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Time Series Forecasting Models Involving
Power Transformations

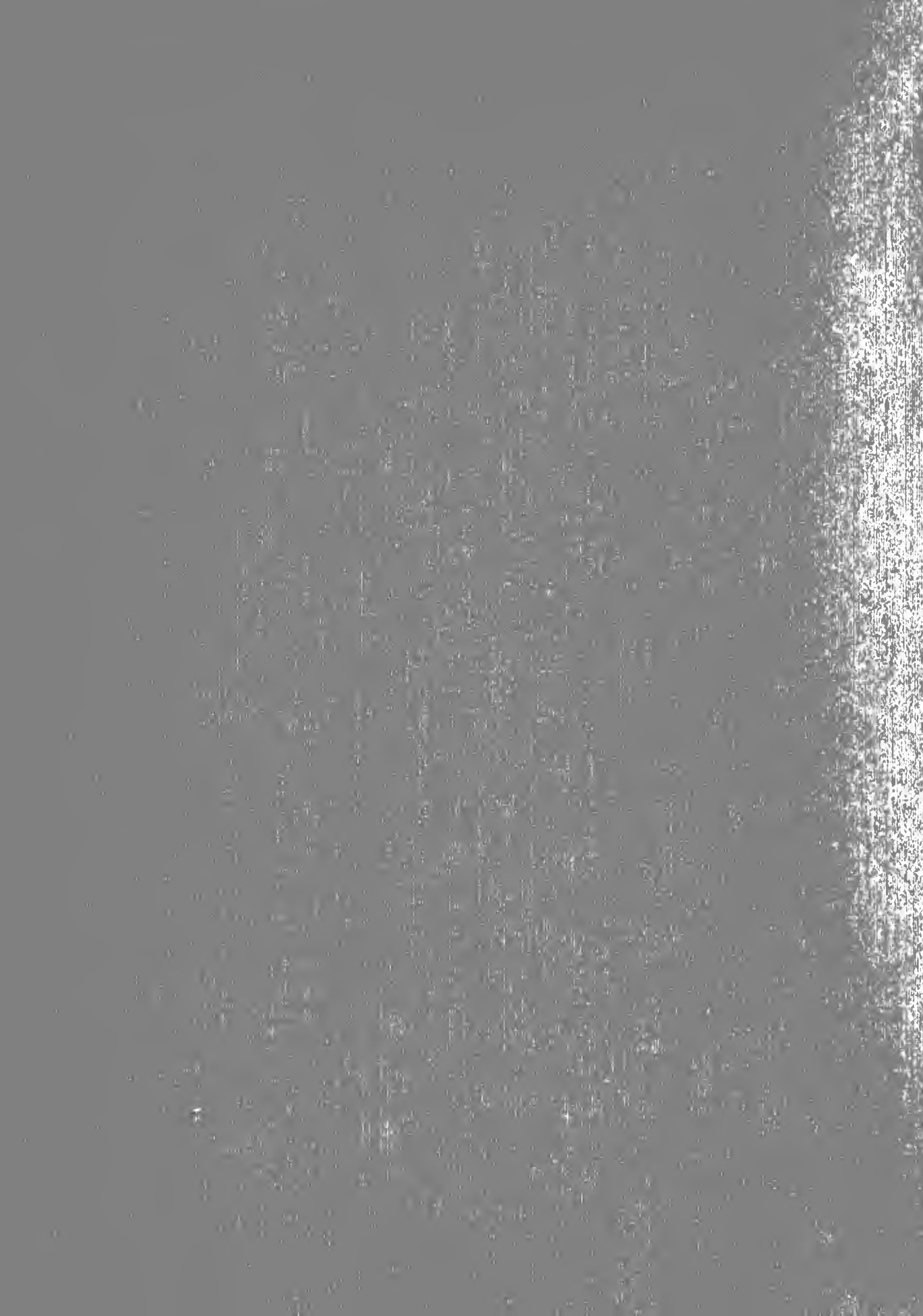
William S. Hopwood
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Time Series Forecasting Models
Involving Power Transformations

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TIME SERIES FORECASTING MODELS
INVOLVING POWER TRANSFORMATIONS

Abstract

In this paper we discuss procedures for overcoming some of the problems involved in fitting autoregressive integrated moving average forecasting models to time series data, when the possibility of incorporating an instantaneous power transformation of the data into the analysis is contemplated. The procedures are illustrated using series of quarterly observations on corporate earnings per share.

1. Power Transformations and ARIMA Models

Box and Jenkins (1970) described in detail a methodology for fitting to an observed time series, X_t , or ARIMA (p,d,q) model

$$(1-\phi_1 B-\dots-\phi_p B^p)(1-B)^d X_t = (1-\theta_1 B-\dots-\theta_q B^q) a_t \quad (1.1)$$

where B is a back-shift operator on the index of the time series, so that $B^j X_t \equiv X_{t-j}$. In (1.1), a_t is taken to be a zero-mean, fixed variance, non-autocorrelated process, known as "white noise". For seasonal time series, with period s, a multiplicative seasonal ARIMA model of the form

$$(1-\phi_1 B-\dots-\phi_p B^p)(1-\phi_1 B^s-\dots-\phi_p B^{ps})(1-B)^d(1-B^s)^D X_t \\ = (1-\theta_1 B-\dots-\theta_q B^q)(1-\theta_1 B^s-\dots-\theta_q B^{qs}) a_t \quad (1.2)$$

is frequently fitted. Box and Jenkins discussed, in detail, an iterative model building strategy, involving model selection, estimation and checking, for fitting to data models of the class (1.1) or (1.2). At the selection stage, based on statistics calculated from the data, a specific model from the general class is chosen for subsequent analysis. Next, using efficient statistical methods, the unknown parameters of the initially selected model are estimated. Finally, checks on the adequacy of representation of the chosen model to the data are carried out. Any inadequacies revealed at this stage may suggest an alternative model; and the model building cycle is iterated until a satisfactory form is achieved. Box and Jenkins show how forecasts of future values of the time series can be obtained from a fitted model.

Box and Jenkins discuss very briefly, as a possibility for obtaining a model with homogeneous error variance, fitting ARIMA models, not necessarily

to the original series, but to a series derived from a member of the class of power transformations analysed by Box and Cox (1964). In this more general model, X_t in (1.1) or (1.2) is replaced by $X_t^{(\lambda)}$, where

$$\begin{aligned} X_t^{(\lambda)} &= (X_t^\lambda - 1) / \lambda && (\lambda \neq 0) \\ \log X_t &&& (\lambda = 0) \end{aligned} \tag{1.3}$$

and λ is regarded as an extra parameter to be estimated. Interest in this model was perhaps first stimulated by the discussion following Chatfield and Prothero (1973), particularly the comments of Box and Jenkins (1973). Chatfield and Prothero analysed a series of monthly sales data. After first taking logarithms of the observations, these authors built, following the strategy of Box and Jenkins, a seasonal ARIMA model. However, the forecasting performance of the achieved model was felt to be unsatisfactory. Several discussants of this paper suggested that this was a result of the inappropriateness of the logarithmic transformation, and that superior forecasts could be obtained if the more general class of power transformations were to be incorporated in the model. Subsequently, in a book of case studies, Jenkins (1979) has emphasised the potential utility of the power transformation in building time series forecasting models.

In the remainder of this paper we will discuss procedures for fitting ARIMA forecasting models, allowing for the possibility of instantaneous power transformations. In particular we will discuss necessary modifications to the usual selection, estimation, checking and forecasting procedures. Our interest in this problem arose from a study of a large collection of quarterly time series of corporate earnings per share, the results of which are reported in Hopwood et al (1981). A good deal of

recent interest in the accounting literature has focussed on procedures for forecasting such series. The Financial Accounting Standards Board (1978), in their conceptual framework project, has emphasized the importance of earnings forecasts. The construction of ARIMA forecasting models for earnings series has been discussed by, for example, Foster (1977), Griffin (1977) and Lorek (1979). Much of this research has concentrated on two questions: do corporate earnings streams have a common structure? (that is, can one find a single model from the general autoregressive integrated moving average class which predicts well for a wide range of corporations?); and, how do the forecasts from time series models compare with those of financial analysts and management? Some discussion on the latter point is contained in Abdel-khalik and Thompson (1977-78), Brown and Rozeff (1978) and Collins and Hopwood (1980).

Although the point had not previously been noted in the accounting literature, it became clear, in the early stages of our study, that, for a great many series in our sample, there was strong evidence of the desirability of a data transformation to induce homogeneity of error variance. It was in response to this phenomenon that we examined the problems to be discussed in subsequent sections of this paper.

2. Model Selection

Following Box and Jenkins (1970), specific models from the general classes (1.1) or (1.2) have generally been chosen on the basis of sample autocorrelations and partial autocorrelations of a series and its low order differences. However, when we further consider the possibility of an instantaneous power transformation, the initial choice of a model is complicated by the fact that the autocorrelation structure of the

transformed series, $X_t^{(\lambda)}$, and its differences, is not independent of the choice of the transformation parameter λ of (1.3). Thus, for example, if the sample autocorrelations and partial autocorrelations of the raw data and its differences are employed in the usual way to suggest values for p , d and q in (1.1), the chosen model may not be adequate to describe the linear properties of $X_t^{(\lambda)}$ for an "appropriate" λ . This point is established theoretically by Granger and Newbold (1976), while a numerical example in Nelson and Granger (1979) shows that it can be practically important.

Of course, the analyst is not irretrievably committed to the initially chosen model. It is possible that any seriously inadequate specification will be detected at the model checking stage, and subsequently rectified. However, it is certainly sensible strategy to seek as reliable an initial specification as possible. Accordingly, we have found it valuable to work with an elaboration of the usual model selection procedure, based on a preliminary estimate of the transformation parameter λ . Our approach is based on the approximation of the underlying true model, by an autoregressive model of moderate order, since pure autoregressive models are inexpensively estimated.

The preliminary estimate, $\hat{\lambda}^*$, of λ is obtained by estimating by least squares, for a grid of values of λ , the k th order autoregressions

$$X_t^{(\lambda)} = \sum_{j=1}^k \beta_j X_{t-j}^{(\lambda)} + e_t \quad (2.1)$$

where, in (2.1), e_t is an error term. Provided k is chosen sufficiently large, the usual residual variance, $\hat{\sigma}_{e,\lambda}^2$, derived from fitting (2.1),

provides a reasonable estimate of the variance of the white noise error, a_t , of (1.1) or (1.2). In practice, for the earnings series we examined, it was found that fixing k at 8 was adequate for our purposes. A further elaboration that might prove useful would base the choice of autoregressive order on some automatic criterion, such as AIC (Akaike 1974) or CAT (Parzen 1974). The initial estimate of λ is then that value $\hat{\lambda}^*$ which, over the grid of chosen values, maximizes

$$g(\lambda) = - \frac{n}{2} \log \hat{\sigma}_{e,\lambda}^2 + (\lambda-1) \sum_{t=1}^n \log X_t \quad (2.2)$$

where n is the length of the series, and the second term on the right hand side of (2.2) is the logarithm of the Jacobian of the transformation (1.3).

Sample autocorrelations and partial autocorrelations are then calculated for $X_t^{(\hat{\lambda}^*)}$ and its appropriate differences, and these are employed in the usual way to select an appropriate model. This modification is very easily incorporated into existing model selection routines and, since (2.1) is estimated by ordinary least squares, the additional computational cost is very small.

To illustrate our approach, we analyse series of 96 quarterly earnings figures for two corporations, Weyerhaeuser Inc. and Freeport Minerals. Fitting autoregressions (2.1) with k fixed at 8, using the criterion (2.2), and searching over a grid of width 0.05, yielded respective initial transformation parameter estimates $\hat{\lambda}^*$ of -0.19 and -0.54 for the two series. For the Weyerhaeuser data, the sample autocorrelations of the transformed series indicated that a single non-seasonal differencing seemed to be sufficient to induce stationarity. The first twelve sample autocorrelations and partial autocorrelations of the differenced series are

shown in the upper third of exhibit 1. For comparison, the middle third of this table shows the same quantities for the first differences of the untransformed series. It is noticeable that the magnitudes and patterns of the two sets of sample autocorrelations are quite different, particularly for low lags. Thus, it is doubtful that the same model would be identified had the initial estimate of the transformation parameter not been obtained. Using the figures in the upper third of exhibit 1, we tentatively entertain the model

$$(1-\phi B^4)(1-B)X_t^{(\lambda)} = (1-\theta B)a_t \quad (2.3)$$

In fact, when this model was estimated, we obtained, as the maximum likelihood estimate of the transformation parameter, $\hat{\lambda} = -0.28$. The lower third of exhibit 1 shows the sample autocorrelations and partial autocorrelations of the first differences of $X_t^{(\hat{\lambda})}$. These are very close to those in the upper third of the table, suggesting that the initial estimate of the transformation parameter provides an adequate basis for model selection.

Insert Exhibit 1 about here

For the Freeport Minerals series, the sample autocorrelations of the initially transformed data indicated the desirability of both a non-seasonal and seasonal differencing factor to induce stationarity. The upper third of exhibit 2 shows the first twelve sample autocorrelations and partial autocorrelations for the appropriately differenced series. The middle third of this table shows the corresponding quantities for the untransformed series. The most important difference between these

two sets of statistics is at lag 1, where, for the untransformed series, the sample autocorrelation is very small. From the upper third of the table, we tentatively identified the model

$$(1-B)(1-B^4)X_t^{(\lambda)} = (1-\theta B)(1-\theta B^4)a_t \quad (2.4)$$

When the model (2.4) was estimated by maximum likelihood, the estimate of the transformation parameter was $\hat{\lambda} = -0.39$. The lower third of exhibit 2 shows the sample autocorrelations and partial autocorrelations for $(1-B)(1-B^4)X_t^{(\hat{\lambda})}$. Once again these are very close to the figures in the upper third of the table, suggesting that our procedure provides a sound basis for model selection.

Insert Exhibit 2 about here

Taken together with our experience in analysing other data sets, these examples suggest that our proposed model selection strategy can be very useful when transformations are employed.

3. Parameter Estimation

Autoregressive-moving average models are most commonly estimated through one or other of the two least squares procedures described by Box and Jenkins (1970). Ansley et al (1977) show how to extend these procedures to deal with models involving power transformations. More recently, however, interest has centered on exact maximum likelihood estimation. A closed form expression for the likelihood function was given by Newbold (1974), while Ansley (1979) presents a computationally efficient algorithm. For the models (1.1) and (1.2) simulation evidence

in Ansley and Newbold (1980) suggests that maximum likelihood estimation may be preferable to least squares, particularly in seasonal models.

Accordingly, we employ exact maximum likelihood to estimate our models. A convenient algorithm can be derived by incorporating the approach of Ansley et al (1977) into the framework of Ansley (1979). The details are very straightforward, but algebraically tedious, and so will not be set out here. The likelihood function can be maximized numerically. Estimated standard errors for the parameter estimates are obtained, in the usual way, from the estimated information matrix.

For the model (2.3), fitted to the Weyerhaeuser data, the parameter estimates (with estimated standard errors in brackets) were

$$\hat{\phi} = 0.38 (0.10); \hat{\theta} = 0.29 (0.11); \hat{\lambda} = -0.28 (0.18)$$

For the model (2.4) for earnings of Freeport Minerals, our estimates were

$$\hat{\theta} = 0.23 (0.11); \hat{\theta} = 0.88 (0.09); \hat{\lambda} = -0.39 (0.21)$$

4. Model Checking

Checks on the adequacy of representation of fitted models of the form (1.1) or (1.2) have generally, following Box and Jenkins (1970), proceeded along one of two lines. More elaborate models can be considered by testing against an alternative involving additional parameters. Also, the assumption that the error terms, a_t , are white noise can be checked through examination of the residual autocorrelations from the fitted model. In fact, as noted for example by Newbold (1980), these two approaches to model checking are not necessarily distinct. The same tests may result whichever perspective is adopted. Recent developments in time series model checking are surveyed in Newbold (1982).

When power transformations are incorporated in the model, no new principles are involved in developing appropriate checks on model adequacy. Once again, we can fit a more elaborate model or examine the residual autocorrelations from the estimated models. Exhibits 3 and 4 show the residual autocorrelations from the ARIMA models estimated for earnings per share of Weyerhaeuser Inc. and Freeport Minerals. Given that the data series each contained 96 observations, these autocorrelations do not seem unduly large, and provide little evidence on which to question the adequacy of the originally chosen models.

Insert Exhibits 3 and 4 about here

The modified portmanteau statistics (Ljung and Box 1978)

$$Q = n(n+2) \sum_{k=1}^{12} (n-k)^{-1} \hat{r}_k^2$$

are, respectively, 12.73 and 9.91. Neither is significant at the 10 percent level. Given this, and the individually low residual autocorrelation values, we conclude that our estimated models should provide an adequate base for forecasting future earnings of these corporations.

5. Forecasting

Having fitted an ARIMA model to the time series $X_t^{(\lambda)}$, one can, standing at time n , compute h -steps ahead forecasts of $X_{n+h}^{(\lambda)}$ in the usual way. Forecasts of the untransformed quantity X_{n+h} could then be obtained by applying the inverse transformation. However, as pointed out by Granger and Newbold (1976), these will not in general be minimum mean squared error predictions. Nelson and Granger (1979) show how minimum mean squared error

forecasts can be achieved, on the assumption that the power transformation yields a model with normally distributed white noise errors.

6. Empirical Studies

We know of just two studies in which the value of including a power transformation in time series models has been checked, in terms of the resulting forecast performance, over a number of real data sets..

Nelson and Granger (1979) considered 21 published economic time series. Forecasting models were built, with and without the use of power transformations, and predictions were evaluated over a hold-out period. The results obtained were rather mixed and the authors concluded that "the evidence when using actual data is that the extra inconvenience, effort and cost is such as to make the use of these transformations not worthwhile."

Hopwood et al (1981) examined 50 quarterly time series of corporate earnings per share. Thus, while these authors considered more series than Nelson and Granger, the scope of coverage was far narrower. In this particular study, however, judged by the criterion of forecasting accuracy, it was found that incorporating power transformations into the model proved, on the average, to be worthwhile. A noticeable improvement in forecast quality tended to follow when a transformation parameter was included in the ARIMA model. Hopwood et al also concluded that the indiscriminate use of the logarithmic transformation in their seasonal models was a poor strategy. This finding tends to reinforce the point made in the discussion of Chatfield and Prothero (1973).

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EXHIBIT 1. SAMPLE AUTOCORRELATIONS (r_K) AND PARTIAL AUTOCORRELATIONS ($\hat{\phi}_{KK}$) OF $(1 - B)x_t^{(\lambda)}$ FOR WEYERHAEUSER DATA¹

λ	K	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{\lambda}^*$	r_K	-.33	.07	-.20	.30	-.11	-.01	-.26	.25	-.24	.05	-.17	.23
		.11	.11	.11	.11	.11	.11	.11	.12	.12	.12	.12	.12
= -0.19	$\hat{\phi}_{KK}$	-.33	-.04	-.21	.20	.06	-.05	-.25	.05	-.21	-.13	-.10	.05
		.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11
1	r_K	.03	-.28	-.05	.33	.01	-.20	-.27	.09	-.15	-.01	-.08	.07
		.11	.11	.11	.11	.11	.11	.11	.12	.12	.12	.12	.12
	$\hat{\phi}_{KK}$.03	-.28	-.03	.27	-.04	-.07	-.28	-.04	-.33	.10	-.06	.08
		.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11
$\hat{\lambda}$	r_K	-.35	.07	-.20	.30	-.11	-.00	-.25	.26	-.24	.05	-.18	.24
		.11	.11	.11	.11	.11	.11	.11	.12	.12	.12	.12	.12
= -0.28	$\hat{\phi}_{KK}$	-.35	-.05	-.22	.19	.06	-.04	-.24	.05	-.20	-.13	-.11	.06
		.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11

¹The rows of numbers directly below the auto/partial autocorrelations represent the approximate sample standard errors.

EXHIBIT 2. SAMPLE AUTOCORRELATIONS (r_K) AND PARTIAL AUTOCORRELATIONS ($\hat{\phi}_{KK}$)

OF $(1 - B)(1 - B^4)x_t^{(\lambda)}$ FOR FREEPORT MINERALS DATA¹

λ	K	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{\lambda}^*$	r_K	-.34	.00	.11	-.39	.15	.08	.15	-.15	.01	-.06	-.04	.05
		.11	.11	.11	.11	.12	.12	.12	.12	.12	.12	.12	.12
= -0.54	$\hat{\phi}_{KK}$	-.34	-.13	.07	-.38	-.13	.05	.31	-.20	-.12	-.01	.19	-.18
		.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11
1	r_K	.04	.00	-.09	-.38	.06	-.03	.12	-.02	-.05	.02	-.10	.11
		.11	.11	.11	.11	.12	.12	.12	.12	.12	.12	.12	.12
	$\hat{\phi}_{KK}$.04	-.00	-.10	-.37	.09	-.05	.06	-.18	.01	-.00	-.05	.04
		.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11
$\hat{\lambda}$	r_K	-.31	.01	.10	-.40	.15	.07	.15	-.14	-.00	-.05	-.05	.06
		.11	.11	.11	.11	.12	.12	.12	.12	.12	.12	.12	.12
= -0.39	$\hat{\phi}_{KK}$	-.31	-.10	.07	-.39	-.11	.07	.30	-.22	-.12	.01	.19	-.17
		.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11

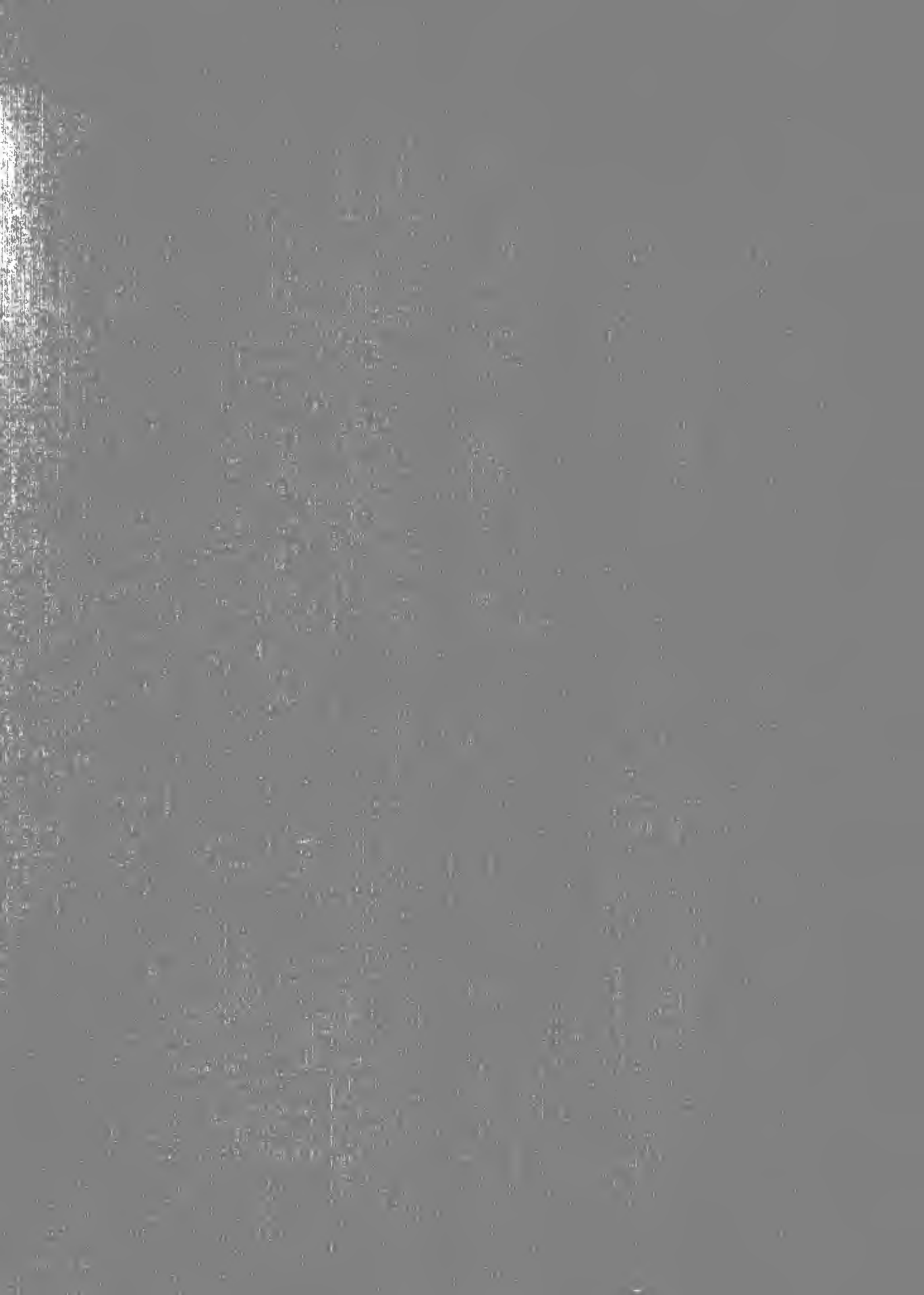
¹The rows of numbers directly below the auto/partial autocorrelations represent the approximate sample standard errors.

EXHIBIT 3. RESIDUAL AUTOCORRELATIONS (\hat{r}_k) FROM MODEL FITTED TO WEYERHAEUSER DATA


k	1	2	3	4	5	6
\hat{r}_k	.03	.07	-.02	-.02	.03	-.10
<hr/>						
k	7	8	9	10	11	12
\hat{r}_k	-.20	.01	-.21	-.05	-.12	.04

EXHIBIT 4. RESIDUAL AUTOCORRELATIONS FROM MODEL FITTED TO
 FREEPORT MINERALS DATA

k	1	2	3	4	5	6
\hat{r}_k	-.04	.17	.04	.06	.02	.16
<hr/>						
k	7	8	9	10	11	12
\hat{r}_k	.12	-.07	-.06	-.06	-.10	-.04





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