O'REILLY ESTATE BUILDING, ST. LOUIS, MO.

A. B. Groves, Architect   Murch Bros. Construction Co., Contractors

G. S. Bergendahl, M. AM., Soc. C. E., Engineer, St. Louis Representative, Mushroom System

THE THEORY OF THE FLEXURE AND STRENGTH

OF

RECTANGULAR FLAT PLATES

APPLIED TO

REINFORCED CONCRETE FLOOR SLABS

BY

HENRY T. EDDY, C. E., Ph. D., Sc. D.
PROFESSOR OF MATHEMATICS AND MECHANICS, COLLEGE OF ENGINEERING,
AND DEAN OF THE GRADUATE SCHOOL, EMERITUS, UNIVERSITY OF MINNESOTA.
CHAIRMAN OF BOARD OF EXAMINERS FOR THE MUSHROOM SYSTEM
APPOINTED BY THE COMMISSIONER OF BUILDINGS
CHICAGO, JUNE 15, 1912

PRICE $5.00

ROGERS & COMPANY, MINNEAPOLIS
1913

PREFACE

In reviewing the history of every type of structural work, we find the designing engineer influenced in his first attempts at any new type of structure by his knowledge of the practical forms of construction with which he is already familiar. In fact until very recent times precedent was the engineer's sole guide. It was to be expected therefore that the pioneers in concrete-steel construction should follow and closely imitate timber and structural steel construction. In following this type, the idea has been to build up the structure as a whole by assembling and joining together a number of independent elements or units: whereas concrete, with or without reinforcement, is a kind of material that is best suited by its nature to construction in monolithic form. But when the attempt has been made to treat such structures theoretically, preconceived ideas have led to the effort to treat them by analysing them into separate members and computing the strength of these arbitrarily selected units, assumed to act independently as they do in steel structures. Such treatment has led to errors of as much as two or three hundred percent in the computation of slabs with two way reinforcement supported on four sides, and to errors of four hundred percent in case of continuous flat slab construction such as occurs in the mushroom system.

When we consider the fact that fire losses in Canada and the United States amount each year to half a billion dollars, and that the question of commercial economy determines whether buildings shall be built of fireproof and incombustible materials such as reinforced concrete, or of inflammable materials such as are used in timber construction, it is at once evident how important it is to the general public to be able to determine on theoretically correct principles whether safe fireproof buildings can be built at approximately the same or less cost than combustible ones. In case of any uncertainty on this question, the designer is compelled for safety to employ materials in such lavish amounts as to render cost prohibitive.

The failure of engineers and mathematicians generally to apply the mathematical theory of elasticity to the new material concrete-steel, has led to considerable controversy between practical constructors of experience, and theoretical engineers without such experience.

Marsh in his treatise on Reinforced Concrete, edition of 1905, Part V, p. 209, makes the following remarks upon this subject:—

> "When properly combined with metal, concrete appears to gain properties which do not exist in the material when by itself, and although much has been done by various experimenters in recent years to increase our knowledge on the subject of the elastic behaviour of reinforced concrete, we are still very far from having a true perception of the characteristics of the composite material.
>
> It may be that we are wrong from the commencement in attempting to treat it after the manner of structural ironwork, and that although the proper allowances for the elastic properties of the dual material is an advancement on the empirical formulae at first employed, and used by many constructors at the present time, yet we may be entirely wrong in our method of treatment.
>
> The molecular theory, i. e. the prevention of molecular deformation by supplying resistances of the reverse kind to the stresses on small particles, may prove to be the true method of treatment for a composite material such as concrete and metal. This theory is the basis of the Cottacin construction which certainly produces good results and very light structures, and M. Considere's latest researches on the subject of hooped concrete are somewhat on these lines."

In this statement, Marsh undoubtedly has in mind the great discrepancy between the results of tests and of computations of multiple way reinforcement.

The empirical formulas of Hennebique, a pioneer in this field and one of its most extensive investigators, give numerical results at variance with the usual published theories for two way reinforcement, while the empirical formulas published by Turner in 1908 exhibit an equally radical divergence.

It has remained apparently for Dr. Eddy to discover the reason for this great discrepancy by a rigid application of the mathematical theory of elasticity to the problem presented by multiple way reinforcement.

The clearing up of the mathematical difficulties with which the theoretical engineer has heretofore struggled in dealing with reinforced concrete, will lead to its more general adoption by the elimination of uncertainty in design; and will lead to the adoption of those types which are safest to erect, and those which possess a degree of toughness, due to their monolithic construction, which is lacking in types that merely imitate older forms of timber and steel construction.

The record of the Mushroom system in the successful construction of between one and two thousand buildings, without accident to the workmen, and without failure to make good the guarantee of test capacity, can be accounted for only on the ground that scientifically designed multiple way reinforcement is inherently safer to erect, and more reliable at all stages of construction than other types. The theory given in detail which accounts for its economy and safety, should, we believe be of interest to the profession at large, and should prove of value to the practicing engineer in checking designs for his clients.

The endorsement of this theory by the undersigned does not constitute a license to use the patented type, though the extension of the theory to older types which are not patented will undoubtedly lead to the closest competition with it along legitimate lines. Any loss to the patentee arising from such competition, will, in the writer's judgment, be more than counterbalanced from a commercial standpoint, by the increased safety and corresponding popularity of reinforced concrete as a material of construction.

The invention covered by the claims of the broad patent just mentioned includes much more than merely the standard mushroom system. This is, however, the one form of all others of the patent, which the writer prefers by reason of certain practical advantages which it possesses. Its arrangement of parts whereby a continous multiple way reinforced flat slab is supported by a large cantilever column head integral with the column and embedded in the slab so as to resist tensile stresses both radial and circumferential in zones near the top of the slab over and around the columns, and in the zones near the bottom of the slab toward the center of the panels, will be fully discussed in succeeding pages. This discussion, which deals with the mushroom system primarily, is intended as an advance chapter or two of a more comprehensive treatise on Concrete-Steel Construction, in which it is intended to treat somewhat fully concrete columns, beam and slab construction, wall panels, etc., as well as flat slabs, and to introduce the results of experimental work now under way to determine the value of Poisson's ratio for different combinations of steel and concrete.

This treatise will then represent the joint efforts of a professional mathematician accustomed to treating these problems, and a professional builder and designer of reinforced concrete with many years of practical experience behind him.

The price charged for this booklet will be credited in return for it, on the larger treatise which the authors intend to complete as soon as the magnitude of the task will permit.

C. A. P. TURNER.

CONTENTS

TISCHERS CREEK BRIDGE, DULUTH, MINN.
Spans are 26 feet longtitudinally
This type is built with spans up to 50′0″

View of Reinforcement in Place
TISCHERS CREEK BRIDGE, DULUTH, MINN.
Designed by C. A. P. Turner   Geo. H. Lounsbury, Contractor

FLAT SLAB FLOORS

1. The superiority of flat slab floors supported directly on columns, over other forms of construction when looked at from the standpoint of lower cost, better lighting, greater neatness of appearance, and increased safety and rapidity of construction, is so generally, or rather so universally conceded as to render any reliable information relative to the scientific computation of stresses in this type of construction of great interest. Heidenreich, in his Engineer's Pocket Book on Reinforced Concrete, page 89, classifies this type as floors without beams and girders—"Mushroom System."

Since "mushroom," as applied to concrete, is an arbitrary or fanciful term, and indeed, almost a contradictory one, a word of explanation as to its origin may be of interest. The term was originated by C. A. P. Turner, of Minneapolis, and applied to his flat plate construction, more particularly because of the fancied resemblance to the mushroom, of the column and column head reinforcement of that particular form of his flat plate construction which he seemed to prefer by reason of certain practical advantages. Another fancied resemblance is the rapidity of erection, comparable to the over-night growth of the mushroom. Here the resemblance ceases, since the construction, once erected, is enduring and permanent.

The Mushroom System is a continuous flat plate of concrete supported directly on columns, and reinforced in such a maner that circular and radial tensile stresses concentric with the column are provided for by metal reinforcement in the tension zone above the columns, and similar provision is made for tensile stresses in the lower portion of the slab concentric with the center of the panel, diagonally between the columns. Since all forces in a plane may be resolved into equivalent components along any pair of axes at right angles to each other, it is possible to provide reinforcement to resist any horizontal tensile stresses in the slab by various arrangements of intersecting belts of rods at zones where these stresses occur. All arrangements of this kind are by no means equally effective. A system of wide reinforcing belts from column to column combined with a system of radial and ring rods to constitute a large, substantial cantilever mushroom head at the top of each column

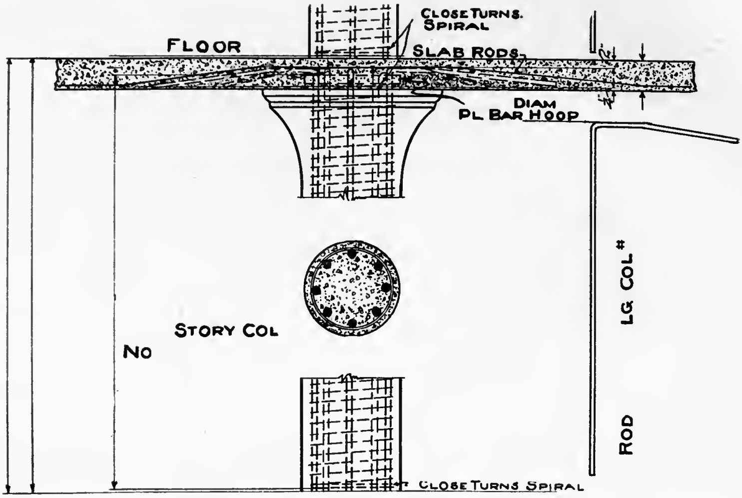

Fig. 1. Vertical Section of Standard Mushroom Head showing position of Radial and Ring Rods, and Slab Rods, Vertical and Horizontal Sections of Spirally Hooped Column, with Plain Bar Hoop Collar Band, Vertical Reinforcing Rods and Elbow Rods.

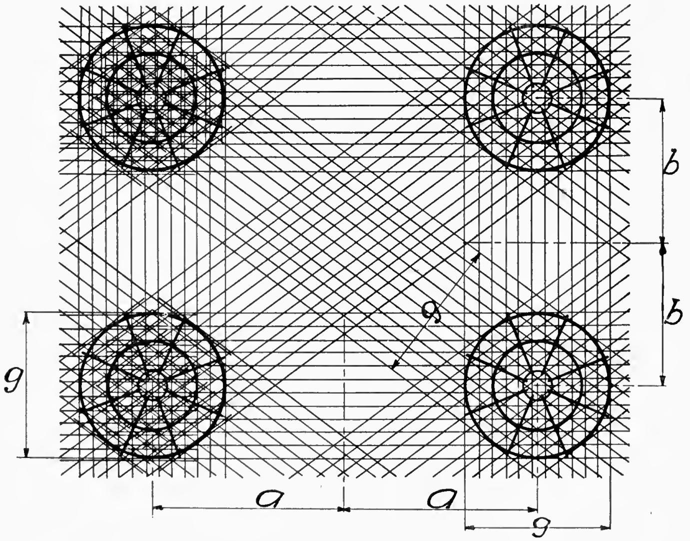

Fig. 2. Plan of Reinforcement in Standard Mushroom System. Radial and Ring Rods, Collar Band and Slab Rods. Diameter of Head $= g = 7/16(a+b)$.

provides a very effective and economical arrangement for controlling the distribution of the stresses in the slab, and furnishes the resistance necessary to support these stresses by placing the steel where it is most needed. It not only has the same kind of advantage that the continuous cantilever beam has over the simple girder for long spans, but combines with it the kind of superiority that the dome has over the simple arch by reason of circumferential stresses called into play, which adds greatly to the carrying capacity of the slab.

In the standard mushroom type, which is quite fully discussed in this paper, the heavy frame work, concentric with the column, supports the slab reinforcement at a fixed elevation, furnishes a high degree of resistance to shear, and secures a high degree of safety during construction. It extends as a cantilever approximately one fourth of the way to the next column as shown in Figs. 1 and 2 on page 2. Arranged upon the radial rods of this frame rest two or more large hoops and upon these rest the wide spreading belts of rods which extend both directly and diagonally from column to column. Over the columns these belts lie near the upper surface of the slab, but they run near the lower surface as they approach points midway between columns.

The cantilever slab thus formed, not only has the same advantages for this form of construction that the cantilever construction has for long span bridges, but it causes the slab to have greater stiffness and gives it greater resistance to shear in the neighborhood of the columns; it removes the locus of zero bending moment to a much greater distance from the column than would otherwise be the case, thus dimininishing the area of that part of the slab which tends to become concave on its upper face and enlarging the convex area.

The cantilever frame-work further, not only moves the locus of zero bending outward from the column, but it also fixes the locus of zero bending moment at a known position so that it does not vary with increase and decrease of the load or change of the load from one span to an adjacent span as would be the case were the mass of metal in the frame and its stiffness largely reduced. This is accomplished as follows:

The locus of zero bending moments is fixed by the dip of the reinforceing rods as they leave the upper surface of the slab near the edge of mushroom and pass below the neutral surface to a level near the bottom of the slab. Such change of tensile resistance in the slab necessarily localizes at these points the zero bending moments.

In addition to the advantages just mentioned, which are of so self-evident a character as to be readily appreciated even by the layman, there is another of such an obscure and apparently inexplicable a nature that it was for years denied as incredible and regarded as non-existent by practical builders, and engineers as well, unless they had opportunity to be convinced of its reality by experiment. I refer here to the additional strength and stiffness which is imparted to a belt of rods in a given direction in a slab by another belt at right angles to the first belt, or at various angles with it. This should be designated as slab action proper in distinction from cantilever action. It depends for its amount upon the value of Poisson's ratio of lateral contraction to direct elongation in the slab, and is the basis of the so called circumferential stresses, which make the strength and stiffness of such reinforced flat slabs much greater than they are estimated to be when these are neglected, as they usually have been. This mistaken view has in the past constituted the most serious obstacle to the adoption of this form of structure, and has been the ground of conscientious opposition to its introduction on the part of consulting engineers. It is the object of this investigation to remove if possible all reasonable uncertainty as to the rational theory of this form of structure.

The following partial bibliography of this subject may be useful to those unfamiliar with what has been done in this field.

Concrete Steel Construction, (305pp)
By C. A. P. Turner, M. Am. Soc. C. E.
816 Phoenix Bldg., Minneapolis, 1909.

Reinforced Concrete Construction, (259pp)
By Turneaure and Maurer, University of Wisconsin
Wiley, N. Y., 1907.

Concrete, Plain and Reinforced, (483pp)
By Taylor and Thompson,
Wiley, N. Y., 1911.

Trans. Am. Soc. C. E.
Vol. LVI. June 1906.

Engineering News:—
Oct. 4, 1906, p. 361.
Feb. 18, 1909, p 176.
Dec. 23, 1909, p. 694.

Engineering Record:—
March 28, 1908, p 374.
May 2, 1908, p 575.
Oct. 10, 1909, p 411.
April 3, 1909, p 408.
April 10 ,1909, p 492.

NOTATION

2. All lengths and areas are measured in inches, and all weights in pounds.

A = area of cross section of steel reinforcement per unit width of slab, in case it be assumed to be replaced by a uniform sheet of equal weight.

A_1 = area of cross section of all the rods in one side belt.

A_2 = area of cross section of all the rods in one diagonal belt.

a = one half the longer side of a panel from center to center of columns.

b = one half the shorter side of a panel.

B = the shortest distance along one side of a panel from the edge of a column cap to the edge of the next cap.

C_1 and C_2 are constants depending on the relative lengths of the sides of any panel, which reduce to unity for any square panel.

D_1 = the deflection of the middle of the longer side of the panel below the edge of the cap.

D_2 = the deflection of the center of the panel below the edge of the cap.

d = the effective thickness of the slab at any point, being the vertical distance from the center of action of the reinforcement to the compressed surface of the concrete.

d_1 = the vertical distance from the center of the rods in the side belt at mid span to the top surface of the concrete.

d_2 = the distance at the center of the panel from the center of the rods in the second or upper diagonal belt to the top of the concrete.

d_3 = the distance at the edge of the cap from the center of the third belt of rods from the top, to the compressed surface of the concrete.

E or E_s = Young's modulus for steel = 3×10^7.

E_c = Young's modulus for concrete.

e_1 = elongation in steel parallel to long side belt.

e_2 = elongation in steel parallel to short side belt.

e_1 = elongation in steel parallel to diagonal belt.

F = modulus of elastic resistance to shearing.

f_s = Ee = intensity of actual stress in steel.

f_c = intensity of stress in concrete.

g = $7/16\ (a+b)$ = the diameter of the mushroom head and width of belts.

h = the total actual thickness of concrete slab.

id = vertical distance from center of tension of steel to neutral surface of slab.

jd = vertical distance from center of tension in steel to center of compression in concrete.

kd = vertical distance from neutral surface to compressed surface of concrete, hence $i + k = 1$.

K = Poisson's ratio of lateral contraction to longitudinal stretching for reinforced concrete slabs.

L_1 = $2a$ = long side of panel between column centers.

L_2 = $2b$ = short side of panel between column centers.

l = distance from collar band at top of column to edge of cap.

m_1 = true moment of resistance of the tensile stresses in steel parallel to the long side per unit of width of slab.

m_2 = true moment of resistance of steel parallel to short side per unit of width.

m_1 and m_2 = apparent moments per unit of width of forces applied parallel to the long and short sides respectively.

n = the apparent moments per unit of width of the equal twisting couples parallel to either side.

p_1 = intensity of the forces applied parallel to the long side.

p_2 = ditto for short side.

p = intensity of stress in extreme fiber of radial rods.

q = load on slab in pounds per square inch.

R_1 and R_2 = the radii of curvature of vertical sections of the slab parallel to the long and short sides respectively.

s_1 and s_2 = the vertical shearing stresses per unit of width of slab respectively perpendicular to the long and short sides of the slab.

s = the intensity of vertical shearing stress in radial rods.

t = either of the equal horizontal tangential or shearing stresses parallel to the sides of the panel.

t	=	the thickness of a radial rod.
u and v	=	deformations parallel to the long and short sides respectively.
V	=	total vertical shearing stress in radial rod.
$x\ y\ z$	=	horizontal and vertical coordinates parallel to sides of panel.
$\triangle z$	=	difference of two vertical coordinates.
z_1 and z_2	=	deflections of radial rods.
δ	=	sign of partial differential.
$\frac{\delta z}{\delta x}$	=	partial differential coefficient of z with respect to x.

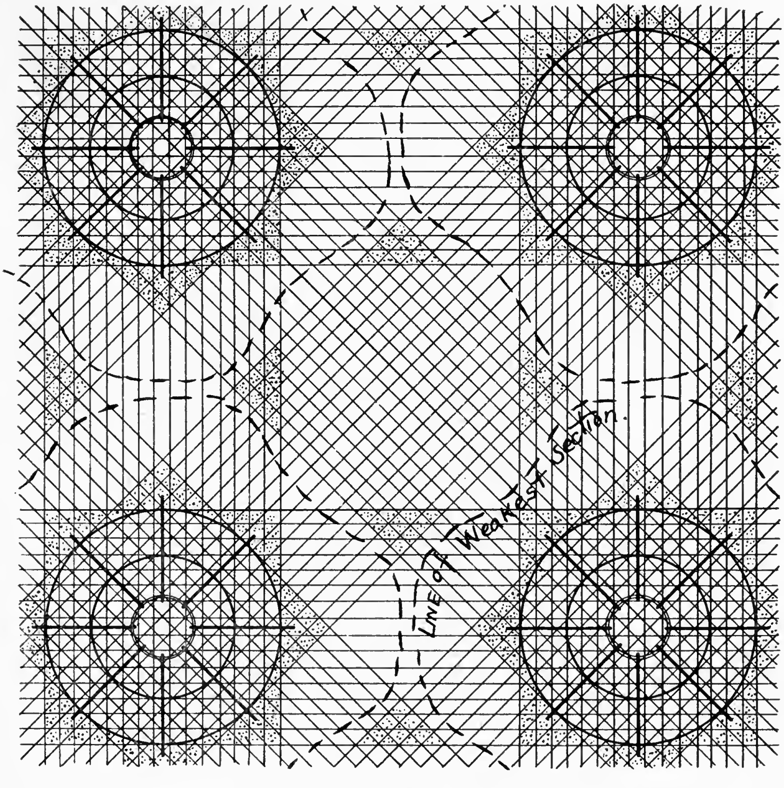

Fig. 3. Plan of Reinforcement Mushroom System.
Square Panel, $g=\frac{1}{2}L$ (as drawn).
Line of Ultimate Weakness.

3. As preliminary to a general investigation of the rational analysis of the flat slab, it seems desirable in the first place to make a brief exposition of the relationship between the true bending moments and the apparent bending moments in the flat slab as follows:

The fundamental equations of extensional stress and strain in thin flat plates and slabs, established a generation ago and accepted by Grashof* and by all authorities on the subject since then, may be written in the forms:

$$\left.\begin{aligned} Ee_1 &= p_1 - Kp_2 \\ Ee_2 &= p_2 - Kp_1 \end{aligned}\right\} \dots\dots\dots\dots (1)$$

$$\left.\begin{aligned} (1 - K^2)p_1 &= E(e_1 + Ke_2) \\ (1 - K^2)p_2 &= E(e_2 + Ke_1) \end{aligned}\right\} \dots\dots\dots\dots (1a)$$

in which p_1 and p_2 are the external applied or apparent stresses per unit of area of cross section of the plate, or of the reinforced slab, which act parallel to the axes of x and y respectively if these latter lie in the neutral plane of the slab; and e_1 and e_2 are extensometer elongations of plate or slab reinforcement per unit of length parallel to x and y respectively. E is Young's modulus of elasticity, and K is Poisson's ratio of lateral contraction to linear elongation. Any piece of material which is subjected to stress, and is of such shape that more than one of its dimensions is considerable, as compared with its remaining dimension, must have its stresses and strains considered with reference to lateral contraction. This is the case in plates and slabs, as it is not in case of rods and beams.

In the above equations Ee_1 and Ee_2 are the true stresses per square inch of section of reinforcement acting along lines parallel to x and y respectively, whatever p_1 and p_2 may be. These latter are the cause of true stresses, but are not themselves the values of the true stresses, as they are in case of rods, etc., where one dimension only is large.

These equations show that the elongation e_1 in the direction of x is not dependent alone upon the tension p_1 applied in that direction, for it is diminished by any tension acting along y, but is increased by any compression acting along y. It thus appears that any tension p_2 along y assists the piece in resisting elongation along x and makes it able to endure safely a larger applied stress p_1 with the same degree of safety, *i. e.*, with the same percentages of elongation or true stress. But it is also equally true that any compression of amount p_2 reduces the safe value of p_1 which may be applied to

*Theorie der Elasticität und Festgkeit, F. Grashof Berlin 1878.

it. These principles are not in accordance with those which hold in ordinary computations for rods and bars, whose lateral dimensions are small compared with their lengths, and whose lateral stresses are negligible. This divergence between the true stresses as shown by actual deformations, and the apparent or applied stresses, is a fruitful source of error in the attempted computation of slabs.

Equations (1) in their present form apply to simple extensional or compressive stresses and strains but may be extended to apply to bending of slabs in the following manner:

Take A as the cross section of the reinforcement per unit of width of slab when the actual reinforcement is regarded as distributed into a thin sheet of uniform thickness, and let jd be the vertical distance from the center of the reinforcement to the center of compressional resistance of the concrete regarded as a fraction j of d, d being the distance from the center of the steel to the top of the slab. Then

$$\mathrm{m}_1 = Ap_1\,jd, \text{ and } \mathrm{m}_2 = Ap_2\,jd, \quad\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

are the apparent bending moments per unit of width of slab, of the applied apparent stresses p_1 and p_2, tending when positive, to cause lines which before bending are straight and parallel to x and y respectively, to become concave upwards.

$$\text{Again } m_1 = Ee_1\,Ajd, \text{ and } m_2 = Ee_2\,Aid, \quad\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

are the true bending moments of the actual resistance stresses in the reinforcement per unit of width of slab, as shown by extensometer strains in the steel parallel to the axes of x and y respectively.

Multiply equations (1) thru by Ajd and substitute the values given in equations (2) and (3), from which we obtain the following relations between the true and apparent bending moments in the slab.

$$\left.\begin{aligned} m_1 &= \mathrm{m}_1 - K\mathrm{m}_2 \\ m_2 &= \mathrm{m}_2 - K\mathrm{m}_1 \end{aligned}\right\} \quad\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\left.\begin{aligned} (1 - K^2)\mathrm{m}_1 &= m_1 + Km_2 \\ (1 - K^2)\mathrm{m}_2 &= m_2 + Km_1 \end{aligned}\right\} \quad\ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

These equations bring out in a striking manner the essential divergence of the correct theory of slab action from that of beam action in which latter case we have the well known equations

$$\mathrm{m}_1 = m_1, \text{ and } \mathrm{m}_2 = m_2$$

i. e., in beams the moment of the applied forces is equal to the moment of the internal resistance, which is not true in slabs.

All attempts to base computations of the deflection of slabs upon beam action are therefore necessarily erroneous. Such computations are inapplicable and misleading, hence deflections and stresses in slabs cannot be correctly computed by any form of simple or compound beam theory.

Equations (4) show:

1st That at points where m_1 and m_2 are of the same sign, (as for example in the convex part of the mushroom near the columns and also near the center of the panel) the true bending moments m_1 and m_2, which determine the actual stresses in the reinforcement are less than the apparent bending moments, which latter have been ordinarily assumed, according to the beam theory, to determine those stresses.

2nd That the compressive stresses in the concrete around the column cap are determined on the same principles as the tensile stresses and are consequently reduced in accordance with the value of K by a considerable percentage below values corresponding to m_1 and m_2 of the beam theory.

3rd That at points where m_1 and m_2 have different signs, as they have for example in the middle part of the span at the side of the panel directly between mushroom heads, the values of the true bending moments are larger than the apparent moments as found by the beam theory.

4th One deduction from this (which is also confirmed by extensometer tests) is, that in slabs having equal side and diagonal belts of reinforcing rods the greatest actual extensions and true stresses in the steel occur at the mid points of those reinforcing rods which run directly between the mushroom heads parallel to the sides of the panel, and do not occur at the center of the panel where m_1 and m_2 have their greatest values. Further, the true stresses in the reinforcement are not so large at the edge of the column caps as at the points just indicated. Neither of these conclusions is in accordance with the beam theory as implied in ordinary formulas such as have been frequently adopted in practice in computing slabs.

5th In making any statement or specification respecting the bending moments at any point of a slab, it is essential to state which bending moments are contemplated, the true bending moments or the apparent moments, with the understanding that the true bending moments only are to be used in determining cross sections and stresses of steel. Any statement omitting this distinction is ambiguous, and any requirement seeking to proportion cross sections of steel to apparent stresses and apparent moments is incorrect.

4. Poisson's ratio K plays an important rôle in the theory of flat slabs and plates, as is evident from equations (1) and (4). Few attempts have been made to determine K by directly measuring the amount of the lateral contraction accompanying the elongation of test specimens, and, were such measurements made, the relative dimensions of the cross section of the specimen would need to be considered as affecting in a very complicated way the true value of K to be derived from observation. Reliable determinations of K usually depend upon observations of Young's modulus of elasticity E and the shearing modulus of elasticity F.

It is proven in the general theory of the deformation of isotropic elastic solids that all the elastic properties of any such solid are determined without excess or defect by its values of E and F, and that Poisson's ratio is a function of E and F expressed by the equation

$$K + 1 = \tfrac{1}{2} E/F \dots\dots\dots\dots (5)$$

There is evidence to show that for concrete K is approximately 0.1*. For steel it is known that $K = 0.3$ nearly.

Now it is evident that a horizontal slab of reinforced concrete, in which the reinforcement consists of rods, differs from one in which its reinforcement is considered to be a simple uniform sheet of metal in this, that the former has much less shearing rigidity in resisting horizontal forces than the latter, for in it all stresses transmitted from one band or belt of rods to any other belt crossing it are transmitted thru concrete only, as is not the case if the reinforcement consists of a continuous sheet. It is evident therefore that the value of K which must be employed in applying the foregoing equations to reinforced concrete slabs must exceed 0.3, the value required in case the reinforcement is a sheet of steel.

This analysis of the conditions affecting the value of K for a reinforced flat slab differs radically from assuming at ramdon that because $K = 0.3$ for steel alone and $K = 0.1$ possibly, for concrete alone, that therefore some intermediate value of K may be correct for these two materials combined in a slab. Such an assumption is merely a blind guess and has no rational basis.

As already partly stated, the view here put forth is this: Since in any homogeneous, isotropic, elastic material the experimental values of E and F perfectly define all its elastic properties, and since we are evidently at liberty to assume our flat slab as sufficiently fine grained in its structure to act nearly like a slab constructed of some sort of homogeneous materials, it will be possible to determine

* Turneaure and Maurer's Reinforced Concrete Construction 2nd Ed. 1907, p. 210.

certain mean values of E and F which will define its elastic properties. It is moreover evident that in a slab, where two kinds of elastic solids are combined as they are here, the mean value of F for the combination is much more affected by the concrete than is E, which latter may be taken as that applying to the steel alone, and, consequently as unchanged by the combination. It is otherwise, however, with F, because the arrangement of the combination is such as to require the assumption of a value of F lying somewhere between that for steel and that for concrete. Since the latter value is much less than the former, the mean value of F is smaller than for steel alone.

This reasoning and other independent theoretical and kinematical considerations have led to the same conclusion, viz: that the correct value of K for the slab is larger than 0.3.

Assuming $E = 30{,}000{,}000$, we may compute corresponding values of K and F from (5) as follows:—

If $K = 0.1$, $F = 13{,}600{,}000$
If $K = 0.3$, $F = 11{,}600{,}000$
If $K = 0.5$, $F = 10{,}000{,}000$

Were a perfectly complete and accurate mathematical theory of the flat slab at our disposal, we might consider every experimental test of the deflection of such a slab, and every extensometer measurement of its reinforcing rods as an experiment for determining the numerical value of K, since deflections and extensions would then all be known functions of K. Having brought such a rational theory to a somewhat satisfactory degree of perfection, the writer finds that, in the light of all known tests of slabs, the value that best satisfies all conditions is $K = 0.5$ . . . . (6)

It is possible that this value of the constant K for slabs may need some slight modifications hereafter, but for the present this may be regarded as substantially correct for mushroom slabs. It may be found necessary to assume a somewhat different value for other forms of structure, as for example, beam and girder construction. That, however, must be determined later. Moreover it must be said that this value of K applies to tests made upon slabs from 2 to 4 months old, and under loads which have been applied to such relatively soft concrete as this for a period of usually not longer than one or two days, and of an intensity such as to cause a maximum stress in the steel of from 10,000 to 16,000 lbs. per square inch. Less loads on better cured concrete, or longer time under load, may show considerable deviation from this value of K.

How important a factor K is in slab theory is evident on considering equations (4) which show that in a square panel uniformly loaded the true moments as shown by the elongations of the reinforcing rods at the center of the panel and over the centers of the columns are only one half the corresponding apparent moments derived from considering the moments required to hold the applied forces in equilibrium, this being on the assumption of course that $K = 0.5$.

5. In order to derive the general differential equation of shears and moments in any rectangular panel in an extended horizontal plate or slab, take the axes of x and y in the neutral plane of the plate and parallel respectively to the longer and shorter sides of the panel with the origin at its center before flexure occurs, and assume that they remain fixed with reference to the points of support of the panel. Then during flexure the center of the panel and all other points of the slab or plate not in contact with the fixed points of support will attain some deflection z, of amount to be determined later. Take z positive downwards.

Then $\delta x \delta y$ is the horizontal area of an element of the slab bounded by vertical planes, and if d be the effective thickness of the slab or plate, the areas of the sides of this element which are respectively perpendicular to x and y are $d\delta y$ and $d\delta x$, while $d\delta x\delta y$ is the volume of the element.

We proceed to obtain the equations of equilibrium of this element of the slab as follows:—

Let s_1 and s_2 be the total vertical shearing stresses per unit of width of slab for sections perpendicular to x and y respectively. In case these shears are variable, as they are in a continuously loaded slab, they respectively contribute elementary forces tending to move the element vertically, of the following amounts:

$$\frac{\delta s_1}{\delta x}\delta y\delta x, \qquad \text{and} \qquad \frac{\delta s_2}{\delta y}\delta x\delta y$$

Assume that the slab carries a uniformly distributed load of q pounds per square unit of area. Then the load upon the elementary area $\delta x\delta y$ is $q\delta x\delta y$, and the equation of equilibrium of the vertical forces acting on the element reduces to this:

$$\frac{\delta s_1}{\delta x} + \frac{\delta s_2}{\delta y} + q = 0 \qquad \ldots\ldots\ldots\ldots (7)$$

in which s_1 and s_2 are taken as positive when they are such as would be produced in the slab by the loading q in case it were supported at the origin only.

Let m_1 and m_2 be the apparent moments per unit of width of slab of the applied forces which tend to bend those lines in the slab which before bending are parallel to x and y respectively. Take them as positive when they tend to make those lines respectively concave upwards. These are the moments obtained by multiplying the total applied tension per unit of width of slab by the vertical distance jd from the center of the reinforcement of the slab to the center of compression in the concrete as given in (2). These moments are not identical in a slab with the true resisting moments m_1 and m_2 in the same directions, which latter are the moments obtained by multiplying jd by the actual tension in the steel per unit of width of slab, which last is to be correctly computed by taking the product of the area of steel per unit of width and its elongation multiplied by E its modulus of elasticity as shown in (3).

Again, let n be the twisting moment per unit of width of vertical section of slab cut by planes perpendicular to either x or y, and acting about either x or y, which moment n is regarded as due to the variation of the vertical shearing stress s_1 when y varies, and to the variation of s_2 when x varies. The moment n is held in equilibrium by horizontal shearing stresses in these same sections, which are opposite in sign above and below the neutral surface. Let t be the total horizontal shearing stress per unit of width of slab in the reinforcement on one side of the neutral plane, then:

$$\mathrm{n} = \mathrm{t}\ A\ j\ d \qquad \text{(8)}$$

At any point xy this horizontal shearing stress t must be the same for the section perpendicular to x, as for the section perpendicular to y, because in every state of stress the tangential components are equal and of opposite sign on any two planes mutually at right angles. Consequently the moment n is the same about x as about y, as has been assumed in (8).

It is implicitly assumed in (2) and (3) that the concrete on the same side of the neutral plane as the reinforcement is ineffective and that its resistance is negligible, so that on that side the resistance of the reinforcement alone counts. This condition actually occurs only after a state of quite considerable stress obtains, and of itself affords a sufficient reason why the formulas based on it fail of accurately representing deflections and elongations at small loads and low stress.

The elementary couples acting on the vertical faces of the element which are in equilibrium with those arising from the shearing stresses are:—

$$\left(\frac{\delta\, m_1}{\delta\, x} + \frac{\delta\, n}{\delta\, y}\right) \quad \delta x \delta y \quad \text{about } y, \text{ and}$$

$$\left(\frac{\delta\, m_2}{\delta\, y} + \frac{\delta\, n}{\delta\, x}\right) \quad \delta x \delta y \quad \text{about } x;$$

while those arising from the shears themselves are:—

$$s_1 \ \delta x\ \delta y \quad \text{and} \quad s_2\ \delta x\ \delta y.$$

Consequently the equations of equilibrium of the couples acting on the element reduce to the following:

$$\left.\begin{aligned} \frac{\delta\, m_1}{\delta\, x} + \frac{\delta\, n}{\delta\, y} + s_1 = 0 \\ \frac{\delta\, m_2}{\delta\, y} + \frac{\delta\, n}{\delta\, x} + s_2 = 0 \end{aligned}\right\} \ldots\ldots\ldots\ldots (9)$$

Differentiate equations (9) with respect to x and y respectively and substitute in (7), and we obtain

$$\frac{\delta^2\, m_1}{\delta\, x^2} + 2\frac{\delta^2\, n}{\delta x\, \delta y} + \frac{\delta^2\, m_2}{\delta\, y^2} = q \ldots\ldots\ldots\ldots (10)$$

which is a general differential equation of the apparent moments of the applied forces which exist in a uniformly loaded slab in terms of rectangular coordinates. From it the differential equation of the deflections may be derived as follows:—

6. To obtain the general differential equation of the deflections of a slab, note that from geometrical considerations such as are familiar in the theory of beams we have

$$R_1\, e_1 = i\, d = R_2\, e_2 \ldots\ldots\ldots\ldots (11)$$

in which R_1 and R_2 are the radii of curvature of sections of the neutral surface by vertical planes parallel to x and y respectively; and id is the distance from the center of the reinforcement to the neutral surface. In equations ($1a$) replace p_1 and p_2 by values given in (2), and e_1 and e_2 by values taken from (11) and we have:—

$$\left.\begin{aligned}(1-K^2)\mathrm{m}_1 &= E\,A\,ijd^2\left(\frac{1}{R_1}+\frac{K}{R_2}\right)\\[2ex](1-K^2)\mathrm{m}_2 &= E\,A\,ijd^2\left(\frac{1}{R_2}+\frac{K}{R_1}\right)\end{aligned}\right\}\ldots\ldots\ldots\ldots(12)$$

But from the theory of curvature

$$\frac{1}{R_1} = \pm\frac{\delta^2\ z}{\delta\ x_2},\ \text{and}\ \frac{1}{R_2} = \pm\frac{\delta^2\ z}{\delta\ y^2}\ldots\ldots\ldots\ldots(13)$$

Also write for brevity $\quad I = A\ i\ j\ d^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$

Then we have from (12), (13) and (14):

$$\left.\begin{aligned}(1-K^2)\mathrm{m}_1 &= \pm EI\left(\frac{\delta^2\ z}{\delta\ x^2}+K\frac{\delta^2\ z}{\delta\ y^2}\right)\\[2ex](1-K^2)\mathrm{m}_2 &= \pm EI\left(\frac{\delta^2\ z}{\delta\ y^2}+K\frac{\delta^2\ z}{\delta\ x^2}\right)\end{aligned}\right\}\ldots\ldots(15)$$

By the fundamental equations of elasticity we also have

$$\mathrm{t} = Fe_3 = F\left(\frac{\delta u}{\delta y}+\frac{\delta v}{\delta x}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

in which F is the shearing modulus, e_3 is the horizontal shearing deformation of the reinforcement for two vertical planes one unit apart horizontally, and

$$u = \pm\, i\ d\,\frac{\delta z}{\delta x}\ ,\quad v = \pm\, i\ d\,\frac{\delta z}{\delta y}\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

are the deformations along x and y respectively, due to the vertical distance id of the reinforcement from the neutral surface.

From (16) by help of (17) we have

$$\mathrm{t} = \pm\, 2\ F\ i\ d\ \frac{\delta^2\ z}{\delta x \delta y}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

In (18) replace F by its value obtained from (5), and then substitute the resulting value of t in (8):—
we then have

$$n = \frac{E\ I}{1 + K} \cdot \frac{\delta^2\ z}{\delta x \delta y} \dots\dots\dots\dots (19)$$

From (15) and (19) obtain values of the second differential coefficients of the moments appearing in (10), which on being introduced into (10), transform that equation into the required general differential equation of deflections as follows:—

$$\frac{\delta^4\ z}{\delta\ x^4} + 2\frac{\delta^4\ z}{\delta x^2 \delta y^2} + \frac{\delta^4\ z}{\delta\ y^4} = \frac{(1 - K^2)}{EI} q \dots\dots (20)$$

which is a partial differential equation of the fourth order that must be satisfied by the coordinates $x\ y\ z$ of the neutral surface of any uniform plate or slab initially flat, when deflected by the application of a uniformly distributed load of intensity q, and supported in any manner whatever.

It may be shown that any deviations from strict accuracy by reason of local stretching of the neutral surface (here neglected) are small compared with corresponding deviations in beam theory.

7. The solution of the general differential equation of deflections (20) for the case of a horizontal slab carrying a uniformly distributed load and supported on rows of columns placed in rectangular array and having the points of support all on the same level, will now be considered.

The integration or solution of (20) would, since it is a partial differential equation, introduce arbitrary functions of the independent variables x and y whose forms would need to be so determined as to cause the solution to satisfy the conditions imposed by the position and character of the supports at certain points, or along certain lines. It would be possible to expand these functions in terms of ascending whole powers and products of x and y, and, in case the supports are symmetrically situated with respect to the axes, the expansions will contain no odd powers of x or y, because the value of z must remain unchanged by changes of sign of either x or y, or both x and y. Any form of polynomial expansion which satisfies (20), and also all the conditions of any given case, must be the correct solution for that case, for, the solution of any given case must be unique.

Instead therefore of carrying thru the tedious analytical development involved in solving (20) mathematically and then applying it to the case we are treating, we shall at once write down the form of solution that applies to the case in hand and verify the fact that it satisfies (2) and all the required geometrical conditions. It will therefore be the solution sought for, which might also have been obtained by the somewhat intricate analytical processes involved in the intregation of such differential equations as (20).

Assuming at first that the slab is unlimited in extent and uniform thruout in the distribution of its reinforcement and loading, and that the parallel rows of supporting columns divide the slab into equal rectangular panels, we shall find a solution in which every panel is deformed precisely in the same manner as every other. Modifications made later will render it possible to take account of variations and irregularities in the distribution and arrangement of the reinforcement, and to estimate to some extent at least the effect of loading only one or more panels.

Let $2a$ be the length and $2b$ be the breadth of a panel; then the equation of its neutral surface, referred to axes parallel to its sides and to an origin fixed in space at the center of the neutral surface of the panel before deflection, is:—

$$48\,EIz = q(1 - K^2)\left[(a^2 - x^2)^2 + (b^2 - y^2)^2\right] \quad \ldots\ldots (21)$$

This is the correct solution of (20) not only because it satisfies (20), as it will be found to do by trial, (and just as many other functions of x and y do also) but it also satisfies all the other conditions required by the case proposed, viz.:

1st $z = 0$ when both $x = \pm a$ and $y = \pm b$;
because there must be no deflection at these points of support which are on the same level as the origin.

2nd $dz/dx = 0$, when $x = 0$, and also when $x = \pm a$; as well as $dz/dy = 0$, when $y = 0$, and also when $y = \pm b$; because straight lines drawn in space to touch the slab across its edges, and across its mid sections parallel to those edges, must all be horizontal by reason of the symmetry of the slab on each side of its edges and mid sections. That these conditions hold is evident from the following equations derived from (20):

$$\left.\begin{aligned} \frac{\delta z}{\delta x} &= \frac{(1-K^2)}{12\,E\,I}\,q\,x\,(x^2 - a^2) \\ \frac{\delta z}{\delta y} &= \frac{(1-K^2)}{12\,E\,I}\,q\,y\,(y^2 - b^2) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

It is of interest to note that the sections of this surface made by all vertical planes parallel to the axes of y, *i. e.*, by x = constant, are precisely the same except in position, since their equations differ by a constant only. The same is true of sections parallel to x. It thus appears, that, in a square panel where $a = b$, the surface may be regarded as a ruled surface described by using the two of these curves on a pair of parallel sides of the panel as directrices and a third one of these curves as a ruler sliding on the first two in such a manner as to remain parallel to the other pair of parallel sides.

The deflections at the center of the panel and middles of the sides are:

$$\begin{aligned} &\text{At } x = 0 = y, && 48\, E\, I\, z = q\,(1-K^2)\,(a^4 + b^4) \\ &\text{At } x = \pm a,\ y = 0, && 48\, E\, I\, z = q\,(1-K^2)\,b^4 \\ &\text{At } x = 0,\ y = \pm b, && 48\, E\, I\, z = q\,(1-K^2)\,a^4 \end{aligned}$$

so that in a square panel the center deflection is twice the mid edge deflection.

Differentiating equations (22) we have by help of (11), (13), (14) and (3):

$$\left.\begin{aligned} e_1 &= \frac{id}{R_1} = \pm\, i\, d\, \frac{\delta^2 z}{\delta x^2} = \frac{(1-K^2)}{12\, E\, A\, j\, d}\, q\,(3x^2 - a^2) \\ e_2 &= \frac{id}{R_2} = \pm\, i\, d\, \frac{\delta^2 z}{\delta x^2} = \frac{(1-K^2)}{12\, E\, A\, j\, d}\, q\,(3y^2 - b^2) \end{aligned}\right\} \ldots\ldots (23)$$

$$\left.\begin{aligned} m_1 &= \frac{(1-K^2)}{12}\, q\,(3x^2 - a^2) \\ m_2 &= \frac{(1-K^2)}{12}\, q\,(3y^2 - b^2) \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (23a)$$

in which the ambiguous signs are to be so taken that m_1 and m_2 in (15) will be positive at $x = 0 = y$, and negative at $x = \pm a$ and $y = \pm b$.

From (23) it appears that extensions vanish and contra-flexure occurs at lines lying in vertical planes whose equations are

$$x = \pm \tfrac{1}{3}\, a\sqrt{3} \quad \text{and} \quad y = \pm \tfrac{1}{3}\, b\sqrt{3} \ldots\ldots\ldots\ldots\ldots (24)$$

It thus appears that the slab is subdivided by these lines (24) drawn parallel to the edges into a pattern which consists of a rectangle occupying the middle part of each panel, of a size $\frac{2}{3}\, a\sqrt{3}$ by

$\frac{2}{3} b\sqrt{3}$, *i. e.*, of the same relative dimensions as the panel itself, and bounded by lines (24), which rectangle is concave upward thruout.

On all four sides of this central rectangle are rectangles of saddle shaped curvature directly between the central rectangles of adjoining panels, while each point of support is situated in a rectangle which is convex upward over its entire area, of dimensions

$$2a(1 - \tfrac{1}{3}\sqrt{3}) \text{ by } 2b(1 - \tfrac{1}{3}\sqrt{3}).$$

From (22) we obtain the equation

$$\delta^2 z \,/\, \delta x \delta y = 0 \dots\dots\dots\dots (25),$$

hence by (18) and (19) it follows that

$$\mathrm{t} = 0 = \mathrm{n}, \dots\dots\dots\dots (26),$$

from which it appears that there is no horizontal shear in the steel, and no twisting moment in vertical planes perpendicular to x or y. This would be otherwise evident from considerations of symmetry. It will be shown that this is not true of all other vertical planes.

Again from (15) and (23) we have

$$\left.\begin{aligned} \mathrm{m}_1 &= \tfrac{1}{12} q \left[3x^2 - a^2 + K(3y^2 - b^2)\right] \\ \mathrm{m}_2 &= \tfrac{1}{12} q \left[K(3x^2 - a^2) + 3y^2 - b^2\right] \end{aligned}\right\} \dots\dots\dots (27)$$

in which we have omitted the sign $\pm$ as superfluous.

From (9) by help of (26) and (27), we have

$$-\mathrm{s}_1 = \frac{\delta \mathrm{m}_1}{\delta x} = \tfrac{1}{2} q\, x, \quad \text{and} \quad -\mathrm{s}_2 = \frac{\delta \mathrm{m}_2}{\delta y} = \tfrac{1}{2} q\, y \dots\dots (28)$$

from which it appears that any strip of the panel parallel to x or y, and one unit wide exerts a shear at its ends such as it would if it were an isolated beam loaded uniformly with an intensity of $\frac{1}{2}q$ per unit of length. According to this, a total shear of $q\ a\ b$, which is one fourth of the total load carried by the panel, appears at each edge of the panel, this total shear on each edge being uniformly distributed along it.

It is seen therefore that the form of solution which we are investigating implicitly assumes that at each edge of the panel there is some auxiliary form of structure that will bear the shears coming to it from each side and at the same time assume the curvatures and deflections contemplated in (21). This will immediately engage our further attention.

8. In order to investigate more fully the deflections, stresses and strains in the side belts of any panel directly between the mushroom heads, let us consider the results just reached somewhat more fully.

The conclusion drawn from (28) was, that a panel with reinforcement distributed with perfect uniformity thruout would require to be supported by a narrow auxiliary girder extending from column to column along each side, and of such resisting moment as to take on, under its load, the precise curvature required by the neutral surface in (21), which curvature must be produced by a uniformly distributed load of $2\,q\,a\,b$, one half of it coming from each of the two panels beside it.

It seems then, that up to this point, we have in reality been treating the theory of the continuous uniform slab with specially designed continuous beams supporting its edges, without as yet investigating those beams in detail. But since no such beams in fact exist under the flat slab, it is clear that the side belts of the slab lying directly between the extended heads of the columns must discharge the functions which would be discharged by the auxiliary beams just spoken of. Such functions must necessarily be added to those already discharged by those belts in supporting the loading which rests directly upon them. In order that this may occur in a manner readily amenable to analysis, the extended stiffened headings of the columns which constitute the mushrooms should in general be approximately of the diameter required to support the ends of a belt of reinforcing rods forming a flat beam which fills the width along the edge of two adjacent panels between the two lines of contra-flexure on each side of that edge, as given in (24).

This requires that the mushroom head should have a width of at least $(1 - \frac{1}{3}\sqrt{3}) = .423$ of the width of the slab between columns. For reasons that will appear later, it is current practice to make these heads not less than $\frac{7}{16} = .437$ of this width.

The lines of contra-flexure in (24) have a fixity of position, (in a flat slab constructed with mushroom heads of this size and stiffness,) under single panel loads, that does not exist in a uniform slab, or where the headings are not so stiff. It may be readily shown by Mohr's theorem respecting deflection curves as second moment polygons, that where there are large sudden changes in the magnitude of the moment of inertia I, such as exist in this case at the lines of contra-flexure at the edges of the mushroom, the lines of contra-flexure remain fixed. But in systems where the diameter of the head is smaller than given above, or its stiffness is much reduced, these lines may be removed to greater distances from the center in loaded panels surrounded by those not loaded than when all are loaded, thereby increasing the deflections and stresses in a single loaded panel over that of a uniformly loaded slab of many panels.

The lines of contra-flexure in (24) separate the slab into areas which are largely independent of each other, since no bending moments are propogated from one to another. The only forces crossing these lines of section are the total vertical and horizontal shearing stresses. The horizontal shears (which are unimportant so far as deflections go) will be considered later so far as may be necessary, but the vertical shears found by (28) are of prime importance. Let us then consider one of these side belts.

In any extended slab with its panels all loaded uniformly thruout, the vertical shear must vanish at all points along sections made by vertical planes thru the centers of columns at each side of any panel, as appears by reason of symmetry of loads. Let the edges of the side belts be situated at some given distances, say x_1 and y_1 on each side of the centers of all the panels, where x_1 and y_1 are not necessarily the values of x and y given in (24), altho those values are also included in this supposition. Then by (28) there is a uniformly distributed vertical shear of intensity $\frac{1}{2}\, q\, y_1$ along the edge of the belt at $y = y_1$, even tho the reinforcement in the side belt may be greater than that in the central rectangle, for the deviations caused by the irregularity of its distribution may be regarded as unimportant and practicably negligible.

It may then be assumed that any side belt parallel to x must carry, in addition to that already provided for in (21), a total loading of $q\, y_1$ per unit of length, uniformly distributed along the two edges that are parallel to x. Now since the width of this belt is $2(b - y_1)$, the load already provided for in (21) is $\frac{1}{2}q$ per unit of area, or $q(b\text{-}y_1)$ per unit of length parallel to x, which added to that arising from the shears just mentioned makes a sum total of $q\, b$ per unit of length of belt, which it will be noticed is independent of the width of the belt. In other words, any such belt must support a load of one fourth of the total load on the two panels of which it forms a part, or one half of all that lies between the panel center lines which are parallel to it on either side. This in effect transfers the entire loading of the slab to the side belts by the agency of the shearing stresses. It does this in such a way that one half of the total loading of the entire slab is carried by one set of side belts, and the other half by a second set which crosses the first at right angles.

In those parts of the slab area where these sets of belts cross, forming the heading of the columns, the loading is superposed also.

The preceeding investigation of the shears at the edges of side belts and their loading is independent of their width and of the position of the lines of contra-flexure, but their width will be assumed in what follows to be determined by the position of those lines as shown in (24) on account of the independence of action of belts of their width, as previously explained, where it was shown that no bending moments are propogated across those lines.

The question now arises, how the vertical shears at the edges of the side belts are distributed across their width and carried by them. Since by symmetry of loading, etc., there is no vertical shear at the edge of the panel where $y = b$, the shear must diminish from each edge of a belt to zero at that line. If it be assumed to diminish uniformly, that is equivalent in its action to a uniformly distributed load on the belt, which may be assumed in computation to replace the shears at the edges. Whether it will be so distributed or not depends upon the stiffness of the mushroom head and the smallness of its flexure. Extensometer measurements on the rods of the side belt of the floor slab of the St. Paul Bread Company Building by Prof. Wm. H. Kavanaugh show beyond question that in the mushroom system the load is so distributed. Other extensometer measurements to which the writer has access also show that in systems in which the heading of the column is not so stiff as this the distribution of loading cannot be taken as uniform over the side belts.

Now the belt parallel to x was shown to carry a load per unit of length of $q\,b$ and to have a width $2(b - y_1)$, in general, or a width $2b(1 - \frac{1}{3}\sqrt{3})$ for the belt between the lines of contraflexure; hence the intensity of the loading on this belt is $q\,b/2(b - y_1)$, instead of q, as it would be in a uniformly loaded panel duly supported at its edges by beams from column to column. Let ΣA, designate the area of the effective right cross section of the steel in the entire width of a side belt regarded as forming a single sheet of metal of the width of the belt; then $\Sigma A/2(b - y_1)$ is the effective right cross section per unit of width of belt, and we may write (14) in the form

$$I = i\,j\,d^2\,\Sigma A/2(b - y_1) \qquad \text{(29)}$$

We shall consider admissible values of ΣA later.

Since the deflection of the side belts may be taken independently of the rest of the slab, let those values for the intensity of loading and the moment of inertia (29) be introduced into (21).

We then obtain an expression for the law governing the deflection of that part of the side belts parallel to x which lies between the mushroom heads, and is bounded by lines of contra-flexure, viz:

$$z = \frac{(1 - K^2)\, q\, b}{48\, E\, i\, j\, d^2\, \Sigma A} [\, (a^2 - x^2)^2 + (b^2 - y^2)^2 \,] \ldots\ldots (30)$$

with a corresponding equation for the side belts parallel to y, which may be obtained by replacing $q\, b$ in (30) by $q\, a$. Call this second equation (31). Now (30) and (31) would hold thruout the entire length of these belts from column to column were they entirely separate from each other and from the diagonal belts where they cross each other. It will be necessary later to obtain the equation which holds true where these belts cross and combine with each other.

9. Practical formulas for the stresses in the steel and concrete of side belts between the lines of contra-flexure will now be obtained from (30) and (31).

In order to do this, consider the summation in (30) expressing the effective cross section of the steel in the mid area of the side belt regarded as forming a single uniform sheet, that mid area being bounded on all sides by lines of contra-flexure.

It is to be noticed that the factor $(1 - K^2)$ of (30) takes into account the fact that the lattice of rods forming the reinforcement is less effective than the same amount of metal in the form of a sheet, the only question left being this: Will the great irregularity of distribution of the reinforcement in this area cause it to act differently to any noticeable extent from the manner in which the same amount of metal would act were it possible to distribute it uniformly over the entire area? There are strong reasons which go to sustain the view that this irregularity of distribution is negligible in the standard mushroom slab, at least for loads less than those that stress the steel below the yield point, or do not stress the concrete for too long a time while it is imperfectly cured. On examining a diagram of the reinforcing rods of a slab made with square panels of such proportions that the width of the belts is one half the distance between columns, then the pattern previously mentioned into which it would be divided by these belts will be seen to consist of equal squares whose edges are equal to the width of the belts, with one central square in each panel concave upwards, and one half of each of the saddle shaped squares which border it, also lying within the same panel, and one quarter of each of the four convex squares at the head of each of the columns at the corners of the panel, also lying within the same panel, see Fig. 3, page 7.

Each side square will be found in this case to have double (or two belt) reinforcement over one half or its area, single belt reinforcement over a diamond occupying one fourth of its mid area, and triple reinforcement over four triangular areas along its sides which together cover one fourth of the square. This gives a mean value of $\Sigma A = 2 A_1$ in which A_1 is the total right cross section of the rods in the side belt.

The belts in the standard mushroom are, however, not so wide as this, since that system simply requires that the edges of the side and diagonal belts intersect in a single point, Fig. 2, instead of forming four areas of triple reinforcement on the sides. This makes the width of the singly reinforced diamond sufficient to just reach across the side belt. In this practical case we find that very approximately

$$\Sigma A = 1.5 A_1 \qquad (32)$$

in which, as before, A_1 is the total right cross section of the side belt in square inches. It is evidently impossible for this single side belt of rods which crosses the diamond, to elongate without a corresponding equal elongation of the double reinforcement on all its sides, or at least it is impossible for readjustments to take place in any short time such as will make these direct deformations within the diamond larger than those in the areas along side of it, or before somewhat more permanent deformations have taken place in the concrete.

In cases where the column heads are smaller than the standard, and the side belts still narrower, not only may ΣA become much less than $1.5 A_1$ but the belt become so weakened near the central diamond as to render it very questionable whether the irregularity of distribution of steel in the area considered may be safely disregarded. Diminution of the size of the heading thus not only diminishes cantilever action, but reduces the effective resistance of the reinforcing steel. Not much diminution of the size of head would be required to reduce the value of ΣA to an amount as small as A_1.

Introducing the estimate given in (32) for the standard mushroom into (30) we derive by (23), (23a) and (3), for that part of the side belt parallel to x between $x = +\frac{1}{3} a\sqrt{3}$ and $x = -\frac{1}{3} a\sqrt{3}$,

$$\left.\begin{aligned} f_s &= E e_1 = \pm E i d \frac{\delta^2 z}{\delta x^2} = \pm \frac{(1-K^2) q b}{18 j d A_1}(3x^2 - a^2) \\ M_1 &= 1.5 A_1 j d f_s = \pm \frac{(1-K^2)}{12} q b (3x^2 - a^2) \end{aligned}\right\} \qquad (33)$$

in which M_1 is the total true moment of resistance of the side belt, f_s is the true stress per square unit of the reinforcement in the side belt, and $1.5\ A_1$ is the effective right cross section of the reinforcement. This is independent of y as before noted, showing that the values of f_s and e_1 are the same for one rod as for another, but they attain their greatest values at the mid length where $x = 0$. If units be pounds and inches, and we assume $j = 0.91$ for the very small percentage of reinforcement of the standard mushroom system, then by (33) and (6) the practical formulas for design are:

$$\left.\begin{aligned} f_s &= \frac{3\ q\ a^2\ b}{4 \times 18 \times 0.91\ d_1\ A_1} = \frac{W\ L}{175\ d_1\ A_1} \\ M_1 &= 1.5\ A_1\ j\ d_1\ f_s = \frac{W\ L}{128} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (34)$$

in which f_s is the true stress in the steel, and M_1 is the true bending moment of the effective cross section $1.5\ A_1$ of the steel in the entire belt as shown by the elongation (at mid span) of the rods in a side belt of length L, where L is either $2a$ or $2b$, and $W = 4\ q\ a\ b$ is the total load on the panel in pounds, where d_1 is the vertical distance from the center of the rods in the single belt at mid span to the top surface of the slab.

While the values obtained from (34) are conservative for $j = 0.91$, corresponding to a percentage of reinforcement for one belt of less than 0.25%, (34) should be regarded merely in the light of a specimen equation for that percentage, and any slab where the percentage differs materially from that assumed value should be submitted to separate computation in the same manner.

Values of j are given for beams by Turnearue and Maurer in their "Reinforced Concrete Construction," page 57, for different percentages of reinforcement on the straight line theory, which latter is now accepted usage. As already stated, standard mushroom design makes the percentage of reinforcement for warehouse floors where the panels are, say 20′ x 20′, as low as 0.25% or less, at the middle of the side belts, reckoned on the beam theory. But in heavier and larger construction it may reach 0.33%.

We have taken the mean available steel in the belt as $1.5\ A_1$, hence the mean slab reinforcement will not be less than 1.5 x 0.23 = 0.4% in the side belt areas between lines of contra-flexure.

In case we assume the ratio of E_s for steel to E_c for concrete to be 15, as is often prescribed, we find the above stated value of j as a good mean value, which will be less in cases where the percentage of steel is greater. The small percentage of steel and great relative thickness of concrete is one of the distinguishing features of the standard mushroom design.

We may write (34) in the form:

$$f_s = \frac{W\,L}{175\,d_1\,A_1}, \text{ and } M_1' = A_1\,j\,d\,f_s = \frac{W\,L}{192} \quad \ldots . (34a)$$

in which M_1' is the true bending moment of the actual cross section A_1 at mid belt. We have written this modification of (34), not for use in design, but merely for the purpose of instituting a comparison with empirical formulas obtained by Mr. Turner to express the results of numerous tests made by him. On pages 26 and 28 of his "Concrete Steel Construction" he has given equations expressing the values of stresses and moments in mushroom slabs which in our notation may be written as follows:—

$$M_1' = A_1\,j\,d\,f_s = \frac{W\,L}{200}, \text{ and } f_s = \frac{W\,L}{200 \text{ x } 0.85\,d\,A_1} = \frac{W\,L}{170\,d\,A_1} \quad . (35)$$

in which he has assumed 0.85 as a mean value of j.

It is seen that equations (34a), obtained from rational theory alone, are in practical agreement with (35), which were deduced from experimental tests of mushroom slabs, where the numerical coefficient introduced is entirely empirical.

As will be seen later, (34) is the equation which ultimately controls the design of the slab reinforcement; so that the agreement of these two entirely independent methods of establishing this fundamental equation cannot but be regarded with great satisfaction as affording a secure basis for designs that may be safely guaranteed by the constructor, as has been the custom in constructing standard mushroom slabs.

The slab theory here put forth diverges so radically from the results of beam theory that we introduce here the following comparative computation of the smallest values of true bending moment and stress in steel, which can be obtained by beam theory for the side belt parallel to x, as follows:—

That part of the side belt between the lines of contra-flexure is simply supported at its ends by shearing stresses, and so may be taken to be a simple beam resting on supports at these end lines.

Hence the true stress f_s and the true bending moment M' at the middle of this simple uniformly loaded beam may be computed from the equation,

$$M' = A_1 j d f_s = \tfrac{1}{8} W' L' \quad \text{.......................} (36)$$

in which M' is the total moment of resistance.

A_1 is the total right cross section of the reinforcement, W' is the total uniformly distributed load, and L' is the length of the beam. The length of the simple beam in that case is evidently the distance along x between lines of contra-flexure, viz, $L' = \frac{2}{3} a \sqrt{3} = \frac{1}{3} L \sqrt{3}$, where L is the edge of the panel, and the total load at most will be that already proven to be carried by the side belt viz, $q\,b$ per unit of length, or a total for a span L' of $W' = q\,b\,L' = \frac{2}{3} q\,a\,b \sqrt{3} = \frac{1}{6} W \sqrt{3}$ where $W = 4\,qab$ is the total load on the panel, hence

$$M' = A_1 j d f_s = \frac{W\,L}{48} \quad \text{.......................} (36a)$$

It thus appears that according to simple beam theory the true stress, or the cross section of steel required in the belt, is four times that obtained by slab theory as shown by (34*a*). Since (34*a*) is in good accord with experimental tests, this comparison justifies the statements made near the beginning of this paper respecting the inapplicability of beam theory to the computation of slab design.

The floor of the St. Paul Bread Co. Building, previously mentioned, is a rough slab 6″ thick, and has panels 16′ x 15′, with ten 3/8″ round rod reinforcement in each belt, built for a design load of 100 pounds per square foot; constructed in winter and frozen, the final test was not made until the end of its first summer after unusually complete curing, such as might make the value of K given in (6) not entirely applicable. In one long side belt, extensometer measurements were made at the mid span on three rods, (1) a middle rod, (2) an intermediate rod and (3) an outside rod of the belt, with the following results for the given live load in pounds per square foot:

Live Loads	108.4#	316.8#	416.8#
$f_s = E\,e_1$ (1)	7650	15000	17940
" (2)	7080	14190	16470
" (3)	7320	13920	17160
Average	7350	14370	17200
f_s by (34)	5000	14440	19000

The observed results are seen to be in excellent agreement with those computed from (34) for the heavier loads, while any disagreement is on the safe side. Agreement is not expected for light loads.

The accuracy and applicability of (34) and preceeding formulas is dependent on the fixity of the lines of contra-flexure (24) which were previously stated to be practically immovable because of the sudden large change of the moment of resistance of the slab at those lines. That fact may be put in a more definite and convincing form than has been done so far. Consider for a moment that form of continuous cantilever bridge where there are joints between the cantilevers over the successive piers (which are in the form of a letter T) and the intermediate short spans which connect the extremities of the cantilevers. At such joints the resisting moments vanish, and they form in a sense artificially fixed points of contra-flexure. The same thing approximately occurs at the edge of the mushroom, because there the reinforcing steel rapidly dips down from a level above the neutral plane to one below it, and the sign of the moment of resistance changes thru zero at that edge.

Furthermore, it may be proper to state in this connection that the foregoing theory has been developed in consonance with the general principles of elasticity, and that somewhat different conditions and relations are thought to exist when the steel at the middle of the side belts reaches its yield point, as it does in advance of the rest of the reinforcement. As the yield point is reached equations (34) no longer hold; for, as will be seen more clearly later, the single belt of reinforcing steel, which crosses the circumference of an approximately circular area of radius $L/2$ about the center of each column, will everywhere reach the yield point at practically the same instant, and if loaded much beyond this will develop a continuous line of weakness there. The equations that hold in this case will be approximately those due to the actual cross section A_1 of the belt, in place of (34), which contain the effective cross section, viz:

$$\left.\begin{aligned} f_s &= \frac{3\,q\,a^2\,b}{4 \times 12 \times 0.91\,d_1\,A_1} = \frac{W\,L}{117\,d_1\,A_1} \\ M_1 &= A_1\,j\,d_1 f_s = \frac{W\,L}{128} \end{aligned}\right\} \qquad (37)$$

which may be regarded as expressing the relations that exist at the limit of the elastic strength of the slab and the beginning of permanent deformation, tho not necessarily of collapse.

The percentage of reinforcement in standard mushroom slabs is small enough to make their elastic properties depend upon the resistance of the steel. The stresses in the concrete may then be be computed from those in the steel, but many uncertainties attend any such computation. It is usage, fixed by the ordinances of the building codes of most cities to require the application of the so called "straight line theory" in such computations, not because that will give results which will be verified by extensometer tests of compressions in the concrete, for it will not, but because it is definite and on the side of safety. Furthermore it is usually prescribed that the ratio of the modulus of elasticity of steel divided by that of concrete shall be assumed to be 15, where the moduli are unknown by actual test of the materials. This is usually far from a correct value. The consequence is that the results of computation of the stresses in concrete are highly artificial in character, and should not be expected to be in agreement with extensometer tests. With this understanding the computed stress in the concrete at the middle of the side belt will be found as follows:—

Let id be the distance from the center of the steel to the neutral plane. (It happens to be more convenient in this investigation to use this distance id here and in our previous formulas than to introduce the distance from the neutral axis of the slab to the compressed surface of the concrete, as is done by many writers, under the designation $k\,d$. These quantities are so related that $i + k = 1$).

Then, as is well known from the geometry of the flexure of reinforced concrete beams, in case tension of concrete is disregarded,

$$f_c = \frac{k}{i} \cdot \frac{E_c}{E_s} f_s \qquad (38)$$

where the subscripts c and s refer to concrete and to steel respectively.

Applying (38) to the greatest computed stress $f_s = 19000$ in the St. Paul Bread Co's Building, gives a computed stress $f_c = 492$; but taking the greatest observed stress $f_s = 17940$ gives $f_c = 465$ lbs. per sq. inch, as the greatest computed compressive stress in the concrete at the middle of the side belt, if $i = 0.72$.

The tensile stress across the middle of the side belt at the extreme fiber of its upper surface is fixed by the curvature of the vertical sections of the slab in planes that cut the side belt at right angles. As stated previously all such planes make cross sections of the side belt that are identical in shape. That is a consequence of the conclusion reached previously, that all the rods in the side belt are subjected to equal tensions. The curvature of these sections is controlled by the stiffness of the mushroom heads, which is so great as to make the curvature very small. No considerable tensile cross stresses are consequently to be apprehended; but in case the stiffness of the head were to be decreased, stresses might arise such as to develop longitudinal cracks over the middle rod of the side belts.

10. In order to obtain practical formulas for the deflections and stresses in the steel thruout the areas at and near the tops of the columns where all the belts cross each other, and lying between lines of contra-flexure, we shall have recourse to (30) and (31) which are here superimposed on each other, and combined together. Were there no steel here in addition to the side belts, that superposition could be correctly effected by writing a value of z whose numerator would be the sum of the numerators of (30) and (31), for that would superpose the loads of the two side belts, and thus place the total required loading upon this area as previously explained; and then by writing for a denominator the sum of the denominators of (30) and (31), for that would superpose and combine the resistance of all the steel in both belts. But such a result would leave out of account the reinforcement arising from the diagonal rods, and the radial and ring rods, which should also be reckoned in as furnishing part of the resistance.

Supposing this additional steel to be distributed in this area in the same manner as is that of the side belts, a supposition which is very close to the fact, we may write

$$z = \frac{(1 - K^2)\, q\, (a + b)}{48\, E\, i\, j\, d^2\, \Sigma\, A} \left[(a^2 - x^2)^2 + (b^2 - y^2)^2\right] \quad (39)$$

in which ΣA is the cross section of the total reinforcement in this area regarded as forming a uniform sheet, i and j stand for mean values that have to be determined by the percentage of reinforcement and its position, while d is the mean distance of the center of action of the steel above the lower compressed surface of the concrete at the point xy.

We may conservatively assume in the standard mushroom that the center of action of the steel is at the center of the third layer of rods from the top, as will appear more clearly later. This defines d, which we shall consequently designate by d_3.

It remains therefore to estimate the amount of the total reinforcement ΣA, and then find mean values of i and j.

In case of reinforcing rods which are all of them continuous over a head without laps, the percentage of reinforcement falls only slightly below 4 times that at the middle of a side belt; but on the other hand were none of them continuous for more than one panel and each lap reached beyond the center of the column to the edge of the mushroom, the percentage of reinforcement would not be less than 7 times that at the middle of a side belt, and to this must be added that due to the steel in the radial and ring rods. Thus the percentage of reinforcement here may be varied not only by reason of the larger or smaller number of laps over each mushroom, but by reason of the length of the laps, from perhaps 3.75 to 7 times that at the middle of a side belt. For standard mushroom construction using long rods, it may be taken conservatively as a 4.25 times that at the middle of a side belt.

It is impossible to make an estimate that will be accurate for all cases, but commonly the 8 radial rods of a 20′ x 20′ panel are equivalent in amount to a single 1–1/8″ round rod, or a 1″ square bar circumscribing the area under consideration, that is to 4 square inches of additional reinforcement to be distributed in the width of a single side belt.

The two rings rod, of which the larger is commonly 7/8″ round, and the smaller 5/8″ round, may be taken to increase the reinforcement of this area by at least one square inch of cross section, giving all told some five square inches of cross section additional, equivalent forty-five 3/8″ round rods, or twenty-one 1/2″ rods. It thus appears that the increased reinforcement from this source reaches from 2 to 4 times A_1, and we may safely assume a mean total reinforcement over this area of

$$\Sigma A = 7.5\, A_1 \qquad (40)$$

of which the center of action may be pretty accurately stated to be at the middle of the third layer of reinforcement rods from the top.

In the standard design of mushroom floors for warehouses with panels about 20′ x 20′, the mean percentage of reinforcement for a single belt A_1 being about 0.23%, may be taken by (40) for a reinforcement 7.5 A_1 as

$7.5 \times 0.23 + = 1.75\%$ The corresponding value of j is 0.83, and we shall have

$$j\, \Sigma\, A = 0.83 \times 7.5\, A_1 = 6A_1 \qquad (41)$$

As previously stated, these equations (containing estimated mean numerical values) are given as a specimen computation for the purpose of making comparisons. In actual design, computations like these should be made which introduce the exact values appearing in the design under consideration.

We now derive from (39) and (40) by the help of (23) the following equations for this area where the belts all cross:—

$$\left.\begin{aligned} f_s &= Ee_1 = \pm Eid_3 \frac{\delta^2 z}{\delta x^2} = \frac{\pm(1 - K^2)\, q\, (a + b)}{90\, j\, d_3\, A_1}(3x^2 - a^2) \\ M_1 &= 7.5\, A_1\, j\, d_3\, f_s = \frac{(1 - K^2)}{12} q\, (a + b)\, (3x^2 - a^2) \end{aligned}\right\} \quad ..(42)$$

in which j and d_3 are less than in (33) and (34), as has been stated previously.

Apply (42) to find the stresses at the edge of the column cap on the long side L_1.

Let $B = 2x$ be the shortest distance along the middle of the side belt parallel to x between the edges of the caps of two adjacent columns, and introduce the values $j = 0.83$, $K = 0.5$, and $W = 4qab$, then;

$$\left.\begin{aligned} f_s &= \frac{W\, L_1\, (L_1 + L_2)\quad (3B^2 / L_1^2 - 1)}{800\, d_3\, A_1\, L_2} \\ M_1 &= 7.5\, A_1\, j\, d_3\, f_s = \frac{W\, L_1\, (L_1 + L_2)\quad (3B^2 / L_1^2 - 1)}{128\, L_2} \end{aligned}\right\} \quad . (43)$$

in which 7.5 A_1 is the effective cross section of the steel in this area, and M_1 is the true resisting moment of the steel derived from the elongation, and d_3 is as stated after (39).

Take the case of a square panel, and assume the diameter of the column cap to be $0.2L_1$, then $B = 0.8L_1$ and (43) reduce to:

$$\left.\begin{aligned} f_s &= \frac{W\, L_1}{435\, d_3\, A_1} \\ M_1 &= 7.5\, A_1\, j\, d_3\, f_s = \frac{W\, L_1}{70} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

It will be readily seen that if d_3 in (44) is more than 0.4 of the vertical distance designated by d_1 in (34), (as it must be) then the stress f_s in (34) at the middle of the side belt exceeds f_s in (43) at the edge

of the cap. But this does not prove that the stress in the concrete at the edge of the cap is less than that at the middle of the side belt, for, the value of i in (37) at the middle of the side belt is about $2/3$ and at the edge of the cap about $1/2$, as will be seen by consulting Turneaure and Maurer, page 57, for values of i corresponding to the values of j at these points. Hence, using these values of i, if primes be used to designate the stress at the edge of the cap, we have by (38), $\quad f_c'/f_c = 2f_s'/f_s$ . . . . (45)
from which it is seen that the stress f_s' at the edge of the cap must be only half that in the side belt in order that the corresponding stresses in the concrete may be equal. But ordinarily $2f_s' > f_s$, and so $f_c' > f_c$. The stress in the concrete at the edge of the cap will be computed from that of the steel found in (44) by using (38), in which if we put $i = K = \frac{1}{2}$, we have $f_c' = f_s'/15$, as the computed value of the stress at the edge of the cap.

Tests have seemed to show that much higher compressive stresses may be safely permitted in the concrete around column caps where there is compression in two directions, than in the extreme fiber of a beam where compression takes place in one direction only. A like principle applied to the extreme fiber at the middle of the side belt where tension exists at right angles to the compression would show that there only a low value should be permitted in compression.

In order to compare the greatest stresses in the steel across the mushroom with that at the middle of the side belts in a square panel let $B = L_1 = L_2$ in (43), then the stress in a section thru the column center along the edges of the panel over the mushroom area is found from the following equations:

$$\left.\begin{aligned} f_s &= \frac{W\,L_1}{200\,d_3\,A_1} \\ M_1 &= 7.5\,A_1\,jd_3f_s = \frac{W\,L_1}{32} \end{aligned}\right\} \qquad (46)$$

which are to be compared with (34), from which it appears that if d_3 in (46) is more than $7/8$ of d_1 in (34), the stress in the steel across the mushroom is less than at the center of the side belts. In any case these stresses are so nearly equal that the inadvisability of decreasing the steel in the mushroom head below standards indicated above is evident. However, some of the steel at the edge of the mushroom especially the outer hoop is at such level in this right section of the head as possibly to assist the concrete in bearing compressive stresses. Such a large portion of this section, moreover, falls within the cap, that no question of its stability and safety need arise, in case the collar band of the column is sufficient to resist the comparatively small tensions of the radial rods.

It will be noticed that in order to make f_s and f_c as small as possible in this area d_3 must be made as large as possible, *i. e.*, the steel at the edge of the cap must be raised as near the top of the slab as possible. Neglect of this is to invite failure and weakness such as has overtaken certain imitators of the mushroom system.

A final remark is here in place respecting the values of j and d_3 in this area. The stresses f_s and f_c diminish very rapidly towards the lines of contra-flexure, where they vanish, and the fact that the steel also rapidly increases its distance from the top of the slab at the same time might be regarded at first thought as requiring some modification of the assumptions we have made as to the values of j and d_3, which are approximately correct at the edge of the cap where the steel is placed as near the top surface as due covering will permit. But the fact is this: the only consideration of importance is the one respecting the position of the steel in that part of this area where the moments and stresses are large. The effect of the position of the steel near the lines of contra-flexure is negligible, and the fact that the amount of reinforcement may be somewhat smaller near these lines than elsewhere may also be neglected, so that the mean effective reinforcement previously estimated is likely to be an underestimate rather than the reverse. Further, the fact that the slab is practically clamped horizontally either at the edge of the cap or the edge of the superposed column, instead of at its center as assumed in our formulas, renders the results given thus far slightly too large.

Good average values of the size of steel used in the standard mushroom system of medium span would make the radial rods $9/8''$ round, the outer ring rod $7/8''$ round, the inner ring rod $5/8''$ and the belt rods $3/8''$ round. The importance of having the belt rods small is that for a given thickness of slab the smaller these rods are the larger is d in both (34) and (43) and consequently the smaller is f_s and A_1.

11. In attempting to consider the stresses in the diagonal rods of the central rectangle between the side belts of a panel, it will be noticed, as stated before, that no true bending moments are propogated across the vertical planes or lines of contra-flexure (24) which

bound it, and since the vertical shearing stresses at these lines are uniformly distributed along them, as already shown, (28), there are no true twisting moments in these planes. The curvatures of this rectangle will consequently depend upon its own loading and the resistance of its own moment of inertia, regarded as uniformly distributed, independently of that of other parts of the slab.

Hence (21) may be correctly applied to this area, regardless of the values which I (and q) may assume elsewhere, provided only that the values of I in other areas may be assumed to have constant values thruout those areas, and, further, that those areas are symmetrically disposed, so that all central rectangles have one and the same given value of I thruout, all side belts also have one given value of I, and the mushroom heads have a given value also, each of these three sorts of areas being independent. The truth of this proposition has been heretofore tacitly assumed in applying (21) to these latter areas as has been done.

It will be seen however, that the values of z obtained from such diverse equations express deflections of any point xy on the supposition that all the areas considered have the same value of I; but these separate equations, each with its own peculiar value of I, can be used separately to find the difference of level $z_1 - z_2$ between any two points $x_1\,y_1$ and $x_2\,y_2$ which lie in an area where I may be regarded as constant. We shall return to this point when we come to the derivation of practical deflection formulas.

For convenience in computing stresses in the rods of the diagonal belt, let the direction of the coordinates be changed so that in square panels they will lie along the diagonals which make angles of 45° with those used thus far. In (21) let

$$x = \tfrac{1}{2}\sqrt{2}(x' + y'), \qquad y = \tfrac{1}{2}\sqrt{2}(x' - y'), \text{ then}$$

$$z = \frac{(1 - K^2)\,q\,g}{24\,E\,i\,j\,d^2\Sigma A}\left[a^4 - a^2(x'^2 + y'^2) + x'^2y'^2 + \tfrac{1}{4}(x'^2y'^2)^2\right]\ldots.(47)$$

in which the panel is square and the axes of x' and y' lie along its diagonals, while the value of $\Sigma A / g$ is the effective cross section per unit of width of all the reinforcement in this area regarded as a single uniform sheet of metal, and $g = 7/8\ a$, is the width of a diagonal belt, and is equal to the diameter of the mushroom head. In rectangular panels $g = 7/16\ (a + b)$.

From (34) we have

$$\frac{\delta z}{\delta x'} = \frac{(1-K^2)\,q\,g}{24\,E\,i\,j\,d^2\,\Sigma A}\,[\,x'(x'^2 + 3y'^2) - 2a^2x'\,] \dots\dots (48)$$

$$e_1' = e_2' = -i\,d\,\frac{\delta^2 z}{\delta x^2} = -id\,\frac{\delta^2 z}{\delta y'^2} = \frac{(1-K^2)\,q\,g}{24\,E\,j\,d\,\Sigma A}\,[2a^2 - 3(x'^2 + y'^2]\,(49)$$

and $$\frac{\delta^2 z}{\delta x'\,\delta y'} = \frac{(1-K^2)\,q\,g\,x'\,y'}{4\,E\,i\,j\,d^2\,\Sigma A} \dots\dots (50)$$

These expressions satisfy (20) as they should, for (20) is independent of the directions of the rectangular axes x and y.

From (49) it appears that $e_1 = 0 = f_s$, on the circumference of the circle $x'^2 + y'^2 = \frac{2}{3}a^2$, which passes thru the points where the lines of contra-flexure intersect.

By (19), which holds for any rectangular axes, and by (50), we find

$$n' = \tfrac{1}{4}(1-K)\,q\,x'\,y' \dots\dots (26)'$$

From $(26)'$ it appears that in sections by all vertical planes parallel to the diagonals, the twisting increases uniformly with the distance from the diagonal.

Hence by (9) we have

$$\left.\begin{aligned} -s_1' &= \left(\frac{\delta m_1'}{\delta x'} + \frac{\delta n'}{\delta y'}\right) = \tfrac{1}{2}qx' \\ -s_2' &= \left(\frac{\delta m_2'}{\delta y'} + \frac{\delta n'}{\delta x'}\right) = \tfrac{1}{2}qy' \end{aligned}\right\} \dots\dots (28)'$$

It thus appears that the same law holds for vertical shearing stresses on planes parallel to the diagonals, as holds in (28) for planes parallel to the edges of the panel.

In standard mushroom designs the edges of the diagonal belts intersect on or very near to the edges of the side belts. That makes the middle half of the central square to be covered by double belting, and the remainder of it by single belting, so that $\Sigma A = 1.5A_2$ or perhaps $1.6\,A_2$, and the mean value of A, the reinforcement per unit of width of slab here, is to be found by dividing this by the width of a belt, which is $7/8\,a$. We should then find $A = 1.5\,A_2/7/8\,a = 1.7\,A_2/a$. But this mean value of A is not its mean effective value for this area, because the reinforcement is so disposed as to furnish the larger values of I in the central diamond just where the largest true applied moments and stresses occur. The mean value of A in the central diamond is $2A_2/7/8\,a = 2.3A_2/a$. The mean effective value lies between these two extremes, probably nearer the latter than the former. A similar question was discussed in connection with (40) and (41). We shall assume as the mean effective reinforcement in this central rectangle,

$$A = 2A_2/a, \text{ and } I = 2A_2\, i\, j\, d_2^2/a$$

or in case of rectangular panels

$$I = 4A_2\, i\, j\, d_2^2/(a+b) \qquad \text{(51)}$$

In case of rectangular panels the term $2a^2$ in (49) should be replaced by $a^2 + b^2$ as a mean value to make it depend the dimensions of the panel symmetrically, as it must. Making these substitutions in (49) we have at $x' = 0 = y'$ the center of the panel.

$$\left.\begin{aligned} f_s &= Ee' = \frac{W(L_1+L_2)(L_1^2+L_2^2)}{1024\, L_1 L_2 A_2\, j\, d_2} = \frac{C_1 W L_1}{256\, A_2\, j\, d_2} \\ M_1' &= 2A_2\, j\, d_2\, f_s = \frac{W(L_1+L_2)(L_1^2+L_2^2)}{512\, L_1 L_2} = \frac{C_1 W L_1}{128} \end{aligned}\right\} \qquad \text{(52)}$$

where $C_1 = \frac{1}{4}(L_1/L_2 + 1)(1 + L_2^2/L_1^2)$. Take $j = 0.89$.
If $1 > L_2/L_1 > 0.75$ then $1 < c_1 < 1.042$, hence C_1 varies less than 5% while L_2/L_1 varies by 25% between its extreme permissible values. C_1 may ordinarily be taken as unity, or may be found with sufficient precision by interpolation between the values just given.

The steps by which these equations (52) were deduced may not seem conclusive, since they are not rigorous. They need be only good, working approximations for the purpose for which they will be here used, viz, to show that the stresses at the center of the panel are less than those at the mid span of the side belts in case $A_1 = A_2$.

The value of d_2 in (52) is less than d_1 in (34), but always more than 90% of it. We may define d_2 as the vertical distance from the center of the second and upper of the two diagonal belts to the top surface of the concrete. We may assume $d_2 = 0.9d_1$ and $j = 0.89$ in (52), and then we may compare these stresses for a square panel as follows:—

$$f_s' = \frac{175}{205} f_s \qquad \text{(53)}$$

where f_s' refers to the center of the panel. Even were the smaller value for the mean reinforcement, $1.7\, A_2/a$, used in deriving (52) and (53), the stress given by these equations would not exceed that given by (34). The compressive stress f_c in the concrete at the center of the panel may readily reach a dangerous value in case the forms are removed too soon. It should therefore be carefully considered in each case. Here, we have an approximate value of $i = 2/3$ and (38) then becomes $f_c = f_s/30$ with no possible assistance from steel reinforcement since that is all on the bottom of the slab. An estimate that the elastic stress in the steel at the center of the panel does not much exceed 80% of that at the middle of the side belt cannot be far from the truth.

While this is undoubtedly the fact, it will appear on further consideration that local stresses and strains which exist at incipient failure are of such magnitude as to make the weakest points of the diagonal belts to lie ultimately not at the center, but, instead, just outside the diamond where they cross each other.

Take the standard case where the central diamond reaches just across to the side belts. For square panels imagine a circle to be drawn concentric with each column of radius $L/2$. Any circle at a column will be tangent to the edges of four diagonal belts across the tops of the four columns adjacent to it, and then the octagon circumscribing it, whose sides cut at right angles all the belts that cross this column head, intersects but a single belt of rods as every point of its perimeter. It is evident that, so far as reinforcement is concerned, such a line or section cuts less steel per unit of perimeter than any other regular figure concentric with the column and that the reinforcement is entirely symmetrically disposed about the column center, so that in case of equal diagonal and side belts, it would be impossible from their geometry to distinguish the one from the other by anything inside the octagon. That fact would make it inherently probably that the stresses and strains of the rods where they cross any one side of this octagon should be approximately the same ultimately as in those that cross any other side, whether they be rods in a diagonal belt or in a side belt. And what will be attempted to be shown immediately is that ultimately the stresses and strains in these several belts approach equality. If that should be established, it will follow from the conclusion already reached as to the excess of the stresses and strains of the side belt over those at the center of the panel, that ultimately those at the edges of the ocatgon exceed those in the same rods at the center of the panel.

The qualification implied above in affirming that this is what will occur ultimately, is for the purpose of conveying the idea that this is the approximate distribution of stresses and strains which will take place when the slab is sufficiently loaded to bring the steel at the middle of the side belt to the yield point. At less stress than this there is so much lag in the distribution of the effect of loading that it penetrates to the various parts of the slab unequally.

Taking up now the deferred proof that the diagonal rods where they cross the edge of the octagon are subject ultimately to the same local stresses and strains as the direct rods of the side belts; note that these diagonal rods lie in a triangular area between two side belts, which latter experience equal elongations e_1 in directions at right angles to each other. The edges of the triangle in which the single layer of diagonal rods lie are continuous with the side belts and necessarily experience the same elongations, which are propogated from the side belts into the triangle by the agency of horizontal shears on its edges. Such equal elongations at right angles imply the same elongation in every direction in the triangle, as appears from the fundamental properties of equal principal stresses and strains. Hence we have the same elongations along the diagonal rods as along the rods of the side belts at the edges. The existence of an ultimate stress and strain in the diagonal belt equal to that in the side belt would require that the cross sections A_2 and A_1 of the two belts should be equal, altho so far as the elastic value of f_s at the center of the panel is concerned A_2 might be less than A_1, as has been already shown in (52) and (53). The relationships of stress, load, etc., for this ultimate condition, have been already given in (37).

Besides the stresses and strains in the diagonal belts, just investigated, those due to the local stretching (arising from the deflections themselves) exert their greatest effect on the rods of the diagonal and side belts just in the region of the line of weakest section, and partly because of that fact. While these local stresses may not exceed 10% in addition to those already present, their existence should prevent any thought of taking ΣA larger than A_1 in (37) when deriving the ultimate stresses at the yield point. Similar results may be formulated to cover cases where g is greater or less than $7/16\ L$.

It is perhaps desirable at this point to consider a little more at length the matter of local stretching in a slab. It is impossible for a continuous flat floor slab to undergo the deflections which we are treating, consisting of convexities, concavities, etc., without local stretching to allow this to occur. A floor slab of many panels does not undergo any change of its total linear dimensions which would account for these corrugations. A continuous beam under flexure would have its extremities drawn toward each other. But not so to any such extent with a slab. Such contractions are resisted by local circumferential strains which result in true stresses. An investigation of such stresses leads to the conclusion just stated that in general they cannot exceed 10% of the ordinary stresses due to slab bending when they are left out of the consideration. For this reason a single panel alone will not function precisely in the same way as a panel in a floor of many panels.

12. Actual deflections are distances which any given points of a slab sink down by reason of the application of a given load, and their theoretical values are to be computed by help of the formulas which have been developed for z in the various areas into which the panel has been divided.

We shall now make a slight modification in our definition of the level of the origin of coordinates, and shall take it at the upper or lower plane surface of the flat slab before flexure, in which surface the axes of x and y are assumed to lie. It is of no consequence whether it be the upper or the lower surface which is assumed, the equations will be the same in either case. The reason for this new definition of the position of the origin is this: Each kind of partial area into which the slab has been supposed to be subdivided has its neutral surface at a different depth in the slab, and so it does not furnish a single suitable level from which to reckon deflections, as does the upper or lower surface of the slab. None of the equations which have been derived in this paper will undergo any modification by reason of this change of definition. It has been assumed that each kind of area has a separate value of I which remains constant thruout, so that the neutral surfaces of different areas do not join at their edges. As previously explained this is of no consequence mechanically by reason of the zero true moments that exist at these edges. The modification just introduced avoids the geometrical perplexities arising from this discontinuity of neutral surfaces.

Deflections in the side belt area between the lines of contraflexure (24) are to be found from (30), or (31), and (32). To find the deflection or difference of level in the mid side belt between $x = 0$, $y = b$, and $x = \frac{1}{3} a\sqrt{3}$, $y = b$, substitute these values in (30), take $i = 0.71$, $j = 0.91$, $k = 0.5$ and subtract the value z at the second point from that at the first point, which gives the following value of the deflection of the one point below the other:

$$\triangle z_1 = \frac{W L_1^3}{10.7 \text{ x } 10^{10} d_1^2 A_1} \qquad \text{(54)}$$

in which h_1 is the vertical distance from the center of the single belt of rods at the mid span of the side belt to the effective top of the slab, considering the strip fill or other concrete finish at its effective value.

In the same manner take the difference of level in the central rectangle bounded by the lines of contraflexure between the center point at $x=0$, $y=0$ and the corner $x=\frac{1}{3}a\sqrt{3}$, $y=\frac{1}{3}b\sqrt{3}$ by using (21) and (51) and introducing the values $i = 2/3$, $j = 0.89$, etc., and $C_2 = 1/4(L_1/L_2 + 1)(1 + L_2^4/L_1^4)$, then:

$$\triangle z_2 = \frac{C_2 W L_1^3}{6.56 \times 10^{10} d_2^2 A_2} \qquad \ldots\ldots (55)$$

in which A_2 is the cross section of one diagonal belt and h_2 is the vertical distance from the center of the upper or second diagonal belt to the effective upper surface of the panel at its center.

On evaluating C_2 above, we find

when $1 > L_2/L_1 > 0.75$

then $1 > C_2 > 0.77$

hence we may with sufficient accuracy for practical purposes assume

$$C_2 = L_2/L_1 \qquad \ldots\ldots (56)$$

Deflections in the mushroom area between lines of contraflexure (24) are to be derived from (39) (40) and (41) by introducing $i = \frac{1}{2}$, $j = 0.83$, $k = 0.5$ and $\Sigma A = 7.5 A_1$. Assuming the diameter of the cap to be $0.2L_1$ we have, at its edge where $x = 0.8a$, $y = b$, from (39)

$$z = \frac{W L_1^3 (L_1/L_2 + 1)}{19.1 \times 10^{10} d_3^2 A_1} \left(\frac{36}{100}\right)^2 \qquad \ldots\ldots (57)$$

The value of z at the edge of the mushroom area, where $x = \frac{1}{3}a\sqrt{3}$, $y = b$, is to be obtained from (57) by replacing the last factor by $4/9$; and the deflection between the edge of the cap and the edge of the mushroom obtained by taking the difference of these quantities is as follows:

$$\triangle z_3 = \frac{W L_1^3 (L_1/L_2 + 1)}{60 \times 10^{10} d_3^2 A_1} \qquad \ldots\ldots (58)$$

in which h_3 is the vertical distance of the center of the third layer of reinforcing rods over the edge of the cap above the bottom surface of the slab.

Similar expressions may be obtained for the values of z and $\triangle z$ on the side parallel to y, where $x = a$ at $y = 0.8b$, and $y = \frac{1}{3} b\sqrt{3}$, by exchanging L_1 and L_2 in (57) and (58).

Take half the sum of (57) and the corresponding values so obtained at $x = a$, $y = 0.8b$, as the value of z at the edge of the cap where it is intersected by the diagonal of the panel, viz.

$$z = \frac{W\,(L_1 + L_2)\,(L_1^4 + L_2^4)}{38.2 \text{ x } 10^{10}\, L_1\, L_2\, d_3^2\, A_1} \left(\frac{36}{100}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots (59)$$

and subtract this from the value of z on the diagonal at the corner of the mushroom area where $x = \frac{1}{3} a\sqrt{3}$, $y = \frac{1}{3} b\sqrt{3}$ and we have

$$\triangle z_4 = \frac{C_2\, W\, L_1^3}{12.5 \text{ x } 10^{10}\, d_3^2\, A_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

as the deflection along the diagonal between the edge of the cap and the intersection of the lines of contraflexure, in which C_2 and h_3 are as previously defined.

$$\left.\begin{aligned} \text{Let} \quad D_1 &= \triangle z_1 + \triangle z_3 \\ \text{and} \quad D_2 &= \triangle z_2 + \triangle z_4 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

in which D_1 is the deflection of the mid point of the side belt below the edge of the cap, and D_2 is the deflection of center of the panel below the edge of the cap.

The proportionate deflections of these points are obtained by dividing by the spans, viz: D_1 / L_1 and $D_2 / \sqrt{L_1^2 + L_2^2}$.

13. Estimated proportionate deflections may be obtained from (61) under such circumstances as to convey reliable information respecting what may be reasonably expected. Let h = the total thickness of the slab. The limiting values of the thickness of standard mushroom construction are expressed as follows:

$$L_1 / 20 > h > L_1 / 35, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

and assuming that the reinforcing rods are 1/2″ rounds with 1/2″ covering of concrete we shall have from the definitions of d_1, d_2 and d_3, already given

$$h = d_1 + 0.75 = d_2 + 1.25 = d_3 + 1.75 \quad \ldots\ldots\ldots (63)$$

Substituting these in (62) etc. we have

$$\left.\begin{aligned} L_1/20 - 0.75 &> d_1 > L_1/35 - 0.75 \\ L_1/20 - 1.25 &> d_2 > L_1/35 - 1.25 \\ L_1/20 - 1.75 &> d_3 > L_1/35 - 1.75 \end{aligned}\right\} \quad \ldots\ldots\ldots (64)$$

If it be assumed that we are dealing with medium sized panels about 20′ x 20′ (64), may be written in the form:—

$$(1 - 0.062)\ L_1/20 > d_1 > (1 - 0.02)\ L_1/35$$
$$(1 - 0.1)\ L_1/20 > d_2 > (1 - 0.036)\ L_1/35$$
$$(1 - 0.15)\ L_1/20 > d_3 > (1 - 0.05)\ L_1/35$$

or,

$$\left.\begin{aligned} \frac{0.94}{20} &> \frac{d_1}{L_1} > \frac{0.98}{35} \\ \frac{0.90}{20} &> \frac{d_2}{L_1} > \frac{0.964}{35} \\ \frac{0.85}{20} &> \frac{d_3}{L_1} > \frac{0.95}{35} \end{aligned}\right\} \dots\dots\dots\dots (65)$$

In (54), (55), (58) and (60) replace $W\ L_1$ by its value given in (34), viz, $175\ d_1\ A_1\ f_s$, and we have

$$\left.\begin{aligned} \triangle z_1 &= \frac{L_1^2\ f_s}{6.11 \times 10^8\ d_1} \\ \triangle z_2 &= \frac{C_2\ d_1\ L_1^2\ A_1\ f_s}{3.75 \times 10^8\ d_2^2\ A_2} \\ \triangle z_3 &= \frac{d_1\ L_1^2\ (L_1/L_2 + 1)\ f_s}{34.3 \times 10^8\ d_3^2} \\ \triangle z_4 &= \frac{C_2\ d_1\ L_1^2\ f_s}{7.14 \times 10^8\ d_3^2} \end{aligned}\right\} \dots\dots\dots\dots (66)$$

in which f_s is the greatest stress in the steel, *i. e.*, at the mid side belt, employed here to express deflections instead of expressing them in terms of panel load as was done previously.

Introduce into (66) the numerical values given in (65) which will then express limiting values of deflection for medium spans. For simplicity let $L_1 = L_2$ then:

$$\left.\begin{aligned} 287 &> \frac{L_1 f_s}{10^5 \Delta z_1} > 170 \\ 162 &> \frac{L_1 f_s}{10^5 \Delta z_2} > 106 \\ 660 &> \frac{L_1 f_s}{10^5 \Delta z_3} > 451 \\ 275 &> \frac{L_1 f_s}{10^5 \Delta z_4} > 188 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (67)$$

By (61) we have the proportionate deflection of the side and diagonal belts as follows:—

$$\left[\frac{1}{287} + \frac{1}{660}\right]\frac{f_s}{10^5} < \frac{D_1}{L_1} < \left[\frac{1}{170} + \frac{1}{451}\right]\frac{f_s}{10^5},$$

$$\left[\frac{1}{162} + \frac{1}{275}\right]\frac{f_s}{10^5\sqrt{2}} < \frac{D_2}{L_1\sqrt{2}} < \left[\frac{1}{106} + \frac{1}{188}\right]\frac{f_s}{10^5\sqrt{2}}$$

$$\left.\begin{aligned} \frac{f_s}{200 \times 10^5} &< \frac{D_1}{L_1} < \frac{f_s}{123.4 \times 10^5} \\ \frac{f_s}{141.4 \times 10^5} &< \frac{D_2}{L_1\sqrt{2}} < \frac{f_s}{95.9 \times 10^5} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (68)$$

$$\left.\begin{aligned} \text{If } f_s = 16000, \quad & \frac{1}{884} < \frac{D_2}{L_1\sqrt{2}} < \frac{1}{600} \\ \text{If } f_s = 24000, \quad & \frac{1}{590} < \frac{D_2}{L_1\sqrt{2}} < \frac{1}{400} \\ \text{If } f_s = 32000, \quad & \frac{1}{442} < \frac{D_2}{L_1\sqrt{2}} < \frac{1}{300} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (69)$$

Larger spans then $20'$, or smaller steel than $1/2''$ round, or $L_2 < L_1$ will reduce the above values somewhat, while smaller spans or larger steel will increase these values, all of which can in each case be submitted to calculation by the methods here developed.

To recur at this point to the expression for the deflection D_2 in terms of the panel load W by help of (55), (60) and (61)

$$D_2 = \frac{C_2\, W\, L_1^3}{10^{10}\, A_1}\left[\frac{1}{6.56\, d_2^2} + \frac{1}{12.5\, d_3^2}\right] \qquad (70)$$

By (65) we find

$$\frac{90}{85} > \frac{d_2}{d_3} > \frac{96.4}{95}$$

and using this inequality to eliminate d_3 from (70) we find after reduction

$$\frac{C_2\, W\, L_1^3}{4.46 \text{ x } 10^{10}\, d_2^2\, A_1} < D_2 < \frac{C_2\, W\, L_1^3}{4.33 \text{ x } 10^{10}\, d_2^2\, A_1}$$

from which we may write as a mean value

$$D_2 = \frac{C_2\, W\, L_1^3}{4.4 \text{ x } 10^{10}\, d_2^2\, A_1} \qquad (71)$$

The empirical deflection formula given on page 29 of Turner's Concrete Steel Construction, when written in these units, is

$$D_2 = \frac{W\, L_1^3}{4.84 \text{ x } 10^{10}\, d_2^2\, A_1} \qquad (72)$$

This is identical with (71) when $C_2 = 0.909$, and diverges from it slightly for other admissible values of C_2. The practical agreement of (71) and (72) affords a second confirmation of the theoretical deductions made thus far, and this taken in conjunction with the practical identity of formulas (34) and (35), the theoretical and empirical expressions for the maximum tensile stresses in the reinforcement, furnishes what on the theory of probabilities may be regarded as so strong a probability of the general trustworthiness of the entire theory as to exclude any rational suppositition to the contrary.

The various formulas for stresses and for deflections which have been developed in this paper have been obtained under the express proviso that the panel under consideration was assumed to be one of a practically unlimited number of equal panels constituting a continuous slab, all of which are loaded uniformly and equally. The question at once arises as to the amount and kind of deviations from these formulas which will occur by reason either of discontinuity of slab or loading, such as occurs at the outside panels of a slab or at panels surrounded partly or entirely by others not loaded. The answer to this question depends very largely upon the construction of the flat slab itself.

In the standard mushroom construction it has been found that the stresses and deflections of any panel are almost entirely independent of those in surrounding panels. This is due to the fact that the mushroom head is an integral part of the supporting column in such a manner that it is impossible for it to tilt appreciably over the column under the action of any eccentric or unequal loading of panels near it. When single panels have been loaded with test loads, no appreciable deflections have been discoverable in surrounding panels, and no greater stresses and deflections have been discovered than were to be expected in case surrounding panels were loaded also. Future careful investigation of this may reveal measureable effects of this kind, but they must be small.

A like statement cannot be made of other systems of flat slab construction where the reinforcement over the top of the column is not an integral part of the column reinforcement itself. Tests on these systems have shown clearly the effects of the tipping of the part of the slab on the top of the column, and lack of stiffness of head, in the increase of the deflection of the single loaded panel over the deflection to be expected in case of multiple loaded panels, and especially in the disturbance of the equality of the stress in the otherwise equal stresses in the rods of the side belts. Such distrubance, by increasing the stress in part of these rods, would necessitate larger reinforcement in the side belts of such systems than would be required in mushroom slabs. The great stiffness of the mushroom head is also of prime importance in taking care of accidental and unusual strains liable to occur in the removal of forms from under insufficiently cured slabs.

14. In considering the design of the ring rods and radial cantilever rods of the mushroom head, it should be borne in mind that they occupy a position in such close proximity to the level of the neutral surface as to prevent them from being subjected to severe tensile or compressive stresses by reason of the bending of the slab as a whole. Their principal function as slab members is to resist shearing stresses and the bending stresses due to

local bending. Their total longitudinal stresses are too small in comparison to require consideration.

Let a cylindrical surface be imagined to be drawn concentric with a column to intersect the slab, then the total vertical shearing stress which is distributed on the surface of intersection is equal to the total panel load W diminished by the amount of that part of the panel load lying inside the cylinder. If the cylinder be not large, the total shear may be taken as approximately equal to W itself.

It is evident that the smaller the diameter may be that is assumed for this cylinder, the greater will be the intensity of the vertical shear on its surface and that for two reasons: First, because the total load thus carried to the column will be greater the smaller the diameter, and second because the surface over which the total shear will be distributed decreases with its diameter.

The result of this is that the dangerous section for shear is the cylindrical surface at the edge of the cap. For cylinders smaller than this the increased vertical thickness of the cap diminishes the intensity of the shear. We proceed therefore to consider the manner in which the total vertical shearing stress of approximately W in amount is distributed in the material of the cylindrical surface at the edge of the cap.

In a beam or slab the horizontal shearing stresses due to bending reach a maximum at the neutral surface. It is a fundamental condition of equilibrium that shearing stresses on planes at right angles shall be equal, and it is this condition that determines the distribution of the vertical shears, which are at right angles to the horizontal shears resulting from bending the slab as a whole. From this we have the well known fact that the vertical shear varies from zero at the upper and lower surfaces to a maximum at the neutral surface, and this is necessarily the manner in which the total shear is distributed at the edge of the cap. The top belt of rods will be subjected to comparatively small shearing stresses, and successive layers of rods will be under larger and larger shearing stresses by reason of their greater nearness to the neutral surface, while the total shear borne by the radial rods near the neutral surface will be much larger than that upon the others. The shearing stress in the concrete will need to be considered also.

It is to be noticed that all the steel of the belts and mushroom head act together without the necessity of supposing large compressive stresses in the concrete to transmit vertical forces, because the belts of reinforcement rest directly upon each other, and these in turn upon the ring rods and radial rods, all in metallic contact with each other, in the mushroom head, and so they transmit and adjust the distribution of stresses within the system to a very large extent independently of the concrete.

We can then safely assign moderate values of the shearing stress to each of the elements that constitute the slab at the edge of the cap, with the assurance that they will each play a part in general accordance with the distribution which has been already explained.

The mushroom is constructed of great strength and stiffness not merely to effect the results which have appeared previously in the course of the investigation but also to ensure the stability of the slab in case of unexpected or accidental stresses due to the too early removal of the forms, before the slab is well cured, at a time when the only load to which it is subjected is due to the weight of the structure itself.

The working load to be assumed in designing the mushroom may be taken as the dead load of a single slab plus the design load, provided sufficiently low values of the shearing stresses be assumed in the cross sections of steel and concrete at the edge of the cap for the support of this working load, as follows:

For slabs having a thickness of $h = L/35$ a mean working shearing stress of 2000 lbs. per square inch at the right cross section of each reinforcing rod which crosses the edge of the cap, a mean shearing stress of 40 lbs. per square inch in the vertical cylindrical section of the concrete at the edge of the cap, and 8000 lbs. per square inch of right cross section of each radial rod.

For slabs having a thickness of $h = L/20$ the intensities just given may be safely increased by 50 per cent for reasons that will be explained later. For slabs of intermediate thickness increase the intensities proportionately.

These values are sufficiently low to enable the structure to support itself before the concrete is very thoroughly cured, and the head so designed will be found after it is well cured to be so proportioned as to carry safely a test load of double the live and dead loads for which it was designed.

In this connection it seems desirable to investigate what takes place in case of overloading and incipient failure of an insufficiently cured slab, or one unduly weakened by thawing of partially frozen concrete. Suppose that under such circumstances a shearing crack were formed extending completely thru the head at the edge of the cap, and we wish to investigate the stresses and behavior of the rods that cross the crack at which shearing deformation has begun to take place. Designate the position of the crack by X.

The total vertical shearing stress on a radial rod at X is the sum of two parts found as follows: First, the vertical reaction at the top of a column is made up of the vertical reaction of the concrete core of the column and the reactions of its vertical reinforcing rods. Call the vertical reaction of one of these rods V_1. The rod is bent over radially and V_1 expresses also the amount of the vertical shear in that rod where it starts out radially from the column. Between this point and X for a distance which measures usually from 9 to 12 inches, the rod experiences the supporting pressure of the concrete in the cap under it to a total amount which we will designate by V_2. The total shear in the radial rod at X will then amount to

$$V = V_1 + V_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

provided we neglect the weight of that small part of the actual load of the slab which lies directly over this piece of the rod and may be regarded as resting upon it. This portion of the radial rod of length l is a cantilever fixed at one end in the top of the column, and carrying a load V at the other end with a supporting pressure underneath of total amount V_2 whose intensity is greatest at X and gradually decreases along l from X to the fixed end. The rod has a point of contraflexure and zero moment at X. The portion of the rod outside the crack has a fixed point in the slab at the place where it supports the inner ring rod, at a distance from X which should not exceed l as just defined. Similar conditions hold for this length; *i. e.* there will be a totol shear in the radial rod at a point just inside the inner ring, rod due to its total shear outside this ring rod and to the vertical loading imparted to it by the ring rod itself. To this must be added the downward pressure of the concrete between the inner ring rod and X. All these, together, constitute the total shear $-V$ at X, in equilibrium with the reaction $+V$ already obtained at that point.

We shall discuss separately the action of V_1 and V_2 upon a radial rod. A load V_1 at the end of a cantilever of length l causes a deflection of amount $z_1 = \frac{1}{3} V_1 l^3 / EI \ldots\ldots\ldots\ldots\ldots\ldots (74)$
in which $I = \pi t^4 / 64$ where t = the thickness of the rod.

Also $V_1 = s_1 A$, $\quad A = \pi t^2 / 4$

in which s_1 = the mean shearing stress per square unit of cross section and A is the cross section of the rod. Hence

$$s_1 = 3 z_1 E t^2 / 16 l^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

which shows that so far as V_1 is concerned, for any given displacement z_1 the shearing stress carried per square unit of rod will be proportional to the square of its diameter, and up to its permissible limiting shearing resistance, each unit of section of such a rod will be effective in proportion to the square of its diameter. For economical construction, this will require the radial rods to be few and large, rather than numerous and small. The bending moment is greatest at the distance l from X and amounts to $V_1\, l$. The stress in the extreme fiber due to the bending moment $V_1\, l$ in the rod is

$$p_1 = V_1\, l\, t/2I = 8s_1\, l/t \qquad (76)$$

This equation shows that the stress in the extreme fiber is so very large at the fixed end of the rod compared with the shear at X that so far as V_1 is concerned the rod will suffer permanent deformation by bending long before there is any danger of its shearing. V_1 is so large compared with V_2 that this conclusion will not be altered when we come to consider the combined action of V_2.

Incipient failure of this kind will therefore cause distortion and sag without collapse. In case such sag as occurs in this case is detected underneath the head around the cap, the slab should be blocked up at once and the concrete picked out at all parts showing facture. This should then be refilled with a stronger concrete which will set rapidly. Such repair should not weaken the slab.

Whenever the intensity with which a radial rod presses upon the concrete at the edge of a crack at X passes the compressive strength f_c of the concrete, it must begin to yield. At this instant we shall have a pressure of the concrete against the rod which gradually diminishes as we pass along the rod from X to the distance l, where it becomes zero. We shall assume that the pressure diminishes uniformly with this distance. This may not be precisely correct, but cannot be much in error. If the shear V_2 at X is the sole cause of this pressure, then $V_2 = \frac{1}{2}\, t\, l\, f_c$, and $\frac{1}{3}\, V_2\, l = \frac{1}{6}\, t\, l^2\, f_c$ is the bending moment in the rod at the distance l, due to V_2 at X and the pressure distributed along l.

It will be found that these produce a deflection

$$z_2 = 3\, f_c\, l^4/20EI = 0.3\, l^3\, V_2/EI \qquad (77)$$

a unit shear of

$$s_2 = V_2/A = z_2\, E\, t^2/4.8\, l^3 \qquad (78)$$

and a stress on the extreme fiber at a distance l amounting to

$$p_2 = V_2\, t\, l/3I = 16s_2\, l/t \qquad (79)$$

It thus appears that the equations expressing the action of V_2 are precisely similar to those for V_1, differing only in their numerical coefficients, and consequently all the statements as to the resistance of the radial rods under the action of V_1 hold for the action of V_1 and V_2 together in the case of given initial deformations, $z_1 = z_2$ at X.

While the preceding investigation has, in order to make ideas explicit, ostensibly assumed a crack at X and an initial small shearing deformation at X, the investigation applies equally well to the elastic shearing deformation of the concrete at the dangerous section in which case the total shearing stress will consist of an additional componenent due to the resistance of the concrete, which however may for additional safety be neglected. If the assumed deformation be confined within limits so small that the concrete is able to endure it without cracking then the preceding investigation may properly be applied to it. It is right here that the thickness of the radial rods is able to render its most effective service, for it appears from (75) and (78) that any permissible intensity of shear may be developed in the radial rods by making them of suitable thickness, even tho the deflection be kept within the elastic limits of the concrete.

As already stated we must not overlook the fact that the major stresses here are those under the head of V_1, which are due to the direct metallic contacts of the steel rods resting one upon another, where large stresses are transmitted and pass independently of the concrete except for the distortions of the steel which meet resistance, and the secondary reactions such as have been treated in a single aspect while investigating the action of V_2.

It is due to this fact that large shearing stresses may be safely borne by the slab at and near the edge of the cap, which the concrete mostly escapes, it merely furnishing some lateral stiffening to the steel. On this principle the outer ring rod should have a cross section not much less than one half that of the radial rods on which it rests. For, this arrangement provides for the transferal to the radial rods of all the shear the ring rod is able to carry, it being in double shear compared with the radial rod it rests on.

It is impossible to determine the cross section of the inner ring rod, with the same definiteness as that of the radial rods, but that is unimportant. Its position has already been fixed as not more than l from the edge of the cap, where l is the distance from the top hoop or collar band of the column to the edge of the cap.

The vertical shearing stresses may be regarded as sufficiently resisted outside the mushroom by the concrete alone. The critical cylindrical surface separating those areas where the shear may be assumed to be safely carried by concrete alone, from those areas where the steel may be relied on to carry as much of the shear as may be required, should evidently be taken somewhat inside the outer ring rod, but just where is of no particular consequence.

The supposition of the existence of a crack at X, either actual or potential, on which our computation of the stresses in the radial rods has been based, is sufficiently satisfactory so far as the rods themselves are concerned; but it seems desirable to consider in more detail the phenomena attending the development of the stresses in the concrete at and near the edge of the cap, especially in soft concrete when the limit of its compressive resistance is reached in this region.

The horizontal compressive resistance of the concrete at the lower surface of the slab is that already treated in (38), and it is our present object to consider how that is to be combined with the vertical supporting pressures under the radial rods, and with the horizontal and vertical shears in the slab due to bending. These latter are greatest in the neutral surface, as has been previously stated, and according the general theory of stresses are equivalent to, and may be replaced by, a compression and a tension in the material respectively at 45° with the vertical (and mutually at right angles) of the same intensity as the shear. It is evident that the combination and resultant of these three compressive stresses would form the dangerous element in the stress, since the single tensile element would be relatively unimportant, and it would find assistance in its resistance from the steel running in a direction thru the concrete such as to afford it substantial support. This direction is that of the straight lines on the surface of a right cone whose vertex is above the center of the column and whose slope is 1 to 1.

Consider now two of the elements of the compression in the concrete around the cap, viz, the horizontal compression which is a maximum at the lower surface and zero at the neutral surface, and that due to shear which is parallel to the sides of a right cone with vertex downward, whose sides have an upward and outward slope of 1 to 1, while its intensity is so distributed that it is zero at the bottom of the slab and greatest at the neutral surface. It appears consequently that the lines of greatest compression in the concrete due to the combination of these two elements of compression would lie in vertical planes on a bowl or saucer-shaped surface that is horizontal at the edge of the cap and inclined at a slope of 45° at the neutral surface; and if the concrete were to crush under these stresses alone, the surface of fracture would have the shape indicated instead of that of the cylindrical surface previously assumed. This change would not, however, materially affect the computations we have made of stresses in steel; it merely serves to fix more definitely the position of the points of contra-flexure of the radial rods.

But there is still one further element or component of the total compression in the concrete to be considered and combined with those just treated in order to arrive at the resultant or total compression. This componenent is that due to the concentrated pressures underneath each of the radial rods. These rods are at some distance apart circumferentially and so do not exert a pressure that is uniformly distributed circumferentially. Any concentrated stress, such as that in the concrete supporting a rod, diffuses itself in the material in such a manner that its intensity rapidly diminishes with the distance from the surface of the rod, in accordance the same law as exists in case of centers of attraction. Since the supporting compression under the rods is vertical, we can imagine the lines of greatest compression in the concrete, when this component is combined with those already mentioned, to lie in vertical planes on a bowl or saucer-shaped surface which has as many indentations or scollops around its edge as there are radial rods, at which indentations the slope of the sides is such more nearly vertical than a slope of 45°. At such parts of the surface the intensity is also more severe, and especially is this the case if the slab is thin so that the concentrated pressure has small opportunity to distribute itself by radiating into a considerable body of material before it reaches the bottom of the slab. It thus comes about that thick slabs are enabled to carry safely larger intensities of shearing stress around the cap than can thin slabs, which is in accordance with and in justification of the statements already made as to permissible shears around the cap.

The resulting surface of fracture due to shear and compression around the cap would be of irregular conical shape starting from the edge of the cap and extending thru the entire thickness of the slab, were this not interfered with in the upper part of the slab by the mat of reinforcing rods, which are so tenacious as to tear to pieces and fracture the upper surface to a considerable distance in all directions whenever any such fracture occurs around the column.

Nevertheless such fracture as here described does not under any ordinary circumstances result in a dangerous collapse of the slab, or one that cannot be repaired without much difficulty, for, the radial rods and the reinforcing rods will at most have suffered some individual deformation by bending and are still far from being broken. This will become evident later where an experimental attempt to load a full-sized slab to failure is described in detail, and full account of the results reached is explained and illustrated.

It is stated on good authority that in experience with many hundreds of buildings constructed on this system, no case of shear failure or even of incipient shear failure or fracture has occurred in a well cured slab near the column and while a few cases of incipient failure have occurred in floors where forms were prematurely removed, no injury or fatality has resulted therefrom to any person.

It appears that the line of weakest section in the cured slab of the standard mushroom type is that discussed previously in obtaining (37) and shown in Fig. 3 page 7. This is brought out later by a test to destruction of a fairly well cured slab. The line of weakest section in a partly cured slab is on the other hand not definitely fixed, but may be and sometimes is, shearing weakness near the column as has been discussed and pointed out. Provision against such weakness or carelessness is a safeguard which, while costing a small amount in the matter of steel, is an insurance against serious accident well worth the investment involved. It is secured by making the radial and ring rods sufficiently stiff and strong.

15. This section will be devoted to a consideration of the mushroom system, and to several more or less similar flat slab systems, in order to comment on the modifications in mechanical action that are produced by the particular modifications of the arrangement of the reinforcement in these systems.

Fig. 1, page 2 represents the section of a standard mushroom head by a vertical plane thru the axis of the column. In this the elbow rods are shown, the vertical portions of which are embedded for such distances as may be necessary in the columns or are themselves column rods. One of these is represented separately at the right side of the Fig. They are confined just under the elbow at the top of the column by a steel neck band, and are bent over at the elbow to extend radially into the slab. This bent over portion is formed to scale as to length and slopes in accordance with the size and thickness of the slab in which it is to be used, in such a way that when the ring rods and four layers of slab rods rest upon it and are tied in place, the top of the upper layer will be

0.75 inch below the top of the slab at a distance of the thickness of the slab outside the edge of the cap, and at the same time the extremities of the radial rods will be 0.5 inch above the bottom of the slab. In order to accomplish this, the radial portions of these rods must be nearly horizontal over the cap, and have a suitable slope outside the cap as shown in Fig. 1.

Fig. 3, page 7, shows the ground plan of the reinforcement of the mushroom slab when the panel is square so that $L_1 = L_2 = 2a = 2b$. In this Fig. the diameter of the mushroom head is assumed to be of the extreme size $g = L/2$, a size which would increase the cantilever beyond that in usual practice to an extent not adopted except in the case of very unusual intensity of loading. It will be observed that the areas where the reinforcement consists of a single belt or layer are thereby rendered small, and the slab action due to the mutual lateral action of belts which cross each other exists over nearly the whole slab.

In Fig. 2, the dimensions of the rectangular sides are so taken that $L_1/L_2 = 0.75$, which is assumed to be the limiting or smallest value of that ratio for constructional purposes. Further, the diameter of the mushroom is made as small as will permit the reinforcing belts to cover the entire panel, viz. $g = 7\,(a + b)/16$. For example if $L_1 = 20$, and $L_2 = 15$, we have $g = 7.65+$. This may be considered to represent standard practice, where the edges of the diagonal belts intersect on the edges of the side belts. This was the case assumed for treatment in deriving the formulas of the preceeding investigations. Those formulas could be modified to apply to larger values of g, by taking lines of contra-flexure at the edges of the head nearer the panel center than given by (24), and by taking larger values of the effective cross section of steel than those employed in (32), (40) and (51).

Now it is evident that systems similar to this may differ from it in several ways:—

1st. The design of the frame-work at the top of the column may be different from this without any change in the belts of reinforcing rods. It is hardly possible for any other form of framework to be substituted for this which will exhibit the same rigidity of connection between it and the column as do the elbow rods embedded in the column and bent over radially in the slab so as to make the column and slab integral with each other by means of this common reinforcement. Any reduction of the stiffness of connection between column and frame-work of head results in increased tipping of the head under eccentric loading of the slab.

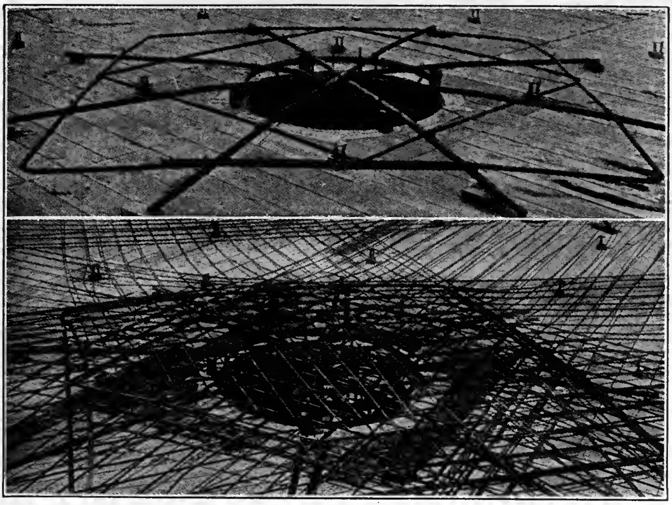

Fig. 4.

Eccentric loading is any loading of one panel differently from another. Tipping of the head increases some deflections at the expense of others, and increased stresses in some of the reinforcing rods at the expense of others, and so requires some additional reinforcement. Such a frame-work is illustrated in Fig. 4, which merely rests upon the top of the column without the support of metallic connection with the vertical column rods. It consequently affords less resistance to tipping under eccentric loads than when stiffened by such metallic connection.

2nd. The ground plan of the reinforcing belts may remain unchanged but part only of the belt rods may be carried at the top of the slab over the column head, while the rest of them are carried thru under the head at the bottom of the slab. This modification of design, when a sufficient number of rods go over the head to resist the negative bending moments there, is very uneconomical of steel, because in the case where they all go over the head, it is the fact that altho the mean tension of the steel is not so great as at mid span, nevertheless, by reason of the overlapping of the belts in crossing, the stresses in the rods at the top reach a value not much less than at mid span, and cannot be safely diminished in number. It thus appears that the rods carried thru on the bottom are largely superfluous. Of these two mats of rods at top and bottom, one of them is necessarily in tension and the other in compression. But it is a mistake to use steel to resist compression when concrete can be better used for this purpose. The lower mat is superfluous for this reason.

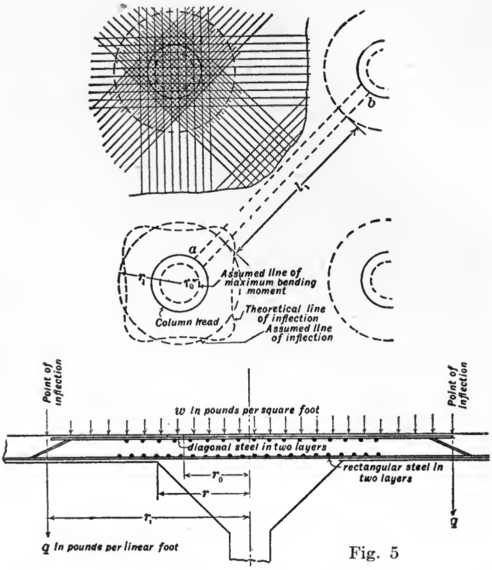

Fig. 5

There is still another and, if possible, more serious objection to this arrangement of rods to form a mat or double layer of rods at the top and at the bottom of the slab near the columns. This is because they are too far removed from each other in the slab for the elongations of the steel in one mat to be resisted by lateral contractions in the other. The reinforcement does not therefore conspire to produce the slab action expressed by Poisson's ratio, which requires that the interacting steel concerned should lie approximately in the same zone or level.

This arrangement is illustrated in Fig. 5, copied from Taylor and Thompson's Concrete Plain and Reinforced, p. 484. In this design the size of the head is small enough to reduce the width of the belts so greatly that not only are the areas where we have a single layer of rods on the plan much enlarged, but we find that nowhere do more than two layers lie in metallic contact with each other, and the areas where even this occurs are limited to one relatively small square over each column, and one of equal size at the middle of each panel. The remaining areas are subject to the law of single rod reinforcement, where we must assume lateral action to be such as greatly to diminish K for the combination, a fact very injurious to the efficiency of the reinforcement. This as has been said, is due partly to the smallness of the head and partly to the separation of the layers between the top and the bottom of the slab.

3rd. Another modification of design without change of ground plan is that where the rods that are carried over the head at the top of the slab are given a sudden steep dip at the line of contraflexure to carry them to the bottom of the slab at that line. This is also illustrated in Fig. 5. Such sudden bends or kinks anywhere in the rods may give rise to very serious fractures because of straightening out under tension, especially when the forms are removed. Such bends give rise to great differences of stress in the extreme fibers of the rods, thus diminishing their resistance also. All sudden bends in rods embedded in concrete should be sedulously avoided as tending very effectively to crack the concrete, whether the rods are part of the belts or in the frame-work of the head, as shown in Fig. 3, in which are many such angles and elbows unsupported except by concrete, and therefore objectionable.

It seems fair to conclude that the cracks shown in the plan of the floor of the Deere & Webber Company Building, Minneapolis, tested by Mr. Arthur R. Lord, and occuring along the edges of some of the loaded panels at the upper surface, where none usually appear, were due to the elbows in the frame work of the head, like that in Fig. 4, in conjunction with the comparatively small resistance to bending in a vertical plane offered by the rods forming this projecting elbow.

In the mushroom head the only bend permitted is that at the elbow of the radial rods where a strong steel neck band prevents any such bad effect as has just been pointed out.

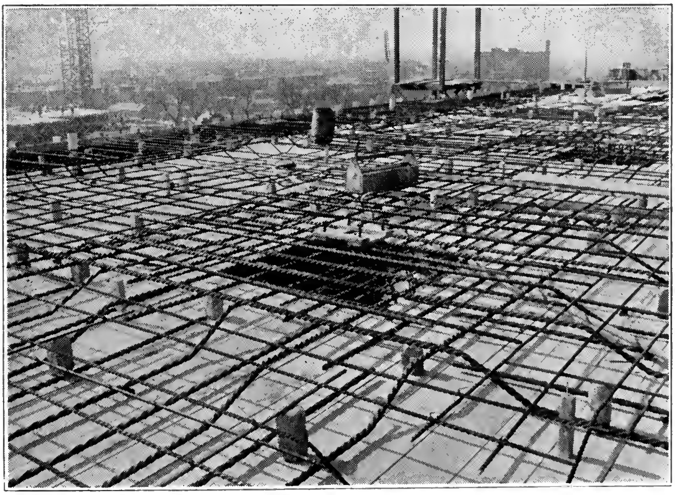

Fig. 6

4th. We may notice a form of design in which the diagonal belts are omitted and the entire panel is covered by rods parallel to the sides of the panel. This, while apparently very different in ground plan from those just considered does not differ from it materially in principle. It is clear that the lattice pattern of the web in this case is in many parts of the panel not woven so close as where diagonals exist, while in other parts of the mesh the number of layers in contact with each other has been decreased. Experimental results do not as yet enable us to determine with certainty whether Poisson's ratio for this combination is as great as for the mushroom. Upon that depends in part the relative efficiency of the two arrangements. A form of this design is seen in Fig. 6.

The maximum deflections at the center of a loaded panel of the system of Fig. 6, would occur when the panels touching its four sides were also loaded. In this particular it differs from a loaded panel in a mushroom slab which would theoretically have its deflection slightly decreased by loading surrounding panels, tho this is too insignificant to have been observed as yet.

Deflections shown by tests of this system of two way reinforcement are wholly inconsistent with simple beam theory, and can only be explained on the basis of slab theory. Nevertheless, some of its advocates attempt to design its reinforcement and compute its strength on the basis of beam theory, which actual deflections show to be untenable. Such attempts should be entirely abandoned as erroneous and misleading.

All considerations which have been discussed under the three previous counts are to be taken as applying equally to this plan of arranging the reinforcing rods, especially as to carrying of part of the belts thru on the bottom surface at columns.

5th. Another element of design is the relative number of rods in the side and diagonal belts. We have previously adduced reasons to show that in a square panel the same number of rods is required ultimately in the diagonal belts as in the side belts, tho for stresses less than the yield point of the steel, it would be possible to diminish the number of rods in the diagonal belts somewhat. Equation (34) shows that for equal stresses in the steel of the side belts the number of rods should have the same ratio as the lengths of the sides.

A different rule from this has been erroneously proposed, viz., that the ratio of the number of rods in the side belts should be equal to the ratio of the cubes of their lengths. The only foundation for this rule is that according to the beam strip theory as developed in Marsh's Reinforced Concrete, p. 283, a rectangular plate carried by a level rigid support around its perimeter, would divide the load per unit of area which is carried by two unit-wide rectangular strips that cross each other, as the fourth power of their lengths, and hence would carry to the edges of the rectangle loads proportional to the cubes of the lengths of those edges. Were this so, the case of a horizontal rigid support around the entire perimeter of the panel is wholly different from support on columns at the corners, and such a rule would be wholly inapplicable therefore to a floor slab so supported. This rule was, however, evidently adopted in the design of the Larkin Building, Chicago, as shown by a photograph of its reinforcement in place before the concrete was poured, to which the writer has access and published in Cement Era for February, 1913. The very exhaustive tests of this building made by the Concrete Steel Products Company of Chicago, and published in the Cement Era, for January 1913, show that this ratio of rods caused the stresses for the larger loads to be more than twice as great at the middle of the short side belts as at the middle of the long side belts. This was assuredly an uneconomical distribution of steel, since correct design would require these stresses to be equal, when in fact one exceeded the other by 120 to 140 per cent. This discrepancy would be largely rectified by making the number of rods directly proportional to the lengths of the sides, as required by (34).

It also appears that the diameter of the mushroom head and the width of belts of slab rods in the Larkin Building is less than the limiting size in the standard mushroom system, viz. $g = 7(a+b)/16$. This makes the intersection of the diagonal belts fall nearer the center of the panel than the edges of the side belts. The very considerable effect of a very inconsiderable change of this width has been mentioned on p. 25. The result would be that the steel would for this reason be far less effective, and its resistance would be more nearly in accordance with (37) than with (34), a loss of perhaps 25 to 30% in its effectiveness.

16. This section will be devoted to a specimen computation applying several of the preceeding formulas to a floor slab of practically the same dimensions and reinforcement as one or two recently designed and now under construction (1913).

Long Side $L_1 = 28' \times 12 = 336''$.

Short side $L_2 = 25'\ 10'' = 310''$.

Thickness of rough slab, $h = 10'' = L/33.6$.

By (56) $C_2 = L_2/L_1 = 0.9$ nearly.

Diameter of head $g = 7\ (L_1 + L_2)/32 = 141''$.

Diameter of cap $L_1 — B = 0.2L_1 = 67''$. $B = 0.8L_1 = 268.8''$.

Each belt has 25 — $7/16''$ round rods.

Cross section of each belt, $A = 25 \text{ x } 0.15+ = 3.76$ sq. inches.

Depth of center of mid side belt with $\frac{1}{2}$ inch concrete covering, $d_1 = 10 — 0.5 — 0.2 = 9.3''$.

Depth of center of second layer slab rods at panel center, $d_2 = 10 — 0.5 — 0.64 = 8.86''$

Depth of bottom surface below third layer of slab rods at edge of cap with $\frac{3}{4}''$ covering, $d_3 = 10 — 0.75 — 1.1 = 8.15''$.

Design load per square foot = 150 lbs.

Dead load per square foot = 130 lbs.

Panel load, $W = 280 \text{ x } 28 \text{ x } 25\ 5/6 = 202{,}550$ lbs.

A maximum tension is found in the slab rods at the middle of the long side belt, and is to be computed from (34) as follows:

$$f_s = \frac{202550 \text{ x } 336}{175 \text{ x } 9.3 \text{ x } 3.76} = 11{,}120 \text{ lbs. per sq. inch.....(80)}$$

Any other loading within elastic limits of the steel would produce proportionate stresses.

The tension in the steel at the center of the panel is computed by (52), as follows:

$$f_s = \frac{1.02 \text{ x } 202550 \text{ x } 336}{256 \text{ x } 3.76 \text{ x } 0.89 \text{ x } 8.86} = 9{,}145 \text{ lbs. per sq. in..(81)}$$

The radial tension at the edge of the cap is by (43),

$$f_s = \frac{202550 \text{ x } 336 \text{ x } 646\ (3 \text{ x } 0.64 — 1)}{800 \text{ x } 8.15 \text{ x } 3.76 \text{ x } 310} = 5320 \text{ lbs. per sq.in. (82)}$$

The circumferential tension at the vertical section thru the center of the column at the end of the long side may be computed by placing $B = L_1$ in (43), and we obtain,

$$f_s = \frac{202550 \times 336 \times 646 \times 2}{800 \times 8.15 \times 3.76 \times 310} = 11{,}570 \text{ lbs. per sq. in.} \quad (83)$$

as the mean computed intensity of stress in each of these rods, regardless of its distance from the center of the column. This stress may be reduced by increasing the number of laps over the head. The result in (83) is, however, an over-estimate of the tension across the top of the head because the head is integral with the cap of the column where compressions in the concrete are no longer confined merely to the thickness of the slab but take in a much greater depth of concrete in the cap. This in effect puts the neutral surface at a lower level throughout the cap and by thus increasing the lever arm of the reinforcement reduces its tension and deformation. This will react upon the rest of the reinforcement in such a manner as practically to make the stresses smaller than given by (83) because the mean lever arm will have increased. In fact the greatest stress in these rods will be that given by (80), instead of (83).

The compression in the concrete lengthwise of the longer side belt at its middle is to be computed from (38) and (80) as follows: By taking the percentage of belt reinforcement at 0.3%, the corresponding value of $i = 0.72$, and $E_s/E_c = 15$:

$$f_c = \frac{0.28 \times 11120}{0.72 \times 15} = 288 \text{ lbs. per sq. in.} \quad (84)$$

The compression at the center of the panel where the percentage of slab reinforcement may be conservatively assumed at 0.6% and $i = 0.66$ may be computed thus:

$$f_c = \frac{9145}{2 \times 75} = 305 \text{ lbs. per sq. in.} \quad (85)$$

The compression at the edge of the cap lengthwise of the side belt is uncertain in the absence of exact information as to the laps in the slab rods over the head. Assume that one-half the rods are lapped over each head, and that we take six belts as the reinforcement of the slab, the percentage then is 1.8% and $i = \frac{1}{2}$, then,

$$f_c = \frac{5{,}320}{15} = 355 \text{ lbs. per sq. in.} \quad (86)$$

For reasons already given in discussing the circumferential tensions in the head, it appears that any computation of the circumferential compressions in the concrete on the basis of (38) would be incorrect and subject to large errors of as much possibly as 50%. That this is the fact appears evident when we consider the large mass of concrete in the cap which must be actually diminished in lateral dimensions before the slab which is integral with it can be subjected to true stresses of equal intensity, and consider also that near the edges of the head the radial rods and the outer ring rods approach the lower surface sufficiently to afford reinforcement to resist compression. It is consequently unnecessary to look further than (86) in computing the greatest compression in the concrete.

As previously stated, computations based on (38) are highly artificial and arbitrary in their character, since they assume the straight line theory as well as an arbitrary value of the ratio of Young's moduli for steel and concrete. Furthermore, concrete in compression in both circumferential and radial directions at the same time, as it is at the edge of the cap, is known to resist with safety compressive stresses of greater intensity than when in simple compression in one direction.

If a test load of twice the design load, viz., in this case of 300 lbs. per square foot, be placed upon the slab, the deflections which will be produced by the addition of this total load of 217,000 lbs. may be computed as follows:

$$\text{By (54),} \quad \triangle z_1 = \frac{217000 \text{ x } 336^3}{10.7 \text{ x } 10^{10} \text{ x } 9.3^2 \text{ x } 3.76} = 0.237 \ldots. (87)$$

$$\text{By (55),} \quad \triangle z_2 = \frac{0.89 \text{ x } 217000 \text{ x } 336^3}{6.56 \text{ x } 10^{10} \text{ x } 8.86^2 \text{ x } 3.76} = 0.378 \ldots. (88)$$

$$\text{By (58),} \quad \triangle z_3 = \frac{217000 \text{ x } 336^3 \text{ x } 2.084}{60 \text{ x } 10^{10} \text{ x } 8.15^2 \text{ x } 3.76} = 0.115 \ldots. (89)$$

$$\text{By (60),} \quad \triangle z_4 = \frac{0.89 \text{ x } 217000 \text{ x } 336^3}{12.5 \text{ x } 10^{10} \text{ x} 8.15^2 \text{ x } 3.76} = 0.235 \ldots. (90)$$

By (61), $D_1 = 0.352$, and $D_2 = 0.613$. . . . . (91)

$$\frac{D_1}{L_1} = \frac{1}{960}, \text{ and } \frac{D_2}{\sqrt{L_1^2 + L_2^2}} = \frac{1}{745} \quad \text{. . . . . (92)}$$

Any loading differing from this would produce deflections proportionate to its intensity.

In this specimen floor slab, which is near the limit of least thickness permissible in the standard mushroom system, viz., $d = L_1/35$, it is clear that the design load brings stresses to bear upon its reinforcement which are very moderate in their intensity indeed. It is also evident that were the slab to be loaded with a test load of such amount that the total load sustained would be twice the dead load of the slab itself plus twice the design or live load, viz. 560 lbs. per square foot, none of the steel would be stressed up to the yield point, and the first failure would take place by cracking the concrete, tho the steel would still prevent sudden failure and collapse. Altho the slab is relatively so thin the deflections are also very small for so large a span.

It has not yet been so generally recognized as it should be that a thin construction, such as a flat slab is, should not be expected to show so small proportionate deflections as is required in girders.

The observed results of quite a number of tests of mushroom slab floors are to be found on pp. 32 and 44 of Turner's Concrete Steel Construction. These are there compared with results computed according to Turner's empirical formula, which translated into our present notation has been reproduced in equation (72). The observed and computed results show a very close agreement. The results given by (72) are in close agreement, as has been seen, with those derived from (61).

Some of these test slabs present peculiarities of reinforcement such as need to be individually considered in order to make exact computations of their deflections. It is thought that the specimen computation already given will afford sufficiently guidance in the methods to be employed.

Having considered the stresses and deflections of a slab which is near the minimum thickness for the standard mushroom system, viz. $L_1/35$, it will be instructive to consider a specimen or two near the maximum thickness $L_1/20$.

Tischers Creek Bridge, Duluth

Test of Tischers Creek Bridge with 30 ton construction cars, each loaded with 20 tons of rails
Deflection less than one twenty thousandth part of the span

Take for example the bridge over Tischer's Creek, Duluth, shown in the cuts on page VII and page 66. It is supported on three rows of columns crossing the gorge, at a distance apart of 27 feet center to center of columns, the two street car tracks being over the side belt that lies along the center line of the bridge lengthwise. Each of these rows consist of six columns lengthwise of the bridge, at a distance apart of 26 feet from center to center, so that

$$L_1 = 27 \text{ x } 12 = 324''$$
$$L_2 = 26 \text{ x } 12 = 312''$$

The size of the mushroom heads and width of the belts is 12 feet, which is in excess of $7\ (L_1 + L_2)/32 = 139\ 1/8'' = 11.6'$, thus giving great stiffness. The object to be obtained by maximum thickness and large head is to secure great stiffness and so reduce vibrations as well as decrease deflections. There are twenty 9/16 inch round slab rods in each belt, or a total cross section in each belt of $A_1 = 5$ square inches of metal. The slab is 15″ deep at its thinnest part at the gutter on each side of the roadway, and the steel is kept down to that level throughout the slab, altho at the crown of the roadway under the tracks and over the center row of columns the slab is 5″ thicker, or 20″, with the same thickness over the side rows of columns where the sidewalks are. The mean thickness is somewhat in excess of $L_2/20$. This makes $d_1 = 19''$ for the short side belts, $d_1 = 17''$ for the long side belts and $d_3 = 14''$ approximately for the heads. The design load per square foot = 150 pounds. The dead load of the slab per square foot = 300 pounds. Hence $W = 450 \text{ x } 26 \text{ x } 27 = 315{,}900$ pounds. The effective cross section of slab steel is so great by reason of large heads that instead of (34) we may take

$$f_s = \frac{W\ L}{200\ d_1\ A_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)'$$

For the long side belt this gives $f_s = 6{,}033$ pounds per square inch. The total load imposed on the slab might be made six times as great without causing the steel to reach its yield point, and the live load might become 900 pounds per square foot without causing f_s to exceed 16,000 pounds.

This slab was tested as shown in the cut, page 66, by running two construction cars loaded with 20 tons of rails each over the bridge at the same time along one track of the short side belt 26 feet long. Weight of each car = 60,000 pounds. Weight of rails 40,000 pounds. Total weight of train = 200,000 pounds extending over several spans. The deflections was too small to be discovered by observations with level and rod. It is useless to attempt to compute the deflection of this slab under the test load because the four steel rails of the railway tracks across the bridge were so fastened to the steel cross ties which were embedded in the concrete as to make the rails a part of the reinforcement of the slab. They furnish a cross section of reinforcement equal perhaps to $7A_1$, which would effectually bar the application of our deflection formulas and reduce deflections to very small quantities.

In so thick a slab as this the action of any contemplated load is widely distributed by the slab itself, and such loads, as well as all shocks and vibrations are largely dissipated or absorbed by the body of slab itself without causing observable local stresses as they do in steel structures.

VIEW OF REINFORCING STEEL

Flat Slab Bridge, Denver, Colo. Spans 43 ft. 6 in. Carries Heavy Interurban Cars

The Curtis Street bridge, Denver, Colorado, is one of four bridges across Cherry Creek, shown by the cut on page 68, constructed on the mushroom system It has three rows of three columns each crossing the stream, the middle column of each row in mid stream with spans of 42 feet between columns centers lengthwise of the bridge, thus obstructing the waterway as little as possible. It has a width of 28 feet between column centers. The slab is 17 inches thick at the gutters, 26.5 inches at the sidewalks outside the gutters, and 21″ over the center row of columns. The sidewalk is stiffened with fourteen 3/8″ round rods lengthwise just below its top surface as supplementary reinforcement, and there is an outside parapet giving added stiffness. There are also three stiffening rods 24″ apart across the bridge midway between columns. There are three ring rods, and the width of the belts is 16′. This is in excess of $7\ (L_1 + L_2)/32 = 183.75'' = 15\ 5/16'$. The heads are exceptionally stiff each having twelve 1 3-8″ round radial rods. Each belt has twenty-six 5/8″ round rods, hence $A_1 = 26 \text{ x } 0.3 = 8$ square inches nearly.

$L_1 = 42 \text{ x } 12 = 504''$, $L_2 = 28 \text{ x } 12 = 336''$.

The dead load per square foot = 300 pounds.

The design load per square foot = 150 pounds.

$W = 450 \text{ x } 42 \text{ x } 28 = 529{,}200$ pounds.

$d_1 = 20''$ for long side belt.

Compute the stress in the steel by (34) modified to (34)′ by reason of exceptional stiffness, and we obtain $f_s = 13{,}320$ pounds.

Compute the central deflection due to a test load of 100 pounds per square foot. Let $d_3 = 16''$. Then in (71) $L_2/L_1 = 2/3$: hence $C_2 = 3/4$, and we have $D_2 = 0.125''$. This is probably considerably in excess of the correct deflection, since the slab is stiffer than the one considered in equation (71), which was derived for 20 foot spans. More correct values are to be computed from (54), (58) and (61). Moreover for such comparatively light stresses in the concrete, the deflections, as we have seen previously fall short of those computed by the formula, which agrees with experiment for stresses nearer the yield point of the steel. $D_2 = 0.125''$ is less than one four-thousandth of the span, and the deflection under the working load would undoubtedly be less than one sixth-thousandth of the span.

A word is here in place respecting working stresses and the factor of safety in the reinforcement of slabs, to the effect that the same values of these quantities in slabs affords a greater degree of security than in ordinary structural steel construction, and that occurs for several reasons:

1st. Steel rods such as are used in slabs have a higher yield point by perhaps 25% than the steel of other structural members. Furthermore, it is quite possible and desirable to use a higher carbon steel for these rods than the mild steel necessarily used in structural work, where it must be manipulated in such ways that high carbon steel cannot be used. But in these rods which suffer no usage tending to impair their condition, there is good reason to use a steel of higher yield point and greater ultimate strength. This yield point may readily be 70% greater than that of ordinary mild steel for structural purposes.

2nd. Rods embedded in concrete do not yield as do bare single rods in a testing machine or elsewhere by the formation of a neck and drawing out at that point. The concrete embedment prevents that.

3rd. In a reinforcement consisting of multiple parallel rods acting together, no single rod can become overstrained and yield to any appreciable extent before bringing into play adjacent rods. This makes the construction tough, and not liable to sudden collapse, as well as obviates concentration of stresses thus ensuring a high degree of security.

17. This section will be devoted to a detailed consideration of a test to destruction of two slabs, 12′ x 12′ between column centers, constructed for experimental purposes. The tests were made by Professor Wm. H. Kavanaugh, in November and December, 1912, and the results he obtained, together with a mathematical discussion based upon them, will be here given. One slab was constructed in accordance with the plans and specifications of the U. S. Patent No. 698,542 issued to O. W. Norcross for a slab for flooring of buildings, and the other was a Turner Mushroom slab under U. S. Patent No. 1,003,384. The test serves to bring out in a striking manner not only how two slabs, which present a superficial resemblance in the plan of arrangement of reinforcement, differ from an experimental and practical standpoint, but it also makes evident their radical divergence of action mechanically and mathematically.

That two slabs of the same span, thickness and amount of reinforcement should on test show that one of them was more than twenty times as stiff, and more than five times as strong as the other, and that the failure of the weaker one was a sudden and complete collapse, with little or no warning to the inexperienced eye, while the other gave way by slowly pulling apart little by little, thus gradually getting out of shape without any final break down, are phenomena that deserve the close attention of the designer, and are of the highest interest scientifically as well as practically. The enormous differences in the deflections and in the stresses in the reinforcement as shown by extensomoter measurements, and in the character of the failure in respect of safety and its relation to the line or zone of weakest section, as well as in the difference of design loads and breaking loads amounting to 500%, all illustrate what scientific design will accomplish and what results are possible by an ingenious arrangement of the reinforcement.

These slabs were each of the same thickness, viz 6″, and were supported by columns placed at the corners of a square 12′ x 12′ from center to center of columns. The slabs projected 2′ to 3′ beyond the centers of the columns on each side, and had precisely the same number and size of reinforcing rods in each belt, viz eleven 3/8 inch round rods. The concrete was of a 1 : 2 : 4 mix, and while only about four weeks old at the time of the test, it had been poured warm and kept warm by steam heat under such unusually favorable conditions as to have become well cured at the time of the test. The steel used showed by test a stress at yield point of 51,000 to 55,000 pounds per square inch, and an ultimate strength of 76,000 to 80,000 pounds, with an elongation of twenty to twenty-five per cent.

The first slab was made in accordance with the specifications of the Norcross patent already referred to except that belts of rods were substituted for the netting mentioned by the patentee. This design was selected as one of the two for this comparative test, not because it is a good design, or one that any engineer would to-day care to employ, but because it exhibits, according to the express intention of the patentee, simple tension on its lower surface, everywhere between columns, and simple compression everywhere on its upper surface between columns; this being in direct contrast to the other design, which is arranged not only to resist direct tensions over the supports, which the first does not, but also to resist circumferential stresses both around the supports and around the panel centers, as any truly continuous flat slab must.

This test may then be viewed in the light of an experimental demonstration of the difference between a reinforced flat slab constructed in accordance with the beam theory and one constructed in accordance with correct slab theory, where true and apparent moments differ radically as shown at the beginning of this investigation, but are wholly contradictory to any form of simple or continous beam theory. This test may be regarded as settling once for all the question of applying simple beam theory to a cantilever flat slab, reinforced throughout practically its entire area with a lattice of rods crossing each other and in contact. It shows that it is impossible to compute the deflections of such a slab by beam theory. Furthermore this impossibility makes it certain that the stresses in such a slab cannot be computed by beam theory, for to do this is to commit an inconsistency such as has heretofore too often been committed, but one which should hereafter be carefully avoided.

Norcross in his patent already referred to describes his construction as consisting "essentially, of a panel of concrete having metallic network encased therein, so as to radiate from the posts on which the floor rests....... The posts are first erected, and a temporary staging built up level with the tops of posts. Strips of wire netting are then laid loosely in place on top of the staging.... The concrete is then spread upon or moulded in place on the staging to enclose the metallic network. In practice I have sometimes laid the concrete in layers of different quality, the lower layer of the floor which encloses the wire being laid with the best concrete available....... If the forces acting upon a section of flooring supported between two posts be analyzed it will be found that the tendency of the floor section to sag between its supports will cause the lower layers of the flooring to be under tension while the upper layers of the flooring will be under compression, these stresses being, of course, the greatest at the top and bottom layers, respectively."

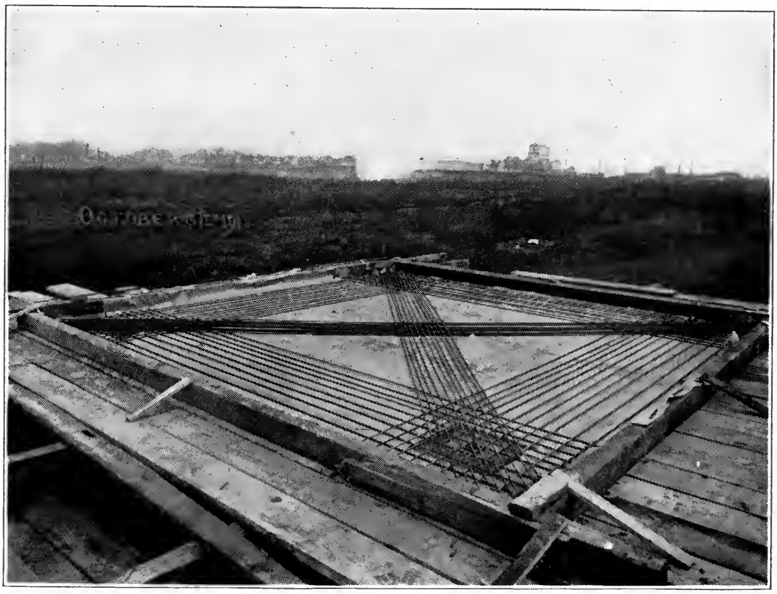

Fig. 7. Reinforcement of Norcross Slab

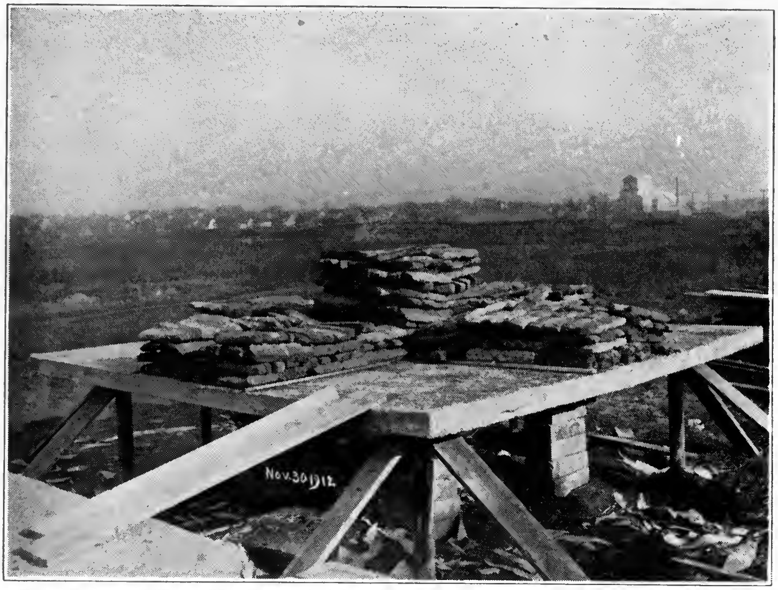

Fig. 8. Norcross Slab Carrying Load 3

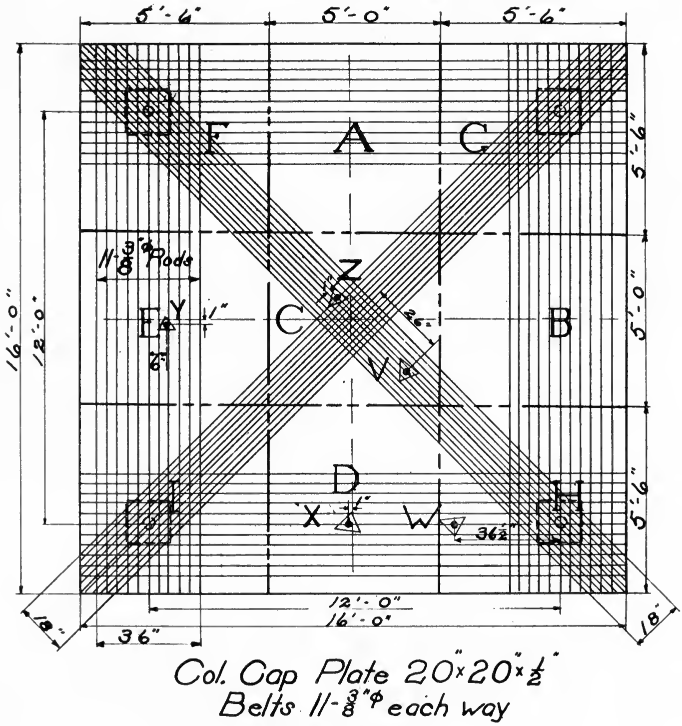

Fig. 9. Norcross Slab

Table 1. Loads on Norcross slab in pounds

No.	1		2		3		4	
Areas	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total
A	114.1	3138	228.7	6288	228.7	6288	228.7	6288
B	114.1	3138	228.7	6288	228.7	6288	228.7	6288
C	114.1	2852	240.1	6002	456.8	11420	456.8	11420
D	114.1	3138	228.7	6288	228.7	6288	228.7	6288
E	114.1	3138	228.7	6288	228.7	6288	228.7	6288
F							250	7560
G			Corner Areas Not Loaded.				250	7560
H							250	7560
I							250	7560
Slab	60	15404	121	31154	143	36572	276	66812

The number and arrangement of the reinforcing rods in the Norcross experimental slab, (eleven 3/8″ round rods in each side and diagonal belt) is clearly shown in the view of Oct. 31st, Fig. 7, which shows the forms ready for pouring the concrete. Steel plates 20″ x 20″ x 0.5″ carry the rods and rest on the tops of the columns, which last in this case consisted of steel pipes about $5\frac{1}{2}''$ in diameter filled with concrete and embedded at their lower ends in large concrete blocks. A vertical central bolt in the concrete at the upper end of each pipe permitted the plates to be firmly secured to the tops of the columns. The view of Nov. 30th, Fig. 8, clearly shows the manner of placing the pig iron on the slab for load 3. This slab is 16′ x 16′. The loading at first covered an area having the form of a Greek cross whose central square was five feet on a side with arms 5′ 6″ long, as represented in accompanying diagram of loaded areas A, B, C, D, E, Fig. 9, and of amounts shown in Table 1.

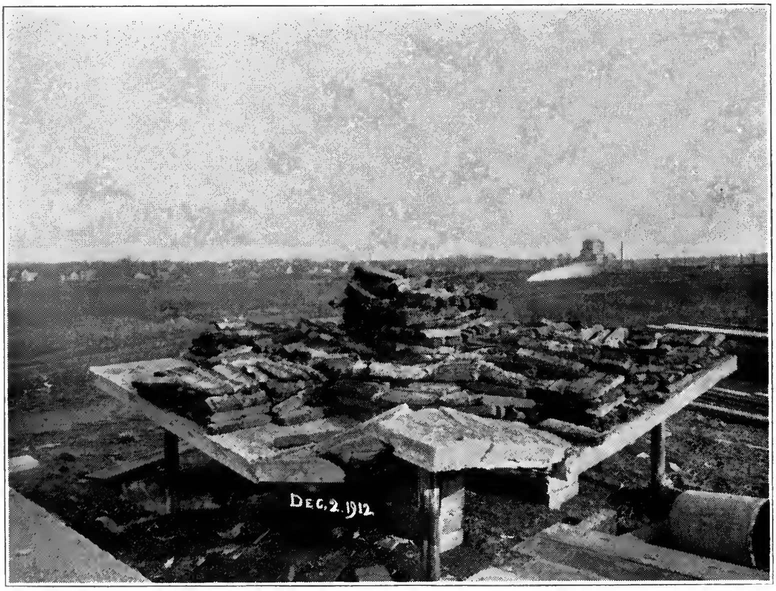

Fig. 10. Collapse of Norcross Slab

When 10,000 pounds had been piled on the central part of the slab in addition to load No. 4, of 66,812 pounds, the slab suddenly failed. In anticipation of such failure timber blocking had been placed under the slab to prevent its falling more than possibly ten or twelve inches.

Fig. 11. Collapse of Norcross Slab

The two views of Dec. 2d, Fig. 10 and Fig. 11, show the condition of the slab after removing part of the final loading in order to render the nature of the failure visible. Careful extensometer measurements of the elongations of the steel rods at the middle of the side and diagonal belts were made under the action of loads 1, 2, 3 and 4, and also similar extensometer measurements in the concrete both on the top and the bottom of the slab along the center line of the side and diagonal belts near those edges of two of the steel plates which were nearest the center of the belts. Besides these, certain other measurements of the concrete were made at right angles to the diagonals. Deflections were also measured under these loads at the middle of the diagonal belt and of two of the side belts at V, W, X, Y, Z.

These measurements all show beyond question that the side and diagonal belts act like simple beams in this form of construction, since the stresses in the steel and concrete on the under side of the slab in the direction of the rods is invariably tensile, while the stresses in the same directions on top of the slab are always compressive. It was the avowed intention of Norcross to reinforce the slab in this manner since he regarded the upper part of the slab as being subjected everywhere to compression and the lower part to tension only, as stated in his specifications as already quoted.

The following computation, Table 2, shows a good approximate agreement of the results of this test with the beam theory of flexure, assuming for simplicity that the stiff steel supporting plate and interlacing of the ends of the belts diminishes the effective span of the side belts by 12″, and the diagonals in the same proportion, and further assuming that the loading was all applied at the middle of the side and diagonal belts.

The extensometer measurements made were for a length of 8″, consequently the stress in the steel per square inch would be computed thus:

$f_s = 1/8$ (elongation in 8″) x 30,000,000;.......$(1)_1$

and, this being known from observation, it will be possible to compute the load W carried by the beam in which the given elongation occurs, as follows:

The bending moment due to a concentrated load W at the middle of a beam of length L is $M = \frac{1}{4}\ W\ L$,.................$(2)_1$ and the equal moment of resistance of the reinforcement by which it is held in equilibrium is $M = A\ j\ d\ f_s$..................$(3)_1$ in which A is the total cross section of the steel in the belt = 11 x 0.11 = 1.215 sq. in., and the distance from the center of the steel to the center of compressive resistance of the concrete is assumed to be, $j\ d$ = 0.9 x 5.75

when d = 5.75 is taken as the distance from the center of action of the steel to the top of the slab,

Hence $W = 4\ A\ j\ d\ f_s/L$......................$(4)_1$

is the load required to cause the stress f_s in the steel. In the side belts we assume the span L to be 132″, and in the diagonals $132\sqrt{2}$.

In Table 2, which follows, it will be noticed that loading No. 1 is too small to develop sufficient elongations or deflections to overcome the initial compressions in the concrete in which the reinforcement is embedded, so that the load carried by the steel is only about one half of the actual load, the other half being evidently carried by the concrete in which it is embedded. This is in complete accord with other similar experiments. But in case of loads No. 2 and No. 3, where the steel is stressed close to the yield point, the sum of the loads as shown by the stresses in the steel is very close to the total actual load. It is assumed that these total actual loads are carried by the various belts in the same proportion as the computed loads, since there is no other way of dividing the total load between the belts. This may be stated mathematically, as follows:

Table 2, Loads and deflections of the Norcross Slab computed on the simple beam theory from the observed enlongations of the steel reinforcement, compared with the observed loads and deflections.

	Load No.	1	2	3	4
Side Belts	Average observed elongation in 8″	.00205	.00746	.00922	.01133
	f_s in pounds by $(1)_1$	7690	27980	34575	42500
	W_1 computed by $(4)_1$	1465	5330	6590	....
Diag. Belts	Average observed elongation in 8″	.00195	.00722	.00939	.01229
	f_s in pounds by $(1)_1$	7313	27070	35200	45900
	W_2 computed by $(4)_1$	985	3650	4750	....
$4W_1$	Load in 4 side belts comp. by $(4)_1$	5860	21320	26360	....
$2W_2$	Load in 2 diagonal belts comp. by $(4)_1$	1970	6700	9490	....
$4W_1 + 2W_2$	Total load on Slab Comp. by $(4)_1$	7830	28020	35850	....
$4W'_1 + 2W'_2$	Total actual load on slab observed	15404	31154	36572	....
Side Belts	W'_1 actual load by $(5)_1$	2880	5800	6690	....
	D_1 mid deflection by $(6)_1$	0.186	0.37	0.43	....
	D'_1 observed mid deflection	0.128	0.41	0.52	....
Diag. Belts.	W'_2 actual load by $(5)_1$	1940	3970	4820	....
	D_2 mid deflection by $(6)_1$	0.35	0.72	0.865	....
	D'_2 observed mid deflection	0.22	0.707	1.02	....

Let W_1 = the computed load on a side belt.
and W_2 = the computed load on a diagonal belt.
Let W_1' = the actual load on a side belt.
and W_2' = the actual load on a diagonal belt.
Then $4W_1 + 2W_2$ = total computed load on slab.
and $4W_1' + 2W_2'$ = total actual load on slab.

$$\text{Then } \frac{4\,W_1' + 2\,W_2'}{4\,W_1 + 2\,W_2} = \frac{W_1'}{W_1} = \frac{W_2'}{W_2}, \quad \ldots\ldots\ldots\ldots \quad (5)_2$$

from which W_1' and W_2' can be computed, W_1, W_2 and $4W_1' + 4W_2'$ being already known.

The stresses in the steel under load No. 4, are so far beyond the yield point as to make computation useless. Having found the actual distribution of loading W_1' and W_2' the center deflections of the belts have been computed by simple beam theory from the formula.

$$D_2 = \frac{W'\,L^3}{48\,E\,A\,i\,j\,d^2} \quad \ldots\ldots\ldots\ldots \quad (6)_1$$

in which $i\,d$ = the distance from the steel to the neutral axis and the value of j has been assumed to be 0.69; W' is the actual load on the belt and L is its span as previously stated.

It appears from Table 2, that the effect of the reinforcement is accounted for to a reasonably close approximation by considering the belts to act as a combination of simple beams, at least within the range of loading near the yield point of the steel.

It appears that the steel reached its yield point under a total load on the slab of from 15 to 18 tons and final collapse occured under a total load of a little over twice the latter amount not distributed uniformly but piled more in the general form of a pyramid.

It was observed that the application of the relatively small loading on the corner areas F, G, H, I, had a very injurious effect upon the slab, tending to break it across the tops of the columns.

The results of the test may be summarized in the Norcross system as follows:

1st. This slab is of the simple beam type, and the test shows no cantilever action and no circumferential slab action.

2nd. The narrow belts running diagonally leave large areas without reinforcement, and there is consequently no provision for resisting circumferential tensions as required in slab action.

3rd. The concrete showed compressive stresses on the upper surface of the slab in the direction of all the reinforcing rods.

4th. The concrete showed tension at the bottom surface in the direction of all the reinforcing rods, in agreement with Norcross' own analysis.

5th. This slab deflected 1.6″ under 33 tons and then broke down completely under 38 tons.

6th. The first crack appeared under a load of 15 tons and deflection of 0.7″.

7th. The slab, not being reinforced on the top surface over the columns, inevitably cracks at a column when the slab is loaded around the column.

8th. At failure the steel had passed its yield point. The percentage of reinforcement in the diagonal belt if we regard the belt as about 18″ wide is very nearly 1%, but since a width of concrete somewhat greater than that may be assumed to act with this steel, the percentage of reinforcement is somewhat less than 1%. Similarily, the side belts of width 36″ have a reinforcement less than 0.5%. The full strength of the steel in both belts was developed by the concrete, which fact demonstrates that the concrete was of high grade and well cured. The steel was also of good standard quality, and the test was therefore in every way fair to the Norcross slab, since it was so loaded as to cause the stresses in the side and diagonal belts to be practically equal, thus using the steel most economically. The slab failed because the steel yielded near the middle of the spans, thus causing the concrete above the steel to crack and break.

The second slab was made according to the Turner Mushroom System, under the patent already referred to.

Since all forces in a plane may be resolved into components along any pair of axes at right angles to each other it is possible to provide reinforcement to resist any horizontal tensile stresses in the slab by various arrangements of intersecting belts of rods at zones where these stresses occur. The combination of such belts with radial and ring rods to constitute a large and substantial cantilever mushroom head at the top of each column affords a very effective and economical arrangement for controlling the distribution of the stresses in the slab, and it places the reinforcement where it is most needed. It not only has the same kind of advantage that the continuous cantilever beam has over the simple girder for long spans, but combines with it the kind of superiority that the dome has over the simple arch by reason of circumferential stresses called into play, which greatly adds to the carrying capacity of the slab.

Fig. 12. Reinforcement of Mushroom Slab

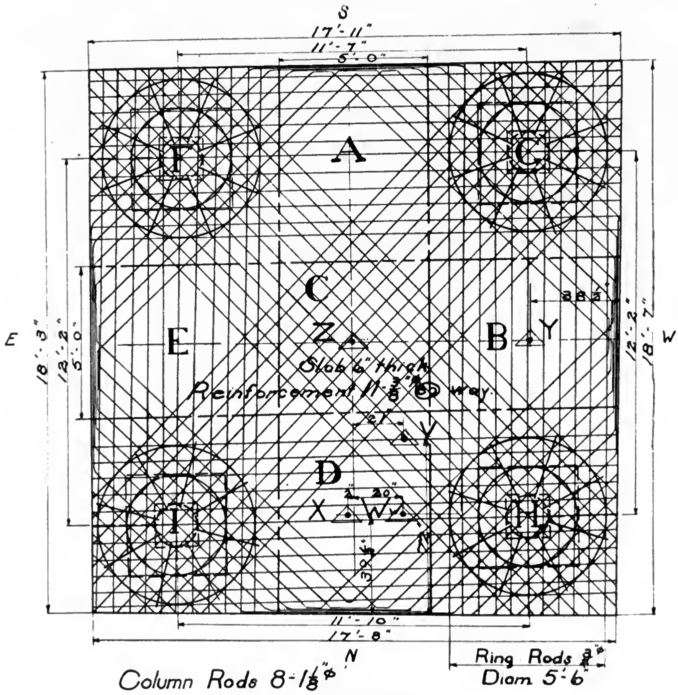

Fig. 13. Mushroom Slab

The mushroom test slab was six inches thick, and was supported on four 18″ by 18″ square reinforced concrete columns distance 12′ from center to center. These had square capitals, 42″ x 42″. The slab was appromimately 18′ x 18′, and the diameter of the outer ring rod of the Mushroom was 66″, while the inner ring was 42″. These were supported on eight 1-1/8″ round radial column rods.

Fig. 14. Mushroom Slab, Load 4.

This will be clearly understood from the view dated October 31st, Fig. 12, which shows the reinforcement and forms ready for pouring the concrete. The remaining views are explained by their accompanying legends.

The diagram of loaded areas for the mushroom slab Fig. 13, is like that already given for the Norcross slab in every particular except that the size of the mushroom slab being 18′ x 18′, while the Norcross slab was 16′ x 16′, the arms of the Greek cross in the mushroom slab are each 5′ 6″ long and 5′ wide.

Table 3. Loads on Mushroom Slab in pounds.

Load	1		2		3		4		5	
Area	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total
A	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
B	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
C	100.8	2532	201.6	5040	418.3	10458	861.8	21546	1261.8	31546
D	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
E	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
				Corner	areas not	loaded.				
Slab	48.2	15624	96.4	31248	113	36666	237	76835	348	112836

Table 3—Cont. Loads on Mushroom Slab in pounds

Load	6		7		8		9		10	
Area	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total
A	736	23922	722.9	25122	899	29222	899	29222	899	29222
B	736	23922	772.9	25122	899	29222	899	29222	899	29222
C	1480.2	37006	1569.4	39236	1889.4	47236	2689.4	67236	3329.6	83236
D	736	23922	772.9	25122	899	29222	899	29222	899	29222
E	736	23922	772.9	25122	899	29222	899	29222	250	29222
F	....	....	250	10560	250	10560	250	10560	250	10560
G	....	....	250	10560	250	10560	250	10560	250	10560
H	....	....	250	10560	250	10560	250	10560	250	10560
I	....	....	250	10560	250	10560	250	10560	250	10560
Slab	409	132695	564.5	181965	637	206365	700	226365	748	242365

Fig. 15. Mushroom Slab, Load 7.

Fig. 16. Mushroom Slab, Load 9.

The accompanying Table 3, exhibits the loads per square foot of each of the subsidiary areas shown in the diagram as also the total loads on each of those areas. The view of Dec. 3, Fig. 14, shows load 4, and that of Dec. 13, Fig. 15, load 7, while that of Dec. 16, Fig. 16, shows load 9.

Elongations of steel were measured by Berry extensometers in two of the side belts and in one of the diagonal belts until the yield point of the steel was reached at load No. 8. Deflections were also measured. In Table 4, these will be considered so far as they relate to the middle points of the belts. Loads 8, 9, 10, are of great interest as exhibiting the behavior of the slab under excessive loads, showing, as they do, yielding and large permanent deformation without dangerous collapse.

By (52) the uniformly distributed load per square foot of panel area when the stress in the diagonal belt is f_s is found for a square panel from the expression

$$144q = w = W/144 = \frac{256\, j\, d_2\, A}{144\, L} f_s \qquad \text{(52a)}$$

which applied to this slab gives us

$$w = \frac{256 \times 0.89 \times 5.125 \times 1.215}{144 \times 144} f_s = f_s/14.6 \qquad \text{(52b)}$$

The values of this uniformily distributed load w is tabulated in table 4, for each of the observed values of the f_s in the diagonal belts. The values of w so computed tend to become identical, in case of the heavier loads, with the loads per square foot on the central area C, as might reasonably be expected, w being the uniformly distributed load which is equivalent so far as the stress on the diagonal belt is concerned to the action of the actual loads which are not uniformly distributed.

How compute by (54), (55), (58), (60) and (61), the deflections at the mid side belt and at center of the panel, due to a uniform load. These results are given in Table 4, and accord closely with those actually observed, as they should, because the irregularity of distribution does not produce deflections that differ much from the equivalent uniform load as computed above.

In these computations it is assumed that $d_1 = 5.5''$, $d_2 = 5.125''$, $d_3 = 4''$

The double set of values under loads 4 and 5 is due to the fact that readings were had under load 4, immediately after the load was applied, and again 7 days later before applying load 5. The second set of readings were the larger as shown. The second set of readings under load 5, were taken four days subsequently to the first set.

It appears from Table 4, that the observed results are accounted for by the slab theory to a good degree of approximation up to the yield point of the steel.

Fig. 17. Comparative Deflections of Norcross and Mushroom Slabs.

A graphical representation of the experimental observations in the deflections at the points V, W, X, Y, Z, of the two slabs is found in Fig. 17, which shows in a striking manner how small the loads and how great the deflections were in the Norcross slab on the one hand, and how large the loads and how small the deflections were in the mushroom slab on the other hand.

Table 4, Loads and Deflections of the Mushroom Slab computed on the Slab Theory from observed Enlongations of the Steel Reinforcement, compared with the Observed Loads and Deflections.

	Load No.	1	2	3	4	5	6	7
Diag. Belt	Obs. elong. of mid rod in 8″......	.00026	.00058	.00071	.00140 .00270	.00373 .00610	.00628	.00708
	f_s by $(1)_2$........	975	2175	2662	5250 10120	14000 22900	23550	26550
	w the effective load per sq.ft. by (52b)	65.5	149	182	356 692	958 1550	1610	1816
	D_2 center deflection comp. by (61)	.0206	.046	.057	.111 .215	.300 .488	.50	.573
	D_2' obs. mid deflection........	.0170	.0353	.0463	.121 .251	.305 .408	.439	.517
Side Belts	Ave. obs. elong. in 8″............	.00017	.00031	.00039	.00126 .00272	.00401 .00607	.00681	.00758
	f_s by $(1)_1$.......	637.5	1160	1460	4725 10200	15000 12762	25500	28400
	D_1 mid deflection computed by (61)	.0095	.0215	.027	.052 .111	.140 .228	.231	.268
	D_1' mean of obs. deflection.......	.0100	.0202	.0250	.0653 .155	.173 .241	.256	.303

It will be seen from Tables 1 and 3, that the first three loads were practically the same for both slabs. In the Norcross slab load 3, of 18 tons, stressed the steel up to the yield point, but in the mushroom slab the stress was so small, (being in fact less than ten per cent of the former) as probably not to remove all the compression from the concrete in which it was embedded. Indeed the load on the latter slab became five times as much, 90 tons, before its steel approached the yield point, at which time it was carrying about twice the load which caused the complete failure of the Norcross slab.

Moreover the deflection of the Norcross slab under load 3, was twenty-two times that of the mushroom slab under the same load. This result is in full accord with slab theory which shows that the central deflection of a continuous diagonal beam with fixed ends uniformly loaded with one sixth of the total load on the slab and having the same thickness and reinforcement as the diagonal belt, would have more than six times the central deflection of the slab, while the stress in its steel would be three or four times as much. This gives a measure of the effect of slab action.

By the phrase "slab action" we designate the increased strength and stiffness of the slab by reason of its resistance to circumferential stresses around the columns and around the center of the panel.

Furthermore, if this continuous beam be compared with a simple beam uniformly loaded and having the same reinforcement, the latter would have five times the deflection of the continuous beam, or thirty times that of the slab, while the stress in the steel would be one and one-half times that in the continuous beam, and six or seven times that in the slab. This last exhibits the effect of cantilever action combined with slab action.

The apparent discrepancy between the observed ratio of deflections in these two slabs of 22 and the just computed deflections of 30, is to be accounted for by the fact that the computation assumed equal spans, whereas the Norcross span was assumed to be diminished from 144″ to 132″ by the column plate. A reduction of the span of this amount will change the computed deflections in the ratio of $144^3 : 132^3 :: 30 : 23$ which is in practical agreement with the observed result of 22.

By the phrase "cantilever action" we designate the increased strength and stiffness which is due to the continuity of the beam or slab at its supports so that it is convex upwards at such points.

While the concentration of the loading toward the middle of the panel, such as was the case in this test, may prevent any precise agreement of these numerical estimates based on uniform loading with the results of the tests, they cause the general agreement shown in the tables and tend strongly to sustain our confidence in the validity of the analysis from which these concordant approximate estimates are obtained.

The amazing difference in the strength and stiffness of these two slabs, which contain practically the same amount of concrete and steel, is due to the difference of principle of their construction, which may be summarized for the mushroom system by considering its slab action and its cantilever action under the following counts, viz:

1st. Circumferential slab stresses are most economically and effectively provided for by the ring rods around the column heads.

2nd. The size of the mushroom heads is such as to make the belts so wide as to provide reinforcement over the entire area of the slab, thus securing slab action in the central part of the panel where the belts lie near the lower surface.

3rd. The reinforcing belts cover a wide zone at the top of the slab over the columns and mushroom head, which thus provides resistance to tension, and ensures effective cantilever and slab action.

4th. Concrete is thus stressed in compression at the bottom of the slab for a wide zone around the columns.

5th. Under a load equal to the breaking load of the Norcross slab, amounting to thirty-eight tons, the mushroom slab deflected at first only 1/8″, but after exposure to rain and great changes of temperature for seven days had somewhat softened the concrete the deflection increased to 1/4″.

6th. The first crack appeared underneath the edge of the slab across the side belt under load No. 5, of fifty-six tons, with a center deflection of 0.4″ and an average deflection at the middle of side belts of 0.25″.

7th. No cracks appeared on the upper side of slab at the edge, nor were any seen elsewhere, until load No. 7, of 90 tons was applied, when the yield point of the steel was evidently nearly or quite reached, giving a center deflection of 1/2″.

Fig. 18. Failure of Mushroom Slab.

Fig. 19. Failure of Mushroom Slab. Load Removed.

8th. The slab carried its final load of over 120 tons for twenty-four hours without giving way. It demonstrated the impossibility of its sudden failure by gradually yielding until it reached a final deflection of some nine inches, as seen in the views of Dec. 17th and 24th, Figs. 18 and 19.

9th. While the slab steel in each belt was the same as in the Norcross slab, the crossing of the belts increased the percentage of slab reinforcement so much above that of the simple belt reinforcement that stress in the steel did not pass the yield point and the failure was largely due to the giving way of the concrete around the cap, but partly to some yielding at the line of weakest ultimate resistance, both of which statements are confirmed by the view of Dec. 24th, Fig. 19, where the removal of the loading permits the irregular circular line previously mentioned to be made out at a distance from the center of each column of somewhat less than $L/2$.

Less steel is required in this system than in the Norcross slab for the same limiting stresses. Since the steel in this slab did not pass the yield point any greater percentage of reinforcement would be useless and would not increase the strength of the slab. It has been found that good practice requires a percentage of steel dependent in the following manner upon the thickness of the slab:

$$\text{If } d = L/35 \text{ the belt reinforcement } = 0.2\%$$
$$\text{If } d = L/24 \text{ the belt reinforcement } = 0.3\%$$
$$\text{If } d = L/20 \text{ the belt reinforcement } = 0.4\%$$

Comparision of the steel in the test slabs:	Norcross.	Mushroom.
Size of slab	16′ x 16′	18.4′ x 17.8′
Area of slab	256 sq. ft.	328 sq. ft.
Length of 3/8″ rods in the slab	1188 ft.	1450 ft.
Weight of 3/8″ rods in the slab	446 lbs.	544 lbs.
Weight of Plates or Heads in the slab	268 lbs.	435 lbs.
Total weight of steel in the slab	714 lbs.	979 lbs.
Weight of steel per square foot of slab	2.8 lbs.	3 lbs.

Area of Panel 12 x 12 ft.	144 sq. ft.	144 sq. ft.
Length of slab rods per panel	638 ft.	638 ft.
Weight of slab rods per panel	239 lbs.	239 lbs.
Weight in plates or heads per panel	67 lbs.	109 lbs.
Total weight of steel per panel	306 lbs.	348 lbs.
Weight of steel per square foot of panel.	2 1/8 lbs.	2 5/12 lbs.

SUGGESTIONS REGARDING THE CONSTRUCTION AND FINISH OF FLOOR SLABS

By C. A. P. TURNER

18. THE EXECUTION OF WORK: Construction work of any kind involves a great responsibility, not only on the part of the designer, but also on the part of those in charge of the work, and that responsibility is for the safety of those erecting the work.

Perhaps the construction of no type of building is so free from hazard and risk to the lives of those erecting it as reinforced concrete construction when scientifically designed and intelligently executed.

During the last ten or twelve years, the manufacturers of Portland Cement, have through improvements in methods of manufacture and great reduction in cost, placed this material on the market at such reasonable rates that it has given a remarkable impetus to the construction of concrete work in all lines. Since, as a material of construction, it has but recently come into general use, it is not surprising that a large part of the engineering and architectural profession have not yet become so familiar with its characteristics, but that designs lacking in conservatism from a scientific standpoint have been frequently made, and this combined with the execution of the work by unskilled contractors, has resulted in a number of instances in needless sacrifice of life and large property losses, such as a more thorough knowledge and study of the characteristics of the material should entirely prevent.

It would be neglect of duty to fail even in this short discussion to call attention pointedly to those properties and characteristics of concrete which must be known and appreciated by the engineer and constructor in order that he may avoid the serious disasters into which those ignorant or forgetful of them have been too frequently led.

THE HARDENING OF CONCRETE: Concrete may be defined as an artificial conglomerate stone in which the coarse aggregate or space-filler is held together by the cement matrix. The cement should conform to the Standard Specifications for Cement, recommended by the American Society for Testing Materials.

The contractor and architect should, at least, see to it that the cement is finely ground, and that it meets the requirements of the boiling test. This last may be readily made by forming pats of the cement of $3\frac{1}{2}$ to 4 inches in diameter on a piece of glass, kneading them thoroughly with just enough moisture to make them plastic, so that they will hold their shape without flowing, and taper to a thin edge. Store the pats under a moist cloth at a temperature of sixty-five to seventy-five degrees Fahr. for a period of 24 hours. Then place the pats in a kettle or pan of cold water, and after raising the temperature of the water to the boiling point, continue boiling for a period of four hours. If the pats do not then show cracks, and if they harden without cracking or disintegrating, the constructor may be satisfied that the cement is suitable for use in the work. Coarse grinding reduces the sand-carrying capacity of the cement, and its consequent efficiency.

The function assigned to the concrete element in the combination of reinforced concrete is to resist compressive stresses in bending; but when first mixed the concrete is nothing more than mud, and in order for it to become the hard, rigid material necessary to fulfill its function in the finished work it must evidently pass in the process of hardening thru all stages and varying degrees of hardness from mud and partly cured cement to the final stage of hard, rigid material. This curing or hardening being a chemical process, does not occur in any fixed period of time, save and except the temperature conditions are absolutely constant. Hence the time at which forms may be safely removed is not to be reckoned by a given number of days, but rather it must be determined by the degree of hardness attained by the cement. In other words, during warm summer weather, concrete may become reasonably well cured in twelve or fifteen days. If the weather, however, is rainy and chilly, it may not become cured in a month. In the cold, frosty weather of the spring and autumn, unless warm water is used in the mix, the concrete may require two or three months to become thoroughly cured, while by heating the mixing water, whenever the temperature is below 50 degrees Fahr., the concrete will harden approximately as it does during the more favorable season.

Concrete which has been chilled by the use of ice cold water, or that has become chilled within the first day or two of the time it is cast, has this peculiarity, that it is very difficult indeed for the most expert to determine when it is in such condition that it will retain its shape after the removal of the forms. Once having been chilled in the early stages, it goes through consecutive stages of sweating with temperature changes, and during these periods it sometimes happens that the concrete diminishes in compressive strength, and if the props are removed it sags and gets out of shape. Such deformation will generally result in checks and fine cracks, though there may not be any serious diminition of the ultimate strength. These checks may be prevented as explained above by the simple method of heating the mixing water whenever the temperature has dropped below 50 degrees Fahr. In colder weather, that is below the freezing point, not only must the water be heated, but as a rule the sand and stone too, also a little salt may be advantageously used. The work must then be properly housed and kept warm for at least three weeks subsequent to pouring.

Use of Salt in Cold Weather: We have mentioned the use of salt in cold weather. The action of salt is two-fold: It retards the setting and thus enables us to use water heated to a higher temperature than we could use without salt. It also lowers the freezing point. Should the concrete then be frozen at the subsequent sweating period which occurs with a rise in temperature, the salt retains the necessary moisture for crystallization because of its affinity for moisture, thus preventing the softened concrete from drying out and disintegrating through lack of moisture to enable it to crystalize and harden properly. The amount of salt to be used is about a cup to the sack of cement with the temperature from 18 to 20 degrees Fahr. If the temperature is below this, increase the amount of salt, and when working below zero Fahr., use not less than two cups of salt to the bag of cement.

Pouring Concrete: Bad work frequently results from improper pouring, or casting of the work. In filling the forms, the lowest portion of the forms should be filled first. A column should be filled from the center and not from the side of the cap. Filling from the center will insure a clean smooth face when the forms are removed. Filling from the side will frequently give a bad surface because the mortar will flow into the center of the column through the hooping, leaving the coarse aggregate with voids unfilled at the outside. As more concrete is then poured in, the voids between the core and the out side portion will become filled, and the soft mortar will not be able to flow back to completely fill the voids between the hooping and the casing. Where the spacing of the hooping is wide, this is not so important, but it becomes very important where the spiral used has close spacing. It is better to cast the column and mushroom frame complete, continuing to pour the concrete over the center of the column so that it always flows from the column into the Mushroom slab rather than the reverse. All splices must be made in a vertical plane, in a beam preferably at the middle of the span, and in a slab at a center line of a panel.

Test for Hardness in Warm Weather: We have pointed out that the criterion governing the safe removal of forms is the hardness or rigidity of the concrete. A test of hardness in concrete not frozen may be made by driving a common eight-penny nail into it; the nail should double up before penetrating more than half an inch. The concrete should further be hard enough to break like stone in knocking off a piece with the hammer. Noting the indentation under a blow with the hammer, gives a fair idea of its condition to those having experience.

Subcentering, as provided in the appended specification, is a desirable method of preventing deformation, where the use of the forms is desired for upper stories before the concrete is fully cured.

Test for Hardness in Cold Weather: Concrete freshly mixed and frozen hard will not only sustain itself but carry a large load in addition, until it thaws out and softens, when collapse in whole or in part is inevitable. Partly cured concrete if frozen, sweats and softens with a rise in temperature, hence in cold weather there is danger of mistaking partly cured concrete made rigid by frost for thoroughly cured material. In fact the only test that can be depended upon with certainty in cold, frosty weather, is to dig out a piece of concrete, place a sample on a stove or hot radiator, and note whether, as the frost is thawed out of it, it sweats and softens. This gives the builder and engineer a perfectly conclusive test of the condition of the concrete as to whether it is cured or merely stiffened up by frost.

Lap of Reinforcement over Supports: Thoroughly tying the work together by ample lap in the reinforcement is a prime requisite for safety in any form or type of construction. This general precaution insures toughness, and prevents instantaneous collapse, should the workman exercise bad judgment in premature removal of forms.

Responsibility of the Engineer: The steps which it is possible for the engineer to take in securing safe construction are limited in the first place to the production of a conservative design, and one which will present toughness, so that its failure under overload or under premature removal of the forms will be slow and gradual. This he can do, and this we believe he is morally bound to do. On the other hand, he cannot design reinforced concrete work which will hold its shape without permanent deformation, unless it is properly supported until the concrete has had time under proper conditions to become thoroughly cured.

Concrete in setting shrinks, and sometimes cracks by reason of this shrinkage, particularly when it hardens rapidly, as it does in hot weather. This shrinkage sets up certain stresses in the concrete, which, combined with temperature changes, occasionally manifest themselves by subsequent cracks in the work. Such checks or cracks do not of necessity indicate weakness, providing the concrete is hard and rigid, since the steel is intended to take the tensile stresses and the concrete the compressive. Such checks sometimes cause an unwarranted lack of confidence in the safety and stability of the work arising from the common lack of familiarity with the characteristics of the material. For example, the owner of a frame building would never imagine it to be unsafe because he found a few season checks in the timber. He is sufficiently familiar with the seasoning of timber to understand how these checks occur, and that in most instances they do not mean a loss of strength, since, as the timber hardens by thoroughly drying out, it becomes stronger, as a rule, to an amount in excess of any slight weakness which might be developed by ordinary season cracks or checks. So in concrete, when the general public becomes more familiar with its characteristics they will regard as far less important than they now do, checks which are produced by temperature and shrinkage stresses, or possibly by slight unequal settlement of supports.

Proper and Improper Methods of Floor Finish: In concrete work there are a number of small defects which occur through failure to properly manipulate the material, for which the designer of the engineering part of the work is frequently censured improperly. For example, cases have occurred where a good splice was not secured owing to the fact that in very hot weather the stone aggregate became heated in the sun and was not properly cooled down before mixing the concrete, and so the water dried out too quickly, while the heat in the stone caused the cement to set so rapidly that a good splice to the previous work could not be made.

The worst trouble, however, which has been observed, is that resulting from poor surface finish of floors. Improper methods in common practice are of two different kinds. One is the attempt to finish the work approximately at the time it is cast, making the surface finish integral with the slab. The difficulty with this method of finishing lies in the fact that as soon as the columns are cast in the story above, unequal moisture conditions are produced around the foot of the column owing to the excess of moisture in the column; thus the concrete in the surface of the slab around and near the foot of the column is expanded by the excess moisture, and it ultimately shrinks, and leaves a series of spider web cracks as it dries out. This will occur to a greater or less extent depending on the humidity of the surrounding atmosphere during the curing or drying out of the floor. If the weather is dry these checks will be very pronounced indeed, though they will not be very deep. If it is rainy and damp, and the floor is kept soaked all the time, they may be nearly or quite lacking.

Another objection to this method of finish is that unusual precautions must be taken to protect the floor before the centering can be placed for a story above, and regardless of the method used to protect it the floor usually becames scarred and deeply scratched before the work is complete, leaving a surface difficult to satisfactorily repair.

Another method which leads to bad results is the following: The rough slab is cast, and the centering removed in due time, the slab cleaned and the finish coat applied in a sloppy or plastic form, flowed in place, screeded to approximate surface, and then allowed to partly set, so that the finishers can get on the floor and trowel it down. A floor finished in this manner looks well when the work is new. It does not wear well but dusts badly, pits and rapidly grows rough and ragged under trucking.

The correct method of applying floor finish is as follows: The finish coat should be not less than 1 and 1/4 inches to 1 and 1/2 inches in thickness. It should be applied after the rough slab has been fairly well cured. The surface of the rough slab should be thoroughly cleaned of dirt and *laitance* and thoroughly soaked with water. Then the floor finish, a mixture preferably of one part of cement to one and one-half sand (the sand a silicious sand with grains from 1/8 inch down, if such can be secured), should be thoroughly mixed with just enough water to make an extremely stiff paste, one which will hold its form if squeezed in the hand, but one which will not run or flow, and will need a fair amount of tamping to bring the moisture to the surface. This concrete, so mixed, should be applied to the rough slab in blocks of from four to five feet square, first grouting the rough slab with a neat cement grout, then tamp until the moisture is brought to the surface, level up and trowel immediately. The cement finish should not be mixed more rapidly than it can be applied, so that the cement will not be killed by taking a partial set before troweling, which is what occurs where the finish is applied sloppy, and the workmen wait for it to partly harden before they can get on it to trowel. A finish applied as just stated will stand severe usage and last for several years without showing appreciable evidence of pitting, dusting, or undue wear.

The addition of ground iron ore, to the amount of twenty pounds to the barrel of cement, appears to improve the finish and give it a more pleasing color.

Checks in cement finish have no relation whatever, as a rule, to the strength of the work. They will invariably occur in the cement finish where the finish coat is too thin. When it is less than 1 inch or 3/4 inch at one part of the floor with 1 1/4 inches or 1 1/2 inches at another, the surface will invariably check and crack badly if applied at a sloppy consistency and allowed to partly cure before it is polished down. We know of no type of construction where there has not been much trouble with finished surfaces in such buildings as have come under our observation. But experience has shown us that these troubles are needless, and can be avoided by the proper handling and application of the finishing coat.

It is difficult indeed to re-educate those who profess to be cement finishers, whose experience has been largely in sidewalk finish, or work of that character, to appreciate the necessity for a different method of executing work in a building; but when this has been accomplished the owner will have the use of a floor finish free from the unpleasant defects above pointed out.

Strips and Strip Fill for Wood Floors: The proper time for the application of the strips and fill is immediately after the rough slab has become sufficiently hardened to work upon it, for the reason that at this time the strips may be spiked to the partially hardened concrete and wedged up or lined up to the desired level without difficulty. Then the strip fill can be put in with the same rig that is used to cast the floor slab.

The writer prefers the strip fill of the same mixture as the slab except where the loads are so light that increased strength and stiffness are of no importance. Then a one to three and one-half, four, or even five, mix will answer the purpose. No natural cement or lime should be used in the mixture, since when it is used, trouble almost invariably follows, caused by its extremely slow hardening and its retention of moisture until hardening takes place. This moisture frequently swells and expands the flooring to such an extent that it springs away from the fastenings, thereby necessitating the entire relaying of the floors. Conservative practice accordingly is to use Portland Cement alone, which will dry out far quicker than any natural cement or brown lime.

APPENDIX

STANDARD SPECIFICATION FOR REINFORCED CONCRETE FLOORS

By C. A. P. TURNER, Consulting Engineer
Minneapolis, Minn.

Reinforcement. Reinforcement shall be of sizes of bars shown on the accompanying plans and details which form a part of this specification.

All reinforcing metal shall be of medium open hearth or Bessemer steel, meeting the requirements of the Manufacturers' Standard Specifications, in composition, ultimate strength, ductility and elastic limit, and the required bending basis. Hard grade may be used for slab rods only.

Bending. Bending shall preferably be done cold. If the column rods are heated and blacksmith work is done, care must be exercised that the steel is not burned in the operation, otherwise it will be condemned by the engineer.

Cement. Cement shall be of good quality of Portland Cement, of a brand which has been upon the market and successfully used for at least four years, meeting the requirements of the specification adopted by the American Society for Testing Materials.

The contractor shall give the owner the opportunity to test all cement delivered, and shall furnish the use of testing machine for this purpose.

The cement shall be delivered in good condition and properly protected under suitable cover after delivery on the premises so that it may not be damaged by moisture.

Sand. Sand used in the concrete work shall be clean and coarse, meeting the requirements and approval of the engineer and architect.

Stone. Stone used shall be sound, hard stone, free from lumps of clay and other soft unsatisfactory material, or hard smelter slag may be used. In size it shall be crushed to pass a 1-inch ring, for slabs and columns, and shall be screened free from dirt and dust.

Concrete. All concrete shall be mixed in a standard batch machine to the consistency of brick mortar, so that it will flow slowly and require only puddling around the reinforcement.

Concrete shall be thoroughly mixed in the following proportions: one part cement, meeting the requirements of the standard specifications; two parts clean, coarse sand free from clay, loam or other impurities; and four parts crushed stone or clean gravel.

The concrete shall be poured in the low portions of the forms first. That is, it shall be poured directly into the column boxes, beam boxes, etc., before it is poured on the slab. It shall be so placed that it will be forced to flow as little as possible to get to the required position, since by flowing, the cement is readily separated from the mixture.

Splices. Splices in beams or slabs are to be made in a vertical plane, preferably in the center of the panel or beam.

Proportions. Each sack of cement shall be considered equivalent to one cubic foot in volume, and the mixture of the cement, sand and stone used in the concrete shall be proportioned by volume on this basis and as hereinafter specified.

Concrete for footings, columns, beams and rough slabs throughout shall consist of a mixture of one cement, two sand and four of crushed stone.

For the retaining walls, the concrete mixture shall be one cement, three sand and five parts of stone.

Concrete in which the cement has attained its initial set shall not be used on the work. Concrete, however, which has slopped out of the mixer, if cleaned up within a short time, not over every half hour, may be put back in the mixer, and after being thoroughly mixed again with water may be used on the work.

Forms. All forms for the reinforced concrete shall be substantially made and true to line. Any irregularities due to defective workmanship in this respect, shall be made good as directed by the architect, by dressing down the finished work, or removal and properly replacing it in case that it cannot be satisfactorily done.

A fair quality of lumber, preferably 1x6 square edge fencing shall be used for the slab forms. This lumber shall be dressed on the side next to the concrete except where plaster is specified by the architect for office finish, in which case the rough side of the boarding shall be placed upwards, next to the concrete.

Column Forms. Column forms shall be made up with plank not less than 1½ inches thick and stayed at intervals not more than 18 inches vertically between bands or straps and shall fit closely at the corner joints, or the forms may be made of sheet metal.

Removal of the Forms. Forms shall not be removed under the most favorable conditions, prior to two weeks' time, and under less favorable conditions where the temperature is lower than 50° until the concrete is hard and rigid.

The superintendent will keep in mind the fact that it is not the number of days time which has elapsed since placing the concrete which shall determine the earliest removal of the forms, but rather how rapidly the concrete has thoroughly cured and hardened and that the concrete may be readily stiffened up by cold and frost which, when it thaws, will sweat and fail to maintain the desired form.

Sub-Centering. Where a series of floors are cast one above the other, sub-centering of substantial posts about 10 feet centers shall be kept in place until there are at least two supporting slabs that are well cured and hard so that the concrete may not be overstained in the early stages of hardening.

Placing and Inspection of Reinforcement. BEFORE COMMENCING THE CONCRETE WORK, the reinforcement shall be properly placed and inspected by the architect or the engineer representing the owner, and not until after this inspection and approval may the work of casting the floor proceed.

The floor slab rods shall be wired together to hold them in the position as shown on the plans. Special attention being given to placing the rods in belts of the width of the mushroom frame and fairly uniform spacing, although this is of less importance than keeping to the general distribution through the full width of the belts of reinforcement.

In placing the floor slab rods, all those running from column to column directly on one side of a panel shall be placed first, then those running at right angles, next all those in one diagonal belt, and then those in the other diagonal.

Where a belt of slab rods runs parallel to a wall place one rod at bottom on forms. Then see that belts normal and diagonally are placed, following up with slab rods parallel to the wall on the top of normal and diagonal belts.

In wiring the rods together it is desirable to use No. 16 soft annealed wire, taking a piece, say a yard long, fastening an intersection, then carry the wire diagonally to the next intersection, taking a half hitch and proceed until this piece is used up and making the end fast. Then start with a new piece and proceed as before.

Two lines of ties, crossing and normal to the intersecting belts at the center will hold these rods in position very nicely.

A similar tie across the parallel belts, and a suitable number of fastenings around the mushroom head are required to hold the bars in position.

Floor Finish. The finish coat on the rough slab shall not be less than 1 inch thick, and the rough slab shall be prepared for its reception as follows:

The slab shall be thoroughly scrubbed with a steel brush and water,and then after it has been thoroughly cleaned from dirt and laitance it shall be kept wet for at least six hours. The surface shall then be coated with neat cement grout and the finish coat applied.

The finish coat shall consist of a mixture of one cement to one and one-half clean, coarse sand. The finish coat shall be mixed with just enough water to make a very stiff paste and not enough to make it soft and sloppy. It shall be tamped in place and troweled to a smooth finish.

Mixing the material wet and sloppy renders it necessary to wait until the material hardens somewhat before it is possible to polish it down. In allowing it to partly harden the finisher is then obliged to break up the surface of partly hardened cement which results in a finished surface that will dust badly, pit readily and wear rough under subsequent use, so that this method should not be employed.

This finish coat is to be blocked off in squares along the center line of columns, and joints shall be made in this coat between panel joints at five to six foot intervals.

Conduits. Before casting the concrete, the concrete contractor shall see that the electric contractor has placed the necessary conduits for the wires. These shall be kept above the reinforcement wherever they come in the center of a panel, the idea being to have these conduit pipes above the steel and dip down into the socket at the junction, or to use a special deep socket which would be prefered by the engineer.

These conduit pipes should be carried below the level of the reinforcement around the mushroom heads where the reinforcement is of necessity near the top of the slab.

Depositing Concrete in Warm Weather. When the concrete is deposited in temperatures above 70 ° Fahr., the slab shall be thoroughly wet down twice a day for two days after it has been cast. Any preliminary shrinkage cracks which occur on the surface of the slab due to too rapid drying shall be immediately filled with liquid cement grout.

Any concrete work indicating that it has not been thoroughly mixed in the required proportions shall be dug out and replaced as directed by the engineer and architect.

Placing Concrete in Cold Weather. Where the temperature is below 45° Fahr., the water shall be heated to a temperature of at least 110°. Where the temperature is below 30° Fahr., artifical heat shall be used to assist in curing the concrete, and this must be continued until such a time as the slab is thoroughly cured and dry throughout.

Pouring Concrete. In the mushroom system concrete shall be poured over the center of the column until the column is filled. Then the pouring shall be continued until the mushroom and mushroom frame is filled up so that the concrete will flow from the column toward the center of the slab and not from the center of the slab toward the column. In this way solid concrete without joints and planes of imperfect bond will be secured around and in the vicinity of column heads, where it is most needed.

Test. No test shall be made until the concrete is thoroughly cured, is dry, hard and rigid throughout. Ninety days of good drying weather at a temperature above 60° Fahr., either natural or artificial, shall be the criterion as to when the test of double the working capacity can be reasonably made.

General. It is the general intent of this specification to require first class work in all particulars, and work unsatisfactory to the engineer and architect representing the owners shall be made good by the contractor as they direct.

PRINTED BY
HEYWOOD
MINNEAPOLIS

LIST OF ONE HUNDRED BUILDINGS SELECTED FROM MORE THAN A THOUSAND DESIGNED ON THE

MUSHROOM SYSTEM

1906	Johnson-Bovey Co's. Bldg	Minneapolis, Minn.
1906	Hoffman Building	Milwaukee, Wis.
1907	Bostwick Braun Bldg	Toledo, Ohio
1907	Lindeke Warner Bldg	St. Paul, Minn.
1907	Hamm Brewery Bldg	" " "
1907	Smythe Building	Wichita, Kans.
1907	Forman Ford Bldg	Minneapolis, Minn.
1907	Grellet Collins Bldg	Philadelphia, Pa.
1907	Parsons Scoville Bldg	Evansville, Ind.
1907	Born Building	Chicago, Ill.
1908	South Dakota State Capitol	Pierre, S. D.
1908	Merchants Ice & Cold Storage Bldg	Cincinnati, Ohio
1908	St. Mary's Hospital	Kansas City, Mo.
1908	John Deere Plow Co	Omaha, Nebr.
1908	Minn. State Prison Bldgs ...(6)	Stillwater, Minn.
1908	Ripley Apartments	Tacoma, Wash.
1908	Velie Motor Bldg	Moline, Ill.
1908	Park Grant Morris Bldg	Fargo, N. D.
1909	Con P. Curran Bldg	St. Louis, Mo.
1909	Manchester Biscuit Co's. Bldg	Fargo, N. D.
1909	Blue Line Transfer & Storage Bldg	Des Moines, Ia.
1909	Cutler Hardware Bldg	Waterloo, Ia.
1909	Mass. Cotton Mills	Boston, Mass.
1909	McMillan Packing Co	St. Paul, Minn.
1909	Vancouver Ice and Cold Storage Co	Vancouver, Bldg.
1909	Omaha Fireproof Storage Bldg	Omaha, Nebr.
1909	J. I. Case Bldg	Oklahoma City, Okla.
1909	Tibbs Hutchings & Co	Minneapolis, Minn.
1909	Snead Mfg. Bldg	Louisville, Ky.
1909	New England Sanitary Bakery Bldg	Decatur, Ill.
1909	International Harvester Bldg	Milwaukee, Wis.
1909	Congress Candy Co	Grand Forks, N. D.
1910	Y. M. C. A. Bldg	Winnipeg, Man.
1910	West Publishing Co's. Bldg	St. Paul, Minn.
1910	Beatrice Creamery Bldg	Lincoln, Nebr.
1910	Iten Biscuit Co	Omaha, Nebr.
1910	Turner Moving & Storage Bldg	Denver, Colo.
1910	Congress Realty Co's. Bldg	Portland, Me.
1910	Sniders & Abrahams Bldg	Melbourne, Australia
1910	Strong & Warner Bldg	St. Paul, Minn.
1910	Lexington High School Bldg	St. Paul, Minn.
1910	Weicker Transfer & Storage Bldg	Denver, Colo.
1910	Chehallis County Court House	Montesano, Wash.
1910	Missouri Glass Co's. Bldg	St. Louis, Mo.
1910	Industrial Bldg	Newark, N. J.
1910	Revel & Wagner Bldg	Little Rock, Ark.
1910	Jobst Bethard Bldg	Peoria, Ill.
1910	International Harvester (Keystone Works)	Sterling, Ill.

1910	Patterson Hotel	Bismarck, N. D.
1910	O'Neil Bldg	Akron, Ohio
1911	Lindsay Bldg	Winnipeg, Man.
1911	King George Hotel	Saskatoon, Sask.
1911	Northern Cold Storage Bldg	Duluth, Minn.
1911	Leighton Supply Co	Fort Dodge, Ia.
1911	Kinsey Bldg	Toledo, Ohio
1911	Lozier Motor Bldg	Detroit, Mich.
1911	Mullin Warehouse Bldg	Cedar Rapids, Ia.
1911	Griggs Cooper & Co	St. Paul, Minn.
1911	Swift Canadian Co's. Bldgs	Vancouver, B. C.
1911	McKenzie Bldg	Brandon, Man.
1911	Swift Canadian Co's. Bldg	Fort William, Ont.
1911	Commerce Bldg	St. Paul, Minn.
1911	Experimental Eng. Bldg. Univ. of Minn	Minneapolis, Minn.
1911	St. Paul Bread Co's. Bldg	St. Paul, Minn.
1911	Rust Parker Martin Bldg	Duluth, Minn.
1912	Woodward Wight Co. Ltd. Bldg	New Orleans, La.
1912	International Harvester Co's. Bldg	Fort William, Ont.
1912	H. W. Johns-Manville Bldgs.... (3)	Finderne, N. J.
1912	Cooledge Bldg	Atlanta, Ga.
1912	Lawrence Leather Co's. Bldg	Lawrence, Mass.
1912	Sears, Roebuck & Co	Dallas, Texas
1912	Vineburg Bldg	Montreal, Quebec
1912	Imperial Tobacco Co	Montreal, Quebec
1912	Richards Pinhorn Bldg	Denver, Col.
1912	Kinney & Levan Co. Bldg	Cleveland, Ohio
1912	Standard Oil Co. Bldgs... (2)	Cleveland, Ohio
1912	Silver Sunshine Bldgs... (2)	Cleveland, Ohio
1912	Commercial Improvement Co's. Bldg	Columbus, O.
1912	Moore Department Store Bldg	Memphis, Tenn.
1912	Main Eng. Bldg. Univ. of Minn	Minneapolis, Minn.
1912	Honeyman Hardware Bldg	Portland, Ore.
1912	Revillon Wholesale Hardware Bldg	Edmonton, Alta.
1912	Calgary Furniture Co's. Bldg	Calgary, Alta.
1912	Willoughby Sumner Bldg	Saskatoon, Sask.
1912	U. S. Post Office	Minneapolis, Minn.
1912	Motor Mart Bldg	Sioux City, Ia.
1912	Finch Van Slyke & McConville Bldg	St. Paul, Minn.
1912	Hudson Bay Co's. Warehouse	Winnipeg, Man.
1912	Snell Bldg	Moose Jaw, Sask.
1913	Y. M. C. A. Bldg	Vancouver, B. C.
1913	Reynolds Tobacco Factory Bldg	Winston Salem, N. C.
1913	Ford Motor Bldg	Memphis, Tenn.
1913	Ford Motor Bldg	Los Angeles, Cal.
1913	G. Sommers & Co. Bldg	St. Paul, Minn.
1913	Knickerbocker Bldg	Los Angeles, Cal.
1913	Trinity Auditorium Bldg	Los Angeles, Cal.
1913	U. S. Alumium Co's. Bldg	Pittsburg, Pa.
1913	Gordon Fergusen Co's. Bldg	St. Paul, Minn.
1913	S. H. Kress & Co's. Bldg	Houston, Tex.
1913	"Los Muchachos" Bldg	San Juan, Porto Rico