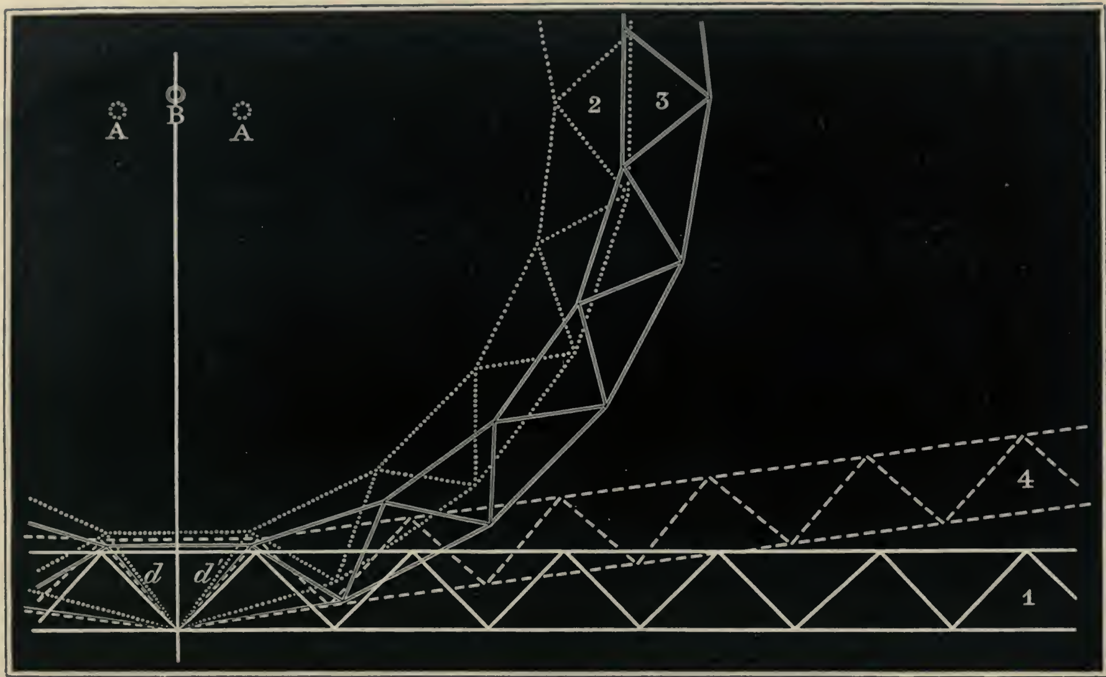
FRONTISPIECE.

PLATE I.



THE
THEORY OF STRAINS
IN
GIRDERS AND SIMILAR STRUCTURES,
WITH
OBSERVATIONS ON THE APPLICATION OF THEORY TO PRACTICE
AND
TABLES OF THE STRENGTH AND OTHER PROPERTIES OF MATERIALS.

BY

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Prius quàm incipias, consulto; et ubi consulueris, mature facto opus est.

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PREFACE TO THE FIRST EDITION.

The following pages have been written at various times during such brief intervals of leisure as the author could spare from his professional duties. They are for the most part the result of experience combined with theory; it is therefore hoped that they may supply the student with what has long been a want in Engineering literature, namely, *a Handbook on the Theory of Strains and the Strength of Materials*, giving practical methods for calculating the strains which occur in girders and similar structures. The theory of transverse strain has, indeed, been incidentally treated by writers on Mechanical Philosophy; their researches, however, have been confined to strains in plain girders, or to a few brief remarks on the more elementary forms of trussing, which, without further development, are of little practical use, and but too frequently afford a pretext for the ill-concealed contempt which so-called practical men sometimes entertain for theoretic knowledge.

A thorough acquaintance with the theory of strains and the strength and other properties of materials forms the basis of all sound engineering practice, and when this is wanting, even natural constructive talent of a high order is frequently at fault, and the result is either excess and consequent waste of material, or, what is still more disastrous, weakness in parts where strength is essential. The time has gone by when practical sagacity formed the sole qualification for high engineering success. Before the improvement of the steam engine gave rise to a new profession there were indeed some memorable names on the roll of engineers, generally self-taught mechanics, whom great natural ability had raised to pre-eminence in their profession; but practice which was formerly excusable, or even worthy of the highest commendation, would, now that knowledge has increased, be properly described as culpable waste, arising either from prejudice or ignorance.

The usual resource of the merely practical man is precedent, but the true way of benefiting by the experience of others is not by blindly following their practice, but by avoiding their errors as well as extending and improving what time and experience have proved successful. If one were asked what is the difference between an engineer and a mere craftsman, he would well reply, that the one merely executes mechanically the designs of others, or copies something which has been done before without introducing any new application of scientific principles, while the other moulds matter into new forms suited for the special object to be attained; and though experience and practical knowledge are essential for this, he lets his experience be guided and aided by theoretic knowledge, so as to arrange and proportion the various parts to the exact duty they are intended to fulfil.

> Then prove we now with best endeavour
> What from our efforts yet may spring;
> He justly is despised who never
> Did thought to aid his labours bring.
> For this is art's true indication,
> When skill is minister to thought;
> When types that are the mind's creation
> The hand to perfect form has wrought.

The well-educated engineer should combine the qualifications of the practical man and of the physicist, and the more he blends these together, making each mould and soften what the other would seem to dictate if allowed to act alone, the more will his works be successful and attain the exact object for which they are designed. The engineer should be a physicist, who, in place of confining his operations to the laboratory or the study, exerts his energies in a wider field in developing the industrial resources of nature, and compelling mere matter to become subservient to the wants and comforts and civilization of the human race.

CONTENTS.

CHAPTER I.

INTRODUCTORY.

CHAPTER II.

FLANGED GIRDERS WITH BRACED OR THIN CONTINUOUS WEBS.

CASE II.—FLANGED SEMI-GIRDER LOADED UNIFORMLY.

CASE III.—FLANGED GIRDER SUPPORTED AT BOTH ENDS AND LOADED AT AN INTERMEDIATE POINT.

CASE IV.—FLANGED GIRDER SUPPORTED AT BOTH ENDS AND LOADED AT IRREGULAR INTERVALS.

CASE V.—FLANGED GIRDER SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

CHAPTER III.

TRANSVERSE STRAIN.

CHAPTER IV.

GIRDERS OF VARIOUS SECTIONS.

CASE II.—SEMI-GIRDERS LOADED UNIFORMLY.

CASE III.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED AT AN INTERMEDIATE POINT.

CASE IV.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

CHAPTER V.

BRACED GIRDERS WITH PARALLEL FLANGES AND WEBS FORMED OF ISOSCELES BRACING.

CHAPTER VII.

BRACED GIRDERS WITH OBLIQUE OR CURVED FLANGES.

CHAPTER VIII.

DEFLECTION.

Class I.—Girders whose sections are proportioned so as to produce uniform strength.

Class II.—Girders whose section is uniform throughout their length.

CASE I.—SEMI-GIRDERS OF UNIFORM SECTION LOADED AT THE EXTREMITY.

CASE II.—SEMI-GIRDERS OF UNIFORM SECTION LOADED UNIFORMLY.

CASE III.—GIRDERS OF UNIFORM SECTION SUPPORTED AT BOTH ENDS AND LOADED AT THE CENTRE.

CASE IV.—GIRDERS OF UNIFORM SECTION SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

CHAPTER IX.

CONTINUOUS GIRDERS.

CHAPTER XI.

ANGLE OF ECONOMY.

CHAPTER XII.

TORSION.

CHAPTER XIII.

STRENGTH OF HOLLOW CYLINDERS AND SPHERES.

CHAPTER XIV.

CRUSHING STRENGTH OF MATERIALS.

CHAPTER XV.

PILLARS.

CHAPTER XVI.

TENSILE STRENGTH OF MATERIALS.

CHAPTER XVII.

SHEARING-STRAIN.

CHAPTER XVIII.

ELASTICITY AND SET.

CHAPTER XIX.

TEMPERATURE.

CHAPTER XX.

FLANGES.

CHAPTER XXI.

WEB.

CHAPTER XXII.

CROSS-BRACING.

CHAPTER XXIII.

CROSS-GIRDERS AND PLATFORM.

CHAPTER XXIV.

COUNTERBRACING.

CHAPTER XXV.

DEFLECTION AND CAMBER.

CHAPTER XXVI.

DEPTH OF GIRDERS AND ARCHES.

CHAPTER XXVII.

CONNEXIONS.

CHAPTER XXVIII.

WORKING STRAIN AND WORKING LOAD.

CAST-IRON.

WROUGHT-IRON.

STEEL.

TIMBER.

FOUNDATIONS, STONE, BRICK, MASONRY, CONCRETE.

WORKING LOAD ON RAILWAYS.

WORKING LOAD ON PUBLIC BRIDGES AND ROOFS.

CHAPTER XXIX.

ESTIMATION OF GIRDER-WORK.

EXAMPLE 1.

EXAMPLE 2.

EXAMPLE 3.

CHAPTER XXX.

LIMITS OF LENGTH OF GIRDERS.

CHAPTER XXXI.

CONCLUDING OBSERVATIONS.

APPENDIX.

THE

THEORY OF STRAINS IN GIRDERS

AND

SIMILAR STRUCTURES.

CHAPTER I.

INTRODUCTORY.

1. Strain—Tension—Compression—Transverse strain—Shearing-strain—Torsion.—On the application of force all bodies change either form or volume, or both together. Forces considered with reference to the internal changes they tend to produce in any solid are termed strains* and may be classified as follows:—

Tensile strains,	producing fracture by	tearing asunder.
Compressive do.,		crushing.
Transverse do.,		breaking across.
Shearing do.,		cutting asunder.
Torsional do.,		twisting asunder.

This five-fold division is made for convenience merely, for the strength of any material, in whatever manner it may be employed, depends ultimately on its capability of sustaining strains which tend either to tear its parts asunder or to crush them together. It is therefore of essential importance to know the ultimate resistance to tension or compression which each material possesses, and thence deduce those strains which may be safely imposed in practice. To this end various experimenters have devoted their attention;

* It will be useful for the student to know that some writers apply the term *stress* to what I have termed *strain* in the text, that is, to the combination of internal forces or reactions which the particles of any body exert in resisting the tendency of external forces to produce alteration of form, and they apply the term *strain* to what I call *deformation*, that is, to alteration of form resulting from stress.

in the United Kingdom, none with more perseverance or success than the late Eaton Hodgkinson, Esq., to whose life-long labours we are mainly indebted for the physical investigations on which calculations of the strength of structures are based.

2. Unit-strain—Inch-strain—Foot-strain.—Wherever English measures are used, tensile and compressive forces are measured by the number of tons or pounds strain on the square inch or square foot. It will be convenient, however, to have some short expression for the strain on the unit of sectional area, irrespective of any particular measure of length or weight, and I have adopted the term *Unit-strain* to denote this quantity, and the words *Inch-strain* or *Foot-strain* to express the strain per square inch or square foot, as the case may be. The unit-strains of tension and compression are represented indifferently by the symbol f, unless it be desirable to distinguish them, in which case the unit-strain of compression is represented by the symbol f'. Thus, if F be the total strain in any bar whose area $= a$, we have

$$\mathsf{F} = af. \tag{1}$$

Ex. 1. If the crushing unit-strain of cast-iron be 42 tons per square inch, what weight will crush a short solid pillar 9 inches in diameter?

$$\text{Here,} \quad a = \frac{9 \times 9 \times \pi}{4} = 63{\cdot}6 \text{ inches,}$$

$$f = 42 \text{ tons.}$$

$$\textit{Answer.} \ \mathsf{F} = af = 63{\cdot}6 \times 42 = 2{,}671 \text{ tons.}$$

Ex. 2. If the tearing unit-strain of beech be 11,500 pounds per square inch, what force in tons will tear asunder a tie-beam 15 inches square?

$$\text{Here,} \quad a = 15 \times 15 = 225 \text{ square inches,}$$

$$f = 11{,}500 \text{ lbs.}$$

$$\textit{Answer.} \ \mathsf{F} = 225 \times \frac{11{,}500}{2{,}240} = 1{,}155 \text{ tons.}$$

3. Elasticity—Cubic elasticity—Linear elasticity.—Besides the strains of tension and compression another matter claims attention, namely, the alteration of length or, in other words, the *elongation* and *shortening* of the material subject to strain. *Elasticity* is the property which all bodies under the influence of external force possess to a greater or less degree of perfection of returning to their original volume or form after the force has been withdrawn. Thus we have *Cubic elasticity* or elasticity of volume, and *Linear elasticity* or elasticity of form. Fluids possess elasticity of volume, but not of form. Solids possess both, but linear elasticity alone demands our attention in questions relating to the strength of materials.

4. Elastic stiffness and Elastic flexibility are correlative terms which express the strength or weakness of the elastic reaction of the fibres of any elastic solid, whether that reaction be due to tensile or compressive strains, applied separately or in combination so as to produce flexure or torsion. Thus, glass is elastically stiff, indian-rubber elastically flexible. In general, the terms *Stiffness* and *Flexibility* are not restricted to elastic solids, but express merely the relative amount of resistance to change of form, whether the material returns to its original shape or not after the force is withdrawn. In this sense copper is stiffer than lead, but neither is elastic, or but very slightly so. Elasticity should not, as in popular language, be confounded with a wide *range* of elastic flexibility. Glass, for instance, is both stiff and elastic, whereas indian-rubber, though very flexible, is less perfectly elastic than glass, that is, it returns with less exactness to its original form after being strained. Again, a thin spring of tempered steel is both elastic and flexible. In popular language, however, indian-rubber is said to be more elastic than glass or steel, because its *range* of elastic flexibility exceeds that of either.

5. Ductility—Toughness—Brittleness.—*Ductility* is the reverse of elasticity and is the property of retaining a permanent change of form after the force which produced it has been removed, and the wider the range over which a body can be altered in shape the more ductile it is said to be. Gold, for instance, is one of the most ductile of metals, as it can be drawn out into extremely fine wire or hammered into leaves of extraordinary thinness. *Toughness* consists in the union of tenacity with ductility. *Brittleness* is incapability of sustaining rapid changes of form without fracture, and is opposed to toughness. Low-Moor iron, for instance, is tough; a bar of it can be twisted into a knot without breaking;

but highly tempered steel is brittle; though more tenacious than iron, it breaks short without any sensible change of length; it is not ductile; it will not stretch under strain. Sealing-wax also is brittle; though more ductile than iron under prolonged pressure, it is not tenacious and will not bear a sudden change of shape without fracture. Accurately speaking we may doubt if there is such a thing as a perfectly elastic solid, for Mr. Hodgkinson's investigations seem to prove that there is no strain, however slight, which will not after its removal leave a permanent, though perhaps to ordinary tests an inappreciable, alteration of length in any of the materials on which he experimented. In other words, every material is more or less ductile.* This view, however, is not held by some authorities.

6. Set—Influence of duration of strain.—When the unit-strain is considerable the defect of elasticity becomes very apparent in some materials, especially in ductile metals, for they do not return to their original length when released from strain, but are sensibly elongated or shortened, as the case may be, by a certain amount which varies according to the nature of the material and the force applied. This residual elongation or shortening is called the *Set*, and is not sensibly increased by subsequent applications of the same unit-strain which first produced it. It should be observed, however, that the ultimate set is not instantaneously produced on the application of force. Iron, and possibly all materials, take time, more or less prolonged, to adapt themselves to new conditions of strain. Hence, a rapidly applied force may snap a brittle bar without producing any very perceptible change in its length.

7. Hooke's law—Law of elasticity—Limit of elasticity.—It is evident that the elastic reaction of any material is equal to the force producing extension or compression, and it has been proved by experiment that the following law of uniform elastic reaction, expressed by Hooke in the phrase " ut tensio sic vis," and generally

* *Report of the Commissioners appointed to inquire into the application of Iron to Railway Structures*, 1849, App. A, p. 1. Also, *Experimental Researches on the Strength and other Properties of Cast-iron*, by E. Hodgkinson, pp. 381, 409, 486.

known as the *Law of elasticity*, though perhaps not accurately true of any solid, is practically true of the materials used in construction. *When any material is strained either by a tensile or a compressive force, the elastic reaction of the fibres (equal to the applied force) is proportional to their increment or decrement of length*, provided the alteration of length does not exceed a certain limit beyond which the above stated law ceases to apply, and the change of length, no longer regular, increases for each additional unit of strain more rapidly than the reaction due to the elasticity of the fibres; this produces set and ultimately rupture. Experience has proved that the safe working strain of any material does not exceed its sensible limit of uniform elastic reaction, generally called the *limit of elasticity;* indeed, it generally lies considerably within it. The limit of elasticity may also be defined to be the greatest strain that does not produce an appreciable set. It will be seen hereafter that with some materials, such as glass, there is no limit of elasticity short of rupture, as they are elastic up to the breaking point and apparently take no set when the strain is removed.

8. Coefficient of elasticity, E—Table of coefficients.—The coefficient of elastic reaction, or briefly, the *Coefficient of elasticity*,* is represented by the symbol **E**, and is the weight (in lbs.) requisite to elongate or shorten a bar whose transverse section equals a superficial unit (one square inch) by an amount equal to its length, on the imaginary hypothesis that the law of elasticity holds good for so great a range. In assuming that the coefficient of elasticity is the same for compression and extension I have followed Navier,† but some writers on the strength of materials seem to overlook the fact that, if the law of elasticity be rigidly exact, a given force of compression will shorten any material by the same proportion of its original length that an equal tensile force will extend it. In practice the coefficient of elastic compression will generally be found to differ slightly from that of elastic tension.

If a bar whose length $= l$ be extended or compressed within

* Called also the *Modulus of elasticity*.

† *Résumé des Leçons données à l'École des Ponts et Chaussées*, p. 41.

the limits of elasticity by a strain of f lbs. per square inch, the increment or decrement of length λ is expressed by the following relation,

$$\frac{\lambda}{l} = \frac{f}{\mathsf{E}}$$

whence, $$\mathsf{E} = \frac{fl}{\lambda} \qquad (2)$$

Ex. How much will an inch-strain of 5 tons stretch a bar of wrought-iron whose length equals 10 feet?

Here (see table following), $\mathsf{E} = 24{,}000{,}000$ lbs.,
$f = 5$ tons,
$l = 10$ feet.

Answer. $\lambda = \frac{fl}{\mathsf{E}} = \frac{5 \times 2{,}240 \times 10 \times 12}{24{,}000{,}000.} = {\cdot}056$ inches.

It is obvious that the coefficient of elasticity should be deduced from experiments in which the applied unit-strain lies within the limit of elastic reaction. It should also be noted whether the material has been previously *stretched* by excessive strain; otherwise the results will be anomalous. The following table contains the coefficients of elasticity of various materials, derived chiefly from experiments on transverse strain:—

Description of Material.	Coefficient of Elasticity in lbs. per square inch. E	Authority.
METALS.		
Brass (cast), - - - - -	8,930,000	Tredgold.
Gold (drawn), - - - - -	11,564,000	Wertheim.
Do. (annealed), - - - - -	7,943,000	do.
Gun metal (copper 8, tin 1), - - -	9,873,000	Tredgold.
Iron (cast, from transverse strain), - -	18,400,000	do.
Do. (do., from direct tension or compression), -	12,000,000	Hodgkinson.
Do. (wrought), - - - - -	24,000,000	do.
Lead (cast), - - - - -	720,000	Tredgold.
Platina thread, - - - - -	24,240,000	Wertheim.
Do. (annealed), - - - - -	22,070,000	do.
Silver (drawn), - - - - -	10,465,000	do.
Do. (annealed), - - - - -	10,155,000	do.
Steel, - - - - - -	29,000,000	Young.
Do., - - - - - -	31,000,000	Fairbairn.
Tin (cast), - - - - -	4,608,000	Tredgold.
Zinc (cast), - - - - -	13,680,000	do.
TIMBER.		
Acacia (English growth), - - -	1,152,000	Barlow.
Ash, - - - - - -	1,644,800	do.
Beech, - - - - - -	1,353,600	do.

Description of Material.	Coefficient of Elasticity in lbs. per square inch. E	Authority.
TIMBER—*continued.*		
Birch (American black), - - -	1,477,000	Barlow.
Do. (common), - - - - -	1,644,800	do.
Box (Australia), - - - -	2,155,200	Trickett.
Deal (Christiana), - - - -	1,589,600	Barlow.
Do. (Memel), - - - - -	1,603,600	do.
Elm, - - - - - -	699,840	do.
Fir (Mar Forest), - - - -	645,360	do.
Do. (do., another specimen), - - -	869,600	do.
Do. (New England), - - - -	2,191,200	do.
Do. (Riga), - - - - -	1,328,800	do.
Do. (do., another specimen), - - -	990,400	do.
Do. (Memel, across the grain), - - -	42,500	Bevan.
Do. (Scotch. do.), - - - -	24,600	do.
Greenheart, - - - - -	2,656,400	Barlow.
Iron bark (Australia), - - - -	1,669,600	Trickett.
Larch, - - - - - -	616,320	Barlow.
Do. (another specimen), - - -	1,052,800	do.
Mahogany (Honduras), - - -	1,596,000	Tredgold.
Norway spar, - - - - -	1,457,600	Barlow.
Oak (Adriatic), - - - - -	974,400	do.
Do. (African), - - - - -	2,305,400	do.
Do. (Canadian), - - - - -	2,148,800	do.
Do. (Dantzic), - - - - -	1,191,200	do.
Do. (English), - - - - -	1,451,200	do.
Do. (do. inferior), - - - -	873,600	do.
Pine (Pitch), - - - - -	1,225,600	do.
Do. (Red), - - - - -	1,840,000	do.
Do. (do.), - - - - -	1,200,000	Clark.
Do. (American yellow), - - -	1,600,000	Tredgold.
Poon, - - - - - -	1,689,600	Barlow.
Spotted gum (Australia), - - -	1,942,000	Trickett.
Stringy bark (do.), - - - -	1,375,600	do.
Teak, - - - - - -	2,414,400	Barlow.
STONES.		
Marble (White), - - - -	2,520,000	Tredgold.
Quartz Rock (Holyhead, across lamination), -	4,598,000	Mallet.
Do. (do., parallel to lamination),	545,000	do.
Slate (Welsh), - - - - -	15,800,000	Tredgold.
Do. (Westmoreland), - - - -	12,900,000	do.
Do. (Scotch), - - - - -	15,790,000	do.
Do. (Portland), - - - - -	1,533,000	do.
MISCELLANEOUS.		
Whalebone, - - - - -	820,000	Tredgold.
Bone of Beef, - - - - -	2,320,000	Bevan.

Barlow, *Barlow on the Strength of Materials.*

Bevan, *Philosophical Magazine*, 1826, Vol. lxviii., pp. 111, 181.

Clark, *The Britannia and Conway Tubular Bridges*, p. 463.

Fairbairn, *Report of British Association*, 1867.

Hodgkinson, *Report of Commissioners appointed to inquire into the application of Iron to Railway Structures*, 1849, pp. 108, 172.

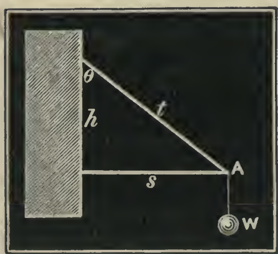
Mallet, *Philosophical Transactions*, 1862, p. 671.
Tredgold, *Tredgold on the Strength of Cast-iron.*
Young, *idem.*
Wertheim, *Résistance des Matériaux, par M. Morin*, p. 46.

9. Mechanical laws—Resolution of forces.—The investigation of transverse strains may be reduced to the three following fundamental laws in mechanics:—

If three forces acting at the same point balance (are in equilibrium), three lines parallel to their directions will form a triangle the sides of which are proportional to the forces. Also, *If two out of three forces which balance meet, the third passes through their point of intersection.*

Hence, it follows that, if we know the magnitude and direction of two intersecting forces, we can find both the magnitude and direction of their resultant; and if the directions of any two components into which a single known force is resolved be given, the amount of these components can be found. Thus, the weight **W**, Fig. 1, is supported by an oblique tie and a horizontal strut. The weight and the strains in the tie and strut meet at **A**, and may be represented by the triangle *h t s*. Let the sides of the triangle be as the numbers 3, 4 and 5; then, if $\mathbf{W} = 3$ tons, t will sustain a tension of 5 tons, and s a thrust or compression of 4 tons. Calling the angle the tie makes with the vertical line θ, the relation between these three forces may be algebraically expressed as follows:—

Fig. 1.

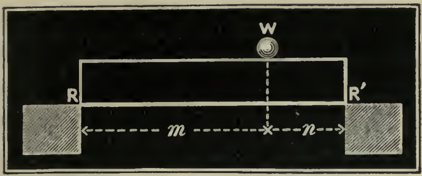


$$t = \mathbf{W} \sec \theta \qquad\qquad s = \mathbf{W} \tan \theta$$

10. The Lever.—*If a weight rest upon a beam supported by two props at its extremities, these props react with two upward pressures whose sum is equal to the weight, and by the principle of the lever the portion of the weight sustained by either prop is to the whole weight as the remote segment is to the whole beam.*

Thus, in Fig. 2, if $\mathbf{W} = 10$ tons and the segments are as 3 : 2, the

Fig. 2.



reaction of the left abutment, R = 4 tons; that of the right, R′ = 6 tons. Calling the segments *m* and *n*, these relations may be algebraically expressed as follows:—

$$R + R' = W, \qquad R = \frac{n}{m+n} W, \qquad R' = \frac{m}{m+n} W.$$

It is obvious that this principle is not affected by any bracing of the beam within itself, provided it merely rests on the points of support.

11. Equality of moments.—*When any number of forces acting in the same plane on a rigid body balance (are in equilibrium), the sum of the moments of the forces tending to turn it in one direction round any given point is equal to the sum of the moments of those tending to turn it in the opposite direction.* Also, *when any number of forces acting in the same plane have a single resultant, the sum of the moments of each force round a given point is equal to the moment of their resultant.**

Thus, in Fig. 2, taking moments round the right abutment, $R \times \overline{m+n} = W n$; the amount of R′ vanishes, since R′ passes through the point round which the moments are taken.

On these three mechanical laws—the *Resolution of Forces*, the law of the *Lever* and the *Equality of Moments*—are founded all the following investigations of the strength of materials when subject to transverse strain.

12. Beam—Girder—Semi-girder.—The term *Beam* is generally applied to any piece of material of considerable scantling, whether subject to transverse strain or not; as for example, "Collar-beam," "Tie-beam," "Bressummer-beam;" the two former being subject to longitudinal strains of compression and tension respectively, and the latter to transverse strain. The term *Girder* is, however, restricted to beams subject to transverse strain and

* The moment of a force round a given point is the product of the force by the perpendicular let fall on its direction from the point.

exerting a vertical pressure merely on their points of support. This term was originally applied to the main beams of floors, but has now become universally adopted by engineers. A *Semi-girder* is a cantilever, that is, a beam fixed at one extremity only and subject to transverse strain; in addition to its vertical pressure it exerts a tendency to overthrow the wall or other structure to which it is attached.

13. Flanged girder—Single-webbed girder—Double-webbed or Tubular girder—Box girder—Tubular bridge.— In the term *Flanged girder* are included not only iron girders of the ordinary **I** form, but also all girders which consist of one or two flanges united to a vertical web, whether the latter be continuous as in plate girders, or open-work as in lattice and bowstring girders. Flanged girders are again subdivided into Single-webbed and Double-webbed or Tubular. A *single-webbed girder* is one whose flanges are connected by a single vertical web. Thus, we have "Single-webbed cast-iron girders," "Single-webbed plate girders," "Single-webbed lattice girders," "Single-webbed bowstring girders," &c. A *Double-webbed* or *Tubular girder* is one whose flanges are connected by a double vertical web, continuous or open-work as the case may be. Small tubular girders formed of continuous plates are sometimes called *Box girders*. A *Tubular bridge* is merely a tubular girder of such large dimensions that the roadway passes through the tube.

In the following theoretic investigations all girders are assumed to be horizontal and without weight, unless otherwise stated.

CHAPTER II.

FLANGED GIRDERS WITH BRACED OR THIN CONTINUOUS WEBS.

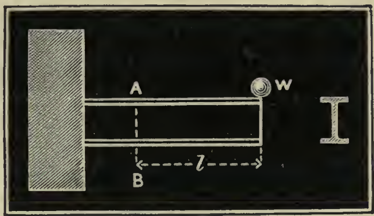
14. Transverse-strain — Shearing-strain. — The formulæ investigated in this chapter are, unless otherwise expressed, applicable to all flanged girders whose webs are formed of bracing, or if continuous, yet so thin that the transverse strength of the web as an independent rectangular girder may be neglected without sensible error. Our knowledge of the strains in this vertical web when continuous is still imperfect. Analogy indeed leads us to conclude that they follow laws similar to those which hold good in braced girders, but in the absence of experimental proof this is to a certain degree conjecture—a conjecture, however, which I feel confident my readers will share after they have had the patience to read through this book.

The mode in which a load affects a girder may be thus analysed. From experience we learn that the load bends the girder downwards and develops longitudinal strains of tension and compression in the flanges. If the semi-girder, represented in Fig. 3, be supposed divided into vertical slices or transverse sections of small thickness, the weight tends to shear or separate the section on which it immediately rests from the adjoining one. The lateral connexion of the sections, however, prevents this separation, and the second section is drawn down by a vertical force equal to the weight which tends to shear it from the third section and so on. Thus, *a vertical force equal to the weight is transmitted from section to section as far as the point of support.* This vertical strain has been aptly named the *Shearing-strain ;* but few writers, until the last few years, have noticed the practical results which follow from the fact that this force can be communicated from section to section only through the medium of some diagonal strain. Respecting the exact directions of the strains which this shearing force develops in a continuous web we know nothing positively; it is probable that they assume various directions crossing each other like close lattice-work, some vertical, some diagonal, perhaps some curved. However this may be, we know that certain of them must be diagonal, since the weight, which is a vertical force, produces strains in the flanges, which are longitudinal, through the medium of the web, which in fact fulfils the part of bracing in a lattice girder. The reader will perceive that we have really three sets of forces to deal with, namely, horizontal, vertical, and diagonal forces. The latter, however, may be resolved into horizontal and vertical components, and thus we have at present only horizontal and shearing forces to consider, recollecting that *the shearing-strain of any transverse section of a girder means the total vertical strain transmitted through that section, including in the term shearing strain the vertical components of diagonal strains.*

15. Horizontal strains in braced or thin continuous webs may be neglected.—When the vertical web of a girder with horizontal flanges is open-work like latticing, the shearing-strain is altogether transmitted through the bracing, the flanges being capable of conveying strains in the direction of their length only; but when the web is continuous, as in a plate-girder, there can be no doubt that a certain amount of shearing-force acts upon the flanges also, so inconsiderable, however, that we may practically neglect it. If, however, one or both flanges are curved, the whole or a considerable portion of the shearing-strain is conveyed through that part of the flange which is sloped, the amount depending upon its angle of inclination. In this case the web has less duty to perform than if the flanges were horizontal, and its sectional area may therefore be reduced. It will also be observed that the diagonal strains developed by the shearing force in a continuous web have horizontal components within the web itself, and consequently, a continuous web aids the flanges to a certain extent, for those parts of the web which adjoin the flanges share the horizontal strains in the latter, and this flange action of the web is greater the thicker the web is. When, however, the web is very thin, the total amount of this flange action of the web is small compared with the strain in the flanges themselves and may therefore be neglected without introducing any serious error. In this chapter all horizontal strains in the web are neglected.

CASE I.—FLANGED SEMI-GIRDER LOADED AT THE EXTREMITY.

Fig. 3.



16. Flanges—At any cross section the horizontal components of strain in the flanges are equal and of opposite kinds—Strength of flanged girders varies directly as the depth and inversely as the length.

Let **W** = the weight,

l = the distance of any cross section **AB** from **W**,

d = the depth of the girder at this cross section,

T = the horizontal strain of tension in the top flange at **A**,

C = the horizontal strain of compression in the bottom flange at **B**.*

The segment **A B W** is held in equilibrium by the weight **W**, the horizontal forces of tension and compression in the flanges at **A** and **B**, and the shearing and horizontal strains in the web at **A B**. Since these forces balance, the sum of the moments of those which tend to turn **A B W** round any point in one direction is equal to the sum of those which tend to turn it round the same point in the opposite direction (**11**). If the point lie in the cross section **A B**, the moment of the shearing force will be cipher, since its direction passes through this point. Neglecting the

* When the flanges are oblique, **T** and **C** represent the horizontal components of their longitudinal strains. The vertical components are a portion of the shearing-strain.

horizontal strain in the web when continuous, and taking moments round **A** and **B** successively, we obtain the following relations:—

$$\mathsf{W}\,l = \mathsf{T}\,d = \mathsf{C}\,d \tag{3}$$

whence,

$$\mathsf{T} = \mathsf{C} \tag{4}$$

that is, *at any cross section the horizontal component of tension in one flange is equal to the horizontal component of compression in the other.*

If **F** represent the horizontal strain in either flange indifferently, we have from eq. 3

$$\mathsf{W} = \frac{\mathsf{F}\,d}{l} \tag{5}$$

$$\mathsf{F} = \frac{\mathsf{W}\,l}{d} \tag{6}$$

Eq. 5 proves that *the weight which a flanged girder is capable of supporting varies directly as the depth and inversely as the length.*

When both flanges are horizontal, we have from eq. 4

$$af = a'f' \tag{6_*}$$

where a and f represent the sectional area and unit-strain of the upper flange, and a' and f' those of the lower flange. Hence, when both flanges are horizontal, *the unit-strains in the flanges are to each other inversely as the areas.*

Ex. 1. A semi-girder, 9 inches deep, supports 7 tons at its extremity; what is the strain in each flange at 12 feet from the load?

Here, $\mathsf{W} = 7$ tons,
$l = 12$ feet,
$d = 9$ inches.

Answer (Eq. 6). $\mathsf{F} = \frac{\mathsf{W}\,l}{d} = \frac{7 \times 12 \times 12}{9} = 112$ tons.

If 4 tons per square inch be a safe working strain in the flanges, the sectional area of each flange should $= \frac{112}{4} = 28$ square inches.

Ex. 2. If the flange be 15 inches wide and $1\frac{1}{2}$ inches deep, what will be the inch-strain?

Here, $a = 22{\cdot}5$ square inches,
$\mathsf{F} = 112$ tons.

Answer. $f = \frac{\mathsf{F}}{a} = \frac{112}{22{\cdot}5} = 5$ tons inch-strain nearly.

Ex. 3. A wrought-iron semi-girder is 7 feet long and 11 inches deep, and each flange is 4 inches wide and ½ an inch thick; what weight at the end will break it across, the tearing inch-strain of wrought-iron being 20 tons?

$$\text{Here, } \mathsf{F} = af = 4 \times {\cdot}5 \times 20 = 40 \text{ tons,}$$
$$d = 11 \text{ inches,}$$
$$l = 7 \text{ feet.}$$
$$\textit{Answer} \text{ (Eq. 5).} \quad \mathsf{W} = \frac{\mathsf{F}\,d}{l} = \frac{40 \times 11}{7 \times 12} = 5{\cdot}24 \text{ tons.}$$

17. Girder of greatest strength—Areas of horizontal flanges should be to each other in the inverse ratio of their ultimate unit-strains.—The distribution of a given amount of material in the flanges, so as to produce the girder of greatest strength, occurs when both flanges are simultaneously on the point of rupture, for if either flange contain more material than is required to sustain its proper strain when the other gives way, it can spare some of the surplus material to strengthen the other. When both flanges are on the point of rupture, f and f' are the ultimate unit-strains of tension and compression, and since $\frac{a}{a'} = \frac{f'}{f}$, it follows that, to ensure the greatest strength with a given amount of material in a girder with horizontal flanges, *the sectional areas of the flanges should be to each other inversely as their ultimate unit-strains*—a result amply confirmed by experience.

18. Shearing-strain—The web should contain no more material than is requisite to convey the shearing-strain—The quantity of material in the web of girders with parallel flanges is theoretically independent of their depth.—*The shearing-strain is the same at each vertical section of the semi-girder and equals* W (**14**). If the flanges are parallel this strain is transmitted from section to section of the web (**15**), which should therefore have the same sectional area throughout and be sufficiently strong to transmit the shearing-strain to the wall or point of support. *The web should also for economical reasons contain no more material than is requisite to transmit the shearing-strain*, for any surplus material, if placed in the flanges, would increase the strength of the girder more than if it were to remain in the web, since its leverage to sustain horizontal strains would be thereby increased. This will appear clearer when the reader has perused the succeeding chapters. From these considerations it follows that *the quantity of material required in the web of a girder with parallel flanges is theoretically independent of the depth.*

19. Girder of uniform strength—Economical distribution of material.—A girder of uniform strength is one in which all parts, both flanges and web, are duly proportioned to the strain which they have to bear, *i.e.*, are equally capable of sustaining the particular strain which is transmitted through them. If such a girder were perfect, there is no reason why any one part should fail before another, since the train in each part is the same sub-multiple of the ultimate or breaking-strain of that part. The girder of uniform strength is obviously the most economical also in its proportions, for no part has a wasteful excess of material; *the tensile or compressive unit-strain is constant throughout the entire length of each flange respectively, and the shearing-unit-strain in each section of the web is the same as in every other section.*

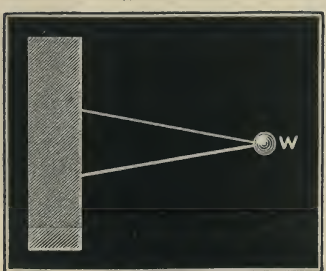
20. Flange-area of semi-girder of uniform strength when the depth is constant.—From eq. 6 we have when both flanges are horizontal,

$$f = \frac{\mathrm{W}l}{ad} \tag{7}$$

where f and a express the unit-strain and sectional area of either flange indifferently at a distance l from the extremity.

In a girder of uniform strength f is constant for all values of l, and the quantity $\frac{l}{a}$, to which f is proportional (since by hypothesis the depth d is uniform), will be constant for every value of l; consequently a, that is, the area of each flange, will vary as l, and if the depth of the flange be uniform, its breadth will vary as l, and the plan of the flange will be triangular, as in Fig. 4.

Fig. 4.—Plan.



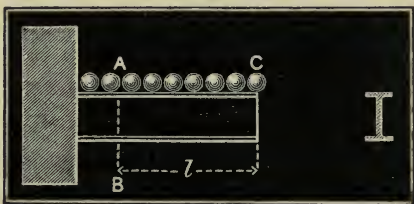
21. Depth of semi-girder of uniform strength when the flange-area is constant.—If, however, one flange be sloped, f

Fig. 5.—Elevation.

and a in eq. 7 apply to the horizontal flange only; hence, if its sectional area and unit-strain remain uniform, d will vary directly as l, and the side elevation of the girder will be triangular as in Fig. 5. The strain in the oblique flange exceeds that in the horizontal flange in the ratio of their lengths (**9**). This is due to the shearing-strain, which is entirely transmitted through the oblique flange in addition to a horizontal strain of the same amount as that in the horizontal flange, and the longitudinal strain in the oblique flange is their resultant. In this case the web has no duty to perform and may therefore be omitted, the girder becoming the simplest form of truss, viz., a triangle.

CASE II.—FLANGED SEMI-GIRDER LOADED UNIFORMLY.

Fig. 6.



22. Flanges.—Let w = the load per unit of length,

l = the distance of any cross section **A B** from the end of the girder,

d = the depth of the girder at this cross section,

W = wl = the load on **A C**,

F = the total horizontal strain exerted by either flange at **A** or **B**, that is, the horizontal component of the longitudinal strain if the flange is oblique.

The forces which keep **A B C** in equilibrium are the weights uniformly distributed along **A C**, the horizontal strains of tension and compression in the flanges at **A** and **B**, and the shearing and horizontal strains in the web at the plane of section **A B**. If the web be continuous and very thin, we may, as in the previous case, neglect the moments of the horizontal strains in the web as insignificant compared with those of the other horizontal forces. The sum of the moments round **A** or **B** of each weight in the length l is equal to the sum of the weights multiplied by the distance of their centre of gravity from **A** or **B** (**11**), that is, their collective moments $= wl\frac{l}{2}$. Equating this to the amount of the horizontal strain in either flange round **A** or **B**, we obtain the following relations:—

$$w\frac{l^2}{2} = \mathsf{F}d \tag{8}$$

$$\mathsf{W} = wl = \frac{2\mathsf{F}d}{l} \tag{9}$$

$$\mathsf{F} = \frac{wl^2}{2d} = \frac{\mathsf{W}l}{2d} \tag{10}$$

Ex. 1. A cast-iron semi-girder, 8 feet long and 13 inches deep, supports a uniform load of 1 ton per running foot; what area should the top flange have at the abutment in order that its inch-strain may not exceed 1·5 tons?

Here, $w = 1$ ton per foot,
$l = 8$ feet,
$d = 13$ inches,
$f = 1{\cdot}5$ tons.

From eq. 10, $\mathsf{F} = \frac{wl^2}{2d} = \frac{1 \times 8 \times 8 \times 12}{2 \times 13} = 29{\cdot}5$ tons.

Answer (eq. 1). $a = \frac{\mathsf{F}}{f} = \frac{29{\cdot}5}{1{\cdot}5} = 19{\cdot}7$ inches.

Ex. 2. The lattice-bridge at the Boyne Viaduct is in three spans. Each side span is 140 feet 11 inches long and 22 feet 3 inches deep. The permanent load supported by one main girder of a side span equals 0·68 tons per running foot, and the gross sectional area of its lower flange over each pier is 127 inches. On one occasion an extraordinary load in the centre span depressed it to such an extent as to raise the ends of the side spans off the abutments, thus forming each side span into a semi-girder. What was the compressive inch-strain in the lower flange at the piers?

Here, $w =$ 0·68 tons per foot?
$l = 140{\cdot}92$ feet,
$d =$ 22·25 feet,
$a = 127$ inches.

Answer (eq. 10). $f = \frac{\mathsf{F}}{a} = \frac{wl^2}{2ad} = \frac{{\cdot}68 \times 140{\cdot}92 \times 140{\cdot}92}{2 \times 127 \times 22{\cdot}25} = 2{\cdot}4$ tons inch-strain.

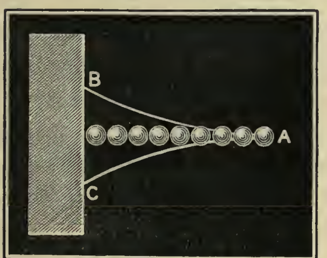
23. Web—Shearing-strain.—When a semi-girder is uniformly loaded the shearing-strain at any cross section is equal to the sum of the weights between it and the extremity of the girder, since this is the pressure transmitted through that section to the wall (**14**). *The shearing-strain therefore equals wl, and varies directly in proportion to the distance from the extremity of the girder, that is, directly as the ordinates of a triangle.* When the flanges are parallel, nearly all the shearing-strain passes through the web, and its sectional area should for economical reasons vary in this ratio also, for any excess of material in the web beyond that required to transmit the shearing-strain is valuable only for horizontal strains, and would act with greater leverage, and therefore with greater effect, if placed in the flanges.

24. Flange-area of semi-girder of uniform strength when the depth is constant.—From eq. 10 we have, when both flanges are horizontal

$$f = \frac{wl^2}{2ad} = \frac{\mathrm{W}l}{2ad} \tag{11}$$

where a and f represent the area and unit-strain of either flange indifferently at a distance l from the extremity. If the girder be of uniform strength, the unit-strain in each flange will be uniform throughout its length, and the quantity $\frac{l^2}{a}$, to which f is proportional, will be constant, that is, the sectional area of each flange will vary as l^2. Hence, if the depth of the flange be uniform, its breadth will vary as l^2, and the plan of the flange will, if symmetrical, be bounded by two parabolas whose common vertex is at **A**, Fig. 7, with the axis perpendicular to the length of the girder.

Fig. 7.—Plan.



25. Depth of semi-girder of uniform strength when the flange-area is constant.—If one flange be horizontal and the other curved, f and a, in eq. 11, apply to the horizontal flange only; hence, if its sectional area be constant and if the girder be of

Fig. 8.—Elevation.

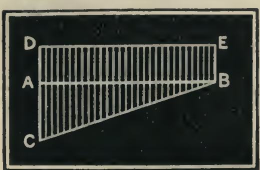
uniform strength, d will vary as l^2, and the side elevation of the girder will be bounded by a parabola whose vertex is at **A**, Fig. 8, with its axis vertical. In this case it may be shown that the whole shearing-strain passes through the curved flange, and the web has no duty to perform unless the load rest upon the horizontal flange, in which case pillars, represented by vertical lines (or suspension rods if Fig. 8 be inverted with the weights beneath), are requisite for conveying the pressure of each successive weight to the curved flange.

26. Strain in curved flange.—The longitudinal strain in the curved flange is the resultant of the shearing-strain and a horizontal compression, the latter being equal to the tension in the horizontal flange. If therefore, the lines **A** 1, **A** 2, **A** 3, &c., Fig. 9, represent the shearing-strains at different points, and if the horizontal line **A B** represent **F** (or the uniform horizontal compression), then the sloped lines **B** 1, **B** 2, **B** 3, &c., will represent the longitudinal strains in the curved flange at these several points (**9**).

Fig. 9.

27. Semi-girder loaded uniformly and at the extremity also, shearing-strain.—If, in addition to a uniformly distributed load, the semi-girder support a weight **W′** at its extremity, the shearing-strain at any section will equal **W′** + wl. Consequently, when the flanges are parallel, the area of the web should increase in arithmetical ratio as it approaches the wall and may be represented by the ordinates of a truncated triangle. If, for instance, the line **A B**, Fig 10, represent the length of a uniformly loaded semi-girder, and if **A C** represent the whole distributed load, that is, the shearing-strain at the wall, then the ordinates of the triangle **A B C** will represent the shearing-strain at each point. Now, let an additional weight **W′** be

Fig. 10.—Shearing-strain.



suspended from the end of the girder at B, then, if B E represent this weight, the ordinates of the rectangle A D E B will represent the shearing-strains produced by it alone; and when the girder supports both it and the uniform load, the collective shearing-strains are represented by the ordinates of the trapezium C D E B.

28. Flange-area of semi-girder of uniform strength loaded uniformly and at the end when the depth is constant.— When both flanges are horizontal and the semi-girder supports a uniformly distributed load in addition to the weight W′ at its extremity, we have from eqs. 7 and 11,

$$f = \frac{2W'l + wl^2}{2ad} \qquad (12)$$

Where a and f represent the area and unit-strain of either flange indifferently at a distance l from the extremity. If the semi-girder be of uniform strength, f will be constant and a will vary as $l\,(2W' + wl)$, and, if the depth of the flange be uniform, its breadth will vary in the same ratio. Consequently, the plan of the flanges will, if symmetrical, be bounded by a pair of parabolas, differing however, from Fig. 7 in the position of their vertices.

29. Depth of semi-girder of uniform strength loaded uniformly and at the end when the flange-area is constant.— If, however, one flange be horizontal and the other curved, f and a, in eq. 12, apply to the horizontal flange only; hence, if its area be uniform, d will vary as $l\,(2W' + wl)$, and the elevation of the girder will be bounded by a parabola.

Ex. A semi-girder, 44·7 feet long and 22·25 feet deep, supports a uniformly distributed load of 1·82 tons per running foot in addition to a weight of 161·6 tons at the extremity. What is the inch-strain on the net section of the tension flange at the point of support, its gross area being 132·6 inches, but reduced by rivet-holes to the extent of $\frac{2}{9}$ths?

$$\text{Here,}\quad W' = 161{\cdot}6 \text{ tons},$$
$$l = 44{\cdot}7 \text{ feet},$$
$$d = 22{\cdot}25 \text{ feet},$$
$$w = 1{\cdot}82 \text{ tons per foot},$$
$$a = \frac{7 \times 132{\cdot}6}{9} = 103{\cdot}13 \text{ square inches.}$$

Answer (eq. 12). $f = \dfrac{2 \times 161{\cdot}6 \times 44{\cdot}7 + 1{\cdot}82 \times 44{\cdot}7 \times 44{\cdot}7}{2 \times 103{\cdot}13 \times 22{\cdot}25} = 3{\cdot}94$ tons inch-strain.

CASE III.—FLANGED GIRDER SUPPORTED AT BOTH ENDS AND LOADED AT AN INTERMEDIATE POINT.

Fig. 11.

30. Flanges.—Let W = the weight, dividing the girder into segments containing respectively m and n linear units,

$l = m + n$ = the length of the girder,

d = the depth at any given cross section **A B**,

x = the distance of this cross section from the end of the segment in which it occurs,

F = the horizontal strain exerted by either flange at **A** or **B**, that is, the horizontal component of the longitudinal strain if the flange be oblique.

On the principle of the lever (**10**), the reaction of the left abutment $= \frac{n}{l}\mathsf{W}$, and **A B C** is held in equilibrium by this reaction of the left abutment, the horizontal flange-strains at **A** and **B**, the shearing-strain in the cross section **A B**, and the horizontal strains in the web when continuous. Neglecting these latter when the web is thin, and taking the moments of the other forces round **A** or **B**, we obtain the following relations:—

$$\frac{n}{l}\mathsf{W}\,x = \mathsf{F}\,d \qquad (13)$$

$$\mathsf{W} = \frac{\mathsf{F}dl}{nx} \qquad (14)$$

$$\mathsf{F} = \frac{nx\mathsf{W}}{dl} \qquad (15)$$

31. Maximum flange-strains occur at the weight.—If the cross section be taken at the weight, $x = m$, and eqs. 14 and 15 become

$$\mathsf{W} = \frac{\mathsf{F}dl}{mn} \tag{16}$$

$$\mathsf{F} = \frac{mn\mathsf{W}}{dl} \tag{17}$$

x attains its greatest value when it equals m; hence, comparing eqs. 15 and 17, we find that the horizontal strain at any point in either flange attains its greatest value when the weight rests there.

32. Concentrated rolling load, maximum strains in flanges are proportional to the rectangle under the segments.—If W is a rolling load and the flanges are parallel, the maximum strain at any point in either flange occurs when the load is passing that point and is proportional to mn, that is, to the rectangle under the segments.

33. Weight at centre.—This rectangle attains its greatest value when the weight is at the centre, in which case eqs. 16 and 17 become

$$\mathsf{W} = \frac{4\mathsf{F}d}{l} \tag{18}$$

$$\mathsf{F} = \frac{l\mathsf{W}}{4d} \tag{19}$$

Ex. 1. A cast-iron girder is 26 feet long and $27\frac{1}{2}$ inches deep, and the area of the bottom flange $= 16 \times 3 = 48$ inches. If the tearing inch-strain of cast-iron be 7 tons, what weight laid on the middle of the girder will break it across by tearing the bottom flange, omitting any strength which may be derived from the web?

Here, $l = 26$ feet,
$d = 27{\cdot}5$ inches,
$f = 7$ tons inch-strain,
$a =$ the area of the bottom flange $= 48$ inches,
$\mathsf{F} = fa = 7 \times 48 = 336$ tons.

Answer (eq. 18). $\mathsf{W} = \frac{4\mathsf{F}d}{l} = \frac{4 \times 336 \times 27{\cdot}5}{12 \times 26} = 118{\cdot}5$ tons nearly.

Ex. 2. In an experiment made by Mr. G. Berkley,* a small double-flanged cast-iron girder was broken by 18 tons in the centre. The following were the dimensions:—

Effective length, $l = 57$ inches,
Total depth, $d = 5{\cdot}125$ inches,
Area of top flange, $a_1 = 2{\cdot}33 \times 0{\cdot}31 = 0{\cdot}72$ sq. inches,
Area of bottom flange, $a_2 = 6{\cdot}67 \times 0{\cdot}66 = 4{\cdot}4$ sq. inches,
Thickness of web, $= 0{\cdot}266$ inches.

* Proc., I. C. E., Vol. xxx., p. 254.

What was the inch-strain in each flange at the centre of the girder at the moment of fracture?

Ans. (eq. 19). Inch-strain in top flange $f' = \frac{lW}{4a_1d} = \frac{57 \times 18}{4 \times 0{\cdot}72 \times 5{\cdot}125} = 69{\cdot}5$ tons.

$$\text{Inch-strain in lower flange } f = \frac{lW}{4a_2d} = \frac{57 \times 18}{4 \times 4{\cdot}4 \times 5{\cdot}125} = 11{\cdot}37 \text{ tons.}$$

It is not recorded which flange failed first, but as the tensile strength of the metal was proved by direct experiment to be very high, namely, 13·94 tons per square inch, and as the inch-strain in the bottom flange fell considerably short of this, the girder probably failed by the crushing of the top flange, the inch-strain in which, however, was so unusually high, even for cast-iron, that this flange no doubt derived considerable aid from the web.

Ex. 3. In an experiment recorded by Sir William Fairbairn,* a girder, cast from a mixture of Gartsherrie, Dundyvan and Hæmatite Irons, 27 feet 4 inches long, 18 inches deep, and whose lower flange was 10 inches wide and 1½ inch thick, was broken by a weight of 29½ tons in the centre. What was the inch-strain at the centre of the lower flange at the moment of rupture, supposing that it derived no aid from the web which was ¾ inch thick?

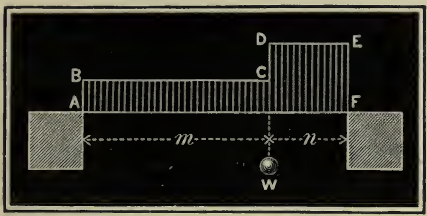
$$\begin{aligned} \text{Here,}\quad l &= 27{\cdot}33 \text{ feet,}\\ d &= 1{\cdot}5 \text{ feet,}\\ a &= 15 \text{ sq. inches,}\\ W &= 29{\cdot}5 \text{ tons.}\end{aligned}$$

$$\textit{Answer (eq. 19).}\ f = \frac{lW}{4ad} = \frac{27{\cdot}33 \times 29{\cdot}5}{4 \times 15 \times 1{\cdot}5} = 8{\cdot}96 \text{ tons.}$$

34. Web, shearing-strain.—*The shearing-strain in each segment is uniform throughout that segment and equals the pressure which is transmitted through it to the abutment* (**14**). Thus, in Fig. 11, the shearing-strain at $AB = \frac{n}{l}W =$ the reaction of the left abutment. This shearing-strain is uniform throughout the left segment, while that in the right segment is also uniform and equals $\frac{m}{l}W$. When both flanges are horizontal, nearly all the shearing-strain is transmitted through the web (**15**), and each segment should have its web of uniform area adequate to sustain a shearing-strain equal to the reaction of the adjacent abutment. This may be represented graphically as follows:—let the line

* Application of Iron to building purposes, p. 171.

Fig. 12.



A F represent the length of the girder, divided by **W** into the segments m and n, and let the ordinate **A B** represent the reaction of the left abutment, $= \frac{n}{l}\mathbf{W}$, and let **F E** represent the reaction of the right abutment, $= \frac{m}{l}\mathbf{W}$; then the ordinates of the rectangle **A B C W** will represent the shearing-strains at each point in the left segment, and those of the rectangle **W D E F** will represent the shearing-strains at each point in the right segment. The sectional area of the web should therefore be proportional to these ordinates when both flanges are horizontal. When a single weight is at the centre of the girder, the rectangles become equal, and, if both flanges are horizontal, the section of the web should be uniform throughout its whole length, as it sustains a uniform shearing-strain $= \frac{\mathbf{W}}{2}$.

35. Single fixed load, flange-area of girder of uniform strength when the depth is constant.—When both flanges are horizontal, we have from eq. 15,

$$f = \frac{nx\mathbf{W}}{adl} \tag{20}$$

where f and a represent the unit-strain and area of either flange at a distance x measured from the abutment. When the girder is of uniform strength, f is constant throughout each flange, and a will vary as x. Hence, if the depth of the flange be uniform, its width will vary as x, and the plan of the flange will be two triangles united at their bases, as in Fig. 13.

Fig. 13.—Plan.

Ex. 1. A girder (see Fig. 11), 50 feet long and 4 feet deep, supports a load of 16 tons

at 9 feet from one end; what should be the area of the top flange in the middle of the girder so that the inch-strain may not exceed 4 tons?

Here, $W = 16$ tons,
$l = 50$ feet,
$d = 4$ feet,
$f = 4$ tons inch-strain,
$n = 9$ feet,
$x = 25$ feet.

Answer (eq. 20). $a = \dfrac{nxW}{dfl} = \dfrac{9 \times 25 \times 16}{4 \times 4 \times 50} = 4\frac{1}{2}$ square inches.

Ex. 2. What is the strain in either flange at the load?

Here, $m = 41$ feet.

Answer (eq. 17). $F = \dfrac{mnW}{dl} = \dfrac{41 \times 9 \times 16}{4 \times 50} = 29{\cdot}5$ tons.

Ex. 3. What is the shearing-strain in each segment?

Answer. The segments are respectively 9 and 41 feet long, and the shearing-strain throughout the shorter segment $= \dfrac{41}{50} \times 16 = 13{\cdot}12$ tons, and that throughout the longer segment $= \dfrac{9}{50} \times 16 = 2{\cdot}88$ tons.

36. Single fixed load, depth of girder of uniform strength when the flange-area is constant.—If, however, one flange be horizontal and the other sloped, f and a, in eq. 20, apply to the horizontal flange only, and if its area be uniform, d will vary as x, and the elevation of the girder will be a triangle whose apex is at the weight, Fig. 14. In this case the shearing-strain is transmitted through the oblique flange; the web may therefore be omitted and the girder becomes the simplest form of truss. The longitudinal strain in the oblique flange may be calculated according to the principle explained in **9**. When the weight rests upon the horizontal flange, a strut h is required of sufficient strength to support W and transmit its weight to the apex.

Fig. 14.—Elevation.

37. Concentrated rolling load, shearing-strain.—If the weight be a rolling load, the shearing-strain in either segment varies directly as the length of the other segment (**34**). Consequently, it attains its greatest value at each point just as the weight passes, when it suddenly changes both in amount and in the direction in which it is transmitted, to the right or left abutment as the case may be. In this case the maximum shearing-strain at each section is proportional to its distance from the farther abutment and, if both flanges be horizontal, the area of the web should increase in the same ratio also—*i.e.*, as the ordinates of the figure **A B C D E**, Fig. 15, in which the horizontal line **A B** represents the length of the girder, and each of the vertical lines **A E** and **B C** represents the weight of the passing load.

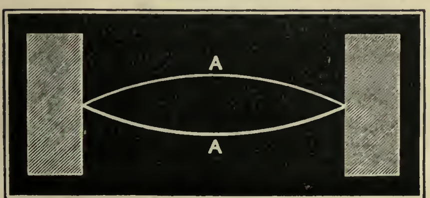
Fig. 15.—Shearing-strain.

38. Concentrated rolling load, flange-area of girder of uniform strength when the depth is constant.—In the case of a single load traversing a girder both of whose flanges are horizontal, we have at the place the weight is passing, from eq. 17,

$$f = \frac{mn\mathbf{W}}{adl} \tag{21}$$

where a and f represent the area and maximum unit-strain of either flange at the weight, and m and n represent the lengths of the two segments into which the weight divides the girder at the moment of passing. If the girder be of uniform strength, f will be constant throughout each flange, and a will vary as the rectangle mn. Hence, if the depth of the flange be uniform, its breadth will vary as mn also, and the plan of the flange, if symmetrical, will be formed by the overlap of two parabolas whose vertices are at **A A**, Fig. 16.

Fig. 16.—Plan.



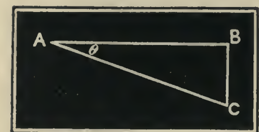
39. Concentrated rolling load, depth of girder of uniform strength when the flange-area is constant.—If, however, one flange be horizontal and the other curved, f and a apply to the

Fig. 17.—Elevation.

horizontal flange only, and, if its section be uniform, d will vary as mn. Hence, the elevation of the curved flange will be a parabola whose axis is vertical and its vertex at **A**, Fig. 17.

40. Concentrated rolling load, strain in curved flange—Section of curved flange.—The maximum longitudinal strain at any point in the curved flange of Fig. 17, *i.e.*, the strain when the weight rests over that point, may be thus obtained. Eq. 17 proves that the horizontal component of this longitudinal strain is equal to the strain in the horizontal flange at the same cross section; it is therefore a known quantity, and the longitudinal strain may be found from it as follows:—Let the line **A B**, Fig. 18, represent **F**,

Fig. 18.

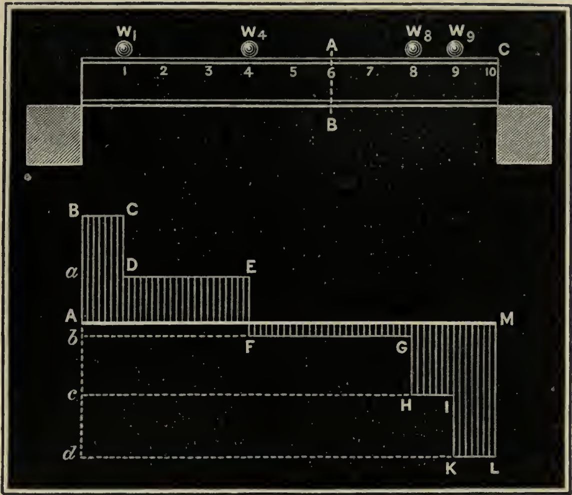


i.e., the horizontal component; draw **A C** parallel to the tangent of the curve at the given point, and draw **B C** perpendicular to **A B**; then **A C** will represent the maximum longitudinal strain at the given point, and **B C** will represent its vertical component, or that portion of the shearing-strain which is transmitted through the curved flange (**9**); the remainder of the shearing-strain passes through the web, which indeed prevents the girder from assuming a form similar to Fig. 14, a result that would occur were the curved flange flexible like a chain and the web absent.

From what has just been stated it appears that the longitudinal strain in the curved flange from a single rolling load $=$ **F** $sec\,\theta$ where θ represents the inclination of the flange to a horizontal line, and its sectional area should increase therefore as it approaches the abutments in proportion to $sec\,\theta$, since, by hypothesis, **F** is constant.

CASE IV.—FLANGED GIRDER SUPPORTED AT BOTH ENDS AND LOADED AT IRREGULAR INTERVALS.

Figs. 19 and 20.



41. Flanges.—When several weights rest upon a girder, the strain at any point in either flange is equal to the sum of the strains due to each weight acting separately. An example in which numbers are mixed with symbols will illustrate the method of calculation better than symbols alone. Let the girder represented in Fig. 19 be divided into any convenient number of equal parts or units of length, say 10; and let it be loaded with any number of weights of different magnitudes, say 4, placed at irregular intervals, as in the figure.

Let $\mathbf{W}_1$, $\mathbf{W}_4$, $\mathbf{W}_8$, $\mathbf{W}_9$. = the several weights,

l = the length of the girder (divided into 10 units),

d = the depth at any given cross section **AB**, measured in the same units as l,

F = the horizontal strain exerted by either flange at **A** or **B**, that is, the horizontal component of the longitudinal strain if the flange be oblique.

On the principle of the lever, the reaction of the right abutment

$$= \frac{1}{l}\mathsf{W}_1 + \frac{4}{l}\mathsf{W}_4 + \frac{8}{l}\mathsf{W}_8 + \frac{9}{l}\mathsf{W}_9$$

$$= \frac{1}{l}(\mathsf{W}_1 + 4\,\mathsf{W}_4 + 8\,\mathsf{W}_8 + 9\,\mathsf{W}_9),$$

and the segment **A B C** is held in equilibrium by the reaction of the right abutment acting upwards, the weights W_8 and W_9 pressing downwards, the horizontal flange-strains at **A** and **B**, the shearing-strain in the cross section **A B**, and the horizontal strains in the web when continuous. Neglecting the latter when the web is thin, and taking moments round **A** or **B**, we have

$$\mathsf{F}d = \frac{4}{l}(\mathsf{W}_1 + 4\,\mathsf{W}_4 + 8\,\mathsf{W}_8 + 9\,\mathsf{W}_9) - 2\,\mathsf{W}_8 - 3\,\mathsf{W}_9$$

arranging, we have

$$\mathsf{F} = \frac{1}{ld}(4\,\mathsf{W}_1 + 16\,\mathsf{W}_4 + 12\,\mathsf{W}_8 + 6\,\mathsf{W}_9).$$

If the weights are of equal magnitude, this becomes

$$\mathsf{F} = \frac{38\,\mathsf{W}}{ld} = \frac{3.8\,\mathsf{W}}{d}.$$

42. Web, shearing-strain.—Bearing in mind the definition given in **14**, it will be apparent that *the shearing-strain at any cross section = those portions of the weights in the left segment which are conveyed to the right abutment minus those portions of the weights in the right segment which are conveyed to the left abutment.* Thus, in the foregoing example,

the shearing-strain at **A B** $= \frac{1}{l}(\mathsf{W}_1 + 4\,\mathsf{W}_4 - 2\,\mathsf{W}_8 - \mathsf{W}_9).$

The shearing-strain may also be derived from another consideration as follows. The vertical forces acting on the right segment **A B C** are:—the reaction of the right abutment acting upwards, the weights W_8 and W_9 pressing downwards, and the shearing-strain at **A B**. The only other forces are horizontal, namely, the horizontal components of the flange-strains at **A** and **B**; consequently, the vertical forces must balance each other, for otherwise there would be motion, and we may therefore define the shearing-strain at any cross section to be *the algebraic sum of the external* *forces on either side of the section, forces acting upwards being positive and those acting downwards being negative.* For example, we have the shearing-strain at **A B** = the reaction of the right abutment minus the intermediate weights W_8 and W_9

$$= \frac{1}{l}(W_1 + 4W_4 + 8W_8 + 9W_9) - (W_8 + W_9)$$

$$= \frac{1}{l}(W_1 + 4W_4 - 2W_8 - W_9)$$

as before. If the weights are of equal magnitude, this becomes $\frac{2W}{l} = 0{\cdot}2\,W$.

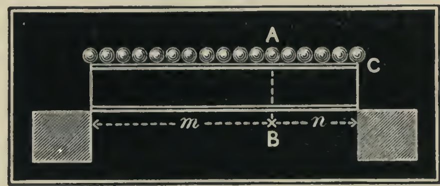
The shearing-strain with irregular loading may be represented graphically as follows:—Using the same example as before, let the line **A M**, Fig. 20, represent the length of the girder, and let the ordinates **A B** and **M L** represent to a scale of weights the shearing-strains at the ends, that is, the reactions of the abutments; then **B**d will equal the sum of all the weights; mark off **B**a, ab, bc and cd respectively equal to W_1, W_4, W_8 and W_9, and draw horizontal lines through these points till they intersect vertical lines drawn through the weights. The ordinates of the stepped figure **A B C D E F G H I K L M**, indicated by lines of shading, will represent the shearing-strains in the web, and the line **E F** shows where they part to the right and left.

Ex. A girder, 267 feet long and 22 feet 3 inches deep, supports three locomotives, weighing 25 tons each, at points whose distances from the left abutment are respectively 19, 75 and 230 feet. What are the flange-strains and the shearing-strain at 180 feet from the left abutment?

Answer. The reaction of the right abutment $= \frac{19 + 75 + 230}{267} \times 25 = 30{\cdot}34$ tons, and the strain in either flange at 180 feet from the left abutment $= \frac{30{\cdot}34 \times 87 - 25 \times 50}{22{\cdot}25}$ $= 62{\cdot}45$ tons. The shearing-strain at the same point $= 30{\cdot}34 - 25 = 5{\cdot}34$ tons.

CASE V.—FLANGED GIRDER SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

Fig. 21.



43. Flanges.—Let l = the length of the girder,

d = the depth of the girder at any given cross section **A B**,

w = the load per unit of length,

$\mathsf{W} = wl$ = the whole load,

F = the horizontal strain exerted by either flange at **A** or **B**, that is, the horizontal component of the longitudinal strain if the flange be oblique,

m and n = the segments into which the section **A B** divides the girder.

The forces which keep **A B C** in equilibrium are the reaction of the right abutment, $= \frac{wl}{2}$, the weights uniformly distributed along **A C**, $= wn$, the horizontal strains of compression and tension in the flanges at **A** and **B**, the shearing-strain in the plane of section **A B**, and the horizontal strains in the web when continuous. Neglecting these latter forces when the web is thin, and taking the moments of the remainder round either **A** or **B**, we have (**11**)—

$$\frac{wl}{2}n - wn\frac{n}{2} = \mathsf{F}d \tag{22}$$

whence

$$\mathsf{F} = \frac{wmn}{2d} = \frac{mn\mathsf{W}}{2dl} \tag{23}$$

and

$$\mathsf{W} = \frac{2\mathsf{F}dl}{mn} \tag{24}$$

Ex. A wrought-iron plate girder, 50 feet long and 4 feet deep, supports a uniformly distributed load of 32 tons; what is the strain in either flange at 9 feet from one end?

$$\begin{aligned} \text{Here,} \quad W &= 32 \text{ tons}, \\ l &= 50 \text{ feet}, \\ d &= 4 \text{ feet}, \\ m &= 9 \text{ feet}, \\ n &= 41 \text{ feet}. \end{aligned}$$

$$\textit{Answer} \text{ (eq. 23).} \quad F = \frac{mnW}{2dl} = \frac{9 \times 41 \times 32}{2 \times 4 \times 50} = 29{\cdot}5 \text{ tons.}$$

If 4 tons per square inch be a safe strain, the area of the flange should $= \frac{29{\cdot}5}{4} = 7{\cdot}4$ square inches.

44. Strains at centre of girder.—At the centre of the girder $m = n = \frac{l}{2}$ and we have from eq. 23,

$$F = \frac{Wl}{8d} = \frac{wl^2}{8d} \tag{25}$$

and

$$W = \frac{8Fd}{l} \tag{26}$$

Ex. 1. A segment of either side span of the Boyne Viaduct, 101·2 feet long and 22·25 feet deep, supports a uniform load of 1·68 tons per running foot; what is the strain at the centre of either flange?

$$\begin{aligned} \text{Here,} \quad l &= 101{\cdot}2 \text{ feet}, \\ d &= 22{\cdot}25 \text{ feet}, \\ w &= 1{\cdot}68 \text{ tons per running foot.} \end{aligned}$$

$$\textit{Answer} \text{ (eq. 25).} \quad F = \frac{wl^2}{8d} = \frac{1{\cdot}68 \times 101{\cdot}2 \times 101{\cdot}2}{8 \times 22{\cdot}25} = 96{\cdot}6 \text{ tons.}$$

Ex. 2. The Conway tubular bridge is 412 feet long from centre to centre of bearings, and 23·7 feet deep from centre of top cells to centre of bottom cells at the centre of the bridge. The weight of wrought-iron in one tube, 412 feet long, is 1,147 tons, which, however, is not quite uniformly distributed, as the sectional area of the tube is greater at the centre than at the ends in the ratio of $\frac{14}{11}$. Making an extra allowance for this, and adding the weight of the permanent way and the light galvanized iron roof, we may assume the total permanent load to be equivalent to 1,250 tons uniformly distributed. What is the permanent strain in either flange at the centre of the girder from this dead load?

$$\textit{Answer} \text{ (eq. 25).} \quad F = \frac{Wl}{8d} = \frac{1{,}250 \times 412}{8 \times 23{\cdot}7} = 2{,}716 \text{ tons.}$$

The gross area of the top flange at the centre of the bridge is 645 square inches; that of the bottom or tension flange is 536 square inches. If we assume that the weakening effect of rivet holes in the tension flange is equivalent to the aid which the continuous webs gives the flange, which is the same thing as if we suppose the gross area of the flange available for tension, we have the permanent tensile inch-strain at the centre of the lower flange $= \frac{2,716}{536} = 5 \cdot 067$ tons. The collective area of the two sides, *i.e.*, of the web, at the centre of the bridge, is 257 square inches, and it will be shown in Chap. IV. that a continuous web theoretically aids the flanges as much as if one-sixth of its area were added to each flange. Assuming then that $\frac{257}{6}$, $= 43$ square inches, are added to the compression flange, we have its permanent inch-strain $= \frac{2,716}{645 + 43} = 3 \cdot 948$ tons. These calculations, it will be observed, are based on the hypothesis that the web gives its full theoretical aid to the flanges, which is much too liberal an allowance to make in reality. A train-load of $\frac{3}{4}$ ton per running foot, $= 309$ tons uniformly distributed over one line of way, will increase the permanent unit-strains by nearly one-fourth, or more accurately, the inch-strain in the tension flange at the centre of the bridge will $= 6 \cdot 32$ tons and that in the compression flange will $= 4 \cdot 924$ tons.

Ex. 3. What are the flange-strains in one of the Conway tubes from the permanent load at the quarter-spans where the depth from centre to centre of cells $= 22 \cdot 25$ feet?

$$\begin{aligned} \text{Here,} \quad W &= 1,250 \text{ tons}, \\ l &= 412 \text{ feet}, \\ d &= 22 \cdot 25 \text{ feet}, \\ m &= \frac{l}{4}, \\ n &= \frac{3l}{4}, \end{aligned}$$

Answer (eq. 23). $F = \frac{mnW}{2dl} = \frac{3lW}{32d} = \frac{3 \times 412 \times 1250}{32 \times 22 \cdot 25} = 2,170$ tons.

The gross area of the top flange at each quarter-span $= 566$ square inches, that of the bottom or tension flange $= 461$ square inches. If we assume, as before, that the aid which the continuous sides theoretically give the tension flange compensates for the weakening effect of rivet holes, we have the permanent tensile inch-strain in the lower flange at each quarter-span $= \frac{2,170}{461} = 4 \cdot 707$ tons.

The area of both sides of the tube together at each quarter-span $= 241$ square inches, and if we assume, as before, that one-sixth of this, or the full theoretic amount, aids the compression flange, we have its permanent inch-strain at each quarter-span $= \frac{2,170}{566 \times 40} = 3 \cdot 581$ tons. On comparing the unit-strains in the flanges at the quarter-spans with those at the centre of the tube we find that they are nearly equal, and that the girder is therefore, as regards the flanges, a girder of very nearly uniform strength.

Ex. 4. One of the large tubes of the Britannia Bridge is 470 feet long from centre to centre of bearings, and 27·5 feet deep from centre to centre of flange cells at the middle of the span, and its weight is 1,587 tons. What was the strain in either flange at the centre while it was an independent girder and before it was connected with the other tubes?

$$\textit{Answer} \text{ (eq. 25).} \quad \mathsf{F} = \frac{\mathsf{W}l}{8d} = \frac{1{,}587 \times 470}{8 \times 27{\cdot}5} = 3{,}390 \text{ tons.}$$

The gross areas of the top and bottom flanges at the centre of the span are respectively 648 and 585 square inches, and if we concede, as before, that the theoretic aid which the webs give the tension flange is a sufficient compensation for the weakening effect of rivet holes, we have the inch-strain in the lower or tension flange $= \frac{3{,}390}{585} = 5{\cdot}795$ tons.

The area of both sides at the middle of the span = 302 square inches, and adding, as before, the full theoretic proportion of one-sixth in aid of the compression flange, we have the compressive unit-strain in the upper flange $= \frac{3{,}390}{648 \times 50} = 4{\cdot}856$ tons. The student is cautioned that it is not safe practice to assume what has been claimed by some advocates of continuous versus braced webs, and which has been conceded above, namely, that so large a proportion as one-sixth of the web really aids each flange, especially in large plate girders such as the tubular bridges. Hence, the unit-strains in examples 2, 3, and 4 are doubtless below the reality.

45. A concentrated load produces the same strain in the flanges as twice the load uniformly distributed.—Comparing eqs. 17 and 23, we find that the horizontal strain at any point in either flange from a single weight resting there is double that which would be produced by the same load uniformly distributed. This, however, does not apply to the web.

46. Web, shearing-strain.—When the load is symmetrically arranged on each side of the centre, *the shearing-strain at the centre of the girder is cipher, and at any other cross section it equals the sum of the weights between it and the centre.* This will appear evident from the consideration that the shearing-strain at any section is the pressure which is transmitted to the abutment through that section (**14**). Hence, with a uniformly distributed load, the shearing-strain is proportional to the distance from the centre of the girder, where it is cipher, and increases towards the ends, where it equals $\frac{\mathsf{W}}{2}$, as the ordinates of a triangle. This may be represented graphically,

Fig. 22.—Shearing-strain.

as in Fig. 22, where the line **A B** represents the length of the girder, and the ordinates **A C** and **B E** represent the reactions of each abutment, $=\frac{W}{2}$; connecting **C** and **E** with the centre at **D**, the ordinates of the figure **A C D E B** will represent the shearing-strains at each point along the girder. When both flanges are horizontal, the sectional area of the web ought for economical reasons to vary in the ratio of these ordinates, for any surplus material would be more valuable for sustaining horizontal strains if placed in the flanges, as its leverage would be thereby increased.

Ex. 1. What is the shearing-strain in the web at each end of the girder in the first example in **44**?

Answer. Shearing-strain $= \frac{wl}{2} = \frac{1 \cdot 68 + 101 \cdot 2}{2} = 85$ tons.

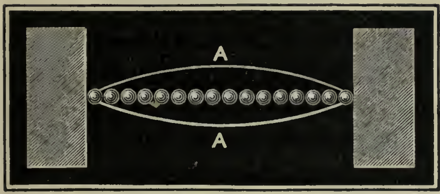
Ex. 2. The iron work of one of the Conway tubes, 400 feet long in the clear span, weighs 1,112 tons; adding 400 tons for weight of permanent way, roof and a passing train, we have a total load of 1,512 tons, of which one-fourth, = 378 tons, is the shearing-strain at each end of each side where the web is about 19 feet high and $\frac{5}{8}$ inch thick. Consequently, its gross section = 142·5 square inches, but as the vertical edges of the plates are pierced by one-inch rivet holes, three inches apart centres, their net section is one-third less, or 95 square inches, and the shearing-strain at the joints when a heavy train is passing is about 4 tons per square inch of net section. In this example no credit has been given to the outside plates of the cellular flanges, which doubtless contribute their quota of strength to withstand shearing-strain.

47. Flange-area of girder of uniform strength when the depth is constant.—From eq. 23 we have, when both flanges are horizontal,

$$f = \frac{wmn}{2ad} \tag{27}$$

where a and f represent the area and unit-strain of either flange at any section which divides the girder into segments containing m and n linear units. If the girder be of uniform strength, f will be constant throughout each flange (**19**), and a will vary as mn. Hence, if the depth of the flange be uniform,

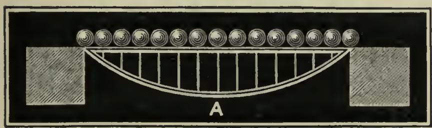
Fig. 23.—Plan.



its width will vary as *mn*, and the plan of the flange will, if symmetrical, be formed by the overlap of two parabolas whose vertices are at **A A**, Fig. 23.

48. Depth of girder of uniform strength when the flange-area is constant.—If, however, the depth of the girder vary while the area of the horizontal flange remains uniform, *d* will vary as *mn*. Hence, the elevation of the curved flange will be

Fig. 24.—Elevation.



a parabola whose axis is vertical, with its vertex at **A**, Fig. 24. In this case it may be shown that the whole shearing-strain passes through the curved flange, and that therefore no web is required for diagonal strains. When, however, the load rests upon the horizontal flange, pillars, represented by vertical lines (or suspension rods, if Fig. 24 be inverted), are required to convey the vertical pressure of each weight to the curved flange. The longitudinal strain in the curved flange increases towards the points of support and may be found by the method explained in **26.**

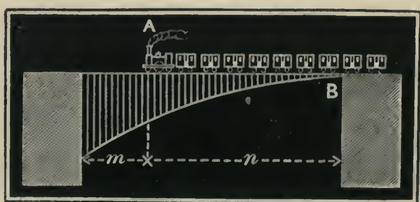
49. Suspension bridge—Curve of equilibrium.—The horizontal flange, Fig. 24, prevents the ends of the curved flange from approaching each other ; the same effect may be produced by fastening the ends of the curved flange to the abutments, in which case, the load being suspended below the curved flange, we have the suspension bridge for a uniform horizontal load. The curve which an unloaded chain of uniform section assumes from its own weight is the catenary, which, however, differs but slightly from a parabola when the ratio of the deflection to the span does not exceed that commonly adopted for suspension bridges, viz., $\frac{1}{15}$

If Fig. 24 be inverted and the horizontal flange replaced by solid abutments, to keep the arch from spreading, we have the arch of equilibrium for a uniform horizontal load, and when the arch has merely its own weight to support, the inverted catenary becomes the arch of equilibrium. Every change in the position of a load alters the form of the curve of equilibrium, *whose horizontal component is uniform throughout the whole curve;* for it is obvious that, if the horizontal strain at one point of a flexible chain exceed that at another point, the intermediate portion will move towards that side on which the stronger pull is exerted, so as to conform to the position of equilibrium. A suspension bridge, being flexible, accommodates itself to each change of load, assuming at each moment the position of equilibrium for the particular load to which it is temporarily subjected; but neither the rigid flanges of a girder, nor the voussoirs of a stone arch, can thus suit themselves to the changing position of the load. The web of the former, and the spandril walls of the latter, are therefore requisite to enable a rigid structure to sustain a variable load without fracture, which they do by converting what would otherwise be transverse strains in the arch or flanges into longitudinal ones.

CASE VI.—FLANGED GIRDER SUPPORTED AT BOTH ENDS AND TRAVERSED BY A TRAIN OF UNIFORM DENSITY.

50. Passing train of uniform density—Shearing-strain—Flanges.—When a distributed rolling load, such as a railway train, traverses a girder, the shearing-strain throughout the unloaded segment may be found as follows. Let the train be of uniform density per running foot, and its total length not less than that of the girder.

Fig. 25.—Shearing-strain.



Let l = the length of the girder,
d = the depth of the girder at **A**, in front of the train,
w = the weight of the train per unit of length,
m and n = the segments into which the front of the train divides the girder, n being the loaded segment,
R = the reaction of the left or unloaded abutment, *i.e.*, the shearing-strain in the segment m.
F = the horizontal strain exerted by either flange at **A**, that is, the horizontal component of the longitudinal strain if the flange be oblique.

The girder is held in equilibrium by the upward reaction of each abutment and the downward pressure of the train. This latter = wn, which we may conceive collected at its centre of gravity whose distance from the right abutment $=\frac{n}{2}$ (**11**). Taking moments round this abutment, we have $\mathbf{R}l = wn\frac{n}{2}$. Hence,

$$\mathbf{R} = \frac{wn^2}{2l} \tag{28}$$

This is the shearing-strain throughout the unloaded segment, since it is transmitted through every section between the front of the train and the left abutment (**14**). As the train moves forward, the shearing-strain in front increases as the square of the loaded segment, and varies therefore as the ordinates of a parabola, the ordinates being represented by the vertical lines of shading in Fig. 25, with the vertex at **B**.

The flange-strain in front of the train may be easily found by taking moments round either flange at **A**, when we have

$$\mathbf{F} = \frac{\mathbf{R}m}{d} = \frac{wmn^2}{2dl} \tag{29}$$

51. Maximum strains in web occur at one end of a passing train.—It can be easily proved that the shearing-strain at any point **A** is greater when the load covers the longer segment than when it covers the whole girder. In the latter case the load is uniformly distributed all over, and the shearing-strain at $\mathbf{A} = \frac{w(n-m)}{2}$ (**16**), but when the load covers the greater segment only, the shearing-strain at A $= \dfrac{wn^2}{2(m+n)}$. Subtracting the former from the latter quantity, we obtain the following result. The shearing-strain at the end of a passing train of uniform density covering the greater segment exceeds that produced by a load of equal density, but extending over the whole girder, by a quantity equal to $\dfrac{wm^2}{2l}$, where m represents the shorter and unloaded segment. It will be observed that this excess is equal to the shearing-strain throughout the unloaded segment whenever the train covers the lesser segment only.

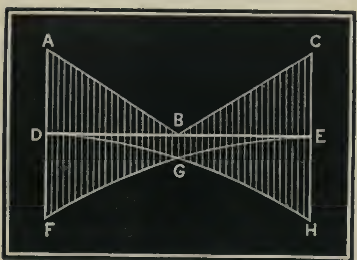
Ex. A railway girder is 90 feet in length, and the heaviest train weighs $1\frac{1}{4}$ tons per running foot; what is the maximum shearing-strain from this train at 15 feet from one end? This will occur when the train covers the greater segment, and we have

$$l = 90 \text{ feet,}$$
$$m = 15 \text{ feet,}$$
$$n - 75 \text{ feet,}$$
$$w = 1{\cdot}25 \text{ tons.}$$

Answer (eq. 28). $\mathrm{R} = \dfrac{wn^2}{2l} = \dfrac{1{\cdot}25 \times 75 \times 75}{2 \times 90} = 37{\cdot}95$ tons.

52. Uniform load and passing train, shearing-strain.—Let D E, Fig. 26, represent a railway girder, and let the ordinates D A and E C represent the shearing-strains at its extremities from a load uniformly distributed over its whole length, such as the permanent bridge-load. Draw A B and C B to the centre of D E and the ordinates of the figure D A B C E will represent the shearing-strains at each point due to this uniformly distributed load (**46**). Again, let D E and E H represent the shearing-strains at the extremities from the greatest rolling load of uniform density (say engines), when covering the whole girder. Draw the parabolas D G H and E G F, and the ordinates of the figure D F G H E will represent the greatest shearing-strains due to this maximum rolling load. The ordinates

Fig. 26.—Shearing-strain.



of the two figures combined, namely **A B C H G F**, will represent the greatest possible shearing-strains to which the girder is liable whatever may be the position of the rolling load.*

53. Maximum strain in flanges occur with load all over.— The horizontal strains in the flanges attain their greatest value when the load covers the whole girder, for the strain at each point equals the sum of those produced by each weight acting separately, and is consequently diminished by the removal of any one weight; the same result may be obtained by comparing equations 23 and 29, when we find that the flange-strain in front of a train is less than when the train covers the whole girder in the ratio of $\frac{n}{l}$, where n represents the segment covered by the train.

54. Area of a continuous web calculated from the shearing-strain—Quantity of material in a continuous web.— When the flanges are parallel, the theoretic area of a continuous web may be calculated from the shearing-strain by the following rule:—

$$\text{Sectional area of web} = \frac{\text{Shearing-strain}}{\text{Unit-strain}}$$

in which the unit-strain is the safe unit-strain for shearing. This gives the minimum thickness, which, however, is often much less than a due regard for durability requires; neither does this rule give an adequate idea of the additional material required for stiffening the web against buckling, of which more hereafter.

Ex. A single-webbed plate girder, 50 feet long and 4 feet deep, supports a uniformly distributed load of 32 tons; what is the theoretic thickness of the web, if 4 tons per square inch be a safe shearing unit-strain? The shearing-strain at each end = 16 tons, and the theoretic section of the web = $\frac{16}{4}$ = 4 square inches; but as the depth of the girder is 4 feet, the thickness of the web would be only $\frac{4}{48} = \frac{1}{12}$th inch, which is altogether too thin for safe practice. The second example in **46**, however, shows that the rule is applicable to the Conway tubular bridge.

On comparing **34**, **37**, **46**, and **52**, we find that when a girder with parallel flanges and a continuous web is loaded in the manner described below, where

l = the length, and

f = the safe unit-strain for shearing force,

* Appendix to Paper on Lattice Beams. By W. B. Blood, Esq., *Proc. I. C. E.*, Vol. xi.. p. 9.

the theoretic quantity of material in the web should be as follows:—

Kind of load.	Theoretic quantity of material in a continuous Web.	Proportional numbers
Fixed central load . . . = W	$\frac{Wl}{2f}$	12
Concentrated rolling load = W	$\frac{3Wl}{4f}$	18
Uniformly distributed load = W	$\frac{Wl}{4f}$	6
Distributed rolling load . = W	$\frac{7Wl}{24f}$	7

55. Depth and length for calculation.—In calculating the flange-strains of girders with continuous webs, the extreme depth may be taken as the depth for calculation whenever the web is neglected; but when a continuous web is taken into account, or when the web is formed of bracing, the depth may be measured from the upper to the lower intersection of the web with the flanges, at which points the flanges are assumed to be concentrated. Girders with cellular flanges are, however, exceptions to the foregoing rule, as in these the depth for calculation is measured from centre of upper cells to centre of lower cells.

The length for calculation should be measured from centre to centre of bearings, which may be called the *effective length* of a girder, and will always be greater than the clear span and less than the total length.

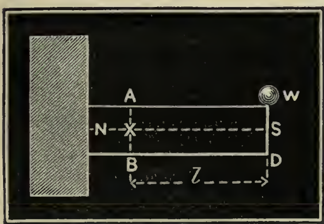
Ex. The depth of the Boyne lattice girder for calculation is measured from root to root of flange angle irons, and equals 22·25 feet—see plate IV. The extreme depth of the Conway tube at the centre is 25·42 feet, but as the cellular flanges are each 1·75 feet deep, the depth for calculation is 23·67 feet. The extreme length of the Conway tube is 424 feet, the clear span between the supports is 400 feet, and the effective length for calculation is 412 feet, the bearings at each end being 12 feet in length.

CHAPTER III.

TRANSVERSE STRAIN.

56. Transverse strain.—Let Fig. 27 represent a semi-girder of any form whatever of cross section, loaded at the extremity with the weight **W**, and let $l =$ the distance of **W** from any plane of section **A B**. We know from experience that whenever a semi-girder such as that described is subject to transverse strain, deflection takes place, the upper edge being extended and the lower edge compressed. This longitudinal elongation and shortening are not confined to the outside fibres merely, but affect those in the interior of the girder, their change of length becoming less and less in direct proportion as their distance from the edge increases, as is proved by the lines **A B** and **W D** remaining straight after deflection. Experiments also prove that the amount of deflection is proportional to the bending weight, provided the limits of elastic reaction of the extreme upper and lower fibres are not exceeded (**7**).*

Fig. 27.



57. Neutral surface.—The surface of unaltered length, **N S**, at or near the centre of the girder, where extension ceases and compression begins, is called the *Neutral surface*—a term calculated to produce a false impression that this part of a girder is free from all strain, whereas, as has been already stated (**14**), the weight, which is a vertical force, could not produce longitudinal strains in the fibres except through the medium of certain diagonal strains, which, as will be shown hereafter, act probably with their greatest intensity in the vicinity of the neutral surface. *The Neutral surface of any girder is, therefore, that surface along which the resultant*

* Morin, pp. 122, 138.

of the horizontal components of all the diagonal forces equals cipher; and according to this definition it may be said to exist in diagonally braced girders, in those at least in which the systems of triangulation are numerous. The reader will find his physical conceptions of these diagonal strains much clearer after he has studied the action of diagonal bracing in succeeding chapters.

58. Neutral axis—Centres of strain—Resultant of horizontal forces in any cross section equals cipher.— The line at **X**, perpendicular to the plane of the figure, and formed by the intersection of the neutral surface with any cross section of the girder, is called the Neutral line, or more generally, the *Neutral axis* of that particular section. *The Neutral axis of any section is, therefore, the line of demarcation between the horizontal elastic forces of tension and compression exerted by the fibres in that particular section of the girder.* For these tensile and compressive forces we may substitute their resultants.

Let $\mathbf{T}$ = the resultant of the horizontal tensile forces above the neutral axis,

$\mathbf{C}$ = the resultant of the horizontal compressive forces below the neutral axis,

δ = the distance between the points of application of these resultants,

called the *Centres of strain*, or for distinction's sake, the *Centres of tension and compression.* The segment **A B W D** is held in equilibrium by the weight **W**, the horizontal resultants **T** and **C**, and the shearing-strain at the section **A B**. Taking moments round the centres of compression and tension successively, we have

$$\mathbf{W}l = \mathbf{T}\delta = \mathbf{C}\delta \tag{30}$$

whence

$$\mathbf{T} = \mathbf{C} \tag{31}$$

Thus, in every girder of whatsoever form, *the resultant of all the horizontal forces in any cross section equals cipher*, or in other words, *the horizontal forces in any cross section balance each other*, a result which has been already proved in the case of flanged girders (eq. 4).

We may arrive at the same conclusion from the following consideration. Suppose a loaded girder to rest on rollers at both ends so as to be perfectly free to move in a horizontal direction. If we consider the forces acting at any cross section we find that they may be resolved into three series, the first of which is vertical, viz., the shearing-strain; the second is horizontal, tending to thrust the segments apart, and the third is likewise horizontal, tending to draw them together. These horizontal forces must balance; otherwise the girder would separate at the section, since by hypothesis the segments are free to move horizontally on the points of support.

59. Moment of resistance, M.—Bending moment.—The sum of the moments of the horizontal elastic forces in any transverse section round any point whatsoever is called the *Moment of forces resisting rupture*, or more briefly, the *Moment of resistance* of that particular section.* Representing the moment of resistance by the symbol $\mathbf{M}$, we have for a semi-girder loaded at the extremity,

$$\mathbf{W}l = \mathbf{M} \tag{32}$$

where $l =$ the distance of $\mathbf{W}$ from the transverse section. It will be observed that the moment of resistance of any particular section is constant, no matter round what point the moments of the horizontal forces may be taken, since the sum of the tensile forces is equal to the sum of the compressive forces, so that they form a couple. The product $\mathbf{W}l$ is called the *Bending moment* of the weight, and eq. 32 may be expressed in general terms as follows:—*The moments of the external forces on either side of any given section of a girder which tend to produce rotation round any point in that section are equal to the moments of the horizontal elastic reactions in the same section which resist rotation*, or briefly, the bending moment round any section $=$ the moment of resistance.

The general case of a girder of any form of cross section is similar to that of a flanged girder whose flanges are at the centres of horizontal strain, and the formulæ in the several cases of flanged girders in the previous chapter would be applicable to this general case, if we only knew the resultants of the horizontal tensile and compressive strains and also the distance between their points of application.

* Called also the *Moment of rupture.*

60. Coefficient of rupture, S.—The following method is frequently adopted for calculating the breaking weight of solid rectangular or solid round girders, though applicable to other forms also, and possesses the advantage of being founded on general reasoning independently of any assumption relating to the laws of elastic reaction or of direct experiments on the tensile and compressive strength of materials, which generally require special apparatus and are therefore less easily made than experiments on transverse strength. We have just seen (eq. 30), that the relation between the weight, length, horizontal elastic forces and distance between the centres of strain of a semi-girder fixed at one end and loaded at the other, is expressed by the equation

$$\mathsf{W} = \frac{\mathsf{F}\delta}{l}$$

in which F represents indifferently the sum of the horizontal elastic forces, either above or below the neutral axis, and is therefore proportional in girders of similar section to the number of horizontal fibres in the girder, that is, to its sectional area; $\delta =$ the distance between the centres of strain, and is evidently proportional to the depth, and $l =$ the length. Hence, we obtain the following relations for a

Semi-girder loaded at the extremity.—

$$\mathsf{W} = \frac{ad\mathsf{S}}{l} \tag{33}$$

$$\mathsf{S} = \frac{l\mathsf{W}}{ad} \tag{34}$$

in which $\mathsf{W} =$ the breaking weight,
$a =$ the sectional area,
$d =$ the depth,
$l =$ the length,

and S is a constant, which must be determined for each material by finding experimentally the breaking weight of a girder of known dimensions and similar in section to that whose strength is required. The constant S is called the *Coefficient of transverse rupture*, or more briefly, the *Coefficient of rupture** of that particular material and

* Sometimes called the *Modulus of rupture.*

section from which it is derived, and equals the breaking weight of any semi-girder of similar section in which the quantity $\frac{ad}{l} = 1$.

By reasoning similar to that adopted in the several cases of Chapter II., we have the following formulæ for girders supported and loaded in various ways:—

61. Semi-girder loaded uniformly.

$$W = \frac{2ad\mathrm{S}}{l} \tag{35}$$

$$S = \frac{l\mathrm{W}}{2ad} \tag{36}$$

62. Girder supported at both ends and loaded at an intermediate point, the segments containing m and n linear units, and l representing the length, $= m + n$.

$$W = \frac{adl\mathrm{S}}{mn} \tag{37}$$

$$S = \frac{mn\mathrm{W}}{adl} \tag{38}$$

63. Girder supported at both ends and loaded at the centre.

$$W = \frac{4ad\mathrm{S}}{l} \tag{39}$$

$$S = \frac{l\mathrm{W}}{4ad} \tag{40}$$

64. Girder supported at both ends and loaded uniformly.

$$W = \frac{8ad\mathrm{S}}{l} \tag{41}$$

$$S = \frac{l\mathrm{W}}{8ad} \tag{42}$$

65. Table of coefficients of rupture.—These formulæ, though generally restricted in practice to solid rectangular and solid round girders, are also applicable to girders of any form, provided they are similar in section to the experimental girder from which the coefficient S for that form is derived. In each class we must obtain the coefficient of rupture for its particular section by experimentally breaking a model girder. This has been done for certain forms of section and the results are given in the following tables which contain the values of S, or the coefficients of rupture, which in the case of square or round sections are the breaking weights of solid semi-girders whose length, depth, and breadth are each one inch, fixed at one end and loaded at the other. Hence, when using these coefficients in the preceding equations, *all the dimensions should be in inches.* The reader may easily satisfy himself that the value of **S** is constant for all rectangular sections of the same depth from the consideration, that any number of rectangular girders of equal depth placed side by side have the same collective strength as the single girder which they would become if united laterally. Hence $\frac{\mathbf{W}}{a}$ has the same value for the multiple girder as for one of its component girders, and therefore, from eq. 34, **S** is the same in both.

MATERIAL.	Value of S in tons.	Authority.
CAST-IRON.		
Small rectangular bars (not exceeding one inch in width), -	3·40	Clark
Large rectangular bars (three inches wide), - - -	2·25	„
Small round bars, - - - - - -	2·00	„
Circular tubes of uniform thickness, - - - -	2·85	„
Square tubes of uniform thickness, - - - -	3·42	„
WROUGHT-IRON.		
New rectangular bars whose deflection limits their utility, -	3·82	„
Rectangular bars previously strained by bending them while hot and straightening them when cold, and employed in the direction in which they were straightened, - -	5·58	„
New round bars, - - - - - -	2·25	„
Circular welded tubes of uniform thickness (boiler tubes), -	5·23	„
Circular riveted tubes of plate iron with transverse joints double riveted, - - - - - -	3·26	„
Rolled I girders with flanges of equal area, about - -	4·60	—
T iron, with the flange above, about - - -	4·00	—
Do., with the flange below, about - - - -	3·83	—
STEEL (Rectangular bars).		
Hammered Bessemer steel for tyres, axles and rails, -	9·53	Kirkaldy
Rolled Bessemer steel for tyres, axles and rails, - -	8·57	„
Hammered crucible steel for tyres and axles, - -	11·00	„
Rolled crucible steel for axles, - - - -	8·80	„
Average of a large number of specimens of Cast, Bessemer, and Shear steel, strained only as far as the limit of elasticity, - - - -	6·00	Fairbairn

Clark, *Britannia and Conway Tubular Bridges*, pp. 436, 743.
Fairbairn, *Report of the British Association for 1867.*
Kirkaldy, *Experiments on Steel by a Committee of Civil Engineers, 1868.*

WOOD.

Solid Rectangular Girders and Semi-Girders.

DESCRIPTION OF WOOD.	Initials of Experimenters.	Specific Gravity.	Value of S in lbs.
Acacia, - - - - - -	B.	710	1,867
Ash, English, - - - - -	B.	760	2,026
„ American, - - - - -	D.N.	626	1,795
„ „ swamp, - - - -	D.	925	1,165
„ „ black, - - - -	D.	533	861
Beech, English, - - - - -	B.	696	1,556
„ American, white, - - - -	D.	711	1,380
„ „ red, - - - -	D.N.	775	1,739
Birch, Common, - - - - -	B.	711	1,928
„ American, black, - - - -	B.D.N.	670	2,061
„ „ yellow, - - - -	D.	756	1,335
Box, Australian, - - - - -	T.	1,280	2,445
Bullet Tree, Demerara, - - - -	B.Y.	1,052	2,692
Cabacally, - - - - -	B.	900	2,518
Canada Balsam, - - - - -	D.	548	1,123
Cedar, Bermuda, - - - - -	N.Y.	748	1,443
„ Guadaloupe, - - - -	N.	756	2,044
„ American, white, - - - -	D.	354	766
„ of Lebanon, - - - -	D.	330	1,493
Crab Wood, Demerara, - - - -	Y.	648	1,875
Deal, Christiana, - - - - -	B.	689	1,562
Elm, English, - - - - -	B.D.	579	782
„ Canada Rock, - - - -	D.N.	725	1,970
Fir, Mar Forest, - - - - -	B.	698	1,232
„ Spruce, - - - - -	M.	503	1,346
„ „ American, black, - - -	D.	772	1,036
Greenheart, Demerara, - - - -	B.Y.	985	2,615
Hemlock, - - - - - -	D.	911	1,142
Hickory, American, - - - -	D.N.M.Y.	831	2,129
„ Bitter Nut, - - - -	D.	871	1,465
Iron Bark, Australia, - - - -	T.	1,211	2,288
Iron Wood, Canada, - - - -	D.	879	1,800
Kakarally, Demerara, - - - -	Y.	1,223	2,379
Larch, - - - - - -	B.D.M.	556	1,335
„ American, or Tamarack, - - -	D.	433	911
Lignum Vitæ, - - - - -	N.	1,082	2,013
Locust, Demerara, - - - -	B.	954	3,430
Mahogany, Nassau, - - - -	M.N.Y.	668	1,719
Mangrove, Bermuda, black, - - -	N.	1,188	1,699
„ „ white, - - -	N.	951	1,985
Maple, soft Canada, - - - -	D.	675	1,694
Norway Spar, - - - - -	B.	577	1,474
Oak, Adriatic, - - - - -	B.M.	855	1,471
„ African, - - - - -	B.D.M.N.	988	2,523
„ American, live, - - - -	N.	1,160	1,862
„ „ red, - - - -	D.N.	952	1,687
„ „ white, - - - -	B.D.M.N.	779	1,743
„ Dantzic, - - - - -	B.M.	720	1,518
„ English, - - - - -	B.D.M.N.	829	1,694
„ Italian, - - - - -	M.	796	1,688
„ Lorraine, - - - - -	M.	796	1,483
„ Memel, - - - - -	M.	727	1,665

DESCRIPTION OF WOOD.	Initials of Experimenters.	Specific Gravity.	Value of S in lbs.
Pine, American red, - - - -	B.D.M.N.Y.	576	1,527
" " pitch, - - - -	B.D.	740	1,727
" " white, - - - -	D.N.Y.	432	1,229
" " yellow, - - - -	B.D.M.	508	1,185
" Archangel, - - - - -	M.	551	1,370
" Dantzic, - - - - -	M.	649	1,426
" Memel, - - - - -	M.	601	1,348
" Prussian, - - - - -	M.	596	1,445
" Riga, - - - - -	B.M.	654	1,383
" Virginian, - - - - -	M.	590	1,456
Poon, - - - - - -	B.M.	673	1,954
Sneezewood, South Africa, - - -	N.	1,066	3,305
Spotted Gum, Australia, - - - -	T.	1,035	2,006
Stringy Bark, Australia, - - - -	T.	937	1,818
Teak, - - - - - -	B.M.N.	729	2,108
Wallaba, Demerara, - - - -	Y.	1,147	1,643
Yellow Wood, West Indies, - - -	N.	926	2,103

The coefficients for wood are chiefly taken from the *Professional Papers of the Corps of Royal Engineers*, Vol. v. The initial letters refer to the following experimenters:—B, Barlow; D, Denison; M, Moore; N, Nelson; T, Trickett; Y, Young; two or more letters signify that the tabulated number is the mean result of the experimenters whom they represent.

The reader should observe that the foregoing values of **S** for timber are derived from selected samples of small scantling, perfectly free from knots and other imperfections that cannot be avoided in large timber, and the few experiments recorded on the latter indicate that the values of **S** must be reduced to very little more than one-half (·54 times,) those given in the table when applied to girders of large size, such as occur in ordinary practice.

STONE.

Solid Rectangular Girders and Semi-Girders.

DESCRIPTION OF STONE.	Value of S in lbs.	Authority.
GRANITES.		
Ballynocken, Co. Wicklow, coarse and loosely aggregated, - - - - - - -	109	Wilkinson
Golden Hill, Blessington, Co. Wicklow, coarse, -	76	"
Golden Ball, Co. Dublin, largely crystalline, - -	182	"
Killiney, Co. Dublin, felspathic, - - -	270	"
Kingstown, Co. Dublin, - - - -	346	"
Newry, Co. Down, syenitic, - - - -	340	"
Taylors' Hill, Galway, felspar red and greenish, -	407	"

DESCRIPTION OF STONE.	Value of S in lbs.	Authority.
SANDSTONES AND GRITS.		
Green Moor, Yorkshire blue stone, - - -	335	G. Rennie
„ „ white stone, - - -	359	„
Caithness, Scotland, - - - - -	857	„
Irish sandstones from various localities, - -	57 to 1,095	Wilkinson
LIMESTONES.		
Listowel Quarry, Kerry, - - - - -	414	Wilkinson
Ballyduff Quarry, Tullamore, King's County, -	351	„
Woodbine Quarry, Athy, Co. Kildare, - -	283	„
Finglass Quarry, Co. Dublin, - - - -	291	„
SLATES.		
Valencia Island, Kerry, on edge of strata, - -	1,091	Wilkinson
„ „ on bed of strata, - -	951	„
Glanmore, Ashford, Co. Wicklow, on bed of strata, -	1,097	„
Killaloe, Tipperary, on bed of strata, - - -	1,233	„
„ „ on edge of strata, - -	1,037	„
Welsh slate, - - - - - - -	1,961	G. Rennie
BASALTS AND METAMORPHIC ROCKS.		
Hornblende Schist, Glenties, Donegal, - -	556	Wilkinson
Moore Quarry, Ballymena, Antrim, crystalline, hornblendic and felspathic, - - - -	531	„

Wilkinson, *Practical Geology and Ancient Architecture of Ireland.*
G. Rennie, *Barlow on the Strength of Materials*, p. 187.

The foregoing table contains a very small selection from Mr. Wilkinson's experiments on the transverse strength of Irish stones, and in addition to these the reader will find in his book a vast number of most valuable details relating to the crushing strength and other properties of building materials throughout Ireland.

66. Strength of stones, even of the same kind, is very variable.—Mr. Wilkinson's experiments were made on stones 14 inches long, with sides 3 inches square; the distance between the bearings was exactly 12 inches, and the pressure was applied on the top in the centre of each stone by a saddle one inch wide. "The result of these experiments shows the average strength of the principal rocks to be in the following order:—Slate rock, basalt, limestone, granite, and sandstone. The great variation which exists in the different rocks, and even in the quality of the same kind of stone, serves to show the caution which should be used in their selection and the value to be attached to the records of actual experiments."

Ex. 1. In an experiment made by the author, a wrought-iron bar, 4 inches deep and $\frac{3}{4}$ inch wide, had a weight of 1,568 lbs. hung from one end, the other end being rigidly fixed. It commenced bending at 2 ft. 8 in. from the load, at a part which had been previously softened in the fire and allowed to cool slowly. What is the value of S?

$$\text{Here,}\quad \begin{aligned} \mathsf{W} &= 1{,}568 \text{ lbs.,} \\ l &= 32 \text{ inches,} \\ d &= 4 \text{ inches,} \\ a &= 3 \text{ square inches.} \end{aligned}$$

$$\textit{Answer } \text{(eq. 34).}\quad \mathsf{S} = \frac{l\mathsf{W}}{ad} = \frac{32 \times 1{,}568}{3 \times 4 \times 2{,}240} = 1{\cdot}86 \text{ tons.}$$

Comparing this with the tabular value of S for "new rectangular bars whose deflection limits their utility," it would appear that the useful strength of bars rendered ductile by annealing is only one-half that of new bars fresh from the rolls. This result is confirmed by two of Mr. Hodgkinson's experiments on annealed wrought-iron bars heated to redness and allowed to cool slowly.—See Appendix to *Report of the Commissioners on the Application of Iron to Railway Structures*, pp. 45, 46.

Ex. 2. The teeth of a cast-iron wheel are 3·5 inches long, 2·3 inches thick, and 7 inches wide; what is the breaking weight of a tooth?

$$\text{Here,}\quad \begin{aligned} l &= 3{\cdot}5 \text{ inches,} \\ d &= 2{\cdot}3 \text{ inches,} \\ a &= 16{\cdot}1 \text{ square inches,} \\ \mathsf{S} &= 2{\cdot}25 \text{ tons.} \end{aligned}$$

$$\textit{Answer } \text{(eq. 33).}\quad \mathsf{W} = \frac{ad\mathsf{S}}{l} = \frac{16{\cdot}1 \times 2{\cdot}3 \times 2{\cdot}25}{3{\cdot}5} = 23{\cdot}8 \text{ tons.}$$

Ex. 3. A round wrought-iron shaft, 5 feet long and supported at the extremities, sustains a transverse strain of 30 tons at 14 inches from one end; what should its diameter be when on the point of yielding?

$$\text{Here,}\quad \begin{aligned} \mathsf{W} &= 30 \text{ tons,} \\ l &= 5 \text{ feet,} \\ m &= 14 \text{ inches,} \\ n &= 46 \text{ inches,} \\ \mathsf{S} &= 2{\cdot}25 \text{ tons.} \end{aligned}$$

From eq. 38, $ad = \frac{mn\mathsf{W}}{l\mathsf{S}} = \frac{14 \times 46 \times 30}{60 \times 2{\cdot}25} = 143{\cdot}1$ inches; but $ad = \frac{\pi d^3}{4}$, whence

$$\textit{Answer.}\quad d = \sqrt[3]{\frac{4 \times 143{\cdot}1}{3{\cdot}1416}} = 5{\cdot}7 \text{ inches.}$$

Ex. 4. In an experiment made by Mr. Anderson, a piece of memel fir, 2 inches deep and $1\frac{15}{16}$ inches wide, was securely fixed at one extremity, the projecting part being 2 feet long. It sustained a load of 504·5 lbs. at the end for twenty hours without breaking right across. This load, however, upset the timber on the lower or compression side next the fulcrum. What is the value of S derived from this experiment?

$$\text{Here, } \mathsf{W} = 504{\cdot}5 \text{ lbs.}$$
$$l = 24 \text{ inches,}$$
$$d = 2 \text{ inches,}$$
$$b = 1{\cdot}94 \text{ inches.}$$

$$\textit{Answer} \text{ (eq. 34). } \mathsf{S} = \frac{l\mathsf{W}}{ad} = \frac{24 \times 504{\cdot}5}{1{\cdot}94 \times 2 \times 2} = 1{,}560 \text{ lbs.}$$

This value of S exceeds that given in the table, namely, 1,348 lbs. The piece of memel in this experiment was, however, remarkably straight-grained and well seasoned, and consequently above the average.

Ex. 5. A horizontal gaff of red American pine, 15 inches square, is hinged to a mast at the inner end and suspended by a chain 9 feet from the outer end. What weight will it safely bear at the extremity? In this example the outer segment is a semi-girder 9 feet long, and we have

$$a = 15 \times 15 \text{ inches,}$$
$$d = 15 \text{ inches,}$$
$$l = 9 \times 12 \text{ inches,}$$
$$\mathsf{S} = 1{,}527 \text{ lbs.}$$

$$\textit{Answer} \text{ (eq. 33). } \mathsf{W} = \frac{ad\mathsf{S}}{l} = \frac{15 \times 15 \times 15 \times 1{,}527}{9 \times 12 \times 2{,}240} = 21{\cdot}3 \text{ tons.}$$

For temporary purposes, and *if the timber be perfectly sound*, one-fourth of this, or 5·3 tons, will be the safe quiescent load. If, however, the load, though temporary, is hoisted up and down and therefore liable to produce jerks, one-sixth, or 3·5 tons, will be the safe load, but if the timber be exposed to the weather and in frequent strain, one-tenth, or 2·13 tons, will be the proper working load.

67. Strength of similar girders—Limit of length.—It appears from the foregoing investigations that the strength of similar girders varies as the square of their linear dimensions, for $\frac{l}{d}$, in eqs. 33 to 42, is constant in similar girders, and consequently the breaking weight W varies as the area a. The weight of the girder itself, however, varies as al, *i.e.*, as the cube of its linear dimensions. If this weight, which we shall call G, equal $\frac{1}{n}$th of the breaking weight, we have the breaking weight of girders loaded uniformly (eqs. 35 and 41),

$$\mathsf{W} = \frac{\mathsf{K}ad\mathsf{S}}{l} = n\mathsf{G}$$

in which $\mathsf{K} = 2$ for a semi-girder and 8 for a girder supported at both ends. The breaking weight W' of a similar girder n times longer is as follows:—

$$\mathsf{W}' = \frac{n^2\mathsf{K}ad\mathsf{S}}{l} = n^3\mathsf{G}$$

where $n^3\mathsf{G}$ is the weight of the second girder. Hence, *if the weight of any girder is* $\frac{1}{n}$*th of its breaking weight, a similar girder n times longer will break from its own weight.* This defines the theoretic limit of length of similar girders. The same idea may be usefully expressed in the following terms:—*The unit-strains of similar girders from their own weight will vary directly as any of their linear dimensions.* From this it also follows that, *the weights of similar girders are as the cubes of their unit-strains.*

Ex. 1. The Conway tubular girder, 412 feet long, sustains from its own weight a tensile inch-strain of nearly 5 tons in the lower flange at the centre of the bridge; what is the length of a similar girder whose tensile inch-strain is 7 tons?

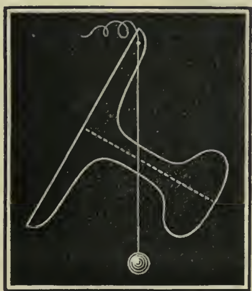
Answer. Length $= \frac{412 \times 7}{5} = 577$ feet.

Ex. 2. The weight of the Conway tube is 1,147 tons; what will be the weight of the larger girder?

Answer. Weight $= 1{,}147 \times \frac{7^3}{5^3} = 3{,}147$ tons.

68. Neutral axis passes through the centre of gravity—Practical method of finding the neutral axis.—If the law of uniform elastic reaction hold good in girders subject to transverse strain, the horizontal elastic reaction exerted by each fibre will be in proportion to the extension or compression of the fibre, that is, in direct proportion to its distance from the neutral axis (**56**). Its amount will also be proportional to the sectional area of the fibre, and if the variable distance from the neutral axis be called y, and the sectional area $d\sigma$ (differential of σ), then the elastic force of the fibre may be represented by $yd\sigma$ multiplied by a constant, and F, or the sum of the horizontal elastic forces on either side of the neutral axis, equals $\int yd\sigma$, taken within proper limits and multiplied by the same constant. This integral for the horizontal elastic forces on the upper side of the neutral axis is equal to the similar expression for the horizontal elastic forces on the lower side (eq. 31). Now this equality is also the condition which determines the position of the centre of gravity of the section. Hence, it follows that, *when the fibres are not strained beyond the limit of uniform elastic reaction, the neutral axis of any cross section of a girder passes through its centre of gravity*, and we have the following practical rule for finding the position of the neutral axis where the section is unsymmetrical, as in T iron, or in girders with unequal flanges. Cut a model of the cross section of the girder out of card-board or thin sheet metal and find its centre of gravity by means of a plumb-bob or by balancing it on a knife-edge. This will give the position of the neutral axis of the girder quite accurately enough for practical purposes.

Fig. 28.

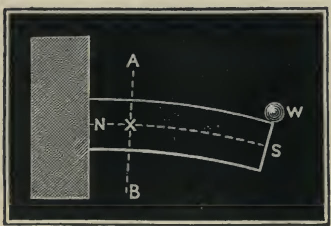


CHAPTER IV.

GIRDERS OF VARIOUS SECTIONS.

69. Moment of resistance.—The following method of investigating the strength of girders of any form whatsoever of cross section is based on the assumption that the law of uniform elastic reaction is true, that is, that the horizontal fibres exert forces which are proportional to their change of length, and therefore directly proportional to their distance from the neutral axis, an hypothesis which is sensibly true so long as the strains do not exceed those which are considered safe in practice, and which lie considerably within the limits of uniform elastic reaction (**56**). Suppose a girder composed of longitudinal fibres of infinitesimal thickness, and let us consider the horizontal elastic forces developed by the weight **W** in any cross section **A B**,

Fig. 29.



Let **M** = the moment of resistance of the section **A B** (**59**),

d = the depth of the girder,

y = the variable distance of any fibre in the section **A B**, either above or below the neutral axis,

β = the breadth of the section at the distance y from the neutral axis, and consequently a variable, except in the case of rectangular sections,

f = the horizontal unit-strain exerted by fibres in the same section at a given distance c from the neutral axis,

c = a known distance, either above or below the neutral axis, of fibres which exert the horizontal unit-strain f.

According to our assumption, the unit-strain in any other fibres at a distance y from the neutral axis will be $\frac{fy}{c}$. Let the depth of the latter fibres $= dy$ (differential of y); then the total horizontal force exerted by the fibres in the breadth β will $= \frac{f}{c}\beta y dy$. The moment of this force round the neutral axis $= \frac{f}{c}\beta y^2 dy$, and the integral of this quantity will be the sum of the moments of all the horizontal elastic forces in the section **A B** round its neutral axis, that is, the *moment of resistance* of the section in question (**59**). Representing this as before by the symbol **M**, we have

$$\mathbf{M} = \frac{f}{c}\int \beta y^2 dy \qquad (43)$$

in which the integral must be taken within proper limits for each form of cross section and may be readily found for those sections which occur in practice in the following manner.*

70. Let $h_1 =$ the distance of the top of the girder above the neutral axis,

$h_2 =$ the distance of the bottom of the girder below the neutral axis.

The expression for the moment of resistance becomes

$$\mathbf{M} = \frac{f}{c}\int_0^{h_1} \beta y^2 dy + \frac{f}{c}\int_0^{h_2} \beta y^2 dy \qquad (44)$$

in which β, if variable, must be expressed in terms of y.

71. M for sections symmetrically disposed above and below the centre of gravity.—When the material is symmetrically disposed above and below the centre of gravity, the neutral axis bisects the depth (**68**), and if $d =$ the depth, we have $h_1 = h_2 = \frac{d}{2}$, and

$$\mathbf{M} = \frac{2f}{c}\int_0^{\frac{d}{2}} \beta y^2 dy \qquad (45)$$

The values of **M** for the usual forms of cross section are as follows, recollecting that $f =$ the unit-strain in fibres whose distance from the neutral axis $= c$.

* The reader will recognize the integral $\int \beta y^2 dy$ as that which expresses the *Moment of Inertia* of the cross section round its neutral axis, represented by the symbol I.

72. M for a solid rectangle.

Let b = the breadth,

d = the depth.

In the case of a rectangle, $\beta = b$ and is therefore constant, and we have from eq. 45,

$$\mathrm{M} = \frac{2bf}{c}\int_0^{\frac{d}{2}} y^2 dy = \frac{bd^3 f}{12c} \tag{46}$$

73. M for a solid square with one diagonal vertical.

Let a = the semi-diagonal,

b = the side of the square.

The variable breadth β, expressed in terms of y, $= 2(a - y)$; substituting this value in eq. 45, we have

$$\mathrm{M} = \frac{4f}{c}\int_0^a (a - y)\, y^2 dy$$

Integrating and reducing,

$$\mathrm{M} = \frac{a^4 f}{3c} = \frac{b^4 f}{12c} \tag{47}$$

The moment of resistance of a square, it will be observed, is the same whether the sides or one diagonal be vertical.

74. M for a circular disc.

Let r = the radius.

The variable breadth β, expressed in terms of y, becomes $2\sqrt{r^2 - y^2}$; substituting this value in eq. 45, we have

$$\mathrm{M} = \frac{4f}{c}\int_0^r \sqrt{r^2 - y^2}.y^2 dy.$$

Integrating and reducing,

$$\mathrm{M} = \frac{\pi f r^4}{4c} \tag{48}$$

75. M for a circular ring of uniform thickness.

Let r = the external radius,

r_1 = the internal radius.

The moment of resistance of a ring is equal to that of the external circle minus that of the internal one, and we have from eq. 48,

$$\mathrm{M} = \frac{\pi f}{4c}(r^4 - r_1{}^4) \tag{49}$$

If $t =$ the thickness of the ring, $r_1 = r - t$; whence, by substitution in eq. 49,

$$\mathsf{M} = \frac{\pi f}{4c}(4r^3t - 6r^2t^2 + 4rt^3 - t^4)$$

If the thickness be small compared with the radius the last three terms may be neglected, and we have

$$\mathsf{M} = \frac{\pi f r^3 t}{c} \tag{50}$$

76. M for an elliptic disc with one axis horizontal.

Let $b =$ the horizontal semi-axis,
$d =$ the vertical semi-axis.

The equation of an ellipse whose origin is at the centre is $\frac{x^2}{b^2} + \frac{y^2}{d^2} = 1$;

hence, the variable $\beta = 2x = 2\frac{b}{d}\sqrt{d^2 - y^2}$; substituting this value of β in eq. 45, we have for the moment of resistance of an elliptic disc round its horizontal axis,

$$\mathsf{M} = \frac{4bf}{cd}\int_0^d \sqrt{d^2 - y^2}.y^2dy.$$

Integrating and reducing,

$$\mathsf{M} = \frac{\pi b f d^3}{4c} \tag{51}$$

77. M for an elliptic ring with one axis horizontal.

Let b = the external horizontal semi-axis,
b_1 = the internal horizontal semi-axis,
d = the external vertical semi-axis,
d_1 = the internal vertical semi-axis.

If the ring be of uniform thickness, as generally occurs in practice, both the external and internal curves cannot be true ellipses; when however the ring is thin, we may assume that the ellipse passing through the extremities of the internal axis is equidistant from the external ellipse, and that the moment of resistance of the ring is equal to that of the external minus that of the internal ellipse; whence (eq. 51), we have for the moment of resistance of an elliptic ring round its horizontal axis,

$$\mathsf{M} = \frac{\pi f}{4c}(bd^3 - b_1{d_1}^3) \tag{52}$$

If $t =$ the thickness of the tube, $b_1 = b - t$ and $d_1 = d - t$; substituting these values in eq. 52, expanding, and neglecting the terms in which the higher powers of t occur, we have when the thickness of the tube is small compared with its axis-minor,

$$\mathsf{M} = \frac{\pi d f^2 t}{4c}(3b + d) \qquad (53)$$

78. Two classes of flanged girders.—The term "flanged girder," as has been already remarked (**13**), includes rectangular tubes and braced girders as well as the ordinary single-webbed plate girder. The sides of a tube, the braced web of a lattice girder, and the continuous web of a plate girder—all perform the same duty of conveying the vertical pressure of the load (shearing-strain) to the points of support, developing at the same time longitudinal strains in the flanges. It is obvious, therefore, that the sides of the tube are equivalent to the web of the single-webbed girder, which is the form best suited for calculating the moment of resistance.

Flanged girders may be subdivided into two classes.

1st. Those in which the web is formed of bracing, or, if continuous, yet so thin that the horizontal strains developed in it are insignificant compared with those in the flanges. This class has been already investigated in Chapter II.

2nd. Those in which the web is continuous and so thick that the horizontal strains in it are of considerable value, in which case the web acts as a thin rectangular girder, enabling the flanged girder to sustain a greater load than is due merely to the sectional area of its flanges. In either case it will be sufficiently accurate for practical purposes if we suppose the mass of each flange concentrated at one point or centre of strain, which may be assumed to coincide with the intersection of the web and flanges (**55**).

79. M for the section of a flanged girder or rectangular tube, neglecting the web.—

Let $a_1 =$ the area of the upper flange,

$a_2 =$ the area of the lower flange,

$a_3 =$ the area of the web above the neutral axis,

$a_4 =$ the area of the web below the neutral axis,

h_1 = the height of the web above the neutral axis,
h_2 = the height of the web below the neutral axis,
$d = h_1 + h_2$ = the depth of the web,
$\mathsf{A} = a_1 + a_2$ = the area of both flanges together.

From eq. 44 the moment of resistance of the flanges alone

$$\mathsf{M} = \frac{f}{c}(a_1h_1^2 + a_2h_2^2) \tag{54}$$

If we neglect the web, the neutral axis passes through the centre of gravity of the two flanges (**68**), and we have $h_1 = \frac{a_2d}{\mathsf{A}}$ and $h_2 = \frac{a_1d}{\mathsf{A}}$; hence, by substitution,

$$\mathsf{M} = \frac{a_1a_2d^2f}{\mathsf{A}c} \tag{55}$$

80. M for the section of a flanged girder or rectangular tube, including the web.—When, however, the horizontal strains in the web are too considerable to be safely neglected, the moment of resistance of the web, derived from eq. 44, must be added to that just obtained for the flanges (eq. 54), when we have

$$\mathsf{M} = \frac{f}{c}\left\{\left(a_1 + \frac{a_3}{3}\right)h_1^2 + \left(a_2 + \frac{a_4}{3}\right)h_2^2\right\} \tag{56}$$

81. M for the section of a flanged girder or rectangular tube with equal flanges, including the web.—If the flanges have equal areas, the neutral axis will be in the middle of the depth, in which case $h_1 = h_2 = \frac{d}{2}$, and eq. 56 becomes

$$\mathsf{M} = \frac{d^2f}{12c}(6a + a') \tag{57}$$

where a = the area of either flange,
a' = the area of the web.

The moment of resistance of a rectangular tube with flanges of equal area may also be obtained from eq. 46 by subtracting the moment of resistance of the inner from that of the outer rectangle as follows :—

$$\mathsf{M} = \frac{f}{12c}(bd^3 - b_1d_1^3) \tag{58}$$

where b = the external breadth,
b_1 = the internal breadth,

d = the external depth,
d_1 = the internal depth.

82. M for the section of a square tube of uniform thickness, either with the sides or one diagonal vertical.—From eqs. 46 or 47,

$$\mathbf{M} = \frac{f(b^4 - b_1{}^4)}{12c} \tag{59}$$

where b = the external breadth of the tube,
b_1 = the internal breadth of the tube.

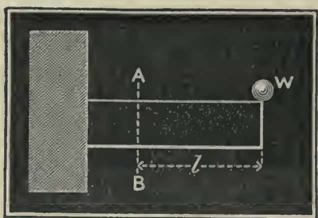
If t = the thickness of the tube, $b_1 = b - 2t$; substituting this value in eq. 59, expanding, and neglecting the terms in which the higher powers of t occur, we have when the thickness is small compared with the breadth of the tube,

$$\mathbf{M} = \frac{2fb^3t}{3c} \tag{60}$$

When the value of **M** is known for any particular section of girder we can easily find the value of the weight **W** in terms of f, or *vice versâ*, as explained in the following cases:—

CASE I.—SEMI-GIRDERS LOADED AT THE EXTREMITY.

Fig. 30.



83. Let **W** = the weight at the extremity,
l = the distance of **W** from any cross section **A B**,
M = the moment of resistance of the section **A B**.

The forces which keep the segment **A B W** in equilibrium are the weight **W**, the shearing-strain at **A B**, and the horizontal elastic forces developed in the same section. Taking the moments of all these forces round the neutral axis we have (eq. 32),

$$\mathbf{W}l = \mathbf{M} \tag{61}$$

84. Solid rectangular semi-girders.—

Let b = the breadth,

d = the depth,

From eqs. 46 and 61,

$$\mathsf{W}l = \frac{fbd^3}{12c}$$

where f = the unit-strain in fibres whose distance from the neutral axis = c.*

If, however, f = the unit-strain in the extreme fibres, $c = \frac{d}{2}$, and we have

$$\mathsf{W} = \frac{fbd^2}{6l} \tag{62}$$

Ex. A piece of teak, 2 inches deep and $1\frac{15}{16}$ inches wide, is fixed as a semi-girder at one extremity; what weight hung 2 feet from the point of attachment will break it across, the crushing inch-strain of dry teak being 12,000 lbs.?

Here, l = 2 feet,
b = 1·94 inches,
d = 2 inches,
f = 12,000 lbs.

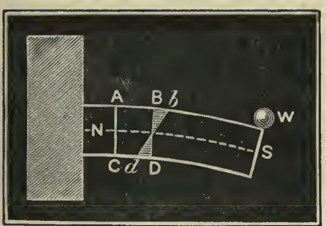
Answer. $\mathsf{W} = \frac{fbd^2}{6l} = \frac{12{,}000 \times 1{\cdot}94 \times 2 \times 2}{6 \times 24} = 647$ lbs.

The crushing strength of teak being considerably less than its tearing strength, rupture will probably ensue from the crushing of the fibres on the compressed side.

85. Geometrical proof.—Eq. 62 may be easily deduced from geometrical consideration as follows:—

Let the rectangle **A B C D**, Fig. 31, represent in an exaggerated degree the side view of any small transverse slice whose breadth before deflection = **A B**. Suppose the upper edge after deflection stretched out to the length **A**b, and the lower edge compressed to **C**d; then the lines of shading in the two little triangles will represent

Fig. 31.



* When **W** = the breaking load, the unit-strain f has been called by some writers the *modulus of rupture* of the material, but when **W** is the working load, it has been called the *working modulus*. This must not, however, be confounded with the *coefficient or modulus of rupture*, S, and it is better to restrict the expression to the latter coefficient.

the alteration of length of the intermediate fibres, **N S** being the neutral surface which divides the section into equal parts (**56**). The sum of the horizontal forces exerted by the fibres in either the upper or the lower half of the section is equal to the product of the half section by the mean unit-strain of the fibres, and if $f=$ the unit-strain in the extreme fibres, then $\frac{f}{2}$ is the mean unit-strain of all the fibres, for it equals the unit-strain exerted by the fibres lying mid-way between the neutral surface and either the upper or the lower edge. The total strain of tension in the upper half and that of compression in the lower half are, therefore, each equal to $\frac{f}{2} \times \frac{bd}{2}$, where b and d represent the breadth and depth of the section. Moreover, since the horizontal elastic forces in the various fibres are proportional to the lines of shading in the two triangles (**7**), the centres of tension and compression (**58**) coincide with their centres of gravity, and their distance apart therefore $= \frac{2}{3}d$. Hence, taking moments round either centre of strain, we have as before,

$$\mathrm{W}l = \frac{fbd^2}{6}$$

86. Solid square semi-girders with one diagonal vertical—Solid square girders with the sides vertical are 1·414 times stronger than with one diagonal vertical.—If one diagonal is vertical, we have from eqs. 47 and 61,

$$\mathrm{W}l = \frac{fb^4}{12c}$$

where $f =$ the unit-strain in fibres whose distance from the neutral axis $= c$.

If, however, $f =$ the unit-strain in the extreme fibres, $c = \frac{b}{\sqrt{2}}$, and we have,

$$\mathrm{W} = \frac{fb^3}{8{\cdot}5l} \qquad (63)$$

Comparing eqs. 62 and 63, we find that the transverse strength of a solid square girder with the sides vertical $= \frac{8\cdot5}{6} = 1\cdot414$ times the strength of the same girder with the diagonal vertical.*

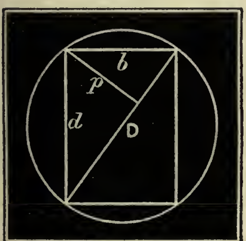
The strength of square semi-girders in the direction of their diagonals may be investigated in a different manner as follows. Let Fig. 32 represent a cross section of the girder, and let the line **AB** represent the shearing force acting downwards. We may conceive this replaced by its components **AC** and **AD** parallel to the sides of the girder. Since the section is square, each component will equal $\frac{\mathsf{AB}}{\sqrt{2}}$.

Fig. 32.

Now the force **AC** will produce tension in the side parallel to **AD**, and the force **AD** will produce tension in the side parallel to **AC**; the corner will therefore sustain twice the strain that either component alone would produce, that is, it will sustain a strain which would be produced by a force equal to $\frac{2\,\mathsf{AB}}{\sqrt{2}}, = 1\cdot414\ \mathsf{AB}$, acting in the direction of one side, which result agrees with that already obtained.

87. Rectangular girder of maximum strength cut out of a cylinder.—It is sometimes required to cut a rectangular girder of maximum strength out of round timber.

Fig. 33.



Let D = the diameter of the log,

b = the breadth of the girder of maximum strength,

d = its depth.

From eq. 62, the strength of a rectangular girder is maximum when bd^2 is maximum, or, since $d^2 = \mathsf{D}^2 - b^2$, when $b\mathsf{D}^2 - b^3$ is maximum. Equating the differential coefficient of this quantity to cipher, we have

* Barlow's experiments on battens of elm, ash and beech, 2 inches square and 36 inches long, do not corroborate the theory in the text, for the strength of the elm was the same whether fixed erect or diagonally, whereas it was found that ash and beech were both a little weaker in the latter position.—*Strength of Materials*, p. 143.

$$b^2 = \frac{1}{3}D^2$$

from which we derive the following rule. Erect a perpendicular, p, at one-third of the length of the diameter, and from the point where this perpendicular intersects the circumference draw two lines, b and d, to the extremities of the diameter; then $b^2 = \frac{1}{3}D^2$ *

88. Solid round semi-girders.

Let $r =$ the radius.

From eqs. 48 and 61,

$$Wl = \frac{\pi f r^4}{4c}$$

where $f =$ the unit-strain in fibres whose distance from the neutral axis $= c$.

If, however, $f =$ the unit-strain in the extreme fibres, $c = r$, and

$$W = \frac{\pi f r^3}{4l} \tag{64}$$

89. Solid square girders are 1·7 times as strong as the inscribed circle, and 0·6 times as strong as the circumscribed circle.—Comparing eqs. 62 and 64, we find that the strength of a solid square girder is 1·7 times that of the solid inscribed cylinder, whereas its strength is only $\frac{5·66}{9·42} = 0·6$ times that of the solid circumscribed cylinder.†

90. Hollow round semi-girders of uniform thickness.

Let r = the external radius,

r_1 = the internal radius.

From eqs. 49 and 61,

$$Wl = \frac{\pi f}{4c}(r^4 - r_1{}^4)$$

where $f =$ the unit-strain in fibres whose distance from the neutral axis $= c$.

* *Euclid*, Book vi.; Cor., prop. 8.

† In Barlow's experiments on very fine specimens of Christiana deal, the breaking weight of girders 4 feet long and 2 inches square, supported at the ends and loaded in the middle, was 1,116 lbs. The breaking weight of round girders of the same length and 2 inches in diameter was 772 lbs. The ratio of these breaking weights = 1·45, not 1·7, which the theory in the text gives.—*Barlow*, p. 142.

If, however, f = the unit-strain in the extreme fibres, $c = r$, and

$$W = \frac{\pi f}{4rl}(r^4 - r_1^4) \tag{65}$$

If, moreover, the thickness of the tube be small compared with the radius, we have from eqs. 50 and 61,

$$W = \frac{\pi f r^2 t}{l} \tag{66}$$

where t represents the thickness of the tube.

Ex. A tubular crane post of plate iron is 11 feet high and 2·4 feet diameter at the base. The load on the crane produces a bending-strain equivalent to 20 tons acting at right angles to the top of the post; what should be the thickness of the plating at the base in order that the inch-strain may not exceed 3 tons?

$$\begin{aligned} \text{Here,}\quad W &= 20 \text{ tons,} \\ l &= 11 \text{ feet,} \\ r &= 1{\cdot}2 \text{ feet,} \\ f &= 3 \text{ tons per square inch.} \end{aligned}$$

Answer (eq. 66). $t = \frac{Wl}{\pi f r^2} = \frac{20 \times 11}{3{\cdot}1416 \times 3 \times 12 \times (1{\cdot}2)^2} = 1{\cdot}35$ inches.

91. Centre of solid round girders nearly useless.—The centre or core of a cylindrical girder may be removed without sensibly diminishing its transverse strength; for it appears, from eqs. 64 and 65, that the strengths of two cylinders of equal diameters, one solid and the other hollow, are as $\frac{r^4}{r^4 - r_1^4}$, in which r and r_1 are the external and internal radii; let $r = nr_1$, then the ratio becomes $\frac{n^4}{n^4 - 1}$; if, for example, $n = 2$, the loss of strength in the hollow cylinder amounts to only $\frac{1}{16}$th of that of the solid cylinder while the saving of material amounts to $\frac{1}{4}$th. For this, among other reasons, cylindrical castings, such as crane posts, should be made hollow.

92. Hollow and solid cylinders of equal weight.—It may also be shown that the transverse strength of a thin hollow cylinder is to that of a solid cylinder of equal weight as the diameter of the former is to the radius of the latter. By eqs. 66 and 64, the ratio of the strength of a hollow to that of a solid cylinder $= \frac{4r^2 t}{r_1^3}$, in which r and t represent the radius and thickness of the hollow cylinder, and r_1 represents the radius of the solid cylinder; since by hypothesis the two cylinders are of equal weight, we have $2rt = r_1^2$; whence, by substitution, the ratio of strength becomes $\frac{2r}{r_1}$, that is, as the diameter of the hollow cylinder is to the radius of the solid cylinder.

93. Solid elliptic semi-girders.

Let b = the horizontal semi-axis,

d = the vertical semi-axis.

From eqs. 51 and 61, we have,

$$Wl = \frac{\pi f b d^3}{4c}$$

where f = the unit-strain in fibres whose distance from the neutral axis = c.

If, however, f = the unit-strain in the extreme fibres, $c = d$, and

$$W = \frac{\pi f b d^2}{4l} \qquad (67)$$

94. Hollow elliptic semi-girders.

Let b = the external horizontal semi-axis,

b_1 = the internal horizontal semi-axis,

d = the external vertical semi-axis,

d_1 = the internal vertical semi-axis,

From eqs. 52 and 61 we have

$$Wl = \frac{\pi f}{4c}(bd^3 - b_1 d_1^3)$$

where f = the unit-strain at the distance c from the neutral axis.

If, however, f = the unit-strain in the extreme fibres, $c = d$, and

$$W = \frac{\pi f}{4dl}(bd^3 - b_1 d_1^3) \qquad (68)$$

If, moreover, the thickness of the tube is small compared with the shorter axis, we have from eqs. 53 and 61,

$$W = \frac{\pi f d t}{4l}(3b + d) \qquad (69)$$

where t = the thickness of the tube.

95. Flanged semi-girder or rectangular tube, taking the web into account.

Let a_1 = the *net* area of the top flange,

a_2 = the area of the bottom flange,

a_3 = the area of the web above the neutral axis,

a_4 = the area of the web below the neutral axis,

h_1 = the distance of the top flange above the neutral axis,

h_2 = the distance of the bottom flange below the neutral axis,

f = the unit-strain in fibres whose distance from the neutral axis = c.

From eqs. (56) and (61), we have

$$\mathsf{W} = \frac{f}{cl}\left\{\left(a_1 + \frac{a_3}{3}\right)h_1{}^2 + \left(a^2 + \frac{a_4}{3}\right)h_2{}^2\right\} \qquad (70)$$

96. Flanged semi-girder or rectangular tube with equal flanges.—If the flanges are equal, we have from eqs. 57 and 61,

$$\mathsf{W}l = \frac{fd^2}{12c}(6a + a')$$

where d = the depth of web,

a = the area of either flange,

a' = the area of the web,

f = the unit-strain in fibres whose distance from the neutral axis = c.

If f = the unit-strain in either flange, $c = \frac{d}{2}$, and we have

$$\mathsf{W} = \frac{fd}{l}\left(a + \frac{a'}{6}\right) \qquad (71)$$

In the case of a rectangular tube with equal flanges, the following equation, derived from eqs. 58 and 61, may be used instead of eq. 71,

$$\mathsf{W} = \frac{f}{6dl}(bd^3 - b_1 d_1{}^3) \qquad (72)$$

where b = the external breadth,

b_1 = the internal breadth,

d = the external depth,

d_1 = the internal depth,

f = the unit-strain in the extreme fibres, in which case $c = \frac{d}{2}$.

97. Square tubes with vertical sides.—If the tube is square with vertical sides of uniform thickness, we have from eq. 72,

$$W = \frac{f}{6bl}(b^4 - b_1^4) \tag{73}$$

If, moreover, the thickness of the tube is small compared with its breadth, we have from eqs. 60 and 61,

$$W = \frac{4fb^2t}{3l} \tag{74}$$

where t = the thickness of the side of the tube.

98. Square tubes with diagonal vertical—Square tubes of uniform thickness with vertical sides are 1·414 times stronger than with one diagonal vertical.—If one diagonal of the square tube is vertical, the sides being of equal thickness, we have from eqs. 59 and 61,

$$Wl = \frac{f}{12c}(b^4 - b_1^4)$$

where f = the unit-strain at the distance c from the neutral axis.

If f = the unit-strain in the extreme fibre, $c = \frac{b}{\sqrt{2}}$, and we have

$$W = \frac{f}{8{\cdot}5bl}(b^4 - b_1^4) \tag{75}$$

If, moreover, the thickness of the tube is small compared with its breadth, we have from eqs. 60 and 61,

$$W = \frac{{\cdot}94fb^2t}{l} \tag{76}$$

where t = the thickness of the side of the tube.

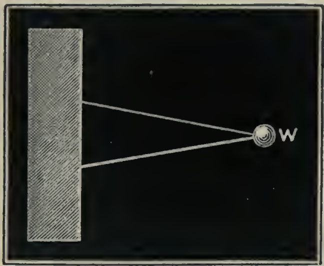
Comparing eqs. 73 and 75, we find that the strength of a square tube of uniform thickness, with the sides vertical, equals 1·414 times the strength of the same tube with the diagonal vertical.

99. Square tubes of uniform thickness with vertical sides are 1·7 times as strong as the inscribed circle of equal thickness, and 0·85 times as strong as the circumscribed circle of equal thickness—Square and round tubes of equal thickness and weight are of nearly equal strength.—Comparing eqs. 74 and 66, we find that the strength of a square tube with vertical sides is to that of a round tube of equal thickness and whose diameter equals the side of the square (inscribed circle) as $\frac{16}{9{\cdot}42} = 1{\cdot}7$; whereas the strength of the square tube with vertical sides is to that of a round tube of equal thickness but whose diameter equals the diagonal of the square (circumscribed circle,) as $\frac{8}{9{\cdot}42} = 0{\cdot}85$. It also appears that the strength of the circumscribed circle is twice that of the inscribed circle of equal thickness. If square and round tubes are of equal thickness and weight, their peripheries will be equal, that is, $4b = 2\pi r$, or $b = \frac{\pi}{2}r$; substituting this value for b in eq. 74, and comparing the result with eq. 66, we find that the relative strength of square tubes with vertical sides and round tubes of equal weight and thickness $= \frac{\pi}{3} = 1{\cdot}0472$, or very nearly a ratio of equality, the square tube being very slightly stronger than the other. When semi-girders are subject to transverse strain in various directions like crane posts, the round tube is generally preferable to a square tube of equal weight, as the latter is much weaker in the direction of the diagonals (**98**). Nevertheless, rectangular tubes of plate iron, with strong angle iron in the corners, form very efficient crane posts.

100. Value of web in aid of the flanges.—The strength of a girder with equal flanges and continuous web, in which full credit is given to the web for the horizontal strains which it sustains, is equal to the strength derived from the flanges alone plus that derived from the web acting as an independent rectangular girder. Eqs. 5 and 71 prove that *a continuous web, in a girder with flanges of equal area, does theoretically as much duty in aid of the flanges as if one-sixth of the web were added to each flange and the web were made of bracing.* In girders with unequal flanges, the centre of gravity, and therefore the neutral surface, is closer to the large flange; consequently the small flange will derive more benefit from a continuous web than the large one.

101. Plan of solid rectangular semi-girder of uniform strength, depth constant.—From eq. 62, the unit-strain in the

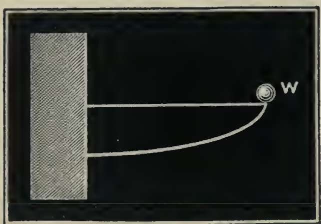
Fig. 34.—Plan.



extreme fibres of a solid rectangular semi-girder $f = \frac{6l\mathbf{W}}{bd^2}$. If the semi-girder be of uniform strength (**19**), f will be constant, and consequently the quantity $\frac{l}{bd^2}$, to which f is proportional, will also be constant. Hence, if the depth of the girder be uniform, b will vary as l, that is the plan of the girder will be triangular, Fig. 34.

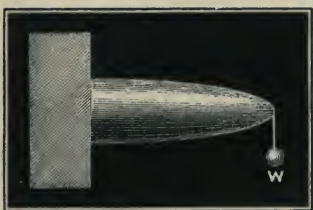
102. Elevation of solid rectangular semi-girder of uniform strength, breadth constant.—If, however, the breadth be uniform, d^2 will vary as l, and if the top of the girder be horizontal the bottom will be bounded by a parabola whose vertex is at $\mathbf{W}$ and its axis horizontal, Fig. 35.

Fig. 35.—Elevation.



103. Solid round semi-girder of uniform strength.—From eq. 64, the unit-strain in the extreme fibres of a solid round semi-girder $f = \frac{4l\mathbf{W}}{\pi r^3}$. If its strength be uniform, r^3 will vary as l, and the semi-girder will be a solid formed by the revolution of a cubic parabola round a horizontal axis, Fig. 36. The beak of an anvil is a rude approximation to this form of semi-girder.

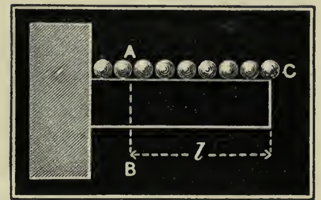
Fig. 36.



104. Hollow round semi-girder of uniform strength.—From eq. 66, the unit-strain in the extreme fibres of a thin round tube $f = \frac{l\,\mathbf{W}}{\pi r^2 t}$. If its strength be uniform, f will be constant and r^2t will vary as l. When the thickness is constant, r^2 will vary as l, and a hollow semi-girder, formed by the revolution of a parabola round a horizontal axis, will result. This, for instance, is the theoretic form for a hollow crane post of plate iron; the circumscribing cone, however, is preferable in practice, as it is more easily constructed.

CASE II.—SEMI-GIRDERS LOADED UNIFORMLY.

Fig. 37.



105. Let $l =$ the distance of any cross section, **AB**, from the extremity of the girder,

$w =$ the load per linear unit,

$\mathbf{W} = wl =$ the sum of the weights resting on **AC**,

$\mathbf{M} =$ the moment of resistance of the section **AB**.

The forces which keep **ABC** in equilibrium are the weights uniformly distributed along **AC**, the shearing-strain at **AB**, and the horizontal elastic forces developed in the same section. Taking the moments of all these forces round the neutral axis of the section **AB**, and recollecting that the sum of the bending moments of the separate weights is equivalent to the moment of a single weight equal to their sum and placed at their centre of gravity (**11**), we have (**59**),

$$\mathbf{W}\frac{l}{2} = \frac{wl^2}{2} = \mathbf{M} \qquad (77)$$

106. Solid rectangular semi-girders.—From eqs. 46 and 77, we have

$$\mathbf{W} = \frac{fbd^2}{3l} \qquad (78)$$

in which b and d represent the breadth and depth of the girder, and $f =$ the unit-strain in the outer fibres at top and bottom, in which case $c = \frac{d}{2}$.

107. Solid round semi-girders.—From eqs. 48 and 77,

$$W = \frac{\pi f r^3}{2l} \tag{79}$$

where $r =$ the radius, and $f =$ the unit-strain in the extreme fibres at top and bottom, in which case $c = r$.

108. Hollow round semi-girders of uniform thickness.—From eqs. 49 and 77,

$$W = \frac{\pi f}{2lr}(r^4 - r_1{}^4) \tag{80}$$

in which r represents the external, and r_1 the internal radius, and $f =$ the unit-strain in the extreme fibres at top and bottom. If, moreover, the thickness, t, is inconsiderable compared with the radius, we have from eqs. 50 and 77,

$$W = \frac{2\pi f r^2 t}{l} \tag{81}$$

109. Flanged semi-girders or rectangular tubes, taking the web into account.—From eqs. 56 and 77,

$$W = \frac{2f}{cl}\left\{\left(a_1 + \frac{a_3}{3}\right)h_1{}^2 + \left(a_2 + \frac{a_4}{3}\right)h_2{}^2\right\} \tag{82}$$

where $a_1 =$ the *net* area of the top flange,
$a_2 =$ the area of the bottom flange,
$a_3 =$ the area of the web above the neutral axis,
$a_4 =$ the area of the web below the neutral axis,
$h_1 =$ the distance of the top flange above the neutral axis,
$h_2 =$ the distance of the bottom flange below the neutral axis,
$f =$ the unit-strain in fibres whose distance from the neutral axis $= c$.

If the flanges are equal and if $f =$ the unit-strain in either flange, in which case $c = \frac{d}{2}$, we have from eqs. 57 and 77,

$$W = \frac{2df}{l}\left(a + \frac{a'}{6}\right) \tag{83}$$

where $a =$ the area of either flange,
$a' =$ the area of the web,
$d =$ the depth of the web.

110. Plan of solid rectangular semi-girder of uniform

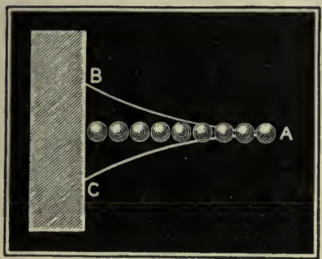
strength, depth constant.—From eq. 78, the unit-strain in the outer fibres of a solid rectangular semi-girder loaded uniformly,

$$f = \frac{3wl^2}{bd^2}$$

in which w represents the load on the unit of length, $= \frac{\mathsf{W}}{l}$.

When the strength of the girder is uniform throughout its whole length (**19**), the quantity $\frac{l^2}{bd^2}$, to which f is proportional, is constant, and, if d be uniform, b will vary as l^2, and the plan of the girder will, if symmetrical, be bounded by two parabolas whose common vertex is at **A** with the axis vertical, Fig. 38.

Fig. 38.—Plan.



111. Elevation of solid rectangular semi-girder of uniform strength, breadth constant.—If, however, the breadth be uniform, d will vary as l, and the elevation of the girder will be triangular.

112. Solid round semi-girder of uniform strength.—From eq. 79, the unit-strain in the extreme fibres of a solid round semi-girder loaded uniformly,

$$f = \frac{2wl^2}{\pi r^3}$$

If the strength be uniform, r^3 will vary as l^2, and the semi-girder will be a solid formed by the revolution of a semi-cubic parabola round a horizontal axis.

113. Hollow round semi-girder of uniform strength.—From eq. 81, the unit-strain in the extreme fibres of a thin round tube,

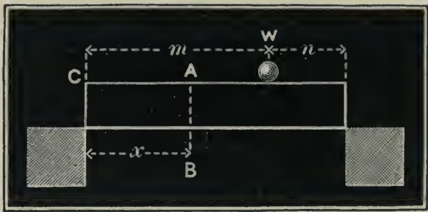
$$f = \frac{wl^2}{2\pi r^2 t}$$

If the strength be uniform, r^2t will vary as l^2. Hence, if t be constant, r will vary as l, and the tube will be conical.

The strength of semi-girders of other sections loaded uniformly may be obtained by multiplying the corresponding values of **W** in the previous case by 2.

CASE III.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED AT AN INTERMEDIATE POINT.

Fig. 39.



114. Let $\mathbf{W}$ = the weight, dividing the girder into segments containing respectively m and n linear units,

$l = m + n =$ the length of the girder,

$x =$ the distance of any cross section $\mathbf{AB}$ from that end of the girder which is remote from $\mathbf{W}$,

$\mathbf{M} =$ the moment of resistance of the section $\mathbf{AB}$.

On the principle of the lever, the reaction of the left abutment $= \frac{n}{l}\mathbf{W}$, and the segment $\mathbf{ABC}$ is held in equilibrium by this reaction, the shearing-strain at $\mathbf{AB}$, and the horizontal elastic forces developed in the same section. Taking the moments of all these forces round the neutral axis of the section $\mathbf{AB}$, we have (**59**),

$$\frac{n}{l}\mathbf{W}\,x = \mathbf{M} \tag{84}$$

When $f =$ the unit-strain in the extreme fibres at top or bottom, $c =$ the distance of the top or bottom from the neutral axis, and we have the following expressions for the strength of each class of girder.

115. Solid rectangular girders.—

Let $b =$ the breadth,

$d =$ the depth.

From eqs. 46 and 84,

$$\mathbf{W} = \frac{fbd^2l}{6nx} \tag{85}$$

If both the weight and cross section are at the centre of the girder, $x = n = \frac{l}{2}$, and

$$\mathbf{W} = \frac{2fbd^2}{3l} \tag{86}$$

116. Solid round girders.—From eqs. 48 and 84,

$$W = \frac{\pi f l r^3}{4nx} \tag{87}$$

in which $r =$ the radius.

If both the weight and cross section are at the centre,

$$W = \frac{\pi f r^3}{l} \tag{88}$$

117. Hollow round girders of uniform thickness.—From eqs. 49 and 84,

$$W = \frac{\pi f l}{4nrx}(r^4 - r_1{}^4) \tag{89}$$

where r and r_1 represent the external and internal radii.

If both the weight and cross section are at the centre,

$$W = \frac{\pi f}{lr}(r^4 - r_1{}^4) \tag{90}$$

If the thickness, t, is inconsiderable compared with the radius, we have from eqs. 50 and 84,

$$W = \frac{\pi f l r^2 t}{nx} \tag{91}$$

If, moreover, the weight and cross section are at the centre,

$$W = \frac{4\pi f r^2 t}{l} \tag{92}$$

Ex. A cylindrical tube of riveted boiler-plate, 0·095 inch thick, 27 feet long between supports, 24·2 inches diameter, and weighing 0·4295 tons, was torn through a riveted joint in the bottom by a weight of 4·857 tons at the centre (Clark, p. 92). What was the tearing-strain per square inch in the bottom plate?

Here, $W = 4{\cdot}857 + 0{\cdot}21475 = 5{\cdot}072$ tons,
$l = 27$ feet,
$r = 12{\cdot}1$ inch,
$t = 0{\cdot}095$ inch.

Answer (eq. 92). $f = \frac{Wl}{4\pi r^2 t} = \frac{5{\cdot}072 \times 27 \times 12}{4 \times 3{\cdot}1416 \times \overline{12{\cdot}1}|^2 \times 0{\cdot}095} = 9{\cdot}4$ tons.

118. Flanged girders or rectangular tubes, taking the web into account.—From eqs. 56 and 84,

$$W = \frac{fl}{cnx}\left\{\left(a_1 + \frac{a_3}{3}\right)h_1{}^2 + \left(a_2 + \frac{a_4}{3}\right)h_2{}^2\right\} \tag{93}$$

where $a_1 =$ the area of the upper flange,

$a_2 =$ the *net* area of the lower flange,

$a_3 =$ the area of the web above the neutral axis,

$a_4 =$ the area of the web below the neutral axis,

$h_1 =$ the height of the web above the neutral axis,

$h_2 =$ the height of the web below the neutral axis,

$f =$ the unit-strain in fibres whose distance from the neutral axis $= c$.

Ex. What is the unit-strain of compression in the upper flange at the centre of the girder described in Ex. 2 (**33**), supposing the web taken into account? From a full-sized card-board section of the girder it appears that the centre of gravity, that is, the neutral axis of the section, (**68**), is 3·57 inches below the intersection of the upper flange with the web, and we have,

$$a_1 = 0{\cdot}72 \text{ square inches,}$$
$$a_2 = 4{\cdot}4 \text{ square inches,}$$
$$a_3 = 3{\cdot}57 \times {\cdot}266 = {\cdot}95 \text{ square inches,}$$
$$a_4 = 0{\cdot}585 \times {\cdot}266 = {\cdot}156 \text{ square inches,}$$
$$h_1 = 3{\cdot}57 \text{ inches,}$$
$$h_2 = 0{\cdot}585 \text{ inches,}$$
$$c = 3{\cdot}57 \text{ inches,}$$
$$l = 57 \text{ inches,}$$
$$n = x = \frac{l}{2},$$
$$\mathsf{W} = 18 \text{ tons at the centre.}$$

From eq. 93., $$18 \text{ tons} = \frac{4f'}{3{\cdot}57 \times 57}\left\{\left({\cdot}72 + \frac{{\cdot}95}{3}\right)\times(3{\cdot}57)^2 + \left(4{\cdot}4 + \frac{{\cdot}156}{3}\right)\times({\cdot}585)^2\right\}$$

Solving this equation for the unit-strain in the compression flange, we have,

Answer. $f' = 61{\cdot}5$ tons per square inch.

Comparing this with Ex. 2 (**33**), we see that taking the web into account has reduced the inch-strain in the compression flange from 69·5 to 61·5 tons, or 8 tons per square inch.

If the flanges are equal and $f =$ the unit-strain in either flange, we have from eqs. 57 and 84,

$$\mathsf{W} = \frac{fdl}{nx}\left(a + \frac{a'}{6}\right) \qquad (94)$$

in which $a =$ the area of either flange,

$a' =$ the area of the web,

$d =$ the depth from centre to centre of flange.

If, moreover, the weight and cross section are at the centre,

$$\mathsf{W} = \frac{4fd}{l}\left(a + \frac{a'}{6}\right) \qquad (95)$$

119. Plan of solid rectangular girder of uniform strength, depth constant.—From eq. 85, the unit-strain in the extreme fibres of a solid rectangular girder,

$$f=\frac{6nxW}{bd^2l}$$

When the strength of the girder is uniform, the quantity $\frac{x}{bd^2}$, to which f is proportional, will be constant. Hence, if the depth, d, is uniform, b will vary as x, and the plan of the girder will be two triangles joined at their bases, Fig. 40.

Fig. 40.—Plan.

120. Elevation of solid rectangular girder of uniform strength, breadth constant.—If, however, the breadth be uniform, d^2 will vary as x, and if the top of the girder is horizontal, the bottom will be bounded by two parabolas which intersect underneath the weight, with horizontal axes and their vertices at the extremities of the girder, Fig. 41.

Fig. 41.—Elevation.

121. Solid round girder of uniform strength.—From eq. 87, the unit-strain in the extreme fibres of a solid round girder,

$$f=\frac{4nxW}{\pi lr^3}.$$

If the strength be uniform, r^3 will vary as x, and the girder will be formed by two spindles joined at their base, each spindle being produced by the revolution of a cubic parabola round its axis.

122. Hollow round girder of uniform strength.—From eq. 91, the unit-strain in the extreme fibres of a thin hollow cylinder,

$$f=\frac{nxW}{\pi lr^2t}.$$

In a girder of uniform strength, the quantity $\frac{x}{r^2t}$, to which f is proportional, will be constant; hence, if t be uniform, r^2 will vary as x, and the girder will be formed by two hollow spindles joined at their bases, each spindle being generated by the revolution of a parabola round its axis. This, for instance, is the form which the hollow axis of a transit instrument should theoretically have, though a double cone is preferred in practice from its greater facility of construction.

123. Concentrated rolling load, plan of solid rectangular girder of uniform strength when the depth is constant—Elevation of same when the breadth is constant.—If W be a *single moving load*, the maximum strain at each point will occur as the load passes that point, for x attains its greatest value when it equals m; hence, from eq. 85, the unit-strain in the extreme fibres of the section where the weight occurs,

$$f = \frac{6mn\mathrm{W}}{bd^2l} \tag{96}$$

If the strength of the girder be uniform, $\frac{mn}{bd^2}$ will be a constant quantity, and if d be uniform, b will vary as the rectangle under the segments; hence, the plan of the girder, if symmetrical, will be bounded by two overlapping parabolas whose vertices are at **A A**, Fig. 42. If, however, the breadth be uniform, d^2 will vary as mn and the elevation of the girder will be a semi-ellipse, Fig. 43.

Fig. 42.—Plan.

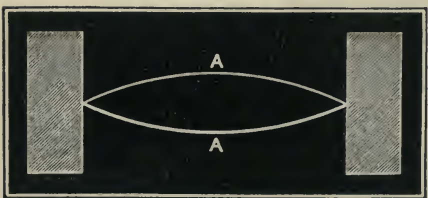
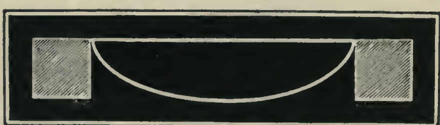
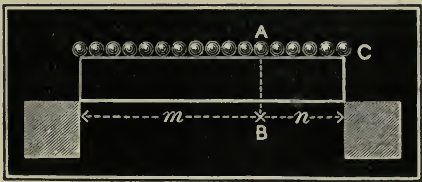


Fig. 43.—Elevation.



CASE IV.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

Fig. 44.



124. Let l = the length of the girder,
w = the load per linear unit,
$\mathbf{W} = wl$ = the whole load,
m and n = the segments into which any given cross section **A B** divides the girder,
$\mathbf{M}$ = the moment of resistance of the section **A B**.

The forces which hold **A B C** in equilibrium are the reaction of the right abutment, $= \frac{wl}{2}$, the weights uniformly distributed along **A C**, $= wn$, the shearing-strain at **A B**, and the horizontal elastic forces in the same section. Taking the moments of all these forces round the neutral axis of **A B**, we have (**59**),

$$\frac{mnw}{2} = \mathbf{M} \tag{97}$$

Multiplying the left side of the equation by $\frac{l}{l}$, we have

$$\frac{mn\mathbf{W}}{2l} = \mathbf{M} \tag{98}$$

When f = the unit-strain in the extreme fibres at top or bottom of the section, c = the distance of the top or bottom from the neutral axis, and we have the following expressions for the strength of each class of girder.

125. Solid rectangular girders.

Let b = the breadth,
d = the depth.

From eqs. 46 and 98,

$$W = \frac{fbd^2l}{3mn} \tag{99}$$

If the cross section is at the centre, $m = n = \frac{l}{2}$, and

$$W = \frac{4bd^2f}{3l} \tag{100}$$

126. Solid round Girders.—From eqs. 48 and 98,

$$W = \frac{\pi flr^3}{2mn} \tag{101}$$

in which $r =$ the radius.

If the section is at the centre, $m = n = \frac{l}{2}$, and

$$W = \frac{2\pi fr^3}{l} \tag{102}$$

127. Hollow round girders of uniform thickness.

Let r = the external radius,

r_1 = the internal radius.

From eqs. 49 and 98,

$$W = \frac{\pi fl}{2mnr}(r^4 - r_1{}^4) \tag{103}$$

At the centre of the girder $m = n = \frac{l}{2}$, and

$$W = \frac{2\pi f}{lr}(r^4 - r_1{}^4) \tag{104}$$

If the thickness, t, is inconsiderable in comparison with the radius, we have from eqs. 50 and 98,

$$W = \frac{2\pi fr^2tl}{mn} \tag{105}$$

If, moreover, the plane of section is at the centre,

$$W = \frac{8\pi fr^2t}{l} \tag{106}$$

128. Flanged girders or rectangular tubes, taking the web into account.—From eqs. 56 and 98,

$$W = \frac{2fl}{cmn}\left\{\left(a_1 + \frac{a^3}{3}\right)h_1{}^2 + \left(a_2 + \frac{a^4}{3}\right)h_2{}^2\right\} \tag{107}$$

Where a_1 = the area of the upper flange,
a_2 = the *net* area of the lower flange,
a_3 = the area of the web above the neutral axis,
a_4 = the area of the web below the neutral axis,
h_1 = the height of the web above the neutral axis,
h_2 = the height of the web below the neutral axis,
f = the unit-strain in fibres whose distance from the neutral axis = c.

If the flanges are equal, and if f = the unit-strain in either flange, $c = \frac{d}{2}$, and we have from eqs. 57 and 98,

$$W = \frac{2dfl}{mn}\left(a + \frac{a'}{6}\right) \tag{108}$$

in which a = the area of either flange,
a' = the area of web,
d = the depth of the web.

At the centre, $m = n = \frac{l}{2}$, and eq. 108 becomes

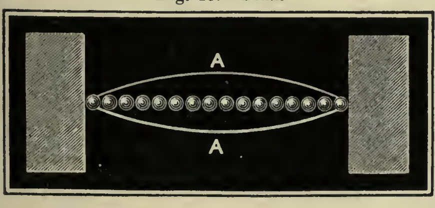
$$W = \frac{8df}{l}\left(a + \frac{a'}{6}\right) \tag{109}$$

129. Plan of solid rectangular girder of uniform strength when the depth is constant.—From eq. 99, the unit-strain in the extreme fibres of a solid rectangular girder,

$$f = \frac{3mnW}{bd^2l}$$

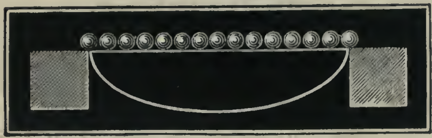
When the strength of the girder is uniform, and the material consequently disposed in the most economical manner, the unit-strain f will be uniform (**19**), and the quantity $\frac{mn}{bd^2}$, to which it is proportional, will be constant. Hence, if the depth, d, be uniform, b will vary as mn, and the plan of the girder, if symmetrical, will be formed by the overlap of two parabolas whose vertices are at **A A**, Fig. 45.

Fig. 45.—Plan.



130. Elevation of solid rectangular girder of uniform strength when the breadth is constant.—If, however, the breadth be uniform, d^2 will vary as mn, and the elevation of the girder will be a semi-ellipse, Fig. 46.

Fig. 46.—Elevation.



131. Discrepancy between experiments and theory—Shifting of neutral axis—Longitudinal strength of materials derived from transverse strains erroneous.—The student will naturally conclude that the formulæ investigated in the present and preceding chapters should give identical, or nearly identical, results when they are applied to the same girder; that, for instance, the breaking weight of a solid rectangular semi-girder, calculated by eq. 33, should closely agree with its breaking weight calculated by eq. 62; and, if our theory were complete, this would no doubt be the case. To test its accuracy, let us compare these two equations, when we obtain this result,

$$\mathsf{S} = \frac{f}{6}$$

that is, the value of S for solid rectangular girders of any given material should equal one-sixth of the ultimate tearing or crushing strength of that material, according as it yields by tearing or crushing. In many instances, however, this will be found to be far from the truth; for example, the value of S for small rectangular bars of cast-iron = 3·4 tons (**65**), and 6 times this, = 20·4 tons, far exceeds the tensile strength of ordinary cast-iron, which is about 7 or 8 tons per square inch. It must, indeed, be confessed that the law of elasticity ceases to be applicable when we approach the limits of rupture; and that the formulæ for *solid* girders investigated in the present chapter give their breaking weight much under what it really is for many materials, and this discrepancy will probably be found more marked in those whose ultimate tearing strain differs widely from their ultimate crushing strain. Greater confidence, however, may be placed in the formulæ relating to hollow and flanged girders.

Mr. Hodgkinson endeavours to explain this discrepancy by a change in the position of the neutral axis as soon as the limit of elastic reaction of the horizontal fibres has been passed, and gives some reasons for this hypothesis derived from experiments on cast-iron, in his *Experimental researches on the strength of Cast-iron*, p. 384. This seems a plausible hypothesis, for if the neutral axis of a solid rectangular cast-iron girder approach its compressed edge as the weight increases, and after the limit of tensile elasticity has been passed by the fibres along the extended edge, we shall have a larger proportion than one-half the girder subject to tension, and consequently the total horizontal tensile strain may exceed that derived from our theory, which assumes that the neutral axis always passes through the centre of gravity of the cross section (**68**). Mr. Hodgkinson concludes from his experiments that the neutral axis of a rectangular girder of cast-iron divides the depth in the proportion of $\frac{1}{5}$ or $\frac{1}{6}$ at the time of fracture, that is, that the compressed section is to the extended section nearly in the inverse proportion of the compressive to the tensile strength of the material. This view is corroborated by experiments made by Duhamel,* who found that sawing through the middle of small timber girders to $\frac{3}{4}$ths of their depth from the upper or compression surface, and inserting a thin hardwood wedge in the gap, did not diminish their ultimate strength, and also by similar experiments made by the elder Barlow,† which seem to indicate that the neutral axis in rectangular girders of timber is very nearly at $\frac{5}{8}$ths of the depth, and in rectangular bars of wrought-iron at about $\frac{1}{4}$th of the depth from the compressed surface at the time of fracture.

Mr. W. H. Barlow, however, controverts Mr. Hodgkinson's conclusions in two papers which will be found at page 225 of the Philosophical Transactions for 1855, and at page 463 of the Transactions for 1857. In the former of these papers Mr. Barlow gives the results of micrometrical measurements on two cast-iron rectangular girders, each 7 feet long, 6 inches deep and 2 inches thick, which he subjected to transverse strain; his inference from these experiments is that the neutral axis does not shift its position, and this view seems in accordance with experiments made long ago by Sir D. Brewster who transmitted polarized light through a little

* *Morin*, p. 120. † *Strength of Materials*, pp. 126, 133.

rectangular glass girder 6 inches long, 1·5 inch broad, and 0·28 inch thick; when this was bent by transverse pressure, the neutral surface remained in the centre, and colours due to strain were developed above and below it in curved lines, which may perhaps aid the physicist in investigating the strains in continuous webs.* Unless, however, the tensile and compressive elasticities of glass are materially different near the point of rupture, as they are in cast-iron when approaching its limit of tensile strength, this experiment does not throw much light on the subject. The whole question, it must be confessed, is one of great difficulty, and may require numerous experiments before it can be satisfactorily solved. One practical inference, however, is of great importance, namely, that *the tearing and crushing strengths of materials derived from experiments on the transverse strength of solid girders are often erroneous*, and have even led astray men of such capacity as Tredgold.

132. Transverse strength of thick castings much less than that of thin castings.—In some experiments made by Captain (now Colonel Sir Henry) James, as a member of the Royal Commission for inquiring into the application of iron to railway structures, it was found that the central part of bars of iron planed was much weaker to bear a transverse strain than bars cast of the same size.† He states that "it was found by planing out ¾-inch bars from the centre of 2-inch square and 3-inch square bars, that the central portion was little more than half the strength of that from an inch bar, the relation being as 7 to 12." In page 111 of the same report, Mr. Hodgkinson showed that rectangular bars of cast-iron, cast 1, 2, and 3 inches square, laid upon supports 4½ feet, 9 feet, and 13½ feet asunder, were broken by weights of 447 lbs., 1394 lbs., and 3043 lbs. respectively. These weights, divided by the squares of the lengths, should give equal results; the quotients, however, were as 447, 349, and 338 respectively. Mr. Hodgkinson attributed this falling off and deviation from theory partly to the defect of elasticity, which he had always found in cast-iron, but principally to the superior hardness of the smaller castings.‡

* *Encycl. Metrop.*, Art. Light, par. 1090. † *Iron Report*, 1849, App. B., p. 250.

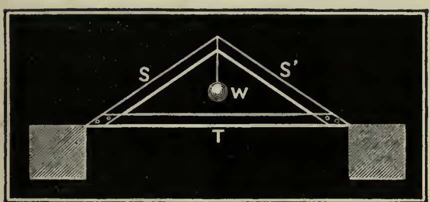
‡ *Phil. Trans.*, 1857, p. 867.

CHAPTER V.

GIRDERS WITH PARALLEL FLANGES AND WEBS FORMED OF ISOSCELES BRACING.

133. Object of bracing.—The primary object of bracing is to convert transverse strains into others which act in the direction of the length of the material employed and tend either to shorten or extend it, according as the material performs the function of a strut or tie. This object is attained by dividing the structure into one or more triangles; for since the triangle, or some modification of it, is the only geometric figure which possesses the property of preserving its form unaltered so long as the lengths of its sides remain constant, it is, therefore, that which is best adapted for structures in which rigidity is essential for stability. Hence, three pieces at least are required to constitute a braced structure. Take, for instance, the common roof truss which is an example of one of the simplest forms of bracing, Fig. 47. The weight **W** is transmitted through a pair of struts **S** and **S′**, to the walls. As, however, the oblique thrust of the struts would tend to overthrow the walls, it is necessary to connect their feet by a tie-beam **T**.

Fig. 47.



The strains in the different parts may be derived from the principle enunciated in **9**.

The class of girders which I purpose investigating in this chapter is that in which the flanges are parallel and connected by diagonals which form one or more systems of *isosceles* triangles. This class of bracing includes girders whose web consists of a single system of triangles, such as "Warren's" girder, as well as girders whose web consists of two or more systems of equal-sided triangles, such as isosceles "Lattice" girders.

Definitions.

134. Brace.—The term *Brace* includes both struts and ties.

135. Apex.—The intersection of a brace with either flange is called an *Apex*.

136. Bay.—The portion of a flange between two adjacent apices is called a *Bay*.

137. Counterbraced brace.—A brace is said to be *counterbraced* when it is capable of acting either as a strut or as a tie.

138. Counterbraced girder.—A girder is said to be *counterbraced* when it is rendered capable of supporting a moving load. This may be effected either by counterbracing the existing braces, or by adding others

139. Symbols.—The symbol $+$, placed before a number which represents a strain, signifies that the strain is compressive; the symbol —, signifies that the strain is tensile.

Axioms.

140. *The strain in each brace or bay is uniform throughout its length and acts in the direction of the length only.* This will be obvious if we consider a braced girder to be an assemblage or framework of straight bars connected with each other by pins passing through their extremities merely.

141. *A brace cannot undergo tension and compression simultaneously.*

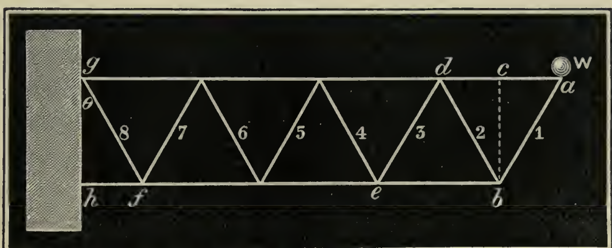
142. *If several weights, acting one at a time, produce in any brace strains of the same kind, either all tensile or all compressive, the strain produced by all these weights acting together will equal in amount the sum of those produced by each weight acting separately.*

143. *If several weights, acting one at a time, produce in any brace strains of different kinds, some tensile, some compressive, the strain resulting from all these weights acting together will equal the algebraic sum of all the strains;* in other words, *their resultant will equal the difference between the sum of the tensile and the sum of the compressive strains.*

144. *A uniformly distributed load may without sensible error be assumed to be grouped into weights resting on the apices, each apex* *supporting a weight equal to the load resting on the adjoining half bays.* This view is evidently correct if each bay be connected with the adjoining bays and diagonals by a single pin at their intersection, as in "Warren's" girder. In this case each loaded bay is a short girder covered by a uniform load, the vertical pressure of which is transferred to the adjoining diagonals. In addition to the transverse strain, each bay sustains a longitudinal strain which it transmits to the adjacent bays, from which, however, it derives no aid to its transverse strength on the principle of continuity. In practice, the cross girders, on which the flooring rests, generally occur at the apices, so that no bay is subject to transverse strain except from its own weight.

CASE I.—SEMI-GIRDERS LOADED AT THE EXTREMITY.

Fig. 48.



145. Web.—Let W = the load at the extremity of the girder,

Σ = the strain in any diagonal,

F = the strain in any given bay of either flange,

n = the number of diagonals between the centre of the given bay and the weight,

θ = the angle which the diagonals make with a vertical line.

The weight W is supported by the first diagonal and the upper flange, the former sustaining compression, the latter tension. At a three forces meet and balance; namely, the weight, the horizontal tension of the upper flange and the oblique thrust of diagonal 1;

their relative amounts may therefore be represented by the sides of the triangle *abc* (**9**). Hence, the tension in the first bay of the upper flange is to W as *ac* is to *cb*, that is, $\mathsf{F} = \mathsf{W}\tan\theta$, and the compression in the first diagonal is to W as *ab* is to *cb*, that is, $\Sigma = \mathsf{W}\sec\theta$. The tension of *ad* is transmitted throughout the upper flange to its connexion with the abutment, but the compression in diagonal 1 is resolved at *b* into its components in the directions of diagonal 2 and the lower flange, producing tension in the former and compression in the latter. Thus, there are three forces in equilibrium meeting at *b*, and their relative amounts may be represented to the same scale as before by the sides of the triangle *edb*; whence, the tension in diagonal 2 equals the compression in diagonal 1, and the compression in the first bay of the lower flange equals twice the tension in the first bay of the upper flange, $= 2\mathsf{W}\tan\theta$.

In this way it may be shown that all the diagonals are strained equally, but by forces alternately tensile and compressive, while the flanges receive at each apex equal increments of strain each equal to $2\mathsf{W}\tan\theta$. The general formulæ for the strain in any diagonal is therefore

$$\Sigma = \mathsf{W}\sec\theta \qquad (110)$$

Ex. If $\theta = 45°$, $\sec\theta = 1{\cdot}414$, and we have $\Sigma = 1{\cdot}414\ \mathsf{W}$.*

146. Flanges.—Since the flanges receive at each apex successive increments of strain, each equal to $2\mathsf{W}\tan\theta$, the resultant strains in the successive bays, being the sum of these successive increments, increase as they approach the abutment in an arithmetic progression whose difference $= 2\mathsf{W}\tan\theta$; they are, therefore, for any given bay proportional to the number of diagonals between it and the load, and we have,

$$\mathsf{F} = n\mathsf{W}\tan\theta \qquad (111)$$

where n represents the number of diagonals between the centre of any given bay and the weight (**20**).

Ex. In the last bay of the upper flange of Fig. 48, $n = 7$, and if $\theta = 45°$, $\tan\theta = 1$, and we have $\mathsf{F} = 7\mathsf{W}$.

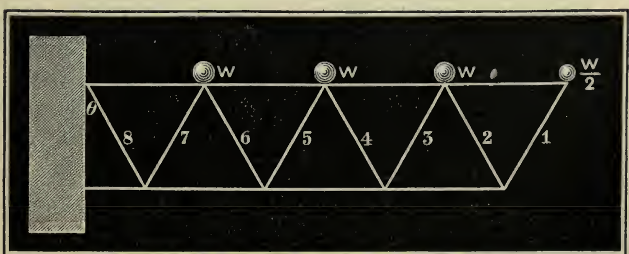
* See the table in Chap. xi. for the numerical values of the tangents and secants of different angles.

The tension in the last diagonal may be resolved at g into a vertical force pressing downwards through the abutment, and a horizontal force tending to pull the abutment towards the weight. The relative amounts of these three forces may be represented by the sides of the triangle fgh; whence, the vertical pressure $= \mathsf{W}$, and the horizontal force $= \mathsf{W} tan\theta$; the latter, added to the tension in the last bay of the upper flange, gives the total horizontal force exerted by the upper flange to pull the abutment towards **W**. It will be observed that the horizontal thrust of the lower flange against the abutment is equal and opposite to the pull of the upper flange, so that they form a couple whose tendency is to overturn the abutment on its lower edge next the weight.

147. Strains in braced webs may be deduced from the shearing-strain.—When the flanges are parallel and the bracing consists of a single system of triangulation, the strain in any brace is equal to the shearing-strain multiplied by $sec\theta$. Hence, the strains in the bracing might be deduced from the shearing-strain in the web calculated in the manner explained in **18**. The method of the resolution of forces just described is, however, better calculated to give a correct perception of the properties of diagonal bracing, and it has, moreover, the advantage of being applicable to lattice girders as well as those whose bracing consists of a single system of triangles.

CASE II.—SEMI-GIRDERS LOADED UNIFORMLY.

Fig. 49.



148. Web.—Let W = the weight of so much of the load as covers one bay, *i.e.*, the weight resting on each apex of the loaded flange (**144**),

n = the number of these weights between any given diagonal and the outer end of the girder,

Σ = the strain in the given diagonal,

F = the strain in any bay of either flange,

θ = the angle which the diagonals make with a vertical line.

The weight on the apex farthest from the abutment equals $\frac{W}{2}$, since it is assumed to support the load spread over the outer half bay, while the load spread over the half bay next the abutment is assumed to rest on the apex in contact with the abutment and may therefore be neglected. If each weight be supposed acting alone, it would, as in Case I., produce strains of equal amount, but of opposite kinds, in each diagonal between its point of application and the abutment, without affecting that part of the girder which lies outside it; consequently, when the whole load is applied, each diagonal sustains the sum of the strains produced by the several weights which occur between it and the outer end of the girder (**23, 142**) and we have

$$\Sigma = nW sec\theta \qquad (112)$$

Ex. The value of n for diagonal 5 is $2\frac{1}{2}$; if $\theta = 45°$, $sec\theta = 1{\cdot}414$, and we have $\Sigma = 3{\cdot}535$ W.

149. Strains in intersecting diagonals.—When the apex of any pair of diagonals supports a weight, W, the strain in that diagonal which is nearer the abutment exceeds that in the more remote by $W sec\theta$. But when an apex does not support a weight (those, for instance, in the lower flange of Fig. 49), the strains in the two diagonals are equal in amount but of opposite kinds.

150. Increments of strain in flanges.—In the case of semi-girders loaded uniformly, the *increments* of strain at the apices increase as they approach the wall in an arithmetic ratio whose difference = $2W tan\theta$, and the resultant strains in each bay consequently increase in a much more rapid ratio, viz., as the square of their distance from the outer end of the girder (see eq. 11).

151. Resultant strains in flanges.—The resultant strains in the bays may be represented by equations if desirable. For the loaded flange,

$$\mathsf{F} = \left\{ m\,(m - 1) + \tfrac{1}{2} \right\} \mathsf{W} tan\theta \qquad (113)$$

For the unloaded flange,

$$\mathsf{F} = m^2 \mathsf{W} tan\theta \qquad (114)$$

where m represents the number of the bay measured along its own flange from the outer end of the girder. These equations are obtained by summation; their proof will afford instructive practice to the student.

152. General law of strains in horizontal flanges of braced girders.—The strains in the flanges may also be derived from the following law, which is applicable to all braced girders or semi-girders with horizontal flanges, no matter how loaded, or whether the bracing be isosceles, or the triangulation be single or lattice. *The increment of strain developed in the flange at any apex is equal to the algebraic sum* (*i.e.*, *the resultant*,) *of the horizontal components of the strains in the intersecting diagonals.* Keeping this in our recollection, we may readily exhibit on a rough diagram—first, the strains in the diagonals; secondly, their horizontal components at the apices; and lastly, the successive sums of these components, that is, the total strains in the several bays of each flange.

Ex. Let Fig. 50 represent such a diagram, the load being on the upper flange.

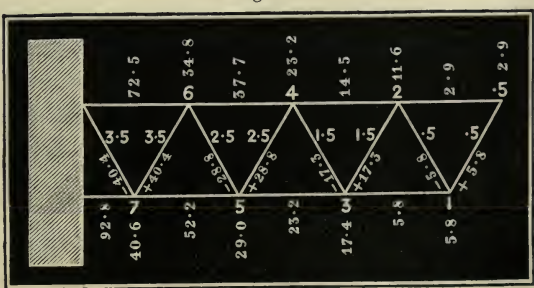
Let $\mathsf{W} = 10$ tons,

$\theta = 30°$,

$Sec\,\theta = 1{\cdot}154$,

$Tan\,\theta = 0{\cdot}577$.

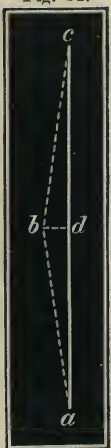
Fig. 50.



The horizontal numbers attached to the diagonals are the coefficients n in eq. 112; these multiplied by $W sec\theta$ give the strains in each diagonal (see the numbers written alongside). The horizontal numbers at each apex are obtained by adding the coefficients of the two intersecting diagonals, and when multiplied by $W tan\theta$ give the horizontal components of the strains in the diagonals, *i.e.*, the increments of flange-strain at each apex (see the vertical numbers at each apex). Finally, the successive additions of these increments give the resultant strains in each bay (see the vertical numbers at the centre of each bay). These may be checked by eqs. 113 and 114; thus, in the 3rd bay of the upper flange, $F = (3 \times 2 + \frac{1}{2}) \times 10 \times \cdot 577 = 37 \cdot 5$ tons, which differs merely in the decimals from the number obtained by the diagram.

153. Lattice web has no theoretic advantage over a single system—Practical advantage of lattice web—Long pillars.— If two or more systems of triangulation be substituted for the single system just described, we have a lattice girder; and here I may remark that lattice bracing has no theoretic advantage over a single system of triangulation; its advantages are entirely of a practical nature, consisting in the frequent support which the tension diagonals give to those in compression, and which both afford the flanges. Long pillars are serious practical difficulties, owing to their tendency to flexure, and lattice tension bars subdivide the struts, which would otherwise be long unsupported pillars, into a series of shorter pillars and hold them in the direction of the line of thrust. That this does not injuriously affect the tension diagonals will be evident, when we reflect that the longitudinal strain produced in a tension diagonal by the deflection of a strut crossing it at right angles, *in the plane of the girder*, bears the same ratio to the weight on the strut, as twice the versine of the deflection curve bears to the length of the half strut—an amount quite inappreciable in practice. If, for instance, a strut adc, Fig. 51, be ten feet long, and if its central deflection under strain, bd, equal half-an-inch (an amount much greater than occurs in practice), the transverse force in the direction of bd, which will sustain the thrust due to deflection, is to the longitudinal pressure as $\frac{2bd}{dc}$, that is, it is only $\frac{1}{60}$th of the weight passing through the pillar; so that in most cases a stout wire in tension would be sufficiently strong to keep the pillar from

Fig. 51.



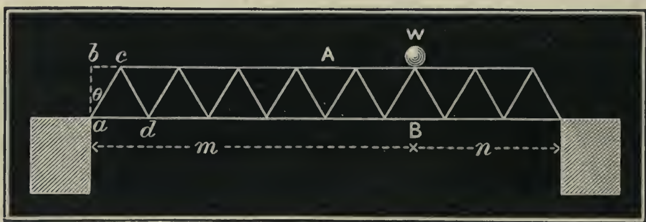
deflecting in the plane of the girder. Again, if the force requisite to resist the tendency of a strut to deflect *at right angles to the plane of the girder* were supplied altogether by a tension brace, the longitudinal strain in that brace would equal the weight on the strut, but it does not follow that this strain is developed in the tension brace. In fact, the force with which the ends of the tension brace are pulled asunder is practically independent of the strut, for the increase in the strain on the tension brace is only due to the difference between the lengths *bc* and *dc*. These considerations show that a moderate lateral force will keep a long pillar from bending, and the apprehension of long compression bars yielding by flexure in the plane of the girder, or producing undue strains in the tension bars, need not deter us from applying lattice bracing to girders exceeding in length any girder bridge hitherto constructed. They also explain the otherwise anomalous strength and rigidity of plate girders and lattice girders whose webs are formed merely of thin plates or thin bars. Such modes of construction are, however, more or less defective. The struts should be formed of angle, T, or channel iron, or the material should be thrown into some other form than that of a thin bar, which is quite unsuitable for resisting flexure at right angles to the plane of the web. A very effective method of stiffening thin compression bars has been applied to tubular lattice girders. It consists of a species of light internal cross-bracing between the opposite compression bars of the double web; this stiffens them at right angles to the plane of the web, while the tension braces keep them from deflecting in the plane of the web (see Plate IV.)

154. Multiple and single triangulation compared—Lattice semi-girders loaded uniformly.—The effect of latticing, compared with a single system of triangulation, is, as far as theory is concerned, merely to distribute the load over a greater number of apices, and consequently to reduce the strain in each of the original diagonals in proportion to the increased number of systems; for, since the several systems are, as we have just seen, practically independent of each other, each diagonal sustains the strain due to those weights alone which are supported on the apices of the system to which it belongs. Eq. 112 will, therefore, give the strain in any brace of a lattice semi-girder loaded uniformly, observing that the coefficient n will now express the number of those weights alone which are supported by that system to which the brace in question belongs, and which occur between it and the outer end of the semi-girder. The strains in the flanges of a lattice semi-girder increase less abruptly than when one system of triangulation is adopted, and are most conveniently obtained by a diagram similar to Fig. 50.

155. Girder balanced on a pier.—The case of a girder balanced midway on a pier is obviously included in the preceding cases, since each segment is a semi-girder.

CASE III.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED AT AN INTERMEDIATE POINT.

Fig. 52.



156. Web.—Let W = the weight, dividing the girder into segments containing respectively m and n bays,

$l = m + n =$ the number of bays in the span,

$\Sigma =$ the strain in any diagonal,

F = the strain in any bay of either flange,

$\theta =$ the angle which the diagonals make with a vertical line,

$x =$ the number of diagonals between any bay and either abutment, measured from the centre of the bay.

On the principle of the lever (**10**), the reaction of the right abutment $= \frac{m}{l}\mathbf{W}$, and that of the left abutment $= \frac{n}{l}\mathbf{W}$. Since the flanges are capable of transmitting strains in the direction of their length only (**140**), they cannot transfer vertical pressures to the abutments; $\frac{m}{l}\mathbf{W}$ must therefore be transmitted through the diagonals on the right side of **W** to the right abutment, while $\frac{n}{l}\mathbf{W}$ pass through the diagonals on the left side of **W** to the left abutment. These quantities are in fact the shearing-strains described in **34**, that is, they are the vertical components of the strains in the diagonals of each segment. The actual strain in any diagonal is to its vertical component as the length of the diagonal is to the depth of the girder, or, calling the angle of inclination of a diagonal to a vertical line θ, we have the strain in each diagonal in the right segment,

$$\Sigma = \frac{m}{l}\mathbf{W} sec\theta \tag{115}$$

in the left segment,

$$\Sigma = \frac{n}{l}\mathbf{W} sec\theta \tag{116}$$

The diagonals which intersect at the weight are both subject to the same kind of strain, while the strains in the diagonals of each segment are alternately tensile and compressive. If the weight be at the centre of the girder all the diagonals will be equally strained.

157. Flanges.—The tensile strain in the second diagonal, *cd*, is resolved at *c* into its components in the directions of the top flange and the first diagonal. The former $= \frac{2n}{l}\mathbf{W} tan\theta$, and is transmitted throughout the flange as far as **W**, receiving at the intervening apices successive increments of strain each equal to $\frac{2n}{l}\mathbf{W} tan\theta$. At **W** these horizontal strains are met and balanced by a similar series of horizontal increments developed at each apex to the right of **W** and acting in the opposite direction to the first series. The strains in the lower flange may be found in a similar manner, for the thrust of the first diagonal, *ac*, is resolved at *a* into a vertical pressure on the abutment, $= \frac{n}{l}\mathsf{W}$, and a horizontal tensile strain in the lower flange which acts as a tie. As these three forces which meet at *a* balance, their relative amounts may be represented by the sides of the dotted triangle *abc*; hence, the horizontal strain in the first bay of the lower flange $= \frac{n}{l}\mathsf{W}\tan\theta$, which is transmitted throughout the flange as far as the bay underneath W, receiving at each intervening apex successive increments each equal to $\frac{2n}{l}\mathsf{W}\tan\theta$. Beneath W these strains are met and balanced by the reverse series generated at the several apices in the right segment.

The resultant strain in any bay of either flange equals the sum of the increments generated at the several apices between it and the abutment of the segment in which it occurs. If the bay be in the right segment and x be measured from the right abutment,

$$\mathsf{F} = \frac{mx}{l}\mathsf{W}\tan\theta \tag{117}$$

If the bay be in the left segment and x be measured from the left abutment,

$$\mathsf{F} = \frac{nx}{l}\mathsf{W}\tan\theta \tag{118}$$

The maximum strains in the flanges occur at W and are represented by the equation

$$\mathsf{F} = \frac{2mn}{l}\mathsf{W}\tan\theta \tag{119}$$

Ex.—See Fig. 52.

$$\begin{aligned} \text{Let } \theta &= 30°, \\ l &= 8, \\ m &= 5{\cdot}5, \\ n &= 2{\cdot}5, \\ \sec\theta &= 1{\cdot}154, \\ \tan\theta &= 0{\cdot}577. \end{aligned}$$

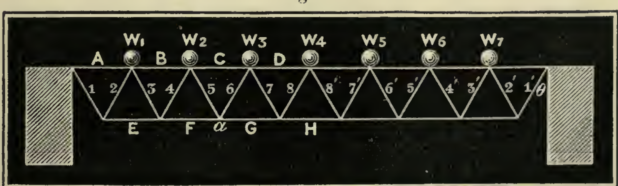
From eqs. 115 and 116, the strains in each diagonal of the right segment = 0·7934 W, and those in each diagonal of the left segment = 0·3606 W. From eq. 118 the compressive strain in bay A = 1·4425 W, and the tensile strain in bay B = 1·9834 W.

158. Concentrated rolling load.—If the weight be a rolling load, the strains in the diagonals will vary according to its position, changing from tension to compression and *vice versâ*, as it passes each apex (**37**). If the upper flange supports the load, the maximum compression in any diagonal occurs when the weight is passing its upper extremity, and the maximum tension when passing the adjoining apex at that side to which the diagonal slopes downwards. If the lower flange supports the load, the maximum tensile strain in any diagonal occurs when the weight is passing its lower end, and the maximum compressive strain when passing the adjoining apex on that side to which the diagonal slopes upwards. The maximum strain in any bay of the unloaded flange occurs when the moving load is in the vertical line passing through that bay, as may be seen from eqs. 117 or 118, for mx and nx are at their maximum when they become mn (**32**). The maximum strain in any bay of the loaded flange occurs when the passing load rests on the adjoining apex on the side next the centre, for the product mn, in eq. 119, is greater for this apex than for the adjoining apex on the side remote from the centre.

159. Lattice girder traversed by a single load.—In this case the strains in the diagonals may be calculated by eqs. 115 and 116, for the reasoning by which these equations were deduced is equally applicable to lattice girders. It will also be observed that only one system of triangulation is strained at a time, *i.e.*, supposing the load to rest on a single apex, which, however, is seldom the case, as generally two or more adjacent apices are loaded together.

CASE IV.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

Fig 53.



160. Web.—Let W = the weight of so much of the load as covers one bay, *i.e.*, the weight resting on each apex of the loaded flange,

l = the number of bays in the span,

n = the number of weights between any given diagonal and the centre of the girder,

Σ = the strain in the given diagonal,

F = the strain in any bay of either flange,

θ = the angle which the diagonals make with a vertical line.

If the load be uniformly distributed so that an equal weight rests upon each apex, the strains in the diagonals gradually increase from the centre toward the ends. Any two diagonals equally distant from the centre sustain all the intermediate load. If they are tension diagonals, the weight is suspended as it were between them; if they are compression diagonals it is supported by them as oblique props. Each diagonal conveys, therefore, to the abutment the pressure of the weights between it and the centre, and the sum of these weights constitutes its vertical component or shearing-strain (**46**). Hence, we have for a uniform load,

$$\Sigma = n\mathsf{W}sec\theta \tag{120}$$

161. Flange-strains derived from a diagram.—The strain in the flanges may be derived from the law stated in **152** by the aid of a rough diagram, as explained in the following example:—

Ex. 1. Let Fig. 54 represent one-half of a girder 80 feet long, with the bracing formed of 8 equilateral triangles, and supporting a uniform load of half a ton per running foot. From these data we have

$$\mathsf{W} = 5 \text{ tons},$$
$$\theta = 30°,$$
$$l = 8,$$
$$tan\theta = 0{\cdot}577,$$
$$sec\theta = 1{\cdot}154,$$
$$\mathsf{W}tan\theta = 2{\cdot}885 \text{ tons},$$
$$\mathsf{W}sec\theta = 5{\cdot}770 \text{ tons}.$$

Fig. 54.

The horizontal numbers attached to the diagonals are the coefficients n, in eq. 120; these, multiplied by $W sec\theta$, give the strains in the several diagonals (see the numbers written alongside them). The horizontal numbers at each apex are the sums of the coefficients attached to the intersecting diagonals; these multiplied by $W tan\theta$ give the horizontal components of the strains in the diagonals, that is, the increments of flange-strain at each apex (see the numbers written in a vertical direction at each apex). Finally, the successive additions of these increments give the resultant strains in the flanges (see the numbers written in a vertical direction at the centre of each bay).

Ex. 2. Let Fig. 53 represent a girder 80 feet long, with the bracing formed of 8 right-angled triangles, and supporting a uniform load of half a ton per running foot.

$$\text{Here} \quad W = 5 \text{ tons},$$
$$\theta = 45^\circ,$$
$$l = 8,$$
$$tan\theta = 1,$$
$$sec\theta = 1{\cdot}414,$$
$$W tan\theta = 5 \text{ tons},$$
$$W sec\theta = 7{\cdot}07 \text{ tons}.$$

The strains in tons are as follows :—

DIAGONALS, . .	1	2	3	4	5	6	7	8
Strains in tons (eq. 120),	−24·7	+24·7	−17·7	+17·7	−10·6	+10·6	− 3·5	+ 3·5
FLANGES, . . .	A	B	C	D	E	F	G	H
Strains in tons, . .	+17·5	+47·5	+67·5	+77·5	− 35	− 60	− 75	− 80

162. Web, second method.—The strains in the diagonals may also be obtained by forming a table of the strains which each weight would produce if acting separately, and then taking as the resultant strain from all acting together the sum or difference of the tabulated strains, according as they are of the same or opposite kinds. Thus, diagonal 4, Fig. 53, is subject to compressive strains from all the weights except the first; the resultant strain is therefore found by subtracting the tensile strain produced by the first weight from the sum of the compressive strains produced by the remaining six weights (**143**). This method, as applied to the first example in **161**, is exhibited in the annexed table, the numerals in the first column of which represent the diagonals, and the letters in the upper row the weights, in order of position. The numbers found at the intersection of a diagonal with a weight represent in tons the strain produced in that diagonal by the weight in question (see eq. 115). The last column contains the strains which the load produces when distributed uniformly all over. These are obtained by adding algebraically the several horizontal rows, and the strains thus obtained should agree with those derived from eq. 120.

Diagonals.	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Strains in diagonals due to a uniform load.
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
1	− 5·1	− 4·3	− 3·6	− 2·9	− 2·2	− 1·4	− ·72	− 20·2
2	+ 5·1	+ 4·3	+ 3·6	+ 2·9	+ 2·2	+ 1·4	+ ·72	+ 20·2
3	+ 0·7	− 4·3	− 3·6	− 2·9	− 2·2	− 1·4	− ·72	− 14·4
4	− 0·7	+ 4·3	+ 3·6	+ 2·9	+ 2·2	+ 1·4	+ ·72	+ 14·4
5	+ 0·7	+ 1·4	− 3·6	− 2·9	− 2·2	− 1·4	− ·72	− 8·7
6	− 0·7	− 1·4	+ 3·6	+ 2·9	+ 2·2	+ 1·4	+ ·72	+ 8·7
7	+ 0·7	+ 1·4	+ 2·2	− 2·9	− 2·2	− 1·4	− ·72	− 2·9
8	− 0·7	− 1·4	− 2·2	+ 2·9	+ 2·2	+ 1·4	+ ·72	+ 2·9

It will be observed that, when once the strain produced by W_7 in diagonal 1 is obtained, all the other strains may be derived from it by addition.

163. Increments of strain in flanges.—The flanges receive successive increments of strain at each apex as they approach the centre where the maximum strains occur. These increments *diminish* as they approach the centre in an arithmetic progression whose difference $= 2W tan\theta$. Hence, the strains in the bays might be expressed by an equation; they may, however, be more conveniently found by the aid of a rough diagram, as already described in **161**.

164. Strains in flanges calculated by moments.—The strains in any given bay may also be obtained by taking moments round the apex immediately above or below it. To obtain the strain in bay **C**, Fig. 53, for example, take moments round the apex *a*. The left segment of the girder is held in equilibrium by the reaction of the left abutment (= 17·5 tons), the two first weights, $\mathbf{W}_1$ and $\mathbf{W}_2$, the horizontal tension in **C**, and the strains at *a*. Taking moments round the latter point, we have

$$\mathbf{F}d = 17{\cdot}5 \times 2{\cdot}5b - 5\,(1{\cdot}5 + 0{\cdot}5)b,$$

where $\mathbf{F}$ = the strain in the flange at **C**,
b = the length of one bay,
d = the depth of the girder.

If $\theta = 45°$, $b = 2d$, and we have $\mathbf{F} = 67{\cdot}5$ tons, as in ex. 2, (**161**).

This method is, it will be perceived, merely a modification of that described in **43**. It is sometimes convenient for checking results obtained by the resolution of forces.

165. Girder loaded unsymmetrically.—If the load be distributed in an unsymmetrical manner, the strains produced by each weight acting separately should first be tabulated, and then the resultant strains may be obtained as indicated in **162**.

166. Girder loaded symmetrically.—If the central part of a symmetrically loaded girder be free from load, the central braces may be removed without affecting the strength of the structure. If, for example, the girder represented in Fig. 53 support only $\mathbf{W}_1$, $\mathbf{W}_2$, $\mathbf{W}_6$, $\mathbf{W}_7$, the braces in the interval, 5, 6, 7, 8, 8′, 7′, 6′, 5′, may be removed. If the central weight alone be wanting, then braces 7, 8, 8′, 7′, may be removed.

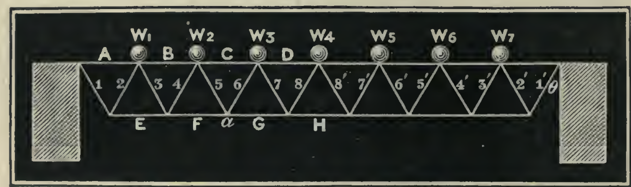
167. Strains in end diagonals and bays.—When the load is symmetrical, each of the end diagonals sustains a strain equal to one-half the load multiplied by $sec\theta$, and the extreme bays of the longer flange sustain a strain equal to one-half the load multiplied by $tan\theta$. Consequently, when $\theta = 45°$, the strain in each of these extreme bays equals half the load.

168. Strains in intersecting diagonals—General law of

Strains in intersecting diagonals of isosceles bracing with parallel flanges.—When two diagonals intersect at a *loaded* apex of a girder loaded uniformly, the strain in that diagonal which is more remote from the centre exceeds that in the other by $\mathsf{W}\sec\theta$. The following law is applicable to all girders with parallel flanges and isosceles bracing whether single or lattice; *when two diagonals intersect at an unloaded apex, no matter how the load may be distributed, the strains in the two diagonals are equal in amount, but of opposite kinds.*

CASE V.—GIRDERS SUPPORTED AT BOTH ENDS AND TRAVERSED BY A TRAIN OF UNIFORM DENSITY.

Fig. 55.



169. Web.—Let W = the weight of so much of the uniformly distributed load as covers one bay, *i.e.*, the permanent load resting on each apex,

W' = the weight of so much of the passing load as covers one bay, *i.e.*, the passing weight on each apex,

l = the number of bays in the span,

n = the number of apices loaded by the passing load between any given diagonal and either abutment,

Σ = the strain in the given diagonal due to the permanent load,

Σ' = the maximum strain in the given diagonal due to the passing load,

θ = the angle the diagonals make with a vertical line.

The strains in the diagonals vary according to the position of the passing train, not only in amount, but also in kind. If, for instance, W_1 alone rests upon the girder, diagonal 4 is subject to tension. If now W_2 be added, its tendency will be to produce compression in diagonal 4, that is, a strain of an opposite kind to that produced by W_1, and the true strain which this diagonal sustains, when both weights rest upon the girder, is equal to the difference of the strains produced by each weight acting separately (**143**). The third, fourth, fifth, sixth, and seventh weights tend to increase the compression in diagonal 4, while the first weight alone tends to produce tension. Consequently, the maximum compression in this diagonal takes place when all the weights except the first rest upon the girder, and the maximum tension occurs when all the weights are removed except the first. The same result may be arrived at in any particular case by means of a table of strains, such as that in **162**, where we find at the intersection of diagonal 4 and W_1, that this weight produces a tension of 0·7 tons in the diagonal, while each of the remaining weights produces compression. When all the weights rest upon the girder, the first and last produce no effect on diagonal 4, since the strains due to these weights are equal and have opposite signs. In fact, these weights are supported exclusively by the flanges and the last pair of diagonals at each end, and, as far as they alone are concerned, all the intermediate diagonals might be omitted. If, however, W_1 be removed, the eighth part of W_7 is transmitted to the left abutment, and consequently increases the compression in diagonal 4 by the strain found in the table at the intersection of W_7 and 4. If, on the other hand, W_7 be removed, the eighth part of W_1 is transmitted to the right abutment, diminishing the compression in diagonal 4 by the strain found at the intersection of W_1 and 4. In a similar manner we find from the table that any other diagonal, 7 for instance, sustains the greatest amount of compression when the first, second, and third weights alone rest upon the girder, and the greatest tension when these are removed and the other weights remain.

170. Maximum strains in web—Strains in intersecting

diagonals.—*The maximum strain in any diagonal occurs when the passing train covers only one segment* (**51**); and in general terms, *the maximum tensile strain in any diagonal occurs when the passing train covers the segment from which the diagonal slopes upwards, and the maximum compressive strain when it covers the segment towards which the diagonal slopes upwards.* When a pair of diagonals meet at the *unloaded* flange, the strains in the two diagonals are equal in amount but of opposite kinds, and the maximum tensile strain in one is equal to the maximum compressive strain in the other, and *vice versâ* (**168**).

171. Permanent load—Absolute maximum strains.—In all the foregoing investigations the weight of the girder and roadway has been left out of consideration, but in practice the permanent load materially modifies the strains, especially in bridges of large span where the ratio of the permanent to the passing load is considerable. If the supported load be uniformly distributed, its weight may be added to that of the structure, provided the latter be also uniform, and the calculations made for their combined weights as already explained for uniform loads. But when the load moves, the strains in the bracing produced by the weight of the permanent structure will be increased or diminished, or even a strain of an opposite kind produced, according to the position of the passing load. In order to obtain the absolute maximum strains to which the bracing is liable under these circumstances, we must calculate—first, the strains produced by the permanent structure alone, and afterwards the maximum strains, both tensile and compressive, due to the passing load alone. These latter, when added to, or subtracted from, the strains produced by the permanent load, according as they are of the same or opposite kinds, will give the absolute maximum strains to which each brace is liable in any position of the passing load.

172. Web, first method.—Perhaps the simplest method of obtaining the strains in the diagonals from a passing train is by forming a table of strains produced by each weight acting separately, as in **162**. Then adding, first the tensile, and afterwards the compressive, strains in each horizontal row, we obtain the required maximum strains of each kind.

Ex. The following example of a girder of eight bays will illustrate this method of calculating the absolute maximum strains when the bridge is traversed by a load of uniform density whose length is not less than the span. Let Fig. 55 represent a railway girder, 80 feet long and 5 feet deep, the bracing of which is formed of 8 right-angled isosceles triangles, with the roadway attached to the upper flange. Let the permanent bridge-load equal half a ton per running foot, and the greatest passing train of uniform density equal one ton per foot; we then have

$$\begin{aligned} W &= 5 \text{ tons from the permanent load,} \\ W' &= 10 \text{ tons from the passing train,} \\ l &= 8, \\ \theta &= 45° \\ tan\theta &= 1, \\ sec\theta &= 1{\cdot}414, \\ W sec\theta &= 7{\cdot}07 \text{ tons,} \\ \frac{W'}{l} sec\theta &= 1{\cdot}77 \text{ tons,} \\ (W + W')\, tan\theta &= 15 \text{ tons.} \end{aligned}$$

Diagonals.	W_1'	W_2'	W_3'	W_4'	W_5'	W_6'	W_7'	C'	T'	Σ	C	T
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
1	−12·4	−10·6	−8·9	−7·1	−5·3	−3·5	−1·8	...	−49·6	−24·7	...	−74·3
2	+12·4	+10·6	+8·9	+7·1	+5·3	+3·5	+1·8	+49·6	...	+24·7	+74·3	...
3	+ 1·8	−10·6	−8·9	−7·1	−5·3	−3·5	−1·8	+ 1·8	−37·2	−17·7	...	−54·9
4	− 1·8	+10·6	+8·9	+7·1	+5·3	+3·5	+1·8	+37·2	− 1·8	+17·7	+54·9	...
5	+ 1·8	+ 3·5	−8·9	−7·1	−5·3	−3·5	−1·8	+ 5·3	−26·6	−10·6	...	−37·2
6	− 1·8	− 3·5	+8·9	+7·1	+5·3	+3·5	+1·8	+26·6	− 5·3	+10·6	+37·2	...
7	+ 1·8	+ 3·5	+5·3	−7·1	−5·3	−3·5	−1·8	+10·6	−17·7	− 3·5	+ 7·1	−21·2
8	− 1·8	− 3·5	−5·3	+7·1	+5·3	+3·5	+1·8	+17·7	−10·6	+ 3·5	+21·2	− 7·1

The numbers in the first column represent the diagonals, and the seven first letters in the upper row the passing weights, in order of position. The numbers found at the intersection of a diagonal with a weight represent the strains produced in the diagonals by the passing load resting on each apex separately; these are derived from eqs. 115 and 116. The columns marked C' and T' contain the maximum strains of compression and tension which the passing load can produce; they are obtained by adding, first the compressive, and afterwards the tensile, strains in each row in the first part of the table. The column marked Σ contains the strains due to the uniform permanent load; these are derived from eq. 120. Finally, the two last columns, marked C and T, contain the absolute maximum strains which the combination of permanent and passing loads can produce; these are obtained by adding algebraically column Σ to columns C' and T' respectively. If one ton per foot be the greatest passing load to which the girder is liable, the strains in the bracing can never exceed these absolute maximum strains.

173. Flanges.—The maximum strains in the flanges occur when the passing load covers the whole girder (**53**).

In our example this occurs when the girder supports a uniformly distributed load of 1·5 tons per running foot, equivalent to 15 tons at each apex. The strains in the several bays are given in the following table; they are obtained by the aid of a diagram, as described in **161**.

Bays,	A	B	C	D	E	F	G	H
Strains in tons,	+ 52·5	+ 142·5	+ 202·5	+ 232·5	− 105	− 180	− 225	− 240

174. Counterbracing.—On examining the two last columns of the table in **172**, it will be seen that diagonals 7 and 8 are the only braces which are liable to both tensile and compressive strains. Consequently, the four central diagonals alone require to be counterbraced (**137**); whereas, if the permanent load had been left out of consideration, all the diagonals except the extreme pair at each end would require counterbracing; and if, on the other hand, the strains from the passing load had been calculated on the supposition of its being a uniformly distributed, in place of a passing load, none of the diagonals would require counterbracing.

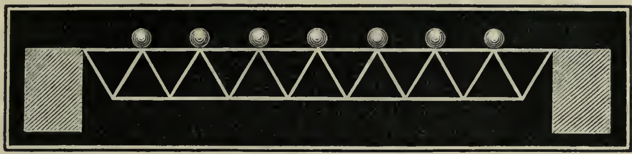
175. Permanent load diminishes counterbracing.—In bridges of large span, the permanent load will materially diminish the amount of counterbracing that would be required if the passing load alone had to be provided for; and when the span is very large, it will be more accurate to consider the permanent load as resting, part on the upper, and part on the lower flange. In small spans this nicety of calculation may be neglected, since the cross road-girders and roadway, with the flange to which they are attached, form the greater portion of the permanent load.

176. Web, second method.—The maximum strains in the diagonals due to a passing train of uniform density may be expressed by equations similar to those given in the preceding cases, for which purpose it is necessary to divide girders into two classes.

Class A.

Girders in which the extreme apices of the loaded flanges are each distant one *whole bay* from the abutments, as in Fig. 56.

Fig. 56.



From eq. 115 the strain in any diagonal from the passing weight at—

$$\text{The 1st apex} = \frac{W'}{l} sec\theta,$$

$$\text{2nd apex} = 2\frac{W'}{l} sec\theta,$$

$$\text{3rd apex} = 3\frac{W'}{l} sec\theta,$$

*   *   *   *

$$nth \text{ apex} = n\frac{W'}{l} sec\theta,$$

where n represents the number of loaded apices between the diagonal and one abutment. The maximum strain is equal to the sum of these separate strains; hence,

$$\Sigma' = (1 + 2 + 3 + \ldots n)\frac{W'}{l} sec\theta,$$

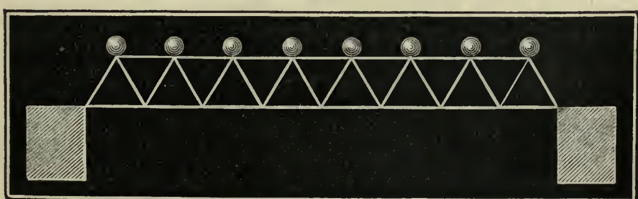
or by summation,

$$\Sigma' = \frac{n\,(n+1)}{2} \times \frac{W'}{l} sec\theta. \qquad (121)$$

Class B.

Girders in which the extreme apices of the loaded flange are each distant one *half-bay* from the abutment, as in Fig. 57.

Fig. 57.



The strain in any diagonal from the passing weight at—

$$\text{The 1st apex} = \frac{W'}{2l} sec\theta,$$

$$\text{2nd apex} = 3\frac{W'}{2l} sec\theta,$$

$$\text{3rd apex} = 5\frac{W'}{2l} sec\theta,$$

* * * *

$$nth \text{ apex} = (2n - 1)\frac{W'}{2l} sec\theta.$$

Adding these together, we have the strain due to the passing load,

$$\Sigma' = (1 + 3 + 5 + \ldots \overline{2n - 1})\frac{W'}{2l} sec\theta,$$

or by summation,

$$\Sigma' = \frac{n^2}{2} \times \frac{W'}{l} sec\theta. \tag{122}$$

Eq. 122 proves that the strains in the diagonals produced by a passing load are proportional to the square of the loaded segment (**50**).

Ex. The following example of a girder of 8 bays with equilateral triangles, belonging to *Class A*, will illustrate this method of calculating the maximum strains produced by a passing train of uniform density sufficiently long to extend over the whole bridge. Let the girder be 80 feet long, the permanent load 0·5 tons per running foot, and the passing load of greatest density (say engines) one ton per foot; we then have, using the same notation as before,

$$W = 5 \text{ tons from the permanent load},$$
$$W' = 10 \text{ tons from the passing train},$$
$$l = 8,$$
$$\theta = 30°,$$
$$tan\theta = 0·5773,$$
$$sec\theta = 1·154,$$
$$W sec = 5·77 \text{ tons}$$
$$\frac{W'}{l} sec\theta = 1·442 \text{ tons}$$
$$(W + W')\, tan\theta = 8·66 \text{ tons}.$$

Diagonals	$\frac{n(n+1)}{2}$	Σ	C′	T′	C	T
		Tons.	Tons.	Tons.	Tons.	Tons.
1	− 28	− 20·2	...	− 40·4	...	− 60·6
2	− 0	+ 20·2	+ 40·4	...	+ 60·6	...
3	− 21	− 14·4	+ 1·4	− 30·3	...	− 44·7
4	− 1	+ 14·4	+ 30·3	+ 1·4	+ 44·7	...
5	− 15	− 8·7	+ 4·3	− 21·6	...	− 30·3
6	− 3	+ 8·7	+ 21·6	− 4·3	+ 30·3	...
7	− 10	− 2·9	+ 8·7	− 14·4	+ 5·8	− 17·3
8	− 6	+ 2·9	+ 14·4	− 8·7	+ 17·3	− 5·8

The numerals in the first column represent the diagonals (see Fig. 55). The second column contains the coefficients for each diagonal, $\frac{n(n+1)}{2}$ in eq. 121, n being measured alternately from the right and the left abutment. Column Σ contains the strains produced by the permanent bridge-load; these are calculated by eq. 120. Columns C′ and T′ contain the maximum strains produced by the passing load; these are calculated by the aid of the second column and eq. 121 (see **170**). Finally, the two last columns contain the absolute maximum strains of either kind in the bracing, taking both permanent and passing loads into consideration; these are obtained by adding columns C′ and T′ algebraically to column Σ. The strains in the flanges are as follows (**161**) :—

Bays,	A	B	C	D	E	F	G	H
Strains in tons,	+30·3	+82·3	+117·0	+134·2	−60·6	−103·9	−129·9	−138.6

CASE VI.—LATTICE GIRDERS SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

177. Approximate rule for strains in lattice web.—It has been already shown (**154**) that the effect of increasing the number of diagonals, so as to form a lattice girder, is merely to distribute the load over a greater number of apices and thus diminish the strain in each diagonal in proportion to the increased number of systems. This suggests the following approximate rule for finding the strains in the bracing of lattice girders. *Calculate the strains on the supposition that there is only one system of triangles. These divided by the number of systems will give the strains in the corresponding lattice diagonals.* As, however, more exact methods of calculation are of easy application, they are preferable to a rule which is merely approximate.

178. Web—Flanges.—In the case of a uniform load the strains in the bracing may be calculated by eq. 120, observing that the

Fig. 58.

coefficient n will represent in a lattice girder the number of those weights which occur between any given diagonal and the centre of the girder, and which rest only on the apices belonging to its own system of triangulation. This assumes that the strains from weights belonging to different systems, but at equal distances on opposite sides of the centre, such as W_5 and W_{11} in Fig. 58, do not pass through the intermediate diagonals, but merely through the flanges and those diagonals of their respective systems which occur between them and the abutments. This is the simplest way of calculating the strains due to a uniform load, but they may also be calculated for each system separately (**162**), in which case the strains in the diagonals will differ somewhat from those obtained by the first method. The strains in the flanges are most conveniently obtained by the aid of a diagram of strains (**161**).

Ex. The following example of a lattice girder, 80 feet long and 10 feet deep, with four systems of right-angled triangles, *i.e.*, 16 bays, will illustrate the mode of calculation (see Fig. 58). If the uniform load equal half a ton per running foot, we have,

$$\mathsf{W} = 2{\cdot}5 \text{ tons} = \text{the weight on each apex},$$

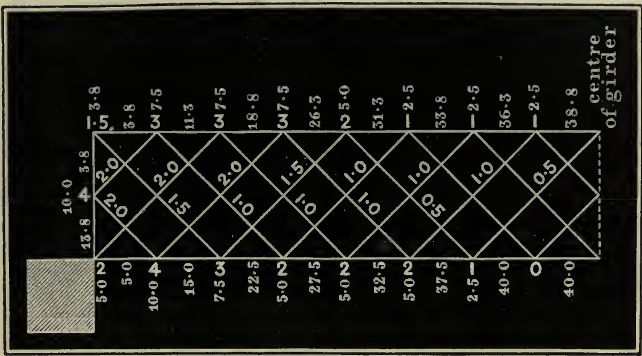
$$\theta = 45°,$$

$$\mathsf{W} sec\theta = 3{\cdot}535 \text{ tons},$$

$$\mathsf{W} tan\theta = 2{\cdot}5 \text{ tons},$$

n = the number of weights belonging to its own system between any given diagonal and the centre of the girder.

Fig. 59.



The numbers attached to the diagonals in the preceding diagram of strains are the coefficients n, in eq. 120; these multiplied by $W sec\theta$ give the strains in the diagonals, as in the following table, the upper row of which represents the diagonals in order of position (see Fig. 58), and the lower row the corresponding strains in tons :—

Diagonals.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Strains in tons.	+7·1	+7·1	+7·1	+5·3	+3·5	+3·5	+3·5	+1·8	..	..	..	—1·8	—3·5	—3·5	3·5	—5·3	—7·1

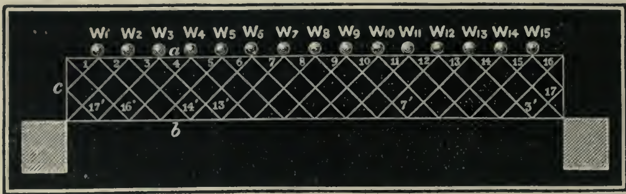
The horizontal numbers at the apices are obtained by adding the coefficients of the intersecting diagonals. These numbers multiplied by $W tan\theta$ are the increments of strain in the flanges (see the vertical figures at each apex). Finally, the successive additions of these increments give the resultant strains in the flanges in tons (see the vertical figure at the centre of each bay).

CASE VII.—LATTICE GIRDERS SUPPORTED AT BOTH ENDS AND TRAVERSED BY A TRAIN OF UNIFORM DENSITY.

179. Web, first method.—Perhaps, the simplest method of obtaining the strains in the case of a passing train is to tabulate the strains produced by each weight separately, and thence infer what condition of the load will produce the maximum strains in each diagonal (**162**).

Ex. The following example of a lattice girder, 80 feet long and 10 feet deep, with 4 systems of right angled triangles, will illustrate this method (see Fig. 60) :—

Fig. 60.



Let the permanent bridge-load equal half a ton per running foot and the passing train equal one ton per running foot. From these data we have,

$$\mathsf{W} = 2{\cdot}5 \text{ tons at each apex from the permanent load,}$$

$$\mathsf{W}' = 5{\cdot}0 \text{ tons at each apex from the passing train,}$$

$$l = 16 = \text{the number of bays in the span,}$$

$$\theta = 45°,$$

$$\mathsf{W}\sec\theta = 3{\cdot}535 \text{ tons,}$$

$$\frac{\mathsf{W}'}{l}\sec\theta = 0{\cdot}442 \text{ tons,}$$

$$\frac{\mathsf{W}+\mathsf{W}'}{l}\tan\theta = 0{\cdot}47 \text{ tons.}$$

The upper row in the table on p. 115 represents the passing weights, and the first column represents the diagonals. The next fifteen columns contain the strains produced in the diagonals by each weight acting separately; these are derived from eqs. 115 and 116. The next two columns, marked C′ and T′, contain the maximum strains of compression and tension produced by the passing load; these are obtained by adding the strains of compression and tension in each row separately. The column headed Σ contains the strains produced by the permanent load; it is copied from the previous example in **178**. Finally, the last two columns, marked C and T, contain the absolute maximum strains which the combined passing and permanent loads can produce; these are obtained by adding column Σ to columns C′ and T′ successively. From this table it appears that diagonals 9, 10, and 11 are subject to both compression and tension; consequently, the six central diagonals require counterbracing. The maximum strains in the flanges occur when the passing load extends uniformly over the whole girder (**53**); they may be obtained by means of a diagram of strains as explained in **178**. In this example the flange-strains are three times greater than in the example in **178**, for the passing load per running foot equals twice the permanent load.

180. End pillars.—The end pillars of lattice girders are sometimes subject to transverse strain from the horizontal components of the diagonals which intersect them midway between the flanges. This transverse strain is, however, of slight amount, as it is merely a differential quantity, being due to the excess of the strain in the

Diagonals	W'_1	W'_2	W'_3	W'_4	W'_5	W'_6	W'_7	W'_8	W'_9	W'_{10}	W'_{11}	W'_{12}	W'_{13}	W'_{14}	W'_{15}	C′	T′	Σ	C	T
																Tons.	Tons.	Tons.	Tons.	Tons.
1	+6·6	...	...	...	+4·9	...	...	...	+3·1	...	...	...	+1·3	...	...	+15·9	...	+7·1	+23·0	...
2	...	+6·2	...	...	...	+4·4	...	...	...	+2·7	...	...	...	+0·9	...	+14·2	...	+7.1	+21·3	...
3	...	...	+5·7	...	...	...	+4·0	...	...	...	+2·2	...	...	...	+0·4	+12·3	...	+7·1	+19·4	...
4	...	...	...	+5·3	...	...	...	+3·5	...	...	...	+1·8	...	...	...	+10·6	...	+5·3	+15·9	...
5	−0·4	...	...	...	+4·9	...	...	...	+3·1	...	...	...	+1·3	...	...	+ 9.3	− 0·4	+3·5	+12·8	...
6	...	−0.9	...	...	...	+4·4	...	...	...	+2·7	...	...	...	+0·9	...	+ 8·0	− 0·9	+3·5	+11·5	...
7	...	...	−1·3	...	...	...	+4·0	...	...	...	+2·2	...	...	...	+0·4	+ 6·6	− 1·3	+3·5	+10·1	...
8	...	...	...	−1·8	...	...	...	+3·5	...	...	...	+1·8	...	...	...	+ 5·3	− 1·8	+1·8	+ 7·1	...
9	−0·4	...	...	...	−2·2	...	...	...	+3·1	...	...	...	+1·3	...	...	+ 4·4	− 2·6	...	+ 4·4	− 2·6
10	...	−0·9	...	...	...	−2·7	...	...	...	+2·7	...	...	...	+0·9	...	+ 3·6	− 3·6	...	+ 3·6	− 3·6
11	...	...	−1·3	...	...	...	−3·1	...	...	...	+2·2	...	...	...	+0·4	+ 2·6	− 4·4	...	+ 2·6	− 4·4
12	...	...	...	−1·8	...	...	...	−3·5	...	...	...	+1·8	...	...	...	+ 1·8	− 5·3	−1·8	...	− 7·1
13	−0·4	...	...	...	−2·2	...	...	...	−4·0	...	...	...	+1·3	...	...	+ 1·3	− 6·6	−3·5	...	−10·1
14	...	−0·9	...	...	...	−2·7	...	...	...	−4·4	...	...	...	+0·9	...	+ 0·9	− 8·0	−3·5	...	−11·5
15	...	...	−1·3	...	...	...	−3·1	...	...	...	−4·9	...	...	...	+0·4	+ 0·4	− 9·3	−3·5	...	−12·8
16	...	...	...	−1·8	...	...	...	−3·5	...	...	...	−5·3	...	...	...	...	−10·6	−5·3	...	−15·9
17	−0·4	...	...	...	−2·2	...	...	...	−4·0	...	...	...	−5·7	...	...	...	−12·3	−7.1	...	−19·4

tension diagonals over those in compression, or *vice versâ*. In Fig. 60, for example, the vertical component of the diagonals meeting at c is transmitted through the lower half of the pillar to the abutment in addition to any pressure which it may receive from the upper half. Their horizontal component, however, tends to deflect the pillar outwards or inwards, according as the strain in the compression or tension diagonal is in excess, and this transverse strain converts the pillar into a vertical girder whose abutments are the flanges. This excess does not attain its greatest value when the girder is uniformly loaded; for since the load is on the upper flange, the tension in diagonal 17′ equals the compression in diagonal 3, and, on examining the preceding table, we find that the greatest excess of strain in diagonal 1 over that in diagonal 3 occurs when all the apices of the system to which the former diagonal belongs are loaded, while those of the latter are free from load. This of course is a condition of load which is very unlikely to occur in practice, but it is quite possible that passing weights may rest on two apices of the first system, say $\mathbf{W}_1$ and $\mathbf{W}_5$, while the apices belonging to the other system are free from load. This might occur, for instance, if a pair of engines or heavy wagons were to cross with a proper interval between them. If this were to occur in our example, the horizontal component of the strain in diagonal 1 would $= \dfrac{(15 + 11)\,\mathbf{W}'}{l} \tan\theta = 8{\cdot}1$ tons. The pillars ought accordingly to be designed with adequate strength to meet such transverse strains, as well as those of compression in the direction of their length.

181. Ambiguity respecting strains in lattice bracing.—When a lattice girder contains three or more systems of triangles, a slight ambiguity may occur respecting the strains if the load be disposed on both sides of the centre. Take for example $\mathbf{W}_7$ and $\mathbf{W}_9$, Fig. 60, which belong to different systems, but rest on apices equally distant from the centre; the whole of $\mathbf{W}_7$ may be transmitted to the left abutment through diagonals 7, 13′, 3 and 17′, and the whole of $\mathbf{W}_9$ to the right abutment through diagonals 7′, 13, 3′ and 17, without producing strains in the other diagonals of either system, which indeed might be safely removed as far as these weights are concerned. The method of calculation described in **178** assumes this to be the case. But again, $\frac{7}{16}$ths of W_7 may be transmitted to the right abutment, and $\frac{9}{16}$ths to the left, through the diagonals of its own system, and similarly with W_9 (**10**). This is assumed to be the case for the passing load in the example in **179**. Hence, there is a slight ambiguity respecting the strains, as they may go in either way, or partly in one, partly in the other, just as it is impossible to say how much pressure is transmitted through any one leg of a four-legged table. If, however, the girder be strong enough to sustain the strain in whichever way it can be conveyed the safety of the structure is secured, and practically there is a very slight difference in the resulting strains whichever method of calculation is adopted. It may be thought that the "principle of least action" will necessarily determine the direction of the strains, *i.e.*, that they will take that direction in which the work done is a minimum; practically, however, a slight inaccuracy in the exact length of the bars will doubtless determine the direction they will take. It ought also to be admitted that a structure will stand as long as it has not exhausted the whole of its possible conditions of stability, and it is therefore sufficient assurance that any structure will stand if we prove that a certain state of stability can be realised.

182. Flange-strains calculated by moments.—When calculating the strain in any bay of a lattice girder by the method of moments (**164**), we must not neglect the moments of the strains in the diagonals. That part of the girder represented in Fig. 60, for instance, which is to the left of a line drawn through bays *a* and *b*, is held in equilibrium by the reaction of the left abutment, the weights W_1, W_2, and W_3, the horizontal forces at *a* and *b*, and the oblique forces in diagonals 4, 5, 13′ and 14′. The moments of the former pair of diagonals are opposed to those of the latter pair, but they seldom balance exactly. Hence, the strains in two bays vertically over each other are rarely precisely the same in value, but differ by an amount equal to the horizontal component of the strains in the diagonals which are intersected by a line joining them; this, indeed, is true whether the bays lie vertically over each other or not, and is merely a modification of the law stated in **58**. Again, it would be erroneous to expect that the strains in the bays of braced girders when uniformly loaded must necessarily agree precisely with those obtained by eqs. 23 or 25. In some cases it happens that they do so agree, but in general they are only close approximations. This arises from our assuming that the load in braced girders is concentrated at the apices, in place of being uniformly distributed. In Fig. 60, for instance, the load on the extreme half-bays is assumed to rest directly over the pillars, while that on the two central half-bays is assumed to rest exactly on the central apex; consequently, these portions of the load are neglected in calculating the central strains in the flanges by the method of moments. If, however, the moments be calculated on the supposition that these loads act at their centres of gravity, *i.e.*, at a distance from the pillars equal to a quarter-bay, and at a distance from the centre also equal to a quarter-bay, the strain at the centre will agree with that obtained by eq. 25.

183. Web, second method.—The strains in the bracing of lattice girders subject to passing loads of uniform density may be expressed by an equation obtained in the following manner:—

Let $\mathbf{W}'$ = the passing weight on each apex,

l = the number of bays in the span (= 16 in Fig. 61),

k = the number of systems of triangles, *i.e.*, the number of bays in the base of one of the primary triangles (= 6 in Fig. 61),

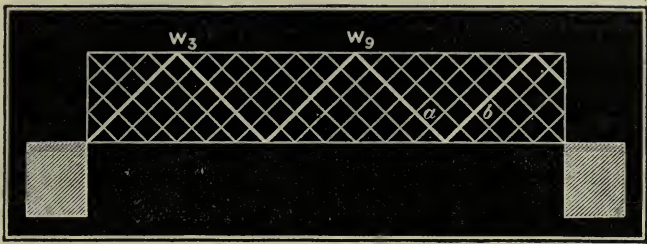
Σ' = the maximum strain which any given diagonal sustains from the passing load,

n = the number of bays between the given diagonal and one of the abutments, measured along the loaded flange,

p = the integral number of times that its own system occurs between the given diagonal and the same abutment, measured also along the loaded flange (= the integral part of $\frac{n}{k}$),

θ = the angle which the diagonals make with a vertical line.

Fig. 61.



Suppose the load traversing the upper flange of Fig. 61; diagonal a sustains the maximum compressive strain when W_3 and W_9 rest upon the girder, and in general, each brace will sustain the maximum strain when the passing load covers only one segment—which segment may be easily seen by inspection (**170**)—but the strain it sustains is due to those weights alone which rest on the apices of its own system. If, for example, there are n bays between the top of diagonal a and the left abutment, then, on the principle of the lever, the portion of W_9 which is transmitted to the right abutment through $a = \frac{n}{l}W'$; and of $W_3 = \frac{n-k}{l}W'$. The maximum compressive strain in diagonal a is equal to the sum of these quantities multiplied by $\sec\theta$, and equals $(n + \overline{n-k})\frac{W'}{l}\sec\theta$; and in general, the maximum strain in any given diagonal due to the passing load,

$$\Sigma' = (n + \overline{n-k} + \overline{n-2k} + \overline{n-3k} + \dots \overline{n-pk})\frac{W'}{l}\sec\theta,$$

or summing these up,

$$\Sigma' = \left(n - \frac{pk}{2}\right).(p+1)\frac{W'}{l}\sec\theta. \tag{123}$$

The maximum tension in $a =$ the maximum compression in b (**170**), and Σ' will represent compressive or tensile strains according as the load traverses the upper or lower flange.

Ex. Let Fig. 62 represent a lattice girder 80 feet long and 5 feet deep, whose bracing consists of two systems of right angled triangles with the load traversing the upper flange.

Fig. 62.

Let the permanent bridge-load equal half a ton per running foot, and the heaviest passing train of uniform density equal one ton per foot. Then we have,

$$\mathsf{W} = 2{\cdot}5 \text{ tons at each apex from the permanent load,}$$
$$\mathsf{W}' = 5 \text{ tons at each apex from the passing train,}$$
$$\theta = 45^\circ,$$
$$l = 16,$$
$$k = 2,$$
$$\mathsf{W}sec\theta = 3{\cdot}54 \text{ tons,}$$
$$\frac{\mathsf{W}'}{l}sec\theta = 0{\cdot}44 \text{ tons,}$$
$$(\mathsf{W} + \mathsf{W}')\,tan\theta = 7{\cdot}5 \text{ tons.}$$

The strains in tons are given in the following table, the numbers in the first column of which represent the diagonals in Fig. 62. The 2nd, 3rd, and 4th columns are the coefficients in eq. 123, from which the maximum strains produced by the passing load, columns C′ and T′, are derived. The strains produced by the permanent bridge-load (column Σ) are obtained from eq. 120, observing that the coefficient n in that equation now represents the number of weights belonging to its own system which occur between any given diagonal and the centre of the girder (**178**). The last two columns, C and T, give the absolute maximum strains due to both permanent and passing loads; these are obtained by adding columns C′ and T′ successively to column Σ.

Diagonals.	n	p	$\left(n-\frac{pk}{2}\right)(p+1)$	C′	T′	Σ	C	T
				Tons.	Tons.	Tons.	Tons.	Tons.
1	15	7	64	+ 28·2	...	+ 14·2	+ 42·4	...
2	14	7	56	+ 24·6	...	+ 12·4	+ 37·0	...
3	13	6	49	+ 21·6	— 0·4	+ 10·6	+ 32·2	...
4	12	6	42	+ 18·5	— 0·9	+ 8·9	+ 27·4	...
5	11	5	36	+ 15·8	— 1·8	+ 7·1	+ 22·9	...
6	10	5	30	+ 13·2	— 2·6	+ 5·3	+ 18·5	...
7	9	4	25	+ 11·0	— 4·0	+ 3·5	+ 14·5	— 0·5
8	8	4	20	+ 8·8	— 5·3	+ 1·8	+ 10·6	— 3·5
9	7	3	16	+ 7·0	— 7·0	...	+ 7·0	— 7·0
10	6	3	12	+ 5·3	— 8·8	— 1·8	+ 3·5	— 10·6
11	5	2	9	+ 4·0	— 11·0	— 3·5	+ 0·5	— 14·5
12	4	2	6	+ 2·6	— 13·2	— 5·3	...	— 18·5
13	3	1	4	+ 1·8	— 15·8	— 7·1	...	— 22·9
14	2	1	2	+ 0·9	— 18·5	— 8·9	...	— 27·4
15	1	0	1	+ 0·4	— 21·6	— 10·6	...	— 32·2
16	0	0	0	...	— 24·6	— 12·4	...	— 37·0

The maximum strains in the flanges occur when the passing load covers the whole girder. They are most conveniently obtained by the aid of a diagram, as described in **178**, and are given in the following table, the letters in the upper rows of which refer to the bays in Fig. 62. The figures in the lower row represent the strains in tons.

Bays, . .	A	B	C	D	E	F	G	H
Strains in tons,	+ 26·3	+ 78·8	+ 123·8	+ 161·3	+ 191·3	+ 213·8	+ 228·8	+ 236·3
Bays, . .	I	J	K	L	M	N	O	P
Strains in tons,	— 30	— 82·5	— 127·5	— 165	— 195	— 217·5	— 232·5	— 240

The compressive strain in each of the end pillars is equal to the vertical component (shearing-strain) of the end tension diagonal plus the load resting on the last half-bay; it reaches its maximum when the girder is loaded all over, in which case it equals 26·25 + 3·75 = 30 tons on each pillar.

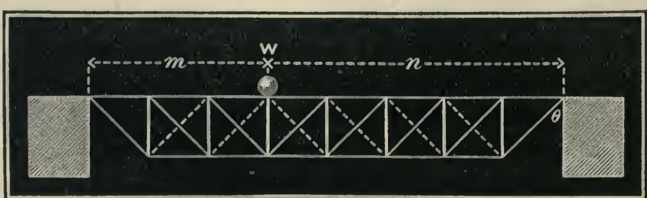
CHAPTER VI.

GIRDERS WITH PARALLEL FLANGES CONNECTED BY VERTICAL AND DIAGONAL BRACING.

184. Introductory.—In the preceding chapter our attention was confined to that form of braced web which consists of isosceles triangles. There is, however, another class of bracing in common use which consists of right-angled triangles, the braces being alternately vertical and oblique. Besides its employment in the webs of girders, this species of bracing is extensively used in scaffolding and for stiffening the platforms of suspension bridges, but more especially for horizontal cross-bracing between the flanges of large girder bridges, so as to strengthen them against side pressure, whether arising from the wind or other sources. The ordinary form of plate girder is, as will be shown hereafter, a modification of this form of bracing. Since the verticals may act as struts, and the diagonals as ties, or *vice versâ*, each of the following cases might be subdivided; this, however, is unnecessary, as in each case it will be evident on inspection whether any given brace is designed to act as a strut or a tie.

CASE I.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED AT AN INTERMEDIATE POINT.

Fig. 63.



185. Let W = the weight, dividing the girder into segments containing respectively m and n bays,
$l = m + n$ = the number of bays in the span,
θ = the angle between the diagonal and vertical braces,
Σ = the strain in a diagonal brace,
Σ' = the strain in a vertical brace.

On the principle of the lever, $\frac{m}{l}\mathsf{W}$ is transmitted to the right abutment through the bracing of the right segment (**10**). Hence, the strain in each vertical of the right segment,

$$\Sigma' = \frac{m}{l}\mathsf{W} \tag{124}$$

Similarly in the left segment,

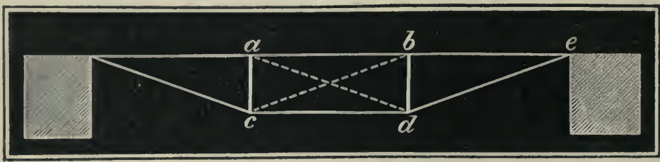
$$\Sigma' = \frac{n}{l}\mathsf{W} \tag{125}$$

These strains in the verticals are identical with the shearing-strain of **34**. The strains in the diagonals are the same as in Case III. of the preceding chapter, that is, they equal the foregoing strains in the verticals multiplied by $sec\theta$ (see eqs. 115 and 116). The strains in the flanges may be found by the aid of a rough diagram of coefficients in the diagonals (**152**), or more simply, by adding the successive increments at the apices, each of which is equal to $\frac{m}{l}\mathsf{W}tan\theta$ or $\frac{n}{l}\mathsf{W}tan\theta$, according as the apex lies to the right or left of W.

186. Single moving load.—If the load move, the girder must be counterbraced (**138**); this may be effected either by counterbracing the existing braces, or by adding a second series of diagonals. In the latter case there will always be certain braces not acting when the load is in any given position; thus, when the weight rests as represented in Fig. 63, and the verticals are in compression, the dotted diagonals are free from strain.

187. Trussed beam.—The trussed beam of the gantry or travelling crane, Fig. 64, is a familiar example of vertical and diagonal bracing. It is, however, seldom counterbraced by the

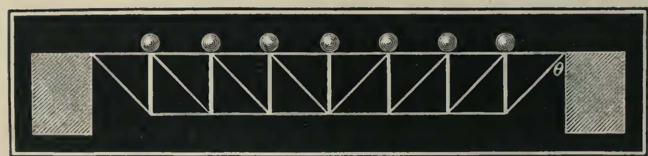
Fig. 64.



dotted diagonals; hence, when the weight rests on a, the tension rod cde tends to straighten itself and thrust b upwards. This is counteracted by the stiffness of the horizontal beam, abe, which is generally formed of a whole balk of timber. Fig. 64 when counterbraced is a simple form of girder for small bridges.*

CASE II.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

Fig. 65.



188. By reasoning similar to that used in Case IV. of the preceding chapter, it may be shown that each brace sustains a strain which is due to all the weights between it and the centre of the girder.

Let W = the weight resting on each apex,

n = the number of weights between any given brace and the centre of the girder,

θ = the angle between the diagonal and vertical braces,

Σ = the strain in a diagonal,

Σ' = the strain in a vertical.

* The railway bridge over the Wye, near Chepstow, erected by the late Mr. Brunel, is an example of this truss on a gigantic style. (See *Clark on the Tubular Bridges*, p. 101). The road, however, is attached to the lower flange, but in small bridges it is usual to place the truss upwards, like Fig. 64 inverted, for this arrangement leaves greater headway beneath, and as the truss forms part of the hand-rail, it answers a double purpose.

The strain in each vertical equals the shearing-strain of **46**, that is,

$$\Sigma' = n\mathsf{W} \tag{126}$$

The strain in each diagonal,

$$\Sigma = n\mathsf{W}sec\theta \tag{127}$$

The increment of strain at each apex $= n\mathsf{W}tan\theta$ where $n =$ the number of weights between the diagonal which intersects that apex and the centre; the successive additions of these increments will give the resultant strains in the several bays.

CASE III.— GIRDERS SUPPORTED AT BOTH ENDS AND TRAVERSED BY A TRAIN OF UNIFORM DENSITY.

Fig. 66.

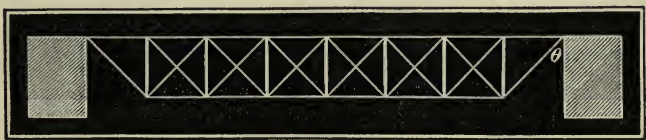
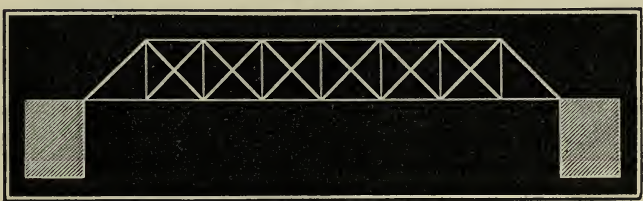


Fig. 67.



189. Web.—When the load traverses the upper flange, each vertical, if acting as a strut (Fig. 66), sustains the maximum strain when the passing load rests on its own apex and on those between it and the farther abutment: if acting as a tie (Fig. 67), when its own apex is free from load and those between it and the farther abutment are loaded.

When the load traverses the lower flange, each vertical, if acting as a strut (Fig. 66), sustains the maximum strain when its own apex is free from load and those between it and the farther abutment are loaded; if acting as a tie (Fig. 67), when its own apex and those between it and the farther abutment are loaded.

The maximum strain in any diagonal, if in tension (Fig. 66), occurs when the load rests on each apex between it and the abutment from which it slopes *upwards;* if in compression (Fig. 67), when the load rests on each apex between it and the abutment from which it slopes *downwards* (**170**).

Let $\mathbf{W}$ = the passing weight on each apex,

n = the number of weights resting on the girder in the foregoing cases of maximum strain,

l = the number of bays in the span,

θ = the angle between the diagonal and vertical braces,

Σ = the maximum strain in a diagonal,

Σ' = the maximum strain in a vertical.

The maximum strain in any vertical is represented by the following arithmetical series:—

$$\Sigma' = (1 + 2 + 3 + 4 + \ldots . n)\frac{\mathbf{W}'}{l}$$

$$\Sigma' = \frac{n(1+n)}{2} \cdot \frac{\mathbf{W}'}{l} \qquad (128)$$

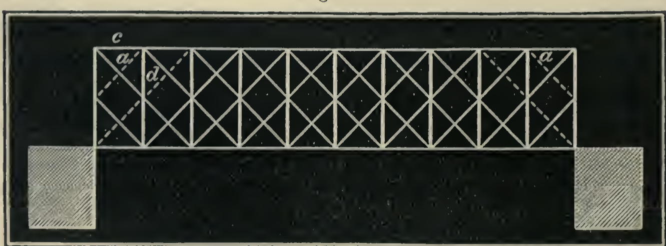
Similarly, the maximum strain in any diagonal,

$$\Sigma = \frac{n(1+n)}{2} \cdot \frac{\mathbf{W}'}{l} sec\theta \qquad (129)$$

The absolute maximum strains in girders subject to both fixed and passing loads are found by tabulating the strains produced by each class of load separately, and then adding or subtracting them according as they are of the same or of opposite kinds (**171**).

CASE IV.—LATTICE GIRDERS SUPPORTED AT BOTH ENDS AND TRAVERSED BY A TRAIN OF UNIFORM DENSITY.

Fig. 68.



190. Web.—In this form of latticing the verticals are generally constructed so as to act as struts and the diagonals as ties, in which case the dotted diagonals are theoretically unnecessary.

Let $\mathbf{W}'$ = the passing weight on each apex,

l = the number of bays in the span (= 10 in Fig. 68),

k = the number of systems of right-angled triangles, *i.e.*, the number of bays in the base of one of the primary right-angled triangles (= 2 in Fig. 68),

$\mathbf{T}$ = the maximum tensile strain which any given diagonal sustains from the passing load,

n = the number of bays between the foot of the given diagonal and that abutment from which it slopes upwards,

p = the integral number of times that its own (right-angled) system occurs between the foot of the diagonal and the same abutment, = the integral part of $\frac{n}{k}$,

θ = the angle between the diagonal and vertical braces.

It may be shown by reasoning similar to that employed in **183**, that the maximum tensile strain in any diagonal,

$$\mathbf{T} = \left(n - \frac{pk}{2}\right) . \ (p+1)\frac{\mathbf{W}'}{l} \sec\theta \tag{130}$$

The maximum compression in any vertical equals the maximum tension in one of the conterminous diagonals divided by $\sec\theta$. If the load traverses the upper flange, take the diagonal intersecting at bottom on the side remote from the centre. If the load traverse the lower flange, take the diagonal intersecting it at top on the side next the centre.

191. End pillars—Ambiguity respecting strains in faulty designs.—In this form of latticing the end pillars are subject to a severer transverse strain than in the isosceles latticing described in the preceding chapter (**180**). In the present case the end pillars must be made sufficiently strong to sustain the horizontal components of *all* the diagonals which intersect them between the flanges. This inconvenience may be remedied by introducing short diagonal struts, such as *a*, *a*, Fig. 68, which will relieve the end pillars of a certain, though indefinite, amount of transverse strain, and at the same time diminish the compression in the bay *c* and the vertical *d*. Both diagonals and verticals are occasionally constructed so as to act indifferently either as struts or ties; in such designs calculation is at fault, for the strains may pass through the isosceles system of triangles alone, or through the vertical and diagonal system alone, or partly through one and partly through the other. In such designs there will generally be found a certain waste of material.

CHAPTER VII.

BRACED GIRDERS WITH OBLIQUE OR CURVED FLANGES.

192. Introductory—Calculation by diagram.—The class of braced girders to which our attention has been directed in the two preceding chapters is characterized by the parallelism of the flanges. We have seen that the strains in each part vary according to the position of the load, and that they may be calculated by simple formulæ with a degree of accuracy which leaves nothing further to be desired. I now propose investigating braced girders, one or both of whose flanges are oblique or curved. The **A** truss and the bowstring girder may be taken as the chief representatives of this class, which also includes cranes of various kinds, crescent girders and the braced arch. Formulæ for strains are unsuited to this species of bracing on account of the various inclinations of the several parts of the structure. Instead, we have recourse to carefully constructed diagrams in which strains are represented to scale, by the aid of which, however, a degree of accuracy is attainable which is practically nearly as perfect as that obtained by the application of formulæ to the girders described in previous chapters.*

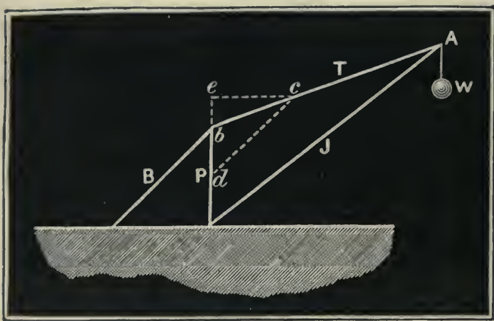
CASE I.—BENT SEMI-GIRDERS LOADED AT THE EXTREMITY.

193. Derrick crane.—The derrick crane, Fig. 69, consists of a revolving post **P**, a jib **J**, a chain or tie-bar **T**, and two back-stays, one of which is shown at **B**, the other, lying in a plane at right angles to that of the figure, is not represented, being hidden by the post. The derrick crane is generally made of wood. It is simple in con-

* Curved flanges are assumed to be polygonal, *i.e.*, formed of straight lines joining the apices (**144**).

struction and easily erected. Hence, it is well adapted for temporary works, as also for quarries or other situations where the back-stays do not interfere with the traffic. At the peak **A**, three forces meet, viz., the downward pull of **W**, the tension of the tie-bar **T**, and the oblique thrust of the jib **J**. Since these three forces are in equilibrium, their relative amounts may be represented by the sides of the triangle **PTJ** (**9**).

Fig. 69



Hence, the tension of the tie-bar $= \frac{\mathsf{T}}{\mathsf{P}}\mathsf{W}$, and the compression of the jib $= \frac{\mathsf{J}}{\mathsf{P}}\mathsf{W}$.

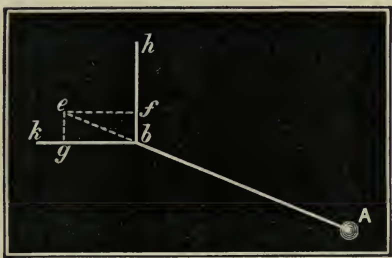
If the chain pass along **T**, and so over a pulley at *b* down to the chain barrel bolted to the foot of the post, it relieves the tie-bar of an amount of tension equal to that in the chain, namely, **W** divided by the number of falls in the hanging part of the chain.*

If, however, the chain pass along the jib, the compression of the latter is increased by an amount equal to the tension of the chain. The tension in **T** being known, the strains in the post and back-stays, which are its components, are easily found. This operation is most conveniently performed by the aid of a skeleton diagram (Fig. 69) drawn accurately to scale. Let the jib and one back-stay lie in the same plane. Lay off *bc* by scale to represent the tension in $\mathsf{T}\left(= \frac{\mathsf{T}}{\mathsf{P}}\mathsf{W}\right)$, and draw *cd* parallel to **B**; then *cd*, measured by the same scale, will represent the tension in the back-stay, and *bd* the compression of the post. In this case the second back-stay is

* This is not accurately true, for the friction of the blocks, pulleys, &c., increases or diminishes the tension of the chain, according as the weight happens to be raised or lowered.

free from strain, but when the jib does not lie in the same plane with either back-stay, both back-stays are subject to strain; to a less degree, however, than in the case already considered, as will appear from the following considerations. Let Fig. 70 represent a plan of the crane, *bh* and *bk* being the horizontal projections of the back-stays, and *b* A that of the tie-bar and jib; let *be* represent the horizontal component of the tension in the tie-bar (equal *ce* in Fig. 69), then *bf* and *bg* will represent the horizontal components of the strains in the back-stays, and hence, the strains in the back-stays can be found. It is obvious, however, that either *bf* or *bg* will attain its maximum when the tie-bar lies in the same plane with one of the back-stays. Hence, the former case, in which the jib and one back-stay lie in the same plane, is sufficient for us to consider when calculating the requisite strength of the stays.

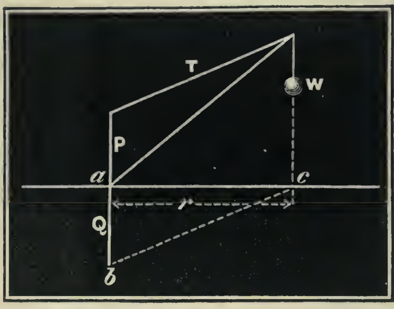
Fig. 70.



The strain in the post attains its greatest value when the plane of the tie-bars and jib bisects the angle between the back-stays, for then the sum of *bf* and *bg* is maximum, and consequently, the sum of the vertical components of the strains in the stays is maximum also. But the strain transmitted through the post is equal to the sum of these vertical components + or — the vertical component of the tension in the tie-bar, according as the latter slopes downwards or upwards from the head of the post. The back-stays act sometimes as struts, sometimes as ties, and when the jib is swung round, so as to lie alongside one of the back-stays, the latter will sustain its maximum compression, equal to the maximum tension produced when the jib and stay lie in the same plane. The radius of the circle described by the jib, or the range of the derrick, is generally capable of adjustment by lengthening or shortening the tie-bar, which is then a chain attached to a small auxiliary crab-winch fastened to the post near the working barrel, in which case the working chain passes along the jib. This form of derrick is convenient for setting masonry, as its range is equal to a circle described by the jib when nearly horizontal, in which position moreover the crane is most severely strained.

194. Wharf crane.—The wharf crane, unlike the derrick crane, has no back-stays. Consequently, the post is subject to transverse strain from the oblique pull of the tie-bar; it is in fact a semi-girder fixed in the ground and loaded at the extremity. The strains in the tie-bar and jib are calculated in the same way as for the derrick crane. The bending moment (**59**) of the post attains its greatest value at its intersection with the ground, and equals the horizontal component of the tension in T multiplied by the height of the post above ground. It may, however, be more conveniently found as follows:—

Fig. 71.



The whole crane above *ac* (the ground line,) is a bent semi-girder held in equilibrium by the weight and the elastic forces at *a* (in this case vertical). Taking moments round either the centre of tension or the centre of compression at *a* (**58**), we have the bending moment $= Wr$, where $r =$ the radius of the circle described by the jib. From this it follows that the transverse strain at *a* is not affected by increasing the height of the post, which, however, diminishes the strains in the jib and the tie-bar, and is so far attended with advantage; neither is it affected by raising or lowering the peak of the jib in the same vertical line. It also follows that the transverse strain on the post is increased when the weight is farther out than the circle described by the jib, for the leverage of W is then increased and attains its greatest value when the chain is at right angles to the jib. If the post be fixed in the ground, the frame, to which the jib, tie-bar and wheelwork are attached, is generally suspended by a cross head from the top of the post which forms a pivot round which the cross-head turns. In this form of crane the weight is transmitted from the pivot through the whole length of the post in addition to the longitudinal strains to which as a semi-girder it is liable, and the section of the post should theoretically be circular (**99**), since it may be equally strained in all directions.* When the post revolves on its axis, the jib and wheelwork are bolted to it and all move together on a pivot at the toe-plate b. In this case the post should be double-flanged. The underground portion is subject to a vertical compression equal to the weight (viz., the difference of the vertical components of the strains in the jib and tie-bar,) in addition to the longitudinal strain derived from its acting as a semi-girder. When the post moves round its axis, friction rollers may be advantageously placed between the post and a curb plate which is secured to the masonry at a.

To find the amount and direction of the pressure at the toe, join b with a point c vertically beneath W. The whole structure is balanced by three forces, viz., the weight W, the horizontal pressure against the curb plate at a, and the pressure on the toe at b. The two former forces pass through c; consequently, the latter intersects them at the same point (**9**). Hence, the sides of the triangle abc represent the relative amounts of these forces, and we have the horizontal component of the oblique pressure at b equal $\frac{r}{\mathsf{Q}}\mathsf{W}$. The vertical components equals W, which is otherwise evident.

195. Bent crane.—This form of semi-girder has been adopted for wharf cranes where head-room is required close to the post. The flanges may be equi-distant, as in Fig. 72, though a more pleasing form is produced by bringing them closer together as they approach the peak.†

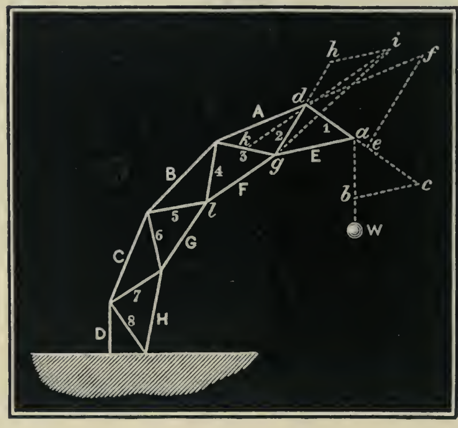
The weight W is supported by diagonal 1 and the first bay in the lower flange E, producing tension in the former and compression

* Square tubular posts built of boiler plates with angle iron at the corners form very simple and efficient posts for small cranes not exceeding four or five tons.

† Tubular cranes of this form were first made with plate webs by Sir Wm. Fairbairn (*Proc. Inst. M. E.*, Part I., 1857), and the braced web was first adopted by William Anderson, Esq., in a six-ton crane erected for the Government at the Pigeon House Fort, near Dublin. Mr. Anderson also designed a very fine twenty-ton bent crane, with plate webs, for the Russian Government, 60 feet high, and 31′·6″ radius. (*Trans. Inst. C. E. of Ireland*, Vols. vi. and vii.)

in the latter. The tension of diagonal 1 is resolved at *d* into its components in the direction of **A** and diagonal 2. The resultant of the strains in diagonal 2 and **E**, found by a triangle of force, is resolved at *g* into its components in the directions of the third diagonal and **F**. In a similar manner the resultant of the strains in diagonal 3 and **A** is resolved into its components in diagonal 4 and **B**, and so on throughout the girder.

Fig. 72.



An example (see Fig. 72) will illustrate this fully, and the student is recommended to work it out for himself by the aid of a diagram drawn accurately to a scale of not less than five feet to one inch. The strains may be represented to a scale of ten tons to one inch, though in many cases a larger scale will be found preferable.* The flanges are equi-distant, forming quadrants of two circles whose radii are respectively 20 and 24 feet. The inner flange is divided into four equal bays, on which stand equal isosceles triangles, and a weight of 10 tons is suspended from the peak. Draw *ab* vertically and equal to 10 tons measured on the scale representing strains, and draw *bc* parallel to **E** so as to meet the diagonal 1 produced ; *bc* and *ac* represent the strains in **E** and diagonal 1, and measure on the scale

* Rolling parallel rules, 15 or 18 inches in length, will be found useful for laying off parallel lines of strain.

of strains + 10·8 tons and — 13·1 tons respectively. Next, take *de* equal 13·1 tons (= *ac*), and draw *ef* parallel to diagonal 2, so as to meet **A** produced; *ef* and *df* represent the strains in diagonal 2 and **A**, and measure + 18·8 tons and — 21·7 tons respectively. Next, produce diagonal 2 so that *gh* may equal 18·8 tons (= *ef*), and draw *hi* parallel to **E** and equal 10·8 tons (= *bc*); *ig* is the resultant of the strains in diagonal 2 and **E**, and is transmitted through **F** and diagonal 3. Draw *ik* parallel to **F**; *ik* and *kg* will represent the strains in **F** and diagonal 3, and measure + 30·5 tons and — 5·4 tons respectively. Proceeding in this manner, we obtain the strains given in the following table:—

Bracing, . . .	1	2	3	4	5	6	7	8
Strains in tons, . .	—13·1	+18·8	—5·4	+21·4	+3·2	+20·5	+11·2	+8·8
Flanges, . . .	A	B	C	D	E	F	G	H
Strains in tons, . .	—21·7	—39·7	—51·3	—49·0	+10·8	+30·5	+45·3	+52·8

196. Calculation by moments.—It is prudent to check the calculation by diagram by computing the strains in some of the bays by the method of moments. That portion of the crane which extends above **B***l*, for instance, is held in equilibrium by the tension in **B**, the weight **W**, and the forces which meet at *l*. Taking moments round the latter point, we obtain the strain in **B**. In this example, **B***l* measures 3·55 feet, and the horizontal distance of *l* from **W** measures 14·12 feet; hence, we have

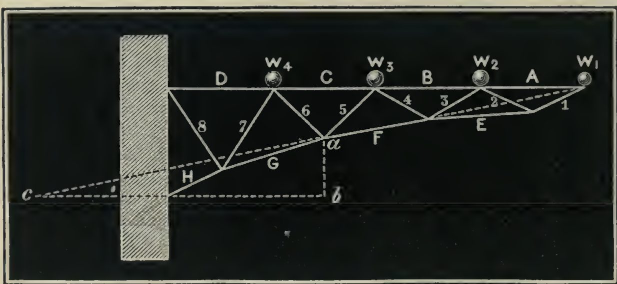
$$3·55 \times \text{strain in } \mathbf{B} = 14·12 \times 10 \text{ tons};$$

whence, the strain in **B** = 39·8 tons, which agrees closely with the former result. When only one system of triangulation is adopted, the strains in the flanges may be obtained in this manner by moments, and those in the diagonals may afterwards be obtained by decomposing the strains in the flanges. This method is perhaps more simple in practice than that first described, and has a farther advantage that errors do not accumulate.

197. Lattice webs not suited for powerful bent cranes.—The chief merit claimed for the bent crane is the large amount of head-room it allows underneath the jib, which enable boilers or other bulky articles to be brought close up to the peak. This merit, however, is balanced, and in many cases more than balanced, by the greater simplicity of the ordinary wharf crane. The lattice web is not well suited for bent cranes exceeding 10 tons, as the diagonal bars become so wide, and leave so little open space, that plating may be advantageously substituted for bracing.

CASE II.—THE BRACED SEMI-ARCH.

Fig. 73.



198. Swing bridge.—This form of semi-girder is a modification of the previous case, in which the radius of the upper flange becomes infinite; it is suitable for swing bridges, in which case the end next the abutment is prolonged backwards with parallel flanges and loaded at the inner extremity with a counterpoise weight to balance the projecting part. This backward continuation resembles the semi-girder described in Case I., Chap. V. In order to obtain the maximum strains when a concentrated load or a passing train traverses the girder, we must first calculate the strains produced by the weight on each apex separately, and tabulating these, we can find what position of the load, if it be concentrated, or what weights, if there are several, will produce maximum strains in each part of the structure, and the methods of calculation described in the preceding case are applicable to this one also.

199. Single triangulation.—When, however, there is but one system of triangles in the bracing, the following plan is more simple in practice, and as errors do not accumulate, it is less liable to inaccuracy. Suppose a weight resting on the extremity of the girder; on examining the forces which hold any portion $\mathsf{C}a\mathsf{W}_1$ in equilibrium, we find that two of them, viz., the weight and the horizontal tension in **C** pass through W_1; consequently, the third force, viz., the resultant of the strains in bay **G** and diagonal 6 also passes through W_1 (**9**). In the same way it can be shown that the resultants at each of the other lower apices pass through W_1. If the weight rest on any other apex, W_2 for example, the resultant strains produced by it at each lower apex pass through W_2; or, to express this more generally, *the resultant strain at each apex in the lower flange from a weight at any apex in either flange will pass through the intersection of the horizontal flange with a vertical line drawn through the weight, provided there be but one system of triangulation.* Again, since the horizontal flange transmits no vertical strains, the weight must be conveyed to the wall through these resultant strains at each lower apex. Their vertical components are in fact the shearing-strain and equal to the weight; hence, knowing both their directions and their vertical components, we can find their amounts. Thus, the resultant strain at a from W_1 may be found as follows:—Draw a vertical line ab, equal (by a scale of strains) to W_1, and draw bc horizontally till it meet $\mathsf{W}_1 a$ produced; ac is the required resultant, and may be resolved into its components in bay **G** and diagonal 6. The strain in the latter may next be resolved at W_4 in the directions of bay **D** and diagonal 7. The former component is the *increment* of horizontal strain at the apex, and when added to the sum of the preceding increments gives the resultant strain in **D**. The strains in the other parts may be obtained in a similar manner.

200. Example.—The following example, Fig. 73, in which the strains have been worked out on a diagram drawn to a scale of 5 feet to one inch, will be found useful practice for the student. The projecting portion of the girder is 40 feet long, and 10 feet deep at the wall, with a circular lower flange which has a horizontal tangent two feet below the extremity of the girder. Consequently, the versine of the arch is 8 feet, and its radius 104 feet. The load is uniform and equal to one ton per running foot, which for calculation is supposed collected into weights of 10 tons at each upper apex except the outer one, which has only 5 tons, or the load which rests on half a bay. The strains have been calculated for each weight separately.

		W_1	W_2	W_3	W_4	Uniform Load.	Max. Comp^n.	Max. Tension.
		Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
BRACING.	1	+ 12·7	...	...	...	+ 12·7	+ 12·7	...
	2	− 8·0	...	...	...	− 8·0	...	− 8·0
	3	+ 5·9	+ 19·0	...	...	+ 24·9	+ 24·9	...
	4	− 0·3	− 9·8	...	...	− 10·1	...	− 10·1
	5	+ 0·2	+ 7·3	+ 14·0	...	+ 21·5	+ 21·5	...
	6	+ 2·7	− 1·1	− 7·5	...	− 5·9	+ 2·7	− 8·6
	7	− 2·3	+ 0·9	+ 6·3	+ 11·7	+ 16·6	+ 18·9	− 2·3
	8	+ 3·1	+ 1·7	− 2·8	− 7·2	− 5·2	+ 4·8	− 10·0
FLANGES.	A	− 11·7	...	...	...	− 11·7		
	B	− 24·1	− 16·1	...	...	− 40·2		
	C	− 24·7	− 29·7	− 9·9	...	− 64·3		
	D	− 21·6	− 30·8	− 18·5	− 6·2	− 77·1		
	E	+ 19·3	...	...	...	+ 19·3		
	F	+ 25·2	+ 25·2	...	...	+ 50·4		
	G	+ 23·9	+ 31·8	+ 15·9	...	+ 71·6		
	H	+ 21·4	+ 32·1	+ 21·4	+ 10·7	+ 85·6		

The reader will perceive that the strain produced in bay **H** by $\mathbf{W}_4$ is half that produced by $\mathbf{W}_3$, and one-third of that produced by $\mathbf{W}_2$, and in general, the strains produced by the different weights in any given bay will be sub-multiples of the strain produced by the most remote weight, for they are proportional to the leverage of the weights round the apex above or below the given bay. This check on the accuracy of the work is, however, applicable only in the case of a single system of triangulation. The strains in girders of this form are not always such as might perhaps be expected at first sight; $\mathbf{W}_1$, for instance, produces compression in both diagonals 6 and 8, and in bay **D** a strain of less amount than in bay **C**. These apparent anomalies occur when the resultant at the lower apex, *ac* for example, passes altogether *above* the lower flange.

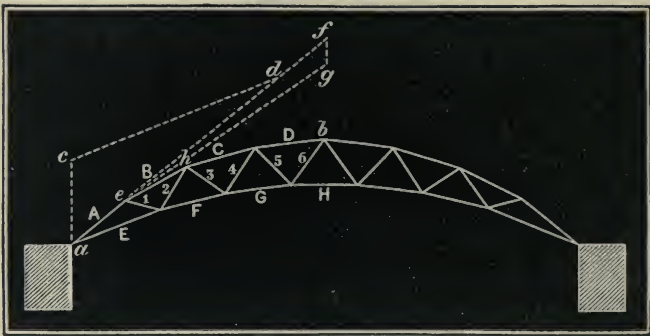
201. Lattice semi-arch—Triangular semi-girder.—When two or more systems of triangulation are introduced, the strains in one system produce strains in the others in consequence of the curvature of the arched flange, and this renders the calculations more tedious than would otherwise occur. This remark applies to all arched girders with lattice webs. In this particular case the calculations would be much simpler if the girder were triangular with a straight lower flange, since each bay would communicate its strain directly to the adjoining bay without affecting the diagonals at their junction, but this form of semi-girder has the disadvantage of being somewhat unsightly in appearance, which in some cases might prevent its adoption, whatever merits, and they are considerable, it may possess in other respects.*

202. Inverted semi-arch.—When head-room beneath is required, we may invert the girder represented in Fig. 73, so that it will resemble one-half of a suspension-bridge. By so doing we change the strains in kind, but not in amount.

* A large iron swing bridge, a drawing of which appeared in the *Illustrated London News* for October 12, 1861, has been constructed at Brest, in France; it is formed of two triangular semi-girders with vertical and diagonal bracing.

CASE III.—CRESCENT GIRDER.

Fig. 74.



203. Suitable for roofs—Flanges.—Frequent modifications of the crescent girder occur in the roofs of our railway stations and crystal palaces, to which its graceful outline and lightness of appearance impart an air of elegance which no other form possesses to the same degree. It may also be employed for bridges where greater headway is required beneath the centre than at the abutments. I shall, however, merely investigate the strains produced by a load symmetrically disposed on both sides of the centre, such as a roof principal generally sustains. When the girder is subject to a partial or a passing load, the more general method of calculating the strains due to each weight separately, and which is investigated in the next case, becomes necessary. The horizontal strains at the centre of the flanges are equal and of opposite kinds; their amount depends upon the central depth of the girder and may be found by the method of moments as follows:—

Let W = the load symmetrically distributed,

l = the span,

d = the central depth from flange to flange = $b\mathsf{H}$,

l' = the distance of the centre of gravity of each half load measured from the centre of the girder,

T = the tension at the centre of the lower flange,

C = the compression at the centre of the upper flange.

The half girder, $ab\mathsf{H}$, is held in equilibrium by the reaction of the left abutment $\left(= \frac{\mathsf{W}}{2}\right)$, by the left half load (which we may conceive collected at its centre of gravity), and by the horizontal strains of compression and tension at b and H. Taking moments round each of these latter points successively, we have $\frac{\mathsf{W}}{2}\left(\frac{l}{2} - l'\right) = \mathsf{T}d = \mathsf{C}d$; whence,

$$\mathsf{T} = \mathsf{C} = \frac{\mathsf{W}(l - 2l')}{4d} \tag{131}$$

This, which is merely a particular form of eq. 25, proves that the strains at the centre do not depend upon the height of the lower flange above the chord line, but upon the depth of the girder from flange to flange. The method of calculating the strains in other parts of the girder consists in working by the resolution of forces from either abutment, whose reaction is a known quantity, towards the centre. The following examples, which have been worked out on a diagram drawn to a scale of 5 feet to one inch, and with strains represented by 4 tons to one inch, will explain this clearly.

204. Example 1.—The span of the girder, Fig. 74, is 80 feet; the versines of the flanges respectively 10 and 16 feet; both flanges are circular and each flange is divided into equal bays, with the exception of the extreme bays of the lower flange, which are each half as long again as the other bays. The load is supposed equal to 8 tons distributed, so that each apex sustains a weight of one ton; hence, the reaction of each abutment equals 4 tons, of which, however, half a ton is at once balanced by the weight of the first half bay of the roof which rests directly on the wall-plate. Consequently, the resultant of the forces in A and E = 3·5 tons pressing downwards on the wall. Draw ac = 3·5 tons, and draw cd parallel to E until it meets A produced. The lines ad and cd represent the strains in A and E, and measure by scale + 12·25 tons and — 10·43 tons respectively. Next, lay off $ef = ad$, and draw fg vertically equal to one ton, that is, equal to the weight at the first apex. The line eg is the resultant of the strain in A and the weight at e, and the strains in **B** and diagonal 1 are its components, and can therefore be found by resolving eg in their directions. Similarly, the resultant of **E** and diagonal 1 may be resolved in the directions of **F** and diagonal 2. At h we must find the resultant of *three* forces, viz., the strain in **B**, the strain in diagonal 2, and the weight resting on the apex. From this resultant the strains in **C** and diagonal 3 are derived, and so on to the centre. The following table contains these strains:—

BRACING, . . .	1	2	3	4	5	6		
Strains in tons, . .	−2·4	−1·05	−1·36	−0·91	−1·04	−1·0		
FLANGES, . . .	A	B	C	D	E	F	G	H
Strains in tons, . .	+12·3	+13·5	+13·1	+12·9	−10·4	−11·7	−12·2	−12·2

The accuracy of the work may be checked by comparing the strain in **H** with the central strain in the flanges obtained by the method of moments. As the distance of the centre of gravity of the half load from the centre of the girder is unknown, the most convenient method for obtaining the leverage of the weights is by accurately measuring on the diagram the distance of each weight from the centre. Doing this, and taking moments round the centre of either flange, we have

$$6{\cdot}15\ \mathbf{F} = 40 \times 3{\cdot}5 \text{ tons} - (31{\cdot}4 + 21{\cdot}6 + 11{\cdot}1)$$

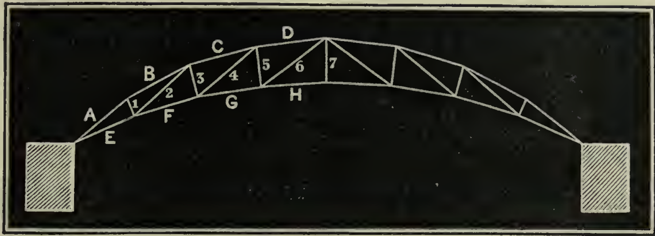
whence, the strain at the centre of either flange,

$$\mathbf{F} = 12{\cdot}34 \text{ tons},$$

in place of 12·2 tons, an amount of discrepancy which is immaterial. The central depth by which **F** is multiplied has been obtained by measurement, and is, it will be observed, slightly in excess of 6 feet, arising from the central bay of the lower flange being a straight line, and therefore slightly farther from the upper flange than the arc of which it is the chord.

205. Example 2.—Flange-strains nearly uniform with symmetric loading.—The girder, represented in Fig. 75, has the same span, depth and versine as the preceding example, but the mode of bracing is similar to that described in Chapter VI. Each flange is divided into eight equal bays and every alternate brace is nearly radial to the lower flange.

Fig. 75.



The strains due to a load of one ton at each apex of the upper flange are as follows:—

BRACING, . . .	1	2	3	4	5	6	7	
Strains in tons, . .	—1·75	+0·6	—1·65	+0·45	—1·7	+0.2	—1·4	
FLANGES, . . .	A	B	C	D	E	F	G	H
Strains in tons, . .	+13·7	+12·7	+12·6	+12·6	—11·8	—12·3	—12·6	—12·7

The horizontal strain at the centre of either flange equals 12·68 tons. Checking this as before by the method of moments, we have

$$6\,\mathsf{F} = 40 \times 3{\cdot}5 \text{ tons} - (31{\cdot}4 + 21{\cdot}6 + 11{\cdot}1)$$

whence, the strain at the centre of either flange,

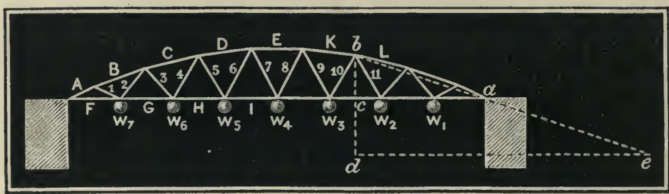
$$\mathsf{F} = 12{\cdot}65 \text{ tons.}$$

In the previous examples it will be observed that the strains are nearly uniform throughout the flanges, and that the bracing has comparatively little work to do. Hence, the crescent girder seems well fitted for large roofs, the loading of which, with the exception of wind pressure, is generally symmetrically distributed.

206. Ambiguity in the strains of a crescent girder when resting on more than two points.—This class of girder is occasionally constructed with equi-distant flanges, in which case it is essential for accurate calculation that the girder rest on two points only, either the extremities of the inner, or the extremities of the outer, flange; otherwise we cannot say how much pressure any one point sustains, just as the pressure on any one leg of a four-legged table is indefinite. The girder in fact becomes an arched rib and partakes of the uncertainty of the arch as regards the direction of the line of thrust.

CASE IV.—BOWSTRING GIRDER.

Fig. 76.



207. Concentrated load.—Let a single weight W_3 rest upon one of the apices which divides the girder into segments containing respectively m and n segments. On the principle of the lever, the pressure on the right abutment $= \frac{m}{m+n}W_3$, and that on the left $= \frac{n}{m+n}W_3$. This latter quantity is the resultant of the strains in bays **A** and **F**, which can therefore be obtained from it by a diagram of strains. Again, the strains in **B** and diagonal 1 may be derived from that in **A**, and by resolving the strain in diagonal 1 in the directions of diagonal 2 and bay **G**, we obtain the strain in the former and the horizontal increment of strain developed at the first apex of the lower flange. This increment, added to the strain in **F**, gives the total strain in **G**. The resultant of the strains in **B** and diagonal 2 is also the resultant of those in **C** and diagonal 3, which can therefore be derived from it, and so on.

208. Passing load—Example—Little counterbracing required in bowstring girders of large size.—When the load is a concentrated passing load or a train, we must tabulate the strains produced by the weight on each apex separately, and thence deduce what position of the load produces maximum strains. It will be found that the maximum strains in the flanges occur when the train covers the whole girder, and that they are of nearly uniform magnitude throughout each flange, while the maximum strains in the diagonals increase as they approach the centre, just the reverse of what occurs in the webs of girders with horizontal flanges. The following example, Fig. 76, will illustrate fully the mode of calculating the strains in this important form of girder. They have been worked out on a diagram drawn to a scale of 5 feet to one inch. The span is 80 feet, divided into 8 equal bays, and the bow is a circular arc whose versine equals 10 feet, but, as there is no apex at the crown, the central depth of the inscribed polygon, measured by scale, equals 9·85 feet in place of 10 feet. The load is supposed to traverse the lower flange and to be of uniform density, equal to one ton per running foot, which is equivalent to 10 tons at each apex.

		W_1	W_2	W_3	W_4	W_5	W_6	W_7	Uniform Load.	Max. Compn.	Max. Tensn.
		Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
Bracing.	1	—0·39	— 0·8	— 1·2	— 1·6	— 2·0	— 2·3	— 2·7	—11·0	...	—11·0
	2	+0·23	+ 0·5	+ 0·7	+ 0·9	+ 1·1	+ 1·4	—11·4	— 6·6	+4·8	—11·4
	3	—0·56	— 1·1	— 1·7	— 2·2	— 2·8	— 3·4	+ 4·8	— 7·0	+4·8	—11·8
	4	+0·51	+ 1·0	+ 1·5	+ 2·0	+ 2·6	— 8·6	— 4·3	— 5·3	+7·6	—12·9
	5	—0·90	— 1·8	— 2·7	— 3·6	— 4·5	+ 4·7	+ 2·4	— 6·4	+7·1	—13·5
	6	+0·88	+ 1·8	+ 2·6	+ 3·5	— 6·9	— 4·6	— 2·3	— 5·0	+8·8	—13·8
	7	—1·40	— 2·8	— 4·2	— 5·6	+ 4·2	+ 2·8	+ 1·4	— 5·6	+8·4	—14·0
Flanges.	A	+2·82	+ 5·6	+ 8·5	+11·3	+14·1	+16·9	+19·7	+78·9		
	B	+3·08	+ 6·2	+ 9·2	+12·3	+15·4	+18·5	+21·6	+86·3		
	C	+3·47	+ 6·9	+10·4	+13·9	+17·3	+20·8	+10·4	+83·2		
	D	+4·11	+ 8·2	+12·3	+16·4	+20·5	+13·7	+ 6·8	+82·0		
	E	+5·11	+10·2	+15·3	+20·4	+15·3	+10·2	+ 5·1	+81·6		
	F	—2·52	— 5·0	— 7·6	—10·1	—12·6	—15·1	—17·6	—70·5		
	G	—3·01	— 6·0	— 9·0	—12·0	—15·0	—18·1	—13·1	—76·2		
	H	—3·62	— 7·2	—10·9	—14·5	—18·1	—15·9	— 7·9	—78·1		
	I	—4·46	— 8·9	—13·4	—17·8	—17·1	—11·4	— 5·7	—78·8		

On examining the foregoing table we observe that, when the permanent (uniform) load is equal to, or less than, the passing load, a large number of the diagonals require counterbracing; in this example, for instance, diagonals 4, 5, 6, 7, and their counterparts at the other side of the centre, require counterbracing. If, however, the permanent load be much greater than the passing load, it may happen that the diagonals will always be in tension and thus relieve the engineer of one difficulty in large girders, namely, that of providing against flexure in long struts. Hence, the bowstring girder seems well suited for large spans. On examining the table we also find that all the intermediate strains are multiples of those in the columns under either W_1 or W_7. They agree also in sign with their sub-multiples. This arises from the reaction of each abutment being directly proportional to the length of the remote segment, and indicates a speedy method of filling up the table, viz., by calculating on a diagram the strains produced by the two extreme weights and thence deriving those due to all the intermediate weights.

209. Calculation by moments.—When there is only one system of triangulation, the work may be checked by calculating the strains in some of the bays by the method of moments. Thus, in the central bay E, the strain

$$\mathsf{F} = \frac{35 \times 40 - 10 \times 60}{9{\cdot}85} = 81{\cdot}2 \text{ tons compression,}$$

a close approximation to the amount in the table, as the discrepancy is only 0·4 tons, or $\frac{1}{203}$rd of the whole. Having found the strains in the flanges by the method of moments, the strains in any pair of intersecting diagonals may be found by decomposing the strains in the two adjoining bays.

210. Uniformly distributed load, little bracing required—Absolute maximum strains.—If a uniform horizontal load be suspended by vertical rods from a circular bow, the diagonal bracing will scarcely come into action, and the tension throughout the string will be very nearly uniform, for a small arc of a circle differs but slightly from the parabola which a chain (inverted arch) assumes when loaded uniformly per horizontal foot (**49**). In this case the horizontal component of strain is nearly uniform throughout the bow and equals the compression at the crown, or the tension in the string. The vertical component at the springing is equal to the half load, and at any other point it equals the half load supported above the level of that point. The longitudinal compression at any point in the bow is the resultant of these horizontal and vertical components, and would be strictly tangential to the curve if it were a parabola, *i.e.*, the curve of equal horizontal thrust for a uniform horizontal load. The bow forms a considerable item of the total weight of a bridge of large span, and the annexed method of calculating the strains will be found more accurate than one which supposes the whole permanent load resting on the lower flange:—

1°. Calculate the maximum strains in both flanges and bracing produced by the passing load of greatest uniform density, as already explained.

2°. Calculate the strains produced by the permanent load which rests on the lower flange, including in this the string, roadway and bracing. These may be obtained by proportion from the strains produced by the passing load when the latter covers the whole bridge.

3°. Calculate the (nearly) uniform strain produced throughout the bow and string by the weight of the former (eq. 25). If greater accuracy is required the longitudinal strains in the bow may be obtained by the method explained in **26**.

Having these arranged in a tabular form, we can easily find the absolute maximum strains which each part sustains. The 2nd and 3rd of the foregoing calculations may be replaced by the method described in the preceding case for calculating the strains due to a permanent load, without however simplifying the operation in practice.

211. Single triangulation, second method of calculation.— When the bracing of a bowstring girder consists of a single system of triangulation, as in Fig. 76, the strains may be calculated by a method similar to that described in **199**. Suppose, for example, that $\mathbf{W}_3$ alone rests upon the girder, dividing the lower flange into segments containing respectively m and n bays; the segment abc is held in equilibrium by three forces, viz., the reaction of the right abutment, the horizontal tension at c, and the resultant of the strains in K and diagonal 10. The two former meet at a; consequently, the third, the resultant at b, passes through the same point (**9**). Again, since the lower flange is horizontal, it cannot convey a vertical pressure to the abutment; hence, $\frac{m}{m+n}\mathsf{W}_3$ (= the reaction of the abutment,) must be conveyed through the bow and diagonals to the right abutment, forming the vertical component of the resultant at each upper apex. This suggests the following method of calculating the strains. Draw bd vertically equal to $\frac{m}{m+n}\mathsf{W}_3$, and draw de horizontally till it meets ba produced; be represents the resultant at b, and hence we can find its component in K and diagonal 10, or in L and diagonal 11. The same reasoning will apply if all the apices to the left of W_3 are loaded, in which case diagonals 10 and 11 will sustain the maximum strains of tension and compression which a passing train can produce in them. At the several apices in the bow over the *unloaded* segment resultant strains will be developed, each of which will pass through a and have the same vertical component, viz., the reaction of the right abutment, provided there be but one system of triangles. In the case of the train, bd will represent $\frac{\mathsf{W}}{m+n}(1+2+3+4+5) = \frac{15}{8}\mathsf{W}$, since there are 5 loaded apices in the left segment and 8 bays in the span. This operation must be repeated at each apex of the bow.

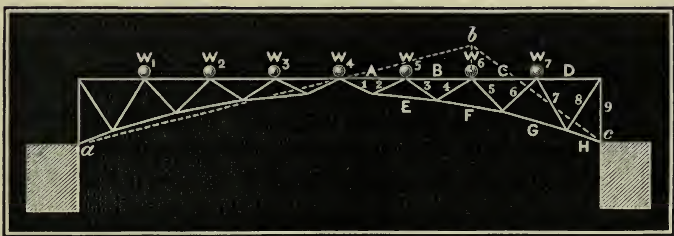
The maximum strains in the diagonals of the example in **208** are calculated by this method and are given in the annexed table. They agree closely with those previously obtained:—

Diagonals.	Maximum compression.	Maximum tension.
	Tons.	Tons.
1	...	— 11·0
2	+ 4·7	— 11·4
3	+ 4·8	— 11·8
4	+ 7·6	— 12·8
5	+ 7·1	— 13·6
6	+ 8·7	— 13·6
7	+ 8·4	— 14·0

212. Inverted bowstring, or fish-bellied girder—Bow and invert, or double-bow.—The methods of calculating the strains of the bowstring girder are also applicable to its inverse—the fish-bellied girder, *i.e.*, the arc in tension with a horizontal flange in compression, as well as the lenticular girder compounded of the two, *i.e.*, a bow and invert connected by bracing, such as the Royal Albert Bridge, Saltash. Examples of these forms are, however, comparatively rare, except in cast-iron girders and beams of steam engines, but the fish-bellied girder is sometimes used for cross road-girders.

CASE V.—THE BRACED ARCH.

Fig. 77.



213. Law of the lever applicable to the braced arch.—Properly speaking, the braced arch is not a girder, since it exerts an oblique thrust against the abutments (**12**), but it resembles a girder in so many respects that the investigation of its strains may fitly be considered in this chapter. In the braced arch the upper flange is usually horizontal and supports the roadway. Both flanges are in general subject to compression throughout their whole length, and the lower one exerts an oblique pressure against the abutments. In this respect the braced arch resembles its prototype, the stone arch, while it also resembles the girder in its capability of sustaining transverse strain. The horizontal components of the pressures against the abutments are equal and in opposite directions; equal—since, if the horizontal reaction of one abutment exceed that of the other, the arch will move towards that side which exerts the weaker thrust, a thing manifestly impossible. We may therefore conceive a horizontal tie substituted for the horizontal reaction of the abutments, and the arch will then follow the laws of girders, exerting a vertical pressure only on the points of support. The principle of the lever (**10**) is, consequently, applicable to this form of bracing, and hence we can find the direction and amount of the thrust against either abutment for each position of the load. Theoretically, the lower flange of the arch represented in Fig. 77 should not be continued across the crown of the arch, for if it were, the strains in every part would be uncertain, since the central bay of this flange would be subject to tensile strains of indefinite amount, varying with the load and temperature, and modifying therefore to an unknown extent the horizontal reaction of the abutments. To illustrate this, let us suppose for a moment that the reaction of the abutments is replaced by a tie-bar; we then have three unknown horizontal forces, viz., compression in the top flange, tension in the lower flange at the crown, and tension in the tie-bar; also three known vertical forces, viz., the weight and the vertical reaction of each abutment. Now, it is evident that we cannot determine the three unknown forces by the method of moments from these data, and we must therefore get rid of the difficulty by supposing the lower flange discontinued at the crown, which, indeed, is not far from the truth in practice, for the two flanges generally merge into one, and the less in depth is the line of junction of the two semi-arches, *i.e.*, the depth of the arch at the crown, the nearer will the following theory and practice agree.

Let us now consider the effect of a single weight $\mathbf{W}_6$. The left semi-arch is subjected to two forces only, viz., the pressure of the other semi-arch at the crown and the reaction of the left abutment at a. Since equilibrium exists, these forces are equal and opposite; consequently, the reaction of the left abutment acts in the direction $a\mathbf{W}_4$. Again, the whole arch is balanced by the weight $\mathbf{W}_6$ and the reactions of the abutments. The weight and the reaction of the left abutment intersect at b; consequently, that of the right abutment passes through the same point (**9**). Resolving $\mathbf{W}_6$ in the directions *ba* and *bc*, we obtain these reactions, and once they are known, we can work from the abutments towards the weight by the resolution of forces and thus find the strains produced by $\mathbf{W}_6$ throughout the arch. Performing similar operations for each weight, and tabulating the results, we can obtain the maximum strains of each kind produced in every part of the structure. Those produced in the arch represented in Fig. 77, by weights of 10 tons at each apex, are given in the following table. The arch is 80 feet in span with a rise or versine of 8 feet, and the depth measured from the springing to the upper flange is 10 feet. The upper flange is divided into 8 equal bays, and the bracing consists of a series of isosceles triangles of which these bays form the bases.

		W_1	W_2	W_3	W_4	W_5	W_6	W_7	Uniform Load.	Max. Compn.	Max. Tensn.
		Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
BRACING.	1	+3·2	+ 6·4	+ 9·6	+12·7	— 9·6	— 6·4	— 3·2	+12·7	+31·9	—19·2
	2	—2·0	— 4·0	— 6·0	— 8·0	+ 6·0	+ 4·0	+ 2·0	— 8·0	+12·0	—20·0
	3	+1·5	+ 3·0	+ 4·5	+ 5·9	+14·5	— 3·0	— 1·5	+24·9	+29·4	— 4·5
	4	—0·07	— 0·1	— 0·2	— 0·3	— 9·7	+ 0·1	+ 0·07	—10·2	+ 0·2	—10·4
	5	+0·05	+ 0·1	+ 0·15	+ 0·2	+ 7·1	+13·9	— 0·05	+21·4	+21·5	— 0·1
	6	+0·7	+ 1·4	+ 2·1	+ 2·7	— 3·2	— 8·9	— 0·7	— 5·9	+ 6·9	—12·8
	7	—0·6	— 1·2	— 1·8	— 2·3	+ 2·7	+ 7·5	+12·3	+16·6	+22·5	— 5·9
	8	+0·8	+ 1·6	+ 2·4	+ 3·1	— 0·6	— 4·4	— 8·0	— 5·1	+ 7·9	—13·0
	9	—0·7	— 1·4	— 2·1	— 2·8	— 0·4	+ 3·6	+ 6·8	+ 3·0	+10·4	— 7·4
FLANGES.	A	+2·0	+ 4·0	+ 6·0	+ 8·1	+24·0	+16·0	+ 8·0	+68·1	+68·1	...
	B	—1·1	— 2·2	— 3·3	— 4·4	+17·1	+22·4	+11·2	+39·7	+50·7	—11·0
	C	—1·2	— 2·4	— 3·6	— 4·7	+ 3·8	+12·6	+11·3	+15·8	+27·7	—11·9
	D	—0·4	— 0·8	— 1·2	— 1·7	+ 0·3	+ 2·3	+ 4·2	+ 2·7	+ 6·8	— 4·1
	E	+4·8	+ 9·7	+14·5	+19·3	—14·5	— 9·7	— 4·8	+19·3	+48·3	—29·0
	F	+6·3	+12·6	+18·9	+25·2	+ 6·3	—12·6	— 6·3	+50·4	+69·3	—18·9
	G	+6·0	+12·0	+18·0	+23·9	+13·9	+ 3·9	— 6·0	+71·7	+77·7	— 6·0
	H	+5·3	+10·7	+16·0	+21·4	+16·0	+10·7	+ 5·3	+85·4	+85·4	...

214. Strains in the braced arch loaded symmetrically resemble those in the semi-arch—Portions of the flanges liable to tensile strains from unequal loading.—On examining the preceding table it will be observed that the strains produced in the right semi-arch by W_1, W_2, and W_3 are sub-multiples of those produced by W_4; this arises from the circumstance, that the reactions of the right abutment from the weights on the left semi-arch act all in the same direction, viz., cW_4, and are proportional to the distance of each weight from the left abutment. Hence, having calculated the strains produced by W_4, we can deduce thence the strains produced by the three other weights. On comparing this table with that in **200**, we find that the strains produced by a symmetrical load in the diagonals and lower flange of the braced arch and semi-arch are identical. If the weight of the structure be small compared with that of the moving load, some of the bays may sustain tensile strains from the latter. These are the end bays of the upper flange and the central bays of the lower flange.

215. Calculation by moments—Calculation of strains in a latticed arch impracticable, except when the load is symmetrical.—When there is only one system of triangulation, the strains may be calculated by the method of moments in the manner already explained in **209**, and it is always desirable thus to check calculations made by the aid of diagrams. When there are two or more systems of triangulation, that is, when the web is latticed, the strength may be calculated by working out the strains from the weights towards the abutments, provided the load is disposed symmetrically on each side of the centre, but when the weights are distributed in an irregular manner this is not possible, and accurate calculation seems out of the question, for then more than two braces meet at the abutment, and we cannot say how the reaction of the abutment, when decomposed, is divided between them.

216. Flat arch, or arch with horizontal flanges.—If the radius of the lower flange be infinite, both flanges will be horizontal, and this flat arch will resemble girders of the ordinary form. Fig. 57, but with their lower flanges severed at the centre so as to exert a lateral thrust against the abutments. When the load is uniform, this thrust will equal the central compression in the upper flange. This modification of the braced arch possesses some qualities which merit our attentive consideration. In the first place the quantity of material required for its lower flange is less than in girders of the usual form, for the increments of strain increase as they approach the abutments, and it is therefore more economical to convey them *from*, than *towards*, the centre; and again, the heavier parts of the lower flange are near the abutments instead of near the centre, which is a matter of some importance in very large girders whose own weight forms the greater portion of the total load.

217. Rigid suspension bridge.—When inverted, the braced arch becomes a rigid suspension bridge. Other modifications might be suggested, such as the crescent girder inverted, with a horizontal roadway suspended beneath. The railway bridge over the Donau Canal in Vienna, 83·44 mètres long, is constructed on this latter system. There are two suspension chains on each side formed of flat links and equi-distant, one above the other, with bracing between; a trussed platform for the rails is suspended beneath by vertical rods in the usual manner. The chains being equi-distant, and therefore hung from four points, there must be an ambiguity in the strains, as already explained in **206**.

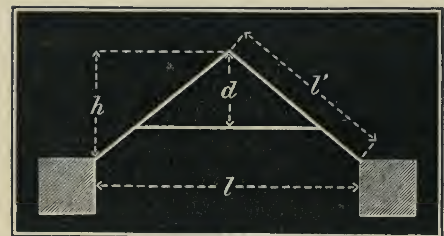
218. Triangular arch.—If the lower flange of the braced arch be formed of two straight bars meeting at the centre like the letter **A**, so that the arch becomes two braced triangles, the calculations as well as the construction will be much simplified, especially where multiple systems of bracing are employed. This arrangement has some great practical merits, its chief objection being the inelegance of its outline, which, however, will be an immaterial objection in many situations.

219. Cast-iron arches.—The spandrils of cast-iron arches frequently consist of vertical or radial struts without any diagonal bracing whatever. This form of arch resembles the common suspension bridge inverted; and since the spandrils do not brace the flanges together so as to change their transverse into longitudinal strains, but resemble in their action the rungs of a ladder placed on its side, it is necessary to make the flanges sufficiently deep to act as girders and sustain the transverse strain when the moving load causes the line of thrust to pass outside the rib or curved flange (**49**). Unless very massive, iron arches with vertical spandrils may be expected to be more subject to vibration and deflection than those with braced spandrils.

CASE VI.—THE BRACED TRIANGLE.

220. The common A roof.—In the common **A** roof, the span of which seldom exceeds 40 feet, each pair of rafters is kept from exerting a lateral thrust against the wall by a tie-beam, which is often placed a few feet above the wall-plate for the sake of the head-room which this arrangement allows. Consequently, each pair of rafters with their tie-beams constitutes a simple truss which supports so much of the roof as lies between two adjacent pairs of rafters.

Fig. 78.



Let W = the weight uniformly distributed over each pair of rafters,

l = the span of the roof,

l' = the length of each rafter,

d = the height of the ridge above the tie-beam, *i.e.*, the depth of the truss;

h = the height of the ridge above the wall-plates,

T = the tension in the tie-beam.

Each rafter is held in equilibrium by the uniformly distributed weight of the roof (equivalent to $\frac{\mathsf{W}}{2}$ acting downwards at the middle of the rafter), the upward reaction of the wall-plate, $\left(=\frac{\mathsf{W}}{2}\right)$, the horizontal thrust of the opposite rafter at the ridge and the horizontal tension of the tie-beam. Taking the moments of these forces round the ridge, we have,

$$\frac{W}{2} \times \frac{l}{2} - \frac{W}{2} \times \frac{l}{4} = Td$$

whence,
$$T = \frac{lW}{8d}$$

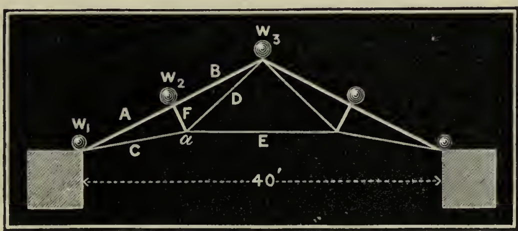
By taking moments round the foot of the rafter it may be shown that the horizontal thrust of the rafters against each other at the ridge $= T$. This investigation of the horizontal strains in a simple trussed girder is, it will be perceived, merely a repetition of that given in **43** (eq. 25). Each rafter is subject to transverse strains as a girder and to longitudinal compression as a pillar. The transverse strains are produced by the components of W and of T at right angles to the rafter. The former $= \frac{lW}{4l'}$ distributed uniformly. The latter $= \frac{h}{l'}T = \frac{hlW}{8dl'}$ applied at the intersection of the rafter and tie-beam. Hence, the transverse strength of the rafter may be calculated by eqs. 100 and 85, or perhaps, more conveniently by eqs. 41 and 37. The longitudinal component of W compresses the rafter like a pillar, and accumulates gradually from the ridge, where it equals cipher, to the wall-plate, where it equals $\frac{hW}{2l'}$. The longitudinal component of $T = \frac{lT}{2l'} = \frac{l^2W}{16dl'}$; it compresses that part of the rafter which lies between the ridge and tie-beam, and is balanced by the longitudinal component of the thrust of the opposite rafter at the ridge. When the tie-beam is placed high, for the sake of room beneath, d is shortened and T increased in the same proportion. The transverse strain and deflection of the rafter is, however, increased in a higher ratio, for not only is the component of T at right angles to the rafter increased, but its bending moment also, in consequence of its acting nearer to the centre of the rafter and farther from the wall-plate, which acts the part of an abutment. When rafters are in danger of sagging from their great length, a horizontal *collar-beam* is attached midway between the ridge and the tie-beam.

This collar-beam resists the tendency of the rafters to approach each other and is subject to compression, in which case each rafter is a continuous girder supported at both ends and at the collar-beam, and subject to a transverse pressure from the roofing material equal to $\frac{l\mathsf{W}}{4l'}$ distributed uniformly. If the tie-beam connect the feet, and the collar-beam the centres, of each pair of rafters, $\frac{5}{8}$ths of this pressure is sustained by the collar-beam, the remaining $\frac{3}{8}$ths being supported by the thrust of the opposite rafter and the reaction of the wall-plate (eq. 169). Hence, $\frac{5l\mathsf{W}}{32l'}$ is the pressure against the collar-beam, measured at right angles to the rafter; resolving this horizontally, we have the longitudinal compression of the collar-beam $= \frac{5l\mathsf{W}}{32h}$. A collar-beam increases the tension of the tie-beam, and this tension may be found when the strain in the collar-beam is known by taking moments round the ridge.

The foregoing investigation is only an approximation to the truth. The longitudinal strains produced in the rafter by the forces acting at its ends will modify the longitudinal strains due to the transverse forces, and an accurate investigation would be very complicated, if not altogether impracticable, for we cannot say how much of these longitudinal strains pass through the tension fibres or lower side of the rafter, and how much pass through its compression fibres or upper side. If there be any tendency in the rafter to sag, the probability is that they will pass altogether through the compression fibres, and therefore the upper side of the rafter should be strong enough to sustain the longitudinal strains produced by the end forces in addition to the longitudinal strain due to the transverse components of the load and tie-beam; but in general it is unnecessary to take these longitudinal compression strains into consideration, for when rafters fail they commonly give way on the under side which is in tension. Of course, if the sag be very considerable, so that a line joining the ridge and wall-plate passes above the rafter, the longitudinal compression will increase the strain in the tension flange in proportion to the versine of the deflection.

221. The A truss.—Fig. 79 represents a simple form of braced triangle, often used for iron roofs where the span does not exceed 40 feet. The strains in the several parts may be conveniently obtained by finding the reaction of either abutment and working thence towards the centre, as explained in the following example, which has been calculated by the aid of a diagram drawn to a scale of 5 feet = 1 inch, and with a scale of weights of 1 ton = 1 inch.

Fig. 79.



The span is 40 feet, the depth of the truss 8 feet, and the height of the ridge above the wall-plate 10 feet. The load is 8 tons uniformly distributed, for which we may substitute its equivalent, namely, the load on a whole bay, or 2 tons, concentrated at each apex, and the load on half a bay, or 1 ton, at each abutment. The reaction of the left abutment = 4 tons, of which 1 ton is immediately balanced by the weight, W_1, concentrated there, leaving 3 tons to be resolved in the directions of **A** and **C**, the strains in which are respectively + 10·35 tons and — 9·38 tons. The vertical pressure of W_2 is supported by **A** and **F**, and when resolved in their directions produces + 0·9 and + 1·78 tons respectively; the former being a downward thrust is opposed to the upward thrust already existing in **A**; consequently, the difference, = + 9·45 tons, is the thrust transmitted upwards through **B**. At *a* we have two known forces, namely, the tension in **C** and the thrust in **F**; finding their resultant, and decomposing it again in the directions of **D** and **E**, we have the strains in these bars = — 4·64 tons and — 5·06 tons respectively. The following table gives the strains in the left half truss in a collected form.

FLANGES AND BRACING.	A	B	C	D	E	F
Strains in tons.	+ 10·35	+ 9·45	— 9·38	— 4·64	— 5·06	+ 1·78

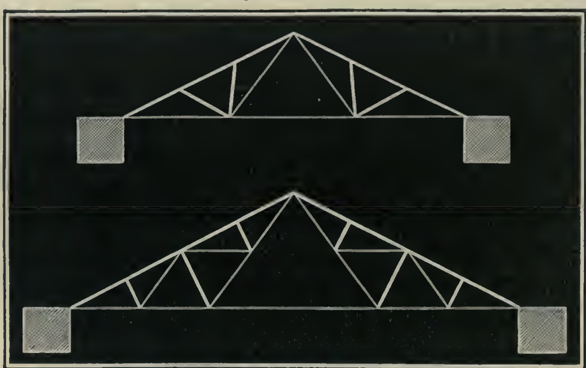
The accuracy of the work may be checked by the method of moments as follows. The external forces acting on the left half-truss are the reaction of the left abutment acting upwards and the weights $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_3$ acting downwards. The internal forces which resist these are the thrust of the opposite half-truss at the ridge and the pull of the central tie-rod below; taking moments round the ridge, and calling the tension in the tie-rod $\mathbf{T}$, we have,

$$4 \times 20 - (1 \times 20 + 2 \times 10) = \mathbf{T} \times 8$$

whence, $\mathbf{T} = 5$ tons, which shows that a trifling error of ·06 tons has been made in the calculation by diagram.

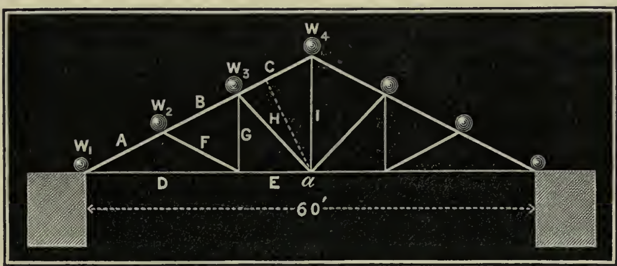
Fig. 80 represents another form of braced triangle suited for spans between 30 and 60 feet. The method of calculation is so similar to that just described that an example is unnecessary. In both trusses the most important part of the bracing is in tension, and they have therefore a light and graceful appearance.

Figs. 80 and 81.



The form of truss represented in Fig. 81 may be used for spans between 50 and 100 feet, and, if desirable, the secondary trussing may be carried out to a much greater extent than in the figure, so as to cover far wider spans. A braced triangle of the type represented in Fig. 82 may also be used up to very large spans indeed. Different modes of calculating the strains have been suggested, but the method of working by the resolution of forces from either abutment towards the centre seems the most satisfactory, as illustrated in the following example, which has been calculated by the aid of a diagram drawn to a scale of 5 feet to one inch.

Fig. 82.



The span and depth are 60 feet and 15 feet respectively, and the load distributed uniformly over the rafters, *i.e.*, the upper flange, = 12 tons, which is equivalent to 2 tons concentrated at each of the apices and 1 ton at each abutment. The upward reaction of the left abutment = 6 tons, of which 1 ton is at once balanced by $\mathbf{W}_1$, and the remaining 5 tons, being decomposed in the directions of **A** and **D**, produce a thrust of + 11·19 tons in **A**, and a pull of — 10 tons in **D**. At the next apex, $\mathbf{W}_2$ (= 2 tons,) is supported by **A** and **F** in equal proportions, as they form the sides of an isosceles triangle, and its components in their directions are each = + 2·24 tons; that in the direction of **A** reduces its upward thrust to + 8·95 tons which is transmitted onwards through **B**, while the thrust in **F** produces a tension of — 1 ton in **G** and reduces the pull in **D** so that a tension of only — 8 tons is transmitted through **E**. At $\mathbf{W}_3$ we have its downward pressure (= 2 tons,) added to the downward pull of **G** (= 1 ton,) which gives a total vertical pressure of 3 tons at this apex; this, when resolved in the directions of **H** and **B**, produces a tension of — 2·83 tons in **H** and reduces the upward thrust in **B** so that only + 6·71 tons is transmitted through **C**. Resolving the downward thrust in **H** in the directions of **E** and **I**, we obtain a pull of — 2 tons in **I**, to which should be added a corresponding pull from the right half of the truss, so that the total tension in **I** = — 4 tons. We may check the accuracy of the calculation by finding the strain in **C** by the method of moments, as follows. The segment, $\mathbf{W}_1\,\mathbf{C}a$, is held in equilibrium by the external bending forces, namely, the upward reaction of the left abutment, the downward pressures of $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_3$, and the resisting forces in the structure itself, namely, the thrust in **C** and the various forces meeting at a; taking moments round a, and measuring the distance $\mathbf{C}a$ by scale, = 13·43 feet, we can find the thrust in **C** by the following equation,

$$\mathbf{F} \times 13{\cdot}43 = 6 \times 30 - (1 \times 30 + 2 \times 20 + 2 \times 10)$$

Where **F** represents the strain in **C**; hence,

$$\mathbf{F} = \frac{90}{13{\cdot}43} = 6{\cdot}7 \text{ tons},$$

or nearly exactly the same as before.

The following table gives the strains in a collected form.

FLANGES AND BRACING	A	B	C	D	E	F	G	H	I
Strains in tons.	+11·19	+8·95	+6·71	—10·0	—8·0	+2·24	—1·0	+2·83	—4·0

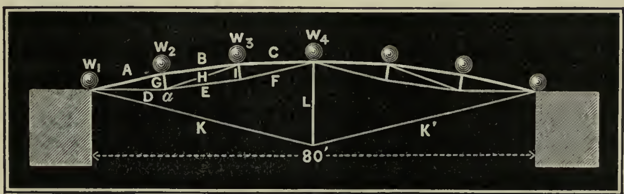
The roofing material generally rests directly on laths and purlins, which are again supported by the upper, or oblique, flange. Consequently, unless the purlins rest directly over an apex, each bay of the upper flange is subject to a transverse strain from the pressure of the purlins which cross it, in addition to a longitudinal thrust which it sustains as a member of the truss, and its strength must be made sufficient to bear this double strain. It will be seen hereafter that its continuity across the apices adds materially to the strength of the rafter.

The arrangement of the bracing may be varied so as to put the verticals in compression and the diagonals in tension, and sometimes the tie is raised at the centre so as to form a low triangle and give more head-room beneath; this of course diminishes the effective depth of the truss, but it has the advantage of shortening the length of the struts.

CASE VII.—THE SUSPENSION TRUSS.

222. Suited for domed roofs.—This form of truss is generally employed for supporting low-domed roofs resting on circular walls, in which case the trusses intersect each other at the centre

Fig. 83.



and have a common central strut beneath the crown of the dome. Each half of the bow, or upper flange, is strengthened by a secondary truss **D E F**. At first sight there seems some ambiguity about the strains, inasmuch as three braces intersect at the abutment, and we cannot say how the reaction of the latter is distributed among them. On a little consideration, however, the matter is simple; let us confine our attention to the external forces which keep the secondary truss, **A B C D E F**, in equilibrium, and taking their moments round the centre of the roof, we have the moment of the tension in the string **K** equal to the upward moment of the reaction of the left abutment minus the downward moments of W_2 and W_3. We can thus find the tension in the string, and knowing this and the reaction of the abutment, we can readily find their resultants in **A** and **D**, and from these again derive the strains in the other braces. The following example will illustrate this clearly. It has been worked out by the aid of a diagram drawn to a scale of 5 feet to one inch. Let Fig. 83 represent a suspension truss, 80 feet in span, 5 feet in depth from the crown to a horizontal line joining the wall-plates, and 15 feet in total depth. The bow is divided into 6 equal bays, and the secondary truss has been formed by making **D** a horizontal line, and the short struts **G** and **I** parallel to the radial line which would pass through the centre of **B**; thus **A** = **B** = **C** = **E**, and **G** = **I**, and **D** = **F**. Let the weight of a sector of the circular roof supported by the half-truss, **A B C L K**, = 9 tons, which is divided among the apices in proportion to the area of the sector supported by each bay and, assuming that the sector is a triangle, we shall have the weights at the several apices as follows:—

$$\left.\begin{aligned} \mathbf{W}_1 &= 2\tfrac{3}{4} \text{ tons}, \\ \mathbf{W}_2 &= 4 \quad ,, \\ \mathbf{W}_3 &= 2 \quad ,, \\ \mathbf{W}_4 &= \tfrac{1}{4} \quad ,, \end{aligned}\right\} = 9 \text{ tons}.$$

Since $\mathbf{W}_1$ rests directly on the wall-plate, we may leave it out of consideration in calculating the longitudinal strains in the truss, though it will be necessary to consider it subsequently when calculating the transverse strength of **A** as an independent girder supporting directly its proper share of distributed roof-load. The secondary truss, **A B C D E F**, is held in equilibrium by

1°. The oblique pull in the tie **K**,

2°. The upward reaction of the abutment, $= \mathbf{W}_2 + \mathbf{W}_3 + \mathbf{W}_4 = 6\tfrac{1}{4}$ tons,

3°. The downward pressures of $\mathbf{W}_2$ and $\mathbf{W}_3$,

4°. $\mathbf{W}_4$, the thrust of the central strut **L**, and that of the opposite half-truss,—all three intersecting at the crown.

If we take moments round the crown we get rid of the three latter forces, but to do this we must find by scale

the leverage of	**K**	round the crown	= 14·58 feet,
do.	$\mathbf{W}_2$	do.	= 26·85 feet,
do.	$\mathbf{W}_3$	do.	= 13·45 feet.

Taking moments round the crown, we have the

$$\text{tension in } \mathbf{K} = \frac{6{\cdot}25 \times 40 - (4 \times 26{\cdot}85 + 2 \times 13{\cdot}45)}{14{\cdot}58} = 7{\cdot}93 \text{ tons}.$$

We now know two of the forces meeting at the abutment, namely, the upward reaction of the abutment, $= 6\frac{1}{4}$ tons, and the tension in **K**, $= 7\cdot93$ tons. Finding the resultant of these, and decomposing it in the directions of **A** and **D**, we find the compression in **A** $= + 21$ tons, and the tension in **D** $= - 12\cdot86$ tons. At $\mathbf{W}_2$ four forces meet, namely, the thrust in **A**, the weight $\mathbf{W}_2$, the thrust in **G** and the thrust in **B**. As we know the two former forces we can find their resultant, and decomposing it in the directions of **G** and **B**, we find the strains in these equal to $+ 2\cdot25$ tons and $+ 20\cdot44$ tons respectively. At *a*, four forces meet, namely, the tension in **D**, the thrust in **G**, and the tensions in **E** and **H**.* The two former are known, and finding their resultant and decomposing it, we get the strain in **E**, $= - 9\cdot6$ tons, and that in **H**, $= - 3\cdot2$ tons. Proceeding thus, we find all the strains which are given in the following table.

FLANGES AND BRACING.	A	B	C	D	E	F	G	H	I	K	L
Strains in tons.	+ 21	+20·44	+17·06	−12·86	− 9·6	−9·63	+2·25	− 3·2	+ 1·15	−7·93	+3·8

The compression in the central pillar, **L**, is that due to both sides of the primary truss, and should equal the vertical resultant of the strains in the tie bars **K** and **K′**. This will be a check on the accuracy of the work. This form of truss is that generally used for supporting the roofs of gasholders; the truss, however, does not come into action unless the holder is empty, for when it is charged with gas (the pressure of which sometimes reaches 5 inches of water), the upward pressure of the gas is greater than the weight of the roof and lifts both it and the sides of the gasholder, and an explosion would, no doubt, sometimes occur, were it not for the domed shape of the roof which resists internal pressure like the ends of an egg-ended boiler.

* The diagonal **H** is required because $\mathbf{W}_2$ exceeds $\mathbf{W}_3$; in practice, however, it is generally omitted.

CHAPTER VIII.

DEFLECTION.

CLASS 1.—*Girders whose sections are proportioned so as to produce uniform strength.*

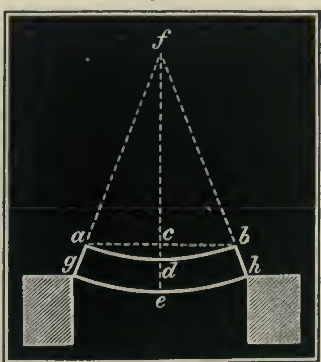
223. Deflection curve circular in girders of uniform strength—Amount of deflection not materially affected by the web.—The equations generally used for calculating the deflections of loaded girders are based on the assumption that the section of the girder is uniform throughout its entire length, that is, that there is the same amount of material at the centre as at the ends. In scientifically constructed girders, however, this is not the case. Each part is duly proportioned to the maximum strain which can pass through it, so that no material is wasted; and when this occurs in a girder with horizontal flanges and a uniformly distributed load, that is, the load which produces the maximum strain in the flanges, these latter will, as has been already shown in **47**, taper from the centre, where their section is greatest, towards the ends as the ordinates of a parabola. The girder is then said to be of uniform strength, because the unit-strain in each flange is uniform throughout the whole length of the flange and no part has an excess of material, or is unduly strained beyond the rest (**19**). Now, as the contraction and elongation are according to Hooke's law proportional to the unit-strain, so long as it does not exceed the limits which are considered safe in practice (**7**), the contraction per running foot of the upper flange will be uniform throughout its length, and the extension per running foot of the lower flange will likewise be uniform throughout its length; and this uniform contraction and elongation must produce a circular deflection, since the circle is the only curve that is due to a uniform cause. At first sight it may be thought that the continuous web of the plate girder, or the braced web of the lattice girder, will seriously affect the amount of the deflection curve; but it can be readily shown by carefully constructed diagrams, in which the alterations of length due to the load are drawn to a highly exaggerated scale, that the construction of the web has scarcely any influence on the curvature so long as the unit-strains in the flanges are unaltered in amount by the method of construction, and it is only when this is the case that a fair comparison can be instituted between the rival girders.

Fig. 1, Plate I., represents one-half of a diagonally braced girder of the simplest form, namely, a girder with one system of triangles before the load rests upon it. Every part is then in its normal state, and the girder will be horizontal. Now, suppose that a uniform load deflects it and shortens each bay of the top, or compression, flange by a certain quantity, while it lengthens each bay of the lower, or tension, flange to a similar extent; and further, let us suppose that the diagonals are alternately shortened and lengthened by equal amounts, according as they are struts or ties. Fig. 2 now represents the girder; the deflection curve forms a segment of a circle whose centre is at **A**, a little to the left of the vertical line drawn through the middle of the girder. Next, suppose that the flanges are compressed and extended as in Fig. 2, but that the diagonals remain of their original length as in Fig. 1, that is, that their length is not affected by the load. Fig. 3 is the result, which it will be perceived, is circular and differs but slightly from Fig. 2, having its centre, however, at **B**, in the vertical line drawn through the middle of the girder. It may at first seem strange that **A**, the centre of Fig. 2, is not in the vertical line passing through the middle of the girder. This is due to the circumstance that, with a uniform load, the two central diagonals, d and d', are subject to the same strain, either both lengthened or both shortened, while all the other diagonals are alternately lengthened and shortened. Hence, a very slight angle is produced at the centre, as shown in Fig. 4, where the flanges are unaltered as in Fig. 1, while the diagonals are alternately lengthened and shortened as in Fig. 2. Considering, however, the exaggerated scale of the diagrams, Fig. 4 is practically horizontal when compared with Figs. 2 or 3, and the chief effect of this common change in the length of the two central diagonals is to throw the centre of each half of the girder in Fig. 2 a little to the right or left of the middle line. These diagrams give very interesting results; they show that the curvature of flanged girders is practically independent of change of form in the web, and almost entirely due to the shortening of the upper, and the elongation of the lower, flange; and a further inference may be derived from them, viz., that deflection is practically unaffected by the nature of the web, whether it be formed of plates or lattice bars, provided that the unit-strains in the flanges are not increased or diminished by a different formation of web. Consequently, if there be two girders of equal length and depth, one a lattice, the other a plate girder, having the same unit-strains transmitted throughout their respective flanges, they will both deflect to the same extent.

224. Formula for the deflection of circular curves—Deflection of similar girders when equally strained varies as their linear dimensions.—The circumstance of the curve of a loaded girder of uniform strength being circular enables us to find a very simple equation for calculating its deflection.

Let *adbgeh*, Fig. 84, represent a girder supported at both ends and of uniform strength for the load, which generally occurs when the load is uniformly distributed.

Fig. 84.



Let $l = adb =$ the length of the girder,
$d = de =$ the depth,
$\mathbf{R} = af =$ the radius of curvature,
$\lambda = geh - adb =$ the difference in length of the flanges after deflection,
$\mathbf{D} = cd =$ the central deflection.

Since the deflection is very small compared with the radius of curvature, we may assume $cf = af = \mathbf{R}$, and $ab = adb = l$; then (*Euclid*, prop. 35, book iii.),

$$\mathbf{D} = \frac{l^2}{8\mathbf{R}}.$$

By similar triangles, $$\mathbf{R} = \frac{dl}{\lambda}$$

whence, by substitution, $$\mathbf{D} = \frac{\lambda l}{8d} \qquad (132)$$

in which the value of λ is known, as it depends on the coefficients of elasticity of the flanges and the strains to which they are subject. This equation for the deflection curve confirms the previous investigation, for the depth, d, is the only quantity in the equation which can be affected by a change in the length of the diagonals, and it is obvious that a slight change in the value of d will not affect that of $\mathbf{D}$ to any appreciable extent.

It follows from equation 132 that when similar girders sustain the same unit-strains in their flanges, their deflections will vary directly as any of their linear dimensions.

Ex. 1. The length and depth for calculation of the Conway tubular bridge are respectively 412 feet and 23·7 feet, and it appears from ex. 2 (**44**) that the inch-strains in the lower and upper flanges at the centre of the bridge from the permanent load are 5·067 tons and 3·948 tons respectively; what is the central deflection on the supposition that the flanges are of uniform strength, which is very nearly true? The coefficient of elasticity of wrought-iron is 24,000,000 lbs. = 10,714 tons per square inch; consequently, it contracts or extends $\frac{1}{10714}$th of its length for each ton per square inch, and we have the following data :—

$$l = 412 \text{ feet},$$
$$d = 23{\cdot}7 \text{ feet},$$
$$\lambda = \frac{412}{10{,}714}(5{\cdot}067 + 3{\cdot}948) = {\cdot}347 \text{ feet}.$$

Answer (eq. 132). $\mathbf{D} = \dfrac{\lambda l}{8d} = \dfrac{{\cdot}347 \times 412}{8 \times 23{\cdot}7} = {\cdot}754 \text{ feet} = 9{\cdot}048 \text{ inches}.$

The mean deflection of the two tubes immediately on removal of the platform was 8·04 inches, and 8·98 inches after taking a permanent set due to strain. When the permanent way was added and after 12 month's use, the deflection of the second tube in the month of January was 10·3 inches. The deflection in hot weather would doubtless be somewhat less. The deflection, from additional weight placed at the centre, was ·01104 inch for each ton. (*Clark*, p. 662.)

Ex. 2. The length and depth for calculation of one of the large tubes of the Britannia bridge are respectively 470 and 27·5 feet, and from ex. 4 (**44**), the inch-strains at the centre from the weight of the tube as an independent girder were 5·795 and 4·856 tons in the lower and upper flanges respectively. What was the central deflection? Using the same coefficient of elasticity as before, we have,

$$l = 470 \text{ feet},$$
$$d = 27{\cdot}5 \text{ feet},$$
$$\lambda = \frac{470}{10{,}714}(5{\cdot}795 + 4{\cdot}856) = {\cdot}467 \text{ feet.}$$

Answer (eq. 132). $\mathsf{D} = \dfrac{\lambda l}{8d} = \dfrac{{\cdot}467 \times 470}{8 \times 27{\cdot}5} = 1$ foot nearly.

The mean deflection of the two tubes of the up line, immediately on removing the platform, was 11·75 inches; the mean deflection after being raised was 12·57 inches. (*Clark*, p. 673.)

Ex. 3. A wrought-iron girder of uniform strength is 84 feet long and 7 feet deep. A certain load produces a deflection of 1·2 inches at the centre; what are the unit-strains in the flanges from this load? From equation 132, we have,

$$\lambda = \frac{8d\mathsf{D}}{l} = \frac{8 \times 7 \times 1{\cdot}2}{84} = {\cdot}8 \text{ inches.}$$

The inch-strains in both flanges together $= \dfrac{{\cdot}8 \times 10{,}714}{84 \times 12} = 8{\cdot}55$ tons, which when divided between the two flanges inversely as their sectional areas, will give the inch-strain in each flange due to the given load.

Class 2.—*Girders whose section is uniform throughout their length.*

225. The following investigations are based on the law of uniform elastic reaction, and are therefore only applicable to girders whose strains lie within the limits of elasticity (**7**).

Let W = the bending weight,

M = the moment of resistance of the horizontal elastic forces at any given cross section of the girder (**59**),

x = the horizontal distance of the same section from the left abutment,

y = the vertical distance of any fibre in the section, either above or below the neutral axis,

β = the breadth of the section at the distance y from the neutral axis, and consequently a variable, except in the case of rectangular sections,

f = the horizontal unit-strain exerted by fibres in the given section at a distance c from its neutral axis,

c = the distance from the neutral axis of horizontal fibres which exert the unit-strain f,

$\mathbf{I}$ = the moment of inertia of any cross section round its neutral axis, and consequently, a constant quantity throughout the whole length of the girder when the latter is of uniform section,

$\mathbf{R}$ = the radius of curvature,

$\mathbf{E}$ = the coefficient of elasticity.

It has already been shown (eq. 43) that **M**, the moment of the horizontal elastic forces of any cross section round its neutral axis, may be expressed by the equation,

$$\mathbf{M} = \frac{f}{c}\int \beta y^2 dy$$

provided the horizontal fibres are not strained beyond their limit of elastic reaction. When the girder is of uniform section throughout its length, the integral $\int \beta y^2 dy$, being a definite integral, will be a constant throughout the girder, and as it happens to express the moment of inertia of the cross section round its neutral axis (**69**), we may substitute for this integral the symbol **I**, when we have

$$\mathbf{M} = \frac{f}{c}\mathbf{I} \tag{133}$$

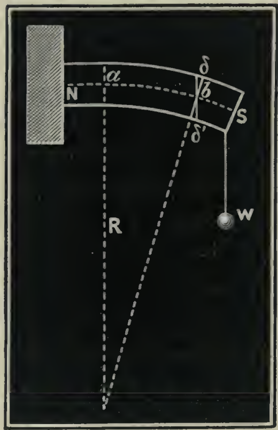
In order to transform this equation into one involving the co-ordinates of the deflection curve, we must substitute for the three variables, **M**, f and c, their values in terms of the co-ordinates x and y. Let us first deal with f and c.

Fig. 85 represents a deflected semi-girder, whose neutral surface is **N S**.

Let ab = a unit of length,

δ and δ' = the increment and decrement in length of a linear unit of the extreme fibres after deflection.

Fig. 85.



When the horizontal strains do not exceed the limits of elasticity, we have the following relation,

$$\frac{f}{c}=\frac{\mathsf{E}}{\mathsf{R}}$$

Substituting this in eq. 133, we have the moment of resistance,

$$\mathsf{M}=\frac{\mathsf{E}}{\mathsf{R}}\mathsf{I} \qquad (134)$$

From the principles of the differential calculus we know that, where the deflection is small compared with the length of the curve,

$$\frac{1}{\mathsf{R}}=-\frac{d^2y}{dx^2} \text{ nearly,}$$

whence, by substitution in eq. 134, we have,

$$\mathsf{M}=-\mathsf{E}\,\mathsf{I}\frac{d^2y}{dx^2} \qquad (135)$$

in which M is a positive or negative moment according as the upper flange is in compression or tension, y being measured downwards. This equation expresses the moment of resistance of the horizontal elastic forces at any section of a girder in terms of the ordinates of the deflection curve, the coefficient of elasticity, and the moment of inertia of the cross section round its neutral axis. In order to solve eq. 135, there still remains before integration to substitute for the variable M its value in terms of the ordinates of the deflection curve, which may be derived from the leverage of the weight, observing that the moments of forces are to be taken as positive or negative according as they tend to compress or extend the upper flange. To effect this substitution we must consider each case separately, and after integration, the value of **I**, which is a different constant for each form of section, may be obtained by multiplying the values of **M**, already determined in (**71**) and the succeeding articles, by $\frac{c}{f}$ (eq. 133).

CASE I.—SEMI-GIRDERS OF UNIFORM SECTION LOADED AT THE EXTREMITY.

226. Let $\mathbf{W}$ = the load at the extremity,
l = the length of the semi-girder,
x = the abscissa of the deflection curve measured from the fixed end,
y = the ordinate of the deflection curve measured downwards,
$\mathbf{D}$ = the deflection at the extremity,
$\mathbf{M}$ = the moment of resistance of the horizontal elastic forces at any given section, whose distance from the fixed ends = x (**59**),
$\mathbf{I}$ = the moment of inertia of any cross section,
$\mathbf{E}$ = the coefficient of elasticity.

Taking moments round the neutral axis of the given section, we have,

$$\mathbf{M} = -\mathbf{W}(l - x)$$

Substituting this in eq. 135, we have,

$$\mathbf{E}\,\mathbf{I}\frac{d^2y}{dx^2} = \mathbf{W}(l - x)$$

Integrating,

$$\mathbf{E}\,\mathbf{I}\frac{dy}{dx} = \mathbf{W}\left(lx - \frac{x^2}{2}\right) + \text{constant}.$$

The constant = 0, for when $x = 0$, $\frac{dy}{dx}$ also = 0, since the tangent of the curve is horizontal at the fixed end. Integrating again, and determining that the new constant = 0, from the consideration that $y = 0$ when $x = 0$, we have,

$$\mathsf{E\,I}\,y = \mathsf{W}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \tag{136}$$

This is the equation of the deflection curve, y being the deflection at any point whose distance from the fixed end equals x.

At the extremity where $x = l$, $y = \mathsf{D}$, and we have,

$$\mathsf{E\,I\,D} = \mathsf{W}\frac{l^3}{3}$$

whence,

$$\mathsf{D} = \frac{\mathsf{W}l^3}{3\mathsf{EI}} \tag{137}$$

227. Solid rectangular semi-girders—Deflection of solid square girders is the same with the sides or one diagonal vertical.—Let b = the breadth and d = the depth. From eqs. 46, 133, and 137,

$$\mathsf{D} = \frac{4\mathsf{W}l^3}{\mathsf{E}bd^3} \tag{138}$$

Comparing eqs. 46 and 47, we find that the deflection of solid square girders is the same whether the diagonal or one side be vertical. Their strength, however, is not the same (**86**).

Ex. The piece of Memel timber, described in Ex. 4 (**66**), deflected 0·66 inch from a load of 336 lbs. hung at its extremity; what is the value of E?

Here, $\mathsf{W} = 336$ lbs.,
$l = 24$ inches,
$b = 1{\cdot}94$ inches,
$d = 2$ inches,
$\mathsf{D} = 0{\cdot}66$ inch.

Answer (eq. 138). $\mathsf{E} = \dfrac{4\mathsf{W}l^3}{\mathsf{D}bd^3} = 1{,}800{,}000$ lbs.

228. Solid round semi-girders.—Let r = the radius. From eqs. 48, 133, and 137,

$$\mathsf{D} = \frac{4\mathsf{W}l^3}{3\mathsf{E}\pi r^4} \tag{139}$$

229. Hollow round semi-girders of uniform thickness.—Let t = the thickness of the tube, supposed small in proportion to its radius r. From eqs. 50, 133, and 137,

$$\mathsf{D} = \frac{\mathsf{W}l^3}{3\mathsf{E}\pi r^3 t} \tag{140}$$

230. Semi-girders with parallel flanges.—When the web is formed of bracing, or if continuous, is yet so thin that we may safely neglect the support it gives the flanges, we have from eqs. 55, 133, and 137,

$$\mathsf{E} = \frac{\mathsf{WA}l^3}{3\mathsf{E}a_1a_2d^2} \tag{141}$$

where $\mathsf{A} = a_1 + a_2 =$ the sum of the areas of the two flanges, and $d =$ the depth of the web.

When the web is taken into account and the flanges are of equal area,

let a = the area of either flange,

a' = the area of the web.

From eqs. 57, 133, and 137,

$$\mathsf{D} = \frac{4\mathsf{W}l^3}{\mathsf{E}(6a + a')d^2} \tag{142}$$

231. Square tubes of uniform thickness, with the sides or one diagonal vertical.—From eqs. 59, 133, and 137,

$$\mathsf{D} = \frac{4\mathsf{W}l^3}{\mathsf{E}\,(b^4 - b_1{}^4)} \tag{143}$$

where b and b_1 are the external and internal breadths.

If the thickness of the tube be small compared with the breadth, we have from eqs. 60, 133, and 137,

$$\mathsf{D} = \frac{\mathsf{W}l^3}{2\mathsf{E}b^3t} \tag{144}$$

in which t represents the thickness of one side.

CASE II.—SEMI-GIRDERS OF UNIFORM SECTION LOADED UNIFORMLY.

232. Let l = the length of the semi-girder,

x = the abscissa of the deflection curve measured from the fixed end,

y = the ordinate of the deflection curve measured downwards,

w = the load per unit of length,

$\mathsf{W} = wl$ = the whole load,

D = the deflection at the extremity,
M = the moment of resistance of the horizontal elastic forces at any given section, whose distance from the fixed end = x (**59**),
E = the coefficient of elasticity.

Taking moments round the neutral axis of the given section, we have,

$$\mathsf{M} = -\frac{w}{2}(l-x)^2$$

Substituting this in eq. 135, we have,

$$\mathsf{EI}\frac{d^2y}{dx^2} = \frac{w}{2}(l-x)^2$$

Integrating,

$$\mathsf{EI}\frac{dy}{dx} = -\frac{w}{6}(l-x)^3 + \text{constant.}$$

When $x = 0$, $\frac{dy}{dx} = 0$ also; hence, the constant equals $\frac{wl^3}{6}$. Substituting this value and integrating again,

$$\mathsf{EI}y = \frac{w}{24}(l-x)^4 + \frac{wl^3x}{6} + \text{constant.}$$

Determining the second constant by the consideration that $y = 0$ when $x = 0$, we have,

$$\mathsf{EI}y = \frac{w}{24}(l-x)^4 + \frac{wl^3x}{6} - \frac{wl^4}{24}$$

At the extremity where $x = l$, $y = \mathsf{D}$, and we have,

$$\mathsf{D} = \frac{wl^4}{8\mathsf{EI}} = \frac{\mathsf{W}l^3}{8\mathsf{EI}} \tag{145}$$

233. Deflection of a semi-girder loaded uniformly equals three-eighths of its deflection with the same load concentrated at its extremity.—Comparing eqs. 145 and 137, we see that the deflection of a semi-girder loaded uniformly is to its deflection with the same load concentrated at the extremity as $\frac{3}{8}$. Hence, to obtain the deflections of the various classes of semi-girders in the case of a uniform load, we have merely to multiply the formulæ in the preceding case by $\frac{3}{8}$, recollecting that **W** will now represent the uniformly distributed load.

CASE III.—GIRDERS OF UNIFORM SECTION SUPPORTED AT BOTH ENDS AND LOADED AT THE CENTRE.

234. Let $l =$ the length of the girder,

$x =$ the abscissa of the deflection curve measured from the left end of the girder,

$y =$ the ordinate of the deflection curve measured downwards,

$\mathsf{W} =$ the load at the centre,

$\mathsf{D} =$ the deflection at the centre,

$\mathsf{M} =$ the moment of resistance of the horizontal elastic forces at any given section whose distance from the left end $= x$ (**59**),

$\mathsf{E} =$ the coefficient of elasticity.

Taking moments round the neutral axis of the given section, we have

$$\mathsf{M} = \frac{\mathsf{W}x}{2}$$

Substituting this in eq. 135, we have,

$$\mathsf{E\,I}\,\frac{d^2y}{dx^2} = -\frac{\mathsf{W}x}{2}$$

Integrating,

$$\mathsf{E\,I}\,\frac{dy}{dx} = -\frac{\mathsf{W}x^2}{4} + \text{constant}.$$

To determine the constant, we must recollect that the tangent of the curve is horizontal at the centre; hence, $\frac{dy}{dx} = 0$ when $x = \frac{l}{2}$, and the constant $= \frac{\mathsf{W}l^2}{16}$; substituting this,

$$\mathsf{E\,I}\,\frac{dy}{dx} = \frac{\mathsf{W}}{4}\left(\frac{l^2}{4} - x^2\right)$$

Integrating again, and observing that the second constant $= 0$ from the consideration that $y = 0$ when $x = 0$, we have,

$$\mathsf{E\,I}\,y = \frac{\mathsf{W}}{4}\left(\frac{l^2x}{4} - \frac{x^3}{3}\right)$$

which is the equation of the deflection curve.

At the centre where $x = \frac{l}{2}$, $y = \mathsf{D}$, and we have,

$$\mathsf{D} = \frac{\mathsf{W}l^3}{48\mathsf{EI}} \tag{146}$$

235. Solid rectangular girders.—From eqs. 46, 133, and 146,

$$\mathsf{D} = \frac{\mathsf{W}l^3}{4\mathsf{E}bd^3} \tag{147}$$

in which b and d represent the breadth and depth of the girder.

Ex. From the mean of five experiments made by Mr. Hodgkinson on Blaenavon cast-iron, No. 2,* it appears that the breaking weight and ultimate deflection of a rectangular bar 13 feet 6 inches between points of support, 3 inches wide and $1\frac{1}{2}$ inch deep, are respectively 819 lbs. and 10·46 inches; what is the value of the coefficient of transverse elasticity at the limit of rupture?

Here, $\mathsf{W} = 819$ lbs.
$l = 13{\cdot}5$ feet,
$b = 3$ inches,
$d = 1{\cdot}5$ inches,
$\mathsf{D} = 10{\cdot}46$ inches.

Ans. (eq. 147). $\mathsf{E} = \frac{\mathsf{W}l^3}{4\mathsf{D}bd^3} = \frac{819 \times (13{\cdot}5 \times 12)^3}{4 \times 10{\cdot}46 \times 3 \times (1{\cdot}5)^3} = 8{,}200{,}000$ lbs. per square inch.

The deflection of the same bar when loaded with 260 lbs., which was within the limit of elasticity, was 2 inches. What was its coefficient of elasticity within this limit?

Here, $\mathsf{W} = 260$ lbs.
$\mathsf{D} = 2$ inches.

Answer. $\mathsf{E} = \frac{260 \times (13{\cdot}5 \times 12)^3}{4 \times 2 \times 3 \times (1{\cdot}5)^3} = 13{,}600{,}000$ lbs.

The reader should be informed that this coefficient of transverse elasticity of Blaenavon iron is less than that of average cast-iron, especially when mixed.

236. Solid round girders.—From eqs. 48, 133, and 146,

$$\mathsf{D} = \frac{\mathsf{W}l^3}{12\mathsf{E}\pi r^4} \tag{148}$$

in which r represents the radius.

237. Hollow round girders of uniform thickness.—From eqs. 50, 133, and 146,

$$\mathsf{D} = \frac{\mathsf{W}l^3}{48\mathsf{E}\pi r^3 t} \tag{149}$$

in which t represents the thickness of the tube, supposed small in proportion to its radius r.

* See *Report of Com.* p. 69.

238. Girders with parallel flanges.— When the vertical web is formed of bracing, or if continuous, yet so thin that it affords but slight assistance to the flanges in sustaining horizontal strains, its stiffness as an independent girder may be neglected, and we have from eqs. 55, 133, and 146,

$$\mathbf{D} = \frac{\mathbf{WA}l^3}{48\mathbf{E}a_1a_2d^2} \tag{150}$$

in which $\mathbf{A} = a_1 + a_2 =$ the sum of the areas of the top and bottom flanges, and $d =$ the depth of the web.

When the web is taken into account, and the flanges are of equal area, from eqs. 57, 133, and 146,

$$\mathbf{D} = \frac{\mathbf{W}l^3}{4\mathbf{E}(6a + a')d^2} \tag{151}$$

in which $a =$ the area of one flange and $a' =$ that of the web.

239. The deflections of girders of other forms of section may be obtained in a similar manner from eqs. 133 and 146 by substituting for $\mathbf{M}$ the corresponding values given in Chap. IV.

CASE IV.—GIRDERS OF UNIFORM SECTION SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

240. Let $l =$ the length of the girder,

$w =$ the load per linear unit,

$\mathbf{W} = wl =$ the whole load,

$x =$ the abscissa of the deflection curve measured from the left end of the girder,

$y =$ the ordinate of the deflection curve measured downwards,

$\mathbf{D} =$ the deflection at the centre,

$\mathbf{M} =$ the moment of resistance of the horizontal elastic forces at any given section whose distance from the left end $= x$ (**59**),

$\mathbf{E} =$ the coefficient of elasticity.

Taking moments round the neutral axis of the given section, we have,

$$\mathsf{M} = \frac{w}{2}(lx - x^2)$$

Substituting this in eq. 135, we have

$$\mathsf{EI}\frac{d^2y}{dx^2} = -\frac{w}{2}(lx - x^2) \tag{152}$$

Integrating,

$$\mathsf{EI}\frac{dy}{dx} = -\frac{w}{2}\left(\frac{lx^2}{2} - \frac{x^3}{3}\right) + \text{constant.}$$

When $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$, and the constant becomes $\frac{wl^3}{24}$; substituting this,

$$\mathsf{EI}\frac{dy}{dx} = \frac{w}{2}\left(\frac{x^3}{3} - \frac{lx^2}{2} + \frac{l^3}{12}\right)$$

Integrating again, and observing that the second constant $= 0$ from the consideration that $y = 0$ when $x = 0$,

$$\mathsf{EI}\,y = \frac{w}{24}(x^4 - 2lx^3 + l^3x)$$

which is the equation of the deflection curve.

At the centre where $x = \frac{l}{2}$, $y = \mathsf{D}$, and we have,

$$\mathsf{D} = \frac{5wl^4}{384\,\mathsf{EI}} = \frac{5\mathsf{W}l^3}{384\,\mathsf{EI}} \tag{153}$$

241. Central deflection of a girder loaded uniformly equals five-eighths of its deflection with the same load concentrated at the centre.—Comparing eqs. 153 and 146, we find that the central deflection of a girder loaded uniformly is $\frac{5}{8}$ths of the deflection if the same load were concentrated at the centre. This has been corroborated by experiments by M. Dupin on rectangular girders of oak.*

242. Solid rectangular girders.—From eqs. 46, 133, and 153,

$$\mathsf{D} = \frac{5wl^4}{32\mathsf{E}bd^3} = \frac{5\mathsf{W}l^3}{32\mathsf{E}bd^3} \tag{154}$$

where b and d represent the breadth and depth of the girder.

* *Morin*, p. 140.

Comparing eqs. 46 and 47, we find that the deflection of solid square girders is the same, whether one side or the diagonal be vertical. The former, however, is theoretically 1·414 times stronger than the latter (**86**).

243. Solid round girders.—From eqs. 48, 133, and 153,

$$\mathsf{D} = \frac{5wl^4}{96\,\mathsf{E}\pi r^4} = \frac{5\mathsf{W}l^3}{96\,\mathsf{E}\pi r^4} \tag{155}$$

where r represents the radius of the cylinder.

244. Hollow round girders of uniform thickness.—From eqs. 50, 133, and 153,

$$\mathsf{D} = \frac{5wl^4}{384\,\mathsf{E}\pi r^3 t} = \frac{5\mathsf{W}l^3}{384\,\mathsf{E}\pi r^3 t} \tag{156}$$

where $r =$ the radius, and $t =$ the thickness of the tube, supposed small in comparison with the radius.

245. Girders with parallel flanges.—When the web is formed of bracing, or if continuous, yet so thin that its strength as an independent girder may be neglected, we have from eqs. 55, 133, and 153,

$$\mathsf{D} = \frac{5\mathsf{A}wl^4}{384\,\mathsf{E}a_1 a_2 d^2} = \frac{5\mathsf{A}\mathsf{W}l^3}{384\,\mathsf{E}a_1 a_2 d^2} \tag{157}$$

where $\mathsf{A} = a_1 + a_2 =$ the sum of the areas of top and bottom flanges, and $d =$ the depth of the web.

If the web be taken into account and if the flanges have equal areas, from eqs. 57, 133, and 153,

$$\mathsf{D} = \frac{5wl^4}{32\,\mathsf{E}(6a + a')d^2} = \frac{5\mathsf{W}l^3}{32\,\mathsf{E}(6a + a')d^2} \tag{158}$$

where $a =$ the area of one flange, and $a' =$ that of the web.

246. Discrepancy between coefficients of elasticity derived from direct and from transverse strain.—The coefficients of elasticity derived from experiments on transverse strain do not always agree with those derived from direct longitudinal tension or compression; they vary also with different forms of cross section, as exhibited in the following table, which contains the coefficients of transverse elasticity of cast and wrought-iron girders of the more usual forms of cross section.

	Material.	Value of E, the coefficient of transverse elasticity per square inch.		
	Cast-Iron.	lbs.		tons.
1.	Rectangular bars of simple cast-irons, .	15,200,000	=	6,785
2.	Do. do. mixed do., .	18,892,000	=	8,434
3.	Rectangular, circular, or elliptical tubes, .	12,215,000	=	5,453
4.	Double-flanged girders, ...	13,200,000	=	5,893
	Wrought-Iron.			
5.	Double-flanged rolled beams, for floors, &c.,	16,360,000 to 21,570,000	=	7,304 to 9,630
6.	Single-webbed double-flanged plate girders, riveted, ...	14,316,000	=	6,391
7.	Tubular plate girders, ...	23,610,000	=	10,541
8.	Conway tubular bridge, ...	18,754,000	=	8,372

1. *Experimental Researches*, p. 404.

2. 3. 4. 6. 7. 8. *Morin*, pp. 260, 264, 269, 299, 322, 323.

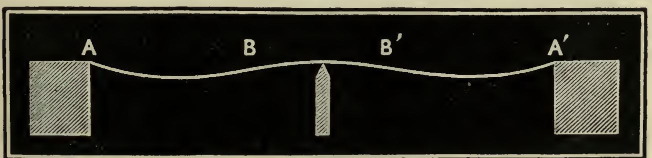
5. *Idem*, p. 293, *and Mr. W. Anderson.*

CHAPTER IX.

CONTINUOUS GIRDERS.

247. Continuity—Contrary flexure—Points of inflexion.— A girder is said to be continuous when it overhangs its bearings, or is sub-divided into more than one span by one or more intermediate points of support. When a loaded girder is balanced on a single pier at or near its centre, like the beam of a pair of scales, the upper flange is subject to tension, the lower one to compression, and the girder becomes curved with the convex flange uppermost. If, however, the same girder be supported at its extremities, the pier being removed, the strains in the flanges are reversed, the upper flange being now compressed and the lower one extended, and in this case the convex flange is underneath. If, while in this latter position, we replace the central pier so as to form two spans, the girder becomes continuous and partakes of the nature of both the independent girders; each flange is in part extended, in part compressed, and the curve becomes a waved line. Let Fig. 86 represent a continuous girder of two spans uniformly loaded.

Fig. 86.



The central segment BB′ resembles the independent girder in the first case, namely, when balanced over a pier; the extreme segments, AB, B′A′, resemble it in the second case, since one end of each rests upon an abutment and the other end is supported by the central segment, which thus sustains besides its own proper load an additional weight suspended from each extremity, equal to the half load on each of the end segments. The points **B**, **B′**, where the curvature alters its direction, are called the *points of contrary flexure*, or more briefly, the *points of inflexion*. The curves of the end and central segments have common tangents at these points, and here the strains in the flanges change from tension to compression, and *vice versâ*. Exactly at these points the strains in the flanges are cipher; consequently, the flanges might be severed there without altering the conditions of equilibrium in any respect. In fact, a continuous girder may be regarded as formed of independent girders connected merely by chains at the points of inflexion. In braced girders the bracing acts as the chain, in others the continuous web.

248. Passing load.—For the investigation of the strains in a continuous girder it is necessary—first, to find the points of inflexion, and afterwards to calculate the strains in the separate segments on the principles already laid down for independent girders. A passing load complicates the question, for its effect is to alter the position of the points of inflexion, and consequently the lengths of the component segments; if, for instance, a passing train covers the left span, its deflection will be increased and that of the right span diminished, or even altogether removed, if the passing load be sufficiently heavy to lift the right end off the abutment **A′**. The effect of this partial loading on the points of inflexion will be to bring **B** nearer to, and remove **B′** farther from, the central pier, and this is that disposition of the load which gives the greatest length to the segment **AB**; it is necessary, therefore, in the case of a passing load to find this new position of the points of inflexion and calculate the strains in **A B** as an independent girder of this maximum length. Of course, the same calculations will suit **B′A′** when it is of maximum length, that is, when the right span only is loaded. The central segment, **B B′**, becomes of maximum length when the load is uniformly distributed over the whole girder, and the points of inflexion have to be determined under this condition of the load also. Having thus calculated the strength of each part when subject to the load which produces the maximum strain in the flanges of that part, we may assume that there is sufficient strength for any other disposition of the load, since the motion of the points of inflexion is restricted within these limits. The reaction of either abutment is equal to half the load on the adjacent segment; thus, the reaction of the left abutment equals half the load resting upon **A B**. The reaction of the pier equals the load resting upon the central segment, **B B′**, plus the sum of the reactions of the two abutments.

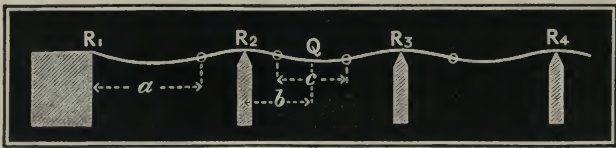
249. Experimental method of finding the points of inflexion—The depth of a girder does not affect the position of the points of inflexion.—The following method of finding the points of inflection depends partly on theory, partly on experiment, and is applicable to continuous girders containing any number of spans. Take a long rod of clean yellow pine or other suitable material to represent the continuous girder, and let it be supported at intervals corresponding to the spans of the real girder. Next, load this model uniformly all over, or each span separately, or in pairs, or make any other disposition of the load which can occur in practice. Now, it is clear that, if the model and its load be a tolerably accurate representation of the girder and its load, the points of inflection of the former will correspond with those of the latter; they might therefore be at once obtained by projecting the curves of the model on a vertical plane. It is difficult, however, to do this so as to determine the points of inflection with the requisite accuracy, for the exact place where the curvature alters is never very precisely defined to the eye. The pressures on the points of support may, however, be measured with considerable accuracy, taking the precaution of keeping them all in the same horizontal line, as a slight error in their level would seriously affect the curvature and lengths of the component segments. We shall assume therefore that the reactions of the points of support have been thus found experimentally.*

Let Fig. 87 represent a continuous girder containing any number of spans, each loaded uniformly, and let *o*,*o*,*o*, &c., represent

* It is a safe precaution to measure the pressures on the points of support with the rod turned upside down as well as erect, and then take the mean measurement as the true result.

successive points of inflection, the intervals between which are called segments.

Fig. 87.



Let R_1, R_2, R_3, &c. = the reactions of the successive points of support as found by experiment,

l, l', &c. = the lengths of the successive spans,

w, w', &c. = the loads per linear unit on each span,

a, b, c = the lengths of certain parts of the girder, as represented in the figure,

Q = the centre of the third segment.

R_1, the reaction of the left abutment, is equal to half the load on the first segment a, whence, $R_1 = \frac{aw}{2}$, and

$$a = \frac{2R_1}{w} \tag{159}$$

This equation gives the distance of the first point of inflexion from the left abutment, since R_1 is known from experiment.

R_2, the reaction of the first pier, is equal to the load resting on the girder as far as Q minus the reaction of the first abutment; that is, $R_2 = wl + w'b - R_1$, whence,

$$b = \frac{R_1 + R_2 - wl}{w'} \tag{160}$$

Again, taking moments round either flange at Q, which is now a known point, we have,

$$Fd = R_1(l + b) + R_2 b - wl\left(\frac{l}{2} + b\right) - \frac{w'b^2}{2}$$

in which F = the strain in either flange at Q, and d = the depth of the girder; but from eq. 25 we have,

$$Fd = \frac{w'c^2}{8}$$

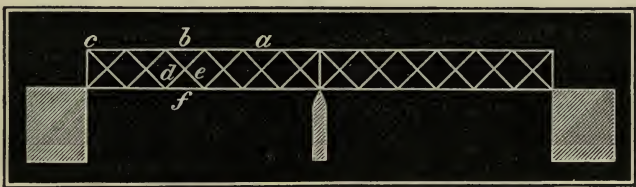
c being the length of the third segment, as marked in the figure; substituting this value for $\mathsf{F}d$ and arranging, we have,

$$c = \sqrt{\frac{8}{w'}\left\{\mathsf{R}_1(l+b) + \mathsf{R}_2 b - wl\left(\frac{l}{2}+b\right) - \frac{w'b^2}{2}\right\}} \quad (161)$$

The distance of the second point of inflexion from the first pier $= b - \frac{c}{2}$, and so on. It will be observed that the depth of the girder does not enter into these equations, and therefore does not affect the position of the points of inflexion.

250. Practical method of fixing the points of inflexion—Economical position of points of inflexion.—I shall here briefly describe a method by which the points of inflexion of braced girders may be fixed in any particular bay at will, so that there may be no uncertainty respecting their position, or so that they may, if desirable, be made to assume that position which is most advantageous for economy in the flanges.

Fig. 88.



Let Fig. 88 represent a continuous lattice girder capable of free horizontal motion on the points of support. Suppose that the point of inflexion, as determined by theory, is at a, but that it is desirable to fix it at b, that is, to make that part of the upper flange which lies between a and b subject to tension in place of compression. This may be effected by severing the flange at b, and lowering the end of the girder on the left abutment slightly, so as just to separate the parts at b. The left segment, cb, will then assume the condition of an independent girder supported at one extremity by the abutment and at the other by the oblique forces in diagonals d and e. The upper flange from c to b will undergo compression, from b to some corresponding point in the second span, tension. Further, the operation of fixing the point of inflexion in the upper flange determines its position in the lower one also, for, when the former is severed at *b*, the only horizontal forces acting upon the segment *cbf* are the strains in the lower flange at *f* and the horizontal component of the strains in diagonals *d* and *e*. This component must therefore be exactly equal and opposite to the strain at *f*, otherwise, the left segment, *cbf*, will move either to the right or left, since by hypothesis it is free to move horizontally on the abutment (**58**). Hence, it is evident that the point of inflexion in the lower flange is not far from *f*, probably not farther than the adjoining bay. Its position is determined by the condition that *the horizontal component of the strains in the diagonals intersected by a line joining the points of inflexion in the two flanges is equal to cipher.* Thus, by leaving any particular bay in one of the flanges of a continuous girder of two spans permanently severed, we have the point of inflexion in that span fixed under all conditions of the load; and when this is determined, we can find the strains in the flanges over the pier, and thence deduce the position of the point of inflexion in the second span. If the severed flange be united when any given load rests upon the girder, though the point of inflexion will move with every change of load, yet it will return to its original position whenever a similar load rests on the girder in the same position as when the flange was first severed.

If there be three spans, the central span may have both points of inflexion fixed independently of each other, and these again will determine the corresponding points in the side spans. The operation is safe in practice, as was proved at the Boyne Viaduct, where the points of inflexion in the centre span were fixed by severance in those bays in which theory had previously indicated their probable existence.* The most economical arrangement in theory for the flanges of a large girder of one span uniformly loaded consists in forming points of inflexion at the quarter-spans. In this case the end segments of the upper flange must be held back by land chains, as in suspension bridges, while those of the lower flange exert a

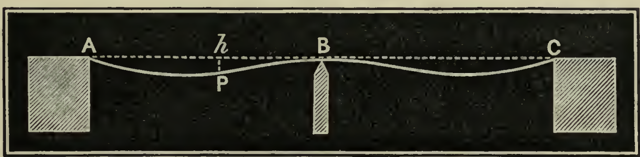
* See *Description of the Boyne Viaduct* in the Appendix.

horizontal thrust against the abutments like the flat arch (**216**). The two extreme segments of the girder thus form semi-girders, while the central segment is an independent girder suspended between them by the web.

The following theoretic investigations respecting continuous girders are based on the assumption that the material is perfectly elastic, and that the girder is of uniform section throughout its whole length.

CASE I.—CONTINUOUS GIRDERS OF TWO EQUAL SPANS, EACH LOADED UNIFORMLY THROUGHOUT ITS WHOLE LENGTH.*

Fig. 89.



251. Pressures on points of support—Points of inflexion—Deflection.—Let $l = \mathsf{AB} = \mathsf{BC} =$ the length of each span,

$w =$ the load per linear unit of AB,

$w' =$ the load per linear unit of BC,

$\mathsf{R}_1, \mathsf{R}_2, \mathsf{R}_3 =$ the reactions of the three points of support A, B and C, respectively,

$x = \mathsf{A}h =$ the horizontal distance of any point P from the left abutment,

$y = h\mathsf{P} =$ the deflection at that point,

$\mathsf{M} =$ the moment of resistance of the horizontal elastic forces at P (**59**),

$\beta =$ the inclination to the horizon of the tangent to the curve at B,

* See Mr. Pole's paper on the "Investigation of general formulæ applicable to the Torksey bridge," *Proc. Inst. C. E.*, Vol. ix., p. 261.

I = the moment of inertia of any cross section round its neutral axis, and consequently, a constant quantity throughout the whole length of the girder when the section of the latter is uniform from end to end,

E = the coefficient of elasticity.

The forces which hold the segment **AP** in equilibrium are the reaction of the left abutment, R_1; the load wx uniformly distributed over **AP**; the vertical shearing-strain at **P**, and the horizontal elastic forces at the same place. Taking the moments of these forces round the neutral axis at **P**, we have,

$$\mathsf{M} = \mathsf{R}_1 x - \frac{wx^2}{2} \tag{162}$$

Substituting for **M** its value in eq. 135,

$$\mathsf{EI}\frac{d^2y}{dx^2} = \frac{wx^2}{2} - \mathsf{R}_1 x.$$

Integrating this, and determining the constant by the consideration that $\frac{dy}{dx} = tan\beta$ when $x = l$, we have,

$$\mathsf{EI}\left(\frac{dy}{dx} - tan\beta\right) = \frac{w}{6}(x^3 - l^3) - \frac{\mathsf{R}_1}{2}(x^2 - l^2)$$

Integrating again, and determining the second constant by the consideration that $y = 0$ when $x = 0$, we have,

$$\mathsf{EI}\,(y - x\,tan\beta) = \frac{w}{6}\left(\frac{x^4}{4} - l^3x\right) - \frac{\mathsf{R}_1}{2}\left(\frac{x^3}{3} - l^2x\right) \tag{163}$$

which is the equation of the deflection curve from **A** to **B**.

At the point **B**, $x = l$ and $y = 0$; substituting these values in eq. 163, we have,

$$tan\beta = \frac{l^2}{24\,\mathsf{EI}}(3wl - 8\,\mathsf{R}_1) \tag{164}$$

Applying a similar process to the second span, and remembering that the angle β must in this case have a contrary sign, we have,

$$tan\beta = \frac{l^2}{24\,\mathsf{EI}}(8\,\mathsf{R}_3 - 3w'l) \tag{165}$$

Again, taking moments round **B**, we have,

$$\mathsf{R}_1 l - \frac{wl^2}{2} = \mathsf{R}_3 l - \frac{w'l^2}{2} \tag{166}$$

also,

$$\mathsf{R}_1 + \mathsf{R}_2 + \mathsf{R}_3 = (w + w')l \tag{167}$$

By solving these last four simultaneous equations we obtain the reactions of the points of support, as follows:—

$$\mathsf{R}_1 = \frac{7w - w'}{16} l \tag{168}$$

$$\mathsf{R}_2 = \frac{5}{8}(w + w')\, l \tag{169}$$

$$\mathsf{R}_3 = \frac{7w' - w}{16} l \tag{170}$$

At the points of contrary flexure the horizontal forces become cipher. Hence, the distance of the point of inflexion in the left span from **A** may be obtained from eq. 162, by making $\mathsf{M} = 0$ and substituting for R_1 its value in eq. 168, as follows:—

$$x = \frac{2\mathsf{R}_1}{w} = \frac{7w - w'}{8w} l \tag{171}$$

Similarly, the distance of the point of inflexion in the right span measured from **C**,

$$x' = \frac{2\mathsf{R}_3}{w'} = \frac{7w' - w}{8w'} l \tag{172}$$

The deflection y, in the left span, may be derived from eq. 163 by substituting for $\tan\beta$ its value in eq. 164, as follows:—

$$y = \frac{wx}{24\mathsf{EI}}\left\{x^3 - l^3 + \frac{4\mathsf{R}_1}{w}(l^2 - x^2)\right\} \tag{173}$$

The value of I for each form of cross section may be obtained from **71** and the succeeding articles by the aid of eq. 133.

The maximum strains in the flanges occur over the pier, and half way between the abutments and the points of inflexion, and when the latter are known, may be easily determined on the principles laid down in the second and fourth chapters for calculating the strains in independent girders; see eqs. 12 and 23 for girders with braced webs; or 70, 82 and 107 for girders with continuous webs.

252. Both spans loaded uniformly.—If both spans have the same load per running foot, $w = w'$, and we have

$$R_1 = R_3 = \frac{3}{8}wl \tag{174}$$

$$R_2 = \frac{5}{4}wl \tag{175}$$

The distance of each point of inflexion from the near abutment,

$$x = \frac{3}{4}l \tag{176}$$

Ex. The Torksey bridge is a continuous girder bridge in two equal spans, and was erected by Mr. Fowler to carry the Manchester, Sheffield and Lincolnshire Railway over the river Trent. Each span is 130 feet long in the clear, with a double line of railway between two double-webbed plate main girders with cellular top flanges. These main girders are 25 feet apart, with single-webbed plate cross-girders, 14 inches in depth and 2 feet apart, attached to the lower flanges. The extreme depth of each main girder is 10 feet. The depth from centre to centre of flanges is 9 feet $4\frac{5}{8}$ inches, or $\frac{1}{14}$th of each span. The gross sectional area of each top flange at the centre of each span is 51 inches, and the net area of each lower flange is about 55 inches. The thickness of each side of the web at the centre of each span is $\frac{1}{2}$ inch, increasing to $\frac{5}{8}$ inch at the abutments and central pier.

The load on each span of 130 feet was estimated as follows :—

	Tons.	Tons.
Rails and chairs, ...	8	
Timber platform, ...	15	
Cross-girders, ...	27	177
Ballast, 4 inches thick, ...	35	
Two main girders, ...	92	
Rolling load, as agreed upon by Mr. Fowler and Capt. Simmons (Government Inspector), ...		195
Total distributed load, ...		372 tons.

The strength of the Torksey bridge as a continuous girder was calculated by Mr. Pole from the following data :—

The length of each span = 130 feet = 1,560 inches.

The total distributed load on the first span = 400 tons, or for each girder 200 tons.

The distributed load on the second span = 164 tons, or for each girder 82 tons.

The coefficient of elasticity is taken equal to 10,000 tons for a bar one inch square.

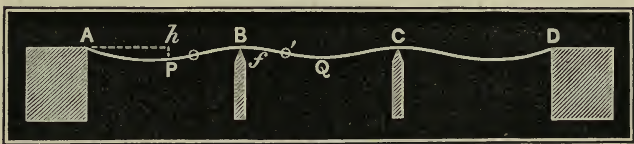
By eqs. 168, 169, and 170, the pressures of one main girder on the points of support are as follows :—

$$R_1 = \ \ 82{\cdot}375 \text{ tons.}$$
$$R_2 = 176{\cdot}250 \text{ tons.}$$
$$R_3 = \ \ 23{\cdot}375 \text{ tons.}$$

By eq. 171, the distance of the point of inflection in the loaded span is 22 feet 11 inches from the centre pier. The moment of inertia = 372,500 by Mr. Pole's calculation. The distance of the top plates from the neutral axis = 64 inches; that of the bottom plates from the same axis = 56 inches, and the maximum strains in the flanges of the longer segment, 107 feet long, are 4·55 tons compression per square inch of gross area in the top flange, and 4 tons tension per square inch of net area in the bottom flange. The deflection, with 222 tons distributed over one span, was 1·26 inches.

CASE II.—CONTINUOUS GIRDERS OF THREE SYMMETRICAL SPANS LOADED SYMMETRICALLY.*

Fig. 90.



253. Pressure on points of support—Points of inflexion—Deflection.—Let Q be the centre of the centre span,

AB = CD = l = the length of each side span,

AQ = nl,

w = the load per linear unit on each side span,

w' = the load per linear unit on the centre span,

R_1 = the reaction of either abutment, A or D,

R_2 = the reaction of either pier, B or C,

x = Ah = the horizontal distance of any point P from the left abutment.

y = hP = the deflection at this point,

M = the moment of resistance of the horizontal elastic forces at P (**59**),

* For the elegant investigation in **253** and **254** the author is indebted to William B. Blood, Esq., sometime Professor of Civil Engineering in Queen's College, Galway.

β = the inclination to the horizon of a tangent to the curve at **B** or **C**,

I = the moment of inertia of any cross section round its neutral axis, and consequently, a constant quantity throughout the whole length of the girder when the section of the latter is uniform,

E = the coefficient of elasticity.

It can be shown by the same process of reasoning as that adopted in **251** that the equation of equilibrium for any point **P** in the side span, **AB**, is

$$\mathsf{M} = \mathsf{R}_1 x - \frac{wx^2}{2} \tag{177}$$

whence, as before,

$$tan\beta = \frac{l^2}{24\mathsf{EI}}(3wl - 8\,\mathsf{R}_1) \tag{178}$$

The equation of equilibrium for any point in the centre span is

$$\mathsf{M} = \mathsf{R}_1 x + \mathsf{R}_2(x - l) - wl\left(x - \frac{l}{2}\right) - \frac{w'}{2}(x - l)^2 \tag{179}$$

Substituting for **M** its value in eq. 135,

$$\mathsf{EI}\frac{d^2y}{dx^2} = wl\left(x - \frac{l}{2}\right) + \frac{w'}{2}(x - l)^2 - \mathsf{R}_1 x - \mathsf{R}_2(x - l)$$

Integrating, and determining the constant by the consideration that $\frac{dy}{dx} = tan\beta$ when $x = l$, we have,

$$\mathsf{EI}\frac{dy}{dx} = \mathsf{EI}\;tan\beta + \frac{w}{2}lx(x - l) + \frac{w'}{6}(x - l)^3 - \frac{\mathsf{R}_1 + \mathsf{R}_2}{2}(x^2 - l^2) + \mathsf{R}_2 l(x - l) \tag{180}$$

which is the equation of the deflection curve from **B** to **C**.

Since $\frac{dy}{dx} = 0$ when $x = nl$, we have,

$$tan\beta = \frac{l^2}{\mathsf{EI}}\left\{-\frac{l}{2}\left(n(n-1)w + \frac{(n-1)^3}{3}w'\right) + \frac{n^2-1}{2}(\mathsf{R}_1 + \mathsf{R}_2) - (n-1)\,\mathsf{R}_2\right\} \tag{181}$$

also

$$\mathsf{R}_1 + \mathsf{R}_2 = l\left\{w + (n-1)\,w'\right\} \tag{182}$$

From eqs. 178, 181, and 182, we obtain the reactions of the points of support, as follows:—

$$\mathsf{R}_1 = l\,\frac{(1\cdot5n - 1\cdot125)\,w - (n-1)^3 w'}{3n-2} \tag{183}$$

$$\mathsf{R}_2 = l\,\frac{(1\cdot5n - 0\cdot875)\,w + (n^3 - 2n + 1)w'}{3n-2} \tag{184}$$

The distance of the point of inflexion in either side span from the abutment is obtained from eq. 177 by making $\mathsf{M} = 0$.

$$x = \frac{2\mathsf{R}_1}{w} \tag{185}$$

The distances of the points of inflexion in the centre span from **A** are obtained from eq. 179 by making $\mathsf{M} = 0$, substituting for R_1 its value in eq. 182, and solving the resulting quadratic, as follows:—

$$x = l\left\{n \pm \sqrt{n^2 - 1 + \frac{w}{w'} - \frac{2\mathsf{R}_2}{w'l}}\right\} \tag{186}$$

The equation for the deflection of the side spans is the same as eq. 173. That for the deflection at the centre of the centre span where $x = nl$, is obtained by integrating eq. 180 and determining the constant by the consideration that $y = 0$ when $x = l$, as follows:—

$$\mathsf{E\,I}\,y = \frac{wl^4}{12}(2n^3 - 3n^2 + 1) + \frac{w'l^4}{24}(n-1)^4 - \frac{\mathsf{R}_1 + \mathsf{R}_2}{6}\,l^3(n^3 - 3n + 2)$$
$$+ \frac{\mathsf{R}_2 l^3}{2}(n-1)^2 + \mathsf{E\,I}\,tan\beta l\,(n-1) \tag{187}$$

The value of **I** for each form of cross section may be obtained from **71** and the following articles by the aid of eq. 133.

254. Three spans loaded uniformly.—If the girder be loaded uniformly throughout the three spans, $w = w'$, and the pressures on the point of support become

$$\mathsf{R}_1 = wl\left\{\frac{n^3 - 3n^2 + \frac{3n}{2} + 0\cdot125}{2 - 3n}\right\} \tag{188}$$

$$\mathsf{R}_2 = wl\left\{\frac{n^3 - \frac{n}{2} + 0\cdot125}{3n - 2}\right\} \tag{189}$$

The distance of the point of inflexion in each side span from the abutment is as before:—

$$n = \frac{2\mathbf{R}_1}{w} \tag{190}$$

The distances of the points of inflexion in the centre span from **A** are as follows:—

$$x = l\left\{n + \sqrt{n^2 - \frac{2\mathbf{R}_2}{wl}}\right\} \tag{191}$$

If the radicle in eqs. 186 or 191 vanish, there will be no strain at **Q**, and the centre span will be cambered throughout. If the value of $\mathbf{R}_1$ in eqs. 183 or 188 be negative, the ends of the girder will be lifted off the abutments, owing to the excess of load on the centre span.*

255. Maximum strains in flanges.—The maximum strains in the flanges occur as follows:—in the side spans when the passing load covers both side spans, leaving the centre span free from load; in the centre span, when the passing load covers it alone, leaving both side spans free from load; and over either pier, when the passing load covers the centre span and the adjacent side span, leaving the remote side span free from load. When the lengths of the component segments are determined, the strains in the flanges may be calculated by eqs. 12 and 23 if the girders are diagonally braced, or by eqs. 70, 82 and 107 if they are plate girders. The hypothesis of the load being symmetrically disposed on either side of the centre prevents us from finding the points of inflexion when the segment over either pier is of maximum length; we have, however, a close approximation to its maximum length in the case of a passing load covering all three spans, and if desirable, a small extra allowance may be made for greater security. When the maximum length of the segment over either pier is thus determined, the calculation for the strains in its flanges are made as indicated in previous chapters, recollecting that each of these pier segments supports not only its own proper load, but also the weight of half the adjoining segments with their load, suspended from its extremities by the vertical web.

* The reader is referred to the description of the Boyne lattice bridge in the Appendix for a practical example of the application of the foregoing formulæ.

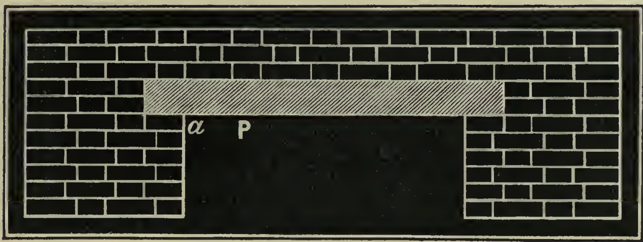
256. Maximum strains in web—Ambiguity in calculation.— Though we obtain by these means the maximum strains of either kind to which the flanges are subject, it does not follow that we have also got the maximum strains in the web. Let *o*, for example, in Fig. 90, be the point of inflexion when the segment A*o* is of maximum length. Now this segment does not remain of this maximum length while a train is passing from A to B, that is, while the maximum strains are being produced in the web of A*o*; the point of inflexion is much closer to A when the train first comes upon the bridge (especially if the centre span happens to be traversed at the same time by another train), and gradually moves forward towards B as the train advances. It is incorrect therefore to calculate the maximum strains in the web on the hypothesis that A*o* is the length of the segment while the load advances. The maximum strain in a diagonal, at P for instance, takes place when the load covers A P, but the point of inflexion is then really nearer A than the point *o* is, and the maximum strain in the diagonal at P is therefore greater than if we assume the segment constant in length during the advance of the train. A similar or even greater uncertainty occurs in the centre span, for there neither end of the segment is fixed.

257. Permanent load, shearing-strain.—When a continuous girder supports a fixed load, the strains in the web are not modified at the points of inflexion. The horizontal strains in the flanges change from tension to compression, or *vice versâ*, at these points, but the vertical or diagonal strains are transmitted through the web just as if no points of inflexion existed. The effect of contrary flexure is merely this; the horizontal increments of strain developed in the flanges pull from the piers in place of thrusting towards the centres of the component segments, and *vice versâ*. Hence, when a continuous girder of three, five, or any uneven number of spans, is symmetrically loaded, the strains throughout the web of the centre span are the same as if the centre span were an independent girder supported at its extremities. This perhaps will be made clearer from the consideration that the shearing-strain at any section in the centre span, when the points of inflexion are symmetrical, is equal to the weight between the section and the centre of the span, and this is the case whether there be any point of inflexion or not. Thus, the shearing-strain at any point f, Fig. 90, is equal to the load on fo' + that on $o'\mathbf{Q}$; but if the central span were an independent girder, resting on abutments at **B** and **C** and uniformly loaded, the shearing-strain at f would equal the load on $f\mathbf{Q}$, that is, it would be the same as before.

258. Advantages of continuity—Not desirable for small spans with passing loads, or where the foundations are insecure.—The advantage of continuity arises from two causes; first, from the smaller amount of material required in the flanges; secondly, from the removal of a certain portion of their weight from the central part of each span to a position nearer the piers. The latter is but a trifling advantage in continuous girders of moderate spans, say under 150 feet, which support heavy passing loads, for the part so removed forms but a small proportion of the total weight. In the case of a fixed load, however, the saving from this cause is considerable; but when the load is a passing train the advantages of continuity are liable to be over-rated, especially in girders of small spans, for on a little reflection it will be evident that, when the points of inflexion move under the influence of the passing load, a greater amount of material is required than if their position remained stationary, and this moreover introduces the necessity of providing for both tension and compression in those parts of the flanges which lie within the range of the points of inflexion; this latter objection is perhaps of little consequence when wrought-iron is the material employed. A subsidence of any of the points of support of a continuous girder will cause a change of strain whose amount it is quite impossible to foresee, and which may seriously injure the structure or perhaps render it dangerous. Hence, continuous girders should be avoided where the foundations of the piers are insecure. In bridges of large span, where the permanent load constitutes the greater portion of the whole weight, the advantage of continuity is very considerable. The position of each point of inflexion alters but little with a passing load, and a considerable portion of the permanent weight, which would otherwise rest at, or near the centre, of each span, is brought close to the points of support.

CASE III.—GIRDERS OF UNIFORM SECTION IMBEDDED AT BOTH ENDS AND LOADED UNIFORMLY.

Fig. 91.



259. Strain at centre theoretically one-third, and strength theoretically once and a half, that of girders free at the ends.—When both ends of a girder are built into a wall so as to be rigidly imbedded there, the tangent to the girder at its intersection with the wall is horizontal, and the strains closely resemble those which occur in the centre span of a continuous girder of three spans when the load is so disposed that the tangents over the piers are horizontal.

Let $l =$ the span from wall to wall,

$w =$ the load per linear unit,

$\mathsf{M}' =$ the moment of resistance of the horizontal elastic forces at the intersection of the girder with the wall (**59**),

$\mathsf{M} =$ the moment of resistance of the horizontal elastic forces at any cross section P,

x and $y =$ the co-ordinates of P, measured from a as origin,

$\mathsf{I} =$ the moment of inertia of any cross section round its neutral axis,

$\mathsf{E} =$ the coefficient of elasticity.

Taking moments round P (eq. 135),

$$\mathsf{M} = -\mathsf{E}\,\mathsf{I}\,\frac{d^3y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2} - \mathsf{M}' \qquad (192)$$

Integrating, and determining that the constant $= 0$ from the consideration that $\frac{dy}{dx} = 0$ when $x = 0$,

$$\mathsf{E}\,\mathsf{I}\,\frac{dy}{dx} = \frac{wx^3}{6} - \frac{wlx^2}{4} + \mathsf{M}'x$$

Making $x = l$, we have $\frac{dy}{dx} = 0$, and

$$\mathbf{M}' = \frac{wl^2}{12}$$

Substituting this value in eq. 192, we have,

$$\mathbf{M} = -\frac{w}{2}\left(x^2 - lx + \frac{l^2}{6}\right)$$

At the points of inflexion, $\mathbf{M} = 0$, and we have $x^2 - lx + \frac{l^2}{6} = 0$, whence,

$$x = l\left(\frac{1}{2} \pm \frac{1}{\sqrt{12}}\right) = \cdot 211l \text{ or } \cdot 789l \tag{193}$$

The length of the middle segment $= \cdot 578l$, and if the girder be a flanged girder, the central strain in either flange (eq. 25) $= \frac{(\cdot 578)^2 wl^2}{8d} = \frac{\cdot 334wl^2}{8d} = \frac{wl^2}{24d}$, in which $d =$ the depth of the girder. This central strain is just $\frac{1}{3}$rd of what it would be were the ends merely resting on the wall, in place of being built therein. From eq. 12, we find that the strain in either flange at the wall $= \frac{wl^2}{12d}$, which is just double the strain at the centre of the flanges, and $\frac{2}{3}$rds of what would be the central strain from the same load if the girder were merely resting on the walls. From this it follows, that the strength of a girder of uniform section imbedded firmly at both ends and loaded uniformly is theoretically once and a half that of the same girder merely supported at the ends, and that the points of greatest strain are at the intersections with the wall.

CASE IV.—GIRDERS OF UNIFORM SECTION IMBEDDED AT BOTH ENDS AND LOADED AT THE CENTRE.

260. Strain at centre theoretically one-half, and strength theoretically twice, that of girders free at the ends.— Let $\mathbf{W} =$ the load at the centre of the girder, and let the other symbols remain as before.

Taking moments round $\mathbf{P}$ (eq. 135),

$$\mathbf{M} = -\mathbf{EI}\frac{d^2y}{dx^2} = \frac{\mathbf{W}}{2}x - \mathbf{M}' \tag{194}$$

Integrating, and determining that the constant = 0 from the consideration that $\frac{dy}{dx} = 0$ when $x = 0$,

$$\mathbf{EI}\frac{dy}{dx} = \mathbf{M}'x - \frac{\mathbf{W}}{4}x^2$$

Making $x = \frac{l}{2}$, we have $\frac{dy}{dx} = 0$, and

$$\mathbf{M}' = \frac{\mathbf{W}l}{8}$$

Substituting this value in eq. 194, we have,

$$\mathbf{M} = \frac{\mathbf{W}}{2}\left(x - \frac{l}{4}\right)$$

At the points of inflexion $\mathbf{M} = 0$, and we have their distance from the walls,

$$x = \frac{l}{4} \qquad (195)$$

The length of the middle segment $= \frac{l}{2}$, and its central strain is just $\frac{1}{2}$ of what it would be if the ends of the girder were not imbedded in the wall but merely resting thereon. The strain at the wall also is equal to the central strain; consequently, the strength of a girder of uniform section imbedded firmly at both ends and loaded at the centre is theoretically twice that of the same girder merely supported at the ends. Mr. Barlow's experiments on timber, however, do not corroborate this theory, as he found the strength of an imbedded beam loaded at the centre to be only $1\frac{1}{2}$ times that of a free beam, and fracture always took place at the centre, the ends being comparatively little strained.* Our theory is doubtless defective in supposing that the horizontal fibres at the wall are in the same state of strain as if the girder were really a continuous girder in three spans, for in the latter case the girder is bent downwards in each of the side spans, whereas, when imbedded in the walls, the ends which correspond to these side spans are horizontal, and consequently, the points of inflexion are really nearer to the walls than in a truly continuous girder.

* *Strength of Materials*, pp. 32, 136.

CHAPTER X.

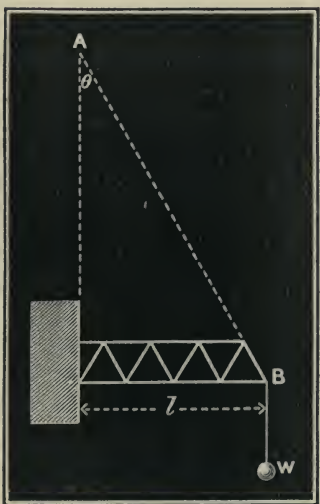
QUANTITY OF MATERIAL IN BRACED GIRDERS.

CASE I.—SEMI-GIRDERS LOADED AT THE EXTREMITY, ISOSCELES BRACING.

261. Web.

Let W = the weight at the extremity,
l = the length of the semi-girder,
d = its depth,
θ = the angle the diagonals make with a vertical line,
f = the unit-strain,
Q = the cubical quantity of material in the diagonals,
Q' = the cubical quantity of material in either flange.

Fig. 92.



The cubical quantity of material required for the diagonal bracing is equal to the sum of the products of the length and section of each brace. When the triangles are isosceles and the load is a single weight, the section, if proportional to the strain, is the same for all the diagonals, and the quantity of material is therefore equal to the product of their aggregate length by their common section. The line $\overline{\mathsf{AB}}$, Fig. 92, is equal in length to the sum of the several diagonals; expressing its length in terms of l and θ, we have

$$\overline{\mathsf{AB}} = l.cosec\theta$$

The section of each brace is equal to the total strain passing through it divided by the unit-strain, $= \frac{\mathbf{W} sec\theta}{f}$ (eq. 110). Multiplying this by the foregoing value for the length, we have,

$$\mathbf{Q} = \frac{\mathbf{W}l}{f} sec\theta \,.\, cosec\theta \tag{196}$$

262. Flanges.—The quantity of material in the flanges is most conveniently deduced from the principles stated in Chapter II. as follows:—The sectional area of either flange at the wall $= \frac{\mathbf{W}l}{df}$ (eq. 7), and when the girder is of uniform strength gradually diminishes towards the extremity as the ordinates of a triangle (**20**). Hence, the quantity of material in one flange equals its sectional area at the wall multiplied by $\frac{l}{2}$, and we have,

$$\mathbf{Q}' = \frac{\mathbf{W}l^2}{2df} \tag{197}$$

CASE II.—SEMI-GIRDERS LOADED UNIFORMLY, ISOSCELES BRACING.

263. Web, length containing a whole number of bays.—
Let $\mathbf{W}$ = the total weight resting on the girder,

n = the number of bays in the longest flange, supposed a whole number, and the other symbols as in Case I.

When the bracing is formed of isosceles triangles the length of one bay equals $2d.tan\theta$, whence,

$$l = 2nd.tan\theta. \tag{198}$$

The quantity of material that the weight at any given apex would require in the bracing, if it alone were supported by the girder, may be obtained from eq. 196 by substituting for $\mathbf{W}$ and l the load resting on the apex $\left(= \frac{\mathbf{W}}{n}\right)$, and the distance of the weight from the wall. The quantity required for the whole load is equal to the sum of the quantities required for the separate weights. Hence, recollecting that the weight on the last apex equals half that on each of the other apices (**144**), we have, when there is no half bay in the length, that is, where n is a whole number,

$$\mathsf{Q} = \frac{\mathsf{W}}{fn} 2d.tan\theta \left\{ (1 + 2 + 3 + \ldots n) - \frac{n}{2} \right\} sec\theta.cosec\theta$$

$$= \frac{\mathsf{W}}{f} nd.tan\theta.sec\theta.cosec\theta.$$

Substituting for $nd.tan\theta$ its value in eq. 198, we have,

$$\mathsf{Q} = \frac{\mathsf{W}l}{2f} sec\theta.cosec\theta \qquad (199)$$

264. Web, length containing a half-bay.—When the length contains a half-bay, the quantity of material in the bracing, derived from eq. 196,

$$\mathsf{Q} = \frac{\mathsf{W}l}{2f} sec\theta.cosec\theta + \frac{\mathsf{W}d^2}{2fl} sec^2\theta.tan\theta. \qquad (200)$$

265. Flanges.—From eq. 11 the area of either flange at the wall $= \frac{\mathsf{W}l}{2fd}$, and diminishes towards the extremity as the ordinates of a parabola, but from the well-known properties of the parabola the area of **A B C**, Fig. 7, equals one-third of the circumscribed rectangle. Hence, the quantity of material in either flange equals its area at the wall multiplied by $\frac{l}{3}$, that is,

$$\mathsf{Q}' = \frac{\mathsf{W}l^2}{6fd} \qquad (201)$$

CASE III.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED AT AN INTERMEDIATE POINT, ISOSCELES BRACING.

266. Quantity of material in the web is the same for each segment.—Let W = the weight resting on the girder,

l = its length, and the other symbols as in Case I.

Let the weight divide the girder into segments containing respectively m and n linear units, as in Fig. 52. The strains throughout the girder will in no respect be altered if we conceive it inverted, resting on a pier at W, and loaded with $\frac{m}{l}\mathsf{W}$ at the right extremity, and with $\frac{n}{l}\mathbf{W}$ at the left. Each segment will then become a semi-girder loaded at its extremity. Hence, the quantity of material in the bracing of each segment $= \frac{mn\mathbf{W}}{fl} sec\theta . cosec\theta$ (eq. 196). The quantity of the material in the bracing of both segments together is equal to twice this, that is,

$$\mathbf{Q} = \frac{2mn\mathbf{W}}{fl} sec\theta . cosec\theta \tag{202}$$

If the weight be at the centre, equation 202 becomes

$$\mathbf{Q} = \frac{\mathbf{W}l}{2f} sec\theta . cosec\theta \tag{203}$$

267. Flanges.—From eq. 20, the sectional area of either flange at the point where the weight rests $= \frac{mn\mathbf{W}}{fdl}$, and diminishes gradually towards each extremity as the ordinates of a triangle (**35**). Hence, the quantity of material in one flange equals its area at the weight multiplied by $\frac{l}{2}$, and we have,

$$\mathbf{Q}' = \frac{mn\mathbf{W}}{2fd} \tag{204}$$

If the weight be at the centre, eq. 204 becomes,

$$\mathbf{Q}' = \frac{\mathbf{W}l^2}{8fd} \tag{205}$$

CASE IV.—GIRDERS SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY, ISOSCELES BRACING.

268. Web, length containing an even number of bays.—
Let $\mathbf{W}$ = the total weight on the girder,
l = the length, and the other symbols as in Case I.

In order to avoid unnecessary minuteness in this case I shall first assume that the number of bays in the half-length is a whole number, in other words, that the length contains an even number of bays. Let us consider each half of the girder by itself; the vertical forces which act upon each half are the upward reaction of its abutment, and the downward pressure of the weights between the abutment and the centre. The former pressure, if acting alone, would require a certain amount of material for the bracing, obtained by eq. 196, while the weights, leaving the reaction of the abutment out of consideration, would require an amount of material which may be obtained from eq. 199. The latter forces tend to relieve the strain produced by the reaction of the abutment; consequently, the true quantity of material required is equal to the difference of the amounts which would be required were each set of forces to act independently of the other. Hence, subtracting eq. 199 from 196, and bearing in mind that W and l have twice the value they had in the semi-girder, we have the quantity of material in the web of the whole girder,

$$\mathsf{Q} = \frac{\mathsf{W}l}{4f} \sec\theta . \operatorname{cosec}\theta \tag{206}$$

that is, half the quantity that would be required if all the weight were concentrated at the centre.

269. Web, the length containing an odd number of bays.—If the half-length contain a half-bay, the quantity of material in the bracing is obtained by subtracting eq. 200 from eq. 196, that is,

$$\mathsf{Q} = \frac{\mathsf{W}l}{4f} \sec\theta . \operatorname{cosec}\theta - \frac{\mathsf{W}d^2}{fl} \sec^2\theta . \tan\theta \tag{207}$$

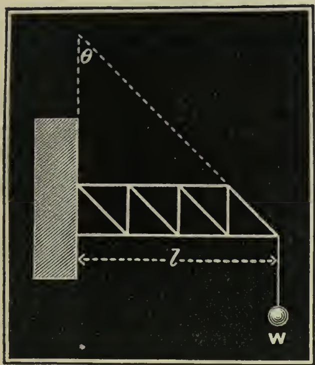
270. Flanges.—From eq. 25 the sectional area of either flange at the centre of the girder $= \frac{\mathsf{W}l}{8fd}$, and diminishes towards either end as the ordinates of a parabola (**47**). But the area of Fig. 23 equals two-thirds of the circumscribed rectangle; hence, the quantity of material required for either flange equals its central section multiplied by $\frac{2}{3}l$, and we have,

$$\mathsf{Q}' = \frac{\mathsf{W}l^2}{12fd} \tag{208}$$

which is two-thirds of the quantity that would be required if all the weight were concentrated at the centre.

CASE V.—SEMI-GIRDERS LOADED AT THE EXTREMITY, VERTICAL AND DIAGONAL BRACING.

Fig. 93.



271. Web.—When every alternate brace is vertical, as in Fig. 93, we must divide the material in the web into two parts, namely, that in the vertical, and that in the diagonal bracing.

Let $\mathbf{Q}$ = the quantity of material in the diagonals,

$\mathbf{Q}''$ = the quantity of material in the verticals, and the other symbols as before.

The quantity of material required for the diagonal bracing is as before (eq. 196),

$$\mathbf{Q} = \frac{\mathbf{W}l}{f} \sec\theta \,.\, \operatorname{cosec}\theta \tag{209}$$

The strain transmitted through each vertical $= \mathbf{W}$; hence, its sectional area $= \frac{\mathbf{W}}{f}$. Multiplying this by the aggregate length of the verticals ($= l.\cot\theta$), we have,

$$\mathbf{Q}'' = \frac{\mathbf{W}l}{f} \cot\theta. \tag{210}$$

CASE VI.—BOWSTRING GIRDERS UNIFORMLY LOADED.

272. Flanges.—When a bowstring girder is uniformly loaded, the strains are nearly uniform and equal throughout both flanges (**210**); hence, we can find a close approximation to the quantity of material by multiplying the length of each flange by its sectional area.

Let $\mathbf{W}$ = the total weight uniformly distributed over the girder,

l = the length of the string,

nl = the length of the bow,

$d =$ the depth of girder at the centre,
$\mathsf{Q}' =$ the quantity of material in the string,
$\mathsf{Q}'' =$ the quantity of material in the bow,
$f =$ the unit-strain.

The strain at the centre of either flange $= \dfrac{\mathsf{W}l}{8d}$ (eq. 25); hence, the sectional area of the flange $= \dfrac{\mathsf{W}l}{8df}$; multiplying this latter quantity by the respective lengths of the string and bow, we have

$$\mathsf{Q}' = \frac{\mathsf{W}l^2}{8df} \tag{211}$$

$$\mathsf{Q}'' = \frac{n\mathsf{W}l^2}{8df} \tag{212}$$

273. The following table contains the corresponding values of $\dfrac{d}{l}$ and n, the depth being expressed in fractional parts of the length

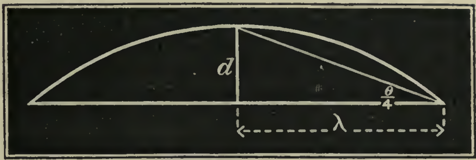
$\frac{d}{l}$	n
$\frac{1}{4}$	1·158
$\frac{1}{6}$	1·073
$\frac{1}{8}$	1·040
$\frac{1}{10}$	1·027
$\frac{1}{12}$	1·019
$\frac{1}{14}$	1·014
$\frac{1}{16}$	1·010

n, or the ratio of the length of the bow to the length of the string, is thus found.

Let $\lambda =$ the half span $= \dfrac{l}{2}$,

$r =$ the radius of the bow,
$\theta =$ the angle the bow subtends at the centre of the circle.

Fig. 94.



$$n = \frac{\text{length of bow}}{l} = \frac{r\theta}{2\lambda} \qquad (a)$$

also, $$(2r - d)\, d = \lambda^2$$

whence, $$r = \frac{\lambda^2 + d^2}{2d}$$

again, $$\frac{d}{\lambda} = tan\,\frac{\theta}{4}$$

whence, $$\theta = 4 tan^{-1}\frac{d}{\lambda}$$

Substituting in eq. (a) these values for r and θ, we have,

$$n = \frac{\lambda^2 + d^2}{d\lambda}.\,tan^{-1}\frac{d}{\lambda} = \left(\frac{\lambda}{d} + \frac{d}{\lambda}\right).\,tan^{-1}\frac{d}{\lambda} \qquad (213)$$

whence we can obtain the values of n corresponding to different values of $\frac{d}{l}$.

274. Quantity of material in the bracing independent of depth—Weights of railway girders up to 200 feet span are nearly as the squares of their length.—The reader will observe that the depth of the girder does not enter into those equations which express the quantity of material required in the bracing, whereas it enters into the denominator of those which express the quantity of material in the flanges. Hence, we conclude that altering the depth of braced girders does not affect the amount of bracing (**18**); but the quantity of material in the flanges varies inversely as the depth, and consequently, the deeper a girder is made the greater will be the economy, theoretically speaking. In practice, the additional material required to stiffen long struts generally defines the limit to which this increase of depth can be judiciously extended; but of this in succeeding chapters.

It will also be observed that, when the ratio of depth to length is constant, the quantity of material varies as $\mathbf{W}l$, or if $\mathbf{W}$ varies as l, as l^2. Consequently, when such girders are of small weight compared to the load, and when the latter is proportional to the length, the weight of the girders will vary very nearly as the square of their length—which rule is approximately true for railway girders up to 200 feet span.

CHAPTER XI.

ANGLE OF ECONOMY.

275. Angle of Economy for Isosceles bracing is 45°.—On examining those equations in the last chapter which express the quantity of material required for the vertical web of girders whose bracing consists of isosceles triangles, we find that they may all be expressed by one general equation,

$$\mathsf{Q} = \mathsf{K} sec\theta . cosec\theta$$

in which K for each case is a constant quantity depending upon the length, weight, and unit-strain. Q is therefore proportional to the variable quantity $sec\theta . cosec\theta$, or to its equivalent, $\frac{2}{sin 2\theta}$, which is a minimum when $\theta = 45°$. This proves that the angle of 45° is the most economical inclination for the diagonals of isosceles bracing, and it is to be observed that certain of the diagonals being in compression, and therefore practically requiring a greater amount of material to stiffen them than others, does not materially affect this conclusion; for, let the compression diagonals take m times the quantity of material they would require on the supposition that they were subject to tension in place of compression, then, since every alternate diagonal is in compression when the load is stationary, the foregoing expression becomes

$$\mathsf{Q} = \frac{m+1}{2} \mathsf{K} sec\theta . cosec\theta$$

but the variable part of this expression is $sec\theta . cosec\theta$ as before, and therefore the angle of economy is 45°.*

276. Angle of economy for vertical and diagonal bracing is 55°.—The angle of economy in girders with vertical and diagonal bracing differs from that in girders whose webs are formed of isosceles triangles. From eqs. 209 and 210, we find that the quantity of material in the bracing may be expressed as follows:—

$$\mathsf{Q} + \mathsf{Q}'' = \mathsf{K} (sec\theta . cosec\theta + cot\theta).$$

* Mr. Bow first drew attention to the fact that 45° is the angle of economy for isosceles bracing; see his *Treatise on Bracing*. Edinburgh, 1851.

It is necessary to equate the differential coefficient of the bracketed part of this equation to cipher in order to find the value of θ which makes $\mathbf{Q} + \mathbf{Q}''$ a minimum. Doing so, we have,

$$cosec\theta.sec\theta.tan\theta - sec\theta.cosec\theta.cot\theta - cosec^2\theta = 0,$$

dividing by $cosec\theta.sec\theta$ and transposing,

$$tan\theta = 2cot\theta$$

whence,

$$tan\theta = \sqrt{2}, \text{ and } \theta = 54° \ 44' \ 8{\cdot}2'' = 55° \text{ nearly},$$

which therefore is the angle of economy for this form of bracing, and has moreover the merit of forming lozenge-shaped openings, which have a more agreeable appearance than square ones.

277. Isosceles more economical than vertical and diagonal bracing.—The superior economy of the isosceles over the vertical and diagonal system of bracing will be now apparent, for the quantity of material required in the latter exceeds that in the former by an amount never less than $\mathbf{Q}''$, and exceeds $\mathbf{Q}''$ when θ differs from 45°.

278. Trigonometrical functions of θ.—The following table contains the value of different trigonometrical functions of θ.

Angle of bracing, θ.	$sec\theta$.	$sec\theta.cosec\theta$.	$cot\theta$.	$sec\theta.cosec\theta + cot\theta$.	$tan\theta$.
20°	1·064	3·11	2·747	5·857	·364
25°	1·103	2·61	2·144	4·754	·466
30°	1·154	2·31	1·732	4·041	·577
35°	1·221	2·13	1·428	3·557	·700
40°	1·305	2·03	1·192	3·222	·839
45°	1·414	2·00	1·000	3·000	1·000
50°	1·515	2·03	·839	2·869	1·192
55°	1·743	2·13	·700	2·829	1·428
60°	2·000	2·31	·577	2·886	1·732
65°	2·369	2·61	·466	3·076	2·144
70°	2·924	3·11	·364	3·474	2·747

279. Relative economy of different kinds of bracing—Continuous web theoretically twice as economical as a braced web.—By means of this table we can at once compare the relative economy of different descriptions of bracing as follows:—

Values of θ.	Value of Q.	Comparative quantities of material required in web.
Isosceles bracing, - - $\theta = 45°$	$Q = 2{\cdot}00\ K$	100
Ditto (Warren's girder), - $\theta = 30°$	$Q = 2{\cdot}31\ K$	115·5
Vertical and diagonal bracing, $\theta = 55°$	$Q \times Q'' = 2{\cdot}83\ K$	141·5

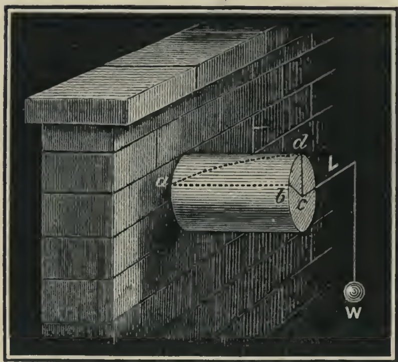
From this it appears, that equilateral bracing ("Warren's girder") requires 15½ per cent., and vertical and diagonal bracing of the best form requires 41½ per cent., more material in the web than isosceles bracing at an angle of 45°.

If we compare equations 203 and 206 with the equations in **54** which represent the theoretic quantity of material in a continuous web, we find that the most economical form of braced web, namely, isosceles bracing at an angle of 45°, requires just double the quantity of material that the continuous web requires if made as thin as theory alone would indicate. In practice, however, the braced web is generally the most economical, as will be shown hereafter in the chapter on the web.

CHAPTER XII.

TORSION.

Fig. 95.



280. Twisting moment.—Let one end of a horizontal shaft be rigidly fixed and let the free end have a lever, **L**, attached at right angles to the axis. A weight, **W**, hung at the end of this lever, will twist the shaft round its axis and fibres, such as *ab*, originally longitudinal and parallel to the axis, will now assume a spiral form, *ad*, like the strands of a rope. Radial lines, such as *cb*, in any cross section, will also have moved through a certain angle, *bcd*, which experiments prove to be proportional,

1°. to *ab*, the distance of the section from the fixed end,

2°. to **L**, the length of the lever,

3°. to **W**, the weight,

provided the shaft be not twisted beyond its limit of elastic reaction. If we consider any two consecutive transverse sections of the shaft, we find that the one more remote from the fixed end will be twisted round a little in advance of the other, and this movement tends to wrench asunder the longitudinal fibres by one of the sections sliding past the other. This wrenching action, it will be observed, closely resembles shearing from transverse pressure

(**14**). It is clear that, the farther the fibres are from the axis the greater will be the arc through which they are twisted, and the greater, therefore, will be their elastic resistance to wrenching, and the greater also will be the leverage which they will exert, and we may conceive, at least in shafts of circular, polygonal, or square sections, the elastic reactions of the fibres replaced by a resultant equal to their sum and applied in a linear ring round the axis, whence, we have the *twisting moment* of the weight,

$$\mathsf{W\,L} = \mathsf{F}\delta$$

where F = the annular resultant of all the elastic reactions,
δ = the mean distance of this annular resultant from the axis of the shaft.

F is proportional in shafts of different sizes, but similar in section, to the number of fibres in the cross section, that is, in solid shafts to the square of the diameter, and δ is evidently proportional to the diameter. Hence, we obtain the following relations.

281. Solid round, square, or polygonal shafts—Coefficient of torsional rupture, T.—

$$\mathsf{W} = \frac{\mathsf{T}d^3}{\mathsf{L}} \tag{214}$$

$$d = \sqrt[3]{\frac{\mathsf{WL}}{\mathsf{T}}} \tag{215}$$

where W = the breaking weight by torsion,
L = the length of the lever, measured from the centre of the shaft,
d = the diameter of the shaft, if round; or its breadth, if square or polygonal,

and T is a constant, which must be determined for each material by finding experimentally the breaking weight of a shaft of known dimensions and similar in section to that whose strength is required. The constant, T, may be called the *Coefficient or modulus of torsional rupture* of that particular material and section from which it is derived, and equals the breaking weight of a shaft of similar section in which the quantity $\frac{d^3}{\mathsf{L}} = 1$.

282. Hollow shafts of uniform thickness.—The number of fibres in the cross section of a hollow shaft is proportional to the product of the diameter by the thickness, and we have,

$$W = \frac{Td^2t}{L} \qquad (216)$$

where $t =$ the thickness of the tube and the other symbols are as before.

283. Coefficients of torsional rupture for solid round shafts.—The following table contains the values of T, or the coefficients of torsional rupture, for solid round shafts; these are the breaking weights of shafts one inch in diameter and whose length, L, is also one inch; hence, in using these coefficients in the preceding equations, all the dimensions should be in inches.

COEFFICIENTS OF TORSIONAL RUPTURE FOR SOLID ROUND SHAFTS.

MATERIAL.	Initial of Experimenter.	Value of T in lbs.
Cast-iron, - - - - - - -	D	5,400
Wrought-iron, - - - - - -		9,800
Steel, Bessemer, - - - - - -	K	15,000
Do., Crucible, hammered, - - - -	K	17,000
Ash, - - - - - - - -	B	274
Elm, - - - - - - - -	B	274
Larch, - - - - - - - -	B	190 to 333
Oak, - - - - - - - -	B	451
Red Pine, - - - - - - -	B	98 to 157
Spruce Fir, - - - - - - -	B	118

B. Bouniceau, *Rankine's Machinery and Millwork*, p. 479.
D. Dunlop, *Tredgold on the Strength of Cast-iron*, p. 99.
K. Kirkaldy, *Experiments on Steel and Iron by a Committee of Civil Engineers.*

Ex. 1. From experiments made by Mr. Kirkaldy for a "Committee of Civil Engineers," it appears that 3,300 lbs. at the end of a 12-inch lever will twist asunder a round bar of Bessemer steel 1·382 inch in diameter; what is the value of T?

$$\text{Here,}\quad \mathsf{W} = 3{,}300 \text{ lbs.},$$
$$\mathsf{L} = 12 \text{ inches},$$
$$d = 1{\cdot}382 \text{ inches}.$$

Answer (eq. 214). $\mathsf{T} = \frac{\mathsf{WL}}{d^3} = \frac{3{,}300 \times 12}{\overline{1{\cdot}382}|^3} = 15{,}005 \text{ lbs.}$

Ex. 2. What should be the diameter of a wrought-iron screw-propeller shaft, the length of the crank being 13 inches and the pressure 15,000 lbs., taking 8 as the factor of safety?

$$\text{Here,}\quad \mathsf{W} = 15{,}000 \text{ lbs.},$$
$$\mathsf{L} = 13 \text{ inches},$$
$$\mathsf{T} = 9{,}800 \text{ lbs.}$$

Answer (eq. 215). $d = \sqrt[3]{\frac{8\mathsf{WL}}{\mathsf{T}}} = \sqrt[3]{159} = 5{\cdot}42$ inches.

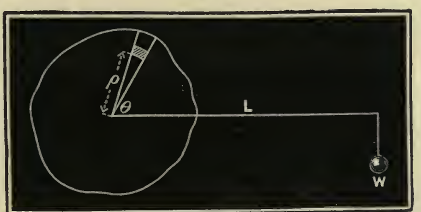
Ex. 3. What should be the diameter of a wrought-iron crane shaft, the radius of the wheel being 16 inches, and the pressure at its circumference 300 lbs., taking 10 as the factor of safety?

$$\text{Here,}\quad \mathsf{W} = 300 \text{ lbs.},$$
$$\mathsf{L} = 16 \text{ inches},$$
$$\mathsf{T} = 9{,}800 \text{ lbs.}$$

Answer (eq. 215). $d = \sqrt[3]{\frac{10\mathsf{WL}}{\mathsf{T}}} = \sqrt[3]{4{\cdot}9} = 1{\cdot}7$ inches.

284. Moment of resistance of torsion.—The following more exact method of investigating torsional strain resembles that applied to transverse strain in **69**, and, like it, is based on the assumption that the law of uniform elastic reaction is true, that is, that the fibres exert elastic forces which resist twisting in proportion to their change of length, and (in circular sections at least) directly therefore as their distance from the central axis. Suppose the shaft composed of longitudinal fibres of infinitesimal thickness, and let us confine our attention to any given cross section represented by Fig. 96.

Fig. 96.



Let $\mathbf{W}$ = the weight producing torsion at the end of the lever $\mathbf{L}$,

$\mathbf{L}$ = the length of the lever, measured from the axis of the shaft,

ρ = the distance of any fibre in the given cross section, measured radially from the axis,

f = the torsional unit-strain exerted by fibres in the same section at a distance c from the axis, that is, the resistance of the fibres to being twisted or shorn asunder referred to a unit of sectional surface,

c = the distance from the axis at which the unit-strain f is supposed to be exerted,

θ = the angle between the line ρ and a horizontal diameter of the section,

r = the radius vector of the curve which bounds the given section.

According to our assumption the torsional unit-strain exerted by fibres at the distance ρ from the axis will $= \frac{f\rho}{c}$; if the thickness of a little element of these fibres measured radially $= d\rho$ (differential of ρ,) and if its width $= \rho d\theta$, the area of the element, shaded in the figure, will $= \rho d\rho d\theta$, and the resisting force exerted by it will $= \frac{f}{c}\rho^2 d\rho d\theta$; the moment of this round the axis $= \frac{f}{c}\rho^3 d\rho d\theta$, and the integral of this, within proper limits, is the sum of the moments round the axis of all the elastic forces in the given section which resist torsion, called the *Moment of resistance to torsion* of that particular section, and this balances $\mathbf{W}\,\mathbf{L}$, or the twisting moment of $\mathbf{W}$. We can obtain the moment of resistance of the little triangle in the figure by integrating the foregoing expression from $\rho = 0$ to $\rho = r$. Doing this, we find the moment of resistance of the little triangle $= \frac{f}{4c} r^4 d\theta$, and therefore the moment of resistance of the whole section can be obtained by integrating this from $\theta = 0$ to $\theta = 2\pi$, as follows,

$$\mathbf{W}\,\mathbf{L} = \frac{f}{4c}\int_0^{2\pi} r^4 d\theta \qquad (217)$$

285. Solid round shafts.—In the case of round shafts the radius vector r is constant, whence, from eq. 217,

$$\text{W L} = \frac{\pi f r^4}{2c} \tag{218}$$

If $f =$ the torsional unit-strain exerted by fibres at the circumference, $c = r$, and we have,

$$\text{W L} = \frac{\pi f r^3}{2} \tag{219}$$

286. Hollow round shafts.—The moment of resistance of a ring is equal to that of the outer circle minus that of the inner one, whence, from eq. 218,

$$\text{W L} = \frac{\pi f}{2c}(r^4 - r_1{}^4)$$

Where $r =$ the external radius,
$r_1 =$ the internal do.

If $f =$ the torsional unit-strain exerted by fibres at the circumference, $c = r$, and we have,

$$\text{W L} = \frac{\pi f}{2r}(r^4 - r_1{}^4) \tag{220}$$

If $t =$ the thickness of the ring, $r_1 = r - t$, whence, by substitution,

$$\text{W L} = \frac{\pi f}{2r}(4r^3 t - 6r^2 t^2 + 4rt^3 - t^4)$$

If the thickness be small compared with the radius, the last three terms may be neglected, and we have,

$$\text{W L} = 2\pi f r^2 t = 6{\cdot}28\, f r^2 t \tag{221}$$

We may perhaps get a clearer conception of the strains in a hollow round shaft by imagining the tube to be formed of a series of diagonal bars forming right-handed coils in one direction, and crossed by other bars forming left-handed coils in the opposite direction, so as to produce a spiral lattice tube, in which, however, the bars in each series are so close together as to touch each other, side by side, and thus form two continuous tubes. The effect of twisting this double tube will be to extend one set of coils and compress the other in the direction of their length, and this will tend to make the tension coils collapse inwards towards the axis of the tube, and force the compression coils outwards, but these tendencies, being equal and in opposite directions, will balance each other. We may go further and imagine the coils springing at an angle of 45° from any given cross section of the tube, and therefore at right angles to each other, and if we suppose that the same piece of material can sustain without injury strains of tension and compression passing through it at right angles to each other, we have the section opposed to either tension or compression $= \frac{2\pi rt}{\sqrt{2}} = \sqrt{2}\pi rt$

where r = the radius of the tube,

t = the thickness of the tube.

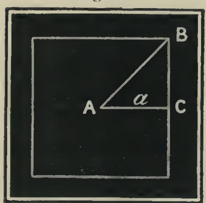
If f = the unit-strain of tension or compression indifferently, we have the twisting moment of the weight,

$$\mathrm{WL} = 2\pi f r^2 t$$

which is the same as equation 221.

287. Solid square shafts.—

Fig. 97.



Let a = half the side of the square.

The radius vector $r = a \sec\theta$ as far as one quarter extends, that is, from $\theta = 0$ up to $\theta = \frac{\pi}{4}$; hence, carrying the integral over the triangle **A B C**, and multiplying by 8 to complete the whole section, we have from eq. 217,

$$\mathrm{WL} = \frac{8fa^4}{4c}\int_0^{\frac{\pi}{4}} \sec^4\theta \, . \, d\theta = \frac{2fa^4}{c}\int_0^{\frac{\pi}{4}} \sec^2\theta \, . \, d\tan\theta$$

$$= \frac{2fa^4}{c}\int_0^{\frac{\pi}{4}} (1 + \tan^2\theta) \, . \, d\tan\theta = \frac{2fa^4}{c}(1 + \frac{1}{3}),$$

and finally,

$$\mathrm{WL} = \frac{8fa^4}{3c}$$

if f = the torsional unit-strain exerted by the extreme fibres in the corners, $c = \sqrt{2}a$, and we have,

$$\mathrm{WL} = \frac{8fa^3}{3\sqrt{2}} \tag{222}$$

If d = the side of the square, eq. 222 becomes,

$$\mathrm{WL} = \frac{fd^3}{4{\cdot}243} = 0{\cdot}236fd^3 \tag{223}$$

Comparing eqs. 219 and 222, we find that the moments of resistance to torsion of the solid square shaft and the solid inscribed circle are in the ratio of $\frac{8\sqrt{2}}{3\pi} = 1{\cdot}2$.

The foregoing theory of the strength of square shafts is based on the hypothesis that the ratio $\frac{f}{c}$ is a constant quantity at different points of the cross section, but this is true for circular sections only, and Professor Rankine gives the following equation for the strength of solid square shafts on the authority of M. de St. Venant, who has investigated the subject theoretically with great care.

$$\mathbf{W\,L} = 0{\cdot}281 f d^3 \tag{224}$$

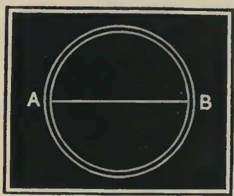
This, it will be observed, makes the strength of a solid square shaft nearly 20 per cent. higher than eq. 223.

CHAPTER XIII.

STRENGTH OF HOLLOW CYLINDERS AND SPHERES.

288. Hollow cylinders—Elliptic tubes.—The strains in hollow cylinders from fluid pressure, either within or without, may be investigated as follows.

Fig. 98.



Let d = the diameter of the cylinder,
t = the thickness of metal,
p = the fluid pressure on each unit of surface (generally in lbs. or tons per square inch),
f = the tangential unit-strain, either of tension or compression, according as p is internal pressure tending to burst the cylinder, or external pressure tending to make it collapse.

Let Fig. 98 represent a thin slice or cross section of a cylinder, the thickness of the slice being one unit measured at right angles to the plane of the paper, and let **A B** represent an imaginary plane through the diameter. Suppose the lower half of the fluid below this plane converted into a solid like ice—an hypothesis which will not affect the conditions of equilibrium in any way—then, the pressure exerted by the upper half of the fluid on the surface, **A B**, of the lower half is obviously equal to pd, and this pressure tends to separate the upper half of the cylinder from the lower half by tearing the metal at **A** and **B**. Hence, the tensile strain at either **A** or **B** $= pd$, that is,

$$2ft = pd \tag{225}$$

The compressive strain due to external pressure, of the same intensity as before, is equal and opposite to the tensile strain just found, for we may conceive the solidified half cylinder removed and a strong plate **A B** substituted for it, in which case the pressure on the under surface of the plate will balance that on the outside of the upper semi-cylinder as before. The same result may be arrived at in another way. Let a cylinder subject to internal pressure, as in the first case, be immersed in a larger vessel, and let fluid be forced into the latter until its pressure equals that within the cylinder, in which case the previous tangential tensile strain due to internal pressure will be cancelled, since the pressures inside and out balance each other. Now, let the fluid inside the cylinder be withdrawn and, the balance being destroyed, a tangential compressive strain will result, equal and opposite to the tensile strain which existed before the cylinder was immersed.

Ex. What should be the thickness of the plates of a cylindrical boiler, 6 feet in diameter and worked to a pressure of 50 lbs. steam per square inch, in order that the working tensile strain may not exceed 1·67 tons per square inch of gross section?

Here, $d = 72$ inches,

$p = 50$ lbs. per square inch of surface,

$f = 1{\cdot}67$ tons $= 3741$ lbs. per square inch of section.

Answer (eq. 225). $t = \frac{pd}{2f} = \frac{50 \times 72}{2 \times 3741} = {\cdot}481$ inch.

Supposing the material equally capable of resisting tension and compression, the strength of a cylinder subject to external pressure, like the flue of a Cornish boiler, is theoretically the same as if it were subject to an equal internal pressure. Practically, however, the strength is much less, owing to the flue not being a perfect circle in cross section. If the outside shell be not a perfect circle, the tendency of internal pressure will be to render it more so, whereas, with the flue, the tendency will be to increase the defect and cause collapse, and Sir William Fairbairn has deduced from an extensive series of experiments the following empirical rule for calculating the strength of wrought-iron tubes, such as boiler flues, within the limits of length which occur in ordinary practice.*

$$p = 806{,}300 \frac{t^{2 \cdot 19}}{ld} \qquad (226)$$

where $p =$ the collapsing pressure in lbs. per square inch of surface,

$t =$ the thickness of the metal in inches,

$l =$ the length of the tube in feet,

$d =$ the diameter in inches.

* Useful Information for Engineers, 2nd series.

Ex. What is the collapsing pressure of a flue 10 feet long, 36 inches in diameter, and composed of ½ inch iron plates ?

$$\text{Here,} \quad t = 0{\cdot}5 \text{ inch,}$$

$$ld = 36 \times 10 = 360,$$

$$\log p = \log 806{,}300 + 2{\cdot}19 \log 0{\cdot}5 - \log 360$$

$$= 5{\cdot}9064967 + 2{\cdot}19 \times \bar{1}{\cdot}69897 - 2{\cdot}5563025 = 2{\cdot}6909385.$$

Answer. $p = 491$ lbs.

The safe working pressure for a land boiler would be $\frac{491}{6} = 82$ lbs.; for an ordinary marine boiler in which salt water is used, $\frac{491}{8} = 61$ lbs.

It will be observed that the strength varies inversely as the length, and Sir William Fairbairn found that "by introducing rigid angle or T iron ribs (in practice from 8 to 10 feet apart,) round the exterior of the flue, we vertically decrease the length and increase the strength in the same proportion. Two or three such rings on the flues of boilers, constructed of plates equal in thickness to those of the shell, will usually render the resistance to collapse equal to the bursting pressure of any other part of the boiler." It was also found that the ordinary longitudinal lap-joints in boiler flues were weaker than butt joints in the ratio of about 7 to 10, and Sir William Fairbairn recommends that tubes required to resist external pressure should be formed with longitudinal butt joints with covering strips outside riveted to both plates.

Elliptical tubes are obviously very weak for resisting external pressure, and it appears from Sir William Fairbairn's experiments that their strength is the same as that of the osculating circle at the flattest part of the ellipse; thus, if a and b are the major and minor semi-axes of the ellipse, the diameter of the cylinder of equal strength will equal $\frac{2a^2}{b}$. If, for example, the ellipse be 6×4 feet, the diameter of the cylinder of equal strength will equal $\frac{2 \times 3^2}{2} = 9$ feet.

289. Cylinder ends.—The flat ends of cylinders sustain a total pressure equal to their area multiplied by the pressure per unit of surface, that is,

$$\text{total end pressure} = \frac{\pi p d^2}{4} \qquad (227)$$

where $p =$ the pressure per square unit of surface,
$d =$ the diameter.

This end pressure is sustained by the rivets or bolts which connect the ends of the cylinder to the sides, and if $t =$ the thickness of metal in the latter, the longitudinal tensile unit-strain in the cylinder,

$$f = \frac{\pi p d^2}{4} \div \pi d t = \frac{pd}{4t} \tag{228}$$

Comparing this with eq. 225, we find that the longitudinal unit-strain in a cylinder is one-half the tangential unit-strain. If the cylinder be a boiler with internal flues, the end area is diminished by the sectional area of the flues, which latter moreover support a large share of the end pressure, so that the longitudinal unit-strain in the shell is greatly reduced. Stay rods connecting the ends above the flues reduce this longitudinal strain still more, so that little anxiety need be felt about the transverse joints of the shell giving way. The longitudinal joints of the shells of high-pressure boilers are generally double-riveted and the cross joints either single or double-riveted.

290. Hollow spheres.—We may conceive, as in the case of the cylinder already investigated, an imaginary plane passing through the centre of the sphere and dividing it into two equal parts. The fluid pressing on the surface of this plane tends to tear asunder the sphere along the circle formed by its intersection with the plane. Hence, if

$d =$ the diameter of the sphere,
$t =$ the thickness of metal,
$p =$ the fluid pressure per square unit of surface,
$f =$ the tangential unit-strain,

we have,

$$\pi f d t = \frac{\pi p d^2}{4}$$

reducing,

$$4ft = pd \tag{229}$$

Comparing this with eq. 225, we find that a sphere is twice as strong as a cylinder of the same diameter and thickness of metal, and that therefore the ends of egg-ended boilers are their strongest part.

CHAPTER XIV.

CRUSHING STRENGTH OF MATERIALS.

291. Nature of compressive strain.—In most of the foregoing theoretic investigations it has been tacitly assumed that the tensile or compressive strength of any material is proportional to its sectional area, whatever that may be. This, however, is not always true of compressive strains, and one of the first difficulties which the student encounters, when seeking to reduce theory to practice, is the necessity of providing in struts or pillars not only against absolute crushing of the material, which in reality rarely occurs, but more especially against flexure and buckling, to resist which a greater amount of material is generally required than theory alone might seem to indicate. To understand the matter clearly we must recollect that the mode in which a pillar fails varies greatly, according as it is long or short in proportion to its diameter. A very short pillar—a cube, for instance, of wrought-iron, timber, or stone—will bear a weight nearly sufficient to upset, to splinter, or to crush it into powder; while a still shorter pillar—such as a penny, or other thin plate of ductile metal—will often bear an enormous weight, far exceeding that which the cube will sustain, the interior of the thin plate being prevented from escaping from beneath the pressure by the surrounding particles. Alluding to his experiments on copper, brass, tin, and lead, Mr. Rennie observes:—"When compressed beyond a certain thickness, the resistance becomes enormous,"* and I have observed the same thing in a very marked degree when experimenting on cubes of cast zinc which slowly spreads out like a plastic material as the strain increases. We can thus conceive how stone or other materials in the interior of the globe withstand pressures that would crush them into powder at the surface, merely because there is no room

* *Phil. Trans.*, 1818, p. 126.

for the particles to escape from the surrounding pressure. A long thin pillar on the other hand, such as a walking cane, will yield by flexure long before it is crushed, and if the bending be carried so far as to break the pillar, the fracture will resemble that due to transverse strain. Hence, it is convenient to subdivide the results of compressive strain into flexure and crushing.

292. Flexure—Crushing—Buckling—Bulging—Splintering. —*Flexure* is the bending or deflection of a pillar whose length is very considerable in proportion to its thickness or diameter.

Crushing may be subdivided into buckling, bulging, and splintering.

(*a.*) *Buckling* is the undulation, wrinkling, or crumpling up, usually of a thin plate of a malleable material. Buckling is frequently preceded by flexure; when, for instance, long tubes of plate-iron are compressed longitudinally, they first deflect, and finally fail by the buckling or puckering of a short piece on the concave side.

(*b.*) *Bulging* is the upsetting or spreading out under pressure of ductile or fibrous materials, such as lead, wrought-iron and timber, also of many semi-ductile crystalline metals, such as cast-brass or zinc.

(*c.*) *Splintering* is the splitting off in fragments of highly crystalline, fibrous, or granular materials, such as cast-iron, glass, timber, stone and brick; the splintering of granular and vitreous materials is often abrupt and terminates in their being crushed to powder, while even the most crystalline metals are to some extent ductile and therefore bulge slightly before they splinter. Again, some materials, such as glass, form numerous prismatic splinters; others, like cast-iron, form two or more wedge-shaped or pyramidal splinters, the plane of separation being oblique to the line of pressure.

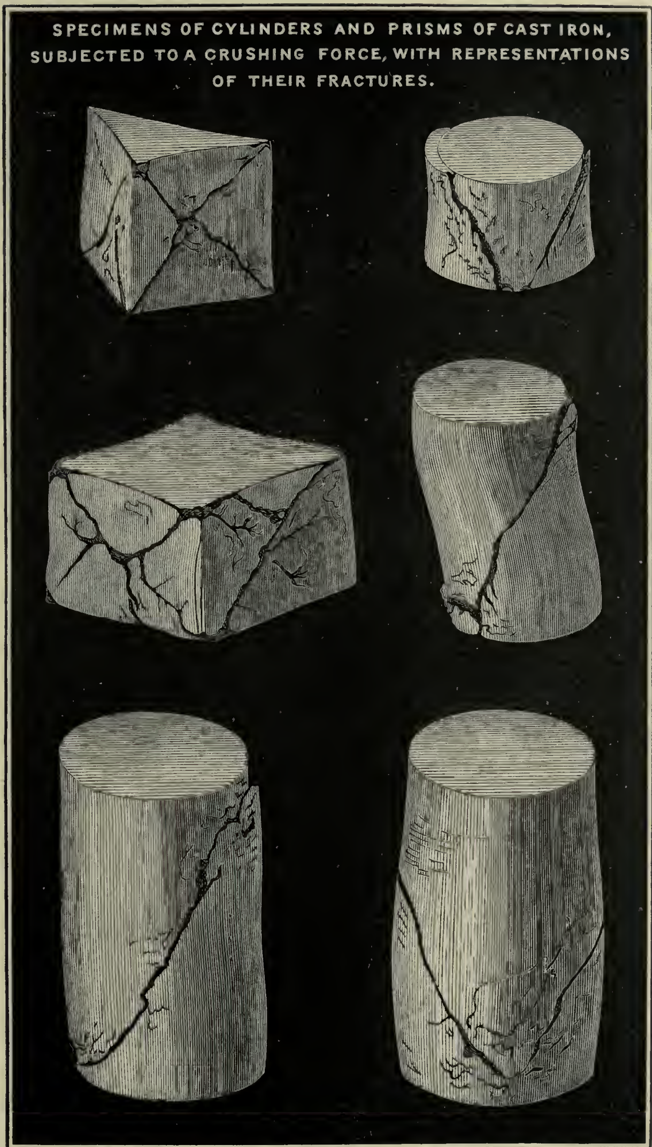
293. Crushing strength of short pillars—Angle of fracture.—It has been found by experiment that the strength of short pillars of any given material, all having the same diameter, does not vary much, provided the length of the pillar is not less than one, and does not exceed four or five diameters; and the weight which will just crush a short prism whose base equals one square unit (generally a square inch), and whose height is not less than one or one and a half, and does not exceed four or five diameters, is called the *crushing strength* of the material experimented upon. When the height of a solid prism lies within these limits "fracture is (generally) caused by the body becoming divided diagonally in one or more directions. In this case the prism, in cast-iron at least, either does not bend before fracture, or bends very slightly; and therefore the fracture takes place by the two ends of the prism forming cones or pyramids, which split the sides and throw them out; or, as is more generally the case in cylindrical specimens, by a wedge sliding off, starting at one of the ends, and having the whole end for its base; this wedge being at an angle which is constant in the same material, though different in different materials (see Plate II.). In cast-iron the angle is such that the height of the wedge is somewhat less than $\frac{3}{2}$ of the diameter. In timber, like as in iron and crystalline bodies generally, crushing takes place by wedges sliding off at angles with their base which may be considered constant in the same material; hence, the strength to resist crushing will be as the area of fracture, and consequently as the direct transverse area, since the area of fracture would, in the same material, always be equal to the direct transverse area, multiplied by a constant quantity."* In other words, eq. 1 is applicable to short pillars, and their crushing strength is equal to their transverse section multiplied by the crushing unit-strain of the material. If the length exceeds four or five times the diameter, "the body bends with the pressure, and though it may break by sliding off as before, the strength is much decreased. In cases where the length is much greater than as above, the body breaks across, as if bent by a transverse pressure."†

From the foregoing observations the reader will perceive that the crushing unit-strain of any material should be derived from experiments on prisms whose height is not less than the length of the wedge, nor so great that the prism will deflect. Mr.

* *Experimental Researches on the Strength of Cast-iron*, by E. Hodgkinson, pp. 319, 323.

† *Idem*, p. 321.

PLATE II.


SPECIMENS OF CYLINDERS AND PRISMS OF CAST IRON, SUBJECTED TO A CRUSHING FORCE, WITH REPRESENTATIONS OF THEIR FRACTURES.

Hodgkinson seems to have preferred prisms whose height equalled two diameters, and in Table I. it will be seen that prisms of cast-iron, whose height equalled one diameter, generally bore more than those whose height equalled two diameters. If, however, the material, like glass and some limestones, do not form wedge-shaped but longitudinal splinters, it seems probable that, within considerable limits, the height of the specimen will not materially affect its crushing strength. Experimenters on stone have generally used cubes; Mr. Hodgkinson's practice, however, seems preferable. If the length of pillars never exceeded four or five diameters, all we need do to arrive at the strength of any given pillar would be to multiply its transverse area in square units by the tabulated crushing strength of that particular material. It rarely happens, however, that pillars are so short in proportion to their length, and hence, we must seek some other rule for calculating their strength when they fail, not by actual crushing, but by flexure. If we could insure the line of thrust always coinciding with the axis of the pillar, then the amount of material required to resist crushing merely would suffice, whatever might be the ratio of length to diameter. But practically it is impossible to command this, and a slight error in the line of thrust produces a corresponding tendency in the pillar to bend. With tension-rods, on the contrary, the greater the strain the more closely will the rod assume a straight line, and, in designing their cross section, it is only necessary to allow so much material as will resist the tensile strain. This tendency to bend renders it necessary to construct long pillars, not merely with sufficient material to resist crushing, supposing them to fail from that alone, but also with such additional material, or bracing, as may effectually preserve them from yielding by flexure. In masonry, heavy timber framing, or similar massive structures, the desired effect is produced by mere bulk of material, which insures the line of thrust always lying at a safe distance within the limits of the structure. In hollow pillars the same result is obtained by removing the material to a considerable distance from the line of thrust, which, though it may deviate slightly from the axis of the pillar, yet will not pass beyond its circumference. When the pillar is neither tubular nor solid, one of the forms of section represented in Fig. 99 is generally adopted.

Fig. 99.

However, before treating about flexure, it seems desirable to give the crushing strengths of short prisms of various materials and afterwards show how these are modified by increasing the length of the prism.

CAST-IRON.

294. Crushing strength of cast-iron.—Table I. contains the results of experiments by Mr. Hodgkinson "on the crushing strength of cylinders of cast-iron of various kinds; the diameters of the cylinders being turned to $\frac{3}{4}$ inch each, and the heights being $\frac{3}{4}$ and $1\frac{1}{2}$ inches respectively. In both cases the height was so small that the specimen could not bend before crushing. Before each experiment was commenced, a very thin sheet of lead was laid over and under the specimen, to prevent any small and unavoidable irregularity between its flat surface and those of the parallel steel discs between which it was to be crushed."*

TABLE I.—CRUSHING STRENGTH OF CAST-IRON.

Description of iron.	Height of specimen.	Crushing weight per square inch of section.	Mean.
	inch.	lbs. tons.	lbs. tons.
Low Moor iron, No. 1 - - -	$\frac{3}{4}$ $1\frac{1}{2}$	64534 = 28·809 56445 = 25·198	60489 = 27·004
Do. No. 2 - - -	$\frac{3}{4}$ $1\frac{1}{2}$	99525 = 44·430 92332 = 41·219	95928 = 42·825
Clyde iron, No. 1 - - - -	$\frac{3}{4}$ $1\frac{1}{2}$	92869 = 41·459 88741 = 39·616	90805 = 40·537

* *Report of the Commissioners appointed to inquire into the application of iron to railway purposes*, 1849, App. A., pp. 12, 13.

TABLE I.—CRUSHING STRENGTH OF CAST-IRON—*continued.*

Description of iron.	Height of specimen.	Crushing weight per square inch of section.	Mean.
	inch.	lbs. tons.	lbs. tons.
Clyde iron, No. 2 - - - -	$\frac{3}{4}$ $1\frac{1}{2}$	109992 = 49·103 102030 = 45·549	106011 = 47·326
Do. No. 3 - - - -	$\frac{3}{4}$ $1\frac{1}{2}$	107197 = 47·855 104881 = 46·821	106039 = 47·339
Blaenavon iron, No. 1 - - -	$\frac{3}{4}$ $1\frac{1}{2}$	90860 = 40·562 80561 = 35·964	85710 = 38·263
Do. No. 2—1st sample -	$\frac{3}{4}$ $1\frac{1}{2}$	117605 = 52·502 102408 = 45·717	110006 = 49·109
Do. No. 2—2nd sample -	$\frac{3}{4}$ $1\frac{1}{2}$	68559 = 30·606 68532 = 30·594	68545 = 30·600
Calder iron, No. 1 - - - -	$\frac{3}{4}$ $1\frac{1}{2}$	72193 = 32·229 75983 = 33·921	74088 = 33·075
Coltness iron, No. 3 - - -	$\frac{3}{4}$ $1\frac{1}{2}$	100180 = 44·723 101831 = 45·460	101005 = 45·091
Brymbo iron, No. 1 - - -	$\frac{3}{4}$ $1\frac{1}{2}$	74815 = 33·399 75678 = 33·784	75246 = 33·592
Brymbo iron, No. 3 - - -	$\frac{3}{4}$ $1\frac{1}{2}$	76133 = 33·988 76958 = 34·356	76545 = 34·171
Bowling iron, No. 2 - - -	$\frac{3}{4}$ $1\frac{1}{2}$	76132 = 33·987 73984 = 33·028	75058 = 33·508
Ystalyfera Anthracite iron, No. 2 -	$\frac{3}{4}$ $1\frac{1}{2}$	99926 = 44·610 95559 = 42·660	97742 = 43·635
Yniscedwyn Anthracite iron, No. 1 -	$\frac{3}{4}$ $1\frac{1}{2}$	83509 = 37·281 78659 = 35·115	81084 = 36·198
Do. No. 2 -	$\frac{3}{4}$ $1\frac{1}{2}$	77124 = 34·430 75369 = 33·646	76246 = 34·038
Mean of the foregoing 16 irons - - - - - -			86284 = 38·519
Mr. Morries Stirling's iron, 2nd quality*	$\frac{3}{4}$ $1\frac{1}{2}$	125333 = 55·952 119457 = 53·329	122395 = 54·640
Do. 3rd quality†	$\frac{3}{4}$ $1\frac{1}{2}$	158653 = 70·827 129876 = 57·980	144264 = 64·403

* Composed of Calder No. 1 hot-blast, mixed and melted with about 20 per cent. of malleable iron scrap.

† Composed of No. 1 hot-blast Staffordshire iron from Ley's Works, mixed and melted with about 15 per cent. of common malleable iron scrap.

Table II. contains the "crushing weights of short cylinders of different kinds of cast-iron, cut from the bars, $2\frac{1}{2}$ inches diameter previously used (in experiments on pillars), and now turned to be $\frac{3}{4}$ inch diameter nearly, and $1\frac{1}{2}$ inch high. The results are means from three or four experiments on each kind of iron. The specimens were usually cut out of the iron between the centre and the circumference of the bar, denominated the medium part. In several cases they were cut out of the centre of the bar, and sometimes out of the circumference."*

TABLE II.—CRUSHING STRENGTH OF CAST-IRON.

Description of iron.	Diameter of specimen.	Crushing weight per square inch of section.
	inch.	lbs. tons.
Old Park iron, No. 1. Medium, - - - - - - - - - -	·747	88070 = 39·32
Old Park iron, No. 1. Centre, - - - - - - - - - -	·747	74653 = 33·33
Derwent iron, No. 1. Medium, - - - - - - - - - -	·747	97160 = 43·37
Coltness iron, No. 1. Medium, - - - - - - - - - -	·747	63048 = 28·14
Blaenavon iron, No. 1. Medium, - - - - - - - - - -	·748	70909 = 31·66
Level iron, No. 1. Medium, - - - - - - - - - -	·749	68217 = 30·45
Carron iron, No. 1. Medium, - - - - - - - - - -	·750	68509 = 30·58
London Mixture. Medium, - - - - - - - - - -	·749	80923 = 36·08
Calder iron, No. 1. Medium, - - - - - - - - - -	·750	84648 = 37·79
Portland iron, No. 1. Medium, - - - - - - - - - -	·748	94802 = 42·32

* *Philosophical Transactions*, 1857, p. 889.

TABLE II.—CRUSHING STRENGTH OF CAST-IRON—*continued.*

Description of iron.	Diameter of specimen.	Crushing weight per square inch of section.
Old Hill iron, No. 1. Medium, - - - - - - - - - - -	inch. ·749	lbs. tons. 54761 = 24·45
Low Moor iron, No. 2. Medium, - - - - - - - - -	·748	77489 = 34·59
Low Moor iron, No. 2. Centre, - - - - - - - - - -	·742	66407 = 29·65
Blaenavon iron, No. 3. Medium, - - - - - - - - -	·737	83517 = 37·28
Blaenavon iron, No. 3. Centre, - - - - - - - - - -	·747	76643 = 34·22
Second London Mixture. Medium. From 2½ inch pillar, as all above have been, -	·747	95338 = 42·56
Second London Mixture. Centre. From 2½ inch pillar, as all above have been, -	·747	78451 = 35·02
Second London Mixture. Medium. From 1½ inch pillar, - - - - -	·750	111080 = 49·59
Second London Mixture. Centre. From 1½ inch pillar, - - - - - -	·750	104071 = 46·46
Low Moor iron, No. 2. From a hollow pillar 4 inches diameter and ½ inch thick. The height of the first two specimens was ·72 inch, and of the last 1·502 inch, - - - - - - -	·421	87502 = 39·06
Low Moor iron, No. 2. From the thin ring of a hollow pillar about 3½ inches diameter. Height of specimens ·53 inch, - - -	·299	115993 = 51·78
Low Moor iron, No. 2. From the thin ring of a hollow pillar about 3½ inches diameter. Height of Specimens ·53 inch, - - -	·296	110212 = 49·20
Mean of the foregoing 22 irons, - - - - - - -		84200 = 37·6

From the experiments recorded in the two foregoing tables it appears that the average crushing strength of simple cast-irons does not exceed 38 tons per square inch; the strength of mixtures, however, is higher and may in general be taken at 42 tons per square inch, though occasionally it reaches 50 tons. Repeated meltings seem to have the effect of increasing the crushing strength of cast-iron (See Chap. XVI.).

295. Hardness and crushing strength of thin castings greater near the surface than in the heart—Crushing strength of thin greater than that of thick castings.—Mr. Hodgkinson found that "of the different irons tried in the experiments on pillars, whether solid or hollow, the external part of the casting was always harder than that near to the centre, and the iron of the external ring of a hollow casting was very hard, the hardness increasing with the thinness. Thus, in solid pillars $2\frac{1}{2}$ inches diameter of Low Moor iron, No. 2 (Table II.), the crushing force per square inch of the central part was 29·65 tons, and that of the intermediate part near to the surface was 34·59 tons, whilst the external ring, $\frac{1}{2}$ inch thick, of a hollow cylinder 4 inches diameter, of which the outer crust had been removed, was crushed with 39·06 tons per square inch; and external rings of the same iron, thinner than half an inch, required from 49·2 to 51·78 tons per square inch to crush them. These facts show the great superiority of hollow pillars over solid ones of the same weight and length."* Hence, removing the skin of a *thin* casting reduces its strength to resist compression.

296. Hardness and crushing strength of thick castings at the surface and in the heart not materially different.—"To ascertain whether the internal strength of larger pillars varied in the same manner as that of smaller ones, a cylindrical casting was made 5 inches diameter and 15 inches long. It was cast vertically, from Blaenavon iron. Through the axis of this cylinder, a slab, 15 inches long, 5 inches broad, and about 1 inch thick, was taken. Across the middle of this slab five cylinders, $1\frac{1}{2}$ inch long and $\frac{3}{4}$ inch diameter, were obtained at equal distances from each other, the middle one

* *Phil. Trans.*, 1857, p. 890.

being in the centre, and the outer ones as near as possible to the sides. Comparing the results of the experiments (on crushing these cylinders) it appears that the external part of the casting was somewhat stronger than the internal. But the variation was only from 62 to 66 (62,444 to 65,739 lbs. per square inch), and therefore much less than was obtained from the smaller masses." From this and other experiments on small cylinders cut out of a slab of Derwent iron, No. 1, cast 9 inches square and 12 inches long, "it appears that the difference of hardness between the external and internal parts of a large casting is much less than in a small one, and may frequently be neglected."* For the safe working strain on cast-iron see Chap. XXVIII.

WROUGHT-IRON.

297. Crushing strength of wrought-iron—12 tons is the limit of compressive elasticity of wrought-iron.—The crushing strength of wrought-iron varies with the hardness of the iron, but ordinary wrought-iron is completely crushed, *i.e.*, bulged, with a pressure of from 16 to 20 tons per square inch, and when the pressure exceeds 12 or 13 tons, Mr. Hodgkinson found that "in most cases it cannot be usefully employed, as it will sink to any degree, though in hollow cylinders it will sometimes bear 15 or 16 tons per square inch."† The point at which compressive set sensibly commences, that is, the limit of compressive elasticity, is about 12 tons per square inch. For the safe working strain in practice see Chap. XXVIII.

STEEL.

298. Crushing strength of steel—21 tons is the limit of compressive elasticity of steel.—The following table contains the results of experiments on the crushing weights of cylinders of cast-steel by Major Wade,‡ U.S. Army:—

* *Phil. Trans.*, 1857, pp. 891, 892.

† *Com. Rep.*, p. 121.

‡ *Reports of Experiments on the Strength and other Properties of Metals for Cannons*, by Officers of the Ordnance Department, U.S. Army, p. 258. Philadelphia, 1856.

TABLE III.—CRUSHING STRENGTH OF CAST-STEEL.

Kind of cast-steel.	No. of sample.	Length.	Diameter.	Crushing weight per square inch of section.
		inch.	inch.	lbs.
Not hardened, - - - - - -	1	1·021	·400	198,944
Hardened; low temper; chipping chisels, - -	2	·995	·402	354,544
Hardened; mean temper; turning tools, - -	3	1·016	·403	391,985
Hardened; high temper; tools for turning hard steel,	4	1·005	·405	372,598

NOTE—All the samples of steel tested were cut from the same bar. No. 1 remained unchanged, as made at the steel factory. Nos. 2, 3, and 4, were all hardened, and the temper afterwards drawn down in different proportions.

Table IV. contains the results of experiments made by Mr. Kirkaldy for the "Steel Committee," on the crushing strength of carefully turned cylinders of steel 1·382 inches in diameter (= 1·5 square inches area), and whose height equalled 4 diameters, the steel being intended for tyres, axles, and rails.*

TABLE IV.—LIMIT OF COMPRESSVIE ELASTICITY OF CRUCIBLE AND BESSEMER STEEL BARS.

Kind of steel.	Crushing weight per square inch at which sensible set commenced, *i.e.*, Limit of compressive elasticity.
	tons.
Crucible steel, hammered, -	22·92
Do. rolled, - -	18·75
Bessemer steel, hammered, -	21·79
Do. rolled, - -	18·08
Mean, - - - - - -	20·38

Shorter cylinders of the same kinds of steel of the same sectional area, but only one diameter in height, were subjected to a crushing weight of 200,000 lbs. per square inch, the result being that they

* *Experiments on Steel and Iron by a Committee of Civil Engineers*, 1868–70.

bulged but did not crack; the average contraction of length (ultimate compressive set) under this strain was for crucible steel 32 per cent., and for Bessemer steel 38 per cent., of the original length. From 31 experiments made subsequently by the same committee at Woolwich Dockyard, on the compression of bars of crucible, Bessemer, and cast-steel, 10 feet long and $1\frac{1}{2}$ inches diameter, the maximum and minimum limits of compressive elasticity were 27 and 15 tons respectively, and the average was 21·35 tons per square inch, which agrees sufficiently closely with the mean of the experiments in Table IV. to allow us to assume 21 tons to be the practical limit of compressive elasticity of average steel.

The reader will find in Chap. XVI. additional experiments by Sir William Fairbairn on the crushing strength of various kinds of steel. For the safe working load see Chap. XXVIII.

VARIOUS METALS.

299. Crushing strength of copper, brass, tin, lead, aluminium-bronze, zinc.—The following table contains the results of experiments by Mr. G. Rennie on the crushing strength of $\frac{1}{4}$ inch cubes of different metals.*

TABLE V.—CRUSHING STRENGTH OF VARIOUS METALS.

Description of metal.	Crushing weight on a $\frac{1}{4}$ inch cube.
	lbs.
Cast-copper crumbled with - - - - -	7318
Fine yellow brass reduced $\frac{1}{10}$th, with - - - -	3213
Do. do. $\frac{1}{2}$, with - - - -	10304
Wrought-copper reduced $\frac{1}{16}$th, with - - - -	3427
Do. do. $\frac{1}{8}$th, with - - - -	6440
Cast-tin do. $\frac{1}{16}$th, with - - - -	552
Do. do. $\frac{1}{3}$rd, with - - - -	966
Cast-lead do. $\frac{1}{2}$, with - - - -	483

Alluding to these ductile metals, Mr. Rennie observes:—"The experiments on the different metals give no satisfactory results. The difficulty consists in assigning a value to the different degrees

* *Phil. Trans.*, 1818, p. 125.

of diminution. When compressed beyond a certain thickness, the resistance becomes enormous." The crushing weight of aluminium bronze, according to Professor Rankine, is 59 tons per square inch. In my own experiments I found that cast-zinc will spread out to any degree under severe pressure, but it will bear 5 or 6 tons per square inch without any very appreciable change of shape.

TIMBER.

300. Crushing strength of timber—Wet timber not nearly so strong as dry.—The following table contains the results of experiments by Mr. Hodgkinson on the crushing strength of various kinds of timber, "the force being applied in the direction of the fibre."*

TABLE VI.—CRUSHING STRENGTH OF TIMBER.

Description of wood.	Crushing weight per square inch of section.	
	Wood in the ordinary state of dryness.	Wood very dry.
	lbs.	lbs.
Alder, - - - -	6,831	6,960
Ash, - - - - -	8,683	9,363
Baywood, - - - -	7,518	7,518
Beech, - - - -	7,733	9,363
Birch, American, - - -	...	11,663
Birch, English, - - -	3,297	6,402
Cedar, - - - -	5,674	5,863
Crab, - - - - -	6,499	7,148
Deal, red, - - - -	5,748	6,586
Deal, white, - - - -	6,781	7,293
Elder, - - - -	7,451	9,973
Elm, - - - - -	...	10,331
Fir, Spruce, - - - -	6,499	6,819
Hornbeam, - - - -	4,533	7,289
Larch (fallen two months), - -	3,201	5,568
Mahogany, - - - -	8,198	8,198
Oak, Quebec, - - - -	4,231	5,982
Oak, English, - - - -	6,484	10,058
Oak, Dantzic (very dry), - -	...	7,731
Pine, pitch, - - - -	6,790	6,790
Pine, yellow (full of turpentine), -	5,375	5,445
Pine, red, - - - -	5,395	7,518
Plum, wet, - - - -	3,654	...
Plum, dry, - - - -	8,241	10,493
Poplar, - - - -	3,107	5,124
Sycamore, - - - -	7,082	...
Teak, - - - - -	...	12,101
Walnut, - - - -	6,063	7,227
Willow, - - - -	2,898	6,128

* *Phil. Trans.*, 1840, p. 429.

"The results in the first column were in each case a mean from about three experiments upon cylinders of wood turned to be one inch diameter, and two inches long, flat at the ends. The wood was moderately dry, being such as is employed in making models for castings. The second column gives the mean strength, as before, from similar specimens, after being turned and kept drying in a warm place two months longer. The lengths of these latter specimens were, in some instances, only one inch, which reduction would increase the strength a little. But the great difference frequently seen in the strength, as given by the two columns, shows strongly the effect of drying upon wood, and the great weakness of wet timber, *it not having half the strength of dry*"—a consideration of much importance in works under water. For the safe working load on timber see Chap. XXVIII.

STONE, BRICK, CEMENT, AND GLASS.

301. Crushing strength of stone and brick.—The following table contains the crushing strength of stone and brick. For the safe working load see Chap. XXVIII.

TABLE VII.—CRUSHING STRENGTH OF STONE AND BRICK.

Description of stone.	Specific gravity.	Crushing weight per square inch.	Authority.
GRANITES.		lbs.	
Aberdeen, blue kind, - - - - -	2·625	10914	Rennie.
Peterhead, hard close grained, - -	...	8283	"
Cornish, - - - - - -	2·662	6356	"
Killiney, Co. Dublin, very felspathic, -	...	10780	Wilkinson.
Kingstown, do., grey colour, - -	...	10115	"
Blessington, Co. Wicklow, coarse and loosely aggregated, - - - - - -	...	3630	"
Ballyknocken, Co. Wicklow, coarse, micaceous,	...	3173	"
Newry, slightly syenitic, - - -	...	13440	"
Mount Sorrel granite, - - -	2·675	12861	Fairbairn.
SANDSTONES AND GRITS.			
Arbroath pavement, - - - - -	...	7884	Buchanan.
Caithness do. - - - - -	...	6493	"
Dundee sandstone or Brescia, - - -	2·530	6630	Rennie.
Craigleith white freestone, - - -	2·452	5487	"
Bramley Fall, near Leeds (with and against strata)	2·506	6059	"
Derby Grit, a red friable sandstone, - -	2·316	3142	"
Ditto, from another quarry, - - -	2·428	4345	"
Yorkshire paving (with and against strata), -	2·507	5714	"
Red sandstone, Runcorn (17 feet per ton), -	...	2185	L. Clark.
Quartz rock, Holyhead (across lamination), -	...	25500	Mallet.
Ditto (parallel to lamination), - -	...	14000	"

TABLE VII.—CRUSHING STRENGTH OF STONE AND BRICK—*continued.*

Description of stone.	Specific gravity.	Crushing weight per square inch.	Authority.
OOLITES.		lbs.	
Portland stone, - - - - -	2·423	3729	Rennie.
Ditto, another specimen, - - -	2·428	4570	,,
MARBLES.			
Marble, statuary, - - - - -	...	3216	,,
Ditto, white statuary, not veined, - -	2·760	6058	,,
Ditto, white Italian, veined, - - -	2·726	9681	,,
Ditto, black Brabant, - - - -	2·697	9219	,,
Ditto, Devonshire red, variegated, - -	...	7428	,,
Ditto, Kilkenny black, - - -	...	15120	Wilkinson.
Ditto, black Galway, from Menlo quarry, -	...	20160	,,
LIMESTONES.			
Limestone, compact, - - - - -	2·584	7713	Rennie.
Ditto, black compact, Limerick, - -	2·598	8855	,,
Ditto, Purbeck, - - - - -	2·599	9160	,,
Ditto, magnesian, Anston, stone of which Houses of Parliament are built, - -	...	3050	Fairbairn.
Ditto, Anglesea (13½ cubic feet per ton), -	...	7579	L. Clark.
Ditto, Listowel quarry, Kerry, - -	...	18043	Wilkinson.
Ditto, Ballyduff quarry near Tullamore, King's Co.	...	11340	,,
Ditto, Woodbine quarry near Athy, Kildare, -	...	14350	,,
Ditto, Finglas quarry, Co. Dublin, - -	...	16940	,,
Chalk, - - - - - -	...	501	Rennie.
SLATES.			
Valencia Island, Kerry, - - - -	...	10943	Wilkinson.
Killaloe quarry, Tipperary, on bed of strata, -	...	26495	,,
Do. do. on edge of strata, -	...	15225	,,
Glanmore, Ashford, Wicklow, on bed of strata	...	21315	,,
Do. do. on edge of strata	...	12740	,,
BASALTS AND METAMORPHIC ROCKS.			
Whinstone, Scotch, - - - - -	...	8270	Buchanan.
Felspathic greenstone, from Giant's Causeway,	...	17220	Wilkinson.
Hornblendic greenstone, Galway, Co. Galway,	...	24570	,,
Moore quarry, Ballymena, Antrim, crystalline and hornblendic, - - -	...	20552	,,
Grauwacke, from Penmaenmawr, - -	2·748	16893	Fairbairn.
BRICKS.			
Pale red, - - - - - -	2·085	562	Rennie.
Red brick, - - - - - -	2·168	808	,,
Yellow-face baked Hammersmith paviors, -	...	1002	,,
Yellow-faced burnt Hammersmith paviors, -	...	1441	,,
Fire brick, Stourbridge, - - -	...	1717	,,
Buckley Mountain brick, N. Wales, - -	...	2130	L. Clark.
Brickwork set in cement (bricks not of a hard description) - - - -	...	521	,,

Buchanan, *Practical Mechanic's Journal*, Vol. ii., p. 285.
L. Clark, *The Britannia and Conway Tubular Bridges*, p. 365.
Fairbairn, *Useful information for Engineers*, second series, p. 136.
Mallet, *Philosophical Transactions*, 1862, p. 671.
Rennie, *Philosophical Transactions*, 1818, p. 131.
Wilkinson, *Practical Geology and Ancient Architecture of Ireland.*

The following table gives the results of experiments made by Mr. Grant with a hydraulic press on the crushing strength of various kinds of brick.*

TABLE VIII.—CRUSHING STRENGTH OF COMMON BRICK AND BRICKS MADE OF PORTLAND CEMENT.

Description of brick.	Length.	Breadth	Thickness.	Area exposed to pressure.	Weight. Dry.	Weight. Wet.	Crushing weight per brick.
	ins.	ins.	ins.	ins.	lbs.	lbs.	tons.
Gault clay, pressed, - - -	8·75	4·125	2·75	36·09	5·13	6·47	40·04
Gault clay, wire cut, - -	9·00	4·125	2·75	37·125	5·86	6·85	32·70
Gault clay, perforated, - -	9·00	4·375	2·625	39·375	4·95	5·76	46·40
Suffolk brimstones, - - -	9·00	4·5	2·625	40·5	6·18	7·14	34·94
Stock, - - - - - -	9·00	4·125	2·625	37·125	5·0	5·57	38·74
Fareham red, - - - -	8·75	4·125	2·625	36·09	6.55	7·52	90·40
Staffordshire blue (pressed, with frog), - - - -	8·75	4·125	2·75	36·09	7·82	7·90	111·04
Staffordshire blue(rough, without frog), - - - -	8·75	4·125	2·75	36·09	7·75	7·81	117·92
Portland Cement bricks, neat, compressed, and kept in air 12 months, - - -	9·00	4·5	3·0	40·50	9·51	9·76	96·60
Do. kept in water 12 months, -	...	...	...	...	...	...	132·62
Portland Cement and sand, 1 to 4, compressed and kept in air 12 months, - - -	...	...	...	...	8·79	9·51	43·60
Do. kept in water 12 months, -	...	...	...	...	...	...	29·92
Portland Cement and sand, 1 to 6, compressed and kept in air 12 months, - - -	...	...	...	...	8·43	9·38	30·28
Do. kept in water 12 months, -	...	...	...	...	...	...	11·24

302. Mode of fracture of stone.—In Mr. Clark's experiments "the sandstones gave way *suddenly*, and without any previous cracking or warning. After fracture the upper portion generally retained the form of an inverted square pyramid, very symmetrical, the sides bulging away in pieces all round. The limestones formed *perpendicular* cracks and splinters a considerable time before they crushed." Mr. Rennie observes, "it is a curious fact in the rupture of amorphous stones, that pyramids are formed, having for their base the upper side of the cube next the lever, the action of which displaces the sides of the cubes, precisely as if a wedge had operated between them." Mr. Wilkinson remarks, "The results of the (one inch) cubes experimented on show the strongest stones to be the basalts, primary limestones, and slates. Of the limestones, the primary limestones and compact hard calp are the strongest; and the light dove-coloured and fossiliferous

* *Proc. Inst. C.E.*, Vol. xxxii.

limestones are among the weakest. The strength of the sandstones, like their mineral aggregation, is very variable."

The strength of stones, though bearing the same name and presenting the same lithological characters, is so variable in different localities, that, when any building of importance is proposed, it is prudent to test the strength of the stone by actual experiment rather than trust to books for the information required. In my own experiments, I find that with granite and limestones the first crack may be expected to take place with from one-half to two-thirds of the ultimate crushing weight.

303. Crushing strength of rubble masonry.—Professor Rankine states that "the resistance of *good coursed rubble* masonry to crushing is about four-tenths of that of single blocks of the stone that it is built with. The resistance of *common rubble* to crushing is not much greater than that of the mortar which it contains."* For the safe working load on masonry see Chap. XXVIII.

304. Crushing strength of Portland cement, mortar and concrete.—The following table contains the results of experiments by Mr. Grant on the crushing strength of Portland cement and cement mortar.†

TABLE IX.—CRUSHING STRENGTH OF PORTLAND CEMENT AND CEMENT MORTAR.

Description of cement or mortar.		Crushing weight per square inch.
		lbs.
Portland cement, neat, - -	made 3 months.	3795
1 Portland Cement to 1 pit sand, -		2491
ditto 2 ditto, -		2004
ditto 3 ditto, -		1436
ditto 4 ditto, -		1331
ditto 5 ditto, -		959
Portland cement, neat, - -	made 6 months.	5388
1 Portland Cement to 1 sand, - -		3478
ditto 2 ditto, - -		2752
ditto 3 ditto, - -		2156
ditto 4 ditto, - -		1797
ditto 5 ditto, - -		1540
Portland cement, neat, - -	made 9 months.	5984
1 Portland Cement to 1 pit sand, -		4561
ditto 2 ditto, -		3647
ditto 3 ditto, -		2393
ditto 4 ditto, -		2208
ditto 5 ditto, -		1678

* *Civil Engineering*, p. 387. † *Proc. Inst. C. E.*, Vol. xxv.

In these experiments the specimens were made into bricks $9 \times 4{\cdot}25 \times 2{\cdot}75$ inches, and exposed to the pressure of a hydraulic press on their flat surface of $9 \times 4{\cdot}25$ inches $= 38{\cdot}25$ square inches. The results would doubtless have been somewhat different if they had been cubes. Each specimen showed signs of giving way with considerably less pressure than that which finally crushed it, the average ratio of the weight which produced the first crack to that which finally crushed it being nearly as $\frac{5}{8}$.

The following table gives the strength of lime mortar 18 months old, on the authority of Rondelet.*

TABLE X.—CRUSHING STRENGTH OF LIME MORTAR 18 MONTHS OLD.

Description of mortar.	Crushing weight per square inch.
	lbs.
Mortar of lime and river sand, - -	436
The same, beaten, - - - -	596
Mortar of lime and pit sand, - - -	578
The same, beaten, - - - -	800
Mortar of cement and pounded tiles, - -	677
The same, beaten, - - - -	929
Mortar made with pounded sandstone, -	417
Mortar made with puzzolana from Naples and Rome mixed, - - - -	521
The same, beaten, - - - -	758

Fifteen years later these experiments were repeated, when mortars of lime and sand were found to have increased in strength about $\frac{1}{8}$th, and mortars of cement or puzzolana about $\frac{1}{4}$th.

The following tables give the results of some of Mr. Grant's experiments with a hydraulic press on the crushing strength of concrete blocks, made of Portland cement and ballast in various proportions, set and kept in air for one year, also set and kept in water for the same time.†

* Navier, *Application de la Mécanique*, p. 8.

† *Proc. I. C. E.*, Vol. xxxii.

TABLE XI.—PORTLAND CEMENT CONCRETE BLOCKS OF BALLAST, set and *kept in Air* for One Year, also set and *kept in Water* for the same time.

Size of Block—12″ × 12″ × 12″. Compressed.

Proportions.	Weight in lbs.			Weight of each Block in lbs.		Crushed at tons.		Remarks.
	Cement.	Sand and Gravel.	Water.	Kept in Air.	Kept in Water.	Air.	Water.	
1 to 1	59·36	66·96	16·00	137·60	147·25	107·0*	170·50	* Exceptional.
2 „ 1	42·64	96·40	12·00	142·60	152·50	149·0	160·0	
3 „ 1	32·00	108·56	10·00	145·25	152·25	113·0	115·50	
4 „ 1	25·84	116·96	8·80	145·75	152·50	103·0	108·50	
5 „ 1	21·28	120·24	8·00	142·10	150·95	89·0	99·50	
6 „ 1	18·08	122·48	8·00	141·56	150·00	80·50	91·0	
7 „ 1	15·84	125·04	7·60	141·70	150·20	75·0	80·50	
8 „ 1	14·08	127·04	7·60	142·30	150·80	61·50	76·0	
9 „ 1	12·64	128·64	7·20	142·10	151·50	54·0	68·50	
10 „ 1	11·36	128·88	6·80	142·00	150·00	48·50	48·0	
Size of Block—6″ × 6″ × 6″. Compressed.								
1 to 1	7·42	8·37	2·00	17·50	18·04	38·0	33·60	
2 „ 1	5·33	12·05	1·50	17·78	18·97	43·0	34·50	
3 „ 1	4·00	13·57	1·25	18·28	19·35	30·0	35·50	
4 „ 1	3·23	14·62	1·10	18·28	18·71	30·0	28·00	
5 „ 1	2·66	15·03	1·00	18·26	18·98	24·50	35·50	
6 „ 1	2·26	15·31	1·00	17·90	18·60	20·40	19·60	
7 „ 1	1·98	15·63	·95	17·85	18·85	16·50	16·0	
8 „ 1	1·76	15·88	·95	17·86	18·90	13·50	13·50	
9 „ 1	1·58	16·08	·90	17·78	19·0	12·0	11·00	
10 „ 1	1·42	16·11	·85	17·68	18·70	10·50	10·50	
Size of Block—6″ × 6″ × 6″. Not Compressed.								
1 to 1	7·12	8·04	1·92	16·44	17·60	30·0	37·50	
2 „ 1	4·90	11·09	1·38	17·57	18·03	38·50	36·00	
3 „ 1	3·56	12·11	1·11	17·75	18·98	24·0	28·00	
4 „ 1	2·85	12·92	·97	17·84	18·28	28·0	27·00	
5 „ 1	2·33	13·18	·87	17·90	18·73	24·0	23·50	
6 „ 1	2·00	13·49	·88	17·35	18·30	18·20	17·00	
7 „ 1	1·77	14·02	·85	17·32	17·90	14·0	12·50	
8 „ 1	1·60	14·51	·85	17·38	17·95	12·50	11·00	
9 „ 1	1·43	14·59	·80	17·40	17·97	10·0	9·00	
10 „ 1	1·26	14·35	·75	17·20	17·50	8·0	7·00	

It will be observed that the concrete which was compressed was considerably stronger than that not compressed. In my own practice I always have concrete carefully rammed, and when it forms the matrix for large rubble stone the concrete is compressed between the stones with iron tamping tools having T shaped ends about 5 inches long. This permits it to be mixed stiff with but little water, and, when thus solidly rammed, the stones will generally break sooner than the concrete in which they are imbedded. In one of Mr. Grant's experiments a twelve-inch cube of concrete, made with blue Lias lime and Thames ballast 1 + 6, 10 months old and kept in water, bore 6 tons per square foot, or 93 lbs. per square inch. A similar cube of Lias concrete, but made with Bramley Fall chippings 1 + 6, in place of ballast, and also kept in water 10 months, bore 20·4 tons per square foot, or 317 lbs. per square inch.* For the safe working load on concrete see Chap. XXVIII.

305. Crushing strength of glass.—The following table contains the crushing strength of glass from experiments by Sir Wm. Fairbairn and Mr. Tate.†

TABLE XII.—CRUSHING STRENGTH OF ANNEALED GLASS BARS.

Kind of Glass.	Sp. gravity.	Crushing weight per square inch.
		lbs. tons.
Best flint glass annealed rod, drawn out when molten to about ¾ inch diameter, - - - -	3·0782	27582 = 12·313
Common green glass ditto ditto, - -	2·5284	31876 = 14·227
White crown glass ditto ditto, - -	2·4504	31003 = 13·840

"The specimens were crushed almost to powder from the violence of the concussion, when they gave way; it, however, appeared that the fractures occurred in vertical planes, splitting up the specimen in all directions; cracks were noticed to form some time before the specimen finally gave way; then these rapidly increased in number, splitting the glass into innumerable irregular prisms of the same height as the cube; finally, these bent or broke, and the pressure, no longer bedded on a firm surface, destroyed the specimen." Seven cubes were also cut from the centre of large lumps of glass, and crushed. Their resistance was less than that of the drawn rods in the ratio of $\frac{5}{8}$, possibly because they were less perfectly annealed than the drawn rods, and also because the external skin of the latter gave them some extra strength (**295**).

* *Proc. I. C. E.*, Vol. xxv., p. 110.

† *Philosophical Transactions*, 1859, p. 213.

CHAPTER XV.

PILLARS.

306. Very long thin pillars.—The law which determines the flexure of very long thin pillars may be investigated theoretically as follows:—Let Fig. 100 represent a pillar of uniform section throughout, not fixed at the ends, very long in proportion to its breadth, and just on the point of failing from flexure.

Fig. 100.

Let W = the deflecting weight,

D = the lateral deflection at the centre.

M = the moment of resistance of the longitudinal elastic forces (**59**),

b = the breadth of the pillar,

d = its diameter or least lateral dimension,

l = its length,

f = the longitudinal unit-strain in the extreme fibres in a horizontal section across the middle of the pillar,

λ = the difference in length between the convex and the concave edges of the pillar,

C = the resultant of all the longitudinal forces of compression in the concave side at the plane of section,

T = the resultant of all the longitudinal forces of tension in the convex side at the plane of section,

E = the coefficient of elasticity.

The upper half of the pillar is held in equilibrium by three sets of vertical forces—viz., the weight, acting in the chord-line of the curve; the longitudinal tensile strains in the convex side at the middle section; and the longitudinal compressive strains in the concave side, also at the middle section. When the pillar is very long in proportion to its width, and the deflection therefore considerable, even though the curvature be small,* we may assume D equal to the distance from the chord-line to either the centre of tensile or the centre of compressive strains. Taking moments round either of these points indifferently, we have

$$\mathsf{W\,D} = \mathsf{M} \text{ nearly,} \tag{a}$$

Again, assuming that the deflection curve is a circle, from which it can differ but slightly, we have from eq. 132,

$$\mathsf{D} = \frac{\lambda l}{8d} \text{ nearly,} \tag{b}$$

whence, by substitution in eq. (a), we have,

$$\mathsf{W} = \frac{8d\mathsf{M}}{\lambda l} \tag{c}$$

Further, recollecting that λ is equal to the contraction of the concave plus the extension of the concave edge, we have from eq. 2,

$$\lambda = \frac{2fl}{\mathsf{E}}$$

Substituting this in eq. (c), we have

$$\mathsf{W} = \frac{4d\mathsf{EM}}{fl^2} \tag{230}$$

Replacing M by its values in **71** and the succeeding sections and recollecting that the ratio $\frac{d}{f}$ in eq. 230 is equal to the ratio $\frac{2c}{f}$ in the 46th and succeeding equations, we obtain the following values for the strength of long pillars† of various sections:—

* Mr. Hodgkinson's experiments show that this investigation is not applicable to pillars whose length is less than fifty diameters if made of cast-iron, or eighty diameters if made of wrought-iron.

† Calling the diameter unity, it may be shown that the lateral deflection of a very long pillar per unit of its length = ¼th of the shortening of the concave side, or ¼th of the extension of the convex side, per linear unit, in the following manner :—

Let R = the radius of curvature,

δ = the lateral deflection of a unit of length,

λ' = the longitudinal shortening or extension per linear unit,

and the other symbols as before ;

$$\text{from } (b),\quad \mathsf{D} = \frac{\lambda l}{8d} \text{ or, since } d = \text{unity, } = \frac{\lambda' l^2}{4}$$

$$\text{also,}\quad \delta = \frac{1}{8\mathsf{R}},\ \text{and } \mathsf{D} = \frac{l^2}{8\mathsf{R}}$$

$$\therefore \delta = \frac{\mathsf{D}}{l^2} = \frac{\lambda'}{4}$$

307. Long solid rectangular pillars—Long solid round pillars—Long hollow round pillars—Strength of long pillars depends on the coefficient of elasticity.—From equations 46 and 230 we have for long solid rectangular pillars,

$$\mathsf{W} = \frac{2\mathsf{E}bd^3}{3l^2} \tag{231}$$

where $d =$ the shortest side.

From equations 48 and 230 we have for long solid round pillars,

$$\mathsf{W} = \frac{\pi\mathsf{E}d^4}{8l^2} \tag{232}$$

where $d =$ the diameter of the pillar.

From equations 49 and 230 we have for long hollow round pillars

$$\mathsf{W} = \frac{\pi\mathsf{E}(d^4 - d_1{}^4)}{8l^2} \tag{233}$$

where $d =$ the external diameter,

$d_1 =$ the internal diameter.

These equations prove that the strength of very long square or round pillars varies as the fourth power of their diameter divided by the square of their length, and the longer the pillar is in proportion to its diameter, the closer will these equations represent the truth; in such pillars the neutral surface will not lie far from the central axis, and the deflecting weight, W, will be small compared to that which would crush a very short pillar of the same diameter. It is also to be observed that the strength of very long pillars depends, not on the strength of the material, but on E, which represents its stiffness and capability of resisting flexure. This theoretic result agrees with the fact that, although a short round pillar of cast-iron will bear a much greater weight than a similar pillar of wrought-iron, because the crushing strength of cast-iron is from two to three times greater than that of wrought-iron, yet a solid wrought-iron pillar over 26 diameters in length will support a greater weight than a similar one of cast-iron, because the coefficient of elasticity of wrought-iron is considerably higher than that of cast-iron (**332**).

308. Strength of similar long pillars are as their transverse areas—Weights of long pillars of equal strength and similar in section, but of different lengths, are as the squares

of their lengths.—These equations also prove that the strengths of similar long pillars are as the squares of any linear dimension, that is, as their transverse areas; while their weights are as the cubes of any linear dimension. Further, if the strengths of long pillars of similar section remain constant while their lengths vary, their transverse areas will vary as their lengths, and their weights therefore will vary as the squares of their lengths.

309. Weight which will deflect a very long pillar is very near the breaking weight.—It appears from eq. (b) that, if a very long pillar be bent in different degrees, D will vary as λ, that is, as f (7); and, from eq. (a), $\mathsf{W} = \frac{\mathsf{M}}{\mathsf{D}}$, which is constant, since M also varies as f; hence it follows, that W, the weight which keeps the pillar bent, is nearly the same whether the flexure be greater or less. This statement would be accurately true were it not that the assumptions on which eqs. (a) and (b) are based and the law of elasticity are only approximate. It will, however, agree very closely with experiment when the pillar is long enough to allow D to be considerable, even though the curvature be small. From this it follows, that any weight which produces moderate flexure in a very long pillar will also be very near the breaking weight, as a trifling additional load will bend the pillar very much more, and strain the fibres beyond what they can bear. This theoretic result is in accordance with the following observation of Mr. Hodgkinson:—"From the first experiment on long hollow pillars with rounded ends, it was evident that so little flexure of the pillar was necessary to overcome its greatest resistance (and beyond this a smaller weight would have broken it), that the elasticity of the pillars was very little injured by the pressure, if the weight was prevented from acting upon the pillar after it began to sink rapidly, through its greatest resistance being overcome."*

As all the longitudinal forces at the middle of the pillar balance, we have the following equation:—

$$\mathsf{C} = \mathsf{T} + \mathsf{W}.$$

This enables us to predict how a very long pillar will fail, whether

* *Phil. Trans.*, 1840, p. 411.

by the convex side tearing asunder, or by the concave side crushing. A long wrought-iron pillar, for instance, may be expected to fail on the concave side, because its power to resist compression, *i.e.*, bulging, is less than that to resist extension. A long pillar of cast-iron, on the contrary, will probably fail by the convex side tearing asunder, because the compressive strength of cast-iron greatly exceeds its tenacity. This is corroborated by Mr. Hodgkinson's experiments on long hollow cast-iron pillars which "seldom gave way by compression."*

310. Pillars divided into three classes according to length.—Our knowledge of the laws of the resistance of pillars to flexure, though perhaps not so satisfactory in a theoretic point of view as might be desired, is, however, owing to Mr. Hodgkinson's able investigations, aided by the liberality of Sir William Fairbairn, the late Mr. R. Stephenson and the Royal Society, practically far enough advanced to enable us to predict with considerable accuracy the strength of pillars of the usual forms. The results of these investigations are here given; the reader who desires more detailed information respecting the experiments, is referred to Mr. Hodgkinson's original papers,† in which he divides pillars into three classes according to length:—

1°. *Short pillars*, whose length (if cast-iron, under four or five diameters) is so small compared with their diameter that they fail by actual crushing of the material, not by flexure; the strength of these has been already investigated in the previous chapter.

2°. *Long flexible pillars*, whose length is so great (if cast-iron, thirty diameters and upwards when both ends are flat, fifteen diameters and upwards when both ends are rounded,) that they fail by flexure like girders subject to transverse strain, the breaking weight being far short of that required to crush the material when in short pieces.

* *Phil. Trans.*, 1840, p. 409.

† *Report of the British Association*, Vol. vi.—*Philosophical Transactions*, 1840 and 1857.—*Experimental Researches on the strength and other properties of Cast-Iron.* By E. Hodgkinson, F.R.S. London, 1846.—*Report of the Commissioners appointed to inquire into the application of Iron to Railway Structures*, 1849.

3°. *Medium, or short flexible pillars*, whose length is such that, though they deflect, yet the breaking weight is a considerable portion of that required to crush *short* pillars. This class includes all pillars which are intermediate in length between those in the first two classes, and they may be said to fail partly by flexure and partly by crushing.

In the following remarks the passages in inverted commas are verbatim extracts from Mr. Hodgkinson's writings.

LONG PILLARS WHICH FAIL BY FLEXURE; LENGTH, IF BOTH ENDS ARE FLAT AND FIRMLY BEDDED, EXCEEDING 30 DIAMETERS FOR CAST-IRON AND TIMBER, AND 60 DIAMETERS FOR WROUGHT-IRON.

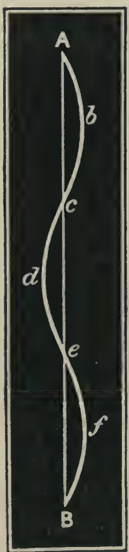
311. Long pillars with flat ends firmly bedded are three times stronger than pillars with round ends.—"In all *long* pillars of the same dimensions, the resistance to fracture by flexure is about three times greater when the ends of the pillars are flat and firmly bedded, than when they are rounded and capable of turning."—*Exp. Res.*, p. 332. From this it follows, that pillars like the jib of a crane would be much stronger if their ends were fixed; there is, however, a practical advantage sometimes in having them jointed for the purpose of altering the range or height of the jib.

312. Strength of pillars with one end round and the other flat is a mean between that of a pillar with both ends round and one with both ends flat.—"The strength of a pillar, with one end round and the other flat, is the arithmetical mean between that of a pillar of the same dimensions with both ends rounded, and with both ends flat. Thus, of three cylindrical pillars, all of the same length and diameter, the first having its ends rounded, the second with one end rounded and one flat, and the third with both ends flat, the strengths are as 1, 2, 3, nearly."—*Exp. Res.*, p. 332. This law applies to medium as well as to long pillars, but in the medium pillars the strength of those with flat ends varies from 3 to 1·5 times that of those with rounded ends, or less according as we reduce the number of times which the length exceeds the diameter.—*Phil. Trans.*, 1840, pp. 389, 421.

313. A long pillar with ends firmly fixed is as strong as a pillar of half the length with round ends.—" A *long* uniform pillar, with its ends firmly fixed, whether by discs or otherwise, has the same power to resist breaking as a pillar of the same diameter, and half the length, with the ends rounded or turned so that the force would pass through the axis."—*Exp. Res.*, p. 332.

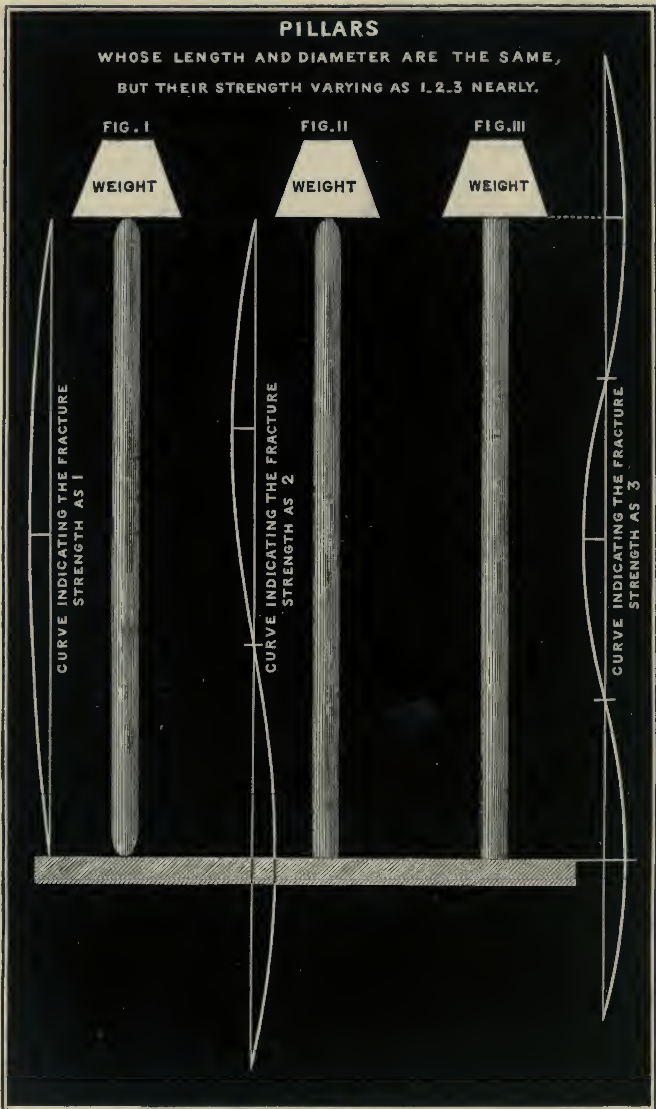
Of this fact Mr. Hodgkinson offers the following explanation :— "Suppose a long uniform bar of cast-iron were bent by a pressure at its ends so as to take the form A*bcdef*B, where all the curves A*bc*, *cde*, *ef*B, separated by the straight line A*ce*B, would be equal, since the bar was supposed to be uniform. The curve having taken this form, suppose it to be rendered immovable at the points *b* and *f*, by some firm fixings at those points. This done, it is evident we may remove the parts near to A and B, without at all altering the curve *bcdef* of the part of the pillar between *b* and *f*, and consider only that part. The part *bf*, which alone we shall have to consider, will be equally bent at all the points *b*,*d*,*f*. The points *c* and *e* too are points of contrary flexure, consequently the pillar is not bent in them. These points are unconstrained except by the pressure which forces them together, and the pillar might be reduced to any degree in them, provided they were not crushed or detruded by the compressing force. These points may then be conceived as acting like the rounded ends of the pillars, and the part *cde* of the pillar, with its ends *c* and *e* rounded, will be bearing the same weight as the whole pillar *bcdef* of double the length with its ends, *bf*, firmly fixed."—*Phil. Trans.*, 1857, p. 855.

Fig. 101.



314. Hodgkinson's laws apply to cast-iron, steel, wrought-iron, and wood.—" The preceding properties were found to exist in *long* pillars of steel, wrought-iron and wood," as well as cast-iron. They apply only to pillars whose length is so great in proportion to their diameter that the breaking unit-strain of the pillar is far short (for cast-iron not exceeding one-fourth) of the crushing unit-strain of the material.—*Exp. Res.*, pp. 333, 341.

PLATE III.


PILLARS
WHOSE LENGTH AND DIAMETER ARE THE SAME,
BUT THEIR STRENGTH VARYING AS 1.2.3 NEARLY.
FIG. I
FIG. II
FIG. III
WEIGHT
WEIGHT
WEIGHT
CURVE INDICATING THE FRACTURE STRENGTH AS 1
CURVE INDICATING THE FRACTURE STRENGTH AS 2
CURVE INDICATING THE FRACTURE STRENGTH AS 3

315. Position of fracture in long cast-iron pillars.—Long uniform cast-iron pillars with both ends round break in one place only—the middle; those with both ends flat in three—at the middle and near each end; those with one end round and one flat, at about one-third of the distance from the round end. Plate III. represents the curves indicating the form of flexure in each class of pillar.—*Phil. Trans.*, 1857, p. 858.

316. Discs on the ends add but little to the strength of flat-ended pillars.—Cast-iron pillars with discs on their ends are somewhat stronger than those with merely flat ends, but the difference of strength is trifling.—*Phil. Trans.*, 1840, p. 391.

317. Enlarging the diameter in the middle of solid pillars increases their strength slightly.—"In all the (solid cast-iron) pillars with rounded ends, those with increased middles were stronger than uniform pillars of the same weight, the increase being about one-seventh of the weight borne by the former." This increase of strength was more marked in pillars with rounded ends than in those with discs, for "in the pillars with discs, those with the middle but little increased had no advantage, with regard to strength, over the uniform ones. But the pillars with the middle diameter half as great again as the end ones bore from one-eighth to one-ninth more than uniform pillars of the same weight with discs upon the ends."—*Phil. Trans.*, 1840, p. 395.

318. Enlarging the diameter in the middle or at one end of hollow pillars does not increase their strength.—In hollow (cast-iron) pillars of greater diameter at one end than the other, or in the middle than at the ends, it was not found that any additional strength was obtained over that of uniform cylindrical pillars."—*Exp. Res.*, p. 349.

319. Solid square cast-iron pillars yield in the direction of their diagonals.—Solid "square (cast-iron) pillars do not bend or break in a direction parallel to their sides, but to their diagonals, nearly."—*Exp. Res.*, p. 331.

320. Long pillars irregularly fixed lose from two-thirds to four-fifths of their strength.—"A (long) pillar irregularly fixed, so that the pressure would be in the direction of the diagonal, is reduced to one-third of its strength, the case being nearly similar to that of a (long) pillar with rounded ends, the strength of which has been shown to be only $\frac{1}{3}$rd of that of a pillar with flat ends."—*Exp. Res.*, p. 350. And in two experiments on long solid cast-iron pillars with the ends formed so that the pressure would not pass through the axis, but in lines one-fourth of the diameter and one-eighth of the diameter respectively from one side, the breaking weights were little more than one-half that of a pillar of the same dimensions with the ends turned so that the force would pass through the axis, that is, their strength was reduced to about $\frac{1}{3}$th of that of a similar flat-bedded pillar.—*Phil. Trans.*, 1840, pp. 413, 449.

321. Strength of similar long pillars is as their transverse area.—The strength of similar long pillars is nearly as the area of their transverse section. As derived from Mr. Hodgkinson's experiments on cast-iron, the strength varied as the 1·865 power of the diameter or any other linear dimensions.—*Exp. Res.*, p. 346. This has already been proved theoretically in **308**.

CAST-IRON PILLARS.

322. Hodgkinson's rules for solid or hollow round cast-iron pillars whose length exceeds 30 diameters.—The following formulæ have been deduced by Mr. Hodgkinson from his experiments to represent the breaking weights of pillars with both ends *flat* and *well bedded*, and whose lengths exceed 30 diameters.* If the ends are rounded or otherwise insecurely bedded, the breaking weight given by the formulæ must be divided by 3 (**311**).

Let W = the breaking weight in tons,
l = the length of the pillar in feet,
d = the external diameter in inches,
d_1 = the internal diameter of hollow pillars in inches,
m = a coefficient varying with the quality of the cast-iron, and derived from experiment.

Long solid round pillars of cast-iron.

$$W = m\frac{d^{3.5}}{l^{1.63}} \tag{234}$$

* *Phil. Trans.*, 1857, pp. 862, 872.

Long hollow round pillars of *Low Moor* cast-iron, No. 2.*

$$W = 42{\cdot}347 \frac{d^{3.5} - d_1^{\,3.5}}{l^{1.63}} \tag{235}$$

Ex. What is the breaking weight of a solid round cast-iron pillar 10 feet long and 2 inches in diameter? From table I., $m = 42{\cdot}6$ tons,

$$\textit{Answer} \text{ (eq. 234), } W = m\,\frac{2^{3.5}}{10^{1.63}} = 42{\cdot}6\,\frac{11{\cdot}314}{42{\cdot}66} = 11{\cdot}3 \text{ tons.}$$

If the pillar be not very securely fixed at the ends, the breaking weight will $= \frac{11{\cdot}3}{3} = 3{\cdot}77$ tons, and the safe load in practice will be $\frac{1}{6}$th of this $= {\cdot}63$ tons, provided the pillar is not subject to vibration, in which case the safe load will be only $\frac{1}{12}$th $= 0{\cdot}314$ tons.

The three following tables contain the values of the coefficient m, derived from experiments on solid pillars of cast-iron 10 feet long and $2\frac{1}{2}$ inches diameter, with their ends flat; also the powers of diameters and lengths of pillars.—*Phil. Trans.*, 1857, pp. 872 and 860.

TABLE I.—Coefficients m in eq. 234 (representing the strength of a pillar 1 foot long and 1 inch in diameter.

Description of iron.	Value of coefficient m.
	lbs. tons.
Old Park iron, No. 1. Stourbridge—cold blast, - - - - -	111858 = 49·94
Derwent iron, No. 1. Durham—hot blast, - - - - - -	105079 = 46·91
Portland iron, No. 1. Tovine, Scotland—hot blast, - - - - -	104098 = 46·47
Calder iron, No. 1. Lanarkshire—hot blast, - - - - - -	104137 = 46·49
London mixture. One-half old plate iron, and one-half Calder iron, - -	92862 = 41·46
Level iron, No. 1. Staffordshire—hot blast, - - - - -	94202 = 42·05

* "The pillars from this iron were cast 10 feet long, and from $2\frac{1}{2}$ to 4 inches diameter, approaching in some degree, as to size, to the smaller ones used in practice." —*Proc. Roy. Soc.*, Vol. viii., p. 319.

TABLE I.—COEFFICIENTS m in eq. 234—*continued.*

Description of iron.	Value of coefficient m.
Coltness iron, No. 1. Edinburgh—hot blast, - - - - - -	90119 = 40·23
Carron iron, No. 1. County of Stirling—hot blast, - - - - -	89949 = 40·16
Blaenavon iron, No. 1. South Wales—cold blast, - - - - -	86114 = 38·44
Old Hill iron, No. 1. Staffordshire—cold blast, - - - - -	75270 = 33·60
Second London mixture. One-third No. 1 best Scotch pig-iron, and two-thirds old metal, -	104623 = 46·21
Low Moor iron, No. 2. Yorkshire—cold blast, - - - - - -	90674 = 40·48
Blaenavon iron, No. 3. South Wales—cold blast, - - - - -	92329 = 41·22
Mean of 13 irons, - - - - - -	95486 = 42·6

TABLE II.—POWERS OF DIAMETERS, OR $d^{3·5}$.

$1·0^{3·5} = 1·0000$	$4·25^{3·5} = 158·26$	$6·8^{3·5} = 819·94$
$1·25^{3·5} = 2·1837$	$4·3^{3·5} = 164·87$	$6·9^{3·5} = 862·92$
$1·5^{3·5} = 4·1335$	$4·4^{3·5} = 178·68$	$7·0^{3·5} = 907·49$
$1·75^{3·5} = 7·0898$	$4·5^{3·5} = 193·305$	$7·1^{3·5} = 953·68$
$2·0^{3·5} = 11·314$	$4·6^{3·5} = 208·76$	$7·2^{3·5} = 1001·53$
$2·1^{3·5} = 13·4205$	$4·7^{3·5} = 225·08$	$7·25^{3·5} = 1026·08$
$2·2^{3·5} = 15·7935$	$4·75^{3·5} = 233·58$	$7·3^{3·5} = 1051·07$
$2·25^{3·5} = 17·086$	$4.8^{3·5} = 242·295$	$7·4^{3·5} = 1102·33$
$2·3^{3·5} = 18·452$	$4·9^{3·5} = 260·43$	$7·5^{3·5} = 1155·35$
$2·4^{3·5} = 21·416$	$5·0^{3·5} = 279·51$	$7·6^{3·5} = 1210·17$

TABLE II.—POWERS OF DIAMETERS, OR $d^{3.5}$—*continued.*

$2.5^{3.5} = 24.705$	$5.1^{3.5} = 299.57$	$7.7^{3.5} = 1266.83$
$2.6^{3.5} = 28.340$	$5.2^{3.5} = 320.635$	$7.75^{3.5} = 1295.85$
$2.7^{3.5} = 32.3425$	$5.25^{3.5} = 331.56$	$7.8^{3.5} = 1325.35$
$2.75^{3.5} = 34.488$	$5.3^{3.5} = 342.74$	$7.9^{3.5} = 1385.78$
$2.8^{3.5} = 36.733$	$5.4^{3.5} = 365.91$	$8.0^{3.5} = 1448.15$
$2.9^{3.5} = 41.533$	$5.5^{3.5} = 390.18$	$8.25^{3.5} = 1612.83$
$3.0^{3.5} = 46.765$	$5.6^{3.5} = 415.58$	$8.5^{3.5} = 1790.47$
$3.1^{3.5} = 52.4525$	$5.7^{3.5} = 442.14$	$8.75^{3.5} = 1981.66$
$3.2^{3.5} = 58.617$	$5.75^{3.5} = 455.87$	$9.0^{3.5} = 2187.00$
$3.25^{3.5} = 61.886$	$5.8^{3.5} = 469.89$	$9.25^{3.5} = 2407.11$
$3.3^{3.5} = 65.283$	$5.9^{3.5} = 498.86$	$9.5^{3.5} = 2642.61$
$3.4^{3.5} = 72.473$	$6.0^{3.5} = 529.09$	$9.75^{3.5} = 2894.12$
$3.5^{3.5} = 80.212$	$6.1^{3.5} = 560.60$	$10.0^{3.5} = 3162.28$
$3.6^{3.5} = 88.5235$	$6.2^{3.5} = 593.43$	$10.25^{3.5} = 3447.73$
$3.7^{3.5} = 97.433$	$6.25^{3.5} = 610.35$	$10.5^{3.5} = 3751.13$
$3.75^{3.5} = 102.12$	$6.3^{3.5} = 627.61$	$10.75^{3.5} = 4073.14$
$3.8^{3.5} = 106.965$	$6.4^{3.5} = 663.18$	$11.0^{3.5} = 4414.43$
$3.9^{3.5} = 117.15$	$6.5^{3.5} = 700.16$	$11.25^{3.5} = 4775.66$
$4.0^{3.5} = 128.00$	$6.6^{3.5} = 738.59$	$11.5^{3.5} = 5157.54$
$4.1^{3.5} = 139.55$	$6.7^{3.5} = 778.51$	$11.75^{3.5} = 5560.74$
$4.2^{3.5} = 151.835$	$6.75^{3.5} = 799.03$	$12.0^{3.5} = 5985.96$

TABLE III.—POWERS OF LENGTHS, OR $l^{1.63}$.

$1^{1.63} = 1.$	$7\frac{1}{2}^{1.63} = 26.6901$	$16^{1.63} = 91.7731$
$2^{1.63} = 3.0951$	$8^{1.63} = 29.6508$	$17^{1.63} = 101.305$
$2\frac{1}{2}^{1.63} = 4.4529$	$9^{1.63} = 35.9265$	$18^{1.63} = 111.197$
$3^{1.63} = 5.9939$	$10^{1.63} = 42.6580$	$19^{1.63} = 121.442$
$4^{1.63} = 9.5798$	$11^{1.63} = 49.8276$	$20^{1.63} = 132.032$
$5^{1.63} = 13.7823$	$12^{1.63} = 57.4203$	$21^{1.63} = 142.961$
$6^{1.63} = 18.5518$	$13^{1.63} = 65.4226$	$22^{1.63} = 154.223$
$6\frac{1}{4}^{1.63} = 19.8282$	$14^{1.63} = 73.8225$	$23^{1.63} = 165.812$
$7^{1.63} = 23.8512$	$15^{1.63} = 82.6093$	$24^{1.63} = 177.723$

323. Hodgkinson's rules for solid or hollow round cast-iron pillars of medium length, *i.e.*, pillars whose length is less than 30 diameters, with both ends flat and well bedded.—" The formulæ above (eqs. 234, 235) apply to all (cast-iron) pillars whose length is not less than about 30 times the external diameter; for pillars shorter than this, it will be necessary to modify the formulæ by other considerations, since in these shorter pillars the breaking weight is a considerable proportion of that necessary to crush the pillar. Thus, considering the pillar as having two functions, one to support the weight, and the other to resist flexure, it follows that when the material is incompressible (supposing such to exist), or when the pressure necessary to break the pillar is very small, on account of the greatness of its length compared with its lateral dimensions, then the strength of the whole transverse section of the pillar will be employed in resisting flexure; when the breaking pressure is half of what would be required to crush the material, one half only of the strength may be considered as available for resistance to flexure, whilst the other half is employed to resist crushing; and when, through the shortness of the pillar, the breaking weight is so great as to be nearly equal to the crushing force, we may consider that no part of the strength of the pillar is applied to resist flexure."—*Exp. Res.*, p. 337. Acting on this view, Mr. Hodgkinson devised the following formula for the ultimate strength of medium pillars of cast-iron and timber whose length is less than 30 diameters, with both ends flat and well bedded.

$$\mathsf{W}' = \frac{\mathsf{W}c}{\mathsf{W} + \frac{3}{4}c} \tag{236}$$

where $\mathsf{W} =$ the breaking weight in tons derived from the formulæ for long pillars, on the hypothesis that the pillar yields by flexure alone,

$c =$ the crushing weight of a short length of the pillar, *i.e.*, its sectional area multiplied by the crushing unit-strain of the material in tons,

$\mathsf{W}' =$ the real breaking weight of the medium pillar in tons, from the combined effects of flexure and crushing

Ex. 1. What is the breaking weight of a solid pillar of Blaenavon iron, No. 3, 9 feet long and 6 inches in diameter, with flat ends carefully bedded, and whose crushing strength = 37·3 tons per square inch?

From Table I., $m = 41{\cdot}2$ tons,

$$c = 37{\cdot}3 \times 28{\cdot}3 = 1056 \text{ tons,}$$

$$\text{from eq. 234,} \quad W = 41{\cdot}2 \frac{529}{36} = 605 \text{ tons.}$$

$$\textit{Answer}, \text{(eq. 236). Breaking weight, } W' = \frac{605 \times 1056}{605 + 792} = 457 \text{ tons.}$$

If intended for a warehouse, the greatest load in practice should not exceed $\frac{1}{6}$th of this, = 76 tons, and that only when the ends are adjusted with the greatest care, so as to have a very uniform bearing; when this is not the case the effect will be the same as if the ends were rounded, in which case the breaking weight will be much less (**312**), probably only $\frac{W'}{2} = \frac{457}{2} = 228{\cdot}5$ tons, of which $\frac{1}{6}$th, or the safe working load, will = 38 tons.

Ex. 2. What is the breaking weight of a hollow flat-bedded pillar of the same iron, of the same height and external diameter, and whose internal diameter = 4 inches?

On examining Table II. (**294**), we find that the crushing strength of Blaenavon iron, No. 3, medium, = 37·3 tons per inch, while that of Low Moor, No. 2, medium, = 34·6 tons. We may therefore assume that the coefficient in eq. 235 for hollow cylinders of Blaenavon iron is the same as that for Low Moor.

$$\text{Here,} \quad c = 37{\cdot}3 \times 15{\cdot}7 = 586 \text{ tons,}$$

$$\text{from eq. 235,} \quad W = 42{\cdot}35 \frac{529 - 128}{36} = 472 \text{ tons nearly.}$$

$$\textit{Answer}, \text{(eq. 236). Breaking weight, } W' = \frac{472 \times 586}{472 + 440} = 303 \text{ tons,}$$

of which $\frac{1}{6}$th, or the working load, = 50·5 tons, *i.e.*, when the ends are fitted with extreme care; otherwise, $\frac{W'}{12} = 25{\cdot}25$ tons, is a sufficient load in ordinary practice.

324. A slight inequality in the thickness of hollow cast-iron pillars does not impair their strength materially—Rules for the thickness of hollow cast-iron pillars.—Referring to castings of unequal thickness, Mr. Hodgkinson remarks:—"In experiments upon hollow pillars it is frequently found that the metal on one side is much thinner than that on the other; but this does not produce so great a diminution in the strength as might be expected, for the thinner part of a casting is much harder than the thicker, and this usually becomes the compressed side."—*Phil. Trans.*, 1857, p. 862.

In practice, neither the excess or want of thickness should exceed 25 per cent. of the average thickness; if, for instance, a hollow pillar is specified to be 1 inch in thickness, then in no place should the metal be less than $\frac{3}{4}$ inch or more than $1\frac{1}{4}$ inch thick. General Morin gives the following rule, based on the founder's experience, for the *minimum* thickness of ordinary hollow cast-iron pillars*:—

Height of pillar in feet,	7 to 10	10 to 13	13 to 20	20 to 27
Minimum thickness in inches,	·5	·6	·8	1·0

Another practical rule is to make the thickness of metal in no case less than $\frac{1}{12}$th of the diameter of the pillar.

325. + and H shaped pillars.—A cast-iron pillar of the + form of section, "as in the connecting rod of a steam engine, the ends being movable, is very weak to bear a strain as a pillar, and indeed less than half the strength of a hollow cylindrical pillar of the same weight and length, rounded at the ends."—*Phil. Trans.*, 1857, p. 893.—*Exp. Res.*, p. 350.

A cast-iron pillar of the H form of section with rounded ends was found to be "considerably stronger than the preceding, but much weaker than a hollow cylinder of the same weight." Their relative strengths, according to Mr. Hodgkinson's experiments, were in the following proportions, all the pillars being of the same weight and length and rounded at the ends.—*Phil. Trans.*, 1840, pp. 413, 449.

Hollow cylindrical pillar, ...	100
H shaped pillar, ...	75
+ shaped pillar, ...	44

326. Relative strength of round, square, and triangular solid cast-iron pillars.—From a comparison of Mr. Hodgkinson's experiments it appears that long solid square cast-iron pillars are about 50 per cent. stronger than solid cylindrical pillars of the same length and of diameters equal to the sides of the squares, whereas their area, *i.e.*, their weight, is only 27 per cent. greater. This is equivalent to saying that the breaking unit-strain of a long solid square cast-iron pillar is 1·178 times that of the inscribed circular pillar of

* *Résistance des Matériaux*, p. 110.

equal length.—*Phil. Trans.*, 1840, pp. 431, 437. Solid triangular pillars of cast-iron with flat ends are somewhat stronger than those with either circular or square sections.—*Phil. Trans.*, 1857, p. 893. Their relative strengths, according to Mr. Hodgkinson's experiments, were in the following proportions, all the pillars being of the same weight and length:—

Long solid round pillar, ...	100
„ square „	93
„ triangular „	110

From this it follows that for practical purposes the round pillar is the most economical form of solid cast-iron pillar, since the shape of the triangle will generally prohibit its use.

327. Gordon's rules for pillars.—Professor Gordon has deduced from Mr. Hodgkinson's experiments the following convenient formulæ for the strength of pillars:—

Let f = the breaking weight per square unit of section, *i.e.*, the breaking unit-strain,
r = the ratio of length to diameter,
a and b = constants depending on the material and the section of the pillar.

1°. Pillars with both ends flat and bedded with extreme care.

$$f = \frac{a}{1 + br^2} \tag{237}$$

2°. Pillars with both ends jointed or imperfectly fixed.

$$f = \frac{a}{1 + 4br^2} \tag{238}$$

328. Solid or hollow round cast-iron pillars.—The values of the coefficients in Gordon's formulæ for solid or hollow cast-iron pillars are as follows:—

$$a = 36 \text{ tons}, \qquad b = \frac{1}{400}.$$

The following table has been calculated from these equations, and shows at a glance the breaking weight per square inch of solid or hollow round cast-iron pillars of various ratios of length to diameter.

TABLE IV.—FOR CALCULATING THE STRENGTH OF SOLID OR HOLLOW ROUND CAST-IRON PILLARS.

Ratio of length to diameter.		5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Breaking weight in tons per square inch.	Both ends flat and bedded with extreme care.	33·9	28·8	23	18	14	11	8·9	7·2	5·9	5·0	4·2	3·6	3·1	2·7	2·4	2·1
	Both ends jointed or imperfectly fixed.	28·8	18	11	7·2	5·0	3·6	2·7	2·1	1·7	1·4	1·15	·97	·83	·72	·63	·55

Ex. 1. What is the breaking weight of a solid round cast-iron pillar, 10 feet long and 2 inches in diameter? Here, the ratio of length to diameter = 60, and, if both ends are flat and bedded with extreme care or otherwise securely fixed, the corresponding breaking weight per square inch = 3·6 tons; multiplying this by the sectional area, we have,

Answer, Breaking weight $= 3{\cdot}1416 \times 3{\cdot}6 = 11{\cdot}3$ tons,

which agrees with the example in **322** calculated by Hodgkinson's rule.

If the ends are jointed or imperfectly fixed, we have,

Answer, Breaking weight $= 3{\cdot}1416 \times {\cdot}97 = 3{\cdot}05$ tons,

and the working load should in general not exceed one-sixth of this, = ·51 tons.

Ex. 2. What is the breaking weight of a hollow round cast-iron pillar 9 feet long, 6 inches external, and 4 inches internal, diameter? Here, the ratio of length to diameter = 18, and, if both ends are flat and bedded with extreme care, the corresponding breaking weight per square inch = 20 tons; multiplying this by the sectional area, = 15·7 square inches, we have,

Answer, Breaking weight $= 15{\cdot}7 \times 20 = 314$ tons.

If the ends are jointed, or are not flat bedded with extreme care, the breaking weight per square inch = 8·5 tons and we have

Answer, Breaking weight $= 15{\cdot}7 \times 8{\cdot}5 = 133{\cdot}45$ tons,

of which one-sixth, = 22·24 tons, will be the safe working load when free from vibration, as in a grain store; if the pillar supports a factory floor with machinery in motion, one-eighth, = 16·68 tons, will be a sufficient load; but if the pillar forms a moving part of an engine, then one-tenth, = 13·34 tons, or even less, will be the proper working load. The reader will observe that Gordon's rule in this example gives results which agree tolerably closely with the 2nd example in **323** calculated by Hodgkinson's rule.

Ex. 3. What is the breaking weight of a solid round cast-iron pillar, 9 feet long and 6 inches in diameter, with both ends solidly imbedded? Here, the ratio of length to diameter = 18, and the corresponding breaking weight per square inch is 20 tons, and we have,

$$\textit{Answer}, \quad \text{Breaking weight} = \frac{3{\cdot}1416 \times 6 \times 6 \times 20}{4} = 565{\cdot}5 \text{ tons.}$$

This, it will be observed, is nearly 24 per cent. higher than the 457 tons in example 1, (**323**); no doubt, because Professor Gordon's rule applies to average mixed irons, which are in general stronger than simple irons, such as Blaenavon.

329. Solid or hollow rectangular cast-iron pillars.—It appears from Mr. Hodgkinson's experiments that the breaking unit-strain of a long solid square cast-iron pillar is 1·178 times that of the inscribed circular pillar of equal length (**326**), and, guided by this, we may modify Gordon's formulæ to suit rectangular pillars by making $r =$ the ratio of length to least breadth, and

$$b = \frac{1}{500}$$

The following table has been calculated on this basis, and gives the breaking weight per square inch of solid or hollow rectangular cast-iron pillars of various ratios of length to breadth.

TABLE V.—For Calculating the Strength of Solid or Hollow Rectangular Cast-iron Pillars.

Ratio of length to least breadth.		5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Breaking weight in tons per square inch.	Both ends flat and bedded with extreme care.	34·3	30	24·8	20	16	12·9	10·4	8·6	7·1	6·0	5·1	4·4	3·8	3·3	2·9	2·6
	Both ends jointed or imperfectly fixed.	30	20	12·9	8·6	6·0	4·4	3·3	2·6	2·1	1·7	1·4	1·2	1·0	·90	·78	·69

Ex. 1. What is the breaking weight of a solid cast-iron pillar, 10 feet long and 2 inches square? Here, the ratio of length to breadth = 60, and, if both ends are securely fixed, the corresponding breaking weight per square inch = 4·4 tons; multiplying this by the area, we have,

Answer, Breaking weight = 4 × 4·4 = 17·6 tons.

If the ends are imperfectly fixed, we have,

Answer, Breaking weight = 4 × 1·2 — 4·8 tons.

Of which, in general, one-sixth, = ·8 tons, will be the proper working load.

Ex. 2. What is the breaking weight of a hollow cast-iron pillar, 9 feet long, 6 inches square, with metal one inch thick? Here, the ratio of length to breadth = 18, and, if both ends are flat and bedded with extreme care, the corresponding breaking weight per square inch = 21·84 tons. Multiplying this by the area, = 20 square inches, we have,

Answer, Breaking weight = 20 × 21·84 = 436·8 tons.

If the ends are not very carefully bedded, the breaking weight per square inch = 10·02 tons, and we have,

Answer, Breaking weight = 20 × 10·02 = 200·4 tons, of which one-sixth, = 33·4 tons, will be the safe working load for ordinary warehouses, when free from vibration.

For the safe working load on cast-iron pillars see Chap. XXVIII.

WROUGHT-IRON PILLARS.

330. Solid wrought-iron pillars.—Professor Gordon's formulæ in **327** may be applied to solid rectangular wrought-iron pillars by giving the coefficients the following values,

$$a = 16 \text{ tons} \qquad b = \frac{1}{3000}$$

The following table has been calculated from these formulæ, and gives the breaking weight per square inch of solid rectangular wrought-iron pillars of various ratios of length to least breadth.

TABLE VI.—For Calculating the Strength of Solid Rectangular Wrought-iron Pillars.

Ratio of length to least breadth,		5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
Breaking weight in tons per square inch.	Both ends flat and bedded with extreme care, - -	15·8	15·5	15·	14·1	13·2	12·3	11·3	10·4	9·5	8·7	7·7	7·3	6·6	6·08	5·6	5·1
	Both ends jointed or imperfectly fixed,	15·5	14·1	12·3	10·4	8·7	7·3	6·1	5·1	4·3	3·7	3·2	2·76	2·4	2·1	1·9	1·7

Ex. 1. What is the breaking weight of a solid square pillar of wrought-iron, 10 feet long and 2 inches square? Here, the ratio of length to breadth = 60, and the corresponding breaking weight per square inch, if both ends are very securely fixed, = 7·3 tons; multiplying this by the sectional area, we have,

Answer, Breaking weight = 4 × 7·3 = 29·2 tons.

If the ends are jointed or imperfectly fixed, we have,

Answer, Breaking weight = 4 × 2·76 = 11·04 tons,

of which one-fourth, = 2·76 tons, will be the safe working load if the pillar be free from vibration, but if liable to shocks like the jib of a crane, one-sixth, = 1·84 tons, will be enough. If, however, the bar forms a moving part of machinery, such as the connecting rod of a steam engine, one-twelfth, = ·92 tons, will generally be a sufficient load.

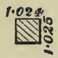
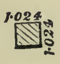
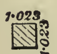
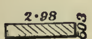
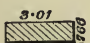
Ex. 2. What is the breaking weight of a rectangular pillar of wrought-iron, 10 feet long, and whose sectional area = 4 × 3 inches, with the ends securely riveted to a fixed structure? Here, the ratio of length to least breadth = 40, and the corresponding breaking weight per square inch = 10·4 tons; multiplying this by the area, we have,

Answer, Breaking weight = 4 × 3 × 10·4 = 124·8 tons.

Of this, one-fourth, = 31·2 tons, will be sufficient in practice for a stationary load, and that only when the ends are rigidly secured.

The following table, arranged in a convenient form by Mr. G. Berkley, M.I.C.E., contains the results of experiments on the compressive strength of solid rectangular wrought-iron bars, with their ends perfectly flat and well-bedded, which were made under Mr. Hodgkinson's supervision during the experimental inquiry respecting the Britannia and Conway tubular bridges.*

TABLE VII.—HODGKINSON'S EXPERIMENTS ON SOLID RECTANGULAR WROUGHT-IRON PILLARS.

Form of section.	Length.	Least breadth.	Sectional area.	Ratio of length to least breadth.	Breaking weight.	Breaking weight per square inch of area.
	inches. 90·0	inches. 1·024	sq. ins. 1·049	88·0	lbs. 10,236 tons 4·57	lbs. 9,753 tons 4·354
	60·0	1·024	1·0486	58·6	18,106 „ 8·083	17,268 „ 7·709
	30·0	1·023	1·0475	29·3	26,530 „ 11·843	25,327 „ 11·307
	15·0	1·023	1·0465	14·6	36,162 „ 16·144	34,554 „ 15·426
	7·5	1·023	1·0465	7·3	50,946 „ 22·744	48,682 „ 21·733
	3·75	1·023	1·0465	3·65	Bore 23·549 tons, = 22·5 tons per sq. in., without fracture.	... ...
	120·0	·503	1·498	238·56	1,222 „ ·545	8,157 „ ·363
	120·0	·766	2·306	156·6	7,793 „ 3·479	3,379 „ 1·5

* *Proc. Inst. C. E.*, Vol. xxx.

TABLE VII.—HODGKINSON'S EXPERIMENTS ON SOLID RECTANGULAR WROUGHT-IRON PILLARS—*continued.*

Form of section.	Length.	Least breadth.	Sectional area.	Ratio of length to least breadth.	Breaking weight.	Breaking weight per square inch of area.
	inches.	inches.	sq. ins.		lbs.	lbs.
2·995 ·995	120·0	·995	2·975	120·0	12,735 tons 5·685	4,280 tons 1·91
3·0 1·51	120·0	1·51	4·53	80·0	46,050 „ 20·558	10,165 „ 4·537
2·983 ·5023	90·0	·5023	1·498	179·0	3,614 „ 1·613	2,410 „ 1·076
3·005 ·9955	90·0	·9955	2·9915	90·0	29,619 „ 13·22	9,912 „ 4·425
3·0 1·53	90·0	1·53	4·59	59·0	91,746 „ 40·958	19,987 „ 8·923
5·86 ·995	90·0	·995	5·8307	90·0	54,114 „ 24·158	9,280 „ 4·143
2·98 ·507	60·0	·507	1·511	118·0	8,469 „ 3·78	5,604 „ 2·502
2·98 ·507	60·0	·507	1·498	119·28	8,496 „ 3·792	5,653 „ 2·523
3·01 ·767	60·0	·767	2·309	78·0	29,955 „ 13·372	12,969 „ 5·79
3·01 ·995	60·0	·995	2·995	60·0	54,114 „ 24·158	18,067 „ 8·066
5·84 ·996	60·0	·996	5·8166	60·0	102,946 „ 45·958	17,698 „ 7·901
2·986 ·5026	30·0	·5026	1·5011	60·0	25,299 „ 11·294	16,853 „ 7·524
3·01 ·763	30·0	·763	2·297	39·0	63,786 „ 28·476	27,767 „ 12·396
3·0 ·996	30·0	·996	2·988	30·0	88,610 „ 39·558	29,655 „ 13·239

I have made the following abstract from the foregoing experiments in order to show how closely they corroborate Gordon's formulæ when applied to solid rectangular wrought-iron pillars.

TABLE VIII.—Table derived from Hodgkinson's Experiments on Solid Rectangular Wrought-iron Pillars carefully bedded.

Proportion of length to least breadth, . . .	7	15	30	40	60	80	90	120	160	180
Breaking weight per square inch in tons, . .	22	15	12	10	7·5	5	4·3	2·2	1·5	1

The breaking unit-strain of solid round wrought-iron pillars is probably from 15 to 20 per cent. less than those given in Table VI. for rectangular pillars.

331. Solid wrought-iron pillars stronger than cast-iron pillars when the length exceeds 15 diameters.—Comparing Tables V. and VI. which represent the relative strengths of solid rectangular cast and wrought-iron pillars, we find that a cast-iron pillar with round ends is stronger than one of wrought-iron when the length is under 15 diameters, but above that ratio, wrought-iron is the stronger of the two, thus corroborating the theoretic result previously arrived at in **307**.

332. Pillars of angle, tee, channel and cruciform iron.—Mr. Unwin has deduced from experiments made by Mr. Davies of the Crumlin Works the following values for the coefficients of Gordon's formulæ in **327**, when applied to pillars of angle, tee, channel and cruciform wrought-iron.*

$$a = 19 \text{ tons}, \qquad b = \frac{1}{900}$$

In each of these sections the least diameter for calculation is to be measured in that direction in which the pillar is most flexible. This may be found by taking the shortest diameter of a rectangle or triangle circumscribed about the section. The following tables exhibit the results of Mr. Davies' experiments reduced to a convenient form by Mr. Berkley.†

* *Iron Bridges and Roofs*, p. 50.

† *Proc. Inst. C. E.*, Vol. xxx.

TABLE IX.—DAVIES' EXPERIMENTS ON ANGLE-IRON PILLARS.

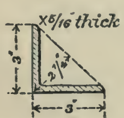
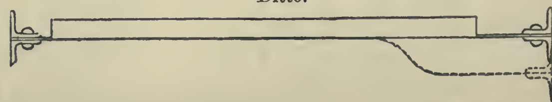
Form of section.	Length.	Least diameter.	Sectional area.	Thickness of metal.	Ratio of length to least diameter.	Ratio of least diameter to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.	Observations.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.	
	18·0	2·25	1·78	·3125	8·0	7·2	70,134 tons 31·30	39,401 tons 17·589	1 foot 6 inches. 
Ditto.	36·0	2·25	1·78	·3125	16·0	7·2	62,989 „ 28·12	35,381 „ 15·795	3 feet. 
Ditto.	48·0	2·25	1·78	·3125	21·3	7·2	52,483 „ 23·429	29,484 „ 13·162	4 feet. 
Ditto.	60·0	2·25	1·78	·3125	26·7	7·2	42,000 „ 18·75	23,595 „ 10·533	5 feet. 
Ditto.	60·0	2·25	1·78	·3125	26·7	7·2	27,440 „ 12·25	15,415 „ 6·88	Ditto. 

TABLE X.—DAVIES' EXPERIMENTS ON TEE AND CHANNEL IRON PILLARS.

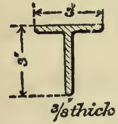
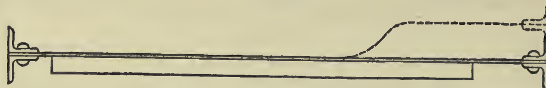
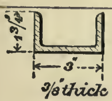
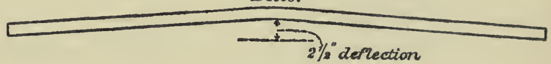
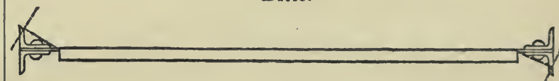
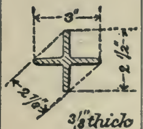
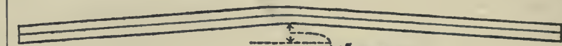
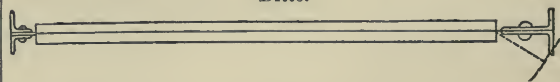
Form of section.	Length.	Least diameter.	Sectional area.	Thickness of metal.	Ratio of length to least diameter.	Ratio of least diameter to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.	Observations.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.	
 3/8" thick	60·0	3·0	2·1	·375	20·0	8·0	62,989 tons 28·12	29,994 tons 13·39	5 feet. 
Ditto.	60·0	3·0	2·1	·375	20·0	8·0	42,000 „ 18·75	20,000 „ 8·92	Ditto. 
CHANNEL IRON.									
 3/8" thick	60·0	1·75	2·15	·375	34·2	4·6	38,304 tons 17·1	17,815 tons 7·953	Ditto.  2½" deflection
Ditto.	60·0	1·75	2·15	·375	34·2	4·6	31,360 „ 14·0	14,586 „ 6·5	Ditto. 

TABLE XI.—DAVIES' EXPERIMENTS ON CRUCIFORM-IRON PILLARS.

Form of section.	Length.	Least diameter.	Sectional area.	Thickness of metal.	Ratio of length to least diameter.	Ratio of least diameter to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.	Observations.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.	
	60·0	2·125	1·88	·375	28·2	5·66	38,304 Tons 17·1	20,379 Tons 9·097	5 feet. 
Ditto.	60·0	2·125	1·88	·375	28·2	5·66	34,944 " 15·6	18,587 " 8·3	Ditto. 

333. Resistance of long plates to flexure.—An isolated plate under compression may be regarded as a wide rectangular pillar, or as a number of square pillars placed side by side, and it will therefore follow the laws of pillars so far as deflection at right angles to its plane is concerned. Hence, the ultimate resistance of *long* unsupported plates to flexure is theoretically as the cube of the thickness multiplied by the breadth and inversely as the square of the length. Mr. Hodgkinson found that this closely agreed with his experiments on plates whose length exceeded 60 times their thickness, and which were so long that they failed by flexure with strains not exceeding 9 tons per square inch (see Table VII.).* If, however, the plates form the sides of a tube, this rule does not apply, since in that case they yield by buckling or wrinkling of a short length and not by flexure, being held in the line of thrust by the adjacent sides which enable them to bear a greater unit-strain than if not so supported along their edges.

334. Strength of rectangular wrought-iron tubular pillars is independent of their length within certain limits.—When the length of a rectangular wrought-iron tubular pillar does not exceed 30 times its least breadth, it fails by the bulging or buckling of a short portion of the plates, not by flexure of the pillar as a whole, and within this limit the strength of the tube seems nearly independent of its length. It is quite possible that the ratio of length to breadth of rectangular wrought-iron tubes might be considerably greater than 30 without very materially affecting their strength, but the recorded experiments do not extend sufficiently far to determine this point.

335. Crushing unit-strain of wrought-iron tubes depends upon the ratio between the thickness of the plate and the diameter or breadth of the tube—Safe working-strain of rectangular wrought-iron tubes.—The crushing unit-strain of a wrought-iron tubular pillar is generally greater the thicker the plates are in proportion to the diameter or breadth of the tube, and in most of the experimental rectangular tubes which sustained a compression of 10 tons per square inch or upwards the thickness

* *Com. Rep.*, p. 119.

of the plate was not less than one-thirtieth of the breadth of the tube. In the last experiment recorded in Table XII., a square tube, 8 feet long, 18 inches in breadth, and made of ½-inch plates united by angle-irons in the corners, sustained a compressive strain of 13·6 tons per square inch. Unfortunately there were no further experiments made on tubes thus strengthened at the angles. From this and other experiments, but especially from one made during the construction of the Boyne Viaduct to test the strength of a braced pillar, and which is described in the appendix at the end of this volume, I infer that the strongest form of rectangular cell to resist buckling is one in whose angles the chief part of the material is concentrated, making the sides of plating or lattice work to withstand flexure of the angles, in which case the sides act the part of the web, and the angles act as the flanges of a girder.

From what has been said we may conclude that a rectangular plate-iron tubular pillar, whose length does not exceed 30 times its least breadth and whose greatest breadth does not exceed 30 times the thickness of the plates, will sustain a breaking weight of not less than 12 tons per square inch, especially if the corners are strengthened by stout angle-iron. When the ends of such pillars are properly fixed, as in the compression flange of a girder, experience sanctions a working-strain of 4 tons per square inch in ordinary girder-work, and 3 tons in crane-work where shocks may be expected.

I have deduced the foregoing conclusions respecting tubular pillars chiefly from experiments conducted under Mr. Hodgkinson's supervision during the experimental inquiry respecting the Conway and Britannia tubular bridges. The following tables exhibit the results of these experiments reduced to a convenient form by Mr. G. Berkley,* and the reader can judge for himself how far the experiments warrant the foregoing conclusions.

* *Proc. Inst. C. E.*, Vol. xxx.

TABLE XII.—EXPERIMENTS ON WROUGHT-IRON RECTANGULAR TUBULAR PILLARS.

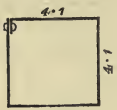
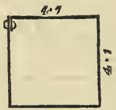
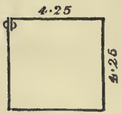
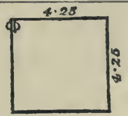
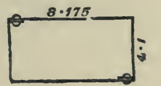
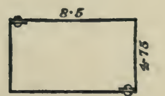
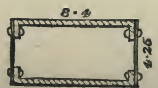
Form of section.	Length.	Least breadth.	Sectional area.	Thickness of metal.	Ratio of length to least breadth.	Ratio of greatest unsupported breadth to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.
	120·0	4·1	·504	·03	29·26	136·0	5,534 tons 2·47	10,980 tons 4·9
Ditto	60·0	...	...	...	14·6	136·6	5,803 „ 2·59	11,513 „ 5·14
Ditto	30·0	...	...	...	7·3	136·6	6,251 „ 2·79	12·402 „ 5·537
	120·0	4·1	1·02	·06	29·26	68·0	19,646 „ 8·77	19,260 „ 8·598
	120·0	4·25	1·484	·083	28·23	51·0	37,354 „ 16·675	25,171 „ 11·237
Ditto	60·0	...	1·52	·085	14·1	50·0	35,850 „ 16·0	23,584 „ 10·529
Ditto	30·0	...	...	...	7·0	50·0	41,674 „ 18·6	27,417 „ 12·24

TABLE XII.—EXPERIMENTS ON WROUGHT-IRON RECTANGULAR TUBULAR PILLARS—*continued.*

Form of section.	Length.	Least breadth.	Sectional area.	Thickness of metal.	Ratio of length to least breadth.	Ratio of greatest unsupported breadth to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.
	120·0	4·25	2·395	·134	28·23	31·7	51,690 tons 23·076	21,585 tons 9·636
Ditto	90·0	...	...	...	21·1	31·7	55,562 " 24·8	23,201 " 10·358
	120·0	4·1	1·532	·061	29·26	134 & 67·0	23,289 " 10·392	15,201 " 6·786
Ditto	92·0	...	...	...	22·4	134 & 67·2	24,843 " 11·09	16,215 " 7·239
Ditto	28·0	...	...	...	6·8	134 & 67·2	24,395 " 10·89	15,921 " 7·108
	120·0	4·75	7·326	·264	25·3	32·2 & 18·0	197,163 " 88·0	26,913 " 12·015
	120·0	4·25	6·89	Channel ·26 Plate ·126	28·23	32·3	206,571 " 92·22	29,981 " 13·384

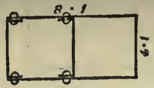
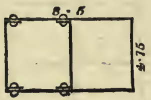
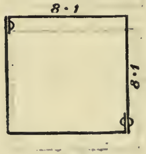
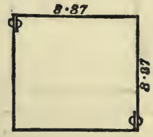
	120·0	4·1	1·885	·059	29·26	70·0	43,673 „ 19·497	23,169 „ 10·343
Ditto	91·5	...	...	...	22·3	69·0	45,451 „ 20·29	24,111 „ 10·764
Ditto	44·0	...	...	...	10·7	68·3	41,259 „ 18·415	21,889 „ 9·772
Ditto	19·5	...	...	...	4·7	68·3	49,035 „ 21·89	26,010 „ 11·61
	120·0	4·75	8·347	·25	25·3	19·0	did not fail 207,915 „ 92·819	not broken 24,909 „ 11·12
	120·0	8·1	2·070	·06	14·8	135·0	27,545 „ 12·296	13,276 „ 5·926
Ditto	92·0	...	...	...	11·3	135·0	27,531 „ 12·20	13,301 „ 5·938
	120·0	8·37	4·926	·139	14·33	60·0	100,395 „ 44·819	20,364 „ 9·09

TABLE XII.—EXPERIMENTS ON WROUGHT-IRON RECTANGULAR TUBULAR PILLARS—*continued.*

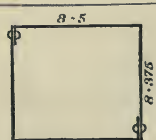
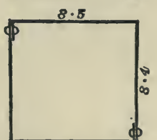
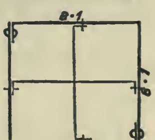
Form of section.	Length.	Least breadth.	Sectional area.	Thickness of metal.	Ratio of length to least breadth.	Ratio of greatest unsupported breadth to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.
	120·0	8·375	7·737	·219	14·33	38·2	198,955 tons 88·819	25,716 tons 11·48
	120·0	8·4	8·466	·245 ·238	14·33	35·7	not crushed 225,835 „ 100·78	not crushed 26,675 „ 11·9
	121·0	8·1	3·55	·0637	14·94	63·6	70,070 „ 31·281	19,732 „ 8·809
Ditto	118·0	8·1	...	...	14·57	63·6	82,027 „ 36·619	23,100 „ 10·312
Ditto	60·0	...	...	...	7·4	63·6	82,411 „ 36·8	23,208 „ 10·361
Ditto	28·0	...	...	...	3·4	63·6	85,771 „ 38·29	24,153 „ 10·783
	96·0	18·0	50·0	·5	5·33	36·0	1,523,200 „ 680·0	30,464 „ 13·6

TABLE XIII.—EXPERIMENTS ON WROUGHT-IRON ROUND TUBULAR PILLARS.

Form of section.	Length.	Diameter.	Sectional area.	Thickness of metal.	Ratio of length to diameter.	Ratio of diameter to thickness of metal.	Breaking weight	Breaking weight per square inch of area.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.
	120·0	1·495	·444	·1	80·0	15·0	6,514 tons 2·9	14,673 tons 6·55
Ditto	60·0	...	...	...	40·0	15·0	13,860 „ 6·187	31,180 „ 13·92
Ditto	30·0	...	...	...	20·0	15·0	15,204 „ 6·787	34,220 „ 15·277
Ditto	120·0	1·964	·61	·104	60·0	18·8	14,156 „ 6·319	23,206 „ 10·359
Ditto	60·0	...	...	...	30·5	18·8	20,332 „ 9·076	33,299 „ 14·866
Ditto	30·0	...	...	...	15·3	18·8	22,572 „ 10·076	36,980 „ 16·509
Ditto	120·0	2·49	·804	·107	47·8	23·27	23,958 „ 10·695	29,798 „ 13·30

TABLE XIII.—EXPERIMENTS ON WROUGHT-IRON ROUND TUBULAR PILLARS—*continued.*

Form of section.	Length.	Diameter.	Sectional area.	Thickness of metal.	Ratio of length to diameter.	Ratio of diameter to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.
	60·0	2·49	·804	·107	24·1	23·27	28,244 tons 12·6	35,100 tons 15·67
Ditto	30·0	...	...	...	21·0	23·27	29,364 „ 13·1	36,489 „ 16·29
Ditto	120·0	2·35	1·605	·242	51·0	9·7	34,516 „ 15·40	21,572 „ 9·63
Ditto	30·0	2·383	1·651	·246	12·5	9·7	54,666 „ 24·404	33,107 „ 14·78
Ditto	30·0	2·343	1·407	·21	12·8	11·1	53,770 „ 24·0	38,214 „ 17·06
Ditto	29·0	2·373	1·554	·231	12·2	10·27	57,354 „ 25·6	36,906 „ 16·476
Ditto	120·0	2·34	1·435	·216	51·28	10·8	31,828 „ 14·20	22,179 „ 9·90
Ditto	60·0	2·35	1·472	·22	25·5	10·6	43,180 „ 19·276	29,330 „ 13·094
Ditto	60·0	2·335	1·371	·205	25·7	11·4	41,164 „ 18·376	29,998 „ 13·392
Ditto	30·0	2·335	1·435	·205	12·8	11·4	52,588 „ 23·476	36,639 „ 16·357

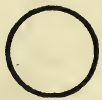
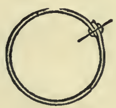
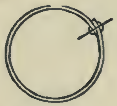
Ditto	30·0	...	1·435	·205	12·8	11·4	50,796 „ 22·676	35,389 „ 15·799
Ditto	29·0	2·343	1·358	·202	12·3	11·6	53,770 „ 24·0	39,569 „ 17·665
Ditto	120·0	3·0	1·35	·15	40·0	20·0	37,356 „ 16·676	27,671 „ 12·353
Ditto	90·0	3·035	1·414	·168	29·6	18·0	42,122 „ 18·8	29,789 „ 13·299
Ditto	28·0	3·00	1·414	·153	9·3	19·6	52,874 „ 23·6	37,392 „ 16·693
Ditto	120·0	4·05	1·707	·14	29·6	29·0	47,212 „ 21·076	27,657 „ 12·346
Ditto	90·0	4·052	1·613	·121	22·2	30·9	53,770 „ 24·0	33,331 „ 14·88
	120·0	4·06	1·9	·155	29·6	26·1	49,900 „ 22·276	26,263 „ 11·724
	120·0	6·36	2·54	·13	18·9	49·0	91,402 „ 40·80	35,985 „ 16·064
Ditto	90·0	6·366	2·547	·13	14·1	48·9	106,122 „ 47·375	41,664 „ 18·60

TABLE XIII.—EXPERIMENTS ON WROUGHT-IRON ROUND TUBULAR PILLARS—*continued.*

Form of section.	Length.	Diameter.	Sectional area.	Thickness of metal.	Ratio of length to diameter.	Ratio of diameter to thickness of metal.	Breaking weight.	Breaking weight per square inch of area.
	inches.	inches.	sq. ins.	inches.			lbs.	lbs.
	120·0	6·18	1·8	·094	19·4	65·0	60,075 tons 26·819	33,375 tons 14·899
Ditto	60·0	6·175	1·799	·101	9·7	61·1	69,002 „ 30·8	38,355 „ 17·123
Ditto	30·0	6·125	1·799	·098	4·9	62·5	74,411 „ 33·214	41,361 „ 18·465
	89·0	4·00	2·879	·243	22·2	16·5	74,988 „ 33·476	26,046 „ 11·628
Ditto	89·0	4·00	2·897	·25	22·2	16·0	76,780 „ 34·276	26,503 „ 11·832
Ditto	89·0	4·00	2·873	·242	22·2	16·5	79,916 „ 35·676	27,816 „ 12·418
Ditto	60·0	3·995	2·895	·245	15·0	16·3	86,922 „ 38·8	30,024 „ 13·404
Ditto	60·0	3·995	2·848	·241	15·0	16·5	98,122 „ 43·8	34,453 „ 15·381
Ditto	28·0	4·0	2·845	·25	7·0	16·0	136·202 „ 60·8	47,844 „ 21·36
Ditto	28·0	4·026	2·85	·25	6·95	16·0	138·442 „ 61·8	48,576 „ 21·68

STEEL PILLARS.

336. Solid Steel Pillars.—Mr. B. Baker gives the following values for the co-efficients in Gordon's formulæ in **327**, when applied to solid steel pillars.*

Solid round pillars.	Mild Steel . .	$a = 30$ tons . .	$b = \dfrac{1}{1400}$
	Strong Steel . .	$a = 51$ tons . .	$b = \dfrac{1}{900}$
Solid rectangular pillars.	Mild Steel . .	$a = 30$ tons . .	$b = \dfrac{1}{2480}$
	Strong Steel . .	$a = 51$ tons . .	$b = \dfrac{1}{1600}$

Ex. 1. What is the breaking weight of a mild cast-steel pillar, 10 feet long and 2 inches in diameter, securely fixed at both ends? Here, the ratio of length to diameter = 60, and we have, from eq. 237, the inch-strain,

$$f = \frac{30}{1 + \dfrac{60 \times 60}{1400}} = 8{\cdot}4 \text{ tons};$$

multiplying this by the sectional area, we have,

Answer, Breaking weight = $3{\cdot}1416 \times 8{\cdot}4 = 26{\cdot}39$ tons.

If the pillar is jointed at the ends, we have from eq. 238,

$$f = \frac{30}{1 + \dfrac{4 \times 60 \times 60}{1400}} = 2{\cdot}658 \text{ tons};$$

multiplying this by the area as before, we have,

Answer, Breaking weight = $3{\cdot}1416 \times 2{\cdot}658 = 8{\cdot}35$ tons,

of which one-fourth, = 2·09 tons, will be a sufficient load when the pillar is free from vibration or shocks.

Ex. 2. What is the breaking weight of a mild cast-steel pillar, 10 feet long and 2 inches square, securely fixed at both ends? Here, the ratio of length to breadth = 60, and we have, from eq. 237, the inch-strain,

$$f = \frac{30}{1 + \dfrac{60 \times 60}{2480}} = 12{\cdot}245 \text{ tons};$$

multiplying this by the sectional area, we have,

Answer, Breaking weight = $4 \times 12{\cdot}245 = 49$ tons nearly.

If the pillar is jointed at the ends, we have from eq. 238,

$$f = \frac{30}{1 + \dfrac{60 \times 60}{620}} = 4{\cdot}405 \text{ tons};$$

multiplying this by the area as before, we have,

Answer, Breaking weight = $4 \times 4{\cdot}405 = 17{\cdot}62$ tons,

of which one-fourth, = 4·405 tons, will be a sufficient load for pillars free from shocks.

* *Strength of Beams*, pp. 207, 209.

TIMBER PILLARS.

337. Square is the strongest form of rectangular timber pillar—Hodgkinson's rules for solid rectangular timber pillars.—It appears from Hodgkinson's experiments that the strength of long round or square timber pillars is nearly as the fourth power of the diameter or side divided by the square of the length. Also, "of rectangular pillars of timber it was proved experimentally that the pillar of greatest strength, where the length and quantity of material are the same, is a square."*

Hodgkinson gives the following rules for the strength of timber pillars with both ends flat and well bedded and whose lengths exceed 30 diameters.†

Let $\mathbf{W}$ = the breaking weight in tons,
l = the length of the pillar in feet,
d = the breadth in inches,

Long square pillars of Dantzic oak (dry).—

$$\mathbf{W} = 10{\cdot}95\,\frac{d^4}{l^2} \tag{239}$$

Long square pillars of Red deal (dry).—

$$\mathbf{W} = 7{\cdot}8\,\frac{d^4}{l^2} \tag{240}$$

Long square pillars of French oak (dry).‡—

$$\mathbf{W} = 6{\cdot}9\,\frac{d^4}{l^2} \tag{241}$$

When timber pillars are less than 30 diameters in length, they come under the class of medium pillars, and their strength may be calculated by eq. 236, the value of $\mathbf{W}$ being computed by one of the equations just given. To find the strength of a rectangular pillar, find as above the breaking weight of a square pillar whose side is equal to the short side of the rectangle; this multiplied by the ratio of the long to the short side will give the breaking weight of the rectangular pillar.

Ex. 1. What is the breaking weight of a pillar of white deal, 9 feet long, 11 inches wide and 3 inches thick? Looking at the table in **300**, we find that the crushing

* *Exp. Res.*, p. 351.

† *Phil. Trans.*, 1840, pp. 425, 426.

‡ The crushing strength of French oak, according to Rondelet, = 6,336 lbs. per square inch.—*Phil. Trans.*, 1840, p. 427.

strength of white deal is about 1·2 times that of red deal, from which we may conclude that the strength of a long square pillar of white deal, derived from eq. 240, is as follows:—

$$W = 1{\cdot}2 \times 7{\cdot}8 \frac{d^4}{l^2}$$

From this, the breaking weight of a pillar 9 feet long and 3 inches square = $1{\cdot}2 \times 7{\cdot}8 \frac{3^4}{9^2} = 9{\cdot}36$ tons, and we have for a pillar 11 inches wide,

$$\textit{Answer}, \quad \text{Breaking weight} = \frac{11 \times 9{\cdot}36}{3} = 34{\cdot}32 \text{ tons.}$$

If the pillar be not very securely fixed at the ends, its breaking weight will = $\frac{34{\cdot}32}{3}$ = 11·44 tons (**311**), of which $\frac{1}{4}$th, = 2·86 tons, will be a sufficient working load for temporary purposes; and $\frac{1}{8}$th, = 1·43 tons, for permanent use where protected from the weather.

Ex. 2. What is the breaking weight of a strut of red deal, 26 feet long and 13 inches square? If the strut were long enough to give way chiefly by flexure (over 30 diameters in length), its breaking weight, from eq. 240, would be

$$W = 7{\cdot}8 \frac{13^4}{26^2} = 329{\cdot}5 \text{ tons,}$$

and if the strut were short enough (under 10 to 15 diameters in length), to give way by crushing alone, its breaking weight would equal its sectional area multiplied by the tabulated crushing strength of red deal in the table in **300**, that is,

$$c = \frac{13^2 \times 5748}{2240} = 434 \text{ tons.}$$

As the strut is a *medium-sized* pillar, we have the true breaking weight, from eq. 236,

$$\textit{Answer}, \quad W' = \frac{Wc}{W + \frac{3}{4}c} = \frac{329{\cdot}5 \times 434}{329{\cdot}5 + 325{\cdot}5} = 218{\cdot}3 \text{ tons,}$$

that is, provided the ends are very carefully bedded; but if they are liable to rough adjustment, as in the cross struts of a cofferdam, from which this example has been taken, the breaking weight will probably be about $\frac{1}{2}$ the above, = 109 tons (**312**), and the safe working load for this kind of temporary work will be one-fourth of this again, = 27·25 tons.

338. Rondelet's and Brereton's rules for timber pillars.—Rondelet deduced the following rule from his experiments on the compression of oak and fir.* Taking the force which would crush a cube as unity, the force requisite to break a timber pillar with fixed ends whose height is—

12 times the thickness, will be			- -	$\frac{5}{6}$
24	,,	,,	,, - -	$\frac{1}{2}$
36	,,	,,	,, - -	$\frac{1}{3}$
48	,,	,,	,, - -	$\frac{1}{6}$
60	,,	,,	,, - -	$\frac{1}{12}$
72	,,	,,	,, - -	$\frac{1}{24}$

* Navier; *Application de la Mécanique*, p. 200.

Rondelet also found that timber pillars do not begin to yield by flexure until their length is about ten times their least lateral dimensions. This rule is easily applied, as illustrated by the following examples:—

Ex. 1. What is the breaking weight by Rondelet's rule of a white deal pillar, 9 feet long, 11 inches wide, and 3 inches thick, with the ends very carefully secured? From the table in **300** the crushing strength of white deal = 6781 lbs. per square inch, and the crushing strength of a very short length of the pillar is therefore 11 × 3 × 6781, = 223,773 lbs. As the length of the plank is 36 times its least width, we have according to Rondelet's rule,

$$\textit{Answer}, \quad \text{Breaking weight} = \frac{223{,}773}{3} = 74{,}591 \text{ lbs.} = 33{\cdot}3 \text{ tons,}$$

which differs but slightly from its strength calculated by Hodgkinson's rule in ex. 1, **337**.

Ex. 2. What is the breaking weight of a red deal strut 26 feet long and 13 inches square, with both ends securely fixed? In ex. 2, **337**, we found that the breaking weight of a short length of the strut was 434 tons, and as the real length = 24 diameters, Rondelet's coefficient is $\frac{1}{2}$; consequently we have,

$$\textit{Answer}, \quad \text{Breaking weight} = \frac{434}{2} = 217 \text{ tons,}$$

which is almost identical with the strength calculated by Hodgkinson's rule in the example referred to.

Mr. R. P. Brereton states that "in experiments made with large timbers, with lengths of from ten to forty times the thickness, he had found that timber 12 inches square and 10 feet long bore a weight of 120 tons; when 20 feet long it bore 115 tons; when 30 feet long 90 tons; and when 40 feet long it carried 80 tons."*

Plotting the curve of Mr. Brereton's experiments we get the following:—

TABLE XIV.—For calculating the strength of Rectangular Pillars of Fir or Pine Timber.

Ratio of length to least breadth	10	15	20	25	30	35	40	45	50
Breaking weight in tons per square foot of section, - -	120	118	115	100	90	84	80	77	75

This is probably the most useful rule yet published for the strength of large pillars of soft foreign timber with their ends

* *Proc. Inst. C. E.*, Vol. xxix., p. 66.

adjusted in the ordinary manner, that is, without any special precautions.

Ex. 1. What is the breaking weight of a red deal strut, 26 feet long and 13 inches square? Here, the ratio of length to side is 24, and the breaking weight in the table for this ratio is 103 tons per square foot; consequently, for 13 inches square,

$$\textit{Answer},\ \text{Breaking weight} = \frac{13 \times 13 \times 103}{12 \times 12} = 121 \text{ tons, nearly.}$$

This answer, it will be observed, approximates very closely to the 109 tons obtained by Hodgkinson's rule in ex. 2, **337**.

Ex. 2. A pillar of ordinary memel timber, 20 feet long and 13 inches square, was broken in a proving machine with 136 tons. What is its breaking weight computed by the foregoing rule? Here, the ratio of length to side is 18·5, and the corresponding breaking weight from the table = 116 tons per square foot.

$$\textit{Answer},\ \text{Computed breaking weight} = \frac{13 \times 13 \times 116}{12 \times 12} = 136 \text{ tons.}$$

STONE PILLARS.

339. Influence of the height and number of courses in stone columns.—From Rondelet's experiments it would appear that when three cubes of stone are placed on top of each other, their crushing strength is little more than half the strength of a single cube.* Vicat, however, attributes this result to imperfect levelling and the absence of mortar or cement in the joints, and he found from experiments on plaster prisms carefully bedded, that the strength of a monolithic prism, whose height is h, being represented by unity, we have the strength of prisms:—

Of 2 courses	and of the height	h	$= 0·930$
Of 4	,,	$2h$	$= 0·861$
Of 8	,,	$4h$	$= 0·834$

even without the interposition of mortar. He concludes that the division of a column into courses, each of which is a monolith, with carefully dressed joints and properly bedded in mortar, does not sensibly diminish its resistance to crushing; but he intimates that this does not hold good when the courses are divided by vertical joints.†

340. Crushing strength of Rollers and Spheres.—From M. Vicat's experiments it appears that the strength of cylinders employed as rollers between two horizontal planes is proportional

* Morin, p. 72. † *Idem*, p. 76.

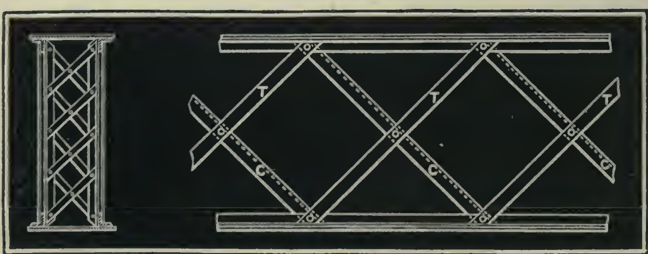
to the product of their axis by the diameter, and that the strength of spheres to resist crushing is proportional to the square of their diameter. If the strength of a cube be represented by unity, that of the inscribed cylinder standing on its base will be 0·80; that of the same cylinder on its side will be 0·32; and that of the inscribed sphere will be 0·26.*

BRACED PILLARS.

341. Internal Bracing—Example.—One of the chief practical difficulties which occur in bridges of large span is the combination of lightness with stiffness in long struts, such as the compression bars of the web. The internal bracing represented in Fig. 102 is a modification of the bracing so familiar in scaffolding. It is now in common use for the compression bars of lattice girders, and the bracing of iron piers, and as it unites the requisite qualities of strength and lightness in an eminent degree, it is worth devoting some space to investigating the nature of the strains in this form of pillar.

The diagram represents the cross section and side elevation of a

Fig. 102.



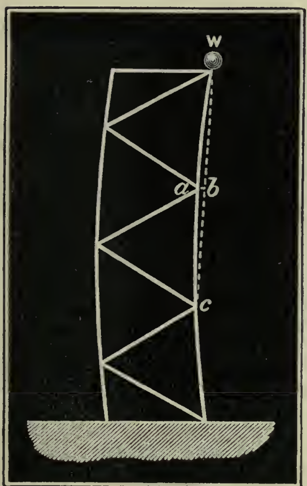
lattice tubular girder of simple construction. The tension diagonals (marked T,) intersect the compression diagonals (marked C,) at moderate intervals, and keep them from deflecting, especially in the plane of the girder. It is obvious, however, that long compression bars, even though formed of angle or tee iron, have but little stiffness in themselves, and we cannot trust to the tension bars

* Morin, pp. 75, 82.

keeping them in the line of thrust at right angles to the plane of the girder, for the tension bars may not always be in a sufficient state of strain (**153**). Hence, it is desirable, at least in long pillars, to connect each pair of compression bars by internal cross-bracing, as shown in the section. The strains to which a braced pillar is subject may be investigated in the following manner, which, though rude, is yet sufficiently approximate for practical purposes:—

Let Fig. 103 represent a pillar which has become deflected, either from the weight resting more on one side than on the other, or from defective construction, or from accident.

Fig. 103.



Let W = the weight resting on one side,

$\mathsf{D} = ab$ = the lateral deflection in the interval of two bays,

$l = \mathsf{W}a = ac$ = the length of one bay,

R = the radius of curvature of the deflected pillar,

P = the resultant of the strains in $\mathsf{W}a$ and ac, *i.e.*, the nearly horizontal pressure produced on the two braces intersecting at a, in consequence of the weight being transmitted through a curved pillar.

At the apex, a, three forces balance, viz., the nearly vertical pressures (each = W,) in the two adjacent bays, and their resultant P. Hence, we have $\mathsf{P} = \frac{2\mathsf{DW}}{l}$; but $\mathsf{D} = \frac{l^2}{2\mathsf{R}}$, therefore,

$$\mathsf{P} = \frac{\mathsf{W}l}{\mathsf{R}} \qquad (242)$$

The pillar may therefore be regarded as a girder, each of whose flanges is subject to a longitudinal pressure equal to W, in addition to having a weight P resting on each apex. Hence, the strains in the bracing may be found by the methods already explained in

Chapters V. and VI. If the pillar have a tendency to assume an **S** form, the strains developed in the internal bracing in one loop of the curve may, to some extent, neutralize those produced in the other. If, however, the pressure on one side exceed that on the other by any known or assumed quantity, then their difference of length, and the corresponding deflection, may be obtained as explained in the chapter on deflection, but in practice, errors of workmanship will almost always exceed the amount of deflection produced by a difference of pressure and experience must dictate the requisite allowance. Let, for example, a pillar with internal bracing, composed of two systems of right-angled triangles, similar to that represented in Fig. 102, be 30 feet long and two feet wide, and let each bay be 2 feet in length, in which case there will be 15 bays in each side, and let the total load on the pillar = 40 tons, or 20 tons on a side. Now, suppose that the maximum error of workmanship amounts to half an inch of lateral deflection in the centre of the pillar, in which case **R** will equal 2,700 feet, then the pressure **P**, produced at each apex by a vertical pressure of 20 tons on each side of the pillar, is as follows:—

$$\mathsf{P} = \frac{\mathsf{W}l}{\mathsf{R}} = \frac{20 \times 2}{2700} \text{ tons} = 33{\cdot}2 \text{ lbs.}$$

As there are 14 apices in each system of bracing, *i.e.*, 7 on each side, the strain in each of the end braces $= \dfrac{33{\cdot}2 \times 14 \times 1{\cdot}414}{2}$ $= 328{\cdot}6$ lbs. (eq. 120). We thus see that the strain in the internal bracing is comparatively trifling, and that the difficulty of providing against flexure in long compression braces is not so formidable as might have been supposed. It will be observed that the internal bracing develops longitudinal strains in the side bars at each apex. These increments are, however, insignificant compared with the pressure due to the weight.

342. Each bay of a braced pillar resembles a pillar with rounded ends—Compression flanges of girders resemble braced pillars.—In braced pillars the side bars must be made stiff enough to resist flexure for the length of one bay between the apices of the internal bracing. Each bay cannot, however, be regarded as a pillar of this length firmly fixed at the ends, but rather as one with rounded ends, since it might assume a waved form like the letter S, consecutive bays deflecting in opposite directions. This remark also applies to the compression flanges of girders. The vertical webs preserve them from deflecting in a vertical plane; the cross-bracing between the flanges performs the same service in a horizontal plane, and the compression diagonals, especially if they are braced pillars, also convey a large share of rigidity from the tension flanges and roadway to the compression flanges. The failure of the latter, therefore, as far as flexure is concerned, is thus generally confined to the short length of one bay.

343. Strength of braced pillars is independent of length within certain limits—Working strain.—From Hodgkinson's experiments on plate-iron tubular pillars, it seems highly probable that the strength of braced pillars is also within considerable limits independent of their length, for internal bracing will generally be made somewhat stronger than theory alone might require (**334**).

In my own practice I adopt 4 tons per square inch of gross section (excluding, of course, the cross bracing,) for the working-strain of wrought-iron braced pillars in ordinary girder-work. In crane-work, where shocks may occur, 3 tons per square inch is enough. In both cases the ends of the pillar are supposed to be firmly fixed by construction.

CHAPTER XVI.

TENSILE STRENGTH OF MATERIALS.

344. Nature of tensile strain.—The tendency of tensile strain is to draw the material into a straight line between the points of attachment, and, unless its shape alters very suddenly or the mode of attachment is defective, so as to produce indirect strain, each transverse section will sustain a uniform unit-strain throughout its whole area; eq. 1 is, therefore, applicable to ties without any other practical correction than this, that if the material be pierced with holes, such as rivet or bolt holes in iron, or knots in timber, the effective area for tension in any transverse section is not the gross, but the net area which remains after deducting the aggregate area of all the holes or imperfections which occur in that particular transverse section.

CAST-IRON.

345. Tensile strength.—The following table contains the results of Mr. Hodgkinson's experiments on the tensile strength of various kinds of British cast-iron.* Those samples whose specific gravity are given are the same irons as those whose crushing strengths have been already stated in Table I., **294**.

TABLE I.—TENSILE STRENGTH OF CAST-IRON.

Description of iron.	Specific gravity.	Tearing weight per square inch of section.
		lbs. tons.
Carron iron (Scotland), No. 2, hot-blast, - - -		13,505 = 6·03
Ditto, do., cold-blast, - - - -		16,683 = 7·45
Ditto, No. 3, hot-blast, - - - -		17,755 = 7·93
Ditto, do., cold-blast, - - - -		14,200 = 6·35
Devon iron (Scotland), No. 3, hot-blast, - - - -		21,907 = 9·78

* *Experimental Researches on the Strength and other Properties of Cast-Iron*, by Eaton Hodgkinson, p. 310. Also, *Report of the Commissioners appointed to inquire into the application of Iron to Railway Structures*, 1849, p. 9.

TABLE I.—TENSILE STRENGTH OF CAST-IRON—*continued.*

Description of iron.	Specific gravity.	Tearing weight per square inch of section.
		lbs. tons.
Buffery iron (near Birmingham), No. 1, hot-blast,		13,434= 6·00
Ditto, do., cold-blast,		17,466= 7·80
Coed-Talon iron (North Wales), No. 2, hot-blast,		16,676= 7·45
Ditto, do., cold-blast,		18,855= 8·40
Low Moor iron (Yorkshire), No. 3,		14,535= 6·50
Mixture of iron,		16,542= 7·39
Low Moor iron, No. 1,	7·074	12,694= 5·667
Ditto, No. 2,	7·043	15,458= 6·901
Clyde iron (Scotland), No. 1,	7·051	16,125= 7·198
Ditto, No. 2,	7·093	17,807= 7·949
Ditto, No. 3,	7·101	23,468=10·477
Blaenavon iron (South Wales), No. 1,	7·042	13,938= 6·222
Ditto, No. 2, first sample,	7·113	16,724= 7·466
Ditto, No. 2, second sample,	7·051	14,291= 6·380
Calder iron (Lanarkshire), No. 1,	7·025	13,735= 6·131
Coltness iron (Edinburgh), No. 3,	7·024	15,278= 6·820
Brymbo iron (North Wales), No. 1,	7·071	14,426= 6·440
Ditto, No. 3,	7·037	15,508= 6·923
Bowling iron (Yorkshire), No. 2,	6·989	13,511= 6·032
Ystalyfera Anthracite iron (South Wales), No. 2,	7·119	14,511= 6·478
Yniscedwyn Anthracite (South Wales), No. 1,	7·034	13,952= 6·228
Yniscedwyn Anthracite, No. 2,	7·013	13,348= 5·959
Mean of the foregoing 27 irons,	- -	15,679= 7·00
Mr. Morries Stirling's iron, denominated 2nd quality,*	7·165	25,764=11·502
Mr. Morries Stirling's iron, denominated 3rd quality,†	7·108	23,461=10·474

* Composed of Calder, No. 1, hot blast, mixed and melted with about 20 per cent. of malleable iron scrap.

† Composed of No. 1, hot-blast, Staffordshire iron, from Ley's works, mixed and melted with about 15 per cent. of common malleable iron scrap.

From these experiments it appears that the average tensile strength of simple British irons is 7 tons per square inch. The strength of mixed irons, however, often reaches 9 or 10 tons, while that of some American cast-iron is nearly double of this.

346. Cold-blast rather stronger than hot-blast iron—Mixtures stronger than simple irons.—On comparing the tenacity of hot and cold-blast iron in the first part of the foregoing table, it will be observed that, with one exception, the cold-blast irons are rather stronger than the hot-blast irons of the same make. This is confirmed by experiments made in the United States, where, since 1840, hot-blast iron has been condemned for ordnance purposes.* The following are the conclusions which the late Mr. Robert Stephenson deduced from a series of experiments on the transverse strength of cast-iron bars, made preparatory to the commencement of the high level bridge at Newcastle.

1. Hot-blast is less certain in its results than cold-blast.

2. Mixtures of cold-blast are more uniform than those of hot-blast.

3. Mixtures of hot and cold-blast give the best results.

4. Simple samples do not run so solid as mixtures.

5. Simple samples sometimes run too hard, and sometimes too soft for practical purposes.†

Having regard to the fact that hot-blast is now in general use, and that it seems to improve some kinds of iron, probably those of a hard nature, the best plan for the engineer to adopt is to specify the test which he requires the iron to stand and let the founder bear the responsibility of producing the required result.

347. Re-melting, within certain limits, increases the strength and density of cast-iron.—Re-melting cast-iron seems to have an important effect in increasing its density as well as in

* *Report on the Strength and other Properties of Metals for Cannon.* By Officers of the Ordnance Department, U.S. Army. Philadelphia, 1856, p. 338.

† *Rep. of Iron Com.*, App., p. 389.

improving its tensile and transverse strength, as appears from the following experiments by Major Wade on proof bars of No. 1 Greenwood pig-iron thrice re-melted:*—

TABLE II.—EXPERIMENTS ON THE TENSILE AND TRANSVERSE STRENGTH OF RE-MELTED CAST-IRON.

	Density.	Tearing weight per square inch.	Coefficient of transverse rupture, S
		lbs.	lbs.
Crude pig-iron, - - - -	7·032	15,129	5,290
Do. re-melted once, - -	7·086	21,344	6,084
Do. do. twice, - -	7·198	30,107	7,322
Do. do. three times, -	7·301	35,786	9,448

In summing up the results of his experiments on re-melting cast-iron, Major Wade observes, "the softest kinds of iron will endure a greater number of meltings with advantage than the higher (more decarbonized) grades, and it appears that when iron is in its best condition for casting into proof bars of small bulk, it is then in a state which requires an additional fusion to bring it up to its best condition for casting into the massive bulk of cannon. In selecting, and preparing iron for cannon, we may, therefore, proceed by repeated fusions, or by varying the proportions of the different grades, until the maximum tenacity in proof bars is attained; the iron will then be in its best condition for being again melted and cast into cannon."

Experiments made by Sir William Fairbairn, for the British Association, though on a much more limited scale than those by Major Wade, also prove the advantage to be derived from repeated fusions.† One ton of No. 3 Eglinton hot-blast iron was melted 18 times successively, each time under similar conditions of fusion,

* *Rep. on Metals for Cannon*, pp. 242, 249.

† *Application of Iron to Building Purposes*, p. 60.

and proof bars, 5 feet long and 1 inch square, were cast each time, and broken by transverse strain, the distance between the supports being 4 feet 6 inches. The results are given in the following table:—

TABLE III.—EXPERIMENTS ON THE TRANSVERSE AND CRUSHING STRENGTH OF RE-MELTED CAST-IRON.

No. of meltings.	Specific gravity.	Mean breaking weight of bars exactly 1 in. square, and 4 feet 6 inches between supports.	Mean ultimate deflection.	Power to resist impact.	Crushing weight per square inch.
		lbs.	inches.		tons.
1	6·969	490·0	1·440	705·6	44·0
2	6·970	441·9	1·446	630·9	43·6
3	6·886	401·6	1·486	596·7	41·1
4	6·938	413·4	1·260	520·8	40·7
5	6·842	431·6	1·503	648·6	41·1
6	6·771	438·7	1·320	579·0	41·1
7	6·879	449·1	1·440	646·7	40·9
8	7·025	491·3	1·753	861·2	41·1
9	7·102	546·5	1·620	885·3	55·1
10	7·108	566·9	1·626	921·7	57·7
11	7·113	651·9	1·636	1066·5	69·8
12	7·160	692·1	1·666	1153·0	73·1
13	7·134	634·8	1·646	1044·9	66·0 *
14	7·530	603·4	1·513	912·9	95·9
15	7·248	371·1	0·643	238·6	76·7
16	7·330	351·3	0·566	198·5	70·5
17	Lost.	...	...	...	...
18	7·385	312·7	0·476	148·8	88·0

In these experiments it will be observed that the transverse strength increased up to the 12th melting, after which it fell off in a marked degree.

348. Prolonged fusion, within certain limits, increases the strength and density of cast-iron.—The improvement due to prolonged fusion is shown by the following experiments by Major Wade on Stockbridge iron of the 2nd fusion.†

* The cube did not bed properly upon the steel plates, otherwise it would have resisted a much greater force—probably 80 or 85 tons per square inch.

† *Rep. on Metals for Cannon*, pp. 40, 44.

TABLE IV.—EXPERIMENTS ON PROLONGED FUSION.

	Density.	Tearing weight per square inch.	Coefficient of transverse rupture, S
		lbs.	lbs.
Iron in fusion ½ hour, -	7·187	17,843	7,126
Do. do. 1 „ -	7·217	20,127	8,778
Do. do. 1½ „ -	7·250	24,387	10,083
Do. do. 2 „ -	7·279	34,496	11,614

In some experiments made at Woolwich Arsenal by Mr. F. J. Bramwell, it was found that fusion for 3¾ hours increased the tensile strength of No. 1 Acadian cold-blast iron, from Nova Scotia, from 7·5 to 10·8 tons per square inch, or nearly 50 per cent. This when cooled was re-melted with an equal proportion of the original No. 1 iron and the tensile strength of bars cast immediately upon re-melting was 11 tons, and after 4 hours fusion, 18·5 tons per square inch.*

On this subject Mr. Truran makes the following observations†:—"The composition of the original grey pig-iron doubtless influences, in a very great measure, the amount of improvement obtained with different periods of fusion. A refining of the iron takes place; and the quantity of alloyed matters oxidized and removed will vary with the character of the pig-iron. Carbon is a principal ingredient in cast-iron; and a long exposure, equally with repeated meltings, offers a ready method of burning it away. The reverberating column of gases in the re-melting furnace contains a proportion of free oxygen, which combines with the carbon to form carbonic acid; but since the oxygen is in contact only with the surface of the metal, its removal requires numerous fusions, or the maintenance in fusion for a long period. Repeated fusions of the

* *Proc. I. C. E.*, Vol. xxii., p. 559.

† *The Useful Metals and their Alloys*, pp. 215, 217. London: 1857.

iron are attended with a heavy waste of material, which goes far to compensate for the increase of strength. The tensile strength, as influenced by the size of the masses and rapidity of cooling, varies with the condition of the iron previous to casting. If the refining process, by lengthened fusion or numerous re-meltings, be carried too far, the resulting product will be of a hard, brittle quality; and when cast into small articles, be chilled to that extent as to be incapable of working with steel cutting-tools. Cast into larger articles, however, and cooled more slowly, a maximum tenacity may be developed, and the texture of the iron be found of a character to bear cutting-tools on its surface. Continuing the operation too long also produces a thickening of the molten iron, until it is of too great a consistence for the proper filling of the moulds, and the prevention of air cavities in the body of the casting. The burning away of the carbon is attended with a loss of fluidity; and this defect occurring, there is no remedy short of introducing further portions of the original crude iron, to restore, by mixing, a certain degree of fluidity."

349. Tensile strength of thick castings of highly decarbonized iron greater than that of thin ones—Annealing small bars of cast-iron diminishes their density and tensile strength.—It has been already shown (**132**) that the transverse strength of thin castings exceeds that of thick ones, and it might naturally be thought that this was always due to greater tensile strength in the smaller castings. This, however, seems to be disproved by the following experiments by Major Wade, of the United States army, who found that small castings in vertical dry sand moulds had a less tensile strength than large gun castings similarly moulded and cast at the same time.* The diminution of tensile strength in the small bars amounted to nearly 5 per cent., while their transverse strength was 14 per cent. greater than that of bars cut from the guns, as is shown in the following table:—

* *Report on Metals for Cannon*, p. 45.

TABLE V.—Comparison of Proof Bars cut from the body of the Gun, with those cast at the same time in separate vertical dry Sand Moulds, showing the difference in the same iron, caused by slow cooling in large masses, and more rapid cooling in small Castings.

Guns.	Coefficient of transverse rupture, S		Tearing weight per square inch.		Specific gravity.	
	Bar cut from gun.	Bar cast separate.	Bar cut from gun.	Bar cast separate.	Bar cut from gun.	Bar cast separate.
	lbs.	lbs.	lbs.	lbs.		
6-pounder gun, -	8,415	9,880	30,234	29,143	7·196	7·263
6-pounder gun, -	9,233	9,977	31,087	30,039	7·278	7·248
8-inch gun, - -	8,575	10,176	26,367	24,583	7·276	7·331
Mean, - - -	8,741	10,011	29,229	27,922	7·250	7·281
Proportional, -	1·000	1·145	1·000	·955	1·000	1·004

"These results," observes Major Wade, "show that the transverse strength is augmented by rapid cooling in small castings, and that the tensile strength is increased by slow cooling in large masses. The differences in specific gravity are less marked; but it is somewhat higher in the small castings cooled rapidly." This conclusion, however, must be qualified by further statements of the same author at pp. 234 and 268; where, in allusion to similar experiments, he says:—"Such results happen only in cases where the iron is very hard. As a general rule, the tenacity of the common sorts of foundry iron is increased by rapid cooling. In this case the condition of the iron when cool was *too high*—that is to say, the process of decarbonization had been carried too far—for a maximum strength, when cooled rapidly, in small masses; although it was in its best condition for casting into a large mass, where it must cool slowly. As iron of high density, when cast into bars of small bulk, is liable to become unsound and to contain small cavities, this cause may account, in some measure, for the diminished tensile strength in bars of high density." Major Wade found that annealing small bars of cast-iron invariably diminished both their density and tenacity.* American cannon iron, the reader will

* *Report on Metals for Cannon*, p. 234.

observe, is much stronger and denser than ordinary English cast-iron, the mean tensile strength of a large number of American guns cast in 1851 being 37,774 lbs., or nearly 17 tons per square inch.*

350. Indirect pull greatly reduces the tensile strength of cast-iron.—Mr. Hodgkinson found "that the strength of a rectangular piece of cast-iron, drawn along the side, is about one-third, or a little more, of its strength to resist a central strain."† In proving specimens of cast-iron in a testing machine it is essential that the strain pass exactly through the axis of the specimen, otherwise the apparent will be much less than the real tensile strength.

351. Cast-iron not suited for tension.—Cast-iron is liable to air-holes, internal strains from unequal contraction in cooling and other concealed defects which often seriously reduce its effective area for tension and, as its tenacity is only about one-third of that of wrought-iron, the latter material or steel should be preferred for tensile strains whenever practicable. For these reasons cast-iron is seldom used in the form of a tie-bar. It frequently occurs, however, in tension in the lower flanges of girders with continuous webs, for the safe working strain in which see Chap. XXVIII.

WROUGHT-IRON.

352. Tensile strength of wrought-iron—Fractured area—Ultimate set.—We are indebted to Mr. David Kirkaldy for an exceedingly valuable series of experiments on the tensile strength of wrought-iron and steel, made by means of a lever testing machine at the works of Messrs. Robert Napier and Sons, Glasgow.‡ The following tables contain abstracts of the more important results of these experiments. The column headed "Tearing weight per square inch of fractured area" gives the breaking weight per square inch of the area when reduced by the specimen drawing out under proof. The ratio of this to the "tearing weight per square inch of

* *Report on Metals for Cannon*, p. 276.

† *Ex. Res.*, p. 312.

‡ *Experiments on Wrought-iron and Steel*, by David Kirkaldy, Glasgow, 1863.

original area" indicates the quality of the iron, whether ductile or the reverse. The soft and ductile irons draw out to a small "fractured area," and consequently have a very high unit-strain referred to it, whereas the hard irons stretch but little under proof, and therefore have a comparatively low unit-strain referred to the same standard. The last column, headed "Ultimate elongation or tensile set after fracture," gives the ratio of the increment of length after fracture to the original length before fracture, in the form of a percentage of the latter. The figures in this column are greater or less according as the material is more or less ductile, and consequently, this "set after fracture" is a test of the toughness and ductility of the iron under proof. In my own practice I find that the "set after fracture" is more easily measured than the "fractured area," and that it is a very convenient test of the ductility and toughness of the iron.

TABLE VI.—TENSILE STRENGTH OF WROUGHT-IRON BARS.

NOTE.—All the pieces were taken *promiscuously* from engineers' or merchants' stores, except those marked *samples*, which were received from the makers.

District.	Names of the Makers or Works.	Description.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
			lbs.	lbs.	per cent.
Yorkshire.	LOW MOOR, - -	Rolled bars, 1 inch square,	60,364	117,147	24·9
	Do. - -	Rolled bars, 1 inch round,	61,798	131,676	26·5
	Do. - -	Rolled bars, 11/16 inch, for rivets,	60,075	125,775	20·5
	Do. - -	Planed from 1 inch square bars,	60,245	114,410	23·8
	Do. - -	Forged from 1¼ inch round bars,	66,392	115,040	20·2
	BOWLING, - -	Rolled bars, 1 inch round,	62,404	114,220	24·4
	Do. - -	Turned from 1¼ inch round bars,	61,477	120,229	26·0
	FARNLEY, - -	Rolled bars, 1 inch round,	62,886	127,425	25·6

TABLE VI.—TENSILE STRENGTH OF WROUGHT-IRON BARS—*continued.*

District.	Names of the Makers or Works.	Description.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
			lbs.	lbs.	per cent.
Staffordshire.	J. BRADLEY and Co., Ⓛ (Charcoal) - -	*Samples.* Rolled bars, 1 inch round,	57,216	146,521	30·2
	Do. B. B., Scrap, -	Rolled bars, 1 inch round,	59,370	123,805	26·6
	Do. S C ♔ -	Rolled bars, $\frac{3}{4}$ inch, for rivets,	56,715	112,336	22·5
	Do. do. - -	Rolled bars, $\frac{7}{8}$ inch round,	62,231	97,575	22·2
	G. B. THORNEYCROFT & Co.,	Rolled bars, $\frac{13}{16}$ inch, for rivets,	59,278	99,595	22·4
	LORD WARD, TNS L ♔ W.R-O	Rolled bars, $\frac{11}{16}$ inch, for rivets,	59,753	95,724	18·6
	MALINSLEE, ♔ BEST, -	Rolled bars, $\frac{3}{4}$ inch × 1 inch,	56,289	88,300	21·4
	BAGNALL, ♔ J. B. -	Rolled bars, $1\frac{1}{4}$ inch round,	55,000	75,351	17·3
	Do. do. -	Do. do., turned down to 1 inch,	55,381	80,638	19·1
Lancashire	ULVERSTON RIVET, BEST,	Rolled bars, $\frac{3}{4}$ inch round,	53,775	104,680	21·6
	MERSEY Co., BEST, -	Forged from $\frac{3}{4}$ inch square bars,	60.110	86,295	16·9
Lanarkshire.	GOVAN, EX. B. BEST, -	Rolled bars, $\frac{3}{4}$ inch square,	56,655	99,000	19·1
	Do. do. -	Rolled bars, $\frac{3}{4}$ inch round,	57,591	95,248	17·3
	Do. do. -	Rolled bars, $1\frac{1}{8}$ inch round,	58,358	97,821	23·8
	Do. do. -	Rolled bars, 1 inch round,	59,109	98,527	22·3
	Do. do. -	Rolled bars, $\frac{7}{8}$ inch round,	58,169	101,863	19·2
	Do. do. -	Rolled bars, $\frac{3}{4}$ inch round,	57,400	92,880	17·6
	GOVAN, B. BEST, -	Rolled bars, $1\frac{1}{8}$ inch round,	60,879	84,770	17·0
	Do. do. -	Rolled bars, 1 inch round,	62,849	88,550	19·1
	Do. do. -	Rolled bars, $\frac{7}{8}$ inch round,	61,341	96,442	20·0
	Do. do. -	Rolled bars, $\frac{3}{4}$ inch round,	64,795	97,245	17·3
	Do do. -	Rolled bars, $\frac{3}{4}$ inch round,	59,548	95,706	16·9
	GOVAN, ✱ -	Rolled bars, $1\frac{1}{8}$ inch round,	58,326	78,139	16·7
	Do. do. -	Rolled bars, 1 inch round,	59,424	79,373	16·4

TABLE VI.—TENSILE STRENGTH OF WROUGHT-IRON BARS—*continued.*

District.	Names of the Makers or Works.	Description.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
			lbs.	lbs.	per cent.
Lanarkshire—*continued.*	GOVAN, ✳ -	Rolled bars, $\frac{7}{8}$ inch round,	63,956	88,512	15·8
	Do. do. -	Rolled bars, $\frac{3}{4}$ inch round,	61,887	95,319	18·8
	GLASGOW, B. BEST, -	Rolled bars, 1 inch round,	58,885	97,548	23·2
	Do. do. -	Rolled bars, $\frac{15}{16}$ inch round,	58,910	97,559	21·3
	Do. do. -	Forged from 1 inch rolled bars,	59,045	80,053	20·9
	Do. do. -	Rolled bars, $1\frac{1}{4}$ inch round,	54,579	85,012	20·3
	Do. do. -	Do., do., turned down to 1 inch,	55,533	86,590	21·3
	Do. do. -	Do., do., forged down to 1 inch,	56,112	81,508	18·6
	Do. do. -	Rolled bars, $\frac{5}{8}$ inch round,	59,300	99,612	20·0
	GLASGOW BEST RIVET, -	Rolled bars, $\frac{7}{8}$ inch round,	57,092	96,205	23·7
	COATBRIDGE, BEST RIVET,	Rolled bars, $\frac{3}{4}$ inch round,	61,723	96,267	21·6
	ST. ROLLOX, BEST RIVET,	Rolled bars, $\frac{13}{16}$ inch round,	56,981	77,383	16·6
	R. SOLLOCH E. BEST, -	Rolled bars, $\frac{11}{16}$ inch, for rivets,	57,425	96,959	17·7
	◇ GOVAN, ◇ -	Rolled bars, $1\frac{1}{4}$ inch round,	57,598	114,866	24·8
	Do. do. -	Do., do., turned down to 1 inch,	57,288	116,869	25·6
	Do. do. -	Do., do., forged down to 1 inch,	57,095	112,705	23·1
	Do. do. -	Rolled bars, 1 inch round,	58,746	113,700	25·2
	Do. do. -	Rolled bars, $\frac{5}{8}$ inch round,	58,199	116,549	21·4
	DEMDYVAN (Common), -	Rolled bars, $1\frac{1}{4}$ inch round,	51,327	54,100	6·3
	Do. do. -	Do., do., turned down to 1 inch,	55,995	63,280	11·1
	Do. do. -	Rolled bars, $1\frac{1}{4}$ inch, forged down,	54,247	60,856	7·3
	Do. do. -	Rolled bars, 1 inch round,	53,352	58,304	6·8
	BLOCHAIRN, B. BEST, -	Rolled bars, 1 inch round,	56,141	90,313	21·3
	BLOCHAIRN, BEST RIVET, -	Rolled bars, $\frac{3}{4}$ inch round,	59,219	89,279	19·4
	PORT DUNDAS, EX. B. BEST,	Rolled bars, $1\frac{1}{8}$ inch round,	54,594	85,563	20·6
	GOVAN, Puddled Iron, -	Rolled bars, $\frac{1}{2} \times 2\frac{1}{4}$ inch, forged down,	46,771	48,057	3·4

TABLE VI.—TENSILE STRENGTH OF WROUGHT-IRON BARS—*continued.*

District.	Names of the Makers or Works.	Description.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
			lbs.	lbs.	per cent.
South Wales.	YSTALYFERA, Puddled Iron,	Rolled bars, $\frac{3}{4} \times 2\frac{3}{4}$ inch, forged down,	29,626	29,818	0·6
	Do. do. -	Do., do., strips cut off,	38,526	39,470	2·0
Hammered.	HAMMERED SCRAP IRON, -	— —	53,420	94,105	24·8
	BUSHELED IRON FROM TURNINGS, - -	— —	55,878	72,531	16·6
	Cut out of a CRANK SHAFT of Hammered Scrap Iron, 14″ wide, and reduced to the required shape in the lathe, not on the anvil, - -	Length of shaft, -	47,582	59,003	21·8
		Across shaft, - -	44,758	50,971	16·8
	Do. do.	Lengthways, - -	43,759	56,910	20·5
		Crossways, - -	38,487	42,059	8·4
	HAMMERED ARMOUR PLATE, 16′ 6″ × 3′ 9″ × 4½″, cut off the end and turned down, - - -	Crossways, - -	38,868	44,611	11·7
		Do. - -	36,824	39,085	6·4
Swedish.	Per ECKMAN AND Co., R F, Gothenburg,	flat tilted bars, Strips cut off, -	47,855	121,065	27·8
	Do. do.	flat tilted bars, Forged round, -	48,232	150,760	26·4
Russian.	PRINCE DEMIDOFF, C C ND,	flat tilted bars, Strips cut off, -	49,564	73,118	13·3
	Do. Do.	flat tilted bars, Forged round, -	56,805	77,632	15·3
Foreign.	SWEDISH, OC	flat tilted bars, Strips cut off,	48,933	141,702	17·0
	Do. ♔ ♔ W	flat tilted bars, Strips cut off,	43,509	77,349	15·3
	Do. ♔ C	flat tilted bars, Strips cut off,	42,421	63,632	15·2
	RUSSIAN, I-O P 3	flat tilted bars, Strips cut off,	59,096	68,047	6·0
	SWEDISH, OC	flat tilted bars, Forged down,	50,262	188,731	18·7
	Do. ♔ C	flat tilted bars, Forged down,	41,251	98,510	14·8
	Do. ♔ ♔ W	flat tilted bars, Forged down,	44,230	83,851	15·8
	RUSSIAN, I-O P 3	flat tilted bars, Forged down,	51,466	67,907	7·5

TABLE VII.—TENSILE STRENGTH OF ANGLE IRON.

NOTE.—All the pieces were taken *promiscuously* from engineers' or merchants' stores, except those marked *samples*, which were received from the makers.

District.	Names of the Makers or Works.	Thick.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
			lbs.	lbs.	per cent.
Yorkshire,	FARNLEY,	$\frac{9}{16}$	61,260	104,468	20·9
Lanarkshire,	GLASGOW Best Scrap,	$\frac{5}{8}$	56,094	71,764	15·0
	GLASGOW Best Best,	$\frac{9}{16}$	55,937	70,706	15·4
	Do. do.	$\frac{5}{8}$	55,520	62,373	8·5
	Do. do.	$\frac{9}{16}$	53,300	65,770	12·8
	Do. do.	$\frac{5}{8}$	51,800	64,962	12·7
Staffordshire,	ALBION (crown) Best,	$\frac{5}{8}$	56,157	69,367	14·0
	ALBION Best,	$\frac{5}{8}$	52,159	67,695	14·1
	Do. do.	$\frac{11}{16}$	51,467	60,675	11·2
	EAGLE Best Best,	$\frac{5}{8}$	54,727	71,441	13·7
	EAGLE,	$\frac{21}{32}$	50,056	58,545	8·8
Durham,	CONSETT Best Best,	$\frac{1}{2}$	53,548	65,554	12·6
	CONSETT Ship Angle Iron,	$\frac{9}{16}$	50,807	58,201	5·8

TABLE VIII.—TENSILE STRENGTH OF WROUGHT-IRON STRAPS AND BEAM IRON.

NOTE.—All the pieces were taken *promiscuously* from engineers' or merchants' stores, except those marked *samples*, which were received from the makers.

District.	Names of the Makers or Works.	Thick.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
			lbs.	lbs.	per cent.
Lanarkshire,	GLASGOW, Ship Beam,	$\frac{3}{8}$ $\frac{1}{2}$	55,937	67,606	10·79
	DUNDYVAN, Ship Strap,	$\frac{1}{2}$ $\frac{11}{16}$	55,285	63,635	8·03
	MOSSEND, Ship Strap,	$\frac{9}{16}$ 1	45,439	50,459	5·18
Staffordshire,	THORNEYCROFT, Ship Strap,	$\frac{1}{2}$	52,789	59,918	8·03
S. Wales,	DOWLAIS, Ship Beam,	$\frac{1}{2}$	41,386	45,844	4·82

TABLE IX.—TENSILE STRENGTH OF WROUGHT-IRON PLATES.

NOTE.—All the pieces were taken *promiscuously* from engineers' or merchants' stores, except those marked *samples*, which were received from the makers. L denotes that the strain was applied lengthways of the plate; C, crossways.

District.	Names of the Makers or Works.	Thick.	See note above.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
				lbs.	lbs.	per cent.
Yorkshire,	LOWMOOR, - -	$\frac{5}{16}$	L	52,000	64,746	13·2
			C	50,515	57,383	9·3
	BOWLING, - -	$\frac{3}{8}$	L	52,235	61,716	11·6
			C	46,441	50,009	5·9
	FARNLEY, - -	$\frac{3}{8}$	L	56,005	68,763	14·1
			C	46,221	53,293	7·6
	Do. - -	$\frac{1}{4}$	L	58,487	70,538	10·9
			C	54,098	59,698	5·9
	Do. - -	$\frac{3}{4}$	Samples L	58,437	83,112	17·0
			C	55,033	68,961	11·3
Durham,	CONSETT, - -	$\frac{3}{4}$	L	51,245	59,183	8·93
			C	46,712	52,050	6·43
	Do. Best Best, -	$\frac{7}{16}$ & $\frac{14}{16}$	L	49,120	55,472	8·0
			C	46,755	50,000	5·2
	Do. do. -	$\frac{7}{16}$ & $\frac{9}{16}$	L	53,559	62,346	11·5
			C	45,677	48,358	4·0
Staffordshire,	J. BRADLEY & Co., S. C. ♔	$\frac{1}{2}$	L	55,831	67,406	12·5
			C	50,550	55,206	5·5
	Do. L F do. -	$\frac{3}{8}$ to $\frac{1}{2}$	L	56,996	66,858	13·0
			C	51,251	56,070	5·9
	Do. ,, do. -	$\frac{3}{8}$ to $\frac{1}{2}$	L	55,708	65,652	10·7
			C	49,425	54,002	5·1
	T. WELLS, Best Best ♔	$\frac{5}{16}$ to $\frac{8}{16}$	L	47,410	51,521	4·0
			C	46,630	48,348	3·4
	K B M	$\frac{5}{16}$	L	46,404	51,896	6·1
			C	44,764	47,891	4·3
	MALINSLEE, Best ♔	$\frac{3}{8}$	L	52,572	62,131	8·6
			C	50,627	55,746	5·8
	G. B. THORNEYCROFT, Best D W Best, -	$\frac{15}{16}$	L	54,847	62,747	11·2
			C	45,585	47,712	4·6
	J. WELLS ♔ B. Best, -	$\frac{10}{16}$ & $\frac{12}{16}$	Samples L	45,997	51,140	6·7
			C	49,311	54,842	7·0
	LLOYDS, FOSTER, & Co., Best, -	$\frac{6}{16}$ to $\frac{7}{16}$	L	44,967	49,162	5·3
			C	44,732	48,344	4·6

TABLE IX.—TENSILE STRENGTH OF WROUGHT-IRON PLATES—*continued.*

District.	Names of the Makers or Works.	Thick.	See note above.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
				lbs.	lbs.	per cent.
Shropshire,	SNEDSHILL ♔ Best, -	$\frac{5}{16}$ to $\frac{7}{16}$	L C	52,362 43,036	61,581 45,300	9·6 2·8
Lanarkshire,	MOSSEND, Best Best, -	$\frac{3}{8}$	L C	43,433 41,456	46,038 43,622	3·3 2·9
	GLASGOW, Best Boiler,	$\frac{3}{8}$ to $\frac{11}{16}$	L C	53,849 48,848	60,522 52,252	9·3 4·6
	Do. Ship, -	$\frac{3}{16}$ to $\frac{11}{16}$	L C	47,773 44,355	49,816 45,343	3·65 2·11
	Do. Best Best, -	$\frac{7}{16}$ to $\frac{11}{16}$	L C	45,626 41,340	48,208 42,430	4·34 2·37
	Do. do. -	$\frac{1}{4}$ to $\frac{3}{4}$	L C	53,399 41,791	59,557 43,614	8·95 2·63
	Do. Best Scrap,	$\frac{5}{8}$	L	50,844	58,412	10·5
	Makers' stamp uncertain,	$\frac{4}{16}$ to $\frac{11}{16}$	L C	47,598 40,682	53,182 43,426	5·9 2·5
	GOVAN, Best, - -	$\frac{1}{4}$ to $\frac{1}{2}$	L C	43,942 39,544	45,886 40,624	3·4 1·4
	◇ GOVAN ◇ -	$\frac{13}{32}$	L C	54,644 49,399	66,728 54,020	11·6 6·5

353. Tensile strength of wrought-iron, mean results.—The following short table contains the mean results of Mr. Kirkaldy's experiments on the tensile strength of wrought-iron:—

TABLE X.—TENSILE STRENGTH OF WROUGHT-IRON, MEAN RESULTS.

	lbs.		tons.	
188 bars, rolled, - - - - -	57,555	=	$25\frac{3}{4}$	
72 angle-iron and straps, - - -	54,729	=	$24\frac{1}{2}$	
167 plates, lengthways, - - - -	50,737	=	22·65	$21\frac{3}{4}$
160 plates, crossways, - - - -	46,171	=	20·6	

In my own experience I find that the common brands of plate-iron which are manufactured for girder-work and ship-building are about 10 per cent. weaker than the mean results in the foregoing table, and that their set after fracture, lengthways, rarely exceeds 5 per cent. of the total length; also that Staffordshire and North of England iron are generally tougher than Scotch iron.

354. Kirkaldy's conclusions.—Mr. Kirkaldy sums up the results of his experimental inquiry in the following concluding observations, which the student should study carefully:—

1. The breaking strain does *not* indicate the quality, as hitherto assumed.
2. A *high* breaking strain may be due to the iron being of superior quality, dense, fine, and moderately soft, or simply to its being very hard and unyielding.
3. A *low* breaking strain may be due to looseness and coarseness in the texture, or to extreme softness, although very close and fine in quality.
4. The contraction of area at fracture, previously overlooked, forms an essential element in estimating the quality of specimens.
5. The respective merits of various specimens can be correctly ascertained by comparing the breaking strain *jointly* with the contraction of area.
6. Inferior qualities show a much greater variation in the breaking strain than superior.
7. Greater differences exist between small and large bars in coarse than in fine varieties.
8. The prevailing opinion of a rough bar being stronger than a turned one is erroneous.
9. Rolled bars are slightly hardened by being forged down.
10. The breaking strain and contraction of area of iron plates are greater in the direction in which they are rolled than in a transverse direction.
11. A very slight difference exists between specimens from the centre and specimens from the outside of crank shafts.
12. The breaking strain and contraction of area are greater in those specimens cut lengthways out of crank shafts than in those cut crossways.
13. The breaking strain of steel, when taken alone, gives no clue to the real qualities of various kinds of that metal.
14. The contraction of area at fracture of specimens of steel must be ascertained as well as in those of iron.
15. The breaking strain, *jointly* with the contraction of area, affords the means of comparing the peculiarities in various lots of specimens.
16. Some descriptions of steel are found to be very hard, and, consequently, suitable for some purposes; whilst others are extremely soft, and equally suitable for other uses.
17. The breaking strain and contraction of area of *puddled-steel* plates, as in iron plates, are greater in the direction in which they are rolled; whereas in *cast*-steel they are less.
18. Iron, when fractured suddenly, presents invariably a crystalline appearance; when fractured slowly, its appearance is invariably fibrous.

19. The appearance may be changed from fibrous to crystalline by merely altering the shape of specimen, so as to render it more liable to snap.

20. The appearance may be changed by varying the treatment, so as to render the iron harder and more liable to snap.

21. The appearance may be changed by applying the strain so suddenly as to render the specimen more liable to snap, from having less time to stretch.

22. Iron is less liable to snap the more it is worked and rolled.

23. The "skin" or outer part of the iron is somewhat harder than the inner part, as shown by appearance of fracture in rough and turned bars.

24. The mixed character of the scrap-iron used in large forgings is proved by the singularly varied appearance of the fractures of specimens cut out of crank shafts.

25. The texture of various kinds of wrought-iron is beautifully developed by immersion in dilute hydrochloric acid, which, acting on the surrounding impurities, exposes the metallic portion alone for examination.

26. In the fibrous fractures the threads are drawn out, and are viewed externally, whilst in the crystalline fractures the threads are snapped across in clusters, and are viewed internally or sectionally. In the latter cases the fracture of the specimen is always at right angles to the length; in the former it is more or less irregular.

27. Steel invariably presents, when fractured slowly, a silky fibrous appearance; when fractured suddenly, the appearance is invariably granular, in which case also the fracture is always at right angles to the length; when the fracture is fibrous, the angle diverges always more or less from 90°.

28. The granular appearance presented by steel suddenly fractured is nearly free of lustre, and unlike the brilliant crystalline appearance of iron suddenly fractured; the two combined in the same specimen are shown in iron bolts partly converted into steel.

29. Steel which previously broke with a silky fibrous appearance is changed into granular by being hardened.

30. The little additional time required in testing those specimens, whose rate of elongation was noted, had no injurious effect in lessening the amount of breaking strain, as imagined by some.

31. The rate of elongation varies not only extremely in different qualities, but also to a considerable extent in specimens of the same brand.

32. The specimens were generally found to stretch equally throughout their length until close upon rupture, when they more or less suddenly drew out, usually at one part only, sometimes at two, and, in a few exceptional cases, at three different places.

33. The ratio of ultimate elongation may be greater in short than in long bars in some descriptions of iron, whilst in others the ratio is not affected by difference in the length.

34. The lateral dimensions of specimens forms an important element in comparing either the rate of, or the ultimate, elongations—a circumstance which has been hitherto overlooked.

35. Steel is reduced in strength by being hardened in water, while the strength is vastly increased by being hardened in oil.

36. The higher steel is heated (without, of course, running the risk of being burned) the greater is the increase of strength, by being plunged into oil.

37. In a highly converted or hard steel the increase in strength and in hardness is greater than in a less converted or soft steel.

38. Heated steel, by being plunged into oil instead of water, is not only considerably *hardened*, but *toughened* by the treatment.

39. Steel plates hardened in oil, and joined together with rivets, are fully equal in strength to an unjointed soft plate, or the loss of strength by riveting is more than counterbalanced by the increase in strength by hardening in oil.

40. Steel rivets, fully larger in diameter than those used in riveting iron plates of the same thickness, being found to be greatly too small for riveting steel plates, the probability is suggested that the proper proportion for iron rivets is not, as generally assumed, a diameter equal to the thickness of the two plates to be joined.

41. The shearing strain of steel rivets is found to be about a fourth less than the tensile strain.

42. Iron bolts, case-hardened, bore a less breaking strain than when wholly iron, owing to the superior tenacity of the small proportion of steel being more than counterbalanced by the greater ductility of the remaining portion of iron.

43. Iron highly heated and suddenly cooled in water is hardened, and the breaking strain, when gradually applied, increased, but at the same time it is rendered more liable to snap.

44. Iron, like steel, is softened, and the breaking strain reduced, by being heated and allowed to cool slowly.

45. Iron subject to the cold-rolling process has its breaking strain greatly increased by being made extremely hard, and not by being "consolidated," as previously supposed.

46. Specimens cut out of crank-shaft are improved by additional hammering.

47. The galvanizing or tinning of iron plates produces no sensible effects on plates of the thickness experimented on. The result, however, may be different, should the plates be extremely thin.

48. The breaking strain is materially affected by the shape of the specimen. Thus the amount borne was much less when the diameter was uniform for some inches of the length than when confined to a small portion—a peculiarity previously unascertained, and not even suspected.

49. It is necessary to know correctly the exact conditions under which any tests are made before we can equitably compare results obtained from different quarters.

50. The startling discrepancy between experiments made at the Royal Arsenal, and by the writer, is due to the difference in the shape of the respective specimens, and not to the difference in the two testing machines.

51. In screwed bolts the breaking strain is found to be greater when old dies are used in their formation than when the dies are new, owing to the iron becoming harder by the greater pressure required in forming the screw thread when the dies are old and blunt than when new and sharp.

52. The strength of screw-bolts is found to be in proportion to their relative areas, there being only a slight difference in favour of the smaller compared with the larger sizes, instead of the very material difference previously imagined.

53. Screwed bolts are not necessarily injured, although strained nearly to their breaking point.

54. A great variation exists in the strength of iron bars which have been cut and welded; whilst some bear almost as much as the uncut bar, the strength of others is reduced fully a third.

55. The welding of steel bars, owing to their being so easily burned by slightly overheating, is a difficult and uncertain operation.

56. Iron is injured by being brought to a white or welding heat, if not at the same time hammered or rolled.

57. The breaking strain is considerably less when the strain is applied suddenly instead of gradually, though some have imagined that the reverse is the case.

58. The contraction of area is also less when the strain is suddenly applied.

59. The breaking strain is reduced when the iron is frozen; with the strain gradually applied, the difference between a frozen and unfrozen bolt is lessened, as the iron is warmed by the drawing out of the specimen.

60. The amount of heat developed is considerable when the specimen is suddenly stretched, as shown in the formation of vapour from the melting of the layer of ice on one of the specimens, and also by the surface of others assuming tints of various shades of blue and orange, not only in steel, but also, although in a less marked degree, in iron.

61. The specific gravity is found generally to indicate pretty correctly the quality of specimens.

62. The density of iron is *decreased* by the process of wire-drawing, and by the similar process of cold rolling, instead of *increased*, as previously imagined.

63. The density in some descriptions of iron is also decreased by additional hot-rolling in the ordinary way; in others the density is very slightly increased.

64. The density of iron is decreased by being drawn out under a tensile strain, instead of increased, as believed by some.

65. The most highly converted steel does not, as some may suppose, possess the greatest density.

66. In cast-steel the density is much greater than in puddled-steel, which is even less than in some of the superior descriptions of wrought-iron.

The foregoing extracts afford the reader but a meagre idea of Mr. Kirkaldy's laborious researches, and the student who seeks more detailed information regarding his experiments, or the instruments and method he adopted in testing specimens, is referred to his book on the subject.

355. Strength of iron plates lengthways 10 per cent. greater than crossways—Removing skin of wrought-iron does not injure its tensile strength.—From Table X. it appears that the average strength of wrought-iron plates drawn in the direction of their length is about ten per cent. greater than when drawn across the grain. The "set after fracture" is also much greater in the direction of the fibres. This agrees with Mr. Clark's experiments* as well as with my own experience. With reference to the effect of removing the outer skin or glaze on rolled iron, Mr. Kirkaldy observes, "The generally received opinion, that by removing the 'skin' the relative strength was greatly reduced, or that a *rough* bar was much stronger than one *turned* to the same diameter, is proved to be erroneous."†

356. Bar and angle iron are tougher and stronger than plates—Boiler plates—Ship plates—Hard iron unfit for shipbuilding.—Both bar and angle iron are tougher and stronger than plate iron, and from Table X. it appears that bars of ordinary sizes are nearly 14 per cent. stronger than plates; perhaps this does not apply to bars of large section, say three inches in diameter and upwards. The great demand for iron ships has given rise to the manufacture of a cheap quality of plate iron called "ship" or "boat" plates; this iron is generally inferior in strength and toughness to "boiler" plates, and is often so hard and brittle that its set after fracture does not exceed two or three per cent. of the length, even with the grain, while its tensile strength is frequently less than eighteen tons per square inch. There can be no greater mistake than to suppose that hard iron is fit for ships. Iron plates which are tough and ductile like copper will, when struck, often escape with a mere dint or bulge, whereas hard iron under the same circumstances will crack or tear, especially along a line of rivet holes.

357. Large forgings not so strong as rolled iron—Annealing reduces the tensile strength of small iron, but increases its ductility—Annealing injurious to large forgings—Very prolonged annealing injurious to all wrought-iron—Excessive strain renders iron brittle.—It is generally believed that large forgings are less tenacious than small ones. About this, however, there is some difference of opinion, and the subject requires further experiments before it can be definitively settled.‡ Large forgings certainly require greater manufacturing skill than small ones, and it is probable that large forgings, such as

* Clark *on the Tubular Bridges*, p. 377.

† *Expts.*, p. 27.

‡ See discussion on Mr. Mallet's paper on the Coefficients of Elasticity and Rupture in Massive Forgings.—*Proc. Inst. C. E.*, Vol. xviii., p. 296.

shafts for marine engines, are somewhat weaker in tensile strength than bar or plate iron to which the rolling process imparts a fibrous structure; this view seems to be confirmed by Mr. Kirkaldy's experiments on hammered iron in Table VI. Annealing small iron reduces its tensile strength (**354** 44), though it increases its ductility and toughness, which are sometimes more important qualities. For instance, it is a good practice to anneal old crane chains which have become brittle by overstraining, and thus render them less liable to snap from sudden jerks. Annealing large forgings is injurious, as it produces a crystalline structure, the reverse of fibrous, and very prolonged annealing of small sized iron seems to have a similar bad effect.* If an iron bar be torn asunder several times in succession, its tensile strength each time will apparently increase, because it first gives way at the weakest point, next time at the second weakest, and so on; but though several applications of the tearing strain do not diminish its ultimate strength to resist a steady pull, they take the ductility or stretch out of the iron and render it hard and brittle and therefore liable to snap from sudden shocks. For the safe working load of wrought-iron see Chap. XXVIII.

IRON WIRE.

358. Tensile strength of iron wire—Annealing iron wire reduces its tensile strength.—From Mr. Telford's experiments it appears that the strength of iron wire $\frac{1}{10}$th inch diameter = 36 tons per square inch.† The strength of the iron wire used by Mr. Roebling at the Niagara Falls suspension bridge was nearly 100,000 lbs. (= 44·6 tons) per square inch. This wire measures 18·31 feet per lb., and is "small No. 9 Gauge, 60 wires forming one square inch of solid section."‡

The following table contains the results of experiments made by M. Seguin on iron-wire of different sizes and qualities.§

* *Morin*, p. 47.

† Barlow *on the Strength of Materials*, p. 283.

‡ *Papers and Practical Illustrations of Public Works of Recent Construction, both British and American.* Weale: 1856. pp. 16, 18.

§ *Résumé des leçons sur l'application de la Mécanique.* Par M. Navier. Bruxelles, 1839, p. 30.

TABLE XI.—TENSILE STRENGTH OF IRON WIRE.

Description of Wire.	Diameter.	Tearing weight per square millimetre.
	millimetres.	kilogrammes.
Iron wire from Bourgogne, No. 8, unequally annealed,	1·172	38·2
Idem, No. 7, carefully annealed, - - -	1·062	36·1
Idem, No. 18, not annealed, - - -	3·366	58·8
Idem, No. 7, not annealed, - - - -	1·062	73·7
Fil de l'Aigle, employed for carding, - -	0·2294	89·8
Passe-perle, rather soft, - - - -	0·5917	85·7
Wire from a factory in Besançon—		
No. 1, soft, - - -	0·6188	86·1
2, soft, - - -	0·7078	87·0
3, brittle, - - -	0·7327	80·8
4, brittle, - - -	0·838	76·6
5, very brittle, - -	0·9115	72·3
6 - - - -	1·022	76·1
7 - - - -	1·08	71·2
8, very brittle, - -	1·123	67·3
9, rather brittle, - -	1·293	69·8
10, very soft, - -	1·435	64·8
11, very soft, - -	1·476	58·6
12 - - - -	1·691	55·5
13 - - - -	1·8	57·2
14, very soft, without elasticity,	2·072	49·3
15 - - - -	2·226	51·9
16, very soft, - -	2·489	63·9
17, flawed, - -	2·695	68·1
18 - - - -	3·087	84·0
19 - - - -	3·492	78·2
20 - - - -	4·14	65·7
21 - - - -	4·812	62·5
22, very brittle, - -	5·449	67·7
23, soft, - -	5·942	62·6

NOTE.—A millimetre equals very nearly ·04 = $\frac{1}{25}$th inch; and kilogrammes per square millimetre may be converted into tons per square inch by multiplying by 0·635.

That annealing iron wire seriously impairs its tensile strength may be inferred from the foregoing experiments.

STEEL.

359. Tensile strength, ultimate set and limit of elasticity of steel.—The following table contains the results of experiments on the tensile strength and other properties of steel bars 50 inches long and 1·382 inch diameter (= 1·5 sq. inch), made by Mr. Kirkaldy for the "Steel Committee," the samples being carefully turned down from two-inch square bars.*

TABLE XII.—TENSILE STRENGTH AND LIMIT OF ELASTICITY OF STEEL BARS.

Kind of Steel.		Tearing weight per square inch.		Ultimate elongation, or tensile set after fracture.	Limit of tensile elasticity.
		tons.		per cent.	tons.
CRUCIBLE STEEL.					
Hammered,	Tyres,	35·51	38·19	9·17	20·62
	Axles,	40·94		8·72	25·56
	Rails,	38·14		2·96	19·64
Rolled,	Axles,		30·62	10·56	18·75
BESSEMER STEEL.					
Hammered,	Tyres,	35·09	33·93	11·1	23·30
	Axles,	33·47		12·1	21·87
	Rails,	33·24		12·8	21·43
Rolled,	Tyres,	32·09	31·99	18·8	19·19
	Axles,	32·22		19·0	17·85
	Rails,	31·67		16·0	20·09
Mean,			33·68	12·12	20·83

Table XIII. contains the results of additional experiments made

* *Experiments on Steel and Iron by a Committee of Civil Engineers*, 1868–70.

by the same Committee at Woolwich Dockyard on various descriptions of steel bars 10 feet long and $1\frac{1}{2}$ inch diameter.

TABLE XIII.—TENSILE STRENGTH AND LIMIT OF ELASTICITY OF STEEL BARS.

Kind of Steel.	What the steel was intended for.	Tearing weight per square inch.	Ultimate elongation, or tensile set after fracture.	Limit of tensile elasticity.
		tons.	per cent.	tons.
Crucible cast steel from Swedish bar iron, chisel temper, -	...	52·76	5·29	26·00
Crucible cast steel, - -	...	51·01	7·29	25·50
Cast steel, - - -	Tyres - -	43·48	4·74	26·00
Ditto, - - -	Piston rods, &c. -	41·85	1·12	27·00
Crucible steel, - - -	...	40·54	4·13	20·50
Ditto, - - -	Gun barrels -	38·51	7·95	16·83
Hammered crucible cast steel, -	...	37·05	13·54	25·00
Crucible steel, - - -	...	35·47	9·63	20·00
Bessemer steel, - - -	Faggoted, hammered & rolled	35·40	11·13	19·50
Cast steel, - - -	Piston rods, &c. -	33·65	0·89	26·75
Rolled crucible cast steel, -	...	34·43	2·02	20·50
Bessemer steel, - - -	...	34·19	11·90	20·00
Ditto, - - -	...	33·63	11·48	17·50
Ditto, - - -	Tyres and axles -	33·66	13·61	16·50
Mean, - - - -	...	38·97	7·48	21·97

Table XIV. gives the results of experiments by Sir William Fairbairn on the mechanical properties of steel.*

* *Brit. Ass. Rep.*, 1867.

TABLE XIV.—COEFFICIENTS OF RUPTURE, AND THE TENSILE AND CRUSHING STRENGTHS OF STEEL.

MANUFACTURERS.	Coefficient of transverse rupture at the limit of elastic reaction. S	Tearing weight per square inch.		Ultimate elongation, or tensile set after fracture.	Greatest crushing weight laid on per square inch.		Corresponding contraction, or set due to compression.
	tons.	lbs.	tons.	per cent.	lbs.	tons.	per cent.
MESSRS. BROWN & CO.							
Best cast-steel from Russian and Swedish iron, for turning-tools,	6·326	68,404	30·53	·25	225,568	100·7	25·3
Do. milder,	6·326	91,520	40·85	1·50	"	"	26·3
Cast-steel from Swedish iron, for tools,	6·958	106,714	47·64	1·00	"	"	18·3
Do. milder, for chisels,	6·134	116,183	51·86	3·62	"	"	29·3
Do. mild, for welding,	6·134	110,055	49·13	3·31	"	"	24·3
Bessemer steel,	5·297	91,972	41·05	19·62	"	"	40·3
Specimen of double shear steel from Swedish iron,	5·527	92,555	41·31	5·43	"	"	44·3
Do., foreign bar, tilted direct,	5·394	76,474	34·27	13·56	"	"	49·3
English tilted steel, made from English and foreign pigs,	5·170	59,538	26·57	21·06	"	"	55·3
C. CAMMELL & CO.							
Specimen of cast-steel, termed "Diamond Steel,"	7·504	110,055	49·13	1·53	"	"	23·3
Do. do. termed "Tool Steel,"	5·904	109,072	48·69	1·50	"	"	26·3
Do. do. termed "Chisel Steel,"	7·413	120,398	53·75	2·50	"	"	31·3
Do. do. termed "Double Shear Steel,"	5·132	96,665	43·15	2·37	"	"	30·3
Bar of hard Bessemer steel,	4·588	89,121	39·78	20·87	"	"	43·3
Do. soft Do.	4·988	81,483	36·37	20·43	"	"	49·3
MESSRS. NAYLOR, VICKERS, & CO.							
Cast-steel, called "Axle Steel,"	5·742	88,665	39·58	16.25	"	"	42·3
Do. do. "Tyre Steel,"	5·505	91,520	40·85	9·00	"	"	38·8
Do. do. "Vickers' Cast-steel, Special,"	7·856	134,145	59·87	1·00	"	"	15·3
Do. do. "Naylor & Vickers' Cast-steel,"	7·358	118,066	52·70	1·75	"	"	18·3
S. OSBORNE.							
Specimen of best tool, cast-steel,	5·432	98,942	44·17	0·93	"	"	20·3
Do. best chisel, do.	6·400	123,686	55·21	3·18	"	"	24·3
Sates-cup, shear-blades, and boiler-makers' steel,	4·691	115,849	51·71	2·12	"	"	25·3
Best cast-steel for taps and dies,	6·037	98,790	44·10	1·68	"	"	26·3
Toughened cast-steel for shafts, &c.,	5·559	103,116	46·03	5·25	"	"	32·3
Specimen of best double shear-steel,	4·329	87,931	39·25	2·43	"	"	32·3
Extra best tool, cast-steel,	6·860	85,724	38·26	0·43	"	"	19·3
Cast-steel for boiler plates,	5·671	111,676	49·85	13·50	"	"	33·3

TABLE XIV.—Coefficients of Rupture, and the Tensile and Crushing Strengths of Steel—*continued.*

Manufacturers.	Coefficient of transverse rupture at the limit of elastic reaction. S	Tearing weight per square inch.		Ultimate elongation, or tensile set after fracture.	Greatest crushing weight laid on per square inch.		Corresponding contraction, or set due to compression.
	tons.	lbs.	tons.	per cent.	lbs.	tons.	per cent.
H. Bessemer.							
Specimen of hard Bessemer steel,	6·882	103,085	46·02	1·87	,,	,,	22·3
Do. milder do.	5·317	88,175	39·36	20·00	,,	,,	44·3
Do. soft do.	4·778	78,606	35·09	19·12	,,	,,	47·3
Sanderson, Brothers.							
Bar of cast-steel from Russian iron suitable for welding,	5·539	83,484	37·26	2·25	,,	,,	39·8
Specimen of double-shear steel,	4·808	107,940	48·18	3·31	,,	,,	30·3
Do. single do.	6·780	107,182	47·84	2·81	,,	,,	28·3
Bar of faggot-steel, welded,	5·572	75,199	33·57	1·25	,,	,,	32·3
Specimen of drawn bar, not welded,	4·907	103,960	46·41	3·43	,,	,,	33·3
Messrs. Turton & Sons.							
Steel intended for the manufacture of cups,	5·392	100,155	44·71	2·75	,,	,,	28·3
Do. do. drills,	6·625	87,552	39·08	1·06	,,	,,	19·3
Do. do. cutters,	6·718	95,372	42·57	1·37	,,	,,	24·3
Do. do. turning tools,	6·337	80,273	35·02	0·12	,,	,,	26·3
Do. do. machinery,	6·576	102,915	45·94	1·43	,,	,,	23·3
Do. do. punches,	5·440	102,567	45·79	1·62	,,	,,	25·3
Do. do. mint dies,	5·861	106,237	47·42	2·87	,,	,,	29·3
Do. do. dies,	5·570	87,471	39·04	0·87	,,	,,	27·3
Do. do. taps,	5·788	97,994	43·74	1·87	,,	,,	27·3
Specimen of double shear-steel,	4·561	73,266	32·70	0·81	,,	,,	29·3
Mean,	6·0	106,848	47·7				

The mean coefficient of elasticity, derived from experiments on deflection under moderate transverse strain, = 31,000,000 lbs. per square inch. The coefficients of rupture in the second column were derived from experiments on the transverse strength of bars 1 inch square, 4 feet 6 inches long between supports, and loaded at the centre up to the elastic limit. The tearing and the crushing experiments were subsequently made on portions of the same bars which had been previously strained by transverse pressure. The ultimate elongations were taken on eight-inch lengths of bar. Thirty-two of the bars supported a compression of 100·7 tons per square inch without undergoing any sensible fracture, whilst twenty-three bars were more or less fractured with this pressure, but all of them bulged laterally under pressure, some, it will be perceived by the last column, to a very considerable extent.

Tables XV. and XVI. contain the principal results of Mr. Kirkaldy's experiments on the tensile strength of steel bars and plates.* His "conclusions" respecting steel will be found in **354**.

TABLE XV.—TENSILE STRENGTH OF STEEL BARS.

NOTE.—All the pieces were taken *promiscuously* from engineers' or merchants' stores, except those marked *samples*, which were received from the makers.

District.	Names of the Makers or Works.	Description.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or set after fracture.
			lbs.	lbs.	per cent.
Sheffield.	T. TURTON AND SONS, Cast Steel for Tools (from Acadian Iron),	All forged from rolled bars by the same smith, reheated after hammering and allowed to cool gradually.	132,909	139,124	5·4
	THOMAS JOWITT, Cast Steel for Tools,		132,402	151,857	5·2
	Do. do., Cast Steel for Chisels,		124,852	150,243	7·1
	Do. do., Cast Steel for Drifts,		115,882	147,570	13·3
	T. JOWITT, Double Shear Steel,		118,468	147,396	13·5
	BESSEMER (tool), *samples*,		111,460	143,327	5·5
	WILKINSON, (L) Blister Steel,		104,298	132,472	9·7
	T. JOWITT, Cast Steel for Taps,		101,151	142,070	10·8
	T. JOWITT, Spring Steel,	Forged from $\frac{3}{4}$ inch rolled bars,	72,529	95,490	18·0
	MOSS AND GAMBLES, Cast Steel for Rivets, *samples*,	Rolled bars, $\frac{3}{4}$ inch round,	107,286	158,013	12·4
	NAYLORS, VICKERS, AND Co., Cast Steel for Rivets,	Rolled bars, $\frac{3}{4}$ inch round,	106,615	158,785	8·7
	SHORTRIDGE, HOWELL, AND Co., Homogeneous Metal,	Rolled bars, $\frac{9}{16}$ inch, for rivets,	90,647	142,920	13·7
	Do., do.,	Forged, - -	89,724	121,212	11·9
Liverpool.	MERSEY Co., Puddled Steel,	Forged, - -	71,486	110,451	19·1
Glasgow.	BLOCHAIRN, Puddled Steel,	Rolled bars, -	70,166	84,871	11·3
	Do., do.,	Forged from slabs,	65,255	80,370	12·0
	Do., do.,	Forged from rolled bars,	62,769	71,231	9·1
Prussia.	KRUPP, Dusseldorf, Cast Steel for Bolts,	Rolled bars, round,	92,015	139,434	15·3

* *Expts. on Wrought Iron and Steel.*

TABLE XVI.—TENSILE STRENGTH OF STEEL PLATES.

NOTE.—All the pieces were taken *promiscuously* from engineers' or merchants' stores, except those marked *samples*, which were received from the makers. L denotes that the strain was applied *lengthways* of the plate; C, *crossways.*

District.	Names of the Makers or Works.	Thick.	Description.	Tearing weight per square inch of original area.	Tearing weight per square inch of fractured area.	Ultimate elongation, or tensile set after fracture.
		inch.		lbs.	lbs.	per cent.
Sheffield.	T. TURTON AND SONS, Cast Steel,	$\frac{1}{4}$	*Samples* L C	94,289 96,308	100,063 111,811	5·71 9·64
	NAYLOR, VICKERS, AND Co., Cast Steel,	$\frac{1}{4}$	*Samples* L C	81,719 87,150	104,232 112,018	17·50 17·32
	MOSS AND GAMBLES, Cast Steel,	$\frac{3}{16}$ & $\frac{4}{16}$	*Samples* L C	75,594 69,082	105,554 112,546	19·82 19·64
	SHORTRIDGE, HOWELL, AND Co., Homogeneous Metal,	$\frac{3}{16}$	L C	96,280 97,150	114,106 114,300	8·61 8·93
	Do., do.,	$\frac{3}{8}$	C	96,989	113,305	14·4
	Do., Second Quality, -	$\frac{1}{4}$	*Samples* L C	72,408 73,580	81,823 78,245	5·93 3·21
Liverpool.	MERSEY Co., Puddled Steel (Ship Plates),	$\frac{2}{16}$ & $\frac{3}{16}$	L C	101,450 84,968	109,552 91,746	2·79 1·25
	MERSEY Co., Puddled Steel "Hard,"	$\frac{1}{4}$	*Samples* L C	102,593 85,365	107,827 89,116	4·86 3·30
	Do. "Mild," do., -	$\frac{1}{4}$	*Samples* L C	77,046 67,686	88,240 73,634	6·16 5·72
	Do. do. (Ship Plates), -	$\frac{9}{32}$	L	71,532	77,520	3·57
Glasgow.	BLOCHAIRN, Puddled Steel, -	$\frac{3}{16}$	*Samples* L C	102,234 84,398	108,079 87,877	3·60 2·68
	Do., do. (Boiler Plates),	$\frac{5}{16}$	L C	96,320 73,699	107,614 76,646	8·22 4·14

360. Steel plates often deficient in uniformity and toughness—Punching as compared with drilling greatly reduces the tensile strength of steel plates; strength generally restored by annealing—Annealing equalizes different qualities of steel plates.—From the foregoing table it appears that the difference of strength lengthways and crossways is often much greater in steel than in iron plates, amounting to nearly 20 per cent. in some specimens. The reader will also observe that the ultimate tensile set of steel plates is in general small compared with that of the tougher kinds of iron in Table IX. This indicates the direction to which manufacturers of steel should direct their attention, as for many purposes, especially shipbuilding, toughness and ductility are quite as essential as great tensile strength (**356**). Sometimes steel plates are so brittle as to fly in pieces under the hammer, or split in punching, and thick plates are said to possess this undesirable quality to a greater degree than thin ones, and occasionally they fly without any apparent cause whatever shortly after they have been riveted in place. Complaints also are made of want of uniformity of texture, some plates of a lot being all that could be desired, while others of the same lot may be hard and brittle. Owing to this uncertainty the manufacture of steel plates seems still in a transition state, and consequently, engineers and shipbuilders have not made use of the material to the extent to which its superior tensile strength seems to destine it.

It appears from papers on the treatment of steel, read at the annual meeting of the Institution of Naval Architects in April, 1868, that steel plates, such as are now sometimes used in shipbuilding, may be obtained of a tensile strength of from 30 to 35 tons per square inch. Punching, as compared with drilling, reduced the strength of Bessemer steel plates 33 per cent. It was found, however, that annealing these punched Bessemer plates restored them to their original strength. In other experiments on mild puddled steel plates the loss of strength from punching was 21 per cent., and there was no benefit from subsequent annealing. With mild crucible steel plates the loss of strength from punching was 7 per cent., and the gain of annealed over unannealed was 14 per cent. Annealing was also recommended to equalize the strength of steel, as in a batch of plates sent in by the same manufacturer the plates sometimes greatly differ, and a bath of molten lead was recommended as a cheap and certain mode of annealing. It was also stated that enlarging the die when punching steel, so as to give the die a large clearance, as much as $\frac{3}{16}$th inch, round the punch and make a taper hole, gave a great advantage with Bessemer steel, amounting to 25 per cent., but in experiments on iron plates it was found that a greater clearance than the usual one of $\frac{1}{16}$th inch rather injured the iron. Mr. Krupp says with regard to the treatment of cold cast-steel boiler plates:—"In working the plates cold, all sharp turns, corners, and edges must be avoided or removed. The surfaces of cuts and rivet-holes must, before bending and riveting, be worked and rounded off as neatly as possible, so that no rough and serrated places remain after cutting and punching." He also recommends as a general rule that the plates should be thoroughly and equally annealed at a *dark-red* heat after every large operation, and that they should certainly have such annealing at the conclusion of all operations. The directions given by him as to bending hot are as follows:—"The plates should be heated, preparatory to bending, to a heat *not exceeding* a bright cherry-red. Also the greatest possible portion of the surface should be heated, and not merely the edge, and even, where practicable, the whole plate should be equally heated. By this means the strains which arise from local heating and cooling, and which are much greater in cast-steel plates, on account of their higher absolute and reflex density, than in iron, are, by the general heating of the plate, more equably distributed. The thickest and toughest plates can be broken by local heating, bending and cooling. Bends which cannot be completed in one, or at most in two consecutive heatings, must be made gradually and equably over the whole extent to be operated on." In bending, for example, to an angle of 90°, the whole plate should first be bent through about one-third of the angle, then through another third, and finally to the complete angle:—"After the whole of these operations, the plate is to be equably annealed at a *dark-red* heat, which will thus equalize the strains caused by the previous working."* For the safe working-strain of steel see Chap. XXVIII.

* *Reed on Shipbuilding.*

STEEL WIRE.

361. Tensile strength of steel wire.—In experiments made for the Atlantic Telegraph the strength of steel wire ·095 inch diameter was 1950 lbs., while that of special charcoal wire of the same size was 750 lbs.*

VARIOUS METALS AND ALLOYS.

362. Tensile strength of various metals and alloys.—The following table contains the tensile strength of various metals and alloys by several experimenters.

TABLE XVII.—TENSILE STRENGTH OF VARIOUS METALS AND ALLOYS.

Description of Metal.	Specific gravity.	Initials of Experimenters.	Tearing weight per square inch.
			lbs. tons.
Aluminium Bronze, - - -	—	Rk.	73,000 = 32·59
Brass, Fine Yellow Cast, - -	—	R.	17,968 = 8·02
Do., Wire, - - - -	—	D.	91,325 = 40·77
Copper, Wrought, reduced per hammer, -	—	R.	33,792 = 15·08
Do., do., in bolts, - -	—	K.	47,936 = 21·40
Do., Cast, - - - -	—	R.	19,072 = 8·51
Do., do., Lake Superior, - -	8,672	W.	24,252 = 10·82
Do., Sheet, - - - -	—	N.	30,016 = 13·4
Do., Wire, not annealed, - -	8,741	M. D.	77,504 = 34·6
Do., do., annealed, - - -	8,741	M. D.	32,144 = 14·35
Gun Metal or Bronze, hard, - -	—	R.	36,368 = 16·23
Do., mean of 83 gun-heads, - -	8,523	W.	29,655 = 13·24
Do., mean of 5 breech-squares, -	8,765	W.	46,509 = 20·76
Do., mean of 32 small bars cast in same moulds with guns,	8,584	W.	42,019 = 18·76
Do., small bars cast separately in { iron moulds, -	8,953	W.	37,688 = 16·82
Do., small bars cast separately in { clay do., -	8,313	W.	25,783 = 11·51
Do., in finished guns, - - -	—	W.	23,108 to 52,192 = 10·3 to 23·3

* Fairbairn's *Useful Information for Engineers*, third series, p. 282.

TABLE XVII.—TENSILE STRENGTH OF VARIOUS METALS AND ALLOYS—*continued.*

Description of Metal.	Specific gravity.	Initials of Experimenters.	Tearing weight per square inch.
			lbs. tons
Yellow Metal, Patent,	—	K.	49,185 = 21·9
Lead, Cast,	—	R.	1,824 = 0·81
Do., Sheet,	—	N.	1,926 = 0·86
Soft Solder, 2 parts tin to 1 lead by weight,	—	RK.	7,500 = 3·35
Tin, Cast,	—	R.	4,736 = 2·11
Do., Banco,	7,297	W.	2,122 = 0·95
Do.,	—	M. D.	2,845 = 1·27
Zinc, Cast,	—	S.	2,993 = 1·336

D. Dufour, *Application de la Mécanique*, Navier. Brussels, 1839, p. 35.
M. D. Minard et Desormes, *idem*, pp. 34, 36.
N. Navier, *idem*, p. 36.
K. Kingston, *Barlow on the Strength of Materials*, p. 211.
R. Rennie, *Philosophical Transactions for* 1818, p. 126.
RK. Rankine's *Machinery*, p. 464.
S. Stoney.
W. Wade, *Reports on Metals for Cannon*, pp. 281, 288, 289, 290, 295.

363. Gun-metal or bronze—High temperature at casting injurious to bronze.—The proportion of tin to copper in the bronze gun-metal on which Major Wade experimented was 1 to 8, and the great diversity in its tenacity seems attributable to defective homogeneity in the alloy, some parts containing more tin than others, and consequently having a smaller tenacity. A high temperature at casting is injurious to the quality of bronze, as it seems to facilitate the separation of the metals, and small bars are stronger than large castings, probably because the former solidify more suddenly and are thereby not allowed a sufficient time for a division of the alloy into separate compounds. Bronze guns are cast on end in flask moulds, with the breech downwards, and a large extra head of metal above the muzzle to ensure sufficient liquid pressure. Breech-squares, being at the bottom of the moulds, are subject to a much higher pressure than the gun-heads which are at the top, and they are consequently both stronger and denser than the latter. The small bars cast in the gun mould are stronger than those cast separately, probably in consequence of their being under greater pressure, and because they were fed, as they solidified, from the mass of the gun with which they communicated. Major Wade also attributes their superiority to the annealing process they underwent after solidification, from the proximity of the large mass of the gun.*

364. Alloys of copper and tin.—The following table contains the results of experiments made by Robert Mallet, Esq., F.R.S., on the physical properties of certain alloys of copper and tin.†

TABLE XVIII.—PHYSICAL PROPERTIES OF ALLOYS OF COPPER AND TIN.

COPPER AND TIN.				
Chemical Constitution.	Composition by weight per cent.	Specific gravity.	Tearing weight per square inch.	Commercial Title.
			tons.	
10 Cu + Sn	84·29 + 15·71	8·561	16·1	Gun Metal.
9 Cu + Sn	82·81 + 17·19	8·462	15·2	Gun Metal.
8 Cu + Sn	81·10 + 18·90	8·459	17·7	Gun Metal, tempers best.
7 Cu + Sn	78·97 + 21·03	8·728	13·6	Hard Mill Brasses, &c.
Cu + Sn	34·92 + 65·08	8·056	1·4	Small bells, brittle.
Cu + 3 Sn	15·17 + 84·83	7·447	3·1	Speculum Metal of Authors.
Sn	0 + 100	7·291	2·5	Tin.

NOTE.—"The ultimate cohesion was determined on prisms of 0·25 of an inch square, without having been hammered or compressed after being cast. The weights given are those which each prism just sustained for a few seconds before rupture."

TIMBER.

365. Tensile strength of timber.—The following table contains the results of experiments by various authorities on the

* *Report on Metals for Cannon*, pp. 296, 299.

† *On the Construction of Artillery*, p. 82.

tensile strength of timber drawn in the direction of the fibres. For the safe working-strain see Chap. XXVIII.

TABLE XIX.—TENSILE STRENGTH OF TIMBER LENGTHWAYS.

Description of Wood.	Tearing weight per square inch.	Authority.
	lbs.	
Alder,	13,900	Muschenbroeck.
Apple,	19,500	Bevan.
Ash,	16,700	Do.
Do.	17,000	Barlow.
Beech,	11,500	Do.
Do.	17,300	Muschenbroeck.
Do.	22,000	Bevan.
Birch,	15,000	Do.
Box,	20,000	Barlow.
Cane,	6,300	Bevan.
Cedar,	11,400	Do.
Chesnut, Spanish,	13,300	Rondelet.
Do.	10,500	Bevan.
Do., Horse,	12,100	Do.
Cypress,	6,000	Muschenbroeck.
Deal, Christiana,	12,900	Bevan.
Elder,	10,000	Muschenbroeck.
Elm,	14,400	Bevan.
Fir,	12,000	Barlow.
Hawthorn,	10,000	Bevan.
Holly,	16,000	Do.
Jugeb,	18,500	Muschenbroeck.
Laburnum,	10,500	Bevan.
Lance Wood,	23,400	Do.
Larch,	10,220	Rondelet.

TABLE XIX.—TENSILE STRENGTH OF TIMBER LENGTHWAYS—*continued.*

Description of Wood.	Tearing weight per square inch.	Authority.
	lbs.	
Larch, - - - - -	8,900	Bevan.
Lemon, - - - - -	9,250	Muschenbroeck.
Lignum Vitæ, - - - -	11,800	Bevan.
Locust-tree, - - - -	20,100	Muschenbroeck.
Mahogany, - - - - -	8,000	Barlow.
Do. - - - - -	16,500 to 21,800	Bevan.
Maple, - - - - -	17,400	Do.
Mulberry, - - - - -	10,600	Do.
Do. - - - - -	12,500	Muschenbroeck.
Oak, English, - - - -	10,000	Barlow.
Do., do. - - - -	14,000 to 19,800	Bevan.
Do., French, - - - -	13,950	Rondelet.
Do., Black Bog, - - - -	7,700	Bevan.
Orange, - - - - -	15,500	Muschenbroeck.
Pear, - - - - -	9,800	Barlow.
Pine, Pitch, - - - -	7,650	Muschenbroeck.
Do., Norway, - - - -	14,300	Bevan.
Do., do. - - - -	7,287	Rondelet.
Do., Petersburg, - - -	13,300	Bevan.
Plane, - - - - -	11,700	Do.
Plum, - - - - -	11,800	Muschenbroeck.
Pomegranite, - - - -	9,750	Do.
Poplar, - - - - -	5,500	Do.
Do. - - - - -	7,200	Bevan.
Quince, - - - - -	6,750	Muschenbroeck.
Sycamore, - - - - -	13,000	Bevan.
Tamarind, - - - - -	8,750	Muschenbroeck.

TABLE XIX.—Tensile Strength of Timber Lengthways—*continued.*

Description of Wood.	Tearing weight per square inch.	Authority.
	lbs.	
Teak, - - - - -	15,000	Barlow.
Do., old, - - - - -	8,200	Bevan.
Walnut, - - - - -	8,130	Muschenbroeck.
Do. - - - - -	7,800	Bevan.
Willow, - - - - -	14,000	Do.
Yew, - - - - -	8,000	Do.

Barlow, *Barlow on the Strength of Materials*, p. 23.
Muschenbroeck, *idem*, p. 4.
Bevan, *Philosophical Magazine*, 1826, Vol. lxviii., pp. 270, 343.
Rondelet, *Tredgold's Carpentry*, 4th edition, p. 41.

Comparing the foregoing table with Table VI. (**300**), we see that the tensile strength of most kinds of wood is much greater than their compressive strength.

366. Lateral adhesion of the fibres.—The following table gives the lateral adhesion of the fibres, that is, the tensile strength of timber across the grain, in which direction it is much weaker than lengthways.

TABLE XX.—Tensile Strength of Timber Crossways.

Description of Wood.	Tearing weight per square inch.	Authority.
	lbs.	
Fir, Memel, - - - -	540 to 840	Bevan.
Do., Scotch, - - - -	562	Do.
Larch, - - - - -	970 to 1,700	Tredgold.
Oak, - - - - -	2,316	Do.
Poplar, - - - - -	1,782	Do.

Bevan, *Philosophical Magazine*, 1826, Vol. lxviii., p. 112.
Tredgold, *Tredgold's Carpentry*, p. 42.

STONE, BRICK, MORTAR, CEMENT, GLASS.

367. Tensile strength of stone.—As stone is rarely employed in direct tension, there are but few experiments on its tensile strength, and it would be desirable to have these corroborated.

TABLE XXI.—TENSILE STRENGTH OF STONE.

Name of Material.	Tearing weight per square inch.	Authority.
	lbs.	
Arbroath Pavement, - - - -	1,261	Buchanan.
Caithness do. - - - -	1,054	Do.
Craigleith Stone, - - - - -	453	Do.
Hailes, - - - - - -	336	Do.
Humbie, - - - - - -	283	Do.
Binnie, - - - - - -	279	Do.
Redhall, - - - - - -	326	Do.
Whinstone, - - - - -	1,469	Do.
Marble, White, - - - - -	722	Do.
Do., do. - - - - -	551	Hodgkinson.

Buchanan, *Practical Mechanics' Journal*, Vol. i., pp. 237, 285.
Hodgkinson, *Tredgold on the Strength of Cast-iron*, p. 287.

368. Tensile strength of Plaster of Paris and Lime mortar.—

TABLE XXII.—TENSILE STRENGTH OF PLASTER OF PARIS AND LIME MORTAR.

Name of Material.	Tearing weight per square inch.	Authority.
	lbs.	
Plaster of Paris, - - - - -	71	Rondelet.
Mortar of Quartzose Sand and eminently Hydraulic Lime, well made, - - - - -	136	Vicat.
Mortar of Quartzose Sand and ordinary Hydraulic Lime, well made, - - - - -	85	Do.
Mortar of Quartzose Sand and ordinary Lime, well made, - - - - - -	51	Do.
Mortar badly made, - - - -	21	Do.

Rondelet, *Navier's Application de la Mécanique*, p. 13.
Vicat, *idem.*

369. Tensile strength of Portland cement and cement mortar—Organic matter or loam very injurious to cement mortar.—The following tables showing the tensile strength of cements and cement mortar are taken from Mr. Grant's valuable papers on the Strength of Cement in the Proceedings of the Institution of Civil Engineers, Vols. xxv. and xxxii. Proof samples of cement are generally made into -shaped bricks with rounded shoulders and $1\frac{1}{2}$ inches square, $= 2{\cdot}25$ square inches area, at the waist; these are immersed in water as soon as the cement sets, and they remain immersed till the time of testing.

Artificial Portland cement is made of chalk and clay in certain definite proportions, carefully mixed together in water. The mixture is then run off into reservoirs where it settles, and, after attaining sufficient consistency to handle, it is artificially dried and calcined in kilns at a high temperature, the calcination being carried to the verge of vitrification. The calcined cement is ground in the ordinary way between millstones, and for the sake of economy its fineness should be such that not more than 10 per cent.

is stopped by a sieve the meshes of which are $\frac{1}{50}$th of an inch in diameter, for the coarser particles act to a great degree like inert grains of sand and consequently reduce the value of the cement.

TABLE XXIII.—METROPOLITAN MAIN DRAINAGE—PORTLAND CEMENT, SEVEN DAY TESTS, from 1866 to 1871.

Names of Manufacturers and Agents.	Quantity in bushels.	Average weight per bushel.	Number of tests.	Average breaking weight on area = 2·25 square inches.
		lbs.		lbs.
Formby, - - - -	31,581	118·27	550	862·01
Booth, - - - -	12,464	119·75	80	846·50
Lee and Co., - - -	512	120·00	10	839·00
Burham Brick and Cement Company, - - - -	320,716	113·54	3,705	825·73
Casson and Co., Agents, - -	5,200	114·50	50	816·80
Knight, Bevan, and Sturge, -	19,429	114·52	820	803·38
Robins and Co. (Limited), -	68,880	118·00	620	795·31
White and Co., - - -	60	119·00	10	791·70
Burge and Co., Agents, - -	4,500	113·00	30	789·30
Hilton, - - - -	103,453	117·17	1,300	786·99
Beaumont, Agent, - -	40	116·00	10	765·00
Lavers, Agent, - - -	12,002	116·17	160	706·97
Weston, - - - -	600	120·00	10	666·40
Young and Son, Agents, -	200	117·00	10	655·80
Coles and Shadbolt, - -	240	107·00	10	580·00
Tingey, - - - -	6,300	115·50	100	564·27
Harwood and Hatcher, Agents, -	3,040	117·78	30	408·03
Generally, - -	589,217	115·23	7,505	806·63 = 358·5 per square inch.

NOTE—1 cubic foot = ·779 bushels.
1 bushel = 1·283 cubic feet.

TABLE XXIV.—Results of Experiments with Portland Cement, weighing 112 lbs. per bushel, mixed with different proportions of Sand, showing the Breaking Weight on a sectional area of 2·25 square inches.

1 Month.	6 Weeks.	2 Months.	6 Months.	12 Months.	Proportion of sand to cement.
lbs.	lbs.	lbs.	lbs.	lbs.	
306·0	383·0	407·5	505·5	541·0	3 to 1
403·5	397·5	411·0	479·0	554·5	4 to 1
Broke winding up.	246·0	269·5	439·0	482·0	5 to 1
133·5	189·5	221·0	273·0	319·0	6 to 1
159·0	186·0	215·0	280·5	368·0	7 to 1
103·0	143·0	140·5	282·5	352·5	8 to 1

Organic matter or loam in the sand are very detrimental to the strength of cement mortar, and clean sharp sand, quite free from argillaceous matter, will give the best result. Portland cement bears a much greater proportion of coarse than of fine sand, and cement mortar should be mixed rapidly and not be triturated under edge stones, as is a common practice with lime mortar. It is also very essential that bricks or porous stone, which are to be set in cement, should be previously well soaked in water, as dry materials absorb moisture from the mortar and prevent it from setting properly.

TABLE XXV.—Results of Experiments with Portland Cement weighing 123 lbs. to the imperial bushel, gauged neat, and with an equal proportion of clean Thames Sand. The whole of the specimens were kept in water from the time of their being made till the time of testing.

Age.	On area = 2·25 square inches.	
	Neat Cement.	1 of Cement to 1 of Sand.
	Average breaking test of 10 experiments.	Average breaking test of 10 experiments.
	lbs.	lbs.
7 Days - -	817·1	353·2
1 Month - -	935·8	452·5
3 Months - -	1055·9	547·5
6 Ditto - -	1176·6	640·3
9 Ditto - -	1219·5	692·4
12 Ditto - -	1229·7	716·6
2 Years - -	1324·9	790·3
3 Ditto - -	1314·4	784·7
4 Ditto - -	1312·6	818·1
5 Ditto - -	1306·0	821·0
6 Ditto - -	1308·0	819·5
7 Ditto - -	1327·3	863·6

TABLE XXVI.—Southern Outfall Works, Crossness. Summary of Portland Cement Tests, from 1862 to 1866, showing generally increase of strength with increased specific gravity.

Number of bushels.	Average weight per bushel.	Tearing weight on area = 2·25 square inches; 7 days old.	Number of bushels.	Average weight per bushel.	Tearing weight on area = 2·25 square inches; 7 days old.
	lbs.	lbs.		lbs.	lbs.
1,800	106	472·6	12,500	119	777·9
5,800	107	592·3	18,530	120	732·3
26,166	108	650·1	15,144	121	705·6
37,036	109	646·6	5,000	122	716·6
20,820	110	708·3	5,428	123	673·6
6,900	111	693·8	13,400	124	819·9
13,812	112	687·5	5,400	125	816·2
10,610	113	701·5	1,800	126	657·2
24,224	114	699·7	1,800	127	864·6
16,240	115	705·5	3,600	128	916·6
27,400	116	768·3	1,820	129	920·2
26,800	117	718·4	1,800	130	913·9
23,306	118	644·1			

370. Tensile strength of Roman cement—Natural cements generally inferior to the artificial Portland.—The following tables contain the results of Mr. Grant's experiments on the tensile strength of Roman cement. This cement is much weaker than Portland, and inferior qualities are apt to vegetate and crumble away, especially if mixed with loamy sand. Roman cement is a natural cement, derived from argillo-calcareous, kidney-shaped stones, called "Septaria," belonging to the Kimmeridge and London clay, generally gathered on the sea-shore near the mouth of the Thames, though sometimes dug out of the ground. Natural cements are found in various places at home and abroad and, though generally inferior in strength to artificial Portland, are very useful in their way.

TABLE XXVII.—Results of Experiments with neat Roman Cement, manufactured by Messrs. J. B. WHITE and BROTHERS.

Time kept immersed in water.	On Area — 2·25 square inches.		
	Minimum breaking test.	Maximum breaking test.	Average breaking test.
	lbs.	lbs.	lbs.
7 Days - -	170	240	202·0
14 Ditto - -	160	190	173·0
21 Ditto - -	170	205	186·5
1 Month - -	246	291	260·3
3 Months - -	307	344	322·5
6 Ditto - -	442	502	472·7
9 Ditto - -	313	520	471·1
12 Ditto - -	596	680	643·1
2 Years - -	577	610	546·3
3 Ditto - -	522	647	603·8
4 Ditto - -	600	658	632·2
5 Ditto - -	582	662	627·4
6 Ditto - -	603	711	666·4
7 Ditto - -	646	780	708·7

TABLE XXVIII.—TABLE of the Results of 250 Experiments with Roman Cement and Sand. Cement manufactured by Mr. JAMES R. BLASHFIELD. March, 1864.

Age and Time immersed in water.	On a Sectional area = 2·25 square inches.																	
	Neat Cement.			1 to 1 Sand.			1 to 2 Sand.			1 to 3 Sand.			1 to 4 Sand.			1 to 5 Sand.		
	Min. breaking test.	Max. breaking test.	Average breaking test.	Min. breaking test.	Max. breaking test.	Average breaking test.	Min. breaking test.	Max. breaking test.	Average breaking test.	Min. breaking test.	Max. breaking test.	Average breaking test.	Min. breaking test.	Max. breaking test.	Average breaking test.	Min. breaking test.	Max. breaking test.	Average breaking test.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
7 Days -	95	150	120·5	35	75	47·5	5*	10*	7·0*	10†	10†	10·0†	10†	10†	10†	—	—	—
14 Ditto -	145	209	169·9	53	79	65·6	22	57	42·8	8*	30*	19·2*	—	—	—	—	—	—
21 Ditto -	128	171	155·2	54	89	74·2	37	56	45·9	8	25	17·4	—	—	—	—	—	—
1 Month	343	378	358·2	75	91	81·2	8	54	41·9	—	—	—	—	—	—	—	—	—
3 Months	160	274	220·4	32	160	121·9	72‡	126‡	91·75‡	—	—	—	40§	54§	45·5	29‖	48‖	37·71‖
6 Ditto -	227	300	252·5	296	337	314·3	—	—	—	—	—	—	—	—	—	—	—	—
9 Ditto -	169	300	251·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12 Ditto -	216	300	268·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

* Five of these would not bear the minimum strain. † Eight do. ‡ Two do. § Six do. ‖ Three do.

371. Tensile strength of Keene's, Parian, and Medina cements.—The following tables contain the results of Mr. Grant's experiments on the tensile strength of Keene's, Parian and Medina cements. The two former are chiefly used for internal decoration. Keene's cement is made by soaking plaster of Paris in alum water, then re-burning and grinding it; Parian cement is made by mixing gypsum with borax in powder, then calcining the mixture and grinding it. Medina is a natural cement with rather more lime than Roman cement, and is inferior in strength to Portland cement, which, as already stated, is an artificial mixture of chalk and clay. Quick-setting Medina is useful for pointing the joints of marine masonry which have been set in Portland cement. It hardens rapidly and prevents the rising tide from washing the slower setting Portland out of the joints before it has had time to harden sufficiently to resist the action of water in motion.

TABLE XXIX.—Results of 120 Experiments with Keene's Cement, manufactured by Messrs J. B. WHITE and BROTHERS; and Parian Cement, manufactured by Messrs. FRANCIS and SONS.

Age and time immersed in water.	Average breaking test on area = 2·25 square inches.			
	Keene's Cement.		Parian Cement.	
	In water.	Out of water.	In water.	Out of water.
	lbs.	lbs.	lbs.	lbs.
7 Days - - -	543·9	546·0	595·1	642·3
14 Ditto - - -	486·9	585·8	600·8	671·2
21 Ditto - - -	503·0	579·4	543·4	696·6
1 Month - -	490·2	584·2	544·3	746·7
2 Months - -	454·7	648·4	500·7	725·6
3 Ditto - - -	508·8	720·5	521·1	853·7

TABLE XXX.—Results of 100 Experiments with Medina Cement, manufactured by Messrs. FRANCIS, BROTHERS, 1864.

Age and time immersed in water.	On area = 2·25 square inches.		
	Minimum breaking test.	Maximum breaking test.	Average breaking test.
	lbs.	lbs.	lbs.
7 Days - -	83	100	92·1
ditto (2nd Series)	195	235	211·0
14 Days - -	238	335	303·4
21 ditto - -	274	332	298·0
1 Month - -	210	346	306·0
3 Months - -	420	468	448·8
6 ditto - -	376	438	412·4
9 ditto - -	438	507	457·2
12 ditto - -	456	527	476·9
2 Years - -	235	328	276·0
3 ditto - -	200	342	275·5
4 ditto - -	236	430	287·8
5 ditto - -	245	395	307·0
6 ditto - -	309	475	365·0
7 ditto - -	335	440	377·5

372. Adhesion of Plaster of Paris and Mortar to brick or stone.—Rondelet states that the adhesive strength of plaster of Paris to brick or stone is about two-thirds of its tensile strength, and that its adhesion is greater for millstone and brick than for limestone, and diminishes greatly with time; he also states that the adhesion of lime mortar to stone or brick exceeds its tensile strength and increases with time.*

The following table gives the results of experiments by Mr. Grant on the tensile strain required to separate bricks cemented together in blocks of 4, one on top of the other, with Portland cement and lime mortars, at the end of 12 months.†

* Navier, *Application de la Mécanique*, p. 13. † *Proc. Inst. C. E.*, Vol. xxxii.

TABLE XXXI.—Tensile strain required to separate Bricks cemented together with Portland Cement and Lime Mortars, at the end of 12 Months.

Description of Brick.	Area of bed separated in square inches.	Weight in lbs.		Tensile strain in lbs.									Set in air or water.
				Portland Cement and Sand.						Lime and Sand.			
		Dry.	Wet.	Neat.	1 to 1.	2 to 1.	3 to 1.	4 to 1.	5 to 1.	1 to 2. Blue Lias.	1 to 2. Dorking.	1 to 2. Chalk.	
Gault clay, pressed, -	36·09 ...	5·13 ...	6·47 ...	1631·4 1673·4	1576·4 1642	871 1708	988·2 1264	735·2 750·3	{ Broke in drilling. }	1448·0 ...	1520·5 ...	638·7 ...	Air. Water.
Gault clay, wire cut, -	37·125 ...	5·86 ...	6·85 ...	2440 1679	1558·3 1408·3	Not set. Not set.	1037 Not set.	791 Not set.	772 Not set.	Not set. ...	Not set. ...	679·4 ...	Air. Water.
Gault clay, perforated, -	39·375 ...	4·95 ...	5·76 ...	3898·8 3011	2982 2721	2231·6 1834·4	1663·2 1561·4	1088·6 933·2	812·6 641	1763 ...	1349·3 ...	547 ...	Air. Water.
Suffolk brimstones, -	40·5 ...	6·18 ...	7·14 ...	3275·2 3231	2346 2484	2247 2238	2030·6 1934·4	1930·6 1869	1337 1154·2	2372 ...	1788·2 ...	428·6 ...	Air. Water.
Stock, - - -	37·125 ...	5·0 ...	5·57 ...	2816·8 3445	2264 2455	1571·8 2009	1516·2 1580·8	825·8 1259·6	792·6 845·6	1280·6 ...	1479·3 ...	423·8 ...	Air. Water.
Fareham red, - -	36·09 ...	6·55 ...	7·52 ...	4538 4449	2990 2237	2064 1917	1499 1128	1236 1145	1040 653	1919 ...	1743 ...	449 ...	Air. Water.
Staffordshire blue, (pressed, with frog), -	36·09 ...	7·82 ...	7·90 ...	2563·4 2749	2008 1329	1844 1304	1535 1053	1183 1016	752 718	1102·7 ...	3030·6 ...	Not set. ...	Air. Water.
Staffordshire blue, (rough, without frog),	36·09 ...	7·75 ...	7·81 ...	1744·4 1451	1701·6 1048·2	1746 1186	1299·6 1090	988 843	Not set. Not set.	1269·6 ...	1030·8 ...	Not set. ...	Air. Water.

"The pressed gault bricks show the lowest amount of adhesiveness; partly because of their smooth surface, and partly because in making them some oily matter is used for lubricating the dies of the press through which they are passed before being burnt. In the case of the perforated gault bricks the cement-mortar seems to act as dowels, and the results are consequently high. The Suffolk and the Fareham red bricks, which each absorb about a pound of water per brick, adhere much better than the Staffordshire, which are not absorbent. This shows the importance of thoroughly soaking bricks which are to be put together with cement, as dry bricks deprive the cement-mortar of the moisture which is necessary for its setting." Mr. Robertson found that the adhesion of first-class hydraulic mortar, made of blue Lias lime and ground in mortar pans for forty minutes, to blue vitrified Staffordshire bricks, not too highly glazed, was 40 lbs. per square inch, after six months; while to the hardest grey-stocks, although watered, as in practice, the adhesion was only 36 lbs., or 10 per cent. less. To soft "place" bricks, the adhesion was only 18 lbs., or 55 per cent. less than to blue bricks.*

373. Grant's conclusions.—The following conclusions are the result of Mr. Grant's numerous experiments on cement during the execution of the Southern Metropolitan Main Drainage Works:—

1. Portland cement, if it be preserved from moisture, does not, like Roman cement, lose its strength by being kept in casks, or sacks, but rather improves by age; a great advantage in the case of cement which has to be exported.

2. The longer it is in setting, the more its strength increases.

3. Cement mixed with an equal quantity of sand is at the end of a year approximately three-fourths of the strength of neat cement.

4. Mixed with two parts of sand, it is half the strength of neat cement.

5. With three parts of sand, the strength is a third of neat cement.

6. With four parts of sand, the strength is a fourth of neat cement.

7. With five parts of sand, the strength is about a sixth of neat cement.

8. The cleaner and sharper the sand, the greater the strength.

9. Very strong Portland cement is heavy, of a blue-grey colour, and sets slowly. Quick setting cement has, generally, too large a proportion of clay in its composition, is brownish in colour, and turns out weak, if not useless.

10. The stiffer the cement is gauged, that is, the less the amount of water used in working it up, the better.

* *Proc. Inst. C. E.*, Vol. xvii., p. 420.

11. It is of the greatest importance, that the bricks, or stone, with which Portland cement is used, should be thoroughly soaked with water. If under water, in a quiescent state, the cement will be stronger than out of water.

12. Blocks of brick-work, or concrete, made with Portland cement, if kept under water till required for use, would be much stronger than if kept dry.

13. Salt water is as good for mixing with Portland cement as fresh water.

14. Bricks made with neat Portland cement are as strong at from six to nine months as the best quality of Staffordshire blue brick, or similar blocks of Bramley Fall stone, or Yorkshire landings.

15. Bricks made of four parts or five parts of sand to one part of Portland cement will bear a pressure equal to the best picked stocks.

16. Wherever concrete is used under water, care must be taken that the water is still. Otherwise, a current, whether natural or caused by pumping, will carry away the cement, and leave only the clean ballast.

17. Roman cement, though about two-thirds the cost of Portland, is only about one-third its strength, and is therefore double the cost, measured by strength.

18. Roman cement is very ill adapted for being mixed with sand.

374. Tensile strength of glass—Thin plates of glass stronger than stout bars—Crushing strength of glass is 12 times its tensile strength.—

TABLE XXXII.—Tensile Strength of Glass.

Description of Glass.	Tearing weight per square inch.	Authority.
	lbs. tons.	
Glass Tubes and Rods, - - -	3,527 = 1·57	Navier.
Annealed Flint Glass Rod, - -	2,413 = 1·07	Fairbairn and Tate.
Common Green Glass Rod, - -	2,896 = 1·29	Do.
White Crown Glass Rod, - -	2,546 = 1·14	Do.

Fairbairn and Tate, *Philosophical Transactions*, 1859, p. 216.
Navier, *Résumé des leçons sur l'application de la Mécanique*, p. 37.

In their experiments on the resistance of thin glass globes to internal pressure, Sir William Fairbairn and Mr. Tate found that the tenacity of glass in the form of thin plates is 5,000 lbs. per square inch, or about twice that of glass in the form of bars, on which they observe:—"The tensile strength is much smaller in the case of glass fractured by a direct strain in the form of bars, than when burst by internal pressure in the form of thin globes. This difference is, no doubt, mainly due to the fact that thin plates of this material generally possess a higher tenacity than stout bars, which, under the most favourable circumstances, may be but imperfectly annealed." "The ultimate resistance of glass to a crushing force is about 12 times its resistance to extension"* (**305**).

CORDAGE.

375. Tensile strength of cordage.—The following table gives the sizes, weights, and strength of different kinds of best Bower cables employed in the British Navy.† The strength was determined by the chain-testing machine in Woolwich Dockyard, in which the strain is measured by levers.

TABLE XXXIII.—TENSILE STRENGTH OF BOWER CABLES.

Best Bower hempen cables, 100 fathoms.				Number of threads in each.	Tearing weight by experiment.		
Circumference.	Weight.						
Inches.	Cwt.	qrs.	lbs.		Cwt.	qrs.	lbs.
23	96	2	27	2,736	114	0	0
22	89	0	12	2,520	89	0	0
21	80	0	22	2,268			
18	58	2	6	1,656	63	0	0
14½	38	0	21	1,080	40	0	0

The next table "shows the mean results of 300 trials made by Captain Huddart. It shows the relative strength or cohesive power of each kind of rope, taking as a standard of comparison $\frac{1}{10}$th of a circular inch, equal to an area of ·078 or nearly $\frac{1}{13}$th of a square inch. It shows that ropes formed by the warm register are stronger than those made up with the yarns cold; because the heated tar is more fluid, and penetrates completely between every fibre of hemp, and because the heat drives off both air and moisture,

* *Phil. Trans.*, 1859, pp. 216, 246.

† Barlow *on the Strength of Materials*, p. 260.

so that every fibre is brought into close contact by the twisting and compression of the strand; the tar thus fills up every interstice, and the rope becomes a firmly agglutinated elastic substance almost impermeable to water. But, although rope so made is both stronger and more durable, it is less pliable, and therefore the cold registered rope is more generally used for crane work, where the rope must be wound round barrels, or passed through pulleys."*

TABLE XXXIV.—Tensile Strength of Tarred Hemp Rope.

Size of Ropes.		Tearing weight, made by the old method.				Tearing weight, made by the register.			
Girth.	Diameter.	Of common staple Hemp.	Per $\frac{1}{10}$ of a circular inch in area.	Of the best Petersburg Hemp.	per $\frac{1}{10}$ of a circular inch in area.	Cold Register.	Per $\frac{1}{10}$ of a circular inch in area.	Warm Register.	Per $\frac{1}{10}$ of a circular inch in area.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
3	0·95	5,050	561	6,030	670	7,380	935	8,640	960
3½	1·11	6,784	554	8,669	707	11,165	911	11,760	906
4	1·27	8,768	548	10,454	653	13,108	819	15,360	960
4½	1·43	10,308	504	12,440	614	16,325	806	19,440	960
5	1·59	13,250	530	15,775	631	20,500	820	24,000	960
5½	1·75	15,488	512	18,604	614	24,805	820	29,040	960
6	1·91	18,144	504	21,616	600	24,520	820	33,120	920
6½	2·07	20,533	486	23,623	559	34,645	820	40,554	959
7	2·24	22,932	468	27,342	558	40,188	819	47,040	960
7½	2·39	24,975	444	30,757	546	46,125	820	54,000	960
8	2·54	26,880	421	32,000	500	52,480	820	61,430	960

Note.—$\frac{1}{10}$th of a circular inch = ·078, or nearly $\frac{1}{13}$th of a square inch.

The proof-strain of rope which is given in Table XXXVII. is about one-half its tearing weight.

* Glynn's *Rudimentary Treatise on the Construction of Cranes and Machinery*, pp. 93, 94.

376. Strength and weight of Cordage—English rule—French rule.—By the old ropemakers' rule the square of the girth in inches multiplied by four gave the ultimate or breaking strength of the rope in cwts., and it was a good rule for small cordage, up to 7 inches in girth. The square of the girth divided by four was considered to represent the weight of a fathom in pounds.* The old ropemakers' rule for strength is equivalent to 2·51 tons per square inch of section. The French rule, as given by Morin,† allows 2·79 tons per square inch for the tearing weight of tarred hemp cordage.

377. Working strain of Cordage.—Cordage rapidly deteriorates by use and exposure to the weather, and when passed round barrels or pulleys the outer strands are subject to greater strains than those next the barrel. For this reason, as well as in order to diminish useless work, the diameters of pulleys and barrels should be made as large as practicable. Experience alone can estimate the proper allowance to be made for wear and friction, which latter is sometimes excessive in badly made blocks, and after deducting this allowance from the original tearing strength, one-fourth of the remainder is a sufficient load for continued strain, and one-third for merely temporary purposes, though workmen often apply one-half. A common practical allowance for friction in ordinary tackles is one-third of the theoretic amount; if, for example, the tackle consists of an upper and lower block with three pulleys in each block, there will be 6 parts to the rope and the theoretic pull on each part will $= \frac{W}{6}$; the foregoing rule, however, makes the pull on each part $= \frac{1·33W}{6}$, and the rope should therefore be one-third stronger than if friction had not existed.

CHAINS.

378. Stud-link or Cable chain.—Close-link or Crane chain—Long open-link or Buoy chain—Middle-link chain.—

* Glynn's *Rudimentary Treatise*, p. 92.

† *Résistance des Matériaux*, p. 41.

Stud-link chain is chiefly used for ships' cables, and derives its name from the cast-iron stud or stay which is inserted across the shorter diameter of each oval link to keep the sides from closing together under heavy strains. It also prevents the chain from kinking, to which long links without stays are liable. *Short* or *close-link* chain, called also *rigging* or *crane* chain, is that in common land use. It is well adapted for crane work where flexibility is essential to enable the chain to pass freely round barrels and pulleys. Long *open-link* chain without studs is used for permanent mooring cables, where flexibility is a secondary object, and where lightness is desirable, as in the case of light-ships or beacon buoys. *Middle-link* chain is occasionally used; its link is intermediate in length between those of the close and open-link chains.

The standard proportions of the links of the different kinds of chain are as follows, in terms of the diameter of the bar of iron:—

	Extreme length.	Extreme width.
Stud-link,	6 diameters.	3·6 diameters.
Close-link,	5 do.	3·5 do.
Open-link,	6 do.	3·5 do.
Middle-link,	5·5 do.	3·5 do.
End-links,	6·5 do.	4·1 do.

End-links are the links which terminate each 15-fathom length of chain; they are longer and wider than the common links in order to allow the joining shackles to pass through, and they require therefore to be made of stouter iron, generally 1·2 diameters of the common links.

379. Tensile strength of stud-chain.—The following table contains the results of experiments on the tensile strength of stud-chain made by Mr. William Smale, leading man of the test house in Her Majesty's Dockyard, Woolwich.* Mr. Smale found that the average tearing weight of good round bars of one inch diameter was 19 tons, = 24·19 tons per square inch of section, their greatest strength being about 20 tons, = 25·46 tons per square inch of section.

* *Report from the Select Committee on Anchors, &c.* (Merchant Service), 1860. Appendix, pp. 151, 152.

TABLE XXXV.—TENSILE STRENGTH OF STUD-CHAIN.

Size of Chain.	Length of each piece.		Number of pieces tested.	Mean tearing weight.	Government proof strain.	Ratio of tearing to proof strain.	Area of Bar.	Tearing weight per square inch of each side of link.	
in.	ft.	in.		tons.	tons.		sq. in.	tons.	
$\frac{5}{8}$	24	0	6	9·58	7·00	1·37	·307	15·6	Manufactured by various contractors for the Government.
$\frac{3}{4}$	,,		6	13·51	10·125	1·33	·442	15·3	
1	,,		6	24·25	18·00	1·35	·785	15·4	
$1\frac{1}{8}$	,,		6	29·54	22·75	1·30	·994	14·9	
$1\frac{1}{2}$	,,		6	59·58	40·50	1·47	1·767	16·9	
$1\frac{3}{4}$	,,		6	74·125	55·125	1·34	2·405	15·4	
$1\frac{7}{8}$	,,		6	92·88	63·25	1·47	2.761	16·8	
2	,,		3	99·54	72·00	1·38	3·141	15·8	
$\frac{7}{8}$	2	0	20	20·38	13·75	1·48	·601	16·9	
$1\frac{3}{4}$	Single links		30	78·70	55·125	1·42	2·405	16·3	Made in Woolwich Dockyard.
			Mean	—	—	1·39		15·9	

Messrs. Brown, Lenox, & Co., inform me that they have found by experience that the average breaking strain of stud-link chain, up to $2\frac{1}{4}$ inches, is from 900 to 1,000 lbs. per circular $\frac{1}{8}$th of an inch of the diameter of the bar—equivalent to from 16·37 to 18·19 tons per square inch of each side of the link. This is for cables of *good quality*, much chain being made of a description of iron that will stand the proof and but little more. Hence, stud-chain is about $\frac{2}{3}$rds as strong as bar iron of the same sectional area as both sides of the links together; in other words, the bar loses about 33 per cent. of its strength by being converted into a link.

Ex. A one-inch stud-chain contains 64 circular $\frac{1}{8}$ths, and, if of good quality, its tearing weight should equal $64 \times 900 = 57{,}600$ lbs. $= 25·7$ tons. The tearing weight of two round bars of good iron, each one inch diameter, should equal $2 \times 19 = 38$ tons.

380. Admiralty Proof-strain for Stud-chain.—By the Chain Cable and Anchor Act of 1871 it is enacted that a maker of or dealer in chain cables or anchors shall not sell, consign, or contract to sell or consign, nor shall any person purchase or contract to purchase any chain cable whatever, or any anchor exceeding 168 lbs., which has not been previously tested and duly stamped, and where any chain cable is brought to a tester for the purpose of being proved, he shall test every fifteen fathoms of it in the manner following; that is to say,

1°. He shall select and cut out a piece of three links from every such fifteen fathoms and shall test that piece by subjecting it to the appropriate breaking strain mentioned in the second schedule to this Act (see the last column in Table XXXVI.):—

2°. If the piece so selected fail to withstand such breaking strain, he shall select and cut out another piece of three links from the same fifteen fathoms, and shall test such piece in like manner:—

3°. If the first or second of such pieces of any fifteen fathoms of cable withstand the breaking strain, he shall then, but not otherwise, test the remaining portion of that fifteen fathoms of cable by subjecting the same to the tensile strain mentioned in the Act of 1864 (see the Admiralty proof-strain in the 7th column of Table XXXVI.):—

4°. He shall not stamp a chain cable as proved which has not been subjected to the breaking and tensile strains in accordance with the provisions of this section, or has not withstood the same.

For stud-chain the Admiralty proof-strain equals 630 lbs. per circular $\frac{1}{8}$th of an inch of the diameter of the bar, equivalent to 11·46 tons per square inch of each side of the link. Hence, this proof-strain for stud-chains is about two-thirds of the ultimate strength of cables of good quality, and one-half the strength of good round bar iron—*i.e.*, the Government proof of a stud-chain is equal to the ultimate strength of the single bar of which it is made, supposing this equals 23 tons per square inch, = 18·064 tons per circular inch.

Ex. A one-inch stud-chain has 1·57 square inches of area in both sides of the link together, and $1{\cdot}57 \times 11{\cdot}46 = 18$ tons = the proof-strain. The ultimate strength of good chain should reach $\frac{3}{2} \times 18 = 27$ tons, and the breaking weight of the single bar should not be less than 18·064 tons, = 23 tons per square inch, and the iron should be tough and fibrous with a "set after fracture" of not less than 15 per cent.

The following table gives the proof-strains and weight per 100 fathoms of stud-chain cables for Her Majesty's Naval Service, also the appropriate breaking strain referred to in the Act of Parliament.

TABLE XXXVI.—ADMIRALTY PROOF-STRAIN AND APPROPRIATE BREAKING-STRAIN FOR CHAIN CABLES.

Diameter of the bar of which the chain is made.	Common Links. Mean length 6 diameters of the bar; not to be over more than one-tenth of a diameter.	Common Links. Mean width 3·6 diameters of the bar; not to be over or under more than one-tenth of a diameter.	Stay Pins, one diameter of the bar at the ends; 0·6 do. at the centre. Weight of each not to exceed,	Weight of 100 fathoms of Cable in 8 lengths, including 4 swivels and 8 joining shackles, not to be exceeded by more than one-fifteenth part for sizes $2\frac{1}{4}$ inch and upwards, and not more than one-twentieth part for sizes under $2\frac{1}{4}$ inch.			Weight of 100 fathoms, with the allowance added.			Admiralty Proof-strain, equal to 630 lbs. per circular $\frac{1}{8}$th inch.	Appropriate breaking strain.
inch.	inch.	inch.	ozs.	cwts.	qrs.	lbs.	cwts.	qrs.	lbs.	tons.	tons.
$2\frac{3}{4}$	$16\frac{1}{2}$	9·9	72	363	0	0	387	0	22	$136\frac{1}{8}$	190·5
$2\frac{1}{2}$	15	9·0	54·69	300	0	0	320	0	0	$112\frac{1}{2}$	157·5
$2\frac{3}{8}$	$14\frac{1}{4}$	8·55	47·5	270	3	0	288	3	6	$101\frac{1}{2}$	141·9
$2\frac{1}{4}$	$13\frac{1}{2}$	8·1	40	243	0	0	259	0	22	$91\frac{1}{8}$	127·5
$2\frac{1}{8}$	$12\frac{3}{4}$	7·65	33·584	216	3	0	227	2	9	$81\frac{1}{4}$	113·7
2	12	7·2	28	192	0	0	201	2	11	72	100·8
$1\frac{7}{8}$	$11\frac{1}{4}$	6·75	23	168	3	0	177	0	21	$63\frac{1}{4}$	88·5
$1\frac{3}{4}$	$10\frac{1}{2}$	6·3	18·76	147	0	0	154	1	11	$55\frac{1}{8}$	77·0
$1\frac{5}{8}$	$9\frac{3}{4}$	5·85	15	126	3	0	133	0	9	$47\frac{1}{2}$	66·5
$1\frac{1}{2}$	9	5·4	11·81	108	0	0	113	1	17	$40\frac{1}{2}$	60·75
$1\frac{3}{8}$	$8\frac{1}{4}$	4·95	9	90	3	0	95	1	4	34	51·0
$1\frac{1}{4}$	$7\frac{1}{2}$	4·5	6·836	75	0	0	78	3	0	$28\frac{1}{8}$	42·0
$1\frac{1}{8}$	$6\frac{3}{4}$	4·05	4·983	60	3	0	63	3	4	$22\frac{3}{4}$	35·5
1	6	3·6	3·5	48	0	0	50	1	16	18	27·0
$\frac{7}{8}$	$5\frac{1}{4}$	3·15	2·344	36	3	0	38	2	10	$13\frac{3}{4}$	20·5
$\frac{3}{4}$	$4\frac{1}{2}$	2·7	1·473	27	0	0	28	1	11	$10\frac{1}{8}$	15·0
$\frac{11}{16}$	$4\frac{1}{8}$	2·475	1·137	22	2	21	23	3	8	$8\frac{1}{2}$	12·75
$\frac{5}{8}$	$3\frac{3}{4}$	2·25	·854	18	3	0	19	2	21	7	10·5
$\frac{9}{16}$	$3\frac{3}{8}$	2·025	·622	15	0	21	15	3	22	$5\frac{1}{2}$	8·25
$\frac{1}{2}$	3	1·8	·437	12	0	0	12	2	11	$4\frac{1}{2}$	6·75
$\frac{7}{16}$	$2\frac{5}{8}$	1·575	·293	9	0	21	9	2	16	$3\frac{1}{2}$	5·25

The "appropriate breaking strains" of the heavier chains are almost exactly 16 tons per square inch of each side of the link; for the smaller sizes they are about one ton higher.

Cables generally weigh the full weight allowed, the iron being rolled a little full to allow for waste in the manufacture. Those for the merchant service are usually made in lengths of 15 fathoms each, with joining shackles connecting the lengths together.

381. Close-link chain—Proof-strain.—The Admiralty proof-strain for close-link chain is 420 lbs. per circular $\frac{1}{8}$th of an inch of the diameter of the bar, or two-thirds of the proof for stud-chains; this is equivalent to 7·64 tons per square inch of each side of the link, or nearly one-half the breaking weight of the chain. The following table gives the proof-strain and weight per 100 fathoms of close-link chain, the extreme length of links not to exceed 5 diameters of the bar; it also gives the size and weight of rope of equal strength.

TABLE XXXVII.—Admiralty Proof-strains for Close-link Chain.

Diameter of Chain.	Average weight per 100 fathoms.	Proof-strain, equal to 420 lbs. per circular $\frac{1}{8}$th inch.	Girth of Rope of equal strength.	Weight of Rope per fathom.
inches.	cwt.	tons.	inches.	lbs.
$1\frac{5}{8}$	155	$31\frac{5}{8}$	—	—
$1\frac{1}{2}$	125	27	—	—
$1\frac{3}{8}$	104	$22\frac{5}{8}$	—	—
$1\frac{1}{4}$	86	$18\frac{3}{4}$	—	—
$1\frac{1}{8}$	70	$15\frac{1}{4}$	—	—
1	56	12	10	22
$\frac{15}{16}$	50	$10\frac{1}{2}$	$9\frac{1}{2}$	$19\frac{1}{2}$
$\frac{7}{8}$	42	$9\frac{1}{8}$	9	$17\frac{1}{2}$
$\frac{13}{16}$	35	$7\frac{7}{8}$	$8\frac{1}{4}$	15
$\frac{3}{4}$	32	$6\frac{3}{4}$	$7\frac{1}{2}$	12
$\frac{11}{16}$	25	$5\frac{5}{8}$	7	$10\frac{1}{2}$
$\frac{5}{8}$	21	$4\frac{5}{8}$	$6\frac{1}{4}$	$8\frac{1}{2}$

TABLE XXXVII.—ADMIRALTY PROOF-STRAINS FOR CLOSE-LINK CHAIN—*continued.*

Diameter of Chain.	Average weight per 100 fathoms.	Proof-strain, equal to 420 lbs. per circular $\frac{1}{8}$th inch.	Girth of Rope of equal strength.	Weight of Rope per fathom
$\frac{9}{16}$	16	$3\frac{3}{4}$	$5\frac{1}{2}$	7
$\frac{1}{2}$	13	3	$4\frac{3}{4}$	5
$\frac{7}{16}$	10	$2\frac{1}{4}$	4	$3\frac{3}{4}$
$\frac{3}{8}$	7	$1\frac{5}{8}$	$3\frac{1}{4}$	$2\frac{1}{2}$
$\frac{5}{16}$	5	$1\frac{1}{8}$	$2\frac{1}{2}$	$1\frac{1}{2}$
$\frac{1}{4}$	3	$\frac{3}{4}$	2	—
$\frac{3}{16}$	2	$8\frac{1}{2}$ cwt.	$1\frac{1}{2}$	—

The rope of the foregoing table "is such as is now generally made by machinery at most of the large rope works, but was formerly known as 'Patent Rope,' in which every yarn is made to bear its part of the strain; but if common hand-laid rope be used, the proof-strain must be reduced one-fourth, and in actual work the load should not, at any time, exceed one-half the proof."* It will be observed that the diameter of a close-link chain is approximately one-tenth of the girth of hemp rope of equal strength.

382. Long open-link chain—Admiralty proof-strain—Trinity proof-strain—French proof.—The links of open-link chain are not oval like those of a stud-chain, but parallel-sided, and the open-link chain of the same length of link as the stud-chain is lighter by the weight of the studs. As already observed, it is suited for moorings of a permanent character, such as those of mooring buoys, beacon buoys, or light-ships, which are seldom shifted, and where, consequently, flexibility in passing round chain barrels is a secondary object. Besides its comparative lightness, open-link chain has another advantage over either close-link or stud-chain, for each 15-fathom length of the two latter requires long end links for the purpose of connecting it by joining shackles to the adjoining lengths, and if either of these chains break, a whole

* Glynn on the *Construction of Cranes*, p. 92.

length must be taken out, since there is not room for a shackle to pass through the ordinary close-link or stud-link. When, however, a long-link chain breaks, the links adjoining the fracture can be connected together without taking out a whole 15-fathom length, as a shackle will generally pass through any of the common links. The old Admiralty proof for large open long-link chain without studs was 315 lbs. per circular $\frac{1}{8}$th of an inch, or one-half the proof of stud-chain, as shown in the following table; the links were generally of great length.

TABLE XXXVIII.—Admiralty Proof-strains for Pendant and Bridle Chains.

Diameter of iron.	Proof strain equal to 315 lbs. per circular $\frac{1}{8}$th inch.	
Inches.	Tons.	
$3\frac{1}{2}$	110	Permanent deflection or collapsion of link not to exceed one quarter of an inch.
$3\frac{1}{4}$	95	
3	81	
$2\frac{7}{8}$	74	
$2\frac{3}{4}$	68	
$2\frac{5}{8}$	62	
$2\frac{1}{2}$	56	
2	36	

The following are the proofs which the Elder Brethren of the Trinity House require in testing open-link chains such as are used for mooring light-ships and beacon buoys, as well as close-link rigging or crane chains:—The chains are subjected, in lengths of 15 fathoms, to a strain of 466 lbs. per circular $\frac{1}{8}$th inch of the diameter of the bar, (equivalent to 8·47 tons per square inch of each side of the link, or about one-half the breaking weight of the chain.) This test—which was determined after numerous experiments—is the highest strain to which open-linked chain can be subjected without altering the shape of the link, and is comparatively much more severe than the usual test for chain without studs. In addition to the foregoing limited proof-strain, test pieces, 4 feet long, are cut out of each size of chain and the quality of the iron is ascertained by testing the iron in one link of each length. The remainder of each four-feet length is then torn asunder to test the welding, and its breaking weight must not be less than 16 tons per square inch of each side of the link, or 880 lbs. per circular $\frac{1}{8}$th inch of the diameter of the bar. The lengths of chain from which the test pieces are taken are then made good and re-proved as before.

In the French Marine the proof for stud-chains $\frac{5}{8}$th inch in diameter and upwards equals 10·8 tons per square inch of the bar. For chains less than $\frac{5}{8}$th inch, without studs, the proof is 8·9 tons per square inch.*

383. Working-strain of chains should not exceed one-half the proof-strain.—Mr. Glynn† states that chains "may safely be worked to half the strain to which they have been proved, but not to more." This for stud-chain $= \frac{11{\cdot}46}{2} = 5{\cdot}73$ tons per square inch of each side of the link, or about one-third of the ultimate strength of good chain and one-fourth of that of round bar-iron. For close-link chain this rule allows $\frac{7{\cdot}64}{2}$, $= 3{\cdot}82$ tons per square inch of each side of the link, or about one-fourth of the ultimate strength of common chain and one-sixth of that of bar-iron. When, however, chains are liable to shocks, as in cranes, one-third of the proof-strain, $= 2{\cdot}55$ tons per square inch of each side of the link, will be a sufficient working load.

384. Comparative strength of stud and open-link chain.—I am indebted for the following practical observations to the courtesy of Messrs. Brown, Lenox, & Co., the eminent manufacturers of anchors and chains:—"We are not of opinion that studs increase the strength of chain, or enable it to bear a heavier ultimate breaking strain than if made without them, both descriptions being made of the same length of link. The object of their being used is to prevent collapse of the link, which in open-link chain takes place at a strain considerably below the breaking weight, and, of course, renders the chain unserviceable. They thereby enable chains, made with them, to be used for heavier strains than open-link chain, but do not add to their ultimate

* Morin, *Résistance des Matériaux*, p. 42.

† *Rudimentary Treatise on the Construction of Chains*, p. 91.

strength—indeed, from the experiments we have tried, and the experience we have had, we are inclined to believe that the link without stay-pins almost invariably breaks at a higher strain than stud-chains. The proof for studded chain is the higher, only because a sufficient proof cannot be given to open-link chain before the link spoils its form and becomes rigid. The stay prevents collapse, by which the link is prevented elongating so much, and taking its natural position before its utmost power is exhausted and a break ensues. The link, if sound in the workmanship, will nearly always break near the stay-pin, which is caused by the nip across the stay-pin. If made without stays, it will collapse until it is rigid, and the iron will reach as near as possible the direct line of the strain, or right through the centre of the chain; the sides of the links will incline inwards, and the break will ensue at the nip across the crown of the next link."

385. Weight and strength of bar-iron, stud-chain, close-link chain, and cordage.—The weight of a stud-chain in lbs. per foot is very nearly equal to 9 times the square of the diameter of the bar; for instance, a two-inch stud-chain weighs 36 lbs. per foot nearly. Stud-chain is about $3\frac{1}{2}$ times as heavy as the bar of which it is made:—thus, one fathom of $1\frac{1}{2}$ inch stud-chain weighs about 125 lbs.—a bar 21 feet long would weigh about 124 lbs. Close-link chain is about 4 times as heavy as the bar:—thus, one fathom of $1\frac{1}{2}$ chain weighs about 140 lbs.—a bar 24 feet long would weigh about 141 lbs. Close-link chain is about 12 per cent. heavier than stud-chain made with stay-pins of Government dimensions; large and heavy stays are introduced by some manufacturers into ordinary cables, thereby greatly increasing the useless weight of cast-iron, and enabling the chain to be sold cheaper by weight. The following table shows at a glance the relative weights and strength of bar-iron, stud-chain, close-link chain, and hemp cordage.

TABLE XXXIX.—WEIGHTS AND STRENGTH OF BAR-IRON, CHAIN AND CORDAGE.

	Weight of 100 fathoms: (d = diameter in inches.)	Tearing weight per square inch.	Relative weight of equal lengths of the same ultimate strength; *i.e.*, each length on the point of rupture from the same load.	Safe Working strain per square inch.	Relative weight of equal lengths of the same useful strength; *i.e.*, each length strained to its limit of safe working strain from the same load.
	tons.	tons.		tons.	
Bar-iron, best quality, -	$0{\cdot}70d^2$	24	100	6·0	100
Stud-chain, - - -	$2{\cdot}45d^2$	16 } on each side of link	262	5·73 } on each side of link	184
Close-link chain, - -	$2{\cdot}80d^2$	16 } on each side of link	300	3·82 } on each side of link	314
Hemp Cordage, - -	$0{\cdot}11d^2$	2·51	150	0·63	150

WIRE ROPE.

386. Tensile strength of round iron and steel wire ropes and hemp rope.—The following table shows the strength of iron wire rope and hemp rope, by the eminent American Engineer, J. A. Roebling, Esq.* The breaking weight is given in the American ton of 2,000 lbs.

TABLE XL.—STRENGTH OF ROUND IRON WIRE ROPE AND HEMP ROPE, BY J. A. ROEBLING, C.E.

	Circumference of Wire rope in inches.	Trade number.	Circumference of Hemp rope of equal strength in inches.	Tearing weight in tons of 2,000 lbs.
Fine Wire,	6·62	1	15½	74
	6·20	2	14½	65
	5·44	3	13	54
	4·90	4	12	43·6
	4·50	5	10¾	35
	3·91	6	9½	27·2
	3·36	7	8	20·2
	2·98	8	7	16
	2·56	9	6	11·4
	2·45	10	5	8·64

* *Memoranda on the Strength of Materials,* by J. K. Whildin, New York, p. 9.

TABLE XL.—STRENGTH OF ROUND IRON WIRE ROPE AND HEMP ROPE, BY J. A. ROEBLING, C.E.—*Continued.*

	Circumference of Wire rope in inches.	Trade number.	Circumference of Hemp rope of equal strength in inches.	Tearing weight in tons of 2,000 lbs.
Coarse Wire,	4·45	11	$10\frac{3}{4}$	36
	4·00	12	10	30
	3·63	13	$9\frac{1}{2}$	25
	3·26	14	$8\frac{1}{4}$	20
	2·98	15	$7\frac{1}{4}$	16
	2·68	16	$6\frac{1}{4}$	12·3
	2·40	17	$5\frac{1}{2}$	8·8
	2·12	18	5	7·6
	1·9	19	4·75	5·8
	1·63	20	4	4·09
	1·53	21	3·3	2·83
	1·31	22	2·80	2·13
	1·23	23	2·46	1·65
	1·11	24	2·2	1·38
	0·94	25	2·04	1·03
	0·88	26	1·75	0·81
	0·78	27	1·50	0·56

TABLE XLI.—WEIGHT, STRENGTH, AND WORKING LOAD OF HEMP AND ROUND IRON AND STEEL WIRE ROPES, AS STATED BY THE MAKERS, MESSRS. NEWALL AND CO. OF GATESHEAD-ON-TYNE.

Hemp.		Iron.		Steel.		Equivalent Strength.	
Circumference. Inches.	Lbs. Weight per fathom.	Circumference. Inches.	Lbs. Weight per fathom.	Circumference. Inches.	Lbs. Weight per fathom.	Working Load. Cwt.	Tearing weight. Tons.
$2\frac{3}{4}$	2	1	1	—	—	6	2
—	—	$1\frac{1}{2}$	$1\frac{1}{2}$	1	1	9	3
$3\frac{3}{4}$	4	$1\frac{5}{8}$	2	—	—	12	4
—	—	$1\frac{3}{4}$	$2\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	15	5
$4\frac{1}{2}$	5	$1\frac{7}{8}$	3	—	—	18	6
—	—	2	$3\frac{1}{2}$	$1\frac{5}{8}$	2	21	7
$5\frac{1}{2}$	7	$2\frac{1}{8}$	4	$1\frac{3}{4}$	$2\frac{1}{2}$	24	8
—	—	$2\frac{1}{4}$	$4\frac{1}{2}$	—	—	27	9
6	9	$2\frac{3}{8}$	5	$1\frac{7}{8}$	3	30	10
—	—	$2\frac{1}{2}$	$5\frac{1}{2}$	—	—	33	11
$6\frac{1}{2}$	10	$2\frac{5}{8}$	6	2	$3\frac{1}{2}$	36	12
—	—	$2\frac{3}{4}$	$6\frac{1}{2}$	$2\frac{1}{8}$	4	39	13
7	12	$2\frac{7}{8}$	7	$2\frac{1}{4}$	$4\frac{1}{2}$	42	14
—	—	3	$7\frac{1}{2}$	—	—	45	15
$7\frac{1}{2}$	14	$3\frac{1}{8}$	8	$2\frac{3}{8}$	5	48	16
—	—	$3\frac{1}{4}$	$8\frac{1}{2}$	—	—	51	17
8	16	$3\frac{3}{8}$	9	$2\frac{1}{2}$	$5\frac{1}{2}$	54	18
—	—	$3\frac{1}{2}$	10	$2\frac{5}{8}$	6	60	20
$8\frac{1}{2}$	18	$3\frac{5}{8}$	11	$2\frac{3}{4}$	$6\frac{1}{2}$	66	22
—	—	$3\frac{3}{4}$	12	—	—	72	24
$9\frac{1}{2}$	22	$3\frac{7}{8}$	13	$3\frac{1}{4}$	8	78	26
10	26	4	14	—	—	84	28
—	—	$4\frac{1}{4}$	15	$3\frac{3}{8}$	9	90	30
11	30	$4\frac{3}{8}$	16	—	—	96	32
—	—	$4\frac{1}{2}$	18	$3\frac{1}{2}$	10	108	36
12	34	$4\frac{5}{8}$	20	$3\frac{3}{4}$	12	120	40

387. Tensile strength of flat iron and steel wire ropes and flat hemp rope.

TABLE XLII.—WEIGHT, STRENGTH AND WORKING LOAD OF FLAT HEMP ROPE AND FLAT IRON AND STEEL WIRE ROPES, AS STATED BY THE SAME MAKERS.

HEMP.		IRON.		STEEL.		Equivalent Strength.	
Size in inches.	Lbs. Weight per fathom.	Size in inches.	Lbs. Weight per fathom.	Size in inches.	Lbs. Weight per fathom.	Working Load. Cwts.	Tearing weight. Tons.
$4+1\frac{1}{8}$	20	$2\frac{1}{4}+\frac{1}{2}$	11	—	—	44	20
$5+1\frac{1}{4}$	24	$2\frac{1}{2}+$ „	13	—	—	52	23
$5\frac{1}{2}+1\frac{3}{8}$	26	$2\frac{3}{4}+\frac{5}{8}$	15	—	—	60	27
$5\frac{3}{4}+1\frac{1}{2}$	28	$3+$ „	16	$2+\frac{1}{2}$	10	64	28
$6+1\frac{1}{2}$	30	$3\frac{1}{4}+$ „	18	$2\frac{1}{4}+\frac{1}{2}$	11	72	32
$7+1\frac{7}{8}$	36	$3\frac{1}{2}+$ „	20	„ „	12	80	36
$8\frac{1}{4}+2\frac{1}{8}$	40	$3\frac{3}{4}+\frac{11}{16}$	22	$2\frac{1}{2}+\frac{1}{2}$	13	88	40
$8\frac{1}{2}+2\frac{1}{4}$	45	$4+$ „	25	$2\frac{3}{4}+\frac{3}{8}$	15	100	45
$9+2\frac{1}{2}$	50	$4\frac{1}{4}+\frac{3}{4}$	28	$3+$ „	16	112	50
$9\frac{1}{2}+2\frac{3}{8}$	55	$4\frac{1}{2}+$ „	32	$3\frac{1}{4}+$ „	18	128	56
$10+2\frac{1}{2}$	60	$4\frac{5}{8}+$ „	34	$3\frac{1}{2}+$ „	20	136	60

388. Safe working load of wire rope.—From Table XLI., the safe working load of round hemp or wire rope is a little more than one-seventh of their tearing weight; and from Table XLII., the working load of flat hemp and wire rope is about one-ninth of their tearing weight; and Messrs. Newall and Co. state that "round rope in pit-shafts must be worked to the same load as flat ropes." It also appears from Table XLI. that the length at which a round iron wire rope will break from its own weight is 26,880 feet; the working limit of length therefore, supposing the rope has only its own weight to support, is under 4,000 feet.

MISCELLANEOUS MATERIALS.

389. Tensile strength of bone, leather, whalebone, gutta-percha, glue.—From Bevan's experiments it appears that the tensile strength of bones of horses, oxen and sheep varies from 33,000 to 42,500 lbs. per square inch.*

The following are the results of Mr. H. Towne's experiments on the tensile strength of single leather belts.†

	Tearing weight in lbs. per inch wide.
Through the lace holes, - - -	210
Through the rivet holes, - - -	382
Through the solid part, - - -	675

The thickness being ·219 inch, the tensile strength of the solid leather was 3,082 lbs., = 1·376 tons per square inch. The strengths of new and partially used belts were found to be nearly identical. The maximum working strain may vary from one-fourth to one-third of the tearing weight, *i.e.*, from 52 to 70 lbs. per inch wide of ordinary single belting, but the former is the safer rule. Helvetia leather belting, manufactured by a peculiar process by Messrs. Norris and Co., of Shadwell, London, from fresh Swiss ox hides, is stated to be stronger and more flexible than ordinary tanned English belting, as shown by the following table, which contains the results of Mr. Kirkaldy's experiments.‡

TABLE XLIII.—TENSILE STRENGTH OF LEATHER BELTING.

	English Belting.	Helvetia Belting.
	lbs.	lbs.
Double, 12 inches, - -	14,861	17,622
„ 7 „ - -	6,193	11,089
„ 6 „ - -	5,603	10,456
„ 4 „ - -	4,365	6,207
„ 2 „ - -	2,942	4,237
Single, 10 „ - -	8,846	11,888
„ 5 „ - -	4,060	5,426
„ 4 „ - -	3,248	3,948
„ 3½ „ - -	3,007	3,377

* *Phil. Mag.*, 1826, p. 181. † *Engineer*, Aug., 1868, p. 145.

‡ *The Engineer*, Aug., 1872, p. 125.

Professor Rankine states that the tenacity of raw hide is about once and a half that of tanned leather, and that the tenacity of whalebone is 7,700 lbs. per square inch.* Mr. Box states that the tensile strength of gutta-percha is 1,680 lbs., = ·75 ton, per square inch, and that in belting it will bear about 400 lbs. per square inch.†

Bevan found that the adhesion of common glue to dry ash timber amounted to 715 lbs. per square inch when the glue was freshly made and the season was dry; when the glue had been frequently melted and in the winter season, the adhesion varied from 350 to 560 lbs. per square inch. The tensile strength of solid glue was 4,000 lbs. per square inch.‡

* *Machinery*, p. 475.

† Box *on Millgearing*, p. 69.

‡ *Phil. Mag.*, 1826, Vol. lxviii., p. 112.

CHAPTER XVII.

SHEARING-STRAIN.

390. Shearing in detail—Simultaneous shearing.—The nature of shearing-strain* in the vertical web of girders has been already investigated in the second chapter, and we have frequent examples of the same kind of strain, though on a smaller scale, in rivets or similar connexions which sustain forces tending to cut them across at right angles to their length. For example, the rivet joining the blades of a pair of scissors is subject to a shearing-strain equal to the pressure applied to the handles, plus the resistance of the fabric which is being cut. The latter also is subject to a shearing-strain, differing, however, in character from that which the rivet sustains in consequence of the inclination of the blades which sever only a short length of the fabric at a time. Machines for shearing metals act on this principle, their cutting edges being generally set at an acute angle to each other, so that they shear plates in detail, and thus diminish the effort exerted at each instant of time; in punching machines, however, the whole circumference of the hole is cut at the first effort, and subsequent pressure is merely necessary to overcome friction and push out the burr. The shearing-strains which occur in engineering structures generally resemble that which rivets sustain, where the whole transverse area simultaneously resists shearing. In this case it is clear that the strength of the rivets is proportional to their sectional area; in other words, if F and f represent the total and the unit shearing-strains, eq. 1 will apply to shearing as well as to tensile and compressive forces, provided always that the cutting edges bear simultaneously over the whole surface of the rivet or material under strain.

* Called *Detrusion* by some authors.

391. Shearing strength of cast-iron.—The shearing strength of cast-iron, according to Professor Rankine, is 27,700 lbs. = 12·37 tons per square inch. In my own experiments I have found its shearing strength equal to 8 or 9 tons per square inch, which is substantially the same as its tensile strength.

392. Experiments on punching wrought-iron.—Table I. exhibits the results of experiments made at Bristol by Mr. Jones, "on the force required for punching different sized holes in different thicknesses of plates, up to 1 inch diameter and 1 inch thickness; the force was applied by means of dead weights with a pair of levers giving a total leverage of 60 to 1, so that 1 cwt. in the scale gave a pressure of 3 tons on the punch; the weights were added gradually by a few lbs. at a time until the hole was punched."*

TABLE I.—EXPERIMENTS ON PUNCHING PLATE IRON.

Diameter of hole.	Thickness of plate.	Sectional area cut through.	Total pressure on Punch.	Pressure per square inch of area cut.
inch.	inch.	square inch.	tons.	tons.
0·250	0·437	0·344	8·384	24·4
0·500	0·625	0·982	26·678	27·2
0·750	0·625	1·472	34·768	23·6
0·875	0·875	2·405	55·500	23·1
1·000	1·000	3·142	77·170	24·6

Table II. contains experiments by Mr. C. Little on punching holes in hammered scrap iron with Eastwood's hydraulic shearing press, the force applied being measured by weights hung on the end of the force-pump handle. This method of measurement is not so accurate as that by direct leverage, since the friction of the press is rather an uncertain element in the calculation.†

* *Proc. Inst. Mech. Eng.*, 1858, p. 76.

† *Idem*, p. 73.

TABLE II.—EXPERIMENTS ON PUNCHING HAMMERED SCRAP IRON.

No. of experiment.	Diameter of Punch.	Sectional area cut.		Pressure on Punch.		Remarks.
		Thickness and circumference.	Area.	Total.	Tons per square inch of area cut.	
	ins.	inches.	sq. ins.	tons.	tons.	
1	1	0·51×3·14	1·60	35·8	22·4	22·5 mean.
2	1	0·98×3·14	3·08	69·3	22·6	
3	2	0·52×6·28	3·27	59·7	18·3	19·4 mean.
4	2	0·57×6·28	3·58	70·5	19·7	
5	2	1·06×6·28	6·66	132·8	19·9	
6	2	1·52×6·28	9·55	186·7	19·5	

393. Experiments on shearing wrought-iron.—Table III. contains experiments, also by Mr. Little, with Eastwood's hydraulic shearing press, on the force required to shear bars of hammered scrap and rolled iron presented edgeways and flatways to the cutter.

TABLE III.—EXPERIMENTS ON SHEARING HAMMERED SCRAP BARS AND ROLLED IRON.

No of experiment.	Direction of shearing.	Sectional area cut.		Pressure on Cutters.		Remarks.	
		Thickness and breadth.	Area.	Total.	Tons per square inch of area cut.		
		inches.	sq. ins.	tons.	tons.		Parallel Cutters, *i.e.*, Simultaneous shearing.
7	Flat	0·50×3·00	1·50	33·4	22·3	22·7 mean.	
8	Edge	0·50×3·00	1·50	34·6	23·1		
9	Flat	1·00×3·00	3·00	69·2	23·1	21·5 mean.	
10	Edge	1·00×3·00	3·00	68·1	22·7		
11	Flat	1·00×3·02	3·02	59·7	19·8		
12	Edge	1·00×3·02	3·02	62·1	20·6		
13	Edge	1·80×5·00	10·20	210·6	20·6	Flanged tyre.	

TABLE III.—EXPERIMENTS ON SHEARING WROUGHT-IRON—*continued.*

No. of experiment.	Direction of shearing.	Sectional area cut.		Pressure on Cutters.		Remarks.
		Thickness and breadth.	Area.	Total.	Tons per square inch of area cut.	
		inches.	sq. ins.	tons.	tons.	
14	Flat	0·56×3·00	1·68	21·2	12·6	Inclined Cutters (angle 1 in 8), *i.e.*, Shearing in detail.
15	Edge	0·56×3·00	1·68	33·2	19·7	
16	Flat	0·90×3·37	3·03	27·4	9·0	
17	Edge	0·87×3·32	2·89	57·4	19·8	
18	Flat	1·06×3·02	3·20	50·2	15·7	
19	Edge	1·06×3·02	3·20	67·5	21·1	
20	Flat	1·52×3·03	4·61	83·7	18·2	
21	Edge	1·53×3·03	4·64	93·3	20·1	
22	Flat	1·39×4·50	6·25	89·7	14·3	
23	Edge	1·38×4·50	6·21	111·2	17·9	
24	Flat	1·73×5·30	9·17	153·1	16·7	
25	Edge	1·73×5·30	9·17	207·0	22·6	
26	Flat	1·56×6·00	9·36	140·0	15·0	
27	Edge	1·56×6·00	9·36	172·3	18·4	
28	Square	3·10×3·10	9·61	165·1	17·2	Hammered iron. Shearing in detail.
29	Square	3·10×3·10	9·61	155·5	16·2	Rolled iron.
30	Flat	1·80×5·00	10·20	99·3	9·7	Flanged tyre.
31	Edge	1·80×5·00	10·20	185·5	18·2	Flanged tyre.
32	Edge	1·70×5·25	10·57	179·5	17·0	Flanged tyre.

"In the above experiments of shearing (Nos. 7 to 13 inclusive), cutters with parallel edges were used; but when the ordinary cutter with edges inclined to one another at an angle of 1 in 8 were employed (Nos. 14 to 32 inclusive), the force required in shearing was diminished, and considerably so in the case of the thinner sections when sheared flatways; and as bars are usually sheared flatways, a decided advantage is shown in favour of inclined over parallel cutters. The force in tons per square inch of section cut with the bars

		Flatways. tons.		Edgeways. tons.			
3 $\times 1\frac{1}{2}$ inch	was	18·2	and	20·1	or	10	per cent. less flatways.
$4\frac{1}{2} \times 1\frac{3}{8}$	,,	14·3	,,	17·9	,,	20	,,
3 $\times 1$	,,	15·7	,,	21·1	,,	26	,,
$5\frac{1}{4} \times 1\frac{3}{4}$	,,	16·7	,,	22·6	,,	26	
6 $\times 1\frac{1}{2}$	,,	15·0	,,	18·4	,,	18	,,

"A trial was also made of the force required to shear some hard railway tyres $1\frac{3}{4}$ inch thick, and the result was 185 tons total edgeways, and 99 tons flatways (Nos. 30 and 31). A 3 inch square bar of rolled iron was also tried, and the force required was 155 tons total, against a total of 165 tons required for a hammered bar of the same section (Nos. 28 and 29)."*

During the construction of the Britannia and Conway tubular bridges several experiments were made by means of a lever on the shearing strength of bars of rivet iron $\frac{7}{8}$th inch diameter. "The mean result from these experiments gives 23·3 tons per square inch as the weight requisite to shear a single rod of rivet iron of good quality. The ultimate tensile strength of these same bars was also found to be 24 tons; hence their resistance to single shearing was nearly the same as their ultimate resistance to a tensile strain." Two plates $\frac{5}{8}$th inch thick were also "riveted together by a single rivet $\frac{7}{8}$th inch diameter, and the rivet was sheared by suspending actual weights from the plate; the rivet thus sustained 12·267 tons, or 20·4 tons per square inch. Three plates were then united by a similar rivet, and the rivet was sheared in two places by the centre plate. The ultimate weight suspended from the rivet was 26·8 tons, or 22·3 tons per square inch of section."†

394. Shearing strength of wrought-iron equals its tensile strength.—From these various experiments on punching and shearing, we may infer that the shearing strength of wrought-iron is practically equal to its tensile strength, and that the safe shearing

* *Proc. Inst. Mech. Eng.*, 1858, p. 74.

† Clark *on the Tubular Bridges*, p. 392.

unit-strain for wrought-iron rivets or bolts is practically the same as the safe tensile unit-strain in the plates they connect, *i.e.*, about 5 tons per square inch of section in ordinary girder-work.

395. Shearing strength of rivet steel is three-fourths of its tensile strength.—From Mr. Kirkaldy's experiments it appears that the shearing strength of rivet steel is 63,796 lbs., = 28·48 tons per square inch, the tensile strength of the bar employed being 86,450 lbs., = 38·59 tons per square inch of area.* Hence, the shearing strength of rivet steel is about three-fourths of its tensile strength. The tensile strength of some rivet steel used in one of H.M. ships was 35·93 tons per square inch.† The heads of steel rivets are very apt to fly off, and Lloyd's committee have prohibited their use in shipbuilding.

396. Shearing strength of copper.—From experiments by Mr. Joseph Colthurst on punching plates of wrought-iron and copper with a lever apparatus, it appears that the force required to punch copper is two-thirds of that required to punch iron. "It was observed, that duration of pressure lessened considerably the ultimate force necessary to punch through metal, and that the use of oil on the punch reduced the pressure about 8 per cent."‡

397. Shearing strength of fir in the direction of the grain — Shearing strength of oak treenails.— From Mr. Barlow's experiments on the resistance of fir to drawing out, *i.e.*, shearing, in the direction of the grain, it appears that this amounts to 592 lbs. per square inch, or nearly one-twentieth of the tensile strength of the timber lengthways.§

The following table contains experiments by Mr. Parsons of H.M. dockyard service, on the "transverse strength of Treenails of English oak, used as fastening for planks of 3 and of 6 inches in thickness, and subjected to a cross strain."‖

* *Experimental Inquiry*, p. 71.

† Reed *on Shipbuilding*, p. 382.

‡ *Proc. Inst. of C. E.*, Vol. i., p. 60.

§ Barlow *on the Strength of Materials*, p. 23.

‖ Murray *on Shipbuilding in Iron and Wood*, p. 94.

TABLE IV.—Strength of Treenails of English Oak.

Number of the experiment.	Diameter of the treenails.							
	1 inch.		1¼ inch.		1½ inch.		1¾ inch.	
	Thickness of the plank.							
	3 inches.	6 inches.	3 inches.	6 inches.	3 inches.	6 inches.	3 inches.	6 inches
	T. C.	T. C.	T. C.	T. C.	T. C.	T. C.	T. C.	T. C.
1	1 8	1 7	1 14	2 8	2 0	3 12	3 0	5 10
2	1 7	1 15	2 2	2 2	2 6	2 10	2 10	3 13
3	1 2	1 8	1 17	2 19	2 15	2 10	4 0	4 0
4	1 5½	1 8	2 2	2 2	2 4	3 12	2 8	3 8
5	2 12	1 3	2 2	1 15	2 18	2 5	3 10	4 0
6	2 2	1 7	2 9	2 10	2 6	2 5	3 10	5 8
7	2 4	1 10	2 8	2 10	3 7	2 5	3 5	3 12
8	1 6	2 3	2 7	2 0	2 5	3 0	3 5	3 13
9	1 8	1 8	2 12	2 10	3 0	4 0	4 6	4 13
10	1 2	2 3	2 10	2 15	3 0	4 10	3 8	4 0
11	2 0	2 0	2 7	2 0	3 9	2 18	4 0	3 8
12	1 8	1 7	2 10	2 0	4 2	3 0	4 10	5 0
13	1 16	2 8	2 17	2 0	3 2	3 18	4 2	5 5
Average	1 11	1 13	2 6	2 6	2 16	3 2	3 10	4 6
Total Shearing force in tons.	1·6		2·3		2·95		3·9	
Tons per square inch of section.	2·04		1·88		1·67		1·62	

"In all these experiments where the treenails were evidently good, they gave way gradually. In some of the rejected experiments, however, the treenails certainly did break off suddenly, but then they were evidently, on examination, either of bad or over-seasoned material. In the experiments on treenails, the plank generally moved about half an inch previous to the fracture of the treenail."

From these experiments Professor Rankine deduces,

1. That the shearing strength of English oak treenails across the grain is about 4,000 lbs. per square inch of section.

2. That in order to realize that strength, the planks connected by the treenails should have a thickness equal to about three times the diameter of the treenails.*

* *Civil Engineering*, p. 459.

CHAPTER XVIII.

ELASTICITY AND SET.

398. Limit of Elasticity—Set—Hooke's law of elasticity practically true.—It has been already stated in **5** that Mr. Hodgkinson's experiments led him to infer the non-existence of a definite *elastic limit* within which, if the particles of a substance be displaced, they will return exactly to their original relative positions after the disturbing force is removed. The opposite view was held by Professor Robison, whose opinions are also entitled to great respect. In the article on the "Strength of Materials" in the *Encyclopædia Britannica*, he writes as follows:—"It is a matter of fact that all bodies are in a certain degree perfectly elastic; that is, when their form or bulk is changed by certain moderate compressions or distractions, it requires the continuance of the changing force to continue the body in this new state; and when the force is removed, the body recovers its original form. We limit the assertion to *certain moderate* changes. For instance, take a lead wire of one-fifteenth of an inch in diameter and ten feet long; fix one end firmly to the ceiling, and let the wire hang perpendicular; affix to the lower end an index like the hand of a watch; on some stand immediately below, let there be a circle divided into degrees, with its centre corresponding to the lower point of the wire; now turn this index twice round, and thus twist the wire. When the index is let go, it will turn backwards again, by the wire untwisting itself, and make almost four revolutions before it stops; after which it twists and untwists many times, the index going backwards and forwards round the circle, diminishing, however, its arch of twist each time, till at last it settles precisely in its original position. This may be repeated for ever. Now, in this motion, every part of the wire partakes equally of the twist. The particles are stretched, require force to keep them in their state of extension and recover completely their relative positions. These are all the characters of what the mechanician calls *perfect* elasticity. This is a quality quite familiar in many cases, as in glass, tempered steel, &c., but was thought incompetent to lead, which is generally considered as having little or no elasticity. But we make the assertion in the most general terms, with the limitation to moderate derangement of form. We have made the same experiment on a thread of pipe-clay, made by forcing soft clay through the small hole of a syringe by means of a screw, and we found it more elastic than the lead wire; for a thread of one-twentieth of an inch diameter and seven feet long allowed the index to make two turns, and yet completely recovered its first position. But if we turn the index of the lead wire four times round and let it go again, it untwists again in the same manner, but it makes little more than four turns back again; and after many oscillations, it finally stops in a position almost two revolutions removed from its original position. It has now acquired a new arrangement of parts, and this new arrangement is permanent like the former; and what is of particular moment, it is perfectly elastic. This change is familiarly known by the denomination of a set."*

Whatever opinion the reader may hold regarding the existence or non-existence of a definite elastic limit, experiments prove that Hooke's *Law of Elasticity*, namely, that the elastic reaction of the fibres is proportional to their increment or decrement of length, according as they are subject to tension or compression, is for all practical purposes substantially true of most of the materials used in construction over a very considerable range of strain, extending in some cases even to the breaking weight of the material (7).

CAST-IRON.

399. Decrement of length and set of cast-iron in compression—Coefficient of compressive elasticity.—We are indebted to Mr. Hodgkinson for some valuable experiments on the decrements of length and compressive sets of eight bars of cast-iron, each 10 feet long and 1 inch square nearly. The first pair of bars were Low Moor iron No. 2; the second pair, Blaenavon iron No. 2;

* *Enc. Brit.*, 8th Ed., Vol. xx., p. 749, Art. "Strength of Materials."

the third pair, Gartsherrie iron No. 3; and the fourth pair, a mixture of Leeswood iron No. 3 and Glengarnock iron No. 3, in equal proportions. Table I. contains the mean of these experiments reduced to a convenient unit-strain by Mr. Clark, and I have added in the last column the coefficients of compressive elasticity per square inch, obtained by dividing the original length, viz., 120 inches, by the decrements of length per ton in the second column (8).*

TABLE I.—DECREMENTS OF LENGTH AND COMPRESSIVE SETS OF A CAST-IRON BAR 10 FEET LONG AND 1 INCH SQUARE.

Tons per square inch.	Decrements of length per ton.	Total Decrements of length.	Sets.	E′ The coefficient of Compressive Elasticity per square inch.
	inch.	inch.	inch.	tons.
1	·020338	·020338	·000510	5900
2	·021038	·042077	·002452	5704
3	·021618	·064855	·004340	5551
4	·021369	·085479	·006998	5615
5	·021594	·107872	·009188	5557
6	·021752	·130513	·011798	5517
7	·021950	·153654	·015243	5467
8	·022154	·177235	·018572	5416
9	·022374	·201373	·024254	5363
10	·022477	·224774	·028126	5339
11	·022567	·248237	·032023	5317
12	·022802	·273632	·037653	5262
13	·023014	·299187	·043318	5214
14	·023523	·329330	·052640	5101
15	·023539	·353092	·060905	5098
16	·024409	·390558	·080256	4916
17	·024805	·421695	·086298	4838

Mean, 5,417 tons = 12,134,080 lbs.

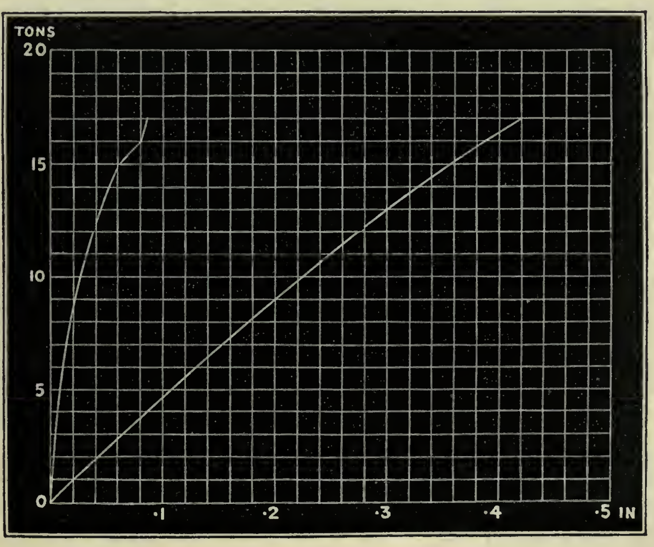
* *Rep. of Iron Com., App.*, p. 63; and Clark *on the Tubular Bridges*, p. 312.

Mr. Hodgkinson makes the following remarks on these experiments:—"The great difficulty of obtaining accurately the decrements and sets from the small weights in the commencement of the experiments, rendered those decrements and sets, particularly the latter, very anomalous; it was found, too, that some of the bars which had been strained by 16 or 18 tons had become very perceptibly undulated. It has not been thought prudent, therefore, to draw any conclusion from bars which have been loaded with more than 14 to 16 tons; and it may be mentioned that the results from 2 to 14 tons are those only which ought to be used in seeking for general conclusions."* (See the mean value of **E**′ in the last column.)

The results of Table I. are exhibited graphically in Fig. 104, where the longer curve refers to the total decrements of length, and the shorter one to the sets. The ordinates represent the weights in column 1, and the abscissas the total decrements of length and sets in columns 3 and 4 respectively of Table I.

Fig. 104.

DECREMENT OF LENGTH AND SET OF CAST-IRON IN COMPRESSION.



* *Rep. of Iron Com., App.*, p. 64.

The uniformity of the curve of decrements shows that there is no abrupt alteration in the compressive elasticity of cast-iron as far as 17 tons per square inch and possibly up to a higher amount.

400. Hodgkinson's formulæ for the decrement of length and set of cast-iron in compression.—The following formula was deduced by Mr. Hodgkinson from his experiments on the four different irons just described to express the relation between the load and the corresponding decrements of length in cast-iron bars 1 inch square and of any length.*

$$\lambda' = l\{\cdot012363359 - \sqrt{\cdot000152853 - \cdot00000000191212\mathrm{W}}\} \quad (243)$$

Where λ' = the decrement of length in inches,

l = the length in inches,

W = the weight in lbs. compressing the bar.

Mr. Hodgkinson expressed the compressive set of bars of Low Moor cast-iron 10 feet long by the following equation†:—

$$\text{Compressive set in inches} = \cdot543\lambda'^2 + \cdot0013. \quad (244)$$

401. Increment of length and set of cast-iron in tension—Coefficient of tensile elasticity.—The following table shows the increments of length and tensile sets of cast-iron bars 10 feet long and 1 inch square, reduced by Mr. Clark from Mr. Hodgkinson's experiments "upon round bars of iron, united together at the ends, so that the whole length, exclusive of the couplings, was 50 feet, except in two instances, where the length was 48 feet 3 inches. There were nine experiments upon these connected lengths, and the experiments were upon four kinds of cast-iron—Low Moor No. 2, Blaenavon No. 2, Gartsherrie No. 3, and a mixture of iron, composed of Leeswood No. 3 and Glengarnock No. 3, in equal proportions. There were two experiments upon each of the simple irons, and three upon the mixture, and the mean results were afterwards reduced to those of 10 feet and 1 square inch exactly." "The bars were suspended vertically, and acted upon directly by weights attached at their lower ends."‡ I have added in the last column the coefficients of tensile elasticity, obtained by

* *Rep. of Iron Com., App.*, p. 109.

† *Idem*, p. 123.

‡ *Idem*, pp. 59, 51; and Clark *on the Tubular Bridges*, p. 379.

dividing the original length, viz., 120 inches, by the increments of length per ton in the second column.

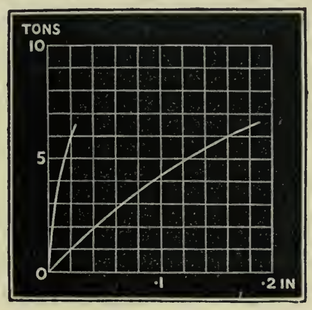
TABLE II.—INCREMENTS OF LENGTH AND TENSILE SETS OF A CAST-IRON BAR 10 FEET LONG AND 1 INCH SQUARE.

Tons, per square inch.	Increments of length per Ton.	Total increments of length.	Sets.	E, The coefficient of Tensile Elasticity per square inch.	
	inch.	inch.	inch.	tons. lbs.	
1	·01976	·01976	·000579	6073 = 13,603,520	
2	·02027	·04155	·001860	5920 = 13,260,800	Mean = 5366 tons = 12,020,960 lbs
3	·02171	·06515	·003954	5528 = 12,382,720	
4	·02318	·09274	·007543	5177 = 11,596,480	
5	·02479	·12397	·012619	4841 = 10,843,840	
6	·02727	·16363	·020571	4400 = 9,856,000	
6½	·02815	·18297	·023720	4263 = 9,549,120	

The mean increment of length per ton for the first 3 tons per square inch equals ·0001715 of the length. The results of Table II. are exhibited graphically in Fig. 105, where the longer curve refers to the total increments of length and the shorter one to the sets. The ordinates represent the weights in column 1, and the abscissas the total increments of length and sets in columns 3 and 4.

Fig. 105.

INCREMENT OF LENGTH AND SET OF CAST-IRON IN TENSION.



The uniformity of the curve of increments shows that there is no abrupt change in the tensile elasticity of cast-iron up to 6·5 tons per square inch, and possibly up to the limit of rupture, the mean of which for the 4 irons experimented on was 7·014 tons per square inch.

By the aid of Tables I. and II. we can easily find approximately the decrement, increment, or set of cast-iron bars of any section.

Ex. The compression flange of a new cast-iron girder, 40 feet long, which has not been previously strained, will be shortened by an inch-strain of 6 tons by an amount equal to $40 \times 0·0130513 = 0·522052$ inch, and its set, or residual decrement of length after the load has been removed, will equal $40 \times 0·0011798 = 0·047192$ inch. If the whole of this set were permanent, which however is problematical, the flange would be permanently shortened by this amount, and on any subsequent application of the same load its new decrement of length would $= 0·522052 - 0·047192 = 0·474860$ inch.

402. Hodgkinson's formulæ for the increment of length and set of cast-iron in tension.—The following formula was deduced by Mr. Hodgkinson from his experiments on the extension of the four different irons just described, to express the relation between the load and the corresponding increments of length in cast-iron bars 1 inch square and of any length.*

$$\lambda = l\{·00239628 - \sqrt{·00000574215 - ·000000000343946\,\mathbf{W}}\} \qquad (245)$$

Where $\lambda =$ the increment of length in inches,

$l =$ the total length in inches,

$\mathbf{W} =$ the weight in lbs. extending the bar.

The tensile set of bars 10 feet long is as follows:—

$$\text{Tensile set in inches} = ·0193\lambda + ·64\lambda^2 \qquad (246)$$

403. Coefficients of tensile, compressive and transverse elasticity of cast-iron different.—On comparing Tables I. and II. it will be observed that, though the mean of the coefficients of compressive elasticity up to 14 tons, and of tensile elasticity up to 5 tons, per square inch are substantially the same, namely, 12,000,000 lbs. per square inch, the several coefficients themselves differ materially, especially as they approach the limit of tensile strength; for instance, at 6 tons per square inch the coefficient of compressive elasticity is 1·25 times that of tensile elasticity. The coefficients of transverse elasticity derived from experiments on a

* *Rep. of Iron Com., App.*, pp. 60, 108.

moderate and on the ultimate deflection of a rectangular bar of Blaenavon iron, broken by transverse pressure, are also different, though they closely approach the limiting coefficients of tensile elasticity in Table II. See ex. in **235**, also **246**.

404. Increment of length and set of cast-iron extended a second time—Relaxation of set—Viscid elasticity.—Mr. Hodgkinson made a second series of experiments on the extension of some parts of the coupled bars which were strained nearly to their breaking point, but had escaped actual rupture at the first trial.* Their total increments of length on the second trial, though very nearly the same as before, were slightly less for the higher loads. It might perhaps be supposed that bars once stretched would not again take a set, provided the second load did not exceed that previously applied. This, however, was not the case, for the barstook sets again, though in general less than before, their mean ultimate set being nearly half that on the first trial. It is very probable that cast-iron, and also other materials, recover a portion of the set when the strain producing it is relaxed for some time—in fact, that there exists a sort of sluggish elasticity, due perhaps to a certain viscidity of the material. Possibly, constant repetitions or long continuation of strain would render the set permanent. Experiments alone can settle these points, which, however, have more interest for the physicist than practical importance for the engineer.

405. Set of cast-iron bars from transverse strain nearly proportional to square of deflection.—The set of cast-iron bars subject to *transverse strain* is nearly proportional to the square of their deflection, though somewhat less, and may be expressed approximately by the following formula deduced by Mr. Hodgkinson from his experiments on rectangular bars of Blaenavon cast-iron bent transversely by a load in the middle.†

$$\text{Transverse set in inches} = \frac{\mathbf{D}^2}{31{\cdot}5} \qquad (247)$$

in which $\mathbf{D}$ represents the deflection of the bar in inches.

* *Rep. of Iron Com.*, *App.*, p. 61.

† *Ibid.*, p. 69.

WROUGHT-IRON.

406. Decrement of length of wrought-iron in compression—Coefficient of compressive elasticity—Elastic limit.— The following table contains the results of experiments by Mr. Hodgkinson on the compression of two wrought-iron bars 10 feet long and 1 inch square nearly, the weights increasing at first by 2 tons and afterwards by 1 ton at a time.*

TABLE III.—DECREMENTS OF LENGTH OF WROUGHT-IRON BARS 10 FEET LONG AND 1 INCH SQUARE NEARLY.

Bar 1. Area of section = 1·025 × 1·025 = 1·0506 square inches.			Bar 2. Area of section = 1·016 × 1·02 = 1·0363 square inches.		
Weight compressing Bar.	Total Decrements of length.	Decrements per ton.	Weight compressing Bar.	Total Decrements of length.	Decrements per ton.
lbs.	inches.	inches.	lbs.	inches.	inches.
5098	·028	—	5098	·027	—
9578	·052	·012	9578	·047	·010
14058	·073	·0105	14058	·067	·010
16298	·085	·012	—	—	—
18538	·096	·011	18538	·089	·011
20778	·107	·011	20778	·100	·011
23018	·119	·012	23018	·113	·013
25258	·130	·011	25258	·128	·015
27498	·142	·012	27498	·143	·015
29738	·154	·012	29738	·163	·020
31978	·174	·020	31978	·190	·027
34218	·214	·040	in ½ hour.	·261	·071
—	—	—	31978	·269	—
—	—	—	in ½ hour.	·282	—
—	—	—	repeated.	·328	—

In the foregoing experiments the total decrements of length

* *Rep. of Iron Com., App.*, p. 122.

increase with considerable uniformity in proportion to the weight, until the pressure reaches the elastic limit of about 12 tons per inch, after which irregular bulging begins, the amount of which, no doubt, will depend on the quality of the iron, the hard and brittle irons bulging less than the tough and ductile kinds. The mean decrement of length per ton per square inch within this elastic limit $= \cdot 0000964 = \frac{1}{10{,}376}$th of the original length. Hence, the coefficient of compressive elasticity of bar iron from Hodgkinson's experiments = 10,376 tons = 23,243,179 lbs. per square inch.* In several experiments made by the "Steel Committee" on the compression of iron bars 10 feet long and $1\frac{1}{2}$ inch diameter, the mean limit of compressive elasticity was 12·32 tons per square inch, and the mean decrement of length within this limit was ·00007725, $= \frac{1}{12{,}945}$th of the original length for each ton, which makes the coefficient of compressive elasticity of these particular bars = 12,945 tons = 29,000,000 lbs. per square inch, or very nearly equal to that of steel.†

407. Increment of length and set of wrought-iron in tension—Coefficient of tensile elasticity—Elastic limit—Effects of cold-hardening and annealing on the elasticity of iron.—Table IV. contains the results of experiments by Mr. Hodgkinson on the extension and set of two bars of *annealed* wrought-iron of the quality denominated "best," reduced to the standard of bars 10 feet long and 1 inch square; their real dimensions were as follows:‡—

	Bar 1.	Bar 2.
Length, - - -	49 feet 2 inches, -	50 feet.
Mean diameter, -	·517 inch, - -	·7517 inch.
Mean area of section,	·2099 square inch,	·44379 square inch.

* *Rep. of Iron Com., App.*, p. 172.

† *Expts. on Steel and Iron.*

‡ *Rep. of Iron Com., App.*, pp. 47, 49.

TABLE IV.—INCREMENTS OF LENGTH AND TENSILE SETS OF TWO ANNEALED "BEST" WROUGHT-IRON BARS, 10 FEET LONG AND 1 INCH SQUARE.

Bar 1.			Bar 2.		
Weight per square inch of section.	Total Increments of length.	Sets.	Weight per square inch of section.	Total Increments of length.	Sets.
lbs.	inches.	inches.	lbs.	inches.	inches.
—	—	—	1262	·00520	—
2668	·00986	—	2524	·01150	—
5335	·02227	—	3786	·01690	·00050
8003	·03407	·000305	5047	·02240	·00060
10670	·04556	·000407	6309	·02772	·00050
13338	·05705	·000509	7571	·03298	·00045
16005	·06854	·000610	8833	·03790	·00050 ?
18673	·07993	·000813	10095	·04300	·00050 ?
21340	·09193	·001525	11357	·04854	—
24008	·10485	·003966	12619	·05370	·00070
26676	·12163	·009966	13880	·05950	—
29343	·15458	·031424	15142	·06480	—
32011	·26744	—	16404	·06980	—
—	·28271 in 5 minutes.	·13566	17666	·07530	·00130
34678	·5148	·36864	18928	·08170	—
37346	1·0995	1·01695	20190	·08740	·00270
Repeated.	1·1949	1·02966	21452	·09310	—
40013	·220 in 5 minutes.	1·093	22713	·09920	·00410
Repeated and left on.	1·411 after 1 hour.	—	23975	·10570	—
"	1·424 after 2 hours.	—	25237	·11250	·00680
"	1·433 after 3 hours.	—	26499	·12040	—
"	1·434 after 4 hours.	—	27761	·12880	·0120
"	1·436 after 5 hours.	—	29023	·14500	—
"	1·437 after 6 hours.	—	30285	·1991	—

TABLE IV.—INCREMENTS OF LENGTH AND TENSILE SETS OF TWO ANNEALED WROUGHT-IRON BARS, 10 FEET LONG AND 1 INCH SQUARE—*continued.*

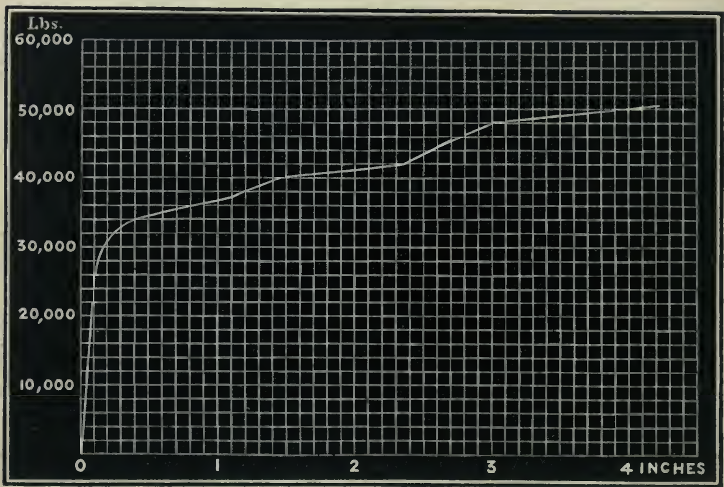
Bar 1.			Bar 2.		
Weight per square inch of section.	Total Increments of length.	Sets.	Weight per square inch of section.	Total Increments of length.	Sets.
lbs.	inches.	inches.	lbs.	inches.	inches.
Repeated and left on.	1·443 after 7 hours.	—	30285	·2007 after 5 minutes.	—
"	1·443 after 8 hours.	—	"	·2018 after 10 minutes.	·0736
"	1·443 after 9 hours.	—	"	·2054 after 15 minutes.	·0774
"	1·443 after 10 hours.	—	Repeated.	·2080 nearly, after 20 minutes.	·0796
42681	2·148 in 5 minutes.	1·983	"	·2096 after 1 hour.	·0814
Repeated.	2·339 in 5 minutes.	—	"	·2366 after bearing the weight 17 hours	·1082
"	2·383 in 10 minutes.	2·212 after 1 hour.	31546	·242 after 5 minutes.	·1083
Repeated.	2·428 after 46 hours.	2·237	Repeated.	·2449 after 5 minutes.	·1111
45348	2·580 after 5 minutes,	2·377	32808	·5506	·4141
Repeated.	2·605 after 1 hour.	—	Repeated.	·7024 after 5 minutes.	·5635
"	2·606 after 2 hours.	—	"	·7966 after 10 minutes.	·6558
"	2·606 after 19 hours.	2·403	"	1·014 after about ½ hour.	·866
48016	2·975 after 5 minutes.	2·733 after 10 minutes.	34070	1·346 after 1 minute.	—
Repeated.	3·019 after 1 hour.	—	"	1·400 after 2 minutes.	—
"	3·029 after 11 hours.	—	"	1·600	1·44
50684	4·195 in 10 minutes.	3·941 in 10 minutes.	Repeated.	1·65 after 1 minute.	—
Repeated.	4·226	—	"	1·786 after 1 hour or less.	1·628
"	4·227 in 7 hours.	—	35332	2·04 after 5 minutes.	1·874
"	4·227 in 12 hours.	—	Repeated.	2·18 after 5 minutes.	2·01
53351 = 23·817 tons.	Broke at one of the "weldings" where there was a slight defect; perhaps a rather smaller weight would have broken it.		"	2·254	2·08
			36594	2·54 after 6 minutes.	—
			37856 = 16·9 tons.	2·894	—
				The loop at the lower end of the rod having broken, the experiment was discontinued.	

From these experiments Mr. Hodgkinson inferred that the coefficient of tensile elasticity = 27,691,200 lbs. = 12,362 tons per square inch.* The limit of tensile elasticity, it will be observed, lies between 11 and 12 tons per square inch.

The relation between the weights and corresponding increments of length of the first bar in the foregoing table are exhibited graphically in Fig. 106, in which the ordinates represent the weights per square inch of section, and the abscissas the corresponding increments of length.

Fig. 106.

INCREMENT OF LENGTH OF WROUGHT-IRON IN TENSION.



The following table is given by Mr. Clark at p. 373 of his work on the Britannia and Conway tubular bridges. Though not expressly so stated, it is probably reduced from Mr. Hodgkinson's experiment on Bar 1 in Table IV.

* *Rep. of Iron Com., App.*; p. 172.

TABLE V.—INCREMENT OF LENGTH AND TENSILE SET OF A NEW WROUGHT-IRON BAR, 10 FEET LONG AND 1 INCH SQUARE.

Tons per square inch.	Observed extension in terms of the length.	Computed extension assumed uniform at $\frac{8}{100000}$ of the length per ton per square inch.	Corresponding extension in fractional parts of the length computed at $\frac{8}{100000}$ per ton per square inch.	Observed set in terms of the length.	Observed set in fractional parts of the length.
1	·0000689	·00008	$\frac{1}{12500}$		
2	·000156	·00016	$\frac{1}{6250}$		
3	·000238	·00024	$\frac{1}{4166}$	·00000213	$\frac{1}{468750}$
4	·000319	·00032	$\frac{1}{3125}$	·00000283	$\frac{1}{350877}$
5	·000399	·00040	$\frac{1}{2500}$	·00000356	$\frac{1}{281030}$
6	·00048	·00048	$\frac{1}{2083}$	·00000427	$\frac{1}{233918}$
7	·00056	·00056	$\frac{1}{1786}$	·00000497	$\frac{1}{201005}$
8	·00064	·00064	$\frac{1}{1562}$	·00000650	$\frac{1}{153846}$
9	·00072	·00072	$\frac{1}{1389}$	·00001201	$\frac{1}{83275}$
10	·00080	·00080	$\frac{1}{1250}$	·00001334	$\frac{1}{74955}$
11	·000896	·00088	$\frac{1}{1136}$	·00003392	$\frac{1}{29484}$
12	·00102	·00096	$\frac{1}{1040}$	·00008368	$\frac{1}{11950}$
13	·00128	·00104	$\frac{1}{961}$	·0002598	$\frac{1}{3848}$
14	·00218 in ten minutes ·00231	·00112	$\frac{1}{893}$	·0011075	$\frac{1}{903}$
15	·00416	·00120	Beyond this weight the permanent set is greater than the computed extension as above.	·002976	$\frac{1}{336}$
16	·00443	·00128		·0003175	$\frac{1}{315}$
17	·00934 in ten minutes ·01015	·00136		·008750	$\frac{1}{114}$
18	·01024 in ten minutes ·01212	·00144		·009170	$\frac{1}{109}$
19	·01785 in ten minutes ·02017	·00152		·018590	$\frac{1}{54}$
20	·02124 in ten minutes ·02146	·00160		·019790	$\frac{1}{50}$
21	·02429 in ten minutes ·02472	·00168		·022310	$\frac{1}{45}$
22	·03400 in ten minutes ·03425	·00176		·031933	$\frac{1}{31}$

The foregoing tables and the diagram show that the increment of length of annealed wrought-iron in tension increases with great uniformity in proportion to the weight, and nearly equals ·00008, $= \frac{1}{12,500}$th of the length for each ton per square inch up to 11 or 12 tons, after which the law suddenly changes, and rapid and rather irregular stretching begins, the amount depending, no doubt, on the quality of the iron, *i.e.*, its hardness or ductility.

Mr. Barlow also made several experiments on bars of wrought-iron, from which he inferred that its limit of tensile elasticity is about 10 tons per square inch, and that it extends $\cdot000096 = \frac{1}{10,417}$th of its length for each ton within this limit.* In experiments made by the "Steel Committee" on 10 feet lengths of iron bars, $1\frac{1}{2}$ inches diameter, the mean limit of tensile elasticity was 12·7 tons per square inch, and the mean increment of length within this limit was $\cdot0000784 = \frac{1}{12,755}$th of the original length for each ton per square inch.

General Morin also made some experiments on fine charcoal iron wire, and found that the process of hardening wire by cold drawing increased its limit of elasticity to about 19 tons per square inch, while the coefficient of elasticity remained the same as that of ordinary bar iron, viz., 12,473 tons per square inch. Annealing iron wire had the effect of reducing its coefficient of tensile elasticity to 10,009 tons per square inch.† We may conclude from these various experiments that the elastic limit and the coefficient of elasticity of wrought-iron vary considerably with the quality and condition of the iron, but for practical purposes we may generally adopt 12 tons as the limit of elasticity, and 24,000,000 lbs., = 10,714 tons per square inch, as the coefficient of elasticity of ordinary plate and bar-iron, either in tension or in compression, though sometimes it may reach 29,000,000 lbs.; the former is equivalent to an

* *Strength of Materials*, p. 315.

† *Proc. Inst. C. E.*, Vol. xxx., p. 261.

alteration of $\frac{1}{10,714}$th = ·000093 of the original length for each ton per square inch.

408. Elastic flexibility of cast-iron twice that of wrought-iron—Law of elasticity truer for wrought than for cast-iron.—Comparing the coefficients of elasticity of cast and wrought-iron, we find that the *elastic flexibility* of cast-iron is nearly twice as great as that of wrought-iron, that is, the alteration of length from the same unit-strain is nearly twice as great in cast as in wrought-iron; in other words, wrought-iron is nearly twice as stiff as cast-iron. On this account a girder of cast-iron will deflect nearly twice as much as a similar one of wrought-iron, provided the flanges of both girders are subject to the same unit-strains. It will also be observed that Hookes' law of the proportionality of the loads to the changes of length they produce is less exact for cast than for wrought-iron within the limits of elasticity.

409. Stiffness of imperfectly elastic materials improved by stretching—Practical method of stiffening wrought-iron bars—Limit of elasticity of wrought-iron equals 12 tons per square inch—Proof-strain should not exceed the limit of elastictiy.—When an imperfectly elastic material has received a permanent set from the application of any weight which is subsequently removed, the material becomes more perfectly elastic than before within the range of strain which first produced the set, and its alteration of length per unit of strain is less than at first. When, for instance, a girder is tested for the first time, its deflection exceeds that produced by a subsequent application of the same load. Hence, the common practice of "stretching" girders by heavy loads before their final inspection. In compound structures, such as lattice girders, some of the initial deflection may, perhaps, be attributed to the separating or closing together of the numerous joints on the first application of a heavy load, though probably the greater portion is due to the straightening of parts in tension originally constructed a little out of line. The ultimate deflection of a bar of soft wrought-iron subject to transverse strain is very considerable, and when the useful load which such a bar will carry is determined by the amount of deflection rather than by its breaking weight, its useful strength, *i.e.*, its stiffness, may be much increased by giving it a considerable camber when at a dull red heat, and afterwards straightening it when cold. Such a bar, as far as deflection in the direction in which it was straightened is concerned, is stronger than before.* For practical purposes the *limit of elasticity* of wrought-iron, as already stated, does not exceed 12 tons per square inch, and though higher strains than this may not in the least diminish its ultimate strength, yet they will take the "stretch" out of the iron and may thus render what was originally tough and ductile metal so hard and brittle as to be seriously injured for many purposes. A tough quality of iron will evidently sustain sudden shocks with greater impunity than brittle iron, and previous over-straining may perhaps thus explain the unexpected rupture of chains with suddenly applied loads considerably below their statical breaking weight. For instance, sudden jerks from surging may double the usual safe working strain of a chain and thus strain it temporarily beyond its limit of elastic reaction. This frequently repeated will produce permanent elongation and render the chain brittle until it has been annealed (**357**). These considerations show that the proof-strain of wrought-iron should not exceed its limit of elasticity.

410. Experiments on elasticity liable to error—Sluggish or viscid elasticity.—Scientific conclusions derived from experiments on the elasticity of materials in which the effect of previous strain is overlooked are evidently worthless, and it should be recollected that time ought to be allowed after each experiment in order to let the material adjust itself to the new condition of strain, especially when the load approaches the limits of rupture, in which case the deformation, or change of form, may continue for a considerable time after the load is laid on, especially if aided by vibration. Referring to the Britannia and Conway Tubular Bridges Mr. Clark observes, "In all the tubes a considerable time elapsed before they attained a deflection which remained constant. Time is an important element in producing the ultimate permanent set

* Clark *on the Tubular Bridges*, p. 449.

in any elastic material; but when the permanent set due to the strain is once attained, the continuance of the same strain induces no further deflection, which is confirmed by the fact, that no subsequent change has occurred in the deflection of the Conway Bridge from two years of use, nor has any increase in the versed sine of the Menai Suspension-bridge taken place in twenty-five years, where the strain is greater than in the plates of the Conway Bridge, and liable to be considerably varied from the oscillation which occurs in gales of wind. The permanent strain in the Britannia Bridge is under three-fifths of that in the Suspension Bridge. The effect of time in producing permanent elongation has been also observed at the High Level Bridge (Newcastle-upon-Tyne), where the wrought-iron tie-chains, which resist the thrust of the arches, although under much less strain than the above, continued to extend for a considerable period before they attained a set at which they remained constant. These motions are so extremely minute that they are only ascertainable in large rigid structures, where they are measured by the corresponding increase of deflection."*

The residual set, after the strain has been removed, also takes time to adjust itself to a permanent condition, and some crude experiments of my own tend to prove that the set of wrought-iron relaxes to a considerable extent, even after the lapse of several days after the strain has been removed.

STEEL.

411. Law of elasticity true for steel—Coefficient and limit of elasticity of steel.—Numerous experiments made by the "Steel Committee" prove that the law of elasticity applies to steel with great exactitude within the limit of elastic reaction which for practical purposes is about 21 tons per square inch both for tension and compression (**298** and **359**). Within this limit the mean decrement of length per ton per square inch from compression $= \cdot 0000743 = \frac{1}{13,459}$th of the original length, and the mean

* Clark *on the Tubular Bridges*, p. 671.

increment from extension = $\cdot 0000764 = \frac{1}{13{,}089}$th of the original length. Taking the mean of these, the coefficient of either tensile or compressive elasticity = 13,274 tons = 29,733,760 lbs. per square inch. From Sir William Fairbairn's experiments on deflection under transverse strain, the coefficient of transverse elasticity = 31,000,000 lbs. (**359**). For practical purposes we may assume 30,000,000 lbs., = 13,393 tons per square inch, as the coefficient of elasticity of steel, which is 25 per cent. greater than the usual coefficient for wrought-iron, though the latter sometimes approaches 29,000,000 lbs., or very closely that of steel.

TIMBER.

412. Limit of elasticity of timber not accurately defined—Coefficient of elasticity depends on the dryness of the timber.—Experiments on timber by MM. Chevandier and Wertheim lead them to form the following conclusions.*

1°. The density of timber appears to vary but slightly with age.

2°. The coefficient of elasticity, on the contrary, diminishes beyond a certain age and depends on the dryness and aspect as well as the nature of the soil in which the trees grow, northerly aspects and dry soils raising the coefficient.

3°. The coefficient of elasticity is not sensibly affected by cutting trees before or after the sap is down.

4°. Properly speaking, there is no true limit of elasticity, as there is always a permanent set along with an elastic elongation.

5°. The limit of elasticity rises with the dryness of the timber, and wet timber takes a permanent set more readily than dry timber.

6°. In timber artificially dried in a stove, the limit of elasticity coincides nearly with the limit of rupture, *i.e.*, such timber takes scarcely any permanent set.

7°. Artificial drying greatly increases the stiffness of timber.

STONE.

413. Vitreous materials take no set.—It is stated by Dr. Robinson that "hard bodies of an uniform glassy structure, or

* Morin, *Résistance des Matériaux*, p. 37.

granulated like stones, are elastic through the whole extent of their cohesion, and take no set, but break at once when overloaded."* It may be doubted whether this is true of all granulated bodies like stones, for Mr. Mallet, referring to his experiments on crushing small cubes of quartz and slate rock from Holyhead, 0·707 inch upon each edge, observes, "the *per-saltum* way in which all the specimens of both rocks yield, in whatever direction pressed, is another noteworthy circumstance. The compressions do not constantly advance with the pressure, but, on the contrary, the rock occasionally suffers almost no sensible compression for several successive increments of pressure, and then gives way all at once (though without having lost cohesion, or having its elasticity permanently impaired), and compresses thence more or less for three or four or more successive increments of pressure, and then holds fast again, and so on. This phenomenon is probably due to the mass of the rock being made up of intermixed particles of several different simple minerals, having each specific differences of hardness, cohesion, and mutual adhesion, and which are, in the order of their resistances to pressure, in succession broken down, before the final disruption of the whole mass (weakened by these minute internal dislocations) takes place. Thus it would appear that the micaceous plates and aluminous clay-particles interspersed through the mass give way first. The chlorite in the slate, and probably felspar-crystals in the quartz-rock, next, and so on in order, until finally the elastic skeleton of silex gives way, and the rock is crushed. It is observable, also, that this successive disintegration does not occur at equal pressures, in the same quality and kind of rock, when compressed transverse and parallel to the lamination."† Hookes' law probably applies up to the limit when the first crushing of the weakest ingredient occurs. What takes place afterwards corresponds with the intermittent way in which wrought-iron in tension stretches once the limit of elasticity has been passed.

* *Encyc. Metr.*, 8th ed., art. "Strength of Materials," Vol. xx., p. 756.

† *Phil. Trans.*, 1862, p. 669.

CHAPTER XIX.

TEMPERATURE.

414. Arches camber, suspension bridges deflect, and girders elongate, from elevation of temperature—Expansion rollers.—Changes of temperature affect bridges very differently according to their mode of construction. An increase of temperature causes the crowns of iron arches which are confined between fixed abutments to rise, and the spandrils to extend lengthways, chiefly along their upper flange or horizontal member; hence, room for longitudinal expansion should be provided by leaving a vertical space between the ends of the spandrils and the masonry of the abutments above springing level. When iron arches extend over several spans, the spandrils of the different spans should not be rigidly connected together like continuous girders, for then their expansion may cause a dangerous crushing strain along the vertical line of junction and throughout the horizontal member, a portion of which strain will, no doubt, be transmitted to the ribs themselves. When, therefore, it is considered desirable to connect together the spandrils of consecutive iron arches, this should be effected by sliding covers, or some similar contrivance, which, though they restrain lateral motion, yet will allow perfect freedom for changes of length. The rise in the crown of one of the cast-iron arches of Southwark Bridge was observed by Mr. Rennie to be about 1·25 inches for a change of temperature of 50°F; the length of the chord of the extrados is 246 feet and its versed sine is 23 feet 1 inch; accordingly, the length of the arch, which is segmental, is 3020·8 inches.* The cast-iron bridge of Charenton, whose span and versed sine are 35 and

* *Trans. Inst. C. E.*, Vol. iii., p. 201.

4 metres respectively, has been observed to rise 14 millimetres (·55 inch) on the side exposed to the west from an elevation of 14°C. in the temperature of the air.*

Stone arches are affected in a similar way to iron arches. With increased temperature the crown rises and joints in the parapets open over the crown, while others over the springing close up. The reverse takes place in cold weather; the crown descends, joints over the springing open and those over the crown close. When stone or iron arches are of large span these movements from changes of temperature will generally dislocate to a certain degree the flagging and pavement of the roadway above. This is very conspicuous in Southwark Bridge.

An increase of temperature causes suspension bridges to deflect, just the reverse of what happens with arches. Girders, which exert only a vertical pressure on the points of support, extend longitudinally under the same influence, and on this account it is usual in long bridges to provide expansion rollers, or, if the span be moderate, sliding metallic surfaces, under one end of each main girder. It may be questioned, however, whether sliding surfaces remain long in working order, and some engineers prefer timber wall-plates beneath the ends of the girder, even when the span reaches 150 feet. In place of being supported by rollers, which are apt to set fast, girders are sometimes hung from suspension links, the pendulous motion of the links affording the requisite longitudinal movement due to change of temperature.† The chains of suspension bridges are generally attached to saddles which rest on rollers on top of the towers; the object of these, however, is rather to compensate for unequal loading than for changes of temperature.

415. Alteration of length from change of temperature—Coefficients of linear expansion.—The coefficient of linear

* Morin, *Résistance des Matériaux*, p. 116.

† Expansion rollers were placed under one end of each principal of the roof over the New-street Station, Birmingham, 212 feet span; the other end was attached to cast-iron columns. The rollers did not move, but the columns rocked 0·01917 inches for each degree Fahrenheit.—(*Proc. Inst. C.E.*, Vol. xiv., p. 261.) Expansion rollers were also placed under one end of each of the crescent-shaped principals of the old Lime-street Station, Liverpool, 153½ feet span, but did not act.—(*Idem*, Vol. ix., p. 207.)

expansion of any material is the fractional part of its length at zero centigrade which it elongates or shortens from a change of one unit of temperature, generally 1°C. The alteration of length for other changes of temperature is expressed by the following equation :—

$$\lambda = nkl \tag{248}$$

Where l = the length of the bar at 0°C.,

k = the coefficient of linear expansion of the material for one degree centigrade,

n = the number of degrees through which the temperature of the bar is raised or lowered,

λ = the increment or decrement of length due to a change of temperature equal to n degrees.

Ex. The total length of the Britannia wrought-iron tubular bridge is 1,510 feet, and an increase of temperature of 26°F. caused an increase of length of $3\frac{1}{4}$ inches, what is the coefficient of linear expansion of the tube for 1°C. ?—(*Clark*, p. 715.)

Here, l = 1510 feet = 18120 inches,

n = 26°F. = 14·44°C.,

λ = 3·25 inches.

$$\textit{Answer},\ k = \frac{\lambda}{nl} = \frac{3{\cdot}25}{14{\cdot}44 \times 18120} = 0{\cdot}000012421 \text{ inch,}$$

which, it will be observed, agrees closely with the coefficient of expansion of wrought-iron in the table below.

The following table contains the coefficients of linear expansion of various materials for one degree centigrade.

TABLE I.—COEFFICIENTS OF LINEAR EXPANSION FOR 1°C.

Description of Material.	Authority.	Coefficients of linear expansion for 1°C.
METALS.		
Antimony,	Smeaton,	·000010833
Bismuth,	Do.	·000013917
Brass (supposed to be Hamburg plate brass),	Ramsden,	·000018554
Do. (English plate, in form of a rod),	Do.	·000018928

NOTE.—One degree Fahrenheit = $\frac{5}{9}$ths of one degree centigrade. To convert a given temperature on Fahrenheit's scale to the corresponding temperature centigrade, subtract 32°., and multiply the remainder by $\frac{5}{9}$. Thus, the *temperature* of 86°F. = 30°C., but a *range* of 86°F. = 48°C., nearly.

TABLE I.—COEFFICIENTS OF LINEAR EXPANSION FOR 1°C.—*continued.*

Description of Material.	Authority.	Coefficients of linear expansion for 1°C.
METALS.		
Brass (English plate, in form of a trough), -	Ramsden, -	·000018949
Do. (cast), - - - - -	Smeaton, -	·000018750
Do. (wire), - - - - -	Do. -	·000019333
Copper, - - - - - -	Laplace & Lavoisier,	·000017122
Do. - - - - - -	Do. -	·000017224
Gold (de départ) - - - -	Do. -	·000014661
Do. (standard of Paris, not annealed), -	Do. -	·000015516
Do. (do. annealed), - -	Do. -	·000015136
Iron (cast), - - - - -	Ramsden, -	·000011094
Do. (from a bar cast 2 inches square), -	Adie, - -	·000011467
Do. (do. ½ an inch square), -	Do. -	·000011022
Do. (soft forged), - - - -	Laplace & Lavoisier,	·000012204
Do. (round wire), - - - -	Do. -	·000012350
Do. (wire), - - - - -	Troughton, -	·000014401
Lead, - - - - - -	Laplace & Lavoisier,	·000028484
Do., - - - - - -	Smeaton, -	·000028667
Palladium, - - - - -	Wollaston, -	·000010000
Platina, - - - - - -	Dulong & Petit, -	·000008842
Do., - - - - - -	Troughton, -	·000009918
Silver (of Cupel), - - - -	Laplace & Lavoisier,	·000019097
Do. (Paris standard), - - - -	Do. -	·000019087
Do., - - - - - -	Troughton, -	·000020826
Solder (white ; lead 2, tin 1), - - -	Smeaton, -	·000025053
Do. (spelter ; copper 2, zinc 1), - -	Do. -	·000020583
Speculum metal, - - - -	Do. -	·000019333
Steel (untempered), - - -	Laplace & Lavoisier,	·000010788

TABLE I.—COEFFICIENTS OF LINEAR EXPANSION FOR 1°C.—*continued.*

Description of Material.	Authority.	Coefficients of linear expansion for 1°C.
METALS.		
Steel, (tempered yellow, annealed at 65°C.), -	Laplace & Lavoisier,	·000012396
Do. (blistered), - - - -	Smeaton, -	·000011500
Do. (rod), - - - - -	Ramsden, -	·000011447
Tin (from Malacca), - - - -	Laplace & Lavoisier,	·000019376
Do. (from Falmouth), - - -	Do. -	·000021730
Zinc, - - - - - -	Smeaton, -	·000029417
TIMBER.		
Baywood, in the direction of the grain, dry, -	Joule, -	·00000461 to ·00000566
Deal, do. do. do. -	Do. -	·00000428 to ·00000438
STONE, BRICK, GLASS, CEMENT.		
Arbroath pavement, - - - -	Adie, - -	·000008985
Brick (best stock), - - - -	Do. - -	·000005502
Do. (fire), - - - - -	Do. - -	·000004928
Caithness pavement, - - - -	Do. - -	·000008947
Cement (Roman), - - - -	Do. - -	·000014349
Glass (English flint), - - - -	Laplace & Lavoisier,	·000008117
Do. (French, with lead),	Do. - -	·000008720
Granite (Aberdeen grey), - - -	Adie, - -	·000007894
Do. (Peterhead red, dry), - - -	Do. - -	·000008968
Do. (do. moist), - - -	Do. - -	·000009583
Greenstone (from Ratho), - - -	Do. - -	·000008089
Marble (Carrara, moist), - - -	Do. - -	·000011928
Do. (do. dry), - - - -	Do. - -	·000006539
Do. (black Galway), - - -	Do. - -	·000004452
Do. (do. softer specimen, containing more fossils),	Do. - -	·000004793

TABLE I.—COEFFICIENTS OF LINEAR EXPANSION FOR 1°C.—*continued.*

Description of Material.	Authority.	Coefficients of linear expansion for 1°C.
STONE, BRICK, GLASS, CEMENT.		
Marble (Sicilian white, moist), - -	Adie, - -	·000014147
Do. (do. dry), - - -	Do. - -	·000011041
Sandstone (from Craigleith quarry), - -	Do. - -	·000011743
Slate (from Penrhyn quarry, Wales), - -	Do. - -	·000010376

Adie ; *Dixon's Treatise on Heat,* p. 35.
Dulong and Petit ; *Pouillet, Eléments de Physique,* p. 221.
Joule ; *Proc. Roy. Soc.,* Vol. ix., No. 28, p. 3.
Laplace and Lavoisier ; *Dixon's Treatise on Heat,* p. 29.
Ramsden ; *idem,* p. 27.
Smeaton ; *Pouillet, Eléments de Physique,* p. 221.
Troughton ; *idem.*
Wollaston ; *idem.*

416. Expansibility of timber diminished, or even reversed, by moisture.—Mr. Joule found that moisture occasioned a marked diminution in the expansibility of timber by heat. After a rod of bay-wood on which he experimented "had been immersed in water until it had taken up 150 grains, making its total weight 882 grains, its coefficient of expansion was found to be only ·000000436. Experiments with the rod of deal, weighing when dry 425 grains, gave similar results; when made to absorb water its coefficient of expansion gradually decreased, until, when it weighed 874 grains, indicating an absorption of 449 grains of water, expansion by heat ceased altogether, and on the contrary, a *contraction* by heat equal to ·000000636 was experienced.*

417. Moisture increases the expansibility of some stones—Raising the temperature produces a permanent set in others.—"In the case of greenstone, and some descriptions of marble, the effect of moisture was to increase the amount of

* *Proc. Roy. Soc.,* Vol. ix., No. 28, p. 3.

expansion; in other instances no effect of this kind was perceptible. Mr. Adie also found that in white Sicilian marble a permanent increase in length was produced every time that its temperature was raised, the amount of increase diminishing each time."*

418. A change of temperature of 15°C. in cast-iron, and 7·5°C. in wrought-iron, are capable of producing a strain of one ton per square inch—Open-work girders in the United Kingdom are liable to a range of 45°C.—The alteration of length of a cast-iron bar within the range of three tons tension and seven tons compression per square inch, which include the ordinary limits of working strain, is about ·000175 of the original length for each ton per square inch, and its coefficient of linear expansion for 1°C. = ·000011467 according to Adie; consequently a change of temperature of about 15°C. (= 27°F.) is capable of developing a force equal to one ton per square inch. Again, if we assume that the alteration of length of a bar of wrought-iron for both tensile and compressive strains = ·000093 of its length for each ton per square inch, its coefficient of expansion for 1°C. being ·000012204, a change of temperature of about 7·5°C. (= 13·5°F.) is capable of developing a force equal to one ton per square inch. Hence, a given change of temperature will develop twice as much force in wrought as in cast-iron. The range of temperature to which open-work bridges through which the air has free access are subject in this country seldom exceeds 45°C. (= 81°F.), for which range wrought-iron alters ·000549, or nearly $\frac{1}{1800}$th of its original length. This change of length is nearly equivalent to that which would be produced by a strain of 6 tons per square inch. The range of temperature of cellular flanges may, however, exceed that mentioned above, as Mr. Clark mentions that the temperature of the Britannia Tubular Bridge, before it was roofed over, differed "widely from that of the atmosphere in the interior, for the top during hot sunshine has been oberved to reach 120°F., and even considerably more; and, on the other hand, a thermometer on the surface of the snow on the tube has registered as low as 16°F."†

* Dixon's *Treatise on Heat*, p. 34.

† *Britannia and Conway Tubular Bridges* p. 71

A familiar instance of the contractile force of wrought-iron in cooling is exhibited in the tires of wheels. "An ingenious application of this force was also made in the case of a gallery in the Conservatoire des Arts et Metiers in Paris, whose walls were forced outwards by some horizontal pressure. To draw them together M. Molard, formerly director of the Museum in that establishment, had iron bars passed across the building, and through large plates of metal bearing on a considerable surface of the external walls. The ends of these bars were formed into screws, and provided with nuts, which were first screwed close home against the plates. Each alternate bar was then elongated by means of the heat of oil lamps suspended from it, and when expanded the nuts were again screwed home. The lamps being removed, the bars contracted, and in doing so drew the walls together. The other set of bars was then expanded in the same manner, their nuts screwed home, and the wall drawn in through an additional space by their contraction. And this series of operations was repeated until the walls were completely restored to the vertical, in which position the bars then served permanently to secure them."*

419. Tubular plate girders are subject to vertical and lateral motions from changes of temperature—Open-work girders are nearly quite free from these movements.—In addition to the longitudinal movements to which all girders are subject from changes of temperature, tubular plate girders move vertically or laterally whenever the top or one side becomes hotter than the rest of the tube. Referring to the Britannia Tubular Bridge, Mr. Clark states that "even in the dullest and most rainy weather, when the sun is totally invisible, the tube rises slightly, showing that heat as well as light is radiated through the clouds. On very hot sunny days the lateral motion has been as much as 3 inches, and the rise and fall 2 inches and $\frac{3}{10}$ths."† These vertical and lateral motions have not been much observed in lattice or open-work girders; no doubt because the air and sunshine have

* Dixon's *Treatise on Heat*, p. 121.

† *Tubular Bridges*, p. 717.

free access to all parts and thus produce an equable temperature throughout the whole structure.

420. Transverse strength of cast-iron not affected by changes of temperature between 16°F. and 600°F.—It appears from Sir William Fairbairn's experiments on the transverse strength of cast-iron at various temperatures from 16°F. upwards, that its strength "is not reduced when its temperature is raised to 600°F., which is nearly that of melting lead; and it does not differ very widely, whatever the temperature may be, provided the bar be not heated so as to be red hot."*

421. Tensile strength of plate-iron uniform from 0°F. to 400°F.—It also appears from Sir William Fairbairn's experiments on wrought-iron at various temperatures that the tensile strength of plates is substantially uniform between 0°F. and 400°F. This result is corroborated by the experiments of the committee of the Franklin Institute appointed to report on the strength of materials employed in the construction of steam boilers. Sir Wm. Fairbairn also found that the strength of the best bar-iron was increased about one-third when the temperature reached 320°F., after which it again diminished.† This, however, seems anomalous, and further confirmation would be desirable.

* Hodgkinson's *Exp. Res.*, p. 378.

† *Useful Information for Engineers*, second series, pp. 114, 124.

CHAPTER XX.

FLANGES.

422. Cast-iron girders.—The compression flange of cast-iron girders is frequently made stronger than is theoretically necessary for the purpose of rendering it sufficiently stiff to resist side pressure, vibration, or other disturbing causes; in a word, to resist flexure. As the average crushing strength of cast-iron is about 5 times its tensile strength, theory indicates the most economical proportion of the compression to the tension flange, when both are horizontal, to be also 1 to 5 (**17**), whereas it is generally made much stronger than this, its area being sometimes one-third of that of the tension flange. Hence, cast-iron girders rarely fail in the compression flange and it is a common practice to calculate their strength, as well as that of wrought-iron girders, from the leverage of the tension flange by the following well-known modification of eq. 18:—

$$\mathbf{W} = \frac{adc}{l} \tag{249}$$

in which $\mathbf{W}$ = the breaking weight at the centre in tons,

a = the *net* area of the tension flange in square inches,

d = the depth of the web at the centre in inches,

l = the length between bearings in inches,

c = a coefficient depending on the material.

For cast-iron double-flanged girders the coefficient $c = 4 \times 7 = 28$, the average tensile strength of simple cast-irons being about 7 tons per square inch. For wrought-iron box girders with equal flanges, $c = 4 \times 20 = 80$, the tensile strength of ordinary plate iron being about 20 tons per square inch. This equation omits any strength derived from the vertical web acting as an independent rectangular girder (**100**); it gives, therefore, too low a result when the area of the web forms a large portion of the total cross section, or when the tensile strength of cast-iron exceeds 7 tons; on the other hand, the formula will give too high a result with narrow plate girders which, if unsupported, generally fail by bending sideways.

423. Cellular flanges.—The closed cell was for some years a favourite form for the compression flange of tubular plate girders, whereas the tension flange was generally made of one or several plates riveted together so as to form practically one thick plate.

Fig. 107.

The adoption of the cell in this instance arose from the impression that it was better adapted than other forms of pillar for resisting flexure, and so no doubt it was when used as a pillar without extraneous support. Its connexion with the continuous web, however, prevents the flange from deflecting in a vertical direction, for at each point along its length it is held rigidly in the direction of the thrust, nor can it escape from this without separating from the side plates, and it is obvious that a very moderate force will hold a pillar in the line of thrust when the flexure is of trifling amount (**153**). It should also be kept in view that the stiffness of a long unsupported plate to resist flexure is proportional to the cube of its thickness (**333**), and consequently, if the top and bottom plates of the cell be riveted together, we have a plate 8 times as stiff as either separately. If to these we add the central plate and the upper half of each side of the cell (so as to leave the depth of girder measured from the centre of the cell to the lower flange unaltered) and the spare angle irons, we have a top flange at least 3 times as thick and therefore 27 times as stiff to resist vertical flexure as the unsupported top of the original cell. Though we do not thoroughly know the laws which govern the buckling of the sides of a tube (**335**), it is evident that the pile of plates possesses a superiority over the cell in this respect. It is, moreover, clear that the lateral stiffness of the flange is scarcely, if at all, affected by using one thick plate of the same width and sectional area as the cell, for, regarding the pile as a girder on its side, we have the adjacent parts of the double web performing the duty of flanges in place of the sides of the cell. One great objection to the cell is this; a large extent of surface is exposed to corrosion and is at the same time difficult of access and therefore liable to be neglected; at the best its preservation is costly, and depends on the amount of care which the painter may feel inclined to bestow on an irksome task, for the proper completion of which he feels but little responsibility since his work is rarely inspected, while during its tedious and unhealthy performance he is obliged to assume an unnatural and fatiguing posture.*

424. Piled flanges—Long rivets not objectionable.— When several plates are built into one pile it may be objected that great length of rivet is required, and that the workmanship is in consequence less sound; but this objection has no real value so far as the riveting is concerned. In parts of the Britannia Tubular Bridge rivets passed through six layers of iron of an aggregate thickness of nearly $3\frac{1}{2}$ inches,† and in the Boyne Viaduct many rivets passed through six and seven plates, and in some parts even nine. As I had forgotten the exact method of manipulating these long rivets at the Boyne Viaduct, I obtained from Mr. Colville,

* A painful soreness of the eyes and tendency to faint are experienced in close cells whenever the stifling vapour of new lead paint is not removed by constant currents of fresh air passing through them. Hence, when the ventilation is defective, the painter must come out at short intervals to breathe the fresh air.

† *Britannia and Conway Tubular Bridges*, p. 575.

the intelligent superintendent of the iron-work, the following details:—

"The longest rivet we had was about 8 inches long and the holes must be well rimed out. The rivets were kept cool, head and point, by dipping in water, and the body of the rivet made very hot, which enabled the workmen to use the cup tool and the heavy hammer at once. Some of the long rivets I had cut out after being riveted, to see what they looked like, and I must say they filled better than I expected, being at top of the piers, which was very difficult to get to. I see no difficulty in riveting such thickness as was at the Boyne Bridge, but it must be with care in the heating of the rivets and using about a 14 ℔. hammer and cup tools. Common light riveting hammers would only upset the rivet at the point and would not fill in the body in such thickness as $4\frac{1}{2}$ to 5 inches." Mr. Clark made some experiments on rivets 12 inches long, most of which "broke at the head in cooling, and it was found necessary to cool the centre part of the rivet artificially previous to inserting them, the head and tail alone remaining red-hot. In this manner the contraction was avoided and the rivets remained sound." This seems to be the reverse of the practice at the Boyne Bridge, but it is probable that in Mr. Clark's experiments the heads of the rivets were damaged by prolonged hammering with light hammers, as he inserted some red-hot rivets 8 feet long in some castings of great strength, which, therefore, could not yield to the tension, and these rivets on cooling remained in all cases perfectly sound and had merely undergone a permanent extension proportionate to the temperature.*

425. Punching and drilling tools.—Careful attention is doubtless required in punching plates so that the holes in the successive layers may coincide, and without proper precaution much trouble and expense would be incurred in subsequent riming out the holes, but this labour may, to a great extent, be avoided by using accurate templates, or when the magnitude of the work warrants such an outlay, by punching machines similar to the Jacquard machine used

* *The Tubular Bridges*, p. 395.

at the Conway Bridge, and subsequently at the Boyne Viaduct and Canada Works, and constructed expressly for the purpose of producing accurate repetitions of any required pattern.* Drilling tools for boring several holes at once have been introduced with much success, as at Charing-cross Bridge. Such tools will often repay their first cost by the saving of manual labour in punching and plating, besides insuring more accurate work, but for ordinary girder-work the common punching machine is the cheapest tool.

426. Position of roadway—Compression flange stiffened by the compression bracing of the web.—The roadway is generally attached to one or other of the flanges, but is sometimes placed midway. The latter position is objectionable, since we then lose the advantage of horizontal rigidity which the roadway imparts to the flange to which it is attached. Moreover, less material is generally required for forming the connexion between the cross-girders and the main girders at the flanges than elsewhere. When local circumstances do not determine the level of the road it may at first sight appear desirable to connect it with the upper or compression flanges, so as to stiffen them against horizontal flexure, and this is generally the best position with shallow girders, as it allows the load to be placed more immediately over the longitudinal axis of each girder and thus dispenses with heavy cross-girders, which is often a very important saving, besides removing any tendency to unequal strain which a one-sided load on the lower flanges might produce. But with large and deep girders, independently of the theoretic consideration that the lower the centre of gravity the more stable the structure, some slight counterbalancing advantage results from connecting the road with the lower flange, as the expense of a parapet is saved and there is a greater appearance of security when a train travels through, instead of over, a tubular bridge. When the roadway is attached to the lower flanges and the depth of girder is not sufficient to admit of cross-bracing between the upper flanges, the horizontal stiffness of the road is communicated to the upper flanges by the internal bracing of the compression

* For a description of this machine see Part 121 of the *Civil Engineers' and Architects' Journal.*

braces when the web is a double-latticed web like Fig. 102, or by vertical angle-iron frames when the web is plated, and in the latter case triangular gussets are sometimes introduced to connect these stiffening frames with the cross-girders. The cross-girders are also occasionally prolonged like cantilevers and their extremities connected by raking struts with the upper flanges, as is usual in the parapets of wooden bridges.

427. Waste of material in flanges of uniform section—Arched upper flange—Waste of material in continuous girders crossing unequal spans.—It frequently happens that the flanges have a greater sectional area near their ends than theory requires, in order to preserve the symmetry of the flange throughout its entire length and avoid injudicious thinning of the material. This source of loss does not exist in the bowstring girder, as in it the strain is nearly uniform throughout each flange. A compromise may be effected between the bowstring girder and that with parallel flanges by arching the upper flange, as in Fig. 108. In this form of girder the strains near the ends of each flange are increased and

Fig. 108.

thus the extra material is utilized at the same time that the strains in the end braces are diminished in consequence of the oblique flange taking a share of their shearing strain. The mode of calculation is the same as for the bowstring girder. For a similar cause to that just mentioned there is sometimes a waste of material in the flanges of continuous girders of uniform depth crossing spans of very unequal length. In this case the segments over the smaller spans are much deeper in proportion to their length than those over the larger spans, and hence a considerable waste of material may arise from carrying the general design of the flanges symmetrically throughout.

428. An excess of strength in one flange does not increase the strength of braced girders, though it may slightly increase the strength of girders with continuous webs.—If the flanges of a braced girder be well proportioned, both flanges will fail simultaneously with the breaking load, and any increase of strength in one flange only does not increase the strength of the girder, but rather diminishes its useful strength by the excess of dead weight. When, however, the web is continuous, an increase of strength is produced by enlarging one of the flanges beyond its due proportion for the following reason:—The unit-strain in the re-enforced flange is less than before; consequently, there is less alteration in its length from strain and the neutral surface approaches closer to it than if the flanges were duly proportioned; hence, a larger proportion of the web aids the weaker flange. The useful strength of the girder, however, is not necessarily increased, since the extra strength thus obtained may merely suffice to support the extra weight of the re-enforced flange (**100**).

429. Bearing surface on the abutments—Working load on expansion rollers.—The area of bearing surface of a girder on the abutments should be sufficient to prevent undue crushing of the wall-plates on top of the abutments. A common rule for cast-iron girders is to make the length of bearing on the abutment equal to the depth of the girder at the middle, say $\frac{1}{15}$th of the span. It does not seem desirable to put a greater pressure on cast-iron expansion rollers than 2 or 3 tons per linear inch, and where the length of a girder does not exceed 150 feet, creosoted timber wall-plates will generally be found preferable to rollers or metallic sliding beds, both of which are apt to become rigid (**414**).

CHAPTER XXI.

WEB.

430. Plate web—Calculation of strains.—In lattice girders the flanges and the compression braces are intersected at short intervals and thus divided into short pillars as far as their tendency to flexure in the plane of the girder is concerned; this support is carried to its extreme limit in plate girders, the characteristic feature of which is the continuity of the vertical connexion (single or double, as the case may be) between the flanges. As the thin webs of plate girders are ill adapted to resist buckling or flexure under compression, it is usual to stiffen them by vertical T or angle irons reaching from flange to flange, like the frames of a ship. On a little consideration it will be obvious that these stiffening frames make the web more rigid at short intervals in vertical lines; thus this method of constructing plate girders resembles the vertical and diagonal bracing investigated in the sixth chapter, and the strains in the web may be approximately calculated in the manner there described, though they are more frequently obtained from the shearing-strain, as explained in **54.** If these frames are placed diagonally in place of vertically, the web will resemble the class of bracing investigated in the fifth chapter and should be treated accordingly.

431. Ambiguity respecting direction of strains in continuous webs—Bracing generally more economical than plating—Minimum thickness of plating in practice—Relative corrosion of metals.—Besides these compressive strains acting in directions more or less defined, there exist in the web of every plate girder diagonal tensile strains which cross the stiffening frames and whose directions are not so clearly defined and doubtless vary to some extent with every position of the load. It thus appears that some portions of the web of plate girders are simultaneously sustaining tension and compression and it might therefore seem at first sight that a continuous web is more economical than one formed of diagonal bracing, since in the former arrangement the same piece of material performs a double duty, which in the diagonal system requires two distinct braces (279). Theoretically this view is correct if it be conceded that one and the same portion of material is capable of sustaining without injury both tensile and compressive strains transmitted through it simultaneously at an angle with each other and, in the absence of direct experiment, there seems some reason for believing this to be the case within the limits of strain which are considered safe in practice. For instance, the shell and ends of a cylindrical boiler with internal flue are subject to tensile strains, the former in two directions at right angles to each other, the latter in various directions, while the flue is subject to tension longitudinally and compression transversely. Again, experiments on the strength of riveted joints have not indicated any source of weakness in the plates other than that due to the reduction of area by the rivet holes or the mode of punching, and if moderate compression does reduce tensile strength, closely riveted joints, such as those of boilers, would be perceptibly weakened by the compression caused by the contraction of the rivets in cooling. Further, in experiments on the tensile strength of iron bars, their ends are frequently grasped by powerful nippers which compress them sufficiently to prevent the bar slipping through, and it seldom breaks where thus compressed, rupture generally taking place near the centre. It seems, therefore, reasonable to infer that a moderate strain of either kind does not affect the ultimate strength of iron to sustain a strain of the other kind at right angles to the former. However this may be, practical reasons prevent plate-iron webs from being so economical as those formed of bracing, except in small or shallow girders, or girders which sustain unusually heavy loads and in which therefore the shearing strain is exceptional, or near the ends of girders of very large span; for unless the plating be reduced in thickness to the extent which theory indicates as sufficient, but which is quite unsuitable for practical reasons, the bars of the braced web will require so much less material than the continuous web of a plate girder as to make the former really the more economical.

One quarter inch may be assumed to be the minimum thickness that experience sanctions for the plating of permanent structures. A thinner plate than this may with care last for years, but few engineers would wish to risk the stability of any important structure on the chance of such frequent attention to prevent corrosion as so great a degree of tenuity would require. Indeed, $\frac{5}{16}$ is quite thin enough for ordinary practice, and $\frac{3}{8}$ or $\frac{1}{2}$ inch if a girder is within the influence of air charged with salt, as when railway bridges cross tidal estuaries. Mr. Mallet gives the relative oxidation of certain metals in moist air as follows:*—

Cast-iron, - - - -	·42
Wrought-iron, - - - -	·54
Steel, - - - - -	·56

He also states at p. 27 of his third report to the British Association in 1843 on the action of air and water upon iron, that in one century the depth of corrosion of Low Moor Plates, as deduced from his experiments, would be—

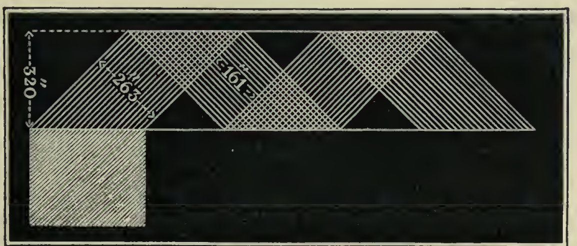
	Inch.
In clear sea water, - -	0·215
In foul sea water, - -	0·404
In clear fresh water only, -	0·035

432. Plating more economical than bracing near the ends of very long girders—Continuous webs more economical in shallow than in deep girders.—When the span is of great extent the opens between the braces towards the ends become smaller from the increased width of the bars and therefore nearly equal to their overlap; hence, there is a certain length of girder beyond which it may be found more economical to form the ends of the web of continuous plating and the intermediate portion of diagonal bracing. The length of girder at whose extremities the same amount of material is required for the web, whether formed of bracing or of plates, depends, among other things, on the ratio

* *On the Construction of Artillery*, p. 138.

of depth to span. In large railway girders, in which this ratio is frequently about 1 to 15, the span beyond which it becomes more economical to substitute plating near the ends in place of bracing lies between 300 and 400 feet. Take, for instance, the single-line railway bridge of 400 feet span, whose weight is calculated in Example 4, in the chapter on the estimation of girder-work. The length is 400 feet and the depth is 26·67 feet, or 1-15th of the length, and the maximum weight, including the permanent load, which the bridge has to support is 1,490 tons distributed uniformly. One-fourth of this, or 372·5 tons, is the shearing-strain supported by the web at each end of each main girder. Now, if the bracing be at an angle of 45°, which is the angle of economy, the strain in the end diagonals will equal the shearing-strain multiplied by 1·414, = 526·7 tons, requiring, at 4 tons per square inch, a gross section of 131·7 square inches.* If the iron be half-inch thick, the width of the end diagonal will equal 263 inches, as in Fig. 109, in which for simplicity only one system of triangulation is represented, since the overlap will be the same whether one or several systems be adopted.

Fig. 109.



It is evident that the overlap of the bars considerably exceeds the open spaces. This example, therefore, has attained the span beyond which it would be more economical to employ plating for the end portions of the web. If $\frac{3}{8}$-inch plating be considered sufficiently thick the limit would of course happen sooner. If, however, the depth were greater than 1-15th of the length, the limit would be

* In consequence of the rivet holes, 4 tons per square inch of gross section is for tensile strain assumed equivalent to 5 tons per square inch of net section.

greater than in our example. It is obvious also, from what has just been stated, that the relative economy of plate webs is greater in shallow than in deep girders; for, if bracing were used, the opens between the braces would be much smaller in the former than in the latter case, and consequently, if these opens be filled up by continuous plating, there will be less waste of material in the shallow than in the deep girder.

433. Greater proportion of a continuous web available for flange-strains in shallow than in deep girders.—That plate girders derive from the continuity of the web some increase of strength over that due to the sectional area of the flanges is certain (**100**), but the amount of horizontal strain which a thin web is capable of transmitting is, in large girders, generally too indefinite to admit of any considerable reduction in the area of the flanges on this account and is, therefore, practically of slight importance, for it seems unlikely that horizontal strains of compression can be transmitted with much energy through the thin continuous web of a deep girder, except in that portion which is close to the flange and therefore stiffened against buckling by its connexion therewith. In shallow plate girders, however, such as those used for the cross-girders of bridges, deck-beams of ships, fire-proof floors, &c., the web generally forms a large portion of the whole section, possesses considerable strength by itself, and is therefore available for horizontal as well as vertical strains. These considerations show that the flanges of a shallow plate girder derives a greater percentage of aid from the web than those of a deep girder.

434. Deflection of plate girders substantially the same as that of lattice girders.—From these considerations it would also appear that the deflection of plate girders is little, if at all, less than that of lattice girders, the length, depth and flange-area being the same in both; for if their flanges be subject to the same unit-strains, their deflections will be alike (**223**). Even assuming that the web does relieve the flanges of horizontal strain to the full extent which theory indicates, the deflection will not be very materially diminished thereby, for it appears from eq. 151 that a continuous web is for horizontal strain equivalent to only $\frac{1}{6}$th of its area placed in each flange. Plate girders, it is true, are generally thought to be stiffer than those with braced webs, and closely latticed girders than those with only one or two systems of triangulation, but I am not aware of accurate comparative experiments on this subject. It is quite possible that when the compression flange has but few points supported by intersecting braces it may assume under strain a slightly undulating line, and therefore be a little shorter than a similar flange held straight at short intervals by close latticing or a plate web; this would of course increase the deflection.

435. Webs of cast-iron girders often add materially to their strength.—The webs of cast-iron girders are usually made much stronger than is required for the mere transmission of the shearing-strain. Hence, they rarely require stiffening ribs, and the web should add to the strength of such girders, calculated merely from the leverage of either flange round the other as a fulcrum, by an amount nearly equal to the breaking weight of the web taken separately. Stiffening ribs are generally to be avoided in cast-iron girders, as they have been found to cause rupture in some instances from unequal contraction of the metal.

436. Minute theoretic accuracy undesirable.—In constructing wrought-iron girders of small span, say under 30 or 40 feet, it is generally more economical to make the lattice bars of one, or at most of two sizes throughout, even though they might be safely reduced in section as they approach the centre. This arises from the expense and trouble of having different templates and a stock of bars of various sizes. It is, therefore, cheaper to have a slight excess of material than go to the nicety of sizes which would be theoretically strong enough. For a similar reason $2\frac{1}{2}$ inches may be assumed to be the minimum useful width for a lattice bar of ordinary railway girders. When of less width it is generally necessary to swell out the rivet holes in the forge, so as to avoid reducing the effective section of the bar and, independently of the bad effect sometimes produced by heating the iron, this process is of course more expensive than cold punching. One result of all this is that the central bracing is generally stronger than theory requires.

437. Multiple and single systems of triangulation compared—Simplicity of design desirable—Ordinary sizes of iron.— This leads to another consideration, viz., the number of systems employed in bracing. It has been already stated in **153** that the practical advantage of a multiple over a single system of triangulation consists in the more frequent support given to the compression bars by those in tension, and by both to the flanges, thus subdividing the parts which are subject to compression into a number of short pillars and restraining them from deflection, chiefly in the plane of the girder. It may also be urged in favour of close latticing, that if an accident, such as an engine running off the line, occurs on a bridge with the braces few and far apart, that in such a case the safety of the whole structure is menaced by the fracture of a single bar, whereas a closely latticed or plate girder is not only freer from this danger, but affords greater security in case of one bar being originally defective, while to the public eye it has the semblance of greater safety, a consideration not altogether to be despised. The number of systems adopted will also depend on the distance between the cross-girders which generally occur at an apex, and on the practical consideration of what sized material is the most economical; and this again will depend on two things, the first cost of iron of small and large scantlings and the subsequent cost of workmanship, which latter item varies much according to the simplicity or complexity of the design. No definite rule can be laid down for all cases, but one consideration of importance should not be overlooked in seeking after apparent economy at the outset. The larger the scantlings and the more simple the method of construction, the smaller is the surface exposed to atmospheric influences and the more easily detected is any corrosion or decay. The chief advantage of masonry is its permanent character. No rust or decay in it requires constant attention or painting and, if well executed at the outset, masonry truly deserves the title of permanent.

It will be useful to recollect that bars or strips are not rolled wider than 9 inches; when a greater width than this is required narrow plates with shorn edges must be used. Plates exceeding 4 feet in width, or 15 feet in length, or containing more than 32 square feet, or weighing more than 4 cwt., are generally charged extra; also **T** or angle iron, the sum of whose sides exceeds 9 or 10 inches. Plates can be rolled up to 7 feet wide, or 30 feet long, or 60 square feet in area, but such sizes are very costly; they increase in thickness by sixteenths of an inch, and are generally called *sheet* iron when less than $\frac{3}{16}$ inch thick. Ordinary angle iron can be got in lengths of from 30 to 36 feet, and up to $6 \times 6 \times \frac{7}{8}$ inches.

438. Testing small girders by a central weight equal to half the uniform load is inaccurate.—Small girders are frequently tested by a central weight equal to half the uniform or passing load which they are expected to carry with safety. Though convenient, this is not altogether a fair trial of the web. Let $\mathsf{W} =$ the proof load in the centre, and $2\mathsf{W} =$ the uniform load. The web of a girder designed to support a central load, W, should be of uniform strength, for it sustains throughout a shearing-strain equal to $\frac{\mathsf{W}}{2}$(**34**). The web of a girder designed for a uniform load, $2\mathsf{W}$, should increase from the centre where the shearing-strain is nil, towards the ends where the strain $= \mathsf{W}$, in proportion to the distance from the centre (**46**); and the web of a girder designed to support a passing load of the same density as the uniform load should increase from the centre towards the ends, where the shearing-strain $= \mathsf{W}$, in the ratio of the square of the distance from the further end (**50**). Consequently, the strain in the centre of the web from a passing load $= \frac{\mathsf{W}}{4}$. It is obvious, therefore, that the web near the centre is subject to a much greater strain from a central load than from a uniform or passing load of twice its weight, whereas at the ends the reverse of this takes place. The importance of these remarks may be practically lessened by the considerations referred to in **436**.

439. Connexion between web and flanges—Uniform strain in flanges—Trough and ⊢⊣-shaped flanges—Rivets preferable to pins—Limit of length of single-webbed girders.—In wrought-iron girders the shearing area of the rivets connecting each brace with the flanges should equal the net section of the brace; otherwise there is a risk of its separating from the flanges at a much lower strain than would destroy the brace. If the web be a continuous plate, the shearing area of the connecting rivets should equal its theoretic horizontal section, *i.e.*, the horizontal net section of a plate whose thickness is that which theory demands; in practice, however, the plate area is generally considerably in excess of what theory requires and hence the rivet area seldom equals its horizontal net section. The trough-shaped section, such as that represented in Plate IV., is a favourite form for the flanges of tubular braced girders as it affords great facilities for attaching the bracing to the flanges. Objections have been raised to the trough with deep vertical plates on the ground that the unit-strain is not constant throughout its whole area, the unconnected edges of the vertical plates being subject to a severer unit-strain than the horizontal plates in consequence of each brace giving off its horizontal component of strain at a point which generally lies nearer the free edge of the vertical plate than the centre of gravity of the whole section. Let us confine our attention to the upper or compression flange, as similar reasoning applies to that in tension. This tendency to excessive local strain is sometimes supposed to show itself by a slight undulation or buckling of the free edge of the vertical plate endeavouring to escape from the line of thrust. This buckling, however, is not necessarily a sign of excessive local compression, but rather of defective stiffness in the lower part of the plate, for if it were stiffened laterally so that it could not escape from the line of thrust, and if the unit-strain along this edge were greater than that in the horizontal plates, the result would be that the whole flange would camber from the shortening of its lower edge. This, however, does not take place, and hence it is reasonable to suppose that the strain is not very unequally distributed throughout the whole section. Undulation certainly is a defect and proves that the plate is not standing up to its work, and therefore not subject to excessive compressive strain; it rather indicates that a small portion of the vertical plate at each apex on the side remote from the centre may be in tension, pulling, instead of thrusting, the flange towards the centre. Vertical plates ought therefore to be thick enough to resist buckling, say $\frac{1}{15}$th of their depth (**335**), or else be stiffened by an angle iron along their free edges. The weight of the trough itself, acting as a series of short girders between the apices, tends to produce local tension in the lower edges of the vertical plates, and so far counteracts excessive compressive strain, and the whole flange being held at short intervals by the bracing resembles a long thin pillar inside a tube; the pillar may undulate slightly and press here and there against the sides of the tube, but the compressive strain may for all practical purposes be considered as being distributed uniformly throughout the whole section of the pillar. The ⊢⊣ section of flange also has its advocates, who maintain that it is free from the objections alleged to lie against the trough section. The practical convenience of the latter, however, will probably enable it to hold its ground against its rival. The student who wishes to learn the views of eminent engineers on this subject is referred to the discussions on "The Charing Cross Bridge" and "Uniform Stress in Girder Work," in the 22nd and 24th Vols. of the *Proceedings of the Institution of Civil Engineers.* The main bracing is sometimes connected to the vertical plates by pins, like those of suspension bridges. Judging, however, from the experience gained at the Crumlin viaduct—where riveting was substituted for pins, after some years' wear and vibration had loosened the latter*—it seems generally desirable to make rigid connexions, and for this purpose riveting is at once the most convenient and effective method. Moreover, pins evidently do not form so firm a termination for a strut as riveting, a matter of great importance in long pillars (**311**). The braces should intersect somewhere in the vertical plate. In very faulty designs they are sometimes arranged so that they do not intersect each other in the flange, but would, if produced, meet considerably outside it, in which case the flange is subject to an injurious cross-strain and is liable to become broken-backed from the compression braces thrusting it upwards while the tension braces pull it down, or *vice versâ.* In some instances this has

* *The Engineer,* November, 1866, p. 384.

produced disastrous results. When the vertical plate is deep enough to give a choice of position, the apex may either be in the middle or rather closer to the upper edge, the latter position being perhaps the better of the two.

The length of single-webbed girders rarely exceeds 150 feet. Indeed, a double web seems desirable when the span exceeds 40 feet, as there can be no doubt that it contributes greatly to the stiffness of the flange plates to be bound by angle iron along both edges when their width exceeds 18 or 20 inches, and, regarding the whole flange as a long unsupported pillar, it is obvious that its resistance to lateral flexure is far greater when the angle irons are along the edges than when they are central.

CHAPTER XXII.

CROSS-BRACING.

440. Weather-bracing—Maximum force of wind—Pressure of wind may be considered as uniformly distributed for calculation.—Cross-bracing generally fulfils two functions; it acts as a horizontal web, holding the compression flanges at short intervals in the line of thrust and thus preserving them from lateral flexure to which all long pillars are liable; it also braces the whole structure in a horizontal plane, stiffening it against vibration and strengthening it to resist the side pressure of the wind just as the vertical web enables the main girders to sustain the downward pressure of the load. When the roadway is attached to the lower flange and the depth of the main-girders is not sufficient to admit of cross-bracing between the upper flanges, the latter must be made sufficiently wide to resist any tendency they may have to deflect sideways under longitudinal compression and their lateral stiffness may be calculated by the laws of pillars, though they are much aided by the internal bracing of latticed webs or the angle iron stiffening frames of plate webs, which convey a large share of rigidity from the roadway to the upper flanges. Under these circumstances the roadway and cross-bracing between the lower flanges have to resist the greater portion of the lateral pressure of the wind whose maximum force in this country may, for the purpose of calculation, be assumed equivalent to a uniform pressure of 25 lbs. per square foot of side surface exposed to its influence. The pressure of the wind is not always, as might be supposed, uniformly exerted along the whole length of a girder. With reference to the effect of violent gales on the Britannia Bridge, Mr. Clark remarks:—"The blow struck by the gale was not simultaneous throughout the length of the tube, but impinged locally and at unequal intervals on all parts of the length which presented a broadside to the gale."* A little further on he remarks:—"The tube, however, on no occasion attained any serious

* *The Tubular Bridges,* p. 455.

oscillation, but appeared, to some extent, permanently sustained in a state of lateral deflection, without time to oscillate in the opposite direction." Hence, the effect of wind may be assumed to be not very different from that of a uniformly distributed load; as a precautionary measure, however, it is desirable to make the central weather-bracing somewhat stronger than would be requisite if the pressure were really uniform.

441. Rouse's table of the velocity and force of wind—Beaufort scale.—The following table of the velocity and corresponding pressure of the wind by Mr. Rouse is given by Smeaton in the *Philosophical Transactions* for the year 1759:—

TABLE I.—ROUSE'S TABLE OF THE VELOCITY AND FORCE OF WIND.

Velocity of the Wind. Miles per hour.	Velocity of the Wind. Feet per second.	Perpendicular force on a square foot, in lbs. avoirdupois.	Common appellations of the Wind.
1	1·47	·005	Hardly perceptible.
2	2·93	·020	Just perceptible.
3	4·40	·044	
4	5·87	·079	Gentle pleasant gale.
5	7·33	·123	
10	14·67	·492	Pleasant brisk gale.
15	22·00	1·107	
20	29·34	1·968	Very brisk.
25	36·67	3·075	
30	44·01	4·429	High winds.
35	51·34	6·027	
40	58·68	7·873	Very high.
45	66·01	9·963	
50	73·35	12·300	A storm or tempest.
60	88·02	17·715	A great storm.
80	117·36	31·490	A hurricane.
100	146·70	49·200	A hurricane that tears up trees, and carries buildings before it, &c.

The following table contains the Beaufort scale which is used in the Navy to represent the force of the wind, but it conveys no information respecting its actual pressure or velocity and is therefore of little use for scientific purposes.

TABLE II.—BEAUFORT SCALE.

0.	Calm.			
1.	Light air, steerage way.			
2.	Light breeze, ship in full sail will go		1 to 2	knots.
3.	Gentle breeze,	do.	3 to 4	do.
4.	Moderate breeze,	do.	5 to 6	do.
5.	Fresh breeze, ship will carry royals.			
6.	Strong breeze, single reefed topsails and topgallant sails.			
7.	Moderate gale, double reefed topsails, jib, &c.			
8.	Fresh gale, triple reefed topsails, &c.			
9.	Strong gale, close reefed topsails and courses.			
10.	Whole gale, will scarcely bear close reefed main topsail and reefed foresail.			
11.	Storm, storm staysails only.			
12.	Hurricane, which no canvas could withstand.			

442. Cross-bracing must be counterbraced—Best form of cross-bracing—Initial strain advantageous.—As the wind may blow on either side of a bridge it is necessary to counterbrace the cross-bracing throughout; hence, the description of bracing described in Chap. VI., with transverse struts and diagonal ties, is well suited for cross-bracing and, in order to make it stiff and come into action before much lateral movement takes place, it is desirable to put a small initial strain on the diagonals. This will tend also to stiffen the whole structure against lateral vibration from loads in motion. The initial strain may be produced by coupling screws, cotters, or similar appliances. When the design does not admit of these the transverse struts may be first riveted in place, and then the diagonals may be riveted while they are temporarily expanded by heat; when cold the whole will be in a state of slight strain. The same effect may be produced in small tubes by laying them on their side so that the cross-bracing may be in a vertical plane; a few weights will then stretch one system of diagonals, and when thus strained the second series may be riveted in place; after the removal of the weights the required degree of initial strain will be produced if the operation has been carefully performed. The sagging of the horizontal tension bars of cross-bracing from their own weight will also aid in producing the required amount of stiffness, provided the bars be supported in a horizontal position while riveting up.

The absence of the initial strain alluded to was strongly marked in the Britannia Bridge, for Mr. Clark remarks:—"The effect of pressure against the side of the tube is very striking; a single person, by pushing against the tube, can bend them to an extent which is quite visible to the eye; and ten men, by acting in unison, and keeping time with the vibrations, can easily produce an oscillation of $1\frac{1}{4}$ inch, the tube making 67 double vibrations per minute."* A severe storm on the 14th of January, 1850, produced oscillations not exceeding one inch. This, however, was before the two tubes were connected together, side by side.

443. Strains produced in the flanges by cross-bracing—End pillars of girders with parallel flanges and bow of bowstring girders are subject to transverse strain.—When there are both upper and lower cross-bracings, each has to sustain one-half the pressure of the wind; consequently, in every gale the compression flange on the weather, and the tension flange on the lee side have their normal strains somewhat increased, while those in the other flanges are diminished to the same extent. This increase and diminution of strain are, however, generally insignificant compared to the strains produced by the load and are, of course, less in open-work girders than in those with solid sides which present a larger unbroken surface to the action of the wind.

When cross-bracing occurs between the upper flanges, the pressure of the wind against the upper half of the girder is transmitted to the abutments or piers through the end pillars which form the terminations of the web immediately over the points of support, at least so much of it as is not conveyed by the web stiffeners to the lower flanges and thence to the abutments.

* *The Tubular Bridges*, p. 717.

These pillars are, therefore, semi-girders as well as pillars, for they are subject not only to vertical compression from the shearing-strains in the main bracing, but to lateral pressures at top tending to overthrow them, which are nearly equal in amount to one-half the total pressure of the wind. Thus, if there be two main girders and four end pillars, each of the latter sustains a transverse pressure at top nearly equal to one-eighth of the pressure of the wind. It is, therefore, desirable to fix the lower ends of these pillars very securely by means of strong iron gussets attached to the masonry, or, if these be inadmissible from the longitudinal expansion of the bridge, to a cross road-girder, which may be made stronger and stiffer than usual for this purpose, so as to resist the racking action of the wind.

The bowstring girder, with roadway attached to the string, does not admit of cross-bracing between the bows throughout their entire length, but only near the centre where there is sufficient headway for carriages beneath. The ends of the bows are, consequently, subject to transverse strains similar to those just described in the case of the end pillars of girders with horizontal flanges.

CHAPTER XXIII.

CROSS-GIRDERS AND PLATFORM.

444. Maximum weight on cross-girders—Distance between cross-girders.—The cross-girders of railway bridges support the platform, ballast, sleepers and rails; and when the interval between them does not exceed that between two adjacent axles of a locomotive, say 6 or 7 feet, the greatest load which each cross-girder has to support is determined by the weight resting on one pair of driving-wheels, which rarely, if ever, exceeds 16 tons, or 8 tons per wheel. Consequently, if the effect of the rails, sleepers and platform in spreading the load over several girders be neglected, each cross-girder, however close they may be together, ought to be capable of sustaining 16 tons if the bridge be made for a single line, and twice this if made for a double line, in addition to the dead weight of platform, ballast and permanent way, and as a train of ordinary locomotives and tenders, that is, the load of maximum density, does not exceed $1\frac{1}{3}$ tons per running foot, it would obviously be the most economical arrangement to place the cross-girders, at all events, not closer together than the above stated distance of 6 or 7 feet.* It may, perhaps, be supposed that cross-girders placed at shorter distances need not be so strong in consequence of the rails, sleepers and platform distributing the load over several cross-girders, and this, no doubt, is to a certain extent correct, and numerous bridges have been constructed on this principle. Government Inspection is now, however, more critical

* The cross-girders of the Boyne Viaduct are 7 feet 5 inches apart, equal to the diagonal of the square formed by the lattice bars of the main-girders. The interval between those of the Britannia and Conway Tubular Bridges is 6 feet.

than formerly, and each cross-girder should be strong enough to sustain the load on the driving wheels of the heaviest engine which can come on the line, inasmuch as the sleepers may decay, joints may occur in the rails close to a cross-girder, or the platform may require renewal and perhaps be altogether removed for this purpose.

445. Rail-girders or keelsons—Economical distance between the cross-girders—Weight of single and double lines—Weight of snow.—When the cross-girders are farther than 3 feet apart (the distance between centres of sleepers) the rails may be supported by shallow longitudinal girders resting on the cross-girders or framed in between them, and in certain cases, especially when the levels permit the cross-girders to be of great depth, these rail-girders may be economically made of considerable length, with the cross-girders placed at long intervals apart, in some cases 20 feet asunder; but care must be taken not to strain the lattice bars of the main girders beyond their safe limit by bringing too great a local pressure on those which intersect at the ends of the cross-girders. The rail-girders may be conveniently made of plating or lattice work, similar in general design to the main girders of small bridges and framed in between the cross-girders. In some cases these rail-girders run above the cross-girders in unbroken lines from end to end of the bridge like the keelsons of a ship. This arrangement requires greater depth from soffit of bridge to rail than the former, and cannot therefore be so frequently adopted. Mr. Wm. Anderson has shown the great economy of placing the cross-girders 12 feet apart or upwards, especially with double line bridges, by means of the following data and estimate based thereon.*

Maximum weight of engine, - -	34 tons,
Maximum load on driving wheels, -	16 tons,
Wheel base, - - - - -	12 feet,
Depth of cross-girders, - - -	$\frac{1}{12}$th of span.

* *Trans. Inst. of C. E. of Ireland,* Vol. viii., 1866.

	Span.	Total load on girders.	Net area of bottom flange.	Weight of girders.	Weight per ft. run of bridge.
SINGLE LINE.	feet.	tons.	sq. in.	lbs.	lbs.
Cross-girders, 3 feet apart, -	14	17·26	6·3	1,206	402
Cross-girders, 12 feet apart, -	14	29·35	10·93	1,700	268·2
Longitudinal rail-girders, -	12	19·54	10·8	1,518	
DOUBLE LINE.					
Cross-girders, 3 feet apart, -	$25\frac{1}{2}$	35·00	11·4	3,654	1,218
Cross-girders, 12 feet apart, -	$25\frac{1}{2}$	58·64	19·2	4,704	645
Longitudinal rail-girders, -	12	38·64	21·6	3,026	

The permanent load of the roadway per running foot, including cross-girders 3 feet apart, sheeting, ballast, sleepers and rails for a single-line bridge, 14 feet wide between main girders (Irish gauge 5′ 3″), he estimates as follows:—

SINGLE LINE BRIDGE.	Weight in tons per running foot of bridge.
Cross-girders, 3 feet apart, - - -	·18
Sheeting of 4-inch planks and bolts for same,	·10
Rails, chairs, spikes, and sleepers (permanent way),	·06
Ballast (from 3 to 4 inches deep), - -	·20
	0·54 tons,

which is equivalent to a load of 86·4 lbs. per square foot of platform. This 0·54 ton is the permanent load of roadway for a single line per running foot, and is exclusive of main girders and cross-bracing, which vary with the span. The similar permanent load of roadway for a double line, $25\frac{1}{2}$ feet between main girders, is about 1·2 ton per running foot, or a little more than double that for a single line, which, however, may be reduced to about 1 ton by placing the cross-girders from 10 to 12 feet apart with rail-girders between.

In cold countries the weight of snow should not be left out of consideration. This has been estimated in America as high as 30 lbs. per square foot over the whole surface of the bridge.

In bridges of moderate span it is generally more economical to place the main-girders immediately beneath the rails; they then act as rail-girders and thus dispense with cross-girders. When, however, there is but little head-room beneath the rails, a modification of the trough girder may be adopted, such as that designed by Mr. Anderson for one of the bridges on the Dublin, Wicklow and Wexford Railway, and represented below.

Fig. 110.

Half Longitudinal Section and half Elevation of Bridge.

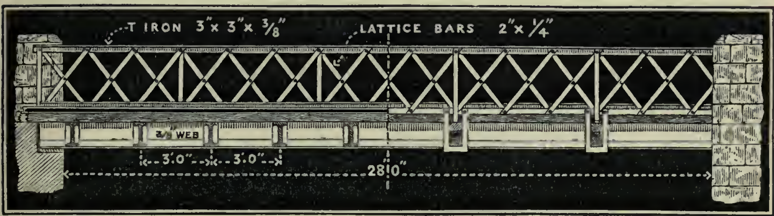
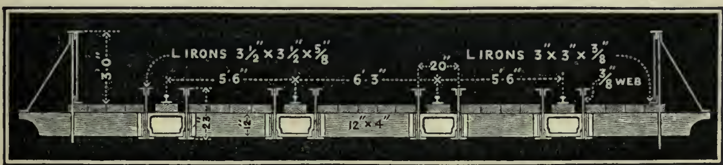


Fig. 111.

Cross Section of Bridge.



Each rail is carried between a pair of plate girders connected by short cast-iron saddles on which the sleeper and rail are laid and to which they can be securely bolted. The girders are thus accessible in every part for cleansing and painting without disturbing the permanent way, and at the same time no water can lodge in any part of the structure.*

446. Regulations of Board of Trade.—The following are the regulations of the Board of Trade respecting the cross-girders and platforms of railway bridges.

1. The heaviest engines in use on railways afford a measure of the greatest moving loads to which a bridge can be subjected.

* *Trans. Inst of C. E. of Ireland,* Vol. viii., p. 45.

This rule applies equally to the main and the transverse girders. The latter should be so proportioned as to carry the heaviest weights on the driving wheels of locomotive engines.

2. The upper surfaces of the wooden platforms of bridges and viaducts should be protected from fire.

3. No standing work should be nearer to the side of the widest carriage in use on the line than 2 feet 4 inches at any point between the level of 2 feet 6 inches above the rails and the level of the upper parts of the highest carriage doors. This applies to all arches, abutments, piers, supports, girders, tunnels, bridges, roofs, walls, posts, tanks, signals, fences and other works, and to all projections at the side of a railway constructed to any gauge.

4. The intervals between adjacent lines of rails, or between lines of rails and sidings, should not be less than 6 feet.

447. Roadways of public bridges—Buckled-plates.— The roadways of iron public bridges are generally formed in one of the four following ways.

1°. Brick arches spring between the lower flanges of the longitudinal or cross-girders as the case may be, and their haunches are levelled up with concrete, over which the pavement is laid. Sometimes a thin layer of tar asphalt is spread over the concrete to prevent surface water from percolating through the brickwork. The span of the arches, that is, the distance between the girders, may vary from 4 to 8 feet, and iron cross-ties are required at moderate intervals to bind the girders together and prevent them from spreading sideways under the thrust of the arches. The weight of a square foot of this roadway, exclusive of girders and cross-ties, may be estimated as follows:—

	lbs.		lbs.
Brickwork, $4\frac{1}{2}$ inches deep, - -	36	if 9 inches deep,	72
Concrete, averaging 4 inches deep, -	47	if 6 do.	70·5
Asphalt, $\frac{1}{2}$ inch deep, - - -	7	- - -	7
Pavement and sand, 9 inches deep, or 12 inches of broken stone, -	110	- - -	110
	200		259·5

2°. Arched wrought-iron flooring plates, $\frac{3}{8}$ to $\frac{1}{2}$ inch thick, are riveted to the upper flanges of the longitudinal girders and their haunches are levelled up with asphalt or concrete, over which the pavement or broken stone is laid as before. These arched plates also require cross-ties to prevent the outside girders from spreading, but the plates themselves may often be made to take an important share in the structure by strengthening the upper, or compression, flanges of the girders, and thus economizing material. The weight per square foot of this roadway, excluding cross-ties, may be estimated as follows:—

	lbs.		lbs.
Arched plates, - - - - - -	20	to	26
Asphalt, averaging 3 inches deep, - -	42	if 4 inches,	56
Pavement or broken stone as before, -	110	- - -	110
	172		192

3°. Flat cast-iron plates, $\frac{3}{4}$ to 1 inch thick with stiffening ribs on the upper surface, are bolted to the upper flanges of the longitudinal girders and then levelled up with asphalt to the top of the ribs, 3 or 4 inches deep, over which the pavement or broken stone is laid as before. The weight per square foot of this roadway is from 20 to 30 lbs. more than in the last case, but no cross-ties are required.

4°. Wrought-iron buckled-plates, $\frac{1}{4}$ to $\frac{5}{16}$th inch thick, are bolted or riveted to the upper flanges of the longitudinal girders and levelled up with concrete or asphalt, over which the broken stone or pavement is laid as before. Angle or tee iron covers are riveted to the cross joints of the plates and support them at frequent intervals like short cross-girders. The weight per square foot of this roadway, including the angle or tee iron covers, is closely the same as in case 2.

The following data respecting Mallet's buckled-plates are derived from the trade circular.

The resistance of square buckled-plates is directly as the thickness and inversely as the clear bearing. A buckled-plate, bolted or riveted down all round, gives double the resistance of the same plate merely supported all round, and if two opposite sides be wholly unsupported, its resistance is reduced in the ratio of 8 to 5. Within the limit of "safe load" the resistance is nearly the same, whether it be upon the crown or uniformly diffused. The stiffness at any point of the plate, as against unequal loading, is as the square of the thickness nearly, and inversely as the curvature. The curvature (unless for special object) should never exceed that which will just prevent the "crippling load" bringing the plate down flat, by compression of the material; less than 2 inches versed-sine of curvature has been found sufficient for $\frac{1}{4}$ inch buckled-plates 4 feet square. A 3 foot square buckled-plate, of ordinary Staffordshire iron $\frac{1}{4}$ in. thick, 2 in. width of fillet, $1\frac{3}{4}$ in. curvature, supported only all round, requires upwards of nine tons diffused over about half the superficies at the crown to cripple it down, and double this, or eighteen tons to cripple it, if firmly bolted or riveted down to rigid framing all round. A similar plate of soft puddled steel has been found to bear nearly double the preceding, or thirty-five tons to the square yard. Mr. Thomas Page, C.E., has proved the buckled-plates of the floor of Westminster new bridge—each averaging 7 feet by 3 feet, $\frac{1}{4}$ inch thick, and $3\frac{1}{2}$ inch curvature—by lowering upon the crown of each a block of granite of seventeen tons weight, which they sustained without injury. In structures exposed to impulsive loads, such as railway or other bridge flooring, one-sixth of these "crippling loads" should not be exceeded for the safe load, nor one-fourth for quiescent loading. The size of buckled-plates formed of one single rolled plate is only limited by the breadth, to which sheet or plate iron can be rolled, at market prices; and the sizes that have been found most advantageous for the majority of purposes are plates of 3 feet and of 4 feet square, or of those widths by the full length of the sheet. Square plates of either of the two ordinary market sizes are always to be preferred, on the ground of economy in prime cost, and in application, and facility in being obtained promptly from the makers. Square plates produce a stronger floor, with a given weight of iron, than any rectangular plate; the resistance of the latter being that nearly of a square plate, whose side is equal to the longer dimension. If rectangular plates be used the longer edge should not be much more in length than twice the shorter. Economy is always consulted by supporting each plate all round—one pair of opposite fillets resting on the girders or joists of the structure, and the joints of the cross fillets supported by an angle iron above, thus forming a lap plate.

TABLE OF STRENGTH, WEIGHT, AND COST OF BUCKLED-PLATES.

No.	Thickness of Plate.		Weight per square yard of Buckled-Plate, excluding the angle iron at the cross-joints.	Weight of an equal surface (1 square yard) of Corrugated Plate of corresponding thickness.	Safe passive load, uniformly diffused per square yard, for three feet square Buckled-Plates.	Safe impulsive load, uniformly diffused per square yard, for three feet square Buckled-Plates.	Cost per superficial yard of Buckled-Plate, at £13 per ton.		Nearest number of square yards in one ton of Buckled-Plates.
	B. W. G.	inch.	lbs.	lbs.	tons.	tons.	*s.*	*d.*	sq. yards.
1	No. 18 =	·048	17·3	20·7	0·27	0·20	2	2	129
2	No. 16 =	·066	23·6	28·3	0·43	0·32	2	10	95
3	No. 12 =	·107	38·7	46·4	0·64	0·48	4	7	57
4	"	1-8	45·0	54·0	1·0	0·75	5	3	49
5	"	3-16	67·5	81·0	2·5	1·7	7	11	33
6	"	1-4	90·0	108·0	4·5	3·0	10	6	24
7	"	5-16	112·5	135·0	6·2	4·7	13	2	20
8	"	3-8	135·0	162·0	9·0	6·8	15	8	16

NOTE.—The safe loads in columns 5 and 6 may be taken at double for buckled-plates of puddled steel.

Nos. 1, 2, and 3—Applicable to roofing, iron house building, and fireproofing, flooring, &c.

Nos. 4 and 5—For the lighter class of bridge and other floors.

Nos. 6 and 7—For the heavier floors of railway and other bridges, and viaducts: No. 6 is that adopted for the new bridge at Westminster, London: No. 7 for bridges in India.

No. 8—Has not hitherto been found necessary in any structures, however heavy.

The working loads on public bridges are given in Chapter XXVIII.

CHAPTER XXIV.

COUNTERBRACING.

448. Permanent or dead load—Passing or live load.— The strains in the web of a braced girder are constant both in amount and kind so long as the load remains stationary. If, however, the load changes its position the strain will alter in amount, and perhaps in kind also, and it is to meet this latter change in the character of the strain that counterbracing is required. Now, a certain portion of the load which every girder sustains is fixed and consists of what I have elsewhere called the "permanent load," or "dead load," including in this term the weight of the whole superstructure, viz., the main girders, cross-girders, cross-bracing, platform, rails, sleepers and ballast. This permanent load produces definite strains in the bracing which remain constant, both in amount and kind, until a further load comes upon the bridge. Let us consider the effect of a moving or "live" load of uniform density, say a train of carriages, traversing a girder with horizontal flanges, and we may chiefly confine our attention to the strains developed in the bracing at either end of the train, as it has been shown in **51** and **170**, that the maximum strains in the bracing from train-loads occur at these points. As the advancing train approaches the centre of the girder the normal strains in the bracing between the centre and the front of the train are diminished, or even reversed, by the passing load. In the latter case each brace attains its maximum reverse strain as the front of the train passes it and counterbracing must be provided accordingly. During the same period, *i.e.*, while the train advances towards the centre, the permanent strains in the second half-girder are receiving gradual increments of their own kind, but each brace in this half does not attain its state of maximum strain until the train has crossed the centre and is so far advanced that its front is passing that particular brace, after which the strain again diminishes till the other end of the train is passing, when the strain is either at its minimum, or, if altered, attains its maximum of the reverse kind to that produced by the permanent load, in which case therefore the brace requires counterbracing.

449. Passing loads require the centre of the web to be counterbraced—Large girders require less counterbracing in proportion to their size than small ones.—The permanent load is usually disposed symmetrically on either side of the centre; consequently, the normal strains in the bracing near the centre are less in amount than in other parts, and it is in the central braces alone that strains of a reverse character are produced by a moving load, requiring counterbracing for some distance on either side of the centre. It is evident that the heavier the permanent load is, the less will be the amount of counterbracing required for a given passing load. It has been already shown in **50** that the shearing-strain (to which the strain in the bracing is proportional) at the end of a passing train $= \frac{w'n^2}{2l}$, where

w' = the passing load per linear unit,
l = the length of the girder,
n = the length covered by the advancing load.

But the shearing-strain at the same point from the permanent load

$$= w\left(\frac{l}{2} - n\right)$$

where w = the permanent load per linear unit, and n and l are as before, n being supposed less than $\frac{l}{2}$. Now, if n be proportional to l in girders of different lengths, the shearing-strain from the passing load will vary as $w'l$, and that from the permanent load as wl; and, since w increases in large girders as some high power of the length, while w' may be considered constant for girders of all sizes, the shearing-strain due to the permanent load will bear a considerably greater ratio to that from the passing load in long than in short girders. Consequently, the proportion which the counterbracing bears to the whole amount of material diminishes rapidly with the span of the girder. The counterbracing terminates where the two shearing-strains are equal, and the point where this occurs may be determined by equating them to each other and solving the resulting equation for n as follows:—

$$\frac{w'n^2}{2l} = w\left(\frac{l}{2} - n\right)$$

Arranging according to powers of n,

$$w'n^2 + 2wln - wl^2 = 0$$

solving for n,

$$n = l\left(\frac{-w \pm \sqrt{w^2 + ww'}}{w'}\right) \tag{250}$$

If, for example, $w = w'$,

$$n = l\,(-1 \pm \sqrt{2}) = \cdot 414l$$

450. Counterbracing of vertical and diagonal bracing—Large bowstring girders require little counterbracing.— Girders with vertical and diagonal bracing, such as that investigated in Chapter VI., may be counterbraced either by making the bracing near the centre capable of acting indifferently as struts and ties, or by adding a second system of diagonals crossing the first. If this counterbracing be carried throughout the whole length of the girder (as in cross-bracing), it is possible by tightening it up to produce an initial strain in the bracing proper, in which case the effect of a load will be to diminish the strain in the counterbracing, which, however, will relapse into its former state of strain as soon as the load is removed (**442**).

I cannot close these observations on counterbracing without drawing attention to one important merit which bowstring girders possess. When the load is uniformly distributed the strains in the bracing are tensile, for the lower flange and load are merely suspended from the bow, which differs but slightly from the curve of equal horizontal thrust and therefore requires but little bracing to keep it in form. Hence, compressive strains are produced in the bracing only under the influence of passing loads; and in large girders, where the permanent load of string and roadway is great compared with the passing load, it may happen that the compressive strains produced by the latter do not exceed the tensile strains which the bracing sustains in its normal state. If, for instance, the permanent load of the lower flange and roadway in the example worked out in **208** were twice as heavy as the passing load, the strains in all the diagonals would be tensile under all circumstances; even if the permanent load were only once and a half as heavy as the passing load, diagonal 6 alone would sustain slight compression. In this case the difficulty of providing against flexure in long compression bars does not arise, and the only part of the structure subject to compression is the bow, which from its large sectional area can be economically constructed of a form suited to resist buckling or flexure.

CHAPTER XXV.

DEFLECTION AND CAMBER.

451. Deflection curve of girders with horizontal flanges of uniform strength is circular.—It has been already shown in Chap. VIII. that the deflection curve of girders with horizontal flanges of uniform strength, that is, girders whose flanges vary in sectional area so that they are subject to the same unit-strain throughout the whole length of each flange respectively, is circular and easily calculated by a simple formula (eq. 132). When, however, the flanges are of uniform section throughout their whole length, and their strength therefore excessive near the ends, the deflection will be somewhat less, and may be calculated by the method explained in **226** and the following articles. When the strength of a girder is not uniform, there is of course a certain waste of material, which, however, cannot always be avoided, although some methods of construction—the cellular flanges of tubular bridges for instance—are more liable to this objection than others, as they cannot in practice be tapered off towards the ends in accordance with theory.

452. Deflection an incorrect measure of strength.—Since the deflection depends not only on the unit-strains in the flanges, but also on the proportion of length to depth, on the coefficient of elasticity of the material, and to some extent on the mode of construction, the popular rule by which the strength is estimated from the deflection alone, though possessing the merit of simplicity, is extremely vague and liable to lead to false conclusions unless when comparing girders of the same length, depth, and material. The deflection of any particular girder, however, is sensibly proportional to the load, provided the strains are within the elastic limit, which they always are in safe practice.

453. Camber ornamental rather than useful—Permanent set after construction.—As the amount of deflection is in practice very small compared with the length of a girder, no appreciable diminution of strength is produced merely by the change from a horizontal line to the deflection curve, for deflection, unless so excessive as to change the vertical reaction of the abutments into an oblique one, is the result, not the cause, of increased strain. A downward curve, or even a truly horizontal line is, however, less pleasing to the eye than a slight camber; hence, it is desirable to give an initial camber somewhat in excess of the calculated deflection, so that when the girder is loaded no perceptible sag may suggest the idea of weakness, even though imaginary. It should also be borne in mind that the various parts of a built girder are put together free from strain and are frequently a little out of line; consequently, when a large girder first supports its own weight, and again, but in a less degree, when it is tested with a heavy load for the first time, there is a certain slight motion from the closing up or stretching out of the various parts accommodating themselves to their new state. A permanent set is the result, which, however, is not necessarily indicative of weakness, provided it is not increased by subsequent loads, which should only produce a temporary deflection. This congenital set sometimes nearly doubles the calculated deflection.

454. Loads in rapid motion produce greater deflection than stationary or slow loads—Less perceptible in large than small bridges—Deflection increased by road being out of order—Railway bridges under 40 feet span require extra strength in consequence of the velocity of trains.—The Commissioners appointed to inquire into the application of iron to railway structures "carried on a series of experiments to compare the mechanical effect produced by weights passing with more or less velocity over bridges, with their effect when placed at rest upon them. For this purpose, amongst other methods, an apparatus was constructed, by means of which a car loaded at pleasure with various weights was allowed to run down an inclined plane; the iron bars which were the subject of the experiment were fixed horizontally at the bottom of the plane, in such a manner that the loaded car would pass over them with the velocity acquired in its descent. Thus the effects of giving different velocities to the loaded car, in depressing or fracturing the bars, could be observed and compared with the effects of the same loads placed at rest upon the bar. This apparatus was on a sufficiently large scale to give a practical value to the results; the upper end of the inclined plane was nearly 40 feet above the horizontal portion, and a pair of rails, 3 feet asunder, were laid along its whole length for the guidance of the car, which was capable of being loaded to about 2 tons; the trial bars, 9 feet in length, were laid in continuation of this railway at the horizontal part, and the inclined and horizontal portions of the railway were connected by a gentle curve. Contrivances were adapted to the trial bars, by means of which the deflections produced by the passage of the loaded car were registered; the velocity given to the car was also measured, but that velocity was, of course, limited by the height of the plane, and the greatest that could be obtained was 43 feet per second, or about 30 miles an hour. A great number of experiments were tried with this apparatus, for the purpose of comparing the effects of different loads and velocities upon bars of various dimensions, and the general result obtained was that the deflection produced by a load passing along the bar was greater than that which was produced by placing the same load at rest upon the middle of the bar, and that this deflection was increased when the velocity was increased. Thus, for example, when the carriage loaded to 1,120 lbs. was placed at rest upon a pair of cast-iron bars, 9 feet long, 4 inches broad, and $1\frac{1}{2}$ inch deep, it produced a deflection of $\frac{6}{10}$ths of an inch; but when the carriage was caused to pass over the bars at the rate of 10 miles an hour, the deflection was increased to $\frac{8}{10}$ths, and went on increasing as the velocity was increased, so that at 30 miles per hour the deflection became $1\frac{1}{2}$ inch; that is more than double the statical deflection. Since the velocity so greatly increases the effect of a given load in deflecting the bars, it follows that a much less load will break the bar when it passes over it than when it is placed at rest upon it, and accordingly, in the example above selected, a weight of 4,150 lbs. is required to break the bars if applied at rest upon their centres; but a weight of 1,778 lbs. is sufficient to produce fracture if passed over them at the rate of 30 miles an hour. It also appeared that when motion was given to the load, the points of greatest deflection, and, still more, of the greatest strains, did not remain in the centre of the bars, but were removed nearer to the remote extremity of the bar. The bars, when broken by a travelling load, were always fractured at points beyond their centres, and often broken into four or five pieces, thus indicating the great and unusual strains they had been subjected to."* These experiments show that a load in rapid motion causes greater deflection than the same load at rest or moving slowly, especially when the moving load is very large compared with the dead weight of the girder. The increase, however, is generally slight in railway practice, and the greater the weight of the structure is to that of the passing train the less will be the increment of deflection due to rapid motion. The difference of deflection caused by a locomotive crossing the central span of the Boyne Viaduct, 264 feet in the clear between supports, at a very slow speed and at 50 miles an hour was scarcely perceptible, and did not exceed the width of a very fine pencil stroke, but the increase of deflection is more marked in bridges of small span, as appears from the following experiments made on the Godstone Bridge, South Eastern Railway, by the Commissioners appointed to inquire into the application of iron to railway structures.† The Godstone is a cast-iron girder bridge, 30 feet in span, with two lines of railway.

	Tons.
Weight of two girders, - - - - -	15
Weight of platform between these girders, - -	10
Weight of half the bridge, *i.e.*, dead load, -	25
Weight of engine, - - - - -	21
Weight of tender, - - - - -	12
Moving load, - - - -	33

* *Iron Com. Report*, p. xi. † *Idem*, *App.*, p. 250.

Velocity in feet per second.	Deflection in decimals of an inch.
0,	·19
22 = 15 miles per hour, - - -	·23
40 = 27·3 do. do. - - -	·22
73 = 49·8 do. do. - - -	·25

Similar results were obtained from the Ewell Bridge, upon the Croydon and Epsom Line. The span of the Ewell Bridge is 48 feet, the dead weight of one-half is 30 tons, and the statical deflection due to an engine and tender, weighing 39 tons, was rather more than one-fifth of an inch. "This was slightly but decidedly increased when the engine was made to pass over the bridge, and at a velocity of about 50 miles per hour an increase of one-seventh was observed. As it is known that the strain upon a girder is nearly proportional to the deflection, it must be inferred that in this case the velocity of the load enabled it to exercise the same pressure as if it had been increased by one-seventh, and placed at rest upon the centre of the bridge. The weight of the engine and tender was 39 tons, and the velocity enabled it to exercise a pressure upon the girder equal to a weight of about 45 tons."*

The fact of slightly increased deflection from rapidly moving loads is also confirmed by Mr. Hawkshaw's experiments with an engine and tender run at a speed of about 25 miles an hour over five compound iron girder bridges on the Wakefield and Goole Railway. These girders varied in span from 55 feet 7 inches to 88 feet 6 inches, and were therefore less affected by rapid loads than the smaller bridges just described. Mr. Hawkshaw inferred that "where the road is in good order the deflection is not much increased by speed, but that where the road is out of order, then there is an increase of deflection." For instance, the road immediately leading on to one of the bridges in question "was considerably depressed in level, so that in running the train over the bridge at speed the whole weight of the train had to be

* *Iron Com. Report*, p. xiv.

suddenly lifted, and this of course had to be sustained by the girders as well as the ordinary weight of the train."*

The conclusions of the Commissioners, as given at p. xviii. of their report, is as follows:—"That as it has appeared that the effect of velocity communicated to a load is to increase the deflection that it would produce if set at rest upon the bridge; also that the dynamical increase in bridges of less than 40 feet in length is of sufficient importance to demand attention, and may even for lengths of 20 feet become more than one-half of the statical deflection at high velocities, but can be diminished by increasing the stiffness of the bridge; it is advisable that, for short bridges especially, the increased deflection should be calculated from the greatest load and highest velocity to which the bridge may be liable; and that a weight which would statically produce the same deflection should in estimating the strength of the structure, be considered as the greatest load to which the bridge is subject."

455. Effect of centrifugal force.—Centrifugal force produces a very slight but appreciable increase of pressure when the load passes rapidly across girders which, though ordinarily level, become deflected by the load, and still more so if they happen to have been built originally hollow in place of being level or cambered. The increased pressure due to this cause is expressed by the following well known equation:—

$$\mathsf{P} = \frac{v^2\mathsf{W}}{g\mathsf{R}} \tag{251}$$

Where P = the pressure due to centrifugal force,
R = the radius of curvature in feet,
W = the load,
v = the velocity in feet per second,
g = the acceleration due to gravity = 32 feet per second.

Ex. 1. A girder bridge 200 feet in span is deflected 0·25 foot below the horizontal line by a certain load, W, at rest; what is the increased pressure due to centrifugal force if W traverses the bridge at the rate of 60 miles an hour?

$$\text{Here,} \quad v = \frac{60 \times 5280}{60 \times 60} = 88 \text{ feet per second.}$$

$$\mathsf{R} = \frac{100 \times 100}{0{\cdot}5} = 20{,}000 \text{ feet.}$$

* *Iron Com. Report, App.*, p. 412.

$$\textit{Answer,} \quad P = \frac{W \times 88 \times 88}{32 \times 20{,}000} = \cdot 0121 W.$$

Ex. 2. If the span were only 100 feet, and the deflection and velocity as before, we would have $R = 5{,}000$ feet, or $\frac{1}{4}$th of its former value, whence,

$$\textit{Answer,} \quad P = \cdot 0484 W = \frac{W}{20} \text{ nearly.}$$

456. Practical methods of producing camber and measuring deflection.—The deflection of a girder supported at both ends is the result of the lower flange being extended while the upper one is shortened, and camber may be produced by the reverse of this, that is, by making the bays of the upper flange slightly longer than those of the lower one when the girder is in process of construction (**223**).

When small girders are under proof, their deflection may be conveniently measured, unless there happens to be a strong wind, by means of a fine wire fastened to one end of the girder and passing over a pulley attached to the other end, where a small weight will keep it in a state of constant tension. The deflections should be read on a scale attached to the girder itself; when measured from an object fixed outside the girder they cannot be depended on, owing to the supports on which the ends of the girder rest being compressed by the weight of the testing load. When great accuracy is not required the deflection of a girder bridge from passing loads may be measured by means of two wooden rods, the bottom of one of which rests on the surface of the ground beneath the bridge, while the top of the second rod is pressed upwards against the soffit of the girder, so that they overlap each other midway; a pencil line is then ruled across both rods, and when the upper one is depressed by a passing load its line will descend slightly, the distance between the two lines giving the deflection of the girder.

CHAPTER XXVI.

DEPTH OF GIRDERS AND ARCHES.

457. Depth of girders generally varies from one-eighth to one-sixteenth of the span—Depth determined by practical considerations.—The depth of large girders, with the exception of triangular trusses, seldom exceeds 1-8th, or is less than 1-16th of the span. For many years the common rule for cast-iron girders was to make the depth 1-15th of the span and this established a precedent for wrought-iron girders, but modern practice has with great advantage increased the ratio, so that from 1-8th to 1-12th are now common proportions for braced girders. As the leverage of the flange is directly as the depth, while the quantity of material in the web is theoretically independent of it, it might be inferred that the deeper the girder the greater the economy (**274**). The practical limit, however, is defined by the extra material required to stiffen long compression bars or thin deep plate webs, nor should we overlook the necessity of having sufficient thickness in the web for durability and sufficient material in the compression flange to keep it from flexure or buckling. The following table contains the principal dimensions of some important Bowstring bridges, which are generally made deeper than girders with horizontal flanges.

TABLE I.—PRINCIPAL DIMENSIONS OF BOWSTRING GIRDER BRIDGES.

No.	Name of Bridge.	Span.	Depth.	Ratio of span to depth.	Construction of Bow.	Construction of String.	Date.	Engineer.
		feet.	feet.					
1.	Royal Albert Bridge at Saltash, Cornwall Railway (Bow and Invert, or Double-Bow),	455	56·25	8·1	An oval tube of plate iron; horizontal axis 16·75 ft., vertical axis 12·25 ft., stiffened by annular diaphragms and longitudinal ribs.	The Invert is formed of flat bar links; the longitudinal main girders are single-webbed and double flanged, 8 ft. deep and 2 ft. wide.	1859.	Brunel.
2.	Bridge over the Thames at Windsor, Great Western Railway,	187	25	7·5	Triangular tube of plate iron, 3·5 ft. wide and 3 ft. deep, with the base above and a vertical plate through the middle.	Double-flanged and single-webbed main girders, 5·9 ft. deep and 2·5 ft. wide.	1849.	Brunel.
3.	Bridge over the Shannon, Midland Great Western Railway, Ireland,	165	20·17	8·2	I-shaped, 2·83 ft. deep and 2 ft. wide.	Single-webbed, $\perp$-shaped main girders, 2·6 ft. deep and 1·5 ft. wide.	1850.	Hemans.
4.	Commercial Road Bridge, Blackwall Extension Railway,	130	8·66	15	Rectangular tube, 1·5 ft. square, the upper plate projecting 8 inches beyond each side.	Formed of flat bar links.	—	Locke.

1. 2. 3. Humber *on Iron Bridge Construction.*
4. Weale's *Supplement to Bridges.*

458. Economical proportion of web to flange—Practical rules.—When a given quantity of material is to be distributed in the most advantageous manner, the thinner the web and the more the material is concentrated in the flanges, the stronger will the girder be, provided the web retains sufficient material for transmitting the shearing-strain; but when, as is frequently the case in small girders, the girder derives a considerable portion of its strength from the web acting as an independent rectangular girder, its thickness being determined from practical considerations, there is a certain depth, depending on the thickness of the web and the relation between the flanges, which will produce a girder of maximum strength. If the flanges are of equal area this depth may be found as follows:—

Let l = the length of the girder,
b = the thickness of the web, as determined by practical considerations,
d = the depth of the girder,
a = the area of either flange,
$a' = bd$ = the area of the web,
$\mathsf{A} = 2a + a'$ = the total sectional area, which is a given quantity.

From equation 71, we have for the weight which an equal-flanged semi-girder loaded at the end will support,

$$\mathsf{W} = \frac{fd}{l}\left(a + \frac{a'}{6}\right)$$

in which f is the unit-strain in either flange. W is maximum when $d\left(a + \frac{a'}{6}\right)$ is maximum, and in order to find what value of d will produce this result we must equate the differential coefficient of $d\left(a + \frac{a'}{6}\right)$ to cipher, first substituting for a and a' their values in terms of d and the constant A, as follows:—

$$\mathsf{W} = \frac{f}{l}\left(\frac{\mathsf{A}}{2}d - \frac{bd^2}{3}\right)$$

Equating the differential coefficient of the term within the bracket to cipher, we have,

$$\frac{\mathsf{A}}{2} - \frac{2}{3}\,bd = 0$$

whence,

$$bd = \frac{3}{4}\,\mathsf{A} \qquad (252)$$

The depth therefore should be such that the web may contain $\frac{3}{4}$ths of the whole amount of material.

The thickness of the webs of wrought-iron plate girders for railway or public bridges should not be less than $\frac{5}{16}$ inch (**431**), while those of cast-iron girders generally vary from 1 to 2 inches. The following rule for the *minimum* thickness of cast-iron webs is given by M. Guettier, a skilful French founder.*

Length of girder.	Minimum thickness of cast-iron Webs.
4 metres, - - -	20 millimetres = 0·8 inches.
5 ,, - - -	25 ,, = 1·0 ,,
6 ,, - - -	30 ,, = 1·2 ,,
8 ,, - - -	35 ,, = 1·4 ,,

Stiffening ribs are sometimes formed at right angles to the webs of cast-iron girders, so as to act as brackets to the flanges, but they are apt to shrink unequally in cooling and produce dangerous cracks in the casting.

459. Depth of iron and stone arches.—The two following tables contain the principal dimensions of some important iron and stone arched bridges. See also the tables relating to cast-iron arches and wrought-iron roofs in Chap. XXVIII.

* Morin, *Résistance des Matériaux*, p. 277.

TABLE II.—PRINCIPAL DIMENSIONS OF SOME IMPORTANT CAST AND WROUGHT-IRON ARCHED BRIDGES.

No.	Name of bridge.	No. of arches.	Width between parapets.	Form of arch.	Material.	Dimensions of centre arch. Span.	Rise or Versine of arch.	Ratio of span to rise.	Form of ribs.	Depth of rib at crown.	Distance between ribs.	Thickness of piers.	Date.	Engineer.
			feet.			feet.	feet.			feet.	feet.	feet.		
1	Southwark Bridge, over the Thames,	3	42	Circular,	Cast-iron,	240	24	10	8 ribs of I section, $3\frac{1}{2}$ inches thick.	6·0	—	24	1819	Rennie.
2	Sunderland Bridge, over the Wear,	1	32	Do.	Do.	236	34	6·9	6 open-work ribs.	5·0	6	—	1796	Wilson.
3	Severn Valley Railway Bridge, Coalbrookdale Railway,	1	25	Do.	Do.	200	20	10	4 ribs of I section, 2 inches thick.	4·0	6	—	—	Fowler.
4	Staines Bridge, over the Thames,	1	—	Do.	Do.	180	16	11·25	Open-work ribs.	—	—	—	1802	Wilson.
5	Victoria Railway Bridge, over the Thames,	4	30·75	Do.	Wrought-iron,	175	17·5	10	6 ribs of I section, arranged in pairs.	3·5	12·5	12·33	1860	Fowler.
6	Tewkesbury Bridge, over the Severn,	1	24	Do.	Cast-iron,	170	17	10	6 open-work ribs.	3·0	—	—	1826	Telford.
7	Rochester Bridge, over the Medway,	3	40	Do.	Do.	170	17	10	8 ribs of I section, 2 inches thick.	4·5	5·87	16·0	—	Cubitt.
8	Craigellachie Bridge, over the Spey,	1	15	Do.	Do.	150	20	7·5	4 open-work ribs.	—	—	—	—	Telford.
9	Westminster Bridge, over the Thames,	7	84·17	Curve parallel to an ellipse,	Cast and wrought-iron,	120	20	6	15 ribs of I section, cast-iron near the springing, wrought-iron at centre of arch.	2·33	5·2	10·5	—	Page.
10	Railway Bridge over the Thames at Richmond,	3	25	Circular,	Cast-iron,	100	12	8·3	Ribs of I section, 2 inches thick.	2·75	8·0	9·0	—	Locke.
11	Lary Bridge, near Plymouth, -	5	24	Elliptical,	Do.	100	14·5	6·9	5 ribs of I section, 2 inches thick.	2·0	6·0	10·0	—	Rendel.
12	Railway Bridge over the Trent, Midland Counties Railway,	3	27	Circular,	Do.	100	10	10	6 ribs of I section.	2·5	—	10·0	—	Vignoles.

1. 2. 4. 8. *Encyc. Brit.*, 8th Ed., art. *Iron Bridges.*
3. 5. *Proc. Inst. C. E.*, Vol. xxvii.
6. 11. *Trans. Inst. C. E.*, Vols. i. and ii.
7. 9. Humber *on Iron Bridge construction.*
10 *Supplement to Bridges*, Weale.
12. Brees' *Railway Practice.*

TABLE III.—PRINCIPAL DIMENSIONS OF SOME IMPORTANT STONE BRIDGES.

No.	Name of bridge.	No. of arches.	Width of bridge between parapets.	Form of centre arch.	Material.	Dimensions of centre arch. Span.	Rise or Versine of arch.	Ratio of span to rise.	Depth of key-stone.	Radius of curvature at crown.	Thickness of the piers.	Date.	Engineer.
			feet.			feet.	feet.		feet.	feet.	feet.		
1	Bridge over the Adda, at Trezzo,	1	—	Circular,	Granite,	251	87·8	2·9	4·0	134	—	14th century.	—
2	Bridge over the Dee, at Chester,	1	33·0	Do.	Sandstone,	200	42·00	4·8	4·0	140	—	1833	Hartley.
3	Vielle Brionde, over the Allier,	1	16·0	Do.	Tufa,	183·25	70·25	2·6	5·25	94·87	—	1454	Grenier & Estone
4	Victoria Bridge, over the Wear. Durham Junction Railway,	4	—	Do.	Pensher sandstone,	160	72	2·2	—	80·5	21·5	—	—
5	London Bridge, - -	5	53·5	Ellipse,	Granite,	152	37·5	4·0	4·75	154	24·0	1831	Rennie.
6	Claix, over the Drac, at Grenoble,	1	—	Circular,	Stone,	150	54	2·8	3·08	79	—	1611	—
7	Gloucester Bridge, over the Severn,	1	35·0	Ellipse,	Sandstone,	150	35	4·3	4·5	160·7	—	1827	Telford.
8	Bridge over the Dora Riparia near Turin,	1	40·0	Circular,	Granite,	147·64	18·045	8·1	4·92	160	—	—	Mosca.
9	Pont-y-tu-Pridd, over the Taff,	1	11·0	Do.	Grey sandstone in Aberthaw lime,	140	35	4	1·5	87·5	—	1750	Edwards.
10	Railway bridge at Maidenhead, over the Thames,	2	28·0	Ellipse slightly pointed,	Brick in cement,	128	24·25	5·2	5·25	169	28·0	1835	Brunel.
11	Neuilly Bridge, over the Seine,	5	48·0	False ellipse,	Saillancourt stone,	127·83	31·83	4	5·25	128	13·83	1774	Perronet.
12	Waterloo Bridge, over the Thames,	9	41·5	Ellipse,	Granite,	120	32	3·8	5·00	112·5	—	1816	Rennie.
13	Old Blackfriars Bridge, over the Thames,	9	42·0	False ellipse,	Portland stone,	100	41·5	2·4	6·58	56	20·0	1771	Mylne.

14	Bridge of the Holy Trinity, over the Arno at Florence,	3	33·75	Slightly pointed,	White marble,	95·25	14·83	6·4	2·75	172·63	26·0	1569	B. Ammannati.
15	Bridge of Jena, over the Seine.	5	42·5	Circular,	Stone,	91·75	10·75	8·5	5·0	103	10·0	1807	Lamande.
16	Bridge over the Lea Cut, London and Blackwall Railway,	1	24·0	Do.	Brick in cement,	87·0	16·0	5·4	4·12	67	—	—	—
17	Darlaston Bridge, Staffordshire,	1	26·5	Do.	Stone,	86·0	13·5	6·4	—	75·2	—	—	—
18	Railway bridge over the Cart at Paisley,	1	25·0	Do.	Freestone,	85·0	18·0	4·7	3·0	59·2	—	—	Locke.
19	Hutcheson Bridge, over the Clyde at Glasgow,	5	—	Do.	Sandstone,	79·0	13·37	5·9	3·5	65	11·0	1833	R. Stephenson.
20	Bridge of St. Maxence, over the Oise,	3	41·5	Do.	Saillancourt stone,	76 67	6·25	12·2	4 67	121	9·5	1784	Perronet.
21	Bridge over the Mersey, Grand Junction Railway,	2	25·0	Do.	Stone,	75·0	14·5	5·1	3·5	55·7	10·0	—	Locke.
22	Staines Bridge, over the Thames,	3	30·0	Do.	Stone,	74·0	9·22	8·0	2·33	79·0	8·25	—	G. Rennie.
23	Wellesley Bridge, over the Shannon at Limerick,	5	41·0	Circular and elliptic,	tone,	70·0	$\begin{cases} 8·5 \\ 17·5 \end{cases}$	$\left.\begin{matrix} 8·2 \\ 4·0 \end{matrix}\right\}$	2·0	$\begin{cases} 76·3 \\ 70·0 \end{cases}$	$\}$ 10·0	—	Nimmo.
24	Bridge over the river Eden at Carlisle,	5	—	Ellipse,	Stone,	65·0	21·0	3·1	3·75	50	9·0	—	Smirke.
25	Bridge over the Lea at Bow,	1	40·0	Do.	Aberdeen granite,	64·0	13·8	4·6	2·5	74·2	—	1839	Walker & Burgess.
26	Bridge near Blisworth, over the London and Birmingham Railway,	1	18·0	Circular,	Stone,	63·0	13·0	4·8	2·5	44·7	—	—	R. Stephenson.
27	Bridge over the Forth at Stirling,	5	32·83	Do.	Greenstone,	60·0	13·5	4·4	2·75	40·0	9·0	—	R. Stephenson.
28	Stanhope-street Bridge, over the London and Birmingham Railway,	2	40·5	Do.	Brick,	25·0	2·5	10·0	2·33	32·5	4·0	—	R. Stephenson.

1. 4. 15. 16. 17. 19. 22. 23. 24. *Bridges by Hann & Hosking.*
2. 5. 6. 8. 25. *Trans. Ins. C. E.*, Vols. i. and iii.
3. 5. 7. 10. 11. 12. 13. 14. 20. *Rud. Treatise on C. Engineering by Law.*
9. *Proc. Ins. C. E.*, Vol. v.
18. 21. 26. 28. Brees' *Railway Practice.*

CHAPTER XXVII.

CONNEXIONS.

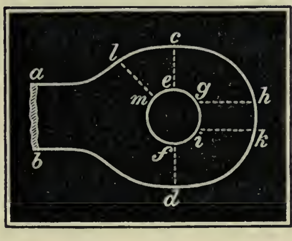
460. Appliances for connecting iron-work—Strength of joints should equal that of the adjoining parts—Screws.— One general rule applies to all jointed structures, namely, that the strength of the whole is limited by that of its weakest part, and accordingly the strength of joints should not be less than that of the parts which they connect. The usual appliances for connecting iron-work may be divided into four classes:—

1. Screws.	3. Gibs and cotters.
2. Bolts or pins.	4. Rivets.

The strain to which the above-mentioned connectors are subject is generally a shearing-strain, and as the strength of iron to resist shearing is practically equal to its tensile strength (**394**), the strength of an iron rivet, bolt, cotter, or screw, is measured by the product of the area subject to shearing multiplied by the tearing unit-strain of the iron. The thread of a screw which is subject to longitudinal tension may be "stripped" or shorn off by the nut; in the case of V threaded screws both nut and screw may be stripped simultaneously midway between the base and vertex of the thread, and the shearing area is approximately measured by the circumference of the screw at base of thread multiplied by half the length grasped by the nut; in the case of square threads the shearing area is the same. From this it follows that the length of the nut should be at least one-half the effective or net diameter of the screw. In practice it is generally made equal to 1 or $1\frac{1}{2}$ times the gross diameter and the diameter of a nut, or bolt-head, or rivet-head is seldom less than twice that of the bolt.

461. Bolts or pins—Proportions of eye and pin in flat links—Upsetting and bearing surface.—A bolt or pin is the simplest appliance for connecting together two pieces of iron, and as the principal considerations connected with a bolt joint also apply to other and more complex forms, I shall devote a short space to its investigation. Take, for example, the joint of a suspension bridge, the chains of which are formed of long flat links connected by pins passing through eyes formed at their ends. Such a joint may fail in six ways.

Fig. 112.



1. By the link tearing through the eye at *cd*, for want of sufficient material to withstand the longitudinal tensile strain. Hence, the sectional area at *cd* should theoretically equal that of the shank at *ab*, but in practice it may be somewhat greater, as the strain is less direct round the eye than in the body of the link.

2. By the end of the link being split along one or two lines, such as *gh* and *ik*, for want of sufficient area to resist the shearing action of the pin. Hence, the combined areas at *gh* and *ik* should theoretically equal that of the shank at *ab*, but in practice be considerably greater, as this part of the eye acts as a short girder whose abutments are *ce* and *fd*; this causes the outer circumference near *h* and *k* to be in severe tension and, therefore, very liable to tear, especially when the "reed" of the iron is open, as is frequently the case with bar and angle iron.

3. By the pin being shorn across. This arises from its diameter being too small. Hence, if the pin be iron and in double-shear, its area should in no case be less than one-half that of the shank at *ab* (**394**).

4. By the pin bending. This also arises from its diameter being too small to afford the requisite stiffness, but ultimate failure may

generally be prevented by the links being kept from spreading asunder by a head and nut on the pin, at the loss, however, of freedom of motion.

5. By the link tearing through the shoulder at *lm*, in consequence of the curvature or change of form being too abrupt to permit the lines of strain in the shank bending gradually round the eye.

6. By the crown of the eye being upset between *g* and *i*. This arises from the bearing surface of the pin being too small in proportion to the longitudinal strain, in which case there is an excessive pressure on each superficial unit at the crown of the eye, whereby the material there is upset, and the sides of the eye at *e* and *f* become first unduly attenuated and then torn, the rent extending from the inside towards the circumference. Sir C. Fox has drawn attention to this latter source of failure in a valuable communication to the Royal Society, in which the following remarks occur:*—"If the pin be too small, the first result on the application of a heavy pull on the chain will be to alter the position of the hole through which it passes, and also to change it from a circular to a pear-shaped form, in which operation the portion of the metal in the bearing upon the pin becomes thickened in the effort to increase its bearing surface to the extent required. But while this is going on, the metal round the other portions of the hole will be thinned by being stretched, until at last, unable to bear the undue strains thus brought to bear upon it, its thin edge begins to tear, and will, by the continuance of the same strain, undoubtedly go on to do so until the head of the link be broken through, no matter how large the head may be; for it has been proved by experiment that by increasing the size of the head, without adding to its thickness (which, from the additional room it would occupy in the width of the bridge, is quite inadmissible), no additional strength is obtained. The practical result arrived at by the many experiments made on this very interesting subject is simply that, with a view to obtaining the full efficiency of a link, *the area of its semi-cylindrical surface bearing on the pin must be a*

* "On the Size of Pins for connecting Flat Links in the Chains of Suspension Bridges."—*Proc. Roy. Soc.*, Vol. xiv., No. 73, p. 139.

little more than equal to the smallest transverse sectional area of its body; and as this cannot, for the reasons stated, be obtained by increased thickness of the head, it can only be secured by giving a sufficient diameter to the pins. That as the rule for arriving at the proper size of pin proportionate to the body of a link may be as simple and easy to remember as possible, and bearing in mind that from circumstances connected with its manufacture the iron in the head of a link is perhaps never quite so well able to bear strain as that in the body, I think it desirable to have the size of the hole a little in excess, and accordingly for a 10 inch link I would make the pin $6\frac{2}{3}$ inch in diameter, instead of $6\frac{1}{2}$ inch, that dimension being exactly two-thirds of the width of the body, which proportion may be taken to apply to every case (where the body and heads are of uniform thickness). As the strain upon the iron in the heads of a link is less direct than in its body, I think it right to have the sum of the widths of the iron on the two sides of the hole 10 per cent. greater than that of the body itself. As the pins, if solid, would be of a much larger section than is necessary to resist the effect of shearing, there would accrue some convenience, and a considerable saving in weight would be effected, by having them made hollow and of steel." Mr. G. Berkley also has made several valuable experiments on the strength of links, from which he concludes that the diameter of the pin should equal $\frac{3}{4}$ths of the width of the shank, while Mr. Brunel in his latest practice adopted the same proportion of pin as Sir C. Fox, but made the curve of the shoulder exceedingly gradual—the radius being 7·6 times the width of the shank—with the object of deflecting the lines of strain along the shank as gradually as possible before passing round the eye, the experiments which were made for the Chepstow and Saltash bridges having led to the belief that strength depended more upon the shape of the shoulder than upon excess of metal about the eye.*

The following table gives these and other proportions adopted by the foregoing authorities in a concise form:—

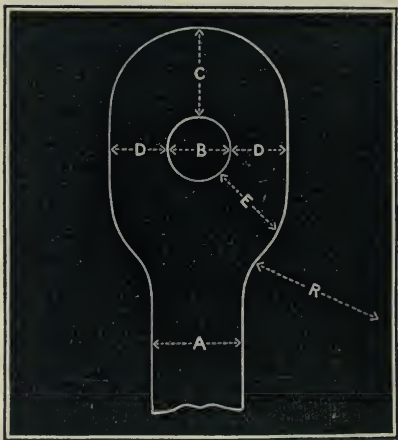
* *Proc. Inst. C. E.*, Vol. xxx., pp. 220 and 271.

TABLE I.—PROPORTIONS OF THE EYES OF FLAT BAR LINKS.

		Fox.	Berkley.	Brunel.
Shank,	A,	1·00	1·00	1·00
Diameter of pin,	B,	·66	·75	·66
End of eye,	C,	—	1·00	·60
Sides of eye,	D + D	1·10	1·25	1·21
Width of shoulder,	E,	—	1·00	—
Radius of shoulder,	R,	—	1·50	7·60

The sides of the eye of an ordinary forged tie rod have usually a collective area equal to 1·5 or 2 times that of the rod.

Fig. 113.



462. Rivets in single and double shear—Proportions of rivets in tension and compression joints—Hodgkinson's rules for the strength of single and double riveting—Injurious effect of punching holes—Relative strength of punched and drilled holes.—The strength of a riveted joint, so far as the rivets are concerned, is proportional to the number of shears to which they are subject, a rivet in double-shear, Fig. 114, being twice as strong as a rivet in single-shear, Fig. 115; so that to make the joints of equal strength, the single-shear joint must have twice as many rivets as the other.

Fig. 114.

Double-Shear.

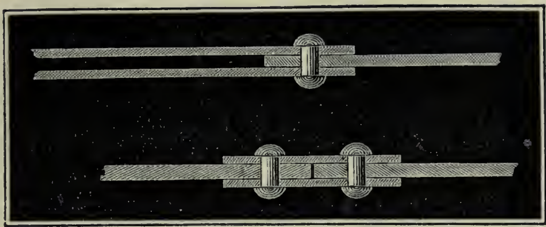
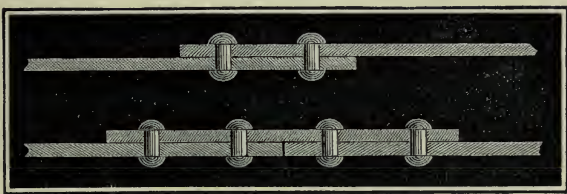


Fig. 115.

Single-Shear.



When a joint connects plates in tension, the aggregate shearing area of the rivets on each side of the joint line multiplied by the safe shearing unit-strain of the rivets should equal the total working strain transmitted through the plates. It thus happens in girder-work that the collective shearing area of the rivets of a well proportioned tension joint is nearly equal to the effective plate area, *i.e.*, the net area of the plates after deducting rivet holes (**394**). In practice the rivet area is generally made about 1-10th greater, in order to compensate for any inequality in the distribution of the strain among the rivets. In steel plating the rivet area, if the rivets are steel, should be one-third greater than the net area of the plates, but the heads of steel rivets are very apt to fly off (**395**). When a joint connects compression plates whose ends do not butt closely against each other, the thrust is transmitted through the covers and tends to shear the rivets across exactly in the same manner as when a tensile strain is transmitted, and the foregoing rule applies here also. If, however, the compression plates have their ends planed square and then brought very carefully into close contact so as to form a "jump" joint, a short cover and one, or at most two, transverse rows of rivets on each side of the joint line will suffice, as the use of the cover in this case is merely to keep the plates in line but not to transmit the thrust. A jump compression joint is erroneously supposed to be stronger than one in which the plates are slightly apart with the covers and rivets duly proportioned as for a tension joint, and engineers are sometimes over-exacting in this respect, expecting water-tight joints when the contractor gets only 18s. or 20s. per cwt. for the girder. A real jump joint with the plates butting along their whole width is rare, as the process of riveting generally draws the plates slightly apart and an interval of a hundredth of an inch is theoretically as bad as a quarter inch. A little caulking of the edges, however, makes all smooth to the eye, and the so-called "jump" joint passes muster. A practical remedy for this is described in **464**.

With respect to the ordinary method of riveting in transverse rows, each row containing the same number of rivets, Mr. Hodgkinson deduced from his experiments that "the strength of plates however riveted together with one row of rivets, was reduced to about one-half the tensile strength of the plates themselves; and if the rivets were somewhat increased in number, and disposed alternately in two rows, the strength was increased from one-half to two-thirds or three-fourths at the utmost."*

Reducing these conclusions to a convenient standard, we have the following rule for the relative strength of lap-joints:—

Strength of the unpunched plate,	-	100
Strength of a double-riveted joint,	-	66
Strength of a single-riveted joint,	-	50

Nearly all experimenters on the subject agree, and my own experience corroborates the fact, that punching reduces the tensile strength of iron to a greater degree than the aggregate area of the metal punched out, and a close examination of the border of each hole shows that it has been subject to a certain degree of violence, which in most cases has injuriously affected the fibre of the iron. Drilling does not damage the metal surrounding the

* *Iron Com. Rep., App*, p. 116.

hole, and it is therefore preferred where the nature of the work will permit the extra cost of drilling over punching. Mr. Maynard inferred from his experiments that drilled plates are 19 per cent. stronger than punched plates. There can be little doubt, however, that the exact percentage will depend—1°., on the condition of the punching tool, *i.e.*, the maintenance of the proper proportion of size between the punch and die; and 2°., on the quality of the iron—a tough coppery iron, like Low Moor, suffering less injury from punching than a hard brittle iron, and thick plates suffering more than thin ones. Mr. Maynard was also led to the conclusion that rivets in drilled holes were 4 per cent. weaker than rivets in punched holes, because the sharp edges of the drilled plates have a tendency to shear off the rivets cleaner than those in the punched plates, and he finally concluded that the difference is 15 per cent. in favour of drilled work when compared with punched work.

463. Covers—Single and double covers compared—Lap-joint.—*The strength of the covers of tension joints, and compression joints where the plates do not butt closely, should equal that of the plates ;* hence, a single cover should resemble a short length of the plate and each side of a double cover be at least half as thick as the plate.

As the quantity of material required for covers forms a very considerable percentage of the plates (12 per cent. and upwards, depending on the length of the plates), it is of great importance that the joints be as few as possible and arranged in the very best manner. This is more especially the case in large girders, where every ton of useless weight requires perhaps several tons in the main girders for its support, as will be shown in a succeeding chapter. For this reason large plates, with few joints, though they may cost extra per ton, will often make a cheaper girder than plates of ordinary sizes with more numerous joints (**437**). In the usual method of cover riveting, two or three transverse rows of rivets are placed on each side of the joint line, each row containing the same number of rivets, and the effective area of the plate, if in tension, is reduced by the aggregate section of the rivet holes in any one row. Hence, it would appear that the fewer rivet holes there are in each transverse row the less is the plate weakened and the more is its material economized. But this again requires several successive rows of rivets in order to provide sufficient rivet area, thus introducing the necessity of long covers, which may more than counterbalance the saving in the plates. The size of the plates therefore will determine to some extent the economical length of the covers as well as the transverse pitch of the rivets.*

The few experiments described in **393** seem to indicate that rivets in single-shear will not withstand so great a unit-strain as rivets in double-shear; this, however, requires confirmation, and good experiments on the strength of various forms of rivet joints are much wanted. From those recorded by Sir William Fairbairn in the appendix to the first series of "Useful Information for Engineers," it appears that, so far as the plates are concerned, a single-cover or lap-joint with only one transverse row of rivets in the lap is considerably weaker (in the experiments about 25 per cent. less) than a double-cover joint of the same theoretic strength, *i.e.*, with the same net area of plates taken across the rivet holes. This arises from the distortion of the single-cover or lap-joint which, yielding in its effort to assume a straight line between the points of traction, bends the plates slightly and makes them liable to tear across the line of rivet holes. When, however, a single-cover or lap-joint had two or more transverse rows of rivets in the lap its strength was not less than that of a double-cover joint of equal plate area. If the plates are kept in a straight line by being riveted to an angle iron or web, like the flange plates of a girder, it is still more likely that the strength of a single-cover joint will be fully equal to that of a double-cover joint of the same theoretic strength, but whenever convenient, the double-cover should be adopted from economical motives, as it gives double-shear to the rivets, and need therefore be only half as long as a single cover with the same rivet area. The common lap-joint represented first in Fig. 115, is, however, an exception to this, as the lap need not be longer than half the single cover represented beneath it.

* The "pitch" is the distance measured from centre to centre of rivets.

464. Tension joints of piles—Compression joints of piles require no covers if the plates are well butted—Cast-zinc joints.— I have already advocated the piling of plates over each other when a large flange area is required, and I have shown that long rivets form no practical objection to this arrangement (**423, 424**). When several plates are riveted together their joints are generally arranged in steps, and the length of each cover equals the lap of one plate multiplied by the number of plates + 1. Thus, in Fig. 116, the pile consists of three plates and the length

Fig. 116.

of each cover equals four laps. The length of lap is generally twice the longitudinal pitch of the riveting. The thickness of the covers of tension piles should be somewhat greater than half that of one plate, for it is clear that when a joint occurs in an upper or lower plate, more than half the tension in that plate will be thrown into the nearest cover. Hence, it is a good rule to make the covers of tension piles not less than $\frac{5}{8}$ths of the thickness of a single plate.

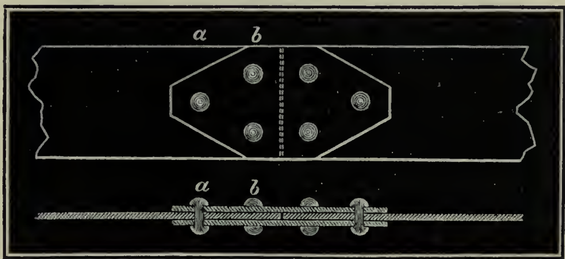
If a pile of several plates be in compression and closely fitted so as to butt against each other, no covers will be required, and great economy will result from this in very large girders, so much so as amply to repay the extra expense of planing the ends of the plates and bringing them carefully into close contact. To ensure this, however, requires considerable attention, for the riveting process has, as already observed, a tendency to open the joints slightly, but cast-zinc, which is a very hard substance, may be usefully employed for running into the compression joints of wrought as well as cast-iron, provided they are sufficiently open to let the molten metal flow freely. The joints of the cast-iron voussoirs of the Bridge of Austerlitz in Paris, finished in 1806, were thus formed,* and in my own practice I have used cast-zinc for filling up the irregular intervals between the ends of the arched ribs of a cast-iron bridge of 96 feet span and the wall-plates from

* *Enc. Brit.*, 8th Ed., art. "Iron Bridges," Vol. xii., p. 581.

which they sprang; in the latter case accurate fitting would have been extremely difficult, if not impossible, and a very satisfactory and close joint was made by slightly warming the parts with a fire of chips "to expel the cold air," as the workmen say, before pouring in the molten zinc. The heat probably expels moisture and assists the flowing of the metal into the narrower crevices. I have also used cast-zinc very successfully for securing crane posts (both cast and wrought-iron) in their foundation plates, where it ensures close contact without the cost of fitting. The following description of this method of forming the joints of a cast-iron arch of 133 feet span on the Pennsylvania Central Railroad occurs at p. 244 of *Haupt on Bridge Construction*:—"The joints were separated to the distance of one-fourth of an inch, and filled with spelter (cast-zinc) poured into them in a melted state; this was very conveniently done by binding a piece of sheet-iron around each joint, and covering it with clay. The material introduced being nearly as hard as the iron itself, and filling all the inequalities of the surface, rendered the connexion perfect." If the space between two plates be very narrow, the joint should be placed in a vertical position so that gravity may aid the flow of the metal, and a little tin added to the zinc is said to render the latter more fluid.

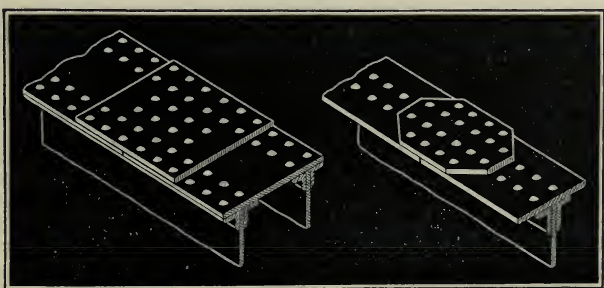
465. Various economical arrangements of tension joints.— The following method of riveting reduces the tensile strength of the parts connected less than that in common use, and possesses the merit of being applicable to plates as well as bars. Its peculiarity consists in diminishing the number of rivets in each row as they recede from the joint-line, and at the same time slightly increasing the thickness of the cover or covers beyond that of the parts connected. Fig. 117 represents this arrangement applied to a bar or narrow plate with double covers. There are eight different ways in which the joint may fail. 1°. By the bar tearing at *a*, where its area is reduced by only one rivet hole. 2°. By both covers tearing at *b*, where each is weakened by two rivet holes; this, however, is compensated for by their united area being somewhat greater than that of the bar. 3°. By the bar tearing at *b* at the same time that the rivet at *a* is double-shorn. 4°. By the rivets on one side of the joint line double-shearing. 5°. By the rivets on the alternate half-faces single-shearing. 6°. By the rivets on one half-face single-shearing while the opposite cover tears at *b*. 7°. By both covers tearing at *a* simultaneously with the rivets double-shearing at *b*. 8°. By both covers tearing at *a* simultaneously with the bar tearing at *b*. If, for example, the plates are 7 inch × $\frac{1}{2}$ inch, connected by two $\frac{5}{16}$th inch covers with $\frac{13}{16}$th inch rivet holes, the net area of the plate at *a* is 3·1 square inches nearly; the double-shearing area of the rivets at one side of the joint line equals 3·1 inches, and the net area of both covers together at *b* is 3·36 inches.

Fig. 117.



Finally, the net area of the plate at *b* together with the double-shearing area of the rivet at *a* equals 3·7 inches. This joint is therefore tolerably well proportioned, while the effective strength of the plates is really reduced by only one rivet hole, viz., that at *a*. A similar plan of joint is applicable to broad plates, Fig. 118.

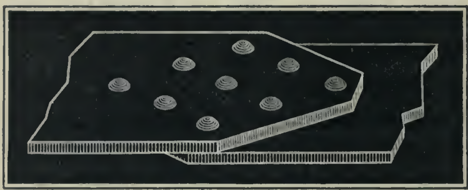
Fig. 118.



When this mode of riveting is applied to a pile of plates, the extra thickness of the covers should be sufficient to compensate for the reduction in the strength of the whole pile caused by the close transverse riveting at the joints.

When bars or plates are lap-jointed the arrangement proposed by Mr. Barton, and represented in Fig. 119, is an excellent one.

Fig. 119.



The diagonal joint running obliquely across the plate is another useful arrangement, and it appears from experiments instituted by Mr. J. G. Wright that the strength of a single-riveted diagonal lap-joint at 45° was 64·7 per cent. of that of the solid plate, whereas the strength of a similar straight joint was only 48·2 per cent., the increase in strength of the diagonal joint being 34 per cent. over the other, that is, the diagonal single-riveted joint was nearly as strong as an ordinary straight double-riveted joint.*

466. Contraction of rivets and resulting friction of plates—Ultimate strength of rivet-joints not increased by friction.— Rivets contract in cooling and draw the plates together with such force that the friction produced between their surfaces is generally sufficient to prevent them from slipping over each other so long as the strain lies within limits which are not exceeded in practice, and when this occurs the rivets are not subject to shearing strain. From experiments made during the construction of the Britannia Tubular Bridge it appears that the value of this friction is rather variable.† In one experiment with a $\frac{7}{8}$th inch rivet passing through three plates, and therefore in double-shear, it amounted to 5·59 tons, in another with a $\frac{7}{8}$th inch rivet and two plates lap-jointed with $\frac{5}{16}$th inch washers next the rivet heads it reached 4·73 tons, while in a third experiment with three plates and $\frac{7}{8}$th inch rivet with $\frac{1}{2}$ inch washers next the rivet heads, making the shank of the

* *Proc. Inst. M. E.*, 1872, p. 77. † Clark *on the Tubular Bridges*, p. 393.

rivet $2\frac{3}{8}$ inch long, the middle plate supported 7·94 tons before it slipped. In these experiments the hole in one or both plates was oval and the sliding took place abruptly. Though the friction of riveted plates may be sufficient to convey the usual working-strain without subjecting the rivets to shearing, it does not follow, nor do experiments indicate, that the *ultimate* strength of a rivet joint is increased by this friction. It is an interesting fact, however, that rivets in ordinary girder-work and plating are subject to a tensile and not a shearing strain.

467. Girder-makers', Boiler-makers' and Shipbuilders' rules for riveting—Chain-riveting.—Joints may fail by each rivet splitting or shearing out the piece of plate in front of itself. Consequently, the minimum theoretic distance of the rivets from the edge of an iron plate or from each other lengthways should be determined by the consideration that the shearing area of the plate (along two lines) between each rivet and the one behind it, or between each rivet in the first row and the edge of the plate, be not less than that of the rivet. If, for example, the rivets in Fig. 117 be $\frac{3}{4}$ inch and the plates $\frac{1}{2}$ inch thick, the shearing area of each rivet (in double-shear) equals 1 square inch nearly,* and the distance of the edge of the rivet holes from the joint line should theoretically not be less than $\frac{1}{2}$ an inch. Practically, however, this is insufficient, for punching tends to burst the edges of the holes if placed so close to each other or to the edge of the plate, especially if the plate be thick or of brittle quality, and in boilers the distance between the holes and the edge of the plate is usually about once the diameter of the rivet. If the distance exceed this it is difficult to make the seam steam-tight by caulking. In girder-work, which does not require caulking like a boiler, this distance is seldom less than $1\frac{1}{2}$ times the diameter of the rivet, and the pitch may vary from $2\frac{1}{2}$ to 5 or even 7 inches, but should not exceed 15 times the thickness of a single plate, from 6 to 12 times being common practice. The rivets in ordinary girder-work range from

* Rivet holes are generally punched from $\frac{1}{32}$nd to $\frac{1}{16}$th inch larger than the nominal size of the rivet, in order that the latter when red hot may pass freely through the hole. Hence, the area of a $\frac{3}{4}$ inch rivet, after riveting, is nearly half a square inch.

$\frac{3}{4}$ to 1 inch and occasionally $1\frac{1}{8}$ inch in diameter. The rivet holes in first-class work are now frequently bored out with drilling machines, so as to avoid the weakening effect of punching on the plates. The great majority of girder-work, however, will probably always be done by the punch, as it does not pay to have the holes drilled unless in large girders where there are frequent repetitions of the same pattern (**425**). The following table shows the usual practice in boiler-work.

TABLE II.—RULES FOR BOILER RIVETING.

Thickness of plate.	Diameter of rivet.	Length of rivet from head.	Central distance of rivets. (Pitch).	Lap in single joints.	Lap in double joints.	Equivalent length of head.
inch.	inch.	inch.	inch.	inch.	inch.	inch.
$\frac{3}{16}$ = ·19	$\frac{3}{8}$ = ·38	$\frac{7}{8}$	$1\frac{1}{4}$ to $1\frac{1}{2}$	$1\frac{1}{4}$	$2\frac{1}{16}$	$\frac{1}{2}$
$\frac{1}{4}$ = ·25	$\frac{1}{2}$ = ·50	$1\frac{1}{8}$	$1\frac{1}{2}$ to $1\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$\frac{5}{8}$
$\frac{5}{16}$ = ·31	$\frac{5}{8}$ = ·63	$1\frac{3}{8}$	$1\frac{5}{8}$ to $1\frac{7}{8}$	$1\frac{7}{8}$	$3\frac{1}{8}$	$\frac{3}{4}$
$\frac{3}{8}$ = ·38	$\frac{3}{4}$ = ·75	$1\frac{5}{8}$	$1\frac{7}{8}$ to $2\frac{1}{4}$	$2\frac{1}{4}$	$3\frac{3}{8}$	$\frac{7}{8}$
$\frac{1}{2}$ = ·50	$\frac{13}{16}$ = ·81	$2\frac{1}{4}$	$2\frac{1}{8}$ to $2\frac{3}{8}$	$2\frac{3}{8}$	$3\frac{3}{4}$	$1\frac{1}{4}$
$\frac{9}{16}$ = ·56	$\frac{7}{8}$ = ·88	$2\frac{1}{2}$	$2\frac{1}{4}$ to $2\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{8}$	$1\frac{3}{8}$
$\frac{5}{8}$ = ·63	$\frac{15}{16}$ = ·94	$2\frac{3}{4}$	$2\frac{1}{2}$ to $2\frac{3}{4}$	$2\frac{3}{4}$	$4\frac{5}{8}$	$1\frac{1}{2}$
$\frac{11}{16}$ = ·69	1 = 1·00	3	$2\frac{3}{4}$ to 3	3	5	$1\frac{5}{8}$
$\frac{3}{4}$ = ·75	$1\frac{1}{8}$ = 1·13	$3\frac{1}{4}$	3 to $3\frac{1}{4}$	$3\frac{1}{4}$	$5\frac{7}{8}$	$1\frac{3}{4}$

NOTE.—The equivalent length of head given in the last column is intended for bat heads, such as are usual in boilers, but if the rivets have cup heads like those in Fig. 117, as is usual in girder-work, the equivalent length of head must be about one-half more than the amount given in the last column. The pitch in girder-work is generally from once and a-half to twice that in column 4.

The boiler-maker's rule is nearly as follows:—For plates less than half an inch thick, the diameter of the rivet = twice the thickness of the plate. For plates more than half an inch thick, the diameter of the rivet = once and a half the thickness of the plate. The pitch of single joints = $2\frac{3}{4}$ to 3 diameters, and that for double joints = $3\frac{1}{2}$ to 4 diameters of the rivet. The lap for single joints = 3 diameters, and that for double joints = 5 diameters of the rivet.

Lloyd's rules for the dimensions of rivets in ship-building are as follows:—

TABLE III.—Lloyd's Rules for Ship Riveting.

Thickness of Plates in inches, -	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$	$\frac{8}{16}$	$\frac{9}{16}$	$\frac{10}{16}$	$\frac{11}{16}$	$\frac{12}{16}$	$\frac{13}{16}$	$\frac{14}{16}$	$\frac{15}{16}$	$\frac{16}{16}$	Rivets to be $\frac{1}{4}$ of an inch larger in diameter in the stem, stern-post and keel.
Diameter of Rivets in inches, -	$\frac{5}{8}$			$\frac{3}{4}$			$\frac{7}{8}$			1			

"The rivets not to be nearer to the butts or edges of the plating, lining pieces to butts, or of any angle iron, than a space not less than their own diameter, and not to be farther apart from each other than four times their diameter, or nearer than three times their diameter, and to be spaced through the frames and outside plating, and in reversed angle iron, a distance equal to eight times their diameter apart. The overlaps of plating, where double riveting is required, not to be less than five and a half times the diameter of the rivets; and where single riveting is admitted, to be not less in breadth than three and a quarter times the diameter of the rivets." The Liverpool rules differ somewhat from Lloyd's and are as follows:—

TABLE IV.—Liverpool Rules for Ship Riveting.

Thickness of Plates in inches, -	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$	$\frac{8}{16}$	$\frac{9}{16}$	$\frac{10}{16}$	$\frac{11}{16}$	$\frac{12}{16}$	$\frac{13}{16}$	$\frac{14}{16}$	$\frac{15}{16}$	$\frac{16}{16}$
Diameter of Rivets in inches, -	$\frac{8}{16}$	$\frac{10}{16}$	$\frac{10}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{13}{16}$	$\frac{14}{16}$	$\frac{14}{16}$	$\frac{15}{16}$	$\frac{16}{16}$	$\frac{17}{16}$	$\frac{18}{16}$
Breadth of lap in seams in inches,												
Single-riveting, -	$1\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{3}{4}$	—	—	—	—	—	—	—	—
Double-riveting, -	3	$3\frac{3}{4}$	$3\frac{3}{4}$	$4\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{1}{4}$	$5\frac{1}{4}$	$5\frac{5}{8}$	6	$6\frac{3}{8}$	$6\frac{3}{4}$
Breadth of butt strip,												
Double-riveting, -	$7\frac{1}{4}$	8	8	10	10	$10\frac{3}{4}$	$11\frac{1}{2}$	$11\frac{1}{2}$	$12\frac{1}{4}$	13	$13\frac{3}{4}$	$14\frac{1}{2}$
Treble-riveting, -	9	$11\frac{1}{2}$	$11\frac{1}{2}$	$13\frac{1}{2}$	$13\frac{1}{2}$	15	16	16	17	18	19	20

"Rivets to be four diameters apart, from centre to centre, longitudinally in seams and vertically in butts, except in the butts where treble riveting is required, where the rivets in the row farthest from the butt may be spaced eight diameters apart, centre to centre. Rivets in framing to be eight times their diameter apart, from centre to centre, and to be of the size required in the above table. All double or treble riveting in butts of plates to be in parallel rows, or what is termed chain riveting. It is recommended that the necks of all rivets be bevelled under the head so as to fill the countersink made in punching, and their heads should be no thicker than two-thirds the diameter of the rivet." It will be observed that the pitch may be one-third as great again in water as in steam joints.

The term "chain-riveting" is applied to riveting in several transverse rows, the rivets being placed longitudinally one behind the other like the links of a chain. It merely means that both the longitudinal and transverse rows of rivets form straight lines, in place of the rivets being zigzag.

468. Adhesion of iron and copper bolts to wood—Strength of clenches and forelocks.—The shearing strength of oak treenails has been already given in **397**. The two following tables are also the results of Mr. Parson's experiments.* "The first of these tables exhibits the adhesion of iron and copper bolts, driven into sound oak, with the usual drift, not clenched, and subject to a direct tensile strain. By drift is meant the allowance made to insure sufficient tightness in a fastening; it is therefore the quantity by which the diameter of a fastening exceeds the diameter of the hole bored for its reception."

* *Murray on Shipbuilding*, p. 94.

TABLE V.—TABLE OF THE ADHESION OF IRON AND COPPER BOLTS DRIVEN INTO SOUND OAK WITH THE USUAL DRIFT, NOT CLENCHED, AND SUBJECTED TO A DIRECT TENSILE STRAIN.

Diameter of the Bolt.	Number of the experiment.	Iron. Length of the Bolt driven into the Wood. Four inches.		Iron. Six inches.		Copper. Four inches.		Copper. Six inches.	
inches.		tons.	cwts.	tons.	cwts.	tons.	cwts.	tons.	cwts.
¼	1	1	13	—		0	18½	—	
	2	2	0	—		0	18	—	
	3	2	2	—		0	19	—	
	4	1	13	—		0	18	—	
⅜	1	2	6	2	12	1	7	2	2
	2	2	4	2	11	1	8	2	2
	3	2	4	2	16	1	10	2	2
	4	2	0	2	10	1	13	2	0
½	1	3	2	3	12	2	10	2	15
	2	3	4	4	0	1	17	3	10
	3	3	0	4	0	2	2	3	1
	4	2	10	4	0	2	5	2	15
⅝	1	3	2	5	5	3	0	4	5
	2	3	0	4	8	3	6	3	18
	3	3	1	4	8	3	6	3	15
	4	3	1	5	0	2	9	3	5
¾	1	3	3	6	0	3	10	5	5
	2	3	2	6	0	3	10	5	5
	3	3	10	5	0	3	10	5	8
	4	3	10	6	0	3	18	4	18
⅞	1	4	10	6	2	4	0	4	13
	2	5	12	5	10	4	0	4	13
	3	3	10	6	11	4	5	4	19
	4	4	10	6	4	4	2	4	19

TABLE V.—TABLE OF THE ADHESION OF IRON AND COPPER BOLTS DRIVEN INTO SOUND OAK WITH THE USUAL DRIFT, NOT CLENCHED, AND SUBJECTED TO A DIRECT TENSILE STRAIN—*continued.*

Diameter of the Bolt.	Number of the experiment.	Iron.				Copper.			
		Length of the Bolt driven into the Wood.							
		Four inches.		Six inches.		Four inches.		Six inches.	
inches.		tons.	cwts.	tons.	cwts.	tons.	cwts.	tons.	cwts.
1	1	5	0	7	2	4	2	5	19
	2	4	7	8	1	4	8	5	0
	3	4	11	6	5	3	15	6	5
	4	4	0	7	0	4	10	5	0

"In Riga fir the adhesion was, on an average, about one-third of that in oak, and in good sound Canada elm it was about three-fourths of that in oak.

"The following table exhibits the strength of clenches and of forelocks as securities to iron and copper bolts, driven six inches, without drift, into sound oak, either clenched or forelocked on rings, and subjected to a direct tensile strain. It gives the diameter of the bolt on which the experiment was made, as well as the number of the experiment:—

TABLE VI.—TABLE OF THE STRENGTH OF CLENCHES AND OF FORELOCKS, AS SECURITIES TO IRON AND COPPER BOLTS, DRIVEN SIX INCHES, WITHOUT DRIFT, INTO SOUND OAK, EITHER CLENCHED OR FORELOCKED ON RINGS, AND SUBJECTED TO A DIRECT TENSILE STRAIN.

Diameter of the Bolt.	Number of the experiment.	Iron.				Copper.			
		Clench.		Forelock.		Clench.		Forelock.	
inch.		tons.	cwts.	tons.	cwts.	tons.	cwts.	tons.	cwts.
$\frac{1}{4}$	1	1	16	0	16	1	0	0	8
	2	1	13	0	14	0	19	0	8
	3	1	9	0	20	1	0	0	7
	4	1	9	0	18	1	0	0	6

TABLE VI.—TABLE OF THE STRENGTH OF CLENCHES AND OF FORELOCKS, AS SECURITIES TO IRON AND COPPER BOLTS, DRIVEN SIX INCHES, WITHOUT DRIFT, INTO SOUND OAK, EITHER CLENCHED OR FORELOCKED ON RINGS, AND SUBJECTED TO A DIRECT TENSILE STRAIN—*continued.*

Diameter of the Bolt.	Number of the experiment.	Iron.				Copper.			
		Clench.		Forelock.		Clench.		Forelock.	
inch.		tons.	cwts.	tons.	cwts.	tons.	cwts.	tons.	cwts.
3/8	1	3	0	1	15	2	10	1	4
	2	3	0	1	8	2	10	1	0
	3	2	16	1	9	2	5	1	2
	4	2	15	1	14	2	9	1	4
1/2	1	4	15	2	11	3	10	1	18
	2	4	10	2	15	3	15	1	18
	3	4	5	2	10	4	0	2	4
	4	4	12	2	12	4	10	1	16
5/8	1	5	18	3	15	6	0	2	13
	2	6	8	3	6	5	15	2	10
	3	6	8	3	0	6	5	2	16
	4	6	0	3	7	5	10	2	10
3/4	1	7	10	3	10	7	0	—	
	2	7	10	3	15	7	0	—	
	3	8	0	3	10	7	5	—	
	4	8	15	3	15	7	8	—	
7/8	1	11	11	5	1	7	16	—	
	2	11	15	5	10	7	16	—	
	3	8	11	4	6	7	12	—	
	4	8	6	4	15	7	5	—	
1	1	12	0	5	18	7	1	—	
	2	12	3	6	18	7	1	—	
	3	11	3	5	12	7	14	—	
	4	11	1	5	2	8	14	—	

"In the experiments on the clenches, the clenches always gave way; but with the forelocks it as frequently occurred that the forelock was cut off as that the bolt broke; and in the cases of the bolt breaking, it was invariably across the forelock hole. According to the tables, the security of a forelock is about half that of a clench. It appears an anomaly that the strength of a clench on copper should be equal to that of one on iron. But, in consequence of the greater ductility of copper, a better clench is formed on it than on iron. Generally the thickness of the fractured clench in the copper was double that in the iron. With rings of the usual width for the clenches, the wood will break away under the ring, and the ring be imbedded for two or more inches before the clench will give way. With the inch copper-bolts, all the rings under the clenches turned up into the shape of the frustrum of a cone, and allowed the clench to slip through at the weights specified.

"Experiments with ring-bolts were made to ascertain the strength of the rings in comparison with the clenches. The rings were of the usual size, viz.: the iron of the ring one-eighth inch less in diameter than that of the bolt. It was found that the rings always carried away the clenches, but that they were drawn into the form of a link with perfectly straight sides. The rings bore, before any change of form took place, not quite one-half the weight which tore off the clenches. It appears that the rings are well proportioned to the strength of the clenches."

469. Adhesion of nails and wood-screws.—"The following abstract of Mr. Bevan's experiments exhibits the relative adhesion of nails of various kinds, when forced into dry Christiana Deal, at right angles to the grain of the wood."*

* *Tredgold's Carpentry*, p. 189.

TABLE VII.—ADHESION OF NAILS OF VARIOUS KINDS IN DRY CHRISTIANA DEAL.

Kind of Nails.	Number to the pound avoirdupois.	Inches long.	Inches forced into the wood.	Pounds required to extract.
Fine sprigs, - -	4,560	0·44	0·0	22
Ditto, - -	3,200	0·53	0·44	37
Threepenny brads, -	618	1·25	0·50	58
Cast-iron nails, - -	380	1·00	0·50	72
Sixpenny nails, - -	73	2·50	1·00	187
Ditto, - -	—	—	1·50	327
Ditto, - -	—	—	2·00	530
Fivepenny, - -	139	2·00	1·50	320

"The force required to draw the same sized nail from different woods averaged as under:—

TABLE VIII.—RELATIVE ADHESION OF SAME NAIL IN DIFFERENT WOODS.

Kind of Wood.	Weight in lbs. required to draw a sixpenny nail, driven in one inch.
Dry Christiana deal, - - - - -	187 lbs.
Dry oak, - - - - - - -	507 „
Dry elm, - - - - - - -	327 „
Dry beech, - - - - - -	667 „
Green sycamore, - - - - - -	312 „
Dry Christiana deal, driven in endways, - -	87 „
Dry elm, driven in endways, - - - -	257 „

"It was further desirable to ascertain the degree of dependence that might be placed on nailing two pieces together, and Mr. Bevan kindly undertook to make some trials. Two pieces of Christiana deal, seven-eighths of an inch thick, were nailed together with two sixpenny nails; and a longitudinal force in the plane of the joint, and consequently at right angles to the direction of the nails, was applied to cause the joint to slide; it required a force of 712 lbs., and the time was 15 minutes; the nails curved a little and were then drawn. Another experiment was made in the same manner with dry oak, an inch thick, in which the force required was 1,009 lbs.; the sixpenny nails curved, and were drawn by that force. Dry sound ash, an inch thick, joined in the same manner by two sixpenny nails, bore 1,220 lbs. 30 minutes without sensibly yielding; but when the stress was increased to 1,420 lbs. the pieces separated with an easy and gradual slide; curving and drawing the nails as before, one of which broke.

"The following experiments on the force necessary to draw screws of iron, commonly called wood screws, out of given depths of wood, were made by Mr. Bevan. The screws he used were about two inches in length, $\frac{22}{100}$ diameter at the exterior of the threads, $\frac{15}{100}$ diameter at the bottom, the depth of the worm or thread being $\frac{35}{1000}$, and the number of threads in one inch $= 12$. They were passed through pieces of wood, exactly half an inch in thickness, and drawn out by the weights stated in the following tables:—

TABLE IX.—RELATIVE ADHESION OF SCREWS IN DIFFERENT WOODS.

Kind of Wood.	Weight required to draw out screws passed through half-inch boards.
Dry beech, - - - - - -	460 lbs.
Ditto ditto, - - - - - -	790 „
Dry sound ash, - - - - - -	790 „
Dry oak, - - - - - - -	760 „
Dry mahogany, - - - - - -	770 „
Dry elm, - - - - - - -	665 „
Dry sycamore, - - - - - -	830 „

"The weights were supported about two minutes before the screws were extracted. He found the force required to draw similar screws out of deal and the softer woods about half the above.

"The force necessary to cause pieces screwed together to slide at the joining, was also determined; the pieces being joined by two screws; the resultant of the force coinciding with the plane of the joint, and in line with the places of the screws. With Christiana deal, seven-eighths of an inch thick, joined by two screws one and five-eighths of an inch in length, and five-fortieths of an inch in diameter within the worm, a load of 1,009 lbs. gradually applied broke both the screws at the line of joint, after elongating the interior of the hole and sliding about six-tenths. With very dry seasoned oak, 1 inch thick, two screws one and five-eighths long, and six-fortieths diameter within the thread, bore 1,009 lbs. for ten minutes without any signs of yielding: with 1,137 lbs. both screws broke in two places; each screw about two-tenths of an inch within each piece of wood; the holes were a little elongated. With dry and sound ash, 1 inch thick, with screws $2\frac{1}{4}$ inches long, passing one quarter of an inch through one of the pieces, the diameter at bottom of the worm seven-fortieths; the load began with was 1,224 lbs.; gradually increased for two hours to 2,661 lbs.; they produced a slow and moderate sliding, not separation, the screws being neither drawn nor broken; but probably would, if not removed on account of night coming on, and putting an end to the experiment."

CHAPTER XXVIII.

WORKING STRAIN AND WORKING LOAD.

470. Working strain—Fatigue—Proof strain—English rule for working strain—Coefficient of safety.—The *working strain* is the strain to which any material is subject in actual practice, but the term, unless accurately defined, is somewhat ambiguous, as it is applied to strains which the material sustains on rare occasions from extraordinary loads, as well as to those to which it is liable in ordinary every-day use. For instance, a railway girder may sustain a constant strain of $3\frac{1}{2}$ tons per square inch from the permanent bridge-load, which rises to $4\frac{1}{2}$ tons when an ordinary train passes, but reaches a maximum of 5 tons with a train of the greatest possible density, such as locomotives; or again, the chains of a suspension bridge may sustain only $2\frac{1}{2}$ tons per square inch from the permanent or dead weight of the structure, while a dense crowd of people may occasionally raise this to 6 tons per square inch. In such cases we have three classes of strains. 1°. The permanent strain, due to the permanent or dead weight of the structure itself, and from which the material suffers what has been termed *fatigue*. 2°. The ordinary working strain, due to ordinary live loads added to the dead weight of the structure. 3°. The maximum working strain, due to the greatest load possible in practice added to the dead weight of the structure, and it is this latter maximum strain which defines the strength of any structure, and which therefore we have to consider in this chapter. The proof load of a bridge is generally equal to the greatest load possible in practice, but the proof strain of separate parts of a structure, such as the individual links of a suspension bridge, is frequently 50 per cent. over their intended maximum working strain when in the structure. As might have been anticipated, different opinions are held respecting the safe unit-strain for each kind of material. English practice generally makes the working strain some sub-multiple of the tearing or crushing strength of the material, while General Morin and others recommend the working strain to be such that the resulting alteration of length shall in no case exceed one-half that which corresponds to the limit of elasticity. Neither rule should be adopted to the exclusion of the other, but as we know the limit of elasticity of but few materials (in fact only wrought-iron and steel), and as those which are not ductile seem to have no very definite limit at all (see Chap. XVIII.), the common English rule seems more generally applicable, and it has the sanction of extensive experience in its favour. The term factor, or coefficient of safety is applied to the ratio of the breaking to the working strain. If, for instance, the tearing inch-strain of plate-iron is 20 tons, and the working inch-strain 5 tons, the coefficient of safety will be 4.

CAST-IRON.

471. Effect of long continued pressure on cast-iron pillars and bars.—To determine the effect of long continued pressure upon cast-iron, Sir Wm. Fairbairn had four pillars cast of Low-Moor iron; the length of each was 6 feet, and the diameter 1 inch, and they were rounded at the ends. The first was loaded with 4 cwt., the second with 7 cwt., the third with 10 cwt., and the fourth with 13 cwt. These weights are respectively 30, 52, 75, and 97 per cent. of the weight which had previously broken another pillar of the same dimensions when the weight was carefully laid on without loss of time. The pillar loaded with 13 cwt. bore the weight between five and six months and then broke; that loaded with 10 cwt. was increasing slightly in flexure at the end of three years; when first taken its deflection was ·230 inch, and after each succeeding year it was ·380, ·380, and ·409. The other pillars, though a little bent, did not alter. In these experiments we see that a cast-iron pillar bore a steady load of one-half its breaking weight for three years without alteration, while the deflection of another pillar with three-fourths of its breaking weight was increasing slightly at the end of the same period.*

To ascertain how far cast-iron bars might be trusted with permanent loads, Sir Wm. Fairbairn made the following experiments also:—"He took bars, both of cold and hot blast iron (Coed Talon, No. 2), each 5 feet long, and cast from a model 1 inch square; and having loaded them in the middle with different weights, with their ends supported on props 4 feet 6 inches asunder, they were left in this position to determine how long they would sustain the loads without breaking. They bore the weights, with one exception, upwards of five years, with small increase of deflection, though some of them were loaded nearly to the breaking point." After that time, however, less care was taken to protect them from accident, and three others were found broken. They were examined, and had their deflections taken occasionally, which are set down in the following Table, which contains the exact dimensions of the bars, with the load upon each.†

* *Experimental Researches* by E. Hodgkinson, p. 351.

† *Idem*, p. 374.

TABLE I.—Experiments on the Strength of Cast-Iron Bars to Resist Long-continued Transverse Strain.

Date of observation.	Temperature of the air at time of observation.	Experiment 1.—Cold blast iron. Depth of bar 1.050 Breadth of bar 1.030	Experiment 2.—Hot blast iron. Depth of bar 1·050 Breadth of bar 1·010	Experiment 3.—Cold blast iron. Depth of bar 1·030 Breadth of bar 1·020	Experiment 4.—Hot blast iron. Depth of bar 1·040 Breadth of bar 1·020	Experiment 5.—Cold blast iron. Depth of bar 1·030 Breadth of bar 1·020	Experiment 6.—Hot blast iron. Depth of bar 1·050 Breadth of bar 1·000	Experiment 7.—Cold blast iron. Depth of bar 1·000 Breadth of bar 1·010	Experiment 8.—Cold blast iron. Depth of bar 1·020 Breadth of bar 1·030	Experiment 9.—Hot blast iron. Depth of bar 1·040 Breadth of bar 1·010
	Fah.	Deflections with a permanent load of 280 lbs. laid upon each.		Deflections with a permanent load of 336 lbs. laid upon each.		Deflections with a permanent load of 392 lbs. laid upon each.		Deflections with a permanent load of 448 lbs. laid upon each.		
1837.										
Mar. 6		—	—	1·267	—	1·684	1·715	1·964	1·410	This broke with 392 lbs.; other hot blast bars were tried, but they were successively broken with 448 lbs.
„ 9	49°	·916	1·043	1·270	1·454	1·694	1·758	2·005	1·413	
„ 11		·930	1·064	1·270	1·461	1·694	1·760	2·005	1·413	
„ 17		—	—	—	—	—	—	2·010	1·413	
April 15	47°	·930	1·078	1·271	1·475	1·716	1·767	2·014	1·422	
May 31	62°	·932	1·082	1·274	1·481	1·725	1·775	Broke after bearing the weight 37 days	1·424	
Aug. 22	70°	·937	1·086	1·288	1·504	1·737	1·783		1·438	
Nov. 18	45°	·942	1·083	1·286	1·499	1·724	1·773		1·431	
1838.										
Jan. 8	38°	·941	1·086	1·288	1·502	1·722	1·773		1·430	
Mar. 12	51°	·945	1·091	1·298	1·505	1·801	1·784		1·439	
June 23	78°	·963	1·107	1·316	1·538	1·824	1·803		1·457	
1839.										
Feb. 7	54°	·950	1·093	1·293	1·524	1·815	1·784		1·433	
July 5	72°	·959	1·104	1·305	1·533	1·824	1·798		1·446	
Nov. 7	50°	·955	1·102	1·303	1·531	1·824	1·796		1·445	
Dec. 9	39°	·956	1·102	1·303	1·531	1·823	1·796		1·445	
1840.										
Feb. 14	50°	·955	1·104	1·305	1·531	1·824	1·797		1·446	
April 27	63°	·954	1·116	1·309	1·519	1·818	1·802		1·445	
June 6	61°	·951	1·112	1·303	1·520	1·825	1·798		1·445	
Aug. 3	74°	·953	1·115	1·305	1·523	1·826	1·801		1·447	
Sept. 14	55°	*1·047	1·115	1·305	*1·613	1·826	1·802		1·447	
1841.										
Nov. 22	50°	1·045	1·115	1·306	1·620	1·829	1·804		1·449	
1842.										
April 19	58°	—	—	1·308	1·620	1·828	1·812		1·449	

On these experiments Mr. Hodgkinson made the following observations:—"Looking at the results of these experiments, and the note upon the first and fourth, it appears that the deflection in each of the beams increased considerably for the first twelve or

* After August 3, 1840, a body seems to have fallen upon the bars of the 1st and 4th Experiment, and this may have increased their deflections.

fifteen months; after which time there has been, usually, a smaller increase in their deflections, though from four to five years have elapsed. The beam in experiment 8, which was loaded nearest to its breaking weight, and which would have been broken by a few additional pounds laid on at first, had not, perhaps, up to the time of its fracture, a greater deflection than it had three or four years before; and the change in deflection in Experiment 1, where the load is less than $\frac{2}{3}$rds of the breaking weight, seems to have been almost as great as in any other; rendering it not improbable that the deflection will, in each beam, go on increasing till it becomes a certain quantity, beyond which, as in that of Experiment 8, it will increase no longer, but remain stationary (**410**). The unfortunate fracture of this last beam, probably through accident, has left this conclusion in doubt." Mr. Hodgkinson inferred from these experiments that cast-iron girders might be safely trusted with one-third of their breaking weight. This conclusion, however, he seems to have subsequently modified, when a member of the Iron Commission in 1849, which reported in favour of not less than one-sixth.

472. Effects of long-continued impact and frequent deflections on cast-iron bars.—The Commissioners appointed to inquire into the application of iron to railway structures, reported as follows on the effects of long-continued impacts and frequent deflections of cast-iron bars:—"A bar of cast-iron, 3 inches square, was placed on supports about 14 feet asunder. A heavy ball was suspended by a wire 18 feet long, from the roof, so as to touch the centre of the side of the bar. By drawing this ball out of the vertical position at right angles to the length of the bar, in the manner of a pendulum, to any required distance, and suddenly releasing it, it could be made to strike a horizontal blow upon the bar, the magnitude of which could be adjusted at pleasure either by varying the size of the ball or the distance from which it was released. Various bars (some of smaller size than the above) were subjected by means of this apparatus to successions of blows, numbering in most cases as many as 4,000; the magnitude of the blow in each set of experiments being made greater, or smaller, as occasion required. The general result obtained was, that when the blow was powerful enough to bend the bars through one-half of their ultimate deflection (that is to say, the deflection which corresponds to their fracture by dead pressure), no bar was able to stand 4,000 of such blows in succession; but all the bars (when sound) resisted the effects of 4,000 blows, each bending them through one-third of their ultimate deflection.

"Other cast-iron bars, of similar dimensions, were subjected to the action of a revolving cam, driven by a steam-engine. By this they were quietly depressed in the centre, and allowed to restore themselves, the process being continued to the extent, even in some cases, of an hundred thousand successive periodic depressions for each bar, and at a rate of about four per minute. Another contrivance was tried by which the whole bar was also, during the depression, thrown into a violent tremor. The results of these experiments were, that when the depression was equal to one-third of the ultimate deflection, the bars were not weakened. This was ascertained by breaking them in the usual manner with stationary loads in the centre. When, however, the depressions produced by the machine were made equal to one-half of the ultimate deflection, the bars were actually broken by less than nine hundred depressions. This result corresponds with and confirms the former.

"By other machinery, a weight equal to half of the breaking weight was slowly and continually dragged backwards and forwards from one end to the other of a bar of similar dimensions to the above. A sound bar was not apparently weakened by ninety-six thousand transits of the weight.

"It may, on the whole, therefore, be said, that as far as the effects of reiterated flexure are concerned, cast-iron beams should be so proportioned as scarcely to suffer a deflection of one-third of their ultimate deflection. And as it will presently appear, that the deflection produced by a given load, if laid on the beam at rest, is liable to be considerably increased by the effect of percussion, as well as by motion imparted to the load, it follows that to allow the greatest load to be one-sixth of the breaking weight, is hardly a sufficient limit for safety even upon the supposition that the beam is perfectly sound.

"In wrought-iron bars no very perceptible effect was produced by 10,000 successive deflections by means of a revolving cam, each deflection being due to half the weight which, when applied statically, produced a large permanent flexure.

"Under the second head, namely, the inquiry into the mechanical effects of percussions and moving weights, a great number of experiments have been made to illustrate the impact of heavy bodies on beams. From these, it appears, that bars of cast-iron of the same length and weight struck horizontally by the same ball (by means of the apparatus above described for long-continued impact), offer the same resistance to impact, whatever be the form of their transverse section, provided the sectional area be the same. Thus a bar, $6 \times 1\frac{1}{2}$ inches in section, placed on supports about 14 feet asunder, required the same magnitude of blow to break it in the middle, whether it was struck on the broad side or the narrow one, and similar blows were required to break a bar of the same length, the section of which was a square of three inches, and, therefore, of the same sectional area and weight as the first.

"Another course of experiments tried with the same apparatus showed, amongst other results, that the deflections of wrought-iron bars produced by the striking ball were nearly as the velocity of impact. The deflections in cast-iron are greater than in proportion to the velocity.

"A set of experiments was undertaken to obtain the effects of additional loads spread uniformly over a beam, in increasing its power of bearing impacts from the same ball falling perpendicularly upon it. It was found that beams of cast-iron, loaded to a certain degree with weights spread over their whole length, and so attached to them as not to prevent the flexure of the bar, resisted greater impacts from the same body falling on them than when the beams were unloaded, in the ratio of two to one. The bars in this case were struck in the middle by the same ball, falling vertically through different heights, and the deflections were nearly as the velocity of impact."*

* *Rep. of Iron Com.*, p. x.

473. Working strain of cast-iron girders—Rule of Board of Trade—Working strain of cast-iron arches—French rule—Proving cast-iron.—The reader will observe that the Commissioners considered one-sixth of the breaking strain hardly a sufficient limit of safety for cast-iron girders when liable to percussion and deflection from moving loads. This inference was, no doubt, influenced by their experiments on bars which were much lighter in proportion to their trial loads than ordinary bridge girders are compared with the loads which traverse them. As a general rule, *one-sixth* of the breaking strain may be taken as the safe working strain for cast-iron girders which are liable to vibration, as in railway or public bridges, but when the load is stationary and free from all vibration, such as water tanks, *one-fourth* of the breaking strain is safe. When, however, cast-iron girders are liable to sudden severe shocks, as in crane posts or machinery, their working strain should not exceed *one-eighth* of their breaking strain. The railway department of the Board of Trade has laid down the following rule for the guidance of engineers in the construction of railways:—"In a cast-iron bridge the breaking weight of the girders should be not less than three times the permanent load due to the weight of the superstructure, added to six times the greatest moving load that can be brought upon it." Notwithstanding this rule, engineers will do well not to design cast-iron girders for railway bridges of less strength than six times the total maximum load, that is, six times the permanent load added to six times the greatest moving load. The reader who desires detailed information respecting the practice of our most eminent engineers during the reign of cast-iron is referred to the evidence attached to the "Report of the Commissioners appointed to inquire into the application of iron to railway structures" in 1849. It seems certain that the transverse strength of thick rectangular cast-iron bars is less than that of thin ones (**132**), but it does not necessarily follow that the strength of large flanged girders is diminished by the massiveness of the casting, or that they are relatively weaker than smaller girders of similar section, for the quality of the iron will, no doubt, materially influence their strength (**348, 349**).

Experiments on a large scale can only decide these questions, which, however, have less importance now than when the Iron Commission sat in 1849, as it is very unlikely that large cast-iron girders will be employed in important works when wrought-iron is available.

Cast-iron can be readily got, on specification, to stand from $7\frac{1}{2}$ to 9 tons per square inch in tension; consequently, the rule of one-sixth allows an inch-strain of from $1\frac{1}{4}$ to $1\frac{1}{2}$ tons for the usual safe tensile working-strain in the lower flanges of cast-iron girders, but this material is quite unfitted for tie-bars for the reasons referred to in **350** and **351**. Cast-iron will safely bear 6 or 7 tons per square inch in compression, provided it be in a form suited to resist flexure; but the effects of flexure will seriously diminish the safe unit-strain for pillars or unbraced cast-iron arches, in which the line of pressure may vary so as to alter the calculated unit-strain very materially, perhaps as much as 50, or even 100 per cent. In practice, the safe working-strain of cast-iron arches rarely exceeds 3 tons per square inch. For instance, the calculated working strain in the Severn Valley Bridge carrying the Coalbrookdale Railway, 200 feet span and 20 feet rise, is between $2\frac{1}{2}$ and 3 tons per square inch,* while that of the centre arch of Southwark Bridge, 240 feet span, is about 2 tons per square inch.

The French ministerial limit of working strain for cast-iron in tension is one kilogramme per square millimetre (= 0·635 tons per square inch), and in compression five kilogrammes per square millimetre (= 3·175 tons per square inch), and the following table, prepared by M. Poirée, engineer of Ponts et Chaussées, illustrates some of the best French practice in cast-iron arches.†

* *Proc. Inst. C. E.*, Vol. xxvii., p. 109.

† Morin's *Résistance des Matériaux*, p. 114.

TABLE II.—EXAMPLES OF WORKING STRAINS IN CAST-IRON BRIDGES IN FRANCE.

Name of Bridge.	Number of arches.	Number of ribs in each arch.	Distance between the ribs.	Method of construction	Weight of one span, including everything approximately.	Span of each arch.	Rise or Versine of arch.	Depth of rib.	Sectional area of all the ribs in one arch.	Pressure per square millimetre from the permanent weight of the structure.	Pressure per square millimetre, adding the occasional live load to the permanent weight of the structure.
			metres.		tons.	metres.	metres.	metres.	s. metres	kils.	kils.
Bridge of Austerlitz, Paris, -	5	7	1·95	Ribs of openwork voussoirs.	623	32·30	3·23	1·25	0·212	3·95	4·40 with a load of 200 kils. per square metre, (=41 lbs. per square foot).
Bridge of Carrousel, Paris, -	3	4	2·80	Ribs of elliptic tubes (système Polonceau).	546	47·00	4·90	0·84	0·38	1·9	2·31 Idem.
Viaduct over the St. Denis Canal, (Chemin de fer du Nord),	1	4	2·10 and 1·30	Idem.	246	31·22	3·45	0·84	0·294	1·03	1·81 with a load of 6,000 kils. per metre run of the bridge (= 1·8 tons per foot run).
Viaduct of Villeneuve-St.-Georges, (Chemin de fer de Lyon),	3	7	1·34	Ribs of I section.	363	15·00	1·50	0·55 at key. 0·70 at springing.	0·241 0·281	1·73 1·88	2·59 2·81 Idem.
Viaduct of the Mée, (Idem), -	3	7	1·34	Idem.	824	40·00	5·00	1·75	0·508	1·81	2·34 Idem.
Viaduct of the Basin of Charenton, (Idem),	2	7	1·34	Idem.	700	35·00	4·00	1·00	0·345	1·81	2·34 Idem.
Viaduct of Bernières, (Chemin de fer de Troyes),	3	6	1·13 and 1·50	Idem.	213	22·00	2·45	0·50	0·22	1·19	1·95 Idem.
Viaduct of Montereau, (Idem), -	4	6	1·13 and 1·50	Idem.	240	24·60	3·13	0·50	0·22	1·19	1·95 Idem.
Viaduct of Nevers, (Chemin de fer du Centre),	7	7	1·31	Idem.	800	42·00	4·55	1·15	0·50	2·00	2·70 Idem.
Viaduct of the Rhone, (Chemin de fer d'Avignon à Marseille),	7	8	1·25	Idem.	1,800	60·00	5·00	1·70	1·016	2·8	3·36 Idem.
Viaduct of la Mulatière, Lyons, (Chemin de fer de St. Etienne and road from Perrache to la Mulatière),	4	9	1·20 and 1·70	Ribs of elliptic tubes (système Polonceau).	600	40·14	4·50	0·09	0·720	1·00	1·44 Idem.

NOTE.—One kilogramme per square millimetre = 0·635 ton per square inch.

The direct tensile strength of cast-iron may be tested in the manner described in **482**, but it is also usual to prove its transverse strength by breaking small rectangular bars made of the same metal and at the same time as the principal castings. The following tests were applied in the case of the cast-iron sleepers provided for the Great Indian Peninsula Railway. "The mixture of metal is to be such as will produce the strongest and toughest castings, and is to be approved as such by the consulting engineer. The contractor must cast twice each day, from the same metal as that used in the sleepers, two duplicate bars 3′ 6″ × 2″ × 1″, and two duplicate castings of the form shown on the contract drawing, and exactly 1″ square for a length of $1\frac{1}{2}$″ in the middle. One of the two bars must be tested on edge, on bearings 3 feet apart, by placing weights on the centre thereof, to ascertain its elasticity and breaking weight; and one of the two castings must be tested in a suitable machine of approved construction to ascertain the tensile strength of the iron. The company's inspector will reject all sleepers cast on any day when each of the bars will not bear 30 cwt. placed on the centre without breaking, or when each bar does not deflect at least 0·29 of an inch before fracture, and when each casting will not bear a tensional strain of $11\frac{1}{2}$ tons per square inch of section. Three sleepers will also be tested each day by a weight of $3\frac{3}{4}$ cwt. falling through 5′ 6″, the same having previously been subjected to blows from the same weight falling through 2′ 0″, 2′ 6″, 3′ 0″, 3′ 6″, 4′ 0″, 4′ 6″, and 5′ 0″ successively after the sand foundation (which shall not be more than 24 inches thick under the centre of the sleeper and laid on a cast-iron bed plate 8 inches thick, and weighing 2 tons,) has been well consolidated to the satisfaction of the consulting engineer or his inspector; and whenever every sleeper so tested does not bear these blows without cracking, or showing other signs of failure, the day's make will be rejected. Immediately after every sleeper is cast, it must be protected in a manner which will satisfy the company's engineer, that the process of cooling will proceed so slowly, that its strength will not in any degree be diminished by too rapid or unequal cooling."* Some engineers consider this proof

* *Proc. Inst. C. E.*, Vol. xxx., p. 225.

rather high, and specify that test bars, 2 × 1 inch, placed edgeways on bearings 3 feet apart, shall support a weight on the centre of 25 cwt., as it appears that sleepers can be obtained which would stand better, as far as blows went, without using so high a bar test as that above described. It is a singular fact that there is an excess of about 16 per cent. in the weight that a 2-inch × 1-inch test-bar will support when cast on edge and proved as cast, over that which it will support when proved with the underside as cast placed at the top as proved, and 8 per cent. over the weight which the same test-bar will support if cast on its side or end, and proved on edge.* Hence, cast-iron girders should be cast with the tension flange downwards in the sand.

474. Working load on cast-iron pillars.—Owing to the want of recorded information it is difficult to assign what proportion of the breaking weight eminent engineers have considered to be the safe working load for cast-iron pillars. The opinions elicited by the Commissioners appointed to inquire into the application of iron to railway structures throw little or no light on the matter, as the evidence was chiefly confined to the strength of girders under transverse strain. Navier† gives 1-5th of the breaking weight as the safe load in practice. Francis,‡ an American engineer, also gives 1-5th; while Morin§ adopts 1-6th. My own experience leads me to recommend that cast-iron pillars supporting loads free from vibration, such as grain, should in general not be loaded with more than 1-6th of their calculated breaking weight. In factories or stores, where strong vibrations from machinery occur, the working load should not exceed 1-8th; and if the pillar be liable to transverse strains, or severe shocks, like those on the ground floors of warehouses where loaded waggons or heavy bales are apt to strike against them, the load should not exceed 1-10th of the breaking weight, or even less when the strength of the pillar depends rather on the transverse strain to which it is liable than

* *Proc. Inst. C.E.*, Vol. xxx., pp. 228, 267.

† *Application de la Mécanique*, p. 204.

‡ *On the Strength of Cast-iron Pillars*, p. 17. New York, 1865.

§ *Résistance des Matériaux*, p. 106.

the weight it has to support. For instance, the pressure of wind against a light open shed, supported by pillars, may produce a transverse strain which will be very severe compared with that due to the mere weight of the roof. The same thing may occur if heavy rolling goods, such as casks or loaves of sugar, are piled up against the pillar in such a manner as to cause horizontal pressure like that of a liquid. It is also necessary to take into consideration the foundations on which the pillars rest, for if these yield unequally, one pillar may sustain much more than its proper share of load. Wrought-iron is gradually superseding cast-iron for struts in machinery; when, however, cast-iron is adopted, it is well that the working load should, at all events, not exceed 1-10th of the calculated breaking load. In all these cases it is essential to consider carefully whether the pillar is flat bedded or very securely fixed at the ends, as a slight imperfection in this respect, either immediate or prospective, will reduce the strength to one-third in long pillars, and somewhat less in medium pillars, and if there is any doubt whatever on this point it will be only common prudence to assume in the calculations that the pillar is imperfectly bedded (**311, 312**). The reader will find practical rules for the thickness of hollow cast-iron pillars in **324**, and examples of calculation from **322** to **329**.

WROUGHT-IRON.

475. Effects of repeated deflections on wrought-iron bars and plate girders.—Sir Henry James and Captain Galton made some experiments in Portsmouth Dockyard for determining the effects produced by repeated deflections on wrought-iron bars.* These experiments were made with cams caused to revolve by steam machinery, which alternately depressed the bars and allowed them to resume their natural position for a great number of times. Two cams were used; one was toothed on the edge so as to communicate a highly vibratory motion to the bar during the deflection; the other, a step cam, first gently depressed the bar and then released it suddenly when the full deflection had been obtained. The depressions were at the rate of from four to seven per minute, and the following table gives the principal results:—

* *Rep. of Iron Com., App. B.*, p. 259.

TABLE III.—EXPERIMENTS ON REPEATED DEFLECTIONS OF WROUGHT-IRON BARS, 2 INCHES SQUARE AND 9 FEET LONG BETWEEN POINTS OF SUPPORT.

No. of experiment.	Amount of deflection in inches.	Number of depressions.	Permanent set in inches.	Remarks.
1	·833	100,000	0·015	Toothed cam.
2	·83	10,000	0·	Step cam.
3	1·00	10,000	0·06	Do.
4	2·00	10	0·30	Do.
		50	0·54	Do.
		100	0·69	Do.
		150	0·84	Do.
		200	0·98	Do.
		300	1·84	Do.

The following experiments were made for the purpose of comparison to determine the deflections due to statical loads at the centre of a similar bar.

TABLE IV.—EXPERIMENTS ON A WROUGHT-IRON BAR, 2 INCHES SQUARE AND 9 FEET LONG BETWEEN POINTS OF SUPPORT, SHOWING THE STATICAL WEIGHTS DUE TO GIVEN DEFLECTIONS, THE WEIGHTS BEING APPLIED AND THE DEFLECTIONS MEASURED AT THE CENTRE.

Deflections in inches.	Weights in lbs.	Permanent set.	Remarks.
·333	507	0	After the bar had 1,950 lbs. on, it suddenly gave way, and although it did not break, no further weight could be applied with certainty.
·666	926	0	
·833	1,121	0	
1·00	1,364	0·054	
1·80	1,950	0·86	

In these experiments two things are worthy of note; first, the largest deflection which did not produce a permanent set appears to be that due to rather more than one-half the statical weight which crippled the bar: secondly, 10,000 depressions with the step cam, causing a deflection of 1 inch, produced almost exactly the same permanent set as the statical weight due to the same deflection of 1 inch.

With the view of arriving "at the extent to which a bridge or girder of wrought-iron may be strained without injury to its ultimate powers of resistance, and to imitate as nearly as possible the strain to which bridges are subjected by the passage of heavy railway trains," Sir William Fairbairn caused a weighted lever to be lifted off and replaced alternately, by means of a water-wheel, upon the centre of a wrought-iron single-webbed plate girder of the usual construction, with double angle-irons and flange-plates riveted on top and bottom respectively. The dimensions of the girder were as follows:*—

Extreme length, - - - -	22 feet.
Length between supports, - -	20 feet.
Extreme depth, - - - -	16 inches.
Weight of girder, - - - -	7 cwt. 3 qrs.

	Square inches.	
Area of top flange, 1 plate, 4 inches $\times \frac{1}{2}$ inch, -	2·00	
,, ,, 2 angle-irons, $2 \times 2 \times \frac{5}{16}$, -	2·30	
		4·30
Area of bottom flange, 1 plate, 4 inches $\times \frac{1}{4}$ inch,	1·00	
,, ,, 2 angle-irons, $2 \times 2 \times \frac{3}{16}$,	1·40	
		2·40
Web, 1 plate, $15\frac{1}{4} \times \frac{1}{8}$ inch, - - - - -		1·90
Total sectional area in square inches, - - -		8·60

The area of the $\frac{1}{2}$ inch rivet holes in the bottom flange, two in each angle-iron and two in the plate, is equal to ·625 square inches, which reduces the effective flange area for tension from 2·4 to 1·775 square inches. The web being continuous gave some aid to the flanges, but as it was composed of 9 short plates with vertical joints and single-riveted covering strips, the amount of aid given to the tension flange probably did not exceed one-half the theoretic aid of a perfectly continuous web (**100**), that is, it probably equalled one-twelfth of the gross area of the web, or

* *Useful Information for Engineers,* third series, p. 301.

0·158 square inches; adding this to the net area of the bottom flange, we have a total of $1·775 + 0·158 = 1·933$ square inches available for tension, and assuming the tearing strength of the iron to have been 20 tons per square inch, and the depth for calculation to be taken from inside to inside of the angle-iron flanges, which measures $14\frac{3}{4}$ inches, we have the breaking weight in the centre, from eq. 18, as follows:—

$$\mathsf{W} = \frac{4\mathsf{F}d}{l} = \frac{4 \times (20 \times 1·933) \times 14·75}{240} = 9·5 \text{ tons.}$$

The compression flange, it will be observed, was much stronger than that in tension, and hence it may be supposed that a larger fraction than one-twelfth of the web should be added to the lower flange (**428**). The extra strength on this account must, however, have been very small and could scarcely raise the breaking weight beyond 10 tons. Sir William Fairbairn, however, calculated the breaking weight at 12·8 tons by an empirical formula derived from the model tube at Millwall. The following table contains a summary of the experiments with the corresponding tensile strains, calculated on the supposition that 10 tons was the true statical breaking weight at the centre, and that 20 tons per square inch was the tearing strength of the iron.

TABLE V.—EXPERIMENTS ON REPEATED DEFLECTIONS OF A SINGLE-WEBBED PLATE-IRON GIRDER, 16 INCHES DEEP AND 20 FEET LONG BETWEEN POINTS OF SUPPORT.

No. of experiment.	Weight on middle of girder.	No. of changes.	Deflection.	Tensile strain per square inch of net area of bottom flange.	Remarks.
	tons.		inches.	tons.	
1	2·96	596,790	0·17	5·92	Above half a million changes, working continuously for two months, night and day, at the rate of about eight changes per minute, produced no visible alteration.
2	3·50	403,210	0·23	7·00	One million changes and no apparent injury.
3	4·68	5,175	0·35	9·36	Permanent set of ·05 inches; broke by the tension flange tearing across a short distance from the middle. None of the rivets loosened or broken.

Girder repaired by replacing the broken angle-irons on each side, and putting a patch over the broken plate equal in area to the broken plate itself.

No. of experiment.	Weight on middle of girder.	No. of changes.	Deflection.	Tensile strain per square inch of net area of bottom flange.	Remarks.
	tons.		inches.	tons.	
4	4·68	158	—	9·36	Apparatus accidentally set in motion; took a large but unmeasured set.
5	3·58	25,742	0·22	7·16	———
6	2·96	3,124,100	0·18	5·92	No increase of deflection or permanent set.
7	4·00	313,000	0·20	8·00	Broke by failure of the tension flange as before, close to the plate riveted over the previous fracture. Total number of changes after repair = 3,463,000.

These experiments seem to indicate that a constantly repeated tensile strain of 6 or 7 tons per square inch will not injure wrought-iron, but, as the actual breaking weight of the girder was not determined after each experiment, we cannot be quite certain whether the strength was really impaired or not by the lesser strains. To carry out the experiment scientifically would have required several girders to be broken by dead weight—one when new, as a standard for comparison; and each of the others after a few million changes of the same amount in any one girder, but of different amounts in successive girders.

476. Net area only effective for tension—Allowance for the weakening effect of punching—Rule of Board of Trade for wrought-iron railway bridges—Tensile working strain of wrought-iron—French rule for railway bridges.—The reader will recollect that the whole area of a riveted plate is not available for tension, but only the unpierced portion which lies between the rivet holes in any line of transverse section; this is called the *net* area of the plate, and on this net area alone the working tensile strain should be calculated. The *effective* tensile area of a punched plate is, indeed, somewhat less than its net area, for the tearing strength of iron is generally injured by punching, especially if there be too great a clearance between the punch and die, or if the iron be brittle and, though it is not the practice, it would be more correct to diminish the gross section by the sum of the rivet holes multiplied by a factor greater than unity, perhaps 1·1, or 1·2. It may, perhaps, be supposed more accurate to add a constant quantity, say $\frac{1}{8}$th inch, to the diameter of each hole in place of adding a percentage, but it is probable that the weakening effect of punching is greater the thicker the plate, and as thick plates have generally larger rivet holes than thin ones, the percentage allowance will be more accurate in practice. Good experiments on this subject are much wanted. Meantime, the weakening effect of punching affords an argument in favour of drilling holes, especially in hard and brittle materials. Punching will probably do little injury to soft and ductile iron, or to mild steel, especially when the latter is subsequently annealed (**462**). The following rule has been laid down by the Board of Trade for the strength of railway bridges. "In a wrought-iron bridge the greatest load which can be brought upon it, added to the weight of the superstructure, should not produce a greater strain on any part of the material than 5 tons per square inch." This rule is now confined to parts in tension, in which case the 5 tons is computed on the *net* area only, while the usual limit of strain in the compression flanges is 4 tons per square inch of *gross* area, and, as the tearing and crushing strengths of ordinary plate iron are respectively 20 and 16 tons per square inch, the foregoing rules are equivalent to stating that one-fourth of the breaking strain is the maximum safe working strain for wrought-iron girders which are subject to vibration like railway bridges, and this is now the recognized English practice. When wrought-iron girders support a dead load, like water tanks or grain lofts, they will safely bear one-third of their breaking strain, but when liable to sudden severe shocks, as in gantries or cranes, the working strain should not exceed one-sixth of the computed breaking strain.

The safe tensile working strain for ordinary bar, angle, or tee iron in girder-work is generally the same as for plates, namely, 5 tons per square inch of net section, but bar iron of extra quality, such as the links of suspension bridges, will safely bear 6 tons per square inch. Special care is taken with the manufacture of this class of iron, and it is customary to prove each link individually to a strain of from 8 to 10 tons per square inch before it is admitted into the suspension chain, the tearing strength of the iron being not less than 24 tons per square inch. For merely temporary purposes wrought-iron will bear safely a tensile strain of 9 tons per square inch, unless when subject to violent shocks, in which case 6 tons will be sufficient.

The French rule for wrought-iron railway bridges is that in no part shall the strain, either of tension or compression, exceed 6 kilogrammes per square millimetre, *i.e.*, 3·81 tons per square inch of gross section.

477. Gross area available for compression—Compressive working strain of wrought-iron—Flanges of wrought-iron girders are generally of equal area.—The total sectional area of a riveted plate is available for compression (flexure being duly provided against), since the thrust is transmitted through the rivet just as if it were a portion of the solid plate, for, if the rivet head be properly hammered up, its shank will upset and fill the hole completely. Even supposing that the rivet do not perfectly fill the hole, an exceedingly small motion of the parts, which must take place before crushing commences, will cause the strain to pass through the shank. In practice, however, the longitudinal contraction of each rivet in cooling will produce an amount of friction between the surfaces riveted together which is generally sufficient to resist any movement so long as the strain lies within the usual working limits (**466**). The crushing strength of wrought-iron is generally taken at 16 tons per square inch (**297**), and the safe limit of compressive working strain in girder-work is, according to ordinary English practice, 4 tons per square inch over the gross area, provided the section is so large that it can without extra material be put into a form suitable for resisting flexure or buckling. This is generally the case with the compression flanges of girders. When, however, a thin sheet, like the web of a plate girder, sustains compression, or when the theoretic section of a strut is small, as in the compression bars of a braced web, it is necessary to add additional material to prevent flexure or buckling. Angle, tee, or channel iron are suitable for plate stiffeners or for short struts; for long struts the plan of internal cross-bracing, represented in Plate IV., may be advantageously adopted, the cross-bracing, of course, not being measured as effective area to resist crushing, since it merely keeps the sides in line, but sustains none of the longitudinal thrust, and in small scantlings it will be prudent to limit the maximum compressive working strain to 3 tons per square inch. The working strain of wrought-iron pillars, when subject to shocks, like the jib of a crane, should not exceed 1-6th of the computed breaking weight; with quiescent loads 1-4th is a safe rule. The reader is referred to **330** and the following articles for the mode of calculating the strength of wrought-iron pillars of various sections.

When wrought-iron arches have braced spandrils, the ribs are free from transverse strain and will safely bear as high longitudinal strains as the flanges of girders, but if the spandrils are not braced, the line of pressure in the ribs may vary under the influence of passing loads and thus double, or even treble the normal working strain (**219**). The extreme compressive strains, produced by the most unfavourable combination of circumstances in the wrought-iron arched ribs of the Victoria Railway Bridge, in four spans of 175 feet each, which was designed by Mr. John Fowler, are said in no case to exceed $4\frac{1}{2}$ tons per square inch.*

The flanges of wrought-iron girders are generally made of equal or nearly equal area, for the deduction for rivet holes in the tension flange is compensated by the higher unit-strain in the net area between the holes which is effective for tensile strain.

478. Shearing working strain—Pressure on bearing surfaces—Knife edges.—The shearing strength of wrought-iron is substantially the same as its tensile strength (**394**), from which it follows that the shearing working strain of iron rivets or bolts in ordinary girder-work may equal 5 tons per square inch of section,

* *Proc. Inst. C.E.*, Vol. xxvii., p. 67.

but, as already stated in **462**, the rivet area of a tension joint is usually about 10 per cent. in excess of what this rule allows, in order to compensate for accidental inequalities in the distribution of strain among the rivets. When calculating the area of a plate web from the total shearing-strain in the manner described in **54**, it is a safe rule to adopt 4 tons per sectional inch of web as the *maximum* shearing unit-strain, but this rule gives no idea of the amount of material requisite for stiffening the web, and which can only be determined by experience in each separate case (**430**). The bearing surface of a round bar, such as the pin or bolt of a flat link, is measured by the product of its diameter by the length of bearing, and it appears from the experiments referred to in **461**, that the statical working pressure on a bearing surface of wrought-iron may equal 1·5 times the safe tensile strain, that is, it may equal 7·5 tons per square inch of bearing surface. The pressure of rivets in double-shear against the middle plate, supposing friction does not affect the bearing pressure (**466**), is often double of this, and the pressure of the links of a chain against each other must also be far greater. The rule of the Board of Trade for the steel knife edges of public chain-testing machines requires that the pressure shall not exceed 5 tons per linear inch of knife edge. In my own practice I have frequently put a pressure of 10 tons on each linear inch, and occasionally 17 tons, and found no bad effects.

479. Working-strain of boilers—Board of Trade rule—French rule.—The working load of fresh water boilers should not exceed one-sixth of their bursting pressure, though locomotives are occasionally worked (very unsafely) to one-fourth. One-seventh of the bursting pressure seems a proper working load for salt water boilers, as they are liable to greater hardship than fresh water boilers. The following table will illustrate these rules in a convenient form, applied to parts in tension; the strains are given in tons per square inch of *gross* area. The method of calculating the strength of boiler flues is explained in Chap. XIII.

TABLE VI.—Tensile Working-strain of Boilers.

	"Best best" boiler plate.			Common boiler plate.		
	Tearing strain per square inch of gross area.	Working strain per square inch of gross area.		Tearing strain per square inch of gross area.	Working strain per square inch of gross area.	
		Fresh water boilers.	Salt water boilers.		Fresh water boilers.	Salt water boilers.
	tons.	tons.	tons.	tons.	tons.	tons.
Wrought-iron plates, unpunched,	22	—	—	20	—	—
Do. do., single-riveted, (strength = 50 per cent. of that of the unpunched plate),	11	1·833	1·57	10	1·667	1·43
Do. do., double-riveted, (strength = 70 per cent. of that of the unpunched plate),	15·4	2·567	2·20	14	2·333	2·00

Some engineers allow for single-riveted joints one-fifth greater working strain than is given in the table, in consequence of the additional strength supposed to be derived from the plates breaking joint with each other, but I am not aware of any experiments which support this view. The oral rule of the Board of Trade Surveyors for marine boilers is that their tensile working strain shall not exceed 6,000 lbs., = 2·678 tons, per square inch of gross section; for example, the working pressure of a cylindrical boiler of $\frac{3}{4}$ inch plates, 12 feet in diameter, and double-riveted along the longitudinal joints, should not exceed 62·5 lbs. per square inch.

General Morin states that according to a French royal decree the working strain of plate-iron in boilers shall not exceed 1·9 tons per square inch.*

480. Working strain of engine-work.—In engine and wheel-work it is generally safe practice to proportion the moving parts so that their working strain shall not exceed one-tenth or one-twelfth of that which would break or cripple them; for instance, the working strain of screw bolts in engine-work is generally limited to about 4,000 lbs. per square inch of net section, and the same rule is applied to piston and connecting rods when in tension

* *Résistance des Matériaux*, p. 20.

merely; when in compression, one ton, or 2,240 lbs. per square inch, is an ordinary rule, though, properly speaking, the safe working strain will depend on the strength of the rod to resist flexure, and will therefore vary, like that of other pillars, with the ratio of length to diameter.

481. Examples of working strain in wrought-iron girder and suspension bridges.—The following tables contain examples of the working strains in some important wrought-iron girder and suspension bridges. Several of the suspension bridges in Table VIII. have toll-gates which prevent the occasional load from reaching so high as 80 lbs. per square foot of platform. There are also regulations to prevent horses or vehicles from going faster than a walking pace. See "Working Load on Public Bridges" near the end of this chapter.

TABLE VII.—EXAMPLES OF WORKING STRAINS IN WROUGHT-IRON GIRDER BRIDGES.

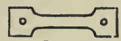
No.	Name of Bridge.	Date.	Engineer.	Working inch-strain in Flanges, from permanent weight of structure.		Working inch-strain in Flanges, when loaded with 1 ton per running foot on each line.		Clear span, or largest span if more than one.	Tearing strength of the iron per square inch.	Observations.
				Tension.	Compression.	Tension.	Compression.			
				tons.	tons.	tons.	tons.	feet.	tons.	
1	Conway.	1849	R. Stephenson.	5·067	3·948	6·32 *	4·924 *	400	19·6	Tubular bridge with plate webs. In calculating the flange strains the webs are taken into account (**44**). * Occasional strain is calculated with $\frac{3}{4}$ ton per running foot of line.
2	Britannia.	1850	Do.	3·1	4·6	—	—	460	19·6	Continuous tubular bridge in 4 spans with plate webs. Strains calculated over land towers, taking webs into account, but not omitting rivet area in the tension flange.
3	Do.	—	—	2·9	2·7	—	—	—	—	Strains calculated at centre of large span, do., do.
4	Newark Dyke.	1852	J. Cubitt.	—	—	5·0	5·0	240$\frac{1}{2}$	—	Warren's girders; top flanges cast-iron tubes; lower flanges wrought-iron flat links; diagonal struts and ties of cast and wrought-iron respectively.
5	Boyne Viaduct.	1855	MacNeill and Barton.	—	—	5·0	4·5	264	20	Wrought-iron lattice girders in 3 continuous spans. Tensile strain calculated for net area of tension flanges.
6	Crumlin Viaduct.	1857	Liddell and Gordon.	—	—	5·75	4·31	148	—	Warren's girder; top flanges rectangular tubes of plate iron; lower flanges wrought-iron bars. Diagonals wrought-iron bars and angle iron.
7	Charing Cross.	1863	Hawkshaw.	—	—	5 †	4 †	154	23 ?	Lattice girders with 4 lines of rail between. † Strain calculated with 1$\frac{1}{4}$ tons per foot on each of four lines. Tearing strength of the iron derived from experiments made with a hydraulic press and therefore perhaps not quite reliable.

[1] Clark *on the Tubular Bridges*, pp. 376, 555, 584, 661, 748.
[2] [3] *Idem*, pp. 376, 587, 785.
[4] *Proc. Inst. C.E.*, Vol. xii., p. 601.
[5] *Idem*, Vol. xiv., p. 443.
[6] *Trans. Inst. C.E. of Ireland*, Vol. vii., p. 97.
[7] *Proc. Inst. C.E.*, Vol. xxii., p. 512, and *Trans. Soc. Eng.*, 1864, p. 170.

TABLE VIII.—EXAMPLES OF WORKING STRAINS IN SUSPENSION BRIDGES.

No.	Name of Bridge.	Date.	Engineer.	Working inch-strain, From permanent weight of structure.	Working inch-strain, when loaded.	Chord of catenary.	Tearing strength of the iron per sq. inch.	Proof strain per sq. inch.	Observations.
				tons.	tons.	feet.	tons.	tons.	
1	Menai.	1826	Telford.	4·21	8·0 if loaded with 80 lbs. per square foot of platform.	580	27	11	Formed of wrought-iron flat bar links.
2	Do. altered after storm of 1839.	1840	Provis.	5·06	8·86 do. do.	—	—	—	About 130 tons of timber and iron-work added for the purpose of strengthening the platform.
3	Hammersmith.	1827	Tierney Clark.	5·38	9·36 do. do.	422¼	—	9	Formed of wrought-iron flat bar links.
4	Pesth.	1849	Do.	5·01	8·11 do. do.	666	—	—	Formed of wrought-iron flat bar links made by Howard and Ravenhill's Patent.
5	Chelsea.	1858	Page.	4·365	8·07 do. do.	348	—	13½	Formed of wrought-iron flat bar links made by Howard and Ravenhill's Patent.
6	Clifton.	1864	Hawkshaw and W. H. Barlow.	2·9	5·03 do. do.	702¼	—	10	Formed of wrought-iron flat bar links. Strain on the suspension rods from maximum load = 4¼ tons per square inch.
7	Argentat.	1829	Vicat.	—	13·4 if loaded with 41 lbs. per square foot of platform.	350½	47	—	12 cables of iron wire No. 18, each wire ·1181 inch in diameter.
8	Niagara.	1855	Roebling.	6·7	8·4 if loaded with 250 tons—viz., a railway train weighing 200 tons, and people and teams weighing 50 tns.	821	44·6	—	Four cables of 10 inches diameter, each composed of 3,640 iron wires of small No. 9 gauge, 18·3 feet to the lb., 60 wires forming one square inch of solid section. Platform is a tube formed of trussed girders, with two floors. The lower floor is used for common travel and the upper one for a single line of railway and side-walks.

1 Drewry *on Suspension Bridges*, pp. 54, 183.
2 *Trans. Inst. C.E.*, Vol. iii., p. 371.
3 Drewry, p. 82, and *Parliamentary Return of Engineers' Reports upon Chelsea Bridge*, 1862, p. 19.
4 *Reports upon Chelsea Bridge*, p. 19, and *Supplement to Theory and Practice of Bridges*. Weale, London.
5 *Reports upon Chelsea Bridge*, pp. 7, 19.
6 *Proc. Inst. C.E.*, Vol. xxvi., p. 243.
7 Drewry, p. 120.
8 *Papers and Practical Illustrations of Public Works of Recent Construction*. Weale.

482. Strength and quality of materials should be stated in specifications—Proof strain of chains and flat bar links—Admiralty tests for plate-iron.—In drawing up specifications for girders, ships, or boiler-work, it is well to specify the tearing strength and quality of the materials. Plates may be tested by tearing asunder samples of the following shape  in a proving machine, several of which are now to be found throughout the kingdom. The amount of elongation of wrought-iron or steel under tensile strain is a test of toughness, a most desirable quality for many purposes, though of little importance in the compression flanges of girders. In my own practice I require the tensile set after fracture (ultimate elongation,) of ship plates and tension plates of girders to be not less than 5 per cent. of their original length, when torn with the grain; at right angles to the grain the set is generally much less, perhaps only 1 or 2 per cent. I also require their tensile strength to be not less than 20 tons per square inch with the grain, and 18 tons across the grain (**352, 353, 356**). In proving cast-iron, care should be taken to round off the arrises of the pin-holes by which the sample is suspended, so that the strain may pass accurately through its axis (**350**). Chains are now tested in proving machines sanctioned by the Board of Trade (**380** to **382**), and it is customary also to prove all the flat bar links of suspension bridges to 9 or 10 tons per square inch, but the proof strain should in no case exceed the limit of elasticity, say 12 tons per square inch, lest the ductility of the iron be impaired and brittleness result (**409**).

The following are the Admiralty tests for wrought-iron ship plates:—

PLATE-IRON (FIRST CLASS).

B.B.

Tensile strain per square inch.	Lengthways, - - - - - -	22 tons.
	Crossways, - - - - - -	18 „

FORGE TEST (*HOT*).

All plates of the first class, of one inch in thickness and under, should be of such ductility as to admit of bending hot, without fracture to the following angles:—

Lengthways of the grain, - - - - -	125 degrees.
Across, - - - - - - - -	90 „

FORGE TEST (*COLD*).

All plates of the first class should admit of bending cold without fracture, as follows :—

With the grain.

1 in. and $\frac{15}{16}$	of an inch in thickness	to an angle of	15	degrees.
$\frac{7}{8}$,, $\frac{13}{16}$	,,	,,	20	,,
$\frac{3}{4}$,, $\frac{11}{16}$	,,	,,	25	,,
$\frac{5}{8}$, $\frac{9}{16}$,, $\frac{1}{2}$	,,	,,	35	,,
$\frac{7}{16}$,, $\frac{3}{8}$	,,	,,	50	,,
$\frac{5}{16}$,, $\frac{1}{4}$	,,	,,	70	,,
$\frac{3}{16}$,, under,	,,	,,	90	,,

Across the grain.

1 in., $\frac{15}{16}$, $\frac{7}{8}$, and $\frac{13}{16}$	of an inch in thickness	to an angle of	5	degrees.
$\frac{3}{4}$ and $\frac{11}{16}$	,,	,,	10	,,
$\frac{5}{8}$, $\frac{9}{16}$, ,, $\frac{1}{2}$	,,	,,	15	,,
$\frac{7}{16}$,, $\frac{3}{8}$	,,	,,	20	,,
$\frac{5}{16}$,, $\frac{1}{4}$	,,	,,	30	,,
$\frac{3}{16}$,, under,	,,	,,	40	,,

PLATE-IRON (SECOND CLASS).

B.

Tensile strain per square inch.	Lengthways, - - - -	20 tons.
	Crossways, - - - -	17 ,,

FORGE TEST (*HOT*).

All plates of the second class of one inch in thickness and under, should be of such ductility as to admit of bending hot, without fracture, to the following angles :—

Lengthways of the grain, - - - - -	90 degrees.
Across, - - - - - - -	60 ,,

FORGE TEST (*COLD*).

All plates of the second class should admit of bending cold without fracture, as follows :—

With the grain.

1 in. and $\frac{15}{16}$	of an inch in thickness	to an angle of	10	degrees.
$\frac{7}{8}$,, $\frac{13}{16}$	,,	,,	15	,,
$\frac{3}{4}$,, $\frac{11}{16}$	,,	,,	20	,,
$\frac{5}{8}$, $\frac{9}{16}$,, $\frac{1}{2}$	,,	,,	30	,,
$\frac{7}{16}$,, $\frac{3}{8}$	,,	,,	45	,,
$\frac{5}{16}$,, $\frac{1}{4}$	,,	,,	55	,,
$\frac{3}{16}$,, under,	,,	,,	75	,,

Across the grain.

$\frac{3}{4}$ in. and $\frac{11}{16}$	of an inch in thickness	to an angle of	5	degrees.
$\frac{5}{8}$, $\frac{9}{16}$,, $\frac{1}{2}$	,,	,,	10	,,
$\frac{7}{16}$,, $\frac{3}{8}$	,,	,,	15	,,
$\frac{5}{16}$,, $\frac{1}{4}$	,,	,,	20	,,
$\frac{3}{16}$,, under,	,,	,,	30	,,

Plates, both hot and cold, should be tested on a cast-iron slab, having a fair surface, with an edge at right angles, the corner being rounded off with a radius of ½ an inch.

The plate should be bent at a distance of from 3 to 6 inches from the edge.

It is intended that all the iron shall stand the forge tests herein named, when taken in four feet lengths, across the grain; and the whole width of the plate, along the grain, whenever it may be necessary to try so large a piece; but a smaller sample will generally answer every purpose.

All plates to be free from lamination and injurious surface defects.

One plate to be taken indiscriminately for testing from every thickness of plate, sent in per invoice, provided they do not exceed fifty in number. If above that number, one for every additional fifty, or portion of fifty.

Where plates of several thicknesses are invoiced together, and there are but few plates of any one thickness, a separate test for plates of each thickness need not be made; but no lot of plates of any one thickness must be rejected before one of that lot has been tested.

"The sample pieces cut from the plate, after having their edges planed, are secured one by one to the cast-iron slab, about 3 or 4 inches from its edge, and are then bent down by moderate blows from a large hammer. The result may be greatly affected by humouring and coaxing on the part of the hammer-man. By striking the iron in the direction of the fibre the workman can make an inferior iron bend with less symptoms of distress than a better iron may exhibit when used more roughly. The same leniency may be shown to the iron by bending it under a steady pressure instead of by blows. The blows should, therefore, be delivered not too lightly, and about square to the surface, and the first signs of fracture should be observed and recorded. The samples for the hot test are heated until they assume an orange colour, and are then bent down to the prescribed angles in the same way as in the cold test."*

STEEL.

483. Working strain for steel—Steel pillars—Admiralty tests for steel plates.—We cannot yet infer from extensive practice what is the safe working strain for steel. Probably one-fourth of the tearing strain, or 8 tons per square inch, is a safe tensile working strain for mild steel plates such as those described in **360**. The most important steel girder bridge which has come

* Reed *on Shipbuilding*, pp. 385, 395.

under my notice is that constructed of puddled steel by Major Adelsköld, of the Royal Swedish Engineers, for the Herljunga and Wenersborg Railway in Sweden. The girder is an inverted bowstring, carrying the railway in one span of 137½ feet over a rapid torrent. "The dimensions are calculated for a strain of 8 tons per square inch, every portion having been tested to 16 tons per square inch before being put in place."* The crushing strength of steel is so high that 12, or even 15 tons, per square inch is perhaps a safe compressive working strain when the material is not permitted to deflect, but when in the form of a solid pillar, the strength of mild steel seems to be only about 1¾ times that of wrought-iron (**336**). Experiments are, however, still wanting to determine this, and, until such are made, it will scarcely be safe to adopt for steel pillars a higher load than 50 per cent. above that which a similar section of wrought-iron would safely carry. The Admiralty tests for steel plates for shipbuilding are as follows:—

Tensile strain per square inch.	Lengthways, - - - -	33 tons.
	Crossways, - - - -	30 „

The tensile strength is in no case to exceed 40 tons per square inch.

FORGE TEST (*HOT*).

All plates of one inch in thickness and under, should be of such ductility as to admit of bending hot, without fracture, to the following angles:—

Lengthways of the grain, - - - - -	140 degrees.
Across the grain, - - - - - -	110 „

FORGE TEST (*COLD*).

All plates should admit of bending cold, without fracture as follows:—

With the grain.	Degrees.	*Across the grain.*	Degrees.
1 inch in thickness to an angle of	30	1 inch in thickness to an angle of	20
⅞ „ „ „	40	⅞ „ „ „	25
¾ „ „ „	50	¾ „ „ „	30
⅝ „ „ „	60	⅝ „ „ „	35
½ „ „ „	70	½ „ „ „	40
7/16 „ „ „	75	7/16 „ „ „	50
⅜ „ „ „	80	⅜ „ „ „	60
5/16 „ „ „	85	5/16 „ „ „	65
¼ „ and under, „	90	¼ „ and under, „	70

The edges should be drilled or sawn, and not punched, in cutting the sample from the plate. In other respects they should be treated as already described for wrought-iron.†

* *The Engineer*, Vol. xxii., p. 240, 1866.
† Reed *on Shipbuilding*, p. 399.

Steel rivets are very brittle and their heads frequently fly off, and accordingly it is usual to unite steel plates with iron rivets, of much larger size, however, than would be required for iron plates of the same thickness.

TIMBER.

484. English, American and French practice—Permanent working strain—Temporary working strain.—The use of timber in important structures is now so rare in the United Kingdom that it is difficult to assign the working strain which English engineers consider safe. At the Landore viaduct, constructed by the late Mr. Brunel of creasoted American pine in compression, with wrought-iron in tension, the timber was generally calculated to bear 373 lbs. per square inch, though in some parts of the structure the strain was allowed to reach 560 lbs., or 50 per cent. more.* At the Innoshannon lattice timber bridge, erected by Mr. Nixon on the Cork and Bandon railway, the *ordinary* working strains in the flanges were 484 lbs. compression, and 847 lbs. tension per square inch. After 16 years' life this bridge was so decayed that it became unsafe and was replaced by a wrought-iron structure in 1862.† In America large timber bridges are still common, and General Haupt, a distinguished American engineer, "has not considered it safe to assign more than 800 lbs. per square inch as a permanent load, and 1,000 lbs. as an accidental load,"‡ and in a paper on American timber bridges, read by Mr. Mosse at the Institution of Civil Engineers in 1863, it is stated that about 900 lbs. per square inch is usually considered by American engineers to be the limit of safe compression for timber framing.§ Navier and Morin, distinguished French authorities, recommend that the working strain of timber should not exceed one-tenth of the breaking strain‖ and, owing to its liability to decay, this rule seems safe practice for structures

* *Proc. Inst. C.E.*, Vol. xiv., p. 500.

† *Trans. Inst. C.E. of Ireland*, Vol. viii., p. 1.

‡ Haupt *on Bridge Construction*, p. 62.

§ *Proc. Inst. C.E.*, Vol. xxii., p. 310.

‖ Navier, p. 103, and Morin, pp. 51, 64, 68.

which are exposed to the weather, but when timber is under cover one-eighth of the breaking strain is a safe working load. For merely temporary purposes a strain of one-fourth of the breaking weight is probably safe, provided there are no shocks, as Mr. Barlow, referring to tensile strain, states that he "left more than three-fourths of the whole weight hanging for 24 or 48 hours, without perceiving the least change in the state of the fibres, or any diminution of their ultimate strength."* With reference to transverse strain, however, Tredgold states that "one-fifth of the breaking weight causes the deflection to increase with time, and finally produces a permanent set,"† and the reader should recollect that the coefficients of rupture of timber, tabulated in **65**, were derived from selected samples of small size and require therefore to be reduced to about one-half when applied to ordinary timber of large size. The method of calculating the strength of timber pillars has been already described in **337** and **338.**

485. Short life of timber bridges—Risk of fire.—In the paper on American timber bridges already referred to, Mr. Mosse states that they do not last in good condition more than 12 or 15 years, the timber being generally unseasoned and shrinking much after being framed. When covered in to protect them from the weather "and cared for, any shrinkage of the braces being immediately remedied, it is believed these bridges will remain in good condition double the usual time, or about twenty-five years." Some of the old Continental bridges, however, lasted much longer than this, but fire seems to be as common an agent of destruction as time in America, where doubtless, the long dry summers give it every advantage.

486. Working load on piles depends more upon the nature of the ground than upon the actual strength of the timber—Working load at right angles to the grain.—As piles in foundations beneath masonry are buried in the ground, which itself supports an uncertain share of the weight of the superstructure, it is impossible to say exactly what weight rests on the pile and how much on the surrounding soil. The piles in the

* Barlow *on the Strength of Materials*, p. 24. † Tredgold's *Carpentry*, p. 57.

foundations of the High Level Bridge at Newcastle, erected by Mr. R. Stephenson, were 40 feet long and driven through sand and gravel till they reached the solid rock. One of these foundation piles was tested with a load of 150 tons, which was allowed to remain several days, and upon its removal no settlement whatever had taken place. The piles are four feet from centre to centre, filled in between with concrete made of broken stone and Roman cement, and the utmost pressure that can come upon a single pile is 70 tons, supposing none of the weight to be carried by the intervening planking and concrete.* The piles in the Royal Border Bridge, erected by Mr. Stephenson over the river Tweed, in 1850, are American elm driven from 30 to 40 feet into gravel and sand; the pressure on each of these is also 70 tons, neglecting any support derived from the intervening soil,† and this is the severest load on piles I find recorded.

Assuming the piles in these two instances to be 15 inches square, and that no part of the weight was supported by the ground between the piles, the pressure does not exceed $\frac{70}{1 \cdot 56} = 45$ tons per square foot, or 700 lbs. per square inch; if, however, the piles were only 12 inches square, the pressure is nearly 1100 lbs. per square inch. Some of the uprights in the lofty scaffolding on which the land spans of the Britannia Bridge were built carried 28 tons per square foot, or 435½ lbs. per square inch. The horizontal timbers, however, were somewhat compressed under this load.‡ The working load on timber piles, surrounded on all sides by the ground, may vary, according to Rondelet, from 427 to 498 lbs. per square inch,§ and Professor Rankine ‖ says:—"It appears from practical examples that the limits of the safe load on piles are as follows:—

"For piles driven till they reach the firm ground, 1000 lbs. per square inch of area of head (= 64·3 tons per square foot).

"For piles standing in soft ground by friction, 200 lbs. per square inch of area of head" (= 12·85 tons per square foot).

* *Encycl. Brit.*, Art. "Iron Bridges," Vol. xii., Part iii., p. 604.

† *Proc. Inst. C.E.*, Vol. x., p. 224.

‡ Clark *on the Tubular Bridges*, p. 549.

§ Morin's *Résistance des Matériaux*, p. 71.

‖ *Manual of Civil Engineering*, p. 602.

Professor Rankine's rule is based on sound principles, for the nature of the ground, and the resistance which it offers to the penetration of the piles, have generally more to do with their safe working load than the strength of the timber has. As far as the latter alone is concerned, we might safely load piles surrounded by the ground with 1-5th of the crushing weight of wet timber, which, according to Hodgkinson's experiments, is equivalent to a load of about 1-10th of the crushing weight of dry timber (**300**). When, however, loaded piles project above the surface of the ground they act in the capacity of pillars, and their strength accordingly should exceed that of piles surrounded by earth. The safe working load of timber at right angles to the grain is about one-third of that lengthways. For instance, 300 ℔s. per square inch is a sufficient load for pine or fir cross-sleepers, and, if we estimate that the pressure from the driving wheel is equal to 8 tons when the engine is running, the bearing surface of the rail in a cross-sleeper road should not be less than from 50 to 60 square inches. Three-fourths of this will probably be sufficient if the sleepers are made of hard wood. A similar rule applies to timber wall-plates, such as those which support the ends of girders.

FOUNDATIONS, STONE, BRICK, MASONRY, CONCRETE.

487. Working load on foundations of earth, clay, gravel and rock.—Professor Rankine states that "the greatest intensity of pressure on foundations in firm earth is usually from 2,500 ℔s. to 3,500 ℔s. per square foot, or from 17 ℔s. to 23 ℔s. per square inch," and that "the intensity of the pressure on a rock foundation should at no point exceed one-eighth of the pressure which would crush the rock."* Foundations should be placed sufficiently deep to protect them from the influence of frost or running water, nor should it be forgotten that excavations and pumping operations in the neighbourhood of buildings frequently cause subsidence of the foundations and superstructure. The following table contains a few examples of heavy pressures on foundations.

* *Civil Engineering*, pp. 380, 377.

TABLE IX.—EXAMPLES OF WORKING LOADS ON FOUNDATIONS.

No.	Name of Bridge.	Date.	Engineer.	Material in foundation.	Loads per square foot.	Observations.
					tons.	
1	Leven and Kent Viaducts, Morecambe Bay, on the Ulverston and Lancaster Railway.	1857	Brunlees.	Sand beneath cast-iron disc piles in an estuary.	4 to 5	Lattice girders resting on cast-iron disc piles, the discs being 2 feet 6 inches in diameter and sunk by forcing water down the centre of the pile. Rubble stone tipped in round the piles prevents the sand from being scoured away by the current.
2	Loch Ken Viaduct on the Portpatrick Railway.	1861	Blyth.	Gravel under the Lake.	$6\frac{1}{4}$	Bowstring girders resting on cast-iron cylinders 8 feet in diameter, and filled with concrete and masonry. A large mound of rubble stone tipped in round the cylinders.
3	Charing Cross Railway Bridge (*London*).	1863	Hawkshaw.	London clay under the Thames.	8	Lattice girders resting on cast-iron cylinders 14 feet diameter below the ground and 10 feet diameter above, filled with Portland cement concrete and brickwork, and sunk from 50 to 70 feet below Trinity high water into the solid London clay. Friction of cylinder not taken into account. Bridge supposed loaded all over with locomotives. The friction in sinking each cylinder amounted to 150 tons. If this be taken into account, it would reduce the pressure on the clay by about 1 ton per square foot.
4	Widening of the Victoria Railway Bridge (*London*).	1866	Fox.	London clay under the Thames.	5	Cast-iron cylinders filled with concrete and brickwork. Friction not taken into account. Bridge supposed loaded all over with locomotives.

[1] *Proc. Inst. C.E.*, Vol. xvii., p. 442. [2] *Ibid.*, Vol xxi., p. 258. [3] *Ibid.*, Vol. xxii., p. 512. [4] *Ibid.*, Vol. xxvii., p. 71.

488. Working load on rubble masonry, brickwork, concrete and ashlar-work.—The crushing strength of building materials has been already given in Chap. XIV. The working load on rubble masonry, brickwork, or concrete, rarely exceeds one-sixth of the crushing weight of the aggregate mass, and this seems a safe practical limit. General Morin, however, states that mortar should not be subject to a greater pressure than one-tenth of its crushing weight.* The ashlar voussoirs of an arch, where the line of thrust may vary considerably from the calculated direction, should not be subjected to a greater (calculated) pressure than one-twentieth of that which would crush the stone. It is safe to apply the same rule to all ashlar-work, as it is very difficult, if not impossible, to command a perfectly uniform pressure throughout the whole bed of each stone, and a slight inequality in the line of pressure may cause splintering or flushing at the joints. Vicat's experiments on plaster prisms (**339**) and the examples of pressure given in the following table, seem to show that the weight on stone columns may sometimes reach as high as one-tenth of the crushing strength of the stone. This, however, is a much severer load than is usual in modern practice and cannot be recommended as very safe.

Ex. What is the safe load per square foot for brickwork in cement, similar to that whose crushing weight is given at p. 238. Here, the crushing weight = 521 lbs. per square inch = 33·5 tons per square foot, and we have,

Answer, Safe working load $= \frac{33 \cdot 5}{6} = 5 \cdot 6$ tons per square foot.

* *Résistance des Matériaux*, p. 51.

TABLE X.—EXAMPLES OF WORKING LOADS ON MASONRY.

No.	Name of the Structure.	Date.	Engineer.	Material.	Load per square foot.	Observations.
					tons.	
1	Pont y tu Pridd Bridge, over the river Taff, in Glamorganshire.[1]	1750	W. Edwards.	Hard grey sandstone rubble, set in Aberthaw lime.	$20\frac{2}{3}$	Single arch 140 feet span. Pressure calculated at crown of arch.
2	Barentin Viaduct, on the Havre Railway.[2]	1845	J. Locke.	Limestone rubble, set in chalk lime.	$3\frac{1}{2}$	Fell from the crushing of the piers, the mortar in the interior being as soft as the first day it was laid. Afterwards rebuilt with brick in hydraulic lime.
3	Lockwood Viaduct, on the Huddersfield and Sheffield Railway.[3]	1849	Hawkshaw.	Flat bedded sandstone rubble, set in Weldon Wood lime.	8	Semicircular arches 30 feet span, with two oblique arches of 40 and 70 feet span respectively. Pressure calculated at base of deepest piers, 7 feet 7 inches thick at the base.
4	Britannia Bridge, Chester and Holyhead Railway.[4]	1850	R. Stephenson.	Anglesey limestone ashlar, set in lime of the same stone.	16	Pressure calculated at base of the Britannia Tower, founded on chlorite schist rock. Crushing weight of the limestone is about 500 tons per square foot. In raising the Conway tube, the pressure on the masonry of the abutments amounted to 50 tons per square foot.
5	Saltash Bridge, Cornwall Railway.[5]	1858	Brunel.	Granite ashlar, set in cement.	10	Pressure calculated at base of middle pier, constructed of ashlar masonry in a wrought-iron cylinder 37 feet diameter and 96 feet high, founded upon rock.
6	Pillars of the dome of St. Peter's (*Rome*).[6]	—	—	Calcareous Tufa, called *Travertin.*	33,330 lbs. = 14·9	Travertin is crushed by about 536,000 lbs., = 239 tons per square foot, or 16 times the working pressure.
7	Pillars of the dome of St. Paul's (*London*).[7]	—	Sir. C. Wren.	Portland stone (oolite limestone).	39,450 lbs. = 17·6	Crushing weight of Portland stone is 537,000 lbs. = 240 tons per square foot, or 13·3 times the working pressure.
8	Pillars of the dome of St. Geneviève (*Paris*).[8]	—	—	Limestone.	60,000 lbs. = 26·8	Crushing weight of this stone is 456,000 lbs., = 203·5 tons per square foot, or 7·6 times the working pressure.
9	Lower courses of the Piers of the Bridge of Neuilly.[9]	—	Perronet.	Limestone.	3,600 lbs. = 1·6	Crushing weight of this stone is 570,000 lbs., = 254 tons per square foot, or 158 times the working pressure.
10	Pillars of the Church of All-Saints (*Angers*).[10]	—	—	Fourneaux stone.	86,000 lbs. = 38·4	Crushing weight of this stone is 880,000 lbs., = 393 tons per square foot, or $10\frac{1}{4}$ times the working pressure.
11	Pillars of the dome of the Pantheon (*Paris*).[11]	—	—	Bagneux stone.	60,000 lbs. = 26·8	Crushing weight of this stone is 496,000 lbs., = 221 tons per square foot, or $8\frac{1}{4}$ times the working pressure. Some of the stones split under the load, which probably is not quite uniformly distributed.
12	Pillar in Chapter House *Elgin*.[12]	—	—	Red sandstone.	40,000 lbs. = 17·9	

[1] *Proc. Inst. C.E.*, Vol. x., p. 241.
[2] *Ibid.*, Vol. xvi., p. 438, and *Weale's Supplement to Bridges*, p. 118.
[3] *Ibid.*, Vol. x., p. 296.
[4] Clark *on the Tubular Bridges*, pp. 370, 542.
[5] *Proc. Inst. C.E.*, Vol. xxi., p. 270.
[6 7 8 9] Mahan's *Elementary Course of Civil Engineering*, p. 28.
[10 11 12] *Encyc. Brit.*, Art. "Stone Masonry," Vol. xx., Part iii., p. 718.

TABLE XI.—EXAMPLES OF WORKING LOADS ON BRICKWORK.

No.	Name of the Structure.	Date.	Engineer.	Material.	Load per square foot.	Observations.
					tons.	
1	Railway Viaduct in Birmingham.	1849	Brunel.	Red Birmingham brick, set in Lias lime mortar.	7	Pressure calculated at top of footing of one of the piers.
2	Charing Cross Railway Bridge (*London*).	1863	Hawkshaw.	London paviours, set in mortar of 1 Portland cement + $2\frac{1}{2}$ sand.	12	Pressure calculated at bottom of the upper length of cylinder 10 ft. in diameter, on the supposition that each of the four lines of rail is loaded with $1\frac{1}{4}$ tons per running foot; see Ex. 3, Table IX.
3	Clifton Suspension Bridge.	1864	Hawkshaw and W. H. Barlow.	Staffordshire blue brick, set in Portland cement.	10	Pressure calculated where chain anchor-plates bear against the brickwork forming the anchorage.
4	Chimney at Adkin's soap works, near Birmingham.	—	—	Brick.	6	Pressure calculated at base. Height 312 feet.
5	Chimney at patent tube works, near Birmingham.	—	—	Do.	$8\frac{1}{2}$	Six large flues cut through base. Height 145 feet.
6	Glass House Cone.	—	—	Brickwork exposed to great heat.	4	Large arches in each of the four sides. Height 75 feet.
7	Chimney at West Cumberland Hæmatite Iron-works.	1867	—	White bricks, set in hydraulic lime.	8	Pressure calculated 2 feet above the ground line. Height above ground line 250 feet. Total height 267 feet.
8	U. S. Light House, "1st order," 160 feet high.	—	—	Brick.	3·7	Pressure calculated on lower courses. Weight that would crush the material = 31·7 tons per square foot, or $8\frac{1}{2}$ times the working load.
9	Merchant's Shot Tower, Baltimore, 246 feet high.	—	—	Do.	6·5	Pressure calculated on lower courses. Crushing weight = 31·7 tons per square foot, or 4·8 times the working load.

[1] *Proc. Inst. C.E.*, Vol. xi., p. 76.
[2] *Ibid.*, Vol. xxii., p. 512, and *Trans. Soc. of Eng.*, 1864, pp. 162, 171.
[3] *Ibid.*, Vol. xxvi., p. 248.
[4 5 6] *Ibid.*, Vol. x., p. 242.
[7] *Trans. Inst. Eng., in Scotland*, Vol. xi., p. 157.
[8 9] *Strength of Materials*, by J. K. Whildin, New York, p. 23.

TABLE XII.—EXAMPLES OF WORKING LOADS ON CONCRETE.

No.	Name of the Structure.	Date.	Engineer.	Material.	Load per square foot.	Observations.
					tons.	
1	Charing Cross Bridge.	1863	Hawkshaw.	Concrete made of Portland cement and Thames gravel, 1+7.	8	See Ex. 3, Table IX.
2	Chimney at West Cumberland Hæmatite Iron Works.	1867	—	Concrete base 3 feet thick, made with hydraulic lime.	2	See Ex. 7, Table XI. Pressure on ground = 1·6 tons per square foot.
3	Base of St. Rollox chimney, Glasgow.		—	Strong concrete or beton, 6 feet thick.	3	450 feet below the summit.

[1] *Proc. Inst. C.E.*, Vol. xxii., p. 515. [2] *Trans. Inst. Eng. in Scotland*, Vol. xi., p. 157.
[3] Rankine's *Civil Engineering*, p. 378.

WORKING LOAD ON RAILWAYS.

489. A train of engines is the heaviest working load on 100-foot railway girders—Three-fourths of a ton per running foot is the heaviest working load on 400-foot girders—Weight of Engines—Girders under 40 feet liable to concentrated working loads.—A train of locomotives, the weight of which generally varies from 1 to 1⅓ tons per running foot, is the heaviest rolling load to which a single-line railway bridge is liable, but it rarely happens in practice that girders are subject to a uniform load of this density, except in short bridges whose length does not exceed that of two engines with their tenders, which may collectively cover from 80 to 100 feet of line. We may therefore safely assume that the maximum strain to which the flanges of railway girders 100 feet in length are subject, does not exceed that due to the permanent bridge-load plus a train-load of from 1 to 1⅓ tons (according to size of engines), per running foot on each line of way. In longer bridges than 100 feet, the train-load per running foot will be less, and in bridges of 400 feet span or upwards, the greatest occasional load can scarcely exceed ¾ ton per running foot on each line, as this is a denser load than that of an ordinary goods train.*

Until lately it has been usual to take one ton per running foot on each line as the ruling load for engines. This, however, is scarcely safe practice, since many engines now exceed this, as shown by the following tables, for the first of which I am indebted to A. M'Donnell, Esq., Locomotive Superintendent of the Great Southern and Western Railway, Ireland, and for the second to J. Ramsbottom, Esq., late Locomotive Superintendent of the London and North Western Railway.

* The following memorandum shows the weight of a train of wagons loaded with sulphur ore on the Dublin, Wicklow and Wexford Railway :—

"Weight of mineral engine loaded, 27 tons.

——————— tender do. 17 do.

Length of engine and tender, buffer to buffer, 44 feet.

Wagon, empty 4 tons, loaded 12 tons ; length 18 feet, out to out of buffers. Two other descriptions of wagons, one 12 feet, and the other 14 feet 6 inches long, taking one ton less and weighing about 5 cwt. less. A mineral train of engine, 20 wagons and van, will weigh about 280 tons and its length will be about 400 feet when buffers are close up ; when running, somewhat longer."

TABLE XIII.—EXAMPLES OF ENGINES ON THE GREAT SOUTHERN AND WESTERN RAILWAY, IRELAND—GAUGE 5′ 3″.

Class of Engine.	No. of Wheels.	Distance from Leading to Middle Wheels.	Distance from Middle to Trailing Wheels.	Wheel base.	Weight on Wheels. Leading.	Weight on Wheels. Middle.	Weight on Wheels. Trailing.	Total weight.	Length over Buffers.	Total weight of Engine and Tender.	Total length of Engine and Tender.	Observations.
		ft. in.	ft. in.	ft in.	tns. cts.	tns. cts.	tns. cts.	tns. cts.	ft. in.	tns. cts.	ft. in.	
Single passenger, 6′ 6″ wheel.	6	7 6	7 4	14 10	9 6	11 8	5 0	25 14	23 0	—	—	2 cwt. of coal in firebox and 6 inches of water in glass.
Passenger, four wheels coupled, 5′ 7″ wheel.	6	6 0	7 11	13 11	9 7	10 0	9 0	28 7	23 3	—	—	2 cwt. of coal in firebox and 6½ inches of water in glass.
Passenger, four wheels coupled, 6′ 6″ wheel.	6	7 0	7 9	14 9	9 12	12 8	10 6	32 6	24 9	—	—	2 cwt. of coal in firebox and 7 inches of water in glass.
Tender for last.	6	5 2	5 2	10 4	6 9	8 12	6 9	21 10	18 4	53 16	43 1	Full tank and 45 cwt. of coal.
Goods, 4 wheels coupled, 5′ 0″ wheel.	6	8 3	6 11½	15 2½	11 6	11 2	5 11	27 19	24 2½	—	—	2 cwt. of coal in firebox and 6 inches of water in glass.
Goods, 6 wheels coupled, 5′ 1¼″ wheel.	6	7 3	8 3	15 6	11 4	10 18	10 18	33 0	24 10½	—	—	3 cwt. of coal in firebox and 6 inches of water in glass.
Tender for last.	6	5 2	5 2	10 4	7 12	6 15	6 11	20 18	18 6	53 18	43 4½	Full tank and 3 tons of coal.
Do., empty.	—	—	—	—	—	—	—	11 2	—	—	—	—
Tank engine, 6 wheels coupled, 4′ 7″ wheel.	6	7 3	7 7	14 10	9 16	12 0	14 14	36 10	29 2	—	—	Full boiler and tanks and 30 cwt. of coal in bunker, but when used for banking the tanks and bunker are seldom quite full and the weight = 34 tons.
Do., empty.	—	—	—	—	9 12	8 7	10 6	28 5	—	—	—	

There are some engines on the Midland Great Western Railway of Ireland which weigh 28 tons, and stand on a 12 feet base. Mr. Price, the Resident Engineer, informs me that they overhang behind and kick dreadfully, and he regards them as the most trying of their stock on the road and girder bridges, especially on small ones.

TABLE XIV.—EXAMPLES OF ENGINES ON THE LONDON AND NORTH WESTERN RAILWAY—GAUGE 4′ 8½″.

Class of Engine.	No. of Wheels.	Distance from Leading to Middle Wheels.	Distance from Middle to Trailing Wheels.	Wheel base.	Weight on Wheels. Leading.	Weight on Wheels. Middle.	Weight on Wheels. Trailing.	Total weight.	Length over Buffers.	Total length of Engine and Tender.	Total weight of Engine and Tender.	Remarks.
		ft. in.	ft. in.	ft. in.	tns. cts.	tns. cts.	tns. cts.	tns. cts.	ft. in.	ft. in.	tns. cts.	
Single passenger, 7′ 6″ wheel.	6	7 7	7 10	15 5	9 8	11 10	6 2	27 0	23 8½	44 7	44 8½	Engine and tender A in working order.
	—	—	—	—	9 9	8 0	6 8	23 17	—	—	—	Do. Empty.
Passenger, four wheels coupled, 6′ 6″ wheel.	6	7 5	8 3	15 8	9 9	11 0	8 15	29 4	25 11½	46 10	46 12½	Engine and tender A in working order.
	—	—	—	—	9 4	9 6	8 15	27 5	—	—	—	Do. Empty.
Single passenger, 7′ wheel.	6	8 4	8 6	16 10	10 19	12 10	6 9	29 18	26 0½	45 6½	47 17	Engine and tender C in working order.
	—	—	—	—	9 9	11 1	5 18	26 8	—	—	—	Do. Empty.
Single passenger, 6′ 6″ wheel.	6	7 9	7 9	15 6	9 0	12 0	6 10	27 10	23 4½	42 10½	45 9	Engine and tender C in working order.
	—	—	—	—	8 0	11 0	5 3	24 3	—	—	—	Do. Empty.
Single passenger, 6′ wheel.	6	6 10	6 6	13 4	7 13	9 3	3 19	20 15	21 7	42 5½	38 3½	Engine and tender A in working order.
	—	—	—	—	7 0	8 12	3 10	19 2	—	—	—	Do. Empty.
Goods, 6 wheels coupled, 5′ wheels.	6	7 3	8 3	15 6	9 14	10 0	7 6	27 0	24 5½	45 4	48 8	Engine and tender B in working order.
	—	—	—	—	8 8	9 14	6 6	24 8	—	—	—	Do. Empty.
Tank, 6 wheels coupled, 5′ wheels.	6	6 11	8 7	15 6	10 11	12 16	9 0	32 7	29 2	—	—	Engine in working order, tank filled partially.
	—	—	—	—	10 0	9 17	8 5	28 2	—	—	—	Do. & tank empty.
Tank, 4 wheels coupled, 5′ wheels.	6	4 8	7 4	12 0	7 7	10 0	9 8	26 15	24 9½	—	—	Engine in working order, tank filled.
	—	—	—	—	5 3	9 16	6 15	21 14	—	—	—	Do. & tank empty.
Tender A, 1,500 gals. of water.	6	6 6	6 0	12 6	6 0	5 7	6 1½	17 8½	20 10½	—	—	Tender in working order.
	—	—	—	—	3 10	3 2½	2 17	9 9½	—	—	—	Do. Empty.
Tender B, 2,000 gals. of water.	6	6 6	6 0	12 6	7 5	7 1	7 2	21 8	20 10½	—	—	Tender in working order.
	—	—	—	—	3 18½	3 7	3 3½	10 9	—	—	—	Do. Empty.
Tender C, 1,500 gals. of water.	6	5 6	5 6	11 0	6 4	5 5	6 10	17 19	19 6	—	—	Tender in working order.
	—	—	—	—	3 15	3 19	3 5	10 19	—	—	—	Do. Empty.

TABLE XV.—EXAMPLES OF STANDARD TYPES OF ENGINES ON VARIOUS ENGLISH RAILWAYS.—(*Proc. Inst. C. E.*, Vol. xxx.)

Name of Railway.	Class of Engine.	No. of Wheels.	Distance from Leading to Middle Wheels.		Distance from Middle to Trailing Wheels.		Wheel base.		Weight on Wheels. Leading.		Weight on Wheels. Middle.		Weight on Wheels. Trailing.		Total weight.	
			ft.	in.	ft.	in.	ft.	in.	tns.	cwt.	tns.	cwt.	tns.	cwt.	tns.	cwt.
Great Eastern, - -	Single passenger, 7′ 1″ wheel.	6	7	0	8	0	15	0	9	11½	11	7¼	8	7	29	5¾
Do.	Goods, 6 wheels coupled, 5′ 3″ wheel.	6	7	7	7	8	15	3	10	18½	11	4	9	11	31	13½
London and South Western,	Passenger, 4 wheels coupled, 6′ 6″ wheel.	6	—		—		14	0	11	0	11	14	11	0	33	14
Do.	Goods, 6 wheels coupled, 5′ 0″ wheel.	6	—		—		14	6	11	0	11	6	11	0	33	6
Do. (Branch lines),	Tank, 4 wheels coupled, 5′ 6″ wheel.	6	—		—		12	6	10	5	10	15	10	5	31	5
Great Northern, - -	Single express passenger, 7′ 1″ wheel.	6	9	6	7	6	17	0	10	8	14	0	8	12	33	0
Great Western (Metropolitan, narrow gauge),	Passenger, 4 wheels coupled, 5′ 0″ wheel.	6	7	3	8	0	15	3	8	0	12	2	12	2	32	4
Do. (broad gauge),	Goods, 6 wheels coupled, 5′ 0″ wheel.	6	7	4	8	10½	16	2½	11	6	11	18	9	2	32	6
Do. (narrow gauge),	Single express passenger, 7′ 0″ wheel.	6	7	8	8	4	16	0	9	10	12	8	7	6	29	4
Do. do.	Goods, 6 wheels coupled, 5′ 0″ wheel.	6	7	4	8	4	15	8	10	10	10	10	8	18	29	18
Do. do.	Passenger, 4 wheels coupled, 6′ 0″ wheel.	6	7	6	8	0	15	6	9	8	9	8	8	8	27	4
Manchester, Sheffield, and Lincolnshire,	Express passenger, 4 wheels coupled, 6′ 0″ wheel.	6	—		—		15	0	8	7	12	7	7	14	28	8

TABLE XV.—EXAMPLES OF STANDARD TYPES OF ENGINES ON VARIOUS ENGLISH RAILWAYS.—*Continued.*

Name of Railway.	Class of Engine.	No. of Wheels.	Distance: from Leading to Middle Wheels.		Distance: from Middle to Trailing Wheels.		Wheel base.		Weight on Wheels: Leading.		Weight on Wheels: Middle.		Weight on Wheels: Trailing.		Total weight.	
			ft.	in.	ft.	in.	ft.	in.	tns.	cwt.	tns.	cwt.	tns.	cwt.	tns.	cwt.
Manchester, Sheffield, and Lincolnshire,	Goods, 6 wheels coupled, 5′ 0″ wheel.	6	—		—		15	4	10	10	11	12	9	7	31	9
Midland, - -	Passenger, 4 wheels coupled, 6′ 6″ wheel.	6	8	0	8	6	16	6	10	18	13	12	10	11	35	1
Do.	Goods, 6 wheels coupled, 5′ 0″ wheel.	6	8	0	8	6	16	6	12	8	12	13	9	12	34	13
Lancashire and Yorkshire,	Passenger, 4 wheels coupled, 5′ 9″ wheel.	6	—		—		14	4	7	12	12	$8\frac{1}{2}$	7	2	27	$2\frac{1}{2}$
Do.	Goods, 6 wheels coupled, 5′ 0″ wheel.	6	—		—		14	10	11	8	11	8	6	$17\frac{1}{4}$	29	$13\frac{1}{4}$
South Eastern, - -	Single express passenger, 7′ 0″ wheel.	6	8	6	8	9	17	3	10	6	12	10	9	0	31	16
Do.	Passenger, 4 wheels coupled, 6′ 0″ wheel.	6	7	5	7	5	14	10	9	10	10	15	10	10	30	15
Do.	Goods, 6 wheels coupled, 4′ 9″ wheel.	6	7	6	7	6	15	0	9	18	10	0	9	14	29	12
London, Chatham, and Dover,	Passenger, 4 wheels coupled, 6′ 6″ wheel.	6	—		—		—		9	10	12	1	12	1	33	12
Do.	Goods, 6 wheels coupled, 5′ 0″ wheel.	6	—		—		15	6	11	1	11	0	10	0	32	1
North London, - -	Passenger Bogie, 5′ 9″ wheel.	8	6 0 / 8	2	8	0	22	2	7 0 / 7	0	16	0	15	0	45	0
Do.	Do., 5′ 3″ wheel.	8	5 8 / 7	$0\frac{3}{16}$	8	0	20	$8\frac{3}{16}$	7 $7\frac{1}{4}$ / 7	$7\frac{1}{4}$	14	5	14	$2\frac{1}{2}$	43	2
Great Western (broad gauge),	Single passenger, 8′ 0″ wheel.	8	4 $9\frac{1}{4}$ / 6	$4\frac{3}{8}$	7	$7\frac{1}{4}$	18	$8\frac{7}{8}$	8 0 / 8	0	12	16	10	10	39	6

Occasional monster engines occur on some railways, generally where the gradients are unusually steep, as illustrated in the following table:—

TABLE XVI.—EXAMPLES OF MONSTER ENGINES ON VARIOUS RAILWAYS.

No.	Railway.	No. of Wheels.	Wheel base. ft.	in.	Weight. tons.	Observations.
1	North London, -	4	—		42	Four wheels coupled.
2	Oldham, - -	6	—		49	Goods engine with 6 wheels coupled; gradient 1 in 27.
3	Brecon & Merthyr,	6	12	0	38	Tank engine; gradient 1 in 38.
4	Vale of Neath, -	8	—		56	Tank engine with 8 wheels coupled; afterwards altered into engine with tender in consequence of the destruction to the permanent way; gradient 1 in 47.
5	Mauritius Railway,	8	15	6	47	Tank engine with eight wheels coupled, 4 feet diameter; gradients 1 in 27.
6	Northern Railway of France,	12	19	8	67½	Tank engine with 4 outside cylinders; wheels coupled together as in two separate six-wheeled coupled engines.
7	Semmering, -	8	—		55¼	—
8	Giovi, - -	8	—		55¼	Four cylinders.
9	Cologne Minden, -	—	11	2	32	—
10	Rhenish, - -	—	13	0	39½	—
11	Do. - -	—	11	0	29	—

[1 2] *Proc. Inst. C.E.*, Vol., xxvi., p. 343, 383.
[3] *Ibid.*, p. 335.
[4] *Ibid.*, pp. 372, 374.
[5] *Ibid.*, p. 384.
[6 7 8] *Ibid.*, pp. 373, 343.
[9 10 11] *Ibid.*, Vol. xxv., p. 436.

It has been already shown in **454** that railway bridges under 40 feet span require extra strength in consequence of high-speed trains increasing their deflection, but besides this they are liable to heavier statical strains than those due to uniform loads of 1, 1¼, or 1⅓ tons per running foot on each line, and their strength should accordingly be greater in proportion than that of girders which exceed this span. If, for instance, a six-wheeled engine, 24 feet long and weighing 32 tons on a twelve-feet wheel base, rest on the centre of a bridge 32 feet in length, the strain in the flanges is obviously greater than would occur if 42·7 tons ($= 32 \times 1\frac{1}{3}$) were distributed uniformly over the whole length of the bridge. A 40-foot bridge would, it is true, have the weight of only one such engine on the centre at a time, and if the load on the middle pair of wheels equal 16 tons, and that on the leading and trailing pairs (6 feet on either side of the centre), equal 8 tons respectively, the equivalent load concentrated at the centre of the bridge is 27·2 tons, or 54·4 tons distributed. If there were three such engines in a row, the pressure might be slightly increased by the weight on the leading and trailing wheels of the extreme engines, each of which would have one pair of wheels, or 8 tons, resting on the bridge within 2 feet of the abutments. This is equivalent to 1·6 tons concentrated at the centre, or 3·2 tons distributed over the bridge. Adding this to the 54·4 tons due to the central engine, we have a total weight equivalent to a distributed load of 57·6 tons, or 1·44 tons per running foot. This arrangement of engines produces the greatest strain at the centre of the flanges. Again, two such engines might stand with their buffers in contact at the centre of the 40-foot bridge, and, though their outer ends would project beyond each abutment, their collective wheel base would cover only 36 feet of the bridge. This arrangement of engines produces greater strains than the former near the ends of the flanges. Indeed, these end strains will in some cases slightly exceed those given by the following rules, but this is compensated for by the flanges being generally made heavier near the ends than theory requires (**427**).

490. Standard working loads for railway bridges of various spans.—The following tables are intended to give the results of the preceding observations in a concise form. They are based on six assumptions:—

1. The working load for railway bridges 400 feet in length and upwards does not exceed $\frac{3}{4}$ ton per running foot on each line.

2. No more locomotives than will cover 100 feet in length follow each other without interruption; hence, the working load per foot diminishes as the span increases from 100 feet up to 400 feet.

3. Engines may be arranged on bridges less than 100 feet long so as to produce greater strains than would be due to the engine load if it were of uniform density; hence, the equivalent working load per foot increases as the span diminishes from 100 feet downwards.

4. Bridges less than 40 feet in span are subject to concentrated loads from single engines, as well as to extra deflection from high-speed trains.

5. The standard locomotive is assumed to be 24 feet long and to have 6 wheels with a 12 feet base; to have half its weight resting on the middle wheels, and one-fourth on the leading and trailing pairs respectively, which are supposed to be at equal distances on either side of the middle wheels.

6. Standard Engines are assumed to weigh 24 tons, 30 tons, and 32 tons, according to their construction. This makes the standard load 1 ton, $1\frac{1}{4}$ ton, or $1\frac{1}{3}$ ton per foot run of single line, according to the weight of the engines which work it, but it is safest to take the higher standards for the railways in Great Britain, as they are so interlaced that engines may pass from one line to another, and it is quite possible that we have not yet arrived at the limit of weight.

BRIDGES FROM 40 TO 400 FEET IN LENGTH.

If the standard working load (the heaviest engine) on a 100-foot bridge weigh 1 ton per foot, while that on a 400-foot bridge weighs ·75 tons per foot, the difference (= ·25 ton per foot) must be gradually distributed among the intervening 300 feet; in other words, the difference for each 10 feet in length $= \frac{\cdot 25}{30} = \cdot 0083$ tons. The differences for the other standards may be found in a similar way, and the following table contains the values of the working loads corresponding to the three standards for bridges of various lengths between 40 and 400 feet.

TABLE XVII.—WORKING LIVE LOADS FOR RAILWAY BRIDGES FROM 40 TO 400 FEET IN LENGTH.

Length of bridge in feet.	Working load in tons per running foot of single line,		
	when the standard load on a 100-foot bridge = 1 ton per foot.	when the standard load on a 100-foot bridge = 1¼ ton per foot.	when the standard load on a 100-foot bridge = 1⅓ ton per foot.
40	1·05	1·35	1·45
50	1·04	1·33	1·43
60	1·03	1·32	1·41
70	1·03	1·30	1·39
80	1·02	1·28	1·37
90	1·01	1·27	1·35
100	**1·00**	**1·25**	**1·33**
120	·98	1·22	1·30
140	·97	1·18	1·26
160	·95	1·15	1·22
180	·93	1·12	1·18
200	·92	1·08	1·14
250	·88	1·00	1·04
300	·83	·92	·94
350	·79	·83	·85
400	**·75**	**·75**	**·75**

BRIDGES UNDER 40 FEET IN LENGTH.

Bridges under 40 feet in length should be strong enough to support a standard engine resting at the centre of the bridge. The following is an approximate method of calculating the value of the working load corresponding to each standard. First, find what load concentrated at the centre of the bridge will produce a strain in the centre of the flanges equivalent to that due to the standard engine; twice this may be taken as the equivalent uniformly distributed load, which again, divided by the span, gives the working load per running foot required, as contained in the following table:—

TABLE XVIII.—Working Live Loads for Railway Bridges under 40 feet in Length.

Length of bridge in feet.	Working load in tons per running foot of single line,		
	when the standard load on a 100-foot bridge = 1 ton per foot.	when the standard load on a 100-foot bridge = $1\frac{1}{4}$ ton per foot.	when the standard load on a 100-foot bridge = $1\frac{1}{3}$ ton per foot.
12	2·0	2·5	2·67
16	1·88	2·34	2·5
20	1·68	2·1	2·24
24	1·5	1·87	2·0
28	1·35	1·68	1·79
32	1·22	1·53	1·62
36	1·11	1·39	1·48

It will be prudent to adopt the highest standard for railway bridges under 40 feet, since loads in rapid motion have a much greater effect on these short bridges than on longer and heavier ones, and if velocities of 50 miles an hour are anticipated, it will be well to add from 10 to 20 per cent. to the above tabulated working loads of bridges under 40 feet (**454**). Short railway girders are so light in proportion to the passing load, that it is a good plan to bed them on thick timber wall plates, which act as elastic cushions and prevent the masonry of the abutments from being shaken to pieces by the vibration of heavy trains.

491. Effect of concentrated loads upon the web.—The weight of a heavy engine may, as already explained, be concentrated within a short wheel base and thus produce a great local pressure on one or two cross-girders, which they again will transmit to one or two points in each main girder. It might even happen in a lattice girder that the intervals of the bracing and cross-girders were such as to throw the load from several successive pairs of wheels on one system of diagonals, which would thus be liable to excessive strain. We have, it is true, some compensation for this; first, in the rigidity of the flanges, platform, sleepers, and rails, all of which help to distribute the weight; and secondly, in the fact that the bracing of the central parts of small girders is for practical reasons generally made stronger than theory requires (**436**), and it will generally be found sufficient to calculate the web strains on the supposition that the passing load is of uniform density, and equal in weight per running foot to the working loads given above.

492. Proof load of railway bridges—English practice—French Government rule.—No definite rule has been yet made by the Board of Trade for the proof load of railway girder bridges, but it is a common practice on the inspection of any important bridge to load each line with as many engines and tenders as the bridge will hold, and measure the corresponding deflection. This proof is generally assumed to vary from 1 ton per running foot on the longer bridges to $1\frac{1}{2}$ ton on the shorter ones; but when a bridge exceeds a certain span, say 150 feet, it is obviously unreasonable to cover it with heavy engines, and ballast wagons may be used along with two or three engines so as to bring the proof load more in accordance with Table XVII.

The following are the French Ministerial regulations for the proof loads of wrought-iron railway bridges:—

a. For bridges under 20 metres each span, a dead load of 5,000 kilogrammes per running metre of each line (= 1·5 tons per running foot).

b. For bridges exceeding 20 metres each span, a dead load of 4,000 kilogrammes per running metre of each line (= 1·2 tons per running foot), but in no case less than 100,000 kilogrammes.

c. In addition to the foregoing proof by dead weight, a train composed of two engines (each weighing with its tender at least 60 tons), and wagons (each loaded with 12 tons), in sufficient number to cover at least one span, is driven across at a speed of from 20 to 35 kilometres (12 to 22 miles) per hour.

d. A second trial is made by driving at a speed of from 40 to 70 kilometres (25 to 43 miles) per hour a train composed of two engines (each with its tender weighing 35 tons), and wagons loaded as in ordinary passenger trains, in sufficient number to cover at least one span.

e. For bridges with two lines the trains are made to traverse each line, at first in parallel, and then in opposite directions so that the trains may meet at the centre.

WORKING LOAD ON PUBLIC BRIDGES AND ROOFS.

493. Men marching in step and running cattle are the severest loads on suspension bridges—A crowd of people is the greatest distributed load on a public bridge—French and English practice—100 lbs. per square foot recommended for the standard working load on public bridges—Public bridges sometimes liable to concentrated loads as high as 12 tons on one wheel.—It is generally considered that infantry marching in step will strain suspension bridges far more severely than any other form of passing load. The actual dead weight of troops on the march is said to be about 35 lbs. per square foot, but this statical load does not represent the true strain due to troops marching in step; on this subject Drewry came to the following conclusions:—"1st, That any body of men marching in step, say at three to three and a half miles per hour, will strain a bridge at least as much as double their weight at rest; and, 2nd, that the strain they produce increases much faster than their speed, but in what precise ratio is not determined. In prudence, not more than one-sixth of the number of infantry that would fill a bridge, should be permitted to march over it in step; and if they do march in step, it should be at a slow pace. The march of cavalry, or of cattle, is not so dangerous; first, because they take more room in proportion to their weight; and secondly, because their step is not simultaneous."* Referring to the Niagara Falls Suspension Bridge Mr. Roebling observes—"In my opinion a heavy train, running at a speed of twenty miles an hour, does less injury to the structure than is caused by twenty heavy cattle under a full trot. Public processions marching to the sound of music, or bodies of soldiers keeping regular step, will produce a still more injurious effect."† A crowd of people constitutes the greatest distributed load on a

* Drewry *on Suspension Bridges*, p. 190.

† *Papers and Practical Illustrations of Public Works*, p. 29. Weale, London.

public bridge, and 15 adults are generally estimated to weigh 1 ton, which gives an average of 149·3 lbs. to each adult. Different statements, however, have been made respecting the number of people that can stand in a given space, and in order to test this I packed twenty-nine Irish artisans and one boy, taken from a forge and fitting shop, and weighing collectively 4,382 lbs. or 146 lbs. per individual, on a weigh-bridge $6' \, 1'' \times 4' \, 10'' = 29{\cdot}4$ square feet. In this experiment the men overhung the edges of the weigh-bridge to a slight extent and gave too high a result, and accordingly, on another occasion I packed 58 Irish labourers, weighing 8,404 lbs. or 145 lbs. a man, in the empty deck-house of a ship, $9' \, 6'' \times 6' \, 0'' = 57$ square feet; this gives a load of 147·4 lbs., or very nearly one man per square foot, and is, I believe, a perfectly reliable experiment. Such cramming, however, could scarcely occur in practice except in portions of a strongly excited crowd, but I have no doubt that it does occasionally so occur. The standard proof load for suspension bridges in France was formerly 200 kilogrammes per square metre, = 41 lbs. per square foot.* This may be a sufficient standard for bridges with gate-keepers at the ends to prevent overcrowding, but it is obviously insufficient for bridges which are free to the public, especially in the vicinity of towns, and modern French practice seems to have raised the standard to 82 lbs. per square foot.† Drewry adopted 70 lbs. per square foot of platform as the greatest load that a public bridge would sustain if covered with people.‡ Tredgold and Professor Rankine estimate the weight of a dense crowd at 120 lbs. per square foot,§ and the late Mr. Brunel is said to have used 100 lbs. in his calculations for Hungerford Suspension Bridge. Mr. Hawkshaw adopted 80 lbs. per square foot for the footpaths of Charing Cross Bridge,‖ and (in conjunction with Mr. W. H. Barlow) 70 lbs. for the Clifton Suspension Bridge,¶ where there are

* Drewry *on Suspension Bridges*, p. 113.

† *Trans. Soc. of Eng. for* 1866, p. 197.

‡ Drewry *on Suspension Bridges*, p. 189.

§ Tredgold's *Carpentry*, p. 169, and Rankine's *Civil Engineering*, p. 466.

‖ *Proc. Inst. C. E.*, Vol. xxii., p. 534.

¶ *Idem*, Vol. xxvi., p. 248.

toll-gates and regulations that carriages and horses shall cross at a walking pace. In my own practice, I adopt 100 lbs. per square foot as the standard working load distributed uniformly over the whole surface of a public bridge, and 140 lbs. per square foot for certain portions of the structure, such, for example, as the footpaths of a bridge crossing a navigable river in a city, which are liable to be severely tried by an excited crowd during a boat race, or some similar occasion. Public bridges are also subject to concentrated loads at single points of quite as severe a character as those to which railway bridges are liable; if, for instance, a marine boiler, a large cannon, an iron girder, a heavy forging or casting be conveyed across a public bridge, the weight resting on a single pair of wheels may reach or even exceed 16 tons. For example, the crank shaft of H.M. armour-plated ship Hercules—weighing, shaft and lorry, about 45 tons on four wheels—was refused a passage across Westminster iron bridge in 1866 for fear of injury to the bridge, and had to be conveyed across Waterloo stone bridge,* and I am informed that even much lighter weights are habitually sent round by the stone bridge. It is necessary therefore to make not only the main ribs and cross-girders, but every part of the sheeting or platform on which the road material rests, strong enough to bear heavy local loads, which, as we have seen in the foregoing instance, may sometimes reach nearly 12 tons on a single wheel.

494. Weight of roofing materials and working load on roofs—Weight of snow—Pressure of wind against roofs.— The following table contains the weights of various roofing materials, exclusive of framing, which is given separately.

* *Engineer*, Vol. xxii., p. 298, Oct., 1866.

TABLE XIX.—WEIGHTS OF VARIOUS ROOFING MATERIALS.

Kind of covering.	Lbs. per square foot of roof surface.
Copper, - - - - -	1·0
Lead, - - - - -	6 to 8
Zinc, 13 to 16 zinc gauge, - - -	1·5 to 2
Corrugated iron, 20 to 16 B. W. G., - -	2·5 to 4
Slating, first quality, - - - -	6 to 7
Do., second quality, - - -	8 to 9
Rendering of Mortar ½ inch thick, - -	5 to 6
Stone slate, - - - - -	24
Plain tiles, - - - - -	18
Pantiles, - - - - -	6·5
Thatch of straw, - - - -	6·5
Ordinary timber framing for slated roofs, -	5 to 6
Boarding ¾ inch thick, - - - -	2·5
Do. 1¼ do., - - - -	4·2
¼ inch glass, exclusive of sash, bars, or frames,	3·5

The following table gives the size and weight of Welsh slating, and the number of squares (100 square feet) of roof each mil. of 1,200 slates will cover, 4 inches being allowed for lap.

TABLE XX.—WEIGHT OF WELSH SLATING.

Kind of slate.		Weight per mil. of 1,200.		1,200 will cover squares of roof.
		1st quality.	2nd quality.	
	in. in.	cwt.	cwt.	
Princesses,	24 × 14	70	90	12
Duchesses,	24 × 12	60	81	10
Marchioness,	22 × 12	55	70	9
Countess,	20 × 10	40	53	7
Viscountess,	18 × 10	36	47	6
Ladies,	16 × 10	31	42	$5\frac{1}{4}$
Do.	16 × 8	25	33	4
Do.	14 × 12	33	44	$5\frac{1}{3}$
Do.	14 × 8	22	27	$3\frac{1}{3}$
Doubles,	13 × 10	25	31	4
Do.	13 × 7	$17\frac{1}{2}$	21	$2\frac{1}{2}$
Do.	12 × 8	$18\frac{1}{2}$	22	$2\frac{1}{2}$

Queens-ton slates are from 27 to 36 inches long and of various breadths; 20 cwt. will cover:—1st quality, 3 to $3\frac{1}{4}$ squares; 2nd quality, $2\frac{3}{4}$ to 3 squares.

The following table contains particulars of some large station roofs.*

* *Proc. Inst. C.E.*, Vols. ix., xiv., xxvii., xxx.

TABLE XXI.—Particulars of various large Station roofs.

Name of Station.	Description of Covering.	Kind of Truss.	Clear span of principal.	Distance between centres of principals.	Versine of principal, measured from wall-plate to crown.	Depth of principal, measured from tie to crown.	Area of tie.	Area of rib in compression.	Weight of each principal.	Estimated weight of covering per square foot of area covered.
			feet.	feet.	feet.	feet.	s. in.	s. in.	tons.	lbs.
Lime-street Station, Liverpool, old roof.	Corrugated iron and glass.	Crescent girder divided into 7 bays by radial struts with diagonal ties.	153·5	21·5	34	12	6·5	14·3	—	—
Charing-cross Station, London.	Boarded and slated and glass.	Crescent girder divided into 9 bays by vertical struts with diagonal ties.	166	35	45	20	14·18	27	27 ?	37
Cannon-street Station, London.	Boarded and slated and glass.	Crescent girder divided into 9 bays by vertical struts with diagonal ties.	190	33·5	60	30	22·14	33·8	47·25	37
New-street Station, Birmingham.	Corrugated iron and glass.	Crescent girder divided into 13 bays by vertical struts with diagonal ties.	212	24	40·5	23	12·6	35	25	20
Lime-street Station, Liverpool, new roof.	Boarded and slated and glass.	—	212	—	—	22·75	33	50·37	44	38
St. Pancras Station, London.	Boarded and slated and glass.	Ogival arch, formed of 2 latticed ribs 6′ deep, meeting at a slight angle at the crown, the tie being formed by the platform beneath the railway, but not reckoned as part of the principal.	240	29·33	107	107	—	46	54	36

When the weight of the covering per square foot, and the distance of the principals apart, are constant for roofs of different spans, the weights of the principals will vary nearly as the squares of the spans (**274**), and if estimated per square of ground, directly as the spans; acting on this rule, Mr. W. H. Barlow states that with an ordinary truss, the distance between the principals being 30 feet, and the covering being boarding, slating and glass, the weight of metal required in the principals can be expressed approximately in tons per square of ground covered (100 square feet), by dividing the span in feet by 320, which gives the following weights for different spans:—

Span of roof in feet.	Weight of Principals in tons, per square of ground covered.
80	·250
120	·375
160	·500
200	·625
240	·750

The previous remarks apply more especially to large roofs whose principals are far apart. In smaller roofs, say under 120 feet span, it is unusual to place the principals farther apart than from 8 to 12 feet, and Mr. Henderson states the results of his experience regarding these in the following terms.*

"If a roof was to be covered with slates, either laid upon iron laths, or upon boarding, for ordinary spans, the principals would be fixed 8 feet apart, from centre to centre; whilst if the roof was to be covered with corrugated iron, either painted or galvanized, the principals would be 12 feet apart, from centre to centre, and purlins of T iron would be used to carry the corrugated iron. The distance of 8 feet apart for the principals, in the former case, was fixed by the fact of that being the greatest limit to which it was safe to go with the ordinary L iron laths, in one case, and $1\frac{1}{4}$ inch boarding in the other. The distance of 12 feet apart, for the principals of roofs covered with corrugated iron was arrived at, by that being about the limit to which purlins of T iron 4 inches deep

* *Proc. Inst. C.E.*, Vol. xiv., p. 268.

could be applied, and from the fact of the same strength which would suffice for principals, placed at distances of 8 feet apart for a slated roof, being also sufficient when placed at 12 feet apart, if the roof was covered with corrugated iron, on account of that covering, with its supports, being so much lighter than a covering of slates with their supports, that expression, 'supports,' being intended to apply only to the laths and the boarding, or purlins, as the case might be.* The four descriptions of coverings, including everything except the principals themselves, might be stated to be of the following values per square (in the year 1855):—

"1st. A covering consisting of L iron laths and slating, including the laths, slates, gutters, skylights, louvre standards and blades, rain-water pipes, glass, and painting complete, at £5 10s. per square.

"2nd. A covering consisting of 1¼ inch beaded boarding, grooved and tongued with iron tongues, including the boarding, slates, gutters, skylights, louvre standards and blades, rain-water pipes, glass and painting complete, at £5 17s. 6d. per square.

"3rd. A covering consisting of T iron purlins and corrugated sheet iron No. 18 B.W.G., painted with four coats on each side, including the purlins, the sheet iron covering, the skylights, the louvre standards and blades, rain-water pipes, glass, and painting complete, at £6 12s. 6d. per square.

"4th. A covering consisting of T iron purlins, and corrugated galvanized sheet iron No. 18 B.W.G., including the purlins, the sheet iron covering, the skylights, the louvre standards and blades, rain-water pipes, glass, and painting complete, at £7 per square.

"The whole of the above calculations were based upon the case of a roof of 60 feet span in the clear, from centre to centre of the shoes, with one-third of the entire surface of covering glazed, and with a raised louvre over the centre, for ventilation. For roofs of 60 feet square, such as the above covering was intended for, the

* "This is perhaps not quite correct, because, although the principals and covering are much lighter, yet in order to make a fair comparison, the same strength ought to be provided for wind and weather; but the truth is, that corrugated iron covering has generally been introduced with a view to economy, and the principals have been made, even comparatively, somewhat lighter and not so strong as for slated roofs."

principals placed in the one case 8 feet apart, from centre to centre, and in the other case 12 feet apart, from centre to centre, would weigh about 18 cwt. and cost about £25. In the one case each principal would serve for about five squares of roofing, measured on plan, and in the other case for about seven squares and a half. It therefore followed, that the weight per square, in the one case, would be about 3 cwt. 2 qrs. 24 lb., and in the other case, about 2 cwt. 1 qr. per square, whilst the cost, in the one case, would be £5 per square, and in the other case, a little more than £3 10s. The average weight of covering, if slating was used, would be about 9 cwts. per square, and if galvanized iron was used it would not exceed 5½ cwts. per square. The foregoing facts, in reference to covering, might be considered to hold good for all cases, where a similar description of roofing was used, with principals 8 feet apart in the one case, and 12 feet apart in the other, and, of course, it would be understood that these dimensions were given as the extreme limits. If the principals were fixed further apart, the strength of the supports of the covering must be increased, and that would augment the expense. For instance, taking a roof where the principals were fixed 24 feet apart, from centre to centre, the purlins would have to be increased in strength to such an extent as would double the price per square for the purlins themselves, but the expense of the other part of the covering would not be altered. As already stated, for a roof of 60 feet span, the principals themselves would weigh 18 cwts. each, and these principals might be used either 8 feet apart or 12 feet apart, according to the covering adopted. For roofs of greater spans the weight of the principals would increase as the squares of the span (the load per superficial foot and the pitch of the rafter being the same), so that the weight of a principal, for a roof of 120 feet span, would be 72 cwts., but of course some trifling alterations in the weight might arise from variations in the details and connexions."

Morin states that snow weighs ten times less than water, and that it may accumulate on roofs to half a metre, or nearly 20 inches in depth, when it will weigh 10 lbs. per square foot.* Mr.

* *Résistance des Matériaux*, p. 382.

Zerah Colburn estimates that the weight of saturated snow on bridges in America is equal to 6 inches of water, or 30 lbs. per square foot over the whole floor of a bridge.* The maximum pressure of wind against bridge girders has been already given in **440** as equivalent to a horizontal pressure of 25 lbs. per square foot of vertical surface. The slope of a roof must greatly diminish this, and it will be sufficient to assume the maximum effort of the wind against a sloped or curved roof to be equivalent to a downward pressure of 20 lbs. per square foot, acting separately on each side. For ordinary roofs in the English climate it will be sufficiently accurate if we calculate their strength on the supposition that they are liable to the following loads:—

1°. A uniform load of 40 lbs. per square foot of ground surface, distributed over the whole roof.

2°. A uniform load of 40 lbs. per square foot of ground surface distributed over the weather side of the roof, and 20 lbs. on the other side which is away from the wind. This 40 lbs. will generally cover the weight of slates, boarding or laths, purlins, framing or principals, snow and wind for roofs under 100 feet in span. For roofs exceeding 100 feet in span, we may assume that the total load is increased by 1 lb. per additional 10 feet—thus, the load for calculation on a 200 feet roof will be—

1°. A uniform load of 50 lbs. per square foot of ground, distributed over the whole roof.

2°. A uniform load of 50 lbs. per square foot of ground plan distributed over one half the roof, and 30 lbs. on the other. When the strength of roof is calculated by the foregoing rules, the working strain in iron tie rods may be as high as 7 tons per square inch of net area, unless they are welded, or unless their section is very small, in either of which cases 5 tons will be enough.

* *Proc. Inst. C. E.*, Vol. xxii., p. 546.

CHAPTER XXIX.

ESTIMATION OF GIRDER-WORK.

495. Theoretic and empirical quantities—Allowance for rivet holes in parts in tension generally varies from one-third to one-fifth of the net section.—Chapter X. contains formulæ for calculating the theoretic amount of material required for braced girders with horizontal flanges, when their length, depth, load and unit-strain are known. In order to render these formulæ of practical use in estimating girder-work, certain large additions, derived from experience, must be added to the theoretic quantities. If, for instance, the girder be made of wrought-iron, the formulæ are based on the supposition that the material is in one continuous piece whose whole section is equally effective for resisting strain. This is not the case in reality, for rivet holes in parts subject to tension, stiffeners in those subject to compression, covers, packing, rivet heads and waste—all require certain additions to the theoretic quantities which experience alone can supply. When the general design is arranged, it is easy to estimate the increased percentage of material arising from the weakening effect of rivet holes in parts subject to tension (**476**). In girder-work the allowance for rivet holes generally varies from one-third to one-fifth of the net sectional area according to the design; the larger allowance of one-third may be required for the tension diagonals of small girders; a medium allowance of one-fourth for the tension diagonals of large girders and the tension flanges of small ones; and an allowance of one-fifth for the tension flanges of large girders.

496. Allowance for stiffeners in parts in compression varies according to their sectional area—Large compression flanges seldom require any allowance for stiffening—Compression bracing requires large percentages.—The additional percentage of material required to withstand flexure or buckling in parts subject to compression is not so easily estimated. It will generally be found to diminish in proportion as the area of the part increases, for when the area is considerable, a stiff form of cross section may be given with little or no extra material. This is frequently the case with the compression flange, especially in large girders. Long compression braces, however, require much extra stiffening and the amount of this varies within considerable limits. In the Boyne Lattice Bridge the extra material required to stiffen the various compression braces varied from 60 to 128 per cent. of the theoretic amount (calculated at 4 tons per square inch) which would have been required to resist crushing merely, if flexure had been left out of consideration, the higher percentages being required in the central diagonals whose scantlings were small, since they had to sustain but slight strains. In bridges above 250 feet span, with two main girders and a double line of railway, a sufficiently close approximation will generally be made if we assume the extra quantity of material to resist flexure in the compression bracing equal to as much again as the theoretic quantity calculated by the formulæ, but when the bridge is designed for a single line of railway this percentage is insufficient; perhaps, in this case twice the theoretic quantity would generally be a safe allowance, as the extra quantity required for stiffening the compression bracing of a single-line bridge is not widely different from that required for the double line.

497. Allowance for covers in flanges varies from 12 to 15 per cent. of the gross section—Estimating girder-work a tentative process.—The allowance for covers will also vary much with the design, long flange-plates requiring fewer covers than short ones (**463** to **465**). In the piled flanges of the Boyne lattice girders, the covers formed about 12 per cent., or nearly 1-8th, of the plates and angle iron. In the cellular flanges of the Conway tubular bridge, the covers of the compression flange formed 5 per cent. of the plates and angle iron, and those of the tension flange 28 per cent.; adding both flanges together, the covers formed about 15 per cent. of the plates and angle iron.*

* Clark *on the Tubular Bridges*, p. 586.

The process of estimating the quantities in any proposed bridge is tentative and depends upon experience, for it is necessary to assume a weight for the permanent bridge-load, and then make the calculations with the various practical allowances above mentioned. Now, the resulting weight from this calculation may not agree with that which has been assumed. In this case the first estimate gives an approximation for a second calculation, and even a third may be necessary where great nicety is required. The following examples will illustrate this method of forming estimates:—

EXAMPLE 1.

498. Double-line lattice bridge 267 feet long.—I shall select for the first example a wrought-iron lattice bridge for a double line of railroad of the same length, depth and width as the central span of the Boyne Lattice Bridge, the weight of which is given in detail in the appendix. As the Boyne Bridge is a continuous girder in three spans, its central span, of course, requires less material than a bridge of equal dimensions which has not the same advantage of continuity.

Let $l = 267$ feet $=$ the length measured from centre to centre of end pillars (**55**),

$d = \frac{l}{12} = 22{\cdot}25$ feet $=$ the depth,

$\theta = 45° =$ the angle of the bracing, whence $\sec\theta . \operatorname{cosec}\theta = 2$ (**278**),

$f = 5$ tons tensile inch-strain of net section,

$f' = 4$ tons compressive inch-strain of gross section,

and let the width of platform between the main girders equal 24 feet as in the Boyne Bridge. Let the maximum passing load equal 1 ton per running foot on each line, $= 534$ tons when covering both lines together, and let us assume that the permanent bridge-load equals 490 tons, which gives the total load supported by the girders as follows:—

$$\mathsf{W} = 534 + 490 = 1024 \text{ tons.}$$

With this load uniformly distributed, the theoretic quantities of material (eqs. 206 and 208) are as follows, 4·6 cubic feet of wrought-iron being assumed equal to 1 ton.

	Tons.
Tension bracing $= \dfrac{1024 \times 267}{4 \times 5 \times 144} = 94{\cdot}93$ cubic feet,* -	20·64
Compression bracing ($= \frac{5}{4}$ths of the tension bracing),	25·80
Tension flange $\left(= \dfrac{l}{3d} \times \text{tension bracing, eq. } 208\right)$,	82·56
Compression flange ($= \frac{5}{4}$ths of the tension flange), -	103·20
Total theoretic weight, - -	**232·20**

The true quantities are obtained from the foregoing by adding the percentages derived from experience, as follows:—

	Tons.	Tons.
Theoretic tension bracing, - - - -	20·64	
Rivet holes, say $\frac{1}{4}$th of net section, - -	5·16	25·80
Theoretic compression bracing, - - -	25·80	
Add as much again for stiffening, - -	25·80	51·60
Theoretic tension flange, - - - -	82·56	
Rivet holes, say $\frac{1}{5}$th of net section, - -	16·51	99·07
Covers of tension flange, say $\frac{1}{8}$th of flange, - -		12·38
Theoretic compression flange, - - - - -		103·20
Covers of compression flange, say $\frac{1}{8}$th of flange, -		12·90
		304·95
Rivet heads, packings, waste (**427, 436**), say 10 per cent.,		30·49
Weight of iron in the main girders, - - -		**335·44**
35 cross-girders, 7 feet 5 inches apart, each 1·32 tons (see Appendix, "Boyne Viaduct"),	46·20	
Cross-bracing, do. do.	17·66	63·86
Weight of iron between end pillars, - - -		**399·30**
6-inch planking of platform 24 feet wide, = 3,204 cubic feet, @ 50 cubic feet per ton,	64·08	
Longitudinal timbers under rails, 12 inches × 6 inches = 534 cubic feet, - - -	10·68	
Barlow rails, 356 yards, @ 100 lbs. per yard, -	15·89	90·65
Permanent bridge-load between end pillars, -		**489·95**

* NOTE.—The theoretic quantity of material in the tension bracing is only one-half that given by eq. 206, which represents the quantity for the whole web.

being 0·05 tons less than that assumed. In order to obtain the total weight of wrought-iron in the bridge, we must add the weight of the 4 end pillars with their 2 lower cross-girders and 2 top cross-girders and gussets (**443**), say 30 tons in all, to the weight of iron between the end pillars; this makes the **total weight of wrought-iron** in the structure = 399·30 + 30 = **429·30 tons.**

In this example we find that 335·44 tons of iron are required in the main girders to support themselves and an additional load of 688·56 tons uniformly distributed. Consequently, each ton of additional load uniformly distributed requires $\frac{355 \cdot 44}{688 \cdot 56} = 0 \cdot 487$ tons of iron in the main girders, and if an additional load of 100 tons of ballast were spread over the platform, we should add 48·7 tons of iron to the main girders to support the weight of this ballast without the unit-strains being increased.

499. Permanent strains — Strains from train-load — Economy due to continuity.—The permanent inch-strains, that is, the inch-strains due to the permanent bridge-load of 489·95 tons, are 2·39 tons tension and 1·91 tons compression; those due to the main girders alone, weighing 335·44 tons, are 1·64 tons tension and 1·31 tons compression, and those due to a train-load of one ton per running foot on each line uniformly distributed are 2·61 tons tension and 2·09 tons compression. The actual weight of iron in the main girders of the long central span of the Boyne Bridge = 297·41 tons; the difference between this and our example = 335·44 — 297·41 = 38·03 tons, which represents the saving effected in the central span of the Boyne Bridge by its connexion over the piers with the side spans. As, however, this connexion causes a certain loss of material in the shorter side spans, the total amount of economy produced by continuity is probably less than that above stated (**258, 427**).

EXAMPLE 2.

500. Single-line lattice bridge 400 feet long.—A wrought-iron lattice bridge for a single line of railway, 400 feet long from centre to centre of end pillars, 25 feet deep and 14 feet wide between main girders, with the bracing at an angle of 45°. Using the same symbols as before, we have,

$$l = 400 \text{ feet},$$
$$d = \frac{l}{16} = 25 \text{ feet},$$
$$\theta = 45°,$$
$$f = 5 \text{ tons tensile inch-strain of net section},$$
$$f' = 4 \text{ tons compressive inch-strain of gross section}.$$

Let the maximum train load equal $\frac{3}{4}$ ton per running foot (**490**), and assuming that the permanent bridge-load equals 1300 tons, we have the total distributed load,

$$\mathbf{W} = 300 + 1300 = 1600 \text{ tons}.$$

The theoretic quantities with their empirical percentages are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \frac{1600 \times 400}{4 \times 5 \times 144} =$		
222·2 cubic feet, @ 4·6 cubic feet per ton, -	48·3	60·4
Rivet holes, say one-fourth of net section, -	12·1	
Theoretic compression bracing (= $\frac{5}{4}$ths of the theoretic tension bracing), - - -	60·4	181·2
Add twice as much for stiffening - - -	120·8	
Theoretic tension flange $= \frac{1600 \times 400 \times 16}{12 \times 5 \times 144} =$		
1,185·18 cubic feet, @ 4·6 cubic feet per ton,	257·6	309·1
Rivet holes, say $\frac{1}{5}$th of net section, - -	51·5	
Covers, say $\frac{1}{8}$th of flange, - - - - - -		38·6
Theoretic compression flange (= $\frac{5}{4}$ths of the theoretic tension flange), - - - - - -		322·0
Covers, say $\frac{1}{8}$th of flange, - - - - - -		40·5
		951·8
Rivet heads, packings, waste, say 10 per cent., -		95·2
Iron in main girders, - - - - - - -		**1047·0**

	Tons.
Cross-girders = 400 × 0·18 tons (**445**), - -	72·0
Cross-bracing, say, - - - - - -	35·0
Weight of iron between end pillars, - - -	**1154·0**
Platform, rails, sleepers and ballast = 400 × 0·36 tons (**445**), - - - - - - -	144·0
Permanent bridge-load between end pillars, -	**1298·0**

being 2 tons less than that assumed. If the 4 end pillars and cross-girders over the abutments weigh 40 tons, the **total weight of wrought-iron** in the bridge = 1,154 + 40 = **1194 tons.**

From this estimate it appears that 1047 tons of iron are required in the main girders to support themselves and an additional load of 553 tons uniformly distributed; consequently, each ton of additional load uniformly distributed requires for its support $\frac{1047}{553} = 1{\cdot}89$ tons in the main girders. If, for instance, the maximum train-load be 1 ton in place of $\frac{3}{4}$ ton per running foot, this uniformly distributed load will amount to 400 tons in place of 300 tons, that is, 100 tons more than has been assumed, and this will require 100 × 1·89 = 189 tons extra iron in the main girders for its support, and the increased total load on the bridge will be 289 tons, or nearly three times the useful addition. The iron in the flanges, including the 10 per cent. for rivet heads, packings and waste, weighs 781·2 tons; the iron in the web, also including the percentage for rivet heads, &c., weighs 265·8 tons; consequently, each ton of useful load uniformly distributed requires $\frac{781{\cdot}2}{553} = 1{\cdot}41$ tons of iron in the flanges, and $\frac{265{\cdot}8}{553} = 0{\cdot}48$ tons in the webs. The inch-strains due to the permanent bridge-load of 1,300 tons between the end pillars are 4·06 tons tension and 3·25 tons compression, while those due to a uniformly distributed train-load of $\frac{3}{4}$ ton per running foot are 0·94 tons tension and 0·75 tons compression.

Example 3.

501. Single-line lattice bridge 400 feet long, as in Ex. 2, but with higher unit-strains.—A wrought-iron lattice bridge of the same dimensions as the last, but in place of the inch-strains being 5 and 4 tons let

$f = 6$ tons tensile inch-strain of net section,

$f' = 5$ tons compressive inch-strain of gross section.

Assuming that the permanent bridge-load = 960 tons, we have the total distributed load,

$$\mathrm{W} = 300 + 960 = 1{,}260 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \dfrac{1260 \times 400}{4 \times 6 \times 144} =$		
145·83 cubic feet, @ 4·6 feet per ton, -	31·7	39·6
Rivet holes, say $\frac{1}{4}$ of net section, - - -	7·9	
Theoretic compression bracing (= $\frac{6}{5}$ths of the theoretic tension bracing), - - -	38·0	152·0
Add three times as much for stiffening,* -	114·0	
Theoretic tension flange $= \dfrac{1260 \times 400 \times 16}{12 \times 6 \times 144} =$		
777·8 cubic feet, @ 4·6 cubic feet per ton,	169·1	202·9
Rivet holes, say $\frac{1}{5}$th of net section, - -	33·8	
Covers, say $\frac{1}{8}$th of flange, - - - - -		25·4
Theoretic compression flange (= $\frac{6}{5}$ths of the theoretic tension flange), - - - - -		202·9
Covers, say $\frac{1}{8}$th of flange, - - - - -		25·4
		648·2
Rivet heads, packings, waste, say 10 per cent., -		64·8
Iron in main girders, - - - - - -		**713·0**

* In this example I allow three times, in place of twice the theoretic amount, because the extra quantity of material required for stiffening the compression bracing is but slightly affected by the adoption of higher unit-strains.

	Tons.
Cross-girders, as in last example, - - - -	72·0
Cross-bracing, say, - - - - - - -	30·0
Weight of iron between end pillars, - - -	**815·0**
Platform, rails, sleepers and ballast, as in last, -	144·0
Permanent bridge-load between end pillars, -	**959·0**

being 1 ton less than that assumed. If the four end pillars and cross-girders over abutments weigh 35 tons, the **total weight of wrought-iron** in the bridge $= 815 + 35 =$ **850 tons.**

The main girders in this example, weighing 713 tons, support themselves and an additional load of 547 tons uniformly distributed. Consequently, each ton of useful load uniformly distributed requires for its support $\frac{713}{547} = 1·304$ tons in the main girders. The inch-strains due to the permanent bridge-load of 960 tons between end pillars $= \frac{6 \times 960}{1260} = 4·57$ tons tension, and $\frac{5 \times 960}{1260} = 3·81$ tons compression, while those produced by a uniformly distributed train-load of $\frac{3}{4}$ ton per running foot are 1·43 tons tension and 1·19 tons compression.

502. Great economy from high unit-strains in long girders—Steel plates.—Comparing this with the preceding example, we find a saving in the main girders equal to 1,047—713 = 334 tons, or nearly 47 per cent. of the lighter bridge. The saving may even be greater than this, since I have neglected any reduction in the weight of the cross-girders due to higher unit-strains. These two examples illustrate the great economy produced in large girders by adopting high unit-strains. In place of the weights of the main girders being in the inverse ratio of the unit-strains, as might be supposed at first sight, we find that they vary in a much higher ratio, at least in large bridges where the main girders form a large proportion of the total load (**67**). Economy from the adoption of high unit-strains will be chiefly marked in the flanges and tension bracing, owing to the necessity of having a certain amount of material to stiffen the compression bracing, no matter how high the ultimate crushing strength of the material may be. Even a better method of riveting or jointing may produce a very important saving in a large girder, by not requiring so may holes in the tension plates, or such large covers at the joints. Mild steel plates, which are now manufactured at a cost not much exceeding that of the better kinds of iron, but about once and a half as strong as the latter, will, doubtless, enable the engineer to construct girders over spans which have been hitherto impracticable. The tensile strength of steel is known; it is to be hoped that satisfactory experiments will be made to determine its stiffness, that is, its strength to resist flexure when in the form of long pillars—an essential element in its application to girder-work (**483**).

503. Suspension principle applicable to larger spans than girders.—We are now in a position to understand how suspension bridges can be built over spans far exceeding those to which rigid girders are applicable, for not only are there no compressive strains in the webs of suspension bridges, but the compression flange of the girder is superseded by land chains, and the structure between the piers is thus relieved of the weight of one flange. Moreover, the material used is generally of such an excellent quality that it is capable of sustaining with safety a higher unit-strain than ordinary plate-iron (**476**), and there is also a less percentage of material required for the joints of suspension chains, as pins passing through eyes in the ends of long bar links supersede the ever-recurring rivets of plated work and the whole intermediate shank of the link is thus available for tension without waste.

EXAMPLE 4.

504. Single-line lattice bridge 400 feet long, with increased depth.—The preceding example illustrates the great economy effected in large girders by the adoption of high unit-strains. Let us now examine the result of a slight increase of depth, all the other dimensions and the unit-strains remaining the same as in Example 2, but in place of the depth being 25 feet, *i.e.*, one-sixteenth of the length, let

$$d = \frac{l}{15} = 26{\cdot}67 \text{ feet.}$$

Assuming the permanent bridge-load to be 1,190 tons, we have the total distributed load,

$$W = 300 + 1190 = 1490 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \dfrac{1490 \times 400}{4 \times 5 \times 144} =$		
207 cubic feet, @ 4·6 feet per ton, - -	45·0	56·2
Rivet holes, say $\frac{1}{4}$th of net section, - -	11·2	
Theoretic compression bracing (= $\frac{5}{4}$ths of the theoretic tension bracing), - - - -	56·2	176·0
Add for stiffening the same as in Ex. 2,* -	120·8	
Theoretic tension flange $= \dfrac{1490 \times 400 \times 15}{12 \times 5 \times 144} =$		
1034·7 cubic feet, @ 4·6 feet per ton, -	225·0	270·0
Rivet holes, say $\frac{1}{5}$th of net section,- - -	45·0	
Covers, say $\frac{1}{8}$th of flange, - - - - -		33·7
Theoretic compression flange (= $\frac{5}{4}$ths of the theoretic tension flange), - - - - -		281·2
Covers, say $\frac{1}{8}$th of flange, - - - - -		35·1
		852·2
Rivet heads, packings, waste, say 10 per cent., -		85·2
Iron in main girders, - - - - - -		**937·4**
Cross-girders, as in Ex. 2, - - - - -		72·0
Cross-bracing, do., - - - - - -		35·0
Weight of iron between end pillars, - - -		**1044·4**
Platform rails, sleepers, and ballast, as in Ex. 2, -		144·0
Permanent bridge-load between end pillars, -		**1188·4**

* In place of adding, as usual, twice the theoretic amount for stiffening, viz., $2 \times 56·2 = 112·4$ tons, I have assumed that this example requires the same quantity as Ex. 2, for though the load in this example is less, yet the length of the compression bracing is greater than in Ex. 2, and the assumption in the text, therefore, will be probably not far from the truth.

being 1·6 tons less than that assumed. If the four end pillars and cross-girders over the abutments weigh 40 tons, the **total weight of wrought-iron** in the bridge $= 1044{\cdot}4 + 40 =$ **1084·4 tons.**

The main girders in this example, weighing 937·4 tons, support themselves and 552·6 tons uniformly distributed. Consequently, each ton of useful load uniformly distributed requires for its support $\frac{937{\cdot}4}{552{\cdot}6} = 1{\cdot}7$ tons nearly in the main girders. The inch-strains due to the permanent bridge-load of 1190 tons between end pillars $= \frac{5 \times 1190}{1490} = 4$ tons tension, and $\frac{4 \times 1190}{1490} = 3{\cdot}2$ tons compression. The inch-strains due to the main girders, weighing 937·4 tons, $= \frac{5 \times 937{\cdot}4}{1490} = 3{\cdot}14$ tons tension, and $\frac{4 \times 937{\cdot}4}{1490} = 2{\cdot}52$ tons compression. The inch-strains due to a train-load of $\frac{3}{4}$ tons per running foot over the whole bridge $= \frac{5 \times 300}{1490} = 1{\cdot}0$ ton tension, and $\frac{4 \times 300}{1490} = 0{\cdot}8$ tons compression.

505. Weights of large girders do not vary inversely as their depth.—Comparing this with Ex. 2, the saving of material in the main girders $= 1047 - 937{\cdot}4 = 109{\cdot}6$ tons. We find therefore that the weights of the girders in these two examples are inversely as the 1·7 power of the depths, but this particular proportion is accidental (**274**).

EXAMPLE 5.

506. Single-line lattice bridge 480 feet long.—A wrought-iron lattice bridge for a single line of railway, 480 feet long from centre to centre of end pillars, 30 feet deep, and 14 feet wide between main girders. Using the same symbols as in Ex. 1, we have,

$l =$ the length $= 480$ feet,

$d =$ the depth $= \frac{l}{16} = 30$ feet,

$\theta = 45° =$ the angle the diagonals make with a vertical line,

$f = 5$ tons tensile inch-strain of net section,

$f' = 4$ tons compressive inch-strain of gross section.

Let the maximum passing load $= \frac{3}{4}$ ton per running foot (**489**), and assuming that the permanent bridge-load weighs 2760 tons, we have the total distributed load,

$$\mathsf{W} = 360 + 2760 = 3120 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \dfrac{3120 \times 480}{4 \times 5 \times 144} = 520$		
cubic feet, @ 4·6 feet per ton, - - -	113·0	141·3
Rivet holes, say $\frac{1}{4}$th of net section, - -	28·3	
Theoretic compression bracing ($= \frac{5}{4}$ths of the		
theoretic tension bracing), - - - -	141·3	423·9
Add twice as much for stiffening,* - -	282·6	
Theoretic tension flange $= \dfrac{3120 \times 480 \times 16}{12 \times 5 \times 144} =$		
2773·3 cubic feet, @ 4·6 feet per ton, - -	602·9	723·5
Rivet holes, say $\frac{1}{5}$th of net section, - -	120·6	
Covers, say $\frac{1}{8}$th of flange, - - - - - -		90·4
Theoretic compression flange ($= \frac{5}{4}$ths of the		
theoretic tension flange), - - - - - -		753·6
Covers, say $\frac{1}{8}$th of flange, - - - - - -		94·2
		2226·9
Rivet heads, packings, waste, say 10 per cent., -		222·7
Iron in main girders, - - - - - - -		**2449·6**
Cross-girders $= 480 \times 0{\cdot}18$ tons (**445**), - - - -		86·4
Cross-bracing,† - - - - - - - - -		50·4
Weight of iron between end pillars, - - -		**2586·4**

* This allowance for stiffening is probably excessive.

† The quantity of cross-bracing is proportional to $\mathsf{W}l$ (eq. 206), where W represents the pressure of the wind against the side of the bridge; if this pressure be assumed proportional to the product of length and depth, which is the case in plate girders, the quantity of cross-bracing in similar girders will vary as l^3. As, however, the side surface of similar lattice girders does not in general increase so rapidly as l^2, and as also the empirical percentages are somewhat less in large than in small bridges, it will probably be nearer the truth to assume that the quantity of cross-bracing is proportional to the square of the length. If, therefore, a bridge 400 feet long (Ex. 2,) requires 35 tons, one 480 feet long will require $35 \times \frac{36}{25} = 50{\cdot}4$ tons.

Weight of iron between end pillars, - - -	**2586·4**
Platform, rails, sleepers, and ballast, = 480 × 0·36 tons (**445**), - - - - - -	172·8
Permanent bridge-load between end pillars, -	**2759·2**

being 0·8 less than that assumed. If the weight of the four pillars and cross-girders at the ends be assumed equal to 70 tons, the **total weight of wrought-iron** in the bridge will equal 70 + 2586·4 = **2656·4 tons.**

The inch-strains due to the permanent bridge-load of 2760 tons between end pillars are $\frac{5 \times 2760}{3120} = 4{\cdot}42$ tons tension, and $\frac{4 \times 2760}{3120} = 3{\cdot}54$ tons compression. The inch-strains due to the main girders, weighing 2449·9 tons, are $\frac{5 \times 2449{\cdot}6}{3120} = 3{\cdot}92$ tons tension, and $\frac{4 \times 2449{\cdot}6}{3120} = 3{\cdot}14$ tons compression. The inch-strains due to a train-load of $\frac{3}{4}$ ton per running foot over the whole bridge $= \frac{5 \times 360}{3120} = 0{\cdot}576$ tons tension, and $\frac{4 \times 360}{3120} = 0{\cdot}46$ tons compression.

507. Waste of material in defective designs.—In this example, 2449·7 tons of iron in the main girders support themselves and an additional load of 670·4 tons uniformly distributed over the bridge. Consequently, each ton of useful load requires for its support $\frac{2449{\cdot}6}{670{\cdot}4} = 3{\cdot}65$ tons of iron in the main girders. This illustrates the great waste of material produced by defective designs for large bridges, since every ton of iron uselessly added involves the necessity of adding 3·65 other tons for its support, making collectively upwards of $4\frac{1}{2}$ tons which might be saved were the design skilfully planned.

EXAMPLE 6.

508. Single-line lattice bridge 480 feet long, as in Ex. 5, but with higher unit-strains.—A wrought-iron lattice bridge of the same dimensions as the last, but in place of the inch-strains being 5 and 4 tons respectively,

Let $f = 6$ tons tensile inch-strain of net section,

$f' = 5$ tons compressive inch-strain of gross section.

Assuming that the permanent bridge-load equals 1710 tons, we have the total distributed load,

$$W = 360 + 1710 = 2070 \text{ tons.}$$

The quantities are as follows (eq. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \dfrac{2070 \times 480}{4 \times 6 \times 144} =$		
287·5 cubic feet, @ 4·6 feet per ton, - -	62·5	78·1
Rivet holes, say $\frac{1}{4}$th of net section, - -	15·6	
Theoretic compression bracing ($= \frac{6}{5}$ths of the theoretic tension bracing), - - - -	75·0	300·0
Add three times as much for stiffening,* -	225·0	
Theoretic tension flange $= \dfrac{2070 \times 480 \times 16}{12 \times 6 \times 144} =$		
1533·3 cubic feet, @ 4·6 feet per ton, -	333·3	400·0
Rivet holes, say $\frac{1}{5}$th of net section, - -	66·7	
Covers, say $\frac{1}{8}$th of flange, - - - - - -		50·0
Theoretic compression flange ($= \frac{6}{5}$ths of the theoretic tension flange), - - - - -		400·0
Covers, say $\frac{1}{8}$th of flange, - - - - - -		50·0
		1278·1
Rivet heads, packings, waste, say 10 per cent., -		127·8
Iron in main girders, - - - - - -		**1405·9**
Cross-girders, as in last example, - - - -		86·4
Cross-bracing, say, - - - - - - -		45·0
Weight of iron between end pillars, - - -		**1537·3**
Platform, rails, sleepers and ballast, as in last, -		172·8
Permanent bridge-load between end pillars, -		**1710·1**

being 0·1 ton greater than that assumed. If the four pillars and

* See note to Ex. 3, p. 532.

cross-girders at the ends weigh 50 tons, the **total weight of wrought-iron** in the bridge will equal 50 + 1537·3 = **1587·3 tons.**

In this example, the main girders, weighing 1405·9 tons, support themselves and an additional load of 664·1 tons uniformly distributed. Consequently, each ton of useful load requires for its support $\frac{1405{\cdot}9}{664{\cdot}1} = 2{\cdot}117$ tons in the main girders. The inch-strains due to the permanent bridge-load of 1710 tons between end pillars $= \frac{6 \times 1710}{2070} = 4{\cdot}96$ tons tension, and $\frac{5 \times 1710}{2070} = 4{\cdot}13$ tons compression. The inch-strains due to the main girders, weighing 1405·9 tons, are $\frac{6 \times 1405{\cdot}9}{2070} = 4{\cdot}08$ tons tension, and $\frac{5 \times 1405{\cdot}9}{2070} = 3{\cdot}4$ tons compression. The inch-strains due to a uniformly distributed train-load of $\frac{3}{4}$ ton per running foot over the whole bridge are $\frac{6 \times 360}{2070} = 1{\cdot}04$ tons tension, and $\frac{5 \times 360}{2070} = 0{\cdot}87$ tons compression.

509. Great economy from high unit-strains in large girders.—The economy effected in large girders by the adoption of high unit-strains is very marked in this example. Compared with the preceding example, the saving amounts to 2656·4 — 1587·3 = 1069·1 tons, or nearly 68 per cent. of the lighter bridge (**502, 67**).

EXAMPLE 7.

510. Single-line lattice bridge 480 feet long, as in Ex. 5, but with increased depth.—The previous example illustrates the great economy in large bridges due to the use of a material capable of sustaining high unit-strains with safety. We shall now examine the effect of a slight increase of depth, all the other dimensions and the unit-strains remaining the same as in Ex. 5. In place of the depth being 30 feet, or $\frac{1}{16}$th of the length, let

$$d = \frac{l}{15} = 32 \text{ feet.}$$

Assuming the permanent bridge-load to be 2435 tons, we have the total distributed load,

$$W = 360 + 2435 = 2795 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \dfrac{2795 \times 490}{4 \times 5 \times 144} =$		
465·8 cubic feet, @ 4·6 feet per ton, - -	101·3	
Rivet holes, say $\frac{1}{4}$th of net section, - -	25·3	126·6
Theoretic compression bracing (= $\frac{5}{4}$ths of the theoretic tension bracing), - - -	126·6	
Add for stiffening the same as in Ex. 5,* -	282·6	409·2
Theoretic tension flange $= \dfrac{2795 \times 480 \times 15}{12 \times 5 \times 154} =$		
2329 cubic feet, @ 4·6 feet per ton, - -	506·3	
Rivet holes, say $\frac{1}{5}$th of net section, - -	101·3	607·6
Covers, say $\frac{1}{8}$th of flange, - - - - - -		76·0
Theoretic compression flange (= $\frac{5}{4}$ths of the theoretic tension flange), - - - - - -		632·9
Covers, say $\frac{1}{8}$th of flange, - - - - - -		79·1
		1931·4
Rivet heads, packings, waste, say 10 per cent., -		193·1
Iron in main girders, - - - - - - -		**2124·5**
Cross-girders, as in Ex. 5, - - - - - -		86·4
Cross-bracing, do., - - - - - -		50·4
Weight of iron between end pillars, - - -		**2261·3**
Platform, rails, sleepers and ballast, as in Ex. 5, -		172·8
Permanent bridge-load between end pillars, -		**2434·1**

being 0·9 ton less than that assumed. If the four pillars and cross-girders at the ends weigh 70 tons, the **total weight of wrought-iron** in the bridge will equal 70 + 2261·3 = **2331·3 tons.**

The main girders, weighing 2124·5 tons, support themselves and

* See note to Ex. 4, p. 535.

670·5 tons uniformly distributed. Consequently, each ton of useful load uniformly distributed requires for its support $\frac{2124 \cdot 5}{670 \cdot 5} =$ 3·17 tons in the main girders. The inch-strains due to the permanent bridge-load of 2434 tons between end pillars $= \frac{5 \times 2434}{2795} =$ 4·35 tons tension, and $\frac{4 \times 2434}{2795} = 3 \cdot 48$ tons compression. The inch-strains due to the main girders, weighing 2124·5 tons = $\frac{5 \times 2124 \cdot 5}{2795} = 3 \cdot 8$ tons tension, and $\frac{4 \times 2124 \cdot 5}{2795} = 3 \cdot 04$ tons compression. The inch-strains due to a train-load of $\frac{3}{4}$ ton per running foot over the whole bridge $= \frac{5 \times 360}{2795} = 0 \cdot 64$ tons tension, and $\frac{4 \times 360}{2795} = 0 \cdot 51$ tons compression.

511. Weights of large girders do not vary inversely as their depth.—Comparing this with Ex. 5, the saving effected in the main girders by a slight increase of depth = 2449·6 — 2124·5 = 325·1 tons. We find also that the weights of the girders in these two examples are inversely as the 2·2 power of their depths (**505**).

EXAMPLE 8.

512. Single-line lattice bridge 600 feet long.—A wrought-iron bridge for a single line of railway, 600 feet long between centres of end pillars, 37·5 feet deep, and 14 feet wide between main girders. Using the same symbols as in Ex. 1, we have,

$l = 600$ feet,

$d = \frac{l}{16} = 37 \cdot 5$ feet,

$\theta = 45°$,

$f = 5$ tons tensile inch-strain of net section,

$f' = 4$ tons compressive inch-strain of gross section.

Let the maximum passing load = $\frac{3}{4}$ ton per running foot, and assuming that the permanent bridge-load weighs 9100 tons, we have the total distributed load,

$$W = 450 + 9100 = 9550 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \frac{9550 \times 600}{4 \times 5 \times 144} =$		
1989·6 cubic feet, @ 4·6 feet per ton, -	432·5	} 540·6
Rivet holes, say $\frac{1}{4}$th of net section, - -	108·1	
Theoretic compression bracing (= $\frac{5}{4}$ths of the theoretic tension bracing), - - -	540·6	} 1081·2
Add as much again for stiffening,* - -	540·6	
Theoretic tension flange $= \frac{9550 \times 600 \times 16}{12 \times 5 \times 144} =$		
10,611 cubic feet, @ 4·6 feet per ton,	2306·7	} 2768·0
Rivet holes, say $\frac{1}{5}$th of net section, - -	461·3	
Covers, say $\frac{1}{8}$th of flange, - - - - -		346·0
Theoretic compression flange (= $\frac{5}{4}$ths of the theoretic tension flange), - - - - -		2883·4
Covers, say $\frac{1}{8}$th of flange, - - - - -		360·8
		7980·0
Rivet heads, packings, waste, say 10 per cent., -		798·0
Iron in main girders, - - - - - -		**8778·0**
Cross-girders $= 600 \times 0·18$ tons (**445**), - -		108·0
Weight of iron between end pillars, - -		**8886·0**
Platform, rails, sleepers, and ballast $= 600 \times 0·36$ tons (**445**), - - - - - -		216·0
Permanent bridge-load between end pillars, -		**9102·0**

being 2 tons in excess of that assumed. No allowance has been made for cross-bracing, for the sectional area of the flanges is so great that they would probably extend over the whole space between the main girders so as to form a tubular bridge, and thus supersede the usual cross-bracing formed of cross-girders

* The quantity of material in the web is so large that it can be thrown into a form suitable for resisting flexure without much extra stiffening; I have therefore added only half the percentage for stiffening that was adopted in most of the preceding cases.

and diagonal tension bars. If the four pillars and cross-girders at the ends be assumed equal to 200 tons, the **total weight of wrought-iron** in the bridge will equal 200 + 8886 = **9086 tons.**

In this example, 8778 tons of iron in the main girders support themselves and an additional load of 772 tons uniformly distributed over the bridge. Consequently, each ton of useful load requires for its support $\frac{8778}{772} = 11{\cdot}37$ tons of iron in the main girders. The inch-strains due to the permanent bridge-load of 9100 tons between end pillars are $\frac{5 \times 9100}{9550} = 4{\cdot}76$ tons tension, and $\frac{4 \times 9100}{9550} = 3{\cdot}81$ tons compression. The inch-strains due to the main girders, weighing 8778 tons, are $\frac{5 \times 8778}{9550} = 4{\cdot}6$ tons tension, and $\frac{4 \times 8778}{9550} = 3{\cdot}67$ tons compression. The inch-strains due to a train-load of $\frac{3}{4}$ ton per running foot over the whole bridge $= \frac{5 \times 450}{9550} = 0{\cdot}235$ tons tension, and $\frac{4 \times 450}{9550} = 0{\cdot}188$ tons compression.

Example 9.

513. Single-line lattice bridge 600 feet long, as in Ex. 8, but with higher unit-strains.—A wrought-iron bridge of the same dimensions as the last, but in place of the inch-strains being 5 and 4 tons,

Let $f = 6$ tons tensile inch-strain of net section,

$f' = 5$ tons compressive inch-strain of gross section.

Assuming that the permanent bridge-load = 3800 tons, we have the total distributed load,

$$\mathbf{W} = 450 + 3800 = 4250 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \frac{4250 \times 600}{4 \times 6 \times 144} =$		
737·8 cubic feet, @ 4·6 feet per ton, - -	160·4	200·5
Rivet holes, say $\frac{1}{4}$th of net section, - -	40·1	

	Tons.	Tons.
Theoretic compression bracing (= $\frac{6}{5}$ths of the theoretic tension bracing), - - -	192·5	577·5
Add twice as much for stiffening, - -	385·0	
Theoretic tension flange $= \frac{4250 \times 600 \times 16}{12 \times 6 \times 144} =$		
3935·2 cubic feet, @ 4·6 feet per ton, -	855·5	1026·6
Rivet holes, say $\frac{1}{5}$th of net section, - -	171·1	
Covers, say $\frac{1}{8}$th of flange, - - - - -		128·3
Theoretic compression flange (= $\frac{6}{5}$ths of the theoretic tension flange), - - - - -		1026·6
Covers, say $\frac{1}{8}$th of flange, - - - - -		128·3
		3087·8
Rivet heads, packings, waste, say 10 per cent., -		308·8
Iron in main girders, - - - - -		**3396·6**
Cross-girders, as in last example, - - - -		108·0
Cross-bracing,* - - - - - - -		78·8
Weight of iron between end pillars, - -		**3583·4**
Platform, rails, sleepers and ballast, as in last example, - - - - - - -		216·0
Permanent bridge-load between end pillars, -		**3799·4**

being 0·6 tons less than that assumed. If the four pillars and cross-girders at the ends weigh 100 tons, the **total weight of wrought-iron** in the bridge will equal $100 \times 3583{\cdot}4 =$ **3683·4 tons.**

In this example the main girders, weighing 3396·6 tons, support themselves and an additional load of 853·4 tons uniformly distributed. Consequently, each ton of useful load requires for its support $\frac{3396{\cdot}6}{853{\cdot}4} = 3{\cdot}98$ tons in the main girders. The inch-strains due to the permanent bridge-load of 3800 tons between end pillars $= \frac{6 \times 3800}{4250} = 5{\cdot}36$ tons tension, and $\frac{5 \times 3800}{4250} = 4{\cdot}47$ tons com-

* See note to Example 5, p. 537.

pression. The inch-strains due to the main girders, weighing 3396·6 tons, are $\frac{6 \times 3396 \cdot 6}{4250} = 4 \cdot 8$ tons tension, and $\frac{5 \times 3396 \cdot 6}{4250} =$ 4·0 tons compression. The inch-strains due to a uniformly distributed train-load of $\frac{3}{4}$ ton per running foot over the whole bridge $= \frac{6 \times 450}{4250} = 0 \cdot 64$ tons tension, and $\frac{5 \times 450}{4250} = 0 \cdot 53$ tons compression.

514. Great economy from high unit-strains in very large girders.—The economy due to the adoption of high unit-strains in girders of great size, whose permanent weight forms by far the larger portion of the total load, is very conspicuous in this example. Compared with the preceding example, the saving amounts to 9086 — 3683·4 = 5402·6 tons, or nearly 147 per cent. of the lighter bridge (**502, 509**).

EXAMPLE 10.

515. Single-line lattice bridge, 600 feet long, as in Ex. 8, but with increased depth.—Let us now examine the effect of a slightly increased proportion of depth to span. In Ex. 8, the depth is $\frac{1}{16}$th of the length; let the proportion now be $\frac{1}{15}$th, and retaining all the other dimensions and unit-strains as before, we have,

$$l = 600 \text{ feet,}$$

$$d = \frac{l}{15} = 40 \text{ feet,}$$

$$\theta = 45^\circ,$$

$$f = 5 \text{ tons tensile inch-strain of net section,}$$

$$f = 4 \text{ tons compressive inch-strain of gross section.}$$

Let the passing load equal $\frac{3}{4}$ ton per running foot, and assuming the permanent bridge-load to equal 6800 tons, we have the total distributed load,

$$\mathbf{W} = 450 + 6800 = 7250 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \frac{7250 \times 600}{4 \times 5 \times 144} =$		
1510·4 cubic feet, @ 4·6 feet per ton, -	328·4	410·5
Rivet holes, say $\frac{1}{4}$th of net section, - -	82·1	

	Tons.	Tons.
Theoretic compression bracing ($=\frac{4}{5}$ths of the theoretic tension bracing), - - -	410·5	951·1
Add for stiffening the same as in Ex. 8,* -	540·6	
Theoretic tension flange $= \frac{7250 \times 600 \times 15}{12 \times 5 \times 144} =$		
7552·1 cubic feet, @ 4·6 feet per ton,	1641·8	1970·2
Rivet holes, say $\frac{1}{5}$th of net section, - -	328·4	
Covers, say $\frac{1}{8}$th of the flange, - - - -		246·3
Theoretic compression flange ($=\frac{5}{4}$ths of the theoretic tension flange), - - - - -		2052·2
Covers, say $\frac{1}{8}$th of the flange, - - - -		256·5
		5886·8
Rivet heads, packings, waste, say 10 per cent., -		588·7
Iron in main girders, - - - - -		**6475·5**
Cross-girders, as in Ex. 8, - - - - -		108·0
Weight of iron between end pillars, - -		**6583·5**
Platform, rails, sleepers and ballast, as in Ex. 8, -		216·0
Permanent bridge-load between end pillars, -		**6799·5**

being 0·5 tons less than that assumed. If the four pillars and cross-girders at the ends weigh 160 tons, the **total weight of wrought-iron** in the bridge will equal 160 + 6583·5 = **6743·5 tons.**

The main girders, weighing 6475·5 tons, support themselves and 774·5 tons uniformly distributed. Consequently, each ton of useful load uniformly distributed requires for its support $\frac{6475 \cdot 5}{774 \cdot 5} = 8 \cdot 36$ tons in the main girders. The inch-strains due to the permanent bridge-load of 6800 tons between end pillars $= \frac{5 \times 6800}{7250} = 4 \cdot 69$ tons tension, and $\frac{4 \times 6800}{7250} = 3 \cdot 75$ tons com-

* See note to Ex. 4, p. 535.

pression. The inch-strains due to the main girders, weighing 6475·5 tons, are $\frac{5 \times 6475 \cdot 5}{7250} = 4 \cdot 47$ tons tension, and $\frac{4 \times 6475 \cdot 5}{7250} = 3 \cdot 57$ tons compression. The inch-strains due to a uniformly distributed train-load of $\frac{3}{4}$ ton per running foot over the whole bridge are $\frac{5 \times 450}{7250} = 0 \cdot 31$ tons tension, and $\frac{4 \times 450}{7250} = 0 \cdot 248$ tons compression.

516. Weights of very large girders vary inversely in a high ratio to their depth.—From this example we see that very considerable economy is effected in girders of great size, whose permanent weight forms the larger portion of the total load, by increasing the ratio of depth to length, even in a slight degree. Compared with Example 8, the saving in the main girders = 8778 — 6475·5 = 2302·5 tons, and the weights of these girders are inversely as the 4·7 power of their depths (**511**).

Example 11.

517. Counterbracing required for passing loads cannot be neglected in small bridges—Single-line lattice bridge 108 feet long.—The examples given in the preceding pages are those of large bridges, exceeding 250 feet in span, in which the permanent bridge-load forms such a large portion of the total load that I have neglected the extra material required for counterbracing the web so as to enable it to meet the maximum strains produced by the passing load when in motion. This is allowable, since the empirical additions for stiffening the compression bracing are probably in excess of those actually required in large girders. In short girders, however, it is necessary to make some allowance in the bracing for the load being in motion, in place of being uniformly distributed, and there is, moreover, a greater proportional waste both in the flanges near the ends, and in the web near the centre, than in large girders (**427, 436**). Hence, the allowance for waste, &c., will be more than 10 per cent. The following example of a wrought-iron lattice bridge for a single line of railway, 108 feet long, 9 feet deep, and 14 feet wide between main girders, will illustrate this. Using the same symbols as in Ex. 1, we have,

$l = 108$ feet,

$d = \dfrac{l}{12} = 9$ feet,

$\theta = 45°$,

$f = 5$ tons tensile inch-strain of net section,

$f' = 4$ tons compressive inch-strain of gross section in the flanges, and 3 tons in the bracing (**477**).

Let the maximum passing load = 1·32 tons per running foot (**490**), and assuming that the permanent bridge-load = 105 tons, we have the total distributed load,

$$\mathbf{W} = 143 + 105 = 248 \text{ tons.}$$

The quantities are as follows (eqs. 206, 208).

	Tons.	Tons.
Theoretic tension bracing $= \dfrac{248 \times 108}{4 \times 5 \times 144} =$		
9·3 cubic feet, @ 4·6 feet per ton, - -	2·02	2·69
Rivet holes, say $\frac{1}{3}$rd of net section, - -	·67	
Theoretic compression bracing, (= $\frac{5}{3}$rds of the theoretic tension bracing), - - - -	3·37	10·11
Add twice as much for stiffening and counter-bracing, - - - - - - - -	6·74	
Theoretic tension flange $= \dfrac{248 \times 108 \times 12}{12 \times 5 \times 144} =$		
37·2 cubic feet, @ 4·6 feet per ton, - -	8·09	10·11
Rivet holes, say $\frac{1}{4}$th of net section, - -	2·02	
Covers, say $\frac{1}{6}$th of the flange,* - - - -		1·68
Theoretic compression flange (= $\frac{5}{4}$ths of the theoretic tension flange), - - - - -		10·11
Covers, say $\frac{1}{6}$th of the flange, - - - -		1·68
		36·38
Rivet heads, packings and waste, say 25 per cent., -		9·09
Iron in main girders, - - - - -		**45·47**

* In large girders it is important to diminish the dead load as much as possible, and it is therefore worth paying extra for large plates so as to diminish the percentage for covers. This, however, is not the case with small girders; hence, the percentage of covers is larger in this than in the preceding examples.

	Tons.
Iron in main girders, - - - - - - - - -	**45·47**
Cross-girders = $108 \times 0{\cdot}18$ tons (**445**), - - -	19·44
Cross-bracing, say, - - - - - - -	1·00
Iron between end pillars, - - - - - -	**65·91**
Platform, rails, sleepers and ballast = $108 \times 0{\cdot}36$ tons (**445**), - - - - - - - -	38·88
Permanent bridge-load between end pillars, -	**104·79**

being 0·21 ton less than that assumed. If the four end pillars weigh 1·5 ton, the **total weight of wrought-iron** in the bridge will equal $65{\cdot}91 + 1{\cdot}5 =$ **67·41 tons.**

In this example, the main girders, weighing 45·47 tons, support themselves and an additional load of 202·53 tons uniformly distributed over the bridge. Consequently, each ton of useful load uniformly distributed requires for its support $\frac{45{\cdot}47}{202{\cdot}53} = 0{\cdot}2245$ tons of iron in the main girders. The inch-strains in the flanges, due to the permanent bridge-load of 105 tons, are $\frac{5 \times 105}{248} = 2{\cdot}12$ tons tension and $\frac{4 \times 105}{248} = 1{\cdot}7$ tons compression. The inch-strains due to the main girders alone, weighing 45·47 tons, are $\frac{5 \times 45{\cdot}47}{248} = 0{\cdot}92$ tons tension, and $\frac{4 \times 45{\cdot}47}{248} = 0{\cdot}73$ tons compression. The inch-strains in the flanges, due to a uniformly distributed train-load of 1·32 tons per running foot over the whole bridge, are $\frac{5 \times 143}{248} = 2{\cdot}88$ tons tension, and $\frac{4 \times 143}{248} = 2{\cdot}3$ tons compression.

518. Error in assuming the permanent load uniformly distributed in large girders—Empirical percentages open to improvement.—In the foregoing examples it has been tacitly assumed that the weight of the main girders is uniformly distributed. This is erroneous, because there is a preponderance of material in the flanges at the centre. It is true that the amount of bracing, both in the web and in the horizontal bracing, increases towards the ends and thus to a great degree compensates for the variation of section in the flanges. Still, the difficulty remains in the case of very large girders whose own weight forms the greater portion of the total load, and this preponderance of flange weight near the centre is the chief reason why single girders are less economical than continuous ones when the span is very great.

The empirical percentages adopted in the foregoing examples may perhaps be objected to, and it must be confessed that they are liable both to criticism and to correction from future experience. I have, however, made the most of the few recorded facts on which dependence can be placed, and would here suggest to my brother engineers that they should, as opportunity occurs, place on record in a tabular form the detailed weights of wrought-iron and steel girders, in order that this branch of our practice may attain that amount of precision that such statistical information alone can supply. In furtherance of this object I have added in the Appendix the detailed weights of the Boyne lattice bridge, which I collected when Resident there, also the details of the Conway tubular plate bridge and a few others. The examples in the present chapter indicate the direction in which improvements in constructive detail may be sought with most prospect of success. In very large girders this is a matter of great importance, for even a very slight diminution of any of the empirical percentages may effect a large amount of economy.

519. Fatigue of the material greater in long than in short bridges.—Though the maximum unit-strains may be the same in two bridges, one long and the other short, the permanent unit-strains, that is, the *fatigue* of the material from the permanent load (**470**), will be much higher in the bridge of great span. Thus, comparing Examples 2 and 11, we find that the fatigue, or permanent inch-strains, of a railway bridge 400 feet long, are 4·06 tons tension and 3·25 tons compression, while the corresponding inch-strains of a bridge 108 feet long, are 2·12 tons tension and 1·7 tons compression. If iron possessed unlimited viscidity, that is, the property of slowly and continuously changing shape, like pitch, under prolonged strains of moderate extent, it seems reasonable to suppose that the longer bridge would fail sooner than the short one, in consequence of its progressive deflection increasing more rapidly. Experience does not favour this hypothesis, for though experiments render it probable that all ductile metals will change shape to an unlimited extent under enormous pressure, in this respect resembling plastic clay, it seems equally certain that no continuous deformation takes place in structures whose unit-strains are kept well within the limits of elasticity (**410**). Again, it is conceivable, nay probable, that severe fatigue (especially if aided by vibration), may so alter the constitution of iron as to weaken parts in tension, either by rendering them brittle or by actually diminishing their tensile strength (**409**). If this were the case within the limits of strain which occur in practice, the longer bridge should still fail first. If, on the other hand, large fluctuations in the amount of strain affect the molecular condition of iron injuriously, and produce a tendency to rupture, then the short bridge should fail sooner. The experiments recorded in Chap. XXVIII. will prevent anxiety in either case when the working strains do not exceed those in usual practice (**471, 472, 475**).

GIRDERS UNDER 200 FEET IN LENGTH.

520. Flanges nearly equal in weight to each other, and web nearly equal in weight to one flange.—When an iron lattice girder of the ordinary proportions of length to depth does not exceed 200 feet in span, the flanges are very nearly equal in weight to each other (**477**), and the web is very nearly equal in weight to one flange. Moreover, the quantity of material in the compression flange is nearly equal to its theoretic central area multiplied by its length; for though, in correct practice, the section of the flange is reduced towards the ends, it so happens that the empirical allowance for covers, rivet heads, packings and waste, that is, the difference between the actual and the theoretic flange, is closely compensated for by assuming that the flange carries its theoretic central area uniformly throughout the whole length. Hence, we have the following empirical formula for the weight of material in the main girders, which will be found convenient in practice.

$$\mathbf{G} = \frac{3al}{4{\cdot}6} = \frac{2}{3}\, al, \text{ nearly.} \tag{253}$$

where $\mathbf{G}$ = the weight of the main girders and end pillars in tons,
a = the theoretic area of both compression flanges together at the centre, in square feet,
l = the length in feet,
$4{\cdot}6$ = the number of cubic feet of wrought-iron in one ton.

For girders loaded uniformly we have (eq. 25),

$$a = \frac{\mathbf{W}l}{8fd}$$

whence, by substitution in eq. 253,

$$\mathbf{G} = \frac{\mathbf{W}l^2}{12fd} \tag{254}$$

where $\mathbf{W}$ = the total distributed load in tons, including the weight of the girder,
l = the length in feet,
d = the depth in feet,
f = the working strain in tons per *square foot* of gross section.*

Ex. In Ex. 11, for instance, $\mathbf{G} = \frac{248\times(108\times108)}{12\times(4\times144)\times9} = 46{\cdot}5$ tons, which is but very slightly less than the former result.

521. Anderson's rule — Weights of lattice and plate girders under 200 feet in length.—I am indebted to William Anderson, Esq., for the following simple rule, derivable from eq. 254, for approximate estimates of railway bridges under 200 feet in length, whose depth is $\frac{1}{12}$th of their length, and whose working inch-strains are 5 tons tension and 4 tons compression. *Multiply the total distributed load in tons by* 4, *and the product is the weight of the main girders, end pillars and cross-bracing in lbs. per running foot.*

Ex. 1. The total distributed load in Ex. 11 equals 248 tons; hence, $4\times248=$ 992 lbs. = the weight of main girders, end pillars and cross-bracing per running foot, and their total weight $= \frac{992\times108}{2240} = 47{\cdot}8$ tons, which agrees very closely with the former result.

* The reader will recollect that the usual tensile working strain of iron, namely, 5 tons per square inch of *net* section, practically requires the same sectional area as the usual compressive working strain of 4 tons per square inch of *gross* section (**477**).

The following table contains the weights of wrought-iron lattice girders for railway bridges up to 200 feet in length, calculated by the foregoing rule for the three different standard working loads described in **490**. In making use of this table, the reader will bear in mind the following conditions:—

a. The working strains in the flanges are 5 tons per square inch of net section for tension, and 4 tons per square inch of gross section for compression.

b. The proportion of depth to length $= \frac{1}{12}$.

c. The dead weight of cross-girders, platform, ballast, sleepers, and rails = 0·54 tons per running foot of single line (**445**).

d. The weight of main girders for a double-line bridge is twice that given in the table for a single-line bridge.

e. It is probable that the weights in the table for the longer bridges, say above 140 feet, are rather in excess of truth, and that those for the shorter bridges, say under 60 feet, are slightly under the truth.

TABLE I.—WEIGHTS OF SINGLE-LINE WROUGHT-IRON LATTICE RAILWAY GIRDERS, THE DEPTH BEING $\frac{1}{12}$TH OF THE LENGTH.

Length of bridge from centre to centre of bearings.	Weight of Main girders, End pillars, and Cross-bracing,		
	when the standard load on a 100-foot bridge = 1 ton per foot.	when the standard load on a 100-foot bridge = 1¼ ton per foot.	when the standard load on a 100-foot bridge = 1⅓ ton per foot.
feet.	tons.	tons.	tons.
12	0·7	0·8	0·84
16	1·14	1·36	1·44
24	2·19	2·59	2·73
32	3·4	4·0	4·2
40	4·9	5·8	6·2
60	11·3	13·4	14·0
80	20·8	24·3	25·5
100	33·5	39·0	40·7
120	49·7	57·6	60·2
140	70·5	80·3	84·0
160	95·4	108·2	112·6
180	125·4	141·6	146·7
200	162·2	180·0	186·7

Ex. 2. What is the weight of iron required for a single-line lattice girder bridge, 140 feet long between bearings, whose depth = 11 feet 8 inches, and whose working inch-strains are the ordinary ones of 5 and 4 tons tension and compression respectively, the standard load being 1¼ tons per foot on a 100-foot bridge? From the table we find that the weight of the main girders, the end pillars and cross-bracing equals 80·3 tons, adding to this the weight of the cross-girders, supposed 3 feet apart, namely, 140 × ·18 = 25·2 tons (**445**), we have the total weight of iron = 105·5 tons.

The following table has been constructed by Mr. Baker, by taking as far as possible the weights of girders actually erected, calculating missing links in the series, rectifying the curves, and interpolating.*

* Baker *on the Strength of Beams*, p. 319.

TABLE II.—WEIGHTS OF WROUGHT-IRON PLATE GIRDERS, the depth being 1-10th of the length, and the working strain 4·5 tons per (gross ?) square inch in tension.

Span in feet.	Load in cwt. per foot run (exclusive of the weight of the girders).										
	10	15	20	25	30	35	40	50	60	70	80
	Weight of Girders in cwt.										
10	5·6	6·5	7·4	8·3	9·2	10·2	11·0	12·8	14·6	16·2	17·5
15	10·0	11·9	13·5	15·2	16·7	18·4	20·2	23·4	26·4	29·2	32·0
20	17·8	20·5	23·4	26·3	29·0	31·7	34·5	40·3	45·6	50·5	55·5
25	26·6	31·0	35·4	39·3	44·2	48·0	52·0	60·8	68·6	76·0	83·5
30	38	44	50	56	62	68	74	86	97	108	118
35	51	58	·66	74	82	89	97	113	129	145	160
40	65	75	85	95	105	115	125	145	166	187	209
45	82	94	106	118	130	142	154	180	207	236	264
50	101	115	130	144	159	173	188	220	254	290	325
55	118	135	152	169	187	204	222	259	298	340	382
60	138	157	177	196	217	237	258	302	348	395	441
65	159	181	204	227	251	275	298	348	400	453	507
70	198	225	253	282	312	342	372	435	500	565	630
75	228	260	292	326	360	394	429	500	575	650	726
80	258	294	331	369	407	446	484	566	650	735	822
85	291	333	375	416	460	502	546	637	732	829	928
90	326	373	420	467	515	563	612	712	818	927	1040
95	365	417	470	523	576	630	686	800	920	1043	1172
100	406	464	522	581	641	701	764	892	1028	1167	1310
110	495	565	636	708	780	855	930	1090	1260	1430	1610
120	595	677	762	848	934	1020	1112	1305	1510	1720	1940
130	705	800	900	1000	1100	1200	1310	1540	1780	2000	2300
140	823	940	1059	1178	1298	1417	1546	1810	2085	2376	2686
150	950	1090	1230	1370	1510	1650	1800	2100	2410	2740	3100
160	1095	1255	1414	1574	1732	1896	2066	2415	2782	3172	3585
170	1250	1430	1610	1790	1970	2160	2350	2750	3180	3630	4100
180	1426	1626	1826	2036	2240	2450	2670	3140	3630	4130	4670
190	1614	1832	2060	2280	2510	2750	3010	3550	4100	4670	5270
200	1810	2050	2300	2550	2800	3070	3370	3980	4600	5230	5900

522. Weights of similar girders under 200 feet span vary nearly as the square of their length—No definite ratio exists between the lengths and weights of very large girders.—An analysis of the foregoing tables shows that the ratio of the weights of similar railway girders from 40 to 200 feet in length vary between the square and the 2·3 power of their lengths (**274**). In Example 2, the main girders, 400 feet long, weigh 1047 tons, and in Example 5, a similar pair of main girders, 480 feet long, weigh 2449·6 tons. These weights are nearly as the 5th power of the lengths. Again, comparing Examples 3 and 6, which differ from the two former merely in having higher unit-strains, we find the weights of the main girders, which are 713 tons and 1405·9 tons respectively, are nearly as the 4th power of the lengths. These comparisons show that no definite ratio exists between the lengths and weights of very large girders, and any argument based on such an assumption must be altogether fallacious.

CHAPTER XXX.

LIMITS OF LENGTH OF GIRDERS.

523. Cast-iron girders in one piece rarely exceed 50 feet in length—Compound girders advisable for greater spans if cast-iron is used.—Cast-iron girders in one piece rarely exceed 50 feet in length, though this is by no means the possible limit of length of single castings, for Mr. Hawkshaw has employed cast-iron in single girders of 86 feet span,* and Sir Wm. Fairbairn mentions a bridge with girders, each 76 feet long in one casting, that were made in England and erected on the Haarlem Railway in Holland.† When cast-iron girders are required of greater length than 40 or 50 feet, it is advisable to truss them with wrought-iron, as cast-iron is ill-suited for resisting tension (**351**). Disastrous results have sometimes attended the use of compound girders, and they acquired a very bad reputation at one time, but the fault lay not so much in the combination of the two materials as in the mode of combination, which sometimes betrayed sad ignorance of the elementary principles on which girders should be constructed, the depth of the trussed girder having been in some instances considerably less at the centre than at the ends.

524. Practical limit of length of wrought-iron girders with horizontal flanges does not exceed 700 feet.—Vested interests and local peculiarities generally determine the spans of large bridges and it may therefore seem useless to attempt solving the question, "What is the practical limit of length of a girder?" Curiosity on this subject is, however, natural, and I may therefore claim indulgence for devoting a short space to investigating a question which, indeed, is not altogether devoid of

* *Proc. Inst. C. E.*, Vol. xiii., p. 474.

† *On the Application of Iron to Building Purposes*, p. 27.

practical utility. When the dimensions, weight and unit-strains of any given girder are known, we can find the length of a similar girder which will barely support itself; for it has been already shown in **67**, that if the weight of a given girder equals $\frac{1}{n}$th of its breaking weight, a similar girder n times longer will just break with its own weight. Thus, in the first example in the previous chapter, a pair of girders whose depth equals 1-12th of their length, 267 feet long and weighing 335·44 tons, sustain from their own weight 1·64 tons tension and 1·31 tons compression per square inch; supposing the tensile and compressive strength of plate iron to be 20 tons and 16 tons per square inch respectively, these working strains are equal to the breaking strains divided by 12·2. Hence, a similar girder 12·2 times longer, or 3257 feet in length, will just break down from its own weight. Now, the length of a similar girder whose working strains are only one-fourth of its ultimate strength will be $\frac{3257}{4} = 814$ feet nearly, which therefore is the extreme possible limit of an iron lattice girder whose depth equals 1-12th of its length, whose inch-strains are 5 tons tension and 4 tons compression, and whose empirical percentages are similar to those in the first example of the preceding chapter. The practical limit is of course far short of this and probably does not exceed 650 feet.

Again, in Ex. 4, the main girders, 400 feet long, whose depth equals 1-15th of their length and which weigh 937·4 tons, sustain 3·14 tons tension and 2·52 tons compression per square inch from their own weight. As these strains are equal to the ultimate strength of ordinary plate iron divided by 6·35, a similar girder 6·35 times longer, or 2540 feet in length, will just break down from its own weight. Hence, the length of a similar girder whose working strains from its own weight are 1-4th of its ultimate strength will be $\frac{2540}{4} = 635$ feet, which therefore is the limiting length of an iron lattice girder whose length equals 15 times its depth, whose inch-strains are 5 tons tension and 4 tons compression, and whose empirical percentages are similar to those adopted in the fourth example of the preceding chapter. The practical limit probably does not exceed 500 feet.

Again, in Ex. 9, the main girders, 600 feet long, whose depth equals 1-16th of their length and which weigh 3396·6 tons, sustain 4·8 tons tension per square inch from their own weight. This equals the ultimate tensile strength of ordinary plate iron divided by 4·16; hence, a similar girder 4·16 times longer, or 2496 feet in length, will just break down from its own weight, and the length of a similar girder whose working tensile inch-strain from its own weight is 6 tons, or $\frac{1}{3 \cdot 333}$ of its ultimate strength, will be $\frac{2496}{3 \cdot 333} =$ 749 feet. This therefore is the limiting length of an iron lattice girder whose tensile inch-strain is 6 tons, whose depth equals 1-16th of the length and whose empirical percentages are the same as those adopted in Ex. 9 of the preceding chapter. The practical limit is, doubtless, below 600 feet.

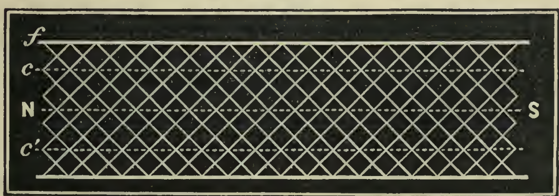
From these few examples we may reasonably infer that, even with the most careful attention to proportion and economy, the practical limit of length of wrought-iron girders with horizontal flanges does not exceed 700 feet. For girders of greater span steel must be employed.

CHAPTER XXXI.

CONCLUDING OBSERVATIONS.

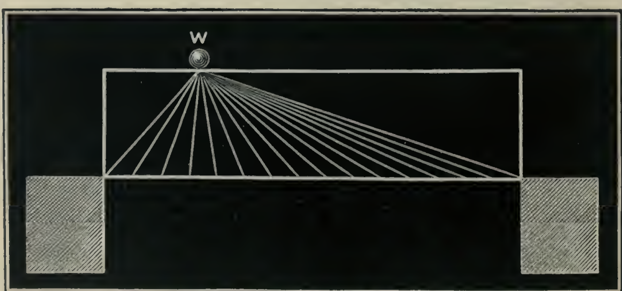
525. Hypothesis to explain the nature of strains in continuous webs.—The reader who has perused the foregoing pages with even slight attention has probably arrived at the conclusion that diagonal strains are not confined to braced girders, but are also developed in every structure which is subject to transverse strain. This follows at once from the mechanical law, that a force cannot change its direction unless combined with another force whose direction is inclined to that of the former. Thus, a vertical pressure cannot produce horizontal strains in the flanges without developing diagonal ones at the same time in the web. The following hypothesis will perhaps give a clearer conception of the nature of the strains in continuous webs. It is offered, however, merely as a conceivable condition of these strains.

Fig. 120.



Let Fig. 120 represent part of a closely latticed girder whose neutral surface, or surface of unaltered length is **N S.** The strain in each diagonal of an ordinary lattice girder is uniform throughout its entire length (**140**). Now, suppose that horizontal stringers are attached to the lattice bars at their first intersections next the flanges, and let us confine our attention to the upper one marked *c*. As soon as the girder deflects under a load, this stringer will become compressed; consequently, it will relieve the upper flange of a certain portion of the horizontal strain which the flange would sustain were the stringer absent, and the unit-strain in the stringer will be to that in the flange as $\frac{Nc}{Nf}$. The part of each diagonal above the stringer will also be relieved of a certain portion of its strain, depending on the horizontal component it yields up to the stringer. Now, conceive similar stringers attached at each horizontal row of lattice intersections above and below the neutral surface, in which case each stringer will sustain horizontal unit-strains directly proportional to its distance from the neutral surface where they are cipher, while, on the other hand, the strains in the diagonals will diminish as they approach the flanges, their *decrements* of strain being cipher at the neutral surface and increasing towards the flanges in the direct ratio of their distance from the neutral surface, provided the stringers are all of equal area. We thus see that the diagonal strains, and therefore the shearing strain in solid girders, or in girders with continuous webs, act with greatest intensity in the neighbourhood of the neutral surface where the horizontal strains are nil, while they act with least intensity at the upper and lower edges where the horizontal strains are most intense. This theory agrees with an instructive experiment made by Mr. Brunel on a single-webbed plate girder, 66 feet long between bearings and 10 feet deep at the centre, in which the web, formed of $\frac{1}{4}$ inch plates with vertical lap joints, gave way by several of these joints near one end tearing open in the neighbourhood of the neutral surface.*

Fig. 121.

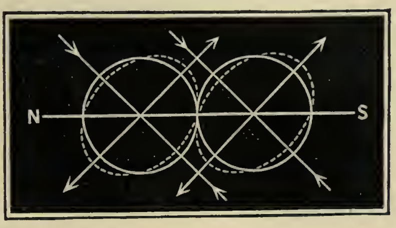


* Clark *on the Tubular Bridges*, p. 437.

When a single weight rests upon a girder with a continuous web, it sends off strains radiating out from the weight in all directions, as represented in Fig. 121, and we may conceive that this first series of diagonal strains are resolved at every point along their length into diagonal and horizontal strains, as in the lattice girder; this second series of diagonal strains being again resolved in a similar manner, and so on, and thus we have horizontal and diagonal strains interlacing at various angles in all girders except those in which they are forced to take definite directions by means of the bracing, but there will probably exist certain lines of maximum strain, either straight or curved, whose directions will vary according to the position and amount of the weight, as well as the flexibility of the material. The student may make some instructive experiments on this subject by the aid of a model girder formed by stretching a web of drawing paper over a light rectangular frame of timber, which will represent the flanges and end pillars. By the aid of little movable wooden struts, to represent verticals, he can vary the directions of the lines of strain to a very considerable extent.

It is not at first sight easy to see how strains are transmitted through the neutral surface, for the particles there are apparently undisturbed in form. It is conceivable, however, that particles which are spherical when free from strain may become elongated by tension in one direction and shortened by compression at right angles to it, so as to assume an oval shape, while horizontal lines parallel to the neutral surface, **N S**, retain their original length, as represented in Fig. 122.

Fig. 122.



526. Strains in Ships.—An iron ship is a large tubular structure, more or less rectangular in section, underneath which the points of support are continually moving, so that, when the waves are high and far apart, the deck and bottom of the vessel are alternately extended and compressed in the same way that the flanges of a continuous girder are near the points of inflexion when traversed by a passing train. The sides of a ship are formed of continuous plating with vertical frames at short intervals, and form very efficient webs; the bottom also is, from its large area, fully adequate to its duty as a flange. The sides and bottom flange of the girder are therefore fully developed, but the upper iron flange is sometimes altogether wanting, or else sadly out of proportion to the remainder of the structure. This deficiency is properly remedied, either by attaching what are technically called stringers to the topsides, or better still, by making the upper deck entirely of iron with a thin sheeting of planks resting on the iron.* Deck stringers are horizontal plates which run continuously fore and aft beneath the planking of the deck. They are seldom more than 3 or 4 feet in width, but in some few cases extend as far as the hatchways. Similar stringers are occasionally riveted to the sides underneath each of the lower decks, and when stringers in the same plane on opposite sides of the ship are connected by diagonal tension braces, the latter, in conjunction with the deck beams, form very efficient cross-bracing, and greatly increase the strength and stiffness of the ship when labouring in a heavy sea. Bulkheads act as gussets or diaphragms, and stiffen the ship transversely by preventing any racking motion from taking place in the direction of their diagonals.

527. Iron and timber combined form a cheap girder—Timber should be used in large pieces, not cut up into planks—Simplicity of design most desirable in girder-work.—Within certain limits of length, one of the cheapest forms of girder is one made of timber in compression with wrought-iron in tension (**187, 221**). The earlier types of wooden lattice bridges had little or no iron in their composition and were characterized by

* The author has built several iron vessels in which tar asphalt is substituted for the timber sheeting.

the small scantlings of the parts, the closeness of the latticing, and in many cases, a want of stiffness both vertically and laterally. This defect was, no doubt, often due to insufficient flange area, but may also be attributed to the small size of the scantlings, and consequent multiplicity of joints. The remedy is obvious. Timber in compression should be used in bulk, and not cut up into thin planks. Laminated arches, it is true, are an apparent exception to this rule, but in reality a laminated beam possesses the aggregate section of its component parts which are bound together so that they act as one solid piece. Even when used in tension, it may be doubtful economy to use several thin planks where one of larger section would suffice. The liability to decay from moisture lodging in the numerous joints is another serious objection to close timber latticing, though this is sometimes diminished by the protection of a roof extending over the whole bridge (**485**).

In conclusion, it may not be amiss to say a few words on designing girders. Simplicity and consequent facility of construction should never be lost sight of. Complicated arrangements are to be deprecated, whether designed to affect some saving more apparent than real, or, as one is sometimes tempted to conjecture, from a craving after novelty. The various parts of girder work should, as much as possible, be repetitions of the same pattern, easily put together and accessible for preservation or repair. Hence, as a rule, closed cells, difficult forgings, curved forms where straight ones would effect the object equally well, and a great variety of sizes to meet excessive theoretic refinement, are to be carefully avoided.

APPENDIX.

BOYNE LATTICE BRIDGE.

528. General description and detailed weights of girder-work.—The Boyne Viaduct carries the Dublin and Belfast Junction Railway across the valley of the River Boyne near Drogheda, and consists of several lofty semi-circular stone arches on the land, and a wrought-iron lattice bridge in three spans over the water, the surface of which is about 90 feet below the girders, so that vessels of considerable tonnage can sail beneath. The girder-work is formed of two lattice double-webbed main girders, having their top flanges connected by cross-bracing, and the lower flanges connected by cross-girders and diagonal ties, so as collectively to form an openwork tubular bridge for a double line of railway, as shown in cross-section in Plate IV. Each main girder is a continuous girder, 3 feet wide and 550 feet 4 inches long, in three spans. The centre span is 267 feet from centre to centre of bearings, and 264 feet long between bearings. Each side span is 140 feet 11 inches long from centre to centre of bearings, and 138 feet 8 inches long between bearings. The flanges are horizontal throughout, and the depth of girder, measured from root to root of angle irons, is 22 feet 3 inches, or 1-12th of the centre span and $\frac{1}{6{\cdot}34}$th of each side span. Each of the terminal pillars is 18 inches broad in elevation and has a bearing surface of $3 \times 1{\cdot}5 = 4{\cdot}5$ square feet; each of the pillars at the ends of the centre span is 3 feet broad in elevation and has a bearing surface of $3 \times 3 = 9$ square feet. The cross-girders are 7 feet 5 inches apart from centre to centre and correspond with the intersections of the lattice bars, which are placed at an angle of 45° and form squares of 5 feet 3 inches on the side. The quantities of material in the girder-work are as follows:—

* For further description, see *Proc. Inst. C.E.*, Vol. xiv.; also, *Proc. Inst. C.E. of Ireland*, Vol. ix.

TABLE I.—Weight of wrought-iron in each side Span, 140 feet 11 inches between centre of bearings and 30 feet wide from out to out.

	Tons.	Tons.
Two top Flanges.		
Plates and angle iron,	27·45	
Covers,	3·57	39·84
Packings,	6·38	
Rivet heads,	2·44	
Two bottom Flanges.		
Plates and angle iron,	27·10	
Covers,	3·84	39·59
Packings,	6·40	
Rivet heads,	2·25	
Two double-latticed Webs.		
Tension diagonals,	10·96	
Compression do.,	27·70	38·79
Rivet heads at intersections,	0·13	
Cross-bracing.		
6 lattice cross-girders connecting top flanges,	3·70	
Horizontal diagonal tension bars (top and bottom) and a longitudinal angle iron stiffener along the centre at top,	5·36	9·16
Rivet heads,	0·10	
Cross-girders.		
18 lattice road-girders, including end gussets,		23·72
Iron between end pillars,		**151·10**
Platform planking,	29·40	
Longitudinal sleepers (double line),	2·45	40·41
Rails and joint plates (Barlow's),	8·56	
Permanent load on one side span, equal to 1·36 tons per running foot for the double line.		**191·51**

TABLE II.—WEIGHT OF WROUGHT-IRON IN THE CENTRE SPAN, 267 FEET BETWEEN CENTRES OF BEARINGS AND 30 FEET WIDE FROM OUT TO OUT.

	Tons.	Tons.
Two top Flanges.		
Plates and angle iron,	79·09	
Covers,	9·38	
Packings,	11·83	105·48
Rivet heads,	5·18	
Two bottom Flanges.		
Plates and angle iron,	82·19	
Covers,	9·85	
Packings,	11·90	109·12
Rivet heads,	5·18	
Two double-latticed Webs.		
Tension diagonals,	30·80	
Compression do.,	51·76	82·81
Rivet heads at intersections,	·25	
Cross-bracing.		
11 lattice cross-girders connecting top flanges,	6·77	
Horizontal diagonal tension bars (top and bottom) and a longitudinal angle-iron stiffener along the centre at top,	10·69	17·66
Rivet heads,	·20	
Cross-girders.		
35 lattice road-girders, including end gussets,		46·13
Iron between end pillars,		**361·20**
Platform planking,	55·57	
Longitudinal sleepers (double line),	4·62	76·39
Rails and joint plates (Barlow's),	16·20	
Permanent load on centre span, equal to 1·64 tons per running foot for the double line.		**437·59**

TABLE III.—Weight of wrought-iron in the Pillars and Cross-girders over supports.

	Tons.	Tons.
Pillars, &c., over each Land Abutment.		
2 terminal pillars at end of one side span, - - -	6·38	
1 lattice cross-girder connecting heads of pillars, - -	3·40	13·23
1 lattice cross-girder and gussets connecting feet of pillars, -	3·45	
Pillars, &c., over south River Pier.*		
2 pillars at south end of centre span, - - - -	15·30	
1 lattice cross-girder connecting heads of pillars, - -	5·24	24·06
1 lattice cross-girder connecting feet of pillars, - - -	1·09	
2 gussets between pillars and pier, - - - -	2·43	
Pillars, &c., over north River Pier.		
2 pillars at north end of centre span, - - - -	15·30	
1 lattice cross-girder connecting heads of pillars, - -	5·24	25·56
1 lattice cross-girder connecting feet of pillars, - - -	5·02	

TABLE IV.—Summary of wrought-iron.

	Tons.
One side span, - - - - - - - -	151·10
Second do., - - - - - - - -	151·10
Centre span, - - - - - - - -	361·20
Pillars, &c., over one land abutment, - - - - -	13·23
Do. ,, second do. - - - - - -	13·23
Do. ,, south river pier, - - - - -	24·06
Do. ,, north river pier, - - - - -	25·56
Total weight of wrought-iron in the 3 spans, - - - 550 feet 4 inches in total length, equal to 1·344 tons per running foot for the double line of railway.	**739·48**

* The pillars are firmly secured to this pier; rollers are used on the north pier and on both abutments.

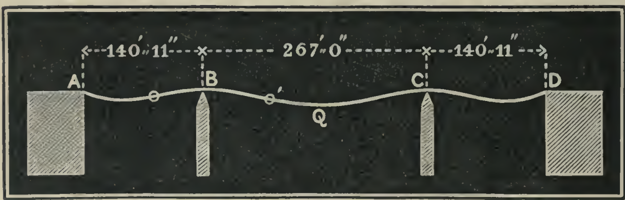
TABLE V.—WEIGHT OF SOLE-PLATES, ROLLERS AND WALL-PLATES.

OVER TWO ABUTMENTS.	Tons.	cwts.	qrs.	lbs.
4 planed cast-iron sole-plates riveted to feet of pillars, -	—	17	2	16
4 planed cast-iron wall-plates resting on the masonry, -	2	11	0	0
2 sets of 4-inch wrought-iron rollers and frames over the north abutment, - - - - - -	—	10	2	0
2 sets of 4½-inch wrought-iron rollers and frames over the south abutment, - - - - - -	—	12	2	26
OVER SOUTH RIVER PIER.				
2 cast-iron sole-plates riveted to feet of pillars, - -	—	19	0	12
2 cast-iron wall-plates resting on the masonry, - -	5	4	0	0
OVER NORTH RIVER PIER.				
2 planed cast-iron sole-plates riveted to feet of pillars, -	—	19	0	12
2 planed cast iron wall-plates resting on the masonry, -	4	13	0	16
2 sets of 5-inch chilled cast-iron rollers and wrought-iron frames, - - - - - - -	1	15	0	16
Total weight of sole-plates, rollers and wall-plates, - - - - - -	**18**	**2**	**1**	**14**

529. Working strains and area of flanges.—The strains produced by the permanent bridge-load, plus one ton of train-load per running foot on each line of way, do not exceed 5 tons tension per square inch of net area, *i.e.*, after deducting the rivet holes, and 4 tons compression per square inch of gross area. The gross sectional area of the top flange of each main girder in the centre of the centre span = 113·5 square inches; the gross area of the bottom flange at the same place = 127 square inches, and its net area = 99 square inches; over the piers, between the centre and side spans, the gross area of the top flange of each main girder = 132·6 square inches, and its net area = 103·4 square inches; the gross area of the bottom flange at the same place = 127 square inches. At the points of inflexion in the centre span, about 40 feet from the piers measured towards the centre of the bridge, the gross area of each flange = 68·5 square inches.

530. Points of inflexion—Pressures on points of support.— The points of inflexion may be obtained by the method explained in **253**, as follows:—

Fig. 123.



Let **Q** be the centre of the centre span, and o and o' the points of inflexion.

Let $l = \mathbf{AB} = \mathbf{CD} = 141$ feet nearly,

$\mathbf{AQ} = nl$, whence $n = \dfrac{274 \cdot 5}{141} = 1 \cdot 95$ nearly,

w = the load per running foot on either side span,

w' = the load per running foot on the centre span,

$\mathbf{R}_1$ = the reaction of either abutment, **A** or **D**,

$\mathbf{R}_2$ = the reaction of either pier, **B** or **C**.

When the bridge supports its own weight only,

$$w = 1 \cdot 36 \text{ tons and } w' = 1 \cdot 64 \text{ tons.}$$

Case 1.

531. Maximum strains in the flanges of the side spans.— These occur when the passing load covers both side spans and the centre span is unloaded (**255**); in which case, assuming that the maximum train-load is equivalent to one ton per running foot on each line of way, we have,

$$w = 3 \cdot 36 \text{ tons and } w' = 1 \cdot 64 \text{ tons.}$$

From equations 183 and 184 the pressures on the points of support are as follows:—

$$\mathbf{R}_1 = 170 \text{ tons.} \qquad \mathbf{R}_2 = 523 \text{ tons.}$$

The positions of the points of inflexion, obtained from equations 185 and 186, are as follows:—

$$\mathsf{A}o = 101{\cdot}2 \text{ feet.} \qquad \mathsf{B}o' = 53{\cdot}2 \text{ feet.}$$

The strain in each of the four flanges midway between **A** and o, *i.e.*, in the centre of the first segment, is 96·6 tons (eq. 25).

Case 2.

532. Maximum strains in the flanges of the centre span.— These occur when the passing load covers the centre span alone, in which case,

$$w = 1{\cdot}36 \text{ tons and } w' = 3{\cdot}64 \text{ tons.}$$

The pressures on the points of support are as follows:—

$$\mathsf{R}_1 = -24{\cdot}6 \text{ tons.} \qquad \mathsf{R}_2 \doteq 704 \text{ tons.}$$

R_1 being negative, signifies that a load of 24·6 tons is required at each end to prevent the girder from rising off the abutments (**254**), and this was actually the case when the bridge was proved with one ton per running foot on each line of the centre span, the side spans being unloaded. The girder was temporarily tied down to the abutments by bolts secured to the masonry, but the bolts drew out and the ends of the girder rose more than an inch above their normal position on the rollers. The weight of a locomotive at each end, however, soon brought them down again. With the lighter working loads which occur in practice this rising off the abutments does not occur. The position of the points of inflexion in the central span is as follows:—

$$\mathsf{B}o' = 40{\cdot}3 \text{ feet,}$$

and the strain in each of the four flanges in the centre at $\mathsf{Q} = 355$ tons (eq. 25). At this place the net area of each lower flange $= 99$ square inches and the tensile inch-strain therefore $= \frac{355}{99} = 3{\cdot}6$ tons.

Case 3.

533. Maximum strains in the flanges over the piers.—The maximum strains over a pier occur when the centre span and the adjacent side span are loaded, and the remote side span is unloaded. We have, however, no formula for this condition of load, but we have a close approximation to it when the passing load covers all three spans (**255**), in which case,

$$w = 3{\cdot}36 \text{ tons and } w' = 3{\cdot}64 \text{ tons.}$$

The pressures on the points of support are as follows:—

$$\mathsf{R}_1 = 107 \text{ tons.} \qquad \mathsf{R}_2 = 853 \text{ tons.}$$

The positions of the points of inflexion are as follows (eqs. 185 and 186):—

$$\mathsf{A}o = 63{\cdot}4 \text{ feet.} \qquad \mathsf{B}o' = 44{\cdot}7 \text{ feet.}$$

The strain in each of the four flanges over the piers = 406·4 tons (eq. 12). The net area of each upper flange at this place = 103·4 square inches and the tensile inch-strain therefore $= \frac{406{\cdot}4}{103{\cdot}4} = 3{\cdot}93$ tons.

534. Points of inflexion fixed practically—Deflection—Camber.—The points of contrary flexure in the centre span were practically fixed in the manner described in **250**. Two joints in the upper flange, 170 feet apart and equi-distant from the piers, were selected for section. The rivets were cut out and drifts temporarily inserted in their place. These drifts were then cautiously struck out with a light hammer, and a slight closing of the joints proved that a certain amount of compression had previously existed in place of perfect freedom from strain. The extreme ends of the side spans were then lowered, one an inch, the other half an inch, which caused the joints to open about $\frac{1}{64}$th of an inch. In this condition it was obvious that no strain was transmitted through the joints, and they were then finally riveted up, the altered levels of the extreme ends of the side spans being maintained by rollers of the proper diameter placed beneath the terminal pillars. Tables VI. and VII. contain the deflections produced by various conditions of load during the first, or Engineer's testing, and the second, or official testing of the bridge by the Government Inspector (**409**).

TABLE VI.—DEFLECTIONS IN INCHES, MEASURED DURING THE FIRST, OR PRIVATE TESTING OF THE BRIDGE, MARCH 28TH, 1855.

NORTH SIDE SPAN.		CENTRE SPAN.					SOUTH SIDE SPAN.			
feet. 89	feet. 44·5	feet. 44·5	feet. 89	feet. 133·5	feet. 89	feet. 44·5	feet. 44·5	feet. 89	Distances measured in feet from centres of River Piers.	
inches. 0·675	inches. 0·475	inches. +0·175	inches. +0·2	inches. +0·175	inches. +0·2	inches. +0·112	inches. 0·5	inches. 0·4	West girder,	138 tons on each line of both side spans, *i.e.*, *maximum* load on both side spans together.*
0·8	0·55	+0·2	+0·25	+0·275	+0·25	+0·2	0·6	0·6	East do.	
+0·45	+0·375	1·0	1·7	2·15	1·85	1·025	0·225	0·3	West do.	271 tons on each line of centre span and 138 tons on each line of south side span, *i.e.*, *maximum* load on centre span and on south side span.
+0·3	+0·233	1·075	1·75	2·075	1·675	0·8	0·15	0·5	East do.	
+0·95	+0·65	2·3	2·4	2·925	2·7	1·6	0·0	0·0	West do.	271 tons on each line of centre span, *i.e.*, *maximum* load on centre span alone.†
+0·525	0·0	1·375	2·4	2·8	2·45	2·025	+1·1	+0·6	East do.	
0·1	0·0	0·3	0·55	0·65	0·65	0·4	0·05	0·1	West do.	Residual set on the 29th, after all the load had been removed (**453**).
0·175	0·1	0·3	0·5	0·6	0·5	0·25	0·05	0·025	East do.	

NOTE.—The sign + prefixed to a deflection denotes that the girder cambered, or rose above its original position.

* With this maximum load on both side spans together, the north-west abutment pillar sank 0·11 inches; the north-east do., 0·25 inches; the south-west do., 0·2 inches; the south-east do., 0·25 inches, in consequence of the compression of some timber packing temporarily placed beneath the pillars.

† With this maximum load on the centre span, the north-west abutment pillar rose 1·25 inches; the north-east do., 1·06 inches; the south-west do., 2·0 inches; the south-east do., 1·62 inches, thus confirming previous calculations (**532**).

TABLE VII.—DEFLECTIONS IN INCHES, MEASURED DURING THE TESTING OF THE BRIDGE BY THE GOVERNMENT INSPECTOR, MARCH 30TH, 1855.

NORTH SIDE SPAN.		CENTRE SPAN.					SOUTH SIDE SPAN.			
feet.	feet.	feet	feet.	feet.	feet.	feet.	feet.	feet.		
89	44·5	44·5	89	133·5	89	44·5	44·5	89	Distances measured in feet from centres of River Piers.	
inches	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.		
0·65	0·5	+0·25	+0·475	+0·5	+0·5	+0·325	0·5	0·7	West girder,	One ton per running foot (= 141·5 tons) on each line of both side spans, *i.e.*, *maximum* load on both side spans together.
0·625	0·45	+0·275	+0·425	+0·5	+0·45	+0·35	0·5	0·7	East do.	
0·3	0·15	0·7	1·3	1·65	0·0	0·0	+0·3	+0·2	West do.	One ton per running foot on each line of north side span and centre span, *i.e.*, *maximum* load on north side span and on centre span.
0·25	0·05	0·75	1·35	1·65	1·425	0·75	+0·3	+0·3	East do.	
+0·25	+0·225	0·85	1·6	1·9	1·6	0·9	+0·3	+0·275	West do.	One ton per running foot (= 267 tons) on each line of centre span, *i.e.*, *maximum* load on centre span alone.*
+0·25	+0·3	0·85	1·55	1·9	1·575	0·8	+0·4	0·35	East do.	
0·3	0·175	0·675	1·25	1·525	1·25	0·625	0·1	0·3	West do.	One ton per running foot on each line of the three spans, *i.e.*, *maximum* load all over the bridge = 1,100 tons.
0·325	0·1	0·65	1·175	1·45	1·175	0·55	0·1	0·35	East do.	
0·1	0·075	0·05	0·1	0·1	0·0	0·0	0·0	0·05	West do.	Residual set, load all removed.
0·1	0·025	0·05	0·075	0·1	0·05	0·0	0·0	0·1	East do.	

NOTE.—The sign + prefixed to a deflection signifies that the girder cambered, or rose above its original level.

* When this maximum load rested on the centre span alone, the extreme ends of the girder were held down by an engine or waggon placed immediately over the first and last road-girders (**532**).

Each span was built on the platform with a camber in order that the sky-line might be nearly horizontal when the bridge was finished (**453**). The camber at the centre of the centre span at different periods was as follows:—

TABLE VIII.—CAMBER AT CENTRE OF THE CENTRE SPAN.

	Inches.
During construction on the platform, - - - - -	3·48
After wedges were struck and bridge was self-supporting, - - -	1·56
After fixing points of inflexion and lowering the ends of the side spans, -	1·80
After the second, or official testing of the bridge, - - - -	0·84
After four months' traffic, - - - - - - -	0·90

535. Experiments on the strength of braced pillars.— The following experiments were made at the Boyne Viaduct in 1854, to determine the strength of one of the compression diagonals of the web which were made of flat bar iron similar to the tension diagonals, but with the addition of internal angle irons and cross-bracing riveted between them as already described in **341**. The theory of braced pillars was then imperfectly understood, and it was determined to test by direct experiment whether this arrangement of internal cross-bracing would enable a bar, thin in proportion to its length, to sustain an endlong pressure like a pillar, such as the compression diagonals should sustain in the bridge. Accordingly, the following experiments were made on one of the smaller compression diagonals which occur near the centre of the centre span, the author being present and recording the results.

EXPERIMENT 1.

The first experimental pillar resembled Fig. 1, Plate V., in every respect, except the lower portion, which was formed as shown in Fig. 4. This pillar, which was 31′ 6″ in length with 4″ × $\frac{5}{8}$″ side bars, was erected in the midst of some timber scaffolding which had been used for a stone hoist. The testing weight was suspended below the wooden framing on which the pillar stood by long suspender rods which were attached to cross pieces of timber resting on the top of the pillar (see Figs. 2 and 3). By this arrangement the pressure was made to pass more accurately through the axis of the pillar than if the testing weight had been heaped up on top; it was also more convenient to load at the lower level. Cross bars *f*, *f*, *f*, were attached to the sides at the same intervals as the latticing in the main girders, and were connected at their ends to the scaffolding, so as to represent the tension diagonals in the bridge; and here I may again remind the reader that the chief advantage of a multiple over a single system of triangulation consists in the more frequent support given by the tension bars to those in compression, as well as by both to the flanges; the parts in compression are in fact subdivided into short pillars, and thus prevented from deflecting in the plane of the girder (**153**). A cord was stretched vertically, in order to get the lateral deflections during the experiment. These were taken at three points, A, B, C, Fig. 1, and the symbols + or — placed before a deflection in the table signifies that it was in the direction of the same sign engraved at the sides of the figure.

TABLE IX.—Lateral Deflections of a braced Pillar.

Date.	Tons.	A.	B.	C.	Remarks.
1854.		inches.	inches.	inches.	
Nov. 16	5	+ 0·03	+ 0·05	+ 0·03	
„	10	0·0	0·0	+ 0·05	
„	15	— 0·03	+ 0·03	+ 0·05	
„	20	— 0·05	0·0	0·0	
„	25	— 0·05	0·0	0·0	
„	30	— 0·05	— 0·03	— 0·04	
„	37½	— 0·06	— 0·07	— 0·06	
„	40	— 0·05	— 0·01	— 0·05	With 40 tons, the side bar at *a*, Fig. 4, bent slightly at right angles to the plane of the figure. The deflection at B seems anomalous; probably a mistake for 0·10.
„	42½	— 0·10	— 0·10	— 0·13	With 42½ tons, the lower part of the pillar at *b*, *b*, became slightly curved, with the convex side towards the — side.
Nov. 17	42½	— 0·10	— 0·10	— 0·13	Left on all night; no change in the morning.
„	45	— 0·10	— 0·15	— 0·16	
„	47½	...	...	...	The side bars gave way, as shown in Fig. 5.

Looking at Fig. 4, it will be seen that about 8 inches in length of each side bar near the ends of the pillar were left without internal angle iron, and when the weight amounted to $47\frac{1}{2}$ tons, this part yielded sideways, as shown in Fig. 5. The area of the two side-bars at the part which failed amounted to 5 square inches; consequently, the compressive strain which passed through them at the moment of yielding equalled $9\frac{1}{2}$ tons per square inch.

Experiment 2.

The pillar in the first experiment failed, as indeed had been anticipated, by the upper part moving sideways past the lower, as if connected to it by hinges. The pillar was taken down, the injured part removed, and the length thus reduced to 28′ 6″. The repaired pillar, Fig. 1, was then replaced within the scaffolding and the following table contains the observations recorded, which include the contraction in length of each side under compressive strain. These latter observations were made by the aid of wooden rods suspended at each side from near the top of the pillar. Each rod was 24′ $8\frac{1}{2}$″ in length from the point of suspension to the index at the lower end, and it will be observed that the contraction of one side exceeds that of the other in a very anomalous manner, which can only be explained by supposing that the timber framing yielded more beneath one side than the other and thus caused a greater strain of compression to pass through that side of the pillar which contracted most.

TABLE X.—Lateral Deflections and Vertical Contraction of a Braced Pillar.

Date.	Tons.	A.	B.	C.	Rod on +side.	Rod on −side.	Observations.
		inches.	inches.	inches.	inches.	inches.	
1854. Nov. 25	30	+0·03	+0·04	+0·01	0·05	0·25	At 30 tons, the side bar at *c* bulged outwards slightly, with a tendency to increase; also a slight hollow was produced at *d*; to remedy this bulging (which seemed to be caused by the unequal compression of the timber packing, that on the + side yielding more than that opposite), a strut was placed against *c*, and the weight was blocked up until the 27th; wedges also were driven between the wooden packings underneath, in order to tighten them up.

TABLE X.—LATERAL DEFLECTIONS, &c.—*continued.*

Date.	Tons.	A.	B.	C.	Rod on + side.	Rod on — side.	OBSERVATIONS.
		inches.	inches.	inches.	inches.	inches.	
Nov. 27	0	...	...	...	0·0	0·20	Load removed, and bulging at *c* removed as nearly as possible by means of a screw-jack which was left in position; opposite side similarly blocked out from staging, and blocks were placed at similar positions at top of pillar, as there appeared a tendency of top to move over to — side.
,,	30	...	...	...	0·05	0·25	
,,	35	...	...	...	0·06	0·27	
,,	40	0·0	—0·01	—0·03	0·08	0·31	Left hanging on all night, wind so strong as to make deflections uncertain.
Nov. 28	40	0·0	—0·37	—0·07	0·09	0·31	The hollow at *d* still well marked and a tendency to deflect towards + side, at the centre of the pillar.
,,	45	0·0	—0·01	—0·07	0·09	0·34	Wind in gusts; 45 tons left hanging on one hour.
,,	50	+0·07	+0·06	—0·03	0·10	0·40	Wind much abated; no visible change.
,,	55	...	...	...	0·10	0·44	Wind so strong as to prevent deflections being taken. No visible change.
,,	60	...	...	...	0·10	0·49	No visible change.
,,	62½	...	...	...	0·105	0·50	Left hanging on all night.
Nov. 29	...	...	...	...	0·11	0·50	No visible change this morning.
,,	65	...	...	...	0·12	0·53	The buckle at centre strongly marked.
,,	70	+0·15	+0·14	+0·03	0·12	0·56	Wind much abated.
,,	72½	...	...	...	0·13	0·60	
,,	75	....	...	...	0·14	0·65	No visible change or upsetting of any part.
,,	77½	...	...	...	0·13	0·69	
,,	80	...	...	...	...	...	Left hanging on all night.
Nov. 30	...	...	...	...	0·14	0·78	In morning.
,,	82½	...	...	...	0·15	0·795	
,,	83½	...	...	...	...	...	Broke down as the additional ton was being laid on, parts *b* and *e*, Fig. 1, giving way. At *e*, both sides of the pillar bent and the internal lattice was completely distorted, the L iron being broken away from the side bar (see Fig. 2).

The sectional area of that part of the pillar which was subject to compression, namely, the side bars and the angle irons, was 7·5 inches. The compression therefore equalled 11 tons per square inch at the period of failure. For a very short portion at *c*, where the bracing ended, the angle irons of the lower cell and that to which the internal lattice bars were connected were not in one continued piece, and the whole weight passed through the unsupported side bars, which were, however, a little thicker here than elsewhere from a weld having been made at that point, so that the area of both side bars together equalled 6 square inches; this short length was therefore subject to a compression of nearly 14 tons per square inch. If we wish to compare the economy of this form of pillar with a tubular one, we must add the cross area of the lattice bars to that of the side bars and angle irons, in order to obtain the strain per sectional inch of material in the whole pillar. The cross area of the lattice bars = 2 inches nearly; adding this to the area of the side bars and angle irons, we have the total sectional area of the braced pillar = $9\frac{1}{2}$ inches, and the compression per square inch of material employed = 8·7 tons. This is a favourable result when compared with those arrived at by Mr. Hodgkinson in his experiments on tubes subject to compression, for if the same amount of iron were thrown into the form of a plated tube, it would have such thin sides that the ultimate crushing inch-strain would probably fall very far short of 8·7 tons (**335**). We may regard the lattice pillar as one side of a tube, in the corners of which the chief part of the material is collected and the sides of which are formed of bracing, connecting and holding the corner pillars in the line of thrust.

536. Experiments on the effect of slow and quick trains on deflection.—The following experiments were made at the Boyne Viaduct to try the effect of slow and quick trains on vibration and deflection:—

April 5th, 1855.—The lateral oscillation at the centre of the centre span from an engine and tender going at the rate of from 30 to 50 miles an hour equalled 0·05 inch on each side, *i.e.*, the total oscillation equalled 0·1 inch. That from a slow engine was scarcely perceptible. The deflection at the centre of the centre span, measured on the same side as the line on which the engine and tender travelled, both for quick and slow speeds equalled ·25″. The same deflection was produced when the engine was brought to a stand at the centre of the centre span. If any difference of deflection with different speeds was perceptible, those deflections which were produced by rapid travelling exceeded the others by a very small amount, perhaps the width of a fine pencil stroke, but for all practical purposes they were identical. On starting the engine from rest at the centre of the bridge the deflection was momentarily increased to a very slight extent. There were about five quick trains, of which one travelled at 48 and the others 50 miles an hour, and about as many slow ones (**454**).

NEWARK DYKE BRIDGE, WARREN'S GIRDER.*

537.—This bridge carries the Great Northern Railway across the Newark Dyke, a navigable branch of the river Trent. It is a skew girder bridge, formed of a single system of equilateral triangles on Warren's principle. Each girder consists of a hollow cast-iron top flange, and a bottom flange, or tie, of wrought-iron flat bar links, connected together by diagonal struts and ties, alternately of cast and wrought-iron, which divide the whole length into a series of equilateral triangles, 18 feet 6 inches long on each side. There are two main girders to each line, between which the train travels on a platform attached to the lower flanges. The length from centre to centre of points of supports is 259 feet, and the clear span between the abutments is 240 feet 6 inches. The depth from centre to centre of flanges is 16 feet, or nearly 1-16th of the length. The permanent weight of bridge for a single line of railway, consisting of two main girders, top and bottom cross-bracing, platform, &c., is as follows:—

	Tons.	Cwts.
Wrought-iron, - - - -	106	5
Cast-iron, - - - - -	138	5
	244	10
Platform, rails, handrail and cornice, -	56	0
Total permanent weight for one line of way,	**300**	**10**

* "Description of the Newark Dyke Bridge."—*Proc. Inst. C.E.*, Vol. xii.

With a load of one ton per running foot the central deflection amounted to $2\frac{3}{4}$ inches. The strain with this load, whether tensile or compressive, is said not to exceed 5 tons per square inch on any part.

CHEPSTOW BRIDGE, GIGANTIC TRUSS.*

538.—This bridge was erected by Mr. Brunel to carry the South Wales Railway across the river Wye near Chepstow. It consists of two gigantic trusses, one for each line of way, 305 feet long and about 50 feet deep, and resembling Fig. 64, p. 124, with this exception, that the roadway is attached to the lower flange. The compression flange of each truss is a round plate-iron tube, 9 feet in diameter and $\frac{5}{8}$th inch thick, with stiffening diaphragms at intervals, and supported by cast-iron arched standards, or end pillars, which rest on the piers. The side girders are plate girders which are divided by the truss into three spans. The weight of iron in one bridge for a single line of railway is as follows:—

	Tons.
298 feet run of tube and butt plates, - -	$127\frac{1}{2}$
Hoops of ditto over piers, - - -	$7\frac{3}{4}$
Side and bottom plates for attachment of main chains, - - - - -	15
Side plates for attachment of counterbracing chains,	$2\frac{1}{4}$
Stiffening diaphragms, 26 feet apart, - -	$4\frac{1}{4}$
Rivet heads, &c., - - - -	$4\frac{3}{4}$
Total weight of one tube (top flange), -	$161\frac{1}{2}$
Main chains, eyes, pins, &c., - - -	105
Counterbracing chains, eyes, pins, &c., - -	23
Vertical trusses, - - - - -	$18\frac{1}{2}$
Total weight of side-bracing, - -	$146\frac{1}{2}$

* *Encyc. Brit.*, Art. "Iron Bridges," and Clark *on the Tubular Bridges*, p. 101.

		Tons.
Side girders, cross-girders, &c.,	-	130
Saddles, collars, &c., at points of suspension,	-	22
		152
Total weight of iron for one line of railway,	-	**460**

CRUMLIN VIADUCT, WARREN'S GIRDER.*

539.—The Crumlin Viaduct is situated on the Newport section of the West-Midland Railway about five miles from Pontypool. The structure is divided by a short embankment into two distinct viaducts of exactly similar construction. The larger viaduct has seven, the smaller three openings of 150 feet from centre to centre of piers. The girders are "Warren's Patent" of 148 feet clear span, but not connected together as in continuous girders. The compression flange is a rectangular plate-iron box or tube, and the tension flange is formed of flat wrought-iron bars; both flanges increase in sectional area from the ends towards the centre. The diagonals form a series of equilateral triangles of angle and bar iron, the section of those in compression being in the form of a cross. The length of each side of the triangle is 16 feet 4 inches. The maximum tensile strain in the diagonals from the permanent load plus a train-load of one ton per running foot was 6·65 tons per square inch of net section when the bridge was first made, the maximum tensile strain in the lower flange from the same load was 5·75 tons per square inch of net section, and in no part did the maximum compression strain from the same load exceed 4·31 tons per square inch of gross section. The viaduct has four girders, two to each line of railway with the road above the girders. The weights for a single line, 150 feet long, were as follows when the bridge was first made, but a very large amount of additional material appears to have been added subsequently for the purpose of strengthening it.†

* *Trans. Inst. C. E. of Ireland*, Vol. vii., p. 97; and Humber *on Bridges*, 1st ed.

† *Engineer*, 1866, Vol. xxii., p. 384.

	Tons.	Cwts.
A pair of main girders, - - -	37	18
Cross-bracing of do., - - - -	3	3
Platform, - - - - -	18	1
Permanent way, - - - -	15	3
Hand-railing, - - - -	9	0
Total permanent weight for one line of way,	**83**	**5**

The tension flange of one girder weighs 5·97 tons, of which 1·5 ton, or one-fourth, was required to make the connexions of the flange.

PUBLIC BRIDGE OVER THE BOYNE, LATTICE GIRDER.*

540.—This bridge crosses the river Boyne at the Obelisk near Drogheda. The main girders are double-webbed lattice girders, 128 feet long, and 10 feet 8 inches deep, or 1-12th of the length. The clear span between the abutments is 120 feet, and the clear width of the roadway, between the inside planes of the lattice bars, is 16 feet 8 inches. Sufficient strength is provided in the main girders to sustain a total load of 3 cwt. per super foot of roadway when the iron in tension is strained up to 5 tons per square inch of net section, and that in compression up to 4 tons per square inch of gross section. The cross-girders are shallow plate girders about 3½ feet apart and capable of supporting a load of 5 cwt. per super foot, the additional strength being given to meet the contingency of a very heavy load resting on each girder in succession with the same working strains as above. The roadway is supported on buckled plates resting on the cross-girders; these plates weigh 67½ lbs. per square yard and have a versine of 2½ inches, four plates being laid in the width of the bridge. A layer of wooden chips, sand and coal tar was first laid so as to cover a little over the level of the crown of the buckled plates and upon this was laid asphalt 8 inches deep, consisting of broken stones, sand and coal tar.

The following table gives the actual weight, the theoretic weight,

* *Trans. Inst. C.E. of Ireland,* Vol. ix., p. 67.

and the percentage of material practically required over the theoretic weight, *i.e.*, the loss of iron due to rivet holes, cover plates, stiffeners and waste.

TABLE XI.—SUMMARY OF MATERIALS IN THE BOYNE OBELISK BRIDGE, 120 FEET SPAN.

	Actual weight.	Theoretic weight.	Percentage lost.
Top flange, in compression, - -	382 cwt.	302 cwt.	26 per cent.
Bottom do., in tension, - - -	382 „	242 „	58 „
End pillars, in compression, - -	41 „	12 „	242 „
Latticing, in compression, - -	152 „	75 „	114 „
Do. in tension, - - -	114 „	71 „	60 „
Total for Main girders, - -	1,071 cwt.	702 cwt.	$52\frac{1}{2}$ per cent.
Hand-rail bars, - - - -	26 „		
35 cross-girders, - - - -	315 „		
70 cast-iron chairs, under ends of last,	44 „		
Buckled plates and side plates for retaining asphalt in place, - -	210 „		
Asphalt, - - - - - -	754 „		
Total weight of bridge, -	**2,420 cwt. = 121 tons.**		

The weight of the main girders is 8·5 cwt. per foot run and that of the roadway 10·8 cwt., forming a total of 19·3 cwt. per foot run. The weight per square foot of roadway surface is ·52 cwt. for the main girders, and ·65 cwt. for the roadway, the total being 1·17 cwt. per square foot. This leaves a balance of 3 — 1·17, = 1·83 cwt. per square foot, for the greatest load, say dense crowds, which in a country bridge can scarcely exceed 100 ℔s. per square foot (**493**). There is, therefore, a considerable margin for deterioration of the iron, which is a wise precaution in a country bridge that is not likely to be painted frequently.

BOWSTRING BRIDGE ON THE CALEDONIAN RAILWAY.*

541.—This bridge was erected by Mr. E. Clark to carry the Caledonian Railway over the Monkland Canal. The arch is partly wrought-iron and partly cast-iron, and the tie or lower flange consists of wrought-iron plates. The total length of the girders is 148 feet, and the depth is 15 feet or about 1-10th of the length. The whole weight of the girders for a double line is 128 tons.

CHARING-CROSS LATTICE BRIDGE.†

542.—This bridge was erected by Mr. Hawkshaw to carry the Charing-Cross Railway across the Thames on the site of the Hungerford Suspension Bridge, the chains of which were removed to Clifton. It comprises nine independent spans, six of 154 feet and three of 100 feet. The leading particulars of one of the 154 feet spans are as follows. The main girders are wrought-iron lattice tubular girders, the web consisting of two systems of nearly right-angled triangles. The tension diagonals are Howard's patent rolled suspension links, and the compression diagonals are forged bars, varying in thickness from $2\frac{1}{2}$ to 3 inches, and united in pairs by zigzag internal cross-bracing. The flanges are formed of horizontal plates in piles, with four vertical ribs attached by angle irons to the horizontal plates, the two outer ribs being 2 feet deep and the two inner ones 21 inches deep. The flanges therefore resemble the usual trough section, but with 4, in place of 2 vertical ribs (**439**). The diagonals have enlarged ends with eyes, and are attached to the vertical ribs by turned pins of puddled steel. In addition to the diagonals already mentioned, there are vertical bars 1 inch thick connecting each pin in the upper flange with that in the flange directly beneath; these vertical bars form diagonals to the squares made by the diagonal bracing and are superfluous (**191**). The extreme length of the main girders is 164 feet, their extreme depth is 14 feet, and the depth from centre to centre of pins is 10 feet 9 inches, but the distance between the

* *Encyclopædia Britannica*, Art. "Iron Bridges," p. 605.

† *Proc. Inst. C. E.*, Vol. xxii.; and *Trans. Soc. Eng. for* 1864.

centres of gravity of the flanges is 12 feet 9 inches, or nearly 1-12th of the clear span, and this seems to have been assumed to be the correct depth for calculating the working strains, which with $1\frac{1}{4}$ ton per foot on each line, are stated to be 5 tons tension per square inch of net section, and 4 tons compression per square inch of gross section. The cross-girders are attached to the under sides of the lower flanges, and project beyond them with cantilever ends which support footpaths 7 feet wide. These cross-girders are 11 feet apart and correspond with the apices of the diagonals in the lower flanges. There are four lines of railway and the width in the clear between the main girders is 46 feet 4 inches. The weight of iron in one main girder, including the end pillars, is as follows:—

	Tons.	Cwts.	Qrs.
Top flange, - - -	70	4	2
Bottom do., - - -	67	15	2
Web, - - - -	46	0	0
End pillars, - - -	6	0	0
Weight of iron in one main girder,	**190**	**0**	**0**

Taking the rolling load at $1\frac{1}{4}$ tons per foot of single line, the maximum distributed load on each main girder is nearly as follows:—

	Tons.
Rolling load on two lines $= 156 \times 2\frac{1}{2}$ tons, - -	390
One main girder, deducting end pillars, - -	184
One half the cross-girders and cantilevers, - -	67
Rails for two lines, - - - - -	7
Timber in the half platform and longitudinals under rails,	41
Load of people on one footpath at 100 lbs. per square foot,	$48\frac{1}{3}$
Total distributed load on one girder, - -	$737\frac{1}{3}$

The foregoing load is exclusive of cornice, hand-rail, fish-plates, bolts, spikes, chairs for rails, hoop-iron tongue and bolts for planking and ballast.

CONWAY PLATE TUBULAR BRIDGE.*

543.—The Conway bridge was erected by Mr. Robert Stephenson

* Clark on the *Tubular Bridges*.

to carry the Chester and Holyhead Railway over the river Conway, in North Wales. It consists of two wrought-iron plate tubular bridges placed side by side, with one line of railway in each tube. The entire length of each tube is 424 feet, the clear span is 400 feet, and the effective length for calculation 412 feet. The external depth at the centre is 25 feet 6 inches, or nearly 1-16th of the length, thence it diminishes gradually towards the ends where it is 22 feet 6 inches. The external width is 14 feet 9 inches; the clear width inside is about 12 feet 6 inches. The tubes are placed 9 feet apart and are not connected in any way.

TABLE XII.—TABULAR STATEMENT OF WROUGHT-IRON WORK IN THE CONWAY BRIDGE—ONE TUBE, SINGLE LINE, LENGTH 424 FEET.

	Upper Flange.	Sides.	Lower Flange.	Summary.
	tons.	tons.	tons.	tons.
Plates, - - -	239	201	242	682
Angle and T-iron, -	115	146	59	320
Covers, - - -	15	22	77	114
Rivet heads, - -	23	24	17	64
Total, -	392	393	395	1180

Plates, 58 per cent.; angle and T-iron, 27 per cent.; covers, 10 per cent.; rivet-heads, 5 per cent.; total, 100.

The following is an analysis of the wrought-iron in one tube 412 feet long, *i.e.*, 6 feet longer at each end than the clear span. This was the length of the tube when floated into its place between the abutments; 6 feet were afterwards added to each end.

TOP FLANGE.

	Tons.	Cwts.	Per cent.
Plates and angle-iron in compression, -	336	0	87·5
Plates and angle-iron acting as covers, -	17	8	4·5
Transverse keelsons, - - -	7	0	2·0
Rivet-heads, - - - -	22	7	6·0
	382	15	100·0

SIDES.

	Tons.	Cwts.	Per cent.
Plates acting as sides, - - -	163	0	43·0
Covers and proportion of T-iron acting as covers, - - - -	90	10	24·0
Gussets, stiffeners, and projecting rib of T-iron engaged in stiffening the sides,	101	16	27·0
Rivet-heads, - - - -	23	15	6·0
	379	1	100·0

LOWER FLANGE.

	Tons.	Cwts.	Per cent.
Plates and angle-iron in tension, -	279	9	72·5
Plates and angle-iron acting as covers, -	76	6	20·0
Transverse keelsons, - - -	14	0	3·5
Rivet-heads, - - - -	15	17	4·0
	385	12	100·0

This makes the **total weight of wrought-iron** in 412 feet of one tube = **1147·4 tons,** or 2·78 tons per running foot for each line. The weight of wrought-iron in each tube, 400 feet long in the clear, is 1112 tons.

Summary of cast-iron work in the Conway Bridge for both lines:—

	Tons.
Castings fixed in the ends of tubes, - -	201
Bed-plates, rollers, &c., - - -	108
Castings fixed in the masonry, - -	325
Total weight of castings for both tubes,	634

The working inch-strains, as already given in Table VII. (**481**), are 6·32 tons tension and 4·924 tons compression with a train-load of $\frac{3}{4}$ ton per foot uniformly distributed.

The mean deflection of the two tubes, immediately after the removal of the platform on which they were built, was 8·04 inches which became 8·98 inches after they took a permanent set due to the strain (**410**). The deflection, from additional weight placed at the centre, is ·01104 inch per ton. The difference of deflection due to change of temperature, between noon and midnight on the 5th of July, 1848, was 1·56 inches (**419**).

BROTHERTON PLATE TUBULAR BRIDGE.*

544.—The Brotherton bridge, on the York and North Midland Railway is a tubular plate bridge with one line of railway in each tube. The span is 225 feet, the depth 20 feet or 1-11th of the span nearly, and the width of each tube between the side plates is 11 feet.

The weight of one tube is as follows:—

Wrought-iron between the bearings, -	198 tons.
Wrought-iron on the bearings, - -	13 ,,
Cast-iron on the bearings, - -	14½ ,,
Cast-iron in rollers and plates, - -	9½ ,,

Total weight of iron for one line of railway, **235 tons.**

The top flange is composed of a single plate in thickness, and no cells whatever have been used either in top or bottom.

545. Size and weights of various materials.—The following tables refer chiefly to the size and weights of various materials, and will be found useful for reference.

* *Encycl. Brit.*, Art. "Iron Bridges," p. 609.

TABLE XIII.—VALUES OF GAGES FOR WIRE AND SHEET METALS IN GENERAL USE, EXPRESSED IN DECIMAL PARTS OF THE INCH.*

Birmingham Wire Gage for Wire, and for Sheet Iron and Sheet Steel.		Birmingham Metal Gage for Sheet Metals, Brass, Gold, Silver, Zinc, &c.		Lancashire Gage for round Steel Wire, and also for Pinion Wire. The smaller sizes are distinguished by numbers. The larger by letters, and called the Letter Gage.					
Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
0000	·454	1	·004	80	·013	40	·096	A	·234
000	·425	2	·005	79	·014	39	·098	B	·238
00	·380	3	·008	78	·015	38	·100	C	·242
0	·340	4	·010	77	·016	37	·102	D	·246
1	·300	5	·012	76	·018	36	·105	E	·250
2	·284	6	·013	75	·019	35	·107	F	·257
3	·259	7	·015	74	·022	34	·109	G	·261
4	·238	8	·016	73	·023	33	·111	H	·266
5	·220	9	·019	72	·024	32	·115	I	·272
6	·203	10	·024	71	·026	31	·118	J	·277
7	·180	11	·029	70	·027	30	·125	K	·281
8	·165	12	·034	69	·029	29	·134	L	·290
9	·148	13	·036	68	·030	28	·138	M	·295
10	·134	14	·041	67	·031	27	·141	N	·302
11	·120	15	·047	66	·032	26	·143	O	·316
12	·109	16	·051	65	·033	25	·146	P	·323
13	·095	17	·057	64	·034	24	·148	Q	·332
14	·083	18	·061	63	·035	23	·150	R	·339
15	·072	19	·064	62	·036	22	·152	S	·348
16	·065	20	·067	61	·038	21	·157	T	·358
17	·058	21	·072	60	·039	20	·160	U	·368
18	·049	22	·074	59	·040	19	·164	V	·377
19	·042	23	·077	58	·041	18	·167	W	·386

* From Holtzapffel's *Mechanical Manipulation.*

TABLE XIII.—VALUES OF GAUGES FOR WIRE AND SHEET METALS IN GENERAL USE, EXPRESSED IN DECIMAL PARTS OF THE INCH—*continued.*

Birmingham Wire Gauge for Iron Wire, and for Sheet Iron and Sheet Steel.		Birmingham Metal Gauge for Sheet Metals, Brass, Gold, Silver, Zinc, &c.		Lancashire Gauge for round Steel Wire, and also for Pinion Wire. The smaller sizes are distinguished by numbers. The larger by letters, and called the Letter Gauge.					
Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.	Mark.	Size.
20	·035	24	·082	57	·042	17	·169	X	·397
21	·032	25	·095	56	·044	16	·174	Y	·404
22	·028	26	·103	55	·050	15	·175	Z	·413
23	·025	27	·113	54	·055	14	·177	A 1	·420
24	·022	28	·120	53	·058	13	·180	B 1	·431
25	·020	29	·124	52	·060	12	·185	C 1	·443
26	·018	30	·126	51	·064	11	·189	D 1	·452
27	·016	31	·133	50	·067	10	·190	E 1	·462
28	·014	32	·143	49	·070	9	·191	F 1	·475
29	·013	33	·145	48	·073	8	·192	G 1	·484
30	·012	34	·148	47	·076	7	·195	H 1	·494
31	·010	35	·158	46	·078	6	·198		
32	·009	36	·167	45	·080	5	·201		
33	·008			44	·084	4	·204		
34	·007			43	·086	3	·209		
35	·005			42	·091	2	·219		
36	·004			41	·095	1	·227		

Column 1 refers to the gauge commonly called the *Birmingham Wire Gauge*, which is employed for iron, brass and other wires, for black steel wire, for sheet iron, sheet steel and various other materials.

The gauge referred to in the second column is called the *Birmingham Metal Gauge* or the *Plate Gauge*, and is employed for most of the sheet metals, excepting iron and steel.

TABLE XIV.—WEIGHT OF A SUPERFICIAL FOOT OF VARIOUS METALS IN LBS.

	THICKNESS BY THE BIRMINGHAM WIRE GAUGE.														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Wrought-Iron,	12·50	12·00	11·00	10·00	8·74	8·12	7·50	6·86	6·24	5·62	5·00	4·38	3·75	3·12	2·82
Copper, -	14·50	13·90	12·75	11·60	10·10	9·40	8·70	7·90	7·20	6·50	5·80	5·08	4·34	3·60	3·27
Brass, -	13·75	13·10	12·10	11·00	9·61	8·93	8·25	7·54	6·86	6·18	5·50	4·81	4·12	3·43	3·10
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Wrought-Iron,	2·50	2·18	1·86	1·70	1·54	1·40	1·25	1·12	1·00	·90	·80	·72	·64	·56	·50
Copper, -	2·90	2·52	2·15	1·97	1·78	1·62	1·45	1·30	1·16	1·04	·92	·83	·74	·64	·58
Brass, -	2·75	2·40	2·04	1·87	1·69	1·54	1·37	1·23	1·10	·99	·88	·79	·70	·61	·55

THICKNESS IN PARTS OF AN INCH.

	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
Wrought-Iron,	2·5	5·0	7·5	10·0	12·5	15·	17·5	20·	25·	30·	35·	40·
Copper, -	2·9	5·8	8·7	11·6	14·5	17·2	20·0	23·2	28·9	34·3	40·4	46·2
Brass, -	2·7	5·5	8·2	11·0	13·7	16·4	19·0	21·8	27·4	32·5	37·9	43·3
Lead, -	3·7	7·4	11·1	14·8	18·5	22·2	25·9	29·6	37·0	44·4	51·8	59·2
Zinc, -	2·3	4·7	7·0	9·4	11·7	14·0	16·4	18·7	23·4	28·1	32·8	37·5

It is useful to recollect that a square foot of plate-iron, $\frac{1}{4}$ inch thick, weighs 10 lbs.

TABLE XV.—WEIGHT OF A LINEAL FOOT OF ROUND AND SQUARE BAR IRON IN LBS.—(Molesworth).

Breadth or diam. in inches.	Square Bars.	Round Bars.	Breadth or diam. in inches.	Square Bars.	Round Bars.	Breadth or diam. in inches.	Square Bars.	Round Bars.
$\frac{1}{4}$	·209	·164	$1\frac{1}{4}$	5·25	4·09	3	30·07	23·60
$\frac{5}{16}$	·326	·256	$1\frac{3}{8}$	6·35	4·96	$3\frac{1}{4}$	35·28	27·70
$\frac{3}{8}$	·470	·369	$1\frac{1}{2}$	7·51	5·90	$3\frac{1}{2}$	40·91	32·13
$\frac{7}{16}$	·640	·502	$1\frac{5}{8}$	8·82	6·92	$3\frac{3}{4}$	46·97	36·89
$\frac{1}{2}$	·835	·656	$1\frac{3}{4}$	10·29	8·03	4	53·44	41·97
$\frac{9}{16}$	1·057	·831	$1\frac{7}{8}$	11·74	9·22	$4\frac{1}{4}$	60·32	47·38
$\frac{5}{8}$	1·305	1·025	2	13·36	10·49	$4\frac{1}{2}$	67·63	53·12
$\frac{11}{16}$	1·579	1·241	$2\frac{1}{8}$	15·08	11·84	$4\frac{3}{4}$	75·35	59·18
$\frac{3}{4}$	1·879	1·476	$2\frac{1}{4}$	16·91	13·27	5	83·51	65·58
$\frac{13}{16}$	2·205	1·732	$2\frac{3}{8}$	18·84	14·79	$5\frac{1}{4}$	92·46	72·30
$\frac{7}{8}$	2·556	2·011	$2\frac{1}{2}$	20·87	16·39	$5\frac{1}{2}$	101·03	79·35
$\frac{15}{16}$	2·936	2·306	$2\frac{5}{8}$	23·11	18·07	$5\frac{3}{4}$	110·43	86·73
1	3·34	2·62	$2\frac{3}{4}$	25·26	19·84	6	120·24	94·43
$1\frac{1}{8}$	4·22	3·32	$2\frac{7}{8}$	27·61	21·68	—	—	—

To convert into weight of other metals, multiply tabular No. for cast-iron by ·93, for steel × 1·02, for copper × 1·15, for brass × 1·09, for lead × 1·47, for zinc × ·92.

TABLE XVI.—Specific Gravity and Weight of a cubic foot of different Woods.*

Kind of Wood, and state.	Specific gravity.	Weight of a cubic foot in pounds.
Abele, dry,	·511 T.	32·00
Acacia (false), green,	·820 E.	51·25
Do., dry,	·791 H.	49·43
Do., dry,	·748 T.	46·75
Do., (three-thorned),	·676 H.	42·25
Alder,	·800 M.	50·00
Do., dry,	·555 E.	34·68
Almond tree,	1·102 H.	68·87
Apple tree,	·793 M.	49·56
Apricot tree,	·789 H.	49·31
Arbor vitæ (Chinese),	·560 H.	35·00
Ash (heart-wood), dry,	·845 P.	52·81
Do., dry,	·832 W.	52·00
Do., young wood, dry,	·811 T.	50·68
Do.	·800 J.	50·00
Do.	·760 B.	47·50
Do. (old tree), dry,	·753 T.	47·06
Do., dry,	·690 E.	43·12
Bay tree,	·822 M.	51·37
Beech (meanly dry),	·854 P.	53·37
Do.	·852 M.	53·25
Do.	·720 H.	45·00
Do.	·696 B.	43·50
Do., dry,	·690 E.	43·12
Birch, dry,	·720 E.	45·00
Box (Dutch),	1·328 M.	83·00
Do., dry,	1·030 J.	64·37
Do.	1·031 P.	64·43
Do. { from	1·024 B.	64·00
{ to	·960 B.	60·00
Do., dry,	·950 W.	59·37
Do., Turkey,	·949 R.	59·31
Brazil wood (red),	1·031 M.	64·43
Canary wood,	·723 R.	45·18
Cedar (Indian),	1·315 M.	82·18
Do. (Canadian),	·753 C.	47·06
Do. (Virginian red), dry,	·650 T.	40·62
Do. (Palestine),	·596 M.	37·25
Do. (American),	·560 M.	35·00
Do. do., seasoned,	·453 C.	28·31
Cedar of Libanus,	·603 H.	37·68
Do. do., dry,	·486 T.	30·37
Cherry tree,	·741 H.	46·31
Do. do., dry,	·672 T.	42·00
Chestnut (sweet), green,	·875 E.	54·68
Do.	·685 H.	42·81
Do. do., dry,	·606 T.	37·95
Do., another specimen, dry,	·535 T.	33·45
Do. (horse),	·657 H.	41·06

Kind of Wood, and state.	Specific gravity.	Weight of a cubic foot in pounds.
Chestnut (horse), dry,	·596 T.	37·28
Do. do., another specimen, dry,	·483 T.	30·18
Cocoa wood,	1·040 M.	65·00
Cork,	·240 M.	15·00
Cowrie,	·579	36·20
Crab tree, meanly dry,	·765 P.	47·81
Cypress,	·655 H.	40·93
Do. (Spanish),	·644 M.	40·25
Deal, white. See fir.		
Do., yellow. See pine.		
Ebony (American),	1·331 M.	83·18
Do. (Indian),	1·209 M.	75·56
Do.	1·108 R.	69·25
Elder tree,	·695 M.	43·43
Elm, green,	·940 C.	58·75
Do.	·693 S.	44·41
Do., seasoned,	·588 C.	36·75
Do.	·553 B.	34·56
Do. (common), dry,	·544 E.	34·00
Do., wych, young tree, green,	·763 E.	47·68
Do. do., dry,	·684 T.	42·75
Filbert tree,	·600 M.	37·50
Fir (Norway spruce),	·512 T.	32·00
Do. (white American spruce),	·465 T.	29·06
Do. (silver green),	·531 Wi.	33·20
Do., dry,	·403 Wi.	25·22
Do. (Scotch). See pine.		
Fustic,	·817 R.	51·06
Hazel,	·606 M.	37·87
Hickery,	·929 S.	58·06
Hornbeam,	·760 H.	47·50
Jasamine (Spanish),	·770 M.	48·12
Juniper wood,	·556	34·75
Laburnum,	·843 T.	52·70
Lance wood,	1·038 L.	64·87
Do. do., dry,	·943 R.	58·93
Larch, green,	·858 Wi.	53·63
Do. (red wood), seasoned	·640 T.	40·00
Do., dry,	·612 Wi.	38·31
Do., dry,	·496 T.	31·00
Do. (white wood), seasoned,	·364 T.	22·75
Lemon tree,	·703	43·93
Letter wood,	1·286 C.	80·37
Lignum vitæ,	1·333 M.	83·31
Do.	1·327 P.	82·93

* Tredgold's *Carpentry*, p. 298.

TABLE XVI.—SPECIFIC GRAVITY AND WEIGHT OF A CUBIC FOOT OF DIFFERENT WOODS—*continued.*

Kind of Wood, and state.	Specific gravity.	Weight of a cubic foot in pounds.	Kind of Wood, and state.	Specific gravity.	Weight of a cubic foot in pounds.
Lime tree,	·604 M.	37·75	Pine (planted Scotch), dry,	·529 T.	33·06
Do.	·564 H.	35 25	Do. (Scotch), dry,	·429 Wi.	26·81
Do.	·480 T.	30·00	Do. (Memel), dry from	·553	34·56
Logwood,	·913 P.	57·06	to	·544 T.	34·00
Mahogany (Spanish), dry	·852 T.	53·30	Do. (Riga), dry, from	·480	30·00
Do. dry,	·816 W.	51·00	to	·466 T.	29·12
Do. (Honduras), dry,	·560 T.	35·00	Do. (Weymouth), dry,	·460 T.	28·75
Maple (Norway),	·795 L.	49·68	Do. (American), dry,	·368 T.	23·00
Do. dry,	·755 P.	47·18	Plane (occidental), dry,	·648 E.	40·50
Do. (common), dry,	·624 T.	32·75	Do. (oriental),	·538 H.	33·62
Medlar tree,	·944 M.	59·00	Plane tree (common). See sycamore.		
Mulberry tree (Spanish),	·897 M.	56·06	Plum tree,	·785 M.	49·06
Oak (live), half seasoned,	1·216 Ch.	76·03	Do.	·663 P.	41·43
Do. (English green),	1·113 C.	69·56	Poona (seasoned),	·635 C.	39·95
Do. (French green),	1·063 Bu.	66·43	Poplar (Spanish, white),	·529 M.	33·06
Do. (Irish bog),	1·046 C.	65·37	Do. (black), dry,	·421 T.	26·31
Do. (evergreen),	·994 H.	62·25	Do. (Lombardy), dry,	·374 E.	24·37
Do. (Adriatic),	·993 B.	62·06	Quince tree,	·705 M.	44·00
Do. (black bog), dry,	·965 R.	60·31	Sassafras,	·482 P.	30·12
Do. (white American), half seasoned,	·908 Ch.	56·75	Satin wood,	·952 R.	59·50
Do. (*Quercus sessiliflora*),	·879 T.	54·97	Saul (Bengal), seasoned,	·994 L.	62·12
Do. (American white),	·840 H.	52·50	Service tree,	·742 H.	46·37
Do. (Provence), seasoned,	·828 D.	51·75	Sissoo (Bengal), seasoned,	·889 L.	55·52
Do. (*Quercus robur*), dry,	·807 T.	50·47	Stinkwood (seasoned),	·681 C.	42·56
Do. (English), seasoned,	·777 C.	48·56	Sycamore,	·645 H.	40·31
Do. (Dantzic), seasoned,	·755 T.	47·24	Do., dry,	·590 E.	36·87
Do. (American), red,	·752 L.	47·00	Teak, dry,	·832 Ch.	52·00
Do. (Riga), dry,	·688 T.	43·00	Do.	·745 B.	46·56
Do. (English), from an old tree, dry,	·625 T.	39·06	Do., seasoned,	·657 C.	41·06
Olive tree,	·927 M.	57·93	Tulip tree,	·477 H.	29·81
Orange tree,	·705 M.	44·06	Vine,	1·237 M.	77·31
Pear tree, dry,	·708 T.	44·25	Walnut tree, green,	·920 E.	57·50
Do.	·646 B.	40·37	Do. (American),	·735 H.	45·93
Pine (American pitch), dry,	·936 T.	58·5	Do. (French),	·671 M.	41·93
Do. (do.), seasoned,	·741 C.	46·31	Do., dry,	·616 T.	38·50
Do. (pinaster), green,	·837 Wi.	52·35	Willow, green,	·619 E.	38·68
Do. (Scotch), green,	·816 Wi.	51·08	Do., dry, from	·568	35·50
Do. (Mar Forest),	·696 B.	43·50	to	·404 T.	25·25
			Yellow wood (seasoned),	·657 C.	41·06
			Yew (Spanish),	·807 M.	50·43
			Do. (Dutch),	·788 M.	49·25
			Do.	·788 H.	48·62

The letters following the specific gravities refer to the authorities—B., Barlow; Bu., Buffon; C., Couche; Ch., from Chapman *on Preservation of Timber;* E., Ebbels; H., from Rondelet's table; J., Jurin; L., Layman; M., Muschenbroek; P., *Philosophical Transactions*, Vol. i., Lowthorp's *Abridgement;* R., Ralph Tredgold; S., Scoresby; T., Tredgold; W., Watson (Bishop); Wi., Wiebeking.

TABLE XVII.—SPECIFIC GRAVITY AND WEIGHT OF A CUBIC FOOT OF VARIOUS MATERIALS.*

Name of the Substance.	Specific gravity.	Weight of a cubic foot in pounds.
Air (atmospheric), -	·0012	·075
Alabaster. See gypsum.		
Ballast, drained, -		97·4
Basalt, from	3·00	187·50
Basalt, to	2·478	154·87
Do. (Fairhead), -	2·95 K.	184·37
Do. (Derbyshire), -	2·921 W.	182·56
Do. (Giant's Causeway),	2·90 K.	181·25
Do. do.	2·864 Br.	179·00
Do. (Rowley rag), -	2·478 K.	154·87
Bees' wax (yellow), -	·965	60·31
Bismuth (cast), -	9·822	613·87
Bitumen, of Judea, -	1·104	69·00
Bone, Beef, - -	2·08†	
Brass (wire drawn), -	8·544	534·00
Do. (plate), - -	8·441 W.	527·56
Do. (cast), - -	8·100 P.	506·25
Brick (common), from	1·557	97·31
Brick (common), to	2·000	125·00
Do. (red), - -	2·168 Re.	135·50
Do. (pale red), -	2·085 Re.	130·31
Do., - - -	1·857 Be.	116·06
Do. (common London stock),	1·841 T.	115·06
Do. paving (English clinker),	1·653 R.	103·31
Do. (Dutch clinker), -	1·482 R.	92·62
Do. (Welsh fire), -	2·408 T.	150·50
Brickwork, about -		95·00
Broken stone. See stone.		
Cement (Roman) and sand in equal parts,	1·817 T.	113·56
Do., alone (cast), -	1·600 R.	100·00
Chalk, from	2·315	144·68
Chalk, to	2·657 Th.	166·06
Do. (Cambridge clunch)	2·657 W.	166·06
Do. (Dorking), -	1·169 R.	116·81
Charcoal from birch, -	·542 K.	33·87
Do. from fir, - -	·441 K.	27·56
Do. from oak, -	·332 K.	20·75
Do. from pine, -	·280 K.	17·50
Clay (potter's), from	1·800	112·50
Clay (potter's), to	2·085 K.	130·31
Do. (common), -	1·919 Be.	119·93
Do., with gravel, -	2·560	160·00
Do., puddling, -		113·35
Do., slate. See slate.		
Coke, - - -	·744 K.	46·50
Coal (Kilkenny), -	1·526 K.	95·37
Do. (Glasgow splint),	1·290 Th.	80·62
Do. (Cannel), -	1·272 Th.	79·50

Name of the Substance.	Specific gravity.	Weight of a cubic foot in pounds.
Coal (Newcastle caking),	1·269 Th.	79·31
Concrete, Ballast and Portland Cement,	4·464	140·00
Copper (British sheet),	8·785 Ha.	549·06
Do. (British cast),	8·607 Ha.	537·93
Earth (common), from	1·520	95·00
Earth (common), to	1·984	124·00
Do. (loamy or strong),	2·016	126·00
Do. (rammed), -	1·584 Pa.	99·00
Do. (loose or sandy), -	1·520	95·00
Firestone. See stone.	1·800	112·50
Flint, from	2·580	161·25
Flint, to	2·630 Th.	164·37
Do. (black Cambridge)	2·592 W.	162·00
Freestone. See stone.		
Glass, white flint, -	3·000	187·50
Do., plate, - -	2·760	172·50
Do., crown, - -	2·520	157·50
Gold, pure cast, -	19·361 Br.	1210·06
Do., standard, -	17·724 Th.	1107·75
Granite, from	2·999	187·47
Granite, to	2·538 K.	158·62
Do. (Guernsey), -	2·999 W.	187·47
Do. (Aberdeen gray),	2·664 R.	166·5
Do. (Cornish), -	2·662 Re.	166·37
Do. (do.), -	2·653 R.	165·81
Do. (Aberdeen red), -	2·643 R.	165·18
Do. (Cornish), - -	2·624 T.	164·00
Gravel, - -	1·749 P.	109·32
Gunpowder (solid), -	1·745	109·06
Do. (shaken), -	·922	57·62
Gypsum (plaster stone),	2·286 W.	142·87
Iron (bar), from	7·600	475·00
Iron (bar), to	7·800 K.	487·50
Do., hammered, -	7·763 M.	485·18
Do., not hammered, -	7·600 M.	475·00
Do. (cast), from	7·600	475·00
Do. (cast), to	7·200 Th.	450·00
Do. (horizontal ditto),	7·113 Re.	444·56
Do. (vertical castings),	7·074 Re.	442·12
Ivory, - - -	1·826 P.	114·12
Lead (milled), - -	11·407 Th.	712·93
Do. (cast), - -	11·352 Br.	709·50
Do., black. See Plumbago.		
Lime, quick, - -	·843 Be.	52·68
Limestone. See stone and marble.		
Loam. See earth, -		
Marble, from	2·840	177·50
Marble, to	2·580	161·25

* Tredgold's *Carpentry*, p. 300.

† Bevan, *Phil. Mag.* 1826, p. 181.

TABLE XVII.—SPECIFIC GRAVITY AND WEIGHT OF A CUBIC FOOT OF VARIOUS MATERIALS—*continued.*

Name of the Substance.	Specific gravity.	Weight of a cubic foot in pounds.	Name of the Substance.	Specific gravity.	Weight of a cubic foot in pounds.
Marble, Parian white,	2·837 K.	177·31	Plumbago, or black lead,	2·267	141·68
Do., veined white, -	2·726 Re.	170·37	Porphyry (green), -	2·875	179·68
Do., Carrara white, -	2·717 K.	169·81	Do. (red), -	2·793	174·56
Do., do. blue, -	2·713 K.	169·56	Potstone, from	3·000	187·50
Do., Italian black, -	2·712 K.	169·50	to	2·768 K.	173·00
Do., Derbyshire entrochal,	2·709 R.	169·31	Puzzolana, from	2·570	160·62
			to	2·850 K.	178·12
Do., Saxon gray, -	2·700 K.	168·75	Quartz (crystallized), -	2·655	165·93
Do., Brabant black, -	2·697 Re.	168·56	Roe-stone. See stone.		
Do., Derbyshire black,	2·690 W.	168·12	Road-grit. See sand.		
Do., Namur black, -	2·682 R.	167·62	Road metal. See stone.		
Do., Sienna yellow, -	2·677 K.	167·31	Rubble masonry, - from		145·00
Do., Pallion brown figured,	2·586 R.	161·62	to		160·00
			Sand (pure quartz), -	2·750	171·87
Marl, from	1·600	100·00	Do., river, - -	1·886 Be.	117·87
to	2·870 Th.	179·37	Do., River Thames (best),	1·638 T.	102·37
Mercury (fluid), -	13·568 Br.	848·00			
Mortar, - -	1·715 Be.	107·18	Do., pit (clean but coarse),	1·610 T.	100·62
Do. of river sand three parts, of lime in paste two parts,	1·615 Ro.	100·93	Do., pit (fine-grained and clean),	1·523 T.	95·18
Do., do., do., well beat together,	1·893 Ro.	118·31	Do., scraped from London roads (road-grit),	1·494 T.	93·37
Do. of pit sand three parts, of lime in paste two parts.	1·588 Ro.	99·25	Do., pit (very fine grained),	1·480 T.	92·50
Do., do., do., well beat together,	1·903 Ro.	118·93	Do., River Thames (inferior), -	1·454 T.	90·87
Do. of pounded tile three parts, of quick-lime two parts,	1·457 Ro.	91·06	Sandstone. See stone.		
			Serpentine, Anglesey green,	2·683 R.	167·68
			Do., blackish green, -	2·574 K.	160·87
Do., do., do., well beat together,	1·663 Ro.	103·93	Do., dark reddish brown,	2·561 K.	160·06
Do., common, of chalk lime, and sand, dry,	1·550 R.	96·87	Silver, pure cast, -	10·474 Br.	654·62
			Do., standard, -	10·312 Th.	644·50
			Slate, Welsh, - -	2·888 K.	180·50
Do., the lining of an antique reservoir near Rome,	1·549 Ro.	96·81	Do., Anglesey, -	2·876 K.	179·75
			Do., Westmoreland, pale blue,	2·791 W.	174·43
Do., from the interior of an old wall, Rome,	1·414 Ro.	88·37	Do., do., dark blue, -	2·781 W.	173·81
			Do., do., pale greenish blue,	2·768 W.	173·00
Do., lime, sand, and hair, used for plastering, dry,	1·384 R.	86·50	Do., do., blackish blue, used for floors,	2·758 W.	172·37
			Do., Welsh rag, -	2·752 K.	172·00
Oolite. See stone, roe.			Do., Westmoreland, fine grained pale blue,	2·732 W.	170·75
Peat, hard,	1·329	83·06			
Pebble (English), -	2·609	163·06			
Pewter, - -	7·248	453·00	Do., Cornwall, greyish blue,	2·512 K.	157·00
Pitch, - - -	1·150 P.	71·87			
Plaster (cast), - -	1·286 Be.	80·37	Stone, Bath (roe-stone),	2·494 K.	155·87
Platina pure, - -	21·531 Th.	1345·68	Do., do.	1·975 R.	123·43

TABLE XVII.—SPECIFIC GRAVITY AND WEIGHT OF A CUBIC FOOT OF VARIOUS MATERIALS—*continued.*

Name of the Substance.	Specific gravity.	Weight of a cubic foot in pounds.	Name of the Substance.	Specific gravity.	Weight of a cubic foot in pounds
Stone, blue lias (limestone),	2·467 R.	154·18	Stone, Portland (roe-stone),	2·423 Re.	151·43
Do., Bromley-fall (sandstone),	2·506 Re.	156·62	Do., do., do., -	2·113 R.	132·06
			Do., pumice, -	·629 R.	39·31
Do., do., - -	2·261 R.	141·31	Do., Purbeck, -	2·680 W.	167·50
Do., Bristol stone, -	2·510	156·87	Do., do., - -	2·599 Re.	162·43
Do., Burford (dry piece),	2·049 P.	128·06	Do., Roach Abbey (magnesian limestone),	1·893 R.	118·31
Do., Caen (calcareous sandstone),	2·108 R.	131·75	Do. (Tottenhoe calcareous sandstone),	1·800 T.	112·50
Do., Clitheroe limestone,	2·686 W.	167·87	Do., Woodstock flagstone,	2·614 K.	163·37
Do., Collalo, white (sandstone),	2·423 Re.	151·43	Do., Yorkshire paving,	2·507 Re.	156·68
Do., do., -	2·040 R.	127·50	Do., do., do., -	2·356 R.	147·25
Do., Craigleith, sandstone,	2·452 Re.	153·25	Stone, limestone broken to go through a two-inch ring,	1·44	90·00
Do., do., - -	2·360 R.	147·50			
Do., Derbyshire (red friable sandstone),	2·346 Re.	146·62	Stonework, mean weight according to Belidor, about		107·00?
Do., Dundee, -	2·530 Re.	158·12			
Do., do., - -	2·517 T.	157·31	Shingle, - -	1·424 Pa.	89·00
Do. (grindstone), -	2·143	133·93	Steel, - from	7·780	486·25
Do., Hedding-stone, lax kind,	2·029 P.	126·81	Steel, - to	7·840 Th.	490·00
			Syenite (Mount Sorrel),	2·621	163·81
Do., Hilton (sandstone),	2·177 R.	136·06	Tile (common plain), -	1·853 R.	116·15
			Do., - - -	1·815 Be.	113·43
Do., Kentish rag, -	2·675 R.	167·18	Tin, hammered, -	7·299 Br.	456·18
Do., Ketton (roe-stone)	2·494 K.	155·87	Do., pure cast, -	7·291 Br.	455·68
Do., do., - -	2·058 R.	128·62	Toadstone (Derbyshire),	2·921 W.	182·56
Do., Kincardine (sandstone),	2·448 T.	153·00	Tufa (Roman), -	1·217 Ro.	76·06
			Water, sea, - -	1·027 Th.	64·18
Do., Limerick (black compact limestone),	2·598 Re.	162·37	Do., rain, - -	1·000	62·50
			Wheat, - -	·64	48·00
Do., Pennarth (limestone),	2·653 W.	165·81	Whinstone (Scotch), -	2·760 W.	172·50
			Wood ashes, - -	·933 P.	58·32
Do., Portland (roe-stone),	2·461 W.	153·81	Wood petrified, -	2·341 P.	146·31
			Zinc, - - -	7·028 W.	439·25

Part of the letters of reference are explained in a note to the preceding table. The rest are as follows:—Be., Belidor; Br. Brisson; Ha., Hatchet; K., from Kirwan's *Mineralogy;* Re., Rennie, *Phil. Magazine*, Vol. liii.; Ro., Rondelet; Th., from Dr. Thomson's *System of Chemistry*, 5th edition; Pa., Pasley, Course of Military Instruction.

TABLE XVIII.—For Converting Tons into Lbs. Avoirdupois.

Tons.	Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.	Lbs.
0·05	112	12	26,880	42	94,080	72	161,280
0·10	224	13	29,120	43	96,320	73	163,520
0·15	336	14	31,360	44	98,560	74	165,760
0·20	448	15	33,600	45	100,800	75	168,000
0·25	560	16	35,840	46	103,040	76	170,240
0·30	672	17	38,080	47	105,280	77	172,480
0·35	784	18	40,320	48	107,520	78	174,720
0·40	896	19	42,560	49	109,760	79	176,960
0·45	1,008	20	44,800	50	112,000	80	179,200
0·50	1,120	21	47,040	51	114,240	81	181,440
0·55	1,232	22	49,280	52	116,480	82	183,680
0·60	1,344	23	51,520	53	118,720	83	185,920
0·65	1,456	24	53,760	54	120,960	84	188,160
0·70	1,568	25	56,000	55	123,200	85	190,400
0·75	1,680	26	58,240	56	125,440	86	192,640
0·80	1,792	27	60,480	57	127,680	87	194,880
0·85	1,904	28	62,720	58	129,920	88	197,120
0·90	2,016	29	64,960	59	132,160	89	199,360
0·95	2,128	30	67,200	60	134,400	90	201,600
1	2,240	31	69,440	61	136,640	91	203,840
2	4,480	32	71,680	62	138,880	92	206,080
3	6,720	33	73,920	63	141,120	93	208,320
4	8,960	34	76,160	64	143,360	94	210,560
5	11,200	35	78,400	65	145,600	95	212,800
6	13,440	36	80,640	66	147,840	96	215,040
7	15,680	37	82,880	67	150,080	97	217,280
8	17,920	38	85,120	68	152,320	98	219,520
9	20,160	39	87,360	69	154,560	99	221,760
10	22,400	40	89,600	70	156,800	100	224,000
11	24,640	41	91,840	71	159,040	101	226,240

TABLE XIX.—For converting Lbs. Avoirdupois into Tons.

Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.	Lbs.	Tons.
0	0·000	775	0·346	23,000	10·268	54,000	24·107	85,000	37·946
25	0·011	800	0·357	24,000	10·714	55,000	24·554	86,000	38·393
50	0·022	825	0·368	25,000	11·161	56,000	25·000	87,000	38·839
75	0·033	850	0·379	26,000	11·607	57,000	25·446	88,000	39·286
100	0·045	875	0·390	27,000	12·054	58,000	25·893	89,000	39·732
125	0·056	900	0·402	28,000	12·500	59,000	26·339	90,000	40·178
150	0·067	925	0·413	29,000	12·946	60,000	26·786	91,000	40·625
175	0·078	950	0·424	30,000	13·393	61,000	27·232	92,000	41·071
200	0·089	975	0·435	31,000	13·839	62,000	27·678	93,000	41·518
225	0·100	1,000	0·446	32,000	14·286	63,000	28·125	94,000	41·964
250	0·112	2,000	0·893	33,000	14·732	64,000	28·571	95,000	42·411
275	0·123	3,000	1·339	34,000	15·178	65,000	29·018	96,000	42·857
300	0·134	4,000	1·786	35,000	15·625	66,000	29·464	97,000	43·303
325	0·145	5,000	2·232	36,000	16·071	67,000	29·911	98,000	43·750
350	0·156	6,000	2·678	37,000	16·518	68,000	30·357	99,000	44·196
375	0 167	7,000	3·125	38,000	16·964	69,000	30·804	100,000	44·643
400	0·179	8,000	3·571	39,000	17·411	70,000	31·250	101,000	45 089
425	0·190	9,000	4·018	40,000	17·857	71,000	31·696	102,000	45·535
450	0·201	10,000	4·464	41,000	18·303	72,000	32·143	103,000	45·982
475	0·212	11,000	4·911	42,000	18·750	73,000	32·589	104,000	46·428
500	0·223	12,000	5·357	43,000	19·196	74,000	33·036	105,000	46·875
525	0·234	13,000	5·804	44,000	19·643	75,000	33·482	106,000	47·321
550	0·246	14,000	6·250	45,000	20·089	76,000	33·929	107,000	47·768
575	0·257	15,000	6·696	46,000	20·535	77,000	34·375	108,000	48·214
600	0·268	16,000	7·143	47,000	20·982	78,000	34·821	109,000	48·660
625	0·279	17,000	7·589	48,000	21·428	79,000	35·268	110,000	49·107
650	0·290	18,000	8·036	49,000	21·875	80,000	35·714	111,000	49·554
675	0·301	19,000	8·482	50,000	22·321	81,000	36·161	112,000	50·000
700	0·313	20,000	8·929	51,000	22·768	82,000	36·607	113,000	50·446
725	0·324	21,000	9·375	52,000	23·214	83,000	37·054	114,000	50·893
750	0·335	22,000	9·821	53,000	23·660	84,000	37·500	115,000	51·339

TABLE XX.—CHANNEL IRON SECTIONS OF VARIOUS THICKNESSES, IN PROPORTION TO THEIR SIZE.

Base.	Sides.	Base.	Sides.	Base.	Sides.	Base.	Sides.
inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
$\frac{3}{4}$	$\frac{5}{16}$ by $\frac{5}{16}$	2	2 by 2	4	$1\frac{5}{8}$ by $1\frac{5}{8}$	$6\frac{5}{16}$	$2\frac{3}{8}$ by $2\frac{3}{8}$
$\frac{3}{4}$	$\frac{3}{8}$,, $\frac{3}{8}$	$2\frac{1}{4}$	$1\frac{1}{4}$,, $1\frac{1}{4}$	4	$1\frac{3}{4}$,, $1\frac{3}{4}$	$6\frac{1}{2}$	3 ,, 3
$\frac{3}{4}$	$\frac{3}{4}$,, $\frac{3}{4}$	$2\frac{3}{8}$	$1\frac{3}{16}$,, $1\frac{3}{16}$	4	3 ,, 3	7	2 ,, 2
$\frac{13}{16}$	$\frac{13}{16}$,, $\frac{13}{16}$	$2\frac{3}{8}$	$1\frac{3}{8}$,, $1\frac{3}{8}$	$4\frac{3}{8}$	2 ,, 2	7	$2\frac{3}{8}$,, $2\frac{3}{8}$
$\frac{7}{8}$	$\frac{3}{8}$,, $\frac{3}{8}$	$2\frac{1}{2}$	$1\frac{1}{4}$,, $1\frac{1}{4}$	$4\frac{3}{8}$	$2\frac{1}{4}$,, $2\frac{1}{4}$	7	$2\frac{3}{4}$,, $2\frac{3}{4}$
$\frac{7}{8}$	1 ,, 1	$2\frac{3}{4}$	$1\frac{1}{4}$,, $1\frac{1}{4}$	$4\frac{1}{2}$	$1\frac{1}{2}$,, $1\frac{1}{2}$	7	3 ,, 3
1	$\frac{1}{4}$,, $\frac{1}{4}$	$2\frac{3}{4}$	$1\frac{3}{8}$,, $1\frac{3}{8}$	$4\frac{1}{2}$	$1\frac{5}{8}$,, $1\frac{5}{8}$	7	$3\frac{1}{2}$,, $3\frac{1}{2}$
1	$\frac{3}{8}$,, $\frac{3}{8}$	3	$\frac{3}{4}$,, $\frac{3}{4}$	$4\frac{1}{2}$	$1\frac{3}{4}$,, $1\frac{3}{4}$	$7\frac{1}{2}$	2 ,, 2
1	1 ,, 1	3	1 ,, 1	$4\frac{1}{2}$	2 ,, 2	$7\frac{1}{2}$	$2\frac{1}{4}$,, $2\frac{1}{4}$
1	$1\frac{1}{4}$,, $1\frac{1}{4}$	3	$1\frac{1}{4}$,, $1\frac{1}{4}$	$4\frac{1}{2}$	$2\frac{1}{2}$,, $2\frac{1}{2}$	$7\frac{1}{2}$	3 ,, 3
$1\frac{1}{8}$	$\frac{3}{8}$,, $\frac{3}{8}$	3	$1\frac{3}{8}$,, $1\frac{3}{8}$	$4\frac{1}{2}$	3 ,, 3	$7\frac{3}{4}$	2 ,, 2
$1\frac{1}{4}$	$\frac{3}{8}$,, $\frac{3}{8}$	3	$1\frac{1}{2}$,, $1\frac{1}{2}$	$4\frac{3}{4}$	$2\frac{5}{16}$,, $2\frac{5}{16}$	$7\frac{3}{4}$	$2\frac{3}{8}$,, $2\frac{3}{8}$
$1\frac{1}{4}$	$\frac{1}{2}$,, $\frac{1}{2}$	3	$1\frac{3}{4}$,, $1\frac{3}{4}$	5	$1\frac{1}{4}$,, $1\frac{1}{4}$	8	$3\frac{3}{8}$,, $3\frac{3}{8}$
$1\frac{1}{4}$	$\frac{5}{8}$,, $\frac{5}{8}$	3	2 ,, 2	5	2 ,, 2	8	$3\frac{1}{2}$,, $3\frac{1}{2}$
$1\frac{1}{4}$	$\frac{3}{4}$,, $\frac{3}{4}$	3	3 ,, 3	5	$2\frac{1}{4}$,, $2\frac{1}{4}$	8	$3\frac{3}{4}$,, $3\frac{3}{4}$
$1\frac{1}{4}$	1 ,, 1	$3\frac{1}{4}$	$1\frac{1}{2}$,, $1\frac{1}{2}$	5	$2\frac{3}{8}$,, $2\frac{3}{8}$	8	4 ,, 4
$1\frac{1}{4}$	$1\frac{1}{8}$,, $1\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{5}{8}$,, $1\frac{5}{8}$	5	$2\frac{3}{4}$,, $2\frac{3}{4}$	8	$4\frac{1}{4}$,, $4\frac{1}{4}$
$1\frac{1}{2}$	1 ,, 1	$3\frac{1}{4}$	2 ,, 2	5	3 ,, 3	$8\frac{1}{4}$	$2\frac{1}{4}$,, $2\frac{1}{4}$
$1\frac{1}{2}$	2 ,, 2	$3\frac{1}{2}$	$1\frac{1}{2}$,, $1\frac{1}{2}$	$5\frac{1}{4}$	$1\frac{1}{2}$,, $1\frac{1}{2}$	$8\frac{1}{2}$	$1\frac{1}{4}$,, $1\frac{1}{4}$
$1\frac{5}{8}$	$\frac{5}{8}$,, $\frac{5}{8}$	$3\frac{1}{2}$	$1\frac{3}{4}$,, $1\frac{3}{4}$	6	$2\frac{7}{16}$,, $2\frac{7}{16}$	9	$2\frac{5}{16}$,, $2\frac{5}{16}$
$1\frac{3}{4}$	$1\frac{1}{4}$,, $1\frac{1}{4}$	$3\frac{3}{4}$	$2\frac{1}{2}$,, $2\frac{1}{2}$	6	$2\frac{1}{2}$,, $2\frac{1}{2}$	9	$3\frac{3}{8}$,, $3\frac{3}{8}$
2	$\frac{3}{4}$,, $\frac{3}{4}$	4	1 ,, 1	6	3 ,, 3	$9\frac{1}{4}$	$3\frac{3}{8}$,, $3\frac{3}{8}$
2	1 ,, 1	4	$1\frac{1}{4}$,, $1\frac{1}{4}$	6	$3\frac{1}{2}$,, $3\frac{1}{2}$	$9\frac{7}{8}$	$3\frac{9}{16}$,, $3\frac{9}{16}$
2	$1\frac{1}{4}$,, $\frac{3}{4}$	4	$1\frac{3}{8}$,, $1\frac{3}{8}$	6	4 ,, 4	10	$3\frac{1}{2}$,, $3\frac{1}{2}$
2	$1\frac{1}{2}$,, $1\frac{1}{2}$	4	$1\frac{1}{2}$,, $1\frac{1}{2}$	$6\frac{1}{4}$	$1\frac{3}{4}$,, $1\frac{3}{4}$		

TABLE XXI.—Rolled Iron Girders.

Depth.	Width of Flanges.	Approximate weight per foot.	Depth.	Width of Flanges.	Approximate weight per foot.
inch.	inch.	lbs.	inch.	inch.	lbs.
$19\frac{3}{4}$	$6\frac{1}{4}$ by $6\frac{1}{4}$	97	10	$5\frac{1}{2}$ by $5\frac{1}{2}$	
16	6 ,, 6	70	10	5 ,, 5	34 to 36
16	$5\frac{1}{2}$,, $5\frac{1}{2}$	60	10	$4\frac{1}{2}$,, $4\frac{1}{2}$	31 ,, 42
$15\frac{3}{4}$	$5\frac{1}{2}$,, $5\frac{1}{2}$	60 to 71	10	4 ,, 4	29 ,, 39
15	$5\frac{1}{2}$,, $5\frac{1}{2}$	59	$9\frac{3}{4}$	$4\frac{3}{4}$,, $4\frac{3}{4}$	38 ,, 42
15	5 ,, 5	70	$9\frac{5}{16}$	$3\frac{5}{8}$,, $3\frac{5}{8}$	20 ,, 28
14	6 ,, 6	60	$9\frac{1}{4}$	$4\frac{1}{2}$,, $4\frac{1}{2}$	28 ,, 36
$13\frac{3}{4}$	6 ,, 6	54 to 56	$9\frac{1}{4}$	$3\frac{7}{8}$,, $3\frac{7}{8}$	24
$13\frac{3}{4}$	$5\frac{1}{2}$,, $5\frac{1}{2}$	54 ,, 62	$9\frac{1}{4}$	$3\frac{3}{4}$,, $3\frac{3}{4}$	24 to 30
$12\frac{3}{16}$	$8\frac{1}{4}$,, $8\frac{1}{4}$	150	$9\frac{1}{4}$	$3\frac{1}{2}$,, $3\frac{1}{2}$	21 ,, 29
12	10 ,, 10	118 to 120	$9\frac{1}{8}$	$3\frac{5}{8}$,, $3\frac{5}{8}$	23 ,, 28
12	6 ,, 6	57 ,, 65	9	$5\frac{1}{2}$,, $5\frac{1}{2}$	38 ,, 40
12	$5\frac{1}{4}$,, $5\frac{1}{4}$	60	9	5 ,, 5	32 ,, $36\frac{1}{2}$
12	5 ,, 5	41 to 60	9	5 ,, 3	28 ,, 32
11	3 ,, 3	30	9	$4\frac{1}{2}$,, $4\frac{1}{2}$	32 ,, 35
$10\frac{5}{8}$	$2\frac{3}{4}$,, $2\frac{3}{4}$	27 to 33	9	4 ,, 4	30 ,, 35
$10\frac{9}{16}$	$2\frac{13}{16}$,, $2\frac{13}{16}$	29	9	$3\frac{3}{4}$,, $3\frac{3}{4}$	
$10\frac{1}{2}$	$5\frac{1}{2}$,, $5\frac{1}{2}$	35 to 37	9	$3\frac{1}{2}$,, $3\frac{1}{2}$	25
$10\frac{3}{8}$	$3\frac{5}{16}$,, $3\frac{5}{16}$	42	9	$2\frac{1}{2}$,, $2\frac{1}{2}$	22
$10\frac{1}{4}$	$5\frac{3}{4}$,, $5\frac{3}{4}$	66	$8\frac{3}{4}$	$3\frac{3}{4}$,, $3\frac{3}{4}$	23 to 27
$10\frac{1}{4}$	$5\frac{1}{8}$,, $5\frac{1}{8}$	35 to 45	$8\frac{5}{8}$	3 ,, 3	20 ,, 28
$10\frac{3}{16}$	$6\frac{7}{16}$,, $6\frac{7}{16}$	85	$8\frac{1}{2}$	5 ,, 5	30 ,, 32
10	8 ,, 8	62 to 63	$8\frac{1}{2}$	$4\frac{7}{8}$,, $4\frac{7}{8}$	28 ,, 29
10	6 ,, 6		$8\frac{1}{2}$	$4\frac{1}{4}$,, $4\frac{1}{4}$	

TABLE XXI.—ROLLED IRON GIRDERS—*continued.*

Depth.	Width of Flanges.	Approximate weight per foot.	Depth.	Width of Flanges.	Approximate weight per foot.
inch.	inch.	lbs.	inch.	inch.	lbs.
8½	4 by 4	32	7	3 by 3	19 to 22
8½	3 ,, 3	25 to 34	7	2⅝ ,, 2⅝	15
8½	2½ ,, 2½	17 ,, 27	7	2½ ,, 2½	14 to 18
8¼	4 ,, 4	45	7	2⅜ ,, 2⅜	
8	5 ,, 5	29 to 34	7	2⅜ ,, 1⅛	9
8	4½ ,, 4½	28 ,, 35	7	2¼ ,, 2¼	14 to 18
8	4¼ ,, 4¼	33 ,, 34	6¼	3⅛ ,, 3⅛	15 ,, 18
8	4 ,, 4	21 ,, 30	6¼	3 ,, 3	18
8	3¾ ,, 3¾	24 ,, 27	6¼	2⅛ ,, 2⅛	11 to 13
8	3½ ,, 3½	26 ,, 30	6¼	2 ,, 2	
8	3 ,, 3	27	6¼	1¾ ,, 1¾	12½ ,, 15
8	2½ ,, 2½	15 to 20	6	6 ,, 6	29 ,, 32
8	2¼ ,, 2¼	20	6	5 ,, 5	25 ,, 31
7¾	2¾ ,, 2¾	24	6	4½ ,, 4½	24
7½	4¾ ,, 4¾	27 to 30	6	4 1/16 ,, 4 1/16	24
7 8/16	2⅜ ,, 1⅛	9 ,, 11	6	4 ,, 4	16 to 19
7⅛	5⅛ ,, 5⅛	42 ,, 45	6	3¾ ,, 3¾	21
7	7 ,, 7	46	6	3½ ,, 3½	17
7	5 ,, 5		6	3¼ ,, 3¼	17
7	4½ ,, 4	27	6	3 ,, 3	13 to 22
7	4 ,, 4	25	6	2⅛ ,, 2⅛	13 ,, 15
7	3⅞ ,, 2½	20 to 26	6	2 ,, 1¼	12 ,, 14
7	3⅝ ,, 3⅝	19 ,, 25	5½	3⅛ ,, 3⅛	18 ,, 20
7	3½ ,, 3½	23 ,, 25	5½	3 ,, 3	11 ,, 15
7	3¼ ,, 3¼	21 ,, 22	5½	2¾ ,, 2¾	
7	3⅛ ,, 3⅛	19 ,, 22	5½	2¼ ,, 2¼	9 ,, 13

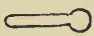
TABLE XXI.—Rolled Iron Girders—*continued.*

Depth.	Width of Flanges.	Approximate weight per foot.	Depth.	Width of Flanges.	Approximate weight per foot.
inch.	inch.	lbs.	inch.	inch.	lbs.
5½	2 by 2	9 to 13	4½	2½ by 2½	11
5½	1¾ „ 1¾		4½	3 „ 3	25
$5\frac{7}{16}$	2⅞ „ 2⅞	15 „ 18	4½	2¼ „ 2¼	12 to 15
5	5 „ 5	24	4	4 „ 4	19 „ 29
5	4½ „ 4½	22 to 24	4	3 „ 3	10
5	2¾ „ 2¾	13	4	2⅜ „ 2⅜	8
5	1¾ „ 1¾	8 to 11	4	2 „ 2	7 to 8
5	3¼ „ 3¼	11 „ 16	4	1¾ „ 1¾	
5	3 „ 3		4	1⅝ „ 1⅝	6 „ 8
4⅞	4 „ 3	15 „ 18	4	1½ „ 1½	7
4¾	4 „ 2¾	14 „ 18	3⅞	2¼ „ 2¼	11
4¾	3⅝ „ 3¼	14 „ 18	3½	1½ „ 1½	7 to 9
4¾	3⅜ „ $2\frac{13}{16}$	14 „ 18	3⅛	1⅝ „ 1⅝	4⅓ „ 6
4¾	2¾ „ 2¾	10	3	3 „ 3	9 „ 11
4¾	1¾ „ 1¾	8 to 10	2½	$1\frac{1}{16}$ „ $1\frac{1}{16}$	6
4⅝	3¾ „ 2¾	15 „ 18	2½	1 „ 1	4
4⅝	3¼ „ 2⅞	14 „ 18	2¼	¾ „ ¾	2¾ to 4
4⅝	1¾ „ 1¾	9 „ 12	1½	1¼ „ ½	2
4½	4 „ 4	15 „ 16	1¼	$\frac{11}{16}$ „ $\frac{11}{16}$	1¾ to 2
4½	3¾ „ 1¾	14 „ 16	$\frac{13}{16}$	½ „ ½	¾
4½	3¼ „ 3¼	15 „ 18			

TABLE XXII.—Deck Beam Iron.

Depth of Beam.	Width of Flange.	Width of Bulb.	Average weight per lineal foot.	Depth of Beam.	Width of Flange.	Width of Bulb.	Average weight per lineal foot.
inch.	inch.	inch.	lbs.	inch.	inch.	inch.	lbs.
16	$6\frac{1}{4}$	$3\frac{1}{4}$	60 to 63	8	$6\frac{1}{4}$	$1\frac{3}{4}$	31
15	$6\frac{1}{4}$	$3\frac{1}{4}$	56 „ 59	8	$5\frac{1}{4}$	$1\frac{7}{8}$	26 to 28
14	$6\frac{1}{4}$	$3\frac{1}{4}$	55 „ 58	8	$4\frac{1}{4}$	$2\frac{1}{2}$	32 „ 33
13	$6\frac{1}{4}$	$3\frac{1}{4}$	54 „ 57	8	4	$2\frac{3}{4}$	24
12	$6\frac{1}{4}$	$3\frac{1}{4}$	54 „ 56	7	5	2	22
11	$6\frac{1}{2}$	$2\frac{1}{4}$	42 „ 44	7	5	$1\frac{3}{4}$	22 „ 25
10	6	$2\frac{1}{8}$	35 „ 37	7	$4\frac{1}{2}$	$1\frac{3}{4}$	25 „ 27
10	$4\frac{1}{4}$	3	30 „ 32	7	4	$2\frac{1}{4}$	21
9	$6\frac{1}{2}$	2	35 „ 37	6	5	$1\frac{1}{2}$	18 to 20
9	$6\frac{1}{4}$	$3\frac{1}{4}$	43 „ 45	6	4	$1\frac{1}{2}$	18 „ 20
9	$5\frac{1}{2}$	2	31 „ 33	6	4	$2\frac{1}{4}$	20
9	$5\frac{1}{2}$	$1\frac{7}{8}$	31 „ 33	5	$4\frac{1}{4}$	3	22 to 23
9	$4\frac{1}{4}$	3	28 „ 29	5	4	$1\frac{1}{2}$	15 „ 16
$8\frac{7}{8}$	5	$1\frac{5}{8}$	31 „ 33	5	4	$1\frac{5}{8}$	14
$8\frac{1}{2}$	$5\frac{1}{2}$	$1\frac{7}{8}$	29 „ 30	4	$3\frac{1}{2}$	$1\frac{1}{4}$	12 to 13

TABLE XXIII.—Plain Bulb Beam Iron of various thicknesses, in proportion to the depth.



$4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, $6\frac{1}{4}$, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, 8, $8\frac{1}{2}$, $8\frac{3}{4}$, 9, $9\frac{1}{4}$, $9\frac{1}{2}$, 10, 11, 12, $12\frac{1}{2}$ inches deep.

TABLE XXIV.—ANGLE IRON SECTIONS OF VARIOUS THICKNESSES, IN PROPORTION TO THE SIZE.

EQUAL SIDED ANGLE IRON.

inch.			inch.			inch.			inch.		
$\frac{3}{8}$	by	$\frac{3}{8}$	$1\frac{3}{8}$	by	$1\frac{3}{8}$	$2\frac{5}{8}$	by	$2\frac{5}{8}$	$4\frac{1}{4}$	by	$4\frac{1}{4}$
$\frac{1}{2}$	,,	$\frac{1}{2}$	$1\frac{1}{2}$	,,	$1\frac{1}{2}$	$2\frac{3}{4}$	,,	$2\frac{3}{4}$	$4\frac{1}{2}$	,,	$4\frac{1}{2}$
$\frac{5}{8}$	,,	$\frac{5}{8}$	$1\frac{5}{8}$	,,	$1\frac{5}{8}$	3	,,	3	$4\frac{3}{4}$	,,	$4\frac{3}{4}$
$\frac{3}{4}$	,,	$\frac{3}{4}$	$1\frac{3}{4}$	,,	$1\frac{3}{4}$	$3\frac{1}{8}$	,,	$3\frac{1}{8}$	5	,,	5
$\frac{7}{8}$	,,	$\frac{7}{8}$	2	,,	2	$3\frac{1}{4}$	,,	$3\frac{1}{4}$	$5\frac{1}{8}$	,,	$5\frac{1}{8}$
1	,,	1	$2\frac{1}{4}$	,,	$2\frac{1}{4}$	$3\frac{1}{2}$	,,	$3\frac{1}{2}$	$5\frac{1}{2}$	,,	$5\frac{1}{2}$
$1\frac{1}{8}$	,,	$1\frac{1}{8}$	$2\frac{3}{8}$	,,	$2\frac{3}{8}$	$3\frac{3}{4}$	,,	$3\frac{3}{4}$	6	,,	6
$1\frac{3}{16}$	,,	$1\frac{3}{16}$	$2\frac{1}{2}$	,,	$2\frac{1}{2}$	4	,,	4	8	,,	8
$1\frac{1}{4}$	,,	$1\frac{1}{4}$									

UNEQUAL SIDED ANGLE IRON.

inch.			inch.			inch.			inch.		
$\frac{3}{8}$	by	$\frac{1}{4}$	$1\frac{1}{4}$	by	$1\frac{1}{8}$	$1\frac{3}{4}$	by	$1\frac{1}{2}$	$2\frac{3}{8}$	by	$1\frac{1}{4}$
$\frac{7}{16}$	,,	$\frac{3}{8}$	$1\frac{3}{8}$	,,	$\frac{9}{16}$	$1\frac{7}{8}$	,,	$1\frac{3}{4}$	$2\frac{3}{8}$	,,	$2\frac{1}{4}$
$\frac{9}{16}$	,,	$\frac{7}{16}$	$1\frac{3}{8}$	,,	1	2	,,	$\frac{5}{8}$	$2\frac{1}{2}$	,,	$1\frac{3}{8}$
$\frac{5}{8}$	,,	$\frac{11}{32}$	$1\frac{3}{8}$	,,	$1\frac{1}{4}$	2	,,	1	$2\frac{1}{2}$	,,	$1\frac{1}{2}$
$\frac{5}{8}$	,,	$\frac{7}{16}$	$1\frac{7}{16}$	,,	$1\frac{3}{16}$	2	,,	$1\frac{1}{4}$	$2\frac{1}{2}$	,,	$1\frac{3}{4}$
$\frac{11}{16}$	,,	$\frac{7}{16}$	$1\frac{1}{2}$	,,	1	2	,,	$1\frac{1}{2}$	$2\frac{1}{2}$	,,	2
1	,,	$\frac{1}{2}$	$1\frac{1}{2}$	,,	$1\frac{1}{4}$	2	,,	$1\frac{3}{4}$	$2\frac{1}{2}$	,,	$2\frac{1}{4}$
1	,,	$\frac{5}{8}$	$1\frac{5}{8}$	,,	$\frac{5}{8}$	$2\frac{1}{4}$	,,	$\frac{7}{8}$	$2\frac{5}{8}$	,,	1
1	,,	$\frac{3}{4}$	$1\frac{3}{4}$	,,	$\frac{9}{16}$	$2\frac{1}{4}$	,,	$1\frac{1}{4}$	$2\frac{5}{8}$	,,	$1\frac{7}{8}$
$1\frac{3}{32}$	,,	$\frac{23}{32}$	$1\frac{3}{4}$	,,	1	$2\frac{1}{4}$	,,	$1\frac{1}{2}$	$2\frac{3}{4}$	,,	$1\frac{1}{8}$
$1\frac{1}{4}$	,,	$\frac{3}{4}$	$1\frac{3}{4}$	,,	$1\frac{1}{4}$	$2\frac{1}{4}$	,,	$1\frac{3}{4}$	$2\frac{3}{4}$	,,	$1\frac{3}{4}$
$1\frac{1}{4}$	,,	1	$1\frac{3}{4}$	,,	$1\frac{3}{8}$	$2\frac{1}{4}$	,,	2	$2\frac{3}{4}$	,,	2

TABLE XXIV.—Angle Iron Sections—*continued.*

UNEQUAL SIDED ANGLE IRON.											
inch.			inch.			inch.			inch.		
2¾	by	2½	3⅝	by	1⅞	5	by	1¼	6½	by	2½
3	,,	1½	3¾	,,	1¾	5	,,	2	6½	,,	3½
3	,,	2	3¾	,,	2½	5	,,	2½	6½	,,	4
3	,,	2¼	3¾	,,	2¾	5	,,	3	6½	,,	5½
3	,,	2½	3⅞	,,	2⅜	5	,,	3¼	7	,,	3
3	,,	2¾	3⅞	,,	2¾	5	,,	3½	7	,,	3½
3⅛	,,	1⅛	4	,,	1½	5	,,	4	7	,,	4
3⅛	,,	2	4	,,	2	5	,,	4¼	7	,,	5
3⅛	,,	2⅜	4	,,	2¼	5	,,	4½	7	,,	5½
3¼	,,	1¾	4	,,	2½	5⅛	,,	4⅛	7½	,,	3½
3¼	,,	1⅞	4	,,	2¾	5½	,,	3	8	,,	2½
3¼	,,	2	4	,,	3	5½	,,	3½	8	,,	3
3¼	,,	2¼	4	,,	3⅛	5½	,,	4	8	,,	3½
3¼	,,	2½	4	,,	3¼	5½	,,	4⅛	8	,,	4
3¼	,,	2¾	4	,,	3½	5½	,,	4½	8	,,	4½
3¼	,,	3	4⅛	,,	3	5⅞	,,	$3\frac{9}{16}$	8	,,	6
3½	,,	1¼	4¼	,,	3⅛	6	,,	2½	8½	,,	4½
3½	,,	1½	4¼	,,	3¼	6	,,	3	8¾	,,	1⅜
3½	,,	1¾	$4\frac{5}{16}$	,,	$2\frac{9}{16}$	6	,,	3½	9	,,	3
3½	,,	2	4⅜	,,	$2\frac{9}{16}$	6	,,	4	9	,,	3½
3½	,,	2¼	4½	,,	2½	6	,,	4½	9	,,	4½
3½	,,	2½	4½	,,	3	6	,,	5	10	,,	3½
3½	,,	2¾	4½	,,	3½	6	,,	5½	10	,,	4
3½	,,	3	4½	,,	4	$6\frac{7}{32}$	,,	3¾	12	,,	3½
$3\frac{9}{16}$	,,	2¾	4¾	,,	3⅛	6½	,,	1⅜	12	,,	8
3⅝	,,	1⅛									

TABLE XXV.—Angle Iron.

ROUND BACKED.

			inch.			inch.		
			$2\frac{3}{8}$	by	$2\frac{3}{8}$	4	by	2
inch.			$2\frac{1}{2}$	,,	$2\frac{1}{4}$	4	,,	$2\frac{1}{2}$
$\frac{7}{8}$	by	$\frac{7}{8}$	$2\frac{1}{2}$	,,	$2\frac{1}{2}$	4	,,	3
1	,,	1	$2\frac{9}{16}$	,,	$2\frac{9}{16}$	4	,,	4
$1\frac{1}{8}$	,,	$1\frac{1}{8}$	$2\frac{5}{8}$	,,	$2\frac{5}{8}$	$4\frac{1}{4}$	,,	$3\frac{1}{4}$
$1\frac{1}{4}$	,,	$1\frac{1}{4}$	$2\frac{3}{4}$	,,	$2\frac{3}{4}$	$4\frac{1}{4}$	,,	$4\frac{1}{4}$
$1\frac{3}{8}$	,,	$1\frac{3}{8}$	$2\frac{7}{8}$	,,	$2\frac{7}{8}$	$4\frac{1}{2}$	,,	$2\frac{1}{2}$
$1\frac{1}{2}$	,,	$1\frac{1}{2}$	3	,,	3	$4\frac{1}{2}$	,,	3
$1\frac{3}{4}$	,,	$1\frac{1}{2}$	$3\frac{1}{8}$	,,	$3\frac{1}{8}$	$4\frac{1}{2}$	,,	$4\frac{1}{2}$
$1\frac{3}{4}$	,,	$1\frac{3}{4}$	$3\frac{1}{4}$	,,	$3\frac{1}{4}$	$4\frac{3}{4}$	,,	$4\frac{3}{4}$
2	,,	2	$3\frac{3}{8}$	,,	$3\frac{3}{8}$	5	,,	5
$2\frac{1}{8}$	,,	$2\frac{1}{8}$	$3\frac{1}{2}$	,,	$2\frac{1}{2}$	6	,,	$2\frac{1}{2}$
$2\frac{3}{16}$	,,	$2\frac{3}{16}$	$3\frac{1}{2}$	,,	3	6	,,	$3\frac{1}{2}$
$2\frac{1}{4}$	,,	$2\frac{1}{4}$	$3\frac{1}{2}$	,,	$3\frac{1}{2}$	$7\frac{5}{16}$	,,	$2\frac{5}{16}$

SQUARE ROOT.

			inch.			inch.		
			$\frac{5}{8}$	by	$\frac{5}{8}$	$1\frac{1}{4}$	by	$1\frac{1}{4}$
inch.			$\frac{3}{4}$	,,	$\frac{3}{4}$	$1\frac{3}{8}$	,,	$\frac{9}{16}$
$\frac{3}{8}$	by	$\frac{1}{4}$	$\frac{7}{8}$	,,	$\frac{7}{8}$	$1\frac{3}{8}$	,,	$1\frac{3}{8}$
$\frac{3}{8}$	,,	$\frac{3}{8}$	1	,,	$\frac{1}{2}$	$1\frac{1}{2}$	,,	$\frac{1}{2}$
$\frac{7}{16}$	,,	$\frac{3}{8}$	1	,,	$\frac{5}{8}$	$1\frac{1}{2}$	,,	1
$\frac{1}{2}$	,,	$\frac{7}{16}$	1	,,	$\frac{3}{4}$	$1\frac{1}{2}$	,,	$1\frac{1}{2}$
$\frac{1}{2}$	,,	$\frac{1}{2}$	1	,,	1	$1\frac{5}{8}$	,,	$\frac{5}{8}$
$\frac{9}{16}$	,,	$\frac{7}{16}$	$1\frac{1}{8}$	,,	$1\frac{1}{8}$	$1\frac{5}{8}$	,,	$1\frac{5}{8}$
$\frac{5}{8}$	,,	$\frac{3}{8}$	$1\frac{1}{4}$	,,	$\frac{5}{8}$	$1\frac{3}{4}$	,,	1
$\frac{5}{8}$	,,	$\frac{7}{16}$	$1\frac{1}{4}$	,,	$\frac{13}{16}$	$1\frac{3}{4}$	,,	$1\frac{3}{4}$
$\frac{11}{16}$	,,	$\frac{11}{16}$	$1\frac{1}{4}$	,,	1	2	,,	$\frac{5}{8}$

TABLE XXV.—ANGLE IRON—*continued.*

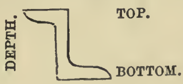
SQUARE ROOT.

inch.			inch.			inch.		
2	by	1	2¼	by	2¼	3	by	3
2	"	1½	2½	"	2½	3¼	"	2½
2	"	2	2¾	"	2¾	3¾	"	2¾
2¼	"	1¼						

BULB ANGLE.

inch.			inch.			inch.		
2	by	1½	2½	by	1½	4	by	2¼
3	"	2	2½	"	2	4	"	3
5	"	2½	3	"	2	4½	"	2½
6	"	3½	3	"	2½	4½	"	3¼
6½	"	3½	3½	"	2½	5	"	3
			4	"	2	5	"	3¼
						5½	"	4
						6	"	3½
						6	"	6

TABLE XXVI.—Z IRON SECTIONS.



Top.	Depth.	Bottom.	Thick.	Top.	Depth.	Bottom.	Thick.
inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
3¼	6½	3¼	½	2¼	3	2¼	7/16
2¾	6	2¾	½	2	2⅝	2	7/16
2½	4¼	2½	½	2	2½	2	5/16 to 7/16
2½	4	2½	⅜	¾	1⅜	9/16	¼
2	4	2	7/16	⅝	1¼	½	3/16
1½	3½	2¼	7/16	½	15/16	¼	⅛
3	3	2¼	7/16				

TABLE XXVII.—T Iron Sections of various thicknesses, in proportion to the size.

TABLE.

LEG.

Table.		Leg.	Table.		Leg.	Table.		Leg.	Table.		Leg.
inch.		inch.	inch.		inch.	inch.		inch.	inch.		inch.
10	by	10	6	by	5½	5¼	by	6	4½	by	4½
8	,,	4⅝	6	,,	5	5¼	,,	3⅝	4½	,,	4
8	,,	4	6	,,	4½	5¼	,,	3½	4½	,,	3¾
7½	,,	4	6	,,	4¼	5¼	,,	2⅝	4½	,,	3⅝
7	,,	7	6	,,	4	5⅛	,,	3½	4½	,,	3½
7	,,	6	6	,,	3½	5⅛	,,	3	4½	,,	3⅜
7	,,	5½	6	,,	3¼	5	,,	8	4½	,,	3
7	,,	5⅛	6	,,	3⅛	5	,,	6	4½	,,	2¾
7	,,	5	6	,,	3	5	,,	5	4½	,,	2½
7	,,	4½	6	,,	2½	5	,,	4	4½	,,	2¼
7	,,	3¼	5⅞	,,	3¼	5	,,	3¾	4½	,,	2
6¾	,,	5	5¾	,,	5	5	,,	3½	4½	,,	1¾
6¾	,,	4	5¾	,,	4	5	,,	3	4 7/16	,,	3⅞
6¾	,,	3¾	5¾	,,	3 11/16	5	,,	2¾	4⅜	,,	1⅞
6½	,,	4½	5¾	,,	3½	5	,,	2½	4¼	,,	4½
6½	,,	3⅜	5¾	,,	3¼	5	,,	2¼	4¼	,,	4
6½	,,	3	5½	,,	4⅞	4¾	,,	4½	4¼	,,	3¾
6⅜	,,	2⅞	5½	,,	4¾	4¾	,,	3¾	4¼	,,	3¼
6¼	,,	8	5½	,,	4⅜	4¾	,,	3½	4¼	,,	3
6¼	,,	6	5½	,,	3½	4¾	,,	3	4¼	,,	2¼
6¼	,,	3⅝	5½	,,	3⅜	4¾	,,	2⅞	4¼	,,	2
6¼	,,	3½	5½	,,	3	4¾	,,	1¾	4¼	,,	1⅞
6	,,	6⅝	5½	,,	2¾	4⅝	,,	3½	4⅛	,,	4
6	,,	6	5⅜	,,	2 5/16	4½	,,	5	4	,,	6

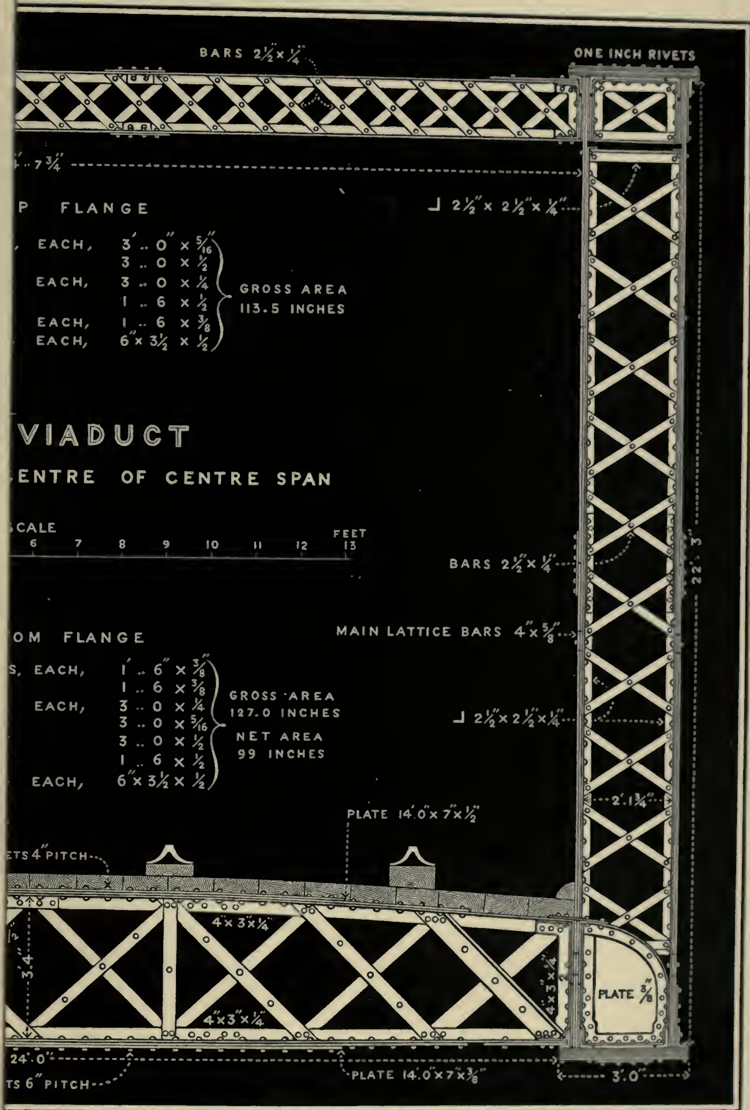
TABLE XXVII.—T IRON SECTIONS—*continued.*

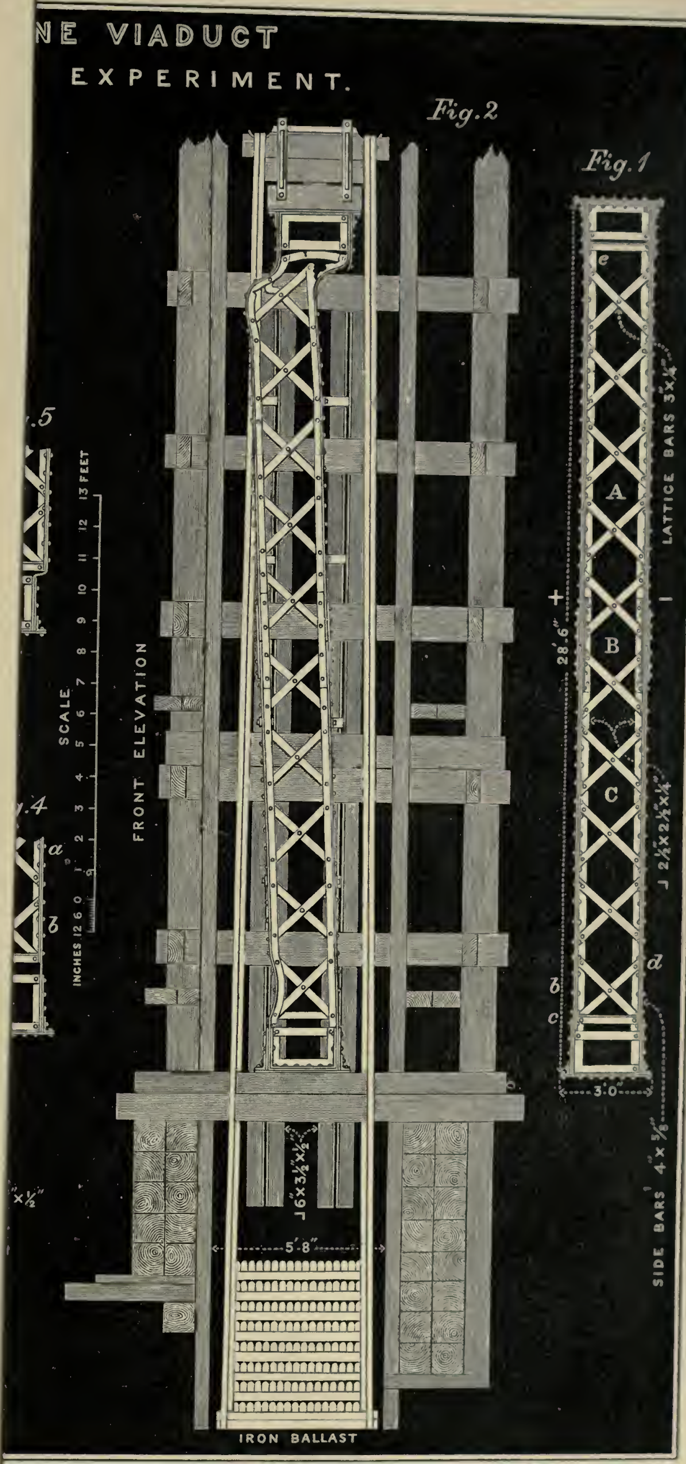
Table.		Leg.	Table.		Leg.	Table.		Leg.	Table.		Leg.
inch.		inch.	inch.		inch.	inch.		inch.	inch.		inch.
4	by	5	3⅛	by	2	2⅝	by	2⅝	2	by	1
4	,,	4½	3	,,	6½	2½	,,	4	1¾	,,	3
4	,,	4	3	,,	6	2½	,,	3½	1¾	,,	2⅝
4	,,	3½	3	,,	5½	2½	,,	3¼	1¾	,,	1¾
4	,,	3	3	,,	5	2½	,,	3	1¾	,,	1½
4	,,	$2\frac{9}{16}$	3	,,	4½	2½	,,	2½	1¾	,,	1
4	,,	2½	3	,,	4	2½	,,	2¼	1⅝	,,	2½
4	,,	$2\frac{5}{16}$	3	,,	3½	2½	,,	2	1⅝	,,	1⅝
4	,,	2¼	3	,,	3¼	2½	,,	1½	1⅝	,,	1½
4	,,	2	3	,,	3	2½	,,	1⅜	1⅝	,,	1⅜
4	,,	1⅞	3	,,	2¾	2½	,,	1¼	1½	,,	2¼
3⅞	,,	3	3	,,	2⅝	2⅜	,,	1	1½	,,	2
3⅞	,,	2⅜	3	,,	2½	2¼	,,	3	1½	,,	1½
3¾	,,	3¾	3	,,	2	2¼	,,	2¾	1½	,,	1⅜
3¾	,,	2	3	,,	1¾	2¼	,,	2¼	1½	,,	1¼
3¾	,,	$1\frac{3}{16}$	3	,,	1⅝	2¼	,,	2	1½	,,	⅞
3½	,,	4½	3	,,	1½	2¼	,,	1¼	1⅜	,,	1½
3½	,,	4	2⅞	,,	3½	2	,,	4	1⅜	,,	1⅜
3½	,,	3½	2⅞	,,	2⅞	2	,,	3½	1⅜	,,	1¼
3½	,,	3¼	2¾	,,	4	2	,,	3	1⅜	,,	1⅛
3½	,,	3	2¾	,,	3½	2	,,	2½	1⅜	,,	1
3½	,,	2½	2¾	,,	3¼	2	,,	2¼	1¼	,,	3
3½	,,	2	2¾	,,	3	2	,,	$2\frac{3}{16}$	1¼	,,	2
3½	,,	1½	2¾	,,	2¾	2	,,	2	1¼	,,	1⅞
3¼	,,	4	2¾	,,	2⅝	2	,,	1¾	1¼	,,	1½
3¼	,,	3¼	2¾	,,	1⅝	2	,,	1½	1¼	,,	1¼
3¼	,,	1¾	2¾	,,	1½	2	,,	1⅛	1¼	,,	1⅜

TABLE XXVII.—┬ Iron Sections—*continued.*

Table.		Leg.	Table.		Leg.	Table.		Leg.	Table.		Leg.
inch.		inch.	inch.		inch.	inch.		inch.	inch.		inch.
$1\frac{1}{4}$	by	1	1	by	$1\frac{1}{2}$	$\frac{7}{8}$	by	$\frac{7}{8}$	$\frac{3}{4}$	by	$1\frac{1}{2}$
$1\frac{1}{4}$	"	$\frac{3}{4}$	1	"	$1\frac{1}{4}$	$\frac{7}{8}$	"	$2\frac{5}{8}$	$\frac{3}{4}$	"	1
$1\frac{1}{8}$	"	$1\frac{1}{8}$	1	"	1	$\frac{7}{8}$	"	$\frac{3}{4}$	$\frac{3}{4}$	"	$\frac{3}{4}$
$1\frac{1}{8}$	"	1	1	"	$\frac{7}{8}$	$\frac{3}{4}$	"	$2\frac{1}{2}$	$\frac{5}{8}$	"	$\frac{5}{8}$
$1\frac{1}{16}$	"	$\frac{15}{16}$	1	"	$\frac{9}{16}$	$\frac{3}{4}$	"	2	$\frac{1}{2}$	"	$\frac{1}{2}$
$1\frac{1}{16}$	"	$\frac{11}{16}$									

TE IV.


BARS 2½"x¼"
ONE INCH RIVETS
. 7¾"
P FLANGE
, EACH, 3'.. 0" x 5/16"
3 .. 0 x ½
EACH, 3 .. 0 x ¼
1 .. 6 x ½
EACH, 1 .. 6 x ⅜
EACH, 6"x 3½ x ½
GROSS AREA
113.5 INCHES
┘ 2½" x 2½" x ¼"
VIADUCT
ENTRE OF CENTRE SPAN
CALE
6 7 8 9 10 11 12 13
FEET
BARS 2½"x ¼"
22'. 3"
OM FLANGE
S, EACH, 1'.. 6" x ⅜"
1 .. 6 x ⅜
EACH, 3 .. 0 x ¼
3 .. 0 x 5/16
3 .. 0 x ½
1 .. 6 x ½
EACH, 6"x 3½ x ½
GROSS AREA
127.0 INCHES
NET AREA
99 INCHES
MAIN LATTICE BARS 4"x ⅝"
┘ 2½"x 2½"x ¼"
2'.1¾"
PLATE 14'.0"x 7"x ½"
ETS 4" PITCH
4"x 3"x ¼"
4"x 3"x ¼"
4"x 3"x ¼
3'.4"
PLATE ⅜"
24'.0"
TS 6" PITCH
PLATE 14'.0"x 7"x ⅜"
3'.0"


NE VIADUCT
EXPERIMENT.
Fig.2
Fig.1
FRONT ELEVATION
SCALE
INCHES 12 6 0 1 2 3 4 5 6 7 8 9 10 11 12 13 FEET
LATTICE BARS 3"×¼"
28'.6"
A
B
C
e
b
c
d
2½"×2½"×¼"
3.0"
SIDE BARS 4"×⅜"
6"×3½"×½"
5'.8"
IRON BALLAST
.5
.4
a
b
"×½"

INDEX.