THEORETICAL NAVAL ARCHITECTURE

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PREFACE

THE present work is an expansion of the Author's "Text-Book of Theoretical Naval Architecture" originally published in 1899. It has been prepared in order to provide students and draughtsmen engaged in Shipbuilders' and Naval Architects' drawing offices with a text-book which should explain the calculations which have continually to be carried out. It is intended also that the work should form a text-book for the various examinations which are held in this subject. The subject of Naval Architecture is continually growing, and it is impossible to deal satisfactorily with it in any one book, but the object of the Author has been to show how all the ordinary ship calculations can be intelligently carried out and to give the student a groundwork of knowledge on which further progress can be based.

A special feature of the book is the large number of examples given in the text and at the ends of the various chapters. By means of these examples, the student is enabled to test his grasp of the principles and processes given in the text. It has been found that this feature is much appreciated, especially by students who have to work without the aid of a teacher.

A bibliography of works on Naval Architecture and cognate subjects is given at the end with short notices as to the scope and value of the various works.

The Author ventures to hope that the present work may be found useful to that large class which desires to obtain a working knowledge of the first principles of Theoretical Naval Architecture.

September, 1922.

E. L. ATTWOOD.

REMARKS ON EDUCATION IN NAVAL ARCHITECTURE

For the bulk of those who study the subject of Naval Architecture, the only instruction possible is obtained in evening classes, and this must be supplemented by private study. The institutions in which systematic instruction in day courses is given are few in number, viz. (1) Armstrong College, University of Durham, Newcastle-on-Tyne; (2) University of Glasgow; (3) University of Liverpool; (4) Royal Naval College, Greenwich; and students who can obtain the advantage of this training are comparatively few in number. An account of the course at Glasgow is to be found in a paper before the I.N.A. in 1889 by the late Prof. Jenkins, and at the Royal Naval College, in a paper before the I.N.A. in 1905 by the writer; see also a paper by Professor Welch on the scientific education of naval architects before N.E. Coast Institution, 1909. There are scholarships to be obtained for such higher education, particulars of which can be had by application to the Glasgow, Liverpool, and Newcastle Colleges, to the Secretary of the Admiralty, Whitehall, S.W., and to the Secretary of the Institution of Naval Architects, Adelphi Terrace, Strand. In these courses it is recognized that the study of other subjects must proceed concurrently with that of Naval Architecture.

The Naval Architect has to be responsible for the ship as a complete design, and in this capacity should have some familiarity with all that pertains to a ship. Thus he should know something of Marine Engineering (especially of propellers); of Electricity and Magnetism; of armour, guns and gun-mountings in warships; of masts, rig, etc., in sailing vessels; of the work of the stevedore in cargo vessels; of questions relating to the docking and undocking of ships; of appliances for loading and unloading of ships; of the regulations of the Registration Societies and the Board of Trade regarding structure, freeboard, and tonnage; of appliances for navigating, as well as having a thorough knowledge of the practical work of the shipyard. In the early stages of a design, the naval architect frequently has to proceed independently in trying alternatives for the desired result, and it is not until the design is somewhat matured that he can call in the assistance of specialists in other departments. The naval architect should, therefore, have an interest in everything connected with the type of ship he has to deal with, and he will continually be collecting data which may be of use to him in his subsequent work.

For the average student of Naval Architecture, in addition to the work he does and observes in the shipyard, mould loft, and drawing office, it is necessary to attend evening classes in Naval Architecture and other subjects. The apprentice should systematically map out his time for this purpose. In the first place, a good grounding should be obtained in mechanical drawing and in elementary mathematics. Both of these subjects are now taught by admirable methods. The drawing classes are usually primarily intended for Engineering students, but this is no drawback, as it will familiarize the student with drawings of engineering details which he will find of considerable service to him in his subsequent work. Some institutions very wisely do not allow students to take up the study of any special subject, as Naval Architecture, until they have proved themselves proficient in elementary drawing and mathematics. The time thus spent is a most profitable investment.

The Board of Education now only hold examinations in two stages, a "lower" and a "higher," see p. 480, but teachers will probably divide the work between these stages, and themselves hold examinations.

We will suppose, then, that a student starts definitely with the lowest class in Naval Architecture. With this subject he should also take up Elementary Applied Mechanics, and, if possible, some Mathematics. The next year may be devoted to the Board of Education Lower Examination in Naval Architecture, with a course in more advanced Applied Mechanics, and a course in Magnetism and Electricity or Chemistry would form a welcome relief. The next year may be devoted to further study in Mathematics, Theoretical and Applied Mechanics, Electricity and Magnetism. The next year may be devoted to another class in Naval Architecture, with more advanced Mathematics, including the Differential and Integral Calculus. This latter branch of mathematics is essential in order to make any progress in the higher branches of any engineering subject. If the student is fortunate enough to live in a large shipbuilding district, he will be able to attend lectures preparing him for the Board of Education Higher Stage Examination in Naval Architecture. A first-class certificate in this stage is worth having, and in preparing for the examination, the student must to a large extent read on his own account, and for that year he will be well advised to devote his whole attention to this subject. Much will depend on the particular arrangements of teaching adopted in a district as to how the work can be best spread over a series of years.

In making the above remarks, the writer wishes to emphasize the fact that a student cannot be said to learn Naval Architecture by merely attending Naval Architecture classes. Teachers in this subject have not the time to teach Geometry, Applied Mechanics, or Mathematics, and unless these subjects are familiar to the student, his education will be of a very superficial nature. Teachers of the subject are always ready to advise students as to the course of study likely to be most beneficial in any given case.

Students are strongly advised to make themselves familiar with the use of the "slide rule," which enables ship calculations to be rapidly performed.

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CHAPTER I.

AREAS, VOLUMES, WEIGHTS, DISPLACEMENT, ETC.

Areas of Plane Figures.

A Rectangle.—This is a four-sided figure having its opposite sides parallel to one another and all its angles right angles. Such a figure is shown in Fig. 1. Its area is the product of the length and the breadth, or AB × BC. Thus a rectangular plate 6 feet long and 3 feet broad will contain—

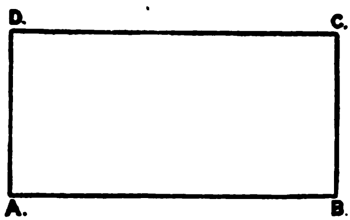


Fig. 1.

$$6 \times 3 = 18 \text{ square feet}$$

and if of such a thickness as to weigh $12\frac{1}{2}$ lbs. per square foot, will weigh—

$$18 \times 12\frac{1}{2} = 225 \text{ lbs.}$$

A Square.—This is a particular case of the above, the length being equal to the breadth. Thus a square hatch of $3\frac{1}{2}$ feet side will have an area of—

$$3\frac{1}{2} \times 3\frac{1}{2} = \frac{7}{2} \times \frac{7}{2} = \frac{49}{4}$$
$$= 12\frac{1}{4} \text{ square feet}$$

A Triangle.—This is a figure contained by three straight lines, as ABC in Fig. 2. From the vertex C drop a perpendicular on to the base AB (or AB produced, if necessary). Then the area is given by half the product of the base into the height, or—

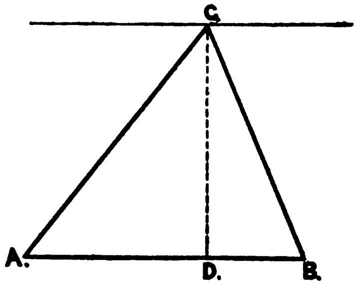


FIG. 2.

$$\tfrac{1}{2}(AB \times CD)$$

If we draw through the apex C a line parallel to the base AB, any triangle having its apex on this line, and having AB for its base, will be equal in area to the triangle ABC. If more convenient, we can consider either A or B as the apex, and BC or AC accordingly as the base.

Thus a triangle of base $5\frac{1}{2}$ feet and perpendicular drawn from the apex $2\frac{1}{4}$ feet, will have for its area—

$$\tfrac{1}{2} \times 5\tfrac{1}{2} \times 2\tfrac{1}{4} = \tfrac{1}{2} \times \tfrac{11}{2} \times \tfrac{9}{4} = \tfrac{99}{16}$$
$$= 6\tfrac{3}{16} \text{ square feet}$$

If this triangle be the form of a plate weighing 20 lbs. to the square foot, the weight of the plate will be—

$$\tfrac{99}{16} \times 20 = 123\tfrac{3}{4} \text{ lbs.}$$

A Trapezoid.—This is a figure formed of four straight lines, of which two only are parallel. Fig. 3 gives such a figure, ABCD.

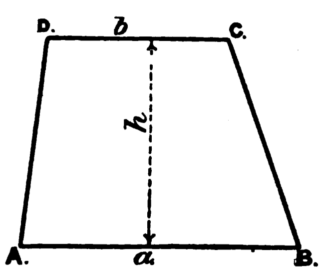


FIG. 3.

If the lengths of the parallel sides AB and CD are a and b respectively, and h is the perpendicular distance between them, the area of the trapezoid is given by—

$$\tfrac{1}{2}(a + b) \times h$$

or one-half the sum of the parallel sides multiplied by the perpendicular distance between them.

Example.—An armour plate is of the form of a trapezoid with parallel sides 8′ 3″ and 8′ 9″ long, and their distance apart 12 feet. Find its weight if 6 inches thick, the material of the armour plate weighing 490 lbs. per cubic foot.

First we must find the area, which is given by—

$$\left(\frac{8' \; 3'' + 8' \; 9''}{2}\right) \times 12 \text{ square feet} = \frac{17}{2} \times 12$$
$$= 102 \text{ square feet}$$

The plate being 6 inches thick $= \frac{1}{2}$ foot, the cubical contents of the plate will be—

$$102 \times \tfrac{1}{2} = 51 \text{ cubic feet}$$

The weight will therefore be—

$$51 \times 490 \text{ lbs.} = \frac{51 \times 490}{2240}$$
$$= 11{\cdot}15 \text{ tons}$$

A Trapezium is a quadrilateral or four-sided figure of which no two sides are parallel.

Such a figure is ABCD (Fig. 4). Its area may be found by drawing a diagonal BD and adding together the areas of the triangles ABD, BDC. These both have the same base, BD. Therefore from A and C drop perpendiculars AE and CF on to BD. Then the area of the trapezium is given by—

$$\tfrac{1}{2}(AE + CF) \times BD$$

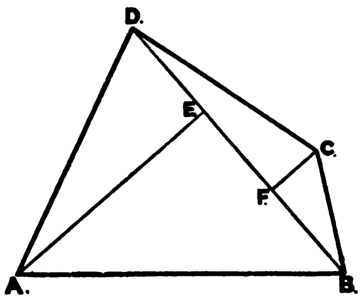


FIG. 4.

Example.—Draw a trapezium on scale $\frac{1}{8}$ inch = 1 foot, where four sides taken in order are 6, 5, 6, and 10 feet respectively, and the diagonal from the starting-point 10 feet. Find its area in square feet.

Ans. 40 sq. feet.

A Circle.—This is a figure all points of whose boundary are equally distant from a fixed point within it called the *centre*. The boundary is called its *circumference*, and any line from the centre to the circumference is called a *radius*. Any line passing through the centre and with its ends on the circumference is called a *diameter*.

The ratio between the circumference of a circle and its diameter is called π,[1] and $\pi = 3{\cdot}1416$, or nearly $\frac{22}{7}$

Thus the length of a thin wire forming the circumference of a circle of diameter 5 feet is given by—

$$\pi \times 5 = 5 \times 3{\cdot}1416 \text{ feet}$$
$$= 15{\cdot}7080 \text{ feet}$$
$$\text{or using } \pi = \tfrac{22}{7}, \text{ the circumference} = 5 \times \tfrac{22}{7}$$
$$= \tfrac{110}{7} = 15\tfrac{5}{7} \text{ feet}$$

The circumference of a mast 2′ 6″ in diameter is given by—

$$2\tfrac{1}{2} \times \pi \text{ feet} = \tfrac{5}{2} \times \tfrac{22}{7}$$
$$= \tfrac{55}{7} = 7\tfrac{6}{7} \text{ feet}$$

The *area of a circle* of diameter d is given by—

$$\frac{\pi \times d^2}{4} \qquad (d^2 = d \times d)$$

Thus a solid pillar 4 inches in diameter has a sectional area of—

$$\frac{\pi \times 4^2}{4} = \tfrac{22}{7} \times 4$$
$$= 12\tfrac{4}{7} \text{ square inches}$$

A hollow pillar 5 inches external diameter and $\frac{1}{4}$ inch thick will have a sectional area obtained by subtracting the area of a circle $4\frac{1}{2}$ inches diameter from the area of a circle 5 inches diameter

$$= \left(\frac{\pi(5)^2}{4}\right) - \left(\frac{\pi(4\frac{1}{2})^2}{4}\right)$$
$$= 3{\cdot}73 \text{ square inches}$$

The same result may be obtained by taking a mean diameter of the ring, finding its circumference, and multiplying by the breadth of the ring.

$$\text{Mean diameter} = 4\tfrac{3}{4} \text{ inches}$$
$$\text{Circumference} = \tfrac{19}{4} \times \tfrac{22}{7} \text{ inches}$$
$$\text{Area} = (\tfrac{19}{4} \times \tfrac{22}{7}) \times \tfrac{1}{4} \text{ square inches}$$
$$= 3{\cdot}73 \text{ square inches as before}$$

[1] This is the Greek letter *pi*, and is always used to denote 3·1416, or $\frac{22}{7}$ nearly; that is, the ratio borne by the circumference of a circle to its diameter.

Trapezoidal Rule.[1]—We have already seen (p. 2) that the area of a trapezoid, as ABCD, Fig. 5, is given by $\frac{1}{2}(AD + BC)AB$, or calling AD, BC, and AB y_1, y_2, and h respectively the area is given by—

$$\frac{1}{2}(y_1 + y_2)h$$

If, now, we have two trapezoids joined together, as in

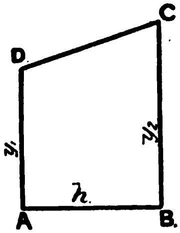


FIG. 5.

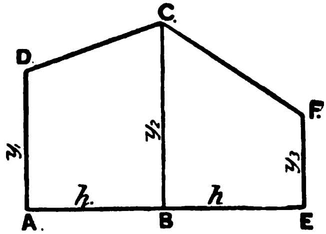


FIG. 6.

Fig. 6, having BE = AB, the area of the added part will be given by—

$$\frac{1}{2}(y_2 + y_3)h$$

The area of the whole figure is given by—

$$\frac{1}{2}(y_1 + y_2)h + \frac{1}{2}(y_2 + y_3)h = \frac{1}{2}h(y_1 + 2y_2 + y_3)$$

If we took a third trapezoid and joined on in a similar manner, the area of the whole figure would be given by

$$\frac{1}{2}h(y_1 + 2y_2 + 2y_3 + y_4) = h\left(\frac{y_1 + y_4}{2} + y_2 + y_3\right)$$

Trapezoidal rule for finding the area of a curvilinear figure, as ABCD, Fig. 7.

Divide the base AB into a convenient number of equal parts, as AE, EG, etc., each of length equal to h, say. Set up perpendiculars to the base, as EF, GH, etc. If we join DF, FH, etc., by straight lines, shown dotted, the area required will very nearly equal the sum of the areas of the trapezoids ADFE, EFHG, etc. Or using the lengths y_1, y_2, etc., as indicated in the figure—

$$\text{Area} = h\left(\frac{y_1 + y_7}{2} + y_2 + y_3 + y_4 + y_5 + y_6\right)$$

[1] The Trapezoidal rule is largely used in France and in the United States for ship calculations.

In the case of the area shown in Fig. 7, the area will be somewhat greater than that given by this rule. If the curve, however, bent towards the base line, the actual area would be somewhat less than that given by this rule. In any case, the closer the perpendiculars are taken together the less will be the error involved by using this rule. Putting this rule into words, we have—

To find the area of a curvilinear figure, as ABCD, Fig. 7, *by means of the trapezoidal rule, divide the base into any convenient number of equal parts, and erect perpendiculars to the base meeting the curve; then to the half-sum of the first and last of these add the sum of all the intermediate ones; the result multiplied by the common distance apart will give the area required*

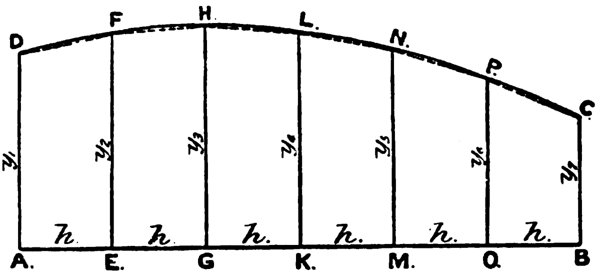


Fig. 7.

The perpendiculars to the base AB, as AD, EF, are termed "*ordinates*," and any measurement along the base from a given starting-point is termed an "*abscissa*." Thus the point P on the curve has an ordinate OP and an abscissa AO when referred to the point A as origin.

Simpson's First Rule.[1]—This rule assumes that the curved line DC, forming one boundary of the curvilinear area ABCD, Fig. 8, is a portion of a curve known as *a parabola of the second order.*[2] In practice it is found that the results given by its application to ordinary curves are very accurate, and it is

[1] It is usual to call these rules Simpson's rules, but the first rule was given before Simpson's time by James Stirling, in his "Methodus Differentialis," published in 1730.

[2] A "parabola of the second order" is one whose equation referred to co-ordinate axes is of the form $y = a_0 + a_1x + a_2x^2$, where a_0, a_1, a_2 are constants.

this rule that is most extensively used in this country in finding the areas of curvilinear figures required in ship calculations.

Let ABCD, Fig. 8, be a figure bounded on one side by the curved line DC, which, as stated above, is assumed to be a parabola of the second order. AB is the base, and AD and BC are end ordinates perpendicular to the base.

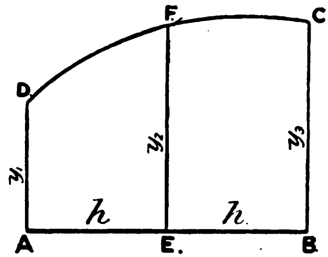


FIG. 8.

Bisect AB in E, and draw EF perpendicular to AB, meeting the curve in F. Then the area is given by—

$$\frac{1}{3}AE(AD + 4EF + BC)$$

or using y_1, y_2, y_3 to represent the ordinates, h the common interval between them—

$$\text{Area} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

Now, a long curvilinear area[1] may be divided up into a number of portions similar to the above, to each of which the above rule will apply. Thus the area of the portion GHNM of the area Fig. 7 will be given by—

$$\frac{h}{3}(y_3 + 4y_4 + y_5)$$

and the portion MNCB will have an area given by—

$$\frac{h}{3}(y_5 + 4y_6 + y_7)$$

Therefore the total area will be, supposing all the ordinates are a common distance h apart—

$$\frac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + y_7)$$

Ordinates, as GH, MN, which divide the figure into the elementary areas are termed "*dividing ordinates.*"

Ordinates between these, as EF, KL, OP, are termed "*intermediate ordinates.*"

[1] The curvature is supposed continuous. If the curvature changes abruptly at any point, this point must be at a dividing ordinate.

Notice that the area must have an *even* number of *intervals*, or, what is the same thing, an *odd* number of *ordinates*, for Simpson's first rule to be applicable.

Therefore, putting Simpson's first rule into words, we have—

Divide the base into a convenient even number of equal parts, and erect ordinates meeting the curve. Then to the sum of the end ordinates add four times the even ordinates and twice the odd ordinates. The sum thus obtained, multiplied by one-third the common distance apart of the ordinates, will give the area.

Approximate Proof of Simpson's First Rule.—The truth of Simpson's first rule may be understood by the following approximate proof:[1]—

Let DFC, Fig. 9, be a curved line on the base AB, and with end ordinates AD, BC perpendicular to AB. Divide AB equally in E, and draw the ordinate EF perpendicular to AB. Then with the ordinary notation—

$$\text{Area} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

by Simpson's first rule. Now divide AB into three equal parts by the points G and H. Draw perpendiculars GJ and HK to the base AB. At F draw a tangent to the curve, meeting GJ and HK in J and K. Join DJ and KC. Now, it is evident that the area we want is very nearly equal to the area ADJKCB. This will be found by adding together the areas of the trapezoids ADJG, GJKH, HKCB.

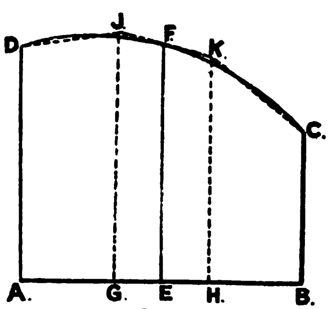


Fig. 9.

$$\text{Area of ADJG} = \tfrac{1}{2}(\text{AD} + \text{GJ})\text{AG}$$
$$\text{,, GJKH} = \tfrac{1}{2}(\text{GJ} + \text{HK})\text{GH}$$
$$\text{,, HKCB} = \tfrac{1}{2}(\text{HK} + \text{BC})\text{HB}$$

[1] Another proof will be found on p. 77. The mathematical proof will be found in Appendix A.

Now, AG = GH = HB = $\frac{1}{3}$AB = $\frac{2}{3}$AE, therefore the total area is—

$$\tfrac{1}{3}\left(\frac{2AE}{3}\right)(AD + 2GJ + 2HK + BC)$$

Now, AE = h, and GJ + HK = 2EF (this may be seen at once by measuring with a strip of paper), therefore the total area is—

$$\frac{h}{3}(AD + 4EF + BC) = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

which is the same as that given by Simpson's first rule.

Application of Simpson's First Rule.—*Example.*—A curvilinear area has ordinates at a common distance apart of 2 feet, the lengths being 1·45, 2·65, 4·35, 6·45, 8·50, 10·40, and 11·85 feet respectively. Find the area of the figure in square feet.

In finding the area of such a curvilinear figure by means of Simpson's first rule, the work is arranged as follows:—

Number of ordinate.	Length of ordinate.	Simpson's multipliers.[1]	Functions of ordinates.
1	1·45	1	1·45
2	2·65	4	10·60
3	4·35	2	8·70
4	6·45	4	25·80
5	8·50	2	17·00
6	10·40	4	41·60
7	11·85	1	11·85

117·00 sum of functions

Common interval = 2 feet

$\frac{1}{3}$ common interval = $\frac{2}{3}$ feet

area = 117 × $\frac{2}{3}$ = 78 square feet

The length of this curvilinear figure is 12 feet, and it has been divided into an *even* number of intervals, viz. 6, 2 feet apart, giving an *odd* number of ordinates, viz. 7. We are consequently able to apply Simpson's first rule to finding its area. Four columns are used. In the first column are placed the numbers of the ordinates, starting from one end of the figure. In the second column are placed, in the proper order, the lengths of the ordinates corresponding to the numbers in the first column. These lengths are expressed in feet and

[1] Sometimes the multipliers used are half these, viz. $\frac{1}{2}$, 2, 1, 2, 1, 2, $\frac{1}{2}$, and the result at the end is multiplied by two-thirds the common interval.

decimals of a foot, and are best measured off with a decimal scale. If a scale showing feet and inches is used, then the inches should be converted into decimals of a foot; thus, 6′ 9″ = 6·75′, and 6′ $3\frac{1}{2}$″ = 6·3′. In the next column are placed Simpson's multipliers in their proper order and opposite their corresponding ordinates. The order may be remembered by combining together the multipliers for the elementary area first considered—

	1	4	1				
			1	4	1		
					1	4	1
or	1	4	2	4	2	4	1

The last column contains the product of the length of the ordinate and its multiplier given in the third column. These are termed the "*functions of ordinates.*" The sum of the figures in the last column is termed the "*sum of functions of ordinates.*" This has to be multiplied by one-third the common interval, or in this case $\frac{2}{3}$. The area then is given by—

$$117 \times \tfrac{2}{3} = 78 \text{ square feet}$$

Simpson's Second Rule.—This rule assumes that the curved line DC, forming one boundary of the curvilinear area

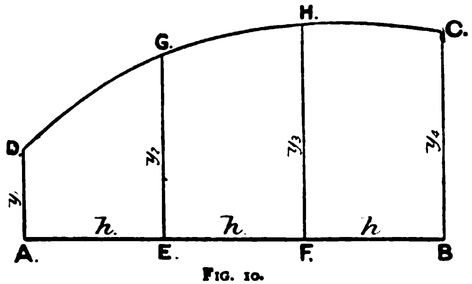


Fig. 10.

ABCD, Fig. 10, is a portion of a curve known as "*a parabola of the third order.*"[1]

Let ABCD, Fig. 10, be a figure bounded on one side by the curved line DC, which, as stated above, is assumed to be

[1] A "parabola of the third order" is one whose equation referred to co-ordinate axes is of the form $y = a_0 + a_1x + a_2x^2 + a_3x^3$, where a_0, a_1, a_2, a_3 are constants.

"*a parabola of the third order.*" AB is the base, and AD and BC are end ordinates perpendicular to the base. Divide the base AB into three equal parts by points E and F, and draw EG, FH perpendicular to AB, meeting the curve in G and H respectively. Then the area is given by—

$$\tfrac{3}{8}\text{AE}(\text{AD} + 3\text{EG} + 3\text{FH} + \text{BC})$$

or, using y_1, y_2, y_3, y_4 to represent the ordinates, and h the common interval between them—

$$\text{Area} = \tfrac{3}{8}h(y_1 + 3y_2 + 3y_3 + y_4)$$

Now, a long curvilinear area[1] may be divided into a number of portions similar to the above, to each of which the above rule will apply. Thus the area of the portion KLCB in Fig. 7 will be given by—

$$\tfrac{3}{8}h(y_4 + 3y_5 + 3y_6 + y_7)$$

Consequently the total area of ABCD, Fig. 7, will be, supposing all the ordinates are a common distance h apart—

$$\tfrac{3}{8}h(y_1 + 3y_2 + 3y_3 + 2y_4 + 3y_5 + 3y_6 + y_7)$$

The ordinate KL is termed a "*dividing ordinate,*" and the others, EF, GH, MN, OP, are termed "*intermediate ordinates.*" This rule may be approximately proved by a process similar to that adopted on p. 8 for the first rule.[2]

Application of Simpson's Second Rule.—*Example.*—A curvilinear area has ordinates at a common distance apart of 2 feet, the lengths being 1·45, 2·65, 4·35, 6·45, 8·50, 10·40, and 11·85 feet respectively. Find the area of the figure in square feet by the use of Simpson's second rule.

In finding the area of such a curvilinear figure by means of Simpson's second rule, the work is arranged as follows :—

Number of ordinate.	Length of ordinate.	Simpson's multipliers.	Functions of ordinates.
1	1·45	1	1·45
2	2·65	3	7·95
3	4·35	3	13·05
4	6·45	2	12·90
5	8·50	3	25·50
6	10·40	3	31·20
7	11·85	1	11·85

103·90 sum of functions

Common interval = 2 feet

$\tfrac{3}{8}$ common interval = $\tfrac{6}{8} = \tfrac{3}{4}$

103·9 × $\tfrac{3}{4}$ = 77·925 square feet

[1] See footnote on p. 7.

[2] See Appendix A for the mathematical proof.

This curvilinear area is the same as already taken for an example of the application of Simpson's first rule. It will be noticed that the number of intervals is 6 or a *multiple of* 3. We are consequently able to apply Simpson's second rule to finding the area. The columns are arranged as in the previous case, the multipliers used being those for the second rule. The order may be remembered by combining together the multipliers for the elementary area with three intervals first considered—

$$\begin{array}{ccccccc} 1 & 3 & 3 & 1 & & & \\ & & & 1 & 3 & 3 & 1 \\ \hline 1 & 3 & 3 & 2 & 3 & 3 & 1 \end{array}$$

or 1 3 3 2 3 3 1

For nine intervals the multipliers would be 1, 3, 3, 2, 3, 3, 2, 3, 3, 1.

The sum of the functions of ordinates has in this case to be multiplied by $\frac{3}{8}$ the common interval, or $\frac{3}{8} \times 2 = \frac{3}{4}$, and consequently the area is—

$$103{\cdot}9 \times \frac{3}{4} = 77{\cdot}925 \text{ square feet}$$

It will be noticed how nearly the area as obtained by the two rules agree. In practice the first rule is used in nearly all cases, because it is much simpler than the second rule and quite as accurate. It sometimes happens, however, that we only have four ordinates to deal with, and in this case Simpson's second rule must be used. When there are six ordinates, neither of the above rules will fit. The following rule gives the area: $\frac{25}{24} \cdot h(\frac{3}{5} \cdot y_1 + y_2 + y_3 + y_4 + y_5 + \frac{3}{5} y_6)$. This may be proved by applying the second rule to the middle four ordinates, and the following 5, 8, −1 rule to the ends.

To find the Area of a Portion of a Curvilinear Area contained between Two Consecutive Ordinates.—Such a portion is AEFD, Fig. 8. In order to obtain this area, we require the three ordinates to the curve $y_1 y_2 y_3$. The curve DFC is assumed to be, as in Simpson's first rule, a parabola of the second order. Using the ordinary notation, we have—

$$\text{Area of ADFE} = \frac{1}{12} h(5y_1 + 8y_2 - y_3)$$

Thus, if the ordinates of the curve in Fig. 8 be 8·5, 10·4, 11·85 feet, and 2 feet apart, the area of AEFD will be given by—

$$\tfrac{1}{12} \times 2(\overline{5 \times 8{\cdot}5} + \overline{8 \times 10{\cdot}4} - 11{\cdot}85) = 18{\cdot}97 \text{ square feet}$$

Similarly the area of EBCF will be given by—

$$\tfrac{1}{12} \times 2(\overline{5 \times 11{\cdot}85} + \overline{8 \times 10{\cdot}4} - 8{\cdot}5) = 22{\cdot}32 \text{ square feet}$$

giving a total area of the whole figure as 41·29 square feet.

Obtaining this area by means of Simpson's first rule, we should obtain 41·3 square feet.[1]

This rule is sometimes known as the "*five-eight*" rule.

Subdivided Intervals.—When the curvature of a line forming a boundary of an area, as Fig. 11, is very sharp, it is found that the distance apart of ordinates, as used for the straighter part of the curve, does not give a sufficiently accurate result. In such a case, ordinates are drawn at a sub-multiple of the ordinary distance apart of the main ordinates.

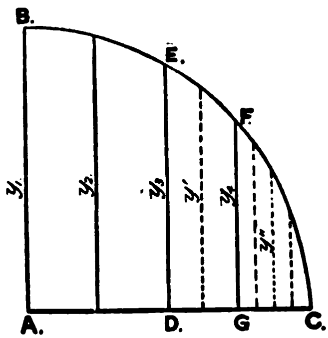


Fig. 11.

Take ABC, a quadrant of a circle (Fig. 11), and draw the three ordinates y_2, y_3, y_4 a distance h apart. Then we should get the area approximately by putting the ordinates through Simpson's first rule. Now, the curve EFC is very sharp, and the result obtained is very far from being an accurate one. Now put in the intermediate ordinates y', y''. Then the area of the portion DEC will be given by—

$$\frac{1}{3}\left(\frac{h}{2}\right)(y_3 + 4y' + 2y_4 + 4y'' + y_5)$$

or we may write this— ($y_5 = c$ at end)

$$\tfrac{1}{3}h(\tfrac{1}{2}y_3 + 2y' + y_4 + 2y'' + \tfrac{1}{2}y_5)$$

The area of the portion ABED is given by—

$$\tfrac{1}{3}h(y_1 + 4y_2 + y_3)$$

[1] See Example 25, p. 41.

or the area of the whole figure—

$$\frac{1}{3}h(y_1 + 4y_2 + 1\frac{1}{2}y_3 + 2y' + y_4 + 2y'' + \frac{1}{2}y_5)$$

Thus the multipliers for ordinates one-half the ordinary distance apart are $\frac{1}{2}$, 2, $\frac{1}{2}$, and for ordinates one-quarter the ordinary dis·ance apart are $\frac{1}{4}$, 1, $\frac{1}{2}$, 1, $\frac{1}{4}$. Thus we diminish the multiplier of each ordinate of a set of subdivided intervals in the same proportion as the intervals are subdivided. Each ordinate is then multiplied by its proper multiplier found in this way, and the sum of the products multiplied by $\frac{1}{3}$ or $\frac{3}{8}$ the whole interval according as the first or second rule is used An exercise on the use and necessity for subdivided intervals will be found on p. 43.

Algebraic Expression for the Area of a Figure bounded by a Plane Curve.—It is often convenient to be able to express in a short form the area of a plane curvilinear figure.

In Fig. 12, let ABCD be a strip cut off by the ordinates AB, CD, a distance Δx apart, Δx being supposed small. Then the area of this strip is very nearly—

$$y \times \Delta x$$

where y is the length of the ordinate AB. If now we imagine the strip to become indefinitely narrow, the small triangular piece BDE will disappear, and calling dx the breadth of the strip, its area will be—

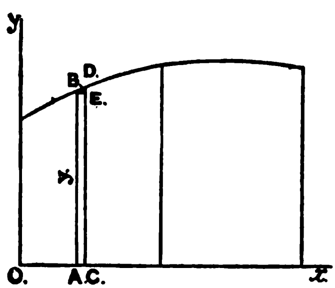


Fig. 12.

$$y \times dx$$

The area of the whole curvilinear figure would be found if we added together the areas of all such strips, and this could be written—

$$\int y \, . \, dx$$

where the symbol $\int$ may be regarded as indicating the sum of all such strips as $y \, . \, dx$. We have already found that

Simpson's rules enable us to find the areas of such figures, so we may look upon the expression for the area—

$$\int y \,.\, dx$$

as meaning that, to find the area of a figure, we take the length of the ordinate y at convenient intervals, and put them through Simpson's multipliers. The result, multiplied by $\frac{1}{3}$ or $\frac{3}{8}$ the common interval, as the case may be, will give the area. A familiarity with the above will be found of great service in dealing with moments in the next chapter.

To find the Area of a Figure bounded by a Plane Curve and Two Radii.—Let OAB, Fig. 13, be such a figure, OA, OB being the bounding radii.

Take two points very close together on the curve PP′; join OP, OP′, and let OP = r and the small angle POP′ = $\Delta\theta$ in circular measure.[1] Then OP = OP′ = r very nearly, and the area of the elementary portion OPP′ = $\frac{1}{2}r(r\,.\,\Delta\theta)$, $r\,.\,\Delta\theta$ being the length of PP′, and regarding OPP′ as a triangle. If now we consider OP, OP′ to become indefinitely close together, and consequently the angle POP′ indefinitely small = $d\theta$ say, any error in regarding POP′ as a triangle will disappear, and we shall have—

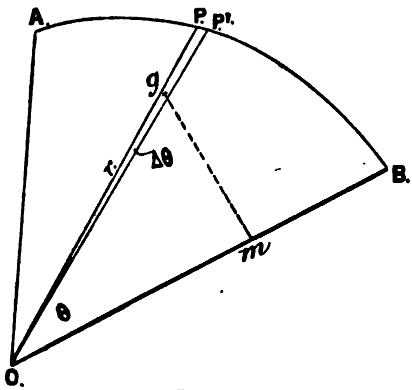


Fig. 13.

$$\text{Area POP}' = \frac{r^2}{2}\,.\,d\theta$$

and the whole area AOB is the sum of all such areas which can be drawn between OA and OB, or—

$$\int \frac{r^2}{2}\,.\,d\theta$$

[1] See pp. 16 and 90.

Now, this exactly corresponds to the algebraic expression for the area of an ordinary plane curvilinear figure, viz.—

$$\int y \, . \, dx \text{ (see p. 15)}$$

y corresponding to $\frac{r^2}{2}$ and dx corresponding to $d\theta$. Therefore divide the angle between the bounding radii into an even number of equal angular intervals by means of radii. Measure these radii, and treat their half-squares as ordinates of a curve by Simpson's first rule, multiplying the addition by $\frac{1}{3}$ the common angular interval *in circular measure.* Simpson's second rule may be used in a similar manner.

The *circular measure of an angle*[1] is the number of degrees it contains multiplied by $\frac{\pi}{180}$, or 0·01745. Thus the circular measure of—

$$90^\circ = \frac{\pi}{2} = \frac{3 \cdot 1416}{2} = 1 \cdot 5708$$

and the circular measure of 15° is 0·26175.

Example.—To find the area of a figure bounded by a plane curve and two radii 90° apart, the lengths of radii 15° apart being 0, 2·6, 5·2, 7·8, 10·5, 13·1, 15·7.

Angle from first radius.	Length of radius.	Square of length.	Simpson's multipliers.	Functions of squares.
0°	0·0	0·0	1	0·0
15°	2·6	6·8	4	27·2
30°	5·2	27·0	2	54·0
45°	7·8	60·8	4	243·2
60°	10·5	110·2	2	220·4
75°	13·1	171·6	4	686·4
90°	15·7	246·5	1	246·5

1477·7 sum of functions

Circular measure of $15^\circ = 0 \cdot 26175$

$$\therefore \text{area} = 1477 \cdot 7 \times \tfrac{1}{3} \times 0 \cdot 26175 \times \tfrac{1}{2}$$
$$= 64\tfrac{1}{2} \text{ square feet nearly}$$

The process is exactly the same as in Simpson's rule for a plane area with equidistant ordinates. To save labour, the squares of the radii are put through the proper multipliers, the multiplication by $\frac{1}{2}$ being performed at the end.

[1] See also p. 90.

Tchebycheff's Rules.—We have discussed above various methods that can be employed for determining the area of a figure bounded by a curved line. The methods that are most largely employed are those known as the "Trapezoidal rule" and "Simpson's first rule." The former is used in France and America,[1] and the latter is used in Great Britain. The trapezoidal rule has a great advantage in its simplicity, but considerable judgment is necessary in its use to obtain good results. Simpson's first rule is rather more complex, but gives exceedingly good results for the areas dealt with in ordinary ship calculations.

In the above rules *the spacing of ordinates is constant.* A rule has been devised for determining the area of a curvilinear figure, in which the multiplier is the same for all ordinates when these ordinates are suitably placed so that the lengths of the ordinates only need adding together to obtain the area of the figure.

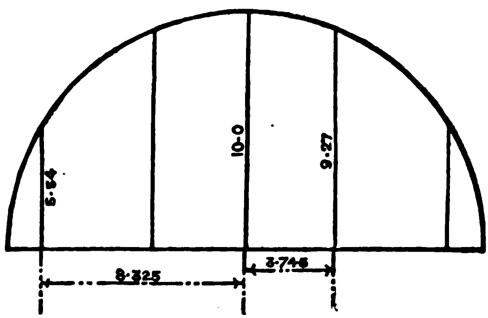


FIG. 13A.

This rule is Tchebycheff's rule,[2] and a much fewer number of ordinates are required than for either the Trapezoidal or Simpson's rules for equally correct results. For instance to find the area of a semicircle 10 feet radius, using five ordinates. These ordinates are placed in the positions shown in Fig. 13A. The lengths of these five ordinates are found to be 5·54, 9·27, 10·0, 9·27, 5·54 feet respectively, and all that is needed is to add these lengths together, multiply the

[1] See a paper read before the American Society of Naval Architects in 1895, by Mr. D. W. Taylor.

[2] See a paper by Mr. C. F. Munday, M.I.N.A., in the *Transactions of the Institution of Naval Architects* for 1899, and a paper by Professor Biles in the *Transactions of Institute of Engineers and Shipbuilders in Scotland* for 1899.

result by the length of the figure, and divide by the number of ordinates. The area by this rule is therefore—

$$39{\cdot}62 \times \frac{20}{5} = 158{\cdot}48 \text{ square feet}$$

The exact area is, of course, 157·08 square feet. Example No. 48, Chapter I., gives an illustration of the number of ordinates it is necessary to use for such a figure when using Simpson's first rule in order to obtain a close approximation to the correct area.

The following table gives the position of ordinates of a curve with reference to the middle ordinate for different numbers of ordinates :—

No. of ordinates used.	Position of ordinates from middle of base in fractions of the half-length of base.								
2					0·5773				
3				0		0·7071			
4				0·1876		0·7947			
5			0		0·3745		0·8325		
6			0·2666		0·4225		0·8662		
7		0		0·3239		0·5297		0·8839	
8		0·1026		0·4062		0·5938		0·8974	
9	0		0·1679		0·5288		0·6010		0·9116
10	0·0838		0·3127		0·5000		0·6873		0·9162

The following example will show how the rule is employed to find the area of the load water-plane of a ship 600 feet long, for which by the ordinary method 21 ordinates would have to be used 30 feet apart. By using 10 ordinates, and drawing them according to the above table, *i.e.* the following distance from amidships both forward and aft, 25·1, 63·8, 150·0, 206·2, 274·9 feet respectively, we obtain the following lengths for the semi-ordinates, commencing from forward : 3·6, 15·0, 25·1, 31·6, 35·1, 35·4, 33·4, 28·8, 22·2, and 11·5 feet respectively. These lengths are added together, and the result is multiplied by the length of the water-plane and divided by the number of ordinates. The result is the half-area of the water-plane, viz.—

$$241{\cdot}7 \times \frac{600}{10} = 14{,}502 \text{ square feet.}$$

The simplification due to this method consists in the fewer number of ordinates necessary and the simple process of addition that is required when the ordinates are measured off.

The method of proof of these rules is given in the Appendix.

Measurement of Volumes.

The Capacity or Volume of a Rectangular Block is the product of the length, breadth, and depth, or, in other words, the area of one face multiplied by the thickness. All these dimensions must be expressed in the same units. Thus the volume of an armour plate 12 feet long, $8\frac{1}{4}$ feet wide, and 18 inches thick, is given by—

$$12 \times 8\frac{1}{4} \times 1\frac{1}{2} = 12 \times \frac{33}{4} \times \frac{3}{2} = \frac{297}{2} = 148\frac{1}{2} \text{ cubic feet.}$$

The Volume of a Solid of Constant Section is the area of its section multiplied by its length. Thus a pipe 2 feet in diameter and 100 feet long has a section of $\frac{\pi 4}{4} = \frac{22}{7}$ square feet, and a volume of $\frac{22}{7} \times 100 = \frac{2200}{7} = 314\frac{2}{7}$ cubic feet.

A hollow pillar 7′ 6″ long, 5 inches external diameter, and $\frac{1}{4}$ inch thick, has a sectional area of—

$$3{\cdot}73 \text{ square inches}$$

$$\text{or } \frac{3{\cdot}73}{144} \text{ square feet}$$

and the volume of material of which it is composed is—

$$\left(\frac{3{\cdot}73}{144}\right) \times \frac{15}{2} = \frac{18{\cdot}65}{96}$$

$$= 0{\cdot}195 \text{ cubic foot}$$

Volume of a Sphere.—This is given by $\frac{\pi}{6} . d^3$, where d is the diameter. Thus the volume of a ball 3 inches in diameter is given by—

$$\frac{\pi}{6} . 27 = \frac{22 \times 27}{42}$$

$$= 14\frac{1}{7} \text{ cubic inches}$$

Volume of a Pyramid.—This is a solid having a base in the shape of a polygon, and a point called its vertex not in the same plane as the base. The vertex is joined by straight lines to all points on the boundary of the base. Its volume is given by the product of the area of the base and one-third the perpendicular distance of the vertex from the base. A *cone* is a particular case of the pyramid having for its base a figure with a continuous curve, and a *right circular cone* is a cone having for its base a circle and its vertex immediately over the centre of the base.

To find the Volume of a Solid bounded by a Curved Surface.—The volumes of such bodies as this are continually required in ship calculation work, the most important cases being the volume of the under-water portion of a vessel. In this case, the volume is bounded on one side by a plane surface, the water-plane of the vessel. Volumes of compartments are frequently required, such as those for containing fresh water or coal-bunkers. The body is divided by a series of planes spaced equally apart. The area of each section is obtained by means of one of the rules already explained. These areas are treated as the ordinates of a new curve, which may be run in, with ordinates the spacing of the planes apart. It is often desirable to draw this curve with areas as ordinates as in Fig. 14, because, if the surface is a fair surface, the curve of areas should be a fair curve, and should run evenly through all the spots; any inaccuracy may then be detected. The area of the curve of areas is then obtained by one of Simpson's rules as convenient, and this area will represent the cubical contents of the body.

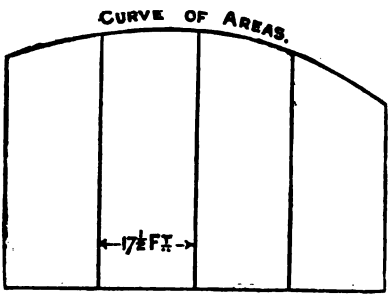


FIG. 14.

Example.—A coal-bunker has sections 17′ 6″ apart, and the areas of these sections are 98, 123, 137, 135, 122 square feet respectively. Find the volume of the bunker and the number of tons of coal it will hold, taking 44 cubic feet of coal to weigh 1 ton.

Areas.	Simpson's multipliers.	Functions of areas.
98	1	98
123	4	492
137	2	274
135	4	540
122	1	122

1526 sum of functions

$$\tfrac{1}{3}\text{ common interval} = \tfrac{1}{3} \times 17\tfrac{1}{2} = \tfrac{35}{6}$$
$$\therefore \text{volume} = 1526 \times \tfrac{35}{6} \text{ cubic feet}$$
$$= 8902 \text{ cubic feet.}$$
and the bunker will hold $\frac{8902}{44} = 202$ tons

The under-water portion of a ship is symmetrical about the fore-and-aft middle-line plane.

We may divide the volume in two ways—

1. By equidistant planes perpendicular to the middle-line plane and to the load water-plane.

2. By equidistant planes parallel to the load water-plane.

The volume as obtained by both methods should be the same, and they are used to check each other.

Examples.—1. The under-water portion of a vessel is divided by vertical sections 10 feet apart of the following areas: 0·3, 22·7, 48·8, 73·2, 88·4, 82·8, 58·7, 26·2, 3·9 square feet. Find the volume in cubic feet. (The curve of sectional areas is given in Fig. 15.)

—— CURVE OF SECTIONAL AREAS. ——

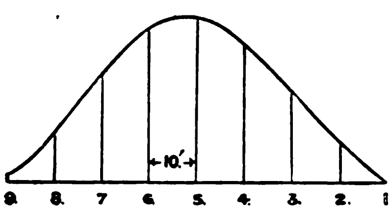


FIG 15.

The number of ordinates being odd, Simpson's first rule can be applied as indicated in the following calculation:—

Number of section.	Area of section.	Simpson's multipliers.	Function of area.
1	0·3	1	0·3
2	22·7	4	90·8
3	48·8	2	97·6
4	73·2	4	292·8
5	88·4	2	176·8
6	82·8	4	331·2
7	58·7	2	117·4
8	26·2	4	104·8
9	3·9	1	3·9

1215·6 sum of functions

$\frac{1}{3}$ common interval $= \frac{10}{3}$

$\therefore$ volume $= 1215·6 \times \frac{10}{3}$

$= 4052$ cubic feet

2. The under-water portion of the above vessel is divided by planes parallel to the load water-plane and $1\frac{1}{2}$ feet apart of the following areas: 944, 795, 605, 396, 231, 120, 68, 25, 8 square feet. Find the volume in cubic feet.

The number of areas being odd, Simpson's first rule can be applied, as indicated in the following calculation:—

Number of water-line.	Area of water-plane.	Simpson's multipliers.	Function of area.
1	944	1	944
2	795	4	3180
3	605	2	1210
4	396	4	1584
5	231	2	462
6	120	4	480
7	68	2	136
8	25	4	100
9	8	1	8

8104 sum of functions

$\frac{1}{3}$ common interval $= \frac{1}{3} \times \frac{3}{2}$

$\therefore$ volume $= 8104 \times \frac{1}{2}$

$= 4052$ cubic feet

which is the same result as was obtained above by taking the areas of vertical sections and putting them through Simpson's rule.

In practice this volume is found by means of a "displacement sheet," or by the "planimeter." See Chapter II. and Appendix A.

Displacement.—The amount of water displaced or put aside by a vessel afloat is termed her "*displacement.*" This may be reckoned as a volume, when it is expressed in cubic feet, or as a weight, when it is expressed in tons. It is usual to take salt water to weigh 64 lbs. per cubic foot, and consequently $\frac{2240}{64} = 35$ cubic feet of salt water will weigh one ton. Fresh water, on the other hand, is regarded as weighing $62\frac{1}{2}$ lbs. per cubic foot, or 36 cubic feet to the ton.[1] The volume displacement is therefore 35 or 36 times the weight displacement, according as we are dealing with salt or fresh water.

If a vessel is floating in equilibrium in still water, the weight of water she displaces must exactly equal the weight of the vessel herself with everything she has on board.

That this must be true may be understood from the following illustrations:—

1. Take a large basin and stand it in a dish (see Fig. 16). Just fill the basin to the brim with water. Now carefully place a smaller basin into the water. It will be found that some of the water in the large basin will be displaced, and water will spill over the edge of the large basin into the dish below. It is evident that the water displaced by the basin is equal in amount to the water that has been caught by the dish, and if this water be weighed it will be found, if the experiment be conducted accurately, that the small basin is equal in weight to the water in the dish—that is, to the water it has displaced.

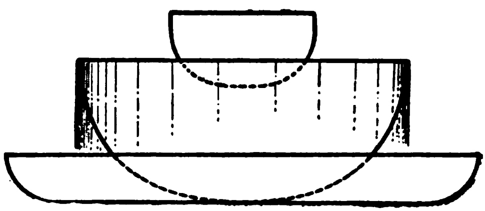


Fig. 16.

2. Consider a vessel floating in equilibrium in still water, and imagine, if it were possible, that the water is solidified, maintaining the same level, and therefore the same density. If now we lift the vessel out, we shall have a cavity left behind which

[1] It is advisable to occasionally test the water at any particular place to obtain the density, which may vary at different states of the tide. Thus we have—at Clydebank the water is 35·87 cubic feet to the ton; at Dundee the water is 1021 ozs. to cubic foot at high water, and 1006 ozs. at low water.

will be exactly of the form of the under-water portion of the ship, as Fig. 17. Now let the cavity be filled up with water. The amount of water we pour in will evidently be equal to the volume of displacement of the vessel. Now suppose that the solidified water outside again becomes liquid. The water we have poured in will remain where it is, and will be supported by the water surrounding it. The support given, first to the vessel and now to the water we have poured in, by the sur-

FIG. 17.

rounding water must be the same, since the condition of the outside water is the same. Consequently, it follows that the weight of the vessel must equal the weight of water poured in to fill the cavity, or, in other words, the weight of the vessel is equal to the weight of water displaced.

If the vessel whose displacement has been calculated on p. 22 is floating at her L.W.P. in salt water, her total weight will be—

$$4052 \div 35 = 115 \cdot 8 \text{ tons}$$

If she floated at the same L.W.P. in fresh water, her total weight would be—

$$4052 \div 36 = 112\tfrac{1}{2} \text{ tons}$$

It will be at once seen that this property of floating bodies is of very great assistance to us in dealing with ships. For, to find the weight of a ship floating at a given line, we do not need to estimate the weight of the ship, but we calculate out from the drawings the displacement in tons up to the given line, and this must equal the total weight of the ship.

Curve of Displacement.—The calculation given on p. 22 gives the displacement of the vessel up to the load-water plane, but the draught of a ship continually varies owing to different weights of cargo, coal, stores, etc., on board, and it is desirable to have a means of determining quickly the displacement at any given draught. From the rules we have already investigated, the displacement in tons can be calculated up to each water-plane in succession. If we set down a scale of mean draughts, and set off perpendiculars to this scale at the places where each water-plane comes, and on these set off on a convenient scale the displacement we have found up to that water-plane, then we should have a number of spots through which we shall be able to pass a fair curve if the calculations are correct.

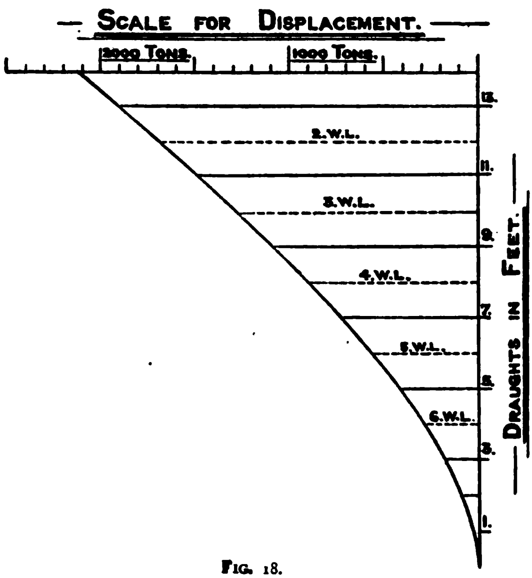


FIG. 18.

A curve obtained in this way is termed a "*curve of displacement*," and at any given mean draught we can measure the displacement of the vessel at that draught, and consequently know at once the total weight of the vessel with everything she has on board. This will not be quite accurate if the vessel is floating at a water-plane not parallel to the designed load water-plane. Fig. 18 gives a "curve of displacement" for a vessel, and the following calculation shows in detail the method of obtaining the information necessary to construct it.

The areas of a vessel's water-planes, two feet apart, are as follows :—

L.W.L.	... ...	7800 square feet.
2 W.L.	... ...	7450 "
3 W.L.	... ...	6960 "
4 W.L.	... ...	6290 "
5 W.L.	... ...	5460 "
6 W.L.	... ...	4320 "
7 W.L.	... ...	2610 "

The mean draught to the L.W.L. is 14′ 0″, and the displacement below the lowest W.L. is 71 tons.

To find the displacement to the L.W.L.

Number of W.L.	Area of water-plane.	Simpson's multipliers.	Function of area.
1	7800	1	7,800
2	7450	4	29,800
3	6960	2	13,920
4	6290	4	25,160
5	5460	2	10,920
6	4320	4	17,280
7	2610	1	2,610
			107,490

$\frac{1}{3}$ common interval $= \frac{1}{3} \times 2$

$\therefore$ displacement in cubic feet $= 107{,}490 \times \frac{2}{3}$

and displacement in tons, salt water $= 107{,}490 \times \frac{2}{105}$

$= 2047$ tons without the appendage

Next we require the displacement up to No. 2 W.L., and we subtract from the total the displacement of the layer between 1 and 2, which is found by using the *five-eight* rule as follows :—

Number of W.L.	Area of water-plane.	Simpson's multipliers.	Function of area.
1	7800	5	39,000
2	7450	8	59,600
3	6960	−1	−6,960
			91,640

Displacement in tons between No. 1 and No. 2 W.L.'s $= 91{,}640 \times \frac{2}{12} \times \frac{1}{35}$

= 436 tons nearly

∴ the displacement up to No. 2 W.L. is 2047 − 436 = 1611 tons without the appendage

The displacement between 1 and 3 W.L.'s can be found by putting the areas of 1, 2 and 3 W.L.'s through Simpson's first rule, the result being 848 tons nearly.

∴ the displacement up to No. 3 W.L. is 2047 − 848 = 1199 tons without the appendage

The displacement up to No. 4 W.L. can be obtained by putting the areas of 4, 5, 6, and 7 W.L.'s through Simpson's second rule, the result being—

819 tons without the appendage

The displacement up to No. 5 W.L. can be obtained by putting the areas of 5, 6, and 7 W.L.'s through Simpson's first rule, the result being—

482 tons without the appendage

The displacement up to No. 6 W.L. can be obtained by means of the five-eight rule, the result being—

201 tons without the appendage

Collecting the above results together, and adding in the appendage below No. 7 W.L., we have—

Displacement up to	L.W.L.	... ...	2118 tons.
" "	2 W.L.	... ...	1682 "
" "	3 W.L.	... ...	1270 "
" "	4 W.L.	... ...	890 "
" "	5 W.L.	... ...	553 "
" "	6 W.L.	... ...	272 "
" "	7 W.L.	... ...	71 "

These displacements, set out at the corresponding draughts, are shown in Fig. 18, and the fair curve drawn through forms the "*curve of displacement*" of the vessel. It is usual to complete the curve as indicated right down to the keel, although the ship could never float at a less draught than that given by the weight of her structure alone, or when she was launched.

Tons per Inch Immersion.—It is frequently necessary to know how much a vessel will sink, when floating at a given water-line, if certain known weights are placed on board, or how much she will rise if certain known weights are removed. Since the total displacement of the vessel must equal the weight of the vessel herself, the extra displacement caused by putting a weight on board must equal this weight. If A is the area

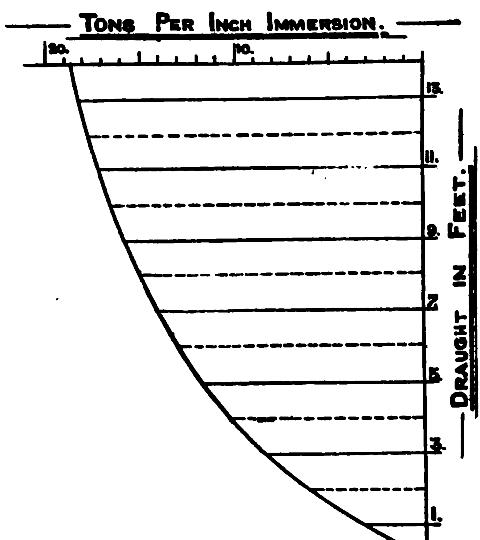


Fig. 19.

of a given water-plane in square feet, then the displacement of a layer 1 foot thick at this water-plane, supposing the vessel parallel-sided in its neighbourhood, is—

$$A \text{ cubic feet}$$

$$\text{or } \frac{A}{35} \text{ tons in salt water}$$

For a layer 1 inch thick only, the displacement is—

$$\frac{A}{35 \times 12} \text{ tons}$$

and this must be the number of tons we must place on board in order to sink the vessel 1 inch, or the number of tons we must take out in order to lighten the vessel 1 inch. This is termed the "*tons per inch immersion*" at the given water-line.[1] This assumes that the vessel is parallel-sided at the water-line for the depth of 1 inch up and 1 inch down, which may, for all practical purposes, be taken as the case. If, then, we obtain the tons per inch immersion at successive water-planes parallel to the load water-plane, we shall be able to construct a "*curve of tons per inch immersion*" in the same way in which the curve of displacement was constructed. Such a curve is shown in Fig. 19, constructed for the same vessel for which the displacement curve was calculated. By setting up any mean draught, say 11 feet, we can measure off the "tons per inch immersion," supposing the vessel is floating parallel to the load water-plane; in this case it is $17\frac{1}{2}$ tons. Suppose this ship is floating at a mean draught of 11 feet, and we wish to know how much she will lighten by burning 100 tons of coal. We find, as above, the tons per inch to be $17\frac{1}{2}$, and the decrease in draught is therefore—

$$100 \div 17\tfrac{1}{2} = 5\tfrac{3}{4} \text{ inches nearly}$$

Curve of Areas of Midship Section.—This curve is sometimes plotted off on the same drawing as the displacement curve and the curve of tons per inch immersion. The ordinates of the immersed part of the midship section being known, we can calculate its area up to each of the water-planes in exactly the same way as the displacement has been calculated. These areas are set out on a convenient scale at the respective mean draughts, and a line drawn through the points thus obtained. If the calculations are correct, this should be a fair curve, and is known as "*the curve of areas of midship section.*" By means of this curve we are able to determine the area of the midship section up to any given mean draught.

Fig. 20 gives *the curve of areas of midship section* for the vessel for which we have already determined the displacement curve and the curve of tons per inch immersion.

Coefficient of Fineness of Midship Section.—If we

[1] For approximate values of the "tons per inch immersion" in various types of ships, see Example 55, p. 44.

draw a rectangle with depth equal to the draught of water at the midship section to top of keel, and breadth equal to the

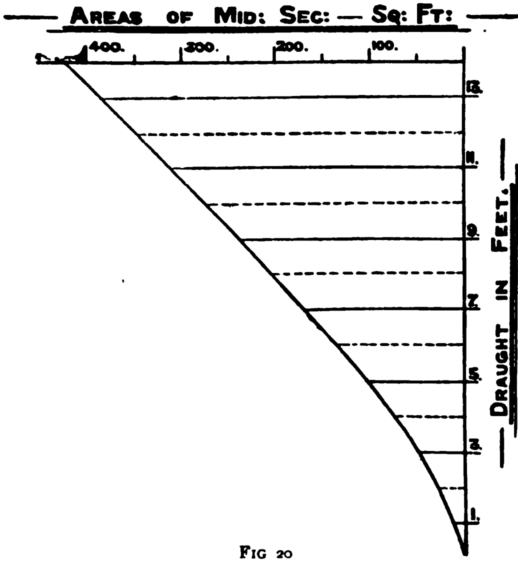


Fig 20

extreme breadth at the midship section, we shall obtain what may be termed the circumscribing rectangle of the immersed midship section. The area of the immersed midship section will be less than the area of this rectangle, and the ratio—

$$\frac{\text{area of immersed midship section}}{\text{area of its circumscribing rectangle}}$$

is termed the *coefficient of fineness of midship section.*

Example.—The midship section of a vessel is 68 feet broad at its broadest part, and the draught of water is 26 feet. The area of the immersed midship section is 1584 square feet. Find the coefficient of fineness of the midship section.

$$\text{Area of circumscribing rectangle} = 68 \times 26$$
$$= 1768 \text{ square feet}$$
$$\therefore \text{coefficient} = \tfrac{1584}{1768} = 0{\cdot}895$$

If a vessel of similar form to the above has a breadth at the midship section of 59′ 6″ and a draught of 22′ 9″, the area of its immersed midship section will be—

$$59\frac{1}{2} \times 22\frac{3}{4} \times 0{\cdot}895 = 1213 \text{ square feet}$$

The value of the midship section coefficient varies in ordinary ships from about 0·85 to 0·95, the latter value being for a section with very full section.

Coefficient of Fineness of Water-plane.—This is the ratio between the area of the water-plane and its circumscribing rectangle.

The value of this coefficient for the load water-plane may be taken as follows :—

For ships with fine ends	0·7
For ships of ordinary form	0·75
For ships with bluff ends	0·85

Block Coefficient of Fineness of Displacement.—This is the ratio of the volume of displacement to the volume of a block having the same length between perpendiculars, extreme breadth, and mean draught as the vessel. The draught should be taken from the top of keel.

Thus a vessel is 380 feet long, 75 feet broad, with 27′ 6″ mean draught, and 14,150 tons displacement. What is its block coefficient of fineness or displacement?

$$\text{Volume of displacement} = 14{,}150 \times 35 \text{ cubic feet}$$

$$\text{Volume of circumscribing solid} = 380 \times 75 \times 27\tfrac{1}{2} \text{ cubic feet}$$

$$\therefore \left.\begin{array}{l}\text{coefficient of fineness of}\\ \text{displacement}\end{array}\right\} = \frac{14150 \times 35}{380 \times 75 \times 27\frac{1}{2}}$$

$$= 0{\cdot}63$$

This coefficient gives a very good indication of the fineness of the underwater portion of a vessel, and can be calculated and tabulated for vessels with known speeds. Then, if in the early stages of a design we have the desired dimensions given, with the speed required, we can select the coefficient of fineness which appears most suitable for the vessel, and so determine very quickly the displacement that can be obtained under the conditions given.

Example.—A vessel has to be 400 feet long, 42 feet beam, 17 feet draught, and 13½ knots speed. What would be the probable displacement?

From available data, it would appear that a block coefficient of fineness of 0·625 would be desirable. Consequently the displacement would be—

$$(400 \times 42 \times 17 \times 0{\cdot}625) \div 35 \text{ tons} = 5100 \text{ tons about}$$

The following may be taken as average values of the block coefficient of fineness of displacement in various types of ships:—

Recent battleships	·60–·65
Recent fast cruisers	·50–·55
Fast mail steamers	·50–·60
Ordinary steamships	·55–·65
Cargo steamers	·65–·80
Sailing vessels	·65–·75
Steam-yachts	·35–·45

Prismatic Coefficient of Fineness of Displacement.—This coefficient is often used as a criterion of the fineness of the underwater portion of a vessel. It is the ratio between the volume of displacement and the volume of a prismatic solid the same length between perpendiculars as the vessel, and having a constant cross-section equal in area to the immersed midship section.

Example.—A vessel is 300 feet long, 2100 tons displacement, and has the area of her immersed midship section 425 square feet. What is her prismatic coefficient of fineness?

$$\text{Volume of displacement} = 2100 \times 35 \text{ cubic feet}$$
$$\text{Volume of prismatic solid} = 300 \times 425 \quad \text{,,}$$
$$\therefore \text{coefficient} = \frac{2100 \times 35}{300 \times 425}$$
$$= 0{\cdot}577$$

Difference in Draught of Water when floating in Sea Water and when floating in River Water.—Sea water is denser than river water; that is to say, a given volume of sea water—say a cubic foot—weighs more than the same volume of river water. In consequence of this, a vessel, on passing from the river to the sea, if she maintains the same weight, will rise in the water, and have a greater freeboard than when she started. Sea water weighs 64 lbs. to the cubic foot, and the water in a river such as the Thames may be taken as weighing 63 lbs. to the cubic foot.[1] In Fig. 21, let the right-hand portion represent the ship floating in river water, and the left-hand portion represent the ship floating in salt water. The distance between the two water-planes will be the amount the ship will rise on passing into sea water.

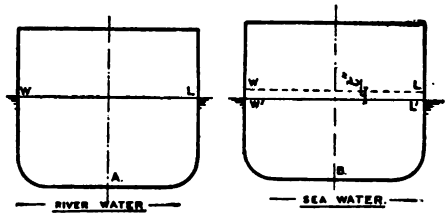


FIG. 21.

Let W = the weight of the ship in tons;

T = the tons per inch immersion at the water-line W'L' in salt water;

t = the difference in draught between the water-lines WL, W'L' in inches.

Then the volume of displacement—

$$\text{in river water} = \frac{W \times 2240}{63}$$

$$\text{in sea water} = \frac{W \times 2240}{64}$$

$$\therefore \text{the volume of the layer} = \frac{W \times 2240}{63} - \frac{W \times 2240}{64}$$

$$= \frac{W \times 2240}{63 \times 64}$$

Now, the volume of the layer also $= t \times T \times \frac{2240}{64}$; therefore we have—

$$t \times T \times \frac{2240}{64} = \frac{W \times 2240}{63 \times 64}$$

$$\text{or } t = \frac{W}{63T} \text{ inches}$$

[1] See also Example 56, p. 44.

This may be put in another way. A ship, if floating in river water, will weigh $\frac{1}{64}$ less than if floating to the same water-line in salt water. Thus, if W is the weight of the ship floating at a given line in salt water, her weight if floating at the same line in river water is—

$$\frac{1}{64}\text{W less}$$

and this must be the weight of the layer of displacement between the salt-water line and the river-water line for a given weight W of the ship. If T be the tons per inch for salt water, the tons per inch for river water will be $\frac{63}{64}$T. Therefore the difference in draught will be—

$$\frac{1}{64}\text{W} \div \frac{63}{64}\text{T} = \frac{\text{W}}{63\text{T}} \text{ inches, as above}$$

Sinkage caused by a Central Compartment of a Vessel being open to the Sea.—Take the simple case of a box-shaped vessel, ABCD, Fig. 22, floating at the water-line WL.

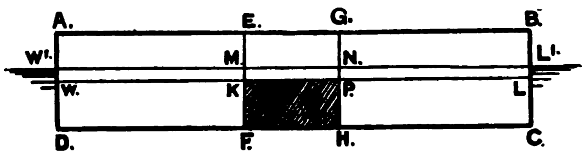


FIG. 22.

This vessel has two water-tight athwartship bulkheads in the middle portion, EF and GH. A hole is made in the bottom or side below water somewhere between these bulkheads. We will take a definite case, and work it out in detail to illustrate the principles involved in such a problem.

Length of box-shaped vessel	...	...	100	feet.
Breadth " "	...	...	20	"
Depth " "	...	...	20	"
Draught " "	...	...	10	"
Distance of bulkheads apart	...	...	20	"

If the vessel is assumed to be floating in salt water, its weight must be—

$$\frac{100 \times 20 \times 10}{35} = \frac{20000}{35} \text{ tons}$$

Now, this weight remains the same after the bilging as before, but the buoyancy has been diminished by the opening of the compartment KPHF to the sea. This lost buoyancy must be made up by the vessel sinking in the water until the volume of displacement is the same as it originally was. Suppose W′L′ to be the new water-line, then the new volume of displacement is given by the addition of the volumes of W′MFD and NL′CH, or, calling *d* the new draught of water in feet—

$$(40 \times 20 \times d) + (40 \times 20 \times d) = 1600d \text{ cubic feet}$$

The original volume of displacement was—

$$100 \times 20 \times 10 = 20{,}000 \text{ cubic feet}$$
$$\therefore 1600\, d = 20{,}000$$
$$\therefore d = \frac{200}{16} = 12'\ 6''$$

that is, the new draught of water is 12′ 6″, or the vessel will sink a distance of 2′ 6″.

The problem may be looked at from another point of view. The lost buoyancy is 20 × 20 × 10 cubic feet = 4000 cubic feet; this has to be made up by the volumes W′MKW and NL′LP, or the area of the intact water-plane multiplied by the increase in draught. Calling *x* the increase in draught, we shall have—

$$80 \times 20 \times x = 4000$$
$$x = \frac{4000}{1600} = 2\tfrac{1}{2} \text{ feet}$$
$$= 2'\ 6''$$

which is the same result as was obtained above.

If the bilged compartment contains stores, etc., the amount of water which enters from the sea will be less than if the compartment were quite empty. The volume of the lost displacement will then be given by the volume of the compartment up to the original water-line less the volume occupied by the stores.

Thus, suppose the compartment bilged in the above example to contain coal, stowed so that 44 cubic feet of it will weigh one ton, the weight of the solid coal being taken at 80 lbs. to the cubic foot.

1 cubic foot of coal, if solid, weighs 80 lbs.
1 " " as stowed " $\frac{2240}{44} = 51$ lbs.

Therefore in every cubic foot of the compartment there is—

$\frac{51}{80}$ cubic feet solid coal
$\frac{29}{80}$ " space into which water will find its way

The lost buoyancy is therefore—

$$\frac{29}{80} \times 4000 = 1450 \text{ cubic feet}$$

The area of the intact water-plane will also be affected in the same way; the portion of the water-plane between the bulkheads will contribute—

$$\frac{51}{80} \times 20 \times 20 = 255 \text{ square feet to the area}$$

The area of the intact waterplane is therefore—

$$1600 + 255 = 1855 \text{ square feet}$$

The sinkage in feet is therefore—

$$\frac{1450}{1855} = 0·78, \text{ or } 9·36 \text{ inches}$$

In the case of a ship the same principles apply, supposing the compartment to be a central one, and we have—

$$\left.\begin{array}{c}\text{Sinkage of vessel}\\\text{in feet}\end{array}\right\} = \frac{\text{volume of lost buoyancy in cubic feet}}{\text{area of intact water-plane in square feet}}$$

In the case of a compartment bilged which is not in the middle of the length, change of the trim occurs. The method of calculating this for any given case will be dealt with in Chapter IV.

In the above example, if the transverse bulkheads EF and GH had stopped just below the new water-line W'L', it is evident that the water would flow over their tops, and the vessel would sink. But if the tops were connected by a water-tight flat, the water would then be confined to the space, and the vessel would remain afloat.

Velocity of Inflow of Water into a Vessel on Bilging.—

Let A = area of the hole in square feet;

d = the distance the centre of the hole below the surface in feet;

v = initial rate of inflow of the water in feet per second.

Then $v = 8\sqrt{d}$ nearly.

and consequently the volume of water passing through the hole per second $\Big\} = 8\sqrt{d} \times$ A cub. ft.

Thus, if a hole 2 square feet in area, 4 feet below the water-line, were made in the side of a vessel, the amount of water, approximately, that would flow into the vessel would be as follows:—

$$\text{Cubic feet per second} = 8 \times \sqrt{4} \times 2$$
$$= 32$$
$$\text{Cubic feet per minute} = 32 \times 60$$
$$\text{Tons of water per minute} = \frac{32 \times 60}{35}$$
$$= 54{\cdot}85$$

Weights of Materials.—The following table gives average weights which may be used in calculating the weights of materials employed in shipbuilding:—

Steel	490 lbs. per cubic foot.
Wrought iron	480 " "
Cast iron	445 " "
Copper	550 " "
Brass	530 " "
Zinc	445 " "
Gunmetal	528 " "
Lead	712 " "
Elm (English)	35 " "
" (Canadian)	45 " "
Fir (Dantzic)	36 " "
Greenheart	72 " "
Mahogany	40–48 " "
" (for boats) ...	35 " "
Oak (English)	52 " "
" (Dantzic)	47 " "
" (African)	62 " "
Pine (Pitch)	40 " "
" (red)	36 " "
" (yellow)	30 " "
Teak	53 " "

It follows, from the weights per cubic foot of iron and steel given above, that an iron plate 1 inch thick weighs 40 lbs. per square foot, and a steel plate 1 inch thick weighs 40·8 lbs. per square foot.

The weight per square foot may be obtained for other thicknesses from these values, and we have the following:—

Thickness in inches.	Weight per square foot in pounds.	
	Iron.	Steel.
$\frac{1}{4}$	10	10·2
$\frac{3}{8}$	15	15·3
$\frac{1}{2}$	20	20·4
$\frac{5}{8}$	25	25·5
$\frac{3}{4}$	30	30·6
$\frac{7}{8}$	35	35·7
1	40	40·8

It is convenient to have the weight of steel per square foot when specified in one-twentieths of an inch, as is the case in Lloyd's rules—

Thickness in inches.	Weight per square foot in pounds.	Thickness in inches.	Weight per square foot in pounds.
$\frac{1}{20}$	2·04	$\frac{11}{20}$	22·44
$\frac{2}{20}$	4·08	$\frac{12}{20}$	24·48
$\frac{3}{20}$	6·12	$\frac{13}{20}$	26·52
$\frac{4}{20}$	8·16	$\frac{14}{20}$	28·56
$\frac{5}{20} = \frac{1}{4}$	10·20	$\frac{15}{20} = \frac{3}{4}$	30·60
$\frac{6}{20}$	12·24	$\frac{16}{20}$	32·64
$\frac{7}{20}$	14·28	$\frac{17}{20}$	34·68
$\frac{8}{20}$	16·32	$\frac{18}{20}$	36·72
$\frac{9}{20}$	18·36	$\frac{19}{20}$	38·76
$\frac{10}{20} = \frac{1}{2}$	20·40	$\frac{20}{20} = 1$	40·80

EXAMPLES TO CHAPTER I.

1. A plate has the form shown in Fig. 23. What is its weight if its weight per square foot is 10 lbs.? *Ans.* 95 lbs.

2. The material of an armour plate weighs 490 lbs. a cubic foot. A certain plate is ordered 400 lbs. per square foot: what is its thickness?

Ans. 9·8 inches.

3. Steel armour plates, as in the previous question, are ordered 400 lbs. per square foot instead of 10 inches thick. What is the saving of weight per 100 square feet of surface of this armour?

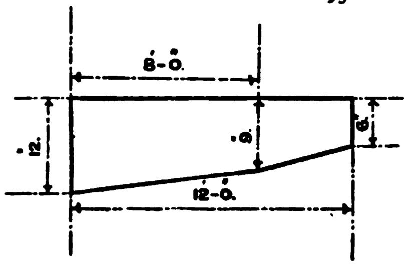


FIG. 23.

Ans. 833 lbs., or 0·37 ton.

4. An iron plate is of the dimensions shown in Fig. 24. What is its area? If two lightening holes 2′ 3″ in diameter are cut in it, what will its area then be?

Ans. 33¾ square feet; 25·8 square feet.

5. A hollow pillar is 4 inches external diameter and ⅜ inch thick. What is its sectional area, and what would be the weight in pounds of 10 feet of this pillar if made of wrought iron?

Ans. 4·27 square inches; 142 lbs.

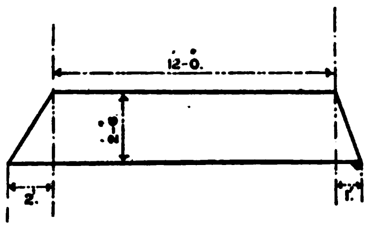


FIG. 24.

6. A steel plate is of the form and dimensions shown in Fig. 25. What is its weight? (A steel plate ⅝ inch thick weighs 25·5 lbs. per square foot.)

Ans. 1267 lbs.

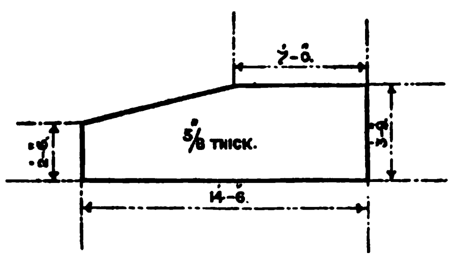


FIG. 25.

7. A wrought-iron armour plate is 15′ 3″ long, 3′ 6″ wide, and 4½ inches thick. Calculate its weight in tons.

Ans. 4·29 tons.

8. A solid pillar of iron of circular section is 6′ 10″ long and 2¼ inches in diameter. What is its weight?

Ans. 90¼ lbs.

9. A Dantzic fir deck plank is 22 feet long and 4 inches thick, and tapers in width from 9 inches at one end to 6 inches at the other. What is its weight?

Ans. 165 lbs.

10. A solid pillar of iron is 7′ 3″ long and 2⅜ inches diameter. What is its weight?

Ans. 143 lbs.

11. The total area of the deck plan of a vessel is 4500 square feet. What would be the surface of deck plank to be worked, if there are—

4 hatchways, each 4′ × 2½′
2 ,, ,, 10′ × 6′

and two circular skylights, each 4 feet in diameter, over which no plank is to be laid?

Ans. 4314·86 square feet.

12. A pipe is 6 inches diameter inside. How many cubic feet of water will a length of 100 feet of this pipe contain?

Ans. 19·6 cubic feet.

13. A mast 90 feet in length and 3 feet external diameter, is composed of 20 lb. plating worked flush-jointed on three T-bars, each 5″ × 3″ × 15½ lbs. per foot. Estimate the weight, omitting straps, and rivet heads.

Ans. 9½ tons nearly.

14. A curve has the following ordinates, 1′ 4″ apart: 10·86, 13·53, 14·58, 15·05, 15·24, 15·28, 15·22 feet respectively. Draw this curve, and find its area—

(1) By Simpson's first rule;
(2) By Simpson's second rule.

Ans. (1) 116·07 square feet; (2) 116·03 square feet.

15. The semi-ordinates in feet of a vessel's midship section, starting from the load water-line, are 26·6, 26·8, 26·8, 26·4, 25·4, 23·4, and 18·5 feet respectively, the ordinates being 3 feet apart. Below the lowest ordinate there is an area for one side of the section of 24·6 square feet. Find the area of the midship section, using—

(1) Simpson's first rule;
(2) Simpson's second rule.

Ans. (1) 961 square feet; (2) 960·7 square feet.

16. The internal dimensions of a tank for holding fresh water are 8′ 0″ × 3′ 6″ × 2′ 6″. How many tons of water will it contain?

Ans. 1·94.

17. The *half*-ordinates of a deck plan in feet are respectively 1¼, 5½, 10¼, 13½, 14¾, 14½, 12½, 9, and 3½, and the length of the plan is 128 feet. Find the area of the deck plan in square yards.

Ans. 296.

18. Referring to the previous question, find the area in square feet of the portion of the plan between the ordinates 1¼ and 5½.

Ans. 106·7.

19. The half-ordinates of the midship section of a vessel are 22·3, 22·2, 21·7, 20·6, 17·2, 13·2, and 8 feet in length respectively. The common interval between consecutive ordinates is 3 feet between the first and fifth ordinates, and 1′ 6″ between the fifth and seventh. Calculate the total area of the section in square feet.

Ans. 586·2 square feet.

20. Obtain the total area included between the first and fourth ordinates of the section given in the preceding question.

Ans. 392·8 square feet.

21. The semi-ordinates of the load water-plane of a vessel are 0·2, 3·6, 7·4, 10, 11, 10·7, 9·3, 6·5, and 2 feet respectively, and they are 15 feet apart. What is the area of the load water-plane?

Ans. 1808 square feet.

22. Referring to the previous question, what weight must be taken out of the vessel to lighten her 3½ inches?

What additional immersion would result by placing 5 tons on board?

Ans. 15 tons; 1·16 inch.

23. The "tons per inch immersion" of a vessel when floating in salt water at a certain water-plane is 44·5. What is the area of this plane?

Ans. 18,690 square feet.

24. A curvilinear area has ordinates 3 feet apart of length 9·7, 10·0, and 13·3 feet respectively. Find—

(1) The area between the first and second ordinates.
(2) The area between the second and third ordinates.
(3) Check the addition of these results by finding the area of the whole figure by Simpson's first rule.

25. Assuming the truth of the five-eight rule for finding the area between two consecutive ordinates of a curve, prove the truth of the rule known as Simpson's first rule.

26. A curvilinear area has the following ordinates at equidistant intervals of 18 feet: 6·20, 13·80, 21·90, 26·40, 22·35, 14·70, and 7·35 feet. Assuming that Simpson's first rule is correct, find the percentage of error that would be involved by using—

(1) The trapezoidal rule;
(2) Simpson's second rule.

Ans. (1) 1·2 per cent.; (2) 0·4 per cent.

27. A compartment for containing fresh water has a mean section of the form shown in Fig. 26. The length of the compartment is 12 feet. How many tons of water will it contain?

Ans. 17 tons.

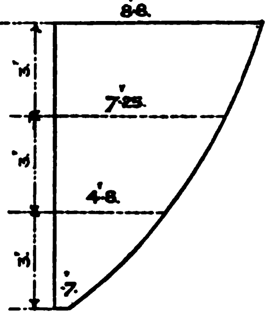


Fig. 26.

28. A compartment 20 feet long, 20 feet broad, and 8½ feet deep, has to be lined with teak 3 inches in thickness. *Estimate* the amount of teak required in cubic feet, and in tons.

Ans. 365 cubic feet; 8·6 tons.

29. The areas of the water-line sections of a vessel in square feet are respectively 2000, 2000, 1600, 1250, and 300. The common interval between them is 1½ foot. Find the displacement of the vessel in tons in salt water, neglecting the small portion below the lowest water-line section.

Ans. 264¼ tons.

30. A series of areas, 17′ 6″ apart, contain 0·94, 2·08, 3·74, 5·33, 8·27, 12·14, 16·96, 21·82, 24·68, 24·66, 22·56, 17·90, 12·66, 8·40, 5·69, 3·73, 2·61, 2·06, 0 square feet respectively. Find the volume of which the above are the sectional areas.

Ans. 3429 cubic feet.

31. Show how to estimate the change in the mean draught of a vessel in going from salt to river water, and *vice versâ*.

A vessel floats at a certain draught in river water, and when floating in sea water without any change in lading, it is found that an addition of 175 tons is required to bring the vessel to the same draught as in river water. What is the displacement after the addition of the weight named?

Ans. 11,200 tons.

32. The vertical sections of a vessel 10 feet apart have the following areas: 10, 50, 60, 70, 50, 40, 20 square feet. Find the volume of displacement, and the displacement in tons in salt and fresh water.

Ans. 2966 cubic feet; 84·7 tons, 82·4 tons.

33. A cylinder is 500 feet long, 20 feet diameter, and floats with the axis in the water-line. Find its weight when floating thus in salt water. What weight should be taken out in order that the cylinder should float with its axis in the surface if placed into fresh water?

Ans. 2244 tons; 62 tons.

34. A vessel is 500 feet long, 60 feet broad, and floats at a mean draught of 25 feet when in salt water. Make an approximation to her draught when she passes into river water. (Coefficient of displacement, 0·5; coefficient of L.W.P., 0·6.)

Ans. 25′ 4″.

35. A piece of teak is 20 feet long, 4½ inches thick, and its breadth tapers from 12 inches at one end to 9 inches at the other end. What is its weight, and how many cubic feet of water would it displace if placed into fresh water (36 cubic feet to the ton)?

Ans. 348 lbs.; 5½ cubic feet about.

36. The area of a water-plane is 5443 square feet. Find the tons per inch immersion. Supposing 40 tons placed on board, how much would the vessel sink?

State any slight error that may be involved in any assumption made. If 40 tons were taken out, would the vessel rise the same amount? What further information would you require to give a more accurate answer?

Ans. 12·96 tons; 3·1 inches nearly.

37. Bilge keels are to be fitted to a ship whose tons per inch is 48. The estimated weight of the bilge keels is 36 tons, and the volume they occupy is 840 cubic feet. What will be the increase of draught due to fitting these bilge keels?

Ans. ½ inch.

38. The tons per inch of a vessel at water-lines 2 feet apart are 19·45, 18·51, 17·25, 15·6, 13·55, 10·87, and 6·52, the lowest water-line being 18 inches above the underside of flat keel. Draw the curve of tons per inch immersion to scale, and estimate the number of tons necessary to sink the vessel from a draught of 12 feet to a draught of 13′ 6″.

Ans. 344 tons.

39. The steamship *Umbria* is 500 feet long, 57 feet broad, 22′ 6″ draught, 9860 tons displacement, 1150 square feet area of immersed midship section. Find—

(1) Block coefficient of displacement.
(2) Prismatic „ „
(3) Midship-section coefficient.

Ans. (1) 0·538; (2) 0·6; (3) 0·896.

40. The steamship *Orient* is 445 feet long, 46 feet broad, 21′ 4½″ draught mean; the midship section coefficient is 0·919, the block coefficient of displacement is 0·621. Find—

(1) Displacement in tons.
(2) Area of immersed midship section.
(3) Prismatic coefficient of displacement.

Ans. (1) 7763 tons; (2) 904 square feet; (3) 0·675.

41. A steam yacht is 144 feet long, 22′ 6″ broad, 9 feet draught; displacement, 334 tons salt water; area of midship section, 124 square feet. Find—

(1) Block coefficient of displacement.
(2) Prismatic " "
(3) Midship-section coefficient.

Ans. (1) 0·4; (2) 0·655; (3) 0·612.

42. Find the displacement in tons in salt water, area of the immersed midship section, prismatic coefficient of displacement, having given the following particulars: Length, 168 feet; breadth, 25 feet; draught, 10′ 6″; midship-section coefficient, 0·87; block coefficient of displacement, 0·595.

Ans. 750 tons; 228·5 square feet; 0·685.

43. A vessel in the form of a box, 100 feet long, 10 feet broad, and 20 feet deep, floats at a draught of 5 feet. Find the draught if a central compartment 10 feet long is bilged below water.

Ans. 5′ 6⅔″.

44. In a given ship, pillars in the hold can be either solid iron 4⅝ inches diameter, or hollow iron 6 inches diameter and half inch thick. Find the saving in weight for every 100 feet length of these pillars, if hollow pillars are adopted instead of solid, neglecting the effect of the solid heads and heels of the hollow pillars.

Ans. 1·35 ton.

45. What is the solid contents of a tree whose girth (circumference) is 60 inches, and length is 18 feet?

Ans. 35·8 cubic feet nearly.

46. A portion of a cylindrical steel stern shaft casing is 12⅝ feet long, 1¼ inch thick, and its external diameter is 14 inches. Find its weight in pounds.

Ans. 2170 lbs.

47. A floating body has a water-plane whose semi-ordinates 25 feet apart are 0·3, 8, 12, 10, 2 feet respectively, and every square station is in the form of a circle with its centre in the water-plane. Find the volume of displacement ($\pi = \frac{22}{7}$).

Ans. 12,414 cubic feet.

48. A quadrant of 16 feet radius is divided by means of ordinates parallel to one radius, and the following distances away: 4, 8, 10, 12, 13, 14, 15 feet respectively. The lengths of these ordinates are found to be 15·49, 13·86, 12·49, 10·58, 9·33, 7·75, and 5·57 feet respectively. Find—

(1) The exact area to two places of decimals.
(2) The area by using only ordinates 4 feet apart.
(3) The area by using also the half-ordinates.
(4) The area by using all the ordinates given above.
(5) The area as accurately as it is possible, supposing the ordinate 12·49 had not been given.

Ans. (1) 201·06; (2) 197·33; (3) 199·75; (4) 200·59; (5) 200·50.

49. A cylindrical vessel 50 feet long and 16 feet diameter floats at a constant draught of 12 feet in salt water. Using the information given in the previous question, find the displacement in tons.

Ans. 231 tons nearly.

50. A bunker 24 feet long has a mean section of the form of a trapezoid, with length of parallel sides 3 feet and 4·8 feet, and distance between them 10·5 feet. Find the number of tons of coal contained in the bunker, assuming

1 ton to occupy 43 cubic feet. If the parallel sides are perpendicular to one of the other sides, and the side 4·8 feet long is at the top of the section, where will the top of 17 tons of coal be, supposing it to be evenly distributed?

(This latter part should be done by a process of trial and error.)

Ans. 22·8 tons; 2′ 3″ below the top.

51. The sections of a ship are 20 feet apart. A coal-bunker extends from 9 feet abaft No. 8 section to 1 foot abaft No. 15 section, the total length of the bunker thus being 132 feet. The areas of sections of the bunker at Nos. 8, 11, and 15 are found to be 126, 177, and 145 square feet respectively. With this information given, estimate the capacity of the bunker, assuming 44 cubic feet of coal to go to the ton. Stations numbered from forward.

Ans. 495 tons.

52. The tons per inch immersion at water-lines 2 feet apart are 18·09, 16·80, 15·15, 13·15, 10·49, and 6·48. The draught of water to the top water-line is 11′ 6″, and below the lowest water-line there is a displacement of 75·3 tons. Find the displacement in tons, and construct a curve of displacement.

Ans. 1712 tons.

53. A tube 35 feet long, 16 feet diameter, closed at the ends, floats in salt water with its axis in the surface. Find approximately the thickness of the tube, supposed to be of iron, neglecting the weight of the ends.

Ans. 0·27 foot.

54. Find the floating power of a topmast, length 64 feet, mean diameter 21 inches, the wood of the topmast weighing 36 lbs. per cubic foot.

(The floating power of a spar is the weight it will sustain, and this is the difference between its own weight and that of the water it displaces. In constructing a raft, it has to be borne in mind that all the weight of human beings is to be placed *on* it, and that a great quantity of provisions and water may be safely carried *under* it. For instance, a cask of beef slung beneath would be 116 lbs., above 300 lbs. See "Sailor's Pocket-book," by Admiral Bedford.)

Ans. 4310 lbs.

55. Show that the following approximate values may be taken for the "tons per inch immersion" in salt water at the load draught:—

(1) For ships with fine ends ... ... $\frac{1}{600} \times L \times B$
(2) „ of ordinary form ... ... $\frac{1}{560} \times L \times B$
(3) „ with bluff ends ... ... $\frac{1}{500} \times L \times B$,

L and B being the length and breadth respectively of the load water-plane.

56. Show that a vessel passing from water of density d' into water of density d (d' being greater than d) will decrease her freeboard by $\frac{W}{T} \cdot \frac{d'-d}{d}$ inches, where W is the displacement in tons and T the tons per inch immersion when in the denser water.

A vessel 400 feet long, 45 feet broad, floats in Belfast water (1011 ozs. to a cubic foot) at a draught of 21′ 2½″. By how much will the freeboard be increased when in salt water (1025 ozs. to a cubic foot)? (Coefficient of fineness of displacement, 0·62; coefficient of fineness of L.W.P., 0·75.)

Ans. 2·9 inches.

CHAPTER II.

MOMENTS, CENTRE OF GRAVITY, CENTRE OF BUOYANCY, DISPLACEMENT TABLE, PLANIMETER, ETC.

Principle of Moments.—The *moment* of a force about any given line is the product of the force into the perpendicular distance of its line of action from that line. It may also be regarded as the *tendency to turn* about the line. A man pushes at the end of a capstan bar (as Fig. 27) with a

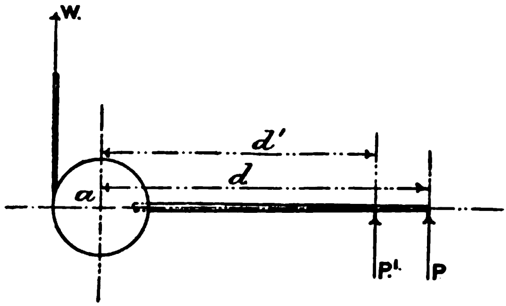


FIG. 27.

certain force. The tendency of the capstan to turn about its axis is given by the force exerted by the man multiplied by his distance from the centre of the capstan, and this is the *moment* of the force about the axis. If P is the force exerted by the man in pounds (see Fig. 27), and *d* is his distance from the axis in feet, then—

The moment about the axis = P × *d* foot-lbs.

The same moment can be obtained by a smaller force with a larger leverage, or a larger force with a smaller leverage, and the moment can be increased:—

(1) By increasing the force;

(2) By increasing the distance of the force from the axis.

If, in addition, there is another man helping the first man, exerting a force of P′ lbs. at a distance from the axis of d' feet, the total moment about the axis is—

$$(P \times d) + (P' \times d') \text{ foot-lbs.}$$

We must now distinguish between moments tending to turn one way and those tending to turn in the opposite direction.

Thus, in the above case, we may take a rope being wound on to the drum of the capstan, hauling a weight W lbs. If the radius of the drum be a feet, then the rope tends to turn the capstan in the opposite direction to the men, and the moment about the axis is given by—

$$W \times a \text{ foot-lbs.}$$

If the weight is just balanced, then there is no tendency to turn, and hence no moment about the axis of the capstan, and leaving out of account all consideration of friction, we have—

$$(P \times d) + (P' \times d') = W \times a$$

The most common forces we have to deal with are those caused by gravity, or the attraction of bodies to the earth. This is known as their weight, and the direction of these forces must all be parallel at any given place. If we have a number of weights, W_1, W_2, and W_3, on a beam at A, B, and C (Fig. 28),

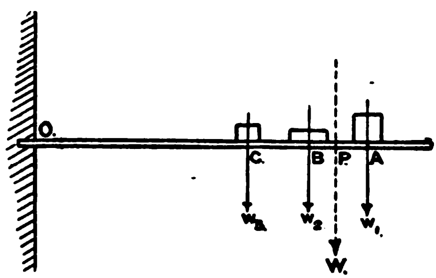


Fig. 28.

whose end is fixed at O, the moment of these weights about O is given by—

$$(W_1 \times AO) + (W_2 \times BO) + (W_3 \times CO)$$

This gives the tendency of the beam to turn about O, due to the weights W_1, W_2, and W_3 placed upon it, and the beam must be strong enough at O in order to resist this tendency, or, as it is termed, the *bending moment.* Now, we can evidently place a single weight W, equal to the sum of the weights W_1, W_2, and W_3, at some point on the beam so that its moment about O shall be the same as that due to the three weights. If P be this point, then we must have—

$$W \times OP = (W_1 \times OA) + (W_2 \times OB) + (W_3 \times OC)$$

or, since $W = W_1 + W_2 + W_3$

$$OP = \frac{(W_1 \times OA) + (W_2 \times OB) + (W_3 \times OC)}{W_1 + W_2 + W_3}$$

Example.—Four weights, 30, 40, 50, 60 lbs. respectively, are placed on a beam fixed at one end, O, at distances from O of 3, 4, 5, 6 feet respectively. Find the bending moment at O, and also the position of a single weight equal to the four weights which will give the same bending moment.

$$\begin{aligned}\text{Bending moment at O} &= (30 \times 3) + (40 \times 4) + (50 \times 5) + (60 \times 6)\\ &= 90 + 160 + 250 + 360\\ &= 860 \text{ foot-lbs.}\\ \text{Total weight} &= 180 \text{ lbs.}\\ \therefore \text{position of single weight} &= \tfrac{860}{180} = 4\tfrac{7}{9} \text{ feet from O}\end{aligned}$$

Centre of Gravity.—The single weight W above, when placed at P, has the same effect on the beam at O as the three weights W_1, W_2, and W_3. The point P is termed the *centre of gravity* of the weights W_1, W_2, and W_3. Thus we may define the centre of gravity of a number of weights as follows :—

The centre of gravity of a system of weights is that point at which we may regard the whole system as being concentrated.

This definition will apply to the case of a solid body, since we may regard it as composed of a very large number of small particles, each of which has a definite weight and occupies a definite position. A homogeneous solid has the same density throughout its volume; and all the solids with which we have to deal are taken as homogeneous unless otherwise specified.

It follows, from the above definition of the centre of gravity, that if a body is suspended at its centre of gravity, it would be perfectly balanced and have no tendency to move away from any position in which it might be placed.

To Find the Position of the Centre of Gravity of a number of Weights lying in a Plane.—Two lines are drawn in the plane at right angles, and the moment of the system of weights is found successively about each of these lines. The total weight being known, the distance of the centre of gravity from each of these lines is found, and consequently the position of the centre of gravity definitely fixed.

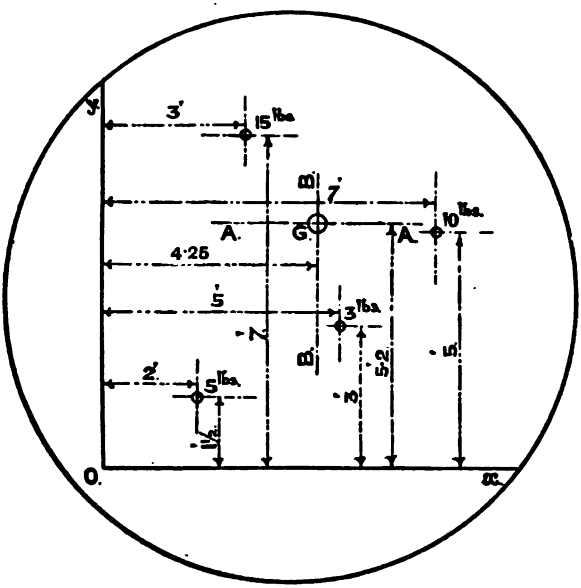


FIG. 29.

The following example will illustrate the principles involved: Four weights, of 15, 3, 10, and 5 lbs. respectively, are lying on a table in definite positions as shown in Fig. 29. Find the position of the centre of gravity of these weights. (If the legs of the table were removed, this would be the place where we should attach a rope to the table in order that it should remain horizontal, the weight of the table being neglected.)

Draw two lines, Ox, Oy, at right angles on the table in any convenient position, and measure the distances of each of the weights from Ox, Oy respectively: these distances are indicated in the figure. The total weight is 33 lbs. The moment of the weights about Ox is—

$$(15 \times 7) + (3 \times 3) + (10 \times 5) + (5 \times 1{\cdot}5) = 171{\cdot}5 \text{ foot-lbs.}$$

The distance of the centre of gravity from Ox $= \frac{171{\cdot}5}{33} = 5{\cdot}2$ feet

If we draw a line AA a distance of 5·2 feet from Ox, the centre of gravity of the weights must be somewhere in the line AA.

Similarly, we take moments about Oy, finding that the moment is 150 foot-lbs., and the distance of the centre of gravity from Oy is—

$$\frac{140}{33} = 4{\cdot}25 \text{ feet}$$

If we draw a line BB a distance of 4·25 feet from Oy, the centre of gravity of the weights must be somewhere in the line BB. The point G, where AA and BB meet, will be the centre of gravity of the weights.

Centres of Gravity of Plane Areas.—A plane area has length and breadth, but no thickness, and in order to give a definite meaning to what is termed its centre of gravity, the area is supposed to be the surface of a thin lamina or plate of homogeneous material of uniform thickness. With this supposition, the centre of gravity of a plane area is that point at which it can be suspended and remain in equilibrium.

Centres of Gravity of Plane Figures.

Circle.—The centre of gravity of a circle is obviously at its centre.

Square and Rectangle.—The centre of gravity of either of these figures is at the point where the diagonals intersect.

Rhombus and Rhomboid.—The centre of gravity of either of these figures is at the point where the diagonals intersect.

Triangle.—Take the triangle ABC, Fig. 30. Bisect any two sides BC, AC in the points D and E. Join AD, BE. The point G where these two lines intersect is the centre of gravity of the triangle. It can be proved that the point G is situated so that DG is one-third DA, and EG is one-third EB. We therefore have the following rules:—

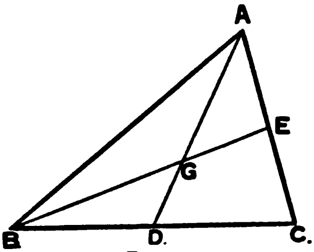


Fig. 30.

1. *Bisect any two sides of the triangle, and join the points thus obtained to the opposite angular points. Then the point in which these two lines intersect is the centre of gravity of the triangle.*

2. *Bisect any side of the triangle, and join the point thus obtained with the opposite angular point. The centre of gravity of the triangle will be on this line, and at a point at one-third its length measured from the bisected side.*

Trapezium.—Let ABCD, Fig. 31, be a trapezium. By joining the corners A and C we can divide the figure into two triangles, ADC, ABC. The centres of gravity, E and F, of these triangles can be found as indicated above. Join EF. The centre of gravity of the whole figure must be somewhere in the line EF. Again, join the corners D and B, thus dividing the figure into two triangles ADB, CDB. The centres of gravity, H and K, of these triangles can be found. The centre of gravity of the whole figure must be somewhere in the line HK; therefore the point G, where the lines HK and EF intersect, must be the centre of gravity of the trapezium.

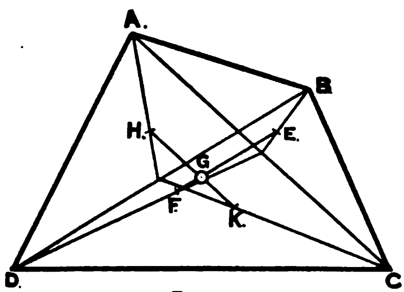


Fig. 31.

The following is a more convenient method of finding the centre of gravity of a trapezium.

Let ABCD, Fig. 32, be a trapezium. Draw the diagonals AC, BD, intersecting at E. In the figure CE is greater than

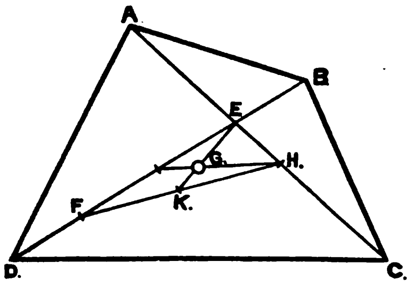


FIG. 32.

EA, and DE is greater than EB. Make CH = EA and DF = EB. Join FH. Then the centre of gravity of the triangle EFH will also be the centre of gravity of the trapezium ABCD.[1]

(A useful exercise in drawing would be to take a trapezium on a large scale and find its centre of gravity by each of the above methods. If the drawing is accurately done, the point should be in precisely the same position as found by each method.)

To find the Centre of Gravity of a Plane Area by Experiment.—Draw out the area on a piece of card-board or stiff paper, and cut out the shape. Then suspend the cardboard as indicated in Fig. 33, a small weight, W, being allowed to hang plumb. A line drawn behind the string AW must pass through the centre of gravity. Mark on the cardboard two points on the string, as A and B, and join. Then the centre of gravity must lie on AB. Now suspend the cardboard by another point, C,

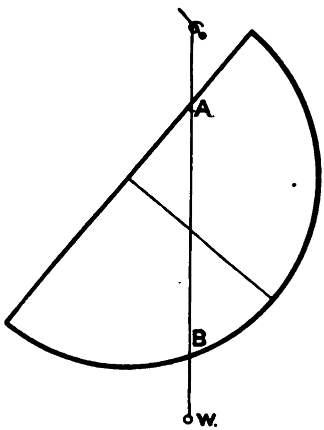


FIG. 33.

[1] See Example 22, for C.G. of a trapezoid.

as in Fig. 34, and draw the line CD immediately behind the string of the plumb-bob W. Then also the centre of gravity must lie on the line CD. Consequently it follows that the point of intersection G of the lines AB and CD must be the centre of gravity of the given area.

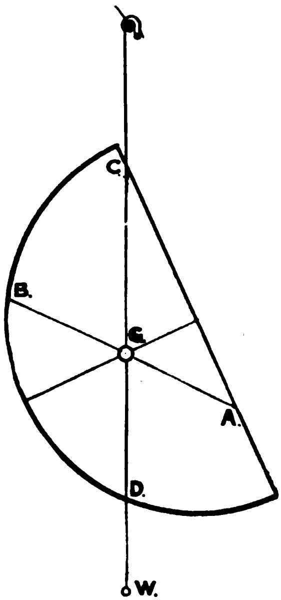


FIG. 34.

Example.—Set out the section of a beam on a piece of stiff paper, and find by experiment the position of its centre of gravity, the beam being formed of a bulb plate 9 inches deep and ½ inch thick, having two angles on the upper edge, each 3" × 3" × ½".

Ans. 3 inches from the top.

Centres of Gravity of Solids formed of Homogeneous Material.

Sphere.—The centre of gravity of a sphere is at its centre.

Cylinder.—The centre of gravity of a cylinder is at one-half its height from the base, on the line joining the centres of gravity of the ends.

Pyramid or Cone.—The centre of gravity of a pyramid or cone is at one-fourth the height of the apex from the base, on the line joining the centre of gravity of the base to the apex.

Moment of an Area.

The geometrical moment of a plane area relatively to a given axis, is the product of its area into the perpendicular distance of its centre of gravity from the given axis. It follows that the position of the centre of gravity is known relatively to the given axis if we know the geometrical moment about the axis and also the area, for the distance will be the moment divided by the area. It is usual to speak of the moment of an area about a given axis when the geometrical moment is really meant.

To find the Position of the Centre of Gravity of a Curvilinear Area with respect to one of its Ordinates.—Let AEDO, Fig. 35, be a plane curvilinear area, and

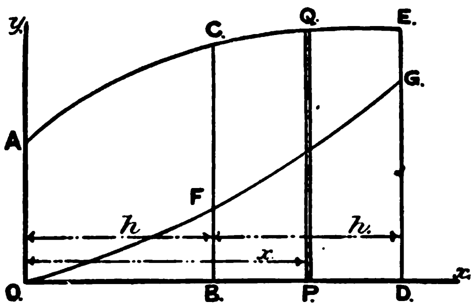


FIG. 35.

we wish to find its centre of gravity with respect to the end ordinate, OA. To do this, we must first find the moment of the total area about OA, and this divided by the area of the figure itself will give the distance of the centre of gravity from OA. Take any ordinate, PQ, a distance of x from OA, and at PQ draw a strip Δx wide. Then the area of the strip is $y \times \Delta x$ very nearly, and the moment of the strip about OA is $(y \times \Delta x)x$ very nearly.

If now Δx be made indefinitely small, the moment of the strip about OA will be—

$$y \,.\, x \,.\, dx$$

Now, we can imagine the whole area divided up into such strips, and if we added up the moments about OA of all such strips, we should obtain the total moment about OA. Therefore, using the notation we employed for finding the area of a plane curvilinear figure on p. 14, we shall have—

$$\textit{Moment} \text{ of the total area about OA} = \int y \,.\, x \,.\, dx$$

The expression for the *area* is—

$$\int y \,.\, dx$$

and this is of the same form as the expression for the moment. Therefore, instead of y we put yx through Simpson's rule in the ordinary way, and the result will be the moment about OA. Set off on BC a length BF = BC × h, and on DE a length DG = DE × $2h$. Then draw through all such points a curve, as OFG.[1] Any ordinate of this curve will give the ordinate of the original curve at that point multiplied by its distance from OA. The area of an elementary strip of this new curve will be $y \cdot x \cdot dx$, and the total area of the new curve will be $\int y \cdot x \cdot dx$, or the moment of the original figure about OA. Therefore, to find the moment of a curvilinear figure about an end ordinate, we take each ordinate and multiply it by its distance from the end ordinate. These products, put through Simpson's rule in the ordinary way, will give the moment of the figure about the end ordinate. This moment divided by the area will give the distance of the centre of gravity of the area from the end ordinate.

Example.—A midship section has semi-ordinates, 1′ 6″ apart, commencing at the L.W.L., of length 8·60, 8·10, 6·95, 4·90, 2·75, 1·50, 0·70 feet respectively. Find the area of the section and the distance of its C.G. from the L.W.L.

Number of ordinates.	Length of ordinates.	Simpson's multipliers.	Function of ordinates.	Number of intervals from No. 1 ord.	Products for moment.
1	8·60	1	8·60	0	0·0
2	8·10	4	32·40	1	32·40
3	6·95	2	13·90	2	27·80
4	4·90	4	19·60	3	58·80
5	2·75	2	5·50	4	22·00
6	1·50	4	6·00	5	30·00
7	0·70	1	0·70	6	4·20
			86·70	Function of moment	175·20

The half-area will be given by $86{\cdot}70 \times (\frac{1}{3} \times 1{\cdot}5) = 43{\cdot}35$ square feet
and the whole area is 86·70 „

The arrangement above is adopted in order to save labour in

[1] If the original curve is a parabola of the second order, this curve will be one of the third order, and it can be proved that Simpson's first rule will integrate a parabola of the third order.

finding the moment of the area. In the fourth column we have the functions of the ordinates, or the ordinates multiplied successively by their proper multipliers. In the fifth column is placed, not the actual distance of each ordinate from the No. 1 ordinate, but the *number of intervals* away, and the distance apart is brought in at the end. In the sixth column the products of the functions in column 4 and the multipliers in column 5 are placed. It will be noticed that we have put the ordinates through Simpson's multipliers first, and then multiplied by the numbers in the fifth column after. This is the reverse to the rule given in words above, which was put into that form in order to bring out the principle involved more plainly. The final result will, of course, be the same in either case, the method adopted giving the result with the least amount of labour, because column 4 is wanted for finding the area. The sum of the products in column 6 will not be the moment required, because it has to be multiplied as follows: First, by one-third the common interval, and second, by the distance apart of the ordinates.

$$\left.\begin{array}{l}\text{The moment of the half-area}\\ \text{about the L.W.L.}\end{array}\right\} = 175{\cdot}20 \times \left(\tfrac{1}{3} \times 1\tfrac{1}{2}\right) \times 1\tfrac{1}{2}$$

$$= 131{\cdot}4$$

and the distance of the C.G. of the half-area from the L.W.L. is—

$$\text{Moment} \div \text{area} = \frac{131{\cdot}4}{43{\cdot}35} = 3{\cdot}03 \text{ feet}$$

It will be noticed that we have multiplied both columns 4 and 6 by one-third the common interval, the distance of the C.G. from No. 1 ordinate being obtained by—

$$\frac{175{\cdot}20 \times (\frac{1}{3} \times 1{\cdot}5) \times 1{\cdot}5}{86{\cdot}70 \times (\frac{1}{3} \times 1{\cdot}5)}$$

The expression $\frac{1}{3} \times 1{\cdot}5$ is common to both top and bottom, and so can be cancelled out, and we have—

$$\frac{175{\cdot}20 \times 1{\cdot}5}{86{\cdot}70} = 3{\cdot}03 \text{ feet}$$

The position of the centre of gravity of the half-area with regard to the L.W.L. is evidently the same as that of the whole area.

When finding the centre of gravity of a large area, such as a water-plane of a vessel, it is usual to take moments about the middle ordinate. This considerably simplifies the work, because the multipliers in column 5 are not so large.

Example.—The semi-ordinates of the load water-plane of a vessel 395 feet long are, commencing from forward, 0, 10·2, 20·0, 27·4, 32·1, 34·0, 33·8, 31·7, 27·6, 20·6, 9·4. Find the area and the distance of its C.G. from the middle ordinate.

In addition to the above, there is an appendage abaft the last ordinate, having an area of 153 square feet, and whose C.G. is 5·6 feet abaft the last ordinate. Taking this appendage into account, find the area and the position of the C.G. of the water-plane.

Number of ordinates.	Length of ordinates.	Simpson's multipliers.	Function of ordinates.	Number of interval from mid. ord.	Product for moment.
1	0·0	1	0·0	5	0·0
2	10·2	4	40·8	4	163·2
3	20·0	2	40·0	3	120·0
4	27·4	4	109·6	2	219·2
5	32·1	2	64·2	1	64·2
6	34·0	4	136·0	0	566·6
7	33·8	2	67·6	1	67·6
8	31·7	4	126·8	2	253·6
9	27·6	2	55·2	3	165·6
10	20·6	4	82·4	4	329·6
11	9·4	1	9·4	5	47·0
			732·0		863·4

The half-area will be given by—

$$732 \cdot 0 \times (\tfrac{1}{3} \times 39 \cdot 5) = 9638 \text{ square feet}$$

The fifth column gives the number of intervals away from the middle ordinate, and the products are obtained for the forward portion adding up to 566·6, and they are obtained for the after portion adding up to 863·4. This gives an excess aft of 863·4 − 566·6 = 296·8. The distance of the C.G. abaft the middle ordinate is then given by—

$$\frac{296 \cdot 8 \times 39 \cdot 5}{732 \cdot 0} = 16 \cdot 01 \text{ feet}$$

The area of both sides is 19,276 square feet.

The second part of the question takes into account an appendage abaft No. 11 ordinate, having an area of 153 square feet.

The total area will then be—

$$19{,}276 + 153 = 19{,}429 \text{ square feet}$$

To find the position of the C.G. of the whole water-plane, we take moments about No. 6 ordinate, the distance of the C.G. of the appendage from it being—

$$197{\cdot}5 + 5{\cdot}6 = 203{\cdot}1 \text{ feet}$$

Moment of main area abaft No. 6 ordinate = 19,276 × 16·01 = 308,609
" appendage " " = 153 × 203·1 = 31,074
∴ total moment abaft No. 6 ordinate = 308,609 + 31,074
= 339,683

and the distance of the centre of gravity of the whole area abaft No. 6 ordinate $\Big\} = \frac{339683}{19429} = 17{\cdot}48$ feet

To find the Position of the Centre of Gravity of a Curvilinear Area contained between Two Consecutive Ordinates with respect to the Near End Ordinate.—The rule investigated in the previous paragraph for finding the centre of gravity of an area about its end ordinate fails when applied to such a case as the above. For instance, try the following example:—

A curve has ordinates 10, 9, 7 feet long, 4 feet apart. To find the position of the centre of gravity of the portion between the two first ordinates with respect to the end ordinate.

Ordinates.	Simpson's multipliers.	Functions.	Multipliers for moment.	Products for moment.
10	5	50	0	0
9	8	72	1	72
7	−1	−7	2	−14
		115		58

Centre of gravity from the end ordinate would be—

$$\frac{58 \times 4}{115} = 2\tfrac{2}{115} \text{ feet}$$

Now this is evidently wrong, since the shape of the curve is such that the centre of gravity ought to be slightly less than 2 feet from the end ordinate.

We must use the following rule:—

To ten times the middle ordinate add three times the near end ordinate and subtract the far end ordinate. Multiply the *remainder by one-twenty-fourth the square of the common interval. The product will be the moment about the end ordinate.*

Using y_1, y_2, y_3, for the lengths of the ordinates, and h the common interval, the moment of the portion between the ordinates y_1 and y_2 about the ordinate y_1 is given by—

$$\frac{h^2}{24}(3y_1 + 10y_2 - y_3)$$

We will now apply this rule to the case considered above.

Ordinates.	Area.		Moment.	
	Simpson's multipliers.	Functions.	Simpson's multipliers.	Functions.
10	5	50	3	30
9	8	72	10	90
7	− 1	− 7	− 1	− 7
		115		113

$$\text{Moment} = 113 \times \tfrac{16}{24}$$
$$\text{Area} = 115 \times \tfrac{4}{12}$$

Therefore distance of the centre of gravity from the end ordinate is—

$$\frac{113 \times \frac{16}{24}}{115 \times \frac{4}{12}} = \frac{113 \times 2 \times 3}{115 \times 3}$$
$$= \tfrac{226}{115} = 1{\cdot}965 \text{ feet}$$

This result is what one might expect by considering the shape of the curve.

To find the Position of the Centre of Gravity of a Curvilinear Area with respect to its Base.—Let DABC, Fig. 36, be a plane curvilinear area. We wish to find the distance of its centre of gravity from the base DC. To do this, we must first find the moment of the figure about DC and divide it by the area. Take any ordinate PQ, and at PQ draw a consecutive ordinate giving a strip Δx wide. Then the area of the strip is—

$$y \times \Delta x \text{ very nearly}$$

and regarding it as a rectangle, its centre of gravity is at a distance of $\frac{1}{2}y$ from the base. Therefore the moment of the strip about the base is—

$$\frac{1}{2}y^2 \times \Delta x$$

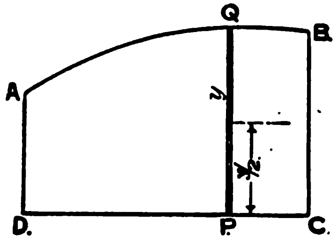


FIG. 36.

If now we consider the strip to be indefinitely thin, its moment about the base will be—

$$\frac{1}{2}y^2 . dx$$

and the moment of the total area about the base must be the sum of the moments of all such strips, or—

$$\int \frac{1}{2}y^2 . dx$$

This expression for the moment is of the same form as that for the area, viz. $\int y . dx$. Therefore, instead of y we put $\frac{1}{2}y^2$ [1] through Simpson's rule in the ordinary way, and the result will be the moment of the area about DC.

Example.—An athwartship coal-bunker is 6 feet long in a fore-and-aft direction. It is bounded at the sides by two longitudinal bulkheads 34 feet apart, and by a horizontal line at the top. The bottom is formed by the inner bottom of the ship, and is in the form of a curve having vertical ordinates measured from the top of 12·5, 15·0, 16·0, 16·3, 16·4, 16·3, 16·0, 15·0, 12·5 feet respectively, the first and last ordinates being on the bulkheads. Find—

(1) The number of tons of coal the bunker will hold.

(2) The distance of the centre of gravity of the coal from the top.

The inner bottom is symmetrical either side of the middle line, so we need only deal with one side. The work is arranged as follows:—

Ordinates.	Simpson's multipliers.	Functions of ordinates.	Squares of ordinates.	Simpson's multipliers.	Functions of squares.
16·4	1	16·4	269	1	269
16·3	4	65·2	266	4	1064
16·0	2	32·0	256	2	512
15·0	4	60·0	225	4	900
12·5	1	12·5	156	1	156

Function of area 186·1 Function of moment 2901

[1] This assumes that Simpson's first rule, which will most probably be used, will correctly integrate a parabola of the fourth order, which can be shown to be the case for all practical purposes.

$$\text{Common interval} = 4{\cdot}25 \text{ feet}$$
$$\text{Half-area of section} = 186{\cdot}1 \times \tfrac{1}{3} \times 4{\cdot}25 \text{ square feet}$$
$$\text{Volume of bunker} = 186{\cdot}1 \times \frac{4{\cdot}25 \times 2 \times 6}{3} \text{ cubic feet}$$
$$\text{Number of tons of coal} = 186{\cdot}1 \times \tfrac{17}{44}$$
$$= 72 \text{ tons}$$
$$\text{Moment of half-area below top} = 2901 \times \frac{1}{2} \times \frac{4{\cdot}25}{3}$$
$$\text{And distance of C.G. from the top} = \frac{\text{moment}}{\text{area}}$$
$$= \frac{2901 \times \frac{1}{2} \times \frac{4{\cdot}25}{3}}{186{\cdot}1 \times \frac{4{\cdot}25}{3}} = \frac{2901}{186{\cdot}1} \times \frac{1}{2}$$
$$= 7{\cdot}8 \text{ feet.}$$

In the first three columns we proceed in the ordinary way for finding the area. In the fourth column is placed, not the half-squares, but the squares of the ordinates in column 1, the multiplication by $\frac{1}{2}$ being brought in at the end. These squares are then put through Simpson's multipliers, and the addition of column 6 will give a function of the moment of the area about the base. This multiplied by $\frac{1}{2}$ and by $\frac{1}{3}$ the common interval gives the actual moment. This moment divided by the area gives the distance of the centre of gravity we want. It will be noticed that $\frac{1}{3}$ the common interval comes in top and bottom, so that we divide the function of the moment 2901 by the function of the area 186·1, and then multiply by $\frac{1}{2}$ to get the distance of centre of gravity required.

It is not often required in practice to find the centre of gravity of an area with respect to its base, because most of the areas we have to deal with are symmetrical either side of a centre line (as water-planes), but the problem sometimes occurs, the question above being an example.

To find the Position of the Centre of Gravity of an Area bounded by a Curve and Two Radii.—We have already seen (p. 15) how to find the area of a figure such as this. It is simply a step further to find the position of the centre of gravity with reference to either of the bounding radii. Let OAB, Fig. 13, be a figure bounded by a curve, AB, and two bounding radii, OA, OB. Take any radius OP, the angle BOP being called θ, and the length of OP being called r.

Draw a consecutive radius, OP′; the angle POP′ being indefinitely small, we may call it $d\theta$. Using the assumptions we have already employed in finding areas, the area POP′ = $\frac{1}{2}r^2 . d\theta$, POP′ being regarded as a triangle. The centre of gravity of POP′ is at g, and Og = $\frac{2}{3}r$, and gm is drawn perpendicular to OB, and $gm = \frac{2}{3}r . \sin\theta$ (see p. 91).

$$\left.\begin{array}{c}\text{The moment of the area}\\ \text{POP}' \text{ about OB}\end{array}\right\} = (\tfrac{1}{2}r^2 . d\theta) \times (\tfrac{2}{3}r . \sin\theta)$$

$$= \tfrac{1}{3}r^3 . \sin\theta . d\theta$$

The moment of the whole figure about OB is the sum of the moments of all such small areas as POP′, or, using the ordinary notation—

$$\tfrac{1}{3}\int r^3 . \sin\theta . d\theta$$

This is precisely similar in form to the expression we found for the area of such a figure as the above (see p. 15), viz.—

$$\tfrac{1}{2}\int r^2 . d\theta$$

so that, instead of putting $\frac{1}{2}r^2$ through Simpson's rule, measuring r at equidistant angular intervals, we put $\frac{1}{3}r^3 . \sin\theta$ through the rule in a similar way. This will be best illustrated by the following example:—

Example.—Find the area and position of centre of gravity of a quadrant of a circle with reference to one of its bounding radii, the radius being 10 feet.

We will divide the quadrant by radii 15° apart, and thus be able to use Simpson's first rule.

Number of radius.	Length of radius.	(Radius)².	Simpson's multipliers.	Product for area.	(Radius)³.	Angle from first radius.	Sin of angle.	Product $r^3 \times \sin\theta$.	Simpson's multipliers.	Product for moment.
1	10	100	1	100	1000	0	0·0	0	1	0
2	10	100	4	400	1000	15	0·258	258	4	1,032
3	10	100	2	200	1000	30	0·500	500	2	1,000
4	10	100	4	400	1000	45	0·707	707	4	2,828
5	10	100	2	200	1000	60	0·866	866	2	1,732
6	10	100	4	400	1000	75	0·965	965	4	3,860
7	10	100	1	100	1000	90	1·000	1000	1	1,000

Function of area 1800 Function of moment 11,452

$$\text{The circular measure of } 180^\circ = \pi = 3{\cdot}1416$$

$$\text{,, \quad ,, \quad } 15^\circ = \frac{3{\cdot}1416}{12}$$

$$\therefore \text{ area} = 1800 \times \tfrac{1}{2} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)$$

$$= 78{\cdot}54 \text{ square feet}$$

$$\text{Moment of area about the first radius} = 11{,}452 \times \frac{1}{3} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)$$

therefore distance of centre of gravity from the first radius is—

$$\text{Moment} \div \text{area} = \frac{11{,}452 \times \frac{1}{3} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)}{1800 \times \frac{1}{2} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)}$$

$$= \frac{11452 \times 2}{1800 \times 3} = 4{\cdot}24 \text{ feet}$$

The exact distance of the centre of gravity of a quadrant from either of its bounding radii is $\frac{4}{3\pi}$ times the radius, and if this is applied to the above example, it will be found that the result is correct to two places of decimals, and would have been more correct if we had put in the values of the sines of the angles to a larger number of decimal places.

Centre of Gravity of a Solid Body which is bounded by a Curved Surface and a Plane.—In the first chapter we saw that the finding the volume of such a solid as this was similar in principle to the finding the area of a plane curve, the only difference being that we substitute areas for simple ordinates, and as a result get the volume required. The operation of finding the centre of gravity of a volume in relation to one of the dividing planes is precisely similar to the operation of finding the centre of gravity of a curvilinear area in relation to one of its ordinates. This will be illustrated by the following example:—

Example.—A coal-bunker has sections 17′ 6″ apart, and the areas of these sections, commencing from forward, are 98, 123, 137, 135, 122 square feet respectively. Find the volume of the bunker, and the position of its centre of gravity in a fore-and-aft direction.

Areas.	Simpson's multipliers.	Functions of areas.	Number of intervals from forward.	Products for moments.
98	1	98	0	0
123	4	492	1	492
137	2	274	2	548
135	4	540	3	1620
122	1	122	4	488
		1526		3148

$$\text{Volume} = 1526 \times \tfrac{1}{3} \times 17\tfrac{1}{2} = 8902 \text{ cubic feet}$$
$$\text{moment} = 3148 \times \tfrac{1}{3} \times 17\tfrac{1}{2} \times 17\tfrac{1}{2}$$
$$\therefore \left.\begin{array}{l}\text{distance of centre of gravity}\\ \text{from forward end}\end{array}\right\} = \frac{3148 \times 17\cdot5}{1526} = 36\cdot2 \text{ feet}$$

It is always advisable to roughly check any result such as this; and if this habit is formed, it will often prevent mistakes being made. The total length of this bunker is 4 × 17′ 6″ = 70 feet, and the areas of the sections show that the bunker is fuller aft than forward, and so, on the face of it, we should expect the position of the centre of gravity to be somewhat abaft the middle of the length; and this is shown to be so by the result of the calculation. Also as regards the volume. This must be less than the volume of a solid 70 feet long, and having a constant section equal to the area of the middle section of the bunker. The volume of such a body would be 70 × 137 = 9590 cubic feet. The volume, as found by the calculation, is 8902 cubic feet, thus giving a coefficient of $\frac{8902}{9590} = 0{\cdot}93$ nearly, which is a reasonable result to expect.

Centre of Buoyancy.—The centre of buoyancy of a vessel is the centre of gravity of the underwater volume, or, more simply, the centre of gravity of the displaced water. This has nothing whatever to do with the centre of gravity of the ship herself. The centre of buoyancy is determined solely by the shape of the underwater portion of the ship. The centre of gravity of the ship is determined by the distribution of the weights forming the structure, and of all the weights she has on board. Take the case of two sister ships built from the same lines, and each carrying the same weight of cargo and floating at the same water-line. The centre of buoyancy of each of these ships must necessarily be in the same position. But suppose they are engaged in different trades—the first, say, carrying a cargo of steel rails and other heavy weights, which are stowed low down. The second, we may suppose, carries a cargo of homogeneous materials, and this has to be stowed much higher than the cargo in the first vessel. It is evident that the centre of gravity in the first vessel must be much lower down than in the second, although as regards form they are precisely similar. This distinction between the centre of buoyancy and the centre of gravity is a very important one, and should always be borne in mind.

To find the Position of the Centre of Buoyancy of a Vessel in a Fore-and-aft Direction, having given the Areas of Equidistant Transverse Sections.—The following example will illustrate the principles involved :—

Example.—The underwater portion of a vessel is divided by transverse sections 10 feet apart of the following areas, commencing from forward : 0·2, 22·7, 48·8, 73·2, 88·4, 82·8, 58·7, 26·2, 3·9 square feet respectively. Find the position of the centre of buoyancy relative to the middle section.

Number of station.	Area of section.	Simpson's multipliers.	Functions of area.	Number of intervals from middle.	Product for moment.
1	0·2	1	0·2	4	0·8
2	22·7	4	90·8	3	272·4
3	48·8	2	97·6	2	195·2
4	73·2	4	292·8	1	292·8
5	88·4	2	176·8	0	761·2
6	82·8	4	331·2	1	331·2
7	58·7	2	117·4	2	234·8
8	26·2	4	104·8	3	314·4
9	3·9	1	3·9	4	15·6

Function of displacement 1215·5   Function of moment } 896·0

$$\text{Volume of displacement} = 1215{\cdot}5 \times \tfrac{10}{3}$$
$$\text{excess of products aft} = 896{\cdot}0 - 761{\cdot}2 = 134{\cdot}8$$
$$\text{moment aft} = 134{\cdot}8 \times \tfrac{10}{3} \times 10$$

$$\text{C.B. abaft middle} = \frac{134{\cdot}8 \times \frac{10}{3} \times 10}{1215{\cdot}5 \times \frac{10}{3}}$$
$$= \frac{134{\cdot}8 \times 10}{1215{\cdot}5} = 1{\cdot}11 \text{ feet}$$

The centre of gravity of a plane area is fully determined when we know its position relative to two lines in the plane, which are generally taken at right angles to one another. The centre of gravity of a volume is fully determined when we know its position relative to three planes, which are generally taken at right angles to one another. In the case of the under-water volume of a ship, we need only calculate the position of its centre of gravity relative to (1) the load water-plane, and (2) an athwartship section (usually the section amidships), because, the two sides of the ship being identical, the centre of gravity of the displacement must lie in the middle-line longitudinal plane of the ship.

Approximate Position of the Centre of Buoyancy.—In vessels of ordinary form, it is found that the distance of the centre of buoyancy below the L.W.L. varies from about $\frac{8}{20}$ to $\frac{9}{20}$ of the mean draught to top of keel, the latter being the case in vessels of full form. For yachts and vessels of unusual form, such a rule as this cannot be employed.

Example.—A vessel 13′ 3″ mean draught has her C.B. 5·34 feet below L.W.L.

Here the proportion of the draught is—

$$\frac{5 \cdot 34}{13 \cdot 25} = 0 \cdot 403 = \frac{8 \cdot 06}{20}$$

This is an example of a fine vessel.

Example.—A vessel 27′ 6″ mean draught has her C.B. 12·02 feet below L.W.L.

Here the proportion of the draught is—

$$\frac{12 \cdot 02}{27 \cdot 5} = \frac{8 \cdot 75}{20}$$

This is an example of a fuller vessel than the first case.

Morrish's Approximate Formula for the Distance of the Centre of Buoyancy below the Load Water-line.[1]

Let V = volume of displacement up to the load-line in cubic feet;

A = the area of the load water-plane in square feet;

d = the mean draught (to top of keel) in feet.

[1] See a paper in *Transactions of the Institution of Naval Architects*, by Mr. S. W. F. Morrish, M.I.N.A., in 1892.

Then centre of buoyancy below L.W.L. $= \frac{1}{3}\left(\frac{d}{2} + \frac{V}{A}\right)$

This rule gives exceedingly good results for vessels of ordinary form. In the early stages of a design the above particulars would be known as some of the elements of the design, and so the vertical position of the centre of buoyancy can be located very nearly indeed. In cases in which the stability of the vessel has to be approximated to, it is important to know where the C.B. is, as will be seen later when we are dealing with the question of stability.

The rule is based upon a very ingenious assumption, as follows:—

In Fig. 36A, let ABC be the curve of water-plane areas, DC being the mean draught *d*. Draw the rectangle AFCD. Make DE $= \frac{V}{A} =$ D say. Draw EG parallel to DA cutting the diagonal FD in H. Finish the figure as indicated. *Then the assumption*

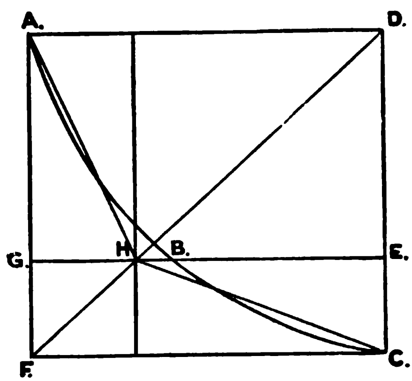


Fig. 36A.

made is that the C.G. of the area DAHC is the same distance below the water-line as the C.G. of DABC which latter, of course, gives the distance below the water-line of the centre of buoyancy. It is seen by inspection that the assumption is a reasonable one.

DAHC and DABC have the same area as we now proceed to show. The rectangles AH and HC are equal, so that the triangles AGH and HEC are equal, and therefore—

$$\text{Area of AHCD} = \text{area of AGED}$$

The latter gives the volume of displacement, as it is a rectangle having sides equal to A and $\frac{V}{A}$ respectively. The area of DABC also gives the volume of displacement, so that DAHC and DABC are equal.

We now have to determine the distance of the C.G. of DAHC below the water-line.

$$\frac{\text{Area AGH}}{\text{Area AGED}} = \frac{\frac{1}{2} \times \text{AG} \times \text{GH}}{\text{AG} \times \text{AD}} = \frac{1}{2} \cdot \frac{\text{GH}}{\text{AD}}$$

$$= \frac{1}{2} \cdot \frac{\text{GF}}{\text{AF}} = \frac{1}{2} \cdot \left(\frac{d - \text{D}}{d}\right)$$

d being the draught.

$$\therefore \text{Area AGH} = \frac{1}{2}\left(\frac{d - \text{D}}{d}\right) \times \text{rectangle AGED}$$

We may regard the figure DAHC made up by taking away AGH from the rectangle AE and putting it in the position HEC. The shift of its C.G. downwards is $\frac{1}{3} . d$. Therefore the C.G. of the whole figure will shift downwards, using the principle explained in p. 100, a distance x, given by—

$$\text{AGED} \times x = \text{AGH} \times \frac{d}{3}$$

or putting in the value found above for the area of AGH, we have—

$$x = \frac{1}{2} . \left(\frac{d - \text{D}}{d}\right) . \frac{d}{3} = \frac{1}{6}(d - \text{D})$$

The C.G. of AGED is a distance $\frac{\text{D}}{2}$ below the water-line. Therefore the C.G. of DAHC is below the water-line, a distance—

$$\frac{\text{D}}{2} + \frac{1}{6}(d - \text{D}) = \frac{1}{3}\left(\frac{d}{2} + \frac{\text{V}}{\text{A}}\right)$$

which is Morrish's approximation to the distance of the C.B. below the water-line.

The Area of a Curve of Displacement divided by the Load Displacement gives the Distance of the Centre of Buoyancy below the Load Water-line.—This is an interesting property of the curve of displacement. The demonstration is as follows:—

Let OBL, Fig. 36B, be the curve of displacement of a vessel constructed in the ordinary way, OW being the mean draught and WL being the displacement in tons.

Take two level lines AB, A'B', a short distance apart, Δx say. Call the area of the water-plane at the level of AB, A square feet, and the distance of this water-plane below the WL, z. The volume between AB and A′B′ is $A \times \Delta z$, or supposing they are indefinitely close together $A \times dz$. The moment of this layer about the WL is $A \times dz \times (z + \frac{1}{2} . dz) = A . z . dz$, neglecting the

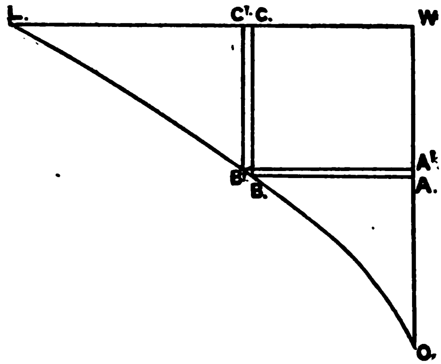


FIG. 36B.

square of the small quantity dz. The distance of the C.B. below WL is the sum of all such moments divided by the displacement volume,

$$\text{or} \quad \frac{\int A . z . dz}{35 \times WL}$$

Now the difference between the lengths of A′B′ and AB is the weight of the water between these level lines or $\frac{1}{35} . A . dz$. The area of the whole figure is given by the summatim of all such areas as the strip B′C, which has a length z and a breadth $\frac{1}{35} . A dz$. Area of figure is therefore $\frac{1}{35} \int A . z . dz$, and this divided by the displacement is—

$$\frac{\int A . z . dz}{35 \times WL}$$

which is the expression found above for the distance of the C.B. below the WL.

Example.—Draw a curve of displacement for all draughts of a cylindrical vessel 20 feet diameter and 150 feet long, and find *by using the curve* the distance of the C.B. from the base when floating (*a*) at 10 feet level draught, (*b*) at 15 feet level draught.

Ans. (*a*) 5·76 feet; (*b*) 8·25 feet.

If a new curve be drawn having for ordinates the area of the curve of displacement at respective levels, it may readily be shown that the tangent to this curve at any draught will intersect the scale of draughts at the height of the centre of buoyancy. This new curve is a curve of moments of displacement up to each level line about such level line. By constructing such a curve in the graphic method of finding displacement (see later), considerable simplification of the process is obtained. Thus, in Fig. 39, AHL is the curve of displacement. By integrating this curve (still by the graphic method), a curve of moments of displacements is obtained, the ordinate of which on BR will be the moment of displacement BL about the L.W.L. This moment, divided by the displacement BL, gives the distance of the C.B. below the L.W.L. This may be checked by drawing the tangent to the new curve, as seen above. In a similar manner, in Fig. 40 the difference between the areas of BB in the fore and after bodies divided by the total displacement gives the fore and aft position of the C.B. with reference to No. 6 station.

Displacement Sheet.[1]—We now proceed to investigate the method that is very generally employed in practice to find the displacement of a vessel, and also the position of its centre of buoyancy both in a longitudinal and a vertical direction. The calculation is performed on what is termed a "*Displacement Sheet*" or "*Displacement Table*," and a specimen calculation is given at the end of the book for a single-screw tug of the following dimensions:—

Length between perpendiculars	74′ 0″
Breadth moulded	14′ 6″
Depth moulded	8′ 3″
Draught moulded forward	5′ 5″
,, ,, aft	6′ 2″
,, ,, mean	5′ 9½″

The sheer drawing of the vessel is given on Plate I. This drawing consists of three portions—the body plan, the half-breadth plan, and the sheer. The sheer plan shows the ship in side elevation, the load water-line being horizontal, and the keel, in this case, sloping down from forward to aft. The ship is supposed cut by a number of transverse vertical planes, which are shown in the sheer plan as straight lines, numbered 1, 2, 3, etc. Now, each of these transverse sections of the ship has a definite shape, and the form of each half-section to the outside of frames is shown in the body-plan, the sections being numbered as in the sheer. The sections of the forward end form what is termed the "*fore-body*," and those of the after end the "*after-body*." Again, the ship may be supposed to be cut by a series of equidistant horizontal planes, of which the

[1] For displacement sheet with combination of Simpson's first rule and Tchebycheff's rule, see Appendix A.

load water-plane is one. The shape of the curve traced on each of these planes by the moulded surface of the ship is given in the half-breadth plan, and the curves are numbered A, 1, 2, 3, etc., to agree with the corresponding lines in the sheer and body plan. Each of these plans must agree with the other two. Take a special station, for example, No. 4. The breadth of the ship at No. 4 station at the level of No. 3 water-plane is O*a'* in the body-plan, but it is also given in the half-breadth plan by O*a*, and therefore O*a* must exactly equal O*a'*. The process of making all such points correspond exactly is known as "*fairing.*" For full information as to the methods adopted in fairing, the student is referred to the works on "Laying-off" given below.[1] For purposes of reference, the dimensions of the vessel and other particulars are placed at the top of the displacement sheet. The water-lines are arranged on the sheer drawing with a view to this calculation, and in this case are spaced at an equidistant spacing apart of 1 foot, with an intermediate water-line between Nos. 5 and 6. The number of water-lines is such that Simpson's first rule can be used, and the multipliers are, commencing with the load water-plane—

$$1 \quad 4 \quad 2 \quad 4 \quad 1\tfrac{1}{2} \quad 2 \quad \tfrac{1}{2}$$

The close spacing near the bottom is very necessary to ensure accuracy, as the curvature of sections amidships of the vessel is very sharp as the bottom is approached, and, as we saw on p. 13, Simpson's rules cannot accurately deal with areas such as these unless intermediate ordinates are introduced. Below No. 6 water-plane there is a volume the depth of which increases as we go aft, and the sections of this volume are very nearly triangles. This volume is dealt with separately on the left-hand side of the table, and is termed an "*appendage.*"

In order to find the volume of displacement between water-planes 1 and 6, we can first determine the areas of the water-planes, and then put these areas through Simpson's rule. To find the area of any of the water-planes, we must proceed in the ordinary manner and divide its length by ordinates so that

[1] "Laying Off," by Mr. S. J. P. Thearle; "Laying Off," by Mr. T. H. Watson; "Laying Off," by Messrs. Attwood and Cooper.

Simpson's rule (preferably the first rule) can be used. In the case before us, the length is from the after-edge of the stem to the forward edge of the body post, viz. 71 feet, and this length is divided into ten equal parts, giving ordinates to each of the water-planes at a distance apart of 7·1 feet. The displacement-sheet is arranged so that we can put the lengths of the semi-ordinates of the water-planes in the columns headed respectively L.W.L., 2W.L., 3W.L., etc., the semi-ordinates at the several stations being placed in the same line as the numbers of ordinates given at the extreme left of the table. The lengths of the semi-ordinates are shown in italics. Thus, for instance, the lengths of the semi-ordinates of No. 3 W.L., as measured off, are 0·05, 1·82, 4·05, 5·90, 6·90, 7·25, 7·04, 6·51 5·35, 2·85, and 0·05 feet, commencing with the forward ordinate No. 1, and these are put down in italics[1] as shown beneath the heading 3 W.L. in the table. The columns under the heading of each W.L. are divided into two, the semi-ordinates being placed in the first column. In the second column of each water-line is placed the product obtained by multiplying the semi-ordinate by the corresponding multiplier to find the area. These multipliers are placed at column 2 at the left, opposite the numbers of the ordinates. We have, therefore, under the heading of each water-line what we have termed the "*functions of ordinates*," and if these functions are added up, we shall obtain what we have termed the "*function of area*."

Taking No. 3 W.L. as an instance, the "*function*" of its area is 144·10, and to convert this "*function*" into the actual area, we must multiply by one-third the common interval to complete Simpson's first rule, *i.e.* by $\frac{1}{3} \times 7·1$; and also by 2 to obtain the area of the water-plane on both sides of the ship. We should thus obtain the area of No. 3 W.L.—

$$144·10 \times \tfrac{1}{3} \times 7·1 \times 2 = 689·07 \text{ square feet}$$

The functions of the area of each water-plane are placed at the bottom of the columns, the figures being, starting with the L.W.L., 163·70, 155·36, 144·10, 128·74, 105·67, 87·27 and 60·97. To get the actual areas of each of the water-planes,

[1] In practice, it is advisable to put down the lengths of the semi-ordinates in some distinctive colour, such as red.

we should, as above, multiply each of these functions by $\frac{1}{3} \times 7.1 \times 2$. Having the areas, we could proceed as on p. 26 to find the volume of displacement between No. 1 and No. 6 water-lines, but we do not proceed quite in this way; we put the "*functions of areas*" through Simpson's rule, and multiply afterwards by $\frac{1}{3} \times 7.1 \times 2$, the same result being obtained with much less work. Below the "functions of areas" are placed the Simpson's multipliers, and the products 163·70, 621·44, etc., are obtained. These products added up give 1951·83. This number is a function of the volume of displacement, this volume being given by first multiplying it by one-third the vertical interval, *i.e.* $\frac{1}{3} \times 1$; and then by $\frac{1}{3} \times 7.1 \times 2$, as seen above. The volume of displacement, between No. 1 W.L. and No. 6 W.L. is therefore—

$$1951.83 \times (\tfrac{1}{3} \times 1) \times (\tfrac{1}{3} \times 7.1) \times 2 = 3079.5 \text{ cubic feet}$$

$$\left.\begin{array}{c}\text{and the displacement in}\\ \text{tons (salt water)}^{1}\end{array}\right\} = \frac{3079.5}{35} = 87.98 \text{ tons}$$

We have thus found the displacement by dividing the volume under water by a series of equidistant horizontal planes; but we could also find the displacement by dividing the under-water volume by a series of equidistant vertical planes, as we saw in Chapter I. This is done on the displacement sheet, an excellent check being thus obtained on the accuracy of the work. Take No. 4 section, for instance: its semi-ordinates, commencing with the L.W.L., are 6·40, 6·24, 5·90, 5·32, 4·30 3·40, and 2·25 feet These ordinates are already put down opposite No. 4 ordinate. If these are multiplied successively by the multipliers, 1, 4, 2, 4, 1½, 2, ½, and the sum of the functions of ordinates taken, we shall obtain the "*function of area*" of No. 4 section between the L.W.L. and 6 W.L. This is done in the table by placing the functions of ordinates immediately below the corresponding ordinate, the multiplier being given at the head of each column. We thus obtain a series of horizontal rows, and these rows are added up, the results being placed in the column headed "*Function of areas.*" Each of these functions multiplied by one-third the common

[1] Thirty-five cubic feet of salt water taken to weigh one ton.

interval, *i.e.* $\frac{1}{3} \times 1$, and then by 2 for both sides, will give the areas of the transverse sections between the L.W.L. and 6 W.L. ; but, as before, this multiplication is left till the end of the calculation. These functions of areas are put through Simpson's multipliers, the products being placed in the column headed "*Multiples of areas.*" This column is added up, giving the result 1951·83. To obtain the volume of displacement, we multiply this by $(\frac{1}{3} \times 1) \times 2 \times (\frac{1}{3} \times 7{\cdot}1)$. It will be noticed that we obtain the number 1951·83 by using the horizontal water-lines and the vertical sections; and this must evidently be the case, because the displacement by either method must be the same. The correspondence of these additions forms the check, spoken of above, of the accuracy of the work. We thus have the result that the volume of displacement from L.W.L. to 6 W.L. is 3079·5 cubic feet, and the displacement in tons of this portion 87·98 tons in salt water. This is termed the "Main solid," and forms by far the greater portion of the displacement.

We now have to consider the portion we have left out below No. 6 water-plane. Such a volume as this is termed an "*appendage.*" The sections of this appendage are given in the body-plan at the several stations. The form of these sections are traced off, and by the ordinary rules their areas are found in square feet. We have, therefore, this volume divided by a series of equidistant planes the same as the main solid, and we can put the areas of the sections through Simpson's rule and obtain the volume. This calculation is done on the left-hand side of the sheet, the areas being placed in column 3, and the functions of the areas in column 4. The addition of these functions is 49·99, and this multiplied by $\frac{1}{3} \times 7{\cdot}1$ gives the volume of the appendage in cubic feet, viz. 118·3; and this volume divided by 35 gives the number of tons the appendage displaces in salt water, viz. 3·38 tons. The total displacement is thus obtained by adding together the main solid and the appendage, giving 91·36 tons in salt water. The displacement in fresh water (36 cubic feet to the ton) would be 88·8 tons.

The sheer drawing for this vessel as given on Plate I. was drawn to the frame line, *i.e.* to the moulded dimensions of the ship; but the actual ship is fuller than this, because of the outer bottom plating, and this plating will contribute a small amount to the displacement, but this is often neglected. Some sheer drawings, on the other hand, are drawn so that the lines include a mean thickness of plating outside the frame line, and when this is the case, the displacement sheet gives the actual displacement, including the effect of the plating. For a sheathed ship this is also true; in this latter case, the displacement given by the sheathing would be too great to be neglected. When the sheer drawing is drawn to the outside of sheathing, or to a mean thickness of plating, it is evident that the ship must be laid off on the mould loft floor, so that, when built, she shall have the form given by the sheer drawing.

We now have to find the position of the centre of buoyancy both in a fore-and-aft and in a vertical direction. (It must be in the middle-line plane of the ship, since both sides are symmetrical.) Take first the fore-and-aft position. This is found with reference to No. 6 station. The functions of the areas of the sections are 0·5, 23·055, etc., and in the column headed "Multiples of areas" we have these functions put through Simpson's multipliers. We now multiply these multiples by the number of intervals they respectively are from No. 6 station, viz. 5, 4, etc., and thus obtain a column headed "Moments." This column is added up for the fore body, giving 1505·43, and for the after body, giving 1913·02, the difference being 407·59 in favour of the after body. To get the actual moment of the volume abaft No. 6 station, we should multiply this difference by $(\frac{1}{3} \times 1)$ for the vertical direction, $(\frac{1}{3} \times 7·1)$ for the fore-and-aft direction, and by 2 for both sides, and then by 7·1, since we have only multiplied by the number of intervals away, and not by the actual distances, or $407·59 \times (\frac{1}{3} \times 1) \times (\frac{1}{3} \times 7·1) \times 2 \times 7·1$. The volume, as we have seen above, is given by—

$$1951·83 \times (\tfrac{1}{3} \times 1) \times (\tfrac{1}{3} \times 7·1) \times 2$$

The distance of the centre of gravity of the main solid from No. 6 station will be—

$$\text{Moment} \div \text{volume}$$

But on putting this down we shall see that we can cancel out, leaving us with—

$$\frac{497.59 \times 7.1}{1951.83} = 1.48 \text{ feet}$$

which is the distance of the centre of gravity of the main solid abaft No. 6 station. The distance of the centre of gravity of the appendage abaft No. 6 station is 4·0 feet; the working is shown on the left-hand side of the table, and requires no further explanation.[1] These results for the main solid and for the appendages are combined together at the bottom; the displacement of each in tons is multiplied by the distance of its centre of gravity abaft No. 6 station, giving the moments. The total moment is 143·73, and the total displacement is 91·36 tons, and this gives the centre of gravity of the total displacement, or what we term the *centre of buoyancy*, C.B., 1·57 feet abaft No. 6 station.

Now we have to consider the vertical position of the C.B., and this is determined with reference to the load water-line. For the main solid the process is precisely similar to that adopted for finding the horizontal position, with the exception that we take our moments all below the load water-plane, the number of intervals being small compared with the horizontal intervals. We obtain, as indicated on the sheet, the centre of gravity of the main solid at a distance of 2·21 feet below the L.W.L. For the appendage, we proceed as shown on the left-hand side of the sheet. When finding the areas of the sections of the appendage, we spot off as nearly as possible the centre of gravity of each section, and measure its distance below No. 6 W.L. If the sections happen to be triangles, this will, of course, be one-third the depth. These distances are placed in a column as shown, and the "functions of areas" are respectively multiplied by them, *e.g.* for No. 4 station the function of the area is 5·92, and this is multiplied by 0·22, the distance of the centre of gravity of the section of the appendage below No. 6 W.L. We thus obtain a column which, added up, gives a total of 13·78. To get the actual moment, we only have to

[1] For the vertical C.G. of the appendage Morrish's rule gives a good approximation.

multiply this by $\frac{1}{3} \times 7 \cdot 1$. The volume of the appendage is $49 \cdot 99 \times (\frac{1}{3} \times 7 \cdot 1)$. So that the distance of the centre of gravity of the whole appendage below No. 6 W.L. is given by moment ÷ volume, or $\frac{13 \cdot 78}{49 \cdot 99} = 0 \cdot 27$ feet, and therefore the centre of gravity of the appendage is 5·27 feet below the L.W.L. The results for the main solid and for the appendage are combined together in the table at the bottom, giving the final position of the C.B. of the whole displacement as 2·32 feet below the L.W.L.

It will be of interest at this stage to test the two approximations that were given on p. 65 for the distance of the C.B. below the L.W.L. The first was that this distance would be from $\frac{8}{20}$ to $\frac{9}{20}$ of the mean draught to top of keel (*i.e.* the mean moulded draught). For this vessel the distance is 2·32 feet, and the mean moulded draught is 5′ 9½″, or 5·8 feet, and so we have the ratio $\frac{2 \cdot 32}{5 \cdot 8}$, or exactly $\frac{8}{20}$. The second approximation (Morrish's), p. 65, was—

$$\frac{1}{3}\left(\frac{d}{2} + \frac{V}{A}\right)$$

All these are readily obtainable from the displacement sheet, and if worked out its value is found to be 2·29 feet. This agrees fairly well with the actual result, 2·32 feet, the error being 3 in 232, or less than 1½ per cent.

For large vessels a precisely similar displacement-sheet is prepared, but it is usual to add in the effect of other appendages besides that below the lowest W.L. Specimen calculations are given on Tables II. and IIIa. at the end of the book.

In the former the ordinates are to a mean thickness of plating. In the latter the moulded surface is used, and the displacement of the shell plating added as an appendage, being obtained by Denny's formula given on page 86.

Graphic or Geometrical Method of calculating Displacement and Position of Centre of Buoyancy.—There is one property of the curve, known as the "*parabola of the second order*" (see p. 6), that can be used in calculating by a graphic method the area of a figure bounded by such a curve. Let BFC, Fig. 37, be a curve bounding the figure ABCD, and suppose the curve is a "*parabola of the second order.*" Draw the ordinate EF midway between AB and DC; then the following is a property of the curve BFC:—the area of the segment BCF is given by two thirds the product of the deflection GF and the base AD, or—

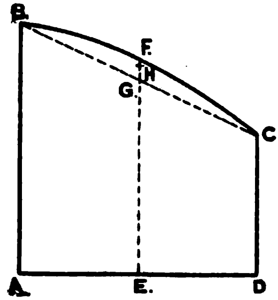


FIG. 37.

$$\text{Area BCF} = \tfrac{2}{3} \times \text{GF} \times \text{AD}$$

Make $\text{GH} = \tfrac{2}{3}\text{GF}$. Then—

$$\text{Area BCF} = \text{GH} \times \text{AD}$$

Now, the area of the trapezoid ABCD is given by AD × EG, and consequently—

$$\text{The area ADCFB} = \text{AD} \times \text{EH}$$ [1]

Thus, if we have a long curvilinear, we can divide it up as for Simpson's first rule, and set off on each of the intermediate ordinates two-thirds the deflection of the curve above or below the straight line joining the extremities of the dividing ordinates. Then add together on a strip of paper all such distances as EH right along, and the sum multiplied by the

[1] This property may be used to prove the rule known as Simpson's first rule. Call AB, EF, DC respectively y_1, y_2, y_3. Then we have—

$$\text{EG} = \frac{y_1 + y_3}{2} \text{ and } \text{FG} = y_2 - \text{EG}$$

$$\therefore \text{FG} = y_2 - \frac{y_1 + y_3}{2}$$

$$\text{HG} = \tfrac{2}{3}y_2 - \frac{y_1 + y_3}{3}$$

$$\text{EH} = \text{EG} + \text{GH}$$

$$= \left(\frac{y_1 + y_3}{2}\right) + \left(\frac{2y_2 - y_1 - y_3}{3}\right)$$

$$= \tfrac{1}{6}(y_1 + 4y_2 + y_3)$$

and calling $\text{AE} = h$, we have—

$$\text{Area ADCFB} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

which is the same expression as given by Simpson's first rule.

distance apart of the dividing ordinates, as AD, will give the area required. Thus in Fig. 38, AB is divided into equal parts

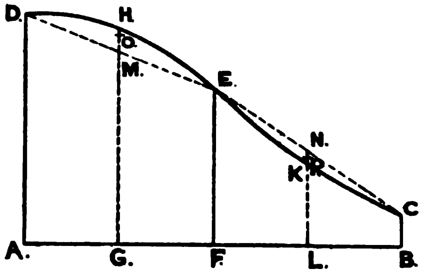


FIG. 38.

as shown. D and E are joined, also E and C; MO is set off $= \frac{2}{3}$HM, and NP is set off $= \frac{2}{3}$NK. Then—

$$\text{Area ADEF} = \text{AF} \times \text{GO}$$
$$\text{and area FECB} = \text{FB} \times \text{LP}$$
$$\text{and the whole area ABCD} = \text{AF} \times (\text{GO} + \text{LP})$$

We can represent the arèa ABCD by a length equal to GO + LP on a convenient scale, if we remember that this length has to be multiplied by AF to get the area. This principle can be extended to finding the areas of longer figures, such as water-planes, and we now proceed to show how the displacement and centre of buoyancy of a ship can be determined by its use. The assumption we made at starting is supposed to hold good with all the curves we have to deal, *i.e.* that the portions between the ordinates are supposed to be "*parabolas of the second order.*" This is also the assumption we make when using Simpson's first rule for finding displacement in the ordinary way.

Plate I. represents the ordinary sheer drawing of a vessel, and the underwater portion is divided by the level water-planes shown by the half-breadth plan. The areas of each of these planes can be determined graphically as above described, the area being represented by a certain length obtained by the addition of all such lengths as GO, etc., Fig 38, the interval being constant for all the water-planes. Let AB, Fig. 39, be set vertically to represent the extreme moulded draught of the vessel. Draw BC at right angles to AB, to represent on a convenient scale the area of the L.W.L. obtained as above. Similarly, DE, FG are set out to represent on the same scale the areas of water-planes 2 and 3, and so on for each water-plane. A curve drawn through all such points as C, E, and G

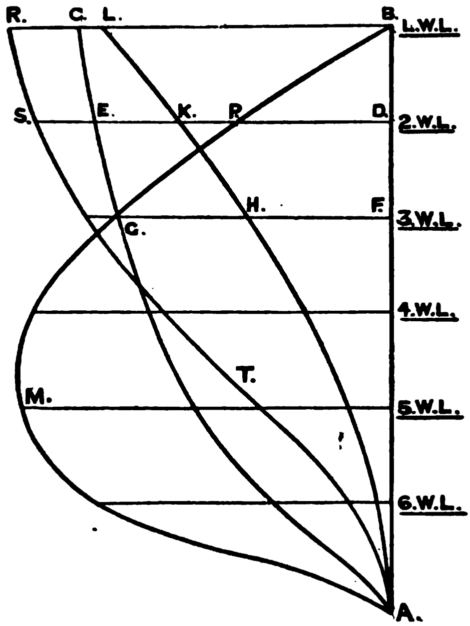


Fig. 39.

will give a "*curve of areas of water-planes.*" Now, the area of this curve up to the L.W.L. gives us the volume of displacement up to the L.W.L., as we have seen in Chapter I., and we can readily find the area of the figure ABCEG by the graphic method, and this area will give us the displacement up to the L.W.L. Similarly, the area of ADEG will give the displacement up to 2 W.L., and so on. Therefore set off BL to represent on a convenient scale the area of the figure ABCE, DK on the same scale to represent the area ADEG, and so on. Then a curve drawn through all such points as L, K will give us a "*curve of displacement*," and the ordinate of this curve at any draught will give the displacement at that draught, BL being the load displacement.

We now have to determine the distance of the centre of buoyancy below the L.W.L., and to find this we must get the *moment* of the displacement about the L.W.L. and divide this by the volume of displacement below the L.W.L. We now construct a curve, BPMA, such that the ordinate at any draught represents the area of the water-plane at that draught multiplied by the depth of the water-plane below the L.W.L. Thus DP represents on a convenient scale the area of No. 2 water-plane multiplied by DB, the distance below the L.W.L. The ordinate of this curve at the L.W.L. must evidently be zero. This curve is a curve of "*moments of areas of water-planes*" about the L.W.L. The area of this curve up to the L.W.L. will evidently be the moment of the load displacement about the L.W.L., and thus the length BR is set out to equal on a convenient scale the area of BPMA. Similarly, DS is set out to represent, on the same scale, the area of DPMA, and thus the moment of the displacement up to 2 W.L about the L.W.L. These areas are found graphically as in the preceding cases. Thus a curve RSTA can be drawn in, and BR ÷ BL, or moment of load displacement about L.W.L. ÷ load displacement, gives us the depth of the centre of buoyancy for the load displacement below the L.W.L.

Exactly the same course is pursued for finding the displacement and the longitudinal position of the centre of buoyancy, only in this case we use a curve of areas of transverse sections instead of a curve of areas of water-planes, and we get the moments of the transverse areas about the middle ordinate. Fig. 40 gives the forms the various curves take for the fore body. AA is the "curve of areas of transverse sections;" BB is the "curve of displacement" for the fore body, OB being the displacement of the fore body. CC is the curve of "moments of areas of transverse sections" about No. 6 ordinate; DD is the curve of "moment of displacement" about No. 6 ordinate, OD being the moment of the fore-body displacement about No. 6 ordinate. Similar curves can be drawn for the after body, and the difference of the moments of the fore and after bodies divided by the load displacement will give the distance of the centre of buoyancy forward or aft of No. 6 ordinate, as the case may be. The total displacement must be the same as found by the preceding method.[1]

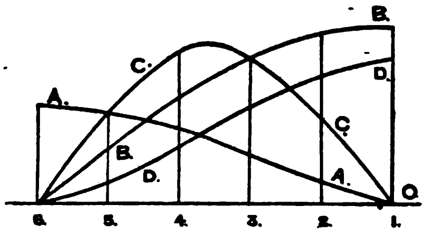


FIG 40.

Method of finding Areas by Means of the Planimeter.—This instrument is frequently employed to find the area of plane curvilinear figures, and thus the volume of displacement of a vessel can be determined. One form of the instrument is shown in diagram by Fig. 41. It is supported at three places: first, by a weighted pin, which is fixed in position by being pressed into the paper; second, by a wheel, which actuates a circular horizontal disc, the wheel and disc both being graduated; and third, by a blunt pointer. The instrument is placed on the drawing, the pin is fixed in a convenient position, and the pointer is placed on a marked spot A on the boundary of the curve of which the area is required. The reading given by the wheel and disc is noted. On passing round the boundary of the area with the pointer (the same way as the hands of a clock) back to the starting-point, another reading is obtained. The difference of the two readings is *proportional* to the area of the figure, the multiplier required to convert the difference into the area depending on the instrument and on the scale to which the figure is drawn. Particulars concerning the necessary multipliers are given with the instrument; but it is a good practice to pass round figures of known area to get accustomed to its use.

[1] This method may be considerably simplified by using the property of the curve of displacement given on p. 66.

By the use of the planimeter the volume of displacement of a vessel can very readily be determined. The body plan is taken, and the L.W.L. is marked on. The pointer of the instrument is then passed round each section in turn, up to the L.W.L., the readings being tabulated. If the differences of the readings were each multiplied by the proper multiplier, we should obtain the area of each of the transverse sections, and so, by direct application of Simpson's rules, we should find the

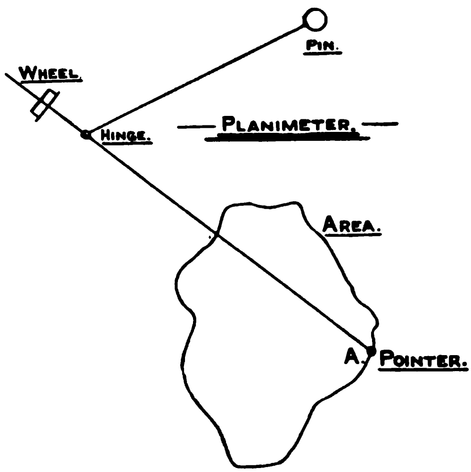


FIG. 41.

required volume of displacement. Or we could put the actual difference of readings through Simpson's multipliers, and multiply at the end by the constant multiplier.

It is frequently the practice to shorten the process as follows: The body-plan is arranged so that Simpson's first rule will be used, *i.e.* an odd number of sections is employed. The pointer is passed round the first and last sections, and the reading is recorded. It is then passed round *all* the even sections, 2, 4, 6, etc., and the reading is recorded. Finally, it is passed round *all* the odd sections except the first and last, viz. 3, 5, 7, etc., and the reading is put down. The differences of the readings are found and put down in a column. The first difference is multiplied by 1, the next difference is multiplied by 4, and the last by 2. The sum of these products is then multiplied for Simpson's first rule, and then by the proper multiplier for the instrument and scale used. The work can conveniently be arranged thus:

Numbers of sections.	Readings.	Differences of readings.	Simpson's multipliers.	Products.
Initial reading	5,124	—	—	—
1, 21	5,360	236	1	236
2, 4, 6, 8, 10, 12, 14, 16, 18, 20 ...	18,681	13,321	4	53,284
3, 5, 7, 9, 11, 13, 15, 17, 19	31,758	13,077	2	26,154
				79,674

The multiplier for the instrument and scale of the drawing used and to complete the use of Simpson's first rule is $\frac{8}{5}$; so that the volume of displacement is $79,674 \times \frac{8}{5}$ cubic feet, and the displacement in tons is $79,674 \times \frac{8}{5} \times \frac{1}{35} = 2732$ tons.

There are two things to be noticed in the use of the planimeter: first, it is not necessary to set the instrument to the exact zero, which is somewhat troublesome to do; and second, the horizontal disc must be watched to see how many times it makes the complete revolution, the complete revolution meaning a reading of 10,000.

It is also possible to find the vertical position of the centre of buoyancy by means of the planimeter. By the method above described we can determine the displacement up to each water-line in succession, and so draw in on a convenient scale the ordinary curve of displacement. Now we can run round this curve with the planimeter and find its area. This area divided by the top ordinate (*i.e.* the load displacement) will give the distance of the centre of buoyancy below the load-line (see p. 67).

To find the centre of buoyancy in a fore-and-aft direction, it is necessary to tabulate the differences for each section, and treat these differences in precisely the same way as the "functions of areas of vertical sections" are treated in the ordinary displacement sheet.

Method of approximating to the Area of the Wetted Surface[1] by "Kirk's" Analysis.—The ship is assumed to be represented by a block model, shaped as shown in Fig. 42, formed of a parallel middle body and a

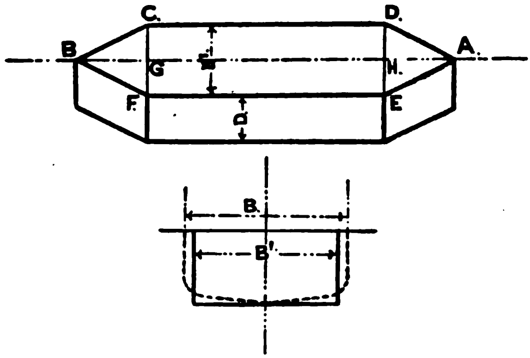


FIG. 42.

tapered entrance and run which are taken as of equal length. The depth of the model is equal to the mean draught, and the length of the model is equal to the length of the vessel. The breadth is not equal to the breadth of the vessel, but is equal to area of immersed midship section $\div$ mean draught. The displacement of the model is made equal to that of the vessel. We then have—

$$\left.\begin{array}{l}\text{Volume of displace-}\\ \quad\text{ment}\end{array}\right\} = \text{V, say}$$

$$= \text{AG} \times \text{area of midship section}$$

$$\text{or AG} = \frac{\text{V}}{\text{area of midship section}}$$

$$\therefore \left.\begin{array}{l}\text{length of entrance}\\ \quad\text{or run}\end{array}\right\} = \text{length of ship} - \frac{\text{V}}{\text{area of midship section}}$$

$$= \text{L} - \frac{\text{V}}{\text{B}' \times \text{D}}$$

[1] The area of wetted surface can be closely approximated to by putting a curve of girths (modified for the slope of the level lines, see p. 244) through Simpson's rule.

where L = length of ship;
B′ = breadth of model;
D = mean draught.

Having found these particulars, the surface of the model can be readily calculated.

$$\text{Area of bottom} = AG \times B'$$
$$\text{Area of both sides} = 2(GH + 2AE) \times \text{mean draught}$$

The surface of a model formed in this way approximates very closely to the actual wetted surface of the vessel. It is stated that in very fine ships the surface of the model exceeds the actual wetted surface by about 8 per cent., for ordinary steamers by about 3 per cent., and for full ships by 2 per cent.

By considering the above method, we may obtain an approximate formula for the wetted surface—

$$\text{Area of bottom} = \frac{V}{D}$$
$$\text{Area of sides} = 2L'D$$

where L′ is the length along ADCB. Then—

$$\text{Surface} = 2L'D + \frac{V}{D}$$

This gives rather too great a result, as seen above; and if we take—

$$\text{Surface} = 2LD + \frac{V}{D}$$

we shall get the area of the wetted surface slightly in excess, but this will allow for appendages, such as keels, etc.

Since V = k . LBD, where k is the block coefficient of displacement, we may write—

$$\text{Surface} = 2LD + k \,.\, LB$$

Approximate Formulæ for finding Wetted Surface. —Mr. Denny gives the following formula for the area of wetted surface :—

$$1 \cdot 7 LD + \frac{V}{D}$$

which is seen to be very nearly that obtained above.

Mr. Taylor, in his work on "Resistance and Propulsion of Ships," gives the following formula :—

$$15 \cdot 5 \sqrt{WL}$$

where W is the displacement in tons.

The following formula for the wetted surface is used at the experimental tank at Haslar—

$$S = V^{\frac{2}{3}}\left(3 \cdot 4 + \frac{L}{2V^{\frac{1}{3}}}\right)$$

One of these formulæ can be used to find the area of wetted surface to calculate the displacement of the skin plating, as is necessary when the sheer drawing is drawn to the moulded surface of the ship. See Brown's Displacement Sheet in Appendix, in which Denny's approximation is used.

EXAMPLES TO CHAPTER II.

1. A ship has the following weights placed on board :—

20 tons ...	...	100 feet before amidships
45 ,, ...	...	80 ,, ,,
15 ,, ...	...	40 ,, ,,
60 ,, ...	...	50 feet abaft ,,
40 ,, ...	...	80 ,, ,,
30 ,, ...	...	110 ,, ,,

Show that these weights will have the same effect on the trim of the ship as a single weight of 210 tons placed 15$\frac{5}{7}$ feet abaft amidships.

2. Six weights are placed on a drawing-board. The weights are 3, 4, 5, 6, 7, 8 lbs. respectively. Their distances from one edge are 5, 4½, 4, 3½, 3, 2 feet respectively, and from the edge at right angles, ½, ¾, 1, 1½, 2, 2½, feet respectively. The drawing-board weighs 6 lbs., and is 6 feet long and 3 feet broad. Find the position where a single support would need to be placed in order that the board should remain horizontal.

Ans. 3·27 feet from short edge, 1·58 feet from long edge.

3. An area bounded by a curve and a straight line is divided by ordinates 4 feet apart of the following lengths: 0, 12·5, 14·3, 15·1, 15·5, 15·4, 14·8, 14·0, 0 feet respectively. Find—

(1) Area in square feet.
(2) Position of centre of gravity relative to the first ordinate.
(3) Position of the centre of gravity relative to the base.

Ans. (1) 423 square feet; (2) 16·27 feet; (3) 7·24 feet.

4. A triangle ABC has its base BC 15 feet long, and its height 25 feet. A line is drawn 10 feet from A parallel to the base, meeting AB and AC in D and E. Find the distance of the centre of gravity of DBCE from the apex.

Ans. 18·57 feet.

5. The semi-ordinates of a water-plane in feet, commencing from the after end, are 5·2, 10·2, 14·4, 17·9, 20·6, 22·7, 24·3, 25·5, 26·2, 26·5, 26·6, 26·3, 25·4, 23·9, 21·8, 18·8, 15·4, 11·5, 7·2, 3·3, 2·2. The distance apart is 15 feet. Find the area of the water-plane, and the position of the centre of gravity in relation to the middle ordinate.

Ans. 11,176 square feet; 10·15 feet abaft middle.

6. Find the area and transverse position of the centre of gravity of "half" a water-line plane, the ordinates in feet being 0·5, 6, 12, 16, 12, 10, and 0·5 respectively, the common interval being 15 feet.

Ans. 885 square feet; 6·05 feet.

7. The areas of sections 17′ 6″ apart through a bunker, commencing from forward, are 65, 98, 123, 137, 135, 122, 96 square feet respectively. The length of bunker is 100 feet, and its fore end is 1′ 6″ forward of the section whose area is 65 square feet. Draw in a curve of sectional areas, and obtain, by using convenient ordinates, the number of cubic feet in the bunker, and the number of tons of coal it will contain, assuming that 43 cubic feet of coal weigh 1 ton. Find also the position of the C.G. of the coal relative to the after end of the bunker.

Ans. 272 tons; 46½ feet from the after end.

8. The tons per inch in salt water of a vessel at water-lines 3 feet apart, commencing with the L.W.L., are 31·2, 30·0, 28·35, 26·21, 23·38, 19·5, 12·9. Find the displacement in salt and fresh water and the position of the C.B. below the L.W.L., neglecting the portion below the lowest W.L. Draw in the tons per inch curve for salt water to a convenient scale, and *estimate* from it the weight necessary to be taken out in order to lighten the vessel 2′ 3¼″ from the L.W.L. The mean draught is 20′ 6″.

Ans. 5405 tons; 5255 tons; 8·01 feet; 847 tons.

9. In the preceding question, calling the L.W.L. 1, find the displacement up to 2 W.L., 3 W.L., and 4 W.L., and draw in a curve of displacement from the results you obtain, and check your answer to the latter part of the question.

10. The tons per inch of a ship's displacement at water-lines 4 feet apart, commencing at the L.W.L., are 44·3, 42·7, 40·5, 37·5, 33·3. Find number of tons displacement, and the depth of C.B. below the top W.L.

Ans. 7670 tons; 7·6 feet.

11. The ship in the previous question has two water-tight transverse bulkheads 38 feet apart amidship, and water-tight flats at 4 feet below and 3 feet above the normal L.W.L. If a hole is made in the side 2 feet below the L.W.L., how much would the vessel sink, taking the breadth of the L.W.L. amidships as 70 feet? Indicate the steps where, owing to insufficient information, you are unable to obtain a perfectly accurate result.

Ans. 8 inches.

12. The areas of transverse sections of a coal-bunker 19 feet apart are

respectively 63·2, 93·6, 121·6, 108·8, 94·8 square feet, and the centres of gravity of these sections are 10·8, 11·6, 12·2, 11·7, 11·2 feet respectively below the L.W.L. Find the number of tons of coal the bunker will hold, and the vertical position of its centre of gravity (44 cubic feet of coal to the ton).

Ans. 174·3 tons; 11·68 feet below L.W.L.

13. A vessel is 180 feet long, and the transverse sections from the load water-line to the keel are semicircles. Find the longitudinal position of the centre of buoyancy, the ordinates of the load water-plane being 1, 5, 13, 15, 14, 12, and 10 feet respectively.

Ans. 106·2 feet from the finer end.

14. Estimate the distance of the centre of buoyancy of a vessel below the L.W.L., the vessel having 22′ 6″ mean moulded draught, block coefficient of displacement 0·55, coefficient of fineness of L.W.L. 0·7 (use Morrish's formula, p. 65).

Ans. 9·65 feet.

15. A vessel of 2210 tons displacement, 13′ 6″ draught, and area of load water-plane 8160 square feet, has the C.B., calculated on the displacement sheet, at a distance of 5·43 feet below the L.W.L. Check this result.

16. The main portion of the displacement of a vessel has been calculated and found to be 10,466 tons, and its centre of gravity is 10·48 feet below the L.W.L., and 5·85 feet abaft the middle ordinate. In addition to this, there are the following appendages:—

	tons.		
Below lowest W.L.	263,	24·8 ft. below L.W.L.,	4·4 ft. abaft mid. ord.
Forward	5,	12·0 ,, ,,	202 ft. forward of mid. ord.
Stern	16,	2·8 ,, ,,	201 ft. abaft mid. ord.
Rudder	16,	17·5 ,, ,,	200 ,, ,,
Bilge keels ...	20,	20 ,, ,,	0 ,, ,,
Shafting, etc. ...	18,	15 ,, ,,	140 ,, ,,

Find the total displacement and position of the centre of buoyancy.

Ans. 10,804 tons; C.B. 6·5 abaft mid. ord., 10·86 ft. below L.W.L.

17. The displacements of a vessel up to water-planes 4 feet apart are 10,804, 8612, 6511, 4550, 2810, 1331, and 263 tons respectively. The draught is 26 feet. Find the distance of the centre of buoyancy below the load water-line.

Ans. 10·9 feet nearly.

18. The load displacement of a ship is 5000 tons, and the centre of buoyancy is 10 feet below the load water-line. In the light condition the displacement of the ship is 2000 tons, and the centre of gravity of the layer between the load and light lines is 6 feet below the load-line. Find the vertical position of the centre of buoyancy below the light line in the light condition.

Ans. 4 feet, assuming that the C.G. of the layer is at half its depth.

19. Ascertain the displacement and position of the centre of buoyancy of a floating body of length 140 feet, depth 10 feet, the forward section being a triangle 10 feet wide at the deck and with its apex at the keel, and the after section a trapezoid 20 feet wide at the deck and 10 feet wide at the keel, the sides of the vessel being plane surfaces; draught of water may be taken as 7 feet.

Ans. 238 tons; 56·3 feet before after end, 3 feet below water-line.

20. Show by experiment or otherwise that the centre of gravity of a

quadrant of a circle 3 inches radius is 1·8 inches from the right angle of the quadrant.

21. A floating body has a constant triangular section, vertex downwards, and has a constant draught of 12 feet in fresh water, the breadth at the water-line being 24 feet. The keel just touches a quantity of mud of specific gravity 2. The water-level now falls 6 feet. How far will the body sink into the mud?

Ans. 4 feet 11¼ inches.[1]

22. Show that the C.G. of a trapezoid, as ABCD, Fig. 5, is distant $\frac{h}{6}\left(\frac{y_2 - y_1}{y_2 + y_1}\right)$ from the middle of the length h. The C.G. must be on a line joining the centres of the parallel sides. Thus the position of the C.G. is fully determined. This will also apply to a figure in which the parallel sides are not perpendicular to one of the other sides.

23. Apply Morrish's rule to find the C.G. of a semicircle, and state the error involved if the semicircle is 20 feet radius.

Ans. 0·19 foot.

24. For the lower appendage of the ship displacement sheet, Table I., find the vertical C.G. by using Morrish's rule.

25. Describe the process of finding the area and position of the C.G. of a plane figure by radial integration, and apply it to find these in the case of a rectangle 8 feet wide and 12 feet long.

26. A vessel is 300 feet × 36½ feet × 13¼ feet draught, 2135 tons displacement. Find the area of wetted surface by each of the formula given on p. 86.

Ans., Denny, 12,421 square feet.
Taylor, 12,400 square feet.
Froude, 12,350 square feet.

The student is advised to take a sheer drawing and obtain a close approximation to the wetted surface by putting the half girths to water-line through Simpson's rule. Then to apply the above formulæ and see what the comparative results are.

27. Show, by means of the result in question 22, that the C.G. of a trapezoid in relation to the parallel sides is given by the construction of Fig. 174.

28. Having given the mean ordinate p of a trapezoid and the distance x of its C.G. from the larger end, show that the lengths of the parallel sides are—

$$\left(4p - \frac{6px}{h}\right) \text{ and } \left(\frac{6px}{h} - 2p\right)$$

where h is the length.

[1] This example is worked out at the end of Appendix A.

CHAPTER III.

CONDITIONS OF EQUILIBRIUM, TRANSVERSE METACENTRE, MOMENT OF INERTIA, TRANSVERSE BM, INCLINING EXPERIMENT, METACENTRIC HEIGHT, ETC.

Trigonometry.—The student of this subject will find it a distinct advantage, especially when dealing with the question of stability, if he has a knowledge of some of the elementary portions of trigonometry. The following are some properties which should be thoroughly grasped:—

Circular Measure of Angles.—The degree is the unit generally employed for the measurement of angles. A right angle is divided into 90 equal parts, and each of these parts is termed a "*degree.*" If two lines, as OA, OB, Fig. 43, are inclined to each other, forming the angle AOB, and we draw at any radius OA an arc AB from the centre O, cutting OA, OB in A and B, then length of arc AB ÷ radius OA is termed the *circular measure* of the angle AOB. Or, putting it more shortly—

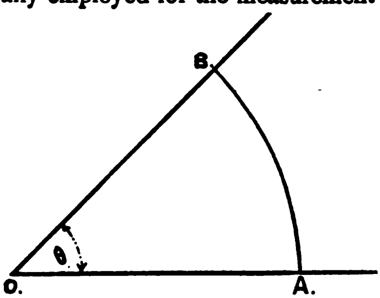


FIG. 43.

$$\text{Circular measure} = \frac{arc}{radius}$$

$$\left.\begin{array}{l}\text{The circular measure of four right}\\ \quad\text{angles, or 360 degrees}\end{array}\right\} = \frac{\text{circumference of a circle}}{\text{radius}}$$

$$= 2\pi$$

$$\text{The circular measure of a right angle} \Big\} = \frac{\pi}{2}$$

Since 360 degrees = 2π in circular measure, then the angle whose circular measure is unity is—

$$\frac{360}{2\pi} = 57{\cdot}3 \text{ degrees}$$

The circular measure of 1 degree is $\frac{2\pi}{360} = 0{\cdot}01745$, and thus the circular measure of any angle is found by multiplying the number of degrees in it by 0·01745.

Trigonometrical Ratios,[1] *etc.*—Let BOC, Fig. 44, be any angle; take any point P in one of the sides OC, and draw PM perpendicular to OB. Call the angle BOC, θ.[2]

PM is termed the perpendicular.

OM is termed the base.

OP is termed the hypotenuse.

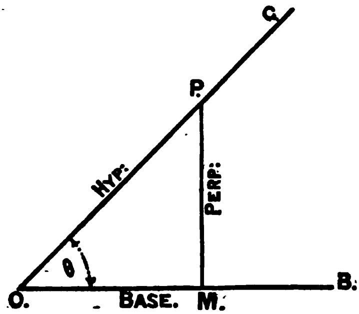


FIG. 44.

Then—

$$\frac{PM}{OP} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \textit{sine } \theta, \text{ usually written } \textit{sin } \theta$$

$$\frac{OM}{OP} = \frac{\text{base}}{\text{hypotenuse}} = \textit{cosine } \theta, \text{ usually written } \textit{cos } \theta$$

$$\frac{PM}{OM} = \frac{\text{perpendicular}}{\text{base}} = \textit{tangent } \theta, \text{ usually written } \textit{tan } \theta$$

These ratios will have the same value wherever P is taken on the line OC.

[1] An aid to memory which is found of assistance by many in learning these ratios is—

*Sin per*plexes *hyp*ocrites
Cos of *base hyp*ocrisy.

[2] θ is a Greek letter (*theta*) often used to denote an angle.

$$\text{We can write } \sin\theta = \frac{\text{per.}}{\text{hyp.}}$$

$$\cos\theta = \frac{\text{base}}{\text{hyp.}}$$

$$\text{and also } \tan\theta = \frac{\sin\theta}{\cos\theta}$$

There are names for the inversions of the above ratios, which it is not proposed to use in this work.

For small angles, the value of the angle θ in circular measure is very nearly the same as the values of sin θ and tan θ. This will be seen by comparing the values of θ, sin θ, and tan θ for the following angles:—

Angle in degrees.	Angle in circular measure.	Sin θ.	Tan θ.
2	0·0349	0·0349	0·0349
4	0·0698	0·0697	0·0699
6	0·1047	0·1045	0·1051
8	0·1396	0·1392	0·1405
10	0·1745	0·1736	0·1763

Up to 10° they have the same values to two places of decimals, and for smaller angles the agreement in value is closer still.

Tables of sines, cosines, and tangents of angles up to 90° are given in Appendix B.

Conditions that must hold in the Case of a Vessel floating freely, and at Rest in Still Water. — We saw in Chapter I. that, for a vessel floating in still water, the weight of the ship with everything she has on board must equal the weight of the displaced water. To demonstrate this, we imagined the cavity left by the ship when lifted out of the water to be filled with water (see Fig. 17). Now, the upward support of the surrounding water must exactly balance the weight of the water poured in. This weight may be regarded as acting downwards through its centre of gravity, or, as we now term it, the centre of buoyancy. Consequently, the upward support of the water, or the buoyancy, must act through the centre of buoyancy. All the horizontal pressures of the water on the surface of the ship must evidently balance among themselves. We therefore have the following forces acting upon the ship:—

(1) The weight acting downwards through the C.G.;

(2) The upward support of the water, or, as it is termed, the buoyancy, acting upwards through the C.B.;

and for the ship to be at rest, these two forces must act in the same line and counteract each other. Consequently, we also have the following condition:—

The centre of gravity of the ship, with everything she has on board, must be in the same vertical line as the centre of buoyancy.

If a rope is pulled at both ends by two men exerting the same strength, the rope will evidently remain stationary; and this is the case with a ship floating freely and at rest in still water. She will have no tendency to move of herself so long as the C.G. and the C.B. are in the same vertical line.

Definition of Statical Stability.—The statical stability of a vessel is the tendency she has to return to the upright when inclined away from that position. It is evident that under ordinary conditions of service a vessel cannot always remain upright; she is continually being forced away from the upright by external forces, such as the action of the wind and the waves. It is very important that the ship shall have such qualities that these inclinations that are forced upon her shall not affect her safety; and it is the object of the present chapter to discuss how these qualities can be secured and made the subject of calculation so far as small angles of inclination are concerned.

A ship is said to be in *stable equilibrium* for a given direction of inclination if, on being slightly inclined in that direction from her position of rest, she tends to return to that position.

A ship is said to be in *unstable equilibrium* for a given direction of inclination if, on being slightly inclined in that direction from her position of rest, she tends to move away farther from that position.

A ship is said to be in *neutral or indifferent equilibrium* for a given direction of inclination if, on being slightly inclined in that direction from her position of rest, she neither tends to return to nor move farther from that position.

These three cases are represented by the case of a heavy sphere placed upon a horizontal table.

1. If the sphere is weighted so that its C.G. is out of the centre, and the C.G. is vertically below the centre, it will be in *stable equilibrium*.

2. If the same sphere is placed so that its C.G. is vertically above the centre, it will be in *unstable equilibrium*.

3. If the sphere is formed of homogeneous material so that its C.G. is at the centre, it will be in *neutral or indifferent equilibrium*.

Transverse Metacentre.—We shall deal first with transverse inclinations, because they are the more important, and deal with inclinations in a longitudinal or fore-and-aft direction in the next chapter.

Let Fig. 45 represent the section of a ship steadily inclined

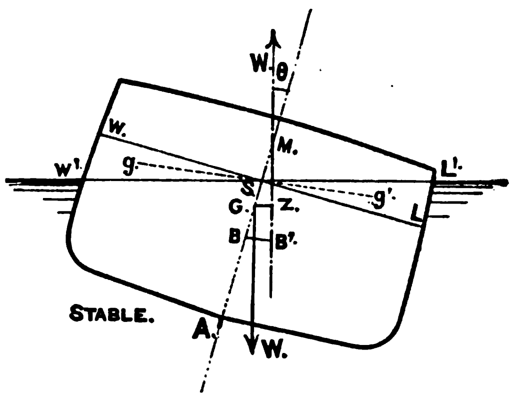


Fig. 45.

at a small angle from the upright by some external force, such as the wind. The vessel has the same weight before and after the inclination, and consequently has the same volume of displacement. We must assume that no weights on board shift, and consequently the centre of gravity remains in the same position in the ship. But although the total volume of displacement remains the same, the shape of this volume changes, and consequently the centre of buoyancy will shift from its original position. In the figure the ship is represented by the section, WAL being the immersed section when upright, WL being the position of the water-line on the ship. On being inclined, W′L′ becomes the water-line, and W′AL′ represents tbe immersed volume of the ship, which, although different in shape, must have the same volume as the original immersed volume WAL.

The wedge-shaped volume represented by WSW′, which has come out of the water, is termed the "*emerged*" or "*out*" wedge. The wedge-shaped volume represented by LSL′, which has gone into the water, is termed the "*immersed*" or "*in*" wedge. Since the ship retains the same volume of displacement, it follows that the volume of the emerged wedge WSW′ is equal to the volume of the immersed wedge LSL′. It is only for small angles of inclination that the point S, where the water-lines intersect, falls on the middle line of the vessel. For larger angles it moves further out, as shown in Fig. 77.

Now consider the vessel inclined at a small angle from the upright, as in Fig. 45. The new volume of displacement W′AL′ has its centre of buoyancy in a certain position, say B′. This position might be calculated from the drawings in the same manner as we found the point B, the original centre of buoyancy; but we shall see shortly how to fix the position of the point B′ much more easily.

B′ being the new centre of buoyancy, the upward force of the buoyancy must act through B′, while the weight of the ship acts vertically down through G, the centre of gravity of the ship. Suppose the vertical through B′ cuts the middle line of the ship in M; then we shall have two equal forces acting on the ship, viz.—

(1) Weight acting vertically down through the centre of gravity.

(2) Buoyancy acting vertically up through the new centre of buoyancy.

But they do not act in the same vertical line. Such a system of forces is termed a *couple*. Draw GZ perpendicular to the vertical through B'. Then the equal forces act at a distance from each other of GZ. This distance is termed the *arm* of the couple, and the *moment* of the couple is W × GZ. On looking at the figure, it is seen that the couple is tending to take the ship back to the upright. If the relative positions of G and M were such that the couple acted as in Fig. 46, the couple would tend to take the ship farther away from the upright; and again, if G and M coincided, we should have the forces acting in the same vertical line, and consequently no

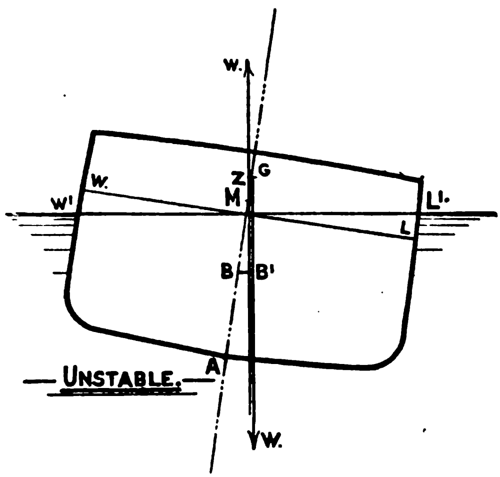


FIG. 46.

couple at all, and the ship would have no tendency to move either to the upright or away from it.

We see, therefore, that for a ship to be in *stable* equilibrium for any direction of inclination, it is necessary that the point M be above the centre of gravity of the ship. This point M is termed the *metacentre*. We now group together the three conditions which must be fulfilled in order that a ship may float freely and at rest in stable equilibrium—

(1) The weight of water displaced must equal the total weight of the ship (see p. 23).

(2) The centre of gravity of the ship must be in the same vertical line as the centre of gravity of the displaced water (centre of buoyancy) (see p. 93).

(3) The centre of gravity of the ship must be below the metacentre.

For small transverse inclinations, M is termed the *transverse metacentre*, which we may accordingly define as follows:—

For a given plane of flotation of a vessel in the upright condition, let B be the centre of buoyancy, and BM the vertical through it. Suppose the vessel inclined transversely through a very small angle, retaining the same volume of displacement, B′ being the new centre of buoyancy, and B′M the vertical through it, meeting BM in M. Then this point of intersection, M, is termed the *transverse metacentre.*

There are two things in this definition that should be noted: (1) the angle of inclination is supposed very small, and (2) the volume of displacement remains the same.

It is found that, for all practical purposes, in ordinary ships the point M does not change in position for inclinations up to as large as 10° to 15°; but beyond this it takes up different positions.

We may now say, with reference to a ship's initial stability or stability in the upright condition—

(1) If G is below M, the ship is in stable equilibrium.

(2) If G is above M, the ship is in unstable equilibrium.

(3) If G coincides with M, the ship is in neutral or indifferent equilibrium.

We thus see how important the relative positions of the centre of gravity and the transverse metacentre are as affecting a ship's initial stability. The distance GM is termed the *transverse metacentric height*, or, more generally, simply the *metacentric height.*

We have seen that for small angles M remains practically in a constant position, and consequently we may say GZ = GM . sin θ for angles up to 10° to 15°, say. GZ is the arm of the couple, and so we can say that the moment of the couple is—

$$W \times GM . \sin \theta$$

If M is above G, this moment tends to right the ship, and we may therefore say that the *moment of statical stability* at the angle θ is—

$$W \times GM \cdot \sin \theta$$

This is termed the *metacentric method* of determining a vessel's stability. It can only be used at small angles of inclination to the upright, viz. up to from 10 to 15 degrees.

Example.—A vessel of 14,000 tons displacement has a metacentric height of $3\frac{1}{2}$ feet. Then, if she is steadily inclined at an angle of 10°, the tendency she has to return to the upright, or, as we have termed it, the moment of statical stability, is—

$$14{,}000 \times 3 \cdot 5 \times \sin 10^\circ = 8506 \text{ foot-tons}$$

We shall discuss later how the distance between G and M, or the metacentric height, influences the behaviour of a ship, and what its value should be in various cases; we must now investigate the methods which are employed by naval architects to determine the distance for any given ship.

There are two things to be found, viz. (1) the position of G, the centre of gravity of the vessel; (2) the position of M, the transverse metacentre.

Now, G depends solely upon the vertical distribution of the weights forming the structure and lading of the ship, and the methods employed to find its position we shall deal with separately; but M depends solely upon the form of the ship, and its position can be determined when the geometrical form of the underwater portion of the ship is known. Before we proceed with the investigation of the rules necessary to do this, we must consider certain geometrical principles which have to be employed.

Centre of Flotation.—*If a floating body is slightly inclined so as to maintain the same volume of displacement, the new water-plane must pass through the centre of gravity of the original water-plane.* In order that the same volume of displacement may be retained, the volume of the immersed wedge SLL_1, Fig 47, must equal the volume of the emerged wedge SWW_1. Call y an ordinate on the immersed side, and y' an ordinate on the emerged side of the water-plane. Then the areas of the sections of the immersed and emerged wedges are respectively (since $LL_1 = y \cdot d\theta$, $WW_1 = y' \cdot d\theta$, $d\theta$ being the small angle of inclination)—

$$\tfrac{1}{2}y^2 \cdot d\theta, \qquad \tfrac{1}{2}(y')^2 \cdot d\theta$$

and using the notation we have already employed—

$$\text{Volume of immersed wedge} = \tfrac{1}{2}\int y^2 \cdot d\theta \cdot dx$$
$$\text{,, emerged ,,} = \tfrac{1}{2}\int (y')^2 \cdot d\theta \cdot dx$$

and accordingly—

$$\tfrac{1}{2}\int y^2 \cdot d\theta \cdot dx = \tfrac{1}{2}\int (y')^2 \cdot d\theta \cdot dx$$
$$\text{or } \tfrac{1}{2}\int y^2 \cdot dx = \tfrac{1}{2}\int (y')^2 \cdot dx$$

But $\tfrac{1}{2}\int y^2 \cdot dx$ is the moment of the immersed portion of the water-plane about the intersection, and $\tfrac{1}{2}\int (y')^2 \cdot dx$ is the moment of the emerged portion of the water-plane about the intersection (see p. 59); therefore the moment of one side of the water-plane about the intersection is the same as the moment of the other side, and consequently the line of intersection passes through the centre of gravity of the water-plane. The centre of gravity of the water-plane is termed the *centre of flotation*. In whatever direction a ship is inclined, transversely, longitudinally, or in any intermediate direction, through a small angle, the line of intersection of the new water-plane with the original water-plane must always pass through the centre of flotation. For transverse inclinations of a ship the line of intersection is the centre line of the water-plane; for longitudinal inclinations the fore-and-aft position of the centre of flotation has to be calculated, as we shall see when we deal with longitudinal inclinations.

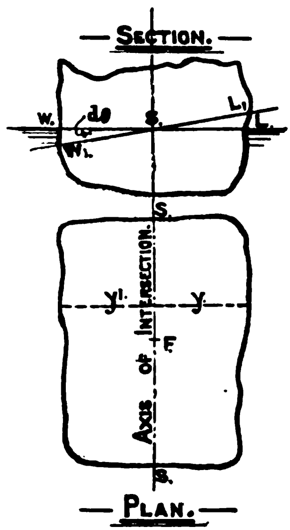


FIG. 47.

Shift of the Centre of Gravity of a Figure due to the Shift of a Portion of the Figure.—In Fig. 48 the figure PQRSTU is made up of the two portions PQTU and QRST, with centres of gravity at g and g' respectively. Let a, a' be the areas, the whole area $a + a' = A$. Then the C.G. of the whole area is at G, such that $a \times gG = a' \times g'G$, or $\frac{gG}{g'G} = \frac{a'}{a}$, *i.e.* the C.G. divides the line joining g and g' *inversely* as the areas. If now the portion QRST is shifted to the position UTVW with C.G. g'', the C.G. of the new combination PQWV is on the line gg'' at G' such that—

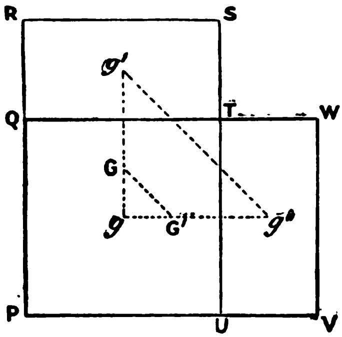


FIG 48.

$$\frac{gG'}{g''G'} = \frac{a'}{a} = \frac{gG}{g'G}.$$

Therefore by the properties of triangles GG' is parallel to $g'g''$.

Also
$$\frac{GG'}{g'g''} = \frac{gG}{gg'}.$$

Now, taking moments about g we have $a \times gg' = A \times gG$

$$\therefore \frac{GG'}{g'g''} = \frac{a}{A}$$

$$\text{or GG', the shift of the C.G.} = \frac{a}{A} \times g'g''$$

or the whole area multiplied by its shift equals the small area multiplied by its shift, and these shifts are in parallel directions. Also for the horizontal shift, $A \times gG' = a \times gg''$, and for the vertical shift, $A \times Gg = a \times g'g$. The above proof is perfectly general, although a simple figure has been taken by which its truth may be readily seen. It applies equally to the shift of weights.

2/2/40

The uses that are made of this will become more apparent as we proceed, but the following examples will serve as illustrations :—

Example.—A vessel weighing W tons has a weight w tons on the deck. This is shifted transversely across the deck a distance of d feet, as in Fig. 49. Find the shift of the C.G. of the vessel both in direction and amount.

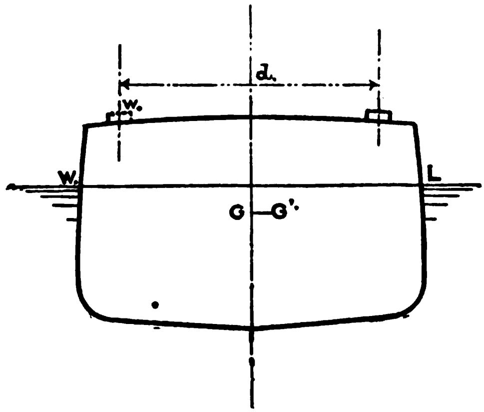


FIG. 49.

G will move to G′ such that GG′ will be parallel to the line joining the original and final positions of the weight w;

$$\text{and GG}' = \frac{w \times d}{W}$$

If $w = 70$ tons, $d = 30$ feet, W = 5000 tons, then—

$$\text{GG}' = \frac{70 \times 30}{5000} = \frac{21}{50} \text{ feet} = 0{\cdot}42 \text{ foot}$$

Example.—In a vessel of 4000 tons displacement, suppose 100 tons of coal to be shifted so that its C.G. moves 18 feet transversely and $4\frac{1}{2}$ feet vertically. Find the shift of the C.G. of the vessel.

The C.G. will move horizontally an amount equal to $\frac{100 \times 18}{4000} = 0{\cdot}45$ ft.

and vertically an amount equal to $\frac{100 \times 4{\cdot}5}{4000} = 0{\cdot}11$ ft.

Moment of Inertia.—We have dealt in Chapter II. with the *moment* of a force about a given point, and we defined it as the product of the force and the perpendicular distance of its line of action from the point; also the moment of an area about a given axis as being the area multiplied by the distance of its centre of gravity from the axis. We could find the moment of a large area about a given axis by dividing it into a number of small areas and summing up the moments of all these small areas about the axis. In this we notice that the area or force is multiplied simply by the distance. Now we have to go a step further, and imagine that each small area is multiplied by the *square of its distance* from a given axis. If all such products are added together for an area, we should obtain not the simple moment, but what may be termed the

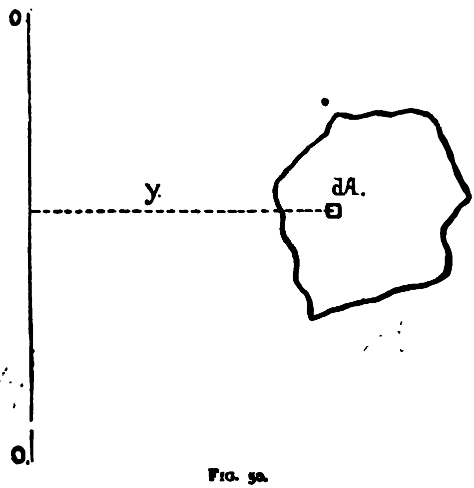


FIG. 59A.

moment of the second degree, or more often the *moment of inertia* of the area about the given axis.[1] We therefore define the moment of inertia of an area about a given axis as follows:—

[1] This is the geometrical moment of inertia. Strictly speaking, moment of inertia involves the mass of the body. We make here the same assumption that we did in simple moments (p. 49), viz. that the area is the surface of a very thin lamina or plate of homogeneous material of uniform thickness.

Imagine the area divided into very small areas, and each such small area multiplied by the square of its distance from the given axis; then, if all these products be added together, we shall obtain the moment of inertia of the total area about the given axis.

Thus in Fig. 50, let OO be the axis. Take a very small area, calling it dA, distance y from the axis. Then the sum of all such products as $dA \times y^2$, or (using the notation we have employed) $\int y^2 \cdot dA$, will be the moment of inertia of the area about the axis OO.

To determine this for any figure requires the application of advanced mathematics, but the result for certain regular figures are given below.

It is found that we can always express the moment of inertia, often written I, of a plane area about a given axis by the expression—

$$nAh^2$$

where A is the area of the figure;

h is the depth of the figure perpendicular to the axis;
n is a coefficient depending on the shape of the figure and the position of the axis.

First, when the axis is through the centre of gravity of the figure parallel to the base, as in Figs. 51 and 52—

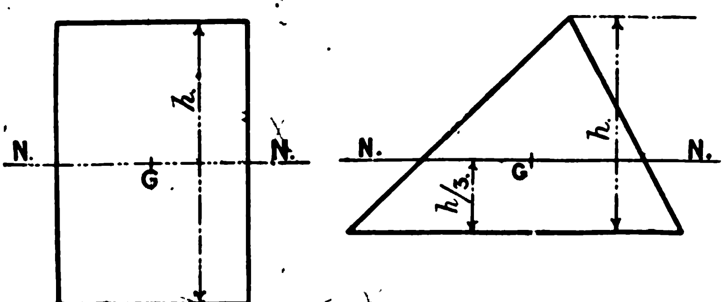


FIG. 51. FIG. 52.

for a circle $n = \frac{1}{16}$, so that $I = \frac{1}{16}Ah^2$
for a rectangle $n = \frac{1}{12}$, „ $I = \frac{1}{12}Ah^2$
for a triangle $n = \frac{1}{18}$, „ $I = \frac{1}{18}Ah^2$

Second, when the axis is one of the sides—

$$\text{for a rectangle } n = \tfrac{1}{3}, \text{ so that } I = \tfrac{1}{3}Ah^2$$
$$\text{for a triangle } n = \tfrac{1}{6}, \quad \text{,,} \quad I = \tfrac{1}{6}Ah^2$$

Example.—Two squares of side a are joined to form a rectangle. The I of each square about the common side is—

$$\tfrac{1}{3}(a^2)a^2 \qquad (a^2 = \text{area})$$

the I of both about the common side will be the sum of each taken separately, or—

$$\tfrac{2}{3}a^4$$

If, however, we took the whole figure and treated it as a rectangle, its I about the common side would be—

$$\tfrac{1}{12}(2a^2)(2a)^2 = \tfrac{2}{3}a^4 \qquad (\text{area} = 2a^2)$$

which is the same result as was obtained before.

To find the moment of inertia of a plane figure about an axis parallel to and a given distance from an axis through its centre of gravity.

Suppose the moment of inertia about the axis NN passing through the centre of gravity of the figure (Fig. 53) is I_0, the area of the figure is A, and OO, the given axis, is parallel to NN and a distance y from it. Then the moment of inertia (I) of the figure about OO is given by—

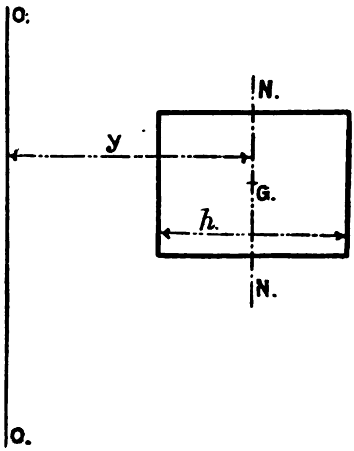


Fig. 53.

$$I = I_0 + Ay^2$$

The moment of inertia of an area about any axis is therefore determined by adding to the moment of inertia of the area about a parallel axis through the centre of gravity, the product of the area into the square of the distance between the two axes. We see from this that the moment of inertia of a figure about an axis through its own centre of gravity is always less than about any other axis parallel to it.

Example.—Having given the moment of inertia of the triangle in Fig. 52 about the axis NN through the centre of gravity as $\frac{1}{18}Ah^2$, find the moment of inertia about the base parallel to NN.

Applying the above rule, we have—

$$\begin{aligned} I &= \tfrac{1}{18}Ah^2 + A\left(\frac{h}{3}\right)^2 \\ &= \tfrac{1}{6}Ah^2 \end{aligned}$$

which agrees with the value given above for the moment of inertia of a triangle about its base.

Example.—Find the moment of inertia of a triangle of area A and height h about an axis through the vertex parallel to the base.

Ans. $\frac{1}{2}Ah^2$.

Example.—A rectangle is 4 inches long and 3 inches broad. Compare the ratio of its moment of inertia about an axis through the centre parallel to the long and short sides respectively.

Ans. 9 : 16.

Example.—A square of 12 inches side has another symmetrical square of half its area cut out of the centre. Compare the moments of inertia about an axis through the centre parallel to one side of, the original square, the square cut out, the remaining area.

Ans. As 4 : 1 : 3, the ratio of the areas being 4 : 2 : 2.

This last example illustrates the important fact that if an area is distributed away from the centre of gravity, the moment of inertia is very much greater than if the same area were massed near the centre of gravity.

To find the Moment of Inertia of a Plane Curvilinear Figure (as Fig. 36, p. 59) **about its Base.**—Take a strip PQ of length y and breadth (indefinitely small) dx. Then, if we regard PQ as a rectangle, its moment of inertia about the base DC is—

$$\tfrac{1}{3}(y \,.\, dx)y^2 = \tfrac{1}{3}y^3 \,.\, dx \qquad (y \,.\, dx = \text{area})$$

and the moment of inertia of the whole figure about DC will be the sum of all such expressions as this; or—

$$\int \tfrac{1}{3}y^3 \,.\, dx$$

that is, we put the third part of the cubes of the ordinates of the curve through either of Simpson's rules. For the water-plane of a ship (for which we usually require to find the moment of inertia about the centre line), we must add the moment of inertia of both sides together: and, since these are symmetrical, we have—

$$I = \tfrac{2}{3}\int y^3 \,.\, dx \qquad (y = \text{semi-ordinate of water-plane}).$$

In finding the moment of inertia of a water-plane about the centre line, the work is arranged as follows :—

Number of ordinate.	Semi-ordinates of water-plane.	Cubes of semi-ordinates.	Simpson's multipliers.	Functions of cubes.
1	0·05	—	1	—
2	4·65	101	4	404
3	10·05	1015	2	2,030
4	14·30	2924	4	11,696
5	16·75	4699	2	9,398
6	17·65	5498	4	21,992
7	17·40	5268	2	10,536
8	16·20	4252	4	17,008
9	13·55	2488	2	4,976
10	9·65	899	4	3,596
11	3·65	49	1	49
				81,685

Common interval = 28 feet

Moment of inertia = $81{,}685 \times \frac{2}{3} \times \frac{28}{3} = 508{,}262$ [1]

The semi-ordinates are placed in column 2, and the cubes of these are placed in column 3. It is not necessary, in ordinary cases, to put any decimal places in the cube; the nearest whole number is sufficient. A table of cubes is given in the Appendix for numbers up to 50, rising by 0·05. These cubes are put through Simpson's multipliers in the ordinary way, giving column 5. The sum of the functions of cubes has to be treated as follows: First there is the multiplier for Simpson's rule, viz. $\frac{1}{3} \times 28$, and then the $\frac{2}{3}$ of the expression $\frac{2}{3}\int y^3 . dx$, which takes into account both sides. The multiplier, therefore, is $\frac{2}{3} \times \frac{28}{3}$, and the sum of the numbers in column 5 multiplied by this will give the moment of inertia required.

Approximation to the Moment of Inertia of a Ship's Water-plane about the Centre Line.—We have seen that for certain regular figures we can express the moment of inertia about an axis through the centre of gravity in the form nAh^2, where n is a coefficient varying for each figure. We can, in the same way, express the I of a water-plane area

[1] This calculation for the L.W.P. is usually done on the displacement sheet. Brown's displacement sheet provides a column for each water-line except the two lowest.

about the centre line, but it is not convenient to use the area as we have done above. We know that the area can be expressed in the form—

$$k \times L \times B$$

where L is the extreme length;
B „ „ breadth;
k is a coefficient of fineness;

so that we can write—

$$I = nLB^3$$

where n is a new coefficient that will vary for different shapes of water-planes. If we can find what the values of the coefficient n are for ordinary water-planes, it would be very useful in checking our calculation work. Taking the case of a L.W.P. in the form of a rectangle, we should find that n = 0·08, and for a L.W.P. in the form of two triangles, n = 0·02.

These are two extreme cases, and we should expect for ordinary ships the value of the coefficient n would lie between these values. This is found to be the case, and we may take the following approximate values for the value of n in the formula $I = nLB^3$:—

For ships whose load water-planes are extremely fine	...	0·04
„ „ „ „ moderately fine	...	0·05
„ „ „ „ very full ...	...	0·06

For the water-plane whose moment of inertia we calculated above, we have, length 280 feet, breadth 35·3 feet, and I = 508,262 in foot-units. Therefore the value of the coefficient n is—

$$\frac{508262}{280 \times (35 \cdot 3)^3} = 0 \cdot 041$$

Formula for finding the Distance of the Transverse Metacentre above the Centre of Buoyancy (BM).—We have already discussed in Chapter II. how the position of the centre of buoyancy can be determined if the underwater form of the ship is known, and now we proceed to discuss how the distance BM is found. Knowing this, we are able to fix the position of the transverse metacentre in the ship.

Let Fig. 45, p. 94, represent a ship heeled over to a very small angle θ (much exaggerated in the figure).

B is the centre of buoyancy in the upright position when floating at the water-line WL.

B' is the centre of buoyancy in the inclined position when floating at the water-line W'L'.

v is the volume of either the immersed wedge LSL or the emerged wedge WSW'.

V is the total volume of displacement.

g is the centre of gravity of the emerged wedge.

g' is the centre of gravity of the immersed wedge.

Then, using the principle given on p. 100, BB' will be parallel to gg', and—

$$BB' = \frac{v \times gg'}{V}$$

since the new displacement is formed by taking away the wedge WSW' from the original displacement, and putting it in the position LSL'.

Now for the very small angle of inclination, we may say that—

$$\frac{BB'}{BM} = \sin\theta$$

$$\text{or } BB' = BM \sin\theta$$

so that we can find BM if we can determine the value of $v \times gg'$, since V, the volume of displacement, is known.

Let Fig. 54 be a section of the vessel; wl, $w'l'$, the original and new water-lines respectively, the angle of inclination being very small. Then we may term wSw' the emerged triangle, and lSl' the immersed triangle, being transverse sections of the emerged and immersed wedges, and ww', ll' being for all practical purposes straight lines. If y be the half-breadth of the water-line at this section, we can say $ww' = ll' = y \sin\theta$, and the area of either of the triangles is—

$$\tfrac{1}{2}y \times y \sin\theta = \tfrac{1}{2}y^2 \sin\theta$$

Let a, a' be the centres of gravity of the triangles wSw', lSl' respectively; then we can say, seeing that θ is very small, that

$ad' = \frac{2}{3}y$, since the centre of gravity of a triangle is two-thirds the height from the apex. The new immersed section being regarded as formed by the transference of the triangle *w*S*w'*

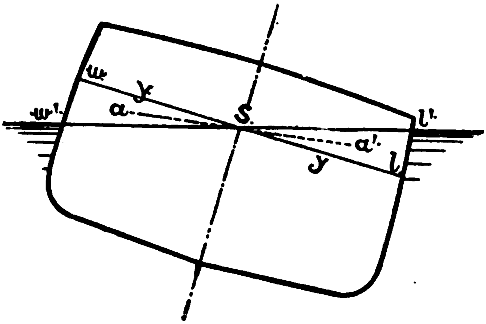


FIG. 54.

to the position occupied by the triangle *l*S*l'*, the moment of transference is—

$$(\tfrac{1}{2}y^2 \sin \theta) \times \tfrac{4}{3}y = \tfrac{2}{3}y^3 \sin \theta$$

and for a very small length *dx* of the water-line the moment will be—

$$\tfrac{2}{3}y^3 \sin \theta \,.\, dx$$

since the small volume is $\frac{1}{2}y^2 \sin \theta \,.\, dx$, and the shift of its centre of gravity is $\frac{4}{3}y$. If now we summed all such expressions as this for the whole length of the ship, we should get the moment of the transference of the wedge, or $v \times gg'$. Therefore we may say, using the ordinary notation—

$$v \times gg' = \int \tfrac{2}{3}y^3 \sin \theta \,.\, dx$$
$$= \tfrac{2}{3} \sin \theta \int y^3 \,.\, dx$$

therefore we have—

$$BB' = BM \sin \theta = \frac{v \times gg'}{V} = \frac{\frac{2}{3} \sin \theta \int y^3 \,.\, dx}{V}$$

$$\text{or } BM = \frac{\frac{2}{3}\int y^3 \,.\, dx}{V}$$

But the numerator of this expression is what we have found to be the moment of inertia of a water-plane about its centre line, y being a semi-ordinate; therefore we can write—

$$BM = \frac{I}{V}$$

We have seen, on p. 105, how the moment of inertia of a water-plane is found for any given case, and knowing the volume of displacement, we can then determine the distance BM, and so, knowing the position of the C.B., fix the position of the transverse metacentre in the ship.

Example.—A lighter is in the form of a box, 120 feet long, 30 feet broad, and floats at a draught of 10 feet. Find its transverse BM.

In this case the water-plane is a rectangle 120′ × 30′, and we want its I about the middle line. Using the formula for the I of a rectangle about an axis through its centre parallel to a side, $\frac{1}{12}Ah^2$, we have—

$$I = \frac{1}{12} \times 3600 \times 900 \qquad (h = 30)$$
$$= 270{,}000$$

$$V, \text{ the volume of displacement, } = 120 \times 30 \times 10 = 36{,}000$$

$$\therefore BM = \frac{270{,}000}{36{,}000} = 7 \cdot 5 \text{ feet}$$

Example.—A pontoon of 10 feet draught has a constant section in the form of a trapezoid, breadth at the water-line 30 feet, breadth at base 20 feet, length 120 feet. Find the transverse BM.

Ans. 9 feet.

It will be noticed that the water-plane in this question is the same as in the previous question, but the displacement being less, the BM is greater. M is therefore higher in the ship for two reasons. BM is greater and B is higher in the second case.

Example.—A raft is formed of two cylinders 5 feet in diameter, parallel throughout their lengths, and 10 feet apart, centre to centre. The raft floats with the axes of the cylinders in the surface. Find the transverse BM.

We shall find that the length does not affect the result, but we will suppose the length is l feet. We may find the I of the water-plane in two ways. It consists of two rectangles each $l' \times 5'$, and their centre lines are 10 feet apart.

1. The water-plane may be regarded as formed by cutting a rectangle $l' \times 5'$ out of a rectangle $l' \times 15'$;

$$\therefore I = \frac{1}{12}(l \times 15) \times 15^2 - \frac{1}{12}(l \times 5) \times 5^2$$
$$= \frac{1}{12}l(15^3 - 5^3)$$
$$= \frac{3250}{12}l$$

this being about a fore-and-aft axis at the centre of the raft.

2. We may take the two rectangles separately, and find the I of each about the centre line of the raft, which is 5 feet from the line through the centre of each rectangle. Using the formula—

$$I = I_0 + Ay^2$$
$$= \frac{1}{12}(l \times 5)5^2 + (l \times 5)5^2$$
$$= \frac{1625}{12}l$$

and for both rectangles the moment of inertia will be twice this, or $\frac{2240}{15}l$, as obtained above.

We have to find the volume of displacement, which works out to $\frac{275}{16}l$ cubic feet. The distance BM is therefore—

$$\frac{2240}{15}l \div \frac{275}{16}l = 13{\cdot}8 \text{ feet}$$

Example.—A raft is formed of three cylinders, 5 feet in diameter, parallel and symmetrical throughout their lengths, the breadth extreme being 25 feet. The raft floats with the axes of the cylinders in the surface. Find the transverse BM.

The moment of inertia of the water-plane of this raft is best found by using the formula $I = I_0 + Ay^2$ for the two outside rectangles, and adding it to I_0, the moment of inertia of the centre rectangle about the middle line. We therefore have for the whole water-plane $I = \frac{4125}{4}l$, where l = the length; and the volume of displacement being $\frac{825}{28}l$, the value of BM will be 35 feet.

Approximate Formula for the Height of the Transverse Metacentre above the Centre of Buoyancy.[1]—The formula for BM is—

$$BM = \frac{I}{V}$$

We have seen that we may express I as $n\text{LB}^3$, where n is a coefficient which varies for different shapes of water-planes, but which will be the same for two ships whose water-planes are similar.

We have also seen that we may express V as $k\text{LBD}$, where D is the mean moulded draft (to top of keel amidships), and k is a coefficient which varies for different forms, but which will be the same for two ships whose under-water forms are similar. Therefore we may say—

$$BM = \frac{n \times L \times B^3}{k \times L \times B \times D}$$
$$= a \cdot \frac{B^2}{D}$$

where a is a coefficient obtained from the coefficients n and k. Sir William White, in the "Manual of Naval Architecture," gives the value of a as being between 0·08 and 0·1, a usual value for merchant ships being 0·09. The above formula shows very clearly that the breadth is more effective than the draught in determining what the value of BM is in any given case. It will also be noticed that the length is not brought in.

[1] For another approximation see Ex. 59 in Appendix.

The ship for which the moment of inertia of a water-plane was calculated on p. 106, had a displacement of 1837 tons up to that water-plane. The value of BM is therefore—

$$\frac{508262}{1837 \times 35} = 7.91 \text{ feet}$$

The breadth and mean draught were 35·3 and 13¼ feet respectively. Consequently the value of the coefficient *a* is 0·084.

To prove that a Homogeneous Log of Timber of Square Section and Specific Gravity 0·5 cannot float in Fresh Water with One of its Faces Horizontal.—The log having a specific gravity of 0·5 will float, and will float with half its substance immersed. The condition that it shall float in stable equilibrium, as regards transverse inclination, in any position is that the transverse metacentre shall be above the centre of gravity.

Let the section be as indicated in Fig. 55, with side length 2*a*. And suppose the log is placed in the water with one side of this section horizontal. Then the draught-line will be at a distance *a* from the bottom, and the log, being homogeneous, *i.e.* of the same quality all through, will have its C.G. in the middle at G, at a distance also of *a* from the bottom. The centre of buoyancy will be at a distance of $\frac{a}{2}$ from the bottom. The height of the transverse metacentre above the centre of buoyancy is given by—

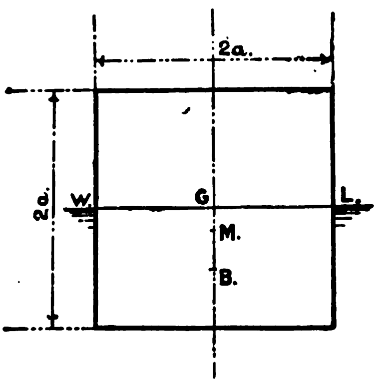


FIG. 55.

$$\text{BM} = \frac{\text{I}}{\text{V}}$$

where I = moment of inertia of water-plane about a longitudinal axis through its centre
V = volume of displacement in cubic feet.

Now, the water-plane of the log is a rectangle of length l and breadth $2a$, and therefore—

$$\text{its I} = \frac{1}{12} l \, . \, 2a(2a)^2 = \frac{8}{12} la^3$$
$$\text{and V} = l \, . \, 2a \, . \, a = 2la^2$$
$$\therefore \text{BM} = \frac{8}{12} la^3 \div 2la^2 = \frac{1}{3} a$$
$$\text{But BG} = \frac{1}{2} a$$

therefore the transverse metacentre is below the centre of gravity, and consequently the log cannot float in the position given.

If, now, the log be assumed floating with one corner downward, it will be found by a precisely similar method that—

$$\text{BG} = 0{\cdot}471a$$
$$\text{and BM} = 0{\cdot}943a$$

Thus in this case the transverse metacentre is above the centre of gravity, and consequently the log will float in stable equilibrium.

It can also be shown by similar methods that the position of stable equilibrium for all directions of inclination of a cube composed of homogeneous material of specific gravity 0·5 is with one corner downwards.

Metacentric Diagram.—We have seen how the position of the transverse metacentre can be determined for any given ship floating at a definite water-line. It is often necessary, however, to know the position of the metacentre when the ship is floating at some different water-line; as, for instance, when coal or stores have been consumed, or when the ship is in a light condition. It is usual to construct a diagram which will show at once, for any given mean draught which the vessel may have, the position of the transverse metacentre. Such a diagram is shown in Fig. 56, and it is constructed in the following manner: A line W_1L_1 is drawn to represent the load water-line, and parallel to it are drawn W_2L_2, W_3L_3, W_4L_4 to represent the water-lines Nos. 2, 3, and 4, which are used for calculating the displacement, the proper distance apart, a convenient scale being $\frac{1}{4}$ inch to 1 foot. A line L_1L_4 is drawn cutting these level lines, and inclined to them at an angle of 45°. Through the points of intersection L_1, L_2, L_3, L_4, are drawn vertical lines as shown. The ship is then supposed to float successively at these water-lines, and the position of the centre of buoyancy and the distance of the transverse metacentre above the C.B.

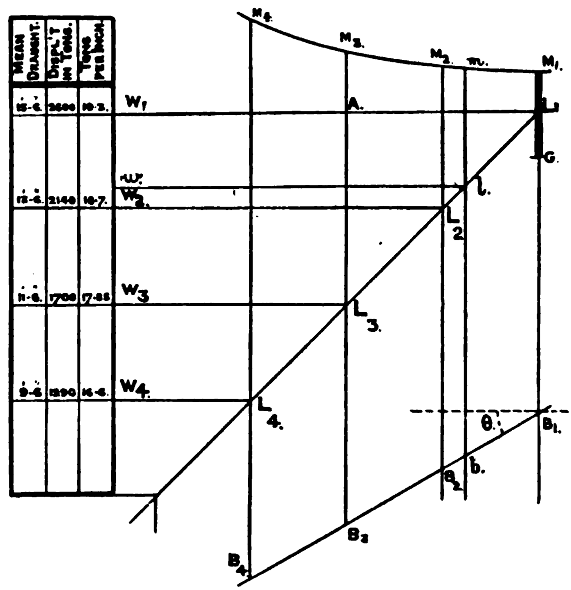


FIG. 56.

calculated for each case. The methods employed for finding the position of the C.B. at the different water-lines have already been dealt with in Chapter II. On the vertical lines are then set down from the L.W.L. the respective distances of the centres of buoyancy below the L.W.L. Thus L_1B_1 is the distance when floating at the L.W.L., and AB_3 the distance when floating at No. 3 W.L. In this way the points B_1, B_2, B_3, B_4 are obtained; and if the calculations are correct, a fair line can be drawn passing through all these spots as shown. Such a curve is termed the *curve of centres of buoyancy*. It is usually found to be rather a flat curve, being straight near the load-line condition. The distance BM for each water-line is then set up from B_1, B_2, B_3, B_4 respectively, giving the points M_1, M_2, M_3, M_4. A curve can then be drawn through these points, which is termed *the curve of transverse metacentres*. Now, suppose the ship is floating at some intermediate water-line—say *wl*: through *l*, where *wl* cuts the 45° line, draw a vertical cutting the curves of centres of buoyancy and metacentres in *b* and *m* respectively. Then *m* will be the position of the transverse metacentre of the ship when floating at the water-line *wl*. It will be noticed that we have supposed the ship to float always with the water-plane parallel to the L.W.P.; that is to say, she does not alter trim. For water-planes not parallel to the L.W.P. we take the mean draught (*i.e.* the draughts at the fore-and-aft perpendiculars are added together and divided by 2), and find the position of M on the metacentric diagram for the water-plane, parallel to the L.W.P., corresponding to this mean draught. Unless the change of trim is very considerable, this is found to be correct enough for all practical purposes. Suppose, however, the ship trims very much by the stern,[1] owing to coal or stores forward being consumed, the shape of her water-plane will be very different from the shape it would have if she were floating at her normal trim or parallel to the L.W.P.; generally the water-plane will be fuller under these circumstances, and the moment of inertia will be greater, and consequently M higher in the ship, than would be given on the metacentric diagram. When a ship is inclined, an operation that will be described later, she is frequently in an unfinished condition, and trims considerably by the stern. It is necessary to know the position of the transverse metacentre accurately for this condition, and

[1] This would be the case in the following: A ship is designed to float at a draught of 17 feet forward and 19 feet aft, or, as we say, 2 feet by the stern. If her draught is, say, 16 feet forward and 20 feet aft, she will have the same mean draught as designed, viz. 18 feet, but she will trim 2 feet more by the stern.

consequently the metacentric diagram cannot be used, but a separate calculation made for the water-plane at which the vessel is floating.

On the metacentric diagram is placed also the position of the centre of gravity of the ship under certain conditions. For a merchant ship these conditions may vary considerably owing to the nature of the cargo carried. There are two conditions for which the C.G. may be readily determined, viz. the light condition, and the condition when loaded to the load-line with a homogeneous cargo. The *light condition* may be defined as follows: No cargo, coal, stores, or any weights on board not actually forming a part of the hull and machinery, but including the water in boilers and condensers. The draught-lines for the various conditions are put on the metacentric diagram, and the position of the centre of gravity for each condition placed in its proper vertical position. The various values for GM, the metacentric height, are thus obtained.

On the left of the diagram are placed, at the various water-lines, the mean draught, displacement, and tons per inch.[1]

There are two forms of section for which it is instructive to construct the metacentric diagram.

1. A floating body of constant rectangular section.

2. A floating body of constant triangular section, the apex of the triangle being at the bottom.

1. For a body having a constant rectangular section, the moment of inertia of the water-plane is the same for all draughts, but the volume of displacement varies. Suppose the rectangular box is 80 feet long, 8 feet broad, 9 feet deep. Then the moment of inertia of the water-plane for all draughts is—

$$\frac{1}{12}(80 \times 8) \times 8^2 = \frac{10240}{3}$$

The volumes of displacement are as follows :—

Draught	6 inches	... ...	$V = 80 \times 8 \times \frac{1}{2}$	cubic feet
,,	1 foot	... ...	$V = 80 \times 8$	,,
,,	2 feet	... ...	$V = 80 \times 8 \times 2$	,,
,,	4 ,,	... ...	$V = 80 \times 8 \times 4$	,,
,,	7 ,,	... ...	$V = 80 \times 8 \times 7$	,,
,,	9 ,,	... ...	$V = 80 \times 8 \times 9$	,,

[1] For a specimen metacentric diagram, see Example 40, Chap. III. Specimen diagrams for various types of ships are given in the Author's "War Ships."

and the values of BM are therefore as follows :—

Draught 6 inches	... ...	BM = 10·66 feet
" 1 foot	... ...	BM = 5·33 "
" 2 feet	... ...	BM = 2·66 "
" 4 "	... ...	BM = 1·33 "
" 7 "	... ...	BM = 0·76 "
" 9 "	... ...	BM = 0·59 "

The centre of buoyancy is always at half-draught, so that its locus or path will be a straight line,[1] and if the values obtained above are set off from the centres of buoyancy at the various water-lines, we shall obtain the curve of transverse metacentres as shown in Fig. 57 by the curve AA, the line BB being the corresponding locus of the centres of buoyancy.

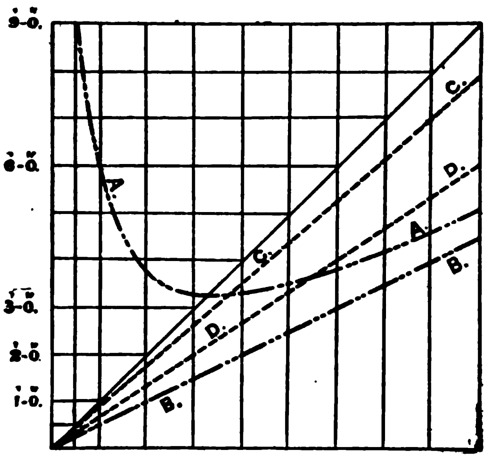


Fig. 57.

2. For a floating body with a constant triangular section, the locus of centres of buoyancy is also a straight line because it is always two-thirds the draught above the base.[1] Suppose the triangular section to be 10 feet broad at the top and 9 feet deep, the length of the body being 120 feet. In this case we must calculate the moment of inertia of each water-plane and the volume of displacement up to each. The results are found to be as follows :—

[1] This may be seen by finding a few spots on this locus.

Draught 1 foot	... ...	BM = 0·20 feet
" 2 feet	... ...	BM = 0·41 "
" 4 "	... ...	BM = 0·82 "
" 6 "	... ...	BM = 1·23 "
" 9 "	... ...	BM = 1·85 "

These values are set up from the respective centres of buoyancy, and give the locus of transverse metacentres, which is found to be a straight line, as shown by CC in Fig. 57, DD being the locus of centres of buoyancy.

Approximation to Locus of Centres of Buoyancy on the Metacentric Diagram.—We have seen (p. 65) how the distance of the centre of buoyancy below the L.W.L. can be approximately determined. The locus of centres of buoyancy in the metacentric diagram is, in most cases, very nearly straight for the portion near the load-line, and if we could obtain easily the direction the curve takes on leaving the position for the load water-line, we should obtain a very close approximation to the actual curve itself. It might be desirable to obtain such an approximation in the early stages of a design, when it would not be convenient to calculate the actual positions of the centre of buoyancy, in order to accurately construct the curve.

Let θ be the angle the tangent to the curve of buoyancy at the load condition makes with the horizontal, as in Fig. 56;

A, the area of the load water-plane in square feet;

V, the volume of displacement up to the load water-line in cubic feet;

h, the distance of the centre of buoyancy of the load displacement below the load water-line in feet.

Then the direction of the tangent to the curve of buoyancy is given by—

$$\tan\theta = \frac{Ah}{V} \text{ (for proof see later.)}$$

Each of the terms in the latter expression are known or can be readily approximated to,[1] and we can thus determine the inclination at which the curve of centres of buoyancy will start, and this will closely follow the actual curve.[2]

[1] See Example 39, p. 143, for a further approximation.

[2] See a paper by the late Professor Jenkins read before the Institution of Naval Architects in 1884.

In a given case—

$$A = 7854 \text{ square feet}$$
$$h = 5{\cdot}45 \text{ feet}$$
$$V = 2140 \times 35 \text{ cubic feet}$$

so that—

$$\tan \theta = \frac{7854 \times 5{\cdot}45}{2140 \times 35}$$
$$= 0{\cdot}572 \text{ [1]}$$

Finding the Metacentric Height by Experiment. Inclining Experiment.—We have been dealing up to the present with the purely geometrical aspect of initial stability, viz. the methods employed and the principles involved in finding the position of the transverse metacentre. All that is needed in order to determine this point is the form of the underwater portion of the vessel. But in order to know anything about the vessel's initial stability, we must also know the vertical position of the centre of gravity of the ship, and it is to determine this point that the *inclining experiment* is performed. This is done as the vessel approaches completion, when weights that have yet to go on board can be determined together with their final positions. Weights are shifted transversely across the deck, and by using the principle explained on p. 100, we can tell at once the horizontal shift of the centre of gravity of the ship herself due to this shift of the weights on board. The weight of the ship can be determined by calculating the displacement up to the water-line she floats at, during the experiment. (An approximate method of determining this displacement when the vessel floats out of her designed trim

[1] The best way to set off this line is to set off a horizontal line of 10 feet long (on a convenient scale), and from the end set down a vertical line 5·72 feet long on the same scale. This will give the inclination required, for $\tan \theta = \frac{\text{per.}}{\text{base}} = \frac{5{\cdot}72}{10} = 0{\cdot}572$.

This remark applies to any case in which an angle has to be set off very accurately. A table of tangents is consulted and the tangent of the required angle is found, and a similar process to the above is gone through.

will be found on p. 152.) Using the notation employed on p. 100, and illustrated by Fig. 49, we have—

$$GG' = \frac{w \times d}{W}$$

Now, unless prevented by external forces, it is evident that the vessel must incline over to such an angle that the centre of gravity G′ and the centre of buoyancy B′ are in the same vertical line (see Fig. 58), and, the angle of inclination being small,

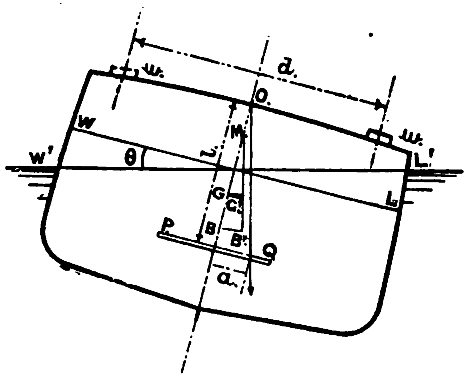


FIG. 58.

M will be the transverse metacentre. If now we call θ the angle of inclination to the upright, GM being the "metacentric height"—

$$\tan \theta = \frac{GG'}{GM}$$

$$GM = \frac{GG'}{\tan \theta}$$

$$= \frac{w \times d}{W \times \tan \theta}$$

using the value found above for GG′. The only term that we do not yet know in this expression is tan θ, and this is found in the following manner: At two or three convenient positions in the ship[1] (such as at bulkheads or down hatchways) plumb-bobs are suspended from a point in the middle line of the ship, and at a convenient distance from the point of suspension a horizontal batten is fixed, with the centre line of the ship marked on it, as shown by PQ in Fig. 58. Before the ship is inclined, the plumb-line should coincide, as nearly as possible, with the centre-line of the ship—that is to say, the ship should be practically upright. When the ship is heeled over to the angle θ, the plumb-line will also be inclined at the same angle, θ, to the original vertical or centre line of the ship, and if l be the distance of the horizontal batten below the point of suspension O in inches, and a the deviation of the plumb-line along the batten, also in inches, the angle θ is at once determined, for—

$$\tan \theta = \frac{a}{l}$$

so that we can write—

$$\mathrm{GM}^{*} = \frac{w \times d}{\mathrm{W} \times \frac{a}{l}}$$

In practice it is convenient to check the results obtained, by dividing the weight w into four equal parts, placing two sets on one side and two sets on the other side, arranged as in Fig. 59.

The experiment is then performed in the following order:—

(*a*) See if the ship is floating upright, in which case the plumb-lines will coincide with the centre of the ship.

(*b*) The weight (1), Fig. 59, is shifted from port to starboard on to the top of weight (3) through the distance d feet, say, and the deviations of the plumb-lines are noted when the ship settles down at a steady angle.

(*c*) The weight (2) is shifted from port to starboard on to the top of weight (4) through the distance d feet, and the deviations of the plumb-line noted.

(*d*) The weights (1) and (2) are replaced in their original positions, when the vessel should again resume her upright position.

[1] If two positions are taken, one is forward and the other aft. If three positions are taken, one is forward, one aft, and one amidships.

* This depends on the assumption that M is a fixed point for the heel obtained, and this is true for ordinary ships. It fails, however, in the case of a vessel of very small or of zero metacentric height. See examples in Appendix A.

(*e*) The weight (3) is moved from starboard to port, and the deviations of the plumb-lines noted.

(*f*) The weight (4) is moved from starboard to port, and the deviations of the plumb-lines noted.

With the above method of conducting the experiment,[1] and using two plumb-lines, we obtain eight readings, and if three plumb-lines were used we should obtain twelve readings. It is important that such checks should be obtained, as a single result might be rendered quite incorrect, owing to the influence of the hawsers, etc. A specimen experiment is given on p. 123, in which two plumb-lines were used. The deviations obtained

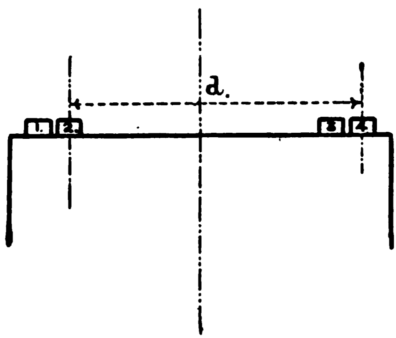


FIG. 59.

are set out in detail, the mean deviation for a shift of $12\frac{1}{2}$ tons through 36 feet being $5\frac{5}{32}$ inches, or the mean deviation for a shift of 25 tons through 36 feet is $10\frac{5}{16}$ inches.

Precautions to be taken when performing an Inclining Experiment.—A rough estimate should be made of the GM expected at the time of the experiment; the weight of ballast can then be determined which will give an inclination of about 4° or 5° when one-half is moved a known distance across the deck. The weight of ballast thus found can then be got ready for the experiment.

A *personal* inspection should be made to see that all weights likely to shift are efficiently secured, the ship cleared of all

[1] There is a slight rise of G, the centre of gravity of the ship, in this method; but the error involved is inappreciable.

free water, and boilers either emptied or run up quite full. Any floating stages should be released or secured by very slack painters.

If possible a fine day should be chosen, with the water calm and little wind. All men not actually employed on the experiment should be sent ashore. Saturday afternoon or a dinner hour is found a convenient time, since then the majority of the workmen employed finishing the ship are likely to be away.

The ship should be hauled head or stern on to the wind, if any, and secured by hawsers at the bow and stern. When taking the readings, these hawsers should be slacked out, so as to ensure that they do not influence the reading. The ship should be plumbed upright before commencing.

An account should be taken, with positions of all weights to be placed on board to complete, of all weights to be removed, such as yard plant, etc., and all weights that have to be shifted.

The following is a specimen report of an inclining experiment :—

Report on Inclining Experiment performed on "——" on ——, 189–, at ——. Density of water — cubic feet to the ton.

Draught of water	16′ 9″ forward.
" "	22′ 10″ aft.
Displacement in tons at this draught	5372

The wind was slight, and the ship was kept head to wind during the experiment. Ballast used for inclining, 50 tons. Lengths of pendulums, two in number, 15 feet. Shift of ballast across deck, 36 feet.

	Deviation of pendulum in 15 feet.	
	Forward.	Aft.
Experiment 1, $12\frac{1}{2}$ tons port to starboard	$5\frac{1}{8}''$	$5\frac{1}{8}''$
" 2, $12\frac{1}{2}$ " "	$10\frac{1}{4}''$	$10\frac{3}{8}''$
Ballast replaced, zero checked	right	right
Experiment 3, $12\frac{1}{2}$ tons starboard to port	$5\frac{1}{4}''$	$5\frac{1}{8}''$
" 4, $12\frac{1}{2}$ " "	$10\frac{1}{8}''$	$10\frac{1}{4}''$

The condition of the ship at the time of inclining is as defined below :—

Bilges dry.
Water-tanks empty.

No water in boilers, feed-tanks, condensers, distillers, cisterns, etc.
Workmen on board, 66.
Tools on board, 5 tons.
Masts and spars complete.
No boats on board.
Bunkers full.
Anchors and cables, complete and stowed.
No provisions or stores on board.
Engineers' stores, half on board.
Hull complete.

The mean deviation in 15 feet for a shift of 25 tons through 36 feet is $10\frac{5}{16}$ inches = 10·312 inches.

$$\therefore \text{GM} = \frac{25 \times 36 \times 15 \times 12}{10 \cdot 312 \times 5372} = 2 \cdot 92 \text{ feet}$$

The ship being in an incomplete condition at the time of the inclining experiment, it was necessary to take an accurate account of all weights that had to go on board to complete, with their positions in the ship, together with an account of all weights that had to be removed, with their positions. The total weights were then obtained, together with the position of their final centre of gravity, both in a longitudinal and vertical direction. For the ship of which the inclining experiment is given above, it was found that to fully complete her a total weight of 595 tons had to be placed on board, having its centre of gravity 11 feet before the midship ordinate, and 3·05 feet below the designed L.W.L. Also 63 tons of yard plant, men, etc., had to be removed, with centre of gravity 14 feet abaft the midship ordinate, and 15 feet above the designed L.W.L. The centre of buoyancy of the ship at the experimental water-line was 10·8 feet abaft the midship ordinate, and the transverse metacentre at this line was calculated at 3·14 feet above the designed L.W.L.

We may now calculate the final position of the centre of gravity of the completed ship as follows, remembering that in the experimental condition the centre of gravity must be in the same vertical line as the centre of buoyancy. The vertical position of G in the experimental condition is found by subtracting the experimental GM, viz. 2·92 feet, from the height of the metacentre above the L.W.L. as given above, viz. 3·14 feet.

	Tons.	Above L.W.L.		Below L.W.L.		Abaft amidships.		Before amidships.	
		Lever.	Moment.	Lever.	Moment.	Lever.	Moment.	Lever.	Moment.
Weight of ship at time of experiment ...	5372	0·22	1182	—	—	10·8	58,017	—	—
Weight to go on board to complete	595	—	—	3·05	1813	—	—	11	6545
	5967		1182		1813		58,017		6545
Weight to be taken from ship	63	15	945	—	—	14·0	882	—	—
	5904		237		1813		57,135		6545
					237		6,545		
					1576		50,590		

The final position of the centre of gravity of the ship is therefore—

$$\frac{1576}{5904} = 0{\cdot}266 \text{ feet below the L.W.L.}$$
$$\frac{50590}{5904} = 8{\cdot}57 \text{ feet abaft amidships}$$

the final displacement being 5904 tons.

The mean draught corresponding to the displacement can be found by the methods we have already dealt with, and corresponding to this draught, we can find on the metacentric diagram the position of the transverse metacentre. In this case the metacentre was 2·73 feet above the L.W.L., and consequently the value of GM for the completed condition was—

$$2{\cdot}73 + 0{\cdot}266 = 2{\cdot}996 \text{ feet}$$

or say, for all practical purposes, that the transverse metacentric height in the completed condition was 3 feet.

It is also possible to ascertain what the draughts forward and aft will be in the completed condition, as we shall see in the next chapter.

Values of GM, the "Metacentric Height."—We have discussed in this chapter the methods adopted to find for a given ship the value of the transverse metacentric height GM. This distance depends upon two things: the position of G, the centre of gravity of the ship; and the position of M, the transverse metacentre. The first is dependent on the vertical distribution of the weights forming the structure and lading of the ship, and its position in the ship must vary with differences in the disposition of the cargo carried. The transverse metacentre depends solely upon the form of the ship, and its position can be completely determined for any given draught of water when we have the sheer drawing of the vessel. There are two steps to be taken in finding its position for any given ship floating at a certain water-line.

1. We must find the vertical position of the centre of buoyancy, the methods adopted being explained in Chapter II.

2. We then find the distance separating the centre of buoyancy and the transverse metacentre, or BM, as explained in the present chapter.

By this means we determine the position of M in the ship.

The methods of estimating the position of G, the centre of gravity for a new ship, will be dealt with separately in Chapter VII.; but we have already seen how the position of G can be determined for a given ship by means of the inclining experiment. Having thus obtained the position of M and G in the ship, we get the distance GM, or the metacentric height.

The following table gives the values of the metacentric height in certain classes of ships. For fuller information reference must be made to the works quoted at the end of the book.

Type of ship.	Values of GM.
Harbour vessels, as tugs, etc.	15 to 18 inches
Modern protected cruisers	2 to 2½ feet
Modern British battleships	4 to 5 feet
Older central citadel armourclads ...	4 to 8 feet
Shallow-draught gunboats for river service	12 feet
Merchant steamers (varying according to the nature and distribution of the cargo)	1 to 3 feet
Sailing-vessels	3 to 3½ feet

The amount of metacentric height given to a vessel is based largely upon experience with successful ships. In order that a vessel may be "*stiff*," that is, difficult to incline by external forces—as, for example, by the pressure of the wind on the sails—the metacentric height must be large. This is seen by reference to the expression for the moment of statical stability at small angles of inclination from the upright, viz.—

$$W \times GM \sin \theta \text{ (see p. 98)}$$

W being the weight of the ship in tons; θ being the angle of inclination, supposed small. This, being the moment tending to right the ship, is directly dependent on GM. A "*crank*" ship is a ship very easily inclined, and in such a ship the metacentric height is small. For steadiness in a seaway the metacentric height must be small.

There are thus two opposing conditions to fulfil—

1. The metacentric height GM must be enough to enable the ship to resist inclination by external forces. This is especially the case in sailing-ships, in order that they may be able to stand up under canvas without heeling too much. In the case of the older battleships with short armour belts and unprotected ends, sufficient metacentric height had to be provided to allow of the ends being riddled, and the consequent reduction of the moment of inertia of the water-plane.

2. The metacentric height must be moderate enough (if this can be done consistently with other conditions being satisfied) to make the vessel steady in a seaway. A ship which has a very large GM comes back to the upright very suddenly after being inclined, and consequently a vessel with small GM is much more comfortable at sea, and, in the case of a man-of-war, affords a much steadier gun platform.

In the case of sailing-ships, a metacentric height of from 3 to $3\frac{1}{2}$ feet is provided under ordinary conditions of service, in order to allow the vessel to stand up under her canvas. It is, however, quite possible that, when loaded with homogeneous cargoes, as wool, etc., this amount cannot be obtained, on account of the centre of gravity of the cargo being high up in the ship. In this case, it would be advisable to take in water or other ballast in order to lower the centre of gravity, and thus increase the metacentric height.

In merchant steamers the conditions continually vary on account of the varying nature and distribution of the cargo carried, and it is probable that a GM of 1 foot should be the minimum provided when carrying a homogeneous cargo (consistently with satisfactory stability being obtained at large inclinations).[1] There are, however, cases on record of vessels going long voyages with a metacentric height of less than 1 foot, and being reported as comfortable and seaworthy. Mr. Denny (*Transactions of the Institution of Naval Architects*, 1896) mentioned a case of a merchant steamer, 320 feet long (carrying a homogeneous cargo), which sailed habitually with a metacentric height of 0·6 of a foot, the captain reporting her behaviour as admirable in a seaway, and in every way comfortable and safe.

It is the practice of one large steamship company to lay down that the metacentric height in the loaded condition is no greater than is required to secure that the metacentric height in the light condition is not negative.

Effect on Initial Stability due to the Presence of Free Water in a Ship.—On reference to p. 123, where the inclining experiment for obtaining the vertical position of the centre of gravity of a ship is explained, it will be noticed that special attention is drawn to the necessity for ascertaining that no free water is allowed to remain in the ship while the experiment is being performed. By free water is meant water having a free surface. In the case of the boilers, for instance, they should either be emptied or run up quite full. We now proceed to ascertain the necessity for taking this precaution. If a compartment, such as a ballast tank in the double bottom, or a boiler, is run up quite full, it is evident that the water will have precisely the same effect on the ship as if it were a solid body having the same weight and position of its centre of gravity as the water, and this can be allowed for with very little difficulty. Suppose, however, that we have on board in a compartment, such as a ballast tank in the double bottom, a quantity of water, and the water does not completely fill the

[1] Mr. Pescod, before the North-East Coast Institution of Engineers and Shipbuilders, 1903, dealt with the minimum GM for small vessels. He there states that it is generally recognized that the GM of cargo vessels should not be less than 0·8 foot provided that a righting arm of like amount is obtained at 30 to 40 degrees.

tank, but has a free surface, as *wl*, Fig. 60.[1] If the ship is heeled over to a small angle θ, the water in the tank must adjust itself so that its surface *w'l'* is parallel to the level water-line W'L'. Let the volume of either of the small wedges *wsw'*, *lsl'* be v_0, and *g*, *g'* the positions of the ircentres of gravity, *b*, *b'* being the centres of gravity of the whole volume of water in the upright and inclined positions respectively. Then, if V_0 be the total volume of water in the tank, we have—

$$V_0 \times bb' = v_0 \times gg'$$

$$\text{and } bb' = \frac{v_0}{V_0} \times gg'$$

and *bb'* is parallel to *gg'*. Now, in precisely the same way as we found the moment of transference of the wedges WSW', LSL', in Fig. 45, we can find the moment of transference of the small wedges *wsw'*, *lsl'*, viz.—

$$v_0 \times gg' = i \times \theta$$

where *i* is the moment of inertia of the free surface of the water in the tank about a fore-and-aft axis through *s*; and θ is the circular measure of the angle of inclination.

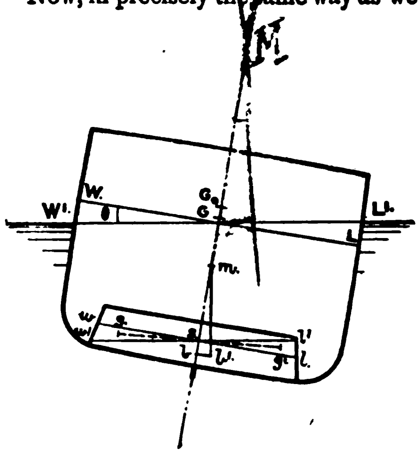


Fig. 60.

Substituting this value for $v_0 \times gg'$, we have—

$$bb' = \frac{i \times \theta}{V_0}$$

Draw the new vertical through *b'*, meeting the middle line in *m*; then—

$$bb' = bm \times \theta$$

[1] Fig. 60 is drawn out of proportion for the sake of clearness.

and consequently—

$$bm \times \theta = \frac{i \times \theta}{V_0}$$

$$\text{and } bm = \frac{i}{V_0}$$

Now, if the water were solid its centre of gravity would be at b both in the upright and inclined conditions, but the weight of the water now acts through the point b' in the line $b'm$, and its effect on the ship is just the same as if it were a solid weight concentrated at the point m. So that, although b is the *actual centre of gravity* of the water, its effect on the ship, when inclined through ever so small an angle, is the same as though it were at the point m, and in consequence of this the point m is termed the *virtual centre of gravity* of the water.[1] This may be made clearer by the following illustrations:—

1. Suppose that one instant the water is solid, with its centre of gravity at b, and the following instant it became liquid. Then, for small angles of inclination, its effect on the ship would be the same as if we had raised its weight through a vertical distance bm from its actual to its virtual centre of gravity.

2. Imagine a pendulum suspended at m, with its bob at b. On the ship being inclined to the small angle θ, the pendulum will take up the position mb', and this corresponds exactly to the action of the water.

We thus see that the centre of gravity of the ship cannot be regarded as being at G, but as having risen to G_0, and if W_0 be the weight of water in tons $= \frac{V_0}{35}$ (the water being supposed salt), we have—

$$W \times GG_0 = W_0 \times bm$$

$$= \frac{V_0}{35} \times bm$$

and therefore—

$$GG_0 = \frac{V_0 \times bm}{35W} = \frac{V_0}{V} \times bm \quad (V = \text{volume of displacement})$$

[1] See a paper by Mr. W. Hök, at the Institution of Naval Architects, 1895, on "The Transverse Stability of Floating Vessels containing Liquids, with Special Reference to Ships carrying Oil in Bulk." See also a paper in the "Transactions of the Institution of Engineers and Shipbuilders in Scotland for 1889," by the late Professor Jenkins, on the stability of vessels carrying oil in bulk.

But we have seen that—

$$bm = \frac{i}{V_0}$$

and therefore—

$$GG_0 = \frac{V_0}{V} \times \frac{i}{V_0} = \frac{i}{V}$$

The new moment of stability at the angle θ is—

$$W \times G_0M \times \sin\theta = W \times (GM - GG_0) \sin\theta$$
$$= W \times \left(GM - \frac{i}{V}\right) \sin\theta$$

the metacentric height being reduced by the simple expression $\frac{i}{V}$. We notice here that the *amount* of water does not affect the result, but only the *moment of inertia of the free surface.* The necessity for the precaution of clearing all free water out of a ship on inclining is now apparent. A small quantity of water will have as much effect on the position of the centre of gravity, and therefore on the trustworthiness of the result obtained, as a large quantity of water, provided it has the same form of free surface. If a small quantity of water has a large free surface, it will have more effect than a very large quantity of water having a smaller free surface.

If the liquid contained is other than the water the vessel is floating in, the loss of metacentric height is $\frac{\rho . i}{V}$, where ρ is the specific gravity of liquid compared with outside water, and V the total volume of displacement.

Example.—A vessel has a compartment of the double bottom at the middle line, 60 feet long and 30 feet broad, partially filled with salt water. The total displacement is 9100 tons, and centre of gravity of the ship and water is 0·26 feet below the water-line. Find the loss of metacentric height due to the water having a free surface.

We have here given the position of the centre of gravity of the ship and the water. The rise of this centre of gravity due to the mobility of the water is, using the above notation—

$$\frac{i}{V}$$

$$\text{and } i = \tfrac{1}{12}(60 \times 30) \times (30)^2$$
$$= 5 \times (30)^3$$

Since the free surface is a rectangle 60 feet long and 30 broad

$$\text{and } V = 9100 \times 35 \text{ cubic feet}$$

therefore the loss in metacentric height $= \frac{5 \times 30^3}{9100 \times 35} = 0·424$ feet

Metacentric Diagrams for Simple Figures.—1. *A rectangular box.*—This is dealt with on p. 116.

$$BM = \frac{1}{12} \cdot \frac{B^2}{D} \qquad B = \text{breadth} \qquad D = \text{draught},$$

and M from base is $\frac{D}{2} + \frac{1}{12} \cdot \frac{B^2}{D}$

By the methods of the calculus this is found to be a minimum when $D^2 = \frac{1}{6} . B^2$, *i.e.* when M is in the W.L. or where it crosses the 45° line.

The M curve is a hyperbola referred to the vertical at zero draught, and the C.B. line as axes, having the equation—

$$x . y = \frac{\sqrt{5}}{24} \cdot B^2$$

the axes being asymptotes.

2. *A vessel with a triangular section, vertex down.*—This is dealt with on p. 117, where it is seen that the M curve is straight—

$$BM = \frac{1}{6} \cdot \frac{B^2}{D} . \qquad M \text{ from base} = \frac{2}{3} . D + \frac{1}{6} \cdot \frac{B^2}{D}$$

$$\therefore \frac{M \text{ from base}}{D} = \frac{2}{3} + \frac{1}{6} \cdot \frac{B^2}{D^2} = \frac{2}{3} + \frac{4}{6} \tan^2 a = \text{constant}$$

$$\frac{B}{2D} = \tan a, \text{ where } a \text{ is the semi-vertical angle}$$

i.e. M curve is a straight line, making an angle θ, with the base line such that $\tan \theta = \frac{2}{3} (1 + \tan^2 a)$.

3. *Vessel with parabolic section.*—A parabola has the equation referred to axes at the vertex, $y^2 = 4ax$, *i.e.* for x draught breadth at waterline is $2y = 4\sqrt{ax}$ (Fig. 60A).

The C.G. of a parabola is $\frac{3}{5}$ the depth, so that the C.B. locus is a straight line making an angle with the base of $\tan^{-1}(\frac{3}{5})$. Area of parabola $= \frac{2}{3}$. circumscribing rectangle.

$$BM = \frac{\frac{1}{12} \cdot (2y) \cdot 4y^2}{\frac{2}{3} \cdot 2y \cdot D} = \frac{1}{2} \cdot \frac{y^2}{D} = 2a = \text{constant}$$

i.e. the locus of metacentres in metacentric diagram is straight and parallel to the C.B. locus.

4. *Vessel with circular section* (Fig. 60B).—In this case the metacentre is always at the centre, so that the M curve is a straight line at mid depth.

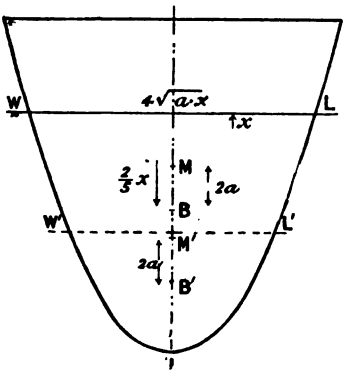


FIG. 60A.

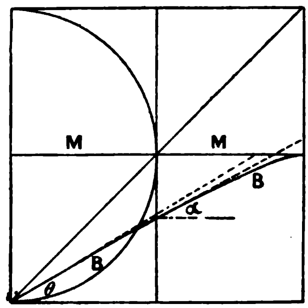


FIG. 60B.

The B curve is a flat curve starting at an angle θ with the base, such that $\theta = \tan^{-1}(\frac{3}{8})$, since there the circle may be regarded as a parabola.

For mid depth B below W.L. is $\frac{4a}{3\pi}$ (a being radius), and the inclination of tangent is an angle α such that $\tan \alpha = \cdot 54$ by the formula given on p. 118.

When completely immersed the curve finishes as a tangent to the M curve.

Curves of Buoyancy, etc.—The *surface of buoyancy* for a given displacement is the surface traced out by the centre of buoyancy as the vessel takes up all possible positions while maintaining that displacement.

The *surface of flotation* is the surface traced out by the centre of flotation under the same conditions.

The *curve of buoyancy* is the curve traced out on the transverse vertical plane by the projection of the centre of buoyancy as the ship is continually revolved about a longitudinal axis fixed in direction while maintaining the same displacement. This curve is also termed an *isovol.*

The *curve of flotation* is the curve traced out by the projection of the centre of flotation under the same conditions.

The *pro-metacentre* is the intersection of any two consecutive lines of action of buoyancy, as M′ in Fig. 60E. When consecutive lines do not intersect the pro-metacentre is the intersection of one of them with the common perpendicular. For a condition of equilibrium this intersection of consecutive lines of buoyancy is the *metacentre.*

The *metacentric* is the locus of pro-metacentres.

The following are definitions of various sorts of equilibrium:—

(1) Rotation in a given direction only.

(*a*) *Stable equilibrium*—for a given direction of inclination when, on being slightly displaced in that direction from its position of rest, the vessel tends, on being released, to go back to that position.

(*b*) *Unstable equilibrium* is as (*a*), only that the vessel moves further from the position of rest.

(*c*) *Indifferent* or *neutral equilibrium*—the vessel neither tends to return to or to go further from the position of rest.

(*d*) *Mixed Equilibrium*—if stable for one direction of inclination and unstable for the opposite direction.

(2) Rotation in all directions.

(*a*) *Absolute equilibrium*—when only stable or unstable for any direction of inclination.

(*b*) *Relative stability*—when stable in some directions and unstable in others.

Thus, in a ship—

(i) If the C.G. is below the transverse metacentre M_T, she is absolutely stable.

(ii) If the C.G. is above the longitudinal metacentre M_L, she is absolutely unstable.

(iii) If the C.G. is between M_T and M_L, she has relative stability, being stable for longitudinal inclinations and unstable for transverse inclinations.

The above definitions are well illustrated by a floating cube of s.g. $\frac{1}{2}$.

(*a*) When floating with a face horizontal, the cube is absolutely unstable.

(*b*) With one corner downwards, the cube has absolute stability.

In going from one point on the surface of buoyancy to the consecutive point, B to B′, $BB' = \frac{(v \times gg')}{V}$, and BB′ is parallel to gg'. Hence, in the limit BB′ is parallel to the water-plane, so that the tangent plane at any point to the surface of buoyancy is parallel to the corresponding water-plane, and the normal to it through the point of contact gives the line of action of buoyancy. The surface must be wholly convex to the tangent plane or wholly concave to some interior point. Similar reasoning will also apply to the curve of buoyancy.

For a position of equilibrium, the line of action of the buoyancy must pass through the C.G.; therefore, the number of positions of equilibrium that a body can take up is equal to the number of normals that can be drawn from the C.G. to the surface of buoyancy. For a given direction of inclination the number of positions of equilibrium equals the number of normals that can be drawn from the C.G. to the curve of buoyancy.

When BG thus drawn is a *minimum*, the equilibrium is *stable*.
When BG thus drawn is a *maximum*, the equilibrium is unstable;

for the stability is the same as that of the curve of buoyancy rolled along a smooth horizontal plane, the weight being concentrated at the C.G. In moving from one position of equilibrium to another, if the C.G. has to be raised we have stable equilibrium, *i.e.* $B'G > BG$. If unstable, similarly $B'G < BG$.

The centre of curvature of the surface of buoyancy is what we have termed the *pro-metacentre*, and the radius of curvature is given by $R = \frac{I}{V}$ where I is the moment of inertia of the water-plane about an axis through its C.G. perpendicular to the plane of rotation, and V is the volume of displacement. This is proved exactly as in Chap. III. for the upright BM.

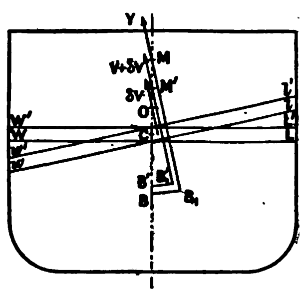


FIG. 60C.

Leclert's theorem for the radius of curvature of the curve of flotation.

In Fig. 60C WL and W'L' are consecutive water-lines for the upright *wl*, *w'l'*, when inclined to a small angle, the increment of displacement being ΔV. Then when inclined the buoyancy V acts through M, and that of ΔV through O, the centre of curvature of the curve of flotation. B and B' are the upright C.B.'s and M' the metacentre for the water-line W'L', and $V + \Delta V$ acts through M' for a small inclination.

Taking moments about C, the C.G. of the layer ΔV, we have—

$$(V \times CM) + (\Delta V \times CO) = (V + \Delta V)CM'$$

or $$V(BM - BC) + (\Delta V \times CO) = (V + \Delta V)(B'M' - B'C)$$

now $$V \times BC = (V + \Delta V)B'C$$

so that $$(V \times BM) + (\Delta V \times CO) = (V + \Delta V)B'M'$$

or $$I + (\Delta V \times CO) = I + \Delta I$$

i.e. $OC = \frac{dI}{dV}$ in the Limit, which is the expression for the radius of curvature of the curve of flotation usually called r.

It can be readily shown that if a weight be added at the point O the *moment of initial stability is not changed.* For ordinary ships parallel-sided at the water-line dI is zero or practically so, so that O is in the water-line. We may therefore say that, generally speaking, if a weight is added above the water-line it will diminish the stability; if added at the water-line there is no change in the stability; if added below the water-line the stability is increased.

Examples.—(i.) r for a body of rectangular section, $r = 0$.

(ii.) r for a body of triangular equilateral section, angle 2θ.
 (*a*) corner downwards, $r = d \tan^2\theta$ (d = draught).
 (*b*) corner upwards, $r = -c \tan^2\theta$ (c being distance of water-line from vertex).

(iii.) r for a circular section, radius a, $r = a \cos\theta$.
 (2θ being angle subtended at the centre by the water-line.)

(iv.) Show that an added weight to keep the *metacentric height* constant should be placed the same distance from G as O is distant from M.

(v.) In Example (ii.) (*b*) above, if $\tan\theta = \frac{3}{4}$ and depth is 40′ then if the draught is less than 14·4 ft. a small addition of ballast to the base of the triangle will make the body *less stable*, but at greater draughts the stability increases with the addition of ballast.

Geometry of the Metacentric Diagram.

1. *Tangent to the curve of C.B.*—In Fig. 60D, let θ be the inclination of this locus to the horizontal at water-line WL.

Then for an increment of displacement ΔV and of draught Δy the C.B. will rise an amount $\frac{h}{V} . \Delta V = \frac{A . h}{V} . \Delta y$.

$\tan\theta = \frac{\text{rise of B}}{\Delta y} = \frac{A \cdot h}{V}$ where A = area of water-plane,

h = C.B. below water-line,

V = volume displacement.

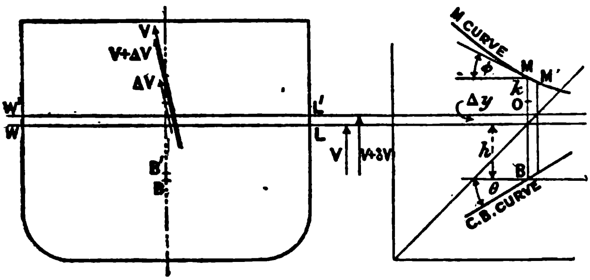


FIG. 60D.

Examples.—(i.) for a box-shaped vessel, $\tan\theta = 0{\cdot}5$.
(ii.) for a triangular section, $\tan\theta = 0{\cdot}66$.
(iii.) for a circular section at half depth } $\tan\theta = 0{\cdot}54$.
For ordinary ships it is found that $\tan\theta = 0{\cdot}55$ about.

2. *Tangent to the curve of metacentres.*—The increment of volume ΔV, for a small inclination, has its line of action through O, the centre of curvature of the curve of flotation. Let OM = *k*—

then $\quad (V + \Delta V)MM' = \Delta V \times OM. = \Delta V \times k \quad$ (Fig. 60C)

or $\quad MM' = \frac{A \cdot k}{V} \cdot \Delta y.$

If ϕ be the angle the tangent to the M curve makes with the horizontal, then

$$\tan\phi = \frac{\text{rise or fall of M}}{\Delta y} = \frac{A \cdot k}{V} \text{ in the limit}$$

If $k = 0$, then $\tan\phi = 0$ and the M curve is horizontal, *i.e.* when the M curve is horizontal, the points M and O coincide. This is otherwise obvious, as the added buoyancy will act through M, which is therefore fixed in height for a small increment of draught. In a box-shaped ship the M curve is horizontal when $D^2 = \frac{1}{6} B^2$.

Co-ordinates of the Centre of Buoyancy referred to axes through the upright C.B.

In Fig. 60E, *x* and *y* are the co-ordinates of B, the C.B. at angle θ. For an increment of angle $d\theta$, B′ is the new C.B. and

$x + dx, y + dy$ the co-ordinates, BM′, B′M′ the normals at B and B′ intersect in M′ the *pro-metacentre*, and BM′ = B′M′ = R, and $R = \frac{I}{V}$. BB′ = R . $d\theta$, and dx = BB′ . cos θ, dy = BB′ . sin θ,

$$\therefore dx = R . \cos\theta . d\theta \qquad dy = R . \sin\theta . d\theta$$

and

$$x = \int R . \cos\theta . d\theta \qquad y = \int R . \sin\theta . d\theta.$$

Curves of R cos θ, R . sin θ can be drawn on a θ base and integrated up to the various angles. Thus, x and y can be obtained and so the curve of buoyancy drawn in. The righting arm at angle θ is given by GZ = x . cos $\theta + y$ sin θ − B_0G . sin θ.

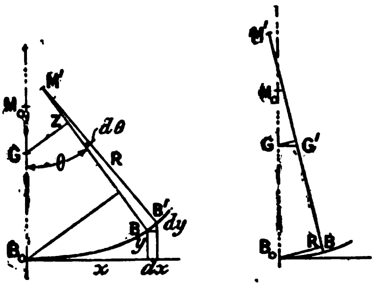


FIG. 60E.

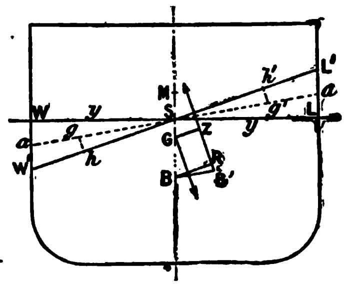


FIG. 60F.

This is the French method of calculating stability due to M. Reech—

For a box so long as wall sided $R = B_0M_0 . \sec^3\theta$ so that

$$x \quad \int B_0M_0 . \sec^3\theta \cos\theta \, d\theta = B_0M_0 . \tan\theta$$
$$y = \int B_0M_0 \sec^3\theta . \sin\theta \, d\theta$$
$$= \int B_0M_0 . \sec^2\theta \tan\theta \, d\theta = \tfrac{1}{2} B_0M_0 . \tan^2\theta$$

This is the solution of question 35 in the Appendix.
Question 36 is solved as follows :—

G′ the new C.G. will lie on BM′ and $GG' = \frac{w \times d}{W}$

$$GG' \cos\theta = x \cos\theta + y \sin\theta - B_0G \sin\theta$$
$$GG' = x + y . \tan\theta - B_0G . \tan\theta$$
$$= B_0M_0 . \tan\theta + \tfrac{1}{2} B_0M_0 . \tan^3\theta - B_0G . \tan\theta$$
$$= GM_0 . \tan\theta + \tfrac{1}{2} B_0M_0 . \tan^3\theta$$

Also $$GG' = \frac{w \times d}{W}$$

$$\therefore \tan^3\theta + 2 . \left(\frac{GM_0}{B_0M_0}\right) \tan\theta - \left(\frac{w \times d}{W \times B_0M_0}\right) = 0$$

Stability of a Wall-sided Vessel.

In Fig. 60F, $BR = \frac{v \times hh'}{V}$

$$v = \tfrac{1}{2}.y^2.\tan\theta$$
$$hh' = \tfrac{2}{3}\text{ the projection of } aa \text{ on to } W'L'$$
$$= \tfrac{2}{3}\text{ the projection of } SL + La \text{ on to } W'L'$$
$$= \tfrac{2}{3}(y.\cos\theta + \tfrac{1}{2}.y.\tan\theta.\sin\theta)$$
$$\therefore v \times hh' = \tfrac{2}{6}(y^3.\sin\theta + \tfrac{1}{2}.y^3.\tan^2\theta.\sin\theta)$$
$$= \tfrac{2}{6}.y^3.\sin\theta(1 + \tfrac{1}{2}.\tan^2\theta)$$
$$\frac{v \times hh'}{V} = BM.\sin\theta(1 + \tfrac{1}{2}\tan^2\theta) \text{ taking a unit length}$$
$$= BR \text{ also.}$$
$$GZ = BR - BG.\sin\theta$$
$$= BM\sin\theta + \tfrac{1}{2}BM.\tan^2\theta\sin\theta - BG\sin\theta$$
$$= \sin\theta(GM + \tfrac{1}{2}BM.\tan^2\theta).$$

This can be used to construct the curve of stability (see Chap. V.) so far as the ship is wall-sided above and below water, and can be used to check the cross curve at 15° say obtained by the Integrator.

This formula may be used to determine the angle to which a ship with negative metacentric height will loll over, for GZ will then be zero, and we have—

$$\tan\theta = \pm\sqrt{2.\frac{GM}{BM}}$$

and the metacentric height when at the angle θ is $2\frac{GM}{\cos\theta}$. (See example 37 in Appendix.)

Examples to Chapter III.

1. Find the circular measure of $5\tfrac{1}{2}°$, $10\tfrac{1}{4}°$, $15\tfrac{3}{4}°$.

Ans. 0·09599; 0·17889; 0·27489.

2. Show that sin 10° is one-half per cent. less in value than the circular measure of 10°, and that tan 10° is one per cent. greater in value than the circular measure of 10°.

✓ 3. A cylinder weighing 500 lbs., whose centre of gravity is 2 feet from the axis, is placed on a smooth table and takes up a position of stable equilibrium. It is rolled along parallel to itself through an angle of 60°. What will be the tendency then to return to the original position?

Ans. 866 foot-lbs.

✓ 4. Find the moment of inertia about the longest axis through the centre of gravity of a figure formed of a square of side $2a$, having a semicircle at each end.

Ans. $\left(\frac{16 + 3\pi}{12}\right)a^4$.

5. Find the moment of inertia of a square of side $2a$ about a diagonal.

Ans. $\frac{4}{3}a^4$.

6. A square has a similar square cut out of its centre such that the moment of inertia (about a line through the centre parallel to one side) of the small square and of the portion remaining is the same. What proportion of the area of the original square is cut out?

Ans. 0·71 nearly.

7. A vessel of rectangular cross-section throughout floats at a constant draught of 10 feet, and has its centre of gravity in the load water-plane. The successive half-ordinates of the load water-plane in feet are 0·5, 6, 12, 16, 15, 9, 0; and the common interval 20 feet. Find the transverse metacentric height.

Ans. 8 inches.

8. A log of fir, specific gravity 0·5, is 12 feet long, and the section is 2 feet square. What is its transverse metacentric height when floating in stable equilibrium in fresh water?

Ans. 0·47 foot.

9. The semi-ordinates of a water-plane 34 feet apart are 0·4, 13·7, 25·4, 32·1, 34·6, 35·0, 34·9, 34·2, 32·1, 23·9, 6·9 feet respectively. Find its moment of inertia about the centre line.

Ans. 6,012,862.

10. The semi-ordinates of the load water-plane of a vessel are 0, 3·35, 6·41, 8·63, 9·93, 10·44, 10·37, 9·94, 8·96, 7·16, and 2·5 feet respectively. These ordinates being 21 feet apart, find—

(1) The tons per inch immersion.
(2) The distance between the centre of buoyancy and the transverse metacentre, the load displacement being 484 tons.

Ans. (1) 7·73 tons; (2) 5·2 feet nearly.

11. The semi-ordinates, 16·6 feet apart, of a vessel's water-plane are 0·2, 2·3, 6·4, 9·9, 12·3, 13·5, 13·8, 13·7, 12·8, 10·6, 6·4, 1·9, 0·2 feet respectively, and the displacement up to this water-plane is 220 tons. Find the length of the transverse BM.

Ans. 20·6 feet.

12. A vessel of 613 tons displacement was inclined by moving 30 cwt. of rivets across the deck through a distance of 22′ 6″. The end of a plumb-line 10 feet long moved through $2\frac{1}{4}$ inches. What was the metacentric height at the time of the experiment?

Ans. 2·93 feet.

13. The semi-ordinates of a ship's water-plane 35 feet apart are, commencing from forward, 0·4, 7·12, 15·28, 21·88, 25·62, 26·9, 26·32, 24·42, 20·8, 15·15, 6·39 feet respectively. There is an after appendage of 116 square feet, with its centre of gravity 180 feet abaft the midship ordinate. Find—

(1) The area of the water-plane.
(2) The tons per inch immersion.
(3) The distance of the centre of flotation abaft amidships.
(4) The position of the transverse metacentre above the L.W.L., taking the displacement up to the above line as 5372 tons, and the centre of buoyancy of this displacement 8·61 feet below the L.W.L.

Ans. (1) 13,292 square feet; (2) 31·6 tons; (3) 14·65 feet; (4) 3·34 feet.

14. A ship displacing 9972 tons is inclined by moving 40 tons 54 feet

across the deck, and a mean deviation of $9\frac{1}{4}$ inches is obtained by pendulums 15 feet long. Find the metacentric height at the time of the operation.

Ans. 4·18 feet.

15. A ship weighing 10,333 tons was inclined by shifting 40 tons 52 feet across the deck. The tangent of the angle of inclination caused was found to be 0·05. If the transverse metacentre was 4·75 feet above the designed L.W.L., what was the position of the centre of gravity of the ship at the time of the experiment?

Ans. 0·73 foot above the L.W.L.

16. A vessel of 26 feet draught has the moment of inertia of the L.W.P. about a longitudinal axis through its centre of gravity 6,500,000 in foot-units. The area of the L.W.P. is 20,000 square feet, the volume of displacement 400,000 cubic feet, and the centre of gravity of the ship may be taken in the L.W.P. Approximate to the metacentric height.

Ans. $5\frac{1}{4}$ feet.

17. Prove the rule given on p. 62 for the distance of the centre of gravity of a semicircle of radius a from the diameter, viz. $\frac{4}{3\pi}a$, by finding the transverse BM of a pontoon of circular section floating with its axis in the surface of the water.

(M in this case is in the centre of section.)

18. Take a body shaped as in Kirk's analysis, p. 84, of length 140 feet; length of parallel middle body, 100 feet; extreme breadth, 30 feet; draught, 12 feet. Find the transverse BM.

Ans. 5·7 feet.

19. A vessel of 1792 tons displacement is inclined by shifting 5 tons already on board transversely across the deck through 20 feet. The end of a plumb-line 15 feet long moves through $5\frac{1}{4}$ inches. Determine the metacentric height at the time of the experiment.

Ans. 1·91 feet.

20. A vessel of displacement 1722 tons is inclined by shifting 6 tons of ballast across the deck through $22\frac{1}{2}$ feet. A mean deviation of $10\frac{1}{2}$ inches is obtained with pendulums 15 feet long. The transverse metacentre is 15·28 feet above the keel. Find the position of the centre of gravity of the ship with reference to the keel.

Ans. 13·95 feet.

21. The ship in the previous question has 169 tons to go on board at 10 feet above keel, and 32 tons to come out at 20 feet above keel. Find the metacentric height when completed, the transverse metacentre at the displacement of 1859 tons being 15·3 feet above keel.

Ans. 1·8 feet.

22. A vessel of 7000 tons displacement has a weight of 30 tons moved transversely across the deck through a distance of 50 feet, and a plumb-bob hung down a hatchway shows a deviation of 12 inches in 15 feet. What was the metacentric height at the time of the operation?

Ans. 3·21 feet.

23. A box is 200 feet long, 30 feet broad, and weighs 2000 tons. Find the height of the transverse metacentre above the bottom when the box is floating in salt water on an even keel. *Ans.* 12·26 feet.

24. Show that for a rectangular box floating at a uniform draught of d feet, the breadth being 12 feet, the distance of the transverse metacentre above the bottom is given by $\frac{d^2 + 24}{2d}$ feet, and thus the transverse metacentre is in the water-line when the draught is 4·9 feet.

25. A floating body has a constant triangular section. If the breadth at the water-line is $\sqrt{2}$ times the draught, show that the curve of metacentres in the metacentric diagram lies along the line drawn from zero draught at 45° to the horizontal, and therefore the metacentre is in the water-line for all draughts.

26. A floating body has a square section with one side horizontal. Show that the transverse metacentre lies above the centre of the square so long as the draught does not much exceed 21 per cent. of the depth of the square. Also show that as the draught gets beyond 21 per cent. of the depth, the metacentre falls below the centre and remains below until the draught reaches 79 per cent. of the depth; it then rises again above the centre of the square, and continues to rise as long as any part of the square is out of the water.

(This may be done by constructing a metacentric diagram, or by using the methods of algebra, in which case a quadratic equation has to be solved.)

27. Show that a square log of timber of 12 inches side, 10 feet long, and weighing 320 lbs., must be loaded so that its centre of gravity is more than 1 inch below the centre in order that it may float with a side horizontal in water of which 35 cubic feet weigh 1 ton.

28. A prismatic vessel is 70 feet long. The section is formed at the lower part by an isosceles triangle, vertex downwards, the base being 20 feet, and the height 5 feet; above this is a rectangle 20 feet wide and 5 feet high. Construct to scale the metacentric diagram for all drafts.

29. A vessel's load water-plane is 380 feet long, and 75 feet broad, and its moment of inertia in foot-units about the centre line works out to 8,000,000 about. State whether you consider this a reasonable result to obtain, the water-plane not being very fine.

30. Find the value of the coefficient a in the formula $\mathrm{BM} = a\frac{\mathrm{B}^2}{\mathrm{D}}$ referred to on p. 111, for floating bodies having the following sections throughout their length :—

(*a*) Rectangular cross-section.
(*b*) Triangular cross-section, vertex down.
(*c*) Vertical-sided for one half the draught, the lower half of the section being in the form of a triangle.

Ans. (*a*) 0·08; (*b*) c·16; (*c*) 0·11.

For ordinary ships the value of a will lie between the first and last of these.

31. A lighter in the form of a box is 100 feet long, 20 feet broad, and floats at a constant draught of 4 feet. The metacentric height when empty is 6 feet. Two bulkheads are built 10 feet from either end. Show that a small quantity of water introduced into the central compartment will render the lighter unstable in the upright condition.

32. At one time, in ships which were found to possess insufficient stability, girdling was secured to the ship in the neighbourhood of the water-line. Indicate how far the stability would be influenced by this means.

33. A floating body has a constant triangular section. If the breadth at the water-line is equal to the draught, show that the locus of metacentres in the metacentric diagram makes an angle with the horizontal of about 40°.

34. A cylinder is placed into water with its axis vertical. Show that if the centre of gravity is in the water-plane, the cylinder will float upright if the radius ÷ the draught is greater than $\sqrt{2}$.

35. In a wholly submerged body show that for stable equilibrium the centre of gravity must lie below the centre of buoyancy.

36. A floating body has a constant triangular section, vertex downwards, and has a constant draught of 12 feet, the breadth at the water-line

being 24 feet. The keel just touches a quantity of mud, specific gravity 2. The water-level now falls 6 feet: find the amount by which the metacentric height is diminished due to this.[1] *Ans.* $2\frac{3}{4}$ feet about.

37. A floating body of circular section 6 feet in diameter has a metacentric height of 1·27 feet. Show that the centre of buoyancy and centre of gravity coincide, when the body is floating with the axis in the surface.

38. It is desired to increase the metacentric height of a vessel which is being taken in hand for a complete overhaul. Discuss the three following methods of doing this, assuming the ship has a metacentric diagram as in Fig. 56, the extreme load draught being 15 feet :—

(1) Placing ballast in the bottom.
(2) Removing top weight.
(3) Placing a girdling round the ship in the neighbourhood of the water-line.

39. Show that the angle θ in Fig. 56 is between 29° and 30° for a vessel whose coefficient of L.W.P. is 0·75, and whose block coefficient of displacement is 0·55. In any case, if these coefficients are denoted by n and k respectively, show that $\tan\theta = \frac{1}{2} + \frac{n}{6k}$ approximately (use Morrish's formula, p. 65).

40. From the following information construct the metacentric diagram, using a scale of $\frac{1}{8}$ inch = 1 foot, and state the metacentric height and draught in the three conditions given.

Draught.	Displacement in tons.	Tons per inch.	C.B. below 19-foot WL.	BM.
21′ 9″	5256	27·1	6·25′	8·85′
19′ 0″	4383	26·48	7·8′	10·4′
16′ 3″	3527	25·37	9·35′	12·2′
13′ 6″	2714	23·84	10·9′	14·5′

(1) Deep load 1000 tons coal 5030 tons, C.G. 0·3 feet below 19′ WL.
(2) Normal load 400 " 4430 " 0·35 " " "
(3) Light condition 3915 " 0·3 " above "

Ans. (1) 2·9′, 21′ $0\frac{1}{2}$″; (2) 2·95′, 19′ $1\frac{3}{4}$″; (3) 2·4′, 17′ 6″.

41. A weight of 10 tons is shifted 40 ft. transversely across the deck of a vessel having a compartment partially filled with salt water, the free surface of this water being 25 ft. long by 50 ft. uniform breadth. Calculate the heel in degrees, having given displacement of vessel 8000 tons, C.G. of ship and contained water 15 ft. above keel. Transverse M. $16\frac{1}{2}$ ft. above keel. (Durham B.Sc. 1910).

The *actual* GM is $1\frac{1}{2}$ feet, but the *virtual* GM is less than this by the amount $\frac{i}{V}$, where i is the moment of inertia of free water surface about a longitudinal axis through its C.G.

$$i = \tfrac{1}{12} . 25 . 50 . 50^2 \qquad V = 8000 \times 35$$

so that $\frac{i}{V} = 0·93$ ft. The *virtual* GM is therefore 0·57 ft., θ being angle of heel.

$$W \times GM \times \sin\theta = w \times d$$

$$\text{or } \sin\theta = \frac{10 \times 40}{8000 \times 0·57} = 0·088$$

$$\text{and } \theta = 5 \text{ degrees.}$$

[1] This example is worked out at the end of the Appendix.

CHAPTER IV.

LONGITUDINAL METACENTRE, LONGITUDINAL BM, CHANGE OF TRIM.

Longitudinal Metacentre.—We now have to deal with inclinations in a fore-and-aft or longitudinal direction. We do not have the same difficulty in fixing on the fore-and-aft position of the centre of gravity of a ship as we have in fixing its vertical position, because we know that if a ship is floating steadily at a given water-line, the centre of gravity must be in the same vertical line as the centre of buoyancy, by the conditions of equilibrium laid down on p. 93. It is simply a matter of calculation to find the longitudinal position of the centre of buoyancy of a ship when floating at a certain water-line, if we have the form of the ship given, and thus the fore-and-aft position of the centre of gravity is determined.

We have already dealt with the inclination of a ship in a transverse direction, caused by shifting weights already on board across the deck; and in a precisely similar manner we can incline a ship in a longitudinal or fore-and-aft direction by shifting weights along the deck in the line of the keel. The *trim* of a ship is the difference between the draughts of water forward and aft. Thus a ship designed to float at a draught forward of 12 feet, and a draft aft of 15 feet, is said to trim 3 feet by the stern.

We have, on p. 97, considered the definition of the transverse metacentre, and the definition of the longitudinal metacentre is precisely analogous.

For a given water-line WL of a vessel, let B be the centre of buoyancy (see Fig. 61), and BM the vertical through it.

Suppose the trim of the vessel to change slightly,[1] the vessel retaining the same volume of displacement, B′ being the new centre of buoyancy, and B′M the vertical through it, meeting

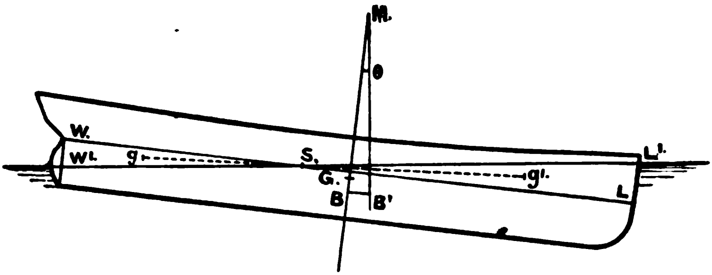


FIG. 61.

BM in M. Then the point M is termed the *longitudinal metacentre.*

The distance between G, the centre of gravity of the ship, and M, the longitudinal metacentre, is termed the *longitudinal metacentric height.*

Formula for finding the Distance of the Longitudinal Metacentre above the Centre of Buoyancy.— Let Fig. 62 represent the profile of a ship floating at the water-line W′L′, the original water-line being WL. The original trim was AW − BL; the new trim is AW′ − BL′. The change of trim is—

$$(AW - BL) - (AW' - BL') = WW' + LL'$$

i.e. *the change of trim is the sum of the changes of draughts forward and aft.* This change, we may suppose, has been caused by the shifting of weights from aft to forward. The inclination being regarded as small, and the displacement remaining constant, the line of intersection of the water-planes WL, W′L′ must pass through the centre of gravity of the water-plane WL, or, as we have termed it, the centre of flotation, in accordance with the principle laid down on p. 98. This centre of flotation will usually be abaft the middle of length, and this introduces a complication which makes the calculation for the longitudinal metacentre more difficult than the corre-

[1] Much exaggerated in the figure.

sponding calculation for the transverse metacentre. In this latter case, it will be remembered that the centre of flotation is in the middle line of the water-plane.

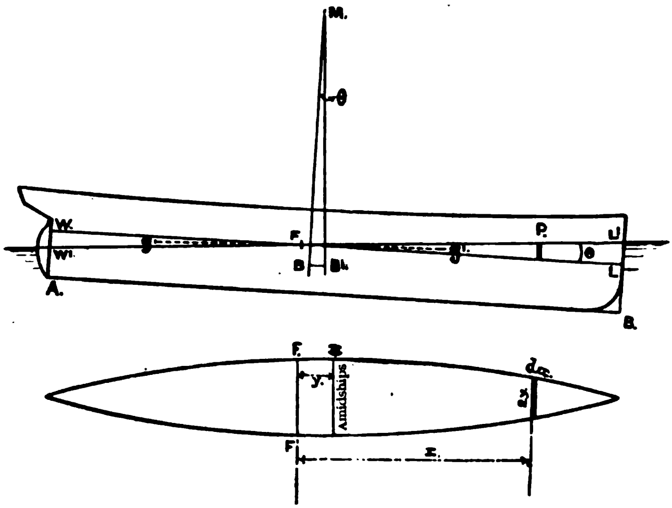


FIG. 62.

In Fig. 62—

Let B be the centre of buoyancy when floating at the water-line WL;

B′, the centre of buoyancy when floating at the water-line W′L′;

FF, the intersection of the water-planes WL, W′L′;

v, the volume of either the immersed wedge FLL′ or the emerged wedge FWW′;

g, g', the centres of gravity of the wedges WFW′, LFL respectively;

V, the volume of displacement in cubic feet;

θ, the angle between the water-lines WL, W′L′, which is the same as the angle between BM and B′M (this angle is supposed very small).

We have, using the principle laid down on p. 100—

$$v \times gg' = V \times BB'$$

$$\text{or BB}' = \frac{v \times gg'}{V}$$

$$\text{But BB}' = \text{BM} \times \theta \text{ (}\theta\text{ is in circular measure)}$$

$$\therefore \text{BM} \times \theta = \frac{v \times gg'}{V}$$

The part of this expression that we do not know is $v \times gg'$, or the moment of transference of the wedges. At P take a small transverse slice of the wedge FLL′, of breadth in a fore-and-aft direction, dx; length across, $2y$; and distance from F, x. Then the depth of the slice is—

$$x \times \theta$$

$$\text{and the volume is } 2y \times x\theta \times dx$$

This is an elementary volume, analogous to the elementary area $y \, . \, dx$ used in finding a large area. The moment of this elementary volume about the transverse line FF is—

$$2yx \, . \, \theta \, . \, dx \times x$$
$$\text{or } 2yx^2 \, . \, \theta \, . \, dx$$

If we summed all such moments as this for the length FL, we should get the moment $v \times \text{F}g'$, and for the length FW, $v \times \text{F}g$, or for the whole length, $v \times gg'$; therefore, using our ordinary notation—

$$v \times gg' = \int 2yx^2 \, . \, \theta \, . \, dx$$
$$= 2\theta \int yx^2 \, . \, dx \text{ (}\theta\text{ being constant)}$$

We therefore have—

$$\text{BM} \times \theta = \frac{2\theta \int yx^2 \, . \, dx}{V}$$

$$\text{or BM} = \frac{2\int yx^2 \, . \, dx}{V}$$

Referring to p. 103, it will be seen that we defined the moment of inertia of an area about a given axis as—

$$\int dA \times y^2$$

where dA is a small elementary area;

y its distance from the given axis.

Consider, now, the expression obtained, $2\int yx^2 \, . \, dx$. The elementary area is $2y \, . \, dx$, and x is its distance from a transverse axis passing through the centre of flotation. We may therefore say—

$$BM = \frac{I_0}{V}$$

where I_0 is the moment of inertia of the water-plane about a transverse axis passing through the centre of flotation. It will be seen at once that this is the same form of expression as for the transverse BM.

The method usually adopted for finding the moment of inertia of a water-plane about a transverse axis through the centre of flotation is as follows[1]:—

We first find the moment of inertia about the ordinary midship ordinate. If we call this I, and y the distance of the centre of flotation from the midship ordinate, we have, using the principle given on p. 104—

$$I = I_0 + Ay^2$$
$$\text{or } I_0 = I - Ay^2$$

The method actually adopted in practice will be best understood by working the following example.

Numbers of ordinates.	Semi-ordinates of L.W.P.	Simpson's multipliers.	Products for area.	Multipliers for moment.	Products for moment.	Multipliers for moment of inertia.	Products for moment of inertia.
1	0·0	½	0·0	5	0·0	5	0·0
1½	1·37	2	2·74	4½	12·33	4½	55·49
2	2·67	1½	4·01	4	16·04	4	64·16
3	4·87	4	19·48	3	58·44	3	175·32
4	6·31	2	12·62	2	25·24	2	50·48
5	6·85	4	27·40	1	27·40	1	27·40
					139·45		
6	7·21	2	14·42	0		0	—
7	7·15	4	28·60	1	28·60	1	28·60
8	6·87	2	13·74	2	27·48	2	54·96
9	6·33	4	25·32	3	75·96	3	227·88
10	5·08	1½	7·62	4	30·48	4	121·92
10½	3·56	2	7·12	4½	32·04	4½	144·18
11	0·71	½	0·35	5	1·75	5	8·75
			163·42 $= S_1$		196·31		959·14 $= S_3$
					139·45		
					56·86 $= S_2$		

[1] This calculation for the L.W.P. is usually performed on the displacement sheet.

In column 2 of the table are given the lengths of semi-ordinates of a load water-plane corresponding to the numbers of the ordinates in column 1. The ordinates are 7·1 feet apart. It is required to find the longitudinal BM, the displacement being 91·6 tons in salt water.

The distance apart of the ordinates being 7·1 feet, we have—

$$\text{Area} = 163{\cdot}42 \times (\tfrac{1}{3} \times 7{\cdot}1) \times 2$$
$$= 773{\cdot}5 \text{ square feet}$$

$$\left.\begin{array}{r}\text{Distance of centre of gravity of}\\ \text{water-plane abaft No. 6 ordinate}\end{array}\right\} = \frac{56{\cdot}86 \times 7{\cdot}1}{163{\cdot}42} = 2{\cdot}47 \text{ feet}$$

(the stations are numbered from forward).

The calculation up to now has been the ordinary one for finding the area and position of the centre of gravity. Column 4 is the calculation indicated by the formula—

$$\text{Area} = 2\int y \, . \, dx$$

Column 6 is the calculation indicated by the formula—

$$\text{Moment} = 2\int yx \, . \, dx$$

It will be remembered that in column 5 we do not put down the actual distances of the ordinates from No. 6 ordinate, but the number of intervals away; the distance apart of the ordinates being introduced at the end. By this means the result is obtained with much less labour than if column 5 contained the actual distances. The formula we have for the moment of inertia is $2\int y \, . \, x^2 \, . \, dx$. We follow a similar process to that indicated above; we do not multiply the ordinates by the square of the actual distances, but by the square of the number of intervals away, leaving to the end the multiplication by the square of the interval. Thus for ordinate No. 2 the actual distance from No. 6 is $4 \times 7{\cdot}1 = 28{\cdot}4$ feet. The square of this is $(4)^2 \times (7{\cdot}1)^2$. For ordinate No. 4 the square of the distance is $(2)^2 \times (7{\cdot}1)^2$. The multiplication by $(7{\cdot}1)^2$ can be done at the end. In column 7 is placed the number of intervals from No. 6, as in column 5; and if the products in column 6 are multiplied successively by the numbers in column 7, we shall obtain in column 8 the ordinates put through Simpson's rule, and also multiplied by the square of the number of intervals from No. 6 ordinate. The whole of column 8 is added up, giving a result 959·14. To obtain the moment of inertia about No. 6 ordinate, this has to be multiplied as follows :—

(*a*) By one-third the common interval to complete Simpson's rule, or $\frac{1}{3} \times 7\cdot1$.

(*b*) By the square of the common interval, for the reasons fully explained above.

(*c*) By two for both sides.

We therefore have the moment of inertia of the water-plane about No. 6 ordinate—

$$959\cdot14 \times (\tfrac{1}{3} \times 7\cdot1) \times (7\cdot1)^2 \times 2 = 228,858$$

The moment of inertia about a transverse axis through the centre of flotation will be less than this by considering the formula $I = I_0 + Ay^2$, where I is the value found above about No. 6 ordinate, and I_0 is the moment of inertia we want. We found above that the area $A = 773\cdot5$ square feet, and $y = 2\cdot47$ feet;

$$\therefore I_0 = 228,858 - (773\cdot5 \times 2\cdot47^2)$$
$$= 224,139$$ [1]

The displacement up to this water-plane is 91·6 tons, and the volume of displacement is—

$$91\cdot6 \times 35 = 3206 \text{ cubic feet}$$

$$\text{The longitudinal BM} = \frac{I_0}{V}$$

$$= \frac{224139}{3206} = 69\cdot9 \text{ feet}$$

Approximate Formula for the Height of the Longitudinal Metacentre above the Centre of Buoyancy.—The following formula is due to M. J. A. Normand, M.I.N.A.,[2] and is found to give exceedingly good results in practice :—

Let L be the length on the load water-line in feet;

B, the breadth amidships in feet;

[1] See note at end of chapter, p. 167.

[2] See "Transactions of the Institution of Naval Architects," 1882.

V, the volume of displacement in cubic feet;

A, the area of the load water-plane in square feet.

Then the height of the longitudinal metacentre above the centre of buoyancy—

$$H = 0{\cdot}0735\frac{A^2 \times L}{B \times V}$$

In the example worked above, the breadth amidships was 14·42 feet; and using the formula, we find—

$$H = 67{\cdot}5 \text{ feet nearly}$$

This compares favourably with the actual result of 69·9 feet. The quantities required for the use of the formula would all be known at a very early stage of a design and a close approximation to the height H can thus very readily be obtained. A formula such as this is useful as a check on the result of the calculation for the longitudinal BM.

We may also obtain an approximate formula in the same manner as was done for the transverse BM on p. 111. Using a similar system of notation, we may say—

$$\left.\begin{array}{l}\text{Moment of inertia of L.W.P. about a trans-}\\ \text{verse axis through the centre of flotation}\end{array}\right\} = n' \times L^3 \times B$$

n' being a coefficient of a similar nature to n used on p. 107.

$$\text{Volume of displacement} = k \times L \times B \times D$$

$$\therefore H = \frac{n' \times L^3 \times B}{k \times L \times B \times D}$$

$$= b\frac{L^2}{D}$$

where b is a coefficient obtained from the coefficients n' and k. Sir William White, in the "Manual of Naval Architecture," says, with reference to the value of b, that "the value 0·075 may be used as a rough approximation in most cases; but there are many exceptions to its use." If this approximation be applied to the example we have worked, the mean moulded draught being 5·8 feet—

The value of H = 65 feet

This formula shows very clearly that the length of a ship is more effective than the draught in determining the value of the longitudinal BM in any given case. For vessels which have an unusual proportion of length to draught, the values of the longitudinal BM found by using this formula will not be trustworthy.

To estimate the Displacement of a Vessel when floating out of the Designed Trim.—The following method is found useful when it is not desired to actually calculate the displacement from the drawings, and a close approximation is sufficiently accurate. Take a ship floating parallel to her designed L.W.L.; we can at once determine the displacement when floating at such a water-line from the curve of displacement (see p. 25). If now a weight already on board is shifted aft, say, the ship will change trim, and she will trim more by the stern than designed. The new water-plane must pass through the centre of gravity of the original water-plane, or, as we have termed it, the centre of flotation, and

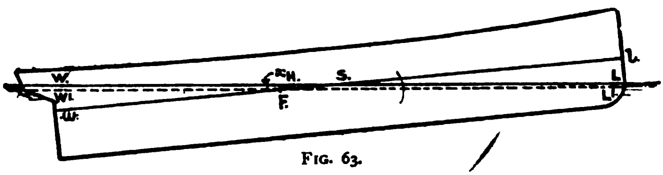


FIG. 63.

the displacement at this new water-line will be the same as at the original water-line. Now, when taking the draught of water a vessel is actually floating at, we take the figures set up at or near the forward and after perpendiculars. These draughts, if not set up at the perpendiculars, can be transferred to the perpendiculars by a simple calculation. The draughts thus obtained are added together and divided by two, giving us the mean draught. Now run a line parallel to the designed water-line at this mean draught, as in Fig. 63, where WL represents the actual water-line, and *wl* the line just drawn. It will not be true that the displacement of the ship is the same as that given by the water-line *wl*. Let F be the centre of flotation of the water-line *wl*, and draw W′L′ through F parallel to WL. Then the actual displacement will be that up to W′L′, which is nearly the same as that up to *wl*, with the displacement of the layer WW′L′L added. The displacement up to *wl* is found at once from the curve of displacement. Let T be the tons per inch at *wl*, and therefore very nearly the tons per inch at W′L′ and WL. SF, the distance the centre of flotation of the water-plane *wl* is abaft the middle of length, is supposed known, and equals *d* inches, say. Now, the angle between *wl* and WL is given by—

$$\tan\theta = \frac{wW + lL}{\text{length of ship}}$$
$$= \frac{\text{amount out of normal trim}}{\text{length of ship}}$$

But if x is the thickness of layer in inches between W′L′ and WL, we also have in the triangle SFH—

$$\tan\theta = \frac{x}{d} \text{ very nearly (for small angles } \tan\theta = \sin\theta \text{ very nearly)}$$

and accordingly x may be determined. This, multiplied by the tons per inch T, will give the displacement of the layer.[1]

The following example will illustrate the above :—

Example.—A vessel floats at a draught of 16′ 5½″ forward, 23′ 1½″ aft, the normal trim being 2 feet by the stern. At a draught of 19′ 9½″, her displacement, measured from the curve of displacement, is 5380 tons, the tons per inch is 31·1 tons, and the centre of flotation is 12·9 feet abaft amidships. Estimate the ship's displacement.

The difference in draught is 23′ 1½″ − 16′ 5½″ = 6′ 8″, or 4′ 8″ out of trim. The distance between the draught-marks is 335 feet, and we therefore have for the thickness of the layer—

$$12 \times 12{\cdot}9 \times \frac{56}{335 \times 12} = 2{\cdot}15 \text{ inches}$$

The displacement of the layer is therefore—

$$2{\cdot}15 \times 31{\cdot}1 = 67 \text{ tons}$$

The displacement is therefore—

$$5380 + 67 = 5447 \text{ tons nearly}$$

Change of Trim due to Longitudinal Shift of Weights already on Board.—We have seen that change

[1] This may be reduced to a formula, set as an example in Appendix A. No. 2, viz. extra displacement for 1 foot extra trim $= 12\,\frac{T \times y}{L}$, y being centre of flotation abaft amidships in feet.

of trim is the sum of the change of draughts forward and aft, and that change of trim can be caused by the shift of weights on board in a fore-and-aft direction. We have here an analogous case to the inclining experiment in which heeling is caused by shifting weights in a transverse direction. In Fig. 64, let w be

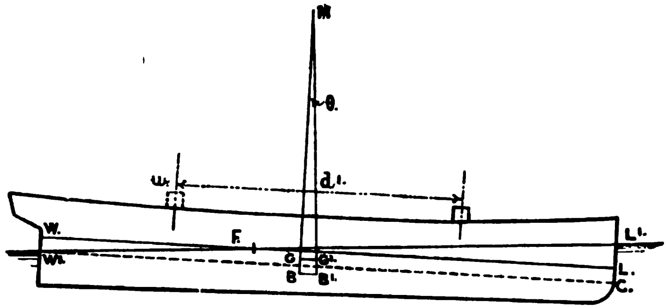


FIG. 64.

a weight on the deck when the vessel is floating at the water-line WL, G being the position of the centre of gravity. Now suppose the weight w to be shifted forward a distance of d feet. G will, in consequence of this, move forward parallel to the line joining the original and final positions of w, and if W be the displacement of the ship in tons, G will move to G′ such that—

$$GG' = \frac{w \times d}{W}$$

Now, under these circumstances, the condition of equilibrium is not fulfilled if the water-line remains the same, viz. that the centre of gravity and the centre of buoyancy must be in the same vertical line, because G has shifted to G′. The ship must therefore adjust herself till the centre of gravity and the centre of buoyancy are in the same vertical line, when she will float at a new water-line, W′L′, the new centre of buoyancy being B′. The original vertical through G and B meets the new vertical through G′ and B′ in the point M, and this point will be the longitudinal metacentre, supposing the change of trim to be small, and GM will be the longitudinal metacentric height. Draw W′C parallel to the original water-line WL.

meeting the forward perpendicular in C. Then, since CL = W'W, the change of trim WW' + LL' = CL' = x, say. The angle of inclination of W'L' to WL is the same as the angle between W'L' and W'C = θ, say, and—

$$\tan \theta = \frac{CL'}{\text{length}} = \frac{x}{L}$$

But we also have—

$$\tan \theta = \frac{GG'}{GM}$$

therefore, equating these two values for tan θ, we have—

$$\frac{x}{L} = \frac{GG'}{GM}$$

$$= \frac{w \times d}{W \times GM}$$

using the value obtained above for GG'; or—

$$\left.\begin{array}{l} x, \text{ the change of trim due to the} \\ \text{moment of transference of the} \\ \text{weight } w \text{ through the distance } d, \end{array}\right\} = \frac{w \times d}{W \times GM} \times L \text{ feet}$$

or—

$$\text{The change of trim in inches} = \frac{12 \times w \times d \times L}{W \times GM}$$

and the *moment to change trim* 1 *inch is*—

$$w \times d = \frac{W \times GM}{12 \times L} \text{ foot-tons}$$

To determine this expression, we must know the vertical position of the centre of gravity and the position of the longitudinal metacentre. The vertical position of the centre of gravity will be estimated in a design when dealing with the metacentric height necessary, and the distance between the centre of buoyancy and the centre of gravity is then subtracted from the value of the longitudinal BM found by one of the methods already explained. The distance BG is, however, small compared with either of the distances BM or GM and any small error in estimating the position of the centre of gravity cannot appreciably affect the value of the moment to change trim one inch. In many ships BM approximately equals the length of the ship, and therefore GM also; we may therefore say that in such ships the moment to change trim 1 inch = $\frac{1}{12}$ the displacement in tons. For ships that are long in proportion to the draught, the moment to change trim 1 inch is greater than would be given by this approximate rule.

In the ship for which the value of the longitudinal BM was calculated on p. 148, the centre of buoyancy was $2\frac{1}{4}$ feet below the L.W.L., the centre of gravity was estimated at $1\frac{1}{4}$ feet below the L.W.L.; and the length between perpendiculars was 75 feet.

$$\therefore \text{GM} = 69 \cdot 9 - 1$$
$$= 68 \cdot 9 \text{ feet}$$

and the moment to change trim 1 inch $= \dfrac{91 \cdot 6 \times 68 \cdot 9}{12 \times 75}$

$$= 7 \cdot 01 \text{ foot-tons}$$

the draughts being taken at the perpendiculars.

Example.—A vessel 300 feet long and 2200 tons displacement has a longitudinal metacentric height of 490 feet. Find the change of trim caused by moving a weight of 5 tons already on board through a distance of 200 feet from forward to aft.

Here the moment to change trim 1 inch is—

$$\frac{2200 \times 490}{12 \times 300} = 300 \text{ foot-tons nearly}$$

The moment aft due to the shift of the weight is—

$$5 \times 200 = 1000 \text{ foot-tons}$$

and consequently the change of trim *aft* is—

$$\frac{1000}{300} = 3\frac{1}{3} \text{ inches}$$

Approximate Formula for the Moment to change Trim 1 *inch.*—Assuming Normand's approximate formula for the height of the longitudinal metacentre above the centre of buoyancy given on p. 151—

$$\text{H} = 0 \cdot 0735 \frac{\text{A}^2 \times \text{L}}{\text{B} \times \text{V}}$$

we may construct an approximate formula for the moment to change trim 1 inch as follows.

We have seen that the moment to change trim 1 inch is—

$$\frac{\text{W} \times \text{GM}}{12 \times \text{L}}$$

We can write $W = \frac{V}{35}$ and assume that, for our purpose,

$$BM = GM = 0 \cdot 0735 \frac{A^2 \times L}{B \times V}$$

Substituting this in the above formula, we have—

$$\left.\begin{array}{l}\text{Moment to change} \\ \text{trim 1 inch}\end{array}\right\} = \frac{V}{35 \times 12 \times L} \times \left(0 \cdot 0735 \frac{A^2 \times L}{B \times V}\right)$$

$$\text{or } 0 \cdot 000175 \frac{A^2}{B}$$

For further approximations, see Example 18, p. 173.

Applying this to the case worked out in detail on p. 148—

Area of L.W.P. = A = 773·5 square feet
Breadth = B = 14·42 feet

so that the moment to change trim 1 inch approximately should equal—

$$0 \cdot 000175 \frac{(773 \cdot 5)^2}{14 \cdot 42} = 7 \cdot 26 \text{ foot-tons}$$

the exact value, as calculated on p. 156, being 7·01 foot-tons.

It is generally sufficiently accurate to assume that one-half the change of trim is forward, and the other half is aft. In the example on p. 156, if the ship floated at a draught of 12′ 3″ forward and 14′ 9″ aft, the new draught forward would be—

$$12' \; 3'' - 1\tfrac{2}{3}'' = 12' \; 1\tfrac{1}{3}''$$

and the new draught aft would be—

$$14' \; 9'' + 1\tfrac{2}{3}'' = 14' \; 10\tfrac{2}{3}''$$

Referring, however, to Fig. 64, it will be seen that when, as is usually the case, the centre of flotation is not at the middle of the length, WW′ is not equal to LL′, so that, strictly speaking, the total change of trim should not be divided by 2, and one-half taken forward and the other half aft. Consider the triangles FWW′, FLL′; these triangles are similar to one another, and the corresponding sides are proportional, so that—

$$\frac{\mathrm{WW'}}{\mathrm{WF}} = \frac{\mathrm{LL'}}{\mathrm{LF}}$$

and both these triangles are similar to the triangle W′CL′. Consequently—

$$\frac{\mathrm{WW'}}{\mathrm{WF}} = \frac{\mathrm{LL'}}{\mathrm{LF}} = \frac{\mathrm{CL'}}{\mathrm{W'C}} = \frac{\text{change of trim}}{\text{length}}$$

$$\therefore \mathrm{WW'} = \frac{\mathrm{WF}}{\text{length}} \times \text{change of trim}$$

$$\text{and } \mathrm{LL'} = \frac{\mathrm{LF}}{\text{length}} \times \text{change of trim}$$

that is to say, the proportion of the change of trim either aft or forward, is the proportion the length of the vessel abaft or forward of the centre of flotation bears to the length of the vessel. Where the change of trim is small, this makes no appreciable difference in the result, but there is a difference when large changes of trim are under consideration.

For example, in the case worked out on p. 156, suppose a weight of 50 tons is moved through 100 feet from forward to aft; the change of trim caused would be—

$$\tfrac{5000}{300} = 16\tfrac{2}{3} \text{ inches}$$

The centre of flotation was 12 feet abaft the middle of length. The portion of the length abaft the centre of flotation is therefore $\frac{138}{300}$ of the length. The increase of draught aft is therefore—

$$\tfrac{138}{300} \times \tfrac{50}{3} = 7\tfrac{2}{3} \text{ inches}$$

and the decrease of draught forward is—

$$\tfrac{162}{300} \times \tfrac{50}{3} = 9 \text{ inches}$$

instead of $8\frac{1}{3}$ inches both forward and aft. The draught forward is therefore—

$$12'\ 3'' - 9'' = 11'\ 6''$$

and the draught aft—

$$14'\ 9'' + 7\tfrac{2}{3}'' = 15'\ 4\tfrac{2}{3}''$$

It will be noticed that the mean draught is not the same as before the shifting, but two-thirds of an inch less, while the displacement remains the same. This is due to the fact that, as the ship increases her draught aft and decreases it forward, a fuller portion of the ship goes into the water and a finer portion comes out.

Effect on the Trim of a Ship due to adding a Weight of Moderate Amount.—If we wish to place a weight on board a ship so that the vessel will not change trim, we must place it so that the upward force of the added buoyancy will act in the same line as the downward force of the added weight. Take a ship floating at a certain water-line, and imagine her to sink down a small amount, so that the new waterplane is parallel to the original water-plane. The added buoyancy is formed of a layer of parallel thickness, and having very nearly the shape of the original water-plane. The upward force of this added buoyancy will act through the centre of gravity of the layer, which will be very nearly vertically over the centre of gravity of the original water-plane, or, as we have termed it, the centre of flotation. We therefore see that to place a weight of moderate amount on a ship so that no change of trim takes place, we must place it vertically over or under the centre of flotation. The ship will then sink to a new water-line parallel to the original water-line, and the distance she will sink is known at once, if we know the tons per inch at the original water-line. Thus a ship is floating at a draught of 13 feet forward and 15 feet aft, and the tons per inch immersion is 20 tons. If a weight of 55 tons be placed over or under the centre of flotation, she will sink $\frac{55}{20}$ inches, or $2\frac{3}{4}$ inches, and the new draught will be 13′ $2\frac{3}{4}$″ forward and 15′ $2\frac{3}{4}$″ aft.

It will be noticed that we have made two assumptions, both of which are rendered admissible by considering that the weight is of moderate amount. First, that the tons per inch does not change appreciably as the draught increases, and this is, for all practical purposes, the case in ordinary ships. Second, that the centre of gravity of the parallel layer of added buoyancy is in the same section as the centre of flotation. This latter assumption may be taken as true for small changes in draught caused by the addition of weights of moderate amount; but for large changes it will not be reasonable, because the centres of gravity of the water-planes are not all in the same section, but vary for each water-plane. As a rule, water-planes are fuller aft than forward near the L.W.P., and this more so as the draught increases; and so, if we draw on the profile of the sheer drawing a curve through the centres of gravity of water-planes parallel to the L.W.P., we should obtain a curve which slopes somewhat aft as the draught increases. We shall discuss further the methods which have to be adopted when the weights added are too large for the above assumptions to be accepted.

We see, therefore, that if we place a weight of moderate amount on board a ship at any other place than over the centre of flotation, she will not sink in the water to a water-line parallel to the original water-line, but she will change trim as well as sink bodily in the water. The change of trim will be forward or aft according as the weight is placed forward or aft of the centre of flotation.

In determining the new draught of water, we proceed in two steps:—

1. Imagine the weight placed over the centre of flotation, and determine the consequent sinkage.

2. Then imagine the weight shifted either forward or aft to the assigned position. This shift will produce a certain moment forward or aft, as the case may be, equal to the weight multiplied by its longitudinal distance from the centre of flotation. This moment divided by the moment to change trim 1 inch as calculated for the original water-plane will give the change of trim.

The steps will be best illustrated by the following example:—

A vessel is floating at a draught of 12′ 3″ forward and 14′ 6″ aft. The tons per inch immersion is 20; length, 300 feet; centre of flotation, 12 feet abaft the middle of length; moment to change trim 1 inch, 300 foot-tons. A weight of 30 tons is placed 20 feet from the forward end of the ship. What will be the new draught of water?

The first step is to see the sinkage caused by placing the weight over the centre of flotation. This sinkage is 1½ inches, and the draughts would then be—

12′ 4½″ forward, 14′ 7½″ aft

Now, the shift from the centre of flotation to the given position is 142 feet, so that the moment forward is 30 × 142 foot-tons, and the change of trim by the bow is—

$$\frac{30 \times 142}{300}, \text{ or } 14\tfrac{1}{4} \text{ inches nearly}$$

This has to be divided up in the ratio of 138 : 162, because the centre of flotation is 12 feet abaft the middle of length. We therefore have—

$$\text{Increase of draught forward } \tfrac{162}{300} \times 14\tfrac{1}{4}'' = 7\tfrac{3}{4}'' \text{ say}$$
$$\text{Decrease of draught aft } \tfrac{138}{300} \times 14\tfrac{1}{4}'' = 6\tfrac{1}{2}'' \text{ say}$$

The final draughts will therefore be—

$$\text{Forward, } 12'\ 4\tfrac{1}{2}'' + 7\tfrac{3}{4}'' = 13'\ 0\tfrac{1}{4}''$$
$$\text{Aft, } 14'\ 7\tfrac{1}{2}'' - 6\tfrac{1}{2}'' = 14'\ 1''$$

Effect on the Trim of a Ship due to adding a Weight of Considerable Amount.—In this case the assumptions made in the previous investigation will no longer hold, and we must allow for the following:—

1. Variation of the tons per inch immersion as the ship sinks deeper in the water.

2. The centre of flotation does not remain in the same transverse section.

3. The addition of a large weight will alter the position of G, the centre of gravity of the ship.

4. The different form of the volume of displacement will alter the position of B, the centre of buoyancy of the ship, and also the value of BM.

5. Items 3 and 4 will alter the value of the moment to change trim 1 inch.

As regards 1, we can obtain first an approximation to the sinkage by dividing the added weight by the tons per inch immersion at the original water-line. The curve of tons per inch immersion will give the tons per inch at this new draught. The mean between this latter value and the original tons per inch, divided into the added weight, will give a very close approximation to the increased draught. Thus, a vessel floats at a constant draught of 22′ 2″, the tons per inch immersion being 44·5. It is required to find the draught after adding a weight of 750 tons. The first approximation to the increase of draught is $\frac{750}{44 \cdot 5} = 17$ inches nearly. At a draught of 23′ 7″ it is found that the tons per inch immersion is 45·7. The mean tons per inch is therefore $\frac{1}{2}(44 \cdot 5 + 45 \cdot 7) = 45 \cdot 1$, and the increase in draught is therefore $\frac{750}{45 \cdot 1} = 16 \cdot 63$, or $16\frac{5}{8}$ inches nearly. This assumes that the ship sinks to a water-plane parallel to the first water-plane. In order that this can be the case, the weight must have been placed in the same transverse section as the centre of gravity of the layer of displacement between the two water-planes. We know that the weight and buoyancy of the ship must act in the same vertical line, and therefore, for the vessel to sink down without change of trim, the added weight must act in the same vertical line as the added buoyancy. We can approximate very closely to the centre of gravity of the layer as follows: Find the centre of flotation of the original W.P. and that of the parallel W.P. to which the vessel is supposed to sink. Put these points on the profile drawing at the respective water-lines. Draw a line joining them, and bisect this line. Then this point will be a very close approximation to the centre of gravity of the layer. A weight of 750 tons placed as above, with its centre of gravity in the transverse section containing this point, will cause the ship to take up a new draught of 23′ 6$\frac{5}{8}$″ with no change of trim.

We can very readily find the new position of G, the centre of gravity of the ship due to the addition of the weight. Thus, suppose the weight of 750 tons in the above example is placed with its centre of gravity 16 feet below the C.G. of the ship; then, supposing the displacement before adding the weight to be 9500 tons, we have—

$$\text{Lowering of G} = \frac{750 \times 16}{10250}$$
$$= 1{\cdot}17 \text{ feet}$$

We also have to take account of 4. In the case we have taken, the new C.B. below the *original water-line* was 9·7 feet, as against 10·5 feet in the original condition, or a rise of 0·8 foot.

For the new water-plane we have a different longitudinal BM, and, knowing the new position of B and of G, we can determine the new longitudinal metacentric height. From this we can obtain the new *moment to change trim* 1 *inch*, using, of course, the new displacement. In the above case this works out to 950 foot-tons.

Now we must suppose that the weight is shifted from the assumed position in the same vertical line as the centre of gravity of the layer to its given position, and this distance must be found. The weight multiplied by the longitudinal shift will give the moment changing the trim either aft or forward, as the case may be. Suppose, in the above case, this distance is 50 feet forward. Then the moment changing trim by the bow is—

$$750 \times 50 = 37{,}500 \text{ foot-tons}$$

and the approximate change of trim is—

$$37{,}500 \div 950 = 39\tfrac{1}{2} \text{ inches}$$

This change of trim has to be divided up in the ordinary way for the change of draught aft and forward. In this case we have—

$$\text{Increase of draught forward} = \tfrac{217}{400} \times 39\tfrac{1}{2} = 21\tfrac{1}{2} \text{ inches say}$$
$$\text{Decrease of draught aft} = \tfrac{183}{400} \times 39\tfrac{1}{2} = 18 \text{ inches say}$$

We therefore have for our new draughts—

$$\text{Draught aft, } 22'\ 2'' + 16\tfrac{5}{8}'' - 18'' = 22'\ 0\tfrac{5}{8}''$$
$$\text{Draught forward, } 22'\ 2'' + 16\tfrac{5}{8}'' + 21\tfrac{1}{2}'' = 25'\ 4\tfrac{1}{8}''$$

For all ordinary purposes this would be sufficiently accurate; but it is evidently still an approximation, because we do not take account of the new GM for the final water-line, and the consequent new moment to change trim 1 inch. These can be calculated if desired, and corrections made where necessary.

To determine the Position of a Weight on Board a Ship such that the Draught aft shall remain constant whether the Weight is or is not on Board.[1]— Take a ship floating at the water-line WL, as in Fig. 65. If a weight w be placed with its centre of gravity in the transverse section that contains the centre of flotation, the vessel will very nearly sink to a parallel water-line W′L′.[2] This, however, is not what is required, because the draught aft is the distance WW′ greater than it should be. The weight will have to be

[1] See also Examples 25, 26 in Appendix.

[2] Strictly speaking, the weight should be placed with its centre of gravity in the transverse section that contains the centre of gravity of the zone between the water-lines WL and W′L′.

moved forward sufficient to cause a change of trim forward of WW' + LL', and then the draught aft will be the same as it originally was, and the draught forward will increase by the amount WW' + LL'. This will be more clearly seen, perhaps, by working the following example :—

It is desired that the draught of water aft in a steamship (particulars given below) shall be constant, whether the coals

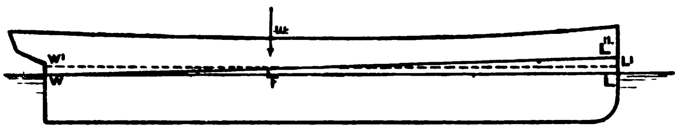


FIG. 65.

are in or out of the ship. Find the approximate position of the centre of gravity of the coals in order that the desired condition may be fulfilled: Length of ship, 205 feet; displacement, 522 tons (no coals on board); centre of flotation from after perpendicular, 104·3 feet; longitudinal BM, 664 feet; longitudinal GM, 661·5 feet; tons per inch, 11·4; weight of coals, 57 tons.

From the particulars given, we find that—

$$\left.\begin{array}{l}\text{Moment to change}\\ \text{trim 1 inch}\end{array}\right\} = \frac{661 \cdot 5 \times 522}{12 \times 205} = 140 \text{ foot-tons}$$

The bodily sinkage, supposing the coals placed with the centre of gravity in the transverse section containing the centre of flotation, will be $\frac{57}{11 \cdot 4} = 5$ inches. Therefore the coals must be shifted forward from this position through such a distance that a change of trim of 10 inches forward is produced. Accordingly, a forward moment of—

$$140 \times 10 = 1400 \text{ foot-tons}$$

is required, and the distance forward of the centre of flotation the coals require shifting is—

$$\frac{1400}{57} = 24 \cdot 6 \text{ feet}$$

Therefore, if the coals are placed—

$$104{\cdot}3 + 24{\cdot}6 = 128{\cdot}9 \text{ feet}$$

forward of the after perpendicular, the draught aft will remain very approximately the same as before.

Change of Trim caused by a Compartment being open to the Sea.—The principles involved in dealing with a problem of this character will be best understood by working out the following example :—

A rectangular-shaped lighter, 100 feet long, 40 feet broad, 10 feet deep, floating in salt water at 3 feet level draught, has a collision bulkhead 6 feet from the forward end. If the side is broached before this bulkhead below water, what would be the trim in the damaged condition?

Let ABCD, Fig. 66, be the elevation of the lighter, with a

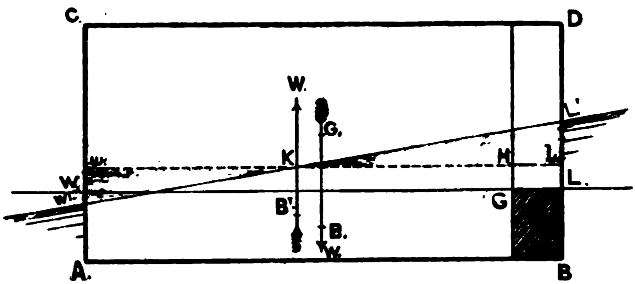


FIG. 66.

collision bulkhead 6 feet from the forward end, and floating at the level water-line WL. It is well to do this problem in two stages—

1. Determine the amount of mean sinkage due to the loss of buoyancy.

2. Determine the change of trim caused.

1. The lighter, due to the damage, loses an amount of buoyancy which is represented by the shaded part GB, and if we assume that she sinks down parallel, she will settle down at a water-line wl such that volume wG = volume GB. This will determine the distance x between wl and WL.

For the volume $wG = wH \times 40$ feet $\times x$
and the volume $GB = GL \times 40$ feet $\times 3$ feet

$$\therefore x = \frac{40 \times 6 \times 3}{94 \times 40} = \frac{18}{94} \text{ feet}$$
$$= 2\tfrac{1}{4} \text{ inches nearly}$$

2. We now deal with the change of trim caused.

The volume of displacement $= 100 \times 40 \times 3$ cubic feet

$$\text{The weight of the lighter} = \frac{100 \times 40 \times 3}{35} = \frac{2400}{7} \text{ tons}$$

and this weight acts down through G, the centre of gravity, which is at 50 feet from either end.

But we have lost the buoyancy due to the part forward of bulkhead EF, and the centre of buoyancy has now shifted back to B′ such that the distance of B′ from the after end is 47 feet. Therefore we have W, the weight of lighter, acting down through G, and W, the upward force of buoyancy, acting through B′. These form a couple of magnitude—

$$W \times 3 \text{ feet} = \frac{2400}{7} \times 3 = \frac{7200}{7} \text{ foot-tons}$$

tending to trim the ship forward.

To find the amount of this trim, we must find the moment to change trim 1 inch—

$$= \frac{W \times GM}{12 \times L}$$

using the ordinary notation.

Now, GM very nearly equals BM;

$$\therefore \text{moment to change trim 1 inch} = \frac{\frac{2400}{7}}{12 \times 100} \times BM$$
$$= \frac{2}{7} \times BM$$
$$BM = \frac{I_0}{V}$$

where I_0 = the moment of inertia of the intact water-plane about a transverse axis through its centre of gravity;
and V = volume of displacement in cubic feet.

$$I = \frac{1}{12}(94 \times 40) \times (94)^2$$
$$V = 12{,}000$$
$$\therefore BM = \frac{40 \times (94)^2}{144000}$$

$$\text{and moment to alter trim 1 inch} = \frac{2 \times 40 \times (94)^2}{7 \times 144000}$$
$$= 66 \text{ foot-tons nearly}$$
$$\therefore \text{the change of trim} = \frac{7200}{7} \div 66$$
$$= 15\tfrac{1}{2} \text{ inches}$$

The new water-line W′L′ will pass through the centre of gravity of the water-line *wl* at K, and the change of trim aft and forward must be in the ratio 47 : 53; or—

$$\text{Decrease of draught aft} = \frac{47}{100} \times 15\tfrac{1}{2} = 7\tfrac{1}{4} \text{ inches}$$
$$\text{Increase of draught forward} = \frac{53}{100} \times 15\tfrac{1}{2} = 8\tfrac{1}{4} \text{ inches}$$

therefore the new draught aft is given by—

$$3'\ 0'' + 2\tfrac{1}{4}'' - 7\tfrac{1}{4}'' = 2'\ 7''$$

and the new draught forward by—

$$3'\ 0'' + 2\tfrac{1}{4}'' + 8\tfrac{1}{4}'' = 3'\ 10\tfrac{1}{2}''$$

The correctness of this result may be seen by finding the displacement and position of the C.B. of the new volume of displacement. The displacement will be found equal to the original displacement, and the C.B. will be found to be 50 feet from the after-end or the same as the C.G.

(For a more difficult example of a similar nature, see No. 34, Appendix A.)

If the sums of the columns 4, 6, and 8 in the table on p. 148 are called S_1, S_2, S_3, then Area $= S_1 \times \frac{2}{3} . h$, and the distance of the C.G. of water-plane from No. 6, viz.:

$$x = \frac{S_2}{S_1} \times h;\ I_{11} = S_3 \times 2 \times \frac{h^3}{3}$$

I_0 the moment of inertia required.

$$= I_{11} - A . x^2 = 2 . \frac{h^3}{3}\left[S_3 - \frac{S_2^2}{S_1}\right]$$

This method saves some work as compared with the above and is used in Brown's displacement sheet given in the Appendix. In the above case

$$I_0 = 2 \cdot \frac{7 \cdot 1^3}{3}\left[959 \cdot 14 - \frac{(56 \cdot 86)^2}{163 \cdot 42}\right]$$

This worked out by the aid of the 4 fig. logs. gives 224,300.

To find the Longitudinal Position of the C.G. of a Ship.—If a ship is floating at the trim assumed for the ordinary calculations, this is a simple matter, as the C.G. must be in the same vertical line as the C.B., and the longitudinal position of the C.B. is readily found for the draught at which the ship is floating. If, however, the ship is floating out of her normal trim, the following gives a close approximation to the position of the C.G.

Suppose the ship is trimming by the stern at the water-line WL, as in Fig. 66A. The water-line cutting off the same displacement is not *wl* at the same mean draught as WL, but *w'l'* passing through F, the centre of flotation. The excess

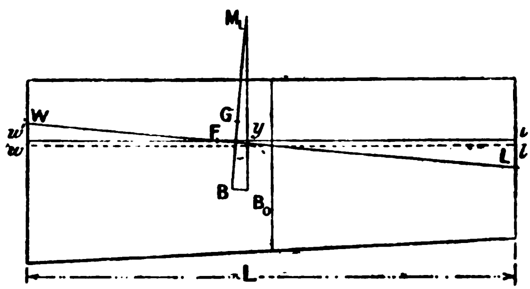


FIG. 66A.

displacement over that corresponding to the mean draught is $12 \times y \times T \times \theta$, where T is the tons per inch, y the C.F. abaft amidships in feet, θ is the angle between WL and *wl* (*i.e.* change of trim ÷ length). The C.B. of the displacement corresponding to *w'l'* is at B_0, and can readily be determined. The ship, in trimming to the water-line WL, may be said to pivot about a transverse axis through F, and the volume Fl'L shifts to Ww'F. Then it can readily be shown that the sternward shift of the C.B. from B_1 to B is $BM_L \times \theta$, BM_L being the longitudinal BM corresponding to $w'l'$ or WL. The C.G. of the ship must be in the line BM_L perpendicular to WL, and therefore G abaft mid-length = B abaft mid-length $-(BG \times \theta)$.

If the ship trims by the bow, the C.B. shifts *forward* $BM_L \times \theta$, and the C.G. is B abaft mid-length $+(BG \times \theta)$.

Change of Trim due to passing from Salt to River Water, or vice-versâ.—If the C.F. is vertically over the C.B. there will be no change of trim. If W is weight, and V and V′ are the volumes of displacement in salt water and river water (say 35 and 35·6 cubic feet to the ton respectively), then $V = W \times 35$, $V' = W \times 35 \cdot 6$. Let $V' - V = v$.

The vessel floating at the line WL in sea water B and G must be in the same vertical. Supposing the ship to sink down parallel to W′L′ in river water, the shift aft of the C.B. is $\frac{v}{V + v} \cdot b$, where b is the fore and aft separation of the C.B. and C.F., and the moment changing trim $= W \times \frac{v}{V + v} \times b$. The following example will illustrate the above.

Example.—A vessel with rectangular sections is 300 feet long, 30 feet broad, and floats in salt water at a draught of 15 feet forward and 20 feet aft. C.G. in WL. Determine the draughts forward and aft on going into river water 63 lbs. to cubic foot.

Bodily sinkage $= \frac{W}{63\ T} = 3 \cdot 34$ inches.

Volume of layer $= 300 \times 30 \times 3 \cdot 34 \times \frac{1}{12} = 2505$ cubic feet.
C.B. and ∴ C.G. abaft amidships = 7·15 feet.
C.F. from amidships = nil.
Moment to change trim one inch = 525 feet tons.
$W = 4500$ tons $V = 157{,}500$ $v = 2505$.

Therefore shift (forward in this case) of C.B. $= \frac{2505}{160005} \times 7 \cdot 15 = 0 \cdot 11$ ft.

The C.G. and the new C.B. are therefore 0·11 feet apart, and the moment to change trim aft is 4500 × 0·11 feet tons, and the change of trim is 1 inch, say.

The draughts are therefore 15′ 3·34″ − 0·5″ = 15′ 2·84″.
" " " 20′ 3·34″ + 0·5″ = 20′ 3·84″.

Draught of a Vessel when Launched.—It is frequently necessary to make a close approximation to the draught forward and aft of a vessel on the occasion of launching, and in addition to the ordinary hydrostatic curves given in Fig. 153 it is necessary to obtain the weight of the vessel on the stocks and the position of the C.G. of this weight, both in a longitudinal and a vertical direction. The weight will enable the mean draught to be fixed, taking into account the density of the water. At this draught the position of the longitudinal metacentre is known, from which the longitudinal GM can be found, and then the moment to change trim 1 inch. The longitudinal centre of buoyancy at the assumed draught can be found readily, and the moment changing trim is determined by multiplying the weight and the longitudinal distance between the centre of buoyancy and the centre of gravity.

The following example will illustrate the methods to be adopted—

A box-shaped vessel, 400 *feet by* 70 *feet, floating when at designed draught at* 22 *feet forward and* 24 *feet aft, weighs before launching* 6400 *tons, and the position of the centre of gravity is* 10 *feet abaft amidships and* 3 *feet below L.W.L. What will be her draught when launched into salt water?*

The mean draught is 8 feet, and assume she floats parallel to the L.W.L., 7 feet forward and 9 feet aft. At this waterline the C.B. is readily calculated to be 8·3 feet abaft amidships and 19·0 feet below L.W.L. The longitudinal BM at this assumed waterline works out to 1,666 feet, and the longitudinal GM 1650 feet, since BG is 16 feet. The moment to change trim 1 inch is 2200 foot tons. The horizontal separation of the C.G. and the C.B. is $10 - 8·3 = 1·7$ feet, so that the change of trim is $\frac{(6400 \times 1·7)}{2200} = 5$ inches aft. In this case, seeing that the centre of flotation is amidships, this 5 inches is divided equally forward and aft, so that the draught when launched is 6 ft. 9½ in. forward and 9 ft. 2½ in. aft.

The principal difficulty in such an estimate is the determination of the longitudinal C.G.

Information for use when Docking in a Floating Dock.—When a ship is to be docked in a floating dock, especially if the weight is close to the lifting capacity of the dock, it is necessary to place the ship in the dock so that its C.G. is at the centre of length of the dock in order that when lifted the dock shall be on an even keel.

Information in the following form is now provided to H.M. ships to enable the position of the C.G. to be closely approximated to knowing the draughts forward and aft. This information can be readily calculated from the sheer drawing by using the principles of the present chapter.

Mean draught between draught marks.	Corresponding displacement at normal trim.	Longitudinal position of centre of buoyancy relative to bulkhead 132.	Movement of the C.B. for every foot change of trim from normal trim by bow or stern.	Tons per inch.
ft. in.	tons.	feet.	feet.	tons.
31 6	25,900	15·7 forward	1·29	83
28 6	22,930	17·1 ,,	1·42	82
25 6	20,000	19·0 ,,	1·58	81

Thus, suppose the above ship is drawing 28 ft. forward and 31 ft. 6 in. aft, the C.B. at the mean draught of 29 ft. 9 in. even keel is 16·4 ft. forward of 132, and this will move aft $3·5 \times 1·36 = 4·75$ ft. for the 3 ft. 6 in. trim by the stern. The C.B. (and therefore the C.G. very nearly) is therefore $16·4 - 4·75 = 11·65$ ft. forward of 132 station, and this point should as nearly as possible be placed at the centre of length of the dock.

Examples.—(i) In the above ship if the draught forward is 25 ft. 6 in., and the draught aft 32 ft., estimate the position the ship should be placed relative to the dock.

Ans. 7·8′ forward of 132 should be well with dock centre.

(ii) In the above ship if the draught forward is 30 ft., and aft 25 ft., estimate the position the ship should be placed relative to the dock.

Ans. 25′ forward of 132 should be well with dock centre.

EXAMPLES TO CHAPTER IV.

✓1. A ship is floating at a draught of 20 feet forward and 22 feet aft, when the following weights are placed on board in the positions named :—

Weight in tons.			Distance from C.G. of water-plane in feet.	
20	...	...	100	before
45	...	...	80	before
60	...	...	50	abaft
30	...	...	10	abaft

What will be the new draught forward and aft, the moment to change trim 1 inch being 800 foot-tons, and the tons per inch = 35?

Ans. 20′ 5⅝″ forward, 22′ 3″ aft.

✓2. A vessel 300 feet long, designed to float with a trim of 3 feet by the stern, owing to consumption of coal and stores, floats at a draught of 9′ 3″ forward, and 14′ 3″ aft. The load displacement at a mean draught of 13′ 6″ is 2140 tons; tons per inch, 18½; centre of flotation, 12½ feet abaft the middle of length. Approximate as closely as you can to the displacement.

Ans. 1775 tons.

✓3. A vessel is 300 feet long and 36 feet beam. Approximate to the moment to change trim 1 inch, the coefficient of fineness of the L.W.P. being 0·75.

Ans. 319 foot-tons.

✓4. A light-draught stern-wheel steamer is very approximately of the form of a rectangular box of 120 feet length and 20 feet breadth. When fully laden, the draught is 18 inches, and the centre of gravity of vessel and lading is 8 feet above the water-line. Find the transverse and longitudinal metacentric heights, and also the moment to change trim one inch.

Ans. 13·47 feet, 791½ feet; 56¼ foot-tons.

✓5. A vessel is floating at a draught of 12′ 3″ forward and 14′ 6″ aft. The tons per inch immersion is 20; length, 300 feet; centre of flotation, 12 feet abaft amidships; moment to change trim 1 inch, 300 foot-tons. Where should a weight of 60 tons be placed on this vessel to bring her to an even keel.

Ans. 123 feet forward of amidships.

✓6. What weight placed 13 feet forward of amidships will have the same effect on the trim of a vessel as a weight of 5 tons placed 10 feet abaft the forward end, the length of the ship being 300 feet, and the centre of flotation 12 feet abaft amidships.

Ans. 30·4 tons.

✓7. A right circular pontoon 50 feet long and 16 feet in diameter is just half immersed on an even keel. The centre of gravity is 4 feet above the bottom. Calculate and state in degrees the transverse heel that would be produced by shifting 10 tons 3 feet across the vessel. State, in inches, the change of trim produced by shifting 10 tons longitudinally through 20 feet.

Ans. 3 degrees nearly; 25 inches nearly.

8. Show why it is that many ships floating on an even keel will increase the draught forward, and decrease the draught aft, or, as it is termed, go down by the head, if a weight is placed at the middle of the length.

9. Show that for vessels having the ratio of the length to the draught about 13, the longitudinal B.M. is approximately equal to the length. Why should a shallow draught river steamer have a longitudinal B.M. much greater than the length? What type of vessel would have a longitudinal B.M. less than the length?

10. Find the moment to change trim 1 inch of a vessel 400 feet long, having given the following particulars: Longitudinal metacentre above centre of buoyancy, 446 feet; distance between centre of gravity and centre of buoyancy, 14 feet; displacement, 15,000 tons.

Ans. 1350 foot-tons.

11. The moment of inertia of a water-plane of 22,500 square feet about a transverse axis 20 feet forward of the centre of flotation, is found to be 254,000,000 in foot-units. The displacement of the vessel being 14,000 tons, determine the distance between the centre of buoyancy and the longitudinal metacentre.

Ans. 500 feet.

12. In the preceding question, if the length of the ship is 405 feet, and the distance between the centre of buoyancy and the centre of gravity is 13 feet, determine the change of trim caused by the longitudinal transfer of 150 tons through 50 feet.

Ans. $5\frac{3}{4}$ inches nearly.

13. A water-plane has an area of 13,200 square feet, and its moment of inertia about a transverse axis $14\frac{1}{2}$ feet forward of its centre of gravity works out to 84,539,575 in foot-units. The vessel is 350 feet long, and has a displacement to the above water-line of 5600 tons. Determine the moment to change trim 1 inch, the distance between the centre of gravity and the centre of buoyancy being estimated at 8 feet.

Ans. 546 foot-tons.

14. The semi-ordinates of a water-plane of a ship 20 feet apart are as follows: 0·4, 7·5, 14·5, 21·0, 26·6, 30·9, 34·0, 36·0, 37·0, 37·3, 37·3, 37·3, 37·3, 37·2, 37·1, 36·8, 35·8, 33·4, 28·8, 21·7, 11·5 feet respectively. The after appendage, whole area 214 square feet, has its centre of gravity 6·2 feet abaft the last ordinate. Calculate—

(1) Area of the water-plane.
(2) Position of C.G. of water-plane.
(3) Transverse B.M.
(4) Longitudinal B.M.
(Volume of displacement up to the water-plane 525,304 cubic feet.)

Ans. (1) 24,015 square feet; (2) 18·2 feet abaft middle ordinate; (3) 17·16 feet; (4) 447·6 feet.

15. The semi-ordinates of the L.W.P. of a vessel $15\frac{1}{2}$ feet apart are, commencing from forward, 0·1, 2·5, 5·3, 8·1, 10·8, 13·1, 15·0, 16·4, 17·6, 18·3, 18·5, 18·5, 18·4, 18·1, 17·5, 16·6, 15·3, 13·3, 10·8, 7·6, 3·8 feet respectively. Abaft the last ordinate there is a portion of the water-plane, the half-area being 27 square feet, having its centre of gravity 4 feet abaft the last ordinate. Calculate the distance of the longitudinal metacentre above the centre of buoyancy, the displacement being 2206 tons.

Ans. 534 feet.

16. State the conditions that must hold in order that a vessel shall not change trim in passing from river water to salt water.

17. A log of fir, specific gravity 0·5, is 12 feet long, and the section is 2 feet square. What is its longitudinal metacentric height when floating in stable equilibrium?

Ans. 16·5 feet nearly.

18. Using the approximate formula for the moment to change trim 1 inch given on p. 157, show that this moment will be very nearly given by $30 \cdot \frac{T^2}{B}$, where T is the tons per inch immersion, and B is the breadth.

Show also that in ships of ordinary form, the moment to change trim 1 inch approximately equals $\frac{1}{10000} \cdot L^2B$.

CHAPTER V.

STATICAL STABILITY, CURVES OF STABILITY, CALCULATIONS FOR CURVES OF STABILITY, INTEGRATOR, DYNAMICAL STABILITY.

Statical Stability at Large Angles of Inclination. Atwood's Formula.—We have up to the present only dealt with the stability of a ship at small angles of inclination, and within these limits we can determine what the statical stability is by using the metacentric method as explained on p. 98. We must now, however, investigate how the statical stability of a ship can be determined for large angles of inclination, because in service it is certain that she will be heeled over to much larger angles than 10° to 15°, which are the limits beyond which we cannot employ the metacentric method.

Let Fig. 67 represent the cross-section of a ship inclined to a large angle θ. WL is the position on the ship of the original water-line, and B the original position of the centre of buoyancy. In the inclined position she floats at the water-line W′L′, which intersects WL in the point S, which for large angles will not usually be in the middle line of the ship. The volume SWW′ is termed, as before, the "*emerged wedge*," and the volume SLL′ the "*immersed wedge*," and g, g' are the positions of the centres of gravity of the emerged and immersed wedges respectively. The volume of displacement remains the same, and consequently these wedges are equal in volume. Let this volume be denoted by v. The centre of buoyancy of the vessel when floating at the water-line W′L′ is at B′, and the upward support of the buoyancy acts through B′; the downward force of the weight acts through G, the centre of gravity of the ship. Draw GZ and BR perpendicular to the vertical through B′, and gh, $g'h'$ perpendicular to the new water-line W′L′. Then the moment of the couple tending to right the ship is W × GZ, or, as we term it, the *moment of statical stability*. Now—

$$GZ = BR - BP$$
$$= BR - BG \sin \theta$$

so that the moment of statical stability at the angle θ is—

$$W(BR - BG \, . \sin \theta)$$

The length BR is the only term in this expression that we do not know, and it is obtained in the following manner. The

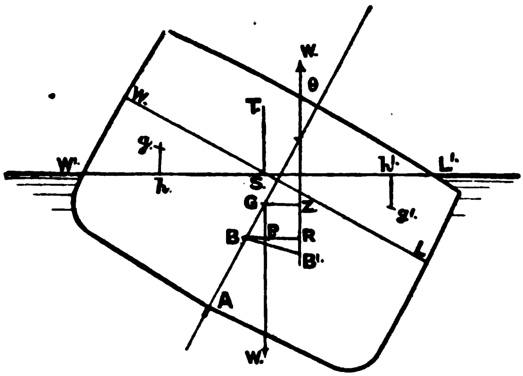


Fig. 67.

new volume of displacement W'AL' is obtained from the old volume WAL by shifting the volume WSW' to the position LSL', through a *horizontal* distance *hh'*. Therefore the *horizontal* shift of the centre of gravity of the immersed volume from its original position at B, or BR, is given by—

$$BR = \frac{v \times hh'}{V}$$

(using the principle discussed on p. 100). Therefore the moment of statical stability at the angle θ is—

$$W\left(\frac{v \times hh'}{V} - BG \, . \sin \theta\right) \text{ foot-tons}$$

This is known as "Atwood's formula."

$$\text{The righting arm or lever} = \frac{v \times hh'}{V} - BG \,.\, \sin \theta$$

If G is below B, as may happen in special cases—

$$GZ = \frac{v \times hh'}{V} + BG \,.\, \sin \theta$$

If we want to find the length of the righting arm or lever at a given angle of heel θ, we must therefore know—

(1) The position of the centre of buoyancy B in the upright condition.

(2) The position of the centre of gravity G of the ship.

(3) The volume of displacement V.

(4) The value of the moment of transference of the wedges parallel to the new water-line, viz. $v \times hh'$.

This last expression involves a considerable amount of calculation, as the form of a ship is an irregular one. The methods adopted will be fully explained later, but for the present we will suppose that it can be obtained when the form of the ship is given.

Curve of Statical Stability.—The lengths of GZ thus obtained from Atwood's formula will vary as the angle of heel increases, and usually GZ gradually increases until an angle is reached when it obtains a maximum value. On further inclination, an angle will be reached when GZ becomes zero, and, further than this, GZ becomes negative when the couple W × GZ is no longer a couple tending to right the ship, but is an upsetting couple tending to incline the ship still further. Take H.M.S. *Captain*[1] as an example. The lengths of the lever GZ, as calculated for this ship, were as follows:—

At 7	degrees, GZ =	$4\frac{1}{4}$	inches	At 35	degrees, GZ =	$7\frac{3}{4}$	inches
,, 14	,, ,, =	$8\frac{1}{2}$	,,	,, 42	,, ,, =	$5\frac{1}{4}$	,,
,, 21	,, ,, =	$10\frac{3}{4}$	,,	,, 49	,, ,, =	2	,,
,, 28	,, ,, =	10	,,	,, $54\frac{1}{2}$	,, ,, =	*nil*	

Now set along a base-line a scale of degrees on a con-

[1] The *Captain* was a rigged turret-ship which foundered in the Bay of Biscay. A discussion of her stability will be found in "Naval Science," vol. 1.

venient scale (say $\frac{1}{4}$ inch = 1 degree), and erect ordinates at the above angles of the respective lengths given. If now we pass a curve through the tops of these ordinates, we shall obtain what is termed a "*curve of statical stability*," from which we can obtain the length of GZ for any angle by drawing the ordinate to the curve at that angle. The curve A, in Fig. 68, is the curve so constructed for the *Captain*. The angle

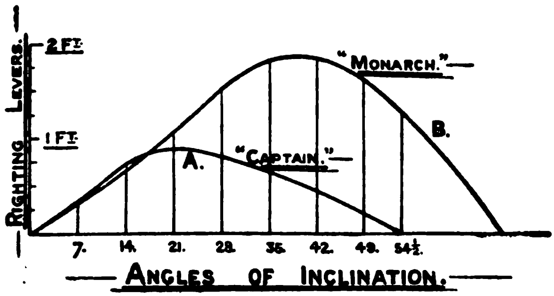


FIG. 68.

at which GZ obtains its maximum value is termed the "*angle of maximum stability*," and the angle at which the curve crosses the base-line is termed the "*angle of vanishing stability*," and the number of degrees at which this occurs is termed the "*range of stability*." If a ship is forced over beyond the angle of vanishing stability, she cannot right herself; GZ having a negative value, the couple operating on the ship is an upsetting couple.

In striking contrast to the curve of stability of the *Captain* is the curve as constructed for H.M.S. *Monarch*.[1] The lengths of the righting levers at different angles were calculated as follows:—

At	7	degrees,	GZ	=	4	inches
„	14	„	„	=	$8\frac{1}{4}$	„
„	21	„	„	=	$12\frac{1}{4}$	„
„	28	„	„	=	$18\frac{1}{4}$	„
„	35	„	„	=	$21\frac{3}{4}$	„

[1] The *Monarch* was a rigged ship built about the same time as the *Captain*, but differing from the *Captain* in having greater freeboard. See also the volume of "Naval Science" above referred to.

At	42 degrees,	GZ	= 22	inches
"	49 "	"	= 20	"
"	$54\frac{1}{2}$ "	"	= $17\frac{1}{2}$	"
"	$69\frac{1}{2}$ "	"	= *nil*	

The curve for this ship, using the above values for GZ, is given by B, Fig. 68. The righting lever goes on lengthening in the *Monarch's* case up to the large angle of 40°, and then shortens but slowly; that of the *Captain* begins to shorten at about 21° of inclination, and disappears altogether at $54\frac{1}{2}°$, an angle at which the *Monarch* still possesses a large righting lever.

Referring to Atwood's formula for the lever of statical stability at the angle θ, viz.—

$$GZ = \frac{v \times hh'}{V} - BG \cdot \sin\theta$$

we see that the expression consists of two parts. The first part is purely geometrical, depending solely upon the *form* of the ship; the second part, BG . sin θ, brings in the influence of the position of the *centre of gravity of the ship*, and this depends on the distribution of the weights forming the structure and lading of the ship. We shall deal with these two parts separately.

(1) Influence of *form* on curves of stability.

(2) Influence of *position of centre of gravity* on curves of stability.

(1) We have here to take account of the form of the ship above water, as well as the form of the ship below water. The three elements of form we shall consider are draught, beam, and freeboard. These are, of course, relative; for convenience we shall keep the draught constant, and see what variation is caused by altering the beam and freeboard. For the sake of simplicity, let us take floating bodies in the form of boxes.[1] The position of the centre of gravity is taken as constant. Take the standard form to be a box:—

Draught	21	feet.
Beam	$50\frac{1}{2}$	"
Freeboard	$6\frac{1}{2}$	"

[1] These illustrations are taken from a paper read at the Institution of Naval Architects by Sir N. Barnaby in 1871.

The curve of statical stability is shown in Fig. 69 by the curve A. The deck-edge becomes immersed at an inclination of $14\frac{1}{2}^\circ$, and from this angle the curve increases less rapidly than before, and, having reached a maximum value, decreases, the angle of vanishing stability being reached at about 38°.

Now consider the effect of adding $4\frac{1}{2}$ feet to the beam, thus making the box—

Draught	21	feet.
Beam	55	"
Freeboard	$6\frac{1}{2}$	"

The curve is now given by B, Fig. 69, the angle of vanishing stability being increased to about 45°. Although the

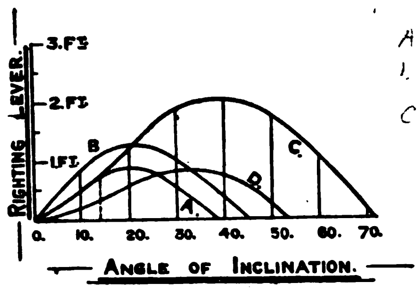


FIG. 69.

position of the centre of gravity has remained unaltered, the increase of beam has caused an increase of GM, the metacentric height, because the transverse metacentre has gone up. We know that for small angles the lever of statical stability is given by GM . sin θ, and consequently we should expect the curve B to start as shown, steeper than the curve A, because GM is greater. There is a very important connection between the metacentric height and the slope of the curve of statical stability at the start, to which we shall refer hereafter.

Now consider the effect of adding $4\frac{1}{2}$ feet to the freeboard of the original form, thus making the dimensions—

Draught	21	feet.
Beam	$50\frac{1}{2}$	"
Freeboard	11	"

The curve is now given by C, Fig. 69, which is in striking contrast to both A and B. The angle of vanishing stability is now 72°. The curves A and C coincide up to the angle at which the deck-edge of A is immersed, viz. $14\frac{1}{2}°$, and then, owing to the freeboard still being maintained, the curve C leaves the curve A, and does not commence to decrease until 40°.

These curves are very instructive in showing the influence of beam and freeboard on stability at large angles. We see—

(*a*) An increase of beam increases the initial stability, and therefore the slope of the curve near the origin, but does not greatly influence the area enclosed by the curve or the range.

(*b*) An increase of freeboard has no effect on initial stability (supposing the increase of freeboard does not affect position of the centre of gravity), but has a most important effect in lengthening out the curve and increasing its area. The two bodies whose curves of statical stability are given by A and C have the same GM, but the curves of statical stability are very different.

(2) We now have to consider the effect on the curve of statical stability of the position of the centre of gravity. If the centre of gravity G is *above* the centre of buoyancy B, as is usually the case, the righting lever is *less* than $\frac{v \times hh'}{V}$ by the expression BG . sin θ. Thus the deduction becomes greater as the angle of inclination increases, because sin θ increases as θ increases, reaching a maximum value of $\sin \theta = 1$ when $\theta = 90°$; the deduction also increases as the C.G. rises in the ship. Thus, suppose, in the case C above, the centre of gravity is raised 2 feet. Then the ordinate of the curve C at any angle θ is diminished by $2 \times \sin \theta$. For 30°, $\sin \theta = \frac{1}{2}$, and the deduction is there 1 foot. In this way we get the curve D, in which the range of stability is reduced from 72° to 53° owing to the 2-feet rise of the centre of gravity.

It is usual to construct these curves as indicated, the ordinates being righting levers, and not righting moments. The righting moment at any angle can be at once obtained by multiplying the lever by the constant displacement. The real curve of statical stability is of course a curve, the ordinates of which represent righting *moments.* This should not be lost sight of, as the following will show. In Fig. 70 are given the

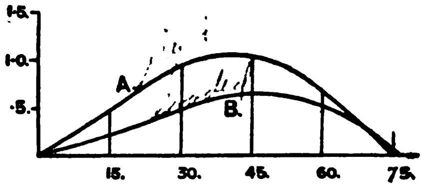


FIG. 70.

curves of righting levers for a merchant vessel in two given conditions, A for the light condition at a displacement of 1500 tons, and B for the load condition at a displacement of 3500 tons. Looking simply at these curves, it would be thought that the ship in the light condition had the better stability; but in Fig. 71, in which A represents the curve of

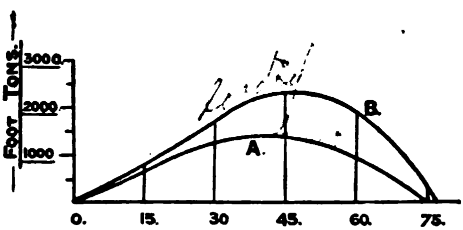


FIG. 71.

righting moments in the light condition, and curve B the curve of righting moments in the load condition, we see that the ship in the light condition has very much less stability than in the load condition.

We see that the following are the important features of a curve of statical stability :—

(*a*) Inclination the tangent to the curve at the origin has to the base-line;

(*b*) The angle at which the maximum value occurs, and the length of the righting lever at this angle;

(*c*) The range of stability.

The angle the tangent at the origin makes with the base-line can be found in a very simple manner as follows: At the angle whose circular measure[1] is unity, viz. 57·3°, erect a perpendicular to the base, and make its length equal to the metacentric height GM, for the condition at which the curve has to be drawn, using the same scale as for the righting levers (see Fig. 72). Join the end of this line with the origin, and the curve as it approaches the origin will tend to lie along this line.

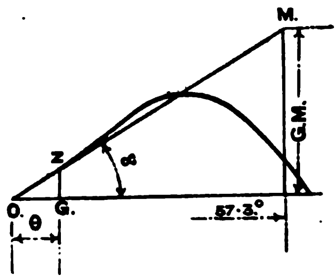


FIG. 72.

The proof of this is given below.[2]

Specimen Curves of Stability.—In Fig. 73 are given some specimen curves of stability for typical classes of ships.

A is the curve for a modern British battleship of about $3\frac{1}{2}$ feet metacentric height. The range is about 63°.

B is the curve for the American monitor *Miantonomoh.* This ship had a low freeboard, and to provide sufficient stability a very great metacentric height was provided. This is shown by the steepness of the curve at the start.

C is the curve for a merchant steamer carrying a miscellaneous cargo, with a metacentric height of about 2 feet. In

[1] See p. 91.

[2] For a small angle of inclination θ, we know that $GZ = GM \times \theta$, θ being in circular measure;

$$\text{or } \frac{GZ}{\theta} = \frac{GM}{1}$$

If now we express θ in degrees, say $\theta = \phi°$, then—

$$\frac{GZ}{\phi°} = \frac{GM}{\text{angle whose circular measure is } 1}$$

$$\text{or } \frac{GZ}{\phi°} = \frac{GM}{57·3°}$$

If α is the angle OM makes with the base, then—

$$\tan \alpha = \frac{GM}{57·3°} = \frac{GZ}{\phi°}$$

and thus the line OM lies along the curve near the origin.

this ship there is a large righting lever even at 90°. It must be stated that, although this curve is typical for many ships, yet

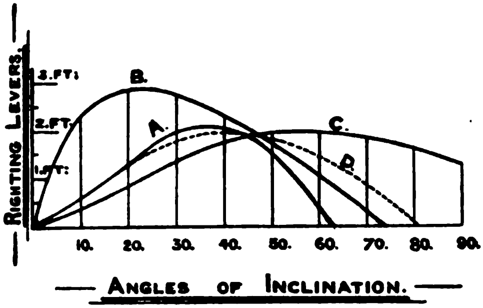


FIG. 73.

the forms of the curves of stability for merchant steamers must vary considerably, owing to the many different types of ships and the variation in loading. Fig. 74 gives curves of stability

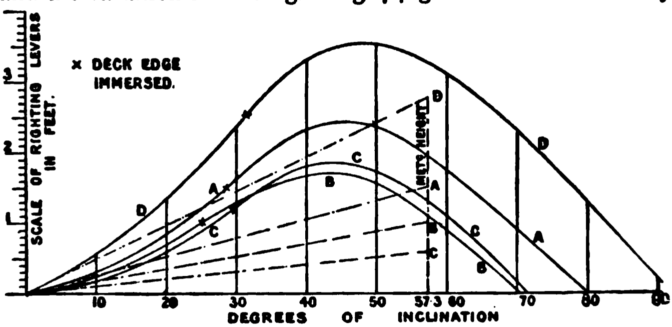


FIG. 74.

for several conditions of the T.S.S. *Smolensk*, 470′ × 58′ × 37′ (Mr. Rowell, I.N.A., 1905). They may be taken as typical curves of a modern steamship of the highest class. A is the curve for the load condition, in which the lower holds are filled with 1200 tons of cargo, and the 'tween decks are filled with 600 tons of cargo homogeneously stowed. All coal, stores, and water are assumed on board. The GM is 1·5 feet, and the range is 80°. B assumes the cargo is all homogeneous, the GM being reduced to 1 foot. The range is rather over 70°. C is for the same cargo as B, but all coal is consumed except 200 tons in bottom of bunkers, and half stores and fresh water only remain on board. The GM is only 0·6 foot, but the lighter draught has the effect of lengthening out the curve to a range of $71\frac{1}{2}°$. D is the condition when the vessel is "light," having the ballast and reserve feed tanks full, and the bunkers full of coal. The GM is 2·8 feet, and the range is over 90°. It will be noticed that the tangent at the origin has been drawn in each case at the angle α such that $\tan \alpha = \frac{GM}{57·3°}$.

The curves E and F in Fig. 74A have been prepared to illustrate the effect of raising the centre of gravity of ship when

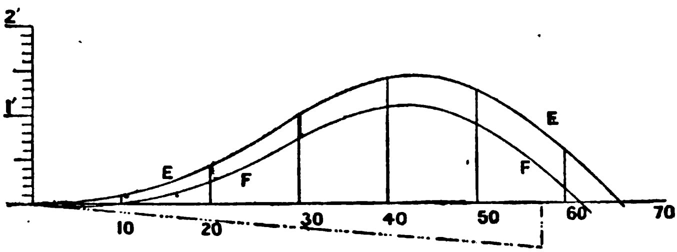


FIG. 74A.

in condition C. If the centre of gravity is raised 0·6 foot by a different disposition of the cargo, the GM is zero, and the curve of stability starts at a tangent to the base line. At all angles the GZ is reduced from that in condition C by 0·6 sin θ, so that we get the curve E, in which the range is 66°. If now we suppose the centre of gravity of ship lifted still higher, viz. 0·5 foot, the vessel has a negative GM in the upright condition, and is therefore in equilibrium, which is, however, unstable. This is shown by the way the curve starts at the origin at an angle of $\alpha = -\tan^{-1}\frac{0·5}{57·3}$. The ship will heel until at 10° the centre of gravity and new CB again get into the same vertical line and the ship is in equilibrium. This time, however, the ship is in *stable* equilibrium, and has a positive GM, so that the ship will loll over to 10°, and there be perfectly safe in calm weather, as is shown by the way the curve of stab·lity stretches out to a range of 62°. At sea it would be advisable to fill up some of the ballast tanks, to improve the stability of the vessel in either of the conditions E and F.

Ships do occasionally get into the condition represented in F, Fig. 74A. The following has reference to the S.S. *Leo*, which capsized in 1895 (taken from Captain Owen's book "Aids to Stability"). This ship left port with a cargo of barley and wheat and 40 tons of coal on deck. Her freeboard was high, no ballast being taken on board, and she had a list of 10° to starboard or port, showing a negative GM. There was some loose water in the bilges which the pump suctions could not touch, as the ship had a list on one side or the other. The ship listing to starboard, the engineers, to reduce the list, used most of the coal from the starboard bunkers. This, however, had the effect of raising the centre of gravity of the ship still further, and increasing her instability when forced to the upright. The wind and sea both now acted on the starboard side, so that she returned to the upright and then lurched over to port. The effect of the motion of the ship, and the force of the wind sending the ship over to leeward, caused her to go far beyond her natural position of equilibrium, say 10° to 15°, and this was helped by the loose water rushing across. Consequently such an angle was reached that the shifting boards gave way and the grain got over to the inclined side, and the ship went right over. The remedy in such a case would undoubtedly have been to fill the water-ballast tanks, so that the ship had a positive GM in the upright condition.

A ship may start her voyage with a small positive GM, but, owing to consumption of coal, etc., during the voyage, she may get a list owing to a negative GM in the upright condition.

The most comfortable ship at sea is one with a small GM, and if this is associated with such a position of the centre of gravity and such a freeboard that the curve gives a good maximum GZ and a good range, say like A in Fig. 74, we have most satisfactory conditions of comfort and seaworthiness. For small-cargo vessels it is generally recognized that the GM should not be less than 0·8 foot, provided that a righting arm of like amount is obtained at 30° to 40°.

For warships the conditions of stability are special. Here, although the high freeboard is conducive to a good area of the curve of stability and a large range, as seen in Fig. 69, curve C, the conditions of design lead to a high position of the centre of gravity, because of the disposition of guns and armour. This discounts the effect of freeboard, as seen by curve D in Fig. 69.

D, in Fig. 73, is the curve of stability for a sailing-ship having a metacentric height of $3\frac{1}{2}$ feet.

The curve of stability for a floating body of circular form is very readily obtainable, because the section is such that the upward force of the buoyancy always acts through the centre of the section, as shown in Fig. 75. The righting lever at any angle θ is GM . sin θ, where G is the centre of gravity, and M the centre of the section. Taking the GM as two feet, then the ordinates of the curve of stability are 0, 1·0, 1·73, 2·0, 1·73, 1·0, at intervals of 30°.

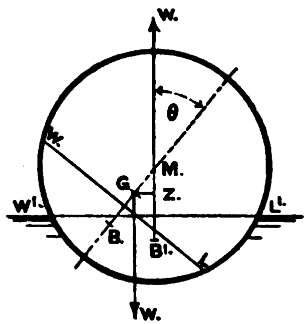


FIG. 75.

The maximum occurs at 90°, and the range is 180°. The curve is shown in Fig. 76. A similar curve is obtained for a submarine boat, the ordinate at angle θ being BG . sin θ, and the range 180°.

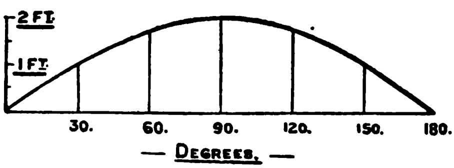


FIG. 76.

Calculations for Curves of Stability.—We now proceed to investigate methods that are or have been adopted in practice to determine for any given ship the curve of righting levers. The use of the integrator is now very general for doing this, and it saves an enormous amount of work; but, in order to get a proper grasp of the subject, it is advisable to understand the methods that were in use previous to the introduction of the integrator.

In constructing and using curves of stability, certain assumptions have to be made. These may be stated as follows:—

1. The sides and deck are assumed to be water-tight for the range over which the curve is drawn.

2. The C.G. is taken in the same position in the ship, and consequently we assume that no weights shift their position throughout the inclination.

3. The trim is assumed to be unchanged, that is, the ship is supposed to be constrained to move about a horizontal longitudinal axis fixed in direction only, and to adjust herself to the required displacement without change of trim.

It is not possible in this work to deal with all the systems of calculation that have been employed; a selection only will be given in this chapter. For further information the student is referred to the *Transactions of the Institution of Naval Architects*, and to the work by Sir E. J. Reed on the "Stability of Ships." The following are the methods that will be discussed:—

1. Blom's mechanical method.
2. Barnes' method.
3. Direct method (sometimes employed as a check on other methods).
4. By Amsler's Integrator and Cross-curves of stability.
5. Benjamin's distorted sections.
6. Tabular method (used at Messrs. John Brown).
7. Mr. Hök's method (given later).

1. **Blom's Mechanical Method.**—Take a sheet of drawing-paper, and prick off from the body-plan the shape of each equidistant section[1] (*i.e.* the ordinary sections for displacement), and cut these sections out up to the water-line at which the curve of stability is required, marking on each section the

[1] In settling the sections to be used for calculating stability by any of the methods, regard must be had to the existence of a poop or forecastle the ends of which are watertight, and the ends of these should as nearly as possible be made stop points in the Simpson's rule.

middle line. Now secure all these sections together in their proper relative positions by the smallest possible use of gum. The weight of these represents the displacement of the ship. Next cut out sections of the ship for the angle at which the stability is required, taking care to cut them rather above the real water-line, and gum together in a similar manner to the first set. Then balance these sections against the first set, and cut the sections down parallel to the inclined water-line until the weight equals that of the first set. When this is the case, we can say that at the inclined water-line the displacement is the same as at the original water-line in the upright condition. This must, of course, be the case as the vessel heels over. On reference to Fig. 67, it will be seen that what we want to find is the line through the centre of buoyancy for the inclined position, perpendicular to the inclined water-line, so that if we can find B′ for the inclined position, we can completely determine the stability. This is done graphically by finding the centre of gravity of the sections we have gummed together, and the point thus found will give us the position of the centre of buoyancy for the inclined condition. This is done by successively suspending the sections and noting where the plumb-lines cross, as explained on p. 51. Having then the centre of buoyancy, we can draw through it a line perpendicular to the inclined water-line, and if we then spot off the position of the centre of gravity, we can at once measure off the righting lever GZ. A similar set of sections must be made for each angle about 10° apart, and thus the curve of stability can be constructed.

2. **Barnes's Method of calculating Statical Stability.**—In this method a series of tables are employed, called Preliminary and Combination Tables, in which the work is set out in tabulated form. Take the section in Fig. 77 to represent the ship, WL being the upright water-line for the condition at which the curve of stability is required. Now, for a small transverse angle of inclination it is true that the new water-plane for the same displacement will pass through the centre line of the original water-plane WL, but as the angle of inclination increases, a plane drawn through S will cut off a volume of displacement sometimes greater and sometimes less than the

original volume, and the actual water-line will take up some such position as W′L′, Fig. 77, supposing too great a volume to be cut off by the plane through S. Now, we cannot say straight off where the water-line W′L′ will come. What we have to do is this: Assume a water-line *wl* passing through S; find the volume of the assumed immersed wedge *l*SL, the volume of the assumed emerged wedge *w*SW, and the area of the assumed water-plane *wl*. Then the difference of the volumes of the wedges divided by the area of the water-plane will give the thickness of the layer between *wl* and the correct water-plane, supposing the difference of the volumes is not too great. If this is the case, the area of the new water-plane is

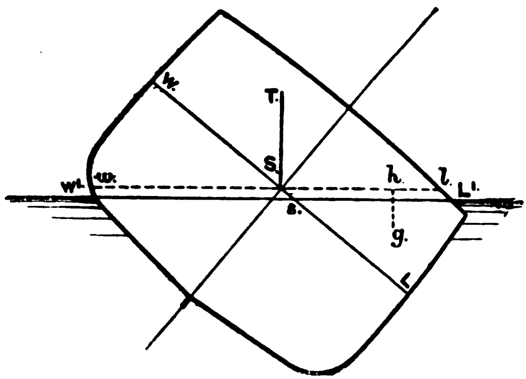


FIG. 77.

found, and a mean taken between it and the original. In this way the thickness of the layer can be correctly found. If the immersed wedge is in excess, the layer has to be deducted; if the emerged wedge is in excess, the layer has to be added.

To get the volumes of either of the wedges, we have to proceed as follows: Take radial planes a convenient angular interval apart, and perform for each plane the operation symbolized by $\frac{1}{2}\int y^2 . dx$, *i.e.* the half-squares of the ordinates are put through Simpson's rule in a fore-and-aft direction for each of the planes. Then put the results through Simpson's rule, using the circular measure of the angular interval. The result will be the volume of the wedge at the particular angle. For proof of this see below.[1]

The results being obtained for the immersed and emerged wedges, we can now determine the thickness of the layer. This work is arranged as follows: The preliminary table, one table for each angle, consists of two parts, one for the immersed wedge, one for the emerged wedge. A specimen table is given on p. 192 for 30°. The lengths of the ordinates of each radial plane are set down in the ordinary way, and operated on by Simpson's multipliers, giving us a function of the area on the immersed side of 550·3, and on the emerged side of 477·3. We then put down the squares of the ordinates, and put them through the Simpson's multipliers, giving us a result for the immersed side of 17,878, and for the emerged side 14,251. The remainder of the work on the preliminary table will be described later.

We now proceed to the combination table for 30° (see p. 193), there being one table for each angle. The functions of squares of ordinates are put down opposite their respective angles, both for the immersed wedge and the emerged wedge, up to and including 30°, and these are put through Simpson's multipliers. In this case the immersed wedge is in excess, and so we find the volume of the layer to be taken off to be 7839 cubic feet, obtaining this by using the proper multipliers. At the bottom is placed the work necessary for finding the thickness of the layer. We have the area of the whole plane 20,540 square feet, and this divided into the excess volume of the immersed wedge, 7839 cubic feet, gives the thickness of the layer to take off, viz. 0·382 foot, to get the true water-line.

We now have to find the moment of transference of the

[1] The area of the section S/L is given by $\frac{1}{2}\int y^2 . d\theta$, as on p. 15, and the volume of the wedge is found by integrating these areas right fore and aft, or—

$$\tfrac{1}{2}\iint y^2 . d\theta . dx$$

which can be written—

$$\tfrac{1}{2}\iint y^2 . dx . d\theta$$
$$\text{or } \int(\tfrac{1}{2}\int y^2 . dx)d\theta$$

i.e. $\frac{1}{2}\int y^2 . dx$ is found for each radial plane, and integrated with respect to the angular interval.

wedges, $v \times hh'$ in Atwood's formula, and this is done by using the assumed wedges and finding their moments about the line ST, and then making at the end the correction rendered necessary by the layer. To find these moments we proceed as follows: In the preliminary table are placed the *cubes* of the ordinates of the radial plane, and these are put through Simpson's rule; the addition for the emerged and immersed sides are added together, giving us for the 30° radial plane 1,052,436. These sums of functions of cubes are put in the combination table for each radial plane up to and including 30°, and they are put through Simpson's rule, and then respectively multiplied by the cosine of the angle made by each radial plane with the extreme radial plane at 30°. The sum of these products gives us a function of the *sum of the moments* of the assumed immersed and emerged wedges about ST. The multiplier for the particular case given is 0·3878, so that the uncorrected moment of the wedges is 3,391,336,[1] in foot-units, *i.e.* cubic feet, multiplied by feet.

[1] The proof of the process is as follows: Take a section of the wedge S*l*L, Fig. 78, and draw ST perpendicular to S*l*. Then what is required is the moment of the section about ST, and this integrated throughout the length. Take P and P′ on the curved boundary, very close together, and join SP, SP′; call the angle P′S*l*, θ, and the angle PSP′, $d\theta$.

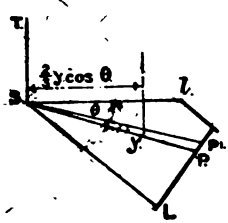


FIG. 78.

Then the area PSP′ $= \frac{1}{2}y^2 . d\theta$ SP $= y$

The centre of gravity of SPP′ is distant from ST, $\frac{2}{3}y . \cos\theta$, and the moment of SPP′ about ST is—

$$\left(\tfrac{1}{2}y^2 . d\theta\right) \times \left(\tfrac{2}{3}y . \cos\theta\right)$$
$$\text{or } \tfrac{1}{3}y^3 . \cos\theta . d\theta$$

We therefore have the moment of *l*SL about ST—

$$\tfrac{1}{3}\int y^3 . \cos\theta . d\theta$$

and therefore the moment of the wedge about ST is—

$$\int\left(\tfrac{1}{3}\int y^3 . \cos\theta . d\theta\right)dx$$
$$\text{or } \tfrac{1}{3}\int\int y^3 . \cos\theta . dx . d\theta$$

i.e. find the value of $\frac{1}{3}\int y^3 . \cos\theta . dx$ for radial planes up to and including the angle, and then integrate with respect to the angular interval. It will be seen that the process described above corresponds with this formula.

PRELIMINARY TABLE FOR STABILITY.

WATER SECTION INCLINED AT 30°.									
Immersed Wedge.									
Number of section.	Ordinates.	Multipliers.[1]	Functions of ordinates.	Squares of ordinates.	Multipliers.[1]	Functions of squares.	Cubes of ordinates.	Multipliers.[1]	Functions of cubes.
1	4·5	½	2·2	20	½	10	91	½	46
2	18·4	2	36·8	339	2	678	6,230	2	12,460
3	28·6	1	28·6	818	1	818	23,394	1	23,394
4	33·8	2	67·6	1142	2	2284	38,614	2	77,228
5	35·5	1	35·5	1260	1	1260	44,739	1	44,739
6	35·6	2	71·2	1267	2	2534	45,118	2	90,236
7	35·6	1	35·6	1267	1	1267	45,118	1	45,118
8	35·6	2	71·2	1267	2	2534	45,118	2	90,236
9	35·6	1	35·6	1267	1	1267	45,118	1	45,118
10	35·6	2	71·2	1267	2	2534	45,118	2	90,236
11	33·8	1	33·8	1142	1	1142	38,614	1	38,614
12	26·9	2	53·8	724	2	1448	19,465	2	38,930
13	14·3	½	7·2	205	½	102	2,924	½	1462
			550·3			17,878			597,817
Emerged Wedge.									
1	4·7	½	2·3	22	½	11	104	½	52
2	13·8	2	27·6	190	2	380	2,628	2	5,256
3	21·4	1	21·4	458	1	458	9,800	1	9,800
4	27·3	2	54·6	745	2	1490	20,346	2	40,692
5	32·8	1	32·8	1076	1	1076	35,288	1	35,288
6	36·7	2	73·4	1347	2	2694	49,431	2	98,862
7	38·2	1	38·2	1459	1	1459	55,743	1	55,743
8	36·5	2	73·0	1332	2	2664	48,627	2	97,254
9	33·6	1	33·6	1129	1	1129	37,933	1	37,933
10	29·3	2	58·6	858	2	1716	25,154	2	50,308
11	23·8	1	23·8	566	1	566	13,481	1	13,481
12	16·9	2	33·8	286	2	573	4,827	2	9,654
13	8·4	½	4·2	71	½	35	593	½	296
			477·3			14,251	Emerged		454,619
							Immersed		597,817
									1,052,436

[1] The multipliers used here are half the ordinary Simpson's multipliers; the results are multiplied at the end by two to allow for this.

COMBINATION TABLE FOR STABILITY.

CALCULATION FOR GZ AT 30°.

Immersed Wedge.									
Inclinations of radial planes.	Functions of ordinates of radial planes.	Functions of squares of ordinates.	Multipliers.	Functions of squares of ordinates for volumes of wedges.	Sums of functions of cubes of ordinates for both sides.	Multipliers.	Products of sums of functions of cubes for both sides.	Cosines of inclinations of radial planes.	Functions of cubes for moments of wedges.
0	—	15,340	1	15,340	974,388	1	974,388	0·8660	843,820
10	—	15,760	4	63,040	990,153	4	3,960,612	0·9397	3,721,787
20	—	16,840	1½	25,260	1,034,251	1½	1,551,377	0·9848	1,527,796
25	—	17,701	2	35,402	1,066,771	2	2,133,542	0·9962	2,125,434
30	550	17,878	½	8,939	1,052,436	½	526,218	1·000	526,218

Immersed wedge	147,991			8,745,065
Emerged wedge	134,522		Multiplier †	0·3878
	13,469		Uncorrected moment	3,391,336
Multiplier *	0·582		Correction for layer	13,875
Volume of layer	7,839 cub. feet.			3,377,461
			Volume of displacement	398,090
			BR	8·48
			BG sin 30° =	5·95
			GZ =	2·53

Longitudinal interval = 30 feet
Circular measure 10° = 0·1745

* Multiplier = ⅓ × 2 × (½ × 30) × (⅓ × 0·1745) = 0·582
† Multiplier = ⅓ × 2 × (⅓ × 30) × (⅓ × 0·1745) = 0·3878
BG = 11·90; sin 30° = 0·5; BG sin θ = 5·95 feet

Emerged Wedge.				
0	—	15,340	1	15,340
10	—	15,157	4	60,628
20	—	14,766	1½	22,149
25	—	14,640	2	29,280
30	477	14,251	½	7,125
				134,522

Area and Position of C.G. of Radial Plane.

	Functions of area.	Functions of moment.
Immersed wedge	550	17,878
Emerged wedge	477	14,250
	1027	3,628

Area = 1027 × 2 × (⅓ × 30)
= 20,540 square feet

C.G. of radial plane on immersed side = $\frac{3628}{1027} \times \frac{1}{2}$ = 1·77 feet

Thickness of layer = $\frac{7839}{20540}$ = 0·382 foot

Note.—The work on the preliminary table may be much simplified by using Tchebycheff's sections: see Appendix A.

We now have to make the correction for the layer. We already have the volume of the layer, and whether it has to be added or subtracted, and we can readily find the position of the centre of gravity of the radial plane. This is done at the bottom of the combination table from information obtained on the preliminary table. We assume that the centre of gravity of the layer is the same distance from ST as the centre of gravity of the radial plane, which may be taken as the case unless the thickness of the layer is too great. If the layer is thick, a new water-line is put in at thickness found, and the area and C.G. of this water-line found. The mean between the result of this and of the original plane can then be used. The volume of the layer, 7839 cubic feet, is multiplied by the distance of its centre of gravity from ST, viz. 1·77 feet, giving a result of 13,875 in foot-units, *i.e.* cubic feet multiplied by feet. The correction for the layer is added to or subtracted from the uncorrected moment in accordance with the following rules:—

If the *immersed* wedge is in excess, and the centre of gravity of the layer is on the *immersed* side, the correction for the layer has to be *subtracted.*

If the *immersed* wedge is in excess, and the centre of gravity of the layer is on the *emerged* side, the correction for the layer has to be *added.*

If the *emerged* wedge is in excess, and the centre of gravity of the layer is on the *emerged* side, the correction for the layer has to be *subtracted.*

If the *emerged* wedge is in excess, and the centre of gravity of the layer is on the *immersed* side, the correction for the layer has to be *added.*

These rules are readily proved, and are left as an exercise for the student.

We, in this case, *subtract* the correction for the layer, obtaining the true moment of transference of the wedges as 3,377,461, or $v \times hh'$ in Atwood's formula. The volume of displacement is 398,090 cubic feet; BG is 11·90 feet; $\sin 30^\circ = 0·5$. So we can fill in all the items in Atwood's formula—

$$GZ = \frac{v \times hh'}{V} - BG \sin \theta$$

$$\text{or } GZ = 2{\cdot}53 \text{ feet}$$

In arranging the radial planes, it has been usual to arrange that the deck edge comes at a stop point in Simpson's first rule, because there is a sudden change of ordinate as the deck edge is passed, and for the same reason additional intermediate radial planes are introduced near the deck edge. In the case we have been considering, the deck edge came at about 30°. The radial planes that were used were accordingly at—

0°, 10°, 20°, 25°, 30°, 35°, 40°, 50°, 60°, 70°, 80°, 90°

These intermediate radial planes lead to rather complicated Simpson's multipliers, and in order to simplify the calculations it is thought to be sufficiently accurate to space the radial planes rather closer, say every 9°. For such a series of radial planes the multipliers for 9°, 18°, 27°, 36°, 54°, 72°, 81°, and 90° are readily obtained by one of Simpson's rules. For 45° the multipliers can be 0·4, 1, 1, 1, 1, 0·4, with the multiplier $\frac{25}{24} \times 0{\cdot}157$. For 63° they can be $\frac{1}{3}$, $\frac{4}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, 0·71, $1\frac{1}{8}$, $1\frac{1}{8}$, $\frac{3}{8}$, with the multiplier 0·157. These can be readily proved; 0·157 is the circular measure of 9°.

Barnes's method of calculating stability has been very largely employed. It was introduced by Mr. F. K. Barnes at the Institution of Naval Architects in 1861, and in 1871 a paper was read at the Institution by Sir W. H. White and the late Mr. John, giving an account of the extensions of the system, with specimen calculations. For further information the student is referred to these papers, and also to the work on "Stability," by Sir E. J. Reed. At the present time it is not used to any large extent, owing to the introduction of the integrator, which gives the results by a mechanical process in much less time. It will be seen that in using this method to find the stability at a given angle, we have to use all the angles up to and including that angle at which the stability is required. Thus a mistake made in the table at any of the smaller angles is repeated right through, and affects the accuracy of the calculation at the larger angles. In order to obtain an independent check at any required angle, we can proceed as follows :—

3. Triangular or Direct Method of calculating Stability.—Take the body-plan, and draw on the trial plane through the centre of the upright water-line at the required angle. This may or may not cut off the required displacement. We then, by the ordinary rules of mensuration, discussed in Chapter I., find the area of all such portions as S*l*L, Fig. 77, for all the sections,[1] and also the position of the centre of gravity, *g*, for each section, thus obtaining the distance S*h*. This is done for both the immersed and emerged wedges. The work can then be arranged in tabular form thus—

Number of section.	Areas.	Simpson's multipliers.	Products for volume.	Levers as S*h*.	Products for moment about ST.
1 2 etc.	A_1 A_2 etc.	1 4 etc.	A_1 $4A_2$ etc.	x_1 x_2 etc.	A_1x_1 $4A_2x_2$ etc.
			S_1		M_1

The volume of the wedge $= S_1 \times \frac{1}{3}$ common interval
The moment of wedge about ST $= M_1 \times \frac{1}{3}$ common interval

This being done for both wedges, and calculating the area of the radial plane, we can find the volume of the layer and the uncorrected moment of the wedges. The correction for the layer is added or subtracted from this, exactly as in Barnes's method, and the remainder of the work follows exactly the methods described above for Barnes's method.

The check spot at 90° is very readily obtained, because the volume and C.G. of the portion above the L.W.L. are readily determined, and we already know the volume and C.B. of portion below the L.W.L. from the displacement sheet. If V is the volume of displacement, and V_1 is one-half the volume above the L.W.L., then $\frac{V}{2} + V_1$ is the volume when at 90° the ship is immersed to the middle-line plane. The C.G. of this

[1] The sections are made into simple figures, as triangles and trapeziums, in order to obtain the area and position of C.G. of each.

volume is readily obtainable. The difference between this and V will give the volume of layer, $\frac{V}{2} - V_1 = V_2$, say, where the layer has to be added. The C.G. of this layer is readily determined, as it will very nearly be that of the middle-line plane of the ship, so that the C.G. of the volume V is found at once, and this gives the GZ at 90°.

There is the disadvantage about the methods we have hitherto described, that we only obtain a curve of stability for one particular displacement, but it is often necessary to know the stability of a ship at very different displacements to the ordinary load displacement, as, for example, in the light condition, or the launching condition. The methods we are now about to investigate enable us to determine at once the curve of stability at any given displacement and any assumed position of the centre of gravity.

4. **Amsler's Integrator. Cross-curves of Stability.** —The Integrator is an extension of the instrument we have described on p. 81, known as the planimeter. A diagram of one form of the integrator is given in Fig. 79. A bar, BB, has a groove in it, and the instrument has two wheels which run in this groove. W is a balance weight to make the instrument run smoothly. There are also three small wheels that run on the paper, and a pointer as in the planimeter. By passing the pointer round an area, we can find—

(1) A number which is proportional to the *area*, *i.e.* a function of the area.

(2) A function of the *moment of the area* about the axis the bar is set to.

(3) A function of the *moment of inertia of the area* about the same axis.

The bar is set parallel to the axis about which moments are required, by means of distance pieces.

(1) is given by the reading indicated by the wheel marked A.
(2) is given by the reading indicated by the wheel marked M.
(3) is given by the reading indicated by the wheel marked I.

The finding of the moment of inertia is not required in our present calculation.

Now let M'LMW represent the body-plan[1] of a vessel inclined to an angle of 30°; then, as the instrument is set, the

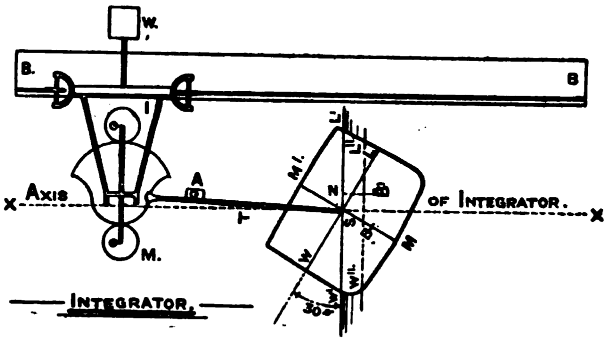


FIG. 79.

axis of moments is the line through S perpendicular to the inclined water-line, and is what we have termed ST. What we want to find is a line through the centre of buoyancy in the inclined position perpendicular to the inclined water-line. By passing the pointer of the instrument round a section, as W'L'M, we can determine its area, and also its moment about the axis ST by using the multipliers; and doing this for all the sections in the body, we can determine the displacement and also the moment of the displacement about ST.[2] Dividing the moment by the displacement, we obtain at once the distance of the centre of buoyancy in the inclined condition from the axis ST. It is convenient in practice to arrange the work in a similar manner to that described for the planimeter, p. 83, and the following specimen calculation for an angle of 30° will illustrate the method employed.[3] Every instrument has multipliers for converting the readings of the wheel A into areas, and those of the wheel M into moments. The multipliers must also take account of the scale used.

[1] The body-plan is drawn for both sides of the ship—the fore-body in black, say, and the after-body in red.

[2] This is the simplest method, and it is the best for beginners to employ; but certain modifications suggest themselves after experience with the instrument. See Example 23 in this chapter.

[3] See Appendix A for calculation, using "Tchebycheff's rule" with the integrator.

30°

Sections.	Areas.				Moments.			
	Readings.	Differences.	Simpson's multipliers.	Products.	Readings.	Differences.	Simpson's multipliers.	Products.
Initial readings	3,146	—	—	—	3900	—	—	—
1 and 17	3,210	64	1	64	3910	− 10	1	− 10
2, 4, 6, 8, 10, 12, 14, and 16	8,859	5649	4	22,596	3124	+ 786	4	3144
3, 5, 7, 9, 11, 13, and 15	14,315	5486	2	10,972	2381	+ 743	2	1486
				33,632				4620

Multiplier for displacement = 0·02

Multiplier for moment = 0·2133

Displacement = 33,632 × 0·02 = 672·6 tons

Moment = 4620 × 0·2133 = 985 foot-tons

$$GZ = \frac{985}{672 \cdot 6} = 1 \cdot 46 \text{ feet}$$

In this case the length of the ship was divided into sixteen equal parts, and accordingly Simpson's first rule can be employed. The common interval was 8·75 feet. The multiplier for the instrument was $\frac{15}{1000}$ for the areas, and $\frac{40}{1000}$ for the moments, and, the drawing being on the scale of $\frac{1}{4}$ inch = 1 foot, the readings for areas had to be multiplied by $(4)^2 = 16$, and for moments by $(4)^3 = 64$. The multiplier for displacement in tons is therefore—

$$\frac{15}{1000} \times 16 \times (\tfrac{1}{3} \times 8 \cdot 75) \times \tfrac{1}{35} = 0 \cdot 02$$

and for the moment in foot-tons is—

$$\frac{40}{1000} \times 64 \times (\tfrac{1}{3} \times 8 \cdot 75) \times \tfrac{1}{35} = 0 \cdot 2133$$

We therefore have, assuming that the centre of gravity is at S—

$$GZ = \frac{985}{672 \cdot 6} = 1 \cdot 46 \text{ feet}$$

Now, this 672·6 tons is not the displacement up to the original water-line WL, and we now have to consider a new conception, viz. *cross-curves of stability*. These are the converse of the ordinary curves of stability we have been considering. In these we have the righting levers at a constant displacement and varying angles. In a cross-curve we have the righting levers for a constant angle, but varying displacement. Thus in Fig. 79, draw a water-line W″L″ parallel to W′L′, and for

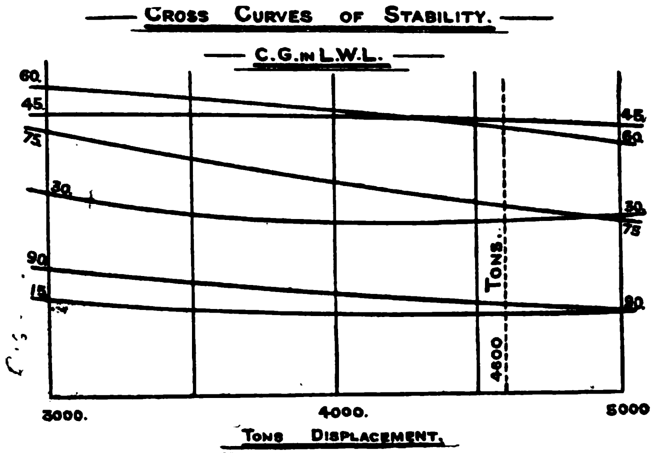


FIG. 80.

the volume represented by W″ML″ find the displacement and position of the centre of buoyancy in exactly the same way as we have found it for the volume W′ML′. The distance which this centre of buoyancy is from the axis gives us the value of GZ at this displacement, supposing the centre of gravity is at S. The same process is gone through for two or more water-lines, and we shall have values of GZ at varying displacements at a constant angle. These can be set off as ordinates of a curve, the abscissæ being the displacements in tons. Such a curve is termed the "cross-curve of stability" at 30°, and for any intermediate displacement we can find the value of GZ at 30° by drawing the ordinate to the curve at this displacement. A similar process is gone through for each angle, the same position for the centre of gravity being assumed all through, and a series of cross-curves obtained. Such a set of cross-curves is shown in Fig. 80 for displacements between 3000 and 5000 tons at angles of 15°, 30°, 45°, 60°, 75°, and 90°. At any intermediate displacement, say at 4600 tons, we can draw the ordinate and measure off the values of GZ, and so obtain the ordinates necessary to construct the ordinary curve of stability at that displacement and assumed position of the centre of gravity. The relation between the cross-curves and the ordinary curves of stability is clearly shown in Fig. 81.

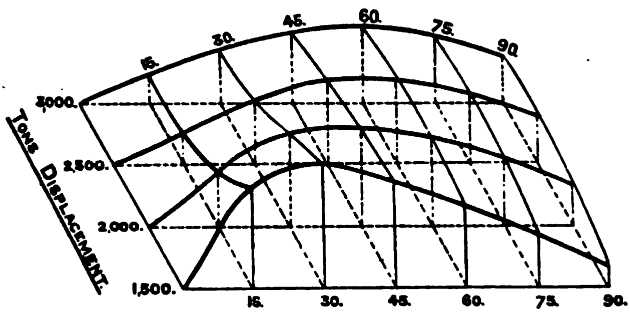


FIG. 81.

We have four curves of stability for a vessel at displacements of 1500, 2000, 2500, and 3000 tons. These are placed as shown in perspective. Now, through the tops of the ordinates at any given angle we can draw a curve, and this will be the cross-curve of stability at that angle.

It will have been noticed that throughout our calculation we have assumed that the centre of gravity is always at the point S, and the position of this point should be clearly stated on the cross-curves. It is evident that the centre of gravity cannot always remain in this position, which has only been assumed for convenience. The correction necessary can readily be made as follows: If G, the centre of gravity, is below the assumed position S, then $GZ = SZ + SG . \sin \theta$, and if G is above S, then GZ = SZ − SG . sin θ for any angle θ. Thus the ordinates are measured from the cross-curves at the required displacement, and then, SG being known, SG sin 15°, SG sin 30°, etc., can be found, and the correct values of GZ determined for every angle.

If an integrator is not available, cross-curves can be calculated by using a modification of Barnes's tables already discussed. Three poles, O, O′, O″, are taken, and Barnes's tables are worked out for each set of radial planes through these poles; but no correction is made for the layer, as for cross-curves we set off the lever for whatever the displacement comes to (see Example 22). For each angle there are thus three spots obtained, and by the method described below tangents are drawn at each of these places to the curves. At 90° the result found for each pole should be the same; at 72° and 81°, say, the spots come very close together and do not give reliable curves. A separate calculation is therefore made for 90° over a wide range of displacement, which can readily be done, and then curves for constant displacement are run in fair, so that auxiliary spots on the cross-curves at 72° and 81° are obtained.

5. Benjamin's Distorted Sections.—This is a method of determining stability by the use of a planimeter, and

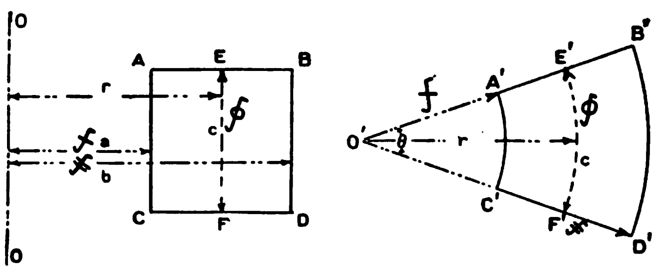


FIG. 81A.

although so far as known it is not much used, it is of considerable interest. It depends on the following principle:—

Take a rectangle ABCD with an axis OO, Fig. 81A, with a line EF distance r from OO. This is turned into a polar diagram, E′F′ being the same length as EF which defines the angle $\theta = \frac{c}{r}$, all distances from the pole O′ being the same as from the axis OO for corresponding points. Such lines as AC and BD become circular arcs A′C′ and B′D′.

Then area $A'B'C'D' = \frac{1}{2}(b^2 - a^2)\theta = \frac{1}{2}(b^2 - a^2)\frac{c}{r}$.

The moment of ABCD about $OO = \frac{1}{2}(b^2 - a^2)c$.

That is, the area of the distorted figure A′B′C′D′ = moment of ABCD about OO $\times \frac{1}{r}$.

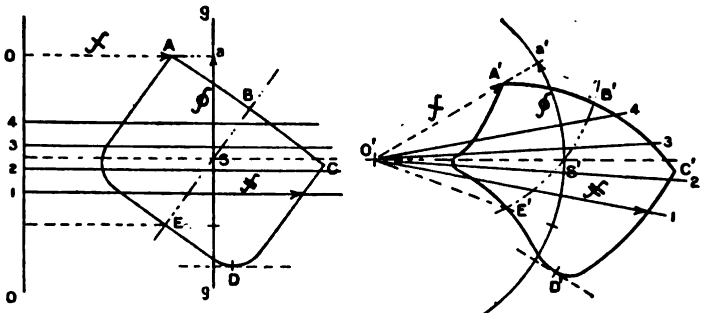


FIG. 81B.

Therefore if A is area of rectangle and *m* the distance of its C.G. from OO, we have—

$$m = A'B'C'D' \times \frac{r}{A}$$

$$= \frac{\text{Planimeter reading } A'B'C'D'}{\text{Planimeter reading } ABCD} \times r$$

Take now in Fig. 81B the section ACD representing the body plan of a ship at a given angle of inclination. The assumed C.G. is S, and the axis *gg* is taken through S for convenience distance *r* from OO. This line becomes a circular arc, radius *r* in the distorted figure. Corresponding points in the section can all be readily fixed. Thus A′ is obtained by making $S'a' = Sa$ and $O'A' = OA$. Several water-lines, 1, 2, 3 and 4, covering the range of displacement are used. The summation of planimeter readings to radial line 1 is then divided by the summation of the planimeter readings to water-line 1, and the result is multiplied by r. This gives the distance of the C.B. of the displacement up to No. 1 W.L. from the axis OO, and the distance from the line through S is at once found. Similarly for water-lines 2, 3 and 4. The displacements up to 1, 2, 3 and 4 are obtained by multiplying the summation of the planimeter readings by the proper multiplier. The above results will enable a cross curve of stability to be drawn in on a base of displacement in the ordinary way, for this particular angle.

This is a method which is available when a planimeter but not an integrator is available, which is often the case, as it is only in large drawing offices that one finds this latter expensive instrument. The method involves a very large amount of drawing work in drawing in all the distorted sections at each angle considered, but the labour is much less than formerly as Tchebycheff sections can be used by which the number of sections is much reduced. Students are strongly recommended as an exercise to work out one cross curve by this method, if they have a planimeter, as a check on other methods that may have been employed. It will be found a most fascinating drawing exercise.

6. Tabular Method of calculating Stability.—The following method of obtaining a cross-curve is a very convenient one to employ; the whole of the work being arranged in tabular form and Tchebycheff's rule being used, the fore and aft integration is easily performed by addition. Take the complete body of a ship, inclined as shown in Fig. 81C, the axis AA being perpendicular to the new water-line, and preferably always taken through the same spot in the middle line, say, the intersection of the M.L. and the L.W.L. It is the practice at Messrs. John Brown and Co., Clydebank, to use this method as an independent check against the results of the Integrator, and the tabular form reproduced in Table IV.[1] is given by the kind permission of Mr. W. J. Luke. Tchebycheff's 10 ordinate rule is used for the fore and aft integration. In the case given, the water-lines are placed 3′ 6″ apart, the lowest one being tangential to the lowest point of the bilge. For each water-line the moment of the area of water-plane

[1] See at the end of the book.

about AA is $\frac{1}{2}[\int y^2 . dx - \int (y')^2 . dx]$, and the area of the water-plane is $[\int y . dx + \int y' . dx]$. Thus, for No. 4 W.L. the sum of ordinates port and starboard side is 296·5, and this is a function of the area of the plane. As regards moment about AA, the starboard side sum of squares is in excess of the port side by 980, which is therefore a function of the moment of the plane on the starboard side. These functions are converted into area and moment, giving 15,860 square feet and 26,216 (square feet × feet). These are obtained in the first part of the table for all the water-lines considered necessary. In the second portion, the areas and moments of water-lines are integrated vertically to 2, 4, 6, 8, 10 water-lines successively as shown. The sum of functions of areas are turned into tons displacement; thus, at 2 W.L.: displacement in tons = $19{,}875 \times \frac{1}{3} \times 3{\cdot}5 \times \frac{1}{35} = 662$ tons. The sum of functions of moments are turned into moment; thus, at 2 W.L. moment in foot tons = $165{,}735 \times \frac{1}{3} \times 3{\cdot}5 \times \frac{1}{35} = 5524$. The division $5524 \div 662 = 8{\cdot}35$ feet, which gives the distance of the C.B. of the displacement to No. 2 W.L. on the starboard side of AA, and thus one spot on the cross-curve at 30° is obtained, viz. 8·35 feet, at a displacement of 662 tons. If the moment on the port side is in excess for any W.L., that is then made negative. Thus, for Nos. 6, 7, 8 in the table the port side is in excess. Similarly, when integrating the water-lines forward and aft, if the section crosses the axis AA, the ordinate is given a negative value. This, of course, will occur more frequently as the angle gets larger.

Tangent to a Cross-curve.—If, as is usual, the cross-curves are drawn to represent righting levers on base of tons displacement, the tangent at any point on a cross-curve has an inclination θ to the base line given by $\tan \theta = \frac{d.\mathrm{GZ}}{d\mathrm{W}}$. If now M is the righting moment in foot-tons, $\mathrm{GZ} = \frac{\mathrm{M}}{\mathrm{W}}$, so that—

$$\tan \theta = \frac{d}{d\mathrm{W}}\left(\frac{\mathrm{M}}{\mathrm{W}}\right) = \frac{1}{\mathrm{W}} \cdot \frac{d\mathrm{M}}{d\mathrm{W}} - \frac{\mathrm{M}}{\mathrm{W}^2} = \frac{1}{\mathrm{W}}\left(\frac{d\mathrm{M}}{d\mathrm{W}} - \frac{\mathrm{M}}{\mathrm{W}}\right)$$

$$= \frac{1}{\mathrm{W}}\left(\frac{d\mathrm{M}}{d\mathrm{W}} - \mathrm{GZ}\right)$$

Now, dM is the increment of moment due to increment of layer dW, so that $\frac{dM}{dW}$ is the distance of the C.G. of radial plane from ST = Sg, say. Therefore $\tan\theta = \frac{1}{W}(Sg - GZ)$. S$g$ is the same distance from ST as the centre of gravity of the radial plane we are dealing with, cutting off the displacement at which we are drawing the tangent.

If r_1, r_2 are ordinates of radial plane on immersed and emerged sides respectively, then with our usual notation—

$$Sg = \frac{1}{2}\frac{\int(r_1^2 - r_2^2)dx}{\int(r_1 + r_2)dx}$$

on the immersed side. These can readily be picked up on the Barnes's tables. If $Sg > GZ$, and g is to the right of S, then $\tan\theta$ is positive; if Sg is to the left of S, $\tan\theta$ is negative. If SG is $<$ GZ, $\tan\theta$ is negative. These tangents must be set off, having in view the scales that are used for righting levers and for displacement.

Dynamical Stability.—The amount of *work* done by a force acting through a given distance is measured by the product of the force and the distance through which it acts. Thus, a horse exerting a pull of 30,000 lbs. for a mile does—

$$30{,}000 \times 1760 \times 3 = 158{,}400{,}000 \text{ foot-lbs. of work}$$

Similarly, if a weight is lifted, the work done is the product of the weight and the distance it is lifted. In the case of a ship being inclined, work has to be done on the ship by some external forces, and it is not always possible to measure the work done by reference to these forces, but we can do so by reference to the ship herself. When the ship is at rest, we have seen that the vertical forces that act upon the ship are—

(1) The weight of the ship acting vertically downwards through the centre of gravity;

(2) The buoyancy acting vertically upwards through the centre of buoyancy;

these two forces being equal in magnitude. When the ship is inclined, they act throughout the whole of the inclination. The centre of gravity is raised, and the centre of buoyancy is lowered. The weight of the ship has been made to move upwards the distance the centre of gravity has been raised, and the force of the buoyancy has been made to move downwards the distance the centre of buoyancy has been lowered. The work done on the ship is equal to the weight multiplied by the rise of the centre of gravity added to the force of the buoyancy multiplied by the depression of the centre of buoyancy; or—

Work done on the ship = weight of the ship multiplied by the vertical separation of the centre of gravity and the centre of buoyancy.

This calculated for any given angle of inclination is termed "*the dynamical stability*" at that angle, and is the work that has to be expended on the ship in heeling her over to the given angle.

Moseley's Formula for the Dynamical Stability at any Given Angle of Inclination.—Let Fig. 67 represent a vessel heeled over by some external force to the angle θ; g, g' being the centres of gravity of the emerged and immersed wedges; gh, $g'h'$ being drawn perpendicular to the new water-line W′L′. The other points in the figure have their usual meaning, BR and GZ being drawn perpendicular to the vertical through B′.

The vertical distance between the centres of gravity and buoyancy when inclined at the angle θ is B′Z.

The original vertical distance when the vessel is upright is BG.

Therefore the vertical separation is—

$$B'Z - BG$$

and according to the definition above—

$$\text{Dynamical stability} = W(B'Z - BG)$$

where W = the weight of the ship in tons.

$$\text{Now, } B'Z = B'R + RZ = B'R + BG \,.\, \cos\theta$$

Now, using v for the volumes of either the immersed wedge or the emerged wedge, and V for the volume of displacement of the ship, and using the principle given on p. 100, we have

$$v \times (gh + g'h') = V \times B'R$$

$$\text{or } B'R = \frac{v \times (gh + g'h')}{V}$$

Substituting in the above value for B'Z, we have—

$$B'Z = \frac{v \times (gh + g'h')}{V} + BG \cos \theta$$

$$\therefore \left.\begin{matrix}\text{the dynamical}\\ \text{stability}\end{matrix}\right\} = W\left[\frac{v \times (gh + g'h')}{V} - BG(1 - \cos \theta)\right]$$

which is known as *Moseley's formula.*

It will be seen that this formula is very similar to Atwood's formula, and it is possible to calculate it out for varying angles by using the tables in Barnes' method of calculating stability. It is possible, however, to find the dynamical stability of a ship at any angle much more readily if the curve of statical stability has been constructed, and the method adopted, if the dynamical stability is required, is as follows:—

The dynamical stability of a ship at any given angle is equal to the area of the curve of statical stability up to that angle (the ordinates of this curve being the actual righting moments).

Referring to Fig. 67, showing a ship heeled over to a certain angle θ, imagine the vessel still further heeled through a very small additional angle, which we may call $d\theta$. The centre of buoyancy will move to B″ (the student should here draw his own figure to follow the argument). B′B″ will be parallel to the water-line W′L′, and consequently the centre of buoyancy will not change level during the small inclination. Drawing a vertical B″Z′ through B″, we draw GZ′, the new righting arm, perpendicular to it. Now, the angle $ZGZ' = d\theta$, and the vertical separation of Z and $Z' = GZ \times d\theta$. Therefore the work done in inclining the ship from the angle θ to the angle $\theta + d\theta$ is—

$$W \times (GZ \, . \, d\theta)$$

Take now the curve of statical stability for this vessel. At the angle θ the ordinate is GZ. Take a consecutive ordinate at the angle $\theta + d\theta$. Then the area of such a strip $= GZ \times d\theta$; but this multiplied by the displacement is the same as the above expression for the work done in inclining the vessel through the angle $d\theta$, and this, being true for any small angle $d\theta$, is true for all the small angles up to the angle θ. But the addition of the work done for each successive increment of inclination up to a given angle is the dynamical stability at that angle, and the sum of the areas of such strips of the curve of statical stability as we have dealt with above is the area of that curve up to the angle θ. Therefore we have the dynamical stability of a ship at any given angle of heel is equal to the area of the ordinary curve of statical stability up to that angle, multiplied by the displacement.

To illustrate this principle, take the case of a floating body whose section is in the form of a circle, and which floats with its centre in the surface of the water. The transverse metacentre of this body must be at the centre of the circular section. Let the centre of gravity of the vessel be at G, and the centre of buoyancy be at B. Then for an inclination through 90° G will rise till it is in the surface of the water, but the centre of buoyancy will always remain at the same level, so that the dynamical stability at $90° = W \times GM$.

Now take the curve of statical stability for such a vessel. The ordinate of this curve at any angle $\theta = W \times GM \, . \sin \theta$, and consequently the ordinates at angles 15° apart will be $W \, . \, GM \, . \sin 0°$, $W \, . \, GM \, . \sin 15°$, and so on; or 0·258 W.GM, 0·5 W.GM, 0·707 W.GM, 0·866 W.GM, 0·965 W.GM, and W.GM. If this curve is set out, and its area calculated, it will be found that its area is $W \times GM$, which is the same as the dynamical stability up to 90°, as found above.

It should be noticed that the angular interval should not be taken as degrees, but should be measured in circular measure (see p. 90). The circular measure of 15° is 0·2618.

The dynamical stability at any angle depends, therefore, on the area of the curve of statical stability up to that angle; and thus we see that the *area* of the curve of stability is of importance as well as the *angle* at which the ship becomes unstable, because it is the dynamical stability that tells us the work that has to be expended to force the ship over. For full information on this subject the student is referred to the "Manual of Naval Architecture," by Sir W. H. White, and Sir E. J. Reed's work on the "Stability of Ships."

Mr. Hŏk's Method of obtaining a Curve of Stability.—In this method the ordinary planimeter is used, and as the use of curves through various spots obtained is a feature of the method, the work is readily checked as one proceeds. The method first obtains the curve of dynamical stability for a given displacement, and then from this curve the curve of statical stability is deduced. The former curve is the integral of the latter, and so the latter is the differential of the former. That is, if H is the dynamical stability at angle θ, then—

$$H = \int_0^\theta . W . GZ . d\theta \quad \text{and} \quad GZ = \frac{1}{W} \cdot \frac{dH}{d\theta}.$$

Take the body prepared with the sections on both sides in the ordinary way inclined, as shown in Fig. 81C. Then, by means of the planimeter, we can determine the displacement up to the various dotted water-lines, and so construct a curve of displacement. A line parallel to the base line and distance away equal to the displacement V, is drawn as shown, and this gives the draught at which we shall cut off the displacement required, and for which we desire the righting lever. The area of *owl* divided by *wl* gives the distance of the C.B. below W.L., so that $wb = owl \div wl$. The area *owl* is readily obtained by the planimeter. Now, G being the assumed position of the centre of gravity, and B′ the new C.B., B being the C.B. when upright, the dynamical stability = W(B′Z − BG), and B′Z − BG can be readily found by measurement. By repeating this for a number of angles, a curve of dynamical stability can be drawn, observing that the W portion can be left out, being constant all through. If h be such

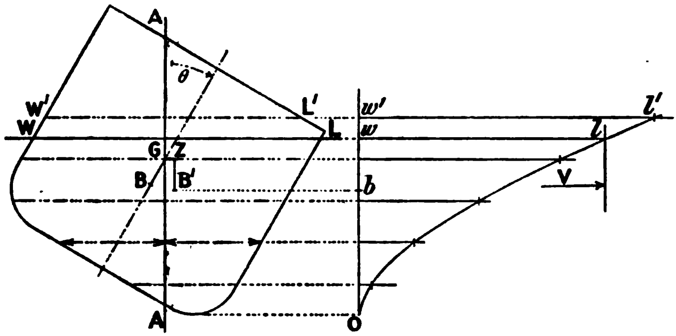


FIG. 81C.

that $H = W . h$, and h_{10}, h_{20}, h_{30}, etc., be the values at 10°, 20°, 30°, etc., and GZ_{10}, GZ_{20}, etc., be the values of the righting arm at 10°, 20°, etc., then at 10° we have—

$$h_{10} = \frac{1}{12} . (0 \cdot 1745)(\overline{5 \times 0} + \overline{8 \times GZ_{10}} - GZ_{20})$$

(0·1745 is the c.m. of 10°)

and $$h_{20} = \frac{1}{3}(0 \cdot 1745)(0 + \overline{4 \times GZ_{10}} + GZ_{20})$$

By these two equations GZ_{10} and GZ_{20} can be obtained. Also $h_{30} = \frac{3}{8}(0 \cdot 1745)(0 + 3\ GZ_{10} + 3\ GZ_{20} + GZ_{30})$, and GZ_{10} and GZ_{20} being known GZ_{30} is found. Thus values of GZ can be determined and the ordinary curve of stability drawn in for the given displacement and assumed C.G. (For all practical purposes $h_{10} = \frac{1}{2} GZ_{10} \times 0 \cdot 1745$, seeing that the ordinary curve of stability is straight near the origin.) This can be done for other assumed displacements, and so a set of cross-curves drawn in for constant angles on a base of displacement, as already explained. All these, of course, assume a constant position for the C.G., and if at any displacement another position of the C.G. has to be allowed for, $GG' \sin \theta$ is added or subtracted, according as the new G' is below or above the old G.

Stability of Self-righting Lifeboats.—The stability of these boats offers several points of interest. The properties of such boats are—

(1) Very large watertight reserve of buoyancy, which renders the boat practically unsinkable.

(2) The water shipped is automatically cleared.

(3) The loss of stability due to shipping water is not sufficient to cause instability.

(4) The boat is unstable when upside down.

(1) For this the ends of the boat have great sheer, and the ends are filled in with air-cases or tanks. These cases are also placed under the deck and at the sides between the deck and seats (see Fig. 81D). The cases are in such numbers that even if some are broached there would still be a sufficient buoyant volume left.

The large buoyant volume at the ends gives great lifting power to the boat when encountering a sea.

(2) The deck of the boat is somewhat higher than the water-line, and in this deck, passing through to the bottom, are eight tubes with *automatic freeing valves*, which allow water to pass down, but not up. These are adjusted so as to drop down with a small pressure of water above. The rise and fall of the boat will soon cause any water on the deck to be discharged.

The deck falls towards midships, and has a "round down," so that water will flow to the valves.

(3) Shipping water on the deck has two effects on the stability, viz.—

(*a*) Will raise the C.G. due to added weight.

(*b*) Will make the *virtual* C.G. higher than the *actual* C.G. with the added water, because of the free surface. (Rise of G $= \frac{i}{V}$, where i is the moment of inertia of free surface, and V is volume displacement with added water).

In order that these two effects may not render the boat unstable—

(i.) An iron keel is fitted to pull C.G. down;

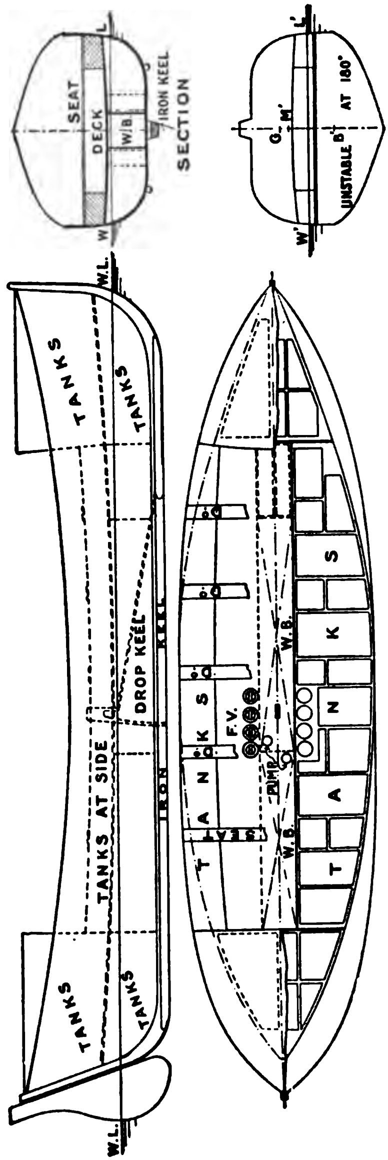


Fig. 81d.—Thirty-five feet self-righting lifeboat.

(ii.) The ends remain intact, so that the lost moment of inertia i shall not be too great.

The GM in this extreme condition should be positive, or otherwise when the boat, after capsizing, came back to the upright she would again capsize, owing to the water on the deck.

This GM will, however, only be small, and as the water-level falls the i will reduce considerably owing to the presence of the side tanks, and the water becoming less in quantity also helps matters.

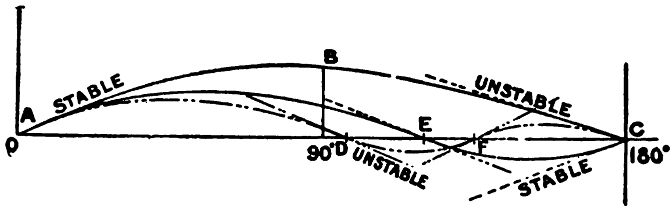


FIG. 81E.

(4) This instability when upside down is the property known as *self-righting*. In this condition, let B′, M′ be C.B. and metacentre respectively, and G the centre of gravity. Then G must be nearer the keel than M′.

The buoyancy being provided largely by the ends, B′ will be a good way from keel, G is also near keel owing to iron keel; therefore B′G is large. Again, B′M′ is not large, because the moment of inertia is provided largely by the ends; therefore we get M′ below G, and the boat is unstable.

It is also necessary that the boat should have a curve of stability as ABC, Fig. 81E, the only positions of equilibrium being at upright and 180°. If the curve of stability were like ADFC, there would be a position of stable equilibrium at F, which would not be desirable. If the curve were as AEC, then the equilibrium at 180° would be stable.

These boats are tested, when fully equipped, with mast up and sail set, by immersing to the gunwale by weights to represent men. The boat is then turned upside down by a chain attached to a crane. The chain is then slipped, when the boat should return to the upright position.

In Fig. 81D the boat is of the type with a drop keel, and provision is made for water ballast. There are eight automatic freeing valves.

Stability of Sailing Vessels—Power to carry sail.—For comparative purposes the sail area is taken as the "plain" or "working" sail, and this is assumed all braced fore and aft. This sail for a ship would include "jib," "fore and main courses," "driver," three "topsails," and three "topgallant sails." The *centre of effort* is assumed as the C.G. of the area of these sails. The *centre of lateral resistance* of the water is taken as the C.G. of the middle line area. The couple heeling the ship is caused by the resultant of the wind pressure and the fluid pressure on the opposite side of the ship. If A is area of sails in square feet, p is pressure in lbs. per square foot, h the vertical distance between the C.E. and the C.L.R., and θ the angle of heel, then the moment of the couple heeling the ship is $\frac{p \times A \times h}{2240}$, and this equals the moment of stability $W \times GM \times \sin\theta$. Taking for comparative purposes a pressure of 1 lb. per square foot (equivalent to a wind of about fourteen knots).

$$\frac{1}{\sin\theta} = \frac{W \times GM}{A \times h} \times 2240.$$

This is termed the *power to carry sail,* and is a measure of the stiffness of a ship. The greater this is, the less a given vessel will incline under a given wind. Sailing merchant vessels have a value 12 to 20; the sloops in the Navy, 12 to 15; smaller values are usual in sailing yachts, as the crew have an influence on the heel by going over to the leeward side.

If a curve of wind moment be constructed on base of angle and plotted on the curve of stability (ordinates representing righting moments), where the two cross will represent where the stability equals the heeling moment, and this is the *angle of steady heel*—in Fig. 81F this is 10°. The area of sail projected on to the vertical plane is $A \times \cos\theta$, and the lever of the moment is $h \times \cos\theta$, so that the heeling moment is $p . A . h . \cos^2\theta$, and from this a curve can be drawn as AD, Fig 81F. If now the ship is supposed upright and exposed to a sudden squall, the work done by the wind will be the area

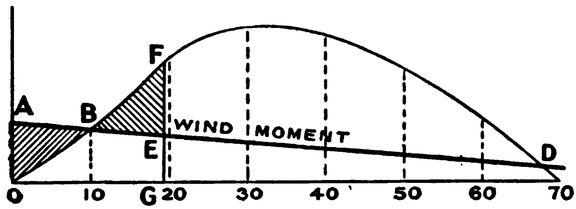


FIG. 81F.

under the wind moment curve. The work done to any angle by the stability will be the area under the curve of stability. At the angle of steady heel, the former is in excess by the amount OAB, and the ship will heel over until the area OAB is equal to the area BEF. If the wind remains constant, the ship will eventually settle to the angle of steady heel—in this case 10°. The area BEDF above the wind curve is thus the reserve the vessel has against further wind pressure, and this area is termed the "*reserve of dynamical stability.*"

If a ship is struck by a squall at the moment of completing a roll to windward, say 10°, the wind moment and the stability of the ship both act together in taking the vessel to the upright, the work being represented by the area OAEF,

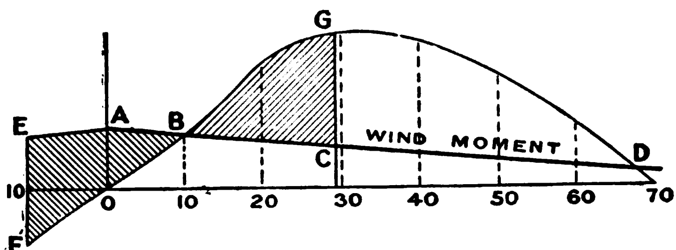


FIG. 81G.

Fig. 81G. It is only after passing the upright that the stability acts against the wind, and the ship must heel to an angle 29°, such that the area FEABO is equal to the area BCG. It is thus seen that a sailing vessel with a low curve of stability, like the *Captain* (Fig. 68), may have insufficient reserve of dynamical stability, and under the above circumstances, might be blown right over. Mr. Wall, I.N.A., 1914, introduced this principle into ordinary ship calculations.

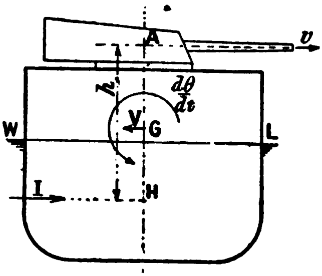


FIG. 81H.

Heel produced by Gun Fire[1] (Fig. 81H).—This problem has to use the principle of momentum. If w and W be the weights of projectiles (and powder) and the ship respectively, v the velocity of the projectile, then on firing, the C.G. of ship will have a backward velocity of V, and if I is the impulsive reaction of the water at rather less than half draught, we have the equation—

$$I = \frac{w}{g} . v - \frac{W}{g} . V.$$

$\frac{w}{g} . v$ being the momentum of the shot, and $\frac{W}{g} . V$ that of the ship.

The angular momentum of the ship is $\frac{W}{g} . k^2 . \frac{d\theta}{dt}$, and this has to be equated to the moment of momentum causing the rotation or—

$$\frac{w}{g} . v . GA + I . HG = \frac{W}{g} . k^2 . \frac{d\theta}{dt}$$

from which—

$$\frac{w}{g} . v . AH = \frac{W}{g} . \left(k^2 . \frac{d\theta}{dt} + V . HG\right)$$

V is practically negligible, so that—

$$\frac{w}{g} . v . AH = \frac{W}{g} . k^2 . \frac{d\theta}{dt}$$

$$T = \pi\sqrt{\frac{k^2}{m . g}}, \text{ from which } k^2 \text{ can be found.}$$

The initial kinetic energy of the ship is $\frac{1}{2} . W . k^2 . \left(\frac{d\theta}{dt}\right)^2$, and if θ is the angle of heel this is equated to the dynamical

[1] See Chap. XII. on The Rolling of Ships for the definitions of T and k.

stability at θ supposing resistances are neglected, *i.e.*

$$\tfrac{1}{2}.W.m.\theta^2,$$

regarding the curve of stability a straight line.

$$\therefore \theta^2 = \frac{k^2}{m.g}\cdot\left(\frac{d\theta}{dt}\right)^2 = \frac{k^2}{m.g}\cdot\left(\frac{w}{W}\cdot\frac{v.AH}{k^2}\right)^2$$

$$\text{or } \theta = \frac{w}{W}\cdot\frac{v.AH}{k^2}\cdot\frac{1}{\sqrt{m.g}} = \frac{w}{W}\cdot\frac{v.AH.\pi}{m.g.T} \text{ circular measure}$$

$$= \frac{w}{W}\cdot\frac{v.AH}{m.g.T}.\ 180 \text{ degrees.}$$

As an example, take a case in which 8 guns are fired on the broadside 25 feet above water, H being 13 feet below water, $w = 1100$ lbs., velocity of discharge 3000 fs., G.M. 5 feet, displacement 18,000 tons, T = 8 seconds.

$$\theta \text{ in degrees} = \frac{1100 \times 3000 \times 38 \times 8 \times 180}{2240 \times 18{,}000 \times 5 \times 32{\cdot}2 \times 8}$$

$$= 3\tfrac{1}{2} \text{ degrees nearly.}$$

Example.—Determine the heel caused by firing simultaneously 4 guns at a muzzle velocity of 1600 feet per second, the weight of projectile and powder being 2375 lbs., the height of guns above centre of lateral resistance being 30 ft., the time of a single oscillation 6 seconds, and the metacentric height $4\tfrac{1}{2}$ feet. Displacement of ship 10,000 tons.

Ans. $4\tfrac{1}{4}$ degrees.

Angle of Heel of a Vessel when Turning.—On putting a vessel's rudder over the pressure on the rudder which acts below the centre of lateral resistance tends to heel the vessel inwards. This inward heeling is very noticeable in the case of destroyers, in which the rudder area is relatively large. In ordinary ships this is only of short duration, and when the the vessel gets on the circle, an outward heel is caused by the centrifugal action, which acts above the centre of lateral resistance (C.L.R.). The moment caused by the product of the centrifugal force and the distance of the C.G. from the C.L.R. is the moment causing the heel, and this is equated to the moment of stability at angle θ. This θ is determined.

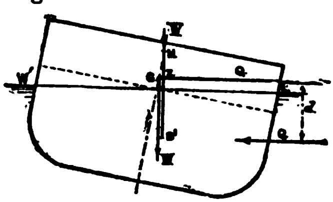


FIG. 811.

The centrifugal force Q caused by a weight W tons moving in a circle radius R feet at speed v feet per second is $\frac{W}{g}\cdot\frac{v^2}{R}$, and the moment of the couple causing heel is $\frac{W}{g}\cdot\frac{v^2}{R}\cdot d$. This is equated to the moment of stability at angle θ, viz. W. GM sin θ, or—

$$\sin\theta = \frac{1}{32.2}\cdot\frac{v^2}{R}\cdot\frac{d}{GM} = 0\text{·}088\left(\frac{V^2}{R}\cdot\frac{d}{GM}\right) \quad \text{(V in knots).}$$

The features therefore which lead to a large heel when turning are (1) high speed, (2) small turning circle, and (3) small metacentric height.

Example.—A vessel whose tactical diameter is 463 yds. at a point on her circular turn has a speed of 15 knots, the draught being 27 ft. and the metacentric height 3·5 ft. Approximate to the angle of heel.

In this case V = 15, R = 695 ft. d = 13 ft. about. GM = $3\frac{1}{2}$ ft.

$$\therefore \sin\theta = 0\text{·}088\,.\,\frac{225}{695}\,.\,\frac{13}{3\text{·}5} = 0\text{·}106, \text{ and } \theta = 6^\circ.$$

Metacentric Height when inclined about an Axis inclined at an Angle a with the longitudinal Middle Line Plane.—We first have to find the moment of inertia of the waterplane about an axis inclined to the principal axes OX and OY, viz. OZ in Fig. 81K. O is the C.G. of waterplane.

Drawing as in figure.

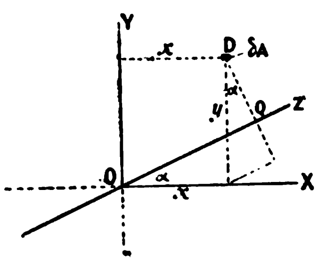


FIG. 81K.

$$I = \Sigma\delta A \times \overline{DQ}^2 \text{ where } \delta A \text{ is an element of area}$$
$$= \Sigma\delta A(y\cos a - x\sin a)^2$$
$$= \Sigma\delta A(y^2\cos^2 a + x^2\sin^2 a - 2xy\sin a\cos a)$$

The last term vanishes on summation since the axes are through the C.G. of plane.

$$\therefore I = \cos^2 a\textstyle\int y^2\,.\,dA + \sin^2 a\int x^2\,.\,dA$$
$$= I_T\,.\,\cos^2 a + I_L\,.\,\sin^2 a$$
$$\therefore BM_a = BM_T\,.\,\cos^2 a + BM_L\,.\,\sin^2 a$$
$$\text{also } BG = BG\,.\,(\cos^2 a + \sin^2 a) \text{ since } \cos^2 a + \sin^2 a = 1$$
$$\therefore GM_a = GM_T\,.\,\cos^2 a + GM_L\,.\,\sin^2 a.$$

Example.—A box-shaped vessel is 80′ long, 20′ wide and floats at a draught of water of 10′. Find the value of the distance between the centre of buoyancy and the metacentre for inclinations about an axis coincident with a diagonal of the rectangular waterplane. (*Honours B. of E.* 1911).

In this case—

$$\alpha = \tan^{-1} \tfrac{1}{4}$$
$$\sin^2 \alpha = \tfrac{1}{17}, \cos^2 \alpha = \tfrac{16}{17}$$
$$BM_T = 3{\cdot}33, \quad BM_L = 53{\cdot}33$$
$$\therefore BM_\alpha = (3{\cdot}33 \times \tfrac{16}{17}) + (53{\cdot}33 \times \tfrac{1}{17}) = 6{\cdot}27 \text{ feet.}$$

Examples to Chapter V.

1. A two-masted cruiser of 5000 tons displacement has its centre of gravity at two feet above the water-line. It is decided to add a military top to each mast. Assuming the weight of each military top with its guns, men, and ready-ammunition supply to be 12 tons, with its centre of gravity 70 feet above the water-line, what will be the effect of this change on—

(1) The metacentric height of the vessel?

(2) The maximum range of stability, assuming the present maximum range is 90°, and the tangent to the curve at this point inclined at 45° to the base-line?

(Scale used, ¼ inch = 1°, ¼ inch = $\frac{1}{10}$ foot GZ.)

Ans. (1) Reduce 0·325 foot, assuming metacentric curve horizontal; (2) reduce range to about 86¾°, assuming no change in cross-curves from 5000 to 5024 tons.

2. The curve of statical stability of a vessel has the following values of GZ at angular intervals of 15°: 0, 0·55, 1·03, 0·99, 0·66, 0·24, and − 0·20 feet. Determine the loss in the range of stability if the C.G. of the ship were raised 6 inches.

Ans. 16°.

3. Obtain, by direct application of Atwood's formula, the moment of stability in foot-tons at angles of 30°, 60°, and 90°, in the case of a prismatic vessel 140 feet long and 40 feet square in section, when floating with sides vertical at a draught of 20 feet, the metacentric height being 2 feet.

4. A body of square section of 20 feet side and 100 feet long floats with one face horizontal in salt water at a draught of 10 feet, the metacentric height being 4 inches. Find the dynamical stability at 45°.

Ans. 171 foot-tons.

5. Indicate how far a vessel having high bulwarks is benefited by them as regards her stability. What precautions should be taken in their construction to prevent them becoming a source of danger rather than of safety?

6. Show from Atwood's formula that a ship is in stable, unstable, or neutral equilibrium according as the centre of gravity is below, above, or coincident with the transverse metacentre respectively.

7. A vessel in a given condition displaces 4600 tons, and has the C.G. in the 19-feet water-line. The ordinates of the cross-curves at this displacement, with the C.G. assumed in the 19-feet water-line, measure as follows: 0·63, 1·38, 2·15, 2·06, 1·37, 0·56 feet at angles of 15°, 30°, 45°, 60°, 75°, and 90° respectively. The metacentric height is 2·4 feet. Draw out the curve of stability, and state (1) the angle of maximum stability, (2) the angle of vanishing stability, and (3) find the dynamical stability at 45° and 90°.

Ans. (1) 50¾°; (2) 100¼°; (3) 3694, 9650 foot-tons.

8. A vessel has a metacentric height of 3·4 feet, and the curve of stability has ordinates at 15°, 30°, 37½°, 45°, and 60° of 0·9, 1·92, 2·02, 1·65, and − 0·075 feet respectively. Draw out this curve, and state the angle of maximum stability and the angle at which the stability vanishes.

Ans. 35½°, 59½°.

9. A vessel's curve of stability has the following ordinates at angles of 15°, 30°, 45°, 60°, and 75°, viz. 0·51, 0·97, 0·90, 0·53, and 0·08 feet respectively. Estimate the influence on the range of stability caused by lifting the centre of gravity of the ship 0·2 foot.

Ans. Reduce nearly 6°.

10. A square box of 18 feet side floats at a constant draught of 6 feet, the centre of gravity being in the water-line. Obtain, by direct drawing or otherwise, the value of GZ up to 90° at say 6 angles. Draw in the curve of statical stability, and check it by finding its area and comparing that with the dynamical stability of the box at 90°.

(Dynamical stability at 90° = 3 × weight of box.)

11. A vessel fully loaded with timber, some on the upper deck, starts from the St. Lawrence River with a list. She has two cross-bunkers extending to the upper deck. She reaches a British port safely, with cargo undisturbed, but is now upright. State your opinion as to the cause of this.

12. Show by reference to the curves of stability of box-shaped vessels on p. 174 that at the angle at which the deck edge enters the water the tangent to the curve makes the maximum angle with the base-line.

13. The curve of stability of a vessel at angles of 15°, 30°, 45°, 60°, 75°, and 90° shows the following values of the righting arm, viz. 0·22, 0·71, 1·05, 1·02, 0·85, and 0·56 feet respectively, the metacentric height being 8 inches and the displacement being 4500 tons. Discuss in detail the condition and behaviour of the ship if 200 tons were removed from a hold 17 feet below the centre of gravity.

(*Assume that the cross-curves from* 4300 *to* 4500 *tons are all parallel to the base-line.*)

14. A vessel of 1250 tons displacement has its centre of gravity 10½ feet above keel. The stability curve (on scale ¼ inch = 1°, ¼ inch = $\frac{1}{10}$ foot) ends as a straight line at 20° slope to the base line, the range being 80°. Find the alteration in metacentric height and range of stability due to taking in 30 tons reserve feed water 3 feet above the keel.

Ans. Increase 0·176 foot in GM.
,, 5° in range.

15. Show that for a wall-sided vessel inclined to an angle θ—

$$GZ = \sin\theta\,(GM + \tfrac{1}{2}BM\,.\tan^2\theta)$$

where GM and BM refer to the upright condition.

16. Show, by using the above formula, that if a wall-sided vessel has a negative metacentric height she will loll over to an angle ϕ such that—

$$\tan\phi = \pm\sqrt{2\,.\frac{GM}{BM}}$$

17. Apply the answer to question 16 to show that the log shown floating in Fig. 55 with a negative G.M. of $\frac{a}{6}$, will take up the position corner downwards if left free.

18. A box-shaped vessel is 100 feet long, 30 feet broad, and 16 feet deep. In the load condition the freeboard is 4 feet and the metacentric height is 6 feet. In the light condition the freeboard is 10 feet and the

metacentric height is still 6 feet. Compare fully the stability of the vessel in these two conditions. (This should be treated in the light of remarks on p. 181.)

19. A vessel 72 feet long floats at 6 feet draught and has $4\frac{1}{2}$ feet freeboard, with sides above water vertical. Determine the GZ at 90°, the C.B. when upright being $3\frac{1}{2}$ feet above keel, the C.G. 1 foot below water-line, the half-ordinates of water-plane being 0·8, 3·3, 5·4, 6·5, 6·8, 6·3, 5·1, 2·8, 0·6, and the displacement 100 tons. *Ans.* + 0·67 foot.

20. A prismatic vessel is 32 feet broad, 13 feet draught, 9 feet freeboard, the bilges being circular arcs of 6 feet radius. GM is 2 feet.

(1) Obtain the first part of curve of stability by formula in question 15 above.

(2) Obtain values of GZ at 45° and 72° by using Barnes's method, using 9° angular intervals.

(3) Obtain GZ at 90° by direct method.

Thus draw in the complete curve of stability.

21. In the above vessel, instead of the sides above water being vertical, they fall in from 1 foot above the water-line to the deck, where the breadth is 24 feet. Obtain the complete curve of stability in a similar manner to the preceding question.

(The curves in these two questions are given in the author's text-book on "Warships," chap. xix.)

22. In obtaining cross-curves by calculation as described above, if v, v' are volumes of emerged and immersed wedges, between the upright water-plane and the radial plane through O, and g, h, g', h', are as in our ordinary notation, show that the righting lever for displacement $V + v' - v$ is given by—

$$\frac{\overline{v \times Oh} + \overline{v'O'h'} - \overline{V . OB \sin \theta}}{V + v' - v}$$

so that, not needing to correct for layer to get the displacement V, we get for example of Barnes's method in this chapter a lever of 2·52 feet at a displacement of 10,158 tons and angle 30°.

23. In using the integrator for stability calculation, the "figure 8" method is frequently employed. This consists in using the displacement MWL with its C.B. B as a basis, and running round sections of wedges in direction SL′L, WW′S. By this means the integrator *adds* up the

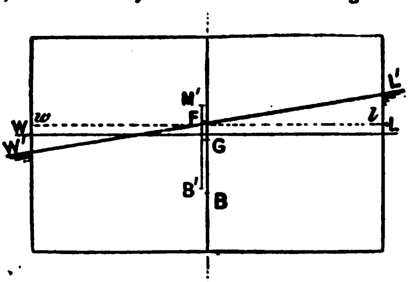


FIG. 81 L.

moments of wedges and *subtracts* the volumes. If w, w' are displacements of in and out wedges, the displacement to W′L′ is W + (area reading ×

proper factor). The GZ at angle θ and displacement $W + w - w'$ is given by—

$$\frac{(\text{moment reading} \times \text{proper factor}) - (W \,.\, BS \,.\, \sin\theta)}{\{W + (\text{area reading} \times \text{proper factor})\}}$$

24. A box-shaped vessel 420 feet long, 72 feet broad, and 24 feet constant draught has a compartment amidships 60 feet long, with a W.T. middle line bulkhead extending the whole depth. Determine the angle of heel caused by the vessel being bilged on one side abreast this bulkhead, the C.G. of the vessel being 23 feet above the keel.

To what height should the transverse bulkheads at the ends of the bilged compartment be carried, so as to confine the water to this part of the vessel?

This is done in two steps: (1) sinkage, (2) heel. See Fig. 81 L.

$$\text{Bodily sinkage} = \frac{\text{volume lost buoyancy}}{\text{area intact W.P.}} = \frac{60 \times 24 \times 36}{390 \times 72} = 1{\cdot}85 \text{ feet.}$$

I of intact W.P. about middle line $= \frac{1}{12} . 360 . 72^3 + \frac{1}{3} . 60 . 36^3 = 12,130,560$

C.F. from middle line as also the C.B. $= 1{\cdot}12$ feet

I_0 of intact W.P. about axis through C.F. $= 12,130,560 - (390 . 72 . \overline{1{\cdot}12^2})$
$= 12,102,560.$

Rise of C.B. being one-half the bodily sinkage $= 0{\cdot}92$ feet.

New B'M' $= \dfrac{12.102.560}{420 \times 72 \times 24} = 16{\cdot}7$ feet.

Vertical distance between G and M' before heeling $= 12 + 0{\cdot}92 + 16{\cdot}7 - 23{\cdot}0$
$= 6{\cdot}62$ feet.

The vessel must heel until G and M' are in the same vertical, so that, θ being the angle of heel,

$$\tan\theta = \frac{1{\cdot}12}{6{\cdot}62} = 0{\cdot}16 \quad \therefore \theta = 9^\circ \text{ nearly.}$$

The height of bulkhead $= 24{\cdot}0 + 1{\cdot}85 + (37{\cdot}12 \times \tan\theta)$
$= 31{\cdot}8$ feet nearly.

25. Example of a similar nature for a box 400 feet by 75 feet by 26 feet, midship compartment with a M.L. bulkhead 50 feet long. C.G. of ship 25 feet from keel.

Ans. $\theta = 12\frac{1}{4}^\circ$, height of bulkhead, $36{\cdot}23$ feet.

26. Example of a similar nature for a box 350 feet by 60 feet by 20 feet, midship compartment 35 feet with a M.L. bulkhead. G.M. = 8 feet.

Ans. $\theta = 6^\circ$, height of bulkhead $24{\cdot}19$ feet.

27. A prismatic vessel 100 ft. long has a transverse section formed of a rectangle, height 10 ft. breadth 20 ft. resting on the top of a semicircle of radius 10 ft. The centre of gravity is 3 ft. above the keel and the draught of water is 10 ft. Find the volume of the correcting layer and the righting moment when the vessel is inclined 45°, the displacement being constant.

(*B. of E.* 1911.)

This is an excellent example to show the application of

Barnes' method, and the solution is accordingly given herewith. (Fig. 81M).

Taking angular intervals of 15° we have the following for

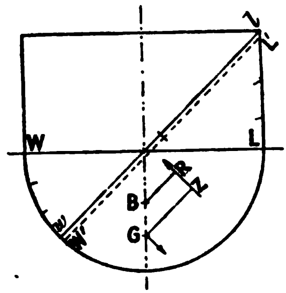


FIG. 81M.

the first part of the combination tables. The preliminary tables are not necessary, seeing that the section of vessel is constant.

	IMMERSED WEDGE.				EMERGED WEDGE.			
Inclination of radial plane.	Ordinates.	Squares.	S.M.	Products.	Ordinates.	Squares.	S.M.	Products.
0	10	100	1	100	10	100	1	100
15	10·3	106	3	318	10	100	3	300
30	11·5	132	3	396	10	100	3	300
45	14·14	200	1	200	10	100	1	100
				1014				800

$$\text{Layer} = (1014 - 800) \times \tfrac{1}{2} \times \tfrac{3}{8} \times 0{\cdot}2617 \times 100 \text{ cubic ft.}$$

$$= \frac{2{\cdot}4 \times 3 \times 0{\cdot}2617 \times 100}{16} = 1050 \text{ cub. ft.}$$

Area of radial plane $= 100 \times 24{\cdot}14$.
Thickness of layer $= 0{\cdot}435$ ft.

The table for finding the uncorrected moment of the wedges is as follows: the cubes of the ordinates, on immersed and emerged sides of each radial plane being added together before putting through the necessary multipliers to satisfy

$$\tfrac{1}{3}\int\int y^3 \,.\, \cos\theta \,.\, dx \,.\, d\theta$$ (see p. 191).

Angle of radial plane.	Sums of cubes of ordinates.	S.M.	Products.	Cos θ (from radial plane).	Products.
0	2,000	1	2,000	0·707	1,414
15	2,093	3	6,279	0·866	5,440
30	2,521	3	7,563	0·966	7,300
45	3,828	1	3,828	1·0	3,828
					17,982

$$\text{Uncorrected moment} = 17{,}982 \times \tfrac{1}{3} \times \tfrac{3}{8} \times 0{\cdot}2617 \times 100$$
$$= 58{,}750 \text{ cubic ft.} \times \text{feet.}$$

The C.G. of the layer is on the immersed side, 2·07 ft., so that the layer correction is 1050 × 2·07 = 2170 cubic ft. × feet. The layer has to be subtracted, seeing that immersed wedge is in excess. The moment is accordingly deducted. The corrected moment is therefore 58,750 − 2170 = 56,580, which is the $v \times hh'$ in Atwood's formula.

$$V = 100 \times \tfrac{22}{7} \,.\, 100 \,.\, \tfrac{1}{2} = 15{,}700 \text{ cubic ft.}$$

$$\text{B below W.L.} = \frac{4}{3\pi} \,.\, \text{radius} = 4{\cdot}25 \text{ ft.}$$

$$\therefore \text{BG is } 2{\cdot}75 \text{ ft.}$$

$$GZ = \frac{v \times hh'}{V} + BG \sin\theta. \quad \text{(G below B.)}$$

$$= \frac{56{,}580}{15{,}700} + (2{\cdot}75 \times 0{\cdot}707)$$

$$= 3{\cdot}6 + 1{\cdot}94 = 5{\cdot}54 \text{ feet.}$$

$$\text{Righting moment} = \frac{15{,}700}{35} \times 5{\cdot}54 = 24{,}800 \text{ foot tons.}$$

CHAPTER VI.

TROCHOIDAL WAVE THEORY.

Trochoidal Wave Theory.—All the observed phenomena of ocean waves, that is, waves in deep water, fit in so well with the trochoidal wave theory that this theory is generally accepted. It is assumed that the waves traverse an ocean of unlimited extent and the depth of water in relation to the dimensions of the wave is sufficiently great so that this also may be assumed unlimited. It will be seen later that the depth of water at which wave motion is practically negligible

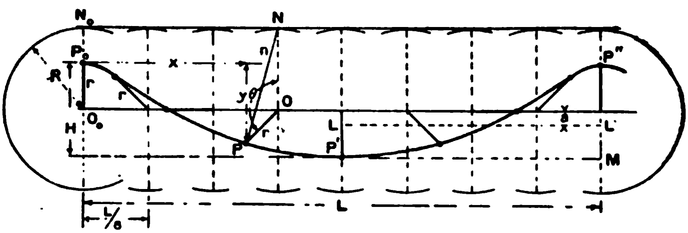


FIG. 81N.

is comparatively small. Sometimes these ideal conditions are fulfilled in ocean waves, but generally several series of waves are superimposed on one another often in different directions, the result being a confused sea. It is necessary in considering the subject to assume a single series of waves.

A trochoid is the curve traced out by a point inside a circle when the circle is rolled along a straight line, and if the circle is below the line it will be found that the resulting curve is sharper at the crest than at the trough (Fig. 81N). If the radius of the rolling circle is R and the circle rolls along the line N_0N, the velocity of the centre of the circle being V, and the radius of the point P within the circle being r, then P traces out a curve which is a trochoid. The length from crest to crest $L = 2\pi R$, and the height from crest to trough $H = 2r$. In Fig. 81 N O_0P_0 is the initial position, and θ is the angle turned through when the centre of the circle arrives at O, and the radius at OP. Then $O_0O = R.\theta$, and if we take the origin at P_0, the axis of x as horizontal, and the axis of y as vertical, then the co-ordinates of P are given by

$$x = R.\theta - r\sin\theta = \frac{L}{2\pi}.\theta - \frac{H}{2}.\sin\theta \quad . \quad . \quad (1)$$

$$y = r - r\cos\theta = \frac{H}{2} - \frac{H}{2}.\cos\theta \quad . \quad . \quad . \quad . \quad (2)$$

As P moves along it revolves about the instantaneous centre N, so that PN is normal to the surface of the trochoid at P.

The velocity of $P = v$ is given by

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

$$= (R - r\cos\theta)^2.\left(\frac{d\theta}{dt}\right)^2 + (r\sin\theta)^2.\left(\frac{d\theta}{dt}\right)^2$$

$$= (R^2 + r^2 - 2Rr\cos\theta)\omega^2,$$

where ω is the angular velocity of OP.

Now if we call $PN = n$, then from the triangle OPN we have $n^2 = R^2 + r^2 - 2Rr\cos\theta$.

$$\therefore v = n.\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Sub-Trochoids.—For the trochoids below the surface the crests and troughs must be in the same vertical line as at the surface. They are all of the same length and described by the same diameter of rolling circle. The variation will be in the position of the rolling line and in the length of the radius arm OP. The radius arm will diminish as we go down, and for two consecutive trochoid surfaces the distance between the lines of orbit centres will be the same as the distance between the rolling lines.

Let the two adjacent trochoids be those through P and P' (Fig. 81 O), the rolling lines being through N and N', and the lines of orbit centres through O and O′, y being measured from some datum line and δy being as shown.

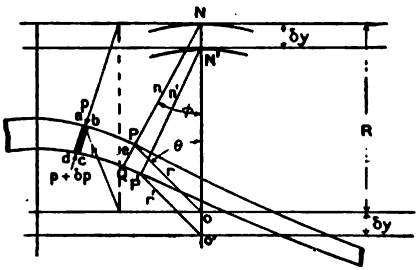


FIG. 81 a.

Then for continuity it is necessary that the water shall continually fill the space between the two trochoids, and if v is the velocity at P and e the breadth of the stream, then the quantity of water passing will equal $v \times e$, and since $v = n\omega$, we have quantity of water passing $= n . e . \omega$, and this must be constant all along the stream and since ω is uniform.

$$\therefore n . e = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Now OP is parallel to O′P′, and in the limit PN may be taken as parallel to P′N′, and we have—

$$PQ + n = n' + NN' . \cos \phi \qquad (PQ = e \quad NN' = \delta y)$$

$$\text{or } e = n' - n + \delta y . \cos \phi$$

$$\text{i.e. } e = \delta n + \delta y \cos \phi \qquad (n' - n = \delta n)$$

$$\text{or } n . e = n . \delta n + n . \delta y . \cos \phi$$

$$\text{Now } n^2 = R^2 + r^2 - 2Rr \cos \theta$$

$$\therefore n . \delta n = r . \delta r - R . \delta r . \cos \theta \qquad (\theta \text{ and R being constant})$$

$$\text{also } n \cos \phi = R - r \cos \theta$$

$$\therefore n . e = r . \delta r - R . \delta r \cos \theta + R . \delta y - r . \delta y . \cos \theta$$

$$= r . \delta r + R . \delta y - \cos \theta \, (R . \delta r + r . \delta y)$$

But ne is constant for all values of θ, and in order that this must be the case—

$$R . \delta r + r . \delta y = 0$$

$$\text{or } \frac{1}{r} . dr = -\frac{1}{R} . dy$$

Integrating, $\log_e r = C - \frac{y}{R}$, and the value of C is obtained by making $r = r_0$ at the surface, when $y = 0$

$$\therefore C = \log_e r_0$$

$$\therefore \log_e r - \log_e r_0 = -\frac{y}{R}$$

and $r = r_0 e^{-y/R}$ (e being the base of Naperian logarithms) (5)

Thus for successive values of y, *i.e.* distances below the centres of the rolling circle of the surface trochoid, we can obtain successive values of r, and it is found that r rapidly decreases with the depth.

Thus for a wave 60 feet long and 12 feet high—

$$R = \frac{L}{2\pi} = 9{\cdot}55 \text{ and } r_0 = 6$$

From (5)—

$$\log_e r = \log_e r_0 - \frac{y}{R} \text{ since } \log_e e = 1$$

or, turning into ordinary logarithms—

$$\log r = \log r_0 - \frac{1}{2{\cdot}3} \cdot \frac{y}{R} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

for $r_0 = 6$ and $y = 2$ $\quad \log r = {\cdot}7782 - {\cdot}091 = {\cdot}6872$

$\quad$ or $r = 4{\cdot}86$

and similarly for $y = 4, 6, 8$ $\quad r = 3{\cdot}94, 3{\cdot}19, 2{\cdot}6$.

Take a wave 600 feet long and 40 feet high. It will be found that at a depth of 400 feet the diameter of the orbit is only about 7 inches, and at a depth of 600 feet the diameter of the orbit is negligible.

Wave Formation.—We can turn the motion of the rolling circle into a wave formation by impressing on the whole a backward velocity V. Then the point O is fixed and points as P revolve with a constant angular velocity ω. All such points as P revolving in circular orbits will give a wave formation, the *formation* travelling with a velocity V. The particles in the crest will move in the same direction as the wave advance and particles in the trough in the opposite direction. *A wave is the passage of motion* and the actual movement of the water is small, it being the wave form which moves along. If a piece of wood be observed floating on a wave it will be seen to oscillate about a mean position all the while the wave is rapidly moving on. Waves may be created in a rope, which will travel along the rope, while it is evident that the rope itself does not travel.

There may be of course a bodily drift of the whole mass of water due to tides, which is a motion on which the wave formation will be superimposed.

The turning of the rolling circle movement into a wave formation by impressing on it a constant velocity does not affect any of the dynamical relations into which we shall have to inquire.

Line of Orbit Centres in Relation to Still Water Level.—Let O_0O be the line of orbit centres, Fig. 81N, and LL a line such that the area of the half trochoid P′P″M equals the area below LL, LLP′M, *i.e.* LL will be the level of still water.

$$\text{Area of half trochoid} = \int_0^\pi x \, . \, dy$$

$$= \int_0^\pi (R\theta - r\sin\theta)\, r\sin\theta \, . \, d\theta$$

$$= \pi \, . \, R \, . \, r - \frac{\pi}{2} \, . \, r^2$$

This must equal area below LL, viz. :—

$$\pi R(r - a)$$

where a is distance of LL below the line of orbit centres from which

$$a = \frac{r^2}{2R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

which is the distance below the line of orbit centres which the trochoid would occupy in still water. See also Fig. 81Q.

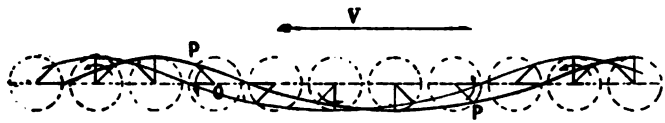


Fig. 81P.

Virtual Gravity and Virtual Upright.—The particle P revolving in its circular orbit is subject to the following forces, viz. :—

(i) Gravity $= m \, . \, g$ downwards (m being mass).
(ii) Centrifugal force in the line OP $= m \, . \, r \, . \, \omega^2$.

The resultant force on the particle must be normal to the surface, the surface being a surface of equal pressure, *i.e.* it will act in the line NP (Fig. 81 o). The triangle OPN is therefore the triangle of forces, and the forces are proportional to the lengths of the sides to which they are parallel, or—

$$\frac{m \, . \, r \, . \, \omega^2}{\text{OP}} = \frac{m \, . \, g}{\text{ON}} = \frac{m \, . \, f}{\text{PN}}$$

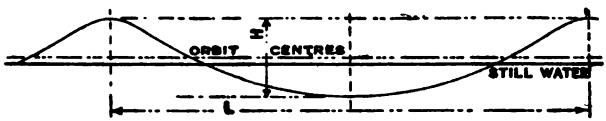


Fig. 81Q.

where mf is the resultant force *in the direction* NP. From this we obtain, since OP $= r$, ON $=$ R, PN $= n$—

$$f = \frac{n}{\text{R}} . g . \quad \ldots \ldots \quad (8)$$

$$\text{and } \omega^2 = \frac{g}{\text{R}}$$

$$\text{from which } \text{V}^2 = g . \text{R} = \frac{g}{2\pi} . \text{L} \quad \left(\text{since} \frac{\text{V}}{\omega} = \frac{\text{L}}{2\pi}\right) \quad (9)$$

This gives the speed of the wave in foot second units, for speed in knots we have—

$$\text{V}_1^2 = 1 \cdot 8\text{L} \qquad \text{or } \text{V}_1 = 1 \cdot 34\sqrt{\text{L}} \quad . \quad . \quad (10)$$

f in (8) is termed the *virtual gravity*, because the particles are acted upon by f instead of the real gravity g. f changes in magnitude and direction all along the wave, being always perpendicular to the surface of the trochoid. The apparent or virtual weight of a body of mass m floating on the surface of a wave therefore varies from $mg \, . \, \frac{\text{R} - r}{\text{R}}$ at the crest to $mg \, . \, \frac{\text{R} + r}{r}$ in the trough. For a wave 600 feet long and 30 feet high the variation is from $\cdot 84g$ in the crest to $1 \cdot 15g$ in the trough.

This is the explanation of the well-known phenomenon of the tenderness of sailing boats on the crest of a wave. The virtual weight is less than the actual, and consequently the righting moment at any angle of inclination is less on the crest than in still water. The wind moment causing heel is not affected, and thus on the crest of a wave a boat, of sufficient stiffness in smooth water, is liable to be blown over to a large angle and possibly capsize.

The virtual gravity will act at right angles to the wave slope, and a small raft will always tend to set itself normal to the wave surface. This normal at any particular instant is termed the *virtual upright*. For a ship which extends well into the wave the virtual upright will be rather different from that at the surface. A wave surface which is normal to each of the virtual upright positions of a ship in a wave is termed the *effective wave slope*, and this is distinctly flatter than the actual surface wave.

Variation of Pressure in a Wave.—For the fluid filaments between adjacent trochoids we not only have to satisfy the condition of continuity but also the condition of lateral equilibrium, *i.e.* the water must be a solid mass with no cavities.

For continuity, as already seen, $n \cdot e =$ constant. Take a small cylinder between adjacent trochoids (Fig. 81 o), of cross section a. If the pressure on the upper face is p, and on the lower face $p + \delta p$, then—

$$\delta p \cdot a = \frac{w}{g} (a \cdot e) \cdot f \qquad a \cdot e \text{ being volume } w = \text{weight per unit mass}$$

$$= \frac{w}{g} \; a \cdot e\left(\frac{n}{R} \cdot g\right) \text{ from (8)}$$

$$\therefore \text{ increment of pressure} = \delta p = \frac{w}{R} \cdot n \cdot e$$

and since $n \cdot e$ is constant along the stream the increment of pressure δp is also constant. That is to say, since the pressure on the surface is constant, on each of the sub-surfaces the pressure is constant, and the trochoidal strip is capable of forming a stream of continuous flow.

The area of a trochoidal strip corresponding to lines of rolling circles δy apart is obtained as follows:—

Area of element $abcd = e \, . \, \delta s$. We have seen that $v = n \, . \, \omega$ $= n \, . \, \frac{d\theta}{dt}$ and $v = \frac{ds}{dt}$

$$\therefore \delta s = n \, . \, \delta\theta$$

and area of element $= n \, . \, e \, . \, \delta\theta$

$$\text{and area of strip} = \int_0^{2\pi} n \, . \, e \, . \, d\theta = 2\pi \, . \, n \, . \, e$$

This is equated to the volume of the corresponding layer in still water, viz. $2\pi R \, . \, \delta y_0$.

$$\therefore n \, . \, e = R \, . \, \delta y_0$$

and since $\delta p = \frac{w}{R} \, . \, n \, . \, e$ we have $\delta p = w \, . \, \delta y_0$.

That is to say, the increment of pressure from one trochoidal surface to the next is the same as the increment between the corresponding layers in still water, and therefore *the pressure at any point in a trochoidal wave is the same as at the corresponding point in still water.*

This is a very important result, and has its application to the case of the buoyancy of a ship on a wave when considering the strength calculations of ships.

Energy stored up in a Wave.—The particles of a wave are revolving in circular orbits each with a lineal velocity $\omega \, . \, r$, and the energy due to this motion is *kinetic energy*. The particles are also lifted above the level they occupied in still water, which is *potential energy.*

The centre of gravity of a trochoidal strip is in the line of orbit centres.

For area of element $= n \, . \, e \, . \, \delta\theta$, and moment about the line of orbit centres distance $y = n \, . \, e \, . \, y \, . \, \delta\theta$.

$\therefore$ centre of gravity of strip from datum line

$$= \frac{\int n \, . \, e \, . \, y \, . \, d\theta}{\int n \, . \, e \, . \, d\theta} = \frac{\int y \, . \, d\theta}{\int d\theta}, \text{ since } n \, . \, e \text{ is constant}$$

$$= \frac{\int_0^{2\pi} r(1 - \cos\theta) \, . \, d\theta}{\int_0^{2\pi} d\theta} = r$$

1. *Kinetic energy.*—The kinetic energy of a particle is—

$\frac{1}{2} \, . \, \frac{w}{g} \, . \, n \, . \, e \, . \, d\theta \, . \, \omega^2 \, . \, r^2$, and for the strip $w \, . \, \pi \, . \, n \, . \, e \, . \, \frac{r^2}{R}$

$$\text{since } \omega^2 = \frac{g}{R}$$

2. *Potential energy*

$$= w \, . \, 2\pi \, . \, n \, . \, e \, . \left(\frac{r^2}{2R}\right) = w \, . \, \pi \, . \, n \, . \, e \, . \, \frac{r^2}{R} \quad . \quad \text{from (7)}$$

so that the energy of the strip is half kinetic and half potential and equals $2\pi \, . \, n \, . \, e \, . \, w \, . \, \frac{r^2}{R}$.

Substituting for $n \, . \, e = r \, . \, \delta r + R \, . \, \delta y$

$$= r \, . \, \delta r - \frac{R^2}{r} \, . \, \delta r$$

$$= \frac{r^2 - R^2}{r} \, . \, \delta r$$

$$\text{Energy} = \frac{2\pi \, . \, w}{R}(r^3 - rR^2)\delta r.$$

For the whole wave this has to be integrated from the surface, where $r = r_0$ to $r = 0$

$$\textit{i.e.} \text{ energy} = \frac{2\pi \, . \, w}{R}\int_0^{r_0}(r^3 - rR^2)dr$$

$$= \frac{1}{8} \, . \, w \, . \, L \, . \, H^2\left(\frac{r_0^2}{2R^2} - 1\right)$$

or changing sign, since energy must be positive, we have—

$$\text{Energy} = \frac{1}{8} \, . \, w \, . \, L \, . \, H^2\left(1 - \frac{r_0^2}{2R^2}\right)$$

Since $\frac{r_0}{R}$ is small, we can say—

$$\text{Energy} = \frac{1}{8} \, . \, w \, . \, L \, . \, H^2 \text{ per unit breadth}$$

The formula given above giving the relation of the speed of a wave to its length may be used to compile the following table :—

Length in feet.	Speed in feet per second.	Speed in knots.	Period, *i.e.* time, the length is travelled.
50	16	9·5	3·1 sec.
100	22·7	13·4	4·4 ,,
200	32·0	19·0	6·2 ,,
400	45·3	26 8	8·8 ,,
600	55·0	32·8	10 8 ,,
800	64·1	37·9	12·5 ,,
1000	71·6	42·4	14·0 ,,
2000	101·5	60·0	19·8 ,,

Slope of Wave Surface.—This is the same as the angle ϕ (Fig. 81 o), and from the triangle OPN $\tan \phi = \frac{r . \sin \theta}{R - r \cos \theta}$. The angle ϕ is a maximum when PN is a tangent to the circle radius OP, and then the slope is given by—

$$\sin a = \frac{r}{R}$$

The trochoidal wave theory as developed above assumes that the fluid is *perfect*, i.e. without viscosity, offering no resistance whatever to change of shape. Water, however, is not a perfect fluid, but the theory fits in so well with all the observed facts that it is generally accepted. It can be shown that the motion in oscillatory waves in a perfect fluid cannot be generated from rest, so that the theory fails to represent the actual motion.

Observations on Waves to determine the velocity, length, and height.

To find the velocity and length (Fig. 81R).

Let V be the speed of the ship.

θ angle of ship with the wave advance.

V_1 speed of waves.

L length of wave (crest to crest).
l distance between observing stations on the ship.

The speed of ship in direction of wave advance is $V \cos \theta$, and speed of ship *relative* to wave advance is $V \cos \theta + V_1$. If t is the time interval of a wave crest as observed at the bow and stern positions perpendicular to wave advance, then—

$$t = \frac{l \cos \theta}{V \cos \theta + V_1}$$

and
$$V_1 = \frac{l - V \cdot t}{t} \cdot \cos \theta$$

If t' be the time interval between successive crests as

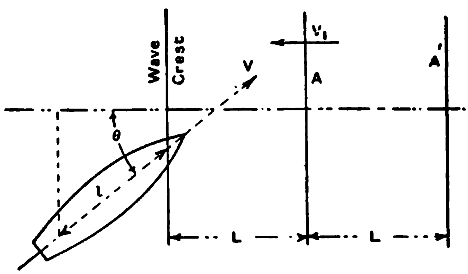


FIG. 81R.

observed at the bow or stern positions perpendicular to the wave advance, then—

$$L = (V \cos \theta + V_1) \cdot t'$$
$$= l \cdot \frac{t'}{t} \cdot \cos \theta$$

If the ship is steaming away from the wave advance, then V in the above is changed to $-V$.

It is found that the calculated velocity in relation to length is generally somewhat smaller than that obtained by observation.

To find the height.—This is more difficult. A position is taken up amidships in the ship so that when the ship is in the trough and upright, successive crests appear to form a chain reaching towards the horizon. The height of the observer's eye above the water-line is then the height of the wave. Such a method is inevitably liable to considerable inaccuracy.

The whole subject of waves is discussed exhaustively by Dr. Vaughan Cornish in his book "Waves of the Sea and Other Water Waves;" see also the "Cantor" lecture, Royal Society of Arts, 1914.

The following table is taken from the above :—

Description of wind.	Beaufort's number for wind force.	Velocity of wind and velocity of wave in statute miles per hour.	Period in seconds $= \frac{V}{3 \cdot 493}$	Length in feet.	Greatest average height.	Length ÷ height.
Strong breeze	6	25	7·2	262	17·5	15·0
Moderate gale	7	31	8·9	404	21·7	18 6
Fresh gale	8	37	10·6	575	25·9	22·2
Strong gale	9	44	12·6	813	30·8	26·4
Whole gale	10	53	15·2	1180	37·1	31·8
Storm . .	11	64	18·3	1720	44·8	38·4
Hurricane	12	77	22·0	2489	—	—

It is seen that as the length of wave increases the ratio $L \div H$ increases, and that the standard ratio of $L \div H = 20$ usually used for poising a ship on a wave for strength calculations cannot fairly be applied to ships longer than about 470 feet.

Principal Formulæ in Trochoidal Wave Theory.—The following are the principal formulæ arising out of the trochoidal wave theory :—

$$L = 2 . \pi . R \qquad r_0 = \frac{H}{2} \qquad V_1^2 = 1 \cdot 8L$$

$$r = r_0 . e^{-y/R} \qquad \log_{10} r = \log_{10} r_0 - \frac{1}{2 \cdot 3} . \frac{y}{R}$$

$$T = 0 \cdot 44 \sqrt{L}$$

Height of centre of orbits of a given particle above the level of that particle is still water $= \frac{r}{2R}$. For the wave surface this distance $= \frac{\pi . H^2}{4L}$.

$$\text{Virtual gravity at crest} = \frac{R - r}{R} \cdot g$$

$$\text{,, \quad ,, \quad trough} = \frac{R + r}{R} \cdot g$$

Where L is length of wave in feet.
H is height of wave in feet.
T is periodic time in seconds.
V_1 is velocity of wave in knots (*i.e.* 6080 feet per hour).
R is radius of rolling circle in feet.
r_0 is radius of tracing arm in feet at surface.
r is radius of tracing arm in feet at depth y.
y being depth of line of orbit centres from that line for the surface trochoid.
g is acceleration due to gravity, *i.e.* 32·2 in foot second units.

The "Smith" Correction due to taking account of the Internal Structure of a Wave.—In the ordinary calculation made for the buoyancy of a ship on a wave, the wave is considered to be a mass of still water, and the pressure at any point that due to its position relative to the surface. We have seen, however, that this is not the case in an actual wave, and it has been proved that the pressure at any point is the same as at the position occupied by the point in still water. Therefore the buoyancy in the crest portion is really less, and the trough position greater than is obtained in the usual manner. This will be referred to in relation to the strength of ships.

Examples.

1. An Atlantic ocean wave is 600 feet long and 40 feet high. Calculate the radii of the orbits at depths of 100, 200, 300, 400, 500 and 600 feet.

Ans. 7·03, 2·49, 0·87, 0·31, 0·11, 0·04 feet.

These results show that even in a wave of large dimensions at a depth less than the length of the wave the motion of the water is practically nil.

2. The successive crests of the wave profile along a ship's side going at speed in still water are observed to be about 300 feet apart. What is the speed of the vessel in knots?

Ans. 23 knots about.

3. What is the speed of a 600 feet wave in knots?

Ans. 33 knots nearly.

4. What is the length of a wave successive crests of which are observed to pass a stationary observer at intervals of 8 seconds?

Ans. 330 feet.

5. A wave is 600 feet long and 40 feet high. Compare the orbital velocity of the particles in the surface with the speed of the wave.

Ans. 11·6 : 55·4, or about $\frac{1}{5}$.

6. What is the *virtual* force of gravity in the crest and trough respectively of a wave 600 feet long and 40 feet high?

Ans. 0·79g, 1·21g.

7. Check the following statement given in Sir W. H. White's "Manual of Naval Architecture":—

The orbits and velocities of the particles of water are diminished by approximately *one-half* for each additional depth below the mid height of the surface trochoid equal to *one-ninth* of a wave length. For example—

Depths in fraction of a wave length .	0	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$
Proportional velocities and diameters	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

CHAPTER VII.

CALCULATION OF WEIGHTS—STRENGTH OF BUTT CONNECTIONS—DAVITS, PILLARS, DERRICKS, AND SHAFT BRACKETS.

Calculations of Weights. — We have discussed in Chapter I. the ordinary rules of mensuration employed in finding the areas we deal with in ship calculations. For any given uniform plate we can at once determine the weight if the weight per square foot is given. For iron and steel plates of varying thicknesses, the weight per square foot is given on p. 38. For iron and steel angles and T bars of varying sizes and thicknesses tables are calculated, giving the weight per lineal foot. Such a table is given on p. 225 for steel angles, etc., the thicknesses being in $\frac{1}{20}$ths of an inch. It is the Admiralty practice to specify angles, bars, etc., not in thickness, but in weight per lineal foot. Thus an angle bar $3'' \times 3''$ is specified to weigh 7 lbs. per lineal foot, and a Z bar $6'' \times 3\frac{1}{2}'' \times 3''$ is specified to weigh 15 lbs. per lineal foot. When the bars are specified in this way, reference to tables is unnecessary. The same practice is employed with regard to plates, the thickness being specified as so many pounds to the square foot.

If we have given the size of an angle bar and its thickness, we can determine its weight per foot as follows: Assume the bar has square corners, and is square at the root, then, if a and b are the breadth of the flanges in inches, and t is the thickness in inches, the length of material t inches thick in the section is $(a + b - t)$ inches, or $\frac{a + b - t}{12}$ feet; and if the bar is of iron, the weight per lineal foot is—

$$\left(\frac{a + b - t}{12}\right) \times 40 \times t \text{ lbs.}$$

WEIGHT OF STEEL ANGLES IN POUNDS PER FOOT

Sum of flanges in inches.	Thickness in Twentieths of an Inch.																			
	1/20	2/20	3/20	4/20	5/20	6/20	7/20	8/20	9/20	10/20	11/20	12/20	13/20	14/20	15/20	16/20	17/20	18/20	19/20	20/20
2	0·332	0·645	0·943	1·22	1·49	1·73	1·96	2·18												
2½	0·417	0·816	1·20	1·56	1·91	2·24	2·56	2·86	3·14											
3	0·501	0·986	1·45	1·91	2·34	2·75	3·15	3·54	3·90	4·25										
3½	0·587	1·15	1·71	2·24	2·76	3·26	3·75	4·22	4·67	5·10	5·51									
4	0·672	1·33	1·96	2·58	3·19	3·77	4·34	4·90	5·43	5·95	6·45	6·94								
4½	0·757	1·50	2·22	2·92	3·61	4·28	4·94	5·58	6·20	6·80	7·39	7·96	8·51							
5	0·842	1·67	2·47	3·26	4·04	4·79	5·53	6·26	6·96	7·65	8·32	8·97	9·62	10·23						
5½	0·927	1·84	2·73	3·60	4·46	5·30	6·13	6·94	7·73	8·50	9·26	10·00	10·72	11·42						
6			2·98	3·94	4·88	5·81	6·72	7·62	8·49	9·35	10·19	11·02	11·83	12·61	13·39					
6½			3·24	4·28	5·31	6·32	7·32	8·30	9·26	10·20	11·13	12·04	12·93	13·81	14·66					
7			3·49	4·62	5·74	6·83	7·91	8·98	10·02	11·05	12·06	13·05	14·04	14·99	15·94	16·86				
7½			3·75	4·96	6·16	7·34	8·51	9·66	10·79	11·90	13·00	14·08	15·14	16·18	17·21	18·22				
8			4·00	5·30	6·59	7·85	9·10	10·34	11·55	12·75	13·93	15·09	16·25	17·37	18·49	19·58	20·66			
8½			4·26	5·64	7·01	8·36	9·70	11·02	12·32	13·60	14·87	16·11	17·35	18·56	19·76	20·94	22·10			
9			4·51	5·98	7·44	8·87	10·29	11·70	13·08	14·45	15·80	17·13	18·46	19·75	21·04	22·30	23·55	24·79		
9½			4·77	6·32	7·86	9·38	10·89	12·38	13·85	15·30	16·74	18·16	19·56	20·95	22·31	23·66	25·00	26·32		
10			5·02	6·66	8·28	9·89	11·48	13·06	14·61	16·15	17·67	19·18	20·67	22·13	23·59	25·02	26·44	27·84	29·24	
10½			5·28	7·00	8·71	10·40	12·08	13·74	15·38	17·00	18·61	20·20	21·77	23·32	24·86	26·38	27·88	29·38	30·85	
11			5·53	7·34	9·13	10·91	12·67	14·41	16·14	17·85	19·54	21·21	22·88	24·51	26·14	27·74	29·33	30·90	32·47	34·00
11½			5·79	7·68	9·56	11·42	13·27	15·10	16·91	18·70	20·48	22·24	23·98	25·70	27·41	29·10	30·77	32·43	34·08	35·70
12			6·04	8·02	9·99	11·93	13·86	15·78	17·67	19·55	21·41	23·26	25·09	26·89	28·69	30·46	32·22	33·97	35·70	37·40

If the bar is of steel, the weight per lineal foot is—

$$\left(\frac{a+b-t}{12}\right) \times 40{\cdot}8 \times t \text{ lbs.}$$

Thus a $3'' \times 3'' \times \frac{3}{8}''$ steel angle bar would weigh 7·17 lbs., and a steel angle bar $3'' \times 3''$ of 7 lbs. per foot would be slightly less than $\frac{3}{8}$ inch thick.

It is frequently necessary to calculate the weight of a portion of a ship's structure, having given the particulars of its construction; thus, for instance, a bulkhead, a deck, or the outer bottom plating. In any case, the first step must be to find the area of plating and the lengths of angle bars. The weight of the net area of the plating will not give us the total weight of the plating, because we have to allow for butt straps, laps, rivet-heads, and in certain cases liners. The method employed to find the allowance in any given case is to take a sample plate and find what percentage the additions come to that affect this plate, and to use this percentage as an addition to the net weight found for the whole. To illustrate this, take the following example:—

A deck surface of 10,335 square feet is to be covered with $\frac{5}{16}$-inch steel plating, worked flush, jointed with single-riveted edges and butts. Find the weight of the deck, allowing 3 per cent. for rivet-heads.

$\frac{5}{16}$-inch steel plates are 12·75 lbs. per square foot, so that the net weight is—

$$\frac{10{,}335 \times 12{\cdot}75}{2240} = 58{\cdot}8 \text{ tons}$$

Now, assume an average size for the plates, say $16' \times 4'$. $\frac{3}{4}$-inch rivets will probably be used, and the width of the edge strip and butt strap will be about 5 inches. The length round half the edge of the plate is 20 feet, and the area of the strap and lap belonging to this plate is—

$$20 \times \tfrac{5}{12} = 8{\cdot}33 \text{ square feet}$$

The percentage of the area of the plate is therefore—

$$\frac{8{\cdot}33}{64} \times 100 = 13 \text{ per cent.}$$

Adding also 3 per cent. for rivet-heads, the total weight is 68·4 tons.

It is usual to add 3 per cent. to allow for the weight of rivet-heads. For lapped edges and butt straps, both double riveted, the percentage[1] comes to about 10 per cent. for laps, $5\frac{1}{2}$ per cent. for butt straps, and 3 per cent. for liners as ordinarily fitted to the raised strakes of plating. No definite rule can be laid down, because the percentage must vary according to the particular scantlings and method of working the plating, etc., specified.

The length of stiffeners or beams required for a given area can be very approximately determined by dividing the area in square feet by the spacing of the stiffeners or beams in feet. For wood decks, 3 per cent. can be added for fastenings.

Example.—The beams of a deck are 3 feet apart, and weigh 22 lbs. per foot run; the deck plating weighs 10 lbs. per square foot, and this is covered by teak planking 3 inches thick. Calculate the weight of a part 54 feet long by 10 feet wide of this structure, including fastenings.
(*S. and A. Exam.* 1897.)

Net area of deck = 54 × 10 = 540
Add for butts and laps 7 per cent. = 37·8
577·8

(Assume single-riveted butt straps and single-riveted laps.)

Weight of plating = 577·8 × 10
= 5778 lbs.
Running feet of beams = $\frac{540}{3}$ = 180
Weight of beams = 180 × 22
= 3960 lbs.[2]
Total weight of plating and beams = 9,738 lbs.
Add 3 per cent. for rivet-heads = 292 „
10,030 „

Weight of teak[3] = 540 × $\frac{53}{4}$ = 7155 lbs.
Add 3 per cent. for fastenings = 215 „
Weight of wood deck 7370 „

Summary.

Plating and beams		10,030 lbs.
Wood deck		7,370 „
	Total ...	17,400 „ = 7·8 tons.

[1] A number of percentages worked out for various thicknesses, etc., will be found in Mr. Mackrow's "Pocket Book."
[2] No allowance made for beam arms, which should be done if a whole deck is calculated.
[3] Teak taken as 53 lbs. per cubic foot.

Use of Curves.—For determining the weight of some of the portions of a ship, the use of curves is found of very great assistance. Take, for instance, the transverse framing of a ship. For a certain length this framing will be of the same character, as, for example, in a battleship, within the double bottom, where the framing is fitted intercostally between the longitudinals. We take a convenient number of sections, say the sections on the sheer drawing, and calculate the weight of the complete frame at each section. Then along a base of length set up ordinates at the sections, of lengths to represent the calculated weights of the frames at the sections. Through the spots thus obtained draw a curve, which should be a fair line. The positions of the frames being placed on, the weight of each frame can be obtained by a simple measurement, and so the total weight of the framing determined. The curve AA in Fig. 82 gives a curve as constructed in this way for the transverse framing below armour in the double bottom of a battleship. Before and abaft the double bottom, where the character of the framing is different, curves are constructed in a similar manner.

Weight of Outer Bottom Plating.—The first step necessary is to determine the area we have to deal with. We

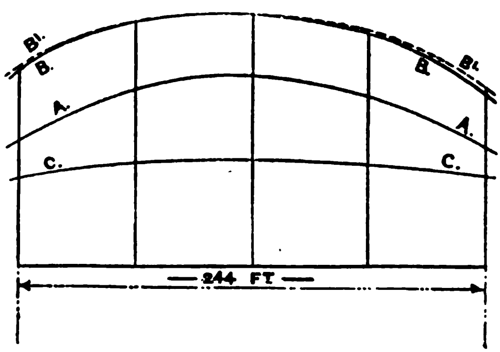


FIG. 82.

can construct a curve of girths, as BB, Fig. 82; but the area given by this curve will not give us the area of the plating, because although the surface is developed in a transverse direction there is no development in a longitudinal direction. (Strictly speaking, the bottom surface of a ship is an undevelopable surface.) The extra area due to the slope of the level lines is allowed for as follows: In plate I., between stations 3 and 4, a line *fg* is drawn representing the *mean slope* of all the level lines. Then the ordinate of the curve of girths midway between 3 and 4 stations is increased in the ratio *fg* : *h*. This done all along the curve will give us a new modified curve of girths, as B′B′, Fig. 82, and the area given by this curve will give a close approximation to the area of the outer bottom of the ship. This is, of course, a net area without allowing for butt straps or laps. Having a modified curve of girths for the whole length, we can separate it into portions over which the character of the plating is the same. Thus, in a vessel built under Lloyd's rules, the plating is of certain thickness for one-half the length amidships, and the thickness is reduced before and abaft. Also, in a battleship, the thickness of plating is the same for the length of the double bottom, and is reduced forward and aft. The curves AA and BB, Fig. 82, are constructed as described above for a length of 244 feet.

Weight of Hull.—By the use of these various methods, it is possible to go right through a ship and calculate the weight of each portion of the structure. These calculable portions for a battleship are—

(1) Skin-plating and plating behind armour.

(2) Inner bottom plating.

(3) Framing within double bottom, below armour, behind armour, and above armour. Outside double bottom, below and above the protective deck.

(4) Steel and wood decks, platforms, beams.

(5) Bulkheads.

(6) Topsides.

There are, however, a large number of items that cannot be directly calculated, and their weights must be estimated by comparison with the weights of existing ships. Such items are stem and stern posts, shaft brackets, engine and boiler bearers, rudder, pumping and ventilation arrangements, pillars, paint, cement, fittings, etc.

It is, however, a very laborious calculation to determine the weight of the hull of a large ship by these means; and more often the weight is estimated by comparison with the ascertained weight of existing ships. The following is one method of obtaining the weight of steel which would be used in the construction of a vessel: The size of the vessel is denoted by the product of the length, breadth, and depth, and for known ships the weight of steel is found to be a certain proportion of this number, the proportion varying with the type of ship. The coefficients thus obtained are tabulated, and for a new ship the weight of steel can be estimated by using a coefficient which has been obtained for a similar type of ship. The weight of wood and outfit can be estimated in a similar manner.

Another method is described by Mr. J. Johnson, M.I.N.A., in the *Transactions of the Institution of Naval Architects* for 1897, in which the sizes of vessels are represented by *Lloyd's old longitudinal number*,[1] modified as follows: In three-decked vessels, the girths and depths are measured to the upper deck

[1] *Lloyd's numbers* (now superseded by the New Rules)—

1. The scantlings and spacing of the frames, reversed frames, and floor-plates, and the thickness of bulkheads are regulated by numbers, which are produced as follows:—

2. For one and two decked vessels, the number is the sum of the measurements in feet arising from the addition of the half-moulded breadth of the vessel at the middle of the length, the depth from the upper part of the keel to the top of the upper-deck beams, with the normal round-up, and the girth of the half midship frame section of the vessel, measured from the centre line at the top of the keel to the upper-deck stringer plate.

3. For three-deck steam-vessels, the number is produced by the deduction of 7 feet from the sum of the measurements taken to the top of the upper-deck beams.

4. For spar-decked vessels and awning-decked steam-vessels, the number is the sum of the measurements in feet taken to the top of the main-deck beams, as described for vessels having one or two decks.

5. The scantlings of the keel, stem, stern-frame, keelson, and stringer plates, the thickness of the outside plating and deck; also the scantlings of the angle bars on beam stringer plates, and keelson and stringer angles in hold, are governed by the *longitudinal number* obtained by multiplying that which regulates the size of the frames, etc., by the length of the vessel.

The measurements for regulating the above scantling numbers are taken as follows:—

1. The *length* is measured from the after part of the stem to the fore part of the stern-post on the range of the upper-deck beams in one, two, and three decked and spar-decked vessels, but on the range of main-deck beams in awning-decked vessels.

In vessels where the stem forms a cutwater, the length is measured from without deducting 7 feet. In spar and awning-deck vessels, the girths are measured to the spar or awning decks respectively. In one, two, and well-decked vessels, the girths and depths are taken in the usual way. Curves are drawn for each type of vessel, ordinates being the weight of iron or steel in tons for vessels built to the highest class at Lloyd's or Veritas, and abscissæ being Lloyd's longitudinal number modified as above. These curves being constructed for ships whose weights are known, it is a simple matter to determine the weight for a new ship of given dimensions. For further information the student is referred to the paper in volume 39 of the *Transactions*.

To calculate the Position of the Centre of Gravity of a Ship.—We have already seen in Chapter III. how to find the C.G. of a completed ship by means of the inclining experiment, and data obtained in this way are found very valuable in estimating the position of the C.G. of a ship that is being designed. It is evident that the C.G. of a ship when completed should be in such a position as to obtain the metacentric height considered necessary, and also to cause the ship to float correctly at her designed trim. Suppose, in a given ship, the C.G. of the naked hull has been obtained from the inclining experiment (that is, the weights on board at the time of the experiment that do not form part of the hull are set down and their positions determined, and then the weight and position of the C.G. of the hull determined by the rules we have dealt with in Chapter III.). The position of the C.G. of hull thus determined is placed on the midship section, and the ratio of the distance of the C.G. above the top of keel to the total depth from the top of keel to the top of the uppermost deck amidships will

the place where the upper-deck beam line would intersect the after edge of stem if it were produced in the same direction as the part below the cutwater.

2. The *breadth* in all cases is the greatest moulded breadth of the vessel.

3. The *depth* in one and two decked vessels is taken from the upper part of the keel to the top of the upper-deck beam at the middle of the length, assuming a normal round-up of beam of a quarter of an inch to a foot of breadth. In spar-decked vessels and awning-decked vessels, the depth is taken from the upper part of the keel to the top of the main-deck beam at the middle of the length, with the above normal round-up of beam.

give us a ratio that can be used in future ships of similar type for determining the position of the C.G. of the hull. Thus, in a certain ship the C.G. of hull was 20·3 feet above keel, the total depth being 34·4 feet. The above ratio in this case is therefore 0·59, and for a new ship of similar type, of depth 39·5 feet, the C.G. of hull would be estimated at 39·5 × 0·59, or 23·3 feet above the keel. For the fore-and-aft position, a similar ratio may be obtained between the distance of the C.G. abaft the middle of length and the length between perpendiculars. Information of this character tabulated for known ships is found of great value in rapidly estimating the position of the C.G. in a new design.

For a vessel of novel type, it is, however, necessary to calculate the position of the C.G., and this is done by combining together all the separate portions that go to form the hull. Each item is dealt with separately, and its C.G. estimated as closely as it is possible, both vertically and in a fore-and-aft direction. These are put down in tabular form, and the total weight and position of the C.G. determined.

In estimating the position of the C.G. of the bottom plating, we proceed as follows: First determine the position of the C.G. of the several curves forming the half-girth at the various stations. This is not generally at the half-girth up, but is somewhere inside or outside the line of the curve. Fig. 83 represents the section AB at a certain station. The curve is divided into four equal parts by dividers, and the C.G. of each of these parts is estimated as shown. The centres of the first two portions are joined, and the centres of the two top portions are joined as shown. The centres of these last-drawn lines, g_1, g_2, are joined, and the centre of the line g_1g_2, viz. G, is the C.G. of the line forming the curve AB, and GP is the distance from the L.W.L. This done for each of the sections will enable us to put a curve, CC in Fig. 82, of distances of C.G. of the half-girths from the L.W.L.[1] We then proceed to find the C.G. of the bottom plating as indicated in the following table. The area is obtained by putting the half-girths (modified as already

[1] This assumes the plating of constant thickness. Plates which are thicker, as at keel, bilge, and sheer, can be allowed for afterwards.

explained) through Simpson's rule. These products are then multiplied in the ordinary way to find the fore-and-aft position of the C.G. of the plating, and also by the distances of the C.G.

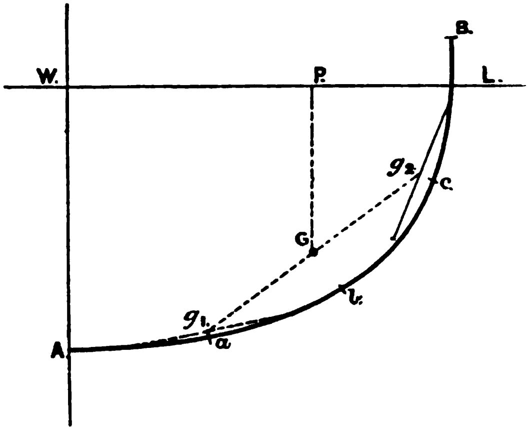


Fig. 83.[1]

of the sections below the L.W.L., which distances are measured off from the curve CC and are placed in column 6. The remainder of the work does not need any further explanation.

CALCULATION FOR AREA AND POSITION OF C.G. OF BOTTOM PLATING FOR A LENGTH OF 244 FEET.

Modified half-girths.	Simpson's multipliers.	Products.	Multiples for leverage.	Products.	C.G. from L.W.L.	Products.
41·5	1	41·5	2	83·0	18·1	751
51·1	4	204·4	1	204·4	21·6	4,415
				287·4		
53·0	2	106·0	0		22·2	2,354
49·6	4	198·4	1	198·4	21·3	4,226
37·5	1	37·5	2	75·0	19·0	712
		587·8		273·4		12,458
				14·0		

[1] The C.G. of wood sheathing, if fitted, can be obtained from this figure by setting off normally to the curve from G the half-thickness of the sheathing.

Common interval = 61 feet

Area both sides = $587 \cdot 8 \times \frac{1}{3} \times 61 \times 2$

= 23,904 square feet

$$\text{C.G. abaft middle of length of plating} = \frac{14 \cdot 0}{587 \cdot 8} \times 61$$

= 1·45 feet

$$\text{C.G. below L.W.L.} = \frac{12,458}{587 \cdot 8} = 21 \cdot 2 \text{ feet}$$

CALCULATION FOR THE POSITION OF THE C.G. OF A VESSEL.

Items.	Tons.	From L.W.L.				From Middle of Length.			
		Below.		Above.		Before.		Abaft.	
		Lever.	Moment.	Lever.	Moment.	Lever.	Moment.	Lever.	Moment.
Equipment—									
Water	25	4·0	100	—	—	—	—	12·0	300
Provisions	30	4·5	135	—	—	25·0	750	—	—
Officers' stores ...	15	2·0	30	—	—	—	—	125·0	1,875
Officers, men, and effects	30	—	—	6·0	180	55·0	1650	—	—
Cables	30	4·0	120	—	—	85·0	2550	—	—
Anchors	10	—	—	15·0	150	90·0	900	—	—
Masts, yards, etc. ...	25	—	—	45·0	1125	—	—	7·0	175
Boats	10	—	—	21·0	210	—	—	20·0	200
Warrant officers' stores	20	1·5	30	—	—	65·0	1300	—	—
Armament	175	—	—	4·0	700	—	—	5·0	875
Machinery	450	4·0	1800	—	—	—	—	33·0	14,850
Engineers' stores ...	50	0·5	25	—	—	—	—	70·0	3,500
Coals	300	0·2	60	—	—	3·0	900	—	—
Protective deck... ...	210	—	—	1·5	315	—	—	15·0	3,150
Hull	1250	—	—	1·5	1875	—	—	11·5	14,375
Total	2630 tons		2300		4555		8050		39,300
					2300				8,050
					2630)2255				2630)31,250
					0·86 ft. above L.W.L				11·88 ft. abaft mid. length.

C.G. above L.W.L. = 0·86

Trans. met. „ „ = 2·97

Trans. met. above C.G. = 2·97 − 0·86

= 2·11 feet.

Calculation for C.G. of a Completed Vessel.—By the use of the foregoing methods we can arrive at an estimate of the weight of hull, and also of the position of its C.G. relative to a horizontal plane, as the L.W.P., and to a vertical athwartship plane, as the midship section. To complete the ship for service, there have to be added the equipment, machinery, etc., and the weights of these are estimated, as also the positions of their centres of gravity. The whole is then combined in a table, and the position of the C.G. of the ship in the completed condition determined.

The preceding is such a table as would be prepared for a small protected cruiser. It should be stated that the table is not intended to represent any special ship, but only the type of calculation.

The total weight is 2630 tons, and the C.G. is 0·86 foot above the L.W.L. and 11·88 feet abaft the middle of length. The sheer drawing enables us to determine the position of the transverse metacentre, and the estimated GM. is found to be 2·11 feet. The centre of buoyancy calculated from the sheer drawing should also be, if the ship is to trim correctly, at a distance of 11·88 feet abaft the middle of length.

Strength of Butt Fastenings.—Fig. 84 represents two plates connected together by an ordinary treble-riveted butt strap. The spacing of the rivets in the line of holes nearest the butt is such that the joint can be caulked and made water-tight, and the alternate rivets are left out of the row of holes farthest from the butt. Such a connection as this could conceivably break in five distinct ways—

1. By the *whole of the rivets* on one side of the butt shearing.

2. By the *plate* breaking through the line of holes, AA, farthest from the butt.

3. By the *butt strap* breaking through the line of holes, BB, nearest the butt.

4. By the *plate* breaking through the middle row of holes, CC, and shearing the rivets in the line AA.

5. By the *strap* breaking through the middle row of holes, CC, and shearing the rivets in the line BB.

It is impossible to make such a connection as this equal to the strength of the unpunched plate, because, although we might

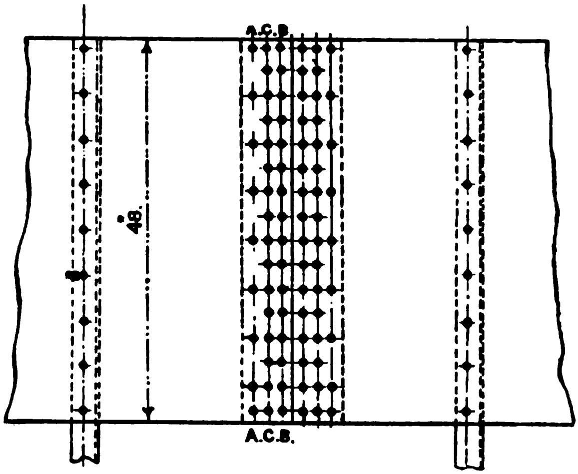


Fig. 84.

put in a larger number of rivets and thicken up the butt strap,

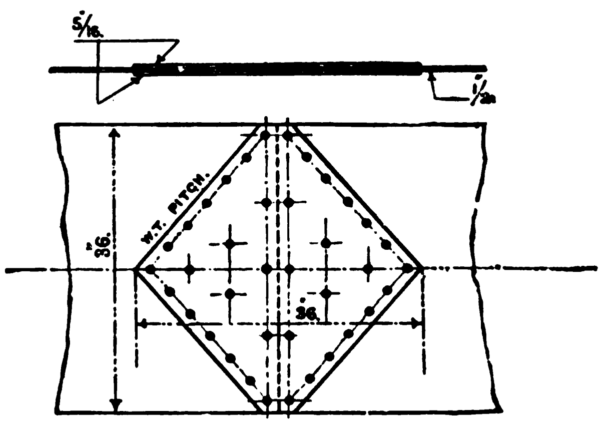


Fig. 85.

there would still remain the line of weakness of the plate through the line of holes, AA, farthest from the butt.

The most efficient form of strap to connect two plates together would be as shown in Fig. 85, of diamond shape. Here the plate is only weakened to the extent of one rivet-hole. Such an efficient connection as this is not required in ship construction, because in all the plating we have to deal with, such as stringers and outer bottom-plating, the plate is necessarily weakened by the holes required for its connection to the beam or frame, and it is unnecessary to make the connection stronger than the plate is at a line of holes for connecting it to the beam or frame. In calculating the strength of a butt connection, therefore, we take as the *standard strength* the strength through the line of holes at a beam or frame, and we so arrange the butt strap that the strength by any of the modes of fracture will at least equal this standard strength.

Experimental Data.—Before we can proceed to calculate the strength of these butt connections, we must have some experimental data as to the tensile strength of plates and the shearing strength of rivets. The results of a series of experiments were given by Mr. J. G. Wildish at the Institution of Naval Architects in 1885, and the following are some of the results given :—

SHEARING STRENGTH OF RIVETS IN TONS.

(*Pan heads and countersunk points.*)

	Single shear.	Double shear.
¾ inch iron rivets in iron plates	10·0	18
¾ " " steel "	8·4	—
¾ inch steel " " "	11·5	21·2
⅞ " " " "	15·25	—
1·0 " " " "	20·25	—

It will be noticed that the shearing strength of the steel rivets of varying sizes is very nearly proportionate to the sectional area of the rivets. Taking the shearing strength of a ¾-inch steel rivet to be 11·5 tons, the strength proportionate to the area would be for a ⅞-inch steel rivet, 15·6 tons, and for a 1-inch steel rivet 20·4 tons. Also, we see that the double shear of a rivet is about 1·8 times the single shear.

The following results were given as the results of tests of mild steel plates :—

Unpunched	28¼ tons per square inch.	
Holes *punched*	22 „ „	or a depreciation of 22 per cent.
Holes *drilled*	29½ tons per square inch.	
Holes *punched* small, and the hole then *countersunk*	29 „ „	

The following give the strength of the material of the plates after being connected together by a butt strap:—

Holes *punched* the full size, the rivets having snap points	24·9 tons per square inch.
Holes *punched* small and then *countersunk*, the rivets being panhead, with countersunk points	28·9 „ „

It appears, from the above results, that if a plate has the holes drilled or has them punched and countersunk in the ordinary way as for flush riveting, the strength of the material is fully maintained. Also that, although punching holes in a plate reduces the strength from 28¼ to 22 tons per square inch, a reduction of 22 per cent., yet when connected by a butt strap, and riveted up, the strength rises to 24·9 tons per square inch, which is only 12 per cent. weaker than the unpunched plate, the process of riveting strengthening the plate.

The following table giving the results of more recent experiments, go to confirm the above figures:—

TABLE OF BREAKING STRESS IN TONS PER SQUARE INCH (OF AREA OF PLATING BEFORE TESTING).

Nature of test.		Mild steel.		High tensile steel.
Unperforated plate		(15) 27·4	(2) 28·0	(5) 34·3
Elongation % in 8 in.		(15) 25 %	(2) 29·8 %	(5) 24·7 %
Plate drilled and unriveted		(8) 29·6	(2) 30·6	(4) 34·1
„ „ riveted		—	(8) 31·6	(12) 36·0
Plate punched not countersunk, unriveted		(16) 26·5	22·1	(6) 26·7
„ „ „ „ riveted .		(33) 29·6	24·7	(12) 25·2
„ „ countersunk and riveted .		(16) 29·6	28·2	(12) 29·0
Single ⅞ in. rivet, countersunk point, tons per rivet	Mild steel rivet	(2) 14·8	15·7	—
	H.T. rivet . .	(2) 21·7	—	(2) 21·0
Double-shear ¾ in. rivet, countersunk point, tons per rivet	Mild steel rivet	—	28·9	—
	H.T. rivet . .	(2) 39·7	—	—

Figures in brackets ( ) denote the number of tests of which the result given is the mean.

In an ordinary butt-strap, with the holes spaced closely together in order to obtain a water-tight pitch for the rivets, it is found that the punching distresses the material in the neighbourhood of the holes, and the strength is materially reduced, as we have seen above, but after riveting the strength is to some extent restored. It was formerly the practice to anneal butt straps of steel plating, but this practice is now discontinued in both Admiralty and Lloyd's practice.

In our calculations of the strength of butt straps, we assume that the strength of the material between the rivet-holes is the same as the strength of the material of the unpunched plate.

Again, the plating, in the cases we have to deal with, has the riveting flush on the outside, and the holes are made with a countersink for this purpose. Here also we can assume that the strength of the material is the same as the strength of the material of the unpunched plate.

The specified tests for the tensile strength of mild steel plates are as follows :—

For ships built for the British Admiralty, not less than 26 and not more than 30 tons per square inch of section.

For ships built to the rules of Lloyd's Register, not less than 28 and not more than 32 tons per square inch of section.

The plates tested above showed a tensile strength of about 28 tons per square inch, or nearly midway between the limits laid down by the British Admiralty. It seems reasonable, therefore, in calculating the ultimate strength of riveted joints, to take as the strength of the material the *minimum* strength to which it has to be tested. Thus, in a ship built for the British Admiralty, we can use 26 tons as the strength per square inch of section, and in a ship built under Lloyd's rules, we can use 28 tons per square inch of section.

The following two examples will illustrate the methods adopted in calculating the strength of butt fastenings :—

1. A steel stringer plate is 48 inches broad and $\frac{7}{16}$ inch thick. Sketch the fastenings in a beam and at a butt, and show by calculations that the butt connection is a good one.

(*S. and A. Exam.*, 1897.)

For a $\frac{7}{16}$-inch plate we shall require $\frac{3}{4}$-inch rivets, and setting these out at the beam, we require 9 rivets, as shown in Fig. 84. The effective breadth of the plate through this line of holes is therefore—

$$48 - 9(\tfrac{3}{4}) = 41\tfrac{1}{4} \text{ inches}$$

and the strength is—

$$41\tfrac{1}{4} \times \tfrac{7}{16} \times 26 = 470 \text{ tons}$$

and this is the standard strength that we have to aim at in designing the butt strap.

(1) As regards the number of rivets. The shearing strength of a ¾-inch rivet being 11·5 tons, the number of rivets necessary to equal the standard strength of 470 tons is—

$$\frac{470}{11 \cdot 5} = 40 \cdot 8, \text{ say } 41 \text{ rivets}$$

If we set out the rivets in the strap as shown in Fig. 84, leaving the alternate rivets out in the line AA, it will be found that exactly 41 rivets are obtained, with a four-diameter pitch. So that, as regards the number of rivets, the butt connection is a good one.

(2) The strength of the plate in the line AA is the same as at the beam, the same number of rivet-holes being punched in each case.

(3) If the strap is $\tfrac{7}{16}$ inch thick, the strength of the strap in the line BB is given by—

$$\{48 - 16(\tfrac{3}{4})\} \times \tfrac{7}{16} \times 26 = 410 \text{ tons}$$

This is not sufficient, and the strap must be thickened up. If made ½ inch thick, the strength is—

$$\{48 - 16(\tfrac{3}{4})\} \times \tfrac{1}{2} \times 26 = 468$$

which is very nearly equal to the standard strength of 470 tons.

(4) The shear of the 9 rivets in the line AA is 103·5 tons, so that the strength of the plate through the line of holes CC and the shear of the rivets in the line AA are—

$$410 + 103 \cdot 5 = 513 \cdot 5 \text{ tons}$$

(5) Similarly, the strength of the strap through the line CC and the shear of the rivets in the line BB are—

$$468 + 184 = 652 \text{ tons}$$

The ultimate strengths of the butt connection in the five different ways it might break are therefore 471½, 470, 468, 513½, 652 tons respectively, and thus the standard strength of 470 tons is maintained for all practical purposes, and consequently the butt connection is a good one.

2. If it were required to so join two plates as to make the strength at the butt as nearly as possible equal to that of the unpierced plates, what kind of butt strap would you adopt?

Supposing the plates to be of mild steel 36 inches wide and ½ inch thick, give the diameter, disposition, and pitch of rivets necessary in the strap.

(*S. and A. Exam.*, 1895.)

The first part of this question has been already dealt with on p. 237. To lessen the number of rivets, it is best to use a double butt strap, as Fig. 85, so as to get a double shear of the rivets. Each of the butt straps should be slightly thicker than the half-thickness of the plate, say $\tfrac{5}{16}$ inch.

The standard strength to work up to is that of the plate through the single rivet-hole at the corner of the strap. ⅞-inch rivets being used, the standard strength is—

$$(36 - \tfrac{7}{8}) \times \tfrac{1}{2} \times 26 = 457 \text{ tons}$$

The single shear of a $\frac{7}{8}$-inch rivet is $15\frac{1}{4}$ tons, and the double shear may be taken as—

$$15 \cdot 25 \times 1 \cdot 8 = 27\tfrac{1}{2} \text{ tons}$$

and consequently the least number of rivets required each side of the butt is—

$$\frac{457}{27 \cdot 5} = 16 \cdot 6, \text{ say } 17 \text{ rivets}$$

The strength of the plate along the slanting row of holes furthest from the butt must be looked into. The rivets here are made with a water-tight pitch, say from 4 to $4\frac{1}{2}$ diameters. If we set out the holes for a strap 2 feet wide, it will be found that the strength is below the standard. A strap 3 feet wide will, however, give a strength through this line of about 465 tons, which is very near the required 457 tons. There are 13 rivets along the edge of the strap, and the inside may be filled in as shown, giving a total number of rivets, each side of the butt, of 19.

For the strength of an assemblage of plating like the outer bottom, we must take the strakes as assisting one another. If two passing strakes are assumed, then we can take a butt with a through strake each side, and see how the strengths by various methods of fracture compare with the standard strength at a frame.

For the strength of plating at a watertight bulkhead, the bulkhead liner is associated with the outside strake and one-half the adjacent inside strakes, and the strength should be brought up to that at an ordinary frame.

Professor Hovgaard, in "Structural Design of Warships," deals very exhaustively with the above. In particular he allows for the reduction of area caused by countersinking and the slightly greater diameter of hole in the plate as compared with the nominal diameter of the rivet.

Strength of Davits.—The size of davits for merchant vessels are usually fixed by the rules of a Registration Society.

The following is the rule adopted by Lloyd's Register, viz.:

For boats and davits of ordinary proportions the diameter in inches is one-fifth of the length of the boat in feet.

Where the height and spread of davits or dimensions of boats are not of ordinary proportions the diameter of davit in inches is found by the formula

$$d = \sqrt[3]{\frac{L \times B \times D}{40}\left(\frac{H}{3} + R\right)}$$

Where L.B.D. are the dimensions of the boat, H is the height and R the outreach from the point of support in feet.

The rule of the British Corporation is of the same form but slightly different, viz.:

$$d = \sqrt[3]{\frac{L \times B \times D}{46}\left(\frac{H}{2} + R\right)}$$

It is usual in H.M. service to test a davit to twice its working load, and this test load is used to calculate the dimensions. If W be the load in tons, r the outreach in inches, then the bending moment is $W \times r$ inch tons, and we apply the formula—

$$\frac{p}{y} = \frac{M}{I} \text{ to find the diameter } d, \ (y = \tfrac{1}{2} . d).$$

$$I = \frac{1}{16} . \frac{\pi . d^2}{4} . d^2, \quad \text{so that } p = \frac{32}{\pi} . \frac{W . r}{d^3} \text{ and}$$

$$d = 2 \cdot 17 \sqrt[3]{\frac{W . r}{p}}$$

Example.—A boat weighing 2 tons is carried in davits with an outreach of 6 ft. 6 in. Determine the diameter of davit, allowing a stress on the material of 5 tons per square inch.

The moment $W \times r = 2 \times 78 = 156$ inch tons, and the diameter is given by

$$d = 2 \cdot 17 \sqrt[3]{\frac{156}{5}} = 6 \cdot 85 \text{ in.}$$

Example.—A boat weighing 1 ton is carried in davits with an outreach of 6 ft. 6 in. Determine the diameter of davit, allowing a stress on the material of 5 tons per square inch.

$$\text{Moment} = 78 \text{ inch tons}$$

$$d = 2 \cdot 17 \sqrt[3]{\frac{78}{5}} = 5 \cdot 4 \text{ in.}$$

This davit was made 5½ in. diameter, and 3¾ in. at head and heel, the bow of davit being flattened out to an oval shape 5½ in. × 4¾ in.

Example.—A davit with outreach of 7 ft. 6 in. is tested to 3 tons. Find the maximum compressive and tensile tresses, the diameter of davit being 7 in.

The max. BM is—

$$3 \times 90 = 270 \text{ inch tons}$$

$$\text{I of cross-section} = \tfrac{1}{16} . \frac{\pi . 7^2}{4} . 49 = \pi . \frac{49 \times 49}{64}$$

$$y = 3 \cdot 5$$

$$\therefore p = \frac{M}{I} . y = \frac{270 \times 3 \cdot 5 \times 64}{\pi \times 49 \times 49} = 8 \text{ tons sq. in.}$$

There is an additional compressive force due to the weight, viz. $3 \div \frac{\pi . 49}{4} = \frac{12}{49 . \pi} = \cdot 08$ tons sq. in. The tensile stress will be diminished by this amount.

$\therefore$ Compressive stress = 8·08 tons sq. in.
Tensile stress = 7·92 tons sq. in.

Davit Diagram.—The following ingenious method of drawing once for all a davit diagram has been devised by J. J. King-Salter, Esq., R.C.N.C. Its use is very simple and obviates the necessity of calculating davit and similar diameters.

If units are taken in lbs. and inches, say a weight of w lbs. and an overhang of r inches, then if a working stress is assumed of 4·5 tons per square inch, the diameter of a davit can be expressed in the simple form—

$$d = \sqrt[3]{\frac{w \times r}{1000}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is seen to depend on the product $w \times r$. It can readily be shown that for a right-angled triangle, where a is the perpendicular from the right angle to the opposite side and b and c are the divisions of that side by the perpendicular, then

$$a^2 = b \times c, \text{ or } a = \sqrt{b . c}$$

In Fig. 85A, above the base at the side is set up a scale of overhang in inches r, and below a scale of weight in lbs. w. Along the base is set off a scale of $\sqrt{w . r}$. Thus for $w =$ 10,000 lbs. and $r = 100$, $\sqrt{w . r} = 1000$. This point must subtend a right angle to the values of w and r taken, and this will determine the scale to use along the base.

Now at various points along the base set up the corresponding value of d from the formula (1) above. Thus where $\sqrt{w . r} = 1000$, $w . r = 1{,}000{,}000$ and $d = 10''$; $d = 5''$ at an abscissa of 354, and so on. Through the spots thus obtained a curve of diameters is drawn as shown.

The method of use is to employ a set square with the sides forming the right angle passing through the values given by the problem fot w and r, with the right angle on the base line.

An ordinate from this point to the curve will give the diameter required. Thus for $w = 4000$ lbs., $r = 70''$, d is found to be 6·6 in. If diameter is 8 in. say, and overhang is 80 in., the load is 6400 lbs., and so on.

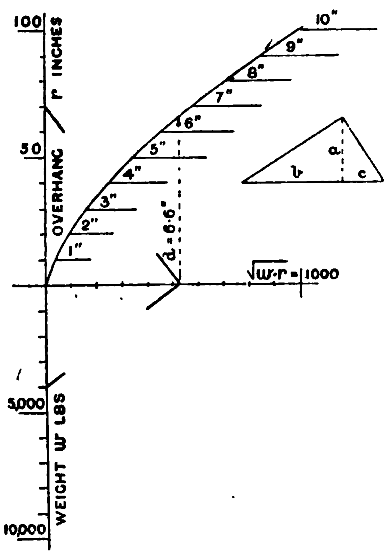


Fig. 85A.

The diagram can be drawn out on a large scale and mounted for general drawing office use.

Pillars.—Gordon's Formula.—The formula usually employed to determine the strength of pillars is that known as Gordon's formula, as follows:—

W is the crippling load, A the cross section, f the stress given in table, n is obtained from $I = n . A . h^2$, where h is the least breadth, c is a coefficient given in the table.

$$W = \frac{A . f}{1 + \frac{l^2}{c . n . h^2}}$$

$n = \frac{1}{12}$ for a rectangular section and $\frac{1}{16}$ for circular section; for a circular hollow section $n = \frac{1}{8}$.

Units are taken as tons and inches.

Material.	f tons sq. in.	c Ends free.	One end fixed.	Both ends fixed.
Wrought iron	16	9,000	18,000	36,000
Mild steel	30	9,000	18,000	36,000
Cast iron	35	1,600	3,200	6,400
Dry timber	3	750	1,500	3,000

This formula is empirical. For a discussion regarding its use the student is referred to such works as Lineham's "Mechanical Engineering."

Example.—A cargo derrick for a vessel is constructed of steel plating $\frac{8}{20}$ in. thick and two T bars 5 in. × 3 in. × $\frac{8}{20}$ in. The jib is 40 ft. long, and the topping lift is led to a point on a mast 32 ft. above heel of derrick. The

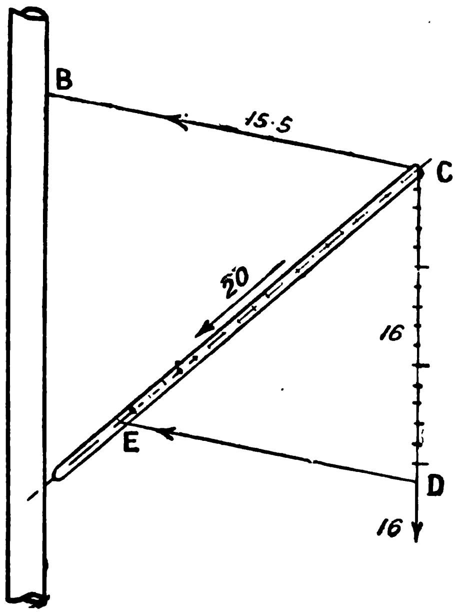


FIG. 85B.

maximum load to be lifted being 16 tons; calculate the approximate diameter of the derrick if the maximum stress on the material is not to exceed 4 tons per sq. in. (neglect effect of T bars).

(*Honours B. of E.* 1909.)

By setting out the triangle of forces CDE at the head of derrick the thrust on derrick is found to be 20 tons, DE being parallel to CB (Fig. 85B).

In the above formula $f = 4$, $c = 9000$, $n = \frac{1}{4}$, $A = \frac{176}{140} \cdot d$, d being the unknown diameter.

$$\therefore \frac{20}{\frac{176}{140} . d} = \frac{4}{1 + \frac{(480)^2}{9000 \times \frac{1}{8} \times d^2}}$$

$$1 + \frac{205}{d^2} = 0·25 . d$$

$$\text{or } 0·25d^3 - d^2 = 205$$

By trial d is found to be 11 inches nearly.

Example.—A wooden derrick 34 ft. long when tested to twice the working load is found to be subject to a thrust of $4\frac{1}{2}$ tons. Determine the diameter, allowing a factor of safety of 6 when being tested.

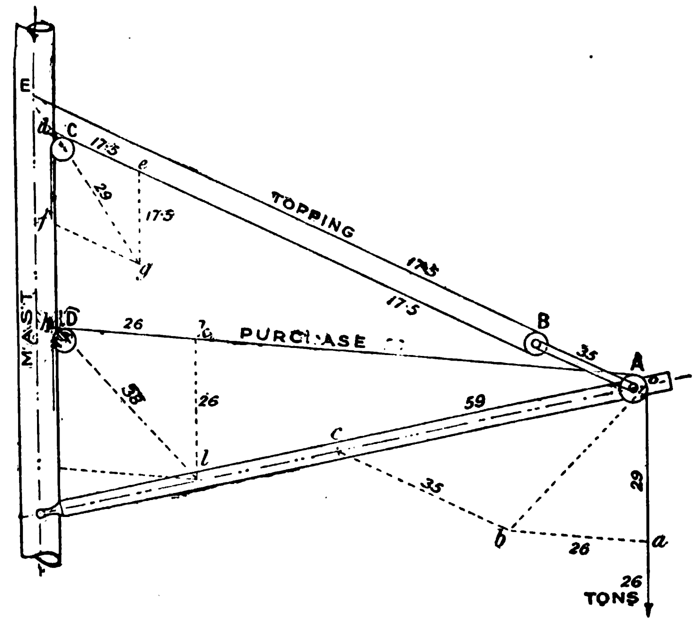


FIG. 85c.

In the above formula, W = 25·5 tons, $f = 3$, $c = 750$, $l = 34 \times 12$, $n = \frac{1}{16}$, d = diameter.

$$\therefore \frac{25·5}{\frac{\pi . d^2}{4}} = \frac{3}{1 + \frac{(34 \times 12)^2}{750 \times \frac{1}{16} \times d^2}}$$

$$\text{or } 1 + \frac{3550}{d^2} = 0·092d^2$$

$$\text{or } 0·092d^4 - d^2 = 3550$$

By trial d is found to be about $14\frac{1}{2}$ in.

Example.—A boat hoisting derrick 60 ft. long, estimated weight 6 tons, is arranged as shown on figure herewith, the purchase being single through sheaves A and D. The topping lift has the fixed part at E, and passes through sheaves B and C. Determine the forces on blocks and ropes and the thrust on derrick, when holding the test load of 26 tons. Determine the diameter of the derrick if formed of $\frac{3}{8}$ in. steel plating, the T bars forming edge strips being neglected, and a factor of safety of 5 being assumed. (Fig. 85C.)

oa is set down = 29 tons, *i.e.* 26 tons plus half weight of derrick, and *ab* is drawn parallel to the purchase AD, and equal to 26 tons. Then *ob* is the resultant force at the head of derrick due to the forces on the purchase and the weight of derrick. *bc* is then drawn parallel to the topping lift. Then *obc* is a triangle of forces, giving for the force on topping lift *bc* = 35 tons and the thrust on derrick *oc* = 59 tons. The force in the link AB is therefore 35 tons, and the two parts of topping lift have each a force of 17·5 tons. The force on the block C is found by drawing the triangle of forces *deg*, *de* = *eg* = 17·5 tons from which force on block is *dg* = 29 tons. Similarly the force on block D is found to be 38 tons.

The length of derrick from pin of sheave to the trunnion is 56·5 feet. In Gordon's formula we have therefore—

$W = 59 \times 5 = 295$ tons, $f = 30$, $c = 9000$, $n = \frac{1}{8}$, d, the diameter, is the unknown, $A = \frac{3}{8} . \pi . d$, $l = 56·5 \times 12$.

We have therefore—

$$295 = \frac{30 \times \frac{3}{8} \times \frac{22}{7} \times d}{1 + \frac{(56·5 \times 12)^2}{9000 \times \frac{1}{8} \times d^2}}$$

$$\text{or } 1 + \frac{410}{d^2} = 0·12d$$

$$\text{or } 0·12d^3 - d^2 = 410$$

from which d is found by trial to be 18½ ins. nearly.

It may be noted that the above derrick was actually made 20 in. diameter, of plating, 14 lbs. per square foot, which allows for the loss due to the rivets in butt strap.

Fig. 85D illustrates the case where electric winches are employed for both topping lift and purchase, and the greater speed of these winches renders more turns of rope necessary.

The test load is 32 tons, being twice the weight of the boat. This with the weight of the block A gives 32·4 tons, which is taken by the three ropes supporting A, giving 10·8 tons to each. To find the force at the topping lift and on the derrick, we draw the diagram of forces, shown on top of figure. *ab* = 36 tons, *i.e.* 32 + 3·6 (half weight of derrick) + 0·4 (weight of block A); *bc* = 10·8 is drawn parallel to the purchase, so that *ac* is the force at the head of the derrick; *cd* is drawn parallel to the topping lift, and *ad* parallel to the derrick. Then *cd* is force on topping lift = 59 tons, and *ad* the thrust on derrick = 60 tons. The topping lift is in four parts, giving 15 tons to each, and the block C has 59 tons. The block D has three parts, viz. 45 tons. The block E has 15 tons along each of the ropes, and as shown by the

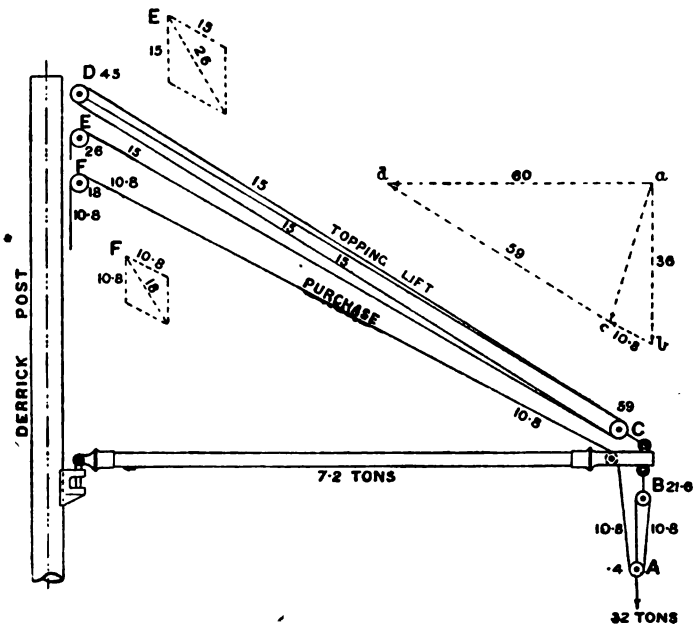


FIG. 85D.

diagram, sustains a force of 26 tons. The block F has 10·8 tons along each of the ropes, and, as shown by the diagram, sustains a force of 18 tons. The strength at the heel of the derrick can be allowed for knowing the thrust to be 60 tons. The test loads of the various parts can now be allowed for, being usually about twice that due to the 32-ton test load applied.

Fig 85E shows an ordinary form of derrick for a cargo vessel, the derrick being supported by a stump mast. The purchase is taken round two single blocks A and B, and thence

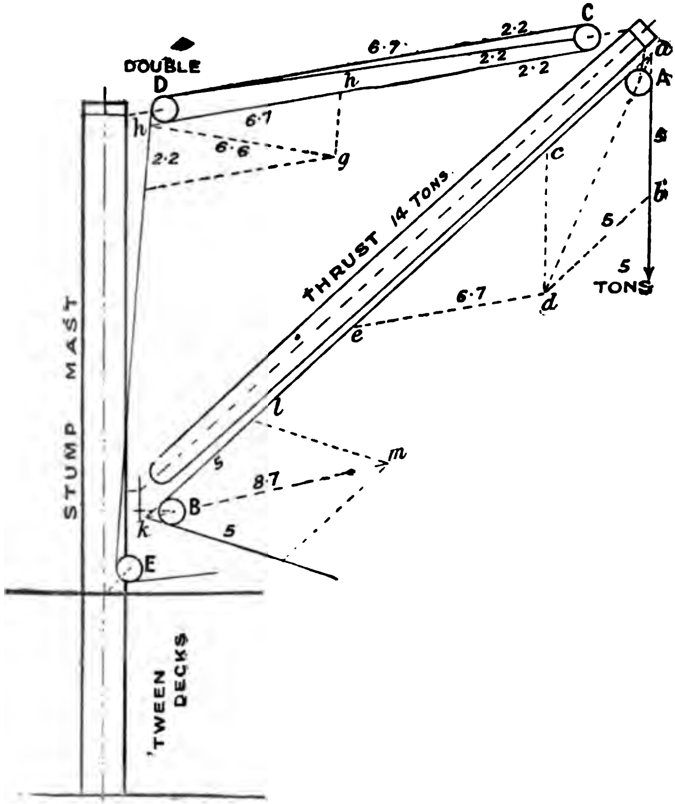


FIG. 85E.

to the winch. The topping lift has a single block at C, a double block at D, and a single block at E, and thence to the winch. Taking a load of 5 tons the triangle of forces *abd* is drawn $ab = ac = 5$, and the resultant force due to the load is $ad = 9$ tons, which is the force on the block A. *de* is drawn parallel to the topping lift. Then $ae = 14$ tons is the thrust on the derrick, and 6·7 tons is the force on block C. This divided into 3 gives 2·2 tons in each portion of the topping lift. The force on block D is obtained by drawing the triangle of forces *fgh*. Similarly the forces on the blocks B and E are obtained. The forces thus obtained give a basis for estimating the strength of all the parts, including the derrick and the stump mast.

Shaft Brackets. (By A. W. Johns, Esq., R.C.N.C.).—The length and diameter of the drum or barrel of a shaft bracket are determined by the requirements of the engineer. The inside diameter is arranged to take the shaft and its

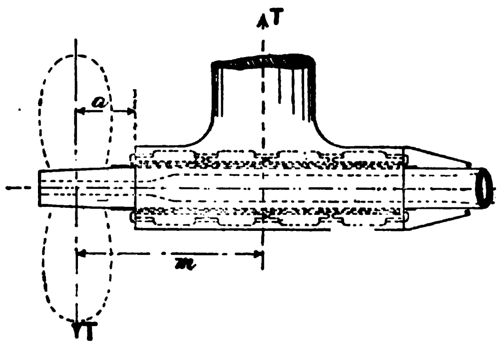


FIG. 85F.

bearings and bushes. The outside diameter is usually from 3 to 6 inches greater than the inside diameter, depending on the size of shaft. The inside surface is generally gulleted to a depth of from 1 to 1½ inches. The length of barrel is governed by the length required by the engineer's bearings in the bracket (see Fig. 85F).

The length of the arms or struts must necessarily depend on the position of the axis of the shaft at the bracket and the shape of the ship in the vicinity. The section of the arms is usually pear-shaped (see Figs. 85G and 108 for examples), with the blunt end forward. The dimensions of the section must be governed by the straining action to which the bracket is subjected. Formerly these dimensions appear to have been determined in a rough-and-ready way from the experience of the designer responsible. Knowing the dimensions in previous cases which on service had proved sufficiently

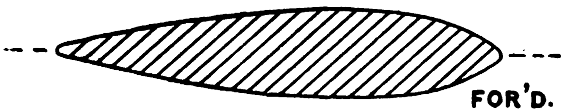


FIG. 85a.

strong, he would vary these dimensions in a new ship according to the variation of the horse-power, or perhaps the size and overhang of the tail shaft. Consequently it will be found that ships of about the same size, horse-power and revolutions produced under different designers, have entirely different dimensions (and weights) of shaft brackets.

At first sight a suitable basis of comparison for such dimensions appears difficult to obtain, but investigation proves that the matter is a comparatively simple one, as is seen by what follows:—

With the centre of gravity of the revolving parts, viz. shaft and propeller, in the axis of rotation, the straining actions which may operate on a bracket are as follows, viz.:—

1. Forces due to the weight of the propeller, shaft and bracket. These are equivalent to a downward force on the bracket, and a bending moment on it, equal to the difference in the moments of weight on the forward and after sides. Both the force and the bending moment may be increased appreciably by the accelerative effect during pitching.

Thus in a given ship 500 feet long pitching in a "single" period of 3 seconds, the maximum acceleration is $250 \times \frac{\pi^2}{T^2} \cdot \times \theta$, where θ is angle of pitching (see Chap. XII. on Rolling).

If $\theta = 4^\circ$, say $\frac{1}{15}$ in circular measure, acceleration = 20 in foot-second units. This added to the acceleration due to gravity gives 52·2 or a *virtual* weight of 1·6 times the actual, and all the forces are increased in this ratio.

2. Forces called into play when pitching or turning due to the gyroscopic action of the propeller and shaft.

3. Forces caused by unequal pressure on the blades of the propeller when the ship is turning. Here, owing to the transverse motion of the stern, the forces on the blades above the horizontal will be different to those below.

If, however, the centre of gravity of the revolving weights is not in the axis of rotation, there will be in addition to the above forces a centrifugal force operating which will tend to bend the shaft where it enters the strut, and will also tend to bend and twist the bracket. At high revolutions, which is the case in turbine machinery, heavy straining actions are set up if a propeller blade is broken or lost.

The following are approximate values of the various forces and moments considered above, worked out for the case of a large cruiser.

	Force on bracket.	Moment on bracket.
(1) Due to weight of propeller, etc.	30 tons	90 foot tons
(1a) Due to weight of propeller, etc., when pitching . .	48 ,,	144 ,,
(2) Gyroscopic action . . .	—	50 ,,
(3) Turning at full speed with full rudder angle . . .	12 ,,	80 ,,
(4) Centrifugal action due to the loss of a propeller blade at full revolutions	100 ,,	500 ,,

It will be seen that the last case produces by far the heaviest straining effect. In addition to straining the bracket, however, the shaft also will be strained, the maximum stress occurring at the section immediately at the after end of the barrel of the bracket. For good design the bracket should be stronger than the shaft, for then the shaft would break at the after end of the boss of shaft bracket, whereas if the bracket were weaker than the shaft the former would first break and the shaft losing its after support would then bend or break with perhaps disastrous effect on the ship. A basis of calculation is therefore obtained by considering the strength of the shaft and making the shaft bracket somewhat stronger.

If T is the force at the propeller (Fig. 85F), producing a bending moment T . a on the shaft which will just bring the material of the shaft to its full working strength, this force T must be employed in determining the dimensions of the arms of the bracket and also in determining the number, size and spacing of the rivets connecting the bracket to the hull. T acting at propeller is equivalent to—

1. A parallel force T acting directly on the bracket, and
2. A moment k . T . m on the bracket, where k has an average value of about 0·65.

If T is caused by centrifugal action, then as the shaft revolves it is always being bent in the same way, but the bracket being fixed the force and moment on it are constantly altering in direction.[1] Bending alone occurs when the line of action of T lies in a plane passing through the axis of shaft and bisecting the angle between the arms. Twisting occurs when the line of action of T is perpendicular to that plane. For other directions of the line of action of T combined bending and twisting occur.

Generally bending alone produces the greatest stress on the shaft arms, and this produces a stress given by—

$$p = \frac{k \,.\, T \,.\, m \,.\, y \,.\, \cos\theta}{2I}$$

where I is the moment of inertia of a right section of the arm about an axis through the geometrical centre and perpendicular to the longer dimension of the section.
y is the distance of the most strained layer from this axis;
and θ is half the angle between the arms.

For ordinary pear-shaped sections, $y = 0{\cdot}55$ R and $I = \frac{1}{27} . R^3 . r$, where R and r are the longer and shorter dimensions of the section of the arm.

Taking, say, 6 tons as the maximum working strength of the shaft, the force T necessary to strain the shaft to this limit can readily be found when a the overhang and D and d the external and internal diameters of the shaft are known. Supposing the shaft bracket is of cast steel and taking $4\frac{1}{2}$ tons as the working strength (5 tons is really allowed, but $\frac{1}{2}$ ton is

[1] The loss of a propeller blade is soon evident, for the ship will vibrate violently if the revolutions of the engine approach the full number.

allowed for the force T acting directly on the bracket), the following relation is obtained—

$$R^2 . r = 0.63 \times \frac{D^4 - d^4}{D} \times \frac{m}{a} \times \cos\theta \quad . \quad . \quad (1)$$

All dimensions being in inches.
Usually θ = about 45° and we then have—

$$R^2 . r = 0.44 \times \frac{D^4 - d^4}{D} \times \frac{m}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If, however, θ is small, we have approximately—

$$R^2 . r = 0.63 \times \frac{D^4 - d^4}{D} \times \frac{m}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

As stated above, the stress produced on the bracket by bending is usually greater than that produced by twisting, but in the case where the angle between the arms is small the stress due to twisting should also be investigated. This can be done as follows:—

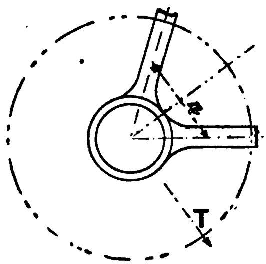


FIG. 85H.

Taking, as in the figure 85H, n as the distance between the centres of the arms, A the area of each arm, q the stress in the arm, the moment resisting twisting is given by $q . A . n$. This must equal $k . T . m$, and hence the stress due to twisting becomes

$$q = \frac{k . T . m}{A . n} \qquad (A = 0.75 R . r)$$

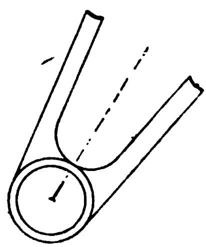


FIG. 85J.

q is a shearing stress and the equation shows that if A is constant q increases as n decreases. Close to the barrel q is a maximum, and is a minimum where the arms enter the hull. For q to be constant A should vary inversely as n. Usually, however, A is kept constant and the arms are run tangential, as in Fig. 85J, instead of radial to the barrel. This has the effect of increasing n near the barrel and diminishing q.

Equations (1), (2), and (3) above can be used to determine the value of $R^2 r$, where the bracket is of cast steel. If the bracket is of other material the working strength of the latter must be substituted for the $4\frac{1}{2}$ tons used above.

It will be noticed that economy of material is obtained by making the ratio $R \div r$ as large as possible, for since the square of R enters into the relation it has far more influence than r, which appears in the first power only. Thus if $R^2 . r = 8000$ and $R = 3r$, then $R = 29$ in. and $r = 9\frac{3}{4}$ in. and $A = 212$. Whereas if $R = 6r$, $R = 36$ in., and $r = 6$ in., and $A = 162$ or a saving in weight of about 25 per cent. There is also an appreciable reduction in resistance. Taylor's experiments with shaft brackets show that resistance in lbs. per foot length of shaft bracket arm is given by—

$$F = \frac{c}{1000}(A + 40)V^2$$

where V = speed in knots

A = area of section in square inches, for values between 40 and 175 sq. inches.

c = a constant depending on the ratio $\frac{R}{r}$.

VALUES OF c.

Ratio $\frac{R}{r}$	3	4	5	6	7	8	9	10	11	12
Value of c	1·88	1·32	1·07	0·94	0·86	0·80	0·76	0·74	0·72	0·71

Further than this there is little doubt that with bracket arms whose ratio $R \div r$ is small, at high speeds a large amount of dead water trails behind the arms reaching to the propeller disc and causing vibration and loss of propeller efficiency as the blades of the propeller enter and leave the dead water. By increasing the ratio $R \div r$ these effects are diminished. Fig. 85E gives the section of the arm of shaft bracket of a recent ship of large power and great speed.

Finally it is interesting to compare the dimensions given by formula (2) above with those adopted in practice in particular cases as the result of experience. The ratio $R \div r$ has been kept the same in the calculation as actually adopted. All dimensions are in inches.

	D	d	m	a	Actually fitted.		Calculated.	
					R	r	R	r
Cruiser .	7¾	0	37	12	14	4	13·3	3·8
,,	13	9	50	21	16	6	16·8	6·3
,,	16	10	56	27	20	6½	21·5	7
,,	16	10	62	27	24	6	24	6
,,	19½	7	78	33	28	8	30·1	8·6
,,	21	11	76	30	32	10	31·2	9·8
Battleship	20	11½	72	30	27	7½	30	8·4
Destroyer	8¼	4½	36	18	13	2	14·3	2·2

The relations in (1), (2), and (3) given above apply strictly to the usual pear-shaped section of arm, but the method indicated can be applied to any particular case and the dimensions calculated.

Examples to Chapter VI.

1. The area of the outer bottom plating of a ship, over which the plating is worked 25 lbs. per square foot, is 23,904 square feet, lapped edges and butt straps, both double-riveted. Estimate the difference in weight due to working the plating with average-sized plates 20′ × 4½′, or with the average size 12′ × 3′.

Ans. About 20 tons.

2. Steel angle bars 3½″ × 3″ are specified to be 8½ lbs. per lineal foot instead of $\frac{7}{16}$ inch thick. Determine the saving of weight per 100 lineal feet.

Ans. 52 lbs.

3. Determine the weight per lineal foot of a steel T-bar 5″ × 4″ × ½″.

Ans. 14·45 lbs.

4. For a given purpose, angle bars of iron 5″ × 3″ × $\frac{8}{16}$″ or of steel 5″ × 3″ × $\frac{8}{20}$″ can be used. Find the saving of weight per 100 feet if steel is adopted.

Ans. 95 lbs.

5. A mast 96 feet in length, if made of iron, is at its greatest diameter, viz. 32 inches, $\frac{8}{16}$ inch thick, and has three angle stiffeners, 5″ × 3″ × $\frac{8}{16}$″. For the same diameter, if made of steel, the thickness is $\frac{18}{20}$ inch, with three angle stiffeners 5″ × 3″ × $\frac{8}{20}$″. Estimate the difference in weight.

Ans. About 1 ton.

6. At a given section of a ship the following is the form: The lengths of ordinates 3 feet apart are 19·6, 18·85, 17·8, 16·4, 14·5, 11·8, 7·35, and 1·0 feet respectively. Estimate the vertical position of the centre of gravity of the curve forming the section, supposing it is required to find the vertical position of the centre of gravity of the bottom plating of uniform thickness.

Ans. About 12¼ feet from the top.

7. The half-girths of the inner bottom of a vessel at intervals of 51 feet are 26·6, 29·8, 32·0, 32·8, and 31·2 feet respectively, and the centres of gravity of these half-girths are 18·6, 20·6, 21·2, 20·0, 17·4 feet respectively below the L.W.L. Determine the area of the inner bottom and the position of its centre of gravity both longitudinally and vertically. If the plating is of 15 lbs. to the square foot, what would be the weight, allowing 14½ per cent. for butts, laps, and rivet-heads.

Ans. 12,655 square feet; 105 feet from 'iner end, 20 feet below the L.W.L.; 97 tons.

8. The whole ordinates of the boundary of a ship's deck are 6·5, 24, 29, 32, 33·5, 33·5, 33·5, 32, 30, 27, and 6·5 feet respectively, and the common interval between them is 21 feet.

The deck, with the exception of 350 square feet, is covered with ⅜ inch steel plating worked flush jointed, with single riveted edges and butts. Find the weight of the plating, including straps and fastenings.

Ans. 45 tons.

9. A teak deck, 2½ inches thick, is supported on beams spaced 4 feet apart, and weighing 15 pounds per foot run. Calculate the weight of a middle-line portion of this deck (including fastenings and beams) 24 feet long and 10 feet wide. *Ans.* 1·65 tons nearly.

10. Taking the net weight of outer bottom plating of a vessel as 1000 tons, *estimate* the saving of weight if the average size of plates is 20 feet by 5 feet as against 18 feet by 3½ feet. (Butt-straps double riveted, lapped edges double riveted, ⅞-inch rivets.) *Ans.* 43 tons about.

11. A longitudinal W.T. bulkhead is bounded at its upper edge by a level deck (having 9-inch beams, 4 feet apart) and at its lower edge by the inner bottom. The depths of the bulkhead at ordinates 61 feet apart are, commencing from forward, 9·0, 16·7, 19·3, 15·4, 9·5 feet respectively. The plating of the bulkhead is 15 lbs. per square foot worked vertically, single riveted, and the stiffening consists of Z bars of 12 lbs. per foot spaced 4 feet apart with intermediate angles of 7 lbs. per foot. There is a single boundary bar of 8·5 lbs. per foot.

Calculate (1) the weight of the bulkhead.
" (2) the distance of C.G. from forward end.
" (3) the distance of C.G. below the deck.

Ans. (1) 39 tons; (2) 120·5 feet; (3) 8 feet.

12. The half ordinates of upper deck of a ship 360 feet long are (1) 0; (2) 9·4; (3) 16·2; (5) 24·4; (7) 28·8; (9) 31·2; (11) 32·4; (13) 32·2; (15) 31·5; (17) 29·6; (19) 24 6; (20) 20·1; (21) 13·8. Over the midship portion (7) to (15) the beams are 24½ lbs. per foot, 4 feet apart, and the plating is 20 lbs. per square foot, with single-riveted edge-strips and double-riveted butt-straps. At the ends the beams are 24½ lbs., 3 feet apart, completely covered with plating 10 lbs. per square foot, lapped and single-riveted. The boundary bar is 3 inches by 3½ inches of 8½ lbs. per foot, and the deck is completely planked with 3-inch teak. Find total weight, neglecting hatches, etc.

Ans. 325 tons about.

CHAPTER VIII.

STRAINS EXPERIENCED BY SHIPS—CURVES OF LOADS, SHEARING FORCE, AND BENDING MOMENT—"SMITH" CORRECTION—EQUIVALENT GIRDER.

Strains experienced by Ships.—The strains to which ships are subjected may be divided into two classes, viz.—

1. *Structural strains*, *i.e.* strains which affect the structure of the ship considered as a whole.

2. *Local strains*, *i.e.* strains which affect particular portions of the ship.

1. *Structural Strains.* — These may be classified as follows:—

(*a*) Strains tending to cause the ship to bend in a fore-and-aft direction.

(*b*) Strains tending to change the transverse form of the ship.

(*c*) Strains due to the propulsion of the vessel, either by steam or sails.

2. *Local Strains.*—These may be classified as follows:—

(*a*) Panting strains.

(*b*) Strains due to heavy local weights, as masts, engines, armour, guns, etc.

(*c*) Strains caused by the thrust of the propellers.

(*d*) Strains caused by the attachment of rigging.

(*e*) Strains due to grounding.

We will now deal with some of these various strains to which a ship may be subjected in a little more detail.

Longitudinal Bending Strains.—A ship may be regarded as a large beam or girder, subject to bending in a fore-and-aft direction. The support of the buoyancy and the distribution of weight vary considerably along the length of a ship, even when floating in still water. Take a ship and imagine she is cut by a number of transverse sections, as in Fig. 86. Each of the portions has its weight, and each has an upward support of buoyancy. But in some of the portions the weight exceeds the buoyancy, and in others the buoyancy exceeds the weight. The total buoyancy of all the sections must, of course, equal the total weight. Now imagine that there is a water-tight bulkhead at each end of each of these portions, and the ship is actually cut at these sections. Then the end portions (1) and (5) have considerable weight but small displacement, and consequently they would sink deeper in the water if left to themselves.[1] In

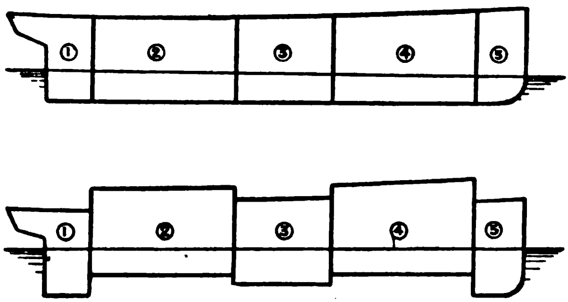


FIG. 86.

the portions (2) and (4), on the other hand, the buoyancy might exceed the weight (suppose these are the fore-and-aft holds, and the ship is light), and if left to themselves they would rise. The midship portion (3) has a large amount of buoyancy, but also a large weight of engines and boilers, and this portion might very well have to sink a small amount if left to itself. In any actual ship, of course, it is a matter of calculation to find how the weight and buoyancy vary throughout the length. This case is somewhat analogous to the case of a beam supported and loaded as shown in Fig. 87. At each point along the beam there is a tendency to bend, caused by the way the beam is loaded and supported, and the beam must be made

[1] Strictly speaking, each portion would change trim if left to itself, but we suppose that the various portions are attached, but free to move in a vertical direction.

sufficiently strong to withstand this bending tendency. In the same way, the ship must be constructed in such a manner as to resist effectually the bending strains that are brought to bear upon the structure.

When a vessel passes out of still water and encounters

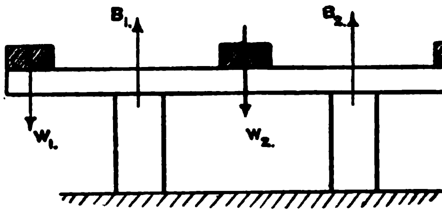
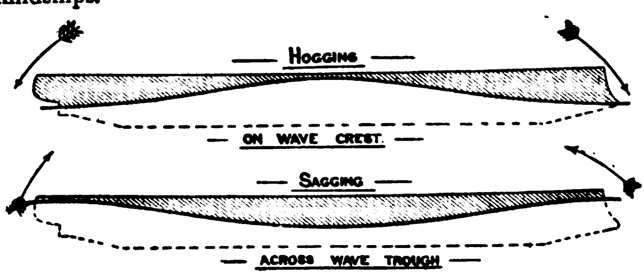


FIG. 87.

waves at sea, the strains to which she is subjected must differ very much from those we have been considering above. Suppose the ship to be end on to a series of waves having lengths from crest to crest or from trough to trough equal to the length of the ship. We will take the two extremes.

(1) The ship is supposed to have the *crest* of the wave amidships.

(2) The ship is supposed to have the *trough* of the wave amidships.



FIGS. 88, 89.

(1) This is indicated in Fig. 88. At this instant there is an excess of weight at the ends, and an excess of buoyancy amidships. The ship may be roughly compared to a beam supported at the middle, with weights at the end.

as in Fig. 90. The consequence is that there is a tendency for the ends to droop relatively to the middle. This is termed *hogging.*

(2) This is indicated in Fig. 89. At this instant there is an excess of weight amidships, and an excess of buoyancy at the ends, and the ship may be roughly compared to a beam supported at the ends and loaded in the middle. as Fig. 91. The consequence is, there is a tendency for the middle to droop relatively to the ends. This is termed *sagging.*

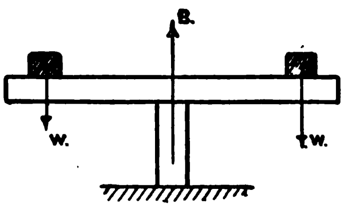


FIG. 90.

We have seen above how the ship may be compared to a beam, and in order to understand how the material should be disposed in order best to withstand the bending strains, we will consider briefly some points in connection with ordinary beams.

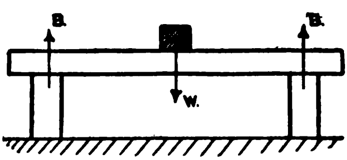


FIG. 91.

Take a beam supported at the ends and loaded at the middle. It will bend as shown exaggerated in Fig. 92. The resistance the beam will offer to bending will depend on the form of the section of the beam. Take a beam having a sectional area of 16 square inches. We can dispose the material in many different ways. Take the following:—

(*a*) 8 inches wide, 2 inches deep (*a*, Fig. 93).
(*b*) 4 inches wide, 4 inches deep (*b*, Fig. 93).
(*c*) 2 inches wide, 8 inches deep (*c*, Fig. 93).
(*d*) 8 inches deep, with top and bottom flanges 5 inches wide and 1 inch thick (*d*, Fig. 93).

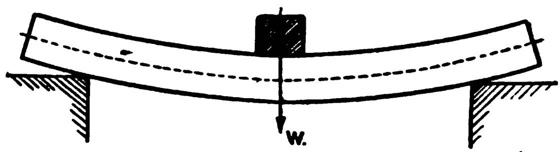


FIG. 92.

Then the resistances of these various sections to bending compare as follows:—

If (*a*) is taken as 1, then (*b*) is 2, (*c*) is 4, and (*d*) is $6\frac{5}{8}$.

We thus see that we can make the beam stronger to resist bending by disposing the material far away from the centre.

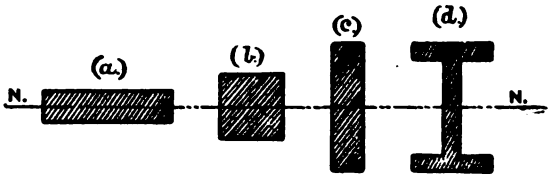


FIG. 93.

The beam (*d*) has $6\frac{5}{8}$ times the strength of (*a*) against bending, although it has precisely the same sectional area. A line drawn transversely through the centre of gravity of the section of a beam is termed the neutral axis.

In the British Standard Sections it will be found that for Z bars and channel bars the flanges are distinctly thicker than the web.

These principles apply equally to the case of a ship, and we thus see that to resist bending strains the material of the structure should be disposed far away from the neutral axis.[1]

For hogging strains, the upper portions of the vessel are in tension and the lower portions are in compression. For sagging strains, the upper portions are in compression and the lower portions are in tension. Thus the portions of the structure that are useful in resisting these hogging and sagging strains are the upper and main decks and stringers, sheer strake and plating below, plating at and below the bilge, both of the inner and outer bottom, keel, keelsons, and longitudinal framing.

Strains tending to change the Transverse Form of the Ship.—Strains of this character are set up in a ship rolling heavily. Take a square framework joined at the corners, and imagine it to be rapidly moved backwards and forwards as a ship does when she rolls. The framework will not break, but will distort, as shown in Fig. 94. There is a tendency to distort in a similar

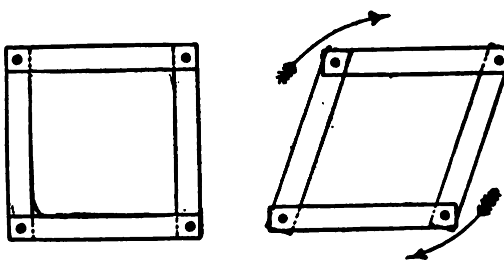


FIG. 94.

way in a ship rolling heavily, and the connections of the beams to the sides, and the transverse structure of the ship, must be made sufficiently strong to prevent any of this racking taking place. Transverse bulkheads are valuable in resisting the tendency to change the transverse form.

A ship, when docked, especially if she has on board heavy weights, as armour or coals, is subjected to severe strains tending to change the transverse form. If the ship is supported

[1] There are other strains, viz. shearing strains, which are of importance (see later).

wholly at the keel, no shores being supposed placed in position, the weight on either side the middle line tends to make the sides drop, and bring the beams into tension. A ship when docked, however, is partially supported by shores as well as at the keel as the water leaves, so that this case is an extreme one.

All recent armoured ships of the Royal Navy have flat bottoms amidships and sit down on three rows of blocks.

Panting.—This term is used to describe the working in and out of the plating, and it is usually found at the fore and after ends of the ship, where the surface is comparatively flat. The forward end especially is subject to severe blows from the sea, and special attention is paid to this part by working special beams and stringers to succour the plating. In vessels built to the rules of Lloyd's Register, the following rules have to be carried out to provide sufficient local strength against panting:—

All stringers, where practicable, to extend fore and aft, and to be efficiently connected at their ends with plates forming hooks and crutches of the same thickness as the floor-plates amidships, and those below the old beams should be spaced about 4 feet apart. In vessels whose longitudinal number[1] is 18,200, or above, an additional hook or crutch should be fitted at the ends of the vessel, between each tier of beams, to the satisfaction of the Surveyors.

Tiers of beams are fitted throughout the depths of peaks not more than 8 feet apart. Extra beams, bracket knees and stringers are fitted abaft the collision bulkhead. Where a vessel has considerable sheer, additional transverse strength is provided. Panting beams and stringers are fitted at the after end of vessel where considered necessary. Sketches of panting arrangements have in all cases to be submitted for approval.

[1] This number is $L \times (B + D)$, where

L is length from forepart of stem to afterpart of sternpost on the range of the upper deck.
B is greatest moulded breadth.
D is depth at mid-length from top of keel to top of beam at side of uppermost continuous deck.

The other local strains mentioned above have to be provided for by special local strengthening.

Longitudinal Bending Strains, etc.—We have seen above the general principles which underlie the subject of the strains that affect the longitudinal portions of the structure of a ship. A ship may be compared to a beam, and it is possible to determine under certain given conditions how this beam is loaded, and the stresses that in consequence are brought on the structure of the ship. Our object now is to show how these may be made the subject of calculation for any given case. It is, however, necessary to discuss in some detail the general subject of beams, and then pass on to the application to actual ships.

In a beam loaded in a given way, say as in Fig. 87, at every point along the beam there is a certain *bending moment*, or tendency to bend, and the distribution of this bending moment along the beam depends on the way the beam is loaded and supported.

Take the simple case of a beam supported at the ends and loaded in the middle with a weight W, the length of the

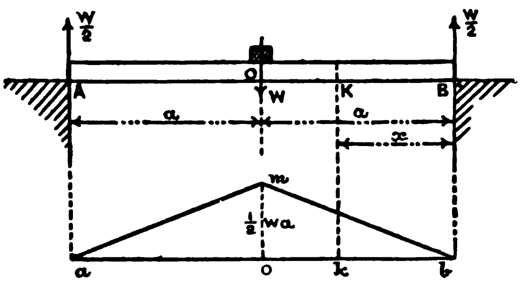


FIG. 95.

beam being *2a* (Fig. 95). At each end there is an upward support $\frac{W}{2}$, and the bending moment at the middle of the beam O or $M_0 = \frac{W}{2} \times a = \frac{1}{2} . W . a$, and the bending moment at any point K, distance x from the end: $M_k = \frac{1}{2} . W . x$. If the value of this bending moment be determined for various points along the beam, we shall be able to draw a line through all the spots as *amb*, which has a maximum ordinate at the centre *o* of $\frac{1}{2}$. W . *a*. This line will give the bending moment at any point along the beam.

Or take the case of a beam supported at the ends and loaded uniformly with the weight *w* per foot run, the total weight

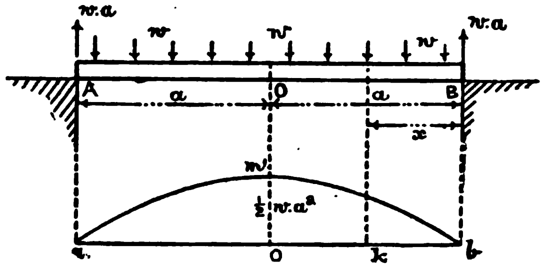


FIG. 96.

being, therefore, 2 . *w* . *a*, as Fig. 96. The support at each end is *w* . *a*. The bending moment at any point K, distance *x* from the end, is $M_k = w . a . x - \frac{1}{2} . w . x^2$. When $x = a$ this bending moment is therefore $\frac{1}{2} . w . a^2$. If a numher of spots be thus obtained throughout the length of the beam, we can draw a curve as *amb*, any ordinate of which will give the bending moment at that point of the beam.

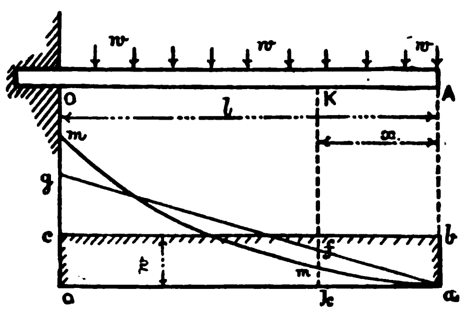


FIG. 97.

Take now the case of a beam supported at one end and loaded uniformly. The load can be graphically represented by a rectangle *oabc*, *ab* = *w* (Fig. 97). At any point K, calculate the area of the rectangle *ob* on one side of *k*, i.e. $w \cdot x$, and set off as an ordinate $kf = w \cdot x$. This is what is termed the "*shearing force*" at K, or the tendency the two consecutive sections of the beam at K have to slide over one another. Doing this all along the beam, we should obtain the line *afg*, the maximum ordinate of which $og = w \cdot l$.

The area of the figure $akf = \frac{1}{2} \cdot w \cdot x^2$, and the bending moment at K also $= \frac{1}{2} \cdot w \cdot x^2$. So that to determine the *bending moment* at any point, we find the *area of the curve of shearing force* up to that point.[1] In this way the curve of bending moment *amm* is constructed, having a maximum ordinate *om* of $\frac{1}{2} \cdot wl^2$. This method of determining the bending moment at any point from the curve of shearing force is of no value in this particular case, but is of assistance when dealing with more complicated cases of loading.

Take, for example, a beam similar to the above, but loaded unevenly along its length, such that the intensity of the load

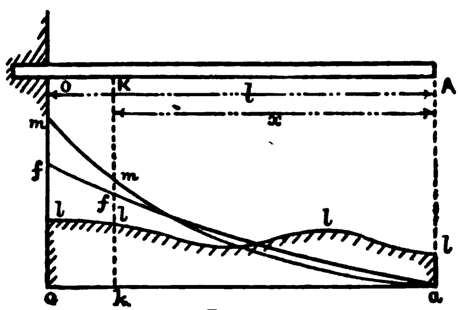


FIG. 98.

at any point is given by the ordinate of the curve *ll*, which we may term a "*curve of loads*," as Fig. 98. Take any point K and determine the area beneath the curve of loads from the point *k* to the end of the beam. This will give the shearing force at K. Doing this all along the beam, we can draw the curve of shearing force *aff*. The area under this curve between

[1] For the proof of this in any general case, see any standard work on "Applied Mechanics," as Cotterill, chap. iii.

k and the end of the beam gives the bending moment at K, and in this manner the curve of bending moment *amm* can be obtained.

Turning now to the case of a ship floating in still water. There will be a certain distribution of the weight and also of the buoyancy. The total weight must, of course, be equal to the total buoyancy, and also the fore-and-aft position of the centre of gravity of the weight must be in the same athwartship section as the centre of buoyancy. But although this is so, the distribution of the weight and buoyancy along the ship must vary from section to section.

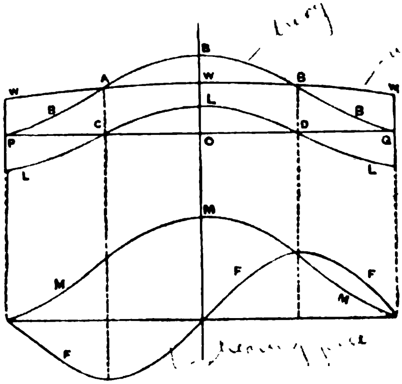


FIG. 99.

Take the case of a vessel floating in still water in which the buoyancy exceeds the weight amidships, and the weight exceeds the buoyancy at the ends. Let BB in Fig. 99 be the "curve of buoyancy." The area under this curve will give the displacement of the vessel, and the fore-and-aft position of the centre of gravity of this area is the same as the fore-and-aft position of the centre of buoyancy.

Also let WW be the "curve of weight." This curve is constructed by taking all the weights between two sections and setting up a mean ordinate to represent the total weight between the sections. This done throughout the length gives a number of spots through which a curve may be drawn as nearly as possible. This curve should be adjusted as necessary to fulfil the conditions stated above, viz. that the area under it shall equal the area of the curve of buoyancy, and the fore-and-aft position of the centre of gravity of the area under it shall be in the same section as that of the curve of buoyancy.

The difference at any point between the ordinates of the two curves WW and BB will give the difference between the weight and the buoyancy at that point. Where the curves cross at A and B, the weight and buoyancy are equal, and the sections at these points are said to be "*water-borne.*"

Now set off ordinates all along, giving the intercept between the curves WW and BB. Set below the base-line where the weight exceeds the buoyancy, and above the base-line where the buoyancy exceeds the weight. In this way we obtain the curve LLL which is the "*curve of loads.*" At the sections where the curve of loads crosses the base-line the ship is water-borne. We now obtain the "*curve of shearing force*" FFF from the curve of loads by finding the area under the curve of loads, as explained above. Also in a similar manner the curve of bending moment MM is obtained by finding the area under the curve of shearing force. The maximum ordinate of this curve will give the greatest bending moment the ship will be subjected to under the assumed conditions.

In constructing curves of bending moment, moments tending to cause "hogging" are put above the base-line, and moments tending to cause "sagging" are put below the base-line. In the case in Fig. 99 the moments are "hogging" throughout the whole length of the vessel.

The area of the curve of loads above the base-line being the same as the area below the base-line, it follows that the ordinate of the shearing force must come to zero at the end. Also the ordinate of the curve of bending moment must be zero at the end, and this constitutes a most effective check on the accuracy of the work.

It is obvious, however, that the strains due to the bending moment in still water are not the worst that in practice will affect the longitudinal structure of the ship. The strains in still water are small in magnitude compared with the strains that may affect the ship at sea. For a ship at sea there are two extreme cases that can be assumed, viz.—

(1) The ship being supposed to be momentarily at rest on the crest of a wave of her own length, the height of the wave being taken some proportion of the length (Fig. 100).

(2) The ship being supposed to be momentarily at rest across the trough of a wave similar to that assumed in (1) (Fig. 101).

It is usual to take the height of the wave from crest to crest, or from trough to trough, $\frac{1}{20}$ the length.* The wave is assumed to be of the form indicated by the "trochoidal theory," but no account is taken of the internal structure of the wave.[1]

Construction of a Trochoidal Wave Profile.—The wave on which a ship is supposed to be momentarily poised has for its profile a curve called a "trochoid." This is a curve traced out by a point inside a circle when the circle is rolled along a straight line. The curve can be traced by its co-ordinates referred to axes through the crest (Fig. 99A), viz.—

$$x = \mathrm{L} \cdot \frac{\theta}{2\pi} - \frac{h}{2} \cdot \sin\theta$$

$$y = \frac{h}{2}(1 - \cos\theta)$$

θ being given the values of 30°, 60°, etc., in circular measure. L is the length of wave from crest to crest, and h is the height from crest to trough. The curve may also be drawn by the

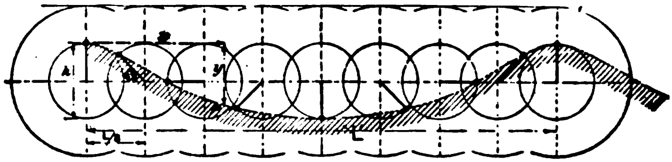


FIG. 99A.

construction indicated in Fig. 99A. It is noticed that the curve is sharper at the crest than in the trough, which is a characteristic feature of sea-waves.

In order to get the displacement of the ship when on the

[1] See the "Manual of Naval Architecture," by Sir W. H. White, chaps. v. and viii.; and a paper at the Institution of Naval Architects, by Mr. (now Sir) W. E. Smith, M.I.N.A., in 1883; see also Chap. VI. and later.

* See table given in Chap. VI. giving heights of waves of various lengths.

wave, it is convenient at each square station to run in a curve of sectional areas. Then, if the profile of the wave is traced and put on the profile of the ship, the area of each section up to the surface of the wave is at once measured off, and these areas integrated throughout the length give the displacement and centre of buoyancy. If these are not correct, a further trial must be made; the ship may have to be trimmed to get the C.B. right, it being essential that this is in the same section as the C.G.

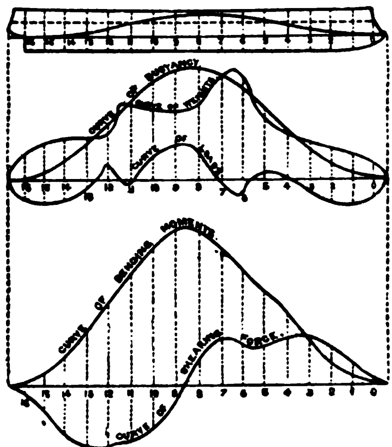


FIG. 100.

With these assumptions we can proceed to construct the "curve of buoyancy" for both cases (1) and (2), and from it and the "curve of weights" we obtain the "curve of loads." Then, by the principles explained above, we can determine the "curve of shearing force" and the "curve of bending moment." In Fig. 100 is given a set of curves for a ship on a wave-crest, and in Fig. 101 a set of curves for the same ship astride a wave-trough.

In any case the maximum bending moment may be expressed in the form $\frac{\text{weight} \times \text{length}}{\text{coefficient}}$, and it is found that the value of this coefficient will not usually fall below 20 for either of the extreme cases taken above. The maximum bending moment in foot-tons for ordinary ships may be generally assumed at from $\frac{1}{20}$ to $\frac{1}{30}$ the product of the length in feet and the displacement in tons. The latter is frequently taken as a standard value. In regard to this matter Mr. Foster King made the following remarks at the I.N.A., 1915: "For all ordinary vessels such as those with which we have to deal, and many extraordinary ones, it has been found that a close approximation to the greatest bending moment obtained from direct calculations is obtained from the formula $\frac{L^2 \times B \times D \times 0{\cdot}8}{35 \times 35}$, where L is the length, B the breadth, and D is winter draught in feet. Twenty years' experience of the application of this formula has shown an agreement of the order of about 5 per cent. with the calculated figures furnished by builders for vessels of the most widely dissimilar character and dimensions and give evidence of its utility as a guide in strength investigations." This maximum bending moment will usually occur somewhere in the vicinity of the midship section.

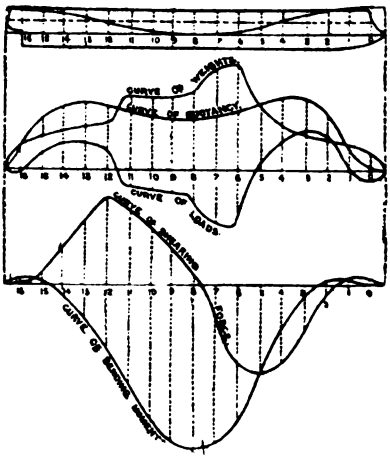


FIG. 101.

The principal labour in drawing the weight curve is that due to the hull weights.

In large warships it has been found that a very close approximation to the distribution of the hull weight is to assume $\frac{2}{3}$ of it distributed as the buoyancy in still water and the remaining $\frac{1}{3}$ as a trapezoid proportioned so that the C.G. of the whole corresponds with that of the hull.

For an ordinary passenger or cargo vessel Sir J. H. Biles gives the following :—

Over the midship third of the length a uniform ordinate of $1{\cdot}195\frac{H}{L}$; at the after end $0{\cdot}653\frac{H}{L}$; at the forward end $0{\cdot}566\frac{H}{L}$, straight lines being drawn from the forward and after ordinates to complete the figure, which can be seen to equal H, the weight of hull.

It is usual to draw the distribution of weight as a series of steps. For a large ship the length can be divided into sections bounded by the main bulkheads, and the weights in each section grouped together and plotted as uniform over each section. Where extreme accuracy is desired, the work can be done in greater detail. Fig. 101A gives the curves of a torpedo-boat

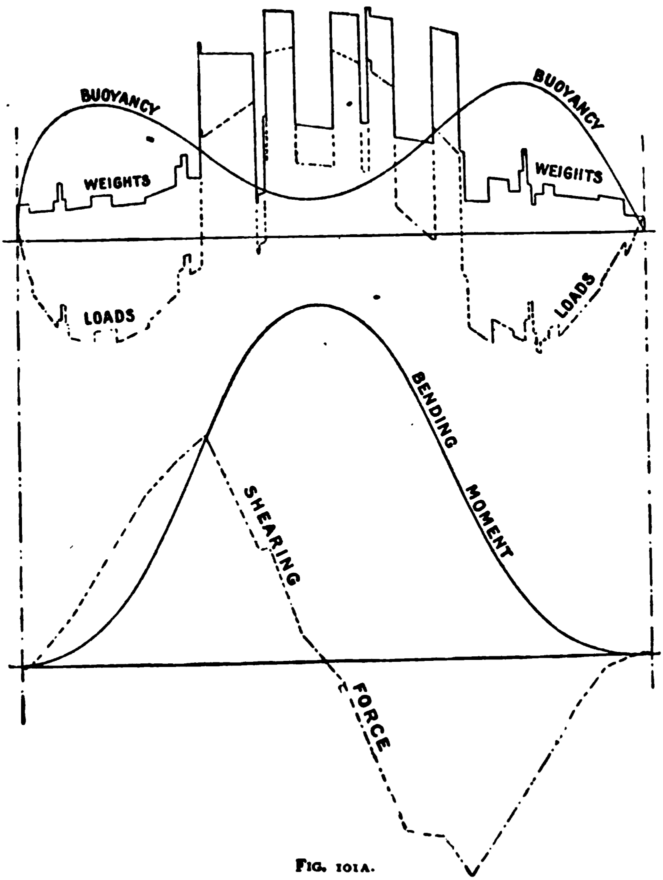


FIG. 101A.

destroyer lying in the trough of a wave of height $\frac{1}{20}$ her length, all the bunkers being full.

The use of a machine called the "integraph" is of great value in getting quickly the curves of shearing force and bending moment from the curves of loads. If the pointer of this machine is run round the boundary of a curvilinear area, the pen traces out a curve the ordinates of which give the area of the original up to corresponding points. This is just what is required in the above calculations. A paper on the integraph was given by Mr. Johnson at the Scottish Shipbuilders in 1904, to which the student is referred for further information. See also a paper by Professor Biles before the I.N.A. in 1905, for an exhaustive discussion on the strength of ships, with special reference to calculations and experiments on H.M.S. *Wolf*.

The "Smith" correction due to taking account of the Internal Structure of a Wave.[1]—The trochoidal wave theory has been dealt with in Chap. VI.

In particular it has been seen that *the pressure at any point in a trochoidal wave is the same as at the point it occupies when in still water.*—Thus in Fig. 101B, along the sub-surface BB, the pressure is the same as at its still-water level *b*, and not due to its distance below the surface trochoid (as is assumed in the standard method of calculation).

We therefore draw the trochoidal profile of the wave we have to deal with and also the sub-surfaces corresponding to lines of orbit centres at distances say 2 feet apart. We then calculate the positions of the corresponding still-water levels, *a*, *b*, *c*, *d*, *e*, *f*.

Take as an illustrative example the wave drawn in Fig. 104E, 60 feet long, and 12 feet high. (This is, of course, an exaggerated ratio of H ÷ L, but has been selected for the sake of clearness). The lines of orbit centres have been taken at 2 feet intervals, and the profile of the surface trochoid can be drawn as already described. In this case $R = \frac{L}{2\pi} = 9{\cdot}55$, and $r_0 = 6$, being one-half of the height of the surface trochoid. In order to draw the sub-surface trochoids we have to find the radii for values of $y = 2, 4, 6$, etc.

$$\text{We have } \log_e r = \log_e r_0 - \frac{y}{R}$$

Turning into ordinary logarithms to the base 10 (see Appendix B.) we have—

[1] See a paper by Mr. (now Sir) W. E. Smith, I.N.A. 1883.

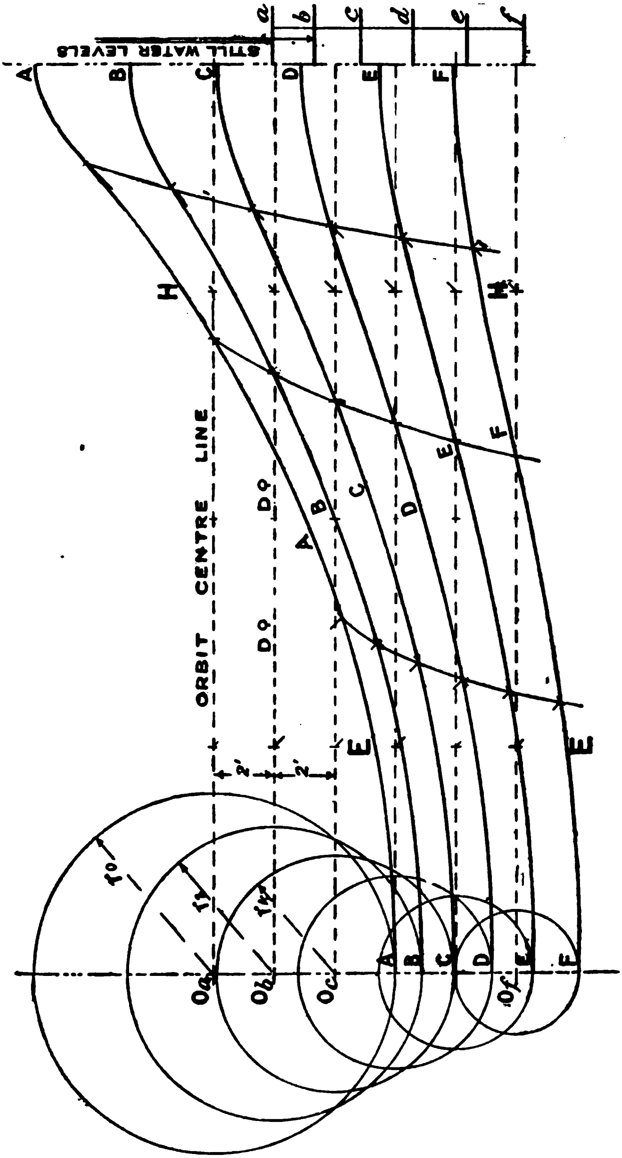

STILL WATER LEVELS
ORBIT CENTRE LINE
D°
D°
H
H
2'
2'
Oa
Ob
Oc
Of
a
b
c
d
e
f
A
B
C
D
E
F

FIG. 101B.

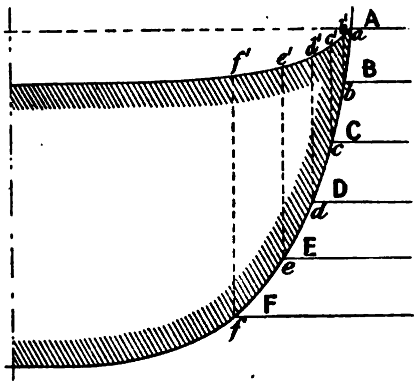
$$\log_{10} r = \log_{10} r_0 - \frac{1}{2 \cdot 3} \cdot \frac{y}{R}$$

Putting in values of r_0 and R, and successive values of $y = 2$, 4, 6, 8, 10, we have for values of r: 4·86, 3·94, 3·19, 2·6 and 2·1. These will be the half heights of the sub-surface trochoids which can then be drawn in as indicated in the figure. The level of still water below the lines of orbit centres is then obtained by putting in the values of r in the expression $\frac{r^2}{2R}$, or 1·88, 1·24, 0·81, 0·53, 0·35, 0·23, which are set down at the side of the figure giving the levels *a*, *b*, *c*, *d*, *e*, *f*. Thus the pressure at any point on the sub-surface DD is that due to the distance of *d* below *a*. At the crest, if the wave pressures were neglected, the pressure at D would be that due to a head of 8·8 feet, whereas really it is only a pressure due to a head of 4·7 feet.

Take now a ship on the wave and consider the section that comes at HH. The levels of the surface A, and the sub-surfaces B, C, D, etc., are placed on the section (Fig. 101C), A, B, C, etc. At the level of B we set up $bb' = ab$, at the level of C, $cc' = ac$, etc., and a curve through $ab'c'$, etc., will give the line to work to to obtain the true value of the buoyancy due to the section. In the case of sections in the crest portion this is less than that up to the level of the wave. Similarly for a section as at EE (Fig. 101D), where the value of the effective buoyancy is greater. This done for sections all along the length will result in a curve of effective areas. When the ship is on the crest this curve is as dotted in (Fig. 101E), and in the trough as dotted in (Fig. 101F), as compared with the full curves obtained in the standard method. These new curves of buoyancy must, of course, satisfy the ordinary conditions, viz. that the displacement and position of the centre of buoyancy are correct.

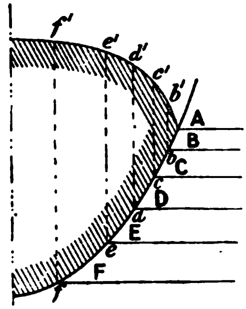
Stress on the Material composing the Section.— Considering now the ship's structure as a girder, a *hogging* moment produces *tension* in the *upper* portion of the girder and *compression* in the *lower* portion of the girder, the reverse being true for a *sagging* moment.

We now have to consider in some detail how a given beam



Section H-H

FIG. 101C.



Section E-E

FIG. 101D.

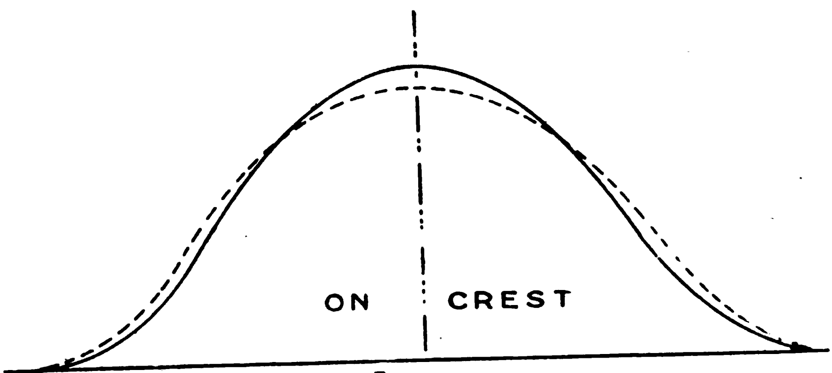


FIG. 101E.

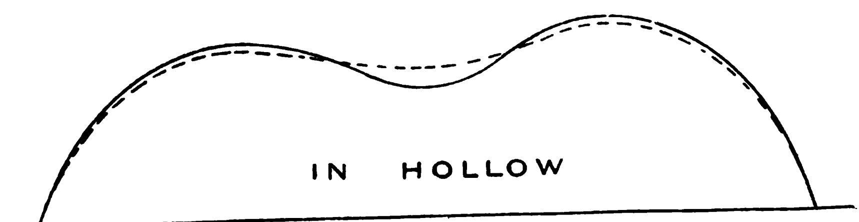


FIG. 101F.

is able to withstand the stresses on its material when subjected to a given bending moment. Take a beam bent as in Fig. 92 and Fig. 102. AB is a longitudinal section and LL is a transverse section of the beam. The upper layers are shortened and the lower layers are lengthened. There must be one intermediate layer which is unaltered in length. This layer is

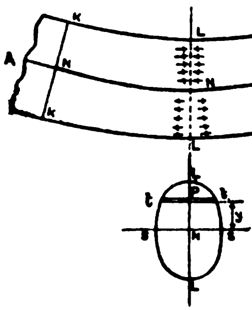


FIG. 102.

called the "neutral surface," and the transverse section SS is called the "neutral axis." This neutral axis can be shown to pass through the centre of gravity of the section.[1] The bending moment at the section LL is resisted by the compressive stresses in the upper layers and the tensile stresses in the lower layers.

It can be shown [1] that the following relation holds :—

$$\frac{p}{y} = \frac{M}{I}$$

where p is the stress in tons per square inch at distance y inches from the neutral axis.

M is the bending moment at this section in inch-tons.

I is the moment of inertia of the section about the neutral axis in inch-units.

It is by this formula that the stress on a particular portion of the section of a beam can be determined, when we know the

[1] See any standard work on "Applied Mechanics," as that by Professor Cotterill, F.R.S.

bending moment at that section, the position of the neutral axis, and the moment of inertia of the section about the neutral axis.

Take, for example, the various sections of beams in Fig. 93. Length of beam 12 feet, beam loaded in the middle with 1 ton (neglecting the weight of the beam).

For (*a*)—

$$y = 1''$$
$$I = \frac{1}{12} \times 16 \times 4 = \frac{64}{12} \text{ in inch-units}$$
$$M = 36 \text{ inch-tons}$$
$$\therefore p = \text{the stress at the top or bottom}$$
$$= 1 \times \frac{36 \times 12}{64}$$
$$= 6\cdot75 \text{ tons per square inch}$$

For (*b*)—

$$y = 2''$$
$$I = \frac{256}{12} \text{ in inch-units}$$
$$M = 36 \text{ inch-tons}$$
$$\therefore p = \text{the stress at the top or bottom}$$
$$= 2 \times \frac{36 \times 12}{256}$$
$$= 3\cdot375 \text{ tons per square inch}$$

For (*c*)—

$$y = 4''$$
$$I = \frac{256}{3} \text{ in inch-units}$$
$$M = 36 \text{ inch-tons}$$
$$\therefore p = \text{the stress at the top or bottom}$$
$$= 4 \times \frac{36 \times 3}{256}$$
$$= 1\cdot6875 \text{ tons per square inch}$$

For (*d*)—

$$y = 4''$$
$$I = \frac{424}{3} \text{ in inch-units}$$
$$M = 36 \text{ inch-tons}$$
$$\therefore p = \text{the stress at the top or bottom}$$
$$= 4 \times \frac{36 \times 3}{424}$$
$$= 1\cdot02 \text{ tons per square inch}$$

Or looking at the question from another point of view, if we say that the stress on the material is not to exceed 10 tons per square inch, then we can determine for each of the sections in Fig. 93 the greatest bending moment to which the beam can be subjected.

For (*a*)— $M = \frac{p}{y} \times I = \frac{10}{1} \times \frac{64}{12} = 53\frac{1}{3}$ inch-tons

For (*b*)— $M = \frac{p}{y} \times I = \frac{10}{2} \times \frac{256}{12} = 106\frac{2}{3}$ inch-tons

For (*c*)— $M = \frac{p}{y} \times I = \frac{10}{4} \times \frac{256}{3} = 213\frac{1}{3}$ inch-tons

For (*d*)— $M = \frac{p}{y} \times I = \frac{10}{4} \times \frac{424}{3} = 353\frac{1}{3}$ inch-tons

It is thus seen that the ratio of the bending moments that these beams can stand is—

$$53\tfrac{1}{3} : 106\tfrac{2}{3} : 213\tfrac{1}{3} : 353\tfrac{1}{3}$$

or

$$1 : 2 : 4 : 6\tfrac{5}{8}.$$

The area of each section is the same, the only difference being in the different distribution of the material of the section with reference to the neutral axis.

We come now to the case of a ship subjected at a particular section to either a "hogging" moment or a "sagging" moment. To determine the stress on any portion of the section, we consider the vessel to be a large beam subjected to a given bending moment, and we apply the formula

$$\frac{p}{y} = \frac{M}{I}$$

There are two things to be found before we can apply this formula to a given section, viz. :—

(i.) The position of the neutral axis, which passes through the centre of gravity of the section.

(ii.) The moment of inertia of the section about the neutral axis.

In considering the strength longitudinally of a section, account must be taken only of such material as actually contributes to the strength through an appreciable length in the vicinity of the section, such as plating of the inner and outer bottom, keel, continuous longitudinals or keelsons, stringers, deck-plating, planking, etc.

A distinction must be made between material in tension and material in compression. In tension, allowance must be made for the material taken away from the plating, etc., for the rivet-holes, but in compression this deduction is unnecessary. It is also usual to consider that wood is equivalent to $\frac{1}{16}$ its area in steel for both tension and compression. For an armoured vessel the armour is not assumed to take any tension, but is assumed to be effective against compression.

We must accordingly have two separate calculations—one for the section under a hogging moment, and one for the section under a sagging moment. The position of the neutral axis and the moment of inertia of the section about the neutral axis will be different for each case.

With reference to the above assumption, the following remarks of Dr. Bruhn (I.N.A., 1899), who has given great attention to this subject, may be noted :—

Dr. Bruhn thinks that the correction for the rivet-holes in the calculation for I is more an act of error than of correction. This correction assumes the structure highly discontinuous, the I and the position of the neutral axis varying *at* the frame and *between* the frames. But the whole theory of bending is based on the assumption that the structure is continuous, and the bending certainly must be continuous. The I should therefore be taken for the *solid section*, and if it is desired to find the stress between the rivets, we may increase the stress in the ratio in which the sectional area is reduced (say $\frac{7}{8}$). This method for other than armoured ships only requires one calculation for the moment of inertia, and when dealing with shearing stresses, there is no more reason for deducting rivet-holes on one side than on the other.

The following form may conveniently be used for calculating the position of the neutral axis and the moment of inertia of the section about the neutral axis, areas being in square inches and levers in feet :—

1 Items.	2 Effective area in square inches.	3 Lever in feet.	4 Moment.	5 Lever in feet.	6 Moment of Inertia.	7 $\frac{1}{12} \times A \times h^2$.
	A		M		I	i

A section of the ship should be drawn out to scale with all the scantlings shown on. An axis is assumed at about one-half the depth of the section. The several items are entered in column 1, the effective areas in column 2, and the distances of the centres of gravity from the assumed axis in feet are entered in column 3. For the items below the axis these levers are negative. We thus obtain column 4, which gives the moment of each item about the axis, and the algebraic sum of this column, M, divided by the addition of column 2, viz. A, gives the distance of the neutral axis from the assumed axis, in feet, say d feet.

We now place in column 5 the same levers as in column 3, and multiplying the moments in column 4 by the levers in column 5, we obtain the areas of the several items multiplied respectively by the *square* of their distances from the neutral axis. Each of these products is, of course, positive. All these added give a total I, say. For the portions of the section which are vertical, an addition is needed for the moment of inertia of the items about axes through their own centres of gravity, viz. $\frac{1}{12} . A . h^2$ (see p. 104). For portions of the section which are horizontal, h is small, and this addition may be neglected. We, therefore, arrive at the moment of inertia of the section about the assumed axis, viz. $I + i = I_A$, say. We now have to transfer this moment of inertia about the assumed axis from that axis to the neutral axis, or $I_0 = I_A - A \times d^2$, as explained on p. 104.

We can now determine the stress on the point of the section farthest from the neutral axis, as this will be the point at which the stress is greatest, by using the formula—

$$\frac{p}{y} = \frac{M}{I}$$

The stresses thus found are only comparative, and must be compared with those for a ship on service found to show no signs of longitudinal weakness. Large ships can bear a stress of 10 tons per square inch, because the standard wave is exceptional. Ships 300 to 400 feet long have stresses 6 to 7 tons per square inch. In the special case of the *Lusitania*, a stress of 10 tons per square inch was allowed when on a wave of her own length of height one-twentieth the length. It is to be observed that such a wave is of quite phenomenal size, and unlikely to be encountered.

The Load Line Committee investigated a standard of strength for vessels up to 600 feet long with a maximum relation of length to depth of 13·5. The results are given in Sir W. S. Abell's paper before the Institution of Naval Architects, 1916.

The values of a factor "f" obtained as follows:—

$$f = \frac{I}{y \times \text{draught} \times \text{breadth}}$$

"I" being moment of inertia of section without deduction of rivet holes,

were tabulated, and for ships 360′ to 600′ in length are as follows:—

Length in feet.	Value of f.
360	10·5
380	11·5
400	12·5
420	13·5
440	14·6
460	15·7
480	16·8
500	18·0
520	19·2
540	20·4
560	21·7
580	23·1
600	24·6

The following remarks were made by the Author in the discussion on H.M.S. *Hood*, I.N.A., 1920: "Although the conditions in regard to the distribution of weight and the ratio of length to depth were different to those in merchant ship practice, we had the strength investigated on the lines of the empirical rules laid down by the Load Line Committee. These rules stopped short at a 600 feet ship, but Mr. Luke kindly gave us the value of the factor 'f' for the *Lusitania*, so that we were able to continue the curve and with but slight extrapolation reached the length of the *Hood*. It was found that the factor for the *Hood* came well above the curve, which provided a satisfactory confirmation of our calculations."

Specimen Calculation for the Moment of Inertia, etc., of Section.—The following specimen calculation for a torpedo-boat destroyer is given as a guide to similar calculations. The depth of the girder was 17·7 feet and the assumed neutral axis was taken as 9 feet above underside of flat keel.

The column for areas was filled in with weight per foot run, the total being turned into areas at the end (f (area) means, proportional to area, *i.e.* a function of area).

Above assumed neutral axis.

One side only.	(1) f(Area) lbs. per ft.	(2) Lever.	(3) f(Moment).	(4) f(I), *i.e.* (2) × (3)	(5) d	(6) f(A . d^2).
Deck stringer	67·5	8·28	559	4630		
Remainder deck	74·38	8·61	640	5510		
Girder coaming	14·0	8·95	123·6	1121	1·75	43
Girder upper angles	8·0	9·24	74·0	683		
Girder deck angles	3·0	8·84	26·5	234		
Girder lower angles	7·0	8·06	56·4	455		
2nd girder plate	4·2	8·17	99·6	813		
2nd girder angles	8·0	8·17				
3rd girder plate	4·2	8·07	98·4	793		
3rd girder angles	8·0	8·07				
Bunker bulkhead top plate	24·0	6·7	160·7	1079	3·0	216
Bunker bulkhead 2nd plate	16·8	3·92	65·9	258·5	2·8	132
Bunker bulkhead 3rd plate	17·1	1·22	20·8	25·5	2·85	139
Gunwale angle	7·0	8·00	56·0	448		
Sheer strake	63·0	5·80	365·3	2120	4·5	1275
Strake below	40·95	1·58	64·7	102	4·55	847
Side stringer	8·97	3·03	27·2	82		
	376·1		2438·1	18,354		12)2652
				+221		221
				18,575		
9/11 for tension	307·9		1995	15,205		

Below assumed neutral axis.

One side only.	$\int$(Area) lbs. per ft.	Lever.	$\int$(Moment).	$\int$(I)	d	$\int(A \cdot d^2)$.
D strake . . .	36·72	2·39	87·7	210	3·8	530
C strake . . .	46·0	5·55	255	1420	2·9	387
B strake . . .	44·55	7·62	340	2590	1·4	87
A strake . . .	55·62	8·58	477	4095	0·8	35
½ Flat keel . .	30·0	9·0	270	2430		
½ Vertical keel .	9·0	8·2	73·8	605	1·5	20
½ Lower angles .	7·0	8·87	62·1	550		
½ Upper angles .	6·0	7·52	45·1	339		
½ Rider plate .	8.33	7·42	61·8	457		
1st long. angles .	6·0	7·4	44·4	328		
2nd long. plates .	5·36	6·9	37·0	255	0·67	2
2nd long. angle .	3·0	7·2	21·6	155		
2nd long. angle .	5.0	6.6	33·0	218		
Lower side stringer . .	8·97	1·37	12·3	17		
Bilge keel plates	10·0	6·6	66·0	435	0·8	6
Bilge keel angles	11·0	6·25	68·7	430		
Bunker bulkhead	16·8	1·47	24·7	36	2·8	132
Bunker bulkhead	32·0	4·73	151·5	717	4·0	512
	341·35		2131·7	15,288		12)1711
				143		143
				15,431		
$\frac{9}{11}$ for tension	279		1743	12,630		

Hogging.—For hogging the full area below axis is taken, and $\frac{9}{11}$ the full area above axis to allow for rivet holes—

$$\int(\text{Area}) = 307·9 + 341·3 = 649·2$$

$$\text{area} = \frac{2}{3·4} \times 649·2 = 381·9 \text{ sq. in.}$$

$$\left.\begin{array}{l}\text{N.A. below assumed}\\ \quad\text{axis}\end{array}\right\} = \frac{2131·7 - 1995}{649·2} = 0·21 \text{ ft.}$$

$$\int(\text{I}) \text{ about assumed axis} = 15,205 + 15,431 = 30,636$$

$$\text{I about N.A.} = \frac{2}{3·4}[30,636 - (649·2 \times \overline{0·21^2})] = 18,000$$

N.A. above keel = 8·79 ft. N.A. below deck = 8·91 ft.

Bending moment = 10,275 ft. tons

$$\text{Tensile stress in deck} = \frac{10,275}{18,000} \times 8·91 = 5·1 \text{ tons sq. in.}$$

$$\left.\begin{array}{l}\text{Compressive stress in}\\ \quad\text{keel}\end{array}\right\} = \frac{10,275}{18,000} \times 8·79 = 5·02 \text{ tons sq. in.}$$

Sagging.—For sagging the full area above axis is taken, and $\frac{9}{11}$ the full area below axis to allow for rivet holes.

$$\int(\text{Area}) = 376{\cdot}1 + 279 = 655{\cdot}1$$

$$\text{area} = \frac{2}{3{\cdot}4} \times 655{\cdot}1 = 385{\cdot}4$$

$$\left.\begin{array}{l}\text{N.A. above assumed}\\ \quad\text{axis}\end{array}\right\} = \frac{2438 - 1743}{655{\cdot}1} = 1{\cdot}06 \text{ ft.}$$

$$\int(\text{I}) \text{ about assumed axis} = 18{,}575 + 12{,}630 = 31{,}205$$

$$\text{I about N.A.} = \frac{2}{3{\cdot}4}[31{,}205 - 655{\cdot}1 \times \overline{1{\cdot}06^2}] = 17{,}920$$

N.A. above keel = 10·06 ft. N.A. below deck = 7·64 ft.

Bending moment = 13,000 ft.-tons

$$\text{Tensile stress in keel} = \frac{13{,}000}{17{,}920} \times 10{\cdot}06 = 7{\cdot}3 \text{ tons sq. in.}$$

$$\left.\begin{array}{l}\text{Compressive stress in}\\ \quad\text{deck}\end{array}\right\} = \frac{13{,}000}{17{,}920} \times 7{\cdot}64 = 5{\cdot}54 \text{ tons sq. in.}$$

Equivalent Girder.—Although not necessary for calculation purposes, it is frequently the practice to draw out for

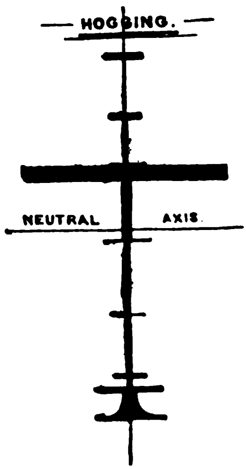


FIG. 103.

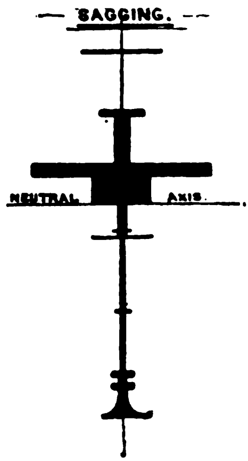


FIG. 104.

the case under consideration a diagrammatic representation of the disposition of the material forming the section. Such a diagram will show at once how the material is disposed relative to the neutral axis, and gives the section of the girder that the ship is supposed to be. Such a diagram is termed the "equivalent girder," and there must be one for hogging and one for sagging, as shown in Fig. 103 and Fig. 104 respectively, which are the equivalent girders for an armoured battleship.

A number of examples of equivalent girders for merchant ships are given in Mr. Foster King's paper, I.N.A., 1913.

Shearing Stresses.—We have seen above that to dispose the material of a ship so as to resist most effectively the stresses due to a bending moment, we must pay special attention to the upper and lower portions of the girder. There are, however, also shearing stresses in a loaded structure which, under certain circumstances, may cause straining action to take place. Professor Jenkins called attention to these stresses in a paper before the I.N.A. in 1890, and their importance has been increased in recent years owing to the great increase in the size of the vessels built.

It is necessary first to deal with the shearing stresses which occur in an ordinary loaded beam. In a beam, besides the bending moment at each section, there is a tendency for each section to slide over the adjacent one. This is measured by the "shearing force." At each point of the section there is a *shearing stress* set up to resist this sliding tendency. It can be shown that such shearing stress is always accompanied by a shearing stress of equal intensity on a plane at right angles.

Consider two consecutive sections of a beam K′ and K″ δx apart (Fig. 104A), at which the bending moments are M and M + δM respec-

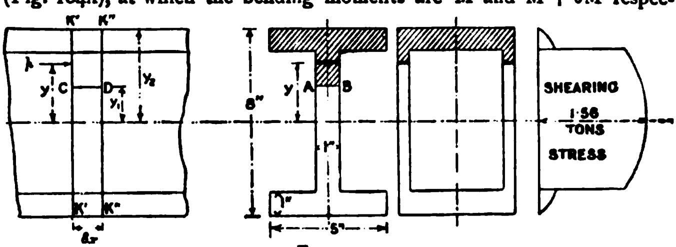


Fig. 104A.

tively. Then, if p is the normal stress at section K′ at a distance y from the neutral axis, and δA is the element of area on which this stress acts, the total normal force on the section above AB is—

$$\int_{y_1}^{y_2} p \, . \, d\text{A} = \int_{y_1}^{y_2} \frac{\text{M}}{\text{I}} \, . \, y \, . \, d\text{A} = \frac{\text{M}}{\text{I}} \int_{y_1}^{y_2} y \, . \, d\text{A}$$

on proceeding to the limit. If now m is the moment of the area above AB about the neutral axis, the normal force is $\frac{M}{I} \cdot m$. At the consecutive section this will be $\frac{M + \delta M}{I} \cdot m$. The difference between these is the resultant horizontal force on the portion of the beam δx long above AB, viz. $\frac{m}{I} \cdot \delta M$. This must be the shearing force causing the section above CD to slide along. If, now, q is the intensity of the shearing stress along CD, and b the breadth of AB, we have $q \cdot b \cdot \delta x = \frac{m}{I} \cdot \delta M$, or $q = \frac{dM}{dx} \cdot \frac{m}{I \cdot b}$ in the limit. Now, $M = \int F \cdot dx$ or $F = \frac{dM}{dx}$, where F is the shearing force on whole section, so that we have $q = \frac{F \cdot m}{I \cdot b}$. This shearing stress is therefore zero at the top and bottom of the section, and it will vary at other points of the section.

To illustrate the variation of this shearing stress, take a beam of I section or hollow section, as Fig. 104A, subjected to a total shearing force of 10 tons. It will be found that the variation of the shearing stress is represented by the right-hand portion of figure. It takes a sudden jump at bottom of flange from 0·25 to 1·24 tons per square inch, because the breadth is suddenly diminished from 5 inches to 1 inch. The maximum value is 1·56 tons at the neutral axis. It is thus possible, in a beam with a thin web, that an excessive shearing stress may be set up, and just at that part of the section where there is no stress due to the bending moment. It is also to be noticed that the shearing strength of steel is about four-fifths the tensile strength.

In a ship the shearing force amidships is usually zero (see Figs. 100, 101), and the shearing force reaches a maximum at about a quarter the length from each end. This, therefore, will be the portion at which shearing strains are likely to be most severe, and the maximum strains will occur in the neighbourhood of the neutral axis, because here the breadth of the section is usually only twice the thickness of the bottom plating. This stress has shown itself by the working of the rivets in these portions of large ships, and the stresses vary also in opposite directions according as the ship is in the trough or on the crest of a wave. It is, therefore, becoming the practice to work treble-riveted fore-and-aft laps at about mid-depth in the fore and after bodies of large ships.[1]

There are also set-up stresses due to bending of the plating owing to the varying pressure of water, and these, together with

[1] Lloyd's Rules say: "In vessels of 480 feet and upwards, with side plating less than 0·84 inch in thickness, the landing edges are to be treble-riveted for one-fourth the vessel's length in the fore and after bodies for a depth of one-third the depth."

the above stresses, have shown themselves by working in the parts above mentioned, which has required the special strengthening referred to. (The subject has been exhaustively discussed by Dr. Bruhn in a paper before the Scottish Institution of Shipbuilders, 1902.)

Principal Stress.—When the material of a beam is subjected to a tensile or compressive stress, together with a shearing stress, these combine together to produce what is termed the "principal stress" at any particular place. It can be shown that if p_n is the ordinary tensile or compressive stress, and q is the shearing stress, then the principal stress p is given by the equation—

$$p(p - p_n) = q^2$$

Thus, take a place immediately beneath the deck of a ship with the stern overhanging in dry dock; $p_n = 3{\cdot}1$, $q = 1{\cdot}42$. Then $p = 3{\cdot}65$ tons, which is seen to be greater than the simple tensile stress.

Unsymmetrical Bending.—In the ordinary investigation we assume that the ship is upright. If a ship is inclined the depth of section is increased, and it may possibly happen that an increased stress would be experienced at the corner of the section.

Let MM (Fig. 104B) be the axis of the bending moment, the ship being heeled to an angle θ. Then M, the bending

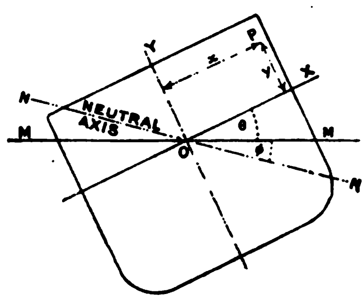


FIG. 104B.

moment, may be resolved into M . cos θ with the axis OX, and M . sin θ with the axis OY, O being the C.G. of section. Each of these will produce stress at P as if it acted alone, and the total stress at P will be—

$$p = \frac{M \cdot y}{I_1} \cdot \cos\theta + \frac{M \cdot x}{I_2} \cdot \sin\theta$$

I_1, I_2 being the moments of inertia about axes OX, OY. The position of the neutral axis is where $p = 0$, or where $-\frac{y}{x} = \frac{I_1}{I_2} \times \tan\theta = \tan\phi$. If the point farthest from NN has co-ordinates y' and x' referred to OX, OY, then the maximum stress is—

$$f = M\left(\frac{y'}{I_1} \cdot \cos\theta + \frac{x'}{I_2} \cdot \sin\theta\right)$$

Professor Biles (Scottish Shipbuilders, 1893-4) gives the results for a ship for all angles from 0° to 90°. He found, for the ship he took, that the maximum stress was reached at 30°, and was there 20 per cent. greater than when the ship was upright.

Examples to Chapter VIII.

1. Determine the maximum stress on the section of an iron bar, 2 inches square and 20 feet long, when supported at the ends and unloaded with one side horizontal. *Ans.* 6000 lbs. per square inch.

2. An iron bar of the same length, and supported as in the previous question, is of circular section, 2 inches diameter. Determine the maximum stress. *Ans.* 8000 lbs. per square inch.

3. A vessel floating in still water is subjected at a certain section to a bending moment of 144 foot-tons. Determine the longitudinal stresses (in pounds per square inch) in the material at top and bottom of this section, assuming the section to be rectangular, 21 feet wide, 10 feet deep, $\frac{1}{2}$" thick, and that the whole of it is effective in resisting stresses.

Ans. 223 lbs.

4. The buoyancy of a vessel is 0 at the ends and increases uniformly to the centre, while the weight is 0 at the centre and increases uniformly to the ends. Draw the curves of shearing force and bending moment, and find the maximum values of these quantities in terms of the displacement and length of the vessel.

Ans. $\frac{1}{4}$ W, $\frac{1}{12}$ W.L.

5. Suppose the skin and plate deck of an iron vessel to have the following dimensions at the midship section, measured at the middle of the thickness of the plates. Find the position of the neutral axis and moment of resistance to bending. Breadth 48', and total depth 24', the bilges being quadrants of 12' radius. Thickness of plate $\frac{5}{8}$" all round, and coefficient of strength $p = 4$ tons.

Ans. Neutral axis 13" above centre of depth.
Moment of resistance to hogging, 32,500 foot-tons.
" " sagging, 39,000 "

(Examples 4 and 5 are from "Applied Mechanics," by Professor Cotterill, F.R.S.)

6. A ship on a wave-crest is subjected at the midship section to a hogging moment of 28,000 foot-tons. The depth of the section is 37·5 feet, and the neutral axis is 18·2 feet from the bottom, the moment of inertia of the section about the neutral axis is 477,778 (square inches × feet²). Determine the maximum compressive and tensile stresses.

Ans. 1·07 tons per square inch compressive at bottom of section.
1·13 tons per square inch tensile at upper part of section.

7. State the maximum bending moment (in terms of weight and length of vessel) in the case of a vessel having the weight uniformly distributed and the curve of buoyancy a parabola. State also the position where these maxima occur.

Ans. S.F. at 0·21L from end $= \dfrac{W}{10·5}$.

B.M. amidships $= \dfrac{WL}{32}$.

8. In question 3, if the ship has a bulwark each side 2 feet high, $\frac{1}{8}$ inch thick, what will then be the maximum stress? Explain the significance of your result as applying to actual ships. *Ans.* 263 lbs.

Increase of depth of section will not necessarily diminish the maximum stress.

$$\frac{p}{y} = \frac{M}{I} \text{ or } \frac{\delta p}{p} = \frac{\delta y}{y} - \frac{\delta I}{I}$$

and δp will be negative, *i.e.* stress diminishes only if $\dfrac{\delta I}{I} > \dfrac{\delta y}{y}$.

This point acquires special importance in vessels with a light continuous superstructure, as—

(1) Boat deck in large cruisers.
(2) Superstructure in merchant vessels.

If the structure is made continuous, it is found that the influence of the increased depth is greater than the increased I, and thus greater stresses are likely to be experienced by the superstructure than it can bear. For this reason, either—

(1) A sliding joint is made, so that the superstructure contributes nothing to the structural strength; or, preferably—
(2) The superstructure is made an integral part of the ship's structure. See a paper by Mr. Montgomerie, I.N.A., 1915.

It does not follow that material added to a section will diminish the maximum stress. We have—

$$\frac{p}{y} = \frac{M}{I}, \text{ or } \frac{\delta p}{p} = \frac{\delta y}{y} - \frac{\delta I}{I}$$

Suppose a small area a is added at a distance f from neutral axis, then this axis will shift:

$$\frac{a \cdot f}{A + a}$$

The new I about old axis $= I + a \cdot f^2$

The new I about new axis $= I + a \cdot f^2 - (A + a)\left(\dfrac{af}{A + a}\right)^2$

$$\therefore \text{ increment of I or } \delta I = a \cdot f^2 - \left(\frac{a^2 \cdot f^2}{A + a}\right) = af^2\left(\frac{A}{A + a}\right)$$

$$\text{and } \delta y = \frac{af}{A + a}\frac{}{}$$

$$\therefore \frac{\delta p}{p} = \frac{af}{A + a}\left(\frac{1}{y} - \frac{Af}{I}\right)$$

$$\text{and } I = A \cdot k^2$$

$$\delta p \text{ is positive if } \frac{1}{y} > \frac{Af}{Ak^2} \text{ or } \frac{f}{k^2}$$

i.e. the stress increases at distance y if the added material is placed less than $\frac{k^2}{y}$ from the neutral axis.

In a rectangular beam, if material be added less than $\frac{1}{6}$ the depth from mid-depth the stress is increased.

If a square inches of plating, placed at a distance of h feet *above* the top of the girder, is such as to give the same stress as before (*i.e.* $\frac{I}{y}$ is the same), show that—

$$a = \frac{\frac{I}{y} \cdot A \cdot h}{A(y + h)^2 + I}$$

This formula was used by Mr. Montgomerie in his paper before I.N.A., 1915, on "The Scantlings of Light Superstructures."

$$\frac{\text{New } I}{\text{New } y} = \frac{I}{y}$$

9. A rectangular vessel is 30 feet broad and 20 feet deep, and has deck plating $\frac{1}{2}$ inch thick, sides and bottom $\frac{3}{8}$ inch. At a certain section it is subjected to a hogging moment of 20,000 tons-feet, and a shearing force of 300 tons. Calculate in tons per square inch—

(1) Maximum tensile and compressive stresses.
(2) Maximum shearing stress.
(3) Principal stress immediately under deck.

Ans. (1) 7·1, 4·9; (2) 1·42; (3) 7·2.

10. In the previous example, what would be the stresses if the deck and bottom were $\frac{3}{8}$ inch thick, and the sides $\frac{1}{2}$ inch?

Ans. (1) 5, 5; (2) 2·62; (3) 5·86.

11. An I beam 8 inches deep and 1 inch thick, with flanges 5 inches wide, overhangs a distance of 5 feet, and a weight of 5 tons is placed at the end. Determine at the point of leaving support (tons per square inch)—

(1) Maximum tensile and compressive stresses.
(2) Maximum shearing stress.
(3) Principal stress immediately below the upper flange.

Ans. (1) 8·5, 8·5; (2) 0·78; (3) 6·46.

12. In a vessel of 10,000 tons displacement and 450 feet long, the maximum bending moment is $\frac{1}{30}$W.L. The depth of midship section is 39 feet, and the neutral axis for hogging is at 0·49 the depth from keel. The I for hogging about the neutral axis is 4000 in foot units. Calculate the maximum tensile and compressive stresses.

Ans. 5·18, 4·97 tons per square inch.

13. A vessel of 3000 tons displacement and 360 feet long has a maximum hogging moment of $\frac{1}{31}$ W.L. The draught is 14½ feet, and the freeboard to stringer is 7½ feet. The neutral axis for hogging is 3·15 feet below the water-line, and the moment of inertia about neutral axis is 73,000 (square inches × feet²). What are the maximum tensile and compressive stresses?

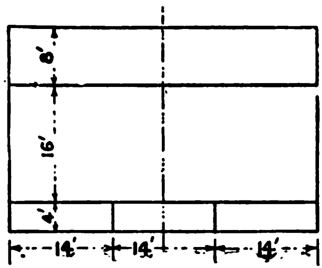
Ans. 5·18, 5·41 tons per square inch.

14. A vessel is designed with a very large overhang at the stern from the cut-up of the keel. Indicate what calculations you would make to see if the ship could safely be dry docked with the stern unsupported. Indicate how you would strengthen such a ship to withstand the strains set up in dry dock.

15. Two similar vessels are respectively 300 feet long, 2135 tons displacement, and 360 feet long, 3000 tons displacement, the depth being nearly the same. Indicate the method you would adopt to ensure the second ship being strong enough, and estimate the increase of maximum bending moment the second ship has to stand as compared with the first.

Ans. About 68 per cent. greater B.M.

16. The effective part of the transverse section of a vessel amidships is represented by the diagram, the vessel being 42 feet broad and 28 feet deep. Find the maximum tensile and compressive stresses when the vessel is subjected to a sagging moment of 60,000 foot-tons. The plating is ½ inch thick and no allowance need be made for rivet holes and laps of plating. (Honours B. of E., 1908.)



This example is worked out below. The best way to proceed is to prepare a table similar to that in this chapter; attention is necessary to the units, the areas being in square inches and the lengths in feet. We therefore have, taking in the first place all distances from the keel—

Items.	Area in sq. in.	Levers in ft.	Moment.	Levers in ft.	Moment of inertia.	$\frac{1}{12}.A.h^2$
Upper deck .	252	28	7,056	28	197,578	—
Main deck . .	252	20	5,040	20	100,800	—
Tank top . .	252	4	1,008	4	4,032	—
Bottom . .	252	—	—	—	—	—
Sides . . .	336	14	4,704	14	65,856	21,952
Girders . .	48	2	96	2	192	64
	1392		17,904		368,458	22,016

$\frac{17904}{1392} = 12{\cdot}86$ ft. C.G. from keel.

$$\begin{array}{r} 368{,}458 \\ 22{,}016 \\ \hline 390{,}474 \end{array}$$ I about keel in sq. in × ft.²

$$\text{I about neutral axis} = 390{,}474 - 1392 \times (12{\cdot}86)^2 = 160{,}224$$

$$\text{stress at keel tensile} = \frac{60000}{160224} \times 12{\cdot}86 = 4{\cdot}81 \text{ tons sq. in.}$$

$$\text{stress at deck compressive} = \frac{60000}{160224} \times 15{\cdot}14 = 5{\cdot}67 \text{ tons sq. in.}$$

CHAPTER IX.

HORSE-POWER, EFFECTIVE AND INDICATED—RESISTANCE OF SHIPS—COEFFICIENTS OF SPEED—LAW OF COMPARISON.

Horse-power.—We have in Chapter V. defined the "work" done by a force as being the product of the force and the distance through which the force acts. Into the conception of work the question of time does not enter at all, whereas "power" involves not only work, but also the time in which the work is done. The unit of power is a "*horse-power*," which is taken as "33,000 *foot-lbs. of work performed in* 1 *minute*," or "550 *foot-lbs. of work performed in* 1 *second*." Thus, if during 1 minute a force of 1 lb. acts through 33,000 feet, the same power will be exerted as if a force of 33 lbs. acts through 1000 feet during 1 minute, or if 50 lbs. acts through 11 feet during 1 second. Each of these will be equivalent to 1 horse-power. The power of a locomotive is a familiar instance. In this case the work performed by the locomotive —if the train is moving at a uniform speed—is employed in overcoming the various resistances, such as the friction of the wheels on the track, the resistance of the air, etc. If we know the amount of this resistance, and also the speed of the train, we can determine the horse-power exerted by the locomotive. The following example will illustrate this point:—

If the mass of a train is 150 tons, and the resistance to its motion arising from the air, friction, etc., amount to 16 lbs. weight per ton when the train is going at the rate of 60 miles per hour on a level plain, find the horse-power of the engine which can just keep it going at that rate.

$$\text{Resistance to onward motion} = 150 \times 16$$
$$= 2400 \text{ lbs.}$$
$$\text{Speed in feet per minute} = 5280$$
$$\text{Work done per minute} = 2400 \times 5280 \text{ foot-lbs.}$$
$$\text{Horse-power} = \frac{2400 \times 5280}{33000}$$
$$= 384$$

In any general case, if—

R = resistance to motion in pounds;

v = velocity in feet per minute;

V = velocity in knots (a velocity of 1 knot is 6080 feet per hour);

then—

$$\text{Horse-power} = \frac{R \times v}{33000}$$
$$= \frac{1}{326}(R \times V) \text{ nearly}$$

The case of the propulsion of a vessel by her own engines is much more complicated than the question considered above of a train being drawn along a level plain by a locomotive. We must first take the case of a vessel being towed through the water by another vessel. Here we have the resistances offered by the water to the towed vessel overcome by the strain in the tow-rope. In some experiments on H.M.S. *Greyhound* by the late Mr. Froude, which will be described later, the tow-rope strain was actually measured, the speed being recorded at the same time. Knowing these, the horse-power necessary to overcome the resistance can be at once determined. For example—

At a speed of 1017 feet per minute, the tow-rope strain was 10,770 lbs. Find the horse-power necessary to overcome the resistance.

$$\text{Work done per minute} = 10{,}770 \times 1017 \text{ foot-lbs.}$$
$$\text{Horse-power} = \frac{10770 \times 1017}{33000}$$
$$= 332$$

Effective Horse-power.—The effective horse-power of a vessel at a given speed is the horse-power required to overcome the various resistances to the vessel's progress at that speed. It may be described as the horse-power usefully employed, and is sometimes termed the "tow-rope" or "tug" horse-power, because this is the power that would have to be transmitted through the tow-rope if the vessel were towed through the water at the given speed. Effective horse-power is often written E.H.P. We shall see later that the E.H.P. is entirely different to the Indicated Horse-power (written I.H.P.), which is the horse-power actually measured at the vessel's engines.

Example.—Find the horse-power which must be transmitted through a tow-rope in order to tow a vessel at the rate of 16 knots, the resistance to the ship's motion at that speed being equal to a weight of 50 tons.

Ans. 5503 H.P.

Experiments with H.M.S. "Greyhound," by the late Mr. William Froude, F.R.S.—These experiments took place at Portsmouth as long ago as 1871, and they settled a number of points in connection with the resistance and propulsion of ships, about which, up to that time, little was known. The thoroughness with which the experiments were carried out, and the complete analysis of the results that was given, make them very valuable; and students of the subject would do well to consult the original paper in the *Transactions of the Institution of Naval Architects for* 1874. A summary of the experiments, including a comparison with Rankine's "Augmented Surface Theory of Resistance," will be found in vol. iii. of *Naval Science*. Mr. Froude's report to the Admiralty was published in *Engineering*, May 1, 1874.

The *Greyhound* was a ship 172′ 6″ in length between perpendiculars, and 33′ 2″ extreme breadth, the deepest draught during the experiments being 13′ 9″ mean. The displacement

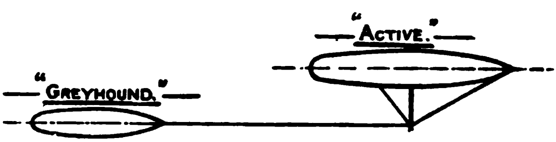


FIG. 105.

corresponding to this mean draught being 1161 tons; area of midship section, 339 square feet; area of immersed surface, 7540 square feet. The *Greyhound* was towed by H.M.S. *Active*. It was essential to the accuracy of the experiments that the *Greyhound* should proceed through undisturbed water, and to avoid using an exceedingly long tow-rope a boom was rigged out from the side of the *Active* to take the tow-rope (see Fig. 105). By this means the *Greyhound* proceeded through water that had not been influenced by the wake of the *Active.* The length of the boom on the *Active* was 45 feet, and the length of the tow-rope was such that the *Greyhound's* bow was 190 feet clear of the *Active's* stern. The actual force on the tow-rope at its extremity was not required, but the "horizontal component." This would be the force that was overcoming the resistance, the "vertical component" being due to the weight of the tow-rope. The horizontal force on the tow-rope and the speed were automatically recorded on a sheet of paper carried on a revolving cylinder. For details of the methods employed and the apparatus used, the student is referred to

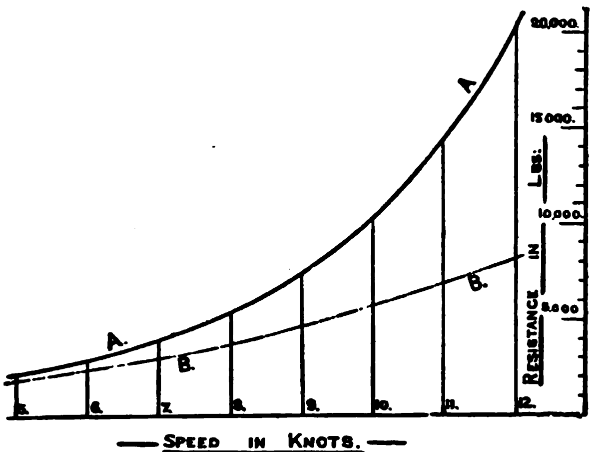


Fig. 106.

the sources mentioned above. The horizontal force on the tow-rope was equal to the nett resistance of the *Greyhound.* The results can be represented graphically by a curve, abscissæ representing speed, and ordinates representing the resistance in pounds. Such a curve is given by A in Fig. 106.

It will be seen that the resistance increases much more rapidly at the higher than at the lower speeds; thus, on increasing the speed from 7 to 8 knots, an extra resistance of 1500 lbs. has to be overcome, while to increase the speed from 11 to 12 knots, an extra resistance of 6000 lbs. must be overcome. Beyond 12 knots the shape of the curve indicates that the resistance increases very rapidly indeed. Now, the *rate* at which the resistance increases as the speed increases is a very important matter. (We are only concerned now with the total resistance.) Up to 8 knots it was found that the resistance was proportional to the *square* of the speed; that is to say, if R_1, R_2 represent the resistances at speeds V_1, V_2 respectively, then, if the resistance is proportional to the square of the speed—

$$R_1 : R_2 :: V_1^2 : V_2^2$$

$$\text{or } \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2}$$

By measuring ordinates of the curve in Fig. 106, say at 5 and 6 knots, this will be found to be very nearly the case. As the speed increases above 8 knots, the resistance increases much more rapidly than would be given by the above; and between 11 and 12 knots, the resistance is very nearly proportional to the *fourth* power of the speed.

The experiments were also conducted at two displacements less than 1161 tons, viz. at 1050 tons and 938 tons. It was found that differences in resistance, due to differences of immersion, depended, not on changes of area of midship section or on changes of displacement, but rather on *changes in the area of wetted surface.* Thus for a reduction of $19\frac{1}{4}$ per cent. in the displacement, corresponding to a reduction of area of midship section of $16\frac{1}{4}$ per cent., and area of immersed surface of 8 per cent., the reduction in resistance was about $10\frac{1}{2}$ per cent., this being for speeds between 8 and 12 knots.

Ratio between Effective Horse-power and Indicated Horse-power. — We have already seen that, the resistance of the *Greyhound* at certain speeds being determined, it is possible to determine at once the E.H.P. at those speeds. Now, the horse-power actually developed by the *Greyhound's* own engines, or the "indicated horse-power" (I.H.P.), when proceeding on the measured mile, was observed on a separate series of trials, and tabulated. The ratio of the

E.H.P. to the I.H.P. was then calculated for different speeds, and it was found that E.H.P. ÷ I.H.P. in the best case was only 0·42; that is to say, as much as 58 per cent. of the power was employed in doing work other than overcoming the actual resistance of the ship. This was a very important result, and led Mr. Froude to make further investigations in order to determine the cause of this waste of power, and to see whether it was possible to lessen it.

The ratio $\frac{\text{E.H.P.}}{\text{I.H.P.}}$ at any given speed is termed the "*propulsive coefficient*" at that speed. As we saw above, in the most efficient case, in the trials of the "*Greyhound*," this coefficient was 42 per cent. For modern vessels with fine lines a propulsive coefficient of 50 per cent. may be expected, if the engines are working efficiently and the propeller is suitable. In special cases, with fine forms and efficient propellers, the coefficient rises higher than this. These values only hold good for the maximum speed for which the vessel is designed; for lower speeds the coefficient becomes smaller. The following table gives some results as given by Mr. Froude. The *Mutine* was a sister-ship to the *Greyhound*, and she had also been run upon the measured mile at the same draught and trim as the *Greyhound*.

Ship	Speed on measured mile in feet per minute.	Resistance due to speed deduced from the towing experiments with *Greyhound*, including an estimate of air-resistance of masts and rigging.	Effective horse-power = resistance $\times \frac{\text{velocity}}{33{,}000}$	Actual indicated horse-power on trial.	Effective horse-power ÷ indicated horse-power.
Greyhound ...	1017	10,770	332·1	786	0·422
	845	6,200	158·7	453	0·350
Mutine	977	9,440	279·5	770	0·363
	757	4,770	109·4	328	0·334

Resistance.—We now have to inquire into the various resistances which go to make up the total resistance which a ship experiences in being towed through the water. These resistances are of three kinds—

1. Resistance due to friction of the water upon the surface of the ship.

2. Resistance due to the formation of eddies.

3. Resistance due to the formation of waves.

1. "*Frictional resistance,*" or the resistance due to the friction of the water upon the surface of the ship. This is similar to the resistance offered to the motion of a train on a level line owing to the friction of the rails, although it follows different laws. It is evident that this resistance must depend largely upon the state of the bottom. A vessel, on becoming foul, loses speed very considerably, owing to the greatly increased resistance experienced. This frictional resistance forms a large proportion of the total at low speeds, and forms a good proportion at higher speeds.

2. *Resistance due to eddy-making.*—Take a block of wood, and imagine it placed a good distance below the surface of a current of water moving at a uniform speed V. Then the particles of water will run as approximately indicated in Fig. 107. At A we shall have a mass of water in a state of

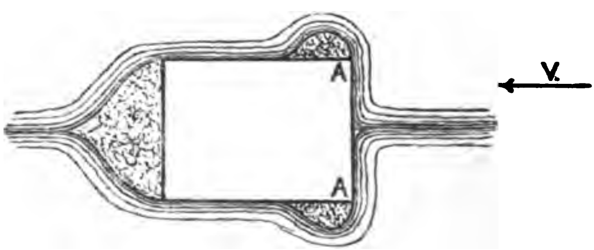


Fig. 107.

violent agitation, and a much larger mass of water at the rear of the block. Such masses of confused water are termed "*eddies,*" and sometimes "*dead water.*" If now we imagine that the water is at rest, and the block of wood is being towed through the water at a uniform speed V, the same eddies will be produced, and the eddying water causes a very considerable resistance to the onward motion. Abrupt terminations which are likely to cause such eddies should always be avoided in vessels where practicable, in order to keep the resistance as low as possible. This kind of resistance forms a very small proportion of the total in well-formed vessels, but in the older vessels with full forms aft and thick stern-posts, it amounted to a very considerable item.

3. *Resistance due to the formation of waves.*—For low speeds this form of resistance is not experienced to any sensible extent, but for every ship there is a certain speed above which the resistance increases more rapidly than would be the case if surface friction and eddy-making alone caused the resistance. This extra resistance is caused by the formation of waves upon the surface of the water.

We must now deal with these three forms of resistance in detail, and indicate as far as possible the laws which govern them.

1. *Frictional Resistance.*—The data we have to work upon when considering this form of resistance were obtained by the late Mr. Froude. He conducted an extensive series of experiments on boards of different lengths and various conditions of surface towed edgewise through water contained in a tank, the speed and resistance being simultaneously recorded. The following table represents the resistances in pounds per square foot due to various lengths of surface of various qualities when moving at a uniform speed of 600 feet per minute, or very nearly 6 knots in fresh water. There is also given the powers of the speed to which the resistances are approximately proportional.

We can sum up the results of these experiments as follows: The resistance due to the friction of the water upon the surface depends upon—

(1) The area of the surface.
(2) The nature of the surface.
(3) The length of the surface.
(4) The density of the water.
and (5) The resistance varies as the n^{th} power of the speed where n varies from 1·83 to 2·16.

Nature of surface.	Length of Surface in Feet.							
	2		8		20		50	
	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.
Varnish	2·00	0·41	1·85	0·325	1·85	0·278	1·83	0·250
Tinfoil	2·16	0·30	1·99	0·278	1·90	0·262	1·83	0·246
Calico	1·93	0·87	1·92	0·626	1·89	0·531	1·87	0·474
Fine sand ...	2·00	0·81	2·00	0·583	2·00	0·480	2·06	0·405
Medium sand ...	2·00	0·90	2·00	0·625	2·00	0·534	2·00	0·488

And thus we can write for a smooth surface in salt water—

$$R = f \cdot \left(\frac{w}{w_0}\right) . S . \left(\frac{V}{6}\right)^{1·83}$$

where R = resistance in pounds;

S = area of surface in square feet;

V = speed in knots relative to still water;

f = a coefficient depending upon the nature and length of the surface;

w = density of salt water;

w_0 = density of fresh water;

w/w_0 = 1·025.

This coefficient f will be the resistance per square foot given in the above table, as is at once seen by making S = 1 square foot, V = 6 knots, and $w = w_0$. It is very noticeable how the resistance per square foot decreases as the length increases. Mr. Froude explained this by pointing out that the leading portion of the plane must communicate an onward motion to the water which rubs against it, and "consequently the portion of the surface which succeeds the first will be rubbing, not against stationary water, but against water partially moving in its own direction, and cannot therefore experience as much resistance from it."

Experiments were not made on boards over 50 feet in length. Mr. Froude remarked, in his report, "It is highly desirable to extend these experiments, and the law they elucidate, to greater lengths of surface than 50 feet; but this is the greatest length which the experiment-tank and its apparatus admit, and I shall endeavour to organize some arrangement by which greater lengths may be successfully tried in open water.

Mr. Froude was never able to complete these experiments as he anticipated. It has long been felt that experiments with longer boards would be very valuable, so that the results could be applied to the case of actual ships.[1]

These experiments show very clearly how important the condition of the surface is as affecting resistance. The varnished surface may be taken as typical of a surface coated with smooth paint, or the surface of a ship sheathed with bright copper, the medium sand surface being typical of the surface of a vessel sheathed with copper which has become foul. If the surface has become fouled with large barnacles, the resistance must rise very high.

In applying the results of these experiments to the case of actual ships, it is usual to estimate the area of wetted surface, and to take the length of the ship in the direction of motion to determine what the coefficient f shall be. See below for E.H.P. due to friction and eddy-making.

Take the following as an example:—

The wetted surface of a vessel is estimated at 7540 square feet, the length being 172 feet. Find the resistance due to surface friction at a speed of 12 knots in salt water, assuming a coefficient of 0·25, and that the resistance varies (*a*) as the square of the speed, and (*b*) as the 1·83 power of the speed.

$$(a)\ \text{Resistance} = 0{\cdot}25 \times 1{\cdot}025 \times 7540 \times \left(\tfrac{12}{6}\right)^2$$
$$= 7728 \text{ lbs.}$$
$$(b)\ \text{Resistance} = 0{\cdot}25 \times 1{\cdot}025 \times 7540 \times \left(\tfrac{12}{6}\right)^{1{\cdot}83}$$
$$= 6870 \text{ lbs.}$$ [2]

---

[1] See discussion on paper by Mr. Baker on Frictional Resistance, I.N.A., 1916.

[2] This has to be obtained by the aid of logarithms.

It is worth remembering that for a smooth painted surface the frictional resistance per square foot of surface is about $\frac{1}{4}$ lb. at a speed of 6 knots.

It is useful, in estimating the wetted surface for use in the above formula, to have some method of readily approximating to its value. Several methods of doing this have been already given in Chapter II., the one known as "Kirk's Analysis" having been largely employed. There are also several approximate formulæ which are reproduced—

(1) Based on Kirk's analysis—

$$\text{Surface} = 2\text{LD} + \frac{\text{V}}{\text{D}}$$

(2) Given by Mr. Denny—

$$\text{Surface} = 1{\cdot}7\text{LD} + \frac{\text{V}}{\text{D}}$$

(3) Given by Mr. Taylor—

$$\text{Surface} = 15{\cdot}5\sqrt{\text{W.L.}}$$

(4) Used at the Experiment Tank at Haslar—

$$\text{Surface} = \text{V}^{\frac{2}{3}}\left(3{\cdot}4 + \frac{\text{L}}{2.\text{V}^{\frac{1}{3}}}\right)$$

L being the length of the ship in feet;
D being the mean moulded draught;
V being the displacement in cubic feet;
W being the displacement in tons.

2. *Eddy-making Resistance.*—We have already seen the general character of this form of resistance. It may be assumed to vary as the square of the speed, but it will vary in amount according to the shape of the ship and the appendages. Thus a ship with a full stern and thick stern-posts will experience this form of resistance to a much greater extent than a vessel with a fine stern and with stern-post and rudder of moderate thickness. Eddy-making resistance can be allowed for by putting on a percentage to the frictional resistance. It is possible to reduce eddy-making to a minimum by paying careful attention to the appendages and endings of a vessel, especially at the stern. Thus shaft brackets in twin-screw ships are often made of pear-shaped

section, as shown in Fig. 85E[1] and Fig. 108. A conical piece is always put at the after end of propeller shafts for this reason.

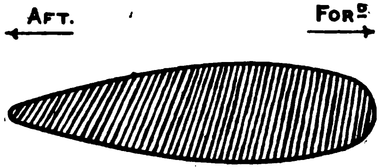


FIG. 108.

The following formula can be used to express the effective horse-power due to surface friction and eddy-making in salt water, viz.:—

$$\text{E.H.P.} = \frac{1}{326} \cdot f \cdot S \cdot V^{2\cdot83}$$

V being in knots.

For the coefficient f, we can take $f = 0\cdot009$ for a length of 500 feet varying to 0·01 for a length of 40 feet. These values are rather greater than would be inferred from Froude's experiments, and include an allowance for eddy-making resistance.

On page 346, a table is given for the E.H.P. due to skin friction, based on Mr. Froude's constants, assuming the skin friction to vary as $V^{1\cdot825}$, from speeds of 10 to 40 knots, and for lengths of 100 to 1000 feet. The reduction of the coefficient as length increases has been allowed for in this table.

Mr. Baker, in his work on "Resistance and Propulsion," gives the following values of f for salt water and for varying lengths in the formula—

$$\text{Frictional resistance in lbs.} = f \cdot S \cdot V^{1\cdot835}$$

where S is wetted surface in square feet

V is speed in knots.

Length in feet	50	75	100	200	300	400	500	700	900
f	0·0096	0·00935	0·0092	0·00898	0·0089	0·00883	0·00877	0·00868	0·0086

[1] See note in "Strength of Shaft Brackets," p. 271, as to resistance of arms.

These values are obtained by assuming that the frictional coefficient of the first 50 feet is the same as that of a 50-foot plank, regardless of the ship's length, and that the remainder of the length has the same frictional resistance as the last foot of the 50-foot plank.

The table given for E.H.P. due to frictional resistance per square foot of wetted surface given on page 332 is calculated from similar figures to the above, and it is suggested as an exercise that some of the figures given be checked. Attention is necessary to the units, as the above is for resistance in lbs., and E.H.P. $= \frac{1}{326} \cdot R \cdot V$, so that

$$E.H.P. = \frac{1}{326} \cdot f \cdot V^{2 \cdot 825}$$

3. *Resistance due to the Formation of Waves.*—A completely submerged body moving at any given speed will only experience resistance due to surface friction and eddy-making provided

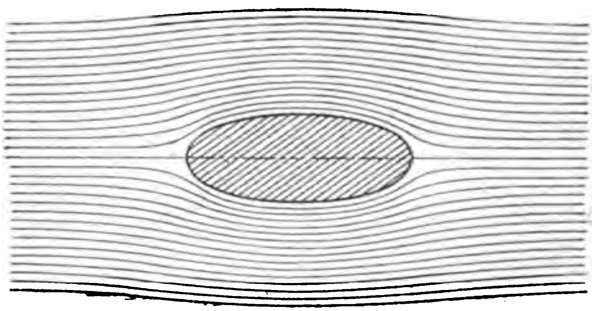


Fig. 109.

it is immersed sufficiently; but with a body moving at the surface, such as we have to deal with, the resistance due to the formation of waves becomes very important, especially at high speeds. This subject is of considerable difficulty, and it is not possible to give in this work more than a general outline of the principles involved.

Consider a body shaped as in Fig. 109 placed a long way below the surface in water (regarded as frictionless), and suppose the water is made to move past the body with a uniform speed V. The particles of water must move past the body in certain lines, which are termed *stream-lines*. These stream-lines are straight and parallel before they reach the body, but owing to the obstruction caused, the particles of water are locally diverted, and follow curved paths instead of straight ones. The straight paths are again resumed some distance at the rear of the body. We can imagine these stream-lines making up the boundaries of a series of stream-tubes, in each of which the same particles of water will flow throughout the operation. Now, as these streams approach the body they broaden, and consequently the particles of water slacken in speed. Abreast the body the streams are constricted in area, and there is a consequent increase in speed; and at the rear of the body the streams again broaden, with a slackening in speed. Now, in water flowing in the way described, any *increase in speed* is accompanied by a *decrease in pressure*, and conversely any *decrease in speed* is accompanied by an *increase in pressure*. We may therefore say—

(1) There is a broadening of all the streams, and attendant decrease of speed and consequent excess of pressure, near both ends of the body; and—

(2) There is a narrowing of the streams, with attendant excess of speed and consequent decrease of pressure, along the middle of the body.

This relation between the velocity and pressure is seen in the draught of a fire under a chimney when there is a strong wind blowing. The excess of the speed of the wind is accompanied by a decrease of pressure at the top of the chimney. It should be noticed that the variations of velocity and pressure must necessarily become less as we go further away from the side of the body. A long way off the stream-lines would be parallel. The body situated as shown, with the frictionless water moving past it, does not experience any resultant force tending to move it in the direction of motion.[1]

[1] This principle can be demonstrated by the use of advanced mathematics. "We may say it is quite evident if the body is symmetrical, that is to say,

Now we have to pass from this hypothetical case to the case of a vessel on the surface of the water. In this case the water surface is free, and the excess of pressure at the bow and stern shows itself by an elevation of the water at the bow and stern, and the decrease of pressure along the sides shows itself by a depression of the water along the sides. This system is shown by the dotted profile of the water surface in Fig. 110, which

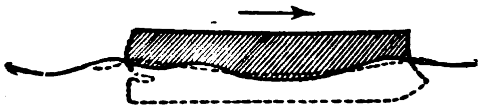


FIG. 110.

has been termed the statical wave. The foregoing gives us the reason for the wave-crest at the stern of the ship. The crest at the bow appears quite a reasonable thing to expect, but the crest at the stern is due to the same set of causes. This disturbance of level at the bow and stern is described by Mr. R. E. Froude as the "forcive" of the actual wave formation. If a stone is thrown into water, the sudden disturbance propagates a series of waves that radiate in all directions. In the case of a ship, the shape of the ship causes the disturbance to form diverging and transverse waves as seen below.

Observation shows that there are two separate and distinct series of waves caused by the motion of a ship through the water: (1) at the bow, and (2) at the stern.

Each of these series of waves consists of (1) a series of diverging waves, the crests of which slope aft, and (2) a series of transverse waves, whose crests are nearly perpendicular to the middle line of the ship.

First, as to the diverging waves at the bow. "The inevitably widening form of the ship at her entrance throws off on each side a local oblique wave of greater or less size according to the speed and obtuseness of the wedge, and these waves form themselves into a series of diverging crests. These waves

has both ends alike, for in that case all the fluid action about the after body must be the precise counterpart of that about the fore body; all the stream-lines, directions, speed of flow, and pressures at every point must be symmetrical, as is the body itself, and all the forces must be equal and opposite" (see a paper by Mr. R. E. Froude, on "Ship Resistance," read before the Greenock Philosophical Society in 1894).

have peculiar properties. They retain their identical size for a very great distance, with but little reduction in magnitude. But the main point is, that they become at once disassociated with the vessel, and after becoming fully formed at the bow, they pass clear away into the distant water, and produce no further effect on the vessel's resistance." These oblique waves are not long in the line of the crest BZ, Fig. 111, and the

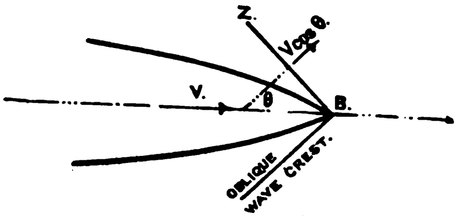


FIG. 111.

waves travel perpendicular to the crest-line with a speed of V cos θ, where V is the speed of the ship. As the speed of the ship increases the diverging waves become larger, and consequently represent a greater amount of resistance.

Besides these diverging waves, however, "there is produced by the motion of the vessel another notable series of waves, which carry their crests transversely to her line of motion." It is this transverse series of waves that becomes of the greatest importance in producing resistance as the speed is pushed to values which are high for the ship. These transverse waves show themselves along the sides of the ship by the crests and troughs, as indicated roughly in Fig. 110. The lengths of these waves (*i.e.* the distance from one crest to the other) bears a definite relation to the speed of the ship. This relation is that the length of the wave varies as the *square of the speed* at which the ship is travelling, and thus as the speed of the ship increases the length from crest to crest of the accompanying series of transverse waves increases very rapidly.

The waves produced by the stern of the ship are not of such great importance as those formed by the bow, which we have been considering. They are, however, similar in character, there being an oblique series and a transverse series.

Interference between the Bow and Stern Transverse Series of Waves.—In a paper read by the late Mr. Froude at the Institution of Naval Architects in 1877, some very important experiments were described, showing how the residuary resistance[1] varied in a ship which always had the same fore and after bodies, but had varying lengths of parallel middle body inserted, thus varying the total length. A strange variation in the resistance at the same speed, due to the varying lengths of parallel middle body was observed. The results were set out as roughly shown in Fig. 112, the resistance being set up on a

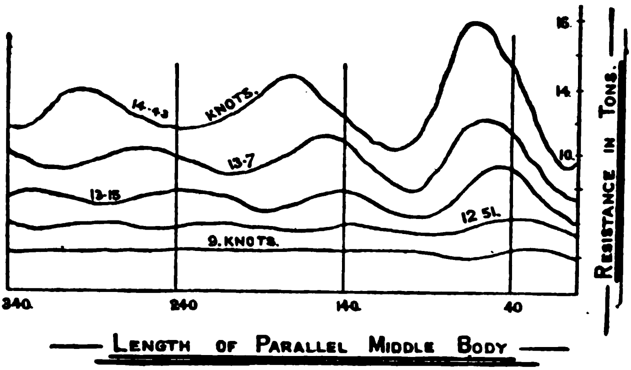


Fig. 112.

base of length of ship for certain constant speeds. At the low speed of 9 knots very little variation was found, and this was taken to show that at this speed the residuary resistance was caused by the diverging waves only.

The curves show the following characteristics:—

(1) The spacing or length of undulation appears uniform throughout each curve, and this is explained by the fact that waves of a given speed have always the same length.

(2) The spacing is more open in the curves of higher speed, the length apparently varying as the square of the speed. This is so because the length of the waves are proportionate to the square of the speed.

[1] Residuary resistance is the resistance other than frictional.

(3) The amplitude or heights of the undulations are greater in the curves of higher speeds, and this is so, because the waves made by the ship are larger for higher speeds.

(4) The amplitude in each curve diminishes as the length of parallel middle body increases, because the wave system, by diffusing transversely, loses its height.

These variations in residuary resistance for varying lengths are attributed to the interference of the bow and stern transverse series of waves. When the crests of the bow-wave series coincide with the crests of the stern-wave series, the residuary resistance is at a *maximum*. When the crests of the bow-wave series coincide with the troughs of the stern-wave series, the residuary resistance is at a *minimum*.

The following formula[1] gives an approximation to the effective horse-power to overcome wave-making resistance, viz.—

$$\text{E.H.P.} = \tfrac{1}{326} \cdot b \cdot \frac{(\text{W})^{\frac{4}{3}}}{\text{L}} \cdot \text{V}^{5}$$

The coefficient *b*, however, has varying values for the same ship owing to the interference above mentioned, so that it is not a formula that can be relied upon. The total formula for E.H.P. can be written—

$$\text{E.H.P.} = \tfrac{1}{326}\left\{f \cdot \text{S} \cdot \text{V}^{2\cdot83} + b \cdot \frac{(\text{W})^{\frac{4}{3}}}{\text{L}} \cdot \text{V}^{5}\right\}$$

where *f* is a coefficient for surface friction and eddy-making appropriate to the length. If 50 per cent. be taken as a standard propulsive coefficient at top speed, to 40 per cent. at 10 knots, say, values of *b* can be determined from trial data in the user's possession which may be useful for estimating purposes. Examples 31 and 32 in Appendix illustrate its use.

The following extracts from a lecture[2] by Lord Kelvin (Sir William Thomson) are of interest as giving the relative influence of frictional and wave-making resistance :—

"For a ship A, 300 feet long, 31½ feet beam, and 2634 tons displacement, a ship of the ocean mail-steamer type, going at 13 knots, the skin resistance is 5·8 tons, and the wave resistance

[1] See Mr. Johns' paper, I.N.A., 1907, for a discussion of "approximate formulæ for determining the resistance of ships," also Prof. Hovgaard, 1908, I.N.A.

[2] Third volume "Popular Lectures and Addresses," 1887.

is 3·2 tons, making a total of 9 tons. At 14 knots the skin resistance is but little increased, viz. 6·6 tons, while the wave resistance is 6·15 tons.

"For a vessel B, 300 feet long, 46·3 feet beam, and 3626 tons, no parallel middle body, with fine lines swelling out gradually, the wave resistance is much more favourable. At 13 knots the skin resistance is rather more than A, being 6·95 tons as against 5·8 tons, while the wave resistance is only 2·45 tons as against 3·2 tons. At 14 knots there is a very remarkable result in the broader ship with its fine lines, all entrance and run, and no parallel middle body. At 14 knots the skin resistance is 8 tons as against 6·6 tons in A, while the wave resistance is only 3·15 tons as against 6·15 tons in A.

"For a torpedo boat, 125 feet long and 51 tons displacement, at 20 knots the skin resistance was 1·2 tons, and the wave resistance 1·1 tons."

Resistance of a Completely Submerged Body.—The conditions in this case are completely different from those which have to be considered in the case of a vessel moving on the surface. In this latter case waves are produced on the surface, as we have seen, but with a completely submerged body this is not so, provided the vessel is immersed sufficiently. We get the clue to the form of least resistance in the shape of fishes, in which the head or forward end is comparatively blunt, while the rear portion tapers off very fine. The reason for the small resistances of forms of this sort is seen when we consider the paths the particles of water follow when flowing past. These paths are termed the stream-lines for the particular form. It will be seen that no eddies are produced for a fish-shaped form, and, as we saw on p. 320, it is the rear end which must be fined off in order to reduce eddy-making to a minimum. This was always insisted on very strongly by the late Mr. Froude, who said, "It is blunt tails rather than blunt noses that cause eddies." A very good illustration of the above is seen in the form that is given to the section of shaft brackets in twin-screw vessels. Such sections are given in Figs. 85E and 108. It will be noticed that the forward end is comparatively blunt, while the after end is fined off to a small radius.

Speed Coefficients.—The method which is most largely employed for determining the I.H.P. required to drive a vessel at a certain speed is by using coefficients obtained from the results of trials of existing vessels. They are based upon assumptions which should always be carefully borne in mind when applying them in actual practice.

1. *Displacement Coefficient.*—We have seen that for speeds at which wave-making resistance is not experienced, the resistance may be taken as varying—

(*a*) With the area of wetted surface;

(*b*) Approximately as the square of the speed;

so that we may write for the resistance in pounds—

$$R = K_1SV^2$$

V being the speed in knots, S the area of wetted surface in square feet, and K_1 being a coefficient depending on a number of conditions which we have already discussed in dealing with resistance.

Now, $E.H.P = \frac{R \times V \times 101}{33000}$, as we have already seen (p. 297). Therefore we may say—

$$E.H.P. = K_2SV^3$$

where K_2 is another coefficient, which may be readily obtained from the previous one. If now we assume that the total I.H.P. bears a constant ratio to the E.H.P., or, in other words, the propulsive coefficient remains the same, we may write—

$$I.H.P. = K_3SV^3$$

K_3 being another new coefficient. S, the area of the wetted surface, is proportional to the product of the length and girth to the water-line; W, the displacement, is proportional to the product of the length, breadth, and draught. Thus W may be said to be proportional to the *cube* of the linear dimensions, while S is proportional to the *square* of the linear dimensions. Take a vessel A, of twice the length, breadth, and draught, of another vessel B, with every linear dimension twice that of the corresponding measurement in B. Then the forms of the two vessels are precisely similar, and the area of the wetted surface of A will be $2^2 = 4$ times the area of the wetted surface of B, and the displacement of A will be $2^3 = 8$ times the displacement of B. The ratio of the linear dimensions will be the cube root of the ratio of the displacements, in the above case $\sqrt[3]{8} = 2$. The ratio of corresponding areas will be the square of the cube root of the ratio of the displacements, in the above case $(\sqrt[3]{8})^2 = 4$. This may also be written $8^{\frac{2}{3}}$. We may accordingly say that for similar ships the area of the wetted surface will be proportional to the two-thirds power of the displacement, or $W^{\frac{2}{3}}$. We can now write our formula for the indicated horse-power—

$$\text{I.H.P.} = \frac{W^{\frac{2}{3}} \times V^3}{C}$$

where W = the displacement in tons;
V = the speed in knots;
C = a coefficient termed the *displacement coefficient*.[1]

If a ship is tried on the measured mile at a known displacement, and the I.H.P. and speed are measured, the value of the coefficient C can be determined, for $C = \frac{W^{\frac{2}{3}} \times V^3}{\text{I.H.P.}}$. It is usual to calculate this coefficient for every ship that goes on trial, and to record it for future reference, together with all the particulars of the ship and the conditions under which she was tried. $W^{\frac{2}{3}}$ can be readily calculated by means of the slide rule, but it is usual to perform the work by the aid of logarithms. A specimen calculation is given here:—

The *Himalaya* on trial displaced 4375 tons, and an I.H.P. of 2338 was recorded, giving a speed of 12·93 knots. Find the "displacement coefficient" of speed.

Here we have—

$$W = 4375$$
$$V = 12{\cdot}93$$
$$\text{I.H.P.} = 2338$$

[1] The coefficients are often termed "Admiralty constants," but it will be seen later that they are not at all constant for different speeds of the same vessel.

By reference to a table of logarithms, we find—

$$\log 4375 = 3{\cdot}6410$$
$$\log 12{\cdot}93 = 1{\cdot}1116$$
$$\log 2338 = 3{\cdot}3689$$
$$\text{so that } \log (4375)^{\frac{2}{3}} = \tfrac{2}{3} \log 4375 = 2{\cdot}4273$$
$$\log (12{\cdot}93)^3 = 3 \log 12{\cdot}93 = 3{\cdot}3348$$
$$\therefore \log\left(\frac{4375^{\frac{2}{3}} \times 12{\cdot}93^3}{2338}\right) = 2{\cdot}4273 + 3{\cdot}3348 - 3{\cdot}3689$$
$$= 2{\cdot}3932$$

The number of which this is the logarithm is 247·3, and accordingly this is the value of the coefficient required.

2. The other coefficient employed is the "*midship-section coefficient.*"[1] If M is the area of the immersed midship section in square feet, the value of this coefficient is—

$$\frac{M \times V^3}{\text{I.H.P.}}$$

This was originally based on the assumption that the resistance of the ship might be regarded as due to the forcing away of a volume of water whose section is that of the immersed midship section of the ship. This assumption is not compatible with the modern theories of resistance of ships, and the formula can only be true in so far as the immersed midship section is proportional to the wetted surface.

In obtaining the $W^{\frac{2}{3}}$ coefficient, we have assumed that the wetted surface of the ships we are comparing will vary as the two-thirds power of the displacement; but this will not be true if the ships are not similar in all respects. However, it is found that the proportion to the area of the wetted surface is much more nearly obtained by using $W^{\frac{2}{3}}$ than by using the area of the immersed midship section. We can easily imagine two ships of the same breadth and mean draught and similar form of midship section whose displacement and area of wetted surface are very different, owing to different lengths and forms. We therefore see that, in applying these formulæ, we must take care that the forms and proportions of the ships are at any rate somewhat similar. There is one other point about these

[1] See note on p. 330.

formulæ, and that is, that the performances of two ships can only be fairly compared at "corresponding speeds."[1]

Summing up the conditions under which these two formulæ should be employed, we have—

(1) The resistance is proportional to the square of the speed.

(2) The resistance is proportional to the area of wetted surface, and this area is assumed to vary as the two-thirds power of the displacement, or as the area of the immersed midship section. Consequently, the ships we compare should be of somewhat similar type and form.

(3) The coefficient of performance of the machinery is assumed to be the same. The ships we compare are supposed to be fitted with the same type of engine, working with the same efficiency. Accordingly we cannot fairly compare a screw steamer with a paddle steamer, since the efficiency of working may be very different.

(4) The conditions of the surfaces must be the same in the two ships. It is evident that a greater I.H.P. would be required for a given speed if the ship's bottom were foul than if it had been newly painted, and consequently the coefficient would have smaller values.

(5) Strictly speaking, the coefficients should only be compared for "corresponding speeds."[2]

With proper care these formulæ may be made to give valuable assistance in determining power or speed for a new design, but they must be carefully used, and their limitations thoroughly appreciated. A good method of recording these coefficients is to plot them on base of $\frac{V}{\sqrt{L}}$. In this way the size of ship is eliminated.

We have seen that it is only for moderate speeds that the resistance can be said to be proportional to the square of the speed, the resistance varying at a higher power as the speed increases. Also that the propulsive coefficient is higher at the maximum speed than at the lower speeds. So if we try a vessel at various speeds, we cannot expect the speed coefficients to remain constant, because the suppositions on which they are

[1] See p. 333. [2] See p. 333.

based are not fulfilled at all speeds. This is found to be the case, as is seen by the following particulars of the trials of H.M.S. *Iris*. The displacement being 3290 tons, and the area of the immersed midship section being 700 square feet, the measured-mile trials gave the following results:—

I.H.P.	Speed in knots.
7556	18·6
3958	15·75
1765	12·5
596	8·3

The values of the speed coefficients calculated from the above are—

	Displacement coefficients.	Mid. sec. coefficients.
18·6 knots	188	595
15·75 "	218	690
12·5 "	243	770
8·3 "	214	677

It will be noticed that both these coefficients attain their maximum values at about 12 knots for this ship, their value being less for higher and lower speeds. We may explain this by pointing out—

(1) At high speeds, although the "propulsive coefficient" is high, yet the resistance varies at a greater rate than the square of the speed, and—

(2) At low speeds, although the resistance varies nearly as the square of the speed, yet the efficiency of the mechanism is not at its highest value.

Corresponding Speeds.—We have frequently had to use the terms "low speeds" and "high speeds" as applied to certain ships, but these terms are strictly relative. What would be a high speed for one vessel might very well be a low speed for another. The first general idea that we have is that the speed depends in some way on the length. Fifteen knots would be a high speed for a ship 150 feet long, but it would be quite a moderate speed for a ship 500 feet long. In trying a model of a ship in order to determine its resistance, it is obvious that we cannot run the model at the same speed as the ship; but there must be a speed of the model "*corresponding*" to the speed of the ship. The law that we must employ is as follows: "*In comparing similar ships with one another, or ships with* *models, the speeds must be proportional to the square root of their linear dimensions."* Thus, suppose a ship is 300 feet long, and has to be driven at a speed of 20 knots; we make a model of this ship which is 6′ 3″ long. Then the ratio of their linear dimensions is—

$$\frac{300}{6 \cdot 25} = 48$$

and the speed of the model corresponding to 20 knots of the ship is—

$$20 \div \sqrt{48} = 2 \cdot 88 \text{ knots}$$

Speeds obtained in this way are termed "*corresponding speeds.*"

Example.—A model of a ship of 2000 tons displacement is constructed on the $\frac{1}{4}$ inch = 1 foot scale, and is towed at a speed of 3 knots. What speed of the ship does this correspond to?

Although here the actual dimensions are not given, yet the ratio of the linear dimensions is given, viz. 1 : 48. Therefore the speed of the ship corresponding to 3 knots of the model is—

$$3\sqrt{48} = 20\tfrac{3}{4} \text{ knots}$$

Expressing this law in a formula, we may say—

$$V = c\sqrt{L}$$

where V = speed in knots;

L = the length in feet;

c = a coefficient expressing the ratio $V : \sqrt{L}$, and consequently giving a measure of the speed.

We may take the following as average values of the coefficient "c" in full-sized ships:—

When $c = 0 \cdot 5$ to $0 \cdot 65$, the ship is being driven at a moderate economical speed;

$c = 0 \cdot 7$ to $1 \cdot 0$, gives the speed of mail steamers and modern battleships;

$c = 1 \cdot 0$ to $1 \cdot 3$, gives the speed of cruisers.

Beyond this we cannot go in full-sized vessels, since it is not possible to get in enough engine-power. This can, however, be done in torpedo-boats and torpedo-boat destroyers, and here we have $c = 1 \cdot 9$ to $2 \cdot 5$. These may be termed excessive speeds.

The remarks already made as to wave resistance gives the reason for the above. For low speeds the wave-making resistance is small. When, however, the speed increases such that the length of the wave is about the length of ship, we have the maximum interference, and the rate of increase of resistance with increase of speed is greatest. If V is speed in knots, the length of accompanying wave is $\frac{V^2}{1 \cdot 8}$; when the wave equals the length of ship, we have $\frac{V}{\sqrt{L}} = 1 \cdot 33$. So that when the ratio $\frac{V}{\sqrt{L}}$ is unity and somewhat above, the resistance is increasing very rapidly. If the speed can be pressed beyond the above, we reach a state of things where the wave is longer than the length of boat, and although the resistance is very high yet it is not *increasing* at so great a rate. This can only be the case in vessels of the destroyer or motor type. The following figures show how the total resistance varies in a typical destroyer:—

Up to 11 knots as second power nearly, at 16 knots as V^3, from 18 to 20 knots as $(V)^{3 \cdot 3}$, at 22 knots as $(V)^{2 \cdot 7}$, at 25 knots as V^2, and at 30 knots as V^2 nearly. The maximum rate of increase is at 18 to 20 knots, and here the accompanying wave approximated to the length of the ship.

Froude's Law of Comparison.—This law enables us to compare the resistance of a ship with that of her model, or the resistances of two ships of different size but of the same form. It is as follows—

If the linear dimensions of a vessel be l times the dimensions of the model, and the resistance of the latter at speeds V_1, V_2, V_3, *etc., are* R_1, R_2, R_3, *etc., then at the "corresponding speeds" of the ship,* $V_1\sqrt{l}$, $V_2\sqrt{l}$, $V_3\sqrt{l}$, *etc., the resistance of the ship will be* $R_1 l^3$, $R_2 l^3$, $R_3 l^3$, *etc.*

In passing from a model to a full-sized ship there is a correction to be made, because of the different effect of the friction of the water on the longer surface. The law of comparison strictly applies to the resistances other than frictional. The law can be used in comparing the resistance of two ships of similar form, and is found of great value when model experiments are not available.

In the earlier portion of this chapter we referred to the experiments of the *Greyhound* by the late Mr. Froude. A curve of resistance of the ship in pounds on a base of speed is given by A, in Fig. 106. In connection with these experiments, a model of the *Greyhound* was made and tried in the experimental tank under similar conditions of draught as the ship, and between speeds *corresponding* to those at which the ship herself had been towed. The resistance of the model having been found at a number of speeds, it was possible to construct a curve of resistance on a base of speed as shown by C in Fig. 113. The scale of the model was $\frac{1}{16}$ full size, and therefore the corresponding speeds of the ship were $\sqrt{16}$, or four times the speed of the model. If the law of comparison

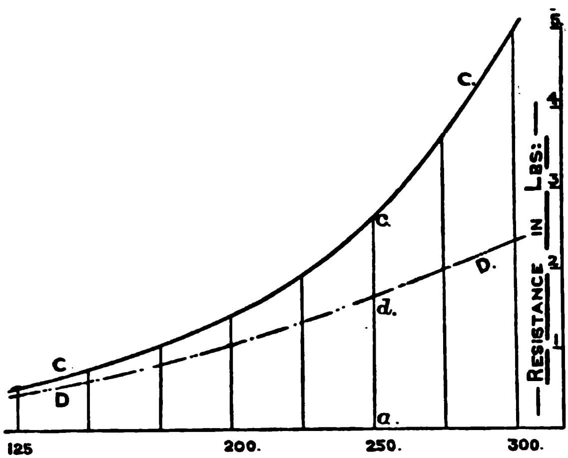


FIG. 113.

held good for the total resistance, the resistance of the ship should have been $16^3 = 4096$ times the resistance of the model at corresponding speeds; but this was not the case, owing to the different effect of surface friction on the long and short surfaces. The necessary correction was made as follows. The wetted surface of the model was calculated, and by employing a coefficient suitable to the length of the model and the condition of its surface, the resistance due to surface friction was calculated for various speeds as explained (p. 319), and a curve drawn through all the spots thus obtained. This is shown by the dotted curve DD in Fig. 113. Thus at 250 feet per minute the total resistance of the model is given by *ac*, and the resistance due to surface friction by *ad*. The portion of the ordinate between the curves CC and DD will give at any speed the resistance due to other causes than that of surface friction. Thus at 250 feet per minute, these other resistances are given by *cd*. This figure shows very clearly how the resistance at low speeds is almost wholly due to surface friction, and this forms at high speeds a large proportion of the total. The wave-making resistance, as we have already seen, is the chief cause of the difference between the curves CC and DD, which difference becomes greater as the speed increases. It is the resistance, other than frictional, to which the law of comparison is intended to apply.

We have in Fig. 106 the curve of resistance, AA, of the *Greyhound* on a base of speed, and in precisely the same way as for the model a curve of frictional resistance was drawn in for the ship, taking the coefficient proper for the state of the surface of the ship and its length. Such a curve is given by BB, Fig. 106. Then it was found that the ordinates between the curves AA and BB, Fig. 106, giving the resistance for the ship other than frictional, were in practical agreement with the ordinates between the curves CC and DD, Fig. 113, giving the resistance of the model other than frictional, allowing for the "*law of comparison*" above stated. That is, at speeds of the ship $\sqrt{16}$, or four times the speeds of the model, the resistance of the ship other than frictional was practically 16^3, or 4096 times the resistance of the model.

These experiments of the *Greyhound* and her model form the first experimental verification of the law of comparison. In 1883 some towing trials were made on a torpedo-boat by Mr. Yarrow, and a model of the boat was tried at the experimental tank belonging to the British Admiralty. In this case also there was virtual agreement between the boat and the model according to the law of comparison. It is now the practice of the British Admiralty and others to have models made and run in a tank. The data obtained are of great value in determining the power and speed of new designs.

For further particulars the student is referred to the sources of information mentioned at the end of the book.

Having the resistance of a ship at any given speed, we can at once determine the E.H.P. at that speed (see p. 297), and then by using a suitable propulsive coefficient, we may determine the I.H.P. at that speed. Thus, if at 10 knots the resistance of a ship is 10,700 lbs., we can obtain the E.H.P. as follows:—

$$\text{Speed in feet per minute} = 10 \times \frac{6080}{60}$$

$$\text{Work done per minute} = 10{,}700 \times \left(10 \times \frac{6080}{60}\right) \text{ foot-lbs.}$$

$$\text{E.H.P.} = \frac{10700 \times \frac{6080}{6}}{33000}$$

$$= 328$$

and if we assume a propulsive coefficient of 45 per cent.—

$$\text{I.H.P.} = \frac{328 \times 100}{45}$$

$$= 729$$

By the use of the law of comparison, we can pass from one ship whose trials have been recorded to another ship of the same form, whose I.H.P. at a certain speed is required. It is found very useful when data as to I.H.P. and speed of existing ships are available. In using the law we make the following assumptions, which are all reasonable ones to make.

(1) The correction for surface friction in passing from one ship to another of different length is unnecessary.

(2) The condition of the surfaces of the two vessels are assumed to be the same.

(3) The efficiency of the machinery, propellers, etc., is assumed the same in both cases, so that we can use I.H.P. instead of E.H.P.

The method of using the law will be best illustrated by the following example:—

A vessel of 3290 tons has an I.H.P. of 2500 on trial at 14 knots. What would be the probable I.H.P. of a vessel of the same form, but of three times the displacement, at the corresponding speed?

$$\text{The ratio of the displacement} = 3$$
$$\therefore \text{ the ratio of the linear dimensions } l = \sqrt[3]{3}$$
$$= 1{\cdot}44$$
$$\therefore \text{ the corresponding speed} = 14 \times \sqrt{1{\cdot}44}$$
$$= 16{\cdot}8 \text{ knots}$$

The resistance of the new ship will be l^3 times that of the original, and accordingly the E.H.P., and therefore the I.H.P., will be that of the original ship multiplied by $l^{\frac{7}{2}} = (1{\cdot}44)^{\frac{7}{2}} = 3{\cdot}6$, and—

$$\text{I.H.P. for new ship} = 2500 \times 3{\cdot}6$$
$$= 9000$$

When ships have been run on the measured mile at progressive speeds, the information obtained is found to be extremely valuable, since we can draw for the ship thus tried a curve of I.H.P. on a base of speed, and thus at intermediate speeds we can determine the I.H.P. necessary. The following example will show how such a curve is found useful in estimating I.H.P. for a new design:—

A vessel of 9000 tons is being designed, and it is desired to obtain a speed of 21 knots. A ship of 7390 tons of similar form has been tried, and a curve of I.H.P. to a base of speed drawn. At speeds of 10, 14, 18, and 20 knots the I.H.P. is 1000, 3000, 7500, 11,000 respectively.

Now, the corresponding speeds of the ships will vary as the square root of the ratio of linear dimension l.

We have—

$$l^3 = \frac{9000}{7390}$$
$$\text{and } l = 1{\cdot}07$$
$$\sqrt{l} = 1{\cdot}035$$

therefore the corresponding speed of the 7390-ton ship is—

$$21 \div 1{\cdot}035 = 20{\cdot}3$$

By drawing in the curve of I.H.P. and continuing it beyond the 20 knots, we find that the I.H.P. corresponding to a speed of 20·3 knots is about 11,700. The I.H.P. for the 9000-ton ship at 21 knots is accordingly—

$$11{,}700 \times l^{\frac{7}{2}} = 11{,}700 \times 1{\cdot}26$$
$$= 14{,}750 \text{ I.H.P. about}$$

Proof of the Law of Comparison.

Take the following symbols:—

- P for force.
- m for mass.
- f for acceleration.
- t for time.
- v for velocity.
- l for length.

Then force P = (mass × acceleration)
$= (m \times f)$
velocity = $(l \div t)$
Acceleration is increase of velocity in unit time, = $(l \div t^2)$
Mass varies as the volume or l^3.
Force, which equals $(m \times f)$, may be written—

$$= \left(l^3 \times \frac{l}{t^2}\right)$$
$$= \left(l^3 \times \frac{l^2}{t^2} \times \frac{1}{l}\right)$$
$$= l^3 \times \frac{v^2}{l}$$

∴ if $\frac{v^2}{l}$ is constant, force will vary as l^3.

Progressive Speed Trials.—It is now the usual practice to run vessels at a series of speeds from a low speed up to the highest speed attainable in order to construct a curve of power, etc., on base of speed. Such a record is of the highest value as data for design purposes, and the information obtained as to slip of propellers will frequently indicate the direction in which improvements may be made. At each speed it is necessary to obtain simultaneously the revolutions, I.H.P.,[1] and speed. The usual practice is to run the ship on a measured-mile course. Fig. 113A shows such a course. Two pairs of posts, AB and

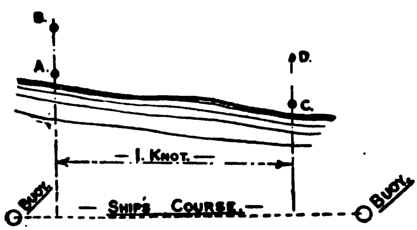


FIG. 113A.

[1] Indicator cards are taken from each piston, showing how the pressure of the steam varies at each point of a revolution. A calculation from these cards enables the I.H.P. to be determined. For turbine engines no corresponding method is available. A method of determining power of turbine machinery has been introduced by Mr. Johnson of Messrs. Denny Bros., Dumbarton, by measuring the torsion of the shaft by electrical instruments. Another method was described by Mr. Gibson of Messrs. Cammell Laird (see I.N.A. for 1907). For a general discussion of the subject see N.E. Coast Inst., 1908.

CD, are placed exactly a knot (6080 feet) apart, and the ship's course is steered at right angles. The time of transit is taken by a chronometer stop-watch. In order to eliminate the effect of tide, several runs are taken both with and against the tide, and the "mean of means" is taken. Thus, suppose a vessel has four runs, and the speeds observed are 15·13, 14·61, 15·66, 14·11 knots respectively. Then the "mean of means" is obtained as follows:—

Speeds.	First means × 2.	Second means × 4.	Mean of means × 8.
15·13			
	29·74		
14·61		60·01	
	30·27		120·05
15·66		60·04	
	29·77		
14·11			

The true mean speed is therefore 120·05 ÷ 8 = 15·006 knots. The ordinary mean of the speeds is 14·88 knots. The same result as the mean of means is got by multiplying by 1, 3, 3, 1 and dividing by 8.

The above is based on the assumption that the speed of tide can be expressed as a quadratic function of the time. That is, if y is speed of tide, then—

$$y = a_0 + a_1 t + a_2 t^2$$

t being the time, a_0, a_1, a_2 being constants.

Thus, when $t = 0$, speed of tide $y_1 = a_0$

$t = t$,, ,, $y_2 = a_0 + a_1 t + a_2 t^2$

$t = 2t$,, ,, $y_3 = a_0 + 2a_1 t + 4a_2 t^2$

$t = 3t$,, ,, $y_4 = a_0 + 3a_1 t + 9a_2 t^2$

If V is the true speed of ship, then, owing to the tide, the speed at intervals of t up and down the mile will be—

$$(V + y_1),\ (V - y_2),\ (V + y_3),\ (V - y_4)$$

or a mean of means of—

$$V + \tfrac{1}{8}(3\overline{y_3 - y_2} - \overline{y_4 - y_1})$$

By substituting in the above values for y_1, etc., this is seen to be equal to V.

If six runs are taken up and down, the mean of means is obtained by multiplying by 1, 5, 10, 10, 5, 1 and dividing by 32, and it is easily shown that if the tide be assumed a *cubic* function of the time, the "mean of means" at equal intervals of time gives the true mean speed.

It is necessary to run measured-mile trials in deep water, or a falling off in speed will be experienced. If the water is not deep, the natural stream-lines are not formed round the ship, and this restriction is a serious cause of resistance. A similar thing is noticed in canals. A conspicuous instance was noticed on the trials of H.M.S. *Edgar.* When tried at Stoke's Bay, with a depth of water of 12 fathoms, 13,260 horse-power was required for $20\frac{1}{2}$ knots. On the deep-sea course between Plymouth and Falmouth, 21 knots was obtained with 12,550 horse-power, or about $\frac{3}{4}$ knot difference for the same power. In consequence of this, trials at high speeds must be carried out on a deep-water course, the finest probably being off Arran, near the Clyde, where the depth of water is very great.

Colonel English's Experimental Method of determining I.H.P. of a New Design by the Use of Models (I.M.E., 1896).—This method of determining the power for a new design is an interesting application of the principles of the present chapter.

Two models are made, one of a known ship, the other of the new design, on such scales that when towed at the *same speed* they shall be at the corresponding speeds proper for each. In the following table the capitals refer to the ships, and the small letters to the models, and the resistance is divided into the frictional and wave-making. It will be remembered that the law of comparison only strictly holds for wave-making resistance.

	Resistance.		Displacement.	Speed.
	Frictional.	Wave-making.		
Actual ship (1) ...	F_1	W_1	D_1	V_1
New design (2) ...	F_2	W_2	D_2	V_2
Model of (1)	f_1	w_1	d_1	v_1
Model of (2)	f_2	w_2	d_2	v_2

By the law of comparison—

$$d_1 = D_1 . \left(\frac{v_1}{V_1}\right)^6 \text{ and } d_2 = D_2 . \left(\frac{v_2}{V_2}\right)^6$$

and if the models are towed at the same speed, $v_1 = v_2$; so that—

$$\frac{d_1}{d_2} = \frac{D_1}{D_2} \cdot \left(\frac{V_2}{V_1}\right)^6$$

This determines the relative scale of models, and—

$$v_1 = v_2 = V_1 \cdot \left(\frac{d_1}{D_1}\right)^{\frac{1}{6}}$$

The total resistance of model (1) $= f_1 + w_1$, and that of model (2) $= f_2 + w_2$. Let $f_2 + w_2 = n(f_1 + w_1)$, say.

The law of comparison indicates that the wave-making resistance varies as the displacement, so that—

$$\frac{w_1}{W_1} = \frac{d_1}{D_1}$$

$$\text{so that } w_2 = n \,.\, W_1 \cdot \frac{d_1}{D_1} + n \,.\, f_1 - f_2$$

We want to get the wave-making resistance of the new design, viz. W_2; we first find w_2 from the above, and we can calculate f_1 and f_2 by the use of appropriate frictional coefficients. To get W_1, we proceed as follows: For the known ship we have data regarding I.H.P. at speed V_1, this can be turned into E.H.P. by the use of a propulsive coefficient, and this E.H.P.

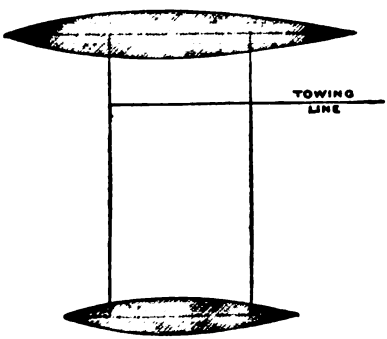


FIG. 113B.

can be turned at once into resistance, which is $(F_1 + W_1)$. The frictional resistance can be calculated by the ordinary rules, and we have left W_1, the wave-making resistance of the known ship. The only part of the above expression we do not know is n. This is obtained by towing the models abreast of one another and adjusting so that they are exactly abreast (Fig. 113B). When this is so, the ratio of the levers determines the ratio n. We thus can determine w_2, and $W_2 = w_2 \cdot \frac{D_2}{d_2}$. F_2 can be calculated, so that $F_2 + W_2$ is determined. This is turned into E.H.P. at the speed V_2, and, using the same propulsive coefficient as before, the I.H.P. is found for the new design at speed V_2. The models were of yellow pine ballasted to desired draught. A small electric motor was used for towing, and when the levers were adjusted so that the models towed abreast, the only measurement necessary was the ratio between the levers.

The method may be made clearer by reference to an example. It is desired to know the I.H.P. to drive a destroyer of 300 tons displacement at a speed of 30 knots, and a known destroyer of 247 tons required 3915 I.H.P. for a speed of 27·85 knots. The model of this vessel was made on a scale of $\frac{1}{20}$, so that the speed corresponding to 27·85 knots was $\frac{27·85}{\sqrt{20}} = 6·23$ knots. The scale of the model of the new ship must be such that 6·23 knots of model corresponds to 30 knots of the ship, giving a scale of $\left(\frac{6·23}{30}\right)^2 = \frac{1}{23·2}$

The wetted surface of known ship was calculated to be 3796 square feet, so that that of model was $3796 \times (\frac{1}{20})^2 = 9·5$. The wetted surface of new ship was 4321 square feet, and of its model $4321 \times \left(\frac{1}{23·2}\right)^2 = 8·02$. Using these values and appropriate values for the coefficient of friction, we have—

F_1 (known ship) $= 0·0094 \times 3796 \times (27·85)^{1·83} = 15{,}720$ lbs.
f_1 (its model) $= 0·01124 \times 9·5 \times (6·23)^{1·95} = 3·15$ lbs.
F_2 (new ship) $= 0·0094 \times 4321 \times (30)^{1·83} = 20{,}500$ lbs.
f_2 (its model) $= 0·01124 \times 8·02 \times (6·23)^{1·95} = 2·66$ lbs.

The propulsive coefficient being assumed as 0·6, we have E.H.P. of known ship $0·6 \times 3915 = 2349$, so that the total resistance of ship was—

$$2349 \times \left(\frac{33{,}000 \times 60}{27 \cdot 85 \times 6080} \right) = 27{,}467 \text{ lbs.}$$

$$\text{therefore } W_1 = 11{,}747 \text{ lbs.}$$

From the towing trial at 6·23 knots, $n = 0 \cdot 811$, so that—

$$w_2 = 0 \cdot 811 \, . \, (\tfrac{1}{20})^2 \, . \, 11{,}747 + 0 \cdot 811 \times 3 \cdot 15 - 2 \cdot 66 = 1 \cdot 08 \text{ lbs.}$$

we therefore have—

$$W_2 = 1 \cdot 08 \times (23 \cdot 2)^3 = 13{,}500 \text{ lbs.}$$

The total resistance of new ship is therefore 34,000 lbs., and assuming the same propulsive coefficient, we have—

$$\text{I.H.P.} = \tfrac{1}{326} \times 34{,}000 \times 30 \times \tfrac{100}{60} = 5220$$

Calculation of E.H.P.—Mr. A. W. Johns gave before the I.N.A., 1907, a table which gives the E.H.P. due to skin friction for a number of speeds and lengths of ship, based on Mr. Froude's constants and on the assumption that the skin friction varies as $V^{1 \cdot 825}$.

If S is the wetted surface in square feet, then E.H.P. $= f . S$, where f has the values given in table, p. 332.

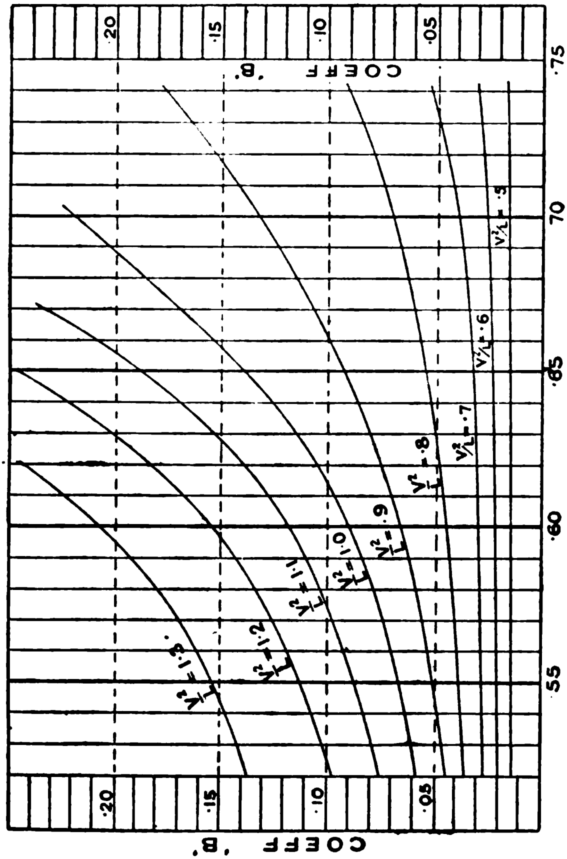
In the same paper he gave a series of curves based on model experiments, reproduced in Fig. 113C, from which, knowing the prismatic coefficient of fineness, the residuary horse-power can be obtained. The curves are drawn for a number of values of $\left(\frac{V}{\sqrt{L}}\right)^2 = \frac{V^2}{L}$ (where L is under-water length) varying from 0·5 to 1·3 on a base of prismatic coefficients varying from 0·52 to 0·74. It is very striking to note how rapidly the residuary horse-power increases, for high values of speed-length ratio, with increase of prismatic coefficient. The prismatic coefficient has been taken with the length P.P., and Mr. Johns states that for merchant ships better results are obtained by increasing the prismatic coefficient by 0·02, this being due to the fact that in such vessels the length P.P. is practically the immersed length of the ship, and not, as in the majority of warships, an appreciably smaller length. In a few ships of exceptionally good form the curves give too great a result, but for ordinary forms of ships the curves give a good approximation to the results obtained from

E.H.P. DUE TO FRICTIONAL RESISTANCE PER SQUARE FOOT OF WETTED SURFACE.

Speed in knots.	Length of ship in feet.								
	100	150	200	300	400	500	600	800	1000
10	0·0188	0·0186	0·0184	0·0183	0·0181	0·0180	0·0179	0·0176	0·0174
11	0·0246	0·0243	0·0241	0·0239	0·0237	0·0235	0·0233	0·0231	0·0228
12	0·0315	0·0312	0·0309	0·0307	0·0304	0·0302	0·0300	0·0296	0·0293
13	0·0397	0·0390	0·0387	0·0384	0·0381	0·0378	0·0375	0·0371	0·0367
14	0·0489	0·0451	0·0478	0·0473	0·0469	0·0466	0·0463	0·0458	0·0453
15	0·0594	0·0585	0·0580	0·0575	0·0570	0·0567	0·0563	0·0557	0·0551
16	0·0713	0·0702	0·0697	0·0690	0·0685	0·0680	0·0675	0·0668	0·0661
17	0·0846	0·0833	0·0827	0·0819	0·0812	0·0807	0·0802	0·0793	0·0784
18	0·0995	0·0979	0·0972	0·0962	0·0955	0·0948	0·0942	0·0931	0·0921
19	0·1159	0·1141	0·1132	0·1121	0·1112	0·1105	0·1098	0·1086	0·1074
20	0·1340	0·1319	0·1308	0·1296	0·1286	0·1277	0·1268	0·1254	0·1241
21	0·1537	0·1514	0·1502	0·1487	0·1476	0·1466	0·1456	0·1440	0·1424
22	0·1753	0·1726	0·1713	0·1696	0·1683	0·1672	0·1661	0·1643	0·1625
23	0·1988	0·1957	0·1942	0·1923	0·1908	0·1895	0·1882	0·1861	0·1841
24	0·2242	0·2207	0·2190	0·2169	0·2152	0·2138	0·2124	0·2101	0·2078
25	0·2516	0·2477	0·2458	0·2434	0·2415	0·2399	0·2383	0·2357	0·2331
26	0·2811	0·2767	0·2746	0·2719	0·2698	0·2680	0·2662	0·2633	0·2604
27	0·3126	0·3078	0·3054	0·3025	0·3001	0·2981	0·2961	0·2929	0·2897
28	0·3466	0·3412	0·3386	0·3353	0·3327	0·3305	0·3283	0·3247	0·3211
29	0·3826	0·3767	0·3738	0·3702	0·3673	0·3649	0·3624	0·3585	0·3545
30	0·4210	0·4145	0·4113	0·4073	0·4041	0·4014	0·3988	0·3944	0·3900
31	0·4624	0·4552	0·4517	0·4473	0·4438	0·4409	0·4379	0·4322	0·4274
32	0·5050	0·4972	0·4934	0·4886	0·4848	0·4816	0·4784	0·4732	0·4680
33	0·5499	0·5414	0·5372	0·5320	0·5278	0·5243	0·5208	0·5151	0·5094
34	0·5995	0·5902	0·5857	0·5800	0·5755	0·5707	0·5679	0·5617	0·5555
35	0·6508	0·6407	0·6358	0·6296	0·6247	0·6206	0·6164	0·6097	0·6030
36	0·7047	0·6938	0·6885	0·6818	0·6765	0·6720	0·6675	0·6603	0·6530
37	0·7611	0·7494	0·7436	0·7364	0·7306	0·7258	0·7209	0·7130	0·7051
38	0·8209	0·8082	0·8020	0·7942	0·7880	0·7828	0·7776	0·7691	0·7606
39	0·8835	0·8698	0·8631	0·8547	0·8480	0·8424	0·8356	0·8276	0·8185
40	0·9490	0·9343	0·9271	0·9181	0·9109	0·9049	0·8989	0·8890	0·8792

NOTE.—The above table has been extended beyond that given in Mr. Johns' paper to include lengths of 1000 feet and speeds up to 40 knots.

model experiments. The curves apply to vessels in which the ratio beam/draught varies from about 2·7 to 2·9. For greater ratios than the latter the curves give results which are smaller



Residuary Horse Power = $B \times W^{\frac{2}{3}}$

Fig. 113c.

than they should be, whilst for smaller ratios than the former the results will be too great.

As an example, take a vessel 500 ft. (P.P.) × 71 ft. × 26 ft. × 14,100 tons, prismatic coefficient 0·582. Under-water length 520 ft.

The approximate wetted surface by Denny's formula—

$$1{\cdot}7\,L\,.\,D + \frac{V}{D} = 1{\cdot}7 \times 500 \times 26 + \frac{14{,}100 \times 35}{26}$$

$$= 41{,}100 \text{ square feet}$$

and by Taylor's formula—

$$15{\cdot}5\,\sqrt{W\,.\,L} = 41{,}200 \text{ square feet}$$

Taking 42,000 and using the coefficients in table, we obtain the following values of E.H.P. due to surface friction, from 16 to 25 knots, viz.: 2860, 3390, 3980, 4620, 5360, 6150, 7020, 7950, 8960, 10,100.

Now going to the curves and erecting an ordinate at 0·582 prismatic coefficient, the values of coefficient at speeds 16·1, 17·65, 19·1, 20·4, 21·6, 22·8, 23·95, 25 knots are measured as 0·015, 0·021, 0·029, 0·042, 0·060, 0·081, 0·104, 0·138, which have to be multiplied by (displacement in

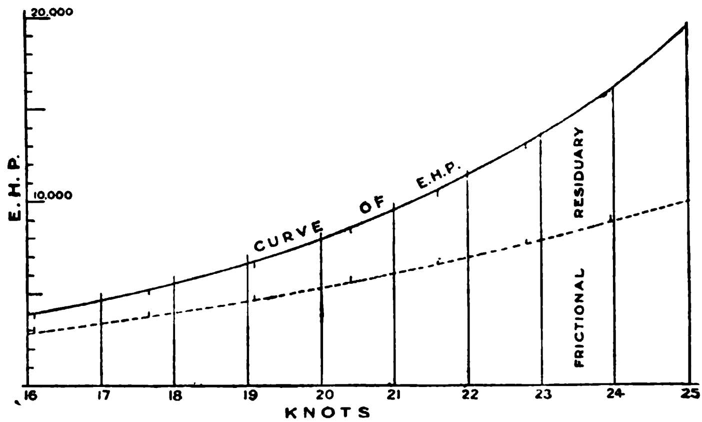


Fig. 113D.

tons)$^{\frac{2}{3}}$, giving us—1040, 1450, 2000, 2900, 4150, 5600, 7200, 9550 residuary horse-power.

The above results, plotted as in Fig. 113D, give us an estimated curve of total E.H.P.

To obtain the Space which must be passed over by a Ship starting from Rest to any Speed short of the Full Speed, supposing the Engines are

exerting the Thrust corresponding to the Maximum Speed.—(*a*) Supposing the resistance is varying as the square of the speed. When a ship is being accelerated through the water there is a certain amount of water accompanying the ship which has to be accelerated as well. This is usually taken (based on the *Greyhound* experiments of Mr. W. Froude) as 20 per cent. of the weight of the ship. The virtual mass to be accelerated is therefore $\frac{6}{5}\cdot\left(\frac{W}{g}\right)$, where g is the acceleration due to gravity (32·2 in foot-second units).

Let R be the resistance of ship at full speed V.
r " " " lower speed v.

Then the force urging the ship is the constant thrust of the propeller = R and the force accelerating the ship is R − r.

Now by the principles of dynamics—

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$\text{or } R - r = \tfrac{6}{5}\cdot\left(\frac{W}{g}\right)\cdot f$$

$$\text{or } f = \tfrac{5}{6}\cdot\frac{R-r}{W}\cdot g.$$

Now

$$f = \text{acceleration} = \frac{dv}{dt}$$

$$= \frac{dv}{ds}\cdot\frac{ds}{dt} = v\cdot\frac{dv}{ds}$$

$$\text{so that } v\cdot\frac{dv}{ds} = \tfrac{5}{6}\cdot\frac{R-r}{W}\cdot g$$

$$\text{or } ds = \tfrac{6}{5}\cdot\frac{W}{g}\cdot\frac{v}{R-r}\cdot dv$$

and on integrating—

$$s = \tfrac{6}{5}\cdot\frac{W}{g}\int_0^{V_1}\frac{v}{R-r}\cdot dv \text{ to speed } V_1 \text{ from rest.}$$

Now on the assumption that resistance varies as the square of the speed—

$$r = R\cdot\frac{v^2}{V^2}$$

$$\text{so that } s = \tfrac{6}{5}\cdot\frac{W}{g\cdot R}\int_0^{V_1}\cdot\frac{v}{\left(1-\frac{v^2}{V^2}\right)}\cdot dv$$

and on integrating—

$$s = \frac{1}{2} \cdot \frac{6}{5} \cdot \frac{W}{g} \cdot \frac{V^2}{R} \cdot \log_e\left(\frac{V^2}{V^2 - V_1^2}\right).$$

and the space from speed V_1 to speed V_2 is—

$$\frac{1}{2} \cdot \frac{6}{5} \cdot \frac{W}{g} \cdot \frac{V^2}{R} \cdot \log_e \cdot \left(\frac{V^2 - V_2^2}{V^2 - V_1^2}\right)$$

(*b*) Without making any assumptions as to the variation of resistance with speed, if we have a curve of I.H.P. on base of speed, we can get a good approximation to the space required to go from one speed to another short of the maximum supposing the full thrust due to the top H.P. is exerted from the start.

Take as an example a vessel of 5600 tons, whose I.H.P. at speeds of 10, 12, 14, 16, 18 and 20 knots are respectively 950, 1640, 2720, 4340, 6660 and 10,060. It is desired to obtain the space required to increase the speed from 10 to 18 knots, supposing the engines are exerting the full thrust corresponding to 20 knots.

Here the virtual weight is $\frac{6}{5} \times 5600 = 6720$ tons, and assuming I.H.P. = 2 E.H.P. all through—

$$(\text{I.H.P.})_v = 2 \times r \times v \times \frac{101 \times 2240}{33{,}000} \quad r \text{ in tons, } v \text{ in knots}$$

$$\text{or } r = \frac{1}{2} \cdot \frac{(\text{I.H.P.})_v}{v} \cdot \frac{1}{6 \cdot 85}$$

$$\therefore R - r = \frac{1}{13 \cdot 7} \cdot \left[\frac{(\text{I.H.P.})_{20}}{20} - \frac{(\text{I.H.P.})_v}{v}\right]$$

Taking v as 10, 12, 14, 16, and 18 knots respectively—

$$R - r \text{ at 10 knots} = \frac{1}{13 \cdot 7}\left(\frac{10{,}060}{20} - \frac{950}{10}\right) = 29 \cdot 8 \text{ tons}$$

$$R - r \text{ at 12 knots} = \frac{1}{13 \cdot 7}\left(\frac{10{,}060}{20} - \frac{1640}{12}\right) = 26 \cdot 8 \text{ tons}$$

$$R - r \text{ at 14 knots} = \frac{1}{13 \cdot 7}\left(\frac{10{,}060}{20} - \frac{2720}{14}\right) = 22 \cdot 5 \text{ tons}$$

$$R - r \text{ at 16 knots} = \frac{1}{13 \cdot 7}\left(\frac{10{,}060}{20} - \frac{4340}{16}\right) = 16 \cdot 9 \text{ tons}$$

$$R - r \text{ at 18 knots} = \frac{1}{13 \cdot 7}\left(\frac{10{,}060}{20} - \frac{6660}{18}\right) = 9 \cdot 7 \text{ tons.}$$

Now $$\frac{ds}{dv} = \tfrac{6}{5} \cdot \frac{W}{g} \cdot \frac{v}{R - r}$$

$$\therefore \left(\frac{ds}{dv}\right)_{10} = \frac{6720}{32 \cdot 2} \cdot \frac{6080}{3600} \cdot \frac{10}{29 \cdot 8} = 118 \cdot 5 \text{ in foot second units}$$

and similarly—

$$\left(\frac{ds}{dv}\right)_{12} = 158, \left(\frac{ds}{dv}\right)_{14} = 220, \left(\frac{ds}{dv}\right)_{16} = 334, \left(\frac{ds}{dv}\right)_{18} = 657.$$

Now $s = \int \frac{ds}{dv} . dv$, so that we can obtain the integration by means of Simpson's first rule having values of $\frac{ds}{dv}$.

We therefore have—

space from 10 to 18 knots

$$= \tfrac{1}{3} \times 3 \cdot 38 . [118 \cdot 5 + 4(158) + 2(220) + 4(334) + 657]$$
$$= 3600 \text{ feet}$$

(3·38 being the equivalent in foot-second units of 2 knots, the interval chosen).

It will have been seen in the above example that special attention is necessary to the units which have been taken as tons, feet and seconds.

The time taken can be obtained in a similar manner by integrating values of $\left(\frac{dt}{dv}\right)$ taken at equal intervals of time.

Example.—The I.H.P. of a vessel of 14,200 tons at 10, 12, 14, 16, 18, and 20 knots are 1750, 3150, 5000, 7600, 10,850 and 15,300. Supposing the vessel is exerting 15,300 I.H.P., how far would the ship travel in going from 10 to 18 knots, and how long would it take.

Ans. 7075 feet. 282 seconds.

Examples to Chapter IX.

1. The *Greyhound* was towed at the rate of 845 feet per minute, and the horizontal strain on the tow-rope, including an estimate of the air resistance of masts and rigging, was 6200 lbs. Find the effective horse-power at that speed.

Ans. 159 E.H.P. nearly.

2. A vessel of 5500 tons displacement is being towed at a speed of 8 knots, and her resistance at that speed is estimated at 18,740 lbs. What horse-power is being transmitted through the tow-rope?

Ans. 460.

3. A steam-yacht has the following particulars given:—

Displacement on trial	176·5 tons
I.H.P. on trial	364
Speed "	13·3 knots

Find the "displacement coefficient of speed."

Ans. 203.

4. A steam-yacht has a displacement of 143·5 tons, and 250 I.H.P is expected on trial. What should the speed in knots be, assuming a displacement coefficient of speed of 196?

Ans. 12·2 knots.

5. The *Warrior* developed 5297 indicated horse-power, with a speed of 14·08 knots on a displacement of 9231 tons. Find the displacement coefficient of speed.

Ans. 233.

6. In a set of progressive speed trials, very different values of the "displacement coefficient" are obtained at different speeds. Explain the reason of this.

7. Suppose we took a torpedo-boat destroyer of 250 tons displacement and 27 knots speed as a model, and designed a vessel of 10,000 tons displacement of similar form. At what speed of this vessel could we compare her resistance with that of the model at 27 knots?

Ans. 50 knots.

8. A ship of 5000 tons displacement has to be driven at 21 knots. A model of this ship displaces 101 lbs. At what speed should it be tried?

Ans. 3 knots.

9. A ship of 5000 tons displacement is driven at a speed of 12 knots. A ship of 6500 tons of similar form is being designed. At what speed of the larger ship can we compare its performance with the 5000-ton ship?

Ans. 12·53 knots.

10. A vessel 300 feet long is driven at a speed of 15 knots. At what speed must a similar vessel 350 feet long be driven in order that their performances may be compared?

Ans. 16·2 knots.

11. A vessel 300 feet long has a displacement on the measured-mile trial of 3330 tons, and steams at 14, 18, and 20 knots with 2400, 6000, and 9000 I.H.P. respectively. What would be the I.H.P. required to drive a vessel of similar type, but of double the displacement, at 20 knots?

Ans. 13,000 I.H.P. about.

12. A vessel of 3100 tons displacement is 270 feet long, 42 feet beam, and 17 feet draught. Her I.H.P. at speeds of 6, 9, 12, and 15 knots are 270, 600, 1350, and 3060 respectively. What will be the dimensions of a similar vessel of 7000 tons displacement, and what I.H.P. will be required to drive this vessel at 18 knots?

Ans. 354 × 55 × 22·3; about 9600 I.H.P.

13. A vessel of 4470 tons displacement is tried on the measured mile at progressive speeds, with the following results:—

Speed.	I.H.P.
8·47	485
10·43	881
12·23	1573
12·93	2117

A vessel of similar form of 5600 tons displacement is being designed. Estimate the I.H.P. required for a speed of 13 knots.

Ans. 2300 I.H.P.

14. Verify the figures given for the coefficients of speed of H.M.S. *Iris* on p. 333.

15. A vessel of 7000 tons requires 10,000 I.H.P. to drive her 20 knots, and the I.H.P. at that speed is varying as the fourth power of the speed. Find approximately the I.H.P. necessary to drive a similar vessel of 10,000 tons at a speed of 21½ knots.

Ans. 16,000 I.H.P.

16. Dr. Kirk has given the following rule for finding the indicated horse-power of a vessel :—

In ordinary cases, where steamers are formed to suit the speed, the I.H.P. per 100 square feet of wetted surface may be found by assuming that, at a speed of 10 knots, 5 I.H.P. is required, and that the I.H.P. varies as the cube of the speed.

Show that this can be obtained on the following assumptions :—

(i.) The resistance can be expressed by the formula $R = f'.\ S.\left(\frac{V}{6}\right)^2$

where $f' = 0{\cdot}265$.

(ii.) The propulsive coefficient assumed to be about 45 per cent.

17. Prove that the Admiralty displacement coefficient of speed is the same for two similar vessels at *corresponding speeds*, supposing that the efficiency of propulsion is the same. What other assumption is made?

18. Draw a curve on base of speed of the Admiralty displacement coefficient of speed for H.M.S. *Drake*, 500 feet long and 14,000 tons, whose curve of I.H.P., based on the trial results, give the following figures :—

Speed in knots	10	12	14	16	18	20	22	24
I.H.P.	1950	3200	4800	7000	10,000	14,800	21,900	31,000
$\frac{W^{\frac{2}{3}} \times V^3}{\text{I.H.P.}}$	299	315	334	341	340	315	284	260

Find also the values of the index n in the formula

$$\text{I.H.P.} = k.V^n \qquad \left(n = \frac{\log \text{I.H.P.}_1 - \log \text{I.H.P.}_2}{\log V_1 - \log V_2}\right)$$

Ans. (11 *k*) 2·71 (13 *k*) 2·63 (15 *k*) 2·83 (17 *k*) 3·03 (19 *k*) 3·73
(21 *k*) 4·11 (23 *k*) 4·0.

19. Using the $W^{\frac{2}{3}}$ coefficient of speed, determine the I.H.P. of a vessel similar to the *Drake*, 555 feet long, at 25 knots.

Ans. 42,600 I.H.P.

(For further examples, see 28, 30, 31, and 32 in Appendix A.)

20. A model 20 feet long, wetted surface 77 square feet, has a resistance of 26·2 lbs. in fresh water at a speed of 5 knots. Calculate the effective horse-power in sea water of a ship having 16 times the linear dimensions

of the model and 20 knots speed (f for model = 0·0104 and for ship 0·0089, speeds in knots).

(*Durham B.Sc.* 1910.)

The frictional resistance of the model is—

$$0·0104 \times 77 \times 5^{1·83} = 15·2 \text{ lbs.}$$

The frictional resistance of the ship at 20 knots is—

$$0·0089 \times 77 \times 16^2 \times 20^{1·83} = 42,100 \text{ lbs.}$$

The residuary resistance of the model at 5 knots is 26·2 − 15·2 = 11 lbs., and that of the ship at the corresponding speed of $5\sqrt{16}$, or 20 knots, is $11 \times 16^3 \times 1·025$ by the law of comparison, allowing for the density of salt water, or 46,200 lbs. nearly.

The total resistance, therefore, of the ship at 20 knots is 42,100 + 46,200 = 88,300 lbs.

The E.H.P. is therefore—

$$88,300 \times 20 \times \frac{6080}{33,000 \times 60} = 5417.$$

21. A vessel on successive runs on the measured mile obtains the following speeds, viz.:—

27·592, 28·841, 27·965, 28·943, 27·777, 28·426

knots respectively.

Obtain :—(i) Ordinary average speed.
(ii) Mean of means of 6 runs.
(iii) „ „ of first 4 runs.
(iv) „ „ of second 4 runs.
(v) „ „ of last 4 runs.

Ans. (i) 28·257; (ii) 28·38; (iii) 28·37; (iv) 28·418; (v) 28·32.

22. In question 18 draw a curve of the $W^{\frac{2}{3}}$ Admiralty coefficient of speed on a base of $V \div \sqrt{L}$.

23. A model, 20 feet long, wetted surface 80 square feet, has a resistance of 28 lbs. in fresh water at a speed of 5 knots.

What is the S.H.P. you estimate is required to propel a ship, 320 feet long, of similar form to the model, at the "corresponding speed"? Coefficients of friction for model and ship may be taken as ·01 and ·009 respectively; speeds in knots.

Ans. About 12,000 S.H.P., assuming 50 per cent. propulsive coefficient.

24. Given the following particulars of a destroyer :—

Length 212 feet, beam 19·75 feet, draught 6·5 feet mean. Wetted surface 3970 sq. feet. Displacement 300 tons. Resistance at 15·8 knots speed = 3·5 tons,

deduce the dimensions, speed and E.H.P. of a cruiser of similar form 765 feet long at the corresponding speed.

Ans. 765′ × 71·2′ × 23·5′ × 14,100 tons. About 30,800 E.H.P. at 30 knots.

(This is the same example as worked out in Fyfe's work on steamship coefficients, etc., where the result obtained is 30,630.)

CHAPTER X.

SCREW PROPULSION.

THIS chapter has been prepared in order to provide an introduction to the subject. The subject is too large to be dealt with adequately in the space at disposal, and for fuller information reference must be made to systematic treatises on the subject. Mr. Fyfe's revised "Steamship Coefficients, Speeds and Powers" (1920) is very exhaustive.

Wake.—We have dealt in the previous chapter with the various resistances which oppose a vessel's progress through the water. These are mainly—frictional and wave making. The friction of the water on the surface of the vessel is the cause of a surrounding zone of water following in the direction of motion and the forward velocity of this zone increases as we go aft. The consequence is that at the stern there is a belt of water having a forward velocity. This velocity is variable in amount and in direction, but may be assumed, in the case of each propeller, to have the same effect as a body of water having a certain uniform velocity forwards. This body of water is termed the *frictional wake.* The speed of the wake is conveniently expressed as a fraction of the speed of the ship, say $x . V$. The wake will have a higher velocity nearer the middle line of the ship than at points farther away. The importance of this wake is due to the fact that the propeller has thus to work in water which has this forward velocity, and therefore the speed of the propeller *through the water* is not the speed of the ship V, but $(1 - x)V = V_1$, say.[1] The propeller derives

[1] In Froude's notation the speed of wake is expressed as a fraction of V_1, say $w . V_1$, so that speed of propeller through the water is $V - wV_1$, or $\frac{V}{V_1} = 1 + w$, w being called the "wake percentage." It follows that $x = \frac{w}{1 + w}$.

increased thrust from this cause, and a single screw will benefit more than twin screws, owing to the fact above mentioned as to the greater velocity of the water nearer the middle line. The frictional wake is caused by the motion of the ship, and the increased thrust may therefore be regarded as the return of a small portion of the energy spent by the ship in overcoming the friction of the water on the surface.

The simple frictional wake above described is complicated by the existence of other factors, viz.:

(*a*) *The stream line wake.* (We have seen that the stream lines closing round a ship tend to a diminution of velocity and an increase of pressure.)

(*b*) *The presence of a wave at the stern.* (If the crest of a wave is over the propeller the particles of water in their orbital motion are moving forwards. If there is a trough the particles are moving backwards.)

The information as to the value of the speed of the wake is scanty, and in systematic propeller design it is necessary to assume some value. Mr. R. E. Froude assumed 10 per cent. of the velocity of the ship as a standard value for the wake, *i.e.* $x = 0{\cdot}1$. The following formulæ have been obtained as the results of Mr. Luke's investigations (I.N.A. 1910):—

$$\text{Twin screws } x = -0{\cdot}2 + 0{\cdot}55 \text{ (block coeff.)} = \frac{w}{1 + w}$$

$$\text{Single screws } x = -0{\cdot}05 + 0{\cdot}5 \text{ (block coeff.)} = \frac{w}{1 + w}$$

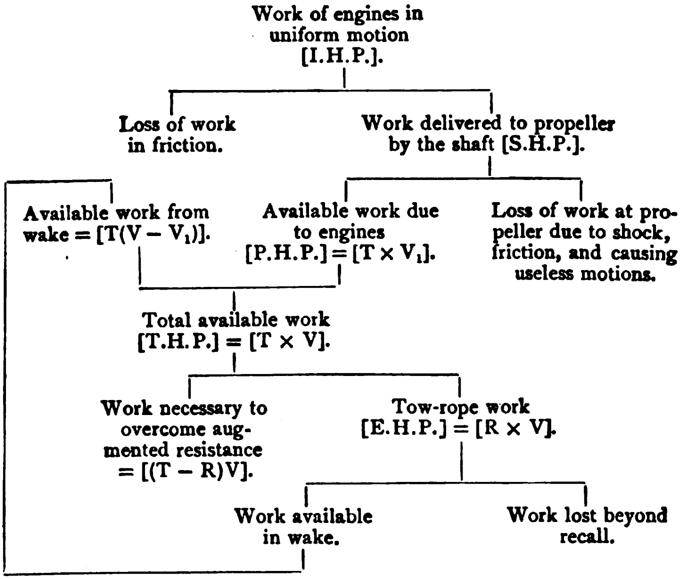
The ratio $V_1 \div V$ represents what may be termed the wake gain factor, and this is $\dfrac{V(1 - x)}{V} = 1 - x$.

Augmentation of Resistance.—Anything which interferes with the natural closing in of the stream lines at the stern of a ship will cause an increase of resistance. The presence of the propeller at the stern is such an interference, and gives rise to an augment of resistance. This will be greater in a single screw than in a twin screw ship, since the propellers in the latter case are further away from the middle line of the ship. It is thus seen that, although a single screw ship stands to gain more from the frictional wake than a twin screw ship, yet it loses more from the augment of resistance.

Thrust Deduction.—Instead of regarding this loss as an augment of resistance, it is preferable to regard it as a loss in the thrust of the propeller. If T be the thrust required to overcome the resistance of the ship plus the augment, and R the thrust required to overcome the resistance only, then $T - R$ is termed the *thrust deduction*, and $T - R = t.T$, so that $R = T(1 - t)$, and $(1 - t)$ is called the thrust deduction factor. The values given above for x may also be used for t.

Hull Efficiency.—The useful work done by the ship is the product of the resistance and the speed, or $R \times V$. The work done by the propeller is the product of the thrust and the speed the propeller passes through the water, or $T \times V_1$. The ratio $(R \times V) \div (T \times V_1)$ is termed the *hull efficiency*, and may be written from the above $\frac{1 - t}{1 - x}$, or $(1 - t)(1 + w)$. The usual value assumed for this is unity, the gain due to the wake being balanced by the increase of thrust due to the augment of resistance. The hull efficiency elements are known to be very sensitive to the *prismatic coefficient of the after body*, and experiments have been made to investigate this subject. Mr. Semple, in his paper "Some Experiments on Full Cargo Ship Models," I.N.A., 1919, gives some interesting results, including also some obtained in U.S.A. by Constructor McEntee.

The following table, taken from Prof. Dunkerley's book on "Resistance and Propulsion," expresses admirably the way the work available in the engines is expended, and what portion of the work is lost beyond recall. The notation is that employed in the present notes.



Horse-Power.—We have seen that the horse-power necessary to overcome the tow-rope resistance of a ship at a given speed is the *Effective Horse-Power*, E.H.P. Leaving out the constants, E.H.P. = R × V. *The thrust horse-power*, *i.e.* T.H.P. = $T \cdot V = \frac{\text{E.H.P.}}{1 - t}$. The *propeller horse-power*, P.H.P., makes allowance for the gain due to the wake $= T \times V_1 = T \cdot V(1 - x)$, so that

$$\text{P.H.P.} = \text{T.H.P.}(1 - x) \quad \text{and} \quad \text{P.H.P.} = \text{E.H.P.}\frac{(1 - x)}{(1 - t)},$$

i.e. E.H.P. = P.H.P. × hull efficiency.

There are certain losses in the propeller due to the frictional and edgewise resistance of the blades and to the rotary motion imparted to the water. The ratio between the P.H.P. and the shaft horse-power, S.H.P., is the measure of the efficiency of the propeller, or P.H.P. ÷ S.H.P. = e. In reciprocating engines the power actually exerted in the cylinders, or I.H.P., is greater than the S.H.P., the relation between the two being the measure of the efficiency of the machinery, or S.H.P. $\div$ I.H.P. = e_m.

The ratio E.H.P. $\div$ I.H.P. is the propulsive coefficient, and tracing through the various stages,

$$\frac{\text{E.H.P.}}{\text{I.H.P.}} = \frac{\text{E.H.P.}}{\text{T.H.P.}} \times \frac{\text{T.H.P.}}{\text{P.H.P.}} \times \frac{\text{P.H.P.}}{\text{S.H.P.}} \times \frac{\text{S.H.P.}}{\text{I.H.P.}}$$
$$= \frac{1 - t}{1} \times \frac{1}{1 - x} \times e \times e_m.$$

Taking a case in which the engine efficiency $e_m = 0{\cdot}85$, propeller efficiency $e = 0{\cdot}65$, x and t each $0{\cdot}15$, the propulsive coefficient is $\frac{0{\cdot}85}{1} \times \frac{1}{0{\cdot}85} \times 0{\cdot}65 \times 0{\cdot}85 = 55{\cdot}25$ per cent.

With turbine machinery it is usual to assume that the propulsive coefficient is the ratio between the E.H.P. and the horse-power being transmitted through the shaft inside the ship. Owing to the high revolutions at which the earlier turbines worked, the propellers connected directly to them had a low efficiency, and it was found (as *e.g.* in the *Lusitania*) that the propulsive efficiency thus defined is about 50 per cent. The use of the geared turbine enables greater all-round efficiency to be obtained, as the turbine can run fast and the propeller run slow.

Cavitation.—The force which pushes the ship along is the reaction from the projection in a sternward direction of the water by the propeller. The momentum of this water, per unit of time, is the measure of the thrust which is transmitted to the ship through the thrust block. The water will not follow up at the back of the blades of the propeller if the thrust is too great and if the velocity of the blades is sufficiently high. This causes a loss of thrust-producing power, and is termed *cavitation*. Mr. Speakman (Scottish Inst. E. and S., 1905), from an analysis of numerous trials, considers that to avoid cavitation the limit of pressure per square inch of *projected surface* should be about 1 lb. for every 1000 feet of circumferential velocity of blade tips. Mr. Sidney Barnaby assigns 11¼ lbs. per square inch as the maximum average thrust per square inch of projected area. These are for an immersion of tip of 12 inches; for each additional foot of immersion $\frac{3}{8}$ lb. per square inch may be added. This figure of $11\frac{1}{4}$ lbs., however, may be exceeded for propellers with turbine machinery, owing to the uniform turning moment.[1]

The thrust of the screw is obtained as follows:—T.H.P. $= \frac{\text{E.H.P.}}{(1 - t)}$; t in the absence of definite information may be taken as 0·1. T.H.P $= \frac{1}{326}$ T × V (V in knots, T in pounds), or thrust $= 326\frac{\text{T.H.P.}}{\text{V}} = 181 \times \frac{\text{I.H.P.}}{\text{V}}$, taking a propulsive coefficient of 0·5. The I.H.P. is that for the screw in question. Taking 11·25 lbs. per square inch of projected area A_p, we have—

$$\left.\begin{array}{l}\text{Projected area of blades}\\ \quad\text{in square feet}\end{array}\right\} = A_p = \frac{181}{11{\cdot}25 \times 144} \times \frac{\text{I.H.P.}}{\text{V}}$$

$$= 0{\cdot}11 \times \frac{\text{I.H.P.}}{\text{V}}$$

The relation between the developed blade area and projected blade area for the Admiralty pattern blade[2] is given by Mr. Barnaby as follows:—

$$\text{Developed area} = \text{projected area}\sqrt{1 + 0{\cdot}425(\text{pitch ratio})^2}.$$

The projected blade area is often expressed as a fraction of the disc area, and Mr. Speakman gives the following as usual values of this ratio:—

Reciprocating machinery (naval practice)—

Large ships . . .	0·33
Destroyers . . .	0·31 – 0·4
Turbines (direct-acting) . .	0·4 – 0·56

The reason for the large value in turbine vessels is the excessive speed of rotation, which causes cavitation unless a large area is given. The diameter is also brought as small as possible in order to avoid excessive velocity of the blade tips, which has, however, been as high as 14,850 feet per minute.

[1] Mr. Baker in his book takes 13 lbs. per square inch to obtain the minimum (developed) area. In Admiralty practice ·75 tons per square foot is taken for ordinary vessels and ·9 tons in high-speed vessels as destroyers.

[2] The Admiralty pattern blade when developed is an ellipse whose major axis is the radius of the propeller and whose minor axis is $\frac{2}{5}$ the major axis. Propellers for turbines have greater width ratio than this.

Pitch.—A screw propeller usually has the driving face (*i.e.* the after side) in the form of a true screw, and the pitch of the screw is defined as the distance this face would advance in one revolution if working in a solid nut. If a variable pitch is used, then there is an equivalent pitch for the whole surface which may be found. The *speed of screw* is the distance it would advance in one minute if working in a solid nut. If N is the revolutions per minute and P the pitch in feet, then speed of screw is P × N feet per minute. The ratio of pitch to diameter (P ÷ D) is the pitch ratio p.

Slip.—The advance of a screw through the water when propelling a ship is not the speed of the ship V, because of the presence of the wake. This speed is $V_1 = (1 - x)V$, where xV is the speed of wake. The difference between the speed of the screw as defined above and its speed forward relative to the water in which it is working is termed the slip, or—

$$\text{slip} = \text{N.P.} - \frac{6080}{60} . V_1 \quad (V_1 \text{ in knots})$$

$$\text{The slip ratio, } s = \frac{\text{N.P.} - \frac{6080}{60} . V_1}{\text{N.P.}}$$

This is the true slip, but as we do not generally know the value of V_1, the *apparent slip* is usually dealt with, being the difference between the speed of screw and the speed of ship.

$$\text{Apparent slip ratio } s_1 = \frac{\text{N.P.} - \frac{6080}{60} . V}{\text{N.P.}} \quad (V \text{ in knots})$$

V_1 being less than V, it follows that the real slip is greater than the apparent slip. Cases are on record in which a *negative* apparent slip has been obtained, which means that the sternward speed of the water from the screw is less than the forward speed of the wake. It is, of course, impossible to have the true slip a negative quantity, as this would involve a thrust being exerted without the projection of water in a sternward direction.

It follows from the above that the true slip and the apparent slip are connected by the following :—

$$s = x + s_1(1 - x).$$

Thus, an apparent slip ratio of 20 per cent. with a wake of 10 per cent. means a true slip of 28 per cent.

We now proceed to deal with the subject of screw propellers in somewhat greater detail.

The Screw Propeller.—The literature devoted to the subject of the screw propeller is enormous, and the subject is a very elusive one, as there are so many variations which complicate the problem. Apart from the "wake" and "thrust deduction" already referred to, the diameter, pitch, revolutions, number and area of the blades, all enter into the problem of the most suitable propeller for a given ship. In the early development of the turbine high revolutions were essential as the propeller was on the same shaft as the turbine. This necessitated a low pitch ratio involving low efficiencies. The appearance of the phenomenon called cavitation forced attention to the influence of blade area on propeller performance.

For high efficiency in the turbine a high rate of revolution is necessary, whereas the converse is true for the propeller. Consequently with the propeller on the same shaft as the turbine a compromise had to be effected and a rate of revolution accepted lower than the most efficient for the turbine and higher than the most efficient for the propeller, in order that the highest overall efficiency might be obtained. With the advent, however, of the "geared" turbine a reversion to the former practice of low revolutions became possible. The propeller could be run at lower revolutions with high efficiency, and the small loss due to the gearing is more than compensated for by the improved performance of the propeller, and the more economical use of the steam in the turbine. Thus, frequently in a new design it is necessary at an early stage to obtain the revolutions and diameter for the propeller giving the best efficiency, and then to see how near the engineer can work to these revolutions, and whether the propeller can be accommodated at the stern of the ship.

Some compromise of course may have to be accepted, but working on these lines, extraordinarily efficient overall results are possible of attainment.

In the best practice propeller dimensions are settled by the naval architect, the marine engineer of course being brought into consultation. The engineer is responsible for the power developed, and the naval architect for speed. The latter must design the form of the ship which determines the resistance and consequently the power required, and it should be his function also to determine the proportions of the propeller which is the instrument through which the power is applied to propel the vessel.

As seen above, the minimum area of blades to avoid cavitation is a matter of great importance, and the effect of variation of area on efficiency has been made the subject of exhaustive experimental investigation by Mr. R. E. Froude (I.N.A., 1908). Similar work has been done by Admiral Taylor in U.S.A. In what follows the results given in Mr. Froude's paper have been used. Three- and four-bladed "elliptical" propellers only have been included. (Students are strongly advised to read Admiral Taylor on this subject. His methods offer some advantages in regard to rapidity of computation.)

Model Experiments on Screw Propellers.—These experiments are of two kinds, viz. :—

(i) *In open water* (*i.e.* the propeller advances behind what Mr. W. Froude called a "phantom" ship). In this way the effect of variation of pitch, area and revolutions can be systematically obtained, and the results plotted and faired and made available for determining the dimensions of a propeller advancing at any speed and slip.

(ii) *Behind the model of the ship* with which the propeller will be associated, in the correct position relative to the hull, but not connected to the model. These experiments are necessary in order to determine the "wake," "thrust deduction," and "hull efficiency," in order that these factors may be assigned when using the "open-water" experimental data.

In regard to (i), if a series of experiments be made on a propeller at a constant linear speed of advance (V_1 say) with varying revolutions, the thrust and turning moment can be measured and plotted on a base of revolutions as Fig. 113E. Now "efficiency" is the ratio between the work due to the thrust and the work due to the turning moment or work delivered divided by work supplied,

$$i.e. \text{ efficiency} = \frac{\text{thrust} \times \text{travel per revolution}}{\text{turning moment} \times 2\pi}$$

and the efficiency can be obtained from the curves of thrust and turning moment and plotted as indicated in Fig. 113E.

If we have such a diagram for speed of advance V_1, suppose at revolutions R_1 the thrust is T_1 and the turning

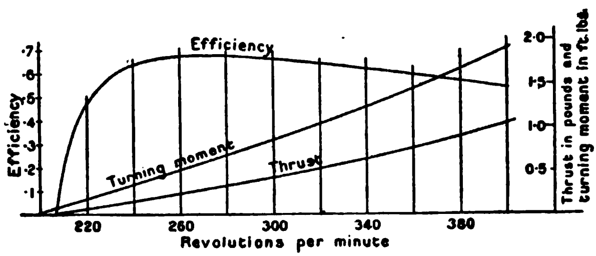


FIG. 113E.

moment M_1. Let the speed of advance be changed to V_2 and the revolutions changed to R_2 such that $R_2 = \frac{V_2}{V_1} \cdot R_1$, then the travel per revolution remains the same and also the slip ratio. The stream line motions will all be unchanged in arrangement, the speeds at all points in the system being simply changed in the ratio of the lineal speed of advance. The direction and relative proportions of all the forces acting in all parts of the propeller whether due to friction or pressure will remain unchanged and be directly proportional to the *square of the lineal speed of advance.* The totals therefore of the resultant thrust and turning moment will be changed in the same ratio and the efficiency will remain the same. Thus a constant linear speed may be used for any propeller and the variations of slip ratio obtained by varying the revolutions.

The character of the efficiency curve may be seen by reference to Fig. 113E. It rises sharply and uniformly at first as the revolutions (and therefore slip) increase and becomes sensibly level in the neighbourhood where the maximum efficiency occurs, after which the descent is gradual. The form of the efficiency curve shows that the efficiency is near the maximum over a large range of slip (see also Fig. 113K).

In going from a model propeller to a larger propeller we have the law that the thrusts and turning forces for a given lineal speed are proportional to the *square of the dimensions*.

In regard to (ii) Fig. 113F is a typical diagram showing the result of a single set of experiments to ascertain the "augment of resistance" or "thrust deduction," and the "wake" for a

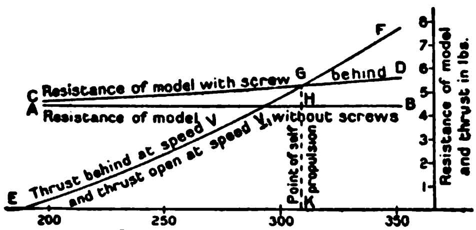


FIG. 113F.

model run at speed V say. The set includes three kinds of experiments, viz.:

(i) On naked model without screws.
(ii) On naked model with screws behind.
(iii) On screws in open water.

At a given speed V the resistance of the model is given by the line AB. The resistance of the model at this speed with the screws working behind will vary slightly with the revolutions, and is represented by the line CD. The difference between AB and CD is the "augment of resistance." The curve EF is now obtained, which is the thrust of the propellers when working behind the model. Now, when the screws are working in open water there will be a speed of advance V_1 at which the thrust curve will follow EF, and then $V - V_1$ will be the speed of the wake, and

$$\frac{V - V_1}{V} = x, \text{ or } \frac{V - V_1}{V_1} = w$$

according as the wake is expressed as a function of the speed of model or as a function of the speed of propellers through the wake water.

The point G where the thrust curve cuts the augmented resistance curve CD gives the point of self-propulsion, and GH is the augment of resistance or the thrust deduction, T − R, and this is expressed as a function of the thrust, so that

$$\frac{GH}{GK} = \frac{T - R}{T} = t, \text{ or } R = T(1 - t).$$

As seen above $\frac{R.V}{T.V_1}$ is the "hull efficiency," and this equals $\frac{1 - t}{1 - x}$, or, as in Froude's notation, $(1 - t)(1 + w)$.

(Mr. Froude's I.N.A. 1883 paper should be consulted if possible in regard to the above. A *précis* of that paper was given by Mr. Luke, I.N.A. 1910.)

The diagram in Fig. 113G was given by Mr. Luke, I.N.A.

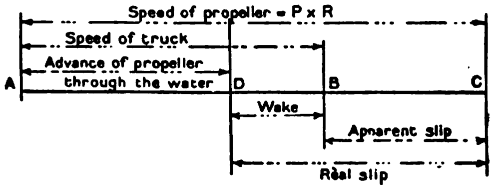


FIG. 113G.

1910, and illustrates most clearly the general principles of the relation between the propeller and the ship.

AB is the speed of the truck carrying the propeller, the propeller being made to revolve at such a rate of revolution that if working in a solid nut (*i.e.* without slip) it would advance at a speed AC. Then BC is the slip, and $\frac{BC}{AC}$ is the slip ratio.

When the propeller is behind the ship the conditions are complicated, because the ship is followed by a belt of water of complex motion, but this can be assumed uniform over the propeller disc as of average speed DB. Then the propeller, being carried along at the former speed over the ground and being revolved as before, would behave as would an "open" propeller with a speed of advance AD and a slip DC. The wake fraction $w = \frac{DB}{AD}$ (Froude's notation), apparent slip is BC, and apparent slip ratio $\frac{BC}{AC}$. Real slip is DC and real slip ratio is $\frac{DC}{AC}$.

The thrust is appropriate to a speed of advance AD and slip DC. The thrust is overcome at a speed AB, and apparent work is Thrust × AB (and not as with an open screw Thrust × AD), and the work "behind" is in excess of that in the "open" by the ratio $\frac{AB}{AD} = 1 + w$, the excess over unity being the wake gain.

Mr. R. E. Froude's Experiments.—Experiments on model propellers are carried out in "open" water, and Mr. Froude's latest experiments, which brought in the effect of variations in area were on propellers 0·8 foot in diameter (1908, I.N.A.).

For any given pitch ratio ($P \div D = p$) it has been found that thrust of propeller varies as—

(i) The square of the speed of advance V_1;
(ii) The square of diameter;
(iii) The disc area ratio (*i.e.* the area of blades divided by the disc area)

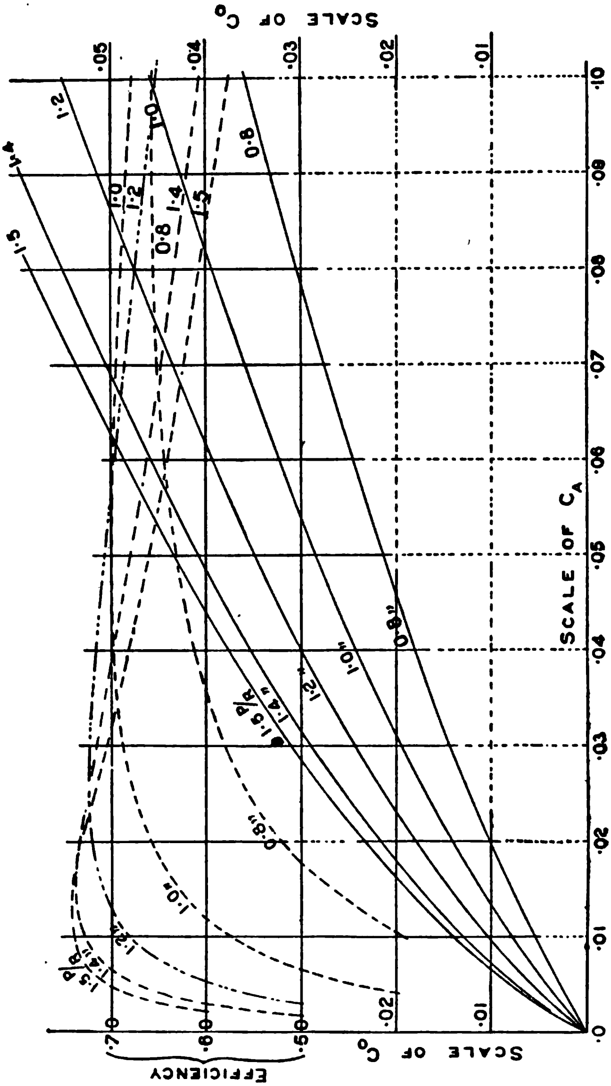
when working at the same slip.

As the propeller horse-power H equals the propeller thrust × V_1, where V_1 is speed of propeller through the water, we have—

$$\frac{H}{D^2 \times V_1^3} = \text{constant}$$

depending on the design of screw and the slip.

Mr. Froude obtained the following formula, bringing in


SCALE OF C_A
SCALE OF C_O
SCALE OF C_O
EFFICIENCY
0
·01
·02
·03
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·05
·06
·07
·08
·09
·10
·01
·02
·03
·04
·05
·5
·6
·7
P/R
1·5 P/R
1·4"
1·2"
1·0"
0·8"
0·8
1·0
1·2
1·4
1·5

FIG. 113H.

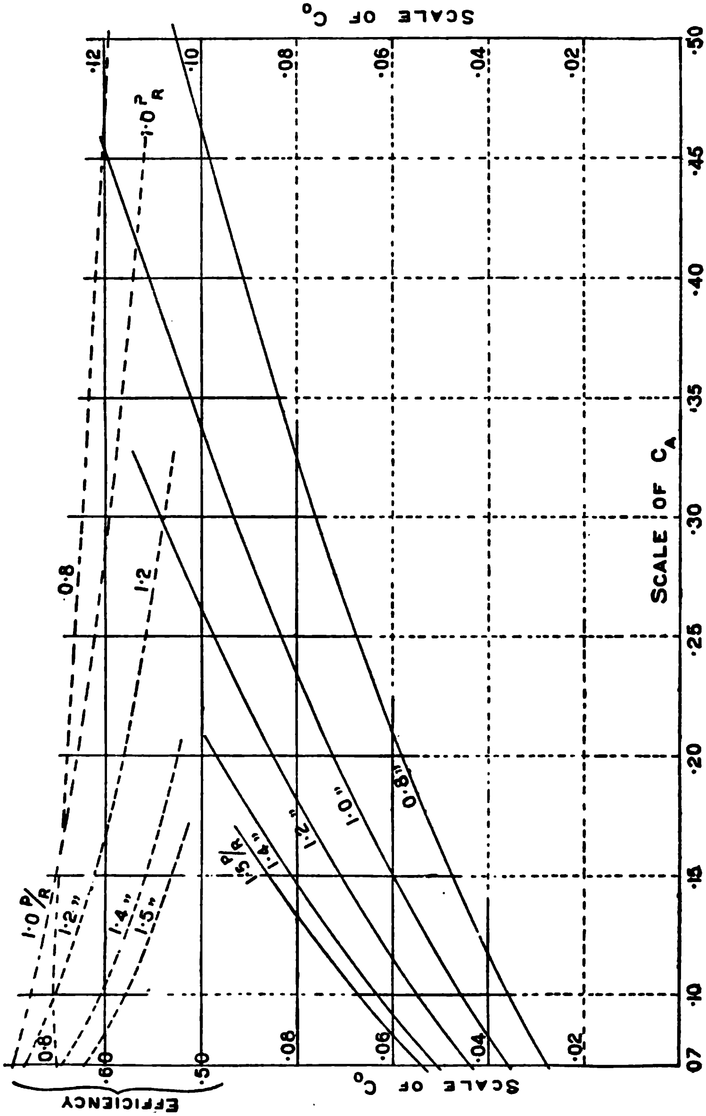

SCALE OF C_0
·12
·10
·08
·06
·04
·02
SCALE OF C_A
·50
·45
·40
·35
·30
·25
·20
·15
·10
·07
1·0 P/R
0·8
1·2
1·0 P/R
1·2 "
1·4 "
1·5 "
1·5 P/R
1·4 "
1·2 "
1·0 "
0·8 "
·08
·06
·04
·02
0·8
·60
·50
EFFICIENCY
SCALE OF C_0

FIG. 113I.

the pitch ratio, p, the slip ratio, s, and a factor "B" for varying disc area ratio—

$$\frac{H}{B \, . \, D^2 \, . \, V_1^3} = \cdot 003216 \, . \, \frac{s(1 - \cdot 08s)}{(1 - s)^2} \, . \, \frac{p + 21}{p}$$

from which a coefficient called C_0 was obtained, for which values have been calculated from the experimental results for different values of p and s, viz.:

$$C_0 = \frac{H}{B \, . \, D^2 \, . \, V_1^3}$$

Again, from the definition of slip-ratio we have—

$$D = \frac{\overline{1 - s} \, V_1}{1 \cdot 013 R \, . \, p}$$

where R is revolutions per minute in hundreds; and substituting for D in equation (1), we have—

$$C_0 \, . \left(\frac{1 - s}{1 \cdot 013}\right)^2 = \frac{H \, . \, R^2}{B \, . \, V_1^5}$$

or say

$$C_A = \frac{H \, . \, R^2}{B \, . \, V_1^5}$$

Curves giving the values of C_0 on base of C_A for varying values of pitch ratio are giving in Figs. 113H and 113I, and the values of efficiency for any value of C_A can also be measured. (These latter are correct for disc area ratio of 0·45, and for three-bladed elliptical propellers. Certain corrections have to be made for other values of disc area ratio and for four-bladed propellers, see later.)

It will be seen that C_A brings in the usually known factors in design, viz. power, revolutions, and speed, and obtaining the corresponding value of C_0 from the curves, the value of the unknown D can be determined, viz.—

$$D = \left(\frac{C_A}{C_0}\right)^{\frac{1}{2}} \, . \, \frac{V_1}{R}$$

The method of the use of these coefficients in design work will perhaps be best understood by taking a concrete example, but before doing so attention is drawn to the curves of efficiency given in Fig. 113K, which are very instructive. They are drawn for pitch ratio values varying from 0·8 to 1·4 on a base of (real) slip ratio, and are all for a standard disc area ratio of 0·45, and for elliptical three-bladed screws.

The curves are all of a similar character, and the efficiency

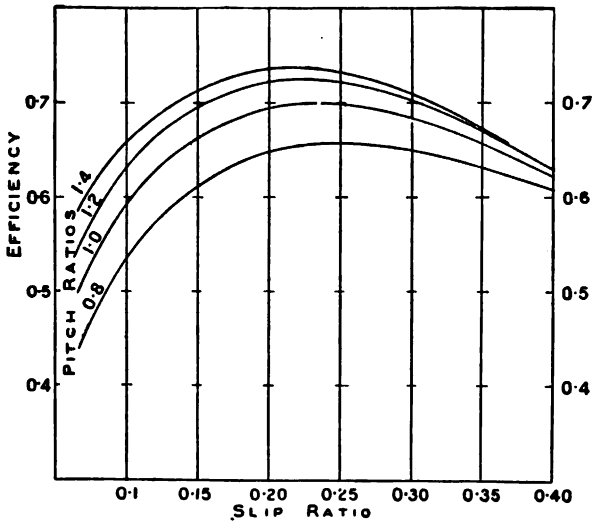


FIG. 113K.

(i) Falls off more slowly from the maximum value for higher slips than for lower slips;
(ii) Falls off as the pitch ratio is reduced; and
(iii) Has maximum values at slip ratio varying from about 22 per cent. to 25 per cent.

There is seen to be a fairly wide range of slip for any given screw where the efficiency is not far from the maximum, and the pitch of a screw should, if possible, be selected so that the (real) slip ratio is near this maximum value, and above rather than below the maximum. It will be seen, therefore, that propeller dimensions may be selected over a fairly wide range without sensible variation of efficiency.

(For four-bladed elliptical propellers a uniform deduction of efficiency of 0·0125 must be made. A correction is also necessary for values of disc area ratio other than 0·45 as given in the table for "B" values below. These corrections, however, do not alter the general characteristics of the efficiency curves.)

The following table gives the values of the "B" factor referred to above for elliptical propellers for varying values of disc area ratio, and the table also gives the corrections in efficiency necessary when other values of disc area ratio than 0·45 are used. There is also the uniform deduction for four-bladed elliptical propellers.

"B" VALUES.

Disc area ratios		0·30	0·40	0·50	0·60	0·70	0·80
"B" for 3 blades		·c978	·1050	·1085	·1112	·1135	·1157
"B" for 4 blades		·1040	·1159	·1227	·1268	·1294	·1318
Efficiency correction for varying pitch ratios	Pitch ratio.						
	0·8	+·01	+·005	−·007	−·028	−·058	−·097
	1·0	+·005	+·002	−·005	−·018	−·035	−·06
	1·2	—	—	−·002	−·01	−·02	−·033
	1·4	—	—	—	−·004	−·008	−·018
	1·5	—	—	—	—	—	−·003
For four-bladed propellers a uniform deduction of efficiency of		·0125					

In applying results to a full-sized propeller, Mr. Froude divides the so-called analysis pitch obtained from the model experiments by 1·02. (Mr. G. S. Baker prefers 1·04 based on later data.)

The area of blades obtained from the disc area ratios is the area obtained by continuing the ellipse to the axis, so that to obtain the actual developed area an allowance is necessary for the boss, and ·20 per cent. is deducted, so that—

$$\text{developed area} = \frac{\pi \cdot D^2}{4} \times \text{disc area ratio} \times \frac{8}{10}$$

Experience has shown that for a given thrust of propeller there is a minimum area necessary to avoid cavitation. The value of thrust in tons ÷ developed area in square feet should not exceed about 0·75 ton per square foot in ordinary vessels rising to about 0·9 ton in high-speed vessels as destroyers. Hence, since thrust in tons $= \frac{H}{6{\cdot}9V_1}$ nearly, the developed area should not be less than about $\frac{H}{5{\cdot}15V_1}$ for ordinary ships, and $\frac{H}{6{\cdot}2V_1}$ in very fast vessels.

(Mr. Baker prefers a higher value than the above figure of 0·75 ton, viz. 0·835 ton per square foot.)

Propeller Design.—Recapitulating the formulæ obtained above, we have—

$$C_A = \frac{R^3 \cdot H}{B \cdot V_1^5} \quad \ldots\ldots\ldots \quad (1)$$

$$C_0 = \frac{H}{B \cdot D^2 \cdot V_1^3} \quad \ldots\ldots \quad (2)$$

$$D = \left(\frac{C_A}{C_0}\right)^{\frac{1}{2}} \cdot \frac{V_1}{R} \quad \ldots\ldots \quad (3)$$

where

H = thrust H.P. per propeller.

V_1 = speed of propeller through the water (*i.e.* V − speed of wake).

R = revolutions per minute in hundreds.

D = diameter in feet.

B = coefficient depending on the disc area ratio.

Regarding H, we have seen that the Thrust H.P. of propeller equals E.H.P. ÷ hull efficiency, and that hull efficiency is usually not far from unity, the gain from the wake being balanced by the thrust deduction. The E.H.P. usually obtained is for the naked hull, and in calculating propeller dimensions it is advisable to add something for the additional resistance caused by appendages and air resistance. (Mr. Baker takes 7 per cent. for these additions.) In the absence of model experiments the E.H.P. may be approximated to from the information given in the preceding chapter (viz. Frictional E.H.P. from the table, and Residuary E.H.P. from Johns' curves). Information is also available in Admiral Taylor's and Professor Sir J. H. Biles' works. If the I.H.P. or S.H.P. is estimated, the E.H.P. can be got by the use of a suitable propulsive coefficient.

Regarding V_1, which is the speed of propeller through the water, this is less than the speed of the ship by the speed of the wake, which is assumed to be uniform. Speed of wake $= x \cdot V$ and $V_1 = V(1 - x)$, where x can be approximated to from the formulæ given as page 356. (It should be noted that in vessels of very high speed as destroyers the value of x is very small, this being probably due to the presence of the wave-trough abreast the propellers.)

We will now proceed to work out a particular problem for a vessel of E.H.P. 20,000 four screws, revolutions per minute 275, speed 25 knots. Assume wake 10 per cent. of speed of ship, so that $V_1 = 25(1 - \frac{1}{10}) = 22 \cdot 5$ knots. Hull efficiency taken as unity. $H = 5000 \times 1 \cdot 07 = 5350$. $R = 2 \cdot 75$. Three-bladed propellers.

Four disc area ratios are taken, viz.: 0·5, 0·6, 0·7 and 0·8, and four values of pitch ratio, viz.: 0·8, 1·0, 1·2 and 1·4, and the work is arranged in tabular form as below. The values of "B" are obtained from the table already given, and C_A can then be calculated for each value of "B." The corresponding values of C_0 can then be scaled off the curves given in Figs. 113H, 113I, and also the efficiencies. These latter are corrected as indicated in the table already given. $\frac{C_A}{C_0}$ can then be found, and thence the values of D. The pitch is (D × pitch ratio ÷ 1·02), and area = $\left(\frac{\pi . D^2}{4} \times \text{disc area ratio} \times \frac{8}{10}\right)$. By a systematic arrangement as shown in table the arithmetical labour is not excessive, and can nearly all be done to a sufficient degree of accuracy by a slide rule.

The results for diameter and pitch for each disc area ratio, and the corresponding corrected efficiencies are then plotted

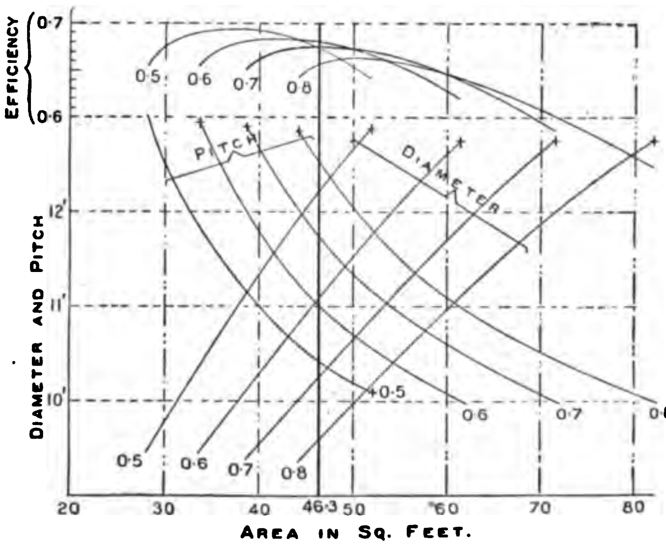


FIG. 113L.

on a base of area as in Fig. 113L. The minimum area for cavitation, viz.: 46·3 square feet, is then drawn in, and the propeller dimensions selected from the right-hand side of this line so as to give the best efficiency. It will be noticed that the highest efficiencies cannot be obtained, as they come on the wrong side of the 46·3 square feet line.

By inspection a propeller of 11·1 feet diameter, 11·1 feet pitch, and 46·3 square feet developed area, appears suitable with an efficiency of 68 per cent. It will be seen that this pitch corresponds to a value of "real" slip of 25·2 per cent. and "apparent" slip of 17 per cent.

Disc area ratio	0·5				0·6				0·7				0·8			
"B" values	0·1085				0·1112				0·1135				0·1157			
$C_A = \frac{H \cdot R^2}{B \cdot V_1^5}$	0·0646				0·0631				0 0618				0·0606			
Pitch ratios	0·8	1·0	1·2	1·4	0·8	1·0	1·2	1·4	0·8	1·0	1·2	1·4	0·8	1·0	1·2	1·4
C_0 from curves	·0261	·034	·0413	·048	·0256	·0336	·0407	·0473	·0253	·033	·04	·046	·0248	·0326	·0396	·0462
Efficiency uncorrected	·648	·697	·69	·655	648	·698	·692	·658	·646	·698	·694	·66	·645	·699	·696	66
Correction	·007	·005	·002	—	·028	·018	·01	·004	·058	·035	·02	·008	·097	·06	·033	·018
Efficiency corrected	·641	·692	·688	·655	·620	·68	·682	·654	·586	·663	·674	·652	·548	·639	·663	·642
$\frac{C_A}{C_0}$	2·48	1·9	1·56	1·35	2·47	1·88	1·55	1·33	2·44	1·87	1·54	1·32	2·44	1·86	1·53	1·31
$D = \left(\frac{C_A}{C_0}\right)^{\frac{1}{2}} \times \frac{22·5}{2·75}$	12·86	11·27	10·2	9·5	12·85	11·21	10·2	9·45	12·8	11·2	10·15	9·4	12·8	11·18	10·12	9·37
$\text{Pitch} = \frac{D \times \text{pitch ratio}}{1·02}$	10·08	11·02	12·0	13·04	10·07	11·0	12·0	12·95	10·04	11·0	11·95	12·9	10·04	10·95	11·9	12·85
$\text{Area} = \frac{\pi \cdot D^2}{4} \times \frac{8}{10} \times \text{disc area ratio}$	52	39·8	32·7	28·2	62·1	47·5	39·2	33·7	72·1	55·2	45·3	38·8	82·5	63·0	51·5	44·2

The formulæ (2) and (3) above may be utilised for approximating to the diameter of propeller, and revolutions from an existing ship where the conditions are similar, and the same pitch ratio and disc area ratio are used. Using small letters to signify the particulars for the model vessel and capital letters for the new vessel, we may say that—

$$D = d \times \sqrt{\frac{v^3}{V^3} \times \frac{\text{I.H.P.}}{\text{i.h.p.}}}$$

$$\text{and } R = r \times \frac{V}{v} \times \frac{d}{D}$$

CHAPTER XI.

PROPULSION BY WIND.

Propulsion by Wind.—The manner in which the force due to the wind, acting on the sails of a ship, produces the motion forward, will now be briefly considered.

The case in which the wind is dead aft, and the vessel has a single sail braced athwartship is simple. The velocity of the wind *relative to the ship* is the speed of the wind, less the speed of the ship, and this *apparent* wind will determine the

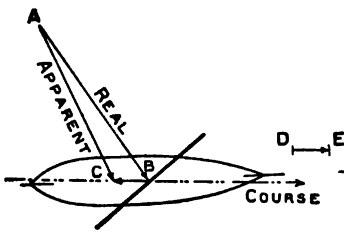


Fig 113M.

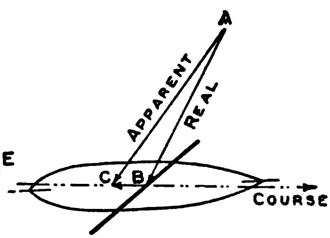


Fig. 113N.

pressure exerted on the sail, which provides the force causing the fore-and-aft motion of the ship.

Whatever the actual direction of the wind, the wind must be combined with the motion of the ship in order to determine the *relative or apparent wind.* Thus, suppose in Fig. 113M AB represents the direction and velocity of the wind, and the ship is moving in the direction of the arrow with a velocity DE on the same scale as AB. Then, if BC is drawn parallel to the direction of motion of the ship and *opposite* to and equal to DE, AC is the *apparent* motion of the wind both in direction and velocity. That is to say, that an observer on board the ship would experience a wind represented by AC.

With the wind abaft the beam the apparent wind will be *less* than the real wind, as in Fig. 113M. With the wind before the beam, the apparent wind will be *greater* than the real wind, as in Fig. 113N.

In each case the motion of the ship is not directly fore and aft, as the actual motion is obtained by compounding the fore-and-aft motion and the "leeway," *i.e.* the sideways motion, which latter, however, is only of small amount. The course of the ship is given by the dotted lines in Figs. 113M and 113N, which make a small angle with the middle line of the ship.

Now, suppose the apparent motion of the wind has been determined, represented both in direction and force by AC, Fig. 113O. Then AC can be resolved into two components, viz.:

AM parallel to the sail, and
CM perpendicular to the sail.

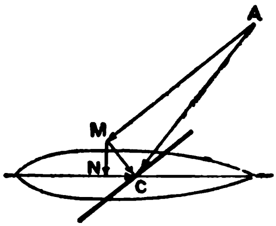


Fig. 113O.

The force CM will not usually act in a horizontal line, and will have three components, viz.:

(i) A vertical force tending to increase immersion;
(ii) A transverse force MN tending to heel the ship and causing leeway; and
(iii) A fore-and-aft force NC which gives the propulsive effect.

A case of steady motion will be reached when NC equals the resistance of the ship, and so long as NC is greater than the resistance, the ship will undergo acceleration or increase of speed.

In this connection it is interesting to note the case of ice-boats (*vide* Sir W. H. White's "Manual of Naval Architecture"). In this case the leeway is very small, and the frictional resistance of the sledge on which the boat runs is exceedingly small even at high speeds. "With the wind varying from a point before the beam to an equal amount abaft the beam, speeds are said to have been reached about equal to twice the real velocity of the wind."

Distribution of Sail Area.—There are three points to consider in the distribution of sail area, viz.:

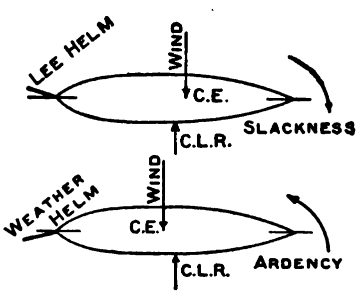
(i) Actual sail area as giving the means of propulsion;
(ii) Vertical distribution of sail area as affecting the angle of heel and consequently the safety of the ship; and
(iii) Longitudinal distribution of sail area as affecting sailing qualities.

Sail Area.—The resistance to onward motion at speeds of sailing ships is almost wholly frictional, and this resistance from one ship to another will vary as the area of wetted surface and of course on the state of the bottoms. For similar ships area of wetted surface will vary as $(\text{Displacement})^{\frac{2}{3}}$. The ratio area of sail $\div W^{\frac{2}{3}}$ is termed the *driving power.*

Vertical Distribution of Sail Area.—This has been dealt with in Chapter V. where power to carry sail was found to be given by the formula—

$$\frac{W \times GM}{A \times h} \times 2240$$

Longitudinal Distribution of Sail Area.—The relative position in a fore-and-aft direction of the "centre of effort" and the "centre of lateral resistance," is important as affecting the sailing qualities of a ship. If the centre of effort is forward of the C.L.R. the bow of the ship tends to fall off from the wind (which is termed "slackness"), and this has to be counteracted by a *lee-helm*, Fig. 113P. If the centre of effort is abaft the C.L.R., the bow of the ship tends to come up to the wind (which is termed "ardency"), and this has to be counteracted by *weather-helm* (Fig. 113Q).



FIGS. 113P AND 113Q.

Of the two qualities of "slackness" and ardency," the former is the more objectionable. The fore-and-aft distance between the centre of effort and the C.L.R. is a matter that can only be determined by comparison with a previous ship regarded as satisfactory. "In ship-rigged vessels and barques the centre of effort is usually from $\frac{1}{14}$th to $\frac{1}{30}$th of the length *before* the C.L.R., $\frac{1}{20}$th being a common value."

The "Manual of Naval Architecture," by Sir W. H. White, contains a chapter on the subject of propulsion by sails in which this subject is very fully discussed.

CHAPTER XII.

THE ROLLING OF SHIPS.

NOTE.—Throughout this chapter, when an angle is called ϕ or Φ it is measured in *degrees;* when it is called θ or Θ it is measured in units of circular measure, so that

$$\phi = \frac{180}{\pi} . \theta$$

$$\Phi = \frac{180}{\pi} . \Theta$$

In dealing with the subject of the rolling of ships, it is necessary to consider first rolling in still water. Although a ship will not under ordinary circumstances roll in still water, yet it is necessary to study this part of the subject before dealing with the more difficult case of the rolling of ships among waves.

Unresisted Rolling in Still Water.—This is a purely theoretical consideration because, even if a ship is rolled in still water, the rolling will sooner or later cease because of the resistances which are set up and which drain the ship of her energy. This energy is *potential* (*i.e.* due to position) at the extremity of each roll, and *kinetic* (*i.e.* due to motion) at the middle of each roll. At intermediate positions the energy of the rolling ship is both potential and kinetic. *Work* has had to be done in the first place to get the ship over, and the ship has then stored up in her a definite amount of potential energy. This energy is gradually dissipated by the various resistances which came into operation until finally the ship comes to rest.

In a ship rolling we cannot fix upon any definite axis about which the oscillation takes place. It appears, however, that the *centre* of *oscillation* or *quiescent point* is not far from the

C.G. of the ship, and this point is usually taken as the centre of the oscillation.

The period of oscillation of a ship from side to side rolling unresistedly in still water through small angles is given by

$$T = \pi\sqrt{\frac{k^2}{m \cdot g}} = 0{\cdot}554\sqrt{\frac{k^2}{m}} \quad . \quad . \quad . \quad (1)$$

Where m is the metacentric height in feet.

g is the acceleration due to gravity in foot-second units, viz. 32·2.

k is the transverse radius of gyration of the ship in feet, defined as follows :—

(The moment of inertia of a body about any axis is found by adding together the product of each weight and the *square* of its distance from the axis. If for a ship this axis is through the C.G., W is the weight and I the moment of inertia, then k is such a quantity that $I = W \times k^2$ and k is the radius of gyration. Expressed in mathematical form $I = W \times k^2 = \Sigma(w \times r^2)$.)

The following is the reasoning leading to the above expression for the period, which may be omitted by students not having a knowledge of the calculus.

The equation of motion of the rolling ship is—

$$\left(\frac{W}{g} \cdot k^2\right)\frac{d^2\theta}{dt^2} = -W \times GZ$$

where GZ is the righting lever in feet.

i.e. $$\left(\begin{matrix}\text{mass moment}\\ \text{of inertia}\end{matrix}\right) \times \left(\begin{matrix}\text{angular}\\ \text{acceleration}\end{matrix}\right) = \left(\begin{matrix}\text{couple causing}\\ \text{the motion}\end{matrix}\right)$$

or $$\frac{d^2\theta}{dt^2} + \frac{g}{k^2} \cdot GZ = 0$$

For ordinary ships the curve of stability for small angles is nearly straight, and we can say $GZ = m \cdot \theta$, so that—

$$\frac{d^2\theta}{dt^2} + \frac{m \cdot g}{k^2} \cdot \theta = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is a differential equation, of which the general solution is—

$$\theta = A \cdot \sin\sqrt{\frac{m \cdot g}{k^2}} \cdot t + B \cdot \cos\sqrt{\frac{m \cdot g}{k^2}} \cdot t \quad . \quad . \quad (3)$$

If $\theta, \frac{d\theta}{dt}, \frac{d^2\theta}{dt^2}$ are all the same after time t, then

$$\sqrt{\frac{m \cdot g}{k^2}} \cdot t = 2\pi \quad \text{or } t = 2\pi\sqrt{\frac{k^2}{m \cdot g}}$$

This is the double oscillation. The period of the *single* oscillation is therefore given by $T = \pi\sqrt{\frac{k^2}{m \cdot g}}$ as stated above.

Equation (3) may now be written

$$\theta = A \cdot \sin\frac{\pi}{T} \cdot t + B \cdot \cos\frac{\pi}{T} \cdot t$$

and if the initial conditions are assumed, such that when $t = 0$, the ship is upright, *i.e.* $\theta = 0$, and the maximum roll of the ship is Θ, this equation becomes

$$\theta = \Theta \sin\frac{\pi}{T} \cdot t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or the angle of heel on a time base is a curve of sines.

The expression for the period is seen to be independent of the angle, and it has been found in actual ships that the period of roll is practically the same for all angles of oscillation when these angles do not exceed 10° to 15° each side of the vertical. This is termed *isochronous* rolling. It is also to be noticed that to make the period *long*, *i.e.* to increase the time of oscillation, it is necessary to

(i) *increase* the radius of gyration and, or
(ii) *decrease* the metacentric height.

An application of the above is seen in the current practice of many merchant vessels. In some trades, voyages have to be undertaken with little or no cargo, owing to the absence of return freights. It is necessary for seaworthiness and the proper immersion of the propellers to sink the vessel by means of water ballast. This ballast has usually been carried in the double-bottom spaces, leading to a low C.G. of the ship, and a large metacentric height. The excessive stability causes a short period, and, in some cases, has not merely rendered the ship uncomfortable, but actually unsafe. The practice, therefore, has grown up of providing spaces for the water in other places. Sometimes deep ballast tanks are provided. In one patent the triangular space at the side beneath the main deck is made into a ballast compartment, and in another the tank top is continued upwards to the deck, forming an inner skin at the side, and in the space thus formed the water can be carried. It is to be observed that such spaces, exclusively devoted to water ballast, are exempt from the measurement for tonnage. The added weight of the ballast produces sufficient immersion for seaworthiness, but does not give excessive stability, and the weight at the sides tends to lengthen the period by increasing the radius of gyration.

The following are the periods of some typical ships. It will be noticed that the heavily armoured battleship of moderate GM has a long period, about 8 sec. In the deck protected cruisers with no side armour a quicker motion is experienced, and in the small classes of war vessel with a relatively large GM there is very quick motion.

H.M.S. *Majestic* (about $3\frac{1}{2}$ ft. GM and great moment of inertia due to side armour)	T = 8 sec.
H.M.S. *Arrogant*, 2nd-class cruiser, deck protected	T = 6 sec.
H.M.S. *Pelorus*, 3rd-class cruiser, deck protected	T = $5\frac{1}{2}$ sec.
Gunboats and Destroyers (small period due to (*a*) small moment of inertia, (*b*) relatively large GM)	T = 2 to 4 sec.
Atlantic liner, small GM	T = 10 to 12 sec.
Passenger yachts	T = 5 to 6 sec.

The formula $T = \pi\sqrt{\frac{k^2}{m \cdot g}}$ fails when the metacentric height is small. In the particular case of a ship with zero metacentric height it gives an infinite period for the roll which is absurd. The problem is possible of solution in a wall-sided vessel, and this was dealt with by Prof. Scribanti at the I.N.A. for 1904. He took three cases, and for each found T_m the period from his formula and T from the formula above $\pi\sqrt{\frac{k^2}{m \cdot g}}$.

In a battleship with 3 ft. GM $\Big\}$ $\frac{T}{T_m} = 1{\cdot}04$, the error being small.

In a liner with 4 in. GM . $\frac{T}{T_m} = 1{\cdot}31$, a considerable error.

In a ship almost zero GM, viz. $\frac{3}{8}$ in. $\Big\}$ $\frac{T}{T_m} = 2{\cdot}98$, a very large error.

For a ship with *zero* metacentric height, assuming wall-sidedness, he found by advanced mathematical analysis[1] the following expression for the period, viz.—

$$T = \frac{5 \cdot 25}{\Theta} \sqrt{\frac{k^2}{g \cdot BM_0}}$$

where Θ is the maximum angle in circular measure.

In the case of a vessel having its curve of stability a curve of sines, like a circular vessel or a submarine, the equation of motion becomes—

$$\frac{d^2\theta}{dt^2} + \frac{\pi^2}{T^2} \cdot \sin \theta = 0$$

This differential equation can be solved by advanced mathematical methods, and the following are the periods of single oscillations for various angles from the upright, taking the period of a small oscillation as unity.

small	30°	60°	90°	120°	150°	180°
1	1·017	1·073	1·183	1·373	1·762	infinite

Thus beyond small angles there is an appreciable lengthening of the period. The range of stability in this case is 180 degrees. If for a ship the curve of stability is of the same character as a curve of sines, it is reasonable to assume that when rolling to angles bearing the same ratio to the range as above a similar lengthening of the period would take place in comparison with a small oscillation. Thus, for a ship with a range of 60° the period for a roll of 20° each side of the vertical would be about 7 per cent. greater than that for small angles. Although the conditions are not exactly similar to the above in the case of ordinary ships the period is lengthened somewhat for large angles, and this departure from isochronous rolling has an important bearing on the safety of a ship when being rolled to large angles in a seaway.

[1] Mr. A. W. Johns, R.C.N.C., gave in *Engineering*, July 18, 1904, a method of approximate solution of this problem.

Forces due to Rolling.—One application of the equation of motion of a rolling ship is to find the maximum force at any point of a ship when rolling. We can say $\frac{d^2\theta}{dt^2} + \frac{\pi^2}{T^2}.\theta = 0$, or the angular acceleration $\frac{d^2\theta}{dt^2} = -\frac{\pi^2}{T^2}.\theta$. This is a maximum when θ is a maximum, *i.e.* at the end of a roll. If we take a ship with a period of 5 sec. rolling through 30°, 15° each side of the vertical, then the angular acceleration at the end of the roll is $\frac{\pi^2}{25} \times 15 \times \frac{\pi}{180} = 0{\cdot}1035$ in foot-second units. The linear acceleration, say 100 feet up, is therefore 10·35 in foot-second units. Force = mass × acceleration $= \frac{w}{g} \times 10{\cdot}35$ $= 0{\cdot}32 \times$ weight, *i.e.* a man 100 feet up would have to hold on with a force one-third his weight at the end of the roll.

The following is an example of a similar nature worked out:—

A topmast 72 feet in length, height of topmast head being 180 feet above water, can be assumed of constant diameter, 15 inches. The ship of 8 seconds period is supposed to roll through 30° each side of the vertical. Make an estimate of the stress on the material of the topmast at its junction with the lower mast, supposing it unsupported by stays.

In Fig. 114, w is the weight of the topmast, F the transverse force at the junction with the mast, L the bending moment, both when at the maximum at the extremity of the roll. Then we have—

(*a*) resolving the forces at right angles to the mast

$$w \,.\, \sin\theta - F = \frac{w}{g}(a+h)\frac{d^2\theta}{dt^2}$$

(*b*) Taking moments about g

$$Fa + L = \frac{w}{g} \,.\, k^2 \,.\, \frac{d^2\theta}{dt^2}$$

$\left(k^2 \text{ is for the topmast and } = \frac{a^2}{3}\right)$

from which the bending moment $= L = wa\left[\sin\theta - \frac{\frac{4a}{3}+h}{g} \,.\, \frac{d^2\theta}{dt^2}\right]$

Taking $\frac{d^2\theta}{dt^2} = -\frac{\pi^2}{T^2} \cdot \sin\theta$, we have—

$$L = wa\sin\theta\left[1 + \left(\frac{4a}{3} + h\right)\cdot\frac{\pi^2}{g \cdot T^2}\right]$$

w is 3520 lbs., taking the wood of topmast as 40 lbs. per cubic foot, $a = 36$, $h = 108$, $\sin\theta = \frac{1}{2}$

$$\therefore L = 111{,}000 \text{ ft. lbs. nearly,}$$

from which, using $\frac{p}{y} = \frac{M}{I}$, the stress at the base of topmast works out to 4000 lbs. per sq. in.

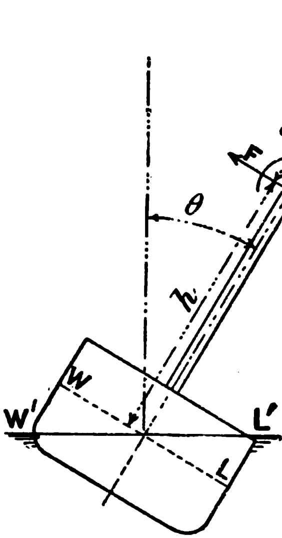


FIG. 114.

Resisted Rolling in Still Water.—It has been found by experiment that the rolling of a ship is practically isochronous, although resistances to the rolling motion are in operation. Experimenters on this subject have actually rolled ships in order to investigate the laws which govern the motion. A small vessel can easily be set rolling by heaving down with tackles from a quay. In a large vessel bodies of men can be run from side to side, their motion being timed to the ship. In the rolling experiments on H.M.S. *Revenge* (I.N.A. 1895), the barbette guns were also trained 15 degrees each side, the guns being run out first to make the C.G. eccentric. When the desired angle of roll was reached, the men and guns were stationed at the middle line, while the observations were being taken.

Observing the angles reached on successive rolls a curve can be constructed as Fig. 115, the abscissæ being numbers of rolls and ordinates the angles reached to port and starboard successively. Such a curve is termed a *a curve of declining angles*. Fig. 116 shows samples of what is termed a *curve of extinction*, which is obtained from the curve of declining angles, the abscissæ representing angles of roll and the ordinates angles lost per swing.

It has been found by analyzing a number of these curves that the *decrement* or angle lost per swing can be expressed as $a\phi + b\phi^2$, where ϕ is the angle of roll in degrees and a and b are coefficients which vary for different ships. Thus, calling $-\Delta\phi$ the decrement, we have—

$$-\Delta\phi = a\phi + b\phi^2$$

Taking Δn as a single roll, we have—

$$-\frac{\Delta\phi}{\Delta n} = a\phi + b\phi^2$$

and in the limit this becomes—

$$-\frac{d\phi}{dn} = a\phi + b\phi^2$$

which is termed the *decremental equation*.

Thus we have—

Inconstant, T = 8 sec. . . . $-\frac{d\phi}{dn} = 0{\cdot}035\phi + 0{\cdot}0051\phi^2$

Devastation, T = 6·75 sec. . $-\frac{d\phi}{dn} = 0{\cdot}072\phi + 0{\cdot}015\phi^2$

Revenge, without bilge keels, T = 7·6 sec. $-\frac{d\phi}{dn} = 0{\cdot}0125\phi + 0{\cdot}0025\phi^2$

Revenge, with bilge keels, T = 7·75 sec. $-\frac{d\phi}{dn} = 0{\cdot}065\phi + 0{\cdot}017\phi^2$

The integral form of the decremental equation is—

$$dn = \frac{d\phi}{a\phi + b\phi^2} \quad \text{or} \quad n = \int_{\phi_2}^{\phi_1} \cdot \frac{1}{a\phi + b\phi^2} \cdot d\phi$$

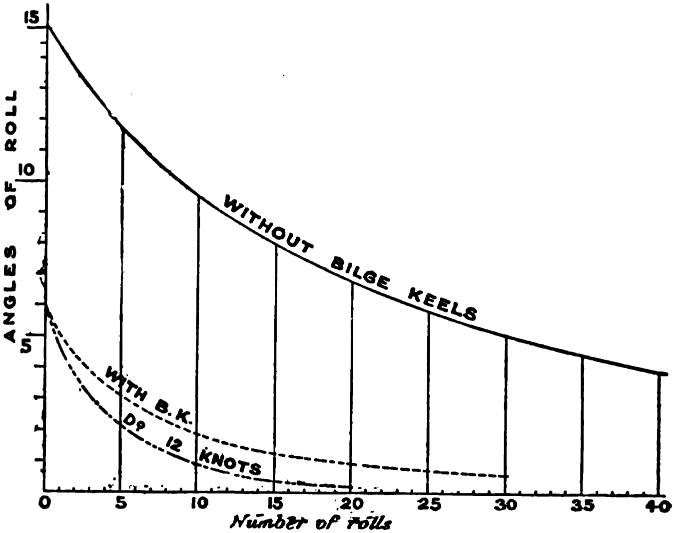


FIG. 115.—CURVES OF DECLINING ANGLES, *Revenge*.

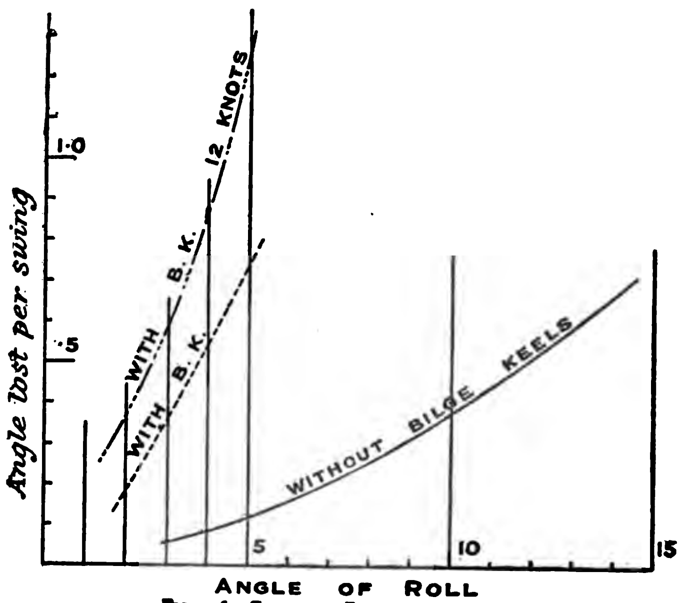


FIG. 116.—CURVES OF EXTINCTION, *Revenge*.

which gives the number of rolls to pass from an angle of roll ϕ_2 to an angle ϕ_1. On integrating this becomes—

$$n = \frac{2\cdot3}{a}\log_{10}\left(\frac{b+\frac{a}{\phi_1}}{b+\frac{a}{\phi_2}}\right)$$

Thus for a ship in which $a = 0\cdot05$, $b = 0\cdot02$, starting from 15 degrees, 13 rolls are necessary before the angle of roll is 2 degrees. For the *Inconstant*, starting from 15 degrees, the successive angles of roll are 13·5°, 12·2°, etc.

If a ship rolls from Θ_1 port to Θ_2 starboard, supposing the curve of stability a straight line, we have—

$$\text{Dynamical stability at } \Theta_1 = \tfrac{1}{2}\,.\,W\,.\,m\,.\,\Theta_1^2$$

$$\text{,, \quad ,, \quad at } \Theta_2 = \tfrac{1}{2}\,.\,W\,.\,m\,.\,\Theta_2^2$$

$$\text{so that the loss of energy} = \tfrac{1}{2}W\,.\,m(\Theta_1^2 - \Theta_2^2)$$

$$= W\,.\,m\,.\,\Theta_m \times \text{decrement}$$

taking $\frac{1}{2}(\Theta_1 + \Theta_2) = \Theta_m$.

If R be the moment of the resistance to rolling at angle θ, the work done by the resistances from Θ_1 to Θ_2 is $\int_{\Theta_2}^{\Theta_1} R\,.\,d\theta$.

We can then equate this work to the loss of energy, since these must be equal, viz.—

$$\int_{\Theta_2}^{\Theta_1} R\,.\,d\theta = W\,.\,m\,.\,\Theta_m \times \text{decrement},$$

and putting in the value of the decrement from the decremental equation above $\left(\text{and remembering that } \Phi = \frac{180}{\pi}\Theta\right)$—

$$\int_{\Theta_2}^{\Theta_1} R\,.\,d\theta = W\,.\,m\,.\,\Theta_m\left(a\,.\,\Theta_m + \frac{180}{\pi}\,.\,b\,.\,\Theta_m^2\right)$$

This was the method adopted by the late Mr. Froude to investigate the laws underlying the resisted rolling of ships.

1 Suppose the moment of resistance varies as the angular velocity, or $R = k_1\,.\,\frac{d\theta}{dt}$. Then assuming, as in unresisted rolling, $\theta = \Theta_1\,.\,\sin\frac{\pi}{T}\,.\,t$, we have—

$$\frac{d\theta}{dt} = \Theta_1\,.\,\frac{\pi}{T}\,.\,\cos\frac{\pi}{T}\,.\,t.$$

The work done from Θ_1 to zero is—

$$\int_0^{\Theta_1} R \, . \, d\theta = \int_0^{\Theta_1} k_1 \, . \, \frac{d\theta}{dt} \, . \, d\theta = \int_0^{\frac{T}{2}} k_1 \left(\frac{d\theta}{dt}\right)^2 . \, dt = \tfrac{1}{4} \, . \, k_1 \, \frac{\pi^2}{T} . \, \Theta_1^2$$

on putting in the above value for $\frac{d\theta}{dt}$ and integrating. Similarly for the other side the work done from o to Θ_2 is $\frac{1}{4} \, . \, k_1 \, . \, \frac{\pi^2}{T} . \Theta_2^2$. So that from Θ_1 to Θ_2 the work done against the resistance is $\frac{1}{2} \, . \, k_1 \, \frac{\pi^2}{T} . \, \Theta_m^2$, putting $\Theta_m^2 = \frac{1}{2}(\Theta_1^2 + \Theta_2^2)$. Equating this to the loss of dynamical stability, viz. $W \, . \, m \, . \, \Theta_m \times$ decrement, we have—

$$\text{decrement} = \tfrac{1}{2} \, . \, k_1 \, . \, \frac{\pi^2}{W \, . \, m \, . \, T} . \, \Theta_m$$

i.e. *a resistance which has a moment proportional to* $\frac{d\theta}{dt}$, *the angular velocity, will give a decrement proportional to the angle of roll.*

2. Suppose the moment of resistance varies as the square of the angular velocity, or $R = k_2 . \left(\frac{d\theta}{dt}\right)^2$. Then, by a similar process to the above and equating the work done by the resistance to the loss of dynamical stability, we get that—

$$\text{decrement} = \tfrac{4}{3} \, . \, k_2 \, . \, \frac{\pi_2}{W \, . \, m \, . \, T^2} . \, \Theta_m^2$$

i.e. *a resistance which has a moment proportional to the square of the angular velocity* $\left(\frac{d\theta}{dt}\right)^2$ *will give rise to a decrement proportional to the square of the angle of roll.*

We thus see that if the resistances to the rolling motion are assumed to vary, partly as $\frac{d\theta}{dt}$ and partly as $\left(\frac{d\theta}{dt}\right)^2$, the decrement is given by—

$$\tfrac{1}{2} \, . \, k_1 \, . \, \frac{\pi^2}{W \, . \, m \, . \, T} . \, \Theta_m + \tfrac{4}{3} \, . \, k_2 \, . \, \frac{\pi^2}{W \, . \, m \, . \, T^2} . \, \Theta_m^2$$

This is of the same form as the decremental equation found to fit the curves obtained from rolling experiments.

Mr. Froude attributed the first term to the formation of waves, and the second to friction and the passage through the water of bilge keels or keel projections (including the flat portions of the ship).

Waves.—The rolling motion of a ship creates waves on the surface of the water, and these waves pass away and require energy for their creation. A wave of very small height represents a considerable amount of energy, and the drain on the energy of the rolling ship is a distinct resistance tending to reduce the rolling motion.

Friction.—This is of small amount, because the surface of a ship is kept smoothly painted to reduce the resistance to steaming to a minimum.

Form of Section.—If a ship has a sharp bilge, the water at the corner has to slip past, and gets a motion opposite to that of the ship. The effect both as regards friction and on bilge keels is therefore greater than if the section were more rounded in form.

Air Resistance.—The resistance of the air to rolling is only small under ordinary circumstances, but it may be made considerable by the use of steadying sails. If a ship with sails set rolls to *windward*, the wind pressure is increased owing to the greater relative velocity, and this the more so the higher up we go. The pressure on the sails therefore is greater when rolling to windward, and the centre of pressure is higher. When rolling to *leeward* the effective pressure is less, and there is a fall in the centre of pressure.

Bilge Keels.—Mr. Froude, in his investigations, took the bilge keels as flat surfaces moving through the water, and by using data obtained from swinging a flat board in water was able to make a calculation for the resistance offered by the bilge keels to the motion (see later for the details of this). It was found, however, that the observed decrement due to the $b \cdot \phi^2$ portion of the decremental equation could not thus be accounted for. Professor Bryan, in a paper before the I.N.A. in 1900, gave further investigations on the subject. Consider the flow of water round a right-angled bend as Fig. 117. The water adjacent to the surface has to come to rest at the corner and change the direction of its flow. Thus along AB we get a diminution of the velocity of the stream lines. With this decrease of velocity there must be associated a rise of pressure both along AB and BC. Taking now the case of bilge keels projecting from the surface as Fig. 118, the ship being supposed to be rolling clockwise. The relative velocity of the ship, and the water along A_2A_1 has to be brought to zero at A_1 and there is caused a rise of pressure along A_2A_1 and similarly along A_4A_3. These pressures will have resultants as

FIG. 117.

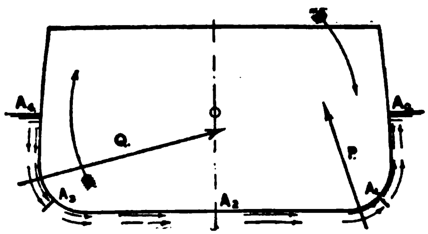


FIG. 118.

P and Q, which with ordinary shaped sections will give a moment tending to stop the motion. The effect will be more pronounced as the section of the ship is sharper, because of the greater relative velocity of the water past the bilge as compared with a round section.

Figs. 115, 116 show very clearly the influence of bilge keels in reducing rolling. It was found in the *Revenge*, starting in each case from 6°, that

without bilge keels	45 to 50 rolls	were	necessary	to reduce	to	2°
with bilge keels	8	"	"	"		2°

Curves are also given, showing the effect of motion ahead on the rolling. In this case the vessel was proceeding into water that was undisturbed by the rolling motion of the ship, and the resistance to rolling was somewhat greater than when the vessel had no onward motion.

It has been found that the addition of bilge keels adds slightly to the period of rolling, in the case of the *Revenge* about 5 per cent.

The following is the investigation regarding the work done by a bilge keel or flat surface, on the assumption that the pressure varies as the square of the speed of the bilge keel through the water.

Let A be the area of one side of the bilge keels in sq. feet,
r the mean distance of the centre of oscillation,
c the coefficient of normal pressure at 1 ft. per sec.: in lbs. per sq. foot.

$$\text{Then pressure} = c \,.\, A \,.\, r^2 \,.\left(\frac{d\theta}{dt}\right)^2 \text{ at any instant.}$$

$$\text{Moment} = c \,.\, A \,.\, r^3 \,.\left(\frac{d\theta}{dt}\right)^2$$

(The k_2 in the former investigation is therefore $c \,.\, A \,.\, r^3$.)

The decrement is given by $\frac{4}{3} \,.\, k_2 \,.\, \frac{\pi^2}{W \,.\, m \,.\, T^2} \,.\, \Theta_m^{\,2}$ for a resistance whose moment is proportional to $\left(\frac{d\theta}{dt}\right)$ as seen above, or—

$$\text{decrement} = \tfrac{4}{3} \,.\, c \,.\, A \,.\, r^3 \,.\, \frac{\pi^2}{W \,.\, m \,.\, T^2} \,.\, \Theta_m^{\,2} \quad \text{(W in lbs.)}$$

Putting W in tons and $T = \pi\sqrt{\frac{k^2}{m \,.\, g}}$ we have

$$\text{decrement} = \frac{c}{1680} \,.\, \frac{A \,.\, r^3 \,.\, g}{W \,.\, k^2} \,.\, \Theta_m^{\,2}$$

which is increased, as one would expect, by increase in the area of the bilge keels and in the lever. It is also noticed that the decrement varies inversely as the I of the vessel, so that the bilge keels are proportionally less effective in a vessel with large I than with small. The decrement also varies as the *square* of the arc of oscillation, so that when large angles are reached, as in a sea way, the influence of bilge keels will be most effective.

Rolling among Waves.—A wave is not the passage of water, but the passage of *motion*. The actual movement of the particles of water composing a wave is small. The *form* moves with considerable speed, but if a piece of wood be observed, it is noticed to oscillate about a mean position. In the generally accepted trochoidal theory the particles of water for deep-sea waves are supposed to move in circular orbits, and the diameters of these orbits decrease as the depth increases. This orbital motion gives rise to centrifugal force and the pressure at the crest of a wave is less than in still water, and at the trough the pressure is greater. The buoyancy, therefore, in the crest portion is less than the normal, and in the trough portion it is greater. This is the explanation of the tenderness of sailing-boats on the crest of a wave. The *virtual* weight is less than the actual, and so the righting moment is reduced as compared with still water. The heeling moment due to the wind is not affected in this way, and so a boat of sufficient stiffness in still water is liable to be blown over on a wave. The *virtual* force of gravity therefore varies at different places on a wave, and its direction also varies, being perpendicular to the wave profile at any particular point. This direction is termed the *virtual upright*, and a small raft will always tend to place its mast along this virtual upright. This has its maximum inclination to the vertical at about a quarter the length of the wave from the crest or trough. A ship rolling amongst waves will at each instant tend to place her masts parallel to a virtual upright, and a surface which is normal to each of these virtual upright positions of a ship in a wave is termed the *effective wave slope*, which is distinctly flatter than the actual observed wave profile.

In dealing with the subject, it is not usual (except for sailing-ships) to consider the variation of the amount of the virtual weight, but allowance must be made for the variation in its direction. Certain assumptions have to be made to bring the problem within the scope of mathematical treatment. These are as follows:—

(*a*) The ship is lying passively broadside to the wave advance.

(*b*) The waves are assumed to be a regular series, identical in size and speed.

(*c*) The waves are assumed long in comparison with the size of the ship.

(*d*) The profile of the wave is taken as a curve of sines.

We have first to express the angle the virtual upright makes to the upright in terms of the time and other known quantities.

Fig. 119 represents the construction of the wave, L being the ½ length, H the height (much exaggerated), x and y the co-ordinates of a point P referred to axes through the trough.

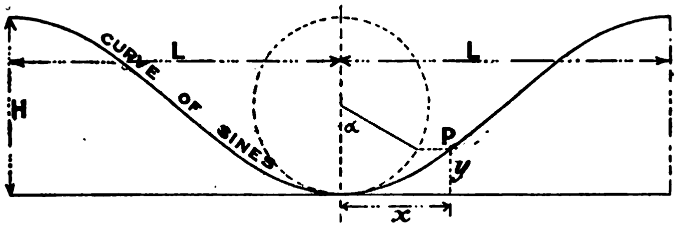


FIG. 119.

This point P is reached in time t, the time from crest to crest being $2T_1$, *i.e.* T_1 is the half period of the wave.

$$\text{Then } a = \frac{\pi}{T_1} . t\,;\ x = \frac{L}{T_1} . t;\ y = \frac{H}{2} - \frac{H}{2} . \cos\frac{\pi}{T_1} . t$$

Therefore
$$\frac{dy}{dx} = \frac{\pi . H}{2 . L} . \sin\frac{\pi}{T_1} . t = \tan\theta_1$$

where θ_1 is the slope at P. The slope being small, we may say that—

$$\theta_1 = \frac{\pi . H}{2L} . \sin\frac{\pi}{T_1} . t$$

which is also the inclination of the virtual upright to the vertical.

From Fig. 120 the equation of motion is, θ being the angle of ship from the vertical—

$$\frac{W}{g} . k^2 . \frac{d^2\theta}{dt^2} + W . m . (\theta - \theta_1) = 0$$

$\theta - \theta_1$ being the angle from the virtual upright,

$$\text{or } \frac{d^2\theta}{dt^2} + \frac{\pi^2}{T^2}(\theta - \theta_1) = 0$$

T being the period in still water. Putting in the value of θ_1 found above—

$$\frac{d^2\theta}{dt^2} + \frac{\pi^2}{T^2}\left(\theta - \frac{\pi . H}{2L} . \sin\frac{\pi}{T_1} . t\right) = 0$$

which is *Froude's general equation for unresisted rolling among waves.*

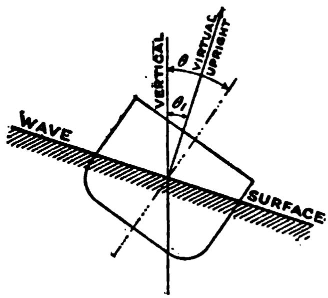


FIG. 120.

Assuming for the initial conditions, *i.e.* when $t = 0$ that the ship is upright and at rest in the wave trough, the solution of this differential equation is

$$\theta = \frac{\Theta_1}{1 - \frac{T^2}{T_1^2}}\left[\sin\frac{\pi}{T_1} . t - \frac{T}{T_1} . \sin\frac{\pi}{T} . t\right]$$

Θ_1 being the maximum wave slope.

1. Take the special case when $T = T_1$, *i.e.* the still-water period of the ship equals the half period of the wave. This is termed *synchronism.* Putting $T = T_1$ in the above equation and using the method of the calculus for dealing with indeterminate forms, we have—

$$\theta = \frac{\Theta_1}{2}\left[\sin\frac{\pi}{T} . t - \frac{\pi}{T} . t \cos\frac{\pi}{T} . t\right]$$

When $t = \frac{T_1}{2}, \frac{3T_1}{2}$, etc. $\theta = \frac{\Theta_1}{2}, -\frac{\Theta_1}{2}$, etc., showing that the inclination of the ship is alternately $\pm$ half the maximum wave slope.

When $t = 0$, T_1 $2T$, etc., we have $\theta = 0, \frac{\pi}{2} . \Theta_1, - \pi\Theta_1, \frac{3\pi}{2}\Theta_1$, etc., *i.e.* for every half wave that passes an additional angle $= \frac{1}{2} . \pi \times$ maximum slope, is given to the roll, and thus *a ship under the given assumptions must inevitably capsize* (see Fig. 121). Thus the *Devastation* with a still-water period of $6\frac{3}{4}''$, if lying broadside with no resistance, to waves of $\frac{1}{2}°$ maximum slope and period $13\frac{1}{2}''$, would increase the roll every half wave by $\frac{1}{2} . \frac{\pi}{2}$ degrees, and in $67\frac{1}{2}$ seconds, or rather over a minute, would reach 8°. Large angles are soon reached also, if there is only approximate synchronism between the ship and the wave. Thus a ship of $5''$ period rolling unresistedly broadside to waves of $4''$ half period with 8° maximum slope will, in successive rolls, reach 11°, $20\frac{1}{2}°$, $27\frac{1}{2}°$.

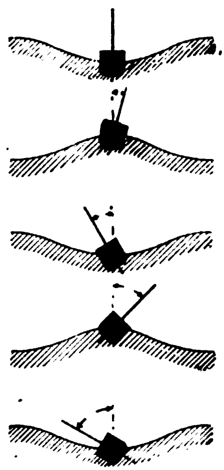


Fig. 121.

These results are borne out by the experience of ships at sea. It has frequently been observed that ships with a great reputation for steadiness occasionally roll heavily at sea. This is due to the fact that a succession of waves has been met with, having a period approximately synchronizing with the double period of the ship. The synchronism may be destroyed by altering the course, since what affects the ship is the *apparent* period of the waves.

2. Suppose the ship has a *very quick* period as compared with that of the wave, so that $\frac{T}{T_1}$ is small. The equation above then reduces to

$$\theta = \Theta_1 . \sin \frac{\pi}{T_1} . t$$

i.e. the ship takes up the motion of the wave and behaves exactly like a raft. The angle of maximum heel will be the maximum slope of the wave.

3. Now take the case in which the period of the ship is *long* compared with that of the wave, *i.e.* $\frac{T_1}{T}$ is small. The equation above can be written

$$\theta = \Theta_1 . \frac{T_1}{T}\left[\sin\frac{\pi}{T}.t - \frac{T_1}{T}.\sin\frac{\pi}{T_1}t\right]$$

This is always small since $\frac{T}{T}$ is small, and the ship will never depart far from the vertical. Thus, to secure steadiness at sea it is necessary to make the still-water period as long as possible. To do this there must be a small metacentric height. Such a ship is *crank*, *i.e.* easily inclined by external forces, but in a sea way is most likely to be steady.

Atlantic storm waves are about 500 to 600 feet in length, and have a period of 10 or 11 seconds (*i.e.* $2 . T_1$). The longest wave recorded had a length of about 2600 feet, and a period of 23 seconds. The battleship and liner, quoted above as having periods of 8 and 10–12 seconds respectively, should therefore prove steady ships in a sea way, as synchronism would only be experienced when meeting with waves of periods 16 to 24 seconds, which are quite exceptional.

Resisted Rolling among Waves.—If we take the critical case of a vessel meeting with waves whose half period is equal to the ordinary period of the ship, then the angle for each swing is increased by $\frac{\pi}{2} . \Phi_1$ as seen above. The decrement due to resistance is given by $a\phi + b\phi^2$ and the increment per swing is therefore $\frac{\pi}{2} . \Phi_1 - (a\phi + b\phi^2)$. The angle of swing will go on increasing until an angle of roll is reached such that $\frac{\pi}{2} . \Phi_1 = a\Phi + b\Phi^2$. The increase due to synchronism is then just balanced by the decrement due to resistance, and we get a steady roll of Φ. We have seen above that when large angles are reached a ship is not isochronous in her rolling, and also that the fitting of bilge keels causes an increase in the period. Therefore, under the actual conditions obtaining, with a synchronous swell the ship will not necessarily capsize, for

(*a*) As large angles are reached the ship departs from isochronous rolling.

(*b*) Resistances come into operation, and there is also the further condition, viz.:

(*c*) A succession of waves of precisely the same period is a very unlikely occurrence.

Apparent Period of Waves.—We have spoken above about the *apparent* period of waves as affecting a ship's rolling. If β is the angle the direction of the ship's advance makes with the crest line of the wave, then if v be the speed of the ship, $v \cdot \sin \beta$ is the speed of the ship against the wave advance; and v_0 being the speed of the wave, the waves will meet the ship at a speed $v_0 + v \cdot \sin \beta$. If T_0 be the actual period of wave, then the apparent period T' is given by $T' = T_0 \div (1 + \frac{v}{v_0} \cdot \sin \beta)$. If β is negative, β' say, *i.e.* the ship travels away from the wave advance, $T' = T_0 \div (1 - \frac{v}{v_0} \cdot \sin \beta')$. Thus in the first case the apparent period is *diminished* and in the second case *increased.*

Graphic Integration of the Rolling Equation.—1. *Unresisted Rolling in Still Water.*—The equation of motion of a ship rolling unresistedly in still water has been seen to be

$$\frac{d^2\theta}{dt^2} + \frac{g}{k^2} \cdot GZ = 0$$

This cannot be mathematically integrated, because there is no relation between GZ, the righting arm and the time. By assuming that $GZ = m \cdot \theta$, a solution can be found leading to the expression, $T = \pi \sqrt{\frac{k^2}{m \cdot g}}$, θ being small. This enables the equation of motion to be written

$$\frac{d^2\theta}{dt^2} + \frac{\pi^2}{T^2} \cdot \frac{GZ}{m} = 0$$

where T is the time of oscillation from side to side, or one half the mathematical period.

By assuming GZ = $m . \sin \theta$ we have the case of a circular ship or a submarine, and the equation can be solved by advanced mathematical methods, the solution for various angles being given on p. 386.

By the process known as "graphic integration" the solution can be accurately found and the process can be extended to the case when resistances operate.

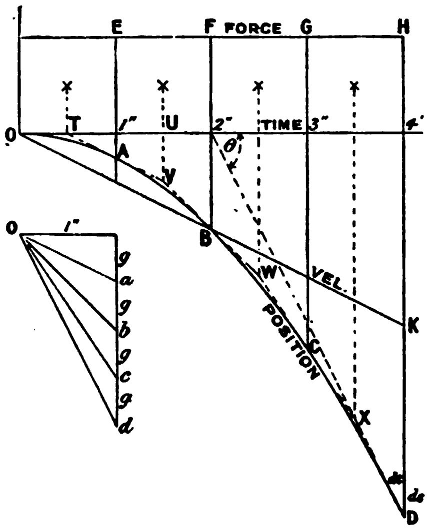


FIG. 122.

To lead up to the subject, take the case of a body falling freely under gravity. The force causing the motion is constant, viz. that due to gravity = P, say. Now force = mass × acceleration, or $P = \frac{w}{g} \cdot f$, and $f = \frac{dv}{dt}$, $\therefore v = \int P . dt$ taking unit mass. If therefore we have a force curve on base of time, Fig. 122 (in this case a straight line), the velocity is found by integrating the force curve. Again $v = \frac{ds}{dt}$ or $s = \int v \cdot dt$, *i.e.* the space or position is found by integrating the velocity curve. Thus, in the figure the body would have fallen the distance 4D in time

$t_{,}$ = 4 seconds. EH being the force curve, OBK the velocity curve, and OBD the position curve.

Now take the reverse process. Having given the position curve OD, at any point, as D, the tangent makes an angle with the base θ such that $\tan\theta = \frac{ds}{dt}$, *i.e.* the velocity. At times o, 1, 2, 3, 4, etc., seconds, the velocities are o, g, $2g$, $3g$, $4g$, etc., *i.e.* $\tan\theta$ has values o, g, $2g$, etc. Setting down as in lower figure, the tangents to the position curve at A, B, C, D, etc., are parallel to Oa, Ob, Oc, Od, etc.

The position curve is the second integral from the force curve, and conversely the force curve is the second derived from the position curve, and the *intersection of tangents at two points of the position curve is below the centre of gravity of the corresponding portion of its second derived*, i.e. *the force curve*. (For proof of this see later.)

Thus to get A we find the C.G. of OE, and at T draw TAV parallel to Oa in the lower figure. For the second interval we draw OAW parallel to Ob, and so on with succeeding intervals, which enables the position curve OABCD to be drawn in.

The equation we have to solve is—

$$\frac{d^2\theta}{dt^2} = \frac{\pi^2}{T^2} \cdot \frac{GZ}{m}$$

dropping the sign,

or

$$\frac{d\omega}{dt} = \frac{\pi^2}{T^2} \cdot \frac{GZ}{m}$$

$\omega = \frac{d\theta}{dt}$, the angular velocity in circular measure.

In a small time Δt the change of angular velocity is therefore

$$\Delta\omega = \frac{\pi^2}{T^2} \cdot \frac{GZ}{m} \cdot \Delta t$$

It is more convenient to use degrees, so that—

$$\Delta\omega = \frac{\pi^2}{T^2} \cdot \frac{180}{\pi} \cdot \frac{GZ}{m} \cdot \Delta t \text{ degrees,}$$

and taking the interval of time $\frac{1}{10} \cdot T$,

$$\Delta\omega = \frac{\frac{180}{\pi} \cdot \frac{GZ}{m}}{1 \cdot 013\ T} \text{ degrees.}$$

If therefore at a certain interval the angular velocity is represented by the slope of the line OB = tan α, Fig. 123, on a base AO = 1·013 T, and BD is set up equal to the mean value of $\frac{180}{\pi}\cdot\frac{GZ}{m}$ over the succeeding interval $\frac{1}{10}\cdot$ T, then the

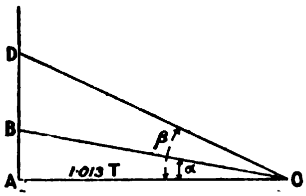


Fig. 123.

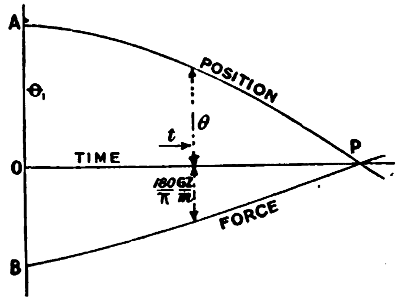


Fig. 124.

slope of OD given by tan β is the angular velocity at the end of the interval $\frac{1}{10}$ T, for

$$\tan\beta = \frac{AB + BD}{AO} = \tan\alpha + \frac{\left(\frac{180}{\pi}\cdot\frac{GZ}{m}\right)}{1{\cdot}013\ T}$$

$$= \frac{d\theta}{dt} + \Delta\omega$$

If the force curve in this case is one where ordinates are $\frac{180}{\pi}\cdot\frac{GZ}{m}$, and if force and position curves are as shown in Fig. 124 on a time base, then at any ordinate at time t the position θ must correspond to the value of $\frac{180}{\pi}\cdot\frac{GZ}{m}$ at that angle. Where these two curves intersect on the base line, as they must do simultaneously, we have the value of the half time of the oscillation supposing we start from Θ_1 the initial angle. At the angle OA there is a definite value for $\frac{180}{\pi}\cdot\frac{GZ}{m}$ = OB, and we now by a process of trial and error have to find the curves AP and BP. In the first place we draw a modified curve of stability on angle base whose ordinates are values of $\frac{180}{\pi} \cdot \frac{GZ}{m}$ (the slope of which is 45° to the base line), as Fig. 125. A convenient scale to use is found to be $\frac{1}{8}'' = 1°$, $\frac{1}{8}'' = 1$ unit of force, and $1'' = \frac{1}{10} \cdot T$.

Set off equal intervals of time $\frac{1}{10}T$ on the base line, 1, 2, etc.

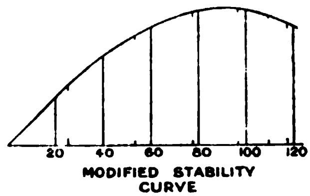


FIG. 125.

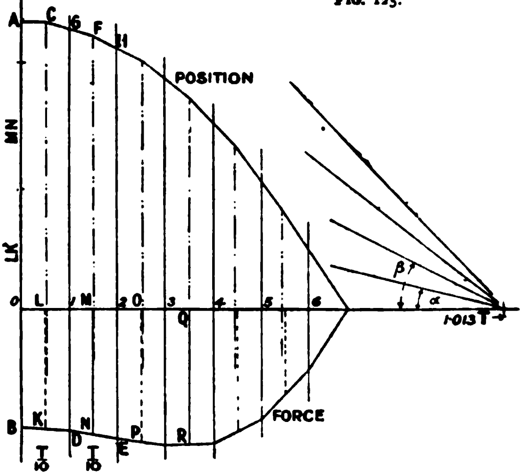


FIG. 125A.—Graphic integration for a simple pendulum.

(Fig. 125A), and mark off OA equal to the initial angle assumed. Then at angle OA the modified curve of stability has the value which is set down OB. Now over the first interval $\frac{T}{10}$ the force will vary (diminishing if we start before the angle of maximum stability and increasing if we start after this angle).

An assumed slope BD is taken for the force curve. The mean value over the interval is LK; the slope given by LK ÷ 1·013 T will give the change of angular velocity in interval $\frac{T}{10}$. Draw AC parallel to base line and the position curve must be a tangent to AC at A, since velocity at starting from the extreme angle is zero. The *c.g.* of the force curve OD is then found and squared up to meet AC in C. From C draw CGF parallel to the slope given by $\tan \alpha = \frac{LK}{1 \cdot 013 T}$. Now we check to see if at the angle 1G the ordinate of the modified curve of stability is 1D. If not, the process must be repeated until an agreement is found. We then proceed to the second interval and guess in DE and find F over the *c.g.* of 1E. FH is drawn parallel to the slope given by $\tan \beta = \tan \alpha + \frac{MN}{1 \cdot 013 T}$. 2H and 2E are again checked as before. In this way, by a process of trial and error, the position curve and the force curve may be obtained and faired in. They must meet simultaneously on the base line.

Fig. 125A shows the diagram worked out for the case of a simple pendulum, or a submarine or circular vessel, for which the equation of motion is

$$\frac{d^2\theta}{dt^2} + \frac{\pi^2}{T^2} \cdot \sin \theta = 0$$

and the ordinate of the modified curve of stability is $\frac{180}{\pi} \cdot \sin \theta$. The initial angle OA is taken as 120°, and it is seen that the curves cross the base line together at an abscissa of 0·685T and double this, viz. 1·37T, is the period of the single swing from 120°, T being the time of a small oscillation. The working of this problem for various initial angles is recommended as an interesting exercise, the results for 30°, 60°, 90°, 120°, 150° should be 1·017, 1·073, 1·183, 1·373, 1·762 times T respectively.

Example.—H.M.S. *Devastation*, with GM of 4 feet, has a curve of stability whose ordinates every 5° are 0, 0·36, 0·69, 0·80, 0·82, 0·78, 0·66, 0·44, 0·20, and range $43\frac{1}{2}°$. The period T for small angles is 6·75 sec. Find the period if heeled to (*a*) 20°, (*b*) 30° from the upright and allowed to roll freely.

Ans. (*a*) 8·1 sec., (*b*) 10·3 sec.

In the above we have to find graphically the C.G. of a trapezoid with reference to an ordinate. This is found as follows: Make AC = $\frac{1}{3}$. AE (Fig. 126), DQ being the middle ordinate. Join CQ and draw SH parallel to the base line, then H is in the same abscissa as the C.G. of the trapezoid.

Proof.—The shift of G. from middle ordinate is due to the shift of the triangle bQS to the position aRQ through a distance of $\frac{4}{3} . h$, $2h$ being the base. The area of triangle bQS is $\frac{1}{2} . h$. PQ. The moment of transference is therefore $\frac{2}{3}$. PQ . h^2. This equals (area of rectangle) × (shift of C.G. $\bar{x}$), or $\frac{2}{3}$. PQ . $h^2 = 2h$. DQ . $\bar{x}$,

or $$\bar{x} = \frac{PQ}{DQ} \cdot \frac{h}{3} = \frac{PH}{CD} \cdot \frac{h}{3} = PH$$

This may also be proved from the result of Example 22, Chap. II., which is left as an exercise.

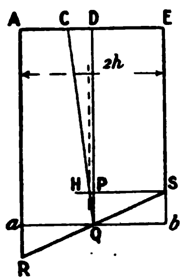


FIG. 126.

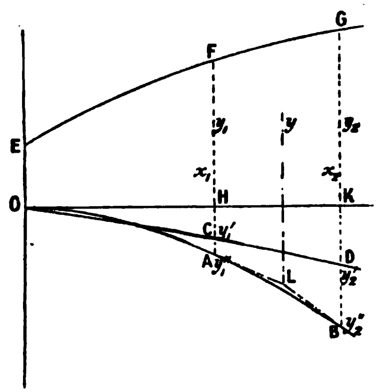


FIG. 127.

Proof that the intersection of tangents at two points of a curve is at the abscissa of the centre of gravity of the corresponding portion of its second derived curve.—Let the curves EFG, OCD, OAB (Fig. 127) be three curves such that any ordinate of OCD is equal to the integral of EFG up to that point, and any ordinate of OAB is equal to the integral of OCD up to that point. *Then the tangents to the curve* OAB *at the points* A *and* B *will intersect at the abscissa of the centre of gravity of the area* HG.

EFG is the second derived from OAB.

Let equation of EFG be $y = f(x)$
" OCD be $y = f'(x)$
" OAB be $y = f''(x)$

Then we have $f'(x) = \int f(x)dx$, and $f''(x) = \int f'(x)dx$

or $f(x) = \dfrac{d}{dx}f'(x)$ and $f'(x) = \dfrac{d}{dx}f''(x)$

Take two abscissæ x_1 and x_2 for which the ordinates of EFG are y_1 and y_2, of OCD are y_1' and y_2', and of OAB y_1'' and y_2''.

$$\text{Then } y_1 = \left(\frac{dy'}{dx}\right)_1 = \left(\frac{d^2y''}{dx^2}\right)_1$$

$$y_2 = \left(\frac{dy'}{dx}\right)_2 = \left(\frac{d^2y''}{dx^2}\right)_2$$

The abscissa $\bar{x}$ of the C.G. of HG is given by

$$\bar{x} = \frac{\int_{x_1}^{x_2} y \,.\, x \,.\, dx}{\int_{x_1}^{x_2} y \,.\, dx} = \frac{\int_{x_1}^{x_2}\left(\frac{dy'}{dx}\right)x \,.\, dx}{\int_{x_1}^{x_2}\left(\frac{dy'}{dx}\right) . \, dx}$$

$$= \frac{\Big[x \,.\, y'\Big]_{x_1}^{x_2} - \int_{x_1}^{x_2} . \, y' \,.\, dx}{\int_{x_1}^{x_2} . \left(\frac{dy'}{dx}\right)dx} = \frac{x_2 y_2' - x_1 y_1' - y_2'' + y_1''}{y_2' - y_1'}$$

$$= \frac{x_2 . \left(\frac{dy''}{dx}\right)_2 - x_1\left(\frac{dy''}{dx}\right)_1 - y_2'' + y_1''}{y_2' - y_1'}$$

Now the tangent to OAB at A has the equation

$$y - y_1'' = \left(\frac{dy''}{dx}\right)_1 (x - x_1)$$

and the tangent at B has the equation

$$y - y_2'' = \left(\frac{dy''}{dx}\right)(x - x_2)$$

Solving for x will give their intersection, or

$$x = \frac{x_2\left(\frac{dy''}{dx}\right)_2 - x_1\left(\frac{dy''}{dx}\right)_1 + y_1'' - y_2''}{\left(\frac{dy''}{dx}\right)_2 - \left(\frac{dy''}{dx}\right)_1}$$

which is the same expression as the abscissa of the centre of gravity of HG found above.

We now have to consider the case of: 2. *Resisted Rolling in Still Water.* We have seen that with resisted rolling the decrement on a single swing can be written

$$-d\theta = \tfrac{1}{2}k_1 \cdot \frac{\pi^2}{W.m.T} \cdot \Theta_m + \tfrac{4}{3} \cdot k_2 \cdot \frac{\pi^2}{W.m.T^2} \cdot \Theta_m^2 \left\{ \begin{matrix}\text{in circular} \\ \text{measure} \\ \text{units}\end{matrix}\right.$$

$$\text{or } -d\phi = \tfrac{1}{2}k_1 \cdot \frac{\pi^2}{W.m.T}\phi_m + \tfrac{4}{3} \cdot \frac{\pi}{180} \cdot k_2 \cdot \frac{\pi^2}{W.m.T^2} \cdot \phi_m^2 \text{ degrees,}$$

i.e. a and b in the decremental equation $-d\phi = a\phi + b\phi_2$ can be written

$$a = \tfrac{1}{2} \cdot k_1 \cdot \frac{\pi^2}{W.m.T}$$

$$b = \tfrac{4}{3} \cdot \frac{\pi}{180} \cdot k_2 \cdot \frac{\pi^2}{W.m.T^2}$$

from which k_1 and k_2 can be determined if a and b are known.

The moment of resistance $= k_1 \cdot \frac{d\theta}{dt} + k_2 \cdot \left(\frac{d\theta}{dt}\right)^2$

Substituting for the unknowns k_1 and k_2 we have—

Moment of resistance

$$= W.m\left[\frac{2.T}{\pi^2} \cdot a \cdot \frac{d\theta}{dt} + \tfrac{3}{4} \cdot \frac{T^2}{\pi^2} \cdot \frac{180}{\pi} \cdot b\left(\frac{d\theta}{dt}\right)^2\right] \qquad \frac{d\theta}{dt}\left\{\begin{matrix}\text{in circular} \\ \text{measure}\end{matrix}\right.$$

$$= W.m.\frac{\pi}{180}\left[\frac{2.T}{\pi^2} \cdot a \cdot \frac{d\phi}{dt} + \tfrac{3}{4} \cdot \frac{T^2}{\pi^2} \cdot b \cdot \left(\frac{d\phi}{dt}\right)^2\right] \qquad \frac{d\phi}{dt}\left\{\begin{matrix}\text{in de-} \\ \text{grees.}\end{matrix}\right.$$

The equation of motion of the rolling ship is now

$$\frac{W}{g} \cdot k^2 \cdot \frac{d^2\theta}{dt^2} + W.m.\left[\frac{2.T}{\pi^2} \cdot a \cdot \frac{d\theta}{dt} + \tfrac{3}{4} \cdot \frac{T^2}{\pi^2} \cdot \frac{180}{\pi} \cdot b\left(\frac{d\theta}{dt}\right)^2\right] + W.GZ = 0$$

or $\frac{d^2\theta}{dt^2} + \frac{\pi^2}{m \cdot T^2}\left[m\left\{\frac{2T}{\pi^2} \cdot a \cdot \frac{d\theta}{dt} + \frac{3}{4} \cdot \frac{T^2}{\pi^2} \cdot \frac{180}{\pi} \cdot b\left(\frac{d\theta}{dt}\right)^2\right\} + GZ\right] = 0$

and for $\frac{d\theta}{dt}$ in degrees we have—

$$\frac{d^2\phi}{dt^2} + \frac{\pi^2}{m \cdot T^2}\left[m \cdot \frac{\pi}{180}\left\{\frac{2T}{\pi^2} \cdot a \cdot \frac{d\phi}{dt} + \frac{3}{4} \cdot \frac{T^2}{\pi^2} \cdot b \cdot \left(\frac{d\phi}{di}\right)^2\right\} + GZ\right] = 0$$

In the former investigation we multiply GZ by $\frac{180}{\pi m}$, and we do the same for the resistance, which becomes

$$\frac{2T}{\pi^2} \cdot a \cdot \frac{d\phi}{dt} + \frac{3}{4} \cdot \frac{T^2}{\pi^2} \cdot b \cdot \left(\frac{d\phi}{dt}\right)^2$$

which is termed the "*resistance indicator.*" This has to be added to the modified force, if the ship is swinging away from the upright when resistance acts with stability in stopping the motion, and subtracted if the ship is swinging towards the upright when resistance acts against the stability.

If $\frac{d\phi}{dt} = 10$ degrees per second, then the ordinate of the resistance indicator is

$$\frac{2T}{\pi^2} \cdot a \cdot 10 + \frac{3}{4} \cdot \frac{T^2}{\pi^2} \cdot b \cdot 100$$

This would be set off as an ordinate at a point C (Fig. 128), such that AC $\div$ 1·013 T = 10 degrees per second, and so on.

Guess in CF resisted and CG unresisted. AD is the mean force over the interval, and we set up OL = AD. Then the slope of LQ gives the angular velocity at the end of interval, MP is therefore the slope of position curve. Then the angle BN should give on the modified stability curve the distance BG, FG being equal to the ordinate of resistance indicator at L. Thus by a process of trial and error we obtain a series of tangents to the position curve and this crosses the base line simultaneously with the stability curve. Proceeding past the upright we should obtain not only the time of the single oscillation but also the angle from the upright to which the vessel rolls. The values of *a* and *b* for the resistance indicator are obtained from the decremental equation for the ship.

As exercises, the "Inconstant" may be taken whose decremental equation gives $a = 0{\cdot}035$, $b = 0{\cdot}0051$. Starting from 30

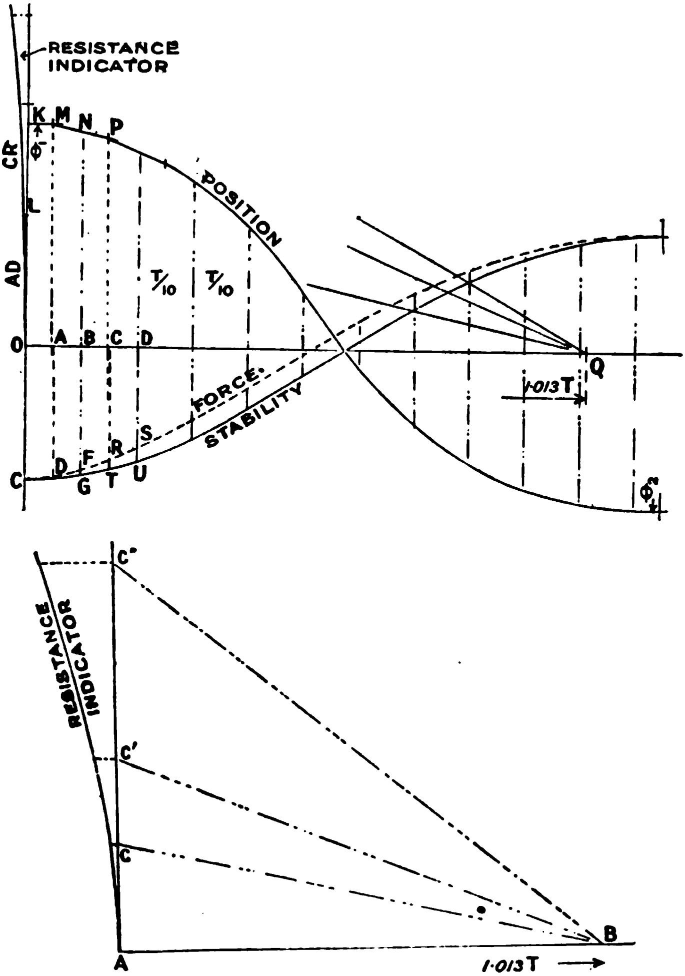


FIG. 128.

the angles reached in successive rolls are 25·2°, 21·6°. Starting from 25° the angles reached are 21·3°, 18·7°. T = 8 sec., GM = 2·3 feet. Stability curve at 10° intervals 0, 0·5, 1·03, 1·7, 2·43, 2·75, 2·61, 2·06, 1·55.

3. *Unresisted Rolling among Waves.*—For a ship broadside on to a given wave the stability at any instant is determined by the angle between the centre line of the ship and the *virtual upright*, *i.e.* the normal to the wave surface. This is the angle between O*a* and O*b* in Fig. 129. We therefore draw on our

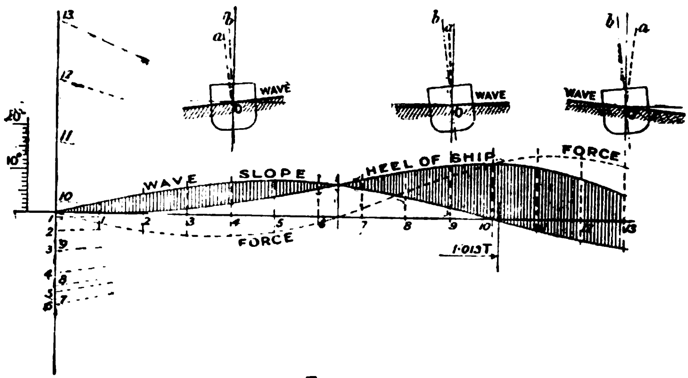


FIG. 129.

base line a curve of wave slope which is taken as a curve of sines—

$$\phi_1 = \Phi_1 \, . \sin \frac{\pi}{T_1} . t$$

Thus for a wave of maximum slope 8° and 8″ half period we have $\phi_1 = 8 \times \sin \frac{180}{8} \cdot t$. This then is the base line from which to measure the angle of inclination to the *virtual upright*. The process is then carried on as before. In Fig. 129 is worked out the case of a ship upright and at rest in the wave trough with straight line stability on a wave of 80° maximum slope, and whose half period T_1 is the same as the time of oscillation T of the ship. These conditions are known from previous investigations to lead to an increment of roll to every half wave of $\frac{\pi}{2} \times 8 = 12\frac{1}{2}°$. The tangents to the position curve are parallel to the lines drawn from 1, 2, 3, etc., to the point given by 1·013T. The force curve is checked interval by interval with the angle from the virtual upright, and must cross the base line at the abscissa of the intersection of the wave slope curve with position curve. If the example in Fig. 129 be continued to 20 intervals or over the complete wave, an angle of 25° would be reached. This is recommended as an exercise. The process can, of course, be applied for a given curve of stability and any assumed conditions for period of ship, period of wave, and maximum slope of wave.

4. *Resisted Rolling among Waves.*—In this case the process is similar, only the effect of the "resistance indicator" is brought in as in 2.

For an exhaustive account of the application of the process of graphic integration, see Sir W. H. White's paper (I.N.A. 1881) on the rolling of sailing ships. In this paper, in addition to resisted rolling among waves, account was taken of (*a*) moment due to pressure of the wind on the sails, and (*b*) the variation of the virtual weight in different portions of the wave, this being necessary as this variation affects the righting moment, while (*a*) is not thus affected.

Pitching.—The expression for the period of pitching of a ship is of a similar form to that for rolling, but we have to use k_1 the radius of gyration of the vessel about a *transverse axis* through the centre of gravity of the vessel, and GM_L the longitudinal metacentric height. This period for a single oscillation is therefore—

$$T_1 = 0·554\sqrt{\frac{k_1}{GM_L}}$$

It would be desirable, if other conditions allowed, to make the period of pitching as small as possible, and ships with the heavy weights concentrated near midships are found to be better sea boats than vessels with heavy weights at the ends.

Examples.

1. A vessel of 13,500 tons displacement has a GM of 3½ feet and a period of 8½ seconds. Find the period of roll when 600 tons of coal are added each side of the vessel in a bunker 21 feet deep and 9 feet wide, the C.G. of the bunkers being 11 feet below the original C.G. of the

ship, and 26 feet out from the middle line. The vessel has a horizontal curve of metacentres over the limits of draughts corresponding to the above conditions.

For the ship originally $T = 0.554\sqrt{\frac{k^2}{m}}$ from which $k^2 = 823$, so that $I = 13,500 \times 823 = 11,100,000$.

The addition of the coal 11′ down pulls down the C.G. of the ship $\frac{1200 \times 11}{14700} = 0.9$ ft., making the GM 4·4 ft.

We now have to calculate the new I about the new C.G., I of coal about old C.G. is given by

$2[(\frac{1}{12} \times 600 \times 9^2 + 600 \times \overline{26}^2) + (\frac{1}{12} \times 600 \times \overline{21}^2 + 600 \times \overline{11}^2)] = 1,008,600$

I of total about old G.C. is accordingly

$$11,100,000 + 1,008,600 = 12,108,600$$

and about the new C.G.

$$12,108,600 - (14,700 \times \overline{0.9}^2) = 12,096,700$$

The new k^2 is therefore

$$\frac{12,096,700}{14,700} = 823 \text{ ft.}^2$$

and the new period is accordingly

$$0.554\sqrt{\frac{823}{4.4}} = 7.6 \text{ secs.}$$

2. A cruiser of 5000 tons has a metacentric height of 2·8 feet, a period of 7 seconds, and a horizontal curve of metacentres. Calculate the period when two fighting tops of 10 tons each are added to the ship at a height of 70 feet above the C.G.

Ans. 7·5 secs.

CHAPTER XIII.

THE TURNING OF SHIPS.

WHEN a ship is moving ahead and the rudder is placed obliquely to the middle line, the streams of water which flow aft relative to the ship are deflected in their course and give rise to a resultant pressure normal to the plane of the rudder, as P in Fig. 130. The calculation of the amount of this normal pressure will be dealt with later, but it may be stated here that it depends on—

(i) The area of the rudder.
(ii) The shape of the rudder.
(iii) The angle at which the rudder is placed to the centre line.
(iv) The *square* of the speed of the water past the rudder.

The area of the rudder is usually expressed in terms of the area of the longitudinal middle line plane of the ship, or approximately the length times the mean draught.

or $$\text{area of rudder} = \frac{1}{m} \cdot \text{L.D.}$$

The value of m varies considerably. In large war vessels it is from 40 to 50, but in exceptional cases, where great manœuvring power was desired, it came out to 33. In the *Lusitania* its value was about 60.

As regards the shape, the pressure for a given area will be appreciably greater for a narrow, deep rudder than for a broad, shallow rudder.

The usual maximum angle to which rudders are put is 35° to the centre line.

In a sailing vessel the speed of the water past the rudder is rather less than the speed of the ship, because there is the *frictional wake.* The friction of the water on the surface of the vessel induces a current of water in the direction of the ship's motion so that at the stern the water a short distance away from the ship has a forward motion. In a sailing ship, in order to get pressure on the rudder, it is necessary that the ship shall be in motion, and such a ship loses her power of steering as she loses way.

In a ship driven by a propeller, although there is the same frictional wake, the action of the propeller sends a stream of water astern, so that such a ship has steerage directly the engines are working, a very great advantage. And this effect will be greater for a ship having the propeller in line with the rudder, as in a single-screw ship and in ships with double rudders like the *Dreadnought*, than in ordinary twin-screw vessels. Although in screw vessels there is the frictional wake mentioned above, the speed of the water past the rudder will be appreciably greater than the speed of the ship, because the speed at which the water leaves the propeller is greater than the speed of the ship, the difference being known as the slip.

In any case it is absolutely necessary for good steering that the water shall get a clean run past the rudder. Vessels with very full sterns have been found to steer very badly.

In a ship having a deadwood in front of the rudder the slackening of the speed of the streams of water gives rise to a side pressure which has a considerable influence in pushing the ship over *at the start*. This is specially noticeable in boats. But for good turning the deadwood is unfavourable, as will be seen later, and in ships designed for great manœuvring power the deadwood is always cut away. See sterns on pages 424, 425.

In Fig. 130 let P be the normal pressure acting on the rudder at C. Introduce at G, the centre of gravity of the ship, two equal and opposite forces of value P parallel to the line of action of P.

Then we have acting on the ship—

(i) A *couple* tending to give angular motion of amount $P \times DG$; and

(ii) a force P acting in the line EG.

The couple is approximately equal to $P \times \frac{L}{2} \times \cos \theta$, and if P is taken to vary as sin θ, the couple will vary as sin $\theta \times \cos \theta$ or sin 2θ, and it can be shown that this is a maximum when $\theta = 45°$. The sine law, however, for the pressure is known to be incorrect, and it is very probable that the usual practice of 35° as the maximum helm angle gives the maximum turning effect. Indeed, it appears quite possible for certain shapes of rudders that angles of helm as low as 25° will give as good turning results as 35°.

The force P acting in the line EG may be resolved into its components.

(i) FG, tending to move the ship bodily sideways. The motion in this direction, however, is small because of the great resistanceof the ship to the side motion.

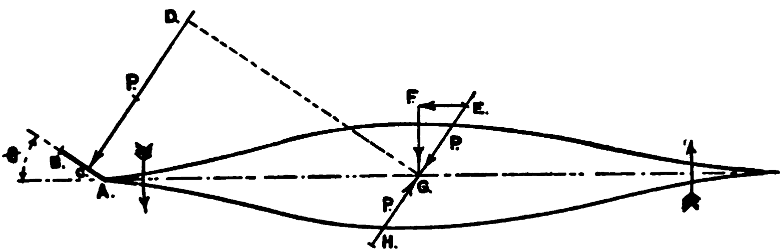


FIG. 130.

(ii) EF, in a fore and aft direction, and this has a sensible effect in checking the speed of the ship.

Path of Ship when Turning.—On putting the rudder over the ship will commence to turn in a spiral path, of which several examples are given later, and this path soon becomes approximately circular. The distance from the position when the helm is put over to the position when the ship is at right angles to her original course is termed the *advance*. The distance from the position when the helm is put over to the position when she has turned through 180° or 16 points is termed the *tactical diameter*.

In a ship thus turning, the middle line of the ship points inside the circular arc and the thrust of the propellers is accordingly in a direction oblique to the path of the ship. The resistance to motion is therefore much greater than when on the straight, and this, together with the fore and aft component due to the rudder, EF in Fig. 130, causes a very considerable reduction of speed. In one case the reduction amounted to quite 50 per cent.

If in Fig. 131 the ship is turning in the path G_1GG_2 passing through the C.G., AF being the centre line of the ship and O the centre of the arc G_1GG_2, then the angle between the tangent

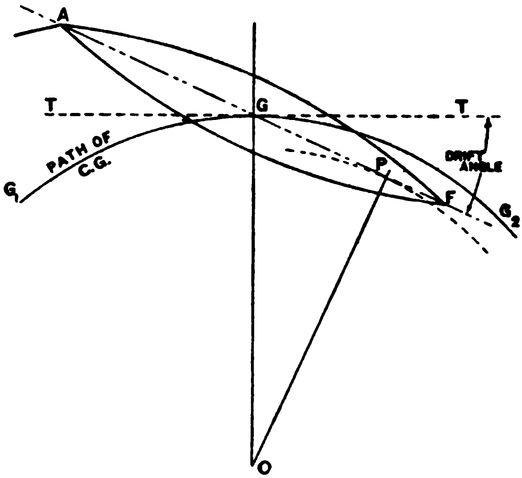


FIG. 131.

GT and the centre line is termed the *drift angle* at the point G. If OP is drawn perpendicular to the centre line there is no drift angle at P, and to an observer on board at P all points of the ship abaft P will appear to be moving to port, and all points forward of P will appear to be moving to starboard. Such a point P is termed a *pivoting point*, as the ship appears to pivot about P.

The features of a ship which influence the turning are principally as follows:

(*a*) Time taken to put the helm over to the maximum angle.
(*b*) Pressure on the rudder.
(*c*) Moment of resistance of the underwater body of the ship to turning.

(*d*) Moment of inertia of the vessel about a vertical axis through the centre of gravity, to which has to be added the mass of water associated with the ship.

(*a*) The introduction of steam steering gear has rendered this item of less importance than formerly. In ships steered by manual power the time taken to put the helm over is considerable, and consequently the possibility of quick manœuvring is small.

The general adoption of balanced rudders has facilitated getting the helm over quickly, as the centre of pressure of the rudder is close to the axis and the moment required to be overcome is comparatively small.

(*b*) The pressure on the rudder depends on various factors, which have already been dealt with above.

(*c*) The ship when turning has angular velocity round the pivoting point P. If we take any portion of the ship at a distance l from P and of area A, the velocity through the water due to the angular motion is $l \cdot \frac{d\theta}{dt}$ and the resistance varies as $A \cdot l^2 \cdot \left(\frac{d\theta}{dt}\right)^2$, and the moment of this about P varies as $A \cdot l^3 \cdot \left(\frac{d\theta}{dt}\right)^2$. This therefore varies as the cube of the length and the square of the angular velocity. This moment at the early stages is small and less than the couple caused by the pressure on the rudder, and consequently the angular velocity increases. A point, however, is reached when the couple due to the pressure on the rudder is equal to the moment of resistance, and then the ship has a constant angular motion.

It is seen by the above that if areas of the ship under water can be omitted where l is greatest, the resistance to the angular movement may be considerably reduced. This is done by the omission of the *deadwood*, or the flat vertical portion of the ship aft, as the pivoting point is usually well forward.

The above may be illustrated by the three turning circles given in Fig. 132 of *Orlando*, *Astræa*, and *Arrogant*. The profiles of the ships are given, from which it will be seen that—

(i) *Orlando* has a square type of rudder, not balanced, and the ship is 300 feet long. The former factor will delay her entry into the circular path.

(ii) *Astræa* has a balanced rudder, and the ship is 320 feet long. The ship gets into the circular path quicker owing to the balanced rudder, but has a larger turning circle owing to the greater length.

(iii) *Arrogant*. Here the rudder area is relatively large, two rudders being fitted. The length is 320 feet, as *Astræa*, but the stern is cut up considerably. The influence of these factors is seen in the very small circle, as compared with the *Orlando* of smaller length and the *Astræa* of the same length.

Many merchant vessels now follow the practice of having balanced rudders with the deadwood aft cut away. In the *Dreadnought* the provision of two rudders with propellers immediately in front and the cut-up shape of the stern (as Fig. 144) resulted in a marked reduction of the turning circle. The following is the comparison with two cruisers of nearly the same length, one having a balanced rudder with no cut-up and the other having a balanced rudder with cut-up :—

	Length in feet.	Rudder.	Cut-up.	Tactical diameter.	
				Yds.	In terms of ship's length.
Powerful	500	Balanced	None, as Fig. 137	1100	6·6
Duke of Edinburgh	480	Do.	As Fig. 143	740	4·63
Dreadnought ...	490	2 No Balanced	As Fig. 144	463	2·84

The influence on the turning of a ship of a propeller acting directly on the rudder is strikingly illustrated by the comparative

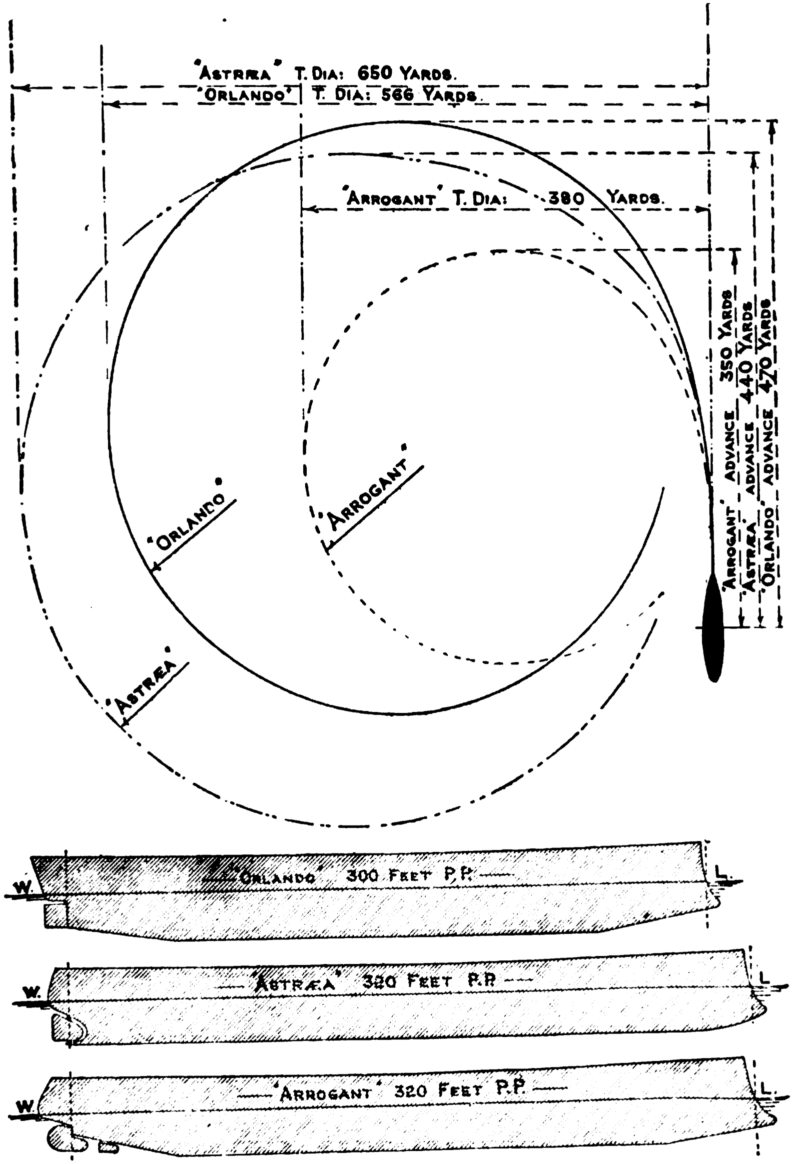

"ASTRÆA" T. DIA: 650 YARDS.
"ORLANDO" T. DIA: 566 YARDS.
"ARROGANT" T. DIA: 380 YARDS.
"ARROGANT" ADVANCE 350 YARDS
"ASTRÆA" ADVANCE 440 YARDS
"ORLANDO" ADVANCE 470 YARDS
"ORLANDO"
"ARROGANT"
"ASTRÆA"
W.
L.
"ORLANDO" 300 FEET P.P.
"ASTRÆA" 320 FEET P.P.
"ARROGANT" 320 FEET P.P.

FIG. 139

turning circles of *Topaze* and *Amethyst*. These were cruisers of similar dimensions, viz. 360′ × 40′ × 14½′ draught, 3000 tons displacement. The shapes of the sterns are given in Fig. 133, from which it will be seen that the former ship was a twin-screw ship and the latter ship a triple-screw ship. This latter was the ship fitted with Parsons' turbines, with small screws running at high revolutions, and had one screw immediately in front of the rudder. The rudders of the two ships were of the same type, viz. balanced, and of much the same area.

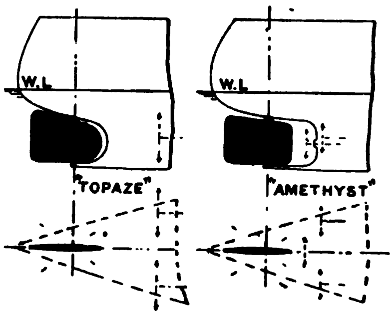


FIG. 133.

The turning circles are given in Fig. 134, from which it is seen that the twin-screw ship has a tactical diameter of 870 yards and the triple-screw ship has a tactical diameter of 550 yards, or 7·25 and 4·6 times the length of ship respectively. It is also seen that the latter ship gets into the circular path much sooner than the former ship. All the conditions are practically identical, except that the ship with the smaller circle has one propeller operating immediately on the rudder.

(*d*) The moment of inertia of a ship about a vertical axis through the C.G. depends on the longitudinal distribution of the weight, which of course is decided upon for other reasons than turning. A ship with great weights at the ends will have a large moment of inertia, and a given turning moment due to the pressure on the rudder will take longer to get the ship into the circle than if the weights were more amidships. (It will be remembered that moment of inertia about any given axis is found by adding together the products of each portion of the weight and the square of its distance from the axis, or $\Sigma . w . l^2$.)

Shapes of Sterns and Rudders.—Fig. 135 shows the ordinary type of rudder fitted to merchant vessels.

In some Atlantic and other liners what is termed the "cruiser type" of stern and rudder is adopted, analogous

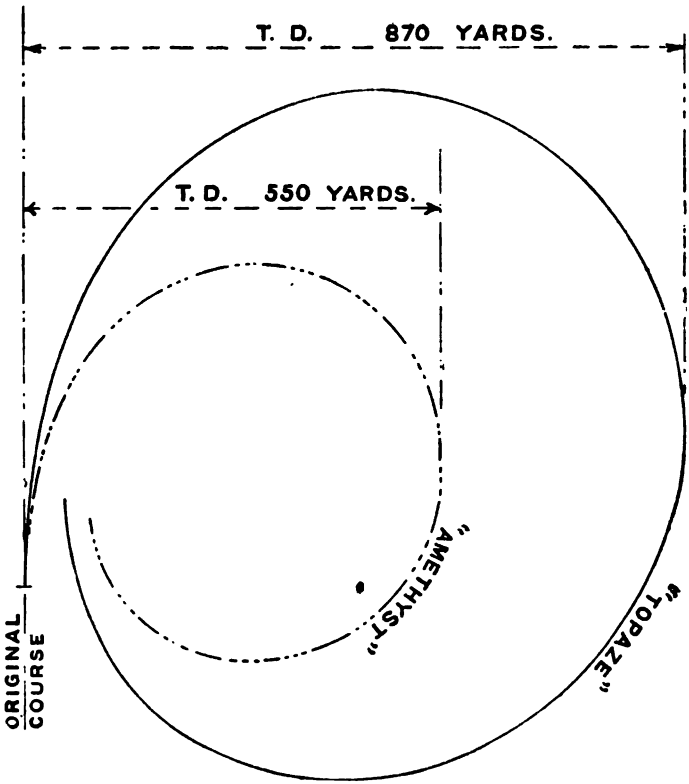


FIG. 134.

to Fig. 143, the principal advantage being the increase of the length of the waterline obtained for a given length over all.

Fig. 136 gives the stern and rudder adopted in the *Aquitania.* The normal shape of overhanging stern above water is obtained, but the deadwood, is cut away and a balanced rudder obtained with the rudder-head below water. This gives the important advantage of having the rudder-head and steering gear under water and less liable to damage due to gun-fire. The vessel was built to be an auxiliary cruiser in case of necessity.

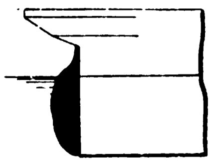


FIG. 135.

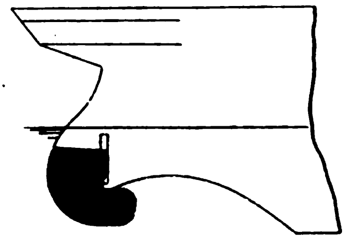


FIG. 136.

Figs. 137 to 145 give a number of different shapes of sterns and rudders adopted in war vessels.

Fig. 137 was adopted for many vessels, including the protected cruisers *Powerful* and *Terrible*. The weight of the rudder is taken inside the ship, a steadying pintle only being provided at the bottom of the sternpost.

Fig. 138 was adopted in the *Arrogant* class, designed as cruisers to company with the Fleet and in which exceptional turning facilities were desired. Two rudders are employed, and the deadwood is cut away.

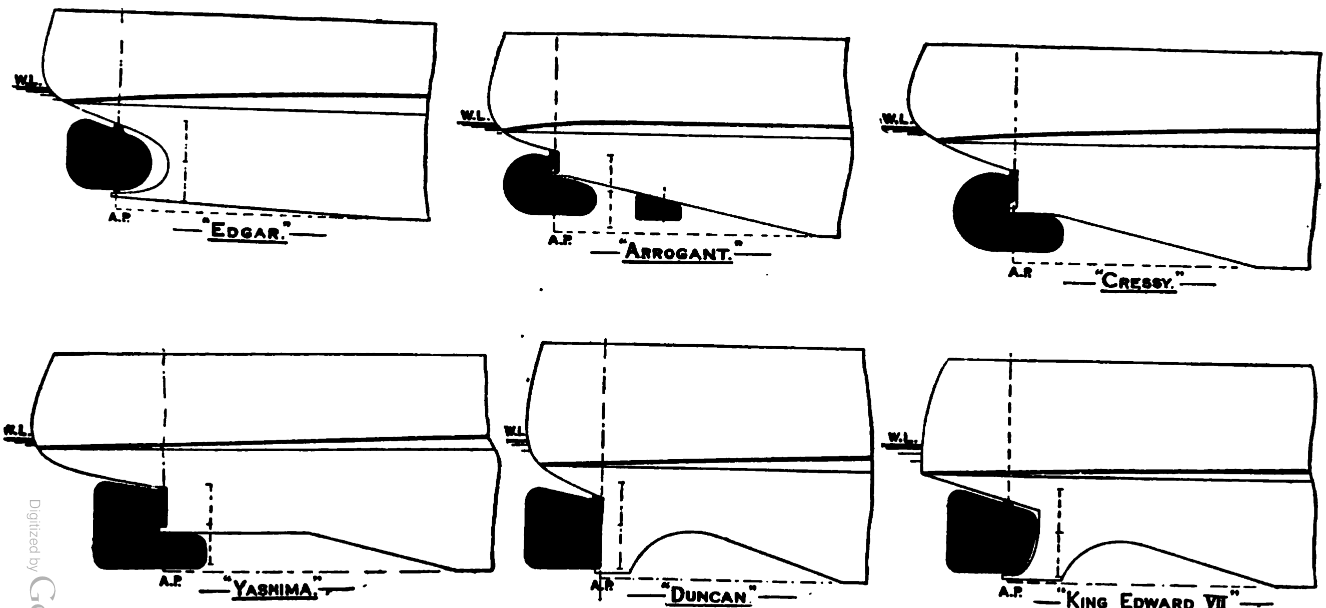
Fig. 139 is on similar lines with a single rudder.

Fig. 140 is for the Japanese battleship *Yashima*, which had very great facility for turning.

Fig. 141 was the type of stern adopted for many battleships; the rudder is approximately square, and is unbalanced. The deadwood is cut away and the sternpost brought down to take the blocks for docking.

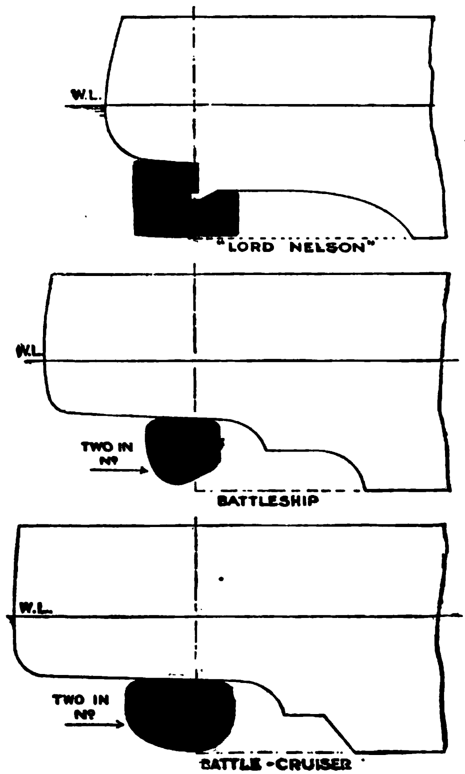
Fig. 142 is the stern adopted in the *King Edward VII.* class, the rudder being partially balanced.

Fig. 143 is the stern of the *Lord Nelson* class, similar to that of the *Yashima*.



FIGS. 137-142.—Shapes of stern and rudder in various ships.

Figs. 144 and 145 show the double rudders employed in most of the *Dreadnought* battleships and battle cruisers. The rudder area by this means was made relatively large, and good powers of turning resulted, in spite of the greatly increased



FIGS. 143, 144, 145.

length as compared with previous ships. In the most recent ships a stern and rudder on the lines of Fig. 143 have been adopted because of the resistance caused by the bossing out to take the heads of the twin rudders.

Turning Trials.—It is usual to carry out systematic turning trials on H.M. ships, and these are put on record for the information of the officers.

The following is the method employed to determine the path of the ship when turning. Two points are selected near the ends of the ship, at a known distance apart, and at these positions a horizontal circle graduated in degrees, etc., is set up with a pointer moveable in a horizontal plane, having sights which can be kept bearing on any given object as the vessel swings round. Two weighted rafts with a flag attachment are dropped overboard about a mile or so apart; it is assumed that these rafts remain stationary relative to the ship.

The ship is brought up to one of these rafts, at the speed desired, so as to pass the raft as nearly as can be judged at

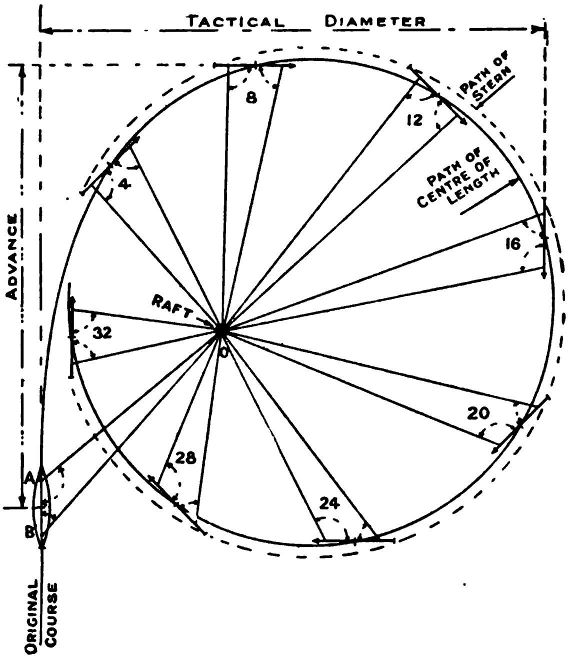


FIG. 146.

the distance of the radius of the turning circle expected away. Shortly before coming broadside on, a signal is made, when the rudder is put over, the course is noted and the time is taken, and also the angles shown at the positions forward and aft, viz. OAB, OBA (Fig. 146), are recorded. These angles with the known distance AB fully determine the triangle OAB and consequently the position of the ship relative to the raft. A similar signal is made at four points (45°), eight points (90°), etc., and corresponding observations taken until the ship has completed the circle. The nine triangles found from this information are then set out on a convenient scale, as shown in Fig. 146, and the path of the ship drawn in. The "tactical diameter" and the "advance" can then be measured off.

Angle of Heel when turning.—On first putting a rudder over, the force on the rudder being usually below the centre of pressure on the hull on the opposite side, the resultant couple will have a tendency to heel the ship inwards, but this tendency is of short duration, as when the ship gets into her circular path centrifugal action comes into play and an outward heel results. It is shown in Chap. V. that this heel θ is given by—

$$\sin\theta = 0.088 . \frac{V^2}{R} . \frac{d}{GM}$$

where V is speed in knots;
R is radius of turning circle;
GM is metacentric height;
d is distance of centre of lateral resistance below the C.G.

A ship, therefore, of high speed, small turning circle, and small metacentric height will be liable to heel considerably when turning at full speed.

Strength of Rudder-heads.—The formula used by the British Corporation is as follows:—

$$d = 0.26 \sqrt[3]{R . A . S^2}$$

where A is area up to water-line in square feet;
R is distance of the C.G. of the area from the pintles;
S is not to be less than 11 knots in vessels of and over 250 feet in length.

In vessels of 100 feet, speed taken as 8 knots. Intermediate lengths at intermediate speeds in proportion.

Lloyd's Rules do not now give a formula, but give diameters of rudder-heads for speeds varying between 10 and 22 knots for different values of A × R as defined above.

Direct Method of determining the Diameter of Rudder-head.—The normal pressure on a rudder of area A square feet at angle of helm θ is usually assumed to be

$$P \text{ in lbs.} = 1{\cdot}12\, A\,.\,v^2\,.\,\sin\theta$$
$$= 3{\cdot}2\, A\,.\,V^2\,.\,\sin\theta$$

where v and V is speed of water past the rudder in ft. per sec. and knots respectively.

It is usual to allow a percentage on to the speed of the ship to allow for the slip of the screw, although at the stern of the ship there is the "frictional wake." About 10 per cent. probably is well on the safe side. V is therefore taken at 1·1 times speed of ship.

In addition to knowing the pressure, it is necessary to know the point at which the centre of pressure acts in order to find the twisting moment about the axis. At 35° the centre of pressure is taken at three-eighths the breadth from the leading edge for a rectangular rudder. For other shapes of rudder the area may be divided approximately into rectangles, or we may adopt the method given later by dividing into a number of strips.

Having obtained the twisting moment (preferably in inch-tons), we equate to the formula

$$T = \frac{1}{16}\,.\,\pi\,.\,f\,.\,d^3$$

where d is diameter of rudder in inches;

f is factor of strength allowed, say—

4 tons for wrought iron,

5 „ cast steel,

3 „ phosphor bronze.

The following example will illustrate the method:—

A rudder is 243 square feet in area, and the centre of pressure is estimated to be 6·12 feet abaft the centre of rudder-head at 35°. If the speed of ship is 19 knots, estimate the diameter of the rudder-head if of cast steel.

$$\text{Pressure in tons} = \frac{3\cdot2}{2240} \times 243 \times 20\cdot9^2 \times 0\cdot574 = 87 \text{ tons}$$

$$\text{Twisting moment} = 87 \times 6\cdot12 \times 12 = 6389 \text{ inch-tons}$$

$$\therefore \tfrac{1}{16} \cdot \pi \cdot 5 \cdot d^3 = 6389, \text{ taking } f = 5$$

from which $d = 18\cdot7$ inches.

Note.—If such a rudder is assumed to be a square and supported by two pintles at the forward edge, one at the bottom and one half-way up, it can be shown that, where W is the total load and l the total depth, that—

$$\left.\begin{array}{ll}\text{Bending moment at head} & = \tfrac{1}{56} \cdot W \cdot l \\ \quad\text{,,} \qquad\text{,,} \qquad \text{mid-depth} & = \tfrac{3}{112} \cdot W \cdot l\end{array}\right\}\text{both of these are small}$$

$$\begin{array}{ll}\text{Force at head} & = \tfrac{13}{56}\, W \\ \text{,,} \quad \text{centre pintle} & = \tfrac{4}{7}\, W \\ \text{,,} \quad \text{lower pintle} & = \tfrac{11}{56}\, W\end{array}$$

The above is an example of where pure twisting only need be considered (as for Fig. 141), but there are other cases to consider—

(i) Rudder-head fixed in direction at sternpost, and the lower part supported at the bottom (as in Figs. 137 and 142).

(ii) Rudder-head fixed in direction at sternpost, and rudder supported about half-way up, the bottom being free (as in Figs. 140 and 143).

(iii) Rudder fixed in sternpost and the lower part unsupported (as in Figs. 144 and 145).

In (i) and (ii) both bending and twisting come into play. In (iii) bending is the determining factor in calculating the diameter of the rudder-head. It is generally assumed that the sternpost holds the rudder-head fixed in position. This gives results well on the safe side.

(1) For the case (i) above, if the rudder is regarded as a beam uniformly loaded, it may be shown that

$$\text{Bending moment at upper end} = \tfrac{1}{8} \times \text{load} \times \text{depth}$$

$$\text{Support at head} = \tfrac{5}{8} \times \text{load}$$

$$\text{,,} \quad \text{,,} \quad \text{heel} = \tfrac{3}{8} \times \text{load.}$$

(2) For the case (ii) above, if the rudder is regarded as a beam loaded at lower half to twice the intensity of the upper

half (*i.e.* the rudder is assumed to be a square with the upper corner cut out), it may be shown that

$$\text{Bending moment at upper end} = \tfrac{1}{16} \times \text{load} \times \text{depth}$$

$$\left.\begin{array}{r} \text{Support at head} = \tfrac{7}{24} \times \text{load} \\ \text{,, \quad ,, heel} = \tfrac{31}{24} \times \text{load} \end{array}\right\} \text{in opposite directions.}$$

(3) For the case (iii) above, the bending moment at the head is found by multiplying the pressure by the distance of the C.G. below the top.

When both bending and twisting have to be considered, we equate $\frac{1}{16} \cdot \pi \cdot f \cdot d^3$ to the *equivalent twisting moment*, viz. $M + \sqrt{M^2 + T^2}$, where M is the bending moment and T the twisting moment.

It may happen that the astern conditions will be the determining factor, because then the centre of pressure is nearer the after edge of rudder and farther from the axis than when the ship is going ahead. The speed astern is usually taken at half the speed ahead. In any case, in designing a rudder, sections must be made at various places besides the rudder-head, and the formula $\frac{p}{y} = \frac{M}{I}$ applied to determine the value of the stress p.

In fixing the shape of a rudder it has to be borne in mind that at *all angles* the centre of pressure should be *abaft the axis*. For angles below 35° the value three-eighths from the leading edge does not apply. The following formula may be used, based on Joessel's experiments for rectangular plates of breadth b.

$$\text{C.P. from leading edge} = 0{\cdot}195\, b + 0{\cdot}305\, b \sin \theta$$

The formula given above, viz. $P = 1{\cdot}12 \cdot A \cdot v^2 \cdot \sin \theta$, is known to be incorrect for small plates moving through water, and the matter was exhaustively considered by Mr. A. W. Johns, R.C.N.C., at the I.N.A. for 1904. The formula, however, has been extensively employed for many years for rudder calculations with satisfactory results. It is to be observed that—

(1) A rudder does not get the full angle at once, so that

the pressure is not in the nature of a shock. We, however, use a coefficient of strength giving a large factor of safety, as if this were the case.

(2) By the time the rudder is over the speed of the ship suffers an appreciable check.

Mr. Denny (I.N.A., 1921), as the result of considerable investigation and experiment, put forward the following empirical formula for the diameter of underhung (spade) rudders in inches :—

$$D = \sqrt[3]{A \times \text{arm} \times V^{1 \cdot 85} \times C}$$

where A = area of rudder surface in square feet.
V = designed speed of ship in knots.
C = constant, say 0·2.
arm = distance of C.G. of rudder surface in feet from the bottom of rudder bearing.

Stress assumed to be 5 tons per square inch.

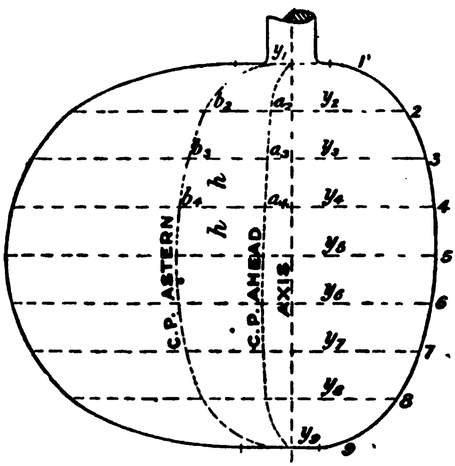


Fig. 147.

The above is only applicable to vessels with no centre propeller.

Centre of Pressure, Calculation of.—For rudder shapes other than rectangular it may be assumed that if the C.G. is x feet from the mid-breadth, the centre of pressure is x feet from the position it would have if the rudder were rectangular. Preferably the following method may be employed :—

Horizontal ordinates are drawn as shown (Fig. 147), common interval h, and $\frac{3}{8}$ the length from forward and after edges is set out on each. Such points represent the centre of pressure of strips of the rudder at the ordinates. Curves of centre of pressure are drawn as shown.

Simpson's Rules then are applied, as indicated in the following table:—

No.	Ordinate.	S. M.	Products for area.	Ahead. C.P. from axis.	Ahead. Product for moment.	Astern. C.P. from axis.	Astern. Product for moment.	Vertical Lever.	Product for moment.
1	y_1	1	y_1	a_1	a_1y_1	b_1	b_1y_1	—	—
2	y_2	4	$4y_2$	a_2	$4a_2y_2$,	b_2,	$4b_2y_2$,	1	$4y_2$
3	y_3	2	$2y_3$	a_3	etc.	etc.	etc.	2	$4y_3$
4	y_4	4	$4y_4$	a_4				3	$12y_4$,
5	y_5	2	$2y_5$	a_5				4	etc.
6	y_6	4	$4y_6$	a_6				5	
7	y_7	2	$2y_7$	a_7				6	
8	y_8	4	$4y_8$	a_8				7	
9	y_9	1	y_9	a_9				8	
			S_1		S_2		S_3		S_4

Area $= S_1 \times \frac{1}{3} \times h$ C.P. ahead $= \frac{S_2}{S_1}$ from axis

C.P. astern $= \frac{S_3}{S_1}$ from axis C.P. below top $= \frac{S_4}{S_1} \times h$

The size of the rudder-head at the steering gear cross-head will be determined by the maximum twisting moment ahead or astern.

The above method may be employed to approximate to the position of the centre of pressure at smaller angles by the use of Joessel's formula given above, in order to ensure that at all angles the centre of pressure is abaft the axis.

Example.

An underhung rudder is 12 feet broad and 7 feet deep, the axis being 4 feet from the forward edge. The forward portion is formed by a semi-circle 3½ feet radius and the after portion is rectangular. The clearance between the ship's bottom and the top edge of rudder is 3 inches. Speed of ship, 23 knots.

Make an estimate of the diameter of the rudder-head and of the maximum twisting moment that should be provided for in the steering gear. (B.Sc., Durham, 1919.)

Ans. About 15⅝ inches; about 440 inch-tons.

Mr. Denny's formula will give a smaller diameter.

CHAPTER XIV.

LAUNCHING CALCULATIONS.

BEFORE starting on these calculations it is necessary to estimate as closely as possible the launching weight of the ship, and also the position of the centre of gravity both vertically and longitudinally. The case of the *Daphne*, which capsized on the Clyde[1] on being launched, drew special attention to the necessity of providing sufficient stability in the launching condition. A ship in the launching condition has a light draught, great freeboard, and high position of the C.G. It is possible, by the use of the principles we have discussed at length, to approximate to the metacentric height, and if this is not considered sufficient, the ship should be ballasted to lower the centre of gravity. It has been suggested that a minimum G.M. of 1 foot should be provided in the launching condition. If the cross-curves of stability of the vessel have been made, it is possible very quickly to draw in the curve of stability in the launching condition, and in case of any doubt as to the stability, this should be done.

It is necessary to prepare a set of launching curves in the case of large heavy ships, in order to see that there is (*a*) no tendency for the ship to "tip," *i.e.* to pivot about the after end of the ways (as in Fig. 148 *c*), in which case damage would probably ensue; and (*b*) to obtain a value for the force which comes on the fore poppets when the stern lifts.

In Fig. 148 *b*, if G is the position of the C.G. of the ship, and B the centre of buoyancy of the immersed portion, then assuming a height of tide that may safely be expected for launching

the moment of the weight abaft the after end of ways $= W \times d$

" " buoyancy " " " " $= w \times d^1$

[1] See *Engineering* (1883) for a report on the *Daphne*, by Sir E. J. Reed.

Then for different portions of the travel down the ways the values of d, w, and d' can be readily found and curves drawn as in Fig. 149, giving values of $(W \times d)$ and $(w \times d)$ on a base of distance travelled. The former will, of course, be a straight line, starting from the point where the C.G. is over the aft end of ways. This line should be below the other curve, and the minimum intercept between them is called the

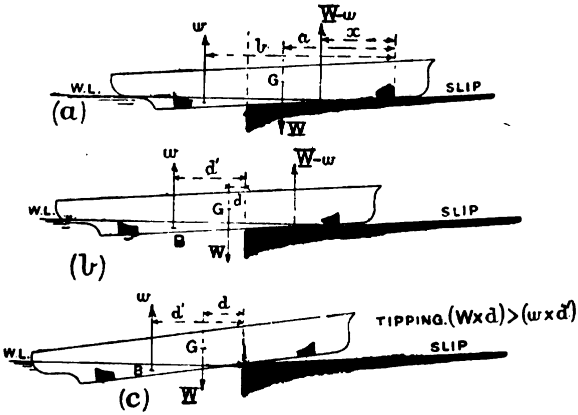


Fig. 148.

"margin against tipping." If it happens that the curves intersect, it shows that a tendency to tip exists, and either (*a*) the ways should be lengthened, or (*b*) ballast placed forward, both of which increase the travel required before G comes over the end of the ways. The buoyancy curve should be drawn out for various heights of tide, in order to know the minimum height of tide on which the ship could be safely launched. This is a point of importance in some shipyards where tides do not always rise as high as expected owing to adverse winds. Ships have been launched successfully which had a tipping moment, but owing to the speed of launching the danger space was safely passed; but this is a risk that few would care to take.

As the ship goes further down the ways a position is reached when the moment of the buoyancy about the fore poppet equals the moment of the weight about the fore poppet. At this point the stern of the ship will begin to lift. If W and w be the values of the weight and buoyancy respectively at this point, then the weight W − w, instead of being taken over the length of ways in contact, is concentrated at the fore poppets. This weight is localized over a short distance both on the ship and on the slip, and it is desirable to know its amount and the position on the slip where it will come.

Values of w are obtained at various points of the travel, and two lines drawn on a base of travel giving values of

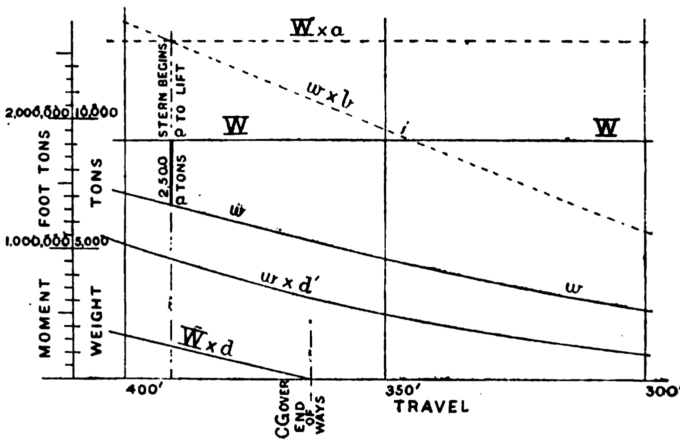


FIG. 149.

W and w, the former being constant. Then if a and b be the distances of the C.G. and the C.B. from the fore poppet (as in Fig. 148 a) at any point of the travel—

moment of weight about fore poppet = W × a
" " buoyancy " " = $w \times b$

Curves are then drawn as in Fig. 149, giving these moments on base of travel, and the point where these curves cross gives the position where the stern begins to lift, and the intercept between the curves of weight and buoyancy at this point, viz. aa, gives the weight on the fore poppets. In this case the weight on the fore poppets was 2500 tons, the launching weight being 9600 tons.

The launching curves for H.M.S. *Ocean* are given in a paper by Mr. H. R. Champness, read before the Institution of Mechanical Engineers in 1899. In that case the weight of the ship was 7110 tons, and the weight on the fore poppets 1320 tons.

The internal shoring of the ship must be specially arranged for in the neighbourhood of the fore poppets, and the portion of the slip under them at the time the stern lifts must be made of sufficient strength to bear the concentrated weight.

Variation of Pressure on the Ways.—In addition to knowing the pressure per square foot when the vessel is on the slip, it is sometimes desirable to know how this pressure varies as the ship goes down the ways. It is quite possible that the pressure might be excessive, and the necessity of strengthening the slip or shoring the ship internally would have to be considered.

The support of the ways at any point of the travel is $W - w$, where W is the weight and w the buoyancy. From this the mean pressure P can be determined. The support of the ways must act at a distance x from the fore end, such that

$$(W - w)x = W.a - w.b$$

where a and b are the distances of the C.G. and C.B. respectively from the fore poppet. Let y be the distance of the centre of pressure from the way ends. There are three cases to consider (see Fig. 150).

(1) If x lies between $\frac{1}{3}l$ and $\frac{2}{3}l$, where l is the length of surface of ways in contact. Then, knowing P the mean ordinate, and assuming that the curve of pressure is a straight line, P_A and P_F, the pressures at the after and forward ends, can be determined. (See Example 28, Chapter II.) If $x = \frac{1}{3}l$ or $\frac{2}{3}l$, then the maximum pressure is 2P, and occurs at the forward or after end as the case may be.

(2) If x is less than $\frac{1}{3}l$, the distribution of pressure is assumed a straight line on a base $= 3x$, and maximum pressure at fore poppet is $\frac{2}{3}.\frac{W - w}{x}$.

(3) If y is less than $\frac{1}{3}l$ we have similarly the maximum pressure at the after end of way $= \frac{2}{3} \cdot \frac{W - w}{y}$.

By this means a curve of pressure at fore poppet may be obtained for all positions of the ship on the slip until the stern

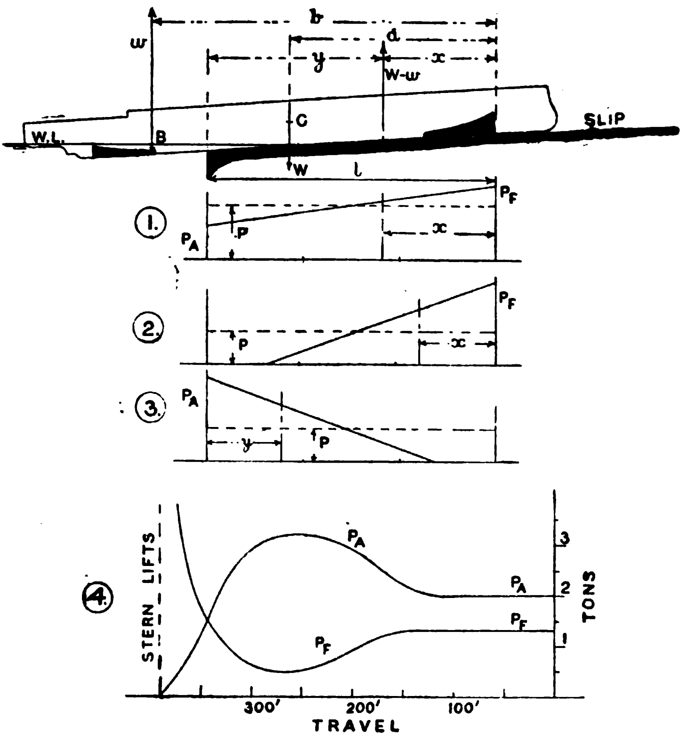


FIG. 150.

lifts, P_F, and similarly for the after ends of ways, P_A, as in (4) Fig. 150.

The maximum pressure at the after end of the ways thus calculated in one ship was $3\frac{1}{4}$ tons per square foot[1] as compared with the mean pressure before launching of 1·7 tons.

The following papers may be consulted in regard to the launching of ships :—

H. R. Champness,	*Ocean*	"I.M.E.", 1899
W. J. Luke,	*Lusitania*	"I.N.A.", 1907
J. Smith,		"I.N.A.", 1909
A. Hiley,		"I.N.A.", 1913
P. A. Hillhouse, W. H. Riddlesworth		"I.N.A.", 1917

Example.—In a certain ship the length of sliding ways was 535 ft. and the breadth 5 ft. 4 ins. The launching weight was 9600 tons with C.G. estimated at 247·5 ft. forward of the after end of sliding ways. Calculate the mean pressure per square foot on the ways, and assuming the pressure to vary uniformly as above, calculate the pressure per square foot at the forward and after ends of sliding ways before launching.

Ans. 1·68 tons; 1·3 tons; 2·0 tons.

(*Note.*—These two latter values are the starting points of the curves in (4) Fig. 150, the latter rising to a maximum value of $3\frac{1}{4}$ tons after a travel of 250 ft. The former curve rises to a high (indeterminate) value when the stern lifts.)

---

[1] In the discussion on the last-mentioned paper it was stated that 4 to 4·5 tons per square foot had been obtained for merchant vessels, 5·5 to 7·25 tons for battleships and cruisers, and as much as 19·5 tons in the case of a small heavily framed vessel launched on an unexpectedly low tide. No damage resulted in any of these cases.

APPENDIX A

Proof of Simpson's First Rule.—Let the equation of the curve referred to the axes Ox, Oy, as in Fig. 35, p. 53, be—

$$y = a_0 + a_1x + a_2x^2$$

a_0, a_1, a_2 being constants; then the area of a narrow strip length y and breadth Δx is—

$$y \times \Delta x$$

and the area required between $x = 0$ and $x = 2h$ is the sum of all such strips between these limits. Considering the strips as being a small breadth Δx, we still do not take account of the small triangular pieces as BDE (see Fig. 12), but on proceeding to the limit, *i.e.* making the strips indefinitely narrow, these triangular areas disappear, and the expression for the area becomes, using the formula of the calculus—

$$\int_0^{2h} y \,.\, dx$$

or, putting in the value for y given by the equation to the curve—

$$\int_0^{2h} (a_0 + a_1x + a_2x^2)dx$$

which equals—

$$\left[a_0x + \tfrac{1}{2}a_1x^2 + \tfrac{1}{3}a_2x^3\right]_0^{2h}$$

which has to be evaluated between the limits $x = 2h$ and $x = 0$. The expression then becomes—

$$2a_0h + 2a_1h^2 + \tfrac{8}{3} \,.\, a_2 \,.\, h^3 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Now, suppose the area $= Ay_1 + By_2 + Cy_3$

$$= Aa_0 + B(a_0 + a_1h + a_2h^2) + C(a_0 + 2a_1h + 4a_2h^2)$$

using the equation to the curve and putting $x = 0$, $x = h$ and $x = 2h$ respectively,

$$\text{Area} = a_0(A + B + C) + a_1h(B + 2C) + a_2 \,.\, h^2(B + 4C) \quad . \quad (2)$$

By a well-known principle of Algebra we can equate the coefficients of a_0, a_1, and a_2 in (1) and (2), so that—

$$A + B + C = 2h$$
$$B \cdot h + 2Ch = 2h^2$$
$$B \cdot h^2 + 4Ch^2 = \tfrac{8}{3} \cdot h^3$$

from which $A = \frac{1}{3} \cdot h$, $B = \frac{4}{3} \cdot h$, $C = \frac{1}{3} \cdot h$

so that the area required is—

$$\tfrac{1}{3} \cdot h(y_1 + 4y_2 + y_3)$$

which is Simpson's First Rule.

It may be shown in a similar manner that Simpson's First Rule will integrate also a curve which is of the third degree, viz.—

$$y = a_0 + a_1 \cdot x + a_2x^2 + a_3 \cdot x^3 \quad . \quad . \quad . \quad . \quad (3)$$

Simpson's First Rule is thus seen to integrate correctly curves both of the second and third degree. It is always used, unless the conditions are such that its use is not possible.

Proof of Simpson's Second Rule.—This may be proved similarly to the above, assuming that the curve has the equation (3) above.

Proof of the Five-eight Minus-one Rule.—The area between y_1 and y_2 is given by

$$\int_0^h y \cdot dx = a_0h + \tfrac{1}{2}a_1h^2 + \tfrac{1}{3} \cdot a_2h^3$$

Assuming this to be equal to $Ay_1 + By_2 + Cy_3$, substituting for y_1, y_2 and y_3, and equating coefficients of a_0, a_1 and a_2, we find—

$$A = \tfrac{8}{12} \cdot h,\ B = \tfrac{8}{12} \cdot h,\ C = -\tfrac{1}{12}h$$

so that the area required is

$$\tfrac{1}{12} \cdot h(5y_1 + 8y_2 - y_3)$$

The area between the ordinates y_2 and y_3 is

$$\tfrac{1}{12}h(5y_3 + 8y_2 - y_1)$$

and adding together, the whole area is

$$\tfrac{1}{3}h(y_1 + 4y_2 + y_3)$$

which is Simpson's First Rule.

Proof of the Three-ten Minus-one Moment Rule (given on p. 58). Assume the equation of the curve is—

$$y = a_0 + a_1x + a_2x^2$$

Then the moment about end ordinate $\Big\} = \int_0^h yx \, . \, dx$

$$= h^2(\tfrac{1}{2}a_0 + \tfrac{1}{3}a_1h + \tfrac{1}{4}a_2h^2) \text{ on integrating}$$

Let the moment $= (Ay_1 + By_2 + Cy_3)$.

Substituting for y_1, y_2, y_3 the values found from the equation to the curve, and equating coefficients of a_0, a_1, a_2, we get—

$$A = \tfrac{3}{24}h^2, \; B = \tfrac{10}{24}h^2, \; C = -\tfrac{1}{24}h^2$$

so that moment $= \frac{1}{24}h^2(3y_1 + 10y_2 - y_3)$

Simple Area Rule for Six Ordinates (for which neither the first nor the second rules can be used).

This is obtained by using the Five-eight Rule for the ends, and Simpson's Second Rule for the middle portion, thus—

$$\begin{array}{cccccc} \frac{5}{12}, & \frac{8}{12}, & -\frac{1}{12}, & -\frac{1}{12}, & \frac{8}{12}, & \frac{5}{12} \\ & \frac{3}{8}, & \frac{9}{8}, & \frac{9}{8}, & \frac{3}{8} & \\ \hline \frac{5}{12}, & \frac{25}{24}, & \frac{25}{24}, & \frac{25}{24}, & \frac{25}{24}, & \frac{5}{12} \end{array}$$

$$\tfrac{25}{24}[0{\cdot}4, \;\; 1, \;\; 1, \;\; 1, \;\; 1, \;\; 0{\cdot}4]$$

Proof of Tchebycheff's Rule with Four Ordinates.—The following is the proof in the case where four ordinates are employed. In solving the equations for eight or ten ordinates imaginary roots are obtained, but the figures obtained for four and five ordinates can be combined together for the two halves of the length giving the figures in the table on page 18.

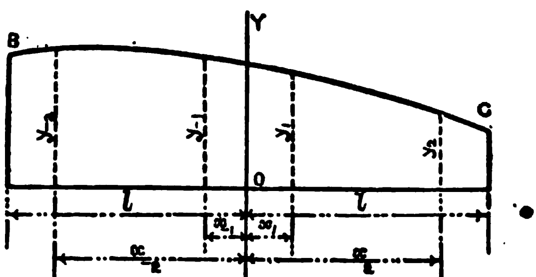


FIG. 151.

Let the curve BC (Fig. 151) be a portion of a parabola whose equation referred to the base and the axis OY is—

$$y = a_0 + a_1 . x + a_2 . x^2 + a_3 . x^3 + a_4 . x^4 \quad . \quad . \quad (1)$$

where a_0, a_1, a_2, a_3, and a_4 are constants.

(For 4 ordinates the curve is taken of the 4th degree.)

(„ *n* „ „ „ *n*th „)

Let $2 . l$ be the length of the area, and select the origin at the middle of the length.

$$\text{Then the area required} = \int_{-l}^{+l} y . dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= 2l\left(a_0 + a_2 . \frac{l^2}{3} + a_4 . \frac{l^4}{5}\right) \quad . \quad . \quad (3)$$

Now let this area $= C \times$ the sum of the 4 ordinates (4)

$$= C \times (y_1 + y_{-1} + y_2 + y_{-2})$$
$$= C \{4a_0 + 2a_2(x_1^2 + x_2^2) + 2a_4(x_1^4 + x_2^4)\} \quad . \quad . \quad . \quad . \quad (5)$$

substituting for y_1, y_{-1}, etc., their values as given by the equation to the curve (1), and taking the ordinates symmetrical about OY.

Equating coefficients of a_0, a_2, a_4 in the equations (3) and (5), we have $C = \frac{l}{2}$ and

$$x_1^2 + x_2^2 = \tfrac{2}{3} . l^2$$
$$x_1^4 + x_2^4 = \tfrac{2}{5} . l^4$$

From these equations we find—

$$x_1 = 0{\cdot}1876l$$
$$x_2 = 0{\cdot}7947l$$

which gives the positions of the ordinates such that the area $= \frac{2 . l}{4}(y_1 + y_{-1} + y_2 + y_{-2})$, *i.e.* the summation of the ordinates is multiplied by the length and divided by the number of ordinates.

Displacement Sheet by Tchebycheff's Rule.—This method may be extended to finding the volume of displacement of a ship, and a table may be employed similar to that on Table I.,[1] and described in Chapter II. There does not appear to be any advantage in applying Tchebycheff's rule in a vertical direction, as the number of water-lines are few in number compared with the number of ordinates usually employed fore and aft; and also by having the waterplanes spaced equally, the displacement and vertical position of the C.B. for the other water-planes can be determined. In the specimen table, therefore, given on Table II.,[1] Tchebycheff's rule is employed for the fore-and-aft integration, and Simpson's first rule for the vertical integration. The figures shown in thick type are the lengths of the semi-ordinates of the various water-lines spaced from amidships as indicated at the top of the sheet. These lengths added up give a function of the area

[1] To be found at the end of the book.

of each water-plane, as 241·7 for the L.W.P. These functions are put through Simpson's rule[1] in a vertical direction, and the addition of these products gives a function of the displacement, viz. 1802·25. This function, multiplied for both rules, etc., as shown, gives the displacement in tons, viz. 16,067 tons.

This result is obtained in another way, as in the ordinary displacement sheet, and an excellent check is thus obtained on the correctness of the calculation. The semi-ordinates of the various sections are put through Simpson's rule, and functions of the areas of the sections are thus obtained, as 255·05 for the section numbered II. These functions are then simply added up, and the same result is obtained as before for the function of the displacement, viz. 1802·25. It will have been noticed that the ordinates at equal distances from the mid-length are brought together; the reason for this will appear as we proceed.

The position of the centre of buoyancy of the main portion with reference to the L.W.P. is obtained in the ordinary way. To obtain the position of the centre of buoyancy of the main portion with reference to the mid-length, we proceed as follows. The functions of areas I. and IA. are subtracted, giving 2·9, and so on for all the corresponding sections. These differences are multiplied by the proportion of the half-length at which the several ordinates are placed, and the addition of the products gives a function of the moment of the displacement about the mid-length. In this case, the function is 36·4755. This, multiplied by the half-length and divided by the function of the displacement, 1802·25, gives the distance of the centre of buoyancy of the main portion abaft midships, 6·07 feet.

The lower appendage is treated in the ordinary way, as shown in the left-hand portion of the table. The reason of this is that the equidistant sections of the ship are usually drawn in on the body plan for fairing purposes, and the areas below the lowest water-line can be readily calculated. The sections at the stations necessary for Tchebycheff's rule would not be placed on the body for this calculation, but the ordinates at the various water-lines would be read straight off the half-breadth plan.

The summary to obtain the total displacement and position of the centre of buoyancy is prepared in the ordinary way, and needs no explanation. The result of this summary is to give the displacement as 16,900 tons, having the centre of buoyancy 11·2 feet below the L.W.L. and 6·52 feet abaft midships.

[1] In this case, instead of 1, 4, 2, 4, 2, 4, 1, the halves of these are used, viz. ½, 2, 1, 2, 1, 2 ½, the multiplication by 2 being done at the end.

Transverse BM.—To determine the moment of inertia of the L.W.P. about the middle line, we place the ordinates of the L.W.P. as shown, cube them and *add* the cubes, the result being 211,999. This is multiplied as shown, giving 8,479,926 as the moment of inertia of the main portion of the L.W.P. about the middle line. Adding for the after appendage, we obtain 8,480,976 as the moment of inertia of the L.W.P. about the middle line in foot-units.

The distance between the centre of buoyancy and the transverse metacentre is given by $\frac{I}{V}$, or—

$$\frac{8,480,976}{16,900 \times 35} = 14.34 \text{ feet}$$

The transverse metacentre is accordingly $14.34 - 11.2 = 3.14$ feet above the L.W.L.

Longitudinal BM.—The position of the centre of gravity of the main portion of the L.W.P. is obtained by taking the differences of corresponding ordinates of the L.W.P. and multiplying these differences by 0.0838, etc., as shown. The addition of these products, 14.6244, treated as shown, gives 18.15 feet as the distance of the centre of gravity of the main portion of the L.W.P. abaft amidships. Adding in the effect of the after appendage, we find that the area of the L.W.P. is 29,144 square feet, and the centre of flotation is 19.53 feet abaft amidships.

To determine the position of the longitudinal metacentre, we need to find the moment of inertia of the L.W.P. about a transverse axis through the centre of flotation. This has to be done in several steps. First we determine the moment of inertia of the main portion about amidships. This is done by taking the sum of corresponding ordinates and multiplying these by $(0.0838)^2$, $(0.3127)^2$, etc., or 0.007, 0.098, etc. The addition of the products, 50.5809, is multiplied by 2 for both sides, by $\frac{600}{10}$ for Tchebycheff's rule and by $(300)^2$, being the square of the half-length, because we only multiplied by the square of the fraction of the half-length the various ordinates are from amidships, and not by the squares of the actual distances. The result gives 546,273,720 in foot-units for the moment of inertia of the main portion of the L.W.P. about the midship ordinate. We add to this the moment of inertia of the after appendage about the midship ordinate, obtaining 539,399,902 in foot-units. This is the moment of inertia of the L.W.P. about the midship ordinate. To obtain the moment of inertia of the L.W.P. about a transverse axis through the centre of flotation, we subtract the product of the area of the L.W.P. and the square of the distance of the centre of flotation abaft amidships. The result is the moment of inertia of the L.W.P. about a transverse axis through the centre of flotation we want, and this divided by the volume of displacement gives the value of the longitudinal BM, 927 feet.

The moment to change trim one inch is obtained in the ordinary way, assuming that the centre of gravity of the ship is in the L.W.L. and that the draught marks are placed at the perpendiculars.

To obtain Cross Curves of Stability by means of the Integrator and using Tchebycheff's Rule.—The rule we have been considering can be used with the integrator to determine the ordinary cross curves of stability in just the same way as with Simpson's rules. In Chapter V. the process of the calculation necessary with the integrator is explained. This calculation may be considerably shortened if Tchebycheff's rule is used instead of Simpson's rule. Not only can fewer sections be used, but the integrator itself performs the summation. In this case a body plan must be prepared, showing the shape of the sections at the distances from amidships required by the rule. Take, for example, a vessel 480 feet long, for which by the ordinary method twenty-one sections would be necessary. By using this rule nine sections will be quite sufficient, and by reference to the table on p. 18 the sections must be placed the following distances forward and aft of amidships, viz. 40·3, 126·9, 144·2, 218·8 feet respectively, the midship section being one of the nine sections.

The multiplier to convert the area readings of the integrator employed into tons displacement was for this case 1·097, and to convert the moment readings into foot-tons of moment was 13·164. All that is necessary, then, having set the body plan to the required angle as in Fig. 79, is to pass round all the nine sections in turn up to the water-line you are dealing with. and put down the initial and final readings. We have, for example—

	Area readings.
Initial	14,198
Final	25,397
	11,199 difference

Displacement in tons = 11,199 × 1·097
= 12,285 tons

	Moment readings.
Initial	5215
Final	5516
	301 difference

$$\text{Moment} = 301 \times 13{\cdot}164 = 3962 \text{ foot-tons}$$

$$GZ = \frac{3{,}962}{12{,}285} = 0{\cdot}32 \text{ feet}$$

It will at once be seen, on comparison with the example given on p. 199, that there is a very great saving of work by using this method. The following table gives the whole of the calculation necessary to determine a cross curve for the above vessel, values of GZ being obtained at four draughts, viz. at the L.W.L., one W.L. above and two W.L.'s below:—

Number of W.L.	Area reading.	Difference.	Displacement.	Moment reading.	Difference.	GZ
Initial ...	1,505	—	—	5,136	—	—
A.W.L. ...	14,198	12,693	13,924	5,215	79	0·07
L.W.L. ...	25,397	11,199	12,285	5,516	301	0·32
2 W.L. ...	35,126	9,729	10,673	6,373	857	1·06
3 W.L. ...	43,301	8,175	8,968	7,261	888	1·30

Displacement Sheet (*used in Messrs. John Brown & Co.'s Drawing Office*).—The displacement table used in the drawing office of Messrs. John Brown & Co., Clydebank, presents several points of interest, and is admirably designed to conveniently contain on one sheet all the calculations necessary for the geometrical features of a ship's lines. This table was devised by Mr. John Black, and I am indebted to Mr. W. J. Luke for permission to reproduce it. The sections are numbered from aft, and the water-planes from below (see Tables III. and IIIA. at end of book).

In Tchebycheff's three-ordinate rule (p. 18), the distance of the ordinates either side of the middle ordinate is 0·707 times the half length of base. If, therefore, we apply this rule five times over for the length, we should set off on either side of Nos. 1, 3, 5, 7, 9 (Fig. 152) a distance of 0·0707 × length, the length being divided into ten equal parts. The addition of the ordinates A, B, C, . . .

O, P, Q, multiplied by $\frac{1}{15}$ length, gives the half area of the water-plane.

The ordinates B, E, H, M, P (Fig. 152) are the following distances from amidships (6, 3, 0, 3, 6)$\frac{L}{15}$. The ordinates A, C, D, F, G, K, L, N, O, Q, are the following distances from amidships, viz. (7·06, 4·94, 4·06, 1·94, 1·06, 1·06, 1·94, 4·09, 4·94, 7·06)$\frac{L}{15}$. These are so close to the integers 7, 5, etc., that without appreciable error the integers 7, 6, 5, 4, etc., can be used for the levers as

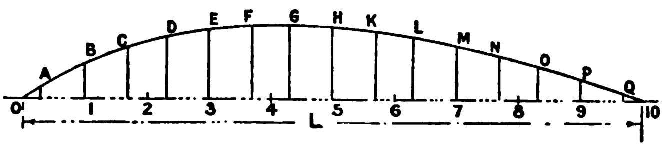


FIG. 152.

in the ordinary displacement sheet. Thus, for any water-line the *addition* of ordinates in column A multiplied by $2 \times \frac{L}{15}$ will give the area. The algebraic sum of column B, divided by the addition of column A and multiplied by $\frac{L}{15}$, gives the distance of the centre of gravity of water-plane from 0 to 10 from midships. Column D is got by multiplying the figures in column B again by the levers, and the addition of the column properly multiplied leads to the longitudinal moment of inertia of water-plane about amidships. This has to be corrected for the after appendage (if any), and then transferred to the centre of flotation, as explained in Chapter IV. From this the longitudinal B.M. is readily obtained for the several water-planes.

In columns C are placed the cubes of the ordinates in columns A, and the addition of these columns leads to the transverse moment of inertia of the water-planes, from which values of the transverse B.M. is obtained for the several water-planes.

The lower appendage is treated by "Thomson's rule,"[1] the sections used being those on the ordinary body plan. The multipliers are obtained as follows :—

[1] It is understood that ordinary sections and Simpson's multipliers are now used in this sheet for the appendage.

	0	½	1	2	3	4	5	6	7	8	9	9½	10
Area 1 to 9	—	—	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	—	—
Areas 0 to 1, 9 to 10	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	—	—	—	—	—	—	—	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$
Whole area	$\frac{1}{3}$	—	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	—	$\frac{1}{3}$
Twice area	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{11}{6}$	2	2	2	2	2	2	2	$\frac{11}{6}$	$\frac{2}{3}$	$\frac{1}{2}$
Area	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{11}{12}$	1	1	1	1	1	1	1	$\frac{11}{12}$	$\frac{1}{3}$	$\frac{1}{4}$

The vertical C.B. of the appendage is obtained by Morrish's rule, viz. $\frac{1}{3}\left(\frac{d}{2} + \frac{v}{a}\right)$, where d is the depth of appendage, v is its volume, and a is the area of No. 1 W.P.

In the combination table the results are grouped together to find the displacement and C.B. up to Nos. 3, 5, 7, and 9 waterplanes.

The displacement is found, in the first place, to the moulded surface of the ship, as is usual outside the Admiralty service; the area of wetted surface is obtained by the formula $S = 1{\cdot}7\, L.D + \frac{V}{D}$, and using a mean thickness of plating, the displacement of the plating is readily obtained, and thus the "full" displacement is obtained.

From the results a series of curves as in Fig. 153 is readily constructed, on base of draught, of displacement, tons per inch, centres of flotation, transverse metacentre, longitudinal metacentre, vertical C.B. fore and aft C.B., moment to change trim one inch, area of midship section and area of wetted surface, and also the various coefficients. To avoid confusion, the curves of vertical C.B. and metacentres are measured from the axis marked L.W.L; those of C.B. and C.F. abaft amidships are measured from the right boundary of the figure. All others are measured from the left boundary. The scales used are appended to all the curves. These curves, when once carefully drawn for a ship, are of great value as records of the features of the ship's form.

Loss of Stability due to Grounding.—When a ship is being docked, the shores cannot finally be set up until the keel takes the blocks all fore and aft. Until this happens there is a portion of the weight taken by the after block (supposing the ship is trimming by the stern), and this becomes a maximum immediately before the ship grounds all fore and aft. Before the shores are set up there is a considerable upward pressure at the keel which might be sufficient, under certain circumstances, to cause instability, and cases are on record in which a ship has fallen over when being dry-docked owing to this cause.

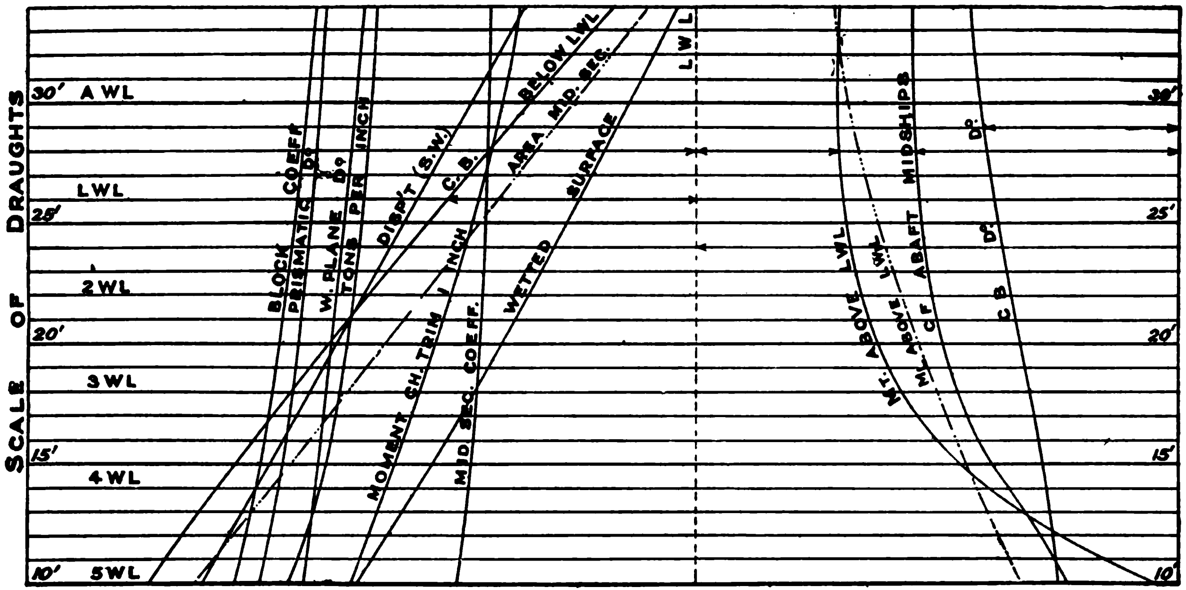

SCALE OF DRAUGHTS
30' A WL
LWL
25'
2 WL
20'
3 WL
15'
4 WL
10' 5 WL
BLOCK COEFF
PRISMATIC D°
W. PLANE D°
TONS PER INCH
DISP'T (S.W.)
MOMENT CH. TRIM 1 INCH
C. B.
BELOW LWL
MID SEC. COEFF.
AREA MID. SEC.
WETTED SURFACE
LWL
MT. ABOVE LWL
ML. ABOVE LWL
CF ABAFT MIDSHIPS
C B
D°
30'
25'
20'
15'
10'

Fig. 153.

In Fig. 154 let the first diagram represent the ship, floating freely, having a small inclination. In the second diagram a portion of the weight, w say, is taken by the blocks. This is equal to the

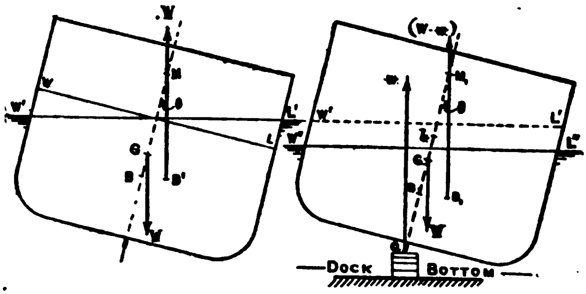


FIG. 154.

displacement between the lines W′L′ and W″L″. If M_1 be the metacentre corresponding to the water-line W″L″, then—

$$\text{Moment of stiffness} = \{(W - w)GM_1 - w \cdot OG\} \sin\theta$$

$$= W \sin\theta\left(GM_1 - \frac{w}{W} \cdot OM_1\right)$$

so that $GM_1 - \frac{w}{W} \cdot OM_1$, which is the virtual metacentric height, must be positive if the ship is to remain stable.

To find w we can proceed as follows:—

1. *Accurately*.—Obtain the displacement and longitudinal position of C.B. when floating freely. At the instant of taking the blocks all along, the moment of buoyancy about after block = moment of weight about after block. This equals moment of buoyancy about after block when floating freely.

Hence we place a profile of the ship on the line of blocks, and draw a series of water-lines parallel to the keel. For each of these calculate the displacement and the longitudinal C.B. Draw out on a scale of draught a curve giving the moment of buoyancy about the after block. Where this crosses the constant line of the moment of weight about after block will give the draught at which the ship will ground, and so the displacement. This deducted from the original displacement gives the pressure on the blocks, and from the above the stability under these conditions can be determined. In a ship with small metacentric height and large trim by the stern, we have a combination of circumstances which would probably cause instability. The course to pursue is to keep the ship under control while any weight is taken by the blocks.

2. *Approximately.*—Suppose the vessel trims t feet by the stern, and let the after block be b feet from the centre of flotation. When the vessel is floating freely, imagine a force Q is applied at the after block just sufficient to bring the vessel to an even keel. $Q = \frac{12 . M . t}{b}$, where M is moment to change trim 1 inch.

The upward force Q will decrease displacement, and the mean draught is reduced by $\frac{Q}{12 . T} = \frac{M . t}{b . T}$ feet, T being tons per inch. Owing, however, to the change of trim, the mean draught is increased by $\frac{t . c}{L}$ feet, where the centre of flotation is c feet abaft amidships. If x is the draught at fore end when floating freely, then the mean draught when just grounding is—

$$x + \frac{t}{2} + \frac{t . c}{L} - \frac{M . t}{b . T}.$$

Theory of the Integrator.—This instrument, shown in diagram in Fig. 79, gives by using suitable multipliers to the results obtained—

(i) Area of a closed figure,
(ii) Moment of a figure about a given axis,
(iii) Moment of inertia of figure about a given axis,

by tracing out the boundary of the figure with the pointer of the instrument.

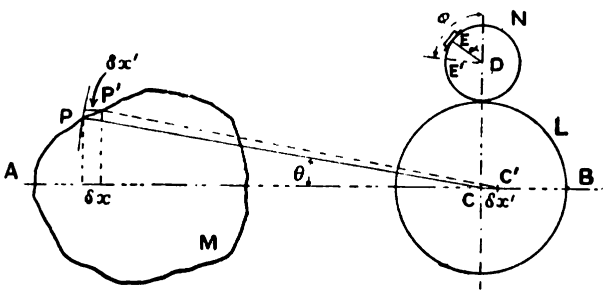


FIG. 155.

In Fig. 155 let M be the closed figure and AB the axis, P is a pointer at the end of an arm PC which is rigidly attached to a circle CL. The centre C of this circle is constrained to move along the line AB. Gearing with L is another circle N, centre D, CD always being perpendicular to AB. At the end of a radius DE of the circle N is a recording wheel capable of rotating about

DE, and this wheel can only record movements perpendicular to DE.

Suppose the ratio of the circles L and N is as $n : 1$. Then for an angular movement θ of L the wheel radius DE will move through $n\theta$, and if when PC is on AB, DE is at an angle α, then when PC is at θ, DE is at an angle $\phi = n\theta + \alpha$.

In going from P to the consecutive point P′ on the curve, separated by a longitudinal interval δx, we have to consider the influence of the recording wheel of two separate movements of PC, viz.—

(i) that due to the angular motion of PC; and

(ii) that due to the horizontal transfer $\delta x'$ of the centre C along AB.

Consider now the influence of these two movements on the wheel—

(i) Since the curve is a closed curve the net result of the angular movement is zero.

(ii) The recording wheel moves $\delta x'$ parallel to AB, and the record on the wheel, *i.e.* its movement perpendicular to DE, is $\delta x' . \cos \phi$,

$$= \delta x' . \cos (n\theta + \alpha),$$

and the total record

$$= \int \cos (n\theta + \alpha) . dx'$$

But $\delta x = \delta x' + \text{CP} . \delta\theta . \sin \theta$.

Hence the total record

$$= \int \cos (n\theta + \alpha) (\delta x - \text{CP} . \sin \theta . d\theta)$$
$$= \int \cos (n\theta + \alpha)\, dx . - \int \text{CP} . \cos (n\theta + \alpha) . \sin \theta . d\theta.$$

CASE I.—Take $n = 1, \alpha = \frac{\pi}{2}$.

The reading is $\int \sin \theta . dx - \int \text{CP} . \sin^2\theta . d\theta$.

The second term vanishes for a complete circuit, and since ordinate of the curve is CP sin θ, the reading is proportional to $\int y . dx$, and therefore to the area.

CASE 2—Take $n = 2, \alpha = 0$.

The reading is $\int \cos 2\theta . dx +$ vanishing terms,

$$= \int (1 - 2 \sin^2 \theta)\, dx$$
$$= \int dx - 2 \int \sin^2 \theta . dx$$
$$= -\frac{2}{c^2} \int y^2 . dx, \text{ since } \int dx = 0$$

i.e. the reading is proportional to the moment of the area about AB.

CASE 3.—Take $n = 3$, $a = \frac{\pi}{2}$.

The reading is $\int \sin 3\theta . dx$ + vanishing terms

$$= \int (3 \sin \theta - 4 \sin^3 \theta) dx$$

$$= \frac{3}{c} . \int y . dx - \frac{4}{c^3} . \int y^3 . dx$$

the first term of which is proportional to area and the second term to the moment of inertia.

Case 3 is little used in ship work. The student on first taking up the use of the integrator is advised to take simple geometrical figures of which the exact area and moment are known, and by this means the accuracy of the instrument may be tested, and, if necessary, any corrections made.

MISCELLANEOUS EXAMPLES.

1. The tons per inch immersion in salt water at a ship's water-planes are as follows, commencing with the L.W.P.: 12·9, 12·4, 11·5, 10·2, 8·0, 6·0, 2·2. The first five water-planes are 21 inches apart, and the last three are 10½ inches apart, the draught being 9 feet to bottom of flat keel.

(*a*) Determine the displacement and the vertical position of the centre of buoyancy to the first three water-planes.

(*b*) Estimate the displacement of the vessel when floating at a draught of 10 feet 1½ inches in water of which 1 cubic foot weighs 63½ lbs.

Ans. (*a*) 1063 tons, 3·77 feet below L.W.L.
797 ,, 4·73 ,, ,,
545 ,, 5·72 ,, ,,
(*b*) 1228 tons.

2. Construct a formula giving the additional displacement, due to 1 foot greater trim aft as compared with the normal trim, in terms of the tons per inch immersion, length between draught-marks, and the distance of the centre of flotation abaft midships.

The vessel in question, No. 1, whose normal draught is 9 feet on an even keel, floats in salt water at a draught of 8 feet 7 inches forward and 9 feet 10 inches aft. Estimate the displacement in tons, the centre of flotation being 7 feet abaft amidships, and the length P.P. 250 feet (draught-marks at perpendiculars).

Ans. $12\frac{T \times y}{L}$, 1100 tons.

3. The vessel in question No. 1 floats at a mean draught of 9 feet 6¾ inches in salt water. While in this condition she is inclined, two plumb-bobs 10 feet long being employed. The following deflections are observed :—

DE, and this wheel can only record movements perpendicular to DE.

Suppose the ratio of the circles L and N is as $n : 1$. Then for an angular movement θ of L the wheel radius DE will move through $n\theta$, and if when PC is on AB, DE is at an angle α, then when PC is at θ, DE is at an angle $\phi = n\theta + \alpha$.

In going from P to the consecutive point P′ on the curve, separated by a longitudinal interval δx, we have to consider the influence of the recording wheel of two separate movements of PC, viz.—

(i) that due to the angular motion of PC; and

(ii) that due to the horizontal transfer $\delta x'$ of the centre C along AB.

Consider now the influence of these two movements on the wheel—

(i) Since the curve is a closed curve the net result of the angular movement is zero.

(ii) The recording wheel moves $\delta x'$ parallel to AB, and the record on the wheel, *i.e.* its movement perpendicular to DE, is $\delta x' . \cos \phi$,

$$= \delta x' . \cos (n\theta + \alpha),$$

and the total record

$$= \int \cos (n\theta + \alpha) . dx'$$

But $\delta x = \delta x' + CP . \delta\theta . \sin \theta$.

Hence the total record

$$= \int \cos (n\theta + \alpha) (\delta x - CP . \sin \theta . d\theta)$$

$$= \int \cos (n\theta + \alpha) \, dx . - \int CP . \cos (n\theta + \alpha) . \sin \theta . d\theta.$$

CASE I.—Take $n = 1, \alpha = \frac{\pi}{2}$.

The reading is $\int \sin \theta . dx - \int CP . \sin^2\theta . d\theta$.

The second term vanishes for a complete circuit, and since ordinate of the curve is CP sin θ, the reading is proportional to $\int y . dx$, and therefore to the area.

CASE 2—Take $n = 2, \alpha = 0$.

The reading is $\int \cos 2\theta . dx$ ... terms,

$= \int (1 - 2 \sin$ ...

$= \int dx - 2 \int$ ...

$= -\frac{2}{c^2} \int y^2$ ... $x = 0$

i.e. the reading is proportional to the moment of the area about AB.

CASE 3.—Take $n = 3$, $a = \frac{\pi}{2}$.

The reading is $\int \sin 3\theta . dx$ + vanishing terms

$$= \int (3 \sin \theta - 4 \sin^3\theta) dx$$

$$= \frac{3}{c} . \int y . dx - \frac{4}{c^3} . \int y^3 . dx$$

the first term of which is proportional to area and the second term to the moment of inertia.

Case 3 is little used in ship work. The student on first taking up the use of the integrator is advised to take simple geometrical figures of which the exact area and moment are known, and by this means the accuracy of the instrument may be tested and, if necessary, any corrections made.

MISCELLANEOUS EXAMPLES.

1. The tons per inch immersion in salt water at a [illegible] are as follows, commencing with the L.W.P.: 12[illegible], 12[illegible], [illegible] 2·2. The first five water-planes are 21 inches apart, [illegible] 10½ inches apart, the draught being 9 feet [illegible]

(*a*) Determine the displacement and the vertical [illegible] of buoyancy to the first three water-planes.

(*b*) Estimate the displacement of the vessel [illegible] of [illegible] feet 1½ inches in water of which 1 cubic foot [illegible]

Ans. (*a*) [illegible]

2. Construct a formula [illegible] a hemi- 1 foot greater trim aft as comp[illegible] the BM tons per inch immersion, [illegible] water. (See of the centre of flotation abaft [illegible]

The vessel in question, [illegible] draught 19 feet, even keel, floats in salt water [illegible] at the L.W.L. is 9 feet [illegible] inches aft. [illegible] flot[illegible] 7 feet abaft [illegible] m[illegible]

[illegible] with the dimensions— [illegible] 19½ feet. If G is the same [illegible] value of GM would you

[illegible] and 111.) *Ans.* 2¾ to 3 feet.

[illegible] has a freeboard to the upper [illegible] above water, and the metacentre locus [illegible] bulwarks causes a rectangular area

		Forward.		Aft.
3 tons through 23 ft. P to S	... ...	3·6"	...	3·5"
6 ,, ,, ,,	... ...	7·15"	...	7·05"

Weights restored, ship came to upright.

3 tons through 23 ft. S to P	... ...	3·55"	...	3·6"
6 ,, ,, ,,	... ...	7·15"	...	7·1"

Estimate the metacentric height at the time.

Ans. 2 feet.

4. A vessel of box form, 150 feet long and 25 feet broad, floats at an even draught of 8 feet, and has a water-tight deck 8½ feet above keel. If a central compartment, 30 feet long, bounded by two transverse bulkheads extending up to the deck, is bilged, what will be (1) the new draught of the vessel; (2) the alteration of the metacentric height?

Ans. 9′ 8½″ nearly; increase nearly 1 foot.

5. A body with vertical sides, the plan being an isosceles triangle 150 feet long and 30 feet broad at the stern, floats in salt water at a constant draught of 10 feet. Determine the displacement when floating at a draught of 9 feet 6 inches forward, 10 feet 6 inches aft—

(*a*) by using formula obtained in question (2) above;

(*b*) by direct calculation, thus verifying (*a*).

6. Obtain a rule for finding the area between two consecutive equidistant ordinates of a curve when three are given. Show that the rule, when used with levers, results in a moment error of $\frac{h^2}{4}$ × (intercept between whole curve and chord), where h is the common interval.

7. The half-ordinates of the water-plane of a ship 320 feet long and of 9500 tons displacement are 1·0, 16·5, 25·0, 29·0, 30·4, 30·6, 30·5, 29·8, 28·1, 24·1, and 15·1 feet. Find the sinkage of the vessel on passing from the Nore to the London Docks (63 lbs. to cubic foot).

Ans. 3·9 inches.

8. If the vessel in the last question draws F 24′ 3″, A 27′ 9″ when at the Nore, find her draughts forward and aft when in the docks, the centre of buoyancy being 5·1 feet abaft middle ordinate and 11 feet below the centre of gravity.

Ans. F 24′ 7½″, A 28′ 0¾″.

9. A cigar-shaped vessel with circular sections floats in salt water with its axis in the surface. The semi-ordinates of the water-plane, 20 feet apart, are, commencing from forward, 0, 3, 6, 8, 7, 4, 1 feet respectively.

Find (1) Tons per inch immersion.
(2) Displacement in tons.
(3) Position of C.F from after end.
(4) Position of C.B. ,, ,,
(5) Transverse BM.
(6) Position of C.B. below W.L.

Ans. (1) 2·76 tons; (2) 157·7 tons; (3) 57 feet; (4) 56·7 feet; (5) 2·84 feet; (6) 2·84 feet.

Note.—Some consideration should be given as to the simplest method of doing this question; (6) should be inferred from (5).

10. A vessel of constant triangular section is 245 feet long, 30 feet broad at the water-line, and floats at 12 feet draught with vertex downwards.

When a weight of 8 tons is moved 30 feet across the deck, a shift of 8 inches is caused on the bob of a 10-feet pendulum. Find the position of the vessel's centre of gravity.

Ans. 17·64 feet from base.

11. Assuming that a barge is of uniform rectangular section, 70 feet long and 20 feet broad, construct the metacentric diagram to scale for all draughts between 2 feet and 10 feet; state the draught for which the height of the metacentre above the keel is lowest, and show that in this condition the metacentre is in the corresponding water-plane.

Ans. 8′ 2″.

12. A right circular cone is formed of homogeneous material, and the tangent of the semi-vertical angle is 0·5. Show that this cone will float in stable equilibrium with vertex down in fresh water so long as the specific gravity of the material is greater than 0·512.

13. A long triangular prism of homogeneous material having the same section as the above floats in fresh water with vertex down. Show that it will float in stable equilibrium so long as the specific gravity of the material is greater than 0·64.

14. A lighter has a constant section 16 feet at the base, 20 feet across the deck, and 10 feet deep. She floats in river water 35·7 cubic feet to the ton at a constant draught of 8 feet, the length being 80 feet. The C.G. when laden to this draught is 6 feet above the base.

Determine the angle of heel caused by taking 5 tons of the cargo out, this cargo being at 6 feet from the base and 6 feet from middle line.

Ans. $2\frac{1}{4}$ to $2\frac{1}{2}$ degrees.

15. What relation exists between the transverse and longitudinal stability of a wholly submerged body?

Discuss the question of submarine navigation from the point of view of longitudinal stability.

16. A lighter with vertical sides is 132 feet long and 30 feet broad for a length amidships of 80 feet. The ends are formed of four circular arcs of 30 feet radius. The draught is 10 feet, and the C.G. at this draught is $7\frac{1}{2}$ feet from the bottom. Determine the metacentric height.

Ans. 4·35 feet.

17. Prove the rule for the distance of the centre of gravity of a hemisphere of radius a from the bounding plane, viz. $\frac{3}{8} . a$, by finding the BM of a sphere floating with its centre in the surface of the water. (See question 17, p. 141.)

18. A ship of length 320 feet, breadth 50 feet, mean draught 19 feet, has a displacement of 4400 tons. The tons per inch at the L.W.L. is 27, BM is 11 feet, and GM is 2·5 feet.

It is proposed to design on similar lines a ship with the dimensions—length 330 feet, breadth 51 feet, mean draught $19\frac{1}{2}$ feet. If G is the same distance above the keel in both ships, what value of GM would you expect in the new ship?

(*Use approximate formulæ on pp.* 66 *and* 111.) *Ans.* $2\frac{3}{4}$ to 3 feet.

19. A vessel of 700 tons displacement has a freeboard to the upper deck of 6 feet. The C.G. is $1\frac{1}{2}$ foot above water, and the metacentre locus is horizontal. A sea breaking over the bulwarks causes a rectangular area

50 feet long and 20 feet wide on the upper deck to be covered with water to a depth of 1 foot. Calculate the loss of metacentric height.

Ans. 1·5 foot.

20. Show that for a vessel wall-sided in the neighbourhood of the water-line, $GZ = (GM + \frac{1}{2}BM \tan^2 \theta) \sin \theta$ at the angle of heel θ.

Use this formula to determine the metacentric height in the upright condition of a box-shaped vessel, 200′ × 35′ × 10′ draught, which is found to loll over to an angle of 5° (see p. 173). *Ans.* − 0·04 foot.

21. A tank, extending across an oil-carrying vessel, is 35 feet wide, 40 feet long, and 10 feet deep. It has an expansion trunk at the middle line 4 feet wide and 6 feet long. The vessel has a displacement of 2000 tons in salt water, and a GM of 2½ feet, the C.G. being 10 feet above the bottom of the tank.

Find the virtual metacentric height when the tank is half full and also when filled. The density of the oil is 0·8 as compared with sea-water, and the metacentric curve is horizontal.

Ans. (1) 1·54 foot; (2) 3·19 feet.

22. y_1, y_2, y_3, y_4, y_5, and y_6 are six consecutive equidistant ordinates of a plane curve: obtain the following expression for the area A of the curve lying between y_1 and y_6, h being the common interval:—

$$A = \tfrac{25}{24}h(0{\cdot}4y_1 + y_2 + y_3 + y_4 + y_5 + 0{\cdot}4y_6)$$

23. A foreign vessel, whose form is not known, has a certain draught at the Nore, the sea-water there being 64 lbs. per cubic foot. Off Greenwich, the water there being 63 lbs. per cubic foot, it is noted that when 100 tons have been unshipped the draught of water is again what it was at the Nore. What is the sea-going displacement of the vessel?

Ans. 6300 tons.

24. A vessel 60 feet broad at water-line has the transverse metacentre 12 feet above C.B., the latter being 10 feet below water. Find the height of metacentre above this water-line when—

(*a*) The beam is increased to 62 feet at the water-line, and in this ratio throughout, the draught being unaltered;

(*b*) The breadths at water-line are increased as above, but the lines fined so as to maintain the original displacement and to raise the C.B. 0·4 foot. *Ans.* (*a*) 2·8 feet; (*b*) 3·64 feet.

25. In a vessel whose moment to change trim one inch is M, tons per inch is T, and centre of flotation from after perpendicular is e times the length between perpendiculars, show that the position for an added weight such that the draught aft shall remain constant is $\frac{M}{e \cdot T}$ feet forward of the centre of flotation, and thus if the C.F. is at mid-length this distance is $2 \cdot \frac{M}{T}$ feet, or, approximately, one-ninth the length in a ship of ordinary form.

26. Show that the distance forward of the after perpendicular at which a weight must be added so that the draught aft shall remain constant is given by *moment of inertia of water-plane about the A.P. divided by the moment of water-plane about the A.P.*

27. A long body of specific gravity 0·5 of homogeneous material floats in fresh water, and has a constant section of the quadrant of a circle of 10 feet radius. Determine the metacentric height when (*a*) corner upwards,

(*b*) corner downwards, and (*c*) when between the positions (*a*) and (*b*). Draw the general shape of the stability curve from zero to 360°, starting with the body corner upwards. (The C.G. of the quadrant is $\frac{4}{3\pi}$ times the radius from each of the bounding radii. Positions of stable and unstable equilibrium occur alternately.)

Ans. (*a*) +2·33 feet; (*b*) +2·33 feet; (*c*) −0·36 foot.

28. Two ships of unequal size are made from the same model. Prove that at the speed at which the resistance varies as the sixth power of the speed, the *same effective horse-power* is required for *both* ships at the *same speed.*

29. A vessel 375 feet between perpendiculars is designed to float at 21 feet F.P., 23 feet A.P. At this draught the displacement is 6500 tons salt water, tons per inch 45, and centre of flotation 13½ feet abaft amidships.

The draught marks are placed on the ship 25 feet abaft the F.P. and 35 feet before the A.P. respectively. Estimate as closely as you can the displacement when the draught marks are observed at the ship, 19′ 6″ forward, 23′ 10″ aft, when floating in water of which 35·7 cubic feet weigh 1 ton.

Ans. 6293 tons.

30. H.M.S. *Pelorus* is 300′ × 36½′ × 13½′ mean draught, 2135 tons displacement, and requires 7000 I.H.P. for 20 knots. On this basis estimate the I.H.P. required for a vessel of similar form, 325′ × 40′ × 15¾′ mean draught, 3000 tons displacement, at 21 knots speed. State clearly the assumptions you make in your estimate.

Among others the following assumptions are made:—
(1) For increased displacement caused by bodily sinkage, the I.H.P. varies as displacement for the same speed.
(2) At the speeds mentioned in question, the I.H.P. is varying as the fourth power of the speed.

Ans. About 10,600 I.H.P.

31. The following formula has been proposed for the E.H.P. of a vessel at speed V knots, viz.—

$$\text{E.H.P.} = \frac{1}{326}\left\{f \,.\, S \,.\, (V)^{2.83} + b \,.\, \frac{(W)^{\frac{2}{3}}}{L} V^6\right\}$$

where S = wetted surface in square feet.
W = displacement in tons.
L = length in feet.
f = a coefficient for surface friction.
b = a coefficient varying with the type of ship.

A vessel 500′ × 70′ × 26½′, draught 14,000 tons, is tried at progressive speeds, and the curve of I.H.P. on base of speed shows the following values, viz. at 10, 12, 14, 16, 18, 20 knots, the I.H.P. is 1800, 3100, 5000, 7500, 11,000, 15,500 respectively.

Assuming the above formula to correctly give the E.H.P., determine the propulsive coefficients at the six speeds given.

(Take $f = 0·009$, $b = 0·2$, $S = 15·5\sqrt{W \times L}$, as p. 262.)

Ans. (10) 46·4 %; (12) 47·1 %; (14) 47·3 %; (16) 48·5 %; (18) 48·9 %; (20) 49·8 %.

32. Using the above formula for E.H.P. (with $f = 0·009$, $b = 0·25$), determine the I.H.P. for speeds of 18 and 19½ knots respectively of a

vessel 350′ × 53½′ × 20′ × 5600 tons, using propulsive coefficients of 45 °/₀ and 47½ °/₀ respectively.

Ans. 7660 I.H.P.; 9740 I.H.P.

33. Draw out the metacentric diagram for all draughts of a square log of 2 feet side, floating with one corner down.

Supposing the log to be homogeneous, determine the limits between which the density must be in order that it shall float thus in stable equilibrium in fresh water.

Ans. Between 0·28 and 0·72.

34. A rectangular vessel is 175 feet long, 30 feet broad, 20 feet deep, and floats at a draught of 8 feet, with a metacentric height of 5 feet. Find the draught forward and aft, and the metacentric height due to flooding an empty compartment between bulkheads 120 feet and 150 feet from the after end. *Ans.* F. 13′ 4⅞″; A. 6′ 7⅛″; 4·2′.

35. In a wall-sided vessel, show that for an angle of heel θ the co-ordinates of the C.B. referred to axes through the C.B. in the upright condition are $x = BM_0 . \tan\theta$; $y = \frac{1}{2}BM_0 . \tan^2\theta$. ($BM_0$ refers to the upright condition.)

36. Using the above, show that a wall-sided vessel will heel to angle θ by shifting a weight w a distance d across the deck, θ being given by the equation—

$$\tan^3\theta + 2 . \left(\frac{GM_0}{BM_0}\right) \tan\theta - 2 . \frac{w \times d}{W \times BM_0} = 0$$

the suffix 0 referring to the upright condition.

Thus, for a zero metacentric height the heel θ is given by—

$$\tan\theta = \sqrt[3]{2 . \frac{w \times d}{W \times BM_0}}$$

37. Show that a wall-sided ship having an initial negative metacentric height will heel to an angle of $\theta = \tan^{-1}\sqrt{\frac{2 . GM_0}{BM_0}}$, and will then have a metacentric height of $2GM_0\sqrt{1 + 2\frac{GM_0}{BM_0}} = 2 . \frac{GM_0}{\cos\theta}$.

38. Prove $\left(\text{by using } BM = \frac{I}{V}\right)$ that the C.G. of a segment of a circle radius a, subtending an angle of 2θ at the centre, is distant from the centre $\frac{2}{3}a . \frac{\sin^3\theta}{\theta - \sin\theta\cos\theta}$, and thus for a semicircle $\left(\theta = \frac{\pi}{2}\right)$ the C.G. is $\frac{4a}{3\pi}$ from centre. (Area of segment is area of sector less area of triangle, or $a^2\theta - a^2\sin\theta\cos\theta$.)

39. A vessel of 300 feet length floats at a draught of 12 feet forward, 15 feet aft. (Tons per inch 18; moment to change trim 1 inch, 295 tons-feet; C.F. 12 feet abaft midships.) It is desired to bring her to a draught not exceeding 12 feet forward and aft. How could this be done?

Ans. Remove 350 tons 42½ feet abaft amidships (account is taken of increase of mean draught due to change of trim.)

40. Draw a curve of displacement for all draughts of a cylindrical vessel of diameter 20 feet and 150 feet long, and find, by using the curve, the distance of the C.B. from the base when floating at a draught of 15 feet.

41. Draw the curve of displacement of a vessel of 14 feet draught having the following displacements up to water-lines 2 feet apart, viz. 2118, 1682, 1270, 890, 553, 272, 71 tons, and by it find the position of the C.B. with reference to the top water-line. Suppose the tons per inch is 18·56, check your result by Morrish's formula.

Ans. 5·45 feet.

42. Prove the rule given on p. 19 for the volume of a sphere, by using Simpson's rules at ordinates, say, ¼ the radius apart.

(An exact result should be obtained, because the curve of areas is a parabola, which Simpson's rule correctly integrates.)

43. A box-shaped vessel 140 feet long, 20 feet broad, 10 feet draught is inclined by shifting 7 tons 15 feet across the deck, and heels to an angle $\tan^{-1}(\frac{1}{4})$. Find the metacentric height (*a*) accurately, (*b*) by ordinary method.

Ans. (*a*) 0·42 foot, (*b*) 0·52 foot.

44. A long iron pontoon, of section 6 feet square and of uniform thickness, floats when empty in sea-water, but lolls over in fresh water. Find the thickness of the iron. (When the M curve is at mid-depth the draughts are 1·268 foot and 4·732 feet. Of these the curve drops for increase of draught only in the former, so that for increase of draught as occurs in fresh water there is a negative metacentric height).

Ans. ⅛ inch.

45. A solid is formed of a right circular cylinder and a right circular cone of the same altitude h on opposite sides of a circular base radius r. It floats with the axis vertical, the whole of the cone and half the cylinder being immersed. Prove that the metacentric height is $\frac{3}{10} \cdot \frac{r^2}{h} - \frac{21}{80} \cdot h$, so that for stable equilibrium r must be greater than $0·934h$.

46. A shallow-draught lightly built vessel is being launched. State the nature of the strains on the structure that will be experienced as she goes down. (From this point of view, the practice of some firms in launching torpedo-boat destroyers is of interest.)

47. A vessel has a list to starboard due to negative metacentric height when upright. It is found that the addition of weights in the 'tween decks on the port side increases the list to starboard. How do you explain this?

48. If a swan or duck is floating in a pond, and reaches down to the bottom for food, why does the bird find it necessary to work with her feet to keep the head down and the tail up?

49. When floating in water, why is it necessary to keep the arms below? If the arms are raised out, what happens, and why?

50. If a certain-sized tin is placed in water it will not float upright. When a certain quantity of water is poured in it floats upright in stable equilibrium. State fully the conditions of stability which lead to this result, bearing in mind the large loss of metacentric height due to the free surface of the water inside.

51. A cube 12 inches side weighs 10·4 lbs. Investigate the stability in fresh water, (*a*) with two faces horizontal, (*b*) with two faces only vertical and one edge downwards, (*c*) with a corner down.

52. In going through the Caledonian Canal, the writer has noticed that the level of the water falls amidships. How do you account for this?

53. If the water in question 36, Chap. III., goes right away with the tide, and the mud is very deep, investigate the stability of the vessel, the original metacentric height being 4 feet.

Ans. Negative GM of ⅓ foot.

54. In a box-shaped vessel, 200 feet long, 30 feet wide, 10 feet draught, and having its C.G. 11·8 feet above the keel, a central transverse compartment 50 feet long (assumed empty) is opened up to the sea. Will this vessel be stable after damage and free from danger in still water with a row of sidelights open, the lower edges of which are 15 feet above the keel?

The GM when intact is 0·7 foot, and when damaged is 0·5 foot, and new draught is $13\frac{1}{3}$ feet, and in the final condition the vessel is all right, but there are intermediate conditions to consider, and the following table gives results of calculations with various depths of water in the middle compartment:—

Height of water in compartment in feet.	Draught of water in feet.	Metacentric height in feet
1	$10\frac{1}{4}$	−0·93
2	$10\frac{1}{2}$	−0·71
3	$10\frac{3}{4}$	−0·51
4	11	−0·32
6	$11\frac{1}{2}$	−0·01
9	$12\frac{1}{4}$	+0·34
$13\frac{1}{3}$	$13\frac{1}{3}$	+0·5

Thus in the early stages the metacentric height is negative, and the ship will "loll," and if the hole through which the water enters is of comparatively small dimensions there would be an appreciable time for the list to develop.

The above is taken from Prof. Welch's paper from the N.E. Coast Institution, 1915, on "The Time Element and Related Matters in some Ship Calculations," to which the reader is referred for a further development of the subject.

55. A water-plane has the following shape each side. The forward half is a curve of versed sines and the after part an ellipse. The exact area is readily obtainable, viz. $\frac{1}{8} . L . B(\pi + 2)$, where L and B are the length and breadth respectively and the centre of gravity abaft midships $\frac{1}{6(\pi + 2)} . L$. Plot these curves for a length of 200 feet and a breadth of 20 feet and see how near the exact results are obtained, (1) by using Simpson's rules for 21 ordinates, (2) by Tchebycheff approximation, using ordinates 2, 5, 7, 10, 12, 15, 17, 20.

56. A water-plane has the following semi-ordinates, viz. 0, 4·7, 8·9, 12·6, 15·9, 18·75, 21·30, 23·35, 25·0, 26·2, 26·95, 27·25, 27·05, 26·5, 25·45, 23·85, 21·55, 18·65, 15·15, 10·85, 5·95, and the displacement to this W.L. is 6060 tons, the ordinates being spaced 26·75 feet apart.

Find the distance between the centre of buoyancy and the transverse metacentre.

Check your result by using the Tchebycheff 4-ordinate rule applied twice over and assuming that the ordinates are sufficiently close to Nos. 2, 5, 7, 10, 12, 15, 17, 20 that the lengths of the latter may be used in the Tchebycheff calculation for an approximate result.

57. Work out the example 14 of Chapter IV. on the assumption of examples 55 and 56.

Ans. (1) 23,944 square feet; (2) 18·16′; (3) 17·1′; (4) 446′.

58. A rectangular pontoon 300′ × 12′ × 4′ draught is divided by a longitudinal bulkhead at the middle line and by four transverse bulkheads equally spaced. A forward compartment on one side is bilged. Find the draughts at the four corners if the original metacentric height is 2 feet.

This is the example worked out in Mackrow and Woollard's pocket-book. The writer prefers to treat it on the "lost buoyancy" principle rather than as an "added weight," as there is free communication with the water outside. As there are a number of pitfalls in the solution, the results at the various stages of the calculation are given—

Lost buoyancy	= 1440 cubic feet.
Bodily sinkage	= 0·445 foot.
New C.B. above keel	= 2·22 feet.
C.G. above keel	= 3 feet.

Taking heel only—

New C.B. and new C.F. from M.L.	= $\frac{1}{8}$ foot.
New GM_T	= 1·92 feet.
θ being angle of keel, tan θ	= 0·175.
∴ sinkage immersed side	= 1·11 feet.
Lift emerged side	= 0·99 foot.

Taking trim only—

New C.B. and new C.F. from midships	= $13\frac{1}{8}$ feet.
New GM_L	= 1466 feet.
ϕ being angle of trim, tan ϕ	= 0·0091.
Sinkage at forward end	= 1·49 feet.
Lift at after end	= 1·24 feet.

Combining bodily sinkage, heel, and trim we have the following results for the draughts at the four corners :—

Ans. Forward, 7·045 feet, 4·945 feet.
Aft, 4·315 feet, 2·215 feet.

This is only a close approximation as the variation in the shape of the water-plane as the vessel heels and trims has not been allowed for.

59. Test the following approximation for the distance between the centre of buoyancy and the transverse metacentre (due to Mr. McCloghrie, R.C.N.C.). $420 \cdot \frac{B.T^2}{W.L.}$

SOLUTION OF QUESTION NO. 21, CHAP. II., AND NO. 36, CHAP. III.

The author has had a number of requests as to the solution of these examples, and as they illustrate an important principle the solution is given below.

The metacentre when floating freely is readily obtained, viz. 16 feet from the base line.

When the water level sinks 6 feet, the lower portion sinks into the mud, say x feet. Then, since the mud has a s.g. of 2, we take in the area O*wl* twice. We equate the new displacement to the old, or—

$$(x + 6)^2 + x^2 = 144$$

from which $x = 4·94$ feet.

For a small inclination one half of the buoyancy of the portion *Owl* will act through *m* the metacentre of *Owl*, and the buoyancy of OW'L' will act through M' the metacentre of OW'L', the portion *Owl* then being

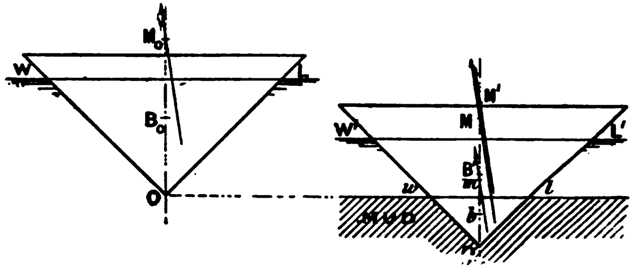


FIG. 156.

included twice. $Om = 6{\cdot}59'$ and $OM' = 14{\cdot}59'$. The total buoyancy will act through a metacentre M such that—

$$(\overline{10{\cdot}94^2} \times 14{\cdot}59) + (\overline{4{\cdot}94^2} \times 6{\cdot}59) = \overline{12^2} \times OM,$$
from which $OM = 13{\cdot}2$ feet.

That is, the new metacentre is 2·8 feet below the original metacentre, and as the C.G. of the ship has not been affected, the loss of metacentric height is 2¾ feet, about.

APPENDIX B

TABLES OF LOGARITHMS, SINES, COSINES, AND TANGENTS, SQUARES AND CUBES.

LOGARITHMS.—For some calculations considerable trouble is saved by using logarithms. One instance of this has been already given on p. 331. A table of logarithms is given on pp. 468, 469, to four places of decimals, which gives sufficient accuracy for ordinary purposes. To the right of the table are given the differences for 1, 2, 3, etc., which enables the logarithms of numbers of four figures to be obtained.

Thus log 2470 = 3·3927. The decimal part is obtained from the table, the whole number being 3, because 2470 is between 1000 and 10,000 (log 1000 = 3, log 10,000 = 4). The log of 2473 is obtained by adding to the above log the difference in the table for 3, viz. 5, *i.e.*—

$$\log 2473 = 3 \cdot 3932.$$

The following are the principal relations in logarithms, viz. :—

$$\log (M \times N) = \log M + \log N$$

$$\log \left(\frac{M}{N}\right) = \log M - \log N$$

$$\log (M)^n = n \, . \log M$$

$$\log \sqrt[n]{M} = \frac{1}{n} \, . \log M$$

Thus multiplication is turned into addition, division is turned into subtraction, the raising to a power is turned into multiplication, and the taking of a root is turned into division.

The decimal portion of a logarithm is always kept positive, and the following are the values of the logarithm of the number 239 for various positions of the decimal point:—

$$\log 23{,}900 = 4{\cdot}3784$$
$$\log 2{,}399 = 3{\cdot}3784$$
$$\log 239 = 2{\cdot}3784$$
$$\log 23{\cdot}9 = 1{\cdot}3784$$
$$\log 2{\cdot}39 = 0{\cdot}3784$$
$$\log 0{\cdot}239 = -1 + 0{\cdot}3784 = \bar{1}{\cdot}3784$$
$$\log 0{\cdot}0239 = -2 + 0{\cdot}3784 = \bar{2}{\cdot}3784$$
$$\log 0{\cdot}00239 = -3 + 0{\cdot}3784 = \bar{3}{\cdot}3784$$

Example.—To find the cube root of 10·75:

$$\log 10{\cdot}75 = 1{\cdot}0315$$
$$\log \sqrt[3]{(10{\cdot}75)} = \tfrac{1}{3}\,(1{\cdot}0315)$$
$$= 0{\cdot}3438$$
$$0{\cdot}3438 = 0{\cdot}3424 + 0{\cdot}0014$$
$$\therefore \sqrt[3]{10{\cdot}75} = 2{\cdot}207$$

Example.—To find the value of $(5{\cdot}725)^{\frac{7}{2}}$:

$$\log 5{\cdot}725 = 0{\cdot}7578$$
$$\log (5{\cdot}725)^{\frac{7}{2}} = \tfrac{7}{2}\,(0{\cdot}7578)$$
$$= 2{\cdot}6523$$
$$\therefore (5{\cdot}725)^{\frac{7}{2}} = 449{\cdot}1$$

Example.—To find the value of $\dfrac{(9231)^{\frac{2}{3}} \times (14{\cdot}08)^3}{5267}$

$$\log (9231)^{\frac{2}{3}} = \tfrac{2}{3}\,(3{\cdot}9652)$$
$$= 2{\cdot}6435$$
$$\log (14{\cdot}08)^3 = 3\,(1{\cdot}1485)$$
$$= 3{\cdot}4455$$
$$\log 5267 = 3{\cdot}7216$$
$$\therefore \log\left[\frac{(9231)^{\frac{2}{3}} \times (14{\cdot}08)^3}{5267}\right] = 2{\cdot}6435 + 3{\cdot}4455 - 3{\cdot}7216$$
$$= 2{\cdot}3674$$

The number of which this is the log is 233

$$\therefore \frac{(9231)^{\frac{2}{3}} \times (14{\cdot}08)^3}{5267} = 233$$

Example.—Find the value of $512 \times 50{\cdot}5 \times 0{\cdot}0037$.

$$\log 512 = 2{\cdot}7093$$
$$\log 50{\cdot}5 = 1{\cdot}7033$$
$$\log 0{\cdot}0037 = \bar{3}\;5682$$
$$\log (\text{product}) = 1{\cdot}9808$$
$$\text{or product required} = 95{\cdot}66$$

Example.—Find the value of $\sqrt[4]{0{\cdot}00765}$.

$$\log 0{\cdot}00765 = \bar{3}{\cdot}8837$$
$$= \bar{4} + 1{\cdot}8837$$
$$\log \sqrt[4]{0{\cdot}00765} = \bar{1} + 0{\cdot}4709$$
$$\therefore \sqrt[4]{0{\cdot}00765} = 0{\cdot}2957$$

Napierian or Hyperbolic Logarithms.

These logarithms, which are also termed "natural," are calculated to the base $e = 2{\cdot}718$, and the following relation holds:

$$\log_e N = 2{\cdot}3 \log_{10} N$$

Ordinary logarithms are calculated to the base 10.

Table of Sines, Cosines, and Tangents.

On pp. 470, 471, is given a table showing the values of the trigonometrical ratios, sines, cosines, and tangents, of angles up to 90°, to three places of decimals, which will be found sufficiently accurate for ordinary purposes.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	31
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE OF NATURAL TANGENTS, SINES, AND COSINES.

Angle.	Tangent.	Sine.	Cosine.	Angle.	Tangent.	Sine.	Cosine.	Angle.	Tangent.	Sine.	Cosine.
° ′				° ′				° ′			
0	0·000	0·000	1·000	30·0	0·577	0·500	0·866	60·0	1·732	0·866	0·500
0·30	·009	·009	·999	30·30	·589	·508	·862	60·30	1·767	·870	·492
1	·017	·017	·999	31	·601	·515	·857	61	1·804	·875	·485
1·30	·026	·026	·999	31·30	·613	·522	·853	61·30	1·842	·879	·477
2	·035	·035	·999	32	·625	·530	·848	62	1·881	·883	·469
2·30	·044	·044	·999	32·30	·637	·537	·843	62·30	1·921	·887	·462
3	·052	·052	·999	33	·649	·545	·839	63	1·963	·891	·454
3·30	·061	·061	·998	33·30	·662	·552	·834	63·30	2·006	·895	·446
4	·070	·070	·998	34	·675	·559	·829	64	2·050	·899	·438
4·30	·079	·078	·997	34·30	·687	·566	·824	64·30	2·097	·903	·431
5	·087	·087	·996	35	·700	·574	·819	65	2·145	·906	·423
5·30	·096	·096	·995	35·30	·713	·581	·814	65·30	2·194	·910	·415
6	·105	·105	·995	36	·727	·588	·809	66	2·246	·914	·407
6·30	·114	·113	·994	36·30	·740	·595	·804	66·30	2·300	·917	·399
7	·123	·122	·993	37	·754	·602	·799	67	2·356	·921	·391
7·30	·132	·131	·991	37·30	·767	·609	·793	67·30	2·414	·924	·383
8	·141	·139	·990	38	·781	·616	·788	68	2·475	·927	·375
8·30	·150	·148	·989	38·30	·795	·623	·783	68·30	2·539	·930	·367
9	·158	·156	·988	39	·810	·629	·777	69	2·605	·934	·358
9·30	·167	·165	·986	39·30	·824	·636	·772	69·30	2·675	·937	·350
10	·176	·174	·985	40	·839	·643	·766	70	2·747	·940	·342
10·30	·185	·182	·983	40·30	·854	·649	·760	70·30	2·824	·943	·334
11	·194	·191	·982	41	·869	·656	·755	71	2·904	·946	·326
11·30	·203	·199	·980	41·30	·885	·663	·749	71·30	2·989	·948	·317
12	·213	·208	·978	42	·900	·669	·743	72	3·078	·951	·309
12·30	·222	·216	·976	42·30	·916	·676	·737	72·30	3·172	·954	·301
13	·231	·225	·974	43	·933	·682	·731	73	3·271	·956	·292
13·30	·240	·233	·972	43·30	·949	·688	·725	73·30	3·376	·959	·284
14	·250	·242	·970	44	·966	·695	·719	74	3·487	·961	·276
14·30	·259	·250	·968	44·30	·983	·701	·713	74·30	3·606	·964	·267
15	·268	·259	·966	45	1·000	·707	·707	75	3·732	·966	·259

TABLE OF NATURAL TANGENTS, SINES, AND COSINES.

Angle.	Tangent.	Sine.	Cosine.	Angle.	Tangent.	Sine.	Cosine.	Angle.	Tangent.	Sine.	Cosine.
15.30	.277	.267	.964	45.30	1.018	.713	.701	75.30	3.867	.968	.250
16	.287	.276	.961	46	1.036	.719	.695	76	4.011	.970	.242
16.30	.296	.284	.959	46.30	1.054	.725	.688	76.30	4.165	.972	.233
17	.306	.292	.956	47	1.072	.731	.682	77	4.331	.974	.225
17.30	.315	.301	.954	47.30	1.091	.737	.676	77.30	4.511	.976	.216
18	.325	.309	.951	48	1.111	.743	.669	78	4.705	.978	.208
18.30	.335	.317	.948	48.30	1.130	.749	.663	78.30	4.915	.980	.199
19	.344	.326	.946	49	1.150	.755	.656	79	5.145	.982	.191
19.30	.354	.334	.943	49.30	1.171	.760	.649	79.30	5.396	.983	.182
20	.364	.342	.940	50	1.192	.766	.643	80	5.671	.985	.174
20.30	.374	.350	.937	50.30	1.213	.772	.636	80.30	5.976	.986	.165
21	.384	.358	.934	51	1.235	.777	.629	81	6.314	.988	.156
21.30	.394	.367	.930	51.30	1.257	.783	.623	81.30	6.691	.989	.148
22	.404	.375	.927	52	1.280	.788	.616	82	7.115	.990	.139
22.30	.414	.383	.924	52.30	1.303	.793	.609	82.30	7.596	.991	.131
23	.424	.391	.921	53	1.327	.799	.602	83	8.144	.993	.122
23.30	.435	.399	.917	53.30	1.351	.804	.595	83.30	8.777	.994	.113
24	.445	.407	.914	54	1.376	.809	.588	84	9.514	.995	.105
24.30	.456	.415	.910	54.30	1.402	.814	.581	84.30	10.385	.995	.096
25	.466	.423	.906	55	1.428	.819	.574	85	11.430	.996	.087
25.30	.477	.431	.903	55.30	1.455	.824	.566	85.30	12.706	.997	.078
26	.488	.438	.899	56	1.483	.829	.559	86	14.301	.998	.070
26.30	.499	.446	.895	56.30	1.511	.834	.552	86.30	16.350	.998	.061
27	.510	.454	.891	57	1.540	.839	.545	87	19.081	.999	.052
27.30	.521	.462	.887	57.30	1.570	.843	.537	87.30	22.904	.999	.044
28	.532	.469	.883	58	1.600	.848	.530	88	28.636	.999	.035
28.30	.543	.477	.879	58.30	1.632	.853	.522	88.30	38.188	.999	.026
29	.554	.485	.875	59	1.664	.857	.515	89	57.290	.999	.017
29.30	.566	.492	.870	59.30	1.698	.862	.508	89.30	114.589	.999	.009
30	.577	.500	.866	60	1.732	.866	.500	90	*inf.*	1.000	0.000

TABLE OF SQUARES AND CUBES OF NUMBERS UP TO 50, RISING BY 0·05.

The following table has been prepared, as squares and cubes of numbers are frequently required in ship calculations. Ordinates usually will not be measured more accurately than to the nearest 0·05; in most cases the nearest decimal point is sufficiently accurate. The squares and cubes are taken to the nearest whole number, which is all that is necessary in ship calculations.

Number.	Squares.	Cubes.	Number.	Squares.	Cubes.	Number.	Squares.	Cubes.
0·05	—	—	1·55	2	4	3·05	9	28
0·10	—	—	1·60	3	4	3·10	10	30
0·15	—	—	1·65	3	4	3·15	10	31
0·20	—	—	1·70	3	5	3·20	10	33
0·25	—	—	1·75	3	5	3·25	11	34
0·30	—	—	1·80	3	6	3·30	11	36
0·35	—	—	1·85	3	6	3·35	11	38
0·40	—	—	1·90	4	7	3·40	12	39
0·45	—	—	1·95	4	7	3·45	12	41
0·50	—	—	2·00	4	8	3·50	12	43
0·55	—	—	2·05	4	9	3·55	13	45
0·60	—	—	2·10	4	9	3·60	13	47
0·65	—	—	2·15	5	10	3·65	13	49
0·70	—	—	2·20	5	11	3·70	14	51
0·75	1	—	2·25	5	11	3·75	14	53
0·80	1	1	2·30	5	12	3·80	14	55
0·85	1	1	2·35	6	13	3·85	15	57
0·90	1	1	2·40	6	14	3·90	15	59
0·95	1	1	2·45	6	15	3·95	16	62
1·00	1	1	2·50	6	16	4·00	16	64
1·05	1	1	2·55	7	17	4·05	16	66
1·10	1	1	2·60	7	18	4·10	17	69
1·15	1	2	2·65	7	19	4·15	17	71
1·20	1	2	2·70	7	20	4·20	18	74
1·25	2	2	2·75	8	21	4·25	18	77
1·30	2	2	2·80	8	22	4·30	18	80
1·35	2	2	2·85	8	23	4·35	19	82
1·40	2	3	2·90	8	24	4·40	19	85
1·45	2	3	2·95	9	26	4·45	20	88
1·50	2	3	3·00	9	27	4·50	20	91

Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.
4·55	21	94	6·80	46	314	9·05	82	741
4·60	21	97	6·85	47	321	9·10	83	754
4·65	22	101	6·90	48	329	9·15	84	766
4·70	22	104	6·95	48	336	9·20	85	779
4·75	23	107	7·00	49	343	9·25	86	791
4·80	23	111	7·05	50	350	9·30	86	804
4·85	24	114	7·10	50	358	9·35	87	817
4·90	24	118	7·15	51	366	9·40	88	831
4·95	25	121	7·20	52	373	9·45	89	844
5·00	25	125	7·25	53	381	9·50	90	857
5·05	26	129	7·30	53	389	9·55	91	871
5·10	26	133	7·35	54	397	9·60	92	885
5·15	27	137	7·40	55	405	9·65	93	899
5·20	27	141	7·45	56	413	9·70	94	913
5·25	28	145	7·50	56	422	9·75	95	927
5·30	28	149	7·55	57	430	9·80	96	941
5·35	29	153	7·60	58	439	9·85	97	956
5·40	29	157	7·65	59	448	9·90	98	970
5·45	30	162	7·70	59	457	9·95	99	985
5·50	30	166	7·75	60	465	10·00	100	1,000
5·55	31	171	7·80	61	475	10·05	101	1,015
5·60	31	176	7·85	62	484	10·10	102	1,030
5·65	32	180	7·90	62	493	10·15	103	1,046
5·70	32	185	7·95	63	502	10·20	104	1,061
5·75	33	190	8·00	64	512	10·25	105	1,077
5·80	34	195	8·05	65	522	10·30	106	1,093
5·85	34	200	8·10	66	531	10·35	107	1,109
5·90	35	205	8·15	66	541	10·40	108	1,125
5·95	35	211	8·20	67	551	10·45	109	1,141
6·00	36	216	8·25	68	562	10·50	110	1,158
6·05	37	221	8·30	69	572	10·55	111	1,174
6·10	37	227	8·35	70	582	10·60	112	1,191
6·15	38	233	8·40	71	593	10·65	113	1,208
6·20	38	238	8·45	71	603	10·70	114	1,225
6·25	39	244	8·50	72	614	10·75	116	1,242
6·30	40	250	8·55	73	625	10·80	117	1,260
6·35	40	256	8·60	74	636	10·85	118	1,277
6·40	41	262	8·65	75	647	10·90	119	1,295
6·45	42	268	8·70	76	659	10·95	120	1,313
6·50	42	275	8·75	77	670	11·00	121	1,331
6·55	43	281	8·80	77	681	11·05	122	1,349
6·60	44	287	8·85	78	693	11·10	123	1,368
6·65	44	294	8·90	79	705	11·15	124	1,386
6·70	45	301	8·95	80	717	11·20	125	1,405
6·75	46	308	9·00	81	729	11·25	127	1,424

Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.
11·30	128	1,443	13·55	184	2,488	15·80	250	3,944
11·35	129	1,462	13·60	185	2,515	15·85	251	3,982
11·40	130	1,482	13·65	186	2,543	15·90	253	4,020
11·45	131	1,501	13·70	188	2,571	15·95	254	4,058
11·50	132	1,521	13·75	189	2,600	16·00	256	4,096
11·55	133	1,541	13·80	190	2,628	16·05	258	4,135
11·60	135	1,561	13·85	192	2,657	16·10	259	4,173
11·65	136	1,581	13·90	193	2,686	16·15	261	4,212
11·70	137	1,602	13·95	195	2,715	16·20	262	4,252
11·75	138	1,622	14·00	196	2,744	16·25	264	4,291
11·80	139	1,643	14·05	197	2,774	16·30	266	4,331
11·85	140	1,664	14·10	199	2,803	16·35	267	4,371
11·90	142	1,685	14·15	200	2,833	16·40	269	4,411
11·95	143	1,706	14·20	202	2,863	16·45	271	4,451
12·00	144	1,728	14·25	203	2,894	16·50	272	4,492
12·05	145	1,750	14·30	204	2,924	16·55	274	4,533
12·10	146	1,772	14·35	206	2,955	16·60	276	4,574
12·15	148	1,794	14·40	207	2,986	16·65	277	4,616
12·20	149	1,816	14·45	209	3,017	16·70	279	4,657
12·25	150	1,838	14·50	210	3,049	16·75	281	4,699
12·30	151	1,861	14·55	212	3,080	16·80	282	4,742
12·35	153	1,884	14·60	213	3,112	16·85	284	4,784
12·40	154	1,907	14·65	215	3,144	16·90	286	4,827
12·45	155	1,930	14·70	216	3,177	16·95	287	4,870
12·50	156	1,953	14·75	218	3,209	17·00	289	4,913
12·55	158	1,977	14·80	219	3,242	17·05	291	4,956
12·60	159	2,000	14·85	221	3,275	17·10	292	5,000
12·65	160	2,024	14·90	222	3,308	17·15	294	5,044
12·70	161	2,048	14·95	224	3,341	17·20	296	5,088
12·75	163	2,073	15·00	225	3,375	17·25	298	5,133
12·80	164	2,097	15·05	227	3,409	17·30	299	5,178
12·85	165	2,122	15·10	228	3,443	17·35	301	5,223
12·90	166	2,147	15·15	230	3,477	17·40	303	5,268
12·95	168	2,172	15·20	231	3,512	17·45	305	5,314
13·00	169	2,197	15·25	233	3,547	17·50	306	5,359
13·05	170	2,222	15·30	234	3,582	17·55	308	5,405
13·10	172	2,248	15·35	236	3,617	17·60	310	5,452
13·15	173	2,274	15·40	237	3,652	17·65	312	5,498
13·20	174	2,300	15·45	239	3,688	17·70	313	5,545
13·25	176	2,326	15·50	240	3,724	17·75	315	5,592
13·30	177	2,353	15·55	242	3,760	17·80	317	5,640
13·35	178	2,379	15·60	243	3,796	17·85	319	5,687
13·40	180	2,406	15·65	245	3,833	17·90	320	5,735
13·45	181	2,433	15·70	246	3,870	17·95	322	5,784
13·50	182	2,460	15·75	248	3,907	18·00	324	5,832

Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.
18·05	326	5,881	20·30	412	8,365	22·55	509	11,467
18·10	328	5,930	20·35	414	8,427	22·60	511	11,543
18·15	329	5,979	20·40	416	8,490	22·65	513	11,620
18·20	331	6,029	20·45	418	8,552	22·70	515	11,697
18·25	333	6,078	20·50	420	8,615	22·75	518	11,775
18·30	335	6,128	20·55	422	8,678	22·80	520	11,852
18·35	337	6,179	20·60	424	8,742	22·85	522	11,930
18·40	339	6,230	20·65	426	8,806	22·90	524	12,009
18·45	340	6,280	20·70	428	8,870	22·95	527	12,088
18·50	342	6,332	20·75	431	8,934	23·00	529	12,167
18·55	344	6,383	20·80	433	8,999	23·05	531	12,247
18·60	346	6,435	20·85	435	9,064	23·10	534	12,326
18·65	348	6,487	20·90	437	9,129	23·15	536	12,407
18·70	350	6,539	20·95	439	9,195	23·20	538	12,487
18·75	352	6,592	21·00	441	9,261	23·25	541	12,568
18·80	353	6,645	21·05	443	9,327	23·30	543	12,649
18·85	355	6,698	21·10	445	9,394	23·35	545	12,731
18·90	357	6,751	21·15	447	9,461	23·40	548	12,813
18·95	359	6,805	21·20	449	9,528	23·45	550	12,895
19·00	361	6,859	21·25	452	9,596	23·50	552	12,978
19·05	363	6,913	21·30	454	9,664	23·55	555	13,061
19·10	365	6,968	21·35	456	9,732	23·60	557	13,144
19·15	367	7,023	21·40	458	9,800	23·65	559	13,228
19·20	369	7,078	21·45	460	9,869	23·70	562	13,312
19·25	371	7,133	21·50	462	9,938	23·75	564	13,396
19·30	372	7,189	21·55	464	10,008	23·80	566	13,481
19·35	374	7,245	21·60	467	10,078	23·85	569	13,566
19·40	376	7,301	21·65	469	10,148	23·90	571	13,652
19·45	378	7,358	21·70	471	10,218	23·95	574	13,738
19·50	380	7,415	21·75	473	10,289	24·00	576	13,824
19·55	382	7,472	21·80	475	10,360	24·05	578	13,911
19·60	384	7,530	21·85	477	10,432	24·10	581	13,998
19·65	386	7,587	21·90	480	10,503	24·15	583	14,085
19·70	388	7,645	21·95	482	10,576	24·20	586	14,172
19·75	390	7,704	22·00	484	10,648	24·25	588	14,261
19·80	392	7,762	22·05	486	10,721	24·30	590	14,349
19·85	394	7,821	22·10	488	10,794	24·35	593	14,438
19·90	396	7,881	22·15	491	10,867	24·40	595	14,527
19·95	398	7,940	22·20	493	10,941	24·45	598	14,616
20·00	400	8,000	22·25	495	11,015	24·50	600	14,706
20·05	402	8,060	22·30	497	11,090	24·55	603	14,796
20·10	404	8,121	22·35	500	11,164	24·60	605	14,887
20·15	406	8,181	22·40	502	11,239	24·65	608	14,978
20·20	408	8,242	22·45	504	11,315	24·70	610	15,069
20·25	410	8,304	22·50	506	11,391	24·75	613	15,161

Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes
24·80	615	15,253	27·05	732	19,793	29·30	858	25,154
24·85	618	15,345	27·10	734	19,903	29·35	861	25,283
24·90	620	15,438	27·15	737	20,013	29·40	864	25,412
24·95	623	15,531	27·20	740	20,124	29·45	867	25,542
25·00	625	15,625	27·25	743	20,235	29·50	870	25,672
25·05	628	15,719	27·30	745	20,346	29·55	873	25,803
25·10	630	15,813	27·35	748	20,458	29·60	876	25,934
25·15	633	15,908	27·40	751	20,571	29·65	879	26,066
25·20	635	16,003	27·45	754	20,684	29·70	882	26,198
25·25	638	16,098	27·50	756	20,797	29·75	885	26,331
25·30	640	16,194	27·55	759	20,911	29·80	888	26,464
25·35	643	16,290	27·60	762	21,025	29·85	891	26,597
25·40	645	16,387	27·65	765	21,139	29·90	894	26,731
25·45	648	16,484	27·70	767	21,254	29·95	897	26,865
25·50	650	16,581	27·75	770	21,369	30·00	900	27,000
25·55	653	16,679	27·80	773	21,485	30·05	903	27,135
25·60	655	16,777	27·85	776	21,601	30·10	906	27,271
25·65	658	16,876	27·90	778	21,718	30·15	909	27,407
25·70	660	16,975	27·95	781	21,835	30·20	912	27,544
25·75	663	17,074	28·00	784	21,952	30·25	915	27,681
25·80	666	17,174	28·05	787	22,070	30·30	918	27,818
25·85	668	17,274	28·10	790	22,188	30·35	921	27,956
25·90	671	17,374	28·15	792	22,307	30·40	924	28,094
25·95	673	17,475	28·20	795	22,426	30·45	927	28,233
26·00	676	17,576	28·25	798	22,545	30·50	930	28,373
26·05	679	17,678	28·30	801	22,665	30·55	933	28,512
26·10	681	17,780	28·35	804	22,786	30·60	936	28,653
26·15	684	17,882	28·40	807	22,906	30·65	939	28,793
26·20	686	17,985	28·45	809	23,028	30·70	942	28,934
26·25	689	18,088	28·50	812	23,149	30·75	946	29,076
26·30	692	18,191	28·55	815	23,271	30·80	949	29,218
26·35	694	18,295	28·60	818	23,394	30·85	952	29,361
26·40	697	18,400	28·65	821	23,517	30·90	955	29,504
26·45	700	18,504	28·70	824	23,640	30·95	958	29,647
26·50	702	18,610	28·75	827	23,764	31·00	961	29,791
26·55	705	18,715	28·80	829	23,888	31·05	964	29,935
26·60	708	18,821	28·85	832	24,013	31·10	967	30,080
26·65	710	18,927	28·90	835	24,138	31·15	970	30,226
26·70	713	19,034	28·95	838	24,263	31·20	973	30,371
26·75	716	19,141	29·00	841	24,389	31·25	977	30,518
26·80	718	19,249	29·05	844	24,515	31·30	980	30,664
26·85	721	19,357	29·10	847	24,642	31·35	983	30,811
26·90	724	19,465	29·15	850	24,769	31·40	986	30,959
26·95	726	19,574	29·20	853	24,897	31·45	989	31,107
27·00	729	19,683	29·25	856	25,025	31·50	992	31,256

Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.
31·55	995	31,405	33·80	1,142	38,614	36·05	1,300	46,851
31·60	999	31,554	33·85	1,146	38,786	36·10	1,303	47,046
31·65	1,002	31,705	33·90	1,149	38,958	36·15	1,307	47,242
31·70	1,005	31,855	33·95	1,153	39,131	36·20	1,310	47,438
31·75	1,008	32,006	34·00	1,156	39.304	36·25	1,314	47,635
31·80	1,011	32,157	34·05	1,159	39,478	36·30	1,318	47,832
31·85	1,014	32,309	34·10	1,163	39,652	36·35	1,321	48,030
31·90	1,018	32,462	34·15	1,166	39,826	36·40	1,325	48,229
31·95	1,021	32,615	34·20	1,170	40,002	36·45	1,329	48,428
32·00	1,024	32,768	34·25	1,173	40,177	36·50	1,332	48,627
32·05	1,027	32,922	34·30	1,176	40,354	36·55	1,336	48,827
32·10	1,030	33,076	34·35	1,180	40,530	36·60	1,340	49,028
32·15	1,034	33,231	34·40	1,183	40,708	36·65	1,343	49,229
32·20	1,037	33,386	34·45	1,187	40,885	36·70	1,347	49,431
32·25	1,040	33,542	34·50	1,190	41,064	36·75	1,351	49,633
32·30	1,043	33,698	34·55	1,194	41,242	36·80	1,354	49,836
32·35	1,047	33,855	34·60	1,197	41,422	36·85	1,358	50,039
32·40	1,050	34,012	34·65	1,201	41,602	36·90	1,362	50,243
32·45	1,053	34,170	34·70	1,204	41,782	36·95	1,365	50,448
32·50	1,056	34,328	34·75	1,208	41,963	37·00	1,369	50,653
32·55	1,060	34,487	34·80	1,211	42,144	37·05	1,373	50,859
32·60	1,063	34,646	34·85	1,215	42,326	37·10	1,376	51,065
32·65	1,066	34,806	34·90	1,218	42,509	37·15	1,380	51,272
32·70	1,069	34,966	34·95	1,222	42,692	37·20	1,384	51,479
32·75	1,073	35,126	35·00	1,225	42,875	37·25	1,388	51,687
32·80	1,076	35,288	35·05	1,229	43,059	37·30	1,391	51,895
32·85	1,079	35,449	35·10	1,232	43,244	37·35	1,395	52,104
32·90	1,082	35,611	35·15	1,236	43,429	37·40	1,399	52,314
32·95	1,086	35,774	35·20	1,239	43,614	37·45	1,403	52,524
33·00	1,089	35,937	35·25	1,243	43,800	37·50	1,406	52,734
33·05	1,092	36,101	35·30	1,246	43,987	37·55	1,410	52,946
33·10	1,096	36,265	35·35	1,250	44,174	37·60	1,414	53,157
33·15	1,099	36,429	35·40	1,253	44,362	37·65	1,418	53,370
33·20	1,102	36,594	35·45	1,257	44,550	37·70	1,421	53,583
33·25	1,106	36,760	35·50	1,260	44,739	37·75	1,425	53,796
33·30	1,109	36,926	35·55	1,264	44,928	37·80	1,429	54,010
33·35	1,112	37,093	35·60	1,267	45,118	37·85	1,433	54,225
33·40	1,116	37,260	35·65	1,271	45,308	37·90	1,436	54,440
33·45	1,119	37,427	35·70	1,274	45,499	37·95	1,440	54,656
33·50	1,122	37,595	35·75	1,278	45,691	38·00	1,444	54,872
33·55	1,126	37,764	35·80	1,282	45,883	38·05	1,448	55,089
33·60	1,129	37,933	35·85	1,285	46,075	38·10	1,452	55,306
33·65	1,132	38,103	35·90	1,289	46,268	38·15	1,455	55,524
33·70	1,136	38,273	35·95	1,292	46,462	38·20	1,459	55,743
33·75	1,139	38,443	36·00	1,296	46,656	38·25	1,463	55,962

Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.
38·30	1,467	56,182	40·55	1,644	66,676	42·80	1,832	78,403
38·35	1,471	56,402	40·60	1,648	66,923	42·85	1,836	78,678
38·40	1,475	56,623	40·65	1,652	67,171	42·90	1,840	78,954
38·45	1,478	56,845	40·70	1,656	67,419	42·95	1,845	79,230
38·50	1,482	57,067	40·75	1,661	67,668	43·00	1,849	79,507
38·55	1,486	57,289	40·80	1,665	67,917	43·05	1,853	79,785
38·60	1,490	57,512	40·85	1,669	68,167	43·10	1,858	80,063
38·65	1,494	57,736	40·90	1,673	68,418	43·15	1,862	80,342
38·70	1,498	57,961	40·95	1,677	68,669	43·20	1,866	80,622
38·75	1,502	58,186	41·00	1,681	68,921	43·25	1,871	80,902
38·80	1,505	58,411	41·05	1,685	69,173	43·30	1,875	81,183
38·85	1,509	58,637	41·10	1,689	69,427	43·35	1,879	81,464
38·90	1,513	58,864	41·15	1,693	69,680	43·40	1,884	81,747
38·95	1,517	59,091	41·20	1,697	69,935	43·45	1,888	82,029
39·00	1,521	59,319	41·25	1,702	70,189	43·50	1,892	82,313
39·05	1,525	59,547	41·30	1,706	70,445	43·55	1,897	82,597
39·10	1,529	59,776	41·35	1,710	70,701	43·60	1,901	82,882
39·15	1,533	60,006	41·40	1,714	70,958	43·65	1,905	83,167
39·20	1,537	60,236	41·45	1,718	71,215	43·70	1,910	83,453
39·25	1,541	60,467	41·50	1,722	71,473	43·75	1,914	83,740
39·30	1,544	60,698	41·55	1,726	71,732	43·80	1,918	84,028
39·35	1,548	60,930	41·60	1,731	71,991	43·85	1,923	84,316
39·40	1,552	61,163	41·65	1,735	72,251	43·90	1,927	84,605
39·45	1,556	61,396	41·70	1,739	72,512	43·95	1,932	84,894
39·50	1,560	61,630	41·75	1,743	72,773	44·00	1,936	85,184
39·55	1,564	61,864	41·80	1,747	73,035	44·05	1,940	85,475
39·60	1,568	62,099	41·85	1,751	73,297	44·10	1,945	85,766
39·65	1,572	62,335	41·90	1,756	73,560	44·15	1,949	86,058
39·70	1,576	62,571	41·95	1,760	73,824	44·20	1,954	86,351
39·75	1,580	62,807	42·00	1,764	74,088	44·25	1,958	86,644
39·80	1,584	63,045	42·05	1,768	74,353	44·30	1,962	86,938
39·85	1,588	63,283	42·10	1,772	74,618	44·35	1,967	87,233
39·90	1,592	63,521	42·15	1,777	74,885	44·40	1,971	87,528
39·95	1,596	63,760	42·20	1,781	75,151	44·45	1,976	87,824
40·00	1,600	64,000	42·25	1,785	75,419	44·50	1,980	88,121
40·05	1,604	64,240	42·30	1,789	75,687	44·55	1,985	88,418
40·10	1,608	64,481	42·35	1,794	75,956	44·60	1,989	88,717
40·15	1,612	64,723	42·40	1,798	76,225	44·65	1,994	89,015
40·20	1,616	64,965	42·45	1,802	76,495	44·70	1,998	89,315
40·25	1,621	65,208	42·50	1,806	76,766	44·75	2,003	89,615
40·30	1,624	65,451	42·55	1,811	77,037	44·80	2,007	89,915
40·35	1,628	65,695	42·60	1,815	77,309	44·85	2,012	90,217
40·40	1,632	65,939	42·65	1,819	77,581	44·90	2,016	90,519
40·45	1,636	66,184	42·70	1,823	77,854	44·95	2,021	90,822
40·50	1,640	66,430	42·75	1,828	78,128	45·00	2,025	91,125

Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.	Numbers.	Squares.	Cubes.
45·05	2,030	91,429	47·05	2,214	104,155	49·05	2,406	118,010
45·10	2,034	91,734	47·10	2,218	104,487	49·10	2,411	118,371
45·15	2,039	92,039	47·15	2,223	104,820	49·15	2,416	118,733
45·20	2,043	92,345	47·20	2,228	105,154	49·20	2,421	119,095
45·25	2,048	92,652	47·25	2,233	105,489	49·25	2,426	119,459
45·30	2,052	92,960	47·30	2,237	105,824	49·30	2,430	119,823
45·35	2,057	93,268	47·35	2,242	106,160	49·35	2,435	120,188
45·40	2,061	93,577	47·40	2,247	106,496	49·40	2,440	120,554
45·45	2,066	93,886	47·45	2,252	106,834	49·45	2,445	120,920
45·50	2,070	94,196	47·50	2,256	107,172	49·50	2,450	121,287
45·55	2,075	94,507	47·55	2,261	107,511	49·55	2,455	121,655
45·60	2,079	94,819	47·60	2,266	107,850	49·60	2,460	122,024
45·65	2,084	95,131	47·65	2,271	108,190	49·65	2,465	122,393
45·70	2,088	95,444	47·70	2,275	108,531	49·70	2,470	122,763
45·75	2,093	95,758	47·75	2,280	108,873	49·75	2,475	123,134
45·80	2,098	96,072	47·80	2,285	109,215	49·80	2,480	123,506
45·85	2,102	96,387	47·85	2,290	109,558	49·85	2,485	123,878
45·90	2,107	96,703	47·90	2,294	109,902	49·90	2,490	124,251
45·95	2,111	97,019	47·95	2,299	110,247	49·95	2,495	124,625
46·00	2,116	97,336	48·00	2,304	110,592	50·00	2,500	125,000
46·05	2,121	97,654	48·05	2,309	110,938			
46·10	2,125	97,972	48·10	2,314	111,285			
46·15	2,130	98,291	48·15	2,318	111,632			
46·20	2,134	98,611	48·20	2,323	111,980			
46·25	2,139	98,932	48·25	2,328	112,329			
46·30	2,144	99,253	48·30	2,333	112,679			
46·35	2,148	99,575	48·35	2,338	113,029			
46·40	2,153	99,897	48·40	2,343	113,380			
46·45	2,158	100,221	48·45	2,347	113,732			
46·50	2,162	100,545	48·50	2,352	114,084			
46·55	2,167	100,869	48·55	2,357	114,437			
46·60	2,172	101,195	48·60	2,362	114,791			
46·65	2,176	101,521	48·65	2,367	115,146			
46·70	2,181	101,848	48·70	2,372	115,501			
46·75	2,186	102,175	48·75	2,377	115,857			
46·80	2,190	102,503	48·80	2,381	116,214			
46·85	2,195	102,832	48·85	2,386	116,572			
46·90	2,200	103,162	48·90	2,391	116,930			
46·95	2,204	103,492	48·95	2,396	117,289			
47·00	2,209	103,823	49·00	2,401	117,649			

APPENDIX C.

SYLLABUS OF EXAMINATIONS IN NAVAL ARCHITECTURE.

The students should be encouraged to make good rough sketches of the different parts of a ship's structure approximately to scale, using squared paper; they should also be impressed with the necessity of noting any detail of work brought before their notice daily in the shipyard. Questions will be set in the examination which require rough sketches of parts of a vessel to be given from memory.

If the class is held in an institution which possesses a testing machine, the students ought to be allowed to use it occasionally to test samples of materials used in shipbuilding.

All students should be provided with suitable scales, set squares, and ship curves, and candidates should bring these to the examination.

Table of logarithms, functions of angles, and useful constants will be provided, and candidates will be restricted to use of these tables, and will not be allowed to bring with them into the examination room any other mathematical or logarithm tables. Slide rules may be used.

Compulsory questions may be set at the examinations.

LOWER EXAMINATION

I. PRACTICAL SHIPBUILDING.—The tests to which the various materials used in shipbuilding are subjected, and the defects to which those materials are liable; the tools and appliances used in ordinary shipyard work, and the general arrangement of blocks, staging, derricks, etc., used on a building slip; plans of flat and vertical keels, inner bottom, shell, deck and other plating; framing, beam, keelson, and stringer plans; watertight and other bulkheads; ceiling and wood decks; pillaring arrangements to secure clear holds, and details of cargo hatchways to meet Lloyd's Rules; rudders, stern frames, and spectacle arrangements for twin-screw ships; bilge keels; supports to engines, boilers, and shafting; masts and derricks; precautions necessary to prevent deterioration of the hull of a ship while building, and while on service; method of docking ships, how they are placed in position and supported.

II. LAYING OFF.—A knowledge of the work carried on in the Mould Loft for the purpose of fairing a set of lines, including traces of keelsons and longitudinals, edges of shell plating, tank margins, ribbands, etc., and transferring the frame and other lines to the scrive board; lifting the bevels and constructing round of beam mould; a ship's block model and the information necessary for its construction; obtaining the dimensions for ordering the shell plating, frames, beams, floors, inner bottom plating, etc.; making and marking ribbands; fairing the edges of shell plating on the frames; making templates or skeleton patterns for stem, sternpost, propeller bracket forgings or castings.

III. DRAWING.—Plotting of curves of displacement, tons per inch immersion, I.H.P., etc., from given data. A rough freehand dimensioned sketch may be given at the examination, requiring candidates to make finished scale drawings, and candidates will be expected to be able to draw, from their own knowledge, the fastenings suitable for connecting together the parts which are the subject of the example.

IV. SHIP CALCULATIONS.—Calculation of the weights of simple parts of a ship's structure; spacing and strength of iron and steel rivets; calculation of the strength of the simple parts of a ship's structure, such as tie plates, butt straps and laps; tons per inch immersion; change of trim, and moment to change trim; change of trim due to moving weights on board, and that due to the addition or removal of weights; the principles and use of Simpson's and other rules for finding the area and position of the centre of gravity of a plane area, and for calculating the position of the centre of buoyancy; graphic methods of finding displacement and position of the centre of buoyancy; curves of displacement and of tons per inch immersion; the fundamental conditions to be fulfilled in order that any body may float freely and at rest in still water; centre of flotation, metacentre, metacentric height, stable and unstable equilibrium; definitions of block, prismatic, water-plane, midship area, and other similar coefficients.

HIGHER EXAMINATION

I. Practical Shipbuilding.—The structural arrangements necessary to resist longitudinal and transverse stresses to which ships are liable in still water and amongst waves, and the arrangements to resist local stresses; description and rough hand sketches of detail fittings of ships, such as anchor and capstan gear, steering gear, and other appliances used in working a ship; davits and fittings in connection therewith; ventilating and coaling arrangements; pumping and draining; the fundamental types of vessels and modifications thereto, the distinctive features of such vessels and consequent effect on freeboard; methods of determining the sizes of structural parts and of detail fittings, making out midship sections to the Rules of the principal classification Societies for various types of vessels; methods of fitting up refrigerating spaces for shipment of frozen and chilled meat, fruit, etc.; construction of oil fuel bunkers, and of vessels for carrying oil; launching arrangements, and the diagrams and curves generally used in connection therewith.

Laying Off.—Expanding the plating of longitudinals and margin plates by the geometric and mocking up methods; expanding stern plating, rudder trunking, and mast plating; obtaining the true shape of a hawse hole in the deck or shell, and similar practical problems; constructing and fairing the form of a twin screw bossing.

III. Ship Calculations.—Displacement sheet and arrangement of calculations made thereon; proofs of Simpson's and other rules for obtaining areas and moments; displacement and deadweight scales; approximate and detailed calculations relating to the weight and position of the centre of gravity of hull; calculations of weight and strength of parts of a ship's structure such as decks, bulkheads, framing, side and bottom plating, etc., also the strength of fittings such as boat davits, derricks, etc.; coefficients of weight of hull, outfit, and machinery for a few of the principal types of ships, also coefficients of position of the centre of gravity of the ships; curves of loads, shearing forces, and bending moments for a ship floating in still water, and amongst waves, also equivalent girder and stress in the material; calculations of the positions of transverse and longitudinal metacentres; consideration of the curves of centres of buoyancy, centres of flotation, and pro-metacentres; the construction and use of metacentric diagrams; Atwood's and Moseley's formulæ, and methods of calculating stability based thereon; the construction and use of curves of stability; inclining experiment and the precautions that must be taken to ensure accuracy; change of draught and trim due to passing from fresh into salt water and vice versa; effect upon trim and stability due to flooding compartments of a ship; effect of free surface on the stability of vessels carrying liquid cargo; methods of determining the size of rudder-heads, and the stresses on rudders balanced and unbalanced; resistance of ships; Froude's experiments on skin friction; Froude's law of comparison for vessels at corresponding speeds; methods of calculating the horse-power to propel a vessel of known form at a given speed; effective horse-power, propulsive coefficient and Admiralty constants, and values of the two last in typical cases; speed of ships on trial, methods adopted and precautions necessary to obtain accurate speed data; progressive trials and their uses; elementary considerations of the oscillations of ships in still water and amongst waves; definitions of a "stiff" and "steady" vessel, and elements of design affecting these qualities; tonnage of ships, how measured, etc.

APPENDIX D.

SPECIMEN QUESTIONS OF THE BOARD OF EDUCATION EXAMINATIONS.

CALCULATIONS.

1. What is the relation which must exist between the weight of a body floating freely at rest in a liquid and the volume of its submerged part? A body of uniform circular transverse section floats freely in sea-water so that the centres of the circular sections are in the water surface. What will its weight be if its length is 100 feet and its diameter 20 feet?

2. Find the area of a half of a ship's water-plane of which the curved form is defined by the following equidistant ordinates spaced 12 feet apart:—

0·1, 5·1, 7·17, 8·75, 10·1, 9·17, 8·05, 6·4, 0·1 feet.

3. By what number would you have to divide the area in square feet of a water-plane in order to obtain the number of tons weight it would be necessary to add to the ship in order to increase her draught one inch in salt water?

4. What is the relative position of the centre of gravity of the weight of a body floating freely at rest in water and the centre of gravity of the volume of the submerged portion of the body? What is the condition necessary for stable equilibrium?

5. What are the weights of a cubic foot of steel, yellow pine, and copper? What is the weight of a hollow steel pillar 10 feet long whose external diameter is 5 inches and internal diameter 4 inches? What is the diameter of a solid pillar of the same weight?

6. Calculate the volume and position of centre of gravity, horizontally and vertically, of a form given by the following ordinates:—

		ft.	ft.	ft.	
No. 1 W.L. ...	0·1	7·17	10·1	8·05	0·1
No. 2 „ ...	0·1	5·66	8·0	6·46	0·1
No. 3 „ ...	0·1	0·1	0·1	0·1	0·1

Horizontal interval, 24 feet; vertical interval, 3 feet.

7. The areas of transverse vertical sections of a solid are 1·2, 61·2, 86·0, 121·0, 96·6, 76·8, 1·2 square feet, at distances apart of 12, 12, 24, 24, 12, and 12 feet, respectively. Find the volume and longitudinal position of the centre of gravity of the solid.

8. What is the transverse metacentre of a ship in the upright position? What is the value of the distance between this metacentre and the centre of buoyancy?

9. A hold beam is formed of two beams, each formed of a $\frac{1}{2}$-inch plate 12 inches deep, and four angles $4'' \times 4'' \times \frac{1}{2}''$: the beams are connected together by a top plate $\frac{1}{2}$ inch thick, extending from the fore edge of the flange of the forward beam to the after edge of the flange of the after beam. Find the weight of 30 feet of such a beam. The frame spacing is 24 inches.

10. The tons per inch immersion of a ship at seven equidistant water-lines 3 feet apart are respectively 31·3, 30·2, 28·5, 26·4, 23·9, 19·6 and 14·2. Find the displacement and the vertical position of the centre of buoyancy. The appendage below the lowest water-plane to be neglected.

11. What conditions have to be fulfilled in order that any body may float freely and at rest in still water? What is the condition necessary for stable equilibrium?

12. State Simpson's second rule.

The equidistant half-ordinates of the load water-plane of a ship in feet are—0·6, 2·9, 9·1, 15·6, 18·0, 18·7, 18·5, 17·6, 15·2, and 6·7 respectively, and the length of the ship is 288 feet. Find the area of the load water-plane and the longitudinal position of the centre of gravity.

13. Describe the process known as the "graphical process," used for finding the displacement and centre of buoyancy of a ship.

14. Write down and briefly explain the rules in common use in ship calculations, for finding the areas of plane surfaces and volumes of displacement.

The semi-ordinates of the boundary of a deck of a vessel are:—0·5, 4·5, 8·8, 10·0, 8·2, 3·8, and 0·4 feet respectively, including the end ordinates. The length of the vessel being 85′, find the area of the deck, and the position of its centre of gravity.

15. A deck of a vessel is composed of flush plating $\frac{8}{20}''$ thick, secured to channel bar beams $8'' \times 4'' \times \frac{1}{2}''$, spaced 3′ 6″ apart.

Calculate the weight of a part of the deck 63′ long by 10′ wide, including rivets, but omitting edge strips and butt straps.

16. What are the curves of "displacement" and "tons per inch immersion," and what are their uses?

The area of a ship's load water-plane is 6050 square feet, the body below is divided by equidistant horizontal sections 3′ apart, whose areas are 5500; 4750; 3500; 2050; 1000; and 250 square feet respectively.

Find the tons per inch at each water-plane, and plot the curve of tons per inch, on the squared paper supplied.

What is the total displacement of the vessel?

17. Explain, in detail, a method of determining graphically the displacement of a vessel of given form.

18. The area of a ship's loadwater-line section is 13,200 square feet, and the areas of other parallel sections 3′ apart, are as follows, viz.:—12,700, 12,000, 11,100, 10,000, 8,200 and 6,000 square feet respectively. Neglecting the volume below the lowest section, calculate (i) the tons per inch immersion at each water-plane, *and* (ii) the total displacement of the vessel.

Construct, on the squared paper supplied, the curve of tons per inch.

19. The half-ordinates of the transverse section of a coal bunker of uniform section are as follows, viz.:—31·0, 31·3, 30·8, 29·0, and 24·6 feet respectively, the ordinates being spaced 5′ apart. The length of the bunker is 25 feet.

On the basis of the coal being stowed only up to the level of the underside of beams, which are 5″ deep and spaced 4′ apart, calculate the weight of coal that can be so carried in the bunker.

20. What is meant by the shear of a rivet? Explain clearly, with sketches, the difference between "single" and "double" shear.

What is the single shear strength of a $\frac{3}{4}''$ diameter mild steel rivet?

Two test bars, of circular section, $\frac{1}{2}''$ diameter, are prepared from the following materials, viz.:—(*a*) mild steel, *and* (*b*) rolled Naval brass, or yellow metal: what breaking force would you expect the testing machine to register when the bars are broken?

What elongation would you expect, in each case, on a length of 2″?

21. Explain why vessels passing from salt water to fresh water change their draught. What condition must be fulfilled in order that a vessel may not change trim in going from fresh to salt water, or *vice versâ*?

A box-shaped vessel is 175′ long, 30′ broad, 20′ deep, and floats at a uniform draught of 8′ in salt water. Calculate the mean draught when the vessel is floating freely in fresh water.

22. Find the displacement up to the 6-feet and 10-feet water-lines of a ship whose form is defined by the following :—

W.L.'s.		Keel.	1 ft.	2 ft.	4 ft.	6 ft.	8 ft.	10 ft.
Nos. of Section.	1	0·1	0·1	0·1	0·1	0·1	0·1	0·1
	1½	0·1	1·4	2·6	4·6	6·7	9·0	11·1
	2	0·1	5·6	8·2	11·5	13·5	14·7	15·3
Distance between sections is 40 feet.	3	0·1	11·1	13·7	15·9	16·7	17·0	16·9
	4	0·1	13·1	15·6	17·1	17·1	17·4	17·4
	5	0·1	10·3	12·6	14·6	15·4	15·8	16·0
	6	0·1	5·7	7·5	9·5	10·7	11·6	12·5
	6½	0·1	1·7	2·7	4·1	5·0	6·0	9·1
	7	0·1	0·1	0·1	0·1	0·1	0·1	0·1

23. Find the vertical and longitudinal position of the centre of buoyancy of the form in the preceding question.

24. Suppose the vessel in Question 54 to be floating at the 10-ft. water-line, and to be inclined transversely through an angle of one in one hundred, by a weight of one ton moved through 30 feet. Find the height of the centre of gravity of the ship above the keel.

(If Questions 22 and 23 have not been done, assume a displacement and height of C.B.)

25. What is the ultimate shearing and tensile stresses of steel rivets and plates respectively? Find the breaking strength of a single butt, double-chain riveted, of a plate 30 inches wide, ½ inch thick, connected by ¾ rivets, spaced 3 inches apart. Find the force necessary to break the plate across a frame-line where the rivets are spaced 6 inches apart.

26. Given the height of the longitudinal metacentre above the centre of gravity, show how you would obtain the moment to trim ship one inch. In a vessel whose distance between draft marks forward and aft is 300 feet, and whose centre of gravity of water-plane is 10 feet abaft centre between draft marks, and whose foot-

tons to trim ship one inch are 400, find the change of draft forward and aft caused by moving 30 tons through 200 feet in a fore and aft direction.

27. A barge 100′ × 20′ × 10′ of rectangular section, is formed of ¼-inch plating on ends, bottom, sides, and deck, and has frames and beams of 4½″ × 4″ × ¼″, spaced 20 inches apart. The ends have stiffeners 2 feet apart, of the scantlings of the frames. A floor-plate, 12″ × ¼″, is on every frame. Find the weight of the hull, assuming that there are no hatches. Suppose the barge to have weights of 10 tons at 10 feet from the stem, 15 tons at 25 feet, 20 tons at 50 feet, 30 tons at 75 feet. Find the longitudinal position of the centre of gravity of the loaded barge.

28. State and prove Simpson's First Rule. State Tchebycheff's Rules for either three, five, or seven ordinates.

29. Prove that in a curve of loads of a ship floating at rest, the integral of the area of the curve from one end up to a chosen point gives the shearing force at that point, and that the integral of the curve of shearing forces over the same part gives the bending at the chosen point.

30. Suppose a curve of buoyancy to be a curve of versed sines, and the corresponding curve of weights to be a common parabola, whose axis is vertical and at the middle of the length. Find the form of the curves of shearing force and bending moment.

31. State and prove Atwood's formula.

32. Describe any method of obtaining a cross-curve of stability.

33. Find the effect upon the draught of water forward and aft of opening to the sea one per cent. of the length of a vessel of rectangular section at any part of the length, supposing this one per cent. to be confined between transverse water tight bulkheads. Explain how the deductions from this result can be made use of to determine the spacing between water tight bulkheads, which shall not be exceeded, in order that when a compartment is flooded, the draft in no case shall exceed a certain specified amount.

34. If B and B_1 be respectively CB's in upright and inclined position, and R be the foot of the perpendicular from B on to the vertical through B_1 in this inclined position, show that B_1R is the integral of BR between the upright and the angle of inclination. Show from this how a curve of CB's can be obtained from a curve of GZ's.

35. A vessel of uniform rectangular section is launched parallel to her keel. What is the form of the curve of tipping and lifting moments, supposing the ship to be deep enough to prevent the upper deck from being immersed? Suppose such a vessel 100 feet

long, 24 feet wide, having its CG at the middle of the length to be launched at a slope of one inch to the foot, with two feet of salt water over the end of the ways, and with a launching weight of 100 tons. What is the maximum pressure on the fore end of the ways (assumed to be at the fore perpendicular)?

36. Suppose a vessel to be instantaneously floating on the crest of a wave of her own length, and of height equal to one-twentieth of the length. What stress would you expect to find with all coal burnt out in—

(1) a battleship of 14,000 tons;
(2) a high-speed Atlantic liner;
(3) a torpedo-boat destroyer?

Give figures for the stress when in the hollow of the same wave with bunkers full.

37. Why would a trochoidal wave cause less stress than that determined in the preceding question? Give results of any calculations you know of, in which the difference due to the wave not being actually at rest is taken into account.

38. Prove that in a ship, whose moment of inertia about a transverse axis through the midship section is the same for the fore end as for the after end, the pitching does not alter the bending moment at the midship section.

39. Find the maximum stress upon a section of a vessel floating upright in still water, and subjected to a bending moment of 1000 foot-tons. The section is rectangular, 20 feet wide, 10 feet deep, and has ½″ plating on deck, bottom, and sides.

Suppose the vessel to be inclined at some known angle, how would you find the maximum stress?

40. What is the Admiralty speed coefficient? What is its value for the vessels named in Question 60 at full speed? How does it vary with speed? What is its value for sea work as compared with trial trips?

41. A model 12 feet long, having a displacement of 1000 lbs., has a resistance of 3 lbs. at 5 feet per second. Find the effective horse-power necessary to drive a vessel of the same form 192 feet long at its corresponding speed. Assume the wetted surface of the model to be 30 square feet, and the frictional resistance of a plane 12 feet long at 5 feet per second to be 0·07 lb. per square foot, and that of a plane 192 feet long to be 0·8 lb. per square foot at 12 knots.

42. What conditions should be fulfilled in a ship to make her easy in her rolling at sea?

43. What is a curve of extinction? How can it be obtained

experimentally? What can be determined from it in relation to the resistance to rolling of a ship?

44. What is the chief cause of vibration in a steamer? What are the subsidiary causes? What precautions are taken to avoid these?

45. Calculate the displacement and vertical position of the centre of buoyancy of a vessel for which the half-ordinates are given below, the distance between the sections being 14 feet, and the keel appendage being 2·6 tons, with centre of buoyancy 4·8 feet below the 5′ 6″ water-line.

Sections.	1′ W.L.	1′ 9″ W.L.	2′ 6″ W.L.	4′ W.L.	5′ 6″ W.L.
1	0·1	0·3	0·8	1·9	2·8
2	2·2	3·9	5·4	6·6	7·0
3	3·4	5·6	6·8	7·4	7·5
4	2·0	3·6	5·1	6·4	6·8
5	0·1	0·2	0·5	1·4	2·8

46. Define change of trim, and moment to change trim one inch.

Obtain an expression for the position in which a weight must be placed on board a ship so as not to increase her maximum draught. Explain clearly why this is not always possible with large weights, and find the limiting weight.

47. What are the curves of displacement and tons per inch immersion, and what are their uses?

The areas of a ship's sections at parallel water-lines 3 feet apart are 9600, 9500, 9000, 7700, 5000, and 2000 square feet. Neglecting the volume below the lowest section, find the tons per inch at each water-plane, and plot the curve of tons per inch. Find also the total displacement.

48. What are the ultimate shearing and tensile stresses of steel rivets and plates respectively?

Two tie plates 24″ wide by ⅜″ thick, are connected together by a lapped joint. Show by calculation the number and sizes of rivets required, indicating how they should be arranged in order that the butt and plate may be nearly of equal strength.

49. Define "centre of flotation," "centre of buoyancy," "metacentre," and "metacentric height."

Determine the distance between the centre of buoyancy and the transverse metacentre of a vessel 72 feet long and 95 tons displacement, floating at a water-plane whose half-ordinates are, 0·8, 3·3, 5·4, 6·5, 6·8, 6·3, 5·1, 2·8, and 0·5.

50. A fore-and aft watertight bulkhead, extending from the tank top to main deck, is 50 feet long and 24 feet deep. Find the total weight of the bulkhead, including stiffeners, connecting angles, etc., having given the following particulars :—

Plating $\frac{5}{16}''$ thick for the lower half depth, and $\frac{1}{4}''$ above, with single-riveted edges and butts; stiffeners alternately $6'' \times 3'' \times 3\frac{1}{2}''$ zed bars of 15 lbs. per foot run, and $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angle bars of 7 lbs. per foot run, spaced 2 feet apart; bounding angles $3\frac{1}{2}'' \times 3''$ of 8·5 lbs. per foot run.

51. State and prove Simpson's second rule for approximating to the area and centre of gravity of a plane surface.

The half-ordinates of a water-plane are 0·2, 1·8, 4·8, 7·4, 5·5, 2·3, and 0·6 feet. The ordinates are spaced 23 feet apart. Find the distance of the centre of gravity of the half water-plane from the middle line.

52. Obtain an expression for the height of the metacentre above the centre of buoyancy in a floating body.

The half ordinates of the water-plane of a vessel, $27\frac{1}{2}$ feet apart are—0·1, 6·9, 10·0, 10·5, 10·1, 7·2, and 0·1 feet respectively. Determine the transverse metacentric height, having given that the displacement to the water-plane (salt water) is 275 tons, and that the centre of gravity of the vessel is 5 feet above the centre of buoyancy.

State the values of the metacentric heights in any two types of ships with which you are acquainted, naming the types of vessels selected.

53. The maximum speed of a vessel is 17 knots, and the rudder, which is 12 feet broad and approximately rectangular in shape, has an area of 200 square feet and a maximum working angle of 35 degrees. Estimate the diameter required for the rudder head if made of cast steel.

How does the case of a balanced rudder differ from that of an ordinary one in the case (*a*) when the ship is going ahead, and (*b*) when she is going astern?

54. What are "cross curves of stability"?

Describe fully how you would construct a set of cross curves of stability for a vessel of known form. Explain clearly the great advantages of having stability calculations recorded in this form.

55. Show how you would estimate the angle of heel to which a ship under sail in still water would be driven, when struck by a squall of known force, (*a*) when the ship is upright and at rest; and

(*b*) when the ship has just completed a roll to windward, when the squall strikes her.

56. Prove that in a curve of loads of a ship floating at rest, the integral of the area of the curve from one end up to a chosen point gives the shearing force at that point.

At the section of a ship at which the shearing force is at a maximum, show how the shearing stress on the material varies, and state under what circumstances this shearing stress would cause straining action to take place.

57. Describe the principles governing the watertight subdivision of war or merchant ships. Is there any legal enforcement for merchant ships?

State briefly the recommendations of the Bulkheads Committee (1890–91).

A barge is of uniform rectangular section, 70 feet long and 20 feet broad, and the draught of water when the vessel is intact is 8 feet. What would be the minimum height of a bulkhead 10 feet from one end of the vessel in order that if the end compartment were flooded, the adjacent compartment should remain dry?

58. Define Statical Stability and Dynamical Stability.

A submarine vessel 140 feet long has a uniform cross section of which the upper part is a semicircle 10 feet in diameter, and the lower a triangle 8 feet deep with vertex downwards. The centre of gravity of the vessel is 6 feet above the keel.

Construct, to scale, the curve of statical stability, and state in foot-tons the dynamical stability at 60 degrees.

59. State what is meant by "effective horse-power," "propulsive coefficient," and "corresponding speeds." State the values of the propulsive coefficients of any two types of vessels with which you are acquainted, naming the types selected. How does the propropulsive coefficient vary with the speed in a particular ship, and why?

A vessel of 1800 tons displacement is propelled at 15 knots by engines of 2500 I.H.P. Estimate the I.H.P. you would consider necessary to drive a vessel of similar model, but of 4000 tons displacement at a speed of 18 knots. What assumptions are made in passing from the one vessel to the other?

60. Describe briefly the causes which produce vibration in the hulls of steamships, and state under what circumstances these vibrations reach a maximum.

(*a*) State whether you consider vibration to be indicative of structural weakness, giving reasons for your answer.

(*b*) How would you attempt to reduce vibration when excessive?

(*c*) At the lowest number of vibrations possible, where would you expect to find the nodal points?

(*d*) What recent modifications in design are known to produce less vibration?

61. Describe fully the method of conducting measured-mile trials and arriving at the measured-mile speed.

State the possible sources of error to which such trials are liable, and how they are reduced.

62. How would you obtain the wetted surface of a ship of known form?

Quote any formula giving a close approximation to the wetted surface.

What use can be made of the wetted surface when obtained?

The wetted surface of a ship of 6000 tons displacement being 25,000 square feet, find the wetted surface of a vessel of similar form, but of 2000 tons displacement.

63. A vessel runs bow-on to a shelving beach; investigate her stability, as compared with her condition when afloat.

A box-shaped vessel, 100 feet long and 20 feet broad, floats at a draught of 6 feet forward and 10 feet aft, the metacentric height being $2\frac{1}{4}$ feet. Find the virtual metacentric height when she just grounds all along on level blocks.

64. Sketch and describe the launching arrangements for a large ship, stating the dimensions of the vessel, declivity of the blocks and launching ways, and the pressure per square foot allowed on the surface of the ways. What is the meaning of the term "camber" as applied to the ground ways, and to what extent is it admissible?

The launching weight of a ship is 2800 tons, its centre of gravity is 9 feet abaft the midship section, and the fore end of the launching cradle is 120 feet before the midship section. When the midship section of the vessel is respectively 0, 10, 20, 30, 40, and 50 feet abaft the after end of the ways, the corresponding buoyancy is respectively 1110, 1310, 1530, 1770, 2030, and 2310 tons, and the distances of the corresponding centres of buoyancy abaft the after end of the ways are respectively 43, 51, 60, $68\frac{1}{2}$, $77\frac{1}{2}$, and $86\frac{1}{2}$ feet.

Construct to scale the corresponding launching diagrams, stating where the stern begins to lift, and the pressure on the fore poppet. Are the ways sufficiently long to prevent tipping?

65. Investigate the value of the metacentric height of a vessel with free water in the hold.

A mud hopper of box form is 200 feet long and 40 feet broad, the mud chamber being the amidships portion 50 feet long. When empty, the draught is 10 feet and the centre of gravity 15 feet above the keel. Find the metacentric heights when (*a*) Empty, (*b*) Discharge-port is open, and (*c*) Chamber is filled to a height of 10 feet with sludge of specific gravity 2.

66. Describe in detail how you would proceed to fix the dimensions and underwater form of a combined passenger and cargo carrying steamer, having given the speed, length of voyage, maximum draught permissible, cargo capacity (both by measurement and dead-weight), number of passengers, and type of vessel. State what you consider satisfactory limits of stability for such a vessel as you select.

67. State and prove Simpson's 1st Rule, for approximating to the area and centre of gravity of a plane surface.

The equidistant half-ordinates of a water-plane being 3·0, 5·4, 7·1, 9·32, 12·2, 14 17, and 19·5 feet respectively, and the length of the base being 84·0 feet, find the area of the water-plane, and the transverse position of the centre of gravity of half the water-plane.

68. The half-ordinates of a portion of a ship's deck, covered with $\frac{4}{20}''$ plating, are 4·2, 9·36, 12·3, 14·84, 16·5, 17·53, and 18·7 feet in length respectively, the common interval being 15 feet.

Calculate the weight of the beams, plating, planking, and fastenings, etc., for this part of the deck, the beams being 8″ × 4″ × $\frac{8}{20}''$ channel bar, spaced 3′ 6″ apart, and the plank being of pitch pine $3\frac{3}{4}''$ thick.

Estimate the cost of laying the deck with planks 6″ wide at $7\frac{1}{2}d.$ per foot run.

69. What is the ultimate shearing and tensile stresses of mild steel rivets and plates respectively?

The shell plating of a vessel is formed of plates 50″ wide and $\frac{11}{20}''$ thick, worked on the raised and sunken system; the spacing of the rivets in the frames are 7 diameters apart, and in the boundary angles of the watertight bulkheads 4 diameters apart. Sketch an arrangement you would make in order that the strength of the shell plating in wake of the bulkheads and ordinary frames shall be approximately the same. Show, by calculations, that your arrangement is a good one.

70. Define "centre of gravity," "centre of buoyancy," "centre of flotation," "metacentre."

A vessel 140′ long, and whose body plan half-sections are squares, floats with its sides upright, and the centres of all the sections lie in the plane of flotation. The lengths of the sides of the sections, including the end ordinates, are 0·8, 3·6, 7·0, 8·0, 6·4, 3·0 and 0·7 feet respectively, equispaced.

Calculate the distance between the centre of buoyancy and the transverse metacentre.

71. Having given the value of six equidistant ordinates of a plane curve, deduce a formula that will give the area of the surface lying between the extreme ordinates and the curve.

Four consecutive polar radii of a curve, taken in order, are 10·9, 11·6, 13·0, and 14·1 feet; the common angular interval between them is 15 degrees. Find the area, in square feet, included between the curve and the extreme polar radii, and prove the rule you use.

72. Obtain an expression giving the height of the longitudinal metacentre above the centre of buoyancy. What use is the information when obtained for any particular vessel?

Draw, to scale, the ordinary metacentric diagram for a vessel whose uniform section throughout her length is a quadrilateral of breadth 50′ at the load line and 25′ at the keel, the draught of water being 20′.

73. Under what circumstances may it be expected that the cargoes of vessels will shift?

In a cargo-carrying vessel, the position of whose centre of gravity is known, show how the new position of the centre of gravity, due to a portion of the cargo shifting, may be found.

A ship of 4800 tons displacement, when fully laden with coals, has a metacentric height of 2·6 feet. Suppose 120 tons of coal to be shifted so that its centre of gravity moves 19 feet transversely and 5 feet vertically, what would be the angle of heel of the vessel, if she were upright before the coal shifted?

74. Prove that for any floating body revolving about an axis fixed in direction, positions of maximum and minimum stability occur alternately.

Investigate all the positions of equilibrium for a square prism of uniform density revolving about a horizontal axis, assuming its density to be three-fourths that of the fluid it is floating in.

75. Quote Moseley's formula for the dynamical stability of a floating body, and prove that the value of the dynamical stability obtained from that formula is identical with that obtained by integrating the curve of statical stability.

A vessel of constant rectangular section is 260′ long, 30′ broad, 30′ deep, and draught of water 15′. The metacentric height of the vessel being 2·5′, find (1) the statical stability, *and* (2) the dynamical stability of the vessel when she is inclined at 45 degrees.

76. A box-shaped vessel 420′ long, 72′ broad, and draught of water 24′, has a compartment amidships 60′ long, with a water-

tight middle line bulkhead extending the whole depth of the vessel. Determine the angle of heel caused by the ship being bilged on one side abreast this bulkhead, the centre of gravity of the vessel being 23′ above the keel.

To what height should the transverse bulkheads at the ends of the bilged compartment be carried, so as to confine the water to this part of the vessel?

If the bilging be caused by a collision, making a hole 1·5 square feet in area at a depth of 18′ below the load water-line of the vessel, in wake of the compartment referred to above, and the pumps be in working order, calculate the capacity of the pumps required to just keep the leak under.

77. Define "freeboard," and state what determines it. Describe the arrangement of the tables giving "freeboard."

How is the statutory deck-line marked?

Distinguish between "flush-deck," "spar-deck," and "awning-deck" vessels. How is the freeboard determined in each case?

78. Prove the relation which exists between the load curve and the curves of shearing force and bending moment.

A vessel 300′ long has a uniform section below water. The weights of hull, machinery, and cargo are 840, 300, and 300 tons respectively. The weight of machinery extends uniformly over ⅓rd of the length amidships, and the weight of cargo extends uniformly over ⅙th of the length from each end. The weight of hull curve is of the form—

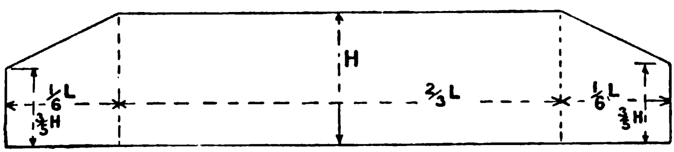


Fig. 157

Draw the curves of shearing force and bending moment and state their maximum values when the vessel is at rest in still water.

79. The effective part of the transverse section of a vessel amidships is represented by the diagram (Fig. 158), the vessel being 42′ broad and 28′ deep.

Find the maximum tensile and compressive stresses when the vessel is subjected to a sagging bending moment of 60,000 foot-tons. The plating shown in the diagram to be taken as $\frac{1}{2}''$ thick, and no allowance need be made for rivet holes or laps of plating.

Assuming I_x and I_y are the moments of inertia of a section about axes at right angles to each other, deduce a formula for finding the stress on the section at any point when a vessel is inclined to an angle to one of the axes.

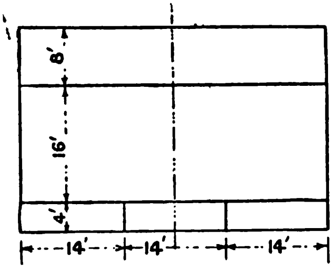


FIG. 158.

80. Enumerate the component parts of the total resistance to propulsion of a ship. What is the relative importance of these component parts at (1) low speeds, *and* (2) at high speeds?

A ship, 290′ long, 45′ beam, 17′ 6″ draught, and 3200 tons displacement, steams $17\frac{1}{2}$ knots. Find the horse-power necessary to overcome frictional resistance, having given that the resistance varies as the 1·83 power of the velocity, and that in fresh water, at a speed of 10 feet per second, the average resistance for a length of 50′ is 0·246 lbs. per square foot, whilst over the last square foot the resistance is 0·232 lbs.

81. Explain in detail how the indicated horse-power for a new ship is estimated.

A model of a vessel, 400′ × 65′ × 24′ draught, of 8560 tons displacement, is run, and the curve of E.H.P. on a base of speed of ship is 3250, 4035, 5020, 6195, and 7660 E.H.P. for 16, 17, 18, 19, and 20 knots respectively. Make an estimate of the I.H.P. of a ship of 16,000 tons, of similar form, for speeds of 20 and 21 knots, and give the dimensions of the new ship.

82. The draught of water, the desired speed, and the load to be carried being given for a new design, state in detail how you would obtain the approximate dimensions of the ship.

Obtain suitable dimensions for a vessel to carry 1100 tons of cargo on a limiting draught of 21′, the speed of the vessel to be 12 knots, with coal sufficient for a voyage of 1500 miles, and 300 tons of passengers and stores.

83. Deduce a formula for the period of a ship whose rolling is unresisted and isochronous.

A vessel of 13,500 tons displacement has a metacentric height of 3·5 feet and a period of 8·5 seconds. Find the period of rolling when 600 tons of coal are added each side of the vessel in a bunker

21′ deep and 9′ wide, the centre of gravity of the bunkers being 11′ below the original centre of gravity of the ship and 26′ out from the middle line. The vessel has a horizontal curve of metacentres over the limits of draught corresponding to the above conditions.

84. Define the terms "effective wave slope" and "virtual upright." Explain under what circumstances the rolling of a vessel amongst waves is likely to be most severe, and state what resistances are in operation to prevent overturning in such critical cases.

What conditions should be fulfilled in a ship to make her easy in her rolling at sea?

85. Discuss the distinctive features of torpedo vessel design. What are the most recent developments in the design of this class of vessel in this country?

What is the effect of depth of water upon the speed of a vessel?

State the deductions that have been made from recent trials with vessels in shallow water.

86. Describe how to construct a set of lines, having given the type of vessel, dimensions, displacement, and the position of the longitudinal centre of buoyancy.

Having obtained the sheer drawing of a vessel, how would you proceed to obtain the structural midship section on the understanding that the vessel is to be built to meet Lloyd's requirements?

87. The given sketch represents part of the after-body of a ship. Calculate the displacement in tons, and the vertical position of the centre of buoyancy, of the form represented by the sketch, between the waterlines *A* and *B*, spaced 10′ 6″ apart, and between the sections *C* and *D* which are 60′ apart. The sketch given is to a scale of ¼″ equals 1 foot.

Three waterlines, at depths of 3′ 6″, 7′ 0″, and 8′ 9″ below *A* waterline, are to be introduced between *A* and *B*, for the purpose of the calculations.

Ordinates are to be measured to the nearest decimal place.

88. Define the terms:—"centre of flotation," "centre of buoyancy," and "metacentre."

A prismatic log of wood, of specific gravity 0·75, whose uniform transverse section is that of an isosceles triangle, floats in water with the base of the section horizontal and vertex upwards. Find the ***maximum*** vertical angle of the section for these conditions to hold.

89. Describe fully the method of making and arranging the various calculations on a displacement sheet, and state fully what information is usually shown thereon.

Explain the relation which exists between a curve of tons per inch and the corresponding curve of displacement, and show how either curve may be derived from the other.

Distinguish between displacement and deadweight scales, and show clearly how each is generally arranged.

90. What conditions must be fulfilled in order that a vessel may not change trim in going from fresh to salt water, or *vice versâ*?

A rectangular vessel, 300′ long and 40′ broad, floats at a draught of 10′ forward and 12′ aft in sea water. Find the draught at which she will float in fresh water weighing $62\frac{1}{2}$ lbs. per cubic foot, the centre of gravity being situated in the original waterline.

91. Define the term "Statical Stability." Show, by means of a diagram, the forces acting on a ship when inclined. What is the "righting lever"?

Sketch a typical statical stability curve, indicating the principal points of importance on it.

92. Deduce a rule for finding the area of a curvilinear figure, by means of 5 ordinates, so spaced that the area of the figure is a multiple of the sum of the ordinates.

Five consecutive polar ordinates of a curve, taken in order, are 5·0, 5·2, 5·7, 6·4 and 7·3 feet respectively, and they are spaced at a common angular interval of 5 degrees. What is the area, in square feet, included between the curve and the extreme polar radii?

93. State fully, and prove, the conditions of equilibrium of a floating body.

Define the terms "stable," "unstable," "neutral," and "mixed" equilibrium.

Show that, in the case of a floating body, the equilibrium is *stable* when the distance between the centre of gravity and centre of buoyancy is a minimum, and *unstable* when that distance is a maximum. Discuss the relation between the number of positions of stable and unstable equilibrium.

94. A vessel is inclined about an axis, in the water-line plane, which makes an angle, other than a right angle, with the longitudinal middle line plane of the ship. Obtain an expression connecting the metacentric height under these conditions with the transverse and longitudinal metacentric heights of the ship.

A box-shaped vessel is 80′ long, 20′ wide, and floats at a draught of water of 10′. Find the value of the distance between the centre of buoyancy and the metacentre, for inclinations about an axis coincident with a diagonal of the rectangular water-plane.

95. Describe, in detail, how an inclining experiment is carried out. What observations are made? Show how to deduce the correct height of the centre of gravity, if loose water was lying in the bilges when the observations were made. How would you determine the amount of ballast to be used on an inclining experiment?

What special calculations would you make, if the vessel at the time of inclining were considerably out of her normal trim?

96. Investigate, and sketch, the metacentric diagram for a vessel of constant parabolic section throughout, and show that in such a vessel the presence of free water in the hold, in any number of compartments, leads to an increase of stiffness.

Draw, roughly, the metacentric diagrams for three distinct types of modern vessels, naming the types chosen. Figure on the diagrams the values of the metacentric heights for the load and light conditions in each case.

97. State, and prove, Atwood's formula for the moment of statical stability of a floating body when inclined at *any* angle from the upright. State clearly how to determine the sign of the moment of the correcting layer.

A prismatic vessel, 100′ long, has a transverse section formed of a rectangle, height 10′ and breadth 20′, resting on the top of a semicircle of radius 10′. The centre of gravity is 3′ above the keel, and the draught of water is 10′. Find the volume of the correcting layer, and the righting moment when the vessel is inclined 45°, the displacement being unaltered.

98. Define "Reserve Dynamical Stability," and explain its importance in the case of sailing ships.

Define "power to carry sail," as applied to sailing ships, and explain clearly why it is usually less in small ships than in large, and why in yachts a small value can generally be safely accepted.

99. Define "Freeboard," and "Range of Stability," and state what determines each.

Explain clearly why, in general, high freeboard is conducive to a long range of stability, and low freeboard to a short range. Show, by simple illustrations, that in certain cases a low freeboard may be associated with a considerable range, and a high freeboard with a short range.

100. Find an expression for the heel produced in a vessel by

flooding a compartment extending to the upper deck, and bounded by two transverse bulkheads and a middle line bulkhead.

A vessel of square transverse section, 40′ broad and deep is 270′ long and floats at a uniform draught of 20′. It has 8 equidistant watertight transverse bulkheads, excluding the ends, and a longitudinal middle-line bulkhead over the midship portion. Find the heel produced by bilging the centre compartment, on one side of the middle line, the original metacentric height of the vessel being 5′.

101. Prove the relation which exists between the curves of loads and shearing forces.

Plot a shearing stress curve for a rectangular beam 12″ deep and 8″ wide, at a section where there is a shearing force of 180 tons. What is the maximum shearing stress at the section?

102. State the assumptions upon which the trochoidal wave-theory is based, and the propositions and conditions which must be fulfilled.

How would you construct a trochoidal wave-profile of given dimensions?

Show clearly how to obtain the supporting force per foot, taking into account wave-pressures. What is the effect upon the maximum stresses caused by taking wave pressures into account, and why?

103. Deduce the equation of motion for a vessel rolling unresistedly in still water. Obtain its solution, making the necessary assumptions. Show that the motion is oscillatory, and deduce a formula giving the period of oscillation of a vessel.

104. A vessel has 12 guns capable of firing on each broadside, the mean height of the centre of guns being 26′ above the waterline, and the draught of water 27′. The ship has a displacement of 22,500 tons, and a metacentric height of 5′. Taking the weight of the shot as 85c lbs., powder 270 lbs., and muzzle velocity of projectile as 2,900 feet per sec., estimate the *maximum* angle of roll of the ship caused by the simultaneous firing of all the guns on the broadside, omitting any resistance to the heel. The period of oscillation of the ship in still water is 9 seconds.

105. Sketch six different types of merchant steamships, naming the several decks and part decks in each case, as well as the name of the type. Explain the particular advantage of each type, and trace the evolution of the modern merchant steamer from the original flush one-deck type.

106. What is the "Admiralty displacement coefficient of speed"? State the assumptions on which it is based.

How is it obtained for any particular vessel, and what use is made of it? What is its value in three distinct types of vessels? Name the types selected.

Show that this coefficient is the same for two similar ships at "corresponding" speeds, supposing that the engines, etc., work with the same efficiency.

What is the value of the coefficient for sea work, as compared with that deduced from trial trips?

107. What are the most important developments, from a designer's point of view, that have taken place in recent years, in any *two* of the following types of vessels, viz.:—

(*a*) Ships of the Mercantile Marine;
(*b*) Motor Boats of high speed;
(*c*) Armoured Ships of War;
(*d*) Torpedo Boat Destroyers?

108. A pontoon raft 40′ long is formed by two cylindrical pontoons 36″ diameter spaced 8′ apart between centres, and is planked over with wood 3″ thick, forming a platform 40′ × 12′.

When laden the raft floats with the cylindrical pontoons half immersed in river water and its centre of gravity when laden is 9″ above the load water-line.

Calculate the transverse and longitudinal metacentric heights.

ANSWERS TO SOME OF THE ABOVE QUESTIONS.

No.

1. 449 tons.

2. 674 square feet.

3. 420.

5. 240 lbs.; 3 inches.

6. 5459 cubic feet; 2·29 feet below No. 1 W.L.; 49·1 feet from fine end.

7. 8280·8 cubic feet; 49·4 feet from fine end.

9. $2\frac{3}{4}$ tons if of steel.

10. 5461 tons, 8·04 feet below L.W.L.

12. 7860 square feet, 167 feet from first ordinate.

22. 797 tons; 1515 tons.

23. 4·38 feet from 10-feet W.L.; 116·6 feet from No. 1 section.

24. 13·49 feet.

25. 297 tons; 346 tons (assuming 28 tons per square inch).

26. 8 inches forward, 7 inches aft.

27. About 50 tons if of steel; 0·2 foot forward of midships.

29. Take two consecutive sections of beam K and K′, distance Δx apart; w = load per foot run; F and F + ΔF are shearing forces at K and K′ respectively; M and M + ΔM are bending moments at K and K′ respectively.

Consider the equilibrium of beam between K and K′.

Vertical forces up, F + ΔF;

,, ,, down, F and $w \times \Delta x$;

$$\therefore F + \Delta F = F + (w \times \Delta x)$$

$$\text{or } \Delta F = w \times \Delta x$$

$$\textit{and } F = \int w\,dx \quad \textit{in the limit.}$$

Also for equilibrium—

$$M + \Delta M = M + (F \times \Delta x)$$

$$\text{or } \Delta M = F \times \Delta x$$

$$\textit{and } M = \int F\,dx \quad \textit{in the limit.}$$

30. The equations to the curves of weight and buoyancy referred to the base-line and one end are as follows:—

Weight—

$$y = 6 \cdot \frac{A}{l^3} \cdot (l \cdot x - x^2)$$

Buoyancy—

$$y = \frac{A}{l}\left(1 - \cos\frac{2\pi}{l} \cdot x\right)$$

A being the area of each, and l the length; from which the curves of shearing force and bending moments may be obtained by a process of successive integration.

Maximum shearing force at about $\frac{3}{10}$ length from either end = $\frac{1}{18}$ *weight* about.

Maximum bending moment amidships = $\frac{1}{82}$ (*weight* × *length*) about.

34. Take consecutive normals to the locus of centres of buoyancy at θ and $\theta + \Delta\theta$, BR′ being perpendicular to the latter normal from B, cutting B_1R in R″. Then RR″ is the increment of

No.

B_1R, *i.e.* $\Delta(B_1R)$ and RR″ also equals $(BR \times \Delta\theta)$; so that—

$$\Delta(B_1R) = BR \times \Delta\theta$$

Proceeding to the limit—

$$d(B_1R) = BR \times d\theta$$

and therefore by integrating $d(B_1R)$—

$$B_1R = \int_0^\theta BR \, . \, d\theta$$

A curve of GZ's enables a curve of BR's to be plotted, and the area of this curve up to a given angle (angles in circular measure) will give B_1R, and so enable the position of the centre of buoyancy at that angle to be obtained.

35. 52 tons about.

37. On account of the orbital motion of the particles of water in a wave, the *virtual* buoyancy is *less* in the *crest* portion and *greater* in the *trough* portion than at the same depth below the surface in still water. Calculations, taking this into account, show that the bending moment is less than when calculating as described in the text.

38. See a paper by the late Mr. T. C. Read, *Inst. Nav. Arch.* for 1890.

39. $1\frac{3}{4}$ tons per square inch about.

41. 360 E.H.P. nearly.

47. 3200 tons.

48. About twenty $\frac{3}{4}$-inch rivets disposed in lozenge-shaped lap.

No.

49. 2·13 feet.

51. 2·67 feet.

52. 2·3 feet.

58. Ordinates of stability curve 1·7 sin θ, range 180°, 270 foot-tons.

59. 7800 I.H.P., assuming I.H.P $\propto V^4$.

62. 12,000 square feet.

63. 0·7 foot.

71. 60·4 square feet.

73. 10 degrees.

76. See example 24, Chapter V.

78. In Fig. 159 are the curves required: $S.F._{max} = 60$ tons, $B.M._{max} = 1740$ foot tons.

79. See example 16, Chapter VII.

80. 1420 about.

$$f = \frac{(50 \times 0{\cdot}246) + (240 \times 0{\cdot}232)}{290} \times 1{\cdot}025 = 0{\cdot}241 \text{ at } 10 \text{ f.s.}$$

81. 10,400, 12,700, 492′ × 80′ × 29·5′.

83. See example at end of Chapter XII.

88. If D is depth of section and θ the semi-vertical angle, then—

B above base $= \frac{2}{3} . D$
M above base $= \frac{2}{3} . D + \frac{1}{3} . D . \tan^2\theta$
G above base $= \frac{1}{3} . D$

Equating the latter two expressions $\tan^2\theta = 1$, from which θ is 45° and the vertical angle should not be less than 90°.

90. A similar example to that worked out at the end of Chapter IV.

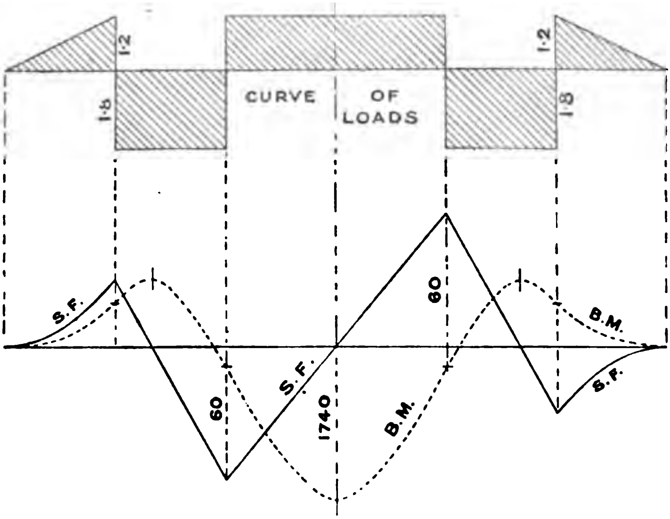


Fig. 159.

No.	
92.	Tchebycheff's rule worked out similarly to that for 4 ordinates in Appendix A.
94.	See end of Chapter V.
97.	This is worked out fully, Example 27, Chapter V.

No.	
100.	Similar to Examples 24, 25, 26, Chapter V.
102.	See "Smith" correction, Chapter VIII.
104.	See end of Chapter V.
108.	12·83′; 111·6′.

BOOKS, ETC., ON "THEORETICAL NAVAL ARCHITECTURE."

"Transactions of the Institution of Naval Architects."

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