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THERMODYNAMICS

OF THE

STEAM-ENGINE

AND OTHER HEAT-ENGINES.

BY

CECIL H. PEABODY,
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PREFACE.

THIS work is designed to give instruction to students in technical schools in the methods and results of the application of thermodynamics to engineering. While it has been considered desirable to follow commonly accepted methods, some parts differ from other text-books, either in substance or in manner of presentation, and may require a few words of explanation.

The general theory or formal presentation of thermodynamics is that employed by the majority of writers, and was prepared with the view of presenting clearly the difficulties inherent in the subject, and of giving familiarity with the processes employed.

In the discussion of the properties of gases and vapors the original experimental data on which the working equations, whether logical or empirical, must be based are given quite fully, to afford an idea of the degree of accuracy attainable in calculations made with their aid. Rowland's determination of the mechanical equivalent of heat has been adopted, and with it his determination of the specific heat of water at low temperatures. The author's "Tables of the Properties of Saturated Steam and Other Vapors" were calculated to accompany this work, and may be considered to be an integral part of it.

The chapters on the flow of gases and vapors and on the injector are believed to present some novel features, especially in the comparisons with experiments.

The feature in which this book differs most from similar works is in the treatment of the steam-engine. It has been deemed advisable to avoid all approximate theories based on the assumption of adiabatic changes of steam in an engine cylinder, and instead to make a systematic study of steam-engine tests, with the view of finding what is actually known on the subject, and how future investigations and improvements may be made. For this purpose a large number of tests have been collected, arranged, and compared. Special attention is given to the investigations of the action of steam in the cylinder of an engine, considerable space being given to Hirn's researches and to experiments that provide the basis for them. Directions are given for testing engines, and for designing simple and compound engines.

Chapters have been added on compressed-air and refrigerating machines, to provide for the study of these important subjects in connection with the theory of thermodynamics.

Wherever direct quotations have been made, references have been given in foot-notes, to aid in more extended investigations. It does not appear necessary to add other acknowledgment of assistance from well-known authors, further than to say that their writings have been diligently searched in the preparation of this book, since any text-book must be largely an adaptation of their work to the needs of instruction.

C. H. P.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
May, 1889.

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THERMODYNAMICS OF THE STEAM-ENGINE.

CHAPTER I.

THERMAL CAPACITIES.

THE object of thermodynamics, or the mechanical theory of heat, is the solution of problems involving the action of heat, and, for the engineer, more especially those problems presented by the steam-engine and other thermal motors. In this work the discussion of the actual nature of heat and the rationale of its various actions will be purposely avoided, and attention will be given rather to the calculation of the results of such actions.

Some conceptions already familiar will appear in a somewhat different light; and some new conceptions will be presented. It will be found that some of the latter are susceptible of more concise definition than others that are now more familiar. Also some methods of operation will be employed which may seem more abstract than those commonly used in engineering. The student will, however, recognize them as methods that he has already learned, and he will gain confidence in them by familiarizing himself with their use.

Effects of Heat.—In general, the action of heat on a body causes a change in all the characteristics of the body, such as the density, tension, temperature, elasticity, refractive index, conductivity, etc. When heat is communicated to or taken from a body, there is a change in the energy of the body. If a body is supposed to be at rest, or if its visible motion is not changed during the operation, and can consequently be disregarded, the change of energy produced by the communication or abstraction of heat may be considered as a property of the body. The energy may in general be divided into potential and kinetic energy, in which case each may be a property of the body.

Characteristic Equation.—The assumption of the mechanical theory of heat is that if any two of the several characteristics or properties of a body be taken as independent variables, any other can be expressed as a function of the two independent variables. If x and y are chosen as the independent variables, and if z is any other characteristic, then

$$z = F(x, y), \quad \text{or} \quad f(x, y, z) = 0.$$

The form of the function is to be determined experimentally; and as yet the necessary experiments have been made for only a few of the numerous functions that may be indicated by using the various characteristics. The most useful characteristic equation is

$$f(p, v, t) = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which p is the pressure, v the volume, and t the temperature.

The pressure is assumed to be a hydrostatic pressure, such as a fluid exerts on the sides of the containing vessel or on an immersed body. We shall always consider the pressure exerted *by* the body rather than *on* the body. The pressure is stated in units of force per unit of area, as pounds per square foot, and is called *specific pressure.*

The *density* of a body is the weight of a unit of volume; for example, a cubic foot of water weighs 62.4 pounds nearly, or its density is 62.4. The reciprocal of the density is the volume occupied by one unit of weight, and is called the *specific volume;* the specific volume of water being $\frac{1}{62.4}$.

The temperature cannot be as satisfactorily defined. The common scales depend on the properties of some substance, such as mercury or air; and it can be shown that the unit is not the same in different parts of the scale. In the course of the work it will be shown that an absolute scale independent of the substances used can be constructed on thermodynamic principles. It will also appear that the scale of the air thermometer coincides very nearly with the thermodynamic scale. Again, the scale of the mercurial thermometer at medium temperature agrees with that of the air thermometer sufficiently well for engineering work. To avoid difficulty that may arise when we are ready to establish the thermodynamic scale, we will abstain from accepting any scale even temporarily.

It will be sufficient to fix the two following ideas of temperature:

(1) Of two bodies in thermal communication (by conduction or radiation), that one which imparts heat to the other is at the higher temperature. If neither gains or loses, they are at the same temperature.

(2) There are certain temperatures, such as the freezing and boiling points of water at atmospheric pressure, which are fixed and can be identified. We may assume that there are a series of such fixed temperatures, natural or artificial, which may be compared to a scale of hardness; and we may say that the temperature of a body coincides with one of them, or is higher than one and lower than another. We may, if we choose, use the air thermometer for the purpose; but in such case the degrees are to be considered as fixed points and not as units of measurement.

We may, however, admit that a true scale is possible, and that temperature may enter into an equation, but that the form of the function remains to be determined.

To give a concrete meaning to the characteristic equation, we may refer to the combined law of Boyle and Gay Lussac for a perfect gas. If we accept for the moment the scale of the air thermometer, the law may be represented by the equation

$$pv = \left(\frac{1}{\alpha} + t\right)R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which R is a constant, α is the coefficient of dilatation of air, and t is the temperature by the air thermometer on the Centigrade scale. Transposing,

$$\left[pv - \left(\frac{1}{\alpha} + t\right)R\right] = 0. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is the characteristic equation for a perfect gas in terms of p, v, and t, in which α may be assumed to be constant.

The value of α is very nearly $\frac{1}{273.7}$. Consequently, if we make $\frac{1}{\alpha} + t = 273.7 + t = T$, we shall have, instead of (3),

$$pv - RT = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The letter T is used to represent what is sometimes called the absolute temperature of the air thermometer above the absolute zero.

Now, air near the freezing point of water expands $\frac{1}{273.7}$ of its volume at freezing for each degree of increase of temperature, and also contracts $\frac{1}{273.7}$ of its volume for each degree below freezing. If the thermometer be formed of a tube of uniform calibre, and the space between freezing and boiling be divided into one hundred parts, and if the division be continued to the closed ends, there will be 273.7 divisions below freezing point. The degrees may be numbered beginning at the closed end; and the temperatures on such scale will be represented by

$$T = 273.7 + t.$$

Now, any equation with three variables may be represented by a geometrical surface, referred to co-ordinate axes, of which surface the variables are the co-ordinates. In the case of a perfect gas which conforms to the equation

$$pv = RT,$$

the surface is such that each section perpendicular to the T is a rectangular hyperbola (Fig. 1).

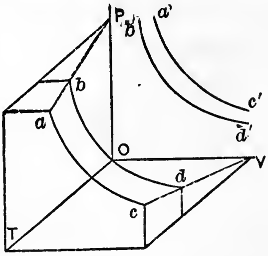

FIG. 1.

Returning now to the general case, and abstaining from adopting any specific scale of temperature, it is apparent that the characteristic equation of any substance may be represented by a geometrical surface referred to co-ordinate axes, since the equation is assumed to contain only three variables; but the surface will in general be less simple in form than that representing the combined law of Boyle and Gay Lussac.

If one of the variables, as t, is given a special constant value, it is equivalent to taking a section perpendicular to the axis of t; and a plane curve will be cut from the surface, which may be conveniently projected on the (p, v) plane.

Sections may be taken perpendicular to the other axes, and a sufficient number of such sections, or, as a substitute, the projections of the intersections, will give a complete description of the surface. The whole process is equivalent to a complete solution of the characteristic equation, but there are few substances of which the properties are sufficiently well known for the purpose.

Other curves in addition to the sections may be drawn on the surface and projected on the (p, v) plane. It is essential sometimes to distinguish between the actual curve and its projection. The reason for choosing the (p, v) plane is that the curves drawn correspond to those drawn by the indicator.

Thermal Capacities.—The amount of heat required to change by unity any quality of a unit of weight of any body under given circumstances is called the thermal capacity corresponding to the given change.

In three cases only have these capacities received special names; *i.e.*, specific heat at constant volume, specific heat at constant pressure, and latent heat of expansion.

Thermal Unit.—Heat is measured in calories, or British thermal units (B. T. U.). A calorie commonly is defined as the heat required to raise one kilogram of water from freezing point to one degree Centigrade; and a British thermal unit, that required to raise one pound from 32° to 33° Fahrenheit. In our work, for reasons that will appear in the discussion of the specific heat of water, we shall choose 62° F. for the standard temperature, and shall define a B. T. U. as the heat required to raise a pound of water from 62° to 63° F.

This statement is subject to the same indefiniteness as the thermal scale from which it is derived. The true value can be defined only after a logical thermal scale is developed.

Specific Heat is the heat required to raise one unit of weight of a substance through one degree of temperature, measured in thermal units. The specific heat of water at the standard temperature is consequently unity. Two specific heats are commonly distinguished; at constant pressure c_p, and constant volume c_v. Both of these specific heats are liable to be variable; consequently, if the amount of heat dQ imparted to a unit of weight of a substance changes the temperature by dt, then the specific heat at a given temperature t is $c = \frac{dQ}{dt}$.

If the change takes place at constant pressure or at constant volume, then

$$c_p = \left(\frac{dQ}{dt}\right)_{p \text{ constant}}, \qquad \text{or} \qquad c_v = \left(\frac{dQ}{dt}\right)_{v \text{ constant}}.$$

Latent Heat of Expansion is the amount of heat required to change the volume of one unit of weight by the amount of one unit.

As this property is also sometimes variable, we may properly write for the latent heat of expansion at the temperature t,

$$l = \left(\frac{dQ}{dv}\right)_{t \text{ constant}}.$$

General Equations of the Effects produced by Heat.—The heat given to a body and which produces a certain change in that body may be a function of p, v, and t. If v and t are chosen for the independent variables, then

$$dQ = \left(\frac{dQ}{dt}\right)_v dt + \left(\frac{dQ}{dv}\right)_t dv. \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

In like manner, with p and t as independent variables,

$$dQ = \left(\frac{dQ}{dt}\right)_p dt + \left(\frac{dQ}{dp}\right)_t dp; \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

and with p and v as independent variables,

$$dQ = \left(\frac{dQ}{dp}\right)_v dp + \left(\frac{dQ}{dv}\right)_p dv. \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

Substituting for $\left(\frac{dQ}{dt}\right)_v$, in equation (5a), the specific heat at constant volume, and for $\left(\frac{dQ}{dv}\right)_t$, the latent heat of expansion,

$$dQ = c_v dt + l dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In like manner the specific heat at constant pressure may be substituted for $\left(\frac{dQ}{dt}\right)_p$ in equation (6a); and for $\left(\frac{dQ}{dp}\right)_t$ may be substituted ~~l~~ m which represents the amount of heat required to be added when the external pressure is increased by unity, a thermal capacity which has not as yet received a name; whence

$$dQ = c_p dt + m dp. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Finally, $\left(\frac{dQ}{dp}\right)_v$ may be represented by n, and $\left(\frac{dQ}{dv}\right)_p$ by o, both being thermal capacities without names, and equation (7a) becomes

$$dQ = n dp + o dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Relations of the Thermal Capacities.—The three equations, (5), (6), and (7), show the changes produced by the addition of an amount of heat dQ to a unit of weight of a substance, the difference coming from the methods of analyzing the changes, but the total amount dQ may be the same in all; consequently the left-hand members may be equated, forming three equations. Thus, equating (5) and (6),

$$c_v dt + l dv = c_p dt + m dp. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From the general characteristic equation we have

$$v = F(p, t),$$

from which, by differentiating, we have

$$dv = \left(\frac{dv}{dt}\right)_p dt + \left(\frac{dv}{dp}\right)_t dp,$$

which substituted in (8) gives

$$c_p dt + m dp = c_v dt + l\left[\left(\frac{dv}{dt}\right)_p dt + \left(\frac{dv}{dp}\right)_t dp\right].$$

$$\therefore\ c_p dt + m dp = \left[c_v + l\left(\frac{dv}{dt}\right)_p\right] dt + l\left(\frac{dv}{dp}\right)_t dp. \quad . \quad . \quad (9)$$

In equation (9) p and t are independent variables, and each may have all possible values; consequently we may equate like coefficients.

$$\therefore\ c_p = c_v + l\left(\frac{dv}{dt}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$l\left(\frac{dv}{dt}\right)_p = c_p - c_v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Again, equating the remaining coefficients,

$$l\left(\frac{dv}{dp}\right)_t = m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

In like manner, by differentiating the general equation,

$$dp = \left(\frac{dp}{dt}\right)_v dt + \left(\frac{dp}{dv}\right)_t dv, \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which substituted in (8) gives

$$c_p dt + m\left[\left(\frac{dp}{dt}\right)_v dt + \left(\frac{dp}{dv}\right)_t dv\right] = c_v dt + l dv,$$

$$\left[c_p\,m + \left(\frac{dp}{dt}\right)_v\right] dt + m\left(\frac{dp}{dv}\right)_t dv = c_v dt + l dv.$$

Equating like coefficients,

$$c_p + m\left(\frac{dp}{dt}\right)_v = c_v, \quad . \quad . \quad . \quad . \quad . \quad (14)$$

or

$$-m\left(\frac{dp}{dt}\right)_v = c_p - c_v. \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Equating (6) and (7),

$$c_p dt + m dp = n dp + o dv.$$

Substituting from

$$dt = \left(\frac{dt}{dv}\right)_p dv + \left(\frac{dt}{dp}\right)_v dp, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$c_p\left(\frac{dt}{dv}\right)_p dv + c_p\left(\frac{dt}{dp}\right)_v dp + m dp = n dp + o dv.$$

Equating coefficients of dv,

$$o = c_p\left(\frac{dt}{dv}\right)_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Finally, equating (5) and (7),

$$c_v dt + l dv = n dp + o dv.$$

Substituting for the value of dt, as above,

$$c_v\left(\frac{dt}{dv}\right)_p dv + c_v\left(\frac{dt}{dp}\right)_v dp + l dv = n dp + o dv.$$

Equating coefficients of dp,

$$n = c_v\left(\frac{dt}{dp}\right)_v. \quad \ldots\ldots\ldots \quad (18)$$

In the preceding work we have six coefficients, c_p, c_v, l, m, n, and o, of which c_p is commonly known for any substance. The coefficient c_v cannot be readily determined directly, but the ratio $\frac{c_p}{c_v} = \kappa$ is known for some substances, especially gases, and from it c_v may be found. The other four are expressed in terms of the specific heats, which for convenience are here assembled:

$$l\left(\frac{dv}{dt}\right)_p = c_p - c_v, \qquad -m\left(\frac{dp}{dt}\right)_v = c_p - c_v,$$

$$n = c_v\left(\frac{dt}{dp}\right)_v, \qquad o = c_p\left(\frac{dt}{dv}\right)_p,$$

$$m = l\left(\frac{dv}{dp}\right)_t.$$

Also we have in general for three variables, as p, v, and t,

$$\left(\frac{dp}{dt}\right)_v \cdot \left(\frac{dv}{dp}\right)_t \cdot \left(\frac{dt}{dv}\right)_p = -1; \quad \ldots\ldots \quad (19)$$

also,

$$\left(\frac{dt}{dp}\right)_v \cdot \left(\frac{dp}{dv}\right)_t \cdot \left(\frac{dv}{dt}\right)_p = -1. \quad \ldots\ldots \quad (20)$$

The relations thus far deduced are merely necessary algebraic relations of the literal functions, and are related to the theory of heat since the forms are those that appear in that theory. They are therefore true, whatever theory be accepted and whatever scale of temperature be adopted.

CHAPTER II.

FIRST LAW OF THERMODYNAMICS.

THE formal statement of the first law of thermodynamics is:

Heat and mechanical energy are mutually convertible, and heat requires for its production and produces by its disappearance a definite number of units of work for each thermal unit.

The mechanical equivalent of heat is designated by J, and the reciprocal by A; so that

$$A = \frac{1}{J}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

The value of the mechanical equivalent of heat given by Joule, and long quoted in all works on heat, is 772 foot-pounds for one B.T.U. In the French system the equivalent of one calorie, corresponding, is taken to be 424 meter-kilograms.

The value of J determined by Rowland at 62° F. or $16\frac{2}{3}°$ C. and reduced to 45° of latitude, is, for

1 B.T.U., 778 foot-pounds;
1 calorie, 426.9 meter-kilograms;

and these values will be used in our work unless the contrary is stipulated.

This law is a special statement of the general law that energy can neither be created nor destroyed, but may be transmuted from one form to another, with a definite number of units in one form, equivalent to a given number in the other. The law is a physical and experimental law, differing essentially from an axiom.

Effects of the Transfer of Heat.—Let a quantity of any substance of which the weight is one unit, *i.e.*, one pound or one kilogram, receive a quantity of heat dQ. It will, in general, experience three changes, each requiring an expenditure of energy. They are: (1) The temperature will be raised, and, by the theory that sensible heat is due to the vibrations of the particles of the body, the kinetic energy will be increased. Let dS represent this change of sensible heat or vibration work in units of work. (2) The mean positions of the particles will be changed; in general the body will expand. Let dI represent the units of work required for this change of internal potential energy, or work of disgregation. (3) The expansion indicated in (2) is generally against an external pressure, and to overcome the same, that is, for the change in external potential energy, there will be required the work dW.

If during the transmission no heat is lost, and if no heat is transformed into other forms of energy, such as sound, electricity, etc., then the first law of thermodynamics gives

$$dQ = A(dS + dI + dW). \quad . \quad . \quad . \quad . \quad . \quad (22)$$

It is to be understood that any or all of the terms of the equation may become zero or may be negative. If all the terms become negative, heat is withdrawn instead of added, and dQ is negative. It is not easy to distinguish between the vibration work and the disgregation work, and for many purposes it is unnecessary; consequently they are treated together under the name of intrinsic energy, and we have

$$dQ = A(dS + dI + dW) = A(dE + dW). \quad . \quad . \quad . \quad (23)$$

The inner work, or intrinsic energy, depends on the state of the body, and not at all on the manner by which it arrived at that state; just as the total energy of a falling body, with reference to a given plane consisting of kinetic energy and potential energy, depends on the velocity of the body and the height above the plane, and not on the previous history of the body.

The external work is assumed to be done against a fluid pressure; consequently

$$dW = pdv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$W = \int_{v_0}^{v_1} pdv, \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where v_1 and v_0 are the final and initial volumes.

This assumption holds good when the substance itself is a fluid; for example, the steam in the cylinder of an engine.

In order to find the value of the integral v, in equation (25), it is necessary to know the manner in which the pressure varies with the volume.

Since the pressure may vary in different ways, the external work cannot be determined from the initial and final states of the body. The heat required to effect a change from one state to another depends on how the change is effected.

Assuming the law of the variation of the pressure and volume to be known, we may integrate thus:

$$Q = A\left(E_1 - E_0 + \int_{v_0}^{v_1} pdv\right). \quad . \quad . \quad . \quad . \quad (26)$$

In order to determine E for any state of a body, it would be necessary to deprive it entirely of vibration and disgregation energy, which of course involves reducing it to a state of absolute cold. Consequently the direct determination is impossible. However, in all our work the substances operated on are changed from one state to another, and in each state the intrinsic energy depends on the state only; consequently the change of intrinsic energy may be determined from the initial and final states only, without knowing the manner of change from one to the other.

All succeeding equations will be arranged to involve differences of energy only, and the hypothesis involved in a separation into vibration and disgregation work avoided.

Thermal Lines.—The external work can be determined only when the relations of p and v are known, or, in general, when the characteristic equation is known. It has already been shown that in such case the equation may be represented by a geometrical surface, on which so-called thermal lines can be drawn representing the properties of the substance under consideration. These lines are commonly projected on the (p, v) plane. It is convenient in many cases to find the relation of p and v under a given condition and represent it by a curve drawn directly on the (p, v) plane.

A number of the thermal lines will be thus represented and discussed.

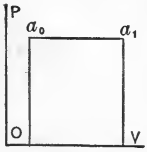

FIG. 2.

Isopiestic lines, or lines of equal pressure.—The change of condition takes place at constant pressure, and consists of a change of volume, as represented in Fig. 2. The tracing point moves from a_0 to a_1, and the volume changes from v_0 to v_1. The work done is represented by the rectangular area under a_0a_1, or by

$$W = p\int_{v_0}^{v_1} dv = p(v_1 - v_0).$$

During the change the temperature may or may not change; the diagram shows nothing concerning it.

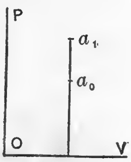

FIG. 3.

Isometric lines, or lines of equal volume.—The pressure increases at constant volume, and the tracing point moves from a_0 to a_1. The temperature usually increases meanwhile. Since dv is zero,

$$W = \int_{v_0}^{v_1} pdv = 0.$$

Isothermal lines, or lines of equal temperature.—The temperature remains constant, and a line is drawn, usually convex, toward the axis OV. The pressure of a mixture of a liquid and its vapor is constant for a given temperature; consequently the isothermal for such a mixture is a line of equal pressure, represented by Fig. 2. The isothermal of a perfect gas, on the other hand, is an equilateral hyperbola, as appears from the law of Boyle, which may be written

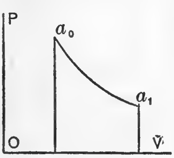

FIG. 4.

$$pv = C. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Isodynamic or isoenergic lines are lines representing changes during which the intrinsic energy remains constant. Consequently all the heat received is transformed into external work. It will be seen later that the isodynamic and isothermal lines for a gas are the same.

Adiabatic Lines.—A very important problem in thermodynamics is to determine the behavior of a body when change of condition occurs without gain or loss of heat; that is, such a change as would occur in a perfectly non-conducting vessel. During the change heat may be transformed into work, or *vice versa;* but no heat is transferred in the form of heat. Direct experiments are very difficult, and are usually approximations; very rapid changes in any vessel are nearly adiabatic, since time is required for conduction and radiation of heat. Rankine gave the name adiabatic to lines representing the volume and pressure during changes that occur without transmission of heat. When there is no transmission of heat, equation (26) becomes

$$0 = Q = A\left(E_1 - E_0 + \int_{v_0}^{v_1} pdv\right);$$

consequently,

$$-(E_1 - E_0) = \int_{v_0}^{v_1} pdv, \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

which shows that work done by the body against external pressure is at the expense of the intrinsic energy. Since the two quantities are numerically equal, the sign may frequently be neglected in numerical problems.

When an adiabatic crosses an isothermal, both being projected on the (p, v) plane, it is the steeper, as shown at a_0, Fig. 5. This is easily shown for substances that expand with the rise of temperature. For, if a body expands in a non-conducting vessel, the external work done is at the expense of the intrinsic energy, as shown by equation (28); and a diminution of the intrinsic energy in general is made up of a loss of potential energy, due to molecular arrangement, and a loss of kinetic energy, due to temperature.

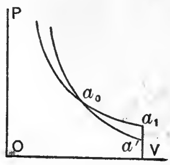

FIG. 5.

Now, a loss of temperature at constant pressure causes a contraction of volume; or, conversely, at constant volume a loss of temperature causes a lowering of pressure. In Fig. 5 an expansion at constant temperature is represented by the isothermal a_0a_1, while an expansion without transmission of heat is represented by the adiabatic a_0a'. The final volume is the same; consequently the pressure represented by a' is less, since the adiabatic expansion is accompanied by a loss of temperature.

Graphical Representations of Change of Intrinsic Energy.—Professor Rankine first used a graphical method of representing a change of intrinsic energy, employing adiabatic lines only, as follows:

Suppose that a substance is originally in the state A (Fig. 6), and that it expands adiabatically; then the external work is done at the expense of the intrinsic energy; hence, if the expansion has proceeded to A_1, the area AA_1a_1a, which represents the external work, also represents the change of intrinsic energy. Suppose that the expansion were to continue indefinitely; then the adiabatic will approach the axis OV indefinitely, and the area representing the work will be included between the curve $A\alpha$ produced indefinitely, the ordinate Aa, and the axis OV; this area will represent all the work that can be obtained by the expansion of the substance; and if it be admitted that during the expansion all the intrinsic energy is transformed into work, so that at the end the intrinsic energy is zero, it represents also the intrinsic energy. In cases for which the equation of the adiabatic can be found, it is easy to show that

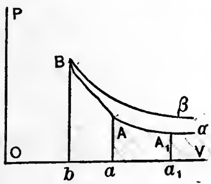

FIG. 6.

$$-E_1 = \int_a^{v_1} pdv$$

is a finite quantity; and in any case, if we admit an absolute zero of temperature, it is evident that the intrinsic energy cannot be infinite. On the other hand, if an isothermal curve were treated in the same way, the area would be infinite, since heat would be continually added during the expansion.

Now suppose the body to pass from the condition represented by A to that represented by B, by any path whatever; that is, by any succession of changes whatever; for example, that represented by the irregular curve AB. The intrinsic energy in the state B is represented by the area $VbB\beta$. The change of intrinsic energy is represented by the area $\beta BbaA\alpha$, and this area does not depend on the form of the curve AB. This graphical process is only another way of stating that the intrinsic energy depends on the state of the substance only, and that change of intrinsic energy depends on the final and initial states only.

Another way of representing change of intrinsic energy by aid of isodynamic lines avoids an infinite diagram. Suppose the change of state to be represented by the curve AB, Fig. 7. Draw an isodynamic line AC through the point A, and an adiabatic line BC through B, intersecting at C. Then the area $ABba$ represents the external work, and the area $bBCc$ represents the change of intrinsic energy; for if the body be allowed

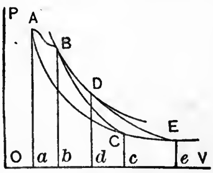

FIG. 7.

to expand adiabatically till the intrinsic energy is reduced to its original amount at the condition represented by A, the external work $bBCc$ will be done at the expense of the intrinsic energy. Since the intrinsic energy is constant for all points on the isodynamic line through A, and in like manner is constant for points on the line through B, there will be the same change of intrinsic energy in passing from a condition represented by any point of the line through A to any point of the line through B; consequently, if through any point, as D of the upper line, an adiabatic DE be drawn, the area $dDEe$ will be equal to $bBCc$, and will equally represent the change of intrinsic energy from the point A to the point B.

Entropy.—If a body have its condition represented by the point e of the isothermal aa_1 (Fig. 8), it will have a definite temperature, which will be the same so long as its condition is represented by some point on aa_1, as, for example, a_1, though the volume and pressure will meanwhile have varied. Should the temperature change, the condition will be represented by some point, as f, on another isothermal bb_1. There will evidently be the

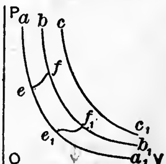

FIG. 8.

same change of temperature in passing from e to f as from e_1

to f_1; that the changes of volume and pressure, external work, and intrinsic energy are different does not affect the statement concerning the temperature. In like manner, it is indifferent how or at what part of the diagram the transfer from bb_1 to cc_1 is accomplished; the same change of temperature must occur.

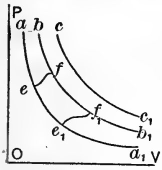

FIG. 9.

Let aa_1, bb_1, and cc_1 represent adiabatic lines. Then if a body having its condition represented by a point on aa_1 experience a change represented by ee_1, it will have neither lost nor gained heat as such, though heat may have been changed into work; or, *vice versa*, any change that can be represented by a portion of an adiabatic line will be subject to the same condition. There is, we see, some property of the body that remains constant during an adiabatic change; and that property is called the entropy. If the substance has its condition represented by f, it will have a different entropy; but for any change represented by a portion of the line bb_1, as ff_1, the entropy will be constant. Just as a change involving the passage from one isothermal to another requires a definite change of temperature, so a change involving the passage from one adiabatic to another involves a definite change of entropy. Thus a passage from e to f involves the same change of entropy as a change from e_1 to f_1; again, the path from e to f is indifferent, and has purposely been represented as irregular.

That the passage from one adiabatic to another under different circumstances may involve different changes of volume and pressure, external work, etc., does not affect the statement concerning the entropy.

An expression for the entropy, as for the different thermal capacities, can be found in some cases. Entropy will be represented by ϕ. It is a property of a body similar to specific volume, specific heat, and latent heat of expansion, but most nearly akin to temperature. It depends on the state of the body and not on the method of the change.

CHAPTER III.

SECOND LAW OF THERMODYNAMICS.

Heat Engines are engines by which heat is transformed into work. All actual engines used as motors go through continuous cycles of operations, which periodically return things to the original conditions. All heat-engines are similar, in that they receive heat from some *source*, transform part of it into work, and deliver the remainder (minus certain losses) to a *refrigerator.*

The source and refrigerator of a condensing steam-engine are the furnace and the condenser. The boiler is properly considered as a part of the engine, and receives heat from the source.

Carnot's Engine.—It is convenient to discuss a simple ideal engine, first described by Carnot; though, from a defect in the theory of heat then accepted, his description was erroneous.

Let P of Fig. 10 represent a cylinder with non-conducting walls, in which is fitted a piston, also of non-conducting material, and moving without friction; on the other hand, the bottom of the cylinder is supposed to be of a material that is a perfect conductor, and which has a zero thermal capacity. There is a non-conducting stand C on which the cylinder can be placed while adiabatic changes take place. The source of heat A at a temperature t is supposed to be so maintained that in operations during which the cylinder is placed on it, and draws heat from it, the temperature is unchanged. The refrigerator B at the temperature t_1 in like manner can withdraw heat from the cylinder when it is placed on it, at a constant temperature.

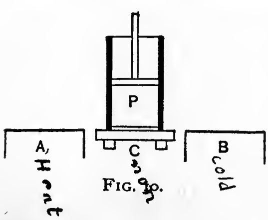

FIG. 10.

Let there be a unit of weight (for example, one pound) of a certain substance in the cylinder at the temperature t of the source of heat. Place the cylinder on the source of heat A (Fig. 10), and let the substance expand at the constant temperature t, receiving heat from the source A.

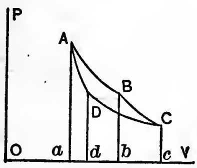

FIG. 11.

all are different

If the first condition of the substance be represented by A (Fig. 11), then the second will be represented by B, and AB will be an isothermal. If E_a and E_b are the intrinsic energies at A and B, and if W_{ab}, represented by the area $aABb$, be the external work, the heat received from A will be

$$Q = A(E_b - E_a + W_{ab}).$$

Now place the cylinder on the stand C (Fig. 10), and let the substance expand adiabatically until the temperature is reduced to t_1, that of the refrigerator, the change being represented by the adiabatic BC (Fig. 11). If E_c is the intrinsic energy at C, then, since no heat passes into or out of the cylinder,

$$0 = A(E_c - E_b + W_{bc}),$$

where W_{bc} is the external work represented by the area $bBCc$. Place the cylinder on the refrigerator B, and compress the substance till it passes through the change represented by CD, yielding heat to the refrigerator so that the temperature remains constant. If E_d is the intrinsic energy at D, then

$$-Q_1 = A(E_d - E_c - W_{cd})$$

is the heat yielded to the refrigerator, and W_{cd}, represented by the area $cCDd$, is the external work, which has a minus sign since it is done on the substances.

The point D is determined by drawing an adiabatic from A to intersect an isothermal through C. The process is completed by compressing the substance while the cylinder is on the stand C (Fig. 10) till temperature rises to t, the change being represented by the adiabatic DA. Since there is no transfer of heat,

$$0 = A(E_a - E_d - W_{da}).$$

Adding together the several equations, member to member,

$$Q - Q_1 = A(W_{ab} + W_{bc} - W_{cd} - W_{da});$$

or, if W be the resulting work represented by the area $ABCD$, then

$$Q - Q_1 = AW;$$

that is, the difference between the heat received and the heat delivered to the refrigerator is the heat transformed into work.

Carnot, in his description of the engine, gives instruction to compress the substance during the third operation, and while in connection with the refrigerator, till all the heat received from the source of heat is yielded, and then to complete the cycle by an adiabatic compression. The caloric theory of heat, assuming it to be a substance, required such a statement, and Carnot compared the difference of temperature to a difference of head of water in hydraulics. In the description now common the operation of the engine corresponds to the first law of thermodynamics.

A Reversible Engine is one that may run either in the usual manner, transforming heat into work, or reversed, describing the same cycle in the opposite direction, and transforming work into heat.

A Reversible Cycle is the cycle of a reversible engine.

Carnot's engine is reversible, the reversed cycle being $ADCBA$ (Fig. 11), during which work is done by the engine on the working substance. The engine then draws from the refrigerator a certain quantity of heat, it transforms a certain quantity of work into heat, and delivers the sum of both to the source of heat.

A Closed Cycle is any cycle in which the final state is the same as the initial state. Fig. 12 represents such a cycle made up of four curves of any nature whatever. If the four curves are of two species only, as in the diagram representing the cycle of Carnot's engine, the cycle is said to be simple. In general, we shall have for a cycle like that of Fig. 12,

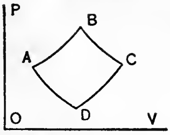

FIG. 12.

$$Q_{ab} + Q_{bc} - Q_{cd} - Q_{da} = \Sigma Q = A \Sigma W$$
$$= A\,(W_{ab} + W_{bc} - W_{cd} - W_{da}).$$

A closed curve of any form may be considered to be the general form of a closed cycle; as that in Fig. 13. For such a cycle we have $\int dQ = A \int dW$, which is one more way of stating the first law of thermodynamics.

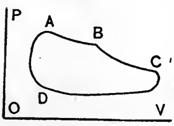

FIG. 13.

It may make this last clearer to consider the cycle of Fig. 14, composed of the isothermals AB, CD, and EG, and the adiabatics BC, DE, and GA. The cycle may be divided by drawing the curve through from C to F. It is indifferent whether the path followed be $ABCDEGA$ or $ABCFCDEGA$ or, again, $ABCFGA + CDEFC$.

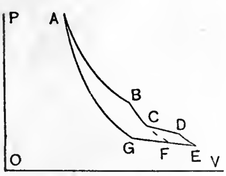

FIG. 14.

Again, an irregular figure may be imagined to be cut into elementary areas by isothermals and adiabatic lines, as in Fig. 15. The summation of the areas will give the entire area, and the summation of the works represented by these will give the entire work represented by the entire area.

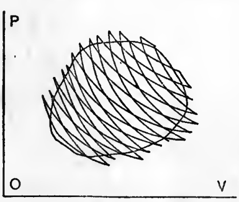

FIG. 15.

The Efficiency of an engine is the ratio of the heat changed into work to the entire heat applied; so that if it be represented by η,

$$\eta = \frac{AW}{Q}. \qquad (29)$$

Carnot enunciated a principle which may be stated as follows:

Carnot's Principle.—*Of all engines working between the same source of heat and the same refrigerator, a reversible engine gives the maximum efficiency.*

For, suppose there are two engines, one A, of any kind whatever, and one R, which is reversible, and, for simplicity, let each take the same quantity of heat Q from the source of heat per unit of time while running direct. The engine R, if reversed, will deliver the same quantity Q of heat per unit of time to the source. Now, if the efficiency of A is greater than that of R, that is, if

$$\frac{AW_a}{Q} > \frac{AW_r}{Q}, \quad \text{or} \quad W_a > W_r,$$

then A, coupled with R, will be able to run R reversed, and at the same time produce available work equal to $W_a - W_r$.

This surplus work can come from the refrigerator only, since the heat taken from the source of heat, and the heat returned to it, in a unit of time are equal. But experience and experiment show that work cannot be so done. Moreover, if it be admitted that surplus work can be done at the expense of the refrigerator, the admission involves the ultimate conclusion that by such a process all the heat might be abstracted from the refrigerator. In general, the efficiency of a non-reversible engine is less than that of a reversible engine. But there is no fundamental reason why it may not approach the efficiency of a reversible engine, or become equal to it.

The Second Law of Thermodynamics is a formal statement of Carnot's principle. It is variously stated, but each statement involves the same principle, which may be considered to be an experimental law.

(1) *All reversible engines, working between the same source of heat and refrigerator, have the same efficiency; i.e.*, the efficiency is independent of the working material.

(2) *A self-acting machine cannot convey heat from one body to*

another at a higher temperature. This is almost equivalent to the convention, that of two bodies, the one to which heat passes by conduction or radiation has the lower temperature.

Carnot's Function.—Taking Carnot's principle, that the efficiency of a reversible engine is independent of the working substance, we thereby eliminate from the expression for the efficiency the variables p and v, the specific pressure and the specific volume, since they are properties of the working substance. The efficiency, therefore, depends only on the temperature of the source of heat, and the difference between that temperature and the temperature of the refrigerator. This statement, in the form of an equation, is

$$\eta = \frac{AW}{Q} = \frac{Q - Q'}{Q} = F(t, t - t'), \quad . \quad . \quad . \quad (30)$$

in which Q is the heat received, and Q' that rejected by the engine, and t and t' are the temperatures of the source of heat and of the refrigerator, on a scale which we have assumed to be possible but, as yet, undetermined.

If the temperature of the refrigerator approaches near that of the source of heat, $Q - Q'$ and $t - t'$ become ΔQ and Δt, and at the limit dQ and dt, so that

$$\frac{dQ}{Q} = F(t, dt). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

But dt is itself a function of t, so that at the limit the efficiency depends on t only.

Multiplying and dividing by dt,

$$\frac{dQ}{Q} = \frac{F(t, dt)}{dt} dt,$$

and considering that the coefficient of dt is a function of t only, equation (31) becomes

$$\frac{dQ}{Q} = f(t)dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$f(t)$ is called Carnot's function, and is represented by μ; its form will depend on the thermometric scale adopted. Had any scale like that of the mercurial or air thermometer been adopted, it would now be necessary to investigate the form of the function, which would be more or less complicated.

Equation (32) is commonly written

$$\frac{dQ}{Q} = \mu dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Absolute Scale of Temperature.—A scale of temperature may now be defined by making $\mu = \frac{1}{T}$, so that

$$\frac{dQ}{Q} = \frac{dt}{T}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

the large T being used instead of t to avoid confusion with the common scales. The scale depends on the efficiency of the reversible engine, and consequently does not depend on the property of any substance. Since the reversible engine is purely ideal, the absolute scale is also ideal; but the correspondence between it and the scale of the air thermometer, with which it agrees very closely, can and has been determined by indirect methods. The scale proposed is justified by the simplicity it introduces into thermodynamic equations, and involves no inconsistency.

The method of defining temperature just stated was first proposed by Sir William Thomson; and the thermometric scale resulting is sometimes called Thomson's absolute scale. He also gives a graphical representation of the scale, which may be taken to be the equivalent of the work establishing Carnot's function.

In Fig. 16, let *ak* and *bi* be two adiabatic lines, and let the substance have its condition represented by the point *a*.

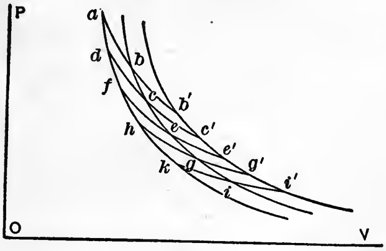

Fig. 16.

Through *a* and *d* draw isothermal lines, then the diagram *abcd* represents the cycle of a simple reversible engine. Draw the isothermal line *fe*, so that the area *dcef* shall be equal to *abcd*; then the diagram *dcef* represents the cycle of a reversible engine, doing the same amount of work per stroke as that engine whose cycle is represented by *abcd*; and the difference between the heat drawn from the source and delivered to the refrigerator, *i.e.*, the heat transformed into work, is the same. The refrigerator of the first engine might serve for the source of heat for the second.

Suppose that a series of equal areas were cut off by isothermal lines, as *fegh*, *hgik*, etc., and suppose there were a series of reversible engines corresponding; then there would be a series of sources of heat of determinate temperatures, which might be chosen to establish a thermometric scale. In order to have the scale correspond with those of ordinary thermometers, one of the sources of heat should be at the temperature of boiling water, and one at that of melting ice; and for the Centigrade scale there should be one hundred, and for the Fahrenheit scale one hundred and eighty such cycles with the appropriate sources of heat, between boiling and freezing point. To establish the absolute zero of the scale the series must be imagined to be continued till the area included between an isothermal and the two adiabatics, continued indefinitely, shall not be greater than one of the equal areas.

The absolute zero thus determined is very nearly identical with that of the air thermometer; and for all engineering purposes one may be used for the other.

Scale of Entropy.—It is convenient to take the areas of Fig. 16 to represent 778 foot-pounds when the Fahrenheit scale is used, that being the equivalent of one thermal unit. Suppose further that a second adiabatic be drawn through $b'c'e'g'i'$, making the area $bb'c'c$ equal to those of the first series; then the points a, b, b', b'', etc., if a series of adiabatics be drawn, represent the conditions of the working substance after the successive addition of t units of heat at constant temperature t. The adiabatics may be numbered ϕ, $\phi + 1$, $\phi + 2$, etc., to ϕ'.

Each one of the areas included between a pair of isothermals and a pair of adiabatics will represent the mechanical equivalent of one thermal unit, provided $abcd$ be chosen as directed above. The proof may be given in the following manner: The two areas, $dcef$ and $bb'c'c$, are equal to $abcd$ by construction. The two engines working on the cycles $abcd$ and $bb'c'c$ each draw the same quantity of heat from the source and reject the same quantity to the refrigerator; for they transform the same quantity of heat into work per stroke, and, working between the same temperatures, they have the same efficiency. The engines working on the cycles $dcef$ and $cc'e'e$ consequently receive the same amount of heat per stroke from their sources of heat; and, since they work between the same temperatures, they must transform the same amount of heat into work, or what is the same thing, the area $cc'e'e$ is equal to $dcef$; and further, all four areas are equal. In the same way the proof may be extended to all areas laid off in a similar method.

A perfect engine working between the isothermals T and T' and the adiabatics ϕ and ϕ' will change into work per stroke the heat

$$(T - T')(\phi' - \phi) = AW = Q - Q', \quad . \quad . \quad (35)$$

in which equation ϕ is the initial entropy of the working substance, and $\phi' - \phi$ is the change of entropy from one adiabatic to the other.

Suppose that T' becomes zero, and that $\phi' - \phi$ becomes $d\phi$, then

$$T d\phi = dQ, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$\therefore \quad d\phi = \frac{dQ}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Efficiency of Reversible Engines.—The efficiency of a reversible engine given by equation (29) may be written

$$\eta = \frac{AW}{Q} = \frac{Q - Q'}{Q}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

But the integration of the equation (34) between limits gives

$$\log_e \frac{Q'}{Q} = \log_e \frac{T'}{T}.$$

$$\therefore \quad \frac{Q'}{Q} = \frac{T'}{T};$$

$$\therefore \quad \eta = \frac{Q - Q'}{Q} = \frac{T - T'}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

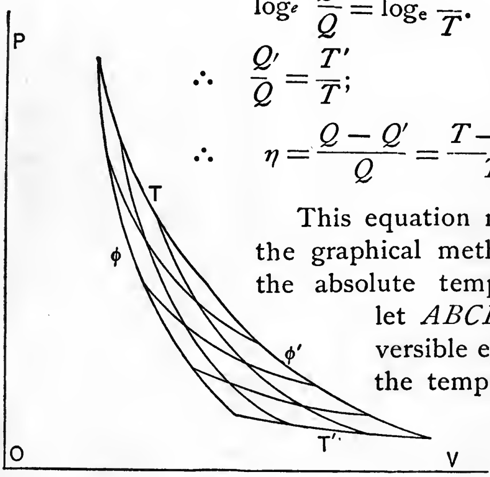

FIG. 17.

This equation may also be derived by the graphical method used in discussing the absolute temperature. In Fig. 17 let $ABCD$ be the cycle of a reversible engine working between the temperatures T and T' and the entropies ϕ and ϕ'. Let intermediate isothermal and adiabatic lines be drawn dividing the cycle into quadrilaterals each one of which represents 778 foot-pounds, or one thermal unit; then it is apparent that the number of these quadrilaterals in the cycle, and the number of thermal units changed into work, is

$$(T - T')(\phi' - \phi).$$

Similarly, the total heat absorbed during the operation represented by AB is

$$T(\phi' - \phi).$$

Consequently the efficiency is

$$\eta = \frac{AW}{Q} = \frac{(T-T')(\phi'-\phi)}{T(\phi'-\phi)} = \frac{T-T'}{T}.$$

Alternative Method.—The method of developing the idea of temperature from the second law has for an advantage the fact that the difficulty of giving an adequate physical definition is made prominent. Some writers, Zeuner, Verdet, and others, prefer, however, to avoid the difficulty by delaying the discussion of temperature; and of the general equations (5), (6), and (7), employ only the latter,

$$dQ = ndp + odv.$$

Equation (23) may be written

$$dQ = A\left\{\left(\frac{dE}{dp}\right)_v dp + \left(\frac{dE}{dv}\right)_p dv + pdv\right\};$$

which, combined with the equation above, gives

$$n = \left(\frac{dE}{dp}\right)_v \quad \text{and} \quad o = \left(\frac{dE}{dv}\right)_p + p.$$

Differentiating,

$$\left(\frac{dn}{dv}\right)_p = \left(\frac{d\left(\frac{dE}{dp}\right)_v}{dv}\right)_p = \frac{d^2E}{dp\,dv};$$

$$\left(\frac{do}{dp}\right)_v = \left(\frac{d\left(\frac{dE}{dv}\right)_p}{dp}\right)_v + 1 = \frac{d^2E}{dv\,dp} + 1.$$

But E depends on the state only, and dE is an exact differential;

$$\therefore \frac{d^2E}{dp\,dv} = \frac{d^2E}{dv\,dp};$$

and by subtraction the preceding equations give

$$\left(\frac{do}{dp}\right)_v - \left(\frac{dn}{dv}\right)_p = 1. \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Now if dQ were an exact differential, so that it could be integrated directly, as would be the case if Q depended on the initial and final states only, we should then have

$$\frac{d^2Q}{dp\,dv} = \frac{d^2Q}{dv\,dp};$$

which may be written

$$\left(\frac{d\left(\frac{dQ}{dv}\right)_p}{dp}\right)_v = \left(\frac{d\left(\frac{dQ}{dp}\right)_v}{dv}\right)_p$$

Performing this operation on equation (7) would give, in such case,

$$\left(\frac{do}{dp}\right)_v = \left(\frac{dn}{dv}\right)_p.$$

A comparison of this last equation with the true equation (40) shows that dQ is not an exact differential, and that equation (7) cannot be integrated directly.

Suppose now that $\frac{1}{S}$ is an integrating factor, such that

$$\frac{dQ}{S} = \frac{n}{S}dp + \frac{o}{S}dv$$

may be integrated directly. The adiabatic equations

$$\phi = \text{const.}, \quad \phi' = \text{const.}, \quad \phi'' = \text{const.}, \quad \text{etc.},$$

represent a series of adiabatic lines, and in like manner the equations

$$\frac{1}{S} = \text{const.}, \quad \frac{1}{S'} = \text{const.}, \quad \frac{1}{S''} = \text{const.}, \text{ etc.},$$

may represent a series of thermal lines.

In Fig. 18 let the cycle *ABCD* be composed of the adiabatic lines *AD* and *CB*, and the lines *AB* and *CD* represented by the equations

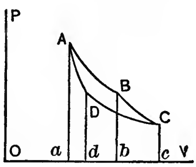

FIG. 18.

$$\frac{1}{S} = \text{const.} \quad \text{and} \quad \frac{1}{S'} = \text{const.}$$

A reversible engine, receiving the heat Q per stroke, and rejecting the heat Q', will have the efficiency

$$\eta = \frac{Q - Q'}{Q} = \frac{AW}{Q}.$$

But $\dfrac{dQ}{S}$ is an exact differential depending on the state only, so that for the entire cycle

$$\int \frac{dQ}{S} = 0.$$

Now, during the operations represented by the adiabatics *AD* and *BC* no heat is transmitted, and during the operations represented by the lines *AB* and *CD*, $\dfrac{1}{S}$ is constant; consequently the integration for the cycle gives

$$\frac{Q}{S} - \frac{Q'}{S'} = 0;$$

$$\therefore \frac{Q - Q'}{Q} = \frac{S - S'}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

That is, the efficiency of an engine working on such a cycle depends on S and S', and on nothing else.

Thus far temperature has not been brought into the discussion, and it may be defined as seems fit. Let the absolute temperature be defined by the equation

$$T = S;$$

then equation (41) becomes

$$\frac{Q - Q'}{Q} = \frac{T - T'}{T},$$

so that the absolute temperature depends on the efficiency of a reversible engine, as in the preceding discussion involving Carnot's function.

Generalization of Carnot's Principle.—Carnot's principle, or the second law of thermodynamics, is sometimes stated by making $\frac{dQ}{T}$ an exact differential, or by writing for a reversible cycle

$$\int \frac{dQ}{T} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

This is immediately evident from equation (37), since ϕ, the entropy, is a property of the substance and depends on the state only. Now in a reversible cycle the substance is in the same state at the end as at the beginning of the cycle; consequently if $d\phi$ be integrated and the limits used are those at the beginning and end of the cycle, that is, are identical, the integral will be equal to zero. At the same time $\frac{dQ}{T}$ will give zero for its integral between the same limits. This may be represented graphically as follows:

Let $ABCDEFGA$, Fig. 19, represent a cycle composed of isothermals and adiabatics; then it may be divided by CE into two simple cycles.

For the cycle $ABCFGA$ we have

$$\frac{Q - Q'}{Q} = \frac{T - T'}{T},$$

whence

$$\frac{Q'}{Q} = \frac{T'}{T}; \quad \therefore \frac{Q'}{T'} = \frac{Q}{T}; \quad \therefore \frac{Q}{T} - \frac{Q'}{T'} = 0.$$

Q being the heat absorbed at the temperature T along the path AB, and Q' the heat rejected at the temperature T' along the path FG.

In like manner for the cycle $CDEFC$ we have

$$\frac{Q''}{T''} - \frac{Q'''}{T'} = 0,$$

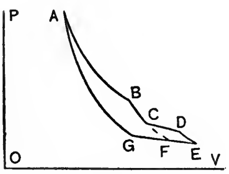

FIG. 19.

and for the entire figure

$$\frac{Q}{T} + \frac{Q''}{T''} - \frac{Q'}{T'} - \frac{Q'''}{T'} = \Sigma \frac{Q}{T} = 0.$$

Any cycle composed of isothermals and adiabatics may in like manner be divided into cycles, for each of which the principle holds, and the summation for the whole cycle will give the same result as above.

If any area be enclosed by any curve whatever, as in Fig. 20, the cycle may be approximately replaced by a complex cycle composed of isothermals and adiabatics only, and for such a cycle we shall have the same result as for the case already discussed. As the curves are drawn nearer together the approximation will be nearer, and at the limit by integration we have

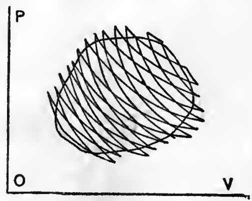

FIG. 20.

$$\int \frac{dQ}{T} = 0.$$

The two laws of thermodynamics may therefore be expressed for closed reversible cycles by the two equations,

$$\int dQ = AW, \quad \int \frac{dQ}{T} = 0.$$

CHAPTER IV.

NON-REVERSIBLE PROCESSES.

SUPPOSE that a body passes from the state represented by the point A to the state represented by B, by some process in which the pressure exerted by the substance, *i.e.*, the specific pressure, is different from the external pressure; then the area $aABb$ represents the external work done.

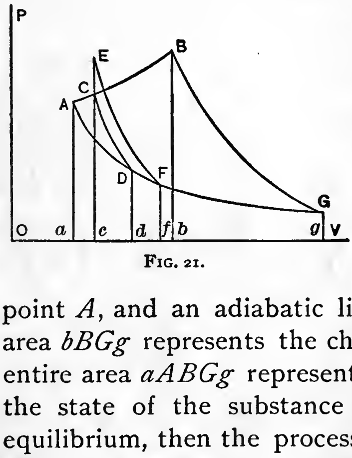

FIG. 21.

The usual method for finding the change of intrinsic energy for a reversible process is to draw an isodynamic line AG through the point A, and an adiabatic line BG through the point B; the area $bBGg$ represents the change of intrinsic energy, and the entire area $aABGg$ represents J times the heat absorbed. If the state of the substance corresponding to B is a state of equilibrium, then the process is equally applicable here. But the case is different for any intermediate point, as C, for it the external work is represented by the area $aACc$, but the change of intrinsic energy is not represented by the area $cCDd$, because the body is not supposed to be in equilibrium. If, for example, a piston moving in a cylinder is suddenly and forcibly withdrawn, the external pressure is less than the specific pressure, and the substance is thrown into commotion. Should the expansion be arrested at the point C and no heat added or abstracted, the body when it arrives at equilibrium will have its state represented by the point E, and the increase of intrinsic energy will be represented by the area $cEFf$. The area $dDCEFf$ will represent the energy due to the mechanical motion or commotion of the substance at the state represented by point C of the process.

If a non-reversible process forms a cycle so that the initial and final states are identical, then the conservation of energy will require that the equation

$$\int dQ = AW$$

must hold. On the other hand, an investigation of the value of $\frac{dQ}{T}$ will show that the integral for the entire cycle is negative. For example, suppose there is a reversible engine working between the temperature T and T' and the entropies ϕ and ϕ'; then it has been shown that the heat changed into work at each stroke is

$$(T - T')(\phi' - \phi) = Q - Q' = AW.$$

A non-reversible engine, working between the same temperatures and taking the same amount of heat per stroke, may have a less efficiency because the working substance has at times a temperature different from that of the source of heat while receiving heat, or from that of the refrigerator while yielding heat. In such case

$$AW_1 = Q_1 - Q_1' < (T - T')(\phi' - \phi);$$

Q_1 and Q_1', the heat received and the heat rejected, being different from the corresponding amounts for the reversible engine, and W_1 being less than W. In like manner, if $(\phi' - \phi)$ becomes $d\phi$, then, in general,

$$dQ_1 < (T - T')\,d\phi;$$

and should the temperature T' approach zero, then $(T - T')$ will approach T, and at the limit

$$dQ_1 < Td\phi.$$

$$\therefore \frac{dQ_1}{T} < d\phi,$$

or, dropping the subscript,

$$\int \frac{dQ}{T} < \int d\phi$$

for a non-reversible cycle,

But, since the cycle is supposed to be complete, the initial and final states of the body are identical; so that, integrating with those states as limits.

$$\int d\phi = 0.$$

Therefore, for a non-reversible cycle,

$$\int \frac{dQ}{T} = -N, \quad \dots \dots \dots \quad (43)$$

in which N may have any value; *i.e.*, it may approach zero.

CHAPTER V.

FUNDAMENTAL EQUATIONS.

Application of the First Law.—Equations (5) and (23) give

$$dQ = A(dE + dW) = c_v dt + l dv; \quad - pdv$$

or, replacing dW by pdv,

$$A(dE + pdv) = c_v dt + l dv;$$

$$\therefore \quad dE = \frac{c_v}{A} dt + \left(\frac{l}{A} - p\right) dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Now E depends on the state of the body only, and not on the method of changing from one condition to another; that is, dE is an exact differential, and consequently

$$\frac{d^2 E}{dt\, dv} = \frac{d^2 E}{dv\, dt},$$

which may be written

$$\left\{ \frac{d\left(\frac{dE}{dt}\right)_v}{dv} \right\}_t = \left\{ \frac{d\left(\frac{dE}{dv}\right)_t}{dt} \right\}_v,$$

in which the partial differential coefficients are those of the equation

$$dE = \left(\frac{dE}{dt}\right)_v dt + \left(\frac{dE}{dv}\right)_t dv.$$

Comparing with equation (44), it appears that

$$\left(\frac{dE}{dt}\right)_v = \frac{c_v}{A} \quad \text{and} \quad \left(\frac{dE}{dv}\right)_t = \left(\frac{l}{A} - p\right) dv,$$

and that consequently

$$\frac{d}{dv}\left(\frac{c_v}{A}\right)_t = \frac{d}{dt}\left(\frac{l}{A} - p\right)_v;$$

$$\therefore \quad \frac{1}{A}\left[\left(\frac{dl}{dt}\right)_v - \left(\frac{dc_v}{dv}\right)_t\right] = \left(\frac{dp}{dt}\right)_v . \quad . \quad . \quad . \quad (45)$$

The combination of equation (23), which expresses the first law of thermodynamics, with equation (5), is the application of the first law to that equation, and equation (45) is the relation between the latent heat of expansion and the specific heat at constant volume which must exist if that law be true.

In a similar manner the first law may be applied to equation (6), as follows:

$$dQ = A(dE + pdv) = c_p dt + mdp.$$

Substituting the value of dv from the equation

$$dv = \left(\frac{dv}{dp}\right)_t dp + \left(\frac{dv}{dt}\right)_p dt,$$

$$A\left[dE + p\left(\frac{dv}{dp}\right)_t dp + p\left(\frac{dv}{dt}\right)_p dt\right] = c_p dt + mdp;$$

$$\therefore \quad dE = \left[\frac{c_p}{A} - p\left(\frac{dv}{dt}\right)_p\right]dt + \left[\frac{m}{A} - p\left(\frac{dv}{dp}\right)_t\right]dp.$$

But

$$\frac{d^2E}{dt\,dp} = \frac{d^2E}{dp\,dt};$$

$$\therefore \quad \frac{d}{dp}\left[\frac{c_p}{A} - p\left(\frac{dv}{dt}\right)_p\right]_t = \frac{d}{dt}\left[\frac{m}{A} - p\left(\frac{dv}{dp}\right)_t\right]_p$$

$$\frac{1}{A}\left(\frac{dc_p}{dp}\right)_t - \left(\frac{dv}{dt}\right)_p - p\left(\frac{d\left(\frac{dv}{dt}\right)_p}{dp}\right)_t = \frac{1}{A}\left(\frac{dm}{dt}\right)_p - p\left(\frac{d\left(\frac{dv}{dp}\right)_t}{dt}\right)_p .$$

But

$$\frac{d^2v}{dp\,dt} = \frac{d^2v}{dt\,dp};$$

$$\therefore\ \frac{1}{A}\left[\left(\frac{dc_p}{dp}\right)_t - \left(\frac{dm}{dt}\right)_p\right] = \left(\frac{dv}{dt}\right)_p, \quad . \quad . \quad . \quad . \quad (46)$$

which is the relation between the thermal capacity m and the specific heat at constant pressure developed by the application of the first law to equation (6).

Again, the same law may be applied to the equation (7).

$$dQ = A(dE + pdv) = ndp + odv;$$

$$\therefore\ dE = \frac{n}{A}dp + \left(\frac{o}{A} - p\right)dv. \quad . \quad . \quad . \quad . \quad (47)$$

Since

$$\frac{d^2E}{dp\,dv} = \frac{d^2E}{dv\,dp},$$

$$\therefore\ \frac{1}{A}\left(\frac{dn}{dv}\right)_p = \frac{1}{A}\left(\frac{do}{dp}\right)_v - 1;$$

$$\therefore\ \frac{1}{A}\left[\left(\frac{do}{dp}\right)_v - \left(\frac{dn}{dv}\right)_p\right] = 1. \quad . \quad . \quad . \quad . \quad (48)$$

Application of the Second Law.—The second law of thermodynamics is expressed by making $\frac{dQ}{T}$ an exact differential. Applying this to equation (5) in the same way as was done with the first law,

$$\frac{dQ}{T} = \frac{c_v}{T}dt + \frac{l}{T}dv.$$

But

$$\frac{d^2 \int \frac{dQ}{T}}{dt\, dv} = \frac{d^2 \int \frac{dQ}{T}}{dv\, dt};$$

$$\therefore \quad \frac{d}{dv}\left(\frac{c_v}{T}\right)_t = \frac{d}{dt}\left(\frac{l}{T}\right)_v;$$

$$\frac{1}{T}\left(\frac{dc_v}{dv}\right)_t = \frac{T\left(\frac{dl}{dt}\right)_v - l}{T^2};$$

$$\therefore \quad \left(\frac{dl}{dt}\right)_v - \left(\frac{dc_v}{dv}\right)_t = \frac{l}{T}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

the relation between l and c_v developed by the application of the second law to equation (5).

Applying to equation (6), we have

$$\frac{dQ}{T} = \frac{c_p}{T}dt + \frac{m}{T}dp,$$

$$\frac{d}{dp}\left(\frac{c_p}{T}\right)_t = \frac{d}{dt}\left(\frac{m}{T}\right)_p;$$

$$\therefore \quad \frac{1}{T}\left(\frac{dc_p}{dp}\right)_t = \frac{T\left(\frac{dm}{dt}\right)_p - m}{T^2};$$

$$\therefore \quad \left(\frac{dc_p}{dp}\right)_t - \left(\frac{dm}{dt}\right)_p = -\frac{m}{T}. \quad . \quad . \quad . \quad . \quad . \quad (50)$$

Again, applying to equation (7),

$$\frac{dQ}{T} = \frac{n}{T}dp + \frac{o}{T}dv,$$

$$\frac{d}{dv}\left(\frac{n}{T}\right)_p = \frac{d}{dp}\left(\frac{o}{T}\right)_v;$$

$$\therefore \quad \frac{T\left(\frac{dn}{dv}\right)_p - n\left(\frac{dt}{dv}\right)_p}{T^2} = \frac{T\left(\frac{do}{dp}\right)_v - o\left(\frac{dt}{dp}\right)_v}{T^2};$$

$$\therefore \quad \frac{1}{T}\left[o\left(\frac{dt}{dp}\right)_v - n\left(\frac{dt}{dv}\right)_p\right] = \left(\frac{do}{dp}\right)_v - \left(\frac{dn}{dv}\right)_p. \quad . \quad . \quad (51)$$

First and Second Laws Combined.—The result of applying both the first and second laws of thermodynamics simultaneously to the fundamental equations is deduced by uniting the equations obtained by applying each separately.

For the equation (5), in terms of c_v and l, the comparison of equations (45) and (49) gives

$$\left(\frac{dp}{dt}\right)_v = \frac{1}{A}\frac{l}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

For equation (6), equations (46) and (50) give

$$\left(\frac{dv}{dt}\right)_p = -\frac{1}{A}\frac{m}{T}. \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Also for equation (7), equations (48) and (51) give

$$A = \frac{1}{T}\left[o\left(\frac{dt}{dp}\right)_v - n\left(\frac{dt}{dv}\right)_p\right]. \quad . \quad . \quad . \quad . \quad (54)$$

Or, substituting the values of n and o from equations (17) and (18),

$$c_p - c_v = AT\left(\frac{dv}{dt}\right)_p \left(\frac{dp}{dt}\right)_v \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Zeuner's Equations.—In his *Mechanische Wärmetheorie*, Zeuner employs the alternative method, so far as to deduce equation (42). Then, instead of assuming that S is the absolute temperature, or giving such a definition of temperature, he assumes that the similarity of the thermodynamic equations to certain gravitation equations indicates an essential similarity, and thereby avoids the second law of thermodynamics. Without discussing his method, there appears no reason why it might not be applied to deduce equations of the same form as those given here. He, however, gives equations of a different form which may readily be deduced from our own, and which it may be convenient to write down here. Comparing equation (47) with

$$dE = \left(\frac{dE}{dp}\right)_v dp + \left(\frac{dE}{dv}\right)_p dv,$$

it is evident that

$$\frac{n}{A} = \left(\frac{dE}{dp}\right)_v ;$$

$$\frac{o}{A} = \left(\frac{dE}{dv}\right)_p + p;$$

forms which were deduced in the alternative method of the second law of thermodynamics. These Zeuner writes:

$$X = \left(\frac{dE}{dp}\right)_v ,$$

$$Y = p + \left(\frac{dE}{dv}\right)_p .$$

Solving equation (54) for o and for n,

$$o = \frac{AT + n\left(\frac{dt}{dv}\right)_p}{\left(\frac{dt}{dp}\right)_v};$$

$$-n = \frac{AT - o\left(\frac{dt}{dp}\right)_v}{\left(\frac{dt}{dv}\right)_p}.$$

Substituting the values successively in equation (7), we have the following:

$$dQ = \frac{1}{\left(\frac{dt}{dp}\right)_v}\left[n\left(\frac{dt}{dp}\right)_v dp + n\left(\frac{dt}{dv}\right)_p dv + ATdv\right].$$

But

$$dt = \left(\frac{dt}{dp}\right)_v dp + \left(\frac{dt}{dv}\right)_p dv;$$

$$\therefore \quad dQ = \left(\frac{dp}{dt}\right)_v [ndt + ATdv];$$

$$dQ = \frac{1}{\left(\frac{dt}{dv}\right)_p}\left[o\left(\frac{dt}{dp}\right)_v dp + o\left(\frac{dt}{dv}\right)_p dv - ATdp\right];$$

$$\therefore \quad dQ = \left(\frac{dv}{dt}\right)_p [odt - ATdp].$$

Zeuner deduces, for his fundamental equations,

$$dQ = A(Xdp + Ydv);$$

$$dQ = \frac{A}{\left(\frac{dt}{dp}\right)}\left[Xdt + \left(\frac{1}{\alpha} + t\right)dv\right];$$

$$dQ = \frac{A}{\left(\frac{dt}{dv}\right)}\left[Ydt - \left(\frac{1}{\alpha} + t\right)dp\right];$$

which may readily be deduced from the equations above.

CHAPTER VI.

PERFECT GASES.

THE characteristic equation of a gas is the algebraic expression of the combined laws of Boyle and Gay Lussac.

$$pv = p_0 v_0 (1 + \alpha t) = p_0 v_0 \alpha \left(\frac{1}{\alpha} + t \right); \quad . \quad . \quad . \quad . \quad (56)$$

$$pv = RT; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

p_0 and v_0 being the specific pressure and specific volume at freezing, and α their coefficient of dilatation at constant pressure.

Coefficient of Dilatation.—Regnault* gives for the dilatation from freezing to boiling point, at Paris, the results:

Hydrogen, ...	0.3667
Atmospheric air, ...	0.3665
Nitrogen, ...	0.3668
Carbonic acid, ...	0.3688

In works on thermodynamics it has been commonly assumed that the coefficient of dilatation for air may be used for all gases, and at all temperatures and pressures, and that, consequently, on the Centigrade scale, α is 0.003665, or very nearly $\frac{1}{273}$. Professor Holman † suggests that as the pressure approaches zero, the coefficient of dilatation of all gases approaches

$$\alpha = \frac{1}{273.7},$$

* Mémoires de l'Institut de France, tome xxi.

† Lecture Notes on Heat, Mass. Inst. Technology.

which agrees with thermodynamic investigations relating to the absolute zero of temperature. On the Fahrenheit scale,

$$\alpha = \frac{1}{492.7}.$$

Specific Volume.—This quantity is determined from the density, which is given for several gases in the following table, at freezing point and at atmospheric pressure, as determined by Regnault.

Weight in grams of one liter:

Atmospheric air, ...	1.293 187
Nitrogen, ...	1.256 167
Oxygen, ...	1.429 802
Hydrogen, ...	0.089 578
Carbonic acid, ...	1.977 414

The specific volumes are as follows. Volumes in cubic meters of one kilogram, at Paris, latitude 48° 50′ 14″; elevation, 60 meters:

Atmospheric air, ...	0.773 2834
Nitrogen, ...	0.796 0724
Oxygen, ...	0.699 3974
Hydrogen, ...	11.163 46
Carbonic acid, ...	0.505 7109

The specific volumes, reduced to the latitude of 45° at sea level, are given in the next table.

Volumes in cubic meters of one kilogram, at 45° of latitude:

Atmospheric air, ...	0.773 5327
Nitrogen, ...	0.796 3291
Oxygen, ...	0.699 6231
Hydrogen, ...	11.167 05
Carbonic acid, ...	0.505 8741

The reduction for the change of the acceleration due to gravity is made by the equation *

$$g = 980.6056 - 2.5028 \cos 2\lambda - 0.000003h, \quad . \quad (58)$$

in which g is the acceleration in centimeters, λ is the latitude, and h is the elevation above the sea in centimeters. One kilogram † is equivalent to 2.20462125 pounds; and one meter, as determined by Professor Rogers, ‡ is equivalent to 39.3702 inches, from which the specific volume in English units may be determined.

Volumes in cubic feet of one pound at 45° of latitude:

Atmospheric air,	12.3909
Nitrogen,	12.7561
Oxygen,	11.2070
Hydrogen,	178.881
Carbonic acid,	8.10324

Specific Pressure.—The weight of one liter of mercury, determined by Regnault, is 13.5959 kilograms; consequently the pressure of one atmosphere, or 760 mm., of mercury on one square meter is

10333 kilograms.

Using the values given above for the kilogram and meter we have, for the English system,

14.6967 pounds per square inch,
2116.32 pounds per square foot.

Value of *R*.—Taking the value of p_0, v_0, and α, at freezing point and under the pressure of 760 mm. of mercury, we have for air,

* Everett's Units and Phys. Const.

† Miller, Phil. Transactions, cxlvi, 1856.

‡ Pro. Am. Acad. of Arts and Sci., 1882–83; also additional observations.

$$\text{French units, } R = \frac{10333 \times 0.77353}{273.7} = 29.20. \quad . \quad (59)$$

$$\text{English units, } R = \frac{2116.3 \times 12.391}{492.7} = 53.22. \quad . \quad (60)$$

The value of R for other gases may be determined in a like manner.

Specific Heat at Constant Pressure.—The specific heat for true gases is very nearly constant, and may be considered to be so for thermodynamic equations. Regnault gives for the mean values for specific heat at constant pressure the following results:

Atmospheric air, ...	0.2375
Nitrogen, ...	0.2438
Oxygen, ...	0.2175
Hydrogen, ...	3.409

Application of the Two Laws of Thermodynamics.—The result of applying the two laws of thermodynamics to equation (7) is given by equation (55),

$$c_p - c_v = AT\left(\frac{dv}{dt}\right)_p \cdot \left(\frac{dp}{dt}\right)_v .$$

Differentiating the characteristic equation (57) for a gas,

$$\left(\frac{dv}{dt}\right)_p = \frac{R}{p}, \qquad \left(\frac{dp}{dt}\right)_v = \frac{R}{v},$$

which, substituted in equation (55), give

$$c_p - c_v = AR. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Specific Heat at Constant Volume.—The specific heat at constant volume has not been determined directly. It is evident from equation (61) that the assumption of the characteristic equation (57), and the assumption that c_p is constant, make c_v also constant. It will be seen subsequently that the ratio of the specific heats may be determined experimentally. The ratio is commonly taken to be

$$\frac{c_p}{c_v} = \kappa = 1.405.$$

Thermal Capacities.—Substituting the values of the partial differential coefficients as deduced from equation (57), in equations (11), (15), (17), and (18), we have, for the values of the thermal capacities for gases,

$$l = \frac{p}{R}(c_p - c_v) = \frac{T}{v}(c_p - c_v); \quad \ldots \quad (62)$$

$$m = -\frac{v}{R}(c_p - c_v) = -\frac{T}{p}(c_p - c_v); \quad \ldots \quad (63)$$

$$n = \frac{v}{R}c_v = \frac{T}{p}c_v; \quad \ldots \quad (64)$$

$$o = \frac{p}{R}c_p = \frac{T}{v}c_p. \quad \ldots \quad (65)$$

Combining these four equations with equation (57) gives

$$l = Ap; \quad \ldots \quad (66)$$

$$m = -Av; \quad \ldots \quad (67)$$

$$n = Av\frac{c_v}{c_p - c_v}; \quad \ldots \quad (68)$$

$$o = Ap\frac{c_p}{c_p - c_v}. \quad \ldots \quad (69)$$

pv = RT

General Equations.—The values of the thermal capacities given by equations (62), (63), (64), and (65), substituted in equations (5), (6), and (7), give

$$dQ = c_v dt + (c_p - c_v)\frac{T}{v}\,dv\,; \quad . \quad . \quad . \quad . \quad (70)$$

$$dQ = c_p dt + (c_v - c_p)\frac{T}{p}\,dp\,; \quad . \quad . \quad . \quad . \quad (71)$$

$$dQ = c_v\frac{T}{p}\,dp + c_p\frac{T}{v}\,dv. \quad . \quad . \quad . \quad . \quad . \quad (72)$$

Just as the first law of thermodynamics was applied to the general equations (5), (6), and (7), by equating them to equation (23), so the first law may be applied to equations (70), (71), and (72) in the same manner. For example, the application of the first law to equation (70) gives

$$dQ = A\,(dE + pdv) = c_v dt + (c_p - c_v)\frac{T}{v}dv\,;$$

$$dE = \frac{c_v}{A}\,dt + \left[\frac{1}{A}(c_p - c_v)\frac{T}{v} - p\right]dv\,;$$

$$\frac{d}{dv}\left(\frac{c_v}{A}\right)_t = \frac{d}{dt}\left[\frac{1}{A}(c_p - c_v)\,\frac{T}{v} - p\right]_v;$$

$$0 = (c_p - c_v)\frac{1}{v} - A\left(\frac{dp}{dt}\right)_v;$$

$$c_p - c_v = Av\left(\frac{dp}{dt}\right)_v;$$

$$c_p - c_v = AR.$$

The application of the same law to each of the other two equations, (71) and (72), gives the same result. The fact that the result is the same as that resulting from the application of both laws of thermodynamics indicates that the characteristic equation for gases implies the second law. An attempt to apply the second law to equations (70), (71), and (72) in the usual manner gives in each case zero equal to zero, which reaffirms the preceding statements in another form.

Isothermal Lines.—The equation to the isothermal line for gas is obtained by making T constant in the general equation, which gives the equation representing Boyle's law:

$$pv = p_0 v_0 = \text{const.}, \quad . \quad . \quad . \quad . \quad . \quad (73)$$

which is the equation to a rectangular hyperbola.

The heat absorbed during an isothermal change is obtained by integrating one of the general equations (70), (71), or (72) with T assumed to be constant, so that dt becomes zero.

Equation (70) gives

$$Q = (c_p - c_v)\, T \int_{v_0}^{v_1} \frac{dv}{v};$$

$$\therefore \; Q = (c_p - c_v)\, T \log_e \frac{v_1}{v_0}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

or, substituting for the value of $c_p - c_v$ from equation (68),

$$Q = ART \log_e \frac{v_1}{v_0} = Ap_0 v_0 \log_e \frac{v_1}{v_0}. \quad . \quad . \quad (75)$$

In like manner equation (71) gives

$$Q = (c_v - c_p)\, T \log_e \frac{p_1}{p_0}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

$$\therefore \; Q = ART \log_e \frac{p_0}{p_1} = Ap_0 v_0 \log_e \frac{p_0}{p_1}; \quad . \quad (77)$$

which equations can be deduced from the preceding by substituting for $\frac{v_1}{v_0}$ from the characteristic equation.

To find the work done, the equation

$$W = \int_{v_0}^{v_1} pdv$$

may be used after substituting for p from the characteristic equation, whence

$$W = p_0 v_0 \int_{v_0}^{v_1} \frac{dv}{v} = p_0 v_0 \log_e \frac{v_1}{v_0}. \quad . \quad . \quad . \quad (78)$$

A comparison of equations (75) and (78) shows that all the heat absorbed is changed into external work; consequently the intrinsic energy remains unchanged during the operation.

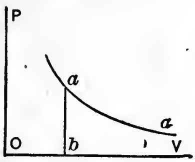

FIG. 22.

The area contained between the axis OV, Fig. 22, the ordinate ab, and the isothermal line $a\alpha$ extended without limit, is

$$W = p_0 v_0 \log_e \frac{\infty}{v_0} = \infty.$$

This may also be seen from the consideration that if heat be continually applied, and all changed into work, there will be a limitless supply of work.

Isoenergic or Isodynamic Lines.—In the discussion of isothermal lines it appears that all the heat received is changed into work; consequently the intrinsic energy remains constant during an isothermal change. From which it is apparent that the isoenergic line is coincident with the isothermal line.

From the importance of the subject, an independent proof will be given that E is a function of t only.

$$dQ = A(dE + pdv);$$

$$dQ = A\left(\frac{dE}{dt}\right)_v dt + A\left[\left(\frac{dE}{dv}\right)_t + p\right]dv.$$

Comparing with equation (71), it is apparent that

$$A\left[\left(\frac{dE}{dv}\right)_t + p\right] = \left(c_p - c_v\right)\frac{T}{v} = AR\frac{T}{v}.$$

$$\therefore \left(\frac{dE}{dv}\right)_t = 0.$$

Again,

$$dQ = A\left[\left(\frac{dE}{dt}\right)_p + p\left(\frac{dv}{dt}\right)_p\right]dt + A\left[\left(\frac{dE}{dp}\right)_t + p\left(\frac{dv}{dp}\right)_t\right]dp.$$

Comparing with equation (72), it is apparent that

$$A\left[\left(\frac{dE}{dp}\right)_t + p\left(\frac{dv}{dp}\right)_t\right] = \left(c_v - c_p\right)\frac{T}{p}.$$

$$A\left[\left(\frac{dE}{dp}\right)_t - \frac{RT}{p}\right] = -A\frac{RT}{p}.$$

$$\therefore \left(\frac{dE}{dp}\right)_t = 0.$$

The importance of the proposition that E is a function of t only, will be apparent in connection with the comparison of the scale of the air thermometer with the thermodynamic scale.

Adiabatic Lines.—During an adiabatic change, for example, the expansion of a gas in a non-conducting cylinder, heat is not communicated to, nor abstracted from, the gas; consequently dQ in equations (70), (71), and (72) becomes zero.

From equation (72),

$$0 = dQ = \frac{c_v}{R}vdp + \frac{c_p}{R}pdv;$$

$$\therefore \frac{c_p}{c_v}\frac{dv}{v} = -\frac{dp}{p};$$

$$\therefore \log_e\left(\frac{v}{v_0}\right)^{\frac{c_p}{c_v}} = \log_e\left(\frac{p_0}{p}\right).$$

The ratio $\frac{c_p}{c_v}$ of the specific heats may be represented by κ, and the above equation may be written

$$\left(\frac{v}{v_0}\right)^\kappa = \frac{p_0}{p}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

$$\therefore \; v^\kappa p = v_0{}^\kappa p_0 = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad (80)$$

From equations (70) and (71),

$$v^{\kappa - 1} T = v_0{}^{\kappa - 1} T_0 = \text{const.}, \quad . \quad . \quad . \quad . \quad . \quad (81)$$

?

$Tp^{\frac{1-\kappa}{\kappa}} = T_0 p_0^{\frac{1-\kappa}{\kappa}}$

$$Tp^{\frac{\kappa - 1}{\kappa}} = T_0 p_0^{\frac{\kappa}{\kappa - 1}} = \text{const.}; \quad . \quad . \quad . \quad . \quad (82)$$

or these last two equations may be deduced from equation (80) by substituting for p or for v from the characteristic equation.

To find the external work, the equation

$$W = \int p dv$$

may be used after substituting for p from equation (79) or (80).

$$W = \int_{v_1}^{v_2} p dv = v_1{}^\kappa p_1 \int_{v_1}^{v_2} \frac{dv}{v^\kappa} = -\frac{p_1 v_1{}^\kappa}{\kappa - 1}\left(\frac{1}{v_2{}^{\kappa - 1}} - \frac{1}{v_1{}^{\kappa - 1}}\right);$$

$$\therefore \; W = \frac{p_1 v_1}{\kappa - 1}\left\{1 - \left(\frac{v_1}{v_2}\right)^{\kappa - 1}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

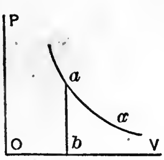

FIG. 23.

In Fig. 23 the area between the axis OV, the ordinate ba, and the adiabatic line $a\alpha$ extended without limit becomes

$$W_1 = \frac{p_1 v_1}{\kappa - 1},$$

and not infinity, as is the case with the isothermal line.

Intrinsic Energy.—Since the external work is done at the expense of the intrinsic energy, the work obtainable by an infinite expansion cannot be greater than the intrinsic energy. If it be admitted that when the volume becomes infinity and the pressure and temperature become zero, the intrinsic energy becomes zero, then we shall have

$$E_1 = W_1 = \frac{p_1 v_1}{\kappa - 1}, \quad \ldots \ldots \quad (84)$$

which gives a method of calculating the intrinsic energy.

Though such a method of calculating the intrinsic energy is subject to an error, from the fact that at zero temperature and pressure the intrinsic energy is not zero, the error is constant for all values of intrinsic energy, and disappears when differences are taken.

Entropy.—From equation (71) we have

$$\frac{dQ}{T} = c_p \frac{dt}{T} + (c_v - c_p) \frac{dp}{p},$$

which, for a reversible cycle, is equal to the differential of the entropy. Consequently we have, on integrating between limits,

$$\phi - \phi_0 = c_p \log_e \frac{T}{T_0} + (c_v - c_p) \log_e \frac{p}{p_0} \cdot \quad \ldots \quad (85)$$

This gives a method of calculating the increase of entropy above that at a certain state; for example, above that at freezing-point and under normal atmospheric pressure.

Similar expressions may be deduced from equations (70) and (72).

Carnot's Cycle.—It has already been shown that the characteristic equation for a gas implies the second law of thermodynamics, but it can be shown in another and more direct way by aid of Carnot's cycle. The demonstration is frequently given in elementary works, and may be useful as an exercise.

The equation to the isothermal line for a gas was deduced from the characteristic equation by making T a constant, and the equation to the adiabatic line was deduced from equation (72) by making dQ equal to zero; in all of which no direct reference was made to the second law of thermodynamics, or the efficiency of a reversible engine. Let us now consider the cycle of Carnot's engine for a perfect gas.

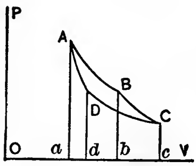

FIG. 24.

We shall have, as in the general case, that the area of $ABCDA$ (Fig. 24) represents the work done; that is,

$$W = W_{ab} + W_{bc} - W_{cd} - W_{da}. \quad \ldots \quad (86)$$

Since AB and DC are isothermal lines,

$$W_{ab} = p_a v_a \log_e \frac{v_b}{v_a}, \quad \ldots \quad (87)$$

$$W_{dc} = p_d v_d \log_e \frac{v_c}{v_d}; \quad \ldots \quad (88)$$

and since BC and DA are adiabatic lines

$$W_{bc} = \frac{p_b v_b}{\kappa - 1} \left\{ 1 - \left(\frac{v_b}{v_c}\right)^{\kappa - 1} \right\}.$$

$$W_{da} = \frac{p_a v_a}{\kappa - 1} \left\{ 1 - \left(\frac{v_a}{v_d}\right)^{\kappa - 1} \right\}.$$

The last two quantities of work are equal, for we have

$$p_b v_b = p_a v_a,$$

because the points a and b are on the same isothermal line; and since a and d are on one adiabatic line, and b and c are on another, we have

$$\left(\frac{v_a}{v_d}\right)^{\kappa - 1} = \frac{T_2}{T_1}, \qquad \left(\frac{v_b}{v_c}\right)^{\kappa - 1} = \frac{T_2}{T_1};$$

T_1 being the absolute temperature of the isothermal line AB, and T_2 of DC.

The last two equations give also

$$\frac{v_a}{v_d} = \frac{v_b}{v_c}. \qquad \therefore \frac{v_a}{v_b} = \frac{v_d}{v_c},$$

and the characteristic equation gives

$$p_a v_a = RT_1, \qquad p_d v_d = RT_2;$$

whence equations (87) and (88) reduce to

$$W_{ab} = RT_1 \log_e \frac{v_b}{v_a},$$

$$W_{dc} = RT_2 \log_e \frac{v_b}{v_a};$$

and these values inserted in equation (86) give

$$W = R(T_1 - T_2) \log_e \frac{v_b}{v_a},$$

To calculate the heat received by the working substance in passing along the isothermal from A to B, we may employ equation (65) with the condition that T shall be constant, and dt shall be zero. Integration between limits gives

$$Q_1 = (c_p - c_v)\, T_1 \log_e \frac{v_b}{v_a}.$$

which, by aid of equation (68), may be reduced to

$$Q_1 = ART_1 \log_e \frac{v_b}{v_a}.$$

Now the efficiency of any engine is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{AW}{Q_1},$$

from which the efficiency of an air-engine, working on Carnot's cycle, is

$$\eta = \frac{AR(T_1 - T_2) \log_e \frac{v_b}{v_a}}{ART_1 \log_e \frac{v_b}{v_a}} = \frac{T_1 - T_2}{T_1}. \quad . \quad . \quad . \quad (89)$$

Comparison of the Air Thermometer with the Absolute Scale.—In connection with the isodynamic line it was shown that the intrinsic energy is a function of the temperature only. This conclusion is deduced from the characteristic equation on the assumption that the scale of the air thermometer coincides with the thermodynamic scale, and affords a delicate method of testing the truth of the characteristic equation, and of comparing the two scales.

The most complete experiments for this purpose were made by Joule and Sir William Thomson, who forced air slowly through a porous plug in a tube in such a manner that no heat was transmitted to or from the air during the process. Also the velocity both before and beyond the plug were so small that the work due to the change of velocity could be disregarded. All the work that would be developed in free expansion from the higher to the lower pressure was used in overcoming the resistance of friction in the plug and so converted into heat, and as none of this heat escaped it was retained by the air itself, the plug remaining at a constant temperature. It therefore appears that the intrinsic energy remained the same, and that a change of temperature indicated a deviation from the assumptions of the theory of perfect gases. The change, though slight, was measurable, and has been used to establish the comparison between the two thermal scales under discussion.

In the discussion of results given by Joule and Thomson * in 1854 they give for the absolute temperature of freezing-point, 273°.7 C. As the result of later † experiments they state that the cooling for a difference of pressure of 100 inches of mercury is represented, on the Centigrade scale, by

$$0^\circ.92\left(\frac{273.7}{T}\right)^2.$$

The following table shows the agreement between this statement and the results of experiment:

FLOW OF AIR THROUGH POROUS PLUG.

Temperature.	Cooling Effect—	
	By Experiment.	By Calculation.
0°	0.92	0.92
7.1	0.88	0.87
39.5	0.75	0.70
92.8	0.51	0.51

From the work of these experiments Rowland ‡ deduced the following comparison of the air thermometer with constant volume, with the absolute thermodynamic scale of temperature.

* Philosophical Transactions, vol. 144, p. 349.
† Ibid., vol. 152, p. 579.
‡ Proceedings of the American Academy, vol. xv. (N. S. viii.) p. 75; 1879.

REDUCTION OF THE AIR THERMOMETER TO THE ABSOLUTE SCALE.

(*Centigrade.*)

Temperature above Freezing.		Correction to Air Thermometer.
Air Thermometer.	Absolute Scale.	
0°	°	°
10	9.9972	− 0.0028
20	19.9952	− 0.0048
30	29.9939	− 0.0061
40	39.9933	− 0.0067
50	49.9932	− 0.0068
60	59.9937	− 0.0063
70	69.9946	− 0.0054
80	79.9956	− 0.0044
90	89.9978	− 0.0022
100	100.000	0.
200	200.037	+ 0.037
300	300.092	+ 0.092
400	400.157	+ 0.157
500	500.228	+ 0.228

Velocity of Sound.—Sound is transmitted through the air in spherical waves, but at a distance from the source of sound the waves are sensibly plane waves, and the progress of the wave is the same as that of a plane wave in a straight tube of uniform section. Let Fig. 25 represent a tube one square meter in section, in which a wave moves with a linear velocity u_0 meters per second; that is, a point at a given phase of the wave, for example, C at the greatest condensation, moves at that velocity.

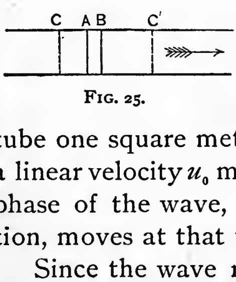

FIG. 25.

Since the wave moves with the velocity u_0, the volume of air disturbed in a unit of time is u_0 cubic meters. If the specific volume in the undisturbed state is v_0, then the weight of air disturbed in a second is

$$w = mg = \frac{u_0}{v_0};$$

m being the mass of air which has the weight w.

Imagine two planes A and B at a small distance apart, which also move with the velocity u_0, so that they remain at the same phase of the wave. Let the absolute velocities of the air at these planes be u_1 and u_2; then the velocities of the air through the planes, that is, the velocities relatively to the planes, is, for A, $u_0 - u_1$, and for B, $u_0 - u_2$. With v_1 and v_2 for the specific volumes at these planes, the weights that pass through the planes A and B per second are

$$\frac{u_0 - u_1}{v_1} \quad \text{and} \quad \frac{u_0 - u_2}{v_2}.$$

Since the phase of that portion of the wave between A and B is constant, the weight of the air between them is also constant, and as much air enters per second as leaves during that time. Again: as, on the whole, the air is not transmitted, but only compressed and rarefied, the whole air disturbed per second must pass through the space between the planes. Therefore,

$$\frac{u_0 - u_1}{v_1} = \frac{u_0 - u_2}{v_2} = \frac{u_0}{v_0} = mg;$$

$$\therefore \; u_1 = u_0 - mv_1g;$$

$$u_2 = u_0 - mv_2g;$$

$$u_1 - u_2 = mg(v_2 - v_1).$$

Now, as the mass m enters the space between the planes with the absolute velocity u_1, and an equal mass leaves with the velocity u_2, consequently there is a change of momentum

$$m(u_1 - u_2);$$

and since this cannot come from the mutual action of the particles, it must come from the difference of pressures at A and B; thus,

$$p_1 - p_2 = m(u_1 - u_2) = m^2g(v_2 - v_1).$$

As the planes A and B approach each other, p_1 and p_2, v_1 and v_2 approach in value, and at the limit

$$dp = - m^2 g dv,$$

$$\frac{dp}{dv} = - m^2 g = - \frac{1}{g} \frac{u_0^2}{v_0^2};$$

the last reduction being obtained by substituting for the value of m from the preceding work. Solving for u_0,

$$u_0^2 = - g v_0^2 \frac{dp}{dv}.$$

The vibrations are so rapid that the changes of state may be assumed to be adiabatic; consequently equation (72) gives

$$0 = dQ = \frac{c_v}{R} vdp + \frac{c_p}{R} pdv;$$

$$\therefore \frac{dp}{dv} = - \frac{c_p}{c_v} \cdot \frac{p}{v} = - \kappa \frac{p}{v}.$$

The planes A and B may be taken at any phase of the wave; for example, at the phase where the pressure and volume are normal, in which case

$$\frac{dp}{dv} = - \kappa \frac{p_0}{v_0}.$$

Substituting in the equation for u_0, we have

$$u_0^2 = g \kappa v_0 p_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

The equation is commonly given in terms of the density, γ, as follows:

$$u_0 = \sqrt{\kappa g \frac{p}{\gamma}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

Ratio of the Specific Heats.—The velocity of sound from direct experiment was found by Moll and Van Beek to be 332.26 meters per second; by Regnault to be 330.70 meters per second. Kayser found from Kundt's dust figures the wave length corresponding to a certain tone, and therefrom deduced the velocity of sound, and gives for the velocity 332.50 meters per second. The true value must be nearly 332 meters per second. Solving equation (94) for κ, and intersecting the known values of $p_0 v_0$ and g for Paris,

$$\kappa = \frac{u_0^2}{g v_0 p_0} = \frac{\overline{332}^2}{9.8092 \times 0.77328 \times 10333}; \quad = 1.332$$

$$\kappa = 1.4063.$$

OK. K.R.

Direct experiments to determine κ may be made as follows: Suppose that a vessel is filled with air at a pressure p_1, while the pressure of the atmosphere is p_0. Let a communication be opened with the atmosphere sufficient to suddenly equalize the pressure; then let it be closed, and let the pressure p_2 be observed after the air has again attained the temperature of the atmosphere. If the first operation is sufficiently rapid it may be assumed to be adiabatic, and we may use equation (79), from which

$$\kappa = \frac{\log p_1 - \log p_0}{\log v_0 - \log v_1}. \quad . \quad . \quad . \quad . \quad . \quad (92)$$

The second operation is at constant volume; consequently the specific volume is the same at the final state as after the adiabatic expansion of the first operation. But the initial and final temperatures are the same; consequently

$$v_1 p_1 = v_0 p_2;$$

$$\therefore \log v_0 - \log v_1 = \log p_1 - \log p_2,$$

which, substituted in equation (92), gives

$$\kappa = \frac{\log p_1 - \log p_0}{\log p_1 - \log p_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

The same experiment may be made by rarefying the air in the vessel, in which case the sign of the second term changes.

Röngten * employed this method, using a vessel containing 70 liters, and measuring the pressure with a gauge on the same principle as the aneroid barometer. Instead of assuming the pressure p_0 at the instant of closing the stop-cock to be that of the atmosphere, he measured it with the same instrument. A mean of ten experiments on air gave

$$\kappa = 1.4053.$$

Again, from equation (68) we have

$$\kappa = \frac{c_p}{c_v} = \frac{1}{1 - \frac{AR}{c_p}};$$

$$\kappa = \frac{1}{1 - \frac{10333 \times 0.77353}{426.9 \times 273.7 \times 0.2375}};$$

$$\kappa = 1.4046.$$

The value of κ already given on page 49 will be used throughout our work, *i.e.*,

$$\kappa = 1.405.$$

Solution of Problems.—The greater part of engineering problems involving gases may be solved by the aid of the characteristic equation

$$pv = RT,$$

* Poggendorff's Annalen, vol. cxlviii. p. 580.

or the equivalent equation,

$$\frac{pv}{T} = \frac{p_0 v_0}{T_0}.$$

In the first of these two equations the specific pressure and volume to be used for English measures are pounds per square foot, and the volume, in cubic feet, of one pound.

For example, let it be required to find the volume of 3 pounds of air at 60 pounds gauge pressure and at 100° F. Assuming a barometric pressure of 14.7 pounds per square inch,

$$v = \frac{53.22\,(460.7 + 100)}{(14.7 + 60)\,144} = 2.773 \text{ cubic feet}$$

is the volume of 1 pound of air under the given conditions, and 3 pounds will have a volume

$$3 \times 2.773 = 8.319 \text{ cubic feet.}$$

The second equation has the advantage that any units may be used, and that it need not be restricted to one unit of weight.

For example, let the volume of a given weight of gas, at 100° C. and at atmospheric pressure, be 2 cubic yards; required the volume at 200° C. and at 10 atmospheres. Here

$$\frac{10v}{473.7} = \frac{1 \times 2}{373.7};$$

$$v = \frac{473.7 \times 2}{10 \times 373.7} = 0.2535 \text{ cubic yards.}$$

EXAMPLES.

1. Find the weight of 4 cubic meters of hydrogen at 30° C. and under the pressure of 800 mm. of mercury. 339 kg

2. Find the volume of 3 pounds of nitrogen at a pressure of 45 pounds to the square inch by the gauge, and at 80° F. 10.34

3. Find the temperature at which one kilogram of air will occupy one cubic meter when at a pressure of 20,000 kilograms per square meter. 411.8° C

4. Find the pressure at which 2 pounds of carbonic acid at freezing-point of water will occupy 3 cubic feet.

5. A gas-receiver having the volume of 3 cubic feet contains half a pound of oxygen at 70° F. What is the pressure?

6. A spherical balloon 20 feet in diameter is to be inflated with hydrogen at 60° F. when the barometer stands at 30.2 inches, so that gas may not be lost on account of expansion when it has risen till the barometer stands at 19.6 inches, and the temperature falls to 40° F. How many pounds and how many cubic feet are to be run in?

7. A gas-receiver holds 14 ounces of nitrogen at 20° C. and under a pressure of 29.6 inches of mercury. How many will it hold at 32° F. and at the normal pressure of 760 mm.?

8. Two cubic feet of air expand at 50° F., from a pressure of 80 pounds to a pressure of 60 pounds, by the gauge. What is the external work?

9. What would have been the external work had the air expanded adiabatically?

10. Find the external work of 2 pounds of air which expands adiabatically until it doubles its volume; the initial pressure being 100 pounds absolute, and the initial temperature 100° F.

11. Find the external work of one kilogram of hydrogen which, starting with the pressure of four atmospheres and with the temperature of 500° C., expands till the temperature becomes 30° C.

12. Find the intrinsic energy of one pound of the several

gases for which the proper data are given, under the standard pressure of one atmosphere and at freezing-point of water.

13. A pound of air has the volume 6 cubic feet under the pressure of 30 pounds absolute to the square inch. Find the intrinsic energy.

14. In example 13, find the increase of entropy above that at atmospheric pressure and at freezing-point.

15. A kilogram of oxygen at the pressure of 6 atmospheres and at 100° C. expands isothermally till it doubles its volume. Find the change of entropy.

16. Suppose a hot-air engine, in which the maximum pressure is 100 pounds absolute, and the maximum temperature is 600° F., to work on a Carnot's cycle. Let the initial volume be 2 cubic feet, let the volume after isothermal expansion be 5 cubic feet, and the volume after adiabatic expansion be 8 cubic feet. Find the external work of one cycle; also the horse-power if the engine is double-acting and makes 30 revolutions per minute.

CHAPTER VII.

SATURATED VAPOR.

OUR knowledge of the properties of saturated vapors is derived mainly from the experiments made by Regnault.* In almost all cases the results of the experiments are stated in form of empirical equations designed to be used for calculating tables; and since such tables are of great value and importance in steam-engineering, it appears advisable to give at some length the data on which those equations are based, so as to show the limits of their application and their degree of accuracy. In some cases the constants of the equations have been recalculated; notably in the case of the pressure of saturated steam, which appeared to be necessary on account of the diversity of the values given by different authors for the constants in the empirical equations used for calculating tables, and on account of the discrepancy between the steam tables in common use, especially on the English system of units.

Pressure of Saturated Vapor.—Regnault's experiments on the temperature of saturated vapor consisted essentially in taking the temperature of the boiling-point of the vapor under varying pressures of the atmosphere, the apparatus being so arranged that the pressure could be varied from a small fraction of an atmosphere to more than twenty atmospheres. The temperature was taken with mercurial thermometers, and the pressures were measured by a mercury column, and, after the necessary corrections were applied and temperatures were reduced to the air thermometer, Regnault selected the results he deemed most trustworthy, and plotted a series of points, and

* Mémoires de l'Institut de France, etc., tome xxvi.

then drew a smooth curve to represent the whole series of experiments.

He then selected points on the experimental curve at regular intervals, and with these points as data he calculated the constants of empirical formulæ for use in calculating the tabular values. The formula selected was of the form

$$\log p = a + b\alpha^n + c\beta^n, \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

in which p is the pressure, and n is the temperature minus the constant temperature t_0 of the lowest limit of the range of temperature to which the formula applies; *i.e.*,

$$n = t - t_0.$$

Let the points through which the curve represented by the equation is to pass be (p_0, t_0), (p_1, t_1), (p_2, t_2), (p_3, t_3), and (p_4, t_4), so chosen that

$$t_1 - t_0 = t_2 - t_1 = t_3 - t_2 = t_4 - t_3.$$

$$\therefore t_2 - t_0 = 2(t_1 - t_0), \quad (t_3 - t_0) = 3(t_1 - t_0), \quad (t_4 - t_0) = 4(t_1 - t_0).$$

Substituting the five known values of p and t in equation (94),

$$\left.\begin{aligned}
\log p_0 &= a + b + c; \\
\log p_1 &= a + b\alpha^{t_1 - t_0} + c\beta^{t_1 - t_0}; \\
\log p_2 &= a + b\alpha^{2(t_1 - t_0)} + c\beta^{2(t_1 - t_0)}; \\
\log p_3 &= a + b\alpha^{3(t_1 - t_0)} + c\beta^{3(t_1 - t_0)}; \\
\log p_4 &= a + b\alpha^{4(t_1 - t_0)} + c\beta^{4(t_1 - t_0)}.
\end{aligned}\right\} \quad . \quad . \quad . \quad (95)$$

Now subtract each equation, member for member, from the one below it, and for convenience let

$$\log p_1 - \log p_0 = y_0, \quad \text{etc.,} \quad \alpha^{t_1 - t_0} = m, \quad \beta^{t_1 - t_0} = n.$$

$$\left.\begin{aligned}
\therefore y_0 &= (m - 1)b + (n - 1)c; \\
y_1 &= (m^2 - m)b + (n^2 - n)c; \\
y_2 &= (m^3 - m^2)b + (n^3 - n^2)c; \\
y_3 &= (m^4 - m^3)b + (n^4 - n^3)c.
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (96)$$

Solving the several equations for c and equating the values,

$$\frac{y_0 - (m - 1)b}{n - 1} = \frac{y_1 - (m^2 - m)b}{n^2 - n}$$

$$= \frac{y_2 - (m^3 - m^2)b}{n^3 - n^2} = \frac{y_3 - (m^4 - m^3)b}{n^4 - n^3} \quad . \quad . \quad . \quad (97)$$

Again, solving for b and equating the values and reducing,

$$\frac{ny_0 - y_1}{n - m} = \frac{ny_1 - y_2}{(n - m)m} = \frac{ny_2 - y_3}{(n - m)m^2}.$$

$$\therefore\ m^2ny_0 - m^2y_1 = mny_1 - my_2 = ny_2 - y_3.$$

$$\therefore\ mny_0 - my_1 = ny_1 - y_2;$$

$$mny_1 - my_2 = ny_2 - y_3.$$

$$\therefore\ mn = \frac{(m+n)y_1 - y_2}{y_0} = \frac{(m+n)y_2 - y_3}{y_1}; \quad . \quad (98)$$

$$m + n = \frac{y_1y_2 - y_0y_3}{y_1^2 - y_0y_2} = M; \quad . \quad . \quad . \quad . \quad . \quad (99)$$

$$mn = \frac{y_2^2 - y_1y_3}{y_1^2 - y_0y_2} = N. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Equations (98) and (99) enable us to calculate numerically the values of M and N from the five given values of log p. Then solving for m and n,

$$m = \frac{M}{2} - \left(\frac{M^2}{4} - N\right)^{\frac{1}{2}};$$

$$n = \frac{M}{2} + \left(\frac{M^2}{4} - N\right)^{\frac{1}{2}}.$$

Solving one of the equations (97) for b,

$$b = \frac{ny_0 - y_1}{n(m-1)-(m^2-m)} = \frac{ny_0 - y_1}{(m-1)(n-m)}. \quad (101)$$

Again, solving the first equation of (96) for c,

$$c = \frac{y_0 - \dfrac{ny_0 - y_1}{n - m}}{n - 1} = \frac{y_1 - my_0}{(n - 1)(n - m)}. \quad . \quad . \quad (102)$$

From the first equation of (95),

$$a = \log p_0 - b - c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

Finally, $\alpha = m^{\frac{1}{t_1 - t_0}}$; . . . . (104)

$$\beta = n^{\frac{1}{t_1 - t_0}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

For temperatures below freezing-point Regnault used the equation

$$p = a + b\alpha^n, \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

which is an equation to a curve passing through three points at equidistant temperatures, and of which the solution is very simple.

Regnault's Data and Equations for Steam.—For equation (106) the data are:

$t_0 = -32°$ C.	$p_0 = 0.32$ mm. of mercury.
$t_1 = -16°$ C.	$p_1 = 1.29$ " "
$t_2 = 0°$ C.	$p_2 = 4.60$ " "

From which Regnault calculated the following equation, by aid of seven-place logarithms:

A. For steam from $-32°$ to $0°$ C.,

$$p = a + b\alpha^n;$$
$$a = -0.08038;$$
$$\log b = 9.6024724 - 10;$$
$$\log \alpha = 0.033398;$$
$$n = 32° - t.$$

Regnault gives three equations of the form given by equation (97), of which the following are the data:

B.	$t_0 = 0°$ C.	$p_0 =$	4.60	mm. of	mercury.
	$t_1 = 25°$ C.	$p_1 =$	23.55	"	"
	$t_2 = 50°$ C.	$p_2 =$	91.98	"	"
	$t_3 = 75°$ C.	$p_3 =$	288.50	"	"
	$t_4 = 100°$ C.	$p_4 =$	760	"	"
C.	$t_0 = 100°$ C.	$p_0 =$	760	"	"
	$t_1 = 130°$ C.	$p_1 =$	2030	"	"
	$t_2 = 160°$ C.	$p_2 =$	4651.6	"	"
	$t_3 = 190°$ C.	$p_3 =$	9426	"	"
	$t_4 = 220°$ C.	$p_4 =$	17390	"	'
D.	$t_0 = -20°$ C.	$p_0 =$	0.91	"	"
	$t_1 = +40°$ C.	$p_1 =$	54.91	"	"
	$t_2 = 100°$ C.	$p_2 =$	760	"	"
	$t_3 = 160°$ C.	$p_3 =$	4651.6	"	"
	$t_4 = 220°$ C.	$p_4 =$	17390	"	"

And from these data he calculated, by aid of seven-place logarithms, the following equations, which are correct at Paris:

B. For steam from 0° to 100° C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 4.7384380;$$
$$\log b = 0.6116485;$$
$$\log c = 8.1340339 - 10;$$
$$\log \alpha = 9.9967249 - 10;$$
$$\log \beta = 0.006865036;$$
$$n = t.$$

C. For steam from 100° to 220° C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 5.4583895;$$
$$\log b = 0.4121470;$$
$$\log c = 7.7448901 - 10;$$
$$\log \alpha = 9.997412127 - 10;$$
$$\log \beta = 0.007590697;$$
$$n = t - 100.$$

D. For steam from − 20° to 220° C.,

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 6.2640348;$$
$$\log b = 0.1397743;$$
$$\log c = 0.6924351;$$
$$\log \alpha = 9.994049292;$$
$$\log \beta = 9.998343862;$$
$$n = t + 20.$$

The temperatures and pressures of saturated steam in the tables given by Regnault were calculated by equations A and B for their respective ranges, but equation D was used instead of C for temperatures above 100° C.

Wishing to attain greater accuracy for meteorological work, Moritz recalculated equation B, using ten-place logarithms and obtained constants that differed but little from those which will be given later. Some of the more recent tables in the French system were calculated by aid of his equation.

Equations for the Pressure of Steam at Paris.—In view of the preceding statements it appeared desirable to recalculate the constants for equations B and C with such a degree of accuracy as to exclude any doubt as to the reliability of the results. Accordingly the logarithms of the five values of p for each equation were taken from Vega's ten-place table, and then the remainder of the calculations were carried on with natural numbers, checking by independent methods, with the following results:

B. For steam from 0° to 100° C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 4.7393622142;$$
$$\log b = 0.6117400190;$$
$$\log c = 8.1320378383 - 10;$$
$$\log \alpha = 9.996725532820 - 10;$$
$$\log \beta = 0.006864675924;$$
$$n = t.$$

C. For steam from 100° to 220 C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 5.4574301234;$$
$$\log b = 0.4119787931;$$
$$\log c = 7.7417476470 - 10;$$
$$\log \alpha = 9.99741106346 - 10;$$
$$\log \beta = 0.007642489113;$$
$$n = t - 100.$$

To show the degree of accuracy attained, the following tables are given:

EQUATION B.

t.	p.	log p from table of logarithms.	log p calculated by equation.
0	4.60	0.66275783[1]7	....
25	23.55	1.3719909115	1.37199097
50	91.98	1.9636934052	1.96369346
75	288.50	2.4601458175	2.46014587
100	760	2.8808135923	2.88081365

EQUATION C.

t.	p.	log p from table of logarithms.	log p calculated by equation.
100	760	2.8808135923	....
130	2030	3 3074960379	3.307496036
160	4651.6	3.6676023618	3.667602359
190	9426	3.9743274354	3.974327428
220	17390	4.2402995820	4.240299575

The results from equation C are quite satisfactory, for the errors come in the ninth place of decimals, and one place of decimals is unavoidably lost in the application of the formula. Equation B was calculated after equation C, and the numerical work was not carried to so large a number of decimal places. For the calculation of tables, the constants are carried to seven places of significant figures only; this gives six places in the result, of which five are recorded in the table.

Pressure of Steam at Latitude 45°,—French System.—It is customary to reduce all measurements to the latitude of 45°, and to sea level. The standard thermometer should then have its boiling and freezing points determined under, or reduced to, such conditions. The value of g, the acceleration due to gravity given by equation (58), is 9.809218 meters at Paris, latitude 48° 50′ 14″, and at an elevation of 60 meters. At 45° and at sea level, $g = 9.806056$; consequently 760 mm. of mercury at 45° latitude give a pressure equal to that of

$$760 \times \frac{980.6056}{980.9218} = 759.755 \text{ mm.}$$

at Paris, and by equation B this corresponds to a temperature of 99°.991 C. In other words, the thermometer which is standard at 45° has each degree 0.99991 of the length of the degree of a thermometer standard at Paris.

Again, we have that the height of a column of mercury at 45° latitude is $\frac{980.9218}{980.6056}$ times the height of a column which will give the same pressure at Paris. Consequently, to reduce equation B to 45° latitude, we have

$$\log p = a + \log \frac{980.9218}{980.6056} - b\alpha^{0.99991\,t} + c\beta^{0.99991\,t}.$$

and for equation C,

$$\log p = a + \log \frac{980.9218}{980.6056} - b\alpha^{(0.99991\,t - 100)} + c\beta^{(0.99991\,t - 100)}$$

$$= a + \log \frac{980.9218}{980.6056} - b\alpha^{-0.009}\,\alpha^{0.99991\,(t-100)}$$
$$+ c\beta^{-0.009}\,\beta^{0.99991\,(t-100)}.$$

The resulting equations are:

B. For steam from 0° to 100° C. at 45° latitude,

$$\log p = a_1 - b\alpha_1^n + c\beta_1^n;$$
$$a_1 = 4.7395022;$$
$$\log b = 0.6117400190;$$
$$\log c = 8.1320378383 - 10;$$
$$\log \alpha_1 = 9.996725827522 - 10;$$
$$\log \beta_1 = 0.006864058103;$$
$$n = t.$$

C. For steam from 100° to 220° C. at 45° latitude,

$$\log p = a_1 - b_1\alpha_1^n + c_1\beta_1^n;$$
$$a_1 = 5.4575701;$$
$$\log b_1 = 0.4120020935;$$
$$\log c_1 = 7.7416788646 - 10;$$
$$\log \alpha_1 = 9.997411296464 - 10;$$
$$\log \beta_1 = 0.007641801289;$$
$$n = t - 100.$$

Pressure of Steam at Latitude 45°,—English System.— To reduce the equations for the pressure of steam, so that they will give the pressures in pounds on the square inch for degrees Fahrenheit, there are required the comparison of measures of length and of weight, the comparison of the scales of the thermometers, and the specific gravity of mercury.

Professor Rogers* gives for the length of the meter, 39.3702 inches. This differs from the value given by Captain Clarke † by an amount that does not affect the values in the tables; his value being 39.370432 inches.

Professor Miller ‡ gives for the weight of one kilogram, 2.20462125 pounds.

Regnault § gives for the weight of one liter of mercury, 13.5959 kilograms.

* Proceedings of the Am. Acad. of Arts and Sciences, 1882–83; also additional observations.

† Proceedings of the Royal Society, vol. xv. 1866.

‡ Philosophical Transactions, cxlvi. 1856.

§ Mémoires de l'Institut de France, vol. xxi.

The degree Fahrenheit is $\frac{5}{9}$ of the degree Centigrade.

Let
$$k = \frac{13.5959 \times 2.204621}{39.3702^2};$$

then equations B and C have for the reduction to degrees Fahrenheit, and pounds on the square inch,

$$\log p = a_1 + \log k - b\alpha_1^{\frac{5}{9}n} + c\beta_1^{\frac{5}{9}n};$$
$$\log p = a_1 + \log k - b_1\alpha_1^{\frac{5}{9}n} + c_1\beta_1^{\frac{5}{9}n}.$$

The resulting equations are:

B. For steam from 32° to 212° F. in pounds on the square inch,

$$\log p = a_2 - b\alpha_2^n + c\beta_2^n;$$
$$a_2 = 3.025908;$$
$$\log b = 0.6117400;$$
$$\log c = 8.13204 - 10;$$
$$\log \alpha_2 = 9.998181015 - 10;$$
$$\log \beta_2 = 0.0038134;$$
$$n = t - 32.$$

C. For steam from 212° to 428° F. in pounds on the square inch,

$$\log p = a_2 - b_1\alpha_2^n + c_1\beta_2^n;$$
$$a_2 = 3.743976;$$
$$\log b_1 = 0.4120021;$$
$$\log c_1 = 7.74168 - 10;$$
$$\log \alpha_2 = 9.998561831 - 10;$$
$$\log \beta_2 = 0.0042454;$$
$$n = t - 212.$$

Other Equations for the Pressure of Steam.—Rankine* gives the following equation for the pressure of saturated steam,

$$\log p = A - \frac{B}{T} - \frac{C}{T^2}, \quad \ldots \ldots \quad (107)$$

* Steam-engine and Other Prime Movers.

in which T is the absolute temperature calculated by the equation

$$T = t + 461^\circ.2 \text{ F}.$$

For pounds on the *square foot* the values of the constants are

$$A = 8.2591; \quad \log B = 3.43642; \quad \log C = 5.59873.$$

For pounds on the *square inch* the only change is

$$A = 6.1007.$$

This equation has the advantage that it may be solved directly for T, a property that Regnault's equations do not have. It gives quite accurate results, and the greater part of English tables of properties of saturated steam are calculated by its aid. The following table will give a comparison between the results from this formula and those from formulæ B and C.

RANKINE'S EQUATION FOR STEAM.

Temperature (Fahrenheit).	Pressure, pounds per square inch.	
	Regnault at 45° latitude.	Rankine.
32°	0.0890	0.083
77	0.4555	0.452
122	1.7789	1.78
167	5.579	5.58
212	14.697	14.70
257	33.711	33.71
302	69.27	69.21
347	129.79	129.8
392	225.56	225.9
428	336.26	336.3

A number of exponential formulæ have been devised, of which the principal advantage is the facility of application. The following, by Magnus, gives pressures in mm. of mercury for degrees Centigrade, and agrees quite well with Regnault's results below 100°, but is not so correct above 100°:

$$p = 4.525 \times 10^{\frac{7.4475\,t}{234.69 + t}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

The table following exhibits the defects of equation (108):

MAGNUS' EQUATION FOR STEAM.

Temperature (Centigrade).	Pressures, mm. of mercury.	
	Regnault at 45° latitude.	Magnus.
0°	4.602	4.525
50	91.98	91.97
100	760.0	759.9
150	3581.9	3627.
200	11664.	12080.

Pressure of Other Vapors.—Regnault * determined also the pressure of a large number of saturated vapors, at various temperatures, and deduced equations for each in the form of equation (94). The equations and the constants as determined by him for the commoner vapors are given in the following table:

PRESSURE OF SATURATED VAPORS.

	$\log p$	a	b	c
Alcohol	$a - b\alpha^n + c\beta^n$	5.4562028	4.9809960	0.0485397
Ether	$a + b\alpha^n - c\beta^n$	5.0286298	0.0002284	3.1906390
Chloroform	$a - b\alpha^n - c\beta^n$	5.2253893	2.9531281	0.0668673
Carbon bisulphide	$a - b\alpha^n - c\beta^n$	5.4011662	3.4405663	0.2857386
Carbon tetrachloride	$a - b\alpha^n - c\beta^n$	12.0962331	9.1375180	1.9674890

	$\log \alpha$	$\log \beta$	n	Limits.
Alcohol	$\bar{1}.99708557$	$\bar{1}.9409485$	$t + 20$	$-20°, +150°$ C.
Ether	0.01445775	$\bar{1}.996877$	$t + 20$	$-20°, +120°$ C.
Chloroform	$\bar{1}.9974144$	$\bar{1}.9868176$	$t - 20$	$+20°, +164°$ C.
Carbon bisulphide	$\bar{1}.9977628$	$\bar{1}.9911997$	$t + 20$	$-20°, +140°$ C.
Carbon tetrachloride	$\bar{1}.9997120$	$\bar{1}.9949780$	$t + 20$	$-20°, +188°$ C.

* Académie des Sciences, Comptes rendus, Tome xxxvi.

Zeuner * states that there is a slight error in Regnault's calculation of the constants for aceton, and gives instead,

$$\begin{aligned}\log\ p &= a - b\alpha^n + c\beta^n;\\ a &= 5.3085419;\\ \log b\alpha^n &= +\,0.5312766 - 0.0026148t;\\ \log c\beta^n &= -\,0.9645222 - 0.0215592t.\end{aligned}$$

Differential Coefficient $\frac{dp}{dt}$.—From the general form of the equation (94), we have

$$\log_e p = \frac{1}{M}a + \frac{1}{M}b\alpha^n + \frac{1}{M}c\beta^n, \quad . \quad . \quad . \quad . \quad (109)$$

M being the modulus of the common system of logarithms. Differentiating,

$$\frac{dp}{pdt} = \frac{1}{M}b \log_e \alpha \,.\, \alpha^n + \frac{1}{M}c \log_e \beta \,.\, \beta^n;$$

or, reducing to common logarithms,

$$\frac{1}{p}\frac{dp}{dt} = \frac{1}{M^2}b \log \alpha \,.\, \alpha^n + \frac{1}{M^2}c \log \beta \,.\, \beta^n;$$

$$\therefore \frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

For saturated steam at 45° latitude, the constants to be used with equation (110) are:

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* Mechanische Wärmetheorie.

French units.

B. For 0° to 100 C., mm. of mercury,

$$\log A = 8.8512729 - 10;$$
$$\log B = 6.69305 - 10;$$
$$\log \alpha_1 = 9.996725828 - 10;$$
$$\log \beta_1 = 0.0068641.$$

C. For 100° to 220° C., mm. of mercury,

$$\log A = 8.5495158 - 10;$$
$$\log B = 6.34931 - 10;$$
$$\log \alpha_1 = 9.997411296 - 10;$$
$$\log \beta_1 = 0.0076418.$$

English units.

B. For 32° to 212° F., pounds on the square inch,

$$\log A = 8.5960005 - 10;$$
$$\log B = 6.43778 - 10;$$
$$\log \alpha_2 = 9.998181015 - 10;$$
$$\log \beta_2 = 0.0038134.$$

C. For 212° to 428° F., pounds on the square inch,

$$\log A = 8.2942434 - 10;$$
$$\log B = 6.09403 - 10;$$
$$\log \alpha_2 = 9.998561831 - 10;$$
$$\log \beta_2 = 0.0042454.$$

It is to be remarked that $\frac{dp}{dt}$ may be found approximately by dividing a small difference of pressure by the corresponding difference of temperature; that is, by calculating $\frac{\Delta p}{\Delta t}$. With a table for even degrees of temperature, we may calculate the value approximately for a given temperature by dividing the difference of the pressures corresponding to the next higher and the next lower degrees by two.

The following table of constants for the several vapors named were calculated by Zeuner from the preceding equations for temperature and pressure of the same vapors:

DIFFERENTIAL COEFFICIENT $\frac{1}{p}\frac{dp}{dt}$.

	Sign.		$\log (A\alpha^n)$	$\log (B\beta^n)$
	$A\alpha^n$	$B\beta^n$		
Alcohol	+	−	$-1.1720041 - 0.0029143t$	$-2.9992701 - 0.0590515t$
Ether	+	+	$-1.3396624 - 0.0031223t$	$-4.4616396 + 0.0145775t$
Chloroform	+	+	$-1.3410130 - 0.0025856t$	$-2.0667124 - 0.0131824t$
Carbon bisulphide	+	+	$-1.4339778 - 0.0022372t$	$-2.0511078 - 0.0088003t$
Carbon tetrachloride	+	+	$-1.8611078 - 0.0002880t$	$-1.3812195 - 0.0050220t$
Aceton	+	+	$-1.3268535 - 0.0026148t$	$-1.9064582 - 0.0215592t$

Heat of the Liquid and Specific Heat.—A preliminary series of experiments convinced Regnault that the specific heat of water at low temperature is unity. To test the specific heat at higher temperatures he ran hot water from a boiler, and at a known temperature, into a calorimeter in which the temperature varied from 8° to 14° C., and the resulting upper temperature varied from 17° to 29° C. Knowing the original weight of water in the calorimeter, the weight run in from the boiler, and the initial and final temperatures in the calorimeter, he calculated the mean specific heat of water between the temperature of the boiler and the final temperatures of the calorimeter. A series of forty such experiments was made, with the temperature of the boiler varying from 108° to 192° C., from which Regnault concluded that the mean specific heat from 0° to 100° is 1.005, and from 0° to 200°, 1.016. The corresponding heat of the liquid, *i.e.*, the heat required to raise one kilogram of water from 0° to a given temperature, t, is:

For 100°, ... 100.5;
" 200°, ... 203.2.

Assuming an equation of the form

$$q = t + At^2 + Bt^3, \quad \ldots \ldots \quad (111)$$

and solving for the two constants by aid of the two known values of q, the following equation, which is commonly used, is deduced:

$$q = t + 0.00002t^2 + 0.0000003t^3. \quad \ldots \quad (112)$$

The specific heat at any temperature is, therefore,

$$c = \frac{dq}{dt} = 1 + 0.00004t + 0.0000009t^2. \quad . \quad (113)$$

These equations are for use with the Centigrade scale; for the Fahrenheit scale, a given temperature may be reduced to the Centigrade scale, and then introduced in the same equations.

The process of making the experiments is really a complex one; for the water in leaving the boiler has work done on it by the steam pressure in the boiler, and it has a certain velocity impressed on it at the same time, and again, in entering the calorimeter, it does work against the atmospheric pressure, and the kinetic energy of its motion is changed into heat. At higher temperatures there is a double change of state; part of the water changes to steam on leaving the boiler, and that steam is condensed again in the calorimeter. It is probable that the error of neglecting the effect of these several actions is inconsiderable.

The degree of accuracy to be accorded to this work is indicated by the fact that Regnault gives four significant figures in stating the data for the calculation of the constants in the equations.

Similar experiments were made to determine the heat of the liquid of other volatile liquids, the results of which were as follows:

HEAT OF THE LIQUID.

Alcohol	$q = 0.54754t + 0.0011218t^2 + 0.000002206t^3$.
Ether	$q = 0.52901t + 0.0002959t^2$.
Chloroform	$q = 0.23235t + 0.0000507t^2$.
Carbon bisulphide	$q = 0.23523t + 0.0000815t^2$.
Carbon tetrachloride	$q = 0.19798t + 0.0000906t^2$.
Aceton	$q = 0.50643t + 0.0003965t^2$.

The specific heat at any temperature may be obtained by differentiating the equations for the heat of the liquid, thereby obtaining equations like equation (113).

Rowland's Experiments.—A series of experiments was made by Rowland * at Baltimore to determine the mechanical equivalent of heat, which gave a delicate method of determining the heat of the liquid and the specific heat.

The apparatus used was similar to that used by Joule, with modifications to give greater certainty of results. The calorimeter was of larger size, and the paddle had the upper vanes curved like the blades of a centrifugal pump, to give a strong circulation up through the centre, past the thermometer for taking the temperatures, and down at the outside. The paddle was driven by a petroleum engine, and the power applied was measured by making the calorimeter into a friction brake, with two arms at which the turning moment was measured. Radiation was made as small as possible, and then was made determinate by use of a water-jacket outside of the calorimeter.

The experiments consisted essentially in delivering a meas-

* Proceedings of the American Academy, vol. xv. (N. S. vii.). 1879.

ured amount of work to the water in the calorimeter, and in noting the rise of temperature produced thereby.

The whole range covered by the experiments was from 2° to 41° C. The results show that 430 kilogrammeters of work are required to raise one kilogram of water from 2° to 3° C. Assuming that the same amount will be required to raise the same weight from 0° to 1°, and from 1° to 2°, the following table has been arranged from Rowland's final table of results:

ROWLAND'S MECHANICAL EQUIVALENT OF HEAT.

Degrees C. 1	Total Number of Meter Kilograms. 2	Mechanical Equivalent of Heat. 3	Heat of the Liquid, Experimental. 4	Heat of the Liquid, Calculated. 5	Degrees C. 1	Total Number of Meter Kilograms. 2	Mechanical Equivalent of Heat. 3	Heat of the Liquid, Experimental. 4	Heat of the Liquid, Calculated. 5
1	430	.....	1.0068	1.007	22	9424	426.1	22.065	22.063
2	860	.....	2.0135	2.014	23	9850	426.0	23.063	23.061
3	1290	.....	3.0204	3.022	24	10277	425.9	24.062	24.059
4	1721	.....	4.0295	4.029	25	10701	425.8	25.055	25.058
5	2150	429.8	5.0339	5.036	26	11128	425.7	26.054	26.053
6	2580	429.5	6.0408	6.040	27	11553	425.6	27.050	27.048
7	3009	429.3	7.0452	7.045	28	11978	425.6	28.045	28.042
8	3439	429.0	8.0520	8.049	29	12399	425.5	29.031	29.037
9	3868	428.8	9.0564	9.054	30	12828	425.6	30.035	30.032
10	4296	428.5	10.059	10.058	31	13253	425.6	31.030	31.027
11	4723	428.3	11.058	11.060	32	13675	425.6	32.018	32.023
12	5151	428.1	12.061	12.061	33	14101	425.7	33.016	33.018
13	5578	427.9	13.060	13.063	34	14527	425.7	34.011	34.014
14	6006	427.7	14.063	14.064	35	14952	425.8	35.008	35.009
15	6433	427.4	15.065	15.066	36	15379	425.8	36.008	36.007
16	6861	427.2	16.064	16.066	37	15805	.....	37.007	37.005
17	7289	427.0	17.066	17.066	38	16231	.....	38.003	38.004
18	7717	426.8	18.068	18.066	39	16657	.....	39.000	39.002
19	8144	426.6	19.068	19.066	40	17083	.....	39.998	40.000
20	8571	426.4	20.068	20.066	41	17508	.....	40.993	......
21	8997	426.2	21.065	21.064					

In the above table, column 1 gives the number of degrees above freezing on the Centigrade scale; column 2 gives the number of kilogrammeters required to raise one kilogram of water from freezing-point to the given temperature; column 3 is Rowland's mechanical equivalent of heat at the given temperature derived from 10° intervals on column 2; column 4 is obtained by dividing the numbers in column 2 by the mechanical equivalent of heat at $16\frac{2}{3}°$ C., or 62° F., from column 3; and column 5 is calculated by considering the specific heat to be constant for each five degrees of temperature. These specific heats were derived from a curve obtained by plotting temperatures for abscissæ, and heats of the liquid for ordinates. The values of the specific heats will be given later in connection with those for higher temperatures.

A review of the preceding table shows that the specific heat at low temperatures varies quite markedly, so that it appeared advisable to investigate the effect of this variation on Regnault's experiments already quoted. This was done quite expeditiously by multiplying the mean specific heat given by him for his several experiments by the true average specific heat for the range of temperature in the calorimeter. This corrected specific heat was then used to calculate the increase of heat from the final temperature of the calorimeter to the temperature of the boiler, and that increase was added to the heat of the liquid from the table to find the heat of the liquid at the temperature of the boiler. The results were then plotted as before, and compared with the heats of the liquid derived from Regnault's mean specific heats uncorrected. The points by the corrected method were a little more regularly arranged than the points obtained by assuming the specific heat to be unity at low temperatures; but the improvement was inconsiderable. The inequality of the specific heat at low temperatures is seldom so much as the unavoidable errors of the method.

It appeared that if the specific heat was assumed to be constant, from 40° to 45°, from 45° to 155°, and from 155° to 200° C., the straight lines thus drawn represented the experimental values as recalculated quite nearly; and further, they represented the uncorrected experimental values more nearly than Regnault's equation.

Specific Heat of Water.—The combination of Rowland's and Regnault's experiments on the heat of the liquid by the method described gives the specific heats set down in the following table:

SPECIFIC HEAT OF WATER.

Range.		Specific Heat.
Centigrade.	Fahrenheit.	
0 to 5	32 to 41	1.0072
5 to 10	41 to 50	1.0044
10 to 15	50 to 59	1.0016
15 to 20	59 to 68	1.
20 to 25	68 to 77	0.9984
25 to 30	77 to 86	0.9948
30 to 35	86 to 95	0.9954
35 to 40	95 to 104	0.9982
40 to 45	104 to 113	1.
45 to 155	113 to 311	1.008
155 to 200	311 to 392	1.046

Standard Temperature.—In the beginning of our work it was stated that we should use 62° F. for our standard temperature; and the reasons for so doing may now be shown. We know actually nothing about the specific heat of water from 0° to 2° C.; consequently, the commonly accepted value of the thermal unit, *i.e.*, the heat required to raise one unit of weight of water from 0° to 1° C., or from 32° to 33° F., is an ideal quantity inferred from the behavior of water at higher temperatures. It is more scientific to take an easily verified quantity for the standard; and there is a practical convenience in choosing 62° F. for the standard temperature, because it is near the mean temperature of the air during experimental work. Therefore, it is near the mean temperature in the calorimeter during ordinary work with that instrument; and the specific heat of water for the range of temperature in the calorimeter may usually be considered to be unity, without error, unless great refinement is desired.

Mechanical Equivalent of Heat.—The mechanical equivalent in meter-kilograms of one calorie at $16\frac{2}{3}$° C., deduced from Rowland's experiments in the third column of the table on page 85, is 427.1.

Since the value given by Joule is commonly quoted, it will be of interest to make a comparison of his latest work (1873) with Rowland's, as given in the following table:

COMPARISON OF ROWLAND'S AND JOULE'S EXPERIMENTS.

Temperature.	Joule's Value at Manchester, English System.	Reduced to the Air Thermometer and to the latitude of Baltimore.		Rowland's Value, Corresponding.
		English.	French.	
14°.7	772.7	776.1	425.8	427.6
12°.7	774.6	778.5	427.1	428.0
15°.5	773.1	776.4	426.0	427.3
14°.5	767.0	770.5	422.7	427.5
17°.3	774.0	777.0	426.3	426.9

The value of g at Baltimore, latitude 39° 17′, is 980.05 centimeters; therefore, reducing to 45° of latitude and at the sea level, the value of J is

$$427.1 \times \frac{980.05}{980.6056} = 426.86,$$

or, as it has been given before,

$$J = 426.9.$$

To reduce to the English system, multiply by $\frac{5}{9}$, and by the length of the meter in feet, so that

$$J = 778.$$

Total Heat.—This term is defined as the heat required to raise a unit of weight of water from freezing-point to a given temperature, and to entirely evaporate it at that temperature. The experiments made by Regnault were in the reverse order; that is, steam was led from a boiler into the calorimeter and there condensed. Knowing the initial and final weights of the calorimeter, the temperature of the steam, and the initial and final temperatures of the water in the calorimeter, he was able, after applying the necessary corrections, to calculate the total heats for the several experiments.

As a conclusion of the work he gives the following values for the total heats:

10°, . . .	610	By equation, 609.6
63°, . . .	625	625.2
100°, . . .	637	
195°, . . .	666	

Assuming an equation of the form

$$\lambda = A + Bt, \quad \ldots \ldots \ldots \quad (114)$$

Regnault calculated the constants from the values given for 100° and 195°, and gives the equation

$$\lambda = 606.5 + 0.305t. \quad \ldots \ldots \quad (115)$$

In order to see the effect of the varying value of the specific heat at low temperatures, the total heats given by experiment were recalculated by a method resembling that used in recalculation of the heats of the liquid, and the results plotted together with Regnault's values uncorrected. The recalculated points were a little more regular than the original ones, and lay nearer the line represented by the equation (115). Especially did the recalculated points for those experiments, for which the true mean specific heat of the water in the calorimeter was nearly unity, lie near that line. It therefore appears that equation (115) represents our best knowledge of the total heat of steam.

For the Fahrenheit scale the equation becomes

$$\lambda = 1091.7 + 0.305\,(t - 32). \quad \ldots \ldots \quad (116)$$

Regnault gives the equations following for other liquids:

Ether.................... $\lambda = 94 + 0.45t - 0.00055556t^2$
Chloroform.............. $\lambda = 67 + 0.1375t.$
Carbon bisulphide........ $\lambda = 90 + 0.14601t - 0.0004123t^2$
Carbon tetrachloride...... $\lambda = 52 + 0.14625t - 0.000172t^2$
Aceton.................. $\lambda = 140.5 + 0.36644t - 0.000516t^2$

Heat of Vaporization.—If the heat of the liquid be subtracted from the total heat, the remainder is called the heat of vaporization, and is represented by r, so that

$$r = \lambda - q. \quad \ldots \ldots \ldots \quad (117)$$

Specific Volume of Liquids.—The coefficient of expansion of most liquids is large as compared with that of solids, but it is small as compared with that of gases or vapors. Again, the specific volume of a vapor is large compared with that of the liquid from which it is formed. Consequently the error of neglecting the increase of volume of a liquid with the rise of temperature is small in equations relating to the thermodynamics of a saturated vapor, or of a mixture of a liquid and its vapor when a considerable part by weight of the mixture is vapor. It is therefore customary to consider the specific volume of a liquid σ to be constant.

Experiments were made by Hirn * to determine the volumes of liquid at high temperatures compared with the volume at freezing-point, by a method which was essentially to use them for the expansive substance of a thermometer. The results are given in the following equations:

SPECIFIC VOLUMES OF HOT LIQUIDS.

		Logarithms.
Water, 100° C. to 200° C. (Vol. at 4° = unity.)	$v = 1 + 0.00010867875t$	6.0361445 − 10
	$+ 0.0000030073653t^2$	4.4781862 − 10
	$+ 0.0000000028730422t^3$	1.4583419 − 10
	$- 0.0000000000066457031t^4$	8.8225409 − 20
Alcohol, 30° C. to 160° C. (Vol. at 0° = unity.)	$v = 1 + 0.00073892265t$	6.8685991 − 10
	$+ 0.0000010552 35t^2$	3.0233492 − 10
	$- 0.0000000092480842t^3$	2.9660517 − 10
	$+ 0.00000000040413567t^4$	0.6065278 − 10
Ether, 30° C. to 130° C. (Vol. at 0° = unity.)	$v = 1 + 0.0013489059t$	7.1299817 − 10
	$+ 0.0000065537t^2$	4.8164866 − 10
	$- 0.000000034490756t^3$	2.5377028 − 10
	$+ 0.00000000033772062t^4$	0.5285571 − 10
Carbon Bisulphide, 30° C. to 160° C. (Vol. at 0° = unity.)	$v = 1 + 0.0011680559t$	7.0674636 − 10
	$+ 0.0000016489598t^2$	4.2172103 − 10
	$- 0.00000000081119062t^3$	0.9091229 − 10
	$+ 0.000000000060946589t^4$	9.7849494 − 20
Carbon Tetrachloride, 30° C. to 160° C. (Vol. at 0° = unity.)	$v = 1 + 0.0010671883t$	7.0282409 − 10
	$+ 0.0000035651378t^2$	4.5520763 − 10
	$- 0.000000014949281t^3$	2.1746202 − 10
	$+ 0.000000000085182318t^4$	9.9303494 − 20

* Annales de Chimie et de Physique, 1867.

Internal and External Latent Heat.—The heat of vaporization overcomes external pressure, and changes the state from liquid to vapor at constant temperature and pressure. Let the specific volume of the saturated vapor be s, and that of the liquid be σ, then the change of volume is $s - \sigma = u$, on passing from the liquid to the vaporous state. The external work is

$$p(s - \sigma) = pu, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

and the corresponding amount of heat, or the external latent heat, is

$$Ap(s - \sigma) = Apu. \quad . \quad . \quad . \quad . \quad . \quad (119)$$

The heat required to do the disgregation work, or the internal latent heat, is

$$\rho = r - Apu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Specific Heats of Water and Steam.—In the general discussion of thermodynamics and the application to gases two kinds of specific heat were used, at constant volume and at constant pressure. In dealing with solids and with liquids below the boiling-point the only specific heat that has been determined is the specific heat at constant pressure: for example, the specific heat of water determined from Rowland's experiments is of that sort. For most solids and liquids the specific heat at constant volume cannot be very different from the specific heat at constant pressure, and it is commonly assumed that they are identical.

If a given substance experience a change of temperature under a specific condition, then the heat required to change the temperature of a unit of weight one degree is the specific heat under that condition. There will, therefore, be as many kinds of specific heat as there are conditions. In dealing with a mixture of a liquid and its vapor, for which the pressure is a function of the temperature only, the specific heat for each is defined under the condition that the pressure shall vary with the temperature, according to the law of saturated vapor given by the general equation (94).

Let c be the specific heat of water under the given conditions. The meaning may be deduced from equation (6) applied to pure liquid,

$$dQ = \left(c_p + m\frac{dp}{dt}\right)dt;$$

$$\frac{dQ}{dt} = c_p + m\frac{dp}{dt}.$$

For water we may use q, the heat of the liquid, for Q, and then

$$c = \frac{dq}{dt} = c_p + m\frac{dp}{dt}. \quad . \quad . \quad . \quad . \quad . \quad (121)$$

The only reason for writing the last equation is to give a clearer conception of the subject, for no experimental work exists for its use. Regnault's experiments probably give c rather than c_p. In any case no attempt is made in thermodynamics to distinguish between the different kinds of specific heat for water.

The specific heat of saturated steam, *i.e.*, the heat that must be given to one unit of weight of steam, when the temperature is raised one degree and the pressure raised the corresponding amount, in order that the steam shall remain dry and saturated, is represented by h. An equation having the form of equation (121) cannot be employed, since the temperature cannot be raised without raising the pressure at the same time. An investigation of other properties of steam will determine the form and value of this function.

General Equation.—A pound or a kilogram of a mixture of a liquid and its vapor consists of a certain part, x, of vapor, and the remainder, $1 - x$, of liquid. The specific volume of the mixture is

$$v = xs + (1 - x)\sigma = (s - \sigma)x + \sigma = ux + \sigma. \quad . \quad . \quad (122)$$

When a mixture of liquid and a vapor receives heat, there is in general an increase of temperature of each component, and a change of part of the liquid into vapor.

$$\therefore\ dQ = hxdt + c(1 - x)dt + rdx. \quad . \quad . \quad (123)$$

Application of the First Law.—Proceeding as we have in our preceding work, equations (24) and (127) give /123

$$dQ = A(dE + pdv) = hxdt + c(1 - x)dt + rdx;$$

$$\therefore\ dE = \frac{1}{A}\Big[hx + c(1 - x)\Big]dt + \frac{r}{A}dx - pdv.$$

But $\quad dv = \left(\frac{dv}{dt}\right)_x dt + \left(\frac{dv}{dx}\right)_t dx;$

$$\therefore\ dE = \left\{\frac{1}{A}\Big[hx + c(1 - x)\Big] - p\left(\frac{dv}{dt}\right)_x\right\}dt + \left[\frac{r}{A} - p\left(\frac{dv}{dx}\right)_t\right]dx.$$

Since $$\frac{d^2E}{dx\,dt} = \frac{d^2E}{dt\,dx},$$

$$\frac{d}{dx}\left\{\frac{1}{A}\Big[hx + c(1 - x)\Big] - p\left(\frac{dv}{dt}\right)_x\right\}_t = \frac{d}{dt}\left[\frac{r}{A} - p\left(\frac{dv}{dx}\right)_t\right]_x.$$

Bearing in mind that h, c, and p are functions of t and not of x, the differentiation gives

$$\frac{1}{A}(h - c) - p\frac{d^2v}{dx\,dt} = \frac{1}{A}\left(\frac{dr}{dt}\right)_x - \left(\frac{dp}{dt}\right)_x\left(\frac{dv}{dx}\right)_t - p\frac{d^2v}{dt\,dx}.$$

But

$$\frac{d^2v}{dx\,dt} = \frac{d^2v}{dt\,dx} \text{ and } \left(\frac{dp}{dt}\right)_x = \frac{dp}{dt} \text{ and } \left(\frac{dr}{dt}\right)_x = \frac{dr}{dt};$$

$$\therefore \frac{1}{A}(h - c) = \frac{1}{A}\frac{dr}{dt} - \frac{dp}{dt}\left(\frac{dv}{dx}\right)_t.$$

From equation (122), σ being constant,

$$\left(\frac{dv}{dx}\right)_t = u;$$

$$\therefore \frac{dr}{dt} + c - h = Au\frac{dp}{dt}. \quad . \quad . \quad . \quad . \quad . \quad (124)$$

Application of the Second Law.—By this law $\frac{dQ}{T}$ is a perfect differential.

$$\therefore \frac{dQ}{T} = \frac{hx + c(1 - x)}{T}dt + \frac{r}{T}dx. \quad . \quad . \quad (125)$$

But
$$\frac{d^2\int\frac{dQ}{T}}{dx\,dt} = \frac{d^2\int\frac{dQ}{T}}{dt\,dx};$$

$$\therefore \frac{d}{dx}\left[\frac{hx + c(1 - x)}{T}\right]_t = \frac{d}{dt}\left(\frac{r}{T}\right)_x; \quad . \quad . \quad (126)$$

$$\therefore \frac{h - c}{T} = \frac{T\frac{dr}{dt} - r}{T^2};$$

$$\therefore \frac{dr}{dt} + c - h = \frac{r}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

First and Second Laws Combined.—The combination of equations (124) and (127) gives

$$r = AuT\frac{dp}{dt}. \qquad (128)$$

Specific Heat of Steam.—Equation (127) solved for h gives

$$h = c + \frac{dr}{dt} - \frac{r}{T}. \qquad (129)$$

Now $$r = \lambda - q = 606.5 + 0.305t - q,$$

and q calculated by the aid of the table on page 87 will have for its value

$$q = \text{const.} + c(t - \text{const.});$$

hence we shall have

$$\frac{dr}{dt} = 0.305 - c;$$

$$\therefore\ h = 0.305 - \frac{r}{T}. \qquad (130)$$

The term $\frac{r}{T}$ may be calculated by aid of equation (115) and the table on page 87, and then the following values of h may be calculated by equation (130):

SPECIFIC HEAT OF STEAM.

	0° C.	50° C.	100° C.	150° C.	200° C.
$h =$	$-$ 1.911,	$-$ 1.461,	$-$ 1.131,	$-$ 0.879,	$-$ 0.676.

The negative sign shows that heat must be abstracted from saturated steam when the temperature and pressure are increased, otherwise it will become superheated. On the other hand, steam, when it suddenly expands with a loss of temperature and pressure, suffers condensation, and the heat thus liberated supplies that required by the uncondensed portion.

Hirn * verified this conclusion by suddenly expanding steam in a cylinder with glass sides, whereupon the clear saturated steam suffered partial condensation as indicated by the formation of a cloud of mist. The reverse of this experiment showed that steam does not condense with sudden compression, as shown by Cazin.

Ether has a positive value for h. As the theory indicates, a cloud is formed during sudden compression, but not during sudden expansion.

The table of values of h for steam shows a notable decrease for higher temperatures, which indicates a point of inversion at which h is zero and above which h is positive, but the temperature of that point cannot be determined by our own experimental knowledge. For chloroform the point of inversion was calculated by Cazin† to be 123°.48, and determined experimentally by him to be between 125° and 129°. The discrepancy is mostly due to the imperfection of the apparatus used, which substituted finite changes of considerable magnitude for the indefinitely small changes required by the theory.

Specific Volume and Density.—Solving equation (128) for u, we have

$$u = \frac{r}{AT} \cdot \frac{1}{\frac{dp}{dt}}, \qquad (131)$$

which gives a method of calculating the increase of volume

* Bulletin de la Société Industr. de Mulhouse, cxxxiii.
† Comptes rendus de l'Académie des Sciences, lxii.

due to vaporization from experimental data. The specific volume s and density γ are thus known from the equations

$$s = u + \sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

$$\gamma = \frac{1}{s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

It is of interest to consider the degree of accuracy that may be expected from this method of calculating the density of saturated vapor. The value of r depends on λ and q; for the first Regnault gives three figures in the data from which the empirical equation is deduced, and the experimental work does not indicate a greater degree of accuracy. The fourth figure, if stated, is likely to be in error to the extent of five units. The value of T is commonly stated in four figures, of which the last may be in error by two units. A, as determined by Rowland, has four figures, the last being uncertain to the extent of one or two units. The differential coefficient $\frac{dp}{dt}$ is deduced from the equations for calculating p; and those equations are derived from data having five places of significant figures. Now each of the equations B and C, for steam at 45° latitude for the English system, gives a pressure of 14.6967 pounds on the square inch; but the specific volume calculated by aid of equation B is 26.550 cubic feet, while equation C gives 26.637 cubic feet. The mean, 26.60, differs from either extreme by about one in seven hundred. This discrepancy is due to the fact that the curves represented by equations B and C meet at the common temperature, 212°, but do not have a common tangent. Since the equations are empirical and not logical, the error or uncertainty is unavoidable, and all calculated specific volumes are affected by a similar uncertainty. The greatest probable error is in determining r, for which it may be about one in one thousand. The error introduced into this equation by using the values of A in common use, that is, 772 instead of 778, is about one in one hundred.

The specific volume and density are commonly calculated in the method given, on account of the great difficulty of the experimental determination, and the error of the method is not greater than that of the other parts of steam tables.

Tate and Fairbairn's Experimental Determination.—The great uncertainty of the direct determination of the density of saturated vapor is due to difficulty of determining when steam is dry and saturated. A small quantity of liquid present, or a slight degree of superheating, will introduce serious errors. Fig. 26 is an ideal representation of the *saturation gauge* used by Tate and Fairbairn* in their experiments to determine this point. *A* and *B* are globes in which there is steam with a small quantity of water, having a communication by means of a tube partially filled with mercury. The globes and tubes are immersed in a bath by which all may be raised to any given temperature. So long as any water exists in both globes the pressure in both will be the same, and the mercury will be at the same level in both legs of the tube. As the temperature rises the water in the globes will be gradually vaporized, till all in *A*, containing the least amount, is changed into steam. At this instant the steam in *A* is dry and saturated, and if the weight and volume are known the density can be found. As the process goes on the steam in *A* becomes superheated, and, the pressure being less than for saturated steam at the same temperature, the mercury at *a* will rise.

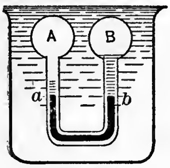

FIG. 26.

In making the experiments it was found advisable to superheat the steam in *A* and then to let the temperature gradually fall, and to take a series of readings of the difference in height of *a* and *b* simultaneously with the readings of the thermometer in the bath, from which the temperature could be inferred at which the steam became saturated. At the same time the steam pressure was taken by a mercury column. The actual apparatus had a different appearance, and was arranged for convenience in observation, and had two forms, one for pressures above that of the atmosphere, and one for pressures below.

* Philosophical Transactions, vol. cl. 1860.

The following table gives a summary of all of the experiments made. The second and third columns give the pressure and temperature of saturation; the fourth gives the relative volume compared with water; the fifth gives the same volume by an empirical formula; and the sixth gives the proportional error of the formula compared with the experimental value.

TATE AND FAIRBAIRN'S EXPERIMENTAL DETERMINATION OF THE DENSITY OF SATURATED STEAM.

	Pressure in Inches of Mercury. P	Maximum Temperature of Saturation. t	Specific Volume from Experiment. V	Specific Volume from Formula. V	Error of Formula.
1	5.35	136.77	8275.3	8183	$-\frac{1}{90}$
2	8.62	155.33	5333.5	5326	$-\frac{1}{762}$
3	9.45	159.36	4920.2	4900	$-\frac{1}{246}$
4	12.47	170.92	3722.6	3766	$+\frac{1}{87}$
5	12.61	171.48	3715.1	3740	$+\frac{1}{149}$
6	13.62	174.92	3438.1	3478	$+\frac{1}{86}$
7	16.01	182.30	3051.0	2985	$-\frac{1}{46}$
8	18.36	188.30	2623.4	2620	$+\frac{1}{874}$
9	22.88	198.78	2149.5	2124	$-\frac{1}{90}$
1′	53.61	242.90	943.1	937	$-\frac{1}{157}$
2′	55.52	244.82	908.0	906	$-\frac{1}{454}$
3′	55.89	245.22	892.5	900	$+\frac{1}{111}$
4′	66.84	255.50	759.4	758	$-\frac{1}{759}$
5′	76.20	263.14	649.2	669	$+\frac{1}{32}$
6′	81.53	267.21	635.3	628	$-\frac{1}{91}$
7′	84.20	269.20	605.7	608	$+\frac{1}{304}$
8′	92.23	274.76	584.4	562	$-\frac{1}{26}$
9′	90.08	273.30	543.2	545	$+\frac{1}{271}$
10′	99.60	279.42	515.0	519	$+\frac{1}{128}$
11′	104.54	282.58	497.2	496	$-\frac{1}{497}$
12′	112.78	287.25	458.3	461	$+\frac{1}{152}$
13′	122.25	292.53	433.1	428	$-\frac{1}{86}$
14′	114.25	288.25	449.6	456	$+\frac{1}{75}$

The formula deduced by the experimenters to represent their work is

$$V = 25.62 + \frac{49513}{P + 0.72}, \quad . \quad . \quad . \quad . \quad (134)$$

in which V is the volume of steam compared with the volume of the water from which it was produced, and P is the pressure in inches of mercury.

Although the experiments were made with great care and all precautions were taken to avoid error, the results are less satisfactory than those obtained by calculation from the other known properties of steam.

To show the comparison between the values of the specific volume determined by the two methods, the following table has been calculated for English units:

SPECIFIC VOLUME OF SATURATED STEAM.

Pressure, Pounds per Square Inch.	Specific Volume, Cubic Feet per Pound.	
	$s = \frac{r}{AT} \cdot \frac{dt}{dp} + \sigma.$	$V = 25.62 + \frac{49513}{P + 0.72}.$
5	73.2	73.2
15	26.2	25.8
25	16.1	15.8
35	11.7	11.4
45	9.3	9.0
55	7.7	7.5

Zeuner's Equation for Internal Latent Heat.—To avoid the laborious calculation of this quantity by the exact methods, Zeuner has proposed the following simple empirical equations for that purpose in the French system:

INTERNAL LATENT HEAT.

Water .... $\rho = 575.40 - 0.791t$
Ether........................ $\rho = 86.54 - 0.10648t - 0.0007160t^2$
Chloroform.................... $\rho = 62.44 - 0.11282t - 0.0000140t^2$
Carbon bisulphide.............. $\rho = 82.79 - 0.11446t - 0.0004020t^2$
Carbon tetrachloride............ $\rho = 48.57 - 0.06844t - 0.0002080t^2$
Aceton........................ $\rho = 131.63 - 0.20184t - 0.0006280t^2$

The following table shows that the equation for water gives a fair degree of approximation:

	0°	50°	100°	150°	200°
By equation (120)..........	575.5	536.3	496.4	457.4	417.4
By empirical equation......	575.4	535.9	496.3	456.8	417.1

Critical Temperature.—The empirical equation, and also the value of ρ in the table above by the exact method, show that the internal latent heat decreases as the temperature rises, and at sufficiently high temperatures it will approach zero. If ρ is made zero in Zeuner's equation for water, the corresponding temperature is 720° C., which indicates that the true point is much beyond the limits of experiments.

The temperature at which ρ becomes zero for any vapor is called the critical temperature, for at that temperature the distinction between the liquid and its vapor vanishes, and above that temperature the vapor or gas cannot be liquefied by pressure alone. It has been proposed to call a substance which is above the critical temperature a gas, and one which is below a vapor.

Experiments on liquids strongly heated in strong glass tubes show that vaporization proceeds gradually as the temperature rises, until a temperature is reached at which the line of demarcation between the liquid and its vapor becomes indistinct. Above that temperature the liquid all disappears, and the tube is full of gas. This is the critical temperature. Avenarius* by this method determined the critical temperature of four liquids. He also selected from Regnault's experiments the data most applicable, and from them deduced equations like those given by Zeuner for the internal latent heat of vapors, and calculated the critical temperature by their aid. The results are as follows:

	Experimental.	Calculated.
Ether	196°.2 C.	196°.8 C.
Carbon bisulphide	276°.1	274°.0
Carbon tetrachloride	292°.5	298°.7
Aceton	246°.1	230°.4

Effect of Pressure on Change of State.—If equation (128) be solved for the differential of the pressure with regard to temperature,

$$\frac{dp}{dt} = \frac{r}{AuT}, \quad \ldots \ldots \quad (135)$$

* Poggendorff's Annalen, cli. 1874.

in which A and T are positive, and r is positive below the critical temperature. Consequently $\frac{dp}{dt}$ is positive or negative, according as s is greater or less than σ. When $\frac{dp}{dt}$ is positive, an increase of pressure causes a rise of the temperature of the change of state, and *vice versa*.

The application of the foregoing to the formation of steam only confirms what we already know, that the temperature of boiling-point increases with the pressure. But an application to the melting of ice is of more interest. To make the application, let s represent the specific volume of water, and σ that of ice, the latter being larger than the former. In this case u is negative, and $\frac{dp}{dt}$ is also negative, showing that an increase of pressure lowers the melting-point, a fact that has been proved by experiment, and which is used in explaining regelation.

Curve of Constant Steam Weight.—It was formerly assumed in the theory of the steam-engine that the interchange of heat between the steam and the iron of the cylinder was by radiation; and further, that the condensation accompanying adiabatic expansion formed a cloud which instigated a rapid interchange of heat, where before little had occurred. The steam-jacket was assumed to impart just heat enough to dissipate this cloud and keep the steam dry. Hence the curve of dry saturated steam was of great importance in the theory of the steam-engine, and it is sometimes drawn on indicator cards instead of the hyperbola. The substitution has no good reason, for the curve is not a better approximation to the curve drawn by an indicator, and is more troublesome to construct.

The action of steam in the engine cylinder has been proved to be quite different, for the interchange of heat is caused by condensation by contact of the steam with the iron, or by evaporation of moisture from it, and the curve of saturated steam no longer plays an important part in the theory of the steam-engine. Still it is of importance as forming the boundary-line between superheated steam and wet steam.

The curve may be represented very closely by an exponential formula resembling that deduced for the adiabatic line for a perfect gas; i.e.,

$$pv^n = p_1 v_1^n = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad (136)$$

Rankine proposed the value $\frac{17}{16}$ for the exponent n, and Zeuner has found that 1.0646 gives still a closer approximation. The actual curve may be drawn by plotting pressures and volumes from a table of the properties of saturated steam.

Exponential Equation.—To find the exponent of an equation representing a curve passing through two points, a_0 and a_1, Fig. 27, take logarithms of both sides of equation (136), and we have

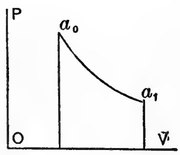

FIG. 27.

$$n \log v + \log p = n \log v_1 + \log p_1.$$

$$\therefore \quad n = \frac{\log p_1 - \log p}{\log v - \log v_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (137)$$

Isothermal Lines.—Since the pressure of saturated vapor is a function of the temperature only, the isothermal line of a mixture of a liquid and its vapor is a line of equal pressures, parallel to the axis of volumes. Steam expanding from the boiler into the cylinder of an engine follows such a line; that is, the steam line of an automatic cut-off engine with ample ports is nearly parallel to the atmospheric line.

The heat required for an increase of volume at constant pressure is

$$Q = r(x_2 - x_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

which may be obtained by integrating equation (123) with the assumption that the temperature is constant, or it may be written directly, since r is the heat of vaporization, and $x_2 - x_1$ is the weight of liquid vaporized.

The work done by the vapor during such an expansion is

$$W = p(v_2 - v_1) = pu(x_2 - x_1). \quad . \quad . \quad . \quad (139)$$

Isodynamic or Isoenergic Lines.—The following method of treating the isodynamic changes of a mixture of a liquid and its vapor is due to Zeuner, and is similar to his method of treating adiabatic changes. It does not give an equation to the curve of pressures and volumes, but it gives the solution of all problems that arise.

The increase of intrinsic energy of the mixture of a liquid and its vapor, above freezing-point, is

$$E = \frac{1}{A}(q + x\rho). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (140)$$

The change of intrinsic energy in passing from one condition to another is

$$E_2 - E_1 = \frac{1}{A}(q_2 - q_1 + x_2\rho_2 - x_1\rho_1). \quad . \quad . \quad (141)$$

When the change is isodynamic, the energy remains the same by definition, and

$$q_2 - q_1 + x_2\rho_2 - x_1\rho_1 = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

which equation, together with the formulæ

$$v_2 = x_2u_2 + \sigma, \quad v_1 = x_1u_1 + \sigma, \quad . \quad . \quad . \quad (143)$$

gives the means of solving all problems.

For example, suppose that the initial and final pressures are given, to find the corresponding volumes; then, x_1 being also known, x_2 may be found from equation (142), and then the volumes may be found by equations (143). On the other hand, if the volumes are given and the pressures are required, the problem can be solved only by approximations.

Assume a probable value p_2' for the final pressure, and calculate the corresponding value of v_2'. From the comparison of v_2 and v_2' assume a second approximate value p_2'', and calculate v_2'', and repeat the process till a sufficiently close approximation is attained. For the first approximation it will commonly be convenient to assume that the pressures are inversely proportional to the volumes.

The direct determination of the heat required and of the external work done by the mixture of liquid and vapor during an isodynamic expansion would require, for convenient work, the calculation of special empirical formulæ or of the tables equivalent to them; which is not justified by the importance of the subject. An approximate solution may be had by aid of an exponential formula determined from the initial and final pressures and volumes; and since the curve represented by such a formula agrees quite well with the actual expansion curve, the approximation is sufficient for the solution of problems.

Suppose we have the formula

$$pv^n = p_1v_1^n = \text{const.};$$

then

$$Q = AW = A\int pdv, \quad . \quad . \quad . \quad . \quad . \quad (144)$$

since the heat received is all changed into external work, the intrinsic energy remaining constant.

Zeuner states that n is 1.0456 when the steam is dry and saturated at the initial state.

Entropy of a Mixture of a Liquid and its Vapor.—From the second law of thermodynamics,

$$d\phi = \frac{dQ}{T};$$

in which ϕ is the entropy, dQ is the heat transmitted, and T is the absolute temperature. Since the entropy depends only on the state of a substance, and not on the method of arriving at that state, the increase of entropy of one unit of weight of a given mixture of a liquid and its vapor, above the entropy of one unit of weight of that liquid at freezing-point of water, may be calculated in the following method:

Suppose that one unit of weight of a liquid be raised from freezing-point to the temperature t, and that a portion x be then changed into vapor. During the first operation the increase of entropy will be

$$\theta = \int_0^t \frac{dq}{T} = \int_0^t \frac{cdt}{T},$$

and during the second operation the increase will be

$$\frac{xr}{T},$$

since the heat is added at a constant temperature t during that operation. The entire increase of entropy will be

$$\frac{xr}{T} + \int_0^t \frac{cdt}{T} = \frac{xr}{T} + \theta.$$

For any other state determined by x_1 and t_1 we shall have, for the increase of entropy above that of liquid at freezing-point,

$$\frac{x_1 r_1}{T_1} + \theta_1 .$$

The change of entropy in passing from one state to another is

$$\phi - \phi_1 = \frac{xr}{T} + \theta - \frac{x_1 r_1}{T_1} + \theta_1 . \quad . \quad . \quad . \quad (145)$$

Entropy of the Liquid.—When the specific heat of a liquid is known in terms of the temperature, the entropy of the liquid is readily calculated. Thus, for ether, the equation is

$$\theta = \int_0^t \left(0.52901 + 0.0005918t\right)dt.$$

The form of the equation for the entropy of water is more easily stated for a special case. For example, at 13° C., it is .

$$1.0072 \log_e \frac{T_5}{T_0} + 1.0044 \log_e \frac{T_{10}}{T_5} + 1.0016 \log_e \frac{T_{13}}{T_0} = 0.04663.$$

Adiabatic Equation for a Liquid and its Vapor.— During an adiabatic change the entropy is constant, so that equation (145) gives

$$\frac{x_1 r_1}{T_1} + \theta_1 = \frac{x_2 r_2}{T_2} + \theta_2 . \quad . \quad . \quad . \quad . \quad . \quad (146)$$

When the initial state, determined by x_1 and t_1 or p_1, is known and the final temperature t_2, or the final pressure p_2, the final value x_2 may be found by equation (146). The initial and final volumes may be calculated by the equations

$$v_1 = x_1 r_1 + \sigma \quad \text{and} \quad v_2 = x_2 r_2 + \sigma.$$

Tables of the properties of saturated vapors commonly give the specific volume s, but

$$s = u + \sigma.$$

Problems in which the initial condition and the final temperature or pressure are given, may be solved directly by aid of the preceding equations. Those giving the final volume instead of the temperature or pressure can be solved only by approximations. An equation to an adiabatic curve in terms of p and v cannot be given, but such a curve for any particular case may be constructed point by point.

Clausius and Rankine independently and at about the same time deduced equations identical with equations (145) and (146), but by methods each of which differed from that given here.

If tables giving θ, the entropy of the liquid, are not at hand, an approximate result may be obtained by considering the specific heat c to be a constant, so that

$$\theta = \int \frac{c\,dt}{T} = c \int \frac{dt}{T} = c \log_e \frac{T_1}{T_2};$$

or equation (146) may be written,

$$\frac{x_2 r_2}{T_2} = \frac{x_1 r_1}{T_1} + \log_e \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (147)$$

In the discussion of the specific heat h of a saturated vapor, it appeared that the expansion of dry saturated steam in a non-conducting cylinder would be accompanied by partial condensation. The same fact may be brought out more clearly at this place. One pound of dry steam at 100 pounds absolute pressure will have the values

$$t_1 = 327°.58 \text{ F.}, \quad r_1 = 884.0, \quad \theta_1 = 0.4733, \quad x_1 = 1.$$

If the final pressure is 15 pounds absolute, we have

$$t_2 = 213°.03 \text{ F.}, \quad r_2 = 965.1, \quad \theta_2 = 0.3143;$$

whence

$$\frac{884.0}{601.28} + 0.4733 = \frac{965.1 x_2}{486.73} + 0.3143;$$

$$\therefore \quad x_2 = 0.8133.$$

On the other hand, h is positive for ether, and partial condensation takes place during compression in a non-conducting cylinder. For example, let the initial condition be

$$t_1 = 10° \text{ C.}, \quad r_1 = 93.12, \quad \theta_1 = 0.0191, \quad x_1 = 1,$$

and let the final conditions be

$$t_2 = 120° \text{ C.}, \quad r_2 = 72.26, \quad \theta_2 = 0.2045,$$

then

$$\frac{93.12}{283.7} + 0.0191 = \frac{72.26 x_2}{393.7} + 0.2045;$$

and $$x_2 = 0.724.$$

Equation (146) applies to all possible mixtures of a liquid and its vapor, including the case of $x_1 = 0$ or the case of liquid without vapor, but at the pressure corresponding to the temperature according to the law of saturated vapor. When applied to hot water, this equation shows that an expansion in a non-conducting cylinder is accompanied by a partial vaporization.

There is some initial state of the mixture such that the value of x shall be the same at the beginning and at the end, though it may vary at intermediate states. To find that value make $x_2 = x_1$ in equation (146) and solve for x_1, which gives

$$x_1 = \frac{\theta_1 - \theta_2}{\frac{r_2}{T_2} - \frac{r_1}{T_1}}. \qquad \qquad (148)$$

The value of x_1 to fulfil the conditions given varies with the initial and final temperatures chosen, but in any case it will not be much different from one half. It may therefore be generally stated that a mixture of steam and water, when expanded in a non-conducting cylinder, will show partial condensation if more than half is steam, and partial evaporation if more than half water. If the mixture is nearly half water and half steam, the change must be investigated to determine whether evaporation or condensation will occur; but in any case the action will be small.

Construction of Adiabatic Lines.—In case a series of adiabatic lines is to be drawn, the following method may be used with advantage. In Fig. 28 draw the line BB parallel to OP, the axis of P, making $OB = \sigma$, so that BB represents the volume of one kilogram of water at all temperatures and pressures. At the point O, which represents the highest pressure to be used, draw $OD = u_1$, corresponding to the given pressure; then D represents the condition when the mixture is all dry saturated steam; i.e., $x = 1$, $v = s$. Though not

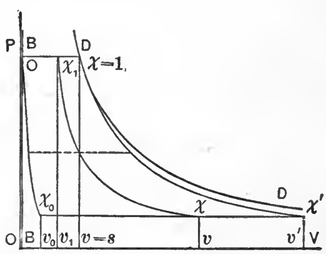

FIG. 28.

essential to the solution of the problem, it is interesting to draw the line of saturated steam *DD*, which forms a boundary between moist steam and superheated steam; a point to the left of *DD*, or between that line and *BB*, represents a mixture of water and steam, and a point to the right of *DD* represents superheated steam.

The points *O* and *D* represent the two extreme cases, pure water and dry steam; and the point x_1, between *O* and *D*, represents a mixture of water and steam. Equation (154) gives for a pressure p the following expressions for x_0, x, and x', corresponding to the initial conditions *O*, x_1, and *D* or $x = 1$:

$$\frac{x_0 r}{T} + \theta = \theta_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (149)$$

$$\frac{xr}{T} + \theta = \frac{x_1 r_1}{T_1} + \theta_1; \quad . \quad . \quad . \quad . \quad (150)$$

$$\frac{x'r}{T} + \theta = \frac{r_1}{T_1} + \theta_1. \quad . \quad . \quad . \quad . \quad (151)$$

Subtracting in succession equation (149) from (150) and (151), we have

$$(x - x_0)\frac{r}{T} = x_1 \frac{r_1}{T_1};$$

$$(x' - x_0)\frac{r}{T} = \frac{r_1}{T_1};$$

$$\therefore \frac{x - x_0}{x' - x_0} = x_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (152)$$

Designating the final volumes by v_0, v, and v', we have

$$v_0 = x_0 u + \sigma, \quad v = xu + \sigma, \quad v' = x'u + \sigma;$$

$$\therefore \frac{v - v_0}{v' - v_0} = \frac{x - x_0}{x' - x_0} = x_1; \quad . \quad . \quad . \quad . \quad (153)$$

$$\therefore v = x_1(v' - v_0) + v_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (154)$$

Now calculate x_0 and x' with the corresponding values of v_0 and v_1 in the usual method, and calculate v by the equation (154), and complete Fig. 28 by plotting the points x_0, x, and x' on the line x_0x' at a distance v_0x_0 from ov, equal to the pressure p.

The equations (149), (151), and (154) are true for any pressure p; for example, the pressure represented by the dotted line on Fig. 28, so that a sufficient number of points may be located and the three curves ox_0, x_1x, and Dx' may be drawn.

Thus far it appears that the labor of constructing the intermediate curve x_1x directly by the usual method would be less, but an advantage will be given by the new method when a large number of intermediate curves corresponding to different initial values of x are to be drawn; equation (153) will then give the several values of x with great rapidity. Fig. 28 is also useful in giving a comprehensive idea of the action of various mixtures of water and steam in a non-conducting cylinder, the proportions being as nearly correct as can be represented by a figure of the size.

External Work during Adiabatic Expansion.—Since no heat is transmitted during an adiabatic expansion, all of the intrinsic energy lost is changed into external work, so that, by equation (141),

$$W = E_1 - E_2 = \frac{1}{A}\left(q_1 - q_2 + x_1\rho_1 - x_2\rho_2\right). \quad . \quad (155)$$

The adiabatic curve cannot be well represented by an exponential equation; for if an exponent be determined for such a curve passing through points representing the initial and final states, it will be found that the exponent will vary widely with different ranges of pressure, and still more with different initial values of x; and that, further, the intermediate points will not be well represented by such an exponential curve, even though it passes through the initial and final points.

This fact was first pointed out by Zeuner, who found that the most important element in determining n was x_1, the initial condition of the mixture. He gives the following empirical formula for determining n, which gives a fair approximation for ordinary ranges of temperature:

$$n = 1.035 + 0.100x_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (156)$$

Rankine* proposed the exponential formula

$$pv^{\frac{10}{9}} = \text{const.}$$

for the expansion of saturated steam in a lagged cylinder without a steam-jacket.

It is probable that this equation was obtained by comparing the expansion lines on a large number of indicator diagrams. It corresponds nearly with the true adiabatic line for x_1, the initial value at cut-off, equal to 0.80. This equation has been largely used in England on account of the esteem in which Rankine's work is held; and as he does not state its origin, it appears to have been regarded as the true adiabatic line for a steam-engine.

There does not appear to be any good reason for using an exponential equation in this connection, for all problems can be solved accurately by the method given, and the action of a lagged steam-engine cylinder is far from being adiabatic. An adiabatic line drawn on an indicator card is instructive, since it shows to the eye the difference between the expansion in an actual engine and that of an ideal non-conducting cylinder; but it can be intelligently drawn only after an elaborate engine test. For general purposes the hyperbola is the best curve for comparison with the expansion curve of an indicator card, for the reason that it is the conventional curve, and is near enough to the curve of the diagrams from good engines to allow a practical engineer to guess at the probable behavior of an engine, from the card alone. It cannot in any sense be considered as the theoretical curve.

EXAMPLES.

1. Calculate the pressure, heat of the liquid, total heat, heat of vaporization, specific volume, etc., at several tempera-

* Steam-engine and Other Prime Movers.

tures for the vapors for which the data and equations are given, and compare with results given in the *Tables of the Properties of Saturated Steam.*

2. Find the exponent for an exponential curve passing through the points $p = 30$, $v = 1.9$, and $p_1 = 15$, $v = 9.6$.

3. Find the exponent for a curve to pass through the points $p = 40$, $v = 2$, and $p_1 = 12$, $v_1 = 6$.

4. In examples 2 and 3, let p be the pressure in pounds on the square inch, and v the volume in cubic feet; find the external work of expansion in each case.

5. Find the external work of expansion of a fluid, following the law given by the equation $pv^{\frac{7}{6}}$, which has the initial volume 3 cubic meters, and the initial pressure 4 atmospheres, and which expands till the pressure becomes one atmosphere.

6. A pound of steam and water at 150 pounds pressure is 0.6 steam; what is the increase of entropy above that of water at 32° F.?

7. A kilogram of chloroform at 100° C. is 0.8 vapor; what is the increase of entropy above that of the liquid at 0° C.? Apply to other vapors for which data are given.

8. The initial condition of a mixture of water and steam is $t = 320°$ F., $x = 0.8$; what is the final condition after adiabatic expansion to 212° F.? Solve with the following values for x: 0.9, 0.6, 0.4, 0.3, 0.0.

9. The initial condition of a mixture of steam and water is $p = 3000$ mm., $x = 0.9$; find the condition after an adiabatic expansion to 600 mm. Apply to other vapors for which data are given.

10. A cubic foot of a mixture of water and steam, $x = 0.8$, is under the pressure of 60 pounds by the gauge. Find its volume after it expands adiabatically till the pressure is reduced to 10 pounds by the gauge; also the external work of expansion.

11. A test of an engine with the cut-off at 0.106 of the stroke, and the release at 0.98 of the stroke, and with 4.5 per cent clearance, gave for the pressure at cut-off 62.2 pounds by the indicator, and at release 6.2 pounds; the mixture in the cylinder at cut-off was 0.465 steam, and at release 0.921 steam.

Find (1) condition of the mixture in the cylinder at release on the assumption of adiabatic expansion to release; (2) condition of mixture on the assumption of hyperbolic expansion, or that $pv = p_1v_1$; (3) the exponent of an exponential curve passing through points of cut-off and release; (4) exponent of a curve passing through the initial and final points on the assumption of adiabatic expansion; (5) the piston displacement was 0.7 cubic feet, find the external work under exponential curve passing through the points of cut-off and release; also under the adiabatic curve.

CHAPTER VIII.

SUPERHEATED STEAM.

A DRY and saturated vapor, not in contact with the liquid from which it is formed, may be heated to a temperature greater than that corresponding to the given pressure for the same vapor when saturated. Such a vapor is said to be superheated. When far removed from the temperature of saturation, such a vapor follows the laws of perfect gases very nearly, but near the temperature of saturation the departure from those laws is too great to allow of calculations by them for engineering purposes.

In the case of superheated steam various provisional characteristic equations have been proposed for use until the necessary experimental investigation shall give the data for a true theory. The theory given here was proposed by Zeuner. It is convenient for calculation and appears to give good results.

Substituting in the characteristic equation for a gas

$$pv = RT,$$

the value of R from equation (61) gives

$$pv = \frac{c_p - c_v}{A} T = \frac{c_p}{A} \cdot \frac{\kappa - 1}{\kappa} \cdot T. \quad . \quad . \quad (157)$$

The form of characteristic equation proposed by Zeuner for superheated steam is

$$pv = \frac{c_p}{A} \cdot \frac{k - 1}{k} \cdot T - Cp^a. \quad . \quad . \quad . \quad . \quad (158)$$

The specific heat at constant pressure c_p is assumed to be constant. k is a constant suggested by the ratio κ of the specific heats of a gas; but it will be shown that the specific heat at constant volume, determined from the equation (158), is a variable, consequently k cannot be the ratio of the specific heats of superheated steam. C and a are constants that are to be determined from the known properties of saturated and superheated steam.

Partial differentiation of equation (158) gives

$$\left(\frac{dt}{dv}\right)_p = \frac{Apk}{c_p(k-1)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (159)$$

$$\left(\frac{dt}{dp}\right)_v = \frac{Avk}{c_p(k-1)} + \frac{ap^{a-1}AkC}{c_p(k-1)} \quad . \quad . \quad . \quad . \quad (160)$$

Application of the First Law.—The application of the first law of thermodynamics by aid of equation (48),

$$\frac{1}{A}\left\{\left(\frac{do}{dp}\right)_v - \left(\frac{dn}{dv}\right)_p\right\} = 1$$

gives another form for $\left(\frac{dt}{dp}\right)_v$. Substituting for o and n in terms of the specific heats gives

$$\frac{d\left[c_p\left(\frac{dt}{dv}\right)_p\right]_v}{dp} - \frac{d\left[c_v\left(\frac{dt}{dp}\right)_v\right]_p}{dv} = A. \quad . \quad . \quad . \quad (161)$$

Substituting the value of $\left(\frac{dt}{dv}\right)_p$ from equation (159), and performing the differentiation indicated,

$$\frac{Ak}{k-1} - \frac{d\left[c_v\left(\frac{dt}{dp}\right)_v\right]_p}{dv} = A;$$

$$\therefore \quad \frac{d\left[c_v\left(\frac{dt}{dp}\right)_v\right]_p}{dv} = \frac{A}{k-1} \quad . \quad . \quad . \quad (162)$$

Integration gives

$$\left(\frac{dt}{dp}\right)_v = \frac{Av}{c_v(k-1)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (163)$$

Application of the Second Law.—Equation (55),

$$c_p - c_v = AT\left(\frac{dv}{dt}\right)_p\left(\frac{dp}{dt}\right)_v,$$

deduced by the successive application of the two laws of thermodynamics, can be most conveniently used in this place. Substituting the values of the partial differential coefficients from equations (159) and (163) gives

$$c_p - c_v = c_p c_v \frac{(k-1)^2}{k}\frac{T}{Apv};$$

$$\therefore \quad \frac{c_p}{c_v} = 1 + c_p\frac{(k-1)^2}{k} \cdot \frac{T}{Apv}, \quad . \quad . \quad . \quad (164)$$

which gives the method of calculating the specific heat at constant volume when c_p and k are known.

Value of the Exponent *a*.—Equating the values of the differential coefficient given by equations (160) and (163),

$$\frac{Av}{c_v(k-1)} = \frac{Avk}{c_p(k-1)} + \frac{ap^{a-1}AkC}{c_p(k-1)}.$$

Substituting for C from equation (158), and for c_v from (164), we have

$$\frac{Av}{c_p(k-1)}\left[1 + c_p\frac{(k-1)^2}{k} \cdot \frac{T}{Apv}\right] = \frac{Avk}{c_p(k-1)}$$
$$+ \frac{ap^{a-1}Ak}{c_p(k-1)}\left[\frac{c_p}{A} \cdot \frac{k-1}{k} \cdot \frac{T}{p^a} - \frac{v}{p^{a-1}}\right];$$

$$\therefore \quad \frac{Av}{c_p(k-1)} + \frac{k-1}{k} \cdot \frac{T}{p} = \frac{Avk}{c_p(k-1)} + a\frac{T}{p} - \frac{aAvk}{c_p(k-1)};$$

$$\therefore \quad \frac{k-1}{k} \cdot \frac{T}{p} - \frac{Av}{c_p} = a\frac{T}{p} - \frac{Av}{c_p} \cdot \frac{k}{k-1};$$

$$\therefore \quad a = \frac{k-1}{k}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (165)$$

Characteristic Equation.—Substituting the value deduced for a in equation (158) gives, for the characteristic equation for superheated steam,

$$pv = \frac{c_p}{A} \cdot \frac{k-1}{k} \cdot T - Cp^{\frac{k-1}{k}} \qquad (166)$$

Thermal Capacities.—From equations (11), (159), and (164),

$$l = c_v\left(\frac{c_p}{c_v} - 1\right)\left(\frac{dt}{dv}\right)_p = c_v c_p \frac{(k-1)^2}{k} \cdot \frac{T}{Apv} \cdot \frac{Apk}{c_p(k-1)};$$

$$\therefore \; l = c_v(k-1)\frac{T}{v}. \qquad (167)$$

From equations (15), (163), and (164),

$$-m = c_v\left(\frac{c_p}{c_v} - 1\right)\left(\frac{dt}{dp}\right)_v = c_v c_p \frac{(k-1)^2}{k} \cdot \frac{T}{Apv} \cdot \frac{Av}{c_v(k-1)};$$

$$\therefore \; -m = c_p \frac{k-1}{k}\frac{T}{p}. \qquad (168)$$

From equations (17) and (160),

$$o = c_p\left(\frac{dt}{dv}\right)_p = \frac{Apk}{k-1}. \qquad (169)$$

From equations (18) and (163),

$$n = c_v\left(\frac{dt}{dp}\right)_v = \frac{Av}{k-1}. \qquad (170)$$

General Equations.—Substituting the values of l, m, n, and o in equations (5), (6), and (7) give, for the general equations for superheated steam,

$$dQ = c_v\left\{dt + (k-1)\frac{T}{v}\,dv\right\}; \qquad (171)$$

$$dQ = c_p\left\{dt - \frac{k-1}{k} \cdot \frac{T}{p}\,dp\right\}; \qquad (172)$$

$$dQ = \frac{A}{k-1}\left\{vdp + kp\,dv\right\}. \qquad (173)$$

It is instructive to compare these equations with the general equations (70), (71), and (72) for perfect gases, which may be written,

$$dQ = \quad c_v \left\{ dt + (\kappa - 1)\frac{T}{v}\,dv \right\}; \quad . \quad . \quad (174)$$

$$dQ = \quad c_p \left\{ dt - \frac{\kappa - 1}{\kappa}\frac{T}{p}\,dp \right\}; \quad . \quad . \quad (175)$$

$$dQ = \frac{A}{\kappa - 1}\left\{ vdp + \kappa p\,dv \right\}. \quad . \quad . \quad . \quad . \quad (176)$$

To obtain equation (176), equation (72) may be written,

$$dQ = c_v \frac{v}{R}\,dp + c_p \frac{p}{R}\,dv;$$

$$\therefore \quad dQ = \frac{Ac_v v}{c_p - c_v} + \frac{Ac_p p}{c_p - c_v}\,dv.$$

(171)?

It is to be remarked that equation (174) is not useful in its present form, since c_v is a variable, but it is written for symmetry in comparison with equations (174), (175), and (176).

Entropy.—Equation (172) gives

$$d\phi = \frac{dQ}{T} = c_p \left\{ \frac{dt}{T} - \frac{k - 1}{k}\frac{dp}{p} \right\}; \quad . \quad . \quad . \quad (177)$$

$$\therefore \quad \phi - \phi_0 = c_p \left\{ \log_e \frac{T}{T_0} - \frac{k - 1}{k}\log_e \frac{p}{p_0} \right\}, \quad . \quad (178)$$

which is to be compared with equation (85) for gases. Equations in terms of v and t, or p and v may be deduced, which will also have the same form as those for gases.

Value of k.—The characteristic equation for superheated steam is intended to apply to all degrees of superheating, approaching, at one limit, the condition of a gas, and at the other, that of saturated vapor. For a mixture of a liquid and its vapor we have, from equation (145),

$$d\phi = d\left(\frac{xr}{T}\right) + \frac{c}{T}\,dt,$$

or, for saturated steam with $x = 1$,

$$d\phi = \frac{c}{T}\,dt + d\left(\frac{r}{T}\right) = \frac{1}{T}\left(cdt + dr - \frac{r}{T}\,dt\right). \quad . \quad (179)$$

Equations (177) and (179) should both be true for dry saturated steam, whence

$$c_p\left(1 - \frac{k-1}{k} \cdot \frac{T}{p} \cdot \frac{dp}{dt}\right) = c + \frac{dr}{dt} - \frac{r}{T}. \quad . \quad . \quad (180)$$

By equation (127) the right-hand member of equation (180) is equal to h, the specific heat of saturated steam; consequently

$$\frac{k-1}{k} = \frac{c_p - h}{\frac{T}{p} \cdot \frac{dp}{dt} \cdot c_p}. \quad . \quad . \quad . \quad . \quad . \quad (181)$$

Numerical Values.—Regnault gives as the results of three experiments on the specific heat of superheated steam at constant pressure

0.48111, 0.48080, 0.47963,

and for the mean value

$$c_p = 0.4805.$$

With this value of c_p and the known values of the other factors, determined from the properties of saturated steam, the following values of k were calculated:

Pressure, pounds on the sq. in.	5	50	100	200	300
k	1.335	1.332	1.330	1.324	1.316

Zeuner assumed for the constant k the value

$$k = \tfrac{4}{3} = 1.333+, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (182)$$

which may be compared with the ratio of the specific heats of air,

$$\kappa = 1.405.$$

With this assumed value of k and the known values of A and c_p the coefficient of T in the characteristic equation (166) becomes:

French system, $$\frac{c_p}{A}\frac{k-1}{k} = B = 51.28;$$

English system, $$\frac{c_p}{A}\frac{k-1}{k} = B = 93.46.$$

The specific volume of saturated steam under atmospheric pressure and at boiling-point is 26.60 cubic feet or 1.661 cubic metres. Solving equation (166) for C,

$$C = \frac{\dfrac{c_p}{A} \cdot \dfrac{k-1}{k} \cdot T - pv}{p^{\frac{k-1}{k}}};$$

and therefore we have—

French system,

$$C = \frac{51.28 \times 373.7 - 10333 \times 1.661}{10333^{\frac{1}{4}}} = 198.4;$$

English system,

$$C = \frac{93.46 \times 672.7 - 2116.32 \times 26.60}{2116.32^{\frac{1}{4}}} = 971.$$

Substituting the constants in the characteristic equation, gives—

French system, $$pv = 51.3T - 198p^{\frac{1}{4}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (183)$$

English system, $$pv = 93.5T - 971p^{\frac{1}{4}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (184)$$

Zeuner's constants for equation (183) differ from those given, since he used 424 for the mechanical equivalent of one calorie, and 273 for the absolute temperature of freezing-point, in this connection and in calculating his tables for saturated steam.

In using these equations for superheated steam it is to be remembered that the pressures are specific pressures, i.e., kilograms per square meter or pounds per square foot, whereas the pressures of saturated steam are commonly stated in millimeters of mercury or in pounds on the square inch.

Specific Heat at Constant Volume.—The specific heat of superheated steam at constant volume may be calculated by applying equation (164) to the case of saturated steam. The following table gives the values obtained at several pressures:

SPECIFIC HEAT OF SUPERHEATED STEAM.

Pressures, pounds per square inch,	5	50	100	200	300
Specific heat, c_v,	0.351	0.348	.346	.344	.341

This table develops the fact already mentioned, that the specific heat of superheated steam at constant volume, deduced from the form of the characteristic equation (166) and the known properties of saturated and superheated steam, is a variable. This conclusion applies properly to steam that is only slightly superheated, whereas our experimental knowledge of the properties of superheated steam relates to steam that is superheated to a marked degree. It is quite as reasonable to suppose that the specific heat at constant volume is constant, as to suppose that the specific heat at constant pressure is constant, as has been assumed. Had the specific heat at constant volume been assumed to be constant, and had the characteristic equation been assumed to have the form

$$pv = BT - Cv^{-b},$$

then the specific heat at constant pressure would have appeared to be variable. A complete set of equations could be worked out under such an assumption that would be on as good a basis as those we have deduced. The form that has been deduced appears to be more useful in engineering work where the pressures are more commonly given than the volumes.

Intrinsic Energy.—The combination of the equation

$$dQ = A(dE + pdv)$$

with equation (173) gives

$$dE = \frac{1}{k-1}(vdp + pdv) = \frac{1}{k-1}d(pv).$$

$$\therefore \quad E - E_0 = \frac{1}{k-1}(pv - p_0v_0). \quad . \quad . \quad . \quad . \quad . \quad (185)$$

In this equation it may be assumed that E_0 is the increase of intrinsic energy of saturated steam at atmospheric pressure, above that of water at freezing-point. From equation (140),

$$AE_0 = q_0 + \rho_0;$$

hence the increase of intrinsic energy of superheated steam, having the pressure p and the volume v, above that of water at freezing-point, is

$$E = \frac{1}{k-1}pv + \frac{1}{A}(q_0 + \rho_0) - \frac{1}{k-1}p_0v_0.$$

Taking the values of q_0, ρ_0, p_0, and v_0 from the tables of the properties of saturated steam at boiling-point, we have—

French system, $$E = \frac{1}{k-1}pv + 476.2; \quad . \quad . \quad . \quad . \quad . \quad (186)$$

English system, $$E = \frac{1}{k-1}pv + 857.2. \quad . \quad . \quad . \quad . \quad . \quad (187)$$

Total Heat.—By total heat of superheated steam is meant the heat required to change one unit of weight of water at freezing-point into superheated steam having a given temperature and pressure. It may be divided into the heat equivalent of the intrinsic energy and the heat equivalent of the external work. The first part may be calculated by equation (186) or equation (187). The second part may be assumed to be

$$Apv.$$

Using the same character that has been used for the total heat of saturated steam,

$$\lambda = \frac{A}{k-1} vp + Apv + \text{const.};$$

$$\therefore \quad \lambda = \frac{Ak}{k-1} vp + \text{const.}$$

Substituting for pv from equation (166)

$$\lambda = c_p\left(T - \frac{C}{B} p^{\frac{k-1}{k}}\right) + \text{const.}, \quad . \quad . \quad . \quad (188)$$

or replacing the constants by their known values, we have—

French system, $\lambda = 0.4805\,(T - 3.86p^{\frac{1}{4}}) + 476.2$; . . (189)

English system, $\lambda = 0.4805\,(T - 1.038p^{\frac{1}{4}}) + 857.2$. . . (190)

Comparison with Experiments.—Experiments on the specific volume of superheated steam were made by Hirn,* from the report of which Zeuner selected the experimental data in the following table. The specific volume has been calculated by aid of equation (183), and placed in the table opposite the experimental results to show the comparison of the characteristic equation with experiment.

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* Théorie Mécanique de la Chaleur.

SPECIFIC VOLUME OF SUPERHEATED STEAM.

Pressure in atmospheres.	Temperature. Centigrade.	Specific Volume. Cubic meters.	
		Hirn's experiments.	Equation (183).
1	118.5	1.74	1.75
1	141	1.85	1.87
3	200	0.697	0.699
4	165	0.4822	0.476
4	200	0.522	0.520
4	246	0.5752	0.577
5	162.5	0.3758	0.376
5	205	0.414	0.418

The following table shows that the characteristic equation for superheated steam applies fairly well to the limiting case of saturated steam. The values in columns 2 and 4 were taken directly from the tables of the properties of saturated steam, and those in column 6 were calculated in the usual manner for saturated steam with $x = 1$. The specific volumes in column 3 were calculated by aid of the empirical equation for the temperatures and pressures taken from tables for saturated steam. Columns 5 and 7 were calculated by equations (184) and (178). The latter equation gives a negative change of entropy for saturated steam for increasing pressure, which is to be taken from the entropy of steam at freezing-point.

APPLICATION OF EQUATION (184) TO SATURATED STEAM.

Absolute pressure, pounds per square inch. 1	Specific Volumes. Cubic feet.		Total Heat.		Entropy.	
	Tabular value. 2	Equation (184). 3	Tabular value. 4	Equation (190) 5	Equation (145). 6	Equation (178). 7
14.7	26.60	26.6	1146.6	1146.6	1.7484	1.752
30	13.59	13.7	1158.3	1158.4	1.6891	1.704
60	7.096	7.12	1171.2	1171.0	1.6340	1.641
100	4.403	4.38	1181.9	1180.3	1.5945	1.598
150	3.011	3.00	1191.2	1190.2	1.5649	1.568
200	2.294	2.30	1198.4	1198.5	1.5446	1.546
300	1.554	1.57	1209.3	1207.2	1.5262	1.517

Adiabatic Line.—Since the entropy remains constant during an adiabatic change, equation (178) gives

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (191)$$

This equation has of course the same form as the corresponding one for a gas, with the essential difference that k is an arbitrary constant, while κ is the ratio of the specific heats. Equations may also be deduced in terms of p and v, or T and v, *i.e.*,

$$v^k p = v_0{}^k p_0 = \text{const.}, \quad . \quad . \quad . \quad . \quad (192)$$

$$T v^{k-1} = T_0 v^{k-1} = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad (193)$$

During an adiabatic expansion, in which external work is done at the expense of the intrinsic energy, the degree of superheating is reduced, and if the expansion be carried far enough the vapor becomes saturated, and then moist. In such case the equation

$$\frac{r_1}{T_1} + \theta_1 + c_p \int \frac{dt}{T} = \frac{x_2 r_2}{T_2} + \theta_2$$

or

$$\frac{r_1}{T_1} + \theta_1 + 0.4805 \log_e \frac{T_s}{T_1} = \frac{x_2 r_2}{T_2} + \theta_2 \quad . \quad . \quad (194)$$

is taken.

For example, let the initial pressure be 100 pounds absolute per square inch, and the initial temperature be 400° F.; required the condition of the steam after an adiabatic expansion to 15 pounds absolute. Here we have

$$t_1 = 327°.6, \qquad r_1 = 884.0, \qquad \theta_1 = 0.4733,$$
$$t_2 = 213°.0, \qquad r_2 = 965.1, \qquad \theta_2 = 0.3143;$$

$$\therefore \quad \frac{884.0}{788.3} + 0.4733 + 0.4805 \log_e \frac{860.7}{788.3} = \frac{965.1 x_2}{673.7} + 0.3143;$$

$$\therefore \quad x = 0.923.$$

Isodynamic or Isoenergic Line.—The equation to this line is obtained from equation (185) by making E equal to E_0, so that

$$pv = p_0 v_0 = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (195)$$

The isodynamic line for superheated steam, like that for a gas, is a rectangular hyperbola.

The combination of equation (195) with the characteristic equation gives

$$BT - Cp^{\frac{1}{4}} = BT_1 - Cp_1^{\frac{1}{4}},$$

$$\therefore \quad T - T_1 = t - t_1 = \frac{C}{B}(p^{\frac{1}{4}} - p_1^{\frac{1}{4}});$$

from which it is evident that the isodynamic line for superheated steam differs from the isothermal.

The external work during an isodynamic change is

$$W = \int pdv = p_1 v_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e \frac{p_1}{p_2}. \quad . \quad . \quad (196)$$

Since all the heat applied is expended in external work,

$$Q = AW. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)$$

Isothermal Line.—The equation to the isothermal line is obtained from the characteristic equation by making T constant, so that

$$pv = p_1 v_1 - C(p^{\frac{1}{4}} - p_1^{\frac{1}{4}}). \quad . \quad . \quad . \quad . \quad (198)$$

The heat applied during an isothermal change is obtained by integrating equation (172) with T constant;

$$\therefore \quad Q = \frac{1}{4} c_p T \log_e \frac{p_1}{p_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

We have also

$$Q = A(E_2 - E_1 + W).$$

If the intrinsic energy of a unit of weight of the fluid in A is E_1, and of that in B is E_2, the energy changed into mechanical work is

$$E_1 - E_2,$$

to which is to be added the mechanical equivalent of the heat applied, or

$$\frac{Q}{A}.$$

Now the gain of kinetic energy of motion, if the velocity changes from w_1 in A to w_2 in B, will be, for each unit of weight of fluid,

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g}.$$

Assuming that all of the work applied produces change of velocity only,

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g} = \frac{Q}{A} + E_1 - E_2 + p_1v_1 - p_2v_2. \quad . \quad . \quad (201)$$

Incompressible Fluids.—The change of volume of most liquids under pressure is so small that it may be neglected, in which case the change of temperature may also be neglected and the intrinsic energy may be assumed to be constant.

Making $Q = 0$, the equation (201) reduces to

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g} = (p_1 - p_2)v_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (202)$$

since $v_1 = v_2$. If the velocity in A is small compared with that in B, we may suppress the subscript and write

$$\frac{w^2}{2g} = (p_1 - p_2)v_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (203)$$

which is the usual equation given in hydraulics. If the difference of pressure is due to a difference of level, or head, h, we have

$$p_1 - p_2 = h\gamma,$$

in which γ is the density, equal to $\frac{1}{v_1}$, so that

$$\frac{w^2}{2g} = h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (204)$$

Flow of Gases.—The most important case for gases is flow without transmission of heat—that is, an adiabatic flow; in which case Q becomes zero. Equation (84) gives

$$E = \frac{pv}{\kappa - 1},$$

so that equation (201) reduces to

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g} = \frac{p_1 v_1}{\kappa - 1} + p_1 v_1 - \left(\frac{p_2 v_2}{\kappa - 1} + p_2 v_2\right);$$

$$\therefore \frac{w_2^2}{2g} - \frac{w_1^2}{2g} = (p_1 v_1 - p_2 v_2)\frac{\kappa}{\kappa - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

Usually the initial velocity is zero, in which case the subscript may be dropped, and we may write

$$\frac{w^2}{2g} = (p_1 v_1 - p_2 v_2)\frac{\kappa}{\kappa - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (206)$$

For an adiabatic transformation

$$p_1 v_1^{\kappa} = p_2 v_2^{\kappa};$$

$$\therefore p_2 v_2 = p_1 v_1 \left(\frac{v_1}{v_2}\right)^{\kappa - 1} = p_1 v_1 \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}$$

$$\therefore \frac{w^2}{2g} = p_1 v_1 \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right]. \quad . \quad . \quad . \quad (207)$$

If the intrinsic energy of a unit of weight of the fluid in A is E_1, and of that in B is E_2, the energy changed into mechanical work is

$$E_1 - E_2,$$

to which is to be added the mechanical equivalent of the heat applied, or

$$\frac{Q}{A}.$$

Now the gain of kinetic energy of motion, if the velocity changes from w_1 in A to w_2 in B, will be, for each unit of weight of fluid,

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g}.$$

Assuming that all of the work applied produces change of velocity only,

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g} = \frac{Q}{A} + E_1 - E_2 + p_1v_1 - p_2v_2. \quad . \quad . \quad (201)$$

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which is the usual equation given in hydraulics. If the difference of pressure is due to a difference of level, or head, h, we have

$$p_1 - p_2 = h\gamma,$$

in which γ is the density, equal to $\frac{1}{v_1}$, so that

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$$E = \frac{pv}{\kappa - 1},$$

so that equation (201) reduces to

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g} = \frac{p_1v_1}{\kappa - 1} + p_1v_1 - \left(\frac{p_2v_2}{\kappa - 1} + p_2v_2\right);$$

$$\therefore \frac{w_2^2}{2g} - \frac{w_1^2}{2g} = (p_1v_1 - p_2v_2)\frac{\kappa}{\kappa - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

Usually the initial velocity is zero, in which case the subscript may be dropped, and we may write

$$\frac{w^2}{2g} = (p_1v_1 - p_2v_2)\frac{\kappa}{\kappa - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (206)$$

For an adiabatic transformation

$$p_1v_1^\kappa = p_2v_2^\kappa;$$

$$\therefore p_2v_2 = p_1v_1\left(\frac{v_1}{v_2}\right)^{\kappa - 1} = p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}$$

$$\therefore \frac{w^2}{2g} = p_1v_1\frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right]. \quad . \quad . \quad (207)$$

The characteristic equation for gases, and the application to it of the first law of thermodynamics, give the two equations

$$pv = RT, \qquad c_p - c_v = AR;$$

$$\therefore p_1 v_1 \frac{\kappa}{\kappa - 1} = p_1 v_1 \frac{c_p}{c_p - c_v} = \frac{c_p T_1}{A};$$

$$\therefore \frac{w^2}{2g} = \frac{c_p T_1}{A}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right]. \quad . \quad . \quad . \quad . \quad . \quad (208)$$

Again,

$$\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = \frac{T_2}{T_1},$$

so that

$$\frac{w^2}{2g} = \frac{c_p}{A}(T_1 - T_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (209)$$

If the area of the orifice is F, then the weight of air discharged per second is

$$G = \frac{Fw}{v_2};$$

or, substituting for w its value from equation (207),

$$G = F\left\{\frac{2gp_1v_1\kappa}{\kappa - 1}\right\}^{\frac{1}{2}}\left\{\frac{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}}{v_2^{\,2}}\right\}^{\frac{1}{2}};$$

or, substituting for v_2^2 its value

$$\left(\frac{p_1}{p_2}\right)^{\frac{2}{\kappa}} v_1^{\,2},$$

$$G = F\left\{\frac{2gp_1\kappa}{v_1(\kappa - 1)}\right\}^{\frac{1}{2}}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}}\right\}^{\frac{1}{2}}. \quad . \quad . \quad (210)$$

Now G will be a maximum when

$$\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}}$$

is a maximum. Differentiating and equating the first differential coefficient to zero gives after reduction

$$\frac{p_2}{p_1} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}.$$

Putting for κ its value 1.405, gives for the ratio of pressures producing the maximum flow

$$\frac{p_2}{p_1} = 0.5274.$$

The equations deduced for the flow of gases can properly be applied only to the flow from one tube into another of smaller diameter when the specific pressure (or, as a substitute therefor, the specific volume, or the temperature) in the smaller tube is known, as well as the initial condition in the large tube. In making experiments on the flow of gases and vapors, it has been customary to allow the fluid in the tube to discharge into the atmosphere or into a reservoir, and to assume that the pressure in the tube is the same as that in the reservoir: and when the lower pressure is less than half the upper pressure such experiments with that assumption show an actual flow greater than the calculated flow and often very much greater.

It was first suggested by Mr. R. D. Napier, in connection with experiments made by him on the flow of steam, that the pressure in the small tube is not necessarily the pressure of the atmosphere or of the reservoir into which it delivers; he further suggested that the pressure in the tube is never less than that pressure which gives a maximum flow.

Professor Fliegner* found that the pressure in the throat of a well-rounded orifice through which air is flowing is never less than 0.57 of the absolute pressure in the reservoir from which the flow takes place. The mean of a large number of

* Der Civilingenieur, vol. xx. p. 14. 1874.

experiments with two well-rounded orifices, 4.085 and 7.314 mm. in diameter at the throat, showed that when the pressure in the reservoir was more than double the pressure of the air, the pressure in the throat was 0.5767 of the pressure in the reservoir. The pressure in the reservoir varied from 3366 mm. of mercury to 808 mm. The number 0.5767 is very nearly equal to $\frac{1}{\sqrt{3}}$, and is to be compared with the ratio for maximum flow given above. In equation (210) substitute for v_1 its value from the equation

$$p_1 v_1 = RT_1,$$

and we have

$$G = F \frac{p_1}{\sqrt{T_1}} \sqrt{\frac{2g}{R}} \left\{ \frac{\kappa}{\kappa - 1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{k}} \right] \right\}^{\frac{1}{2}};$$

in which $\kappa = 1.405$ and $\frac{p_2}{p_1} = 0.5767$;

$$\therefore G = 0.4822\, F \sqrt{\frac{2g}{R}} \cdot \frac{p_1}{\sqrt{T_1}}.$$

For the flow into the atmosphere from a reservoir having a pressure less than twice the atmospheric pressure, Fliegner found the empirical equation

$$G = 0.5622\, F \sqrt{\frac{2g}{R}} \cdot \sqrt{\frac{p_a(p_1 - p_a)}{T_1}}.$$

These equations were found to be justified by a comparison with experiments on the flow of air, made by Fliegner himself, by Zeuner and by Weisbach.

Although these equations were deduced from experiments made on the flow of air into the atmosphere, it is probable that they may be used for the flow of air from one reservoir into another reservoir having a pressure differing from the pressure of the atmosphere.

Fliegner's Equations for Flow of Air.—Introducing the values for g and R in the equations deduced by Fliegner, we have the following equations for the French and English systems of units:

French Units.

$$p_1 > 2p_a, \qquad G = 0.395\, F \frac{p_1}{\sqrt{T_1}};$$

$$p_1 < 2p_a, \qquad G = 0.790\, F \sqrt{\frac{p_a(p_1 - p_a)}{T_1}}.$$

English Units.

$$p_1 > 2p_a, \qquad G = 0.530\, F \frac{p_1}{\sqrt{T_1}};$$

$$p_1 < 2p_a, \qquad G = 1.060\, F \sqrt{\frac{p_a(p_1 - p_a)}{T_1}}.$$

p_1 = pressure in reservoir;
p_a = pressure of atmosphere;
T_1 = absolute temperature of air in reservoir; degrees centigrade, French units; degrees Fahrenheit, English units.

In the English system p_1 and p_a are pounds per square inch, and F is the area of the orifice in square inches, while G is the flow of air through the orifice in pounds per second. If desired, the area may be given in square feet and the pressures in pounds on the square foot, as is the common convention in thermodynamics.

In the French system G is the flow in kilograms per second. The pressures may be given in kilograms per square meter and the area F in square meters; or the area may be given in square decimeters or square centimeters, and the pressures in kilograms on the same unit of area used in connection therewith. If the pressures are in millimeters of mercury, multiply by 13.5959; if in atmospheres, multiply by 10333.

Maximum Velocity of Flow.—According to the kinetic theory of gases, the pressure of a gas on the walls of the containing vessel is due to the impact of the molecules of the gas. To estimate the mean velocity of the molecules Joule* proceeds in the following manner: The weight of one cubic meter of gas is $\frac{1}{v}$, and the pressure which it exerts on each of the six sides of a cubical vessel containing it is p. Suppose that the weight $\frac{1}{v}$ of the gas to be divided into three equal portions, one of which oscillates between each pair of faces of the cube and produces the pressure by impact, first on one and then on the other of the pair. Now, if a body have a velocity equal to g, it will be brought to rest by a force equal to its weight acting on it for one second; and that force acting for two seconds will bring it to rest and then impart to it the same velocity in the opposite direction. In two seconds there will be g impacts on each of the pair of faces, and it will be assumed that the effect of the impacts is equal to that of a pressure equal to $\frac{1}{3gv}$ kilograms on each face; that is, on one square meter. The pressure will vary as the square of the velocity, since both the force required to reverse the velocity and the number of impacts increase with the velocity. Finally, Joule makes

$$u^2 = p \div \frac{1}{3gv},$$

in which u is the mean velocity of the molecules of the gas. This may be written

$$\frac{u}{\sqrt{3}} = \sqrt{gpv} = \sqrt{gRT}.$$

Fliegner assumes that the maximum velocity with which a gas can flow through an orifice is

$$w_{max.} = \sqrt{gRT_1} = 16.9\sqrt{T_1}$$

when the French system of units is used.

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* Memoir Phil. Soc., vol. ix. p. 107.

But if we make $\frac{p_2}{p_1} = 0.5767$ in equation (207), the velocity becomes, for French units,

$$w_{max.} = 17.1\sqrt{T_1}\,.$$

Weisbach's Experiments on the Flow of Air.—Weisbach * gives for the velocity of air through an orifice into the atmosphere

$$\frac{w^2}{2g} = 3p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{3}}\right],$$

and by its use he finds for the coefficient of flow, from his own experiments, the results given in the following tables:

FLOW OF AIR THROUGH AN ORIFICE.

Diameter 1 *centimeter.*

Ratio of pressures $\frac{p_1}{p_2}$	1.05	1.09	1.43	1.65	1.89	2.15
Coefficient	0.555	0.589	0.692	0.724	0.754	0.788

Diameter 2.14 *centimeters.*

Ratio of pressures $\frac{p_1}{p_2}$	1.05	1.09	1.36	1.67	2.01
Coefficient	0.558	0.573	0.634	0.678	0.723

FLOW OF AIR THROUGH A SHORT TUBE.

Diameter 1 *centimeter, length* 3 *centimeters.*

Ratio of pressures $\frac{p_1}{p_2}$	1.05	1.10	1.30
Coefficient	0.730	0.771	0.830

Diameter 1.414 *centimeters, length* 4.242 *centimeters.*

Ratio of pressures $\frac{p_1}{p_2}$	1.41	1.69
Coefficient	0.813	0.822

* Mechanics of Engineering.

Diameter 1 *centimeter, length* 1.6 *centimeters, orifice rounded.*

Ratio of pressures $\frac{p_1}{p_2}$	1.24	1.38	1.59	1.85	2.14
Coefficient	0.979	0.986	0.965	0.971	0.978

Flow of Saturated Vapor.—For a mixture of a liquid and its vapor equation (140) gives

$$E = \frac{1}{A}(q + x\rho),$$

so that equation (201) gives for the adiabatic flow from a receptacle in which the initial velocity is zero,

$$\frac{w^2}{2g} = \frac{1}{A}(q_1 - q_2 + x_1\rho_1 - x_2\rho_2) + p_1v_1 - p_2v_2. \quad . \quad (211)$$

Substituting for v_1 and v_2 from

$$v = xu + \sigma,$$

$$A\frac{w^2}{2g} = q_1 - q_2 + x_1\rho_1 - x_2\rho_2 + Ap_1x_1u_1 - Ap_2x_2u_2 + A\sigma(p_1 - p_2).$$

But

$$\rho + Apu = r;$$

$$\therefore A\frac{w^2}{2g} = x_1r_1 - x_2r_2 + q_1 - q_2 + A\sigma(p_1 - p_2). \quad . \quad (212)$$

The last term of the right-hand member is small, and frequently can be omitted.

The value of x_2 in the tube B, Fig. 29, at a distance from the orifice, can be determined by the equation

$$\frac{x_1r_1}{T_1} + \theta_1 = \frac{x_2r_2}{T_2} + \theta_2;$$

or, if the proper tables are lacking, we may use the approximate form,

$$\frac{x_2 r_2}{T_2} = \frac{x_1 r_1}{T_1} + \log_e \frac{T_2}{T_1}.$$

It is necessary to remember that while the tables commonly give the pressure in pounds on the square inch, or in atmospheres, etc., p_1 and p_2 in the last term of equation (212) are the specific pressures; that is, the pressures in pounds on the square foot, or kilograms on the square meter.

Substituting the first form for x_2, equation (212) gives

$$A\frac{w^2}{2g} = \frac{x_1 r_1}{T_2}(T_1 - T_2) + T_2(\theta_1 - \theta_2) + (q_1 - q_2) + A\sigma(p_1 - p_2). \quad (213)$$

Substituting the second form in equation (212), and neglecting the last term, we have the approximate formula

$$A\frac{w^2}{2g} = \frac{x_1 r_1}{T_1}(T_1 - T_2) + T_2 \log_e \frac{T_1}{T_2} + q_1 - q_2. \quad . \quad (214)$$

The weight of fluid that will pass through an orifice having an area of F square meters or square feet may be calculated by the formula

$$G = \frac{Fw}{x_2 u_2 + \sigma}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

The equations deduced are applicable to all possible mixtures of liquid and vapor, including dry saturated steam and pure hot water. In the first place steam will be condensed in the tube, and in the second water will be evaporated.

If steam blows out of an orifice into the air, or into a large receptacle, and comes to rest, the energy of motion will be turned into heat and will superheat the steam. Steam blowing into the air will be wet near the orifice, superheated at a little distance, and if the air is cool, will show as a cloud of mist, further from the orifice.

Napier's Formulæ for Flow of Steam.—As the result of a large number of experiments made on the flow of steam Mr. R. D. Napier concludes that the pressure in the throat of an orifice from which steam is flowing is never less than that pressure which, compared with the pressure in the reservoir, will give the maximum flow.

The following approximate equations may be used with the English system of units:

$$p_1 = \text{or} > \frac{5}{3} p_a, \qquad G = F \frac{p_1}{70};$$

$$p_1 < \frac{5}{3} p_a, \qquad G = F \frac{p_1}{42} \left\{ \frac{3(p_1 - p_a)}{2p_a} \right\}^{\frac{1}{2}};$$

in which p_1 is the pressure in the reservoir, and p_a is the pressure of the atmosphere, in pounds on the square inch, and G is the flow in pounds per second through an orifice having an area of F square inches.

Rankine* concludes, from an examination of Napier's experiments and a comparison of them with formulæ proposed by him, and a comparison of both with thermodynamic formulæ, that the principle that the pressure in an orifice is never less than that which gives the maximum flow is well substantiated, and that the above equations may be used for rough calculations.

Experiments on Flow of Steam.—The theory of the adiabatic flow of steam should apply to all mixtures of water and steam, including clear water, as from the water space of a boiler. Zeuner† points out an opportunity thus afforded of testing the equations, but states that experiments made by blowing water out of a locomotive boiler gave unsatisfactory results.

Some experiments were made by Mr. B. G. Buttolph‡ in the laboratories of the Institute of Technology on the flow of steam through a brass tube 0.275 of an inch in internal diame-

* The Engineer, vol. xxvii. p. 359. 1869.

† Mechanische Wärmetheorie.

‡ Proceedings Am. Mech. Eng. Soc. 1888.

ter and eight inches long, and having the entrance orifice rounded to reduce contraction. The results are given in the following table:

FLOW OF SATURATED STEAM.

Number of Experiment.	Gauge Pressures. Pounds per square inch. At Entrance.	At Exit.	Difference of Pressures.	Pressure of Atmosphere by Barometer. Pounds.	Flow in Pounds per Hour.
1	69.1	4.4	64.7	14.7	229.0
2	69.6	9.7	59.9	14.7	230.4
3	71.3	14.8	56.6	14.7	242.0
4	69.1	19.4	49.7	....	232.0
5	71.0	24.4	46.5	....	234.5
6	70.3	29.1	41.2	....	229.0
7	72.0	34.2	37.8	14.8	232.0
8	72.0	39.5	32.5	14.8	221.4
9	71.6	44.2	27.4	14.7	216.5

The following table gives the results of some experiments on the flow of steam through an orifice 0.25 of an inch in diameter, in a thin plate, made by Mr. G. P. Aborn * in the laboratories of the Institute.

FLOW OF STEAM THROUGH AN ORIFICE.

Number of Experiment.	Higher Pressure.	Difference of Pressure.	Flow in Pounds per Hour by Tank.
1	71.8	0.92	29.7
2	71.5	1.85	43.1
3	71.9	2.79	52.6
4	71.6	3.89	67.6
5	71.9	5.55	77.6
6	71.8	6.50	84.2
7	71.7	8.07	91.8
8	72.9	9.23	93.9
9	72.5	12.8	110.3
10	73.7	15.9	124.9
11	72.7	21.1	141.5
12	74.2	27.0	156.8
13	71.9	33.7	166.3
14	74.3	41.0	180.7
15	72.7	49.2	187.7
16	72.9	57.0	195.8
17	73.7	64.4	196.9
18	72.0	68.4	197.8

* Thesis, 1886.

Flow of Superheated Steam.—Equation (185) gives for the change of intrinsic energy

$$E_1 - E_2 = \frac{1}{k-1}(p_1v_1 - p_2v_2),$$

so that for an adiabatic flow

$$\frac{w^2}{2g} = (p_1v_1 - p_2v_2)\frac{k}{k-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (216)$$

which, by aid of equation (194), may be reduced to

$$\frac{w^2}{2g} = p_1v_1\frac{k}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]. \quad . \quad . \quad . \quad . \quad (217)$$

Equations (216) and (217) have the same form as the corresponding equations for a gas, since the expression for intrinsic energy has the same form for superheated steam as for a gas.

Substituting for p_1v_1 from the characteristic equation,

$$\frac{w^2}{2g} = 4(BT_1 - Cp^{\frac{1}{4}})\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{4}}\right]. \quad . \quad . \quad . \quad (218)$$

For calculation either equation (217) or (218) can be used, as may be convenient.

EXAMPLES.

1. Find the velocity of flow of air from the pressure of 6 atmospheres in a reservoir to the pressure of 5 atmospheres in the throat of the orifice; also, from 5 to 4 atmospheres, from 4 to 3, and from 3 to 2, the initial temperature in each case being 30° C.

2. Find the weight of air per second that will be discharged from an orifice 1 inch in diameter, from a reservoir having the temperature 60° F. and a pressure of 150 pounds per square inch, into the atmosphere. Calculate also with initial pressures 100, 50, 30, and 20 pounds absolute.

3. Find the weight of saturated steam per second, discharged through an orifice 1 inch in diameter, from a boiler having the

gauge pressure 60 pounds, into the atmosphere. Find also for the following values of x, 90, 80, 60, 50, 40, 20, and for hot water. Calculate also for initial pressures 80, 100, 150, 300 pounds by the gauge.

4. Find the velocity of flow of superheated steam with the initial temperature 600° F. and initial pressure 30 pounds by the gauge, when the pressure in the throat of the orifice is 20 pounds by the gauge.

5. In Example 4 find the weight per second discharged through an orifice 1 inch in diameter.

CHAPTER X.

INJECTORS.

AN injector is an instrument by means of which a jet of steam acting on a stream of water with which it mingles, and by which it is condensed, can impart to the resultant jet of water a sufficient velocity to overcome a pressure that may be equal to or greater than the initial pressure of the steam. Thus, steam from a boiler may force feed-water into the same boiler, or into a boiler having a higher pressure. The mechanical energy of the jet of water is derived from the heat energy yielded by the condensation of the steam-jet. Similar instruments are used in which a jet of steam or air imparts motion to a stream of air, or a jet of water imparts motion to a stream of water, without a change of heat into mechanical energy.

When the reservoir from which water is drawn is below the injector, the injector is called a *lifting injector;* but when the reservoir is above the injector, so that water will flow in under the action of gravity, it is called a *non-lifting injector.*

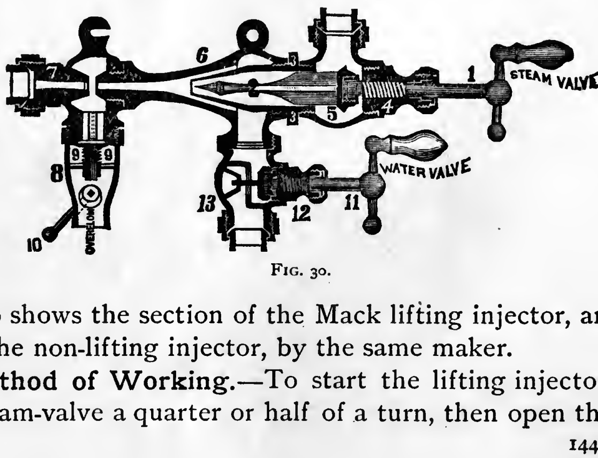


FIG. 30.

Fig. 30 shows the section of the Mack lifting injector, and Fig. 31 of the non-lifting injector, by the same maker.

Method of Working.—To start the lifting injector, open the steam-valve a quarter or half of a turn, then open the valve in the water supply; as soon as water appears at the overflow open the steam-valve until it ceases to overflow.

When the steam-valve is first opened a part of a turn the hollow spindle 5 is farther to the left and closes the orifice 6. A small stream of steam flows past 5, passes through the conical passage, and out at the overflow. The stream of steam flowing past 6 draws air with it from the chamber 5, and the partial vacuum thus produced draws water from the reservoir, which condenses the steam, and with it flows out at the over-

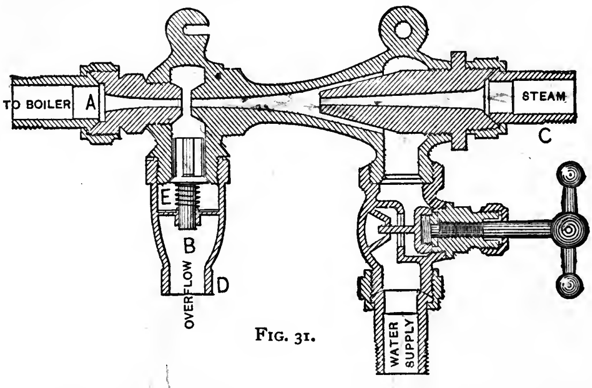


FIG. 31.

flow in a continuous stream. When this stream is well established the steam-valve is opened wide, and a large jet flows past 6, and is condensed in contact with the stream of water. The stream of water flowing through the conical passage has now sufficient velocity to leap across the opening at o and enter the conical passage 7, from whence it passes to the boiler. At the overflow is a valve, held open by a slender spring, which closes when the pressure at o is less than that of the atmosphere, so that air may not be forced into the boiler with the feed-water.

It is customary to have a valve in the steam-pipe above the injector, which is closed when the injector is not working, and which is opened before starting the injector. It is necessary to have a check-valve in the boiler feed-pipe to prevent the water in the boiler from flowing back through the injector when the injector is not working.

The action of the injector may be regulated, within limits, by manipulating the water- or steam-valve, or both. When the pressure of the steam is low or the lift small, it may be necessary to reduce the flow of water by partially closing the water-valve.

To start the non-lifting injector the steam-valve is opened to clear the supply-pipe of condensed water, and then it is closed. The water-valve is opened till water appears at the overflow, upon which the steam-valve is opened till water ceases to run out at the overflow.

It is apparent that the action and the construction of this form are simpler than those of the lifting injector.

Theory of the Injector.—The efficiency of an injector and the proper proportion of its parts cannot be determined entirely from the known properties of steam, the more especially as its action depends on the flow of steam. We shall first study its action under the assumptions that there is no loss from friction and radiation, and that we may use the equations for the adiabatic flow of steam. It will also be assumed that the jet of steam at 6, Fig. 30, is immediately and completely condensed by contact with the stream of water there.

The quantities to be ascertained are:

1. The velocity with which the steam issues from 6, Fig. 30. This depends on the pressure and quality of the steam in the supply-pipe, and the pressure of the orifice 6.

2. The quantity of feed-water that one pound of steam will force into the boiler. This depends on the temperature of the water in the reservoir, the temperatures of the water in the feed-pipe, and the pressure of the steam in the boiler from which steam is drawn and to which water is fed.

3. The velocity with which the stream of water passes the narrowest orifice at 7 on the way to the boiler.

4. The size of the steam and water orifices.

Velocity of the Steam-jet.—Assuming that the flow of steam from the orifice 6 is adiabatic, equation (212) gives

$$A\frac{w^2}{2g} = x_1r_1 - x_2r_2 + q_1 - q_2 + A\sigma(p_1 - p_2),$$

in which p_1 is the pressure of the steam in the supply-pipe, and p_2 the pressure in the orifice 6. x_1, r_1, and q_1 are the quality of the steam, the heat of vaporization, and the heat of the liquid of the steam at the pressure p_1; and x_2, r_2, and q_2 are the corresponding properties at the pressure p_2.

To determine x_2 we have

$$\frac{x_1 r_1}{T_1} + \theta_1 = \frac{x_2 r_2}{T_2} + \theta_2.$$

Quantity of Feed-water per Pound of Steam.—The number of pounds of feed-water y delivered by one pound of steam may be found by assuming only that the losses from friction and radiation may be neglected.

The gain of heat by the feed-water in passing from the temperature t_3 in the reservoir to the temperature t_4 in the feed-pipe is

$$y(q_4 - q_3).$$

The loss of heat in one pound of steam on condensation and reduction to the temperature t_4 is

$$x_1 r_1 + q_1 - q_4.$$

The heat equivalent of the kinetic energy of the jet of water at its smallest section, where the velocity is V, which energy is expended in forcing the water into the boiler, is

$$(1 + y)\frac{AV^2}{2g}.$$

The assumption of no loss of heat gives

$$y(q_4 - q_3) + (1 + y)\frac{AV^2}{2g} = x_1 r_1 + q_1 - q_4. \quad . \quad (219)$$

An approximate value of y can be obtained by neglecting the term containing V, so that

$$y = \frac{x_1 r_1 + q_1 - q_4}{q_4 - q_3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (220)$$

An exact value of y, in the general case, can be obtained by a series of approximations in combination with equation (222), the first approximation being obtained by aid of equation (220).

Velocity of the Water-jet.—Three cases may occur in different forms of injectors:

1. The water in the supply-pipe may have a head S, and the approaching water will have a momentum imparted to it by that head.

2. The water in the supply-pipe may be lifted through a height S, and a corresponding momentum must be imparted to it by the steam-jet.

3. There may be neither lift nor head, and the approaching water will have no momentum.

The first may be considered to be the general case, and may be made to include the other two by making the head negative for one and zero for the other.

The momentum of $1 + y$ pounds of water, at the smallest section of the water-jet, will be the sum of the momentum of one pound of moist steam in the steam-jet, *plus* the momentum imparted to $1 + y$ pounds of feed-water by the head S.

The momentum of $1 + y$ pounds of water in the water-jet is

$$\frac{(1 + y)V}{g}.$$

The momentum of one pound of steam is

$$\frac{w}{g}.$$

Let a be the area of the smallest section of the water-jet; then the force exerted by the head of S water having the density γ on that area is

$$S\gamma a_w.$$

This force acts on a mass

$$\frac{V\gamma a_w}{g}$$

of water per second, and imparts to it a velocity of

$$a_w S\gamma \div \frac{V\gamma a_w}{g} = \frac{Sg}{V}$$

feet per second, so that the momentum imparted to $1 + y$ pounds of water by the head S is

$$\frac{(1+y)}{g} \cdot \frac{Sg}{V} = (1+y)\frac{S}{V}.$$

Hence we have

$$(1+y)\frac{V}{g} = \frac{w}{g} + (1+y)\frac{S}{V};$$

$$\therefore (1+y)\left(V - \frac{Sg}{V}\right) = w;$$

$$\therefore V^2 - \frac{w}{1+y}V = Sg; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (221)$$

$$\therefore V = \frac{w}{2(1+y)} + \sqrt{\frac{w^2}{4(1+y)^2} + Sg}. \quad . \quad (222)$$

For the second case, in which the water is lifted by the injectors through a height h,

$$V = \frac{w}{2(1+y)} + \sqrt{\frac{w^2}{4(1+y)^2} - Sg}. \quad . \quad . \quad (223)$$

For the third case, in which the water enters the injector on the level of the jets, so that the momentum of one pound of steam in the steam-jet imparts the momentum which $1 + y$ pounds of water has in the water-jet, we have

$$\frac{(1+y)V}{g} = \frac{w}{g};$$

$$\therefore V = \frac{w}{1+y}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (224)$$

which may be obtained from equation (221) by making S zero.

Sizes of the Orifices.—Since one pound of steam feeds y pounds of water to the boiler, the steam required per second to feed G pounds of water will be

$$\frac{G}{y}.$$

The specific volume of the moist steam in this orifice is assumed to be

$$v_2 = x_2 u_2 + \sigma,$$

where x_2 is determined by the equation for the adiabatic change from the pressure p_1 in the boiler to the pressure p_2 before the orifice. Consequently the area of the steam-orifice in square feet is

$$a_s = \frac{G(x_2 u_2 + \sigma)}{yw}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

The steam used by an injector is returned to the boiler with the feed-water, so that

$$\frac{G(1 + y)}{y}$$

pounds of water per second pass through the water-orifice. Its area in square feet should therefore be

$$a_w = \frac{G(1 + y)}{\gamma y V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$

The density γ should properly be the weight of water per cubic foot, at the temperature in the feed-pipe, but the ordinary density 62.4 can be used instead.

In all of the foregoing work the English units have been used, but the equations may be applied to problems stated in the French units, kilograms, and meters, without change. They may also be applied to other vapors that behave like steam.

The diameters of the orifices of an injector are commonly given in millimeters or inches and fractions, while the areas by the formulæ are given in square feet or square meters. The reductions are readily made in every case.

PROBLEM.—Required the diameters of the orifices for an injector to deliver 1200 gallons of water per hour; the temperature of the feed-water being 180° F., and that of the water in the reservoir 100° F., while the pressure in the boiler is 45 pounds. Assuming the steam supplied to the injector to be dry, and that the pressure in the steam-orifice is 0.6 of the absolute boiler pressure, we have for the determination of the quality of the steam in the steam-orifice

$$\frac{909.5}{753.2} + 0.4263 = \frac{931.7}{721.2} x_2 + 0.3829;$$

$$\therefore x = 0.9683.$$

The last term of equation (212) may here be neglected, so that

$$\frac{Aw^2}{2g} = 909.5 - 902.1 + 261.6 - 229.7 = 39.3;$$

$$\therefore w = \sqrt{2 \times 32.2 \times 39.3 \times 778} = 1403.$$

The quantity of water delivered per pound of steam is

$$y = \frac{909.5 + 261.6 - 148.5}{148.5 - 68.0} = 12.70.$$

The velocity of the water-jet is

$$V = \frac{1403}{13.70} = 102.4 \text{ ft. per sec.}$$

The injector is required to deliver 1200 gallons an hour, or

$$\frac{1200 \times 231}{1728 \times 60 \times 60} = 0.04456 \text{ cubic ft. per sec};$$

$$\therefore G = 0.04456 \times 62.4 = 2.781 \text{ pounds per sec.}$$

The specific volume of the moist steam in the steam-orifice is

$$x_2 u_2 + \sigma = 0.9683 \times 11.50 + 0.016 = 11.15 \text{ cu. ft.}$$

The area of the steam-orifice is consequently

$$a_s = \frac{2.781 \times 11.15}{12.7 \times 1403} = 0.00174 \text{ sq. ft.,}$$

and the diameter is 0.55 of an inch.

By equation (226) the area of the water-orifice is

$$a_w = \frac{2.781 \times 13.7}{62.4 \times 12.7 \times 102.4} = 0.000469 \text{ sq. ft.,}$$

and the diameter is 0.22 of an inch.

Suppose that the injector was required to lift 20 feet, then the value of y is changed but little, and the values of w and x_2 not at all. The first approximation of V will be

$$V = \frac{1403}{2 \times 13.7} + \sqrt{\left(\frac{1403}{2 \times 13.7}\right)^2 + 20 \times 32.2} = 108.3$$

which gives for the diameter of the water-orifice 0.21 of an inch instead of 0.22, as found before.

Limits to the Action of an Injector.—If the height of a water column equivalent to the pressure p_1 in the boiler, *plus* the height of the lift, be represented by h, then when

$$h < \frac{V^2}{2g}$$

the water will enter the boiler with a residual velocity. If

$$h = \frac{V^2}{2g}, \qquad \qquad (227)$$

the water will enter the boiler without residual velocity. This fixes the limit of the action of the injector, since it will fail to work if

$$h > \frac{V^2}{2g}.$$

Injectors are commonly tested by the makers under the conditions of service, with the pressure in the delivery- or feed-pipe, 10 or 15 pounds above the pressure of the steam supplied, to insure that the water-jet shall be able to overcome the boiler-pressure and the added resistance of valves and pipes. A residual velocity is accompanied by a larger delivery by the injector and a higher temperature, and when the injector is used to feed a boiler is not accompanied by any loss of heat or energy, radiation and friction being neglected.

The height to which an injector will lift cold water depends on the form and proportions of the parts of the injector. Injectors are seldom set to lift more than 20 feet. Since hot water gives off vapor at a pressure depending on its temperature, it cannot be lifted to so great a height as cold water, either by an injector or a pump.

The temperature of feed-water delivered by an injector is limited by the fact that the steam used must be condensed at the pressure existing in front of the steam-orifice. The temperature may be higher with a small lift than with a high lift, and may be increased if the water be supplied under pressure.

The temperature of the water in the reservoir must be low enough to give the range of temperature required to condense the steam used.

A pair of orifices for the steam- and water-jets of an injector which give a considerable residual velocity with a range of temperatures well within the limits, will continue to work if the conditions be changed, but a limit is soon reached beyond which the injector will fail to act. Beyond the limit three cases may arise: (*a*) water will be wasted at the overflow; (*b*) steam will appear at the overflow; (*c*) air will be sucked in at the overflow. In the last case the air drawn in may break the continuity of the water-jet. The other two cases, in addition to the waste of water or steam, are liable to interrupt the action of the injector. They are certain to do so if the overflow-valve is closed after the injector is started.

Fixed Nozzle Injector.—The Mack injector shown by

Figs. 30 and 31 is of this type, that is, the water- and steam-orifices are both fixed. The action of the injector is controlled mainly by regulating the supply of steam at the steam-valve, though the supply of water may be reduced within limits at the water-valve. The overflow-valve allows water or steam to escape, but prevents air from entering. When the steam pressure is low or there is a head, it may be necessary to diminish the supply of water.

An injector of this type has been known to feed water to a boiler with both water- and steam-valve open, and with steam escaping at the overflow.

Giffard Injector.—This is the oldest form of injector in which the steam- and water-orifices were both adjustable. It had a hollow spindle, through which steam was first admitted to induce the flow of water, and this spindle moving inside a hollow cone regulated the supply of steam. The hollow cone was also movable, so that by it the water space surrounding it could be controlled.

Automatic Injectors.—With either of the two preceding examples of injectors, the change of boiler-pressure is liable to require a regulation by hand, lacking which the injector may stop. In some forms the instrument is made automatic, as, for example, the Sellers injector shown by Figs. 32 and 33.

A is the receiving or steam tube, which is opened or closed by the valve X. Through it passes a hollow spindle, to the inside of which steam is admitted by the valve W, which can be opened without raising X from its seat, by moving the spindle until the shoulder just touches that valve. This small motion of the spindle admits steam for raising water till it overflows at P. When this occurs the spindle is drawn back and the steam-valve X opened wide, upon which the action should be complete and water should be forced into the boiler. The rod L is connected to the handle H in such a manner as to close the overflow when the steam-valve is wide open. To diminish the amount of water delivered by the injector the steam-valve may be partially closed.

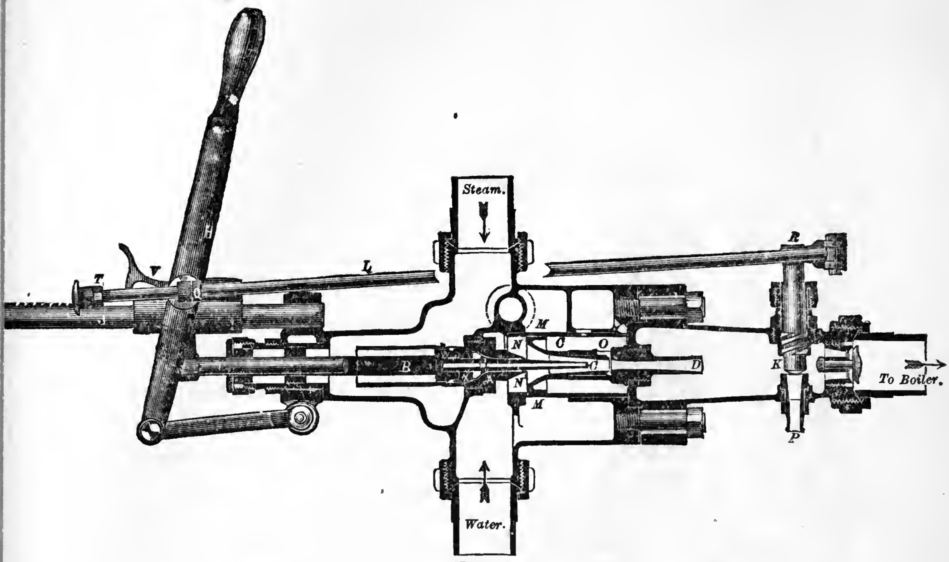

Steam.
R
T
V
H
L
M
N
C
O
G
D
B
A
K
To Boiler.
M
P
Water.

FIG. 32.

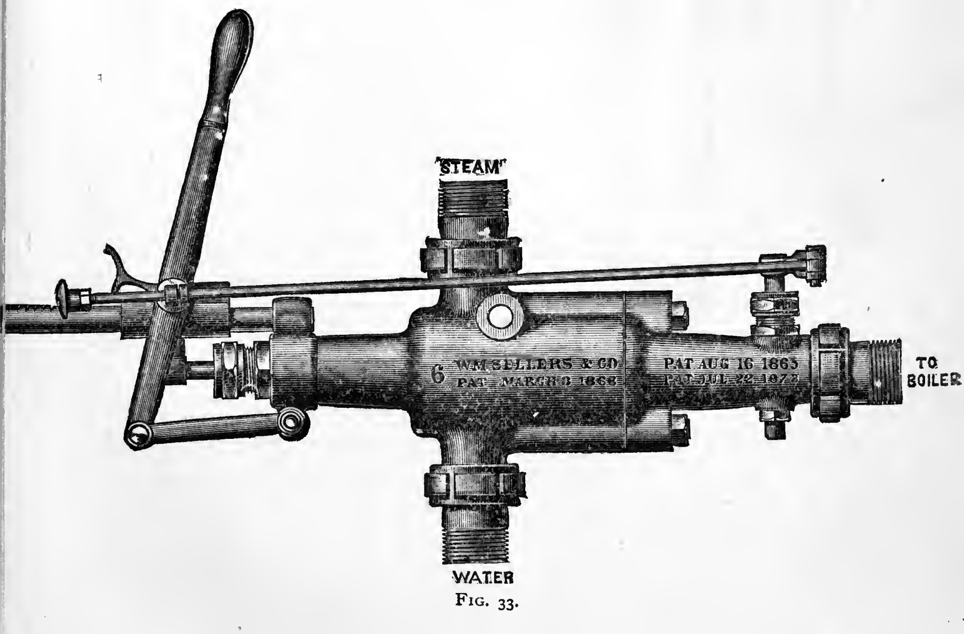

STEAM
6
WM SELLERS & CO
PAT MARCH 3 1868
PAT AUG 16 1865
PAT JUL 22 1873
TO BOILER
WATER

FIG. 33.

The supply of water is controlled automatically by the movable piston *NN*, which moves freely in the cylindrical shell *MM*, and is guided also at the forward end; as shown, it is as far forward as it can go. The impact of the water upon the piston tends to move it forward, and on the other hand water may pass through the orifice at *O*, and produce a pressure tending to move the piston backward. Thus the supply of water is automatically adjusted to the steam.

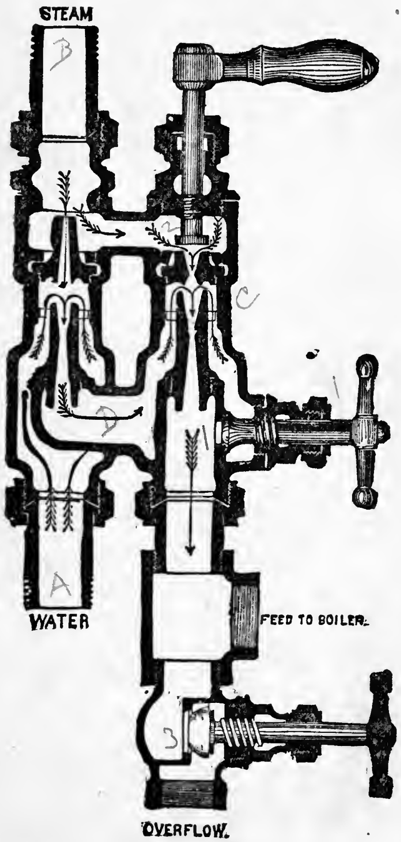


FIG. 34.

Double Injectors. — It was remarked that the temperature of the feed-water could be materially increased in an ordinary single injector by supplying the water under a head. The double injector consists of two parts, each essentially a single injector. The first part, called the lifter, draws water from the reservoir and forces it into a chamber, from which the second part, called the forcer, takes it and forces it to the boiler. The double injector also has the advantage that it is to a large extent self-regulating, since a rise of boiler-pressure which increases the flow of steam through the forcer-jet increases the flow of steam through the lifter-jet in like manner, and thus increases the supply of water, and also increases the pressure in the intermediate chamber.

All double injectors are fixed-nozzle injectors and are lifting injectors.

Fig. 34 shows the Hancock inspirator in section. The steam enters at *B* and flows to the lifter-jet directly beneath, and also supplies the forcer-jet at *C*, where there is a valve for controlling the flow. The water enters at *A* and is delivered by the lifter to the intermediate chamber *D*, from which it is taken by the forcer and delivered to the boiler. At 1 is a valve connecting the intermediate chamber with the delivery, and at 3 is the overflow-valve.

To start the inspirator, close 2, and open 1 and 3. Let on steam, and when water appears at the overflow close 1. Open 2 a quarter of a turn, and then close the overflow, upon which the water will be forced to the boiler. No adjustment is necessary for varying steam-pressure, but the quantity and temperature of the water delivered may be varied by varying the steam- or water-supply.

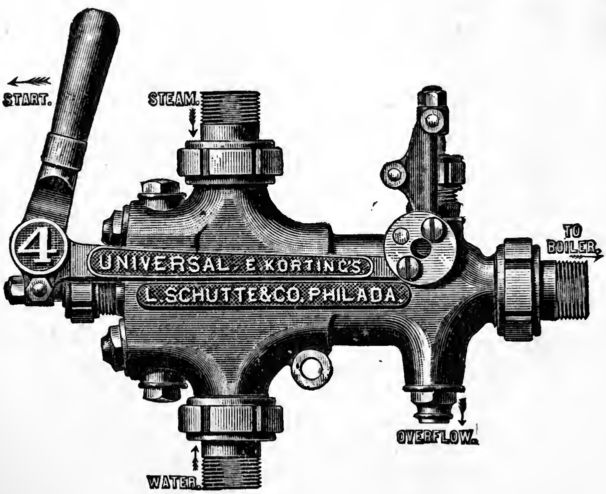


FIG. 35*a*.

The Korting injector is shown by Figs. 35*a* and 35*b*. The arrangement and action of the parts is evident without detailed description. The handle is moved a short distance till the lower or forcer valve, which opens first, has given steam to the forcer, and water appears at the overflow. Then the handle is

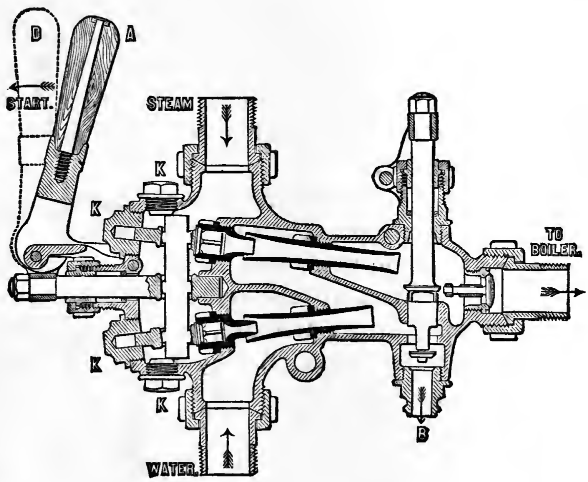


FIG. 35*b*.

pulled back as far as it will go. The overflow to the intermediate chamber closes when the forcer is started.

The Hancock inspirator is also arranged to work by one continuous motion of a handle, when it is applied to locomotives.

Tests of Injectors.—The table opposite gives the results of experiments made on the Sellers self-adjusting injector having the combining tube or water-orifice 6 mm. in diameter at the smallest section.

For each pressure of steam noted in column 1, the water was delivered by the injector into the boiler under approximately the same pressure. The delivery was measured by observing the indications of a water-meter. The pressures in column 8 were obtained by throttling the steam supplied to the injector, and observing the pressure at which it ceased to work, each experiment being repeated several times with precisely

EXPERIMENTS ON A SELLERS INJECTOR.

(Diam. Water-orifice 6 mm.)

Pressure of Steam supplied to Injector, and Pressure against which Water is delivered. *Lbs. per Sq. Inch.*	Delivery in Cubic Feet per Hour. Maximum.	Delivery in Cubic Feet per Hour. Minimum.	Ratio of Minimum to Maximum Delivery.	Temperature, Fahrenheit Degrees. Feed-water.	Temperature, Delivered Water. At Maximum Delivery.	Temperature, Delivered Water. At Minimum Delivery.	Pressure of Steam required to deliver Water against Pressure in Column 1.	Highest Temperature admissible of Feed-water, Fahr. Degrees.
1	2	3	4	5	6	7	8	9
10	75.3	63.6	0.845	66	100	94	3	132
20	82.4	61.2	0.743	66	108	104	9	134
30	94.2	56.5	0.600	66	114	116	16	134
40	100.1	60.0	0.599	66	120	123	22	132
50	108.3	64.7	0.597	66	124	125	27	131
60.	116.5	63.6	0.546	66	127	133	34	130
70	124.8	63.6	0.510	67	130	142	40	130
80	133.0	67.1	0.505	66	134	144	46	131
90	141.3	69.5	0.492	67	136	148	52	132
100	147.2	64.7	0.456	66	140	159	58	132
110	153.0	67.1	0.439	67	144	162	63	132
120	156.6	73.0	0.466	67	148	162	69	134
130	161.2	74.2	0.460	66	150	165	75	130
140	166.0	78.9	0.476	66	153	166	81	126
150	170.7	70.6	0.414	66	157	167	88	121

the same results. The temperatures in column 9 were obtained by gradually heating the water supplied to the injector, and noting the temperature at which it ceased to operate, each temperature recorded being checked by several repetitions of the experiment.

Some experiments were made in the laboratory of the Institute of Technology, by Messrs. Bradlee and Blanchard,* on several styles of injector, of which the results are given in the following table:

* Thesis. 1888.

TESTS OF INJECTORS.

Number.	Name of Injector.	Gauge Pressures. Pounds per Sq. Inch.			Barometer, Pounds per Sq. Inch.	Lift in Feet.	Suction in Inches of Mercury.	Temperatures, Fahrenheit.			Water supplied per Hour, Pounds.	Water delivered per Hour, Pounds.	Steam used per Hour, Pounds.	Water per Pound of Steam.		
		Boiler.	Intermediate.	Delivery.				Supply.	Intermediate.	Delivery.				By Experiment.	Calculated Steam Saturated.	Calculated 3 per cent Priming.
1	Hancock....	72.0	11.5	76	14.7	5.4	4.8	50.5	73.4	133.8	2111.4	2268.2	156.8	13.5	....	12.6
2	"	71.3	12.3	73	14.7	5.0	4.4	44.7	68.5	134.4	2013.4	2172.5	159.1	12.7	....	11.6
3	"	72.5	11.5	74	14.8	6.7	5.9	49.4	73.7	134.2	2170.0	2332.7	162.7	13.3	....	12.4
4	"	71.0	10.8	72	14.8	7.6	6.7	50.1	72.7	134.1	2126.4	2289.7	163.3	13.0	....	12.5
5	"	71.2	10.8	75	14.7	10.2	9.0	51.4	74.7	138.0	2000.0	2160.0	160.0	12.5	....	12.0
6	"	69.6	10.4	71	14.7	10.1	8.9	48.3	71.6	134.4	1983.4	2141.1	157.7	12.6	....	12.2
7	"	69.5	10.5	73	14.7	8.5	7.5	46.9	69.7	132.0	2000.0	2159.0	159.0	12.6	....	12.3
8	"	71.5	11.6	75	15.0	8.5	7.5	56.9	79.8	143.6	2022.5	2181.2	158.7	12.7	....	12.0
9	"	69.1	11.0	73	14.6	8.9	7.9	62.8	86.7	149.7	1962.9	2120.4	157.5	12.5	....	11.9
10	"	23.7	4.4	1.7	14.9	5.2	4.6	54.5	67.7	105.0	1607.8	1677.0	69.2	23.3	21.5	....
11	"	28.2	2.2	2.5	15.0	4.2	3.7	55.2	69.0	109.5	1661.3	1741.8	80.5	20.6	20.0	....
12	Lombard....	71.3	....	72	14.8	7.9	....	60.8	....	190.2	1424.6	1607.7	183.1	7.8	....	7.6
13	"	71.2	....	74	14.8	4.5	....	56.1	....	169.8	1661.3	1840.3	179.0	9.3	....	8.9
14	"	71.8	....	74	14.8	6.7	5.9	58.0	....	180.3	1536.5	1714.3	177.8	8.6	...	8.2
15	Dodge.......	70.3	....	75	14.7	5.4	4.8	61.9	....	144.1	1146.5	1235.0	89.5	13.0	12.9	12.6
16	"	69.8	....	73	14.8	7.8	6.9	61.4	....	150.7	1038.0	1132.2	94.2	11.0	....	11.5
17	"	71.3	....	74	14.9	10.4	9.2	60.0	....	161.3	939.5	1032.6	93.1	10.1	....	10.1
18	"	70.5	...	74	15.0	12.9	11.4	56.8	...	169.5	835.3	925.2	89.9	9.3	....	9.0
19	Hancock....	71.4	11.3	74	14.8	9.9	8.7	69.9	94.0	159.0	1937.6	2101.7	164.1	11.8	....	11.5
20	"	71.0	10.6	75	14.8	12.7	11.2	63.1	88.0	154.2	1871.1	2040.7	169.6	11.0	....	11.3
21	" ...	70.4	9.4	72	14.7	14.3	12.6	56.8	81.4	149.3	1857.6	2022.4	164.8	11.3	...	11.2

Sizes of Orifices.

Hancock:	lifter, steam,	. . . .	0.114
"	forcer, steam,	. . . .	0.206
"	" water,	. . . .	0.149
Lombard:	steam,	. . . .	0.224
"	water,	. . . .	0.164
Dodge:	steam,	. . . .	0.161
"	water,	. . . .	0.131

The experiments 15 to 21, inclusive, were made with improved methods of reducing the evaporation of the hot water delivered by the injector, and the results are more consistent and reliable than the preceding ones. It is apparent that the weight of steam used, which is obtained by taking the difference between the weights of water supplied and delivered, is diminished by the evaporation, and that consequently the experimental quantity of water delivered by one pound of steam is made too large thereby: this explains part of the discrepancy of the first fourteen experiments.

The calculations for Experiment 17 on the Dodge injector, on the assumption that the pressure in the steam-orifice is 0.6 of the absolute boiler-pressure, are as follows:

Pressure in the steam-orifice,

$$0.6 \times 86.2 = 51.7 \text{ pounds.}$$

Quality of steam in the steam-orifice on the assumption of 0.03 priming,

$$\frac{0.97 \times 891.8}{777.7} + 0.4592 = \frac{916x_2}{743.7} + 0.4138;$$

$$x_2 = 0.94.$$

Velocity of the steam in the steam-orifice,

$$\frac{w^2}{778 \times 64.4} = 865 - 861 + 287.3 - 253.3;$$

$$\therefore w = 1398 \text{ feet per second.}$$

Specific volume of steam in steam-orifice,

$$v_2 = 0.94 \times 8.239 + 0.016 = 7.76.$$

Flow of steam in pounds per second,

$$\frac{\pi(0.161)^2 \times 1398}{4 \times 144 \times 7.76} = 0.2547 \text{ pounds};$$

Napier's rule gives 0.2509 pounds.

The flow of steam per hour is 91.7 pounds by the calculation, instead of 93.1 by experiment.

The velocity of the water-jet is

$$V = \frac{1398}{10.1 + 1} = 126 \text{ feet per second.}$$

The head equivalent to 74 pounds per square inch is

$$\frac{74 \times 144}{62.4} = 170.7 \text{ feet.}$$

Adding, in the lift, 10.4 feet, gives 181.1 feet. The velocity required to overcome this head is

$$\sqrt{2 \times 32.2 \times 181.1} = 108 \text{ feet per second.}$$

If it be assumed that the jet in the water-orifice is entirely water, then the velocity calculated from the weight of water delivered per hour and the diameter of the water-orifice is

$$\frac{1032.6}{3600 \times 62.4} \div \frac{\pi(.131)^2}{4 \times 144} = 78 \text{ feet per second.}$$

From these calculations it appears that either the steam is not entirely condensed till after the smallest section of the water-orifice is passed, or else there is a *vena contracta*. A similar result was found by the makers of the Sellers injector for some of the experiments on page 159.

Exhaust-steam Injectors.—It has been pointed out that an injector may be made to deliver water against a pressure higher than the pressure of the steam supplied. An extreme example of this is found in injectors which use the exhaust steam of a non-condensing engine for feeding a boiler. Such an injector must be supplied with cold water: it cannot deliver water at a high temperature, and it carries with it some of the oil from the engine cylinder.

The Injector as a Pump.—The injector is commonly used as a boiler-feeder, in which office it may be advantageous as a feed-water heater, since all the heat not expended as work in forcing the water into the boiler is returned to the boiler with the water, though at a lower temperature. When used as a pump, to deliver water against a head, the heat given to the water is often thrown away, and may be prejudicial; for such work the injector has a very low efficiency.

Acids and solutions are sometimes raised by injectors of special construction made to resist the action of the fluids.

Let the height of one reservoir above the other, through which the water is to be raised, be H; then

$$H = h + s,$$

in which h is the head above the injector and s is the distance the water is lifted. It is desirable that there shall be little or no residual velocity; hence by equation (227)

$$V = \sqrt{2gh}.$$

Equation (221) for this case becomes

$$V^2 - \frac{w}{1 + y} V = - sg;$$

or, substituting for V from the equation above, and reducing

$$1 + y = \frac{w\sqrt{2gh}}{2gh + gs} = \frac{w\sqrt{h}}{\sqrt{2g}\left(h + \frac{s}{2}\right)}. \quad . \quad . \quad (228)$$

An inspection of equation (228) shows that the quantity of water raised per pound of steam increases with w, which depends on the steam-pressure and the quality of the steam, but is independent of the pressure in front of the orifice, when the steam-pressure is more than fifteen pounds by the gauge. The quantity of water raised per pound of steam decreases with the lift h_1 and the suction s. The greatest practical lift is about 26 feet.

PROBLEM.—Required the number of pounds of water raised per pound of steam, through a total height of 100 feet, the injector being 20 feet above the lower reservoir, and the steam pressure being 45 pounds by the gauge.

The velocity of the steam w is 1403 feet per second; consequently by equation (228)

$$1 + y = \frac{1403 \sqrt{80}}{\sqrt{2g}\left(80 + \frac{20}{2}\right)} = 17.4;$$

$$\therefore y = 16.4.$$

If the total height were 50 feet, then the number of pounds of water per pound of steam would be 22.9.

In the first case the work done is

$$16.4 \times 100 = 1640 \text{ foot-pounds,}$$

and in the second

$$22.9 \times 50 = 2290 \text{ foot-pounds.}$$

It consequently appears that it is more economical to raise water through large than through small heights by this method.

The following calculation shows that the consumption of steam by an injector used as a pump is extravagant when compared with a pumping engine. Let the injector be supposed to use one pound of steam per second; in the first case its horse-power is $\frac{1640}{550}$, and the consumption of steam per horse-power per hour is

$$\frac{550 \times 60 \times 60}{1640} = 1200 \text{ pounds.}$$

In the second case the consumption is 1700 pounds per horse-power per hour.

When the injector is used as a boiler-feeder its action is first that of a pump,—in which its efficiency, as has been seen, is small,—and second, that of a feed-water heater; and its second action is useful to such a degree as an independent feed-water heater using boiler steam would be. Feed-water may sometimes be heated by exhaust steam or by hot gases beyond the boiler, in which case heat that would otherwise be wasted is made useful. It is to be remarked that a much greater gain would result from the substitution of a condensing engine, in the first instance, were it possible to do so; and that, in the second instance, it is difficult to properly heat feed-water with the gases that leave an economical boiler.

Water-injector.—Fig. 36 represents a device called a water-injector, in which a small stream of water in the pipe M

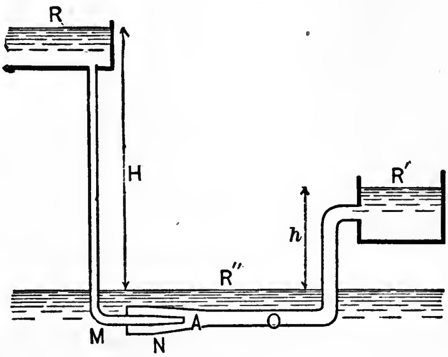


FIG. 36.

flowing from the reservoir R raises water from the reservoir R'' to the reservoir R'.

Let one pound of water from the reservoir R draw y pounds from R'', and deliver $1 + y$ pounds to R'. Let the velocity of the water issuing from A be v; that of the water entering from R'' be v_2 at N; and that of the water in the pipe O be v_1. The equality of momenta gives

$$v + yv_2 = (1 + y)v_1. \quad \quad (229)$$

Let x be the excess of pressure at M above that at N expressed in feet of water; then

$$v_2^2 = 2gx;$$
$$v^2 = 2g(H + x);$$
$$v_1^2 = 2g(h + x).$$

Substituting in equation (229),

$$\sqrt{H + x} + y\sqrt{x} = (1 + y)\sqrt{h + x};$$

$$\therefore y = \frac{\sqrt{H + x} - \sqrt{h + x}}{\sqrt{h + x} - \sqrt{x}}. \quad . \quad . \quad . \quad . \quad (230)$$

It is evident from inspection of the equation (230) that y may be increased by increasing x; for example, by placing the injector above the level of the reservoir so that there may be a vacuum in front of the orifice A.

If the weight G of water is to be lifted per second, then $\frac{G}{y}$ pounds per second must pass the orifice A, G pounds the space at N, and $(1 + y)G$ pounds through the section at O; which, with the several velocities v, v_2, and v_1, give the data for the calculation of the required areas.

PROBLEM.—Required the calculation for a water-injector to raise 1200 gallons of water an hour, $H = 96$ ft., $h = 12$ ft., $x = 4$ ft.

$$\sqrt{x} = \sqrt{4} = 2; \quad \sqrt{H \times x} = \sqrt{100} = 10; \quad \sqrt{h + x} = \sqrt{16} = 4.$$

$$y = \frac{10 - 4}{4 - 2} = 3.$$

The velocities are

$$v = \sqrt{2 \times 32.2 \times 100} = 80.25 \text{ feet per second;}$$
$$v_1 = \sqrt{2 \times 32.2 \times 16} = 32.10 \text{ feet per second;}$$
$$v_2 = \sqrt{2 \times 32.2 \times 4} = 16.05 \text{ feet per second.}$$

1200 gallons an hour $= 0.04452$ cubic feet per second.

The areas are

$$a = \frac{0.04452}{3 \times 80.25} = 0.000185 \text{ square feet;}$$

$$a_1 = \frac{4 \times 0.04452}{3 \times 32.10} = 0.06185 \text{ square feet;}$$

$$a_2 = \frac{0.04452}{16.05} = 0.00277 \text{ square feet.}$$

The diameters corresponding to the velocities v and v_1 are

$$d = 0.18 \text{ of an inch;}$$
$$d_1 = 0.58 \text{ of an inch.}$$

The area a_2 is of annular form, having the area 0.4 of a square inch.

Ejector.—The investigation of the injector used as a pump showed it to be a very wasteful machine, especially when the water was lifted through a small height. The efficiency is much improved by arranging the instrument as in Fig. 37, so that the steam nozzle *A* shall deliver a small stream of water at a high velocity, which, as in the water-injector, delivers a larger stream at a less velocity. Each additional conical nozzle increases the quantity at the expense of the velocity, so that a large quantity of water may be lifted a small height.

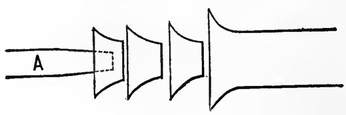


Fig. 37.

Ejectors are commonly fitted in steamships as auxiliary pumps in case of leakage, a service for which they are well fitted, since they are compact, cheap, and powerful, and are used only in emergency, when economy is of small consequence.

Ejector-condensers.—When there is a good supply of cold condensing water, an exhaust-steam injector, using all the steam from the engine, may be arranged to take the place of the air-pump of a jet-condensing engine. The energy of the exhaust steam flowing from the cylinder of the engine to the combining tube, where the absolute pressure is less and where the steam is condensed, is sufficient to eject the water and the air mingled with it against the pressure of the atmosphere, and thus to maintain the vacuum.

PROBLEM.—Suppose that the absolute pressure in the cylinder of an engine is 4 pounds absolute, and the pressure in the steam-orifice of an ejector-condenser is 2 pounds absolute: find the pounds of condensing water per pound of steam and the velocity of the water-jet. Let the exhaust steam contain ten per cent of moisture, then the velocity of the steam-jet is obtained from

$$\frac{Aw^2}{2g} = \frac{0.9 \times 1007.2}{613.8}(153.09 - 126.3) + 587(0.2203 - 0.1754) \\ + 121.4 - 94.4 = 69.2;$$

$$\therefore w = 1300 \text{ feet per second, nearly};$$

$$y = \frac{0.9 \times 1007.2 + 121.4 - 94.4}{68.01 - 28.12} = 22;$$

$$V = \frac{1300}{22 + 1} = 56.$$

The velocity required to overcome the pressure of 12.7 pounds per square inch is 43 feet per second.

EXAMPLES.

1. In the problem given on page 151 suppose that the steam contains 10 per cent of moisture and make all the required calculations.

2. In the same problem let the lift be 10 feet and make all calculations, the steam having 5 per cent of moisture.

3. An injector which feeds a boiler having a pressure of 6 atmospheres takes feed-water at 20° C., and delivers it at 80° C. How many kilograms of steam are fed by one kilogram of water?

4. An injector which feeds a boiler having a pressure of 100 pounds by the gauge delivers 16 pounds of water for each pound of steam: the initial temperature of the feed-water is 40° C.; what is the final temperature? No lift nor head.

5. Suppose an injector is used to supply an ether vaporizer in which the temperature is 100° C., how many kilograms of liquid will be delivered per kilogram of dry vapor, when the initial and final temperatures of the liquid fed are 10° C. and 50° C.?

6. Suppose that an injector supplied with dry steam at 3 atmospheres is used to pump chloroform against a head of 30 metres of the liquid. How many kilograms of liquid will be delivered by one kilogram of steam if the initial and final temperatures of the liquid are 0° and 20° C.?

7. An exhaust steam-injector is supplied with steam at atmospheric pressure. How many pounds of water per pound of steam will it deliver to a boiler at a pressure of 60 pounds by the gauge, the initial and final temperatures of the feed-water being 40° and 80° F.?

8. An injector is used to pump water against a head of 100 feet, it is set 20 feet above the cistern, and is supplied with dry steam at 80 pounds gauge pressure. How many pounds of water will be raised per pound of steam? What is the consumption of steam per horse-power per hour? If the initial temperature of the feed-water is 60° F., what will the final temperature be? What is the efficiency of the instrument as a heat-engine?

CHAPTER XI.

HOT-AIR ENGINES.

Engines of Maximum Efficiency.—In order to have the maximum efficiency, an engine must work on such a cycle that its working substance shall always have the temperature of the source of heat when acquiring heat, and the temperature of the refrigerator when rejecting heat; that is, the engine must be reversible.

The older forms of hot-air engines all had the source of heat at one constant temperature and the refrigerator at another lower constant temperature. To have the maximum efficiency it was required that the working substance should receive heat from external sources at one temperature, and reject heat to external sources at one temperature only.

Carnot's engine is the only simple engine which can fulfil these conditions when air is the working substance. The cycle of that engine has never been adopted in practice, since it involves incompatible requirements; that is, the isothermal changes should be very slow and the adiabatic changes should be very rapid, to make the cycle of an actual engine approximate to the ideal cycle.

By aid of a device called a regenerator or economizer, actual engines have been made which have an ideal cycle of maximum efficiency. Such a cycle is represented by Fig. 38. The curves DC and AB are isothermals, which form those parts of the cycle during which heat is received from the source and rejected to the refrigerator. The curves BC and DA correspond to the adiabatic lines of Carnot's cycle, and must fulfil the one condition, that the heat given to the regenerator during one operation, as that represented by BC, must be equal

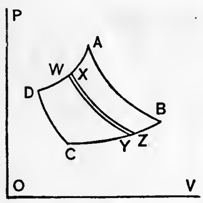


Fig. 38.

to the heat received from the regenerator during the converse operation DA.

The relation between the curves BC and DA may be determined as follows: Let the equations to BC and AD be

$$p = F(v), \qquad p = F_1(v);$$

then by aid of the characteristic equation of the working substance,

$$f(p, v, t) = 0,$$

v may be eliminated, giving

$$p = \phi(T), \qquad p' = \psi(T),$$

for the equations of the curves.

Draw the intermediate isothermals XZ and WY with a difference of temperature dt; then the heat received by one unit of weight of the substance in passing from W to X is

$$dQ = c_p dt + m dp, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (230)$$

and that rejected from Z to Y is

$$dQ = c_p dt + m' dp'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

The conditions of the problem will be fulfilled by making equation (230) equal to equation (231), so that

$$m dp = m' dp'.$$

Substituting for m from equation (53) gives

$$\left(\frac{dv}{dt}\right)_p dp = \left(\frac{dv'}{dt}\right)_p dp'.$$

Deducing the values of the partial differential coefficients from the characteristic equation for a gas, and substituting, we have

$$\frac{R}{p}dp = \frac{R}{p'}dp';$$

$$\therefore \frac{dp}{p} = \frac{dp'}{p'};$$

$$\therefore p = Cp'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (232)$$

That is, the required relation is that the ratio of the pressures at the points cut by any isothermal from the paths DA and BC must be constant.

Stirling's Engine.—This engine was invented in 1816, and was used with good economy for a few years, and then rejected because the heaters, which took the place of the boiler of a steam-engine, burned out rapidly. It is described and its performance given in detail by Rankine in his *Steam-engine*. An ideal sketch is given by Fig. 39. E is a displacer piston filled with non-conducting material, and working freely in an inner cylinder. Between this cylinder and an outer one from A to C is placed a regenerator made of plates of metal, wire screens, or other material, so arranged that it will readily take heat from or yield heat to air passing through it. At the lower end both cylinders have a hemispherical head; that of the outer cylinder is exposed to the fire of the furnace, and that of the inner is pierced with holes through which the air streams when displaced by the plunger. At the upper end there is a coil of pipe through which cold water flows. The working cylinder H has free communication with the upper end of the displacer cylinder, and consequently it can be oiled and the piston may be packed in the usual manner, since only cool air enters it.

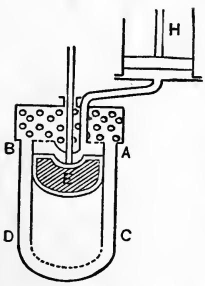


FIG. 39.

In the actual engine the cylinder H is double-acting, and there are two displacer cylinders, one for each end of the working cylinder.

If we neglect the action of the air in the clearance of the cylinder *H* and the communicating pipe, we have the following ideal cycle. Suppose the working piston to be at the beginning of the forward stroke, and the displacer piston at the bottom of its cylinder, so that we may assume that the air is all in the upper part of that cylinder or in the refrigerator, and at the lowest temperature T_2, the condition of one pound of air being represented by the point *D* of Fig. 40. The displacer piston is then moved quickly by a cam to the upper end of the stroke; while the working piston moves so little that it may be considered to be at rest. The air is thus all driven from the upper end of the displacer cylinder through the regenerator, from which it takes up heat abandoned during the preceding return stroke, thereby acquiring the temperature T_1, and enters the lower end of that cylinder. During this process, the line *AD* of constant volume is described on Fig. 40. When this process is complete, the working cylinder makes the forward stroke, and the air expands at constant temperature, this part of the cycle being represented by the isothermal *AB* of Fig. 40. At the end of the forward stroke the displacer piston is quickly moved down, thereby driving the air through the regenerator, during which process heat is given up by the air, into the upper part of the displacer cylinder; this is accompanied by a cooling at constant volume, represented by the line *BC*. The working piston then makes the return stroke, compressing the air at constant temperature, as represented by the isothermal line *CD*, and completing the cycle.

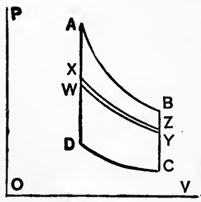


FIG. 40.

To construct the diagram drawn by an indicator, we may assume that in the clearance of the cylinder *H*, the communicating pipe, and refrigerator there is a volume of air which flows back and forth and changes pressure, but remains at the temperature T_2. If we choose, we may also make allowance for a similar volume which remains in the waste spaces at the lower end of the displacer cylinder, at a constant temperature T_1.

In Fig. 41, let $ABCD$ represent the cycle of operations, without any allowance for clearance or waste spaces; the minimum volume will be that displaced by the displacer piston, while the maximum volume is larger by the volume displaced by the working piston. Let the point E represent the maximum pressure, the same as that at A; and the united volumes of the clearance at one end of the working cylinder, of the communicating pipe, of the clearance at the top and bottom of the displacer cylinder, and the volume in the refrigerator and regenerator. Each part of this combined volume will have a constant temperature, so that the volume at different pressures will be represented by the hyperbola EF. To find the actual diagram $A'B'C'D'$, draw any horizontal line, as sy, cutting the true diagram at u and v, and the hyperbola EF at t; make ux and vy equal to st; then x and y are points of the actual diagram. The indicator will draw an oval similar to $A'B'C'D'$ with the corners rounded.

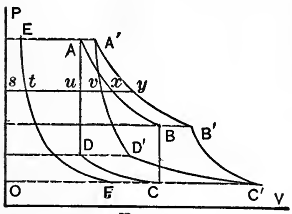


FIG. 41.

To show that the diagram, Fig. 40, fulfils the condition for maximum efficiency, draw an intermediate isothermal XY. Since DA and BC are lines of constant volume,

$$\frac{p_x}{p_d} = \frac{T_x}{T_d} = \frac{T_y}{T_c} = \frac{p_y}{p_c};$$

$$\therefore p_x = \frac{p_d}{p_c} p_y = Cp_y.$$

The diagram in Fig. 42 was reduced from an indicator-card from a recent hot-air engine made on the same principle as Stirling's hot-air engine. To avoid destruction of the lubricant in the working cylinder Stirling found it advisable to connect only the cool end of the displacer cylinder with the working cylinder, and had two displacer cylinders for one working cylinder. It has been found that a good mineral oil can be used to lubricate the displacer piston of the new engine, and that the hot end also of the displacer piston can be advantageously connected with the working cylinders, of which there are two. Thus each working cylinder is connected with the hot end of one displacer cylinder and with the cool end of the other displacer cylinder.

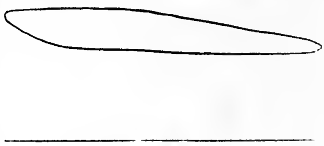


FIG. 42.

The distortion of the diagram Fig. 42 is due in part to the large clearance and waste space, and partly to the fact that the displacer pistons are moved by a crank at about 70° with the working crank.

Ericsson's Engine.—This engine consists essentially of a working cylinder, a compressing pump, and a reservoir. To give a perfect ideal cycle, a regenerator and a refrigerator are required. The pump, which must have a water-jacket which acts as a refrigerator, draws air from the atmosphere at constant pressure, compresses it at constant temperature, and forces it into the reservoir under constant pressure. The pump cycle is represented by the diagram *EDAF* (Fig. 43). The engine draws air from the reservoir through the regenerator, during which process it is heated from the temperature T_2 to T_1; the supply is then cut off by a slide-valve, and the air in the cylinder expands at constant temperature down to the atmospheric pressure. On the return stroke the air is forced from the cylinder at constant pressure through the regenerator, being thereby cooled to the temperature T_2. The engine cycle is represented by the diagram *FBCE*. The diagram of effective work is *ABCD*, which fulfils the condition of maximum efficiency, since *AD* and *BC* are isothermals, and *AB* and *CD* are lines of constant pressure.

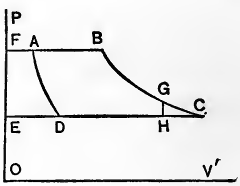


FIG. 43.

The actual engine does not expand down to the atmospheric pressure, so that the diagram is cut short by a line like *GH*. Also, the clearances of the two cylinders introduce irregularities and modifications of the diagram.

Gas-engines.—The various forms of gas-engines in common use are hot-air engines, in which the air is heated by the combustion of gaseous fuel mixed with the air. At full power there is one working stroke for two revolutions, the engines being single-acting. Fig. 44 gives the cycle commonly employed. At the end of the working stroke the pressure suddenly falls to that of the atmosphere, and on the return stroke the gases in the cylinder are expelled at atmospheric pressure. On the forward stroke the new charge of an explosive mixture of gas and air is introduced, and on the return stroke the charge is compressed. These three operations are represented by *CE*, *EC*, and *CD*. At the end of the compression stroke the charge is ignited, and the air and gases are heated at constant volume by a very rapid combustion or explosion. During the forward stroke the gases resulting from the explosion expand, doing work. The two operations completing the cycle are represented by *DA*, *AB*. The cylinder is kept cool by circulation of water to prevent its destruction by the intense heat of the explosion. This cooling influences both expansion and compression curves; also the expansion curve is modified by the fact that the explosion is not instantaneous, but continues throughout nearly the whole of the forward stroke.

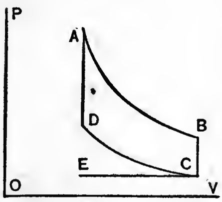


FIG. 44.

The efficiency of this cycle, on the assumption of instantaneous explosion and adiabatic expansion and compression, is easily found. For the works of expansion and compression we have

$$W_{ab} = \frac{v_a p_a}{\kappa - 1}\left\{1 - \left(\frac{v_a}{v_b}\right)^{\kappa - 1}\right\};$$

$$W_{dc} = \frac{v_d p_d}{\kappa - 1}\left\{1 - \left(\frac{v_d}{v_c}\right)^{\kappa - 1}\right\}.$$

But $$v_a = v_d, \quad v_b = v_c, \quad p_a = \frac{T_a}{T_d} p_d,$$

and the heat given to the gases by explosion, if c_v is the mean specific heat of the mixture at constant volume, is

$$Q = c_v(T_a - T_d);$$

$$\therefore \eta = \frac{Av_d p_d \left\{ 1 - \left(\frac{v_d}{v_c}\right)^{\kappa - 1} \right\} \left(\frac{T_a}{T_d} - 1\right)}{(\kappa - 1)c_v(T_a - T_d)};$$

$$\therefore \eta = 1 - \left(\frac{v_d}{v_c}\right)^{\kappa - 1}. \qquad (233)$$

The remarkable conclusion from this last equation is that the efficiency for such a cycle does not depend on the difference of temperatures, but rather on the degree of expansion and compression.

NOTE.—For a full discussion of the theory and practice of gas-engines refer to D. Clerk's Gas-engines.

pa Va = R Ta

CHAPTER XII.

THE STEAM-ENGINE.

Carnot's Cycle.—In the steam-engine the source of heat is ultimately the fire of the furnace, but the heat received by the engine is communicated at the temperature due to the steam-pressure in the boiler. In like manner the lower temperature is that due to the pressure of the vapor in the condenser, or, if none is used, it is the temperature of boiling-point under atmospheric pressure. Consequently heat is received and rejected at constant temperature; and as a regenerator cannot be used, the only cycle of perfect efficiency is Carnot's cycle. The advantage to be derived from the discussion of this cycle is that from it the maximum performance of steam-engines may be calculated from laboratory experiments only, which experiments are susceptible of a degree of refinement impossible with engine experiments. The efficiency of actual engines is always inferior to that of Carnot's cycle, because the cycle of such an engine is incomplete and non-reversible, and there are unavoidable losses from friction, leakage, and loss of heat.

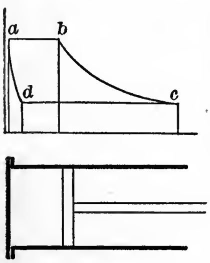


FIG. 45.

Let Fig. 45 represent the cylinder of Carnot's engine, using M pounds of a mixture of steam and water, together with the cycle of operations. Beginning at the point a, the cycle is as follows:

(1) Expansion at constant temperature and pressure, represented by ab, during which some of the water is vaporized, and the heat absorbed is

$$Q_1 = Mr_1(x_b - x_a). \quad . \quad . \quad . \quad . \quad . \quad . \quad (234)$$

(2) Expansion without communication of heat, represented by the adiabatic *bc*, till the temperature is reduced from T_1 to T_2. This expansion is accompanied by a condensation of steam, so that x_b becomes x_c, which may be calculated by the equation

$$\frac{r_2 x_c}{T_2} + \theta_2 = \frac{r_1 x_b}{T_1} + \theta_1. \quad . \quad . \quad . \quad . \quad . \quad (235)$$

(3) Compression at constant temperature and pressure, represented by *cd*, during which steam is condensed and heat is rejected to the amount

$$Q_2 = Mr_2(x_c - x_d). \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$

(4) Compression without communication of heat, represented by *da*, till the temperature is raised from T_2 to T_1 and the cycle is completed. For this operation we have

$$\frac{r_2 x_d}{T_2} + \theta_2 = \frac{r_1 x_a}{T_1} + \theta_1. \quad . \quad . \quad . \quad . \quad . \quad (237)$$

Equations (235) and (237) give

$$x_c - x_d = \frac{T_2}{r_2} \frac{r_1}{T_1} (x_b - x_a); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

$$\therefore Q_2 = M \frac{T_2}{T_1} r_1 (x_b - x_a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (239)$$

The efficiency is therefore

$$\frac{AW}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (240)$$

and the effective work is

$$W = \frac{Q_1}{A} \frac{T_1 - T_2}{T_1} = \frac{Mr_1(x_b - x_a)}{A} \frac{T_1 - T_2}{T_1}. \quad . \quad . \quad (241)$$

It is convenient for calculation to assume that at the beginning of the expansion represented by ab the mixture is all water, and that at the end it is all steam, so that $x_a = 0$ and $x_b = 1$. This assumption gives

$$W = \frac{Mr_1}{A} \frac{T_1 - T_2}{T_1}; \quad . \quad . \quad . \quad . \quad . \quad (242)$$

$$\therefore M = \frac{AW}{r_1} \cdot \frac{T_1}{T_1 - T_2}. \quad . \quad . \quad . \quad . \quad . \quad (243)$$

To find the water evaporated per horse-power per hour in such an engine, make W equal to $60 \times 33,000$ foot-pounds, and substitute for A its value, $\frac{1}{778}$, which gives the expression

$$\frac{60 \times 33000}{778} \cdot \frac{T_1}{r_1(T_1 - T_2)} = 2545 \frac{T_1}{r_1(T_1 - T_2)}. \quad . \quad (244)$$

The following table was calculated by aid of equations (240) and (241). The lower temperature for non-condensing engines is that of boiling-point of water under atmospheric pressure; for condensing engines it depends on the perfection of the vacuum maintained in the condenser, which was assumed in these calculations to be 1.5 pounds of absolute pressure. It should be remarked that the ratio of the steam consumption of two cycles, actual or ideal, is not necessarily the ratio of the efficiencies.

EFFICIENCY AND CONSUMPTION OF A PERFECT STEAM-ENGINE.

Initial Pressure by the Gauge, above the Atmosphere.	Condensing Engines.		Non-condensing Engines.	
	Efficiency $\frac{T_1 - T_2}{T_1}$.	Pounds of Steam per H. P. per Hour.	Efficiency $\frac{T_1 - T_2}{T_1}$.	Pounds of Steam per H. P. per Hour.
15	0.189	14.3	0.053	50.9
30	0.215	12.8	0.084	32.8
60	0.249	11.4	0.124	22.9
100	0.278	10.5	0.157	18.4
150	0.302	9.8	0.186	16.0
200	0.320	9.5	0.207	14.6
300	0.347	9.0	0.238	13.1

Equation (241) shows that the latent heat of evaporation is a measure of the amount of work that one unit of weight of a vapor can do in an engine; the larger the latent heat, the more the work per pound will be. Equation (240) shows that the efficiency does not depend on the latent heat nor on any other property of the working substance, as was shown in the second law of thermodynamics, of which this is a special case.

A comparison of Carnot's cycle for the steam-engine in Fig. 45 with that of Carnot's cycle for an air-engine, Fig. 23, shows that the chief difference is that the isothermal for a mixture of a liquid and its vapor is a straight line, while that of a gas is a hyperbola. In both, the heat is received and rejected during isothermal expansion and isothermal compression only. In the steam-engine the latent heat of vaporization is the property by means of which heat is received and rejected. In the air-engine the latent heat of expansion plays the same part. The fact that the latent heat of vaporization of steam is large, permits the steam-engine cylinder to be smaller than that of a hot-air-engine cylinder, though the true comparison of two such engines should include the weight and bulk of the whole apparatus, including for the steam-engine the boiler and condenser.

The old fallacy that the latent heat of vaporization of steam is the source of a loss that can be avoided by an engine using a substance which, like air, has no change of state, and hence no latent heat of vaporization, is apparent. The latent heat yielded by the steam to the condensing water is indeed large; but it is an unavoidable loss, and an essential action of a heat-engine.

The Actual Engine.—The cycle of all actual steam-engines is quite different from Carnot's cycle, and, as will be seen, it always has a less efficiency. Steam is generated in a boiler at constant pressure, and is carried to the engine through a pipe of more or less length, suffering in transit a loss of heat by radiation and a loss of pressure from friction. This steam is admitted to the cylinder for a portion of the stroke, during which time work is done by an isothermal expansion of the water and steam in the boiler. At the same time the entering steam yields heat to the walls of the cylinder and suffers partial condensation. After a portion of the stroke the supply of the steam is shut off, and the steam in the cylinder expands, doing work. In general, the walls of the cylinder give heat to the water previously condensed, causing a partial reëvaporation, though sometimes the walls receive heat during a part or the whole of the expansion. Near the end of the stroke the communication is opened with the condenser, and the steam quickly falls nearly to the constant pressure maintained in the condenser; when the expansion is carried so far as to reduce the pressure to that of the back pressure, the cycle is complete at this part; otherwise, it is incomplete. During the return stroke the piston expels the steam in the cylinder to the condenser, and the walls yield heat to the steam and water in the cylinder, nearly, if not completely, evaporating all the water remaining. Towards the end of the stroke the communication with the condenser is interrupted, and the steam remaining in the cylinder is compressed, and the temperature and pressure are raised. During the compression heat may be yielded by the steam to the walls, or conversely, depending on the extent of compression, and on whether a steam-jacket is used or not. If the compression be carried so far that the pressure is raised to the initial pressure, the cycle is complete at this part; otherwise, it is not. Just before the end of the stroke steam is admitted in anticipation of the next stroke.

The water resulting from the condensation of steam in the condenser is returned to the boiler by the air-pump and feed-pump, thereby forming a closed cycle of operations. The air-pump is required to remove the air which enters the boiler with the feed-water, and passes with the steam through the engine and into the condenser. If there were no air admitted to the boiler, engine, or condenser, with the feed-water or by leakage, one pump might take the water from the condenser and return it to the boiler. When a surface condenser is used, the same water is returned to the boiler and used continuously, thereby forming a true closed cycle. When a jet condenser is used, the steam and condensing water mingle, and are removed together by the air-pump to the hot well, whence the feed-water is drawn; but the effect is the same as though the same water was returned to the boiler. When no condenser is used, the steam escapes into the air, and the feed-water is drawn from a well or reservoir, the chief difference being then that there is a wide difference of temperature between the steam rejected and the water which replaces it.

The cycle described is clearly non-reversible. Heat is lost by radiation during every part of the process, even when precautions are taken to prevent such loss. Also heat is given to the cylinder at a high temperature during admission, and is restored at a varying but lower temperature during expansion, and the heat given by the walls of the cylinder during the exhaust is thrown into the condenser without compensation.

Considering the furnace, boiler, engine, and condenser as one apparatus, the maximum temperature is that of the fire, and the minimum that of the cooling water; but in considering the cycle of the cylinder, the highest temperature is that due to the pressure in the boiler, and the lowest that due to the pressure in the condenser.

When the expansion and compression are carried far enough so that the cycle is complete, the indicator-diagram is similar to that of Carnot's cycle, Fig. 45; but the expansion and compression curves are not adiabatics. If a non-conducting cylinder could be used, the cycle would still differ from Carnot's cycle, on account of the action of the air- and feed-pumps, and because the feed-water is heated from the lower to the higher temperature in the boiler, instead of having that change of temperature produced by compression.

A complete working theory of the steam-engine should take account of all of the actions named, including the action of the walls of the cylinder, the incomplete expansion and compression, the losses from radiation and friction, the action of the feed- and air-pumps, and the effect of the clearance or waste space at the ends of the cylinder.

Theories have been proposed by Rankine, Clausius, Zeuner, and others, which include most of the actions mentioned, but the exchange of heat between the walls of the cylinder and the steam is in no case included in such theories. Rankine* states the effect of water in the clearance space, and says that the thermal action of the walls of the cylinder has the same effect; but he makes no attempt at an estimation of the effect. Zeuner,† in connection with the discussion of the adiabatic curve of a mixture of water and steam, admits the thermal action of the walls, but asserts that the effect has been over-estimated, and that it is slight. He carries his opinion so far as to assert at the same place, that in locomotives the water mingled with the steam coming from the boiler amounts to from twenty-five to thirty per cent of the whole; that being the percentage required to make the adiabatic curve agree with the actual expansion curve of the indicator-card. In a problem he assumes fifteen per cent of priming. Again, he states that the steam in the clearance space is superheated by the compression, as it would be, if nearly dry, in a non-conducting cylinder. In later writings he has receded in part from this position.

Comparison with the experimental performance of engines shows that these theories are frequently in error by a large amount, which must be due to the neglect of the effect of radiation and of the thermal action of the walls of the cylinder. All investigators from the time of Watt have known that the omission of these actions involved an error, and their failure to include them in their theories is due, in most part, to the lack of experimental data. The theories are to be considered as first approximations, without which the existence and amount of the error would never have been known; and the complete theory is to be obtained by the insertion of the missing quantities in the old theories, which with this exception are correct in principle.

At the present time such a complete theory cannot be stated, because the form even of the factors depending on the thermal action of the walls has not been discovered, and all attempts to introduce such factors have led to unsatisfactory re-

* Steam-engine and other Prime Movers, p. 421.

† Mechanische Wärmetheorie, pp. 421, 486, 499.

sults. The proper direction of investigation appears to be to determine the thermal action of the walls quantitatively, in engines of different forms, and working under different conditions, so that bad construction and methods of working may be avoided, and ultimately a complete theory may be constructed. Though a large amount of experimental information exists, the larger part cannot be used for this purpose, because the experiments, though sufficient for the purposes sought by the experimenters, fail to give certain essential data. This, together with the complexity of the subject, has made all attempts to complete the theory unsatisfactory up to the present time.

Hirn's Analysis.—The best analysis of the action of the steam in the cylinder of an engine is given by Hirn,* and the most complete experiments have been made in accordance, by his direction or under his inspiration. He calls the old form of theories *generic theories*, while his form he calls the *practical theory*. His method cannot be considered as a complete theory, since it does not allow us to predict the action of a new form of engine. It does, however, enable us to determine the real behavior of existing engines.

The clearest statement of Hirn's theory is derived from some memoirs by Zeuner,† in which the equations are deduced by the ordinary methods of thermodynamics, in better form than that given by Hirn himself. The memoirs are written in criticism of Hirn's methods and conclusions, but the equations are accepted by both writers, as in fact they must be, whatever the conclusions from their application may be.

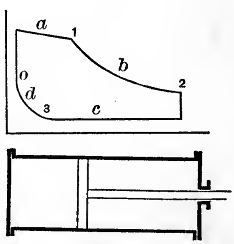


FIG. 46.

Let Fig. 46 represent the cylinder of a steam-engine and the diagram of the actual cycle without lead of admission or release. Let the weight of the mixture of steam and water admitted per stroke be M, of which the part Mx is steam and $M(1-x)$ is water. The condition of the mixture is known

* Théorie Mecanique de la Chaleur, Tome II.

† Revue universelle des Mines, vol. xi. p. 15, vol. xiii. p. 1.

from the pressure p, with which it enters the cylinder, and from x. Let the volume of the clearance be V_0, that of the piston displacement up to cut-off be V_1; let the total piston displacement be V_2, and the volume to be displaced by the piston between compression and the end of the stroke be V_3. Let p_1, p_2, p_3, and p_0 be the pressures at cut-off, at the end of the stroke, at compression, and at admission, while the corresponding values of x are distinguished by the same subscripts. Finally, let the weight of the water and steam in the clearance space during compression be M_0.

The heat required to raise M units of weight of water from freezing-point to the temperature t corresponding to p, and to evaporate the portion Mx, is

$$Q = M(q + xr). \qquad (245)$$

For steam superheated to the temperature t_x,

$$Q = M[q + r + c_p(t_x - t)]. \qquad (246)$$

The internal heat of the mixture in the clearance space at admission is

$$M_0(q_0 + x_0\rho_0).$$

At cut-off the internal heat of the mixture in the cylinder is

$$(M + M_0)(q_1 + x_1\rho_1).$$

During the admission the external work done is W_a, and the walls of the cylinder have absorbed the heat Q_a; hence

$$Q + M_0(q_0 + x_0\rho_0) = AW_a + Q_a + (M + M_0)(q_1 + x_1\rho_1). \quad (247)$$

In case the steam remains superheated up to the point of cut-off, this equation cannot be used, but that condition does not commonly occur in practice. As the heat absorbed by the walls is given a positive sign, the heat yielded should have the negative sign; but it is more convenient to give the positive sign to all, and then in calculation of a problem the numerical values will receive the proper sign to signify in which way the heat passes.

The internal heat of the mixture in the cylinder at the end of expansion is

$$(M + M_0)(q_2 + x_2\rho_2),$$

the external work done is W_b, and the heat yielded by the walls of the cylinder is Q_b. As stated above, the numerical value of Q_b when heat is yielded has the negative sign; if heat is given to the walls during expansion, which may occur if the steam is very strongly superheated, the numerical value is negative, but in such case equation (247) cannot be used. Since no heat comes from sources outside of the cylinder,

$$(M + M_0)(q_1 + x_1\rho_1) = AW_b + Q_b + (M + M_0)(q_2 + x_2\rho_2). \quad (248)$$

During the exhaust the work W_c is done by the engine on the steam, and the walls of the cylinder yield the heat Q_c; the heat carried away by the water resulting from the condensation of steam in the condenser is Mq_4, q_4 being the heat of the liquid corresponding to the pressure in the condenser; the heat carried away by the cooling water is $G(q_k - q_i)$, G being the pounds of cooling water per stroke, and q_i and q_k the heats of the liquid at the initial and final temperatures; the internal heat in the steam caught in the cylinder at compression is

$$M_0(q_3 + x_3\rho_3).$$

Combining these equations, we have

$$(M + M_0)(q_2 + x_2\rho_2) + AW_c$$
$$= Q_c + Mq_4 + G(q_k - q_i) + M_0(q_3 + x_3\rho_3). \quad (249)$$

During compression the work done on the steam is W_d, and the heat transferred to or from the walls of the cylinder is Q_d. The intrinsic energy at the end of compression is

$$M_0(q_0 + x_0\rho_0);$$

$$\therefore M_0(q_3 + x_3\rho_3) + AW_d = Q_d + M_0(q_0 + x_0\rho_0). \quad (250)$$

The four equations (246) to (250) enable us to determine the interchange of heat during each operation of the cycle, provided that a sufficient number of data can be obtained experimentally. The indicator-card furnishes means of determining the work done by or on the steam in each operation, and also gives the pressures from which the values of q and ρ can be found. The weight M may be determined by weighing the feed-water or else the condensed steam, when a surface condenser is used. The weight of cooling water must be determined by direct measurement; also the temperature t_4, t_κ, and t_i must be observed directly. If the entering steam is moist, x must be determined by a calorimeter experiment; if it is superheated, t_x may be observed by a thermometer in the steam-pipe near the engine. The values remaining are M_0, x_0, x_1, x_2, and x_3.

The specific volume of a mixture of steam and water is

$$v = xu + \sigma;$$

or, since the volume occupied by the water in the cylinder is small compared with that occupied by the steam,

$$v = xu, \quad \text{and} \quad V = Mxu. \quad \ldots \ldots \quad (251)$$

If desired, the exact value of v may be carried into the equations, but at the expense of a complication which does not seem necessary in the present state of the subject.

From equation (251),

$$M_0x_0u_0 = V_0; \quad \ldots \ldots \quad (252)$$

$$M_0x_3u_3 = V_0 + V_3; \quad \ldots \ldots \quad (253)$$

$$(M + M_0)x_1u_1 = V_0 + V_1; \quad \ldots \ldots \quad (254)$$

$$(M + M_0)x_2u_2 = V_0 + V_2. \quad \ldots \ldots \quad (255)$$

Substituting in equations (246) to (250), and solving, for the quantities of heat interchanged between the steam and the walls of the cylinder, we have

$$Q_a = Q + M_0 q_0 - (M + M_0) q_1 + V_0 \frac{\rho_0}{u_0} - (V_0 + V_1) \frac{\rho_1}{u_1} - A W_a ; \quad (256)$$

$$Q_b = (M + M_0)(q_1 - q_2) + (V_0 + V_1) \frac{\rho_1}{u_1} - (V_0 + V_2) \frac{\rho_2}{u_2} - A W_b ; \quad (257)$$

$$Q_c = (M + M_0) q_2 - M_0 q_3 - M q_4 - G(q_k - q_i) + (V_0 + V_2) \frac{\rho_2}{u_2}$$
$$- (V_0 + V_3) \frac{\rho_3}{u_3} + A W_c ; \quad (258)$$

$$Q_d = M_0 (q_3 - q_0) + (V_0 + V_3) \frac{\rho_3}{u_3} - V_0 \frac{\rho_0}{u_0} + A W_d . \quad . \quad . \quad . \quad (259)$$

In these last four equations the only quantity on the right-hand side which is not determinable by direct observation is M_0. These equations assume that the steam is always in equilibrium, or that the energy due to velocity, eddies, and commotion is inappreciable.

To determine M_0, Hirn makes the assumption that x_3 is unity, or that the steam is dry at the beginning of compression. In some other experiments, in which the compression is small and the vacuum good, he assumes that M_0 is so small that it may be neglected. The first assumption is one commonly made, and the second cannot cause much error under the conditions given, if the first is allowable. Zeuner points out that if M_0 is taken too small, the equations deduced will show a much greater apparent action of the walls than really occurs, and he states that the assumption of a sufficiently large value of M_0, i.e., of a large enough amount of water in the clearance, will make the terms Q_a, Q_b, Q_c, and Q_d very small.

The total work deduced from the indicator is

$$W = W_a + W_b - W_c - W_d . \quad . \quad . \quad . \quad . \quad . \quad (260)$$

If the heat lost by radiation during one stroke is Q_e and that furnished by condensation of steam in the jacket is Q_j, then

$$Q_a + Q_b + Q_c + Q_d = Q_e - Q_j . \quad . \quad . \quad . \quad . \quad . \quad . \quad (261)$$

Adding the equations (256) to (259), member to member, and using equations (261) and (260), we have

$$Q_e - Q_j = Q - Mq_4 - G(q_k - q_i) - AW, \quad . \quad . \quad (262)$$

which might have been written directly, since it is apparent that if we subtract from the heat supplied the heat rejected and the heat changed into work, the remainder is the heat lost by radiation. When the heat supplied by the jacket is greater than that lost by radiation, both sides of equation (262) become negative. Again, Q_j is zero when a jacket is not used.

Equation (262) gives a method of calculating the heat lost by radiation, a quantity which can be determined directly only when a steam-jacket is used, and which is then subject to uncertainty. It is, however, to be remarked that the equation (262) determines Q_e by subtraction, and therefore it is affected by the accumulated error of the experiment, which may be much larger than the uncertainty of a direct determination when a jacket is used.

If Q_e and Q_j are determined directly, then equation (261) is another equation of condition, so that by its aid equations (251) to (259) may be solved for M_0; or what is more convenient, the equation (262) may be used as a check on the equations for determining the quantities of heat Q_a, Q_b, Q_c, and Q_d, and thus the assumption made in determining M_0, i.e., $x_3 = 1$, may be tested.

The storing of the heat Q_d and the restoring of heat Q_c, both at a varying temperature, are productive of a loss of efficiency; but the most serious loss is due to the direct loss of the heat Q_c, which is thrown into the condenser without compensation. In many engines, Q_c is the most serious cause of low efficiency; it is frequently several times as large as all the heat changed into work. Hirn has proposed to make Q_c a measure of the advantage of various devices, such as superheating, steam-jackets, compounding, etc.

The equations are deduced for an engine having a surface condenser; for one having a jet condenser, q_4 and q_k become identical. For a non-condensing engine equation (258), containing quantities depending on the condenser, cannot be used, but by aid of equation (262) those quantities may be eliminated, giving

$$Q_c = (M + M_0)q_2 - M_0 q_3 + (V_0 + V_2)\frac{\rho_2}{u_2} - (V_0 + V_3)\frac{\rho_3}{u_3}$$
$$- Q - Q_j + Q_e + A(W + W_c), \quad . \quad . \quad . \quad . \quad . \quad (263)$$

which may be used for a non-condensing engine, provided that Q_c and Q_j are both determined by direct observation.

Hirn has used both equation (258) and equation (263) for determining Q_c, to which he attaches so much importance, and thereby obtains a check on his work. The check is, of course, equivalent to determining Q_e, the heat of radiation by equation (262), as well as by direct observation. Or the comparison of results by the two methods may be considered to show the combined error of, first, the observations, and, second, the method of calculating the steam caught in the clearance space. Hirn's original work and that of his school is stated in the numerical solution of special problems; consequently the real meaning of the check from the two methods of calculation is not so clearly presented, and the coincidence of results, from apparently independent methods, is more striking than in the discussion just given.

In certain very important experiments by Hirn and by Hallauer, the compression was nearly if not quite absent, which, with the low absolute pressure in the condenser, made M_0, calculated by the usual method, so small, that it was neglected. This reduced equations (258) and (263) to

$$Q_c = M(q_2 - q_4) - G(q_\kappa - q_i) + (V_0 + V_2)\frac{\rho_2}{u_2} + A W_c . \quad . \quad (264)$$

$$Q_c = Mq_2 + (V_0 + V_2)\frac{\rho_2}{u_2} - Q - Q_j + Q_e + A(W + W_c). \quad (265)$$

The result by equation (264) was considered to be the direct result, and that by equation (265) the proof of the correctness and accuracy of the theory and experiment. The numerical calculation involved some minor differences which could not seriously affect the result. When the amount of the back pressure, or of the compression, forbade the use of these equations, a method nearly equivalent to equations (262) and (263) was employed.

PROBLEM.—The following are the data of a test made on a Harris-Corliss engine in the laboratory of the Institute of Technology, together with the calculation of the results:

Diameter of the engine, ...	8 inches
Stroke, ...	2 feet
Piston displacement: crank end, . .	0.6791 cu. ft.
head end, . .	0.7016 "
Clearance, per cent of piston displacement:	
crank end, ...	3.75
head end, ...	5.42
Boiler-pressure by gauge, ...	77.4 pounds
Barometer, ...	14.8 "
Condition of steam, one per cent of moisture.	
Events of the stroke:	
Cut-off: crank end, ...	0.306 of stroke
head end, ...	0.320 "
Release at end of stroke.	
Compression: crank end, . . .	0.013 of stroke
head end, . . .	0.0391 "
Duration of the test, one hour.	
Total number of revolutions, . . .	3692
Weight of steam used, ...	548 pounds
Weight of condensing water used, .	14,568 "
Temperatures:	
Condensed steam, ...	$t_4 = 141°.1$ F.
Condensing water: cold, . . .	$t_i = 52°.9$ F.
warm, . . .	$t_k = 88°.3$ F.

ABSOLUTE PRESSURES, FROM INDICATOR-DIAGRAMS, AND CORRESPONDING PROPERTIES OF SATURATED STEAM.

	CRANK END.				HEAD END.			
	p	q	ρ	u	p	q	ρ	u
Cut-off.......	83.6	284.6	813.0	5.190	83.3	284.4	813.2	5.207
Release.......	29.2	217.8	864.8	13.924	31.9	222.9	861.8	12.804
Compression..	14.8	181.1	893.2	26.464	14.8	181.1	893.2	26.464
Admission....	21.8	201.5	877.4	18.344	29.8	219.0	863.9	13.664

MEAN PRESSURES, AND HEAT EQUIVALENTS OF EXTERNAL WORKS.

	Crank End.		Head End.	
	Mean Pressures.	Equivalents of Work.	Mean Pressures.	Equivalents of Work.
Admission	87.7	3.369	89.3	3.711
Expansion	44.5	3.877	47.1	4.159
Exhaust	14.8	1.836	14.8	1.847
Compression	18.3	0.0299	21.8	0.1104

VOLUMES, CUBIC FEET.

	Crank End.	Head End.
At cut-off, $V_0 + V_1$	0.2333	0.2626
At release, $V_0 + V_2$	0.7046	0.7396
At compression, $V_0 + V_3$	0.0343	0.0655
At admission, V_0	0.02550	0.03806

At the boiler-pressure, 92.1 pounds absolute, we have

$$r = 888.4, \qquad q = 291.7.$$

The steam used per stroke is

$$M = \frac{548}{2 \times 3692} = 0.0742 \text{ pounds.}$$

The steam caught in the clearance space at compression, on the assumption that the steam is then dry and saturated, is obtained by multiplying the mean volume at that point by the weight of one cubic foot of steam at the pressure at compression;

$$\therefore M_0 = \frac{0.0343 + 0.0655}{2} \times \frac{1}{26.464} = 0.0021 \text{ pounds;}$$

$$M + M_0 = 0.0742 + 0.0021 = 0.0763 \text{ pounds.}$$

The condensing water used per stroke is

$$G = \frac{14568}{2 \times 3692} = 1.973.$$

$$Q = M(xr + q) = 0.0742(0.99 \times 888.4 + 291.7) = 86.903;$$

$$Q_a = Q + M_0 q_0 - (M + M_0) q_1 + V_0 \frac{\rho_0}{u_0} - (V_0 + V_1) \frac{\rho_1}{u_1} - A W_a$$

$$= 86.903 + 0.0021 \times \tfrac{1}{2}(201.5 + 219.0) - 0.0763 \times 284.5$$

$$+ \tfrac{1}{2}\left(0.0255 \times \frac{877.4}{18.344} + 0.0381 \frac{863.9}{13.664}\right)$$

$$- \frac{1}{2}\left(0.2333 \times \frac{813.0}{5.19} + 0.2626 \times \frac{813.2}{5.207}\right) - \frac{3.369 + 3.711}{2}$$

$$= 86.903 + 0.441 - 21.707 + 1.815 - 38.778 - 3.540 = 25.134;$$

$$Q_b = (M + M_0)(q_1 - q_2) + (V_0 + V_1) \frac{\rho_1}{u_1} - (V_0 + V_2) \frac{\rho_2}{u_2} - A W_b$$

$$= 0.763\left(284.5 - \frac{222.9 + 217.8}{2}\right) + 38.778$$

$$- \frac{1}{2}\left(0.7046 \times \frac{864.8}{13.924} + 0.7396 \times \frac{861.8}{12.804}\right) - \frac{3.877 + 4.159}{2}$$

$$= 4.898 + 38.778 - 46.771 - 4.018 = -7.113;$$

$$Q_c = (M + M_0) q_2 - M_0 q_3 + (V_0 + V_2) \frac{\rho_2}{u_2} - (V_0 + V_3) \frac{\rho_3}{u_3} - M q_4$$

$$- G(q_k - q_i) + A W_c$$

$$= 0.0763 \times \frac{222.9 + 217.8}{2} - 0.0021 \times 181.1 + 46.771$$

$$- \frac{0.0343 + 0.0655}{2} \times \frac{893.2}{26.464} - 0.0742 \times 109.3$$

$$- 1.973(56.3 - 21.0) + \frac{1.836 + 1.847}{2}$$

$$= 16.809 - 0.380 + 46.77 - 1.684 - 8.110 - 69.644 + 1.841$$

$$= - 14.397;$$

$$Q_d = M(q_3 - q_0) + (V_0 + V_3) \frac{\rho_3}{u_3} - V_0 \frac{\rho_0}{u_0} + A W_d$$

$$= 0.0021\left(181.1 - \frac{201.5 + 219.0}{2}\right) + 1.684 - 1.815$$

$$+ \frac{0.0299 + 0.1104}{2}$$

$$= - 0.061 + 1.684 - 1.815 + 0.070 = - 0.122;$$

$$Q_a + Q_b + Q_c + Q_d = 25.134 - 7.113 - 14.397 - 0.122 = 3.502 = Q_e.$$

Also, equation (262) for this case gives

$$Q_e = Q - Mq_4 - G(q_k - q_i) - AW$$
$$= 86.903 - 8.110 - 69.644 - (3.735 + 3.846 - 1.841 - 0.070)$$
$$= 86.903 - 8.110 - 5.647 = 3.502.$$

It is to be remembered that the heat lost by radiation and conduction per stroke, when estimated in this manner, is affected by the accumulated errors of observation and computation, which may be a large part of the total value of Q_e as determined.

Dropping superfluous significant figures, we have in B. T. U.

$$Q = 86.9,\quad Q_a = 24.9,\quad Q_b = 6.9,\quad Q_c = 14.4,\quad Q_d = -0.12,\quad Q_e = 3.5.$$

The horse-power of the engine is

$$\frac{778 \times 5.670 \times 3692 \times 2}{60 \times 33000} = 16.35 \text{ H.P.};$$

and the steam per horse-power per hour is

$$\frac{548}{16.35} = 33.5 \text{ pounds.}$$

A consideration of the theory just elaborated will show that the true condition of the steam at compression is of great importance. Zeuner shows that for some of Hallauer's experiments an assumption that $M_0 = M$ will reduce Q_c to zero. In reply Hirn shows that such an assumption does not reduce the other quantities, Q_a, Q_b, and Q_d, to zero, as ought to be the case if there is no interchange of heat between the cylinder and the steam; and Hallauer shows that the behavior of the steam during compression cannot be accounted for on such an assumption. Again, the condition of the exhaust steam may be calculated in some experiments by considering the condenser to be a calorimeter, and such a calculation gave about six per cent of water. If the steam in the cylinder at compression had the same amount of water, the error of considering it dry would be small. Moreover, water which collects in the cold cylinder of an engine when started, quickly passes away, as is indicated by the change of the sound of exhaust from dull and heavy to dry and short. All this shows that a large quantity of water in the clearance space is improbable if not impossible, but it does not entirely justify the assumption that the steam is dry at compression. Experiments made on engines having a large compression may give light on the subject.

Surface Condensers.—The proper discussion of condensers can be given only in connection with the theory of the steam-engine, though it is commonly treated as a separate problem.

Let Fig. 47 represent the cylinder of a steam-engine and a surface condenser. The piston is at the end of the stroke, and the cylinder contains M pounds of a mixture of steam and water, having the conditions determined by t_1 and x_1. The cylinder by the opening of the exhaust-valve is put in communication with surface condenser containing a weight M_2 of steam and water, having the condition determined by t_2 and x_2. We may imagine the process to be divided into two parts: (1) the steam and water in the cylinder and condenser mingle, and are reduced to the temperature t_2; (2) the piston passes to the other end of the stroke and reduces the volume from $V_1 + V_2$ to V_2.

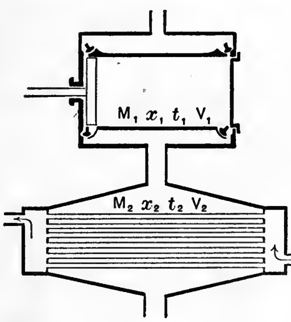


FIG. 47.

The result would be the same if we imagine the steam in the cylinder (1) to be reduced to the temperature t_2 before the exhaust-valve opens, and then (2) after the valve opens to be reduced from the volume $V_1 + V_2$ to V_2.

In the first process the steam and water is reduced, at constant volume, from the temperature t_1 to t_2. To find the heat abstracted, we have

$$dQ = M_1 AdE = M_1[dq + d(x\rho)].$$

But $$x_1u_1 + \sigma = xu + \sigma;$$

$$\therefore dQ = M_1\left[dq + x_1u_1 d\left(\frac{\rho}{u}\right)\right];$$

$$\therefore Q_1 = M_1\left[q_1 - q_2 + x_1u_1\left(\frac{\rho_1}{u_1} - \frac{\rho_2}{u_2}\right)\right]. \quad . \quad . \quad (266)$$

At the end of this process we have a volume $V_1 + V_2$ of steam and water having a weight $M_1 + M_2 = M$, and having a value x_2' that could be found if desired.

The second process reduces the volume from $V_1 + V_2$ to V_2, and the value of x changes again and becomes x_2''. The heat abstracted is

$$Mr_2(x_2' - x_2''). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (267)$$

The volumes before and after compression are

$$V_1 + V_2 = M(x_2'u_2 + \sigma);$$
$$V_2 = M(x_2''u_2 + \sigma);$$

so that the change of volume is

$$V_1 = Mu_2(x_2' - x_2'').$$

But, from the initial condition,

$$V_1 = M_1(x_1u_1 + \sigma);$$

so that, if σ be neglected,

$$M = M_1 \frac{x_1u_1}{u_2(x_2' - x_2'')}.$$

Substituting this in equation (267), we have for the heat abstracted during the second process,

$$M_1x_1u_1\frac{r_2}{u_2} = M_1x_1u_1\left(\frac{\rho_2}{u_2} + Ap_2\right). \quad . \quad . \quad . \quad . \quad (268)$$

The total heat withdrawn during both processes is, therefore,

$$M_1[q_1 - q_2 + x_1(\rho_1 + Ap_2u_1)]. \quad . \quad . \quad . \quad . \quad . \quad (269)$$

But during the exhaust from the cylinder of an engine the walls yield to the steam and water contained the heat Q_c, and this passes into the condenser, so that the total heat to be car ried away by the cooling water is

$$Q_c + [M_1 q_1 - q_2 + x_1(\rho_1 + Ap_2u_1)]. \quad . \quad . \quad . \quad (270)$$

Each unit of weight of cooling water in passing through the condenser will acquire $q_k - q_i$ units of heat; consequently the pounds of cooling water for one pound of mixture of water and steam drawn from the cylinder will be

$$\frac{\frac{Q_c}{M} + q_1 - q_2 + x_1(\rho_1 + Ap_2u_1)}{q_k - q_i}. \quad . \quad . \quad . \quad . \quad (271)$$

If it be assumed that $\frac{Q_c}{M} = 0$, that $x_1 = 1$, and that Ap_2u_1 is the same as Ap_1u_1, then the expression becomes

$$\frac{q_1 - q_2 + r_1}{q_k - q_i} = \frac{\lambda_1 - q_2}{q_k - q_i}. \quad . \quad . \quad . \quad . \quad . \quad (272)$$

The expression commonly used for calculating the cooling water required is

$$\frac{\lambda - t_2}{t_k - t_i},$$

in which λ is the total heat of steam at the pressure in the exhaust pipe.

The expression is deduced by assuming that the exhaust steam is dry and saturated, and that it is condensed and cooled to the temperature t_2 by the cooling water, which thereby gains the temperature $t_k - t_i$.

Some experiments by Hallauer* on a condensing-engine using saturated and superheated steam, show that the exhaust steam in that engine contained from 6 to 12 per cent of moisture. Experiments in the laboratory of the Institute of Tech-

* Bulletin de la Soc. Ind. de Mulhouse, vol. xlvii. 1877.

nology, on non-condensing engines, show in general a less amount of moisture in the exhaust steam. It may be concluded that exhaust steam from an engine is seldom or never superheated, that it usually contains a moderate amount of moisture, and that it is sufficient to assume it to be dry and saturated, for calculation of the condensing water required.

Cooling Surface.—Experiments on the quantity of cooling surface required by a surface-condenser are few and unsatisfactory, and a comparison of condensers of marine engines shows a wide diversity of practice. Seaton* says that with an initial temperature of 60°, and with 120° for the feed-water, a condensation of 13 pounds of steam per square foot per hour is considered fair work. A new condenser in good condition may condense much more steam per square foot per hour than this, but allowance must be made for fouling and clogging, especially for vessels that make long voyages.

Seaton also gives the following table of square feet of cooling surface per indicated horse-power:

Absolute terminal pressure, pounds per sq. in.	Square feet per I. H. P.
30	3
20	2.5
15	2.25
$12\frac{1}{2}$	2.
10	1.8
8	1.6
6	1.5

For ships stationed in the tropics, allow 20 per cent more; for ships which occasionally visit the tropics, allow 10 per cent more; for ships constantly in a cold climate, 10 per cent less may be allowed.

Jet Condenser.—In the jet condenser the cooling water mingles with the steam, and the final temperatures of both become t_k. There are some minor differences that appear in the exact expression for the cooling water required from that for

* Manual of Marine Engineering.

surface condensers; but the difference is slight, and in any case the simple expression

$$\frac{\lambda - q_k}{q_k - q_i}$$

is to be preferred in practice.

The capacity of a jet condenser should not be less than one fourth of that of the cylinder or cylinders exhausting into it, and need seldom be more than one half. One third of the capacity of the cylinder or cylinders is generally sufficient.

Designing Engines.—The only question that is properly discussed here is the probable form of the indicator-diagram, which gives immediately the method of finding the mean effective pressure and, consequently, the size of the cylinder of the engine.

The most reliable way of finding the expected mean effective pressure in the design of a new engine is to measure an indicator-card from an engine of the same or similar type and size, and working under the same conditions.

As it can hardly be expected that a diagram of exactly the required form will be at hand, a diagram like Fig. 48 may be drawn, using the proper cut-off, compression, and clearance. If an indicator-card taken from an engine under similar conditions is attainable, it may be used to determine exponential equations for the expansion and compression curves; usually the exponent will be different for the two curves, and must be determined separately. For ordinary work it is sufficient to use the hyperbola for both curves, and to assume the steam line a and the back-pressure line c to be parallel to the atmospheric line, while the lead of admission and exhaust may be neglected. It is also customary to assume a loss of pressure of one or two pounds between the boiler and the engine, and a back pressure of a like or greater amount above the pressure in the condenser or the pressure of the atmosphere, as the case may be.

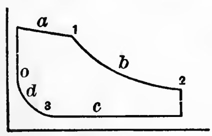


FIG. 48.

If the diagram is drawn to scale, the area and mean effective pressure may be found by measuring it; or, the form of the expansion and compression curves being assumed, the areas under the steam line, the expansion curve, the back-pressure line, and the compression curves may be calculated separately, integrating between limits when necessary, and therefrom the resulting area of the diagram and the mean effective pressure may be determined. Ordinarily, the expansion and compression curves are assumed to be hyperbolæ.

Seaton* gives the following multipliers for finding the mean effective pressure from that calculated by the process described:

MULTIPLIERS FOR FINDING PROBABLE M. E. P., SIMPLE EXPANSIVE ENGINE.

(1) Special valve-gear, or with separate cut-off valve, engine jacketed................................	0.94
(2) Good ordinary valves, large ports, engine jacketed.	0.9–0.92
(3) Ordinary valves and gears as in general practice, unjacketed....................................	0.80–0.85

To estimate the consumption of steam, we may calculate from the pressure and volume at release the weight of steam then present in the cylinder, and in a similar manner the weight of steam caught in the clearance space from the volume and pressure at compression, both under the assumption that the steam is dry and saturated. The difference is the steam exhausted per stroke under the assumption; but to get a fair estimate of the probable consumption, it is necessary to add a fraction of this amount, depending on the style and size of the engine and on the conditions under which it is to run. Sufficient data for this purpose seldom exist; so it is customary to add to the calculated amount one fourth to one third of itself, to get the probable consumption of non-condensing engines of medium size.

PROBLEM.—Required the dimensions of an engine to give 100 horse-power; revolutions, 120; gauge-pressure, 80 pounds; cut-off at $\frac{1}{3}$ stroke; release at end of stroke; compression at $\frac{1}{10}$ stroke, and clearance 5 per cent.

* Manual of Marine Engineering.

Assuming hyperbolic expansion and compression gives for the mean effective pressure

$$0.333 \times 92.7 + 0.383 \times 92.7 \log_e \frac{1.05}{0.383} - 16 \times 0.9$$
$$- 0.15 \times 16 \log_e \frac{0.15}{0.05} = 49.5 \text{ pounds,}$$

if the pressure during admission is 78 pounds, and during exhaust is 1.3 pounds above the atmosphere.

If, further, the stroke of the engine is twice the diameter, then

$$100 = \frac{\frac{\pi d^2}{4} \times \frac{2d}{12} \times 120 \times 2 \times 49.5}{33000};$$

$$\therefore \ d = 12.85, \qquad s = 25.70.$$

The volume of the cylinder will be 1.93 cubic feet, and the terminal pressure will be 33.8 pounds absolute. At 33.8 pounds the density of steam is 0.08234, and at 16 pounds it is 0.04067. The consumption of steam per horse-power per hour, on the assumption of dry steam at release and compression, will be

$$\frac{(0.08234 \times 1.05 - 0.04067 \times 0.15) 1.93 \times 2 \times 120 \times 60}{100} = 22.3 \text{ pounds.}$$

If one third of this quantity be added, then the estimated consumption of steam will be 30 pounds per horse-power per hour.

The calculated dimensions are stated in inches and hundreds, but in practice the engine would be made $12\frac{7}{8}$ inches in diameter by $25\frac{3}{4}$ inches stroke; or possibly the dimensions 13 by 25 would be chosen, since they give nearly the same volume.

EXAMPLES.

1. Find the volume of the cylinder of a double-acting steam engine to give 100 H.P. at 60 revolutions per minute. Assume it to run on Carnot's cycle, to have 150 pounds by the gauge, and 1.5 pounds absolute for the maximum and minimum pressures, and to have the steam dry and saturated at the beginning of the adiabatic expansion.

2. In problem 1, make the minimum pressure 14.7 pounds.

3. Suppose an actual steam-engine, working between the same pressures as in Example 1, to use 18 pounds of steam per hour, and to run without compression. Find the volume of the cylinder if the steam at release contains 20 per cent of moisture.

4. Find the relative sizes of cylinders of perfect heat-engines using water, ether, and carbon tetrachloride, and working between the temperatures of 100° C. and 10° C.

Suggestion: Find the work of Carnot's cycle for one kilogram; find the relative weights to give equal quantities of work, and thence the relatives volumes.

5. Find relative sizes in Example 4 when working between the pressures of 5 atmospheres and $\frac{1}{2}$ of one atmosphere.

6. Make calculations for Hirn's analysis for the experiments given on pages 305 and 334.

7. Find the weight of cooling water for a hundred horse-power engine using 20 pounds of steam per horse-power per hour, the vacuum in the condenser being 26 inches of mercury, the temperature of the condensed water being 120° F., and the temperatures of the cooling water being 60° F. and 110° F.

8. In Example 7, find the area of cooling surface for a surface condenser, the terminal pressure being $12\frac{1}{2}$ pounds.

9. Calculate the problem given on page 201, assuming as the equation of the expansion curve

$$pv^{0.8} = \text{const.},$$

and for the compression curve

$$pv^{1.1} = \text{const.}$$

CHAPTER XIII.

COMPOUND ENGINES.

In a compound engine, steam is admitted from the boiler into a small cylinder, from which it is exhausted above atmospheric pressure. The steam is then admitted to a large cylinder, from which it passes to a condenser. If we assume that the steam from the small cylinder is exhausted into a large receiver, the back pressure in that cylinder and the pressure during the admission to the large cylinder will be uniform. If, further, we assume that there is no clearance in either cylinder, that the back pressure in the small cylinder and the forward pressure in the large cylinder are the same, and that the expansion in the small cylinder reduces the pressure down to the back pressure in that cylinder, the diagram for the small cylinder will be *ABCD*, Fig. 49, and for the large cylinder *DCEFG*. The volume in the large cylinder at cut-off is equal to the total volume of the small cylinder, since the large cylinder takes from the receiver the same weight of steam that is exhausted by the small cylinder, and, in this case, at the same pressure.

The case just discussed is one extreme. The other extreme occurs when the small cylinder exhausts directly into the large cylinder without an intermediate receiver. In such engines the pistons must begin and end their strokes together. They may both act on the beam of a beam engine, or they may act on one crank or on two cranks at 180° apart.

For such an engine, *ABCD*, Fig. 50, is the diagram for the small cylinder. The steam line and expansion line *AB* and *BC* are like those of a simple engine. When the piston of the small cylinder begins the return stroke, communication is opened with the large cylinder, and the steam passes from one to the other, and expands to the amount of the difference of the volume, it being assumed that the communication remains open to the end of the stroke. The back-pressure line *CD* for

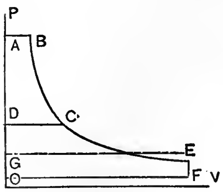


FIG. 49.

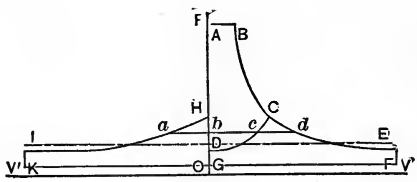


FIG. 50.

the small cylinder, and the admission line *HI* for the large cylinder, gradually fall on account of this expansion. The diagram for the large cylinder is *HIKG*, which is turned toward the left for convenience.

To combine the two diagrams, draw the line *abcd*, parallel to *V'OV*, and from *b* lay off *bd* equal to *ca*; then *d* is one point of the expansion curve of the combined diagram. The point *C* corresponds with *H*, and *E*, corresponding with *I*, is as far to the right as *I* is to the left.

For a non-conducting cylinder, the combined diagram for a compound engine, whether with or without a receiver, is the same as that for a simple engine, which has a cylinder the same size as the large cylinder of the compound engine, and which takes at each stroke the same volume of steam as the small cylinder, and at the same pressure. The only advantage gained by the addition of the small cylinder to such an engine is a more even distribution of work during the stroke, and a smaller initial stress on the crank-pin.

Compound engines may be divided into two classes—those with a receiver and those without a receiver; the latter are called "Woolf engines" on the continent of Europe. Engines without a receiver must have the pistons begin and end their strokes at the same time; they may act on the same crank or on cranks 180° apart. The pistons of a receiver compound engine may make strokes in any order. A form of receiver compound engine with two cylinders, commonly used in marine work, has the cranks at 90° to give handiness and certainty of action. Large marine engines have been made with one small cylinder and two large or low-pressure cylinders, both of which draw steam from the receiver and exhaust to the condenser. Such engines usually have the cranks at 120°, though other arrangements have been made.

In reality, all compound engines have a receiver, or a space between the cylinders corresponding to one, and in no case is the receiver of sufficient size to entirely prevent fluctuation of pressure. In the later marine work the receiver has been made small, and frequently the steam-chests and connecting pipes have been allowed to fulfil that function. This contraction of size involves greater fluctuation of pressure, but for other reasons it appears to be favorable to economy.

Compound Engine without Receiver.—The indicator-cards from a compound engine without a receiver are represented by Fig. 51. The steam line and expansion line of the small cylinder, AB and BC, do not differ from those of a simple engine. At C the exhaust opens, and the steam suddenly expands into the space between the cylinders and the clearance of the large cylinder, and the pressure falls from C to D. During the return stroke, the volume in the large cylinder increases more rapidly than that of the small cylinder decreases, so that the back-pressure line DE gradually falls, as does also the admission line HI of the large cylinder, the difference between these two lines being due to the resistance to the flow of steam from one to the other. At E the communication between the two cylinders is closed by the cut-off of the large cylinder; the steam is thus compressed in the small cylinder and the space between the two cylinders to F, at which the exhaust of the small cylinder closes; and the

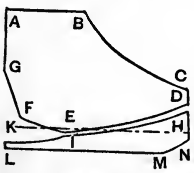


FIG. 51.

remainder of the diagram *FGA* is like that of a simple engine. From *I*, the point of cut-off of the large cylinder, the remainder of the diagram *IKLMNH* is like the same part of the diagram of a simple engine.

The "drop" *CD* at the end of the stroke of the small cylinder, and the difference between the lines *DE* and *HI*, are evident losses of efficiency. The compression *EFG* for the small cylinder, and the accompanying independent expansion *IK* in the large cylinder, are losses of power in the engine; but the compression, as in a simple engine, fills the waste spaces, and in this case mitigates the effect of the "drop." It is apparent that there would be a loss of efficiency in compounding a non-conducting engine, yet under proper circumstances experiment shows an advantage in compounding engines with metallic cylinders.

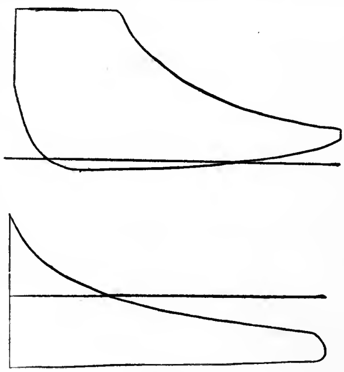


Fig. 52.

Fig. 52 is a diagram taken from a compound pumping-engine at the Lawrence, Mass., water-works.

Compound Engine with Receiver.—In the receiver compound engine with cranks at 90° the cut-off is commonly later than half-stroke, which gives rise to a species of double admission. The diagrams for the small and large cylinders are represented by Fig. 53.

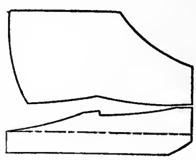


Fig. 53.

When the exhaust of the small cylinder begins, the large piston is at about half-stroke, and communication then exists through the receiver between the two cylinders. The cut-off of the large cylinder closes this communication, and the back pressure rises in the small cylinder until, at about half-stroke, the admission to the other end of the large cylinder makes the back pressure fall, down to the compression in the small cylinder.

The admission to the large cylinder begins at about half-stroke of the small piston, with a free communication between the two cylinders, till the compression begins in the small cylinder. The steam in the receiver then expands with diminishing pressure to about half-stroke, when exhaust at the other end of the small cylinder begins, causing an increase of pressure for the remainder of the admission.

Fig. 53*a* gives diagrams from the high and low pressure cylinders of a receiver compound engine of the yacht Gleam.

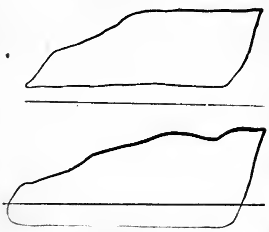


FIG. 53*a*.

The diagrams from three cylinder compound engines, and from other arrangements, have peculiarities that must be investigated separately for each case. All will show a drop at the end of the expansion in the small cylinder, and in general a loss of pressure between the small and large cylinders.

A comparison with the ideal diagram, Fig. 49, will show a loss of efficiency from the drop, and the loss of pressure between the small and large cylinders, and yet, as with the Woolf engine, compounding under proper circumstances is an advantage.

It is customary to attempt the comparison of the expansion in a compound engine to that of a simple engine by an adaptation of the methods of Figs. 49 and 50, making allowance for the clearance and the action of the receiver and of the compression. More or less complication is introduced to meet the difficulties arising, yet the result is not satisfactory. The simplest method, which applies to receiver compound engines, is a modification of Fig. 49. The diagrams are transferred to a common scale, and referred to the same axis of pressure and volume, each being set off to the right of the axis *OP* by the amount of its own clearance. It is customary to complete the process by drawing a rectangular hyperbola in a manner similar to that used for a simple engine.

The whole process is of doubtful utility, the more especially as there appears to be no reason to assume the hyperbola or any other simple curve to represent the actual equivalent expansion in a simple engine. The several gaps that appear in such a combined diagram, due to drop, loss of pressure and compression, seem to show a loss as compared with a simple engine, which may or may not exist. The only way of *knowing* anything about the performance of a type of engine is to make a series of careful tests upon it.

Triple and Quadruple Compound Engines.—The same influences which introduced the compound engines, when the common steam-pressure changed from forty to eighty pounds to the square inch, have brought in the successive expansion through three cylinders, the high-pressure, intermediate, and low-pressure cylinders, now that 125 to 170 pounds pressure are employed. Just as three or more cylinders are combined in various ways for compound engines, so four, five, or six cylinders have been arranged in various manners for triple compound engines; for example, a compound engine with two cylinders may be conveniently changed into a triple compound engine by the addition of a small high-pressure cylinder over each of the existing cylinders.

Quadruple engines with four successive expansions have been employed with high-pressure steam, but with the advisable pressures for present use, the extra complication and friction make it a doubtful expedient.

Horse-power of Compound Engines.—For the first approximation it is customary to calculate the horse-power of a compound engine of any sort, as if the total expansion occurred in the cylinder or cylinders that exhaust into the condenser; and it is assumed that the expansion curve is a rectangular hyperbola.

PROBLEM.—Let the boiler-pressure be 80 pounds by the gauge, or 94.7 pounds absolute; let the back pressure be 4 pounds absolute; and let the total number of expansions be six, so that the volume of steam exhausted to the condenser is six times the volume admitted from the boiler. Neglecting the effect of clearance and compression, the mean effective pressure is

$$94.7 \times \tfrac{1}{6} + 94.7 \times \tfrac{1}{6} \log_e \tfrac{6}{1} - 4 \times 1 = 40.06 = \text{M.E.P.}$$

If the large cylinder is 30 inches in diameter, and the stroke is 4 feet, the horse-power at 60 revolutions per minute is

$$\frac{\pi 30^2}{4} \times 40.06 \times 2 \times 4 \times 60 \div 33000 = 412 \text{ H.P.}$$

Point of Cut-off.—Let the ratio of the volumes of the high and low pressure cylinders of a compound engine be R, let the number of expansions in the small cylinder be e, and let the total number of expansions be E; then

$$E = eR; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273)$$

$$e = \frac{E}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

PROBLEM.—Let the ratio of the cylinders of the engine discussed in the preceding paragraph be 3; with the same stroke the diameters will be $17\frac{5}{16}$ and 30 inches. The number of expansions in the small cylinder will be $\frac{6}{3}$, and the cut-off for that cylinder will be at $\frac{1}{2}$ of the stroke.

The cut-off in the large cylinder has an effect on the distribution of the work and the adjustment of the maximum pressure on each piston, but it does not affect the preliminary calculation of mean effective pressure and horse-power.

Ratio of Cylinders.—In designing compound engines, more especially for marine work, it is deemed important for the smooth action of the engine that the total work shall be evenly distributed upon the several cranks of the engines, and that the maximum pressure on each of the cranks shall be the same, and shall not be excessive. In case two or more pistons act on one crank, the total work and the resultant pressure on those pistons are to be considered; but more commonly each piston acts on a separate crank, and then the work and pressure on the several pistons are to be considered.

If it is desired that the work shall be equally divided between the two cylinders of a receiver compound engine, the ratio of their volumes may be found as follows. Let the initial pressure be p, the receiver pressure p_1, and the pressure in the condenser zero; then, on the assumption that the volumes are inversely as the pressures,

$$p = ep_1 = \frac{E}{R} p_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (275)$$

Let v be the volume of the small cylinder and V that of the large cylinder; then the works done in them may be assumed to be

$$\frac{v}{e} p(1 + \log_e e) - vp_1 \quad \text{and} \quad \frac{V}{R} p_1(1 + \log_e R).$$

Equating these quantities and substituting v for $\frac{V}{R}$,

$$\frac{vp}{e}(1 + \log_e e) - vp_1 = vp_1(1 + \log_e R).$$

Again, substituting for e and p from equations (274) and (275), and reducing,

$$\log_e E = 1 + 2 \log_e R;$$

$$\therefore \log_{10} R = \frac{\log_{10} E - 0.4343}{0.8686}. \quad . \quad . \quad . \quad . \quad . \quad (276)$$

If it is desired to make the maximum pressure on the pistons the same, then we should have

$$pa = p_1 A, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (277)$$

a and A being the areas of the small and large pistons respectively. If the stroke is the same for the two pistons, then the volumes are proportional to the areas, so that equation (277) becomes

$$pv = p_1 V;$$

or, substituting for p from equation (275), and for $\frac{V}{v}$ its value E,

$$R^2 = E. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

Applied to the problem stated above, the ratio of the volumes of the cylinders for six expansions is 2.49 by equation (276) and 2.45 by equation (278).

The method of equation (276) assumes that there is no drop to the receiver; a larger ratio will be accompanied by a drop, and a smaller ratio will cause the steam in the small cylinder to be expanded to a lower pressure than that in the receiver.

In practice both the ratio of the cylinders and the total expansions are assumed, and then the distribution of work and the maximum loads on the crank-pins are calculated, allowing for clearance and compression. Designers of engines usually have a sufficient number of good examples at hand to enable them to assume these data. In default of such data it may be necessary to assume proportions, to make preliminary calculations, and to revise the proportions till satisfactory results are obtained. For compound engines using 80 pounds of steam pressure, the ratio is 1 : 3 or 1 : 4. For triple expansion engines the cylinders may be made to increase in the ratio 1 : 2 or 1 : $2\frac{1}{2}$.

Calculations for Compound Engines.—Instead of deducing equations for the calculations for compound engines, a few problems will be solved to exhibit the method.

EXAMPLE 1.—Boiler-pressure 80 pounds by the gauge, receiver-pressure 18 pounds, pressure in the condenser 4 pounds absolute. Ratio of the cylinders, 3; total expansion, 6. Clearance, 10 per cent for the small cylinder and 8 per cent for the large cylinder. Compression at 0.15 of the stroke for each cylinder.

First Solution.—An approximate solution, neglecting clearance and compression, will first be made.

The cut-off in the small cylinder will be at

$$\frac{3}{6} = \frac{1}{2} \text{ stroke.}$$

The terminal pressure in the small cylinder, on the assumption of hyperbolic expansion, will be

$$\frac{80 + 14.7}{2} = 47.35 \text{ pounds absolute.}$$

The drop to the receiver will be

$$47.35 - (15 + 14.7) = 17.65 \text{ pounds.}$$

The cut-off in the large cylinder is determined by the condition that it must draw the same weight of steam per stroke from the receiver as is delivered to it by the small cylinder. The volume v is discharged by the small cylinder per stroke, which may be assumed to expand to the volume

$$\frac{47.35}{29.7}v$$

in the receiver on account of the drop. The cut-off in the large cylinder is therefore at

$$\frac{47.35}{29.7}v \div V = \frac{47.35}{29.7 \times 3} = 0.509 \text{ of the stroke.}$$

The mean effective pressure in the small cylinder is

$$94.7 \times \tfrac{1}{2}(1 + \log_e 2) - 29.7 = 50.47.$$

The mean effective pressure in the large cylinder is

$$29.7 \times 0.509\left(1 + \log_e \frac{1}{0.509}\right) - 4 = 20.03 \text{ pounds.}$$

The mean effective pressure reduced to the large cylinder is

$$\frac{50.47}{3} + 20.03 = 36.85 \text{ pounds.}$$

This result may be compared with the result obtained on page 209, on the assumption that all the work was done in the large cylinder, i.e., 40.06 pounds.

The division of work between the two cylinders is in the ratio

$$\frac{50.47}{3 \times 20.03} = \frac{1}{1.2}.$$

The drop to the receiver might be reduced by shortening the cut-off of the large cylinder, but such an arrangement would produce a greater inequality in the distribution of work. On the other hand, the work will be more equally distributed with a longer cut-off on the large cylinder, but that would give a larger drop and a more wasteful engine.

The maximum pressure on the small piston is

$$94.7 - 29.7 = 65.0 \text{ pounds,}$$

equivalent to

$$\frac{65.0}{3} = 21.7 \text{ pounds}$$

on the large piston.

The maximum pressure on the large piston is

$$29.7 - 4 = 25.3 \text{ pounds.}$$

Lengthening the cut-off of the large cylinder will increase the pressure on the small piston and diminish that on the large piston.

Second Solution.—A more complete solution with slightly different results can be given, taking account of clearance and compression.

The cut-off in the small cylinder will be taken at one-half stroke, as before.

The terminal pressure is

$$\frac{94.7 \times 0.60}{1.10} = 51.66 \text{ pounds absolute.}$$

The drop to the receiver is

$$51.66 - 29.7 = 21.96 \text{ pounds.}$$

The cut-off for the large cylinder is at

$$\frac{51.66}{29.7 \times 3} = 0.5798 \text{ of the stroke.}$$

The mean effective pressure in the small cylinder is

$$94.7 \times \tfrac{1}{2} + 94.7 \times 0.6 \log_e \frac{1.1}{0.6} - 29.7 \times 0.85$$

$$- 29.7 \times 0.25 \log_e \frac{0.25}{0.10} = 53.65 \text{ pounds.}$$

The mean effective pressure in the large cylinder is

$$29.7 \times 0.5798 + 29.7 \times 0.6598 \log_e \frac{1.08}{0.6598} - 4 \times 0.85$$

$$- 4 \times 0.23 \log_e \frac{0.23}{0.08} = 22.51 \text{ pounds.}$$

The mean effective pressure reduced to the large cylinder is

$$\frac{53.65}{3} + 22.51 = 40.39 \text{ pounds.}$$

The division of work between the two cylinders is in the ratio

$$\frac{53.65}{3 \times 22.51} = \frac{1}{1.26}.$$

The maximum pressures on the small and large pistons are the same as calculated before.

EXAMPLE 2.—Let the data of Example 1 be taken for a non-receiver compound engine. For such an engine the only effect of the cut-off on the large cylinder is to produce a compression in the small cylinder and the space between the two cylinders corresponding to the receiver of a receiver engine. Let the cut-off on the large cylinder occur at 0.70 of the stroke. Let the volume of the intermediate space be 0.10 of the dis placement of the large cylinder, or 0.30 of the volume of the small cylinder.

The terminal pressure in the small cylinder, as in the second solution of Example 1, is 51.66 pounds absolute.

When the exhaust-valve opens, the steam from the small cylinder mingles with that in the intermediate space and in the clearance space of the large cylinder, and a drop occurs. The steam caught by the compression in the clearance of the large cylinder has so small a density that it may be neglected. The effect of the steam in the intermediate space may be estimated in the following manner. Disregarding the steam in the intermediate space, the volume $1.10v$ of steam at the terminal pressure 51.66 may be assumed to occupy the volume

$$(0.3 + 0.1)v + 0.1V + (0.7 + 0.08)V = 3.04v$$

at the cut-off of the large cylinder, and the corresponding pressure is

$$\frac{51.66 \times 1.1}{3.04} = 18.7 \text{ pounds.}$$

After the cut-off occurs the steam in the small cylinder at this pressure is compressed from the volume

$$(0.3 + 0.1)v + 0.1V = 0.7v,$$

which it then occupies, to the volume

$$0.1V + 0.1v = 0.4v,$$

and the final pressure will be

$$18.71 \times \frac{0.7}{0.4} = 32.7 \text{ pounds.}$$

The pressure in both cylinders after the drop has occurred may be assumed to be

$$\frac{32.7 \times 0.1V + 51.66 \times 1.1v}{1.1v + (0.1 + 0.08)V} = \frac{66.64}{1.64} = 40.6 \text{ pounds.}$$

The drop is

$$51.66 - 40.6 = 11.1 \text{ pounds.}$$

The corrected pressure at cut-off of the large cylinder will be

$$\frac{66.64}{3.04} = 21.9 \text{ pounds},$$

instead of 18.7 pounds given above, and it appears unnecessary to make a second approximation.

The mean forward pressure on the small piston is

$$94.7 \times \tfrac{1}{2} + 94.7 \times 0.6 \log_e \frac{1.1}{0.6} = 85.7 \text{ pounds}.$$

The back pressure on the small piston and the forward pressure on the large piston up to the cut-off of the large cylinder is

$$40.6 \times 1.64v \log_e \frac{3.04}{1.64} \div 0.7(V - v) = 29.4 \text{ pounds}.$$

The back pressure on the small piston from the cut-off of the large cylinder to the end of the stroke is properly divided into two parts, the first part ending at the compression of the small cylinder, during which steam is compressed in the small cylinder and its clearance and the intermediate space, and the second part from the compression to the end of the stroke, during which steam is compressed in the small cylinder and its clearance only. For our present purpose it is sufficient to make the calculation on the assumption that the compression of the small cylinder is at the end of the stroke, which gives for the back pressure of this part of the stroke of the small piston

$$21.9 \times 0.7v \log_e \frac{0.7}{0.4} \div 0.3v = 28.6 \text{ pounds}.$$

The mean back pressure on the small piston is therefore

$$0.7 \times 29.4 + 0.3 \times 28.6 = 28.2 \text{ pounds}.$$

The mean effective pressure on the small piston is

$$85.7 - 28.2 = 47.5 \text{ pounds.}$$

The forward pressure on the large piston from cut-off to the end of the stroke is

$$21.9 \times 0.78 \log_e \frac{1.08}{0.78} \div 0.3 = 18.5 \text{ pounds.}$$

The mean forward pressure on the large piston is

$$29.4 \times 0.7 + 18.5 \times 0.3 = 26.1 \text{ pounds.}$$

The mean back pressure on the large piston is

$$4 \times 0.85 + 4 \times 0.23 \log_e \frac{0.23}{0.08} = 4.1 \text{ pounds,}$$

and the mean effective pressure on the same is

$$26.1 - 4.1 = 22 \text{ pounds.}$$

The mean effective pressure reduced to the large cylinder is

$$\frac{47.5}{3} + 22 = 37.8 \text{ pounds.}$$

The division of work between the two cylinders is in the ratio

$$\frac{47.3}{3 \times 22} = \frac{1}{1.4}.$$

The maximum pressure on the small piston occurs at 0.3 of the stroke when the back pressure is a minimum, and is then

$$94.7 - 21.9 = 72.8,$$

which is equivalent to a pressure on the large piston of

$$\frac{72.8}{3} = 24.3 \text{ pounds.}$$

The maximum pressure on the large piston is about 35 pounds.

If the two pistons are fixed to one rod, the maximum pressure, reduced to the large piston, is about 60 pounds. The maximum pressure on the piston of a simple engine using the same steam-pressure and expanding the same number of times will be 90.7 pounds per square inch.

The drop may be made smaller, or if desired it may be made to disappear, by shortening the cut-off of the large cylinder, and experiments show that a gain of efficiency accompanies such a change.

EXAMPLE 3.—If the engine were made with a receiver and with two low-pressure cylinders, each with its own crank, the drop could be avoided, and a good distribution of the work among the three cylinders could be attained. The clearance and compression will be neglected in this solution.

The terminal pressure in the small cylinder, as in the first solution of Example 1, is 47.35 pounds absolute, and this is also the back pressure in that cylinder.

The mean effective pressure in the small cylinder is

$$94.7 \times \tfrac{1}{2}(1 + \log_e 2) - 47.35 = 42.82 \text{ pounds.}$$

To compare this with one of the low-pressure cylinders this result may be divided by $\frac{3}{2}$, giving 28.55 pounds.

The cut-off of each of the low-pressure cylinders must be at $\frac{1}{3}$ stroke.

The mean effective pressure in each of the low-pressure cylinders is

$$47.35 \times \tfrac{1}{3}(1 + \log_e 3) - 4 = 25.79 \text{ pounds.}$$

There is no drop, and the equivalent mean effective pressure reduced to one low-pressure cylinder is

$$\frac{42.83}{3} + 25.79 = 40.06 \text{ pounds,}$$

as found on page 209.

EXAMPLE 4.—A triple-expansion engine using steam at 150 pounds gauge-pressure has the volumes of the cylinders in the ratio $1 : 2\frac{1}{2} : 6\frac{1}{4}$; and the cut-off is at 0.6 of the stroke on the high-pressure and intermediate cylinder and at 0.75 of the stroke on the low-pressure cylinder.

Neglecting the effects of clearance and compression, the total expansion is

$$\frac{1}{0.6} \times 6\tfrac{1}{4} = 10.4;$$

the pressure in the first intermediate receiver is

$$\frac{164.7 \times 0.6}{0.6 \times 2.5} = 65.88 \text{ pounds};$$

and the pressure in the second intermediate receiver is

$$\frac{164.7 \times 0.6}{0.75 \times 6.25} = 21.08 \text{ pounds};$$

while the pressure in the condenser may be taken at 4 pounds absolute.

The mean effective pressure in the high-pressure cylinder is

$$164.7 \times 0.6\left(1 + \log_e \frac{1}{0.6}\right) - 65.88 = 83.41 \text{ pounds}.$$

The mean effective pressure in the intermediate cylinder is

$$65.88 \times 0.6\left(1 + \log_e \frac{1}{0.6}\right) - 21.08 = 38.64 \text{ pounds}.$$

The mean effective pressure in the low-pressure cylinder is

$$21.08 \times 0.75\left(1 + \log_e \frac{4}{3}\right) - 4 = 16.36 \text{ pounds}.$$

The distribution of work among the three cylinders is in the proportion

$$\frac{83.41}{6.25} : \frac{38.64}{2.5} : 16.36 :: 1 : 1.16 : 1.23.$$

The pressures on the crank-pins are in the proportion

$$\frac{164.7 - 65.88}{6.25} : \frac{65.88 - 21.08}{2.5} : 21.08 - 4 :: 1 : 1.14 : 1.08.$$

The drop to the first intermediate receiver is

$$164.7 \times 0.6 - 65.88 = 32.94 \text{ pounds.}$$

The drop to the second intermediate receiver is

$$65.88 \times 0.6 - 21.08 = 12.45 \text{ pounds.}$$

On account of the loss of pressure between the boiler and the engine, and between the engine and the condenser, and of the resistance of valves and passages, the mean effective pressure calculated as in the preceding examples, taking account of clearance and compression, is not realized in practice. The following table of multipliers is given by Seaton * for finding the probable mean effective pressure of compound marine engines:

MULTIPLIERS FOR FINDING PROBABLE M.E.P. COMPOUND ENGINES.

(1) Expansion-valve to H.P. cylinder, large ports, cylinders jacketed.	0.9–0.92
(2) Ordinary slide-valves, good ports, cylinders jacketed...........	0.8–0.85
(3) General practice of merchant service, early cut-off in both cylinders, without expansion-valves or jackets....................	0.7–0.8
(4) Fast-running engines of type and design usually fitted in war ships..	0.6–0.8

To find the probable mean effective pressure, by aid of this table, the mean effective pressure for each cylinder is to be calculated separately, allowing for clearance and compression, and the result multiplied by the proper factor. Or the equivalent mean

* Manual of Marine Engineering.

effective pressure, as calculated in the examples, allowing for clearance and compression, may be multiplied by the factor.

A fair approximation to the probable mean effective pressure of marine engines of the ordinary type can be obtained by calculating the mean effective pressure approximately, on the assumption that the total expansion takes place in one cylinder, not allowing for clearance and compression, and then multiplying successively by 0.96 and by the proper factor from the table.

Hirn's Analysis.—Since the admission to the high-pressure cylinder and the exhaust from the low-pressure cylinder of a compound engine do not differ from the corresponding parts of the cycle of a simple engine, we may apply the equations deduced for the simple engine to the determination of the value of Q_c, the heat rejected from the walls of the cylinder to the condenser.

Hirn and Hallauer, in all of their work, content themselves with determining explicitly this one of the four quantities, Q_a, Q_b, Q_c, and Q_d, but there appears to be no reason why the other three should not be determined also, for compound engines as well as for simple engines, provided a clear idea is obtained of their meanings. Q_a is the heat absorbed by the walls of the high-pressure cylinder during the admission of steam to it; Q_c is the heat rejected to the condenser by the walls of the low-pressure cylinder during its exhaust; and Q_d is the interchange of heat during the compression in the large cylinder. Q_b appears as the heat yielded by the walls of the cylinders during expansion, but it is an incomplete expression for a complicated operation. During the expansion in the small cylinder heat is yielded by its walls to the mixture of water and steam; also during the exhaust heat is yielded by the walls of the small cylinder, so that nearly if not all the water present is vaporized as during the exhaust of a simple engine. In the large cylinder the steam is condensed on the walls during the first part of the admission for a Woolf engine, and probably up to cut-off for a receiver compound engine, and heat is consequently absorbed by the walls of that cylinder, but that heat is given up in large part during the expansion in the remainder of the stroke. The final result is a yielding of heat to the mixture in the cylinder. In good types of compound engines the value of Q_c is small, though there may be a large interchange of heat during the earlier operations, and this fact is the probable explanation of the good efficiency of such engines.

Of the four quantities of work found in the equations for finding the interchange of heat, W_a is the absolute work during admission to the small cylinder, W_c is the negative work of the back pressure, and W_d the negative work of the compression for the large cylinder. W_b is the total work of expansion which may be obtained by the expression

$$W = W_a + W_b + W_c + W_d.$$

Hallauer does not state clearly how W_b is obtained in the work which he gives for compound engines.

The following method is proposed as an extension of Hirn's theory to compound engines, with the hope that by its aid the transfer of heat from the walls of the small cylinder to the walls of the large cylinder, with kindred phenomena, may be calculated after proper experiments are made.

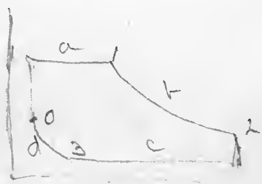
The method can be applied when each cylinder has its own jacket with separate drain, so that the condensation in each and the radiation from each can be determined separately, and when, further, all the data from the condenser can be obtained. For the high-pressure cylinder, as for a non-condensing simple engine, we have

$$Q_a = Q + M_0 q_0 - (M + M_0) q_1 + V_0 \frac{\rho_0}{u_0} - (V_0 + V_1) \frac{\rho_1}{u_1} - AW. \quad (273)$$

$$Q_b = (M + M_0)(q_1 - q_2) + (V_0 + V_1) \frac{\rho_1}{u_1} - (V_0 + V_2) \frac{\rho_2}{u_2} - AW_b. \quad (274)$$

$$Q_c = (M + M_0) q_2 - M_0 q_3 + (V_0 + V_2) \frac{\rho_2}{u_2} - (V_0 + V_3) \frac{\rho_3}{u_3}$$
$$- Q - Q_j + Q_e + A(W + W_c). \quad (275)$$

$$Q_d = M_0 (q_3 - q_0) + (V_0 + V_3) \frac{\rho_3}{u_3} - V_0 \frac{\rho_0}{u_0} + AW_d. \quad . \quad . \quad . \quad . \quad (276)$$



For the low-pressure cylinder the heat received during admission, Q', cannot be obtained directly, but it can be obtained by aid of equation (264), which may be written

$$Q' = Q_e' - Q_j' + Mq_4' + G(q_\kappa' - q_i') + AW' \; ; \quad . \quad . \quad . \quad . \quad . \quad (277)$$

and the four required equations may be written

$$Q_a = Q_e' - Q_j' + M_0'q_0' + Mq_4' - (M + M_0')q_1 - G(q_\kappa - q_i)$$
$$+ V_0' \frac{\rho_0'}{u_0'} - \left(V_0' + V' \frac{\rho_1'}{u_1'}\right) - A(W' + W_a'), \quad . \quad (278)$$

$$Q_b = (M + M_0')(q_1' - q_2') + (V_0' + V_1') \frac{\rho_1'}{u_1'}$$
$$- (V_0' + V_2') \frac{\rho_2'}{u_2'} - AW_b' \; ; \quad . \quad (279)$$

$$Q_c = (M + M_0')q_2' - M_0'q_3' - Mq_4' - G(q_\kappa - q_i)$$
$$+ (V_0' + V_2') \frac{\rho_2'}{u_2'} - (V_0' + V_3') \frac{\rho_3'}{u_3'} + AW_c' \; ; \quad . \quad (280)$$

$$Q_d = M_0'(q_3' - q_0') + (V_0' + V_3') \frac{\rho_3'}{u_3'} - V_0' \frac{\rho_0'}{u_0'} + AW_d'. \quad . \quad . \quad (281)$$

With triple and quadruple expansion engines the following method may be used. The heat rejected by the high-pressure cylinder during exhaust is

$$Q + Q_j - AW - Q_e.$$

This heat passes into the first intermediate cylinder, and from thence with the gain or loss experienced there proceeds to the next cylinder. The sum, or difference, may be taken for Q', the heat brought into that next cylinder per stroke. The same operation may be applied to each successive cylinder, and the result may be checked by the data depending on the condenser.

CHAPTER XIV.

TESTING STEAM-ENGINES.

TESTS of steam-engines are made either to find the cost of power or to study the transformations of heat and work in the engine, though both objects are frequently sought in the same test. The cost of power is commonly stated in pounds of coal, or of combustible, per horse-power per hour. To obtain this cost of power the engine and boiler must be considered as one system, and a test consists essentially in weighing the coal consumed and measuring, in some manner, the work produced in a given time. The power may be measured by aid of steam-engine indicators, by a friction brake, or by a transmission dynamometer; the measurement of the fuel consumed must be done with all the precautions required for an accurate boiler test, and such a test should last at least ten hours. Though a test of this kind will give directly the cost of power of a plant consisting of engine, boiler, etc., or will determine which of two or more plants is the most economical, it does not give the means of distinguishing whether the excellence or defects of a system are due to the engine or the boiler, much less does it enable us to make such an analysis as will show why a given engine or boiler is better than another.

To distinguish between the performance of the engine and of the boiler, it is customary to state the performance of the boiler in pounds of water evaporated per pound of fuel, and that of the engine in pounds of water per horse-power per hour. The evaporative efficiency of a boiler is frequently stated in pounds of water evaporated per pound of fuel, from and at 212° F.; that is, a special thermal unit, equal to 965.8 B.T.U., is employed. For example, if a boiler, for each pound of fuel consumed, takes eight pounds of feed-water at 60° F., and evaporates it into dry steam under a pressure of 100 pounds to the square inch above the atmosphere, then it is assumed that each pound of fuel can evaporate

$$\frac{8 \times 1184.9}{965.8} = 9.8$$

pounds of water at 212° F., and under the pressure of the atmosphere. No attempt has been made to put the steam consumption of engines on as logical a basis, and in general it is necessary to know the type of an engine and the conditions under which it works in order to judge whether its performance is good or not.

It is in general better to make an engine test independent of the boiler test, especially if an attempt is to be made to analyze the transformations of heat and work; and this is the more convenient as an engine test of from one to four hours in length is sufficient under favorable conditions. Even though the immediate object of the test is to ascertain the steam consumption only, as many as possible of the data mentioned in Hirn's analysis, page 185, should be taken and recorded: to wit, the pressure and condition, whether primed or superheated, of the steam supplied to the engine, together with the weight of the same; the weight and initial and final temperatures of the cooling or injection water, and, where a surface condenser is used, the temperature of the water resulting from the condensation of the steam; the vacuum in the condenser and the pressure of the atmosphere; where the engine is compounded, the pressure of the receiver or receivers, when nearly constant, may be taken, and if in any way heat is added to or taken from the steam in a receiver, such heat should be measured if possible; if any cylinder or cylinders have steam-jackets, the pressure and condition of steam supplied to each jacket, and the weight and temperature of the water condensed therein, should be known; indicator-diagrams should be taken at each end of each cylinder, at intervals depending on the length and regularity of the test; the total area of the indicator-diagrams should be measured, and also the areas, down to the line of absolute vacuum, under the lines of admission, to cut-off, expansion, exhaust, and compression. When an engine is steam-jacketed, it is assumed that the condensation in the jacket or jackets when the engine is at rest is a measure of the loss by external radiation, conduction, etc.

Thermometers.—Temperatures are commonly measured by aid of mercurial thermometers, of which three grades may be distinguished. For work resembling that done by the physicist the highest grade should be used, and these must ordinarily be calibrated, and have their boiling and freezing points determined by the experimenter or some qualified person; since the freezing-point is liable to change, it should be redetermined when necessary. For important data good thermometers must be used, such as are sold by reliable dealers, but it is preferable that they should be calibrated or else compared with a thermometer that is known to be reliable. For secondary data or for those requiring little accuracy, common thermometers with the graduation on the stem may be used, but these also should have their errors determined and allowed for. Thermometers with detachable scales should be used only for crude work.

Gauges.—Pressures are commonly measured by Bourdon gauges, and if recently compared with a correct mercury column, these are sufficient for engineering work. The columns used by gauge-makers are commonly subject to minor errors, and are not usually corrected for temperature. It is important that such gauges should be frequently retested. From their convenience, vacuum gauges of the same form are used, even where a mercurial gauge could easily be applied.

The pressure of the atmosphere may be taken with either a mercurial or an aneroid barometer, but if the latter is used its errors must be known. It should be easy to make the barometric errors only a fraction of the unavoidable gauge errors.

Dynamometers.—The standard for measurement of power is the friction brake. For smooth continuous running it is essential that the brake and its band should be freely lubricated with oil, and that the cooling should be done by a stream of water that does not come in contact with the rubbing surfaces. Sometimes the wheel is cooled by a stream of water circulating through it, sometimes the band is so cooled, or both may be. A rubbing surface which is not cooled should be of non-conducting material.

To avoid the increase of friction on the brake-bearings due to the load applied at a single brake arm, two equal arms may be used with two equal and opposite forces applied at the ends to form a statical couple.

With care and good workmanship a friction brake may be made an instrument of precision sufficient for physical investigations, but with ordinary care and workmanship it will give results of sufficient accuracy for engineering work.

All forms of transmission dynamometers should be standardized, and should have their errors determined by comparison with a friction brake.

Indicators.—Our knowledge of the errors of indicators, whether of kind or degree, is very limited. Preliminary experiments seem to show that at moderate speeds, i.e., those that give little or no oscillation of the piston and pencil motion, the largest errors are due to backlash and pencil friction. The latter may be reduced by making the pencil pressure light, but there is no remedy for the former. It probably does not introduce an error of more than one or two per cent in the diagrams taken with good indicators.

It is essential that the reducing motion should be correct, and that the indicator-cord should be short. The communication between the indicator and the cylinder should be short and direct, but if a pipe must be used it should be well wrapped to avoid radiation.

Scales.—Weighing may be done with scales adapted to the load. They should be tested with standard weights.

Weirs and Orifices.—When possible, the quantities of water involved in an engine test should be weighed directly ; and by proper provision of large tanks and scales, and with large valves, large quantities of water may be thus determined. When the water cannot be weighed directly it may be measured in tanks of which the volume is known either from measurement or, preferably, by filling them with weighed water.

When the two preceding methods do not apply, the water may be allowed to flow over a weir or through an orifice, and the volume and weight may be determined by the usual hydraulic methods. If the weirs or orifices are small, the coefficients of flow should be determined by direct experiment.

Steam Consumption.—The steam consumption of an engine is preferably determined by condensing the exhaust steam in a surface-condenser and weighing or gauging the resulting water. A great advantage is that a test an hour or two long is then sufficient.

When the exhaust steam cannot be thus condensed the boiler or boilers supplying the engine may be isolated so that all the steam made must go to the engine, and then the feed-water supplied to the boiler may be weighed or gauged. After the engine has been running long enough to come to its normal condition, the height of the water in the boiler gauge-glass may be noted, all the feed-water during a test of from two to four hours in length may be weighed or measured, and at the end of the test the water in the gauge-glass must be brought to the initial height.

Calorimeters.—When superheated steam is supplied to an engine it is sufficient to take the temperature of the steam in the steam-pipe near the engine. When moist steam is used, the condition of the steam must be determined by a calorimetric experiment. Four kinds of calorimeters will be described out of a large number that have been used by different experimenters and at different times. They are the barrel calorimeter, the Barrus continuous water calorimeter, the Barrus superheated steam calorimeter, and the throttling calorimeter.

The Barrel Calorimeter.—A wooden barrel set on scales is provided with a large valve for emptying it, and provision is made for filling it with cold water, usually from a hydrant pipe, and for bringing the steam to be tested. Some form of stirrer must be used, a good form being a wooden propeller-wheel on a wooden shaft with a hand crank.

The method of making a test is as follows: The barrel is weighed empty, and a suitable quantity of cold water is run in and weighed. The temperature of the cold water should be taken as it enters. The steam-pipe usually terminates in a piece of rubber hose which may be swung into or out of the barrel. When the barrel is nearly filled with cold water, the steam-valve may be opened until all condensed water is blown from the pipe and the hose is warmed up; then the hose may be swung into the barrel and steam may be run into the water till a proper amount is condensed. A preliminary calculation will determine the proper weights of water and steam to give a good range of temperatures in the calorimeter. After the steam is run in, the water in the barrel may be well stirred, and the highest temperature taken as the final temperature.

To eliminate the action of the wood of the barrel, one or more tests are made and rejected, and the times of running in water and steam are made equal, so that the barrel which is already warmed by the preceding test may give up as much heat during one part of the process as it receives during the other part.

If the pressure of the steam is p, and the part of each pound of the mixture which is steam is represented by x, while the initial and final temperatures of the water are t_1 and t_2, and the weights of the water and steam are W and w, then

$$w(xr + q - q_2) = W(q_2 - q_1);$$

$$\therefore x = \frac{W(q_2 - q_1) - w(q - q_2)}{wr}, \quad . \quad . \quad . \quad (282)$$

r and q being the latent heat and heat of the liquid for the pressure p, and q_1 and q_2 being the heats of the liquid for the temperatures t_1 and t_2.

EXAMPLE.—Suppose that 180 pounds of water at the temperature of 60°.2 F. are run into a barrel calorimeter, and that the final temperature of the water in the calorimeter is 103°.6 F., after $7\frac{1}{4}$ pounds of steam at 73.8 pounds by the gauge are run in and condensed. At an absolute pressure of 88.5 pounds, $r = 890.4$, $q = 288.8$; the heats of the liquid at 60°.2 and 103°.6 are 28.32 and 71.6.

$$x = \frac{180(71.6 - 28.32) - 7.25(288.8 - 71.6)}{7.25 \times 890.4} = 0.963;$$

consequently the per cent of priming is 3.7.

It is to be remarked of this kind of calorimeter that satisfactory results are difficult to attain even when every care and precaution are used, and that a small error in determining the weight of steam, which is obtained by subtraction, makes a large difference in the result.

Barrus Continuous Water Calorimeter. — The difficulty of obtaining the weight of steam with sufficient accuracy, which occurs in the use of the barrel calorimeter, is avoided in the use of the continuous water calorimeter represented by Fig. 54. This calorimeter is essentially a small surface condenser of special form, so arranged that the condensed steam is weighed separately from the cooling water.

Steam is brought to the calorimeter by the pipe *j*, with the gauge *i* for giving the pressure. The pipe *a*, which forms the condensing surface, and which may conveniently be made of brass pipe one inch in diameter, should have the joints, above and below, clear of the bucket containing the cooling water. Steam is let into the pipe *a* at full boiler-pressure, and the condensed water gathers in the pipe below, where the water-level is shown at *e*. The height of the water at *e* is kept constant by aid of the valve at *d*, which may have a long wooden handle attached for convenient regulation. At *h* there is a thermometer to determine the temperature of the condensed steam. Since this temperature is only a little less than that due to the boiler-pressure, the condensed water should be led through a cooler like a simple surface condenser, with a separate stream of cooling water, and the cooled water may be collected and weighed on suitable scales.

The cooling water for the calorimeter is brought by the pipe *b* with a valve for regulating the supply, and is led away to a barrel on scales by the pipe *c*, with a valve to regulate the

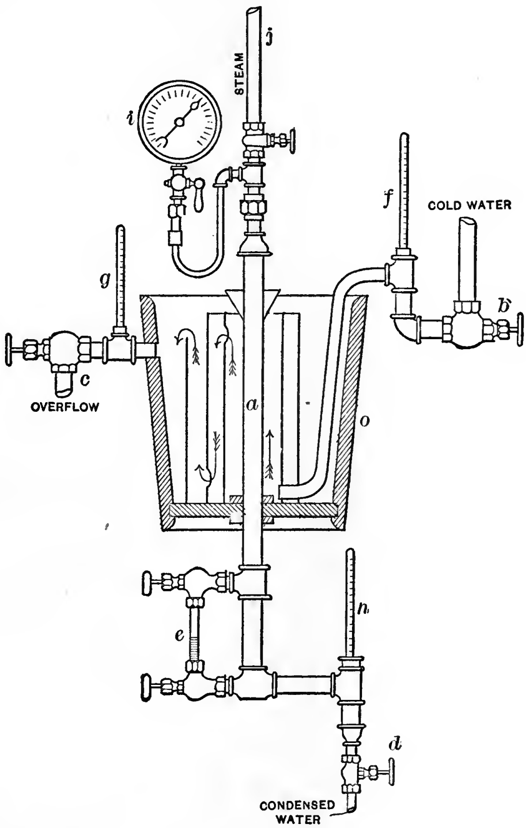


FIG. 54.

height of the water in the bucket. To insure a good circulation and a proper mingling of the cooling water the current is directed through a rubber hose to the bottom of the inner cylinder around the pipe *a*, thence up and into the top of the outer cylinder, thence down and out at the bottom of this cylinder and over a weir at the exit. The temperatures of the cooling water at entrance and exit are taken by the thermometers f and g, which should be reliable to $\frac{1}{20}$ of one degree Fahrenheit.

The pipe j leading to the calorimeter and the pipe containing the condensed steam should be well wrapped as far as to the valve at d. At k there is a brass cone to protect the covering of the pipe from water.

Though not essential, it is convenient to line the bucket with sheet metal.

In preparing for a test the water and steam are let on and properly regulated, and the calorimeter is allowed to run till all parts may be assumed to be at a constant temperature; the cooling water from c and the condensed steam are then directed into the receptacles for weighing, and the time is noted as the beginning of the test. The steam-pressure and the several temperatures are taken at intervals and recorded. At the end of half an hour or an hour the cooling water and condensed water are diverted from the weighing receptacles, and the time is noted as the end of the test. The quantities of the cooling and condensed water can be weighed at the end of the test, or the test may be made continuous for any desired length of time by having two weighing receptacles for each, and filling and emptying them alternately.

The radiation in thermal units per hour must be determined by running the calorimeter without cooling water and with the bucket filled with hair-felt.

In this or any form of calorimeter that is capable of giving accurate results it is essential that the steam-pressure should not change during a test, since a considerable change of pressure will vitiate the results on account of the heat absorbed or yielded by the pipes leading to the condenser.

Let W and w be the weights of the cooling water for the test, and let p be the steam-pressure, and t_3 the final temperature of the condensed steam taken by the thermometer at h, while t_1 and t_2 are the initial and final temperatures of the cooling water; finally, let the radiation during the test be e thermal units.

Then

$$w(xr + q - q_3) = W(q_2 - q_1) + e;$$

$$\therefore x = \frac{W(q_2 - q_1) + e - w(q - q_3)}{wr} \quad . \quad . \quad . \quad . \quad (283)$$

EXAMPLE.—The following are the data of a test made in the laboratory of the Institute of Technology:

Initial temperature of cooling water, . .	37°.49 F.
Final " " " " . .	83°.84 F.
Temperature of condensed steam, . . .	304°.88 F.
Pressure of the atmosphere, ...	14.8 lbs. per sq. in.
Pressure of steam by gauge, ...	72.4 " " " "
Duration of test, ...	40 minutes.
Radiation per hour, ...	180 B. T. U.
Weight of cooling water, ...	573.5 pounds.
" " condensed water, ...	29.89 "

$$x = \frac{573.5(51.91 - 5.53) + 120 - 29.89(287.6 - 274.4)}{29.89 \times 891.2};$$

$$x = 0.988.$$

Per cent of priming, 1.2.

It is apparent that any surface condenser may be used in the same manner, as a calorimeter, except that it is not usually convenient to fill such a condenser with steam at boiler-pressure. Since the wire-drawing of steam in a well-wrapped valve is accompanied with little loss of heat, this need not interfere with such a use of a condenser. In an engine test the quality of exhaust steam flowing to either a jet or a surface condenser can be determined by equation (282) or (283), except that the external radiation cannot always be satisfactorily determined.

Barrus Superheated-steam Calorimeter.—A form of calorimeter devised by Mr. Barrus is shown in Fig. 55, which determines the quality of steam by finding how much heat is required to superheat it.

The steam to be tested comes into the pipe H and passes through a tubular superheater J, and flows out of an orifice at

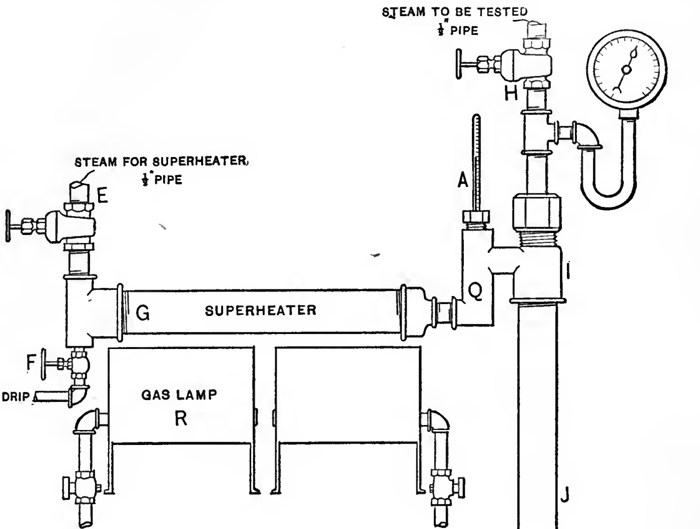
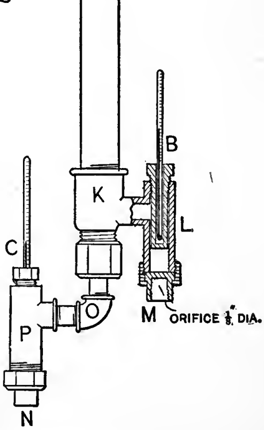


FIG. 55.



M. A separate stream of steam comes in by the pipe E, is strongly superheated by gas lamps in the superheater G, passes around the tubes of the superheater J, and flows out of an orifice at N of the same diameter as that at M. Temperatures are taken by the thermometers at A, B, and C; and the boiler-pressure, which is admitted to all the apparatus, is measured by a gauge. In the calculation it is assumed that the specific heat of superheated steam at all temperatures and pressures is $c_p = 0.48$ as determined by Regnault, and that the same weight of steam will flow out of each of the orifices M and N under the same pressure, though the temperatures are different. The admissibility of the last assumption can be tested for any experiment by condensing and weighing the steam from each orifice separately.

The radiation from this calorimeter may be formed by allowing the superheating steam to flow through the superheater J, while the moist steam to be tested is shut off. The difference between the temperatures given by the thermometers at A and B under such circumstances is due to external radiation, and will be the same under like conditions; let this loss be n degrees. Let the initial temperature of the superheating steam be t_a and the final temperature be t_b. Let the pressure of the steam be p, to which correspond the temperature t and latent heat r. Let the steam, which at first had the temperature t and contained $1-x$ of moisture, leave the orifice M with the temperature t_s. The heat yielded by the superheating steam is $0.48(t_a - t_b)$, of which $0.48n$ is lost by external radiation. The heat gained by the steam under test is $(1 - x)r + 0.48(t_s - t)$.

Consequently,

$$0.48(t_a - t_b - n) = (1 - x)r + 0.48(t_s - t);$$

$$\therefore 1 - x = \frac{0.48[t_a - t_b - n - (t_s - t)]}{r}. \quad . \quad . \quad (184)$$

EXAMPLE.—The following are the data of a test made with this calorimeter in the laboratory of the Institute of Technology:

Pressure of the atmosphere, ...	14.7 pounds;
Gauge-pressure of steam, ...	71.7 "
Final temperature of steam to be tested, ...	331°.8 F.
Initial temperature of superheating steam, . .	417°.4 F.
Final " " " " . .	347°.4 F.
Loss of temperature by radiation, ...	11°.7 F.

$$1 - x = \frac{0.48[417.4 - 347.4 - 11.7 - (331.8 - 317.2)]}{891.7};$$

$$1 - x = 0.024.$$

It was found, by special experiment under the conditions of the experiment, that the radiation of the pipe leading to the calorimeter increased the moisture in the steam 1.2 per cent; consequently the priming is 1.2 per cent.

Throttling Calorimeter.—A simple form of calorimeter, shown by Fig. 56, was devised by the author, which depends on the property that dry steam is superheated by throttling. Steam to be tested is brought in by a wrapped pipe *a*, below which the extension *c* with a drip at the end serves as a pocket to catch the water which may gather on the sides of the pipe. The valve at *b* is opened a slight amount to admit steam to the chamber *A*, and the exit valve at *d* is used to regulate the pressure in the chamber. The temperature in the chamber is taken by a thermometer in a long cup at *e*, and the pressure is taken by the gauge *f*. Let the boiler-pressure be p, and let r and q be the latent heat and heat of the liquid corresponding. Let p_1 be the pressure in the calorimeter, and λ_1 and t_1 the total heat and the temperature of saturated steam at that pressure, while t_s is the temperature of the superheated steam in the calorimeter. Then

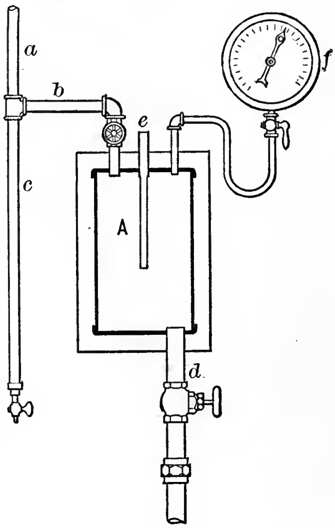


FIG. 56.

$$xr + q = \lambda_1 + c_p(t_s - t_1);$$

$$\therefore x = \frac{\lambda_1 + c_p(t_s - t_1) - q}{r}. \quad . \quad . \quad . \quad . \quad (285)$$

EXAMPLE.—The following are the data of a test made with this calorimeter:

Pressure of the atmosphere, ...	14.8 pounds;
Steam-pressure by gauge, ...	69.8 "
Pressure in the calorimeter, gauge, . .	12.0 "
Temperature in the calorimeter: . . .	268°.2 F.

$$x = \frac{1156.4 + 0.48(268.2 - 243.9) - 286.3}{892.7} = 0.988;$$

Per cent of priming, 1.2.

A little consideration shows that this type of calorimeter can be used only when the priming is not excessive; otherwise the wire-drawing will fail to superheat the steam, and in such case nothing can be told about the condition of the steam either before or after wire-drawing. To find this limit for any pressure, t_s may be made equal to t_1 in equation (285); that is, we may assume that the steam is just dry and saturated at that limit in the calorimeter. Ordinarily the lowest convenient pressure in the calorimeter is the pressure of the atmosphere, or 14.7 pounds to the square inch. The table following has been calculated for several pressures in the manner indicated. It shows that the limit is higher for higher pressures, but that the calorimeter can be applied only where the priming is moderate.

LIMITS OF THE THROTTLING CALORIMETER.

PRESSURE.		Priming.
Absolute.	Gauge.	
300	285.3	0.077
250	235.3	0.070
200	185.3	0.061
175	160.3	0.058
150	135.3	0.052
125	110.3	0.046
100	85.3	0.040
75	60.3	0.032
50	35.3	0.023

When this calorimeter is used to test steam supplied to a condensing engine, the limit may be extended by connecting the exhaust to the condenser. For example, the limit at 100 pounds absolute, with 3 pounds absolute in the calorimeter, is 0.064, instead of 0.046 with atmospheric pressure in the calorimeter.

In case the calorimeter is used near its limit,—that is, when the superheating is a few degrees only,—it is essential that the thermometer should be entirely reliable, otherwise it might happen that the thermometer would show superheating when the steam in the calorimeter was saturated or moist. In any other case a considerable error in the temperature will produce an inconsiderable effect on the result. Thus, at 100 pounds absolute with atmospheric pressure in the calorimeter, 10° F. of superheating indicates 0.035 priming, and 15° F. indicates 0.032 priming. So also a slight error in the gauge-reading has little effect. Suppose the reading to be apparently 100.5 pounds absolute instead of 100, then with 10° of superheating the priming appears to be 0.033 instead of 0.032.

Efficiency of a Steam-engine.—When the performance of an engine is given in pounds of water per horse-power per hour it is necessary to know also the pressure and quality of the steam used, and also to know the temperature of saturated steam at the pressure against which the engine exhausts. The difference of economy to be expected from high-pressure or low-pressure steam, from superheated or wet steam, or from a condensing or non-condensing engine, are specific instances of causes modifying the performance of an engine.

There appears to be no good reason why the performance of an engine should not be stated in thermal units per horse-power per hour, which would enable us to compare directly all forms of engines without allowances or reductions. Such a method also leads at once to the consideration of the true efficiency of an engine.

Suppose that an engine is supplied with M pounds of steam per stroke, having the pressure p and the quality x.

Then the heat in that steam above the heat in the same weight of water at freezing is

$$M(xr + q).$$

Should the steam be superheated and have the temperature t_s, then the heat in the steam is

$$M[c_p(t_s - t) + \lambda].$$

Let the pressure against which the engine exhausts be p_0, at which steam has the temperature t_0 and the heat of the liquid q_0. If the engine is a condensing engine the water in the hot-well, from whence the boiler is fed, can have nearly this temperature, and if the engine is non-condensing, then the feed-water can be raised nearly to this temperature by an exhaust-steam feed-water heater. It may be considered that the heat furnished to the engine per stroke is, for moist steam,

$$M(xr + q - q_0),$$

and that amount of heat must in general be given to the water and steam in the boiler by the fire.

The work done by the steam per stroke, shown by the indicator, is W, the heat equivalent is AW. The efficiency of the engine is consequently

$$\eta_i = \frac{AW}{M(xr + q - q_0)}. \quad . \quad . \quad . \quad . \quad . \quad (286)$$

When C_i, the consumption of steam per indicated horse-power per hour, is stated, the efficiency is

$$\eta_i = \frac{60 \times 33000 \times A}{C_i(xr + q - q_0)}. \quad . \quad . \quad . \quad . \quad . \quad (287)$$

The last two equations give the efficiency of the fluid. The efficiency of the engine, determined by aid of a brake or dynamometer, is found by substituting for C_i, C_n the consumption of steam per net or brake horse-power per hour, so that

$$\eta_n = \frac{60 \times 33000 \times A}{C_n(xr + q - q_0)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (288)$$

Similar equations may be deduced for superheated steam.

The two efficiencies η_i and η_b are to be compared with the maximum efficiency

$$\eta = \frac{T - T_0}{T}$$

of a heat-engine working between the temperatures T and T_0, the absolute temperatures of saturated steam at the pressures p and p_0.

Efficiency of the Boiler.—If the total heat of combustion of the fuel is H thermal units per pound, and if one pound of fuel evaporates m pounds of water from the temperature t_1, which may or may not be equal to t_0, to form steam having the quality x, at the pressure p, then the efficiency of the furnace and boiler is

$$\eta_b = \frac{m(xr + q - q_1)}{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (289)$$

Efficiency of Engine and Boiler.—The efficiency of the engine and boiler combined is the product of the efficiencies of each separately; that is,

$$\eta_o = \eta_n \eta_b .$$

Cost of Power.—The evaporative efficiency of a boiler is commonly given in pounds of water evaporated from and at 212° F., equivalent to 965.8 B. T. U. per pound; so that if m is the actual evaporation as used above, and m_0 the reduced evaporation,

$$m_0 = \frac{m(xr + q - q_1)}{965.8} \quad . \quad . \quad . \quad . \quad . \quad . \quad (290)$$

Here also it would very much simplify matters if thermal units were used; and then, without reduction, the evaporative efficiency could be given in thermal units per pound of fuel, which could be compared directly with the total heat of combustion.

If the consumption of steam per net horse-power per hour is C_n pounds, so that the consumption of heat per horse-power per hour is

$$C_n(xr + q - q_0),$$

then the consumption of coal per horse-power per hour is

$$\frac{C_n(xr + q - q_0)}{m(xr + q - q_1)} = \frac{C_n(xr + q - q_0)}{m_0}. \quad . \quad . \quad (291)$$

Efficiency Test.—The difficulty of arranging for the complete testing of large engines that are in continuous use has led to the proposal of a simple method by which the efficiency given by equation (286) may be found. The engine is indicated to determine AW, the work of the steam per stroke, and the condensing water and condensed steam delivered by the air-pump per stroke of the engine, from the jet condenser, flows in a well-mixed stream over a weir. If G is the injection-water per stroke, and M the steam used per stroke, then this gives

$$M + G = \text{numerical quantity}. \quad . \quad . \quad . \quad (292)$$

The initial and final temperatures t_0 and t_k of the injection-water are taken by the aid of thermometers.

Of the heat

$$M(xr + q - q_0)$$

consumed by an engine per stroke, a part, AW, is changed into work, a part, Q_e, is lost by external radiation, and the remainder, $G(q_k - q_0)$, is carried away by the injection-water, so that

$$M(xr + q - q_0) = AW + Q_e + G(q_k - q_0). \quad . \quad (293)$$

In equation (293), W, q_k, and q_0 are determined directly, r and q are known from the steam-pressure, and x may be determined by calorimetric experiment, or may be known approximately from previous experiments under like conditions; also, Q_e may be known or determined. Under good conditions x is unity or differs but little from unity, and Q_e is small, so that it may be neglected without serious error. We have then, for an approximate result, two equations (292) and (293) with two unknown quantities, and may solve for M directly, and therefrom estimate the steam per horse-power per hour, or by equation (286) may find the efficiency.

The chief advantage of this method is that it need not interfere with the ordinary running of the engine tested, and does not require much time or trouble.

CHAPTER XV.

TESTS OF SIMPLE STEAM-ENGINES.

In this chapter and the three following chapters will be given the data and results of several series of steam-engine tests, which were made to determine the relative economy of different methods of running engines and of different types of engines.

Tests on the Michigan.—In 1861 experiments were made on the engines of the United States paddle-wheel steamer *Michigan*, at Erie, Pa., by a board of naval engineers, and reported by Isherwood,* to determine the advantageous point of cut-off for naval engines of that type.

This vessel had a pair of inclined direct-acting engines, with a jet condenser, and with the form of poppet-valves commonly used on American steamboats, but with the Sickles cut-off gear, by which the cut-off could be varied from the commencement of the stroke to $\frac{4}{9}$ of the stroke; and further, the cut-off could be varied from $\frac{7}{10}$ to $\frac{11}{12}$ of the stroke when the special cut-off gear was disconnected.

The steam-pipe and cylinder sides were covered with felt and lagged with wood; the cylinder-heads were uncovered; the steam-pipes were so inclined that they drained into the cylinders. The main dimensions of the engines were:

Diameter of cylinder, ...	36 in.
Diameter of piston-rod, ...	$3\frac{3}{4}$ in.
Stroke of piston, ...	8 ft.
Piston displacement, allowing for piston-rod,	56.544 cu. ft.
Clearance, ...	3.280 cu. ft.
Net area of steam-valves, ...	114.96 sq. in.
Net area of exhaust-valves, ...	108.38 sq. in.

* Experimental Researches in Steam Engineering.

Diameter of paddle-wheel, ...	$21\frac{1}{2}$ ft.
Number of paddles, ...	16
Width of paddles, ...	31 in.
Length of paddles, ...	14 ft.

There were two rectangular internally fired boilers, with vertical water tubes, each boiler having three furnaces. The boilers were of the type known as Martin boilers, which were much used in the navy.

Manner of making Experiments.—The number of revolutions was recorded by a counter actuated by the engine. The feed-water was measured in a zinc-lined tank that held 70 cubic feet. It was filled from the hot-well through a hose by the bilge-pump; and there was a small hand-pump that was used to bring the level to the reference-mark each time with water from outside the vessel. The water was drawn from the tank by a feed-pump and distributed to the boilers. All connections to the boiler were broken and stopped with iron plates, and the hose from the bilge- and hand-pumps were thrown out of the tank after it was filled. The tank was pumped dry each time, and the feed-pipe emptied. The temperature in the tank was noted when it was half full.

The coal was weighed on scales in equal portions. Refuse was weighed dry on the same scales and in the same manner.

The steam-pressures were measured by a spring-gauge and a siphon mercurial gauge, the indications of which coincided. The vacuum was measured with a similar gauge. The atmospheric pressure was measured by an aneroid barometer, and the temperature was taken from a thermometer attached thereto. Two indicators were attached permanently to the two ends of the cylinder, and were actuated from the air-pump cross-head by a reducing lever.

The temperature of the injection-water was taken by a thermometer on the side of the vessel opposite the hot-well discharge. The temperature of the hot-well was taken by a thermometer immersed in it. The temperature of the external air was taken on deck.

The boilers were fitted with gauge-cocks and glass water-gauges. The only outlet from the boilers was through the blow-off valve; and any leakage would have to pass also a stop-cock and a Kingston valve.

The vessel was secured to the dock, and was housed in, and the power of the engine was expended in paddling the water aft. Each experiment lasted 72 hours, during which the condition of working was not changed in any way. In anticipation of the experiment, the engine was run several hours to bring it to the normal working condition. When all was ready, with average fires and proper steam-pressure and level of water in the glass gauges, the experiment was begun. During the experiment the working of the engine was made as regular as possible, and at the end of the 72 hours all conditions were left as at the beginning. The fires were cleaned at beginning and end; and it was thought that the error from this source for a run of such a length was small. The steam-pressure varied not more than half a pound during the entire experiment.

Indicator-diagrams were taken every half-hour, and each diagram measured separately. During all the experiments the throttle-valve was wide open, and the boiler-pressure was varied so as to keep the initial pressure in the cylinder constant.

At the end of the experiments the paddles were removed from the wheel, and the mean pressure then required to move the engine was used in allowing for the friction of the engine and calculating the net power.

The tightness and freedom from leakage of piston, valves, and stuffing-boxes was determined several times.

The temperature of the gases in the up-take was noted by a high-grade mercurial thermometer, and was about 520° F.

Three sets of observers with regular duties were arranged in watches, each superintended by a chief engineer of the navy.

The data and results of the experiments are given in Table I. The table will be understood from the headings with very little added explanation. In order to make the results of the different experiments directly comparable, the

TABLE I.

TESTS ON THE UNITED STATES STEAMER MICHIGAN.

				Temperatures, Fahrenheit.					Pressures.									
									Inches of Hg.		Pounds per square inch.							
	Duration in hours.	Cut-off.	Revolutions per minute.	External.	Engine-room.	Feed-water.	Injection-water.	Hot-well.	Barometer.	Vacuum in condenser.	Boiler above atmosphere.	Initial absolute.	Cut-off absolute.	End of stroke absolute.	Back pressure absolute.	Mean effective pressure actual.	M.E.P. with back pressure of 2.7 pounds.
1	72	$\frac{11}{12}$	20.609	29	76	66	33	100	30.11	26.5	21.0	34.8	32.2	29.3	4.2	29.8	31.3
2	"	$\frac{7}{10}$	15.561	30	75	60	33	100	29.83	26.1	19.5	33.3	31.4	22.2	3.5	27.6	28.4
3	"	$\frac{4}{9}$	17.282	29	78	62	33	100	29.71	26.3	21.0	34.5	33.0	15.7	3.0	24.1	24.4
4	"	$\frac{3}{10}$	13.690	27	75	62	33	100	30.14	25.8	21.0	34.4	33.4	11.0	3.0	19.9	20.2
5	"	$\frac{1}{4}$	13.868	34	79	55	33	100	29.85	25.8	21.0	34.3	33.3	9.7	2.7	17.4	17.4
6	"	$\frac{1}{6}$	11.168	30	77	57	33	100	29.85	25.6	21.0	34.2	33.2	7.8	2.8	13.6	13.7
7	"	$\frac{4}{45}$	14.100	27	77	59	33	100	29 88	24.1	22.0	34.7	33.0	5.9	3.7	8.8	9.8

	Pressures.		Consumption, pounds per hour.				Horse-power.				Steam per horse-power per hour.					Comparative	
	P'ds p sq. in.						Actual.		Modified.		Actual.		Modified.				
	Net effective press with 2.1 lbs. for frict'n	M.E.P. with hyperbolic expansion.	Feed-water.	Coal.	Per cent of refuse.	Coal per square foot of grate.	Indicated.	Net.	Indicated.	Net.	Indicated.	Net.	Indicated.	Net.	Per cent of water at end of stroke.	Piston speed.	Size of cylinder for equal power.
1	29.2	31.8	12040	1667	6.83	18.522	301.431	280.189	316.604	295.362	**39.942**	42.971	**38.029**	40.763	10.71	330	1.000
2	26.3	28.6	7335	1027	18.53	11.411	210.796	194.757	216.905	200.867	**34.797**	37.662	**33.817**	36.516	15.34	249	1.110
3	22.3	24.9	6762	856.3	6.92	9.515	204.421	186.609	206.966	189.153	**33.078**	36.235	**32.671**	35.748	27.18	277	1.309
4	18.1	20.4	4707	565.8	5.99	6.287	133.712	119.602	135.727	121.617	**35.204**	39.357	**34.681**	38.705	41.66	219	1.613
5	15.3	18.5	4083	469.0	6.31	5.211	118.434	104.140	118.434	104.140	**34.471**	39 202	**34.471**	39.202	39.61	222	1.909
6	11.6	11.5	2761	341.7	5.75	3.796	74.547	63.036	75.095	63.584	**37.039**	43.803	**36.769**	43.425	42.11	179	2.51
7	7.7	9.9	2806	370.4	6.99	4.115	60.900	46.367	67.821	53.288	**46.085**	60.530	**41.382**	52.669	45.10	226	3.79

mean effective pressure and the net effective pressures are given in a modified form, with the assumption of a uniform back pressure of 2.7 pounds, and of a pressure to move the engine unloaded of 2.1 pounds. The modified horse-power and consumption per horse-power per hour are calculated on the same basis from the modified pressures.

Discussion of Results.—All the experiments were made on the starboard engine, the other one being disconnected.

In all the experiments the initial pressure in the cylinder was maintained constant, and the boiler-pressure was varied slightly for that purpose, but not enough to affect the results materially. The speed of the engine could not be controlled at the same time, but the variation from 11 to 20 revolutions could not interfere with the comparability of the results.

The per cent of water in the cylinder at the end of the stroke is not excessive for a cut-off at $\frac{11}{12}$ of the stroke. It increases rapidly to the cut-off at $\frac{4}{9}$, then it remained nearly constant to the cut-off at $\frac{1}{4}$, and then increased again at a cut-off at $\frac{4}{45}$ of the stroke.

The consumption of steam per horse-power per hour is properly a basis of comparison in these tests, since the boiler-pressure did not vary greatly during the tests. Since the variation of the back pressure is due either to a variation in the vacuum or to defects in the valve-gear or steam passages, and since an engine designed for a given expansion may be supposed to have adequate provision for maintaining a good vacuum, and for realizing it in the cylinder, it appears proper to compare the steam consumption corrected for variation of back pressure, and called the modified steam per horse-power per hour in the tables.

The minimum consumption of steam per indicated horse-power per hour was found with a cut-off at $\frac{4}{9}$ of the stroke, but it appears that the consumption is nearly the same for all points of cut-off from $\frac{1}{4}$ to $\frac{7}{10}$ of the stroke.

Comparing the consumptions per net horse-power per hour, the minimum is also at $\frac{4}{9}$ of the stroke in the table, but as the consumption for shorter cut-off increases rapidly and as no experiments were made with the cut-off intermediate between $\frac{4}{9}$ and $\frac{7}{10}$, the most advantageous point of cut-off may be longer than $\frac{4}{9}$ of the stroke; and this seems not improbable, since the consumption with the cut-off at $\frac{7}{10}$ of the stroke exceeds the consumption when the cut-off is at $\frac{4}{9}$ of the stroke by 2 per cent only.

Considering the relative sizes of cylinders for the development of the same power, Isherwood concludes that for naval engines of this type and using saturated steam at a pressure of 20 pounds above the atmosphere, it is advisable to use a cut-off at $\frac{7}{10}$ of the stroke, more especially as a special cut-off gear is not required in such case.

Later experiments on engines using higher pressure of steam show that the advantageous point of cut-off becomes shorter and the number of advisable expansions becomes greater as the pressure increases.

It is instructive to notice that the per cent of water in the cylinder at the end of the stroke increases as the cut-off is shortened, and that with the exception of Experiment 5, which for some reason has a greater consumption of steam than either the test preceding or following, the increase is quite regular. It appears that the condensation and re-evaporation and the exhaust waste in this type of engine, with low-pressure steam, very quickly counteract the gain to be anticipated from expansion.

Tests on the Mackinaw.—The tests on the engine of the United States steamer *Mackinaw* were made by a board of naval engineers* to determine the advantage of using superheated instead of saturated steam, and at the same time an investigation was made to determine the best point of cut-off. The *Mackinaw* was one of a number of paddle-wheel steamers built for special service during the years 1863 and 1864. It had one direct-acting inclined engine, with poppet-valves and a Stevens' cut-off. The engine was furnished with a surface condenser. Steam was supplied by two Martin water-tube boil-

* Experimental Researches in Steam Engineering.

ers, each having five furnaces. The principal dimensions of the engine were:

Diameter of cylinder, ...	4 ft. 10 in.
Stroke of piston, ...	8 ft. 9 in.
Diameter of piston-rod, ...	$6\frac{1}{2}$ in.
Displacement of piston allowing for rod, .	159.5356 cu. ft.
Clearance, ...	13.5254 cu. ft.
Area for admission and exhaust of steam,	393 sq. in.
Diameter of paddle-wheel, ...	26 ft.
Number of paddles, ...	24
Length of paddle, ...	9 ft.
Width of paddle, ...	1 ft. 3 in.
Total grate-area, ...	200 sq. ft.
Total water-heating surface, ...	5036 sq. ft.
Superheating surface in up-takes, . . .	171 sq. ft.

The experiments were made with the boat secured to the dock in the same manner and with the same precautions as those on the *Michigan*. The first five were made with the water-level at the normal height, so that the steam was probably saturated. The last two were made with the water from five to six inches below the upper ends of the vertical water-tubes, so that the steam was superheated.

The first five experiments were intended to be at the same speed of revolution of the paddle-wheels, but the number of revolutions fell to 5.609 per minute when the cut-off was at 0.21 of the stroke in Experiment E, and Experiment A was made with 5.551 revolutions for sake of comparison.

The data and results are given in Table II, and after the discussion of previous work require no explanation beyond that given by the headings.

Discussion of Results.—Isherwood, in reporting these experiments, recalculated the results of the tests B, C, D, and E, on the following assumptions:

(1) That the initial pressure was 50 pounds absolute, and the pressure at cut-off was 47 pounds absolute.

TABLE II.

TESTS ON THE UNITED STATES STEAMER MACKINAW.

	Condition of Steam.	Duration, hours.	Cut-off.	Revolutions per minute.	Temperatures, Fahrenheit.						Pressures.				
					External air.	Engine-room.	Feed-water.	Hot-well.	Injection-water.	Discharge-water.	Inches of mercury. Barometer.	Inches of mercury. Vacuum in condenser.	Pounds per square inch. Initial absolute.	Pounds per square inch. Cut-off absolute.	Pounds per square inch. End of stroke absolute.
A.*	Saturated	48	**0.87**	5.551	53.8	72.6	46.2	103.4	44.5	100.2	30.38	26.49	**43 30**	25.47	22.86
B.	"	48	**0.70**	6 779	44.7	72.3	38.6	103.6	36.6	97.3	30.28	26.36	**41.32**	36.80	26.00
C.	"	48	**0.56**	6.869	39.1	67.3	40.0	100.8	38.0	97.7	30.39	27.00	**43.00**	40.00	23.00
D.	"	48	**0.38**	6.628	43.7	67.0	45.9	102.0	44.0	102.6	30.00	26.05	**49.00**	45.00	19.00
E.	"	48	**0.21**	5.609	53.8	72.6	46.2	103.4	44.5	100.2	30.25	26.58	**53.00**	49.40	15.20
F.†	Superheated	33	**0.65**	15.392	57.1	74.3	51.0	105.0	50.0	96.0	30.22	26.50	**43.97**	28.22	18.33
G.	"	48	**0.23**	15.960	56.3	74.4	52.0	105.0	51.4	99.0	29.99	26.07	**52.00**	47.70	15.70

	Pressures. Pounds per square inch.			Consumption, pounds per hour.				Horse-power.		Consumption per horse-power per hour.				Water evaporated per lb., combustible from 212 degrees.	Per cent of water at end of stroke.
	Back pressure absolute.	Mean effective.	Mean net.	Feed-water.	Coal.	Per cent of refuse.	Coal per sq. ft. grate surface.	Indicated.	Net.	Steam. Indicated horse-power.	Steam. Net horse-power.	Combustible. Indicated.	Combustible. Net.		
A.	2.00	29.20	27.70	8451	904.1	19.85	4.520	225.678	214.085	**37.437**	39.458	3.211	3.385	13.613	21.48
B.	2.02	35.12	33.62	10910	1269.1	18.30	6.346	331.479	317.321	**32.913**	34.382	3.128	3.268	12.376	16.18
C.	1.54	35.57	34.07	10419	1266.2	22.67	6.331	340.183	325.838	**30.628**	31.976	2.878	3.005	12.500	20.63
D.	2.00	34.36	32.86	9609	1075.4	17.60	5.299	317.072	303.240	**30 306**	31.688	2.758	2.884	12.839	31.18
E.	1.84	29.50	28.00	8304	902.1	20.00	4.510	230.379	218.665	**36.044**	37.975	3.132	3.300	13.437	45.66
F.	1.90	29.77	28.27	15688	1599.6	14.00	8.000	637.984	605.839	**24.590**	25.894	2.156	2.271	13.261	5.38
G.	2.15	30.81	29.31	15195	1615.0	14.00	8.075	668.637	651.305	**22.725**	23.330	2.077	2.132	12.711	12.97

* Throttle partially closed. † Steam strongly throttled.

(2) That the back pressure was 2 pounds, and the pressure to overcome the friction of the engine was $1\frac{1}{2}$ pounds.

(3) That the mean total pressure was the same per cent of the initial pressure as it was in the actual experiment.

The results are given in the following table:

	B.	C.	D.	E.
Cut-off,	0.70	0.56	0.38	0.21
Number of expansions,	1.38	1.68	2.33	3.68
Consumption per net horse-power estimated,	33.748	31.948	31.621	38.485
Ratio of cylinders for equal powers,	1.00	1.05	1.24	1.59

An inspection of this table, or of the results in Table II, indicates that the consumption of steam per horse-power per hour is least with the cut-off at 0.38 per cent of the stroke, but that the consumption is but little greater with the cut-off at 0.56 of the stroke. It consequently appears that the advisable cut-off is at half-stroke or a little later, the difference between the results of this series of tests and the results of the tests on the *Michigan* being attributable to the difference of steam-pressure.

The last two tests were made with superheated steam, obtained by running with the water-level in the Martin vertical water-tube boilers below the tops of the tubes, so that of the 4036 square feet of heating surface about 1000 square feet were available for superheating.

In these experiments the number of revolutions per minute was increased by removing a part of the paddles. In Experiment F the steam was strongly throttled, but in Experiment G the throttle-valve was wide open.

If it be admitted that Experiments F and B may be compared, then the gain in consumption of steam per indicated horse-power per hour is

$$\frac{32.913 - 24.59}{32.913} = 0.25.$$

A like comparison of Experiments E and G shows a gain of

$$\frac{36.044 - 22.725}{36.044} = 0.37$$

by the use of superheated steam.

It is to be remembered that the apparent gain from superheating is obtained while the engine is running nearly three times as fast, and exerting nearly three times the power that it did when using moist steam.

Tests on the Eutaw.—The steamer *Eutaw* was one of the same class as the *Mackinaw*, and differed only in that the cylinder had two piston-rods instead of one, and that the boiler was furnished with a tubular superheater in the up-take, so arranged that the engineer could use saturated steam, superheated steam, or a mixture of saturated and superheated steam; more properly, the mixing of the two kinds of steam gave a ready method of controlling the degree of superheating. The piston displacement was 159.2258 cubic feet, but the clearance was the same as for the *Mackinaw*.

The heating surface of the Martin boilers was so efficient that the products of combustion in the up-take were only 40 to 80 degrees above the temperature due to the saturated steam in the boilers, so that to make a provision for superheating the steam efficiently almost all of the water-tubes were removed from one furnace, and the tubular superheater was placed in the space thus provided.

The experiments were made at Washington by a board of naval engineers,* with the vessel secured to the dock. It was intended that the steam-pressure should be the same throughout, and that the wheels should make the number of revolutions per minute that the power would give.

Each experiment lasted 72 hours, and was made with the usual care and precautions required to give reliable results. It is believed that there was no leakage from the boiler nor in the cylinder, and that there was no priming.

With natural draught on the superheater, the temperature of the superheated steam, as in F and G, was about 360 degrees, while the temperature of the saturated steam was 270 degrees, giving 90 degrees of superheating. When the blower was applied to the furnace connected with the superheater the temperature was (H, I, and J) about 390 degrees, showing

* Experimental Researches in Steam Engineering.

TABLE III.

TESTS ON UNITED STATES STEAMER EUTAW.

	Condition.	Cut-off.	Revolutions per minute.	Temperatures, Fahrenheit.							Pressures.			
				External air.	Engine-room.	Fire-room.	Feed-water.	Injection-water.	Discharge water.	Superheated or mixed steam.	Barometer, inches of mercury.	Vacuum in condenser, inches of mercury.	Initial absolute.	Cut-off absolute.
A.	Saturated.	**0.24**	5.492	50.5	76	95.4	50	45.8	75.0	....	30	27	40.28	34.67
B.	"	**0.32**	6.583	76.6	99	124.1	71.8	74.1	104.9	....	29.95	27.58	40.63	36.06
C.	"	**0.50**	8.595	61.4	83.7	121.1	68.1	73.9	105.5	....	30.05	27.18	40.92	37.28
D.	"	**0.50**	7.508	84.6	102.4	143.0	77.5	83.8	105.0	....	30.03	22.22	37.28	34.29
E.	"	**0.58**	9.188	52.7	77.5	108.4	50.0	47.0	78.3	....	29.71	28.00	38.15	35.28
H.	Superheated.	**0.29**	6.483	56.5	81.1	92.5	59.5	57.9	94.8	395.8	30.10	27.99	41.00	35.97
F.	"	**0.32**	6.549	73.6	98.4	119.3	72.0	74.4	103.6	365.9	30.02	26.21	41.10	35.55
G.	"	**0.50**	8.996	74.7	89.3	129.6	73.0	74.7	102.0	357.6	30.03	26.15	41.69	36.97
I.	"	**0.50**	9.154	58.7	86.4	108.8	57.6	48.6	94.3	394.3	30.09	28.50	41.03	36.57
J.	"	**0.58**	9.458	61.0	88.2	116.1	56.8	50.0	93.8	392.0	29.93	28.29	41.16	37.29
M.	Mixed.	**0.29**	6.154	65.5	95.3	103.7	58.2	64.0	89.6	382.1	29.82	27.22	41.14	35.59
K.	"	**0.32**	6.657	72.6	92.6	114.1	71.8	74.1	103.8	331.8	29.94	27.26	40.84	35.16
L.	"	**0.50**	9.045	79.0	96.9	131.5	75.3	81.0	105.0	336.1	29.93	26.55	41.93	37.53
N.	"	**0.50**	8.981	55.4	85.4	114.0	59.6	64.0	77.2	376.5	30.03	27.11	40.83	37.44
O.	"	**0.58**	9.557	47.8	87.5	114.5	50.5	42.9	89.5	369.8	30.02	28.50	41.15	37.66

	Pressures.				Total consumption.			Horse-power.		Steam per horse-power per hour.		Combustible per indicated horse-power per hour.	Water evapor't'd by pound of combustible from 212°.	Coal per square foot of grate per hour.	Per cent of water in cylinder at end of stroke.
	End of stroke absolute.	Back pressure absolute.	Mean indicated pressure.	Mean net press're	Feed-water.	Coal.	Per cent of refuse.	Indicated.	Net.	Indicated H.P.	Net H.P.				
A.	9.41	1.77	20.23	18.73	440376	55100	24.00	154.390	142.942	**39.616**	42.789	3.767	12.249	4.783	55.62
B.	12.60	1.26	24.99	23.49	504260	61600	23.77	228.603	214.881	**30.637**	32.593	2.839	12.329	5.347	37.41
C.	18.88	1.60	31.19	26.69	876670	98100	21.56	372.523	354.608	**32.685**	34.336	2.869	13.059	8.516	30.44
D.	18.39	3.75	27.05	25.55	744534	89300	24.00	282.217	266.567	**36.641**	38.792	3.340	12.473	7.752	31.96
E.	20.43	0.90	32.50	31.00	937702	112800	23.10	414.950	395.800	**31.386**	32.905	2.903	12.593	9.792	24.37
H.	10.28	1.09	23.01	21.51	449765	54200	21.90	207.291	193.780	**30.135**	32.236	2.836	12.274	3.764	43.20
F.	11.90	2.47	23.98	21.48	459516	51400	19.18	218.230	195.479	**29.245**	32.649	2.644	12.636	3.570	36.79
G.	19.11	1.90	31.28	29.78	782834	86900	17.38	391.028	372.277	**27.805**	29.206	2.552	12.444	6.035	17.78
I.	16.58	0.85	31.98	30.48	734919	90800	21.73	406.800	387.720	**25.091**	26.326	2.426	11.964	6.306	21.46
J.	20.50	0.80	34.48	32.98	886195	115200	22.85	453.167	433.453	**27.160**	28.396	2.724	11.544	8.000	17.25
M.	10.72	1.54	22.66	21.16	422820	50800	22.31	193.780	180.953	**30.305**	32.453	2.828	12.390	3.528	40.78
K.	12.24	1.50	24.43	22.93	462142	50600	21.60	225.992	212.117	**28.402**	30.260	2.438	13.333	3.514	33.15
L.	19.16	1.58	32.12	30.62	773858	83100	19.58	403.716	384.863	**26.623**	27.927	2.301	13.176	5.771	15.84
N.	18.57	1.70	30.81	29.31	702481	81700	21.12	384.509	365.789	**25.874**	26.673	2.328	12.590	5.674	11.42
O.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

about 120 degrees of superheating. The highest degree of superheating did not appear to injure the metal of the cylinder.

Discussion of Results.—At the time when these experiments were made, certain steamers plying on the Chesapeake Bay had superheating apparatus, and used strongly superheated steam mixed with moist steam. It was claimed that a great economy was realized from the use of such mixed steam. In reality the whole apparatus was only a means of using superheated steam of which the degree of superheating could be controlled, and a comparison of the consumption of superheated steam per horse-power per hour for Experiments F to J with the consumption for Experiments K to O indicates that the differences, which are sometimes on one side and sometimes on the other, are due to the differing degrees of superheating and to the different initial pressures; furthermore, the differences are all small, and in many cases are within the probable error of the experiments.

The smallest consumption of saturated or moist steam per horse-power per hour was obtained in Experiment B, with a cut-off at 0.32 of the stroke; but the consumption at 0.50 of the stroke being but little larger, it may be concluded as in the discussion of the tests on the *Mackinaw* that the cut-off may be chosen at about half-stroke.

The smallest consumption of superheated steam per horse-power per hour appears to have been obtained with the cut-off at 0.50 of the stroke, but the varying degree of superheating prevents any conclusion on this point. Now the initial condensation of moist steam interferes with the gain to be anticipated from large expansion, and since superheated steam reduces the initial condensation, it might be expected that its use would permit the use of higher degrees of expansion. No experiments can be quoted that are conclusive on this point.

The consumption of moist steam for Experiment D is much greater than that for either Experiment C or Experiment E, and it is noticeable that the back pressure is much greater while the horse-power is less. No explanation is given by Isherwood of these facts, though he includes this test with others in his comparisons, in which, however, he makes allowance for differences of initial pressure and back pressure. If we omit this experiment, the consumption of moist steam per horse-power per hour with the cut-off at half-stroke was 32.7 pounds. The mean consumption of superheated steam for Experiments G, I, L, and N was 26.2 pounds. The gain from the use of superheated steam was therefore

$$\frac{32.7 - 26.2}{32.7} = 0.20 \text{ nearly.}$$

Taking the consumption of fuel per horse-power per hour as the basis of comparison, the real gain was

$$\frac{2.87 - 2.40}{2.87} = 0.16.$$

Dixwell's Tests.—The Harris-Corliss engine, now in the laboratory of the Institute of Technology, was fitted up by Mr. George B. Dixwell * for the purpose of making experiments on superheated steam. At his request a board of three engineers of the United States Navy witnessed a series of experiments made on this engine in 1877 with apparatus belonging to the Institute.

The following is the substance of the report made by the board, of which C. H. Loring, Chief Engineer U. S. N., was the senior officer:

In the apparatus employed steam was taken from the horizontal tubular boilers which supplied steam for heating the buildings of the Institute of Technology, and for other purposes.

The engine is of a well-known Corliss type, of 8 inches diameter and 24 inches stroke of piston. The cut-off was varied by an Allen governor. The entire clearance is $\frac{41}{1000}$ of the space-displacement of the piston.

* Proceedings of the Society of Arts, M. I. T. 1887–88.

The power developed by the engine was absorbed by a friction-brake applied to the fly-wheel.

The exhaust steam from the engine passed to the *calorimeter*, which comprised the following details:

1. A tank built of planks two inches thick, of about 120 cubic feet capacity, containing a system of tubular metallic condensing surfaces, the interior of the latter being in communication with the exhaust-pipe of the engine and with the receiving-tanks hereinafter described. The body of the tank could be filled with water from the city aqueduct, by which heat in the steam discharged from the cylinder into the system of condensing tubes was absorbed. To relieve the walls of the tank from pressure resulting from the expansion of the water in heating, a small vessel of ten feet cubical capacity, two feet high, of wood, was placed above the large tank described above, communicating with the latter by a pipe of two inches diameter. Through this pipe the expanding water could rise to the upper vessel described, which is called the *expansion tank.* The escape of vapor was prevented by a floating cover in the expansion tank, joined to the walls by a flexible diaphragm. The large tank, the expansion tank, and their contents and appendages, stood upon the platform of a Fairbanks scales. Freedom of movement, within sufficiently wide limits, was maintained by fitting the pipe connections of the tank with rubber tubing; and the weighing was accurate within two pounds; the whole weight, tanks, appurtenances, and water being 8100 pounds.

The tank was fitted with thermometers for ascertaining the temperature of the hydrant water entering, and of the water contained. To insure equality of the latter quantity in all parts of the tank chamber, a device for circulating the water was provided, to be worked by hand.

The fall of pressure in the condenser tubes, below that of the atmosphere, was averted by the automatic action of a reverse, or vacuum, valve.

2. The pipe leading from the lower end of the condensing tubes, through which the water resulting from condensation

passed out of the large tank, entered a small tank, which, like the others, was made of 2-inch plank. This tank stood upon the platform of a second Fairbanks scales, and was fitted with a thermometer for ascertaining the temperature of its contents. This tank could be emptied through a pipe leading to the sewer. The pipe connections were, like those of the large tank, flexible, so as to admit of weighing.

The superheating apparatus consisted of a cylindrical boiler, of iron, seven feet long and three feet in diameter, fitted with fifty iron tubes two inches in diameter and five feet long. The latter were fire tubes, vertical, and six inches of the lower end covered with water. The superheater was set in brick-work, in which an annular space allowed the products of combustion to pass downward, around a part of the shell. The furnace was of brickwork, so far removed from the heating surfaces as to prevent direct radiation to them from the fuel. In this vessel the steam from the boiler could be superheated above 600° F.

The superheated steam was delivered to the engine through a $2\frac{1}{2}$-inch pipe. At the receiving end of this pipe a pipe of $1\frac{1}{2}$ inches diameter delivered saturated steam, the admission being regulated so as to govern the temperature of the steam passing through the pipe, which nevertheless remained superheated to a degree measured by a Bulkley pyrometer, placed five or six feet beyond.

A mercurial thermometer was placed close to the steam-chest of the engine, in the steam-pipe, and another Bulkley pyrometer in the clearance space of the cylinder. A mercurial thermometer was also placed at the point last mentioned. During a part of the experiments, a mercurial high-grade thermometer was placed nearly midway of the length of the steam-pipe.

Besides the pipes described, others connecting the engine directly with the generator were fitted. These were cut off from the former at will, by gate-valves made perfectly tight.

An indicator was fitted to each end of the cylinder.

The experiments were made in pairs, as follows:

1. Saturated steam, $\frac{1}{4}$ cut-off; followed by one at the same cut-off with superheating.

2. Saturated steam, $\frac{7}{10}$ cut-off; followed by one at the same cut-off with superheating.

3. Saturated steam, $\frac{1}{2}$ cut-off; followed by one at the same cut-off with superheating.

Diagrams from each end of the cylinder were taken, and readings from the pressure-gauge and thermometers, and of the weighing scales, were registered every five minutes. The large tank was heated, before the beginning of each experiment, to the temperature at which it was desired to close the experiment; then emptied, and weighed empty; then filled with water from the city aqueduct, at the natural temperature, the temperature observed, and the full tank weighed. Throughout each experiment the water in the tank was kept in motion, that the circulation might prevent differences in temperature within it. The temperature and weight of the tank water at the end of the experiment was registered after clearing the condensing-tubes of water. The water delivered into the small receiving-tank was also weighed, and its temperature ascertained every five minutes. From these quantities the total heat of the steam leaving the cylinder is computed.

It was sought to maintain in the cylinder, during each experiment with superheated steam, a temperature 310° F., and an initial pressure of 50 pounds by the gauge.

It will be seen from Table IV, page 261, which contains the averages of all the observations recorded, that this was very nearly accomplished.

It will also be seen that to maintain the above temperature within the cylinder a varied degree of superheating was necessary, accordingly as the cut-off was varied.

After the experiments were completed, the correctness of the instruments used was verified by the very accurate methods of the Institute of Technology. It was then ascertained that some leakage of piston and valves had existed. This leakage affects the *cost* of the power, but not the correctness of the deductions from the data obtained, in their bearings upon the object of the experiments.

Discussion of Results.—The following points are noticeable features of the experiments, and of the action of the apparatus:

1. Throughout all the experiments with saturated steam, considerable variations in the temperature of the cylinder were indicated by the thermometer and the pyrometer during every stroke of the piston. The amplitude of the vibrations of the pyrometer extended over nineteen degree-marks of the dial. But throughout the whole of every stroke of the piston, during the experiments with superheated steam, these instruments constantly indicated a fixed degree of temperature, showing no vibrations whatever.

At the close of the half-stroke and the seven-tenths stroke cut-off experiments with superheated steam, the same instruments showing no vibrations, the cut-off was shortened without change in the superheating. Vibrations of considerable amplitude were presently observed in them.

2. The remarkable fall of temperature of the steam in passing from the superheater to the steam-chest, before entering the latter, being for $\frac{1}{4}$ cut-off, 97°; for $\frac{1}{2}$ cut-off, 49°; for $\frac{7}{10}$, 19°.

3. During experiments with superheated steam the opening of the indicators for preliminary heating was attended by a sudden fall of 15° F. within the cylinder, the temperature gradually rising again as the metal of the indicators became heated.

After using superheated steam, five minutes were required for a fall of 15°, the steam being shut off.

The least consumption of steam, whether moist or superheated, was found with the cut-off at 0.44 of the stroke. The gain in steam consumption from the use of superheated steam, with about 140° F., superheating, was

$$\frac{42.2 - 31.7}{42.2} = 0.25 \text{ nearly};$$

but since fuel was used to superheat the steam the real gain was not so great. The fuel consumption could not be deter-

TABLE IV.

Dixwell's Tests.

	Kind of steam used.	Cut-off.	Ratio of expansion.	Temperature of steam in steam-pipe. By pyrometer near superheater.	By middle thermometer.	By thermometer, near throttle.	Temperature in cylinder. By pyrometer.	By thermometer.	Temperature in exhaust-pipe. By pyrometer.	By thermometer.	Duration of experiment in minutes.	Revolutions per minute.
							Vibrations.	Vibrations.				
1	Saturated.	.217	.247	......	......	302	278 to 297	263 to 267	210	213	127	61.46
2	"	.443	.465	......	......	303	279 to 296	264 to 268	210	213	83	60.41
3	"	.689	.701	... ..	......	303	282 to 300	268 to 273	213	215	63	58.03
4	Superheated.	.218	.248	575	......	478	313	300	217	217	180.3	61.04
5	"	.439	.461	490	486	441	316	306	210	213.5	108	61.44
6	"	.672	.685	425	... ..	406	315	305	212	213	75	59.49

	Boiler-pressure above atmosphere by gauge.	Pressure in cylinder above zero shown by indicator. Initial pressure.	Pressure at cut-off.	Pressure at release.	Pressure at cushion.	Fraction of stroke uncompleted at exhaust closing.	Mean back pressure.	Mean effective pressure.	Per cent of water in cylinder at cut-off.	Per cent of water in cylinder at end of stroke.	Gross effective horse-power.	Water per gross effective horse-power per hour.	Total heat of exhaust steam shown by calorimeter in thermal units above zero.
•1	50.4	65	56.9	18.7	15.7	.01	15.35	20.71	52.2	32.4	7.65	48.2	1046
2	50.2	66.2	59.7	29.5	16	.01	15.65	34.99	35.9	29.3	12.7	42.2	1084.5
3	50.3	66.2	61.3	44.5	16.5	.01	15.8	44.97	27.9	23.9	15.68	45.3	1101.4
4	50.4	65.9	57.4	15.1	15.5	.01	15.15	18.62	27.4	18.3	6.83	35.2	1138.6
5	50	66.6	58.8	26	16	.01	15.35	33.5	13.6	13.6	12.37	31.7	1159.4
6	50.2	66.9	61.1	39.7	15.9	.01	15.45	43.71	8.9	11.5	15.63	35.8	1170.8

[Note.—The above temperatures are the uncorrected indications of the instruments, which, tested under a pressure of 50 lbs. in saturated steam, indicated as follows: Pyrom. in st. pipe, 296; Mid. th. 300; Thermom. near throttle, 303; Cyl. pyrom. 299; Cyl. th. 294; Exhaust pyrom. 296 Th. 296.—*G. B. D.*]

mined in these tests, so the real gain may be estimated as follows. The steam from the boiler probably contained one per cent of moisture, so that each pound of steam may be assumed to have brought to the cylinder of the engine

$$0.99r + q - q_0 = 0.99 \times 901.9 + 272.5 - 183.9 = 981.5 \text{ B. T. U.},$$

in which r and q are the latent heat and heat of the liquid for the temperature of 303° F., and q_0 is the heat of the liquid at 15.65 pounds pressure. On the other hand, the superheated steam brought in per pound

$$c_p(t_s - t) + \lambda - q_0 = 0.48(441 - 299.4) + 1173.2 - 182$$
$$= 1059.2 \text{ B. T. U.},$$

in which c_p is the specific heat of superheated steam at constant pressure, and t_s is the temperature of the superheated steam in the steam-pipe near the throttle-valve; t is the temperature of saturated steam at the pressure in the steam-pipe, and λ is the total heat of steam at that temperature and pressure; while q_0 is the heat of the liquid at the pressure of the back pressure. There appears to be a discrepancy between the boiler-pressure and the initial indicated pressure in each experiment, so that the initial pressure in the cylinder has been taken in this calculation.

The number of thermal units per horse-power per hour furnished to the engine in the test with saturated steam was

$$42.2 \times 981.5 = 41420,$$

while with superheated steam the number of thermal units was

$$31.7 \times 1059.2 = 3358,$$

and the gain from the use of superheated steam was

$$\frac{41420 - 33580}{41420} = 0.19.$$

In the experiments made with saturated steam the per cent of water at cut-off decreased rapidly as the cut-off was lengthened, and the per cent of water at release also decreased, though much less rapidly.

In the experiments made with superheated steam, Experiment 5 shows the same per cent of water at cut-off and at release, which indicates that the condensation and re-evaporation during expansion were equivalent. Experiment 4 shows re-evaporation during expansion, and Experiment 6 shows condensation during expansion.

By aid of the last column of Table IV the per cent of moisture in the exhaust steam may be calculated as follows: The mean back pressure is 15.35 pounds, at which the heat of vaporization is 964.4 and the heat of the liquid is 183, so that if x is the part of the mixture that is steam,

$$964.4x + 183 = 1046;$$

$$\therefore x = 0.895,$$

and the per cent of moisture is 10.5.

The other experiments with saturated steam show a less degree of moisture. The experiments with superheated steam show that the exhaust steam was superheated in the two last experiments, and that it was moist in Experiment 4.

Automatic Cut-off Engines.—At the First Millers' International Exhibition at Cincinnati, June 1880, some competitive tests were made by John W. Hill on three automatic cut-off-engines, condensing and non-condensing, the results of which are of interest since they show the performance of well-made unjacketed simple engines.

Two of the engines were modified forms of the well-known Corliss engine, known as the Reynolds-Corliss and the Harris-Corliss engines. The third engine made by Wheelock had two semi-rotative valves similar to the Corliss valve, one at each end of the cylinder to give admission and exhaust of steam, and also two cut-off valves of similar form, which cut off the supply of steam to the main valve. It had consequently two clearances, one for compression, and a larger one, including the space between the two valves, for expansion.

The dimensions of the engines were as follows:

DIMENSIONS OF ENGINES.

	Reynolds-Corliss.	Harris-Corliss.	Wheelock.
Diameter of cylinder, inches	18.02	18.03	18.26
" " piston-rod, inches	2.8125	2.68	2.9375
" " steam-pipe, inches	5	6	7
" " exhaust-pipe, inches	7	8	8
" " fly-wheel, feet	16	16	16
Area steam-ports, square inches	15.75	13.50	30
Area exhaust-ports, " "	27	24.75	30
Stroke of piston, inches	48	48	48
Weight of engine, exclusive of fly-wheel, pounds	22,180	18,000	9,000
Weight of fly-wheel, pounds	14,694	11,950	12,000
Clearance in decimal of stroke	0.0265	0.0193	0.0235 & 0.0142
Release in decimal of stroke, condensing	0.978	0.940	0.956
" " " non-condensing	0.981	0.969	0.970
Exhaust closure in decimal of stroke, condensing	0.056	0.122	0.056
Exhaust closure in decimal of stroke, non-condensing	0.083	0.120	0.084
Diameter air-pump cylinder, inches	17	9.81	....
Diameter injection-pipe, inches	2	2	2
Diameter overflow-pipe, inches	7	6	3.5
Stroke of air-pump piston, inches	12	12	....

The Wheelock engine had a Bulkley condenser with the head of the condenser set 34 or 35 feet above the level of the hot-well so that no air-pump was required.

The feed-water supplied to the boilers furnishing steam for the tests was weighed in a tank on scales. The quality of the steam was determined by a continuous calorimeter. The condensing water was measured by a water-meter, and though the quantities thus determined are introduced in the tables they cannot be taken with full confidence.

The steam-pressure in the boiler and in the steam-pipe, and the vacuum in the condenser, were taken with gauges. The pressure of the atmosphere was taken with an aneroid barometer.

All temperatures were taken with mercurial thermometers.

Diagrams were taken every fifteen minutes with Thompson indicators at each end of the cylinder, and all other observations were read at the same intervals.

The work of the engines was applied to drive rotary pumps. Attempts were made to find the friction of the engines, and tests of regulation at various loads were also made.

The summary of the results of tests are given in Table V:

TABLE V.

Automatic Cut-off Engine.

	Condensing.			Non-condensing.		
	Reynolds-Corliss.	Harris-Corliss.	Wheelock.	Reynolds-Corliss.	Harris-Corliss.	Wheelock.
Duration of trial..................... hours	10	10	10	9	10	10
Steam-pressure, boilers.............. pounds	**95.8**	**96.1**	**96.3**	**96.6**	**96.3**	**96.3**
" " pipe.............. "	92.5	91.7	91.4	92.5	91.5	91.5
Barometer........................inches hg.	29.72	29.55	29.41	29.75	29.55	29.48
Vacuum in condenser........... " "	25.45	25.67	23.98	....	....	....
Temperature of air.................. Fahr.	84°.2	87°.6	83°.3	87°.4	85°.3	78°.8
" of injection............ "	72.4	75.9	77.2	....	....	....
" of overflow............ "	101.7	97.5	111.7	....	....	....
Revolutions per minute, engine............	75.4	75.8	74.5	75.3	75.8	76.1
" " air-pump.........	59.1	75.8	....	....	....	....
Diagrams.						
Initial pressure........................pounds	91.1	90.1	88.1	90.0	89.5	88.5
Cut-off in decimal of stroke................	**0.124**	**0.119**	**0.131**	**0.160**	**0.136**	**0.170**
Pressure at cut-off.............. ...pounds	86.3	87.0	77.7	84.8	85.9	76.9
Terminal pressure, absolute........ "	15.2	14.6	14.0	17.4	17.0	17.5
Counter-pressure at mid-stroke...... "	4.5	3.4	4.7	0.9	0.4	1.0
Maximum compression pressure.... "	13.6	26.6	28.1	34.7	46.1	44.2
Mean effective pressure............ "	35.4	35.7	33.9	29.8	28.9	29.4
Loads.						
Indicated horse-power....................	162.3	165.6	158.4	137.0	134.3	140.0
Friction of engine.........................	10.6	9.6	7.8	10.3	9.6	8.0
Extra friction due to load, estimated.......	6.1	6.2	6.0	5.1	5.0	5.3
Power absorbed by air-pump.............	3.4	4.7	*0.6	....	....	...
Net effective horse-power................	**143.2**	**145.1**	**143.9**	**121.7**	**119.7**	**126.7**
Coefficient of useful effect..	0.879	0.876	0.909	0.886	0.892	0.905
Calorimeter.						
Condensing-water, per hour........pounds	1427.5	1405.7	1797.5	1303.9	1530.9	1836.3
Condensation-water, " .... "	68.92	50.55	69.20	68.33	54.77	67.70
Temperature of injection............Fahr.	77°.03	78°.07	77°.24	76°.86	77°.87	76°.45
" " overflow............ "	131°.96	121°.87	123°.25	134°.86	119°.27	120°.85
" " condensation........ "	107°.42	97°.86	106°.57	104°.45	98°.67	108°.74
Thermal value of steam......... ...B. T. U.	1243.8	1315.9	1301.7	1211.3	1255.7	1313.1
Relative value of steam....................	0.945	1.000	0.989	....	....	....
Steam Expended.						
Water weighed to boilers	34425	32296	31538	32645	32708	35749
Leakage of tanks........................	26.25	13.	....	20.5	....	....
" of pipe.....	29.5	....	....	15.75	....	....
Correction for variation of water-level.....	−142.8	+285.7	....	....	....	−214.3
Condensation in calorimeter.	620.025	505.500	692.000	615.000	547.750	645.750
Net steam delivered to engine.............	33606	32063	30847	31994	32160	34889
Economy of Engine.						
Steam per indicated horse-power per hour, actual................................	**20.6**	**19.4**	**19.5**	**25.9**	**23.9**	**24.9**
Steam per net horse-power per hour actual.	23.5	22.1	21.4	29.2	26.8	27.5
Steam per indicated horse-power per hour corrected for relative value of steam...	19.5	19.4	19.3	23.9	22.1	24.9
Calculated Economy.						
Steam per horse-power per hour by the diagrams..................................	14.9	13.8	13.9	19.0	18.0	19.8
Condensing Water.						
Water expended per hour..................	103783	104307	76324	....	....	....
Water expended per pound of steam.......	30.9	32.5	24.7	....	....	....

* Power required to raise water for condenser.

Hoadley Portable Engine.—The following are the data and results of a test on a Hoadley portable steam-engine, made at the International Exhibition, at Philadelphia, 1876. The engine was a simple single-cylinder engine, lagged but not jacketed, with a piston slide-valve controlled by an automatic governor, and mounted on the top of an unclothed locomotive boiler. The coal was anthracite of ordinary quality, used without drying.

TEST OF A HOADLEY PORTABLE ENGINE.

Diameter of cylinder,		14.56 in.
" " piston-rod,		2.375 "
Length of stroke,		1.66 ft.
Clearance, fraction of piston displacement: crank end,		0.077
head end,		0.157
Duration of test,		6 hrs. 2 min.
Revolutions per minute,		125.96
Steam-pressure in boiler,		120 lbs.
Initial pressure: crank end,		124.75
head end,		119.9
Absolute pressure at end of stroke: crank end,		23.4
head end,		31.9
Absolute pressure at admission: crank end,		112.5
head end,		117.7
Mean absolute forward pressure: crank end,		58.4
head end,		69.2
Mean absolute back pressure: crank end,		24.7
head end,		25.9
Mean effective pressure: crank end,		33.7
head end,		43.3
Point of cut-off: crank end,		0.1439
head end,		0.2091
Indicated horse-power: crank end,	34.6	
head end,	45.69	
Total,		80.29
Horse-power by brake,		72.72
Friction of engine, horse-power,		7.57
Horse-power by indicator without load,		5.80
Steam per I. H. P. per hour, weighed,		25.61 lbs.
" " brake H. P. per hour, weighed,		28.27 "
" " I. H. P. per hour, indicator,		19.38 "
" " brake H. P. per hour, indicator,		21.4 "
Coal per indicated horse-power per hour,		3.35 "
" " brake " " "		3.69 "

Heating surface of boiler, ...	461.5 sq. ft.
Area of grate, ...	12.75 "
Coal per sq. ft. per hour, ...	21.07 lbs.
Water evaporated per sq. ft. per hour, heating surface, .	4.46 "
Ratio of ashes to coal burned, ...	0.118
Water evaporated per pound of coal, ...	7.65 lbs.
" " " " " combustible, ...	8.68 "

CHAPTER XVI.

TESTS OF SIMPLE AND COMPOUND ENGINES.

THE several series of tests following give the data for the comparison of the performance of simple and compound engines. All the tests except those on the *Gleam* were made by engineers of the United States Navy and the United States Revenue Marine, and consist of tests on the coast-survey steamer *Bache;* of tests on the revenue steamers *Rush*, *Dexter*, and *Dallas;* and of tests on the revenue steamer *Gallatin;* also of tests on the Herreshoff steam-yachts *Leila*, *Siesta*, and on the yacht *Gleam*.

The principal dimensions of the engines and boilers of the United States steamers are given in the following table:

DIMENSIONS OF ENGINES AND BOILERS.

	Bache.	Rush.	Dexter.	Dallas.	Gallatin.
Diameter of cylinders, ins., high pressure	15.98	24	......	......	......
" " " low "	25	38	26	36	34.1
" " piston-rods, ins., high "	2.5	......	......	......	......
" " " " low "	3.625	......	......	......	......
Stroke, inches	24	27	36	30	30
Clearance, per cent, high pressure	4.86	7.887	......	......	.....
" " low "	4.05	5.849	5.37	8.02	6.59
Ratio of cylinders	1 : 2.4398	......	......	......	......
Grate surface, square feet	31.16	57	57	57	55.25
Water-heating surface, square feet	950.10	1572.85	1572.85	1689.24	1805.163
Steam-heating surface, " "	54.32	......	......	......	105.311

Tests on the Bache.—The engine and hull of the United States coast-survey steamer *Bache* were built in 1870 from designs by Mr. Emery,* then consulting engineer to that department. The engine is a direct-acting, inverted, compound engine, with the small cylinder above the large cylinder, both pistons being attached to one piston-rod. The small cylinder

* Journal Franklin Institute, May, 1875.

was not jacketed, but the large cylinder had a steam-jacket on the sides and ends; when working compound the steam for the jacket was taken from the bottom of the small steam-chest; when working as a simple engine, with the small cylinder disconnected, the steam was taken from the main steam-pipe. Suitable pipes and valves were provided so that steam could be supplied directly to the large cylinder, and excluded from the small cylinder, in which case the large cylinder acted as a simple expansive engine. Ordinarily the steam passed from the small to the large cylinder through a large pipe which acted as intermediate receiver. Both cylinders were provided with short slide-valves and independent cut-off valves on the back of the main valves, and the valves of both cylinders were actuated by continuous valve-stems. The engine had a surface condenser. The air-pump was operated by levers from the main cross-head. The circulating pump was of the centrifugal pattern, driven by an independent engine.

During the trials the vessel was secured to the dock.

The feed-water was measured in a tank with two compartments that were filled and emptied alternately. The capacity of each compartment was ascertained by weighing water into it, of the average temperature of the feed-water. Indicator-diagrams were taken every twenty minutes.

The water-level in the boiler was noted every time the feed-water tank was filled, but it did not vary appreciably. The condensed water from the jackets and receiver was collected and weighed separately and returned to the feed-water tank.

In the ninth experiment the coal was weighed, but in the others, which were shorter in duration the water only was weighed. The coal used was anthracite of fair quality.

The data and results of the experiments are given in Table VI.

Tests on the Rush, Dexter, and Dallas.—In 1874 three vessels were built for the United States Revenue Marine, which were designedly alike in all respects, except that the engines were of three distinct types, and the boilers were

TABLE VI.—TESTS ON THE BACHE.

	Manner of operating.	Duration in hours.	Cut-off.		Expansion.		Revolutions per minute.	Temperatures, Fahrenheit.					Vacuum in condens'r, inches.	Barometer, pounds.	Pressures in pounds.			
															Boiler-pressure by gauge.	H. P. cylinder, absol.		
			H. P.	L. P.	H. P.	L. P.		Engine-room.	Fire-room.	Sea-water	Discharge-water.	Hot-well.				Initial.	Terminal.	Cushion.
1	Compound without jacket.	1.833	0.231	0.503	3.750	9.146	42.6	96	84	62	87	98	24	14.72	82	86.72	24.345	28.595
2		2.066	0.335	0.833	2.730	6.658	47.7	96	86.3	62	92	100	24.321	14.72	80.28	91.05	30.47	29.52
3		2.133	0.405	0.779	2.310	5.634	49.3	96	86.3	62	91.3	105.3	24.656	14.72	80.31	87.72	32.32	32.42
4	Compound with jacket.	1.733	0.103	0.820	6.910	16.850	38.9	....	81.4	60	82	110	24.50	14.89	81.38	90.64	20.76	26.39
5		2.066	0.229	0.831	3.77	9.190	48.2	....	80	60	79	104	26.50	14.89	80.33	92.015	27.39	30.03
6		1.933	0.318	0.831	2.86	6.975	53.2	80	78	60	78.5	103	26.50	14.89	80.21	93.47	33.72	36.29
7		1.983	0.396	0.803	2.35	5.732	56.3	76.7	75.3	60	79.7	102	26.562	14.89	80.13	90.30	38.30	35.05
8		7.066	0.398	0.822	2.34	5.707	55.6	83.7	83.8	61.9	90.2	106.6	26.115	14.72	79.96	90.04	36.95	40.94
9		15.233	0.452	0.735	2.09	5.097	53.6	90	85.1	64.7	92.5	125.3	24.39	14.31	79.7	88.78	34.40	37.69
10		2.000	0.554	0.795	1.74	4.240	60.6	78	76.7	60	80.7	115.3	26.5	14.89	79.1	87.30	47.05	42.89
11	Single without jacket.	1.800	......	0.0475	......	11.820	37.3	86.3	84.7	66	96.7	133.7	24	14.64	81	......	......	......
12		1.983	......	0.096	......	7.620	44.9	86.7	84	66	94.3	131.5	23.78	14.64	79.63	......	......	......
13		2.050	......	0.155	......	5.320	47.1	86	83.3	66	93	129.3	24.22	14.64	78.11	......	......	......
14	Single with jacket.	2.100	......	0.042	......	12.620	39.9	84	81	66	91	121.3	24.66	14.64	80.83	......	......	......
15		1.683	......	0.081	......	8.570	46.2	84.7	80.3	66	95	120.7	25.285	14.64	81.07	......	......	......
16		2.116	......	0.163	......	5.110	53.8	75.3	74.8	66	93.8	117	25.52	14.64	79.5	......	......	......
17		1.883	......	0.437	......	2.180	45.3	85	83	66	100.5	137.5	24	14.64	30.88	......	......	......

	Pressures in pounds.								Horse-power.				Water p. ind. H. P. p. hour.			Percentages of water and steam.			
	H.P. cylin., absol.		L. P. cylinder.						Indicated.		Net.		Measured.	By indicator.		St'm con. in jack't & small chest.	St'm condens'd in receiver.	Acct'd for by ind.	
	M. E. P.	Frict'n, estimated.	Initial.	Termin'l.	Cushion.	Back-press're	M. E. P.	Fric'n estimated	Small cylind'r	Total.	Small cylind'r	Total.		H. P. cylind'r	L. P. cylind'r			H. P. cylind'r	L. P. cylind.
1	34.801	0.75	15.72	7.72	5.532	3.360	8.153	2.25	35.586	55.928	34.82	49.548	23.765	15.413	12.699	......	1.5	64.857	53.435
2	43.514	0.75	17.178	9.18	5.97	3.414	9.759	2.25	49.85	77.06	48.874	69.812	23.036	15.671	12.312	......	2.15	68.028	53.45
3	45.137	0.75	18.62	10.02	6.02	3.328	11.276	2.25	53.32	85.81	52.434	78.447	23.21	15.371	12.346	......	1.4	66.224	53.194
4	24.484	0.75	15.89	10.39	6.77	3.690	10.341	2.25	22.852	46.4	22.162	40.59	25.1082	14.553	18.532	6.5	11.2	57.964	73.809
5	35.39	0.75	18.89	11.89	4.89	3.050	13.014	2.25	40.87	77.453	40.005	70.335	20.7133	14.104	15.756	7.0	10.0	68.095	76.067
6	42.34	0.75	21.14	12.93	4.89	3.264	14.484	2.25	54.06	99.181	53.104	91.216	20.332	14.974	14.769	5.1	7.6	73.651	72.643
7	42.938	0.75	23.55	14.23	4.29	3.328	15.88	2.25	58.086	110.514	57.068	102.069	20.3648	15.851	15.251	4.6	6.1	77.835	74.888
8	45.375	0.75	23.68	14.31	5.02	3.360	13.961	2.25	60.562	106.028	59.562	97.701	21.9661	15.854	16.004	5.4	6.9	72.176	72.850
9	43.061	0.75	22.83	12.93	6.26	3.253	14.908	2.25	55.437	102.263	54.472	94.23	22.3798	14.852	14.528	5.6	7.2	66.363	64.916
10	45.33	0.75	28.51	16.89	4.89	3.317	19.34	2.25	65.911	134.53	65.112	125.75	21.169	17.268	16.209	4.0	5.0	81.572	76.569
11	......	......	88.94	10.74	8.84	4.560	21.64	2.25	......	47.24	......	42.329	35.075	......	21.028	......	0.2	......	59.951
12	......	......	90.74	12.34	8.64	4.400	27.27	2.25	......	71.75	......	65.83	29.616	......	17.755	......	0.05	......	55.492
13	......	......	87.39	15.80	9.89	4.400	32.33	2.25	......	89.141	......	82.93	26.247	......	17.352	......	0.2	......	66.111
14	......	......	88.80	10.56	7.84	2.960	23.45	2.25	......	54.838	......	49.574	27.113	......	16.418	4.05	4.45	......	60.554
15	......	......	90.14	11.73	7.64	2.774	27.54	2.25	......	74.618	......	68.518	24.088	......	15.576	1.8	2.07	......	64.663
16	......	......	90.74	16.64	9.03	3.680	36.94	2.25	......	116.015	......	109.37	23.154	......	16.253	2.2	2.8	......	[illegible]
17	......	......	41.54	16.84	8.73	4.720	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

adapted to the engines. The *Rush* had a direct-acting, inverted, compound, receiver engine, with the cranks 90°, thoroughly steam-jacketed, felted, and lagged, designed to use steam at 80 pounds pressure. The small cylinder had an independent cut-off valve on the back of the main valve; the large cylinder had a double-ported valve arranged to cut off at about half-stroke.

The *Dexter* had a single-cylinder, direct-acting, inverted engine, felted and lagged, but not steam-jacketed. The cut-off could be varied by adjustable cut-off plates on the back of the main valve. The steam-pressure was intended to be 70 pounds.

The engine of the *Dallas* was also a single-cylinder, inverted engine, but was intended to carry a steam-pressure of 40 pounds. The cylinder was covered with a non-conducting composition and lagged, but not steam-jacketed. Steam was distributed by a slide-valve with adjustable cut-off plates on the back.

The experiments were made under the direction of Chief Engineer C. H. Loring, U. S. N., and Chas. E. Emery,* Consulting Engineer U. S. R. M., assisted by two chief-engineers of the navy and two of the revenue marine, with a sufficient force of assistants and helpers.

During the experiments the vessels were secured to the dock.

The coal, which was anthracite of fair quality, was sent from the dock in bags, filled to a certain weight, as wanted. The ashes were measured in buckets and then weighed in gross on the wharf. One experiment on each engine was of sufficient length to determine the evaporative efficiency with certainty. The other experiments were shorter, and for them the consumption of water only was determined.

The feed-water was measured in a tank with two compartments, which were alternately filled and emptied, as it came from the condenser and passed to the boiler. The waste from

* Journal Franklin Institute, May, 1875.

leakage, etc., was supplied from a hydrant and charged in the cost.

A number of indicators were compared under steam-pressure with a standard gauge, and a pair selected that were correct at varying pressures. Diagrams were taken every twenty minutes.

The data and results are given in Table VII.

Tests on the Gallatin.—The United States revenue steamer *Gallatin*, built in 1870, was re-engined in 1874 according to designs of Mr. Charles E. Emery, and tested by Chas. H. Loring, U. S. N., and Mr. Emery* at the Boston Navy Yard in Dec. 1874 and Jan. 1875.

The *Gallatin* had a single, inverted, steam-jacketed, direct-acting engine, with a slide-valve set to cut off at two-thirds stroke, and an adjustable cut-off valve on the back of the main valve. The air-pump was operated by levers from the main cross-head. The cooling water for the surface condenser was supplied by a centrifugal pump driven by an independent engine. The boiler, steam-pipes, and cylinder were covered with hair-felt and canvas, and in the engine-room the exposed parts were covered with Russia iron or wood lagging.

Steam for the jackets was ordinarily conducted through a felted pipe from the bottom of the valve-chest to the upper part of the cavity in the cylinder cover. A second pipe leads from the bottom of the cavity in the cover, upward and around to the side jacket, which is in common with the jacket for the bottom of the cylinder. Thus any water which collects in the bottom of the valve-chest or cylinder cover is carried into the main jacket, from which all water is blown into the hot-well through an intermediate vessel provided with a glass gauge.

The boiler was designed to carry a steam-pressure of 60 pounds; during the tests it was worked at 70 pounds part of the time, to compare with tests on the other engines.

The experiments were made with the vessel secured to the wharf.

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* Report of trial.

TABLE VII.—TESTS ON THE RUSH, DEXTER, AND DALLAS.

	Name.	Style.	Duration of experiment, hours.	Cut-off. H. P.	Cut-off. L. P.	Expansion. Small cylinder.	Expansion. Total.	Revolutions per minute.	Temperatures, Fahrenheit scale. External air.	Engine-room.	Sea-water.	Discharge-water.	Hot-well.	Feed-water from tanks.	Vacuum and Barometer, ins. Vacu'm in condenser.	Barometer.	Pressures, lbs. Ab've atm'sph're In boiler.	In intermediate chamb'r
1	Rush.	Compound	55	.35995	.4652	2.4586	6.2157	70.8403	66.23	87.86	59.96	90.56	110.306	114.04	26.495	30.1757	69.06	5.531
2	"	"	6	.5976	.5292	1.5948	4.0301	55.475	66.00	84.23	59 615	93.77	108.77	114.42	26.21	30.147	36.731	4.34
3	Dexter.	High press.	2.9166	.1827		......	4.4573	56.497	67.66	78.428	66.00	99.857	115.74	......	25.862	30.195	68.70	......
4	"	"	1.4166	.2335		......	3.6688	64.3059	64.2	79.6	65.00	104.8	121	......	25.187	30.195	69.286	.....
5	"	"	34.5	.2483		......	3.489	61.0642	63.37	82.27	63.66	102.45	113.60	113.65	25.45	30.143	67.12	......
6	"	"	0.650	.3331		......	2.7239	72.8205	66.285	85.428	65.859	100.14	119.28	......	25.310	30.195	66.42	......
7	"	"	1.3166	.262		......	3.3377	50.8481	66.33	86.00	67.66	99.66	107.33	......	26.10	30.195	40.625	......
8	"	"	1.200	.38114		......	2.4232	55.25	67.00	86.00	66.66	95.66	108.66	......	26.00	30.195	39.9	......
9	"	"	0.9166	.4518		......	2.0845	60.6909	67.00	85.66	67.00	99.00	117.33	......	25.541	30.195	41.875	......
10	Dallas.	Low press'r	1.5166	.13296		......	5.0674	48.6813	69 75	75.25	67.75	103.50	122.00	......	26 08	30.080	35.40	... ..
11	"	"	1.550	.19723		......	3.8936	56.9248	72.00	76.80	68.00	100.00	122.00	......	26.00	30.080	35.286	......
12	"	"	31	.26445		......	3.1341	61.5193	74.35	88.20	68.75	98.93	128.41	122.76	25.20	29.997	31.96	......
13	"	"	1.60	.28774		......	2.9358	64.4791	72.66	76.83	66.00	103.50	128.66	......	25.363	30.080	33.7	......
14	"	"	1.5333	.38585		......	2.3176	63.4674	69.833	73 166	67.666	108.66	134.00	.. ...	24.790	30.080	27.40	......

	Pressures, pounds. Absolute. Initial pressure. H. P.	Initial pressure. L. P.	Terminal pressure. H. P.	Terminal pressure. L. P.	At cushion. H. P.	At cushion. L. P.	Back pressure.		Mean effective pressure.		Estimat'd friction pressure.	Horse-power. Indicated.	Net.	Water per horse-power per hour. Indicated horse-power. By tanks.	By indicator.		Net horse-power.	Relative performances, comparison of water used.	
1	82.2792	23.4622	29.4412	9.222	31.7332	11.1082	9.500	3.4611	29.6848	12.7246	H. P. 2.5	266.547	239.432	18.3836	17.138	13.521	20.4656	Unity	....
2	50 2453	21.9822	27.8506	9.1664	29.0618	8.048	8.2305	3.415	18.8818	12.2997	L. P. 1.5	168.652	145.1745	22.0943	19.736	16.9435	25.6673	1.2018	....
3	80.434		15.344		7.503		3.4487		34.439		3.0	185.872	169.680	23 8572	16.2278		26.1337	1.2977	Unity
4	79 321		17.642		8.536		3.7317		37.1274		"	228.077	205.348	24.1202	17.2408		26.2405	1.3120	1.0110
5	79.2055		16.8746		8.267		3.6550		37.5376		"	218.972	201.472	23.905	16 3303		25.9813	1.3003	1.0020
6	76 976		21.633		10.321		5.2710		42.0285		"	292.370	271.501	24.3131	18.5390		26.1820	1.3225	1.0191
7	52.321		13.134		6.258		3.1570		25.5826		"	124.267	109.694	28.802	18.8462		32.6284	1.5667	1.2073
8	51.371		15.771		8.771		3.6347		30 6649		"	161.848	146.015	28.9356	18 6564		32.0734	1.5739	1.2129
9	53.633		19.008		9.196		4.3137		33.8335		"	196.187	178.794	31.7864	20.3290		34.8786	1.7291	1.3324
10	46 905		9.475		5.065		2.9730		18.5215		2.5	137.962	119.340	26.6866	19 2000		30.8508	1.4516	Unity
11	47.040		11.869		5.194		3.3678		21.4471		"	186.805	165.030	26.9603	20.7230		30.5176	1.4665	1.0102
12	46.5832		12.6810		6.1595		3.9073		23.5255		"	221.447	197.915	26.9447	20.0595		30.1484	1.4657	1.0097
13	45.860		14.008		5.875		4.1101		24.6107		"	242.807	218.137	28.9007	21.2048		32.1693	1.5721	1.0805
14	39.431		14.7783		6.182		4.0911		24.1264		"	234.295	210.017	30.9932	22.7753		34.576	1.6860	1.1614

The coal, which was anthracite of fair quality, was weighed in bags on the wharf and sent on board as needed. The ashes were hoisted in buckets and weighed in bulk on the wharf.

The condensed steam from the surface condenser was measured in the tank with two partitions, which was used in the tests on the revenue steamers already described. The water from the jackets was cooled by passing it through a coil in a ship's distiller, and then weighed in an open tank. After the water was weighed it was delivered to the condenser, and thus was charged in determining the cost of power.

The indicator-diagrams were taken every twenty minutes.

To ascertain the evaporative efficiency of the boiler the machinery was operated continuously for 48 hours with the steam-pressure at 70 pounds.

In all cases when the steam-jacket was not in use, a joint in the pipes was broken to let in air and to detect leakage of steam into the jackets.

In working out the results some of the experiments discovered discrepancies, and when errors in observations were detected such experiments were rejected; but when no errors were found the tests were reported with the others, though all such unsatisfactory tests were made in the first of the work, before the observers were familiar with the work. In the tests numbered 13, 30, 34, 37, 40, 41, and 42, distinguished by an asterisk in the table, some of the indicator-diagrams were faulty, and were corrected by comparison with diagrams taken under like circumstances.

Discussion of Results.—From the data and results given in Tables VI, VII, and VIII may be found the advantage of the use of compound engines, of the use of steam-jackets, and of the use of high-pressure instead of low-pressure steam.

In order to make the comparison more readily the following table is given, in which are collected some of the data and results of those tests that gave the best efficiency under different conditions.

SIMPLE AND COMPOUND ENGINES.

Name of engine.	Method of working.	Boiler-pressure gauge.	Back pressure absolute.	Revolutions per minute.	Total number of expansions.	Indicated horse-power.	Steam per horse-power per hour.	Thermal units per horse-power p. hr.
Bache	Compound with jacket	80.2	3.3	53.2	7.0	99.2	20.3	21700
"	" without "	80.3	3.4	47.7	6.7	69.8	23.0	24500
Rush	" with "	69.1	3.5	70.8	6.2	266.5	18.4	19600
Bache	Simple with jacket	79.5	3.7	53.8	5.1	116.0	23.2	24700
"	" without "	78.1	4.4	47.1	5.3	89.2	26.2	27600
Dexter	" " "	68.7	3.4	56.5	4.5	185.9	23.9	25400
Gallatin	" with "	69.9	4.0	61.1	5.0	306.2	20.7	21900
"	" without "	68.5	3.9	59.9	4.9	279.6	21.9	23200
Reynolds-Corliss	" " "	95.8	4.5	75.4	6.8	162.3	20.6	21800

In the group of tests on the *Gallatin* with the steam-jacket in use and with about 70 pounds boiler-pressure, there are three that have nearly the same consumption, and of these the one was chosen which had nearly the same number of expansions as the other tests in the above table, and with which it should be compared. The test on the Reynolds-Corliss engine was included in the same table, since it is instructive to compare it with the tests on marine engines.

Direct comparison of the consumption of steam per horse-power per hour of the tests given in the table is liable to be misleading, on account of the difference in boiler-pressure and back pressure. The estimated number of thermal units per horse-power per hour has been calculated, on the assumption that the steam in the boiler is dry and saturated, by the expression

$$\lambda - q_0,$$

in which λ is the total heat at the boiler-pressure and q_0 is the heat of the liquid at the back pressure. The back pressure is chosen instead of the pressure in the condenser, since the efficiency of the fluid is to be considered rather than the efficiency of the engines.

The least consumption of steam is shown by the engine of the *Rush*, working compound with both cylinders thoroughly steam-jacketed. This test may properly be compared with the test on the engine of the *Gallatin*, working single, and

TABLE VIII.

TESTS ON THE GALLATIN.

No.	Description of Test.		Duration, hours.	Cut-off fraction of stroke.	Ratio of expansion.	Temperatures, Fahrenheit. External air.	Engine-room.	Sea-wat'r.
1	Steam-jacket not in use		1.9166	0.463	2.02	36	64	35
2			2.25	0.640	1.51	38	70.5	35
3	Steam-jacket in use		2.1	0.468	2.00	36	73.6	35
4			2.1333	0.648	1.49	38.7	69.7	36
5	Steam-jacket supplied with steam at 70 pounds		2.2166	0.525	1.80	25	87.0	32
6			2.2	0.626	1.54	26	71.3	32
7	Steam-jacket not in use.		1.7666	0.114	5.92	43	70	37
8			1.9833	0.139	5.21	36	70	36
9			2.05	0.220	3.73	36.5	70	36.5
10			2.1	0.271	3.16	41.1	62	36.9
11			2.3166	0.326	2.72	40.3	64.1	37
12			2.1667	0.413	2.23	41.8	64	33.8
13*		Throttled	2.5666	0.691	1.41	36	73.3	35
14	Steam-jacket in use.		2.0166	0.105	6.08	19.7	69.5	34.4
15			2.05	0.144	5.07	13.3	80	33
16			2.2	0.155	4.82	19	74	35
17			2.21666	0.172	4.49	13	75.5	33
18			2.0333	0.221	3.71	22	76.5	34.7
19			3.75	0.255	3.32	30.8	69.9	36.2
20			2.0833	0.378	2.40	40.9	65.9	34
21			2.3	0.416	2.21	41	72.3	34
22	Condensing without vacuum.	Steam-jacket not in use.	2.2	0.178	4.37	45	77.3	37
23			2.21666	0.240	3.48	39.5	78	34
24		Steam-jacket in use.	2.05	0.196	4.07	40.4	74.4	....
25			2.1166	0.237	3.52	36	74.6	....
26	Link hauled up.	Without jacket and ind. cut off.	1.91666	0.366	2.47	4	65	33
27		With jacket. With cut-off.	2.06667	0.243	3.45	28.3	71.7	32
28		With jacket. Without cut-off.	1.95	0.383	2.37	14.3	86	33
29	Jacket not in use.		2.5	0.151	4.91	40.4	64.4	33.6
30			2.05	0.200	4.01	37.6	63.3	34
31	Jacket supplied with steam from boiler.		1.8666	0.153	4.87	4	75.6	33.4
32			2.0333	0.212	3.83	6.6	74	32.5
33	Steam-jacket not in use.		23.95	0.173	4.46	20.9	74.6	34.5
34*			2.18333	0.071	7.78	39.8	67.5	36
35			2.05	0.123	5.63	46	73.7	36
36			2.0166	0.150	4.94	44.3	72.3	37
37*			1.9833	0.185	4.25	36	66	34.6
38	Steam-jacket in use.		23.9833	0.171	4.50	37.8	69.4	36
39			2.2166	0.080	7.31	43	70	36
40*			2.0166	0.122	5.68	57.5	65	35.5
41*			2.1166	0.148	4.98	32.5	43.3	35
42*			4.41666	0.173	4.61	48.6	66.4	36
43			1.9333	0.189	4.19	39.5	75	35

TABLE VIII.—*Continued.*

	Temperatures, Fahrenheit.			Pressures—pounds per square inch.								
	Dis-ch'rge w'ter.	Hot-well.	Feed-water in tanks.	Boiler pressure by gauge.	Vacuum in condenser, ins. of mercury.	Barometer	Initial pressu'e absolute.	Terminal pressure absolute.	Cushion pressure absolute.	Back pressure absolute.	Vacuum at half-stroke	Mean effective pressure.
1	75	116.6	115.7	**14.56**	25.54	14.8	25.3	11.1	8.7	3.7	11.7	15.8
2	80.25	124.8	121.8	**12.83**	25	14.8	23.7	13.1	9.4	3.8	11.5	16.1
3	84	118.4	119.3	**15.42**	25.12	14.7	26.3	12.0	9.6	3.6	11.5	16.8
4	78.3	123	123.3	**13.14**	24.88	14.8	24.1	13.8	9.5	4.0	11.2	16.8
5	56.5	115.5	103.7	**13.39**	26.04	15.0	23.9	11.7	9.2	3.0	12.5	15.9
6	56	113.7	112.3	**13.86**	25.5	15.0	25.0	14.2	11.0	3.5	12.0	17.7
7	85	115	103.7	**44.8**	25.9	14.8	56.3	10.2	8.0	3.3	11.8	20.8
8	65	102.5	100.7	**40.9**	26.1	14.8	52.8	10.6	8.0	3.4	11.7	21.0
9	69.7	111.7	107.6	**43.3**	25.9	14.8	55.8	13.2	8.4	3.2	11.9	26.2
10	88.8	128.4	132.4	**39.4**	23.6	14.8	51.8	14.0	10.8	4.4	10.0	26.5
11	72	111.1	108	**39.5**	25.9	14.8	52.3	16.4	8.3	3.7	11.8	30.8
12	106	147.5	138.9	**37.67**	22.7	14.8	50.3	19.3	13.5	5.5	10.4	30.9
13*	79.3	130.0	122.8	**39.0**	24.6	14.6	44.7	21.7	8.3	3.0	11.7	30.4
14	61.7	98.3	102.5	**45.4**	26.1	14.8	58.0	9.8	8.5	3.7	11.3	19.9
15	60.7	112	109.7	**42.8**	26.5	15.1	53.9	10.4	11.4	3.4	11.1	21.7
16	77	109.6	106.9	**41.6**	25.6	14.8	54.6	10.3	8.7	3.5	11.6	21.7
17	61.5	115.5	110.2	**43.9**	26.6	15.1	54.7	11.5	12.2	3.8	12.0	23.6
18	80	115	113.8	**41.3**	25.8	14.8	54.7	11.7	9.8	3.8	11.5	24.6
19	96.7	129.4	123.6	**40.0**	24.6	14.8	52.4	13.5	11.4	4.7	10.7	26.4
20	74.4	111.1	105.8	**36.2**	26.4	14.8	49.3	14.8	14.8	2.9	11.5	28.5
21	85	130	123.5	**37.4**	24.8	14.8	50.4	18.9	10.6	4.5	10.8	31.9
22	....	......	121.7	**70.7**	1.0	14.9	83.7	19.3	41.3	14.9	....	26.5
23	....	......	133.4	**66.2**	1.7	14.7	79.1	21.5	41.8	14.7	....	28.9
24	....	......	126.8	**69.7**	1.0	14.8	82.6	19.7	44.6	14.8	....	27.9
25	....	......	134.4	**67.4**	1.7	14.7	78.8	21.1	45.9	14.8	....	29.0
26	83	141	132.6	**63.8**	24.7	15.1	66.9	19.6	28.3	6.7	10.3	35.0
27	62.7	116.3	112.3	**69.6**	26.0	15.0	70.2	15.1	25.5	5.0	11.9	32.1
28	65.7	121.3	122.3	**59.9**	25.3	15.1	63.6	18.8	28.3	6.0	11.3	34.0
29	69	114.2	109.6	**69.6**	26.0	14.8	82.4	15.7	9.2	3.3	11.0	35.0
30	66	110.6	107.2	**60.4**	25.8	14.8	73.2	16.7	10.5	4.1	11.0	34.7
31	78.8	130	121.1	**71.2**	25.8	15.1	82.6	14.0	16.1	4.2	11.6	31.8
32	75	132	126	**67.0**	25.4	15.1	78.1	15.7	16.9	4.6	11.1	33.4
33	73.4	128.4	123.7	**64.1**	25.3	14.9	76.1	15.4	14.8	4.3	11.3	32.4
34*	75.5	119	113.5	**71.5**	25.2	14.6	83.4	12.5	10.6	4.5	10.3	25.8
35	75.3	120.3	117.1	**68.2**	25.1	14.6	80.8	14.2	10.6	4.7	10.3	30.1
36	76.3	111.5	109.1	**68.5**	25.9	14.8	81.7	15.3	9.9	3.9	11.5	34.0
37*	68.3	112.6	107.6	**61.1**	25.8	14.6	74.0	16.2	10.5	4.0	10.9	34.7
38	72.4	120.5	119.5	**65.4**	25.3	14.7	77.8	15.5	12.3	4.0	11.3	33.4
39	66	114	115.1	**71.6**	25.7	14.8	85.0	12.1	9.9	3.6	11.3	28.0
40*	77.5	122	117.9	**71.8**	25.4	14.8	85.3	15.5	10.5	4.2	11.1	33.6
41*	65.3	111.3	113.8	**69.9**	25.8	14.8	83.5	16.6	10.5	4.0	11.1	36.5
42*	75.4	117.9	120	**68.3**	24.9	14.7	81.7	16.1	12.3	4.2	11.3	35.2
43	75.3	120	121.6	**67.2**	25.0	14.8	80.7	17.5	10.5	4.4	11.1	36.9

TABLE VIII.—*Concluded.*

	Revolutions.			Horse-power.		Water.					Water per horse-power per hour.		
	Total	Per hour.	Per minute.	Indicated.	Net. **	Total water from condenser.	Total water from jack's and steam chest.	Proportion of water from jackets and steam chest.	Water per hour.	Proportion of total water shown by indicator.	Per indicated horse-pow'r measured.	Per net horse-pow'r measured.	Per indicated horse-pow'r by indicator.
1	4606	2403	40.1	87.0	73.2	6722	....	....	3512	0.635	**40.4**	47.9	25.6
2	5517	2452	40.8	90.2	76.1	8949	....	....	3982	0.673	**44.2**	52.3	29.7
3	5199	2476	41.3	95.3	81.1	6716	177	0.026	3175	0.779	**33.3**	39.2	26.0
4	5434	2547	42.5	97.9	83.3	7827	165	0.021	3660	0.871	**37.4**	43.9	32.6
5	5468	2467	41.1	89.7	75.5	6740	292	0.043	3054	0.789	**34.1**	40.4	26.9
6	5580	2536	42.3	102.9	88.4	7849	243	0.031	3588	0.830	**34.8**	40.6	28.9
7	4558	2580	43	122.8	108.1	5618	....	....	3195	0.688	**26.0**	29.6	17.9
8	5264	2654	44.2	127.2	112.0	6745	....	....	3397	0.692	**26.7**	30.3	18.5
9	6245	3046	50.8	182.2	164.8	8980	....	....	4373	0.762	**24.0**	26.5	18.3
10	6318	3009	50.1	182.4	165.2	10039	....	....	4800	0.725	**26.3**	29.1	19.1
11	7787	3361	56.0	236.9	217.7	13468	....	....	5800	0.795	**24.5**	26.6	19.5
12	7248	3345	55.8	236.5	217.3	14474	....	....	6645	0.790	**28.1**	30.6	22.2
13*	8113	3161	52.7	219.5	201.4	16736	....	....	6493	0.882	**29.6**	32.2	26.1
14	5363	2659	44.3	121.1	105.9	5618	516	0.092	2778	0.780	**22.9**	26.2	17.9
15	5665	2763	46.1	137.5	121.4	6731	428	0.066	3295	0.717	**24.0**	27.1	17.2
16	6064	2753	45.9	136.4	120.7	6736	321	0.048	3055	0.777	**22.4**	25.3	17.4
17	6691	3018	50.3	163.3	146.0	8974	416	0.047	4055	0.702	**24.8**	27.8	17.4
18	6003	2952	49.2	166.2	149.3	7846	332	0.042	3852	0.742	**23.2**	25.8	17.2
19	11509	3069	51.2	185.2	167.5	17899	553	0.031	4761	0.721	**25.7**	28.4	18.5
20	6816	3272	54.5	213.0	194.3	11229	300	0.027	5388	0.753	**25.3**	27.7	19.0
21	8038	3495	58.2	255.0	235.0	15654	290	0.019	6763	0.808	**26.5**	28.8	21.4
22	6164	2801	46.7	169.6	153.6	11186	....	....	6165	0.796	**30.0**	33.1	23.9
23	6873	3101	51.7	204.8	187.1	13382	....	....	6901	0.837	**29.4**	32.1	24.6
24	6085	2968	49.5	189.6	172.6	10054	288	0.029	6085	0.940	**25.9**	28.4	22.2
25	6761	3194	53.23	212.2	193.9	12263	429	0.035	6752	0.864	**27.3**	29.9	23.6
26	7128	3719	62.0	297.8	276.5	14500	....	....	7515	0.757	**25.2**	27.2	19.1
27	7268	3517	58.6	258.5	238.4	12334	517	0.042	5946	0.695	**23.0**	24.9	16.0
28	7131	3657	60.9	284.2	263.2	13422	374	0.028	6873	0.783	**24.2**	26.1	18.9
29	9037	3615	60.2	289.2	268.5	15707	416	0.026	6287	0.750	**21.7**	23.4	16.3
30	7402	2611	60.2	286.8	266.2	13470	234	0.017	6578	0.758	**22.9**	24.7	17.4
31	6548	3508	58.5	255.3	235.2	11118	459	0.041	5994	0.660	**23.5**	25.5	15.5
32	7513	3695	61.6	282.5	261.4	14526	301	0.021	7157	0.654	**25.3**	27.4	16.6
33	86941	3630	60.5	268.6	247.9	155413	....	....	6541	0.696	**24.3**	26.4	16.9
34*	6858	3141	52.4	185.1	167.1	10088	....	....	4633	0.699	**25.0**	27.7	17.5
35	6890	3361	56.0	231.4	212.2	11199	....	....	5501	0.716	**23.8**	29.9	17.0
36	7244	3592	59.9	279.6	259.0	12343	....	....	6120	0.748	**21.9**	23.6	16.4
37*	7075	3567	59.5	282.9	262.5	13469	....	....	6786	0.707	**24.0**	25.8	17.0
38	88552	3692	61.5	281.6	260.5	148863	5157	0.035	6208	0.757	**22.0**	23.8	16.7
39	6798	3067	51.1	197.0	179.5	8962	460	0.051	4036	0.765	**20.5**	22.5	15.7
40*	7089	3515	58.6	270.4	250.3	11196	408	0.036	5573	0.811	**20.6**	22.3	16.7
41*	7774	3672	61.2	306.2	285.3	13450	384	0.029	6336	0.797	**20.7**	22.2	16.9
42*	16234	3676	61.3	295.6	274.6	27978	935	0.033	6331	0.847	**21.4**	23.1	18.1
43	7972	4123	68.7	348.0	324.4	14542	453	0.032	7475	0.800	**21.5**	23.0	17.2

** Estimated friction 2.5 pounds per square inch.

with the cylinder thoroughly steam-jacketed. The gain from compounding is

$$\frac{20.7 - 18.4}{20.7} = 0.11 +$$

when the consumptions of steam are compared directly, and is

$$\frac{21900 - 19600}{21900} = 0.11 -$$

when the thermal units per horse-power per hour are compared.

If the test on the *Rush* is compared with the test on the Reynolds-Corliss engine the gain is a little less, but the latter engine has the advantage of much higher boiler-pressure.

The engine of the *Bache*, although it uses steam of 80 pounds boiler-pressure, appears to require nearly ten per cent more steam per horse-power per hour than the *Rush* does, and to use nearly as much steam as the Reynolds-Corliss engine working single and without a steam-jacket. In this comparison it is to be borne in mind that the engine of the *Bache* was a tandem compound engine, having a steam-jacket on the large cylinder only, while the engine of the *Rush* was a receiver compound engine, with the cranks at 90°, and with both cylinders thoroughly jacketed. From the arrangement of the cylinders the radiation was probably much less than the radiation from the engine of the *Bache*. Also, the engine of the *Bache* was considerably smaller than that of the *Rush*. A part of the inferiority of the engine of the *Bache* should be attributed to the forms of the boilers. The boiler of the *Bache* was designed to give high evaporation, consequently the steam in the steam-chimney did not receive much heat from the escaping gases. On the revenue steamers the boilers were designed to give large power for a given space, and the escaping gases in the steam-chimney had a higher temperature, and consequently the steam furnished was probably drier. It appears, therefore, that the larger cost of fuel per pound of steam made when the proportion of heating surface is smaller, may be compensated in part by a smaller cost of power in pounds of steam, on account of superior dryness.

In these experiments, however, the higher evaporative efficiency of the boilers of the *Bache* more than compensated for the greater cost of power in pounds of steam, as the following table will show:

	Bache.	Rush.
Water per horse-power per hour..............	20.332	18.384
Water evaporated per pound of coal	9.131	7.549
Coal per horse-power per hour................	2.227	2.435

The consumption of coal in case of the *Bache* was calculated from the performance of the boiler during the long run (Table VI, Exp. 9), assuming the efficiency of the boiler to be the same.

The gain by compounding in practice is frequently claimed to be twenty per cent, for which Mr. Emery gives the following explanation: In high-pressure condensing engines the pressure is seldom maintained at the point designed. This occurs from two causes—the carelessness of operating the engine, or the imperfect adaptation of the engine to the purpose. No matter what the pressure designed may be, if the engine is designed to work with considerable expansion, the engineer finds that his engine works more smoothly, and with less trouble to himself, with less pressure and less expansion, and for trivial reasons lets his pressure fall or partially closes the throttle, and lengthens the cut-off, and finally believes that it is as well to work in that way all the time. With compound engines there are fewer difficulties in working high-pressure steam, and in most cases it is difficult to keep up the speed with low pressures.

The efficiency of the boilers of the several steamers was tested by one experiment on each, of sufficient length for the purpose. The results are given in the following table:

EFFICIENCY OF BOILERS.

	Coal per sq. foot of grate surface per hour.	Per cent of refuse.	Water evaporated per pound of coal from and at 212°.	Coal per horse-power per hour.
Bache, Table VI, Exp. 9	8.84	19.8	10.281	2.451
Rush, Table VII, Exp. 1	11.388	20.978	8.5675	2.4352
Dexter, Table VII, Exp. 5	12.026	20.291	8.6878	3.1313
Dallas, Table VII, Exp. 12	13.313	20.503	8.7625	3.4267
Gallatin, Table VIII, Exp. 38	15.305	21.609	7.418	3.002

If we compare the tests stated in the table on page 270, it appears that the use of a steam-jacket on the *Bache*, when working single, was accompanied by a gain of

$$\frac{27600 - 24700}{27600} = 0.10 +,$$

while the use of a steam-jacket on the *Gallatin* was accompanied by a gain of

$$\frac{23200 - 21900}{23200} = 0.10 - .$$

In Experiments 29 and 30 on the *Gallatin* the steam-jacket was not in use, but the condensed water was drained from the valve-chest—probably through the pipe which ordinarily supplied the jacket when it was in use. In Experiments 31 and 32 the jacket was supplied with steam directly from the boiler, and presumably the valve-chest was not drained. Comparing these experiments with each other and with Experiment 41, it appears that the draining of the steam-chest was of more importance than the use of a steam-jacket. In some engines the whole supply of steam for the engine is passed through the steam-jacket on the way to the steam-chest; and in such case the steam must suffer condensation in the jacket, and enter the cylinder with more moisture than if it passed directly to the steam-chest. Such engines do not show so good economy as those having a separate supply of steam to the steam-chest.

The gain from the use of a steam-jacket on the large cylinder, only, of a compound engine is shown by comparing the experiments on the *Bache* working compound, with and without the jacket in use. The gain is

$$\frac{24500 - 21700}{24500} = 0.11 + .$$

The experiments do not give the data for the discussion of the saving by the use of a steam-jacket on a compound engine when both cylinders are steam-jacketed. Should we compare the test on the *Bache* without the jacket in use with the test on the *Rush* with both cylinders jacketed, it would appear that there is a great gain from jacketing both cylinders, and that there is a marked gain from applying a steam-jacket to the small cylinder in addition to that on the large cylinder, but from the preceding discussion it is evident that such a comparison would be illusive. It does not appear probable that the jacketing of both cylinders of a compound engine would give a greater gain than the jacketing the cylinder of a simple engine; and if such a conclusion is admissible, then those experiments appear to show that the application of a jacket to the large cylinder only of a compound engine gives as great a gain as the gain from jacketing the cylinder of a simple engine, and that consequently there would be little or no gain from applying a jacket to the small cylinder in addition to one on the large cylinder. On the other hand, Rankine * states that in cases where a steam-jacket has been applied to the small cylinder only the heat thus applied has been found sufficient; but Rankine gives this statement in connection with a wrong theory of the action of a steam-jacket, i.e., that it prevents liquefaction in the mass of the steam during expansion, and he does not quote experiments to substantiate the statement. We must consequently await further experiments on this point.

It may be of interest to state in this connection that a steam-jacket is sometimes applied to the intermediate cylinder only

* Steam-engine, page 396.

of a triple-expansion engine, and that some engineers are in the habit of using the jacket in starting the engine, but not when the engine is working under normal conditions.

The only series of tests that can be used to determine the number of expansions to be used with a compound engine are those on the *Bache*, which show that the best economy was attained with 6 or 7 expansions both when the jacket on the large cylinder was in use, and when it was not in use.

To show the gain from increased steam-pressure, the following table has been made. The greater economy of the engine of the *Gallatin* as compared with that of the *Dexter* may be attributed in part to the larger size of the former.

STEAM-PRESSURE AND CUT-OFF, SIMPLE ENGINES.

	Boiler-pressure gauge.	Cut-off fraction of stroke.	Steam per horse-power per hour.	Thermal units per horse-power per hour.	Relative economy
Michigan	21	0.44	33.1	34400	0.67
Mackinaw	34.3	0.38	30.3	....	....
Eutaw	26.1	0.32	30.6	....	....
Dexter	68.7	0.18	23.9	25400	0.91
Dallas	35.4	0.13	26.9	27500	0.85
Gallatin	14.6	0.46	40.4	42100	0.55
"	43.3	0.22	24.0	25400	0.90
"	68.5	0.15	21.9	23200	1.00

Tests on the Leila, the Siesta, and the Gleam.—The engines of the yachts *Leila* and *Siesta* were built by the Herreshoff Manufacturing Co., and were tested in Narragansett Bay by Mr. Isherwood * in 1880 and 1882. The engine of the yacht *Gleam* was built by the Fore River Engine Co., of Weymouth, Mass., and was tested by Messrs. Roberts † and Sayer of the Class of 1888, Massachusetts Institute of Technology.

The engines of the *Leila* and *Siesta* differed only in size, those of the latter being a little the larger. They each had two cylinders,—one high-pressure and one low-pressure,—with the cranks at right angles. Both cylinders of each engine had

* Reports to the Bureau of Steam Engineering, 1881 and 1883.

† Thesis, 1888.

a plain slide-valve with a cut-off valve on the back of the main valve, the main valves having neither lead nor compression. The cylinders were lagged, but not steam-jacketed. The boiler of the *Leila* was arranged to furnish strongly-superheated steam, but that of the *Siesta* furnished saturated or moist steam.

In the tests on the *Leila* the condensed steam from the surface condenser was measured, as it was delivered by the air-pump, in a standard-gallon measure. The condensed steam from the surface condenser of the *Siesta* was measured in two measuring-tanks of known capacity, which were filled and emptied alternately.

The engine of the *Gleam* had two cylinders with the cranks at right angles. The cylinders were lagged, but not steam-jacketed. Each cylinder had a piston slide-valve actuated by a Joy valve-gear. Steam was furnished by an upright tubular boiler which superheated the steam enough to determine its quality, but not enough to affect the economy of the steam consumption to any marked degree.

The dimensions of the engines of these three yachts are given in the following table:

DIMENSIONS OF YACHT ENGINES.

	Leila	Siesta.	Gleam.	
Number of cylinders	2	2	2	
Diameter, small cylinder, inches	9	10½	9.035	
large cylinder, "	16	18	15.71	
piston-rod, small cylinder, inches	1⅝	1⅝	....	
" large " "	1⅝	1⅝	....	
Stroke, both pistons	18	18	11.96	
			Head-end.	Crank-end.
Displacement, small cylinder, cubic feet	0.652	0.891	0.444	0.443
large " " "	2.034	2.634	1.342	1.340
Clearance, fraction of piston displacement—				
small cylinder	0.088	0.094	0.042	0.045
large cylinder	0.068	0.069	0.024	0.026
Ratio of volumes of cylinders	3.196	2.962	3.026	

The data and the results of the tests on the yachts *Leila*, *Siesta*, and *Gleam* are given in Tables IX, X, and XI.

Discussion of Results.—The tests on the engines of the *Leila*, lettered A to H, were made with substantially the same cut-off and the same number of total expansions. The steam-pressure was diminished progressively, accompanied by a diminution of the amount of superheating and of the speed of revolution. The change from 95 revolutions per minute to 220 revolutions per minute is not enough to produce a marked change of economy; consequently, the regular and marked increase of steam consumption, or of the consumption of thermal units, per horse-power per hour is due to the decrease of pressure and the less amount of superheating.

The remaining experiments were made to determine the effect of cutting-off on the small cylinder only, on the large cylinder only, and of not cutting-off on either cylinder. Also the gain in this engine from compounding.

Comparing the Experiments C and I, which are in other respects alike, the consumptions of thermal units per horse-power per hour, with and without a cut-off on the large cylinder in addition to the cut-off on the small cylinder, are in the ratio of

$$19100 : 21000 :: 1 : 1.099.$$

In the first series it is noticeable that the distribution of work between the two cylinders of the engine is good, and is not greatly affected by the change of pressure and of superheating. In the tests I, J, and K an excessive amount of work is done in the small cylinder. On the other hand, the work done in the large cylinder is excessive in the tests L and M, when the steam is cut off in the large cylinder only.

The total number of expansions is not affected by the cut-off in the large cylinder, and is nearly the same for experiments C and I. The gain from the cut-off on the large cylinder is to be attributed to the diminution of the drop between the high- and low-pressure cylinders and to the better division of the ranges of temperatures.

When the steam was cut off in the large cylinder only the total number of expansions depended on the ratio of the volumes of the cylinders only, and was much less than when the steam was cut off in the small cylinder, and the increased consumption is to be attributed mainly to this cause.

TABLE

TESTS ON

	Manner of Running.	Revolutions per minute.	Cut-off and expansions.			Temperatures, Fahrenheit.					Pressures, pounds on the square inch.			
			Cut-off, small cylinder.	Cut-off, large cylinder.	Total number of expansions.	Of the atmosphere.	Of the sea-water.	Of the feed-water.	Of the super-heated steam.	Of saturated steam at pressure in pipe.	Boiler-pressure by gauge.	Intermediate receiver by gauge.	Barometer, inches of mercury.	Vacuum, inches of mercury.
A	Steam cut off in both cylinders by independent cut-off valves.	221.5	0.4	0.361	**7.12**	42	49	95.0	417	355.2	**129.4**	29.7	30.25	25.83
B		215.9	0.4	0.336	**7.12**	45	49	93.5	416	354.0	**127.2**	29.0	30.25	26.07
C		192.2	0.350	0.335	**7.93**	55	52	76.3	378	340.6	**104.5**	18.2	30.00	25.92
D		181.1	0.335	0.346	**8.21**	50	47	71.0	366	332.0	**91.4**	15.9	29.98	25.49
E		166.5	0.336	0.371	**8.21**	54	51	68.0	350	319.9	**75.0**	10.5	30.03	26.32
F		145.5	0.353	0.368	**7.89**	47	52	64.0	320	302.3	**57.7**	5.5	30.00	26.50
G		111.5	0.358	0.376	**7.79**	47	52	60.0	279	377.1	**32.4**	−0 8	30.00	26.50
H		94.7	0.361	0.349	**7.74**	52	47	64.0	261	361.0	**21.3**	−2.4	30.00	25.18
I	Steam cut off in small cylinder only.	188.1	0.329	....	**8.35**	43	46	76.0	383	340.2	**103.7**	−0.1	30.36	24.41
J		167.3	0.325	....	**8.42**	44	47	69.5	356	319.7	**74.6**	−3.5	30.36	25.18
K		145.9	0.347	....	**7.99**	44	47	64.5	335	303.4	**55.8**	−5.2	30.36	25.58
L	Steam cut off in large cylinder only.	197.9	....	0.297	**3.20**	38	48	88.5	360	308.4	**61.7**	33.4	29.88	25.43
M		122.5	....	0.363	**3.20**	50	47	64.5	304	260.5	**21.0**	8.0	30.06	25.93
N	No cut-off.	189.5	....	....	**3.20**	43	47	91.0	345	304.6	**57.0**	4.9	30.36	25.05
O	Small cylinder disconnected; large cylinder used as a simple engine.	191.5	....	0.335	**2.66**	38	47	96.5	344	292.3	**44.9**	40.7	30.17	25.59
P		147.6	....	0.371	**2.43**	40	47	71.0	304	261.4	**21.4**	19.4	30.19	25.98

In Experiment N the omission of the cut-off on both cylinders gave a fair distribution of the work, but the drop between the cylinders was excessive, and the consumption was larger, and the engine was consequently less economical than when the large cylinder was used alone, as in the Experiment O.

The gain from compounding may be inferred from a comparison of Experiments G and H with Experiments O and P; but a correct conclusion cannot be reached from these experiments, nor from a comparison of any of the tests in the table, since, on the one hand, the economy of the large cylinder used as a simple engine would be improved by an increase of pressure and an increase of the total number of expansions; and, on the other hand, the steam-pressure in Experiments G and H is too low to make compounding advantageous.

IX.

LEILA.

	Pressures, pounds on the square inch. Small cylinder, absolute.						Pressures, pounds on the square inch. Large cylinder, absolute.						Horse-power indicated.			Steam used per horse-power per hour, p'ds.	Thermal units per horse-power per hour	Per cent of steam in the cylinder.		
	Initial pressure.	Pressure at cut-off.	Pressure at end of stroke.	Mean back pressure.	Back press're at begin'g of st'ke	Mean effective pressure.	Initial pressure.	Pressure at cut-off.	Pressure at end of stroke.	Back press're at begin'g of st'ke	Mean back pressure.	Mean effective pressure.	Small cylinder.	Large cylinder.	Aggregate.			Cut-off, small cylinder.	End of stroke, small cylinder.	End of stroke, large cylinder.
A	134.8	118.9	60.8	48.7	44.7	53.8	45.5	32.0	13.3	4.7	6.5	20.4	67.7	82.2	149.9	16.4	18400	98	*102	81
B	130.7	118.6	57.1	46.7	42.5	53.4	44.1	33.0	12.7	4.8	6.1	20.3	65.6	79.6	145.2	16.0	18100	90	*103	80
C	109.0	100.4	40.6	33.2	29.7	44.3	32.0	23.1	9.7	4.1	4.8	14.7	48.4	51.5	99.9	16.7	19100	87	93	76
D	97.6	92.5	38.8	32.3	28.8	39.5	29.6	20.6	8.8	4.1	4.3	13.7	40.7	45.1	85.8	17.4	20000	81	93	72
E	83.7	77.6	32.3	25.3	22.4	33.5	23.8	16.3	7.6	3.3	3.7	10.54	31.7	31.9	63.6	18.7	21400	80	91	74
F	64.6	58.8	26.2	20.0	17.0	26.2	19.0	16.4	6.4	3.3	3.6	8.05	21.7	21.3	43.0	20.9	23900	67	78	64
G	42.5	38.9	18.8	13.7	11.9	17.8	12.2	8.9	5.0	3.3	3.4	4.63	11.3	9.4	20.8	25.0	28400	66	49	45
H	32.3	30.5	16.6	12.6	11.6	12.8	11.1	8.6	5.3	4.0	4.3	3.40	6.9	5.9	12.8	32.7	37000	56	78	83
I	114.2	107.0	43.0	16.8	15 2	63.6	13.6	....	10.9	5.0	5.5	7.50	68.1	25.6	93.7	18.5	21000	87	96	80
J	87.3	80.8	33.5	13.5	12.6	47.7	10.9	....	9.2	4.4	4.9	4.88	45.5	14.8	60.3	20.1	23000	83	96	85
K	66.7	61.0	26.5	11.6	10.7	36.6	8.3	....	7.5	4.4	4.7	3.33	30.4	8.8	39.2	23.9	27300	75	86	80
L	77.5	....	66.4	50.1	44.5	20.1	48.2	38.5	14.2	3.7	5.3	23.6	22.6	85.1	107.7	21.0	23500	....	*113	85
M	33.1	....	31.6	22.5	19.6	18.1	21.4	15.1	7.2	3.5	3.6	9.25	12.6	20.6	33.2	28.6	32300	....	83	64
N	70.6	....	63.1	22.6	20.2	43.9	19.3	....	16.0	4.5	6.0	10.80	47.3	37.2	84.5	28.1	31400	....	*105	86
O	....	....	....	....	....	....	54.5	43.7	17.2	4.3	6.1	28.4	....	98.9	98.9	25.5	28200	....	....	89
P	....	....	....	....	....	....	30.9	24.9	10.9	4.1	4.6	15.8	....	42.4	42.4	32.0	35700	....	....	81

* Superheated.

The tests on the *Siesta* were made primarily to determine the most advantageous cut-off for the large cylinder. Now the cut-off of the large cylinder does not affect the total number of expansions, nor does it affect the aggregate horse-power of the engine seriously. It does affect the pressure in the intermediate receiver, and consequently affects the division of power between the two cylinders, the drop between the cylinders, and the division of the range of temperatures. In comparing tests, those should be chosen in which all essential data except the cut-off of the large cylinder are substantially the same.

The indicator-diagrams for the tests B and C are shown by Figs. 57 and 58. In the first the cut-off of the large cylinder is at 0.251 of the stroke, and the steam in the small cylinder is

TABLE X.—TESTS ON THE SIESTA.

	Revolutions per minute.	Cut-off and expansions.			Temperatures, Fahr.				Barometer, inches of mercury.	Pressures, pounds on the square inch.								
		Cut-off, small cylinder.	Cut-off, large cylinder.	Total number of expansions.	Of the atmosph're	Of the sea-water.	Of the feed-w't'r	Of the engine-room.		Boiler-pressure by gauge.	Intermed'te receiver, gauge.	Absolute pressure in the cond'ns'r	Small cylinder.					
													Initial pressure.	Pressure at cut-off.	Pressure at end of stroke.	Mean back pressure.	B'k pr'ss. at beginning of stroke.	Mean effective pressure.
A	146.5	0.151	0.263	12.9	72	65	73.5	90	30.21	97.1	10.1	2.3	110.0	94.9	22.8	24.0	22.5	28.1
B	169.4	0.226	0.251	9.9	74	62	68.5	94	29.96	87.3	14.5	2.2	102.9	90.7	26.4	28.7	26.4	29.1
C	170 3	0.236	0.465	9.6	70	57	70	93	29.65	87.6	7.8	1.9	105.3	90.1	26.2	22.1	19.9	36.3
D	190.2	0.441	0.208	5.9	78	67	84.5	98	30.16	91.4	28.4	2.9	105.2	83.3	40.0	42.7	39.7	31.4
E	177.9	0.417	0.237	6.2	74	62	74.5	97	29.70	74 0	20.1	2.5	90.5	75.6	34.3	36.9	34.1	27.3
F	178.5	0.401	0.452	6.4	61	58	73.5	86	30 08	75.0	14.0	2.4	82.2	74.0	32.8	26.8	24.6	35.8
G	173.4	0.410	0.846	6.3	61	59	73.5	83	30.05	81.0	2.2	2.2	80.4	74.9	32.9	17.0	16.6	44.4
H	177.1	0.436	0.857	6.0	66	62	76.5	86	29.95	74.8	1.6	2.1	78.3	72.3	30.1	17.0	15.8	45.0
I	176.7	0.448	0.874	5.8	76	66	80	91	30.15	79.3	1.4	2.9	91.3	76.3	34.9	14.1	13.8	50.9
J	193.2	0.882	0.297	3.2	72	65	92	94	30.08	60.7	32.2	3.1	66.7	62.0	55.2	47.4	44.1	18.6
K	185.3	0.877	0.870	3.3	72	61	84	95	29.72	48.7	7.4	3.1	65.6	59.2	52.5	24 0	22.4	38.1
L	184.4	0.877	0.870	3.3	75	64	88	93	30.11	50.5	8.1	3.2	65.0	58.9	52.5	23.8	21.8	39.3

	Pressures, pounds on the sq. in.—large cylinder.						Horse-power, indicated.			Steam per horse-power per hour.	Thermal units per horse-power per hour.	Relative economy.	Percentage of steam in the cylinder.		
	Initial pressure.	Pressure at cut-off.	Pressure at end of stroke.	Mean back pressure.	Back pressure at begin'ng of stroke.	Mean effective pressure.	Small cylinder.	Large cylinder.	Aggregate.				Cut-off, small cylinder.	End of stroke, small cylinder.	End of stroke, large cylinder.
A	26.3	19.5	6.7	3.8	3.7	10.3	32.0	34.7	66.7	17.9	20500	1.08	64	76	67
B	29.7	23.0	7.6	3.8	3.7	11.9	38.3	46.6	84.9	18.9	21600	1.14	70	75	71
C	21.6	13.4	7.2	3.3	3.2	10.2	48.1	40.0	88.1	17.9	20500	1.08	75	78	68
D	47.4	39.6	11.0	5.1	4.9	19.3	46.5	84.8	131.2	16.7	18900	1.00	90	92	82
E	38.0	30.4	9.0	4.3	4.2	15.4	37.8	63.1	100.9	18.5	21000	1.11	86	87	74
F	27.1	17.9	9.5	4.6	4.2	13.5	49.7	55.6	105.3	18.0	20500	1.08	82	85	77
G	15.5	11.5	9.5	4.1	3.8	8.5	59.9	33.0	92.9	20.6	23500	1.24	83	83	75
H	14.7	11.1	9.3	3.8	3.6	8.5	62.0	34 8	96.8	20.6	23400	1.23	84	75	71
I	17.0	12.6	10.8	5.1	4.9	9.1	69.9	37 0	106 9	19.2	21800	1.15	88	84	80
J	50.6	40.4	15.6	5.8	5.2	25.4	27.9	113.0	140.9	20.0	22400	1.18	99	*104	89
K	23.9	17.3	15.1	5.6	5.2	13.6	54.9	58.2	113.1	24.8	27800	1.47	95	95	86
L	24.9	18.3	15.8	5.3	4.6	14.8	56.3	62.8	119.2	23.8	26600	1.40	93	94	86

* Superheated.

TABLE XI.

TESTS ON THE GLEAM.

	Duration, hours.	Revolutions per minute.	Cut-off, per cent of stroke. Small cylinder. Crank end.	Cut-off, Small cylinder. Head end.	Cut-off, Large cylinder. Crank end.	Cut-off, Large cylinder. Head end.	Total number of expansions.	Release, per cent of stroke. Small cylinder.	Release, Large cylinder.	Compress'n, per cent of stroke. Small cylinder.	Compress'n, Large cylinder.	Pressures, pounds per square inch. Boiler pressure by gauge.	Barometer.	Small cylinder, indicated. Initial.	Cut-off.	Release.	Compression.	Back pressure.	Drop.
1	3	117.5	30	25	23	20	10.81	75	80	30	24	112.1	14.73	111.0	97.8	33.1	20.4	17.8	15
2	3	117.5	30	25	23	20	10.81	75	80	30	24	113.1	14.79	109.9	94.8	31.9	18.6	16.3	16
3	3	145.2	45	41	59	47	7.04	81	86	21	15	109.8	14.79	107.7	95.9	44.2	20.7	17.9	26
4	3	147.2	45	41	59	47	7.04	81	86	21	15	112.2	14.56	110.6	98.6	46.0	17.5	16.0	30
5	3	156.9	59	57	70	64	5.22	88	90	15	12	107.3	14.77	104.3	94.8	56.5	17.7	17.0	41
6	3	160.6	59	57	70	64	5.22	88	90	15	12	110.1	14.74	106.8	98.0	57.1	22.3	21.1	35
7	3	164.4	72	66	79	72	4.39	91	94	10	8	104.4	14.78	98.7	91.8	62.8	17.5	17.3	46

	Pressures, pounds per sq. in. Large cylinder. Cut-off.	Release.	Compression.	Back pressure.	Mean effective pressures. Small cylinder. Crank end.	Small cylinder. Head end.	Large cylinder. Crank end.	Large cylinder. Head end.	Indicated horse-power. Small cylinder.	Large cylinder.	Total.	Steam consumed. Total.	Per horse-power per hour.	Per cent of steam shown by indicator. Small cylinder. Cut-off.	Small cylinder. Release.	Large cylinder. Cut-off.	Large cylinder. Release.
1	13.4	− 3.2	− 8.5	− 8.5	33.31	27.71	12.13	9.51	13.7	14.8	28.5	1695.5	19.83	57.5	63.7	39.7	61.0
2	12.4	− 3.9	− 9.7	− 9.6	34.11	28.04	12.70	9.32	13.9	15.1	29.0	1642	18.86	58.6	65.2	41.6	64.4
3	8.2	− 0.2	− 9.0	− 9.0	49.30	48.17	16.03	15.20	27.0	26.4	53.4	2874.5	17.92	73.5	73.5	67.1	69.5
4	7.7	0.7	− 8.7	− 8.7	54.55	52.35	16.33	14.22	30.0	25.9	55.9	3061	18.24	73.2	73.5	61.6	69.1
5	8.4	2.7	− 8.6	− 8.6	64.73	64.55	18.49	18.02	38.7	33.4	70.1	4086	18.88	80.4	79.6	66.1	67.4
6	11.4	4.7	− 7.3	− 7.0	61.26	62.25	19.98	20.71	37.9	38.1	76.0	4070	17.86	83.9	81.1	75.6	76.1
7	11.5	6.5	− 8.6	− 8.0	64.02	69.81	21.92	21.00	42.1	41.1	83.2	4713	18.89	87.1	84.0	78.1	79.4

expanded down to the back pressure in that cylinder. The initial pressure in the large cylinder is the same as the back pressure in the small cylinder at half-stroke, and is greater than

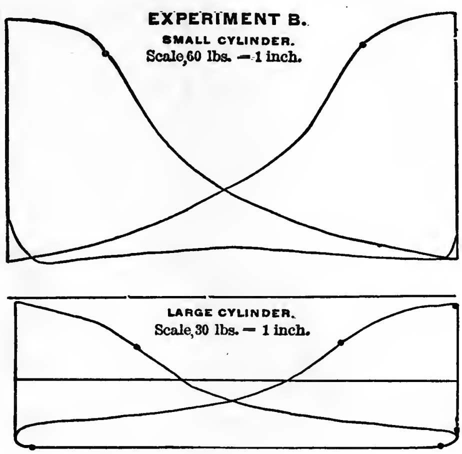

FIG. 57.

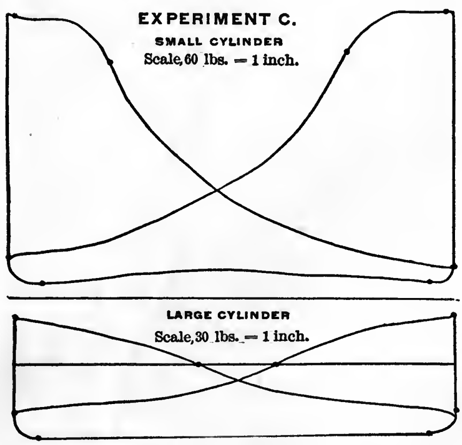

FIG. 58.

the terminal pressure in the small cylinder. This peculiarity of the back pressure in the small cylinder is due to the fact that the cranks of the engine are at right angles. In the test C, which has the cut-off at 0.465 of the stroke, there is a drop of about $4\frac{1}{2}$ pounds from the terminal pressure in the small cylinder to the initial pressure in the large cylinder; owing to the rise of back pressure in the small cylinder at half-stroke, the

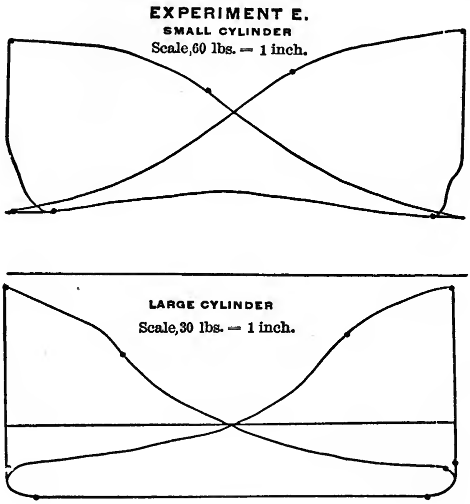

FIG. 59.

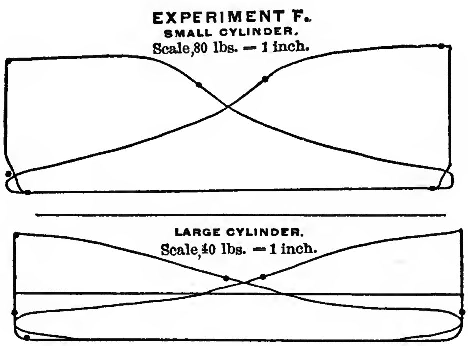

FIG. 60.

drop at the end of the stroke of the small piston is greater than this, as is evident from the diagrams, Fig. 58. The ratio of the consumption per horse-power per hour is

$$21600 : 20500 :: 1.05 : 1$$

in favor of Experiment C.

A similar comparison can be made of the Experiments E and F. In Experiment E the cut-off is at 0.237 of the stroke of the large piston, and the expansion in the small cylinder is carried down to the back pressure. In Experiment F the cut-off is at 0.452 of the stroke of the large piston, and there is a drop of about $4\frac{1}{2}$ pounds from the final pressure in the small cylinder to the initial pressure in the large cylinder, but, as with Experiment C, the drop at the end of the stroke of the small piston is more than that. The ratio of the consumptions is

$$21000 : 20500 :: 1.025 : 1$$

in favor of Experiment F.

These two comparisons show that for this engine under the given circumstances the cut-off on the large cylinder should be at a little less than half-stroke instead of at about quarter-stroke, and that a small drop at the end of the stroke of the small piston is permissible.

The diagrams for the Experiment H are shown by Fig. 61. In it the cut-off of the large cylinder occurred at 0.857 of the

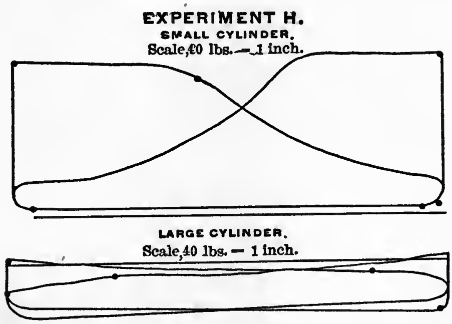

FIG. 61.

stroke, and there was a drop of about $15\frac{1}{2}$ pounds from the final pressure in the small cylinder to the initial pressure in the large cylinder. Compared with Experiment F, the consumptions are in the ratio

$$23400 : 20500 :: 1.14 : 1$$

in favor of the Experiment F.

The conclusion is that a small drop is allowable or even advantageous, but that a large drop is deleterious.

The determination of the most advantageous number of total expansions is made difficult by the varying boiler-pressure. It is, however, apparent that 5.9 expansions in Experiment D gave a better economy than 12.9 expansions in Experiment A, although the steam-pressure in A was higher than in D.

Inspection of Table X shows that six expansions reduce the pressure at the end of the stroke in the large cylinder to about ten pounds absolute. It is probable that a greater number of expansions would not give greater economy for this engine with the given steam-pressure, though this fact cannot be conclusively shown from the tests given.

If Experiment D on the *Siesta*, with 5.9 expansions and using saturated steam, may be compared with Experiment D on the *Leila*, with 8.1 expansions and using superheated steam, then it appears that the small amount of superheating in the latter had less effect than other causes that are commonly considered to be of secondary importance, since the test on the *Siesta* shows the better economy.

The tests on the *Gleam* were made with the boiler-pressure very nearly constant, while the change in the speed of revolution was too small to produce much effect. The throttle-valve was wide open during all of the tests, and the power of the engine was varied by changing the cut-off on both cylinders simultaneously. The tests 1 and 2, 3 and 4, and 5 and 6, were made under similar conditions for each pair. The last test, number 7, was made with full engine-power, at which the boiler was unable to furnish steam without undue forcing. The consumption of steam for the five last experiments is nearly constant, the variation in consumption under like conditions being as great as at the different rates of expansion from 4.39 to 7.04. The consumption for 10.81 appears to be somewhat more than for a less rate of expansion.

Leavitt Pumping-engine.*—Two duty tests of a com-

* E. D. Leavitt, Jr., Boston Soc. Civ. Eng., 1885.

pound pumping-engine at the Boston Main Drainage Works are given here as an example of advanced practice.

The engine is one of a pair of two-cylinder compound beam-engines. The steam-cylinders are inverted, and act on opposite ends of a short beam on a level with the engine-room floor; below the floor are two single-acting plunger-pumps, connected one to each end of the beam, directly under the steam-cylinders. The main shaft carrying the fly-wheel is midway between the lower ends of the steam-cylinders, and the crank is connected to one end of the beam near the point of attachment of the links from the steam-piston and the pump. The steam from each end of the high-pressure cylinder passes through a straight passage to the same end of the low-pressure cylinder, and in these passages are reheaters made of brass tubes supplied with boiler steam, through which the exhaust steam from the high-pressure cylinders passes, and by which it is dried or slightly superheated. The steam for the high-pressure cylinders is not superheated, but during the tests was assumed to be dry and saturated.

The diameter of the high-pressure cylinder is $25\frac{1}{2}$ inches, and that of the low-pressure cylinder is 52 inches; the stroke of each piston and of the pump-plungers is 4 feet.

Each steam-cylinder has four independent valves, operated by cams; the cams which move the admission-valves of the high-pressure cylinder are controlled by a governor. All the other valves have a fixed motion.

The following description of the tests is taken from Mr. Leavitt's paper:

The engine tested is known as Engine No. 3, and was supplied with steam from Boiler No. 2.

The intended duration of each test was twenty-four hours.

The method of making the tests was as follows: Steam was raised in the boiler until the pressure was sufficient to run the engine. The fires were then drawn, the ash-pits carefully cleaned, and new fires were started. It was desired to determine the quantity of water pumped, by actual measurement, in the reservoir at Moon Island, and since stopping the engine would have caused large fluctuations of level in the connecting sewers, and so prevented accuracy of measurement, it was decided to keep the engine running at a constant rate. This was done by furnishing the engine with steam from Boiler No. 3 until a few minutes after the new fires were started, when, by operating the valves rapidly and simultaneously, Boiler No. 3 was shut off, and the engine took steam from No. 2, thus beginning the engine test. The engine counter was read at the instant the test began, and the other necessary observations were taken. The steam-pressure was increased from about 70 pounds at the start, until it reached 100 pounds, at which it was kept constant until near the end of trial, when the fires were burned as low as possible, the steam-pressure dropping in consequence. When the pressure and height of water in the boiler were the same as at the beginning of the experiment, the final observations were taken, and the fires were drawn. The refuse was then spread upon the floor to cool. The unburnt coal was picked from the ashes and weighed. This weight (averaging less than one per cent of the total coal) was deducted from the gross amount of coal charged. The valve between Boiler No. 2 and Boiler No. 3 is supposed to have been tight, but to avoid increasing the duty by any leakage, the pressure in the latter was kept lower than in the former.

The height of the sewage in the pump-well was determined by a float-gauge, tested before each trial; the load on the pump by a mercurial gauge, attached to the force-main of another engine. This gauge represented the height in the pipe-chamber at the end of the force-mains, and, to get the actual pressure pumped against, it was necessary to add the friction in the force-main used. During the second test the actual pressure against which the pumps were working was measured by the elevation of the surface of water in a box at the top of a pipe connected with the force-main a few feet from the pump. A comparison of this gauge with the mercurial one gave a correction for friction in the force-main to use with the first test.

Dry Cumberland coal from the Pocahontas Mine was used during the trial. It was fed to the boiler from a car holding about 1200 pounds. During the first test the car and contents were reweighed at the end of each half-hour, and during the second test after each firing.

The steam-pressures at the boiler and the pressures and vacuum at the engine were determined by Bourdon gauges, which had been previously tested.

The temperature of the steam was taken by a thermometer inserted in the main steam-pipe within a few feet of the boiler. This thermometer was broken, so that readings could not be taken during the second test.

The barometer was an aneroid, placed in the engine-room.

The quantity of water fed to the boiler was measured in the following manner: A barrel, holding about 150 gallons, was placed upon a tested platform-scale, and supplied with cold water from the Cochituate main, and also with condensed water from the reheaters and steam-cylinder jackets. During the second test the exhaust steam from the boiler feed-pump was condensed in a small barrel placed above the weighing barrel, into which it was drawn from time to time.

After having been weighed the water was run into a large tub, from which the feed-pump drew its supply. The measurement of the feed-water was checked by a Worthington water-meter placed between the feed-pump and the feed-water heater.

To ascertain approximately the amount of water returned from the cylinder-jackets and reheaters the amount of cold water used was measured during the second test by a meter placed on the Cochituate supply.

About seven hours after the beginning of the first test a small leak was discovered from a safety-valve on the boiler feed-pipe between the pump and the hot-water meter. After being discovered, the water leaking was caught and returned to the feed-pump tub. For a period of about fourteen hours the leakage was weighed, and the rate so determined was used to make a correction for the time before the leak was discovered. The total amount of this correction was 650 pounds.

On the second test all pipe connections with feed-pipes, boilers, and engine, except those in use, were disconnected to avoid all chance of error from leakage.

Temperatures of the feed-water were taken before and after passing through the feed-water heater by means of thermometers inserted in the feed-pipe.

A thermometer in a tube partially filled with oil was inserted in the flue to ascertain the temperature of the gases beyond the feed-water heater, and on the second test a similar thermometer was placed in the flue between the boiler and heater.

Throughout the trials half-hourly observations were made of the engine-counter, pressure of steam at engine and boilers, vacuum in condenser in inches of mercury, height of water in boiler, height of water in tub holding feed-pump supply, water-meters on boiler feed-pipe and cold-water pipe, barometer, temperature of steam, temperature of gases in flue, and temperature of engine-room. Fifteen-minute readings were taken of the force-main and pump-well gauges, and readings of the feed-water thermometers every ten minutes.

Temperatures of the external air and indicator-diagrams from the steam-cylinders were taken hourly.

A large number of observers were employed, and care was taken to secure accuracy in all of the observations. The more important records were taken independently by assistants.

No calorimeter tests were made to ascertain the quality of the steam. For the purposes of calculation it has been assumed that all of the water was evaporated into dry steam.

RECORD OF TWO DUTY TESTS OF ENGINE NO. 3 (LEAVITT), AT THE BOSTON MAIN DRAINAGE WORKS.

	First Test.	Second Test.
1. Date of trials	Mar. 24–25, '85.	May 1–2, 1885.
2. Time of beginning trial	10.06 A.M.	10.31 A.M.
3. Duration	24h. 43m.	24h. 3½m.
4. Total revolutions	19,526	19,372
5. Revolutions per minute	13.17.	13.42.
6. Displacement of pumps per revolution	226.19 cu. ft.	226.19 cu. ft.
7. Distance from 0 of gauge down to sewage in pump-well	11.68 ft.	15.48 ft.

		First Test.	Second Test.
8.	Height of sewage in pipe-chamber, as given by mercurial gauge, graduated to give equivalent height of column of fresh water	25.76 ft.	26.55 ft.
9.	Pressure in force-main, near the pump, as indicated by column of fresh water at temperature 55° F	....	26.95 ft.
10.	Correction of mercurial gauge for friction in force-main, from data furnished by comparison of No. 8 and No. 9	0.36 ft.	....
11.	Total lift	37.80 ft.	42.43 ft.
12.	Weight of fresh water per cubic foot	62.42 lbs.	62.40 lbs.
13.	Total weight of dry coal consumed	8,307 lbs.	9,478 lbs.
14.	Duty of engine as developed by the trials:		
	1st trial $\dfrac{19526 \times 226.19 \times 37.80 \times 62.42}{83.07} =$	125,450,000*	....
	2d trial $\dfrac{19372 \times 226.19 \times 42.43 \times 62.40}{94.78} =$	....	122,400,000
15.	Mean pressure of steam in boilers	99.4 lbs.	98.6 lbs.
16.	Mean pressure of steam in main steam-pipe near engine	....	96.1 lbs.
17.	Mean vacuum in condenser	28.1 in.	28.0 in.
18.	" atmospheric pressure by barometer	30.18 in.	29.81 in.
19.	" temperature of air in engine-room	67.5 deg.	75.2 deg.
20.	" " of external air	31.7 deg.	40.6 deg.
21.	Total volume of sewage pumped by plunger displacement	33,038,000 gals.	32,778,000 gals.
22.	Total volume of sewage pumped, as actually measured	30,224,000 gals.	31,256,000 gals.
23.	Average slip of pumps	8.5 per cent.†	4.6 per cent.
24.	Indicated horse-power, as determined by the measurement of two sets of cards for each trial	251.5 H. P.	290.2 H. P.
25.	Horse-power in sewage lifted, pump measurement, no allowance for slip	212.9 H. P.	243.5 H. P.
26.	Work done by pump in per cent. of indicated horse-power	84.66 per cent.	83.90 per cent.
27.	Coal burned per hour per indicated horse-power	1.33 lbs.	1.35 lbs.
28.	Steam per indicated horse-power per hour.	13.9	14.2

* To reduce this duty on the first trial to the usual standard, it is necessary to make a correction for the coal used to supply steam to the feed-pumps. Assuming the duty of the feed-pump to be 10,000,000, the corrected duty of the pumping-engine is 122,500,000.

† At the end of the first test it was found that two of the rubber discharge-valves had been torn off, which accounts for the large slip. A study of the question indicated that this would not materially affect the duty, a view which is corroborated by the uniform relation between the indicated and the actual horse-power in the two tests. The loss of action in the pumps, when the valves were less worn, was about 2.5 per cent.

RECORD OF TWO TESTS OF BOILER NO. 2, AT THE BOSTON MAIN DRAINAGE WORKS, MADE IN CONNECTION WITH ENGINE TESTS.

		First Test.	Second Test.
1.	Date of trial	Mar. 24–25, '85	May 1–2, 1885.
1*a*.	Time of beginning trial	9.58 A.M.	10.25 A.M.
2.	Duration of trial	24h. 51m.	24h. 9½m.
	DIMENSIONS AND PROPORTIONS.		
	The general description of the boiler is given in Mr. Leavitt's paper.		
3.	Grate surface	45.5 sq. ft.	45.5 sq. ft.
3*a*.	Area of least draught	5.50 " "	5.50 " "
4.	Water-heating surface	1,826. " "	1,826. " "
5.	Super-heating surface	6. " "	6. " "
5*a*.	Heating surface in feed-water heater	934. " "	934. " "
6.	Ratio of water-heating surface to grate surface	40–1	40–1
	AVERAGE PRESSURES.		
7.	Steam-pressure in boiler by gauge	99.4 lbs.	98.6 lbs.
8.	Absolute steam-pressure	114.2 "	113.2 "
9.	Atmospheric pressure by barometer	30.18 in.	29.81 in.
	AVERAGE TEMPERATURES.		
11.	Of external air	31.7 deg.	40.6 deg.
13.	Of steam	339.0 "	....
14.	Of escaping gases before passing feed-water heater	....	439 deg.
14*a*.	Of escaping gases after passing feed-water heater	183.5 deg.	194.2 deg.
15.	Of feed-water before passing heater	96.5 deg.	120.7 deg.
15*a*.	Of feed-water after passing heater	145.1 deg.	164.1 deg.
15*b*.	Of Cochituate water	38 deg.	46 deg.
	FUEL.		
18.	Dry coal consumed	8,307 lbs.	9,478 lbs.
19.	Total refuse dry { 1st test, 432 lbs.	5.2 per cent.	....
	{ 2d " 497 "	....	5.2 per cent.
20.	Total combustible (weight of coal, item 18, less refuse, item 19)	7,875 lbs.	8,981 lbs.
21.	Dry coal consumed per hour	334.3 lbs.	392.3 lbs.
22.	Combustible consumed per hour	316.9 lbs.	371.8 lbs.
	WATER.		
26.	Total weight of water pumped into boiler and apparently evaporated	86,783 lbs.	98,780 lbs.
26*a*.	Check on above measurement by meter measurement	85,629 lbs.	96,622 lbs.
26*b*.	Per cent less by meter	1.3 per cent.	2.2 per cent.
26*c*.	Feed-water taken from Cochituate main, meter measurement	....	78,836 lbs.

	First Test.	Second Test.
ECONOMIC EVAPORATION.		
28. Water actually evaporated per pound of dry coal, from actual pressure and temperature	10.45 lbs.	10.42 lbs.
29. Equivalent water evaporated per pound of dry coal from and at 212° F.:		
Including f.w.h	12.12 lbs.	11.83 lbs.
Excluding f.w.h	11.60 lbs.	11.35 lbs.
30. Equivalent water evaporated per pound of combustible from and at 212° F.:		
Including f.w.h	12.78 lbs.	12.48 lbs.
Excluding f.w.h	12.23 lbs.	11.98 lbs.
COMMERCIAL EVAPORATION.		
31. Equivalent water evaporated per pound of dry coal, with one sixth refuse, at 70 lbs. gauge-pressure, from temperature of 100° F. = item 33 multiplied by 0.7249:		
Including f.w.h	9.26 lbs.	9.05 lbs.
Excluding f.w.h	8.87 lbs.	8.68 lbs.
RATE OF COMBUSTION.		
32. Dry coal actually burned per square foot of grate surface per hour	7.35 lbs.	8.62 lbs.
RATE OF EVAPORATION.		
33. Water evaporated from and at 212° F.; per sq. ft. of heating surface per hour, excluding f.w.h	2.12 lbs.	2.44 lbs.
COMMERCIAL HORSE-POWER.		
34. On the basis of 30 lbs. of water per hour evaporated from temperature of 100° F. into steam of 70 lbs. gauge-pressure (= $34\frac{1}{2}$ lbs. from and at 212°):		
Including f.w.h	117 H. P.	134 H. P.
Excluding f.w.h	112 H. P.	129 H. P.

CHAPTER XVII.

HIRN'S ANALYSIS.

THE best insight into the actual behavior of steam in the cylinder of an engine is given by Hirn's analysis, and tests giving sufficient data for such an analysis are of special interest and importance, since they indicate why one method of running an engine gives a better result than another.

Hirn* gives the data and the analysis of four such tests, made on engines with and without a steam-jacket, and using moist or superheated steam, which will not be quoted here, since the same ground is covered by later experiments made under his direction or inspiration.

The most notable tests, recorded in Tables XII to XV, are given by Hallauer.† Many of the tests were made by him personally, and all were worked up by him from the original data.

Hallauer's Tests on Simple Engines.—In Table XII are given the data of tests made on an engine designed by Hirn to use superheated steam. It had four independent flat valves moved by cams. These are all the tests given by Hallauer in which the complete data are given; in all other tests some of the data are missing, and the results and conclusions, only, are given here.

In Table XII are given the results of these tests and of tests made on an engine of the Corliss type with a steam-jacket.

Though it is not so stated in Hallauer's mémoires, it appears that during these tests these engines were coupled with another engine, and that the speed was controlled by that engine, with the intention that the point of cut-off and the work of the engine should remain constant during a test. The same

* Théorie mécanique de la chaleur. 1876.

† Bulletin de la Soc. ind. de Mulhouse, vols. xlvii.–liii.; 1877–1883.

method seems to have been employed during most of the tests of compound engines given in Table XIV.

Observations Taken.—(1) The steam consumed was determined by measuring the feed-water in a tank alternately filled and emptied. The level of the water in the boiler was noted evening and morning, and allowance made for the difference; also, the absence of leaks was made certain by applying hydraulic pressure to the boiler, pipes, and so forth.

(2) The water rejected by the air-pump was gauged by allowing it to flow through an orifice in a copper plate under considerable head. The flow through the orifice used was determined by direct experiment under the conditions which obtained during the experiment.

(3) The superheating, when it occurred, was measured directly by a good mercurial thermometer placed in a tube let into the steam-pipe near the engine-cylinder.

(4) The per cent of water mixed with the steam, when saturated steam was used, was determined by calorimetric experiments. In some cases this quantity appears to have been inferred from experiments made at other times under similar conditions.

(5) The rise of temperature of the injection-water in Hallauer's own work was determined by a differential air thermometer reading to one fiftieth of a degree Centigrade.

(6) Other temperatures were taken with mercurial thermometers, or were deduced from the indicated pressures, by aid of tables of the properties of steam.

(7) The work was measured by aid of a good steam-engine indicator, and in some cases by Hirn's flexion pandynamometer, which utilized the beam of the engine as a spring for measuring the force exerted by the steam on the piston. As before indicated, the diagrams thus obtained also gave the pressures at the several interesting points of the stroke, from which the temperatures were determined.

(8) The revolutions per minute, steam-pressure, and other required observations, were taken by aid of proper instruments.

The greater part of Tables XII and XIII is sufficiently intelligible from the preceding account of the observations taken, and from the headings of the columns. The following explanation will make the rest clear:

The real cut-off given in column 5 of both tables is the ratio of the volume of steam in the cylinder at cut-off to the volume at the end of the stroke; it is therefore the reciprocal of the number of expansions.

In Table I, the absolute pressures, in columns 11–18, were measured on the indicator-cards, and the temperatures corresponding were taken from tables of the properties of steam.

The works in the same table were obtained by measuring the appropriate areas on the indicator-card with the polar planimeter. The corresponding quantities of heat were obtained by dividing by 424.

In Table XIII, column 9, the net horse-power for experiments 9–11 is deduced by comparison with other experiments, for which the power was measured by a brake.

In the same table, column 10 shows that the work required to expel the steam from the cylinder varies widely, as compared with the total absolute work during the forward stroke. This is due in part to the varying power of the forward stroke, and in part to the variation in the vacuum maintained in the condenser. To make proper comparisons of different engines, the back pressure should be the same in all, or, as that is seldom possible, they should be reduced to a common back pressure. The simplest and the customary reduction is to assume that the back pressure is zero, or that the vacuum is perfect.

The assumption just made gives rise to a term called total or absolute horse-power, i.e., the horse-power the engine would have if it exhausted into a perfect vacuum, and usually, if there were no compression.

The consumption of dry saturated steam, columns 11–13, is deduced from the actual consumption of superheated or moist steam, by multiplying by the heat required to raise one pound of water from freezing-point to the pressure and condition stated, and then dividing by the total heat of saturated steam at the same pressure. In two cases the steam for total horse-power is the actual weight of superheated steam, and for the same experiments the consumption per net horse-power is not given. Again, the net horse-power is not stated for any of the experiments on the Hirn engine. These several inconsistencies, and others, found in these tables and in Tables XIV and XV, are due to the fact that they are condensed from calculations and tables given by Hallauer in several different mémoires, which, being for specific purposes, included such of the experiments as were convenient. As the details of some of the calculations are not explicitly stated, I have not thought it profitable to supply the omissions.

In the work of Hirn and Hallauer, the heat Q_c, rejected from the walls of the cylinder during exhaust to the condenser, receives special attention, and is the only one of the several quantities Q_a, Q_b, Q_c, and Q_d, which is directly calculated by them. Hallauer's calculations are, however, in such form that the other quantities may be easily deduced from them with good degree of approximation. In Table II I have stated the quantities Q_a and Q_b thus obtained; Q_d is not regarded, as the compression was very slight. For the same reason, and because the engines usually had a good vacuum in the condenser, the weight M_0 is neglected.

In Experiment 8, when the condenser was not used, and where the steam was strongly superheated and much wire-drawn by throttling, the heat rejected to the condenser becomes — 1.2 per cent, a result which is impossible. In this test the condenser was not used, and no check on this quantity could be made. This discrepancy may be due to the fact that the steam was superheated throughout its passage through the engine; or it may be the error of the test.

Hallauer's Method of Calculation.—To show this method and compare it with the theory developed on page 185, an example will be given, using the test on the Hirn engine made Sept. 7, 1875. This test, with some others on the same en-

TABLE XII.

DATA OF TESTS ON HIRN ENGINE.

	Date.	Condition.	Revolutions per minute.	Real cut-off or ratio of expans'n.	Weight of feed-water per stroke, kilos.	Weight of injection-water per stroke, kilos.	Weight of water primed per stroke, kilos.
1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
1	Nov. 18, 1873	Superheated, 230°	30.1736	0.2570	0.3065	9.3500	——
2	Nov. 28, 1873	Saturated,	30.5494	0.2570	0.3732	9.29175	0.0037
3	Aug. 26, 1875	Superheated, 215°	29.969	0.2139	0.2651	8.7291	——
4	Aug. 27, 1875	Superheated, 223°	30.306	0.4539	0.2822	8.5983	——
5	Sept. 7, 1875	Superheated, 195°	29 98	0.1628	0.2240	8.7384	——
6	Sept. 8, 1875	Saturated,	30.41	0.1628	0.2634	8.9132	0.0030
7	Sept. 29, 1875	Superheated, 220°	30.13	0.4539	0.2265	5.9810	——
8	Oct. 28, 1875	Superheated, 220°	30.00	0.2867	0.2714	——	——

	Temperatures of injection-water.		Absolute pressures in kilos per sq. m., and corresponding temperatures of saturated steam in degrees C.							
			Boiler.		Cut-off.		Release.		Back pressure.	
	Initial.	Final.	Pressure.	Temp.	Pressure.	Temp.	Pressure.	Temp.	Pressure.	Temp.
	9.	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**
1	12.6	31.3	48900	150.15	42449	144.96	9355	97.24	3680	73.49
2	11.83	33.65	46380	148.20	37773	140.78	9701	98.24	3670	73.42
3	16.50	33.09	49938	151.00	41415	148 74	7722	92.05	1919	58.86
4	16.50	35.26	48075	150.00	23070	124.00	8417	94.40	1900	58.25
5	16.37	30.42	49680	150.77	39128	142.00	5931	85.00	1881	58.44
6	16.50	32.25	49706	150.77	38339	141.30	5737	84.33	2134	61.15
7	15.85	37.81	50255	152.20	17458	115.80	6457	87.36	1759	57.11
8	——	——	43754	146.20	34333	137.40	11053	101.89	——	——

	Work in meter-kilograms and corresponding heat in calories.										Radiation per stroke 2.5 calories.
	At full pressure.		Of expansion.		Of back press're		Total, absol'te		Real, indicated.		
	Work.	Heat.	Work.	Heat.	Work.	Heat.	Work.	Heat.	Work.	Heat.	
	19.	**20.**	**21.**	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	
1	5531.5	13.01	7020.5	16.52	1787.0	4.205	12552	29.53	10765	25.325	
2	5103.5	12.005	6725.5	15.835	1782.5	4.19	11833	27.84	10050	23.65	
3	4415.0	10.39	6710.0	15.79	932.0	2.19	11125	26.18	10193	23.99	
4	6285.0	14.79	3930.0	9.24	922.0	2.17	10215	24.03	9293	21.86	
5	3315.0	7.80	6085.0	14.31	912.0	2.14	9480	22.11	8487	19.97	
6	3187.0	7.50	5825.0	13.70	1035.0	2.43	9012	21.20	7977	18 77	
7	5225.0	12.29	3070.0	7.22	862.0	2.02	8295	19.51	7433	17.49	
8	5000.0	11.76	6162.0	14.50	5212.0	12.26	11162	26.26	5950	14.00	

TABLE XIII.

RESULTS OF EXPERIMENTS ON SIMPLE EXPANSIVE ENGINES.

	Name of Engine.	Date.	Condition.	Revolutions per minute.	Real cut-off or reciprocal of ratio of expansion.	Absolute pressures in kilos. per square meter. Boiler.	Absolute pressures in kilos. per square meter. Back pressure.	Horse-power. Cheval à vapeur. Indicated.	Horse-power. Cheval à vapeur. Net.	Work of the back pressure in per cent of total work.
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
1	Hirn	Nov. 18, 1873	Superheated, 231°	30.1736	0.2570	48900	3680	144.36		8.3
2	"	Nov. 28, 1873	Saturated	30.5494	0.2570	46380	3670	136.46		9.8
3	"	Aug. 26, 1875	Superheated, 215°	29.969	0.2139	49938	1919	135.77		8.4
4	"	Aug. 27, 1875	*Superheated, 223°	30.306	0.4539	48075	1900	125.17	114.0	8.9
5	"	Sept. 7, 1875	Superheated, 195°	29.98	0.1628	49680	1881	113.08	102.0	9.7
6	"	Sept. 8, 1875	Saturated	30.41	0.1628	49706	2134	107.81	95.0	11.8
7	"	Sept. 29, 1875	† Superheated, 220°	30.13	0.4539	50255	1759	99.53		10.4
8	"	Oct. 28, 1875	‡ Superheated, 220°	30.00	0.2867	43754	....	78.30		46.7
9	Corliss	1878	Saturated, jacketed	50.41	$\frac{1}{11}$		1480	105	92	10.0
10	"	1878	Saturated, jacketed	51.12	$\frac{1}{8}$		1690	137	125	8.8
11	"	1878	Saturated, jacketed	49.34	$\frac{1}{6}$		1840	158	142	8.1

* Throttle-valve partly closed. † Valve nearly closed. ‡ Non-condensing.

	Consumption of equivalent dry saturated steam per horse-power per hour, kilos.			Per cent of water in mixture in the cylinder.			Exchange of heat in per cent of total heat furnished per stroke.						
	Per total horse-power.	Per indicated horse-power.	Per net horse-power.	At cut-off.	At release.	Entrained to condenser.	Absorbed by walls during admission. Q_a	Restored during expansion. Q_b	Rejected to condenser. Q_c	Radiation. Q_e	Furnished by jacket. Q_i	Error by second method in Q_c	Actual values in calories. Q_c
	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.
1	7.000	7.633	8.207	6.50	12.00	4.9	11.0	2.0	7.8	1.2		−0.40	16.61
2	8.449	9.307	10.341	30.40	25.20	9.4	23.9	7.3	15.4	1.0		0.25	37.53
3	6.7519	7.3691	....	0.38	17.50	7.9			9.7	1.4		3.2	17.60
4	7.874	8.655	9.511	†−1.50	13.20	3.11	....	....	10.5	1.3		0.30	20.34
5	6.655	7.370	8.188	24.64	21.38	8.10	22.4	8.4	12.5	1.6		0.05	18.80
6	7.822	8.837	9.929	36.00	35.19	10.63	28.3	5.0	21.6	1.5		−0.20	37.02
7	7.763	8.663	9.844	2.52	15.85	1.10			14.2	1.6		3.20	21.90
8	*6.562	12.315	....	12.00	Dry.	Dry.	....	....	− 1.2	....		....	−1.84
9	7.188	7.983	9.071	38.30	21.70	....	26.4	17.5	12.3	1.65	5.1	1.9	11.21
10	7.236	7.939	8.724	31.70	19.20	. ..	21.1	15.0	9.8	1.3	4.9	1.2	11.14
11	7.307	7.955	8.646	25.30	18.50	....	15.4	10.4	8.0	1.1	4.1	1.1	11.15

* Superheated steam per horse-power per hour. † Superheated.

gine, is calculated twice, in a memoir presented in 1877, and again in one presented in 1878. There are some differences in the two methods, and some of the others show discrepancies that do not appear to be readily explained. In the tables given I have adhered to the earlier calculation, where the entire data are given.

CALCULATION OF TEST ON HIRN ENGINE, Sept. 7, 1875.

Volume of cylinder, including clearance at one end,			0.490 cu. m.
Specific heat of superheated steam,			$c_p = 0.5$
			Calories.
Heat brought by dry vapor in cylinder,	$0.224\,(606.5 + 0.305 \times 150.77) =$		146.15
Heat brought by superheat,	$0.5 \times 0.224\,(195.5 - 150.77) =$		5.01
Total heat brought to cylinder,			151.16
Heat carried away by condensed water,	$0.224 \times 30.42 =$		6.81
Difference, available heat,			144.35
Heat absorbed by condensing water,	$8.7384\,(30.42 - 16.37) =$		122.77
Difference,			21.58
Heat equivalent of external indicated work,		19.97	
Heat lost by radiation, etc.,		2.5	22.47
			-0.89

Per cent of error, $\dfrac{-0.89}{151.16} = -0\%.6.$

The consumption of dry saturated steam per stroke, calculated from the total heat, is $\dfrac{151.16}{652.48} = 0.^{k}2317.$

The indicated horse-power is $\dfrac{2 \times 8487 \times 29.98}{60 \times 75} = 113.08.$

The net horse-power is $113.08 \times 0.90 = 102.$

The consumption of dry saturated steam per horse-power per hour is:

Total horse-power, $\dfrac{0.2317 \times 60 \times 60 \times 75}{9400} = 6.^{k}655;$

indicated horse-power, $\dfrac{0.2317 \times 60 \times 60 \times 75}{8487} = 7.^{k}370;$

net horse-power, $\dfrac{7.370}{0.90} = 8.^{k}188.$

Weight of mixture of steam and water in the cylinder,		$0^k.2240$	
Weight of dry steam at cut-off,	$0.1628 \times 0.490 \times 2.116 =$	0.1688	
Weight of steam condensed during admission,		$= 0.0552$	$= 24\%.6$
Heat yielded during condensation,	$0.0552 \times 506.5 =$	$27^c.96$	
Weight of dry steam at end of stroke,	$0.490 \times 0.3595 =$	$0^k.1761$	
Weight of water in mixture,		$= 0.0479$	$= 21\%.38$
Internal heat at cut-off,	$0.1688 \times 462.68 + 0.224 \times 143.26 =$		110.22
Internal heat at end of stroke,	$0.1761 \times 507.77 + 0.224 \times 85.32 =$		108.56
Difference,			1.66
Heat furnished by superheat,	$(195.5 - 142)\, 0.5 \times 0.224 =$		5.99
Heat furnished by condensation during admission,			27.96
Sum,			35.61
Heat absorbed by work of expansion,		14.31	
Heat lost by radiation, etc.,		2.50	16.81
Q_c, the heat rejected by the walls to the condenser,			$= 18.80$

In per cent of total heat received, $Q_c = \dfrac{18.80}{151.16} = 12\%.5.$

Internal heat at the end of the stroke,	$108^c.56$
Heat equivalent of work of back pressure,	2.14
Heat carried away by condensed water,	$-\ 6.81$
	103.89
Heat acquired by condensing water,	122.77
Difference Q_c,	$= 18.88$

Error in determination of $Q_c = \dfrac{18.88 - 18.80}{151.16.} = 0\%.05.$

Heat yielded by $0^k.224$ dry steam in condensing,

	$0.224\,(624.32 - 30.42) = 133^c.03$
Heat acquired by condensing water,	$= 122.77$
	10.26

Weight of water contained $\dfrac{10.26}{565.75} = 0^c.01815 = \dfrac{0.01815}{0.224} = 8\%.1.$

From the quantities already calculated, the two remaining quantities Q_a and Q_b may be found; Q_d is considered to be zero.

Heat furnished by condensation during admission,	$27^c.96$
Heat furnished by superheat,	5.99
Q_a,	$= 33.95$

$$\frac{33.95}{151.6} = 22\%.4.$$

Heat equivalent of work of expansion,	14.31
Internal heat at cut-off minus internal heat at end of stroke,	1.66
Q_b	= 12.65

$$\frac{12.65}{151.6} = 8\%.4.$$

Hallauer's Tests on Compound Engines.—Table XIV gives the results of experiments made on stationary compound engines of different types. Experiments 1–7 were made on a vertical Woolf beam engine working at Munster. It had the two cylinders side by side acting on the same end of the beam, the small cylinder nearer the columns so that it had a shorter stroke. Both cylinders were jacketed with steam brought from the boiler in a special pipe. The engine was normally controlled by a throttle governor, but during the experiments the governor was disconnected, and the engine was run coupled with another engine which controlled the speed. Experiments 8 and 9 were made on a horizontal Woolf engine having the small cylinder above the large one, and inclined at a small angle, so that both pistons acted on one crank. Both cylinders were jacketed with the steam passing from the boiler to the small cylinder—a method that cannot be recommended, since the condensation in the jacket makes the steam moist as it enters the cylinder. Experiments 10–13 were made on a double vertical Woolf engine coupled to the same shaft; 10 and 12 were made on the left engine, and 11 and 13 were made on the right engine. The engine was controlled by a governor which varied the cut-off of the small cylinder. The cylinders were jacketed with the steam passing to the small cylinders. Experiment 14 was made on a vertical Woolf beam engine, working at Saint-Remy.

Tests on Marine Engines.—Table XV gives the results of experiments on various marine engines. Since the weight of the circulating water could not be determined, the check on the calculation of Q_c could not be obtained.

Experiments 1, 4, and 5 are entered twice in the table,—the first time as calculated with the data given by the experiments, and the second time with modified data which give more con-

TABLE XIV.

Results of Experiments on Stationary Compound Engines.

	Place.	Date.	Dimensions.	Revolutions per minute.	Ratio of expansion.	Absolute pressures, kilos. per sq. m.: Boiler.	Absolute pressures, kilos. per sq. m.: Back pressure, large cylinder.	Horse-power: Indicated.	Horse-power: Net.	Work of back pressure in per cent of total work.
1	Munster.	Sept. 21, 22, 1876	High-pres-	25.3	....	....	....	180.23	....	22.92
2	"	June 20, 21, 1876	sure, diam.	25.0	....	51670	....	246.92	....	....
3	"	Oct. 24, 25, 1876	$0^m.55$, stroke	25.6	....	....	....	284 28	....	....
4	"	Oct. 17, 18, 1876	$1^m.415$. Low-	25.1	....	...	....	346.39	....	19.60
5	"	June 12, 13, 1877	pressure,	25.4	....	42369	....	185.75	....	24.10
6	"	June 21, 22, 1877	diam. $1^m.200$	25.2	....	51670	....	267.85	....	20.52
7	"	July 4, 5, 1877	stroke $2^m.00$,	25.25	....	56837	2930	347.16	....	17.43
8	Factory, Dollfus, Mieg&Cie	Nov., 1876	Diameter, h. p. $0^m.381$,	39.37	6	49600	2530	130	112.08	20.06
9		"	l. p. $0^m.8575$, stroke, 1.297	39.37	6	38380	2950	181	161.00	....
10	Malmer-	1877		26.2	28	56837	1810	143.11	118.38	18.6
11	spach.	"		25.93	25	56837	1750	149.53	124.74	17.5
12	"	"		25.47	13	58903	2260	215.7	185.69	15.6
13	"	"		24.83	13	58903	2180	212.92	183.67	14.9
14	Saint-Remy	....		24.503	19	32220	....	137.00	107.88	9.9
			Ratio cyls.							
15	Woolf*	....	0.147	25	0.131	....	2690	367	....	15.7
16	"	....	0.147	25	0.131	....	1760	178	....	20.0
17	"	....	0.182	25.5	0.0778	....	2150	220	....	14.7
18	"	....	0.182	26.0	0.0385	....	1730	150	....	17.0
19	Compound.	....	0.348	88.5	0.132	....	....	78.5	....	17.6

* Kind of engine.

	Consumption of equivalent dry saturated steam per horse-power per hour, kilos.			Per cent of water.				Exchange of heat in per cent of total heat per stroke.					
	Total horse-power.	Indicated horse-power.	Net horse-power.	Priming from boiler.	At cut-off, small cylinder.	At one-tenth stroke, large cylinder.	End of stroke, large cylinder.	Absorbed during admission. Q_a	Rejected to condenser. Q_c	Error in, by second method. Q_i	Lost by radiation. Q_e	Furnished by jacket. Q_j	Actual value in calories. Q_c
1	7.6605	9.9399	.....	5.5	16.61	30.97	12.52	8.36	6.20	0.24	1.83	8.12	23.74
2	7.7434	9.5619	......	5.0	16.29	32.06	10.32	8.54	5.90	0.25	1.36	6.63	30.27
3	7.3966	9.3984	......	5.0	12.09	31.68	13.75	5.25	6.65	0.43	1.23	6.21	38.10
4	7.6104	9.4663	......	6.0	16.00	30.43	12.33	7.43	5.45	1.26	0.98	6.25	38.94
5	7.3841	9.7299	12.411	2.35	21.17	32.18	7.39	14.44	3.50	3.5	1.81	8.37	13.48
6	6.9452	8.7390	10.357	2.9	14.21	28.44	5.39	8.58	1.32	1.32	1.38	7.24	6.65
7	7.1121	8.6140	9.864	2.8	16.17	28.90	6.60	10.12	3.38	3.38	1.08	6.80	21.89
8	7.290	9.120	10.563	3.0	11.20	....	5.34	6.43	1.19	0.10	2.14	8.05	1.95
9	7.328	8.878	9.975	3.0	10.8	....	4.50	5.56	0.75	0.45	1.59	6.15	1.16
10	6.731	8.273	10.019	5.0	40.0	....	17.60	25.5	7.8	1.6	1.9	6.7	19.34
11	6.821	8.260	9.898	5.0	36.1	....	17.80	22.7	8.2	0.9	1.8	6.7	21.33
12	6.878	8.149	9.465	5.0	23.7	....	17.90	13.72	8.40	3.7	1.23	5.94	31.44
13	6.983	8.210	9.517	5.0	24.7	....	19.5	14.43	9.90	1.6	1.20	5.96	37.80
14	6.840	7.591	9.702	3.0	....	....	....	....	....	....	....	....	...
15	7.042	8.354	......	....	9.4	*8.9	9.9	. ..	4.9	0.7	....	. ..	33.1
16	7.316	9.145	......	....	10.8	10.4	10.6	....	6.4	0.2	....	....	22.9
17	6.883	8.069	......	....	23.2	10.4	17.8	....	10.2	0.8	....	....	38.9
18	6.831	8.228	......	....	39.0	20.1	16.4	....	9.3	0.4	....	....	24.3
19	6.667	7.376	......	....	29.7	15.2	15.9	....	4.5	0.8	....	....	1.6

* Per cent of water at end of stroke, small cylinder for 15–18.

sistent results. In Experiment 1, the steam at cut-off in the small cylinder appears to have 0.2 per cent of water, and at the end of it appears to be dry. Hallauer thinks this improbable even with an efficient steam-jacket, since the steam passages and valve openings are large, so he gives a recalculation in 1*a* which augments the consumption by the amount of

$$\frac{8.416 - 8.179}{8.416} = 2.8 \text{ per cent.}$$

In like manner, Experiment 4 appears to give dry steam at cut-off of the small cylinder, and superheated steam at the end of the stroke. The modified results of 4*a* show augmentation of consumption of 3.5 per cent. From analogy the next experiment has the consumption increased by 1.8 per cent in the second form, 5*a*.

To me, such a change of the original data appears to be questionable, and the modified results are certainly in doubt to an extent equal to the modification made. I have given the original data the preference in the table, but have included the modified results which are used by Hallauer in his comparisons with other engine tests.

Discussion of Results.—The experiments recorded in Table XIII show clearly the effect of superheated steam, and of a steam-jacket on the interchange of heat between the steam in an engine-cylinder and the walls of the cylinder; and show the reason of the economy resulting from these methods of using steam.

When superheated steam is used, the heat Q_a absorbed by the cylinder during admission of steam is furnished in part by the superheat, and consequently there is less initial condensation than when saturated steam is used. At release there is less moisture to be evaporated from the walls of the cylinder, which in turn reduces the amount of Q_a. This is clearly seen from a comparison of Experiments 1 and 2, and 5 and 6, on the Hirn engine. The effect on the value of Q_b, returned by the walls of the cylinder to the steam, is not so well marked. With the cut-off at $\frac{1}{4}$ stroke (Experiments 1 and 2), the use of

TABLE XV.

Experiments on Compound Marine Engines.

	Name.	Condition.	Expansion. Ratio of cylinders.	Expansion. Recip. of expansion in small cylinder.	Expansion. Reciprocal of total expansion.	Revolutions per minute.	Indicated horse-power.	Work of back pressure in per cent of total work.	Back pressure, large piston.
1	Duquesne	Throttle open, long cut-off	0.519	0.725	0.376	80.83	8490	13.5	2880
1a	"		"	"	"	"	"	"	"
2	"	Throttle closed partly	"	"	"	59.33	3180	....	....
3	"	Throttle closed further	"	"	"	44.49	1410	21.5	1590
4	"	Cut-off shortened	"	0.650	0.321	76.67	7200	....	....
4a	"	......	"	"	"	"	"	....	....
5	"	Cut-off shortened	"	0.550	0.285	73.00	6360	11.1	2000
5a	"	......	"	"	"	"	"	"	"
6	"	Cut-off shortened	"	0.225	0.126	62.49	3900	17.7	2460
7	"	Cut-off shortened	"	0.100	0.052	46.55	1665	22.0	1840
8	Vienne	Receiver	0.317	0.660	0.209	75.00	690	13.4	2160
9	Cigale	Receiver	0.309	0.750	0.232	90.00	205	7.5	1780
10	Nièvre	Three cylinders	0.380	0.499	0.189	93.80	740	11.3	2420
11	Mytho	"	0.282	0.601	0.169	44.48	590	16.4	1070
12	"	"	"	"	"	61.40	1350	11.2	1150
13	"	"	"	0.690	0.194	73.16	2200	13.0	1890
14	"	"	"	0.750	0.212	66.00	2590	10.2	1870
15	....	Receiver	*0.82 1.45×0.9	....	....	75.00	689.63	....	....

	Consumption of equivalent dry saturated steam per horse-power per hour, kilos. Total.	Consumption of equivalent dry saturated steam per horse-power per hour, kilos. Indicated.	Per cent of water in the cylinder. Cut-off, small cylinder.	Per cent of water in the cylinder. End of stroke, small cylinder.	Per cent of water in the cylinder. End of stroke, large cylinder.	Exchange of heat in per cent of total heat per stroke. Rejected to condenser. Q_c	Exchange of heat in per cent of total heat per stroke. Furnished by jacket. Q_j	Exchange of heat in per cent of total heat per stroke. Lost by radiation. Q_e	Exchange of heat in per cent of total heat per stroke. Actual value of Q_c in calories.
1	8.179	9.405	0.2	dry	7.1	0.50	1.57	1.23	27
1a	8.416	9.678	2.4	2.2	9.3	1.05	1.53	1.20	103
2	8.744	10.571	7.2	3.8	12.7	5.75	4.05	1.93	176
3	8.832	11.242	4.0	2.7	11.0	3.7	4.7	3.1	72
4	8.114	9.394	0.0	s. h.	8.6	2.9	2.9	1.4	138
4a	8.411	9.737	3.4	0.3	11.7	3.2	2.8	1.3	157
5	7.915	8.907	6.5	1.8	14.4	5.4	3.3	1.6	227
5a	8.056	9.066	8.0	3.4	15.7	7.0	3.3	1.6	290
6	6.861	8.335	24.0	5.4	18.6	9.8	4.0	2.2	278
7	8.154	10.448	49.5	20.5	19.5	12.1	4.3	2.4	244
8	7.513	8.675	10.0	4.6	14.8	6.7	4.8	1.4	28.8
9	7.762	8.390	5.4	3.1	14.7	7.2	4.0	1.4	7.5
10	7.729	8.710	23.8	13.8	32.6	18.7	1.3	2.4	71
11	7.947	9.504	27.1	22.4	30.5	21.9	4.7	3.2	149
12	7.343	8.263	22.3	18.8	29.5	20.6	4.0	2.3	205
13	7.397	8.510	16.4	15.2	25.7	15.8	2.8	1.6	221
14	9.493	8.350	11.8	12.1	21.0	12.3	2.5	1.3	220
15	7.5098	8.6706	7.46	......	13.74	5.74	4.7	1.4	24.9

* Diameters and stroke of cylinder.

superheated steam reduces Q_b, both as compared with the total heat applied and as compared with the value of Q_a; on the other hand, at $\frac{1}{6}$ stroke (Experiments 5 and 6), the reverse is true.

It is noticeable that the actual number of calories rejected by the walls of the cylinder during the exhaust is almost identical in the Experiments 2 and 6, when saturated steam is used. The same thing is true of Experiments 9, 10, and 11, made on the Corliss engine with a steam-jacket and using saturated steam. There is more variation of the actual value of Q_c when superheated steam is used, which may be attributable to the varying degree of superheating.

A steam-jacket reduces the heat rejected by the walls of the cylinder during exhaust in a different way. Especially when the cut-off is short, as in the experiments given in Table XIII, the jacket cannot have much effect on the initial condensation, and almost all of the heat taken by the walls of the cylinder before cut-off is furnished by that condensation. During expansion a very considerable portion of the moisture previously condensed on the walls of the cylinder is evaporated—much more than would be without a jacket; and the heat thus applied by the jacket does work, though with a reduced efficiency. During the exhaust the jacket furnishes a large portion of the heat required to evaporate the moisture on the walls of the cylinder at release; this heat from the jacket is thrown away, and would be entirely wasted were it not true that the walls of the cylinder are not chilled to the same degree as when the jacket is not used, and that the initial condensation is thereby reduced.

The comparison of Experiment 6 with Experiment 11 may be made to illustrate the preceding statements. In Experiment 6, 28.3 per cent of all the heat applied is absorbed by the walls of the cylinder during the admission: of this less than $\frac{1}{5}$ is returned during expansion, and $\frac{3}{4}$ of the heat is thrown out as exhaust waste. In the 11th Experiment, 15.4 per cent of the heat applied is absorbed by the walls of the cylinder. The heat yielded by the walls of the cylinder during expansion is $\frac{2}{3}$ of that absorbed during admission, and $\frac{1}{2}$ of the amount absorbed is thrown out during exhaust. The excess of the heat yielded by the walls of the cylinder during expansion and exhaust over that absorbed during admission, together with the heat lost by radiation, is the measure of heat supplied by the jacket.

To sum up in a few words: It appears that the use of superheated steam reduces the exhaust waste and consequently the initial condensation; while the use of a steam-jacket, by keeping the cylinder hot, reduces the initial condensation and increases the re-evaporation, and consequently reduces the exhaust waste.

Exact conclusions cannot be drawn from the comparative economy of the Hirn engine and the Corliss engine. Yet it may be stated that these experiments show a gain from the use of the steam-jacket of

$$\frac{8.837 - 7.955}{8.837} = 0.10,$$

and a gain from the use of superheated steam of

$$\frac{8.837 - 7.370}{8.837} = 0.166.$$

A comparison of Experiments 1 and 2 shows a gain from superheating of

$$\frac{9.307 - 7.633}{9.307} = 0.18.$$

Hallauer, in estimating the gain from the use of superheated steam, uses the actual consumption of superheated steam and of moist steam, instead of the equivalent dry saturated steam, claiming that the superheating was done by waste gases beyond the boiler; but that is evidence merely that the boiler was not economical.

Inspection of the table shows that in the three tests on the Corliss engine the heat yielded by the walls during expansion is about $\frac{2}{3}$, and the heat rejected during exhaust is about $\frac{1}{2}$, of the heat absorbed during admission. This, taken with the fact already alluded to, shows that the action of the walls was nearly the same during each of these phases, while the cut-off changed from $\frac{1}{11}$ to $\frac{1}{6}$ of the stroke.

Before leaving these experiments, attention should be called to the fact that in the tests on the Hirn engine the per cent of moisture or priming in the exhaust steam has been calculated from the weight and temperatures of the injection-water in the usual calorimetric method. The per cent of priming varies with the method of running the engine in something the same way as does the per cent of the moisture in the cylinder at release; but it is seldom so much as $\frac{1}{3}$ of the latter quantity, and it is never so great as 11 per cent. Attention will be called to this again in connection with the discussion of the question whether the interchange of heat is between the steam and the metal of the cylinder, or between the steam and moisture remaining permanently in the cylinder.

In examining the tests on stationary compound engines in Table XIV, it is noticeable that while the heat absorbed during admission is a considerable fraction of all the heat applied, though in most cases less than with a simple engine, the heat rejected by the walls during exhaust is in all cases small, and in some cases it is insignificant.

The division of the expansion between the two cylinders of a compound engine with the accompanying division of the range of temperature reduces the amount of the interchange of heat between the steam and the walls of the cylinder, but it by no means prevents it. It is possible to make a calculation of Q_b which corresponds to the heat restored by the walls of the cylinder during expansion in a simple engine; but here that quantity represents a complicated change. During the expansion in the small cylinder some of the heat absorbed by the walls of that cylinder before cut-off is restored; during the exhaust from the high-pressure to the low-pressure cylinder heat is rejected from the walls of the former, a part of which is transferred to the walls of the low-pressure cylinder by initial condensation therein; as the expansion goes on in the low-pressure cylinder, before and after cut-off of that cylinder, the lowering pressure is accompanied by re-evaporation of water from its sides, and heat is yielded therefrom. Consequently the calculation corresponding to the finding of Q_b develops positive and negative quantities whose sum is not a measure of the action that actually takes place. The only way of properly investigating this action is that suggested by the application of Hirn's analysis to compound engines on page 222, and the data for it are not given for these tests by Hallauer.

The data of the experiments on marine engines were furnished by M. Widman; the results stated in Table XV were calculated by Hallauer. It was not possible to measure the amount of cooling water used per stroke, consequently the check given by the condenser on the value of Q_c could not be obtained. On the other hand, the use of the surface condenser gave a very exact method of measuring the consumption of steam.

The engine of the *Duquesne* has six cylinders, arranged to form three Woolf engines, with the small cylinder of each above the large cylinder. From the positions of the cylinders the intermediate receiver had a considerable volume. Two series of experiments were made: 1, 2, and 3 show the effects of throttling the steam, and 1, 4, 5, 6, and 7 show the effect of shortening the cut-off. Hallauer has modified the results in a manner already alluded to and has used the second results in his comparisons.

Experiments 1 and 3, Table XV, and 15 and 16, Table XIV, are compared in the following table:

	Duquesne.		Diff.	Woolf.		Diff.
Indicated horse-power	1410	8490		178	367	
Reduction of pressure by throttling	....	20100		9.10	3010	
Percentage of water:						
Small cylinder cut-off	4.0	2.4	1.6	10.8	9.4	1.4
Small cylinder release	2.7	2.2	0.5	10.4	8.9	1.5
Large cylinder release	11.0	9.3	1.7	10.6	9.6	0.7
Consumption in kilos:						
Total horse-power	8.832	8.416	0.416	7.316	7.042	0.274
Indicated horse-power	11.242	9.405	1.837	9.145	8.354	0.791
Heat rejected to condenser	3.7	1.9	1.8	6.4	4.9	1.5

Hallauer considers the parallelism exhibited by this table to be a verification of the tests on the marine engine, in default of the check afforded by measuring the cooling water.

The interchange of heat between the walls of the cylinder and the steam is roughly indicated by the per cent of water in the cylinder at the several points indicated. In the first four experiments, where the power is varied by throttling, these percentages and the value of Q_c, rejected to the condenser, vary in an irregular manner and to a small amount only. On the other hand, the increase of these several quantities is marked and regular when the power is regulated by shortening the cut-off. It is also apparent that the consumption is reduced by increased expansion as far as to eight expansions, though the ratio of the volumes of the cylinders is ill adapted to large expansion, and that it increases rapidly for more than eight expansions.

The performance of the engine of the *Duquesne* with a variable cut-off may be compared with the stationary Woolf engine, Table XIV, Experiments 17 and 18. For example, the marine engine, when exerting 3900 horse-power, with 8 expansions, has 9.8 per cent for the value of Q_c, while the stationary engine, exerting 220 horse-power, with 13 expansions, has $Q_c =$ 10.2 per cent. Again, compare the consumption per total horse-power of the marine engine, when exerting 1665 horse-power with 20 expansions, with that of the stationary engine exerting 150 horse-power with 26 expansions. The latter has the advantage by

$$\frac{8.154 - 6.831}{8.154} = 0.162.$$

Of course some of this may be attributed to the greater degree of expansion of the latter, but the real explanation is to be sought in the methods of obtaining the expansion. The ratio of the cylinders for the stationary engine is 1 : $5\frac{1}{2}$, and the expansions in the small cylinder are 5. On the other hand, the ratio of cylinders for the marine engine is 1 : 2, and the expansions in the small cylinder are 10. This large degree of expansion in the small cylinder of the marine engine causes an initial condensation of 49 per cent, while that of the stationary engine is 39 per cent.

The test 19, in Table XIV, is remarkable for the small consumption of steam per horse-power per hour, though the total horse-power was only 78.5. This engine was a portable compound condensing engine, having a distribution slide-valve and an independent cut-off valve for the small cylinder, and a plain slide-valve for the large cylinder. In 1878 a medal was offered by the Société Industrielle de Mulhouse for the first compound engine constructed in Upper Alsace, that should give one brake horse-power (cheval à vapeur) for less than 9 kilograms of steam. This engine was offered for trial, and was tested by a committee and awarded the medal.

Isherwood * gives, on page 319, the table of data and results of tests transferred to the English system of units.

Mair's Steam-engine Tests.—In Table XVII are given the data and results of a number of tests on large engines of various types, reported by Mr. George Mair,† together with a very complete analysis according to Hirn's method.

In all of these tests the power was measured by indicators that were tested after every trial, and diagrams were taken at intervals of 15 or 20 minutes, which were measured by a planimeter. The mechanical equivalent of heat was assumed to be 772 foot-pounds for facility in use of tables of the properties of steam, though reference is made to Joule's later determinations.

The steam consumption was determined by measuring the feed-water supplied to the boiler. The steam condensed in the steam-jackets was collected and weighed separately. The air-pump discharge was allowed to flow through an orifice under a measured head; the coefficient of discharge for the orifice was determined by direct experiments. The per cent of priming in the steam was determined by calorimetric tests. In the test

* Journal Franklin Inst., vol. cxx., Oct. 1885.

† Proc. of the Inst. of Civ. Engs., vol. lxx. page 313, and vol. lxxix. page 323.

TABLE XVI.

Experiments on a Condensing Compound Engine.

	July 7, 1879. Morning.	July 7, 1879. Afternoon.	July 8, 1879. Morning.	July 8, 1879. Afternoon.
Total Quantities.				
Duration of the experiments, hours	2.99889	4.00139	3.00222	3.24139
Revolutions	16014.	21253.	16211.	17253.
Pounds of feed-water pumped into the boiler	3951.	5379.	3248.	4265.
Pounds of condensing water	73821.	98621.	74039.	78894.
Engine.				
Steam-pressure in boiler in pounds per square inch above the atmosphere	**91.78**	**91.52**	**91.69**	**92.16**
Pressure in the condenser in pounds per square inch above zero	....	....	1.81	1.91
Number of revolutions per minute	89.	88.5	90.	88.7
Position of the throttle-valve	Wide open.	Wide open.	Wide open.	Wide open.
Cut-off, small cylinder	0.42	0.42	0.25	0.42
Release, small cylinder	0.98	0.98	0.98	0.98
Compression, small cylinder	0.925	0.925	0 925	0.925
Cut-off, large cylinder	0.45	0.45	0.45	0.45
Release, large cylinder	0.91	0.91	0.91	0.91
Compression, large cylinder	0.75	0.75	0.75	0.75
Number of times the steam was expanded	6.26	6.26	9.64	6.26
Atmospheric pressure in pounds per sq. inch	14.22	14.22	14.22	14.22
Temperatures.				
Temperature in degrees Fahrenheit of the condensing water when admitted to the condenser	48.3	48.1	48.0	48.0
Temperature in degrees Fahrenheit of the condensing water and water of steam condensation when taken from condenser	95.7	97.7	86.9	96.7
Absolute Steam-pressures in Small Cylinder per Indicator.				
At commencement of stroke	100.8	102.5	99.8	103.0
At point of cut-off	89.3	91.0	88.3	91.5
End of stroke	44.1	44.9	33.4	45.2
Mean back pressure	45.3	46.2	35.3	46.4
At compression	41.6	42.4	34.8	42.6
Indicated pressure	30.2	30.8	27.8	30.7
Absolute Steam-pressures in Large Cylinder per Indicator.				
At commencement of stroke	43.0	43.2	34.5	43.2
Point of cut-off	27.5	27.7	21.16	27.7
End of stroke	12.3	12.5	9.6	12.6
Mean back pressure	3.8	3.8	3.8	3.8
At compression	2.4	2.4	2.4	2.4
Indicated pressure	22.1	22.3	17.0	22.3
Horse-power.				
Indicated horse-power developed in the small cylinder	24.8	25.2	23.1	25.1
Indicated horse-power developed in the large cylinder	52.4	52.5	40.7	52.7
Aggregate indicated horse-power developed by the engine	**77.2**	**77.6**	**63.9**	**77.8**
Horse-power developed by the engine at the friction-brake	67.7	67.5	55.7	67.5
Economic Results.				
Pounds of feed-water per hour per indicated horse-power	**17.7**	**17.3**	**16.9**	**16.9**
Pounds of feed-water per hour per brake horse-power	19.5	19.9	19.4	19.5
Fahrenheit units of heat per hour per indicated horse-power	19110.	19351.	19108.	18924.
Fahrenheit units of heat per hour per brake horse-power	21785.	21960.	21906.	21804.

C the condensation in steam-pipe was drained into buckets and weighed; in the test B the condensation in the steam-pipe flowed to the engine, and an allowance was made depending on the condensation caught during the test C. In all the other tests the steam-pipes drained toward the boilers.

The test A was made on a single cylinder rotative pumping-engine, having a diameter of 45 inches and a stroke of 5 feet 6 inches. The sides and ends of the cylinder were jacketed with boiler steam. The steam was distributed by separate slide-valves near the ends of the cylinder, with expansion-plates adjustable by hand, on the backs of the main valves. A surface condenser was used. Three tests were made on this engine, each of which gave the same result; one test only is therefore given in the table. The engine had been at work nine months when tested; it was carefully examined, and the piston and valves were tight and in good order.

The tests B and C were made on a Woolf beam rotative engine, working a deep-well pump direct from the beam, which had been working six months when tested. The cylinders were 22 inches in diameter by 3 feet 7 inches stroke, and 34 inches diameter by 5 feet 6 inches stroke, and were jacketed with boiler steam on the sides and ends. The steam was distributed by a long slide with equilibrium passages in it, and expansion-plates on the back, worked by eccentrics; there was a jet condenser, and the air-pump discharge was measured. Two tests were made—one with and one without steam in the jackets.

The tests D, E, and F were made on a Woolf beam-engine driving a flour-mill. The cylinders were 24½ inches in diameter by 3 feet 5 inches stroke, and 38 inches diameter by 5 feet 6 inches stroke, and were steam-jacketed on the sides only. The steam was distributed to the high-pressure cylinder by a slide-valve with cut-off plates on the back, and to the low-pressure cylinder by a piston-valve, all being worked by eccentrics. The steam was condensed in a surface condenser. Three tests were made—two with and one without steam in the jackets.

The tests G and H were made on an unjacketed horizontal

TABLE XVII.—MAIR'S INDEPENDENT ENGINE TESTS.

	Type of Engine.	Date of Test.	Volumes in cubic feet. High-pressure cylinder.	Volumes in cubic feet. Low-pressure cylinder.	Volumes in cubic feet. Clearance, high-pressure cylinder.	Volumes in cubic feet. Clearance, low-pressure cylinder.	Volumes in cubic feet. Volume at end of admission.	Injection-water. Temperature, cold.	Injection-water. Temperature, warm.	Injection-water. Pounds per stroke.	Barometer, inches of mercury.	Pressures in lbs. on the sq. inch, absolute. In boiler.	Pressures in lbs. on the sq. inch, absolute. In steam-jackets.	Revolutions per minute.	Piston speed, feet per minute.	Total number of expansions.
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	Single-cylinder beam.	Feb. 1881	60.346	....	1.0	....	8.953	....	70	....	29.6	48	....	14.62	160.82	6.85
B	Compound Woolf beam.	Aug. 1881	9.361	34.67	0.3	0.908	3.823	54.57	98.57	21.98	29.82	58	....	17.84	196.24	9.3
C			9.361	34.67	0.3	0.908	2.257	59.99	85.49	20.48	29.73	62	....	19.62	215.82	15.76
D	Compound Woolf beam.	Dec. 1881	11.053	43.077	0.564	1.431	4.617	....	51.50	....	30.25	84.8	....	33.73	371.03	9.64
E			11.053	43.077	0.564	1.431	4.654	....	54	....	29.8	88.14	....	34.22	376.42	9.56
F			11.053	43.077	0.564	1.431	5.722	...	53	...	29.75	88.95	....	34.52	379.72	7.77
G	Compound horizontal tandem.	Feb. 1882	5.627	18.601	0.395	1.018	1.708	53.1	98.56	5.92	30.67	86.05	....	80.45	683.82	11.48
H			5.627	18.601	0.395	1.018	1.685	53.3	107.54	5.77	30.67	84.77	....	81.51	692.83	11.64
I	Compound receiver beam.	Oct. 1881	13.02	38.66	0.2151	1.035	2.916	50	73.4	25.66	30.20	76	....	23.98	263.78	13.61
K	Cornish.	Nov. 1881	198.18	...	....	....	....	50.45	86.2	129.4	30.04	46.4	....	*12.84	201.33	....
L	Single-cylinder beam engine.	July, 1882	30.377	....	0.558	....	7.140	58	90	....	29.89	56.7	56.68	20.31	223.4	4.33
M			30.377	....	0.558	....	8.05	58	100	....	29.87	55.8	55.8	20.26	222.9	3.84
N	Single-cylinder beam engine.	March, 1883	23.61	....	0.57	....	7.65	57	90	....	29.92	59.4	59.4	20	240	3.16
O			23.61	....	0.57	....	12.37	57	91	....	29.73	59.3	59.3	20	240	1.95
P	Bull engine, double stroke.	July, 1884	†245.7	‡248.72	13.4	†33.26	‡160.8	68.6	112.6	170.0	30.26	55.4	55.4	§12.35	243.6	1.75
Q	Woolf beam engine.	Oct. 1882	24.491	97.814	0.504	1.904	8.341	60.5	107.36	29.63	30.21	67.8	67.84	17.783	284.5	11.95
R			24.491	97.814	0.504	1.904	9.549	60.0	109.2	30.29	30.14	67.2	67.18	17.775	284.4	10.23
S	Woolf beam engine.	January and February, 1884	11.877	48.2	0.406	1.656	3.02	45.7	76.72	17.77	30.4	78.0	78.0	28.305	368	16.5
T			11.877	48.2	0.406	1.656	3.732	45.75	81.31	19.87	30.4	76.0	76.0	28.16	366.1	13.36
U			11.877	48.2	0.406	1.656	3.791	45.5	80.02	21.25	30.06	75.0	116.0	27.48	357.2	13.15

* Strokes. † Steam side. ‡ Exhaust side. § Double strokes.

TABLE XVII.—*Continued.*

	Horse-power.		Percentage of priming in steam.	Percentage of steam passed through jackets.	Percent'ge of wat'r in cylinder.			Pounds of dry steam per horse-power per hour.	Thermal units per h. p. per minute, calculated from 32° F.	Maximum theoretical efficiency.	Actual efficiency.	Ratio of actual to theoretic efficiency.	Weights of steam per stroke.				Heat equivalent of work, thermal units.		
	Indicated.	Absolute.			At cut-off, h. p. cylinder.	At end of stroke, h. p. cylinder.	At end of stroke, l. p. cylinder.						Moist steam passing through cylinders.	Condensation in steam-jackets.	Caught in clearance, h. p. cylinder.	Caught in clearance, l. p. cylinder.	During admission, h. p. cylinder.	During expansion in h. p. cylinder.	During exhaust from h.p. cylinder.
	17	18	19	20	21	22	23	24	24*a*	25	26	27	28	29	30	31	32	33	34
A	101.79	....	4	4	43.8	28.4	....	22.06	429.11	0.281	0.099	0.35	1.2798	0.0540	0.01440	....	55.16	117.00	21.56
B	75.9	...	8½	None	51.1	35.1	38.8	26.62	519.5	0.254	0.082	0.32	1.01059	None	0.01800	0.03070	36.13	39.61	20 33
C	75.2	....	2¼	9.9	40 8	18.1	8.7	17.34	338.8	0.276	0.126	0.46	0.51987	0.05713	0.02200	0.03000	21.89	38.48	15.58
D	267.86	....	2½	7.1	37.6	24.3	17.5	17.73	348.1	0.340	0.122	0.36	1.13074	0.08436	0.08109	0.01212	58.88	57.89	25.44
E	267.94	....	2½	8.3	37.7	21.5	14.3	17.39	341.8	0.339	0.125	0.37	1.08574	0.09284	0.08109	0.01212	60.53	56.47	26.37
F	267.91	....	2½	None	34.0	28.0	35.2	19.24	378.1	0.341	0.113	0.33	1.26834	None	0.07531	0.01716	73.23	50.68	24 3
G	120.30	....	4	None	42.6	25.4	26.3	21.65	425.2	0.281	0.101	0.36	0.27818	None	0.03450	0.02178	13.41	19.07	9.89
H	149.75	....	4	None	43.6	25.5	26.3	20.88	410.2	0.268	0.104	0.39	0.32978	None	0.04146	0.02178	14.87	23.88	12.08
I	127.4	....	4	13.2	34.1	22.3	19 9	14.84	291.0	0.305	0.147	0.48	0.60768	0.08925	0.05913	0.01680	37.49	54.61	31.47
K	146.02	....	3	4.5	....	....	....	24.15	469.5	0.257	0.091	0.35	4.52194	0.20773	....	....	....	....	
L	123.4	147.1	1	4.94	29.3	23.4	....	22.06	430.0	0.265	0.099	0.373	1.08732	0.05555	0.03241	....	64.4	94.4	20.1
M	123.6	154.3	1	None	37.0	27.5	....	26.46	515.9	0.251	0.083	0.33	1.35589	None	0.04835	....	70.2	92.6	24.2
N	119.5	131.5	1	4.9	22.0	17.7	....	21.3	416.3	0.268	0.102	0.38	1.0296	0.0528	0.0197	....	68.0	72.6	10.9
O	122.4	138.1	1	4.4	14.8	14.4	....	23.6	460.7	0.267	0.093	0.348	1.1725	0.0528	0.0231	....	96.9	50.6	14.3
P	174.6	220.4	1	4.1	16.7	14.7	22.8	32.02	624.3	0.233	0.068	0.296	7.3590	0.3158	0.7887	0.2827	*602.7	*236.5	*381.4
Q	212.98	247.3	3	13.4	25 0	13.5	1.5	15.65	306.3	0.253	0.139	0.549	1.42960	0.22211	0.09552	0.06871	91.8	105.1	51.8
R	214.5	271.1	3	11.0	24.3	13.4	2.64	16.58	324 2	0.250	0.132	05.28	1.55950	0.19281	0.17358	0.18042	106	105.1	72.2
S	132.8	151.6	1	11.3	31.0	13.4	15.6	15.51	304.3	0.303	0.140	0.462	0.55726	0.07110	0.09254	0.02333	34.5	53.	27.1
T	171.9	190.6	1	9.7	29.7	16.3	17.4	15.36	301.1	0.295	0.142	0.481	0.82781	0.07838	0.08380	0.02333	41.1	57.6	30.4
U	171.2	190.2	1	10.2	28.9	15.0	12.4	15.12	296.3	0.296	0.144	0.487	0.72971	0.08286	0.08360	0.02275	45.4	56.9	26.6

* Steam side.

TABLE XVII.—*Continued.*

	Heat equivalent of work, thermal units.						Internal heat of mixture in cylinder.							Heat passing through engine per stroke, thermal units.			
	During compression in h.p. cylinder.	Absolute in l. p. cylinder.	During exhaust from l. p. cylinder.	During compression, l. p. cylinder.	Indicated work in h. p. cylinder.	Indicated work in l. p. cylinder.	At cut-off, h. p. cylinder.	At exhaust, h. p. cylinder.	At exhaust, l. p. cylinder.	At beginning of compression, h. p. cylinder.	At end of compression, h. p. cylinder.	At beginning of compression, l. p. cylinder.	At end of compression, l. p. cylinder.	From boiler, dry steam.	From boiler, priming.	Condensation in jacket of h. p. cylinder.	Condensation in jacket of l. p. cylinder.
	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
A	1.80	....	....	....	148.80	....	915.60	1035.30	....	12.88	15.32	....	....	1431.0	13.2	49.60	....
B	1.94	56.98	16.90	2.59	53.47	37.49	683.60	799.70	737.30	19.27	19.45	26.44	32.95	1083.05	22.20	None	None
C	1.49	53.27	12.18	2.49	43.30	38.60	408.08	496.23	539.87	15.21	23.83	21.93	31.88	594.0	3.4	21.0	30.80
D	7.94	98.23	10.84	1.05	83.39	86.34	951.81	1059.03	1036.67	87.02	59.61	9.05	12.84	1299.12	8.09	30.0	45.23
E	7.65	95.13	9.88	0.88	82.98	48.37	913.1	1047.8	1026.33	87.02	51.10	8.44	10.81	1248.27	7.84	33.0	49.32
F	8.55	89.26	13.34	1.10	91.06	74.82	1092.7	1134.6	953.67	80.75	61.06	13.88	18.30	1458.41	9.21	None	None
G	2.44	22.12	8.64	1.67	20.14	11.82	227.3	264.93	244.29	37.07	27.71	20.88	23.26	314.73	3.20	None	None
H	2.76	26.67	9.47	1.84	23.91	15.36	268.98	315.77	287.19	44.53	24.25	20.86	23.26	373.05	3.77	None	None
I	7.14	72.69	12.03	0.60	53.49	60.06	540.80	583.70	546.82	63.54	33.36	10.60	17.89	686.06	6.76	32.43	47.66
K	....	....	....	....	486.07	....	....	....	....	....	....	....	....	5115.10	33.34	191.18	....
L	4.9	....	....	....	129.8	....	937.5	956.8	....	32.8	33.6	....	....	1253.7	2.8	50.1	....
M	8.2	....	....	....	130.4	....	1089.1	1158.8	....	49	41.7	....	....	1570.2	3.5	None	....
N	1.9	....	....	....	127.8	....	947.3	959 9	....	16.7	20.3	....	....	1193.6	2.7	47.5	....
O	2.4	....	....	....	130.8	....	1143.7	1132.6	....	20.8	23.8	....	....	1359.3	3.1	47.5	....
P	*45.8	†351.2	†155.4	†3.3	412.0	192.5	7548.0	*7633.1	†6550.5	*843.1	*723.3	†270.0	†299.4	852.4	19.0	145	140
Q	11.	162.9	32.1	9.1	134.1	121.7	1350.7	1466.5	1573.2	106.8	75.4	72.4	63.1	162.8	11.6	96.8	97.8
R	16.4	203.2	35.3	32.6	122.5	135.2	1541.7	1669.7	1812 0	187.5	110.5	190.7	136.1	1774.4	12.6	70.7	98.2
S	9.3	63.4	12.8	1.4	57.1	49 2	543.3	622.0	529 6	99.7	79.9	24.6	20:2	649.1	1.6	25.4	37.7
T	8.	81.3	12.8	1.4	63.3	67.1	688.3	759.2	675.4	90.2	75.2	24.6	24.3	847.4	2.	24.9	44.7
U	8.	80.2	13.3	1.4	67.7	65.5	696.5	773.0	712.6	92.6	76.6	23.1	24.2	849.4	2.	27.9	43.9

* Steam side. † Exhaust side.

TABLE XVII.—*Concluded.*

	Heat passing through engine per stroke, thermal units.							Interchange of heat between the steam and the walls of the cylinders, per cent.											
	Total heat applied.	Heat retained in condensed steam.	Heat absorbed by injection-water.	Heat equiv. of indicated work.	Radiation, h. p. cylinder.	Radiation, l. p. cylinder.	Error.	Absorb.d'r'g admission, h. p. cylin. Q_a	Restored during expansion, h. p. cylin. Q_b	Rejected during exhaust, h. p. cylinder. Q_c	Absorbed during compression, h. p. cylin. Q_d	Given up during expansion, l. p. cylin. $Q_{b'}$	Rejected during exhaust, l. p. cylinder. $Q_{c'}$	Absorbed during compression, l. p. cylin. $Q_{d'}$	Radiation, h. p. cylin. Q_e	Radiation, l. p. cylinder. $Q_{e'}$	Supplied by jacket, h. p. cylinder. Q_j	Supplied by jacket, l. p. cylinder. $Q_{j'}$	Error.
	52	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
A	1493.8	....	....	....	15.0	....	....	33.8	16.4	19.8	−0.04	....	....	....	1.0	....	3.4	....	...
B	1105.25	67.27	922.68	90.96	6.9	10.0	7.44	39.0	15.0	23.5	0.2	−3.8	25.4	−0.4	0.7	1.0	None	None	0.7
C	649.2	27.8	508.9	81.9	6.9	10.0	13.7	30.8	20.4	11.5	−1.1	10.4	3.3	−1.2	1.1	1.6	3.4	4.7	2.2
D	1382.44	....	....	....	7.0	10.0	....	26.2	12.1	18.3	2.6	9.2	11.5	−0.2	0.5	0.7	2.2	3.3	....
E	1338.43	....	....	....	7.0	10.0	....	25.4	14.5	16.1	3.3	9.4	9.6	−0.1	0.5	0.7	2.5	3.7	....
F	1467.62	....	....	....	7.0	10.0	....	25.2	6.4	20.2	2.0	−3.7	22.4	−0.2	0.5	0.7	None	None	....
G	317.95	18.51	256.32	31.96	1.3	1.9	7.96	35.6	19.3	20.0	4.0	1.8	17.2	−0.7	0.4	0.6	None	None	2.7
H	376.82	24.91	294.87	39.27	1.6	2.4	13.77	36.1	20.1	19.3	3.7	2.1	16.4	−0.4	0.4	0.7	None	None	3.9
I	772.91	25.16	583.83	113.55	7.0	15.72	27.65	19.8	13.0	15.1	5.0	6.7	11.8	−4.5	0.9	2.1	4.3	6.4	3.7
K	5339.62	245.08	4465.85	486.07	48.32	...	94.30	....	....	....	....	....	....	....	0.9	...	3.7	....	1.8
L	1306.6	62.8	...	129.8	8.8	....	....	23.2	8.8	18.0	0.3	....	....	....	0.7	....	4.0	....	....
M	1573.7	92.2	....	130.4	8.8	....	....	30.8	11.0	20.3	0.8	....	....	....	0.6	....	None	....	....
N	1243.8	59.7	....	127.8	7.0	....	....	17.0	7.2	13.1	0.14	....	....	....	0.6	....	4.0	....	....
O	1409.9	69.2	....	130.8	7.0	...	....	10.9	2.9	10.9	0.04	....	....	....	0.5	....	3.6	....	....
P	8828	603.5	7480	604.5	44.0	44.0	52	13.6	3.9	12.9	2.0	−6.9	20.7	−0.3	0.5	0.5	1.8	1.7	0.6
Q	1834	107.7	1388.5	255.8	14.0	24.0	44.2	15.8	12.8	10.2	2.5	19.1	0.4	1.1	0.8	1.4	5.6	5.7	2.5
R	1955.9	111.6	1490.1	257.7	14.0	24.0	58.5	13.5	12.6	9.1	5.1	17.6	0.2	4.8	0.8	1.3	3.8	5.3	3.2
S	713.8	24.9	551.2	100.3	4.5	7.5	25.4	22.2	18.7	10.3	4.2	3.4	12.2	0.9	0.6	1.1	3.7	5.5	3.7
T	919	35.9	706.5	130.4	4.5	7.5	34.2	21.8	14.6	12.1	2.6	3.7	12.8	0.2	0.5	0.8	2.8	5.1	3.9
U	923.2	35.0	733.5	133.2	4.5	7.5	9.5	20.9	15.0	11.3	2.7	6.9	8.5	0.03	0.5	0.8	3.1	4.9	1.1

Woolf engine, of a type very commonly used in factories in Lancashire. The cylinders were $15\frac{3}{4}$ and $28\frac{1}{2}$ inches in diameter by 4 feet 3 inches stroke. The piston speed was about 680 feet per minute and the load was light, so that the steam was much wire-drawn. Three trials were made, during which the boiler-feed and air-pump discharge were measured; the first test was not reported, as the engine was stopped during the test.

The test I was made on a compound beam receiver engine with the cranks at right angles, working pumps directly from the beams. The cylinders were 21 and 36 inches in diameter, and the stroke was 5 feet 6 inches. Both cylinders, with the exception of the high-pressure cylinder and the receiver-covers, were jacketed with boiler steam. The steam was distributed by slides, one at each end of each of the cylinders, with cut-off plates adjustable by hand. A jet condenser was used, and the air-pump discharge was measured. Two tests were made, giving the same result, so that only one was reported.

The test K was made on a Cornish engine working a single-acting piston pump direct from the beam, and having the usual Cornish, double-beat, steam, equilibrium, and exhaust-valves, a single-acting air-pump, and a jet condenser; the cylinder was $68\frac{1}{4}$ inches in diameter by 8 feet stroke, and was jacketed on the sides with boiler steam. The pump delivered its water on the up or steam stroke, so that the preponderance of weight on the pump-pole was only enough to overcome the suction lift. The valves and piston were inspected to assure their tightness before the test. The engine was doing the highest duty at the West Middlesex Waterworks, and was taken as one of the best engines of its type now working. This type of engine was developed in Cornwall, where it was used to pump water from deep mines by a pump-rod hung directly from one end of the beam while the piston was hung from the other end of the beam. It had no fly-wheel, but the pump-rod, beam, and counter-weights made in the aggregate a large reciprocating mass, that absorbed work during the first part of the stroke when the steam-pressure in the cylinder was high, and restored that work and assisted the steam to complete the stroke after it has lost pressure through expansion, during the latter part of the stroke. Such a reciprocating mass is essential to the proper action of the engine with a good degree of expansion. The pumps of the original engines were worked by the weight of the rods during the return or equilibrium stroke, at which time there was free communication between the two ends of the cylinder. The lower end of the cylinder was open to the condenser during the steam-stroke.

The tests L and M were made on a single-cylinder beam rotative pumping-engine, having a diameter of 32 inches and a stroke of 5 feet 6 inches. The cylinder sides and base were jacketed with boiler steam. Steam was distributed by slide-valves at the top and bottom of the cylinder, with cut-off plates, adjustable by hand, on the backs of the main valves. There was a jet condenser, but the air-pump discharge could not be measured.

The tests N and O were made on a single-cylinder beam rotative engine, similar to the one just described, and taking steam from the same boilers. The cylinder was 27 inches in diameter, by 6 feet stroke.

The test P was made on a Bull engine with a cylinder 68 inches in diameter by 10 feet stroke, driving direct a 45-inch plunger-pump, and forcing water to a height of 40 to 55 feet. The valves and gear were of the usual Cornish pattern, and the sides and base of the cylinder were steam-jacketed. This type of engine differs from the Cornish engine in not having a beam, and though the pump-rod is loaded there is seldom sufficient reciprocating mass to allow of much expansion. In the case of the engine tested only $1\frac{3}{4}$ expansions could be obtained. For convenience, the steam-stroke is detailed under the heading of the high-pressure cylinder, and the exhaust-stroke under the heading of the low-pressure cylinder.

The tests Q and R were made on a Woolf beam rotative engine, working a double-acting pump. The cylinders were 29 inches diameter by 5 feet 5 inches stroke, and $47\frac{1}{2}$ inches diameter by 8 feet stroke, and jacketed with steam on the sides and ends. Steam was distributed by slide-valves with adjustable cut-off plates to the high-pressure cylinder; the exhaust-valves are double-beat valves worked by cams.

The interchanges of heat between the steam and the walls of the cylinder in these tests are calculated by a process that is equivalent to that indicated by equations (256) to (259). To make the matter clear, and to explain some of the headings of Table XVII, the entire calculations for the test I are given here.

The engine was compound, steam-jacketed, with an intermediate receiver, and ran at an average speed of 23.98 revolutions per minute and expanded the steam 13.61 times. The absolute boiler-pressure was 76 pounds, and the indicated horse-power 127.4. The air-pump discharge was 25.558 pounds per stroke, the initial and final temperatures being 50°.0 and 73°.4 F.

The weights of water and steam per stroke were:

		Pounds.
Boiler delivery, ...		0.69693
Steam through cylinders, ...	$Mx =$	0.58338
Priming, ...	$M(1 - x) =$	0.02430
Condensation in jackets, ...	$M_j =$	0.08925
Injection-water, ...	$G =$	24.95

The heat brought into the high-pressure cylinder of the engine per stroke is

$$Q = Mx\lambda + M(1 - x)q = 0.58338 \times 1176 + 0.0243 \times 278.58$$
$$= 686.06 + 6.76 = 692.82 \text{ B. T. U.},$$

in which λ is the total heat and q the heat of the liquid corresponding to the pressure of the entering steam. Let r be the heat of vaporization at the same pressure, then the heat supplied by the steam-jackets is

$$Q_j + Q_j' = 0.08925 \times 897.45 = 80.09 \text{ B. T. U.}$$

The total amount of heat delivered to the engine per stroke is

$$Q + Q_j + Q_j' = 772.91 \text{ B. T. U.}$$

The heat retained by the condensed steam at the temperature t_4 of the air-pump discharge is

$$M(t_4 - 32) = 0.60768(73.4 - 32) = 25.16 \text{ B. T. U.}$$

So that the heat used by the engine per stroke was

$$772.91 - 25.16 = 747.75 \text{ B. T. U.}$$

The thermal units per horse-power per minute, calculated from 32° F., was

$$\frac{772.91 \times 2 \times \text{revolutions}}{\text{H. P.}} = \frac{772.91 \times 2 \times 23.98}{127.4} = 291.0 \text{ B. T. U.}$$

The number of pounds of dry steam consumed per horse-power per hour was

$$\frac{772.91 \times 2 \times \text{revolutions} \times 60}{\text{H. P.} \times \lambda} = \frac{772.91 \times 2 \times 2398 \times 60.}{127.4 \times 1176}$$

$$= 14.84 \text{ pounds.}$$

The actual number of pounds of moist steam used per horse-power per hour was

$$\frac{0.69693 \times 2 \times 23.98 \times 60}{127.4} = 15.7 \text{ pounds.}$$

Both of these results show exceptionally high economy of the use of steam at an absolute pressure of 76 pounds.

The density of the steam in the cylinder at different points of the stroke was calculated by aid of an adaptation of Zeuner's * formula

$$\gamma = 0.6061 p^{0.9393},$$

which for English units may be written

$$\log \gamma = 0.9393 \log p - 2.51853.$$

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* Mechanische Wärmetheorie, page 294.

At cut-off in the high-pressure cylinder, for example, the absolute pressure was $p_1 = 64$ pounds; and the density, or weight in pounds of one cubic foot, of dry steam at this pressure, is by the formula $\gamma_1 = 0.1506$ pounds. The volume of steam at cut-off, allowing for clearance, was 2.9161 cubic feet; hence the weight of dry steam at cut-off was

$$2.9161 \times 0.1506 = 0.4392 \text{ of a pound.}$$

By a similar calculation the weight of dry steam caught at the beginning of compression was found to be 0.05913 of a pound. This added to the weight of moist steam per stroke gives for the weight of the mixture in the cylinder

$$0.60768 + 0.05913 = 0.66681 \text{ of a pound.}$$

Consequently the per cent of water in cylinder at cut-off was

$$100 \times \frac{0.66681 - 0.4392}{0.66681} = 34.1 \text{ per cent.}$$

The heat equivalents of the intrinsic energy of the mixture in the cylinder at cut-off, release, compression, and admission are given by the equations

$$I_1 = (M + M_0)(x_1\rho_1 + q_1); \quad . \quad . \quad . \quad . \quad (294)$$
$$I_2 = (M + M_0)(x_2\rho_2 + q_2); \quad . \quad . \quad . \quad . \quad (295)$$
$$I_3 = M_0(x_3\rho_3 + q_3); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (296)$$
$$I_0 = M_0(x_0\rho_0 + q_0); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (297)$$

in which M is the weight of moist steam through the cylinders per stroke, and M_0 is the weight of steam caught at compression, on the assumption that the steam is then dry and saturated; and x is the part of one pound of the mixture that is steam; and ρ and q are the heat equivalent of intrinsic energy and the heat of the liquid, at the several points mentioned.

For the high-pressure cylinder Mr. Mair writes

$$Q_a = Q + I_0 - I_1 - AW_a; \quad . \quad . \quad . \quad . \quad . \quad (298)$$
$$Q_b = I_2 - I_1 + AW_b; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (299)$$
$$Q_c = Q_a + Q_d + Q_j - Q_b - Q_e; \quad . \quad . \quad . \quad (300)$$
$$Q_d = I_3 - I_0 + AW_b; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (301)$$

The equation for Q_c is obtained by aid of the equation

$$Q_a + Q_d + Q_j = Q_b + Q_c + Q_e, \quad . \quad . \quad . \quad . \quad (302)$$

which asserts that the heat absorbed by the cylinder walls during admission and compression, together with the heat given up by the steam in the high-pressure jacket, is equal to the heat yielded during expansion and exhaust, *plus* the heat lost by radiation. For the low-pressure cylinder he gives the equation

$$Q_c + Q_d' + Q_j' = Q_b' + Q_c' + Q_e', \quad . \quad . \quad . \quad (303)$$

which asserts that the low-pressure cylinder walls receive the heat Q_c rejected from the high-pressure cylinder during exhaust, the heat absorbed by the wall during compression, and the heat yielded by the steam condensed in the low-pressure jackets, and that this heat is equal to that yielded by the walls during expansion, during exhaust, and by external radiation. The quantity Q_b' is that complex quantity described on page 222, and it is assumed that it applies to the whole stroke of the low-pressure piston, both before and after cut-off. For the low-pressure cylinder he gives

$$Q_b' = I_2' - (I_2 - I_3 + I_0' + AW_c) + AW_b'; \quad . \quad (304)$$
$$Q_c' = Q_c + Q_d' + Q_j' - Q_b' - Q_e'; \quad . \quad . \quad . \quad . \quad . \quad (305)$$
$$Q_d' = I_3' - I_0' + AW_d'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (306)$$

Equation (306) is of the same form as equation (250), and equation (305) is obtained from equation (261). To find Q_b' it is assumed that the difference between the heat equivalents of the intrinsic energy at release and compression in the high-pressure cylinder is thrown into the low-pressure cylinder, together with the heat equivalent of the absolute work during exhaust from that cylinder, and that to this sum is to be added the heat equivalent of the intrinsic energy of the steam at the end of compression in the low-pressure cylinder.

For the test I we have

$$Q_a = 692.82 + 33.36 - 540.80 - 37.49 = 147.89 \text{ B. T. U.};$$
$$Q_b = 583.70 - 540.80 + 54.61 = 97.51 \text{ B. T. U.};$$
$$Q_d = 63.54 - 33.36 + 7.14 = 37.32 \text{ B. T. U.};$$
$$Q_c = 147.89 + 37.32 + 32.43 - 97.51 - 7.00 = 113.13 \text{ B. T. U.};$$
$$Q_b' = 546.82 - (583.70 - 63.54 + 17.89 + 31.47) + 72.69;$$
$$= 49.99 \text{ B. T. U.};$$
$$Q_d' = 10.60 - 17.89 + 0.60 = -6.69 \text{ B. T. U.};$$
$$Q_c' = 113.13 - 6.69 + 47.66 - 49.99 - 15.72 = 88.39 \text{ B. T. U.}$$

The engine on which test I was made had a jacketed receiver, and in this calculation as well as in Table XVII the condensation in the receiver jacket is added to that in the low-pressure jacket, and the radiation from the receiver is added to that from the low-pressure cylinder.

In the table these several quantities of heat are stated in percentages of the heat used by the engine per stroke, that is, of

$$Q + Q_j + Q_j' - Mq_4,$$

to facilitate the comparison of tests made on different engines.

Discussion of Results.—The effect of a steam-jacket on a single-cylinder engine may be seen by comparing the tests L and M. With a loss of only one pound boiler-pressure, it was found necessary to reduce the expansions from 4.33 to 3.84, in order to obtain the same power without the jacket in M as with the jacket in L. Comparing the B. T. U. per horse-power per minute, the gain from the use of the steam-jacket, and the greater expansion that could then be used, is

$$100 \times \frac{515.9 - 430.0}{515.9} = 16 \text{ per cent.}$$

Comparing the interchanges of heat between the walls of the cylinder, it appears that the use of the steam-jacket reduces the heat Q_c rejected during exhaust, and consequently the initial condensation, as shown by Q_a, while the external radiation is a little larger with a jacket than without.

Comparing the tests B and C made with and without steam-jackets on a compound Woolf engine, there appears to be a gain of

$$100 \times \frac{519.5 - 338.8}{519.5} = 34 \text{ per cent}\cdot$$

but this difference is largely due to the fact that in the test C the steam-pipe was well drained, and in the test B it was not. Comparing the tests E and F, the gain from the use of the steam-jackets and of the larger expansion then possible is

$$100 \times \frac{378.1 - 341.8}{378.1} = 10\tfrac{1}{2} \text{ per cent,}$$

which would probably be increased were the ends of the cylinders jacketed as well as the sides. In this case, with the larger degree of expansion accompanying the use of the jackets, it appears that the initial condensation in the high-pressure cylinder is not greatly affected, and Q_a is nearly the same in both tests; but the heat restored during the expansion in the high-pressure cylinder Q_b, and in the low-pressure cylinder $Q_{b'}$, are both increased, so that Q_c', the exhaust waste, is reduced from 22.4 per cent to 9.6 per cent.

The steam consumption in the test I is remarkably low, showing an excellent adaptation of the engine to its work. It is interesting to compare the interchanges of heat in this test with those in the tests T and U, which give nearly as good an economy, and with other tests giving a poorer economy. It is noteworthy that the exhaust waste Q_c' for the tests of compound engines given in Table XVII cannot be regarded as a measure of the economy of the engine, neither can that engine be said to be working under the best conditions which shows in general the smallest interchanges of heat between the steam and the walls of the cylinder. The best result appears rather to be attained by a judicious or fortunate compromise of the gain from expansion and the loss from condensation and evaporation, and of the amelioration of the latter by the use of steam-jackets in which heat is usefully applied for that purpose, though with a loss of thermodynamic efficiency.

Institute of Technology Tests.—The data and results of tests made on a Harris-Corliss engine in the laboratory of the Massachusetts Institute of Technology are given in Table XVIII. This table is given in part to afford examples to which the equations for Hirn's Analysis on page 192 may be applied by the student.

The engine is an automatic cut-off unjacketed engine using saturated steam. The stroke is 24 inches and the diameter is 8 inches. During the tests the governor was disconnected, the cut-off was fixed, and the speed was regulated by another engine coupled with this engine. Both the condensed steam and the cooling water from a surface condenser were weighed in tanks.

Water in the Cylinder.—As a conclusion of his analysis applied to his own and to Hallauer's engine tests, Hirn * concludes that all the theories of the steam-engine proposed by writers on thermodynamics, based on the hypothesis that the interaction between the steam and the walls of the cylinder is inconsiderable, are liable to be in error to the extent of 50 per cent, and are consequently entirely useless and misleading. In reviewing these experiments and the conclusions from them, Zeuner † developed the equations (256) to (259) substantially as given on page 189; and, in addition to pointing out that the possible effect of eddies and mechanical motion of the steam in general had been neglected, and that the method of calculation given on page 307 is inaccurate, he called attention to the fact that a thin layer of water adhering

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* Théorie mécanique de la Chaleur, vol. ii. p. 68.

† Civil-ingenieur, xxvii.

TABLE XVIII.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY TESTS.

Number.	Duration, minutes.	Total number of revolutions.	Weight of steam used, pounds.	Weight of condensing-water, pounds.	Per cent of priming in steam. $1-x$	Temperatures, degrees Fahrenheit. Condensed steam. t_4	Condensing-water, cold. t_i	Condensing-water, warm. t_k	Barometer, pounds on the square inch.	Boiler-pressure by gauge, pounds. p	Absolute pressures, pounds. Cut-off. p_1 C. E.	H. E.	Release. p_2 C. E.	H. E.	Compression. p_3 C. E.	H. E.	Admission. p_0 C. E.	H. E.
1	30	1839	143	3853	2.	118.6	39.2	74.3	14.8	68.7	77.1	76.3	16.3	16.8	14.8	14.8	24.8	30.8
2	60	3609	327	10077	2.	130.5	54.8	86.0	14.7	73.0	83.7	82.7	17.7	19.7	14.7	14.7	21.7	25.7
3	30	1839	170	4294	2.	118.0	39.2	77.0	14.9	68.5	74.2	77.5	18.6	20.3	14.9	14.9	20.9	24.9
4	45	2672	295	8881	2.	132.8	53.6	132.8	14.8	72.9	77.1	80.5	21.8	23.8	14.8	14.8	23.8	34.8
5	60	3692	548	14568	2.	141.1	52.9	88.3	14.8	77.4	83.6	83.3	29.2	31.9	14.8	14.8	21.8	29.8

Number.	Events of the stroke—per cent of stroke from beginning. Cut-off. C. E.	H. E.	Release. C. E.	H. E.	Compression. C. E.	H. E.	Heat equivalents of work. B. T. U. Admission. AW_a C. E.	H. E.	Expansion. AW_b C. E.	H. E.	Exhaust. AW_c C. E.	H. E.	Compression. AW_d C. E.	H. E.	Interchanges of heat. B. T. U. Heat in M pounds of moist steam. Q	Absorbed by walls during admission. Q_a	Yielded by walls during expansion. Q_b	Yielded by walls during exhaust. Q_c	Absorbed by walls during compression. Q_d	Radiation and error. Q_e
1	10.0	9.0	100	100	2.2	7.7	1.027	0.979	3.119	3.284	1.817	1.773	0.052	0.248	45.12	20.94	12.26	6.43	0 40	2.65
2	12.2	12.2	100	100	1.3	4.6	1.305	1.386	3.285	3.783	1.822	1.820	0.035	0.114	52.61	21.81	11.27	8.99	0.11	1.66
3	15.0	15.0	100	100	0.9	3.4	1.471	1.633	3.202	3.564	1.854	1.868	0.022	0.099	53.60	21.20	11.13	7.43	−0.19	2.45
4	19.5	19.5	100	100	1.5	4.0	2.108	2.217	3.527	3.929	1.830	1.844	0.039	0.116	64.09	23.59	10.55	9.29	−0.32	3.43
5	30.6	32.0	100	100	1.3	3.9	3.369	3.711	3.877	4.159	1.836	1.847	0.0299	0.1104	86.9	25.13	7.11	14.4	−0.122	3.5

to the walls of the cylinder and participating in the changes of temperature, would account for the larger part of the disturbances attributed to the action of the metallic walls of the cylinder. It consequently becomes of great importance to determine M_0, the weight of the mingled water and steam caught in the cylinder at compression. The effect of the mechanical motion of the steam in the cylinder during expansion cannot be considerable, and the errors of the method of calculation employed are insignificant.

To investigate the probable value of M_0, Hallauer* recalculated a number of the tests given in Tables XII and XIII under various assumptions. For the test of Sept. 8, 1875, he finds that the value of Q_c, the exhaust waste, is 37.02 calories, as given in Table XIII, when calculated on the assumptions that the steam caught in the cylinder at compression and the work of compression can both be neglected. Assuming that the steam caught at compression is dry and saturated, and allowing for the work of compression, he finds that $Q_c = 38.52$ calories. The weight of steam caught at compression he finds to be 0.00432 kilogram under this supposition. He then assumes that just enough water is present in the cylinder at compression to give the same intrinsic energy for the mixture at the beginning and end of compression, calculated from the known volumes and pressures at those points. The weight of the steam and water in the cylinder during compression is 0.05082 kilogram on this supposition, and the exhaust waste is $Q_c = 38.17$ calories. The differences of the three values of Q_c thus determined are less than the probable error of the experiment. It should be noted that the weight of steam exhausted from the engine per stroke was 0.2634 kilogram, and that the moisture in this steam was 10.63 per cent, as determined from the initial and final temperatures of the injection-water.

Finally, Hallauer finds that if it be admitted that the weight of water and steam caught in the cylinder at compression is equal to that exhausted per stroke,—an assumption which

* Bulletin de la Soc. Ind. de Mulhouse, vol. lv. 1881.

Zeuner claims to be reasonable, and which would require only a thin layer of water adhering to the cylinder walls,—then he finds that the walls of the cylinder must have yielded 12.22 calories during compression; an amount which may be considered absurd. At the same time he finds that this particular assumption makes $Q_c = 42.66$ calories, or that it involves a larger exhaust waste.

If it be assumed that the interchange of heat between the steam and the walls of the cylinder during compression is inconsiderable in the test of Nov. 28, 1873, Hirn * finds that the equations given by Zeuner give for the interchange of heat between the steam and the walls of the cylinder

$$Q_a = 52.61 \text{ calories} - 68.59G_0;$$
$$Q_b = 21.07 \text{ calories} - 43.29G_0;$$
$$Q_c = 31.5 \text{ calories} - 25.3G_0;$$

the several values of Q being successively for admission, expansion, and exhaust, and G_0 being the weight of water and steam caught in the cylinder at compression, which is assumed to be unknown so that there are four unknown quantities with three equations.

If it be assumed that the steam is dry and saturated at compression, then for this test $G_0 = 0.00112$ kilogram, and

$$Q_a = 52.53 \text{ calories};$$
$$Q_b = 21.02 \text{ calories};$$
$$Q_c = 31.47 \text{ calories}.$$

On the other hand, if it may be assumed that any one of the three quantities Q_a, Q_b, or Q_c is inconsiderable, and can be neglected, then G_0 may be calculated directly. First suppose that Q_b is zero, or that there is no interchange of heat during expansion; then

$$G_0 = 0.487 \text{ kilogram};$$
$$Q_a = 19.21 \text{ calories};$$
$$Q_c = 19.18 \text{ calories}.$$

* Bulletin de la Soc. Ind. de Mulhouse, vol. lii., 1882.

Now the weight of vapor exhausted per stroke is 0.3732 kilogram, so that the assumption requires that the weight of water in the clearance shall exceed the weight of steam exhausted. But while this condition is supposable, it is impossible that the walls of the cylinder should *receive* heat from the steam during exhaust, as is indicated by the positive sign.

If it be assumed that there is no exhaust waste, that is, that $Q_c = 0$, then

$$G_0 = 1.245 \text{ kilograms};$$
$$Q_a = - 32.78 \text{ calories};$$
$$Q_b = + 32.78 \text{ calories};$$

which would require that the walls during admission should yield heat to the entering steam, and that they should absorb heat during expansion.

The conclusion is inevitable, that there is an energetic interchange of heat between the steam and the walls of the cylinder. It also seems probable that there cannot be great error in assuming that the steam is dry and saturated at compression.

CHAPTER XVIII.

VARIOUS STEAM-ENGINE TESTS.

Tests on Donkin Engines.—A large number of efficiency tests were made by Mr. Bryan Donkin, Jr., and Mr. Salter on an engine built for the purpose by Messrs. B. Donkin & Co.,* in which the methods of making the tests and calculations were similar to that outlined on page 242.

The engine was a two-cylinder tandem compound engine; each cylinder was provided with a steam-jacket, which could be supplied with steam in various ways, and the water condensed from each jacket could be drawn off and weighed separately. Each cylinder had the steam distributed by a plain slide-valve, and the small cylinder had a cut-off valve on the back of the main valve.

The heat brought into the cylinder of an engine may be divided into three parts: one part is changed into work, a second part is lost by radiation, and a third part is carried away by the condensing water. In these experiments the part changed into work was determined by taking indicator-diagrams at regular intervals; the power of the engine was also measured by a friction brake. The heat carried away by the condensing water was determined by taking the temperature of the injection-water before it was delivered to the jet-condenser, and the temperature of the mingled condensing water and condensed steam discharged by the air-pump, and by gauging the latter on a weir $1\frac{1}{2}$ inches wide. Allowance was made for the water condensed in the jackets when they were in use. The heat lost by radiation was not determined, but was assumed to

* Engineering, vol. xxi. p. 203; vol. xl. pp. 317 and 342; vol. xlii. pp. 487 and 577.

be small in amount, and nearly if not quite constant. The number of thermal units carried away by the condensing water per indicated horse-power per minute was considered to be the measure of the economy of the engine; that is, it was considered that the most economical method of running the engine was that one in which this quantity was smallest.

The dimensions of the engine were:

Diameter, small cylinder, ...	6 in. minus $\frac{1}{128}$ in.
" large "	10 " " $\frac{1}{64}$ "
Stroke, ...	1 foot.
Piston displacement: small cylinder, . . .	336.4 cu. in.
large " . .	939.1 " "

Clearance, small cylinder:	Cubic inches.	Per cent of small piston displacement.
crank end: clearance ($\frac{5}{8}$ in.),	17.5	5.2
passage,	24.3	7.22
back end: clearance ($\frac{9}{16}$ in.),	17.5	5.2
passage,	24.3	7.22

Clearance, large cylinder:	Cubic inches.	Per cent of large piston displacement.
crank end: clearance, ($\frac{3}{4}$ in.),	67.7	7.2
passage,	26.9	2.86
back end: clearance,	64.2	6.84
passage, ($\frac{5}{8}$ in.),	27.8	2.96
Small-cylinder exhaust passage and large-cylinder steam-chest,	180.5	19.2

The ratio of expansion is calculated as follows: The cubic contents of the low-pressure cylinder + the cover end-clearance and passage + high-pressure exhaust passage and low-pressure steam-chest + the high-pressure cylinder + the front clearance and passage = 1589.8 cubic inches. The cubic contents of the high-pressure front clearance and passage = 41.8 cubic inches.

$$\frac{\text{Total cubic content} = 1589.8}{\text{Cubic content before steam is cut off}} = \text{ratio of expansion.}$$

The experiments to determine the friction of the engine were made with the steam at 40 lbs. pressure in the boiler.

Steam was passed through the jacket of the low-pressure cylinder before it reached the engine, but there was no steam in the jacket of the high-pressure cylinder. The stop-valve was nearly closed, and the governor was connected to the throttle-valve; the cut-off was at about one eighth of the stroke. The brake strap was lifted off the fly-wheel, and when the revolutions were steady three sets of indicator-diagrams were taken. Several experiments were made, and a fair average of the whole, with the speed at 96 revolutions, gave the friction as 1.37 horse-power.

Table XIX gives the data and results of tests made (1) with no steam in either jacket, but with air in both; (2) with

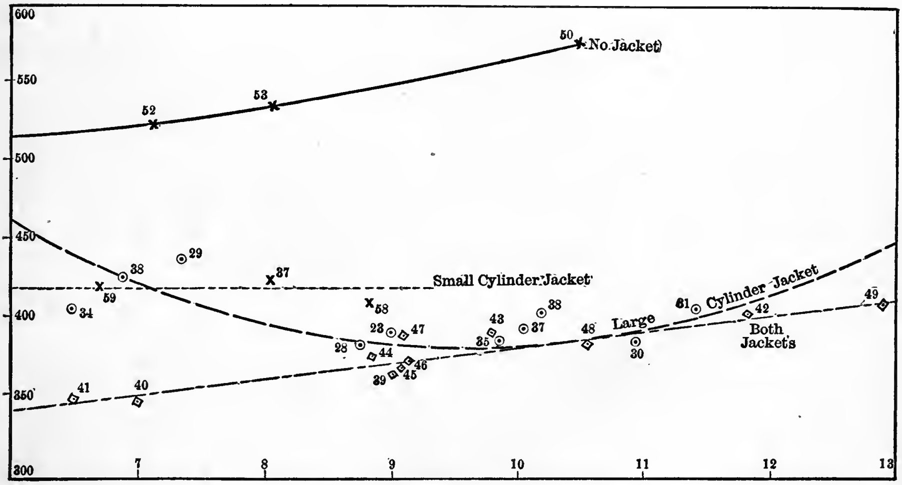

FIG. 62.

steam passed through the jacket of the high-pressure cylinder to the steam-chest of that cylinder, and with air in the low-pressure jacket; (3) with steam in the low-pressure jacket only; (4) with steam passed through the low-pressure jacket, thence through the high-pressure jacket, and thence to the high-pressure steam-chest; (5) with the steam throttled and passed through the low-pressure jacket and thence to the high-pressure steam-chest. The results of the first four series of tests are also represented by Fig. 62, in which curves are drawn with

TABLE XIX.

TESTS ON DONKIN ENGINE.

	Conditions of Test.	Revolutions per minute.	Cut-off, high-pressure cylinder.	Ratio of expansion.	Duration of the experiment, minutes.	Vacuum, inches of mercury.	Horse-power.		Temperature of condensing water.		Pounds of water per min. from condenser.	Pounds of water per minute from jackets.	Thermal units per horse-power p. min.	Kind of lubricant and rate of feeding.
							Indicated.	Brake.	Cold.	Warm.				
56	No jackets on. Boilure-pressure, 43 pds.	98.17	7/16	8.38	30	25.9	7.1	4.91	60.5	103.75	86.25	...	525	None.
53		96.61	½	7.54	30	26.2	8.04	5.79	60.37	104.1	98	...	533	
55		96.18	11/16	5.8	30	16	9.37	6.73	59.5	102.34	135.5	...	619	
54		96.31	15/16	4.45	30	26.1	10.51	7.7	60.75	102.4	145	...	574	
59	High-pressure jacket only. Steam taken from jacket to steam-chest of high-pressure cylinder. Boiler-pressure, 43–44.	97.97	5/16	10.78	30	27	6.72	4.89	59.75	98.42	73	0.475	420	None.
57		97.93	½	7.73	30	27	8.03	5.87	59.25	94.98	94.5	0.52	420	
58		97.5	9/16	7.01	30	27	8.84	6.82	60	95.75	103.5	0.475	418	
38	Low-pressure jacket only. Boiler-pressure, 40 to 41 pds.	102.27	3/16	15.8	30	27.5	6.88	5.11	50.75	87.5	80	0.66	427	None.
91		97.15	5/16	10.78	30	27	7.69	5.83	47	94	66.5	1.75	406	¾ oz. suet per hour.
28		106.8	⅜	9.74	30	27.8	8.72	7.48	48.4	85.6	88.8	0.7	380	
23		104.5	⅜	9.14	30	27.2	9.02	7.31	49.75	101.25	68.25	0.62	388	
35		101.3	7/16	8.38	30	27.6	9.82	8.1	51.5	92.5	92.5	0.64	383	Suet.
30		110.2	½	7.54	30	27.5	10.94	8.82	48	97.3	85.5	0.69	385	
31		106.4	½	7.18	30	27.7	11.44	10.11	51.5	97.23	101.75	0.77	407	
32		99	¾	5.38	30	27.8	12.39	10.89	52	92.6	130.5	0.67	427	
41	Steam passed through low-pressure jacket, thence to high-pressure jacket, thence to valve-chest of high-pressure jacket. Boiler-pressure, 41 to 45 pds.	93.32	¼	13.1	30	27.1	6.5	4.67	52.37	92.18	56.75	0.4	347	None.
40		103.4	¼	13.1	30	26.9	7.01	5.17	54	94.5	60.25	0.56	348	Suet.
44		89.39	7/16	8.38	30	26.8	8.85	6.88	57.37	101.78	74,75	0.23	375	None.
45		99.35	⅜	9.14	30	26.7	9.05	6.95	57.37	102.72	73.25	0.61	367	"
46		99.19	½	7.54	30	26.9	9.08	6.94	57.25	99	82.5	0.7	372	"
48		97.77	9/16	6.7	30	26.6	10.53	8.80	58	100.2	95.75	0.64	385	"
42		97.6	11/16	5.8	30	27.2	11.82	9.76	53.25	89.85	130.65	0.61	405	"
49		97.5	⅞	4.64	30	26.6	12.87	10.7	58.12	100.9	123.5	0.67	410	"
129	Throttling experiments.	97.25	7/16	8.38	30	27.25	8.65	6.81	62.5	93.1	108.06	0.64	382	½ oz. suet per hour.
131	Steam taken through low-pressure jacket to high-pressure steam-chest. Boiler-pressure, 40 to 42 pds.	96.2	7/16	8.38	30	27.5	9.22	6.73	60.25	95.23	105.33	0.62	394	"
127		96.8	½	7.73	30	27.25	9.25	6.77	62.25	96.87	102.52	0.67	384	"
135		96.9	3/16	15.8	30	27	5.79	3.88	59.75	94.7	68.4	0.51	413	"
128		102	None.	....	30	26.4	6.16	4.01	62.37	103.12	68.56	0.58	453	"
132		100.3	"	....	30	26.75	6.23	4.01	61.25	102.6	65.5	0.50	437	"
130		101.2	"	....	30	27.12	6.06	4.05	60.75	104.6	61.37	0.42	444	"
134		103.13	"	....	30	27	5.94	4.12	58.0	95.03	72.2	0.45	450	"

the indicated horse-power as abscissæ and with the number of thermal units per horse-power per minute, carried away by the condensing water, as ordinates. These curves show in a most striking manner the action of steam-jackets on this engine.

In the course of the tests it was found that if an excessive amount of melted suet was fed into the cylinder it formed a non-conducting coat on the wall, and lessened the action of the walls on the steam. Two sets of experiments were made to determine the extent of this effect: in the first, steam was let into the high-pressure jacket only, and suet was fed into the high-pressure steam-chest at regular intervals; in the second set neither jacket was supplied with steam, and suet was fed into the high-pressure steam-chest. The data and results of these tests are shown in Table XX, together with the data and results of tests made by feeding water into the low-pressure cylinder. The results of feeding in suet are also shown by

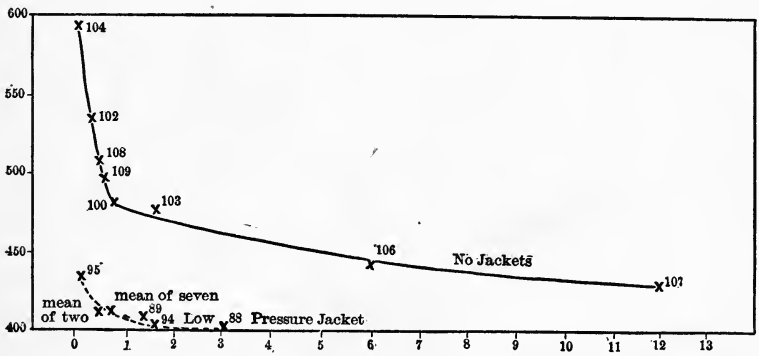

FIG. 63.

Fig. 63, in which the abscissæ are ounces of suet fed per hour, and the ordinates are thermal units per horse-power per minute carried away by the condensing water.

The importance of these experiments on feeding suet and on feeding water into the cylinders of the engine is connected with the question as to the influence of the walls of the cylinder on the steam, and especially with the question as to whether the initial condensation, re-evaporation, and exhaust

TABLE XX.

Tests on Donkin Engine.

	Conditions of the Test.	Revolutions per minute.	Cut-off, high-pressure cylinder.	Ratio of expansion.	Duration of the experiment, minutes.	Vacuum, inches of mercury.	Horse-power. Indicated.	Horse-power. Brake.	Jacket-water, pds. per ind. h. p. per minute.	Thermal units per horse-power per min.	Lubricant. Kind used.	Lubricant. Ounces per hour.	Lubricant. Intervals of feeding, minutes.	Pounds of water fed into large cylinder per minute.	Temp'ture of the water fed into large cylin.
95		96.93	7/16	7.9	30	26.75	9.47	6.78	0.153	441	None.	....	..	....	....
92, 93	Suet fed into high-pressure steam-chest in varying quantities at regular intervals. Low-pressure jacket only. Boiler-pressure, 40 to 44 pounds.	104.47	1/2	7.73	30	26.37	9.70	7.31	0.165	412	Suet.	0.5	2½	....	....
90, 91, 96, 97, 98, 99, 101		96.76	5/16 to 7/16	..	15 to 30	27.25	8.68	6.46	0.112	412	"	0.75	2 to 5	....	....
89		97.27	5/16	10.78	15	27.25	7.48	5.84	0.153	406	"	1.50	5	....	....
94		103.46	3/8	9.74	30	26.4	9.37	7.24	0.162	402	"	1.7	1	....	....
88		97.43	5/16	10.78	15	27.25	7.45	5.85	0.215	402	"	3.	2½	....	....
104	Suet fed into high-pressure steam-chest in varying quantities and at regular intervals. Air in both jackets. Boiler-pressure, 43 to 44 pounds. In tests 106 and 107 suet was fed through grease-cock, in 106 to h. p. cylinder, in 107 to both cylinders.	96.5	7/8	4.7	20	27.25	10.79	6.75	....	593	None.	....	..	....	....
102		96.5	5/8	6.4	15	27.5	8.84	6.75	....	532	Suet.	0.3	5	....	....
108		94.7	5/8	6.4	15	27.3	9.47	6.63	.....	500	Lard oil.	0.5	2½	....	....
109		96.53	5/8	6.15	15	27.22	9.70	6.76	.....	507	Russian tallow.	0.5	2½	....	....
100		96.7	9/16	6.7	30	27.28	9.31	6.77	.....	483	Suet.	0.7	2	....	....
103		96.7	5/8	6.4	15	27.5	8.46	6.83	.....	476	"	1.5	1	....	....
106		96.55	1/2	7.36	20	27.35	8.79	6.76	.....	449	"	6.	1/4	....	....
107		97.6	1/2	7.36	15	27.4	8.5	6.83	.....	433	"	12.	1/4	....	....
116	Water was run continually into the grease-cock on low-pressure cylinder, from which it was drawn into the cylinder at varying rates. In tests 116 and 117, steam was in the low-pressure jacket, but in other tests was in neither jacket.	....	1/2	7.54	20	26.1	9.63	6.7	0.05	405	Suet.	3/4	....	None.	....
117		....	9/16	6.8	15	25.	9.23	6.71	0.09	502	"	3/4	....	0.55	150
118		.....	3/4	5.38	20	27.	9.81	6.79	. ...	504	"	3/4	....	None.	....
121		....	3/4	5.38	20	25.75	8.88	5.84	. ...	571	"	3/4	....	0.4	199
119		....	7/8	4.5	20	26.5	10.05	6.78	.....	559	"	3/4	....	0.5	145
125		....	13/16	5.0	10	25.2	9.04	5.84	.....	564	"	3/4	....	0.56	56
124		....	7/8	4.7	15	25.5	9.23	5.8	.....	645	"	None.	....	1.4	203
128		...	13/16	5.0	15	25.5	8.84	5.81	.....	628	"	3/4	....	1.6	204

waste are caused by the action of the metal of the cylinder or of water remaining permanently in the cylinder.

To further illustrate this method of testing engines, the summary of results of two other tests are inserted. One test was on a mill-engine, and the other on an engine geared to pumps that worked at a slower speed than the engine.

Both engines were similar to the small experimental engine already described. The two cylinders are placed in a line with each other, the high-pressure cylinder being situated next the crank-shaft. The low-pressure cylinder only is jacketed, and the steam is led through this jacket on its way to the valve-chest of the high-pressure cylinder, while the water arising from condensation is carried off by an efficient steam-trap. The distribution of the steam is effected by ordinary slide-valves, that of the high-pressure cylinder having an adjustable expansion-valve at the back. The two main valves are driven by a single eccentric, the spindle for the low-pressure valve being a prolongation of that for the high-pressure cylinder, while a second eccentric drives the expansion-valve as usual. The steam passages are all so arranged that the cylinders are completely drained. The engine is provided with an ordinary injection condenser, and the injection-water is drawn from an adjacent river, no cold-water pump being used.

One engine* is used to drive rag engines at a large paper-mill, and the other† is geared to pumps driven at a slower speed than the engine.

Tests on Donkin Mill-engine.—The object of the test was to ascertain the average horse-power, the quantities of coal and water used, and to account satisfactorily for all of the heat furnished to the engine.

The feed-water was measured by two cylindrical cans into a cast-iron tank; the level of water in the tank was the same at the beginning and end of the test. The temperature of the feed-water was taken every twenty minutes.

Before the experiment commenced all coals were cleared

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* Engineering, November 3, 1871.

† Proceedings of Civ. Engrs., vol. lxvi.

away from the front of the boiler, and into the space thus made the coal to be used on the trial was weighed. The coal used was Powell's Duffryn, and was of excellent quality.

In commencing the experiment at 9.30 A.M., on a signal being given from the engine-house the water-level in the boiler was marked on a scale fixed to the glass of the water-gauge, the pressure of steam was noted, and both fires were at once drawn, with the exception of about a shovelful left in each furnace for relighting. About 12 lbs. of wood were then thrown in, and the firing commenced with the weighed coal, the drawn fires being cleared away. When the fires were drawn the steam stood at 50 lbs. per square inch ; in five minutes it had fallen to 49 lbs. ; but in fifteen minutes it had risen again to 49½ lbs., and in twenty-five minutes it was at 54 lbs. During the day it was kept almost constantly at 53 lbs., scarcely ever varying from this pressure more than a couple of pounds, and the mean of forty-nine observations taken at intervals of twelve minutes showed the pressure last mentioned to be the average throughout the experiment.

At 6.45 P.M., when the experiment was approaching a close, the pressure was 51½ lbs., while at the end of the trial it was 49¾ lbs., or almost exactly the same as it was at the beginning, while the water-level was also precisely the same. On notice being given from the engine-house that the trial was completed, both fires were drawn, and the coal, cinders, etc., taken out and set on one side to cool, while at the same time the ash-pits were cleaned out. When cool, the materials drawn from the fires were passed over a sieve with ¼-inch meshes, and the clinkers picked out by hand, and the weight was then found to be as follows:

Cinders, siftings, clinkers, dirt from ash-pit, 2 cwt. 3 qrs. 20 lbs.

The total amount of coal charged into the furnaces during the trial was 12 cwt., and the quantity consumed was thus 9 cwt. 0 qrs. 8 lbs. = 1016 lbs. plus its proper proportion of the dirt. Of the siftings, one half was judged to be good fuel and the other half dirt, and the total quantity of dirt was thus 13 lbs. siftings + 10 lbs. clinkers + 53 lbs. from ash-pit = 76 lbs. in all, or almost exactly 5.66 per cent. Of this 76 lbs. of dirt, 12 lbs. (a quantity rather below the proper percentage) was taken as belonging to the 239 lbs. of cinders drawn from the fire, and the remainder, 64 lbs., was added to the fuel actually consumed, thus raising the latter to 1080 lbs.

The quantity of water fed into the boiler during the trial was 11,691 lbs., and the evaporation therefore took place at the rate of $\frac{11691}{1080} = 10.82$ lbs. of water per pound of coal.

The observations made in the engine-house were as follows: 1. Every half-hour indicator-diagrams were taken simultaneously from both ends of both cylinders by means of four Richards indicators; 2. Half-hourly readings were taken of the indications of the steam and vacuum gauges, and of the counter with which the engine was provided; 3. An account was kept of the temperature and quantity of water drawn from the steam-jacket; and 4. Observations were taken every quarter of an hour of the quantity and temperature of the water passing off from the condenser. The water discharged by the air-pump was led along a short iron trough fitted with partitions which extended nearly across it. The water on its way down the trough was caused to pass under and over and around the ends of these partitions, and it was thus thoroughly mingled, and the temperature rendered uniform throughout. After escaping the partitions it was discharged over a tumbling bay having a notch 6 in. wide carefully cut in a brass plate, while the head or height of water over the notch was taken by means of a hook gauge. The temperature was taken by a delicate thermometer, on which the water fell in the tumbling bay. The temperature of the water used for injection was also noted at frequent intervals during the day, and thus the rise of temperature in passing through the condenser could be ascertained.

TEST OF DONKIN COMPOUND MILL-ENGINE.

Duration of trial: from 9.30 A.M. to 7.30 P.M., ... =	10 hours.
Mean pressure of steam in boiler-house, ...	53 lbs.
Mean vacuum, ...	$27\frac{1}{4}$ in.
Mean speed of engine in revolutions per minute, ...	46 51

Indicated horse-power—mean results of 84 diagrams:	
Mean indicated power developed in high-pressure cylinder,	32.03 I. H. P.
Mean indicated power developed in low-pressure cylinder,	24.85
Mean total indicated horse-power, ...	56.88
Observations of water from condenser:	
Temperatures:	
Mean initial temperature of injection-water, ...	51°.66
Temperature of water discharged from condenser, . . .	83°.32
Rise of temperature in condenser, ...	31.66
Quantities:	
Mean head over tumbling bay 6 in. wide, taken by a hook-gauge, ...	$2\frac{9}{16}$ in. bare.
Mean discharge per minute, ...	606.5 lbs.
Pound-degrees:	
Pound-degrees of heat discharged from condenser per minute $= 606.5 \times 31.66$, ...	19,202
Pound-degrees per indicated horse-power per minute $= \frac{19,202}{56.88} =$	337.6
Water from trap of steam-jacket:	
Total quantity discharged during ten hours, ...	1,020 lbs.
Quantity discharged per hour, ...	102 "
Feed-water:	
Initial temperature, ...	61°.75
Quantity evaporated during ten hours = 108 cans, weighing each $108\frac{1}{4}$ pounds = ...	11,691 lbs.
Quantity evaporated per hour, ...	1,169.1 "
Quantity evaporated per indicated horse-power per hour,	20.55 "
Quantity evaporated per pound of coal consumed, ...	10.82 "
Coal: Description—Powell's Duffryn:	
Quantity consumed during ten hours, ...	1,080 lbs.
Quantity consumed per square foot of fire-grate per hour,	3.27 "
Quantity consumed per indicated horse-power per hour, .	1.9 "

Test on Donkin Pumping-engine.—In the tests on this engine, in addition to the determination of the efficiency, it was desired to distinguish between the efficiency of the boiler, that of the engine, and that of the pumps. The method of making the tests was similar to that already described for the mill-engine. In this test also the feed-water was measured in cans, and the condensed water from the jacket on the low-pressure cylinder, through which the steam passed on the way from the high- to the low-pressure cylinder, was also measured in cans.

The engine drove, through gearing, two sets of pumps, one of which lifted water from a well into a tank, and the other forced water from this tank into the delivery main. The injection-water was taken from this tank, and consequently the force-pumps delivered so much less water than the lifting-pumps.

On the 29th of March a separate test was made to determine the power required to drive, respectively, the engine and gearing only, and the engine, gearing, and lifting pumps. The distribution of power was found to be—

Engine and gearing, ...	12.15	horse-power.
Lifting pumps, ...	39.31	"
Forcing pumps, ...	35.57	"
Total, ...	87.03	"

The test was continuous, but the calculation has been made for the first five hours and the second five hours, as well as for the entire run of ten hours.

TABLE XXI.

TEST ON DONKIN PUMPING ENGINE.

April 5, 1881.

The experiments were made by Messrs. B. Donkin, Jr., Martin, Salter, and Bacon.

ENGINE.

Class of engine—compound, condensing, horizontal (low-pressure cylinder jacketed).

Name of maker—Bryan Donkin & Co.

Diameters of cylinders $16\frac{17}{32}$ inches and 30 inches: diameter of piston-rods $2\frac{3}{4}$ inches and 4 inches (all from gauges).

Length of strokes, 3 feet.

Lubricant used in cylinders—Engelbert's.

Cylinder lubricated every hour: quantity, about 7 ounces per hour.

BOILER.

Class of boiler—Lancashire.

New in 1872.

Chief dimensions 25 feet long by 6 feet diameter. Two flues, each 2 feet 2 inches in diameter.

Total heating surface, 612 square feet. Boiler and flues clean inside and out.

Grate surface—two grates each 2 feet 2 inches by 6 feet, together 26 square feet.

Blow-off cock tight. Fire doors opened only for firing. Direction of smoke—through tubes, under, split along sides to chimney.

Fires 8 inches thick, stoked ten times each furnace, or every hour.

Pounds of coal per hour per square foot of heating surface, 0.287.

Pounds of water per hour per square foot of heating surface, 2.720.

Temperature of engine-house, $62\frac{1}{2}°$; temperature of outer air, 40°.

	Ten hours.	First five hours.	Second five hours.
Pressure of steam in boiler-house..............lbs.	52	$52\frac{1}{4}$	$51\frac{3}{4}$
Duration of trial, from 9 A.M. to 7 P.M without stoppage..........hrs.	10	5	5
Vacuum in condenser..........ins.	$27\frac{1}{4}$	$27\frac{1}{4}$	$27\frac{1}{4}$
Revolutions per minute..........	55.25	57.35	53.14
Total revolutions run..........	33,148	17,205	15,943
INDICATED H. P.			
Indicated power, high-pressure cylinder..........	46.36	48.55	44.17
Indicated power, low-pressure cylinder..........	40.67	42.66	38.68
Total Indicated H. P..........	87.03	91.21	82.85
WATER FROM CONDENSER.			
Initial temperature of injection-water.........deg.	54.25	54.25	54.25
Temperature of water discharged from air-pump "	85.06	86.27	83.85
Rise of temperature.......... "	30.81	32.02	29.60
Mean discharge per minute..........lbs.	857	865	849
Thermal units per Indicated H. P. per minute.....	303	303	303
WATER FROM STEAM-JACKET.			
Total quantity discharged during ten hours.....lbs.	1,365	710	655
Quantity discharged per hour.......... "	$136\frac{1}{2}$	142	131
" per Indicated H. P. per hour.......... "	1.56	...	...
FEED-WATER.			
Temperature..........deg.	85	84.9	85.1
Quantity evaporated during ten hours..........lbs.	16,731	8,802	7,929
Quantity evaporated per hour.......... "	1,673	1,760	1,586
Quantity evaporated per I. H. P. per hour...... "	19.22	19.30	19.14
Quantity evaporated per lb. of coal consumed, from 85°.......... "	9.52	....	....

The feed-water was measured in a can holding exactly 100 lbs. (special standard) at 80°.

COAL.

Fires drawn at beginning and end, and 28 lbs. of wood used for lighting up, 14 lbs. each fire.

1,792 lbs. put on fires, less 35 lbs. (= half of large cinders drawn from fires) =

	Lbs.
Quantity used during ten hours, ...	1,757
" " per square foot of fire-grate per hour, .	$6\frac{3}{4}$
" " per indicated H. P. per hour, ...	2.02
Weight of dirt and refuse not burnt $94\frac{1}{2}$ lbs. small cinders + 35 lbs. large cinders + 16 lbs. clinkers =	$145\frac{1}{2} = 8\frac{1}{4}$ per cent.

Length of steam-pipe, 55 feet, all covered. Steam-pipes all tight.

All feed-pipes visible and tight.

RESULTS AND EFFICIENCY OF THE WATER-PUMPS, AND COMPARISON OF THE CONTENTS OF THE PUMPS WITH THE WATER PUMPED.

THREE DEEP-WELL OR LOWER PUMPS.

Dimensions and calculated contents:

Pumps 14 inches diameter, mean stroke 2 feet $8\frac{3}{4}$ inches.

Contents of these three pumps 8.755 cubic feet = 546.3 lbs. per revolution.

Mean speed for ten hours = 12.174 revolutions per minute.

Contents per minute = 6,651 lbs.

Quantity actually pumped:

Mean height over 2 feet 3 inches bay in engine-house = 4.395 inches.

Temperature of water 54°.

Quantity = 6,183 lbs. per minute.

Efficiency:

6,183 ÷ 6,651 = 93 per cent water lifted of theoretical contents.

THREE FORCE OR UPPER PUMPS.

Dimensions and calculated contents:

Pumps, 12 inches diameter. Strokes, two 2 feet $1\frac{7}{8}$ inch, one 2 feet 2 inches.

Contents of these three pumps 5.09 cubic feet = 317.6 lbs. per revolution.

Mean speed for ten hours = 20.819 revolutions per minute.

Contents per minute = 6,612 lbs.

Quantity actually pumped:

Same as lower pumps (6,183 lbs.) less 831 lbs. used for injection = 5,352 lbs.

Efficiency:

5,352 ÷ 6,612 = 81 per cent water lifted of theoretical contents.

		Inches.			Lbs.
Comparison of measurement of water in engine-house over 2 feet 3 inches bay, with that at reservoir over two 1 foot 6 inches bays. For about eight hours.	Engine-house,	4.38 height	=		6,155
	Reservoir,	3.38 " }	= 5,421		
		3.27 " }			
	Add injection (water)			831	
	Leak of rising main (measured)			10	6,262

The latter $1\frac{1}{2}$ per cent more nominally than the former.

		Lbs.	Feet.	H. P.	Foot-lbs.
Percentage of foot-pounds of water lifted, of foot-pounds of steam-pistons.	Lower pumps,	6,183	198 lift	= 37.1	(of 33,000)
	Upper "	5,352	148 "	= 24.1	61.2 H. P.
	Steam-pistons,				87.03 "

Foot-pounds of water lifted, per cent of foot-pounds steam-pistons, 70.

Average velocity of water in rising main, 1.3 foot per second.

HEAT ACCOUNT OF THE STEAM-ENGINE IN THERMAL UNITS OR POUND-DEGREES PER MINUTE.

	Units.	Per cent.
Dr. to boiler:		
Feed, 27.888 lbs. × (1,204°.8 — 85° =) 1,119°.8 = . .	31,229	....
Cr.:		
By power, 87.03 I. H. P. × 42.76 = ...	3,721	= 11.9
By waste water from condenser:		
Injection, 831.33 lbs. × (85°.06 — 54°.25 =)30°.81 = 25,613		
Condensed steam in waste water. { Feed, 27.89 lbs. — water from trap 2.30 lbs. = 25.59 lbs. × (85°.06 — 85° =) .06° = ... } = 2		
	25,615	= 82.4
By water from steam-trap 2.30 lbs. × (212° — 85° =)127°=	293	= 0.9
By balance unaccounted for, radiation, etc. = ...	1,600	= 4.8
	31.229	

Major English's Tests.—In November and December, 1886, tests were made by Major Thomas English, R. E.,* to ascertain the most economical method of working some direct-acting, high-pressure, fly-wheel pumping-engines, intended for

* Proceedings of the Inst. M. E. 1887.

use in a hot climate, where fuel was dear, water scarce, and where difficulties of transportation prohibited heavy weights. The first object of the tests was to determine whether, under the circumstances, it was advisable or not to use a surface condenser. The results of the tests shown in Table XXII show that the economy of the engine was small and the consumption of steam was not satisfactory whether the engine was run condensing or non-condensing. In addition to ascertaining the steam consumption, Major English made a very complete analysis of the distribution of the heat, as is shown by the table, and for this reason the tests are of interest. Each engine consisted of a pair of horizontal cylinders, 16 inches diameter by 18 inches stroke, lagged, but not jacketed. Each cylinder drives a differential pump on the prolongation of the piston-rod, with rams 4 inches and $5\frac{3}{4}$ inches diameter, working up to 700 pounds on the square inch. The piston-rods, $2\frac{1}{2}$ inches diameter, pass through both ends of the steam-cylinder, and are connected by a crank-shaft, which has cranks at right angles and a fly-wheel on each end. The engines are sufficiently self-contained to work on a foundation of three timbers bolted together with distance blocks. Each cylinder has a main or distribution slide-valve, and on its back cut-off plates that can be adjusted by hand to give different rates of expansion.

The steam during the tests was furnished by three multitubular boilers of the locomotive type. Two of these boilers were used together to furnish steam for one main engine, an air-pump engine and a feed-pump. The surface of the steam-pipe leading from the boilers to the engine was 141 square feet. During some of the tests this steam-pipe was jacketed with steam from a separate boiler at 140 pounds pressure.

The air-pump and circulating-pump were operated by one auxiliary engine with a steam-cylinder 10 inches in diameter by 14 inches stroke. The feed-pump was a direct-acting Worthington pump. The steam from these two auxiliary engines was condensed in a surface condenser, collected, and weighed. The exhaust from the main engine was in all the tests condensed in a tubular surface condenser, collected, and measured in a tank of known capacity. During the non-condensing tests the air-pump was disconnected and the exhaust was condensed at atmospheric pressure.

The average evaporation during the tests was 7.9 pounds of water per pound of coal; the rate of combustion varied from 6.5 to 12.4 pounds per square foot of grate surface per hour. No priming worth notice appeared at any time.

The indicator-diagrams were taken by a Richards indicator with a 30 spring. The spring was tested by the makers after the tests, and was stated to be correct. The clearance of the engine is seven per cent of the piston displacement.

The data and results of the tests are given in Table XXII, which requires little explanation in addition to that given by the headings.

In columns 11, 12, and 13 are given the heat equivalents of the total work without allowing for back pressure, the effective work, and the work of the back pressure, for each pound of steam supplied.

The distribution of the heat lost at the end of the stroke is shown by columns 14, 15, and 16, calculated from the condition of the steam at release, for each pound of steam supplied. The heat in the steam and the water at release is unavoidably rejected, but that abstracted by the walls of the cylinder corresponds to the exhaust-waste Q_c of Hirn's analysis.

The thermal unit per pound of steam, column 17, is the difference between the total heat of one pound of steam at the initial pressure and of the heat of the liquid at the mean back pressure.

The condensation at the end of the stroke is the ratio of the weight of dry saturated steam required to fill the cylinder up to release, to the actual weight of steam and water then present in the cylinder.

Column 19 gives the number of thermal units that could be converted into work for each pound of steam in a perfect engine; obtained by multiplying the heat supplied (column 17), by the efficiency of a perfect engine working between the

TABLE

MAJOR ENGLISH'S TESTS OF A

Number of Test.	Conditions of Test.		Nominal cut-off.	Number of expansions.	Revolutions per minute.	Absolute pressures.				Indicated horse-power.	Consumption of water.	
						Initial, lbs. per square inch.	Terminal, lbs. per square inch.	Mean forward, lbs. per square inch.	Mean back, lbs. per sq. inch.		Lbs. per indicated horse-power per hour	Lbs. per stroke.
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	Condensing.	Steam-pipe, jacketed.	$\frac{1}{4}$	3.4	40.2	71.3	22.8	43.1	2.9	57.8	34.2	0.205
2			$\frac{1}{8}$	5.8	39.5	89.0	18.9	39.4	3.2	50.3	34.0	0.182
3			$\frac{1}{8}$	5.8	39.7	82.6	17.6	36.1	3.2	46.6	36.1	0.176
4			$\frac{1}{8}$	5.8	40.4	83.3	19.3	38.0	4.2	49.4	38.2	0.194
5			$\frac{1}{16}$	6.7	40.9	87.4	17.0	35.0	2.7	46.9	34.6	0.165
6		Steam-pipe, not jacket'd	$\frac{1}{4}$	3.4	42.2	72.6	23.5	44.1	2.6	62.0	35.4	0.217
7			$\frac{1}{8}$	5.8	40.3	86.5	18.7	38.0	2.5	51.0	34.7	0.183
8			$\frac{1}{8}$	5.8	40.4	83.3	17.7	36.1	3.3	47.2	38.9	0.190
9			$\frac{1}{16}$	6.7	41.2	86.8	16.0	33.5	2.4	45.7	38.8	0.179
10	Non-condensing.	Steam-pipe, jacket'd	$\frac{1}{4}$	3.4	40.4	83.7	28.8	51.8	15.4	52.4	39.6	0.214
11			$\frac{1}{8}$	5.8	40.1	90.9	22.3	41.7	16.5	36.2	41.2	0.155
12			$\frac{1}{16}$	6.7	40.1	88.1	20.0	36.8	16.2	29.6	42.9	0.132
13			$\frac{1}{16}$	6.7	39.3	87.8	20.2	36.6	15.5	29.6	51.0	0.160
14		Steam-pipe, not jacketed.	$\frac{1}{4}$	3.4	39.8	87.6	28.1	52.6	16.6	51.0	41.1	0.220
15			$\frac{1}{8}$	5.8	40.3	91.6	21.9	41.8	16.7	36.0	42.6	0.158
16			$\frac{1}{16}$	6.7	39.5	89.3	20.0	36.7	15.6	29.7	50.5	0.158

initial temperature and the temperature due to the back pressure.

The absolute efficiency, column 20, is the ratio of the heat changed into work to the heat supplied; column 12 and column 17.

The ratio of the efficiencies of the actual engine and of a perfect engine (column 21) is the ratio of the heat changed into work in the actual engine (column 12), and the heat changed into work by a perfect engine (column 19).

Major English gives also diagrams representing the distribution of heat for each of the tests stated in the table. The diagrams are reproduced in Figs. 64 and 65, for the 6th and 14th tests.

The upper part of the figure gives the maximum and minimum diagram taken during the test, with a few intermediate diagrams. The axes are obtained by laying off the

XXII.

STATIONARY STEAM-ENGINE.

Total work, thermal units per lb. of steam supplied.	Effective work, thermal units per lb. of steam supplied.	Back-pressure work, thermal units per lb. of steam supplied.	Thermal units of heat lost at end of stroke.			Thermal units of heat supplied.	Condensation at end of stroke, per cent.	Total work of perfect engine, thermal units per lb. of steam supplied.	Absolute efficiency of actual engine.	Relative efficiency of actual engine in percentage of efficiency of perfect engine.
			Exhaust steam.	Water in cylinder.	Abstracted by cylinder.					
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**
80	75	5	590	38	359	1067	39.3	244	7.0	30.8
82	75	7	543	36	407	1068	43.7	240	7.0	31.3
78	71	7	526	35	427	1066	45.5	233	6.7	30.5
75	66	9	521	32	427	1055	45.5	217	6.3	30.5
80	74	6	550	35	407	1072	43.3	249	6.8	29.3
77	72	5	567	43	384	1071	41.9	237	6.7	30.3
79	72	7	544	40	412	1075	44.1	252	6.6	28.2
72	65	7	489	37	467	1065	49.2	233	6.1	28.0
71	65	5	482	42	482	1077	50.5	254	6.1	25.6
92	65	27	613	11	279	995	31.8	129	6.5	49.6
103	63	40	650	5	235	993	27.2	131	6.4	48.1
107	61	46	684	3	199	993	23.3	129	6.1	47.2
87	51	36	571	5	332	995	36.1	131	5.1	39.0
91	62	29	581	10	310	992	35.1	127	6.2	48.8
100	61	39	621	4	269	994	30.4	131	6.1	45.8
88	51	37	573	5	330	996	35.9	134	5.1	38.1

vacuum line at the proper distance below the atmospheric line, and by laying off the line of zero volume, allowing for clearance. The entire volume, including clearance, is divided into ten equal parts, and ordinates are drawn upon which the absolute pressures are measured to points shown by dots on the ordinates. In the lower part of the figure the total height represents the total thermal units supplied, column 17, Table XXII. The curve *EE* represents the thermal units per pound of steam, changed into work, obtained from the area of the indicated diagram to the left of the ordinate in question. The line *TT* is obtained by adding to the heat changed into effective work the heat per pound of steam required to do the work of the back pressure, which is obtained from the area below the back-pressure line and to the left of the ordinate. The line *TT*, therefore, represents at each ordinate the heat per pound of steam required to do the absolute work up to that point of the stroke of the engine. To obtain the line SS, the fraction x, of a pound of the mixture in the cylinder which was steam, was calculated for each ordinate, from the volume and pressure, and the heat in that steam was calculated by the expression

$$x(r + q - q_0),$$

in which r and q are the heat of vaporization and the heat of the liquid corresponding to the pressure measured on the ordi-

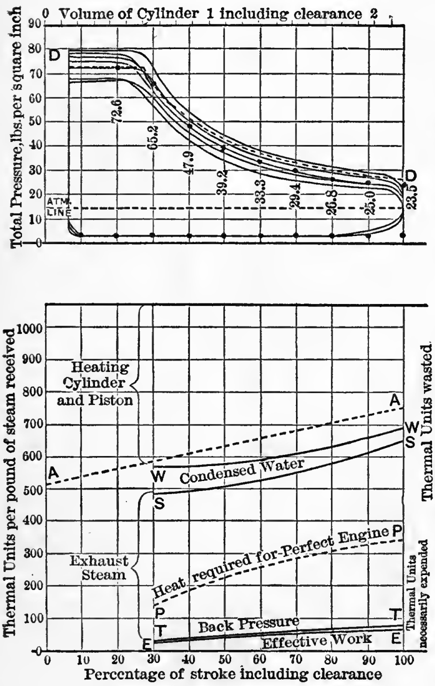


FIG. 64.

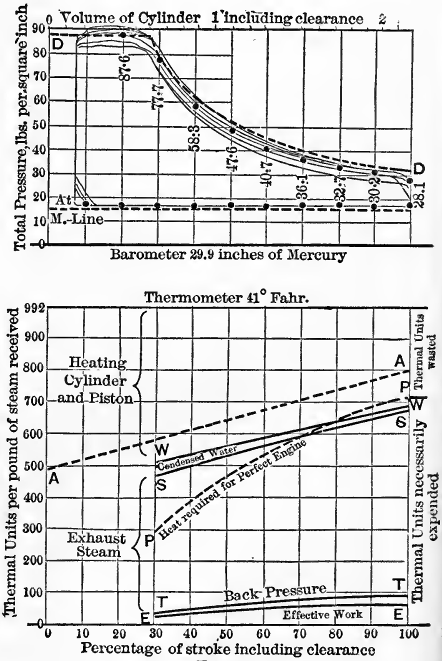


FIG. 65.

nate, and q_0 is the heat of the liquid corresponding to the back pressure. The number of thermal units was then plotted on each ordinate from the line TT, so that the line SS represents the heat per pound of steam, changed into work, plus the heat remaining in the steam. A similar calculation was made for the fraction of a pound of the mixture at each ordinate that was water, using the expression

$$(1 - x)(q - q_0),$$

and the line WW was plotted by laying off the quantities thus found from the line SS; so that the line WW represents the heat per pound of steam, changed into work, plus the heat remaining in the water and steam. The portion of the total heat received per pound of steam, above the line WW, represents the heat absorbed by the walls of the cylinder; this quantity diminishes as the expansion proceeds on account of the re-evaporation.

The dotted curve PP represents the heat theoretically necessary to do the total work shown by the curve TT.

A comparison of all the observed results is shown by Fig. 000, in which the abscissæ represent square feet of surface exposed to the steam throughout the stroke by the steam-passages, cylinder, and piston; the clearance surface measures 5.24 square feet at the commencement, and the total surface 11.98 square feet at the end of the stroke. Ordinates measured downwards from the top of the diagram represent the number of thermal units abstracted from the enclosed steam at any point; and the curves are plotted from such ordinates for each point of the stroke. The palpable convergence of all of these curves to the zero of exposed surface at about 150 thermal units agrees closely with the hypothesis that there is a sudden initial condensation of steam, equivalent, in all the tests on this engine, to the transference of 150 thermal units, or 28.6 thermal units per square foot of exposed clearance surface, to the metal surface of steam-passages, cylinder, and piston; and that this heat is gradually given back again to the steam during the stroke, by re-evaporation. The heat thus regained increases approximately in proportion to the surface exposed; but still leaves in the metal, at the end of the stroke in this engine, an amount of heat equivalent to 0.4 thermal unit for each degree of difference between the temperatures corresponding to the initial and back pressures.

By the aid of this hypothesis the heat abstracted by the walls of the cylinder at each point of the stroke of the piston has been calculated, and is represented by the line *AA* on the Figs. 64 and 65. By the same hypothesis the diagrams rep-

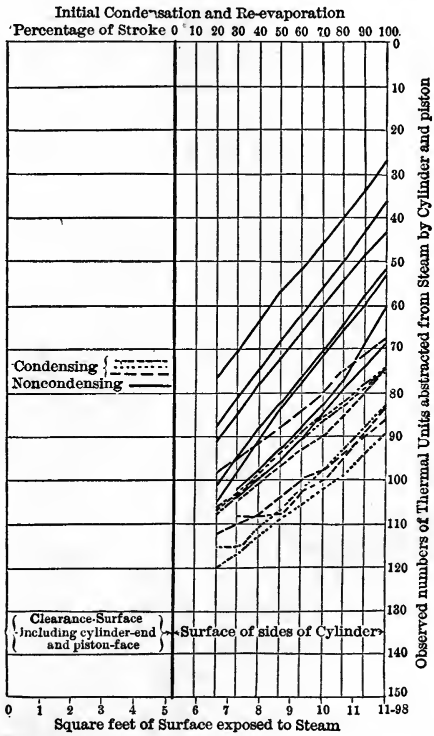


FIG. 66.

resented by the dotted lines in the upper part of Figs. 64 and 65 have been deduced by the reversal of the processes used in laying out the lines *EE* to *WW*. The following table exhibits the correspondence between actual observed quantities, and quantities calculated on this hypothesis.

	Experiment 6.		Experiment 14.	
	Observed.	Calculated.	Observed.	Calculated.
Indicated horse-power	62.0	65.6	51.0	57.1
Water per stroke	0.217	0.211	0.220	0.209
Efficiency, per cent	6.7	7.8	6.2	7.6

Major English draws the following conclusions from the tests:

In order to obtain the best results for any given range of temperature there should be a definite relation between the surface of the steam-passages, the diameter of the cylinder and the length of the stroke; and that in the design of an engine the adjustment of these proportions may be the most important item affecting economy. The following table shows for two different points of cut-off the calculated results of varying the length of the stroke of the engine experimented on, while the diameter of the cylinder, the absolute clearance volume and the clearance surface exposed, remain unaltered; and it will be seen that the same number of expansions may give widely different results as regards the ratio of efficiency and the water consumed per indicated horse-power per hour; and also that, with the same length of stroke, these results are but slightly affected by doubling the number of expansions.

CALCULATED EFFICIENCY AND CONSUMPTION OF STEAM WITH VARYING LENGTH OF STROKE.

Diameter of cylinder, 16 inches. Clearance, 0.143 cubic feet. Clearance surface, 5.24 square feet.	Cut-off, 1.4 inch. Absolute initial pressure, 87.4 pds. Back pressure, 2.7 pounds.			Cut-off, 4.25 inches. Absolute initial pressure, 71.3 pounds. Back pressure, 2.9 pounds.				
Length of stroke, inches	8.0	18.0	21.8	8.0	18.0	21.8	38.6	46.3
Number of expansions	3.47	7.24	8.67	1.68	3.47	4.20	7.24	8.67
Percentage of efficiency	4.8	7.3	8.1	5.3	7.9	8.6	10.3	11.1
Steam per ind. h. p. per hour, pounds	56.0	34.6	31.3	54.8	32.5	30.0	23.0	21.0

Initial Condensation.—Subsequently to the preceding steam-engine tests Major English * made some important experiments to determine the amount of initial condensation directly.

* Proceedings of the Inst. of M. E. 1887.

The experiments were made on a portable engine, 10 inches diameter and 14 inches stroke, jacketed on the sides, but not on the ends. The connecting-rod was disconnected, the piston was rigidly blocked at the end of the cylinder farthest from the crank, and the interior of the cylinder was completely filled with wood and iron, as was also the steam-passage at the crank end. The port at the crank end was filled with a brass plate scraped down to a level with the valve-seat. The port at the head end was left open, and the crank-shaft, eccentric, and valve were driven by another engine. The steam-pressure in the boiler was maintained uniform during a trial, and the regulator was kept open. As a consequence, steam at boiler-pressure was alternately admitted to and exhausted from the clearance space at the head end, once each revolution, for a time corresponding to a cut-off at seven tenths of the stroke.

Each experiment lasted an hour, during which time revolutions were noted by a counter, and indicator-cards were taken. The steam passing through the engine under these conditions was condensed, collected, and weighed. Sixty-four satisfactory tests were made, of which thirty-five were condensing and twenty-nine non-condensing. The steam-pressures were about 45, 30, 20, and 10 pounds above the atmosphere; and the numbers of revolutions were 130, 100, 70, and 50 per minute.

Let γ_1 be the density or weight in pounds of one cubic foot of steam, of the steam up to the point of cut-off, and let t_1 be the temperature; let γ_0 and t_0 be similar quantities at the time when the exhaust-port closes.

Let M be the weight of water per revolution from the condenser. This is made up (1) of the differences of the weights of steam shown by the indicator at cut-off and compression,

$$(\gamma_1 - \gamma_0)V_0,$$

in which V_0 is the volume in cubic feet of the clearance space, including the steam-passages; and (2) of the weight of steam condensed, and not re-evaporated, on the constant surface S_c square feet of the clearance, including the steam-passages. The weight of steam condensed per revolution and not re-evaporated is

$$M - (\gamma_1 - \gamma_0)V_0.$$

Let λ_1 be the total heat of steam at the temperature t_1, and let q_1 be the heat of the liquid; then the thermal units absorbed by the clearance surface are

$$C = (\lambda_1 - q_1)\{M - (\gamma_1 - \gamma_0)V_0\}.$$

These several quantities are given in Tables XXIII and XXIV.

TABLE XXIII.

INITIAL CONDENSATION IN JACKETED CYLINDER OF NON-CONDENSING ENGINE.

1887.	Clearance volume.	Revolutions per second.	Density of steam. Lbs. per cubic ft. Initial.	Density of steam. Lbs. per cubic ft. Exhaust.	Thermal units in one pound of Steam.	Thermal units in one pound of Water.	Water collec'd per revolution.	Net initial condensation by clearance surface. Thermal units. Total per revolution.	Net initial condensation by clearance surface. Thermal units. Per sq. feet of surface at one rev. per second.
	V_0	N	γ_1	γ_2	λ_1	g_1	M	C	$\frac{C\sqrt{N}}{S}$
	Cubic ft.	Revols.	Lb.	Lb.	Units.	Units.	Lb.	Units.	Units.
May 16......	0.035	1.73	0.156	0.043	1173	269	0.0213	15.7	10.3
May 25......	0.035	2.19	0.149	0.042	1172	265	0.0135	8.9	6.6
May 28......	0.035	1.70	0.148	0.043	1172	265	0.0160	11.2	7.3
May 13......	0.035	1.23	0.148	0.043	1172	265	0.0240	18.4	10.2
June 1.......	0.035	1.21	0.146	0.043	1171	263	0.0237	18.3	10.1
May 16......	0.035	1.64	0.146	0.040	1171	263	0.0161	11.3	7.2
June 2.......	0.035	0.92	0.144	0.042	1171	262	0.0251	19.6	9.4
May 13	0.035	1.69	0.140	0.043	1170	260	0.0170	12.4	8.1
May 16......	0.035	1.54	0.126	0.040	1168	254	0.0147	10.7	6.1
May 14......	0.035	1.32	0.118	0.041	1167	248	0.0150	11.3	6.5
May 25	0.035	2.09	0.117	0.040	1166	247	0.0098	6.5	4.7
May 28......	0.035	1.65	0.116	0.039	1166	247	0.0154	11.7	7.5
May 17......	0.035	1.62	0.115	0.042	1166	247	0.0150	11.5	7.3
June 2.......	0.035	0.85	0.115	0.039	1166	247	0.0225	18.2	8.4
June 1.......	0.035	1.25	0.113	0.040	1166	246	0.0128	9.4	5.3
May 14......	0.035	1.30	0.104	0.041	1165	241	0.0120	9.0	5.2
May 17......	0.035	1.66	0.103	0.043	1164	240	0.0149	11.8	7.6
May 25......	0.035	2.05	0.095	0.039	1163	234	0.0069	4.6	3.3
May 14......	0.035	1.25	0.092	0.040	1163	233	0.0110	8.6	4.8
May 31......	0.035	1.69	0.091	0.041	1162	232	0.0124	9.9	6.4
June 1.......	0.035	1.19	0.090	0.040	1162	231	0.0101	7.7	4.2
June 2.......	0.035	0.80	0.089	0.040	1161	230	0.0151	12.5	5.6
May 17.. ...	0.035	1.71	0.089	0.041	1161	230	0.0102	7.9	5.2
May 25......	0.035	2.12	0.071	0.039	1157	216	0.0044	3.1	2.3
May 31......	0.035	1.65	0.071	0.040	1157	216	0.0074	5.9	3.8
May 31......	0.035	1.19	0.070	0.039	1156	214	0.0080	6.4	3.5
May 14......	0.035	1.36	0.068	0.039	1156	213	0.0065	5.1	3.0
May 17......	0.035	1.64	0.067	0.040	1156	212	0.0060	4.7	3.0
June 2.......	0.035	0.85	0.066	0.039	1156	212	0.0081	6.7	3.1

TABLE XXIV.

INITIAL CONDENSATION IN JACKETED CYLINDER OF CONDENSING ENGINE.

1887	Clearance volume.	Revolutions per second.	Density of steam. Lbs. per cubic ft.		Thermal units in one pound of		Water collec'd per revolution.	Net initial condensation by clearance surface. Thermal units.	
			Initial.	Exhaust.	Steam.	Water.		Total per revolution.	Per sq. feet of surface at one rev. per second. $\frac{C\sqrt{N}}{S}$
	V_0	N	γ_1	γ_2	λ_1	g_1	M	C	
	Cubic ft.	Revols.	Lb.	Lb.	Units.	Units.	Lb.	Units.	Units.
June 13......	0.035	1.69	0.146	0.020	1171	263	0.0173	11.7	7.6
June 15......	0.035	1.21	0.145	0.020	1171	263	0.0227	16.6	9.1
June 10......	0.035	1.80	0.143	0.024	1171	262	0.0163	11.0	7.4
June 16......	0.035	0.83	0.143	0.015	1171	262	0.0257	19.3	8.8
Aug. 4.......	0.035	1.95	0.143	0.019	1171	262	0.0203	14.5	10.2
Aug. 3.......	0.035	0.87	0.143	0.012	1171	262	0.0181	12.3	5.7
July 28......	0.038	1.09	0.142	0.028	1171	262	0.0233	17.3	9.0
July 13......	0.038	1.60	0.140	0.023	1170	261	0.0154	10.0	6.3
Aug. 11......	0.038	1.59	0.140	0.018	1170	260	0.0160	10.6	6.6
Aug. 13......	0.038	2.17	0.138	0.025	1170	259	0.0136	8.5	6.3
June 15......	0.035	1.13	0.118	0.017	1167	249	0.0193	14.5	7.7
June 13......	0.035	1.65	0.117	0.019	1166	247	0.0168	12.3	7.9
June 16......	0.035	0.89	0.116	0.015	1166	246	0.0199	15.1	7.1
Aug. 3......	0.035	0.85	0.114	0.012	1166	246	0.0142	9.8	4.5
June 10......	0.035	1.90	0.113	0.019	1166	246	0.0107	6.8	4.7
July 13......	0.038	1.61	0.112	0.021	1166	246	0.0115	7.4	4.7
Aug. 11......	0.038	2.13	0.111	0.015	1165	244	0.0093	5.2	3.8
July 28......	0.038	1.15	0.111	0.025	1165	244	0.0137	9.6	5.2
Aug. 11......	0.038	1.65	0.111	0.015	1165	244	0.0115	7.3	4.7
June 13......	0.035	2.09	0.091	0.016	1161	231	0.0084	5.4	3.9
June 13......	0.035	1.70	0.091	0.016	1161	231	0.0109	7.7	5.0
June 16......	0.035	0.86	0.091	0.014	1161	231	0.0144	10.9	5.1
July 26......	0.038	1.15	0.090	0.020	1161	231	0.0149	11.3	6.1
July 28......	0.038	1.13	0.089	0.017	1161	230	0.0125	9.1	4.8
Aug. 3......	0.035	0.81	0.088	0.012	1161	230	0.0142	10.7	4.8
Aug. 12....	0.038	2.17	0.087	0.015	1161	230	0.0087	5.6	4.1
Aug. 12.....	0.038	1.65	0.083	0.018	1160	225	0.0122	9.1	5.8
June 13......	0.035	2.19	0.071	0.015	1157	216	0.0066	4.3	3.2
June 13......	0.035	1.72	0.070	0.014	1156	215	0.0094	7.0	4.6
July 26......	0.038	1.16	0.070	0.015	1156	215	0.0114	8.8	4.7
July 26......	0.038	0.83	0.070	0.015	1156	215	0.0134	10.6	4.8
Aug. 4......	0.035	0.88	0.066	0.012	1156	213	0.0130	10.5	5.0
Aug. 12.....	0.038	2.13	0.066	0.014	1156	212	0.0060	3.8	2.8
Aug. 4......	0.035	1.13	0.065	0.012	1155	210	0.0103	7.9	4.2
Aug. 11.....	0.038	1.70	0.063	0.013	1155	209	0.0103	7.9	5.2

The results of the experiments indicate that the excess of initial condensation over re-evaporation, in these experiments, varies directly as the initial density and inversely as the square root of the number of revolutions N, per unit of time. Assuming that it varies also as the surface (2 square feet), the last column of the tables gives the amount in thermal units for one square foot and one revolution per second. The results of the experiments are also plotted in Fig. 67, using thermal units

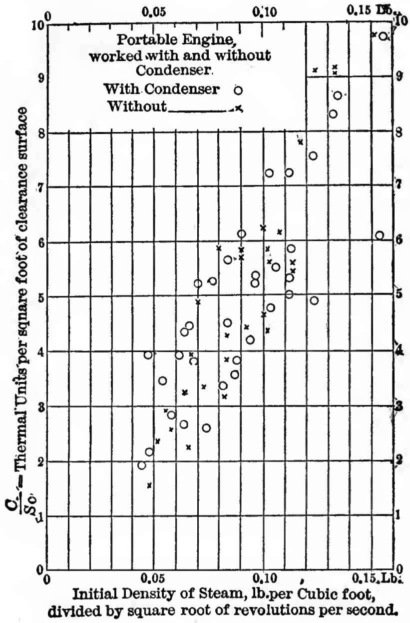


FIG. 67.

per square foot as ordinates and the initial density divided by the square root of the number of revolutions for abscissæ.

The average of the whole series of experiments corresponds with an excess of condensation over evaporation, equivalent to 8.2 thermal units per square foot of clearance surface for steam at 60 pounds pressure absolute.

Major English states that these experiments and his experiments given in Table XXII, and also the experiments on the revenue steamers, pages 270, 273, and 276, may be fairly represented by the following formulæ for the excess of condensation over re-evaporation at any point of the stroke of an engine.

For Jacketed Cylinders.

$$\{\lambda_1 - (xr + q + AW)\}M = \frac{60\gamma_1 S_c}{\sqrt{N}}\left(1 - \frac{\gamma_0 + 0.06}{\gamma_1}\frac{S}{S_c}\right).$$

For Unjacketed Cylinders.

$$\{\lambda_1 - (xr + q + AW)\}M = \frac{80\gamma_1 S_c}{\sqrt{N}}\left(1 - \frac{\gamma_0 + 0.06}{\gamma_1}\frac{S}{S_c}\right).$$

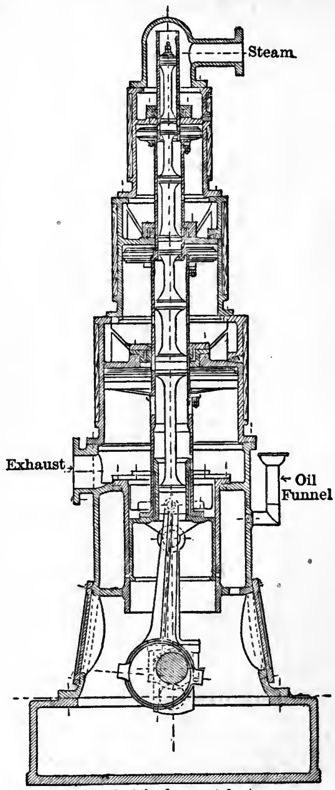
M = excess of pounds of water condensed over re-evaporation;
$xr + q$ = thermal units per pound of the mixture in the cylinder at any given point of the stroke;
AW = heat equivalent of the work done up to that point;
λ_1 = total heat of steam at cut-off;
γ_1 = density of steam at cut-off;
γ_0 = density of steam at compression;
S = surface of cylinder, including clearance up to the given point;
S_c = surface of clearance;
N = revolutions.

Willans' Steam-engine Tests.—In 1887, Mr. Peter Willans * made a large number of tests under various circumstances on an engine of peculiar form invented by him.

This engine is represented by Fig. 68, which gives a vertical section. It has three single-acting pistons of diminishing diameter on a hollow piston-rod, which forms the steam-passages and is provided with ports and piston-valves for admitting and exhausting steam from the several cylinders. The space below each of the two smaller pistons and above the head of the next larger cylinder forms a receiver, into which the steam is exhausted and from which it is drawn by the cylinder below. Below the exhaust space under the large piston is a compression chamber filled with air to insure a constant compression on the piston-rod. The piston

* Proceedings Inst. Civ. Eng., vol. xciii.

working in this chamber serves also as the guide to the connecting-rod, which is made double. The several valves are all on one rod in the hollow piston, and are moved by an eccentric on the crank-pin between the connecting-rod ends. The cut-off is effected by the ports in the hollow piston-rod, running past a ring placed on the cylinder-head, at full piston speed, and can be varied by hand or by a governor. During the tests the cut-off was accomplished by a fixed ring for each test.



Scale 3 inches = 1 foot.
FIG. 68.

The pressures in the receiver spaces between the smallest cylinder and the intermediate cylinder, and between the intermediate cylinder and the largest cylinder, varied to such a degree that there were really five stages of expansion in the engine when running nominally with triple expansion, and three stages when running compound. In Fig. 69 the diagrams from the three cylinders and the two receiver spaces are combined, so as to show the true relations of volumes and pressures.

Indicator-diagrams were taken with a Crosby indicator, and in order to get clear diagrams the main group of experiments were made at 400 revolutions per minute, but in practice it is the habit to run at 500 revolutions per minute.

In comparing these tests it was assumed that the work theoretically due to the heat in the steam was the heat changed into work by an engine working on the cycle represented by

Fig. 70. It was assumed that dry saturated steam was admitted at the absolute pressure p_1 from a to b; that the steam

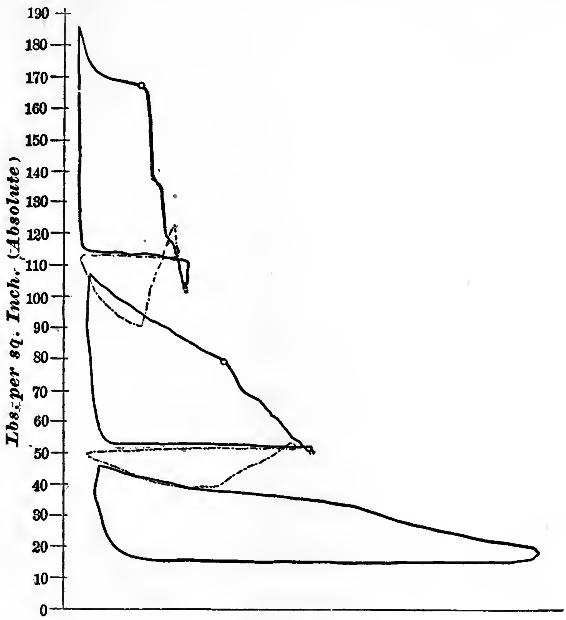


FIG. 69.

was expanded adiabatically from b to c till the absolute pressure became p_2; and that the steam was exhausted against a constant pressure.

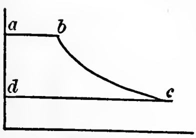


FIG. 70.

The efficiency of this cycle may be calculated as follows: The work of M pounds of steam during admission is

$$W_{ab} = M(p_1 n_1 + p_1 \sigma).$$

The work during expansion by equation (156), page 111, is

$$W_{bc} = \frac{M}{A}(\rho_1 - x_2\rho_2 + q_1 - q_2).$$

The work during exhaust is

$$W_{cd} = M(p_2 x_2 n_2 + p_2 \sigma).$$

The work done by the steam during the cycle is

$$W = \frac{M}{A}(\rho_1 + Ap_1n_1 - x_2\rho_2 - Ap_2n_2x_2 + q_1 - q_2);$$

$$\therefore W = \frac{M}{A}(r_1 - x_2r_2 + q_1 - q_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (307)$$

But by equation (146)

$$\frac{r_1}{T_1} + \theta_1 = \frac{x_2r_2}{T_2} + \theta_2;$$

$$\therefore x_2r_2 = T_2\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right),$$

which, introduced into equation (307), gives

$$W = \frac{M}{A}\left\{\frac{r_1(T_1 - T_2)}{T_1} + T_2(\theta_1 - \theta_2) + q_1 - q_2\right\}. \quad (308)$$

Mr. Willans used 770 for the mechanical equivalent of heat and 461° for the absolute temperature of the zero of the Fahrenheit scale, and gives as an approximation for equation (308)

$$W = \frac{M}{A}\left(\frac{r_1}{T_1} + \frac{T_1 - T_2}{T_1 + T_2}\right)(T_1 - T_2), \quad . \quad . \quad . \quad (309)$$

and for the steam used per horse-power per hour

$$\frac{2571}{\left(\frac{r_1}{T_1} + \frac{T_1 - T_2}{T_1 + T_2}\right)(T_1 - T_2)}, \quad . \quad . \quad . \quad . \quad . \quad (310)$$

which may be compared with equation (243), page 180.

The best ratio of expansion was determined, for any given absolute pressure, by aid of the diagram Fig. 71. The abscissæ represent volumes, and the ordinates the work calculated by equation (309). *Ob* represents the volume of M pounds of steam admitted at the absolute pressure of 50 pounds. 1*a* represents the total work done during the admission, of which $1b$ must be expended in overcoming the back pressure during exhaust, leaving the useful work ab. The lines $2c$, $3e$, etc., represent the total forward work during admission and expansion to 2 times, 3 times, etc., the original volume, of which the portions $2d$, $3f$, etc., represent the work of overcoming the

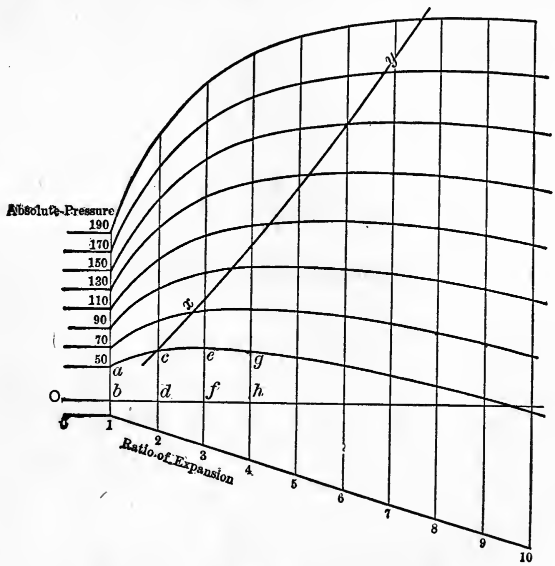


FIG. 71.

back pressure, and the portions cd, ef, etc., represent the useful work. The limit of useful expansion is that at which the useful work becomes a maximum. The line xy is drawn through the points on the curves for the several pressures, beyond which little or no gain is obtained from further expansion. The diagram is for a non-condensing engine exhausting against the pressure of the atmosphere, and it is apparent that much greater expansion would be advisable if the engine were condensing with a small absolute back pressure.

The nominal ratio of expansion in each test was fixed by dividing the volume of steam at the terminal pressure, exhausted from the low-pressure cylinder, by the capacity of the high-pressure cylinder at cut-off, neglecting clearances. The volume exhausted was assumed to be represented by the line CB in Fig. 72. This line CB was assumed to be 0.95 of the stroke, represented by AB.

During the tests the power of the engine was absorbed by a dynamo-machine, the load being adjusted by regulating the resistance in the circuit.

The feed-water was drawn by the feed-pump from a tank holding sufficient water for a test and mounted on a weighing-machine. The weight of the tank and water was noted at the

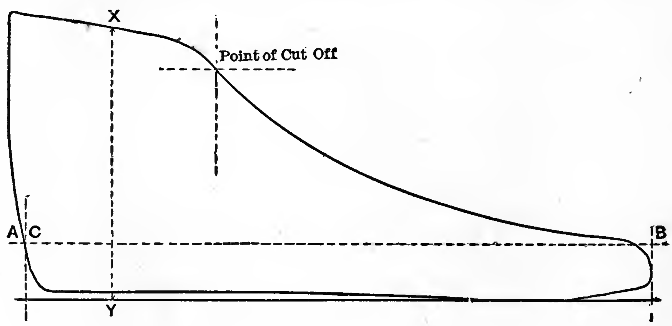


FIG. 72.

beginning and end of the test, at which times the suction of the feed-pump was disconnected. Before beginning a test the water in the boiler was raised above the middle of the glass water-gauge, and the feed-pump was disconnected while the weight was taken. When the water in the glass gauge reached a standard mark at about half the height of the water-gauge, the time of beginning the test was noted, a counter was thrown into gear, the feed-pump was connected and started, indicator-diagrams were taken, and temperatures were read on thermometers. A few minutes before the conclusion of a test the water was raised in the boiler about half an inch above the reference-mark on the glass gauge, and the feed-pump was disconnected and the weight of the tank and water was taken. When the water reached the reference-mark on the gauge the time was noted as the end of the test, and the counter was thrown out of gear. The only source of error was the uncertainty of the height of the water in the glass gauge, which gave an error not exceeding 0.25 per cent.

The pressure in the boiler rarely varied two pounds in any test. Three sets of indicator-diagrams were taken each hour, which were practically identical.

The largest cylinder, nearest the crank, is called the low-pressure or l.-p. cylinder; the other two cylinders are called the high-pressure or h.-p. cylinder, and the h. h.-p. cylinder. The main dimensions of the engine are given in the following table:

Willans Engine.

Stroke,			6 in.
Diameters:	h. h.-p. cylinder,		7 "
	h.-p. cylinder,		10 "
	l.-p. cylinder,		14 "
Net area:	h. h.-p. piston,		34.500 sq. in.
	under side of same,		31.416 "
	h.-p. piston,		71.472 "
	under side of same,		65.973 "
	l.-p. piston,		141.340 "
Capacity of trunk clearance:		h. h.-p.,	11.1 cu. in.
		h.-p.,	15.0 "
		l.-p.,	26.0 "
Capacity of cylinder clearance:		h. h.-p.,	14.8 "
		h.-p.,	30.0 "
		l.-p.,	33.6 "

The data and results of the tests are given in Tables XXV to XXXI.

In Table XXV the ratio of expansion corrected is determined by dividing the volume of steam at the terminal pressure, discharged from the l.-p. cylinder by such a volume of steam at mean admission pressure as would agree with the steam shown by the indicator at the point of cut-off. This method of calculating the expansions takes account of steam

TABLE XXV.

TESTS ON WILLANS ENGINE—SIMPLE, PRESSURE VARIED.

Date of trial.	Duration of trial, minutes.	Revolutions per min., mean during trial.	Point of cut-off in l.-p. cylinder.	Ratio of expansion, corrected.	Temperature of engine-room, Fahrenheit.	Pressures. Barometric, lbs.	Pressures. Boiler, above atmosphere.	Pressures. Cylinder, mean absolute, during admission.	Pressures. Absolute, at point of cut-off.	Pressures. Absolute, at end of stroke.	Pressures. Mean back, absolute.	Pressures. Total mean, referred to l.-p. piston.
1	2	3	4	5	6	7	8	9	10	11	12	13
Dec. 9...	180	393.5	0.604	1.79	68	14.49	36.25	**40.88**	35.7	22.1	14.8	19.6
Dec. 6...	270	408.4	0.437	2.43	65	14.46	51.0	**50.65**	42.8	20.2	14.46	22.62
Dec. 8...	242	409.1	0.339	3.09	65	14.57	74.0	**68.67**	58.2	22.9	15.4	29.14
Nov. 30..	169	403.17	0.296	3.45	64	14.66	85.0	**78.66**	65.4	22.12	15.4	31.06
Nov. 30..	180	400.9	0.264	3.85	58	14.75	97.0	**92.65**	76.3	23.8	15.6	36.83
Dec. 7...	176	397.7	0.2375	4.22	..	14.64	110.0	**98.14**	80.2	23.6	15.4	36.87
Dec. 5...	298	406.16	0.216	4.57	..	14.74	122.0	**106.34**	87.1	23.5	15.0	38.61

Indicated horse-power.	Feed-water used per hour, lbs.	Steam per indicated horse-power per hour. By tank.	Steam per indicated horse-power per hour. By indicator at cut-off.	Steam per indicated horse-power per hour. Required by perfect engine.	Percentage, efficiency.	Lbs. of water collected per hour, l.-p. steam-chest.	Percentage of total feed-water missing. At cut-off, l.-p. cylinder.	Percentage of total feed-water missing. At 0.604 of stroke, l.-p. cylinder.	Percentage of total feed-water missing. At end of stroke, l.-p. cylinder.	Heat units missing. Per hour at cut-off, l.-p. cylinder.	Heat units missing. Per stroke at cut-off, l.-p. cylinder.
14	15	16	17	18	19	20	21	22	23	24	25
16.51	706.0	**42.76**	37.74	34.67	**81.08**	4.6	11.7	11.7	10.4	77.213	3.269
19.77	711.0	**35.96**	29.03	28.62	**79.58**	6.5	19.3	20.53	17.58	126.588	5.166
25.51	830.8	**32.57**	23.92	23.02	**70.6**	2.7	26.5	28.02	19.26	200.544	8.168
26.8	795.2	**29 67**	22.62	21.15	**71.6**	....	23.7	24.1	19.2	170.494	7.048
31.61	850.0	**26.89**	20.21	19.24	**71.5**	8.0	24.8	23.3	18.83	189.140	7.863
31.49	877.7	**27.8**	19.7	18.66	**67.1**	7.2	31.25	27.7	23.44	244.866	10.261
33.55	874.4	**26.0**	18.36	17.9	**68.8**	2.8	29.56	26.7	21.53	229.976	9.436

required to fill clearances, but not of steam condensed during admission.

The cylinder pressure during admission, column 9, is the pressure from the indicator-diagrams, at a point midway between the beginning of the stroke and the point of cut-off.

The total mean pressure referred to the low-pressure cylinder, column 13, is, in Table XXV, simply the mean effective pressure. In the triple and the compound tests it is that mean effective pressure which, acting on the large piston, would give the indicated horse-power of the engine.

In calculating the steam per horse-power per hour shown by the indicator, column 17, there are two clearance spaces to be considered. One is the passage in the trunk between the cut-off ports and the valve-ports, which is filled from the back pressure to the pressure during admission. The other is the true cylinder clearance, which is filled from the pressure at the end of compression to the pressure during admission.

The steam per horse-power per hour, column 18, is calculated for the mean admission pressure by equation (310).

The percentage of efficiency, column 19, is obtained by dividing the steam per horse-power per hour required by a perfect engine by the steam actually used and weighed in the tank. Unless the method of determining this quantity is borne in mind the results in the tables are liable to be misleading.

The heat units missing at cut-off, column 24, are calculated on the assumption that all the steam not accounted for by the indicator at that point is present in the form of water at the temperature which the steam then has.

The heat units missing per stroke, column 25, are inserted to facilitate the comparison between the missing heat and the changes of temperature and surface of the cylinder walls, which are usually supposed to account for it.

The columns added in other tables are those required by the use of the steam in two or three cylinders.

Discussion of Results.—The tests made with the large cylinder only, that is, the simple tests given in Table XXV, show a regular decrease in the steam actually used per horse-power per hour, as the steam pressure and the number of expansions are increased simultaneously, with the revolutions nearly constant at 400 per minute. It is notable that the ratio of the steam shown by the indicator at cut-off to the steam used by a perfect engine is nearly constant, and but little larger than one. This, together with the increased condensation and re-evaporation, explains why the percentage of efficiency, so called, should diminish as the steam pressure and ratio of expansion increase.

TABLE XXVI.

Tests on Willans Engine—Simple Speed Tests.

Date of trial.	Duration of trial.	Revolutions per min., mean during trial.	Point of cut-off in l.-p. cylinder.	Ratio of expansion, corrected.	Temperature at engine-room, Fahrenheit.	Pressure.						
						Barometric, lbs.	Boiler, above atmosphere.	Cylinder, mean absolute, during admission.	Absolute, at point of cut-off.	Absolute, at end of stroke.	Mean back, absolute.	Total mean, referred to l.-p. piston.
1	2	3	4	5	6	7	8	9	10	11	12	13
Dec. 6...	270	408.4	0.437	2.43	65	14.46	51.0	**50.65**	42.8	20.2	14.46	22.62
Dec. 7...	153	200.6	0.437	2.3	60	14.57	44.0	**49.55**	43.6	21.7	15.2	21.72
Dec. 6...	122	110.5	0.437	2.25	50	14.44	40.25	**49.04**	43.8	22.1	14.7	23.13
Dec. 8...	242	409.1	0.339	3.09	65	14.57	74.0	**68.67**	58.2	22.9	15.4	29.14
Dec. 9...	152	205.2	0.339	3.087	66	14.47	66.5	**71.07**	60.1	25.2	15.1	32.0
Dec. 8...	127	112.7	0.339	2.99	68	14.40	62.0	**69.1**	60.6	26.8	14.65	32.9
Nov. 30..	180	400.9	0.264	3.85	58	14.75	97.0	**92.65**	76.3	23.8	15.6	36.83
Nov. 30..	118	223.0	0.264	3.74	..	14.7	84.7	**88.46**	74.6	23.1	15.5	35.1
Dec. 1...	178	122.8	0.264	3.72	66	14.93	80.0	**89.43**	75.6	27.0	15.4	38.0
Dec. 5...	298	406.16	0.216	4.57	..	14.74	122.0	**106.34**	87.1	23.5	15.0	38.61
Dec. 2...	119	223.7	0.216	4.46	..	14.98	112.0	**108.98**	90.0	28.1	15.5	42.8
Dec. 5...	123	138.0	0.216	4.32	..	14.74	105.4	**108.72**	93.1	28.4	15.5	44.31

Indicated horse-power.	Feed-water used per hour, lbs.	Steam per indicated horse-power p. hour, lbs.			Percentage, efficiency.	Lbs. of water collected per hour, l.-p. steam-chest.	Percentage of total feed-water missing.			Heat units missing.	
		By tank.	By indicator at cut-off.	Required by perfect engine.			At cut-off, l.-p. cylinder.	At 0.604 of stroke, l.-p. cylinder.	At end of stroke, l.-p. cylinder.	Per hour at cut-off.	Per stroke at cut-off.
14	15	16	17	18	19	20	21	22	23	24	25
19.77	711.0	**35.96**	29.03	28.62	**79.58**	6.5	19.3	20.53	17.58	126.588	5.166
19.32	389.4	**41.78**	31.79	29.15	**69.7**	8.0	23.9	24.5	17.57	85.931	7.139
5.47	251.8	**46.03**	30.16	29.37	**63.8**	6.0	34.5	34.5	28.8	79.433	11.97
25.51	830.8	**32.57**	23.92	23.02	**70.6**	2.7	26.5	28.02	19.26	200.544	8.168
14.05	483.35	**34.4**	22.55	22.5	**65.4**	5.0	34.44	31.16	23.07	150.973	12.262
7.937	323.1	**40.7**	22.08	22.9	**56.2**	5.0	45.81	41.63	32.71	134.384	19.864
31.61	850.0	**26.89**	20.21	19.24	**71.5**	8.0	24.8	23.3	18.83	189.140	7.863
16.75	465.25	**27.77**	20.8	19.71	**70.9**	....	24.75	23.4	19.52	101.216	7.564
9.98	339.8	**34.05**	19.61	19.64	**57.6**	....	42.5	37.7	28.70	129.168	17.523
33.55	874.4	**26.0**	18.36	17.9	**68.8**	2.8	29.56	26.7	21.53	229.976	9.436
20.49	618.6	**30.19**	17.41	17.68	**57.66**	4.0	42.33	35.58	26.26	232.556	14.338
13.09	408.8	**31.22**	17.41	17.7	**56.6**	3.0	44.5	38.2	30.65	161.254	19.475

Table XXVI gives four groups of three tests each, made on a simple engine with the steam pressure and ratio of expansion nearly constant, and with the speed varying from about 100 to about 400 revolutions per minute. They indicate a regular and notable reduction of steam consumption per horse-power per hour, and a regular gain of efficiency, with a marked reduction of initial condensation.

Four series of tests on the engine running compound were made; those in Table XXVII were made with the speed constant and the number of expansions varied according to the method shown in Fig. 71; those in Table XXVIII were made with the pressure constant and the ratio of expansion varied; those in Table XXIX were made in three groups, in each of which the number of expansions was constant and the pressure varied; those in Table XXX were made in three groups of three each with a varying speed.

In Table XXVII the tests stated in italic numerals were made with the ratio of expansion given by the expression

$$\frac{p}{25},$$

and the others with the ratio of expansions determined by the expression

$$\frac{p - 10}{25},$$

in which p is the absolute steam pressure. The effect of this variation of the number of expansions is shown by Fig. 73; on which are plotted also the consumption for a perfect engine of the type represented by equation (308), and the consumptions of the engine when working simple and when working compound.

As with the simple tests, the consumption of steam per horse-power per hour decreases with the simultaneous increase of steam pressure and expansion; but the percentage of efficiency does not show a notable falling off till the boiler-pressure reaches 130 pounds above the atmosphere.

TABLE XXVII.

TESTS ON WILLANS ENGINE, COMPOUND, PRESSURE AND EXPANSIONS VARIED.

Date of trial.	Duration of trial, minutes.	Revolutions per minute, mean during trial.	Point of cut-off.		Ratio of expansion, corrected.	Temperature of engine-room, Fahrenheit.	Pressure.								
			In high-pressure cylinder.	In low-pressure cylinder.			Barometric, pounds.	Boiler above atmosphere.	Cylinder, mean absolute during admission.	Absolute at point of cut-off, high-pressure cylinder.	Absolute at point of cut-off, low-pressure cylinder.	Absolute at end of high-pressure stroke.	Absolute at end of low-pressure stroke.	Mean back absolute, high-pressure cylinder.	Mean back absolute, low-pressure cylinder.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Oct. 19	180.0	400.0	0.6	0.604	3.35	....	14.97	80.2	78.59	71.21	31.35	45.22	19.3	48.3	15.8
Oct. 19	123.0	401.08	0.6	0.604	3.5	....	14.94	90.5	87.54	79.94	33.7	....	20.8	53.7	15.4
Oct. 12	176.5	397.6	0.523	0.604	4.02	....	14.55	90.4	90.0	81.21	30.0	....	18.9	48.25	15.6
Oct. 14	118.0	401.5	0.523	0.604	4.0	....	14.63	100.5	98.83	88.82	31.3	....	19.6	52.79	15.1
Oct. 20 and Dec. 21	124. and 240.5	405.3	0.470	0.604	4.42	....	14.7	104.6	97.75	88.25	30.27	....	19.2	48.65	14.97
Oct. 20	123.0	402.9	0.470	0.604	4.51	....	14.9	113.85	109.3	95.5	33.2	....	20.2	53.6	15.7
Oct. 21	121	402.66	0.427	0.604	4.9	....	14.96	113.0	108.96	95.0	32.2	....	18.7	48.7	15.7
Oct. 22	181	402.7	0.427	0.604	4.84	....	15.02	124.5	121.27	105.0	32.5	....	20.4	53.4	15.4
Oct. 26	124	404.15	0.392	0.604	5.26	67	14.94	124.8	119.92	104.6	31.15	....	19.7	49.6	15.77
Oct. 27	189	405.5	0.392	0.604	5.33	68	14.78	135.4	130.58	113.3	33.1	....	20.6	54.0	15.4
Oct. 27 and Nov. 28	181.5 and 124	401.9	0.362	0.604	5.87	....	14.72	133.6	129.9	110.13	31.35	...	19.2	50.15	15.2
Oct. 28	184	398.66	0.362	0.604	5.8	73	14.6	143.7	139.7	119.6	32.7	....	20.7	54.7	15.4
Nov. 2	120	405.1	0.336	0.604	6.2	64	14.35	145.2	141.85	120.0	31.8	....	20.0	51.2	15.2
Nov. 3	179	404.05	0.336	0.604	6.15	71	14.14	155.0	149.84	126.1	33.2	48.8	20.2	53.74	15.0
Nov. 12	307	402.1	0.313	0.604	....	...	...	....	...	....	....	....	...	....	...
Nov. 5	177	401.2	0.313	0.604	6.46	67	14.5	165.0	158.5	133.7	33.4	....	20.9	53.7	14.9

TABLE XXVII.—*Continued.*

Pressure.				Indicated horse-power.	Feed-water used per hour, pounds.	Steam per indicated horse-power per hour.			Percentage of efficiency.	Pounds of water collected per hour, low-pressure steam-chest.	Percentage of total feed-water missing.			Heat units missing.	
Mean on high-pressure piston.	Mean on high-pressure piston, under side.	Mean on low-pressure piston.	Total mean referred to low-pressure piston.			By tank.	By indicator at cut-off, high-pressure cylinder.	Required by perfect engine.			At cut-off, high-pressure cylinder.	At cut-off, low-pressure cylinder.	At end of stroke, low-pressure cylinder.	Per hour at cut-off, high-pressure cylinder.	Per stroke at cut-off, high-pressure cylinder.
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
20.24	5.96	16.06	29.08	24.91	652.0	26.17	24.8	21.15	80.8	10.0	5.2	15.17	14.14	30.510	1.271
22.592	7.198	19.156	23.934	29.14	703.9	24.16	22.81	19.86	82.2	9.5	5.0	15.2	14.25	33.804	1.404
25.29	6.29	14.87	30.71	26.14	639.8	24.47	22.61	19.58	80.0	11.5	7.6	17.42	14.53	43.424	1.819
28.05	6.86	17.53	34.93	30.0	690.0	23.0	21.56	18.6	80.8	11.0	6.23	21.25	17.13	38.227	1.586
28.7	6.39	15.5	33.0	28.65	653.4	22.81	20.45	18.7	82.0	11.0	10.2	16.36	14.65	59.948	2.459
32.1	7.2	18.7	38.26	33 0	705.3	21.37	19.34	17.65	82.5	12.0	9.5	16.25	16.59	59.560	2.464
32.531	7.513	16.0	35.95	30.98	663.4	21.41	18.9	17.68	82.5	8.5	11.3	13.3	17.64	66.412	2.748
36.78	7.856	18.0	40.26	34.73	721.3	20.76	18.6	16.8	80.9	13.0	10.5	19.6	17.5	66.261	2.742
35.992	6.654	16.07	37.37	32.33	688.5	21.29	18.6	16.87	79.2	14.5	12.5	18.9	16.07	75.723	3.122
39.0	7.4	18.8	41.85	36.31	736.5	20.35	17.9	16.25	80.0	16.5	11.7	19.1	17.55	75.686	3.110
37.61	6.2	16.8	38.65	33.25	673.27	20.26	17.38	16.26	80.25	14.8	14.2	17.0	17.12	83.637	3.468
41.525	7.983	18.22	42.94	36.64	734.4	20.0	17.17	15.72	78.6	17.5	13.9	19.9	17.54	89.513	3.748
41.0	8.68	17.3	42.02	36.43	714.5	19.64	16.7	15.61	79.4	17.0	14.7	19.76	17.49	91.926	3.781
43.86	7.02	19.2	44.62	38.59	750.8	19.45	16.76	15.23	78.3	15.0	15.1	20.6	20.69	99.180	4.091
44.87	7.28	16.84	42.88	36.95	722.4	19.56	15.87	....	....	16.8	18.8	....	....	....	....
46.02	7.77	19.29	46.04	39.55	759.0	19.19	15.92	14.87	77.4	16.5	17.0	21.3	19.55	112.103	4.657

The reduction of initial condensation by compounding is very noticeable, and is accompanied by a marked reduction of steam consumption at and above 80 pounds absolute. If we may assume that the curve for the compound engine with the expansion $\frac{p}{25}$ can be produced, it would show that compounding ceases to be of value somewhere between 50 and 60

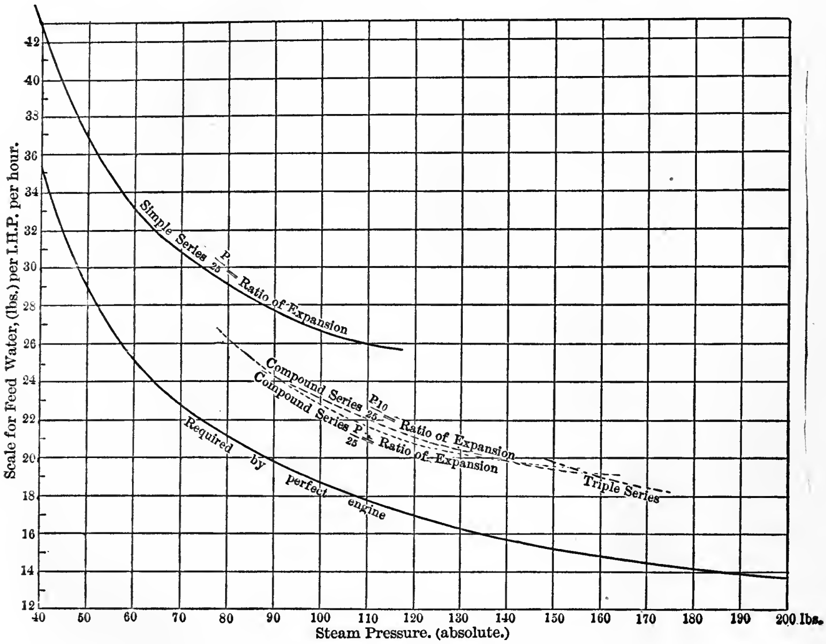


FIG. 73.

pounds absolute, provided that the engine exhausts against atmospheric pressure.

Table XXVIII gives the results of tests made with a constant steam pressure and varying expansion, while Table XXIX gives the results of tests made with fixed rates of expansion and varying pressure. A comparison of these tables with Table XXVII shows that the rates of expansion in the latter are well chosen.

TABLE XXVIII.—Tests on Willans Engine, Compound, Pressure Constant.

Date of trial.	Duration of trial, minutes.	Revolutions per minute, mean during trial.	Point of cut-off.		Ratio of expansion, corrected.	Temperature of engine-room, Fahrenheit.	Pressure.							
			In h. p. cylinder.	In l. p. cylinder.			Barometric, lbs.	Boiler above atmosphere.	Cylin., mean absolute during admission.	Absolute at point of cut-off, h.p. cylinder.	Absolute at point of cut-off, l.p. cylinder.	Absolute at end of stroke, l. p. cylinder.	Mean back absolute, h. p. cylinder.	Mean back absolute, l. p. cylinder.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Oct. 21	186	406.8	0.470	0.604	4.46	....	14.95	134.8	128.85	112.5	36.7	22.7	61.5	15.5
Oct. 26	182	405.0	0.427	0.604	4.85	....	14.99	135.0	129.19	113.0	34.67	21.4	56.3	14.99
Oct. 27	189	405.5	0.392	0.604	5.33	68	14.78	135.4	130.58	113.3	33.1	20.6	54.0	15.4
Oct. 27 & Nov. 28	124 & 181.5	401.9	0.362	0.604	5.87	68	14.72	133.65	129.97	112.2	32.2	19.9	51.0	15.1
Nov. 2	182	402.6	0.336	0.604	6.10	68	14.35	135.0	131.1	111.1	30.9	19.6	48.5	15.4
Nov. 4	180	400.0	0.313	0.604	6.52	72	14.24	135.0	130.09	109.2	29.1	18.0	45.3	14.8
Nov. 28	200	404.45	0.233	0.604	8.38	69	14.745	135.5	128.15	106.4	24.4	15.6	38.5	15.4

Pressure.				Indicated horse-power.	Feed-water used per hour, lbs.	Steam per indicated horse-power per hour.			Percentage efficiency.	Lbs. of water collected per hour, l. p. steam-chest.	Percentage of total feed-water missing.			Heat units missing.	
Mean on h. p. piston.	Mean on h. p. piston, under side.	Mean on l. p. piston.	Total mean referred to l. p. piston.			By tank.	By indicator at cut-off, h. p. cylinder.	Required by perfect engine.			At cut-off, h. p. cylinder.	At cut-off, l. p. cylinder.	At end of stroke, l. p. cylinder.	Per hour at cut-off, h. p. cylinder.	Per stroke at cut-off, h. p. cylinder.
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
39.57	8.95	21.8	45.97	40.03	830.9	20.75	18.9	16.32	78.6	13.0	8.9	20.1	19.27	65.394	2.679
40.14	8.27	19.8	43.95	38.0	778.3	20.48	18.4	16.3	79.5	12.5	10.18	19.8	18.77	69.498	2.86
39.0	7.4	18.8	41.85	36.31	736.5	20.35	17.9	16.25	80.0	16.5	11.7	19.1	17.55	75.686	3.110
37.61	6.2	16.8	38.62	33.25	673.25	20.26	17.38	16.26	80.2	14.87	14.25	17.0	17.12	83.637	3.468
37.8	5.86	16.18	38.01	32.76	654.7	20.0	17.13	16.2	81.0	17.25	14.28	16.0	12.76	82.017	3.395
37.8	6.2	14.4	36.47	31.23	634.6	20.32	16.56	16.26	80.0	14.5	18.4	18.5	17.35	102.989	4.291
32.2	4.5	8.25	26.6	23.03	534.0	23.14	17.37	16.37	70.7	9.25	25.0	18.43	14.47	117.677	4.848

TABLE XXIX.

TESTS ON WILLANS ENGINE, COMPOUND, EXPANSIONS CONSTANT.

Date of trial.	Duration of trial, minutes.	Revolutions per minute, mean during trial.	Point of cut-off. In high-pressure cylinder.	Point of cut-off. In low-pressure cylinder.	Ratio of expansion corrected.	Temperature of engine-room, Fahrenheit.	Pressure. Barometric.	Pressure. Boiler, above atmosphere.	Pressure. Cylinder, mean absolute, during admission.	Pressure. Absolute at point of cut-off, high-pressure cylinder.	Pressure. Absolute at point of cut-off, low-pressure cylinder.	Pressure. Absolute at end of stroke, low-pressure cylinder.	Pressure. Mean back absolute, high-pressure cylinder.	Pressure. Mean back absolute, low-pressure cylinder.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Nov. 8 and Dec. 21	156 and 188	399.9	0.470	0.604	4.51	61	14.609	60.75	**60.98**	53.2	21.5	13.41	31.9	16.0
Dec. 29	237	413.1	0.470	0.604	4.63	48	14.84	75.0	**72.64**	62.1	24.7	15.56	37.1	15.9
Nov. 8	150	399.8	0.470	0.604	4.42	68	14.64	82.3	**81.09**	71.5	25.89	16.2	40.0	15.7
Dec. 30	242	405.7	0.470	0.604	4.57	53	14.8	96.0	**89.8**	77.5	28.6	18.0	43.3	15.4
Oct. 20 and Dec. 21	124 and 240.5	405.3	0.470	0.604	4.42	63	14.7	104.6	**97.75**	84.55	30.27	19.2	47.16	14.97
Oct. 20	123	402.9	0.470	0.604	4.51	....	14.902	113.85	**109.3**	95.5	33.2	20.2	53.6	15.7
Jan. 2	177	409.6	0.470	0.604	4.56	57	14.39	128.2	**120.62**	103.9	34.4	21.7	57.1	14.7
Oct. 21	186	406.8	0.470	0.604	4.46	...	14.95	134.8	**128.85**	112.5	36.7	22.7	61.5	15.5
Dec. 22	180	400.3	0.470	0.604	4.76	56	14.75	130.0	**60.95**	51.2	21.6	13.45	32.0	16.0
Nov. 1	120	421.66	362	0.604	5.57	72	14.34	162.6	**155.44**	135.5	36.0	23.09	58.4	15.8
Nov. 3	179	411.3	336	0.604	6.07	74	14.175	165.0	**157.29**	131.1	34.6	21.27	55.8	14.73
Nov. 5	180	401.2	313	0.604	6.46	67	14.502	165.0	**158.5**	133.7	33.4	20.9	53.7	14.9
Nov. 2	182	402.6	0.336	0.604	6.1	68	14.35	135.0	**131.1**	111.1	30.9	19.6	48.5	15.4
Nov. 2	120	405.1	0.336	0.604	6.2	64	14.35	145.2	**141.85**	120.0	31.8	20.0	51.2	15.2
Nov. 3	180	404.05	0.336	0.604	6.15	71	14.143	155.0	**149.84**	126.1	33.2	20.2	53.74	15.0
Nov. 3	179	411.3	0.336	0.604	6.07	74	14.175	165.0	**157.29**	133.1	34.6	21.27	55.8	14.73

TABLE XXIX.—*Continued.*

Pressure.						Steam per indicated horse-power per hour.					Percentage of total feed-water missing.			Heat units missing.	
Mean, on high-pressure piston.	Mean, on high-pressure piston, under side.	Mean, on low-pressure piston.	Total referred to low-pressure piston.	Indicated horse-power.	Feed-water used per hour, pounds.	By tank.	By indicator at cut-off, high-pressure cylinder.	Required by perfect engine.	Percentage of efficiency.	Pounds collected per hour, low-pressure steam-chest.	At cut-off, high-pressure cylinder.	At cut-off, low-pressure cylinder.	At end of stroke, low-pressure cylinder.	Per hour at cut-off, high-pressure cylinder.	Per stroke at cut-off, high-pressure cylinder.
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
15.6	3.268	6.056	15.45	13.22	438.8	33.17	27.7	24.93	75.15	9.37	17.49	14.3	12.62	70.337	2.931
18.233	4.233	9.041	20.22	17.88	510.3	28.54	24.19	22.2	77.7	14.75	15.25	12.69	9.57	70.525	2.845
23.05	5.14	10.91	24.94	21.35	537.6	25.18	22.61	20.78	82.5	8.25	10.1	16.8	14.88	49.320	2.055
25.58	5.516	12.975	28.462	24.71	608.4	24.62	21.36	19.58	79.5	15.5	13.22	15.94	13.6	72.038	2.959
28.7	6.39	15.5	33.00	28.65	653.45	22.81	20.45	18.7	82.0	11.0	10.2	16.36	14.65	59.948	2.459
32.1	7.2	18.7	38.26	33.0	705.3	21.37	19.34	17.65	82.5	12.0	9.5	16.25	16.59	59.560	2.463
36.0	8.33	20.22	42.28	37.07	798.6	21.54	18.9	16.81	78.0	12.0	12.25	21.6	18.92	86.161	3.505
39.6	8.94	21.8	45.97	40.03	830.9	20.75	18.9	16.32	78.6	13.0	8.9	20.1	19.27	65.394	2.679
15.05	3.058	5.80	14.825	12.705	410.6	32.32	27.35	24.95	77 1	9.0	15.36	8.33	6.26	57.768	2.405
47.8	11	21.0	50.226	45.3	869	19.18	16.67	15.0	78.2	14.5	13.25	22.4	18.54	98.520	3.894
47.28	7.77	20.13	47.53	41.85	801	19.14	16.35	14.9	77.8	14.5	14.6	20.7	18.54	101.213	4.095
46.2	7.77	19.29	46.04	39.55	759	19.19	15.92	14.87	77.4	16.5	17.2	21.3	19.55	112.103	4.656
37.8	5.86	16.18	38.01	32.76	654 7	20.0	17.13	16.2	81	17.25	14.3	16.04	12.76	82.017	3.396
41	8.68	17.3	42.02	36.43	714.5	19.64	16.7	15.61	79.4	17	14.7	19.76	17.49	91.926	3.781
43.86	7.02	19.2	44.62	38.59	750.8	19.47	16.76	15.23	78.3	15	15.3	20.6	20.69	99.180	4.091
47.28	7.77	20.13	47.53	41.85	801	19.14	16.35	14.9	77.8	14.5	14.6	20.7	18.84	101.213	4.095

Table XXX gives the results of tests made with speed varying from 100 to 400 revolutions per minute. These tests, like the simple speed tests, show a reduction of steam consumption at higher speeds and a corresponding improvement in efficiency, due to the reduction of initial condensation. Mr. Willans points out the fact that the total initial condensation in a given time remains nearly constant for these compound speed tests, so that doubling the speed of rotation reduces the initial condensation per stroke one half. He also shows that this does not hold for the simple speed tests.

Table XXXI gives the results of tests made with the engine running with triple expansion. A comparison with the results of Table XXVII shows that it is advisable to use a compound engine below 160 pounds pressure absolute, and a triple-expansion engine above that pressure. The curves of steam consumption in Fig. 73 indicate the same fact. In connection with this conclusion, and when considering the small degree of expansion used even with high-pressure steam, it is to be remembered that this engine exhausted against the atmosphere. Were a condenser to be used with the engine, and a back pressure of two pounds absolute assumed, then the method used for determining the ratio of expansion would give results more nearly in accord with the ordinary practice with compound and triple-expansion engines, which commonly work under such conditions.

To determine the leakage past valves and pistons the engine was blocked in various positions and exposed to the pressure of steam from the boiler; meanwhile the water-level in the boiler was watched, and the fall of water was assumed to be due to leakage and condensation. The largest amount thus determined was 15 pounds per hour—too small a quantity to be satisfactorily determined in that manner.

A separator was at first put on the steam-pipe for the purpose of abstracting the condensation in the steam-pipe and priming from the boiler, but the entire amount collected in an hour was three pounds, therefore the separator was removed.

Afterwards calorimeter tests of the quality of the steam

TABLE XXX.—TESTS ON WILLANS ENGINE, COMPOUND, SPEED TESTS.

Date of trial.	Duration of trial.	Revolutions per minute, mean during trial.	Point of cut-off. In high-pressure cylinder.	Point of cut-off. In low-pressure cylinder.	Ratio of expansion, corrected.	Temperature of engine-room, Fahrenheit.	Pressure. Barometric.	Pressure. Boiler, above atmosphere.	Pressure. Cylinder, mean absolute during admission.	Pressure. Absolute at point of cut-off, high-pres. cylin.	Pressure. Absolute at point of cut-off, low-pressure cylinder.	Pressure. Absolute at end of stroke, low-pressure cylinder.	Pressure. Mean back absolute, high-pressure cylinder.	Pressure. Mean back absolute, l.p. cylinder.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Oct. 19	123	401.08	0.604	0.604	3.5	....	14.94	90.5	87.54	79.94	33.7	20.8	53.7	15.4
Nov. 23	123	210.8	0.604	0.604	3.42	59	14.76	85.16	90.41	81.5	33.5	21.9	51.8	15.5
Nov. 24	150	122.0	0.604	0.604	3.41	....	14.64	83.5	90.84	82.5	33.9	21.9	51.8	15.4
Oct. 20	123	402.9	0.470	0.604	4.51	....	14.9	113.85	109.3	95.5	33.2	20.2	53.6	15.7
Nov. 24	125	211.98	0.470	0.604	4.5	64	14.64	105.0	109.14	92.7	32.4	20.9	50.2	15.2
Nov. 25	116	123.8	0.470	0.604	4.36	60	14.6	102.6	110.8	96.2	33.7	21.4	51.2	15.0
Oct. 27	189	405.5	0.392	0.604	5.33	68	14.78	135.4	130.58	113.3	33.1	20.6	54.0	15.4
Nov. 26	124	216.4	0.392	0.604	5.22	70	14.64	124.0	128.84	110.9	32.9	21.1	51.4	15.6
Nov. 25	150	130.86	0.392	0.604	5.22	63	14.64	120.0	128.79	111.6	34.3	22.6	52.9	15.0

Pressure. Mean on h. p. piston.	Pressure. Mean on h. p. piston, under side.	Pressure. Mean on l. p. piston.	Pressure. Total mean referred to l. p. piston.	Indicated horse-power.	Feed-water used per hour, pounds.	Steam per ind. h.-power p. hour. By tank.	Steam per ind. h.-power p. hour. By indicator at cut-off, h. p. cylinder.	Steam per ind. h.-power p. hour. Required by perfect engine.	Percentage efficiency.	Lbs. of water collected per hour, l. p. steam-chest.	Percentage of total feed-water missing. At cut-off, h. p. cylinder.	Percentage of total feed-water missing. At cut-off, l. p. cylinder.	Percentage of total feed-water missing. At end of stroke, l. p. cylinder.	Heat units missing. Per hour at cut-off, h. p. cylinder.	Heat units missing. Per stroke at cut-off, h. p. cylinder.
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
22.59	7.19	19.15	33.93	29.14	703.9	24.16	22.85	19.86	82.2	9.5	5.0	15.2	14.25	33.804	1.404
27.1	9.71	18.83	37.05	16.73	422.92	25.27	22.07	19.52	77.2	15.6	12.6	24.8	19.35	47.898	3.786
27.94	10.18	19.74	38.39	10.02	270.8	27.02	21.55	19.48	72.0	9.2	20.2	31.2	27.07	48.919	6.682
32.1	7.2	18.7	38.26	33.0	705.3	21.37	19.34	17.65	82.5	12.0	9.5	16.25	16.59	59.560	2.464
35.33	9.0	18.20	40.234	18.26	422.4	23.13	18.42	17.67	76.3	15.0	20.3	26.7	22.77	76.282	6.000
35.15	9.62	19.5	41.73	11.06	273.6	24.73	18.49	17.52	70.8	9.3	25.2	31.4	28.17	61.153	8.230
39.0	7.4	18.8	41.85	36.31	736.5	20.35	17.9	16.25	80.0	16.5	11.7	19.1	17.55	75.686	3.110
40.2	10.83	18.54	43.89	20.33	432.5	21.27	17.19	16.3	76.6	16.9	19.1	26.4	22.59	72.824	5.611
40.86	9.83	20.03	45.24	12.67	300.0	23.67	16.65	16.31	68.8	5.8	29.66	32.6	27.41	78.053	9.94

TABLE XXXI.—TESTS ON WILLANS ENGINE, TRIPLE EXPANSION.

Date of trial.	Duration of trial, minutes.	Revolutions per minute, mean during trial.	Point of cut-off. In h. high-pressure cylinder.	Point of cut-off. In high-pressure cylinder.	Point of cut-off. In low-pressure cylinder.	Ratio of expansion corrected.	Temperature of engine-room, Fahrenheit.	Pressure. Barometric, pounds.	Pressure. Boiler above atmosphere.	Pressure. Cylinder, mean absolute during admission.	Pressure. Absolute at point of cut-off, h. h.-p. cylinder.	Pressure. Absol. at point of cut-off, h.-p. cylinder.	Pressure. Absol. at point of cut-off, l.-p. cylinder.	Pressure. Absolute at end of stroke, h. high-pres. cylinder.	Pressure. Absol. at end of stroke, h.-p. cylinder.	Pressure. Absol. at end of stroke, l.-p. cylinder.	Pressure. Mean back absolute, h.h.-p. cylinder.	Pressure. Mean back absolute, h.-p. cylinder.	Pressure. Mean back absolute, l.-p. cylinder.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Dec. 14 and Jan. 17	178 and 289	405.6	0.694	0.604	0.604	5.97	61.5	14.74	150.15	**151.9**	148.9	70.3	30.35	105.1	46.15	19.52	110.3	49.0	15.35
Dec. 19	241	409.0	0.647	0.604	0.604	6.259	57	14.5	151.9	**149.65**	146.1	68.5	29.2	96.4	43.8	18.5	103.4	47.5	14.8
Jan. 14*	300	401.2	0.647	0.604	0.604	6.158	....	14.99	160.0	**159.49**	156.0	69.0	29.9	102.8	45.4	19.4	107.5	48.4	15.3
Dec. 16	300	408.4	0.606	0.604	0.604	6.507	72	14.47	160.8	**158.07**	155.7	69.7	29.2	96.0	44.4	18.6	105.8	47.2	14.8
Dec. 17†	240	414.85	0.606	0.604	0.604	....	65	14.7	172.0	....	....	....	....	....	....	....	....	....	....
Jan. 12‡	184	400.4	0.606	0.604	0.604	6.52	56	15.04	172.5	**172.54**	168.6	71.4	30.2	104.5	47.1	19.6	110.4	49.2	15.4
Jan. 16§	243	416.46	0.606	0.604	0.604	....	60	14.88	....	....	165.9	71.2	30.5	....	....	19.6	110.4	49.4	15.3

* Prof. Kennedy. † Prof. Unwin. ‡ Mr. Hartwell. § Mr. McFarlane Gray.

Pressure. Mean on h. h.-p. piston.	Pressure. Mean on h. h.-p. piston, under side.	Pressure. Mean on h.-p. piston.	Pressure. Mean on h.-p. piston, under side.	Pressure. Mean on l.-p. piston.	Pressure. Total mean referred to l. p. piston.	Indicated horse-power.	Feed-water used per hour, pounds.	Steam per indicated horse-power per hour. By tank.	Steam per indicated horse-power per hour. By indicator at cut-off, h. h.-p. cylin.	Steam per indicated horse-power per hour. Required by perfect engine.	Percentage efficiency.	Pounds of water collected per hour. H.-p. steam-chest.	Pounds of water collected per hour. L.-p. steam-chest.	Percentage of total feed-water missing. At cut-off, h. h.-p. cylinder.	Percentage of total feed-water missing. At cut-off, h.-p. cylinder.	Percentage of total feed-water missing. At cut-off, l.-p. cylinder.	Percentage of total feed-water missing. At end of stroke, l. p. cylinder.	Heat units missing. Per hour at cut-off, h. h.-p. cylinder.	Heat units missing. Per stroke at cut-off, h. h.-p. cylin.
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
32.94	14.58	22.74	6.69	15.27	41.11	**35.69**	702.6	**19.68**	18.64	15.16	**77.05**	5.04	14.53	5.33	14.84	22.12	18.01	32.122	1.316
33.85	14.1	21.66	6.25	14.266	39.498	**34.58**	669.2	**19.35**	18.29	15.25	**78.8**	7.3	15.75	5.46	11.95	20.48	17.68	31.549	1.285
39.91	15.06	22.04	6.8	15.045	42.424	**36.44**	696.4	**19.11**	18.26	14.57	**76.2**	3.18	15.7	4.43	16.32	23.41	18.57	26.521	1.101
36.71	12.5	22.5	6.64	14.5	40.675	**35.56**	682.6	**19.19**	17.88	14.9	**77.6**	4.5	13.5	6.84	12.06	22.11	18.81	40.000	1.632
39.11	11.359	26.1	7.158	16.454	45.032	**40.041**	749.0	**18.7**	16.77	....	....	3.8	14.6	10.32	11.20	20.64	14.92	....	....
44.533	16.11	23.55	6.39	15.55	44.855	**38.45**	709.5	**18.45**	17.52	14.36	**77.8**	4.5	13.5	5.01	15.33	24.21	19.25	30.345	1.263
43.908	14.680	22.728	6.886	16.63	45.311	**40.42**	745.0	**18.42**	....	....	**77.7**	2.1	15.4	....	....	....	....	....	....

from the boiler were made by blowing a hundred-weight of steam into the water in the iron feed-tank and noting the rise of temperature. The results of the tests are given in the following table, in which w is the weight of the water in the tank and the water equivalent of the iron forming the tank, u is the weight of the steam blown in, p is the absolute pressure of the steam, and t_3 is the corresponding temperature, while t_1 and t_2 are the initial and final temperatures of the water in the tank; x' is the percentage of steam using Regnault's value of the total heat of steam and heat of the liquid; x is the percentage of steam corrected for Bosscha's specific heat for water.

QUALITY OF STEAM USED IN WILLANS ENGINE.

No.	p	t_1	t_2	t_3	w	u	x^1	x	Duration of blow in minutes.	Remarks.
1	105	8.505	24.30	160.0	4416	110.375	99.82	99.96	7.0	Steam blown into warm water.
2	173	23.35	39.3	188.0	4181	109.4	96.87	96.38	7.0	
3	160	11.6	25.81	184.0	4919	110.38	99.27	99.488	8.0	Steam blown in very fast.
4	139	12.00	25.82	178 0	4921	110.2	96.29	96.46	4.0	
5	154	15.6	29.4	183 0	5032	109.97	99.66	99.76	15.0	
6	162	9.10	23.32	185.0	4947	111.1	98.63	98.93	4.5	Water in boiler low.
7	178	9.827	23.501	188.8	5263	112.0	100.4	100.72	12.0	
8	65	10.77	25.19	147.8	4898	112.0	100.8	100.48	9.0	
9	127	8.18	22.73	174.0	4836	110.8	99.63	99.873	9.0	

The diagram in Fig. 74 is given to show the increase of

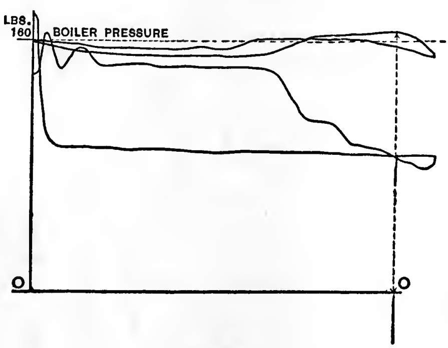


FIG. 74.

the pressure in the steam-chest above the boiler-pressure after cut-off, due to the velocity and the density of the steam at 160 pounds pressure.

The diagrams in Figs. 75, 76, and 77 were taken from the compression chamber between the low-pressure cylinder and the engine-shaft. Fig. 76 was taken under the normal condition with dry air in the compression-cylinder, in which case the compression and expansion curves are scarcely distinguishable, and are both sensibly adiabatic. Fig. 75 was taken when a considerable amount of water was purposely injected into the compression-cylinder, and Fig. 77 was taken with steam in the cylinder instead of air: both indicate an energetic interchange of heat between the fluid and the cylinder walls.

FIG. 75.

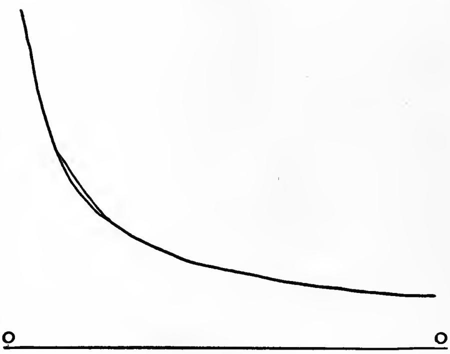


FIG. 76.

Institute of Technology Tests. — The tests recorded in Table XXXII are one series of a large number of tests on simple engines, made in the mechanical engineering laboratory of the Massachusetts Institute of Technology, and forming a part of the regular instruction in that laboratory.

FIG. 77.

TABLE XXXII.—TESTS ON HARRIS-CORLISS ENGINE. LABORATORY OF THE INSTITUTE OF TECHNOLOGY.

No. of test.	Duration min.	Boiler-pressure above the atmosphere, pounds.	Horse-power of the engine.	Revolutions per minute.	Barometer, inches of mercury.	Crank end. Initial pressure.	Crank end. Pressure at cut-off.	Crank end. Pressure at release.	Crank end. Pressure at compression.	Crank end. Per cent cut-off.	Crank end. Per cent release.	Crank end. Per cent compression.	Crank end. M. E. P.	Crank end. Horse-power.	Head end. Initial pressure.	Head end. Pressure at cut-off.	Head end. Pressure at release.	Head end. Pressure at compression.
309	55	70.9	2.916	60.87	29.95	67.5	61.2	−3.2	3.0	2.5	97.0	1.5	5.63	1.014	68.5	57.3	0.0	3.5
310	60	71.0	3.905	61.10	30.34	62.5	58.8	−1.9	0.5	5.0	97.0	2.0	10.17	1.839	66.0	52.5	0.0	2.8
301	45	71.1	5.502	59.31	30.30	69.3	60.0	0.0	1.7	7.0	95.5	2.5	13.53	2.375	70.4	60.4	3.0	2.5
300	55	70.8	5.627	59.74	30.28	68.9	58.7	0.0	3.3	7.5	95.5	2.0	14.13	2.499	70.9	59.9	3.3	5.7
308	55	71.7	6.113	61.36	30.22	56.7	37.7	1.9	2.0	18.0	96.0	1.0	17.38	3.157	56.3	37.7	2.5	2.2
311	55	73.8	6.210	61.45	29.66	55.0	37.8	1.2	2.3	18.0	97.0	1.0	17.02	3.096	53.5	32.5	3.2	2.1
304	55	72.7	6.896	60.43	29.96	68.8	61.1	1.0	2.1	11.0	96.0	1.7	17.82	3.188	70.0	59.2	4.1	2.1
306	55	71.9	7.416	60.54	30.40	68.0	60.0	1.2	1.8	11.0	97.0	2.0	18.45	3.310	69.9	56.2	5.7	3.1
305	55	72.2	7.536	60.03	29.90	71.2	60.3	3.7	3.4	10.0	95.0	2.5	19.13	3.399	69.0	57.0	3.6	1.7
312	60	72.0	8.105	61.20	30.40	57.0	41.6	3.8	1.3	20.5	98.0	1.0	20.97	3.800	57.5	42.7	6.9	1.6
307	55	76.5	9.586	60.67	29.92	68.0	55.7	5.0	0.4	18.5	96.5	1.0	25.61	4.599	70.0	58.3	6.5	1.7
303	55	72.0	10.034	59.44	30.44	67.8	57.9	6.0	2.2	20.0	95.0	1.5	27.58	4.852	71.0	53.4	7.6	3.4
313	60	69.2	10.306	60.80	30.30	64.3	58.9	2.2	0.9	13.0	97.0	1.0	20.64	3.714	67.2	58.7	13.4	2.4
302	55	71.9	11.694	60.36	30.05	68.7	57.8	5.9	4.1	19.0	95.0	2.0	26.92	4.810	70.0	57.3	14.2	4.0

No. of test.	Head end. Per cent cut-off.	Head end. Per cent release.	Head end. Per cent compression.	Head end. M. E. P.	Head end. Horse-power.	Water per H. P. per hour. By tank.	Water per H. P. per hour. By indicator.	Weight of steam per stroke. Crank end. Cut-off.	Crank end. Release.	Crank end. Compression.	Head end. Cut-off.	Head end. Release.	Head end. Compression.	Weight of mixture per revolution.	Per cent of mixture accounted for by indicator as steam. Cut-off.	Release	Diff.	Lbs. re-evap. per horse-power per hour.
309	4.0	97.0	2.5	10.21	1.902	73.14	54.10	.0075	.0205	.0016	.0111	.0269	.0026	.0626	29.7	75.7	46.0	36.1
310	5.5	97.0	3.5	11.05	2.066	57.94	42.90	.0102	.0230	.0015	.0115	.0273	.0028	.0665	32.6	75.6	43.0	26.6
301	8.5	95.5	5.0	17.23	3.127	48.07	27.02	.0127	.0256	.0018	.0170	.0230	.0032	.0811	36.6	71.0	34.4	18.1
300	9.0	95.5	4.0	17.11	3.128	48.08	33.76	.0130	.0256	.0018	.0175	.0325	.0034	.0807	37.9	72.1	34.2	17.2
308	15.0	96.0	2.0	15.74	2.956	46.42	33.90	.0184	.0287	.0014	.0178	.0312	.0022	.0806	44.9	74.3	29.4	14.3
311	20.0	97.0	3.0	16.56	3.114	45.06	33.10	.0183	.0275	.0015	.0200	.0323	.0025	.0799	47.9	74.9	27.0	12.8
304	11.0	96.0	2.5	20.05	3.708	42.16	29.70	.0176	.0271	.0016	.0197	.0335	.0024	.0842	44.3	72.0	27.7	12.2
306	13.5	97.0	3.5	22.16	4.106	42.22	29.90	.0174	.0280	.0017	.0219	.0371	.0024	.0903	43.5	72.1	24.6	12.6
305	14.5	96.0	4.0	22.52	4.137	40.46	28.40	.0162	.0311	.0019	.0233	.0329	.0027	.0893	44.2	71.7	27.5	11.7
312	21.5	98.0	2.0	22.98	4.305	42.20	31.10	.0219	.0326	.0013	.0256	.0395	.0022	.0966	49.2	74.6	25.4	11.1
307	17.5	97.0	2.0	26.86	4.987	38.07	25.90	.0247	.0336	.0012	.0272	.0381	.0022	.1036	50.1	69.2	19.1	7.5
303	20.0	96.0	2.0	28.49	5.182	37.51	25.30	.0273	.0351	.0015	.0284	.0399	.0024	.1094	50.9	68.6	17.7	6.9
313	28.0	97.0	1.6	35.43	6.592	35.69	27.40	.0195	.0296	.0013	.0400	.0512	.0021	.1040	57.2	77.7	20.5	7.5
302	29.0	96.0	3.0	37.27	6.884	37.03	25.30	.0261	.0351	.0019	.0405	.0511	.0028	.1243	53.6	69.4	15.8	6.1

The engine on which the tests were made is the Harris-Corliss engine, referred to on page 333, having a stroke of 24 inches and a diameter of 8 inches. The cylinder is covered with hair felt and wood lagging, but is not steam-jacketed. It is supplied with steam containing from one to two per cent of moisture, as determined by a large number of tests with several kinds of calorimeters.

During the tests the steam used was condensed in a surface condenser, collected and weighed. Indicator-cards were taken at intervals of five minutes simultaneously, from each end of the cylinder, and at the same time the necessary temperatures and pressures were read and recorded, and the revolutions were read on a counter. The data in the tables are determined from the totals or averages of the observations taken during the test.

The errors of all the instruments used during the tests were determined in the laboratory, and corrections were applied when necessary.

To insure regularity of results in this series the following precautions were taken: (1) the governor of the engine was disconnected, the cut-off mechanism was fixed by hand, and the engine was coupled to another engine to control the speed; (2) the steam-pressure was maintained as nearly constant as possible during a single test; (3) the tests of the series all have nearly the same boiler-pressure; (4) during the tests the throttle-valve was wide open. A comparison of the tests of this series with other tests on this engine, in which some of the conditions could not be fulfilled, shows clearly that all are requisite if definite results are to be attained.

Tests of a Worthington High-duty Engine.—In Table XXXIII are given the data and results of two tests made by Prof. Unwin * on a Worthington high-duty duplex compound pumping engine, with compensating cylinders, built by Messrs. Simpson & Co., Pimlico, and operating at the West Middlesex Water-works at Hampton.

The engines of this type have no fly-wheel nor heavy reciprocating mass, but provision is made for cut-off and expansive working of steam in the high-pressure cylinder, by aid of a pair

* Engineering, Dec. 1888.

of compensating cylinders for each piston-rod. The piston-rod carries the two steam-pistons at one end, the pump-plungers at the other end, and near the pumps is operated on by the compensating cylinders, which are swung on trunnions in such a manner that they oppose the motion of the piston-rod at a considerable angle at the beginning of the stroke, offer a less resistance as the stroke proceeds, till at half-stroke they are opposed to each other and mutually counterbalance each other; and during the second half of the stroke, when the steam pressure in the cylinder is reduced by expansion, they restore the work stored during the first half, and help the pump to complete its stroke.

The engines tested are used to pump a large volume of water on a comparatively low lift. The high-pressure pistons are 27 inches in diameter, and the low-pressure pistons are 54 inches in diameter. The maximum stroke is 44 inches; during the test the stroke remained very constant at 43 inches. The main valves of each engine are worked by the other engine, but the independent cut-off valves of each engine are worked from its own piston-rod, and there is an independent control of the compression in the high-pressure cylinder. The engines work directly double-acting ram-pumps, the rams being 40 inches in diameter, and having the same stroke as the steam-pistons. The valves are of India rubber, spring loaded, and the slip is probably small. The compensating pistons are 11 inches in diameter, and are loaded with an air-pressure of 120 pounds per square inch. The pumps lift water from a well communicating with the river and deliver it through two 3-feet mains to reservoirs nine miles distant. The head during the tests, measured by the difference of pressure in the suction- and discharge-pipes, was from 50 ft. to 65 ft., which head was almost entirely expended in overcoming the friction in the mains.

The engine cylinders are completely jacketed, and the steam is also taken through a jacketed reservoir between the cylinders. The jacket-water was discharged through a pipe regulated by a stop-valve and weighed. The condensers are injection-condensers with horizontal air-pumps.

The boilers are single-flue Cornish boilers. Three were used during the trial on October 29th, and four during the trial on November 5th and 6th. The boilers are 28 ft. long and 6 ft. in diameter, with a flue 3 ft. 6 in. in diameter. During the trials on November 5th and 6th the length of the grate was 4 ft. 6 in., making an area of 60 sq. ft.

The coal was weighed on platform scales which had been tested.

The feed-water was supplied from the delivery main at a temperature of 51 degrees. The ordinary feed arrangements for supplying hot water from the jackets and hot-well were disconnected. The feed-water was measured in a gauge-tank, of which the capacity was obtained by weighing in water at the temperature used during the tests, so that no corrections for temperature were required.

The feed-tank delivered by a stop-valve into another tank, from which a small Worthington feed-pump delivered the water into the boilers.

The Worthington pump took its steam from the boilers in use and exhausted into the tank, from which it pumped. The whole of the steam used was therefore recondensed and returned to the boilers.

Of the heat supplied by the boilers to work the feed-pump nearly all was returned to the boilers. A small portion, viz., that due to the useful work of pumping and that lost by radiation from the tank, was no doubt lost. So far, a small error telling against the main engines is introduced.

The water-level at the commencement of each trial in the boiler gauge-glasses was carefully observed, and the water-level was brought to exactly the same marks at the end of the trials. Hence no correction has to be made for difference of level in the boilers. The time at which each tankful was supplied to the boilers was noted, and also the feed-water temperature. Pyrometer observations were made in the flues with two Murries pyrometers. During part of the trial one of these had to be used at temperatures near the bottom of its scale, where the indications are least trustworthy. But the mean of the pyrometer readings is probably not very incorrect. Anemometer observations of the air supplied to each boiler were taken every half-hour during the twenty-four hours' trial, the anemometer having been previously tested.

The air-pump discharge was led into a wooden tank with stilling screens. From this it was discharged through a sharp-edged circular orifice freely into the air. The diameter of the orifice was carefully tested after the trials, and the coefficient of discharge from similar orifices is known to be 0.599. The temperature and head over the orifice was noted every 5 minutes in the first trial, and every $7\frac{1}{2}$ minutes in the second. The temperatures relied on in this report were taken by a fixed zero thermometer, with open scale, recently verified at Kew.

As the stroke is variable, an arrangement of indicating fingers was attached to each engine, and the length of stroke on each engine was noted every quarter of an hour.

The indicated power was taken by four Richards indicators, chosen because they give fairly large diagrams. These indicators were sent to Kensington after the trials and tested under steam. No important error was found at any part of the scale with any of the springs. But with the light springs of the low-pressure cylinder indicators there was a little frictional sticking, or else a little slackness of the parallel motion joints, which under a steady pressure introduced a small uncertainty of indication at one or two points in the range. Probably this would be less still when the indicator-piston was in motion as when drawing a diagram. The indicator-pipes were large, and were clothed. Diagrams were taken every half-hour from all the cylinders, so that there were 128 single diagrams in the eight hours' trial and 384 in the twenty-four hours' trial. All the eight hours' trial diagrams were reduced by planimeter; also all the diagrams taken in the first eight hours of the twenty-four hours' trial, and half of these taken subsequently. The conditions were so constant throughout the trial, and the diagrams so similar, that this was thought sufficient.

The trial of eight hours' duration on October 29th was of the engine only, and no account was taken of the coal burned.

During the trial on the 5th and 6th of November the coal consumption was measured as well as the efficiency of the engines. The engines, as before, had been started in the morning, but before beginning the fires were cleaned and all ashes removed; also all coal was swept from the boiler-house floor. Four boilers were used, and the fires were not drawn; but the condition of the fires was nearly identical at the beginning and at the end of the experiment. The fires were cleaned again about eighteen hours after starting, all the clinker and ash removed being placed in the ash-pits. At the end of the trial the fires were judged to be on the average slightly thicker than at the beginning of the trial. The trial commenced at 10.22 A.M. on the 5th and ended exactly at 10.22 A.M. on the 6th. The stoke-hole floor having been swept clean at the beginning of the trial, the coal was brought in in quantities of about 8 cwt., and the time of finishing each lot was noted. The ash-pits were cleaned before the trial, and afterwards nothing was removed till the end of the trial. The fires were cleaned before the trial began, and again at 4 A.M. on Tuesday morning. The fires were not touched at the end of the trial, but the ash-pits were immediately cleaned, and the whole of the ashes were treated thus:

First the clinkers, including those removed from the fires at 4 A.M. (six hours before the end of the trial), were separated and weighed. The rest of the ashes were sifted through a sieve with $\frac{1}{2}$-in. mesh. All that passed through the sieve is treated as incombustible ash, although probably one third of it is unburned carbon. What did not pass through the sieve is treated as unburned fuel. Analysis in similar cases has shown that the cinders retained by the sieve are almost entirely carbon.

TABLE XXXIII.

Diameter and Areas of Cylinders and Pumps.

		Diameter at 60 degrees F.	Diameter at 316 degrees F.	Area of piston.	Area of rod.	Effective area.	Mean.
		in.	in.	sq. in.	sq. in.	sq. in.	
H. P. cylinder A:	Back....	26.98	27.02	573.4	17.7	555.7	553.5
	front....	26.98	27.02	573.4	23.8	549.6	
H. P. cylinder B:	back....	27.02	27.06	575.1	17.7	557.4	
	front....	27.02	27.06	575.1	23.8	551.3	
L. P. cylinder A:	back....	53.99	54.07	2296.2	7.0	2289.2	2285.1
	front....	53.99	54.07	2296.2	17.7	2278.5	
L. P. cylinder B:	back....	54.02	54.10	2298.7	7.0	2291.7	
	front....	54.02	54.10	2298.7	17.7	2281.0	
Pump-plungers:	back....	39.90	....	1250.0	16.8	1233.2	1241.6
	front....	39.90	....	1250.0	0	1250.0	

Tests of Engines.

Date,	Oct. 29th,	Nov. 5th and 6th.
Duration,	8 hours,	24 hours.
Barometer,	30.166	29.78 in.
"	14.82	14.627 lbs. per sq. in.
Vacuum,	28.04	27.76 in.
"	13.77	13.63 lbs. per sq. in.
Head pumped against,	60.63	53.68 ft.
Total double strokes,	8480	24886
Length of stroke, minimum,	41.96	42.32 in.
maximum,	43.56	43.56 "
Engine A: mean,	42.83	43.06 "
" B: "	43.00	43.05 "
Delivery of pumps, not allowing for slip,	13.598 19580000	13.407 gallons per min. 19305504 gallons in 24h.
Efficiency of machine,	0.8434	0.8495
Total feed-water,	41277.8	108537 lbs.
" jacket-water,	5357	—— "
Double strokes per minute,	17.67	17.282
Boiler-pressure,	75.2	60.29 lbs. per sq. in.
Feed-water per minute,	85.99	75.37 lbs.
Jacket-drains per minute,	11.16	11.77 "
Temperature of steam,	320.06	307.36 degrees F.
Pressure on pump,	26.27	23.26 lbs. per sq. in.
" in compensators,	120.6	120 " " "
Mean pressure in H. P. cylinders	39.23	32.92
" L.P. "	7.42	6.905 " " "

Temperature of injection, . . .	54	49.2 degrees
Temperature of air-pump discharge, ...	81.18	74 965 degrees
Head over orifice, ...	1.9662	1.7033 ft.
Air-pump discharge per minute, .	2777.7	2586 lbs.
Injection-water, ...	2702.9	2522.4 lbs.

INDICATED HORSE-POWER.

Engine A:	H. P. back, ...	37.78			31.662		
	L. P. "	34.19	71.97	143.34	31.145	62.807	128.668
	H. P. front, ...	38.98			34.176		
	L. P. "	32.39	71.37		31.685	65,861	
Engine B:	H. P. back, ...	45.68			35.856		
	L. P. "	32.13	77.81	152.91	28.073	63.929	126.849
	H. P. front, ...	43.93			35.236		
	L. P. "	31.17	75.10		27.684	62.920	

Total indicated horse-power, both engines,	296.25	255.517
	Lbs.	
Total feed per indicated horse-power per hour,	17.41	17.700
Jacket-water ditto, ...	2.26	2.763
Used in cylinders, ...	15.15	14.937
Heat absorbed by injection-water per minute, .	73,460	64,990
Heat retained by condensed steam per minute,	1,958	1,519
Heat retained by jacket-water per minute, . .	2,958	3,020
Total, ...	78,376	69,529
Heat rejected per indicated horse-power per minute, ...	264.6	272.1
Add converted into work, ...	42.7	42.7
Total, ... , ...	307.3	314.8

HEAT PASSING THROUGH ENGINE PER MINUTE PER INDICATED HORSE-POWER.

Thermal units from boiler in saturated steam through cylinders from feed temperature, .	292.0	287.8
Latent heat of jacket steam, ...	33.6	41.45
	325.6	329.25
Heat rejected in air-pump discharge, ...	254.6	260.24
Converted into work, ...	42.7	42.75
Radiation and error, ...	28.3	26.26
	325.6	329.25
Indicated horse-power, ...	296.25	255.517
Pump horse-power, ...	249.84	217.06
Mechanical efficiency, ...	.8434	.8495

Feed per indicated horse-power per hour through cylinders,	15.15	14.937 lbs.
Feed per indicated horse-power per hour through jackets,	2.26	2.763
Piston speed per minute,	126.4	124 ft.

Test of Boilers.

	Lbs.	Lbs.
Gross weight of coal brought into boiler-house,		11,180
Left on floor at end of trial,	99	
Cinders sifted out of ashes,	132	231
Total coal used,		10,949
		= 456.2 lbs. per hour.

	Lbs.
The residue consisted of clinkers,	66
Incombustible ashes,	366
	432

Percentage of clinkers and ashes,	3.9 lbs.
Coal per sq. ft. of grate per hour,	7.24 lbs.
" " " " heating surface per hour,	0.19 "
Coal per indicated horse-power per hour,	1.785 "
Water evaporated per pound of coal from 51°.07 and at 307°.36,	9.914 "
" " " " " " " and at 212°,	11.867 lbs.
Estimated heat of combustion of coal, allowing for 1 per cent of moisture,	14878 B. T. U.
Thermal units per horse-power per minute, estimated from coal consumption,	442.6 B. T. U.
Thermal units per horse-power per minute, estimated from boiler steam (including loss by radiation),	341.1 "
Efficiency of boiler,	0.77
Cubic feet of air per minute by anemometer,	1704
" " " " " pound of coal,	225
Pounds of air per pound of coal,	16.42

Heat used and lost in boilers:	Thermal Units per Indicated Horse-power per Hour.	Per cent.
Total heat due to coal used,	26,557	100
Given to steam.	20,466	77.1
Carried off in furnace gases,	2,657	10.0
Probable loss due to opening fire-doors to stoke,	265	1.0
Due to carbon in ashes,	284	1.1
Radiation and unaccounted for,	2,885	10.8

Duty of engine per 112 lbs. of coal: 24 hours' trial, actual,	106,000,000 ft.-lbs.
8 " " estimated,	106,500,000 "
Corrected for difference of temperature of feed-water from hydrant and from hot-well,	111,500,000 "

CHAPTER XIX.

FRICTION OF ENGINES.

As has been stated in the discussion of the efficiency of the steam-engine, the economic value of an engine is determined by the net or brake horse-power, which is the indicated horse-power less the power used up by the friction of the mechanism of the engine.

Pambour's Method.—It was suggested by Pambour that the friction of the engine could be divided into two parts, one of which remains constant and can be determined by indicating the engine without a load, and the other of which increases with the load and is proportional to it. In form of an equation this becomes

$$F = P_0 + fP, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (311)$$

in which F is the horse-power lost in friction in the engine, P_0 is the power required to run the engine unloaded, and P is the useful or net horse-power, while f is the coefficient for the increase of friction with the load. The work expended on the air-pump is counted with the friction for condensing engines. The efficiency of the mechanism is

$$\frac{P}{F+P} = \frac{P}{P_i} = \frac{P_i - F}{P_i},$$

P_i being the indicated horse-power.

Rankine* states that the unloaded resistance P_0 is equivalent to a pressure of $\frac{1}{2}$ to $1\frac{1}{2}$ pounds to the square inch of the piston; this may be compared with Isherwood's results on pages 265 and 274. He further states that the value of $f = \frac{1}{7}$,

* Steam-engine and Other Prime Movers, p. 423.

proposed by Pambour, is corroborated by general experience.

Alsatian Experiments.—In Tables XXXIV, XXXV, and XXXVI are given the results of tests made by Walther-Meunier and Ludwig,* to determine the friction of a horizontal receiver compound engine, with cranks at right angles and with a fly-wheel, grooved for rope-driving, between the cranks. The piston-rod of each piston extended through the cylinder cover and was carried by a cross-head on guides, and the air-pump was worked from the high-pressure piston-rod. The cylinders each had four plain slide-valves, two for admission and two for exhaust; the exhaust-valves had a fixed motion, but the admission-valves were moved by a cam so that the cut-off was determined by the governor.

The main dimensions of the engine were:

Stroke, ...	1.1	metres.
Diameter: small piston, ...	0.536	"
large piston, ...	0.800	"
piston-rods, ...	0.080	"
Length of connecting-rods, ...	2.475	"
Diameter, air-pump pistons, ...	0.360	"
Stroke, air-pump, ...	0.476	"
Diameter, fly-wheel, ...	6.610	"

The engine during the experiments made 58 revolutions per minute. The air-pump had two single-acting vertical pistons.

Each experiment lasted 10 or 20 minutes, during which the load on the brake was maintained constant, and indicator-diagrams were taken. The experiments with small load on the brake—i.e., No. 9, Table XXXIV; No. 9 and No. 10, Table XXXV; No. 9 and No. 10, Table XXXVI—were difficult and uncertain.

The tests in Table XXXIV were made with the engine working compound. Those in Table XXXV were made with the high-pressure cylinder only in action and with condensation,

* Bulletin de la Soc. Ind. de Mulhouse, vol. lvii. p. 140.

the low-pressure connecting-rod being disconnected. Those in Table XXXVI were made with the high-pressure cylinder in action, without condensation.

TABLE XXXIV.

Tests. Nos.	Horse-power—Chevaux aux vapeur.			Efficiency.
	Effective.	Indicated.	Absorbed by the engine.	
1	248.97	288.45	39.48	0.863
2	238.92	276.88	37.96	0.862
3	228.87	265.62	36.75	0.861
4	208.78	243.72	34.94	0.856
5	188.68	222.73	34.05	0.847
6	168.58	201.48	32.90	0.836
7	148.48	180.44	32.04	0.822
8	128.38	158.12	29.74	0.811
9	108.28	136.07	27.79	0.795

TABLE XXXV.

Tests. Nos.	Horse-power—Chevaux aux vapeur.			Efficiency.
	Effective.	Indicated.	Absorbed by the engine.	
1	128.38	153.12	24.74	0.839
2	118.33	142.—	23.67	0.833
3	108.28	130.89	22.60	0.827
4	98.24	120.06	21.82	0.818
5	88.19	108.96	20.77	0.809
6	78.14	97.45	19.31	0.801
7	68.09	86.32	18.23	0.788
8	58.04	75.72	17.68	0.766
9	47.99	65.46	17.47	0.733
10	37.94	55.19	17.25	0.687

TABLE XXXVI.

Tests. Nos.	Horse-power—Chevaux aux vapeur.			Efficiency.
	Effective.	Indicated.	Absorbed by the engine.	
1	128.38	145.87	17.49	0.880
2	118.33	135.73	17.40	0.871
3	108.28	125.17	16.89	0.865
4	98.24	114.44	16.20	0.858
5	88.19	103.93	15.74	0.848
6	78.14	92.98	14.84	0.840
7	68.09	81.97	13.88	0.830
8	58.04	71.72	13.68	0.809
9	47.99	61.55	13.56	0.779
10	37.94	51.34	13.40	0.738

The results of all of the tests are plotted in Fig. 78 with the effective horse-power for abscissæ, and with the friction horse-power for ordinates. The lines may be taken to represent the several series of tests, and the points where they cross the vertical axis may be considered to give P_0, the indicated power without a load, which was not determined directly.

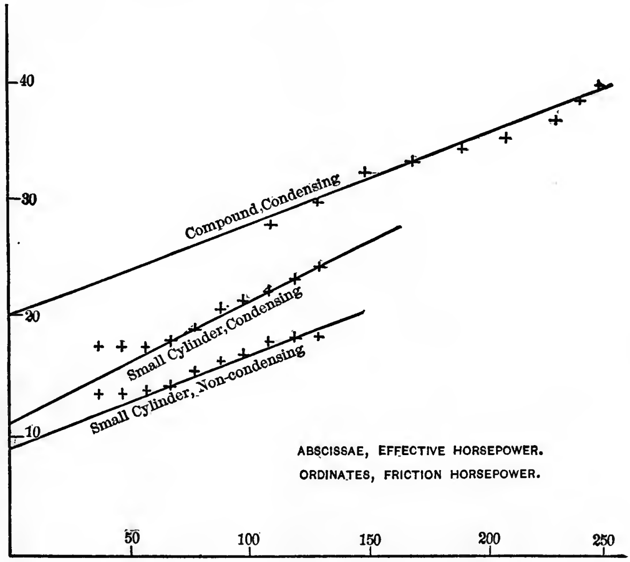


FIG. 78.

Equation (311) for these several series of experiments becomes:

Compound condensing,

$$F = 20 + 0.077P;$$

Small cylinder condensing,

$$F = 11 + 0.107P;$$

Small cylinder non-condensing,

$$F = 9 + 0.062P.$$

When the engine ran with the small cylinder only and with condensation, 27.74 horse-power were consumed by friction and other resistances. Without condensation, and with the air-pump disconnected, 17.49 horse-power were thus consumed. The experimenters therefore considered that 7.25 horse-power were required to run the air-pump. From the mean vacuum they estimated the power required for the air-pump to be 7.38 horse-power.

The best efficiency in each case is found with the largest power. The tables give:

	Efficiency.
Compound condensing, ...	0.863
Small, cylinder condensing, ...	0.839
Small, cylinder non-condensing, ...	0.880

If the power required for the air-pump be deducted from the power absorbed by the engine, then the power used up by the friction of the mechanism divided by the indicated horse-power becomes, for the several cases:

Compound condensing, ...	0.113
Small cylinder, condensing, ...	0.115
Small cylinder, non-condensing, ...	0.12

In the following table are given the results of other tests on engines of various types:

TABLE XXXVII.

FRICTION TESTS OF ENGINES.

Date.	Type of Engine.	Names of Experimenters.	Horse-power —Cheval à vapeur.	Maximum efficiency.
1864	Single cylinder, beam with four valves, using superheated steam.	Leloutre	115	0.908
1867	Woolf, beam	Grosseteste, Hallauer	191.44	0.896
1876	Horizontal Woolf small cylinder inclined	Association Alsacienne; Walther-Meunier, Keller	170.46	0.891
1878	Corliss	Association Alsacienne; Walther-Meunier, Keller	144.82	0.915
1879	Semi-fixed compound, horizontal	Association Alsacienne; Walther-Meunier, Keller	60.	0.876
1884	Colman, horizontal	Association Alsacienne; Walther-Meunier, Ludwig, A. Burghardt	22.26	0.878
1884	Horizontal portable	Association Alsacienne, Ludwig	23.97	0.863
1885	Compound horizontal	Association Alsacienne; Walther-Meunier, Ludwig	59.26	0.896

Thurston's Experiments.—As a result of a large number of tests on non-condensing engines, made under his direction or with his advice, Prof. R. H. Thurston* concludes that, for engines of that type, the friction is independent of the load, and that it can, in practice, be determined by indicating the engine without a load.

TABLE XXXVIII.

FRICTION OF NON-CONDENSING ENGINE.

Straight-line Engine, 8″ × 14″.

No. of Card.	Boiler-pressure.	Revolutions.	Brake H. P.	I. H. P.	Frictional H. P.
1	50	332	4.06	7.41	3.35
2	65	229	4.98	7.58	2.60
3	63	230	6.00	10.00	4.00
4	69	230	7.00	10.27	3.27
5	73	230	8.10	11.75	3.65
6	77	230	9.00	12.70	3.70
7	75	230	10.00	14.02	4.02
8	80	230	11.00	14.78	3.78
9	80	230	12.00	15.17	3.17
10	85	230	13.00	15.96	2.96
11	75	230	14.00	16.86	2.86
12	70	230	15.00	17.80	2.80
13	72	231	20.10	22.07	1.97
14	75	230	25.00	28.31	3.31
15	60	229	29.55	33.04	3.40
16	58	229	34.86	37.20	2.34
17	70	229	39.85	43.04	3.19
18	85	230	45.00	47.79	2.78
19	90	230	50.00	52.60	2.60
20	85	230	55.00	57.54	2.54

Table XXXVIII gives the details of one series of tests. The friction horse-power is small in all the tests, and the variations are small and irregular, and appear to depend on the state of lubrication and other minor causes rather than on the change of load. Much the same result is shown by the tests given in Tables XXXIX and XL, the first on an automatic cut-off engine, and the second on a tandem compound engine.

Distribution of Friction.—As a consequence of his conclusion in the preceding section, Professor Thurston states that

* Trans. of the Am. Soc. of Mech. Engr., vols. viii., ix., and x.

TABLE XXXIX.

FRICTION WITH CHANGE OF LOAD.

Automatic Engine, 12" × 18".

Lansing Iron Works. Steam-pressure, 180 pounds.

No. of Card.	Revolutions per minute.	Total I. H. P.	Brake Load.	Brake H. P.	Frictional H. P.
1	190	11.20	0.	0.	11.20
2	190	11.19	0.	0.	11.19
3	190	10.80	0.	0.	10.80
4	180	8.27	0.	0.	9.27
5	180	8.76	0.	0.	8.76
6	182	13.24	11.5	2.79	10.45
7	182	13.55	11.5	2.79	10.76
8	187	14.60	21.5	5.35	8.25
9	187	16.84	22.5	5.61	11.23
10	180	18.89	23.5	7.89	11.00
11	192	19.43	46.0	11.78	7.65
12	192	20.78	49.0	12.54	8.24
13	192	21.25	49.0	12.54	8.71
14	192	21.82	49.0	12.54	9.28
15	190	25.05	72.5	18.37	6.68
16	192	25.65	72.5	18.56	7.09
17	192	27.53	77.5	19.84	7.69
18	185	36.38	115.5	28.49	7.89
19	185	36.94	120.5	29.72	7.22
20	180	41.27	142.0	34.08	7.19
21	180	41.61	142.0	34.08	7.53
22	180	44.91	150.5	36.12	8.79
23	175	57.44	210.5	49.11	8.33
24	175	58.70	209.5	48.89	9.81

the friction of an engine may be found by driving it from some external source of power, with the engine in substantially the same condition as when running as usual, but without steam in its cylinder, and by measuring the power required to drive it by aid of a transmission dynamometer. Extending the principle, the distribution of friction among the several members of the engine may be found by disconnecting the several members, one after another, and measuring the power required to run the remaining members.

The summary of a number of tests of this sort, made by Prof. R. C. Carpenter and Mr. G. B. Preston, are given in Table XLI. Preliminary tests under normal conditions showed

TABLE XL.

FRICTION WITH CHANGE OF LOAD.

Tandem Compound Engine, 14" and 21" by 20".

No. of Card.	Revolutions.	I. H. P. for both cylinders.	Brake Load.	Brake H. P.	Frictional H. P.
1	165	55.86	123	27.12	28.74
2	168	57.55	131	29.26	28.29
3	138	79.97	284	52.15	27.82
4	152	84.16	290	58.81	25 35
5	168	73.65	202	45.25	28.40
6	160	82.08	257	54.83	27.25
7	165	81.36	252	55.44	25.92
8	165	82.26	244	53.67	28.69
9	159	85.44	289	61.27	24.17
10	159	86.04	292	61.90	24.14
11	183	51.11	83	20.26	30.85
12	174	68.45	145	33.75	34.70
13	160	86.77	263	56.11	30.66
14	162	80.65	256	55.28	25.37
15	158	82.31	261	55.00	27.31
16	160	88.86	298	63.57	25.29
17	156	87.47	283	58.85	28.62
18	136	86.35	333	60.38	25.97
19	130	83.03	353	61.19	21.84
20	156	29.22	0.	0.	29.22
21	150	.....	...	.....	.....
22	156	27.63	0.	0.	27.63
23	158	29.69	0.	0.	29.69
24	190	127.10	...	.....	.....

that the friction of the several engines was practically the same at all loads and speeds.

The most remarkable feature in this table is the friction of the main bearings, which in all cases is large, both relatively and absolutely. The coefficient of friction for the main bearings, calculated by the formula

$$f = \frac{33000 \text{ H. P.}}{pcn},$$

is given in Table XLII. p is the pressure on the bearings in pounds for the engines light, and *plus* the mean pressure on the piston for the engines loaded; c is the circumference of the bearings in feet; n is the number of revolutions per minute; and H. P. is the horse-power required to overcome the friction of the bearings.

TABLE XLI.

Distribution of Friction.

<table>
<tr><th rowspan="2">Parts of Engine.</th><th colspan="5">Percentages of Total Friction.</th></tr>
<tr><th>Straight-line 6×12 Balanced Valve.</th><th>Straight-line 6″×12″ Unbalanced Valve.</th><th>7″×10″ Lansing Iron Works—Traction-Locomotive Valve-gear.</th><th>12″×18″ Lansing Iron Works — Automatic Balanced Valve.</th><th>21″×20″ Lansing Iron Works — Condensing Balanced Valve.</th></tr>
<tr><td>Main Bearings</td><td>47.0</td><td>35.4</td><td>35.0</td><td>41.6</td><td>46.0</td></tr>
<tr><td>Piston and Rod</td><td>32.9</td><td>25.0</td><td>21.0</td><td rowspan="3">49.1</td><td rowspan="3">21.8</td></tr>
<tr><td>Crank Pin</td><td>6.8</td><td>5.1</td><td rowspan="2">13.0</td></tr>
<tr><td>Cross Head and Wrist Pin</td><td>5.4</td><td>4.1</td></tr>
<tr><td>Valve and Rod</td><td>2.5</td><td>26.4</td><td rowspan="2">22.0</td><td rowspan="2">9.3</td><td rowspan="2">21.0</td></tr>
<tr><td>Eccentric Strap</td><td>5.3</td><td>4.0</td></tr>
<tr><td>Link and Eccentric</td><td></td><td></td><td>9.0</td><td></td><td></td></tr>
<tr><td>Air Pump</td><td></td><td></td><td></td><td></td><td>12.0</td></tr>
<tr><td>Total</td><td>100.0</td><td>100.0</td><td>100.0</td><td>100.0</td><td>100.0</td></tr>
</table>

TABLE XLII.

Coefficient of Friction for the Main Bearings of Steam-engines.

Engine.	F. H. P. due to Main Journals.	Weight on Journals in Pounds.	Diameter of Journal in Inches.	Coefficient of Friction, Engine Light.	Coefficient of Friction, Engine Loaded.	Revolutions of Journal per Minute.
6″ × 12″ Straight-line	0.85	1500	3	.10	.06	230
*12″ × 18″ Automatic (L. I. W.)	3.70	2600	5	.19	.05	190
7″ × 10″ Traction (L. I. W.)	0.68	500	$2\frac{3}{4}$	.31	.08	200
21″ × 20″ Condensing (L. I. W.)	3.30	4000	$5\frac{1}{2}$	.09	.04	206

* The 12″ × 18″ automatic engine was new, and gave, throughout, an excessive amount of friction as compared with the older engines of the same class and make.

The large amount of work absorbed by the main bearings and the large coefficient of friction appear the more remarkable from the fact that the coefficient of friction for car-axle journals is often as low as one tenth of one per cent, the difference being probably due to the difference in the methods of lubrication.

The second and obvious conclusion from Table XLI is that the valve should be balanced, and that nine tenths of the friction of an unbalanced slide-valve is unnecessary waste.

The friction of the piston and piston-rod are always considerable, but they vary much with the type of the engine, and with differences in handling. It is quite possible to change the effective power of an engine by screwing up the piston-rod stuffing-box too tightly. The packing of both piston and rod should be no tighter than is necessary to prevent perceptible leakage, and is more likely to be too tight than too loose.

CHAPTER XX.

COMPRESSED AIR.

COMPRESSED air, that is, air at a pressure above that of the atmosphere, is employed for transmitting power from a place where it is conveniently generated to places where it is to be used. The air-blast used in the production of iron and steel is compressed air of moderate pressure; the compressors for such work are called blowing-machines or blowers. Currents of air at slightly greater pressure than that of the atmosphere are used for ventilating mines, buildings, ships, etc., and for producing a forced draught for steam-boilers. Such currents are commonly produced by fans or centrifugal blowers. Air-pumps differ from air-compressors in that they take air from a receptacle in which the pressure is less than that of the atmosphere, compress it, and deliver it against the pressure of the atmosphere.

Power Expended.—The indicator-diagram of an air-compressor with no clearance space is represented by Fig. 79. Air is drawn in at atmospheric pressure in the part of the cycle of operations represented by *dc*, in the part represented by *cb* the air is compressed, and in the part represented by *ba* it is expelled against the higher pressure.

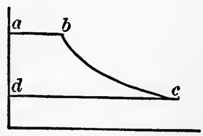


FIG. 79.

If p_1 is the specific pressure and v_1 the specific volume of one pound of air at atmospheric pressure, and p_2 and v_2 corresponding quantities at the higher pressure, then the work done by the atmosphere on the piston of the compressor while air is drawn in is v_1p_1. Assuming that the compression curve *cb* may be represented by an exponential curve having the form

$$pv^n = p_1v_1^n = \text{const.},$$

then the work of compression is

$$\int pdv = p_1v_1^n \int_{v_2}^{v_1} \frac{dv}{v^n} = \frac{p_1v_1}{n-1}\left\{\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right\}$$

$$= \frac{p_1v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}.$$

If the compression is adiabatic, then the exponent becomes

$$\kappa = \frac{c_p}{c_v} = 1.405.$$

The work of expulsion from *b* to *a* is

$$p_2v_2 = p_2v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}.$$

The effective work of the cycle is therefore

$$W = \frac{p_1v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} + p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - p_1v_1\,;$$

$$\therefore\quad W = p_1v_1\frac{n}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (312)$$

Equation (312) gives the work done upon one unit of weight of air, and the pressures and volumes are specific pressures and volumes. If p_0, v_0, and T_0 are the pressure, volume, and temperature under standard conditions, i.e., at atmospheric pressure and at freezing-point, then v_1 may be found from the equation

$$\frac{p_1v_1}{T_1} = \frac{p_0v_0}{T_0} = R.$$

It is frequently convenient to use, instead of equation (312), one for the mean effective pressure that may be found by dividing it by v_1, so that we have

$$\text{M. E. P.} = p_1 \frac{n}{n-1} \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right\}, \quad . \quad . \quad . \quad (313)$$

in which p_1 and p_2 may be stated in any convenient units, such as pounds on the square inch.

Effect of Clearance.—The indicator-diagram of an air-compressor with clearance may be represented by Fig. 80. The end of the stroke expelling air is at a, and the air remaining in the cylinder expands from a to d, till the pressure becomes equal to the pressure of the atmosphere before the next supply of air is drawn in. The expansion curve ad may commonly be represented by an exponential equation having the same exponent as the compression curve cb, in which case the air in the clearance acts as a cushion which stores and restores energy, but does not affect the work done on the air passing through the cylinder. The work of compressing one unit of weight of air in such a compressor may be calculated by aid of equation (312), but the equation (313) for the mean effective pressure cannot be used directly.

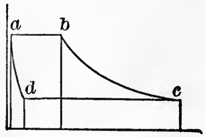


FIG. 80.

The principal effect of clearance is to increase the size of the cylinder required for a certain duty in the ratio of the entire length of the diagram in Fig. 80 to the length of the line dc.

The mean effective pressure may be calculated as for a steam-engine indicator-card, taking account of compression and expansion as shown in Fig. 80, or the mean effective pressure found by equation (313) may be reduced in proportion of the line dc to the length of the diagram in Fig. 80. Or, again, the mean effective pressure by equation (313) may be used with the volume of air actually drawn in per minute, in calculating the horse-power.

Cooling during Compression.—If heat is not withdrawn during compression, the temperature rises according to the law for adiabatic compression. When the maximum pressure p_2 is moderate, as in blowing-machines, there is ordinarily no provision made for cooling either the air or the cylinder, and the compression curve is approximately the adiabatic for air. When the final pressure is considerable, as in the use of compressed air for transmitting power, the high temperatures produced without cooling become troublesome, and, in all but small machines, some provision is made for cooling the air, or the cylinder, or both.

The cylinder may be cooled by a water-jacket, and the air is at the same time cooled, in some degree, by contact with the walls of the cylinder.

The air is most efficiently cooled by injecting water into the cylinder, or by using water freely in the cylinder in some form. By this means the final temperature of the air is much reduced, and the work of compression is also reduced. An inconvenience may sometimes arise from the fact that when water is so used the air delivered is nearly if not quite saturated with moisture.

Moisture in the Cylinder.—If water is not purposely injected into the cylinder of the compressor, the moisture in the air will depend on the hygroscopic condition of the air drawn in by the compressor. Even if the air should be saturated the total and the relative amount of moisture in the cylinder will be insignificant. Thus at 60° F. the pressure of saturated steam is about $\frac{1}{4}$ of a pound on the square inch, and the weight of one cubic foot is about 0.0008 of a pound, while the weight of one cubic foot of air is about 0.08 of a pound. If the air is not saturated the vapor exerts a less pressure than saturated vapor at that temperature, and consequently follows the laws of superheated steam; even if the vapor is at first saturated, it is superheated by compression, and then follows the same laws. Now the adiabatic equation for superheated steam has been shown to be

$$pv^{\frac{4}{3}} = p_1 v_1^{\frac{4}{3}},$$

so that the only effect of the moisture brought into the cylinder by the air is to slightly diminish the exponent of the equation representing the compression curve. This conclusion is probably valid when the cylinder is cooled by a water-jacket.

When water is used freely in the cylinder of a compressor, the air is cooled by contact with the water, and by vaporization of the water. The quantity of moisture in the air, and consequently the weight of the mixture of air and vapor in the cylinder, varies, and the condition of the air during, and at the end of, compression can be determined only when the temperature, volume, and pressure are known. It is commonly assumed that the air is saturated at all times under this condition.

Temperature at the End of Compression.—When the air in the compressor cylinder is dry or contains only the moisture brought in with it, it may be assumed that the mixture of air and vapor follows the law of perfect gases

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1},$$

which, combined with the exponential equation

$$pv^n = p_1 v_1^n,$$

gives

$$\frac{p_2^{\frac{n-1}{n}}}{T_2} = \frac{p_1^{\frac{n-1}{n}}}{T_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (314)$$

from which the final temperature T_2 at the end of compression may be determined when T_1 is known.

When water is used freely in the cylinder of a compressor the final temperature cannot be determined directly. Even if it be assumed that the air is always saturated, and if the exponent for the exponential equation be known, it can be determined only by a series of approximations. In experiments on air-compressors this temperature should be determined directly.

Contraction after Compression.—Ordinarily compressed air loses both pressure and temperature on the way from the compressor to the place where it is to be used. The loss of pressure will be discussed under the head of the flow of air in long pipes; it should not be large, unless the air is carried long distances. This loss of temperature causes a contraction of volume in two ways: first, the volume of the air at a given pressure is inversely as the absolute temperature; second, the moisture in the air, whether brought in by the air or supplied in the condenser, in excess of that which will saturate the air at the lowest temperature in the conduit, is condensed. Provision must be made for draining off the condensed water. The method of estimating the contraction of volume due to the condensation of moisture will be exhibited later in the calculation of a special problem.

Interchange of Heat.—The interchange of heat between the air in the cylinder of an air-compressor and the walls of the cylinder are the converse of those taking place between the steam and the walls of the cylinder of a steam-engine, and are much less in amount. The walls of the cylinder are never so cool as the incoming air nor so warm as the air expelled; consequently the air receives heat during admission and the beginning of compression, and yields heat during the latter part of compression and during expulsion. The presence of moisture in the air increases this effect.

Volume of the Compressor Cylinder.—Let a compressor making n revolutions or $2n$ strokes per minute be required to deliver V_3 cubic units (cubic feet or cubic meters) of air at the absolute temperature T_3 and against the absolute pressure p_3, expressed in convenient units, such as pounds on the square inch or kilograms on the square centimeter. The volume of air drawn in by the compressor per minute at the absolute temperature T_1 and the absolute pressure p_1 can be calculated by the equation

$$\frac{p_1 V_1}{T_1} = \frac{p_3 V_3}{T_3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (315)$$

If the compressor has no clearance the volume of the cylinder in cubic feet or cubic meters will be

$$\frac{V_1}{2n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (316)$$

If the compressor has a clearance, the indicator-diagram will be similar to Fig. 80, and the air in the clearance at the end of the stroke will expand down to the pressure of the atmosphere before the supply-valve will open. Let the clearance be $\frac{1}{m}$ part of the piston displacement. The air in the clearance space will, after expansion from the pressure p_2 to the pressure p_1, occupy

$$\frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

part of the piston displacement. Consequently the piston displacement will be

$$\frac{V_1}{2n}\left\{1+\frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}-\frac{1}{m}\right\}, \quad . \quad . \quad . \quad . \quad (317)$$

expressed in cubic feet or cubic meters.

The pressure in the compressor cylinder when air is drawn in, is always less than the pressure of the atmosphere, and when the air is expelled it is greater than the pressure against which it is delivered. From these causes and from other imperfections the compressor will not deliver the quantity of air calculated from its dimensions, and consequently the volume of the cylinder as calculated, whether with or without clearance, must be increased by an amount to be determined by experiment.

Compound Compressors.—When air is to be compressed from the pressure p_1 to the pressure p_2, but is to be delivered at the initial temperature t_1, the work of compression may be reduced by dividing it between two cylinders, one of which takes the air at atmospheric pressure and delivers it at an intermediate pressure p_2' to a reservoir, from which the other cylinder takes it and delivers it at the required pressure p_2, provided that the air be cooled, at the pressure p_2', between the two cylinders.

The proper method of dividing the pressures and of proportioning the volumes of the cylinders so that the work of compression may be reduced to a minimum may be deduced from equation (312), when there is no clearance or when the clearance is neglected.

The work of compressing one pound of air from the pressure p_1 to the pressure p_2' is

$$W_1 = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p_2'}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}.$$

The work of compressing one pound from the pressure p_2' to p_2 is

$$W_2 = p_2' v_2' \frac{n}{n-1}\left\{\left(\frac{p_2}{p_2'}\right)^{\frac{n-1}{n}} - 1\right\} = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p_2}{p_2'}\right)^{\frac{n-1}{n}} - 1\right\},$$

because the air after compression in the first cylinder is cooled to the temperature t_1 before it is supplied to the second cylinder. The total work of compression is

$$W = W_1 + W_2 = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p_2'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p_2'}\right)^{\frac{n-1}{n}} - 2\right\}, \quad (318)$$

and this becomes a minimum when

$$\left(\frac{p_2'}{p^1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p_2'}\right)^{\frac{n-1}{n}}$$

becomes a minimum. Differentiating with regard to p_2', and equating the first differential coefficient to zero, gives

$$p_2' = \sqrt{p_1 p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (319)$$

Since the air is supplied to each cylinder at the temperature t_1, their volumes should be inversely as the absolute pressures p_1 and p_2'.

When air is compressed to a very high pressure it may be advantageous to carry on the compression in three or more cylinders successively, cooling the air on the way from one cylinder to the next.

Fluid Piston Compressors.—It has been shown that the effect of clearance is to diminish the capacity of the compressor, consequently it should be made as small as possible. With this in view the valves of compressors and blowers are commonly set in the cylinder-heads. Single-acting compressors with vertical cylinders have been made with a layer of water or some other fluid on top of the piston, which entirely fills the clearance space when the piston is at the end of the stroke. An extension of this principle gives what are known as fluid piston compressors. Such a compressor commonly has a double-acting piston in a horizontal cylinder much longer than the stroke of the piston, thus giving a large clearance at each end. The clearance spaces extend upward to a considerable height, and the admission and exhaust valves are placed at or near the top, and the entire clearance space is filled with water. The spaces and heights must be so arranged that when the piston is at one end of its stroke, the water at that end shall fill the clearance and cover the valves, and at the other end the water shall not fall to the level of the top of the cylinder. There are consequently two vertical fluid pistons actuated by a double-acting horizontal piston. It is essential that the spaces in which the fluid pistons act shall give no spaces in which air may be caught as in a pocket, and that there are no projecting ribs or other irregularities to break the surface of the water; and, further, the compressor must be run at a moderate speed.

The water forming the fluid pistons becomes heated and saturated with air by continuous use, and should be renewed. Cooling by a spray of water during compression may be combined advantageously with the use of this form of piston.

Air-pumps used with condensing engines, or for other purposes, may be made with fluid pistons which are renewed by the water coming with the air or vapor. In case the water thus supplied is insufficient, water from without may be admitted, or water from the delivery may be allowed to flow back to the admission side of the pump.

Displacement Compressors.—When a supply of water under sufficient head is available, air may be compressed in suitably arranged cylinders or compressors by direct action of the water on air, compressing it and expelling it by displacement.

Rotary Blowers.—Rotary blowers have one or more rotating parts or pistons, so arranged that as they rotate chambers of varying capacity are formed, which receive the air at atmospheric pressure, compress it and deliver it against the higher pressure. No attempt is made to cool the air, and the clearance should be zero, so that the work of compression may be calculated by equation (312) with $n = \kappa = 1.4$.

Fan Blowers.—The complete theory of centrifugal and other rotating fans cannot be deduced from thermodynamics alone. The work done upon the air may, however, be calculated as follows: Let the pressure and velocity of the air approaching the fan be p_1 and u_1, and of the air leaving the fan p_2 and u_2. Then the intrinsic energy under the initial and final conditions will be, by equation (84),

$$\frac{p_1 v_1}{\kappa - 1}, \qquad \frac{p_2 v_2}{\kappa - 1},$$

and the kinetic energy of one pound under the same conditions will be

$$\frac{u_1^2}{2g}, \qquad \frac{u_2^2}{2g},$$

so that the work done on the air will be

$$W = \frac{u_2^2 - u_1^2}{2g} + \frac{p_2 v_2 - p_1 v_1}{\kappa - 1},$$

or, substituting for v_2 from the equation $p_1 v_1^\kappa = p_2 v_2^\kappa$,

$$W = \frac{u_2^2 - u_1^2}{2g} + \frac{p_2 v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} - p_1 v_1}{\kappa - 1}. \quad . \quad . \quad (320)$$

Tests of Compressors.—From a large number of tests on fluid-piston air-compressors constructed by the Cockerill Works, Seraing, for use at the Mont Cenis tunnel, Mr. John Kraft* has compiled the following table:

PERFORMANCE OF FLUID PISTON IN COMPRESSORS.

Absolute pressure in atmosphere.	Volume of the cylinder to deliver on cubic meter of compressed air.			Work of compression, kilogrammeters.				Friction of piston and piston-rod.		Efficiency of the compressor.
	Volume of air drawn in cubic meters.	Piston displacement, cubic meters.	Loss of vol. in per cent of piston displacem't.	Required to compress air drawn in, at constant temperature.	Required to compress contents of cylin. at constant temperature.	Actual work from indicator-diagram.	Loss from heating in per cent of column 6.	Work applied to compressor through dynamomet.	Friction in per cent of column 7.	
1	2	3	4	5	6	7	8	9	10	11
2	2	2.222	10	14320	15912	17185	8	26293	53	0.544
3	3	3.333	10	34046	37829	43503	15	52204	20	0.652
4	4	4.444	10	57282	63646	70376	20	89360	17	0.647
5	5	5.555	10	83127	92364	113608	23	124969	10	0.665
6	6	6.666	10	111053	123393	155675	26	171023	10	0.649

The first column gives the absolute pressure of the compressed air delivered by the compressor in atmospheres. The second column gives the volume of air that must be drawn in by the compressor to deliver one cubic meter, on the assumption that the air is cooled after compression to the original temperature. The third column gives the piston displacement actually required; this is larger than the volume in column 2, because (*a*) the pressure in the cylinder while the cylinder is filling is less than that of the atmosphere; (*b*) the air is heated as it is drawn into the cylinder; (*c*) the water forming the fluid piston absorbs air at high pressures and gives it up at low pressures; (*d*) some water is injected at each stroke; (*e*) the air during expulsion has a higher pressure in the cylinder than that against which the air is expelled. The total loss from this source, set down in column 4, was determined by numerous experiments. Column 5 gives the work that would be required to deliver one cubic meter of air if there were no loss or imperfection, and if the air were maintained at the original temperature during compression. Column 6 gives the work

* Revue universelle des Mines, 2 Série, Tome vi. p. 301.

required on the assumption that ten per cent of the piston displacement is lost. Column 7 gives the indicated work done on the contents of the cylinder for each cubic meter of air delivered. The loss from the failure to prevent heating during compression is given in column 8. Column 9 gives the work expended on the compressor for each cubic meter of air delivered, determined by experiments on a Prony brake. The loss from friction in per cent of the indicated work is given in column 10. Column 11 gives the ratio of column 5 to column 9, which is the efficiency of the compressor if it is required to deliver air at the original temperature.

For convenience, Mr. Kraft gives the distribution of the work applied to the compressor in the following table:

DISTRIBUTION OF WORK APPLIED TO AIR-COMPRESSOR.

Pressure in Atmospheres.	Useful Work.	Loss from Heating during Compression	Loss from Imperfections of Cycle.	Loss from Friction.
1.	2.	3.	4.	5.
2	0.544	0.0485	0.0600	0.3465
3	0.652	0.1087	0.0724	0.1666
4	0.647	0.1427	0.0712	0.1434
5	0.665	0.1702	0.0732	0.0903
6	0.649	0.1880	0 0720	0.0910

Pernolet* gives the following test of a blowing-engine used to produce the blast for Bessemer converters at Creusot. The engine was a two-cylinder horizontal engine, with the cranks at right angles. The piston-rod for each cylinder extended through the cylinder-head, and actuated a double-acting compressor. The dimensions were:

Diameter, steam-pistons,	1.2	meters.
" air-pistons,	1.5	"
Stroke,	1.8	"
Diameter of fly-wheel,	8.0	"

* L'Air Comprimé, 1876.

At 28 revolutions per minute the following results were obtained:

Indicated horse-power of steam-cylinders, ...	1082
" " of air-cylinders, ...	999
Efficiency, ...	0.92
Temperature of air admitted, ...	10° C.
" " " delivered, ...	60° C.
Pressure of air delivered, meters of mercury above the atmosphere, ...	1.21
Pressure of air in supply-pipe, meters of mercury below the atmosphere, ...	0.023

At 25 revolutions there was no sensible depression of pressure in the supply-pipe.

The air from such a blowing-engine probably suffers little loss of temperature after compression.

Air-pumps.—The feed-water supplied to a steam-boiler usually contains air in solution, which passes from the boiler with the steam to the engine and thence to the condenser. In like manner the injection-water supplied to a jet condenser brings in air in solution. Also, there is more or less leakage of air into the cylinder communicating with the condenser, and into the exhaust-pipe, or the condenser itself. An air-pump must therefore be provided to remove this air and to maintain the vacuum. The air-pump also removes the condensed steam from a surface condenser, and the mingled condensed steam and injection-water from a jet condenser. If no air were brought into the condenser, the vacuum would be maintained by the condensation of the steam by the injection, or the cooling water, and it would be sufficient to remove the water by a common pump; which, with a surface condenser, might be the feed-pump.

The weight of injection-water per pound of steam, calculated by the method on page 196, will usually be less than 20 pounds, but it is customary to provide 30 pounds of injection-water per pound of steam, with some method of regulating the quantity delivered.

It may be assumed that the injection-water will bring in with it one twentieth of its volume of air at atmospheric pressure, and that this air will expand in the condenser to a volume inversely proportional to the absolute pressure in the condenser. The capacity of the air-pump must be sufficient to remove this air, and the condensed steam and injection-water.

An air-pump for use with a surface condenser may be smaller than one used with a jet condenser. In marine work it is common to provide a method of changing a surface into a jet condenser, and to make the air-pump large enough to give a fair vacuum in case such a change should become advisable in an emergency.

Seaton* states that the efficiency of a vertical single-acting air-pump varies from 0.4 to 0.6, and that of a double-acting horizontal air-pump from 0.3 to 0.5, depending on the design and condition; that is, the volume of air and water actually discharged will bear such ratios to the displacement of the pump.

He also gives the following table of ratios of capacity of air-pump cylinders to the volume of the engine cylinder or cylinders discharging steam into the condenser:

RATIO OF ENGINE AND AIR-PUMP CYLINDERS.

Description of Pump.	Description of Engine.	Ratio.
Single-acting vertical	Jet-condensing, expansion $1\frac{1}{2}$ to 2	6 to 8
" "	Surface- " " $1\frac{1}{2}$ to 2	8 to 10
" "	Jet- " " 3 to 5	10 to 12
" "	Surface- " " 3 to 5	12 to 15
" "	" " compound	15 to 18
Double-acting horizontal...	Jet-condensing, expansion $1\frac{1}{2}$ to 2	10 to 13
" " ...	Surface- " " $1\frac{1}{2}$ to 2	13 to 16
" " ...	Jet- " " 3 to 5	16 to 19
" " ...	Surface- " " 3 to 5	19 to 24
" " ...	" " compound	24 to 28

Calculation of an Air-compressor.—Let it be assumed that an air-compressor delivers 100 cubic feet of air at a pres-

* Manual of Marine Engineering.

sure of 50 pounds by the gauge and at 80° F.; also, that the temperature of the air supplied to the compressor is 60° F., that the pressure of the atmosphere is 14.7 pounds, and that there is a loss between the compressor and the point of delivery of two pounds of pressure.

The compressor must draw in per minute

$$\frac{\overset{V_1}{100} \times \overset{p_3}{64.7} \times \overset{T_3}{520.7}}{14.7 \times 540.7\,{}_{T_1}} = \overset{V_3}{424} \text{ cubic feet.}$$

The actual capacity of a fluid-piston compressor is stated to be 0.9 its apparent capacity, so that such a compressor would have for its apparent capacity per minute

$$424 \div 0.9 = 471.1 \text{ cubic feet.}$$

Assuming 20 revolutions or 40 strokes per minute, the piston displacement will be

$$471.1 \div 40 = 11.8 \text{ cubic feet,}$$

or the piston may have a diameter of $24\frac{1}{4}$ inches, and a stroke of 4 feet.

A dry-air compressor will deliver less than the calculated amount of air on account of imperfect action of the valves, heating of the air as it enters, etc.; but there will be no water injected, and consequently none to expel. For comparison with the calculation for the fluid-piston compressor we will assume the actual delivery to be 0.92 of the calculated delivery, allowing for clearance. Consequently the apparent delivery of a compressor taking 424 cubic feet per minute must be

$$424 \div 0.92 = 460.9 \text{ cubic feet.}$$

If the clearance is 0.02 of the piston displacement, then the air in the clearance at 66.7 pounds absolute pressure will occupy

$$0.02\left(\frac{66.7}{14.7}\right)^{\frac{1}{1.4}} = 0.0589$$

of the piston displacement at 14.7 pounds pressure. The piston displacement must consequently be

$$1 + 0.0589 - 0.02 = 1.0389$$

times the displacement for a similar compressor without clearance. The piston displacement per minute will be

$$460.9 \times 1.0389 = 478.8 \text{ cubic feet.}$$

Assuming the compressor to make 60 revolutions per minute, the piston displacement will be

$$478.8 \div 120 = 4 \text{ cubic feet,}$$

or the piston may have a diameter of $15\frac{1}{2}$ inches and a stroke of 3 feet.

The exponent of the equation representing the compression curve for the fluid-piston compressor may be assumed to be 1.2, so that the mean effective pressure will be

$$14.7 \times \frac{1.2}{1.2 - 1} \left\{ \left(\frac{66.7}{14.7}\right)^{\frac{1.2-1}{1.2}} - 1 \right\} = 25.3,$$

and the indicated horse-power will be

$$\frac{25.3 \times 471.1 \times 144}{33000} = 52.0 \text{ H.P.}$$

The mean effective pressure for a dry-air compressor without clearance will be

$$14.7 \times \frac{1.4}{1.4 - 1} \left\{ \left(\frac{66.7}{14.7}\right)^{\frac{1.4-1}{1.4}} - 1 \right\} = 27.8,$$

and the indicated horse-power will be

$$\frac{27.8 \times 460.9 \times 144}{33000} = 55.9 \text{ H.P.}$$

A dry-air compressor with a clearance will have a larger piston displacement, but will absorb no more power, since the work stored and restored by the air in the clearance space does not affect the power required to deliver the given amount of compressed air.

According to Kraft's table, on page 416, the friction of an air-compressor for $4\frac{1}{2}$ atmospheres is about 10 per cent of the gross power expended. Consequently the gross power required to produce 100 cubic feet of air by use of a fluid-piston compressor is

$$52.0 \div 0.90 = 57.8 \text{ H. P.}$$

If the compressor is made with the piston on the same rod as the piston of a steam-engine cylinder, so that engine and compressor form one machine, then it may be assumed that 15 per cent of the indicated horse-power of the engine will be absorbed by the friction of the machine, and the indicated horse-power of the engine will be

$$52.0 \div 0.85 = 61.2 \text{ H. P.}$$

The temperature of the air delivered by the dry-air compressor, found by the equation (314), will be

$$T_2 = 520.7\left(\frac{66.7}{14.7}\right)^{\frac{1.4-1}{1.4}} = 802.1;$$

$$\therefore t_2 = 802.1 - 460.7 = 341^\circ.4 \text{ F.}$$

If the substance in the cylinder of the fluid-piston compressor followed the law of perfect gases, then for it the final temperature would be

$$T_2 = 520.7\left(\frac{66.7}{14.7}\right)^{\frac{1.2-1}{1.2}} = 669.9;$$

$$\therefore t_2 = 669.9 - 460.7 = 209^\circ.2 \text{ F.}$$

But the final temperature cannot be found in this way, since water is vaporized in the cylinder during compression and the weight of the substance operated on is not constant. Now one cubic foot of air at the pressure of 14.7 pounds will, after compression, occupy

$$\left(\frac{14.7}{66.7}\right)^{\frac{1}{1.2}} = 0.2836$$

of one cubic foot. If the temperature 209°.2 be assumed to be the final temperature, and it further be assumed that the air is saturated at that temperature after compression, then the pressure exerted by the vapor will be 13.87 pounds, and the pressure exerted by the air will be

$$66.7 - 13.87 = 52.83 \text{ pounds.}$$

But one cubic foot of air at 14.7 pounds pressure and at 60° F. will at 52.8 pounds pressure and at 209°.2 F. occupy

$$\frac{14.7 \times 669.9}{52.8 \times 520.8} = 0.3582$$

of a cubic foot. Consequently it cannot be that the air after compression is saturated with moisture at 209°.2 F.

Suppose the temperature of the moist air after compression to be 165° F., then the condition of the air may be found as follows: The pressure exerted by the air will be

$$\frac{14.7 \times 625.7}{520.7 \times 0.2836} = 62.29 \text{ pounds,}$$

and the pressure exerted by the vapor of water will be

$$66.7 - 62.29 = 4.41 \text{ pounds,}$$

while the pressure of saturated vapor at that temperature is 5.45 pounds. The weight of one cubic foot of superheated steam at the pressure of 4.41 pounds per square inch, or 635 pounds per square foot, and at the absolute temperature of

625°.7, may be found by aid of equation (184), for superheated steam, to be 0.0118 of a pound. Assuming the weight of the vapor at the temperature of 165° F. to be proportional to the pressure, gives

$$\frac{0.01449 \times 4.41}{5.324} = 0.012 \text{ pound,}$$

which is a sufficient approximation.

Had the temperature been assumed to be 161° F., a similar calculation would give for the pressure of the air 61.88 pounds, and for the vapor 4.82 pounds, while the pressure of saturated vapor at that temperature is 4.84 pounds; that is, the air would be saturated with vapor at that temperature.

The greater part of the vapor in the air at the pressure of 52 pounds by the gauge and at 165° F. will be condensed before the air arrives at the end of the conduit, where the pressure is supposed to be 50 pounds and the temperature 80° F. The volume of air delivered per minute is

$$424 \times 0.2836 = 120.2$$

cubic feet, which by the previous calculation contains 0.012 pound of moisture per cubic foot, or, in all,

$$120.2 \times 0.012 = 1.44 \text{ pounds.}$$

The 100 cubic feet at 80° F. if saturated will contain

$$100 \times 0.001553 = 0.16 \text{ pound,}$$

so that the water condensed per minute is, for 100 cubic feet,

$$1.44 - 0.16 = 1.28 \text{ pounds.}$$

Had the temperature at the point of delivery been the same as that of the air supplied to the compressor, as is common in practice, then nearly all the vapor in the air after compression would have been condensed and withdrawn.

When necessary, the contraction of volume after compression on account of the loss of temperature and accompanying condensation of moisture may be calculated as follows: The pressure exerted by the air was found to be 62.29 pounds, and that exerted by the vapor to be 4.41 pounds. If the moisture were entirely withdrawn at 165°, the air would then exert the entire pressure of 66.7 pounds, and its volume would be

$$\frac{120.2 \times 62.29}{66.7} = 112.3 \text{ cubic feet.}$$

But at the final temperature of 80° the pressure of saturated vapor is 0.5027 of a pound, so that the air exerts a pressure of

$$50 + 14.7 - 0.50 = 64.2 \text{ pounds,}$$

and its volume will be

$$\frac{112.3 \times 540.7 \times 64.2}{625.7 \times 62.29} = 100 \text{ cubic feet.}$$

In this case the calculation gives, as it should, the original datum of the problem, and the calculation is inserted to show the method only. Obviously the final volume of the air delivered by a given compressor, at a given temperature and pressure, may be calculated from the volume drawn into the compressor without calculating the volume delivered at the pressure and temperature near the compressor.

If a so-called dry-air compressor draws air from the atmosphere, and the air is finally used at or near the original temperature of the atmosphere, then that air will ordinarily be saturated. In the problem given, the air supplied to the compressor per minute, if half saturated, will contain

$$\frac{424 \times 0.0008104}{2} = 0.172$$

pound of moisture. If the air is cooled again to 60°, it can contain, when saturated,

$$100 \times .0008104 = 0.081$$

pound of moisture, and the remainder will be condensed. Even at the temperature of 80°, 100 cubic feet of saturated air will contain only

$$100 \times 0.001553 = 0.155$$

pound of moisture, so that the air will, under the suppositions, be saturated, and some moisture will be condensed.

Compressed-air Engines.—Engines for using compressed air differ from steam-engines only in details that depend on the nature of the working fluid. In some instances compressed air has been used in steam-engines without any change; for example, in Fig. 81 the dotted diagram was taken from the cylinder of an engine using compressed air, and the dot-and-dash diagram was taken from the same end of the cylinder when steam was used in it. The full line *ab* is a hyperbola, and the line *ac* is the adiabatic line for a gas drawn through the intersection of the expansion lines of the two diagrams.

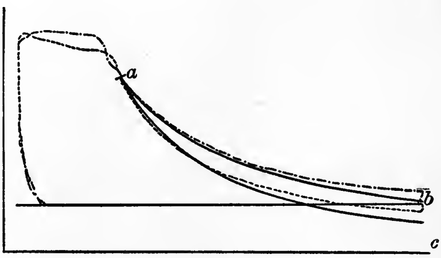


FIG. 81.

Power of Compressed-air Engines.—The probable mean effective pressure attained in the cylinder of a compressed-air engine, or to be expected in a projected engine, may be found in the same manner as is used in designing a steam-engine. In Fig. 82, the expansion curve 1, 2 and the compression curve 3, *o* may be assumed to be adiabatic lines for a gas represented by the equation

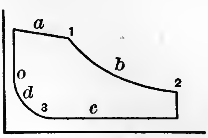


FIG. 82.

$$pv^{\kappa} = p_1 v_1^{\kappa},$$

and the area of the diagram may be found in the usual way, and therefrom the mean effective pressure can be determined. Having the mean effective pressure, the power of a given engine or the size required for a given power may be determined directly.

Let the specific pressure and the specific volume in the supply-pipe be p_3 and v_3, and let the specific pressure and the specific volume in the exhaust-pipe be p_4 and v_4; then, assuming that there are no losses of pressure in the valves and passages, that there is no clearance, and that the expansion is adiabatic, the work of one unit of weight of air is

$$W_1 = p_3v_3 + \int_{v_3}^{v_4} pdv - p_4v_4 = p_3v_3 + \frac{p_3v_3}{\kappa - 1}\left\{1 - \left(\frac{p_4}{p_3}\right)^{\frac{\kappa-1}{\kappa}}\right\} - p_3v_3\left(\frac{p_4}{p_3}\right)^{\frac{\kappa-1}{\kappa}}$$

$$\therefore\ W_1 = p_3v_3\frac{\kappa}{\kappa - 1}\left\{1 - \left(\frac{p_4}{p_3}\right)^{\frac{\kappa-1}{\kappa}}\right\}, \quad . \quad . \quad (321)$$

or, in terms of the final pressure and volume,

$$W_1 = p_4v_4\frac{\kappa}{\kappa - 1}\left\{\left(\frac{p_3}{p_4}\right)^{\frac{\kappa-1}{\kappa}} - 1\right\}. \quad . \quad . \quad (322)$$

When it is more convenient, the mean effective pressure can be obtained by aid of the equation

$$\text{M. E. P.} = p_4\frac{\kappa}{\kappa - 1}\left\{\left(\frac{p_3}{p_4}\right)^{\frac{\kappa-1}{\kappa}} - 1\right\}, \quad . \quad . \quad (323)$$

in which the pressures may be in any convenient units, as, for example, in pounds on the square inch.

If an engine fulfils all of the above conditions except that there is a clearance, then equations (321) and (322) may be used for finding the work developed by one unit of weight of air, provided that the clearance is filled by compression, with air at the admission pressure p_3. The equation (323) for the mean effective pressure cannot be used directly, but it may be used in connection with the volume exhausted per stroke or per minute, in calculating the power of the engine. The diagram from such an engine will be represented by Fig. 83, and the foregoing statement may be put in the following form: The actual mean effective pressure is the mean effective pressure by equation (323), multiplied by the ratio of the line *cd* to the entire length of the diagram. It is apparent that the only effect of clearance is to increase the size of cylinder required for a given purpose, and with it the work lost in friction.

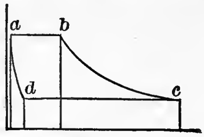


FIG. 83.

Air Consumption.—The air consumed by a given compressed-air engine may be calculated from the volume, pressure, and temperature at cut-off or release, and the volume, temperature, and pressure at compression, in the same way that the indicated consumption of a steam-engine is calculated; but in this case the indicated and actual consumption should be the same, since there is no change of state of the working fluid. Since the intrinsic energy of a gas is a function of the temperature only, the temperature will not be changed by loss of pressure in the valves and passages, and the air at cut-off will be cooler than in the supply-pipe, only on account of the chilling action of the walls of the cylinder during admission, which action cannot be energetic when the air is dry, and probably is not very important when the air is saturated.

Final Temperature.—If the expansion in a compressed-air engine is complete, i.e., if it is carried down to the pressure in the exhaust-pipe, then, assuming that there are no losses of pressure in valves and passages, the final temperature may be found by the equation

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\kappa - 1}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (324)$$

If the expansion is not complete, then the temperature at the end of expansion may be found by the equation

$$\frac{T_r}{T_3} = \left(\frac{V_c}{V_r}\right)^{\kappa - 1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (325)$$

in which V_c is the volume in the cylinder at cut-off and V_r at release, T_r is the absolute temperature at the end of expansion, and T_3 is the temperature at cut-off, assumed to be the same as in the supply-pipe. T_r is not the temperature during back pressure nor in the exhaust-pipe. If the exhaust-valve is opened suddenly at release the air will expand suddenly, and part of the air will be expelled at the expense of the energy in the air remaining—much as though that air expanded behind a piston and the temperature in the cylinder during exhaust, and at the beginning of compression, may be calculated by equation (325). The temperature in the exhaust-pipe will not be so low, for the temperature of the escaping air will vary during the expulsion produced by sudden expansion, and will only at the end of that operation have the temperature T_4, while the energy expended on that air to give it velocity will be restored when the velocity is reduced to that in the exhaust-pipe.

Volume of the Cylinder.—The determination of the volume of the cylinder of a compressed-air engine, which uses a stated volume of air per minute, is the converse of the determination of the air consumed by a given engine, and can be found by a similar process. We may calculate the volume of air at the pressure in the supply-pipe, consumed per stroke by an engine having one unit of volume for its piston displacement, and therefrom find the number of units of volume of the piston displacement for the required engine.

Interchange of Heat.—The interchanges of heat between the walls of the cylinder of a compressed-air engine and the air working therein are of the same sort as those taking place between the steam and the walls of the cylinder of a steam-engine; that is to say, the walls absorb heat during admission and compression, if the latter is carried to a considerable degree, and yield heat during expansion and exhaust. Since the walls of the cylinder are never so warm as the entering air, nor so cold as the air exhausted, the walls may absorb heat during the beginning of expansion, and yield heat during the beginning of compression.

The amount of interchange of heat is much less in a compressed-air engine than in a steam-engine. With a moderate expansion, the interchanges of heat between dry air and the walls of the cylinder are insignificant. Moisture in the air increases the interchanges in a marked degree, but does not make them so large that they need be considered in ordinary calculations.

Moisture in the Cylinder.—The chief disadvantage in the use of moist compressed air—and it is fair to assume that compressed air is nearly if not quite saturated when it comes to the engine—is that the low temperature, experienced when the range of pressures is considerable, causes the moisture to freeze in the cylinder and clog the exhaust-valves. The difficulty may be overcome in part by making the valves and passages of large size. Freezing of the moisture may be prevented by injecting steam or hot water into the supply-pipe or the cylinder, or the air may be heated by passing it through externally heated pipes, or by some similar device. In the application of compressed air to driving street-cars the air from the reservoir has been passed through hot water, and thereby made to take up enough hot moisture to prevent freezing. The study of gas-engines suggests a method of heating compressed air which it is believed has never been tried. The air supplied to a compressed-air engine, or a part of the air, could be caused to pass through a lamp of proper construction to give complete combustion, and the products of combustion passed to the engine with the air. Should such a device be used, it would be advisable that the temperature of the air should be raised only to a moderate degree to avoid destruction of the lubricants in the cylinder, and the combustion at all hazards must be complete, or the cylinder would be fouled by unburned carbon.

Compound Air Engines.—When air is expanded to a considerable degree in a compressed-air engine a gain may be realized by dividing the expansion into two or more stages in as many cylinders, provided that the air can be economically reheated between the cylinders. The heat of the atmosphere or of water at the same temperature may sometimes be used for this purpose. It is not known that machines of this construction have been used. If they were to be constructed, the practical advantages of equal distribution of work and pressure would probably control the ratio of the volumes of the cylinders.

Calculation of a Compressed-air Engine.—Suppose that a compressed-air engine has a diameter of 10 inches and a stroke of 20 inches, that it makes 100 revolutions per minute, and is supplied with air at 80° F. and at 50 pounds gauge-pressure. Let the cut-off be at one-half stroke, the compression at 5 per cent of the stroke, and the clearance 5 per cent of the piston displacement.

The pressure at the end of expansion will be

$$64.7 \times \left(\frac{0.55}{1.05}\right)^{1.4} = 25.8$$

pounds absolute, or 11.12 pounds above the atmosphere. The pressure at the end of compression will be

$$14.7 \times \left(\frac{0.10}{0.05}\right)^{1.4} = 38.8$$

pounds absolute, or 24.1 pounds above the pressure of the atmosphere.

The mean effective pressure will be

$$\text{M. E. P.} = 64.7 \times 0.55 + 64.7 \times (0.55)^{1.4} \int_{0.55}^{1.05} \frac{dv}{v^{1.4}} - 14.7 \times 0.95$$
$$- 14.7 \times (0.1)^{1.4} \int_{0.05}^{0.1} \frac{dv}{v^{1.4}};$$

$$\therefore \text{M. E. P.} = 32.35 + 20.50 - 13.97 - 1.94 = 36.94.$$

If the engine has large ports and automatic cut-off valves the mean effective pressure realized may be assumed to be 0.95 of that calculated, or it will be about 35 pounds.

Assuming the diameter of the piston-rod to be 2 inches, the mean area of the piston will be

$$\frac{2 \times 78.54 - 3.14}{2} = 77 \text{ square inches.}$$

The horse-power will therefore be

$$\frac{35 \times 77 \times 2 \times 100 \times 20}{33000 \times 12} = 13.6 \text{ I. H. P.}$$

If, further, the friction of the engine is assumed to absorb one tenth of the indicated horse-power, the effective horse-power will be $12\frac{1}{4}$.

The temperature of the air at the end of expansion will be

$$540.7 \times \left(\frac{0.55}{1.05}\right)^{0.4} = 417.5;$$

or $-42°$. F. The temperature at the beginning of compression may be assumed to be

$$540.7 \times \left(\frac{14.7}{64.7}\right)^{\frac{0.4}{1.4}} = 354.1;$$

or $-106°$ F. The influence of moisture in the air and of the interchanges of heat will be to increase each of these temperatures.

Were it desired to prevent freezing in the cylinder, then the lowest temperature must be at least 32° F., and the entering air should have at least the temperature of

$$460.7 \times \left(\frac{64.7}{14.7}\right)^{\frac{0.4}{1.4}} = 703.6,$$

or 243°.6 F. It is probable that a less temperature than this will obviate difficulty from freezing.

The piston displacement of the engine will be

$$\frac{77 \times 20}{1728} = 0.891 \text{ cubic foot.}$$

The volume of air caught in the cylinder at compression will be 0.089 of a cubic foot at the pressure of 14.7 pounds, and at the absolute temperature, as calculated, of 354.1. At the temperature 80° F., and at 64.7 pounds absolute pressure, the volume will be

$$0.089 \times \frac{540.7}{354.1} \times \frac{14.7}{64.7} = 0.031 \text{ cubic foot.}$$

Consequently the air consumed per stroke will apparently be

$$0.55 \times 0.891 - 0.031 = 0.459$$

of a cubic foot. But the causes which prevent the realization of the mean effective pressure diminish the consumption of air, so that the consumption per stroke may be assumed to be

$$0.95 \times 0.459 = 0.436$$

of a cubic foot. The consumption per minute will therefore be

$$2 \times 100 \times 0.436 = 87.2 \text{ cubic feet.}$$

Were it required that a compressed-air engine should use 100 cubic feet of air per minute, the piston displacement would obviously be 1.02 cubic feet, and the indicated horse-power would be 15.6, while the effective horse-power would be 14. Comparing this result with the power expended on the fluid-piston air-compressor, it appears that the efficiency of the fluid is

$$15.6 \div 52.0 = 0.30,$$

while the efficiency of the whole apparatus for transferring power is

$$14 \div 57.8 = 0.24.$$

Efficiency of Compressed-air Transmission.—The great defect of compressed air as a means of transmitting power, that is, the small per cent of work realized, is exhibited by the preceding calculation. Though a greater efficiency may be attained by a better choice of pressures and proportions, the result will in all cases be unsatisfactory. Compressed air for this purpose is consequently employed only where power for compression is cheap and abundant, or where there are special reasons for using air. As an example, compressed air is used in mining and tunnelling where the use of steam would be objectionable. It is suggested that compressed air may be used in operating cranes where hydraulic power is objectionable from the liability of freezing water-pipes, and where there is a large loss from condensation of steam in starting and operating only a short time.

Experiments made by M. Graillot* of the Blanzy mines showed an efficiency of from 22 to 32 per cent. Experiments made by Mr. Daniel at Leeds gave an efficiency varying from 0.255 to 0.455, with pressures varying from 2.75 atmospheres to 1.33 atmospheres. An experiment made by Mr. Kraft † gave an efficiency of 0.137 for a small machine, using air at a pressure of five atmospheres without expansion.

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* Pernolet, L'Air Comprimé, pp. 549, 550.

† Revue universelle des Mines, 2 Série, Tome vi.

CHAPTER XXI.

REFRIGERATING MACHINES.

In the discussion of heat-engines it appeared that the simplest cycle described by such an engine is that for Carnot's ideal engine, represented by Fig. 84. When the working substance in the cylinder P is a gas the cycle represented by Fig. 85 is composed of two isothermal lines, AB and CD, and of two adiabatic lines, BC and DA. When the working substance is a mixture of a liquid and its vapor, the two isothermal lines become parallel to the axis OV, but the order of events is in no wise altered.

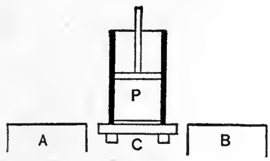

Fig. 84.

When working direct, Carnot's engine takes from the source of heat A a quantity of heat Q, changes a part into mechanical energy, and rejects the remainder Q_1 to the refrigerator B. The efficiency

$$\eta = \frac{AW}{Q} = \frac{Q - Q_1}{Q} = \frac{T - T_1}{T_1}$$

increases with the difference of temperatures of the source of heat and the refrigerator.

If the engine be reversed so that it describes the cycle in the order $ADCBA$, it takes heat from the refrigerator, adds thereto the heat equivalent of the work of the cycle, and delivers the sum to the source of heat, and thus becomes a refrigerating machine. It is apparent that in this action it is desirable to do as little work as possible on the working substance, and that this condition is fulfilled by making the difference of temperatures as small as possible.

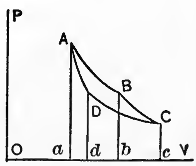

Fig. 85.

In practice it is found convenient to supply to a heat-engine at each stroke a quantity of the working substance at a high temperature, which does work on the piston, and is rejected at a lower temperature. Thus the steam-engine takes steam from the boiler which serves as a source of heat, and, after abstracting some of the heat in the form of work, rejects the steam to the condenser or refrigerator. In like manner refrigerating machines take a supply of the working substance from the refrigerator or refrigerant, do work upon it, and deliver it at a higher temperature to a receptacle which is known as the cooler or condenser, but which takes the place of the source of heat or boiler.

Two forms of refrigerating machines are in common use—air-refrigerating machines and compression-refrigerating machines using a saturated vapor—such as the ammonia-refrigerating or ice-machine.

Air-refrigerating Machine.—The general arrangement of an air-refrigerating machine is shown by Fig. 86. It consists of a compression cylinder A, an expansion cylinder B of smaller size, and a cooler C. It is commonly used to keep the atmosphere in a cold storage-room at a low temperature, and has certain advantages for this purpose, especially on shipboard. The air from the storage-room comes to the compressor at or about freezing-point, is compressed to two or three atmospheres and delivered to the cooler, which has the same form as a surface condenser, with cooling water entering at e and leaving at f. From the cooler the air, usually somewhat warmer than the atmosphere, goes to the expansion cylinder B, in which it is expanded nearly to the pressure of the air and cooled to a low temperature, and then delivered to the storage-room. The inlet valves a, a and the delivery-valves b, b of the compressor are moved by the air itself; the admission-valves c, c and the exhaust-valves d, d of the expansion cylinder are like those of a steam-engine, and must be moved by the machine. The difference between the work done on the air in the compressor, and that done by the air in the expansion cylinder, together with the friction work of the whole machine, must be supplied by a steam-engine or other motor.

The effect of clearance in the compression cylinder, as has been seen in the discussion of air-compressors, is to increase the size required for a certain performance. The exhaust-valves of the expansion cylinder should be so set that the clearance

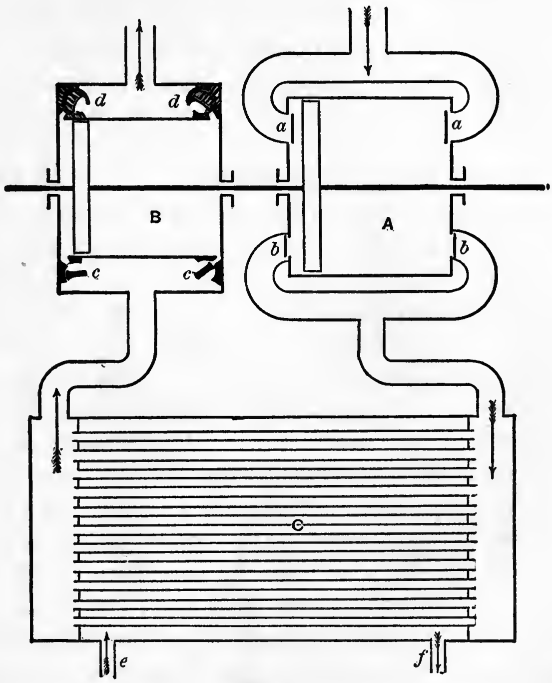

FIG. 86.

shall be filled by compression with air at nearly if not quite the admission pressure, and the cut-off should be such that the air shall expand down to the back pressure. This latter is always of importance for the efficient action of the machine, but if the clearance is small the compression is of less moment.

It is customary to provide the compression cylinder with a water-jacket to prevent overheating, and frequently a spray of water is thrown into the cylinder to reduce the heating and the work of compression. Sometimes the cooler *C*, Fig. 86, is replaced by an apparatus resembling a steam-engine jet condenser, in which the air is cooled by a spray of water. In any case it is essential that the moisture in the air, as well as the water injected, should be efficiently removed before the air is delivered to the expansion cylinder, otherwise snow will form in that cylinder and interfere with the action of the machine. Various mechanical devices have been used to collect and remove water from the air, but air may be saturated with moisture after it has passed such a device. The Bell-Coleman Company use a jet cooler with provision for collecting and withdrawing water, and then pass the air through pipes in the cold room on the way to the expansion cylinder. The cold room is maintained at a temperature a little above freezing-point, so that the moisture in the air is condensed upon the sides of the pipes and drains back into the cooler. The same machine, as made by Menck and Hambrock, is provided with a device for removing moisture from the air, that is shown by Fig. 87. Air from the cooler comes in by the pipe *a*, is distributed by the annular perforated pipe *b*, and passes out to the expansion cylinder by the pipe *c*. The chamber *E* is surrounded by a jacket through which passes the cold air on the way from the expansion cylinder to the cold room. Since the air in the jacket is many degrees below freezing-point the walls of the chamber *E* are quickly covered with frost, which accumulates till a considerable thickness is attained; afterwards the moisture condenses and runs down to the bottom of the chamber, from whence it is withdrawn. A coil of steam-pipe *dd* is provided for thawing ice and snow that may accumulate at the bottom of the chamber. Since the same air is used continuously, being taken from the cold room, chilled and returned, the effect of these devices is to remove the moisture from the air in the cold room and to maintain a cold, dry atmosphere in it, which is well adapted to preserving all kinds of perishable provisions.

When an air-refrigerating machine is used as described the pressure in the cold room is necessarily that of the atmosphere, and the size of the machine is large as compared with its performance. The performance may be increased by running the machine on a closed cycle with higher pressures; for example, the cold air may be delivered to a coil of pipe in a non-freezing salt solution, from which the air abstracts heat through the walls of the pipe and then passes to the compressor to be used over again. The machine may then be used to produce ice, or the brine may be used for cooling spaces or liquids. A machine has been used for producing ice on a small scale, without cooling water, on the reverse of this principle: that is, atmospheric

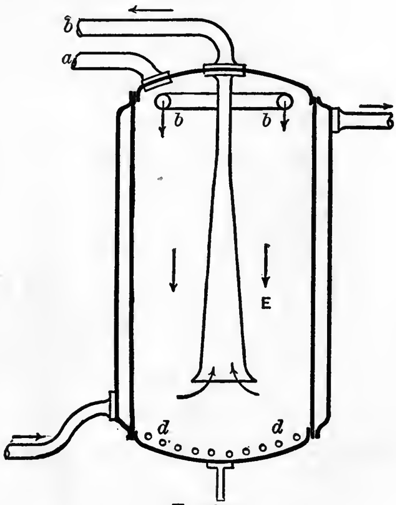

FIG. 87.

air is first expanded and chilled and delivered to a coil of pipe in a salt solution, then the air is drawn from this coil, after absorbing heat from the brine, compressed to atmospheric pressure, and expelled.

Calculation of Air-refrigerating Machine.—The performance of a refrigerating machine may be stated in terms of the number of thermal units withdrawn in a unit of time, or in terms of the weight of ice produced. The latent heat of fusion of ice may be taken to be 80 calories or 744 B. T. U.

Let the pressure at which the air enters the compression cylinder be p_1, that at which it leaves be p_2; let the pressure at cut-off in the expanding cylinder be p_3 and that of the back pressure in the same be p_4; let the temperatures corresponding to these pressures be t_1, t_2, t_3, and t_4, or reckoned from the absolute zero, T_1, T_2, T_3, and T_4. With proper valve-gear and large, short pipes communicating with the cold chamber, p_4 may be assumed to be equal to p_1, and equal to the pressure in that chamber. Also t_1 may be assumed to be the temperature maintained in the cold chamber, and t_3 may be taken to be the temperature of the air leaving the cooler. With a good cut-off mechanism and large passages p_3 may be assumed to be nearly the same as that of the air supplied to the expanding cylinder. Owing to the resistance to the passage of the air through the cooler and the connecting pipes and passages, p_3 is considerably less than p_2.

The expansion in the expanding cylinder may be assumed to be adiabatic, so that

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\kappa - 1}{\kappa}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (326)$$

Were the compression also adiabatic, the temperature t_2 could be determined in a similar manner; but the air is usually cooled during compression, and contains more or less vapor, so that the temperature at the end of compression cannot be determined from the pressure alone, even though the equation of the expansion curve be known.

Let the air passing through the refrigerating machine per minute be M, then the heat withdrawn from the cold room is

$$Q_1 = Mc_p(t_1 - t_4). \quad . \quad . \quad . \quad . \quad . \quad (327)$$

The work of compressing M units of weight of air from the pressure p_1 to the pressure p_2 in a compressor without clearance is

$$W_c = M\left\{ p_1v_1 + \int_{v_2}^{v_1} pdv - p_2v_2 \right\}$$

$$= Mp_1v_1 \frac{n}{n-1}\left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right\}, \quad . \quad (328)$$

provided that the compression curve can be represented by an exponential equation. The work will be the same for a compressor with clearance if the exponent for the equation to the expansion curve is the same as that for the compression curve. If the compression can be assumed to be adiabatic,

$$W_c = Mp_1v_1 \frac{\kappa}{\kappa - 1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right\} = \frac{Mc_p}{A}(t_2 - t_1); \quad . \quad (329)$$

for in such case we have the equations

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}, \qquad AR = C_p - C_v = C_p\frac{\kappa - 1}{\kappa}.$$

The work done by the air in the expanding cylinder should be calculated in the manner used on page 201 in designing a steam-engine, or on page 325 for finding the work of a compressed-air engine. If the expansion and compression are both complete, then the work done by M units of weight of air is

$$W_e = \frac{Mc_p}{A}(t_3 - t_4) \quad . \quad . \quad . \quad . \quad . \quad . \quad (330)$$

The work that must be supplied per minute is

$$W = W_c - W_e,$$

and the net horse-power required is

$$\frac{W}{33000} \text{ H. P.};$$

but the gross horse-power required is much larger, since all the frictional resistances must be overcome, including the friction of both pistons. The proper allowance can readily be determined on a machine with a steam-engine coupled direct, by indicating both of the air-cylinders and the steam-cylinder simultaneously.

The heat carried away by the cooling water is

$$Q = Q_1 + AW. \quad . \quad . \quad . \quad . \quad . \quad . \quad (331)$$

If compression and expansion are both adiabatic, then

$$Q = Mc_p(t_1 - t_4 + t_2 - t_1 - t_3 + t_4) = Mc_p(t_2 - t_3). \quad (332)$$

If the initial and final temperatures of the cooling water are t_i and t_k, and if q_i and q_k are the corresponding heats of the liquid, then the weight of cooling water per minute is

$$G = \frac{Q}{q_k - q_i}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (333)$$

A good supply of cooling water and a method of regulating it should be provided, so that an approximate calculation may be made for any case, under the assumption of adiabatic compression and expansion, by the equation

$$G = \frac{Mc_p(t_2 - t_3)}{q_k - q_i}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (334)$$

The volume of the compression piston displacement, neglecting clearance, is

$$V_c = \frac{Mv_1}{2n} = \frac{Mp_0T_1}{2np_1T_0}v_0 = \frac{MRT_1}{2np_1}, \quad . \quad . \quad . \quad (335)$$

in which n is the number of revolutions per minute, and p_0, v_0, T_0 are the pressure, volume, and absolute temperature, at atmospheric pressure and at freezing-point.

If it be assumed that p_4 is the same as p_1, then the volume of the expanding cylinder, without clearance, may be assumed to be

$$V_e = V_c \frac{T_4}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (336)$$

If the clearance of the compressor is $\frac{1}{m}$ of the piston displacement, then the volume of air in the cylinder when the inlet-valve opens is

$$\frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}},$$

and the volume calculated by equation (335) should be multiplied by

$$1 + \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - \frac{1}{m}. \quad . \quad . \quad . \quad . \quad . \quad (337)$$

If the expansion and compression in the expanding cylinder are complete, the same expression may be used to allow for clearance in that cylinder, making n equal to κ. To allow for loss of pressure in valves and passages and for other imperfections, both of these volumes may be increased by an amount to be determined by experiment. In practice the expansion is seldom carried down to the back pressure in the expanding cylinder, nor is the compression complete, and the volume is smaller than that given by equation (336).

The temperature T_4 may be controlled by the cut-off of the expanding cylinder, and thus the performance of the machine may be varied. As the cut-off is shortened p_2 is increased and T_4 diminished, and this in turn makes V_e smaller compared with V_c.

PROBLEM.—Required the dimensions of an air-refrigerating machine to produce an effect equal to the melting of 200 pounds of ice per hour. Let the pressure in the cold chamber be 14.7 pounds and the temperature 32° F. Let the pressure at cut-off in the expanding cylinder be 29.4 pounds by the indicator or 44.1 pounds absolute. Let the delivery pressure in the compressor be 39.4 pounds by the indicator, i.e., let the loss of pressure in the cooler and passages be 10 pounds. Let the initial and final temperatures of the cooling water be 60° and 80° F., and the temperature of the air from the cooler 90° F. Let the machine make 60 revolutions per minute.

The melting of 200 pounds of ice per hour is equivalent to 28800 B. T. U. per hour, or 480 B. T. U. per minute.

Assuming adiabatic compression and expansion,

$$T_4 = 492.7\left(\frac{14.7}{44.1}\right)^{\frac{0.4}{1.4}} = 360; \quad \therefore t_4 = -100°.7 \text{ F.};$$

$$T_2 = 492.7\left(\frac{54.1}{14.7}\right)^{\frac{4}{1.4}} = 714.9; \quad \therefore t_2 = 254°.2 \text{ F.}$$

The air used per minute is therefore

$$480 \div (32 + 100.7) \times 0.2375 = 12.10 \text{ pounds.}$$

The horse-power of the compression cylinder with adiabatic compression is

$$\frac{W_c}{33000} = \frac{12.1 \times 778 \times 0.2375 \times (254.2 - 32)}{33000} = 15.0 \text{ H. P.}$$

If the compression curve may be represented by

$$pv^{1.2} = \text{const.},$$

then the work of compression will be

$$W_c = 12.1 \times 144 \times 14.7 \times 12.4 \times \frac{1.2}{0.2}\left\{\left(\frac{54.1}{14.7}\right)^{\frac{0.2}{1.2}} - 1\right\}$$
$$= 49230 \text{ foot-pounds,}$$

and the horse-power is therefore 14.0.

The horse-power of the expanding cylinder is

$$\frac{W_e}{33000} = \frac{12.1 \times 778 \times 0.2375 \times (32 + 100.7)}{33000} = 9.0 \text{ H. P.}$$

The net horse-power required is therefore 6 H. P. or 5 H. P., and the indicated horse-power of the steam-cylinder may be assumed to be 8 H. P. or $6\frac{2}{3}$ H. P., according to the manner of the compression.

The volume of the compressor-piston displacement without clearance will be

$$\frac{12.1 \times 12.4}{120} = 1.25 \text{ cubic feet.}$$

The volume of the expanding cylinder under the same condition is

$$1.25 \times \frac{360}{492.7} = 0.91 \text{ cubic feet.}$$

If the clearance of the compressor be assumed to be 0.02, the piston displacement should be

$$1.25 \left\{ 1 + 0.02 \left(\frac{54.1}{14.7}\right)^{\frac{1}{1.2}} - 0.02 \right\} = 1.4 \text{ cubic feet.}$$

If the clearance of the expanding cylinder be assumed to be 0.05, the piston displacement should be

$$0.91 \left\{ 1 + 0.05 \left(\frac{44.1}{14.7}\right)^{\frac{1}{1.4}} - 0.05 \right\} = 0.96 \text{ cubic foot.}$$

If, further, an allowance of ten per cent be made for imperfections, the dimensions may be: diameter, compressor, 17 inches; diameter, expanding cylinder, 14 inches; stroke, 12 inches.

Compression-refrigerating Machine.—The arrangement of a refrigerating machine using a volatile liquid and its vapor is shown by Fig. 88. The essential parts are the compressor *A*, the condenser *B*, the valve *D*, and the vaporizer *C*. The compressor draws in vapor at a low pressure and temperature, compresses it and delivers it to the condenser, which consists of coils of pipe surrounded by cooling water that enters at *e* and leaves at *f*. The vapor is condensed, and the resulting liquid gathers in a reservoir in the bottom, from whence it is led by a small pipe having a regulating valve *D* to the vaporizer or refrigerator. The refrigerator is also made up of coils of pipe, and is immersed in a non-freezing solution of salt, commonly chloride of calcium. The volatile liquid vaporizes and withdraws heat from the surrounding brine, and reduces the temperature

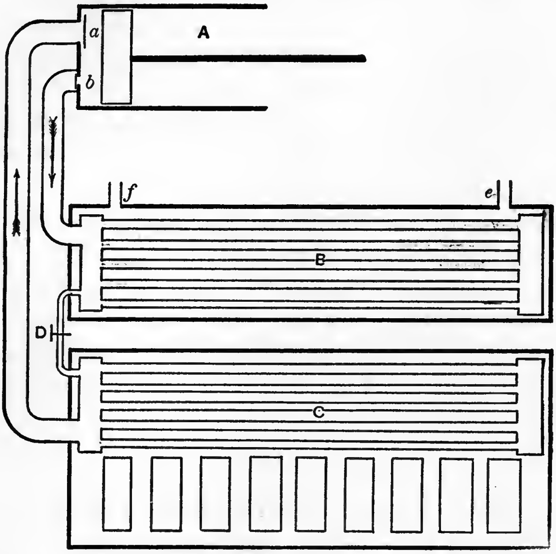

FIG. 88.

below the freezing-point of water. In the figure the machine is represented to be applied to ice-making, the water being introduced and frozen in properly-shaped moulds of thin metal. When the machine is used to cool a room the vaporizer may be made up of a system of pipes arranged to withdraw heat from the air, or brine may be cooled and circulated through such a system of pipes. When brine is used either in ice-making or in cooling, a positive circulation should be given it by a pump or the equivalent.

In Fig. 88 the compressor is represented as single-acting, but for horizontal machines it is commonly made double-acting. Frequently the compressor has two single-acting vertical cylinders driven by a horizontal steam-engine coupled to the shaft. Such compressors sometimes have the clearance filled with oil, of which part is forced through the delivery-valves and allowed to flow back into the cylinder during the inflow of vapor. In any case, it is of great importance that the clearance shall be reduced to the smallest amount possible.

To make the cycle of the machine complete the liquid from the condenser should be allowed to do work in an expansion cylinder like that of an air-refrigerating machine, instead of flowing through the regulating valve D. The size of such a cylinder would be small, and the work recovered, insignificant, so that none of the machines in use are provided with such an expansion cylinder.

Calculation of Compression Machine.—Let the pressure in the condenser be p_1, the temperature t_1, and the heat of the liquid q_1. Let the pressure in the vaporizer or refrigerator be p_2, the temperature t_2, and the total heat of vaporization λ_2. The heat withdrawn from the refrigerant to change one unit of weight of liquid at the temperature t_1 into saturated vapor at the pressure p_2 is

$$\lambda_2 - q_1,$$

so that the heat withdrawn per minute by a machine using M units of weight of the working fluid per minute is

$$Q_1 = M(\lambda_2 - q_1). \quad . \quad . \quad . \quad . \quad . \quad (338)$$

Even though the compressor cylinder be water-jacketed, the walls are at a considerably higher temperature than the entering vapor, and the pressure during admission is a little lower than that in the vaporizer. Also most of the vapors now used in such machines are superheated by adiabatic compression. Therefore it is probable that the vapor is superheated during compression, even though it be moist as it leaves the vaporizer. For an approximate calculation it may be assumed that the pressure in the cylinder during admission is p_2, that the vapor is dry and saturated at the beginning of compression, and that it is compressed adiabatically and superheated during the entire compression. It will be shown that ammonia and sulphur dioxide, when moderately superheated, have the approximate characteristic equation

$$pv = BT - Cp^{\frac{k-1}{k}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (339)$$

and that during an adiabatic change we have the equation

$$\frac{T}{T'} = \left(\frac{p}{p'}\right)^{\frac{k-1}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (340)$$

During the expulsion of vapor from the compressor the pressure in the cylinder is a little higher than in the condenser, but it may be assumed to be the same for our approximate calculation. The temperature of the vapor leaving the compressor and entering the condenser may consequently be calculated by the equation

$$\frac{T_s}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (341)$$

The heat that must be withdrawn by the cooling water is therefore

$$Q = M\{c_p(t_s - t_1) + r_1\}, \quad . \quad . \quad . \quad . \quad . \quad (342)$$

in which c_p is the specific heat of the superheated vapor at constant pressure, and r_1 is the heat of vaporization at the pressure p_1.

If the initial and final temperatures of the cooling water are t_i and t_k, and if q_i and q_k are the corresponding heats of the liquid for water, then the weight of cooling water used per minute is

$$G = \frac{Mc_p(t_s - t_1) + r_1}{q_k - q_i}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (343)$$

For the first approximation the horse-power of the compressor may be calculated by the expression

$$\frac{778 \times M\{c_p(t_s - t_1) + \lambda_1 - \lambda_2\}}{33000}. \quad . \quad . \quad . \quad (344)$$

The power thus calculated should be multiplied by a factor to be found by experiment, in order to find the probable indicated horse-power of the compressor, and the indicated horse-power must be multiplied by another factor to find the power required to drive the machine, or by a factor to find the indicated horse-power of a steam-engine coupled to the shaft driving the compressor.

If the actual pressures in the cylinder of the compressor during admission and delivery are p'' and p', if the specific volume at the beginning of compression is v'', and if the compression and expansion curves may be represented by the equation

$$pv^n,$$

then the horse-power of the compressor may be found by the expression

$$\frac{Mp''v''}{33000} \cdot \frac{n}{n-1}\left\{\left(\frac{p'}{p''}\right)^{\frac{n-1}{n}} - 1\right\}. \quad . \quad . \quad . \quad (345)$$

If the vapor at the beginning of compresssion can be assumed to be dry and saturated, then the volume of the piston displacement of a compressor without clearance, and making N strokes per minute, is

$$V = \frac{Mv_2}{N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (346)$$

To allow for clearance, the volume thus found may be multiplied by the factor

$$1 + \frac{1}{m}\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} - \frac{1}{m},$$

in which $\frac{1}{m}$ is the clearance expressed as a fraction of the piston displacement. The volume thus found is further to be multiplied by a factor to allow for inaccuracies and imperfections.

The vapors used in compression machines are liable to be mingled with air or moisture, and in such case the performance of the machine is impaired. To allow for such action the size and power of the machine must be increased in practice above those given by calculation. It would appear that proper precautions ought to be taken to prevent such action from becoming of importance.

PROBLEM.—Required the dimensions of an ammonia-refrigerating machine to produce 2000 pounds of ice per hour. Let the temperature of the salt solution be 14° F. and the temperature in the condenser 86° F.; let the initial and final temperatures of the cooling water be 60° and 80° F. Let the compressor be double-acting, and let it make 60 revolutions per minute.

The pressures corresponding to the temperatures 14° and 86° are 41.5 and 168.2 pounds, absolute, per square inch, or 26.8 and 153.8 pounds by the gauge. For ammonia, $k = \frac{4}{3}$. Hence by equation (341)

$$T_s = (14 + 460.7)\left(\frac{168.2}{41.5}\right)^{\frac{1}{4}} = 673.7;$$

$$t_s = 213^\circ \text{ F.}$$

If 5 per cent be allowed for ice wasted in removing it from the moulds, and for other losses, the capacity of the machine per minute must be

$$\frac{2100 \times 144}{60} = 5040 \text{ B. T. U.};$$

$$\therefore Q_1 = 5040 = M(\lambda_2 - q_1) = M(554 - 59);$$

$$\therefore M = \frac{5040}{495} = 10.2 \text{ pounds.}$$

The heat withdrawn by the cooling water is

$$Q = M\{c_p(t_s - t_1) + r_1\};$$

$$Q = 10.2\{0.508(213 - 86) + 497\} = 5727 \text{ B. T. U.}$$

The cooling water required per minute is, consequently,

$$G = \frac{5727}{q_k - q_i} = \frac{5727}{48.09 - 28.12} = 287 \text{ pounds.}$$

The horse-power will be, approximately,

$$\frac{778M\{c_p(t_s - t_1) + \lambda_1 - \lambda_2\}}{33000}$$
$$= \frac{778 \times 10.2\{0.508(213 - 86) + 556 - 297\}}{33000} = 77.8.$$

The horse-power of the steam-cylinder may be assumed to be

$$77.8 \div 0.80 = 97.2.$$

On the assumption that the vapor in the compressor is dry and saturated at the beginning of compression, the volume of the piston displacement, not allowing for clearance, is

$$V = \frac{Mv_2}{N} = \frac{10.2 \times 7.05}{2 \times 60} = 5.15 \text{ cubic feet.}$$

If we allow 10 per cent for the effect of clearance and imperfect action, then the volume should be 5.7 cubic feet, or about $20\frac{1}{2}$ inches in diameter by 30 inches stroke.

Fluids Available.—The fluids that have been used in compression-refrigerating machines are ether, sulphurous acid, ammonia, and a mixture of sulphurous acid and carbonic acid, known as Pictet's fluid. The pressures of the vapors of these fluids at several temperatures, and also the pressure of the vapors of methylic ether and carbonic acid, are given in the following table:

PRESSURES OF VAPORS, MM. OF MERCURY.

Temperatures, degrees Centigrade.	Ether.	Sulphur Dioxide.	Methyl-ether.	Ammonia.	Carbon Dioxide.	Pictet's Fluid.
− 30	.....	287.5	576.5	866.1	.....	585
− 20	68.9	479.5	882.0	1392.1	15142	745
− 10	114.7	762.5	1306.6	2144.6	20340	1018
0	184.4	1165.1	1879.0	3183.3	26907	1391
10	286.8	1719.6	2629.0	4574.0	34999	1938
20	432.8	2462.1	3586.0	6387.8	44717	2584
30	634.8	3431.8	4778.0	8701.0	56119	3382
40	907.0	4670.2	......	11595.3	69184	4347

Ether was used in the early compression machines, but at the temperatures maintained in the refrigerator the pressure is small and the specific volume large, so that the machines, like air-refrigerating machines, were either feeble or bulky. Moreover, air was liable to leak into the machine and unduly heat the compressor cylinder. Sulphur dioxide has been used successfully, but it has the disadvantage that sulphuric acid may be formed by the leakage of moisture into the machine, in which case rapid corrosion occurs. Ammonia has been extensively used in the more recent machines with good results. When distilled from an aqueous solution it is liable to contain considerable moisture. As is shown by the table, Pictet's fluid has a pressure at low temperature intermediate between the pressures of sulphur dioxide and ammonia, and the pressure increases slowly with the temperature.

The properties of saturated vapor of ether were determined by Regnault, and are given in Chapter VII. For the other vapors given in the table (except Pictet's fluid) he determined the relations of the temperature and pressure, but not the total heat of vaporization nor the heat of the liquid. He did, however, determine some of the properties of these substances in the gaseous state, or more properly, in the state of superheated vapors. It was first proposed by Ledoux * that the

* Annales des Mines, 1878.

properties of the superheated vapors of sulphur dioxide and ammonia can be represented by equations of the form deduced by Zeuner for superheated steam, and that the application of these equations to the saturated vapors as the limit makes it possible to calculate the properties of the saturated vapors approximately. His equations do not fulfil the conditions given by equation (166), page 118, and do not represent all the properties of the superheated vapors, given by Regnault; consequently new equations have been calculated for both French and English units.

Properties of Sulphur Dioxide.—The specific heat of gaseous sulphur dioxide is given by Regnault* as 0.15438, and the coefficient of dilatation as 0.0039028. The theoretical specific gravity compared with air, calculated from the chemical composition, is given by Landolt and Börnstein† as 2.21295. Gmelin‡ gives the following experimental determinations: by Thomson, 2.222; by Berzelius, 2.247. The figure 2.23 will be assumed in this work, which gives for the specific volume at freezing-point and at atmospheric pressure

$$v_0 = \frac{0.7735327}{2.23} = 0.347$$

cubic metres. The corresponding pressure and temperature are 10333, and 273°.7 C.

Now the coefficient of dilatation is the ratio of the increase of volume at constant pressure, for one degree increase of temperature, to the original volume. Writing the equation (166),

$$pv = \frac{c_p}{A} aT - Cp^a; \quad . \quad . \quad . \quad . \quad . \quad . \quad (347)$$

* Mémoires de l'Institut de France, Tome xxi., xxvi.

† Physikalische-chemische Tabellen.

‡ Watt's translation, p. 280.

we have at 0° C. and 1° C.,

$$p_0 v_0 = \frac{c_p}{A} a T_0 - C p_0^a;$$

$$p_0 v_1 = \frac{c_p}{A} a T_1 - C p_0^a;$$

$$\therefore \frac{v_1 - v_0}{v_0} = \frac{c_p}{A} \cdot \frac{a}{p_0 v_0}.$$

Substituting the known values and solving for a, we obtain 0.212; but the equation obtained from the equation (347) with this figure does not agree well with Regnault's experiments on the compressibility of sulphur dioxide. If, instead, we make

$$a = 0.22,$$

then by equation (347) the coefficient of dilatation becomes 0.00404, and it will be shown later that the equation deduced with this value agrees quite well with the experiments on compressibility.

The coefficient of T in equation (347) is therefore

$$0.15438 \times 426.9 \times 0.22 = 14.5,$$

and the coefficient of p^a is

$$\frac{14.5 \times 273.7 - 10333 \times 0.347}{\overline{10333}^{0.22}} = 48 \text{ nearly};$$

so that the equation becomes

$$pv = 14.5T - 48p^{0.22}. \quad . \quad . \quad . \quad . \quad . \quad (348)$$

Regnault found for the pressures

$$p_1 = 697.83 \text{ mm. of mercury,}$$
$$p_2 = 1341.58 \quad \text{“} \quad \quad \text{“}$$

and at 7°.7 C. the ratio

$$\frac{p_1 v_1}{p_2 v_2} = 1.02088.$$

Reducing the given pressures to kilograms on the square inch, and the temperature to the absolute scale, and applying to equation (347), we obtain instead of the experimental value for the above ratio 1.016.

Regnault gives for the pressure of saturated sulphur dioxide, in mm. of mercury, the equation

$$\begin{aligned}
\log p &= a - b\alpha^n - c\beta^n; \\
a &= 5.6663790; \\
\log b &= 0.4792425; \\
\log c &= 9.1659562 - 10; \\
\log \alpha &= 9.9972989 - 10; \\
\log \beta &= 9.98729002 - 10; \\
n &= t + 28^\circ \text{ C.}
\end{aligned}$$

Applying equations (109) and (110), page 80, to this case,

$$\frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n;$$

$$\begin{aligned}
\log \alpha &= 9.9972989; \\
\log \beta &= 9.98729002; \\
\log A &= 8.6352146; \\
\log B &= 7.9945332; \\
n &= t + 28^\circ \text{ C.}
\end{aligned}$$

The specific volume of saturated sulphur dioxide may be calculated by inserting in equation (347) for the superheated vapor the pressures calculated by aid of the above equation. The results at several temperatures are as follows:

t	-30° C.	0	$+30^\circ$ C.
s	0.8292	0.2256	0.0825

Andréeff * gives for the specific gravity of fluid sulphur dioxide 1.4336; consequently the specific volume of the liquid is

$$\sigma = 0.0007.$$

The value of r, the heat of vaporization, may now be calculated at the given temperatures by equation (128),

$$r = AuT\frac{dp}{dt},$$

in which $$u = s - \sigma.$$

The results are

t	$-30°$ C.	0	$+30°$ C.
r	106.9	97.60	90.54

Within the limits of error of our method of calculation, the value of r may be found by the equation

$$r = 98 - 0.27t.$$

To find the specific heat of the liquid, we may use equation (180), page 120,

$$c_p\left(1 - a\frac{T}{p}\cdot\frac{dp}{dt}\right) = c + \frac{dr}{dt} - \frac{r}{T}.$$

At 0° C. the specific heat is approximately

$$c = 0.4.$$

In English units we have for superheated sulphur dioxide

$$pv = 26.4T - 184p^{0.22},$$

the pressures being in pounds on the square foot, the volumes in cubic feet, and the temperatures in Fahrenheit degrees absolute.

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* Ann. Chem. Pharm. 1859.

For pressures in pounds on the square inch at temperatures on the Fahrenheit scale,

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 3.9527847;$$
$$\log b = 0.4792425;$$
$$\log c = 9.1659562 - 10;$$
$$\log \alpha = 9.9984994 - 10;$$
$$\log \beta = 9.99293890 - 10;$$
$$n = t + 18^\circ.4 \text{ F}.$$

For the heat of vaporization

$$r = 176 - 0.27(t - 32),$$

and for the specific heat of the liquid

$$c = 0.4.$$

Properties of Ammonia.—The specific heat of gaseous ammonia, determined by Regnault, is 0.50836. The theoretical specific gravity compared with air, calculated from the chemical composition, is given by Landolt and Börnstein as 0.58890. Gmelin gives the following experimental determinations: by Thomson, 0.5931; by Biot and Arago, 0.5967. For this work the figure 0.597 will be assumed, which gives for the specific volume at freezing-point and at atmospheric pressure

$$v_0 = \frac{0.7735327}{0.597} = 1.30$$

cubic meters. The coefficient of dilatation has not been determined, and consequently cannot be used to determine the value of a in equation (347). It, however, appears that very consistent results are obtained if a is assumed to be $\frac{1}{4}$, as for superheated steam. The coefficient of T then becomes

$$0.50836 \times 426.9 \times \tfrac{1}{4} = 54.3,$$

and the coefficient of $p^{\frac{1}{4}}$ is

$$\frac{54.3 \times 273.7 - 10333 \times 1.30}{\overline{10333}^{\frac{1}{4}}} = 142;$$

so that the equation becomes

$$pv = 54.3T - 142p^{\frac{1}{4}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (349)$$

The coefficient of dilatation, calculated by the same process as that used in determining a for sulphur dioxide, is 0.00404, which may be compared with that for sulphur dioxide.

Regnault found for the pressures

$$p_1 = 703.50 \text{ mm. of mercury,}$$
$$p_2 = 1435.3 \quad \text{"} \qquad \text{"}$$

and at 8°.1 C. the ratio

$$\frac{p_1v_1}{p_2v_2} = 1.0188,$$

while equation (349) gives under the same conditions 1.0200.

For saturated ammonia Regnault gives the equation

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 11.5043330;$$
$$\log b = 0.8721769;$$
$$\log c = 9.9777087 - 10;$$
$$\log \alpha = 9.9996014 - 10;$$
$$\log \beta = 9.9939729 - 10;$$
$$n = t + 22° \text{ C.};$$

by aid of which the pressures in mm. of mercury may be calculated for temperatures on the Centigrade scale. The differential coefficient may be calculated by aid of the equation

$$\frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n;$$

$$\log A = 8.1635170 - 10;$$
$$\log B = 8.4822485 - 10;$$
$$\log \alpha = 9.9996014 - 10;$$
$$\log \beta = 9.9939729 - 10;$$
$$n = t + 22^\circ \text{ C.}$$

The specific volume of saturated ammonia calculated by equation (339) at several temperatures are

t	-30° C.	0	$+30^\circ$ C.
s	0.9982	0.2961	0.1167

Andréeff gives for the specific gravity of liquid ammonia at 0° C., 0.6364, so that the specific volume of the liquid is

$$\sigma = 0.0016.$$

The values of r at the several given temperatures, calculated by equation (128), are

t	-30° C.	0	$+30^\circ$ C.
r	325.7	300.15	277.5

which may be represented by the equation

$$r = 300 - 0.8t.$$

The specific heat of the liquid, calculated by aid of equation (180), is

$$c = 1.1.$$

In English units the properties of superheated or gaseous ammonia may be represented by the equation

$$pv = 99T - 540p^{\frac{1}{4}},$$

in which the pressures are taken in pounds on the square foot and volumes in cubic feet, while T represents the absolute temperature in Fahrenheit degrees.

The pressure in pounds on the square inch may be calculated by the equation

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 9.7907380;$$
$$\log b = 0.8721769 - 10;$$
$$\log c = 9.9777087 - 10;$$
$$\log \alpha = 9.9997786 - 10;$$
$$\log \beta = 9.9966516 - 10;$$
$$n = t + 7^\circ.6 \text{ F.}$$

The heat of vaporization may be calculated by the equation

$$r = 540 - 0.8(t - 32),$$

and the specific heat of the liquid is

$$c = 1.1.$$

Pictet's Fluid.—Attention has already been called to the mixture of sulphur dioxide and carbon dioxide known as Pictet's fluid, which was adopted by Pictet for use in his refrigerating machines after an extended investigation. The desirable properties are stated by him to be:

1. The tension of the vapor should be greater than that of sulphur dioxide and less than that of ammonia. The boiling-point under atmospheric pressure should be about -20°C.

2. The tension of the vapor in the condenser at about $+30^\circ$ C. should be between 7 and 8 atmospheres.

3. The fluid should be incombustible.

4. The fluid should not attack metals.

5. The fluid should have such a chemical composition that changes of volatility during use need not be feared.

6. The fluid should be of an unctuous nature, so that oil need not be used on the piston of the compressor.

7. The fluid should be inexpensive.

An investigation of the properties of various fluids shows that the addition of oxygen to any compound, whether it entered in solution or into chemical combination, diminished the volatility. Thus carbon monoxide boils at $-140°$ C., while carbon dioxide boils at $-75°$ C.; sulphur dioxide boils at $-10°$ C., and anhydrous sulphurous acid boils at $+32°$, while the hydrate boils at $+326°$ C. Other examples are given by Pictet showing the same result. As a result of experiments on the mixture of sulphur dioxide and carbon dioxide he gives the following table of boiling-points. The formulæ express the proportion of the elements in the mixtures, but are not to be taken to represent chemical compounds.

	Boiling-point.		Boiling-point.
$C_{40}O_{82}S$	$-71°$	CO_6S_2	$-15°$
$C_{30}O_{62}S$	$-54°$	CO_8S_3	$-12°$
$C_{20}O_{42}S$	$-41°$	$CO_{10}S_4$	$-9°.5$
$C_{10}O_{22}S$	$-26°$	$CO_{12}S_5$	$-8°.6$
CO_4S	$-19°$	$CO_{14}S_6$	$-8°$
		$CO_{16}S_7$	$-7°.5$

The mixture, which may properly be expressed by the formula $CO_2 + SO_2$, was found to fulfil all the seven desirable properties, and, as is shown by the table on page (451), the pressure increases less rapidly with the temperature than for simple vapors, so that while the pressure at $-30°$ C. is double that of sulphur dioxide, the pressure at $+30°$ C. is a little less than that of that fluid. This remarkable property appears to be due to the increased solvent action of the two fluids on each other at higher pressures, which acts to diminish the mechanical work of compression from one temperature to another; for example, in the compressor of a refrigerating machine.

Absorption Refrigerating Apparatus.—Fig. 89 gives an ideal diagram of a continuous absorption refrigerating apparatus. It consists of the following essential parts: (1) the generator *B*, containing a concentrated solution of ammonia in water, from which the ammonia is driven by heat; (2) the condenser *C*, consisting of a coil of pipe in a tank, through which cold water is circulated; (3) the valve *V*, for regulating the pressures in *C* and in *I*; (4) the refrigerator *I*, consisting of a coil of pipe in a tank containing a non-freezing salt solution; (5) the absorber A, containing a dilute solution of ammonia, in which the vapor of ammonia is absorbed; and (6) the pump P for transferring the solution from the bottom of A to the top of B; there is also a pipe connecting the bottom of B with the top of A. It is apparent that the condenser and refrigerator or vaporizer correspond to the parts B and C of Fig. 88, and that the absorber and generator take the place of the com-

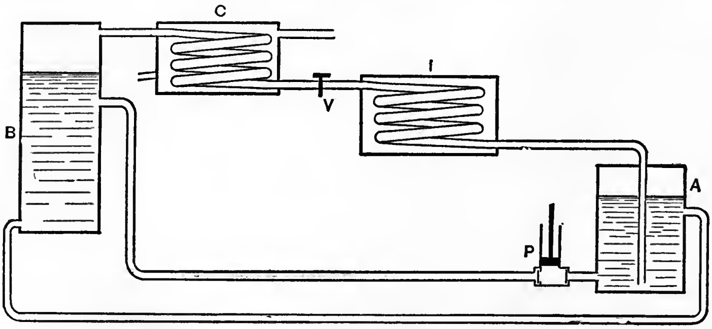

FIG. 89.

pressor. The pipes connecting A and B are arranged to take the most concentrated solution from A to B, and to return the solution from which the ammonia has been driven, from B to A. In practice the generator B is placed over a furnace, by which heat is applied to drive off the ammonia. Also, arrangements are made for transferring heat from the hot liquid flowing from B to A to the cold liquid flowing from A to B. As the ammonia is distilled from water in B the vapor driven off contains some moisture, which causes an unavoidable loss of efficiency.

The earliest absorption apparatus, made by Carré, consisted of a cylindrical receptacle containing a solution of ammonia, and acting alternately as generator and absorber, in open communication through a pipe with a vessel of double conical form, acting alternately as condenser and refrigerator. In use, the generator was placed on a furnace and the condenser in a tank of cold water, and the ammonia driven off from the solution condensed between the inner and outer conical surfaces of the condenser. When a sufficient amount of liquid ammonia had collected, the vessel containing the solution was transferred from the furnace to the cold-water tank, and became thereby changed into the absorber. The condenser at the same time became the vaporizer or refrigerator, and after receiving a mould containing water to be frozen, was securely wrapped with non-conducting material. Apparatus of this kind is only fitted for work on a small scale, and is inefficient.

An adaptation of Carré's apparatus has been used in refrigerator cars for carrying perishable freight. In the car are placed two receptacles—one containing liquid ammonia, which maintains a low temperature by vaporization; and the other containing water, to absorb the ammonia as it is formed. At the end of the route, or when necessary, the receptacles are recharged—one with liquid ammonia and the other with fresh water. The ammonia in the rejected solution is regained by distillation.

Vacuum Refrigerating Apparatus.—A form of absorption apparatus uses water for the volatile liquid and concentrated sulphuric acid for the absorbent. From the fact that vapor of water at freezing-point has a very low tension, such apparatus are called vacuum apparatus.

The first apparatus of this kind was designed for freezing water in carafes, and consisted of a good air-pump, and a receptacle containing oil of vitriol. The carafe, well wrapped in non-conductor, was attached to a pipe leading to the sulphuric acid receptacle, the pump was worked till a good vacuum was produced, and the acid was stirred to present fresh acid to the vapor which rapidly streamed from the water at the low pressure produced. The vaporization of about one sixth of the weight of the water was found to be sufficient to freeze the remainder.

An ideal sketch of a continuous vacuum apparatus is shown by Fig. 90. At B is an air-pump capable of producing a vacuum of one or two mm. of mercury, in the chamber AC. At H there is a tank of concentrated sulphuric acid, from which a spray is delivered at J. The acid absorbs the vapor found in the chamber at the low pressure existing there, gathers in the tank *J*, and flows out through the pipe *K*, which is of sufficient length to deliver the acid against atmospheric pressure

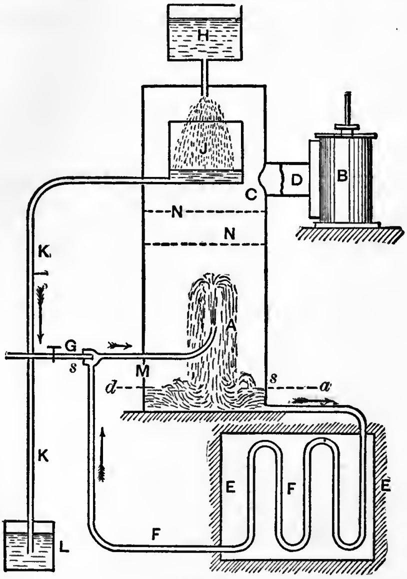

FIG. 90.

in the tank *L*. The dilute acid is reconcentrated and returned to the tank *H*. At *G* is a pipe supplying fresh water, which passes through the water-injector *s*, and throws a jet of salt solution into the chamber at *A*. The finely divided jet loses fresh water by vaporization, is chilled, and gathers in the bottom of the chamber. The salt solution flows through the pipe *F* in the cold chamber *EE*, taking up heat on the way, and is again thrown into the chamber with a fresh supply of water from the pipe *G*. At *N* and *N* are screens to prevent splashing of water into the upper part of the chamber.

Schröter's Tests of Refrigerating Machines.—Professor M. Schröter* made a number of tests on various forms of refrigerating machines, in the years 1885–1887, to determine the efficiency of such apparatus in practice. From his report the following tests have been taken:

* Untersuchungen an Kältemaschinen.

Test of a Bell-Coleman Machine.—An air-refrigerating machine, constructed under the Bell-Coleman patents, was tested at an abattoir in Hamburg, where it was used to maintain a low temperature in a storage-room. The machine is horizontal, and has the pistons for the expansion and compression cylinders on one piston-rod, the expansion cylinder being nearer the crank. Power is furnished by a steam-engine acting on a crank at the other end of the main shaft, and at right angles to the crank driving the air-pistons. Both the steam-cylinder and the expansion cylinder have distribution slide-valves, with independent cut-off valves. The main dimensions are given in the following table:

DIMENSIONS BELL-COLEMAN MACHINE.

	Steam Cylinder.		Compression Cylinder.		Expansion Cylinder.	
	Head end.	Crank end.	Head end.	Crank end.	Head end.	Crank end.
Diameter of piston, cm.	53	53	71	71	53	53
" " piston-rod, cm.	8.1	6.9	9.0	6.8	9.0	9.0
Stroke, m.	0.605	0.605	0.605	0.605	0.605	0.605
Clearance, per cent of piston displacement.	5.9	5.8	1.4	1.4	8.9	8.9

Water is sprayed into the compression cylinder, and the air is further cooled by passing through an apparatus resembling a steam-engine jet condenser, after which it is dried by passing it through a system of pipes in the cold room before it passes to the expansion cylinder.

In the tests indicators were attached to each end of the several cylinders, and the temperature of the air was taken at entrance to and exit from each of the air-cylinders. Specimens of the indicator-diagrams from the air-cylinders show, for the compressor, a slight reduction of pressure during admission and some irregularity during expulsion, and for the expansion cylinder, a little wire-drawing at cut-off, and a good expansion and compression, though neither are complete. No attempt was made to measure the amount and temperatures of the cooling water.

The data and results of the tests and the calculations are given in Table XXXV.

TABLE XXXV.

TESTS ON BELL-COLEMAN MACHINE.

Number of Test	I.	II.	III.
Duration in hours	6	1.63	2.92
Revolutions per minute	65.05	61.2	63.5
Temperatures of air, degrees Centigrade:			
At entrance to compression cylinder	19.3	17.5	19.1
At exit from " "	27.3	26.8	27.2
At entrance to expansion "	19.0	16.6	19.1
At exit from " "	−47.0	−47.0	−47.0
Mean effective pressure, kgs. per sq. cm.:			
Steam-cylinder: head end	2.263	2.336	2.343
crank end	2.239	2.294	2.301
Compression cylinder: head end	1.900	1.861	1.870
crank end	1.869	1.825	1.906
Expansion cylinder: head end	1.592	1.589	1.626
crank end	1.615	1.594	1.624
Indicated horse-power:			
Steam-cylinder	85.12	82.35	85.71
Compression cylinder	128.85	118.55	126.01
Expansion cylinder	60.10	56.12	59.46
Mean pressure during expulsion from compression cylinder, kgs.	3.35	3.25	3.40
Mean pressure during admission to expansion cylinder, kgs.	2.82	2.83	2.84
Difference	0.53	0.42	0.56
Calculation from compression diagram:			
Absolute pressure at end of stroke, kgs	1.04	1.04	1.04
Absolute pressure at opening of admission-valve, kg.:			
Head end	0.783	0.788	0.764
Crank end	0.765	0.749	0.765
Volume at admission, per cent of piston displacement:			
Head end	6.15	5.95	6.03
Crank end	8.50	8.41	7.91
Weight of air discharged per stroke, kg.:			
Head end	0.2744	0.2764	0.2750
Crank end	0.2716	0.2742	0.2730
Weight of air discharged per revolution, kg	0.546	0.551	0.548
Calculation from expansion diagram:			
Absolute pressure at release, kgs.:			
Head end	1.32	1.31	1.33
Crank end	1.45	1.44	1.46
Absolute pressure at compression, kgs.:			
Head end	1.14	1.14	1.17
Crank end	1.20	1.19	1.22
Volume at release, per cent of p. d.:			
Head end	104.65	104.7	104.8
Crank end	106.1	106.3	106.4
Volume at compression, per cent of p. d.:			
Head end	16.5	16.0	16.6
Crank end	19.8	19.6	20.6
Air used per stroke, kg.:			
Head end	0.234	0.233	0.238
Crank end	0.254	0.254	0.255
Air used per revolution	0.488	0.487	0.493
Difference of weights, calculated by compression and expansion diagrams, kg	0.058	0.064	0.055
In per cent of the former	10.6	11.6	10.0
Mean weight of air per revolution, kg	0.514	0.519	0.520
Elevation of temperature at constant pressure, degrees Centigrade	66.3	64.5	66.1
Heat withdrawn per hour, calories	371	354	363

Tests of Compression Machines.—In Table XXXVI are given the data and results of tests on three refrigerating machines on the Linde system using ammonia, and of a machine on Pictet's system using Pictet's fluid. The tests on machines used for making ice were necessarily of considerable length,

TABLE XXXVI.

TESTS ON COMPRESSION REFRIGERATING MACHINES.

Number.	System of the machine.	Application.	Dimensions of the steam cylinder.			Dimensions of the compression cylinder.			Duration of test.	Revolutions per minute, steam engine.	Mean effective pressure. Steam cylinder, kgs. per sq. cm.		Indicated horse-power of steam cylinder.	Revolutions per minute, compressor.	Mean effective pressure compressor, kgs. per sq. cm.	
			Diameter of piston, mm.	Diameter of piston-rod, mm.	Stroke, mm.	Diameter of piston, mm.	Diameter of piston-rod, mm.	Stroke, mm.			Head end.	Crank end.			Head end.	Crank end.
1	Linde.	Ice-making.	371.25	55.5	800	325	63	540	36 min.	68.70	2.13	1.97	53.6	64.8	....	....
2	"	"	"	"	"	"	"	"	34 "	65.40	2.94	2.38	66.1	59.8	4.01	3.84
3	"	Cooling fresh water.	400	....	602	250	55	420	106 "	....	....	....	....	54.7	5.29	5.45
4	"	"	"	....	"	"	"	"	50 "	....	....	....	....	55.1	5.42	5.66
5	"	"	"	....	"	"	"	"	46 "	....	....	....	....	59.1	5.44	5.62
6	"	"	"	....	"	"	"	"	35 "	....	....	....	....	49.6	5.46	5.58
7	"	Cooling brine.	330	52	740	"	"	"	3 hours.	....	....	....	26.1	65.15	3.08	3.12
8	"	" "	"	"	"	"	"	"	3 "	....	....	....	34.5	65.8	4.24	4.28
9	Pictet.	Ice-making.	450	68	900	430	65	900	8.5 "	64.2	2.19	2.38	91.2	64.2	1.33	1.49
10	"	"	"	"	"	"	"	"	11.08 "	64.7	2.28	2.42	94.5	64.7	1.55	1.77
11	"	"	"	"	"	"	"	"	9.83 "	64.5	2.41	2.53	99.2	64.5	1.70	1.87
12	"	"	"	"	"	"	"	"	4.00 "	64.0	....	....	....	64.0	1.91	2.17

TABLE XXXVI.—*Continued.*

Number.	Indicated horse-power of compressor.	Absolute pressures of vapor, kilos. per sq. centimeter.				Cooling water.				Ice produced per hour.					Temperature of water or brine cooled.		Heat withdrawn calories.	
		In compressor during expulsion.	In condenser.	In compressor during admission.	In vaporizer.	Cubic meters per hour.	Initial temperature.	Final temperature.	Heat carried away per hour.	Temperature of water supplied, degrees C.	Gross product per hour, kilos.	Net product per hour, kilos.	Per compressor horse-power, per hour, gross, kilos.	Per compressor horse-power, per hour, net, kilos.	At entrance.	At exit.	Heat withdrawn per hour.	Per compressor horse-power per hour.
1	....	....	6.99	....	2.76	....	...	...	....	9.0	1803	1663	....	...	−4.4	−4.4	162631	4444
2	45.9	9.58	9.31	2.50	2.64	....	....	....	....	8.3	1597	1456	34.8	31.7	−5.9	−5.9	144209	3120
3	26 27	....	13.66	....	4.85 4.55	....	11.19	22.56	....	....	....	....	....	....	11.19	2.95	85339	3249
4	27.30	....	14.06	....	4.90 4.53	9.179	11.2	23.58	113819	....	....	....	....	....	11.2	2.38	91620	3367
5	29.23	....	14.11	....	4.91 4.55	....	11.2	23.04	....	....	....	....	....	....	11.2	2.24	89777	3072
6	24.49	....	13.78	....	4.27 4.83	6.097	11.1	26.10	91455	....	....	....	....	....	11.1	4.71	79909	3263
7	18.1	8.13	7.87	2.36	2.63	18.30	8.77	12.41	66630	....	....	....	....	....	−9.50	−9.97	66680	3684
8	25.8	10.68	10.41	2.97	3.24	6.958	8.82	20.45	80921	....	....	....	....	....	−3.1	−4.1	80624	3086
9	52.01	3.77	3.22	0.45	0.82	30.7	10.15	14.0	118195	11.3	876	789	16.8	15.2	−18.2	−18.2	87074	1674
10	61.70	4.11	3.50	0.63	1.03	30.7	10.1	15.95	179595	11.3	1544	1396	25.0	22.6	−10.0	−10.0	147143	2385
11	66.42	4.23	3.62	0.73	1.15	30.7	10.15	17.20	216435	11.3	1868	1722	28.2	25.9	−9.7	−9.7	175225	2638
12	75.02	5.81	5.11	0.67	1.06	8.36	10.3	31.1	173888	11.3	1545	1391	20.6	18.5	−6.05	−6.05	146929	1958

but the tests on machines used for cooling liquids would be of shorter duration.

The cooling water when measured was gauged on a weir or through an orifice. In the tests 3 to 6 on a machine used for cooling fresh water the heat withdrawn was determined by taking the temperatures of the water cooled, and by gauging the flow through an orifice, for which the coefficient of flow was determined by direct experiment. The heat withdrawn in the tests 7 and 8 was estimated by comparison with the tests 3 to 6. The net production of ice in the tests 1 and 2 was determined directly; and in the test 2 the loss from melting during the removal from the moulds was found by direct experiment to be 8.45 per cent. By comparison with this, the loss by melting in the first test was estimated to be 7.7 per cent. The gross production of ice in the refrigerator was calculated from the net production by aid of these figures. In the tests 9 to 12 on the Pictet machine the gross production was determined from the weight of water supplied, and the net production from the weight of ice withdrawn.

A separate experiment on the machine used for cooling brine gave the following results for the distribution of power:

Total horse-power, ...	57.1
Power expended on compressor, ...	19.5
" " " centrifugal pump, ...	9.8
" " " water-pump, ...	3.6

The centrifugal pump was used for circulating the brine through a system of pipes used for cooling a cellar of a brewery. The water-pump supplied cooling water to the condenser and for other purposes.

A similar test on the Pictet machine gave:

Power of engine alone, ...	7.9 H. P.
" " engine and intermediate gear, . .	16.6 " "
" " engine, gear, and pump, ...	20.0 " "

From the above data the following table was arranged for the several tests on this machine:

INDICATED AND EFFECTIVE WORK.

Number of Test.	9	10	11	12
Indicated work without compressor	19.9	20.0	20.0	19.9
" " engine alone	7.9	8.0	7.9	7.9
Effective work of steam-engine	77.1	80.1	84.6	94.4
Indicated work of steam-engine	91.2	94.5	99.2	109.8
Mechanical efficiency of steam-engine	0.84	0.85	0.85	0.85
Power absorbed by intermediate gearing	11.2	11.2	11.2	11.2
Power absorbed by compressor	65.9(?)	68.9	73.4	83.2
Indicated power of compressor	52.0	61.7	66.4	75.0
Mechanical efficiency of compressor	0.79(?)	0.89	0.90	0.90

Test of an Absorption Machine.—The principal data and the results of a test made by Professor J. E. Denton,* on an absorption ammonia-refrigerating machine, are given in Table XXXVII. The machine is applied to chill a room of about 400,000 cubic feet capacity at a pork-packing establishment at New Haven, Conn. In connection with this test the specific heat of the brine, which served as a carrier of heat from the cold room to the ammonia, was determined by direct experiment. The brine chilled and the cooling water used were measured with meters, which were afterwards tested under the conditions of the experiment.

TABLE XXXVII.

TEST OF AN ABSORPTION MACHINE.—SEVEN DAYS' CONTINUOUS TEST, SEPT. 11–18, 1888.

Average pressures above atmosphere in lbs. per sq. in.	Generator		150.77
	Steam		47.70
	Cooler		23.69
	Absorber		23.4
Average temperatures in Fahrenheit degrees.	Atmosphere in vicinity of machine		80
	Generator		272°
	Brine	Inlet	21.205
		Outlet	16.16
	Condenser	Inlet	54½
		Outlet	80
	Absorber	Inlet	80
		Outlet	111
	Heater	Upper outlet to generator	212
		Lower " " absorber	178
		Inlet from absorber	132
	Inlet from generator		272°
	Water returned to main boilers from steam coil		260

* Trans. Am. Soc. Mech. Eng., vol. x., May, 1889.

Average range of temperatures Fahr. degrees.	Condenser	$25\frac{1}{2}$
	Absorber	31
	Brine	5.13
Brine circulated per hour.	Cubic feet	1,633.7
	Pounds	119,260
Specific heat of brine		0.800
Cooling capacity of machine in tons of ice per day of 24 hours		40.67
Steam consumption per hour, to volatilize ammonia, and to operate ammonia pump, lbs.		1,986
British thermal units.	Eliminated: Per pound of brine	4.104
	Eliminated: Total per hour	481,260
	Of refrigerating effect per pound of steam consumption	243
	Rejected: At condenser, per hour	918,000
	Rejected: At absorber "	1,116,000
	Per pound of steam: On entering generator coil	1,203
	Per pound of steam: On leaving generator coil	271
	Consumed by generator per lb. of steam condensed	932
Condensing water per hour, in lbs.		36,000
Equivalent ice production per pound of coal, if one pound of coal evaporates ten pounds of steam at boiler		17.1
Calories, refrigerating effect per kilogramme of steam consumed.		135
Approximate coil surface in sq. ft.	Condensing coil	870
	Absorber "	350
	Steam "	200
Sizes, in inches, of duplex pumps.	Ammonia pump: Dia. steam cyl.	9
	Ammonia pump: " ammonia cyl.	$3\frac{5}{8}$
	Ammonia pump: Stroke	10
	Brine ": Dia. steam cyl.	$9\frac{1}{2}$
	Brine ": " brine "	8
	Brine ": Stroke	10
Total revolutions per minute.	Ammonia pump, one	22
	Brine pump, two	70
Effective stroke of pumps, part of full stroke		0.8

7525