{4}{47}$	$\frac{4}{46}$	$\frac{4}{45}$
0	6.0	5.9	5.8	5.6
1	18.0	17.6	17.2	16.9
2	30.0	29.4	28.8	28.1
3	42.0	41.1	40.2	39.4
4				

	$\frac{3}{48}$	$\frac{3}{47}$	$\frac{3}{46}$	$\frac{3}{45}$
0	8.0	7.8	7.7	7.5
1	24.0	23.5	23.0	22.5
2	40.0	39.2	38.3	37.5
3				

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	0.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	0.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	0.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	0.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	0.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	0.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	0.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	0.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	0.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	0.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	0.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	0.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	0.49 689	9.97 902	4	20
41	9.48 252	40	9.50 355	43	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	0.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	0.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	0.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	0.49 471	9.97 882	4	15
46	9.48 450	40	9.50 572	44	0.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	0.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	0.49 254	9.97 861	4	10
51	9.48 647	39	9.50 789	44	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998		9.51 178		0.48 822	9.97 821		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	45	44	43
1	4.5	4.4	4.3
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41	40
1	4.2	4.1	4.0
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

	39	5	4	3
1	3.9	0.5	0.4	0.3
2	7.8	1.0	0.8	0.6
3	11.7	1.5	1.2	0.9
4	15.6	2.0	1.6	1.2
5	19.5	2.5	2.0	1.5
6	23.4	3.0	2.4	1.8
7	27.3	3.5	2.8	2.1
8	31.2	4.0	3.2	2.4
9	35.1	4.5	3.6	2.7

	$\frac{5}{43}$	$\frac{4}{45}$	$\frac{4}{44}$
0	4.3	5.6	5.5
1	12.9	16.9	16.5
2	21.5	28.1	27.5
3	30.1	39.4	38.5
4	38.7	—	—
5			

	$\frac{4}{43}$	$\frac{3}{45}$	$\frac{3}{44}$
0	5.4	7.5	7.3
1	16.1	22.5	22.0
2	26.9	37.5	36.7
3	37.6	—	—
4			

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	60
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	0.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	0.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	0.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	4	50
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	0.48 181	9.97 759	5	45
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	40
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	35
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	5	30
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	0.47 339	9.97 674	4	25
36	9.50 374	37	9.52 703	42	0.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	0.47 255	9.97 666	4	23
38	9.50 449	37	9.52 787	42	0.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	20
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	0.47 047	9.97 645	5	18
43	9.50 635	38	9.52 995	42	0.47 005	9.97 640	4	17
44	9.50 673	37	9.53 037	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	15
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	5	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	0.46 756	9.97 615	5	11
50	9.50 896	37	9.53 285	42	0.46 715	9.97 610	4	10
51	9.50 933	37	9.53 327	41	0.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	0.46 550	9.97 593	4	6
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	5	5
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	0.46 385	9.97 576	5	2
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264		9.53 697		0.46 303	9.97 567		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	43	42	41
1	4.3	4.2	4.1
2	8.6	8.4	8.2
3	12.9	12.6	12.3
4	17.2	16.8	16.4
5	21.5	21.0	20.5
6	25.8	25.2	24.6
7	30.1	29.4	28.7
8	34.4	33.6	32.8
9	38.7	37.8	36.9

	39	38	37
1	3.9	3.8	3.7
2	7.8	7.6	7.4
3	11.7	11.4	11.1
4	15.6	15.2	14.8
5	19.5	19.0	18.5
6	23.4	22.8	22.2
7	27.3	26.6	25.9
8	31.2	30.4	29.6
9	35.1	34.2	33.3

	36	5	4
1	3.6	0.5	0.4
2	7.2	1.0	0.8
3	10.8	1.5	1.2
4	14.4	2.0	1.6
5	18.0	2.5	2.0
6	21.6	3.0	2.4
7	25.2	3.5	2.8
8	28.8	4.0	3.2
9	32.4	4.5	3.6

	$\frac{5}{43}$	$\frac{5}{42}$	$\frac{5}{41}$
0			
1	4.3	4.2	4.1
2	12.9	12.6	12.3
3	21.5	21.0	20.5
4	30.1	29.4	28.7
5	38.7	37.8	36.9

	$\frac{4}{43}$	$\frac{4}{42}$	$\frac{4}{41}$
0			
1	5.4	5.2	5.1
2	16.1	15.8	15.4
3	26.9	26.2	25.6
4	37.6	36.8	35.9

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	′
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	**60**
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	0.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	0.46 098	9.97 545	4	55
6	9.51 484	36	9.53 943	41	0.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	0.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	0.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	0.45 894	9.97 523	4	**50**
11	9.51 666	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	0.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	0.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	0.45 691	9.97 501	4	45
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	0.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	**40**
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	0.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	4	35
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	0.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	5	**30**
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	4	25
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	5	**20**
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	15
46	9.52 916	35	9.55 554	39	0.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	0.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	0.44 288	9.97 344	4	**10**
51	9.53 092	34	9.55 752	39	0.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	0.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	0.44 090	9.97 322	5	5
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	4	1
60	9.53 405		9.56 107		0.43 893	9.97 299		**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	37	36	35
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	5	4
1	3.4	0.5	0.4
2	6.8	1.0	0.8
3	10.2	1.5	1.2
4	13.6	2.0	1.6
5	17.0	2.5	2.0
6	20.4	3.0	2.4
7	23.8	3.5	2.8
8	27.2	4.0	3.2
9	30.6	4.5	3.6

	5/41	5/40	5/39
0	4.1	4.0	3.9
1	12.3	12.0	11.7
2	20.5	20.0	19.5
3	28.7	28.0	27.3
4	36.9	36.0	35.1
5			

	4/41	4/40	4/39
0	5.1	5.0	4.9
1	15.4	15.0	14.6
2	25.6	25.0	24.4
3	35.9	35.0	34.1
4			

'	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.53 405	35	9.56 107	39	0.43 893	9.97 299	5	60
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59
2	9.53 475	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	0.43 776	9.97 285	5	57
4	9.53 544	34	9.56 264	39	0.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	0.43 697	9.97 276	5	55
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	5	54
7	9.53 647	35	9.56 381	39	0.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 262	5	52
9	9.53 716	35	9.56 459	39	0.43 541	9.97 257	5	51
10	9.53 751	34	9.56 498	39	0.43 502	9.97 252	4	50
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	5	49
12	9.53 819	35	9.56 576	39	0.43 424	9.97 243	5	48
13	9.53 854	34	9.56 615	39	0.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	35	9.56 693	39	0.43 307	9.97 229	5	45
16	9.53 957	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	38	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	0.43 113	9.97 206	5	40
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	0.42 996	9.97 192	5	37
24	9.54 229	34	9.57 042	39	0.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	4	35
26	9.54 297	34	9.57 120	38	0.42 880	9.97 178	5	34
27	9.54 331	34	9.57 158	39	0.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	38	0.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 433	33	9.57 274	38	0.42 726	9.97 159	5	30
31	9.54 466	34	9.57 312	39	0.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	38	0.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	0.42 611	9.97 145	5	27
34	9.54 567	34	9.57 428	38	0.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	25
36	9.54 635	33	9.57 504	39	0.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	0.42 457	9.97 126	5	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	39	0.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	38	0.42 342	9.97 111	4	20
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	5	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	39	0.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	38	0.42 151	9.97 087	4	15
46	9.54 969	34	9.57 887	38	0.42 113	9.97 083	5	14
47	9.55 003	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	0.41 961	9.97 063	4	10
51	9.55 136	33	9.58 077	38	0.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	4	5
56	9.55 301	33	9.58 267	37	0.41 733	9.97 035	5	4
57	9.55 334	33	9.58 304	38	0.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1
60	9.55 433		9.58 418		0.41 582	9.97 015		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'

P. P.

	40	39	38
1	4.0	3.9	3.8
2	8.0	7.8	7.6
3	12.0	11.7	11.4
4	16.0	15.6	15.2
5	20.0	19.5	19.0
6	24.0	23.4	22.8
7	28.0	27.3	26.6
8	32.0	31.2	30.4
9	36.0	35.1	34.2

	37	35	34
1	3.7	3.5	3.4
2	7.4	7.0	6.8
3	11.1	10.5	10.2
4	14.8	14.0	13.6
5	18.5	17.5	17.0
6	22.2	21.0	20.4
7	25.9	24.5	23.8
8	29.6	28.0	27.2
9	33.3	31.5	30.6

	33	5	4
1	3.3	0.5	0.4
2	6.6	1.0	0.8
3	9.9	1.5	1.2
4	13.2	2.0	1.6
5	16.5	2.5	2.0
6	19.8	3.0	2.4
7	23.1	3.5	2.8
8	26.4	4.0	3.2
9	29.7	4.5	3.6

	$\frac{5}{40}$	$\frac{5}{39}$	$\frac{5}{38}$
0	4.0	3.9	3.8
1	12.0	11.7	11.4
2	20.0	19.5	19.0
3	28.0	27.3	26.6
4	36.0	35.1	34.2
5			

	$\frac{5}{37}$	$\frac{4}{39}$	$\frac{4}{38}$
0	3.7	4.9	4.8
1	11.1	14.6	14.2
2	18.5	24.4	23.8
3	25.9	34.1	33.2
4	33.3	—	—
5			

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	′
0	9.55 433	33	9.58 418	37	0.41 582	9.97 015	5	60
1	9.55 466	33	9.58 455	38	0.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	4	58
3	9.55 532	32	9.58 531	38	0.41 469	9.97 001	5	57
4	9.55 564	33	9.58 569	37	0.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55
6	9.55 630	33	9.58 644	37	0.41 356	9.96 986	5	54
7	9.55 663	32	9.58 681	38	0.41 319	9.96 981	5	53
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	37	0.41 243	9.96 971	5	51
10	9.55 761	32	9.58 794	38	0.41 206	9.96 966	4	50
11	9.55 793	33	9.58 832	37	0.41 168	9.96 962	5	49
12	9.55 826	32	9.58 869	38	0.41 131	9.96 957	5	48
13	9.55 858	33	9.58 907	37	0.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	33	9.58 981	38	0.41 019	9.96 942	5	45
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44
17	9.55 988	33	9.59 056	38	0.40 944	9.96 932	5	43
18	9.56 021	32	9.59 094	37	0.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	33	9.59 168	37	0.40 832	9.96 917	5	40
21	9.56 118	32	9.59 205	38	0.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	37	0.40 757	9.96 907	4	38
23	9.56 182	33	9.59 280	37	0.40 720	9.96 903	5	37
24	9.56 215	32	9.59 317	37	0.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	35
26	9.56 279	32	9.59 391	38	0.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	37	0.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32
29	9.56 375	33	9.59 503	37	0.40 497	9.96 873	5	31
30	9.56 408	32	9.59 540	37	0.40 460	9.96 868	5	30
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26
35	9.56 568	31	9.59 725	37	0.40 275	9.96 843	5	25
36	9.56 599	32	9.59 762	37	0.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	36	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	37	0.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	20
41	9.56 759	31	9.59 946	37	0.40 054	9.96 813	5	19
42	9.56 790	32	9.59 983	36	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	37	0.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16
45	9.56 886	31	9.60 093	37	0.39 907	9.96 793	5	15
46	9.56 917	32	9.60 130	36	0.39 870	9.96 788	5	14
47	9.56 949	31	9.60 166	37	0.39 834	9.96 783	5	13
48	9.56 980	32	9.60 203	37	0.39 797	9.96 778	6	12
49	9.57 012	32	9.60 240	36	0.39 760	9.96 772	5	11
50	9.57 044	31	9.60 276	37	0.39 724	9.96 767	5	10
51	9.57 075	32	9.60 313	36	0.39 687	9.96 762	5	9
52	9.57 107	31	9.60 349	37	0.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	36	0.39 614	9.96 752	5	7
54	9.57 169	32	9.60 422	37	0.39 578	9.96 747	5	6
55	9.57 201	31	9.60 459	36	0.39 541	9.96 742	5	5
56	9.57 232	32	9.60 495	37	0.39 505	9.96 737	5	4
57	9.57 264	31	9.60 532	36	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	37	0.39 432	9.96 727	5	2
59	9.57 326	32	9.60 605	36	0.39 395	9.96 722	5	1
60	9.57 358		9.60 641		0.39 359	9.96 717		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	33	32	31
1	3.3	3.2	3.1
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	6	5	4
1	0.6	0.5	0.4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

	6	5	5
	37	38	37
0			
	3.1	3.8	3.7
1			
	9.2	11.4	11.1
2			
	15.4	19.0	18.5
3			
	21.6	26.6	25.9
4			
	27.8	34.2	33.3
5			
	33.9	—	—
6			

	5	4	4
	36	38	37
0			
	3.6	4.8	4.6
1			
	10.8	14.2	13.9
2			
	18.0	23.8	23.1
3			
	25.2	33.2	32.4
4			
	32.4	—	—
5			

P. P.

'	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.57 358	31	9.60 641	36	0.39 359	9.96 717	6	**60**
1	9.57 389	31	9.60 677	37	0.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	0.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	0.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	0.38 996	9.96 665	5	**50**
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	6	45
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	**40**
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	35
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	**30**
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	25
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	0.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	**20**
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	0.37 744	9.96 483	6	15
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	**10**
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	6	9
52	9.58 949	30	9.62 504	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	5
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 403		**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'

P. P.

	37	36	35
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	6	5
1	2.9	0.6	0.5
2	5.8	1.2	1.0
3	8.7	1.8	1.5
4	11.6	2.4	2.0
5	14.5	3.0	2.5
6	17.4	3.6	3.0
7	20.3	4.2	3.5
8	23.2	4.8	4.0
9	26.1	5.4	4.5

	$\frac{6}{36}$	$\frac{6}{35}$
0		
1	3.0	2.9
2	9.0	8.8
3	15.0	14.6
4	21.0	20.4
5	27.0	26.2
6	33.0	32.1

	$\frac{5}{37}$	$\frac{5}{36}$	$\frac{5}{35}$
0			
1	3.7	3.6	3.5
2	11.1	10.8	10.5
3	18.5	18.0	17.5
4	25.9	25.2	24.5
5	33.3	32.4	31.5

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.59 188	30	9.62 785	35	0.37 215	9.96 403	6	60
1	9.59 218	29	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	0.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	36	0.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	0.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	0.37 039	9.96 376	6	55
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	0.36 969	9.96 365	5	53
8	9.59 425	30	9.63 066	35	0.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	0.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	0.36 865	9.96 349	6	50
11	9.59 514	29	9.63 170	35	0.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	0.36 795	9.96 338	5	48
13	9.59 573	29	9.63 240	35	0.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 632	29	9.63 310	35	0.36 690	9.96 322	6	45
16	9.59 661	29	9.63 345	34	0.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	0.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	0.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	0.36 516	9.96 294	5	40
21	9.59 808	29	9.63 519	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	0.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	0.36 343	9.96 267	5	35
26	9.59 954	29	9.63 692	34	0.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	0.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	35	0.36 170	9.96 240	6	30
31	9.60 099	29	9.63 865	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	0.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	0.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	0.35 997	9.96 212	5	25
36	9.60 244	29	9.64 037	35	0.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	0.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	0.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175	34	0.35 825	9.96 185	6	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	0.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	0.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	0.35 654	9.96 157	6	15
46	9.60 532	29	9.64 381	34	0.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	0.35 483	9.96 129	6	10
51	9.60 675	29	9.64 552	34	0.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	0.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	0.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	6	5
56	9.60 818	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	0.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	6	1
60	9.60 931		9.64 858		0.35 142	9.96 073		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	36	35	34
1	3.6	3.5	3.4
2	7.2	7.0	6.8
3	10.8	10.5	10.2
4	14.4	14.0	13.6
5	18.0	17.5	17.0
6	21.6	21.0	20.4
7	25.2	24.5	23.8
8	28.8	28.0	27.2
9	32.4	31.5	30.6

	30	29	28
1	3.0	2.9	2.8
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

	$\frac{6}{36}$	$\frac{6}{35}$	$\frac{6}{34}$
0			
1	3.0	2.9	2.8
2	9.0	8.8	8.5
3	15.0	14.6	14.2
4	21.0	20.4	19.8
5	27.0	26.2	25.5
6	33.0	32.1	31.2

	$\frac{5}{35}$	$\frac{5}{34}$
0		
1	3.5	3.4
2	10.5	10.2
3	17.5	17.0
4	24.5	23.8
5	31.5	30.6

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.60 931	29	9.64 858	34	0.35 142	9.96 073	6	60
1	9.60 960	28	9.64 892	34	0.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	0.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	0.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	0.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	0.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	0.34 803	9.96 017	6	50
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	0.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	0.34 634	9.95 988	6	45
16	9.61 382	29	9.65 400	34	0.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	0.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	0.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	0.34 465	9.95 960	6	40
21	9.61 522	28	9.65 568	34	0.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	0.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	0.34 297	9.95 931	6	35
26	9.61 662	27	9.65 736	34	0.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	0.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	0.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	0.34 163	9.95 908	6	31
30	9.61 773	27	9.65 870	34	0.34 130	9.95 902	5	30
31	9.61 800	28	9.65 904	33	0.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	0.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	0.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	0.33 962	9.95 873	5	25
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	0.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	0.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	34	0.33 796	9.95 844	5	20
41	9.62 076	28	9.66 238	33	0.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	0.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	0.33 629	9.95 815	5	15
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	0.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	0.33 463	9.95 786	6	10
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	0.33 232	9.95 745	6	3
58	9.62 541	27	9.66 801	33	0.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	5	1
60	9.62 595		9.66 867		0.33 133	9.95 728		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	34	33
1	3.4	3.3
2	6.8	6.6
3	10.2	9.9
4	13.6	13.2
5	17.0	16.5
6	20.4	19.8
7	23.8	23.1
8	27.2	26.4
9	30.6	29.7

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

	6/34	6/33	5/34
0			
1	2.8	2.8	3.4
2	8.5	8.2	10.2
3	14.2	13.8	17.0
4	19.8	19.2	23.8
5	25.5	24.8	30.6
6	31.2	30.2	—

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	60
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	0.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	0.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	50
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	0.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	0.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	0.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	0.32 476	9.95 609	6	40
21	9.63 159	27	9.67 556	33	0.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	0.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	0.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	35
26	9.63 292	27	9.67 719	33	0.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	0.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	0.32 183	9.95 555	6	31
30	9.63 398	27	9.67 850	32	0.32 150	9.95 549	6	30
31	9.63 425	26	9.67 882	33	0.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	0.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	0.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	0.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	33	0.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	0.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	0.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	0.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	20
41	9.63 689	26	9.68 206	33	0.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	0.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	0.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	0.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	0.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	10
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	0.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	33	32
1	3.3	3.2
2	6.6	6.4
3	9.9	9.6
4	13.2	12.8
5	16.5	16.0
6	19.8	19.2
7	23.1	22.4
8	26.4	25.6
9	29.7	28.8

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	7	6	5
1	0.7	0.6	0.5
2	1.4	1.2	1.0
3	2.1	1.8	1.5
4	2.8	2.4	2.0
5	3.5	3.0	2.5
6	4.2	3.6	3.0
7	4.9	4.2	3.5
8	5.6	4.8	4.0
9	6.3	5.4	4.5

	7/32	6/32	5/33
0			
1	2.3	2.7	3.3
2	6.9	8.0	9.9
3	11.4	13.3	16.5
4	16.0	18.7	23.1
5	20.6	24.0	29.7
6	25.1	29.3	—
7	29.7	—	—

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	0.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	0.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	6	46
15	9.64 571	25	9.69 298	31	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	0.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	0.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	0.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	0.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	0.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	32	0.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	0.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	0.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	0.30 385	9.95 211	7	35
26	9.64 851	26	9.69 647	32	0.30 353	9.95 204	6	34
27	9.64 877	25	9.69 679	31	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	0.30 290	9.95 192	7	32
29	9.64 927	26	9.69 742	32	0.30 258	9.95 185	6	31
30	9.64 953	25	9.69 774	31	0.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	32	0.30 195	9.95 173	6	29
32	9.65 003	26	9.69 837	31	0.30 163	9.95 167	7	28
33	9.65 029	25	9.69 868	32	0.30 132	9.95 160	6	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	0.30 068	9.95 148	7	25
36	9.65 104	26	9.69 963	32	0.30 037	9.95 141	6	24
37	9.65 130	25	9.69 995	31	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	32	0.29 974	9.95 129	7	22
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	32	0.29 911	9.95 116	6	20
41	9.65 230	25	9.70 121	31	0.29 879	9.95 110	7	19
42	9.65 255	26	9.70 152	32	0.29 848	9.95 103	6	18
43	9.65 281	25	9.70 184	31	0.29 816	9.95 097	7	17
44	9.65 306	25	9.70 215	32	0.29 785	9.95 090	6	16
45	9.65 331	25	9.70 247	31	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	7	14
47	9.65 381	25	9.70 309	32	0.29 691	9.95 071	6	13
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	32	0.29 628	9.95 059	7	11
50	9.65 456	25	9.70 404	31	0.29 596	9.95 052	6	10
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	7	9
52	9.65 506	25	9.70 466	32	0.29 534	9.95 039	6	8
53	9.65 531	25	9.70 498	31	0.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	0.29 471	9.95 027	7	6
55	9.65 580	25	9.70 560	32	0.29 440	9.95 020	6	5
56	9.65 605	25	9.70 592	31	0.29 408	9.95 014	7	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	32	0.29 315	9.94 995	7	1
60	9.65 705		9.70 717		0.29 283	9.94 988		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	32	31
1	3.2	3.1
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	26	25	24
1	2.6	2.5	2.4
2	5.2	5.0	4.8
3	7.8	7.5	7.2
4	10.4	10.0	9.6
5	13.0	12.5	12.0
6	15.6	15.0	14.4
7	18.2	17.5	16.8
8	20.8	20.0	19.2
9	23.4	22.5	21.6

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	$\frac{7}{32}$	$\frac{7}{31}$	$\frac{6}{32}$
0			
1	2.3	2.2	2.7
2	6.9	6.6	8.0
3	11.4	11.1	13.3
4	16.0	15.5	18.7
5	20.6	19.9	24.0
6	25.1	24.4	29.3
7	29.7	28.8	—

	P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.65 705	24	9.70 717	31	0.29 283	9.94 988	6	**60**
1	9.65 729	25	9.70 748	31	0.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	0.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	0.29 127	9.94 956	7	55
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935	31	0.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	0.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	0.28 972	9.94 923	6	**50**
11	9.65 976	25	9.71 059	31	0.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	0.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	0.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	0.28 847	9.94 898	7	46
15	9.66 075	24	9.71 184	31	0.28 816	9.94 891	6	45
16	9.66 099	25	9.71 215	31	0.28 785	9.94 885	7	44
17	9.66 124	24	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	0.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	0.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	6	**40**
21	9.66 221	25	9.71 370	31	0.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	0.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	0.28 569	9.94 839	7	37
24	9.66 295	24	9.71 462	31	0.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	7	35
26	9.66 343	25	9.71 524	31	0.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	0.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	0.28 383	9.94 799	6	31
30	9.66 441	24	9.71 648	31	0.28 352	9.94 793	7	**30**
31	9.66 465	24	9.71 679	30	0.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	0.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	0.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	0.28 198	9.94 760	7	25
36	9.66 586	24	9.71 833	30	0.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	0.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	0.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	0.28 045	9.94 727	7	**20**
41	9.66 706	25	9.71 986	31	0.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	0.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	0.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	7	15
46	9.66 827	24	9.72 140	30	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	0.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	0.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	0.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	0.27 738	9.94 660	6	**10**
51	9.66 946	24	9.72 293	30	0.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	0.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	0.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	0.27 585	9.94 627	7	5
56	9.67 066	24	9.72 445	31	0.27 555	9.94 620	6	4
57	9.67 090	23	9.72 476	30	0.27 524	9.94 614	7	3
58	9.67 113	24	9.72 506	31	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	0.27 463	9.94 600	7	1
60	9.67 161		9.72 567		0.27 433	9.94 593		**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	25	24	23
1	2.5	2.4	2.3
2	5.0	4.8	4.6
3	7.5	7.2	6.9
4	10.0	9.6	9.2
5	12.5	12.0	11.5
6	15.0	14.4	13.8
7	17.5	16.8	16.1
8	20.0	19.2	18.4
9	22.5	21.6	20.7

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	7/30	6/31	6/30
0	2.1	2.6	2.5
1	6.4	7.8	7.5
2	10.7	12.9	12.5
3	15.0	18.1	17.5
4	19.3	23.2	22.5
5	23.6	28.4	27.5
6	27.9	—	—
7			

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.67 161	24	9.72 567	31	0.27 433	9.94 593	6	60
1	9.67 185	23	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	0.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	0.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	0.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	7	54
7	9.67 327	23	9.72 780	31	0.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	0.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51
10	9.67 398	23	9.72 872	30	0.27 128	9.94 526	7	50
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	6	49
12	9.67 445	23	9.72 932	31	0.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	0.26 977	9.94 492	7	45
16	9.67 539	23	9.73 054	30	0.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	0.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	0.26 856	9.94 465	7	41
20	9.67 633	23	9.73 175	30	0.26 825	9.94 458	7	40
21	9.67 656	24	9.73 205	30	0.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	0.26 765	9.94 445	7	38
23	9.67 703	23	9.73 265	30	0.26 735	9.94 438	7	37
24	9.67 726	24	9.73 295	31	0.26 705	9.94 431	7	36
25	9.67 750	23	9.73 326	30	0.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	0.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	0.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	0.26 554	9.94 397	7	31
30	9.67 866	24	9.73 476	31	0.26 524	9.94 390	7	30
31	9.67 890	23	9.73 507	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	0.26 373	9.94 355	6	25
36	9.68 006	23	9.73 657	30	0.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	15
46	9.68 237	23	9.73 957	30	0.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	0.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	0.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	10
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	0.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	0.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	0.25 655	9.94 189	7	1
60	9.68 557		9.74 375		0.25 625	9.94 182		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	31	30	29
1	3.1	3.0	2.9
2	6.2	6.0	5.8
3	9.3	9.0	8.7
4	12.4	12.0	11.6
5	15.5	15.0	14.5
6	18.6	18.0	17.4
7	21.7	21.0	20.3
8	24.8	24.0	23.2
9	27.9	27.0	26.1

	24	23	22
1	2.4	2.3	2.2
2	4.8	4.6	4.4
3	7.2	6.9	6.6
4	9.6	9.2	8.8
5	12.0	11.5	11.0
6	14.4	13.8	13.2
7	16.8	16.1	15.4
8	19.2	18.4	17.6
9	21.6	20.7	19.8

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	7/31	6/31	6/30
0	2.2	2.6	2.5
1	6.6	7.8	7.5
2	11.1	12.9	12.5
3	15.5	18.1	17.5
4	19.9	23.2	22.5
5	24.4	28.4	27.5
6	28.8	—	—
7			

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	60
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59
2	9.68 603	22	9.74 435	30	0.25 565	9.94 168	7	58
3	9.68 625	23	9.74 465	29	0.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	30	0.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	55
6	9.68 694	22	9.74 554	29	0.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	30	0.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 762	22	9.74 643	30	0.25 357	9.94 119	7	51
10	9.68 784	23	9.74 673	29	0.25 327	9.94 112	7	50
11	9.68 807	22	9.74 702	30	0.25 298	9.94 105	7	49
12	9.68 829	23	9.74 732	30	0.25 268	9.94 098	8	48
13	9.68 852	23	9.74 762	29	0.25 238	9.94 090	7	47
14	9.68 875	22	9.74 791	30	0.25 209	9.94 083	7	46
15	9.68 897	23	9.74 821	30	0.25 179	9.94 076	7	45
16	9.68 920	22	9.74 851	29	0.25 149	9.94 069	7	44
17	9.68 942	23	9.74 880	30	0.25 120	9.94 062	7	43
18	9.68 965	22	9.74 910	29	0.25 090	9.94 055	7	42
19	9.68 987	23	9.74 939	30	0.25 061	9.94 048	7	41
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	40
21	9.69 032	23	9.74 998	30	0.25 002	9.94 034	7	39
22	9.69 055	22	9.75 028	30	0.24 972	9.94 027	7	38
23	9.69 077	23	9.75 058	29	0.24 942	9.94 020	8	37
24	9.69 100	22	9.75 087	30	0.24 913	9.94 012	7	36
25	9.69 122	22	9.75 117	29	0.24 883	9.94 005	7	35
26	9.69 144	23	9.75 146	30	0.24 854	9.93 998	7	34
27	9.69 167	22	9.75 176	29	0.24 824	9.93 991	7	33
28	9.69 189	23	9.75 205	30	0.24 795	9.93 984	7	32
29	9.69 212	22	9.75 235	29	0.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	30	0.24 736	9.93 970	7	30
31	9.69 256	23	9.75 294	29	0.24 706	9.93 963	8	29
32	9.69 279	22	9.75 323	30	0.24 677	9.93 955	7	28
33	9.69 301	22	9.75 353	29	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	23	9.75 411	30	0.24 589	9.93 934	7	25
36	9.69 368	22	9.75 441	29	0.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	30	0.24 530	9.93 920	8	23
38	9.69 412	22	9.75 500	29	0.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	23	9.75 558	30	0.24 442	9.93 898	7	20
41	9.69 479	22	9.75 588	29	0.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	30	0.24 383	9.93 884	8	18
43	9.69 523	22	9.75 647	29	0.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	30	0.24 295	9.93 862	7	15
46	9.69 589	22	9.75 735	29	0.24 265	9.93 855	8	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	30	0.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	29	0.24 148	9.93 826	7	10
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	8	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	30	0.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	29	0.24 031	9.93 797	8	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	5
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	30	0.23 944	9.93 775	7	3
58	9.69 853	22	9.76 086	29	0.23 914	9.93 768	8	2
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1
60	9.69 897		9.76 144		0.23 856	9.93 753		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	30	29	23
1	3.0	2.9	2.3
2	6.0	5.8	4.6
3	9.0	8.7	6.9
4	12.0	11.6	9.2
5	15.0	14.5	11.5
6	18.0	17.4	13.8
7	21.0	20.3	16.1
8	24.0	23.2	18.4
9	27.0	26.1	20.7

	22	8	7
1	2.2	0.8	0.7
2	4.4	1.6	1.4
3	6.6	2.4	2.1
4	8.8	3.2	2.8
5	11.0	4.0	3.5
6	13.2	4.8	4.2
7	15.4	5.6	4.9
8	17.6	6.4	5.6
9	19.8	7.2	6.3

	8/30	8/29
0	1.9	1.8
1	5.6	5.4
2	9.4	9.1
3	13.1	12.7
4	16.9	16.3
5	20.6	19.9
6	24.4	23.6
7	28.1	27.2
8		

	7/30	7/29
0	2.1	2.1
1	6.4	6.2
2	10.7	10.4
3	15.0	14.5
4	19.3	18.6
5	23.6	22.8
6	27.9	26.9
7		

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	′
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	0.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	8	55
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	50
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	45
16	9.70 245	22	9.76 609	30	0.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	0.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	0.23 275	9.93 606	7	40
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	8	39
22	9.70 375	21	9.76 783	29	0.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	0.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	7	35
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	0.22 985	9.93 532	7	30
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	0.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	0.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	0.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	8	25
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	0.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	0.22 726	9.93 465	8	21
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	7	20
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	0.22 610	9.93 435	8	17
44	9.70 846	21	9.77 418	29	0.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	8	15
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	28	0.22 495	9.93 405	8	13
48	9.70 931	21	9.77 533	29	0.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	10
51	9.70 994	21	9.77 619	29	0.22 381	9.93 375	8	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	0.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	0.22 266	9.93 344	7	5
56	9.71 100	21	9.77 763	28	0.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	0.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 123	9.93 307		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	30	29	28
1	3.0	2.9	2.8
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

	7/30	7/29	7/28
0	2.1	2.1	2.0
1	6.4	6.2	6.0
2	10.7	10.4	10.0
3	15.0	14.5	14.0
4	19.3	18.6	18.0
5	23.6	22.8	22.0
6	27.9	26.9	26.0
7			

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.71 184	21	9.77 877	29	0.22 123	9.93 307	8	**60**
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59
2	9.71 226	21	9.77 935	28	0.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	29	0.21 980	9.93 269	8	55
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	8	54
7	9.71 331	21	9.78 077	29	0.21 923	9.93 253	7	53
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	8	52
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51
10	9.71 393	21	9.78 163	29	0.21 837	9.93 230	7	**50**
11	9.71 414	21	9.78 192	28	0.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	29	0.21 780	9.93 215	8	48
13	9.71 456	21	9.78 249	28	0.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	29	0.21 723	9.93 200	8	46
15	9.71 498	21	9.78 306	28	0.21 694	9.93 192	8	45
16	9.71 519	20	9.78 334	29	0.21 666	9.93 184	7	44
17	9.71 539	21	9.78 363	28	0.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	7	41
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	8	**40**
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	7	38
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	8	37
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36
25	9.71 705	21	9.78 590	28	0.21 410	9.93 115	7	35
26	9.71 726	21	9.78 618	29	0.21 382	9.93 108	8	34
27	9.71 747	20	9.78 647	28	0.21 353	9.93 100	8	33
28	9.71 767	21	9.78 675	29	0.21 325	9.93 092	8	32
29	9.71 788	21	9.78 704	28	0.21 296	9.93 084	7	31
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	8	**30**
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29
32	9.71 850	20	9.78 789	28	0.21 211	9.93 061	8	28
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	7	27
34	9.71 891	20	9.78 845	29	0.21 155	9.93 046	8	26
35	9.71 911	21	9.78 874	28	0.21 126	9.93 038	8	25
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	8	24
37	9.71 952	21	9.78 930	29	0.21 070	9.93 022	8	23
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	7	22
39	9.71 994	20	9.78 987	28	0.21 013	9.93 007	8	21
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	**20**
41	9.72 034	21	9.79 043	29	0.20 957	9.92 991	8	19
42	9.72 055	20	9.79 072	28	0.20 928	9.92 983	7	18
43	9.72 075	21	9.79 100	28	0.20 900	9.92 976	8	17
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16
45	9.72 116	21	9.79 156	29	0.20 844	9.92 960	8	15
46	9.72 137	20	9.79 185	28	0.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13
48	9.72 177	21	9.79 241	28	0.20 759	9.92 936	7	12
49	9.72 198	20	9.79 269	28	0.20 731	9.92 929	8	11
50	9.72 218	20	9.79 297	29	0.20 703	9.92 921	8	**10**
51	9.72 238	21	9.79 326	28	0.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7
54	9.72 299	21	9.79 410	28	0.20 590	9.92 889	8	6
55	9.72 320	20	9.79 438	28	0.20 562	9.92 881	7	5
56	9.72 340	20	9.79 466	29	0.20 534	9.92 874	8	4
57	9.72 360	21	9.79 495	28	0.20 505	9.92 866	8	3
58	9.72 381	20	9.79 523	28	0.20 477	9.92 858	8	2
59	9.72 401	20	9.79 551	28	0.20 449	9.92 850	8	1
60	9.72 421		9.79 579		0.20 421	9.92 842		**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	21	20
1	2.1	2.0
2	4.2	4.0
3	6.3	6.0
4	8.4	8.0
5	10.5	10.0
6	12.6	12.0
7	14.7	14.0
8	16.8	16.0
9	18.9	18.0

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

	8/30	8/29	8/28
0	1.9	1.8	1.8
1	5.6	5.4	5.2
2	9.4	9.1	8.8
3	13.1	12.7	12.2
4	16.9	16.3	15.8
5	20.6	19.9	19.2
6	24.4	23.6	22.8
7	28.1	27.2	26.2
8			

P. P.

32°

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	60
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	0.20 365	9.92 826	8	58
3	9.72 482	20	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	8	55
6	9.72 542	20	9.79 747	29	0.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	0.20 140	9.92 763	8	50
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	45
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	40
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	0.19 805	9.92 667	8	38
23	9.72 883	19	9.80 223	28	0.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	0.19 749	9 92 651	8	36
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	35
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	30
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	25
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555	9	24
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	0.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	0.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	20
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	0.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	0.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	9	15
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	0.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	0.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975	28	0.19 025	9.92 441	8	10
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	0.18 970	9.92 425	9	8
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	5
56	9.73 533	19	9.81 141	28	0.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	0.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	0.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	0.18 776	9.92 367	8	1
60	9.73 611		9.81 252		0.18 748	9.92 359		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	21	20	19
1	2.1	2.0	1.9
2	4.2	4.0	3.8
3	6.3	6.0	5.7
4	8.4	8.0	7.6
5	10.5	10.0	9.5
6	12.6	12.0	11.4
7	14.7	14.0	13.3
8	16.8	16.0	15.2
9	18.9	18.0	17.1

	9	8	7
1	0.9	0.8	0.7
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

	$\frac{8}{29}$	$\frac{8}{28}$	$\frac{7}{28}$
0	1.8	1.8	2.0
1	5.4	5.2	6.0
2	9.1	8.8	10.0
3	12.7	12.2	14.0
4	16.3	15.8	18.0
5	19.9	19.2	22.0
6	23.6	22.8	26.0
7	27.2	26.2	—
8			

P. P.

'	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	'
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	60
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	55
6	9.73 727	20	9.81 418	27	0.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	0.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	0.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	0.18 472	9.92 277	8	50
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	0.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	0.18 334	9.92 235	8	45
16	9.73 921	19	9.81 693	28	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	0.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	0.18 197	9.92 194	8	40
21	9.74 017	19	9.81 831	27	0.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	0.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	0.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	0.18 059	9.92 152	8	35
26	9.74 113	19	9.81 968	28	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	0.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	0.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	28	0.17 922	9.92 111	9	30
31	9.74 208	19	9.82 106	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	0.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	0.17 785	9.92 069	9	25
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	0.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	28	0.17 648	9.92 027	9	20
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	27	0.17 565	9.92 002	9	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	0.17 511	9.91 985	9	15
46	9.74 493	19	9.82 517	27	0.17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	0.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	0.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	28	0.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	0.17 319	9.91 925	8	8
53	9.74 625	19	9.82 708	27	0.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	0.17 265	9.91 908	8	6
55	9.74 662	19	9.82 762	28	0.17 238	9.91 900	9	5
56	9.74 681	19	9.82 790	27	0.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	0.17 129	9.91 866	9	1
60	9.74 756		9.82 899		0.17 101	9.91 857		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'

P. P.

	28	27
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	20	19	18
1	2.0	1.9	1.8
2	4.0	3.8	3.6
3	6.0	5.7	5.4
4	8.0	7.6	7.2
5	10.0	9.5	9.0
6	12.0	11.4	10.8
7	14.0	13.3	12.6
8	16.0	15.2	14.4
9	18.0	17.1	16.2

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	$\frac{9}{28}$	$\frac{9}{27}$	$\frac{8}{27}$
0	1.6	1.5	1.7
1	4.7	4.5	5.1
2	7.8	7.5	8.4
3	10.9	10.5	11.8
4	14.0	13.5	15.2
5	17.1	16.5	18.6
6	20.2	19.5	21.9
7	23.3	22.5	25.3
8	26.4	25.5	—
9			

34°

'	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	'
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	60
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	0.16 965	9.91 815	9	55
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	50
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	9	45
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	0.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	40
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	0.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	9	35
26	9.75 239	19	9.83 605	27	0.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	0.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	0.16 287	9.91 599	8	30
31	9.75 331	19	9.83 740	28	0.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	0.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	8	20
41	9.75 514	19	9.84 011	27	0.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	9	15
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	0.15 746	9.91 425	9	10
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	0.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	9	5
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	9	1
60	9.75 859		9.84 523		0.15 477	9.91 336		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'

P. P.

	28	27	26
1	2.8	2.7	2.6
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	19	18
1	1.9	1.8
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	9 / 28	8 / 28	8 / 27
0	1.6	1.8	1.7
1	4.7	5.2	5.1
2	7.8	8.8	8.4
3	10.9	12.2	11.8
4	14.0	15.8	15.2
5	17.1	19.2	18.6
6	20.2	22.8	21.9
7	23.3	26.2	25.3
8	26.4	—	—
9			

P. P.

55°

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	**60**
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	0.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	45
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	0.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	0.14 941	9.91 158	9	**40**
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	0.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	0.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	35
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	0.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	0.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	**30**
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	25
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	**20**
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18
43	9.76 625	17	9.85 674	26	0.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	15
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14
47	9.76 695	17	9.85 780	27	0.14 220	9.90 915	9	13
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	**10**
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	0.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	0.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6
55	9.76 835	17	9.85 993	27	0.14 007	9.90 842	10	5
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	0.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	0.13 900	9.90 805	9	1
60	9.76 922		9.86 126		0.13 874	9.90 796		**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	10	9	8
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

	10/27	10/26
0	1.4	1.3
1	4.0	3.9
2	6.8	6.5
3	9.4	9.1
4	12.2	11.7
5	14.8	14.3
6	17.6	16.9
7	20.2	19.5
8	23.0	22.1
9	25.6	24.7
10		

	9/27	9/26
0	1.5	1.4
1	4.5	4.3
2	7.5	7.2
3	10.5	10.1
4	13.5	13.0
5	16.5	15.9
6	19.5	18.8
7	22.5	21.7
8	25.5	24.6
9		

P. P.

'	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	60
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	10	50
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	9	45
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	40
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	10	35
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	9	31
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	10	21
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	20
41	9.77 626	17	9.87 211	27	0.12 789	9.90 415	10	19
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	15
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	10	10
51	9.77 795	17	9.87 475	26	0.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	5
56	9.77 879	17	9.87 606	27	0.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	0.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	0.12 315	9.90 244	9	1
60	9.77 946		9.87 711		0.12 289	9.90 235		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	'

P. P.

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17	16
1	1.8	1.7	1.6
2	3.6	3.4	3.2
3	5.4	5.1	4.8
4	7.2	6.8	6.4
5	9.0	8.5	8.0
6	10.8	10.2	9.6
7	12.6	11.9	11.2
8	14.4	13.6	12.8
9	16.2	15.3	14.4

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	$\frac{9}{27}$	$\frac{9}{26}$
0	1.5	1.4
1	4.5	4.3
2	7.5	7.2
3	10.5	10.1
4	13.5	13.0
5	16.5	15.9
6	19.5	18.8
7	22.5	21.7
8	25.5	24.6
9		

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	60
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	55
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	9	40
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	35
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	9	31
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	10	30
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	0.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	0.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	25
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	0.11 241	9.89 849	9	20
41	9.78 625	17	9.88 786	26	0.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	10	15
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	10	10
51	9.78 788	17	9.89 046	27	0.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	9	5
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	17	16
1	1.7	1.6
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	$\frac{10}{27}$	$\frac{10}{26}$
0	1.4	1.3
1	4.0	3.9
2	6.8	6.5
3	9.4	9.1
4	12.2	11.7
5	14.8	14.3
6	17.6	16.9
7	20.2	19.5
8	23.0	22.1
9	25.6	24.7
10		

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	60
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	0.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	50
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	45
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	40
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	0.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	35
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	11	32
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	10	31
30	9.79 415	16	9.90 061	25	0.09 939	9.89 354	10	30
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	0.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	0.09 680	9.89 254	10	20
41	9.79 589	16	9.90 346	25	0.09 654	9.89 244	11	19
42	9.79 605	16	9.90 371	26	0.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	0.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	0.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	0.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	0.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	0.09 422	9.89 152	10	10
51	9.79 746	16	9.90 604	26	0.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	0.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	0.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	10	5
56	9.79 825	15	9.90 734	25	0.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	0.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	11	2
59	9.79 872	15	9.90 811	26	0.09 189	9.89 060	10	1
60	9.79 887		9.90 837		0.09 163	9.89 050		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	17	16	15
1	1.7	1.6	1.5
2	3.4	3.2	3.0
3	5.1	4.8	4.5
4	6.8	6.4	6.0
5	8.5	8.0	7.5
6	10.2	9.6	9.0
7	11.9	11.2	10.5
8	13.6	12.8	12.0
9	15.3	14.4	13.5

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

	10 / 26	10 / 25	9 / 26
0	1.3	1.2	1.4
1	3.9	3.8	4.3
2	6.5	6.2	7.2
3	9.1	8.8	10.1
4	11.7	11.2	13.0
5	14.3	13.8	15.9
6	16.9	16.2	18.8
7	19.5	18.8	21.7
8	22.1	21.2	24.6
9	24.7	23.8	—
10			

P. P.

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	**60**
1	9.79 903	15	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	16	9.90 889	25	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	26	0.09 086	9.89 020	11	57
4	9.79 950	15	9.90 940	26	0.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 981	15	9.90 992	26	0.09 008	9.88 989	11	54
7	9.79 996	16	9.91 018	25	0.08 982	9.88 978	10	53
8	9.80 012	15	9.91 043	26	0.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51
10	9.80 043	15	9.91 095	26	0.08 905	9.88 948	11	**50**
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49
12	9.80 074	15	9.91 147	25	0.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	26	0.08 828	9.88 917	11	47
14	9.80 105	15	9.91 198	26	0.08 802	9.88 906	10	46
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 136	15	9.91 250	26	0.08 750	9.88 886	11	44
17	9.80 151	15	9.91 276	25	0.08 724	9.88 875	10	43
18	9.80 166	16	9.91 301	26	0.08 699	9.88 865	10	42
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	11	41
20	9.80 197	16	9.91 353	26	0.08 647	9.88 844	10	**40**
21	9.80 213	15	9.91 379	25	0.08 621	9.88 834	10	39
22	9.80 228	16	9.91 404	26	0.08 596	9.88 824	11	38
23	9.80 244	15	9.91 430	26	0.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36
25	9.80 274	16	9.91 482	25	0.08 518	9.88 793	11	35
26	9.80 290	15	9.91 507	26	0.08 493	9.88 782	10	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	11	33
28	9.80 320	16	9.91 559	26	0.08 441	9.88 761	10	32
29	9.80 336	15	9.91 585	25	0.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	26	0.08 390	9.88 741	11	**30**
31	9.80 366	16	9.91 636	26	0.08 364	9.88 730	10	29
32	9.80 382	15	9.91 662	26	0.08 338	9.88 720	11	28
33	9.80 397	15	9.91 688	25	0.08 312	9.88 709	10	27
34	9.80 412	16	9.91 713	26	0.08 287	9.88 699	11	26
35	9.80 428	15	9.91 739	26	0.08 261	9.88 688	10	25
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	25	0.08 209	9.88 668	11	23
38	9.80 473	16	9.91 816	26	0.08 184	9.88 657	10	22
39	9.80 489	15	9.91 842	26	0.08 158	9.88 647	11	21
40	9.80 504	15	9.91 868	25	0.08 132	9.88 636	10	**20**
41	9.80 519	15	9.91 893	26	0.08 107	9.88 626	11	19
42	9.80 534	16	9.91 919	26	0.08 081	9.88 615	10	18
43	9.80 550	15	9.91 945	26	0.08 055	9.88 605	11	17
44	9.80 565	15	9.91 971	25	0.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	0.08 004	9.88 584	11	15
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	10	14
47	9.80 610	15	9.92 048	25	0.07 952	9.88 563	11	13
48	9.80 625	16	9.92 073	26	0.07 927	9.88 552	10	12
49	9.80 641	15	9.92 099	26	0.07 901	9.88 542	11	11
50	9.80 656	15	9.92 125	25	0.07 875	9.88 531	10	**10**
51	9.80 671	15	9.92 150	26	0.07 850	9.88 521	11	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	25	0.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	26	0.07 773	9.88 489	11	6
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	10	5
56	9.80 746	16	9.92 279	25	0.07 721	9.88 468	11	4
57	9.80 762	15	9.92 304	26	0.07 696	9.88 457	10	3
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	11	2
59	9.80 792	15	9.92 356	25	0.07 644	9.88 436	11	1
60	9.80 807		9.92 381		0.07 619	9.88 425		**0**
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	16	15
1	1.6	1.5
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

	11 / 26	11 / 25
0	1.2	1.1
1	3.5	3.4
2	5.9	5.7
3	8.3	8.0
4	10.6	10.2
5	13.0	12.5
6	15.4	14.8
7	17.7	17.0
8	20.1	19.3
9	22.5	21.6
10	24.8	23.9
11		

P. P.

40°

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.80 807	15	9.92 381	26	0.07 619	9.88 425	10	60
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	55
6	9.80 897	15	9.92 535	26	0.07 465	9.88 362	11	54
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	0.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	50
11	9.80 972	15	9.92 663	26	0.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48
13	9.81 002	15	9.92 715	25	0.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	0.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	11	45
16	9.81 047	14	9.92 792	25	0.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	26	0.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	0.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	0.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	40
21	9.81 121	15	9.92 920	25	0.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	0.07 055	9.88 191	11	38
23	9.81 151	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	0.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	0.06 978	9.88 158	10	35
26	9.81 195	15	9.93 048	25	0.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	0.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	25	0.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	0.06 876	9.88 115	10	31
30	9.81 254	15	9.93 150	25	0.06 850	9.88 105	11	30
31	9.81 269	15	9.93 175	26	0.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	25	0.06 773	9.88 072	11	27
34	9.81 314	14	9.93 252	26	0.06 748	9.88 061	10	26
35	9.81 328	15	9.93 278	25	0.06 722	9.88 051	11	25
36	9.81 343	15	9.93 303	26	0.06 697	9.88 040	11	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	0.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	25	0.06 594	9.87 996	11	20
41	9.81 417	14	9.93 431	26	0.06 569	9.87 985	10	19
42	9.81 431	15	9.93 457	25	0.06 543	9.87 975	11	18
43	9.81 446	15	9.93 482	26	0.06 518	9.87 964	11	17
44	9.81 461	14	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	0.06 467	9.87 942	11	15
46	9.81 490	15	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 505	14	9.93 584	26	0.06 416	9.87 920	11	13
48	9.81 519	15	9.93 610	26	0.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	25	0.06 364	9.87 898	11	11
50	9.81 549	14	9.93 661	26	0.06 339	9.87 887	10	10
51	9.81 563	15	9.93 687	25	0.06 313	9.87 877	11	9
52	9.81 578	14	9.93 712	26	0.06 288	9.87 866	11	8
53	9.81 592	15	9.93 738	25	0.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	26	0.06 237	9.87 844	11	6
55	9.81 622	14	9.93 789	25	0.06 211	9.87 833	11	5
56	9.81 636	15	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 651	14	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	0.06 135	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 084	9.87 778		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

	11/26	10/26	10/25
0			
	1.2	1.3	1.2
1			
	3.5	3.9	3.8
2			
	5.9	6.5	6.2
3			
	8.3	9.1	8.8
4			
	10.6	11.7	11.2
5			
	13.0	14.3	13.8
6			
	15.4	16.9	16.2
7			
	17.7	19.5	18.8
8			
	20.1	22.1	21.2
9			
	22.5	24.7	23.8
10			
	24.8	—	—
11			

P. P.

49°

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	′
0	9.81 694	15	9.93 916	26	0.06 084	9.87 778	11	60
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	0.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	0.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	55
6	9.81 781	15	9.94 069	26	0.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	0.05 880	9.87 690	11	52
9	9.81 825	14	9.94 146	25	0.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	0.05 829	9.87 668	11	50
11	9.81 854	14	9.94 197	25	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	0.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	0.05 752	9.87 635	11	47
14	9.81 897	14	9.94 273	26	0.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	0.05 701	9.87 613	12	45
16	9.81 926	14	9.94 324	26	0.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350	25	0.05 650	9.87 590	11	43
18	9.81 955	14	9.94 375	26	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	25	0.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	0.05 574	9.87 557	11	40
21	9.81 998	14	9.94 452	25	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	0.05 523	9.87 535	11	38
23	9.82 026	15	9.94 503	25	0.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	0.05 472	9.87 513	12	36
25	9.82 055	14	9.94 554	25	0.05 446	9.87 501	11	35
26	9.82 069	15	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	0.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	0.05 345	9.87 457	11	31
30	9.82 126	15	9.94 681	25	0.05 319	9.87 446	12	30
31	9.82 141	14	9.94 706	26	0.05 294	9.87 434	11	29
32	9.82 155	14	9.94 732	25	0.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	0.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	0.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	0.05 192	9.87 390	12	25
36	9.82 212	14	9.94 834	25	0.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	0.05 116	9.87 356	11	22
39	9.82 255	14	9.94 910	25	0.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	26	0.05 065	9.87 334	12	20
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	0.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	0.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	0.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	0.04 938	9.87 277	11	15
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	0.04 887	9.87 255	12	13
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	0.04 810	9.87 221	12	10
51	9.82 424	15	9.95 215	25	0.04 785	9.87 209	11	9
52	9.82 439	14	9.95 240	26	0.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	0.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	0.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	0.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	26	0.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	0.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	0.04 582	9.87 119	12	1
60	9.82 551		9.95 444		0.04 556	9.87 107		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	12	11
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

	12/26	12/25	11/25
0	1.1	1.0	1.1
1	3.2	3.1	3.4
2	5.4	5.2	5.7
3	7.6	7.3	8.0
4	9.8	9.4	10.2
5	11.9	11.5	12.5
6	14.1	13.5	14.8
7	16.2	15.6	17.0
8	18.4	17.7	19.3
9	20.6	19.8	21.6
10	22.8	21.9	23.9
11	24.9	24.0	—
12			

P. P.

42°

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.82 551	14	9.95 444	25	0.04 556	9.87 107	11	60
1	9.82 565	14	9.95 469	26	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	25	0.04 505	9.87 085	12	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	0.04 455	9.87 062	12	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	26	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	0.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	0.04 328	9.87 005	12	51
10	9.82 691	14	9.95 698	25	0.04 302	9.86 993	11	50
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	0.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	0.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	0.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825	25	0.04 175	9.86 936	12	45
16	9.82 775	13	9.95 850	25	0.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	26	0.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	25	0.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	0.04 048	9.86 879	12	40
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	0.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	0.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	0.03 922	9.86 821	12	35
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	0.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155	25	0.03 845	9.86 786	11	32
29	9.82 955	13	9.96 180	25	0.03 820	9.86 775	12	31
30	9.82 968	14	9.96 205	26	0.03 795	9.86 763	11	30
31	9.82 982	14	9.96 231	25	0.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	0.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	0.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25
36	9.83 051	14	9.96 357	26	0.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	0.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	0.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	0.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	0.03 541	9.86 647	12	20
41	9.83 120	13	9.96 484	26	0.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	0.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	0.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	0.03 414	9.86 589	12	15
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	0.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	0.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	0.03 288	9.86 530	12	10
51	9.83 256	14	9.96 738	25	0.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	0.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	0.03 186	9.86 483	11	6
55	9.83 310	14	9.96 839	25	0.03 161	9.86 472	12	5
56	9.83 324	14	9.96 864	26	0.03 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	0.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	0.03 060	9.86 425	12	1
60	9.83 378		9.96 966		0.03 034	9.86 413		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	13
1	1.4	1.3
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	11
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

	12/26	11/26	11/25
0			
1	1.1	1.2	1.1
2	3.2	3.5	3.4
3	5.4	5.9	5.7
4	7.6	8.3	8.0
5	9.8	10.6	10.2
6	11.9	13.0	12.5
7	14.1	15.4	14.8
8	16.2	17.7	17.0
9	18.4	20.1	19.3
10	20.6	22.5	21.6
11	22.8	24.8	23.9
12	24.9	—	—

P. P.

47°

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.		P. P.
0	9.83 378	14	9.96 966	25	0.03 034	9.86 413	12	60	
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59	
2	9.83 405	14	9.97 016	26	0.02 984	9.86 389	12	58	
3	9.83 419	13	9.97 042	25	0.02 958	9.86 377	11	57	
4	9.83 432	14	9.97 067	25	0.02 933	9.86 366	12	56	
5	9.83 446	13	9.97 092	26	0.02 908	9.86 354	12	55	
6	9.83 459	14	9.97 118	25	0.02 882	9.86 342	12	54	
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	12	53	
8	9.83 486	14	9.97 168	25	0.02 832	9.86 318	12	52	
9	9.83 500	13	9.97 193	26	0.02 807	9.86 306	11	51	
10	9.83 513	14	9.97 219	25	0.02 781	9.86 295	12	50	
11	9.83 527	13	9.97 244	25	0.02 756	9.86 283	12	49	
12	9.83 540	14	9.97 269	26	0.02 731	9.86 271	12	48	
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47	
14	9.83 567	14	9.97 320	25	0.02 680	9.86 247	12	46	
15	9.83 581	13	9.97 345	26	0.02 655	9.86 235	12	45	
16	9.83 594	14	9.97 371	25	0.02 629	9.86 223	12	44	
17	9.83 608	13	9.97 396	25	0.02 604	9.86 211	11	43	
18	9.83 621	13	9.97 421	26	0.02 579	9.86 200	12	42	
19	9.83 634	14	9.97 447	25	0.02 553	9.86 188	12	41	
20	9.83 648	13	9.97 472	25	0.02 528	9.86 176	12	40	
21	9.83 661	13	9.97 497	26	0.02 503	9.86 164	12	39	
22	9.83 674	14	9.97 523	25	0.02 477	9.86 152	12	38	
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37	
24	9.83 701	14	9.97 573	25	0.02 427	9.86 128	12	36	
25	9.83 715	13	9.97 598	26	0.02 402	9.86 116	12	35	
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34	
27	9.83 741	14	9.97 649	25	0.02 351	9.86 092	12	33	
28	9.83 755	13	9.97 674	26	0.02 326	9.86 080	12	32	
29	9.83 768	13	9.97 700	25	0.02 300	9.86 068	12	31	
30	9.83 781	14	9.97 725	25	0.02 275	9.86 056	12	30	
31	9.83 795	13	9.97 750	26	0.02 250	9.86 044	12	29	
32	9.83 808	13	9.97 776	25	0.02 224	9.86 032	12	28	
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27	
34	9.83 834	14	9.97 826	25	0.02 174	9.86 008	12	26	
35	9.83 848	13	9.97 851	26	0.02 149	9.85 996	12	25	
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24	
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23	
38	9.83 887	14	9.97 927	26	0.02 073	9.85 960	12	22	
39	9.83 901	13	9.97 953	25	0.02 047	9.85 948	12	21	
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	20	
41	9.83 927	13	9.98 003	26	0.01 997	9.85 924	12	19	
42	9.83 940	14	9.98 029	25	0.01 971	9.85 912	12	18	
43	9.83 954	13	9.98 054	25	0.01 946	9.85 900	12	17	
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16	
45	9.83 980	13	9.98 104	26	0.01 896	9.85 876	12	15	
46	9.83 993	13	9.98 130	25	0.01 870	9.85 864	13	14	
47	9.84 006	14	9.98 155	25	0.01 845	9.85 851	12	13	
48	9.84 020	13	9.98 180	26	0.01 820	9.85 839	12	12	
49	9.84 033	13	9.98 206	25	0.01 794	9.85 827	12	11	
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	10	
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9	
52	9.84 072	13	9.98 281	26	0.01 719	9.85 791	12	8	
53	9.84 085	13	9.98 307	25	0.01 693	9.85 779	13	7	
54	9.84 098	14	9.98 332	25	0.01 668	9.85 766	12	6	
55	9.84 112	13	9.98 357	26	0.01 643	9.85 754	12	5	
56	9.84 125	13	9.98 383	25	0.01 617	9.85 742	12	4	
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3	
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2	
59	9.84 164	13	9.98 458	26	0.01 542	9.85 706	13	1	
60	9.84 177		9.98 484		0.01 516	9.85 693		0	
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′	P. P.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	13
1	1.4	1.3
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	11
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

	13/26	13/25	12/25
0	1.0	1.0	1.0
1	3.0	2.9	3.1
2	5.0	4.8	5.2
3	7.0	6.7	7.3
4	9.0	8.7	9.4
5	11.0	10.6	11.5
6	13.0	12.5	13.5
7	15.0	14.4	15.6
8	17.0	16.3	17.7
9	19.0	18.3	19.8
10	21.0	20.2	21.9
11	23.0	22.1	24.0
12	25.0	24.0	—
13			

′	L. Sin.	d.	L. Tan.	c. d.	L. Cot.	L. Cos.	d.	
0	9.84 177	13	9.98 484	25	0.01 516	9.85 693	12	60
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	26	0.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	25	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	13	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	12	55
6	9.84 255	14	9.98 635	26	0.01 365	9.85 620	12	54
7	9.84 269	13	9.98 661	25	0.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	13	52
9	9.84 295	13	9.98 711	26	0.01 289	9.85 583	12	51
10	9.84 308	13	9.98 737	25	0.01 263	9.85 571	12	50
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	13	48
13	9.84 347	13	9.98 812	26	0.01 188	9.85 534	12	47
14	9.84 360	13	9.98 838	25	0.01 162	9.85 522	12	46
15	9.84 373	12	9.98 863	25	0.01 137	9.85 510	13	45
16	9.84 385	13	9.98 888	25	0.01 112	9.85 497	12	44
17	9.84 398	13	9.98 913	26	0.01 087	9.85 485	12	43
18	9.84 411	13	9.98 939	25	0.01 061	9.85 473	13	42
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	12	41
20	9.84 437	13	9.98 989	26	0.01 011	9.85 448	12	40
21	9.84 450	13	9.99 015	25	0.00 985	9.85 436	13	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	12	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	26	0.00 910	9.85 399	13	36
25	9.84 502	13	9.99 116	25	0.00 884	9.85 386	12	35
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	13	34
27	9.84 528	12	9.99 166	25	0.00 834	9.85 361	12	33
28	9.84 540	13	9.99 191	26	0.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	25	0.00 783	9.85 337	13	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	30
31	9.84 579	13	9.99 267	26	0.00 733	9.85 312	13	29
32	9.84 592	13	9.99 293	25	0.00 707	9.85 299	12	28
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	13	27
34	9.84 618	12	9.99 343	25	0.00 657	9.85 274	12	26
35	9.84 630	13	9.99 368	26	0.00 632	9.85 262	12	25
36	9.84 643	13	9.99 394	25	0.00 606	9.85 250	13	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12	23
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	13	22
39	9.84 682	12	9.99 469	26	0.00 531	9.85 212	12	21
40	9.84 694	13	9.99 495	25	0.00 505	9.85 200	13	20
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12	19
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	13	18
43	9.84 733	12	9.99 570	26	0.00 430	9.85 162	12	17
44	9.84 745	13	9.99 596	25	0.00 404	9.85 150	13	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	12	15
46	9.84 771	13	9.99 646	26	0.00 354	9.85 125	13	14
47	9.84 784	12	9.99 672	25	0.00 328	9.85 112	12	13
48	9.84 796	13	9.99 697	25	0.00 303	9.85 100	13	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	26	0.00 253	9.85 074	12	10
51	9.84 835	12	9.99 773	25	0.00 227	9.85 062	13	9
52	9.84 847	13	9.99 798	25	0.00 202	9.85 049	12	8
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	13	7
54	9.84 873	12	9.99 848	26	0.00 152	9.85 024	12	6
55	9.84 885	13	9.99 874	25	0.00 126	9.85 012	13	5
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4
57	9.84 911	12	9.99 924	25	0.00 076	9.84 986	12	3
58	9.84 923	13	9.99 949	26	0.00 051	9.84 974	13	2
59	9.84 936	13	9.99 975	25	0.00 025	9.84 961	12	1
60	9.84 949		0.00 000		0.00 000	9.84 949		0
	L. Cos.	d.	L. Cot.	c. d.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25	14
1	2.6	2.5	1.4
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	13/26	13/25
0	1.0	1.0
1	3.0	2.9
2	5.0	4.8
3	7.0	6.7
4	9.0	8.7
5	11.0	10.6
6	13.0	12.5
7	15.0	14.4
8	17.0	16.3
9	19.0	18.3
10	21.0	20.2
11	23.0	22.1
12	25.0	24.0
13		

	12/26	12/25
0	1.1	1.0
1	3.2	3.1
2	5.4	5.2
3	7.6	7.3
4	9.8	9.4
5	11.9	11.5
6	14.1	13.5
7	16.2	15.6
8	18.4	17.7
9	20.6	19.8
10	22.8	21.9
11	24.9	24.0
12		

P. P.

TABLES XVII., XVIII.

NATURAL TRIGONOMETRIC FUNCTIONS.

Natural Sines.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
0°	.0000 00	.0029 09	.0058 18	.0087 27	.0116 35	.0145 44	.0174 52	**89°**	2.9
1	.0174 52	.0203 6	.0232 7	.0261 8	.0290 8	.0319 9	.0349 0	88	2.9
2	.0349 0	.0378 1	.0407 1	.0436 2	.0465 3	.0494 3	.0523 4	87	2.9
3	.0523 4	.0552 4	.0581 4	.0610 5	.0639 5	.0668 5	.0697 6	86	2.9
4	.0697 6	.0726 6	.0755 6	.0784 6	.0813 6	.0842 6	.0871 6	85	2.9
5	.0871 6	.0900 5	.0929 5	.0958 5	.0987 4	.1016 4	.1045 3	**84**	2.9
6	.1045 3	.1074 2	.1103 1	.1132 0	.1160 9	.1189 8	.1218 7	83	2.9
7	.1218 7	.1247 6	.1276 4	.1305 3	.1334	.1363	.1392	82	2.9
8	.1392	.1421	.1449	.1478	.1507	.1536	.1564	81	2.9
9	.1564	.1593	.1622	.1650	.1679	.1708	.1736	80	2.9
10	.1736	.1765	.1794	.1822	.1851	.1880	.1908	**79**	2.9
11	.1908	.1937	.1965	.1994	.2022	.2051	.2079	78	2.9
12	.2079	.2108	.2136	.2164	.2193	.2221	.2250	77	2.8
13	.2250	.2278	.2306	.2334	.2363	.2391	.2419	76	2.8
14	.2419	.2447	.2476	.2504	.2532	.2560	.2588	75	2.8
15	.2588	.2616	.2644	.2672	.2700	.2728	.2756	**74**	2.8
16	.2756	.2784	.2812	.2840	.2868	.2896	.2924	73	2.8
17	.2924	.2952	.2979	.3007	.3035	.3062	.3090	72	2.8
18	.3090	.3118	.3145	.3173	.3201	.3228	.3256	71	2.8
19	.3256	.3283	.3311	.3338	.3365	.3393	.3420	70	2.7
20	.3420	.3448	.3475	.3502	.3529	.3557	.3584	**69**	2.7
21	.3584	.3611	.3638	.3665	.3692	.3719	.3746	68	2.7
22	.3746	.3773	.3800	.3827	.3854	.3881	.3907	67	2.7
23	.3907	.3934	.3961	.3987	.4014	.4041	.4067	66	2.7
24	.4067	.4094	.4120	.4147	.4173	.4200	.4226	65	2.7
25	.4226	.4253	.4279	.4305	.4331	.4358	.4384	**64**	2.6
26	.4384	.4410	.4436	.4462	.4488	.4514	.4540	63	2.6
27	.4540	.4566	.4592	.4617	.4643	.4669	.4695	62	2.6
28	.4695	.4720	.4746	.4772	.4797	.4823	.4848	61	2.6
29	.4848	.4874	.4899	.4924	.4950	.4975	.5000	60	2.5
30	.5000	.5025	.5050	.5075	.5100	.5125	.5150	**59**	2.5
31	.5150	.5175	.5200	.5225	.5250	.5275	.5299	58	2.5
32	.5299	.5324	.5348	.5373	.5398	.5422	.5446	57	2.5
33	.5446	.5471	.5495	.5519	.5544	.5568	.5592	56	2.4
34	.5592	.5616	.5640	.5664	.5688	.5712	.5736	55	2.4
35	.5736	.5760	.5783	.5807	.5831	.5854	.5878	**54**	2.4
36	.5878	.5901	.5925	.5948	.5972	.5995	.6018	53	2.3
37	.6018	.6041	.6065	.6088	.6111	.6134	.6157	52	2.3
38	.6157	.6180	.6202	.6225	.6248	.6271	.6293	51	2.3
39	.6293	.6316	.6338	.6361	.6383	.6406	.6428	50	2.3
40	.6428	.6450	.6472	.6494	.6517	.6539	.6561	**49**	2.2
41	.6561	.6583	.6604	.6626	.6648	.6670	.6691	48	2.2
42	.6691	.6713	.6734	.6756	.6777	.6799	.6820	47	2.2
43	.6820	.6841	.6862	.6884	.6905	.6926	.6947	46	2.1
44	.6947	.6967	.6988	.7009	.7030	.7050	.7071	45	2.1
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cosines.

Natural Sines.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
45°	.7071	.7092	.7112	.7133	.7153	.7173	.7193	**44°**	2.0
46	.7193	.7214	.7234	.7254	.7274	.7294	.7314	43	2.0
47	.7314	.7333	.7353	.7373	.7392	.7412	.7431	42	2.0
48	.7431	.7451	.7470	.7490	.7509	.7528	.7547	41	1.9
49	.7547	.7566	.7585	.7604	.7623	.7642	.7660	40	1.9
50	.7660	.7679	.7698	.7716	.7735	.7753	.7771	**39**	1.9
51	.7771	.7790	.7808	.7826	.7844	.7862	.7880	38	1.8
52	.7880	.7898	.7916	.7934	.7951	.7969	.7986	37	1.8
53	.7986	.8004	.8021	.8039	.8056	.8073	.8090	36	1.7
54	.8090	.8107	.8124	.8141	.8158	.8175	.8192	35	1.7
55	.8192	.8208	.8225	.8241	.8258	.8274	.8290	**34**	1.6
56	.8290	.8307	.8323	.8339	.8355	.8371	.8387	33	1.6
57	.8387	.8403	.8418	.8434	.8450	.8465	.8480	32	1.6
58	.8480	.8496	.8511	.8526	.8542	.8557	.8572	31	1.5
59	.8572	.8587	.8601	.8616	.8631	.8646	.8660	30	1.5
60	.8660	.8675	.8689	.8704	.8718	.8732	.8746	**29**	1.4
61	.8746	.8760	.8774	.8788	.8802	.8816	.8829	28	1.4
62	.8829	.8843	.8857	.8870	.8884	.8897	.8910	27	1.4
63	.8910	.8923	.8936	.8949	.8962	.8975	.8988	26	1.3
64	.8988	.9001	.9013	.9026	.9038	.9051	.9063	25	1.3
65	.9063	.9075	.9088	.9100	.9112	.9124	.9135	**24**	1.2
66	.9135	.9147	.9159	.9171	.9182	.9194	.9205	23	1.2
67	.9205	.9216	.9228	.9239	.9250	.9261	.9272	22	1.1
68	.9272	.9283	.9293	.9304	.9315	.9325	.9336	21	1.1
69	.9336	.9346	.9356	.9367	.9377	.9387	.9397	20	1.0
70	.9397	.9407	.9417	.9426	.9436	.9446	.9455	**19**	1.0
71	.9455	.9465	.9474	.9483	.9492	.9502	.9511	18	0.9
72	.9511	.9520	.9528	.9537	.9546	.9555	.9563	17	0.9
73	.9563	.9572	.9580	.9588	.9596	.9605	.9613	16	0.8
74	.9613	.9621	.9628	.9636	.9644	.9652	.9659	15	0.8
75	.9659	.9667	.9674	.9681	.9689	.9696	.9703	**14**	0.7
76	.9703	.9710	.9717	.9724	.9730	.9737	.9744	13	0.7
77	.9744	.9750	.9757	.9763	.9769	.9775	.9781	12	0.6
78	.9781	.9787	.9793	.9799	.9805	.9811	.9816	11	0.6
79	.9816	.9822	.9827	.9833	.9838	.9843	.9848	10	0.5
80	.9848	.9853	.9858	.9863	.9868	.9872	.9877	**9**	0.5
81	.9877	.9881	.9886	.9890	.9894	.9899	.9903	8	0.4
82	.9903	.9907	.9911	.9914	.9918	.9922	.9925	7	0.4
83	.9925	.9929	.9932	.9936	.9939	.9942	.9945	6	0.3
84	.9945	.9948	.9951	.9954	.9957	.9959	.9962	5	0.3
85	.9962	.9964	.9967	.9969	.9971	.9974	.9976	**4**	0.2
86	.9976	.9978	.9980	.9981	.9983	.9985	.9986	3	0.2
87	.9986	.9988	.9989	.9990	.9992	.9993	.9994	2	0.1
88	.9994	.9995	.9996	.9997	.9997	.9998	.9998	1	0.1
89	.9998	.9999	.9999	1.0000	1.0000	1.0000	1.0000	0	0.0
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cosines.

Natural Tangents.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
0°	.0000 0	.0029 1	.0058 2	.0087 3	.0116 4	.0145 5	.0174 6	**89°**	2.9
1	.0174 6	.0203 6	.0232 8	.0261 9	.0291 0	.0320 1	.0349 2	88	2.9
2	.0349 2	.0378 3	.0407 5	.0436 6	.0465 8	.0494 9	.0524 1	87	2.9
3	.0524 1	.0553 3	.0582 4	.0611 6	.0640 8	.0670 0	.0699 3	86	2.9
4	.0699 3	.0728 5	.0757 8	.0787 0	.0816 3	.0845 6	.0874 9	85	2.9
5	.0874 9	.0904 2	.0933 5	.0962 9	.0992 3	.1021 6	.1051 0	**84**	2.9
6	.1051 0	.1080 5	.1109 9	.1139 4	.1168 8	.1198 3	.1227 8	83	2.9
7	.1227 8	.1257 4	.1286 9	.1316 5	.1346	.1376	.1405	82	3.0
8	.1405	.1435	.1465	.1495	.1524	.1554	.1584	81	3.0
9	.1584	.1614	.1644	.1673	.1703	.1733	.1763	80	3.0
10	.1763	.1793	.1823	.1853	.1883	.1914	.1944	**79**	3.0
11	.1944	.1974	.2004	.2035	.2065	.2095	.2126	78	3.0
12	.2126	.2156	.2186	.2217	.2247	.2278	.2309	77	3.1
13	.2309	.2339	.2370	.2401	.2432	.2462	.2493	76	3.1
14	.2493	.2524	.2555	.2586	.2617	.2648	.2679	75	3.1
15	.2679	.2711	.2742	.2773	.2805	.2836	.2867	**74**	3.1
16	.2867	.2899	.2931	.2962	.2994	.3026	.3057	73	3.2
17	.3057	.3089	.3121	.3153	.3185	.3217	.3249	72	3.2
18	.3249	.3281	.3314	.3346	.3378	.3411	.3443	71	3.2
19	.3443	.3476	.3508	.3541	.3574	.3607	.3640	70	3.3
20	.3640	.3673	.3706	.3739	.3772	.3805	.3839	**69**	3.3
21	.3839	.3872	.3906	.3939	.3973	.4006	.4040	68	3.4
22	.4040	.4074	.4108	.4142	.4176	.4210	.4245	67	3.4
23	.4245	.4279	.4314	.4348	.4383	.4417	.4452	66	3.5
24	.4452	.4487	.4522	.4557	.4592	.4628	.4663	65	3.5
25	.4663	.4699	.4734	.4770	.4806	.4841	.4877	**64**	3.6
26	.4877	.4913	.4950	.4986	.5022	.5059	.5095	63	3.6
27	.5095	.5132	.5169	.5206	.5243	.5280	.5317	62	3.7
28	.5317	.5354	.5392	.5430	.5467	.5505	.5543	61	3.8
29	.5543	.5581	.5619	.5658	.5696	.5735	.5774	60	3.8
30	.5774	.5812	.5851	.5890	.5930	.5969	.6009	**59**	3.9
31	.6009	.6048	.6088	.6128	.6168	.6208	.6249	58	4.0
32	.6249	.6289	.6330	.6371	.6412	.6453	.6494	57	4.1
33	.6494	.6536	.6577	.6619	.6661	.6703	.6745	56	4.2
34	.6745	.6787	.6830	.6873	.6916	.6959	.7002	55	4.3
35	.7002	.7046	.7089	.7133	.7177	.7221	.7265	**54**	4.4
36	.7265	.7310	.7355	.7400	.7445	.7490	.7536	53	4.5
37	.7536	.7581	.7627	.7673	.7720	.7766	.7813	52	4.6
38	.7813	.7860	.7907	.7954	.8002	.8050	.8098	51	4.7
39	.8098	.8146	.8195	.8243	.8292	.8342	.8391	50	4.9
40	.8391	.8441	.8491	.8541	.8591	.8642	.8693	**49**	5.0
41	.8693	.8744	.8796	.8847	.8899	.8952	.9004	48	5.2
42	.9004	.9057	.9110	.9163	.9217	.9271	.9325	47	5.4
43	.9325	.9380	.9435	.9490	.9545	.9601	.9657	46	5.5
44	.9657	.9713	.9770	.9827	.9884	.9942	1.0000	45	5.7
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cotangents.

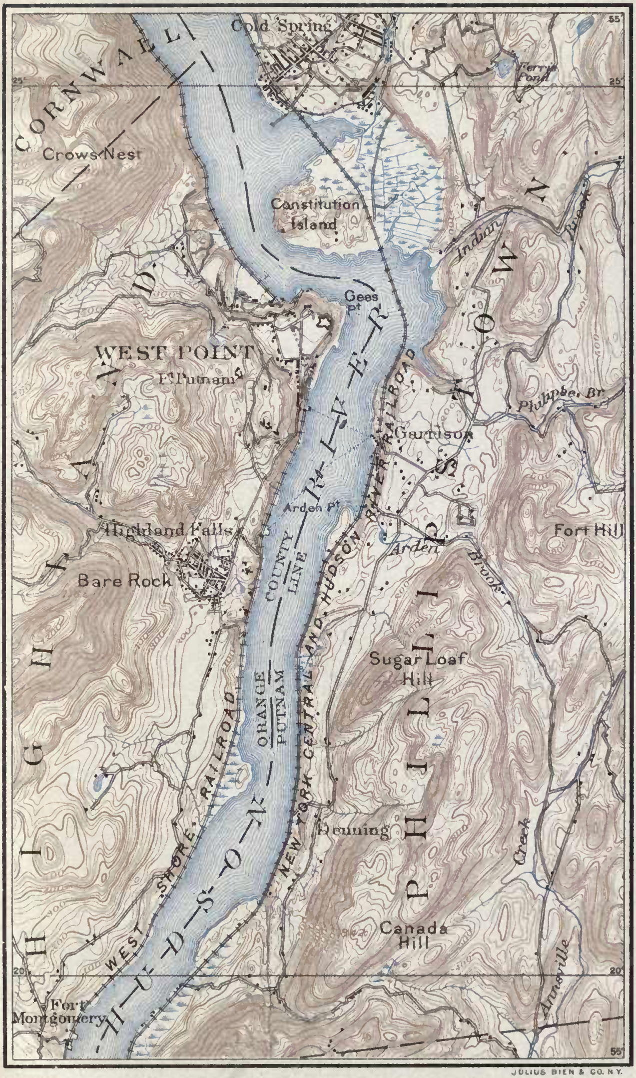
Natural Tangents.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′.
45°	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	**44°**	5.9
46	1.0355	1.0416	1.0477	1.0538	1.0599	1.0661	1.0724	43	6.1
47	1.0724	1.0786	1.0850	1.0913	1.0977	1.1041	1.1106	42	6.4
48	1.1106	1.1171	1.1237	1.1303	1.1369	1.1436	1.1504	41	6.6
49	1.1504	1.1571	1.1640	1.1708	1.1778	1.1847	1.1918	40	6.9
50	1.1918	1.1988	1.2059	1.2131	1.2203	1.2276	1.2349	**39**	7.2
51	1.2349	1.2423	1.2497	1.2572	1.2647	1.2723	1.2799	38	7.5
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37	7.9
53	1.3270	1.3351	1.3432	1.3514	1.3597	1.3680	1.3764	36	8.2
54	1.3764	1.3848	1.3934	1.4019	1.4106	1.4193	1.4281	35	8.6
55	1.4281	1.4370	1.4460	1.4550	1.4641	1.4733	1.4826	**34**	9.1
56	1.4826	1.4919	1.5013	1.5108	1.5204	1.5301	1.5399	33	9.6
57	1.5399	1.5497	1.5597	1.5697	1.5798	1.5900	1.6003	32	10.1
58	1.6003	1.6107	1.6212	1.6319	1.6426	1.6534	1.6643	31	10.7
59	1.6643	1.6753	1.6864	1.6977	1.7090	1.7205	1.7321	30	11.3
60	1.7321	1.7437	1.7556	1.7675	1.7796	1.7917	1.8040	**29**	12.0
61	1.8040	1.8165	1.8291	1.8418	1.8546	1.8676	1.8807	28	12.8
62	1.8807	1.8940	1.9074	1.9210	1.9347	1.9486	1.9626	27	13.6
63	1.9626	1.9768	1.9912	2.0057	2.0204	2.0353	2.0503	26	14.6
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25	15.7
65	2.1445	2.1609	2.1775	2.1943	2.2113	2.2286	2.2460	**24**	16.9
66	2.2460	2.2637	2.2817	2.2998	2.3183	2.3369	2.3559	23	18.3
67	2.3559	2.3750	2.3945	2.4142	2.4342	2.4545	2.4751	22	19.9
68	2.4751	2.4960	2.5172	2.5386	2.5605	2.5826	2.6051	21	21.7
69	2.6051	2.6279	2.6511	2.6746	2.6985	2.7228	2.7475	20	23.7
70	2.7475	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	**19**	
71	2.9042	2.9319	2.9600	2.9887	3.0178	3.0475	3.0777	18	
72	3.0777	3.1084	3.1397	3.1716	3.2041	3.2371	3.2709	17	
73	3.2709	3.3052	3.3402	3.3759	3.4124	3.4495	3.4874	16	
74	3.4874	3.5261	3.5656	3.6059	3.6470	3.6891	3.7321	15	
75	3.7321	3.7760	3.8208	3.8667	3.9136	3.9617	4.0108	**14**	
76	4.0108	4.0611	4.1126	4.1653	4.2193	4.2747	4.3315	13	
77	4.3315	4.3897	4.4494	4.5107	4.5736	4.6382	4.7046	12	
78	4.7046	4.7729	4.8430	4.9152	4.9894	5.0658	5.1446	11	
79	5.1446	5.2257	5.3093	5.3955	5.4845	5.5764	5.6713	10	
80	5.6713	5.7694	5.8708	5.9758	6.0844	6.1970	6.3138	**9**	
81	6.3138	6.4348	6.5606	6.6912	6.8269	6.9682	7.1154	8	
82	7.1154	7.2687	7.4287	7.5958	7.7704	7.9530	8.1443	7	
83	8.1443	8.3450	8.5555	8.7769	9.0098	9.2553	9.5144	6	
84	9.5144	9.7882	10.0780	10.3854	10.7119	11.0594	11.4301	5	
85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	14.3007	**4**	
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750	19.0811	3	
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	28.6363	2	
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039	57.2900	1	
89	57.2900	68.7501	85.9398	114.5887	171.8854	343.7737	∞	0	
	60′	50′	40′	30′	20′	10′	0′	Angle.	

Natural Cotangents.

PLATE I

JULIUS BIEN & CO. N.Y.



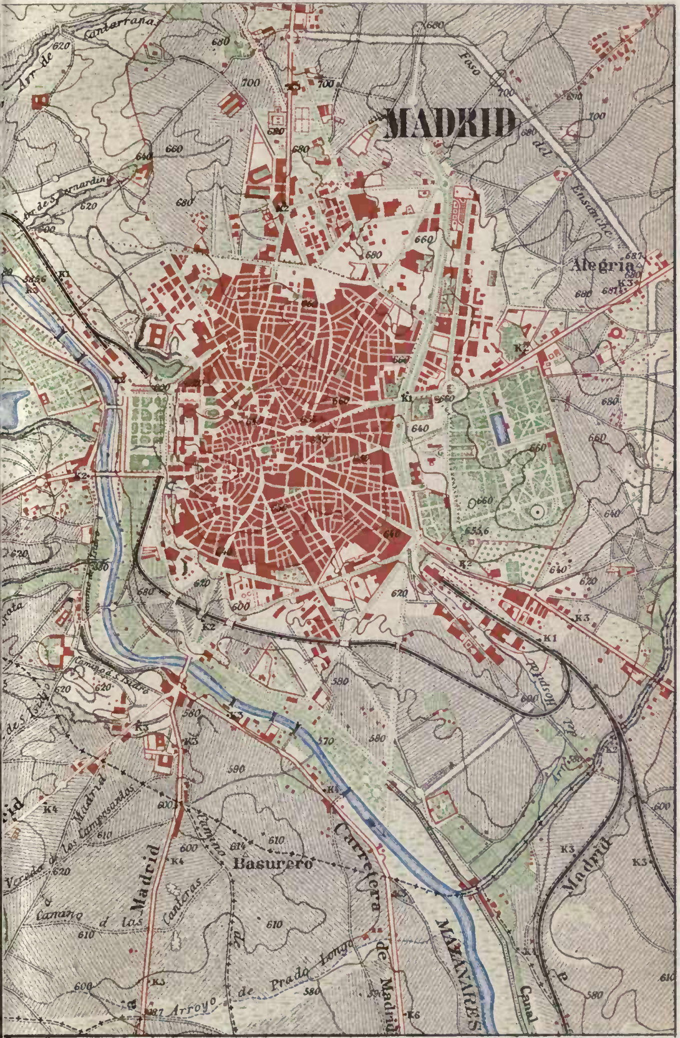
JULIUS BIEN & CO. N.Y.

Initial Meridian – Madrid

Scale

0 500 1000 20

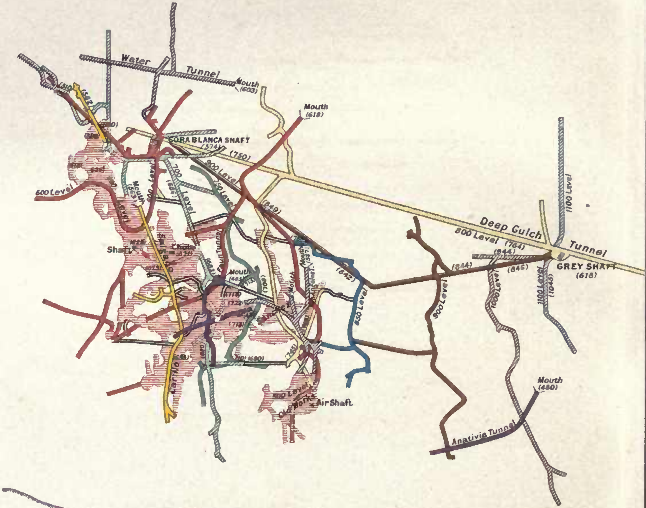
PLATE III.

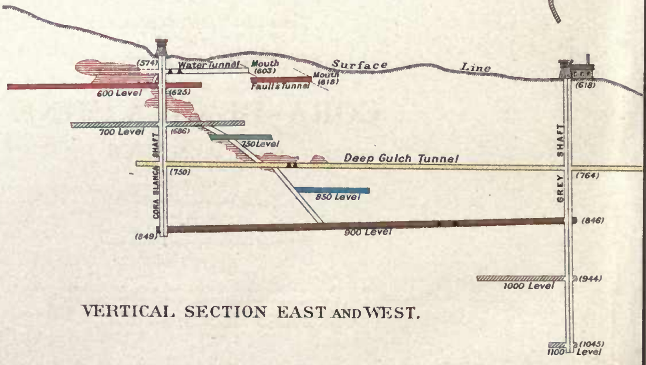


JULIUS BIEN & CO. N.Y.

1 / 0,000

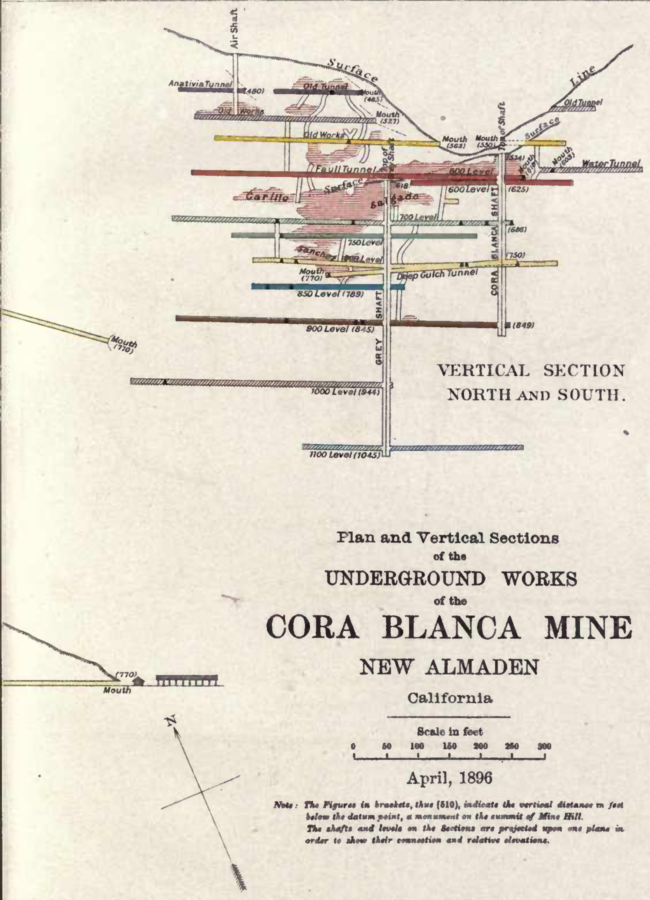
3000 4000 Meters





VERTICAL SECTION EAST AND WEST.

PLATE IV.



Plan and Vertical Sections
of the
UNDERGROUND WORKS
of the

CORA BLANCA MINE

NEW ALMADEN

California

Scale in feet

0 50 100 150 200 250 300

April, 1896

Note: The Figures in brackets, thus (510), indicate the vertical distance in feet below the datum point, a monument on the summit of Mine Hill. The shafts and levels on the Sections are projected upon one plane in order to show their connection and relative elevations.

JULIUS BIEN & CO. N.Y.

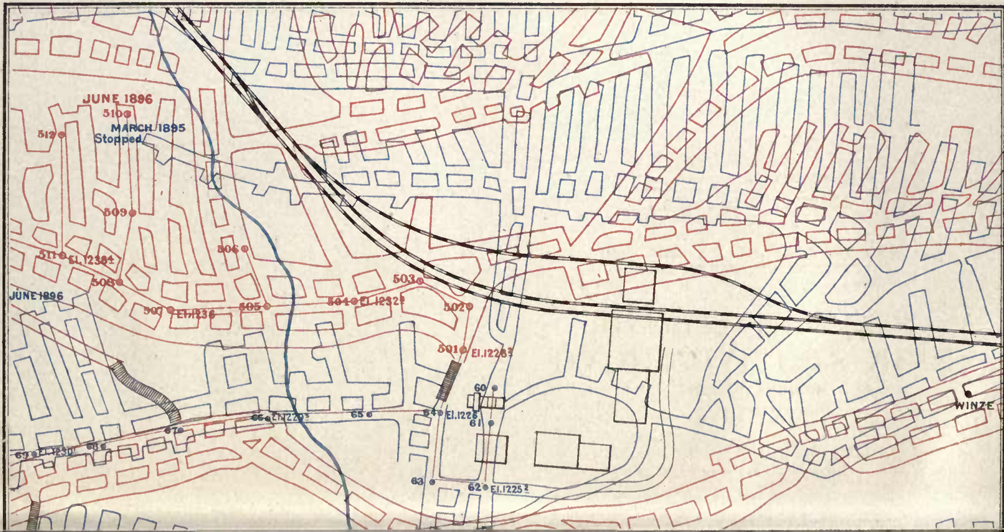

JUNE 1896
510
MARCH 1895
Stopped
512
509
506
511
508
503
JUNE 1896
505
504
502
501
60
61
64
El.1226
65
66
67
68
69
63
62
WINZE

PLATE V.

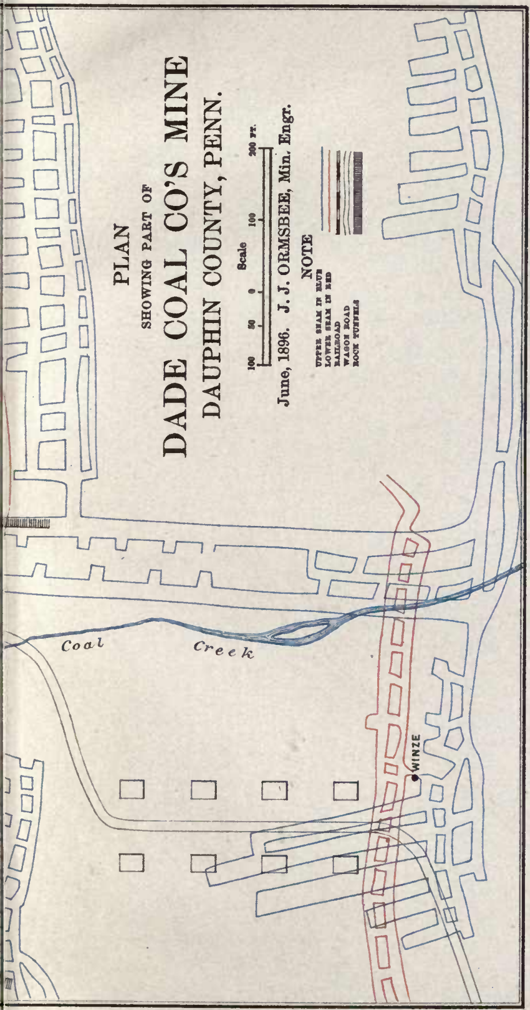
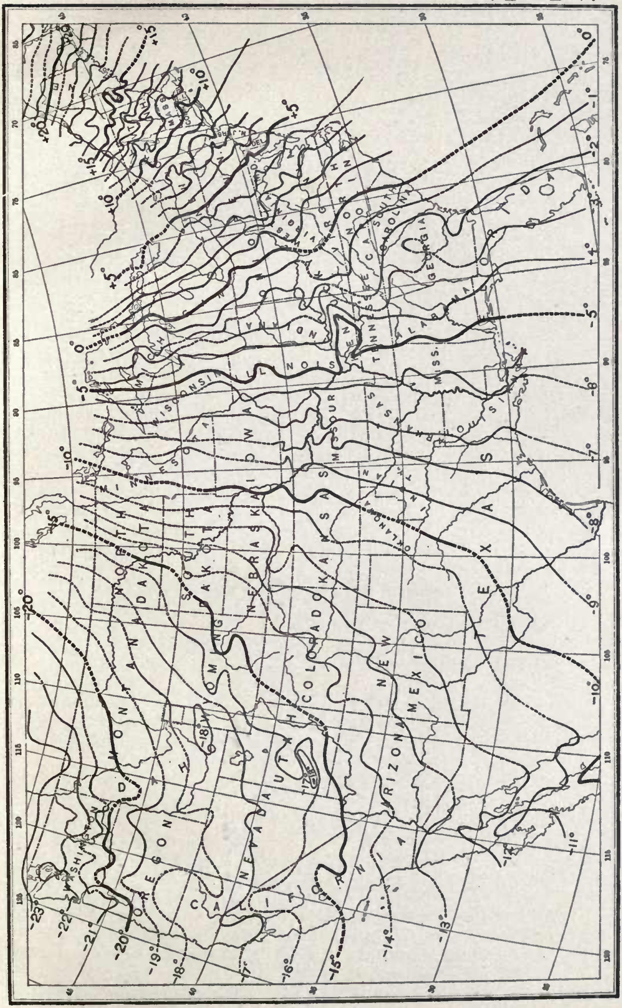

PLAN
SHOWING PART OF
DADE COAL CO'S MINE
DAUPHIN COUNTY, PENN.
Scale
100 50 0 100 200 FT.
June, 1896. J. J. ORMSBEE, Min. Engr.
NOTE
UPPER SEAM IN BLUE
LOWER SEAM IN RED
RAILROAD
WAGON ROAD
ROCK TUNNELS
Coal Creek
WINZE

PLATE VI



WISCONSIN
ILLINOIS
GEORGIA
MISS.
COLORADO
NEBRASKA
WYOMING
ARIZONA
NEW MEXICO
NEVADA
OREGON
TEXAS
OKLAHOMA

INDEX.

Typography by J. S. Cushing & Co., Norwood, Mass.