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Trading Mechanisms, Speculative Behavior of Investors,
and the Volatility of Prices: Spot Versus Futures

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TRADING MECHANISMS, SPECULATIVE BEHAVIOR OF INVESTORS, AND THE VOLATILITY OF PRICES: SPOT VERSUS FUTURES

ABSTRACT

This paper compares the volatility of spot prices (dealership market) with that of futures prices (auction market) to test the implications of different trading mechanisms for the volatility of prices. First, a natural estimator of the volatility is used. Using the intraday data of the major Market Index and its futures prices, we show that the volatility of opening prices is higher than that of closing prices not only in the spot market but in the futures market, and that the intraday volatility patterns are U-shaped in both markets. Of particular interest is that futures prices do not appear to be as volatile as spot prices when the natural estimator of volatility is used, to the contrary of the conventional wisdom. We argue that the different volatility patterns during the day are not necessarily due to the different trading mechanisms, auction market versus dealership market. Instead, after developing a simple theoretical model of speculative prices, we show that at least part of the different volatility patterns during the day may be attributable to speculative behavior of investors based on heterogeneous information. In addition, we further investigate the volatilities of spot and futures prices using a temporal estimator of price volatility as an alternative to the natural estimator. Based on the temporal estimator, we cannot find any systematic pattern of volatilities during the day in both spot and futures markets, and that futures prices appear to be more volatile than spot prices. Thus, we argue that futures prices may be said to be more volatile than spot prices in terms of how quickly the price moves beyond a given unit price level, but not in terms of how much the price changes during a given unit time interval. Some policy implications are also discussed.

TRADING MECHANISMS, SPECULATIVE BEHAVIOR OF
INVESTORS, AND THE VOLATILITY OF PRICES:
SPOT VERSUS FUTURES

I. Introduction

One of the important factors that affect the volatility of
security prices may be the microstructure of the market or the trading
procedures and practices. For example, a recent study, Amihud and
Mendelson (1987), examines the effects of the mechanism by which
securities are traded on their price behavior: the dealership market
versus the clearing house. Acknowledging that the opening transac-
tions in the New York Stock Exchange represent the outcome of a call
auction trading procedure, whereas trading at the close is carried out
at prices that are set or affected by the exchange's market-makers,
Amihud and Mendelson (1987) compares the volatility of the opening
prices with that of the closing prices, using the 30 NYSE stocks which
constituted the Dow Jones Industrial Index for February 8, 1982, to
February 18, 1983. The empirical results show that the volatility is
much higher at the opening than at the closing. Based upon the
results, they argue that the variance is higher in a clearing house
compared to the dealership market.

In light of the results, on one hand, one may expect a higher
volatility in the futures market than in the spot market since the
former is characterized by the clearing open outcry auction while the
latter is basically a dealership market, even though there are some
factors unique to each market. This logic might have led to the
conclusions of the numerous studies after the "crash of October 1987"

by the SEC report, the Presidential Task Force (Brady), General Accounting Office, and the Joint Task Force (Treasury, Federal Reserves, SEC and CFTC). All of these studies have a common theme of using circuit breakers or trading halt to control the market volatility. However, a thorough examination of investors behavior for the given trading mechanism of the markets seems to be a natural step before we impose any change or further regulation on the microstructure of the market. On the other hand, one should not expect different volatilities between opening and closing times in the futures market since it is a clearing auction market from the opening to the closing.

The purpose of this study is multi-fold. First, we compare the volatility of spot prices with that of futures prices to test the implications of different trading mechanisms for the volatility of prices. Initially, following previous studies, a conventional estimator of the volatility is used which is the variance of the changes in observed prices over fixed time intervals (e.g., see Amihud and Mendelson (1987), Wood, McInish and Ord (1985), Admati and Pfleiderer (1988) and MacKinlay and Ramaswamy (1988)). We will call this conventional measure of volatility hereafter the natural volatility, following Cho and Frees (1988). Using intraday data, we estimate the volatilities of prices from closing to closing, from opening to opening, and for 30 minute intervals during the day. Second, in an attempt to explain the observed patterns of the volatility during the day, we develop a simple theoretical equilibrium model for security prices when investors trade the securities for short-term speculative profits based on their imperfect or heterogeneous information, and

derive some implications of the model for the price volatility during the day, in particular, around the opening time. Since the model utilizes speculators' behavior for short-term profits, it is particularly useful to explain the volatility patterns during the trading and nontrading hours during the day. Third, to provide further insights into the volatility of spot and futures prices, we adopt a temporal estimator of volatility, developed by a recent study, Cho and Frees (1988), which examines the time required for the prices to move beyond a given unit price interval using the concept of the so-called first passage time. The temporal estimator would be particularly useful when the intraday data is used.

The paper is organized as follows. Section II compares the volatilities of spot and futures prices using the natural estimator and discusses some policy implications. Section III develops a theoretical model for security prices when investors trade the securities based on their heterogeneous information, and provides partial answers for the observed volatility patterns. Section IV provides the results of the temporal estimator of volatility. Concluding remarks are contained in Section V.

II. Volatilities of Spot and Futures Prices

A. Data

This paper used all intraday spot and futures prices of the Major Market Index (MMI) over the period July 23, 1984 to July 15, 1986. The MMI is a price-weighted index of 20 blue-chip stocks, including 15 of the 30 Dow Jones industrials. The data base includes every transaction as reported for the futures contract and the values of the spot

index occurring one to four times every minute of the trading day, so that a percentage change is available for each minute of trading at minimum. For futures prices, the most actively traded MMI futures contracts were used. In general, the most actively traded futures contract was the nearby contract except for the delivery month when the next contract became most actively traded. Also, following Cornell (1985), all holidays and the days following holidays were excluded from the sample. For the sample period, there were 16 holidays, six on Monday, two on Tuesday, two on Wednesday, three on Thursday and three on Friday. MMI futures contracts were traded between 8:45 a.m. and 3:15 p.m. while the MMI stocks were traded between 9:00 a.m. and 3:00 p.m. in Chicago time before October 1, 1985. But since October 1, 1985, both exchanges have opened 30 minutes earlier.

B. Natural Estimator of Volatility

First, using the natural estimator of volatility, we compared spot prices with futures prices. Table 1 presents the ratio of the variance of the open-to-open returns to the variance of the close-to-close returns. The daily returns were measured by $\log(P_t/P_{t-1})$. In both futures and spot markets, the open-to-open returns appear to be more volatile than the close-to-close returns. For the spot market, this result is consistent with Amihud and Mendelson (1987), even though the magnitude of the ratio of the variances is different from ours. The average ratio of $Var(R_0)/Var(R_c)$ for 30 stocks in Amihud and Mendelson (1987) is 1.20. The difference between their results and ours lies in the fact that they deal with individual stocks whereas ours is for a portfolio. Based on these results, one may be

tempted to conclude that the different volatility of prices between the opening and closing time is due to the different trading procedures, open auction market at the opening versus dealership market at the closing.

Insert Table 1 about here

However, the results on the futures contract suggest that the trading mechanism is not necessarily the reason for the different volatilities between the opening and closing times. Unlike the spot market, the futures market is essentially an open outcry auction market from the opening to the closing. Nevertheless, the opening prices appear to be more volatile than the closing prices, like the spot market. Note also that the ratio of the variances in the futures market is even larger than that in the spot market.

Also, in an attempt to test the possibility of the day of the week effect, we compared the volatilities of spot and futures prices on each day of the week. Table 2 presents the results. It appears that there is no substantial difference of volatility behavior across days of the week in both spot and futures markets. Also, the opening prices appear to be more volatile than the closing prices except Thursday and Friday for spot prices and Thursday for futures prices, which, in general, confirms our previous results. However, contrary to the conventional wisdom in the markets, the spot market appears to be more volatile than the futures market in both opening and closing prices, with one exception in each category, Friday in the opening prices and Tuesday in the closing prices.

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Insert Table 2 about here
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We also estimated the volatilities of spot and futures prices for 30 minute intervals during the day. Table 3 and Figure 1 present the results. Although the CBT closed at 3:15 p.m., the last 15 minutes were discarded to match the spot market's closing time. The intraday volatility patterns appear to be U-shaped, which is consistent with previous studies (e.g., Wood, McInish and Ord (1985)). The volatility of prices in both spot and futures markets is very high posterior to the opening time and declines until noon and goes up prior to the closing time.

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Insert Table 3 and Figure 1 about here
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Of particular interest is that futures prices do not appear to be as volatile as spot prices on average throughout the day.[1] In fact, spot prices appear to be more volatile than futures prices from the opening to around noon. As pointed out by MacKinlay and Ramaswamy (1988), if arbitrageurs maintain the link between the two markets, the variances of spot and futures prices should be equal. Note, also that trading of the MMI stocks are not necessarily synchronous. If some of the stocks were not traded for a short period of time, the volatility of the index was likely to be lower than the case where all of the stocks were traded simultaneously. Thus, the volatility of the index reported here would be a conservative measure. The results in sum suggest that the different volatilities during the day are not necessarily due to the different trading mechanism.

The results in this section have some important implications for regulating the futures market. Since the inception of stock index futures in the early 1980s, the headlines of financial media have often singled out the stock-index futures contracts as the villains, when the stock market experienced unusually volatile swings in prices, rather than attributing the volatility to the fundamental factors that influence the market. A good example is the market crash on October 19, 1987, when the Dow Jones Industrial Average plunged by more than 500 points. While the causes for the crash are still controversial, the majority of people in the Wall Street as well as regulators have blamed speculative behavior of investors in the futures market. As a result, a number of proposals have been suggested to impose some curbs on trading the stock index futures contracts such as circuit breaker system (e.g., Brady's report (1988) and the Joint Task Force report (1988)).[2] The bottom line of such a proposal is that futures prices are too volatile. However, the results in this section lead us to believe the necessity of reconsidering the proposal.

Also, the analogy that the various reports use for the justification of circuit breakers is misleading. They state that if a machine or some other mechanical man made operation is going to get out of control, the best way to keep it under control is to "pull the plug." This may work very well for controlling machines because the machine cannot anticipate the plug being pulled. Whereas financial markets are able to anticipate trading halts or market closure. In fact, the existence of the closing of trading may cause increases in volatility because market participants may want to get their trade complete

before the trading is halted, thereby if participants anticipate a
trading halt they may increase the level of trading to beat the clo-
sure of the market. The increased volatility prior to the closing
time we observed in this section may confirm those activities of
investors. We now show in the next section that the intraday vola-
tilities we have observed may be attributable, at least in part, to
speculative behavior of investors who trade the securities for short-
term speculative profits based on their heterogeneous information.

III. A Model for Speculative Prices

In this section, we attempt to show, in a simple market setting,
that the high volatility around the opening time and the decrease of
the volatility posterior to the opening time in both markets may be
attributable to speculative behavior of investors based on heteroge-
nous information that is created in nontrading hours.

Let us consider a simplified world in which there are large but
equal number N of buyers and sellers. Prior to a release of new
information, investors will have the incentive to privately learn
about the nature of forthcoming information. Unless private infor-
mation is perfect across all investors, they will trade securities
based on their private (heterogeneous) information and thus new in-
formation may be reflected in security prices prior to its public
release. If new information is publicly released at discrete points
in time, investors may speculate between consecutive time points of
public information. However, as the time until public information
approaches zero, investors' expectations will be more homogeneous and,

therefore, speculative opportunities will disappear with the disclosure of public information.

Denote buyers' and sellers' expected security prices by $\bar{v}$ and $\underline{v}$, respectively, where $\bar{v} > \underline{v}$. In addition, let us assume that buyers' and sellers' preferences are characterized by the following short-term trading profit functions, respectively:

$$u_1 = \bar{v} - c \text{ for buyer}$$
$$u_2 = d - \underline{v} \text{ for seller,}$$

(1)

where c is the striking price plus trading cost (for buyer) and d represents the striking price less trading cost (for seller).[3] For analytic convenience, each buyer and seller is assumed to trade exactly one share of the security if and only if his or her expected profit is non-negative (i.e., $Eu_1 \geq 0$ and $Eu_2 \geq 0$). Buyers are indexed by the trading cost s, where $k(s) = 1/a(\bar{v} - \underline{v})$ is the probability density of s with support $o < s \leq a(\bar{v} - \underline{v})$ and a is a positive constant. On the other hand, sellers are indexed by the trading cost t, where $\ell(t) = 1/a(\bar{v} - \underline{v})$ is the probability density of t with support $o < t \leq a(\bar{v} - \underline{v})$. Thus, $Nk(s)$ and $N\ell(t)$ represent the number of buyers of type s and the number of sellers of type t, respectively, and the trading costs are smaller if and only if buyers' and sellers' expectations are more homogeneous. Note that buyers and sellers are identically distributed with respect to their trading costs. Since buyers in one period may become sellers in another period and vice versa, the identical distribution assumption appears reasonable.

Assume that buyers (sellers) know the probability distribution of the security prices x (y) that sellers (buyers) are willing to sell (pay) for. Denote by $F(x)$ and $G(y)$ the probability distribution functions of x and y with support $\underline{x} \leq x \leq \overline{x}$ and $\underline{x} \leq y \leq \overline{x}$ for buyers and sellers, respectively.[4]

Then, the optimal buyer behavior can be described as follows. Consider a buyer of type s, and suppose that Q is the smallest of the x values that he/she has observed. If an additional seller is randomly sampled, the gross increase of his/her expected profit is

$$\phi(Q) = \int_{\underline{x}}^{Q} (Q-x)dF(x) = \int_{\underline{x}}^{Q} F(x)dx. \qquad (2)$$

Since the additional sample costs s, the buyer will keep trying to get more information until $x \leq Q(s)$, where the critical value of $Q(s)$ is determined by the marginality condition

$$s = \phi(Q(s)). \qquad (3)$$

Since $\phi(Q)$ is increasing:

$$\phi'(Q) = F(Q) > 0 \qquad (4)$$

for all $Q > \underline{x}$, buyers are likely to search more information in the average if and only if the costs are smaller.

Following the same logic, the optimal strategy of a seller of type t can be described as: keep trying if an $x < R(t)$ is observed; and trade with the buyer if he/she is willing to pay an $x \geq R(t)$, where the threshold $R(t)$ solves

$$t = \int_{R(t)}^{\overline{x}} [y - R(t)] dG(y) \qquad (5)$$

for all $0 < t \leq a(\overline{v} - \underline{v})$. Define

$$\psi(R) = \int_{R}^{\overline{x}} [y-R] dG(y) = \int_{R}^{\overline{x}} [1-G(y)] dy \qquad (6)$$

for all $\underline{x} \leq R \leq \overline{x}$. Then, the above condition can be written as

$$t = \psi(R(t)) \qquad (7)$$

for all t. Since $\psi(R)$ is decreasing:

$$\Psi'(R) = -[1-G(R)] < 0 \qquad (8)$$

for all $R < \overline{x}$, sellers are likely to search more information in the average if and only if the costs are smaller.

Theorem 1. If a market equilibrium exists, and it is characterized by distribution functions $F(x)$ and $G(x)$ with support $\underline{x} \leq x \leq \overline{x}$, the following must then hold (see Appendix A for proof).

   (a) $\underline{v} \leq \underline{x} < \overline{x} \leq \overline{v}$;

   (b) $\int_{\underline{x}}^{\overline{x}} F(x) dx \leq a(\overline{v} - \underline{v})$;

   (c) $\int_{\underline{x}}^{\overline{x}} [1 - G(x)] dx \leq a(\overline{v} - \underline{v})$;

   (d) $\{\int_{\underline{x}}^{\overline{x}} [1 - G(y)] dy\} \cdot F'(x) = 1 - G(x)$;

(e) $\left\{ \int_{\underline{x}}^{\overline{x}} F(y)dy \right\} \cdot G'(x) = F(x);$

for all $\underline{x} \leq x \leq \overline{x}.$

Theorem 2. If the trading cost parameter a is not too large ($\pi a \leq 2$), then market equilibrium exists and is characterized by the following (see Appendix B for proof):

(a) $F(x) = \sin r(x - \underline{x});$

(b) $G(x) = 1 - \cos r(x - \underline{x});$

(c) $\overline{x} - \underline{x} = \frac{\pi}{2r};$

for all $\underline{x} \leq x \leq \overline{x}$, where

$$\underline{v} \leq \underline{x} \leq \frac{1}{4-\pi} \cdot \text{Min}\{(2-\pi+\pi a)\underline{v} + (2-\pi a)\overline{v}, \ (4-\pi)[(1-a)\overline{v} + a\underline{v}]\} \quad (9)$$

and

$$(1-a)\underline{v} + a\overline{v} - \underline{x} \leq \frac{1}{r} \leq \frac{2}{\pi-2} \cdot \text{Min}\{(1-a)\overline{v} + a\underline{v} - \underline{x}, \ \frac{\pi-2}{2} a(\overline{v}-\underline{v})\} \quad (10)$$

The following implications are immediate from Theorems 1 and 2: i) The price dispersion ($\overline{x} - \underline{x}$) is larger if either trading costs are larger (i.e., a is larger) or traders' expectations get more heterogeneous (i.e., $\overline{v} - \underline{v}$ gets larger); ii) The price dispersion is positively related to the trading cost parameter ($a(\overline{v} - \underline{v})$).

Volatility of Prices Posterior to Opening

We have shown that equilibrium prices prior to disclosure of public information (let us call it the "speculative price" as opposed

to the "normal price" subsequent to public information) critically depends on speculators' behavior due to heterogeneous information. In order to examine the volatility of speculative prices relative to that of normal prices, let us consider an arbitrary but fixed period from the opening time $\tau = 0$ to time $\tau = 1$. For analytic tractability, we assume that the relevant state for the period is realized at $\tau = 0$, but that the state information is to be made public at $\tau = 1$. Denote by v the equilibrium stock price that is to be realized subsequent to public information at $\tau = 1$.

Consider the speculative market at time τ ($0 < \tau < 1$). Since the price v is not likely to be known to investors at the time τ, some investors' estimates of v may be larger than others' unless private information is homogeneous. Denote by $\underline{v}(\tau)$ and $\overline{v}(\tau)$ speculative sellers' and buyers' estimates of v at time $\tau < 1$, respectively. The speculation market is then active at τ if and only if $\underline{v}(\tau) < \overline{v}(\tau)$.

In order to avoid indeterminate striking prices for speculative trading, let us assume that speculative sellers are Stackelberg leaders:[5] a seller of type t sets his/her selling price at the individual reservation price R(t) and waits until a buyer who is willing to pay the price arrives. Given this assumption, the speculative price or actual striking price x is distributed by $F(x|\tau)$ at time τ, and thus its mean can be written as

$$E(x|\tau) = \int_{\underline{x}(\tau)}^{\overline{x}(\tau)} x \, dF(x|\tau) = \underline{x}(\tau) + \frac{1}{r(\tau)} \left(\frac{\pi}{2} - 1\right) \qquad (11)$$

where the quantities $\underline{x}(\tau)$ and $r(\tau)$ satisfy (9) and (10).

Let us define

$$b(\tau) = E(x|\tau) - v \text{ and } z(\tau) = x(\tau) - E(x|\tau) \tag{12}$$

for $0 < \tau < 1$. The quantity $b(\tau)$ measures the deviation of the average speculative price from the price subsequent to public infor- mation at the opening time. We shall call $b(\tau)$ speculators' average bias. Depending upon the reliability of their private information, speculators may be biased positively ($b(\tau) \geq 0$) or negatively ($b(\tau) \leq 0$). On the other hand, the variable $z(\tau)$ measures the deviation of an actual striking price from its mean. We shall call $z(\tau)$ speculators' trading risk component. Note that the trading risk $z(\tau)$ is "small" if and only if the price dispersion $\overline{x}(\tau) - \underline{x}(\tau)$ of speculative prices $x(\tau)$ is small.

Then, the speculative price $x(\tau)$ at time τ ($0 < \tau < 1$) can be written as:

$$x(\tau) = v + b(\tau) + z(\tau). \tag{13}$$

Furthermore, the trading risk $z(\tau)$ can be characterized by[6]

$$E[z(\tau)] = 0 \text{ and } Var[z(\tau)] = \frac{\pi - 3}{[\tau(\tau)]^2} \tag{14}$$

where

$$0 < \frac{1}{\tau(\tau)} \leq a[\overline{v}(\tau) - \underline{v}(\tau)]. \tag{15}$$

Equation (13) implies that speculators are exposed to three risks: the volatility of the normal price v (i.e., the intrinsic risk of the security); the risk due to errors of heterogeneous information ($b(\tau)$);

and the trading risk of speculators $(z(\tau))$. If the relevant state of the firm for the time period $0 < \tau < 1$ is realized at $\tau = 0$, the intrinsic risk component v is not variable during $0 < \tau < 1$. Nevertheless, the variance of the speculative price $x(\tau)$ can be positive due to the bias $b(\tau)$ and the trading risk $z(\tau)$.

As a result, the speculative price would be more volatile than the normal price subsequent to disclosure of public information. In other words, the volatility decreases posterior to disclosure of public information after the market opens. In practice, the precise time when public information becomes available is extremely difficult to determine from the time of the day trading takes place. However, judging from the intraday patterns of the volatility in the previous section, the heterogeneous information created in nontrading hours seems to affect the price around the opening time and it seems to take about 30-45 minutes for the market to reach the normal price.

IV. Temporal Estimator of Volatility

In the previous sections, we have shown that the volatility of prices around the opening time is high relative to other times and, in general, it is U-shaped during the day, and that spot prices appear to be more volatile than futures prices unlike the conventional wisdom. Also, we have shown that the speculative behavior of investors based on heterogenous information created during the nontrading hours might provide at least partial answers to the high volatility of prices around the opening time. However, the question is yet to be answered why futures prices have been acknowledged, in general, as more volatile than spot prices if the variance of futures prices is lower than

that of spot prices as we observed in the previous section. We may find one possible reason in different speed of information adjustment in different markets due to some reasons such as different transaction costs.

Cho and Frees (1988) proposes a temporal estimator of stock price volatility as an alternative to the natural estimator. The estimator comes from the notion of how quickly the price changes rather than how much the price changes. In other works, the more volatile stock price should move quicker, and hence the so-called first passage time should be shorter than the less volatile stock. While the natural estimator focuses on how much the price changes during a given unit time interval, the temporal estimator focuses on how quickly the price moves beyond a given unit price level. It is shown that the temporal estimator has desirable asymptotic properties, including consistency and asymptotic normality. The brief outline of the temporal estimator is as follows.

Assume that the true stock prices are log-normally distributed and the observed stock prices are continuously monitored. That is, the true stock price is assumed to be $P(t) = P(0)\exp(\sigma B(t) + \mu t)$, $t \geq 0$. Here $P(0)$ is a known constant, μ and σ are unknown parameters, and $\{B(t); t \geq 0\}$ is standard Brownian Motion over $[0,\infty]$. The observed stock price is assumed to be $\hat{P}(t) = [P(t)/d] \cdot d$, where d is a known constant. For example, $d = 1/8$ on the New York Stock Exchange.

Based on the notion of the first passage time and these assumptions, they construct a consistent estimator along with its asymptotic sampling distribution. First, they define the sequence of stopping

time random variables $\{\tau_n\}_{n=1}^{\infty}$ by $\tau_n = \{$first time $t > \tau_{n-1}$ such that $P(t)/P(\tau_{n-1}) \notin [(1+d)^{-1}, (1+d)]\}$, where $\tau_0 = 0$. Thus, $\{\Delta\tau_n\}$, where $\Delta\tau_n = \tau_{n+1} - \tau_n$, is an i.i.d. sequence of random variables.

Besides, they introduce two functions: $g_1(x) = \{1+(1+d)^{-x-1}\}^{-1}$ and $g_2(x) = \log(1+d)(2g_1(x)-1)$. Applying Theorem 3.6 of Siegmund (1985), they derive some important relationships between the parameters, μ and σ, and the expected time between price changes, $E\tau$. That is, if $\mu \neq 0$, then $\Pr\{P(\tau) = 1 + d\} \equiv p = g_1(2\mu/\sigma^2)$ and $E\tau = \mu^{-1}g_2(2\mu/\sigma^2)$. The temporal estimator is suggested then as

$$\hat{\sigma}^2 = 2\hat{\mu}_2/g_2^{-1}(\hat{\mu}_2\bar{\tau}_n) = 2\hat{\mu}_2/g_2^{-1}[n^{-1}\log P(\tau_n)], \tag{16}$$

where

$$\hat{\mu}_2 = \tau_n^{-1}\log[P(\tau_n)]$$

$$\bar{\tau} = \tau_n/n.$$

Their simulation study shows that measurement errors in the time of price changes are more likely to induce less biases than measurement errors in the magnitude of price changes. It is also shown that the natural estimator does not become better as one adds more observations per day.

The temporal estimator is particularly useful in this study since we use intraday transaction data. We measured the temporal estimators of volatility for both spot and futures prices for $d = 1/8$ using Eq. (16). Figures 2, 3, 4, and 5 represent daily temporal volatilities of spot and futures prices, for July 23, 1984 - December, 1984, January,

1985 - June, 1985, July 1985 - December, 1985, January, 1986 -
July 15, 1986, respectively.

Insert Figures 2, 3, 4 and 5 about here

Table 4 and Figure 6 show the results on the temporal estimators
of volatility of both spot and futures prices for 30 minute intervals
during the day. In contrast to the results of the natural estimator,
futures prices appear to be more volatile than spot prices, and any
systematic pattern of the volatility does not seem to exist during the
day. It is also clear that the futures price moves more quickly than
the spot price to reach a given unit price level. Thus, as pointed
out earlier, information adjustment in the futures market seems to be
faster than in the spot market, which may be due to lower transaction
costs in the futures market. This may be able to explain why the
futures price has been conceived by the market participants to be more
volatile than the spot price.

Insert Table 4 and Figure 6 about here

Summing up the results of natural and temporal estimates of vola-
tility, we can imagine the following patterns of spot and futures
prices over time.
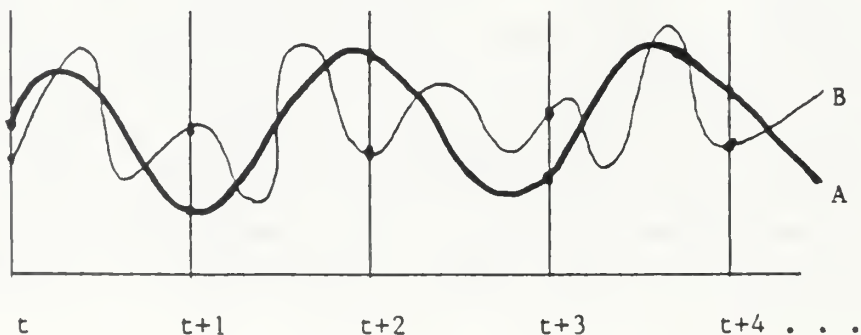
It is obvious that if we use the natural estimator of volatility, the series A is more volatile than the series B. On the other hand, if we use the temporal estimator, the series B is more volatile than the series A. Assuming that the unit time is 30 minutes, it is conceivable that the series A and B correspond to the spot and futures prices, respectively, i.e., the futures price is more (less) volatile than the spot price in terms of the temporal (natural) estimator.

V. Conclusion

It has been generally well-known that opening prices are more volatile than closing prices in the stock market, and that the intraday volatility pattern is U-shaped. The different volatilities around the opening and closing times have often been attributed to the different trading mechanisms by which the prices are determined: the opening transactions in the NYSE represent the outcome of an auction trading procedure, whereas closing prices are determined by the market-makers.

This paper investigates the volatilities of spot and futures prices in an attempt to compare the two different trading mechanisms: the futures market is characterized by the clearing open outcry auction from the opening to the closing while the stock market is basically a dealership market except the opening time. One might expect higher volatility in futures prices than in spot prices and should not expect different volatilities between opening and closing times in the futures market if the different volatility behavior between the opening and the closing is solely due to different trading mechanisms. In

fact, it has been a conventional wisdom that futures prices are more volatile than spot prices, and thus more severe restrictions should be imposed on futures trading. Using the intraday data of the Major Market Index and its future prices, this paper shows that the volatility of opening prices is higher than that of closing prices not only in the spot market but in the futures market, and that the intraday volatility patterns are U-shaped in both markets. Of particular interest is that futures prices do not appear to be as volatile as spot prices when the natural estimator of volatility is used, which is not consistent with the conventional wisdom. Thus, we argue that the different volatility patterns during the day are not necessarily due to the different trading mechanisms, auction market versus dealership market. Instead, we show that the different volatilities during the day, at least in part, may be attributable to speculative behavior of investors based on heterogeneous information.

In addition, we further investigate the volatilities of spot and futures prices using the temporal estimator of price volatility, proposed by Cho and Frees (1988) as an alternative to the natural estimator. When the temporal estimator is adopted, we are not able to find any systematic pattern of volatilities during the day in both spot and futures markets, and futures prices appear, in general, to be more volatile than spot prices. Based on these results of natural and temporal estimators of price volatility, we argue that futures prices may be said to be more volatile than spot prices in terms of how quickly the price moves beyond a given unit price level, but not in terms of how much the price changes during a given unit time interval.

This paper has some important policy implications. For example, the common idea of using circuit breakers recommended by the various reports after the Market Crash in October 1987 should be reconsidered. The lower natural variance and higher temporal variance of futures prices than spot prices suggest that information may be more quickly reflected in prices in the futures market than in the spot market but the absolute changes of futures prices during a given unit time interval are on average lower than those of spot prices. This result is consistent with the view that the futures market can play a positive economic role. Besides, the increased volatility prior to the closing of the markets suggests that trading halts or market closure may cause increases in volatility rather than decreases. A further investigation of trading mechanism is certainly needed before such regulation as circuit breakers is imposed on futures trading. In particular, the question of how much and in what direction the volatility of futures prices leads to the volatility of spot prices deserves a further study.

FOOTNOTES

[1] We also estimated the volatilities for 30 minute intervals for each day of the week. Since the results are not different depending on the day of the week, they are not reported in the paper. However, they are available from the authors upon request.

[2] Other proposals include tightening the price limits of futures prices, increasing margin requirements, and using the opening price as the settlement price.

[3] The trading cost is a general term that includes transaction costs and the cost involved in acquiring information and searching the best striking price.

[4] Note that since buyers and sellers are to be matched on a one-to-one basis, both x and y values must have the same support.

[5] This assumption is just for simplicity. We may assume alternatively that speculative buyers are Stackelberg leaders. Since the equilibrium speculative price must be distributed between the two distribution functions $F(x)$ and $G(x)$, all of the results should be intact with only minor changes even when neither buyers nor sellers are Stackelberg leaders.

[6] From (12), $x(\tau) = v + [E(x|\tau - v] + [x(\tau) - E(x|\tau)]$. Thus (13) is obvious. In addition, $Var[z(\tau)] = Var[x(\tau)] = \int_{\underline{x}}^{\overline{x}} x^2 dF(x|\tau) - E(x|\tau)^2$, where $F(x|\tau) = \sin[r(\tau)(x-\underline{x}(\tau)]$ for $\underline{x}(\tau) \leq x \leq \overline{x}(\tau)$ from Theorem 2, and $E(x|\tau)$ is given by (11). Therefore, $Var[z(\tau)] = (\pi-3)/[r(\tau)]^2$ can be easily derived.

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Table 1

Comparison of Volatility of Open-to-Open
Returns and Close-to-Close Returns*

	$\dfrac{Var(R_0)}{Var(R_c)}$	$\dfrac{Min(R_0)}{Min(R_c)}$	$\dfrac{Max(R_0)}{Max(R_c)}$
MMI (Futures)	1.0967	1.076	0.92059
MMI (Spot)	1.0384	1.0	0.96237

*$Var(R_0)$ and $Var(R_c)$ are the variances of the open-to-open returns and the close-to-close returns, respectively. The return is measured by log (P_t/P_{t-1}); Min and Max represent the minimum and maximum returns, respectively.

Table 2

Comparison of Volatilities Between
Spot and Futures Markets for Each Day of the Week*

(July 23, 1984 – July 15, 1986)

	$\dfrac{\text{Var(O-S)}}{\text{Var(C-S)}}$	$\dfrac{\text{Var(O-F)}}{\text{Var(C-F)}}$	$\dfrac{\text{Var(O-S)}}{\text{Var(O-F)}}$	$\dfrac{\text{Var(C-S)}}{\text{Var(C-F)}}$
Monday	1.047961	1.009623	1.076383	1.029007
Tuesday	1.170940	1.174713	1.143622	0.999704
Wednesday	1.122372	1.127335	1.033475	1.071018
Thursday	0.916231	0.938465	1.046324	1.041716
Friday	0.900601	1.069829	0.891513	1.042032
Average	1.031891	1.081314	1.038263	1.03270

*Var(O-S) and Var(C-S) represents the variance of weekly open-to-open
returns and close-to-close returns in the spot market. Var(O-F) and
Var(C-F) are the counterparts in the futures market.

Table 3

Natural Estimator of Intraday Volatility[a]

Time	Observation	$\hat{\sigma}^2$(Spot)	$\hat{\sigma}^2$(Futures)
8:30 – 9:00[b]	198	1.687 E^{-5}	0.767 E^{-5}
9:00 – 9:30	495	1.113 E^{-5}	0.605 E^{-5}
9:30 – 10:00	496	0.392 E^{-5}	0.283 E^{-5}
10:00 – 10:30	496	0.333 E^{-5}	0.266 E^{-5}
10:30 – 11:00	496	0.325 E^{-5}	0.285 E^{-5}
11:00 – 11:30	496	0.280 E^{-5}	0.259 E^{-5}
11:30 – 12:00	496	0.240 E^{-5}	0.241 E^{-5}
12:00 – 12:30	496	0.308 E^{-5}	0.308 E^{-5}
12:30 – 13:00	496	0.305 E^{-5}	0.304 E^{-5}
13:00 – 13:30	496	0.394 E^{-5}	0.425 E^{-5}
13:30 – 14:00	496	0.490 E^{-5}	0.493 E^{-5}
14:00 – 14:30	496	0.841 E^{-5}	0.855 E^{-5}
14:30 – 15:00[b]	496	0.870 E^{-5}	0.877 E^{-5}

[a]The volatility is measured by the variance of $\log(P_t/P_{t-1})$ and the unit time interval is 30 minutes. If P_t was not available at exact time t, the price closest to t was used.

[b]Since the spot market opened at 9:00 in Chicago time before October 1, 1985, the volatility for 8:30 – 9:00 was estimated using the data only since October 1, 1985. Also, although the futures market closed at 3:15 p.m., the futures prices for the last 15 minutes were discarded to match the spot prices.

Table 4

Temporal Estimator of Intraday Volatility

| | Spot Market | | | Futures Market | | |
Time	$\overline{\sigma}^2$	Min	Max	$\overline{\sigma}^2$	Min	Max
8:30 - 9:00*	0.006830	0.000445	0.342250	0.149058	0.000797	7.771123
9:00 - 9:30	0.012112	0.000486	0.542024	0.019437	0.000668	1.070025
9:30 - 10:00	0.013497	0.000733	0.257501	0.144130	0.000842	11.38132
10:00 - 10:30	0.020661	0.000685	0.524652	0.077486	0.000773	7.747582
10:30 - 11:00	0.020300	0.000751	1.070679	0.137171	0.000902	10.81968
11:00 - 11:30	0.022686	0.000683	0.521450	0.147900	0.000811	10.96116
11:30 - 12:00	0.023127	0.000896	0.785974	0.064182	0.000995	5.683590
12:00 - 12:30	0.021072	0.000890	0.295222	0.058764	0.000807	6.784774
12:30 - 13:00	0.017042	0.000680	0.299914	0.056839	0.000728	7.891387
13:00 - 13:30	0.016385	0.000411	0.416991	0.047883	0.000803	5.171524
13:30 - 14:00	0.011846	0.000497	0.228864	0.064754	0.000653	10.65167
14:00 - 14:30	0.009006	0.000289	0.166973	0.020151	0.000393	1.659474
14:30 - 15:00*	0.009102	0.000456	0.170667	0.033843	0.000426	4.086381

*Since the spot market opened at 9:00 in Chicago time before October 1, 1985, the volatility for 8:30 - 9:00 was estimated using the data only since October 1, 1985. Also, although the futures market closed at 3:15 p.m., the futures prices for the last 15 minutes were discarded to match the spot prices.

Figure 1
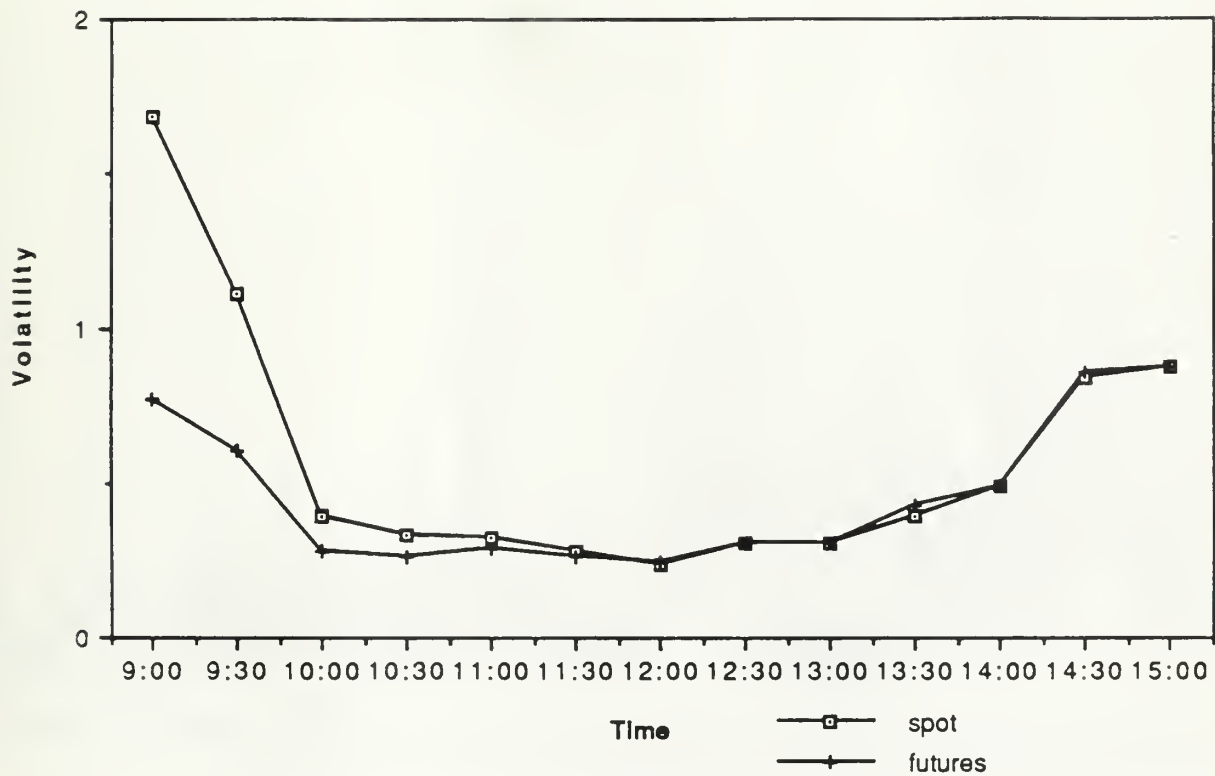
Intraday Natural Variance
(July 23, 1984 - July 15, 1986)

Figure 2

Daily Temporal Volatility of Prices
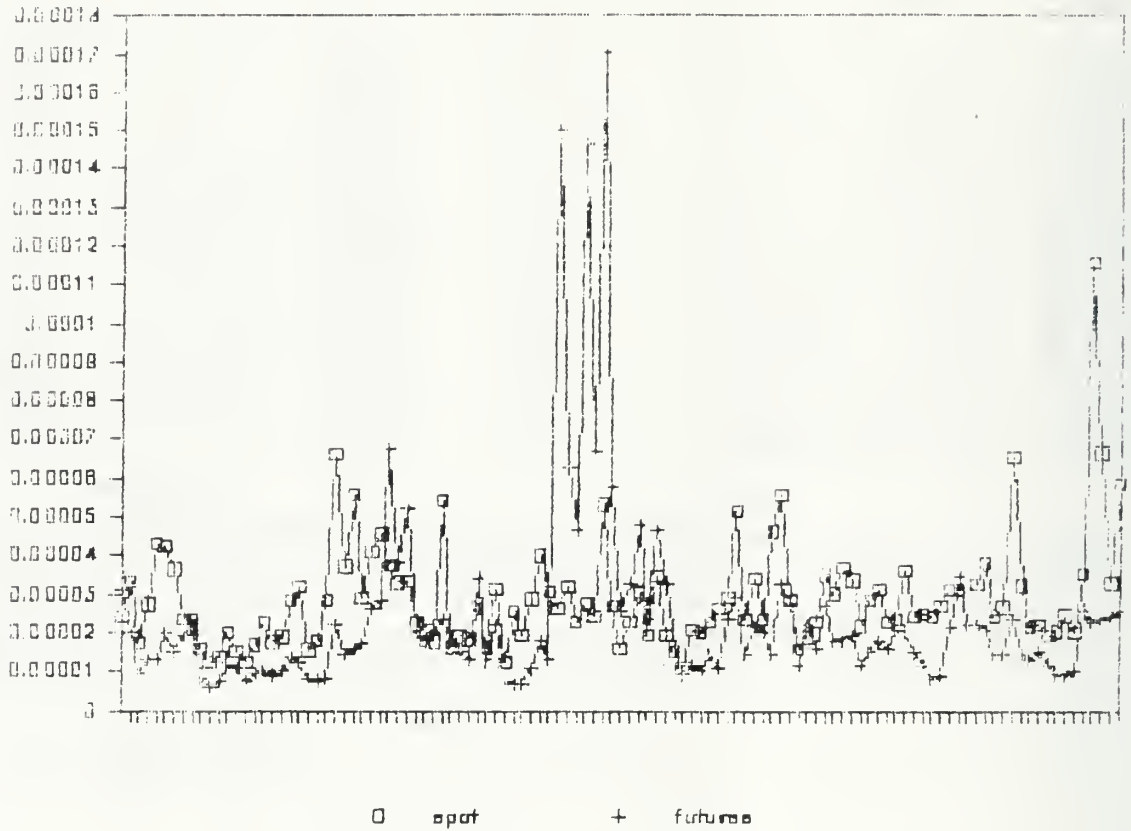(July 23, 1984 - December 31, 1984)

Figure 3

Daily Temporal Volatility of Prices
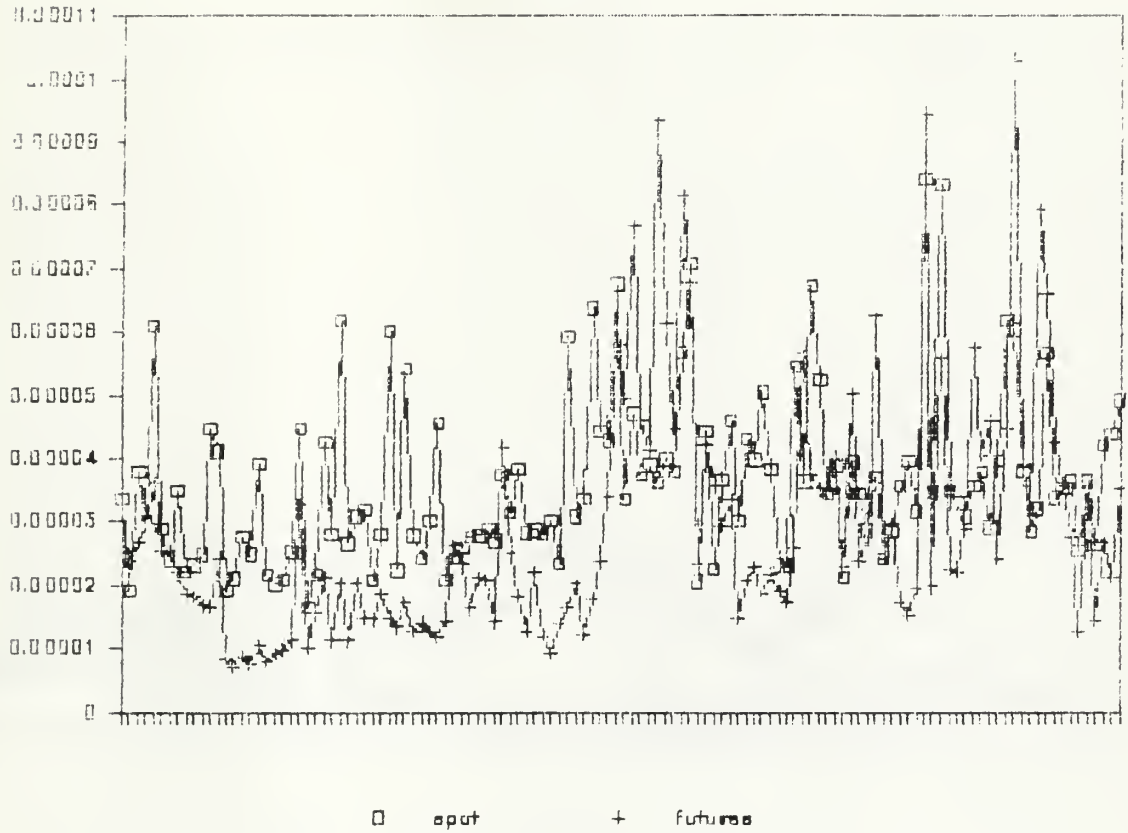### (January, 1985 - June, 1985)

Figure 4

Daily Temporal Volatility of Prices
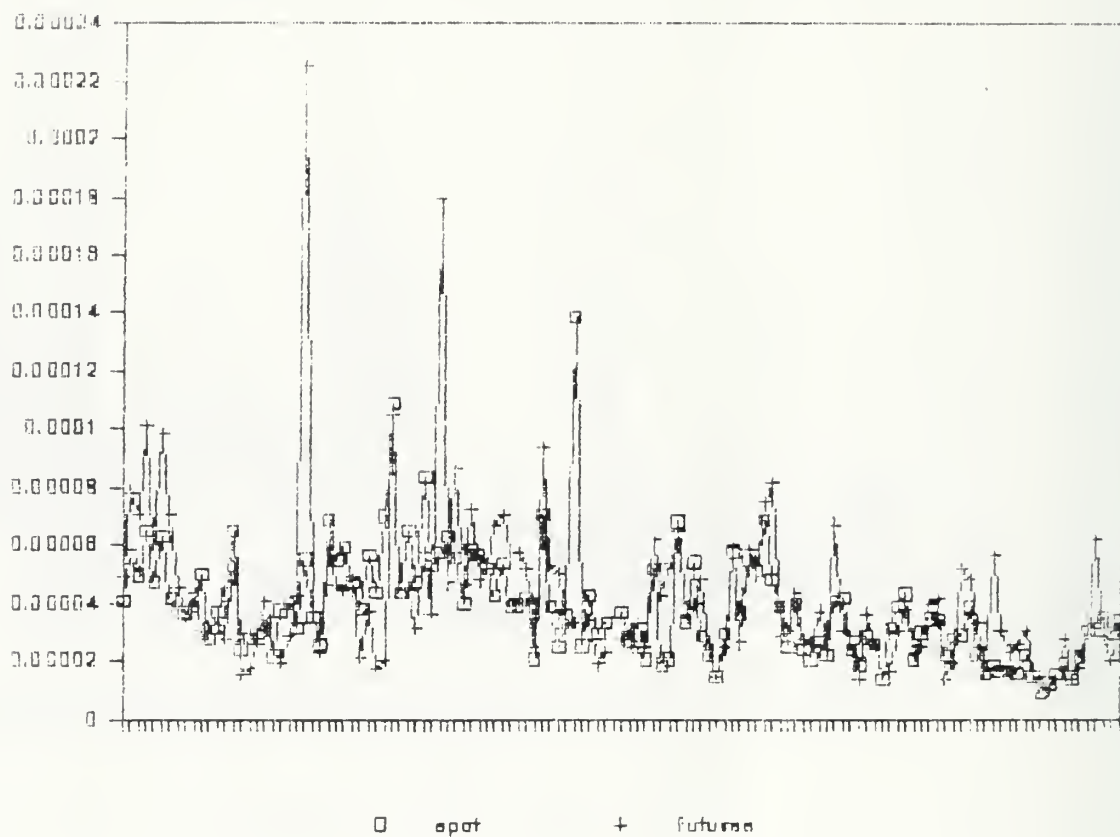(July, 1985 - December, 1985)

Figure 5

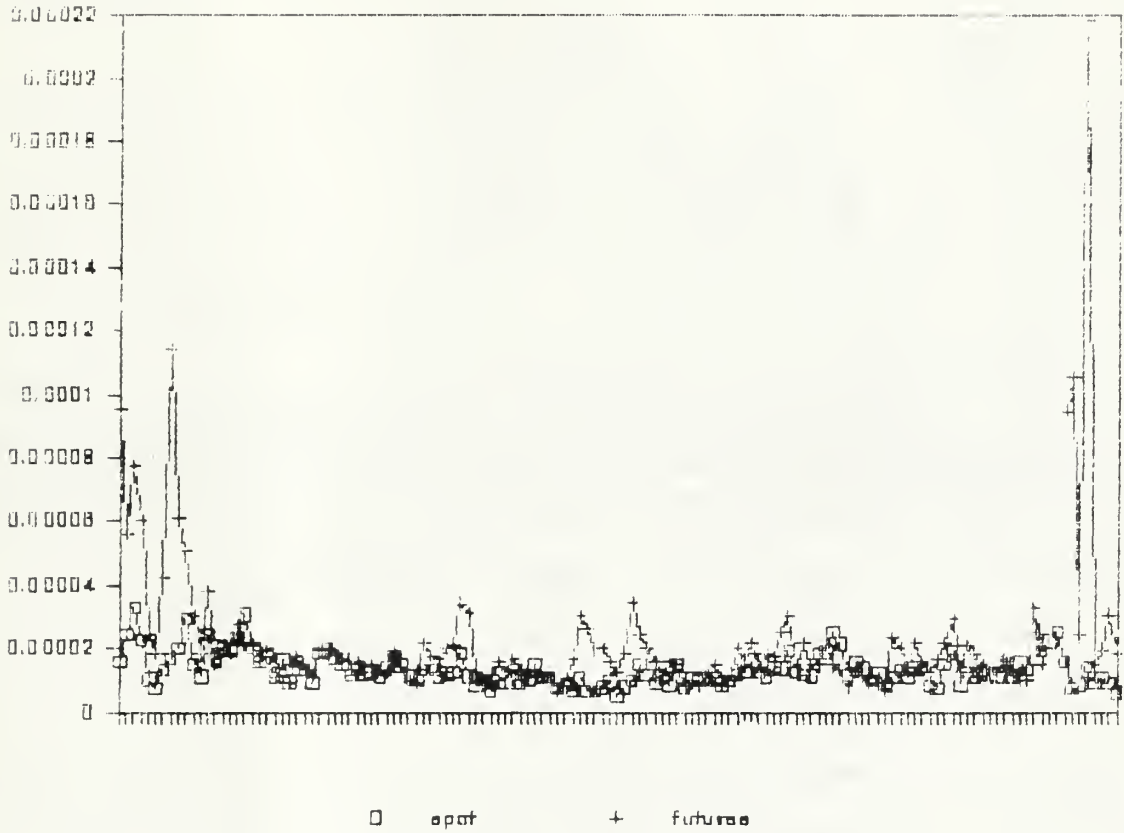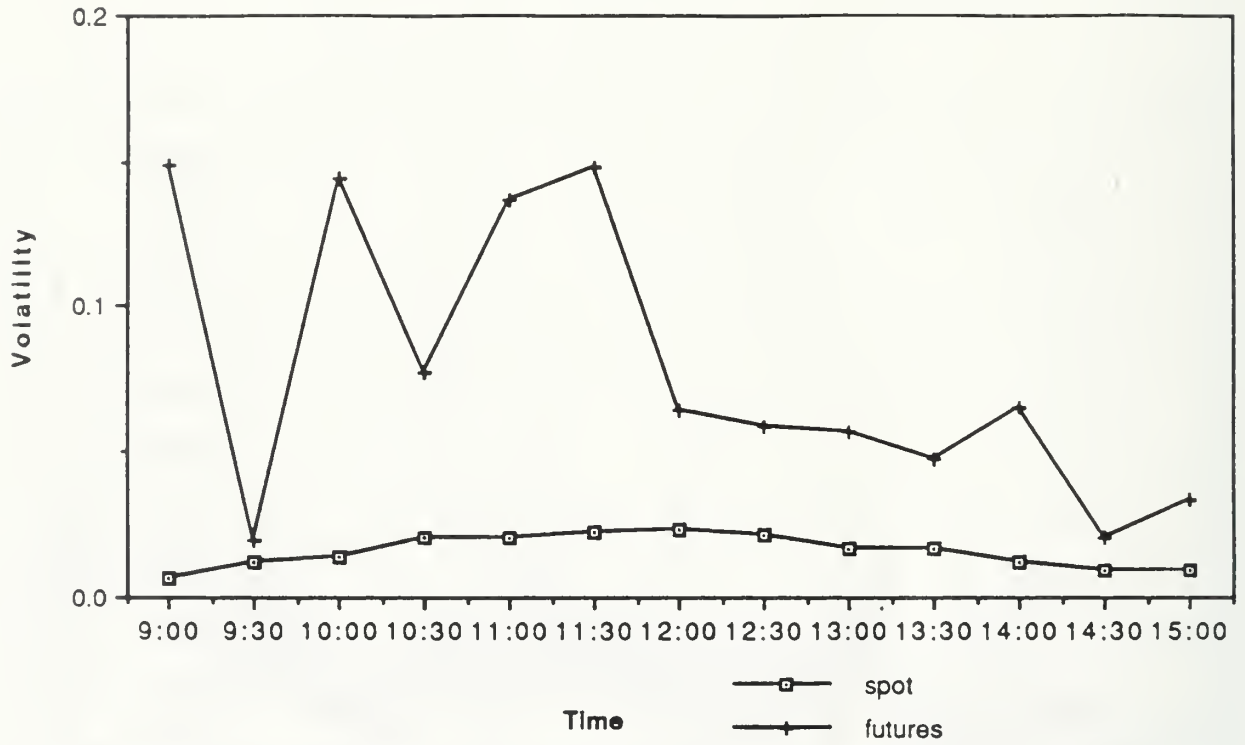### Daily Temporal Volatility of Prices
(January, 1986 - July 15, 1986)

Figure 6

Intraday Temporal Variance
(July 23, 1984 - July 15, 1986)

Appendix A

Proof of Theorem 1

Result (a) is obvious since buyers' and sellers' expected trading
profits must be non-negative. Turning to result (b), assume that it
does not hold:

(A1) $\qquad a(\overline{v} - \underline{v}) < \phi(\overline{x}) = \int_{\underline{x}}^{\overline{x}} F(x)dx.$

Since $\phi(Q)$ is strictly increasing by (4), this implies

(A2) $\qquad Q[a(\overline{v} - \underline{v})] < \overline{x}$

(apply the strictly increasing inverse $\phi^{-1}(\cdot)$ of $\phi(\cdot)$ to (A1) and then
use (3)). It then follows from (A2) that no buyers are going to pay
the prices x with $Q[a(\overline{v} - \underline{v})] < x \leq \overline{x}$: a contradiction to the fact
that $\underline{x} \leq x \leq \overline{x}$ is the support of $G(x)$. In other words, condition (b)
must hold in equilibrium. The proof of result (c) is similar.

Now we consider result (d). Let x be fixed such that $\underline{x} < x \leq \overline{x}$.
$F(x)$ is generated by the set of the reservation prices $R(t)$ of sellers
of type t with $R(t) < x$ conditional upon the constraint that $\underline{x} \leq R(t) \leq \overline{x}$:

$$F(x) = \text{Prob}\{R(t) < x | \underline{x} \leq R(t) \leq \overline{x}\}.$$

Since $R(t) < x$ if and only if $t > \psi(x)$ by (7) and (8), we have

$$F(x) = \frac{1}{\{\psi(\underline{x})/[a(\overline{v} - \underline{v})]\}} \int_{\psi(x)}^{\psi(\underline{x})} \ell(t)dt = 1 - \frac{\psi(x)}{\psi(\underline{x})}$$

for all $\underline{x} < x \leq \overline{x}$. Differentiating the above with respect to x and
then using (6) and (8), we can easily obtain (d). Since the proof of
result (e) is similar, this completes the proof of Theorem 1.

Appendix B

Proof of Theorem 2

Consider the system of differential equations:

(B1) $A F'(x) = 1 - G(x)$ and $B G'(x) = F(x)$

where

$$A = \int_{\underline{x}}^{\overline{x}} [1 - G(y)]dy \text{ and } B = \int_{\underline{x}}^{\overline{x}} F(y)dy$$

(cf. (d) and (e) in Theorem 1). Eliminating $F(x)$ from (B1), we have

(B2) $AB \, G''(x) + G(x) = 1.$

Setting

(B3) $r = \dfrac{1}{\sqrt{AB}},$

we can write the solution of (B2) in the form:

$$G(x) = 1 - c_1 \cos(rx - c_2)$$

for all $\underline{x} \leq x \leq \overline{x}$, where c_1 and c_2 are integration constants. It then
follows from (B1) that $F(x)$ must have the form:

$$F(x) = Brc_1 \sin(rx - c_2)$$

for all $\underline{x} \leq x \leq \overline{x}.$ Since $F(\overline{x}) = G(\overline{x}) = 1$, we must then have

(B4) $Br = 1; \ r\overline{x} - c_2 = n\pi + \dfrac{\pi}{2}; \text{ and } c_1 = (-1)^n$

where n is an integer. Since $Br = 1$, (B3) yields

(B5) $A = B = \frac{1}{r}$.

Since $F(\underline{x}) = G(\underline{x}) = 0$, we also have

(B6) $r\underline{x} - c_2 = m\pi$; and $n - m =$ an <u>even</u> integer.

Since $F(x) \geq 0$ and $G(x) \leq 1$ for all $\underline{x} \leq x \leq \overline{x}$, however, the quantities $(r\overline{x} - c_2)$ and $(r\underline{x} - c_2)$ cannot differ by more than $\frac{\pi}{2}$. Thus, $n - m = 0$ and (B6) can be rewritten as

(B6)' $r\underline{x} - c_2 = n\pi$.

As a result, we can obtain

$$rx - c_2 = r(x - \underline{x}) + (r\underline{x} - c_2) = r(x - \underline{x}) + n\pi$$

and results (a)-(c) thus follow.

Turning to results (9) and (10), note that

(B7) $0 < \frac{1}{r} \leq (\overline{v}-\underline{v}) \cdot \text{Min}[a, \frac{2}{\pi}]$

(cf. (a) and (b) in Theorem 1 and result $\overline{x} - \underline{x} = \frac{\pi}{2r}$). Also, observe that buyers of type s with $\frac{1}{r} < s \leq a(\overline{v}-\underline{v})$ and sellers of type t with $\frac{1}{r} < t \leq a(\overline{v}-\underline{v})$ search the market <u>only once</u> since

$$\int_{\underline{x}}^{\overline{x}} x dF(x) = \overline{x} - \frac{1}{r}, \text{ and } \int_{\underline{x}}^{\overline{x}} y dG(y) = \underline{x} + \frac{1}{r},$$

In view of the facts that

$$Eu_1(s) = \overline{v} - s - \int_{\underline{x}}^{\overline{x}} x dF(x) = \overline{v} - s - \underline{x} - (\frac{\pi}{2} - 1)\frac{1}{r}$$

for all $\frac{1}{r} < s \leq a(\overline{v}-\underline{v})$, and that $Eu_1(s) \geq 0$, we must have

(B8) $\qquad (\frac{\pi}{2} - 1)\frac{1}{r} \leq \overline{v} - a(\overline{v}-\underline{v}) - \underline{x} = (1-a)\overline{v} + a\underline{v} - \underline{x}.$

Similarly, we obtain from $Eu_2(t) \geq 0$ for $t = a(\overline{v}-\underline{v})$ that

(B9) $\qquad \frac{1}{r} \geq a\overline{v} + (1-a)\underline{v} - \underline{x}.$

In other words, we must determine endogenously the quantities r and $\underline{x}$ such that conditions (B7), (B8) and (B9) are satisfied. While tedious, it is not difficult to show that results (9) and (10) follow from (B7), (B8) and (B9). The proof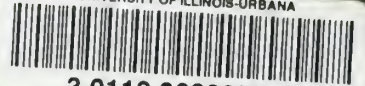 of Theorem 2 is herewith completed.