THE

TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

VOL. XIX.

DUBLIN:

PRINTED BY M. H. GILL,

PRINTER TO THE ROYAL IRISH ACADEMY.

SOLD BY HODGES & SMITH, DUBLIN:

AND BY T. & W. BOONE, LONDON.

MDCCCXLIII.

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// TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

I. *Researches on the Nature and Constitution of the Compounds of Ammonia.*

By Robert Kane, M.D., M.R.I.A., *Superintendant of the Laboratory and Professor of Chemistry to the Apothecaries' Hall of Ireland; Professor of Natural Philosophy to the Royal Dublin Society.*

Read April 9th, May 14th, and May 28th, 1838.

PART I.

ON THE SULPHATES AND NITRATES OF MERCURY, PARTICULARLY THE SUBSALTS FORMED BY AMMONIA.

HAVING shown in a former memoir that by the action of ammonia on the chlorides of mercury, there came into operation the principle which had been found by Dumas and Liebig to regulate the constitution of so many interesting bodies of organic origin,—that is to say, that by the elimination of one equivalent of hydrogen from the ammonia, and the union of the remaining hydrogen and nitrogen with the metal, there was generated an amide,—it became of importance to follow out into other combinations of the metallic salts with ammonia, an investigation which had led, in the few cases already studied, to such novel and interesting results. It is intended in the present memoir to investigate the functions of the ammoniacal elements of the mercurial subsalts, a department, of which, notwithstanding the labours of many chemists, our knowledge has remained imperfect, from circumstances similar to those which had led, in the same hands, to the conflicting opinions as to the nature of white precipitate already noticed.

In addition to the ammoniacal subsalts of mercury, there are described in the present paper the sub-sulphate and the sub-nitrates of the black and red oxides. And as the necessity of a new examination of these compounds may not appear to those who have not themselves studied the chemistry of the salts in detail, I may state, that in order to ascertain the part which the ammonia plays in the subsalts formed by its means, it became necessary to establish a comparison with the ordinary subsalts most analogous in composition; and on searching through the analyses of the mercurial subsalts already recorded, I found the testimonies so conflicting, and the results so imperfect, that I was obliged to commence the subject as if it had been actually new.

In the former memoir I assumed as the atomic weight of mercury the number 202.8, which supposes the corrosive sublimate to be a bi-chloride. This opinion I have since found reason to alter, from evidences, partly derived from the results contained in the present paper, and partly from other sources; I have therefore now adopted the Berzelian number 101.4, by which the calomel is looked upon as a sub-chloride, and sublimate as containing an equivalent of each ingredient. It will be found that by this arrangement the formulæ of these classes of compounds become much more simple than on the plan of the larger number, to which however they can easily be reduced.

Without occupying attention by any unnecessary prefatory matter, I shall pass at once to the analytical results.

I. OF THE SULPHATES OF THE RED OXIDE OF MERCURY.

Before commencing the study of the action of ammonia on the sulphates of mercury, I considered it proper to satisfy myself, by actual analyses, of the composition of these bodies, particularly with reference to the possible existence of water as one of their constituents, and the more so, as from the conflicting statements of chemists with regard to the nature of turpeth mineral, it was not unlikely that a source of error not previously unveiled might exist. As, however, my results have confirmed the ordinary view of the composition of these bodies, I will not detail any of the methods I employed, but merely state the absolute numerical results.

An analysis of neutral sulphate of mercury gave

	Experiment.	Theory $HgO.SO_3$
Sulphuric acid	= 26.72	26.82
Oxide of mercury	= 72.98	73.18
	99.70	100.00

Three analyses of turpeth mineral gave

	I.	II.	III.
Sulphuric acid	= 10.89	10.87	11.08
Oxide of mercury	= 88.71	89.24	88.76
	99.60	100.11	99.84

The theory of $HgO.SO_3 + 2HgO$ should give

Sulphuric acid	= 10.91
Oxide of mercury	= 89.09

I would not have brought forward even this notice of the numbers I obtained, were it not that from the high authority by which some of the incorrect results had been supported, and their insertion in some of the most approved elementary books, it might have appeared objectionable to make any one of the various formulæ given the foundation of a chain of reasoning, without having first established by experiment its superiority over the rest.

II. OF AMMONIA SUB-PERSULPHATE OF MERCURY.

When persulphate of mercury is treated by water of ammonia, it is converted into a white powder, which appears to be almost insoluble in water. In general, on the first addition of the water of ammonia, there is some turpeth mineral formed, which however gradually disappears, and the product is an uniformly white powder. This reaction takes place more rapidly by boiling, but the nature of the result is the same. If turpeth mineral be boiled, or treated in the cold with water of ammonia, it is converted into the same white substance, as shall be proved by the analyses subjoined. The existence of this white ammoniacal subsulphate was noticed by Fourcroy, but he made no analysis of it, nor has it ever been, at least to my knowledge, subjected to an accurate investigation.

This substance is heavy; it is not decomposed by water, which, however, dissolves some traces of it. When heated it becomes brown, exhales traces of ammonia, much water and nitrogen, and there finally remains sulphate of the black oxide of mercury, which by a further heat gives its usual products of decomposition. This powder is soluble in nitric and muriatic acids. When diffused through water, and treated by sulphuretted hydrogen, the mercury is all thrown down as sulphuret, while the liquor remains perfectly neutral, and gives by evaporation sulphate of ammonia.

I shall speak of this substance always as ammonia-turpeth, a name short, and not involving any theory, and therefore the best calculated for use.

To analyze this compound, the following methods were pursued:

A. 5.072 grammes ammonia-turpeth were dissolved in muriatic acid, and precipitated by muriate of barytes. The sulphate of barytes formed was washed until the water passed quite pure; it was then carefully dried and ignited, and weighed, when corrected for the ashes of the filter, = 1.223 gramme, or 24.11 per cent., containing 8.28 of sulphuric acid.

The liquors filtered off the sulphate of barytes were treated by sulphuret of hydrogen, and the sulphuret of mercury was collected on a filter, and carefully dried until it ceased to lose weight; when dried there was

$$\left.\begin{array}{lcl}\text{Sulphuret and filter} & = & 5.835 \\ \text{Filter} & = & 0.910\end{array}\right\} \text{4.925 Hg.S.}$$

giving Hg.S = 96.9 per cent., or 83.69 mercury.

B. 10.375 grammes of sulphate of mercury were boiled with a considerable excess of ammonia, until completely converted into ammonia-turpeth, which was collected on a filter after the whole had been allowed to cool.

The powder was washed until the liquor ceased to give appreciable traces of sulphuric acid; it was then dried by a temperature of 212°, and weighed

$$\left.\begin{array}{lcl}\text{Powder and filter} & = & 8.590 \\ \text{Filter} & = & 0.361\end{array}\right\} \text{8.229 ammonia-turpeth.}$$

To the filtered liquor and washing was added an excess of muriate of barytes, it having been first acidulated by muriatic acid. The sulphate of barytes was collected on a filter and washed, as long as the liquors passed through containing muriatic acid; it was then dried and ignited. The ashes of the filter having been allowed for, it weighed 6.112 grammes.

The liquors remaining contained a trace of mercury, which precipitated gave 0.220 of Hg. s. Therefore 100 of sulphate of mercury gave

Ammonia-turpeth	=	79.31
Sulphate of barytes	=	58.91
∵ Sulphuric acid	=	20.245
And sulphuret of mercury	=	2.10 equivalent to 1.81 of mer-

cury, giving 1.96 oxide and 2.68 sulphate.

There had therefore been decomposed 100—2.68 of the sulphate, and 100 of sulphate completely converted into ammonia-turpeth should give

Ammonia-turpeth	=	81.48
Sulphuric acid	=	20.06

The sulphuric acid in 100 of Hgo. so_3 is 26.82, of which 20.06 is almost exactly three-fourths, for $\frac{3}{4} \cdot 26.82 = 20.115$. Therefore in the ammonia-turpeth is contained all the mercury and one-fourth of the sulphuric acid; its composition therefore comes out,

Mercury	=	67.83	81.48, or	83.25	100.00
Sulphuric acid	=	6.76		8.29	
Other matters	=	6.89		8.46	

C. 7.317 grammes of ammonia-turpeth were diffused through water, and decomposed by a current of sulphuretted hydrogen. The sulphuret of mercury was collected on a filter, and dried carefully, until it ceased to lose weight.

Filter and sulphuret	=	7.422	7.067, or
Filter	=	0.355	

Sulphuret = 96.58 per cent., containing 83.35 mercury.

The clear liquor reacted neutral; it was evaporated in a water-bath to perfect dryness, and the capsule, with the residual sulphate of ammonia, carefully weighed; the salt then cleared out without loss, and the capsule tared; the salt was then again weighed on the tared slip of paper, on which it had been collected, and the second not differing from the first weighing by a milligramme, certainty of accuracy was obtained.

The sulphate of ammonia weighed 0.988 gramme, corresponding to 13.5 per cent., and consisting of

Sulphuric acid	=	8.18	
Ammonia	=	3.48	13.50
Water	=	1.84	

Tabulating the results of these three methods, there is obtained for ammonia-turpeth

		A.	B.	C.	Mean.
Sulphuric acid	=	8.28	8.29	8.18	8.25
Mercury	=	83.69	83.25	83.35	83.43
Ammonia	=			3.48	3.48
Oxygen and loss	=				4.84

The positive values obtained by analysis give the proportions in ammonia-turpeth to be:

1 atom of sulphuric acid.
1 atom of ammonia.
4 atoms of mercury.

But for the oxidation of the mercury there would be required (as, from the solubility of ammonia-turpeth in muriatic acid, the whole of the mercury is proved to be in percombination) oxygen = 6.582, a quantity which is altogether excluded by the sum of the values of the other ingredients, which leave room for only 4.84 of oxygen. Now this number is almost exactly three-fourths of 6.582, since $\frac{3}{4} . 6.582 = 4.937$; and we have consequently the most complete evidence that the fourth atom of metal is combined with some other negative radical than oxygen. If one conceives that in this ammonia-turpeth the azote and hydrogen exist as amidogene, the formula falls in accurately with the experimental results, for there is

SO_3	=	40.16	8.27	Analysis	=	8.25
NH_2	=	16.14	3.32		=	3.27
$4\,Hg$	=	405.60	83.47		=	83.43
$3\,O$	=	24.00	4.94		=	5.05
		485.90	100.00			100.00

By this formula 100 of sulphate of mercury should give 81.30 of ammonia-turpeth, while in experiment B there was obtained 81.48.

It will be seen that the formula $HgO.\,SO_3 + 2\,HgO + Hg\,NH_2$ is completely analogous to that for the yellow powder formed by the action of water on white precipitate, if we write the sulphate of mercury as $Hg.\,SO_4$; then there is

$$Hg.\,Cl + 2\,HgO + Hg\,NH_2, \text{ and}$$
$$Hg.\,SO_4 + 2\,HgO + Hg\,NH_2.$$

We shall have occasion, hereafter, to advert to this type of a remarkable class of combinations.

III. ACTION OF AMMONIA ON SULPHATE OF BLACK OXIDE OF MERCURY.

When the sulphate of the black oxide of mercury is treated by cold or boiling water no reaction occurs indicating the formation of a basic salt; it would therefore appear as if there existed but one sulphate of the black oxide.

When sulphate of the black oxide of mercury is treated by water of ammonia there is obtained a dark grey powder, which, when heated, gives water, ammonia, sulphurous acid, oxygen, and mercury. It is thus indicated to be a basic salt, containing ammonia; but great difficulty was found in tracing accurately the proportions in which complete decomposition occurred.

To determine the nature of this grey compound, the following method was adopted:—A weighed portion of sulphate of black oxide of mercury, was treated by an excess of water of ammonia, until the reaction appeared to be complete, and a uniform dark grey powder was produced. It was then collected on a filter, and the liquors, which contained but a mere trace of mercury, were mixed and acidulated by muriatic acid, and precipitated by muriate of barytes. The sulphate of barytes was then collected and dried, and having been ignited, with its filter, weighed, and the correction for ashes made.

The results of five experiments of this kind are given in the subjoined table, the details being omitted, in consequence of my not intending to use these results as bases for induction, and therefore it not being necessary to specify the particulars of each case:

100 of $\underline{Hg}.\,O + SO_3$	A.	B.	C.	D.	E.
Grey Powder . .	Not determined.	83.08	92.5	90.22	88.89
Free SO_3	13.85	11.75	Not determined.	8.35	9.96

The sulphate of the black oxide of mercury is, when prepared by double decomposition, anhydrous, and is composed of

$$\left.\begin{array}{lcr} \text{Mercury} & = & 80.80 \\ \text{Oxygen} & = & 3.18 \\ \text{Sulphuric acid} & = & 16.02 \end{array}\right\} 100.0$$

But, from the extensive limits, within which the quantity of the sulphuric acid removed by the ammonia, is contained, it would be improper to assert positively by what formula the result should be expressed. I consider that by the action of the ammonia a certain quantity of a per-compound may have been formed, and thus have given rise to the variable nature of the result. The results A and B, however, tend to induce me to look upon the grey compound, when pure, as having the composition $\underline{Hg}O.SO_3 + 2\underline{Hg}O + \underline{Hg}NH_2$, and bearing the same relation to the ammonia-turpeth, that the powder formed by water of ammonia on calomel, bears to white precipitate. If one might hazard a conjecture, the other results would indicate a tendency to a limit in the decomposition, when the half of the sulphuric acid had been removed, and thus there may be a body also grey coloured $\underline{Hg}O.SO_3 + \underline{Hg}NH_2$, or rather $\underline{Hg}SO_4 + \underline{Hg}NH_2$, similar to $\underline{Hg}Cl + \underline{Hg}NH_2$, as described in the former paper.

I did not follow up any analysis of the grey powder, because it was evident, from the variable nature of the circumstances affecting its formation, that no result could be obtained, so closely true, as to prove either for or against the question of the function of the ammonia, or indeed the quantity of the latter constituent (never more than three per cent.), that might have been therein contained. It is necessary therefore, on this point, to allow of the temporary guidance of the analogical evidence, which we derive from the more fixed results of the analyses of corresponding compounds.

IV. OF THE NITRATES OF THE RED OXIDE OF MERCURY.

We owe to the younger Mitscherlich an examination of the nitrates of mercury, which constitutes, up to the present day, all our knowledge regarding them. The singularity of the results to which he arrived, rendered their repetition of importance, and the more so, as the doubts which had been thrown upon the correctness of his analyses of the ammonia-nitrates, by Soubeiran, rendered it necessary to confirm his formulæ before they could be assumed as data in an investigation like the present.

There can be obtained but one crystallized nitrate of the peroxide of mercury: this salt is formed in small prisms, which deliquesce, except in a very dry room; when dried between folds of blotting paper, the crystals taste metallic, but not acid. These crystals are decomposed by water, but only a portion of the mercury is thrown down as a pale yellow powder, whilst the liquor becomes acid. If the supernatant liquor be evaporated, the excess of acid is driven off, and there crystallizes, on cooling, the same salt as had been previously dissolved.

To analyze this salt, the same method was pursued as had been employed by Mitscherlich, and with exactly the same result. As the analyses were but confirmatory of his accuracy, I shall not enter into their details. The formula of this crystallized pernitrate of mercury is $HgO.NO_5 + HgO + 2HO$, and in numbers:

2 atoms of oxide of mercury	=	202.80
1 of nitric acid	=	54.14
2 of water	=	18.00
		274.94

It is well known that this salt is decomposed by water, but there still remains some doubt as to the constitution of the subnitrate thus generated. From the variable appearance it presents, according to the method by which it has been obtained, it evidently is not of constant nature; and it is generally stated by systematic writers, that by washing it can be completely resolved into nitric acid and oxide of mercury. Of this nitrous turpeth, as it has been generally termed, two quantitative analyses have been recorded, of which the results follow:

		Oxide of Mercury.	Nitric Acid.	Reference.	
Braancamp	=	88.0	12	An. Chim.	54
Grouvelle	=	88.97	11.03	An. Ch. et Phys.	19

These results coinciding so closely, and leading immediately to the formula $NO_5 + 4HgO$, might appear to be conclusive, but several circumstances induced me to consider a new examination necessary. Thus, all other analyses made by Braancamp were inaccurate by four or five per cent., a result to be partly attributed to the imperfect state of analytical chemistry at the time he wrote; and also, it appeared from the evidently inconstant nature of the subnitrates obtained by water, that the stages of its production required to be closely studied. In addition I had observed that nitrous turpeth, when heated, always yielded some liquid nitric acid; this fact should introduce water as one of its constituents, which the results obtained by Braancamp and Grouvelle necessarily exclude.

A quantity of crystallized nitrate of mercury was treated by water, and the undissolved portion washed by warm water until the washings no longer reacted acid. It then appeared as a fine yellow powder, very heavy, not acted on by cold water, but converted into a brownish red powder by boiling water, which dissolved out the soluble nitrate of mercury, not affecting blue cabbage paper. When this powder is heated, it gives much red fumes and a quantity of liquid nitric acid, and there remains red oxide of mercury, which by a stronger heat is decomposed. As, by avoiding the use of boiling water, this powder was obtained apparently similar in appearance and properties at different times, it was selected for analysis.

A. 5.458 grammes of this powder were dissolved in muriatic acid, and treated by proto-chloride of tin. There were obtained 4.170 grammes of metallic mercury, giving 76.40 HG per cent.

B. 5.513 grammes of a portion prepared at a different time were dissolved in muriatic acid diluted with a good deal of water, and precipitated by sulphuretted hydrogen; there were obtained

$$\left.\begin{array}{lll}\text{Filter and sulphuret} & = & 5.935\\ \text{Filter} & = & 0.932\end{array}\right\} = 5.003\ \text{HgS},$$

giving mercury = 78.31 per cent.

C. A portion of the yellow powder having been treated by boiling water, and having assumed a brownish red colour, was dissolved in muriatic acid, and precipitated by sulphuretted hydrogen. Thus analyzed, 4.975 of this powder gave 4.919 sulphuret of mercury, corresponding to 85.33 mercury per cent.

D. A quantity was boiled for a long time, until it had been converted into a brick red powder, which was analyzed by solution in muriatic acid and the separation of the mercury by proto-chloride of tin; from 7.746 grammes were obtained 6.673 mercury, or 86.17 per cent.

No matter how far the boiling might be carried, I could not reduce the powder to the state of pure red oxide. The residual powder dried always gave by heat red fumes, and also liquid nitric acid, but in constantly decreasing proportion. I consequently considered it unnecessary to press the series of analyses further.

The analyses A and B give the result $NO_5 . HO + 3 Hg O$ pretty closely, the theoretical numbers being

NO_5	=	54.14	63.14	16.13	Hg	=	77.74
HO	=	9.00			O	=	6.13
$3 Hg$	=	304.20	328.20	83.87	NO_5	=	13.83
$3 O$	=	24.00			HO	=	2.30
			391.34	100.00			100.00

and I am disposed to consider such as being the real composition of the yellow sub-pernitrate prepared by water not boiling. It will be at once seen that this formula assimilates completely the sub-nitrate of mercury with those of copper and of bismuth, the nature of which has been lately elucidated by the experiments of Graham.

With regard to the red sub-nitrate prepared by boiling water, I am inclined to look upon it, in like manner, as having a definite composition, because, whilst the specimen used in analysis C had been boiled but for a few minutes, and that used in analysis D for some hours, their composition appeared to be quite the same. When heated, this red subsalt certainly yields a trace of water, besides nitrous acid fumes; but this water is in such small quantity that it might be considered as hygrometric. The quantity of mercury obtained, may serve equally well for one or other of two formulæ, thus:

		$NO_5 + 6 Hg O$	$NO_5 . HO + 7 Hg O$	
Nitric acid	=	7.62	6.52	
Oxide of mercury	=	92.38	92.39	= 100.0
Water	=		1.09	

Although I have always found this red powder to give a trace of water, yet I incline strongly to the first of the above formulæ, to which I shall refer when treating of some analogous ammonia compounds.

As the composition assigned by Grouvelle to the sub-pernitrate falls within the limits of the two bodies which have been just described, it may be supposed that he had examined a mixture of them, and not a pure substance; this idea I consider probably to be true.

By the action of water on the crystallized pernitrate it is resolved into yellow sub-pernitrate and an acid-reacting salt, which, when evaporated, yields, as was already mentioned, the same crystallizable nitrate, whilst the excess of acid passes off. There takes place, therefore, a division of the mercury into two portions, one of which passes into solution, whilst the other is left in the insoluble yellow powder. The salt in solution does not appear to crystallize, but to give, on concentration, nitric acid and the crystallized basic salt of Mitscherlich. The proportion of mercury which remains in the solution approximated, in my trials, to one-third of that precipitated, and the action of water may be explained by the following formula:

$$2\,(NO_5 + 2\,Hgo + 2\,HO) = \{HO.NO_5 + 3\,Hgo\} + \{Hgo.NO_5 + 3\,HO\}$$

The crystalline pernitrate being considered as a double salt, which is decomposed by water into its constituents. It may evidently be likewise considered as a simple salt, the sum of the number of atoms of hydrogen and mercury remaining still four, but capable of indefinite replacement within that limit.

The proportions of mercury and nitric acid in solution, after the precipitation of the yellow basic salt by water, must be quite definite, and should, if isolated, produce a salt $Hgo\,.\,NO_5\,.\,HO + 2\,HO$, corresponding to the ordinary nitrates of copper and bismuth, but which may be so easily decomposed as to be uncrystallizable. Moreover, if we look to the very general tendency to the formation of bodies containing four equivalents of mercury, it will appear not impossible but that a type of basic nitrates $Hgo.NO_5\,.\,Hgo + 2\,Hgo$ may really exist, and on which Grouvelle may have happened to alight, although I could not, even after many trials, succeed in preparing it.

Thus there should be a series of salts:

$Hgo\,.\,NO_5\,.\,HO + 2\,HO$. uncrystallizable.
$Hgo\,.\,NO_5\,.\,Hgo + 2\,HO$. ordinary salt.
$Hgo\,.\,NO_5\,.\,HO + 2\,Hgo$. yellow basic salt.
$Hgo\,.\,NO_5\,.\,Hgo + 2\,Hgo$. Grouvelle's basic salt.

and also

$Hgo\,.\,NO_5\,.\,Hgo + 4\,Hgo$. red basic salt.

V. OF THE AMMONIA SUB-PERNITRATES OF MERCURY.

It has been long known, that, by adding water of ammonia to a solution of pernitrate of mercury, there is obtained a fine white powder, which has been examined by Mitscherlich and Soubeiran, with results, however, so discrepant, as not to allow us to draw any conclusion whatsoever from them.

Almost immediately on commencing the examination of this reaction, I found that the nature of the precipitate obtained was liable to considerable variation, and that very trivial alterations in the conditions, under which the ammonia was added, changed the proportion of quicksilver by four or five in the hundred,—limits including the values obtained by the above-mentioned chemists. It therefore became probable that, as in the case of white precipitate, the existence of two or more different bodies had led to the discrepancies in the statements of those chemists; and by paying minute attention to the circumstances which influence their formation, I was led to detect the existence of three distinct ammoniacal subnitrates, as prepared by mere precipitation. The circumstances which influence the nature of the precipitate are, the concentration of the mercurial solution, its degree of acidity, the strength of the water of ammonia, the excess of one or other reagent, and the temperature. By slight changes of these, there are produced modifications of composition, and frequently an imperfect change from one to the other form takes place. In addition to these three precipitated compounds, there are two others obtained by crystallization, of which one had been examined by the younger Mitscherlich, and the other was met with first in the course of these investigations.

Ammonia Sub-pernitrate, No. 1.—When a dilute, and not very acid solution of pernitrate of mercury is treated by weak water of ammonia, (taking care not to add an excess of the latter, and the solution being cold,) there is obtained a pure milk-white precipitate, not granular, which remains suspended for a considerable time. This precipitate, collected on a filter, may be exposed to a heat of boiling water without change, and is consequently easily dried.

When this powder is heated, it becomes yellow, and gives azote, ammonia, then red fumes, and finally oxygen and quicksilver. If boiled with water, it becomes granular and heavier, deposits itself more easily, and has lost, in some degree, its pure white colour. The water remains neutral, but is found to hold some nitrate of ammonia in solution.

On analysis, this powder yielded precisely the same results as had been obtained by the younger Mitscherlich; on that account I shall not insert the details of the methods, which in great part resembled those already described in the analyses of ammonia-turpeth, but shall merely note the quantities of mercury and other constituents obtained.

In three analyses there resulted:

		I.	II.	III.
Mercury	=	76.50	76.84	75.9
Nitric acid	=	12.66		
Ammonia	=	4.01		

These three portions had been prepared and analyzed at different periods.

The formula $NO_5 + NH_3 + 3\,Hgo$ gives

3 atoms mercury	=	304.20	76.17
3 „ oxygen	=	24.00	6.01
1 „ nitric acid	=	54.14	13.54
1 „ ammonia	=	17.14	4.28
		399.48	100.00

Mitscherlich's result was

Mercury	=	75.55
Nitric acid	=	14.33
Ammonia	=	4.68

There can, therefore, be no doubt of this being really the composition of the substance, and if we compare it with the yellow sub-pernitrate, we shall observe a very curious analogy. Thus the water in the common subnitrate is replaced by ammonia, that is, by amide of hydrogen, so that the basic function which has been so elegantly shown by Mitscherlich and Graham to belong to water, appears to be enjoyed in a certain degree by ammonia also. This is shown, and the nature of this white substance very elegantly proved, by an experiment well calculated for class illustration: if some of the water subnitrate be put into a solution of nitrate of ammonia, and boiled for a moment, the white powder is rapidly formed, and the liquor will be found to be strongly acid. Thus,

$$(HO\,.\,NO_5 + 3Hgo) + NO_5\,NH_3 = (NH_3.\,NO_5 + 3\,Hgo) + HONO_5.$$

Of the Ammonia Subnitrate, No. 2.—It having been found that, by boiling the former powder with water, it altered in its appearance, and became much heavier and more granular, it was natural to expect from it a different constitution. If the solutions of nitrate of mercury and of ammonia be mixed, while hot, or if they be boiled after mixture, the same modification is produced; and as Soubeiran had been led astray by the effects of boiling white precipitate, it might be inferred that his discordant results arose from his operating with hot solutions in this case also. The powder, thus prepared, gives the same results of decomposition as the former; potash, even boiling, exerts no action on either, giving out no ammonia, and no oxide of mercury separating. The following analyses were made:

A. 7.185 grammes were dissolved in muriatic acid, and the solution precipitated by sulphuretted hydrogen. The sulphuret produced weighed 6.766, or 94.17 per cent., containing 81.24 of mercury.

B. 7.353 of another portion were dissolved in muriatic acid, and the mercury precipitated by proto-chloride of tin. There were obtained 5.978 grammes, being 81.28 per cent.

When this powder, diffused through water, is treated by sulphuretted hydrogen, there is formed sulphuret of mercury, and the liquor contains neutral nitrate of ammonia.

From these results, and the quantity of quicksilver coinciding so closely with that obtained by Soubeiran, there is no doubt but that the substance is the same as that upon which he operated.

The formula given by Soubeiran is $NO_5 + NH_3 + 4HgO$, which gives the numbers

4 Hg	=	405.60	79.71
4 O	=	32.00	6.29
NO_5	=	54.14	10.63
NH_3	=	17.14	3.37
		508.88	100.00

He however obtained 80.08 mercury per cent., or more than he should by his formula; and he proved that the nitric acid and ammonia could not exist in the powder as common nitrate of ammonia. Indeed he expressly states that the clearing up the nature of the function played by ammonia in these combinations should be left to a future period in science. Under these circumstances there can be no doubt but that the true formula for Soubeiran's subnitrate is as follows: $HgO . NO_5 + 2HgO + HgAd$, which gives

$4Hg$	=	405.60	81.13
$3O$	=	24.00	4.81
NO_5	=	54.14	10.83
NH_2	=	16.14	3.23
		499.88	100.00

This compound resembles those already described containing chlorine and sulphuric acid.

By using strong nitrate of mercury, and a considerable excess of a strong solution of ammonia, I have on two occasions obtained a yellowish white precipitate, yielding between 84 and 85 per cent. of mercury, and containing nitric acid and ammonia in the proportions of one equivalent of each. I have not, however, discovered the circumstances under which this third modification may be generated at will, for in trying often to form it, sometimes by hot liquors, at other times using the solutions cold, I have obtained the substances previously described, or else mixtures of them. The existence, however, of a yellowish white powder containing more mercury than either, is certain, and I consider its formula to be probably

$$(HgO . NO_5 + 4HgO + HgAd),$$

I shall not, however, dwell upon it more; the relation which it holds to the red sub-pernitrate is quite evident.

The Crystalline Ammonia Subnitrate.—Mitscherlich had observed that if the ammonia subnitrate of mercury be boiled with an excess of ammonia, and nitrate of ammonia be added, a portion of the powder dissolves, and the liquor, when it cools, yields, according as the excess of ammonia passes off, small crystalline plates of a pale yellow colour. I have verified this observation, but I did not analyze those plates, because I could form but a very small quantity of them; and having found in all cases that Mitscherlich's analyses were remarkably good, I considered that in the case of these crystals, which I found great difficulty in preparing, I might rely upon his accuracy. He found these crystals to be $NH_3 . NO_5 + 2HgO$. But while I believe the numbers to be true, I do not consider that to be the rational formula. These crystals are formed by the solution of Soubeiran's subnitrate in nitrate of ammonia, and the formula is

$$(HgNO_6 + 2\,HgO + HgAd) + (NH_4O \,.\, NO_5),$$

which is equal to twice

$$(NH_3 \,.\, NO_5 + 2\,HgO).$$

That such is its constitution will be clearly shown from the study of the body next to be described.

When Soubeiran's ammonia subnitrate is boiled in a strong solution of nitrate of ammonia it is dissolved in considerable quantity, and the liquor being filtered while hot, deposits, on cooling, small but very brilliant needles, which after some time lose their lustre, and become dull and opaque, an appearance which the salt, when rapidly formed from a very strong solution, occasionally possesses from the commencement. This salt, after it has been once dried, cannot be again brought into contact with water without decomposition; its constituents are reproduced, the nitrate of ammonia dissolving, and Soubeiran's subnitrate being left undissolved. These circumstances rendered a few analyses sufficient for determining its composition.

A. 6.061 grammes of this salt were diffused through water, and decomposed by a current of sulphuretted hydrogen gas. The sulphuret of mercury was collected on a filter, and having been carefully dried, weighed 4.187, corresponding to 69.08 sulphuret and 59.60 mercury per cent. The liquor and washings, evaporated to dryness, in a water-bath, gave 2.173 of nitrate of ammonia, therefore 35.85 per cent.

B. 5.973 of a quantity prepared at a different time were dissolved in muriatic acid, and treated by sulphuretted hydrogen. The sulphuret was cautiously dried until it ceased to lose weight, and amounted to 4.010, giving 67.13 sulphuret, and 57.99 mercury per cent.

Hence there is

Mercury, mean value	=	58.79
Nitric acid	=	24.17
Ammonia	=	7.65

If we divide these numbers by the atomic weights of the bodies, and reduce them to a standard, we shall find that there are almost exactly three atoms of nitric acid, three of ammonia, and four of mercury.

The formula $3(NH_4O \cdot NO_5) + 4Hg O$ gives

$4Hg$	=	405.60	59.78
$4O$	=	32.00	4.72
$3NH_3$	=	51.42	7.58
$3NO_5$	=	162.42	23.94
$3HO$	=	27.00	3.98
		678.44	100.00

I do not consider the rational formula of this compound so simple as should appear from the above expression. It is most likely to contain the ammoniacal subnitrate ready formed; it being decomposed by contact with water, and yielding that substance. If the mercury be as Soubeiran's subnitrate, the formula presents a curious relation; thus,

$$3(NH_4O \cdot NO_5) + 4Hg O =$$
$$(NO_5 \cdot Hg O + 2Hg O + Hg Ad) + 2(NO_5 \cdot HO + 2HO + HAd).$$

The facility with which this salt may be formed by heating red oxide of mercury with nitrate of ammonia might be used as an argument for the former view.

VI. OF THE NITRATES OF THE BLACK OXIDE OF MERCURY.

In the memoir to which I have had so frequently occasion to refer, George Mitscherlich described two crystallized proto-nitrates of mercury, and gave detailed analyses of them. I have had occasion to confirm his results, and I consequently consider the composition of these two salts as well established. I shall not describe any of my own analyses of them, but merely insert the formulæ derived from the numbers of Mitscherlich, in order that the substances, next to be examined, may be compared with them.

The salt obtained in transparent rhombs from an acid liquor has the formula $(\underline{Hg}O + NO_5) + 2HO$, and consists of

Black oxide of mercury	=	74.54	100.00
Nitric acid	=	19.09	
Water	=	6.37	

When this salt is digested with more black oxide of mercury, or when an acid solution of it is left standing on an excess of mercury, the crystals which are deposited are opaque and white, they are generally rhombic prisms. The second (dimorphous) variety described by Mitscherlich I have not analyzed. Their formula is $3\underline{Hg}O + 2NO_5 + 3HO$, and their composition

Black oxide of mercury	=	82.40	
Nitric acid	=	14.08	100.00
Water	=	3.52	

I shall hereafter point out some reasons for considering this to be a double salt.

It had been long since remarked that these crystallizable salts were decomposed by water, but great discordance had arisen among chemists as to the nature of the subsalts thus produced. On treating the crystallized nitrates by cold water there remains undissolved a white powder, which as long as the supernatant liquid is acid retains its colour, but if it be washed it becomes yellow. Further, if it be boiled, the brilliancy of the colour is injured, and by long-continued boiling it is converted into a grey powder, which, according to some writers, must be considered as a basic salt. These various phenomena it is necessary to study in detail.

The white powder, which is formed by the first action of water, I could never obtain in a form justifying any inference from an analysis of it. It is evident that, without freeing it from the liquor holding in solution a quantity of another salt, it would be useless to examine it; and on the other hand, by washing, the change from white to yellow cannot be avoided; it was thence necessary to consider the yellow subsalt, as being the product to which attention should be paid.

This yellow sub-protonitrate of mercury can be easily prepared: the white precipitate of which I spoke may be washed with cold water repeatedly, until it is converted into a bright lemon-yellow powder; by the use of warm water the change may be much accelerated, and the materials may be even boiled for some time without danger, provided that the liquors be not too often changed. The limit is known to have been passed when the brilliant yellow is dimmed by the supervention of a greyish shade. By a very cautious addition of a weak solution of potash, the quantity obtainable from the soluble salt may be very much increased, but the specimens thus prepared are seldom so completely bright and pure as where water alone has been employed in its preparation.

This salt, when heated, gives out red fumes and drops of liquid nitric acid, and leaves red oxide of mercury, which by a further application of the heat is decomposed. It is insoluble in water, and by boiling, is changed into a grey powder, which by the lens is seen to consist chiefly of quicksilver in the metallic state, and the liquor is found to contain some mercury, as nitrate of the red oxide.

Grouvelle has published the results of an analysis of this subnitrate. It is to be regretted that this chemist communicates no details as to his methods, since without them the degree of confidence which should be given to his results cannot be easily ascertained. He states this subnitrate, whether prepared by water or by potash, to consist of

$$\left.\begin{array}{lcr} \text{Black oxide, 2 atoms} & = & 88.6 \\ \text{Nitric acid, 1 atom} & = & 11.4 \end{array}\right\} 100$$

These are the numbers given by theory, and it is very much to be condemned that a chemist should publish that he established a formula by analysis, without giving the details of a single experiment, or stating how close to the theoretic numbers he had actually arrived. In fact I considered that the composition of this body required to be determined, as if it had been perfectly untried.

The following analyses were made to determine its composition :

A. 6.305 grammes of a quantity prepared by hot water, without boiling, gave, treated by proto-chloride of tin, 5.217 mercury, or 82.74 per cent.

B. 4,927 grammes of a quantity prepared by cold water, gave, when treated by proto-chloride of tin, 4.086 mercury, or 82.93 per cent.

C. 6.513 grammes of a different portion was dissolved in muriatic acid, and the liquid much diluted; it was then decomposed by sulphuretted hydrogen, and the sulphuret collected, carefully dried, and weighed with the filter. There was obtained 6.312 sulphuret, or 96.91 per cent., containing 83.7 mercury.

It is abundantly evident that this salt contains some water as constitutional, for when heated, it always yields, in addition to the red fumes, a dew of liquid nitric acid. Assuming, therefore, the nitric acid to exist in the salt combined with an equivalent of water, we obtain the formula $NO_5 . HO + 2\underline{Hg}O$, which gives

$2\underline{Hg}$	=	405.60	83.67
$2O$	=	16.00	3.30
NO_5	=	54.14	11.17
HO	=	9.00	1.86
		484.74	100.00

and which is abundantly confirmed by the reactions of the body and by the quantity of mercury, which analysis indicated it to contain.

When a solution of proto-nitrate of mercury has been kept for a long time, there are frequently deposited in it a fine lemon-yellow crystalline salt, of great brilliancy. I have never seen the crystals larger than pins' heads, and they have been always too closely aggregated to allow of an accurate determination of their form. They react, in every respect, similarly to the powder just described, and their composition was determined by the following analysis :

6.257 grammes were dissolved in muriatic acid, and the solution having been considerably diluted, was treated by sulphuretted hydrogen. There was obtained 6.038 of sulphuret, being 96.5 per cent., containing 83.28 of mercury. Hence the formation of these crystals is evidently owing to the very gradual deposition of the basic salt from an acid liquor, and they are of the same nature as the powder rapidly prepared.

It will be seen that in this basic salt the law of replacement of water by metallic oxide holds, although the absolute number of atoms is quite different. It was found that the first crystallized nitrate of the black oxide had for its formula

$$\underline{Hg}O \,.\, NO_5 + 2HO;$$

and the yellow basic salt is now proved to be

$$HO \,.\, NO_5 + 2\underline{Hg}O.$$

Moreover the second crystallized salt was shown to be, from Mitscherlich's analyses, as well as my own,

$$2NO_5 + 3\underline{Hg}O + 3HO =$$
$$\{\underline{Hg}O \,.\, NO_5 + 2HO\} + \{HO \,.\, NO_5 + 2\underline{Hg}O\}.$$

Hence, as was before alluded to, there is great reason to suppose the second crystallized proto-nitrate to be a double salt, consisting of the first and of the yellow basic salt, united in the proportion of an equivalent of each.

It would be an exceedingly interesting point to determine whether the three salts thus found to be generated by the replacement of successive equivalents of water by metallic oxide, and *vice versâ*, possess any simple crystallographic relation to one another. It would be highly important to determine, if the elements thus replacing one another influence the crystalline form of the salt, for if the metallic oxide which replaces water belong to the same isomorphous family, there should exist identity of form amongst those salts, provided the sum of the number of equivalents of water and metallic oxide remains the same.

If the yellow subnitrate be boiled with much water, in successive portions, it becomes grey, but that alteration is always accompanied by the separation of metallic mercury and the formation of pernitrate. Likewise, if potash be added to the yellow subnitrate it becomes grey, but there is produced a mixture of black oxide and unaltered salt. Thus no positive limit can be found indicating the existence of a blackish or grey sub-protonitrate of really definite composition, and I consider that Donovan and Grouvelle, who had asserted its existence, had been misled by the properties of a mixture of black oxide or of mercury with the subnitrate just described. Indeed Grouvelle, in his paper on the Basic and Acid Nitrates, does not mention this grey subnitrate at all; but Soubeiran, in discussing the composition of Hannehman's soluble mercury, asserts that it contains the blackish subnitrate described by Grouvelle, of which he gives a formula with numbers, which, by typographical errors, is rendered quite unintelligible, and I have never been able to meet a notice of it elsewhere. It shall be shown, moreover, in the next article, that the nature of Hannehman's mercury is quite different, and hence that ground for supposing a grey sub-protonitrate to exist can no longer hold.

I therefore conclude that there exists but one basic nitrate of the black oxide of mercury, that which may be obtained as a lemon-yellow powder, or in minute crystals of the same colour, and whose formula is $\mathrm{HO.NO_5} + 2\,\underline{\mathrm{Hg}}\mathrm{O}$.

VII. On the Ammoniacal Subnitrate of the Black Oxide of Mercury.

The study of the reaction of water of ammonia on the protonitrate of mercury presents great difficulties, in consequence of the facility with which the most important products of it are liable to change, and the consequent admixture of substances, which have their origin in the secondary decompositions of those at first formed; hence we find very irreconcileable statements put forward as to the nature of the black powder, which is the more immediate product of this action, by one chemist it being looked on as a mere oxide, by another as a subnitrate, whilst the analyses of George Mitscherlich, to whose accuracy I have had occasion so often to bear witness, showed that it did really contain ammonia and nitric acid among its elements. I am inclined to believe that Soubeiran himself now admits the incorrectness of his former statements, since in his Nouveau Traité de Pharmacie, he adopts the results of Mitscherlich, without at all adverting to the conclusions which he had advanced in his own paper on the subject.

When, to a solution of protonitrate of mercury, there is added water of ammonia, the precipitate, which at first is of a velvety black colour, gradually changes, passing through various shades of grey, until it becomes nearly white, and its state of aggregation varies in a similar manner: the portions first formed are heavy, and rapidly deposit, but according as the colour becomes lighter, it remains long suspended, at least the whitish portion, whilst a heavy grey powder falls more quickly down.

Having satisfied myself, by treating portions of these precipitates, of various shades of black and grey, with sulphuretted hydrogen, that the liquor contained, after separation of the quicksilver as sulphuret, nitrate of ammonia neutral, proving that an equal number of equivalents of nitric acid and ammonia were present in the precipitate; and having found, moreover, so great difficulty in decomposing the last portions as to render this method unavailable in obtaining a quantitative result, I resolved to examine minutely the influence which the variations in shade had on the quantity of mercury which the precipitate might contain; a result which very simple considerations will show, to lead to a complete knowledge of the nature of the body under examination.

A dilute solution of pure protonitrate of mercury was taken, and there was added to it a quantity of weak water of ammonia, about one-fourth of what would suffice for its complete decomposition. A considerable mass of a fine glossy black powder fell, which was collected on a filter, washed carefully, and dried at a temperature not exceeding 100° F. To the liquor separated from this first portion was added another quantity of water of ammonia, and thus another portion of precipitate obtained, differing but very little in shade, from the first; this having been likewise collected, the liquor was treated by a third quantity of water of ammonia, by which a precipitate was produced of a dark grey colour; after this had been removed, the remaining liquor was completely decomposed by an excess of water of ammonia, and thus a precipitate of a grey colour was obtained.

There had been thus collected, from the one solution of protonitrate, four portions of precipitate, which had gradually become lighter in colour according as the quantity of ammonia added had increased; numbering them in the order in which they had been prepared, they were subjected to analysis:

A. 7.748 of No. 1, dissolved in muriatic acid, gave, by proto-chloride of tin, 6.374 of mercury, or 82.27 per cent.

B. 9.456 of No. 1 gave, treated in a similar manner, 7.791 mercury, or 82.39 per cent.

C. 6.403 of No. 2, dissolved in muriatic acid, and decomposed by proto-chloride of tin, gave 5.410 mercury, or 84.49 per cent.

D. 7.093 of No. 3 gave, by proto-chloride of tin, 6.141 of mercury, or 86.7 per cent.

E. 7.943 of No. 4 gave, similarly treated, 7.067 mercury, or 88.97 per cent.

These results, tabulated, are:

Order of Formation.	Colour.	Mercury in 100.
1	Fine black.	82.27, 82.39
2	Greyish black.	84.49
3	Deep grey.	86.70
4	Grey.	88.97

The result of Mitscherlich's analysis are shown here, in order to understand how far his numbers are reconcileable with mine; he obtained

$$\left.\begin{array}{lcr} \text{Mercury} & = & 85.57 \\ \text{Ammonia} & = & 2.46 \\ \text{Oxygen} & = & 3.38 \\ \text{Nitric acid} & = & 7.32 \end{array}\right\} 98.73.$$

Hence he deduced the formula $NO_5 . NH_3 + 3\,\underline{Hg}O$, which should give

Mercury	=	86.46	100.00
Ammonia	=	2.43	
Oxygen	=	3.43	
Nitric acid	=	7.68	

In any ordinary case, where the error, unavoidable in manipulation, and to which the collection of mercury in the metallic form by proto-chloride of tin, is peculiarly liable, should necessarily tend to diminish the quantities obtained, and consequently reduce the experimental, below the theoretical numbers, his analysis should be considered as completely establishing the formula; but here, there are other circumstances which require to be taken into account, and which will lead us to an opposite conclusion.

It is evident that in the preparation of Hannehman's soluble mercury, there is a tendency to error from the intermixture of a greyish material, and where the whole, or nearly the whole of the solution has been precipitated at once, this intermixture is unavoidable; hence it is only the first portions that can be obtained of the fine black colour which characterizes the pure substance. Now it has been fully proved, that according as the decomposition proceeds, the quantity of mercury in 100 increases in proportion as the colour becomes less deep; and hence the error, in estimating the composition of this body, must be opposite in direction to what generally occurs, and must tend to render the proportion of quicksilver above the truth. Thus, the result of Mitscherlich's theory is almost precisely that obtained in my analysis of specimen No. 3, which was not black, but dark grey; and Mitscherlich himself indicates the powder which he analyzed as grey; he says, "Nach dem Trocknen ist die Farbe des Pulvers grau und es darf sich kein metallisches quecksilber mechanisch herausdrucken lassen;" and also, if the solution which I employed had been precipitated all at once, there should have been a dark grey precipitate, and its composition should have been the mean of the composition of the powders given by the four equal and successive additions of water of ammonia, which average would almost coincide with the result which Mitscherlich obtained. So marked is the production of this whitish matter, that Soubeiran collected and analyzed it, and concluded from his results that it was an ammonia sub-protonitrate with the formula $NO_5 . NH_3 + 4\,\underline{Hg}O$. He now appears to have tacitly abandoned this opinion, and properly, for there is no doubt but that this white powder is a compound of red oxide, and is one or other of the bodies which have been already described in this paper. When treated with iodide of potassium it gives a reddish yellow powder, and it dissolves gently in muriatic acid, without the disengagement of any red fumes indicating a transition to a higher degree of oxydation. At the same time that this white per-compound is formed there is always some metallic mercury set free, which can generally be recognized in the grey specimens by using a lens, but the quantity is seldom so large as to allow of its mechanical separation by the application of pressure only.

From all these circumstances it is evident, that the specimens of Hannehman's soluble mercury, which are of the finest black in colour, are generated under the circumstances most favourable to their perfect purity. And as all the chances of error, except that of analysis, tend to increase the value of mercury, it results, that where the error of manipulation affects all equally, the lowest estimate should be that nearest to the truth. Hence I feel justified in assuming, with some confidence, that the numbers 82.27 and 82.39 are those by which the true formula may be established, and we must therefore consider Hannehman's soluble mercury to be the ammonia sub-nitrate,

$$NH_3 . NO_5 + 2\underline{Hg}O,$$

which should give 82.29 mercury per cent., and evidently corresponds to the yellow subnitrate formed by water, which has been proved to be

$$HO . NO_5 + 2\underline{Hg}O.$$

Note.—It has been very gratifying to me to find that Ullgren, who undertook, under the direction of Berzelius, to control the analyses contained in my first memoir on the Ammonia Compounds, has verified, even to the most minute point, all the results which I then brought forward. I did not receive the Jahresbericht for 1837, containing Berzelius's observations, until this first part of the present memoir had been partly printed, and hence could not earlier introduce any note of the suggestions which he makes. In Germany or Sweden it will not be necessary to adopt the word *amidogene*, as the word *amide* harmonizes better with chlor. cyan. iod. and others; but in English and French it is preferable that there should be a termination, as in cyanogene and oxygene, the final *ide* being in these languanges restricted to binary compounds. I shall, however, for the future adopt his terms of *amidides* and *amidurets*, as I consider them still more expressive of the nature of the bodies, and more directly formed from *amidogene* than the word *amides*.

PART II.

ON THE AMMONIACAL COMPOUNDS OF COPPER AND ZINC, AND ON THE BASIC CHLORIDES AND SULPHATES OF THOSE METALS.

In developing the real nature of the series of quicksilver combinations which contain ammonia or its elements, it was found, that the quantity of the metal present, from its large equivalent number, preponderated so considerably over that of the other constituents of the various bodies analyzed, as to render the absolute exclusion of all theoretical views but that ultimately found correct, extremely difficult, and it was consequently my object, from the commencement, to re-examine in detail the ammonia compounds of certain metals with smaller atomic weights, in order, by an accumulation of numerical facts, to lay the foundation for a true theory of this class of combinations.

The group of metals, the compounds of which are discussed in the present section, is one exceedingly natural, and possessed of characters, particularly in relation to ammonia, which, when compared with those exhibited by quicksilver, should lead the chemist to expect the most remarkable results. Whilst the precipitates given by quicksilver solutions with ammonia are insoluble in an excess of the precipitant, those given by the metals now to be examined easily redissolve, and the peculiar character of the zinc compounds redissolving in an excess of the fixed alcalies, presents a point of contact, the study of which must be of the highest interest.

It will be found that I have connected with the analyses of the ammonia compounds, the examination of a number of basic salts, and of other substances which do not contain ammonia. Generally speaking, I was obliged to occupy myself with these bodies, in order to elucidate difficult passages in the history of the ammonia compounds, and though I have often apparently wandered from my way for the purpose of obtaining either a more elevated point of view, or a more extensive basis for analogical deductions, yet as the discovery of such bodies will be found, I trust, to present so many new facts in science, the proofs of their existence and composition will be given in this memoir, whilst I shall avoid as much as possible entering into any speculations concerning their real nature, as the views to which I have been led by these and previous investigations will require to be developed in a distinct section.

I. OF THE AMMONIACAL SULPHATE OF COPPER.

The composition of this body has been given by Berzelius, and I have found his result to be rigidly correct; I shall therefore not bring forward any details of my own analyses, but assume as true the formula

$$SO_3 . CuO + 2NH_3 + HO.$$

This salt crystallizes in right-rhombic prisms, which are complex macles, and I have not been able to determine the form really belonging to it. The crystal would appear to be produced by a number of rhomboidal plates, uniting at the edges, and leaving very often the centre hollow, but destitute of any other definite cleavage or direction.

When we consider the manner in which this salt is formed, we cannot look upon the oxide of copper as being united with the sulphuric acid. On adding water of ammonia to a solution of sulphate of copper, the action consists in the gradual separation of more and more sulphuric acid from the copper, and when, by an excess of alkali, the precipitate is redissolved, there is nothing in the reaction tending to make the oxide of copper go back again, but rather the reverse. Hence I will apply to this body the formula

$$(NH_3 . HO)\ SO_3 + (NH_3 . CuO);$$

that is, I consider it as being sulphate of ammonia, with which is united oxide of copper and as much more ammonia.

When this substance is exposed to the heat of an oil-bath, or of a carefully regulated spirit-lamp, it gives out ammonia and water, and if the heat be not carried beyond 300°, there remains a fine apple-green powder. When this powder is further heated the result varies according to the manner in which the heat is applied; if rapidly, there is given out ammonia and sulphate of ammonia, whilst sulphate, with oxide and suboxide of copper, remain behind; but if slowly, and that it be not carried beyond 500° F., the remainder of the ammonia can be gotten rid of, and sulphate of copper quite pure will remain behind, there being no water disengaged in this latter period of the process.

To determine the exact nature of this decomposition, the following experiments were made :

A. 1.969 grammes of crystals were reduced to fine powder, and heated, until water ceased to be given off. It was in the state of a fine green powder, which weighed 1.545, or 78.47 per cent.

B. 4.921 of finely powdered crystals were heated in a precisely similar manner; there remained 3.854, or 78.32 per cent.

C. 5.042 grammes treated similarly, gave 3.921, or 77.77 per cent.

D. 2.991 grammes were heated very cautiously, until all ammonia and water were expelled; a mere trace of sulphate of ammonia had formed, and there remained 1.947 of sulphate of copper, or 65.1 per cent., which redissolved almost totally in water.

The theoretical composition of the ammonia sulphate is:

SO_3	=	40.16	32.58
cuO	=	39.60	32.22
$2NH_3$	=	34.28	27.89
HO	=	9.00	7.31
		123.04	100.00

which by heat evidently breaks up into

$cuO.SO_3$	=	64.80	and	HO	=	7.31
NH_3	=	13.95		NH_3	=	13.94
		78.75				21.25

Thus it is demonstrated by experiment, that by the first action of heat all the water of the ammonia sulphate is expelled with half of the ammonia, and there remains the green powder, consisting of sulphate of copper and one equivalent of ammonia, which last, by a further application of the heat, may be driven off. I endeavoured by a cautious application of heat to separate the water without losing the ammonia, but found it impossible to effect it. In this case, therefore, the copper does not exist as amidide, but on referring to the formula

$$(NH_3 . HO)\ SO_3 + (NH_3 . cuO)$$

it is evident that the sulphuric acid is inserted between two equivalent groups, which are related to one another, through the replacement of hydrogen by copper; the acid had been in the crystals more immediately united with that which represents oxide of ammonium, but on the application of heat, the previous affinities were subverted, and, the acid remaining in union with the group of more permanent constitution, the elements of the ammonia and water are set free, the formula of the green powder being

$$(NH_3 . cuO) . SO_3.$$

Graham had already pointed out, that when ammoniacal gas is passed over sulphate of copper at a high temperature, but half an equivalent is absorbed, and hence he hazarded the idea, that the resulting compound might be analogous to an ordinary double sulphate, as

$$cuO . SO_3 + (NH_3 . cuO) SO_3$$

corresponding to

$$cuO . SO_3 + (NH_3 . HO) . SO_3.$$

This body can likewise be obtained when the action of the heat on the ammoniacal sulphate of copper is kept below 400° F.; there are given off three-fourths of the ammonia with the water, and there remains $2(SO_3 . cuO) + NH_3$.

It is well known that the sulphate of copper in the cold absorbs two and a half equivalents of ammonia, and the resulting body warmed loses two, corroborating fully the view originally struck out by Graham, and to which my results lend considerable support.

If the apple-green powder be exposed to the action of damp air it gradually becomes blue, from the absorption of water, but the process is very slow; if, on the other hand, the powder be moistened with a small quantity of water, much heat is evolved, and a full blue colour produced; if there be any water in excess it may be removed by cautious evaporation at a temperature below 100° F., but a large excess produces complete decomposition. To ascertain the quantity of water which in such case combines with the green powder, 2.820 grammes were very slightly moistened, and the excess of water removed by a temperature of 80°. The dry blue powder remaining weighed 3.605, or the green powder had taken 27.8 water per cent., corresponding to three equivalents, and hence the formula $NH_3 . cuO + SO_5$ becomes probably $(NH_3 . HO.) SO_3 + (cuO + 2HO)$.

By the results of the action of a large quantity of water on this green powder are formed sulphate of ammonia, the soluble ammoniacal sulphate of copper, and a bluish green basic sulphate not containing ammonia. In order to understand the reaction it was necessary to analyze this latter :

A. 3.710 grammes gave, dried, a brown powder 3.106, corresponding to 16.28 water per cent. ; this brown powder, dissolved in muriatic acid, and precipitated by chloride of barium, gave sulphate of barytes 1.766 grammes, corresponding to 16.36 of sulphuric acid per cent.

These proportions approximating to those of the common basic sulphate, another analysis was made with more complete accuracy :

B. 5.040 grammes of another specimen gave, dried, a brown powder 4.275, which, dissolved in muriatic acid, and precipitated by chloride of barium, gave 2.678 of sulphate of barytes ; hence the composition

		Theory.		Experiment. A.	Experiment. B.
SO_3	=	40.16	17.13	16.36	17.26
$4CuO$	=	158.40	67.52		
$4HO$	=	36.00	15.35	16.28	15.18
		234.56	100.00		

Thus the basic sulphate resulting from this reaction is the ordinary one, and the analyses given confirm the formula $SO_3 + 4CuO + 4HO$, which had been in some degree doubtful. The decomposition can be thus explained, the water being omitted for the sake of simplicity :

$$\begin{array}{lcr} 3(SO_3 . NH_3) & = & 3NH_3 + \qquad 3SO_3 \\ SO_3 + CuO + 2NH_3 & = & 2NH_3 + CuO + SO_3 \\ SO_3 + 4CuO & = & 4CuO + SO_3 \\ \hline 5(NH_3 . CuO)\, SO_3 & & 5NH_3 + 5CuO + 5SO_3 \end{array}$$

When this salt is heated it does not lose water until the temperature rises to above 300°, but then it loses all, and the brown powder, if exposed to the air, reabsorbs water slowly ; if moistened, it combines with the water rapidly, evolving heat, and regains its original proportion, and also its proper colour.

II. OF A NEW BASIC SULPHATE OF COPPER.

Having found, as in the preceding instance, that by the action of water on the ammoniacal compounds of the metals under examination, there was generated a series of basic salts, I became desirous of re-examining some of those already known, particularly in order to determine the function of the water which they constantly retain. For this purpose I prepared several portions of the sub-sulphate of copper, and I soon perceived, that, according to the quantity of alkali employed in the precipitation, where potash had been used, there were two distinct precipitates produced, the one of the bluish green generally described, the other of a clear grass green, resembling that of hydrated oxide of nickel. When ammonia was employed, the former alone was produced, and the formation of the latter I found to occur where the whole of the copper had been thrown down, but the liquor had not yet begun to react alkaline. It is singular that this basic sulphate had not been observed by any of those chemists who examined the common species. I found it in the first instance accidentally, but I have since seldom failed in preparing it completely pure.

It was analyzed as follows:

A. 7.124 grammes were dried until all traces of watery vapour ceased; there remained a brown powder 5.614, or 78.8 per cent. This was dissolved in muriatic acid, and precipitated by chloride of barium; there was obtained 1.851 of sulphate of barytes, indicating of sulphuric acid 8.94 in 100 of green powder.

B. 3.877 grammes were exposed to a temperature of 300° F. in an oil-bath, until it ceased to give off water, it then weighed 3.460. The oil-bath having been removed, the drying was completed by the spirit-lamp, at a temperature of about 500°, after which there remained 3.042. There had been thus driven off:

In first period	=	10.76 per cent.
In second period	=	10.52
Water in 100 of powder	=	21.28

The composition resulting is:

		Theory.		Experiment. A.	Experiment. B.
SO_3	=	40.16	8.83	8.94	
$8CuO$	=	316.80	68.00		
$12HO$	=	108.00	23.17	21.20	21.28
		464.96	100.00		

When the brown mass resulting from the desiccation of this salt is moistened, it evolves much heat, and combines with a large quantity of water, forming a green mass of a livelier colour than it originally possessed, and becoming always of something more than its former weight. The quantity of water with which it combines varies from 23 to 24 per cent., and hence I attribute the slight deficiency in water shown by analysis, to some of the chemically combined water having been expelled by the very moderate heat applied in drying the precipitate for analysis.

It will be remarked that by 300° F. exactly half of the water is expelled; hence there must be some difference in the degrees of affinity with which the two quantities are retained. From these considerations I am disposed to give to the formulæ for these basic sulphates the following form:

$$CuO \,.\, SO_3 \,.\, CuO + 6(CuO + 2HO),$$

or

$$CuO \,.\, SO_3 \,.\, CuO + 6CuO + 6HO + 6HO;$$

the second group of equivalents of water being expelled by a temperature lower than that necessary for the separation of the remainder.

Thompson had long since pointed out the existence of a basic sulphate of copper containing two equivalents of oxide, and this in its hydrated condition he states to retain two equivalents of water. When this is added to those above described, the series of basic salts follow from the neutral sulphate in the following order:

Real neutral sulphate	=	$Cu.O + SO_3$
Do. with saline water	=	$CuO \,.\, HO + SO_3$
First basic salt, dry	=	$CuO \,.\, CuO + SO_3$
Do. do. hydrated	=	$CuO \,.\, CuO + SO_3 + 2HO$.

Second basic salt, dry	=	$(cuo \,.\, cuo)\, so_3 + 2cuo$
Do. do. hydrated	=	$(cuo \,.\, cuo)\, so_3 + 2cuo + 4HO$
Third basic salt, dry	=	$(cuo \,.\, cuo)\, so_3 + 2cuo + 4cuo$
Do. do. hydrated	=	$(cuo \,.\, cuo)\, so_3 + 2cuo + 4cuo + 6HO + 6HO$

III. OF THE AMMONIACAL CHLORIDE OF COPPER, AND OF THE COMPOUNDS DERIVED FROM IT.

When water of ammonia is added to a solution of chloride of copper, the precipitate which is at first formed redissolves by an excess, and a purple liquid is produced. If this be evaporated there is deposited a bluish flocculent precipitate, and the liquid loses its fine purple colour, and becomes bluish green. If in this condition the solution be set aside to crystallize, the double chloride of copper and ammonium is deposited, which Henry and Cap* have mistaken for the ammonia-chloride, and they have consequently assigned to the latter body a constitution belonging to one of a totally different nature, and which had resulted from its decomposition.

In order to obtain the ammonia-chloride pure and crystallized, a solution of chloride of copper must be taken, nearly saturated when hot, and a stream of ammoniacal gas passed through it, until the precipitate which first appears has been totally redissolved: the mass is kept almost boiling by the heat evolved in the condensation of the ammoniacal gas, and when set aside to cool, the ammonia chloride is deposited in small, but well-marked, octohedrons, or square prisms with pyramidal summits, of a deep blue colour. These crystals must be dried with great care between folds of filtering paper, without the aid of heat, and in a room free from any acid fumes; even with the greatest caution it is difficult to prevent the outer portion of the mass from acquiring a green tinge, arising from loss of ammonia, which will affect in a corresponding degree the analytical results.

Although the existence of this body had been generally admitted by chemists, yet no analysis of it had appeared until that by Henry and Cap; and as it is necessary to disprove their erroneous statement, I will detail those which I performed.

* Journal de Pharmacie, December, 1837.

A. 5.823 grammes of crystals, slightly tarnished, were dissolved in dilute muriatic acid, and treated by sulphuretted hydrogen, until the copper was completely thrown down. The sulphuret of copper was then separated by the filter, and the liquid, with the washings, evaporated in a water-bath. There were obtained 5.211 of sal ammoniac, corresponding to 89.49 per cent., containing 28.83 of ammonia.

B. 4.700 grammes of crystals, dissolved in muriatic acid, and precipitated by caustic potash in excess, gave oxide of copper 1.692, or 36 per cent., containing 28.73 per cent. of copper.

C. 3.594 grammes of crystals were dissolved in an excess of pure nitric acid, and precipitated by nitrate of silver; the chloride of silver formed, collected, well washed, dried, and fused, weighed 4.672, or 130.5 per cent., containing 32.19 chlorine.

Hence there is the formula $cucl + 2NH_3 + HO$, giving

	Theory.			Experiment.
cl =	35.42	32.11		32.19
cu =	31.60	28.65		28.73
$2NH_3$ =	34.28	31.08		28.83
HO =	9.00	8.16	and loss	10.25
	110.30	100.00		100.00

There occurred here a loss of ammonia, which evidently arose from the surface of the crystals having become a little tarnished, and likewise from that which takes place in all evaporations of ammoniacal solutions. Nevertheless, the theoretical and experimental results agree so closely, that there cannot be any doubt of the truth of the formula adopted; it resembles in every respect that of the ammonia-sulphate, and in accordance with the principles explained in the description of that substance, I consider the chlorine to exist in the crystals as sal ammoniac, and the rational formula to be

$$NH_3 . Hcl + cuO.NH_3.$$

When these crystals are exposed to heat, they melt, and ammonia, with watery vapour, is disengaged; I could not succeed in eliminating water without losing ammonia at the same time; in that respect therefore it resembles the ammonia-sulphate. By a temperature of 300° all the oxygen is separated as water, together with one-half of the ammonia, and there remains a fine apple-green powder, resembling very much that from the sulphate, and containing the remainder of the ammonia, with all the chlorine and the copper. Thus, 4.064 of the crystals were heated in an oil-bath, until the disengagement of water and ammonia had ceased; the green powder remaining weighed 3.109, or 76.5 per cent. According to theory, the residue $cl.cu.NH_3$ should weigh 76.3 per cent.; $OH.NH_3$ having been expelled. When this body $cl.cuNH_3$ is exposed to a higher temperature it is decomposed, sal ammoniac sublimes, and sub-chloride of copper remains; there are likewise azote and ammonia given off. The ammonia is retained by so powerful an affinity, that it cannot be expelled by any temperature, without the substance being totally decomposed.

The existence of this body was noticed by Graham, as resulting from the absorption of ammonia by chloride of copper at a high temperature. At ordinary temperatures chloride of copper absorbs three equivalents of ammonia, of which two are easily expelled, but the third is retained more powerfully, and constitutes with the chloride the body just described. We may therefore consider the ammonia-chlorides, formed by water, and by dry ammonia, as corresponding compounds; thus,

$$NH_3\,cu\,cl + NH_3.\,HO.$$
$$NH_3\,cu\,cl + NH_3.\,NH_3;$$

an equivalent of water in the one replacing an equivalent of ammonia in the other, and both, when heated, giving the body $NH_3.cu.cl$, by losing respectively $NH_3.HO$ and $2NH_3$.

IV. OF A NEW BASIC CHLORIDE OF COPPER.

When the body $cl.cuNH_3$ is treated by water it is decomposed; there is dissolved the ammonia-chloride of copper just described, and a quantity of sal ammoniac, and a bluish green powder remains, insoluble in water, and not containing ammonia. When heated it gives off water, and becomes brown; but exposed to the air, it gradually regains a certain quantity of water. Its analysis was effected as follows:

A. 1.901 grammes, dried over a spirit-lamp, gave a chocolate brown powder, which weighed 1.522 grammes, corresponding to 80.06 per cent. This 1.522

were dissolved in nitric acid, and precipitated by nitrate of silver. The chloride of silver produced weighed 0.964, or 50.71 per cent. for the green powder, and containing 12.51 of chlorine.

B. 2.678 grammes, dried over the spirit-lamp, gave, of brown powder, 2.143 or 80.02 per cent., which was boiled in a strong solution of caustic potash, and the oxide of copper washed, until the liquors were perfectly free from traces of free alkali; there was obtained 1.891 of oxide of copper, or 70.61 in 100 of green powder, and containing 56.31 of metal.

Hence the formula $cucl + 4\,cuo + 6\,\text{HO}$ results, which gives

		Theory.		Experiment. A.	Experiment. B.
cl	=	35.42	12.68	12.51	
$5\,cu$	=	158.00	56.55		56.31
$4\,\text{O}$	=	32.00	11.45		
$6\,\text{HO}$	=	54.00	19.32	19.94	19.98
		279.42	100.00		

$$\left.\begin{array}{l} cucl + 4\,cuo \\ 4\,cl\text{H} + 4\,\text{NH}_3 \\ cucl + 2\,\text{NH}_3 \end{array}\right\} = 6.\,cucl.\text{NH}_3 + 4.\text{HO}.$$

The coincidence is quite satisfactory.

This oxychloride differs therefore from that analyzed by Berzelius, in containing, to the same quantity of chloride of copper, one atom more of oxide of copper, and two more of water. The relation between this and the ordinary oxychloride, can be very well shown, by arranging the formulæ of the two in the following manner:

$$\begin{array}{lcl} \text{Common oxychloride} & = & cucl.\,cuo + 2(cuo + 2\,\text{HO}) \\ \text{New oxychloride} & = & cucl.\,cuo + 3(cuo + 2\,\text{HO}) \end{array}$$

V. OF A SECOND NEW BASIC CHLORIDE OF COPPER.

Having prepared, during the course of these researches, a great number of specimens of Brunswick green, I remarked that some, which had been produced by a less perfect precipitation by the alkali employed, were of a much less brilliant colour, and differed markedly in their aspect from the ordinary oxychloride. I consequently submitted these specimens to an examination, from which it results, that there may be prepared, by the action of a base on an excess of solution of chloride of copper, two oxychlorides; that generally formed being the common Brunswick green, with the formula $cucl + 3cuo + 4HO$, but that when a still smaller quantity of base is employed a different substance is produced.

This new oxychloride resembles remarkably in its aspect that last noticed and the sub-sulphate, but can be at once distinguished from Brunswick green by its pale colour; heated it gives out water, and becomes first brown, and leaves finally a black powder. When this powder is moistened it slakes, evolving great heat, and becoming of a very brilliant green colour, brighter than that of Brunswick green. By heat the water reabsorbed may be again expelled, and so repeatedly, without total decomposition taking place.

The analysis of this oxychloride was conducted in the following manner:

A. 12.390 grammes, dried over the spirit-lamp, gave a black powder, weighing 9.725, or 78.49 per cent. These 9.725 were moistened with water, and allowed to assume throughout the rich green colour; the excess of water, which was very slight, was removed by a temperature of 100° F., when the green powder was found to weigh 11.670, having absorbed 16.78 per cent. of water.

B. 5.155 grammes of the green powder, thus formed, were kept at a temperature of 280°, until it ceased to give out watery vapour; it had become chocolate brown, and weighed 4.584. It had lost therefore 11.08 per cent. of the water which it contained.

C. 6.185 grammes of the same green powder, dried at 500° F., gave 5.144 of black powder, or 83.17 per cent.; hence it had lost 16.83 water.

D. The 4.584 of B was dissolved in dilute nitric acid, and precipitated by nitrate of silver, the chloride was collected, washed, and dried, it then weighed 4.099, corresponding to 79.51 per cent., and containing 20.61 of chlorine in the bright green condition.

E. The 5.144 of C was dissolved in dilute muriatic acid, and treated with boiling solution of potash, the oxide of copper which separated was well washed, and collected on a filter, and subsequently ignited. There was obtained 4.112, corresponding to 79.93 per cent., and containing 63.78 of copper.

It consequently follows, that the dry oxychloride is capable of uniting with water in three different proportions; thus,

In pale green powder	100 oxychloride	take	27.4 water.
In bright green do.	100	do.	20.2
In brown do.	100	do.	6.9

But 6.9, 20.2, and 27.4 are nearly as 1, 3, and 4.

From C, D, and E it results, that the dry oxychloride has the composition

	Theory.		Experiment.
cl =	35.42	24.22	23.59
$3cu$ =	94.80	64.84	63.78
$2o$ =	16.00	10.94	10.77
	146.22	100.00	98.14

From the proportion of water, it is evident that the dry oxychloride combines with one, three, and four equivalents in the three different conditions in which it exists, and that hence there are the formulæ

1. $cucl + 2cuo.$
2. $cucl + 2cuo + ho.$
3. $cucl + 2cuo + 3ho.$
4. $cucl + 2cuo + 4ho.$

The discovery of this body leads us to some very interesting relations, in this class of substances; thus, this one is evidently the simplest oxychloride, being related to the crystallized hydrated chloride, as

$$cucl + 2cuo \text{ to } cucl + 2ho;$$

and the first atom of water being so strongly retained, points out the passage through

$$cucl + ho + 2cuo$$

to

$$cucl + cuo + 2cuo,$$

the ordinary oxychloride deprived of water, but both combining with additional quantities of water, and acquiring the brilliant green colour by which they are respectively characterized; and the condition in which this salt retains most water, gives to it a composition which brings to mind the crystallized hydrates of many chlorides of the same class, as

$$cucl + 2\,cuo + 4\,HO.$$

compared with

$$ca.cl + 2\,HO + 4\,HO.$$

and

$$Mgcl + 2\,HO + 4\,HO.$$

I will have occasion to recur to these bodies when speaking of their analogues among the compounds of zinc.

The other new oxychloride of copper, in its dry condition, is analogous to the chlorides which crystallize with four atoms of water, as iron and manganese.

When this oxychloride, in a dry condition, is exposed to a current of ammoniacal gas, an absorption takes place, with the evolution of some heat; but although the current may be continued long after the mass shall have become cold, yet no alteration of colour occurs, the mass remaining brown. If the ammonia be passed over the oxychloride in its hydrated condition, it becomes blue, water is given out, and the whole is evidently decomposed; and if the brown mass be wetted, there is formed a hydrated oxychloride and a blue liquor, showing total decomposition.

Dry ammonia, acting on dry oxychloride, gave the following results:

I. 4.801 of oxychloride absorbed 0.504 ammonia, or 10.4 per cent.

II. 3.970 of oxychloride absorbed 0.436 ammonia, or 11.1 per cent.

These numbers give, for the proportion absorbed, almost exactly one equivalent; and the resulting brown mass has evidently the formula

$$cucl + 2\,cuo + NH_3.$$

According to which 100 should have absorbed 11.8 of ammoniacal gas.

Now putting $NH_3 = HAd$, the relation of this body with those last noticed becomes very remarkable, as we must contemplate the series

$$cucl + 2\,cuo + cuo$$
$$cucl + 2\,cuo + HO$$
$$cucl + 2\,cuo + HAd.$$

in which *cu* and H, O and A*d* mutually replace each other.

VI. OF THE AMMONIA-OXIDE OF COPPER.

I had examined very frequently, and under a great variety of circumstances, the precipitates which are produced by the action of ammonia on solutions of the sulphate, nitrate, and chloride of copper, in order to determine whether compounds similar to those generated under like circumstances with solutions of the quicksilver salts, could be produced. In all such cases, I found the precipitates to be basic salts following certain laws of composition, and not containing ammonia as an element. Indeed a similar result might have been anticipated from what has been already shown in this paper; namely, that the insoluble ammonia-copper compounds are all decomposed by water, giving soluble ammonia compounds and a basic salt destitute of ammonia in its composition.

However, on one occasion, on treating a solution of chloride of copper with ammonia, I obtained a precipitate of a remarkably fine blue colour, approximating to that of the hydrated oxide, or of the refiner's verditer. In the one operation I obtained a sufficient quantity of it for examination, and did not since study the exact circumstances favourable to its production, the specimen I had procured being sufficient to supply my wants, but proceeded at once to determine its properties and composition.

This blue powder is not affected by repeated washings, to which I subjected it, suspecting that its ammoniacal constituent might result from sal ammoniac being attached. It may be heated to 300° F. without being changed, but above that temperature it is rapidly decomposed with a hissing noise. It yields much ammonia, azote, and a large quantity of water, and the residue is red coloured, consisting of a mixture of sub-oxide of copper and of copper in the metallic state. There is no sublimate of sal ammoniac.

Dissolved in dilute nitric acid, this powder gives no precipitate with nitrate of silver. Its elements are therefore ammonia, water, and oxide of copper. The following quantitative analysis was made:

A. 3.410 grammes were dissolved in muriatic acid, and the solution decomposed by sulphuretted hydrogen. The sulphuret of copper having been removed, the liquor and washings were evaporated to perfect dryness in a water-bath, and sal ammoniac, weighing 1.634, was obtained, corresponding to 15.70 per cent. of ammonia.

B. 3.752 grammes were dissolved in dilute muriatic acid, and decomposed by boiling with caustic potash. The oxide of copper precipitated was collected and burned with the filter, it weighed 2.146 or 57.19 per cent.

The difference is evidently the water, and hence the formula

$$3\,cuO + 2\,NH_3 + 6\,HO,$$

which gives

		Theory.		Experiment.
$3\,cuO$	=	118.80	57.37	57.19
$2\,NH_3$	=	34.28	16.55	15.70
$6\,HO$	=	54.00	26.08	27.11
		207.08	100.00	100.00

This result comes sufficiently close to allow of the formula being adopted, but I will not now attempt to arrange it after any theoretical idea. The substance evidently belongs to the same class as the fulminating oxides of silver and mercury, but is still inferior in detonating power even to the latter.

VII. OF THE AMMONIACAL NITRATE OF COPPER.

This salt, the existence and some characters of which have been already noticed by chemists, may be prepared very simply by the same process as that described under the head of the Ammonia-Chloride of Copper, substituting nitrate for the chloride. It crystallizes in a confused mass of minute octohedrons, whose form is with difficulty ascertained. It dissolves easily in water, and on the addition of an acid it yields the ordinary basic nitrate of copper.

When heated, this salt is decomposed in a very remarkable manner: traces of ammonia are evolved, but no water if the salt had been completely dried; black points (oxide of copper) make their appearance, the salt fuses, and if the heat be continued, suddenly explodes with a hissing noise, and the formation of a great cloud of gaseous matter, whilst the inside of the vessel remains lined with oxide of copper. Several attempts were made to manage the decomposition, so as to determine the quantity of the oxide left behind, but without avail; even when the powdered salt was covered in a platinum crucible, with strong nitric or muriatic acid, and then heated, the acid boiled away, and the residual salt underwent its explosive change, as if no such means had been applied.

No quantitative analysis of this body has been recorded, and as from the remarkable circumstances of its decomposition by heat, it is of great importance that its composition should be accurately known, the following analysis was made:

5.982 grammes were introduced into a globe with a strong solution of potash; from this globe there passed a bent tube, dipping into a tall jar containing water with muriatic acid. The mass in the globe was boiled until all the ammonia had been set free, and more than one-half of the liquor had distilled over. The fluid in the jar was then carefully evaporated in a water-bath to dryness, and the sal ammoniac obtained was found to weigh 4.717 or 78.85 per cent., containing 25.23 of ammonia.

The liquor remaining in the globe was diluted with water, and, when cold, filtered; the oxide of copper remaining weighed 1.856 grammes, corresponding to 31.03 per cent.

These numbers give the formula $cuO.NO_5 + 2NH_3$, by which there should have been obtained

	Theory.		Experiment.
cuO =	39.60	30.94	31.03
NO_5 =	54.14	42.28	
$2NH_3$ =	34.28	26.78	25.23
	128.02	100.00	

Since during the process for the formation of the ammonia-nitrate, the first stage consists in the production of the ordinary subnitrate of copper, the nitric acid in the ultimate product must unavoidably be considered as united with ammonia, and hence the above empirical formula must, in assuming a rational form, become

$$(NH_3.HO)\,NO_5 + cuNH_2,$$

from which it follows, that the copper in this compound is united with amidogene.

It is now easy to explain the various circumstances in which this body differs from the similarly constituted ammonia-chloride and sulphate just described. By the application of heat, the evolution of ammonia and water cannot occur, since the second group $cuNH_2$ is not of a nature precisely to replace it. The trace of ammonia whieh is evolved arising probably from a partial expulsion of HNH_2 by $CuNH_2$. The salt resists deeomposition almost eompletely, until the nitrate of ammonium melts, and commenees to be deeomposed, when the sudden burning of the amidogene and copper, in the oxygen of the nitrous oxide formed, gives rise to the explosive reaetion whieh distinguishes this body.

In order to plaee in a still elearer point of view the peculiar nature of this body, I shall refer briefly to some observations whieh I have made on the ammonia-nitrate of silver diseovered by George Mitscherlich. On analyzing it he obtained the formula $NO_5 + AgO + 2NH_3$, and I have verified his result, having obtained from it 52.46 of silver, whilst his formula indieates 52.83. This formula is evidently quite similar to that given by the ammonia-nitrate of eopper; and here also the aetion of the ammonia consists in the separation of the oxide of silver in the first stage and its solution afterwards, when the ammonia has been added in exeess. Giving to the formula, therefore, its true rational construction, it beeomes

$$(NH_3 . HO)\,NO_5 + AgNH_2,$$

and the propriety of this view is supported by a very eurious reaetion of this body, whieh George Mitseherlich does not appear to have observed.

When heated this salt fuses very readily, and gives out a mixture of azote and ammonia, whilst silver is separated in the metallic form, and by rolling about the fused mass in the tube, a mirror surfaee is produeed, as beautiful as that obtained by nitrate of silver with ammonia-aldehyd. When the tube eools, the melted mass solidifies, and is found to be nitrate of ammonia. This I eonsider to be a convineing proof of the existenee of an amidide of silver in this salt; its easy reduction, the simultaneous liberation of the elements of amidogene, and the nitrate of ammonium being set free, unaltered, if the heat be not raised too high, render the peeuliar nature of this body too remarkable to be mistaken. Now in the analogous eopper eompound, the amidide of copper is not so easily deeomposed, its elements remain united until the nitrate of ammonium begins to yield, and then a rapid combustion, alike of eopper and amidogene, takes place in the oxygen of the nitrous oxide formed.

Although the ammonia-eopper element of the ammonia-sulphate of eopper cannot be freed from water, yet in the silver salts, the ammonia-sulphate, and its congeners, which, in the hands of Eilard Mitscherlich, have beeome one of the most beautiful instances of isomorphism, crystallize without that equivalent of water, and the empirical formulæ

$$so_3 + Ago + 2NH_3$$
$$cro_3 + Ago + 2NH_3$$
$$seo_3 + Ago + 2NH_3$$

assume from the above principles the form

$$\left.\begin{array}{l} so_3 \\ cro_3 \\ seo_3 \end{array}\right\} + NH_3 . HO + AgNH_2 .$$

Of these I have re-examined only the sulphate, and that without observing any fact new in its history.

When chloride of silver is dissolved in water of ammonia, rhomboidal tables are produced, white and opaque, consisting of an ammonia-chloride ; they lose ammonia, however, immediately on being removed from the solution, and hence their quantitative analysis became impossible.

VIII. OF THE AMMONIA-CHLORIDE OF ZINC.

When water of ammonia is added to a solution of chloride of zinc, the white precipitate of basic chloride which is at first produced, soon redissolves, and a colourless liquor is obtained, from which, by evaporation at a moderate temperature, crystals may be obtained. These crystals, however, according to circumstances, present very different appearances, and possess quite different properties and composition, and hence the proper methods of obtaining each variety, in a state fit for accurate examination, must be noticed. The plan which I found most successful was to take a strong and hot solution of chloride of zinc in water, to pass into it a stream of gaseous ammonia, until the precipitate was completely redissolved, and filtering very rapidly, in order to separate any traces which might yet remain of turbidity from undissolved material, to allow the whole to cool. A substance, in very minute, but brilliant plates, of a peculiarly soft and talcy feel, and pearly lustre, is deposited, while the liquor cools ; but after it has cooled, then by further evaporation a completely different salt is formed, which crystallizes in stellated groups of square prisms of a brilliant vitreous lustre, and hard to the feel. These two salts I shall indicate as the tabular and the prismatic ammonia-chlorides.

To analyze the tabular ammonia-chloride of zinc, the following method was employed :

A. 3.374 grammes were dissolved in dilute nitric acid, and precipitated by nitrate of silver added in excess; the chloride of silver formed was collected, carefully washed, and dried; it weighed 4.295 grammes, equivalent to 127.3 per cent., containing 31.40 per cent. of chlorine.

From the circumstances of the preparation of this substance, it necessarily follows, that, as in the corresponding copper-salt, the number of atoms of metal is equal to, and that of the ammonia double that of the chlorine ; hence the above determination of the chlorine was fully sufficient to determine the composition of the whole. Thus the formula $zncl + 2NH_3 + HO$ gives

		Theory.		Experiment.
zn	=	32.30	29.10	
cl	=	35.42	31.89	31.40
$2NH_3$	=	34.28	30.90	
HO	=	9.00	8.11	
		111.00	100.00	

Thus the composition of this body corresponds in every particular to that of the ammonia-chloride of copper; and guided by similar considerations, I shall arrange its constituents according to theory, as

$$NH_3 . HCl + NH_3 . znO.$$

When this body is heated it gives out water and ammonia, and the result obtained confirms fully the analytical result above described. Thus,

3.739 of this tabular ammonia-chloride, heated to 300°, until all evolution of ammonia and of water had ceased, left a white powder, weighing 2.900, or 77.56 per cent.

In another experiment, 4.457 kept in a temperature of 300°, until the evolution of water and of ammonia had ceased, left 3.426 of white matter, corresponding to 76.87 per cent. But from theory there should be, supposing the reaction similar to what has been observed in the copper series,

zncl	=	60.99	and	NH_3	=	15.45
NH_3	=	15.45		HO	=	8.11
		76.44				23.56

By the loss of NH_3HO there is produced the substance $NH_3 . zncl$, which remains behind. When this powder is farther heated it fuses into a clear colourless, or very slightly yellow liquid, emitting ammonia; by cooling, this matter congeals into a mass like gum; it shall be examined more minutely a little farther on.

The form and external characters of the prismatic ammonia-chloride of zinc have been already given; its analysis was effected as follows:

A. 2.851 grammes were dissolved in dilute nitric acid, and precipitated by nitrate of silver; the chloride, collected, washed, and dried, weighed 4.550, or 160 per cent., containing 39.47 chlorine.

B. 3.540 grammes were dissolved in dilute muriatic acid, and precipitated by carbonate of soda; the precipitate was collected, and carefully washed, and having been dried, was ignited with its filter; the residual oxide of zinc, allowing for the ashes of the filter, weighed 1.573, or 44.43, containing 35.61 of metallic zinc.

Hence in this compound likewise, the zinc and chlorine are in the proportion of atom to atom; it contains likewise water and ammonia, and calculating from the formula $2(clzn) + 2NH_3 + HO$, there is found

		Theory.		Experiment.
2*cl*	=	70.84	39.64	39.47
2*zn*	=	64.60	36.14	35.61
$2NH_3$	=	34.28	19.18	24.92
HO	=	9.00	5.04	
		178.72	100.00	100.00

Thus this prismatic ammonia-chloride differs from the tabular salt in containing, united with the same quantity of ammonia and water, double the quantity of chloride of zinc, and it has evidently been produced by the dissipation during the evaporation of the liquors, of one-half of the ammonia and combined water which the tabular salt had contained. Hence the true nature of this salt may be best represented as a compound of chloride of zinc with the tabular salt, thus,

$$zncl + NH_3 . HCl + NH_3\, zno,$$

and recollecting the frequent replacements of water of crystallization by $NH_3 = HAd$, some remarkable relations present themselves, as

$$zncl + NH_3 . HCl + HAdHO,$$
$$zncl + NH_3 . HCl + 2HO;$$

and again,

$$cucl + NH_3HCl + 2HO.$$

Hence this prismatic salt assimilates itself very remarkably to the double chlorides of zinc, copper, and ammonium, with water of crystallization; a view which is additionally strengthened by the effects of heat upon this body.

When this ammonia-chloride is heated it emits watery vapour and ammonia, and fuses into a transparent mass, which resists a considerable temperature. This residue, on cooling, forms a mass like pale amber, having but little or no traces of crystalline arrangement, but fissured in every direction like starred glass. To determine the proportion of water and ammonia lost in this reaction, the following experiments were made.

A. 3.250 grammes of prismatic ammonia-chloride gave 2.758 of transparent gummy-looking mass, corresponding to 84.81 per cent.

B. 12.435 grammes gave, similarly treated, 10.748, or 86.47 per cent.

From these results the nature of the substance remaining may be very simply calculated: all the water is driven off, and as much ammonia as may be necessary to account for the weight lost; hence, there result

$zncl$	=	75.78	and	NH_3	=	9.59
NH_3	=	9.59		HO	=	5.04
		85.37				14.63

Hence it is evident that precisely the half of the ammonia is driven off with all the water, forming the elements of oxide of ammonium, and there remains the remainder of the ammonia, with the chloride of zinc, thus arranged:

$$zncl + (NH_3 . zn\, cl;)$$

wherein the body $NH_3 . zncl$, already noticed, is united with chloride of zinc, constituting an anhydrous double chloride, analogous to that of zinc and of ammonium or potassium.

When the body $NH_3 . zncl$ is heated by itself, it gradually loses ammonia, and fuses into the same gummy-looking substance; but the numerical results being similar to those already noticed, it is not necessary to occupy space with them, the more so, as the elimination of the ammonia, by itself, does not take place so clearly as where the portion to be separated is associated with the equivalent quantity of water.

This gummy body, when heated strongly, nearly to redness, boils, but does not emit ammonia; on the contrary, it volatilizes unchanged, and condenses in amber-looking drops, possessing all its original characters. If it be heated, however, with dry lime, there is an immediate and copious evolution of ammonia; when treated by water it is decomposed; there dissolves ammonia-chloride, probably in the prismatic form, and a white powder remains, which is an oxychloride of very remarkable constitution. The same oxychloride is produced by the action of water on the white powder $NH_3 . zncl$, and I shall consequently treat of the properties and composition of this oxychloride without further reference to which of these ammonia zinc-chlorides it had been obtained from.

IX. OF THE OXYCHLORIDE OF ZINC OBTAINED BY THE ACTION OF WATER ON $NH_3 zncl$ OR $NH_3 + 2zncl$.

The substance thus obtained is a very light milk-white powder, tasteless, and insoluble in water; when heated it gives out water, and if ignited, it yields some vapours of chloride of zinc, and is completely decomposed; water subsequently poured upon it, extracting some of the chloride of zinc, and leaving a still more basic combination. The quantity of water which this oxychloride retains is very variable, as a very slight difference in the temperature used in drying it may change, very considerably, the proportion of water with which it may be combined. A quantity prepared by acting with water on $NH_3 . zncl$, and dried at a temperature of about 180° F., gave the following result:

A. 2.404 grammes, dried, until all escape of watery vapour had ceased, gave 2.043 of residue, which had a greyish shade. These 2.043 were dissolved in dilute nitric acid, and precipitated by nitrate of silver; the chloride of silver, collected and dried, weighed 0.975, being 40.56 per cent., containing 10.01 of chlorine.

The quantity of water lost was 0.361, corresponding to 15.02 per cent.

But the chlorine being as chloride of zinc, and the remainder of the deficiency being oxide of zinc, the composition of the whole may be easily calculated, and there is found

$$\begin{aligned} cl.\,10.01 + zn\ \ 9.13 &= 19.14 = zncl \\ \mathrm{o}.\,13.07 + zn\ 52.77 &= 65.84 = zno \\ &\ \ \ 15.02 = \mathrm{HO} \\ &\ \ \ \overline{100.00} \end{aligned}$$

But $\frac{52.77}{9.13} = 5.78$. q. p. 6. And $\frac{10}{15} = \frac{35.42}{53.2}$, or 6HO. Hence the empirical formula is $zncl + 6\,zno + 6\,\mathrm{HO}$.

When this oxychloride is dried at the temperature of the air, it retains a much larger quantity of water, in fact nearly double as much, since quantities of the powder so prepared, gave, when dried, from 23.5 to 23 per cent. of water. To establish an accurate proportion, however, the following analysis was made:

B. 2.078 of the oxychloride, prepared by the action of water on the body $\mathrm{NH_3}.zncl$, and dried without exposure to heat, gave, when dried by the spirit-lamp, 1.590 of a greyish residue, corresponding to 76.51 per cent.; hence 23.49 water.

The residue was dissolved in nitric acid, and precipitated by nitrate of silver; the chloride of silver produced was collected, washed, and dried, when it weighed 0.690, or 33.21 per cent., containing 8.29 per cent. of chlorine.

The zinc being determined in the same manner as that before described, there results that to the same chloride and oxide of zinc, there were in this body united ten atoms of water in place of six; and hence the formulæ of these oxychlorides are:

	Dried at 212°.		Experiment.
cl =	35.42	9.74	10.01
$7\,zn$ =	226.10	62.20	
6 o =	48.00	13.20	
6 HO =	54.00	14.86	15.02
	363.52	100.00	

Dried in the open Air.				Experiment.
cl	=	35.42	8.86	8.29
7 *zn*	=	226.10	56.59	
6 o	=	48.00	12.01	
10 HO	=	90.00	22.54	23.49
		399.52	100.00	

When this oxychloride, dried, but not too much heated, has been exposed to the air, 100 parts of it gradually absorb about 15 of water, corresponding to four equivalents, and which cannot be expelled by the temperature of boiling water. It therefore appears to form in this proportion likewise a hydrate of definite composition.

When a solution of chloride of zinc is decomposed by ammonia, added in such excess as that part of the precipitate at first formed shall be redissolved, there is a hydrated oxychloride produced, which I have found to be in all respects identical with that just described. It has the same amylaceous look and feel, the same lightness, and, as shall be now shown, the same composition.

C. 4.60 grammes of this oxychloride, dried merely at ordinary temperatures, were heated over a spirit-lamp, until all evolution of watery vapour had ceased; there remained the greyish dry oxychloride, weighing 3.510, corresponding to 76.3 per cent., or 23.7 of water. The 3.510 residue was dissolved in dilute muriatic acid, and precipitated by solution of carbonate of soda; the precipitate of carbonate of zinc was washed carefully and ignited, when it left a pure oxide of zinc, weighing 3.237, or 70.22 per cent., containing 56.28 of metallic zinc.

D. A quantity taken from the same filter, was dried at 212°: it had the same appearance as the former. Of this 3.165 dried, left 2.690 of residue, giving 85.0 per cent. and 15.0 of water. The residue was dissolved in dilute nitric acid, and precipitated by nitrate of silver; the chloride formed, collected, and fused, weighed 1.223, or 38.64 per cent., containing 9.53 of chlorine. Hence this oxychloride was composed of

Dried at 60°.			Dried at 212°.		
cl	=		*cl*	=	9.53
zn	=	56.28			
HO	=	23.70	HO	=	15.00

Which agree with the results of the theoretical formulæ given for the oxychloride last examined. When the ammonia employed is not sufficient to precipitate all the zinc, the oxychloride formed is differently constituted from that just described, and is the same with that described by Schindler, and which is analogous to the ordinary oxychloride of copper. Schindler, however, appears to have dried the specimens which he analyzed at 212°, for I have found this oxychloride to retain four equivalents of water at 100° F. Its formula is then

$$zncl + 3\,zno + 4\,\text{HO}.$$

As the same result, except in the estimate of the water, had been obtained by Schindler, I will not enter into any details of my verifications of his results. I have, however, obtained another oxychloride, which, in a less hydrated condition, had been noticed by Schindler likewise. I prepared it by adding to a solution of chloride of zinc, caustic potash liquor, until it began to react alkaline. The process by which Schindler had obtained it, almost necessarily produced the separation of the water it should contain; thus he evaporated chloride of zinc until it had lost a certain proportion of muriatic acid, and then diluted with much water the remaining sirupy liquor. The formula which he obtained was

$$zncl + 9\,zno + 3\,\text{HO}.$$

This oxychloride, as formed in my experiments, scarcely differs from those already described, in its external appearance; when heated it yields water in the same manner. The analysis of it merely, therefore, need be given in detail.

A. 1.790 grammes, dried over the spirit-lamp, gave 1.384 of residue, or 77.32 per cent.

B. 2.131, treated in the same manner, gave 1.646, or 77.24 per cent.

C. The 1.384 of residue, exposed to the air, gradually absorbed water, and became 1.485; therefore the quantity of water absorbed was to the original quantity as 101 to 406, or nearly as one to four.

D. 3.030 of dried oxychloride were dissolved in dilute nitric acid, and precipitated by nitrate of silver; the chloride produced weighed 0.938 grammes, or 30.96 per cent., equivalent to 23.93 per cent. for the hydrated oxychloride, which contains 5.921 of chlorine.

Hence is derived the formula $zncl + 9\,zno + 14\,\text{HO}$, by which there should be

	Theory		Experiment.	
cl =	35.42	6.37	5.92	
$10zn$ =	323.00	58.11		
$9o$ =	72.00	12.95		
$14HO$ =	126.00	22.67	22.68	22.76
	556.42	100.00		

And the dry oxychloride, $zncl + 9zo$, absorbs four equivalents of water, assuming nearly the eondition in whieh it had been examined by Sehindler. The quantity of water found by analysis is intermediate between three and four atoms, but I eonsider that the method used was most likely to lead to an error by defieieney of absorption than by exeess, and henee I adopt four as the quantity re-absorbed. Then there is given the formula $zncl + 9zno + 4HO$.

There are thus found to exist at least three different oxyehlorides of zine, eaeh of whieh may be obtained eombined with various proportions of water.

In order to be able to traee the eonnexion between these oxyehlorides, and to aseertain the relation in whieh they stand to the hydrated neutral ehlorides of the same family, they may be arranged in the following manner :

$$\left.\begin{array}{l} \text{A. 1. } zncl + zno + 2zno + 2HO \\ \text{2. } zncl + zno + 2zno + 4HO \end{array}\right\} \text{Hydrates of } zncl + 3zno.$$

$$\left.\begin{array}{l} \text{B. 1. } zncl + 6zno + 4HO \\ \text{2. } zncl + 6zno + 6HO \\ \text{3. } zncl + 6zno + 10HO \end{array}\right\} \text{Hydrates of } zncl + 6zno.$$

$$\left.\begin{array}{l} \text{C. 1. } zncl + 9zno + 4HO \\ \text{2. } zncl + 9zno + 14HO \end{array}\right\} \text{Hydrates of } zncl + 9zno.$$

The oxyehloride A and its hydrates eonform to the type of the Brunswiek green and of the oxyehloride of mereury. Elsewhere the nature of this type will be diseussed.

The oxyehloride B, in its dry form, is evidently the basie eompound eorresponding to the ehlorides, with six atoms of water of crystallization, and henee

$$zncl + 6zno$$

eorresponds to

$$\text{MgCl} + 6\text{HO}$$
$$\text{Ca.Cl} + 6\text{HO}$$

and other cases, of which the chloride of hydrogen is the most remarkable.

When water is saturated with muriatic acid gas, the solution being kept at the temperature of 32° F., it acquires a specific gravity of 1.2109, and then contains in 100 parts 42.43 of gas, by Edmund Davy's determination. If the water be retained only at 60° the absorption does not proceed so far, the specific gravity reaching only about 1.192, and the liquor containing only 38.38 of chloride of hydrogen in the 100. Thompson found the strongest liquid acid to be 1.203, and to contain 40.66 per cent. of gas. Now, if we calculate the number of equivalents of water which these results indicate as combining with one of chloride of hydrogen, we shall find

In the acid of 1.2109 . $\frac{\text{ClH}}{\text{HO}} = \frac{36.42}{49.4}$ and $\frac{49.4}{9} = 5.5$

In the acid of 1.192 . $\frac{\text{ClH}}{\text{HO}} = \frac{36.42}{58.5}$ and $\frac{58.5}{9} = 6.5$

In the acid of 1.203 $\frac{\text{ClH}}{\text{HO}} = \frac{36.42}{53.15}$ and $\frac{53.15}{9} = 5.91$

Scarcely any doubt can remain, therefore, that in the strongest liquid muriatic acid, the chloride of hydrogen combines with six equivalents of water, and that it is hence analogous to

$$\text{Ca.Cl} + 6\text{HO}$$

and to $\text{ZnCl} + 6\text{ZnO}$.

This strong hydrated chloride of hydrogen cannot be heated without escape of gas, and if it be distilled, the boiling point gradually rises until it reaches 230° F. (110° C.) when it ceases to change, and the liquid subsequently distils unaltered. If a weaker acid be distilled, it loses water until the boiling point rises to the same degree, when acid of the same strength distils, as in the former instance. This acid, with a constant boiling point, has a specific gravity of 1.094, and contains 19.19 per cent. of real acid by Davy's estimate, and 20.44 by Thompson's; hence the proportion is, taking the mean of their results,

$$\frac{\text{ClH}}{\text{HO}} = \frac{19.82}{80.18} = \frac{36.42}{147.3} \text{ and } \frac{147.3}{9} = 16.35.$$

Hence this acid, with constant boiling point, is composed of $\text{H}cl + 16\text{HO}$, and its formula may properly be considered as

$$\text{H}.cl + 6\text{HO} + 10\text{HO}.$$

corresponding to

$$zncl + 6\,zno + 10\text{HO}.$$

the hydrated-oxychloride, which has been described.

X. OF THE AMMONIA-SULPHATES OF ZINC.

This salt was prepared by passing ammoniacal gas through a strong and hot solution of sulphate of zinc, until the whole of the sub-sulphate precipitated had been redissolved. The liquor, on cooling, deposited a flocculent mass, in semi-crystalline grains resembling starch; and if the liquor be evaporated, or kept hot, the separation of this substance continues; when, however, the solution is allowed to cool, and then having been filtered, is left to spontaneous evaporation, it remains clear; and small, but perfectly distinct crystals are deposited, which remain bright while moist, but effloresce, and become opaque almost immediately on being dried and left in the open air. These two bodies contain alike, sulphuric acid, oxide of zinc, ammonia, and water, but the quantity of the constituents is not the same; I shall therefore describe them separately, commencing with the crystallized ammonia sulphate.

When this salt is heated it gives water and ammonia, and there remains sulphate of zinc; if the heat be very gently applied, all ammonia may be expelled, and the residual sulphate of zinc will be quite pure; but if the salt be suddenly heated, a quantity of sulphate of ammonia is produced, and the sulphate of zinc remaining is mixed with oxide.

As this salt, from the manner of its formation, must contain two equivalents of ammonia to one of the sulphate of zinc, the analysis of it became very simple, as it was to be directed specially to the examination of the quantity of water which it might contain.

In efflorescing this salt does not lose ammonia. To determine its composition, 3.701 of clear crystals, dried between folds of blotting-paper, were heated at first very gently, but finally to ignition. On the first application of the heat the salt fused, and emitting water and ammonia, left a perfectly white residue of sulphate of zinc, weighing 2.023, corresponding to 54.66 per cent. Its composition, therefore, is:

Sulphate of zinc	=	54.66
Volatile matter	=	45.34

and

$$\frac{54.66}{45.34} = \frac{80.50 = zno.so_3}{66.77 = 34.28\,NH_3 + 32.49}$$

Consequently the water is $\frac{32.49}{9} = 3.61$ equivalents, and as the salt is efflorescent, the true number is probably four.

When these crystals have been left in the open air for some time they lose altogether their transparency, but retain their form, assuming the milky lustre of the crystals of nitrate of lead. When these milky crystals are heated they melt, and are decomposed with precisely the same phenomena as the transparent ones, leaving a sulphate of zinc redissolving completely in water.

3.030 of these crystals, so treated, gave 1.818 of sulphate of zinc, or 60 per cent.; hence,

$$\text{as } \frac{60}{40} = \frac{80.5}{53.66} \text{ and } 53.66 - 34.28 = 19.38.$$

The quantity of water had evidently been reduced to one-half by efflorescence, no ammonia having been lost, as was ascertained by experiment.

In the decomposition of this salt by heat, the ammonia and water go off together to the end, and this is easily seen, as the material lost is exactly $2(NH_3.HO)$.

By the first application of the heat it was mentioned that the salt fused after it had lost a certain proportion of gas and water; this fused mass, on cooling, solidifies into a mass like gum, which may be again melted, and the remaining ammonia and water expelled, as above described. In order to ascertain whether the fusion of the mass occurred at any definite point in the process of decomposition, a quantity of the effloresced salt was heated until completely fused, the lamp was then removed, and the weight of the residual gummy-looking material determined,—it amounted to 80.29 per cent.; and hence it results that the quantity of volatile matter lost had been exactly half of the entire amount, thus,

Sulphate of zinc	=	60.00	Gummy residue.
a. Volatile matter	=	20.29	
b. Volatile matter	=	19.71	
Effloresced salt	=	100.00	

From these results follow the formulæ

Transparent crystals	=	$zno.SO_3+2NH_3+4HO$
Effloresced crystals	=	$zno.SO_3+2NH_3+2HO$
Gummy mass	=	$zno.SO_3+NH_3+HO$
Which gives by heat	=	$zno.SO_3$

I shall, before proceeding further, return to the examination of the flocculent substance which was deposited from the hot solution of the ammonia-sulphate. It cannot be redissolved in water, which distinguishes it from the transparent crystalline salt; when heated it fuses, and is decomposed with the escape of water and ammonia, as is the case with the substance already described. It was analyzed as follows:

5.033 of this flaky substance was heated until all escape of water or of ammonia had ceased; there remained 3.821 of sulphate of zinc, corresponding to 75.92 per cent., and

$$\frac{75.92}{24.08}=\frac{80.50}{25.53}, \text{ and } 25.53-17.14=8.39, \text{ or nearly } 9.$$

Hence the formula is $zno.SO_3+NH_3+HO$.

These flakes have therefore the same composition as the gummy mass obtained by melting the crystalline salt, and this circumstance proves that the gummy mass is really a definite chemical compound, which could not have been so positively shown from the method by which it had been prepared.

When the crystalline salt is kept for some time at a temperature of from 80° to 100° F. it gradually falls down into a white powder, all traces of crystalline structure having totally disappeared; during this decomposition, water alone escapes, as turmeric paper left on the surface of the powder is not at any period affected. When this powder is heated to about 212°, it gives out water and ammonia, which continues up to a certain point, but in order to finish the expulsion of the water, the temperature must be raised until the mass has become fused;

after that time, the continuance of the heat occasions the loss of more ammonia, but no more water is disengaged. Unless the heat be very accurately managed, sulphite of ammonia is apt to make its appearance before the last portions of the ammonia have been expelled; with care, however, a sulphate of zinc almost completely soluble in water may be obtained.

To determine more closely what occurs in the case just noticed, 4.238 grammes of the powder formed by the efflorescence, at 100°, of the crystals were heated until the sulphate of zinc remained pure behind; it weighed 2.800, or 66.07 per cent.

4.385 of the same powder were heated until it had fused, and the escape of water had ceased, great care being taken to seize the precise time, and to avoid the application of any unnecessary heat; the residual mass weighed 3.470, or 79.13 per cent.

$$\text{Now } \frac{66.07}{33.93} = \frac{80.50}{41.34}, \text{ and } 41.34 = 32.34 + 9, \text{ q. p. } 2\,NH_3 + HO.$$

The proportion of ammonia being a little less than two atoms.

Again, the second experiment gives

$$\left.\begin{array}{ll} zno.SO_3 & 66.07 \\ \text{Ammonia} & 13.06 \end{array}\right\} = 79.13 \text{ of residual fused mass;}$$

and

$$\frac{66.07}{13.06} = \frac{80.50}{15.91}, \text{ or nearly } \frac{80.50}{17.14} = \frac{zno\,SO_3}{NH_3}.$$

The effloresced powder was therefore

$$zno\,SO_3 + 2NH_3 + HO,$$

corresponding to the crystallized ammonia-sulphate of copper, and by heat it loses $NH_3 . HO$, and there is formed

$$SO_3 . znoNH_3,$$

being precisely the same as in the copper series. This effloresced powder put into water dissolves almost without residue, provided the water be free from carbonic acid.

The reasoning which I employed concerning the rational formulæ of the ammoniacal compounds of copper, applying with equal force to those of zinc, I will not repeat it, but arrange the results just now described, in accordance with those views.

1. The crystalline salt $= (NH_3 . HO) SO_3 + ZnO . (NH_3 . HO) + 2 HO$.
2. The effloresced crystals $= (NH_3 . HO) SO_3 + ZnO (NH_3 . HO)$.
3. The effloresced powder $= (NH_3 . HO) SO_3 + ZnO . NH_3$.
4. The flakey substance $= (NH_3 . ZnO) . SO_3 + HO$.
5. The fused mass from 3 $= (NH_3 . ZnO) . SO$.

I will not enter into the consideration of any of the interesting relations which the arrangement of this series of bodies must suggest, except to point out in the ordinary sulphate of ammonia, the anomaly of the crystallization of which, with an atom of water, is so curious, the analogue of the bodies 4 and 5. Thus there is

$$(NH_3 HO.) SO_3 + HO \text{ and } (NH_3 . HO) SO_3$$

as there is

$$(NH_3 . ZnO) SO_3 + HO \text{ and } (NH_3 ZnO) SO_3.$$

When discussing the theory of these bodies in another section, I shall have occasion to recur to these results.

XI. OF A NEW BASIC SULPHATE OF ZINC.

When the bodies (4) or (5) are treated by water they are decomposed, the body (1) dissolves, a quantity of sulphate of ammonia is likewise formed, and the insoluble matter is so definite and marked in its composition, that it must be regarded as a new basic sulphate of zinc. It is white, insoluble in water, when heated it gives water, and leaves a white powder behind. It was analyzed as follows:

2.594 grammes, dried by a spirit-lamp, gave 1.950, or 75.18 per cent., having lost 24.82 water.

The residual sub-sulphate was boiled with solution of carbonate of soda, and the carbonate of zinc collected on a filter, dried, and ignited; the oxide of zinc remaining weighed 1.635, or 64.22 per cent. Hence the composition

Sulphuric acid	=	10.96
Oxide of zinc	=	64.22
Water	=	24.82

2.544, dried, gave 1.957, from whence

Dry sub-sulphate	=	75.88
Water	=	24.12

The dry mass, exposed to the air, absorbed water, and became 2.137, or 8.40 per cent., having taken up almost exactly one-third of the quantity of water it had lost.

These results point out the formula

$$SO_3 + 6\,ZnO + 10\,HO,$$

which should give

SO_3	=	40.16	10.79
$6\,ZnO$	=	241.80	65.02
$10\,HO$	=	90.00	24.19
		371.96	100.00

There are two sub-sulphates of zinc already known, of which the one $SO_3 + 4\,ZnO$ has been described by Schindler, Kuhn, and Graham. It appears to combine with variable proportions of water, from two to ten equivalents, but most commonly is to be found with four. The second has been examined by Schindler alone, who gave its formula as $SO_3 + 8\,ZnO + 2\,HO$. I have not had an opportunity of verifying this result, but I consider the correctness of his analysis as being very probable. The same chemist showed that there may be formed a soluble compound of $SO_3 + 2\,ZnO$, which, however, is destroyed when dried. Hence the series of basic sulphates of zinc may be thus arranged:

Real neutral sulphate	=	$ZnO \,.\, SO_3$.
Salt with saline water	=	$(ZnO.\,HO)\,SO_3$.
Soluble salt of Schindler	=	$(ZnO \,.\, ZnO)\,SO_3$.
Common crystals	=	$(ZnO \,.\, HO)\,SO_3 + 6\,HO$.
Hyperbasic salt, dry	=	$(ZnO \,.\, ZnO)\,SO_3 + 6\,ZnO$.
Common basic salt, dry	=	$(ZnO \,.\, ZnO)\,SO_3 + 2\,ZnO$.
Do. with water—Schindler	=	$(ZnO \,.\, ZnO) \,.\, SO_3 + 2\,ZnO + 2\,HO$.
New basic salt, dry	=	$(ZnO \,.\, ZnO)\,SO_3 + 4\,ZnO$.
Do. with water	=	$(ZnO \,.\, ZnO)\,SO_3 + 4\,ZnO + 10\,HO$.

The law of replacement being precisely what was already shown in the copper series, but still more complete from the discovery of $SO_3 + 6\,ZnO$.

PART III.

ON THE THEORY OF THE AMMONIACAL COMBINATIONS.

On the accession to science of any considerable body of new facts, we should carefully examine how far they tend to modify our ideas of the nature and intimate structure of the bodies to which they relate, and of the forces to the action of which these bodies are subjected, and by remodelling our views in accordance with the ideas thus obtained, we should endeavour after a closer approximation to that truth, the attainment of which is the object of all scientific labour.

A body, possessing so many interesting properties as ammonia, standing as it were, on the confines of mineral and of organic chemistry, and forming the connecting link between them, must even, on its own account, and still more from the remarkable variety of classes of combinations into which it enters, occupy a prominent place in the general theory of chemistry, and the grounds of any proposed alteration in our views concerning it should be examined with the attention due to the importance of the subject. I shall therefore lay before chemists, for discussion, some views of its nature and laws of combination, differing in many important particulars from those hitherto received, which have been suggested to me by the researches on the various classes of compounds of ammonia contained in the present and former papers. These views are connected in a very remarkable manner with those concerning which the opinions of chemists have been so long divided; it will be seen, in fact, that the principles of the theory which I propose, embrace all that was vital in former hypotheses; and it may be almost considered as an argument for its sufficiency, if not actual truth, that in the development of these views is exemplified the ordinary course of advancing knowledge, when the once conflicting elements of rival theories are found forced into coalition by the grasp of some generalization of a higher order.

Before commencing the explanation of my own views, I shall briefly describe the essential principles of the previous theories of ammonia.

A.—The oldest view:

1. That ammonia NH_3 is an independant base, saturating acids and forming salts.

If, as Dulong proposed, all acids be regarded as hydrogen compounds, thus $SO_3 + HO$ as $SO_4 + H$, similar to ClH, the old view explains the main requisite in all theories of ammonia, the presence of water in the salts formed by the oxygen acids. Sulphate of ammonia becomes $SO_4 . H + NH_3$, like $Cl.H + NH_3$.

B.—The theory of Berzelius:

1. That the ammoniacal amalgam contains a body, NH_4, which is metallic, combines with oxygen, and then may replace potash in combination.
2. That when NH_3 combines with HCl. the NH_3 takes H, and forms NH_4, with which the chlorine combines.
3. That the water in the ammoniacal salts with the oxyacids converts NH_3 into $NH_4 + O$.

C.—The amide theory, as left by Dumas and Berzelius:

1. There was assumed a hypothetic body, NH_2, which replaced chlorine and oxygen in certain organic combinations.
2. Potassium or sodium heated in ammonia, liberated therefrom as much hydrogen as from water, and formed amidide of potassium or of sodium.

Ammonia is in no place called amidide of hydrogen by Berzelius or by Dumas, nor is NH_3 ever written $NH_2 + H$, but Dumas may have had that idea indistinctly in his mind when he said that it was perhaps possible that as hydrogen forms hydracids with some bodies, so it might produce hydrobases by its union with others. He may have meant that hydrogen formed ammonia, a hydrobase, by uniting with NH_2 amidogene, but he much more probably referred to the combination of the hydrogen at once with nitrogen; his adherence to the common, but incorrect ideas of the nature of the hydrogen bodies in general having completely prevented him from seeing the true position of ammonia and its compounds.

The insufficiency of these views may be very briefly pointed out; thus,

A.—The oldest view.

1. It applies only to the common ammoniacal salts, but does not attempt any explanation of the nature of the numerous other classes of ammonia compounds.

2. It states merely that NH_3 acts as a base, but does not explain its relation to ordinary bases which are metallic oxides, nor the points in which the ammoniacal salts differ from the metallic salts of the same acid.

B.—The Berzelian view.

1. It does not assign any proper function or place to ammonia itself, which might be absolutely dropped out of the theory without loss. This view, therefore, leaves unexplained all combinations of ammonia with bodies which do not contain hydrogen.

2. That $NH_3 + H.Cl$ becomes $NH_4 + Cl$, is purely hypothetical, and highly improbable, the ammonia not exercising any apparent affinity for hydrogen, while that of chlorine for hydrogen is very strong. Hence the duty of proving the change in position of the fourth atom of hydrogen rests with the Berzelian theory, and has not been yet performed.

C.—The amide theory.

1. Our knowledge of the amidogene combinations has been acquired almost exclusively since the theories just noticed had been proposed, and consequently what is now the most important principle in a complete theory, the connexion of the ammonium and of the amidogene compounds with those containing ammonia itself had no place therein. Hence all former theories are insufficient, from the ground that the new facts gained by the study of the metallic amidides cannot be explained by or included within the principles upon which they rest.

I shall now describe, in a series of propositions, the principles of the theory which I advocate, and then taking each proposition by itself, will sum up the evidence derived from experimental results, by which I consider its validity to be established.

Prop. I.—That the so called hydracids are not really such; that hydrogen, in all its forms of combination, is analogous to certain metals of the electro-positive class, and its compounds react like theirs under similar circumstances.

II.—That ammonia NH_3 is amidide of hydrogen $NH_2 + H$, and resembles in some respects the oxide, in others the chloride of the same positive element.

III.—That NH_2 amidogene may combine with metals, and that the metallic amidides have a singular tendency to combine with the chlorides or oxides of the same metal, or of a metal of the same family, and thus form bodies resembling the chloro-oxides, chloro-sulphurets, or oxysulphurets.

IV.—That $NH_3 = NH_2 + H$ amidide of hydrogen can perform the same functions in combination as water, oxide of hydrogen, whether as basic water, or water of crystallization, and likewise can replace the water termed saline in certain salts by Graham.

V.—That the so called oxide of ammonium NH_4O is oxy-amidide of hydrogen $NH_2 . H + HO$, and that sal ammoniac is chloro-amidide of hydrogen $NH_2 . H + HCl$.

VI.—That the ordinary ammonia salts ally themselves to the salts of the copper and zinc class, which contain two equivalents of oxide.

VII.—That if chlorine could be separated from sal ammoniac, the residual NH_4 should be regarded as $NH_2 + 2H$, sub-amidide of hydrogen, as when by removing the chlorine from white precipitate, the sub-amidide of mercury, $NH_2 + 2Hg$, formed by the action of water of ammonia on calomel, should remain.

Prop. I.—*Of the general positive Nature of the Compounds of Hydrogen.*

In a memoir which was published in 1831 in the Dublin Journal of Medical and Chemical Science, I pointed out that the general bearing of the properties of the compounds of hydrogen should induce us to assign to those bodies a totally different position from that which the names of hydrogen-acids previously assigned to most of them would appear to warrant. Thus that, whilst we found hydrogen to manifest immensely superior electro-positive energies to those of gold, platinum, or sulphur, it was quite unphilosophical to suppose, that when all of these bodies were combined with chlorine, the hydrogen should be that least capable of diminishing the negative power of the chlorine. I showed that from the considerations which are suggested to us by a fair comparison of the properties of the oxides, chlorides, sulphurets, &c. of hydrogen, with those of the similar compounds of the metals, it became quite necessary to allow, that although in some cases, as where water united with potash or lime, the hydrogen body may perform the negative function, yet in the vast majority of cases the part played by it in combination is that of positive constituent.

I shall refer to the memoir above quoted for the details of the views which I then brought forward; previously to that time Mitscherlich had already suggested, that in the hydrated acids the water acted as a base, but this, from the indifferentism of water in the generality of chemical actions, could not be considered as leading to any thing like the general principle which formed the subject of my paper. Since that period, although no writer has broadly reproduced this theory of the hydrogen combinations, yet the progress of research has gradually lent to it the most efficient support, by the discovery of classes of bodies identifying in the strictest manner the chemical relations of hydrogen, and of certain of the more positive among the metals. The beautiful investigations of Graham on water as a constituent of salts, particularly those illustrating the conversion of the neutral into the basic condition by the replacement of the hydrogen by a metal of the magnesian family, has shown that in its relations to oxygen at least no line of distinction can be drawn between hydrogen and the metals which with it constitutes so natural a group.

Passing to the other compounds of hydrogen, there will be found in the series of researches on the zinc and copper families, a variety of instances in which the chloride of hydrogen is represented with remarkable closeness by the chlorides of copper or of zinc. The examination of the various oxychlorides of zinc, in their dry and hydrated conditions, which presents to us the perfect analogues of the chloride of hydrogen in its two stable conditions of definite combination with water, points out an identity of action liable to little objection. Like the chloride of hydrogen also, chloride of zinc is caustic, and when concentrated reddens litmus, so that the peculiarly acid character of affecting that reagent is to be found well developed in bodies to which, under any circumstances of ordinary language, the name of acid could scarcely be applied.

The relation of chloride of zinc to ammoniacal gas is likewise very remarkable, as indicating the general similarity of action between the hydrogen and zinc compounds: the volatility of the ammonia-chloride of zinc, the permanent nature of the ammonia-chloride of copper, indicate a closeness of union between the metallic chloride and the ammoniacal gas, which brings those bodies into very intimate connexion indeed with sal ammoniac.

As this proposition will receive from the evidence of several of the succeeding ones a great deal of additional support, I will not here enter into any further evidence in favour of it. Every fact which, in the course of these researches, became the subject of examination, has tended to strengthen my confidence in the truth of the general principle which the additions to science from the recent investigations of other chemists have likewise uniformly tended to confirm.

Prop. II.—*That ammonia* NH_3 *is amidide of hydrogen, and should be written* $NH_2 + H$.

The re-examination of the results of Gay Lussac, Thenard, and Davy, on the action of potassium on ammoniacal gas, gave to the interesting views of Dumas, arising from the discovery of oxamide, a stability and importance which must be considered as the origin of all subsequent investigations in that extensive field. When we allow for the various sources of error to which, from the easy decomposition of the resulting bodies, the quantitative determinations of the hydrogen evolved from the ammonia is exposed, we shall find in the experiments of those exact chemists a complete proof that potassium liberates from ammonia precisely the same quantity of hydrogen as from water, and hence that the element remaining united with the potassium is amidogene. The idea of ammonia being itself a base differing essentially in constitution from the oxides of hydrogen or of the metals, prevented the distinguished discoverer of oxamide from tracing in the action of potassium on ammonia, the rational constitution of the latter, and although he recognized completely the identity of function performed by the metal in the one case, and the carbonic oxide in the other, yet it is evident, from the tenor of his observations on all occasions, that he looked upon the abstraction of the equivalent of hydrogen as subverting the constitution of the ammonia, and that the amidogene resulting did not stand in any natural relation to the ammoniacal gas employed.

Notwithstanding the remarkable cases discovered and examined by Henry Rose, in which the combinations of ammonia with the various classes of salts appeared to correspond so closely with the same salts containing water of crystallization, whence, taken in connexion with the existence of the amidides of potassium and sodium, the symmetricity of NH_3 and OH might be inferred, and the form $NH_2.H$ given to the former; yet, until the discovery of the composition of white precipitate, and of the similar bodies which I examined, and which was fundamental to all these researches, instances of the resolution of ammonia into amidogene and hydrogen, independent of all destructive action, had not become sufficiently positive and unexceptionable to lead any chemist to express the opinion of its being really amidide of hydrogen, ranking with the oxide and chloride of the same element. This view, however, results almost unavoidably from those experiments, although I myself did not finally adopt it until by the development of the nature of the other quicksilver combinations with ammonia, the complete identification of the principle of action of oxygen and amidogene, particularly as exerted in the two classes of water and of ammonia sub-salts, left no room in my mind for any other hypothesis.

The objection to the assumption of the existence of an hypothetic body, amidogene, which might be supposed to weigh powerfully against the general acceptance of this theory, is deprived of a great deal of its force when we come to examine it with somewhat more care. In order to arrive at an idea of the actual nature of ammonia, and of the position it is suited to occupy in the general scheme of chemical reactions, we must investigate the laws of its affinities, and study accurately the analogies which it presents in its combinations, with those of other bodies of simpler constitution, and the history of which is as yet better understood. From these data must our conclusions be drawn, and decompositions, frequently of an accidental character, and mostly dependant on the peculiar manner in which the affinities of the decomposing body may be exerted, should be considered of but secondary importance, and subordinate to the study of the general history of the substance, although still suited, under proper limitations, to guide us usefully in our course. It is right that the exertions of chemists should be directed to effect the isolation of amidogene, and it is to be hoped that the same success which crowned the beautiful researches of Gay Lussac on Prussic acid, will reward their efforts; but even should this radical, like those of so many of the most important series in organic chemistry, for a longer time elude our grasp, it is proper and just to assume it to exist, if we, by so doing, can obtain a more satisfactory explanation of phenomena, and link together classes of facts previously disconnected and obscure.

Prop. III.—*That amidogene may combine with metals, and that the metallic amidides have a singular tendency to combine with the chlorides or oxides of the same metals.*

The formation of the amidides of potassium and sodium, gives sufficient proof of the first part of this proposition, and there have been found in the researches on the ammoniacal combinations of quicksilver, numerous instances of the truth of the latter principle. Thus white precipitate must be looked upon as a compound of chloride and amidide of mercury, and the black substance formed by the action of water of ammonia on calomel must be composed of sub-chloride, united to the sub-amiduret of the same metal. More complex examples are furnished by the yellow powder

$$\mathrm{Hg}\,cl + 2\,\mathrm{Hg}\mathrm{O} + \mathrm{Hg}\,.\,\mathrm{NH}_2,$$

and the bodies

$$\mathrm{Hg}\,\mathrm{SO}_4 + 2\,\mathrm{Hg}\mathrm{O} + \mathrm{Hg}\,\mathrm{NH}_2$$
$$\mathrm{Hg}\,\mathrm{NO}_6 + 2\,\mathrm{Hg}\mathrm{O} + \mathrm{Hg}\,\mathrm{NH}_2.$$

In the copper family there exist some examples equally remarkable, but which shall be referred to particularly under a distinct head.

Prop. IV.—*That amidide of hydrogen can perform the same functions in combination as oxide of hydrogen, whether as basic water, as water of crystallization, or as the water termed saline by Graham.*

In the most perfect cases of substitution, where the substances belong to strictly isomorphous groups, the similarity of properties and structure existing through the several classes of bodies formed by the mutually replacing elements, assumes an exactness to which no parallel is found in the instances with which the history of the ammoniacal bodies has supplied us; yet amongst the combinations described in the preceding sections, analogies and relations have been observed of such closeness, as to give to the truth of the proposition now in question the highest probability.

A vast number of bodies, such as oxygen-salts, chlorides, iodides, &c., exposed to the action of ammoniacal gas, absorb a considerable quantity thereof, and it is afterwards found that different portions of this ammonia are retained with various degrees of force: the greater part being, generally speaking, expellable by the temperature of boiling water, whilst the remainder clings to the substance with a much higher power, sometimes not being separable, unless the constitution of the body be completely broken up. This fact finds a complete parallel in the relative degree of affinity with which water is retained by ordinary salts and acids. Thus the retention of the basic water by oxalic and common tartaric acids, and the greater affinity of the last atom of water in the sulphates of the magnesian class find in the compounds of ammonia their analogous combinations, and one of the most embarrassing circumstances in the present investigation arises from the fact of the relation of ammonia and water being so close, that where the ammoniacal bodies are soluble in water, they cannot be brought into contact with it without an exchange of position occurring to a considerable extent, and the body crystallizing in a state containing both water and ammonia. Thus, whilst by passing dry ammonia over chloride of copper, the body $CuCl + 3NH_3$ may be obtained, the result of treating a solution of chloride of copper by ammonia is $CuCl + 2NH_3 + HO$, in which the third equivalent of ammonia has evidently given place to one of water; and though the copper, as I have already shown, is separated from the chlorine, however by means of heat both bodies yield $CuClNH_3$; the one losing $2NH_3$, the other $HO.NH_3$. Thus, through the whole class of soluble ammonia-copper and zinc combinations, the water replaces, in the first instance, the metallic constituent, and partly the ammonia itself, and it is only when by the application of heat the water with some ammonia has been expelled, that we arrive at the real combinations of the metallic compound with amidide of hydrogen.

The basic nitrates of mercury being insoluble, furnish one of the most striking examples to be found of the replacement of water in its basic condition by ammonia. It was proved that the basic nitrates stood in the same relation to the neutral salts as that which Graham had pointed out for the nitrates of the magnesian class; and I showed, in the same section, that the ammonia sub-nitrates were so constituted, that the nitric acid and oxide of mercury remained the same, whilst the water of the ordinary sub-salts was replaced by the ammonia thus: Ad representing NH_2. amidogene.

The yellow sub-nitrate of the red oxide is

$$HO.NO_5 + 3HgO.$$

The ammonia sub-nitrate of the red oxide is

$$HAd.NO_5 + 3HgO.$$

The sub-nitrate of the black oxide is

$$HO.NO_5 + 2\underline{Hg}O.$$

The ammonia sub-nitrate of the black oxide is

$$HAd.NO_5 + 2\underline{Hg}O.$$

These examples establish, in this case, the complete similarity of action of hydrogen, whether combined with oxygen or amidogene.

In the second part of the present memoir will be found a remarkable instance of the replacement of water by ammonia. There was described a new chlor-oxide of copper, $cucl + 2cuo$; this unites with water, forming a brown powder,

$$cucl + 2cuo + HO,$$

evidently analogous to dry Brunswick green,

$$cucl + 2cuo + cuo;$$

but it also unites with dry ammonia to form a brown powder,

$$cucl + 2cuo + HAd;$$

under which form the replacement of HO by cuo, and of both by HAd, is evidently shown.

When once the principle of ammonia being considered as amidide of hydrogen, has been steadily brought before the mind, the nature of a vast class of combinations, the functions of the ammonia in which had previously presented great difficulty, is at once cleared up. Thus the combinations of ammonia with the chlorides of tin, of antimony, of phosphorus, &c. are at once seen to resemble those which many of the same bodies enter into with water, in equally definite proportions; thus $sncl_2 + HAd$ is a white solid body, and $sncl_2 + HO$ is equally white and solid. The compounds of the chlorides and oxysalts of the magnesian class of metals present a parallelism still more close, and to which, after some time, I shall again refer.

A class of bodies, the nature of which has frequently given occasion to discussion, is the combinations of the oxygen acids with dry ammonia. Of these, the most remarkable and the most accurately studied is that with sulphuric acid, and I shall consider it in these observations as the type of the whole class.

There are two opinions of the nature of this body,—first, that which vaguely considering ammonia as a base *per se*, looks upon the existence of two classes of ammoniacal salts, one merely of ammonia, the other of oxide of ammonium, as possible, and enumerates this and other similar bodies in the former group; second, that which considers the sulphuric acid and ammonia as being mutually decomposed, and water being formed, an amidide to be produced, with which the water remains united. Thus there is $SO_3 + NH_3$ or $SO_2.NH_2 + OH$.

From the latter view, although supported by the high sanction of Dumas and many others, I must dissent. We have no reason to suppose water to be contained in the compound in this eliminated form; and unless we find no other legitimate method of explaining its origin and properties, an hypothesis of that kind should not be resorted to.

Previous to discussing the first point of view, I must make some observations as to the view of ammonia being an independent base. This phrase has had its origin in the earliest age of organic chemistry, when the volatile ranking with the fixed alcalies, chemists were contented with the observation that there were salts of ammonia, as there were salts of potash and soda, without recognizing accurately any difference of type of constitution amongst them. The progress of analysis, however, pointed out the presence of water in all ordinary ammoniacal salts of the oxygen acids, and hence the notion of the independent basic power of ammonia became almost forgotten. Indeed, if one examines what is said by systematic writers on the combinations of the dry acids with dry ammonia, it will be found that no definite or distinct idea of their nature has been formed; that they are grouped together to separate them from the real ammoniacal salts, which are said to contain ammonium, but that no opinion of their intimate constitution has been hazarded even by Berzelius. In fact in order to understand their nature, our opinions as to the words *acid* and *base* must be reviewed. We can no longer look upon oxygen as being the sole negative element of basic bodies, since sulphur identifies itself with it in all its principles of action, and the analogy has been extended with some justice even to chlorine, iodine, and bromine. Hence there can be no doubt but that amidogene, which relates itself to oxygen so closely in a multiplicity of instances, may form the negative element in combinations of this kind, and as water, oxide of hydrogen, acts as a base, so may ammonia as amidide of hydrogen. The difference between the vague old idea of ammonia as an alkali, and the definite principle of the basic power of amidide of hydrogen will be at once felt; in fact the alkali, the body which resembles and replaces in combination the other alkalies, potash and soda, is not ammonia, but ammonia and water, not amidide of hydrogen, but oxide of ammonium, (of Berzelius). Whilst the amidide of hydrogen, ammonia alone, is analogous to, and replaces oxide of hydrogen, or the oxides of the magnesian class of metals. It is this distinctness in the point of view which will enable us to apply this principle in a useful manner.

Now, taking the instance before described, there is

$$H.O + SO_3, \text{ similar to } H.Ad + SO_3;$$

and the circumstance of the latter not precipitating barytes water, or chloride of barium, is at once seen to result from the heterogeneiety of the negative ingredients in the two cases; because, arranging the formulæ according to Dulong's view, to which the opinions of chemists now so generally incline, there is

$$H + SO_4 \text{ and } H + AdSO_3;$$

and the formation of $Ba.SO_4$, which results naturally in the former case, becomes complicated and difficult in the latter. In fact the body $AdSO_3$ is quite distinct from any thing belonging to sulphuric acid, and can only give origin to it from a complete destruction of the powerful affinities by which it was at first produced.

This view of the basic action of ammonia, and of its relation to acids, will be found to lead to considerations of the highest interest to organic chemistry, but which it would be improper to introduce here, in the detail which alone could be of use.

Prop. V.—*That the so called oxide of ammonium,* NH_4O, *is oxyamidide of hydrogen, and that sal ammoniac is chlor-amidide of hydrogen.*

The only reason which has been advanced in support of the Berzelian ammonium theory, is the beautiful symmetry with which the ammoniacal and potash salts are by it invested, and that as the similarity and replacing power of $OH.NH_3$ and OK constituted one of the best authenticated facts in the doctrine of isomorphism, it was but reasonable to suppose the corresponding portions of those symbols, HNH_3 and K to belong to the same class. The circumstance also of the ammoniacal amalgam preserving so perfectly a metallic appearance, although its density becomes so wonderfully diminished, lent to the idea of the existence of a metal (ammonium) powerful support; and there is indeed nothing in the theory which I now bring forward to negative the leading principles of that view, by the adoption of which so great simplicity had been conferred on the history of the ammonia salts. Thus according to my ideas, as well as in the Berzelian view, the $clNH_4$ replaces clK, and ONH_4 replaces OK in combination, and also NH_4, if isolated, should be considered as fulfilling the functions of K; but in the theory now proposed an additional step is made, by which we are conducted to a closer and more distinct view of the inner constitution of these bodies.

When we place in contact two substances both compound, and which mutually combine, in order to judge of the mode in which these elements unite, we must examine the nature of the affinities by which a breaking up of the original constitution might be effected, and likewise those which would tend to maintain the two constituents in their primitive condition, and allow merely of their union with one another. On these circumstances, and by the general mode of reaction of the new substances formed, must the construction of its rational formula be founded. If we contemplate the reaction of dry chloride of hydrogen and amidide of hydrogen, when brought into contact, we shall not be able to trace any tendency in the latter to deprive the chlorine of the hydrogen with which it is united; on the contrary, we find the affinity of chlorine for hydrogen so preponderating, that ammonia, by its agency, may be reduced to simple azote. It is therefore contrary to all first principles of chemical affinity to believe, that in the combination of the chloride with the amidide of hydrogen, all the hydrogen can exist in one group of the formula, whilst chlorine alone constitutes the other; since, if we had amidogene or ammonium isolated, there can be no doubt but that chlorine could take hydrogen from both. That assumption could only become justifiable if rendered necessary by strongly corroborating facts, and it will be found that no facts at all sufficiently in point can be brought forward.

Regarding ammonia as amidide of hydrogen, its union with chloride of hydrogen becomes but a particular case, although one of the most important, of the general tendency of chlorides, oxides, and amidides of the same or of similar radicals, to unite and form double chlor-oxides, chlor-amidides, or oxamidides. In fact, if we look to the formation of white precipitate by corrosive sublimate and water of ammonia, it will be seen that the decomposition and combinations are on each side quite symmetrical; thus, there is

$$2Hgcl + 2HAd = (Hgcl + HgAd) + (Hcl + HAd).$$

The two resulting compounds, white precipitate and sal ammoniac, being strictly bodies of the same type, one containing quicksilver and the other hydrogen.

I sought very frequently to obtain sal ammoniac combined with water of crystallization, in order to produce a parallel to the compound

$$\mathrm{H}g cl + 2\,\mathrm{H}g\mathrm{O} + \mathrm{H}g\mathrm{A}d,$$

but unsuccessfully. Yet if we consider the close relations of hydrogen and copper, and of oxygen and amidogene, we will find in the bodies

$$cu cl + 2\,cu\mathrm{O} + \mathrm{HO}$$
$$cu cl + 2\,cu\mathrm{O} + cu\mathrm{O}$$
$$cu cl + 2\,cu\mathrm{O} + \mathrm{HA}d$$

similar cases, in the same way as quicksilver, gives

$$\mathrm{H}g cl + 2\,\mathrm{H}g\mathrm{O} + \mathrm{H}g\mathrm{O};$$

and also the soluble ammonia chloride of copper, whether written

$$cu cl + 2\,\mathrm{HA}d + \mathrm{HO},$$

or

$$(\mathrm{H}cl + \mathrm{HA}d) + (cu\mathrm{O}.\mathrm{HA}d),$$

presents analogies fully supplying the place of hydrated sal ammoniac.

To sal ammoniac itself the copper and zinc series affords numerous analogues. Thus, the perfectly definite and well characterized bodies,

1. $cu cl + \mathrm{HA}d.$
2. $zn cl + \mathrm{HA}d.$
3. $\mathrm{N}i cl + \mathrm{HA}d.$

correspond to

$$\mathrm{H}.cl + \mathrm{HA}d;$$

whilst we find for the ordinary compound

$$cu cl + (\mathrm{H}.cl + \mathrm{HA}d) + 2\,\mathrm{HO},$$

the body

$$zn cl + (zn cl + \mathrm{HA}d)$$

and also

$$zn cl + (zn cl + \mathrm{HA}d) + (\mathrm{HO}.\mathrm{HA}d),$$

or else

$$zn cl + (\mathrm{H}cl + \mathrm{HA}d) + (zn\mathrm{O}.\mathrm{HA}d).$$

These analogies are so remarkable, that any detailed comment on them is unnecessary.

Since the oxide of ammonium of Berzelius possesses a definite constitution only in the salts of oxygen acids with which it may unite, the superior simplicity and distinctness of the present view becomes still more remarkable in its case than in the former. We have seen that in combination with oxides the amidide of hydrogen or of the metals assumes, even in the simplest cases, very complicated formulæ; thus, the

Oxamidide of mercury is

$$\text{H}g\text{A}d + 2\text{H}g\text{O} + 3\text{HO}.$$

Oxamidide of copper is

$$2\,cu\text{A}d + cu\text{O} + 6\text{HO}.$$

Oxamidide of gold is

$$2\,\text{A}u\text{A}d_3 + \text{A}u\text{O}_3 + 6\text{HO}.$$

When, therefore, we come to examine the constitution of water of ammonia, a similarly large number of molecules may be expected to be contained in its equivalent group, and in the fact of all the oxamidides above described, and also that of silver, the analysis of which I was obliged to abandon, being the most dangerous and explosive bodies, we may trace the source of a facility of decomposition in the oxamidides of hydrogen, which prevents us from obtaining even the degree of definite constitution which has been found to exist in the hydrates of the chloride of hydrogen, although the approximation in the strongest water of ammonia to the formula $\text{NH}_3 + 4\text{HO}$ cannot be overlooked; and therein also we find the explanation of the want of success in obtaining, in an isolated form, the oxide of ammonium, which has always been, and must continue, an objection to the Berzelian theory.

The transition from the view of the constitution of sal ammoniac just described, to the corresponding theory of the salts with oxygen acids, is very simple, and will not require much exposition. Giving to the oil of vitriol the formula $\text{SO}_4 + \text{H}$, it will at once result that hydrogen combinations of that form should as easily unite with the amidide of hydrogen as with any of the corresponding oxides; and hence the ordinary sulphate of ammonia becomes $\text{H.SO}_4 + \text{HA}d$, the nitrate of ammonia $\text{HNO}_6 + \text{HA}d$. In its common form the sulphate of ammonia assumes two equivalents of water, and becomes $H.SO_4 + HAd.HO$, with which very many analogues will be found. Thus in the magnesian class we find the sulphate of copper uniting with ammonia in a similar manner to form the body $cuSO_4. + HAd$. In nickel there is $Ni.SO_4 + HAd$; and in the zinc combinations there is not merely $znSO_4 + HAd$, but $znSO_4 + HAd.HO$, resembling in constitution the ordinary sulphate of ammonia. It is very much to be regretted that the circumstance of water decomposing these bodies prevents the question of their isomorphism with the ordinary ammonia salts from being fully determined, but it is not improbable that future research may enable some instances to be examined.*

* In the Jahresbericht for 1837, (17th year,) page 139, Berzelius, in commenting on the interesting results of Heinrich Rose on the combinations of dry sulphuric acid and the chlorides of the alkaline metals, &c., speaks of the combination of sulphuric acid and sal ammoniac in the following terms, which, that work being but little circulated in Ireland, I shall here translate, as the opinions of that eminent chemical philosopher must affect considerably the judgment of chemists concerning the views which I have proposed.

" These facts are of great theoretical interest. They appear, if not expressly to answer, at least to give indications for the solution of a great variety of questions. That, for example, whether sal ammoniac consists of muriatic acid and ammonia, or of the metallic body, ammonium, and chlorine. The great analogy between chloride of potassium and sal ammoniac seems to me to speak plainly enough in this question, but distinguished chemists appear not to approve of this evidence, and prefer the former view as the more probable. If we consider the action of dry sulphuric acid on sal ammoniac as a new form of the question put in order to compel an answer, the answer given must negative the view of hydrochlorate of ammonia. Dry sulphuric acid, combined with ammonia, cannot be expelled by muriatic acid, and consequently has a greater affinity for it than the latter. It is hence clear, that if muriatic acid were present in sal ammoniac it should be expelled by the dry sulphuric acid. On the contrary, however, the acid unites with the sal ammoniac, and forms a body, which in all its relations corresponds to the compounds of the acid with the chlorides of potassium and sodium, and it is only by a higher temperature being applied that decomposition sets in, and there are formed dry sulphate of ammonia and free hydrochloric acid. My view may be rather keen-edged, but it appears to me that these experiments of Rose's declare with positive openness the sal ammoniac to be chloride of ammonium, and not hydro-chlorate of ammonia."—Page 141.

If we look upon the relation between ammonia and chloride of hydrogen as being in accordance with the old view, that of acid to base, then the criticism of Berzelius must be considered as possessing very considerable accuracy and force. But it has been my great object in the present section to show, that our views in this respect require a profound alteration. When we apply to the explanation of Rose's results the lights which we receive, in addition, from the change in our point of view, and that we consider the oxyamidide and chloro-amidide of hydrogen as related to each other, like

PROP. VI.—*That the ordinary ammonia salts ally themselves to the salts of the copper and zinc class, which contain two equivalents of oxide.*

The subject of this proposition is one of the most remarkable which I have been induced to adopt in the course of these researches, and the nature of the evidence in its favour will require a cautious and detailed examination of the individual instances of replacement by which it is supported.

I have pointed out already, briefly, that all those ammonia-copper, zinc, and nickel combinations which are formed by solution in water, must be looked upon as combinations of ordinary ammoniacal salts with metallic oxide and amidide of hydrogen, as well as occasionally still more water, at least in their crystallized condition. As the establishment of this principle becomes of great importance, I shall again sum up the proofs of it, and notice one or two examples, which were not at that time alluded to. The progress of the reaction, in which at first a pure ammoniacal salt and a basic metallic compound is always formed, indicates the nature of the resulting body very remarkably; and when we consider that the bodies generated by dry ammoniacal gas were in all cases quite different, the evidence becomes almost complete; likewise, where we find that in the quicksilver compounds the formation of the ammonia-quicksilver body occurs from the commencement, and we cannot trace any stage at which the deposition of a sub-

chloride and oxide of potassium, it appears quite natural that sal ammoniac should combine with acids, as chloride of potassium does in some instances, and that there should be $SO_3 + (HCl + HAd)$ as there is $SO_3 + (HO + HAd)$ equivalent to $2CrO_3 + KO$ and $2CrO_3 + KCl$. On this view there is no reason for the expulsion of chloride of hydrogen as being the weaker acid, but by heat the expulsion of HCl can easily be understood. We cannot, by heating $SO_3 + HO.HAd$, expel HO, without other effects complicating the result; but the reaction in the case of $SO_3 + HCl.HAd$ takes place with greater ease and completeness. The compound $SO_3 + HCl$, formed by Aimé, though not analyzed, evidently resembles $SO_3 + HO$; and by the addition of ammonia a compound of an equivalent character should be produced. Another similar case is the brown powder, $SO_3 + (CuO + HCl)$, which, when heated, gives $SO_3 + CuO$ and $H.Cl$, as there are $SO_3 + CuO.HO$ and $SO_3 + CuO.HAd$, which give precisely similar results. Berzelius appears to have understood from my description, that when dry ClH is passed over dry $CuO.SO_3$, the brown mass becomes moist from free sulphuric acid; that, however, is not the fact, water is set free only when the sulphate of copper is not dry; the brown mass does not fume nor grow damp; it does not give any indication of free acid. The body $SO_3 + CuO.ClH$ is perfectly definite and well characterized.

stance free from ammonia has occurred, some fundamental distinction must necessarily be drawn between the resulting ammonia bodies of the mercurial series and those containing copper, zinc, or nickel.

A remarkable example of this kind is furnished by nitrate of silver. When dry ammonia is passed over nitrate of silver it is absorbed in quantity, but by the application of a moderate heat it can be all again expelled. If an excess of water of ammonia be added to nitrate of silver there is obtained the crystalline compound analyzed by Mitscherlich and myself, and which, when heated, gives common nitrate of ammonia, metallic silver, and the elements of amidogene. Thus there are two bodies,

$$1.\ \mathrm{A}g\mathrm{O}\,.\,\mathrm{NO}_5 + 3\mathrm{NH}_3.$$
$$2.\ \mathrm{HO.NO_5.NH_3} + \mathrm{A}g.\mathrm{A}d.$$

And in the latter case the formation of the common ammoniacal salt and of the metallic amidide becomes quite manifest.

Recurring to the constitution of the ammonia sulphate of copper, there is

$$1.\ \mathrm{HO.SO_3.HA}d + \mathrm{C}u\mathrm{O.HA}d.$$

In the zinc series there is

$$2.\ \mathrm{HO.SO_3.HA}d + \mathrm{Z}n\mathrm{O.HA}d + 2\mathrm{HO}.$$

In the nickel series,

$$3.\ \mathrm{HO.SO_3.HA}d + \mathrm{N}i\mathrm{O.HA}d + \mathrm{HO}.$$

Here a property is found fully displayed, which in the ordinary ammonia salts is either latent, or else but feebly manifested, except when in combination; that is, the power of combining with water of crystallization, or with a group of equivalents of the same type, and capable of representing such. If we set out from the common sulphate of potash, and one form of sulphate of ammonia, quite anhydrous, the second gives to us in $\mathrm{HO.SO_3.HA}d + \mathrm{HO}$ the commencement of the series, the completion of which, for the ordinary salts of ammonia, must be sought in the common alums, where there is

$$(\mathrm{HO.SO_3.HA}d + 6\mathrm{HO}) + (\mathrm{A}l_2\mathrm{O}_3.3\mathrm{SO}_3 + 18\mathrm{HO}),$$

and in which $\mathrm{KO.SO_3}$ is similarly circumstanced.

The complex group, partly metallic oxide and partly ammonia, which occupics one portion of the formula, leads naturally to the study of still more remarkable cases of the operation of these principles.

The bodies

1. $HO.SO_3.HAd + HO.$
2. $HO.SO_3.HAd + ZnO.$
3. $CuO.SO_3.HAd.$
4. $ZnO.SO_3.HAd.$

and in the quicksilver compounds,

5. $HgO.NO_5.HAd + 2HgO.$
6. $HgO.NO_5.HgAd + 2HgO.$
7. $HgO.SO_3.HgAd + 2HgO.$

present to our view a series passing from common sulphate of ammonia to ammonia turbith, in which the successive stages of replacement of hydrogen by metal are so connected, and follow so naturally, that it appears to me very difficult to refuse consent to the proposition that the latter members are constituted on the type of the former, and consequently that we may have forms of ammonia salts, in which the oxygen and amidogene are combined, not with hydrogen, but with metal, and in which, therefore, the peculiarly basic character should preponderate.

If we now for a moment contemplate the formula of a double ammonia sulphate of that class, whose history has been cleared up by Graham, it will be found that some considerations of a most interesting nature will result from their relations to the group last noticed. The double sulphate of copper and ammonia is

$$HO.SO_3.HAd + CuO.SO_3 + 4HO.$$

Graham had himself suggested the following form for the ammonia sulphate of copper described by Rose,

$$CuO.SO_3.HAd + CuOSO_3 + 4HAd,$$

but only as a speculation, the state of our knowledge of the ammonia compounds then not allowing the proper demonstration of its truth. The majority of sulphates absorb, however, a whole number of equivalents of ammonia, thus there is

1. $zno . so_3 . HAd + 2 HAd.$
2. $cdo . so_3 . HAd + 2 HAd.$
3. $Nio . so_3 . HAd + 2 HAd.$
4. $coo . so_3 . HAd + 2 HAd.$

evidently corresponding to

5. $zno . so_3 . HO + 2 HO$, &c.

The close relation which has been thus shown to exist between the most intimately united portion of the amidide of hydrogen and of the constitutional water of the magnesian class of sulphates, may be rendered still more remarkably evident from the following examples.

Anthon has discovered a peculiarly hydrated condition of the sulphate of zinc, which has the formula $zno . so_3 + 3\frac{1}{2} HO$. It crystallizes in rhomboids, of which the exact form has not been determined; this salt appears to be produced under circumstances not yet completely known, but it would be most interesting to ascertain exactly its crystalline admeasurement. I consider that the halving of the equivalent of water in this salt results from precisely the same law as the absorption of half an equivalent of ammonia by dry sulphate of copper, and that its formula should be

1. $zno . so_3 . HO + zno . so_3 + 6 HO$,

the ammonia zinc sulphate being

2. $HO . so_3 . HAd + zno so_3 + 6 HO$.

In this salt, as I could not produce it at will, it was impossible to determine whether the half atom of water was more powerfully retained, so as to give the dry double salt

3. $zno . so_3 . HO + zno . so_3$,

as there is

4. $cuo . so_3 . HAd + cuo so_3$;

but every thing would lead us to suppose it to be in a state of combination differing from the rest.

In a family of the salts differing but very little from the ordinary alums, there will be found some very remarkable examples of the similarity of action of two equivalents of a magnesian protoxide, with oxide of kalium, or ammonia and water. This family was discovered by Klauer, who formed double sulphates of alumina with the protoxides of iron and nickel, with magnesia and oxide of zinc; and lately one of the most remarkable examples of this class, a double sulphate of alumina and protoxide of manganese was found forming a thick bed on the coast of Africa, and brought to this country, where it has been analyzed by Apjohn and by myself.

The general formula of this class, as has been accurately determined with the manganese and zinc members, is as follows: $RO =$ protoxide,

$$1.\ (RO . SO_3 . HO) + (Al_2O_3 + 3\,SO_3) + 24\,HO,$$

resembling accurately

$$2.\ (HO . SO_3 . HAd) + (Al_2O_3 + 3\,SO_3) + 24\,HO,$$

which it further assimilates itself to in taste and solubility.

The relation of the water of these alums to heat is very remarkable, and indicates very accurately the nature of their constitution. Thus by a temperature of 212°, eighteen equivalents of water are lost; by a heat of 300° there are given out six more; but the expulsion of the remaining equivalent requires a temperature equal to the melting point of lead, indicating the intenseness of the power with which it is retained; in fact the zinc alum may be looked on as composed of ordinary sulphate of zinc and ordinary sulphate of alumina,

$$\begin{array}{c} ZnO . SO_3 . HO + 6\,HO \\ Al_2O_3 . 3\,SO_3 + 18\,HO \\ \hline ZnO . SO_3 . HO + Al_2O_3 . 3\,SO_3 + 24\,HO. \end{array}$$

The form $MnO . SO_3 . HO + 6\,HO$ is not the ordinary condition of proto-sulphate of manganese, but it also can be obtained with that quantity of water.

The preceding considerations showing, with considerable probability, that two equivalents of an oxide of the magnesian class may replace in combination, and even affect isomorphism with an equivalent of the alkaline group, it may be proper to inquire how far evidence capable of illustrating the theory under examination can be collected from amongst the numerous species of minerals which are supposed to present cases of replacement of an alkali by an earth. In such cases the substitution may take place in two ways, which renders the demonstration of its occurrence much more difficult than it might at first be supposed; in the first class, the substituting equivalents being oxides of the same, in the second they being oxides of different bases. Thus in the former, two atoms of lime, magnesia, or of water; in the latter, one of lime and one of water, or one of magnesia and one of water, likewise lime and magnesia without water, lime and protoxide of iron, &c. The complication thus arising must render new researches for the determination of the point not only necessary, but very difficult; and hence, although I would look very sanguinely to a re-examination of the harmatome and zeolitic groups for a great accession to our accurate knowledge of this department of science, I have not been able to deduce from analyses at present recorded any definite results, except in one instance, which, however, in itself may be almost looked upon as conclusive.

This example consists in the group of minerals consisting of natrolite, mesolite, and scolezite, which constitute one of the best instances of isomorphism that has been as yet found, and are related to each other in constitution in a very simple manner: the natrolite being a hydrated silicate of soda and alumina, the scolezite being a hydrated silicate of lime and alumina, and the mesolite, probably a product of the crystallizing of the two together, being intermediate in constitution. Now the formulæ accurately given by analyses for the pure species are, thus,

$$\mathrm{N}ao\,.\,\mathrm{S}io_3 + \mathrm{A}l_2o_3\,.\,\mathrm{S}io_3 + 2\,\mathrm{HO}$$

and

$$\mathrm{C}ao\,.\,\mathrm{S}io_3 + \mathrm{A}l_2o_3\,.\,\mathrm{S}io_3 + 3\,\mathrm{HO}, \text{ or better,}$$
$$\mathrm{C}ao\,.\,\mathrm{HO}\,.\,\mathrm{S}io_3 + \mathrm{A}l_2o_3\,.\,\mathrm{S}io_3 + 2\,\mathrm{HO}.$$

Here the equivalency of $\mathrm{C}ao\,.\,\mathrm{HO}$ to $\mathrm{N}ao$ is most remarkable, and certainly must be allowed to go a great way towards confirming the views regarding the nature of the compounds of ammonia, from which the analogy of $\mathrm{NH_2.H}$ to $\mathrm{H.O}$, and hence to $\mathrm{C}ao$ or $\mathrm{M}go$, and of $\mathrm{NH_4O}$ to $\mathrm{C}ao\,.\,\mathrm{HO}$, &c. was first arrived at.*

* Since the above views were completely formed, and the memoir read, I was singularly struck by finding in the Elemente der Crystallographie of Gustav Rose, the same view suggested of the replacement of soda, not by lime, but by its hydrate. Intending to commence an examination of the zeolitic group under the point of view noticed above, I began by the study of their crystallographic relations, to which I had not before applied myself, and selected his work as the system best adapted to my purpose. In speaking of the composition of wernerite, (page 158,) the following passage occurs, which, as the work is not very common in Ireland, I shall translate. "The above

During the examination of the various classes of compounds of ammonia, which the objects of these researches rendered necessary, a variety of results were obtained, which are calculated to throw light on the relation in which the ammoniacal salts stand to the ordinary basic salts of the same acid, and likewise to illustrate the connexion between the corresponding so called neutral and basic salts. In the cases of the nitrates of mercury, my observations have the effect of extending to that metal the law discovered by Graham for the nitrates of the metals of the magnesian class, but as that distinguished chemist has not deduced any general idea of the constitution of the basic sulphates from his observations, I shall briefly suggest such ideas as have occurred to me from my own investigations.

The general principle that the transition from the neutral to the basic condition in salts takes place by the replacement of water by metallic oxide, has, as I conceive, received the fullest confirmation; but I do not consider that the corresponding substitution of water for metallic oxide, which exists so markedly in the sub-nitrates of copper and bismuth can be looked upon as forming a general law. Thus there certainly does not appear the same perfect symmetry between

$$\mathrm{HO}\,.\,\mathrm{NO}_5 + 3\,\mathrm{H}g\mathrm{O} \text{ and } \mathrm{H}g\mathrm{O}\,.\,\mathrm{NO}_5 + \mathrm{H}g\mathrm{O} + 2\,\mathrm{HO}$$

as between

$$cu\mathrm{O}\,.\,\mathrm{NO}_5 + 3\,\mathrm{HO} \text{ and } \mathrm{HO}\,.\,\mathrm{NO}_5 + 3\,cu\mathrm{O}\,;$$

and although I do not possess absolute proof of the existence of a sub-nitrate

formula is that which Hartwell has established. According to his analyses, lime and soda replace one another in indeterminate proportions, and are consequently placed in the formula under one another, as isomorphous bodies, although there is not as yet known any positive example of the isomorphism of lime and soda. The sulphate of soda or thenardite does not appear to be isomorphous with anhydrite, and the analyses of mesotype by Gehlen and Fuchs, show perfectly that lime and soda may replace each other, but that in this case, the quantity of water in the compound also changes, so that one atom of soda can be isomorphous only with an atom of lime + an atom of water, which must consequently be assumed in all other zeolites where lime and soda appear to replace each other, as, for example, in the chabazies." It is singularly interesting to find, that starting from an origin apparently so remote as the composition of white precipitate, I have been gradually conducted to the development of the same principle as had already, though unknown to me, been announced, even though but as a suggestion, by an authority so deservedly high in chemistry and mineralogy as Gustav Rose.

having the four equivalents of oxide all alike, yet I cannot consider such an arrangement as being excluded.

Indeed an idea which was suggested to me by the mercurial nitrates is, that the constitution of the nitrates may be better shown by writing the formula of their class as follows :

$$\mathrm{RO} \,.\, \mathrm{NO_5} \,.\, \mathrm{RO} + 2\,\mathrm{RO}.$$

R being either water or metallic oxide. There is then

$$\mathrm{H}g\mathrm{O} \,.\, \mathrm{NO_5} \,.\, \mathrm{H}g\mathrm{O} + 2\,\mathrm{HO},$$
$$\mathrm{H}g\mathrm{O} \,.\, \mathrm{NO_5} \,.\, \mathrm{HO} + 2\,\mathrm{H}g\mathrm{O},$$

and the red basic nitrate,

$$\mathrm{H}g\mathrm{O} \,.\, \mathrm{NO_5} \,.\, \mathrm{H}g\mathrm{O} + 2\,\mathrm{H}g\mathrm{O} + 2\,\mathrm{H}g\mathrm{O}.$$

and still further,

$$\mathrm{H}g\mathrm{O} \,.\, \mathrm{NO_5} \,.\, \mathrm{HA}d + 2\,\mathrm{H}g\mathrm{O},$$
$$\mathrm{H}g\mathrm{O} \,.\, \mathrm{NO_5} \,.\, \mathrm{H}g\mathrm{A}d + 2\,\mathrm{H}g\mathrm{O}.$$

As in the copper and bismuth nitrates, no water whatsoever can be separated without a corresponding quantity of acid being set free, it is difficult to ascertain whether one atom of the water is more firmly attached to the acid than the other two; but in the case of nitrate of magnesia, Graham has found that two of the equivalents of water may be separated much more easily than the third, and hence its formula should be, as in the quicksilver series,

$$\mathrm{M}g\mathrm{O} \,.\, \mathrm{NO_5} \,.\, \mathrm{HO} + 2\,\mathrm{HO}.$$
$$\mathrm{M}g\mathrm{O} \,.\, \mathrm{NO_5} \,.\, \mathrm{HO} + 2\,\mathrm{M}g\mathrm{O}.$$

Thus connecting still further mercury with the metals of the magnesian class. This form of expression for the nitrates connects them much more closely with the sulphates than the older view, and the equivalent to the right of the acid evidently replaces the saline water of the magnesian sulphates. Thus a sulphate of that group is generally, though not always,

$$\mathrm{RO} \,.\, \mathrm{SO_3} \,.\, \mathrm{HO} + 2\,n\mathrm{HO},$$

n being a whole number.

In the sulphates the most common form of basic constitution approaches still more closely to the type of basic nitrates than in the neutral state ; thus,

1. $cuo \,.\, so_3 \,.\, cuo + 2cuo.$
2. $zno\, so_3 \,.\, zno + 2zno.$

which are those most easily formed, and most permanent.

A great number of circumstances conspire to render the derivation of the basic sulphates of the magnesian class, from the neutral condition, exceedingly complicated. Thus the neutral salts crystallize with quantities of water variable within very extensive limits, and the proportion of metallic oxide by which it may be replaced, is subject to fluctuations equally wide: moreover, the replacement of the water by metallic oxide may be but partial, and hence the different hydrated conditions in which the basic salts exist. From these causes may be deduced the possible existence of a very extensive series of basic sulphates varying considerably in type, and subject only to the one restriction, that in all their different conditions the sum of the equivalents of water and metallic oxide shall always be equal to the sum of the same constituents in some one of the forms in which the neutral salt may crystallize. So that the general expression of the class becomes

$$\text{RO} \,.\, \text{SO}_3 \,.\, \left.\begin{matrix}\text{R}\\\text{H}\end{matrix}\right\} \text{O} + 2n \left.\begin{matrix}\text{R}\\\text{H}\end{matrix}\right\} \text{O}.$$

$\left.\begin{matrix}\text{R}\\\text{H}\end{matrix}\right\}$ indicating the sum of the mutually replacing elements. In the synopsis of the analytical results of the basic sulphates contained in the sections on the copper and zinc compounds, the instances given can be so immediately compared with the above expression, that it is not necessary to reinsert them here.

Although the general form of the crystallized chlorides of the magnesian group of metals, as was well shown by Graham, consists in the adhesion of pairs of equivalents of water, yet in the construction of the basic chlorides or chloroxides the form pointed out for the nitrates and some basic sulphates is adopted; thus, the ordinary chloride of copper, $cucl + 2\,\text{HO}$, cannot be obtained in combination with more water, but the tendency to assume the fourth molecule is shown in its basic forms, thus it may become

$$cucl + 2\,cuo,$$

and thence

$$cucl + 2\,cuo + cuo$$
$$cucl + 2\,cuo + \text{HO}$$
$$cucl + 2\,cuo + \text{HA}d$$

as has been already noticed in another point of view.

In quicksilver there is the oxychloride

$$\text{hg}cl + 2\,\text{hg}o + \text{hg}o,$$

and then

$$\text{hg}cl + 2\,\text{hg}o + \text{hg}\,\text{A}d$$

evidently corresponding; but in most instances the basic chlorides follow, like the sulphates, the form of the hydrated neutral conditions, and hence there is

$$zncl + 6\,zno$$
$$cucl + 4\,cuo$$
$$cucl + 2\,cuo$$

as there are two, four, or six atoms of water in the crystallized conditions of various chlorides.

Prop. VII.—*That if chlorine could be separated from sal ammoniac, the residual* NH_4 *should be regarded as* $NH_2 + 2\,H$, *sub-amidide of hydrogen, as when by removing the chlorine from white precipitate, the sub-amidide of mercury,* $NH_2 + 2\,Hg$, *formed by the action of water of ammonia on calomel, should remain.*

The discussion of this proposition leads to some considerations as to the nature of the so called compound radicals, which of late years have played so distinguished a part in the progress of chemical philosophy. The views which I shall put forward I offer with considerable hesitation, as not resting directly upon experimental evidence, but resulting from the peculiar manner in which my researches have induced me to contemplate the nature of those hypothetic bodies.

The fundamental idea that a compound body might so manifest its affinities as to simulate the properties of an undecompounded substance, received its first conception, as well as proof, from the beautiful discovery of cyanogen by Gay Lussac, which continues even up to the present day the most glaring instance of the truth, as well as the most excellent example of the nature of the theory of compound radicals.

The extension of the principle involved in the very existence of cyanogen, to explain the constitution of classes of bodies of organic origin presenting strong analogies to the cyanides, although the compound radicals of their series could not be successfully isolated, gave to the theory of organic chemistry great clearness and consistency, and was indeed philosophically just, since from the facility of decomposition of cyanogen in a variety of ways, we must infer that many bodies of similar nature may be so much more easily decomposed, that in our ordinary modes of operating on them their preservation becomes impossible, precisely as the existence of cyanogen had escaped the acuteness of Proust, of Berthollet, and others, who had experimented on prussic acid at former times. I therefore do not hesitate to place the theory of compound radicals amongst the greatest benefits which chemistry has lately received, and hope with confident expectation for the addition of very many new examples to the list, hitherto restricted to cyanogen and mellon.

But what is the constitution of a compound radical? does it consist of a group, beyond which we cannot go without reducing it to its merely undecomposable constituents? or has it, again, a symmetricity of constitution like the whole mass from which it had been eliminated. I shall not touch upon this question as affecting cyanogen, benzoyl, or similar bodies, limiting myself altogether to the examination of how far our ideas of the nature of ammonium may be affected by that point of view.

In sal ammoniac, the chlorine is certainly united with a body which replaces potassium, and if we could discover circumstances under which the chlorine might be transferred to another substance, leaving all the hydrogen and azote undisturbed, then the ammonium would be isolated; but let us examine what this ammonium should be. The sal ammoniac is chlor-amidide of hydrogen. If the chlorine were removed, the amidogene should remain combined evidently with twice as much hydrogen as constitutes ammonia, and this body, sub-amidide of hydrogen, might well be able to represent in combination, and to combine with, metals. This partial participation in metallic properties is found in other sub-combinations, as in the sub-oxides of copper and of mercury, and hence the generation of the ammoniacal amalgam, its low specific gravity, the sub-amidide of hydrogen being probably gaseous: an extension of this view might illustrate the condition of the isomorphism of two equivalents of one oxide with one of another, (as pointed out in the alums and certain minerals in the last proposition,) the former, perhaps, assuming the form O (ROR): the sub-oxide represented in the brackets relating itself as a compound radical to the oxygen outside. Hence, likewise, a consideration of the problem, whether a second oxide be a combination of metal with oxygen, or of oxygen with the first oxide, which I must consider as decided by the circumstance of the atomic weight containing one or two equivalents of oxygen. Thus I look upon the study of the salts of mercury as decisive upon the red oxide of that metal being protoxide, but the examination of the compounds of manganese assigns to the black oxide the form $(\mathrm{M}n\mathrm{O}) + \mathrm{O}$.

A remarkable fact in the history of the alkaline salts suggests an extension of the views here discussed, which is thrown out as a speculation, and to which I do not wish to attach otherwise importance. The sulphate of ammonia may be written on the ammonium theory, $\mathrm{SO_3} + \mathrm{O}.(\mathrm{NH_4})$, or $\mathrm{SO_3} + \mathrm{O}(\mathrm{HA}d\mathrm{H})$; and the ammonium being a basic amidide, it results that the ammoniacal salts are all basic salts; hence the condition which the salts of the magnesian class may be made artificially to assume is that naturally belonging to those of the ammoniacal series. Now as the ammonia and potash salts assimilate so completely, the speculation may be hazarded, that research will discover in potassium a structure analogous to that which I have argued to exist in the so called ammonium, and the result may show that the reason of the alkalies not producing basic salts, arises from the circumstance of their salts being already basic in their common form.

SUPPLEMENTARY NOTE

ON A COMPOUND HITHERTO CONSIDERED AS WHITE PRECIPITATE.

Some time since I learned that Professor Woehler had found that the white precipitate in the possession of some Hanoverian apothecaries differed in many important particulars from that which formed the subject of my researches, as well as of the experiments of verification made by Ullgren. The body in question had been prepared by precipitating a solution of sal alembroth by potash in the cold. The precipitate which is produced, resembles externally the true white precipitate so completely as to have been always taken for it, and hence in many pharmacopœias this process is given for preparing white precipitate for medicinal purposes. It is, however, quite different in its nature, and as its analysis is of importance as well in a practical as in a theoretical point of view, the following brief description of its nature is subjoined :

When heated it fuses into a clear liquid, giving off at the same time azote and ammonia, but no water, if the precipitate had been completely dried. The fused substance sublimes ultimately in a mass partly transparent like gum, and partly white and opaque. When the sublimed mass is treated with water, it in part dissolves, calomel remaining undissolved, the solution is neutral, and on examination is found to contain sal ammoniac and sublimate. If this new white precipitate be boiled in water there results the same yellow powder, which is produced by boiling the genuine white precipitate; but the sal ammoniac is formed in the liquor in much larger quantity.

The methods of analysis pursued were precisely the same as those described in the memoir on white precipitate, and consequently it is unnecessary to repeat the details of them here. The results of three analyses were:

		I.	II.	III.
Mercury	=	65.42	66.27	65.74
Chlorine	=	22.05	22.70	22.95
Ammonia	=	10.65	11.01	10.94
		98.12	99.98	99.63

These numbers lead directly to the formula $\mathrm{H}g\mathrm{c}l + \mathrm{NH}_3$, which should give

$\mathrm{H}g$	=	101.40	65.86
$\mathrm{c}l$	=	35.42	23.01
NH_3	=	17.14	11.13
		153.96	100.00

This body may therefore be looked on as consisting of an atom of sublimate and one of ammonia. Now the result of passing ammonia over sublimate is to generate a white substance, $2\,\mathrm{c}l\mathrm{H}g + \mathrm{NH}_3$, which is evidently a kind of double chloride, $\mathrm{H}g\mathrm{c}l + \mathrm{H}g\mathrm{A}d\mathrm{H}.\mathrm{c}l$, similar to many bodies already noticed in these researches, as

$$\mathrm{z}n\mathrm{c}l + \mathrm{z}n\mathrm{A}d.\mathrm{H}\mathrm{c}l$$
$$\mathrm{c}u\mathrm{SO}_4 + \mathrm{c}u\mathrm{A}d.\mathrm{HSO}_4$$

This body is likewise of interest, as standing midway between sal ammoniac and the real white precipitate, and serving to link bodies apparently so dissimilar still more closely to the principles of the theory of the ammonia compounds developed in the present memoir; the chlor-amidide of hydrogen, $\mathrm{H}cl + \mathrm{HA}d$, and the chlor-amidide of mercury, $\mathrm{H}gcl + \mathrm{H}g\mathrm{A}d$, being connected by the intermediate chlor-amidide of mercury and hydrogen, $\mathrm{H}gcl + \mathrm{HA}d$.

I had remarked long since, that by the addition of sal ammoniac to the water in which the real white precipitate is boiled, its decomposition, or at least the formation of the yellow powder is prevented. The white precipitate remains white, but its nature is totally altered; it is converted altogether into the new compound, and it will be seen that its composition would be represented, supposing it to be formed by the union of the two substances which had been put in contact, for $2\,(\mathrm{H}gcl.\mathrm{NH}_3) = (\mathrm{H}gcl.\mathrm{H}g\mathrm{A}d + \mathrm{H}cl.\mathrm{HA}d)$.

Such a mode of representing its nature would likewise explain its various properties, but I prefer the view first described, and look upon this body as simply expressed by $\mathrm{H}gcl + \mathrm{HA}d$. I would propose for it the empirical name of Woehler's white precipitate, and if one founded on composition be deemed necessary, that of the hydrargyro-chlor-amidide of hydrogen.

A reaction which I have lately observed, and which as a remarkable property of white precipitate, is worthy of being noticed, is, that when the chlor-amidide of mercury is boiled with an excess of chloride of copper, it is totally converted into sal alembroth, and there results brunswick green. The reaction appears to be as follows:

$$8\,cucl + 3(\mathrm{H}gcl + \mathrm{H}g\mathrm{A}d) + 6\,\mathrm{HO} = 2\,(cucl + 3\,cu\mathrm{O}) + 3(2\,\mathrm{H}gcl + \mathrm{NH}_4cl).$$

In the sal alembroth thus produced the proportions of its ingredients are different from those of the more common form: the sublimate containing twice as much chlorine as the sal ammoniac. It is, however, quite definite, and can be easily procured by dissolving together sublimate and sal ammoniac in the proper quantities. It crystallizes in two forms, one rhomboidal, the other in long silky needles; in the former condition the salt is dry, in the latter it retains an equivalent of water. Frequent analyses gave for their composition the formulæ

Rhomboidal state $= 2\,\mathrm{H}gcl + \mathrm{NH}_4cl.$

Fibrous state $= 2\,\mathrm{H}gcl + \mathrm{NH}_4cl + \mathrm{HO}.$

The ordinary form being, as is well known, $\mathrm{H}gcl + \mathrm{NH}_4cl + \mathrm{HO}$.

II. *Description of the Cydippe Pomiformis mihi,* (*Beroë Ovatus,* Flem.) *with Notice of an apparently undescribed Species of Bolina, also found on the Coast of Ireland. By* Robert Patterson, Esq., *Member of the Natural History Society of Belfast.*

Read 10th December, 1838.

IT is proposed to give in the present paper some account of the appearance, organization, economy, and habits of a Beroë, not uncommon on our Irish coast, in the hope that such details may prove interesting with regard to the species described, and may be of some value as illustrative of the family to which it belongs.

These observations were commenced in the month of May, 1835, at which time I resided in the immediate vicinity of the small sea-port town of Larne, in the County of Antrim. My lodging was situated on the small peninsula termed the Corran,* and nearly midway between the two stations, whence ferry-boats ply to the opposite peninsula of Island Magee. Through the narrow channel, across which these boats are continually plying, the tide runs with great rapidity into Larne Lough. Hence I had, by means of the ferry-boats, an easy mode of taking, at all hours during the day, the small *Medusæ* and *Crustacea,* which the flow of the tide placed within reach of a small canvass towing net. As the Beroës could thus with facility be procured, and were to me highly attractive, my sitting-room, for between two and three weeks, was never without some of them. They were kept in glass jars, the water in which was changed twice each day. The particulars which I then observed, were published in the Edinburgh New Philosophical Journal for January, 1836, and reasons adduced for regarding the species as distinct from the Beroë Pileus, the only tentaculated Beroë then regarded as British.

* This word in the Irish language signifies "Reaping Hook," to which implement the little peninsula has a striking resemblance in form.

The ensuing summer I again visited the same locality, and had the pleasure of taking a Beroë similar in size to the one formerly described, but exhibiting very conspicuously an arrangement of whitish coloured vessels, which had their origin near the lower part of the stomach, and branched off to the several bands of cilia, one vessel running out to each band, and joining it not very far from the centre.

On the 24th of June, 1837, I was again at the Corran, and succeeded not only in taking three Beroës, exhibiting this singular structure, but with the assistance of a friend was enabled to have it delineated. These drawings were unfortunately mislaid before any more finished representation could be executed.

On the 8th of the next month, in Strangford Lough, I again took an individual of the same description, but the circumstances under which I was then placed, prevented its being subjected to any critical examination.

Its occurrence on so many different occasions excited the hope that it would again be met with; and when at the beginning of June, 1838, I returned to my former lodgings at the Corran, I felt desirous of being able to observe its peculiarities, and ascertain its species. This desire was augmented by a careful perusal of Doctor Fleming's paper, read before the Wernerian Society of Edinburgh 18th November, 1820, in which he describes a Beroë, subsequently designated, in his History of Bristish Animals, *B. ovatus.* This animal appeared to be furnished with vessels similar to those I had observed, but it differed from mine in being destitute of tentacula. The following is an extract from Dr. Fleming's description:

" The tube which conducts from the mouth to the centre of the body, and is prolonged in its axis to the summit, had on each side a compressed organ adhering to its walls. These terminated in the centre, each in an ovate head, apparently containing air. Immediately below each head, there were numerous twisted vessels, some of which contained a reddish fluid. The tube which descended from the summit, as it approached the centre, suddenly expanded, and sent off a branch to a vesicle on each side, after which it appeared to unite with the one from the mouth. Each of the lateral vesicles terminated below in a blind cavity, which contained a glandular body, to the upper surface of which several white threads were attached. The upper extremity of each vesicle was open, and terminated on the surface on each side, in the space between two ribs.

From each side of the vesicle, near its connexion with the central vessel, there arose a tube, which after dividing, sent a branch to each contiguous rib. The cavity of these tubes, at their union with the ribs, appeared to be filled with a whitish coloured pulp. Each rib is furnished with a tube uniting with it near the middle. In consequence of this peculiar structure, I could easily observe the water enter the tube at the summit, pass into the lateral vesicles, and go out at their external openings; and in some cases the motion of the current was reversed."

On the 10th of June, 1838, I had an opportunity, for the first time, of examining, under a lens, one of the Beroës exhibiting the peculiar ramiform structure already noticed. The animal was lying, like that observed by Dr. Fleming, with the mouth downwards, and evidently in an exhausted state. To my great satisfaction I observed the particles of fluid in motion, nearly in the manner that author has described, and in the vessels close to the stomach could observe there were two currents flowing in opposite directions. The same was visible in the whitish coloured vessels, going out to the bands of cilia. It was not apparent in the "lateral vesicles," they were filled with water, which moved at times backwards and forwards, but did not exhibit the active and continuous current presented by the other parts. That water issued from them was, however, obvious, by the effect visible on the fluid adjoining the terminal aperture, and exterior to the body of the animal. While examining one of the "glandular" bodies, I noticed that it did not always retain the same appearance, but was capable of expansion and contraction, and that on one occasion it was extended almost to the surface of the animal, moving within one of the "lateral vesicles," and approaching its external orifice. I waited in hopes that both of these "glandular" bodies would be still more fully thrown out, and would prove to be tentacula; but the inertness of the animal prevented at that time the fulfilment of my expectation. Next morning another Beroë was taken, vigorous and perfectly uninjured, and with the whitish ramiform vessels equally conspicuous as in the previous specimen. In the course of a few minutes it unfolded to my view its graceful and ever varying tentacula, furnished with delicate filaments, and exhibiting a ceaseless variety of outline.

The presence of the tentacula removes the animal from the genus Beroë of Fleming to the Pleurobrachia of the same author. His inaccuracy in the present instance was occasioned by his description having been drawn up from an examination of a single individual, "found in the Frith of Tay, in a pool left by the tide." The animal was then in an exhausted state, when the tentacula would naturally be retracted within the body. Dr. Fleming referred this species doubtfully to the ovata of Baster, but as they form not only distinct species, but belong to different genera, it is necessary to substitute another appellation, and as such I propose "Pomiformis."

Lesson, in his paper "Sur les Beroides,"* conjectures that the Beroë described by Fleming, might eventually be found, as is now the case, to belong to those bearing tentacula. The words in which this idea is expressed are the following :—" Peut-etre est ce au Cydippe globuleux qu'appartient l'espece trouvée par le Dr. Fleming (Mem. Soc. Wer. t. iii. p. 400,) dans le detroit de Tay, et qui n'avait point de prolongemens."

Having ascertained the identity of the Irish Beroë with that of Fleming, my next object was to have such drawings and descriptions prepared as would distinguish it from the species I had formerly described, and which in size and external appearance it precisely resembled. For this purpose I brought up with me in sea-water to Belfast, three of each, and hastened to a friend, to whose pencil I had been indebted on similar occasions. On my arrival it was found that the white radiating vessels of the C. Pomiformis, which had been so conspicuous when the animal was first taken, were scarcely perceptible. I took the earliest opportunity of procuring a further supply, but found that at the end of a few hours these distinguishing whitish coloured vessels were no longer visible. Knowing, however, their situation, I examined them under a lens, and though the vessels had lost their whiteness, saw in each the circulation of the fluid going on as usual. I then took eleven of the Beroës, in which no apparatus of the kind was conspicuous, and on subjecting them to a similar scrutiny, had the satisfaction of discovering that the same structure existed in all, and consequently that Dr. Fleming's Beroë, of the capture of which we have no record, save that of a single individual in 1820, was identical with that which I had taken so frequently, during successive years, at the entrance to Larne Lough.

In bringing together, under several heads, the observations made at various

* Annales des Sciences Naturelles, tome v.

times on this Beroë, it is necessary to make frequent reference to its congener, the Pleurobrachia Pileus, Flem., Beroë Pileus of Lamarck, Cydippe Pileus Eschscholtz, Cydippe Globuleux, Blainville and Lesson, that the several points of accordance or of difference may be enumerated as the description proceeds. Dr. Grant's interesting and valuable paper "On the Nervous System of Beroë Pileus, Lam., and on the Structure of its Cilia,"* has rendered that species well known to naturalists, and furnished a standard, with which the one here recorded may with facility be compared.

In size it is from two to nine lines in length, and about a third less in breadth. The general form is oval, but in some it is nearly globose, and in others flattened towards the poles, and similar in shape to an orange. The difference is to be attributed to a contractile power possessed by the animal, and not to any permanent diversity in form. The body is transparent and colourless, with the exception of the reddish coloured intestinal vessels noticed by Dr. Fleming, and which present a different aspect in different individuals.

The eight bands to which the cilia are attached extend about three-fourths of the distance from the mouth to the anus, but approach more nearly to the latter, and diminish in breadth towards either extremity. In C. Pileus there are about forty in each band; in C. Pomiformis the number in some individuals amounted only to fifteen, and in none which I observed did it exceed twenty-seven. Along each band a cord or slight ridge extends, dividing it longitudinally into two equal parts. The filaments on each band consist therefore of two parcels, which in general move simultaneously, although each portion possesses a separate and independent power of motion.

Dr. Grant remarks, that the cilia of C. Pileus are the largest he had yet met with in any animal, and states that "they are not single fibres, but consist of several short straight transparent filaments, placed parallel to each other in a single row, and connected together by the skin of the animal, like the rays supporting the fins of a fish. Viewed with the aid of a lens, the parallel fibres appeared like transparent tubes, sometimes a little detached from each other at their free extremities by injury done to the connecting membrane, and at these parts the isolated spines projected stiffly outwards. When the fins were quite

* Trans. Zool. Soc. vol. i. p. 9.

entire, the membrane connected the tubular rays to their extremity, where the fin presented a slightly rounded outline."

In C. Pomiformis the cilia present appearances very dissimilar to the above. In many individuals the filaments are not connected by any membrane, but appear numerous, flat, tapering, and slightly recurved towards the extremity. In others they are covered by a transparent membrane, divided as usual into two equal parts, and showing in each half but one or two divisions. They never exhibit an entire and unbroken surface, nor a continuous and regular margin. It was natural to suppose that a membranous covering might originally have existed in all, but had been abraded or torn, and thus caused the apparent diversity which the cilia exhibited in the number of their subdivisions; but this conjecture was shaken, by observing that specimens of less than the average size, and which might be presumed to be young, presented the same want of uniformity. The dissimilarity which prevails in this particular among different Beroës, will however be better estimated by a glance at the annexed figures, than by any detailed description.

The entire cilia, never for more than a moment remain perfectly at rest, unless when the animal is in a very exhausted state, and may hence be presumed to be organs of respiration as well as of locomotion. Sometimes, however, those of one or two continuous bands will vibrate, while all the remainder are still; or be at rest, while all the others are in motion. At times a slow vibration will commence at one extremity of a band, and pass along it, like the wave which can be impelled along an extended piece of cloth, or like the undulations of a fluid. Hence it is obvious that the Beroë can direct the aqueous currents which pass along the base of the cilia into any particular band, and can regulate at pleasure the velocity of their undulations. In the larger species, which I have named Bolina Hibernica, these currents are very conspicuous, and may be seen under* each band, one ascending, the other descending at the same time with great regularity.

* The size of this ciliograde varies from little more than half an inch to nearly two inches diameter; its figure is diversiform, being nearly round, oval, or cylindrical, but most generally somewhat compressed. The lobes at each side of the mouth, at times very protuberant, giving to the animal a rudely cordate form, like the Mnemie de Schweiger. (Vid. Blainville, pl. 8, fig. 4.) The surface smooth.

There are eight rows of cilia, the alternate ones much shorter than the others. The cilia are

Dr. Sharpey remarks, "in the Beroë, and others of a similar form, the cilia* point towards the closed extremity of the body, so that the opposite or open end is carried forward." In the two species which have fallen under my observation, the cilia, when at rest, point not to the closed but to the open extremity of the body, and as they strike downwards towards the closed extremity, the animal is propelled forward in the contrary direction.

The tentacula of these animals were, next to the cilia, the most attractive parts of their organization. They were seldom displayed immediately after the

detached, flexible, tapering, pointing upwards towards the mouth. At the upper extremity of each of the shorter bands is a circular orifice, with a ciliated margin. From each of these four apertures issues a singular aliform or auriform appendage; these are regarded by Mertens as tentacula covered with skin. Their appearance is extremely beautiful, both from their transparency and from the numerous minute delicate pointed cilia along their edges. Their aspect was ever changing. When first viewed they were pointed, erect, and hollowed longitudinally, so as to form a miniature representation of the ears of a horse. At other times they extended horizontally from the body of the animal, or were seen hanging loosely down like the ears of a lap-dog, or curved like the petals of the Martagon lily.

Between the 6th and the 18th of June, 1838, I took thirty-two specimens in a canvass towing-net, at the entrance to Larne Lough, County of Antrim. It had not fallen under my observation during any of the three previous summers, during which I had paid occasional visits to the same locality; nor was it met with after the date mentioned. On showing to Robert Ball, Esq. of Dublin, and William Thompson, Esq. of Belfast, several drawings of it taken from living specimens, I had the satisfaction of learning from these gentlemen its occurrence on other portions of the coast, it having been found by them at the island of Lambay, near Dublin, on the 1st of June, 1838, (or about the same time it was observed on the Antrim coast,) by Mr. Thompson in Strangford Lough, on the 3rd of July, (where it was in vain sought for by the writer on the 7th of August;) and by Mr. Ball a single specimen was taken at Youghal in June, 1837.

My object in making known, at the present time, its existence on the Irish coast, is to enable me to refer to it for the purpose of comparison and contrast with the C. Pomiformis. At a future period I hope to bring forward a detailed account of its structure and economy. Meantime I refer it, though with some doubt, to the genus Bolina of Mertens, (Mem. Acad. Imp. des Sciences St. Petersbourg, t. ii. p. 513,) established by him as a connecting link between the Callianyræ and the true Beroës; and as it has not been recorded as British—as it is distinct from the two species of Bolina described by Mertens—and is not noticed by any other continental writer, to whose works I have had access, I propose to give it provisionally the specific name Hibernica. If undescribed, this title will record the locality where it was at first observed; if already known, it will prove a convenient synonym, indicative of its occurrence on the Irish coast.

* Article "Cilia" in Cyclopædia of Anatomy and Physiology.

Beroës had been taken from the net, or while the glass vessel in which they were kept was crowded by the number it contained. When, however, not more than five or six were placed there, the tentacula were thrown out to their fullest extent, and were occasionally above six times the longest diameter of the body. In two instances they even exceeded these proportions; for a Beroë of less than five lines in diameter, exhibited them four inches in length, and one not exceeding six lines in diameter protruded them to the extent of five inches, as actually measured by a rule applied to the side of the glass vessel, from the top of which the tentacula extended downwards. Dr. Grant, in the paper already quoted, remarks,—"They extend from two curved tubes, placed near the sides of the stomach, which pass obliquely downwards and outwards, to terminate between two of the bands, at some distance above the mouth. * * * These tubes have a sigmoid form, and are shut and somewhat dilated at their upper extremity." In the Irish species the tubes are not curved in the form described, and their external orifice is at some distance, not from the mouth, but from the anus, agreeing in this particular with Blainville's description of their position.* The tentacula in both "consisted of two thin white filaments, round, and tapering to a very fine extremity." "Along their whole course they present," says Dr. Grant, "minute equidistant filaments, extending from their lower margin, which coil themselves up in a spiral manner, and adhere close to the tentacula, when they are about to be withdrawn into their sheaths or tubes." The filaments were in some individuals not less than half an inch in length, and of a delicate pinkish colour; and even so many as fifty may occasionally be reckoned on a single tentaculum. Most accurately has Dr. Grant remarked, "The tentacula are often thrown out from their tubes to their full extent by one impulse, and the slow uncoiling of the slender serpentine filaments from their margin, is then very beautiful; when coiled up they appeared like very minute tubercles along the side of the tentaculum." Of course, in particular points of view, they presented a moniliform appearance; and sometimes, while the filaments on the upper half of the tentaculum were seen under this aspect, those in the lower half were like delicate hairs or cilia, waving from the edge. In this respect, however, they were incessantly varying, and the tentacula, at the same time, were continually

* Manuel d'Actinologie, p. 150.

assuming new aspects, being retracted either separately or together, and thrown out in the same diversified manner. It is scarcely possible to convey, by any description, an idea of the beauty and diversity of their forms. They seem endued with exquisite sensibility, which, however, is not always equally delicate. At times the slightest touch will cause a tentaculum to be drawn back into its tube, with a sudden jerk; at other times it is apparently unfelt. The Beroës never seemed poised, or supported in the water by their tentacula. In one instance, however, they were extended to the bottom of the vessel, where they seemed to act as suckers, and formed fixed points, whence the animal rose and fell at pleasure, and appeared as if moored by these delicate and novel cables, the mouth being retained in the usual erect position.

What are the functions of these singular organs, is a natural inquiry. My friend, Robert Ball, Esq. of Dublin, states, that he regards them as organs of prehension. This is the view taken by Blainville, when he speaks of them as "servant pour attirer vers la bouche la proie qui s'y est attachée, probablement par une mâtière glutineuse."* Though unable to offer any more plausible conjecture, I cannot consider this opinion correct, as applied to the present species, as during all my observations I have never seen them thus employed, and from the comparative proximity of the orifices whence they issue to the anal extremity, the tentacula float behind the animal, and never approach the mouth, except at those times when the Beroë permits itself gradually to sink without reversing its previous position in the water.

"The mouth and œsophagus," as Dr. Grant remarks, "are wide; and the latter continues so to the stomach, which extends to the centre of the body. * * * There are four prominent membranous lobes placed around the mouth, which the animal can retract at pleasure." In the present species the appearance of four lobes arises from two membranous plates, which unite along their edges at either side, and are capable of being extended, so as to inclose an almost circular space. In general, however, they are so nearly together that they present very different appearances in different positions. The upper edge of each membrane is divided into two semi-circular lobes, and these are constantly vary-

* Manuel, p. 151.

ing, both in the extent to which they are protruded and that to which they are distended. It is seldom they are porrected to their full extent, but, when so, they produce so great a change in the oval form which the animal generally presents, that they make its outline appear like a miniature representation of one of those old fashioned bottles which we see in the pictures of the Flemish school.

The only food I have ever been able to detect in the stomach has been small crustacea of different kinds. The first of these was an undescribed species, since named by my friend Robert Templeton, Esq. R. A., *Anomalocera Pattersonii.** It was one line in length, and its bright green colour, contrasted beautifully, when in the stomach of the Beroë, with the crystalline transparency of the body, in which it was enclosed. In some instances two of these crustacea were visible in the stomach of one Beroë. The second I observed was a species of Zoea, on which Mr. Templeton also bestowed the specific appellation above mentioned.† Besides some other Zoea, I have distinguished some of the Gammaridæ. One of this family appeared to be half the length of the Beroë, and lay across the interior of the stomach, slightly bent, and when first observed was still living, and occasionally shifting its position. By a note in Trans. Ent. Society, vol. ii. p. 40, I learn that "M. Risso mentions his finding *phronima sedentaria* in the interior of a Beroë."

If, however, the Beroës feed upon small crustacea, they in turn furnish a supply of food to creatures more powerful than themselves. I have seen two of them swallowed by the Actinia Gemmacea,‡ in the course of twenty minutes. Next morning portions of the bands of cilia and more solid parts of the Beroës were observed rolled together, and adhering, with some darkish coloured pellets, to the filaments of the Actinia, whence after some time they were thrown off. On another occasion I took a small Medusa of the genus Callirhoë, (of a species undescribed by Lamarck,) and placed in the glass vessel with it a Beroë, which had been taken at the same time. While the latter was swimming round the glass, with that lively and graceful movement for which it is so remarkable, it came in contact with the filiform tentacula attached to the arms of its companion. The arms instantly closed, and the Beroë was a prisoner. I endeavoured to separate them, and for this purpose moved them about, by pushing them with a

* Trans. Ent. Society, vol. ii. part 1, p. 34.

† Vol. ii. part 2, p. 114.

‡ Johnston's Hist. Brit. Zoophytes, p. 214.

a camel-hair pencil, but without effect. In about half an hour afterwards, when I again observed them, they were asunder, the Beroë swimming about, and the cilia of its bands vibrating as briskly as usual. It had not, however, escaped uninjured from its captor. The Callirhoë had taken from the body of the Beroë a portion which extended transversely across three of the bands, and longitudinally for about the one-third of its entire length. The being who had suffered this mutilation seemed, however, quite unconscious of its misfortune, moved about in every respect as before, and for four days, during which I afterwards kept it, seemed to possess all its powers in unimpaired activity.

To this instance of apparent insensibility to pain may be added one illustrative of the extent to which the principle of vitality, or of vital irritability, seems diffused throughout every portion of its frame. On one occasion two Beroës were taken after a storm, with some of the cilia abraded, and other parts of the body shattered and even torn. Any of the cilia, however, which were attached to these mutilated parts, retained all their former mobility unimpaired. The most damaged of these Beroës was then cut with a pair of scissors into several pieces, and each part exhibited in its cilia the same undiminished rapidity of movement. One of these portions was again subdivided into parts so minute as to possess only one or two cilia on each, yet no change in the ceaseless motion of these extraordinary organs took place. Thirty-three hours after this minute subdivision, several of them were vibrating as usual; and, at the expiration of forty-two hours, the two cilia belonging to one fragment showed undiminished activity.

If a longitudinal incision be made in the body of a Beroë when dead, and the watery particles allowed gradually to evaporate, the bands of cilia and the tentacula will appear as if painted in a confused manner on the surface whereon the body has been placed, and when perfectly dry can be removed by a touch, as completely as if they had never formed a portion of animated existence.

Although, from this circumstance, it is obvious that the quantity of solid matter which enters into the composition of their bodies, must be extremely trifling, they possess a greater degree of firmness and consistency than is generally supposed. Frequently have some of them dropped from my net into the boat when about transferring them to the glass vessels in which they were kept; and, at such times, I have invariably lifted them in my fingers, and placed them with their companions, without their having received any apparent injury. If the finger be pressed against one recently dead, the Beroë will not, by such a pressure, be changed into a broken and shapeless mass. It will, on the contrary, by its smoothness and elasticity, slide from beneath the finger. In this respect it formed a singular contrast to the Bolina Hibernica, which could scarcely be removed without injury, and when taken in the hand appeared a shapeless mass of jelly. Some of the continental writers do not appear to have noticed this difference in the consistency of different Beroës, and have applied to the entire family, observations which are only correct when applied to particular species. Thus Lesson describes them as "peu consistant se brisant aisement à la moindre pression;"* and Blainville, under the genus Cydippe, introduces the observation of Othon Fabricius: "C'est un des plus jolis animaux qu'il soit possible de voir; mais aussi l'un des moins consistance, car à peine est il touché, qu'il est brisé et reduit en morceaux."†

From the inconsiderable quantity of solid material which enters into the body of the Beroës, and the rapid circulation of water, which is apparent throughout their frame, we would naturally suppose that any tinge which the body might accidentally acquire would be extremely fugitive. It was found, however, to be much less so than *a priori* would have been expected. My attention was drawn to this peculiarity by the circumstance of all my glass vessels being one evening occupied by Beroës and Crustacea, so as to compel me to place a small Medusa in a tin vessel, which chanced to be rusted at the seams. Next morning the colourless appearance of the animal was changed to a bright yellow, which appeared to pervade every part, and doubtless arose from the oxide of iron diffused through the sea water. This tint remained during the entire day, although the animal was transferred to pure sea water. Wishing to try if the vessels of the Beroë would become distinct, if filled with some coloured fluid, from which the animal could suddenly be withdrawn, and viewed through the usual transparent medium of sea water, I placed a Beroë in a weak infusion of saffron. At the end of twenty minutes its colour had undergone a perceptible change. I allowed it, however, to remain immersed for about six or seven hours, when it had assumed a bright yellow hue. It was then placed in pure sea water, but

* Annales des Sciences, tome v. p. 236.

† Manuel, p. 151.

retained its yellow colour for twenty-four hours afterwards; and though it gradually became fainter, it was very perceptible even at the expiration of forty-eight hours.

Lamarck observes, "Les Beroës sont tres-phosphoriques; ils brillent pendant la nuit, comme autant de lumières suspendues dans les eux; et leur clarté est d'autant plus vive que leurs movemens sont plus rapides."* Blainville, in his general remarks on the family of "les Ciliogrades," describes them as "agitant continuellement les cils dont leur corps très contractile est pourvu, organes qui jouissent les la faculté phosphorescente au plus haut degré;"† thus attributing the effect to the action of the cilia, rather than to any innate power possessed by the animal. That at least one British Beroë was endowed with a high degree of phosphorescence, was established by Dr. Macartney's description of B. Fulgens, taken by him in Hearne Bay, coast of Kent.‡ The same species was observed by the late John Templeton, Esq., "floating in with the waves on the shore of Dundrum Bay," County of Down.§ The phosphorescent quality does not, however, seem to prevail universally; at least I have never been able to detect its presence, though I have frequently for that purpose taken a glass containing Beroës into a darkened room. My hope of observing it was renewed by the following passage in a paper by Mr. F. D. Bennett,‖ "Fresh water appears to act as a powerful and permanent stimulus on marine Noctilucæ. Those which have intervals of repose from their phosphorescence, immediately emit their light when brought in contact with fresh water, and this fact was very strikingly exhibited in the Pyrosomata. * * * * When also the same Molluscs were mutilated, or so near death as to refuse to emit light upon irritation in sea water, immersing them in fresh water produced at least a temporary revival of their brightest gleam; indeed I have always felt assured that the contact of fresh water, in a darkened room, would ever elicit the luminous power of a marine creature, were the latter of a luminous nature." Acting on the suggestion here given, I took some Beroës into a darkened room, and transferred them to a jar

* Animaux sans Vertebres, tome ii. p. 469. † Manuel, p. 143.

‡ Phil. Trans. 1810, p. 264.

§ Mag. Nat. Hist. vol. ix. p. 303. In the same article the following occurs: "Beroë Mull. Pileus Gm. occasionally detected in our deep bays." We cannot from this brief record determine whether the C. Pileus or Pomiformis is the species alluded to.

‖ Proceedings Zool. Soc., June 13, 1837.

of fresh water. No luminosity ensued; and hence if Mr. Bennett's inference be applicable to the Beroë, I may feel warranted in concluding that the C. Pomiformis is not possessed of any luminous property.

But although the experiment failed, so far as the object for which it was performed was concerned, it was not utterly fruitless, for it showed the effect produced on the Beroës by immersion in fresh water. The moment they came into contact with the fluid, the action of the cilia ceased, or was limited to two or three irregular strokes, and the animal sank, apparently lifeless, to the bottom of the jar. If instantly removed, and replaced in sea water, the cilia began again to vibrate, but had acquired a degree of opacity they had not previously possessed, and the entire body seemed in some degree contracted, and less transparent than before. If a Beroë be plunged into boiling water or alcohol, the instantaneous change from its ordinary diaphanous appearance is very conspicuous.

The ovaries in the specimen examined by Dr. Grant "consisted of two lengthened clusters of small spherical gemmules of a lively crimson colour, extending along the sides of the intestine and stomach." In above five hundred individuals of the present species, which I have had in different years the opportunity of observing, between May and October, these crimson gemmules were totally wanting. In the glass jars in which they were kept, a glutinous substance might occasionally be seen, in some cases in contact with the tentacula* of the animal. In it were numerous small bright transparent gemmules, which I thought might be ova. This conjecture was verified, by placing under the powerful microscope of my friend, Dr. J. L. Drummond, portions of the body of a Beroë, from which most of the watery particles had been evaporated. We then distinctly saw the colourless ova, which were similar to those I had formerly seen in the jars. On one occasion, in the glass vessels in which some specimens of B. Hibernica were kept, I observed two glutinous strings, one about three, and the other about five inches in length, and both containing numerous ova, ranged at irregular intervals, and sometimes disposed in clusters.

* When treating of the genus Eucharis of Peron, to which the present species would belong, Lesson remarks, with a note of interrogation, "De ce retrécissement sur les côtés partent deux prolongemens cirrhigères, portant peut-être les ovaires?"—*Annales des Sciences Naturelles*, tome v. p. 252.

Dr. Grant, in speaking of the nervous system of C. Pileus, states, that he could perceive, at a short distance above the mouth, "a double transverse filament of a milky white colour, * * * * which formed a continuous circle round the body. In the middle of the space, however, between each of the bands of cilia, these cords presented a small knot or ganglion, so that there were eight ganglia in the course of this ring." Never having been able to observe these cords and ganglia in the C. Pomiformis, I took a number of specimens, some living and others recently dead, and placed them under the microscope already mentioned. But although Dr. Drummond, whose eye was well accustomed to microscopic examination, gave his valuable assistance, we were unsuccessful in detecting their presence.

A transparent membrane extends across a portion of the lower extremity of the body. It is entirely superficial, and may, perhaps, be of use in giving greater strength and stability to that part of the animal. This, however, is merely a conjecture, which I am at present unable to confirm or to correct by the opinion of others, as the membrane does not appear to have been noticed by any previous observer.

The Beroë is most usually described as swimming with its mouth downwards. Thus Blainville informs us, "Il nage peu obliquement, l'anus ou l'extrémite arrondie en haut, et trainant ses deux longs cirrhes comme deux queues."* Audouin and Milne Edwards, in like manner, state, "Il existe dans l'axe des Beroës une cavité qui va d'un pole à l'autre, et qui communique au-dehors à l'aide d'une ouverture inféricure, qu'on peut considerer comme l'avant bouche."† The words of Lesson convey a very different idea: "Dans l'eau leur position est très oblique ou presque horizontale."‡ It is with the mouth downwards that the C. Pileus is figured by Dr. Grant, and his description consequently bears reference to the animal as seen in that position. In this particular the C. Pomiformis is the reverse of its congener, the usual position of the mouth being uppermost, except when the animal is in a state of exhaustion, when it either rests on its mouth, or lies languidly on its side, at the bottom of the glass. At other times, when fresh and vigorous, its movements are lively, animated, varied, and inces-

* Manuel, p. 150.

† Quoted by Lesson, Annales des Sciences, tome v. p. 240.

‡ Annales des Sciences, tome v. p. 237.

sant. Sometimes it is seen rising to the surface of the water with a slow and equable motion, like that of a balloon, then gradually descending, the mouth being retained in its usual erect position. Next ascending with rapidity, and turning the mouth downwards, or revolving on the transverse axis of the body; and then abandoning all these modes of progression, revolving on its longitudinal axis, the body being vertical, and in this position twirling round and round the vessel. When the movements of the body are thus varied, how great must be the variety of motion in the cilia by which the body is propelled!

When the movements of the Beroës were thus diversified, it may be imagined they afforded highly pleasing objects for contemplation, especially as they displayed in the sunshine a splendid iridescence, caused by the action of the cilia in the water. To the various persons whom I met in the ferry-boats, plying between the Corran and Island Magee, their existence had been previously unknown. They seemed to be delighted no less by the novelty than by the beauty of their appearance, and not unfrequently compared the action of the cilia to that of the paddles of a steam-boat.

The C. Pomiformis, as now described, differs from the C. Pileus in the number and structure of its cilia, the position of the tentacula, the form of their sheaths, the want of colour in the ova, the inconspicuous structure of the nervous system, the existence of a transverse membrane at the anus, and the position in which the body is held when vigorous and unexhausted. I do not include in these distinctive characters the intestinal vessels which convey the fluid to the several bands of cilia, as it is possible that further investigation may prove that a somewhat similar arrangement prevails in both.*

When we contemplate the delicacy of structure displayed by the Beroës, we are prompted to inquire how they escape destruction from the turbulent element in which they live. On this subject Lesson remarks, "On doit supposer qu'ils augmentent leur pesanteur spécifique pour se précipiter à une certaine profondeur, là ou la mer est calme, et ou les lames sourdes, se font moins sentir."†

* Nov. 22, 1838. I have this day, for the first time, had access to the observations and researches of Mertens on the Acalepha of the Beroë family, (Memoires de l'Acad. Imp. des Sciences de S. Petersbourg, tome ii. p. 479,) and am glad to find the above opinion confirmed by the authority of that author. In his illustrative plates, drawn from living specimens, the ramiform vessels going out to the bands of cilia are figured in several different species.

† Annales des Sciences, tome v. p. 243.

So far as their absence from the surface during stormy weather may be regarded as corroborative of this observation it is correct; but the procedure appears to be insufficient to defend them when near the coast from serious and often fatal injury. On this subject I would refer to the diary published by me in the Edinburgh New Philosophical Journal for January, 1836, as to the weather of the early part of May, 1835, considered in connexion with the number of Beroës taken at various intervals during the same period.

That they are more abundant in some seasons than in others, may be inferred from the fact, that in the beginning of May, 1835, I took, in crossing the ferry from the Corran to Island Magee and returning, so many as thirty-five. In the same locality, in the apparently more genial month of June, 1838, the greatest number I took in any one of twelve crossings, between the 5th and the 30th of that month, was seven. On the 10th of September, however, in the same year, and in the same place, I took the unusual number of forty-one. All of these were small in size, the largest not exceeding four lines in length.

Nearly a month later than this, I placed my net, &c. in the hands of my friend Mr. W. Thompson, who, in the prosecution of his researches into our marine productions, was going out for a day's dredging in the Belfast Lough. In the evening he gave me the unexpected pleasure of seeing nearly eighty Beroës, all of the present species, and rendered still more acceptable by the following note:

"The entire of these were taken between ten and half-past twelve o'clock this forenoon, the day being very calm and bright for the season; the wind easterly. The towing net was first placed in the water opposite to Holywood; about three quarters of an hour afterwards, near to Craig-a-vade, it was found to contain twenty specimens. In five minutes more thirty-six were taken, in the next ten minutes eight, and in another quarter of an hour fifteen."

The ensuing day, 6th October, my friend Mr. G. C. Hyndman, while engaged in similar pursuits, employed my net with even greater success, and in the same locality took nearly one hundred individuals, all of them similar to the above.

The present species appears to be extensively diffused around the Irish coast. It has been taken at the Giant's Causeway by Mr. Hyndman; in the Loughs of Larne, Belfast, and Strangford, by the author, as already mentioned; in the Bay of Dublin, outside of Kingstown Harbour,* and at Lambay Island, by Mr. Thompson and Mr. Ball, and by the latter gentleman at Youghal.

In conclusion it may be remarked, that the species now described combines the characters in Fleming's definition of the genus Beroë: "body with vertical ciliated ribs; tubular vessels traverse the axis of the body with lateral and terminal apertures;" and those in his genus Pleuro-brachia, "body sub-orbicular, with eight ciliated ribs and two ciliated arms, one on each side."

As it seems desirable to place under our view the distinctive characters of C. Pileus and Pomiformis, as detailed in the present paper, I subjoin a brief definition of each. They are the only British species at present referrible to the genus Cydippe of Eschscholtz.

C. Pileus.—Tentacula issuing near the mouth; cilia fin-like, with slightly rounded outline; ovaries crimson; nervous system, whitish cords, and ganglia.

C. Pomiformis.—Tentacula issuing near the anus; cilia divided; ovaries colourless; nervous system inconspicuous.

* The specimens there captured were exhibited before the Natural History Section of the British Association in Dublin.—See 4th vol. of Reports, p. 72.

I am informed by Mr. Ball, that Mr. Bergin of Dublin has preserved some of these animals in a solution of acetate of alumina for fifteen months. In alcohol they have generally fallen to pieces in the course of a few weeks, or become so contracted as to be valueless as specimens.

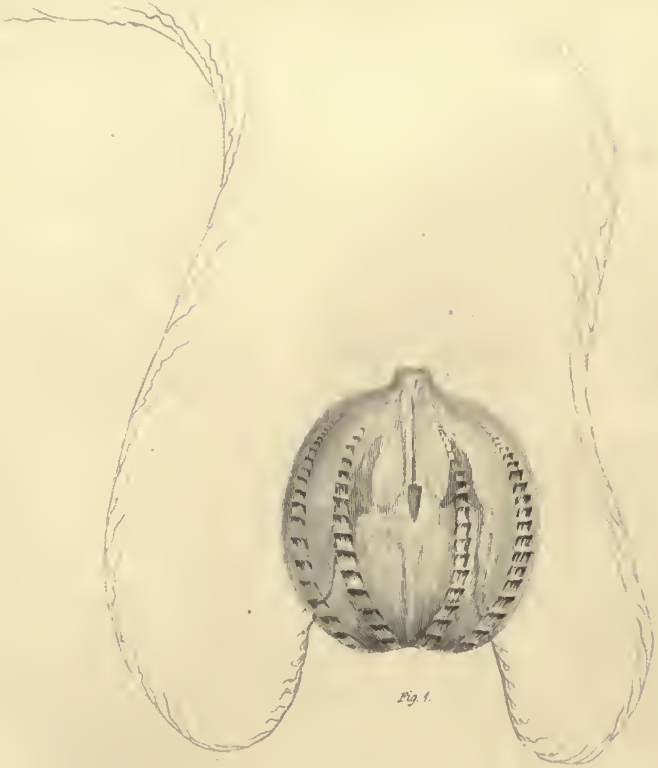


Fig. 1.

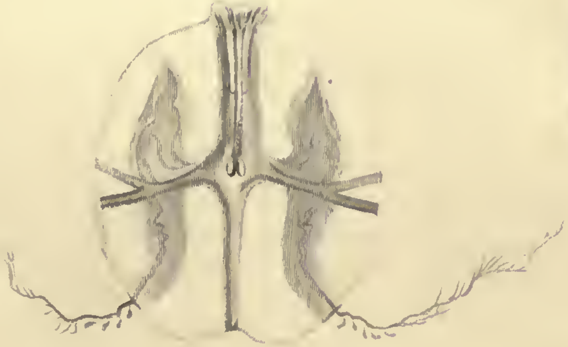


Fig. 2.

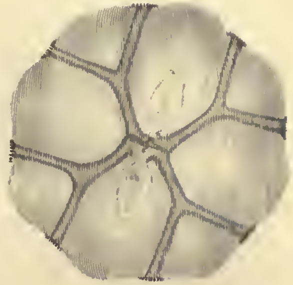


Fig. 3.

Fig. 5.

Fig. 6

Fig. 7.

Fig. 4.

Fig. 8.

R.J. Clarke Delt

J. Kirkwood. Sc.

EXPLANATION OF THE PLATE

ILLUSTRATIVE OF THE APPEARANCE AND STRUCTURE OF CYDIPPE POMIFORMIS.

Fig. 1.—Magnified representation of this Beroë in the act of revolving on the longitudinal axis of the body.

2.—Internal structure, exhibiting the form and position of the sheaths of the tentacula, and arrangement of the ramiform vessels communicating with the several bands of cilia.

3.—The same vessels as viewed vertically; the dotted figures mark the outline presented in this position by the sheaths of the tentacula.

4.—Anal extremity of the body, with the transverse transparent membrane, and part of the several bands of cilia.

5.—Membranous lobes surrounding the mouth.

6.—Cilia represented in motion.

7.—Cilia of another individual when at rest.

8.—Cilia of C. Pileus; copied, for comparison, from the Zoological Transactions, vol. i. plate 2.

For the original drawings, which are taken from living specimens, I am indebted to the kindness of my relative, B. J. Clarke, Esq. of La Bergerie, Portarlington, with whom I had the pleasure of repeating many of the observations recorded in the present paper.

The Beroë is represented of about three times its natural diameter.

III. *On the Longitude of the Armagh Observatory, given by fifteen Chronometers of Arnold and Dent, &c. By the* Rev. Dr. Robinson, M.R.I.A., &c.

Read 10th December, 1838.

THE determination of this important element is at least as difficult as essential; and whatever be the care of the astronomer it often happens that after years of observations have elapsed, the result still remains in some degree uncertain. The various methods of determining arcs of longitude have each their peculiar causes of error. When the methods of signals can be employed with only one intermediate station, it is decidedly the best; but obviously the measurement of large distances is in most cases impracticable, and when many stations intervene the accumulated errors may attain a serious magnitude. The expense of this process, and the number of assistants required, are also frequently very serious objections.

The longitudes assigned by geodetic operations depend on an assumed figure of the earth, whose constants are not well known, whose very existence is problematical; and even if correct, it will differ from the Astronomical longitude whenever local attractions deflect the direction of gravity to the east or west of the theoretic vertical.

The mere observation of an occultation is the most satisfactory that can be imagined in common cases; but there is uncertainty enough in deducing from it a longitude, caused by the doubtful nature of some elements that enter the calculation. It is affected by errors in the tabular place of the moon, which are not totally corrected when the declination has been actually observed, as only one limb can be taken, and that is affected by irradiation. It is influenced by the error of the tabular semidiameter, and still more of the horizontal parallax, which is to a certain extent hypothetical, whether given by theory or deduced from observation. And lastly, it depends on the assumed distance of the spectator from the earth's centre, a quantity computed on the hypotheses of its spheroidal figure and given compression, but which in strictness ought to be investigated by independent research.

In cases when the apparent tract of the star is very oblique to the moon's limb, its irregularities present a new source of error; and the final result is, that though the observations may be certain to a tenth of a second, the longitudes deduced may differ several seconds, and the truth can only be attained by a mean of many, taken under circumstances differing as much as possible.

The method of transits of the moon and lunar stars, though it afford an easy and pretty accurate approximation, is affected by the influence of irradiation, which I believe to vary not only with the telescope, but also with atmospheric changes. The *personal equation* is also different in some instances, for the planet and the stars, as I infer from the fact, that the transits observed by my late assistant gave the longitude five seconds of time less than those observed by myself after his death. In this method, therefore, it is necessary not merely to have observations of each limb, but to multiply the stations of comparison, that among the variety of observers and telescopes a kind of mean result may be obtained.

The determination by chronometers depends on the perfection of these machines, and in particular on their rate being unchanged by the agitation of a long journey. This, strictly speaking, is never the case, though it is sometimes very nearly accomplished, and its effect will disappear from the mean of the results obtained in going and returning, if the circumstances of the two journies are nearly similar.

Unfortunately it rarely happens that an astronomer has the power of making these experiments on a sufficient scale; but such an opportunity seemed to Sir William Hamilton and myself to present itself, in consequence of Mr. Dent's chronometric visit to Paris, and the yet more remarkable notice, read at the Newcastle Meeting of the British Association, of the Chronometric Longitude of Sir Thomas Brisbane's Observatory. Mr. Dent not merely promised us every assistance, but when, having obtained the consent of the authorities of our respective observatories, we proceeded to make the necessary pecuniary arrangements, he treated the matter as one of science, not of commerce, and not only took on himself the expense and risk of the journey, but came in person.

The chronometers which he placed at our disposal were fifteen, of which twelve were those that had been used in the determinations of Paris and

Makerstown. These latter were rated for some days at the Royal Observatory, Greenwich, and on September 20th were delivered to Mr. Dent. The remaining three were timed by the pupils of the Marine School at Greenwich, on the same day. They were packed in two boxes, and kept steady by a stuffing of horse hair, which to me at least appeared a very insufficient guard against the concussions of their rapid journey, but it seems to have been effectual. Much of this journey was performed with the marvellous rapidity of modern improvement, yet it may be questioned whether a slower passage would not have been more favourable; for the jarring of the railroad is severe, and the peculiar vibration of a steam-vessel I know to be very liable to disturb the performance of a chronometer. In this instance, of the total distance travelled, 275 miles were sea, 190 in Ireland in the common cars or stage-coaches, and the rest, amounting to 500, were performed on railways.

On the morning of September 22, the watches were compared at the Dublin Observatory, with the transit clock, by Sir William Hamilton, his assistant Mr. Thomson, and Mr. Dent himself; and on that of the following day, at Armagh, by Mr. Dent and myself. As Mr. Dent's time was precious, and I attach little or no importance to *stationary* rates, he started on the evening of the 24th, after we had again each compared the watches; and revisiting Dublin on the following day, and again making the comparisons, he sailed in the evening for Liverpool. The watches were finally returned to Greenwich, and compared by Mr. Main with the transit clock on the 27th, shortly after noon.

In making these comparisons, the Dublin astronomers appear to have taken beats of the watches, and divided the seconds of the sidereal clock. Mr. Dent took beats of the clock, and divided those of the watch, and I waited for coincidences and separation of the beats,—far the most accurate, but also far the most tedious mode of comparison. My results were, however, almost identical with Mr. Dent's.

Mr. Main, I believe, used the same method; for entire and half seconds only appear in his comparison, as must be the case when the watches beat twice in the second.

If we denote by E the correction of a watch when leaving the eastern, W that when arriving at the western station, I the interval of the watch's time between the two comparisons, and R its rate, (+ when losing, because it increases the positive correction,) we obviously have

$$L = E - W + R \times I,$$

and accenting the letters for the return,

$$L = E' - W' - R' \times I'.$$

If we suppose $R = R'$, that is, either the rate unchanged on the road, or similarly disturbed in the two journeys, then we have

$$R = \frac{(E' - W') - (E - W)}{I + I'} \qquad (1)$$

which may be called the *travelling rate*, and is given by subtracting from the watches' change between the two eastern comparisons the change between the two western, and dividing by the difference of the intervals; and this obviously is the rate which should be used.

We have also

$$2L = E' - W' + E - W + R \times (I - I') \qquad (2)$$

from which it is obvious, that if the times employed in going and returning are equal, or nearly equal, the effect of an error in the assumed rate is insensible in the mean of the two.

As the expression of R assumes that the longitudes obtained going and returning are equal, it is obvious that when the *travelling rate* is applied, it is useless to compute them separately.

If we suppose that $E - W$ requires a correction ϵ, whether caused by errors in the comparisons, or by accidental disturbance on the journey, then we obtain a value of R by eq. (1), which requires the correction

$$\rho = \frac{\epsilon' - \epsilon}{I + I'}$$

and the correction of the mean longitude given by eq. (2)

$$dL = \frac{\epsilon I' + \epsilon' I}{I + I'}$$

which in general will differ but little from that which occurs if we use stationary rates,

$$d\text{L}'' = \tfrac{1}{2}\epsilon + \tfrac{1}{2}\epsilon'$$

Errors caused by the journey produce opposite effects going and returning, and as the disturbances may be expected to be nearly equal in the two cases, it is highly probable that their effect on the mean longitude is insensible in such a case as the present.

Having premised so much as to the principles of the process, I annex its elements. The first column of the following table contains the number and distinguishing letter of the watch; the second its correction at the epoch of its own time given in the third; the fourth and fifth are for the return.

GREENWICH OBSERVATORY.

Arnold and Dent 1034	A	$+ 4^m.57^s.36$	$20^d.0799$	$+ 5^m.17^s.92$	$27^d.0278$
„ 1042	B	− 0.1.58	20.0847	+ 0.13.61	27.0218
„ 965	C	+ 3.11.15	20.0809	+ 3.24.89	27.0219
„ 910	D	+ 0.19.53	20.0014	+ 0.48.81	27.0271
„ 718	E	+ 3.32.60	20.0805	+ 3.44.14	27.0278
„ 1663	F	+ 0.9.33	20.0833	+ 0.30.81	27.0295
„ 1155	G	− 0.0.90	20.0833	− 0.18.76	27.0322
„ 978	H	+ 6.3.82	20.0788	+ 6.23.82	27.0315
„ 995	I	+ 2.38.37	20.0815	+ 2.49.79	27.0321
„ 1152	K	+ 0.56.77	20.0380	+ 0.53.09	27.0343
„ 1153	L	+ 5.25.91	20.0789	+ 5.59.05	27.0329
„ 777	M	+ 1.55.36	20.0820	+ 2.2.27	27.0369

GREENWICH NAVAL SCHOOL, REDUCED TO THE OBSERVATORY.

Arnold and Dent 820	N	+ 1.15.20	20.2083	+ 1.11.90	26.9722
„ 1017	O	− 0.22.70	20.2083	− 0.14.40	26.9722
„ 1045	P	+ 0.25.30	20.2083	+ 0.49.10	26.9722

DUBLIN OBSERVATORY.

A	— $20^m.18^s.15$	$21^d.9661$	— $20^m.8^s.72$	$25^d.0028$
B	— 25.19.22	21.9715	— 25.12.00	25.0076
C	— 22.6.00	21.9725	— 22.0.04	25.0062
D	— 24.53.50	21.9770	— 24.39.36	25.0090
E	— 21.44.66	21.9780	— 21.39.58	25.0076
F	— 25.6.92	21.9837	— 24.56.49	25.0104
G	— 25.27.30	21.9864	— 25.34.74	25.0125
H	— 19.11.95	21.9871	— 19.3.17	25.0083
I	— 22.39.63	21.9912	— 22.33.91	25.0118
K	— 24.25.35	21.9951	— 24.26.60	25.0139
L	— 19.46.04	21.9951	— 19.31.80	25.0111
M	— 23.23.22	22.0000	— 23.20.44	25.0146
N	— 24.6.83	22.0021	— 24.7.87	25.0163
O	— 25.41.70	22.0079	— 25.38.19	25.0180
P	— 24.49.39	22.0097	— 24.39.68	25.0183

The correction of the Dublin transit clock was on the 21st $= + 35^s.75$ by *α* Lyræ and *α* Aquarii, and its rate $= + 0^s.25$, using the places of the Nautical Almanac. It is confirmed by *α* Aquarii, Fomalhaut and *α* Pegasi on the 22nd.

The correction on the 24th $= + 36^s.53$ by *α* Cygni, *α* Aquarii, *α* Pegasi, and *α* Andromedæ.

ARMAGH OBSERVATORY.

A	— $21^{m}.29^{s}.10$	$22^{d}.9563$	— $21^{m}.26^{s}.95$	$23^{d}.9724$
B	— 26.31.61	22.9625	— 26.29.35	23.9770
C	— 23.18.33	22.9608	— 23.16.92	23.9761
D	— 26.3.46	22.9645	— 25.58.94	23.9786
E	— 22.57.05	22.9639	— 22.55.87	23.9778
F	— 26.17.89	22.9674	— 26.14.82	23.9815
G	— 26.44.10	22.9683	— 26.46.61	23.9826
H	— 20.23.35	22.9647	— 20.20.58	23.9787
I	— 23.52.59	22.9677	— 23.50.88	23.9819
K	— 25.39.57	22.9697	— 25.40.73	23.9843
L	— 20.55.21	22.9683	— 20.51.17	23.9816
M	— 24.36.72	22.9722	— 24.35.92	23.9857
N	— 25.20.74	22.9739	— 25.21.41	23.9876
O	— 26.53.93	22.9760	— 26.53.96	23.9904
P	— 25.59.82	22.9770	— 25.58.61	23.9904

The correction of the Armagh transit clock on September 21st was = — $30^{s}.45$ by α and β Lyræ; ζ, γ, α and β Aquilæ; and α Cygni.

On Sept. 23rd, by the same stars, it = — $31^{s}.23$, and the rate = — $0^{s}.38$.

Hence I derive the following longitudes of Armagh; each being the mean of those coming and returning:

A	+ $26^{m}.35^{s}.39$
B	36.32
C	35.46
D	35.03
E	34.69
F	36.17
G	35.74
H	35.56

I	35.69
K	35.12
L	35.28
M	35.06
N	34.67
O	35.12
P	35.99
Mean . .	26.35.44

The consistency of these results is very remarkable, the probable error of their mean being less than $0^s.1$, but its agreement with the longitude given by other means is not less striking.

From solar eclipses* of 1826 and 1836, and 19 occultations, 24 in all, I found

$$+ 26.35.58.$$

Three others were doubtful, as the star-paths were nearly tangents to the moon; but as the sum of the coefficients produced by the uncertainties of parallax and declination is nearly $= 0$, I add their mean with the weight of two $= 34^s.15$, and the occultation longitude is

$$+ 26^m.35^s.47.$$

The longitudes deduced from lunar transits give

Greenwich, 7 of first L. . .	26 36.26
„ 3 of second L. . .	32.04
	26.34.15
Dublin, 30 of first L. . . .	35.16
„ 9 of second L. . .	42.49
	26.38.82
Cambridge, 11 of first L. . .	38.79
„ 3 of second L. . .	28.38
	26.33.58

* The beginning in 1826; the beginning and end in 1836; and the beginning and end of the Annulus.

Königsberg, 8 of first L. . .	38.17
„ 4 of second L. . .	26.69
	26.32.43
Paris, 2 of first L. . . .	42.06
„ 3 of second L. . . .	32.42
	26.37.24

The differences are considerable, but I think the mean

26.35.64

must be very near the truth.

I have had few chronometric results previous to Mr. Dent's visit, and those obtained with my pocket-watch, Sharp, 1760, during my visits to London, &c. under unfavourable circumstances. They are :

Greenwich, 2 pair	26.35.44
Kensington, 3 do.	34.54
Edinburgh, 1 do.	36.04
	26.35.09

But the weights of these being much less than those of the results obtained with Mr. Dent's watches, can only be considered as depressing a little the mean of them.

On the whole, therefore, I am not inclined to change the quantity which some years since I gave to Mr. Stratford for insertion in the Nautical Almanac,

+ 26.35.50.

The differences between me and Dublin are :

A	+ $1^{m}.14^{s}.31$
B	14.84
C	14.56
D	14.63
E	14.30
F	14.57
G	14.39
H	14.30

I	14.91
K	14.18
L	14.13
M	14.46
N	13.73
O	13.95
P	14.55
Mean . .	+ 1.14.39

If each of these be subtracted from the corresponding longitude of Armagh, we obtain that of Dublin, such as would be given on the system of computation employed. But I have found by a direct comparison the longitude of Dublin :

A . .	+ $25^m.21^s.08$
B	21.48
C	20.90
D	20.40
E	20.39
F	21.60
G	21.35
H	21.26
I	20.78
K	20.94
L	21.15
M	20.60
N	20.94
O	21.17
P	21.44
Mean . .	+ 25.21.08

It is, I think, evident, that the original longitude of the Bishop of Cloyne, 25.21.00 is the true one. That illustrious astronomer had latterly increased this a second, probably induced by the result of lunar transits; but though I am sure he would not have done this without weighty reasons, yet I think the evidence of these chronometers would have been considered by him irresistible.

The geodetic difference of longitudes is, as I have already said, altogether unconnected with this inquiry, but in the ensuing summer I hope that we shall be able to lay before the Academy a determination of the differences between Dublin and Armagh, by means of Rocket signals, for which the Honourable Board of Ordnance have afforded us most ample means, though unfortunately too late in the autumn to be available this year. By the valuable aid of Lieutenant Larcom, I trust we shall be enabled to perform this interesting operation in the most satisfactory way; and by extending the same system to Mr. Cooper's Observatory at Markree, we shall have an arc of longitude measured in the most perfect manner, entirely across the island.

ARMAGH OBSERVATORY,
Nov. 9, 1838.

IV. *On the difference of Longitude between the Observatories of Armagh and Dublin, determined by Rocket Signals. By the* Rev. T. R. Robinson, D.D., *Member of the Royal Irish Academy, and other Philosophical Societies.*

Read 24th June, 1839.

In the communication respecting the Chronometric Longitudes of Armagh and Dublin, which I had the honor of submitting to the Academy last winter, I mentioned that it was our intention to determine the difference of our meridians by rocket signals; this has since been performed, and has given results which are the subject of this paper.

The method of signals is the most obvious of all, and under favourable circumstances, the most accurate. In it, the time of one place is transported to another, not by any machine, imperfect in its performance, and disturbed by that very transporting; the chronometer in it is light. If the appearance used for a signal be instantaneous, the only known source of error is in the determination of the Observatory time, which equally affects all other longitude methods. It appears to have been first used by the celebrated Picart, in a journey to Denmark, for the purpose of ascertaining the true position of Tycho's Observatory. He caused a fire to be kindled on the tower of the Observatory of Copenhagen, which was occasionally covered by a screen, and the time of its disappearance noted there, as well as by an observer at the ruins of Uraniburg. The distance is not more than seventeen miles, and there must have been some difficulty in covering the fire rapidly, as, from a passage in another of Picart's works, it appears to have been three feet in diameter. If, instead of a fire, one of Drummond's lights, placed in the focus of two Fresnel's lenses, directed to the stations, were suddenly covered by a hood, we should have a signal visible at any distance; which, besides being perfect in its nature, might serve to remove a doubt which has sometimes occurred to me. The impression of a luminous object remains for one or two-tenths of a second on the eye: is this duration the same for all persons? Is there a corresponding delay in the perception of light at its first appearance; or, does the mind take instantaneous cognizance of the action on the retina? If not, is the interval of time required the same for every observer? The beautiful experiments of Mr. Wheatstone* show that we can see an object whose visibility lasts only the millionth part of a second; but our perception of it may not be synchronous with its appearance. All of this which concerns the astronomer might be decided by observing the reappearance of the light, as well as its vanishing. The management of chemical apparatus on a mountain summit is, however, no easy matter, and Lieut. Larcom, R. E., has suggested an application of the heliostat, which offers the same results: directing its beam to one station, but diverting a portion to the other by a second mirror, suitably placed, the same occultation and reappearance may be effected with the utmost facility. The necessary apparatus was ready, and if there had been enough of sunshine in May, I should have reported on the performance of it; but I hope that before these longitude operations are completed, I shall have another opportunity.

No more mention of fire signals occurs in the annals of astronomy till 1735, when De La Condamine proposed to measure an arc of longitude by means of the flash of cannon; taking the idea, in all probability, from the ridiculous project of Whiston. As the signals are generally given on mountains, where cannon are of difficult conveyance, his proposal is scarcely less absurd; but it was made practicable four years after by Cassini and Lacaille, who used the powder without the artillery. Stationed on mountains, in the south of France, 110 miles apart, these astronomers observed the flash of ten pounds of powder fired at an intermediate point, and deduced, though but imperfectly, the difference of longitude. Besides the imperfection of their means of getting the time, the quantity of powder used was excessive, and its flame must have lasted one or two seconds. Even with so small a quantity as half a pound, this inconvenience is felt: Professor Santini complains that the signals given with this quantity, at Monte Baldo, in 1824, were not instantaneous, the inflammation lasting $\frac{1}{5}$ of a second. It must, however, be observed, that this is more remarkable when the powder is unconfined, than when fired in ordnance, or in the head of a rocket.

* Philosophical Transactions, 1834, p. 591.

Nor is such a quantity as ten pounds at all necessary in respect of visibility. Von Zach found that even so little as four ounces was seen at 150 miles, by the reflection of its light from the air, the flash itself being below the horizon; and that it was visible at 140 in the twilight:* and the French observers† state, that at twenty-seven miles one-eighth of an ounce can be seen with the naked eye. These are important as guiding facts; at the same time, the superior clearness of the air in the central parts of Germany should be kept in mind.

This method was again forgotten till Von Zach revived it at the beginning of this century. It has since been extensively used in Germany,‡ and by the French and Italian astronomers in the measurement of an arc of longitude between Marennes and Fiume.§ Where the localities of the line afford fit stations, this method is very satisfactory; but, where mountains of the requisite height, and in proper places, are wanting, a sufficient elevation must be obtained by art. I am not prepared to say how far it might be possible to obtain this by "Captive balloons," though the fates of Pilatre de Rozier and Madame Blanchard are strong arguments against the union of ærostation and pyrotechny.‖ The use of rockets in such cases was proposed by Robins, in 1749, and was practised by the elder Wollaston, and some other astronomers, near London, in 1775. More lately it was used on a large scale by the French, between Brest and Strasbourg, and by a commission of French and English, between Greenwich and Paris. The first is briefly described in the elegant notice by Major Sabine, given in the Quarterly Journal, vol. xxiii.; and that part which was done in 1824 is given with sufficient detail in the Memorial du Depot de la Guerre, vol. iii., to enable us to appreciate its value. It seems to have been unsuccessful, as out of 300 signals, on each branch of the arc, only six transmissions in the first attempt occurred on one branch, and none in the other; and on the second trial, out of 360, only thirty-six on the first. It is possible that this may

* Correspondence Astronomique, vol. iii., p. 437.

† Nicollet Con. des Tems, 1829, p. 381.

‡ For details of some of these by Littrow, see Cor. Astron., vol. vii. p. 257.

§ Con. des Tems, and Plana, Arc du Parallele Moyen.

‖ Howitzer shells were tried by the French, but rejected, as the flash was not sufficiently bright; their fragments would, I think, be very dangerous to those who give the signals, and the howitzer not easily managed on a mountain.

have been owing to the bad quality of the rockets employed, as they are said to be similar to those furnished for the English operation, which proved defective, a large proportion of them bursting. They were, in fact, overloaded, the signals being given with eight ounces of powder; and it seems that in attempting to make them able to carry this to the requisite elevation, the limit of strength was approached rather too closely. None of the distances are excessive. That (La Heve, St. Clair) which in the first line barred all transmission, is but seventy-one statute miles; it however required an elevation of 680 yards, which probably many of the rockets did not reach. Colonel Bonne, who reports this, attributes the failure to the fog which rests on the Seine, as the line of sight crossed this river seven times; and seems to think that in all such operations, the passing large surfaces of water should be avoided. Before adopting this conclusion, we should remember that in 1825, when the line was changed, and when no distance exceeded fifty-two miles, no greater success was obtained. Perhaps sufficient attention was not paid to the selection of clear nights for the signals; as every astronomer is aware that sometimes small stars can be seen almost to the horizon, while in ordinary good observing weather, this is by no means the case. *When such favourable circumstances are noticed at the observatories*, which are the extremities of the chain, a transmission of signals by numerous intermediate posts, should run along the line as a notice to fire the rockets, and thus success may be insured by a moderate expenditure of blue lights and patience.

The operations on the arc between Greenwich and Paris are described by Sir John Herschel in the Philosophical Transactions for 1826, with his usual precision and elegance: the memoir explains the method of successive signals with peculiar clearness, and in particular illustrates the method of using the broken sets to the best advantage. The distances here also were moderate, the greatest (La Canche, Lignieres) being only fifty-six miles; yet the success was not very great, ten complete transmissions being obtained only on four nights out of twelve, by 120 signals at each of the three stations. It is however evident, that Colonel Bonne's opinion of the difficulty of passing water does not hold with respect to sea; for, while 109 of the Wrotham signals were seen at 26 miles, ninety-two of those at La Canche, at fifty-two miles, were visible.

These operations were not followed up in Great Britain for several years, but in 1834 the British Association expressed a wish that the longitudes of Cambridge, Oxford, Edinburgh, Dublin and Armagh should be determined by the method of signals, and by chronometers. For this object it appointed a committee from its astronomical members, and gave them authority to apply to Government for any assistance that might be necessary. Of this Sir William Hamilton and myself are members; and I am happy to say that its work has commenced in Ireland. As far as the chronometric part is concerned, there is, perhaps nothing to be desired, except the personal equation of the Greenwich observers, which will be determined when an opportunity offers; and though the signal-measure, which is the subject of the present communication, relates to the smallest of the arcs, it is important, both on its own account, and as a means of training us for more extensive lines.

The Observatories of Armagh and Dublin are situated very unfavourably for the signal-method, there being no point visible from both. About four miles south of the first, a range of hills rises from 600 to 1000 feet above its level; but these are shut out from the view of Dublin, by a ridge about twelve miles to the north of it, 500 feet high. Even with powerful rockets it was not easy to clear these barriers; but our difficulties were removed by the aid, and, I may add, encouragement which we received from our friend Lieutenant Larcom. He not only gave us whatever information we required, but added a personal attention to the details of our work, without which it would, perhaps, have failed. Among other matters for which we have to thank him, was a diagram, in which he laid down the observatories, and all the mountains which could possibly serve as signal stations. To each was annexed its height, distance, azimuth at each observatory, altitude affected by the average terrestrial refraction; and when the line of sight was thrown up by an intervening ridge, the height there, and the elevation at which it passed the summit of the station, and which, of course, it was necessary that the rocket should clear, after allowing for refraction.* This

* It is really wonderful how completely every undulation of the ground has been registered in the Survey. The altitudes sent to me, which must have been computed from the general sections, agree with observation in the most extraordinary way. A fact of another kind will show such members as may not be acquainted with these things the precision of the Ordnance Survey. I set a telescope to the azimuth given for Slieve Gullion, and ascended the intervening hill with a theodolite, which I moved till, by signal from the Observatory, it was in the line; then I took, with

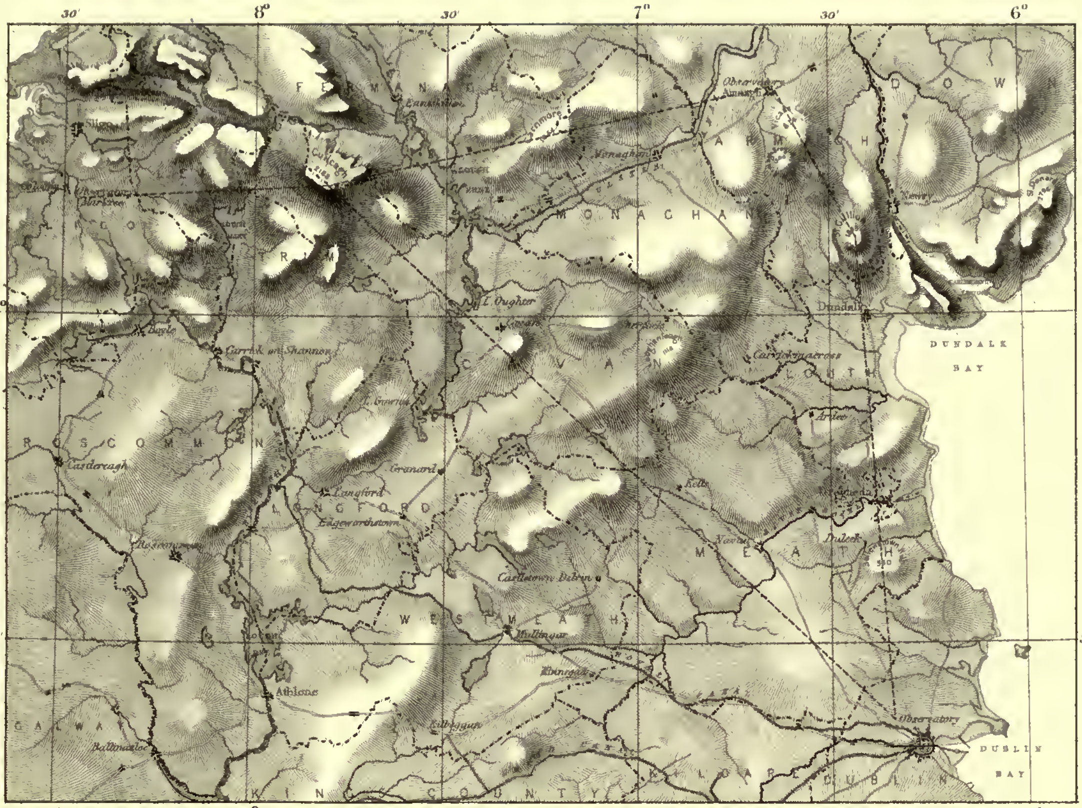
showed at once that our choice lay between two—Loughanleagh, in the county of Cavan, and Slieve Gullion, at the southern extremity of Armagh. The first would divide the distance better, but as its line passes through the smoke of the town of Armagh, the other was adopted.

Its summit, 1893 feet above the sea, is occasionally visible at Dublin, but is 800 feet below my view, the distances being 50.9 and 18.2 miles, as shewn in the annexed map, for which I am obliged to Lieutenant Larcom ; the section beneath shows the character of the intervening land. From this, the necessary size of rockets can be inferred; the pound rocket ($1^{in}.7$ diameter) rises 1400 feet, on an average, but cannot carry four ounces of powder, while it is evident from Sir J. Herschel's paper, that the two-pounder ($2^{in}.1$ diameter) is quite sufficient. These projectiles, when of such a size, require extreme care in the details of manufacture ; and, if ill made, are not merely uncertain, but actually dangerous ; and the case seeming of sufficient importance to authorize an application to Government, I made an application to the Board of Ordnance, stating the nature of my work, and requesting a supply of rockets. My reliance on that liberality which I have always found in the Government, when the importance of any scientific object is duly laid before them, was not disappointed, and I have much pleasure in acknowledging the kindness with which the Master-General, Sir Hussey Vivian, and the other members of the Board attended to me ; not merely giving the rockets, but tents for the firing party, and other matters which were necessary, but which I had in the first instance overlooked.* I may add, that as a measure of precaution against the interference of curious visitors, two of the police were placed at my disposal ; it was, however, unnecessary, as, though great crowds of the peasantry were attracted by an exhibition so new to them, they shewed every disposition to oblige and assist.

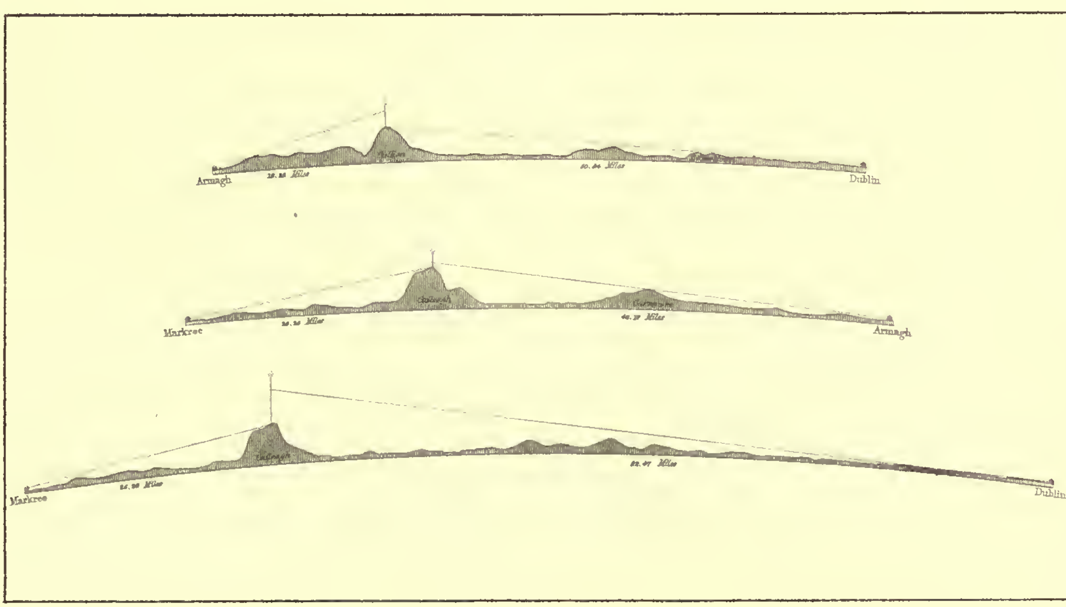
Having made all requisite preparations, I proceeded, on the 13th of May, to

the theodolite, the angle between the telescope and the pile on the mountain top, where our rockets were to be fired ; it proved 180°. 0′. 0″., or the three points were in one right line.

* The rockets were remarkably good ; *not one* burst, which certainly is a singular contrast to the French rockets in Sir J. Herschel's and Colonel Bonne's operations. Their average rise, on the only evening that I measured it, was 800 yards ; they had, however, only four ounces of powder, but the part of the case which contained it weighed six ounces more, so that they actually carried a greater weight than the French.



Scale 20 Miles to One Inch



Vertical Scale increased 10 times

MAP AND SECTIONS

SHEWING THE RELATIVE POSITIONS OF THE OBSERVATORIES OF

DUBLIN, ARMAGH AND MARKREE.

1840

establish my party at the mountain. This month was found by the officers of the Survey favourable for their work, and I knew it to be equally so for astronomical observations. On arriving, I found all difficulty removed by the kindness of Dr. Campbell, the rector of Forkhill, who had, with the hospitality for which he is remarkable, even in Ireland, provided such assistance that we were able to have the tents pitched, and the stores arranged within a couple of hours ; nor was his attention bounded with this, but continued during the whole of our operations.*

The wind blew furiously from the N.W., and next day the snow fell several inches deep on the mountain. I had not reckoned on such weather, but the sky was clear at intervals ; and I knew that even a gale will not affect the ascent of a well proportioned rocket. I therefore left my eldest son, Mr. T. A. Robinson, in command of the party, with directions to commence firing at ten, and give a signal every five minutes, as far as twenty, unless the night was decidedly cloudy. It would have been better to have arranged signals with him, but in my uncertainty of the quality of the rockets, I was desirous to economize them as much as possible.

Sir W. Hamilton (H) and myself (R) had arranged a list of stars to be observed daily, and, as I have stated, Lieutenant Larcom had given us the means of directing our instruments to the mountain with astronomical precision. The signals were, in fact, visible at Dublin, when the weather was fine, by the naked eye, but this could not be trusted to in moonlight or cloud, and they were observed there with Sharp's equatorial, whose telescope, by Cauchoix, has an object glass of flint-glass and quartz, $5^{in}.2$ aperture, with a power of 54. The time was noted by Arnold's clock. At Armagh the locality permitted the use of more instruments. My assistant, Mr. Edmondson (E), observed, by the transit clock, with a $3\frac{1}{2}$ feet achromatic, by Tulley, of $3^{in}.2$ aperture, power 30, placed at the

* The tents were pitched at the cairn, which is the trigonometrical point of the Survey. It is of great size, and contains a sepulchral chamber, in the form of a cross. The peasantry open it with great reluctance, and close it as soon as possible, believing it the dwelling of a sorceress, one of whose feats is given in Miss Brooke's Relics of Irish Poetry. Afterwards, when the weather became still more tempestuous, they were moved about 600 yards northward, near the lake which is found on this lofty summit. This new position is about 100 feet lower, but the rockets were much too powerful to make this of any consequence ; they might in fact have been fired in the valley of Forkhill, had I been aware of their excellence.

southern window of the transit room. I had intended to use my great reflector, with a power of 70, but the rapid motion of the rockets across the field* of view, and the oblique movements of the equatorial, $2^h. 15^m.$ from the meridian embarrassed me, and after losing a few, I betook myself to its finder, $2\frac{3}{4}^{in}$ aperture, power 18, with a field of $1\frac{1}{2}$ degrees, which proved quite satisfactory. The clock is by Sharp, with a mercurial pendulum. Mr. Robert Finlay (F) was to observe with Troughton's equatorial, $2\frac{3}{4}^{in}$ aperture, power 75, but as the field of view is narrow, and from not being accustomed to such instruments, he was even more embarrassed than I; he also was driven to the finder, which is a common affair, with an aperture of an inch. The clock has a gridiron pendulum.

The equatorial clocks were compared with the transit clock by chronometers, before and after the observations of each night; and as the simple reduction of these indications to sidereal time is not likely to involve any mistake, the observations are given in sidereal time, as it seems needless to occupy valuable space by setting down the actual clock times noted. They are as follow:

May 14, 1839, cloudy, high wind, fourteen rockets fired.

	ARMAGH.	DUBLIN.
No. 1.	R } E } Seen, but not observed.	. . . .
No. 2.	R } E } Seen.	. . . .
No. 3.	R Seen.	
	E $13^h. 36^m. 24^s. 68$	H $13^h. 37^m. 39^s. 10$
	E observed with the naked eye.	
No. 4.	R } E } Seen.	H . $42^m. 32^s. 10$
No. 5.	R } E } Ditto.	H . 48 25 10

* They rose, on an average, a degree of declination above the boundary of view, while the field is but 38 minutes.

	ARMAGH.		DUBLIN.
No. 6.	R 13h. 51m. 51s. 45 E	. . . .	H 13h. 53m. 5s. 30
No. 7.	R . 56 46 75 E	. . . .	H . 58 0 10
No. 8.	R 14 1 51 74 E	. . . .	H 14 3 6 10
No. 9.	R . 6 28 74 E	. . . .	H . 7 44 00
No. 10.	R . 11 28 24 E	. . . .	H . 12 43 10
No. 11.	R . 16 18 43 E . . 18 67	. . . .	H . 17 33 10
	Marked doubtful at Dublin.		
No. 12.	R . 21 34 23 E . . 34 67	. . . .	H . 22 49 10
No. 13.	R, E Lost in cloud,	. . . . .	H . 28 51 10
	Doubtful at Dublin.		
No. 14.	R 14 31 56 23 E . . 56 66	. . . .	H 14 33 10 60

The flash, at lighting the rockets, was seen at Dublin; the train, as well as the explosion, (which was instantaneous,) was visible by the naked eye at Armagh.

On May 16th, thirteen rockets were fired, but the evening became rainy, and many were missed.

	ARMAGH.					DUBLIN.				
No. 1.	R	13^h.	35^m.	31^s.	80					
	E	.	.	31	67	H	13^h.	36^m.	46^s.	62
	F									
No. 2.	R	.	40	48	30					
	E	.	.	47	67	H	.	42	2	12
	F	.	.	47	33					
No. 3.	R	.	45	41	79					
	E	.	.	46	67	H	.	.	.	.
	F									

R noted the disappearance of the train in the cloud, which was sudden. E suspected the explosion. H saw train but not explosion, and did not note the time of disappearance, which, however, may sometimes give a good result.

No. 4.	R	.	50^m.	40^s.	89				
	E	.	.	40	67	H	.	.	.
	F	.	.	42	23				

The rocket turned before exploding, and was not seen in Dublin.

No. 5.	R	.	55^m.	57^s.	79				
	E	.	.	57	67	H	.	.	.
	F	.	.	57	43				

Faint, not seen in Dublin.

No. 6.	R	14^h.	1^m.	14^s.	08	
	E	.	.	13	66	. . . .
	F	.	.	13	39	
No. 7.	R	.	5	45	78	
	E	.	.	45	65	. . . .
	F	.	.	45	99	
No. 8.	R					
	E	.	10	48	56	. . . .
	F					

	ARMAGH.	DUBLIN.
No. 9.	R Lost in cloud.	
	E . 15^{m}. 44^{s}. 66	. . . .
	F	

No. 10. Lost in heavy rain, though it was clear at the mountain.

No. 11.	R . 25^{m}. 55^{s}. 75	
	E . . 55 66	T . 27^{m}. 10^{s}. 12
	F . . 55 48	

Observed at Dublin by Mr. Thompson, Sir Wm. Hamilton's assistant.

No. 12.	R . 30^{m}. 39^{s}. 75	
	E . . 39 36	T . 31^{m}. 54^{s}. 12
	F . . 39 28	
No. 13.	R . 35 23 94	
	E . . 23 55	H . 36 38 12
	F . . 23 22	

The rocket-stand was moved, as the fury of the gale made it impossible to remain at the cairn, and all work was impracticable till the 20th, when it was fine on the mountain, but there was much haze below, strongly illuminated by the moon; and some annoyance from flying clouds. Twenty rockets were fired.

No. 1.	R 13^{h}. 52^{m}. 38^{s}. 79	
	E . . 38 63	T 13^{h}. 53^{m}. 53^{s}. 70
	F . . 39 18	

Faint at Armagh.

No. 2.	R . 57^{m}. 43^{s}. 79	
	E . . 43 63	T . 58 58 32
	F . . 44 18	
No. 3.	R 14^{h}. 2^{m}. 14^{s}. 92	
	E . . 15 16	T 14 3 29 82
	F	

No. 4. R and T saw train but not explosion.

	ARMAGH.					DUBLIN.
No. 5.	Train seen, but not flash,				T	13^m. 37^s. 70
No. 6.	R	17^m.	36^s.	96		
	E				T	Train, but not flash.
	F		37	22		
No. 7.	R	22	24	13		
	E		23	77	T	Not seen.
	F		24	22		
No. 8.	R	27	19	53		
	E		19	27	T	Train, but not flash.
	F		19	22		
No. 9.	Train seen, but not flash,				T	33^m. 29^s. 70
No. 10.	R	36^m.	55^s.	73		
	E		55	77	T	38 10 10
	F		55	71		
No. 11.	R	42	1	93		
	E		6	78	T	43 17 70
	F					

R noted the disappearance in the cloud. T appears to have taken the same. E was a suspicion. R used the large reflector for the next three.

No. 12.	R	47^m.	9^s.	13		
			9	08	T	48^m. 23^s. 70
			9	20		
No. 13.	R	52	9	03		
	E		9	48	T	53 23 70
	F		9	20		

Barking of dogs troublesome at Armagh.

No. 14.	R	57^m.	25^s.	13		
	E		24	78	T	58 39 70
	F		25	20		

	ARMAGH.					DUBLIN.				
No. 15.	R	15^h.	2^m.	17^s.	93					
	E	.	.	17	78	T	15^h.	3^m.	31^s.	70
	F	.	.	18	19					
No. 16.	R	.	7	24	43					
	E	.	.	24	68	T	.	8	38	70
	F	.	.	24	69					
No. 17.	R	.	12	22	63					
	E	.	.	22	78	T	.	13	36	70
	F	.	.	22	99					
No. 18.	R	.	17	41	91					
	E	.	.	41	28	T	.	18	55	70
	F	.	.	41	69					

This did not rise into the field of R's telescope, but was noted as above by another person at the same clock, with the naked eye.

No. 19.	R	.	22^m.	17^s.	84					
	E	.	.	17	68	T	.	23^m.	31^s.	70
	F	.	.	18	07					
No. 20.	R	15	27	16	93					
	E	.	.	16	79	T	15	28	31	20
	F	.	.	17	17					

On the 21st, twenty rockets were fired.

No. 1.	Not seen.					T	13^h.	57^m.	35^s.	38
No. 2.	R	14^h.	1^m.	35^s.	87					
	E	.	.	35	73	T	14	2	50	38
	F	.	.	35	62					
No. 3.	R	.	6	15	87					
	E	.	.	16	23	T	.	7	30	38
	F	.	.	16	02					

		ARMAGH.				DUBLIN.
No. 4.	R	11m. 25s. 36	}	T	.	12m. 39s. 38
	E	25 73				
	F	25 61				
No. 5.	Not seen.			T	.	17 59 88
No. 6.	Not seen.			H	.	22 37 38

H notes that it seemed to last from 36s. to 38s. ; it was probably the train seen through an opening in the cloud.

No. 7.	Not seen.			T	.	27m. 54s. 38
No. 8.	R	31m. 39s. 82	}	T Saw train but no flash.		
	E					
	F					
No. 9.	R	36 33 83	}	H	.	37m. 48s. 38
	E	34 23				
	F	34 71				
No. 10.	R	41 41 03	}	H	.	42 55 38
	E	41 13				
	F	40 71				
No. 11.	R	47 0 33	}	H	.	48 14 38
	E	0 14				
	F	0 77				

R noted this as low. F lost it for a time, but saw the flash.

No. 12.	R	51m. 45s. 23	}	H	.	52 59 88
	E	45 44				
	F	45 67				
No. 13.	R	56 41 02	}	H	.	57 55 38
	E	41 04				
	F	40 76				

No. 14. Exploded before it rose to its full height and was not visible at Armagh.

ARMAGH.						DUBLIN.				
No. 15.	R	15h.	6m.	51s.	02					
	E	.	.	51	24	T	15h.	8m.	5s.	38
	F	.	.	51	76					

This also exploded at less than the usual elevation.

No. 16.	R	.	11m.	52s.	72					
	E	.	.	52	75	H	.	13	7	38
	F	.	.	52	76					
No. 17.	R	.	17	21	61					
	E	.	.	22	25	T	.	18	35	88
	F	.	.	22	24					

At Armagh the rocket disappeared in cloud, but passed through it, and the train and explosion were well seen.

No. 18.	R	.	21m.	53s.	81	T	.	23m.	7s.	38
	E	.	.	53	65	H	.	.	8	38
	F	.	.	53	74					

H observed with a night-glass, held in the hand, but is unquestionably right.

No. 19.	R	.	26m.	43s.	29					
	E	.	.	43	25	H	.	27m.	58s.	38
	F	.	.	43	73					
No. 20.	R	.	31	44	49					
	E	.	.	44	45	H	.	31	59	08
	F	.	.	44	73					

In consequence of the miscarriage of a letter, there was no firing on the 22nd, the only perfectly fine night of the whole period; and though nine were fired on the 23rd, of which six were seen here, none were visible at Dublin. The moon was now so nearly full, and so low, that it became difficult to see the rockets at Armagh: and the results already obtained proved so satisfactory, that it was thought needless to repeat the signals from this station. Indeed, bad as the weather was, it was as favourable as that which has succeeded it.

As the most important part of longitude measures is the determination of the Observatory time, I annex the transit observations, and the clock corrections deduced from them.

The instrument at Armagh is $5\frac{1}{4}$ feet focal length, and 3.8 inches aperture, power 160; its axis was examined by the level daily, and its meridional position constantly verified by two marks, which being exactly adjusted to the meridian, would also detect any error of collimation, if it existed. This was insensible, as also is shown by six reversions made on May 25th, for the purpose of verifying the equality of the pivots, the difference of which is given by them $= 0^s.0004$, in fact, evanescent. At the same time their figure was tried by examining the inclination at every twenty degrees from the northern to the southern horizon; but though tenths of seconds of space can be estimated on the level, no error could be found. The transits were, except in two instances, taken by Mr. Edmondson.

At Dublin, they were taken by Mr. Thompson: the instrument has six feet focal length, and four inches aperture, power = 100. The inclination of its axis was found by the level, on the 8th, 17th, 22nd, and 23rd, $= +2''.18$; its meridional position by nine observations of Polaris, from April 30 to May 22, and its error of collimation by four of the same star, on May 20th, reversing between the wires, from which it appears that the observed transits require the correction,

$$-0^s.5371 + 0^s.6134 \text{ tang } \delta - 0^s.1059 \text{ secant } \delta.$$

The clock corrections are deduced from the places of Encke's Jahrbuch, which for γ Ursæ, and some other stars, agree better with our observations than those of the Nautical Almanac.

DUBLIN.					ARMAGH.			
DATE.	STAR.	OBSERVED TRANSIT.	NO. WIRES.	CLOCK CORRECTIONS.	STAR.	OBSERVED TRANSIT.	NO. WIRES.	CLOCK CORRECTIONS.
May 12. ⊙					Sirius,	$6^h. 38^m. 5^s. 26$	9	-2^s 19
					Procyon R,	7 30 54 72	9	−2 02
					Pollux R,	7 35 30 14	9	−2 21
					α Hydræ,	9 19 43 40	9	−2 03
					Regulus,	9 59 50 81	9	−1 99
					β Leonis,	11 40 54 36	9	−1 89
					γ Ursæ,	11 45 24 91	8	−1 79
					Polaris, S.P.	13 1 5 48	3	−1 25
					Spica,	13 16 47 13	9	−1 80
					Level	+ 1″ 45		
„ 13. ☽	β Leonis,	$11^h. 40^m. 57^s. 34$	5	$-4^s.32$	β Leonis,	11 40 54 06	9	−1 65
	Polaris, S.P.	13 1 27 00	1	−5 68	γ Ursæ,	11 45 24 52	9	−1 50
	Spica,	13 16 50 32	5	−4 19	Polaris, S.P.	13 1 6 90	3	−0 96
	η Ursæ,	13 41 18 91	5	−4 20	Spica,	13 16 46 61	9	−1 31
					Level	+ 1″ 45 lowered axis.		
„ 14. ♂	γ Ursæ,	11 45 27 10	5	−3 99	Procyon,	7 30 53 97	9	−1 23
					Pollux,	7 35 29 33	9	−1 36
					Regulus,	9 59 50 26	9	−1 40
					Level	+ 0″ 50		
„ 15. ☿	Rigel,	5 6 53 15	3	−4 38	Capella,	5 4 49 60	7	−1 53
	α Hydræ,	9 19 46 60	5	−4 50	Rigel,	5 6 49 58	3	−1 58
	Regulus,	9 59 53 94	5	−4 58	Sirius,	6 38 4 63	6	−1 58
					Pollux,	7 35 29 56	3	−1 63
					Level	+ 0″ 85		
„ 16. ♃	β Leonis,	11 40 57 76	5	−4 77	Procyon,	7 30 54 02	9	−1 29
	γ Ursæ,	11 45 27 72	5	−4 62	Pollux,	7 35 29 39	3	−1 43
					Regulus,	9 59 50 28	2	−1 43
					β Leonis,	11 40 53 94	9	−1 44
					γ Ursæ,	11 45 24 54	8	−1 34
					Level	+ 0″ 22		
„ 17. ♀	Polaris, S.P.	13 1 27 00	1	−3 15				
	Spica,	13 16 50 86	5	−4 76				
„ 19. ⊙					Regulus,	9 59 49 73	9	−0 90
					β Leonis,	11 40 53 32	7	−0 84
					γ Ursæ,	11 45 23 83	6	−0 65
					Spica,	13 16 46 22	9	−0 87
					Level	− 0″ 12		

DUBLIN.					ARMAGH.			
DATE.	STAR.	OBSERVED TRANSITS.	NO. WIRES.	CLOCK CORRECTIONS.	STAR.	OBSERVED TRANSIT.	NO. WIRES.	CLOCK CORRECTIONS.
May 20. ☽	Capella,	5h. 4m. 53s. 39	5	—5s.17	Capella,	5h. 4m. 48s. 50	9	—0s.35
	Rigel,	5 6 53 93	4	—5 16	Sirius,	6 38 3 52	9	—0 49
	Procyon,	7 30 58 52	5	—5 24	Procyon,	7 30 53 01	9	—0 31
	Pollux,	7 35 33 61	4	—5 35	Regulus,	9 59 49 16	8	—0 36
	β Leonis,	11 40 58 24	5	—5 29	β Leonis,	11 40 52 74	4	—0 28
	γ Ursæ,	11 45 28 02	4	—5 05	γ Ursæ,	11 45 23 33	8	—0 21
	Polaris, S.P.	13 1 29 50	2	—4 08	Spica,	13 16 45 59	9	—0 24
	reversed,	. . 37 33	2	—5 08				
	α Serpentis,	15 36 28 86	5	—5 19	Level	+ 0″ 15		
„ 21. ♂					Sirius,	6 38 3 15	3	—0 14
	α Coronæ,	15 28 0 46	5	—5 09	Spica,	13 16 45 15	9	+0 19
	α Serpentis	15 36 28 84	5	—5 16	η Ursæ,	13 41 14 40	9	+0 25
					Level	+ 0″ 42		
„ 22. ☿	Capella,	5 4 53 17	5	—4 94	Capella,	5 4 47 72	6	+0 46
	Rigel,	5 6 53 80	2	—5 03	Rigel,	5 6 47 52	3	+0 52
	β Tauri,	5 16 12 63	2	—5 11	β Tauri,	5 16 6 80	9	+0 35
	Sirius,	6 38 8 98	5	—5 12	Sirius,	6 38 2 67	9	+0 27
	Procyon,	7 30 58 06	5	—4 79	Regulus,	9 59 48 26	9	+0 53
	Pollux,	7 35 33 36	5	—5 12	β Leonis,	11 40 51 75	3	+0 70
	Regulus,	9 59 54 28	5	—4 99	γ Ursæ,	11 45 22 42	8	+0 69
	Polaris, S.P.	13 1 33 10	3	—6 57	Spica,	13 16 44 67	8	+0 68
					η Ursæ,	13 41 13 83	8	+0 87
					Level	+ 0″ 44		

Hence I deduce the clock corrections :

May 14,	Dublin,	= — 4s. 25	at	11h. 56m.		
	Armagh,	= — 1 34	„	15 0		
„ 16,	Dublin,	= — 4 65	„	12 47		
	Armagh,	= — 1 35	„	15 0		
„ 20,	Dublin,	= — 5 22	„	13 22		
	Armagh,	= — 0 20	„	15 50		
„ 21,	Dublin,	= — 5 13	„	13 38		
	Armagh,	= + 0 26	„	15 50		

It will be observed that both clocks were accelerated at the 15th; this was chiefly caused by a fall of the barometer of three-fourths of an inch (Memoirs Ast. Soc., vol. v. p. 125). The mercurial pendulum of my clock is accelerated $0^s.37$ by a fall of one inch; the coefficient for the gridiron pendulum which belongs to the Dublin clock is probably greater, but as the effect is only differential, it seemed unnecessary to allow for it.

The differences of longitude given by the signals are as follows:

DATE.	NO.	R.	E.	F.	MEAN.
May 14.	3	. . .	$1^m.14^s.42$		Mean of R (8) $1^m.14^s.45$
	6	$1^m.13^s.85$	. . .		E (4) . 14 30
	7	. 13 35	. . .		
	8	. 14 36	. . .		
	9	. 15 26	. . .		
	10	. 14 86	. . .		
	11	. 14 67	. 14 43		
	12	. 14 87	. 14 43		
	14	. 14 37	. 13 94		
„ 16.	1	1 14 82	1 14 95	. . .	Mean of R (5) 1 14 31
	2	. 13 82	. 14 45	$1^m.14^s.79$	E (5) . 14 64
	11	. 14 37	. 14 46	. 14 64	F (4) . 14 79
	12	. 14 37	. 14 76	. 14 84	
	13	. 14 18	. 14 57	. 14 90	
„ 20.	1	. 14 91	. 15 07	. 14 52	Mean of R (14) 1 14 45
	2	. 14 53	. 14 69	. 14 14	or omitting the two doubtful
	3	. 14 90	. 14 66	. . .	R′ (12) 1 14 40
	10	. 14 37	. 14 33	. 14 39	Mean of E (13) . 14 40
	11	. 15 77?	. . .	. . .	Mean of F (12) . 14 12
	12	. 14 57	. 14 62	. 14 50	
	13	. 14 67	. 14 22	. 14 50	
	14	. 14 57	. 14 92	. 14 50	
	15	. 13 77	. 13 92	. 13 51	
	16	. 14 27	. 14 02	. 14 01	
	17	. 14 07	. 13 92	. 13 71	
	18	. 13 79?	. 14 42	. 14 01	
	19	. 13 86	. 14 02	. 13 63	
	20	. 14 27	. 14 41	. 14 03	
„ 21.	2	. 14 51	. 14 65	. 14 76	Mean of R (14) 1 14 47
	3	. 14 51	. 14 15	. 14 36	Mean of E (14) . 14 41
	4	. 14 02	. 13 65	. 13 77	Mean of F (14) . 14 24
	9	. 14 55	. 14 15	. 13 67	
	10	. 14 35	. 14 25	. 14 67	

DATE.	NO.	R.	E.	F.	MEAN.
May 21.	11	$1^m.14^s.05$	$1^m.14^s.24$	$1^m.13^s.61$	
	12	14 65	. 14 44	. 14 21	
	13	. 14 36	. 14 34	. 14 62	
	15	. 14 36	. 14 14	. 13 62	
	16	. 14 66	. 14 63	. 14 62	
	17	. 14 27	. 13 63	. 13 64	
	18	. 14 57	. 14 73	. 14 64	
	19	. 15 09	. 15 13	. 14 65	
	20	. 14 59	. 14 63	. 14 35	

Were we to suppose the results of each night of equal weight, and take the arithmetical mean, we should find,

$$\text{R} = 1^m.\ 14^s.\ 44$$
$$\text{E} = \ .\ 14\ \ 44$$
$$\text{F} = \ .\ 14\ \ 38$$

but this condition cannot be assumed; for a greater number of signals are observed on some nights, and the clock correction is concluded with unequal probability. The probable error of the difference of observed times is, denoting by ϵ that of the transit of a single star supposed the same at each observatory (as it is at Armagh and Dublin in fact), and by s the number of stars,

$$= \pm \epsilon \sqrt{\frac{1}{s} + \frac{1}{s'}}$$

If the number of rockets be r, and the probable error of the observation of one at both observatories be $\pm \epsilon m$, that of the mean of the night is $\pm \frac{\epsilon m}{\sqrt{r}}$, and therefore that of the night's result

$$(\epsilon) = \pm \epsilon \times \sqrt{\frac{1}{s} + \frac{1}{s'} + \frac{m^2}{r}}$$

By examining these results, I find $\epsilon = \pm 0^s.065$ and $\epsilon m = \pm 0^s.23$ for R and E, F being greater, and hence the probable weight of each night

$$\text{w} = \frac{1}{\frac{1}{s} + \frac{1}{s'} + \frac{12}{r}} \quad *$$

To apply this, the Dublin correction on the 14th is derived from one star, and the mean of three on the preceding, and two on the following day. I assume $s = 3$.

At Armagh $s' = 3$.

On the 16th, two stars, and the mean of three preceding and one following give $s = 3$; $s' = 5$.

On the 20th, $s = s' = 7$.

On the 21st, two and the mean of seven and seven give $s = 9$; at Armagh, four and the mean of seven and nine give $s' = 11$.

Hence, calling the decimals of the second of a result L, we have

May 14,	w = 0.46154	WL = 0.20769	R
	0.27273	0.08182	E
May 16,	0.34091	0.10568	R
	Same	0.21818	E
	0.22059	0.17426	F
May 20,	0.875	0.39375	R
	0.77778	0.31040	R′
	0.82727	0.33091	E
	0.61765	0.07412	F

* This expression of w shows, that with us the flash can be observed with about the same precision as the appulse of a star to a wire; but a more important deduction may be made respecting the method by successive signals. As each of these *adds* to the denominator of w a term $\frac{12}{r}$ their number diminishes it rapidly. Thus on the 20th, if, as in the Paris and Greenwich arc, we had employed two intermediate stations, it would have been but 0.37 of its actual value, even supposing the transmission perfect. I am therefore decidedly of opinion, that stations of transmission should be made absolute stations, when it is possible, by furnishing them with transit instruments: this guards against failure, and scarcely lessens the value of the result. Thus in the case supposed, w is 0.33, but it will be obvious that in Sir J. Herschel's operation, had this been done, instead of the ten complete results which he obtained, he would have got at least ninety.

May 21,	0.94414	0.44375	R
	Same,	0.38710	E
	0.74356	0.17845	F

The final means are, therefore,

$$\text{R} = 1^{\text{m}}.\ 14^{\text{s}}. + \frac{1^{\text{s}}.15087}{2.62159} = 1^{\text{m}}.\ 14^{\text{s}}.\ 439$$

$$\text{R}' = 1 \quad 14 \quad + \frac{1.06752}{2.52437} = 1 \quad 14 \quad 423$$

$$\text{E} = 1 \quad 14 \quad + \frac{1.01801}{2.38505} = 1 \quad 14 \quad 427$$

$$\text{F} = 1 \quad 14 \quad + \frac{0.42683}{1.58180} = 1 \quad 14 \quad 270$$

The result F has obviously far less weight than the other two, which must be attributed not merely to Mr. Finlay's total want of practice in such observations, but also to the small optical power of his telescope. Though it differs but little from the others, I think it best to omit it, and consider the mean of R′ and E as the definitive result

$$1^{\text{m}}.\ \ 14^{\text{s}}.\ \ 425.$$

But had I used it and retained the two omitted on May 20th, this would be only $0^{\text{s}}.\ 03$ less, and identical with the result given by Mr. Dent's chronometers.

These, however, require a correction for what is called the *Personal Equation* of the transit observers. It may appear strange that two practised observers should not observe the passage of a star over a spider's line at the same instant, but the fact is undoubted, and the difference is not of a decimal or two, but in the case of perhaps the first of European astronomers, it exceeds a second. The cause is unknown, but as from its being almost invariably independent of the declination, it appears not to originate in the eye, the probability is, that it is caused by some exercise of thought in associating the indications of the ear to those of the eye. In most cases it is constant for many years in the same individual; in some, probably by carelessness, it goes on increasing.

The usual method of determining its amount is thus: the observer, E, observes the transit of a star at the first wires, and T at the remainder. Each wire is then reduced to the centre; this is repeated for many stars. If they agree, there is no personal equation; otherwise, it is their difference. Or they may observe entire transits alternately on one night, and again inversely on a subsequent one, each taking the stars which the other had previously examined. The clock rates deduced from these will be ultimately too great, and too little, by the personal equation, which, therefore, is half their difference. Or, lastly, by a method shown to me many years since by Sir James South, which I prefer, as enabling the astronomer to decide several questions connected with the subject.* This requires an equatorial, whose micrometer wires are to be separated any quantity, I^s, and set parallel to the meridian. Let P, the personal equation, be the correction to be added to E, the time observed by one, to reduce it to T, that by the other; then

$$T^s - E^s - P^s = I^s \times \text{secant}\ \delta\,;$$

then move the equatorial, by its horary movement, into another position, and repeat the process till a sufficient number be obtained; then let the order of observing be inverted, and we have

$$E'^s + P^s - T'^s = I^s \times \text{secant}\ \delta\,;$$

and hence we find

$$2P^s = S\,(T^s - E^s) - S\,(E'^s - T'^s).$$

If the equatorial were very much out of adjustment, and the hour angle considerable, this process might require a correction, which, however, presents no difficulty. Far from the meridian a correction for refraction might also be required, but such circumstances will always be avoided.

I sent Mr. Edmondson to Dublin for the purpose of making such a comparison, which, after much delay by rainy weather, he effected on August 18th. Sharp's equatorial was used for the observations.

* In particular as to the moon. In many cases, I believe, the personal equation for this planet is different from that for stars; and that even for the first and second limbs it is not always equal. The bearing of this on the longitude method, by moon culminating stars, is evident, as also the mode of ascertaining its influence and amount.

With 71 Aquilæ, $\delta = -1^\circ\ 40'$ by 16 pairs,

$$\left.\begin{array}{l} \text{E}^s - \text{T}^s = 24^s\ 287 \\ \text{T}' - \text{E}' = 24\ \ 581 \end{array}\right\} \text{P} = +\,0^s.147$$

25 Aquarii, $\delta = +\,1^\circ\ 31$ by 17 pairs, with another opening of the wires,

$$\left.\begin{array}{l} \text{E} - \text{T} = 20^s\ 088 \\ \text{T}' - \text{E}' = 20\ \ 412 \end{array}\right\} \text{P} = +\,0.162$$

Another set of 14 pairs,

$$\left.\begin{array}{l} \text{E} - \text{T} = 20^s\ 053 \\ \text{T}' - \text{E}' = 20\ \ 371 \end{array}\right\} \text{P} = +\,0.154$$

63 Aquarii, $\delta = -\,5^\circ\ 6'$, 16 pairs,

$$\left.\begin{array}{l} \text{E} - \text{T} = 20^s\ 100 \\ \text{T}' - \text{E}' = 20\ \ 444 \end{array}\right\} \text{P} = +\,0.172$$

Again 15 pairs,

$$\left.\begin{array}{l} \text{E} - \text{T} = 20^s\ 207 \\ \text{T}' - \text{E}' = 20\ \ 613 \end{array}\right\} \text{P} = +\,0.203$$

The mean of the seventy-eight pairs is $+\,0^s.167$, or Mr. Thompson observes so much later than Mr. Edmondson. I regret that the moon was not observable. They tried the sun's second limb, and found by 14 pairs $\text{P} = +\,0.225$.

Hence, our true difference of longitude is by

Rocket signals .	1^m.	14^s.	258
Chronometers . .		14	220

I stated that it appeared unnecessary to continue the signals at Slieve Gullion; and this, I hope, will be admitted in reference to the object proposed, the determination of the arc of longitude between Dublin and Armagh.

As, however, calculating on the number of failures in the French rockets, I had got more than proved to be required, it is my intention to employ the remainder in a way, which, while it verifies the present work, will determine the position of another point, likely to become of great importance, the Observatory of E. J. Cooper, Esq., at Markree ; which, not merely from the magnificence of its instruments, but the intention of its possessor to make it a permanent establishment, merits this distinction. It will be seen, on referring to the map, that the high mountain Cultiagh, in Leitrim, has been selected with this view : it is visible from Markree, barely hid from Armagh by Cairnmore; and, though eighty-two miles from Dublin, yet, as 1700 feet above its summit will reach the view at that place, this, also, is completely within the scope of these rockets. If there be any fine weather in autumn, I hope to perform this then ; and, afterwards it will be our object to connect the Irish observatories with those of Scotland and England. Several points in Antrim are visible from Armagh, and also from the west coast of Scotland : and if the method of successive signals were employed, there is no difficulty in reaching Edinburgh. But for reasons already given, I would use this only as a last resource, and then make the intermediate stations absolute, which, if they are chosen at primary points of the triangulation, is likely to give very useful geodetic information.

But in the present instance I conceive it quite possible, by using large rockets, to effect the junction with one signal station. The mountain Goatfell, in the Island of Arran, has been chosen as the station. Its height is 2865 feet, and if the rockets can add to this 3300, they will be in view both here and at Edinburgh, the distances being 105 and 86 miles.

That this can be accomplished is certain, for a few which I made recently, no heavier than those which have been described, rose, with four ounces of powder, 4500 feet; and if the Board of Ordnance continue their powerful aid to us, I am confident of success.*

Similar rockets will, I think, also connect immediately Oxford with Dublin. If fired on Plinlimmon, 1500 feet will bring them within view of the latter, and also of the other, probably, unless the circumstances of the ground in its vicinity forbid it. But as to this I have not yet consulted my geodetic Mentor. If, however, it be necessary to observe them from one of the neighbouring hills,

* Since this was written, the Board have granted my application for a supply of rockets capable of ascending to the required height.

that is scarcely an objection, if it be so near the observatory that time can be transmitted *certainly* by powder signals, as they can be multiplied to any extent.

The junction of Oxford with Greenwich is a matter of no difficulty.

T. R. ROBINSON.

ARMAGH OBSERVATORY.

V. *On the Direction and Mode of Propagation of the electric Force traversing interposed Media. By* George J. Knox, Esq., A. M., M.R.I.A.

Read February 11, 1839.

WHATEVER theory be adopted to explain the passage of the electric force traversing an intervening fluid or solid substance not undergoing electrolyzation, —whether we suppose it to originate in an inductive influence affecting the circumambient ether of each particle of the substance in the line of direction of the force, in whose alternate states of induction and equilibrium consists the passage of the electric current, (the rapidity of such changes constituting its intensity,) while the vibratory motion produced in the particles of the ether on each successive return to a state of equilibrium causes the phenomena of the light and heat developed; or whether we adopt the gross conception of the passage of a fluid; still it is important to determine if the electric force passes along the surface of the interposed substance, or through the interior of its mass.

Dr. Faraday* has shown that water will convey a feeble current of electricity, without undergoing electrolyzation. To determine whether, under such circumstances, it will convey an electrical current along its surface or through its substance, a glass tube, ten feet long, and half an inch internal diameter, bent in the centre twice at right angles, was filled with distilled water. Two copper wires, twenty feet long, having platina wires soldered to their extremities, were inserted in barometer tubes of six feet in length, the platina wires being sealed in the tubes within half an inch of their extremities. The other ends of the copper wires were connected with a delicate galvanometer, and a constant battery of successively one, two, four, &c. pair of elements.

On immersing the platina wires in the liquid, their relative distances from each other should decrease if the current passes through the water, but should

* Series VIII. (970.)

increase if it passes along the surface, the deflexion of the galvanometer indicating the path. With one pair of elements there was no deflexion of the galvanometer; with two pair of elements there was a slight deflexion visible through a lens, which increased slightly on immersing the platina wires in the liquid. With four pair of elements, a deflexion of two degrees took place when the platina wires were on the surface of the water ; a deflexion of four degrees when they were immersed to the bottom of the tubes. As the number of alternations in the battery increased, so did proportionably the comparative deflexions of the galvanometer : the experiments proving that water, whether undergoing electrolization or not, conveys an electric current *through its substance*, and not *along its surface*, and that the decomposition of the water is an effect produced by the passage of the electricity when of sufficient intensity, and not the necessary consequence of its passage.

A similar experiment having been tried with phosphorus melted under spirits of wine, (being a non-conductor,) it was found to obey the same law with water ; that is, to convey the current through its substance.*

To determine whether the metals followed the same law, I suspended from the top of the new patent shot tower at Waterloo-bridge a leaden pipe, 170 feet long, and three-fourths of an inch internal diameter, through which was drawn an insulated copper wire, 180 feet long, one extremity of which being soldered to the inside of the end of the pipe, this end was sealed with fused metal, and to its external surface was soldered a copper wire of the same length as the former; round the tube, at its orifice, was twisted a copper wire ten feet long. The insulated wire being connected with a constant battery of one pair of elements in contact with one pole of an exceedingly delicate galvanometer, (constructed by Mr. E. M. Clarke of the Lowther Arcade,) the other pole of the galvanometer was brought successively in contact with the extremities of the uninsulated wires. The deflexion was greater when the current passed along the wire connected with the orifice of the tube, (although here the contact was not so good,) than when it passed along that soldered to the sealed extremity.

Again, the uninsulated wires being connected with separate galvanometers,

* It was unnecessary to try similar experiments with the analogous bodies, sulphur, selenium, and iodine.

so as to allow the current of electricity to pass along either of the uninsulated wires alone, or to be distributed between both, it was found (as well as could be determined by transposing the galvanometers,) to have divided itself into two equal currents flowing along both wires.

From the first experiment we may infer that a current of electricity passes with greater facility along the surface of a metal than through the interior of its mass, although we cannot hereby infer that it could not pass through the interior of the metal, when this is the only road open for its transit.*

To the experiments with phosphorus it might be objected that its capability for conducting an electric current is due to the presence of water, of which some have supposed that it could not be entirely deprived, although the experiments of Sir H. Davy, wherein he obtained hydrogen and oxygen from sulphur and phosphorus by heating them in contact with potassium and sodium, and by submitting them to the electrolytic action of a powerful galvanic battery, did not prove that they were united with the basis of these substances in such proportions as to form water, nor indeed does he appear to have entertained such an opinion himself. His opinion of the nature of sulphur was, that it was "a compound of small quantities of oxygen and hydrogen, with a large quantity of a basis, that produces the acids of sulphur in combustion, and which, on account of its strong attraction for other bodies, will probably be difficult to obtain in its pure form."† To put the question beyond any further doubt, I will mention some experiments which I tried in the Laboratory of the Royal Dublin Society in the year 1837, having had, through the kindness of Professor Davy, a galvanic battery of sixty pair of plates, five inches square, put at my disposal.

When fused phosphorus, sulphur, selenium and iodine, were submitted separately to the action of this battery charged with a strong acid solution, they conveyed the electrical current freely during the whole time, giving a spark whenever contact was broken; yet at the end of two hours they showed not the

* The high conducting power of mercury for electricity renders it almost impossible to determine, by this method, whether metals in the *fluid* state obey the same laws of conduction as when in the solid state. If they do not, it is highly probable there is a general law, that *all solids conduct along their surface, and all fluids through their substance.* The investigation of such general law I propose to continue in another paper.

† Bakerian Lecture, 1809.

slightest trace of decomposition, no gas being evolved at either pole, which would have been the case had there been any water present.

Having by these experiments shown the *direction* of propagation of the electric force, I will now consider the source from which it originates in the voltaic pile, the mode of its transfer, and its sustaining principle.

Sir H. Davy's* opinion that the contact of the metals was the *primum mobile* of voltaic excitement, having been proved by Dr. Faraday† to be erroneous, chemists are now pretty generally agreed that the electrical force developed in the voltaic pile is due altogether to chemical action, concerning which there are different opinions; of these, I will mention two, which are the most applicable to the present argument—Dr. Faraday's‡ and Mr. Becquerel's.§ The former supposes that the development of electricity is due to decomposition alone, and in no case to the chemical union of bodies, while the latter contends that it is due to both, and in proof of his opinion shows that when an alkali unites with an acid, with a neutral salt, and in fact with any solution whose natural state is with regard to it electrically negative, a current of electricity will flow from the alkali to that solution. Sir H. Davy‖ has taken a different view of these experiments from Mr. Becquerel, supposing that the electric current is produced by the action of the acid or alkali upon the platinum plates; but the latter has shown that the electrical current is produced equally when no such action could take place, the platinum poles being placed in separate cups filled with water.¶

The accuracy then of Mr. Becquerel's experiments having been fully established, the question arises, how are we to reconcile them with other well known contradictory facts? such as for instance those of Sir H. Davy,**—solid potash and sulphuric acid combining in an isolated platinum crucible, and causing no electrical development. Again, a plate of copper and of sulphur, when heated, have their electrical states increased until chemical action begins, when they cease.

* Phil. Trans., Bakerian Lecture, 1826.

† Eighth Series, (880).

‡ Eighth Series, (927) (928).

§ Tom. ii. from page 77 to 81.

‖ Phil. Trans., Bakerian Lecture, 1826.

¶ He might have added another experiment, free from all objections—namely, the increased intensity consequent upon an increased number of alternations of acid and alkali.

** Phil. Trans., Bakerian Lecture, 1807.

The simplest and clearest course, and that most reconcileable with the laws of statical electricity, seems to me to be :—to consider that no electrical development is caused by the *union* of an alkali with an acid, (the electricity being thereby disguised,) but that, at the *instant before* the union takes place, the particles of the alkali and of the acid, being in opposite electrical states, affect their surrounding particles by induction, causing thereby a feeble current of electricity to circulate from the acid through the galvanometer to the alkali, which supposition is borne out by the fact that a dry acid and alkali, when in contact, show opposite electrical states.

The same arguments apply equally well with regard to thermo-electricity. The contact of two metals produces in them opposite electrical states. Their chemical union in an isolated vessel gives no electrical development; thus a "solid amalgam of bismuth and lead become liquid when mixed together, without producing any electrical effect."* Again, "a thin plate of zinc placed upon a surface of mercury, and separated by an insulating body, is found to be positive, the mercury negative; but when kept together a sufficiently long time to *amalgamate*, the compound gives no signs of electricity."*

These experiments explain why the *contact* of the two extremities of metallic wires, constituting a closed circuit, should, as the potash and nitric acid just mentioned, produce an induced electric current. That the electric states of different metals in contact, when excited by heat, do not follow the law of their natural electrical states, and change on increase of temperature, is no argument against the explanation I have given, for upon what this change in the electrical excitation produced by heat depends, whether upon a peculiar arrangement of the crystalline parts of the metal, or of their compound elementary particles, we are as yet perfectly ignorant.

That the same general law of the contact of metals and of fluids applies equally (although in an inferior degree, owing to their want of conducting power) to the contact of the gases, may be shown by the experiment of Dr. Faraday (Sixth Series) of the union of hydrogen and oxygen by a plate of platinum; the electrical force, which circulates by the interposed platinum plate, facilitating the union of the two gases.†

* Phil. Trans., Bakerian Lecture, 1807.

† Aqueous solutions of different gases, when brought into contact, have been found to produce electrical currents.

To return to the source of the voltaic force in the battery. Zinc, when placed in contact with a dry acid, has been found to become positively electrified. When the zinc plate has been immersed in the acid solution, being positive, it attracts oxygen, by union with which its electrical state is disguised, while the hydrogen, set free in a highly positive electrical state, reacts upon the oxide of zinc, rendering it negative by induction. The platinum wire connecting the positive solution with the negative zinc plate, reduces all for the moment to a state of equilibrium, so that the electricity becomes disguised, not transferred bodily from the platinum to the zinc; which state of equilibrium is no sooner restored than it is destroyed, the zinc regaining its positive state, and the oxide being removed by the acid.

If we consider then what takes place, we shall perceive that the zinc plate undergoes alternate states of induction and equilibrium, as do likewise the particles of the solution between the zinc and platinum plates, and, in fine, the platinum plate itself, and that as the number of alternations of zinc and platinum increases, the electrical energy of the zinc plate increases, as does also *the rapidity of its oxidation and deoxidation, and as a consequence the rapidity of change of induction and equilibrium upon which the intensity of the current depends.*

The decomposition of the electrolyte may be considered to be the effect produced by two forces acting upon its particles; the attraction of the poles* of the battery (whether they be metal, water, or air) *originating*, while the electrical states induced upon the particles give the *direction* to the electrolytic action.

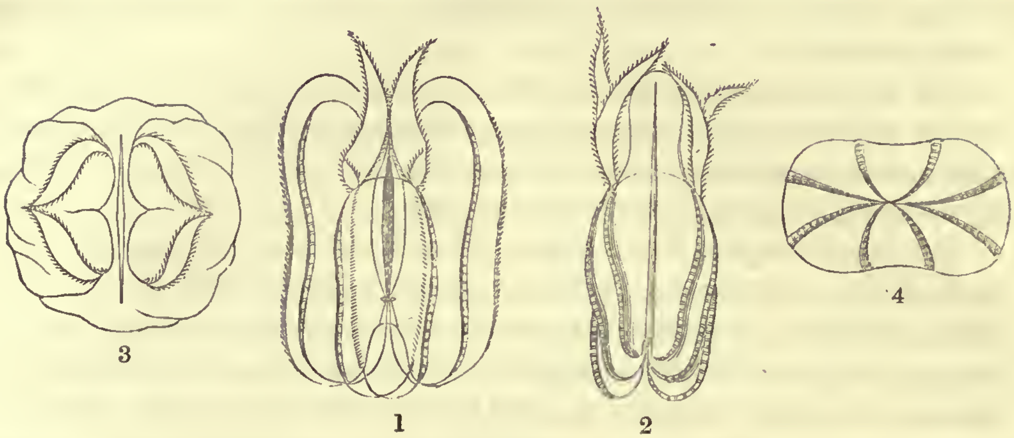
From what has been said above, we may, I think, presume that an electric current originates in a natural electro-inductive power of bodies when brought into *contact*, and is continued by alternate states of induction and equilibrium, the *rapidity* of change of state constituting its *intensity*. And inasmuch as the accumulation of the electric ether on the surface of the particles by the inductive

* In place of *poles*, I should more properly have said *electrodes*, their bounding surfaces. It follows, as a consequence of the theory, that the particles of oxygen in contact with the electrodes should be attracted by, and set free from, those electrodes upon each alternation of the states of induction and equilibrium; and that, when the induced state has not sufficient energy to overcome the affinities already engaged, the current of electricity passes without producing electrolyzation. For a different explanation, vid. Dr. Faraday's Series of Researches, 493, 494, 495, 534, 535, 536, 537, 807.

force, and its recession on each return to a state of equilibrium produces what may be called an oscillation in the ether, the theory may be otherwise stated thus:—the mass of oscillating ether which surrounds the particles constitutes the quantity, while the rapidity of the oscillations constitutes the intensity of an electric current.

The late experiments of Dr. Faraday upon induction (Eleventh Series) shewing that an insulated body (the particles of bodies may be presumed to be such) cannot receive an absolute charge of electricity, but only an inductive charge, afford a strong argument in favour of my views.

The theory proposed in this paper, and deduced from the experiments of Sir H. Davy, given in his Bakerian Lectures, is an extension of the views therein developed, reconciles the contact with the chemical theory, and reduces to the laws of *statical* electricity *all* the phenomena of electricity in motion. I will now endeavour to show how the law of the definite nature of electro-chemical decomposition, so beautifully developed by Dr. Faraday, follows as a consequence from this theory. Were the particles of all bodies endued with the same quantity of electricity, and of the same density, it is evident from the laws of statical electricity, that no one body could have an attraction or repulsion for another; consequently, it is an evident fact, that the quantity and density of the electric ether varies in different bodies; and as, from the theory above stated, electricity never leaves the particles, but merely (to use the words of statical electricity) accumulates upon the surface, and returns, it follows that the electrical states of the particles of bodies are constant and unalterable, and therefore it is obvious that the law discovered by Dr. Faraday follows as a consequence from this hypothesis, which is at once clear and simple, which includes all the phenomena, and is but a reference of the laws of *statical* electricity to the *particles* of bodies in place of their *masses*.



VI. *On the Bolina Hibernica. By* ROBERT PATTERSON, Esq., *Member of the Natural History Society of Belfast.*

Read November 11, 1839.

IN a paper on the Cydippe pomiformis, read before the Royal Irish Academy in December, 1838, and published in the present volume,* the occurrence on our coast of another species of ciliograde was mentioned, its figure described, and some particulars respecting its economy brought forward. The present is intended as a sequel to the former communication respecting this animal, the Bolina Hibernica.

The specimens from an examination of which I am enabled to give the particulars here recorded, were obtained the 11th of July, 1839, when I was

EXPLANATION OF THE FIGURES.

Fig. 1. Front view. 3. Anterior portion viewed from above.
2. Lateral view. 4. Posterior portion seen from beneath.

* *Ante,* page 91.

lodging at Bangor, county of Down ; and such was their abundance on that day, that in thc course of twenty-five minutes, one hundrcd and twenty-six individuals were taken in the bay by means of two small canvass towing nets. On several occasions, both beforc and after that date, my efforts to obtain specimens were totally unsucccssful.

The general movement of the animal appears more deliberate, or less vivacious than that of the Cydippe pomiformis, though always graceful and varied. The spiral motion on an axis, mentioned by Mertens as the mode of locomotion, may occasionally be seen, but is not habitual. Like Cydippe pomiformis, it generally swims in an erect position, with the mouth upwards. Its increase of power does not seem proportionate to its incrcase of size, for a small medusa of the genus Geryonia of Cuvier, which chanced to be thrown into the glass, attached its peduncle to a Bolina from twelve to sixteen times its own bulk, and with great apparent ease towed it round the vessel, reminding the spectator of a pigmy steam tug towing a stately merchantman.

This species of beroe is extremely suseeptible of injury, and henee, when any number are taken, some are sure to be found in a shattered state, perhaps, with so much as one-half of the body torn away. Any of the cilia detaehed from the body, along with a small piece of skin, will continue to vibrate for many hours; this is particularly apparent in the four tentaeula, and in the four eiliated rings or orifices, from which these organs are protruded. In both, we do not merely behold marginal eilia in rapid and continuous motion, but their number and variety of position is such, that the mutilated part to whieh they belong, is moved about with the briskness and activity whieh we are apt to regard as charaeteristic of a perfeet and vigorous animal. Under each of the bands of eilia, two aqueous currents are easily discernible, one ascending, and one descending with great regularity.*

The tentaeula were formerly mentioned as "extremely beautiful in appearance, both from their transparency, and from the numerous minute, delieate, pointed eilia along their edges." Their great attraction, however, is their versatility of form. They may be seen pointed, erect, and hollowed longitudi-

* In a communication on C. Pileus, made by Mr. Garner, at the late meeting of the British Association, it was stated that such currents are occasioned by the action of minute internal cilia, placed on the parietes of the vessels.

nally like the ears of a horse, or somewhat funnel shaped, and occasionally either flattened or concave, with the extremity rounded. At times their position is horizontal, at others they hang "loosely down like the ears of a lap-dog, or are curved like the petals of the martagon lily."

A whitish cord-like body extends round the orifice of the mouth; another round each of the four apertures, whence the tentacula issue. From each of the longer bands of cilia, a similar cord of a whitish milky colour, extends over the lobes at the mouth, touches the one first mentioned, and is continued to the four orifices already noticed, one going to each.* These orifices are connected in a similar manner with each, those on the same side of the body by a straight cord, those on opposite sides by an arched one, which adapts itself to the expansions or contractions of the body. The cords from all the bands converge near the anal extremity.

The two prominent lobes adjoining the mouth, and which sometimes constitute one-fifth of the entire length of the animal, are not permanent in their form, but vary not only in the regularity of their outline, but also in the extent to which they are distended, and at times, especially when the animal is in an exhausted state, become so reduced in size as to be scarcely perceptible.

During the time the drawings were in progress, specimens of the animal were kept in glass vessels of various dimensions, for the convenience of reference and examination, and one of these containing several individuals, was placed on the mantle piece, adjoining to some glasses filled with garden flowers. On looking at these through the transparent body of the Bolina, the flowers were seen so distinctly, that the several kinds were at once recognised, and the parts of fructification in some campanulate corollas, were with ease distinguished.

On taking a glass containing one of these beroes into a dark room, no luminosity was apparent, but on its being shaken, transient gleams of light were emitted. The animal was then taken and plunged in a glass of fresh water, which appeared instantaneously filled with innumerable small bright globules

* The following passage in Jones's "Outline of the Animal Kingdom," occurs in treating of the Beroeform species of Ciliograde Acalephæ. "From both extremities of the digestive cavity, arise vascular vessels, one surrounding the oral, and the other the anal portions of the body: from these two rings eight double vessels arise, which run longitudinally from one pole to the other of the creature, beneath each of the cartilaginous ribs, upon which the cilia are placed."—p. 75.

of fire, all in motion and rapidly disappearing; and on a light being brought, the Bolina was found lying lifeless at the bottom. In glasses containing a few individuals, flashes of light were given out, sufficient to render the figures on the dial plate of a watch visible for a moment, but too transient to allow the hour to be observed. Two large opaque vessels, each containing twenty or thirty individuals, were next subjected to examination in the dark cellar in which they had been placed. On agitating the first of these, light of a pale green tinge seemed instantly to diffuse itself through the water. On doing the same with the second, the whole contents of the vessel became lighted up so completely, as to render all the adjacent objects visible for a moment. On stirring it round, the animals were seen like lamps suspended in the water, to which their own radiancy imparted* a milder and fainter effulgence. On touching them with the hand, light was invariably given out with increased brilliancy, the bands, and every portion of the cilia being distinctly exhibited, with a splendid greenish lustre as beautiful as it was evanescent. It was impossible to behold these bodies of innocuous fire, floating amid the brightness which they themselves diffused, and not feel, that to convey an adequate idea of their beauty, would be a task more fitted for the imagery of the poet, than the language of the naturalist.

Being obliged to leave Bangor early next morning, the sea water in one of the larger vessels was not changed during the day, and in consequence of this neglect, I found, on my return at night, that all its occupants had died. The water, owing to their decomposition, then presented a discoloured milky appearance, and emitted a peculiar and disagreeable odour. On being agitated in the dark, no light was given out, thus proving that the luminosity of the previous evening was peculiar to the living animal, and was not extended to the putrescence of its decaying parts. This species, and the Beroe fulgens of Macartney, taken by J. Templeton, Esq., on the Down coast, are the only Irish ciliogrades in which the luminous power has hitherto been observed.

Being desirous of ascertaining if the present species had been recognized in any other localities, I exhibited the accompanying figures at the late meeting

* "Ils brillent pendant la nuit, comme autant de lumières suspendues, dans les eux."—*Lamarck.*

of the British Association in Birmingham, and solicited information on the subject. It was unknown to any of the naturalists then present; and my friend Edward Forbes, Esq., who communicated a valuable paper "on the Ciliogrades of the British Seas," pronounced it to be distinct from any of the eight species enumerated by him.

As it does not appear to have been previously recorded, either by British or Continental writers, the specific name Hibernica, before applied provisionally, may now be regarded as permanent. It would be premature to say the same of its generic title; for although it agrees with the Bolina of Mertens more nearly than with any other at present defined or figured, we recognise in the diminished size of the lobes, and in the more extended portion of the longer bands occupied by cilia, a still nearer approach to the true beroes; so that it is possible when we attain a more extended knowledge of the various species of ciliogrades, the present may be referred to an intermediate genus, yet to be established, or ranked with some of those now existing, under one common and comprehensive appellation.

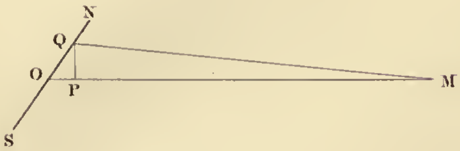
The localities in which it has hitherto been observed are, Larne Lough, county of Antrim, (R. Patterson); Bangor Bay, (R. Patterson); Strangford Lough, county of Down, (W. Thompson); Lambay Island, county of Dublin, (R. Ball, and W. Thompson); and Youghal Harbour, county of Cork, (R. Ball).

The present species is not likely to be confounded with either of its two congeners,—B. elegans, of a pink colour, found in the South Sea, or B. septrionalis, clear bluish, taken in Beering's Straits. The following brief specific description may suffice to distinguish it from other British ciliogrades.

Bolina Hibernica. Form variable, generally ovate, rounded, and compressed. Hyaline, lobes contractile, and not more than one-fifth of the entire length of the animal. Longer bands, ciliated nearly to their apex.

For the accurate figures by which the present paper is illustrated, I am indebted to the skill and kindness of Miss Masson of Bangor. A much greater number would, however, be requisite to convey an adequate idea of the diversiform aspect of the animal, especially with regard to the inflated appearance occasionally presented by the upper portion of the body.

1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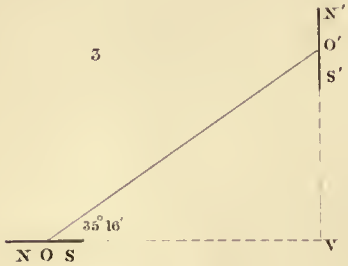
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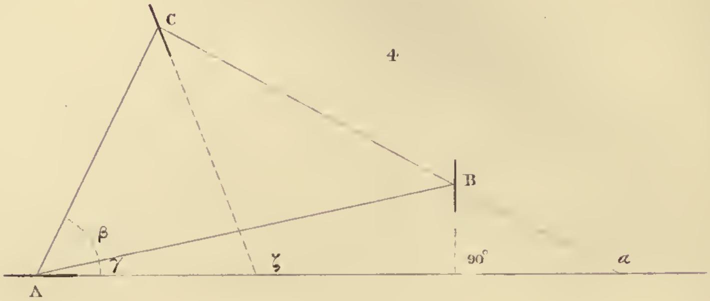
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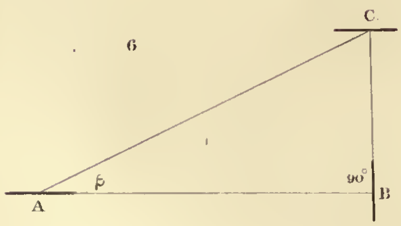
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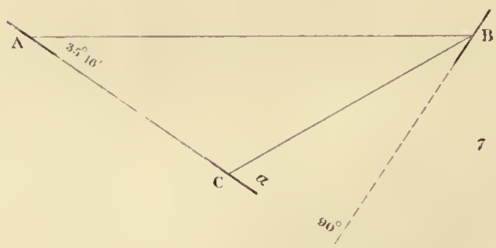
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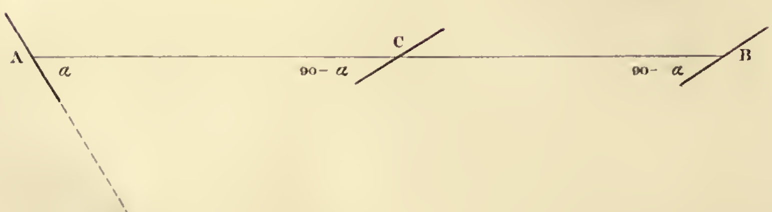
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VII. *On the mutual Action of Permanent Magnets, considered chiefly in reference to their best relative Position in an Observatory. By the Rev.* HUMPHREY LLOYD, A.M., *Fellow of Trinity College, and Professor of Natural Philosophy in the University of Dublin,* F.R.S., V.P.R.I.A., *Honorary Member of the American Philosophical Society.*

Read February 11, 1839.

IT is a problem of much importance, in connexion with the arrangement of a Magnetical Observatory, to determine the relative position of the magnets in such a manner, that their mutual action may be either absolutely null, or, at the least, readily calculable.

As a preliminary step to the solution of this problem, it is necessary that we should know the direction and intensity of the resultant force exerted by a magnet upon an element of free magnetism placed in any manner with respect to it. This question has been already solved by Biot, on the supposition that the action of a magnet is equivalent to that of *two* forces of equal intensity, one attractive, and the other repulsive, emanating from two *definite points* or poles. There is no difficulty in generalizing the problem, and in obtaining a solution independent of this particular hypothesis.

The middle point O, of the magnet NS, (Fig. 1) being taken as the origin of coordinates, and the line connecting it with the magnetic element M as the axis of abscissæ, the distance, MQ, of that element from any point (x, y) of the axis of the magnet-bar is

$$\sqrt{(a-x)^2+y^2},$$

the distance OM being denoted by a. Hence, if m denote the quantity of free magnetism in the magnetic element M, q the corresponding quantity in a given elementary portion of the magnet at Q, the force exerted by the latter on the former is

$$\frac{m\,q}{(a-x)^2+y^2};$$

the law of the force being similar to that of gravity, i. e. directly as the product of the *magnetic* masses, and inversely as the square of their distance. Let this force be resolved in the direction of the axes of coordinates. The portion parallel to the axis of x is

$$\frac{mq\,(a-x)}{\{(a-x)^2+y^2\}^{\frac{3}{2}}},$$

and that parallel to the axis of y is

$$\frac{m\,q\,y}{\{(a-x)^2+y^2\}^{\frac{3}{2}}};$$

and the sums of these portions, taken throughout the entire length of the magnet, are the components of the total action.

Let the distance OQ $= r$, and the angle MOQ $= \phi$,

$$x = r\cos\phi, \qquad y = r\sin\phi;$$

and substituting, the components of the force exerted by Q on M are

$$\frac{mq\,(a-r\cos\phi)}{(a^2-2\,ar\cos\phi+r^2)^{\frac{3}{2}}}, \qquad \frac{m\,q\,r\sin\phi}{(a^2-2\,ar\cos\phi+r^2)^{\frac{3}{2}}}.$$

Hence if X and Y denote the components of the total force exerted by the magnet NS on M, we have

$$X = m\int_{-l}^{+l}\frac{(a-r\cos\phi)\,q\,d\,r}{(a^2-2\,ar\cos\phi+r^2)^{\frac{3}{2}}}, \quad Y = m\int_{-l}^{+l}\frac{\sin\phi\,q\,r\,d\,r}{(a^2-2\,ar\cos\phi+r^2)^{\frac{3}{2}}}; \qquad (1)$$

l being half the length of the magnet. The quantity q being an unknown function of r, it is manifest that the integration of these formulæ cannot be effected in finite terms.

If we develop the trinomial factor

$$\left(a^2 - 2\,ar\cos\phi + r^2\right)^{-\frac{3}{2}} = a^{-3}\left(1 - 2\frac{r}{a}\cos\phi + \frac{r^2}{a^2}\right)^{-\frac{3}{2}},$$

it is manifest that the quantity within the brackets will be expressed by a series ascending by the powers of $\frac{r}{a}$; and that accordingly the preceding integrals may be developed in series of the form

$$\frac{m}{a^2}\left\{U_0 + \frac{U_1}{a} + \frac{U_2}{a^2} + \frac{U_3}{a^3} + \text{\&c.}\right\},$$

in which the coefficient of the general term is

$$U_m = V\int_{-l}^{+l} q r^m\, dr,$$

V being a function of the constant angle ϕ. Now, if the distribution of free magnetism be symmetric on either side of the centre, the alternate coefficients, U_0, U_2, U_4, &c. vanish, the values of q being *equal*, with *opposite signs*, at the corresponding distances $r = \pm s$. We have therefore, in this case,

$$\left.\begin{aligned} X &= \frac{m}{a^2}\left(\frac{A_1}{a} + \frac{A_3}{a^3} + \frac{A_5}{a^5} + \text{\&c.}\right), \\ Y &= \frac{m}{a^2}\left(\frac{B_1}{a} + \frac{B_3}{a^3} + \frac{B_5}{a^5} + \text{\&c.}\right); \end{aligned}\right\} \quad (2)$$

the two series descending according to the odd powers of a.

When the length of the magnet is small, in comparison with the distance a, these series converge rapidly, and, for most purposes, the first term affords a sufficient approximation to the actual value. We have then, approximately,

$$X = \frac{m A_1}{a^3}, \qquad Y = \frac{m B_1}{a^3}; \quad (3)$$

and denoting the total force by R, and the angle which it makes with the axis of abscissæ by ω,

$$\tan \omega = \frac{B_1}{A_1}, \qquad R = \frac{m\sqrt{A_1^2 + B_1^2}}{a^3}. \tag{4}$$

Now, stopping at the first dimension of $\frac{r}{a}$ in the development of the trinomial factor,

$$\left(1 - 2\frac{r}{a}\cos\phi + \frac{r^2}{a^2}\right)^{-\frac{3}{2}} = 1 + 3\frac{r}{a}\cos\phi, \quad q.p.$$

and substituting, we find

$$A_1 = 2\cos\phi \int_{-l}^{+l} qrdr = 2M\cos\phi, \quad B_1 = \sin\phi \int_{-l}^{+l} qrdr = M\sin\phi;$$

putting, for abbreviation,

$$\int_{-l}^{+l} qrdr = M.$$

Finally, substituting these values in (3) and (4)

$$X = \frac{2Mm}{a^3}\cos\phi, \qquad Y = \frac{Mm}{a^3}\sin\phi; \tag{5}$$

$$\tan\omega = \tfrac{1}{2}\tan\phi, \qquad R = \frac{Mm}{a^3}\sqrt{1 + 3\cos^2\phi}. \tag{6}$$

The theorems expressed by the formulæ (6) were taken by Biot as the basis of his well-known theory of terrestrial magnetism.

If we desire to push the approximation further, we must include (in the development of the trinomial factor) the terms involving $\frac{r^3}{a^3}$. We thus find

$$A_3 = 2M_3\cos\phi\,(5\cos^2\phi - 3), \qquad B_3 = \tfrac{3}{2}M_3\sin\phi\,(5\cos^2\phi - 1);$$

in which we have made, for abridgment,

$$\int_{-l}^{+l} qr^3dr = M_3.$$

Hence the components of the force are

$$\left.\begin{aligned} X &= \frac{2m}{a^3}\cos\phi\left\{M_1 + \frac{M_3}{a^2}(5\cos^2\phi - 3)\right\}, \\ Y &= \frac{m}{a^3}\sin\phi\left\{M_1 + \frac{3M_3}{2a^2}(5\cos^2\phi - 1)\right\}; \end{aligned}\right\} \quad (7)$$

the integral involving the first dimension of r being denoted, for distinction by M_1.

When $\phi = 0$, these values become

$$Y = 0, \qquad X = \frac{2m}{a^3}\left(M_1 + \frac{2M_3}{a^2}\right);$$

and the resultant force is, consequently, directed in the connecting line.

When $\phi = 90^0$, we find

$$X = 0, \qquad Y = \frac{m}{a^3}\left(M_1 - \frac{3M_3}{2a^2}\right);$$

and the force is altogether perpendicular to the joining line.

Returning to the approximate formulæ (5), it is easy to deduce the *directive force*, or the moment of the action exerted by one magnet on another, the length of each being supposed small in comparison with the distance between them. In this, and other similar applications of the formulæ, we may consider the distance a, and the angle ϕ, as the *same* for all the elements of the magnet acted upon; the variations of these quantities being of the order of those which we have already neglected in this approximation.

Let us assume that the two magnets NS and N′S′ (Fig. 2) are in the same horizontal plane, and that the magnet acted on, N′S′, is capable of motion in that plane round an axis passing through its centre of gravity. Let X and Y denote, as before, the components of the force exerted by the former upon any element of free magnetism, q', situated at the point Q′ of the latter. These forces being directed in the line OQ′, and in the line perpendicular to OQ′, respectively, their moment to turn the magnet N′S′ round its centre of motion O′, is

$$\text{O}'\text{Q}'\,(X \sin \text{N}'\text{Q}'\text{O} - Y \cos \text{N}'\text{Q}'\text{O}).$$

Now the angle Q′OO′ being very small, we may (in the same order of approximation as before) put OO′ for OQ, NOO′ for NOQ′, and N′O′O for N′Q′O; and accordingly, denoting the distances OO′ and O′Q′ by a and r', and the angles NOO′ and N′O′O by ϕ and ϕ', we have (5)

$$X = \frac{2\,Mq'}{a^3}\cos\phi, \qquad Y = \frac{Mq'}{a^3}\sin\phi;$$

M being the moment of free magnetism of the acting magnet, as already defined. Hence the moment of these forces to turn the magnet N′S′ is

$$\frac{Mq'r'}{a^3}\left\{2\cos\phi\sin\phi' - \sin\phi\cos\phi'\right\} = \frac{Mq'r}{2\,a^3}\left\{\sin(\phi+\phi') - 3\sin(\phi-\phi')\right\};$$

and multiplying by dr', and integrating, the total moment is

$$\frac{MM'}{2a^3}\left\{\sin(\phi+\phi') - 3\sin(\phi-\phi')\right\}, \tag{8}$$

in which M' denotes the moment of free magnetism of the second magnet, or the value of the integral $\int q'r'dr'$, taken throughout its entire length.

Let us apply this result to the case of the mutual action of two horizontal magnets, the axis of one which, NS, lies in the magnetic meridian, while that of the other, N′S′, is perpendicular to it (Fig. 3). Such is the position of the magnets in the instruments used in determining the *declination*, and the horizontal component of the *intensity* of the earth's magnetic force.

The moment of the force exerted by the second magnet on the first is in this case (8)

$$\frac{MM'}{2\,a^3}(1 - 3\cos 2\,\phi);$$

since $\phi + \phi' = 90°$. Hence, that this moment may be nothing, we must have

$$\cos 2\,\phi = \frac{1}{3}. \tag{9}$$

Accordingly the *mean direction* of the first magnet will be undisturbed by the second, when the line connecting their centres is inclined to the magnetic meridian at the angle $\phi = 35° 16'$. Mr. Weber has already arrived at this result by other methods.

With respeet to the *deviations* of the magnet from its mean position, (or the apparent variations of the dcelination,) it is manifest that they will be increased or diminished in a *given ratio*, the aetion of the second magnet on the first being in the same direetion as that of the earth, and therefore altering the directive force in a given ratio. The true variations will therefore be obtained from the apparent, simply by multiplying by a eonstant eoefficient.

The reciprocal aetion of the first magnet on the seeond, however, is not dirccted either in the magnetie meridian, or in the line perpendicular to it, and the second magnet is therefore disturbed by the first. With two magnets, accordingly, it is impossible to neutralize the effects of mutual aetion.

Now let a *third* magnet be introdueed; and let us suppose, in the first instanee, that this magnet is *fixed*, being destined only for the purposes of correetion. We have, in this ease, only to consider the forces exerted upon the first and second magnets.

Let A, B, C, (Fig. 4) be the three magnets—of which A is the declination bar, having its axis in the magnetie meridian; B the horizontal intensity bar, whose axis is perpendieular to the magnetic meridian; and C the third, or eorreeting bar, the azimuth of whose axis is arbitrary. Lines being supposed drawn joining the eentres of these magnets, lct the sides of the triangle opposite to the points A, B, C, be denoted by a, b, c, respectively, and the angles whieh these lines form with the magnetie meridian by α, β, γ; let the angle which the axis of the third magnet C makes with the same meridian be denoted by ζ; and finally, let the magnetie moments of the three magnets be A, B, C.

The forees exerted by the magnet B, upon any element m of the magnet A, in the direction AB, and in the direetion perpendicular to AB, respeetively, are (5)

$$+\frac{2Bm}{c^3}\sin\gamma, \qquad -\frac{Bm}{c^3}\cos\gamma;$$

the magnetism of m being supposed to be *northern*, and the positive and negative signs being employed in the usual conventional manner. Let these forees be resolved each into two, in the magnetic meridian, and perpendicular to the magnetic meridian. The former components are

$$+\frac{2Bm}{c^3}\sin\gamma\cos\gamma, \qquad +\frac{Bm}{c^3}\sin\gamma\cos\gamma;$$

and the latter

$$+\frac{2Bm}{c^3}\sin^2\gamma, \qquad -\frac{Bm}{c^3}\cos^2\gamma.$$

Again, the forces exerted by C upon the element m of A, in the direction AC, and in the direction perpendicular to AC, are

$$+\frac{2Cm}{b^3}\cos(\zeta-\beta), \qquad -\frac{Cm}{b^3}\sin(\zeta-\beta);$$

and the resolved portions of these forces in the magnetic meridian are

$$+\frac{2Cm}{b^3}\cos(\zeta-\beta)\cos\beta, \qquad +\frac{Cm}{b^3}\sin(\zeta-\beta)\sin\beta;$$

while the components perpendicular to the magnetic meridian are

$$+\frac{2Cm}{b^3}\cos(\zeta-\beta)\sin\beta, \qquad -\frac{Cm}{b^3}\sin(\zeta-\beta)\cos\beta.$$

Accordingly, the conditions of the complete equilibrium of the forces exerted by B and C on A, are

$$\frac{C}{b^3}\left\{2\cos(\beta-\zeta)\cos\beta-\sin(\beta-\zeta)\sin\beta\right\}+3\frac{B}{c^3}\sin\gamma\cos\gamma=0.$$

$$\frac{C}{b^3}\left\{2\cos(\beta-\zeta)\sin\beta+\sin(\beta-\zeta)\cos\beta\right\}+\frac{B}{c^3}(2\sin^2\gamma-\cos^2\gamma)=0.$$

In like manner, the forces exerted by the magnet A upon any element m of the magnet B, in the direction AB, and in the direction perpendicular to AB, respectively, are

$$+\frac{2Am}{c^3}\cos\gamma, \qquad +\frac{Am}{c^3}\sin\gamma.$$

And the forces exerted by C upon the same element, in the direction BC, and in the direction perpendicular to BC, are

$$-\frac{2\,Cm}{a^3}\cos(\alpha-\zeta), \qquad -\frac{C\,m}{a^3}\sin(\alpha-\zeta).$$

Resolving these forces, as before, in the direction of the magnetic meridian, and in the direction perpendicular to it, and making the sum of the resolved parts in each direction equal to nothing, the equations of equilibrium are found to be

$$\frac{C}{a^3}\left\{2\cos(\alpha-\zeta)\cos\alpha-\sin(\alpha-\zeta)\sin\alpha\right\}+\frac{A}{c^3}(2\cos^2\gamma-\sin^2\gamma)=0,$$

$$\frac{C}{a^3}\left\{2\cos(\alpha-\zeta)\sin\alpha+\sin(\alpha-\zeta)\cos\alpha\right\}+3\frac{A}{c^3}\sin\gamma\cos\gamma=0.$$

If we resolve the trigonometric products, and make, for abridgment,

$$\frac{A}{C}=P, \quad \frac{B}{C}=Q, \quad \frac{a}{c}=p, \quad \frac{b}{c}=q,$$

the four equations of equilibrium become

$$3\cos(2\beta-\zeta)+\cos\zeta+3\,Q\,q^3\sin 2\gamma=0, \tag{10}$$

$$3\sin(2\beta-\zeta)+\sin\zeta+Q\,q^3(1-3\cos 2\gamma)=0, \tag{11}$$

$$3\cos(2\alpha-\zeta)+\cos\zeta+P p^3(1+3\cos 2\gamma)=0, \tag{12}$$

$$3\sin(2\alpha-\zeta)+\sin\zeta+3\,P p^3\sin 2\gamma=0; \tag{13}$$

of which (10) and (12) relate to the forces in the magnetic meridian, and (11) and (13) to those perpendicular to it. The ratios p and q are functions of the angles α, β, γ, ζ, expressed by the formulæ:

$$p=\frac{\sin(\beta-\gamma)}{\sin(\alpha-\beta)}, \qquad q=\frac{\sin(\alpha-\gamma)}{\sin(\alpha-\beta)}. \tag{14}$$

The complete solution of the problem is contained in the preceding equations; and it follows, in general, that they may be satisfied by means of the four arbitrary angles, α, β, γ, ζ,—and consequently the desired equilibrium produced by suitably determining the positions of the three magnetic bars, whatever (within certain limits) be their relative intensities.

In the case which we have at present in view,—that is, when the third magnet is merely used as a counteracting power,—its intensity may be taken at pleasure; and accordingly one of the ratios, P or Q, is disposable, as well as the four angles. It follows from this, as there are but four conditions to be fulfilled, that one of the five quantities abovementioned remains arbitrary; and the nature of the problem obviously suggests that this should be the angle γ, which determines the position of the line connecting the two principal magnets, and that the conditions of equilibrium should be fulfilled by means of the other variables, which determine the position and force of the subsidiary magnet.

Let us suppose, for example, that it has been chosen to take the line connecting the magnets A and B coincident with the magnetic meridian; or that

$$\gamma = 0.$$

The equations (10, 11, 12, 13) thus become

$$3 \cos (2\beta - \zeta) + \cos \zeta = 0,$$
$$3 \sin (2\beta - \zeta) + \sin \zeta = 2\, Q q^3,$$
$$3 \cos (2\alpha - \zeta) + \cos \zeta = -4\, P p^3,$$
$$3 \sin (2\alpha - \zeta) + \sin \zeta = 0.$$

From the first and fourth we have, at once,

$$\frac{\frac{1}{3} + \cos 2\beta}{\sin 2\beta} = -\tan \zeta = \frac{\sin 2\alpha}{\frac{1}{3} - \cos 2\alpha}.$$

Another relation between the angles α and β may be inferred from the second and third of the foregoing equations, from which we obtain, by division and substitution,

$$\frac{\frac{1}{3} - \cos 2\alpha}{\sin 2\beta} = \frac{1}{2} \frac{Q q^3}{P p^3} = \frac{1}{2} \frac{B}{A} \cdot \frac{\sin^3 \alpha}{\sin^3 \beta}.$$

From this and the preceding equation, the values of α and β may be obtained by elimination. These angles being known, ζ is given by means of either of the expressions for tan ζ above written; and one of the ratios, Q or P, by the second or third equation, the other remaining arbitrary.

We have hitherto considered the third magnet as fixed, and serving only to complete the equilibrium of the forces arising from the mutual action of the other two. This magnet may, however, be a *moveable* one, and its movements serve to exhibit the changes of one of the magnetic elements. In fact, three independent variables are required, in order to determine completely the terrestrial magnetic force, (or its changes,) in direction and intensity; and, accordingly, whatever elements be taken as the basis of this determination, three separate instruments will be, in general, requisite for their observation. In this case, then, it becomes necessary to consider the action of the first and second magnet on the third.

The third magnet employed in the Dublin Magnetical Observatory, is intended for the determination of the variations of the *vertical component* of the earth's magnetic intensity. It is a horizontal magnet, supported on knife edges, and capable of motion in a vertical plane. The plane passing through the centres of the three magnets being horizontal, the axes of the magnets necessarily lie in the same plane; and, consequently, the action of the first and second on the third is directed in that plane. Let this force be resolved into two, one in the direction of the axis of the magnet, and the other perpendicular to it. It is obvious that the latter component can have no effect on the position of the magnet, being at right angles to the plane in which it is constrained to move; we may, therefore, confine our attention to the former,—that is, to the resolved part of the force in the direction of the magnet.

Using the same notation as before, the forces exerted by the magnet A, upon any element m of the magnet C, in the direction AC, and in the direction perpendicular to AC, respectively, are (5)

$$+\frac{2Am}{b^3}\cos\beta, \qquad +\frac{Am}{b^3}\sin\beta;$$

and the resolved parts of these forces in the direction of the axis of the magnet C are

$$+\frac{2Am}{b^3}\cos\beta\cos(\zeta-\beta), \qquad +\frac{Am}{b^3}\sin\beta\sin(\zeta-\beta).$$

In like manner, the forces exerted by B upon the same element m of C, in the direction BC, and in the direction perpendicular to BC, are

$$+\frac{2\,B\,m}{a^3}\sin\alpha, \qquad +\frac{B\,m}{a^3}\cos\alpha;$$

and the resolved parts in the direction of the axis of C are

$$+\frac{2\,B\,m}{a^3}\sin\alpha\cos(\alpha-\zeta), \qquad +\frac{B\,m}{a^3}\cos\alpha\sin(\alpha-\zeta).$$

Making the sums of these resolved parts equal to nothing, and performing the same reductions as before, the condition of equilibrium of the forces exerted upon the magnet C, in the direction of its axis, is expressed by

$$P\,p^3\left\{3\cos(2\beta-\zeta)+\cos\zeta\right\}+Q\,q^3\left\{3\sin(2\alpha-\zeta)+\sin\zeta\right\}=0. \quad (15)$$

For the conditions of equilibrium of the disturbing forces exerted upon the *three* magnets, A, B, C, by their mutual action, we must combine equation (15) with the four equations (10, 11, 12, 13) already given; and, as there are but four arbitrary angles, it follows that complete equilibrium is not attainable, except for determinate values of the relative forces of the magnets.

It fortunately happens that, for the special purposes which we have here in view, we may, without inconvenience, dispense with one of the conditions of equilibrium,—that, namely, of the forces exerted upon the magnet B resolved in the direction of the magnetic meridian. This condition, (which is expressed by equation (12)) being left unfulfilled, it follows from (13) that the resultant force exerted upon the magnet B by the other two, will be directed in the magnetic meridian itself, and will therefore conspire with, or directly oppose, the force exerted by the earth on the same magnet. Consequently the changes of position of the magnet bar, (which, in this instrument, are proportional to the changes of force divided by the total force,) are thereby only diminished or increased in a *constant ratio*,—namely, the ratio of the force of the earth to the sum or difference of that force and the resultant force of the two magnets. The changes sought are therefore obtained simply by multiplying by a constant coefficient. Accordingly, the four equations (10, 11, 13, 15) being fulfilled, the disturbing action exerted upon the magnets A and C will be completely balanced; and, with respect to that exerted upon the magnet B, its effect may be at once eliminated from the results, by altering in a suitable manner the constant in the formula of reduction.

It follows at once from the equations (10, 13, and 15) that

$$\sin 2\gamma = 0\,; \tag{16}$$

and therefore that $\gamma = 0$, or $\gamma = 90°$. The line connecting the magnets A and B must therefore be *parallel* or *perpendicular* to the magnetic meridian. Substituting the former of these values, equations (10, 11, 13) become

$$3\cos(2\beta - \zeta) + \cos\zeta = 0, \tag{17}$$

$$3\sin(2\beta - \zeta) + \sin\zeta = 2\,Q\,q^3, \tag{18}$$

$$3\sin(2\alpha - \zeta) + \sin\zeta = 0\,; \tag{19}$$

in which $q = \frac{\sin\alpha}{\sin(\alpha - \beta)}$. Equation (15) is rendered identical. When we make $\gamma = 90°$, the only difference is, that the second member of (18) becomes $\frac{4\,Q\cos^3\alpha}{\sin^3(\alpha - \beta)}$, instead of $\frac{2\,Q\sin^3\alpha}{\sin^3(\alpha - \beta)}$. It is easy to see in what manner we should proceed for the purpose of eliminating among these equations; the final equation, however, will be one of much complexity.

In the application of the original formulæ it will often occur that we are not at liberty to consider the four angles, α, β, γ, ζ, as *all* arbitrary, some circumstance connected with the locality determining one or more of these quantities, or establishing one or more relations among them.

Let us suppose, in the first place, that there are but *three* arbitrary quantities, so that we can satisfy but three of the equations of condition. We shall select for that purpose the equations (10, 11, 13), leaving (15) unfulfilled, as well as (12). This being done, the disturbing action exerted upon the magnet C remains unbalanced; but, as the effective part of this action is directed in the axis of the magnet itself in its mean position, it does not alter that position, but merely diminishes or increases the deviations from it in a given ratio. In the case of this magnet therefore, as in that of the magnet B, the effect of the disturbing action may be allowed for, by a suitable alteration in the coefficient by which the changes of angle are multiplied.

In order to illustrate this, and at the same time to apply the formulæ in a very important case, let it be required that the centres of the three magnets shall be situated in the *same right line.* This condition is expressed by the relations

$$\alpha = \beta = \gamma;$$

the two equations being equivalent to a *single* condition, inasmuch as one of them is a consequence of the other. Substituting in the formulæ (10, 11, 13), and expanding, they become

$$(\tfrac{1}{3} + \cos 2\alpha) \cos \zeta + \sin 2\alpha \sin \zeta + Q q^3 \sin 2\alpha = 0, \qquad (20)$$

$$(\tfrac{1}{3} - \cos 2\alpha) \sin \zeta + \sin 2\alpha \cos \zeta + Q q^3 (\tfrac{1}{3} - \cos 2\alpha) = 0, \qquad (21)$$

$$(\tfrac{1}{3} - \cos 2\alpha) \sin \zeta + \sin 2\alpha \cos \zeta + P p^3 \sin 2\alpha = 0. \qquad (22)$$

Dividing (20) by (21), we find, on reduction,

$$\cos \zeta = 0, \quad \text{and therefore } \zeta = 90^\circ. \qquad (23)$$

Accordingly the plane in which the magnet c is constrained to move must be *perpendicular to the magnetic meridian.*

Now, making $\zeta = 90^\circ$ in the three equations (20, 21, 22), the two former are found, of course, to be identical; and we have

$$1 + Q q^3 = 0, \qquad \tfrac{1}{3} - \cos 2\alpha + P p^3 \sin 2\alpha = 0.$$

From the first of these we obtain

$$q = \frac{-1}{\sqrt[3]{Q}} = -\sqrt[3]{\frac{C}{B}}; \qquad (24)$$

which determines the place of the centre of the intermediate magnet c. Again, in virtue of the relation $p + q = 1$, there is

$$p = 1 + \sqrt[3]{\frac{C}{B}}, \qquad P p^3 = \frac{A}{C}\left(1 + \sqrt[3]{\frac{C}{B}}\right)^3 = \frac{A}{B}\left(\sqrt[3]{\frac{B}{C}} + 1\right)^3,$$

Wherefore putting, for abbreviation,

$$k = \frac{A}{B}\left(\sqrt[3]{\frac{B}{C}} + 1\right)^3, \qquad (25)$$

the second equation becomes $(\tfrac{1}{3} - \cos 2\alpha) + k \sin 2\alpha = 0$; and we find

$$\tan a = -\tfrac{3}{4} k \pm \sqrt{\tfrac{9}{16} k^2 + \tfrac{1}{2}}\,; \tag{26}$$

which determines the azimuth of the line connecting the three magnets. This arrangement of the magnets is represented in Fig. 5.

This is, in many respects, a very advantageous disposition. The disturbing forces exerted upon the magnet A are in complete equilibrium, so that this magnet (which is that employed in *absolute* determinations of declination and intensity) may be used as if it were insulated; and, with respect to the magnets B and C, the effect of the disturbing forces is corrected by a simple change of a coefficient. As to the Observatory itself, one long and narrow room, about forty-eight feet in length, and sixteen feet in breadth, will suffice; the *bearing* of the axis of the room, along which the three magnets are to be disposed, being determined by (25, 26). The magnet A should be so far from one end as to allow a space of eight or nine feet in a direction perpendicular to the magnetic meridian, on either side, for experiments of deflection; the magnet B may be close to the other end. The place of the intermediate magnet will be determined by (24).*

Having considered the case in which three only, of the four variables, are arbitrary, it remains to examine that in which there are but *two* disposable quantities; the other two being either absolutely determined, or else connected with the rest by given relations.

We can satisfy, in this case, but two of the equations of equilibrium; and we shall select for that purpose (11) and (13), which express the conditions of equilibrium of the forces exerted upon the magnets A and B in the direction perpendicular to the magnetic meridian. These being fulfilled, the resultant action on each of these magnets is directed in the magnetic meridian itself, and therefore conspires with, or directly opposes, the force of the earth. Hence the *mean position* of the magnet A is unaltered; and the *changes of position* of

* These dimensions have reference to magnets whose directive power is about the same as in those employed in the Dublin Magnetical Observatory. The magnet bars, A and B, are here of the same size—each 15 inches in length, $\tfrac{7}{8}$ of an inch in breadth, and $\tfrac{1}{4}$ of an inch in thickness; they are of course magnetized, as nearly as possible, to saturation. The magnet C is 12 inches in length, but much smaller than A and B in its other dimensions.

both magnets are merely diminished or increased in a *constant ratio*,—namely, in the ratio of the force of the earth to the sum or difference of that force and the resultant force of the magnets. Lastly, it appears from what has been already said, that the mean position of the magnet c is likewise unchanged by the disturbing action, and that its variations of position are only altered is a constant ratio. The effect of the disturbing forces, therefore, is in every case readily allowed for.

As an example of this case of the general problem, let it be required that the three magnets shall be in the same right line, that line being no longer arbitrary, as before, but determined. The two equations (11) and (13) are in this case reduced to (21) and (22). Dividing the former by the latter, we have

$$\frac{P\,p^3}{Q\,q^3} = \frac{\frac{1}{3} - \cos 2\alpha}{\sin 2\alpha}, \quad \text{and} \quad \frac{p}{q} = \sqrt[3]{\frac{Q}{P}\left(\frac{\frac{1}{3} - \cos 2\alpha}{\sin 2\alpha}\right)}. \qquad (27)$$

This equation, in which the second member is known, determines the place of the centre of the intermediate magnet. Denoting the second member, for abridgment, by r, we have $p = q\,r$, $p + q = 1$; whence

$$p = \frac{r}{1 + r}, \qquad q = \frac{1}{1 + r}. \qquad (28)$$

It is manifest from (27) that we cannot have $\cos 2\alpha = \frac{1}{3}$, or $\sin 2\alpha = 0$, and accordingly that the angle α cannot have any of the values 0°, 90°, or 35° 16′, otherwise the intermediate magnet would be infinitely near one of the extremes.*

To determine the azimuth, ζ, of the plane of the intermediate magnet, we divide either of the original equations (21) or (22) by $\sin 2\alpha$, and substitute for

* In order that the intermediate magnet should be *equally distant* from the other two, the angle α must have one of the values determined by the equation

$$\frac{\frac{1}{3} - \cos 2\alpha}{\sin 2\alpha} = \frac{P}{Q} = \frac{A}{B}, \quad \text{or } \tan\alpha = \frac{3}{4}\frac{A}{B} \pm \sqrt{\frac{9}{16}\frac{A^2}{B^2} + \frac{1}{2}}.$$

When $A = B$, or the forces of the extreme magnets equal, this becomes

$$\tan\alpha = \frac{3 \pm \sqrt{17}}{4}\left(= 1.781, \text{ or} = -0.281\right);$$

and the corresponding values of α are $+$ 60° 41′, and $-$ 15° 41′.

$\frac{\frac{1}{3} - \cos 2\alpha}{\sin 2\alpha}$ its value $\frac{Pp^3}{Qq^3}$ above deduced. We thus obtain

$$\frac{\cos \zeta}{Pp^3} + \frac{\sin \zeta}{Qq^3} + 1 = 0.$$

Whence

$$\tan \zeta = \frac{-mn \pm \sqrt{m^2 + n^2 - 1}}{n^2 - 1}; \tag{29}$$

in which we have put, for abridgment,

$$m = \frac{1}{Pp^3} = \frac{Cc^3}{Aa^3}, \qquad n = \frac{1}{Qq^3} = \frac{Cc^3}{Bb^3}. \tag{30}$$

This solution becomes impossible when $m^2 + n^2 < 1$, or

$$\frac{1}{(Aa^3)^2} + \frac{1}{(Bb^3)^2} < \frac{1}{(Cc^3)^2}.$$

The formulæ (11) (13) suggest of themselves many other cases of easy solution. Thus, if it be assumed that $\gamma = 0$, $\alpha = 90$, or the line connecting A and B coincident with the magnetic meridian, and the line connecting B and C perpendicular to it, equation (13) gives $\zeta = 0$. Substituting in (11), it becomes $3 \sin 2\beta = 2\,Qq^3$, or, since in this case $q = \frac{1}{\cos \beta}$,

$$\sin \beta \cos^4 \beta = \tfrac{1}{3}\,Q;$$

from which the angle β is determined. This disposition of the magnets is represented in Fig. 6.

The equilibrium is fulfilled in this case independently of the value of P, or of the relative forces of the magnets A and C: the reason of this is evident. On the other hand, the solution requires that Q shall not exceed a small limit; for the first member of the preceding equation is a maximum, when $\tan \beta = \frac{1}{2}$, and substituting, the greatest possible value of Q is $\frac{48}{25\sqrt{5}} = 0.859$.

Again, if we have $\cos 2\gamma = \frac{1}{3}$, $\beta = 0$, (11) gives $\zeta = 0$, as before; and (13) becomes $3 \sin 2\alpha + 2\sqrt{2}\,Pp^3 = 0$. But $p = -\frac{\sin \gamma}{\sin \alpha} = \frac{-1}{\sqrt{3} \sin \alpha}$, and substituting,

$$\sin^4 \alpha \cos \alpha = \frac{\sqrt{2}}{9\sqrt{3}} P;$$

from which the angle α is determined. This arrangement is represented in Fig. 7.

The conditions of equilibrium are here satisfied independently of Q. As to P, it cannot exceed the limit determined by making the first member of the preceding equation a maximum. This gives $\tan \alpha = 2$; and, for the greatest value of P, $\frac{144}{25}\sqrt{\frac{3}{10}} = 3.155$.

VIII. *On the Constant of Refraction determined by Observations with the Mural Circle of the Armagh Observatory. By the* Rev. T. R. Robinson, D. D., *Member of the Royal Irish Academy, and other Philosophical Societies.*

Read 11th January, 1841.

IT may, perhaps, appear presumptuous in me to approach a subject which has already occupied so many of the greatest masters of mathematical science, and in the opinion of many is exhausted. But if we look without prejudice at the labours of Laplace, Bessel, Ivory, and Plana, besides many others of less renown, and carry our examination a little beyond the mere analytical work, we shall find that the problem of astronomical refraction has not been rigorously solved by theory, and I am even inclined to think never can be. All it appears to me that theory can be expected to perform, is the supplying astronomers with ready means of approximating to tables of refraction, which shall satisfy their observations; and on the other hand they are bound to remember, that such tables, however carefully verified for one observatory, may be defective when tried at another.

For in fact it is universally assumed in these investigations, that the atmosphere is arranged, with the surfaces of equal density spherical and concentric to the earth; this gives the differential of refraction in function of the density and distance from the centre. Now, firstly, this fundamental hypothesis is not even approximately true. Near the earth, the surfaces of equal temperature (and therefore of equal density) must depend on the figure of the ground; the air over a hill must be very differently circumstanced in respect of heat, from that at the same height over a deep valley. Forests, large bodies of water, and the vicinity of cities must exert a similar disturbing influence, and that to an extent which cannot be neglected. In a set of hourly observations, made some years since on the altitude of my meridian mark, I found an increase of refraction, amounting sometimes to 13″, when the intervening valley was overshadowed by clouds, though the meteorological indications at the observatory remained the same. But how much greater would the disturbance of a star have been whose light must have passed through many miles subject to these anomalies? For we have no reason to suppose that they are confined to the immediate vicinity of the earth's surface; they must extend as far as the clouds, (whose existence shews an irregular distribution of heat,) or at least six miles high; more than three times the height of Quito, at which Bouguer found the refraction only two-thirds of what it is at the level of the sea. Some remarkable facts respecting the variation of terrestrial refraction, when the ground is covered with snow, and immediately after sunset, are given by Struve, in his Gradmessung, but one still more in point is mentioned by the Rev. G. Fisher,* in the Appendix to Parry's Second Voyage, page 175. He found, while observing at Igloolik, that at temperatures of from 20° to 30° below Zero, and at an altitude of 3° 8′, the refractions of Sirius were about a minute less when observed over open sea to the south-east, than over land covered with snow or ice, to the south-west. The existence of these local anomalies can only be ascertained by low refractions; and therefore theory is in such cases unavailing.

But secondly, even were the hypothesis on which the differential equation of refraction is based strictly true, yet that equation cannot be integrated without assuming a relation between its variables, their real relation being unknown. Philosophers have been guided in this, either by supposed conformity to the law of nature, or by facilities of integration; but in both cases their results cannot be supposed to have any value except as far as they are confirmed by observation, and therefore all must be pronounced alike empirical. But at low altitudes observations are both difficult and uncertain, and therefore it is by no means easy to pronounce on the results of a given hypothesis; so that besides that lately published by Biot (but which I believe has not yet been applied to construct refraction tables) there are at least four of high authority; that of Newton, as modified by Bessel, supposing the temperature uniform, but changing the modulus of atmospheric elasticity by an experimental co-efficient; that of Simpson,

* To whom I am indebted for much valuable information respecting the important observations published there, and indeed for my acquaintance with the book itself.

assuming the density to decrease uniformly as the height increases; that of Laplace, expressing the density by a product of two factors, representing the preceding hypotheses, and that of Ivory, supposing it as $\left(1 - \frac{r - a}{5l}\right)^4$.* Now these are obviously mere arbitrary assumptions, and the verifications which some of them are supposed to receive by exhibiting the decrease of temperature at a small elevation, and the barometric formula for heights, are worth little; the first being unknown at any given place,† and the second being a consequence of any law which will make the temperature decrease nearly uniformly within a few thousand feet. The slightest attention to meteorological facts will show that there cannot be any general formula expressing the density in terms of *the height alone*, and that even could it be found for one place by experiment, it would be entirely inapplicable to any other. It is certain, that between the tropics there is an ascending current of heated air, replaced by a stream of cooler from the north, while it flows towards the poles, descending in its turn and giving out its heat; and it is therefore equally certain that the law of atmospheric temperature must depend on the latitude. It is not impossible, that in the arctic regions we may find a uniform temperature, or even an increase on ascending. Such must indeed be the case, if there be any truth in the conclusions of Fourier, or Poisson, respecting the temperature at the termination of our atmosphere; for if with the former we suppose it $= - 58°$ of Fahrenheit, or with the latter, much more elevated, approaching 32°, yet cold below either has been observed by northern travellers. At a given place we might, perhaps, by aeronautic investigations, ascertain the law of decreasing density and temperature, for a certain epoch; but it is highly probable, that this would not obtain when the sun had a different declination, or the weather was different;‡ it is unquestionable, that it would be

* The last appears the best, but it is to be regretted that Mr. Ivory has assumed the use of the internal thermometer, and not given separate reductions for the temperature of the barometer. This last also applies to the very convenient tables of Bessel's Refractions, given by Mr. Airy.

† Because the decrease in free air cannot be the same as that observed on the side of a mountain, and in contact with a mass of matter influenced both by the air and the earth's internal heat.

‡ In the celebrated ascent of Gay Lussac, the temperature at Paris was 87° Fahrenheit, so that the air cannot have been in a normal condition: the meteorological instruments below should have been noted every few minutes, and the times of observation above given. In the published

disturbed by wind, or variations in the hygrometric state of the air. And it must be remembered, that at least three-fourths of the entire refraction are produced in the region which is thus affected; and that in observation we find differences of 15 or 20 seconds in the same star, when the thermometer, barometer, and hygrometer of the observatory shew no change.

It appears to me, therefore, vain to expect an *a priori* solution of the problem of astronomical refraction, and that it will always be necessary to reform by observation whatever tables may be proposed to us. The tables of Bessel or Ivory—(if the refractive and thermometrical constants of the latter were corrected, I should prefer them)—are sufficiently exact for this purpose in the observatories of Europe.* Down to 74° zenith distance, it is known, that the law of density has no sensible effect on the refraction; and in ordinary cases this is sufficient for the astronomer, who seldom observes so near the horizon, because there the fluctuations of a star are so great, that a great number of observations are necessary to give even moderate precision. But he *must* occasionally observe, under such circumstances, comets and planets; and, besides, it is necessary for an accurate determination of the principal constant, that he should go as far from the zenith as is possible, without risking the certainty of his correction. In my latitude, at 74° zen. distance, an error in the constant is only doubled; and the average discordance of observation will be near a second; so that were we limited to the use of stars above this altitude, it would be almost

account it is stated, that the thermometer was steady at 30·75 cent. As light clouds existed far above the balloon there must have been an evolution of heat from their formation. Still it is to be wished that the experiment were repeated.

* In the Arctic regions all the tables fail completely. I give a couple of instances from the Appendix to Parry, already noticed, p. 209. They are Nos. 25 and 29. The first gives from 108 observations, the refraction = 665″.9 at zen. dist. 84°.13′, 82, Bar. 29.79, A. T. + 45, Ext. T. − 35°.9. After correcting for latitude, Bessel's refraction is 18″.72 in defect, Ivory's 13″.27, and mine 20″.71. Again, 32 observations give refraction = 342″.5 at 79°40′. 61, bar. 29.86, A. T. + 45°, E. T. − 26°.7. Here Bessel's is 40″.31 in excess, Ivory 31″.66, and mine 22″.78. It seems to follow from these and similar instances, that in such extreme cases the arrangement of the atmosphere must be regulated by very different laws from those that prevail in more temperate latitudes; and it seems equally obvious, that its influence on refraction commences much nearer the zenith. It is my intention to recur to these Arctic observations in a subsequent communication on the lower refractions.

impossible to determine it to the tenth of a second. But it is practicable to go about 10° lower, by a principle, first, I believe, remarked by Laplace; namely, that the refraction computed on the hypothesis of uniform temperature is greater than the truth, and on the hypothesis of uniformly decreasing density less, and that the mean of the two is nearly exact. For instance, Laplace gives for the horizontal refraction, (T = 32°; barometer, 29.92,)

U. Temp.	2394″.6	288.6
Observed	2106 .0	
Uniform decrease of dens.	1824 .1	281.9

The arithmetical mean = 2109.3; the geometrical = 2090. Ivory finds (T = 50, bar. = 30.00,)

U. T.	2254.5	223.0
French tables . . .	2031.5	
U. D. D.*	1722.7	308.8

In this case the second deviates the most, arith. mean = 1988.6; geometrical = 1970.7.

At zen. dist. 85° 16′.70, T = 54.2, bar. 30.24, I find with Ivory's constant,

U. T.	624.3	3.7
Ivory's first tables . .	620.6	
U. D. D.	615.8	4.8

Henderson found the refraction (by 29 Cape observations of γ Draconis) = 614.10, which, when increased for the difference between Ivory's constant, and Bessel's reduced to the Cape, would become 617.86.

The arithmetical mean = 620·05, the geometrical = 620.03.

Ivory has given a table constructed on the hypothesis of U T for T = 70 and B = 28.85, from which I take, at zen. dist. 86°,

U. T.	653.1	6.5
Ivory	646.6	
U. D. D.	642.5	4.1

Arithmetical mean = 647.80, geometrical 647.77.

* As corrected by Plana (Observations, Int. lxxxvi.) The series for U T is slowly convergent, and the computation would be very troublesome, were it not for the tables of the integral which Bessel gives in the Fundamenta.

Again, zen. dist. 87°,

U. T.	. . . . .	802.5	12.4
Ivory	. . . . .	790.1	
U. D. D.	. . . . .	776.1	14.0

Arithmetical = 789.30 ; geometrical = 789.19.

Lastly, Brinkley gives the comparison of 42 observations of α Lyræ S P with these hypotheses, zenith distance = 87°.42′, T = 35°, B. 29.50,

U. T.	. . . . .	1067″.0	20.5
Observed	. . . .	1046 .5	
U. D. D.	. . . .	1011 .0	35.5

Arithmetical = 1039″ ; geometrical = 1038·6. But it must be remarked, that the temperature is by the internal thermometer, the external being 31.3; the barometer also is $0^{i}.078$ too little ; in respect of the first of which the observed refraction should be lessened 9″.2, and for the second 2″.90.

It is evident that these means are not in error *one-twentieth of the difference between the two hypotheses ;* and, therefore, as far as 85° from the zenith may be depended on as certainly as any table extant.

Laplace used this principle not only in constructing the French tables, but also to show that the refractions above 74° are independent of the law of density. Brinkley, however, showed that the same method could assign them as far as 80°.45 ; the most important of the terms omitted by Laplace in the development of R in tang. θ has at that zen. distance in the two hypotheses the values 2″.60 and 1″.73 ; the arithmetical mean of these *cannot* be 0″.43 wrong, and its error is probably less than 0″.04. The opinion expressed by this great astronomer in his second memoir on refraction, Transactions Royal Irish Academy, vol. xiii. p. 169, that, by the method given there, a table of refractions could be more certainly derived from observation "than from any hypothesis respecting the actual variation of density," probably hindered him from pursuing the present method to its full extent, which, however, may be done with extreme facility.

In his notation, Transactions Royal Irish Academy, vol. xii. p. 83, the equation of refraction is,

$$d\mathrm{R} = \frac{-d\rho \times ab \sin\theta \sqrt{1+b\rho'}}{2r\,(1+b\rho)\sqrt{1+b\rho-\frac{a^2}{r^2}(1+b\rho')\sin^2\theta}}$$

when ρ is the density at the distance r from the centre, ρ' and a, the same quantities at the earth's surface;* $b\rho$ the refractive force of air at the density ρ, and θ the apparent zenith distance.

If we assume,

$$\text{A} = \frac{\sqrt{1+b\rho'}\,\sin\theta}{\sqrt{1+b\rho-(1+b\rho')\sin^2\theta}}$$

Q = refraction if the earth were plane,

$$s = \frac{r-a}{r},$$

Brinkley has shown, page 85, that,

$$d\text{Q} = \frac{-\frac{1}{2}b\text{A}d\rho}{1+b\rho}$$

$$d\text{R} = \frac{(1-s)\,d\text{Q}}{\sqrt{1+(2s-s^2)\times \text{A}^2}}$$

and by developing A we find,

$$\text{A}^n = (\text{tang}\,\theta)^n \times \left\{1 + \frac{n}{2}\,\frac{b(\rho'-\rho)}{\cos^2\theta}\right\},$$

omitting higher powers of b. Developing $d\text{R}$ we have,

* These quantities more strictly relate to the osculating circle, and the constant of a table must be modified accordingly. The quantity $\frac{l}{a}$ is one of these; if we assume the mean radius of curvature as the standard, and the earth's compression $\frac{1}{300}$, then for another latitude,

$$\frac{l}{a'} = \frac{l}{a} \times 1 + 0.0004991 \times \cos 2\text{L}.$$

Laplace has remarked that this should make the refraction to the north and south unequal. In fact, if we suppose the last rays of twilight to be once reflected, and that refraction ceases with reflection, (in which case I find, taking into account the curvature of the ray, which Delambre has neglected, that the height of the reflecting point is 41.536 miles,) and the ray is acted on in the case of horizontal refraction, through 8° 43′ of latitude. The change of the radius of curvature, and the place of its centre, must make a sensible difference in the two refractions, but the effect of the difference of temperature in the two trajectories is perhaps still greater.

The value of l is also inversely as local gravity, and that of b (or of the density corresponding to a given barometric column) directly as it; they must therefore be divided and multiplied respectively by $1 - 0.002695 \times \cos 2\text{L}$.

These corrections may seem minute, but are very sensible in low refractions.

$$d\text{R} = \qquad d\text{Q}$$

$$+\frac{\frac{1}{2}bd\rho}{1+b\rho} \times \begin{cases} s(\text{A}+\text{A}^3) - \frac{3}{2}s^2(\text{A}^3+\text{A}^5) \\ +\frac{1}{2}s^3(\text{A}^3+6\text{A}^5+5\text{A}^7) \\ -\frac{5}{8}s^4(3\text{A}^5+10\text{A}^7+7\text{A}^9) \\ +\frac{3}{8}s^5(\text{A}^5+15\text{A}^7+35\text{A}^9+21\text{A}^{11}) \\ \quad \&\text{c.} \end{cases}$$

From the height of the atmosphere given in the preceding note $=7.53\times l$, it appears that $b^2 s$ is nearly $=s^5$, and, therefore, we need not develope beyond terms of this order, and the equation becomes

$$d\text{R} = d\text{Q}$$

$$+\tfrac{1}{2}bd\rho \times \begin{cases} s\times\frac{\text{tang}}{\cos^2}.\,\theta\,[1+\frac{1}{2}b(\rho'-\rho)\,(1+3\,\text{tang}^2.\,\theta)] \\ -\frac{3}{2}s^2\times\frac{\text{tang}^3}{\cos^2}.\,\theta\,[1+\frac{1}{2}b(\rho'-\rho)\,(3+5\,\text{tang}^2\,\theta)] \\ +\frac{1}{2}s^3\times\frac{\text{tang}^3}{\cos^2}.\,\theta.\,[1+5\text{tang}^2.\theta+\frac{1}{2}b(\rho'-\rho)\,(3+30\text{tang}^2+35\text{tang}^4)] \\ -\frac{5}{8}s^4\times\frac{\text{tang}^5}{\cos^2}.\,\theta[3+7\text{tang}^2\theta+\frac{1}{2}b(\rho'-\rho)(15+70\text{tang}^2\theta+63\text{tang}^4\theta)] \\ +\frac{3}{8}s^5\times\frac{\text{tang}^5}{\cos^2}.\,\theta\,[1+14\,\text{tang}^2+21\,\text{tang}^4\,\theta+\frac{1}{2}b\,(\rho'-\rho)\,(5+ \\ \qquad 105\,\text{tang}^2\,\theta+315\,\text{tang}^4+231\,\text{tang}^6)]. \end{cases}$$

These terms are of the form $s^n d\rho$, and $s^n\rho\, d\rho$.

The hypothesis of uniform temperature is expressed by the equation,

$$\rho = e^{-\frac{as}{l}},$$

giving the density unity at the surface, and evanescent at an infinite height. Between these limits we have,

$$\int_1^0 s^n d\rho = -\frac{l^n}{a^n}\times(n\,.\,n-1\,\ldots\ldots 2\,.\,1)$$

$$\int_1^0 s^n\rho d\rho = -\frac{l^n}{a^n}\frac{(n\,.\,n-1\,\ldots\ldots\ldots 1)}{2^{n+1}}.$$

The hypothesis of uniformly decreasing density gives,

$$\rho = 1 - \frac{as}{2l}$$

$$\int_1^0 s^n d\rho = -\frac{l^n}{a^n} \times \frac{2^n}{n+1}$$

$$\int_1^0 s^n \rho d\rho = -\frac{l^n}{a^n} \times \frac{2^n}{(n+1)(n+2)}.$$

The term $\int s d\rho$, is the same on either hypothesis, being a result of the atmosphere's equilibrium; the coefficients of the higher terms differ, those on the hypothesis U T increasing much more rapidly. $\int s^2 d\rho$ is that which Brinkley added to Laplace's expression, using the arithmetical mean, which gives $\frac{5}{3} \times \frac{l^2}{a^2}$. I have preferred the geometric mean of the separate terms, as giving less weight to U T, which is especially necessary near the limit of convergence.* If we develope Q, pass from sines to arcs, and put μ for $\frac{\sqrt{1+b}-1}{\sin 1''}$, we shall have,

$$\text{R}'' = \mu \times \text{tang}\,\theta$$

$$+ \frac{\mu^2 \sin 1''}{2} \times \text{tang}^3\,\theta + \frac{\mu^3 \sin^2 . 1''}{2} \times \text{tang}^5\,\theta \qquad (\text{Q}'.\ \text{Q}'')$$

$$- \frac{b}{\sin 2''} \times \frac{l}{a} . \frac{\text{tang}}{\cos^2}\,\theta\ [1.00000 + b \times \text{tang}^2\theta\ (1.06698)] \qquad (\alpha.\ \alpha')$$

$$+ \frac{b}{\sin 2''} \times \frac{l^2}{a^2} . \frac{\text{tang}^3}{\cos^2}\,\theta\ [2.44949 + b \times \text{tang}^2\,\theta\ (5.04119)] \qquad (\beta.\ \beta')$$

$$- \frac{b}{\sin 2''} \times \frac{l^3}{a^3} . \frac{\text{tang}^5}{\cos^2}\,\theta\ [8.65117 + b \times \text{tang}^2\,\theta\ (26.92202)] \qquad (\gamma.\ \gamma')$$

$$+ \frac{b}{\sin 2''} \times \frac{l^4}{a^4} \frac{\text{tang}^7}{\cos^2} . \theta\ [38.43867 + b \times \text{tang}^2\,\theta\ (160.08103)] \qquad (\delta.\ \delta')$$

$$- \frac{b}{\sin 2''} \times \frac{l^5}{a^5} . \frac{\text{tang}^9}{\cos^2} . \theta\ [199.22000\ \&c.].$$

* The original intention was to have assumed the terms $= \sqrt{\alpha \text{I} \times \alpha' \text{I}'}$; α and α' being arbitrary factors determined by observation; but as the simple $\sqrt{\text{I} \times \text{I}'}$ was found to satisfy my observations,

The terms β, γ, and δ have nearly the ratio $\frac{4l}{a} \times \text{tang}^2\,\theta$, and therefore the convergence ceases when the fraction $= 1$; or below 85°. Near that limit several of the higher terms are equal with opposite signs, and therefore (Lacroix, III. p. 160) I retain half the two last, which I find give at 85° the same results as a much more extended development, including all affected with b^3 and $\frac{\text{tang}^{13}}{\cos^2}\,\theta$.

This expression may be put into the form given by Brinkley, certainly the most convenient with which I am acquainted,

$$\text{R} = \mu \times \text{tang}\,\theta - \text{C};$$

the last of which quantities can be tabulated with the argument zenith distance, and is, in most cases, independent of the barometer and thermometer.

Their influence is, when necessary, easily allowed for: if a unit of air at 50° become $1 + \epsilon\,(t - 50)$ at $t°$, the quantity $\frac{l}{a}$ must be multiplied by this factor, and that of μ or b divided by it, from which we deduce the change of C for temperature,

$$\text{D} = \epsilon\,(t - 50°) \times [a' + \beta - 2\text{Q}' - 3\text{Q}'' - \gamma],$$

which is always small from the absence of a, the largest of the terms.

this was unnecessary. Assuming Bessel's μ to be 57″.524, and Ivory's 58″.496, my table, when changed for these values, gives at their normal circumstances,

Zen. dist.	R — B.	R — I.
77°	—0″.11	—0″.02
78	—0 .10	—0 .05
79	—0 .11	—0 .07
80	—0 .12	—0 .10
81	—0 .06	—0 .12
82	—0 .08	—0 .19
83	—0 .10	—0 .25
84	—0 .13	—0 .30
85	—0 .28	—0 .42

The difference obviously depending on some slight difference between the values of μ and those used in computing the tables. It is equally evident, that to the zenith distance of 85 the results of the three formulæ are identical for all practical purposes.

If the barometer become $\text{H} + h$, instead of H, the normal pressure, the terms α, β, γ, &c., are to be multiplied by $\frac{\text{H}+h}{\text{H}}$; $\text{Q}', \alpha', \beta'$, &c., by its square, and Q'' by its cube; we find the barometric change of C,

$$\text{E} = \frac{h}{\text{H}} \times [\text{C} + \text{Q}' + 2\text{Q}'' - \alpha' + \beta' \text{ \&c.}].$$

If h be one inch, the value of E at $85^\circ = -2''.34$, so that these corrections can be worked by mental computation.*

* This form of the refraction has the advantage of being easily applicable to the equatorial. In a memoir on this instrument, (Trans. R. I. A. vol. xv.,) I have shewn that most of its corrections depend on an arc of the hour circle passing through the star intercepted between the pole and a perpendicular from the zenith. It is also equal to the intercept between the horizon and equator, whence I call it the horizontal declination. Denoting it by the symbol ζ, the polar distance by D; and being satisfied with the approximation, Refr. in P. Dist. = Refr. in Zen. Dist. $\times$ cosine of angle of position, we have,

$$(\text{R}) = \mu \times \text{tang}(\text{D} - \zeta) - \text{C} \times \frac{\text{tang}(\text{D} - \zeta)}{\text{tang}\,\theta}.$$

C may be put in the form,

$$\frac{\text{tang}}{\cos^2}.\theta\,[q' \sin^2\theta - a + b\,\text{tang}^2\theta - c\,\text{tang}^4\theta \text{ \&c.}],$$

and its resultant in declination,

$$(\text{C}) = \frac{\text{tang}}{\cos^2}(\text{D}-\zeta) \times \frac{\cos^2\zeta}{\sin^2\text{lat}} \times \left\{ \begin{array}{l} [q' \sin^2(\text{D}-\zeta) - a + b\,\text{tang}^2(\text{D}-\zeta) - c\,\text{tang}^4(\text{D}-\zeta)] \\ + q' \cos^2(\text{D}-\zeta)\left(1 - \dfrac{\sin^2\text{lat}}{\cos^2\zeta}\right) \\ + \dfrac{\left(\frac{\cos^2\zeta}{\sin^2 l} - 1\right)}{\cos^2(\text{D}-\zeta)} \times \left[b - c\left(2\,\text{tang}^2(\text{D}-\zeta) + \dfrac{\left(\frac{\cos^2\zeta}{\cos^2(\text{D}-\zeta)} - 1\right)}{\cos^2(\text{D}-\zeta)}\right)\right] \end{array} \right\}$$

The first of these three terms is obviously the value of C taken with the argument $(\text{D} - \zeta)$ instead of θ, and multiplied by $\frac{\cos^2\zeta}{\sin^2\text{lat}}$, of which latter a table for each hour is sufficient. The second is never $= 0''.01$; and the third, which is insensible above 80°, is computed by the formula

$$\frac{\text{tang}}{\cos^4}(\text{D}-\zeta)\frac{\cos^2\zeta}{\sin^2\text{lat}} \times \left(\frac{\cos^2\zeta}{\sin^2\text{lat}} - 1\right)\left[\log^{-1}(6.28162) - \log^{-1}\frac{(3.90574)\left(\frac{\cos^2\zeta}{\sin^2 l} + 1\right)}{\cos^2(\text{D}-\zeta)}\right],$$

which at 85° zenith distance and 6 hours from the meridian, is only 1″58, and (if it be thought

To construct a table of refractions from this formula, we require the numerical values of $\frac{l}{a}$, of μ at some given temperature and pressure, and of ϵ the expansion of air for one degree of Fahrenheit. The last of these has almost universally been taken from Gay Lussac, who found that a unit of any gas or vapour at the freezing point of water, became 1.375 at the boiling point. But the experiments of Rudberg have shown that this number is too great, and that the true increase is 1.365. I have, therefore, used this coefficient, notwithstanding the opinion of some whose authority is of much weight, that even Gay Lussac's number should be increased on account of the moisture of the atmosphere. But the expansion of vapour is the same as of dry air: if water be present, it does indeed seem greater, because heat increases the quantity as well as the bulk of the vapour, and a correction to this effect is necessary to the barometric measurement of heights. In respect of refraction the case is otherwise; aqueous vapour and dry air refract alike under equal pressure and temperature; when, therefore, more vapour is added to the atmosphere, the effect is the same as if so much dry air were added as is equivalent to its tension. Observation leads to the same conclusion; for the illustrious astronomer of Königsberg found that the coefficient which satisfies the variations of refraction is 1.00364.—Tab. Reg. p. lx. The only way in which the hygrometric state of the atmosphere can affect refraction is by changing the value of l, or by varying the arrangement of the strata. The latter of these cannot be taken into account, and the former is, in this climate, insensible within the limits of this inquiry.

The value of l used is that given by Arago and Biot in their experiments on the refractive power of air. They give it for 0 centesimal; but as their experiments were made at the mean temperature 10° cent. or 50° Fahrenheit, the normal temperature of most refraction tables, their result is not affected by the error of Gay Lussac's expansion.

There remains only the refractive power of air, which may be investigated

necessary to employ it) can be computed by the sliding rule. A table of ζ for every minute of the first 6 hours is almost essential to the use of the equatorial, and if my first table and the second $\times \frac{\cos^2\zeta}{\sin^2 l}$ were added to it, the refraction can be as easily computed as on the meridian.

either by direct experiment, as was done by Arago and Biot,* or by astronomical observations. Notwithstanding the well known accuracy of these distinguished philosophers, it seems desirable that their conclusions should be verified by the more refined means of examination, which Arago himself has since indicated. At present, the result appears in excess, giving for μ at 50° and $29^{i}.60$ the value $57''.82$. That which is most generally received is De Lambre's, employed in the French tables, as well as in those of Brinkley and Ivory. It is at the same temperature and pressure $57''.72$, and was deduced from observations made with the repeating circles of Le Noir, so that it would not have much weight now were it not for the confirmation which it seemed to derive from the comparison of simultaneous observations by Brinkley and Brisbane, at Dublin and Paramatta. The sum of the Dublin north polar, and Paramatta south polar distances gives very nearly 180 degrees, and the resulting value of μ is 57.77 ; but it must be remarked, that the temperature used in computation is that by the internal thermometer, which, however necessary at Dublin, may not be so at the other observatory. It is also important to notice, that the Dublin barometer is by no means perfect. I have been enabled to determine its error by comparison with that of the Magnetic Observatory of Trinity College, (by Newman, and differing from mine and the standard of the Royal Society merely in having the cistern of glass.) Observations made during thirteen successive days at 22^{H} give

	BAR.	E. T.	A. T.
Magnetic Observ. .	30.001	41.60	41.60
Astronom. Observ. .	29.625	35.53	37.70

The difference of height of these stations is, according to Captain Larcom, 258.8 feet, and I compute that the actual pressure at the upper station was 29.702; so that the reading there requires the correction + 0.077. Subsequently this has been confirmed by the kindness of Dr. Coulter, who compared two portable barometers, by Cary, with that of the magnetic observatory, very carefully. They were then carried out to the astronomical observatory, compared there, and on their return compared again with the magnetic. From the result of the two sets I deduce the corrections + 0.0770, and + 0.0800, the mean + 0.0785 I consider preferable to the other, and this would reduce the constant 57.72 to

* Memoires des Scavans Etrangers, T. vii.

57.567, a remarkable approximation to that of Bessel. This is, however, for the temperature of the barometer 37°; but it will probably avail for 50° also; as if, on the other hand, the Dublin barometer has a wooden mounting, on the other there is probably a little air in the upper part of the tube which will compensate for its inferior expansion of scale.

Bessel has given for a or $\frac{\frac{1}{2} b}{1+b}$, 57.538 at 48°.75, but the barometer at 50°. He, however, found afterwards, that the refractions of his table require to be multiplied by 1.001779, which would make it at the normal temperature and pressure 57.4993, hence $\mu = 57.524$. This appears to satisfy the Greenwich observations, as well as* those at the Cape of Good Hope; and its unexpected agreement with Brinkley shows how safely it may be depended on. At the same time, the very circumstances of that agreement give additional weight to the opinion which I have already expressed, that every fixed observatory should verify the refractions which it employs, and employ meteorological instruments of the best quality that can be made.

The *observed* refraction of a star below the pole is obviously (omitting degrees)

$$\text{R} = \text{O} - \delta,$$

O being the observed polar distance, δ the assumed declination of the star. Calling $d\text{O}$ and $d\delta$ the corrections which these require, the true refraction is

$$\text{O} - \delta + d\text{O} - d\delta.$$

If we put $\mu \times \text{V}$ for the tabular refraction, we have,

$$\text{V}\,(\mu + d\mu) = \text{R} + d\text{O} - d\delta,$$

Now, the polar point having been determined with an erroneous refraction, all the polar distances require the correction $d\mu \times \text{P}$; and if we determine the declination by observations above the pole, we have,

$$d\text{O} = d\mu \times \text{P}; \quad d\delta = -\,d\mu\,(\text{V}' + \text{P});$$

and hence,

$$\text{R} - \text{V}\mu = d\text{R} = d\mu\,[\text{V} - \text{V}' - 2\text{P}] = d\mu \times \text{K}.$$

* When the necessary corrections for the latitude and the change of the length of the pendulum are applied.

The constants V and V′ must be computed for the mean refraction of each set of observations; P from the annual mean temperature and pressure, as the observations for index correction and latitude extend through the year.

If we observe a star of southern declination, and assume it to have been well determined at some place where it passes near the zenith, we obtain $d\mu$ with a much larger coefficient, for we find in the same way,

$$dR = d\mu\,(V + P) = d\mu \times K.$$

It may be doubted, however, whether anything is gained by the superior magnitude of K; for it is unsafe to argue, as if the results of one set of instruments were identical with those which another would give in the same locality. The refraction used at the southern observatory must also have been carefully verified, as P′ the polar constant is in those existing very considerable.

The process must, of course, be applied to as many stars as possible, both for the sake of accuracy in the final result, and also because the identity of the values of $d\mu$, obtained at different zenith distances, is an evidence of the correctness of the formula used to compute the refraction. Among the various modes of combining the partial results, I prefer that which makes the sum of the squares of errors of observation a minimum; not taking into account those irregular fluctuations to which low stars are liable, caused by momentary changes in $d\mu$, or in the law of density, and, therefore, scarcely coming within this application of the theory of probabilities.* This gives the formula,

$$d\mu = \frac{K \times S\,(dR) + K' \times S\,(dR') \ldots.}{K^2 \times n + K'^2 \times n' \ldots.}$$

The Armagh circle has been described by me in the Memoirs of the Royal Ast. Soc. vol. ix. After using it pretty extensively, during the last six years, I have found no reason to change the favourable opinion of it which is expressed there; and, in particular, find no trace of the evil which Mr. Airy considers probable in circles divided on the face, namely, great and irregular fluctuations of run in the microscopes, (Mem. R. Ast. Soc. vol. x. p. 266.) So far from this, it is remarkably steady in that respect. A change of 30° alters the mean run of the four microscopes from 0″.25 to 0″.47; the utmost force that can be applied

* See on this subject, Bessel Ast. Nachrichten, No. 358.

drawing the instrument from the pier, and pushing it toward it, makes only a change of 0″.02; of 30 sets taken round the circle at different times, the greatest I have found is 0″.75, and the least 0″.00; and during the last three years that at 360° (which equals the mean of the 30 sets) has been within the limits of 0″.25 and 0″.54. In respect of its division, after a careful examination of 288 diameters in four positions, I have stated, that I considered it good; trifling, however, as the resulting error may be, it is obviously always necessary to correct for it when it is known. I have not, however, obtained my corrections in the present instance by the method described in that memoir. The errors which I found were absolutely casual, so that it was impossible to interpolate between them; the individual research of each would have demanded an impracticable sacrifice of time; and even could this have been afforded, the value of the result appears to me at least doubtful. All such modes of examination assume, that the divisions keep the same relative position while the circle is turned through any arc; but it is found in actual experience, both with this and other circles, that occasionally the correction of a diameter varies with its situation to a whole second or even more. I have, therefore, applied twelve equidistant microscopes to the circle; and presuming (as is also shown by the table of errors which I had constructed by my first method of correction) that their mean is free from sensible error, I use it to correct that of the four reading microscopes, in a way as simple as I believe it to be effective. Let M_x, m_x be the means of the reading microscopes, and of the twelve when any number x is at the index. Then, on this supposition, we have,

$$m_x - m_o = \mathrm{M}_x - \mathrm{M}_o + \epsilon(x) - \epsilon(o).$$

We may assume the reading of the four at o to be a zero to which all others are referred, and therefore,

$$\epsilon(x) = (m_x - m_o) - (\mathrm{M}_x - \mathrm{M}_o),$$

which only implies the permanence of the microscopes while the readings are taken. Out of more than 100 of these corrections most are negative, which arises from the zero reading M_o requiring, according to my former mode of examination, a correction of $+$0″.93; about one-fourth of the number differ from this more than $\pm$ 0.49, and in some I have found reason to suspect a minute change depending on the temperature. As, however, it can be determined in a few minutes at the very time of observation, this is of no consequence.

The index correction of this instrument is deduced from observations of Polaris. The star is observed five times near the meridian, and reduced to it by a table computed from the formula,

$$do = \text{A} + \text{A}^2 \times \text{tang}\,\delta \times \sin 1'',$$

where,

$$\text{A} = \frac{\sin \times \cos . \delta}{\sin 1''} \times \text{versine P}.$$

These, compared with the mean places of Bessel brought up by the constants of Baily's catalogue (for the time) and corrected for the term 2☽, give the approximate correction. When conjugate observations (above and below the pole) can be obtained, the mean is independent of any error of the assumed declinations; but at other times the difference between Bessel's place and my own is applied as a correction.* As long as the difference of individual results is manifestly mere error of observation, it is assumed that the mean is the index correction during that period. Its changes are slow, having an annual period, and a given extent of variation during the eight years that the instrument has been used. The most probable cause of this appears to be some influence of temperature on the hill, for the transit instrument, and a telescopic meridian mark about fifty feet south, suffer analogous variations. As the fact is curious, I annex a table of the index corrections during 1839, which will also show that no error can arise from its occurrence.†

* Equal to $+0''.21$ by 700 conjugate observations.

† 1838, Dec. 18,	1839, Feb. 24,	— 4.16	80 obs.
1839, Feb. 24,	„ April 7,	— 4.75	40
„ April 7,	„ „ 24,	— 5.20	50
„ „ 24,	„ May 16,	— 4.19	55
„ May 16,	„ June 3,	— 3.27	115
„ June 3,	„ „ 25,	— 1.63	10
„ „ 25,	„ Sept. 11,	— 0.14	75
„ Sept. 11,	„ Oct. 18,	— 2.39	45
„ Oct. 18,	„ Dec. 29,	— 3.49	105
„ Dec. 29,	1840, Feb. 28,	— 4.27	25

The declinations of those refraction stars which are in the Nautical Almanac were compared with its places, as long as they were given to the second place of decimals. Afterwards, they were reduced by the constants of Baily's catalogue, and compared with its mean places for the year, corrected when necessary for proper motion. The others were taken from that catalogue, and reduced by its precession, corrected for Bessel's last value of n, and for secular variation (computed from its value compared with the precessions given in the Fundamenta). When any of them have been observed at Greenwich, by Airy, the proper motion has been deduced from his results by the formula,

$$\pi = \frac{\text{A} - \text{cat} + \frac{4}{5}(\text{P} - \text{B}) - 1''.053 \times \cos a}{75 + t},$$

where $\text{P} - \text{B}$ is the number found in the last column of the Fundamenta, t the time in years from 1830, and 1.053 the correction for the error in the constant of precession used in that work. When Airy had not observed the star, I use my own declination changed for Bessel's refraction.

The low stars are often neat spectra (that of a Lyræ, I have found 22″ long); sometimes the blue and violet disappear for several seconds, and sometimes, though less frequently, the red, the rest remaining unabsorbed. When the colours are distinctly separated, I take the yellow where it borders on green, which I think a tolerable average for the mean of the spectrum. The star should be carefully watched during its whole transit, for the undulations that produce irregular refraction are often of long duration; and sometimes a star, which is apparently well bisected for several seconds, will leave the wire altogether.

The temperature is observed by a thermometer of Troughton which I found here. I have verified its freezing and boiling points to assure myself that it had not undergone the change said to have occurred in some thermometers. I have also compared it at several points with a standard instrument made for me by Troughton and Simms, in 1834, and think it of equal excellence. It is established at a north window of the eastern tower, about four feet above the centre of the circle, and twelve distant in a horizontal direction, in a semicylinder of polished copper, and an interior one of tin, arranged so as to permit a free circulation of air, but excluding all external radiation. In summer, when the rays of the sun reach the northern side of the tower, a second thermometer is used at a southern window of the same tower, till both agree, which generally is the case an hour after sunset. The internal temperature is also in most cases recorded, from a third standard thermometer attached to the telescope near its centre; but in this observatory it is not to be used in computing refraction. If any error were produced by preferring the external, its amount should be greatest when the difference is greatest, which I do not find to be the case. For instance, among 39 refractions of α Cygni, I find,

9 with I — E from 0° to 3°, mean 2°.37, give diff. from mean — 0″.22.
10 from 3° to 4° diff., mean 3°.39, give — 0″.17
10 from 4° to 5° diff., mean 4°.45, give + 0″.58
10 from 5° to 7°, mean 6°.01, give — 0″.21

In this star, 1° would change the refraction 0″.72.

Among southern stars, 23 of λ Sagittarii.

8 from 0° to 3° mean 2°.16 give — 0″.22
8 from 3° to 5° mean 3°.78 give — 0″.11
7 from 5° to 7° mean 5°.66 give + 0″.33

Here 1° gives a change of 0″.65. In these the discordances obviously have no connexion with the state of the internal thermometer; and the case is the same with other stars.

The barometer used was, till December 4, 1835, a portable one, by Ramsden. It was then replaced by a standard one of Newman, similar to that described by Mr. Baily in the Philosophical Transactions for 1837, p. 431. Mr. Newman states, that the specific gravity of its mercury is 13.545 at 60°, and that the diameter of its tube is $0^{i}.570$. In such a tube the correction for capillary action is nearly insensible; but it happens to be unnecessary here, for a reason given by Laplace, Conn. des Tems, 1829, but not, that I am aware, noticed in any English work. In barometers like this, the scale is terminated at its lower extremity with a point which is brought into contact with the mercury of the cistern; but the surface of the latter is also curved, so that the contact, if near the edge, is made at a surface lower than the real zero. If the distance from the edge be properly assumed, this may be made to counteract the depression above: it is rather too great here, giving only $0^{i}.003$, but the rest is neutralized by the fact, that the contact (if estimated, as I do it, by the meeting of the point and its reflected image) does not take place without a minute depression of the mercury, which is between 0.001 and 0.002.

The refractions have been computed with $\mu = 57.7682$ (Brinkley's reduced to my latitude), and the colatitude 35° 38′ 47″.3. In this climate and this exposed situation, it is not very easy to observe by reflection, and I have not yet definitively settled this element.

With the first division of the circle,	41	pair give	47″. 22
With the second ,,	58	,,	47″. 48
With the third ,,	132	,,	47″. 37
	mean	.	47″. 37

The first and third are corrected for error of division. In the second, three divisions were read at each microscope. It is obvious that these give no reason for changing 47″.3, which had previously been determined with Troughton's equatorial by upwards of 200 pair of observations; and equally so that whatever uncertainty there be, can have no effect.

The following are the results that I have obtained:

45 ω^2 *Cygni.*

Twelve observations (1838. 772) with Brinkley's Constant of Refraction give the Declination for 1830,

$$\delta = +48° \ 23' \ 1''. \ 51.$$

Precession = + 11″. 844; sec var. = + 0″. 212; proper motion = + 0″.033.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.*	OBS. REFRACT.	dR.
1836, Feb. 14.	42.2	43.5	44.2	30.122	77° 10′. 53	256.67	+ 0.01
,, ,, 17.	36.2	38.3	39.1	30.241	77 10 55	256.51	— 4.47
,, ,, 26.	29.7	34.5	35	28.979	77 10 65	252.63	— 0.77
1838, Feb. 7.	37.0	39.5	40.1	29.804	77 10 27	253.50	— 3.07
,, ,, 8.	37.5	39.9	41.4	30.173	77 10 27	255.00	— 4.58
,, ,, 15.	38.8	44.1	45	29.768	77 10 36	250.60	— 4.70
,, ,, 17.	35.5	39.3	40.6	29.367	77 10 35	251.06	— 2.62
,, ,, 23.	31.2	35.6	37.1	29.474	77 10 28	256.20	— 0.76
,, ,, 29.	43.8	46.8	48.3	30.409	77 10 29	256.63	— 1.44

* The figures after the minutes of zenith distance are decimals.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839,	Feb.	9.	37.1	..	43.1	30.084	77° 9′. 89	257.20	— 1.64
,,	,,	12.	36.7	..	40.9	30.046	77 9 90	257.59	— 1.19
,,	,,	14.	35.8	..	40.5	29.733	77 9 93	256.58	— 0.33
,,	,,	17.	34.2	..	39.5	29.915	77 9 95	261.75	+ 2.73
,,	,,	18.	31	..	34.2	29.380	77 9 96	255.97	— 0.17
,,	,,	24.	33.1	..	37.4	29.462	77 10 02	253.11	— 2.59
,,	April	5.	40.9	..	43.7	29.735	77 10 12	253.17	— 0.72
,,	,,	7.	38.1	40.9	42.2	30.091	77 10 07	256.50	— 1.94

$17 \times d\text{R} = -28''.25$ $\qquad$ $d\text{R} = -1''.66$

$\text{K} = 2.8861$ $\qquad$ $d\mu = -0.576$

31. *o Cygni.*

Twelve observations (1838. 533) give

$$\delta = +46^\circ\,13'\,45''.59.$$

Precession $= +10''.648$; sec var. $= +0''.228$; proper motion $= +0''.039$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	March	1.	29.2	34	35	30.193	79° 18′. 92	314.06	— 2.27
,,	,,	14.	34.1	37.3	39.5	30.287	79 18 91	314.91	+ 0.98
,,	,,	23.	32.2	34.5	35.8	29.665	79 19 05	307.47	— 1.38
,,	,,	24.	29	33.3	36.8	29.725	79 19 05	307.50	— 4.04
,,	,,	30.	36.1	38	42.1	29.758	79 19 03	309.29	+ 2.16
,,	April	3.	35	37.8	39	29.429	79 19 03	308.90	+ 4.41
,,	,,	4.	35	38	40.3	29.683	79 19 11	304.79	— 2.36
,,	,,	7.	38.9	41.7	43.2	30.297	79 19 03	309.08	— 1.77
1838,	Feb.	20.	31	34.4	35	29.496	79 18 51	305.76	— 1.88
,,	,,	21.	31.8	35.5	36.6	29.577	79 18 78	307.62	— 0.41
,,	March	6.	38.8	39.7	40.2	29.456	79 18 93	301.81	— 0.42
,,	,,	7.	36.5	38.8	40.3	29.790	79 18 86	305.76	— 1.42
,,	,,	8.	37.9	39.9	41.7	30.176	79 18 79	310.09	— 0.10
,,	,,	17.	35.8	39.1	40.9	29.368	79 18 95	301.48	— 1.78
,,	,,	23.	31.3	35.7	37.3	29.480	79 18 88	309.24	+ 1.86
,,	,,	29.	44.2	47	48.5	30.410	79 18 83	309.43	+ 1.04

$16 \times d\text{R} = -7''.38$ $\qquad$ $d\text{R} = -0''.46$

$\text{K} = 3.7450$ $\qquad$ $d\mu = -0.160$

Capella.

Eighteen observations (1837. 65) give,

$*\delta = + 45° 48' 54''. 12.$

Precession $= + 4.840$; sec var. $= - 0''.627$; proper motion $= - 0''.472$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, June 22.	58.3	63.6	64.8	30.114	79° 44'. 17	307.47	— 0.54
" " 25.	50	56.9	59	30.076	79 44 10	311.98	— 0.43
" July 7.	55.8	60.3	62	30.100	79 44 15	309.64	+ 0.23
" " 8.	58.8	62.9	65	30.019	79 44 18	308.36	+ 1.57
" " 9.	55.3	62	64	29.899	79 44 16	309.66	+ 1.91
" " 13.	56.4	61	64.5	29.472	79 44 28	302.94	+ 0.19
" " 14.	58.3	65.4	64.2	29.544	79 44 11	302.49	+ 0.34
" " 16.	58.8	60.6	62.3	29.917	79 44 22	306.27	— 1.46
" " 26.	60.1	62.1	64.1	29.762	79 44 32	301.23	— 2.22
" " 27.	55.4	59	62	29.571	79 44 30	301.86	— 2.56
" August 5.	48.9	53	55.9	30.150	79 44 11	313.34	— 1.24
" " 6.	52.8	56.9	59	30.239	79 44 17	311.43	— 1.55
" " 7.	53.5	57.8	60.3	30.264	79 44 15	311.58	— 1.28
" " 8.	56.1	59.5	61.8	30.193	79 44 18	309.35	— 1.02
" " 14.	58.2	61.3	64	30.069	79 44 21	307.58	— 0.09
" " 15.	60.9	62.5	64.5	30.079	79 44 25	306.22	+ 0.03
1838, July 25.	52.1	..	59	29.897	79 44 11	307.74	— 2.08
" " 26.	52.4	57.1	58.5	29.678	79 44 20	303.13	— 4.27
" August 4.	56.7	..	62	29.203	79 44 24	299.29	— 0.56
" " 5.	54	58	60	29.008	79 44 27	298.63	— 0.83

$20 \times d\text{R} = - 15''.86$ $\qquad$ $d\text{R} = - 0''. 79$

$\kappa = 3.7318$ $\qquad$ $d\mu = - 0.\ 212$

* Brinkley's δ . . . =	54''.70	Airy (Cambridge,) . .	54''.78
Bessel's, ...	53 .61	Argelander, ...	53 .50
Airy (Greenwich,) . .	53 .40	Mine, ...	54 .31

P.XXI. 157 *Cygni.*

Fifteen observations (1838. 800) give,

*δ for 1838. Jan. 1, $= +45° 42' 55''.74$.

Precession $= +15''.586$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, March 13.	29.2	33.6	34.8	30.211	79° 50'. 72	329.51	— 2.93
„ „ 14.	33	36.8	38	30.277	79 50 69	331.53	+ 1.14
„ „ 24.	28.2	33	35.1	29.726	79 50 81	325.78	— 1.96
„ „ 29.	32.1	35	38	29.535	79 50 84	324.26	+ 1.31
„ „ 30.	34.7	36.5	42.1	29.760	79 50 82	326.20	+ 2.68
„ April 1.	33.8	35	39	29.810	79 50 85	324.39	+ 0.29
„ „ 3.	33	37	38	29.438	79 50 92	320.41	— 0.91
1838, „ 11.	40.5	45	46	29.849	79 50 64	321.39	+ 0.99

$8 \times d\text{R} = +0''.62$ $\qquad$ $d\text{R} = +0''.077$

$\text{K} = 4\ .0544$ $\qquad$ $d\mu = +0\ .019$

22. *Andromedæ.*

Eleven observations (1838. 337) give,

$\delta = +45° 7' 33''.65$.

Precession $= +20''.056$; sec var. $= -0''.009$; proper motion $= +0''.005$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, May, 3.	44.7	46.2	50	29.722	80° 23'. 54	331.12	— 2.21
1838, „ 4.	44.2	48.8	50.1	30.008	80 23 12	334.54	— 3.13
„ „ 5.	48.7	52	53.5	30.200	80 23 12	334.72	— 0.89
„ „ 6.	52.1	54	55.8	30.163	80 23 15	332.73	— 0.11
„ „ 8.	56.5	60	61.8	30.176	80 23 19	329.47	+ 0.56
„ „ 10.	47.1	53.2	55	30.260	80 23 06	338.20	+ 0.90
„ „ 11.	49.1	53.5	55.2	30.132	80 23 15	332.81	— 1.72
1839, April, 17.	37.8	40.8	43.4	29.101	80 22 82	328.10	— 2.83

* This star has not been reduced to 1830, as I am doubtful of Piazzi's place; the right ascension which he gives is also erroneous.

It is rather too faint for subpolar observation here.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839, April, 18.	36.9	43.3	44.2	29.212	80° 22′. 77	331.71	— 1.05
,, ,, 19.	40	42.7	43.9	29.766	80 22 70	336.39	— 0.38
,, ,, 23.	46.8	50.5	51.3	29.916	80 22 79	331.63	— 2.02
,, ,, 24.	44.4	47.4	49.2	29.912	80 22 73	335.20	— 0.05
,, ,, 30.	50.6	53	54	29.818	80 22 87	327.36	— 2.54
,, May 2.	46.1	48.1	53	29.890	80 22 77	332.96	— 0.78
,, ,, 7.	49.8	51	53.1	29.875	80 22 86	327.98	— 3.12
,, ,, 10.	43.2	47.4	49.2	30.124	80 22 76	334.07	— 1.90
,, ,, 12.	44.9	47.9	50.8	30.002	80 22 81	331.43	— 3.30

$17 \times d\text{R} = -24''.57$ $d\text{R} = -1''.44$

$\kappa = 4.1560$ $d\mu = -0''.348$

β Aurigæ.

Nine observations (1837. 675) give

$* \delta = +44° 55' 12''.66.$

Precession $= +1''.132$; sec var. $= -0''.642$; proper motion $= -0.''019$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1833, July 23.	49.9	56.6	..	29.718	80° 37′. 97	339.66	+ 2.07
,, August 1.	56.3	62.2	..	30.348	80 38 00	339.51	— 0.64
,, ,, 2.	55.9	61	..	30.268	80 38 05	336.49	— 2.98
1835, July 29.	55.1	..	..	30.076	80 37 99	331.64	— 6.35
,, ,, 31.	57.7	62	..	29.993	80 37 93	335.03	+ 0.71
,, Aug. 2.	58.2	62	..	29.871	80 37 98	331.45	— 1.66
,, ,, 6.	53.8	61.6	..	29.796	80 37 91	336.55	+ 0.96
,, ,, 30.	57.2	60.6	..	29.858	80 37 97	333.55	— 0.47
1837, July 8.	57.7	63	64.2	30.025	80 37 79	334.92	+ 0.52
,, ,, 9.	54.1	61	63.0	29.896	80 37 73	338.34	+ 1.98
,, ,, 10.	56.3	62.3	65	29.846	80 37 80	334.37	+ 0.22
,, ,, 13.	56	59.8	63.1	29.454	80 37 89	329.11	— 1.10
,, ,, 14.	57.7	65	64	29.544	80 37 85	331.49	+ 0.55

* Airy (Greenwich, 36 and 37) . . 11″. 40 Argelander . . . 11″. 00
,, (Cambridge) . . . . 12 . 35 Mine . . . . . 12 . 76

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 16.	55	60	61.8	29.908	80° 37′. 75	337.77	+ 1.72
„ „ 20.	55 5	61	63	29.934	80 37 86	331.43	— 4.50
„ „ 27.	55.8	58.2	61.5	29.571	80 37 88	330.75	— 0.80
„ August 5.	47.8	52	54.9	30.152	80 37 67	339.71	— 4.35
„ „ 6.	51.5	54.9	57.2	30.239	80 37 75	339.43	— 2.82
„ „ 7.	51.2	55.7	59	30.264	80 37 74	339.63	— 3.14
„ „ 8.	55	58.9	61	30.193	80 37 79	336.78	— 2.43
„ „ 14.	57.1	61	63	30.069	80 37 82	335.27	— 1.07
„ „ 15.	58.8	61.9	63.1	30.081	80 37 81	333.83	— 1.49
„ „ 16.	60.9	63	65	29.971	80 37 89	331.26	— 1.41
„ „ 26.	50.9	55.8	59	29.930	80 37 71	339.54	+ 0.45
„ „ 29.	48.2	54.9	57.3	29.429	80 37 86	333.88	— 1.49
„ „ 31.	50.1	55	57	29.266	80 37 97	330.74	— 1.54
1838, July 25.	50.8	..	58	29.883	80 37 71	338.42	— 0.21
„ „ 26.	51.2	..	57	29.680	80 37 75	336.39	+ 0.30
„ August 4.	55.7	..	61.5	29.205	80 37 86	330.05	+ 2.41
„ „ 5.	53.1	..	59.1	29.013	80 37 98	323.32	— 4.03

$30 \times d\textsc{r} = -30''.59$ $\qquad d\textsc{r} = -1''.02$

$\textsc{k} = 4.2046$ $\qquad d\mu = -0.242$

a Cygni.

Twenty-four observations (1838. 105) give,

$$^{*}\delta = +44° 40' 35''.50.$$

Precession $= +12''.597$; sec var. $= +0''.226$; proper motion insensible.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1836, Feb. 17.	36.2	38.2	38	30.241	80° 51′. 08	359.84	— 2.16
„ „ 26.	29.7	34.5	35.5	28.985	80 51 40	348.59	— 3.39
„ March 7.	34	39.8	40.2	29.166	80 51 49	349.11	— 1.78
1837, March 12.	28.4	33.1	35	29.617	80 51 07	359.68	— 0.80
„ „ 13.	29.2	34	35	30.193	80 51 02	362.03	— 4.85
„ „ 17.	38.1	40.4	41.3	30.206	80 51 09	358.74	— 1.30
„ „ 24.	28.6	34.9	36.8	29.722	80 51 12	357.98	— 3.63
„ „ 29.	32	37.4	38.2	29.530	80 51 20	353.88	— 2.77

* Brinkley's δ . . . . = 36.25
Bessel, ... 34.21
Argelander, ... 35.50
Airy, Cambridge, . . . 35.14
Airy, Greenwich, (36), . = 34.76
Challis (1837,) 35.95
Mine, ... 35.70

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	April	1.	34.6	38	40	29.816	80° 51′ 20	353.63	— 4.48
,,	,,	3.	34	37.5	38	29.438	80 51 27	350.08	— 4.00
,,	,,	4.	34.4	37.8	40.3	29.683	80 51 18	354.39	— 3.08
,,	,,	7.	37.6	40.6	42	30.308	80 51 15	357.57	— 4.79
,,	,,	9.	39	42.1	43.2	30.245	80 51 10	359.99	— 0.77
,,	,,	16.	35	40.2	41	29.558	80 51 23	352.35	— 2.29
,,	,,	17.	42	43.2	44.5	29.764	80 51 23	352.02	+ 0.18
1838,	March	7.	37	39.5	40.1	29.804	80 50 96	352.08	— 3.93
,,	,,	8.	37.3	39.6	41.4	30.173	80 50 86	358.00	— 2.10
,,	,,	17.	35.2	39.6	40.3	29.366	80 51 00	351.13	— 0.77
,,	,,	23.	31	35.5	36.9	29.468	80 50 93	356.10	— 0.45
,,	,,	29.	43.8	46.8	48.3	30.409	80 50 93	357.18	— 0.67
,,	April	11.	41.3	45.4	47	29.830	80 50 97	356.23	+ 3.14
,,	,,	12.	43	46.1	47.6	30.188	80 50 94	357.43	+ 1.56
1839,	Feb.	9.	37.1	..	43.1	30.084	80 50 48	360.18	+ 0.28
,,	,,	12.	36.7	..	40.9	30.046	80 50 50	359.80	+ 0.97
,,	,,	17.	24.2	..	28.2	29.244	80 50 58	356.36	— 0.74
,,	,,	18.	31.7	..	34.2	29.374	80 50 62	354.75	— 0.03
,,	,,	20.	28.9	..	33.5	30.066	80 50 44	365.35	+ 0.10
,,	,,	24.	33.1	..	37.3	29.462	80 50 65	353.85	— 0.86
,,	March	2.	37	..	44	29.856	80 50 66	354.79	— 1.53
,,	,,	3.	40.2	..	43.6	29.820	80 50 71	352.37	— 1.18
,,	,,	17.	34.2	..	39.5	29.915	80 50 67	357.23	— 2.06
,,	,,	25.	36.6	..	42.9	29.424	80 50 83	348.45	— 3.13
,,	,,	27.	41	..	45.4	29.082	80 50 93	341.86	— 2.40
,,	April	5.	40.9	45	43.7	29.735	80 50 85	347.89	— 4.28
,,	,,	6.	38.1	42.8	44.8	30.122	80 50 66	359.57	+ 0.93
,,	,,	7.	38.1	40.9	42.2	30.091	80 50 69	357.61	— 0.78
,,	,,	11.	39.9	43	46	30.442	80 50 62	361.85	+ 0.79
,,	,,	12.	44.2	46.5	47.1	30.270	80 50 72	356.80	— 0.02
,,	,,	19.	44.8	47	47.6	29.708	80 50 86	347.01	— 1.92

$39 \times d\text{R} = -58''.99$ $\quad$ $d\text{R} = -1''.51$

$\text{K} = 4.5685$ $\quad$ $d\mu = -0.331$

46 *Andromedæ.*

Thirteen observations (1838. 083) give,

$$\delta = +44^\circ\ 38'\ 7''.08.$$

Precession $= +19''.065$; sec var. $= -0''.161$; proper motion $= -0''.017$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	May	18.	45.1	49.9	50	30.193	80° 52′. 63	355.18	— 0.18
1838,	May	5.	47.2	50.7	52.2	30.200	80 52 30	352.96	— 0.65
,,	,,	6.	49.9	52.8	54.1	30.165	80 52 34	350.11	— 1.09

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1838,	May 8.	53.7	57.9	60.1	30.172	80° 52′. 36	349.05	+ 0.59
,,	,, 10.	46.0	52.1	53.4	30.261	80 52 22	357.59	+ 2.47
,,	,, 11.	47.1	52	54.5	30.128	80 52 32	351.68	— 1.09
,,	,, 15.	39.4	45.2	47.7	29.688	80 52 31	352.44	— 0.97
,,	,, 23.	48.2	56.7	54.5	29.780	80 52 45	344.70	— 3.26
,,	,, 24.	49.2	53	54.7	29.864	80 52 36	350.03	+ 1.84
1839,	April 23.	45.8	48	50.2	29.912	80 52 02	349.51	— 1.63
,,	May 2.	44.3	49	50	29.884	80 51 98	351.59	— 0.27
,,	,, 6.	45	51	52.5	29.989	80 51 97	352.78	+ 0.30
,,	,, 7.	46	49.9	51.3	29.864	80 52 05	347.81	— 2.63
,,	,, 10.	41	45.8	48	30.136	80 51 88	358.50	+ 1.21
,,	,, 12.	43	46.1	49	29.984	80 51 89	357.08	+ 3.10
,,	,, 21.	44.8	50.2	52	30.050	80 52 00	351.48	— 1.94
,,	,, 22.	42.7	46.2	49.2	30.176	80 51 92	356.59	+ 0.31
,,	,, 25.	48	53.8	55.2	30.028	80 52 04	349.25	— 1.51
,,	,, 26.	48.2	52	54.7	29.987	80 52 05	348.53	— 1.63

$19 \times d\text{R} = -7''.65$ $\qquad$ $d\text{R} = -0''.40$

$\text{K} = 4.4839$ $\qquad$ $d\mu = -0.090$

64 ξ *Cygni.*

Twelve observations (1838. 767) give,

$$\delta = +43° \; 15' \; 11''.98.$$

Precession $= +14''.104$; sec var. $= +0''.219$; proper motion $= +0''.033$.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1838,	March 8.	36.8	39.4	41.1	30.170	82° 15′. 02	417.67	— 3.05
,,	,, 17.	34.6	39.6	40.3	29.377	82 15 18	409.25	— 2.41
,,	,, 23.	30.4	35.0	36.6	29.549	82 15 13	413.15	— 3.52
,,	,, 29.	43.5	46.6	48.1	30.408	82 15 10	416.00	— 1.84
,,	April 8.	42.3	45.1	46	29.460	82 15 32	403.74	— 2.09
1839,	Feb. 20.	28.5	..	33.1	30.060	82 14 52	427.84	+ 1.34
,,	,, 24.	33.1	..	37.2	29.467	82 14 75	415.18	+ 1.16
,,	March 3.	39.3	..	43.5	29.820	82 14 81	413.24	— 0.09
,,	,, 17.	34.9	..	39.5	29.917	82 14 79	417.30	— 1.41
,,	,, 27.	40.8	..	45	29.070	82 15 10	398.47	— 3.27
,,	April 6.	37.4	42.1	44.2	30.125	82 14 79	418.96	— 0.24
,,	,, 7.	37.9	40.7	42	30.089	82 14 86	414.90	— 3.51
,,	,, 11.	39.3	43	45.2	30.440	82 14 72	423.71	+ 1.89
,,	,, 12.	43.3	45.8	46.9	30.270	82 14 92	411.74	— 4.26

$14 \times d\text{R} = -21''.30.$ $\qquad$ $d\text{R} = -1''.52.$

$\text{K} = 5.6710.$ $\qquad$ $d\mu = -0.268.$

17 *Andromedæ.*

Fifteen observations (1838. 801) give,

$$\delta = +42^\circ\ 19'\ 39''.41.$$

Precession $= +19''.883$; sec var. $= +0.051$; proper motion $= +0.042$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, April 16.	31.3	35	35.9	29.578	83° 9′. 22	468.26	+ 1.25
,, ,, 17.	36.8	40.7	42.1	29.875	83 9 23	466.92	+ 2.45
,, ,, 22.	40	42.8	43	29.640	83 9 33	459.87	+ 0.81
,, May 3.	45.2	49.7	50	29.673	83 9 46	452.95	— 1.35
1838, May 5.	49.8	53	54	30.190	83 9 05	455.77	— 1.44
,, ,, 6.	52.8	54.3	55.9	30.156	83 9 09	453.39	— 0.36
,, ,, 8.	58.2	60	62.9	30.180	83 9 13	450.20	+ 1.43
1839, April 17.	37.9	42.8	43.4	29.101	83 8 78	448.57	— 3.56
,, ,, 18.	38.1	42.8	44.5	29.209	83 8 74	451.33	— 2.23
,, ,, 19.	40.2	42	43.8	29.764	83 8 62	458.92	— 1.13
,, ,, 24.	44.9	47.3	49.7	29.916	83 8 59	460.91	+ 2.43
,, May 2.	47	50.2	53.1	29.894	83 8 73	453.09	— 2.04
,, ,, 5.	47	49.7	51.1	29.786	83 8 75	451.66	— 1.91
,, ,, 7.	50.8	52.2	54.2	29.873	83 8 78	450.22	— 0.94

$14 \times d\text{R} = -6''.59$ $\qquad d\text{R} = -0''.47$

$\kappa = 6.2444$ $\qquad d\mu = -0.075$

10 *Ursæ Majoris.*

Twelve observations (1837. 932) give,

$$^*\delta = +42^\circ\ 26'\ 58''.89.$$

Precession $= -13''.522$; sec var. $= -0''.418$; proper motion $= -0''.294$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1835, Aug. 30.	53.9	58.5	..	29.870	83° 5′. 56	444.58	— 0.35
,, Sept. 6.	53.9	56.7	..	29.827	83 5 53	447.82	+ 2.47
,, ,, 8.	46.9	54.7	..	29.509	83 5 48	451.05	+ 5.00
,, ,, 12.	49	53.8	..	29.277	83 5 78	435.41	— 5.45

* Argelander's $\delta = 57''.80$; proper motion $= -0''.286$.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1835,	Sept. 15.	48.3	52.5	..	29.427	83° 5′. 65	441.67	— 1.92
,,	Oct. 3.	46.1	50.2	..	29.227	83 5 64	445.37	+ 1.51
1837,	Aug. 30.	47.2	51.8	54.7	29.252	83 6 06	442.15	— 0.31
1838,	Sept. 9.	44.3	48.8	52.5	30.127	83 5 96	458.33	— 0.28
,,	,, 20.	44.5	..	53.9	29.606	83 6 27	446.88	— 3.86
,,	,, 23.	49.9	..	55.5	29.560	83 6 36	442.07	— 2.77
,,	,, 24.	46.8	..	56.9	29.721	83 6 21	451.16	+ 1.14
,,	,, 25.	45.5	..	54	29.860	83 6 17	453.78	— 0.78
,,	Oct. 4.	45.1	..	55	30.286	83 6 09	460.06	— 0.24
1839,	Sept. 5.	52.9	57	57.9	29.474	83 6 69	434.36	— 5.60
,,	,, 10.	51.3	54.1	56.2	29.888	83 6 54	444.38	— 4.27
,,	,, 11.	48.4	52 2	55.1	29.714	83 6 56	443.35	— 5.44
,,	,, 21.	46.6	51.7	53.5	29.390	83 6 59	444.18	— 1.08
,,	Oct. 2.	43.1	49.7	52	29.620	83 6 49	452.28	— 0.28
,,	,, 4.	42.1	45.1	47	29.888	83 6 36	460.33	+ 2.64
,,	,, 12.	46.8	48	50.1	29.664	83 6 57	448.62	— 1.17
,,	,, 16.	44.2	47.3	47.9	29.582	83 6 59	448 01	— 3.14
,,	,, 17.	41.2	49	49.9	29.956	83 6 51	445.46	— 4.19
,,	,, 18.	43.1	46.8	48.8	29.788	83 6 50	453.58	— 1.68
,,	,, 20.	45.9	48.8	49.4	29.778	83 6 57	450.11	— 2.32

$24 \times d\text{R} = -32''.38$ $\qquad$ $d\text{R} = -1''.35$

$\text{K} = 6.1247$ $\qquad$ $d\mu = -0''.220$

μ Ursæ Majoris.

Ten observations (1838. 235) give,

$\delta = +42° 21' 4''. 05.$

Precession = — 17″.877 ; sec var. = — 0″.236 ; proper motion = — 0″.015.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1835,	Sept. 22.	49.9	52.9	..	28.907	83° 11′. 96	439.21	— 1.13
,,	,, 23.	47.3	52.4	..	29.285	83 11 90	443.02	— 5.51
,,	,, 24.	45	49.6	..	29.727	83 11 66	457.80	+ 0.16
,,	Nov. 22.	39.3	45.6	..	29.411	83 11 86	460.16	+ 1.66
1838,	Sept. 23.	49.8	..	54.8	29.571	83 12 77	446.78	— 4.61
1839,	Sept. 30.	44	50.3	52.3	29.828	83 12 97	458.09	— 3.23
,,	Oct. 2.	42.5	47.1	49.8	29.625	83 12 98	458.14	— 3.86
,,	,, 4.	42.9	45.8	47	29.919	83 12 88	464.49	+ 0.59

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839, Oct. 5.	41.3	47	50.2	30.148	83° 12′. 84	465.82	— 3.09
,, ,, 12.	45.5	47.5	49.2	29.703	83 12 97	461.13	+ 3.15
,, ,, 15.	43.2	48.9	49.1	29.570	83 13 04	457.58	— 0.66
,, ,, 16.	44.2	47.8	47.8	29.610	83 13 08	455.33	— 2.65
,, ,, 17.	39.9	46.1	47.5	29.947	83 12 91	465.57	— 1.75
,, ,, 20.	44.2	46.8	48.5	29.786	83 13 09	455.46	— 5.23
,, ,, 27.	41	46.1	47.3	30.298	83 12 86	470.96	— 0.75
,, Nov. 11.	43.3	45.5	48.1	29.000	83 13 28	448.78	— 0.75
,, ,, 12.	41.9	45	47	29.320	83 13 24	452.70	— 3.19
,, ,, 13.	38.2	42.2	45.5	29.679	83 13 12	459.66	— 5.50

$18 \times d\text{R} = -36''.35$ $\qquad$ $d\text{R} = -2''.02$

$\text{K} = 6.2821$ $\qquad$ $d\mu = -0.321$

ν Persei.

Twelve observations (1838. 416) give,

$$\delta = +42^\circ\, 2'\, 2''.57.$$

Precession $= +11.954$; sec var. $= -0.471$; proper motion $= -0.004$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, June 3.	45.2	..	54.6	29.891	83° 27′. 37	475.51	— 0.42
,, ,, 5.	50	56.9	57.3	30.005	83 27 35	476.63	+ 3.92
,, ,, 13.	52	55.1	57.1	29.500	83 27 66	458.63	— 4.40
,, ,, 14.	52.1	57.1	59	29.735	83 27 57	464.67	— 1.85
,, ,, 23.	62.4	63.8	65.3	30.122	83 27 68	457.66	— 4.70
1838, June 12.	52	54.9	59	29.632	83 27 32	460.78	— 3.94
1839, June 16.	52.9	57.9	58.8	30.144	83 27 05	468.37	— 3.25

$7 \times d\text{R} = -14''.64$ $\qquad$ $d\text{R} = -2''.09$

$\text{K} = 6.5578$ $\qquad$ $d\mu = -0.326$

58 *Aurigæ.*

Twelve observations (1837. 561) give,

$\delta = + 41° \; 58' \; 16''.86.$

Precession = — 3″.376; sec var. = — 0″.613; proper motion = — 0″.138.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1833,	Aug. 14.	47.7	53.3	..	29.708	83° 32′. 89	473.59	— 2.77
1835,	July 29.	53.2	58	..	30.066	83 32 84	475.56	— 0.72
,,	,, 31.	56.5	62	..	29.990	83 32 87	473.71	+ 1.98
,,	Aug. 30.	55.2	59.5	..	29.868	83 32 86	467.04	— 4.14
1837,	July 16.	54.2	59.1	61.2	29.897	83 32 96	470.52	— 2.03
,,	,, 20.	53.7	59	60.9	29.944	83 32 92	473.26	— 0.11
,,	Aug. 5.	46.3	51	53.9	30.152	83 32 72	486.84	+ 2.05
,,	,, 6.	51	53.9	55	30.245	83 32 86	478.50	— 3.03
,,	,, 7.	49.4	54.8	58	30.260	83 32 77	483.46	+ 0.27
,,	,, 15.	57.9	61	63	30.084	83 32 99	470.93	— 0.87
,,	,, 26.	49	54.7	56.5	29.939	83 32 94	475.06	— 3.63
,,	,, 29.	46.5	52	54.6	29.429	83 32 99	471.83	— 1.31
1838,	Aug. 4.	54.8	..	60	29.204	83 33 18	461.39	+ 0.21
,,	,, 11.	56.9	..	62.2	29.764	83 33 09	467.64	— 0.22
,,	,, 12.	56.3	..	61.8	29.840	83 33 07	477.92	— 0.99
,,	,, 13.	51.8	..	58.5	30.060	83 32 92	467.64	+ 0.08
1839,	July 15.	50.1	52.8	57.3	29.853	83 33 00	474.37	— 1.85
,,	,, 19.	51.4	54.4	59.2	29.071	83 33 20	462.91	+ 0.45
,,	,, 24.	52.9	58	59.3	29.578	83 33 15	466.32	— 2.82
,,	,, 27.	52.2	60	61.5	29.636	83 33 07	471.34	+ 0.84
,,	,, 31.	47.8	52	55.2	29.624	83 33 05	472.63	— 2.37
,,	Aug. 2.	56.1	57.1	59.8	29.762	83 33 15	467.52	— 1.25
,,	,, 4.	52.3	57.4	59.9	30.184	83 32 98	477.59	— 1.68
,,	,, 12.	49.1	56	58	30.124	83 32 94	480.66	— 0.84

$24 \times d\text{R} = -\,24''.75$ $\qquad d\text{R} = -\,1''.03$

$\text{K} = 6.5578$ $\qquad d\mu = -\,0.157$

γ Andromedæ.

Twelve observations (1837. 531) give,

$^{*}\delta = +41° 30' 34''. 54.$

Precession $= + 17''.647$; sec var. $= - 0''.260$; proper motion $= - 0''.057$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1836,	May	28.	54.2	59.4	61	30.281	83° 58′. 12	506.19	— 1.47
1837,	"	12.	46.7	51.6	52.1	29.517	83 57 88	499.84	— 2.78
"	"	14.	44 5	51.8	52.8	30.013	83 57 66	513.25	— 0.08
"	"	18.	44.8	48.4	50	30.193	83 57 61	516.38	+ 0.24
"	"	26.	43.5	51.1	51	29.588	83 57 72	510.12	+ 2.89
"	"	27.	50	54.2	54.1	29.800	83 57 84	503.23	— 0.52
"	"	30.	46.9	52.5	53.2	29.837	83 57 74	508.82	+ 1.15
"	June	3.	48.7	54.6	56.8	29.896	83 57 77	506.96	+ 0.40
1838,	May	15.	38.2	44	46.9	29.684	83 57 33	512.64	— 1.91
"	"	17.	39	45.9	47.1	29.716	83 57 32	513.95	— 0.20
"	"	23.	47.4	51.3	53.3	29.785	83 57 54	500.85	— 4.08
"	"	24.	48.3	52.1	53.9	29.870	83 57 42	508.03	+ 0.84
"	"	25.	50.4	54	55.5	29.906	83 57 47	505.28	+ 0.70
"	"	26.	52	54.8	55.9	29.931	83 57 45	506.31	+ 3.17
1839,	May	25.	46.7	51	53.7	30.208	83 57 05	510.21	— 3.21
"	"	26.	46.2	50	53.1	29.988	83 57 13	505.87	— 4.06
"	"	28.	56.2	57.1	58	30.064	83 57 24	499.33	— 3.75
"	"	29.	53 8	56.2	60	30.077	83 57 16	503.79	+ 0.56
"	"	30.	56.1	58	61.2	30.044	83 57 22	500.25	+ 0.02
"	"	31.	57	58.8	62.1	29.916	83 57 27	497.58	— 0.67
"	June	1.	52.8	55.5	59.2	29.786	83 57 21	500.65	+ 1.18
"	"	2.	52.1	56	59.8	29.624	83 57 26	498.09	+ 0.58
"	"	3.	46.9	50.4	52.5	29.500	83 57 20	501.67	+ 0.41

$23 \times d\text{R} = - 10''.59$ $\qquad$ $d\text{R} = - 0''.46$

$\kappa = 7.1337$ $\qquad$ $d\mu = - 0''.065$

* Argelander's δ . . . . $= 35''. 20$ Mine, . . . $34''. 74$
Airy, Greenwich, (1836 and 1837,) 34 11

58 *Persei.*

Eight observations (1837. 198) give,

$\delta = + 40° 54' 24''. 32.$

Precession $= + 8''.071$; sec var. $= - 0''.329$; proper motion $= - 0''.035$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	d_R.
1837, June 11.	50.7	58	58.7	29.506	84° 34′. 35	540.79	— 3.89
„ „ 13.	51.7	55.5	57	29.502	84 34 27	546.15	+ 2.77
„ „ 14.	51.2	55.1	57.2	29.735	84 34 26	547.53	— 0.78
1839, „ 16.	51.6	56.8	58.8	30.144	84 33 87	551.15	— 3.66
„ „ 28.	47.1	50	53.8	29.881	84 33 89	550.96	— 4.89
„ „ 29.	45.9	50.1	54.9	30.102	84 33 72	560.81	— 0.28

$6 \times d_R = - 10''.73$ $\qquad d_R = - 1''.79$

$\kappa = 7.8566$ $\qquad d\mu = - 0.228$

58 *Cygni.*

Twelve observations (1838. 024) give,

$\delta = + 40° 30' 58''. 86.$

Precession $= + 13''. 603$; sec var. $= + 0''. 233$; proper motion $= + 0''. 018$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	d_R.
1837, March 24.	28.5	33.9	35.1	29.722	84° 56′. 49	604.06	— 9.41
„ „ 29.	32.2	36.1	38.2	29.530	84 56 58	599.77	— 4.47
„ April 1.	33.9	37.0	39	29.812	84 56 40	610.71	+ 2.96
1838, March 8.	36.8	39.4	41	30.170	84 56 20	604.29	— 5.79
„ „ 17.	34.6	39.6	40.3	29.377	84 56 38	595.87	— 1.39
„ „ 23.	30.4	35	36.6	29.459	84 56 32	600.31	— 4.61
„ „ 29.	43.5	46.6	48.1	30.408	84 56 35	599.37	— 6.09
1839, April 6.	37.8	42.2	44.2	30.125	84 55 98	606.53	— 0.66
1840, Feb. 26.	31.8	35.8	37	30.357	84 55 43	619.21	— 0.74
„ „ 27.	28.5	34	35.3	30.258	84 55 41	620.33	— 2.47
„ „ 29.	32	35.7	36.5	30.264	84 55 48	615.43	— 2.48
„ March 1.	30.8	33.6	34.9	30.330	84 55 43	618.21	— 2.80
„ „ 2.	34.5	35.7	36.2	30.382	84 55 59	609.34	— 7.60

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1840, March 3.	34.2	35.8	37.2	30.415	84° 55′. 40	621.50	+ 3.82
" " 4.	35.5	37.2	38.2	30.247	84 55 45	618.75	+ 6.38
" " 5.	38.2	38.2	40.2	30.108	84 55 71	603.40	— 2.67
" " 6.	44.2	43.1	43.1	30.249	84 55 84	595.56	— 5.10
" " 9.	40.3	42.8	43.5	30.481	84 55 69	605.34	— 5.13
" " 18.	38.2	42.8	44.5	30.150	84 55 79	600.66	— 3.91
" " 20.	35.6	40.1	43.2	30.380	84 55 75	603.22	— 9.06
" " 23.	36	37.9	40.2	30.261	84 55 71	606.31	— 5.98

$21 \times d\text{R} = -67''.20$ $\qquad$ $d\text{R} = -3''.20$

$\kappa = 8.8831$ $\qquad$ $d\mu = -0.360$

The discordances in the separate values of $d\mu$ have obviously no relation to the zenith distance, or the time of year, and may therefore be regarded as casual.

If we combine them according to the method already assigned, we obtain,

NAME.	NO. OBS.	$nd\text{R} \times \kappa$	$n \times \kappa^2$.	$d\mu$.
45 Cygni.	17	— 81.5223	445.642	—0″.576
31 "	16	— 31.2112	224.400	— 0 160
Capella.	20	— 59.1490	278.526	— 0 212
Pxxi. 157.	8	+ 2.4732	131.505	+ 0 019
22 Andromedæ.	17	— 102.0713	293.630	— 0 348
β Aurigæ.	30	— 128.6188	530.360	— 0 242
α Cygni.	39	— 269.4501	813.976	— 0 331
46 Andromedæ.	19	— 34.3018	382.002	— 0 090
64 Cygni.	14	— 120.7923	450.243	— 0 268
10 Ursæ Majoris.	24	— 198.2566	900.287	— 0 220
17 Andromedæ.	14	— 41.1506	545.895	— 0 075
μ Ursæ Majoris.	18	— 228.3543	710.366	— 0 321
ν Persei.	7	— 93.8717	287.796	— 0 326
58 Aurigæ.	24	— 162.3056	1032.114	— 0 157
γ Andromedæ.	23	— 75.5459	1170.462	— 0 065
58 Persei.	6	— 84.3546	370.351	— 0 228
58 Cygni.	21	— 596.9444	1657.992	— 0 360
Sum . .	317	— 2305.4273	10225.547	

Hence

$$d\mu = -\frac{2305.4273}{10225.547} = -0.2255.$$

The value of μ used in computing the refractions is,

$$\begin{aligned} \mu &= 57.7682\,; \\ d\mu &= -\ \ 0.2255\,; \\ \text{sum} &= 57.5427. \end{aligned}$$

This may perhaps require a correction for the run of the microscopes, which though very small is sensible. From the erection of the circle to July 8, 1837, its effect on the mean of four microscopes was $= -\frac{0''.\,18 \times \text{A}'}{5'\ 0''}$. At this time it was changed by the rough operations necessary in attaching another pair of microscopes, and has been since considered permanent at $+ 0''.\,41 \times \frac{\text{A}'}{5'}$. This is, however, a mean value, being deduced from readings of the four, in 30 equidistant positions of the circle. Hence I found as above

$$d^2\mu = + \frac{38''.2909}{10225.547} = + 0''.0037$$

and

$$\mu = 57''.5464$$

a value whose near approximation to Bessel's $57''.524$, will prove very remarkable, if when I have means of determining the length of the seconds' pendulum here, it should be found little different from that of Königsberg. That observatory is a little north of me, but it is only 90 feet above the Baltic; while this is 211 feet above the sea, and the substratum, dense limestone, so that the local gravity must be nearly alike in both cases.

As to the southern stars, I have used the declinations of the St. Helena catalogue, reduced to Bessel's refractions, by the table given page 22, and those of Professor Henderson. (Mem. R. Ast. Soc. X. 80.) The two are not strictly comparable in respect of refraction, for the St. Helena Observatory, being 700 feet above the sea, and resting on dense volcanic rocks, may be expected to have an excess of gravity above the Cape, and therefore larger refraction. At the latter place I find, by comparing the length of the pendulum with that of Greenwich, that Bessel's refractions should be multiplied by 0.9984; and, in fact, Henderson's observations on refraction shew, that even a greater diminution is required. I have not, however, changed them further than by reducing them to 1830, with the precession, &c., annexed to each star. When possible, the proper motions are deduced by comparison with Airy's Greenwich places.

24. o^2 *Canis Majoris.*

$\delta = -23° \, 35' \, 23''.83$. J. (Johnson).

Precession $= -4''.846$; sec var. $= -0''.352$; proper motion $= +0''.011$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Feb.	18.	34.3	41	41	29.274	77° 52'. 73	269.48	+ 2.37
,,	March	12.	29.8	35.6	36.8	29.575	77 52 85	274.78	+ 1.19
,,	,,	13.	30.7	34.8	36	30.174	77 52 66	276.29	— 2.10
,,	,,	17.	38.6	41.8	43	30.211	77 52 74	271.47	— 2.76
,,	,,	21.	38.8	40.1	41.1	29.712	77 52 79	268.76	— 0.88
,,	,,	23.	32.2	36	37.6	29 663	77 52 76	270.51	— 3.10
,,	,,	24.	32	36.5	38.1	29.727	77 52 77	270.15	— 3.51
1838,	Feb.	8.	38.7	39.8	40.2	28.524	77 53 02	254.86	— 4.14
,,	,,	13.	27	30	31.2	29.479	77 52 86	270.93	— 3.51
,,	,,	20.	31.8	34.7	35.7	29.483	77 52 76	273.16	+ 1.59
,,	,,	21.	32.9	36.1	38	29.583	77 52 82	269.90	— 2.06
,,	March	15.	39.2	44.8	47.2	29.798	77 52 88	267.99	— 2.08
,,	,,	17.	36.2	39.8	41.2	29.344	77 52 93	265.12	— 2.64

$13 \times d\text{R} = -21''.63$ $\qquad d\text{R} = -1''.16$

$\text{K} = 5.2240$ $\qquad d\mu = -0.318$

15 *Argûs.*

* $\delta = -23° \, 49' \, 8''.58$. (J. H.)

Precession $= -10''.051$; sec var. $= -0''.317$; proper motion $= +0''.075$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	March	13.	29.2	34.1	35	30.185	78° 6'. 92	282.69	— 2.54
,,	,,	14.	34.1	37.3	38.4	30.287	78 6 96	280.78	— 2.28
,,	,,	23.	32.2	34.5	35.8	29.657	78 7 03	277.17	— 1.17

* Johnson's δ . . . $= 7''.80$ Henderson's δ . . . $= 9''.36$

Had the first been used, the refractions would be $0''.78$ less; $d\mu = -0''.306$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	April	3.	35	37.8	39	29.429	78° 7′. 11	272.91	— 1.63
,,	,,	4.	35.7	38.7	40.3	29.683	78 7 00	279.65	+ 3.22
1838,	Feb.	20.	31.2	34.4	35	29.496	78 7 08	279.34	+ 2.90
,,	,,	21.	31.8	35	36.9	29.577	78 7 13	276.93	— 0.96
,,	March	17.	35.8	39.1	40.9	29.368	78 7 22	274.61	+ 1.11
1839,	Feb.	20.	29.6	..	34.1	30.066	78 7 20	283.91	+ 0.06
,,	,,	24.	33.8	..	38	29.461	78 7 13	274.69	— 0.86
,,	March	17.	35	..	40	29.907	78 7 12	279.87	+ 0.89
,,	,,	25.	37.9	..	43.9	29.424	78 7 59	269.77	— 3.14
,,	April	5.	40.4	..	44	29.722	78 7 47	272.52	— 1.69
,,	,,	6.	39	44	45.8	30.118	78 7 39	273.57	— 1.05
,,	,,	7.	38.5	40.2	43.2	30.094	78 7 45	276.75	— 2.00
,,	,,	11.	41.8	45.2	47	30.442	78 7 42	274.51	— 5.48

$16 \times d\text{R} = -14''.62$ $\qquad d\text{R} = -0''.91$

$\text{K} = 5.5356$ $\qquad d\mu = -0.165$

16. o^1 *Canis Majoris.*

$\delta = -23^\circ\ 58'\ 35''.82.$ J.

Precession $= -4''.092$; sec var. $= -0''.353$; proper motion $= -0''.059$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Feb.	18.	34.3	41	41	29.274	78° 15′. 70	279.53	+ 3.05
,,	March	12.	29.8	35.6	36.8	29.575	78 15 68	283.13	+ 1.06
,,	,,	13.	30.7	34.8	36	30.174	78 15 71	281.70	— 5.65
,,	,,	17.	38.6	41.8	43	30.211	78 15 76	278.48	— 3.56
,,	,,	23.	32.2	36	37.6	29.663	78 15 76	279.06	— 2.43
,,	,,	24.	32	36.5	38.1	29.727	78 15 73	280.69	— 1.62
1838,	Feb.	8.	38.7	39.8	40.2	28.524	78 15 97	265.28	— 1.72
,,	,,	13.	27	30	31.2	29.479	78 15 72	281.30	— 1.67
,,	,,	20.	31.8	34.7	35.7	29.483	78 15 72	282.64	+ 2.59
,,	,,	21.	32.9	36.1	38	29.583	78 15 79	278.13	— 1.89
,,	March	15.	39.2	44.8	47.2	29.798	78 15 87	275.23	— 3.36
,,	,,	17.	36.2	39.8	41.2	29.344	78 15 91	273.32	— 4.36
,,	,,	23.	33.5	35.2	39.7	29.500	78 15 87	275.28	— 3.91

$13 \times d\text{R} = -23''.47$ $\qquad d\text{R} = -1''.81$

$\text{K} = 5.5514$ $\qquad d\mu = -0.325$

ξ *Argus.*

*$\delta = -24° 26' 17''.90$. (J.)

Precession $= -8''.647$; sec. var. $= -0''.329$; proper motion $= -0''.012$. (A.)

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, March 12.	28.5	33.4	35	29.604	78° 43'. 68	296.33	+ 1.38
,, ,, 13.	29.2	34.1	35	30.182	78 43 62	300.13	— 0.04
,, ,, 14.	34.2	37.6	39	30.287	78 43 68	296.36	— 1.64
,, ,, 29.	32.2	38.6	40	29.521	78 43 59	290.11	— 1.47
,, ,, 30.	36.1	38.2	42	29.757	78 43 61	288.93	— 2.57
1838, Feb. 20.	31.4	34	35.2	29.496	78 43 81	293.98	+ 1.93
,, ,, 21.	31.9	35.2	36.9	29.577	78 43 87	290.84	— 1.68
,, March 29.	45.1	47.1	48.5	30.410	78 43 95	289.59	— 2.93
1839, Feb. 20.	29.6	..	34.1	30.066	78 43 86	297.64	— 1.18
,, March 17.	35.1	..	40.1	29.912	78 44 02	291.56	— 2.24
,, ,, 25.	38.9	..	44.1	29.424	78 44 15	283.90	— 2.83
,, April 5.	40.3	..	44	29.717	78 44 16	284.37	— 4.48
,, ,, 7.	38.9	40.4	43.2	30.094	78 44 07	289.26	— 4.12

$13 \times d\text{R} = -21''.87$ $\qquad$ $d\text{R} = -1''.68$

$\kappa = 5.7931$ $\qquad$ $d\mu = -0.290$

22 λ *Sagittarii.*

† $\delta = -25° 30' 23''.90$. (J.)

Precession $= +1''.528$; sec. var. $= +0''.537$; proper motion $= -0''.291$. (J.)

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 20.	54.7	57.3	61	29.940	79° 46'. 40	308.74	— 1.29
,, ,, 27.	55	57	61.5	29.571	79 46 48	306.47	+ 0.43
,, August 5.	46.9	51	53.9	30.152	79 46 36	313.99	+ 0.53

* Airy (15 observations, 1836-7) . 18''.93

† The declinations of this star are discordant:

Airy (16 in 1837) . . . . . 25''.79 $\qquad$ Maclear . . . . . 24''.45

Johnson ,, ... 23 .90

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Aug.	6.	51.2	54	56.2	30.240	79° 46′. 32	316.17	+ 0.76
,,	,,	7.	50.3	54	58	30.261	79 46 20	313.75	— 2.54
,,	,,	14.	56.8	60	62	30.070	79 46 43	309.84	+ 0.05
,,	,,	15.	58	61	63	30.082	79 46 45	308.51	— 0.84
,,	,,	16.	60.6	62.1	64	29.968	79 46 54	305.71	— 0.95
,,	,,	29.	47.6	52.2	56	29.430	79 46 41	311.51	+ 2.19
,,	,,	31.	48.7	53	56.1	29.275	79 46 49	306.92	— 0.16
1839,	July	15.	50.2	52.8	57.3	29.853	79 46 29	315.08	+ 3.41
,,	,,	19.	51.8	54.5	59.2	29.071	79 46 51	301.84	— 0.68
,,	,,	24.	53.7	59.2	61	29.578	79 46 42	307.17	+ 0.62
,,	,,	28.	52.4	57.2	61.8	29.778	79 46 13	311.02	+ 2.96
,,	,,	31.	48.1	53.8	56.1	29.622	79 46 32	313.22	+ 2.62
,,	Aug.	2.	56.9	57.9	60.1	29.764	79 46 46	305.70	— 0.84
,,	,,	4.	52.9	58.1	60.2	30.186	79 46 30	314.58	+ 0.44
,,	,,	19.	47.1	54.2	56.2	29.960	79 46 29	315.52	+ 0.72
,,	,,	20.	50.8	56	57.8	30.084	79 46 33	313.05	— 0.66
,,	,,	21.	53.2	56.2	58.9	29.932	79 46 36	311.48	+ 0.90
,,	,,	26.	51.2	55.5	59.1	29.620	79 46 39	310.31	+ 1.73
,,	Sept.	5.	55.9	58	61.7	29.428	79 46 50	303.39	— 0.28
,,	,,	11.	51.2	57.2	60	29.736	79 46 42	308.58	— 1.20

$23 \times d\text{R} = +\ 7''.92$ $\qquad$ $d\text{R} = +\ 0''.34$

$\kappa = 6.1035$ $\qquad$ $d\mu = +\ 0.056$

Antares.

* $\delta = -\ 26° \ 2' \ 47''.69.$ (J. and H.)

Precession $= -\ 8''.556$; sec var. $= +\ 0''.487$; proper motion $= -\ 0''.031$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	June	14.	51.2	55.1	57.2	29.735	80° 19′. 80	325.90	— 0.49
,,	,,	15.	50.9	57.1	61.1	30.090	80 19 72	331.25	+ 0.43
,,	July	7.	56.2	59	62.2	30.106	80 19 83	325.18	— 2.27
,,	,,	9.	56.9	63.7	65.4	29.905	80 19 89	321.55	— 3.16
,,	,,	10.	60.1	64.2	66.5	29.846	80 19 93	319.50	— 2.47
,,	,,	16.	56.6	61	63.1	29.923	80 19 85	322.83	— 2.37
,,	,,	18.	57	60.2	63.5	29.612	80 19 81	320.07	— 1.40

* Airy, δ (18 obs. in 36 and 37) . 48″.11 $\qquad$ Argelander . . . . . 46″.50
Henderson . . . . . 48 .68 $\qquad$ Mine . . . . . . 47 .44
Johnson, ... 46 .71

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 20.	57.0	60	63	29.934	80° 19′. 82	325.53	+ 0.55
,, August 5.	51.2	53.8	57.1	30.147	80 19 76	329.79	— 1.56
,, ,, 15.	63	63	66	30.075	80 19 97	320.11	— 2.53
1838, July 1.	53.2	57.3	58.7	29.806	80 19 99	325.10	— 1.23
,, ,, 25.	53.2	57.5	60	29.904	80 20 00	325.21	— 2.15
,, ,, 31.	57.3	..	62	29.812	80 20 01	324.79	+ 1.32
,, August 4.	58.8	..	63	29.192	80 20 14	313.62	— 2.35
1839, June 14.	48.1	53.8	56.2	29.892	80 20 03	331.01	+ 0.21
,, ,, 16.	51.6	56.8	58.8	30.144	80 20 06	329.49	— 1.70
,, ,, 28.	47.1	50	53.8	29.881	80 20 03	331.33	+ 0.66
,, ,, 29.	45.9	50.1	54.9	30.102	80 19 94	337.18	+ 2.54
,, July 9.	52	54.8	58.8	29.370	80 20 26	318.29	— 4.17
,, ,, 10.	53.9	56.6	60.2	29.517	80 20 22	320.71	— 2.00
,, ,, 20.	55.3	57.2	60	29.360	80 20 23	320.04	— 0.07
,, ,, 22.	55.4	59	60.9	29.750	80 20 15	324.57	— 0.78

$22 \times d\text{R} = -24''.99$ $\qquad d\text{R} = -1''.14$

$\text{K} = 6.3200$ $\qquad d\mu = -0.180$

19 δ *Sagittarii.*

$\delta = -29° \ 53' \ 25''.75$ (J).

Precession $= + 0''.884$; sec var. $= + 0''.559$; proper motion $= - 0''.014$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 20.	54.8	57.3	62	29.938	84° 6′. 24	507.06	+ 0.36
,, ,, 27.	55	57	61.5	29.571	84 6 37	499.91	— 4.38
,, Aug. 6.	51.2	54	56.2	30.240	84 6 10	516.16	— 3.91
,, ,, 7.	50.3	54	58	30.261	84 5 95	524.71	+ 3.45
,, ,, 14.	56.8	60	63	30.069	84 6 34	508.72	— 8.85
,, ,, 15.	58.6	61	63	30.082	84 6 30	504.49	— 6.74
,, ,, 16.	60.6	62.1	64.5	29.970	84 6 33	502.80	— 2.47
,, ,, 29.	47.6	52.2	56	29.430	84 6 16	513.58	+ 3.08
,, ,, 31.	48.7	53	56.1	29.275	84 6 27	506.99	+ 0.78
1838, Aug. 4.	55.1	..	60.5	29.200	84 6 45	494.66	— 2.04
,, ,, 14.	52.1	..	60	30.040	84 6 18	511.22	— 4.24
1839, July 15.	50.3	52.8	57.3	29.853	84 6 13	511.52	— 2.81
,, ,, 24.	53.7	59.2	61	29.578	84 6 17	509.15	+ 0.98
,, ,, 31.	48.1	53.8	56.1	29.622	84 6 17	509.50	— 2.34
,, Aug. 11.	50.9	56	58.9	30.162	84 6 07	516.65	— 2.19
,, ,, 19.	47.1	54.2	56.2	29.960	84 6 08	516.41	— 3.34
,, Sept. 5.	55.9	58	61.7	29.428	84 6 32	501.96	+ 1.11
,, ,, 11.	51.2	57.2	60	29.736	84 6 29	504.57	— 7.76

$18 \times d\text{R} = -41''.41$ $\qquad d\text{R} = -2''.30$

$\text{K} = 9.5710$ $\qquad d\mu = -0.241$

34 σ *Sagittarii.*

*$\delta = -26° 29' 55''.31$. (J.)

Precession $= +3''.889$; sec var. $= +0''.532$; proper motion $= -0''.093$.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1838,	Aug. 4.	54.8	..	60	29.204	80° 45′. 20	330.87	— 1.19
„	„ 13.	51.8	..	58.5	30.060	80 45 09	309.30	— 4.91
„	„ 14.	52.9	..	58.2	30.033	80 45 06	340.34	— 2.80
1839,	July 19.	51.4	54	57.3	29.072	80 45 09	333.01	+ 0.37
„	„ 24.	53.7	59.2	61	29.578	80 45 07	334.58	— 5.18
„	„ 28.	51.5	55.7	60	29.776	80 44 97	340.62	— 0.43
„	„ 31.	47.4	51.2	55	29.627	80 44 93	342.88	+ 0.46
„	Aug. 2.	56.1	57.1	59.8	29.762	80 45 06	335.22	— 2.56
„	„ 3.	52.7	56.8	59.2	30.026	80 44 95	341.96	— 1.11
„	„ 4.	52.3	57.4	59.2	30.184	80 44 90	345.06	— 0.12
„	„ 11.	51	56.2	58.2	30.169	80 44 89	345.50	— 0.45
„	„ 12.	49.1	56	58	30.124	80 44 82	350.24	+ 3.50
„	„ 19.	47.7	51.7	56.2	29.960	80 44 92	344.39	— 1.56
„	„ 21.	53.1	55.4	58.1	29.930	80 45 01	339.40	— 2.41
„	„ 24.	54.8	57.7	60	29.745	80 45 00	339.45	+ 1.02
„	„ 26.	51	54	59.1	29.620	80 44 98	341.03	+ 1.40
„	Sept. 5.	54.9	57	60.8	29.442	80 45 10	334.20	— 0.73

$17 \times d\text{R} = -16''.70$ $\qquad d\text{R} = -0''.98$

$\text{K} = 6.7651$ $\qquad d\mu = -0.145$

* This star is doubtful.

δ by Airy (3 observations),	57″.52
Henderson (Edinburgh, 5 obs.), Bessel's Refraction,	54 .56
„ Cape,	58 .11
Maclear, Direct,	58 .17
„ Reflected,	57 .23
Johnson,	55 .31

ε Canis.

$*\delta = -28°\,44'\,45''.35$ (J. and H.)

Precession $= -4''.507$; sec var. $= -0''.333$; proper motion $= -0''.011$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.8	39.6	40.2	29.295	82° 59′. 03	452.35	+ 3.28
„ March 12.	29.3	33.5	36.8	29.575	82 58 95	460.15	+ 1.31
„ „ 13.	30.4	34.7	36	30.177	82 58 89	463.88	— 3.05
„ „ 17.	38.2	41.4	43.1	30.211	82 59 01	453.46	— 6.04
„ „ 23.	32.1	35.2	37.6	29.663	82 59 00	457.54	+ 0.28
„ „ 24.	32	36.5	38.1	29.727	82 59 02	456.63	— 1.73
1838, Feb. 8.	38.7	39.5	40.2	28.524	82 59 43	430.66	— 3.38
„ „ 13.	27	30	31.2	29.478	82 59 02	456.53	— 3.29
„ „ 21.	32.9	36.1	38	29.583	82 59 14	450.18	— 5.18
1839, Feb. 12.	36.2	..	41.2	30.034	82 59 05	459.29	+ 0.50
„ „ 14.	36.1	..	40.9	29.734	82 59 13	454.90	+ 0.50
„ „ 17.	22.7	..	29.8	29.210	82 59 08	458.68	— 1.38
„ „ 18.	29.7	..	34.1	29.400	82 59 15	454.22	— 1.67
„ „ 20.	29.8	..	34.9	30.054	82 58 95	466.47	+ 0.75
„ March 3.	40.2	..	45.5	29.820	82 59 22	452.36	+ 0.74
„ „ 17.	35.2	..	40.5	29.912	82 59 15	457.61	— 0.49

$16 \times d\text{R} = -18''.85.$ $d\text{R} = -1''.18.$

$\text{K} = 8.6376.$ $d\mu = -0.136.$

31 *η Canis Majoris.*

$\dagger\delta = -28°\,58'\,35''.79$ (J.)

Precession $= -6''.642$; sec var. $= -0''.323$; proper motion $= -0''.011$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.8	39.6	40.2	29.295	83° 12′. 90	465.05	+ 2.24
„ March 14.	34.7	38.6	40.8	30.287	83 12 74	477.79	— 0.62
„ „ 17.	38.2	41.4	43.1	30.211	83 12 89	469.29	— 4.19
„ „ 23.	32.1	35.2	37.6	29.663	83 12 85	471.85	+ 0.67

* δ by Airy (26 obs.) . . 46″.38 Henderson, Cape, ... 46″.36

† Henderson's declination is a second greater, but rests on a much less number of observations.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
,, ,, 24.	32	36.5	38.1	29.727	83 12 83	473.37	+ 1.12
,, ,, 30.	37	40.1	42.1	29.756	83 13 00	463.15	— 4.39
1838, Feb. 8.	38.5	39.3	40.2	28.524	83 13 30	444.99	— 1.94
,, ,, 21.	32.4	35.5	38	29.583	83 12 98	473.32	+ 3.62
1839, Feb. 9.	39	..	43.7	30.064	83 13 04	468.45	— 1.86
,, ,, 12.	36.2	..	41	30.040	83 13 04	468.91	— 4.10
,, ,, 14.	35.9	..	40.8	29.733	83 13 11	465.39	— 3.16
,, ,, 17.	22.3	..	29.8	29.220	83 12 93	477.02	+ 2.32
,, ,, 18.	30.7	..	34.1	29.394	83 13 09	467.04	— 1.65
,, ,, 20.	29.9	..	34.3	30.058	83 12 92	477.85	— 2.20
,, March 17.	35.4	..	40.1	29.908	83 13 10	470.39	— 1.47
,, ,, 25.	40.1	..	44.1	29.416	83 13 33	456.61	— 2.76

$16 \times d\text{R} = -18''.37$ $\qquad$ $d\text{R} = -1''.15$

$\text{K} = 8.8592$ $\qquad$ $d\mu = -0''.130$

δ Canis Majoris.

$\delta = -26° 7' 42''. 18.$ (J.)

Precession $= -5''.316$; sec var. $= -0''.340$; proper motion $= +0''.021$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.8	39.6	40.2	29.295	80° 24′. 02	335.17	— 0.90
,, March 12.	29.3	33.5	36.8	29.575	80 23 93	343.39	+ 0.18
,, ,, 13.	30.4	34.7	36	30.177	80 23 88	346.83	— 2.54
,, ,, 14.	34.7	38.6	40.8	30.287	80 23 95	343.81	— 3.55
,, ,, 17.	38.2	41.4	43.1	30.211	80 23 99	340.16	— 3.81
,, ,, 23.	32.1	35.2	37.6	29.663	80 23 94	343.68	+ 1.53
,, ,, 24.	32	36.5	38.1	29.727	80 24 02	338.56	— 4.42
1838, Feb. 8.	38.5	39.3	40.2	28.524	80 24 28	321.45	— 3.17
,, ,, 13.	27	30	31.2	29.479	80 24 00	339.75	— 4.16
,, ,, 20.	31.8	34.7	35.7	29.483	80 24 17	338.76	— 1.70
,, ,, 21.	32.9	36.1	38	29.583	80 24 03	339.11	— 1.57
,, March 15.	39.2	44.8	47.2	29.798	80 24 13	335.69	— 2.74
,, ,, 17.	36.2	39.8	41.2	29.344	80 24 19	338.60	— 1.80
,, ,, 23.	33.5	35.2	39.7	29.500	80 24 09	338.62	— 0.68

$14 \times d\text{R} = -29''.33$ $\qquad$ $d\text{R} = -2''.09$

$\text{K} = 6.5921$ $\qquad$ $d\mu = -0.318$

ζ Canis Majoris.

$\delta = -29° 59' 34''.62$ (J. H.)

Precession $= -1''.205$; sec var. $= -0''.335$; proper motion $= -0''.022$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.7	41.6	41.4	29.264	84° 12′. 13	533.37	+ 3.77
,, March 12.	30	36.4	37	29.562	84 12 09	537.47	— 3.17
1838, Feb. 8.	39	39.8	40.6	28.530	84 12 48	510.98	— 0.89
,, ,, 13.	27.6	30.3	31.8	29.474	84 12 15	539.80	— 2.61
1839, Feb. 10.	42	..	44	30.116	84 12 23	527.50	— 9.27
,, ,, 12.	36.2	..	41.4	30.034	84 12 02	540.77	— 1.01
,, ,, 14.	36.3	..	41.5	29.735	84 12 13	534.72	— 2.63
1840, Feb. 13.	34.8	37	40.2	29.625	84 12 15	534.57	— 2.30
,, ,, 26.	33.8	36.7	40.1	30.370	84 12 14	542.88	— 8.82
,, ,, 28.	32.7	35	37.2	30.234	84 11 96	548.47	— 2.02
,, March 2.	33.5	35	38.5	30.386	84 11 89	553.44	+ 1.33
,, ,, 3.	34.8	36.4	38.4	30.416	84 11 87	554.56	+ 3.43
,, ,, 4.	35.8	38	40	30.254	84 12 09	541.50	— 5.59
,, ,, 5.	38.2	39.7	41.5	30.128	84 12 13	539.00	— 2.90
,, ,, 9.	44.9	44.8	45	30.477	84 12 11	540.30	+ 0.55
,, ,, 17.	42.2	46	49.1	30.214	84 12 16	537.84	— 0.45
,, ,, 18.	41	45.3	49	30.146	84 12 17	537.55	— 0.95

$17 \times d\text{R} = -33''.53$ $d\text{R} = -1''.97$

$\text{K} = 10.0672$ $d\mu = -0''.196$

38. *ζ Sagittarii.*

* $\delta = -30° 6' 49''.15$. (J.)

Precession $= +4''.487$; sec var. $= +0''.543$; proper motion (J.) $= -0''.013$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Aug. 5.	46.1	51	53.7	30.155	84° 18′ 64	538.27	— 2.57
,, ,, 6.	48.8	53.2	55	30.248	84 18 48	546.92	+ 6.15
,, ,, 7.	49.2	54.8	56.2	30.259	84 18 63	538.23	— 0.39

* The proper motion is deduced from J., as Airy's places for 1836 and 1837 differ $2''.68$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Aug.	15.	57.9	60.4	62.8	30.090	84° 18′. 97	518.33	— 8.92
,,	,,	16.	59.2	62	64	29.965	84 18 98	518.13	— 4.12
,,	,,	26.	49	54.7	56.5	29.939	84 18 77	531.06	— 3.10
,,	,,	29.	46.5	52	54.6	29.429	84 18 86	525.97	— 1.99
1838,	Aug.	4.	54.8	..	60	29.204	84 18 94	515.17	+ 1.34
,,	,,	13.	51.8	..	58.5	30.060	84 18 75	527.16	— 5.21
,,	,,	14.	53	..	58.2	30.033	84 18 70	530.00	— 0.42
1839,	July	24.	52.5	57.4	59.2	29.578	84 18 65	526.57	+ 3.79
,,	,,	27.	52.2	60	61.5	29.636	84 18 68	524.82	— 0.46
,,	,,	28	51.5	55.7	60	29.776	84 18 63	528.18	— 0.27
,,	,,	31.	47.4	51.2	55	29.627	84 18 63	528.50	— 1.22
,,	Aug.	2.	56.1	57.1	59.8	29.762	84 18 72	523.14	+ 1.12
,,	,,	3.	52.1	56	59.2	30.026	84 18 57	532.28	+ 0.99
,,	,,	4.	52.1	57.8	59.2	30.184	84 18 62	529.28	— 4.75
,,	,,	11.	51	56.2	58.2	30.169	84 18 61	530.05	— 5.02
,,	,,	12.	49.1	56	58	30.124	84 18 49	537.07	+ 0.74
,,	,,	24.	54.8	57.7	60	29.745	84 18 81	518.55	— 4.74
,,	,,	26.	51	54	59.1	29.620	84 18 68	526.98	+ 1.70
,,	Sept.	5.	54.9	57	60.8	29.442	84 18 85	517.95	+ 0.15
,,	,,	11.	50.8	53.7	58	29.735	84 18 76	522.60	— 5.14

$23 \times d\text{R} = -32''.34$ $\qquad$ $d\text{R} = -1''.41$

$\text{K} = 9.8637$ $\qquad$ $d\mu = -0''.142$

Fomalhaut.

$* \delta = -30° 31'. 15''.26.$ (H. J.)

Precession $= +19''.073$; sec var. $= +0''.135$; proper motion $= -0''.180$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839,	Oct.	12.	44.8	46.8	48.9	29.710	84° 39′. 95	566.45	+ 2.07
,,	,,	17.	39.1	44.9	46.5	29.944	84 39 82	575.00	— 1.11
,,	,,	27.	41.1	45	47	30.293	84 39 70	583.29	+ 3.22
,,	,,	28.	43.1	46.2	48.5	30.412	84 39 94	569.18	— 10.86

* Airy, (Greenwich, 22 obs.) .	16″.00	Johnson, . . .	14″.75
,, (Cambridge, 21) . . .	13 .38	Mine, . . .	14 .35
Henderson, (Cape,) . . .	15 .78	Bessel, (Tab. Reg.)	20 .24

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839,	Nov.	11.	42.9	44	47.3	28.998	84° 40′. 18	555.57	+ 2.57
„	„	12.	40.9	43	46.1	29.332	84 40 17	557.07	— 4.92
„	„	26.	32	35.8	40	29.173	84 39 83	578.74	+ 8.02
„	Dec.	2.	38.2	40.8	42.4	29.758	84 40 00	568.91	— 5.24
„	„	28.	29.8	34.2	37.2	29.762	84 39 82	579.90	— 5.53
1840,	Sept.	28.	47.1	50	51.1	29.016	84 39 83	550.25	+ 2.36
„	„	29.	45.1	47.8	49.1	29.582	84 39 54	568.21	+ 2.97
„	Oct.	2.	42	45	46.1	30.148	84 39 46	574.50	— 1.28
„	„	3.	39.5	47	49.8	30.160	84 39 45	574.35	— 4.67
„	„	4.	40.8	46	46.8	30.119	84 39 47	572.97	— 3.79
„	„	10.	41.8	43.8	45.5	30.210	84 39 42	577.23	+ 0.09
„	„	11.	42.8	45.2	46	30.295	84 39 42	577.20	— 0.26
„	„	12.	45.9	47.5	49	30.405	84 39 52	571.35	— 4.20
„	„	14.	41.2	43.2	45.5	30.208	84 39 31	584.18	+ 6.84
„	Nov.	21.	43.9	42.8	43.8	29.470	84 39 84	556.80	— 4.16
„	„	27.	41.4	42.8	43	30.130	84 39 70	565.65	— 11.08

$20 \times d\text{R} = -28''.96$ $\qquad d\text{R} = -1''.45$

$\text{K} = 10.6207$ $\qquad d\mu = -0.136$

Combining, we obtain,

NAME.	NO. OBS.	$nd\text{R} \times \text{K}$	$n\,\text{K}^2$.	$d\mu$.
o^2 Canis.	13	— 112.9950	354.772	—0″.318
15 Argûs.	16	— 80.9305	490.286	— 0 165
o^1 Canis.	13	— 130.2914	400.634	— 0 325
ξ Argus.	13	— 126.6950	436.280	— 0 290
λ Sagittarii.	23	+ 48.3397	856.812	+ 0 056
Antares.	22	— 157.9369	878.733	— 0 180
δ Canis Maj.	14	— 193.3463	608.381	— 0 318
σ Sagittarii.	17	— 112.9772	778.032	— 0 145
ϵ Canis Maj.	16	— 162.8188	1193.730	— 0 136
η Canis Maj.	16	— 162.7435	1255.767	— 0 130
δ Sagittarii.	18	— 396.3260	1648.873	— 0 241
ζ Canis.	17	— 337.5532	1722.926	— 0 196
ζ Sagittarii.	23	— 318.9921	2237.730	— 0 142
Fomalhaut.	20	— 307.5758	2255.988	— 0 136
Sum . .	241	— 2552.8420	15118.944	

$$\text{and } d\mu = \frac{-2552.842}{15118.944} = -0.1688$$

The correction for run for these stars give,

$$d^2\mu = \frac{-95.5541}{15118.944} = -0.0063$$

and we have,

$$\begin{array}{r} \mu = 57.7682 \\ -\ \ 0.1688 \\ -\ \ 0.0063 \\ \hline 57.5931 \end{array}$$

which agrees so nearly with the determination from sub-polar stars (their difference being only $0''.5$ at Fomalhaut) that there is obviously no necessity for supposing any discrepancy between the northern and southern refractions at this observatory, especially as it would vanish entirely were the Cape declinations not used. If now we take $\mu = 57.546$; the value of $\frac{l}{a}$ reduced to my latitude is 0.00129263, and (using the well-known notation of Mr. Babbage to save space) the equation of refraction becomes for $\tau = 50$, barometer 29.60,

$$\text{R} = \text{tang} . \theta \times \log^{-1} (1.7600151)$$

$$+ \text{tang}^3 . \theta \times \log^{-1} (7.9045751) \{1 + \text{tang}^2 . \theta \times \log.^{-1} (6.44559)\}$$

$$- \frac{\text{tang}}{\cos^2} . \theta \times \log^{-1} (8.8715498) \{1 + \text{tang}^2 . \theta \times \log.^{-1} (6.77484)\}$$

$$+ \frac{\text{tang}^3}{\cos^2} . \theta \times \log^{-1} (6.3720995) \{1 + \text{tang}^2 . \theta \times \log^{-1} (7.06014)\}$$

$$- \frac{\text{tang}^5}{\cos^2} . \theta \times \log^{-1} (4.0315728) \{1 + \text{tang}^2 . \theta \times \log^{-1} (7.23971)\}$$

$$+ \frac{\text{tang}^7}{\cos^2} . \theta \times \log^{-1} (1.7907405) \{1 + \text{tang}^2 . \theta \times \log^{-1} (7.34007)\}$$

From this the following tables have been computed. In the first, the column A contains the logarithm of $\frac{\mu(1+\epsilon(\mathrm{T}-50))}{29.60}$, and B that of $\frac{1+\epsilon'(\tau-50)}{1+\epsilon''(\tau-50)}$, ϵ' the expansion of the brass scale being taken $= 0.000010479$; and ϵ'' that of mercury $= 0.0001$.

The second table contains C, the sum of all the terms except the first, for the argument zen. distance; D = the change of C for one degree increase of temperature; and E its change for one inch rise of the barometer. This last serves also to change C for a slight variation in μ, the constant, for

$$\frac{dc}{d\mu} = \mathrm{E} \times 0.5144$$

and A must be changed by $\log \mu' - \log \mu$.

The refraction is given by

$$\log \mathrm{R}' = \mathrm{A} + \mathrm{B} + \log \text{tang apparent zen. dist.} + \log. \text{bar.}$$
$$\mathrm{R} = \mathrm{R}' - \mathrm{C} - \mathrm{D} \times (\mathrm{T} - 50^\circ) - \mathrm{E} \times (\text{bar.} - 29.60.)$$

Argument of A, external thermometer = T

Argument of B, attached thermometer = τ

Argument of C, D, and E, apparent zenith distance.

Table I.

Ther. = 50° ; bar. = 29.60 inches.

T.	A.	B.	T.	A.	B.	T.	A.	B.
0	0.33343_{94}		31	0.30517_{88}	+ 74	62	0.27864_{83}	− 46
1	0.33249_{94}		32	0.30429_{88}	+ 70	63	0.27781_{83}	− 50
2	0.33155_{94}		33	0.30341_{88}	+ 66	64	0.27698_{82}	− 54
3	0.33061_{94}		34	0.30253_{88}	+ 62	65	0.27616_{82}	− 58
4	0.32968_{94}		35	0.30165_{87}	+ 58	66	0.27534_{83}	62
5	0.32874_{93}		36	0.30078_{87}	+ 54	67	0.27451_{82}	− 66
6	0.32781_{93}		37	0.29991_{87}	+ 50	68	$0{\cdot}27369_{82}$	− 70
7	0.32688_{93}		38	0.29904_{87}	+ 46	69	0.27287_{82}	− 74
8	0.32595_{92}		39	0.29817_{87}	+ 42	70	0.27205_{82}	− 78
9	0.32503_{92}		40	0.29730_{87}	+ 39	71	0.27123_{81}	− 81
10	0.32411_{92}		41	0.29643_{86}	+ 35	72	0.27042_{81}	− 85
11	0.32319_{92}		42	0.29557_{86}	+ 31	73	0.26961_{81}	− 89
12	0.32227_{92}		43	0.29471_{86}	+ 27	74	0.26880_{81}	− 93
13	0.32135_{91}		44	0.29385_{87}	+ 23	75	0.26799_{81}	− 97
14	0.32044_{91}		45	0.29298_{86}	+ 19	76	0.26718_{81}	− 101
15	0.31953_{91}		46	0.29212_{86}	+ 15	77	0.26637_{80}	− 105
16	0.31862_{91}		47	0.29126_{85}	+ 11	78	0.26557_{81}	− 109
17	0.31771_{91}		48	0.29041_{85}	+ 7	79	0.26476_{80}	− 113
18	0.31680_{91}		49	0.28956_{84}	+ 3	80	0.26396_{80}	− 117
19	0.31589_{90}		50	0.28872_{85}	0	81	0.26316_{80}	− 121
20	0.31499_{90}	+ 117	51	0.28787_{84}	− 3	82	0.26236_{80}	− 125
21	0.31409_{90}	+ 113	52	0.28703_{85}	− 7	83	0.26156_{80}	− 129
22	0.31319_{89}	+ 109	53	0.28618_{84}	− 11	84	0.26076_{80}	− 132
23	0.31230_{90}	+ 105	54	0.28534_{85}	− 15	85	0.25996_{80}	− 136
24	0.31140_{89}	+ 101	55	0.28449_{84}	− 19	86	0.25916_{79}	− 140
25	0.31051_{90}	+ 97	56	0.28365_{84}	− 23	87	0.25837_{79}	− 144
26	0.30961_{89}	+ 93	57	0.28281_{84}	− 27	88	0.25758_{79}	− 148
27	0.30872_{89}	+ 89	58	0.28197_{84}	− 31	89	0.25679_{79}	− 152
28	0.30783_{89}	+ 85	59	0.28113_{83}	− 35	90	0.25600_{79}	− 156
29	0.30694_{88}	+ 81	60	0.28030_{83}	− 39	91	0.25521_{79}	− 160
30	0.30606_{89}	+ 78	61	0.27947_{83}	− 42	92	0.25442	− 163

TABLE II.

Z. D.	C.	D.	E.	Z. D.	C.	D.	E.	Z. D.	C.	D.	E.
4°	0.01			76° 20′	4.69_{35}	0.002	0.14	81° 55′	20.82_{60}	0.005	0.63_{1}
10	0.01_{1}			40	5.04_{38}	0.002	0.15	82 0	21.42_{63}	0.005	0.64_{2}
15	0.02_{1}			77 0	5.42_{42}	0.002	0.16	5	22.05_{65}	0.005	0.66_{2}
20	0.03_{1}			20	5.84_{47}	0.002	0.18	10	22.70_{68}	0.006	0.68_{2}
25	0.04_{1}			40	6.31_{52}	0.002	0.19	15	23.38_{70}	0.006	0.70_{3}
30	0.05_{2}			78 0	6.83_{28}	0.002	0.21	20	24.08_{73}	0.006	0.73_{2}
35	0.07_{3}			10	7.11_{30}	0.002	0.21	25	24.81_{76}	0.006	0.75_{2}
40	0.10_{5}			20	7.41_{31}	0.002	0.22	30	25.57_{78}	0.006	0.77_{2}
45	0.15_{1}			30	7.72_{33}	0.002	0.23	35	26.35_{83}	0.006	0.79_{3}
46	0.16_{1}			40	8.05_{35}	0.003	0.24	40	27.18_{85}	0.007	0.82_{3}
47	0.17_{1}			50	8.40_{36}	0.003	0.25	45	28.03_{89}	0.007	0.85_{2}
48	0.18_{1}			79 0	8.76_{39}	0.003	0.26	50	28.92_{93}	0.007	0.87_{3}
49	0.19_{1}			10	9.15_{42}	0.003	0.28	55	29.85_{97}	0.007	0.90_{3}
50	0.20_{1}		0.01	20	9.57_{44}	0.003	0.29	83 0	$30.82_{1.00}$	0.008	0.93_{3}
51	0.21_{2}		0.01	30	10.01_{46}	0.003	0.30	5	$31.82_{1.06}$	0.008	0.96_{3}
52	0.23_{2}		0.01	40	10.47_{49}	0.003	0.31	10	$32.88_{1.10}$	0.008	0.99_{4}
53	0.25_{2}		0.01	50	10.96_{53}	0.003	0.33	15	$33.98_{1.15}$	0.009	1.03_{3}
54	0.27_{2}		0.01	80 0	11.49_{28}	0.003	0.35	20	$35.13_{1.19}$	0.009	1.06_{4}
55	0.29_{3}		0.01	5	11.77_{28}	0.003	0.35	25	$36.32_{1.24}$	0.010	1.10_{4}
56	0.32_{3}		0.01	10	12.05_{29}	0.003	0.36	30	$37.56_{1.31}$	0.010	1.14_{4}
57	0.35_{4}		0.01	15	12.34_{30}	0.004	0.37	35	$38.87_{1.37}$	0.011	1.18_{4}
58	0.39_{4}		0.01	20	12.64_{31}	0.004	0.38	40	$40.24_{1.42}$	0.012	1.22_{5}
59	0.43_{4}		0.01	25	12.95_{33}	0.004	0.39	45	$41.66_{1.50}$	0.013	1.27_{4}
60	0.47_{5}		0.01	30	13.28_{33}	0.004	0.40	50	$43.16_{1.57}$	0.013	1.31_{5}
61	0.52_{6}		0.02	35	13.61_{35}	0.004	0.41	55	$44.73_{1.64}$	0.014	1.36_{5}
62	0.58_{7}		0.02	40	13.96_{35}	0.004	0.42	84 0	$46.37_{1.72}$	0.015	1.41_{6}
63	0.65_{7}		0.02	45	14.31_{36}	0.004	0.43	5	$48.09_{1.80}$	0.016	1.47_{6}
64	0.72_{8}		0.02	50	14.67_{38}	0.004	0.44	10	$49.89_{1.89}$	0.018	1.53_{6}
65	0.80_{11}		0.03	55	15.05_{40}	0.004	0.45	15	$51.78_{1.99}$	0.019	1.59_{6}
66	0.91_{12}		0.03	81 0	15.45_{41}	0.004	0.46_{2}	20	$53.77_{2.09}$	0.022	1.65_{7}
67	1.03_{14}		0.03	5	15.86_{42}	0.004	0.48_{1}	25	$55.86_{2.20}$	0.023_{2}	1.72_{7}
68	1.17_{17}		0.04	10	16.28_{44}	0.004	0.49_{1}	30	$58.06_{2.31}$	0.025_{3}	1.79_{8}
69	1.34_{21}	0.000	0.04	15	16.72_{45}	0.004	0.50_{2}	35	$60.37_{2.45}$	0.028_{3}	1.87_{8}
70	1.55_{25}	0.001	0.05	20	17.17_{47}	0.004	0.52_{1}	40	$62.82_{2.58}$	0.031_{4}	1.95_{9}
71	1.80_{29}	0.001	0.06	25	17.64_{48}	0.005	0.53_{1}	45	$65.40_{2.71}$	0.035_{4}	2.04_{9}
72	2.09_{39}	0.001	0.06	30	18.12_{50}	0.005	0.54_{2}	50	$68.11_{2.89}$	0.039_{5}	2.13_{10}
73	2.48_{49}	0.001	0.08	35	18.62_{52}	0.005	0.56_{2}	55	$71.00_{3.06}$	0.044_{6}	2.23_{11}
74	2.97_{62}	0.001	0.09	40	19.14_{54}	0.005	0.58_{1}	85 0	74.06	0.050	2.34
75	3.59_{78}	0.001	0.11	45	19.68_{56}	0.005	0.59_{1}				
76	4.37_{32}	0.001	0.13	50	20.24_{58}	0.005	0.60_{3}				

Example.

Fomalhaut, zen. dist. 84° 39′. 46; E. T. 42°; bar. 30ⁱ.148; A. T. 46°.1.

$$
\begin{array}{llll}
\text{tang z. D.} & 1.02913 & & \text{C} - 62.56 \\
\text{A.} & 0.29557 & & (\text{D}) + \;\; 0.25 = -8^{7} \times -0.031 \\
\text{B.} \;\; + & \quad\;\; 15 & & (\text{E}) \underline{- \;\; 1.01} = +0.548 \times -1.95 \\
30.148 & \underline{1.47926} & & -63.38 \\
 & 2.80411 \ldots\ldots\ldots & & \underline{636.96} = \text{R}' \\
 & & & 573.58 = \text{R}.
\end{array}
$$

The Reader is requested to make the following Correction:—

Page 223, last line, *for* 1 + *read* 1 —.

IX. *On the Heat developed during the Combination of Acids and Bases. By* THOMAS ANDREWS, M.D., M.R.I.A., *Professor of Chemistry in the Royal Belfast Institution.*

Read 11th January, 1841.

1. IT has been long known that chemical actions are in general accompanied by the evolution or abstraction of caloric. In most cases the change of temperature depends upon the result of the action of different causes, some of which tend to increase, and others to diminish the initial temperature of the reacting bodies. Thus, in the decomposition of a solution of carbonate of soda by concentrated sulphuric acid, the combination of the sulphuric acid with water and with the alcali are two distinct sources of heat, while the separation of the carbonic acid from the soda, and its evolution in the gaseous form, are equally distinct causes of a diminution of temperature. To estimate the influence of each of these circumstances in any particular instance is a problem of great difficulty; and we can only expect to accomplish its complete solution, by confining our investigations, in the first place, to these simpler cases, where the variations of temperature are produced by the operation of one single cause. For this reason, I have confined myself, in this preliminary inquiry, to the examination of the calorific phenomena which occur during the combination of acids and bases with each other, under the most favourable circumstances, for obtaining results free from complication.

2. The experiments to be hereafter described were all performed with very dilute solutions, by which means no correction was required for the heat evolved, when strong solutions of certain acids and alcalies are diluted. The method of operating is easily described. In separate glass vessels solutions of determinate weights were prepared, one containing the quantity of alcali whose power of generating heat was sought, and the other, a little more than the equivalent of

acid required to neutralize the alcali. After the liquids had acquired the same temperature, they were mixed together in the jar containing the alcali, and the increase of heat carefully observed by a delicate thermometer. This process was adopted from the facility of its execution and the uniformity of its results. It is, however, obvious, that a large portion of heat would be absorbed by the glass vessel; and it was, therefore, necessary to establish, by a series of independent experiments, the corrections to be applied to the temperatures thus obtained.

3. As a basis to this whole investigation, the heat developed in the combination of nitric acid and potash was carefully determined. But before describing the method employed, I must anticipate an observation which will be afterwards proved, viz., that the same amount of heat is developed when a given quantity of an alcali is united to an acid, whether the acid added be just sufficient to neutralize the alcali, or be considerably in excess.* The addition of a slight excess of acid does not, therefore, in any way interfere with the results, except in so far as it renders them more uniform and certain, by producing a rapid and complete neutralization of the alcali.

4. A cylindrical vessel of very thin brass was procured, capable of containing rather more than the quantity of liquid employed. Into this vessel was introduced the solution of caustic potash, the weight of which solution was about nine times greater than that of the dilute nitric acid destined to neutralize it. This vessel was so thin that we may assume, without any sensible error, its temperature to have been identical with that of its liquid contents. It weighed 6.63 grammes, which, assuming the specific heat of brass to be .094, is equivalent to 0.623 gr. water.

5. As the weights of the glass and mercury in the bulb and immersed portion of the tube of the thermometer were both unknown, I was obliged to have recourse to a direct experiment, in order to ascertain their equivalent of water. For this purpose, 30 grammes of water (the quantity of liquid usually employed) were introduced into the brass vessel, and the increase of its temperature carefully observed, when the thermometer, previously heated through a

* These observations, as well as others of a similar kind in subsequent parts of this paper, refer always to dilute solutions, such as are employed in these experiments.

certain number of degrees, was suddenly cooled by immersion in it. Denoting by t the loss of heat sustained by the thermometer, and by t' the temperature gained by the liquid, I obtained in different trials the following numbers:

1	2	3
$t = 59^\circ.00$,	$t = 69^\circ.00$,	$t = 72^\circ.00$.
$t' = 0^\circ.90$,	$t' = 1^\circ.00$,	$t' = 1^\circ.15$.

Hence, we deduce for the value of the thermometer in grammes of water,

1	2	3	Mean.
0.47,	0.45,	0.49,	0.47.

6. From the last two results we may therefore conclude, that the brass vessel and thermometer, taken together, are equivalent to 1.09 gr. water.

7. A very important source of error in this and other similar investigations, where the variation of temperature of a liquid requires to be observed with the utmost precision, arises from the cooling influence of the surrounding air during the time occupied by the observation, which, in the experiments I am about to describe, amounted to nearly 1′. Where the increase of temperature does not exceed 2° or 3° Fah., the common method of cooling the liquid before the experiment begins, as many degrees below the temperature of the air as it will afterwards rise above it, may be employed with success; but for greater increments of heat, this process is liable to a serious error, which it is necessary to avoid. In fact, on mixing the liquids together, the thermometer attains, in a very few seconds, almost its ultimate point of elevation, and it occupies at least four-fifths of the entire time in rising through the last half degree. As, therefore, the mixture continues much longer in the upper than in the lower half of its range of temperature, the method just described will necessarily yield results sensibly below the truth.* In practice, this error may be effectually obviated, by reducing the initial temperature of the liquid so far below the temperature of the air, that its final maximum may never reach higher than 2° F. above the same point.

* A similar observation has been made by M. Regnault in his recent and valuable memoir on the "Specific Heats of Simple and Compound Bodies" (Ann. de Chim. t. 63, p. 23); but the error thus induced he corrects by means of an interpolating formula.

8. The strongest nitric acid employed in these experiments contained 13.3 per cent. of real acid, and when one part of such an acid is diluted with nine parts of water, no sensible production of heat can be discovered by the most delicate thermometer. The corresponding solution of caustic potash, containing only 1.3 per cent. of alcali, was of course far beyond the limit of such sources of heat. That simple dilution exercised no influence on the result was further proved, by increasing the weight of the acid liquid, and diminishing that of the alcaline, while, at the same time, the quantities of acid and alcali in each, as also the total weight of both liquids, remained the same; yet such variations in the form of the experiment produced no change whatever in the elevation of temperature observed on mixing them.

9. Having discussed the corrections arising from the form of apparatus, I now proceed to give the details of the fundamental experiment, on the absolute amount of heat evolved in the union of nitric acid and potash. The general accuracy of these results was tested and confirmed by repeating the experiments in the form of a series, in which (the weight of the whole liquid remaining constant) the quantities of the combining substances were taken successively, in the proportions expressed by the numbers 1, 2, 4; and it will be seen that the corresponding increments of temperature bear a similar ratio to each other.

10. Into the brass vessel before described, a solution of caustic potash, containing .0882 gr. of alcali was introduced. It weighed 27.3 gr., which, added to 1.09 gr., the equivalent in water of the vessel and thermometer (6), makes the whole equal to 28.39 gr. water. The acid solution, in a small glass tube, weighed 2.83 gr., and contained .106 anhydrous nitric acid. Thermometer in air stood at 38° F.

Temp. of acid, ...	38°.20	
„ alcaline solution, . .	37°.00	
Mean temp. before mixture, . .	37°.11	
Temp. after mixture, ...	38°.75	
Increase in temp. (31.22 water) .		1°.64

11. The last experiment repeated. Ther. in air 39°.

Temp. of acid, . . .	39°.00
„ alcaline solution .	37°.50
Mean temp. before mixture	37°.64
Temp. after mixture, . .	39°.25
Increase (31.22 gr. water) .	1°.61

12. Alcaline solution weighed 27.2 gr., and contained .1765 gr. of pure potash, or double that in the last experiments. Acid solution weighed 2.85, gr. containing .212 anhydrous nitric acid. Ther. in air 39°.5.

Temp. of acid, . . .	39°.00
„ alcaline solution, .	37°.00
Mean temp. before mixture,	37°.18
Temp. after mixture, . .	40°.40
Increase (31.14 water), . .	3°.22

13. Alcaline solution 26.85 gr., containing .353 potash ; acid liquid 3.25 gr., containing .424 anhydrous nitric acid. Ther. in air 39.3°.

Temp. of acid . . .	39°.70
„ alcaline solutions .	34°.30
Mean. Temp. before mixture,	34°.86
Temp. after mixture, . .	41°.45
Increase (31.19 water) . .	6°.59

14. Reducing these results to the quantity of alcali (.353 gr.) used in last experiment, and to 30 gr. of water, we obtain the following numbers :

1	2	3	4	Mean.
6°.83,	6°.70,	6°.68,	6°.85,	6°.76.

15. This may be otherwise expressed, by stating that 1 gr. of potash, in combining with nitric acid, is capable of heating 85 gr. of water through 6°.76 of Fahrenheit's scale, or, which is the same thing, of heating 574.6 gr. of water through 1°. It must, however, be carefully observed, that in this experiment it

is not pure water, but a weak solution of nitrate of potash, which is actually heated; and the above numbers would therefore require a further correction, in consequence of the difference between the specific heats of these liquids. This correction, however, must be extremely small, from the very dilute solutions obtained: it would probably be within the limit of the errors of observation.

16. Many of the subsequent experiments would have been performed with difficulty in a metallic vessel. I therefore substituted a pretty thick glass jar for the brass vessel, and both solutions were brought as nearly as possible to the temperature of the surrounding air, at the commencement of each observation. In this way, numerous experiments were easily performed, which yielded results comparable with each other, although all below the truth. It was, therefore, necessary to ascertain the absolute loss of heat when the experiment was performed in this way, and whether it was proportional to the elevation of temperature. For this purpose, solutions were prepared containing the same quantity of potash and nitric acid as in the experiments with the brass cylinder.

17. Alcaline solution 27 gr., containing .0882 gr. potash; acid solution 3 gr., containing 1.06 nitric acid.

Temp. rose on mixture, 1°.45.
Another experiment gave 1°.45.

18. Alculine solution 27 gr., containing .1765 potash; acid solution 3 gr., containing .212 nitric acid.

Temp. rose on mixture 2°.90.
Another experiment gave 2°.95.

19. Alcaline solution 27 gr., containing .353 potash; acid solution 3 gr., containing .424 nitric acid.

Temp. rose on mixture 5°.8.
Another experiment gave 5°.8.

20. Alcaline solution 24 gr., containing .353 potash; acid liquid 6 gr., containing .424 nitric acid.

Temp. on mixture rose to 5°.9.

21. Collecting these results, we obtain for the elevation of temperature of

30 gr. of water, in a glass vessel, by the combination of .353 gr. potash with nitric acid:

1	2	3	4	5	6	Mean.
5°.8,	5°.8,	5°.8,	5°.9,	5°.8,	5°.9,	5°.83.

This number differs by 0.93° from the absolute quantity of heat before found, which is the loss of heat by this method of performing the experiment. It also appears from the coincidence of the results obtained with different proportions of alcali, that the loss of heat is proportional to the rise of temperature, and hence the necessary correction for this error is, in all cases, easily made.

22. When the base is insoluble in water, and slowly soluble in the acid, a new element enters into the observation, and requires to be estimated, viz., the cooling of the liquid during the prolonged duration of the experiment. In the observations last described, the thermometer attained its maximum in about 45″ from the time the liquids were mixed, but in the solution of such substances, as magnesia or the oxide of zinc, not less than 2′, or 2½′ will elapse before the liquid becomes transparent, and the thermometer stationary. Even to complete the solution within this period, the liquid requires to be constantly stirred with a glass rod. This circumstance renders these results less precise than those in which the combination occurs instantaneously; but the amount of error thus produced may be estimated, by repeating the same experiment in precisely the same manner, with a solution of caustic potash, containing exactly the quantity of alcali (as deduced by calculation from the foregoing experiments) which should produce the same elevation of temperature as had been obtained with the slowly soluble base. The difference between the increase of heat actually found, and that deduced from calculation, will be equal to the loss of caloric occasioned by the stirring, and length of the experiment; and consequently the required correction for the number obtained by observation. The precise value of this correction will be given hereafter.

23. The general conclusions which I shall endeavour to establish in the subsequent part of this communication, may be enunciated in the form of the three following laws:

LAW 1.—*The heat developed during the union of acids and bases is determined by the base and not by the acid; the same base producing, when* *combined with an equivalent of different acids, nearly the same quantity of heat; but different bases a different quantity.*

LAW 2.—*When a neutral is converted into an acid salt, by combining with one or more atoms of acid, no change of temperature occurs.*

LAW 3.—*When a neutral is converted into a basic salt, by combining with an additional proportion of base, the combination is accompanied with the evolution of heat.*

24. To the first of these laws important exceptions are presented by the peroxide of mercury among the bases, and by the hydrocyanic, and probably the carbonic acid, among the acids; and it is not improbable that more extended investigations will lead to the discovery of other exceptions. The second law has been established by numerous experiments, and can scarcely be said to be liable to any well-marked exception; but I feel much less confidence in enunciating the third, as a general principle, from the very limited number of cases of soluble subsalts in which it was possible to verify its accuracy.

25. In order to obtain results of as much uniformity as possible, the standard alcaline solution was always mixed with rather a greater quantity of acid than was necessary to neutralize it.* The combination was thus effected more rapidly and certainly, than if an attempt had been made to form an exactly neutral compound. That this excess of acid did not produce any sensible difference in the result, will be rendered evident, when the experiments are examined, which will be hereafter cited, in illustration of law second; and, indeed, if no basic compound existed, the numbers obtained were identical, whether an equivalent of base was neutralized by an excess of acid, or a like equivalent of acid neutralized by an excess of base. I have arranged, in distinct tables, the increments of temperature obtained by combining an equivalent of each base with the acids. The equivalents taken were .353 grammes potash, .234 gr. soda, .129 gr. ammonia, .572 gr. barytes, .213 gr. lime, .154 gr. magnesia, .301 gr. oxide of zinc, .834 gr. oxide of lead, .870 gr. oxide of silver, and .810 gr. peroxide of

* In the cases of the phosphoric and arsenic acids, the quantity of acid was just sufficient to convert the alcali into the common phosphate and arseniate; that is, half an equivalent of acid for an equivalent of base. The reason of this will appear again (55). The number for chromic acid is only deduced from an indirect experiment upon the bichromate of potash.

mercury. The entire weight of the solution, after the mixture was made, amounted in every instance to 30 grammes. In the first four tables, the first column of numbers contains the elevation of the thermometer actually observed; and the second, the result corrected for the loss of heat, occasioned by the mode of performing the experiment (21).

26. TABLE I.—*Potash.*

ACID.	FOUND.	CORRECTED.	DIFFERENCE FROM MEAN.
Sulphuric, ...	6°.30	7°.32	+ 0°.80
Nitric, ...	5.83	6.76	+ 0.24
Phosphoric, ...	5.70	6.61	+ 0.09
Arsenic, ...	5.70	6.61	+ 0.09
Hydrochloric, ...	5.65	6.56	+ 0.04
Hydriodic, ...	5.80	6.74	+ 0.22
Boracic, ...	5.60	6.50	− 0.02
Chromic, ...	5.55	6.46	− 0.06
Oxalic, ...	5.70	6.62	+ 0.10
Acetic, ...	5.50	6.39	− 0.13
Formic, ...	5.50	6.39	− 0.13
Tartaric, ...	5.25	6.10	− 0.42
Citric, ...	5.25	6.10	− 0.42
Succinic, ...	5.25	6.10	− 0.42
Mean, ...	. . . .	6.52	

27. TABLE II.—*Soda.*

ACID.	FOUND.	CORRECTED.	DIFFERENCE FROM MEAN.
Sulphuric, ...	6°.40	7°.44	+ 0°.96
Nitric, ...	5.55	6.45	− 0.03
Phosphoric, ...	5.55	6.45	− 0.03
Arsenic, ...	5.60	6.50	+ 0.02
Hydrochloric, ...	5.80	6.74	+ 0.26
Hydriodic, ...	5.70	6.62	+ 0.14
Boracic, ...	5.80	6.74	+ 0.26
Oxalic, ...	5.75	6.68	+ 0.20
Acetic, ...	5.45	6.34	− 0.14
Tartaric, ...	5.10	5.93	− 0.55
Citric, ...	5.10	5.93	− 0.55
Succinic, ...	5.10	5.93	− 0.55
Mean, ...	. . . .	6.48	

28. TABLE III.—*Barytes.*

ACID.	FOUND.	CORRECTED.	DIFFERENCE FROM MEAN.
Nitric, ...	5°.90	6°.85	+ 0°.10
Hydrochloric, ...	5.85	6.79	+ 0.04
Hydriodic, ... , . .	6.00	6.97	+ 0.22
Acetic, ...	5.50	6.39	− 0.36
Mean, ...	. . . .	6.75	

29. Table IV.—*Ammonia.*

Acid.	Found.	Corrected.	Difference from mean.
Sulphuric, ...	5°.45	6°.34	+ 0°.82
Nitric, ...	4.80	5.58	+ 0.06
Arsenic, ...	4.90	5.69	+ 0.17
Hydrochloric, ...	4.80	5.58	+ 0.06
Hydriodic, ...	4.80	5.58	+ 0.06
Oxalic, ...	4.90	5.69	+ 0.17
Acetic, ...	4.70	5.47	— 0.05
Tartaric, ...	4.40	5.11	— 0.41
Citric, ...	4.35	5.05	— 0.47
Succinic, ...	4.40	5.11	— 0.41
Mean, ...	. . . .	5.52	

30. The remainder of the bases examined, being either insoluble or very slightly soluble in water, were added in the solid state to the acid solution, whose weight was always so adjusted as, together with that of the base, to be equal to 30 grammes. The bases were all taken in the anhydrous state, except lime, which dissolves with extreme difficulty even in the dilute acids, unless previously converted into a hydrate. The experiments performed with these bases occupied from 80″ to 100″ longer than those with the soluble alcalis. This renders the application of a new correction necessary. The method of determining the amount of this correction has been already explained (23). In the remaining tables, the first column contains the result as found by experiment; the second, the duration of the observation; the third, the correction applied for the heat lost thereby; the fourth, the corrected result; and the fifth, the difference from the mean.

31. TABLE V.—*Magnesia.*

ACID.	FOUND.	TIME.	COR. TIME.	CORRECTED.	DIFFERENCE FROM MEAN.
Sulphuric, . .	7°.00	2′	0°.30	8°.48	+ 0°.24
Nitric, . . .	6.70	2	0.30	8.13	+ 0.11
Hydrochloric,	6.60	2	0.30	8.11	— 0.13
Mean, . .	. . . .	. . . .	. . . .	8.24	

32. TABLE VI.—*Lime.*

ACID.	FOUND.	TIME.	COR. TIME.	CORRECTED.	DIFFERENCE FROM MEAN.
Nitric, . . .	5°.95	$1\frac{1}{2}'$	0°.25	7°.20	+ 0°.10
Hydrochloric,	5.85	$1\frac{1}{2}$	0.25	7.08	— 0.02
Acetic, . .	5.80	$1\frac{1}{2}$	0.25	7.03	— 0.07
Mean, . .	. . . .	. . . .	. . . . . .	7.10	

33. TABLE VII.—*Oxide of Zinc.*

ACID.	FOUND.	TIME.	COR. TIME.	CORRECTED.	DIFFERENCE FROM MEAN.
Sulphuric, . .	4°.45	2′	0°.20	5°.40	+ 0°.49
Nitric, . . .	3.90	2	0.20	4.76	— 0.15
Hydrochloric,	4.00	2	0.20	4.88	— 0.03
Hydriodic, .	3.50	4	0.45	4.59	— 0.32
Mean, . .	. . . .	. . . .	. . . .	4.91	

34. TABLE VIII.—*Oxide of Lead.*

ACID.	FOUND.	TIME.	COR. TIME.	CORRECTED.	DIFFERENCE FROM MEAN.
Nitric, . . .	3°.45	2′	0°.15	4°.18	+ 0°.20
Acetic, . . .	2.95	3	0.30	3.78	— 0.20
Mean, . .	. . . .	. . . .	. . . .	3.98	

35. The oxide of silver gave, with nitric acid, an increase of temperature of 2°.7 corresponding, when corrected, to an actual elevation of 3°.23.

36. To render the numbers in each table strictly comparable with one another, would require a minute investigation of the influence of every possible source of a variation of temperature in the experiments; such are, differences in the specific heats of the solutions formed, alterations in the density of the liquids after mixture, &c. However, from very dilute solutions being employed, and also, from the results being identical when the strength of the solutions was greatly varied (9), it is probable that the errors arising from such causes could not amount, in most cases, to more than a few hundreths of a degree. Taking, therefore, the results as they appear in the tables, it will be found on examination, that they are in accordance with Law 1, (24). If we refer to the first, second, and fourth tables, as being the most extensive, from the large number of soluble compounds formed by potash, soda, and ammonia, it will be observed, that the sulphuric acid developes from 0°.8 to nearly 1° more than the mean heat given by the other acids, while the tartaric, citric, and succinic acids fall from 0°.4 to 0°.55 short of the same. A minute investigation of the influence of the disturbing sources of heat will, no doubt, discover the cause of these discrepancies; the high numbers for sulphuric acid are probably connected with that acid's well-known property of developing much heat when combined with successive atoms of water. All the other acids develope very nearly the same amount of heat in combining with the same base; the greatest divergences from the mean quantity being in the case of potash, + 0°.24, and — 0°.13; in that of soda, + 0°.26,

— 0°.14 ; and in that of ammonia + 0°.17 and — 0°.05. These differences are almost within the limits of the errors of experiment. In the other tables a similar agreement will be found to exist. Indeed the sulphuric acid does not exhibit in them so wide a discordance from the other acids as before. I must, however, remark that the numbers for the insoluble bases are scarcely so exact as those which are contained in the first four tables.

37. Whether the base be soluble or insoluble in water, the increments of temperature obtained, by combining the same base with different acids, may be compared with each other; but if we wish to discover the relations subsisting between the temperatures developed by different bases, it becomes necessary to take into consideration the heat absorbed by the insoluble bases, in passing from the solid to the fluid state. I am not at present acquainted with any method whereby the heat thus abstracted can be estimated. But the numbers for the insoluble bases, from this cause, will be all too low. We may, therefore, arrange the bases in the following order, in respect to their power of developing heat when combining with the acids; but this arrangement is liable to be disturbed when the value of the unknown quantities shall be determined. It must also be recollected that the potash, soda, barytes and lime were in the state of hydrates before mixture, while the magnesia, oxide of zinc, oxide of lead, and oxide of silver were anhydrous.

Magnesia, ...	$8°.24 + x$
Lime, ...	$7.10 + x'$
Barytes, ...	6.75
Potash, ...	6.52
Soda, ...	6.48
Ammonia, ...	5.52
Oxide of Zinc, . . .	$4.91 + x''$
Oxide of Lead, . . .	$3.98 + x'''$
Oxide of Silver, . . .	$3.23 + x''''$

38. The peroxide of mercury has given results altogether at variance with the preceding. It developes with the nitric and acetic acids nearly the same quantity of heat, but with the hydracids the most singular anomalies occur, as will appear in the next table.

39. Table IX.—*Peroxide of Mercury.*

ACID.	FOUND.	TIME.	CORR. TIME.	CORRECTED.
Nitric, ...	1°.20	2′	0°.05	1°.27
Acetic, ...	1.20	2	0.05	1.27
Hydrochloric, ...	3.80	2	0.20	4.65
Hydrocyanic, ...	5.85	2	0.25	7.10
Hydriodic, ...	9.20	3	0.60	11.40

40. To the last number some objection may be made, as a large excess of hydriodic acid was used to prevent the formation of the insoluble periodide of mercury; but even if we omit it altogether, the other parts of the table exhibit singular discrepancies. It is probable that a more extended investigation will discover other metallic oxides, resembling the peroxide of mercury, in yielding different quantities of heat, when they combine with the hydracids.

41. The hydrocyanic acid stands not less apart from the other acids than the oxide of mercury from the rest of the bases, in its development of heat when forming compounds; and it is remarkable that no analogous property appears with the hydrochloric and hydriodic acids. The hydrocyanic acid used in these experiments was perfectly pure: it was employed immediately after being rectified over chalk, and had no action on vegetable colours. I have collected together the elevations of temperature produced by it, and contrasted them with the mean quantities of heat given by the other acids with each base.

BASE.	HYDROCYANIC ACID.	MEAN OF OTHER ACIDS.	DIFFERENCE.
Potash, ...	1°.45	6°.52	5°.07
Soda, ...	1.45	6.48	5.03
Barytes, ...	1.68	6.75	5.07
Ammonia, ...	0.51	5.52	5.01
Peroxide of Mercury, . .	7.10		

42. Thus the hydrocyanic acid developes with potash, soda, barytes, and ammonia, 5° less than the other acids. On the other hand, it yields no less than 7°.1 in combining with the peroxide of mercury, while the oxyacids produce with the same base, only 1°.27.

43. I now proceed to cite a few experiments in illustration of Law 2; viz., that during the conversion of a neutral into an acid salt, no evolution of heat occurs.

44. 23 gr. of a solution of caustic potash, containing .353 gr. of alcali, were mixed with 7 gr. of a solution of oxalic acid, containing .271 gr. (or 1 equiv.) of acid.

Temp. after mixture rose 5°.7.

45. 31 gr. of a solution of neutral oxalate of potash, containing .624 gr. of the salt, were mixed with 9 gr. of a solution of oxalic acid, containing .271 gr. (1 equiv.) acid.

Temp. after mixture rose 0°.0.

46. The solution of binoxolate of potash, obtained in last experiment, was mixed with 18 gr. of the solution of oxalic acid (2 equiv.)

Temp. rose after mixture 0°.15.

After some time, crystals of quadroxalate of potash began to form, which accounted for the slight elevation of temperature.

47. On adding to a solution of sulphate of potash a second atom of sulphuric acid, the temperature of the mixture rose only 0°.1, although the combination of the first atom had produced 6°.3 of heat.

48. Similar results were obtained with the oxalate, tartrate, and acetate of soda, when converted into the corresponding supersalts; and by neutralizing these acid salts with the base, the same heat was invariably produced as if the excess of acid had existed in a free state. I may cite particularly the case of the bichromate of potash, which gave, when converted into the neutral chromate, a quantity of heat corresponding with that developed by the acids in general with potash, viz., 6°.45. In verifying this principle, care must be taken to select examples where all the compounds are soluble salts; otherwise, the latent heat extricated by the solid precipitate would interfere with, and complicate the

result. It is for this reason that the formation of the bitartrate of potash is accompanied by heat, although none is evolved when the neutral tartrate of soda combines with a second atom of acid.

49. As a farther illustration of the same principle, I am unwilling to omit the description of an interesting experiment, although its complete explanation involves the consideration of a class of phenomena which I have carefully excluded from the present communication. Three solutions were prepared, each containing 25 gr. of liquid; the first, holding in solution .353 gr. of pure potash; the second, .520 gr. of carbonate of potash; and the third, .683 gr. of bicarbonate of potash; consequently the amount of real alcali the same in all. They were then separately neutralized by 5 gr. of a solution of nitric acid, containing a considerable excess of acid, and the two latter solutions were well stirred, to expel, as far as possible, the carbonic acid gas before the final temperature was observed. The elevations of temperature were, for

		DIF.
Pure Potash, . . .	5°.8	
Carbonate of Potash, . .	1.7 —	4.1
Bicarbonate of Potash, . .	0.4	1.3

50. Thus we see that the effect of separating the first atom of carbonic acid, in the gaseous state, from its combinations with the alcali, was to cause the disappearance of 4°.1 of heat; while the separation of the second atom. and its complete expulsion in the gaseous state, produced a further diminution of temperature of only 1°.3. In these observations, two distinct sources of an absorption of caloric exists; one, the separation of the chemical compound into its constituents; the other, the change of one of those constituents from the liquid to the gaseous state. Had both causes acted equally on the second as on the first atom of carbonic acid, we should have obtained with the bicarbonate, as great a diminution of temperature as had occurred with the carbonate, or the thermometer would have sunk 2°.4 instead of rising .4°. But the conversion of the second atom of carbonic acid into the gaseous state is completely effected, while a considerable portion of the first atom remains dissolved in the liquid; and consequently, the striking difference in the result can only be accounted for, on the principle stated in the second law, that the combination, or separation of

the second atom of carbonic acid is attended with no evolution or abstraction of heat.

51. The tribasic, phosphoric, and arsenic acids, in their combinations with the fixed alcalis, present a slight divergence from this law, and at the same time, give results closely coincident with each other. In the following table, the increments of temperature are exhibited which were observed, when solutions, containing the compounds denoted by the first and second members of the expression, were mixed together. The symbol NaO corresponds, as before, to .234 gr. soda, and the entire weight of the solution was 30 grammes.

	FOUND.	CORRECTED.
$(NaO + \frac{1}{2}P_2O_5) + \frac{1}{4}P_2O_5$	0°.40	0°.46
$(NaO + \frac{3}{4}P_2O_5) + \frac{1}{4}P_2O_5$	0°.30	0°.35

52. In other words, the combination of the common phosphate of soda with half as much acid as it already contains produces an increment of temperature of 0°.46; and its complete conversion into the biphosphate, a farther increase of 0°.35. Similar numbers were obtained with the arsenic acid.

	FOUND.	CORRECTED.
$(NaO + \frac{1}{2}As_2O_5) + \frac{1}{4}As_2O_5$	0°.40	0°.46
$(NaO + \frac{3}{4}As_2O_5) + \frac{1}{4}As_2O_5$	0°.35	0°.40

54. The same acid gave with potash,

	FOUND.	CORRECTED.
$(KO + \frac{1}{2}As_2O_5) + \frac{1}{2}As_2O_5$	0°.80	0°.93

55. From these experiments it follows, that during the conversion of the common alcaline phosphates and arseniates into supersalts, a quantity of heat is evolved, which is about one-seventh part of that produced during the formation of those salts themselves. As, however, the alcaline phosphates and arseniates are not strictly neutral in composition, and their solutions have an alcaline reaction, it is, perhaps, scarcely correct to adduce them as exceptions to Law 2. The pyrophosphoric acid, in similar circumstances, scarcely produces any heat;

resembling, in this and its other thermal properties, the ordinary acids. Denoting the pyrophosphoric acid by Pyr. we have,

	FOUND.	CORRECTED.
$(NaO + \frac{1}{2}\ Pyr_2O_5) + \frac{1}{4}Pyr_2O_5$	0°.15	0°.17
$(NaO + \frac{3}{4}Pyr_2O_5) + \frac{1}{4}Pyr._2O_5$	0°.00	0°.00

55. The formation of the alcaline subphosphates and subarseniates, by the direct union of the common phosphates and arseniates, with an additional equivalent of base, is accompanied with a definite evolution of heat. On adding to solutions of these salts, containing the equivalents of alcali before referred to (NaO, .234 gr. KO, .353 gr.), alcaline solutions having half as much base as was already in the salts themselves, I obtained very uniform results.

	FOUND.	CORRECTED.
$(NaO + \frac{1}{2}P_2O_5) + \frac{1}{2}NaO$	1°.7	1°.97
$(NaO + \frac{1}{2}As_2O_5) + {}^1NaO$	1°.7	1°.97
$(KO + \frac{1}{2}As_2O_5) + \frac{1}{2}KO$	1°.7	1°.97
$(NaO + \frac{1}{2}Pyr_2O_5) + \frac{1}{2}NaO$	0°.1	0°.12

56. That the heat produced was connected with the formation of the subsalt, appears distinctly from the circumstance, that a further addition of alcali was not attended with any increase of temperature. The absence of any heat in the case of the pyrophosphate of soda is easily explained on the same principle, as Graham has shown that no subpyrophosphate of soda exists.

57. The formation of these subsalts exercises a remarkable influence on the quantities of heat developed, when the base is neutralized by successive portions of acid. In ordinary cases, the heat evolved in this way is proportional to the quantity of acid added. Thus, on mixing a solution of pure potash with one-fourth, one-half, &c., an equivalent of nitric acid, the elevations of temperature will be one-half, one-fourth, &c. of what is observed when the alcali is completely neutralized. And the same principle I find to hold good, when successive portions of the phosphoric (tribasic) and arsenic acids are added to solutions of the pure alcalis, till the subsalts are formed; but, after that point, a very different law is followed, as will be seen in the next tables:

	FOUND.	CORRECTED.
I. $NaO + \frac{1}{3}P_2O_5$, ...	4°.65	5°.40
$(NaO + \frac{1}{3}P_2O_5 + \frac{1}{6}P_2O_5)$.	0 .90	1 .04
II. $NaO + \frac{1}{3}As_2O_5$, ...	4°.75	5°.51
$(NaO + \frac{1}{3}As_2O_5) + \frac{1}{6}As_2O_5$, .	.85	.99
III. $KO + \frac{1}{3}As_2O_5$, ...	4.80	5°.57
$(KO + \frac{1}{3}As_2O_5) + \frac{1}{6}As_2O_5$, .	.90	1 .04

58. Had the evolutions of heat corresponded with the additions of acid the second numbers would have been one-half of the first in each set of experiments. Hence, the increments of temperature for equal portions of acid are nearly as 2.5 : 1, before and after the formation of the subsalt. The pyrophosphoric acid, on the contrary, presents no similar irregularity, developing equal increments of heat, for equal additions of acid, till the pyrophosphate of soda $(NaO + \frac{1}{2}Pyr_2O_5)$ is formed.

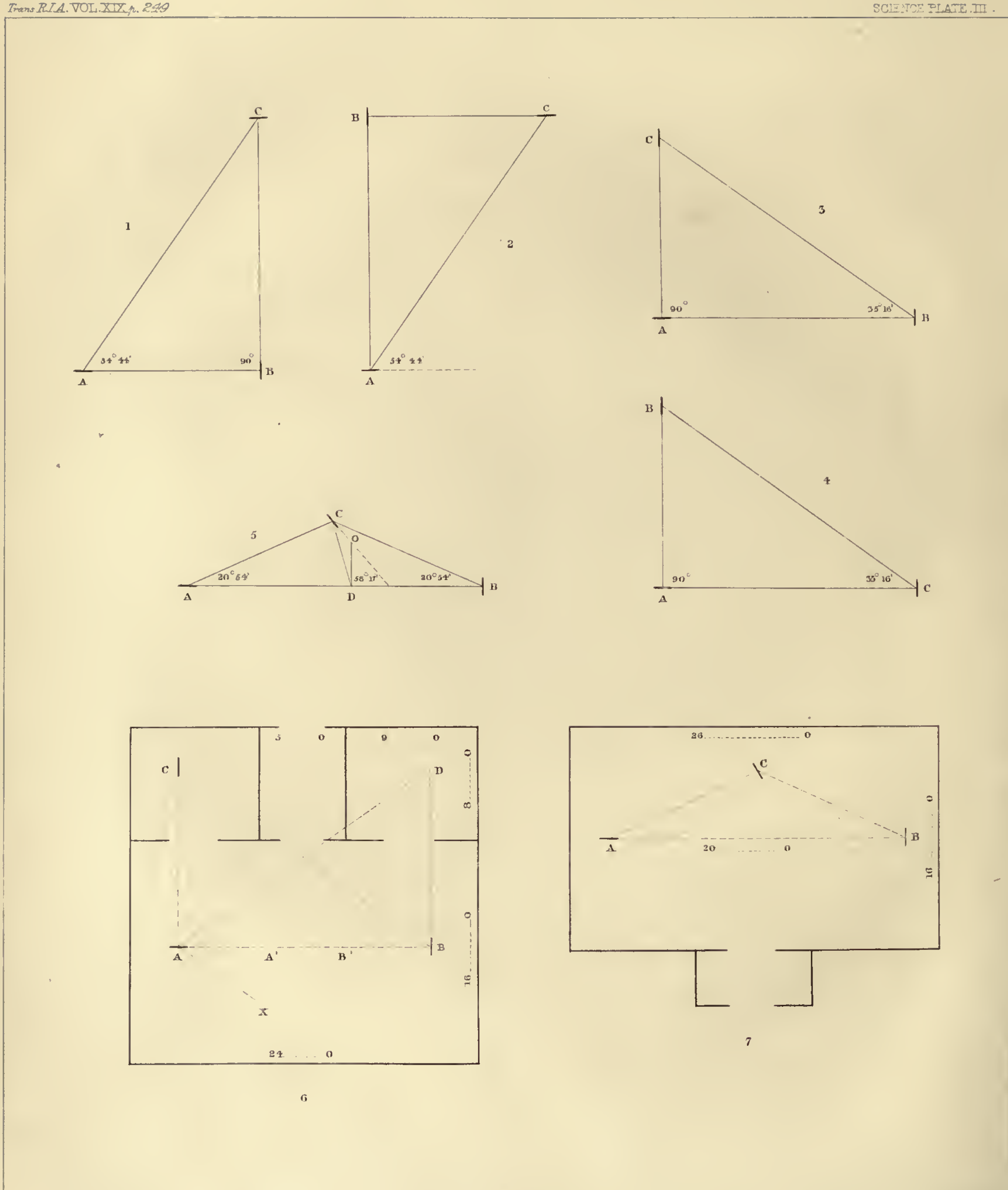
59. It may, perhaps, be prematnre, from such imperfect and limited data, to offer any general observations on the preceding experiments; but I shall, nevertheless, venture to show the accordance of laws second and third, with those general views of the constitution of the salts which have been so ably illustrated by Graham. The conversion of a neutral into an acid salt being in reality the formation of a double salt, is not accompanied by any disengagement of heat; because such combinations as the latter do not evolve heat. No caloric is extricated when the tartrates of potash and soda unite ; and, consequently, none ought to be given off, when the tartrate of soda is combined with the tartrate of water. But, on the other hand, heat is disengaged when the base in the tartrate of water is replaced by soda ; because soda, in its combinations with the acids, evolves much more heat than water. How far the heat evolved in the formation of the different hydrated acids may be the same, is an interesting question not yet determined ; but there can be little doubt that water holds a very low rank among the bases, in reference to its power of generating heat when combining with the acids. On the same principles, and again referring to the observations of Graham, we can understand the cause of the evolution of heat during the

conversion of the neutral phosphates and arseniates into basic salts. In reality, an equivalent of water is here again replaced by an equivalent of alcali, just as occurs in the direct combinations of the acids and alcalis.*

* When the experiments detailed in the foregoing paper were almost completed, I received the 6th No. of *Poggendorff's Annalen*, for 1840, containing the first part of a valuable Memoir, by M. Hess, entitled "Thermo-chemical Researches." The experiments detailed by M. Hess refer principally to the heat developed when sulphuric acid and water combine together—a subject not touched upon in the present paper. He has, however, extended his inquiry to the heat evolved during the combination of sulphuric acid with potash, soda, ammonia, and lime; and also of hydrochloric acid with potash, soda, and ammonia. But the results obtained by M. Hess cannot be immediately compared with those given in this communication, as his experiments were performed with stronger acids, which disengaged heat when diluted with water. The quantity of heat thus extricated, M. Hess has shown to be the same, whether the acid and water be mixed together in presence of a base or alone; and he has likewise furnished accurate data, by means of which the heat derived from this source, in his experiments, may be estimated. Now, assuming with him, as a term of comparison, the number of grammes of water which would be heated through 1° centigrade, by saturating with each alcali 1 gramme of sulphuric acid, or the corresponding equivalent (0.908 gr.) of hydrochloric acid—all taken in the state of very dilute solutions—we deduce from the foregoing tables the numerical results in the first of the following columns; while those in the second are derived from the memoir of M. Hess:

		TABLES.	HESS.
Sulphuric Acid with	Potash,	407	406
	Soda,	413	411
	Ammonia,	352	403
Hydrochloric Acid with	Potash,	364	362
	Soda,	373	368
	Ammonia,	310	318

It is very satisfactory to observe how closely these numbers agree with each other, with the single exception of that which expresses the heat evolved when sulphuric acid and ammonia combine. The cause of this discrepancy I have endeavoured in vain to discover; but it probably depends upon some condition in the experiment of M. Hess, which may have escaped my observation.



J. Kirkwood Sc.

X. *Supplement to a Paper "On the mutual Action of permanent Magnets, considered chiefly in Reference to their best relative Position in an Observatory." By the Rev.* Humphrey Lloyd, *D.D., Fellow of Trinity College, and Professor of Natural Philosophy in the University of Dublin, F.R.S., V.P.R.I.A., Honorary Member of the American Philosophical Society.*

Read April 26, 1841.

In a former paper I have investigated the conditions of equilibrium of the forces exerted upon one another by three magnets, such as those employed in the Dublin Magnetical Observatory, and in the Observatories since established by the British government, in observing the three elements* of the Earth's Magnetic Force. The axes of these magnets being supposed to lie in the same horizontal plane, the forces which they exert upon one another are necessarily directed in that plane; and the conditions of equilibrium of these forces are expressed by *five* equations, the forces exerted upon one of the magnets, in the direction perpendicular to its axis, being destroyed by the reaction of its supports. To fulfil these conditions, there are only *four* arbitrary quantities,—namely, the angles which the lines connecting the centres of the three magnets make with the magnetic meridian, and the azimuth of the axis of one of the magnets. Hence it followed, that complete equilibrium was not attainable, except for determinate values of the relative forces of the magnets. I was, therefore, compelled to select among the conditions of equilibrium, all of which

* These elements are the *declination*, and the *horizontal* and *vertical components* of the force. The magnets employed in observing the first and second of these elements are capable of motion in the *horizontal* plane, the axis of the first being in the magnetic meridian, and that of the second perpendicular to it; the third magnet, being supported on knife-edges, is capable of motion only in a *vertical* plane, and its azimuth is arbitrary.

are not of equal practical value ; and I was thus led to consider some less complete solutions of the problem, in which three, or even two only, of these conditions are satisfied.

But all these solutions are exposed to the objection, that the positions of the magnets which fulfil the conditions are dependent upon their relative forces, and are, therefore, subject to vary along with them :—in other words, that upon any change of these forces, the equilibrium already effected will be destroyed, and a new arrangement of the magnets be required to restore it.

To obviate the inconvenience arising from such a displacement of the magnets employed in the observations, it has been suggested to fulfil the conditions of equilibrium by means of additional magnets, whose positions could be readily altered as the forces varied. To this, however, there are serious objections. In the first place, by thus increasing the number of balancing actions, the chances of error in the positions of the centres of force, as well as the liability to alteration in their intensities, are multiplied ; and, secondly, on account of this liability to change, no absolute measurement could be safely made, without a re-examination of the relative forces of the magnets, and a readjustment of their positions ; so that nothing appears to be gained.

Under all these circumstances, the best course appears to be, to satisfy so many of the conditions of equilibrium, as are capable of being fulfilled *independently of the relative forces of the magnets,* and to apply corrections for the actions which remain unbalanced. In this manner, the changes which the forces of the magnets may undergo, in process of time, will not disturb the equilibrium which has been effected ; and the unbalanced actions, being in definite directions, will admit of being determined by an easy experiment, and allowed for by a simple correction.

In order that any one of the equations of equilibrium* may subsist independently of the ratios of the forces of the magnets, the part which contains one of these ratios, and that which is independent of it, must *separately* vanish, and the five equations are resolved into the following :

* Equations (10, 11, 12, 13, 15), pp. 167, 170.

$$3\cos(2\beta-\zeta)+\cos\zeta=0,\qquad \sin 2\gamma=0; \tag{1}$$

$$3\sin(2\beta-\zeta)+\sin\zeta=0,\qquad 1-3\cos 2\gamma=0; \tag{2}$$

$$3\cos(2\alpha-\zeta)+\cos\zeta=0,\qquad 1+3\cos 2\gamma=0; \tag{3}$$

$$3\sin(2\alpha-\zeta)+\sin\zeta=0,\qquad \sin 2\gamma=0; \tag{4}$$

$$3\cos(2\beta-\zeta)+\cos\zeta=0,\qquad 3\sin(2\alpha-\zeta)+\sin\zeta=0. \tag{5}$$

Now it will be seen, on a little consideration, that of these five pairs of equations, the equations (2) and (3) exclude, each, the other four; so that if we fulfil the condition expressed by (2), or that expressed by (3), in this way, we cannot at the same time satisfy *any* other. On the other hand, each pair of the remaining conditions, expressed by the equations (1, 4, 5), has one equation in common; so that for the fulfilment of these three conditions, *three* equations only are to be satisfied; and these three equations are not only not inconsistent, but even leave one of the angles still undetermined.

These equations are

$$\sin 2\gamma=0, \tag{6}$$

$$3\cos(2\beta-\zeta)+\cos\zeta=0, \tag{7}$$

$$3\sin(2\alpha-\zeta)+\sin\zeta=0. \tag{8}$$

The first of them determines the angle γ; and as the other two contain three arbitrary angles, they may be fulfilled in an infinite variety of ways. Accordingly we must have

$$\gamma=0,\quad \text{or } \gamma=90^\circ; \tag{9}$$

that is, the line connecting the magnets A and B must be *parallel* or *perpendicular to the magnetic meridian.* And the angles, α, β, ζ, which determine the place and azimuth of the third magnet, are connected by the relations,

$$\frac{\frac{1}{3}+\cos 2\beta}{\sin 2\beta}=-\tan\zeta=\frac{\sin 2\alpha}{\frac{1}{3}-\cos 2\alpha}; \tag{10}$$

so that when one of these angles is assumed or given, the other two are determined.

The natural course is to assume the *azimuth* of the magnet C, and thence determine the place of its centre. Let us suppose, then, that the plane of the magnet C is *parallel to the magnetic meridian*, or that

$$\zeta = 0.$$

The equations (7, 8) then give,

$$\cos 2\beta = -\tfrac{1}{3}, \quad \sin 2\alpha = 0;$$

and these two equations, together with (6), solve the problem. As we cannot have $\gamma = 0$, $\alpha = 0$, simultaneously, there are two solutions, namely:

$$\left.\begin{array}{ll} \gamma = 0, & \alpha = 90^\circ, \\ \gamma = 90^\circ, & \alpha = 0, \end{array}\right\} \beta = 54^\circ\, 44'.$$

The corresponding arrangements of the magnets are represented in Figs. 1 and 2.

Again, if the plane in which the magnet C is constrained to move be *perpendicular to the magnetic meridian*, or

$$\zeta = 90^\circ,$$

the equations (7, 8) are then reduced to

$$\sin 2\beta = 0, \quad \cos 2\alpha = \tfrac{1}{3};$$

which, in conjunction with (6), furnish the two solutions:

$$\left.\begin{array}{ll} \gamma = 0, & \beta = 90^\circ, \\ \gamma = 90^\circ, & \beta = 0, \end{array}\right\} \alpha = 35^\circ\, 16'.$$

These arrangements are represented in Figs. 3 and 4.

In estimating the comparative merits of these four arrangements, we should observe that the magnet C is usually much less massive, and therefore less powerful than either of the other two; and, accordingly, that the arrangements represented in Figs. 1 and 4, in which the distance, AB, of the stronger magnets is the shortest side of the triangle ABC, are, on that account, inferior to those represented in Figs. 2 and 3. Of the latter, the arrangement (Fig. 3) is to be preferred, where our object is to diminish as much as possible the residual action upon the declination magnet, A; and, on the other hand, the arrangement (Fig. 2) should be chosen, if we prefer to diminish the action upon the magnet B.

There is still another particular disposition which deserves to be considered: that, namely, in which the magnet C is *equally distant* from the other two. This condition is expressed by the relation,

$$\alpha + \beta = 180°;$$

and eliminating, by means of it, the angle β in (10), we have

$$\frac{\cos 2\alpha + \frac{1}{3}}{\sin 2\alpha} = \frac{\sin 2\alpha}{\cos 2\alpha - \frac{1}{3}};$$

whence $\cos^2 2\alpha - \sin^2 2\alpha = \frac{1}{9}$, $\sin 2\alpha = \pm \frac{2}{3}$, and

$$\alpha = \pm 20° 54'.$$

Again, substituting this value in (10), we have

$$\tan \zeta = \frac{\pm 2}{\sqrt{5} - 1} = \pm 1.6180, \qquad \zeta = 58° 17', \text{ or } = 180° - 58° 17'.$$

Accordingly, the arrangement of the magnets is that represented in Fig. 5, or the reverse arrangement, in which the magnet C is in the corresponding position on the opposite side of the line AB.

Let us now consider, briefly, the corrections required for the residual actions, and the manner in which they are to be experimentally determined.

In virtue of the equations (6) and (7), the action exerted by the magnets B and C upon A, in the magnetic meridian, is null; the disturbing action is, therefore, *perpendicular* to the meridian, and operates only as a *deflecting* force. The amount of the deflection produced by this resultant force is easily determined; for we have only to reverse the magnets B and C simultaneously, and it is obvious that the difference of the readings of the magnet A, in these two positions of the deflecting magnets, is double the deflection sought. In order to eliminate the actual changes of declination which may occur in the interval of the two parts of the observation, simultaneous observations should be made with an auxiliary apparatus in another apartment; or, should such an apparatus be not at hand, the effect of the changes may be got rid of by making a series of readings of the magnet A, with the deflecting magnets alternately in the two positions. The amount of the deflection, thus determined, is to be applied as a correction in measurements of the *absolute declination:* being a constant quantity, or nearly so, its effect upon the *declination changes* may be disregarded. Lastly, there being no disturbing force upon the magnet A, in the magnetic meridian itself, the *absolute horizontal intensity*, determined by experiments of vibration and deflection, according to the method of Gauss, will need no correction.*

On the other hand, the disturbing force exerted upon the magnet B, by the other two, is directed in the magnetic meridian itself, and therefore *conspires with*, or *opposes*, the force of the earth. The correction required for its action is determined with the same facility as in the former case. We have only to reverse the magnets A and C simultaneously, and to note the change of position of the magnet B thereby produced. Half the change, converted into parts of the whole force by multiplying it by a coefficient already known, is the ratio, $\frac{f}{F}$, of the disturbing force to the total force; and, in order to correct for this force, we have only to multiply the observed results by the coefficient $1 \mp \frac{f}{F}$, using the upper sign when the disturbing action conspires with that of the earth, and the lower when it is opposed to it.

Finally, with respect to the magnet C, the disturbing action, being perpendicular to the plane in which the magnet is constrained to move, is destroyed by the reaction of its supports, and no correction is needed.

* The *resultant* of the force of the earth, and of the disturbing action, will of course differ, theoretically, from the former; but, in general, by an inappreciable amount. If X denote the earth's horizontal force, and δ the deflection produced by the disturbing action, the resultant force will be X secant δ. Now, supposing δ to be *two minutes* (which is greater than any amount it can have with magnets of the size of those employed in the Dublin Observatory, and at the distances recommended below) the resultant force will exceed X by the quantity .0000002X.

It may be useful to suggest, in a few words, the form of building adapted to these arrangements.

For the arrangement represented in Fig. 3, the ground-plan of the building may be a square, whose sides (24 feet in length) are parallel and perpendicular to the magnetic meridian, (Fig. 6). This area may be conveniently divided into four parts, viz.: a principal room, 24 feet in length and 16 feet in width; two subordinate rooms, and a vestibule. The principal room should contain the magnets A and B, which may be placed at an interval of 18 feet,* the joining line being the axis of the room. Two pedestals, A′ and B′, (at an interval of $4\frac{1}{2}$ feet), will serve to support the reading telescopes; and the observer's chair may be placed between them. The magnet C should be placed in one of the small rooms, its distance from the magnet A being $\text{AC} = \text{AB} \times \text{tang } 35° 16' = 18 \times 0.707 = 12.73$ feet. In order to diminish, as far as possible, the deflecting force exerted by the magnets B and C upon A, these magnets should have their poles *similarly* placed (i. e. the same pole in each turned to the east); for, in this case, the resulting action is the *difference* of the forces exerted by the separate magnets.

It will be convenient to fix another pedestal, D, for the support of an inclination instrument, in the second of the small rooms, and at the point corresponding to C in the first;—the line BD being perpendicular to the magnetic meridian, and the distance BD = AC. It is manifest that, in this position, the action of the magnets B and C upon a magnetic particle at D will be perpendicular to the magnetic meridian; and will, therefore, have no effect upon the position of the inclination needle, being destroyed by the reaction of its supports. And, in order that the action of the magnet A may be in the same direction, it is only necessary to turn it round, so that its axis may lie in the line AX, which makes with the magnetic meridian an angle BAX = BAD. For $\tan \text{D} = \sqrt{2}$; and $\tan \text{DAX} = \dfrac{2 \tan \text{D}}{\tan^2 \text{D} - 1} = 2\sqrt{2}$; so that $\tan \text{D} = \frac{1}{2} \tan \text{DAX}$, and DB is the

* At this distance, the deflection produced by the magnet B upon A, (the deflecting magnet being of the size and power of those employed in the Dublin Magnetical Observatory), is only about $1\frac{1}{2}$ minutes; and the greater part of this small disturbance will be annulled by the opposing action of the magnet C.

direction of the force exerted by the magnet A (in that position) upon the point D. This temporary adjustment of the magnet A may be at once effected by means of a line drawn on the supporting pedestal; and it is obvious that it may be accomplished without removing the magnet from its stirrup, or interfering in any way with its permanent adjustments.

The building required to receive the magnets, in the arrangement represented in Fig. 5, may be still simpler; consisting only of a single room, 26 feet in length, and 16 feet in width, and having a portico with a second door, to prevent draughts of air, (Fig. 7).

To find a suitable place for the inclination instrument, we have only to determine the point on the line AB, at which the action of the magnet C is perpendicular to AB. Then, the action of the magnet B being perpendicular to AB at every point of this line, the forces exerted by B and C will be perpendicular to the meridian, and will therefore be destroyed by the reaction of the supports; and, in order that the same thing should hold also for the magnet A, we have only to turn that magnet, temporarily, into a position perpendicular to the meridian.

Let D (Fig. 5) be the point sought, and DO a line perpendicular to AB; then the condition requires that $\tan \text{CDO} = \frac{1}{2} \tan \text{OCD}$; or, denoting the angle CDA by x, $\cot x = \frac{1}{2} \tan (x - 58° \, 17')$. Whence, developing and substituting the value of $\tan (58° \, 17')$, we have the following quadratic for the determination of $\tan x$,

$$\tan^2 x - 4.854 \tan x - 2 = 0.$$

Accordingly, $\tan x = 5.236$, or $= -0.382$; and $x = 79° \, 11'$, or $= -20° \, 54'$. Of these solutions the former is that adapted to the present purpose; the latter giving the point A itself.

The pedestal erected at the point D will likewise serve to support the reading telescope of the magnet B, which may be inserted in a groove cut in the top, so as not to interfere with the other instrument. The supporting pedestal of the telescope of the magnet A should be on the line DA, its centre being four or five feet from the point D, so as to admit the observer's chair between the two pedestals.

XI. *Supplementary Researches on the Direction and Mode of Propagation of the electric Force, and on the Source of electrical Development.* By George J. Knox, Esq., A. M., M. R. I. A.

Read May 25th, 1840.

HAVING in my former paper* described some experiments which proved that water and phosphorus convey a current of electricity through their substances, while metals convey the current along their surface, and feeling anxious to discover some general law regarding the direction of propagation in liquid and solid bodies, I have continued the investigation to fluids; not only those which convey the feeble current of the voltaic pile, but to others which require the high intensity of the electrical machine; and although the experiments be few, yet I think that they may be considered to be sufficient to establish the law regarding fluids, that they convey through their substance in all directions alike; an opinion which one would be inclined to adopt previous to experiment, from considering the difference between the nature of liquid and solid bodies, the one having their particles chained down by powerful affinities, which no ordinary electrical force can overcome, while the other, from the perfect mobility of their particles, allow the electric state to be induced upon them with equal facility in one direction as well as another.

That there exists no regular law with regard to solids, appears from the Researches of Dr. Faraday (XI. and XIV. Series), in which he shows, that the lines of induction do not pass *through* metallic bodies (1221), (affording a corroborative proof to mine that they do not convey through their substance), while several solid bodies, such as shell-lac, sulphur, &c. (1228, 1308, 1309, 1310), allow the inductive force to pass through them with greater facility even than air.

* Tran. R. I. A., vol. xix. p. 147; Phil. Mag. vol. xvi. p. 185.

EXPERIMENTS.

The bent glass tube which I employed in my former experiments having been filled alternately with muriatic acid, hydriodic acid, sulphate of copper, and muriate of ammonia, and the circuit being completed by a current from a sustaining battery of one pair of elements, the same law was found to subsist as when water and phosphorus were employed, i. e. that the current passed through their substance and not along their surface. The same likewise took place when the tube was filled with fused chloride of tin, which conducts by electrolysis, and fused periodide of mercury, which conducts by conduction.

To determine whether this law with regard to liquids which convey a galvanic current subsists when non-conducting fluids are employed, I filled the tube alternately with alcohol, naphtha, oil, fused lard, bees' wax, and resin, and having connected one of the insulated wires with the ground, I connected the other with an insulated brass ball, fixed at the distance of four-tenths of an inch from the prime conductor, of a nine inch electrical machine.

ALCOHOL OR NAPHTHA.

When the platinum wires were immersed in the legs of the bent tube until their extremities were placed at the distance of five inches, ten sparks passed in one revolution of the plate; when at two feet distance, eight sparks; when at four feet distance, six sparks.

OIL.

At the distance of five inches, seven sparks passed in one revolution of the plate; at the distance of two feet, four sparks passed; and at the distance of four feet, two sparks passed in one revolution.

FUSED LARD.

At the distance of five inches, two sparks passed in one revolution of the plate; at the distance of two feet, one spark in one revolution; at the distance of four feet, one spark in three revolutions.

BEES' WAX.

At the distance of five inches, one spark passed in one revolution of the plate; at the distance of two feet, one spark in one revolution and a half; at the distance of four feet, one spark in two revolutions.

RESIN.

At the distance of five inches, one spark passed in one revolution of the plate; at the distance of two feet, one spark in two revolutions; at the distance of four feet, one spark in two and three-fourth revolutions.

These latter substances begin to conduct when in the viscid state, and the conducting power increases up to the boiling point.

SOURCE OF ELECTRICAL DEVELOPMENT.

Before reconsidering the source of electrical development, I shall briefly mention the arguments which may be brought forward against the emission, and in favour of the vibratory theory, the former supposing a transference of electricity from particle to particle, the latter assuming that the atoms of matter are encircled with ethereal atmospheres, the atoms of which can oscillate within certain distances. The arguments in favour of this latter theory, independent of such as the mathematician may bring forward, rest upon the hypothesis proposed by Sir H. Davy,* "which, after a lapse of twenty years, continued, as it was in the beginning, to be the guide and foundation of all his researches;" a theory now almost universally received as established—that chemical affinity is an electrical phenomenon, and that the entire subject of chemistry is an illustration of that primary law of electricity, the attraction of oppositely electrical bodies. If the electric forces which cause the attraction of bodies be definite, as they are, being their atomic numbers, how can this be consistent with a theory which supposes that the electricity leaves the particles, allowing them at one moment to contain more electricity than at another, and, consequently, a higher affinity, and a different atomic number?

When two atoms are brought into contact, their electrical ethers, being disturbed, cause a disturbance to take place in the electrical ethers of adjacent atoms, which disturbance should increase until it arrives at a maximum, when combination takes place. The same may be said of the compound atoms or molecules, of the compound molecules or particles, and of the compound particles or bodies *en masse;* and that such a development of electricity by contact of the *latter* does take place, the original experiments of Volta, together with

* Bakerian Lecture, 1807–1826.

the late experiments of Fechner and Peclet, have fully established. Fechner* has proved (having shown that the same experiment was incorrectly tried by Delarive), that when potassium, or sodium, are brought into contact with platinum, electrical development takes place without chemical action. Peclet† has proved that electrical excitation is caused by the contact of platinum and gold, *where chemical action could not take place.* In support of the opposite opinion is the experiment of Delarive,‡ who found, that when chlorine gas is passed through an insulated copper tube, the condenser exhibits electrical development, which, he remarks, decreases when chlorine, unmixed with atmospheric air, is employed, and also when the chemical action between the chlorine and copper is violent; circumstances which admit of a simple explanation by the contact theory, according to which the air receives an electrical charge from the chlorine while in contact with the copper, which charge so much of the gas as *combines* with the copper loses. The same explanation may be given to the experiments of Peclet,§ who has satisfactorily shown that the presence of moisture is necessary in order that the oxidation of the amalgam on the rubber of an electrical machine should develope electricity, the aqueous vapour in this case receiving the charge. *Experiments*, then, having proved, that contact and not chemical action causes the development of electricity, the question arises, how are we to explain the phenomena? When two atoms unite, it is difficult to avoid the conclusion, that the compound atom (molecule) must have oppositely electrical surfaces. Two such polarized molecules approaching cause a disturbance to take place in the electrical ether, which disturbance is propagated by induction to a distance; but when the molecules approach sufficiently near to combine, the two oppositely electrical surfaces of one molecule coming in contact with the two oppositely electrified surfaces of the other, no development of electricity can take place, the electrical states becoming completely disguised; and such a supposition is borne out by every fact in crystallography, which shows that the molecules have poles. The particles being compound molecules should have poles likewise; and when they unite, or chemical combination takes place, there should be no development of electricity; and, consequently, when oxygen unites with zinc (as in

* Phil. Mag. vol. xiii. 1838.

† Annales de Chimie, tom. lxxi. p. 80.

‡ Bib. Univer. N. S. tom. iii.

§ Annales de Chimie, tom. lxxi. p. 83.

the galvanic battery) no development of electricity should take place from their union; but the hydrogen, whose positive pole had been previously united with the negative pole of the oxygen, should induce negative electricity upon the oxide, while the negative pole should induce positive electricity upon the next particle, and so on to the platinum plate.

The greater the number of particles of hydrogen inducing electricity upon the platinum plate, the greater, of course, the quantity of electricity induced upon that plate; the number of particles of hydrogen being the measure of the quantity, whether it was oxygen, chlorine, iodine, or bromine, with which the hydrogen may have been previously in combination; and that such is the case is proved by experiment. That alternate recombinations and decompositions take place has been shown by Grothhus and Faraday.

How beautiful is the analogy which subsists between statical and voltaic electricity when the contact theory is adopted! By friction (lateral contact) between silk and glass opposite electrical states are induced upon each. By the contact of zinc with a dry acid, or alkali, opposite states are induced upon each. When the plate of the electrical machine is put in motion, the prime conductor receives a charge whose intensity is directly as the non-conducting or insulating nature of the glass, and as the distance between the collecting forks and the rubber when the axis is made of glass. When the zinc is placed in contact with the acid, or alkali in *solution*, the charge is allowed to pass from the zinc to the platinum, being in this case a charge by induction, as in the former case it was one by convection; and the intensity varies as the insulating state of the solution, and as the distance between the platinum and zinc, as is proved by the experiments of Delarive,* which show that the water battery charges to a higher intensity than the acid battery, although it takes a longer time than the latter to charge to a given amount. Again, when a small electrical machine is rotated rapidly, while a larger one is rotated slowly, the former will charge to a *given intensity* in a shorter time than the latter, although it never can rise to an *equal intensity*. So in the acid and water batteries, the former, owing to the rapidity of alternations of induction and equilibrium, charges to a *given intensity* in a shorter time than the latter, yet still it never can rise to an *equal intensity*. Similarly may be explained why, when two metals in a solution form a closed

* Bib. Univer. tom. iv. p. 360.

circuit, whatever increases the chemical action upon one more than upon the other, increasing the rapidity of alternate states of induction, produces a charge in a shorter time; and this takes place not only when two different metals are employed, but also, when plates of the same metal being used, a difference of polish or a difference of heat applied alters the chemical action upon one plate more than upon the other. A further analogy is faintly borne out by the following experiments, which may lead to an explanation of some curious facts regarding the alternate increase and decrease of intensity in the voltaic pile, dependent upon the number of alternations, as observed by Delarive* and others. Having connected, by means of insulated copper wires, the insulated conductors of an electrical machine, with two insulated brass balls, the spark that passed between the two balls measured one-fourth of an inch. When the insulated negative conductor of this machine was connected with the insulated prime conductor of another similar one, and its insulated negative conductor with one of the brass balls, and the two machines rotated simultaneously, the length of the discharging spark was increased to one-half; with three electrical machines similarly arranged, the length of the spark which passed was one-third; with four, it returned to one-half; beyond this number no regularity in the length of the discharging sparks was observable. The quantity in the electrical machine increases with the number of collecting forks, when the rubbers and forks are disposed in such a manner, that the latter can receive the greatest quantity of electricity from the excited glass; so in the voltaic pile, the quantity is as the number of particles of hydrogen set free against the surface of the platinum.

The effect which a current of electricity, considered to be a row of particles whose oppositely electrified surfaces are ranged in the same direction, undergoing alternate states of induction and equilibrium, produces upon contiguous particles, should be to induce in them oppositely electrified surfaces, which, in undergoing alternate states of induction and equilibrium, should obviously give rise to a current of electricity in an opposite direction,—and this is agreeable to fact.

To afford an explanation of magnetism, considered as an electrical phenomenon, no theory as yet proposed is adequate. That of Ampère (although exceedingly beautiful) is yet all but physically impossible, for how can we suppose that when the electrical current which magnetizes a steel bar ceases, the electricity in the

* Bib. Univer. Tom. iv. p. 360.

bar continues to revolve round the particles of the steel? Does not the marked difference between iron and other metals, and between steel and soft iron in the same metal, show that magnetism (if electrical) must be a case of statical electricity? What arrangement of electrified bodies may produce such a state of statical power may possibly be within the reach of experiment; but to determine the condition of the electrical ether in a bar of steel, is a question which, as it regards the mutual actions of systems of attracting and repelling points, being far beyond the reach of experiment, requires for its solution a higher, more elegant, and more comprehensive instrument of research, mathematical analysis.

XII.—*On Fluctuating Functions.* *By* Sir William Rowan Hamilton, LL. D., P. R. I. A., F. R. A. S., *Fellow of the American Society of Arts and Sciences, and of the Royal Northern Society of Antiquaries at Copenhagen; Honorary or Corresponding Member of the Royal Societies of Edinburgh and Dublin, of the Academies of St. Petersburgh, Berlin, and Turin, and of other Scientific Societies at home and abroad; Andrews' Professor of Astronomy in the University of Dublin, and Royal Astronomer of Ireland.*

Read June 22nd, 1840.

The paper now submitted to the Royal Irish Academy is designed chiefly to invite attention to some consequences of a very fertile principle, of which indications may be found in Fourier's Theory of Heat, but which appears to have hitherto attracted little notice, and in particular seems to have been overlooked by Poisson. This principle, which may be called the *Principle of Fluctuation*, asserts (when put under its simplest form) the evanescence of the integral, taken between any finite limits, of the product formed by multiplying together any two finite functions, of which one, like the sine or cosine of an infinite multiple of an arc, changes sign infinitely often within a finite extent of the variable on which it depends, and has for its mean value zero; from which it follows, that if the other function, instead of being always finite, becomes infinite for some particular values of its variable, the integral of the product is to be found by attending only to the immediate neighbourhood of those particular values. The writer is of opinion that it is only requisite to develope the foregoing principle, in order to give a new clearness, and even a new extension, to the existing theory of the transformations of arbitrary functions through functions of determined forms. Such is, at least, the object aimed at in the following pages; to which will be found appended a few general observations on this interesting part of our knowledge.

[1.] The theorem, discovered by FOURIER, that between any finite limits, a and b, of any real variable x, any arbitrary but finite and determinate function of that variable, of which the value varies gradually, may be represented thus,

$$fx = \frac{1}{\pi}\int_a^b d\alpha \int_0^\infty d\beta \cos(\beta\alpha - \beta x) f\alpha, \tag{a}$$

with many other analogous theorems, is included in the following form :

$$fx = \int_a^b d\alpha \int_0^\infty d\beta\, \phi(x, \alpha, \beta) f\alpha; \tag{b}$$

the function ϕ being, in each case, suitably chosen. We propose to consider some of the conditions under which a transformation of the kind (b) is valid.

[2.] If we make, for abridgment,

$$\psi(x, \alpha, \beta) = \int_0^\beta d\beta\, \phi(x, \alpha, \beta), \tag{c}$$

the equation (b) may be thus written :

$$fx = \int_a^b d\alpha\, \psi(x, \alpha, \infty) f\alpha. \tag{d}$$

This equation, if true, will hold good, after the change of $f\alpha$, in the second member, to $f\alpha + \mathrm{F}\alpha$; provided that, for the particular value $\alpha = x$, the additional function $\mathrm{F}\alpha$ vanishes ; being also, for other values of α, between the limits a and b, determined and finite, and gradually varying in value. Let then this function F vanish, from $\alpha = a$ to $\alpha = \lambda$, and from $\alpha = \mu$ to $\alpha = b$; λ and μ being included, either between a and x, or between x and b ; so that x is not included between λ and μ, though it is included between a and b. We shall have, under these conditions,

$$0 = \int_\lambda^\mu d\alpha\, \psi(x, \alpha, \infty)\, \mathrm{F}\alpha; \tag{e}$$

the function F, and the limits λ and μ, being arbitrary, except so far as has been above defined. Consequently, unless the function of α, denoted here by $\psi(x, \alpha, \infty)$, be itself $= 0$, it must change sign at least once between the limits $\alpha = \lambda$, $\alpha = \mu$, however close those limits may be ; and therefore must change sign indefinitely often, between the limits a and x, or x and b. A function which thus changes sign indefinitely often, within a finite range of a variable on which it depends, may be called a *fluctuating function.* We shall consider now a class of cases, in which such a function may present itself.

[3.] Let N_α be a real function of α, continuous or discontinuous in value, but always comprised between some finite limits, so as never to be numerically greater than $\pm\mathrm{c}$, in which c is a finite constant; let

$$\mathrm{M}_\alpha = \int_0^\alpha d\alpha\, \mathrm{N}_\alpha ; \tag{f}$$

and let the equation

$$\mathrm{M}_\alpha = \mathrm{a}, \tag{g}$$

in which a is some finite constant, have infinitely many real roots, extending from $-\infty$ to $+\infty$, and such that the interval $\alpha_{n+1} - \alpha_n$, between any one root α_n and the next succeeding α_{n+1}, is never greater than some finite constant, b. Then,

$$0 = \mathrm{M}_{\alpha_{n+1}} - \mathrm{M}_{\alpha_n} = \int_{\alpha_n}^{\alpha_{n+1}} d\alpha\, \mathrm{N}_\alpha ; \tag{h}$$

and consequently the function N_α must change sign at least once between the limits $\alpha = \alpha_n$ and $\alpha = \alpha_{n+1}$; and therefore at least m times between the limits $\alpha = \alpha_n$ and $\alpha = \alpha_{n+m}$, this latter limit being supposed, according to the analogy of this notation, to be the m^{th} root of the equation (g), after the root α_n. Hence the function $\mathrm{N}_{\beta\alpha}$, formed from N_α by multiplying α by β, changes sign at least m times between the limits $\alpha = \lambda$, $\alpha = \mu$, if *

$$\lambda \ngtr \beta^{-1}\alpha_n, \ \mu \nless \beta^{-1}\alpha_{n+m} ;$$

the interval $\mu - \lambda$ between these limits being less than $\beta^{-1}(m+2)\,\mathrm{b}$, if

$$\lambda > \beta^{-1}\alpha_{n-1}, \ \mu < \beta^{-1}\alpha_{n+m+1} ;$$

so that, under these conditions, (β being > 0,) we have

$$m > -2 + \beta\,\mathrm{b}^{-1}(\mu - \lambda).$$

However small, therefore, the interval $\mu - \lambda$ may be, provided that it be greater

* These notations $\ngtr$ and $\nless$ are designed to signify the contradictories of $>$ and $<$; so that "$a \ngtr b$" is equivalent to "a not $> b$," and "$a \nless b$" is equivalent to "a not $< b$."

than 0, the number of changes of sign of the function $\mathrm{N}_{\beta a}$, within this range of the variable a, will increase indefinitely with β. Passing then to the extreme or limiting supposition, $\beta = \infty$, we may say that the function $\mathrm{N}_{\infty a}$ *changes sign infinitely often* within a finite range of the variable a on which it depends; and consequently that it is, in the sense of the last article, a FLUCTUATING FUNCTION. We shall next consider the integral of the product formed by multiplying together two functions of a, of which one is $\mathrm{N}_{\infty a}$, and the other is arbitrary, but finite, and shall see that this integral vanishes.

[4.] It has been seen that the function N_a changes sign at least once between the limits $a = a_n$, $a = a_{n+1}$. Let it then change sign k times between those limits, and let the k corresponding values of a be denoted by $a_{n,1}, a_{n,2}, \ldots a_{n,k}$. Since the function N_a may be discontinuous in value, it will not necessarily vanish for these k values of a; but at least it will have one constant sign, being throughout not <0, or else throughout not >0, in the interval from $a = a_n$ to $a = a_{n,1}$; it will be, on the contrary, throughout not >0, or throughout not <0, from $a_{n,1}$ to $a_{n,2}$; again, not <0, or not >0, from $a_{n,2}$ to $a_{n,3}$; and so on. Let then N_a be never <0 throughout the whole of the interval from $a_{n,i}$ to $a_{n,i+1}$; and let it be >0 for at least some finite part of that interval; i being some integer number between the limits 0 and k, or even one of those limits themselves, provided that the symbols $a_{n,o}$, $a_{n,k+1}$ are understood to denote the same quantities as a_n, a_{n+1}. Let F_a be a finite function of a, which receives no sudden change of value, at least for that extent of the variable a, for which this function is to be employed; and let us consider the integral

$$\int_{a_{n,i}}^{a_{n,i+1}} da\, \mathrm{N}_a\, \mathrm{F}_a. \tag{i}$$

Let F' be the algebraically least, and F'' the algebraically greatest value of the function F_a, between the limits of integration; so that, for every value of a between these limits, we shall have

$$\mathrm{F}_a - \mathrm{F}' \not< 0,\ \mathrm{F}'' - \mathrm{F}_a \not< 0;$$

these values F' and F'', of the function F_a, corresponding to some values $a'_{n,i}$ and $a''_{n,i}$ of the variable a, which are not outside the limits $a_{n,i}$ and $a_{n,i+1}$. Then, since, between these latter limits, we have also

$$\mathrm{N}_a \nless 0,$$

we shall have

$$\left.\begin{aligned} &\int_{a_{n,i}}^{a_{n,i+1}} da\, \mathrm{N}_a (\mathrm{F}_a - \mathrm{F}^{\backprime}) \nless 0; \\ &\int_{a_{n,i}}^{a_{n,i+1}} da\, \mathrm{N}_a (\mathrm{F}^{\backprime\backprime} - \mathrm{F}_a) \nless 0; \end{aligned}\right\} \quad \text{(k)}$$

the integral (i) will therefore be not $< s_{n,i}\, \mathrm{F}^{\backprime}$, and not $> s_{n,i}\, \mathrm{F}^{\backprime\backprime}$, if we put, for abridgment,

$$s_{n,i} = \int_{a_{n,i}}^{a_{n,i+1}} da\, \mathrm{N}_a; \quad \text{(l)}$$

and consequently this integral (i) may be represented by $s_{n,i}\, \mathrm{F}'$, in which

$$\mathrm{F}' \nless \mathrm{F}^{\backprime},\ \mathrm{F}' \ngtr \mathrm{F}^{\backprime\backprime},$$

because, with the suppositions already made, $s_{n,i} > 0$. We may even write

$$\mathrm{F}' > \mathrm{F}^{\backprime},\ \mathrm{F}' < \mathrm{F}^{\backprime\backprime},$$

unless it happen that the function F_a has a constant value through the whole extent of the integration; or else that it is equal to one of its extreme values, $\mathrm{F}^{\backprime}$ or $\mathrm{F}^{\backprime\backprime}$, throughout a finite part of that extent, while, for the remaining part of the same extent, that is, for all other values of a between the same limits, the factor N_a vanishes. In all these cases, F' may be considered as a value of the function F_a, corresponding to a value $a'_{n,i}$ of the variable a which is included between the limits of integration; so that we may express the integral (i) as follows:

$$\int_{a_{n,i}}^{a_{n,i+1}} da\, \mathrm{N}_a\, \mathrm{F}_a = s_{n,i}\, \mathrm{F}_{a'_{n,i}}; \quad \text{(m)}$$

in which

$$a'_{n,i} > a_{n,i},\ < a_{n,i+1}. \quad \text{(n)}$$

In like manner, the expression (m), with the inequalities (n), may be proved to hold good, if N_a be never > 0, and sometimes < 0, within the extent of the integration, the integral $s_{n,i}$ being in this case < 0; we have, therefore, rigorously,

$$\int_{a_n}^{a_{n+1}} da\, \mathrm{N}_a\, \mathrm{F}_a = s_{n,o}\, \mathrm{F}_{a'_{n,o}} + s_{n,1}\, \mathrm{F}_{a'_{n,1}} + \dots + s_{n,k}\, \mathrm{F}_{a'_{n,k}}. \quad \text{(o)}$$

But also, we have, by (h)

$$0 = s_{n,0} + s_{n,1} + \dots + s_{n,k}; \tag{p}$$

the integral in (o) may therefore be thus expressed, without any loss of rigour:

$$\int_{a_n}^{a_{n+1}} da\, \mathrm{N}_a \mathrm{F}_a = s_{n,0} \triangle_{n,0} + \dots + s_{n,k} \triangle_{n,k}, \tag{q}$$

in which

$$\triangle_{n,i} = \mathrm{F}_{a'_{n,i}} - \mathrm{F}_{a_n}; \tag{r}$$

so that $\triangle_{n,i}$ is a finite difference of the function F_a, corresponding to the finite difference $a'_{n,i} - a_n$ of the variable a, which latter difference is less than $a_{n+1} - a_n$, and therefore less than the finite constant b of the last article. The theorem (q) conducts immediately to the following,

$$\int_{\beta^{-1} a_n}^{\beta^{-1} a_{n+1}} da\, \mathrm{N}_{\beta a} \mathrm{F}_a = \beta^{-1} (s_{n,0}\, \delta_{n,0} + \dots + s_{n,k}\, \delta_{n,k}), \tag{s}$$

in which

$$\delta_{n,i} = \mathrm{F}_{\beta^{-1} a'_{n,i}} - \mathrm{F}_{\beta^{-1} a_n}; \tag{t}$$

so that, if β be large, $\delta_{n,i}$ is small, being the difference of the function F_a corresponding to a difference of the variable a, which latter difference is less than β^{-1}b. Let $\pm \delta_n$ be the greatest of the $k+1$ differences $\delta_{n,0}, \dots \delta_{n,k}$, or let it be equal to one of those differences and not exceeded by any other, abstraction being made of sign; then, since the $k+1$ factors $s_{n,0}, \dots s_{n,k}$ are alternately positive and negative, or negative and positive, the numerical value of the integral (s) cannot exceed that of the expression

$$\pm \beta^{-1} \left(s_{n,0} - s_{n,1} + s_{n,2} - \dots + (-1)^k s_{n,k} \right) \delta_n. \tag{u}$$

But, by the definition (l) of $s_{n,i}$, and by the limits $\pm$c of value of the finite function N_a, we have

$$\pm s_{n,i} \gg (a_{n,i+1} - a_{n,i})\, \mathrm{c}; \tag{v}$$

therefore

$$\pm \left(s_{n,0} - s_{n,1} + \dots + (-1)^k s_{n,k} \right) \gg (a_{n+1} - a_n)\, \mathrm{c}; \tag{w}$$

and the following rigorous expression for the integral (s) results:

$$\int_{\beta^{-1}\alpha_n}^{\beta^{-1}\alpha_{n+1}} d\alpha \, \mathrm{N}_{\beta\alpha} \, \mathrm{F}_\alpha = \theta_n \, \beta^{-1} \, (\alpha_{n+1} - \alpha_n) \, \mathrm{c} \, \delta_n \, ; \tag{x}$$

θ_n being a factor which cannot exceed the limits ± 1. Hence, if we change successively n to $n+1, n+2, .. n+m-1$, and add together all the results, we obtain this other rigorous expression, for the integral of the product $\mathrm{N}_{\beta\alpha} \, \mathrm{F}_\alpha$, extended from $\alpha = \beta^{-1} \alpha_n$ to $\alpha = \beta^{-1} \alpha_{n+m}$:

$$\int_{\beta^{-1}\alpha_n}^{\beta^{-1}\alpha_{n+m}} d\alpha \, \mathrm{N}_{\beta\alpha} \, \mathrm{F}_\alpha = \theta \, \beta^{-1} \, (\alpha_{n+m} - \alpha_n) \, \mathrm{c}\delta \, ; \tag{y}$$

in which δ is the greatest of the m quantities $\delta_n, \delta_{n+1}, \ldots$, or is equal to one of those quantities, and is not exceeded by any other; and θ cannot exceed ± 1. By taking β sufficiently large, and suitably choosing the indices n and $n+m$, we may make the limits of integration in the formula (y) approach as nearly as we please to any given finite values, a and b; while, in the second member of that formula, the factor $\beta^{-1} (\alpha_{n+m} - \alpha_n)$ will tend to become the finite quantity $b - a$, and $\theta\mathrm{c}$ cannot exceed the finite limits $\pm \mathrm{c}$; but the remaining factor δ will tend indefinitely to 0, as β increases without limit, because it is the difference between two values of the function F_α, corresponding to two values of the variable α of which the difference diminishes indefinitely. Passing then to the limit $\beta = \infty$, we have, with the same rigour as before:

$$\int_a^b d\alpha \, \mathrm{N}_{\infty\alpha} \, \mathrm{F}_\alpha = 0 \, ; \tag{z}$$

which is the theorem that was announced at the end of the preceding article. And although it has been here supposed that the function F_α receives no sudden change of value, between the limits of integration; yet we see that if this function receive any finite number of such sudden changes between those limits, but vary gradually in value between any two such changes, the foregoing demonstration may be applied to each interval of gradual variation of value separately; and the theorem (z) will still hold good.

[5.] This theorem (z) may be thus written:

$$\lim_{\beta=\infty} \int_a^b d\alpha \, \mathrm{N}_{\beta\alpha} \, \mathrm{F}_\alpha = 0 \, ; \tag{a'}$$

and we may easily deduce from it the following:

$$\lim_{\beta=\infty} \int_a^b d\alpha \, \mathrm{N}_{\beta(\alpha-x)} \mathrm{F}_\alpha = 0; \tag{b'}$$

the function F_α being here also finite, within the extent of the integration, and x being independent of α and β. For the reasonings of the last article may easily be adapted to this case; or we may see, from the definitions in article [3.], that if the function N_α have the properties there supposed, then $\mathrm{N}_{\alpha-x}$ will also have those properties. In fact, if N_α be always comprised between given finite limits, then $\mathrm{N}_{\alpha-x}$ will be so too; and we shall have, by (f),

$$\int_0^\alpha d\alpha \, \mathrm{N}_{\alpha-x} = \int_{-x}^{\alpha-x} d\alpha \, \mathrm{N}_\alpha = \mathrm{M}_{\alpha-x} - \mathrm{M}_{-x}; \tag{c'}$$

in which M_{-x} is finite, because the suppositions of the third article oblige M_α to be always comprised between the limits $\mathrm{a} \pm \mathrm{bc}$; so that the equation

$$\int_0^\alpha d\alpha \, \mathrm{N}_{\alpha-x} = \mathrm{a} - \mathrm{M}_{-x}, \tag{d'}$$

which is of the form (g), has infinitely many real roots, of the form

$$\alpha = x + \alpha_n, \tag{e'}$$

and therefore of the kind assumed in the two last articles. Let us now examine what happens, when, in the first member of the formula (b′), we substitute, instead of the finite factor F_α, an expression such as $(\alpha - x)^{-1} f_\alpha$, which becomes infinite between the limits of integration, the value of x being supposed to be comprised between those limits, and the function f_α being finite between them. That is, let us inquire whether the integral

$$\int_a^b d\alpha \, \mathrm{N}_{\beta(\alpha-x)} (\alpha - x)^{-1} f_\alpha, \tag{f'}$$

(in which $x > a$, $< b$), tends to any and to what finite and determined limit, as β tends to become infinite.

In this inquiry, the theorem (b′) shows that we need only attend to those values of α which are extremely near to x, and are for example comprised between the limits $x \mp \epsilon$, the quantity ϵ being small. To simplify the question, we shall suppose that for such values of α, the function f_α varies gradually in value;

we shall also suppose that $\mathrm{N}_0 = 0$, and that $\mathrm{N}_\alpha \alpha^{-1}$ tends to a finite limit as α tends to 0, whether this be by decreasing or by increasing; although the limit thus obtained, for the case of infinitely small and positive values of α, may possibly differ from that which corresponds to the case of infinitely small and negative values of that variable, on account of the discontinuity which the function N_α may have. We are then to investigate, with the help of these suppositions, the value of the double limit:

$$\lim_{\epsilon = 0} \, . \, \lim_{\beta = \infty} \, . \int_{x-\epsilon}^{x+\epsilon} d\alpha \, \mathrm{N}_{\beta(\alpha - x)} (\alpha - x)^{-1} f_\alpha \, ; \qquad \text{(g}'\text{)}$$

this notation being designed to suggest, that we are first to assume a small but not evanescent value of ϵ, and a large but not infinite value of β, and to effect the integration, or conceive it effected, with these assumptions; then, retaining the same value of ϵ, make β larger and larger without limit; and then at last suppose ϵ to tend to 0, unless the result corresponding to an infinite value of β shall be found to be independent of ϵ. Or, introducing two new quantities y and η, determined by the definitions

$$y = \beta(\alpha - x), \quad \eta = \beta\epsilon, \qquad \text{(h}'\text{)}$$

and eliminating α and β by means of these, we are led to seek the value of the double limit following:

$$\lim_{\epsilon = 0} \, . \, \lim_{\eta = \infty} \, . \int_{-\eta}^{\eta} dy \, \mathrm{N}_y y^{-1} f_{x + \epsilon\eta^{-1} y} \, ; \qquad \text{(i}'\text{)}$$

in which η tends to ∞, before ϵ tends to 0. It is natural to conclude that since the sought limit (g′) can be expressed under the form (i′), it must be equivalent to the product

$$f_x \times \int_{-\infty}^{\infty} dy \, \mathrm{N}_y y^{-1} \, ; \qquad \text{(k}'\text{)}$$

and in fact it will be found that this equivalence holds good; but before finally adopting this conclusion, it is proper to consider in detail some difficulties which may present themselves.

[6.] Decomposing the function $f_{x + \epsilon\eta^{-1} y}$ into two parts, of which one is indedent of y, and is $= f_x$, while the other part varies with y, although slowly, and vanishes with that variable ; it is clear that the formula (i′) will be decomposed into two corresponding parts, of which the first conducts immediately to the expression (k′) ; and we are now to inquire whether the integral in this expression has a finite and determinate value. Admitting the suppositions made in the last article, the integral

$$\int_{-\zeta}^{\zeta} dy\, \mathrm{N}_y\, y^{-1}$$

will have a finite and determinate value, if ζ be finite and determinate ; we are therefore conducted to inquire whether the integrals

$$\int_{-\infty}^{-\zeta} dy\, \mathrm{N}_y\, y^{-1}, \qquad \int_{\zeta}^{\infty} dy\, \mathrm{N}_y\, y^{-1},$$

are also finite and determinate. The reasonings which we shall employ for the second of these integrals, will also apply to the first ; and, to generalize a little the question to which we are thus conducted, we shall consider the integral

$$\int_{a}^{\infty} d\alpha\, \mathrm{N}_\alpha\, \mathrm{F}_\alpha\, ; \tag{l′}$$

F_α being here supposed to denote any function of α which remains always positive and finite, but decreases continually and gradually in value, and tends indefinitely towards 0, while α increases indefinitely from some given finite value which is not greater than a. Applying to this integral (l′) the principles of the fourth article, and observing that we have now $\mathrm{F}_{\alpha'_{n,i}} < \mathrm{F}_{\alpha_n}$, $\alpha'_{n,i}$ being $> \alpha_n$, and α_n being assumed $\not< a$; and also that

$$\pm(s_{n,0} + s_{n,2} + \ldots) = \mp(s_{n,1} + s_{n,3} + \ldots) \not> \tfrac{1}{2}\,\mathrm{bc}\, ; \tag{m′}$$

we find

$$\pm \int_{\alpha_n}^{\alpha_{n+1}} d\alpha\, \mathrm{N}_\alpha\, \mathrm{F}_\alpha < \tfrac{1}{2}\,\mathrm{bc}\left(\mathrm{F}_{\alpha_n} - \mathrm{F}_{\alpha_{n+1}}\right) ; \tag{n′}$$

and consequently

$$\pm \int_{\alpha_n}^{\alpha_{n+m}} d\alpha\, \mathrm{N}_\alpha\, \mathrm{F}_\alpha < \tfrac{1}{2}\,\mathrm{bc}\left(\mathrm{F}_{\alpha_n} - \mathrm{F}_{\alpha_{n+m}}\right). \tag{o′}$$

This latter integral is therefore finite and numerically less than $\frac{1}{2}\,\mathrm{bc}\, \mathrm{F}_{\alpha_n}$, however great the upper limit α_{n+m} may be ; it tends also to a determined value as m

increases indefinitely, because the part which corresponds to values of α between any given value of the form α_{n+m} and any other of the form α_{n+m+p} is included between the limits $\pm \frac{1}{2}$ bc $\mathrm{F}_{\alpha_{n+m}}$, which limits approach indefinitely to each other and to 0, as m increases indefinitely. And in the integral (l′), if we suppose the lower limit a to lie between α_{n-1} and α_n, while the upper limit, instead of being infinite, is at first assumed to be a large but finite quantity b, lying between α_{n+m} and α_{n+m+1}, we shall only thereby add to the integral (o′) two parts, an initial and a final, of which the first is evidently finite and determinate, while the second is easily proved to tend indefinitely to 0 as m increases without limit. The integral (l′) is therefore itself finite and determined, under the conditions above supposed, which are satisfied, for example, by the function $\mathrm{F}_\alpha = \alpha^{-1}$, if α be > 0. And since the suppositions of the last article render also the integral

$$\int_0^a d\alpha\, \mathrm{N}_\alpha\, \alpha^{-1}$$

determined and finite, if the value of a be such, we see that with these suppositions we may write

$$\varpi' = \int_0^\infty d\alpha\, \mathrm{N}_\alpha\, \alpha^{-1}, \tag{p′}$$

ϖ' being itself a finite and determined quantity. By reasonings almost the same we are led to the analogous formula

$$\varpi'' = \int_{-\infty}^0 d\alpha\, \mathrm{N}_\alpha\, \alpha^{-1}; \tag{q′}$$

and finally to the result

$$\varpi = \varpi' + \varpi'' = \int_{-\infty}^\infty d\alpha\, \mathrm{N}_\alpha\, \alpha^{-1}; \tag{r′}$$

in which ϖ'' and ϖ are also finite and determined. The product (k′) is therefore itself determinate and finite, and may be represented by ϖf_x.

[7.] We are next to introduce, in (i′), the variable part of the function f, namely,

$$f_{x+\epsilon\eta^{-1}y} - f_x,$$

which varies from $f_{x-\epsilon}$ to $f_{x+\epsilon}$, while y varies from $-\eta$ to $+\eta$, and in which ϵ may be any quantity > 0. And since it is clear, that under the conditions assumed in the fifth article,

$$\lim_{\epsilon=0} . \lim_{\eta=\infty} . \int_{-\zeta}^{\zeta} dy\, \mathrm{N}_y y^{-1} (f_{x+\epsilon\eta^{-1}y} - f_x) = 0, \tag{s'}$$

if ζ be any finite and determined quantity, however large, we are conducted to examine whether this double limit vanishes when the integration is made to extend from $y=\zeta$ to $y=\eta$. It is permitted to suppose that f_a continually increases, or continually decreases, from $a=x$ to $a=x+\epsilon$; let us therefore consider the integral

$$\int_{\zeta}^{\eta} da\, \mathrm{N}_a \mathrm{F}_a \mathrm{G}_a, \tag{t'}$$

in which the function F_a decreases, while G_a increases, but both are positive and finite, within the extent of the integration.

By reasonings similar to those of the fourth article, we find under these conditions,

$$\pm \int_{a_n}^{a_{n+1}} da\, \mathrm{N}_a \mathrm{F}_a \mathrm{G}_a < \mathrm{bc} \left(\mathrm{F}_{a_n} \mathrm{G}_{a_{n+1}} - \mathrm{F}_{a_{n+1}} \mathrm{G}_{a_n} \right); \tag{u'}$$

and therefore

$$\left.\begin{array}{l} \pm \frac{1}{\mathrm{bc}} \int_{a_n}^{a_{n+m}} da\, \mathrm{N}_a \mathrm{F}_a \mathrm{G}_a < \mathrm{F}_{a_{n+m-1}} \mathrm{G}_{a_{n+m}} - \mathrm{F}_{a_{n+1}} \mathrm{G}_{a_n} \\ \quad + \left(\mathrm{F}_{a_n} - \mathrm{F}_{a_{n+2}} \right) \mathrm{G}_{a_{n+1}} + \left(\mathrm{F}_{a_{n+2}} - \mathrm{F}_{a_{n+4}} \right) \mathrm{G}_{a_{n+3}} + \&c. \\ \quad + \left(\mathrm{F}_{a_{n+1}} - \mathrm{F}_{a_{n+3}} \right) \mathrm{G}_{a_{n+2}} + \left(\mathrm{F}_{a_{n+3}} - \mathrm{F}_{a_{n+5}} \right) \mathrm{G}_{a_{n+4}} + \&c. \end{array}\right\} \tag{v'}$$

This inequality will still subsist, if we increase the second member by changing, in the positive products on the second and third lines, the factors G to their greatest value $\mathrm{G}_{a_{n+m}}$; and, after adding the results, suppress the three negative terms which remain in the three lines of the expression, and change the functions F, in the first and third lines, to their greatest value F_{a_n}. Hence,

$$\pm \int_{a_n}^{a_{n+m}} da\, \mathrm{N}_a \mathrm{F}_a \mathrm{G}_a < 3\,\mathrm{bc}\, \mathrm{F}_{a_n} \mathrm{G}_{a_{n+m}}; \tag{w'}$$

this integral will therefore ultimately vanish, if the product of the greatest values of the functions F and G tend to the limit 0. Thus, if we make

$$\mathrm{F}_{\alpha} = \alpha^{-1}, \quad \mathrm{G}_{\alpha} = \pm (f_{x+\epsilon\eta^{-1}\alpha} - f_x),$$

the upper sign being taken when f_a increases from $a = x$ to $a = x + \epsilon$; and if we suppose that ζ and η are of the forms a_n and a_{n+m}; we see that the integral (t′) is numerically less than $3\,\mathrm{bc}\,a_n^{-1}(f_{x+\epsilon} - f_x)$, and therefore that it vanishes at the limit $\epsilon = 0$. It is easy to see that the same conclusion holds good, when we suppose that η does not coincide with any quantity of the form a_{n+m}, and when the limits of the integration are changed to $-\eta$ and $-\zeta$. We have therefore, rigorously,

$$\lim_{\epsilon = 0} . \lim_{\eta = \infty} . \int_{-\eta}^{\eta} dy\, \mathrm{N}_y\, y^{-1} (f_{x+\epsilon\eta^{-1}y} - f_x) = 0, \tag{x′}$$

nowithstanding the great and ultimately infinite extent over which the integration is conducted. The variable part of the function f may therefore be suppressed in the double limit (i′), without any loss of accuracy; and that limit is found to be exactly equal to the expression (k′); that is, by the last article, to the determined product ϖf_x. Such, therefore, is the value of the limit (g′), from which (i′) was derived by the transformation (h′); and such finally is the limit of the integral (f′), proposed for investigation in the fifth article. We have, then, proved that under the conditions of that article,

$$\lim_{\beta = \infty} . \int_a^b d\alpha\, \mathrm{N}_{\beta(\alpha - x)} (\alpha - x)^{-1} f_\alpha = \varpi f_x; \tag{y′}$$

and consequently that the arbitrary but finite and gradually varying function f_x, between the limits $x = a$, $x = b$, may be transformed as follows:

$$f_x = \varpi^{-1} \int_a^b d\alpha\, \mathrm{N}_{\infty(\alpha - x)} (\alpha - x)^{-1} f_\alpha; \tag{z′}$$

which is a result of the kind denoted by (d) in the second article, and includes the theorem (a) of FOURIER. For all the suppositions made in the foregoing articles, respecting the form of the function N, are satisfied by assuming this function to be the sine of the variable on which it depends; and then the constant ϖ, determined by the formula (r′), becomes coincident with π, that is, with the ratio of the circumference to the diameter of a circle, or with the least positive root of the equation

$$\frac{\sin x}{x} = 0.$$

[8.] The known theorem just alluded to, namely, that the definite integral (r′) becomes $= \pi$, when $\mathrm{N}_\alpha = \sin \alpha$, may be demonstrated in the following manner. Let

$$\mathrm{A} = \int_0^\infty d\alpha \frac{\sin \beta\alpha}{\alpha};$$

$$\mathrm{B} = \int_0^\infty d\alpha \, \frac{\cos \beta\alpha}{1 + \alpha^2};$$

then these two definite integrals are connected with each other by the relation

$$\mathrm{A} = \left(\int_0^\beta d\beta - \frac{d}{d\beta}\right) \mathrm{B},$$

because

$$\int_0^\beta d\beta \mathrm{B} = \int_0 d\alpha \, \frac{\sin \beta\alpha}{\alpha(1 + \alpha^2)},$$

$$-\frac{d}{d\beta}\mathrm{B} = \int_0^\infty d\alpha \, \frac{\alpha \sin \beta\alpha}{1 + \alpha^2};$$

and all these integrals, by the principles of the foregoing articles, receive determined and finite (that is, not infinite) values, whatever finite or infinite value may be assigned to β. But for all values of $\beta > 0$, the value of A is constant; therefore, for all such values of β, the relation between A and B gives, by integration,

$$e^{-\beta}\left\{\left(\int_0^\beta d\beta + 1\right) \mathrm{B} - \mathrm{A}\right\} = \text{const.};$$

and this constant must be $= 0$, because the factor of $e^{-\beta}$ does not tend to become infinite with β. That factor is therefore itself $= 0$, so that we have

$$\mathrm{A} = \left(\int_0^\beta d\beta + 1\right) \mathrm{B}, \text{ if } \beta > 0.$$

Comparing the two expressions for A, we find

$$\mathrm{B} + \frac{d}{d\beta}\mathrm{B} = 0, \text{ if } \beta > 0;$$

and therefore, for all such values of β,

$$\text{B}\, e^{\beta} = \text{const.}$$

The constant in this last result is easily proved to be equal to the quantity A, by either of the two expressions already established for that quantity; we have therefore

$$\text{B} = \text{A}\, e^{-\beta},$$

however little the value of β may exceed 0; and because B tends to the limit $\frac{\pi}{2}$ as β tends to 0, we find finally, for all values of β greater than 0,

$$\text{A} = \frac{\pi}{2}, \; \text{B} = \frac{\pi}{2}\, e^{-\beta}.$$

These values, and the result

$$\int_{-\infty}^{\infty} d\alpha \frac{\sin \alpha}{\alpha} = \pi,$$

to which they immediately conduct, have long been known; and the first relation, above mentioned, between the integrals A and B, has been employed by LEGENDRE to deduce the former integral from the latter; but it seemed worth while to indicate a process by which that relation may be made to conduct to the values of both those integrals, without the necessity of expressly considering the second differential coefficient of B relative to β, which coefficient presents itself at first under an indeterminate form.

[9.] The connexion of the formula (z′) with FOURIER's theorem (a), will be more distinctly seen, if we introduce a new function P_α defined by the condition

$$\text{N}_\alpha = \int_0^\alpha d\alpha\, \text{P}_\alpha, \qquad \text{(a}''\text{)}$$

which is consistent with the suppositions already made respecting the function N_α. According to those suppositions the new function P_α is not necessarily continuous, nor even always finite, since its integral N_α may be discontinuous; but P_α is supposed to be finite for small values of α, in order that N_α may vary gradually for such values, and may bear a finite ratio to α. The value of the first integral of P_α is supposed to be always comprised between given finite limits, so as never to be numerically greater than $\pm\text{c}$; and the second integral,

$$\mathrm{M}_\alpha = \left(\int_0^\alpha d\alpha\right)^2 \mathrm{P}_\alpha, \tag{b''}$$

becomes infinitely often equal to a given constant, a, for values of α which extend from negative to positive infinity, and are such that the interval between any one and the next following is never greater than a given finite constant, b. With these suppositions respecting the otherwise arbitrary function P_α, the theorems (z) and (z′) may be expressed as follows:

$$\lim_{\beta = \infty} . \int_a^b d\alpha \left(\int_0^{\beta\alpha} d\gamma\, \mathrm{P}_\gamma\right) f_\alpha = 0\,; \tag{A}$$

and

$$f_x = \varpi^{-1} \int_a^b d\alpha \int_0^\infty d\beta\, \mathrm{P}_{\beta(\alpha - x)} f_\alpha\,;\ (x > a, < b) \tag{B}$$

ϖ being determined by the equation

$$\varpi = \int_{-\infty}^{\infty} d\alpha \int_0^1 d\beta\, \mathrm{P}_{\beta\alpha}. \tag{c''}$$

Now, by making

$$\mathrm{P}_\alpha = \cos \alpha,$$

(a supposition which satisfies all the conditions above assumed), we find, as before,

$$\varpi = \pi,$$

and the theorem (B) reduces itself to the less general formula (a), so that it includes the theorem of FOURIER.

[10.] If we suppose that x coincides with one of the limits, a or b, instead of being included between them, we find easily, by the foregoing analysis,

$$f_a = \varpi^{\backprime -1} \int_a^b d\alpha \int_0^\infty d\beta\, \mathrm{P}_{\beta(\alpha - a)} f_\alpha\,; \tag{d''}$$

$$f_b = \varpi^{\backprime\backprime -1} \int_a^b d\alpha \int_0^\infty d\beta\, \mathrm{P}_{\beta(\alpha - b)} f_\alpha\,; \tag{e''}$$

in which

$$\varpi^{\backprime} = \int_0^\infty d\alpha \int_0^1 d\beta\, \mathrm{P}_{\beta\alpha}\,; \tag{f''}$$

$$\varpi'' = \int_{-\infty}^{0} d\alpha \int_{0}^{1} d\beta \, \text{P}_{\beta\alpha} \, ; \tag{g''}$$

so that, as before,

$$\varpi = \varpi' + \varpi''.$$

Finally, when x is outside the limits a and b, the double integral in (B) vanishes; so that

$$0 = \int_{a}^{b} d\alpha \int_{0}^{\infty} d\beta \, \text{P}_{\beta(\alpha - x)} f_{\alpha}, \text{ if } x < a, \text{ or } > b. \tag{h''}$$

And the foregoing theorems will still hold good, if the function f_α receive any number of sudden changes of value, between the limits of integration, provided that it remain finite between them; except that for those very values α' of the variable α, for which the finite function f_α receives any such sudden variation, so as to become $= f'$ for values of α infinitely little greater than α', after having been $= f''$ for values infinitely little less than α', we shall have, instead of (B), the formula

$$\varpi' f' + \varpi'' f'' = \int_{a}^{b} d\alpha \int_{0}^{\infty} d\beta \, \text{P}_{\beta(\alpha - \alpha')} f_{\alpha}. \tag{i''}$$

[11.] If P_α be not only finite for small values of α, but also vary gradually for such values, then, whether α be positive or negative, we shall have

$$\lim_{\alpha = 0} . \text{N}_\alpha \alpha^{-1} = \text{P}_0 \, ; \tag{k''}$$

and if the equation

$$\text{N}_{\alpha - x} = 0 \tag{l''}$$

have no real root α, except the root $\alpha = x$, between the limits a and b, nor any which coincides with either of those limits, then we may change f_α to $\frac{(\alpha - x)\,\text{P}_0}{\text{N}_{\alpha - x}} f_\alpha$, in the formula (z′), and we shall have the expression:

$$f_x = \varpi^{-1} \text{P}_0 \int_{a}^{b} d\alpha \, \text{N}_{\infty(\alpha - x)} \text{N}_{\alpha - x}^{-1} f_{\alpha}. \tag{m''}$$

Instead of the infinite factor in the index, we may substitute any large number, for example, an uneven integer, and take the limit with respect to it; we may, therefore, write

$$f_x = \varpi^{-1}\,\mathrm{P}_0 \lim_{n=\infty} \int_a^b da \frac{\int_0^{(2n+1)(a-x)} da\,\mathrm{P}_a}{\int_0^{a-x} da\,\mathrm{P}_a} f_a. \qquad (\mathrm{n}'')$$

Let

$$\int_{(2n-1)a}^{(2n+1)a} da\,\mathrm{P}_a = \mathrm{Q}_{a,n} \int_0^a da\,\mathrm{P}_a\,; \qquad (\mathrm{o}'')$$

then

$$1 + \mathrm{Q}_{a,1} + \mathrm{Q}_{a,2} + \dots + \mathrm{Q}_{a,n} = \frac{\int_0^{(2n+1)a} da\,\mathrm{P}_a}{\int_0^a da\,\mathrm{P}_a}, \qquad (\mathrm{p}'')$$

and the formula (n″) becomes

$$f_x = \varpi^{-1}\,\mathrm{P}_0 \left(\int_a^b da\, f_a + \Sigma_{(n)1}^{\infty} \int_a^b da\, \mathrm{Q}_{a-x,n} f_a \right); \qquad (\mathrm{c})$$

in which development, the terms corresponding to large values of n are small. For example, when $\mathrm{P}_a = \cos a$, then

$$\varpi = \pi,\ \mathrm{P}_0 = 1,\ \mathrm{Q}_{a,n} = 2\cos 2na,$$

and the theorem (c) reduces itself to the following known result :

$$f_x = \pi^{-1} \left(\int_a^b da\, f_a + 2\,\Sigma_{(n)1}^{\infty} \int_a^b da \cos(2na - 2nx) f_a \right); \qquad (\mathrm{q}'')$$

in which it is supposed that $x > a$, $x < b$, and that $b - a \ngtr \pi$, in order that $a - x$ may be comprised between the limits $\pm \pi$, for the whole extent of the integration ; and the function f_a is supposed to remain finite within the same extent, and to vary gradually in value, at least for values of the variable a which are extremely near to x. The result (q″) may also be thus written :

$$f_x = \pi^{-1}\,\Sigma_{(n)-\infty}^{\infty} \int_a^b da \cos(2na - 2nx) f_a\,; \qquad (\mathrm{r}'')$$

and if we write

$$a = \frac{\beta}{2},\ x = \frac{y}{2},\ f_{\frac{y}{2}} = \phi_y,$$

it becomes

$$\phi_y = \frac{1}{2\pi}\,\Sigma_{(n)-\infty}^{\infty} \int_{2a}^{2b} d\beta \cos(n\beta - ny)\,\phi_\beta, \qquad (\mathrm{s}'')$$

the interval between the limits of integration relatively to β being now not greater than 2π, and the value of y being included between those limits. For example, we may assume

$$2a = -\pi, \; 2b = \pi,$$

and then we shall have, by writing a, x, and f, instead of β, y, and ϕ,

$$f_x = \frac{1}{2\pi} \Sigma_{(n)-\infty}^{\infty} \int_{-\pi}^{\pi} da \cos(na - nx) f_a, \tag{t''}$$

in which $x > -\pi$, $x < \pi$. It is permitted to assume the function f_a such as to vanish when $a < 0$, $> -\pi$; and then the formula (t″) resolves itself into the two following, which (with a slightly different notation) occur often in the writings of POISSON, as does also the formula (t″):

$$\tfrac{1}{2}\int_0^{\pi} da f_a + \Sigma_{(n)1}^{\infty} \int_0^{\pi} da \cos(na - nx) f_a = \pi f_x; \tag{u''}$$

$$\tfrac{1}{2}\int_0^{\pi} da f_a + \Sigma_{(n)1}^{\infty} \int_0^{\pi} da \cos(na + nx) f_a = 0; \tag{v''}$$

x being here supposed > 0, but $< \pi$; and the function f_a being arbitrary, but finite, and varying gradually, from $a = 0$ to $a = \pi$, or at least not receiving any sudden change of value for any value x of the variable a, to which the formula (u″) is to be applied. It is evident that the limits of integration in (t″) may be made to become $\mp l$, l being any finite quantity, by merely multiplying $na - nx$ under the sign cos., by $\frac{\pi}{l}$, and changing the external factor $\frac{1}{2\pi}$ to $\frac{1}{2l}$; and it is under this latter form that the theorem (t″) is usually presented by POISSON: who has also remarked, that the difference of the two series (u″) and (v″) conducts to the expression first assigned by LAGRANGE, for developing an arbitrary function between finite limits, in a series of sines of multiples of the variable on which it depends.

[12.] In general, in the formula (m″), from which the theorem (c) was derived, in order that x may be susceptible of receiving all values $> a$ and $< b$ (or at least all for which the function f_x receives no sudden change of value), it is necessary, by the remark made at the beginning of the last article, that the equation

$$\int_0^a da\, \mathrm{P}_a = 0, \tag{w$''$}$$

should have no real root a different from 0, between the limits $\mp(b-a)$. But it is permitted to suppose, consistently with this restriction, that a is < 0, and that b is > 0, while both are finite and determined; and then the formula (m$''$), or (C) which is a consequence of it, may be transformed so as to receive new limits of integration, which shall approach as nearly as may be desired to negative and positive infinity. In fact, by changing a to λa, x to λx, and $f_{\lambda x}$ to f_x, the formula (C) becomes

$$f_x = \lambda \varpi^{-1}\, \mathrm{P}_0 \left(\int_{\lambda^{-1}a}^{\lambda^{-1}b} da\, f_a + \Sigma_{(n)\,1}^{\,\infty} \int_{\lambda^{-1}a}^{\lambda^{-1}b} da\, \mathrm{Q}_{\lambda a - \lambda x, n} f_a \right); \tag{x$''$}$$

in which $\lambda^{-1}a$ will be large and negative, while $\lambda^{-1}b$ will be large and positive, if λ be small and positive, because we have supposed that a is negative, and b positive; and the new variable x is only obliged to be $> \lambda^{-1}a$, and $< \lambda^{-1}b$, if the new function f_x be finite and vary gradually between these new and enlarged limits. At the same time, the definition (o$''$) shows that $\mathrm{P}_0\, \mathrm{Q}_{\lambda a - \lambda x, n}$ will tend indefinitely to become equal to $2\, \mathrm{P}_{2n\lambda(a-x)}$; in such a manner that

$$\lim_{\lambda = 0} \cdot \frac{\mathrm{P}_0\, \mathrm{Q}_{\lambda a - \lambda x, n}}{2\, \mathrm{P}_{2n\lambda(a-x)}} = 1, \tag{y$''$}$$

at least if the function P be finite and vary gradually. Admitting then that we may adopt the following ultimate transformation of a sum into an integral, at least under the sign $\int_{-\infty}^{\infty} da$,

$$\lim_{\lambda = 0} \cdot 2\lambda \left(\tfrac{1}{2} \mathrm{P}_0 + \Sigma_{(n)\,1}^{\,\infty}\, \mathrm{P}_{2n\lambda(a-x)} \right) = \int_0^{\infty} d\beta\, \mathrm{P}_{\beta(a-x)}, \tag{z$''$}$$

we shall have, as the limit of (x$''$), this formula:

$$f_x = \varpi^{-1} \int_{-\infty}^{\infty} da \int_0^{\infty} d\beta\, \mathrm{P}_{\beta(a-x)} f_a; \tag{D}$$

which holds good for all real values of the variable x, at least under the conditions lately supposed, and may be regarded as an extension of the theorem (B), from finite to infinite limits. For example, by making P a cosine, the theorem (D) becomes

$$f_x = \pi^{-1} \int_{-\infty}^{\infty} d\alpha \int_0^{\infty} d\beta \cos(\beta\alpha - \beta x) f_\alpha, \tag{a'''}$$

which is a more usual form than (a) for the theorem of FOURIER. In general, the deduction in the present article, of the theorem (D) from (C), may be regarded as a verification of the analysis employed in this paper, because (D) may also be obtained from (B), by making the limits of integration infinite; but the demonstration of the theorem (B) itself, in former articles, was perhaps more completely satisfactory, besides that it involved fewer suppositions; and it seems proper to regard the formula (D) as only a limiting form of (B).

[13.] This formula (D) may also be considered as a limit in another way, by introducing, under the sign of integration relatively to β, a factor $\mathrm{F}_{k\beta}$ such that

$$\mathrm{F}_0 = 1, \ \mathrm{F}_\infty = 0, \tag{b'''}$$

in which k is supposed positive but small, and the limit taken with respect to it, as follows:

$$f_x = \lim_{k=0} \cdot \varpi^{-1} \int_{-\infty}^{\infty} d\alpha \left(\int_0^{\infty} d\beta \, \mathrm{P}_{\beta(\alpha - x)} \mathrm{F}_{k\beta} \right) f_\alpha. \tag{E}$$

It is permitted to suppose that the function F decreases continually and gradually, at a finite and decreasing rate, from 1 to 0, while the variable on which it depends increases from 0 to ∞; the first differential coefficient F' being thus constantly finite and negative, but constantly tending to 0, while the variable is positive and tends to ∞. Then, by the suppositions already made respecting the function P, if $\alpha - x$ and k be each different from 0, we shall have

$$\left. \begin{aligned} \int_0^{\beta} d\beta \, \mathrm{P}_{\beta(\alpha - x)} \mathrm{F}_{k\beta} = \mathrm{F}_{k\beta} \mathrm{N}_{\beta(\alpha - x)} (\alpha - x)^{-1} \\ - k (\alpha - x)^{-1} \int_0^{\beta} d\beta \, \mathrm{N}_{\beta(\alpha - x)} \mathrm{F}'_{k\beta}; \end{aligned} \right\} \tag{c'''}$$

and therefore, because $\mathrm{F}_\infty = 0$, while N is always finite, the integral relative to β in the formula (E) may be thus expressed:

$$\int_0^{\infty} d\beta \, \mathrm{P}_{\beta(\alpha - x)} \mathrm{F}_{k\beta} = (\alpha - x)^{-1} \psi_{k^{-1}(\alpha - x)}, \tag{d'''}$$

the function ψ being assigned by the equation

$$\psi_\lambda = -\int_0^\infty d\gamma \, \mathrm{N}_{\lambda\gamma} \, \mathrm{F}'_\gamma. \tag{e'''}$$

For any given value of λ, the value of this function ψ is finite and determinate, by the principles of the sixth article; and as λ tends to ∞, the function ψ tends to 0, on account of the fluctuation of N, and because F' tends to 0, while γ tends to ∞; the integral (d''') therefore tends to vanish with k, if α be different from x; so that

$$\lim_{k=0} \cdot \int_0^\infty d\beta \, \mathrm{P}_{\beta(\alpha-x)} \, \mathrm{F}_{k\beta} = 0, \text{ if } \alpha \gtrless x. \tag{f'''}$$

On the other hand, if $\alpha = x$, that integral tends to become infinite, because we have, by (b'''),

$$\lim_{k=0} \cdot \mathrm{P}_0 \int_0^\infty d\beta \, \mathrm{F}_{k\beta} = \infty. \tag{g'''}$$

Thus, while the formula (d''') shows that the integral relative to β in (E) is a homogeneous function of $\alpha - x$ and k, of which the dimension is negative unity, we see also, by (f''') and (g'''), that this function is such as to vanish or become infinite at the limit $k = 0$, according as $\alpha - x$ is different from or equal to zero. When the difference between α and x, whether positive or negative, is very small and of the same order as k, the value of the last mentioned integral (relative to β) varies very rapidly with α; and in this way of considering the subject, the proof of the formula (E) is made to depend on the verification of the equation

$$\varpi^{-1} \int_{-\infty}^{\infty} d\lambda \, \psi_\lambda \, \lambda^{-1} = 1. \tag{h'''}$$

But this last verification is easily effected; for when we substitute the expression (e''') for ψ_λ, and integrate first relatively to λ, we find, by (r'),

$$\int_{-\infty}^{\infty} d\lambda \, \mathrm{N}_{\lambda\gamma} \, \lambda^{-1} = \varpi; \tag{i'''}$$

it remains then to show that

$$-\int_0^\infty d\gamma \, \mathrm{F}'_\gamma = 1; \tag{k'''}$$

and this follows immediately from the conditions (b'''). For example, when P is a cosine, and F a negative neperian exponential, so that

$$\mathrm{P}_\alpha = \cos \alpha, \quad \mathrm{F}_\alpha = e^{-\alpha},$$

then, making $\lambda = k^{-1}(\alpha - x)$, we have

$$\int_0^\infty d\beta \, e^{-k\beta} \cos(\beta\alpha - \beta x) = (\alpha - x)^{-1} \psi_\lambda ;$$

$$\psi_\lambda = \int_0^\infty d\gamma \, e^{-\gamma} \sin \lambda\gamma = \frac{\lambda}{1 + \lambda^2};$$

and

$$\varpi^{-1} \int_{-\infty}^{\infty} d\lambda \psi_\lambda \lambda^{-1} = \pi^{-1} \int_{-\infty}^{\infty} \frac{d\lambda}{1 + \lambda^2} = 1.$$

It is nearly thus that POISSON has, in some of his writings, demonstrated the theorem of FOURIER, after putting it under a form which differs only slightly from the following:

$$f_x = \pi^{-1} \lim_{k=0} \int_{-\infty}^{\infty} d\alpha \int_0^\infty d\beta e^{-k\beta} \cos(\beta\alpha - \beta x) f_\alpha ; \qquad (1''')$$

namely, by substituting for the integral relative to β its value

$$\frac{k}{k^2 + (\alpha - x)^2};$$

and then observing that, if k be very small, this value is itself very small, unless α be extremely near to x, so that f_α may be changed to f_x; while, making $\alpha = x + k\lambda$, and integrating relatively to λ between limits indefinitely great, the factor by which this function f_x is multiplied in the second member of (1'''), is found to reduce itself to unity.

[14.] Again, the function F_α retaining the same properties as in the last article for positive values of α, and being further supposed to satisfy the condition

$$\mathrm{F}_{-\alpha} = \mathrm{F}_\alpha, \qquad (m''')$$

while k is still supposed to be positive and small, the formula (D) may be presented in this other way, as the limit of the result of two integrations, of which the first is to be effected with respect to the variable α:

$$f_x = \lim_{k=0} \cdot \varpi^{-1} \int_0^\infty d\beta \int_{-\infty}^{\infty} d\alpha \, \mathrm{F}_{k\alpha} \mathrm{P}_{\beta(\alpha - x)} f_\alpha . \qquad (\mathrm{F})$$

Now it often happens that if the function f_α be obliged to satisfy conditions which determine all its values by means of the arbitrary values which it may have for a given finite range, from $\alpha = a$ to $\alpha = b$, the integral relative to α in the formula (F) can be shown to vanish at the limit $k = 0$, for all real and positive values of β, except those which are roots of a certain equation

$$\Omega_\rho = 0 ; \tag{G}$$

while the same integral is, on the contrary, infinite, for these particular values of β; and then the integration relatively to β will in general change itself into a summation relatively to the real and positive roots ρ of the equation (G), which is to be combined with an integration relatively to α between the given limits a and b; the resulting expression being of the form

$$f_x = \Sigma_\rho \int_a^b d\alpha \, \phi_{x,\alpha,\rho} f_\alpha . \tag{H}$$

For example, in the case where P is a cosine, and F a negative exponential, if the conditions relative to the function f be supposed such as to conduct to expressions of the forms

$$\int_0^\infty d\alpha \, e^{-h\alpha} f_\alpha = \frac{\psi(h)}{\phi(h)}, \tag{n'''}$$

$$\int_0^{-\infty} d\alpha \, e^{h\alpha} f_\alpha = \frac{\psi(-h)}{\phi(-h)}, \tag{o'''}$$

in which h is any real or imaginary quantity, independent of α, and having its real part positive; it will follow that

$$\left.\begin{aligned} &\int_{-\infty}^{\infty} d\alpha \, e^{-k\sqrt{\alpha^2}} (\cos \beta\alpha - \sqrt{-1} \sin \beta\alpha) f_\alpha \\ &= \frac{\psi(\beta\sqrt{-1} + k)}{\phi(\beta\sqrt{-1} + k)} - \frac{\psi(\beta\sqrt{-1} - k)}{\phi(\beta\sqrt{-1} - k)}, \end{aligned}\right\} \tag{p'''}$$

in which $\sqrt{\alpha^2}$ is $= \alpha$ or $= -\alpha$, according as α is $>$ or $<$ 0, and the quantities β and k are real, and k is positive. The integral in (p'''), and consequently also that relative to α in (F), in which, now,

$$P_\alpha = \cos \alpha, \quad F_\alpha = e^{-k\sqrt{\alpha^2}},$$

will therefore, under these conditions, tend to vanish with k, unless β be a root ρ of the equation

$$\phi(\rho\sqrt{-1}) = 0, \qquad (\mathrm{q}''')$$

which here corresponds to (G); but the same integral will on the contrary tend to become infinite, as k tends to 0, if β be a root of the equation (q'''). Making therefore $\beta = \rho + k\lambda$, and supposing $k\lambda$ to be small, while ρ is a real and positive root of (q'''), the integral (p''') becomes

$$\frac{k^{-1}}{1+\lambda^2}(\mathrm{A}_\rho - \sqrt{-1}\,\mathrm{B}_\rho), \qquad (\mathrm{r}''')$$

in which A_ρ and B_ρ are real, namely,

$$\left.\begin{array}{l} \mathrm{A}_\rho = \dfrac{\psi(\rho\sqrt{-1})}{\phi'(\rho\sqrt{-1})} + \dfrac{\psi(-\rho\sqrt{-1})}{\phi'(-\rho\sqrt{-1})}, \\ \mathrm{B}_\rho = \sqrt{-1}\left(\dfrac{\psi(\rho\sqrt{-1})}{\phi'(\rho\sqrt{-1})} - \dfrac{\psi(-\rho\sqrt{-1})}{\phi'(-\rho\sqrt{-1})}\right); \end{array}\right\} \qquad (\mathrm{s}''')$$

ϕ' being the differential coefficient of the function ϕ. Multiplying the expression (r''') by $\pi^{-1}\,d\beta\,(\cos\beta x + \sqrt{-1}\sin\beta x)$, which may be changed to $\pi^{-1}\,kd\lambda\,(\cos\rho x + \sqrt{-1}\sin\rho x)$; integrating relatively to λ between indefinitely great limits, negative and and positive; taking the real part of the result, and summing it relatively to ρ; there results,

$$f_x = \Sigma_\rho(\mathrm{A}_\rho\cos\rho x + \mathrm{B}_\rho\sin\rho x); \qquad (\mathrm{t}''')$$

a development which has been deduced nearly as above, by POISSON and LIOUVILLE, from the suppositions (n'''), (o'''), and from the theorem of FOURIER presented under a form equivalent to the following:

$$f_x = \lim_{k=0} \cdot \pi^{-1}\int_0^\infty d\beta \int_{-\infty}^{\infty} d\alpha\, e^{-k\sqrt{\alpha^2}}\cos(\beta\alpha - \beta x) f_\alpha; \qquad (\mathrm{u}''')$$

and in which it is to be remembered that if 0 be a root of the equation (q'''), the corresponding terms in the development of f_x must in general be modified by the circumstance, that in calculating these terms, the integration relatively to λ extends only from 0 to ∞.

For example, when the function f is obliged to satisfy the conditions

$$f_{-\alpha} = f_\alpha,\ f_{l-\alpha} = -f_{l+\alpha}, \tag{v'''}$$

the suppositions (n''') (o''') are satisfied; the functions ϕ and ψ being here such that

$$\phi(h) = e^{hl} + e^{-hl},$$

$$\psi(h) = \int_0^l d\alpha\, (e^{h(l-\alpha)} - e^{h(\alpha-l)}) f_\alpha;$$

therefore the equation (q''') becomes in this case

$$\cos \rho l = 0, \tag{w'''}$$

and the expressions (s''') for the coefficients of the development (t''') reduce themselves to the following:

$$\text{A}_\rho = \frac{2}{l} \int_0^l d\alpha \cos \rho\alpha f_\alpha;\ \text{B}_\rho = 0; \tag{x'''}$$

so that the method conducts to the following expression for the function f, which satisfies the conditions (v'''),

$$f_x = \frac{2}{l} \Sigma_{(n)1}^{\infty} \cos \frac{(2n-1)\pi x}{2l} \int_0^l d\alpha \cos \frac{(2n-1)\pi\alpha}{2l} f_\alpha; \tag{y'''}$$

in which f_α is arbitrary from $\alpha = 0$ to $\alpha = l$, except that f_l must vanish. The same method has been applied, by the authors already cited, to other and more difficult questions; but it will harmonize better with the principles of the present paper to treat the subject in another way, to which we shall now proceed.

[15.] Instead of introducing, as in (E) and (F), a factor which has unity for its limit, we may often remove the apparent indeterminateness of the formula (D) in another way, by the principles of fluctuating functions. For if we integrate first relatively to α between indefinitely great limits, negative and positive, then, under the conditions which conduct to developments of the form (H), we shall find that the resulting function of β is usually a fluctuating one, of which the integral vanishes, except in the immediate neighbourhood of certain particular values determined by an equation such as (G); and then, by integrating only in such immediate neighbourhood, and afterwards summing the results, the development (H) is obtained. For example, when P is a cosine, and when the conditions (v''') are satisfied by the function f, it is not difficult to prove that

$$\int_{-2ml-l}^{2ml+l} d\alpha \cos(\beta\alpha - \beta x) f_\alpha = \frac{2\cos(2m\beta l + \beta l + m\pi)}{\cos\beta l} \cos\beta x \int_0^l d\alpha \cos\beta\alpha f_\alpha; \quad (z''')$$

m being here an integer number, which is to be supposed large, and ultimately infinite. The equation (G) becomes therefore, in the present question and by the present method, as well as by that of the last article,

$$\cos\rho l = 0;$$

and if we make $\beta = \rho + \gamma$, ρ being a root of this equation, we may neglect γ in the second member of (z'''), except in the denominator

$$\cos\beta l = -\sin\rho l \sin\gamma l,$$

and in the fluctuating factor of the numerator

$$\cos(2m\beta l + \beta l + m\pi) = -\sin\rho l \sin(2m\gamma l + \gamma l);$$

consequently, multiplying by $\pi^{-1} d\gamma$, integrating relatively to γ between any two small limits of the forms $\mp\epsilon$, and observing that

$$\lim_{m=\infty} \cdot \frac{2}{\pi}\int_{-\epsilon}^{\epsilon} d\gamma \frac{\sin(2ml\gamma + l\gamma)}{\sin l\gamma} = \frac{2}{l},$$

the development

$$f_x = \frac{2}{l}\Sigma_\rho \cos\rho x \int_0^l d\alpha \cos\rho\alpha f_\alpha,$$

which coincides with (y'''), and is of the form (H), is obtained.

[16.] A more important application of the method of the last article is suggested by the expression which Fourier has given for the arbitrary initial temperature of a solid sphere, on the supposition that this temperature is the same for all points at the same distance from the centre. Denoting the radius of the sphere by l, and that of any layer or shell of it by α, while the initial temperature of the same layer is denoted by $\alpha^{-1} f_\alpha$, we have the equations

$$f_0 = 0, f'_l + \nu f_l = 0, \quad (a^{IV})$$

which permit us to suppose

$$f_\alpha + f_{-\alpha} = 0, f'_{l+\alpha} + f'_{l-\alpha} + \nu(f_{l+\alpha} + f_{l-\alpha}) = 0; \quad (b^{IV})$$

ν being here a constant quantity not less than $-l^{-1}$, and f' being the first differential coefficient of the function f, which function remains arbitrary for all values of α greater than 0, but not greater than l. The equations (b^{IV}) give

$$(\beta \cos \beta l + \nu \sin \beta l) \int_{l-a}^{l+a} d\alpha \sin \beta \alpha f_{\alpha} = \qquad (c^{IV})$$

$$(\beta \sin \beta l - \nu \cos \beta l) \int_{a-l}^{a+l} d\alpha \cos \beta \alpha f_{\alpha} - \cos \beta a \, (f_{a+l} + f_{a-l})\,;$$

so that

$$(\rho \sin \rho l - \nu \cos \rho l) \int_{a-l}^{a+l} d\alpha \cos \rho \alpha f_{\alpha} = \cos \rho a \, (f_{a+l} + f_{a-l}), \qquad (d^{IV})$$

if ρ be a root of the equation

$$\rho \cos \rho l + \nu \sin \rho l = 0. \qquad (e^{IV})$$

This latter equation is that which here corresponds to (G) ; and when we change β to $\rho + \gamma$, γ being very small, we may write, in the first member of (c^{IV}),

$$\beta \cos \beta l + \nu \sin \beta l = \gamma \, \{(1 + \nu l) \cos \rho l - \rho l \sin \rho l\}, \qquad (f^{IV})$$

and change β to ρ in all the terms of the second member, except in the fluctuating factor $\cos \beta a$, in which a is to be made extremely large. Also, after making $\cos \beta a = \cos \rho \alpha \cos \gamma \alpha - \sin \rho \alpha \sin \gamma \alpha$, we may suppress $\cos \gamma \alpha$ in the second member of (c^{IV}), before integrating with respect to γ, because by (d^{IV}) the terms involving $\cos \gamma \alpha$ tend to vanish with γ, and because $\gamma^{-1} \cos \gamma \alpha$ changes sign with γ. On the other hand, the integral of $\frac{d\gamma \sin \gamma a}{\gamma}$ is to be replaced by π, though it be taken only for very small values, negative and positive, of γ, because α is here indefinitely large and positive. Thus in the present question, the formula

$$f_x = \frac{1}{\pi} \cdot \lim_{a = \infty} \cdot \int_0^{\infty} d\beta \sin \beta x \int_{l-a}^{l+a} d\alpha \sin \beta \alpha f_{\alpha}, \qquad (g^{IV})$$

(which is obtained from (a''') by suppressing the terms which involve $\cos \beta x$, on account of the first condition (b^{IV}),) may be replaced by a sum relative to the real and positive roots of the equation (e^{IV}) ; the term corresponding to any one such root being

$$\frac{R_{\rho} \sin \rho x}{(1 + \nu l) \cos \rho l - \rho l \sin \rho l}, \qquad (h^{IV})$$

if we suppose $\rho > 0$, and make for abridgment

$$\left.\begin{aligned} \mathrm{R}_\rho = (\nu \cos \rho l - \rho \sin \rho l) \int_{a-l}^{a+l} d\alpha \sin \rho \alpha f_\alpha \\ + \sin \rho a (f_{a+l} + f_{a-l}). \end{aligned}\right\} \qquad (\mathrm{i}^{IV})$$

The equations (b^{IV}) show that the quantity R_ρ does not vary with a, and therefore that it may be rigorously thus expressed :

$$\mathrm{R}_\rho = 2 (\nu \cos \rho l - \rho \sin \rho l) \int_0^l d\alpha \sin \rho \alpha f_\alpha ; \qquad (\mathrm{k}^{IV})$$

we have also, by (e^{IV}), ρ being > 0,

$$\frac{2 (\nu \cos \rho l - \rho \sin \rho l)}{\cos \rho l + l (\nu \cos \rho l - \rho \sin \rho l)} = \frac{2\rho}{\rho l - \sin \rho l \cos \rho l}. \qquad (\mathrm{l}^{IV})$$

And if we set aside the particular case where

$$\nu l + 1 = 0, \qquad (\mathrm{m}^{IV})$$

the term corresponding to the root

$$\rho = 0, \qquad (\mathrm{n}^{IV})$$

of the equation (c^{IV}), vanishes in the development of f_x ; because this term is, by (g^{IV}),

$$\frac{x}{\pi} \int_0^\beta d\beta \left(\beta \int_{l-a}^{l+a} d\alpha \sin \beta \alpha f_\alpha\right), \qquad (\mathrm{o}^{IV})$$

a being very large, and β small, but both being positive ; and unless the condition (m^{IV}) be satisfied, the equation (c^{IV}) shows that the quantity to be integrated in (o^{IV}), with respect to β, is a finite and fluctuating function of that variable, so that its integral vanishes, at the limit $a = \infty$. Setting aside then the case (m^{IV}), which corresponds physically to the absence of exterior radiation, we see that the function f_x, which represents the initial temperature of any layer of the sphere multiplied by the distance x of that layer from the centre, and which is arbitrary between the limits $x = 0$, $x = l$, that is, between the centre and the surface, (though it is obliged to satisfy at those limits the conditions (a^{IV})), may be developed in the following series, which was discovered by FOURIER, and is of the form (H) :

$$f_x = \Sigma_\rho \frac{2\rho \sin \rho x \int_0^l d\alpha \sin \rho \alpha f_\alpha}{\rho l - \sin \rho l \cos \rho l} ; \qquad (\mathrm{p}^{IV})$$

the sum extending only to those roots of the equation (e^{IV}) which are greater than 0. In the particular case (m^{IV}), in which the root (n^{IV}) of the equation (e^{IV}) must be employed, the term (o^{IV}) becomes, by (c^{IV}) and (d^{IV}),

$$\frac{3x}{\pi l^3}\left\{\int_{a-l}^{a+l} d\alpha f_\alpha \, a \, \mathrm{C} - l\,(f_{a+l} + f_{a-l})\, a \, \mathrm{C}\right\}, \qquad (q^{IV})$$

in which, at the limit here considered,

$$\mathrm{C} = \int_0^\infty d\theta \, \frac{\text{vers}\,\theta}{\theta^2} = \frac{\pi}{2}; \qquad (r^{IV})$$

but also, by the equations (b^{IV}), (m^{IV}),

$$\int_{a-l}^{a+l} d\alpha f_\alpha \, \alpha - l\,(f_{a+l} + f_{a-l})\, \alpha = 2\int_0^l d\alpha f_\alpha \, \alpha \, ; \qquad (s^{IV})$$

the sought term of f_x becomes, therefore, in the present case,

$$\frac{3x}{l^3}\int_0^l d\alpha f_\alpha \, \alpha, \qquad (t^{IV})$$

and the corresponding term in the expression of the temperature $x^{-1} f_x$ is equal to the mean initial temperature of the sphere; a result which has been otherwise obtained by POISSON, for the case of no exterior radiation, and which might have been anticipated from physical considerations. The supposition

$$\nu l + 1 < 0, \qquad (u^{IV})$$

which is inconsistent with the physical conditions of the question, and in which FOURIER's development (p^{IV}) may fail, is excluded in the foregoing analysis.

[17.] When a converging series of the form (H) is arrived at, in which the coefficients ϕ of the arbitrary function f, under the sign of integration, do not tend to vanish as they correspond to larger and larger roots ρ of the equation (G); then those coefficients $\phi_{x,\alpha,\rho}$ must in general tend to become fluctuating functions of α, as ρ becomes larger and larger. And the sum of those coefficients, which may be thus denoted,

$$\Sigma_\rho \, \phi_{x,\alpha,\rho} = \psi_{x,\alpha,\rho}, \qquad (\text{I})$$

and which is here supposed to be extended to all real and positive roots of the equation (G), as far as some given root ρ, must tend to become a fluctuating function of α, and to have its mean value equal to zero, as ρ tends to become infinite, for all values of α and x which are different from each other, and are both comprised between the limits of the integration relative to α; in such a manner as to satisfy the equation

$$\int_{\lambda}^{\mu} d\alpha\, \psi_{x,\alpha,\infty} f_{\alpha} = 0, \qquad \text{(K)}$$

which is of the form (C), referred to in the second article; provided that the arbitrary function f is finite, and that the quantities λ, μ, x, α are all comprised between the limits a and b, which enter into the formula (H); while α is, but x is not, comprised also between the new limits λ and μ. But when $\alpha = x$, the sum (I) tends to become infinite with ρ, so that we have

$$\psi_{x,x,\infty} = \infty, \qquad \text{(L)}$$

and

$$\int_{x-\epsilon}^{x+\epsilon} d\alpha\, \psi_{x,\alpha,\infty} f_{\alpha} = f_{x}, \qquad \text{(M)}$$

ϵ being here a quantity indefinitely small. For example, in the particular question which conducts to the development (Y'''), we have

$$\phi_{x,\alpha,\rho} = \frac{2}{l} \cos \rho x \cos \rho \alpha, \qquad (\text{V}^{IV})$$

and

$$\rho = \frac{(2n-1)\pi}{2l}; \qquad (\text{W}^{IV})$$

therefore, summing relatively to ρ, or to n, from $n = 1$ to any given positive value of the integer number n, we have, by (I),

$$\psi_{x,\alpha,\rho} = \frac{\sin \dfrac{n\pi(\alpha - x)}{l}}{2l \sin \dfrac{\pi(\alpha - x)}{2l}} + \frac{\sin \dfrac{n\pi(\alpha + x)}{l}}{2l \sin \dfrac{\pi(\alpha + x)}{2l}}; \qquad (\text{X}^{IV})$$

and it is evident that this sum tends to become a fluctuating function of α, and to satisfy the equation (K), as ρ, or n, tends to become infinite, while α and x are different from each other, and are both comprised between the limits 0 and l. On the other hand, when α becomes equal to x, the first part of the expression

(X^{IV}) becomes $=\frac{n}{l}$, and therefore tends to become infinite with n, so that the equation (L) is true. And the equation (M) is verified by observing, that if $x>0,<l$, we may omit the second part of the sum (X^{IV}), as disappearing in the integral through fluctuation, while the first part gives, at the limit,

$$\lim_{n=\infty} \int_{x-\epsilon}^{x+\epsilon} d\alpha \frac{\sin \frac{n\pi(\alpha-x)}{l}}{2l \sin \frac{\pi(\alpha-x)}{2l}} f_\alpha = f_x. \qquad (Y^{IV})$$

If x be equal to 0, the integral is to be taken only from 0 to ϵ, and the result is only half as great, namely,

$$\lim_{n=\infty} \cdot \int_0^\epsilon d\alpha \frac{\sin \frac{n\pi\alpha}{l}}{2l \sin \frac{\pi\alpha}{2l}} f_\alpha = \tfrac{1}{2} f_0 ; \qquad (Z^{IV})$$

but, in this case, the other part of the sum (X^{IV}) contributes an equal term, and the whole result is f_0. If $x=l$, the integral is to be taken from $l-\epsilon$ to l, and the two parts of the expression (X^{IV}) contribute the two terms $\frac{1}{2}f_l$ and $-\frac{1}{2}f_l$, which neutralize each other. We may therefore in this way prove, *à posteriori*, by the consideration of fluctuating functions, the truth of the development (Y''') for any arbitrary but finite function f_x, and for all values of the real variable x from $x=0$ to $x=l$, the function being supposed to vanish at the latter limit; observing only that if this function f_x undergo any sudden change of value, for any value x' of the variable between the limits 0 and l, and if x be made equal to x' in the development (Y'''), the process shows that this development then represents the semisum of the two values which the function f receives, immediately before and after it undergoes this sudden change.

[18.] The same mode of *à posteriori* proof, through the consideration of fluctuating functions, may be applied to a great variety of other analogous developments, as has indeed been indicated by FOURIER, in a passage of his Theory of Heat. The spirit of POISSON'S method, when applied to the establishment, *à posteriori*, of developments of the form (H), would lead us to multiply, before the summation, each coefficient $\phi_{x,\alpha,\rho}$ by a factor $F_{k,\rho}$ which tends to unity as k tends to 0, but tends to vanish as ρ tends to ∞; and then instead of a *generally fluctuating sum* (I), there results a *generally evanescent sum* (k being evanescent), namely,

$$\Sigma_\rho \, \mathrm{F}_{k,\rho} \, \phi_{x,\alpha,\rho} = \chi_{x,\alpha,k,\rho}, \tag{N}$$

which conducts to equations analogous to (K) (L) (M), namely,

$$\lim_{k=0} \int_\lambda^\mu d\alpha \, \chi_{x,\alpha,k,\infty} f_\alpha = 0\,; \tag{O}$$

$$\lim_{k=0} \chi_{x,x,k,\infty} = \infty\,; \tag{P}$$

$$\lim_{k=0} \int_{x-\epsilon}^{x+\epsilon} d\alpha \, \chi_{x,\alpha,k,\infty} f_\alpha = f_x. \tag{Q}$$

It would be interesting to inquire what form the generally evanescent function χ would take immediately before its vanishing, when

$$\mathrm{F}_{k,\rho} = \epsilon^{-k\rho},$$

and

$$\phi_{x,\alpha,\rho} = \frac{2\rho \sin \rho x \sin \rho \alpha}{\rho l - \sin \rho l \cos \rho l},$$

ρ being a root of the equation

$$\rho l \cot\!an\, \rho l = \text{const.},$$

and the constant in the second member being supposed not greater than unity.

[19.] The development (C), which, like (H), expresses an arbitrary function, at least between given limits, by a combination of summation and integration, was deduced from the expression (m″) of the eleventh article, which conducts also to many other analogous developments, according to the various ways in which the factor with the infinite index, $\mathrm{N}_{\infty(\alpha-x)}$, may be replaced by an infinite sum, or other equivalent form. Thus, if, instead of (o″), we establish the following equation,

$$\int_{(2n-2)a}^{2na} d\alpha \, \mathrm{P}_\alpha = \mathrm{R}_{a,n} \int_0^a d\alpha \, \mathrm{P}_\alpha, \tag{a′}$$

we shall have, instead of (C), the development:

$$f_x = \varpi^{-1} \text{P}_0 \Sigma_{(n)1}^{\infty} \int_a^b d\alpha \, \text{R}_{\alpha - x, n} f_\alpha ; \qquad (\text{R})$$

which, when P is a cosine, reduces itself to the form,

$$f_x = \frac{2}{\pi} \Sigma_{(n)1}^{\infty} \int_a^b d\alpha \cos (\overline{2n - 1} \, . \, \overline{\alpha - x}) f_\alpha, \qquad (\text{b}^V)$$

x being $> a$, $< b$, and $b - a$ being not $> \pi$; and easily conducts to the known expression

$$f_x = \frac{1}{l} \Sigma_{(n)1}^{\infty} \int_{-l}^{l} d\alpha \cos \frac{(2n - 1)\pi(\alpha - x)}{2l} f_\alpha, \qquad (\text{c}^V)$$

which holds good for all values of x between $-l$ and $+l$. By supposing $f_\alpha = f_{-\alpha}$, we are conducted to the expression (y'''); and by supposing $f_\alpha = -f_{-\alpha}$, we are conducted to this other known expression,

$$f_x = \frac{2}{l} \Sigma_{(n)1}^{\infty} \sin \frac{(2n - 1)\pi x}{2l} \int_0^l d\alpha \sin \frac{(2n - 1)\pi\alpha}{2l} f_\alpha ; \qquad (\text{d}^V)$$

which holds good even at the limit $x = l$, by the principles of the seventeenth article, and therefore offers the following transformation for the arbitrary function f_l:

$$f_l = -\frac{2}{l} \Sigma_{(n)1}^{\infty} (-1)^n \int_0^l d\alpha \sin \frac{(2n - 1)\pi\alpha}{2l} f_\alpha . \qquad (\text{e}^V)$$

For example, by making $f_\alpha = \alpha^i$, and supposing i to be an uneven integer number; effecting the integration indicated in (e^V), and dividing both members by l^i, we find the following relation between the sums of the reciprocals of even powers of odd whole numbers:

$$1 = [i]^1 \omega_2 - [i]^3 \omega_4 + [i]^5 \omega_6 - \dots ; \qquad (\text{f}^V)$$

in which

$$[i]^k = i(i-1)(i-2) \dots (i-k+1); \qquad (\text{g}^V)$$

and

$$\omega_{2k} = 2 \left(\frac{2}{\pi}\right)^{2k} \Sigma_{(n)1}^{\infty} (2n - 1)^{-2k} ; \qquad (\text{h}^V)$$

thus

$$1 = \omega_2 = 3\omega_2 - 3.2.1.\omega_4 = 5\omega_2 - 5.4.3\omega_4 + 5.4.3.2.1\,\omega_6, \qquad (\text{i}^V)$$

so that

$$\omega_2 = 1,\ \omega_4 = \tfrac{1}{3},\ \omega_6 = \tfrac{2}{15}. \qquad (k^{v})$$

Again, by making $f_a = a^i$, but supposing $i =$ an uneven number $2k$, we get the following additional term in the second member of the equation (f^{v}),

$$(-1)^k [2k]^{2k} \omega_{2k+1}, \qquad (l^{v})$$

in which

$$\omega_{2k+1} = -2\left(\frac{2}{\pi}\right)^{2k+1} \Sigma_{(n)1}^{\infty} (-1)^n (2n-1)^{-2k-1}; \qquad (m^{v})$$

thus

$$1 = \omega_1 = 2\omega_2 - 2.1\,\omega_3 = 4\omega_2 - 4.3.2\omega_4 + 4.3.2.1\,\omega_5, \qquad (n^{v})$$

so that

$$\omega_1 = 1,\ \omega_3 = \tfrac{1}{2},\ \omega_5 = \tfrac{5}{24}. \qquad (o^{v})$$

Accordingly, if we multiply the values (k^{v}) by $\frac{\pi^2}{8}, \frac{\pi^4}{32}, \frac{\pi^6}{128}$, we get the known values for the sums of the reciprocals of the squares, fourth powers, and sixth powers of the odd whole numbers; and if we multiply the values (o^{v}) by $\frac{\pi}{4}, \frac{\pi^3}{16}, \frac{\pi^5}{64}$, we get the known values for the sums of the reciprocals of the first, third, and fifth powers of the same odd numbers, taken however with alternately positive and negative signs. Again, if we make $f_a = \sin a$, in (e^{v}), and divide both members of the resulting equation by $\cos l$, we get this known expression for a tangent,

$$\tan l = \Sigma_{(n)-\infty}^{\infty} \frac{2}{(2n-1)\pi - 2l}; \qquad (p^{v})$$

which shows that, with the notation (h^{v}),

$$\tan l = \omega_2 l^1 + \omega_4 l^3 + \omega_6 l^5 + \dots; \qquad (q^{v})$$

so that the coefficients of the ascending powers of the arc in the development of its tangent are connected with each other by the relations (f^{v}), which may be briefly represented thus:

$$\sqrt{-1} = (1 + \sqrt{-1}\,\mathrm{D}_0)^{2k-1} \tan 0; \qquad (r^{v})$$

the second member of this symbolic equation being supposed to be developed, and

$D_0^{\,i} \tan 0$ being understood to denote the value which the i^{th} differential coefficient of the tangent of α, taken with respect to α, acquires when $\alpha = 0$; thus,

$$\left. \begin{aligned} 1 = D_0 \tan 0 &= 3D_0 \tan 0 - D_0^{\,3} \tan 0 \\ &= 5D_0 \tan 0 - 10 D_0^{\,3} \tan 0 + D_0^{\,5} \tan 0. \end{aligned} \right\} \qquad (s^V)$$

Finally, if we make $f_\alpha = \cos \alpha$, and attend to the expression (p^V), we obtain, for the secant of an arc l, the known expression:

$$\sec l = \Sigma_{(n)-\infty}^{\ \ \infty} \frac{2(-1)^{n+1}}{(2n-1)\pi - 2l}; \qquad (t^V)$$

which shows that, with the notation (m^V),

$$\sec l = \omega_1 l^0 + \omega_3 l^2 + \omega_5 l^4 + \ldots, \qquad (u^V)$$

and therefore, by the relations of the form (n^V),

$$\sqrt{-1}\,(1 - (\sqrt{-1}\, D_0)^{2k} \sec 0) = (1 + \sqrt{-1}\, D_0)^{2k} \tan 0; \qquad (v^V)$$

thus

$$\left. \begin{aligned} 1 = \sec 0 &= 2D_0 \tan 0 - D_0^{\,2} \sec 0 \\ &= 4D_0 \tan 0 - 4D_0^{\,3} \tan 0 + D_0^{\,4} \sec 0. \end{aligned} \right\} \qquad (w^V)$$

Though several of the results above deduced are known, the writer does not remember to have elsewhere seen the symbolic equations (r^V), (v^V), as expressions for the laws of the coefficients of the developments of the tangent and secant, according to ascending powers of the arc.

[20.] In the last article, the symbol R was such, that

$$\Sigma_{(n)1}^{\ \ n} R_{\alpha,n} = N_{2n\alpha} N_\alpha^{-1}; \qquad (x^V)$$

and in article [11.], we had

$$1 + \Sigma_{(n)1}^{\ \ n} Q_{\alpha,n} = N_{2n\alpha+\alpha} N_\alpha^{-1}. \qquad (y^V)$$

Assume, now, more generally,

$$\nabla_\beta S_{\alpha,\beta} = N_{\beta\alpha} N_\alpha^{-1}; \qquad (z^V)$$

and let the operation ∇_β admit of being effected after, instead of before, the integration relatively to α; the expression (m'') will then acquire this very general form:

$$f_x = \varpi^{-1} \mathrm{P}_0 \nabla_\infty \int_a^b d\alpha \, \mathrm{S}_{\alpha - x, \beta} f_\alpha ; \tag{s}$$

which includes the transformations (C) and (R), and in which the notation ∇_∞ is designed to indicate that after performing the operation ∇_β we are to make the variable β infinite, according to some given law of increase, connected with the form of the operation denoted by ∇.

[21.] In order to deduce the theorems (C), (R), (S), we have hitherto supposed (as was stated in the twelfth article), that the equation $\mathrm{N}_\alpha = 0$ has no real root different from 0 between the limits $\mp (b - a)$, in which a and b are the limits of the integration relative to α, between which latter limits it is also supposed that the variable x is comprised. If these conditions be not satisfied, the factor $\mathrm{N}_{\alpha-x}^{-1}$, in the formula (m″), may become infinite within the proposed extent of integration, for values of α and x which are not equal to each other; and it will then be necessary to change the first member of each of the equations (m″), (C), (R), (S), to a function different from f_x, but to be determined by similar principles. To simplify the question, let it be supposed that the function N_α receives no sudden change of value, and that the equation

$$\mathrm{N}_\alpha = 0, \tag{a^{VI}}$$

which coincides with (w″), has all its real roots unequal. These roots must here coincide with the quantities $\alpha_{n,i}$ of the fourth and other articles, for which the function N_α changes sign; but as the double index is now unnecessary, while the notation α_n has been appropriated to the roots of the equation (g), we shall denote the roots of the equation (a^{VI}), in their order, by the symbols

$$\nu_{-\infty}, \ldots . \nu_{-1}, \nu_0, \nu_1, \ldots . \nu_\infty ; \tag{b^{VI}}$$

and choosing ν_0 for that root of (a^{VI}) which has already been supposed to vanish, we shall have

$$\nu_0 = 0, \tag{c^{VI}}$$

while the other roots will be $>$ or $<$ 0, according as their indices are positive or negative. If the differential coefficient P_α be also supposed to remain always finite, and to receive no sudden change of value in the immediate neighbourhood of any root ν of (a^{VI}), we shall have, for values of α in that neighbourhood, the limiting equation:

$$\lim_{a = \nu} \cdot \mathrm{N}_a (a - \nu)^{-1} = \mathrm{P}_\nu ; \tag{d^{VI}}$$

and P_ν will be different from 0, because the real roots of the equation (a^{VI}) have been supposed unequal. Conceive also that the integral

$$\int_{-\infty}^{\infty} da\, \mathrm{N}_{a+\beta\nu}\, a^{-1} = \varpi_{\nu,\beta} \tag{e^{VI}}$$

tends to some finite and determined limit, which may perhaps be different for different roots ν, and therefore may be thus denoted,

$$\varpi_{\nu,\infty} = \varpi_\nu, \tag{f^{VI}}$$

as β tends to ∞, after the given law referred to at the end of the last article. Then, by writing

$$a = x + \nu + \beta^{-1} y, \tag{g^{VI}}$$

and supposing β to be very large, we easily see, by reasoning as in former articles, that the part of the integral

$$\int_a^b da\, \mathrm{N}_{\beta(a-x)}\, \mathrm{N}_{a-x}^{-1} f_a, \tag{h^{VI}}$$

which corresponds to values of $a - x$ in the neighbourhood of the root ν, is very nearly expressed by

$$\varpi_\nu\, \mathrm{P}_\nu^{-1} f_{x+\nu} ; \tag{i^{VI}}$$

and that this expression is accurate at the limit. Instead of the equation (S), we have therefore now this other equation:

$$\Sigma_\nu\, \varpi_\nu\, \mathrm{P}_\nu^{-1} f_{x+\nu} = \nabla_\infty \int_a^b da\, \mathrm{S}_{a-x,\beta} f_a ; \tag{T}$$

the sum in the first member being extended to all those roots ν of the equation (a^{VI}), which satisfy the conditions

$$x + \nu > a, < b. \tag{k^{VI}}$$

If one of the roots ν should happen to satisfy the condition

$$x + \nu = a, \tag{l^{VI}}$$

the corresponding term in the first member of (T) would be, by the same principles,

$$\varpi^{\backprime}_{\nu}\, \mathrm{P}_{\nu}^{\prime -1} f_{a}, \tag{m^{VI}}$$

in which

$$\varpi^{\backprime}_{\nu} = {}_{\beta=\infty}^{\lim} \int_{0}^{\infty} da\, \mathrm{N}_{a+\beta\nu}\, a^{-1}. \tag{n^{VI}}$$

And if a root ν of (a^{VI}) should satisfy the condition

$$x + \nu = b, \tag{o^{VI}}$$

the corresponding term in the first member of (T) would then be

$$\varpi^{\backprime\backprime}_{\nu}\, \mathrm{P}_{\nu}^{\prime -1} f_{b}, \tag{p^{VI}}$$

in which

$$\varpi^{\backprime\backprime}_{\nu} = {}_{\beta=\infty}^{\lim} \int_{-\infty}^{0} da\, \mathrm{N}_{a+\beta\nu}\, a^{-1}. \tag{q^{VI}}$$

Finally, if a value of $x + \nu$ satisfy the conditions (k^{VI}), and if the function f undergo a sudden change of value for this particular value of the variable on which that function depends, so that $f = f^{\backprime\backprime}$ immediately before, and $f = f^{\backprime}$ immediately after the change, then the corresponding part of the first member of the formula (T) is

$$\mathrm{P}_{\nu}^{\prime -1} (\varpi^{\backprime}_{\nu} f^{\backprime} + \varpi^{\backprime\backprime}_{\nu} f^{\backprime\backprime}). \tag{r^{VI}}$$

And in the formulæ for ϖ_{ν}, $\varpi^{\backprime}_{\nu}$, $\varpi^{\backprime\backprime}_{\nu}$, it is permitted to write

$$\mathrm{N}_{a+\beta\nu}\, a^{-1} = \int_{0}^{1} dt\, \mathrm{P}_{ta+\beta\nu}. \tag{s^{VI}}$$

[22.] One of the simplest ways of rendering the integral (e^{VI}) determinate at its limit, is to suppose that the function P_{a} is of the periodical form which satisfies the two following equations,

$$\mathrm{P}_{-a} = \mathrm{P}_{a},\ \ \mathrm{P}_{a+p} = -\mathrm{P}_{a}; \tag{t^{VI}}$$

p being some given positive constant. Multiplying these equations by da, and integrating from $a = 0$, we find, by (a$''$),

$$\mathrm{N}_{-a} + \mathrm{N}_{a} = 0,\ \ \mathrm{N}_{a+p} + \mathrm{N}_{a} = \mathrm{N}_{p}; \tag{u^{VI}}$$

therefore

$$\mathrm{N}_{p} = \mathrm{N}_{\frac{p}{2}} + \mathrm{N}_{-\frac{p}{2}} = 0, \tag{v^{VI}}$$

and

$$\mathrm{N}_{a+p} = -\mathrm{N}_{a},\ \ \mathrm{N}_{a+2p} = \mathrm{N}_{a},\ \&c. \tag{w^{VI}}$$

Consequently, if the equations (t^{VI}) be satisfied, the multiples (by whole numbers) of p will all be roots of the equation (a^{VI}) ; and reciprocally that equation will have no other real roots, if we suppose that the function P_α, which vanishes when α is any odd multiple of $\frac{p}{2}$, preserves one constant sign between any one such multiple and the next following, or simply between $\alpha = 0$ and $\alpha = \frac{p}{2}$. We may then, under these conditions, write

$$\nu_i = ip, \qquad (x^{VI})$$

i being any integer number, positive or negative, and ν_i denoting generally, as in (b^{VI}), any root of the equation (a^{VI}). And we shall have

$$\int_{-\infty}^{\infty} d\alpha \, N_{\alpha+kp} \, \alpha^{-1} = (-1)^k \varpi, \qquad (y^{VI})$$

k being any integer number, and ϖ still retaining the same meaning as in the former articles. Also, for any integer value of k,

$$P_{kp} = (-1)^k P_0. \qquad (z^{VI})$$

These things being laid down, let us resume the integral (e^{VI}), and let us suppose that the law by which β increases to ∞ is that of coinciding successively with the several uneven integer numbers 1, 3, 5, &c., as was supposed in deducing the formula (c). Then $\beta\nu$ in (e^{VI}) will be an odd or even multiple of p, according as ν is the one or the other, so that we shall have by (x^{VI}), (y^{VI}), the following determined expression for the sought limit (f^{VI}) :

$$\varpi_{\nu_i} = (-1)^i \varpi ; \qquad (a^{VII})$$

but also, by (x^{VI}), (z^{VI}),

$$P_{\nu_i} = (-1)^i P_0 ; \qquad (b^{VII})$$

therefore

$$\varpi_\nu P_\nu^{-1} = \varpi P_0^{-1}, \qquad (c^{VII})$$

the value of this expression being thus the same for all the roots of (a^{VI}). At the same time, in (i^{VI}),

$$f_{x+\nu} = f_{x+ip} ; \qquad (d^{VII})$$

the equation (T) becomes therefore now

$$\Sigma_i f_{x+ip} = \varpi^{-1}\, \mathrm{P}_0\, \nabla_\infty \int_a^b d\alpha\, \mathrm{S}_{\alpha-x,\beta} f_\alpha, \tag{U}$$

β tending to infinity by passing through the successive positive odd numbers, and i receiving all integer values which allow $x+ip$ to be comprised between the limits a and b. If any integer value of i render $x+ip$ equal to either of these limits, the corresponding term of the sum in the first member of (U) is to be $\frac{1}{2}f_a$, or $\frac{1}{2}f_b$; and if the function f receive any sudden change of value between the same limits of integration, corresponding to a value of the variable which is of the form $x+ip$, the term introduced thereby will be of the form $\frac{1}{2}f' + \frac{1}{2}f''$.

For example, when

$$\mathrm{P}_\alpha = \cos\alpha,\ \varpi = \pi,\ p = \pi, \tag{e^{VII}}$$

we obtain the following known formula, instead of (r''),

$$\Sigma_i f_{x+i\pi} = \pi^{-1}\, \Sigma_{(n)-\infty}^{\ \ \infty} \int_a^b d\alpha \cos(2n\alpha - 2nx) f_\alpha; \tag{f^{VII}}$$

which may be transformed in various ways, by changing the limits of integration, and in which halves of functions are to be introduced in extreme cases, as above.

On the other hand, if the law of increase of β be, as in (R), that of coinciding successively with larger and larger even numbers, then

$$\varpi_\nu = \varpi,\ \mathrm{P}_\nu = \mp \mathrm{P}_0, \tag{g^{VII}}$$

and the equation (T) becomes

$$\Sigma_i (-1)^i f_{x+i\pi} = \varpi^{-1}\, \mathrm{P}_0\, \nabla_\infty \int_a^b d\alpha\, \mathrm{S}_{\alpha-x,\beta} f_\alpha. \tag{V}$$

For example, in the case (e^{VII}), we obtain this extension of the formula (b^{V}),

$$\Sigma_i (-1)^i f_{x+i\pi} = \pi^{-1}\, \Sigma_{(n)-\infty}^{\ \ \infty} \int_a^b d\alpha \cos(\overline{2n-1}\,.\,\overline{\alpha-x}) f_\alpha. \tag{h^{VII}}$$

We may verify the equations (f^{VII}) (h^{VII}) by remarking that both members of the former equation remain unchanged, and that both members of the latter are changed in sign, when x is increased by π. A similar verification of the equations (U) and (V) requires that in general the expression

$$\nabla_\infty \int_a^b d\alpha\, \mathrm{S}_{\alpha-x,\beta} f_\alpha \tag{i^{VII}}$$

should either receive no change, or simply change its sign, when x is increased by p, according as β tends to ∞ by coinciding with large and odd or with large and even numbers.

[23.] In all the examples hitherto given to illustrate the general formulæ of this paper, it has been supposed for the sake of simplicity, that the function P is a cosine; and this supposition has been sufficient to deduce, as we have seen, a great variety of known results. But it is evident that this function P may receive many other forms, consistently with the suppositions made in deducing those general formulæ; and many new results may thus be obtained by the method of the foregoing articles.

For instance, it is permitted to suppose

$$\mathrm{P}_\alpha = 1, \text{ if } \alpha^2 < 1; \qquad (\mathrm{k}^{VII})$$

$$\mathrm{P}_1 = 0; \qquad (\mathrm{l}^{VII})$$

$$\mathrm{P}_{\alpha+2} = -\mathrm{P}_\alpha; \qquad (\mathrm{m}^{VII})$$

and then the equations (t^{VI}) of the last article, with all that were deduced from them, will still hold good. We shall now have

$$p = 2; \qquad (\mathrm{n}^{VII})$$

and the definite integral denoted by ϖ, and defined by the equation (r'), may now be computed as follows. Because the function N_α changes sign with α, we have

$$\varpi = 2\int_0^\infty d\alpha\,\mathrm{N}_\alpha\,\alpha^{-1}; \qquad (\mathrm{o}^{VII})$$

but

$$\left.\begin{array}{llll} \mathrm{N}_\alpha = \alpha, & \text{from } \alpha = 0 & \text{to } \alpha = 1; \\ \ldots 2 - \alpha, & \quad\ldots 1 & \quad\ldots\ 3; \\ \ldots \alpha - 4, & \quad\ldots 3 & \quad\ldots\ 4; \end{array}\right\} \qquad (\mathrm{p}^{VII})$$

and

$$\mathrm{N}_{\alpha+4} = \mathrm{N}_\alpha. \qquad (\mathrm{q}^{VII})$$

Hence

$$\int_0^4 d\alpha\,\mathrm{N}_\alpha\,\alpha^{-1} = 6\log 3 - 4\log 4, \qquad (\mathrm{r}^{VII})$$

the logarithms being Napierian; and generally, if m be any positive integer number, or zero,

$$\int_{4m}^{4m+4} da\,\mathrm{N}_a\,a^{-1} = \int_0^4 da\,\mathrm{N}_a\,(a+4m)^{-1}$$

$$= 4m\log(4m) - (8m+2)\log(4m+1)$$

$$+ (8m+6)\log(4m+3) - (4m+4)\log(4m+4)$$

$$= \Sigma_{(k)\,1}^{\ \infty}\,\frac{1-2^{-2k}}{k\,(k+\frac{1}{2})}(2m+1)^{-2k}. \qquad (\mathrm{s}^{VII})$$

But, by (h^{V}),

$$\Sigma_{(m)\,0}^{\ \infty}\,(2m+1)^{-2k} = \tfrac{1}{2}\left(\frac{\pi}{2}\right)^{2k}\omega_{2k}, \qquad (\mathrm{t}^{VII})$$

if k be any integer number > 0; therefore

$$\varpi = \Sigma_{(k)\,1}^{\ \infty}\,\frac{1-2^{-2k}}{k\,(k+\frac{1}{2})}\left(\frac{\pi}{2}\right)^{2k}\omega_{2k}; \qquad (\mathrm{u}^{VII})$$

ω_{2k} being by (q^{V}) the coefficient of x^{2k-1} in the development of $\tan x$. From this last property, we have

$$\Sigma_{(k)\,1}^{\ \infty}\,\frac{\omega_{2k}\,x^{2k}}{k\,(k+\frac{1}{2})} = \frac{4}{x}\left(\int_0^x dx\right)^2 \tan x = \frac{4}{x}\int_0^x dx\log\sec x; \qquad (\mathrm{v}^{VII})$$

therefore, substituting successively the values $x=\frac{\pi}{2}$ and $x=\frac{\pi}{4}$, and subtracting the result of the latter substitution from that of the former, we find, by (u^{VII}),

$$\varpi = \frac{8}{\pi}\left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{4}}\right) dx\log\sec x$$

$$= \frac{8}{\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} dx\log\tan x$$

$$= \frac{8}{\pi}\int_0^{\frac{\pi}{4}} dx\log\cot x. \qquad (\mathrm{w}^{VII})$$

Such, in the present question, is an expression for the constant ϖ; its numerical value may be approximately calculated by multiplying the Napierian logarithm of ten by the double of the average of the ordinary logarithms of the cotangents of the middles of any large number of equal parts into which the first octant may be divided; thus, if we take the ninetieth part of the sum of the logarithms of the cotangents of the ninety angles $\frac{1^0}{4}, \frac{3^0}{4}, \frac{5^0}{4}, \ldots \frac{177^0}{4}, \frac{179^0}{4}$, as given by the ordinary tables, we obtain nearly, as the average of these ninety logarithms, the number 0,5048; of which the double, being multiplied by the Napierian logarithm of ten, gives, nearly, the number 2,325, as an approximate value of the constant ϖ. But a much more accurate value may be obtained with little more trouble, by computing separately the doubles of the part (r^{VII}), and of the sum of (s^{VII}) taken from $m = 1$ to $m = \infty$; for thus we obtain the expression

$$\varpi = 12 \log 3 - 8 \log 4$$

$$+ 2\, \Sigma_{(k)\,1}^{\;\infty} \frac{1 - 2^{-2k}}{k\,(k + \frac{1}{2})} \Sigma_{(m)\,1}^{\;\infty} (2m + 1)^{-2k}, \qquad (\mathrm{x}^{VII})$$

in which each sum relative to m can be obtained from known results, and the sum relative to k converges tolerably fast; so that the second line of the expression (x^{VII}) is thus found to be nearly $= 0{,}239495$, while the first line is nearly $= 2{,}092992$; and the whole value of the expression (x^{VII}) is nearly

$$\varpi = 2{,}332487. \qquad (\mathrm{y}^{VII})$$

There is even an advantage in summing the double of the expression (s^{VII}) only from $m = 2$ to $m = \infty$, because the series relative to k converges then more rapidly; and having thus found $2 \int_8^\infty d\alpha\, \mathrm{N}_\alpha\, \alpha^{-1}$, it is only necessary to add thereto the expression

$$2 \int_0^8 d\alpha\, \mathrm{N}_\alpha\, \alpha^{-1} = 12 \log 3 - 20 \log 5 + 28 \log 7 - 16 \log 8. \qquad (\mathrm{z}^{VII})$$

The form of the function P and the value of the constant ϖ being determined as in the present article, it is permitted to substitute them in the general equations of this paper; and thus to deduce new transformations for portions of arbitrary functions, which might have been employed instead of those given by FOURIER and POISSON, if the discontinuous function P, which receives alternately the values 1, 0, and -1, had been considered simpler in its properties than the trigonometrical function cosine.

[24.] Indeed, when the conditions (t^{VI}) are satisfied, the function P_x can be developed according to cosines of the odd multiples of $\frac{\pi x}{p}$, by means of the formula (y'''), which here becomes, by changing l to $\frac{p}{2}$, and f to P,

$$\mathrm{P}_x = \Sigma_{(n)\,1}^{\ \infty}\, \mathrm{A}_{2n-1} \cos \frac{(2n-1)\,\pi x}{p}, \qquad (\mathrm{a}^{VIII})$$

in which

$$\mathrm{A}_{2n-1} = \frac{4}{p} \int_0^{\frac{p}{2}} d\alpha \cos \frac{(2n-1)\,\pi \alpha}{p}\, \mathrm{P}_\alpha; \qquad (\mathrm{b}^{VIII})$$

the function N_x at the same time admitting a development according to sines of the same odd multiples, namely,

$$\mathrm{N}_x = \frac{p}{\pi} \Sigma_{(n)\,1}^{\ \infty} \frac{\mathrm{A}_{2n-1}}{2n-1} \sin \frac{(2n-1)\,\pi x}{p}; \qquad (\mathrm{c}^{VIII})$$

and the constant ϖ being equal to the following series,

$$\varpi = p\, \Sigma_{(n)\,1}^{\ \infty} \frac{\mathrm{A}_{2n-1}}{2n-1}. \qquad (\mathrm{d}^{VIII})$$

Thus, in the case of the last article, where $p = 2$, and $\mathrm{P}_\alpha = 1$ from $\alpha = 0$ to $\alpha = 1$, we have

$$\mathrm{A}_{2n-1} = \frac{4}{\pi} \frac{(-1)^{n+1}}{2n-1}; \qquad (\mathrm{e}^{VIII})$$

$$\mathrm{P}_x = \frac{4}{\pi}\left(\cos \frac{\pi x}{2} - 3^{-1} \cos \frac{3\pi x}{2} + 5^{-1} \cos \frac{5\pi x}{2} - \dots\right); \qquad (\mathrm{f}^{VIII})$$

$$\mathrm{N}_x = \frac{8}{\pi^2}\left(\sin \frac{\pi x}{2} - 3^{-2} \sin \frac{3\pi x}{2} + 5^{-2} \sin \frac{5\pi x}{2} - \dots\right); \qquad (\mathrm{g}^{VIII})$$

$$\varpi = \frac{8}{\pi}(1^{-2} - 3^{-2} + 5^{-2} - 7^{-2} + \dots); \qquad (\mathrm{h}^{VIII})$$

so that, from the comparison of (w^{VII}) and (h^{VIII}), the following relation results:

$$\int_0^{\frac{\pi}{4}} dx \log \cot x = \Sigma_{(n)\,0}^{\ \infty} (-1)^n (2n+1)^{-2}. \qquad (\mathrm{i}^{VIII})$$

But most of the suppositions made in former articles may be satisfied, without assuming for the function P the periodical form assigned by the conditions (t^{VI}).

For example, we might assume

$$\text{P}_a = \frac{4}{\pi}\int_0^\pi d\theta \sin\theta^2 \cos(2a\sin\theta)\,; \qquad (\text{k}^{VIII})$$

which would give, by (a″), and (b″),

$$\text{N}_a = \frac{2}{\pi}\int_0^\pi d\theta \sin\theta \sin(2a\sin\theta)\,; \qquad (\text{l}^{VIII})$$

$$\text{M}_a = \frac{1}{\pi}\int_0^\pi d\theta \,\text{vers}\,(2a\sin\theta)\,; \qquad (\text{m}^{VIII})$$

and finally, by (r′),

$$\varpi = 2\int_0^\pi d\theta \sin\theta = 4. \qquad (\text{n}^{VIII})$$

This expression (k^{VIII}) for P_a satisfies all the conditions of the ninth article; for it is clear that it gives a value to N_a which is always numerically less than $\frac{4}{\pi}$; and the equation

$$\text{M}_a = 1, \qquad (\text{o}^{VIII})$$

which is of the form (g), is satisfied by all the infinitely many real and unequal roots of the equation

$$\int_0^\pi d\theta \cos(2a\sin\theta) = 0, \qquad (\text{p}^{VIII})$$

which extend from $a = -\infty$ to $a = \infty$, and of which the interval between any one and the next following is never greater than π, nor even so great; because (as it is not difficult to prove) these several roots are contained in alternate or even octants, in such a manner that we may write

$$a_n > \frac{n\pi}{2} - \frac{\pi}{4}, < \frac{n\pi}{2}. \qquad (\text{q}^{VIII})$$

We may, therefore substitute the expression (k^{VIII}) for P, in the formulæ (A), (B), (C), &c.; and we find, by (B), if $x > a$, $< b$,

$$f_x = \pi^{-1}\int_a^b d\alpha \int_0^\infty d\beta \int_0^\pi d\theta \sin\theta^2 \cos\{2\beta(\alpha - x)\sin\theta\} f_\alpha\,; \qquad (\text{r}^{VIII})$$

that is,

$$f_x = \frac{1}{2\pi} \lim_{\beta = \infty} \int_0^\pi d\theta \sin\theta \int_a^b d\alpha \sin\{2\beta(\alpha - x)\sin\theta\}(\alpha - x)^{-1} f_\alpha\,; \quad (\mathrm{s}^{VIII})$$

a theorem which may be easily proved *à posteriori*, by the principles of fluctuating functions, because those principles show, that (if x be comprised between the limits of integration) the limit relative to β of the integral relative to α, in (s^{VIII}), is equal to πf_x. In like manner, the theorem (C), when applied to the present form of the function P, gives the following other expression for the arbitrary function f_x:

$$\left.\begin{aligned} f_x &= \tfrac{1}{2}\int_a^b d\alpha f_\alpha \\ &+ \Sigma_{(n)\,1}^{\ \infty} \int_a^b d\alpha f_\alpha \frac{\int_0^\pi d\theta \sin\theta \sin\big(2(\alpha - x)\sin\theta\big)\cos\big(4n(\alpha - x)\sin\theta\big)}{\int_0^\pi d\theta \sin\theta\sin\big(2(\alpha - x)\sin\theta\big)}\,; \end{aligned}\right\} \quad (\mathrm{t}^{VIII})$$

x being between a and b, and $b - a$ being not greater than the least positive root ν of the equation

$$\frac{1}{\nu}\int_0^\pi d\theta \sin\theta \sin(2\nu\sin\theta) = 0. \quad (\mathrm{u}^{VIII})$$

And if we wish to prove, *à posteriori*, this theorem of transformation (t^{VIII}), by the same principles of fluctuating functions, we have only to observe that

$$1 + 2\,\Sigma_{(n)\,1}^{\ n} \cos 2ny = \frac{\sin(2ny + y)}{\sin y}, \quad (\mathrm{v}^{VIII})$$

and therefore that the second member of (t^{VIII}) may be put under the form

$$\lim_{n = \infty} \int_a^b d\alpha f_\alpha \frac{\int_0^\pi d\theta \sin\theta \sin\big((4n+2)(\alpha - x)\sin\theta\big)}{2\int_0^\pi d\theta \sin\theta \sin\big(2(\alpha - x)\sin\theta\big)}\,; \quad (\mathrm{w}^{VIII})$$

in which the presence of the fluctuating factor

$$\sin\big((4n+2)(\alpha - x)\sin\theta\big),$$

combined with the condition that $\alpha - x$ is numerically less than the least root of the equation (u^{VIII}), shows that we need only attend to values of α indefinitely near to x, and may therefore write in the denominator,

$$\int_0^\pi d\theta \sin\theta \sin\big(2(\alpha - x)\sin\theta\big) = \pi(\alpha - x)\,; \quad (\mathrm{x}^{VIII})$$

for thus, by inverting the order of the two remaining integrations, that is by writing

$$\int_a^b d\alpha \int_0^\pi d\theta \ldots = \int_0^\pi d\theta \int_a^b d\alpha \ldots, \tag{y^{VIII}}$$

we find first

$$\lim_{n=\infty} \int_a^b d\alpha f_\alpha \frac{\sin\left((4n+2)(\alpha - x)\sin\theta\right)}{2\pi(\alpha - x)} = \tfrac{1}{2} f_x, \tag{z^{VIII}}$$

for every value of θ between 0 and π, and of x between a and b; and finally,

$$\tfrac{1}{2} f_x \int_0^\pi d\theta \sin\theta = f_x.$$

[25.] The results of the foregoing articles may be extended by introducing, under the functional signs N, P, a product such as $\beta\gamma$, instead of $\beta\alpha$, γ being an arbitrary function of α; and by considering the integral

$$\int_a^b d\alpha \, \mathrm{N}_{\beta\gamma} \, \mathrm{F}_\alpha, \tag{a^{IX}}$$

in which F is any function which remains finite between the limits of integration. Since γ is a function of α, it may be denoted by γ_α, and α will be reciprocally a function of γ, which may be denoted thus:

$$\alpha = \phi_{\gamma_\alpha}. \tag{b^{IX}}$$

While α increases from a to b, we shall suppose, at first, that the function γ_α increases constantly and continuously from γ_a to γ_b, in such a manner as to give always, within this extent of variation, a finite and determined and positive value to the differential coefficient of the function ϕ, namely,

$$\frac{d\alpha}{d\gamma} = \phi'_\gamma. \tag{c^{IX}}$$

We shall also express, for abridgment, the product of this coefficient and of the function F by another function of γ, as follows,

$$\phi'_\gamma \, \mathrm{F}_\alpha = \psi \tag{d^{IX}}$$

Then the integral (a^{IX}) becomes

$$\int_{\gamma_a}^{\gamma_b} d\gamma \, \mathrm{N}_{\beta\gamma} \, \psi_\gamma; \tag{e^{IX}}$$

and a rigorous expression for it may be obtained by the process of the fourth article, namely

$$\left. \begin{aligned} &\left(\int_{\gamma_a}^{\beta^{-1}a_n} + \int_{\beta^{-1}a_{n+m}}^{\gamma_b}\right) d\gamma \, \mathrm{N}_{\beta\gamma} \, \psi_\gamma \\ &\quad + \theta\beta^{-1}(a_{n+m} - a_n)\, \mathrm{c}\delta\,; \end{aligned} \right\} \qquad (\mathrm{f}^{IX})$$

in which, as before, a_n, a_{n+m} are suitably chosen roots of the equation (g); c is a finite constant; θ is included between the limits ± 1; and δ is the difference between two values of the function ψ_γ, corresponding to two values of the variable γ of which the difference is less than $\beta^{-1}\mathrm{b}$, b being another finite constant. The integral (a^{IX}) therefore diminishes indefinitely when β increases indefinitely; and thus, or simply by the theorem (z) combined with the expression (e^{IX}), we have, rigorously, at the limit, without supposing here that N_0 vanishes,

$$\int_a^b d\alpha \, \mathrm{N}_{\infty\gamma} \, \mathrm{F}_\alpha = 0. \qquad (\mathrm{W})$$

The same conclusion is easily obtained, by reasonings almost the same, for the case where γ continually decreases from γ_a to γ_b, in such a manner as to give, within this extent of variation, a finite and determined and negative value to the differential coefficient (e^{IX}). And with respect to the case where the function γ is for a moment stationary in value, so that its differential coefficient vanishes between the limits of integration, it is sufficient to observe that although ψ in (e^{IX}) becomes then infinite, yet F in (a^{IX}) remains finite, and the integral of the finite product $d\alpha \, \mathrm{N}_{\beta\gamma} \, \mathrm{F}_\alpha$, taken between infinitely near limits, is zero. Thus, generally, the theorem (W), which is an extension of the theorem (z), holds good between any finite limits a and b, if the function F be finite between those limits, and if, between the same limits of integration, the function γ never remain unchanged throughout the whole extent of any finite change of α.

[26.] It may be noticed here, that if β be only very large, instead of being infinite, an approximate expression for the integral (a^{IX}) may be obtained, on the same principles, by attending only to values of α which differ very little from those which render the coefficient (e^{IX}) infinite. For example, if we wish to find an approximate expression for a large root of the equation (p^{VIII}), or to express approximately the function

$$f_\beta = \frac{1}{\pi}\int_0^\pi da \cos(2\beta \sin a), \tag{g^{IX}}$$

when β is a large positive quantity, we need only attend to values of a which differ little from $\frac{\pi}{2}$; making then

$$\sin a = 1 - y^2, \; da = \frac{2dy}{\sqrt{2-y^2}}, \tag{h^{IX}}$$

and neglecting y^2 in the denominator of this last expression, the integral (g^{IX}) becomes

$$f_\beta = \mathrm{A}_\beta \cos 2\beta + \mathrm{B}_\beta \sin 2\beta, \tag{i^{IX}}$$

in which, nearly,

$$\left.\begin{aligned} \mathrm{A}_\beta &= \frac{\sqrt{2}}{\pi}\int_{-\infty}^{\infty} dy \cos(2\beta y^2) = \frac{1}{\sqrt{2\pi\beta}}; \\ \mathrm{B}_\beta &= \frac{\sqrt{2}}{\pi}\int_{-\infty}^{\infty} dy \sin(2\beta y^2) = \frac{1}{\sqrt{2\pi\beta}}; \end{aligned}\right\} \tag{k^{IX}}$$

so that the large values of β which make the function (g^{IX}) vanish are nearly of the form

$$\frac{n\pi}{2} - \frac{\pi}{8}, \tag{l^{IX}}$$

n being an integer number; and such is therefore the approximate form of the large roots a_n of the equation (p^{VIII}) : results which agree with the relations (q^{VIII}), and to which POISSON has been conducted, in connexion with another subject, and by an entirely different analysis.

The theory of fluctuating functions may also be employed to obtain a more close approximation; for instance, it may be shown, by reasonings of the kind lately employed, that the definite integral (g^{IX}) admits of being expressed (more accurately as β is greater) by the following semiconvergent series, of which the first terms have been assigned by POISSON :

$$f_\beta = \frac{1}{\sqrt{\pi\beta}} \Sigma_{(i)\,0}^{\;\infty} [0]^{-i} ([-\tfrac{1}{2}]^i)^2 (4\beta)^{-i} \cos\left(2\beta - \frac{\pi}{4} - \frac{i\pi}{2}\right); \tag{m^{IX}}$$

and in which, according to a known notation of factorials,

$$\left.\begin{array}{l}[0]^{-i}=1^{-1}.\,2^{-1}.\,3^{-1}\ldots.\,i^{-1};\\ [-\tfrac{1}{2}]^{i}=\dfrac{-1}{2}.\dfrac{-3}{2}.\dfrac{-5}{2}\ldots\dfrac{1-2i}{2}.\end{array}\right\} \qquad (\mathrm{n}^{IX})$$

For the value $\beta=20$, the 3 first terms of the series (m^{IX}) give

$$\left.\begin{array}{l}f_{20}=\left(1-\dfrac{9}{204800}\right)\dfrac{\cos 86°49'52''}{\sqrt{20\pi}}+\dfrac{1}{320}\dfrac{\sin 86°49'52''}{\sqrt{20\pi}}\\ \quad=0{,}0069736+0{,}0003936=+0{,}0073672.\end{array}\right\} \qquad (\mathrm{o}^{IX})$$

For the same value of β, the sum of the first sixty terms of the ultimately convergent series

$$f_{\beta}=\Sigma_{(i)\,0}^{\ \ \infty}\,([0]^{-i})^{2}\,(-\beta^{2})^{i} \qquad (\mathrm{p}^{IX})$$

gives

$$\left.\begin{array}{r}f_{20}=+7\ 447\ 387\ 396\ 709\ 949{,}9657957\\ -7\ 447\ 387\ 396\ 709\ 949{,}9584289\\ =+0{,}0073668.\end{array}\right\} \qquad (\mathrm{q}^{IX})$$

The two expressions (m^{IX}) (p^{IX}) therefore agree, and we may conclude that the following numerical value is very nearly correct:

$$\frac{1}{\pi}\int_{0}^{\pi}d\alpha\cos(40\sin\alpha)=+0{,}007367. \qquad (\mathrm{r}^{IX})$$

[27.] Resuming the rigorous equation (W), and observing that

$$\int_{0}^{\infty}d\beta\,\mathrm{P}_{\beta\gamma}=\lim_{\beta=\infty}.\ \mathrm{N}_{\beta\gamma}\,\gamma_{\alpha}^{-1}, \qquad (\mathrm{s}^{IX})$$

we easily see that in calculating the definite integral

$$\int_{a}^{b}d\alpha\int_{0}^{\infty}d\beta\,\mathrm{P}_{\beta\gamma}\,f_{\alpha}, \qquad (\mathrm{t}^{IX})$$

in which the function f is finite, it is sufficient to attend to those values of α which are not only between the limits a and b, but are also very nearly equal to real roots x of the equation

$$\gamma_{x}=0. \qquad (\mathrm{u}^{IX})$$

The part of the integral (t^{IX}), corresponding to values of α in the neighbourhood of any one such root x, between the above-mentioned limits, is equal to the product

$$\frac{f_x}{\gamma'_x} \times \int_{-\infty}^{\infty} d\alpha \, \frac{\mathrm{N}_{\beta\gamma'_x(\alpha-x)}}{\alpha - x}, \tag{v^{IX}}$$

in which β is indefinitely large and positive, and the differential coefficient γ'_x of the function γ is supposed to be finite, and different from 0. A little consideration shows that the integral in this last expression is $= \pm \varpi$, ϖ being the same constant as in former articles, and the upper or lower sign being taken according as γ'_x is positive or negative. Denoting then by $\sqrt{\gamma'^2_x}$ the positive quantity, which is $= + \gamma'_x$ or $= - \gamma'_x$, according as γ'_x is > 0 or < 0, the part (v^{IX}) of the integral (t^{IX}) is

$$\frac{\varpi f_x}{\sqrt{\gamma'^2_x}}; \tag{w^{IX}}$$

and we have the expression

$$\int_a^b d\alpha \int_0^\infty d\beta \, \mathrm{P}_{\beta\gamma_\alpha} f_\alpha = \varpi \Sigma_x \frac{f_x}{\sqrt{\gamma'^2_x}}, \tag{x^{IX}}$$

the sum being extended to all those roots x of the equation (u^{IX}) which are $> a$ but $< b$. If any root of that equation should coincide with either of these limits a or b, the value of α in its neighbourhood would introduce, into the second member of the expression (x^{IX}), one or other of the terms

$$\frac{\varpi' f_a}{\gamma'_a}, \quad \frac{-\varpi'' f_a}{\gamma'_a}, \quad \frac{\varpi'' f_b}{\gamma'_b}, \quad \frac{-\varpi' f_b}{\gamma'_b}; \tag{y^{IX}}$$

the first to be taken when $\gamma_a = 0$, $\gamma'_a > 0$; the second when $\gamma_a = 0$, $\gamma'_a < 0$; the third when $\gamma_b = 0$, $\gamma'_b > 0$; and the fourth when $\gamma_b = 0$, $\gamma'_b < 0$. If, then, we suppose for simplicity, that neither γ_a nor γ_b vanishes, the expression (x^{IX}) conducts to the theorem

$$\Sigma_x f_x = \varpi^{-1} \int_a^b d\alpha \int_0^\infty d\beta \, \mathrm{P}_{\beta\gamma_\alpha} f_\alpha \sqrt{\gamma'^2_\alpha}; \tag{X}$$

and the sign of summation may be omitted, if the equation $\gamma_x = 0$ have only one real root between the limits a and b. For example, that one root itself may then be expressed as follows:

$$x = \varpi^{-1} \int_a^b d\alpha \int_0^\infty d\beta \, \mathrm{P}_{\beta\gamma_\alpha} \alpha \sqrt{\gamma'^2_\alpha}. \tag{z^{IX}}$$

The theorem (X) includes some analogous results which have been obtained by CAUCHY, for the case when P is a cosine.

[28]. It is also possible to extend the foregoing theorem in other ways ; and especially by applying similar reasonings to functions of several variables. Thus, if $\gamma, \gamma^{(1)} \dots$ be each a function of several real variables $a, a^{(1)}, \dots$; if P and N be still respectively functions of the kinds supposed in former articles, while $\mathrm{P}^{(1)}$, $\mathrm{N}^{(1)}, \dots$ are other functions of the same kinds ; then the theorem (W) may be extended as follows :

$$\int_a^b da \int_{a^{(1)}}^{b^{(1)}} da^{(1)} \dots \mathrm{N}_{\infty\gamma} \mathrm{N}^{(1)}_{\infty\gamma^{(1)}} \dots \mathrm{F}_{a, a^{(1)}, \dots} = 0, \tag{Y}$$

the function F being finite for all values of the variables $a, a^{(1)}, \dots$, within the extent of the integrations; and the theorem (X) may be thus extended :

$$\left.\begin{array}{l} \Sigma f_{x, x^{(1)}, \dots} = \varpi^{-1} \varpi^{(1)-1} \dots \int_a^b da \int_{a^{(1)}}^{b^{(1)}} da^{(1)} \dots \int_0^\infty d\beta \int_0^\infty d\beta^{(1)} \dots \mathrm{P}_{\beta\gamma} \mathrm{P}^{(1)}_{\beta^{(1)}\gamma^{(1)}} \dots \\ \qquad \dots f_{a, a^{(1)}, \dots} \sqrt{\mathrm{L}^2} ; \end{array}\right\} \tag{Z}$$

in which, according to the analogy of the foregoing notation,

$$\varpi^{(i)} = \int_{-\infty}^{\infty} da \int_0^1 d\beta \, \mathrm{P}^{(i)}_{\beta a} ; \tag{a$^{\mathrm{X}}$}$$

and L is the coefficient which enters into the expression, supplied by the principles of the transformation of multiple integrals,

$$\mathrm{L} da \, da^{(1)} \dots = d\gamma \, d\gamma^{(1)} \dots ; \tag{b$^{\mathrm{X}}$}$$

while the summation in the first member is to be extended to all those values of $x, x^{(1)}, \dots$ which, being respectively between the respective limits of integration relatively to the variables $a, a^{(1)}, \dots$ are values of those variables satisfying the system of equations

$$\gamma_{x, x^{(1)}, \dots} = 0, \gamma^{(1)}_{x, x^{(1)}, \dots} = 0, \dots . \tag{c$^{\mathrm{X}}$}$$

And thus may other remarkable results of CAUCHY be presented under a generalized form. But the theory of such extensions appears likely to suggest itself easily enough to any one who may have considered with attention the remarks already made ; and it is time to conclude the present paper by submitting a few general observations on the nature and the history of this important branch of analysis.

LAGRANGE appears to have been the first who was led (in connexion with the celebrated problem of vibrating cords) to assign, as the result of a species of interpolation, an expression for an arbitrary function, continuous or discontinuous in form, between any finite limits, by a series of sines of multiples, in which the coefficients are definite integrals. Analogous expressions, for a particular class of rational and integral functions, were derived by DANIEL BERNOUILLI, through successive integrations, from the results of certain trigonometric summations, which he had characterized in a former memoir as being *incongruously true.* No farther step of importance towards the improvement of this theory seems to have been made, till FOURIER, in his researches on Heat, was led to the discovery of his well known theorem, by which any arbitrary function of any real variable is expressed, between finite or infinite limits, by a double definite integral. POISSON and CAUCHY have treated the same subject since, and enriched it with new views and applications; and through the labours of these and, perhaps, of other writers, the theory of the development or transformation of arbitrary functions, through functions of determined forms, has become one of the most important and interesting departments of modern algebra.

It must, however, be owned that some obscurity seems still to hang over the subject, and that a farther examination of its principles may not be useless or unnecessary. The very existence of such transformations as in this theory are sought for and obtained, appears at first sight paradoxical; it is difficult at first to conceive the possibility of expressing a perfectly arbitrary function through any series of sines or cosines; the variable being thus made the subject of known and determined operations, whereas it had offered itself originally as the subject of operations unknown and undetermined. And even after this first feeling of paradox is removed, or relieved, by the consideration that the number of the operations of known form is infinite, and that the operation of arbitrary form reappears in another part of the expression, as performed on an auxiliary variable; it still requires attentive consideration to see clearly how it is possible that none of the values of this new variable should have any influence on the final result, except those which are extremely nearly equal to the variable originally proposed. This latter difficulty has not, perhaps, been removed to the complete satisfaction of those who desire to examine the question with all the diligence its importance deserves, by any of the published works upon the subject. A conviction, doubtless, may be attained, that the results are true, but something is, perhaps, felt to be still wanting for the full rigour of mathematical demonstration. Such has, at least, been the impression left on the mind of the present writer, after an attentive study of the reasonings usually employed, respecting the tranformations of arbitrary functions.

POISSON, for example, in treating this subject, sets out, most commonly, with a series of cosines of multiple arcs; and because the sum is generally indeterminate, when continued to infinity, he alters the series by multiplying each term by the corresponding power of an auxiliary quantity which he assumes to be less than unity, in order that its powers may diminish, and at last vanish; but, in order that the new series may tend indefinitely to coincide with the old one, he conceives, after effecting its summation, that the auxiliary quantity tends to become unity. The limit thus obtained is generally zero, but becomes on the contrary infinite when the arc and its multiples vanish; from which it is inferred by POISSON, that if this arc be the difference of two variables, an original and an auxiliary, and if the series be multiplied by any arbitrary function of the latter variable, and integrated with respect thereto, the effect of all the values of that variable will disappear from the result, except the effect of those which are extremely nearly equal to the variable originally proposed.

POISSON has made, with consummate skill, a great number of applications of this method; yet it appears to present, on close consideration, some difficulties of the kind above alluded to. In fact, the introduction of the system of factors, which tend to vanish before the integration, as their indices increase, but tend to unity, after the integration, for all finite values of those indices, seems somewhat to change the nature of the question, by the introduction of a foreign element. Nor is it perhaps manifest that the original series, of which the sum is indeterminate, may be replaced by the convergent series with determined sum, which results from multiplying its terms by the powers of a factor infinitely little less than unity; while it is held that to multiply by the powers of a factor infinitely little greater than unity would give an useless or even false result. Besides there is something unsatisfactory in employing an apparently arbitrary contrivance for annulling the effect of those terms of the proposed series which are situated at a great distance from the origin, but which do not themselves originally tend to vanish as they become more distant therefrom. Nor is this difficulty entirely removed, when integration by parts is had recourse to, in order to show that the effect of these distant terms is insensible in the ultimate result; because it then becomes necessary to differentiate the arbitrary function; but to treat its differential coefficient as always finite, is to diminish the generality of the inquiry.

Many other processes and proofs are subject to similar or different difficulties; but there is one method of demonstration employed by FOURIER, in his separate Treatise on Heat, which has, in the opinion of the present writer, received less notice than it deserves, and of which it is proper here to speak. The principle of the method here alluded to may be called the *Principle of Fluctuation*, and is the same which was enunciated under that title in the remarks prefixed to this paper. In virtue of this principle (which may thus be considered as having been indicated by FOURIER, although not expressly stated by him), if any function, such as the sine or cosine of an infinite multiple of an arc, changes sign infinitely often within a finite extent of the variable on which it depends, and has for its mean value zero; and if this, which may be called a *fluctuating function*, be multiplied by any arbitrary but finite function of the same variable, and afterwards integrated between any finite limits; the integral of the product will be zero, on account of the mutual destruction or neutralization of all its elements.

It follows immediately from this principle, that if the factor by which the fluctuating function is multiplied, instead of remaining always finite, becomes infinite between the limits of integration, for one or more particular values of the variable on which it depends; it is then only necessary to attend to values in the immediate neighbourhood of these, in order to obtain the value of the integral. And in this way FOURIER has given what seems to be the most satisfactory published proof, and (so to speak) the most natural explanation of the theorem called by his name; since it exhibits the actual process, one might almost say the interior mechanism, which, in the expression assigned by him, destroys the effect of all those values of the auxiliary variable which are not required for the result. So clear, indeed, is this conception, that it admits of being easily translated into geometrical constructions, which have accordingly been used by FOURIER for that purpose.

There are, however, some remaining difficulties connected with this mode of demonstration, which may perhaps account for the circumstance that it seems never to be mentioned, nor alluded to, in any of the historical notices which

POISSON has given on the subject of these transformations. For example, although FOURIER, in the proof just referred to, of the theorem called by his name, shows clearly that in integrating the product of an arbitrary but finite function, and the sine or cosine of an infinite multiple, each successive positive portion of the integral is destroyed by the negative portion which follows it, if infinitely small quantities be neglected, yet he omits to show that the infinitely small outstanding difference of values of these positive and negative portions, corresponding to the single period of the trigonometric function introduced, is of the second order; and, therefore, a doubt may arise whether the infinite number of such infinitely small periods, contained in any finite interval, may not produce, by their accumulation, a finite result. It is also desirable to be able to state the argument in the language of limits, rather than in that of infinitesimals; and to exhibit, by appropriate definitions and notations, what was evidently foreseen by FOURIER, that the result depends rather on the *fluctuating* than on the *trigonometric* character of the auxiliary function employed.

The same view of the question had occurred to the present writer, before he was aware that indications of it were to be found among the published works of FOURIER; and he still conceives that the details of the demonstration to which he was thus led may be not devoid of interest and utility, as tending to give greater rigour and clearness to the proof and the conception of a widely applicable and highly remarkable theorem.

Yet, if he did not suppose that the present paper contains something more than a mere expansion or improvement of a known proof of a known result, the Author would scarcely have ventured to offer it to the Transactions* of the Royal Irish Academy. It aims not merely to give a more perfectly satisfactory demonstration of FOURIER's celebrated theorem than any which the writer has elsewhere seen, but also to present that theorem, and many others analogous thereto, under a greatly generalized form, deduced from the principle of fluctu-

* The Author is desirous to acknowledge, that since the time of his first communicating the present paper to the Royal Irish Academy, in June, 1840, he has had an opportunity of entirely rewriting it, and that the last sheet is only now passing through the press, in June, 1842. Yet it may be proper to mention also that the theorems (A) (B) (C), which sufficiently express the character of the communication, were printed (with some slight differences of notation) in the year 1840, as part of the *Proceedings* of the Academy for the date prefixed to this paper.

ation. Functions more general than sines or cosines, yet having some correspondent properties, are introduced throughout; and constants, distinct from the ratio of the circumference to the diameter of a circle, present themselves in connexion therewith. And thus, if the intention of the writer have been in any degree accomplished, it will have been shown, according to the opinion expressed in the remarks prefixed to this paper, that the development of the important principle above referred to gives not only a new clearness, but also (in some respects) a new extension, to this department of science.

XIII.—*On the Minute Structure of the Brain in the Chipanzee, and of the human Idiot, compared with that of the perfect Brain of Man; with some Reflections on the Cerebral Functions.* By James Macartney, M. D., F. R. S., F. L. S., M. R. I. A., &e. &e.

Read June 27, 1842.

MANY years ago I diseovered, with only a eommon poeket lens, a retieulation of fine white fibres, immediately under the surface of the cerebrum, in birds. This first led me to believe that the medullary fibres, as they are ealled, extended farther, and were more subdivided than had been hitherto supposed. I have sinee been able to demonstrate to medical students, and to several teachers of anatomy, the existenee of those filaments in every part of the brain, by simply moistening the substance of the organ, during the dissection, with a solution of alum in water, whieh has the effect of slightly coagulating, and rendering the finer filaments visible, which, in their natural condition, are transparent. By this means, I have shown that the filaments (which I prefer to eall *sentient*, instead of *white* or *medullary*) everywhere assumed a plexiform arrangement, and that the most delieate and intricate plexusus were to be found inelosed in the grey or coloured substanees of the brain. This faet proves the analogy between the eoloured substanees of the brain, and the ganglia of the nervous system, in whieh there is a elose retieulation of nervous fibres. I have long been in the habit of considering the magnitude and form of the entire brain, and of its several parts, as being merely subservient to the number, extent, and eonnexions of the various plexuses, in whieh, and espeeially in those oeeupying the eoloured substanees, I believe the sensorial powers of the brain to reside.

A Chimpanzee (the pigmy of Tyson) having some months ago died in Dublin, and the disseetion of it having been entrusted to Mr. Wilde, I proposed to him that I should undertake the examination of the animal's brain, in my own manner. Tyson and others had described the bulk, shape, and external appearance of the different parts of this creature's brain, but the intimate structure had never been examined by any anatomist.

I shall now lay before the Academy an account of what I observed in the brain of the Chimpanzee, and likewise in those of two idiots; by which it will appear that the brain in the latter possesses a still lower degree of organization, than in the former animal.

DISSECTION OF THE BRAIN IN THE CHIMPANZEE (SIMIA TROGLODYTES.—LIN.)

The *external form* bore so great a resemblance to the human brain, that, excepting the difference in size, the one might be mistaken for the other. The *convolutions* were as decidedly marked, and the proportions of the cerebellum to the cerebrum were exactly as in man. On the under surface of the brain I observed that the two white pea-shaped bodies, called *corpora candicantia*, were very indistinct; and they did not appear to be, as in man, the continuation of the anterior crura of the *fornix*. The *pons*, which unites the lateral lobes of the cerebellum, was, perhaps, rather flatter than in the human subject, and the fifth pair of nerves entered it, and passed for a little way distinctly, which is so remarkable in the sheep. The *pyramids* did not decussate to any extent; only two superficial bundles of fibres crossed. The *corpora olivaria* did not project distinctly, and the band which surrounds them was not observed. The structure internally of these bodies consisted of white filaments included in grey substance. The branches of the *arbor vitæ* were, perhaps, not so deep, but quite as numerous as in us. The white filaments composing the trunk were not so fine, nor so strictly interwoven, as in man, and therefore they were more easily distinguished. The *corpus fimbriatum* was a long shape, and appeared to be composed chiefly of grey substance, and wanted the denticulated edge. The part called *locus niger*, in the crura of the cerebrum, was a small, greenish-grey mass, of an irregular figure, and less than a pea, instead of the crescentic form, as in man; and it did not mingle with the white fibres of the crus. The *pineal* gland was large. It was removed in making a cast of the ventricles, and lost; it was not, therefore, ascertained whether it had any calcareous matter in it or not. The parts in the lateral ventricles corresponded very nearly with the same in man. The *soft com-*

missure was particularly strong, and held distinct white filaments. The *linea semilunaris* was faintly marked. The two anterior of the *tubercula quadrigemina*, called *nates*, were the smaller. The *fourth ventricle* was much prolonged into the lateral lobes of the cerebellum. The *grey substance* on the floor of the ventricle was not raised into the appearance of two ganglia, and there were no *white striæ*. The sentient or white filaments formed looser or less complicated plexuses, wherever they were examined, than in man, and I could not discover any of the delicate *arborescent filaments* in the base of the *corpora striata*.

DISSECTION OF A FEMALE IDIOT, WITH EXTRAORDINARY BRAIN.

The whole mass of the brain was small, but the front part did not recede. The *convolutions* were rather small, but sufficiently deep for the size of the brain. The *lobes* of the *cerebellum* were not the one-third of the usual size. The *gyri* were scarcely distinguishable, and the divisions were few and shallow. The *arbor vitæ* had but two principal branches, and the sub-divisions of these were few. The anterior part of the lobes was supplied by two clusters of membranous cells, filled with red jelly or albuminous fluid, such as we find substituted for the brain in acephalous fœtuses. The *corpus fimbriatum* was indistinct, wanted the denticulated margin, and the proper structure interiorly, and was not half the proper size. The *pons* was exceedingly small, and its internal structure obscure. The *pyramids* were parallel cylindric forms, and did not appear to decussate. The *corpora olivaria* had little prominence, and the coloured substance was deficient. The *locus niger* was imperfectly formed, and not of a dark colour. The *corpora striata* were very small, as also the white filaments contained in them. The *pineal gland* was rather of a large size, and contained a cluster of round soft bodies, in place of the calcareous granules. In fine, the character of the whole brain was imperfection of intimate structure. The plexuses were not intricate, and the grey substances pale, and not in sufficient quantity. This person had been a patient in the Whitworth Hospital. The account I received of the state of her intellect from the house pupil was, that she was foolish, and that he could never get a rational answer from her. She was extremely ugly, with projecting jaws and teeth, and an idiotic countenance. She was an unmarried woman, but not a virgin, notwithstanding the great deficiency in her organ of amativeness.

DISSECTION OF THE BRAIN OF A MALE IDIOT.

The *cerebrum* was small, and the anterior lobes especially so. The *cerebellum* projected beyond the posterior lobes of the hemispheres. The *convolutions* of the cerebrum were small, particularly those of the anterior lobes on the *left side*,—they were so imperfectly developed, and so closely connected to each other, that they had more the appearance of a tuberculated than of a convoluted surface. The *olfactory nerves* were small, and very deficient in grey substance, indeed all the coloured parts of the brain were rather pale. The *pyramids* could scarcely be distinguished, being extremely small, and confounded in the projection of the corpora olivaria; they did not appear to decussate; the one on the left side was particularly small. The left hemisphere of the brain was smaller than the one on the right side. The *tubercula quadrigemina* were of an equal size, and a grey colour on their surface. The *pineal gland* was large, semi-transparent, and contained very little of the gritty matter. On the surface of the *left* crus of the cerebrum there was a green tinge observed, which, on being cut into, proved to be the *locus niger* in a disorganized and nearly dissolved state. There were no *white striæ* in the fourth ventricle. The *plexus* of white filaments at the roots of the *olfactory nerves* was very plain on the *right* side, but very imperfect on the *left*. The *brain* was tolerably firm. The *spinal marrow* was hard, and the *cerebellum* was soft. The structure, as well as form of the parts in this brain, was imperfect throughout, but most remarkably so on the left side; the want of agreement between the two sides would necessarily impair the functions of the brain.

The first deviations from the perfect brain of man appear to be with respect to the following parts:—The *locus niger, the corpus fimbriatum, the white striæ in the floor of the fourth ventricle, the decussation of the pyramids, the distinction of the anterior crura of the fornix, the corpora olivaria, the degree of intermixture of the sentient or white filaments in the arbor vitæ, the corpora candicantia, and the existence of calcareous granules in the pineal gland.*

It is remarkable, that many of these parts are not found in the first stages of fœtal life, and some of them not until after birth. The pineal gland, according to Meckel, is not perfect until the seventh year of infancy. The same parts, also, first decline, and ultimately disappear in animals, according to their scale of organization; and further, it is chiefly with respect to these parts, that varieties of structure are observed in the brains of different rational human beings. I have found many deviations from the ordinary structure in subjects, without being able to ascertain what peculiarities of character belonged to them when alive; but in one instance, of a deaf and dumb person, the *white striæ* of the fourth ventricle (with which the auditory nerves communicate) were imperfectly formed, were not subdivided, and did not unite with each other. If, therefore, we can ever arive at correct notions of the functions of the brain, it must be by careful dissections of the interior parts of the cerebral organ, and by ascertaining the correspondence between the minute structure, and the endowments and dispositions of the different individuals; taking into account, at the same time, the influence of the various organs of the body, instead of ascribing to certain parts on the surface of the brain, distinct and often opposing faculties, as Gall and Spurzheim have done.

It seems to be particularly absurd to suppose that the cerebellum, a part evidently as highly organized, and of as much importance as the cerebrum itself, should be designed to produce merely the sexual instinct. In animals that have the lateral lobes of the cerebellum very small, or who want them altogether, this instinct is stronger than in man. In those instances which are known of the absence of a part, or one lobe, or the whole cerebellum, no want of the venereal appetite existed; and a case is related of a person in whom the sexual desire was so ungovernable, that mechanic restraint became necessary; and it was found, after death, that both lobes of the cerebellum were wanting in this person. In animals that propagate only at particular seasons of the year, the testicles and ovaries are singularly developed at those periods, and afterwards decline, while at the same time no change takes place in the cerebellum. The abolition of the sexual instinct, by the extirpation of the testes, or of the ovaries, puts it beyond all doubt that this impulse does not originate in any part of the brain.

It would appear that all instincts depend upon the condition and state of feeling in those organs with the functions of which they are immediately connected; thus, the maternal instinct (at least in mammiferous animals) is in a great measure the result of the tension of the mammary glands. As soon as this is removed, by the absorbents carrying off the milk, quadrupeds lose all care and anxiety about their young. The cerebral organ would, perhaps, of all others, be the most unfit for the generation of instincts. The brain is destined to direct or control instinctive feelings, and therefore it cannot create them. If a person attempt to command any instinctive impulse to be felt, he will find it as impossible to do, as to rise from his chair, merely by willing it, without the aid of the muscles.

I have ascertained and demonstrated, by repeated dissections, that all the plexuses of the brain are continuous with each other ; that no part of the nervous system is isolated ; and, consequently, the different parts must exercise a mutual influence on each other. I have proved that the spinal nerves, as well as those of the brain, are not inserted in the same way as the roots of plants penetrate the earth, which has been heretofore believed, but that they are united with the parts from which they are supposed to arise, and that the spinal nerves form a chain of communication with each other, after they enter the spinal marrow. It is in consequence of the integrity of the whole nervous system, that the various sympathies, both natural and morbid, exist between the different organs of the body. If the continuity of the sentient or nervous filaments were to be intercepted at any one place, their functions would be arrested at that point, in the same manner as the division of a nerve, destroys sensation and voluntary motion in the parts to which the nerve is sent.

Some anatomists, it is true, have supposed that the various reticulations of the nerves, and the intermixture of the filaments of the brain, were merely to bring them into contact, and that there was no incorporation of the sentient substances. This opinion is consequent upon another, as ill supported by facts ; namely, that there is a subtile or nervous fluid, which carries impressions made on the nerves to the brain, and thus causes sensation ; and that the same fluid, proceeding from the brain to the muscles, produces voluntary motions. It has never been, however, attempted to explain how this imaginary fluid could become the instrument of sensation or volition, more than the sentient substance itself. For my part, I am satisfied with the knowledge of the undoubted fact, that the peculiar matter which exists in the nerves, and the white filaments of the brain, is endowed with the *power* of *feeling*—a power perfectly distinct from every other in nature ; and I think it is equally obvious that the various modifications of sensorial function we observe are the result, and require for their production, the multitude of subdivisions and re-unions that take place in the sentient filaments of the brain and nerves. Voluntary motion appears to me to be the natural consequence of the connexion between the central part of the nervous system, and the muscles which move in obedience to the will or desire of the individual.

EXPLANATION OF THE PLATES.

Plate I.—*Fig.* 1. Was drawn from an accurate plaster cast of the upper surface of the brain of the Chimpanzee.

Fig. 2. Was taken from the cast of the lower surface of the same brain. Both these figures are of the natural size.

Plate II. Exhibits the different parts as they were found on the inferior surface of the brain of an idiot.

a a. The two lateral lobes of the cerebellum, exceedingly small, and imperfectly formed.

b b. The membranous cells, which held a reddish fluid.

c. The pons or commissure of the cerebellum, also small and imperfect.

d d. The pyramidal bodies.

e e. The olive-shaped bodies, making scarcely any projection.

f f. The olfactory nerves.

g g. The optic nerves.

h h. The third pair of nerves.

The other nerves were not preserved.

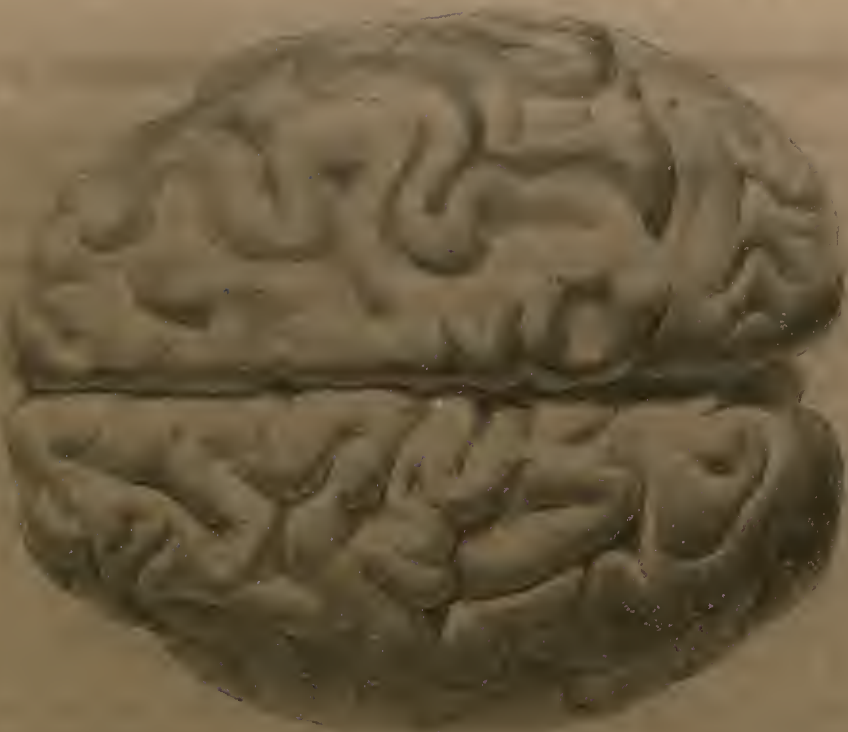
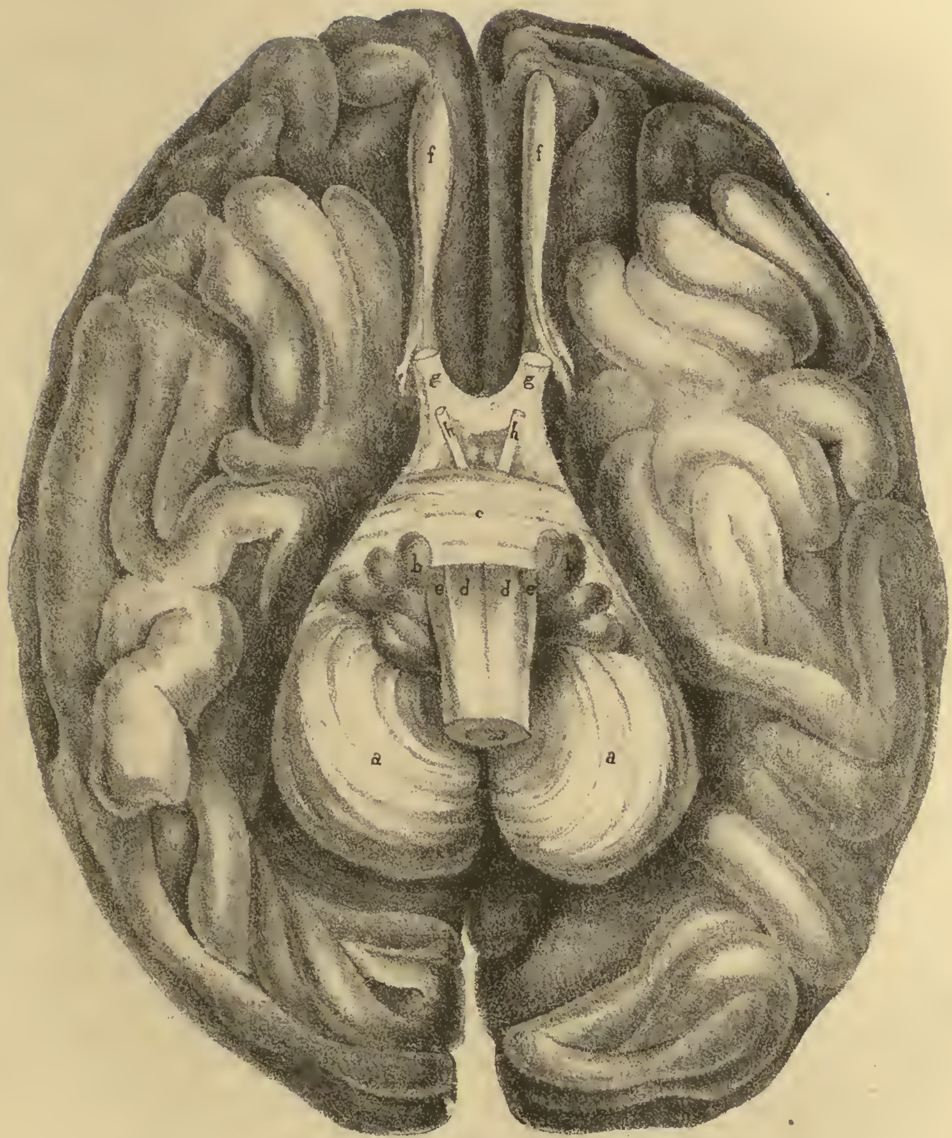


PLATE 2.



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Allen's Litho 16 Trinity St.

XIV.—*On Equations of the Fifth Degree: and especially on a certain System of Expressions connected with those Equations, which Professor Badano* has lately proposed.* By Sir William Rowan Hamilton, LL.D., P.R.I.A., F.R.A.S., *Honorary Member of the Royal Societies of Edinburgh and Dublin; Honorary or Corresponding Member of the Royal or Imperial Academies of St. Petersburgh, Berlin, and Turin, of the American Society of Arts and Sciences, and of other Scientific Societies at home and abroad; Andrews' Professor of Astronomy in the University of Dublin, and Royal Astronomer of Ireland.*

Read 4th August, 1842.

1. LAGRANGE has shown that if α be a given root of the equation

$$\alpha^{n-1} + \alpha^{n-2} + \ldots + \alpha^2 + \alpha + 1 = 0,$$

n being a prime factor of m, and if μ denote for abridgment the quotient

$$\mu = \frac{1.2.3\ldots m}{\left(1.2.3\ldots\frac{m}{n}\right)^n};$$

then the function

$$t = x' + \alpha x'' + \alpha^2 x''' + \ldots + \alpha^{m-1} x^{(m)}$$

has only μ different values, corresponding to all possible changes of arrangement of the m quantities $x', x'', \ldots x^{(m)}$, which may be considered as the roots of a given equation of the m^{th} degree,

$$x^m - \mathrm{A}x^{m-1} + \mathrm{B}x^{m-2} - \mathrm{C}x^{m-3} + \ldots = 0;$$

* Nuove Ricerche sulla Risoluzione Generale delle Equazioni Algebriche del P. Gerolamo Badano, Carmelitano scalzo, Professore di Matematica nella R. Universita di Genova. Genova, Tipografia Ponthenier, 1840.

and that if the development of the n^{th} power of this function t be reduced, by the help of the equation

$$a^n = 1,$$

(and not by the equation $a^{n-1} + \&c. = 0$,) to the form

$$t^n = \xi^{(0)} + a\xi' + a^2\xi'' + \ldots + a^{n-1}\xi^{(n-1)},$$

then this power t^n itself has only $\frac{\mu}{n}$ different values, and the term $\xi^{(0)}$ has only $\frac{\mu}{n(n-1)}$ such values, or is a root of an equation of the degree

$$\frac{1.2.3\ldots.m}{n(n-1)\left(1.2.3\ldots\frac{m}{n}\right)^n},$$

of which equation the coefficients are rational functions of the given coefficients A, B, C, &c.; while $\xi', \xi'', \ldots \xi^{(n-1)}$ are the roots of an equation of the degree $n-1$, of which the coefficients can be expressed rationally in terms of $\xi^{(0)}$ and of the same original coefficients A, ... of the given equation in x.

2. For example, if there be given an equation of the sixth degree,

$$x^6 - \text{A}x^5 + \text{B}x^4 - \text{C}x^3 + \text{D}x^2 - \text{E}x + \text{F} = 0,$$

of which the roots are denoted by $x', x'', x''', x^{IV}, x^V, x^{VI}$, and if we form the function

$$t = x' + ax'' + a^2x''' + a^3x^{IV} + a^4x^V + a^5x^{VI},$$

in which $a = -1$; we shall then have

$$m = 6,\ n = 2,\ \mu = \frac{720}{36} = 20,\ \frac{\mu}{n} = 10,\ \frac{\mu}{n(n-1)} = 10;$$

and the function t will have twenty different values, but its square will have only ten. And if, by using only the equation $a^2 = 1$, and not the equation $a = -1$, we reduce the development of this square to the form

$$t^2 = \xi^{(0)} + a\xi',$$

the term $\xi^{(0)}$ will itself be a ten-valued function of the six quantities $x', \ldots x^{VI}$; and ξ' will be a rational function of $\xi^{(0)}$ and A, namely,

$$\xi' = \text{A}^2 - \xi^{(0)}.$$

3. Again, if with the same meanings of $x', \ldots x^{VI}$, we form t by the same expression as before, but suppose a to be a root of the equation

$$a^2 + a + 1 = 0,$$

then

$$m = 6,\ n = 3,\ \mu = \frac{720}{8} = 90,\ \frac{\mu}{n} = 30,\ \frac{\mu}{n(n-1)} = 15;$$

so that the function t will now have 90 different values, but its cube will have only 30; and if that cube be reduced, by the equation $a^3 = 1$, to the form

$$t^3 = \xi^{(0)} + a\xi' + a^2\xi'',$$

then $\xi^{(0)}$ will be a root of an equation of the fifteenth degree, while ξ' and ξ'' will be the roots of a quadratic equation, the coefficients of this last equation being rational functions of $\xi^{(0)}$, and of the given coefficients A, &c.

4. And if, in like manner, we consider the case

$$m = 5,\ n = 5,\ \mu = 120,\ \frac{\mu}{n} = 24,\ \frac{\mu}{n(n-1)} = 6,$$

so that $x', \ldots x^{V}$ are the roots of a given equation of the fifth degree

$$x^5 - \mathrm{A}x^4 + \mathrm{B}x^3 - \mathrm{C}x^2 + \mathrm{D}x - \mathrm{E} = 0,$$

and

$$t = x' + ax'' + a^2x''' + a^3x^{IV} + a^4x^{V},$$

in which a is a root of the equation

$$a^4 + a^3 + a^2 + a + 1 = 0,$$

then the function t has itself 120 different values, but its fifth power has only 24; and if this fifth power be put under the form

$$t^5 = \xi^{(0)} + a\xi' + a^2\xi'' + a^3\xi''' + a^4\xi^{IV},$$

by the help of the equation $a^5 = 1$, then $\xi^{(0)}$ is a root of an equation of the sixth degree, of which the coefficients are rational functions of A, B, C, D, E, while $\xi', \xi'', \xi''', \xi^{IV}$, are the roots of an equation of the fourth degree, of which the coefficients are rational functions of the same given coefficients A, &c., and of $\xi^{(0)}$.

5. LAGRANGE has shown that these principles explain the success of the known methods for resolving quadratic, cubic, and biquadratic equations; but

that they tend to discourage the hope of resolving any general equation above the fourth degree, by any similar method. And in fact it has since* been shown to be impossible to express any root of any general equation, of the fifth or any higher degree, as a function of the coefficients of that equation, by any finite combination of radicals and rational functions. Yet it appears to be desirable to examine into the validity and import of an elegant system of radical expressions which have lately been proposed by Professor BADANO of Genoa, for the twenty-four values of LAGRANGE's function t^5 referred to in the last article; and to inquire whether these new expressions are adapted to assist in the solution of equations of the fifth degree, or why they fail to do so.

6. In order to understand more easily and more clearly the expressions which are thus to be examined, it will be advantageous to begin by applying the method by which they are obtained to equations of lower degrees. And first it is evident that the general quadratic equation,

$$x^2 - \text{A}x + \text{B} = 0,$$

has its roots expressed as follows:

$$x' = \alpha + \beta, \;\; x'' = \alpha - \beta;$$

α not here denoting any root of unity, but a rational function of the coefficients of the given equation (namely $\frac{1}{2}\text{A}$), and β^2 being another rational function of those coefficients (namely $\frac{1}{4}\text{A}^2 - \text{B}$); because by the general principles of article 1., when $m = 2$ and $n = 2$, we have $\frac{\mu}{n} = 1$, so that the function $(x' - x'')^2$ is symmetric, as indeed it is well known to be.

7. Proceeding to the cubic equation

$$x^3 - \text{A}x^2 + \text{B}x - \text{C} = 0,$$

and seeking the values of the function

$$t^3 = (x' + \theta x'' + \theta^2 x''')^3,$$

in which θ is such that

$$\theta^2 + \theta + 1 = 0,$$

* See a paper by the present writer, "On the Argument of Abel," &c., in the Second Part of the Eighteenth Volume of the Transactions of this Academy.

we know first, by the same general principles, that the number of these values is two, because $\frac{\mu}{n} = 2$, when $m = 3$, $n = 3$. And because these values will not be altered by adding any common term to the three roots x', x'', x''', it is permitted to treat the sum of these three roots as vanishing, or to assume that

$$x' + x'' + x''' = 0;$$

that is, to reduce the cubic equation to the form

$$x'^3 + px' + q = 0.$$

In other words, the function

$$t^3 = (x_1 + \theta x_2 + \theta^2 x_3)^3,$$

in which x_1, x_2, x_3 are the three roots of the equation with coefficients A, B, C, will depend on those coefficients, only by depending on p and q, if these two quantities be chosen such that we shall have identically

$$x^3 - \text{A}x^2 + \text{B}x - \text{C} = (x - \tfrac{1}{3}\text{A})^3 + p\,(x - \tfrac{1}{3}\text{A}) + q.$$

8. This being perceived, and x'' and x''' being seen to be the two roots of the quadratic equation

$$x''^2 + x'\,x'' + x'^2 + p = 0,$$

which is obtained by dividing the cubic

$$x''^3 + px'' - x'^3 - px' = 0,$$

by the linear factor $x'' - x'$; we may, by the theory of quadratics, assume the expressions

$$x'' = \alpha + \beta, \quad x''' = \alpha - \beta,$$

provided that we make

$$\alpha = -\tfrac{1}{2}x', \quad \beta^2 = -\tfrac{3}{4}x'^2 - p,$$

that is, provided that we establish the identity

$$(x'' - \alpha)^2 - \beta^2 = x''^2 + x'\,x'' + x'^2 + p.$$

And, substituting for x', x'', x''', their values as functions of α and β, and reducing by the equation $\theta^2 + \theta + 1 = 0$, we find

$$t^3 = \{-3\alpha + (\theta - \theta^2)\,\beta\}^3 = \alpha' + \beta';$$

in which

$$a' = -27a(a^2 - \beta^2), \quad \beta'^2 = -27\beta^2(9a^2 - \beta^2)^2.$$

But a and β^2 are rational functions of x' and p; and substituting their expressions as such, we find corresponding expressions for a' and β'^2, namely,

$$a' = \frac{27}{2}x'(x'^2 + p), \quad \beta'^2 = \frac{27}{4}(3x'^2 + 4p)(3x'^2 + p)^2.$$

9. Finally, x' is such that

$$x'^3 + px' = -q;$$

and it is found on trial to be possible by this condition to eliminate x' from the expressions for a' and β'^2, obtained at the end of the last article, and so to arrive at these other expressions, which are rational functions of p and q:

$$a' = -\frac{27}{2}q, \quad \beta'^2 = \frac{27}{4}(27q^2 + 4p^3).$$

In this manner then it might have been discovered, what has long been otherwise known, that the function t^3 is a root of the auxiliary quadratic equation

$$(t^3)^2 + 27q\,(t^3) - 27p^3 = 0.$$

And because the same method gives

$$(x' + \theta x'' + \theta^2 x''')(x' + \theta^2 x'' + \theta x''') = 9a^2 + 3\beta^2 = -3p,$$

we should obtain the known expressions for the three roots of the cubic equation

$$x'^3 + px' + q = 0,$$

under the forms:

$$x' = \frac{t}{3} - \frac{p}{t}, \quad x'' = \frac{\theta^2 t}{3} - \frac{\theta p}{t}, \quad x''' = \frac{\theta t}{3} - \frac{\theta^2 p}{t};$$

which are immediately verified by observing that

$$\theta^3 = 1, \ \left(\frac{t}{3}\right)^3 - \left(\frac{p}{t}\right)^3 = -q.$$

The foregoing method therefore succeeds completely for equations of the third degree.

10. In the case of the biquadratic equation, deprived for simplicity of its second term, namely,

$$x'^4 + px'^2 + qx' + r = 0,$$

so that the sum of the four roots vanishes,

$$x' + x'' + x''' + x^{IV} = 0,$$

we may consider x'', x''', x^{IV}, as roots of the cubic equation

$$x''^3 + x'\,x''^2 + (x'^2 + p)\,x'' + x'^3 + px' + q = 0;$$

and this may be put under the form

$$(x'' - a)^3 - 3\eta\,(x'' - a) - 2\epsilon = 0,$$

of which the roots (by the theory of cubic equations) may be expressed as follows:

$$x'' = a + \beta + \gamma, \quad x''' = a + \theta\beta + \theta^2\gamma, \quad x^{IV} = a + \theta^2\beta + \theta\gamma,$$

β, γ, and θ, being such as to satisfy the conditions

$$\beta^3 + \gamma^3 = 2\epsilon, \quad \beta\gamma = \eta, \quad \theta^2 + \theta + 1 = 0.$$

Comparing the two forms of the cubic equation in x'', we find the relations

$$x' = -3a, \quad x'^2 + p = 3\,(a^2 - \eta), \quad x'^3 + px' + q = -a^3 + 3a\eta - 2\epsilon;$$

which give

$$a = -\tfrac{1}{3}x', \quad \eta = -\tfrac{1}{9}\,(2x'^2 + 3p), \quad \epsilon = \tfrac{-1}{54}\,(20x'^3 + 18px' + 27q).$$

Thus, any rational function of the four roots of the given biquadratic can be expressed rationally in terms of a, β, γ; while a, $\beta\gamma$, and $\beta^3 + \gamma^3$, are rational functions of x', p, q; and the function $x'^4 + px'^2 + qx'$ may be changed, wherever it occurs, to the given quantity $-r$.

11. With these preparations it is easy to express, as follows, the function

$$(x' - x'' + x''' - x^{IV})^2,$$

which the general theorems of LAGRANGE, already mentioned, lead us to consider. Denoting it by $4z$, we have

$$z = (-2a + \theta\beta + \theta^2\gamma)^2 = a' + \theta\beta' + \theta^2\gamma';$$

in which

$$a' = 4a^2 + 2\beta\gamma, \quad \beta' = \gamma^2 - 4a\beta, \quad \gamma' = \beta^2 - 4a\gamma:$$

and the three values of z are the three roots of the cubic equation

$$(z-\alpha')^3-3\eta'(z-\alpha')-2\epsilon'=0;$$

in which

$$\alpha'=4\alpha^2+2\eta,$$
$$\eta'=\beta'\gamma'=\eta^2+16\alpha^2\eta-8\alpha\epsilon,$$
$$\epsilon'=\tfrac{1}{2}(\beta'^3+\gamma'^3)=2\epsilon^2-\eta^3-12\alpha\epsilon\eta+48\alpha^2\eta^2-64\alpha^3\epsilon.$$

Substituting for α, η, ϵ, their values, as functions of x', p, q, we find

$$\alpha'={}^-\tfrac{2}{3}p;$$
$$\eta'=\tfrac{1}{9}(-12x'^4-12px'^2-12qx'+p^2);$$
$$\epsilon'=\tfrac{1}{54}(72px'^4+72p^2x'^2+72pqx'+27q^2+2p^3);$$

and eliminating x', by the condition

$$x'^4+px'^2+qx'=-r,$$

we obtain

$$\eta'=\tfrac{1}{9}(12r+p^2);$$
$$\epsilon'=\tfrac{1}{54}(-72pr+27q^2+2p^3).$$

The auxiliary cubic in z becomes therefore

$$(z+\tfrac{2}{3}p)^3-\tfrac{1}{3}(12r+p^2)(z+\tfrac{2}{3}p)+\tfrac{1}{27}(72pr-27q^2-2p^3)=0;$$

that is

$$z^3+2p\,z^2+(p^2-4r)\,z-q^2=0;$$

and if its threc roots be denoted by z', z'', z''', in an order such that we may write

$$z'=\tfrac{1}{4}(x'+x''-x'''-x^{IV})^2=\alpha'+\beta'+\gamma',$$
$$z''=\tfrac{1}{4}(x'-x''+x'''-x^{IV})^2=\alpha'+\theta\beta'+\theta^2\gamma',$$
$$z'''=\tfrac{1}{4}(x'-x''-x'''+x^{IV})^2=\alpha'+\theta^2\beta'+\theta\gamma',$$

we may express the four roots of the biquadratic equation under known forms, by means of the square roots of z', z'', z''', as follows:

$$x'=+\tfrac{1}{2}\sqrt{z'}+\tfrac{1}{2}\sqrt{z''}+\tfrac{1}{2}\sqrt{z'''},$$
$$x''=+\tfrac{1}{2}\sqrt{z'}-\tfrac{1}{2}\sqrt{z''}-\tfrac{1}{2}\sqrt{z'''},$$
$$x'''=-\tfrac{1}{2}\sqrt{z'}+\tfrac{1}{2}\sqrt{z''}-\tfrac{1}{2}\sqrt{z'''},$$
$$x^{IV}=-\tfrac{1}{2}\sqrt{z'}-\tfrac{1}{2}\sqrt{z''}+\tfrac{1}{2}\sqrt{z'''}.$$

It may be noticed also that the present method gives for the product of these three square roots, the expression:

$$\sqrt{z'}.\sqrt{z''}.\sqrt{z'''} = \tfrac{1}{8}(x' + x'' - x''' - x^{IV})(x' - x'' + x''' - x^{IV})$$
$$(x' - x'' - x''' + x^{IV})$$
$$= (-2a + \beta + \gamma)(-2a + \theta\beta + \theta^2\gamma)(-2a + \theta^2\beta + \theta\gamma)$$
$$= -8a^3 + 6a\eta + 2\epsilon = -q;$$

a result which may be verified by observing that, by the expressions given above for a', η', ϵ', in terms of a, η, ϵ, we have the relation

$$z'z''z''' = a'^3 - 3a'\eta' + 2\epsilon' = (-8a^3 + 6a\eta + 2\epsilon)^2.$$

12. In this manner, then, it might have been discovered that the four roots x_1, x_2, x_3, x_4, of the general biquadratic equation

$$x^4 - \text{A}x^3 + \text{B}x^2 - \text{C}x + \text{D} = 0,$$

are the four values of an expression of the form $a + \beta + \gamma + \delta$, in which, a, $\beta^2 + \gamma^2 + \delta^2$, $\beta\gamma\delta$, and $\beta^2\gamma^2 + \gamma^2\delta^2 + \delta^2\beta^2$, are rational functions of the coefficients A, B, C, D, and may be determined as such by comparison with the identical equation

$$(a + \beta + \gamma + \delta - a)^4 - 2(\beta^2 + \gamma^2 + \delta^2)(a + \beta + \gamma + \delta - a)^2$$
$$+ (\beta^2 + \gamma^2 + \delta^2)^2 = 8\beta\gamma\delta(a + \beta + \gamma + \delta - a) + 4(\beta^2\gamma^2 + \gamma^2\delta^2 + \delta^2\beta^2),$$

of which each member is an expression for the square of $2(\beta\gamma + \gamma\delta + \delta\beta)$. It might have been perceived also that any three quantities, such as here β^2, γ^2, δ^2, which are the three roots of a given cubic equation, may be considered as the three values of an expression of the form $a' + \beta' + \gamma'$, in which, a', $\beta'\gamma'$, and $\beta'^3 + \gamma'^3$ are rational functions of the coefficients of that given equation, and may have their forms determined by comparison with the identity,

$$(\alpha' + \beta' + \gamma' - \alpha')^3 - 3\beta'\gamma'(\alpha' + \beta' + \gamma' - \alpha') - \beta'^3 - \gamma'^3 = 0.$$

And finally that any two quantities which, as here β'^3 and γ'^3, are the two roots of a given quadratic equation, are also the two values of an expression of the form $a'' + \beta''$, in which a'' and β''^2 may be determined by comparing the given equation with the following identical form,

$$(\alpha'' + \beta'' - \alpha'')^2 - \beta''^2 = 0.$$

Let us now endeavour to apply similar methods of expression to a system of five arbitrary quantities, or to an equation of the fifth degree.

13. Let, therefore, x_1, x_2, x_3, x_4, x_5, be the five roots of the equation

$$x^5 - \text{A}x^4 + \text{B}x^3 - \text{C}x^2 + \text{D}x - \text{E} = 0, \tag{1}$$

and let x', x'', x''', x^{IV}, x^V, be the five roots of the same equation when deprived of its second term, or put under the form

$$x'^5 + px'^3 + qx'^2 + rx' + s = 0, \tag{2}$$

so that

$$x' + x'' + x''' + x^{IV} + x^V = 0, \tag{3}$$

and

$$x_1 = x' + \tfrac{1}{5}\text{A}, \quad x_2 = x'' + \tfrac{1}{5}\text{A}, \ \&\text{c.} \tag{4}$$

Dividing the equation of the fifth degree

$$x''^5 - x'^5 + p(x''^3 - x'^3) + q(x''^2 - x'^2) + r(x'' - x') = 0, \tag{5}$$

by the linear factor $x'' - x'$, we obtain the biquadratic

$$x''^4 + x'x''^3 + (x'^2 + p)x''^2 + (x'^3 + px' + q)x'' + x'^4 + px'^2 + qx' + r = 0, \tag{6}$$

of which the four roots are x'', x''', x^{IV}, x^V. Hence, by the theory of biquadratic equations, we may employ the expressions:

$$x'' = \alpha+\beta+\gamma+\delta,\ x''' = \alpha+\beta-\gamma-\delta,\ x^{IV} = \alpha-\beta+\gamma-\delta,\ x^V = \alpha-\beta-\gamma+\delta; \tag{7}$$

provided that α, β, γ, δ are such as to satisfy, independently of x'', the condition:

$$\left.\begin{aligned} &(x''-\alpha)^4 - 2(\beta^2+\gamma^2+\delta^2)(x''-\alpha)^2 - 8\beta\gamma\delta(x''-\alpha) + \beta^4+\gamma^4+\delta^4 \\ &\qquad - 2(\beta^2\gamma^2+\gamma^2\delta^2+\delta^2\beta^2) \\ &= x''^4 + x'x''^3 + (x'^2+p)x''^2 + (x'^3+px'+q)x'' + x'^4 + px'^2 \\ &\qquad + qx' + r; \end{aligned}\right\} \tag{8}$$

which decomposes itself into the four following:

$$\left.\begin{aligned} &-4\alpha = x'; \\ &+6\alpha^2 - 2(\beta^2+\gamma^2+\delta^2) = x'^2 + p; \\ &-4\alpha^3 + 4\alpha(\beta^2+\gamma^2+\delta^2) - 8\beta\gamma\delta = x'^3 + px' + q; \\ &+\alpha^4 - 2\alpha^2(\beta^2+\gamma^2+\delta^2) + 8\alpha\beta\gamma\delta + (\beta^2+\gamma^2+\delta^2)^2 - 4(\beta^2\gamma^2+\gamma^2\delta^2+\delta^2\beta^2) \\ &\qquad = x'^4 + px'^2 + qx' + r; \end{aligned}\right\} \tag{9}$$

and, therefore, conducts to expressions for α, $\beta^2+\gamma^2+\delta^2$, $\beta\gamma\delta$, and $\beta^2\gamma^2+\gamma^2\delta^2+\delta^2\beta^2$, as rational functions of x', p, q, r. Again, by the theory of cubic equations, we may write:

$$\beta^2=\epsilon+\kappa+\lambda,\ \gamma^2=\epsilon+\theta\kappa+\theta^2\lambda,\ \delta^2=\epsilon+\theta^2\kappa+\theta\lambda, \tag{10}$$

in which θ is a root of the equation

$$\theta^2+\theta+1=0, \tag{11}$$

while ϵ, $\kappa\lambda$, and $\kappa^3+\lambda^3$ are symmetric functions of β^2, γ^2, δ^2. Making, for abridgment,

$$\beta\gamma\delta=\eta, \quad \kappa\lambda=\iota, \tag{12}$$

we have, by (10) and (11),

$$\kappa^3+\lambda^3=\eta^2-\epsilon^3+3\epsilon\iota, \tag{13}$$

and

$$\beta^2+\gamma^2+\delta^2=3\epsilon, \quad \beta^2\gamma^2+\gamma^2\delta^2+\delta^2\beta^2=3(\epsilon^2-\iota); \tag{14}$$

and, therefore, by (9),

$$\left.\begin{array}{l} -4\alpha=x';\ 6(\alpha^2-\epsilon)=x'^2+p; \\ -4\alpha^3+12\alpha\epsilon-8\eta=x'^3+px'+q; \\ \alpha^4-6\alpha^2\epsilon+8\alpha\eta-3\epsilon^2+12\iota=x'^4+px'^2+qx'+r; \end{array}\right\} \tag{15}$$

conditions which give

$$\left.\begin{array}{l} \alpha={}^-\tfrac{1}{4}x'; \\ \epsilon={}^-\tfrac{1}{48}(5x'^2+8p); \\ \eta={}^-\tfrac{1}{64}(5x'^3+4px'+8q); \\ \iota=+\tfrac{1}{144}(10x'^4+11px'^2+9qx'+p^2+12r). \end{array}\right\} \tag{16}$$

Thus, α, ϵ, η, and ι, on the one hand, are rational functions of x', p, q, r; and, on the other hand, x', x'', x''', x^{IV}, x^V may be considered as functions, although not entirely rational, of α, ϵ, η, ι. In fact, if these four last quantities (denoted to help the memory by four Greek vowels) be supposed to be given, and if, by extraction of a square root and a cube root, a value of κ be found, which satisfies the auxiliary equation

$$\kappa^6-(\eta^2-\epsilon^3+3\epsilon\iota)\kappa^3+\iota^3=0, \tag{17}$$

and then a corresponding value of λ by the condition $\kappa\lambda = \iota$, we shall have $\pm \beta$ by extraction of another square root, since $\beta^2 = \epsilon + \kappa + \lambda$; and may afterwards, by the extraction of a third square root, either find $\pm \gamma$ from the expression $\gamma^2 = \epsilon + \theta\kappa + \theta^2\lambda$, and deduce δ from the product $\beta\gamma\delta = \eta$, or else find $\pm (\gamma + \delta)$ from the expression

$$(\gamma + \delta)^2 = 2\epsilon - \kappa - \lambda + \frac{2\eta}{\beta}; \tag{18}$$

and may then treat x'', x''', x^{IV}, x^{V}, as the four values of $\alpha + \beta + \gamma + \delta$, while $x' = -4\alpha$. Hence any function whatever of the five roots of the general equation (1) of the fifth degree may be considered as a function of the five quantities A, a, ϵ, η, ι; and if, in the expression of that function, the values (16) be substituted for a, ϵ, η, ι, so as to introduce in their stead the quantities x', p, q, r, it is permitted to make any simplifications of the result which can be obtained from the relation (2), by changing $x'^5 + px'^3 + qx'^2 + rx'$, wherever it occurs, to the known quantity $-s$.

14. Consider then the twentyfour-valued function, referred to in a former article, and suggested (as LAGRANGE has shown) by the analogy of equations of lower degrees; namely, t^5, in which

$$t = x_1 + \omega x_2 + \omega^2 x_3 + \omega^3 x_4 + \omega^4 x_5, \tag{19}$$

and

$$\omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0; \tag{20}$$

ω here (and not a) denoting an imaginary fifth root of unity, so that

$$\omega^5 = 1. \tag{21}$$

Observing, that by (4) and (20), x_1, &c. may be changed in (19) to x', &c.; and distinguishing among themselves the 120 values of the function t by employing the notation

$$t_{abcde} = \omega^5 x^{(a)} + \omega^4 x^{(b)} + \omega^3 x^{(c)} + \omega^2 x^{(d)} + \omega^1 x^{(e)}, \tag{22}$$

which gives, for example,

$$t_{12345} = x' + \omega^4 x'' + \omega^3 x''' + \omega^2 x^{IV} + \omega x^{V}; \tag{23}$$

we shall have, on substituting for x' its value $-4a$, and for x'', x''', x^{IV}, x^{V} their values (7), the system of the twenty-four expressions following:

$$\left.\begin{array}{l}
t_{12345} = -5a + \mathrm{B}\beta + \mathrm{C}\gamma + \mathrm{D}\delta\,; \\
t_{13254} = -5a + \mathrm{B}\beta - \mathrm{C}\gamma - \mathrm{D}\delta\,; \\
t_{14523} = -5a - \mathrm{B}\beta + \mathrm{C}\gamma - \mathrm{D}\delta\,; \\
t_{15432} = -5a - \mathrm{B}\beta - \mathrm{C}\gamma + \mathrm{D}\delta\,;
\end{array}\right\} \qquad (24)$$

$$\left.\begin{array}{lllll}
t_{12453} = - & 5a + & \mathrm{B}\gamma + & \mathrm{C}\delta + & \mathrm{D}\beta\,; \\
t_{14235} = - & 5a + & \mathrm{B}\gamma - & \mathrm{C}\delta - & \mathrm{D}\beta\,; \\
t_{15324} = - & - & + & - & \\
t_{13542} = - & - & - & + &
\end{array}\right\} \qquad (25)$$

$$\left.\begin{array}{lllll}
t_{12534} = - & 5a + & \mathrm{B}\delta + & \mathrm{C}\beta + & \mathrm{D}\gamma\,; \\
t_{15243} = - & + & - & - & \\
t_{13425} = - & - & + & - & \\
t_{14352} = - & - & - & + &
\end{array}\right\} \qquad (26)$$

$$\left.\begin{array}{lllll}
t_{12354} = - & 5a + & \mathrm{B}\beta + & \mathrm{C}\delta + & \mathrm{D}\gamma\,; \\
t_{13245} = - & + & - & - & \\
t_{15423} = - & - & + & - & \\
t_{14532} = - & - & - & + &
\end{array}\right\} \qquad (27)$$

$$\left.\begin{array}{lllll}
t_{12543} = - & 5a + & \mathrm{B}\delta + & \mathrm{C}\gamma + & \mathrm{D}\beta\,; \\
t_{15234} = - & + & - & - & \\
t_{14325} = - & - & + & - & \\
t_{13452} = - & - & - & + &
\end{array}\right\} \qquad (28)$$

$$\left.\begin{array}{lllll}
t_{12435} = - & 5a + & \mathrm{B}\gamma + & \mathrm{C}\beta + & \mathrm{D}\delta\,; \\
t_{14253} = - & + & - & - & \\
t_{13524} = - & - & + & - & \\
t_{15342} = - & - & - & + &
\end{array}\right\} \qquad (29)$$

in which we have made, for abridgment,

$$\left.\begin{aligned} \mathrm{B} &= \omega^4 + \omega^3 - \omega^2 - \omega, \\ \mathrm{C} &= \omega^4 - \omega^3 + \omega^2 - \omega, \\ \mathrm{D} &= \omega^4 - \omega^3 - \omega^2 + \omega. \end{aligned}\right\} \tag{30}$$

But also, by (22) and (21),

$$t_{bcdea} = \omega t_{abcde}, \quad t^5_{bcdea} = t^5_{abcde}; \tag{31}$$

making then

$$t_1{}^5_{abcd} = \mathrm{T}_{abcd}, \tag{32}$$

the twenty-four values of the function t^5 will be those of the function T which arise from arranging in all possible ways the four indices 2, 3, 4, 5; that is, they are the fifth powers of the twenty-four expressions (24) ... (29). It is required, therefore, to develope these fifth powers, and to examine into their composition.

15. For this purpose it is convenient first to consider those parts of any one such power, which are common to the three other powers of the same group, (24) or (25), &c., and, therefore, to introduce the consideration of six new functions, determined by the following definition :

$$\mathrm{V}_{abc} = \tfrac{1}{4}\left(\mathrm{T}_{2abc} + \mathrm{T}_{a2cb} + \mathrm{T}_{bc2a} + \mathrm{T}_{cba2}\right); \tag{33}$$

which gives, for example,

$$\left.\begin{aligned} \mathrm{V}_{345} &= (-5a)^5 + 60\,(-5a)^2\,\mathrm{BCD}\beta\gamma\delta \\ &+ 10\,\{(-5a)^3 + 2\mathrm{BCD}\beta\gamma\delta\}\,(\mathrm{B}^2\beta^2 + \mathrm{C}^2\gamma^2 + \mathrm{D}^2\delta^2) \\ &+ 5\,(-5a)\,(\mathrm{B}^4\beta^4 + \mathrm{C}^4\gamma^4 + \mathrm{D}^4\delta^4 + 6\mathrm{B}^2\mathrm{C}^2\beta^2\gamma^2 + 6\mathrm{C}^2\mathrm{D}^2\gamma^2\delta^2 + 6\mathrm{D}^2\mathrm{B}^2\delta^2\beta^2); \end{aligned}\right\} \tag{34}$$

this being (as is evident on inspection) the part common to the four functions T_{2345}, T_{3254}, T_{4523}, T_{5432}, or to the fifth powers of the four expressions in the group (24). By changing β, γ, δ, first to γ, δ, β, and afterwards to δ, β, γ, the expression (34) for V_{345} will be changed successively to those for V_{453} and V_{534}, which, therefore, it is unnecessary to write; and V_{354}, V_{543}, V_{435}, may be formed, respectively, from V_{345}, V_{453}, V_{534}, by interchanging γ and δ. Or, after substituting in (34) for $\beta^2, \gamma^2, \delta^2$, their values (10), and writing η for $\beta\gamma\delta$, it will only be necessary to multiply κ by θ, and λ by θ^2, wherever they occur, in order to change V_{345} to V_{453}; and to repeat this process, in order to change V_{453} to V_{534}: while V_{345}, V_{453}, V_{534}, will be changed, respectively, to V_{354}, V_{543}, V_{435}, by interchanging θ and θ^2, or κ and λ.

16. In this manner it is not difficult to perceive that we may write

$$\left.\begin{aligned} V_{345} &= g + h + i, \\ V_{453} &= g + \theta h + \theta^2 i, \\ V_{534} &= g + \theta^2 h + \theta i, \end{aligned}\right\} \tag{35}$$

and

$$\left.\begin{aligned} V_{354} &= g' + h' + i', \\ V_{543} &= g' + \theta h' + \theta^2 i', \\ V_{435} &= g' + \theta^2 h' + \theta i', \end{aligned}\right\} \tag{36}$$

in which,

$$\left.\begin{aligned} g = g' = &(-5a)^5 + 60\,(-5a)^2\,\eta BCD \\ &+ 10\,\{(-5a)^3 + 2\eta BCD\}\,\epsilon\,(B^2 + C^2 + D^2) \\ &+ 5\,(-5a)\,\epsilon^2\,(B^4 + C^4 + D^4 + 6C^2D^2 + 6D^2B^2 + 6B^2C^2) \\ &+ 10\,(-5a)\,\iota\,(B^4 + C^4 + D^4 - 3C^2D^2 - 3D^2B^2 - 3B^2C^2)\,; \end{aligned}\right\} \tag{37}$$

$$\left.\begin{aligned} h &= k\kappa + l\lambda^2, \quad i = k'\lambda + l'\kappa^2; \\ h' &= k\lambda + l\kappa^2, \quad i' = k'\kappa + l'\lambda^2; \end{aligned}\right\} \tag{38}$$

$$\left.\begin{aligned} k = &\,10\,\{(-5a)^3 + 2\eta BCD\}\,(B^2 + \theta C^2 + \theta^2 D^2) \\ &+ 10\,(-5a)\,\epsilon\,(B^4 + \theta C^4 + \theta^2 D^4 - 3C^2D^2 - 3\theta D^2B^2 - 3\theta^2 B^2C^2)\,; \\ l = &\,5\,(-5a)\,(B^4 + \theta C^4 + \theta^2 D^4 + 6C^2D^2 + 6\theta D^2B^2 + 6\theta^2 B^2C^2)\,; \end{aligned}\right\} \tag{39}$$

and k', l' are formed from k, l, by interchanging θ and θ^2. Hence also, by the same properties of ϵ, η, ι, which were employed in deducing these equations, we have:

$$\left.\begin{aligned} hh' &= k^2\iota + l^2\iota^2 + kl\,(\eta^2 - \epsilon^3 + 3\epsilon\iota)\,; \\ h^3 + h'^3 &= 2(3k^2 - l^2\iota)\,l\iota^2 + (k^2 + 3l^2\iota)\,k\,(\eta^2 - \epsilon^3 + 3\epsilon\iota) + l^3(\eta^2 - \epsilon^3 + 3\epsilon\iota)^2\,; \end{aligned}\right\} \tag{40}$$

and ii', $i^3 + i'^3$ have corresponding expressions, obtained by accenting k and l.

17. If then we make

$$g = H_1 + \sqrt{H_2}, \quad g' = H_1 - \sqrt{H_2}\,; \tag{41}$$

$$h^3 + h'^3 = 2H_3, \quad h^3 - h'^3 = 2\sqrt{H_4}\,; \tag{42}$$

$$i'^3 + i^3 = 2H_5, \quad i'^3 - i^3 = 2\sqrt{H_6}\,; \tag{43}$$

we see that the six functions V may be expressed by the help of square-roots and cube-roots, in terms of these six quantities H, by means of the following formulæ:

$$\left.\begin{aligned} V_{345} &= H_1 + \sqrt{H_2} + \sqrt[3]{H_3 + \sqrt{H_4}} + \sqrt[3]{H_5 - \sqrt{H_6}}; \\ V_{453} &= H_1 + \sqrt{H_2} + \theta\sqrt[3]{H_3 + \sqrt{H_4}} + \theta^2\sqrt[3]{H_5 - \sqrt{H_6}}; \\ V_{534} &= H_1 + \sqrt{H_2} + \theta^2\sqrt[3]{H_3 + \sqrt{H_4}} + \theta\sqrt[3]{H_5 - \sqrt{H_6}}; \end{aligned}\right\} \quad (a)$$

and

$$\left.\begin{aligned} V_{354} &= H_1 - \sqrt{H_2} + \sqrt[3]{H_3 - \sqrt{H_4}} + \sqrt[3]{H_5 + \sqrt{H_6}}; \\ V_{543} &= H_1 - \sqrt{H_2} + \theta\sqrt[3]{H_3 - \sqrt{H_4}} + \theta^2\sqrt[3]{H_5 + \sqrt{H_6}}; \\ V_{435} &= H_1 - \sqrt{H_2} + \theta^2\sqrt[3]{H_3 - \sqrt{H_4}} + \theta\sqrt[3]{H_5 + \sqrt{H_6}}: \end{aligned}\right\} \quad (b)$$

which have accordingly, with some slight differences of notation, been assigned by Professor BADANO, as among the results of his method of treating equations of the fifth degree. We see, too, that the six quantities $H_1, \ldots H_6$, (of which indeed the second, namely, H_2, vanishes), are rational functions of α, ϵ, η, ι; and therefore, by article 13., of x', p, q, r. But it is necessary to examine whether it be true, as Professor BADANO appears to think (guided in part, as he himself states, by the analogy of equations of lower degrees), that these quantities H are all rational functions of the coefficients p, q, r, s, of the equation (2) of the fifth degree; or, in other words, to examine whether it be possible to eliminate from the expressions of those six quantities H, the unknown root x' of that equation, by its means, in the same way as it was found possible, in articles 11. and 9. of the present paper, to eliminate from the correspondent expressions, the roots of the biquadratic and cubic equations which it was there proposed to resolve. For, if it shall be found that any one of the six quantities $H_1, \ldots H_6$, which enter into the formulæ (a) and (b), depends essentially, and not merely in appearance, on the unknown root x'; so as to change its value when that root is changed to another, such as x'', which satisfies the same equation (2): it will then be seen that these formulæ, although true, give no assistance towards the general solution of the equation of the fifth degree.

18. The auxiliary quantities ω, B, C, D, being such that, by their definitions (20) and (30),

$$\left.\begin{aligned} -1+\mathrm{B}+\mathrm{C}+\mathrm{D}&=4\omega^4,\\ -1+\mathrm{B}-\mathrm{C}-\mathrm{D}&=4\omega^3,\\ -1-\mathrm{B}+\mathrm{C}-\mathrm{D}&=4\omega^2,\\ -1-\mathrm{B}-\mathrm{C}+\mathrm{D}&=4\omega, \end{aligned}\right\} \tag{44}$$

while ω, ω^2, ω^3, ω^4 are the four imaginary fifth roots of unity, we shall have, by the theory of biquadratics already explained, the following identical equation:

$$\begin{aligned} &\{(x+1)^2-(\mathrm{B}^2+\mathrm{C}^2+\mathrm{D}^2)\}^2-8\mathrm{BCD}\,(x+1)-4\,(\mathrm{B}^2\mathrm{C}^2+\mathrm{C}^2\mathrm{D}^2+\mathrm{D}^2\mathrm{B}^2)\\ &\qquad=\{(x+1)^2+5\}^2+40\,(x+1)+180, \end{aligned} \tag{45}$$

the second member being equivalent to

$$x^4+4x^3+4^2x^2+4^3x+4^4;$$

we find, therefore, that

$$\mathrm{B}^2+\mathrm{C}^2+\mathrm{D}^2=-5; \quad \mathrm{BCD}=-5; \quad \mathrm{B}^2\mathrm{C}^2+\mathrm{C}^2\mathrm{D}^2+\mathrm{D}^2\mathrm{B}^2=-45; \tag{46}$$

and, consequently,

$$\mathrm{B}^4+\mathrm{C}^4+\mathrm{D}^4=115. \tag{47}$$

Hence, by (37), the common value of g and g', considered as a function of a, ϵ, η, ι, is:

$$g=g'=125\,(-25a^5+50a^3\epsilon-60a^2\eta+31a\epsilon^2-100a\iota+4\epsilon\eta); \tag{48}$$

and if in this we substitute, for the quantities a, ϵ, η, ι, their values (16), or otherwise eliminate those quantities by the relations (15), and attend to the definitions (41) of the quantities H_1 and H_2, we find:

$$\mathrm{H}_1=\frac{125}{12}\,(25x'^5+25px'^3+25qx'^2+25rx'+pq); \tag{49}$$

and, as was said already,

$$\mathrm{H}_2=0. \tag{50}$$

It is therefore true, of *these* two quantities H, that they are independent of the root x' of the proposed equation of the fifth degree, or remain unchanged when that root is changed to another, such as x'', which satisfies the same equation: since it is possible to eliminate x' from the expression (49) by means of the proposed equation (2), and so to obtain H_1 as a rational function of the coefficients of that equation, namely,

$$H_1 = \frac{125}{12}(pq - 25s). \tag{51}$$

Indeed, it was evident *à priori* that H_1 must be found to be equal to *some* rational function of those four coefficients, p, q, r, s, or some symmetric function of the five roots of the equation (2); because it is, by its definition, the sixth part of the sum of the six functions V, and, therefore, the twenty-fourth part of the sum of the twenty-four different values of the function T; or finally the mean of all the different values which the function t^5 can receive, by all possible changes of arrangement of the five roots $x', \ldots x^V$, or $x_1, \ldots x_5$, among themselves. The evanescence of H_2 shows farther, that, in the arrangement assigned above, the sum of the three first of the six functions V, or the sum of the twelve first of the twenty-four functions T, is equal to the sum of the other three, or of the other twelve of these functions. But we shall find that it would be erroneous to conclude, from the analogy of these results, even when combined with the corresponding results for equations of inferior degrees, that the other four quantities H, which enter into the formulæ (a) and (b), can likewise be expressed as rational functions of the coefficients of the equation of the fifth degree.

19. The auxiliary quantities B^2, C^2, D^2, being seen, by (46), to be the three roots z_1, z_2, z_3, of the cubic equation

$$z^3 + 5z^2 - 45z - 25 = 0, \tag{52}$$

which decomposes itself into one of the first and another of the second degree, namely,

$$z - 5 = 0, \quad z^2 + 10z + 5 = 0; \tag{53}$$

we see that one of the three quantities B, C, D, must be real, and $= \pm \sqrt{5}$, while the other two must be imaginary. And on referring to the definitions (30), and remembering that ω is an imaginary fifth root of unity, so that ω^4 and ω^3 are the reciprocals of ω and ω^2, we easily perceive that the real one of the three is D, and that the following expressions hold good:

$$B^2 = -5 - 2D; \quad C^2 = -5 + 2D; \quad D^2 = 5; \tag{54}$$

with which we may combine, whenever it may be necessary or useful, the relation

$$\text{BC} \doteq - \text{D}. \tag{55}$$

If then we make, for abridgment,

$$\zeta = (\theta - \theta^2)\,\text{D} = (\theta - \theta^2)\,(\omega^4 - \omega^3 - \omega^2 + \omega), \tag{56}$$

θ being still the same imaginary cubic root of unity as before, so that

$$\zeta^2 = -15; \tag{57}$$

we shall have, in (39),

$$\left.\begin{aligned} \text{D}^2 + \theta\text{B}^2 + \theta^2\text{C}^2 &= 10 - 2\zeta, \\ \text{D}^4 + \theta\text{B}^4 + \theta^2\text{C}^4 &= -20 + 20\zeta, \\ \text{B}^2\text{C}^2 + \theta\text{C}^2\text{D}^2 + \theta^2\text{D}^2\text{B}^2 &= 30 + 10\zeta; \end{aligned}\right\} \tag{58}$$

and, consequently (because $\text{BCD} = -5$),

$$\left.\begin{aligned} \theta k &= -100\,(5 - \zeta)\,(25a^3 + 2\eta) + 500\,(11 + \zeta)a\epsilon; \\ \theta l &= -2000\,(2 + \zeta)\,a; \end{aligned}\right\} \tag{59}$$

while $\theta^2 k'$ and $\theta^2 l'$ are formed from θk and θl, by changing the signs of ζ. It is easy, therefore, to see, by the remarks already made, and by the definitions (42) and (43), that the quantities H_3, H_4, H_5, H_6, when expressed as rational functions of a, ϵ, η, ι, or of x', p, q, r, will not involve either of the imaginary roots of unity, θ and ω, except so far as they may involve the combination ζ of those roots, or the radical $\sqrt{-15}$; and that H_5 will be formed from H_3, and H_6 from H_4, by changing the sign of this radical. We shall now proceed to study, in particular, the composition of the quantity H_4; because, although this quantity, when expressed by means of x', p, q, r, is of the thirtieth dimension relatively to x', (p, q, and r being considered as of the second, third, and fourth dimensions, respectively), while H_3 rises no higher than the fifteenth dimension; yet we shall find it possible to decompose H_4 into two factors, of which one is of the twelfth dimension, and has a very simple meaning, being the product of the squares of the differences of the four roots x'', x''', x^{IV}, x^{V}; while the other factor of H_4 is an exact square, of a function of the ninth dimension. We shall even see it to be possible to decompose this last function into three factors, which are each as low as the third dimension, and are rational functions of the five roots of the original equation of the fifth degree; whereas it does not appear that H_3, when regarded as a function of the same five roots, can be decomposed into more than three rational factors, nor that any of these can be depressed below the fifth dimension.

20. Confining ourselves then for the present to the consideration of H_4, we have, by (42) and (38), the following expression for the square-root of that quantity:

$$\sqrt{\text{H}_4} = \tfrac{1}{2}(\kappa^3 - \lambda^3)\{k^3 - 3kl^2\kappa\lambda - l^3(\kappa^3 + \lambda^3)\}; \tag{60}$$

and, therefore, by (59), and by the same relations between κ, λ, and ϵ, η, ι, which were used in deducing the formulæ of the sixteenth article, we obtain the following expression for the quantity H_4 itself, considered as a function of a, ϵ, η, ι:

$$\text{H}_4 = 2^{10}5^{18}\{(\eta^2 - \epsilon^3 + 3\epsilon\iota)^2 - 4\iota^3\}\,\text{L}^2; \tag{61}$$

in which we have made, for abridgment,

$$\text{L} = \mu^3 - 3\iota\mu\nu^2 + (\eta^2 - \epsilon^3 + 3\epsilon\iota)\,\nu^3, \tag{62}$$

and

$$\mu = (-5 + \zeta)(5a^3 + \tfrac{2}{5}\eta) + (11 + \zeta)\,a\epsilon,\ \nu = 4\,(2 + \zeta)\,a. \tag{63}$$

Now, without yet entering on the actual process of substituting, in the expression (61), the values (16) for a, ϵ, η, ι; or of otherwise eliminating those four quantities by means of the equations (15), in order to express H_4 as a function of x', p, q, r, from which x' is afterwards to be eliminated, as far as possible, by the equation of the fifth degree; we see that, in agreement with the remarks made in the last article, this expression (61) contains (besides its numerical coefficient) one factor, namely,

$$(\eta^2 - \epsilon^3 + 3\epsilon\iota)^2 - 4\iota^3 = (\kappa^3 - \lambda^3)^2, \tag{64}$$

which is of the twelfth dimension; and another, namely, L^2, which is indeed itself of the eighteenth, but is the square of a function (62), which is only of the ninth dimension: because a, ϵ, η, ι, are to be considered as being respectively of the first, second, third, and fourth dimensions; and, therefore, μ is to be regarded as being of the third, and ν of the first dimension.

21. Again, on examining the factor (64), we see that it is the square of another function, namely, $\kappa^3 - \lambda^3$, which is itself of the sixth dimension, and is rational with respect to x'', x''', x^{IV}, x^{V}, though not with respect to a, ϵ, η, ι, nor with respect to x', p, q, r. This function $\kappa^3 - \lambda^3$ may even be decomposed into six linear factors; for first, we have, by (11),

$$\kappa^3 - \lambda^3 = (\kappa - \lambda)(\kappa - \theta\lambda)(\kappa - \theta^2\lambda); \tag{65}$$

and, secondly, by (10),

$$3\kappa = \beta^2 + \theta^2\gamma^2 + \theta\delta^2, \quad 3\lambda = \beta^2 + \theta\gamma^2 + \theta^2\delta^2, \tag{66}$$

expressions which give

$$\left.\begin{aligned} \kappa - \lambda &= \tfrac{1}{3}(\theta - \theta^2)(\delta^2 - \gamma^2), \\ \kappa - \theta\lambda &= \tfrac{1}{3}(1 - \theta)(\beta^2 - \delta^2), \\ \kappa - \theta^2\lambda &= \tfrac{1}{3}(\theta^2 - 1)(\gamma^2 - \beta^2); \end{aligned}\right\} \tag{67}$$

but also, by (7),

$$\left.\begin{aligned} \delta^2 - \gamma^2 &= \tfrac{1}{4}(x'' - x''')(x^V - x^{IV}), \\ \beta^2 - \delta^2 &= \tfrac{1}{4}(x'' - x^{IV})(x''' - x^V), \\ \gamma^2 - \beta^2 &= \tfrac{1}{4}(x'' - x^V)(x^{IV} - x'''); \end{aligned}\right\} \tag{68}$$

and

$$(\theta - \theta^2)(1 - \theta)(\theta^2 - 1) = (1 - \theta)^3 = -3(\theta - \theta^2); \tag{69}$$

therefore,

$$\left.\begin{aligned} \kappa^3 - \lambda^3 = -2^{-6}3^{-2}(\theta - \theta^2)(x'' - x''')(x'' - x^{IV})(x'' - x^V) \\ (x''' - x^{IV})(x''' - x^V)(x^{IV} - x^V). \end{aligned}\right\} \tag{70}$$

Thus, then, the square of the product of these six linear factors (70), and of the numerical coefficients annexed, is equal to the function (64), of the twelfth dimension, which itself entered as a factor into the expression (61) for H_4; and we see that this square is free from the imaginary radical θ, because, by (11),

$$(\theta - \theta^2)^2 = -3; \tag{71}$$

and that it is a symmetric function of the four roots x'', x''', x^{IV}, x^V, being proportional to the product of the squares of their differences, as was stated in article 19.: so that this square (though not its root) may be expressed, in virtue of the biquadratic equation (6), as a rational function of x', p, q, r; which followed also from its being expressible rationally, by (64), in terms of ϵ, η, ι.

22. Introducing now, in the expression (64), here referred to, the values (16), or the relations (15), we find, after reductions:

$$\left.\begin{aligned}&\kappa^3+\lambda^3=\eta^2-\epsilon^3+3\epsilon\iota=\\&-2^{-6}3^{-3}\{25x'^6+75px'^4+(48p^2+45r)x'^2+27pqx'\\&\qquad\qquad -2p^3+72pr-27q^2\};\end{aligned}\right\}\quad(72)$$

$$\left.\begin{aligned}&(\kappa^3+\lambda^3)^2=(\eta^2-\epsilon^3+3\epsilon\iota)^2=2^{-12}3^{-6}\{625x'^{12}+3750px'^{10}+(8025p^2+2250r)x'^8\\&+1350pqx'^7+(7100p^3+10350pr-1350q^2)x'^6+4050p^2qx'^5\\&+(2004p^4+15120p^2r-4050pq^2+2025r^2)x'^4\\&+(2592p^3q+2430pqr)x'^3\\&+(-192p^5+6732p^3r-1863p^2q^2+6480pr^2-2430q^2r)x'^2\\&+(-108p^4q+3888p^2qr-1458pq^3)x'\\&+4p^6-288p^4r+108p^3q^2+5184p^2r^2-3888pq^2r+729q^4\};\end{aligned}\right\}\quad(73)$$

$$\left.\begin{aligned}&4\kappa^3\lambda^3=4\iota^3=2^{-10}3^{-6}\{1000x'^{12}+3300px'^{10}+2700qx'^9\\&+(3930p^2+3600r)x'^8+5940pqx'^7+(1991p^3+7920pr+2430q^2)x'^6\\&+(3807p^2q+6480qr)x'^5+(393p^4+5076p^2r+2673pq^2+4320r^2)x'^4\\&+(594p^3q+7128pqr+729q^3)x'^3\\&+(33p^5+792p^3r+243p^2q^2+4752pr^2+2916q^2r)x'^2\\&+(27p^4q+648p^2qr+3888qr^2)x'\\&+p^6+36p^4r+432p^2r^2+1728r^3\};\end{aligned}\right\}\quad(74)$$

and, finally,

$$\left.\begin{aligned}&(\kappa^3-\lambda^3)^2=(\eta^2-\epsilon^3+3\epsilon\iota)^2-4\iota^3=\\&-2^{-12}3^{-3}\{125x'^{12}+350px'^{10}+400qx'^9+(285p^2+450r)x'^8\\&+830pqx'^7+(32p^3+790pr+410q^2)x'^6+(414p^2q+960qr)x'^5\\&+(-16p^4+192p^2r+546pq^2+565r^2)x'^4\\&+(-8p^3q+966pqr+108q^3)x'^3\\&+(12p^5-132p^3r+105p^2q^2+464pr^2+522q^2r)x'^2\\&+(8p^4q-48p^2qr+54pq^3+576qr^2)x'\\&+16p^4r-4p^3q^2-128p^2r^2+144pq^2r+256r^3-27q^4\}.\end{aligned}\right\}\quad(75)$$

23. This last result may be verified, or rather proved anew, and at the same time put under another form, which we shall find to be useful, by a process such

as the following. The biquadratic equation (6), of which the roots are x'', x''', x^{IV}, x^{V}, shows that, whatever x may be,

$$\left.\begin{array}{l}(x-x'')\,(x-x''')\,(x-x^{IV})\,(x-x^{V})= \\ x^4+x'x^3+x'^2x^2+x'^3x+x'^4 \\ +p\,(x^2+x'x+x'^2)+q\,(x+x')+r\,;\end{array}\right\} \quad (76)$$

and, therefore, that

$$(x'-x'')\,(x'-x''')\,(x'-x^{IV})\,(x'-x^{V})=5x'^4+3px'^2+2qx'+r. \quad (77)$$

If then we multiply the expression (75) by the square of this last function (77), we ought to obtain a symmetric function of all the five roots of the equation of the fifth degree, namely, the product of the ten squares of their differences, multiplied indeed by a numerical coefficient, namely, $-2^{-12}\,3^{-3}$, as appears from (70) and (71): and consequently an expression for this product itself, that is for

$$\left.\begin{array}{l}\mathrm{P}=(x'-x'')^2\,(x'-x''')^2\,(x'-x^{IV})^2\,(x'-x^{V})^2\,(x''-x''')^2 \\ \quad (x''-x^{IV})^2\,(x''-x^{V})^2\,(x'''-x^{IV})^2\,(x'''-x^{V})^2\,(x^{IV}-x^{V})^2,\end{array}\right\} \quad (78)$$

must be obtained by multiplying the factor $125x'^{12}+$ &c. which is within the brackets in (75), by the square of $5x'^4+3px'^2+2qx'+r$, and then reducing by the condition that $x'^5+px'^3+qx'^2+rx'=-s$. Accordingly this process gives:

$$\left.\begin{array}{l}\mathrm{P}=3125s^4-3750pqs^3 \\ \quad +(108p^5-900p^3r+825p^2q^2+2000pr^2+2250q^2r)\,s^2 \\ \quad -(72p^4qr-16p^3q^3-560p^2qr^2+630pq^3r+1600qr^3-108q^5)\,s \\ \quad +16p^4r^3-4p^3q^2r^2-128p^2r^4+144pq^2r^3+256r^5-27q^4r^2\,;\end{array}\right\} \quad (79)$$

an expression for the product of the squares of the differences of the five roots of an equation of the fifth degree, which agrees with known results. And we see that with this meaning of P, we may write:

$$(\kappa^3-\lambda^3)^2=-2^{-12}\,3^{-3}\,\mathrm{P}\,(5x'^4+3px'^2+2qx'+r)^{-2}. \quad (80)$$

The expression (61) for H_4 becomes, therefore:

$$\mathrm{H}_4=-2^{-2}\,3^{-3}\,5^{18}\,\mathrm{P}\left(\frac{\mu^3-3\iota\mu\nu^2+(\eta^2-\epsilon^3+3\epsilon\iota)\,\nu^3}{5x'^4+3px'^2+2qx'+r}\right)^2; \quad (81)$$

μ and ν having the meanings defined by (63).

24. With respect now to the factor L, which enters by its square into the expression (61), and is the numerator of the fraction which is squared in the form (81), we have, by (62), (63), and (57),

$$\left.\begin{aligned}\mathrm{L} = {} & \tfrac{4}{5}\,(15625a^9 + 24375a^7\epsilon + 3750a^6\eta \\ & - 16125a^5\epsilon^2 + 1500a^5\iota + 3900a^4\epsilon\eta + 7605a^3\epsilon^3 \\ & - 8820a^3\epsilon\iota - 6260a^3\eta^2 - 1290a^2\epsilon^2\eta + 120a^2\eta\iota + 156a\epsilon\eta^2 + 8\eta^3) \\ & + \tfrac{12}{25}\,\zeta\,(15625\,(a^9 - a^7\epsilon) + 3750a^6\eta - 125a^5\epsilon^2 + 15500a^5\iota - 2500a^4\epsilon\eta \\ & + 1125a^3\epsilon^3 - 4500a^3\epsilon\iota - 100a^3\eta^2 - 10a^2\epsilon^2\eta + 1240a^2\eta\iota - 100a\epsilon\eta^2 + 8\eta^3)\,;\end{aligned}\right\} \qquad (82)$$

and when we substitute for a, ϵ, η, ι, their values (16), we find, after reductions, a result which may be thus written:

$$2^6 5^2 \mathrm{L} = 5\mathrm{L}' - \zeta\mathrm{L}''\,; \qquad (83)$$

if we make, for abridgment,

$$\left.\begin{aligned}\mathrm{L}' = {} & 25px'^7 + 275qx'^6 + (135p^2 - 350r)\,x'^5 + 210pqx'^4 \\ & + (141p^3 - 500pr + 385q^2)x'^3 + (93p^2q - 20qr)\,x'^2 + 20pq^2x' - 4q^3\,; \\ \mathrm{L}'' = {} & 1750x'^9 + 2825px'^7 + 2100qx'^6 + (1120p^2 + 1825r)\,x'^5 \\ & + 1615pqx'^4 + (39p^3 + 1060pr + 500q^2)\,x'^3 \\ & + (109p^2q + 620qr)\,x'^2 + 68pq^2x' + 12q^3.\end{aligned}\right\} \qquad (84)$$

With these meanings of L' and L'', the quantity H_4, considered as a rational function of x', p, q, r, may therefore be thus expressed:

$$\mathrm{H}_4 = -\,2^{-14}\,3^{-3}\,5^{14}\,\mathrm{P}\left(\frac{5\mathrm{L}' - \zeta\mathrm{L}''}{5x'^4 + 3px'^2 + 2qx' + r}\right)^2; \qquad (85)$$

P being still the quantity (79), and ζ being still $= \sqrt{-15}$.

25. Depressing, next, as far as possible, the degrees of the powers of x', by means of the equation (2) of the fifth degree which x' must satisfy, we find:

$$\left.\begin{aligned}\mathrm{L}' &= \mathrm{L}'_0 + \mathrm{L}'_1\,x' + \mathrm{L}'_2\,x'^2 + \mathrm{L}'_3\,x'^3 + \mathrm{L}'_4\,x'^4\,; \\ \mathrm{L}'' &= \mathrm{L}''_0 + \mathrm{L}''_1 x' + \mathrm{L}''_2\,x'^2 + \mathrm{L}''_3\,x'^3 + \mathrm{L}''_4\,x'^4\,;\end{aligned}\right\} \qquad (86)$$

in which the coefficients are thus composed:

$$\left.\begin{aligned} L'_0 &= -110p^2s - 4q^3 + 350rs, \\ L'_1 &= -110p^2r + 20pq^2 - 275qs + 350r^2, \\ L'_2 &= -17p^2q - 25ps + 55qr, \\ L'_3 &= +31p^3 - 175pr + 110q^2, \\ L'_4 &= -90pq; \end{aligned}\right\} \tag{87}$$

and

$$\left.\begin{aligned} L''_0 &= -45p^2s + 12q^3 - 75rs; \\ L''_1 &= -45p^2r + 68pq^2 - 350qs - 75r^2; \\ L''_2 &= +64p^2q - 1075ps + 195qr; \\ L''_3 &= -6p^3 - 90pr + 150q^2; \\ L''_4 &= +190pq - 1750s. \end{aligned}\right\} \tag{88}$$

But because, after the completion of all these transformations and reductions, it is seen that the five quantities

$$5L'_0 - \zeta L''_0, \quad 5L'_1 - \zeta L''_1, \quad 5L'_2 - \zeta L''_2, \quad 5L'_3 - \zeta L''_3, \quad 5L'_4 - \zeta L''_4, \tag{89}$$

which become the coefficients of x'^0, x'^1, x'^2, x'^3, x'^4, in the numerator $5L' - \zeta L''$ of the fraction to be squared in the formula (85), are not proportional to the five other quantities

$$r, \; 2q, \; 3p, \; 0, \; 5, \tag{90}$$

which are the coefficients of the same five powers of x' in the denominator of the same fraction, it may be considered as already evident, at this stage of the investigation, that the root x' enters, not only apparently, but also really, into the composition of the quantity H_4.

26. The foregoing calculations have been laborious, but they have been made and verified with care, and it is believed that the results may be relied on. Yet an additional light will be thrown upon the question, by carrying somewhat farther the analysis of the quantity or function H_4, and especially of the factor L; which, though itself of the ninth dimension relatively to the roots of the equation of the fifth degree, is yet, according to a remark made in the nineteenth article, susceptible of being decomposed into three less complicated factors; each of these last being rational with respect to the same five roots, and being only of the third dimension. In fact, we have, by (62), and by (11), (12), (13),

$$\text{L} = (\mu + \kappa\nu + \lambda\nu)(\mu + \theta\kappa\nu + \theta^2\lambda\nu)(\mu + \theta^2\kappa\nu + \theta\lambda\nu); \tag{91}$$

that is, by (10),

$$\text{L} = (\mu - \epsilon\nu + \beta^2\nu)(\mu - \epsilon\nu + \gamma^2\nu)(\mu - \epsilon\nu + \delta^2\nu); \tag{92}$$

in which, by the same equations, and by (63) and (57),

$$\left.\begin{array}{c} \mu - \epsilon\nu = (-5 + \zeta)(5\alpha^3 + \frac{2}{3}\beta\gamma\delta) + (1 - \zeta)\alpha(\beta^2 + \gamma^2 + \delta^2); \\ \nu = (8 + 4\zeta)\alpha; \quad \zeta = \sqrt{-15}. \end{array}\right\} \tag{93}$$

Thus, L is seen to be composed of three factors,

$$\text{L} = \text{M}_1\text{M}_2\text{M}_3, \tag{94}$$

$$\text{M}_1 = \mu - \epsilon\nu + \beta^2\nu, \quad \text{M}_2 = \mu - \epsilon\nu + \gamma^2\nu, \quad \text{M}_3 = \mu - \epsilon\nu + \delta^2\nu, \tag{95}$$

of which each is a rational, integral, and homogeneous function, of the third dimension, of the four quantities α, β, γ, δ, and, therefore, by (7), of the four roots x'', x''', x^{IV}, x^{V}, of the biquadratic equation (6); or finally, by (4), of the five roots x_1, x_2, x_3, x_4, x_5, of the original equation (1) of the fifth degree: because we have

$$x'' = x_2 - \tfrac{1}{5}(x_1 + x_2 + x_3 + x_4 + x_5), \text{ \&c.}; \tag{96}$$

or because

$$\left.\begin{array}{r} 20\alpha = x_2 + x_3 + x_4 + x_5 - 4x_1, \\ 4\beta = x_2 + x_3 - x_4 - x_5, \\ 4\gamma = x_2 - x_3 + x_4 - x_5, \\ 4\delta = x_2 - x_3 - x_4 + x_5. \end{array}\right\} \tag{97}$$

And the first of these three factors of L may be expressed by the following equation:

$$100\text{M}_1 = 5\text{M}'_1 - \zeta\text{M}''_1; \tag{98}$$

in which,

$$\left.\begin{array}{l} \text{M}'_1 = 4x_1^3 - 3x_1^2(x_2 + x_3 + x_4 + x_5) - 2x_1(x_2^2 + x_3^2 + x_4^2 + x_5^2) \\ \qquad - 2x_1(x_2x_3 + x_4x_5) + 6x_1(x_2 + x_3)(x_4 + x_5) \\ +2\{x_2x_3(x_2+x_3)+x_4x_5(x_4+x_5)\} - 3\{x_2x_3(x_4+x_5)+x_4x_5(x_2+x_3)\}; \end{array}\right\} \tag{99}$$

and

$$
\left.\begin{aligned}
\mathrm{M}_1'' = 4x_1^3 - 3x_1^2(x_2+x_3+x_4+x_5) + 2x_1(x_2^2+x_3^2+x_4^2+x_5^2) \\
+ 14x_1(x_2x_3+x_4x_5) - 6x_1(x_2+x_3)(x_4+x_5) \\
- 3\{x_2x_3(x_2+x_3)+x_4x_5(x_4+x_5)\} \\
- \{x_2x_3(x_4+x_5)+x_4x_5(x_2+x_3)\} \\
-\{x_2^3+x_3^3+x_4^3+x_5^3-2(x_2^2+x_3^2)(x_4+x_5)-2(x_4^2+x_5^2)(x_2+x_3)\};
\end{aligned}\right\} \quad (100)
$$

while the seeond faetor, M_2, ean be formed from M_1 by merely interehanging x_3 and x_4; and the third factor M_3 from M_2, by interchanging x_4 and x_5.

27. If, now, we substitute the expression (94) for the numerator of the fraction whieh is to be squared in the formula (81), and transform also in like manner the denominator of the same fraetion, by introdueing the five original roots $x_1, \ldots x_5$, through the equations (77) and (4), we find:

$$
\mathrm{H}_4 = -\frac{2^{-2}\,3^{-3}\,5^{18}\,\mathrm{P}\mathrm{M}_1^2\,\mathrm{M}_2^2\,\mathrm{M}_3^2}{(x_1-x_2)^2(x_1-x_3)^2(x_1-x_4)^2(x_1-x_5)^2}; \qquad (101)
$$

and we see that this quantity eannot be a symmetric funetion of those five roots, unless the produet of the three factors M_1, M_2, M_3 be divisible by the product of the four differenees $x_1-x_2, \ldots x_1-x_5$. But this would require that at least some one of those three factors M should be divisible by one of these four differenees, for example by x_1-x_2; whieh is not found to be true. Indecd, if any one of these factors, for example, M_1, were supposed to be divisible by any one differenee, sueh as x_1-x_2, it is easy to see, from its form, that it ought to be divisible also by eaeh of the three other differences; because, in M_1, we may interchange x_2 and x_3, or x_4 and x_5, or may interchange x_2 and x_4, or x_2 and x_5, if we also interchange x_3 and x_5, or x_3 and x_4: but a rational and integral funetion of the third dimension eannot have four different linear divisors, without being identically equal to zero, which does not happen here. The same sort of reasoning may be applied to the expressions (95), combined with (93), for the three factors M_1, M_2, M_3, considered as funetions, of the third dimension, of a, β, γ, δ; because if any one of these functions eould be divisible by any one of the four following linear divisors,

$$
\left.\begin{aligned}
x_1 - x_2 &= -5a - (\beta+\gamma+\delta), \\
x_1 - x_3 &= -5a - (\beta-\gamma-\delta), \\
x_1 - x_4 &= -5a - (-\beta+\gamma-\delta), \\
x_1 - x_5 &= -5a - (-\beta-\gamma+\delta),
\end{aligned}\right\} \quad (102)
$$

it ought from its form to be divisible by all of them, which is immediately seen to be impossible. The conclusion of the twenty-fifth article is, therefore, confirmed anew; and we see, at the same time, by the theory of biquadratic equations, and by the meanings of ϵ, η, ι, that the denominator of the fraction which is to be squared, in the form (81) for H_4, may be expressed as follows:

$$\left.\begin{aligned} 5x'^4 + 3px'^2 + 2qx' + r &= (x_1 - x_2)(x_1 - x_3)(x_1 - x_4)(x_1 - x_5) \\ &= (5a)^4 - 6\epsilon(5a)^2 + 8\eta(5a) - 3(\epsilon^2 - 4\iota); \end{aligned}\right\} \quad (103)$$

a result which may be otherwise proved by means of the relations (15).

28. The investigations in the preceding articles, respecting equations of the fifth degree, have been based upon analogous investigations made previously with respect to biquadratic equations; because it was the theory of the equations last-mentioned which suggested to Professor BADANO the formulæ marked (a) and (b) in the seventeenth article of this paper. But if those formulæ had been suggested in any other way, or if they should be assumed as true by definition, and employed as such to fix the meanings of the quantities H which they involve; then, we might seek the values and composition of those quantities, $H_1, \dots H_6$, by means of the following converse formulæ, which (with a slightly less abridged notation) have been given by the same author:

$$\left.\begin{aligned} H_1 + \sqrt{H_2} &= \tfrac{1}{3}(V_{345} + V_{453} + V_{534}); \\ H_3 + \sqrt{H_4} &= \tfrac{1}{27}(V_{345} + \theta^2 V_{453} + \theta V_{534})^3; \\ H_5 - \sqrt{H_6} &= \tfrac{1}{27}(V_{345} + \theta V_{453} + \theta^2 V_{534})^3; \end{aligned}\right\} \quad (e)$$

and

$$\left.\begin{aligned} H_1 - \sqrt{H_2} &= \tfrac{1}{3}(V_{354} + V_{543} + V_{435}); \\ H_3 - \sqrt{H_4} &= \tfrac{1}{27}(V_{354} + \theta^2 V_{543} + \theta V_{435})^3; \\ H_5 + \sqrt{H_6} &= \tfrac{1}{27}(V_{354} + \theta V_{543} + \theta^2 V_{435})^3. \end{aligned}\right\} \quad (d)$$

Let us, therefore, employ this other method to investigate the composition of H_4, by means of the equation

$$54\sqrt{H_4} = (V_{345} + \theta^2 V_{453} + \theta V_{534})^3 - (V_{354} + \theta^2 V_{543} + \theta V_{435})^3; \quad (104)$$

determining still the six functions V by the definition (33), so that each shall still be the mean of four of the twenty-four functions T; and assigning still to these last functions the significations (32), or treating them as the fifth powers of twenty-four different values of LAGRANGE's function t, which has itself 120 values: but expressing now these values of t by the notation

$$t_{abcde} = \omega^5 x_a + \omega^4 x_b + \omega^3 x_c + \omega^2 x_d + \omega x_e, \tag{105}$$

which differs from the notation (22) only by having lower instead of upper indices of x; and is designed to signify that we now employ (for the sake of a greater directness and a more evident generality) the five arbitrary roots x_1, &c., of the original equation (1), between which roots no relation is supposed to subsist, instead of the roots x', &c., of the equation (2), which equation was supposed to have been so prepared that the sum of its roots should be zero.

29. Resuming, then, the calculations on this plan, and making for abridgment

$$\mathrm{A} = x_a + x_b + x_c + x_d + x_e, \tag{106}$$

so that $-\mathrm{A}$ is still the coefficient of the fourth power of x in the equation of the fifth degree; making also

$$\mathrm{W}_{abcde} = x_a^4 x_b + 2x_a^3 x_d^2 + 4x_a^3 x_c x_e + 6x_a^2 x_b^2 x_e + 12 x_a^2 x_b x_c x_d, \tag{107}$$

and

$$\mathrm{X}_{bcde} = 5\left(\mathrm{W}_{abcde} + \mathrm{W}_{bcdea} + \mathrm{W}_{cdeab} + \mathrm{W}_{deabc} + \mathrm{W}_{eabcd}\right); \tag{108}$$

we find (because $\omega^5 = 1$), for the fifth power of the combination (105) of the five roots x, the expression:

$$\left.\begin{aligned} t^5_{abcde} = \mathrm{A}^5 &+ (\omega^4 - 1)\,\mathrm{X}_{bcde} + (\omega^3 - 1)\,\mathrm{X}_{cebd} \\ &+ (\omega - 1)\,\mathrm{X}_{edcb} + (\omega^2 - 1)\,\mathrm{X}_{dbec}; \end{aligned}\right\} \tag{109}$$

and, therefore, for the six functions V, with the same meanings of those functions as before, the formula:

$$\left.\begin{aligned} \mathrm{V}_{cde} &= \tfrac{1}{4}\left(t^5_{12cde} + t^5_{1c2ed} + t^5_{1de2c} + t^5_{1edc2}\right) \\ &= \mathrm{A}^5 + (\omega + \omega^4 - 2)\,\mathrm{Y}_{cde} + (\omega^2 + \omega^3 - 2)\,\mathrm{Y}_{dce}; \end{aligned}\right\} \tag{110}$$

in which,

$$4\mathrm{Y}_{cde} = \mathrm{X}_{2cde} + \mathrm{X}_{c2ed} + \mathrm{X}_{de2c} + \mathrm{X}_{edc2}. \tag{111}$$

If then we make

$$\left.\begin{aligned} \mathrm{Y}_{345} &= \mathrm{Y}'_5 + \mathrm{Y}''_5, & \mathrm{Y}_{435} &= \mathrm{Y}'_5 - \mathrm{Y}''_5, \\ \mathrm{Y}_{453} &= \mathrm{Y}'_3 + \mathrm{Y}''_3, & \mathrm{Y}_{543} &= \mathrm{Y}'_3 - \mathrm{Y}''_3, \\ \mathrm{Y}_{534} &= \mathrm{Y}'_4 + \mathrm{Y}''_4, & \mathrm{Y}_{354} &= \mathrm{Y}'_4 - \mathrm{Y}''_4; \end{aligned}\right\} \tag{112}$$

we shall have, by (20) and (30), the following system of expressions for the functions V:

$$\left.\begin{aligned} V_{345} &= A^5 - 5Y'_5 + DY''_5; \\ V_{453} &= A^5 - 5Y'_3 + DY''_3; \\ V_{534} &= A^5 - 5Y'_4 + DY''_4; \end{aligned}\right\} \tag{113}$$

and

$$\left.\begin{aligned} V_{354} &= A^5 - 5Y'_4 - DY''_4; \\ V_{543} &= A^5 - 5Y'_3 - DY''_3; \\ V_{435} &= A^5 - 5Y'_5 - DY''_5; \end{aligned}\right\} \tag{114}$$

D being still $= \omega^4 - \omega^3 - \omega^2 + \omega$, so that D^2 is still $= 5$. We have also the equation:

$$\left.\begin{aligned} & X_{2345} + X_{3254} + X_{4523} + X_{5432} \\ & + X_{2453} + X_{4235} + X_{5324} + X_{3542} \\ & + X_{2534} + X_{5243} + X_{3425} + X_{4352} \\ & = X_{2354} + X_{3245} + X_{5423} + X_{4532} \\ & + X_{2543} + X_{5234} + X_{4325} + X_{3452} \\ & + X_{2435} + X_{4253} + X_{3524} + X_{5342}; \end{aligned}\right\} \tag{115}$$

because the first member may be converted into the second by interchanging any two of the four roots x_2, x_3, x_4, x_5, on which (and on x_1) the functions X depend, and therefore the difference of these two members must be equal to zero; since, being at highest of the fifth dimension, it cannot otherwise be divisible by the function

$$\varpi = (x_2 - x_3)(x_2 - x_4)(x_2 - x_5)(x_3 - x_4)(x_3 - x_5)(x_4 - x_5), \tag{116}$$

which is the product of the six differences of the four roots just mentioned, and is itself of the sixth dimension. We may therefore combine with the expressions (113) and (114) the relations:

$$Y_{345} + Y_{453} + Y_{534} = Y_{354} + Y_{543} + Y_{435}; \tag{117}$$

and

$$Y''_3 + Y''_4 + Y''_5 = 0. \tag{118}$$

30. With these preparations for the study of the functions V, or of any combination of those functions, let us consider in particular the first of the three following factors of the expression (104) for $54\sqrt{H_4}$:

$$\left.\begin{array}{l} v_{345} - v_{354} + \theta^2 (v_{453} - v_{543}) + \theta (v_{534} - v_{435}); \\ v_{345} - v_{543} + \theta^2 (v_{453} - v_{435}) + \theta (v_{534} - v_{354}); \\ v_{345} - v_{435} + \theta^2 (v_{453} - v_{354}) + \theta (v_{534} - v_{543}); \end{array}\right\} \quad (119)$$

θ being still an imaginary cube-root of unity. We find:

$$\left.\begin{array}{l} v_{345} - v_{354} = 5(Y'_4 - Y'_5) - DY''_3; \\ v_{534} - v_{435} = -5(Y'_4 - Y'_5) - DY''_3; \\ v_{453} - v_{543} = 2DY''_3; \end{array}\right\} \quad (120)$$

expressions which show immediately that

$$v_{345} + v_{453} + v_{534} = v_{354} + v_{543} + v_{435}, \quad (121)$$

and, therefore, by (c) and (d), that

$$H_2 = 0,$$

as was otherwise found before. Also,

$$2\theta^2 - \theta - 1 = (\theta - 1)(2\theta + 1) = -(1 - \theta)(\theta - \theta^2); \quad (122)$$

and, consequently, by (120), the first of the three factors (119) is equivalent to the product of the two following:

$$1 - \theta, \quad 5(Y'_4 - Y'_5) - \zeta Y''_3; \quad (123)$$

in which, as before,

$$\zeta = (\theta - \theta^2) D = \sqrt{-15}.$$

But, by (112) and (117),

$$2(Y'_4 - Y'_5) = Y_{534} - Y_{435} - (Y_{345} - Y_{354}) = 2(Y_{534} - Y_{435}) + Y_{453} - Y_{543}, \quad (124)$$

and

$$2Y''_3 = Y_{453} - Y_{543}; \quad (125)$$

so that the first factor (119) may be put under the form:

$$\tfrac{1}{2}(1 - \theta)\{10(Y_{534} - Y_{435}) + (5 - \zeta)(Y_{453} - Y_{543})\}. \quad (126)$$

Besides, by (111), the three differences

$$Y_{cde} - Y_{ced}, \quad Y_{cde} - Y_{edc}, \quad Y_{cde} - Y_{dce}, \quad (127)$$

are divisible, respectively, by the three products

$$(x_2 - x_c)(x_d - x_e), \quad (x_2 - x_d)(x_e - x_c), \quad (x_2 - x_e)(x_c - x_d); \quad (128)$$

and, therefore, the factor (126) is divisible by the product

$$(x_2 - x_3)(x_4 - x_5), \quad (129)$$

the quotient of this division being a rational and integral and homogeneous function of the five roots x, which is no higher than the third dimension, and which it is not difficult to calculate.

31. In this manner we are led to establish an equation of the form:

$$\text{v}_{345} - \text{v}_{354} + \theta^2(\text{v}_{453} - \text{v}_{543}) + \theta(\text{v}_{534} - \text{v}_{435}) = (1-\theta)(x_2 - x_3)(x_4 - x_5)\,\text{n}_1; \quad (130)$$

in which if we make

$$2\text{n}_1 = 10\text{n}'_1 + (5 - \zeta)\,\text{n}''_1, \quad (131)$$

we have

$$\text{n}'_1 = \frac{\text{Y}_{534} - \text{Y}_{435}}{(x_2 - x_3)(x_4 - x_5)}, \quad \text{n}''_1 = \frac{\text{Y}_{453} - \text{Y}_{543}}{(x_2 - x_3)(x_4 - x_5)}. \quad (132)$$

Effecting the calculations indicated by these last formulæ, we find

$$\text{n}'_1 = \tfrac{5}{4}(\text{m}''_1 - \text{m}'_1), \quad \text{n}''_1 = -\tfrac{5}{2}\text{m}''_1, \quad (133)$$

m'_1 and m''_1 being determined by the equations (99) and (100); and, therefore, with the meaning (98) of m_1, we find the relation:

$$\text{n}_1 = -125\text{m}_1. \quad (134)$$

Thus, the first of the three factors (119) may be put under the form:

$$-125(1-\theta)(x_2 - x_3)(x_4 - x_5)\,\text{m}_1; \quad (135)$$

in deducing which, it is to be observed, that the first term, $x_a^4 x_b$, of the formula (107) for w_{abcde}, gives, by (108), the five following terms of x_{bcde}:

$$5x_a^4 x_b + 5x_b^4 x_c + 5x_c^4 x_d + 5x_d^4 x_e + 5x_e^4 x_a; \quad (136)$$

and these five terms of x give, respectively, by (111), the five following parts of Y_{cde}:

$$\left.\begin{array}{l}\frac{5}{4}\,x_1^4\,(x_2+x_c+x_d+x_e),\\ \frac{5}{4}\,(x_2^4\,x_c+x_c^4\,x_2+x_d^4\,x_e+x_e^4\,x_d),\\ \frac{5}{4}\,(x_c^4\,x_d+x_2^4\,x_e+x_e^4\,x_2+x_d^4\,x_c),\\ \frac{5}{4}\,(x_d^4\,x_e+x_e^4\,x_d+x_2^4\,x_c+x_c^4\,x_2),\\ \frac{5}{4}\,(x_e^4+x_d^4+x_c^4+x_2^4)\,x_1;\end{array}\right\} \qquad (137)$$

which are to be combined with the other parts of Y, derived, in like manner, through X, from the other terms of W, and to be submitted to the processes indicated by the formulæ (132), in order to deduce the values (133) of N'_1 and N''_1, and thence, by (131) and (98), the relation (134) between N_1 and M_1, which conducts, by (130), to the expression (135). For example, the first and last of the five parts (137) of Y, contribute nothing to either of the two quotients (132), because those parts are symmetric relatively to x_c, x_d, x_e; but the second part (137) contributes

$$-\tfrac{5}{4}\,(x_2^3+x_2^2\,x_d+x_2\,x_d^2+x_d^3+x_e^3+x_e^2\,x_c+x_e\,x_c^2+x_c^3), \qquad (138)$$

to the quotient

$$\frac{Y_{cde}-Y_{edc}}{(x_2-x_d)\,(x_e-x_c)}, \qquad (139)$$

and

$$+\tfrac{5}{4}\,(x_2^3+x_2^2\,x_e+x_2\,x_e^2+x_e^3+x_c^3+x_c^2\,x_d+x_c\,x_d^2+x_d^3), \qquad (140)$$

to the quotient

$$\frac{Y_{cde}-Y_{dce}}{(x_2-x_e)\,(x_c-x_d)}; \qquad (141)$$

this second part (137) of Y contributes therefore, by (132),

$$-\tfrac{5}{4}\,(x_2^3+x_2^2\,x_3+x_2\,x_3^2+x_3^3+x_4^3+x_4^2\,x_5+x_4\,x_5^2+x_5^3), \qquad (142)$$

to the quotient N'_1, and the same quantity with its sign changed to the quotient N''_1: and the other parts of the same two quotients are determined in a similar manner.

32. The two other factors (119) may respectively be expressed as follows:

$$-125\,(1-\theta^2)\,(x_2-x_4)\,(x_3-x_5)\,M_2, \qquad (143)$$

and

$$-125\,(\theta-\theta^2)\,(x_2-x_5)\,(x_3-x_4)\,M_3; \qquad (144)$$

in which, M_2 and M_3 are formed from M_1, as in the twenty-sixth article; because the second factor (119) may be formed from the first, by interchanging x_3 and x_4, and multiplying by $-\theta^2$; and the third factor may be formed from the second, by interchanging x_4 and x_5, and multiplying again by $-\theta^2$. If then we multiply the three expressions (135) (143) (144) for the three factors (119) together, and divide by three, we find:

$$18\sqrt{H_4} = -5^9(\theta - \theta^2)\varpi M_1 M_2 M_3; \tag{145}$$

ϖ denoting here the product (116) of the six differences of the four roots $x_2, \ldots x_5$. The expression (101) for H_4 itself is therefore reproduced under the form:

$$H_4 = -2^{-2}3^{-3}5^{18}\varpi^2 M_1^{\ 2} M_2^{\ 2} M_3^{\ 2}; \tag{146}$$

and the conclusions of former articles are thus confirmed anew, by a method which is entirely different, in its conception and in its processes of calculation, from those which were employed before.

33. It may not, however, be useless to calculate, for some particular equation of the fifth degree, the numerical values of some of the most important quantities above considered, and so to illustrate and exemplify some of the chief formulæ already established. Consider therefore the equation:

$$x^5 - 5x^3 + 4x = 0; \tag{147}$$

of which the roots may be arranged in the order:

$$x_1 = 2, \quad x_2 = 1, \quad x_3 = 0, \quad x_4 = -1, \quad x_5 = -2; \tag{148}$$

and may (because their sum is zero) be also written thus:

$$x' = 2, \quad x'' = 1, \quad x''' = 0, \quad x^{IV} = -1, \quad x^{V} = -2. \tag{149}$$

Employing the notation (32), in combination with (22) or with (105), we have now:

$$\left.\begin{aligned}
T_{2345} &= (2 + \omega^4 - \omega^2 - 2\omega)^5; \\
T_{3254} &= (2 + \omega^3 - 2\omega^2 - \omega)^5; \\
T_{4523} &= (2 - \omega^4 - 2\omega^3 + \omega^2)^5; \\
T_{5432} &= (2 - 2\omega^4 - \omega^3 + \omega)^5.
\end{aligned}\right\} \tag{150}$$

But $\omega^5 = 1$; therefore,

$$\text{T}_{5432} = (-2 - \omega^4 + \omega^2 + 2\omega)^5, \tag{151}$$

and

$$\text{T}_{2345} + \text{T}_{5432} = 0. \tag{152}$$

Again,

$$\text{T}_{3254} = (1 - \omega^2)^5 (2 - \omega)^5, \quad \text{T}_{4523} = (1 - \omega^3)^5 (2 - \omega^4)^5; \tag{153}$$

and if we make

$$(2 - \omega)^5 = \text{E} - \text{O}, \quad (2 + \omega)^5 = \text{E} + \text{O}, \tag{154}$$

we shall have

$$\text{E} = 32 + 80\omega^2 + 10\omega^4, \quad \text{O} = 80\omega + 40\omega^3 + \omega^5; \tag{155}$$

also,

$$(1 - \omega^2)^5 = -5\omega^2 (1 - \omega^2)(1 - \omega^2 + \omega^4); \tag{156}$$

we find, therefore, by easy calculations,

$$\left.\begin{aligned} (1 - \omega^2)^5\, \text{E} &= 300 + 430\omega - 110\omega^2 - 540\omega^3 - 80\omega^4, \\ (1 - \omega^2)^5\, \text{O} &= 600 + 190\omega - 405\omega^2 - 395\omega^3 + 10\omega^4; \end{aligned}\right\} \tag{157}$$

and by subtracting the latter of these two products from the former, and afterwards changing ω to its reciprocal, we obtain:

$$\left.\begin{aligned} \text{T}_{3254} &= -300 + 240\omega + 295\omega^2 - 145\omega^3 - 90\omega^4, \\ \text{T}_{4523} &= -300 + 240\omega^4 + 295\omega^3 - 145\omega^2 - 90\omega. \end{aligned}\right\} \tag{158}$$

We have, therefore, by (20),

$$\text{T}_{3254} + \text{T}_{4523} = -750; \tag{159}$$

and, consequently, by (33) and (152),

$$\text{V}_{345} = -\frac{375}{2}. \tag{160}$$

34. In like manner, to compute, in this example, the second of the six functions v, we have

$$\left.\begin{aligned} &\text{T}_{2453} = (2 + \omega^4 - \omega^3 - 2\omega^2)^5 = -\text{T}_{3542}; \\ &\text{T}_{4235} = (1 - \omega)^5 (2 + \omega^3)^5, \quad \text{T}_{5324} = (1 - \omega^4)^5 (2 + \omega^2)^5; \end{aligned}\right\} \tag{161}$$

adding then the two products (157) together, and afterwards changing ω to ω^3 and ω^2 successively, we find, by (154):

$$\left.\begin{array}{l} \mathrm{T}_{4235} = 900 + 620\omega^3 - 515\omega - 935\omega^4 - 70\omega^2, \\ \mathrm{T}_{5324} = 900 + 620\omega^2 - 515\omega^4 - 935\omega - 70\omega^3; \end{array}\right\} \quad (162)$$

but, by (20), (30), and (54),

$$2(\omega + \omega^4) = -1 + \mathrm{D}, \quad 2(\omega^2 + \omega^3) = -1 - \mathrm{D}, \quad \mathrm{D}^2 = 5; \quad (163)$$

therefore,

$$\mathrm{T}_{2453} + \mathrm{T}_{3542} = 0, \quad \mathrm{T}_{4235} + \mathrm{T}_{5324} = 2250 - 1000\mathrm{D}; \quad (164)$$

and

$$\mathrm{V}_{453} = \tfrac{1}{2}(1125 - 500\mathrm{D}). \quad (165)$$

35. To compute the third of the functions V, we have, in the present question, the relations:

$$\mathrm{T}_{2534} = -\mathrm{T}_{3254}, \quad \mathrm{T}_{5243} = -\mathrm{T}_{4235}, \quad \mathrm{T}_{3425} = -\mathrm{T}_{5324}, \quad \mathrm{T}_{4352} = -\mathrm{T}_{4523}; \quad (166)$$

and, therefore, by (159) and (164),

$$\mathrm{V}_{534} = -375 + 250\mathrm{D}. \quad (167)$$

For the fourth function V, we have, by processes entirely similar to the foregoing:

$$\left.\begin{array}{l} \mathrm{T}_{2354} = -(1-\omega^3)^5(2+\omega^4)^5, \quad \mathrm{T}_{4532} = -(1-\omega^2)^5(2+\omega)^5, \\ \mathrm{T}_{2354} + \mathrm{T}_{4532} = -2250 - 1000\mathrm{D}; \end{array}\right\} \quad (168)$$

$$\left.\begin{array}{l} \mathrm{T}_{3245} = -(1-\omega^4)^5(2-\omega^2)^5, \quad \mathrm{T}_{5423} = -(1-\omega)^5(2-\omega^3)^5, \\ \mathrm{T}_{3245} + \mathrm{T}_{5423} = +750; \end{array}\right\} \quad (169)$$

$$\mathrm{V}_{354} = -375 - 250\mathrm{D}. \quad (170)$$

For the fifth function V, we have the relations:

$$\mathrm{T}_{2543} = -\mathrm{T}_{2354}; \quad \mathrm{T}_{5234} = -\mathrm{T}_{4325}; \quad \mathrm{T}_{3452} = -\mathrm{T}_{4532}; \quad (171)$$

and, therefore, by (168),

$$\mathrm{V}_{543} = \tfrac{1}{2}(1125 + 500\mathrm{D}). \quad (172)$$

Finally, for the sixth function V, we have

$$\mathrm{T}_{2435} = -\mathrm{T}_{5423}, \quad \mathrm{T}_{4253} = -\mathrm{T}_{3524}, \quad \mathrm{T}_{5342} = -\mathrm{T}_{3245}; \quad (173)$$

and, therefore, by (169),

$$\mathrm{v}_{435} = -\frac{375}{2}. \tag{174}$$

The three first values of v may therefore be thus collected:

$$\tfrac{2}{125}\mathrm{v}_{345} = -3; \quad \tfrac{2}{125}\mathrm{v}_{453} = 9 - 4\mathrm{D}; \quad \tfrac{2}{125}\mathrm{v}_{534} = -6 + 4\mathrm{D}; \tag{175}$$

and the three last values, in an inverted order, may in like manner be expressed by the equations:

$$\tfrac{2}{125}\mathrm{v}_{435} = -3; \quad \tfrac{2}{125}\mathrm{v}_{543} = 9 + 4\mathrm{D}; \quad \tfrac{2}{125}\mathrm{v}_{354} = -6 - 4\mathrm{D}. \tag{176}$$

36. It is evident that these six values of v are of the forms (113) and (114), and that they verify, in the present case, the general relation (121). They show also, by (e) and (d) of article 28., that not only H_2, but H_1, vanishes in this example; the common value of the two sums (121), of the three first and three last values of v, being zero. Accordingly, if we compare the particular equation (147) with the general forms (1) and (2), we find the following values of the coefficients (B, C, D, E, not having here their recent meanings):

$$\mathrm{A} = 0, \quad \mathrm{B} = -5, \quad \mathrm{C} = 0, \quad \mathrm{D} = 4, \quad \mathrm{E} = 0, \tag{177}$$

and

$$p = -5, \quad q = 0, \quad r = 4, \quad s = 0; \tag{178}$$

and therefore the formula (51) gives here

$$\mathrm{H}_1 = 0. \tag{179}$$

We find also, with the same meanings of θ and ζ as in former articles:

$$\left.\begin{aligned} \tfrac{2}{125}(\mathrm{v}_{345} + \theta^2\mathrm{v}_{453} + \theta\mathrm{v}_{534}) &= 3(4\theta^2 - \theta) + 4\zeta; \\ \tfrac{2\theta^2}{125}(\mathrm{v}_{354} + \theta^2\mathrm{v}_{543} + \theta\mathrm{v}_{435}) &= 3(4\theta - \theta^2) + 4\zeta; \end{aligned}\right\} \tag{180}$$

and, therefore, by (e) and (d),

$$\left.\begin{aligned} 2^3\,3^3\,5^{-9}(\mathrm{H}_3 + \sqrt{\mathrm{H}_4}) &= \{3(4\theta^2 - \theta) + 4\zeta\}^3, \\ 2^3\,3^3\,5^{-9}(\mathrm{H}_3 - \sqrt{\mathrm{H}_4}) &= \{3(4\theta - \theta^2) + 4\zeta\}^3; \end{aligned}\right\} \tag{181}$$

equations which give, by (11) and (57):

$$\sqrt{\mathrm{H}_4} = 2^{-2}\,5^{10}(\theta - \theta^2)(23 + 3\zeta); \tag{182}$$

and

$$\mathrm{H}_4 = -2^{-3}\,3^1\,5^{20}(197 + 69\zeta). \tag{183}$$

Let us now compare these last numerical results with the general formulæ found by other methods in earlier articles of this paper.

37. The method of the thirteenth article gives, in the present example,

$$\left.\begin{array}{l} \alpha = -\tfrac{1}{2}, \quad \beta = 1, \quad \gamma = \tfrac{1}{2}, \quad \delta = 0, \quad \epsilon = \tfrac{5}{12}, \quad \eta = 0, \\ \kappa = \frac{4+\theta^2}{12}, \quad \lambda = \frac{4+\theta}{12}, \quad \iota = \kappa\lambda = \tfrac{13}{144}, \\ \kappa^3 + \lambda^3 = \tfrac{35}{864}, \quad \tfrac{1}{2}(\kappa^3 - \lambda^3) = -2^{-5}\,3^{-1}(\theta - \theta^2); \end{array}\right\} \quad (184)$$

and, therefore, by (59),

$$\left.\begin{array}{l} \frac{3\theta k}{250} = 5(1-\zeta), \quad \frac{3\theta l}{250} = 12(2+\zeta), \\ k^3 - 3kl^2\kappa\lambda - l^3(\kappa^3+\lambda^3) = -2^3\,3^1\,5^{10}(23+3\zeta); \end{array}\right\} \quad (185)$$

and, accordingly, if we multiply the last expression (184) by the last expression (185), we are led, by the general formula (60), to the same result for $\sqrt{\mathrm{H}_4}$, and therefore for H_4, as was obtained in the last article by an entirely different method. The general formula (60) may also, in virtue of the equations (13), (59), (62), (63), (70), (116), and (4), be written thus:

$$18\sqrt{\mathrm{H}_4} = -5^9(\theta - \theta^2)\,\varpi\mathrm{L}; \qquad (186)$$

which agrees, by (94), with the general result (145), and in which we have now

$$\varpi = 1\,.\,2\,.\,3\,.\,1\,.\,2\,.\,1 = 12; \qquad (187)$$

while L may be calculated by the definitions (62) and (63), which give, at present, by the values (184) for α, ϵ, η, ι,

$$\mu = \tfrac{5}{6}(1-\zeta), \quad \nu = -2(2+\zeta), \qquad (188)$$

and

$$\mathrm{L} = -\tfrac{15}{8}(23+3\zeta): \qquad (189)$$

and thus we arrive again at the same value of $\sqrt{\mathrm{H}_4}$ as before. The same value of L may be obtained in other ways, by other formulæ of this paper; for example, by those of the 24th and 25th articles, which give, in the present question,

$$\mathrm{L}' = -2^3\,3^1\,5^2\,23; \quad \mathrm{L}'' = +2^3\,3^2\,5^3. \qquad (190)$$

We may also decompose L into three factors M, which are here:

$$\text{M}_1 = -\tfrac{1}{2}(3+4\zeta); \quad \text{M}_2 = \tfrac{1}{2}(3-\zeta); \quad \text{M}_3 = \tfrac{5}{2}; \tag{191}$$

and which conduct still to the same result.

38. An equation of the fifth degree, which, like that here assumed as an example, has all its roots unequal, may have those roots arranged in 120 different ways; and any one of these arrangements may be taken as the basis of a verification such as that contained in the last five articles. But we have seen that no such change of arrangement will affect the value of either H_1 or H_2; and with respect to H_4, which has been more particularly under our consideration in this paper, it is not difficult to perceive that an interchange of any two of the four last roots (x_2, x_3, x_4, x_5, or x'', x''', x^{IV}, x^{V}), of the proposed equation of the fifth degree, will merely change the sign of the square-root, $\sqrt{\text{H}_4}$, in the foregoing formulæ, without making any change in the value of H_4 itself, which has been shown to depend on the first root (x_1 or x') alone. It will, however, be instructive to exemplify this last-mentioned dependence, by applying the foregoing general processes to the case of the equation of the fifth degree (147), the two first roots being made to change places with each other, in such a manner that the order shall now be chosen as follows:

$$x_1 = 1, \quad x_2 = 2, \quad x_3 = 0, \quad x_4 = -1, \quad x_5 = -2, \tag{192}$$

or (since the sum of all five vanishes),

$$x' = 1, \quad x'' = 2, \quad x''' = 0, \quad x^{IV} = -1, \quad x^{V} = -2. \tag{193}$$

We find, for this new case, by calculations of the same sort as in recent articles of this paper, the following new system of equations for the values of the six functions v:

$$\left.\begin{aligned} &\tfrac{2}{125}\text{v}_{345} = 12 + 4\text{D}; \quad \tfrac{2}{125}\text{v}_{453} = -9 - 4\text{D}; \quad \tfrac{2}{125}\text{v}_{534} = -3; \\ &\tfrac{2}{125}\text{v}_{435} = 12 - 4\text{D}; \quad \tfrac{2}{125}\text{v}_{543} = -9 + 4\text{D}; \quad \tfrac{2}{125}\text{v}_{354} = -3; \end{aligned}\right\} \tag{194}$$

in which, D has again the meaning assigned by (30): and, consequently,

$$\left.\begin{aligned} &\tfrac{2\theta^2}{125}(\text{v}_{345} + \theta^2\text{v}_{453} + \theta\text{v}_{534}) = 3(5\theta^2 - 2\theta) - 4\zeta; \\ &\tfrac{2}{125}(\text{v}_{354} + \theta^2\text{v}_{543} + \theta\text{v}_{435}) = 3(5\theta - 2\theta^2) - 4\zeta; \end{aligned}\right\} \tag{195}$$

$$\left.\begin{aligned} 2^4\, 3^3\, 5^{-9} \sqrt{\text{H}_4} &= \{3(5\theta^2 - 2\theta) - 4\zeta\}^3 - \{3(5\theta - 2\theta^2) - 4\zeta\}^3; \\ \sqrt{\text{H}_4} &= 2^{-3}\, 5^9\, 7\, (\theta - \theta^2)(55 - 6\zeta); \end{aligned}\right\} \tag{196}$$

and

$$\text{H}_4 = -2^{-6}\, 3^1\, 5^{19}\, 7^2\, (497 - 132\zeta): \tag{197}$$

results which differ from those obtained with the former arrangement of the five roots of the proposed equation (147), but of which the agreement with the general formulæ of the present paper may be evinced by processes similar to those of the last article.

39. As a last example, if the arrangement of the same five roots be

$$x_1 = 0, \quad x_2 = 1, \quad x_3 = 2, \ x_4 = -1, \quad x_5 = -2, \tag{198}$$

we then find easily that all the six quantities v vanish, and, therefore, that we have, with this arrangement,

$$\sqrt{\text{H}_4} = 0, \quad \text{H}_4 = 0. \tag{199}$$

All these results respecting the numerical values of H_4, for different arrangements of the roots of the proposed equation (147), are included in the common expression:

$$\text{H}_4 = -2^{-4}\, 3^3\, 5^{18} \left(\frac{5\,(72x' + 5x'^3) - 2\zeta\,(38x' - 17x'^3)}{5x'^4 - 15x'^2 + 4}\right)^2; \tag{200}$$

which results from the formula (85), combined with (79) and (86) (87) (88): and thus we have a new confirmation of the correctness of the foregoing calculations.

40. It is then proved, in several different ways, that the quantity H_4, in the formulæ which have been marked in this paper (a), (b), (c), (d), and which have been proposed by Professor BADANO for the solution of the general equation of the fifth degree, is not a symmetric function of the five roots of that equation. And since it has been shown that the expression of this quantity H_4, contains in general the imaginary radical ζ or $\sqrt{-15}$, which changes sign in passing to the expression of the analogous quantity H_6, we see that these two quantities, H_4 and H_6, are not generally equal to each other, as Professor BADANO, in a supplement to his essay, appears to think that they must be. They are, on the contrary, found to be in general the two unequal roots of a quadratic equation, namely,

$$\text{H}_4^2 + \text{Q}\text{H}_4 + \text{R}^2 = 0, \tag{201}$$

in which

$$\text{Q} = -(\text{H}_4 + \text{H}_6) = 2^{-13}\, 3^{-3}\, 5^{15}\, \varpi^2\, (5\text{L}'^2 - 3\text{L}''^2), \tag{202}$$

and

$$\text{R} = \sqrt{\text{H}_4}.\sqrt{\text{H}_6} = -2^{-14}\, 3^{-3}\, 5^{15}\, \varpi^2\, (5\text{L}'^2 + 3\text{L}''^2), \tag{203}$$

ϖ, L′, and L″, having the significations already assigned; and the values of the coefficients Q and R depend essentially, in general, on the choice of the root x', although they can always be expressed as rational functions of that root.

41. It does not appear to be necessary to write here the analogous calculations, which show that the two remaining quantities H_3 and H_5, which enter into the same formulæ (a), (b), (c), (d), are not, in general, symmetric functions of the five roots of the proposed equation of the fifth degree, nor equal to each other, but roots of a quadratic equation, of the same kind with that considered in the last article. But it may be remarked, in illustration of this general result, that for the particular equation of the fifth degree which has been marked (147) we find, with the arrangement (148) of the five roots, the values:

$$H_3 = 2^{-3}\, 3^{-2}\, 5^9 (1809 - 914\zeta), \quad H_5 = 2^{-3}\, 3^{-2}\, 5^9 (1809 + 914\zeta); \quad (204)$$

with the arrangement (192),

$$H_3 = 2^{-2}\, 3^{-2}\, 5^9 (1269 + 781\zeta), \quad H_5 = 2^{-2}\, 3^{-2}\, 5^9 (1269 - 781\zeta); \quad (205)$$

and, with the arrangement (198),

$$H_3 = 0, \quad H_5 = 0. \quad (206)$$

The general decomposition of these quantities H_3 and H_5, into factors of the fifth dimension, referred to in a former article, results easily from the equations of definition (42) and (43), which give:

$$\left.\begin{aligned} 2H_3 &= (h + h')(h + \theta h')(h + \theta^2 h'); \\ 2H_5 &= (i + i')\ (i + \theta i')\ (i + \theta^2 i'). \end{aligned}\right\} \quad (207)$$

And the same equations, when combined with (40) and (38), show that the combinations

$$H_3^{\,2} - H_4 = h^3 h'^3, \quad H_5^{\,2} - H_6 = i^3 i'^3, \quad (208)$$

are exact cubes of rational functions of the five roots of the equation of the fifth degree, which functions are each of the tenth dimension relatively to those five roots, and are symmetric relatively to four of them; while each of these functions, hh' and ii', decomposes itself into two factors, which are also rational functions of the five roots, and are no higher than the fifth dimension.

42. In the foregoing articles, we have considered only those six quantities H

which were connected with the composition of the six functions v, determined by the definition (33). But if we establish the expressions,

$$\left.\begin{array}{llllllll}
\mathrm{T}_{2cde} = \mathrm{V}_{cde} & + & \mathrm{V}'_{cde} & + & \mathrm{V}''_{cde} & + & \mathrm{V}'''_{cde}, \\
\mathrm{T}_{c2ed} = \mathrm{V}_{cde} & + & & - & & - & \\
\mathrm{T}_{de2c} = \mathrm{V}_{cde} & - & & + & & - & \\
\mathrm{T}_{edc2} = \mathrm{V}_{cde} & - & & - & & + &
\end{array}\right\} \quad (209)$$

which include the definition (33), and give,

$$\left.\begin{array}{llllll}
\mathrm{V}'_{cde} = \tfrac{1}{4}(\mathrm{T}_{2cde} & + \mathrm{T}_{c2ed} & - \mathrm{T}_{de2c} & - \mathrm{T}_{edc2}), \\
\mathrm{V}''_{cde} = \tfrac{1}{4}(\mathrm{T}_{2cde} & - & + & - \quad), \\
\mathrm{V}'''_{cde} = \tfrac{1}{4}(\mathrm{T}_{2cde} & - & - & + \quad),
\end{array}\right\} \quad (210)$$

we are conducted to expressions for the squares of the three functions v$'$, v$''$, v$'''$, which are entirely analogous to those marked (a) and (b), and have accordingly been assigned under such forms by Professor BADANO, involving eighteen new quantities, $\mathrm{H}_7, \ldots \mathrm{H}_{24}$; which quantities, however, are not found to be symmetric functions of the five roots of the equation of the fifth degree, though they are symmetric relatively to four of them.

43. In making the investigations which conduct to this result, it is convenient to establish the following definitions, analogous to, and in combination with, that marked (111):

$$\left.\begin{array}{llll}
4\mathrm{Y}'_{cde} = \mathrm{X}_{2cde} & + \mathrm{X}_{c2ed} & - \mathrm{X}_{de2c} & - \mathrm{X}_{edc2}, \\
4\mathrm{Y}''_{cde} = \mathrm{X}_{2cde} & - & + & - \quad , \\
4\mathrm{Y}'''_{cde} = \mathrm{X}_{2cde} & - & - & + \quad ;
\end{array}\right\} \quad (211)$$

for thus we obtain,

$$\left.\begin{array}{llllllll}
\mathrm{X}_{2cde} = \mathrm{Y}_{cde} & + & \mathrm{Y}'_{cde} & + & \mathrm{Y}''_{cde} & + & \mathrm{Y}'''_{cde}, \\
\mathrm{X}_{c2ed} = \mathrm{Y}_{cde} & + & & - & & - & , \\
\mathrm{X}_{de2c} = \mathrm{Y}_{cde} & - & & + & & - & , \\
\mathrm{X}_{edc2} = \mathrm{Y}_{cde} & - & & - & & + & ;
\end{array}\right\} \quad (212)$$

$$\left.\begin{array}{l}
\mathrm{V}'_{cde} = (\omega^4 - \omega)\,\mathrm{Y}'_{cde} + (\omega^3 - \omega^2)\,\mathrm{Y}''_{dce}, \\
\mathrm{V}''_{cde} = (\omega^4 - \omega)\,\mathrm{Y}''_{cde} - (\omega^3 - \omega^2)\,\mathrm{Y}'_{dce}, \\
\mathrm{V}'''_{cde} = (\omega^4 + \omega - 2)\,\mathrm{Y}'''_{cde} - (\omega^3 + \omega^2 - 2)\,\mathrm{Y}'''_{dce}.
\end{array}\right\} \quad (213)$$

Introducing also the following notations, analogous to (112),

$$\left.\begin{aligned}
&Y'_{345} = Y^{\backslash\prime}_5 + Y^{\backslash\prime\prime}_5, \quad Y''_{435} = Y^{\backslash\prime}_5 - Y^{\backslash\prime\prime}_5,\\
&Y'_{453} = Y^{\backslash\prime}_3 + Y^{\backslash\prime\prime}_3, \quad Y''_{543} = Y^{\backslash\prime}_3 - Y^{\backslash\prime\prime}_3,\\
&Y'_{534} = Y^{\backslash\prime}_4 + Y^{\backslash\prime\prime}_4, \quad Y''_{354} = Y^{\backslash\prime}_4 - Y^{\backslash\prime\prime}_4;
\end{aligned}\right\} \tag{214}$$

$$\left.\begin{aligned}
&Y''_{345} = Y^{\backslash\backslash\prime}_5 + Y^{\backslash\backslash\prime\prime}_5, \; Y'_{435} = Y^{\backslash\backslash\prime}_5 - Y^{\backslash\backslash\prime\prime}_5,\\
&Y''_{453} = Y^{\backslash\backslash\prime}_3 + Y^{\backslash\backslash\prime\prime}_3, \; Y'_{543} = Y^{\backslash\backslash\prime}_3 - Y^{\backslash\backslash\prime\prime}_3,\\
&Y''_{534} = Y^{\backslash\backslash\prime}_4 + Y^{\backslash\backslash\prime\prime}_4, \; Y'_{354} = Y^{\backslash\backslash\prime}_4 - Y^{\backslash\backslash\prime\prime}_4;
\end{aligned}\right\} \tag{215}$$

and

$$\left.\begin{aligned}
&Y'''_{345} = Y^{\backslash\backslash\backslash\prime}_5 + Y^{\backslash\backslash\backslash\prime\prime}_5, \; Y'''_{435} = Y^{\backslash\backslash\backslash\prime}_5 - Y^{\backslash\backslash\backslash\prime\prime}_5,\\
&Y'''_{453} = Y^{\backslash\backslash\backslash\prime}_3 + Y^{\backslash\backslash\backslash\prime\prime}_3, \; Y'''_{543} = Y^{\backslash\backslash\backslash\prime}_3 - Y^{\backslash\backslash\backslash\prime\prime}_3,\\
&Y'''_{534} = Y^{\backslash\backslash\backslash\prime}_4 + Y^{\backslash\backslash\backslash\prime\prime}_4, \; Y'''_{354} = Y^{\backslash\backslash\backslash\prime}_4 - Y^{\backslash\backslash\backslash\prime\prime}_4;
\end{aligned}\right\} \tag{216}$$

we find, by (30), results analogous to (113) and (114), namely,

$$\left.\begin{aligned}
&V'_{345} = BY^{\backslash\prime}_5 + CY^{\backslash\prime\prime}_5, \; V'_{435} = BY^{\backslash\backslash\prime}_5 - CY^{\backslash\backslash\prime\prime}_5,\\
&V'_{453} = BY^{\backslash\prime}_3 + CY^{\backslash\prime\prime}_3, \; V'_{543} = BY^{\backslash\backslash\prime}_3 - CY^{\backslash\backslash\prime\prime}_3,\\
&V'_{534} = BY^{\backslash\prime}_4 + CY^{\backslash\prime\prime}_4, \; V'_{354} = BY^{\backslash\backslash\prime}_4 - CY^{\backslash\backslash\prime\prime}_4;
\end{aligned}\right\} \tag{217}$$

$$\left.\begin{aligned}
&V''_{345} = CY^{\backslash\backslash\prime}_5 + BY^{\backslash\backslash\prime\prime}_5, \; V''_{435} = CY^{\backslash\prime}_5 - BY^{\backslash\prime\prime}_5,\\
&V''_{453} = CY^{\backslash\backslash\prime}_3 + BY^{\backslash\backslash\prime\prime}_3, \; V''_{543} = CY^{\backslash\prime}_3 - BY^{\backslash\prime\prime}_3,\\
&V''_{534} = CY^{\backslash\backslash\prime}_4 + BY^{\backslash\backslash\prime\prime}_4, \; V''_{354} = CY^{\backslash\prime}_4 - BY^{\backslash\prime\prime}_4;
\end{aligned}\right\} \tag{218}$$

and

$$\left.\begin{aligned}
&V'''_{345} = DY^{\backslash\backslash\backslash\prime}_5 - 5Y^{\backslash\backslash\backslash\prime\prime}_5, \; V'''_{435} = DY^{\backslash\backslash\backslash\prime}_5 + 5Y^{\backslash\backslash\backslash\prime\prime}_5,\\
&V'''_{453} = DY^{\backslash\backslash\backslash\prime}_3 - 5Y^{\backslash\backslash\backslash\prime\prime}_3, \; V'''_{543} = DY^{\backslash\backslash\backslash\prime}_3 + 5Y^{\backslash\backslash\backslash\prime\prime}_3,\\
&V'''_{534} = DY^{\backslash\backslash\backslash\prime}_4 - 5Y^{\backslash\backslash\backslash\prime\prime}_4, \; V'''_{354} = DY^{\backslash\backslash\backslash\prime}_4 + 5Y^{\backslash\backslash\backslash\prime\prime}_4.
\end{aligned}\right\} \tag{219}$$

And squaring the eighteen expressions (217) (218) (219), we obtain others, for the eighteen functions V'^2, V''^2, V'''^2, which depend indeed on eighteen others of the forms Y, determined by the definitions (211) (214) (215) (216), but which are free, by (54) and (55), from the imaginary fifth root of unity, ω, except so far as that root enters by means of the combination D, of which the square is $= 5$.

44. If, now, we write like Professor BADANO (who uses, indeed, as has been stated already, a notation slightly different),

$$\left.\begin{aligned}
\mathrm{v}'''^{2}_{453} &= \mathrm{H}_{19} + \sqrt{\mathrm{H}_{20}} + \sqrt[3]{\mathrm{H}_{21} + \sqrt{\mathrm{H}_{22}}} + \sqrt[3]{\mathrm{H}_{23} - \sqrt{\mathrm{H}_{24}}};\\
\mathrm{v}'''^{2}_{534} &= \mathrm{H}_{19} + \sqrt{\mathrm{H}_{20}} + \theta\sqrt[3]{\mathrm{H}_{21} + \sqrt{\mathrm{H}_{22}}} + \theta^2\sqrt[3]{\mathrm{H}_{23} - \sqrt{\mathrm{H}_{24}}};\\
\mathrm{v}'''^{2}_{345} &= \mathrm{H}_{19} + \sqrt{\mathrm{H}_{20}} + \theta^2\sqrt[3]{\mathrm{H}_{21} + \sqrt{\mathrm{H}_{22}}} + \theta\sqrt[3]{\mathrm{H}_{23} - \sqrt{\mathrm{H}_{24}}};
\end{aligned}\right\} \qquad (\mathrm{a}''')$$

and

$$\left.\begin{aligned}
\mathrm{v}'''^{2}_{543} &= \mathrm{H}_{19} - \sqrt{\mathrm{H}_{20}} + \sqrt[3]{\mathrm{H}_{21} - \sqrt{\mathrm{H}_{22}}} + \sqrt[3]{\mathrm{H}_{23} + \sqrt{\mathrm{H}_{24}}};\\
\mathrm{v}'''^{2}_{435} &= \mathrm{H}_{19} - \sqrt{\mathrm{H}_{20}} + \theta\sqrt[3]{\mathrm{H}_{21} - \sqrt{\mathrm{H}_{22}}} + \theta^2\sqrt[3]{\mathrm{H}_{23} + \sqrt{\mathrm{H}_{24}}};\\
\mathrm{v}'''^{2}_{354} &= \mathrm{H}_{19} - \sqrt{\mathrm{H}_{20}} + \theta^2\sqrt[3]{\mathrm{H}_{21} - \sqrt{\mathrm{H}_{22}}} + \theta\sqrt[3]{\mathrm{H}_{23} + \sqrt{\mathrm{H}_{24}}};
\end{aligned}\right\} \qquad (\mathrm{b}''')$$

together with twelve other expressions similar to these, and to those already marked (a) and (b), but involving the functions v′ and v″; we shall have, as the same author has remarked, a system of converse formulæ, analogous to (c) and (d), for the determination of the values of the eighteen quantities $\mathrm{H}_7, \ldots \mathrm{H}_{24}$. Among these, we shall content ourselves with here examining one of the most simple, namely the following:

$$\mathrm{H}_{19} = \tfrac{1}{6}\left(\mathrm{v}'''^{2}_{345} + \mathrm{v}'''^{2}_{453} + \mathrm{v}'''^{2}_{534} + \mathrm{v}'''^{2}_{354} + \mathrm{v}'''^{2}_{543} + \mathrm{v}'''^{2}_{435}\right); \qquad (220)$$

for the purpose of showing, by an example, that this quantity is not independent of the arrangement of the five roots of the original equation of the fifth degree.

45. Resuming with this view the equation (147), and the arrangement of the roots (148), we find the following system of the twenty-four values of the function x_{bcde}:

$$\left.\begin{aligned}
&\mathrm{x}_{2345} = -500; \quad \mathrm{x}_{3254} = -90; \quad \mathrm{x}_{4523} = 240; \quad \mathrm{x}_{5432} = 500;\\
&\mathrm{x}_{2453} = 1165; \quad \mathrm{x}_{4235} = -935; \quad \mathrm{x}_{5324} = -515; \quad \mathrm{x}_{3542} = -1165;\\
&\mathrm{x}_{2534} = 90; \quad \mathrm{x}_{5243} = 935; \quad \mathrm{x}_{3425} = 515; \quad \mathrm{x}_{4352} = -240;
\end{aligned}\right\} \qquad (221)$$

$$\left.\begin{aligned}
&\mathrm{x}_{2354} = -620; \quad \mathrm{x}_{3245} = -295; \quad \mathrm{x}_{5423} = 145; \quad \mathrm{x}_{4532} = 70;\\
&\mathrm{x}_{2543} = 620; \quad \mathrm{x}_{5234} = -720; \quad \mathrm{x}_{4325} = 720; \quad \mathrm{x}_{3452} = -70;\\
&\mathrm{x}_{2435} = -145; \quad \mathrm{x}_{4253} = 375; \quad \mathrm{x}_{3524} = -375; \quad \mathrm{x}_{5342} = 295;
\end{aligned}\right\} \qquad (222)$$

which give, by (211),

$$\left.\begin{aligned}
&4\mathrm{y}'''_{345} = -150; \quad 4\mathrm{y}'''_{453} = 1450; \quad 4\mathrm{y}'''_{534} = -1600;\\
&4\mathrm{y}'''_{435} = 150; \quad 4\mathrm{y}'''_{543} = 550; \quad 4\mathrm{y}'''_{354} = -400;
\end{aligned}\right\} \qquad (223)$$

and, therefore, by (216),

$$\left.\begin{array}{lll} 8\mathrm{Y}^{\backprime\backprime\backprime\prime}{}_5 = 0\,; & 8\mathrm{Y}^{\backprime\backprime\backprime\prime}{}_3 = 2000\,; & 8\mathrm{Y}^{\backprime\backprime\backprime\prime}{}_4 = -\,2000\,; \\ 8\mathrm{Y}^{\backprime\backprime\backprime\prime\prime}{}_5 = -\,300\,; & 8\mathrm{Y}^{\backprime\backprime\backprime\prime\prime}{}_3 = 900\,; & 8\mathrm{Y}^{\backprime\backprime\backprime\prime\prime}{}_4 = -\,1200\,; \end{array}\right\} \quad (224)$$

whence, by (219),

$$\left.\begin{array}{lll} \tfrac{2}{125}\mathrm{V}'''_{345} = 3\,; & \tfrac{2}{125}\mathrm{V}'''_{453} = -9+4\mathrm{D}\,; & \tfrac{2}{125}\mathrm{V}'''_{534} = 12-4\mathrm{D}\,; \\ \tfrac{2}{125}\mathrm{V}'''_{435} = -\,3\,; & \tfrac{2}{125}\mathrm{V}'''_{543} = 9+4\mathrm{D}\,; & \tfrac{2}{125}\mathrm{V}'''_{354} = -\,12-4\mathrm{D}\,; \end{array}\right\} \quad (225)$$

and the squares of these six second members are

$$9, \quad 161 \mp 72\mathrm{D}, \quad 224 \mp 96\mathrm{D}, \qquad (226)$$

so that we have, by (220), with this arrangement of the five roots of the equation (147),

$$\mathrm{H}_{19} = 2^{-1}\,3^{-1}\,5^{6}\,197. \qquad (227)$$

But with the arrangement (192), we find, by similar calculations,

$$\left.\begin{array}{lll} \tfrac{2}{125}\mathrm{V}'''_{345} = 6+4\mathrm{D}\,; & \tfrac{2}{125}\mathrm{V}'''_{453} = -\,9-4\mathrm{D}\,; & \tfrac{2}{125}\mathrm{V}'''_{534} = -\,3\,; \\ \tfrac{2}{125}\mathrm{V}'''_{435} = -\,6+4\mathrm{D}\,; & \tfrac{2}{125}\mathrm{V}'''_{543} = 9-4\mathrm{D}\,; & \tfrac{2}{125}\mathrm{V}'''_{354} = +\,3\,; \end{array}\right\} \quad (228)$$

of which the squares are

$$116 \pm 48\mathrm{D}, \quad 161 \pm 72\mathrm{D}, \quad 9\,; \qquad (229)$$

and we have now

$$\mathrm{H}_{19} = 2^{-1}\,3^{-1}\,5^{6}\,11^{1}\,13, \qquad (230)$$

a value different from that marked (227). And, finally, with the arrangement of the roots (198), we find instead of the quantities (225) or (228), the following:

$$\mp\,18 - 8\mathrm{D}, \quad \pm\,6, \quad 0, \qquad (231)$$

of which the squares are

$$644 \pm 288\mathrm{D}, \quad 36, \quad 0, \qquad (232)$$

and give still another value for the quantity H now under consideration, namely,

$$\mathrm{H}_{19} = 2^{1}\,3^{-1}\,5^{7}\,17. \qquad (233)$$

46. The twelve other expressions which have been referred to, as being analogous to (a) and (b), are of the forms:

$$\mathrm{V}'^{2}_{345} = \mathrm{H}_7 + \sqrt{\mathrm{H}_8} + \sqrt[3]{\mathrm{H}_9 + \sqrt{\mathrm{H}_{10}}} + \sqrt[3]{\mathrm{H}_{11} - \sqrt{\mathrm{H}_{12}}}\,; \qquad (\mathrm{a}')$$

$$\mathrm{V}'^{2}_{354} = \mathrm{H}_7 - \sqrt{\mathrm{H}_8} + \sqrt[3]{\mathrm{H}_9 - \sqrt{\mathrm{H}_{10}}} + \sqrt[3]{\mathrm{H}_{11} + \sqrt{\mathrm{H}_{12}}}\,; \qquad (\mathrm{b}')$$

$$V''^2_{534} = H_{13} + \sqrt{H_{14}} + \sqrt[3]{H_{15} + \sqrt{H_{16}}} + \sqrt[3]{H_{17} - \sqrt{H_{18}}}; \qquad (a'')$$

$$V''^2_{435} = H_{13} - \sqrt{H_{14}} + \sqrt[3]{H_{15} - \sqrt{H_{16}}} + \sqrt[3]{H_{17} + \sqrt{H_{18}}}; \qquad (b'')$$

and they give, as the simplest of the expressions deduced from them, the two following, which are analogous to that marked (220):

$$H_7 = \tfrac{1}{6}(V'^2_{345} + V'^2_{453} + V'^2_{534} + V'^2_{354} + V'^2_{543} + V'^2_{435}); \qquad (234)$$

$$H_{13} = \tfrac{1}{6}(V''^2_{345} + V''^2_{453} + V''^2_{534} + V''^2_{354} + V''^2_{543} + V''^2_{435}). \qquad (235)$$

For the case of the equation (147), and the arrangement of roots (148), we find the numerical values:

$$\left.\begin{aligned} &\tfrac{2}{5}V'_{345} = -126B - 7C;\ \tfrac{2}{5}V'_{453} = 202B - 11C;\ \tfrac{2}{5}V'_{534} = 25B + 50C; \\ &\tfrac{2}{5}V''_{435} = -126C + 7B;\ \tfrac{2}{5}V''_{543} = 202C + 11B;\ \tfrac{2}{5}V''_{354} = 25C - 50B; \end{aligned}\right\} \qquad (236)$$

$$\left.\begin{aligned} &\tfrac{2}{5}V'_{435} = -18B + 47C;\ \tfrac{2}{5}V'_{543} = 100B - 175C;\ \tfrac{2}{5}V'_{354} = -61B - 52C; \\ &\tfrac{2}{5}V''_{345} = -18C - 47B;\ \tfrac{2}{5}V''_{453} = 100C + 175B;\ \tfrac{2}{5}V''_{534} = -61C + 52B; \end{aligned}\right\} \qquad (237)$$

which may be obtained, either by the method of article 43., combined with the values (221) (222) of the twenty-four functions X; or by the formulæ (210), combined with the following table:

$$\left.\begin{aligned} &\tfrac{2}{5}T_{2345} = \phantom{150 + {}} -175B - 25C;\ \tfrac{2}{5}T_{2435} = -150 - 11B - 77C; \\ &\tfrac{2}{5}T_{2453} = \phantom{150 + {}} +377B + 89C;\ \tfrac{2}{5}T_{2543} = 450 + 111B + 27C + 200D; \\ &\tfrac{2}{5}T_{2534} = 150 + 77B - 11C;\ \tfrac{2}{5}T_{2354} = -450 - 111B - 27C - 200D; \end{aligned}\right\} \qquad (238)$$

and with the condition, that, if we write for abridgment,

$$T_{bcde} = T^{(0)}_{bcde} + BT'_{bcde} + CT''_{bcde} + DT'''_{bcde}, \qquad (239)$$

we have in general the relations,

$$\left.\begin{aligned} T_{edcb} &= T^{(0)}_{bcde} - BT'_{bcde} - CT''_{bcde} + DT'''_{bcde}; \\ T_{cebd} &= T^{(0)}_{bcde} + CT'_{bcde} - BT''_{bcde} - DT'''_{bcde}. \end{aligned}\right\} \qquad (240)$$

And hence, for the same equation of the fifth degree, and the same arrangement of the roots, we find, by (54) and (55):

$$\left.\begin{aligned} H_7 &= -2^{-2}\,3^{-1}\,5^4\,(10975 + 706D); \\ H_{13} &= -2^{-2}\,3^{-1}\,5^4\,(10975 - 706D). \end{aligned}\right\} \qquad (241)$$

But, for the same equation (147), with the arrangement of the roots (192), we find, by similar calculations, the values:

$$\left.\begin{aligned} H_7 &= -2^{-2}\,3^{-1}\,5^4\,(10975 - 1472D); \\ H_{13} &= -2^{-2}\,3^{-1}\,5^4\,(10975 + 1472D); \end{aligned}\right\} \quad (242)$$

and with the arrangement (198),

$$\left.\begin{aligned} H_7 &= -2^{-2}\,3^{-1}\,5^4\,(10975 + 3832D); \\ H_{13} &= -2^{-2}\,3^{-1}\,5^4\,(10975 - 3832D). \end{aligned}\right\} \quad (243)$$

We see, therefore, that in this example, the difference of the two quantities H_7 and H_{13} is neither equal to zero, nor independent of the arrangement of the five roots of the equation of the fifth degree. However, it may be noticed that in the same example, the sum of the same two quantities H_7 and H_{13} has not been altered by altering the arrangement of the roots; and in fact, by the method of the 43rd article, we find the formula:

$$\left.\begin{aligned} -\tfrac{48}{5}(H_7 + H_{13}) = {} & (X_{2345} - X_{5432})^2 + (X_{2453} - X_{3542})^2 + (X_{2534} - X_{4352})^2 \\ & + (X_{3254} - X_{4523})^2 + (X_{4235} - X_{5324})^2 + (X_{5243} - X_{3425})^2 \\ & + (X_{2354} - X_{4532})^2 + (X_{2543} - X_{3452})^2 + (X_{2435} - X_{5342})^2 \\ & + (X_{3245} - X_{5423})^2 + (X_{5234} - X_{4325})^2 + (X_{4253} - X_{3524})^2; \end{aligned}\right\} \quad (244)$$

of which the second member is in general a symmetric function of the five roots, and gives, in the case of the equation (147), by (221) and (222), the following numerical value, agreeing with recent results,

$$H_7 + H_{13} = -2^{-1}\,3^{-1}\,5^6\,439. \quad (245)$$

47. It seems useless to add to the length of this communication, by entering into any additional details of calculation: since the foregoing investigations will probably be thought to have sufficiently established the inadequacy of Professor BADANO's method* for the general solution of equations of the fifth degree, notwithstanding the elegance of those systems of radicals which have been proposed by that author for the expression of the twenty-four values of LAGRANGE's

* Professor BADANO's rule is, to substitute, in each H, for each power of x', the fifth part of the sum of the corresponding powers of the five roots, $x', .. x^{V}$; and he proposes to extend the same method to equations of all higher degrees.

function t^5. Indeed, it is not pretended that a full account has been given, in the present paper, of the reasons which Professor BADANO has assigned for believing that the twenty-four quantities which have been called H are all symmetric* functions of the five roots of the equation of the fifth degree; and that those quantities are connected by certain relations among themselves, which would, if valid, conduct to the following expression for resolving an equation of that degree, analogous to the known radical expressions for the solution of less elevated equations:

$$\begin{aligned} t^5 = {} & \mathrm{K}_1 + \sqrt{\mathrm{K}_2} + \sqrt[3]{\mathrm{K}_3 + \sqrt{\mathrm{K}_4}} + \sqrt[3]{\mathrm{K}_3 - \sqrt{\mathrm{K}_4}} \\ & + \sqrt{\{\mathrm{K}_5 + \sqrt{\mathrm{K}_6} + \sqrt[3]{\mathrm{K}_7 + \sqrt{\mathrm{K}_8}} + \sqrt[3]{\mathrm{K}_7 - \sqrt{\mathrm{K}_8}}\}} \\ & + \sqrt{\{\mathrm{K}_5 + \sqrt{\mathrm{K}_6} + \theta\sqrt[3]{\mathrm{K}_7 + \sqrt{\mathrm{K}_8}} + \theta^2\sqrt[3]{\mathrm{K}_7 - \sqrt{\mathrm{K}_8}}\}} \\ & + \sqrt{\{\mathrm{K}_5 + \sqrt{\mathrm{K}_6} + \theta^2\sqrt[3]{\mathrm{K}_7 + \sqrt{\mathrm{K}_8}} + \theta\sqrt[3]{\mathrm{K}_7 - \sqrt{\mathrm{K}_8}}\}}. \end{aligned}$$

But it has been shown, in the foregoing articles, that at least some of the relations here referred to, between the twenty-four quantities H, do not in general exist; since we have not, for example, the relation of equality between H_4 and H_6, which would be required, in order to justify the substitution of a single symbol K_4 for these two quantities. It has also been shown that each of these two unequal quantities, H_4 and H_6, in general changes its value, when the arrangement of the five roots of the original equation is changed in a suitable manner: and that H_7, H_{13}, H_{19}, are also unequal, and change their values, at least in the example above chosen. And thus it appears, to the writer of the present paper, that the investigations now submitted to the Academy, by establishing (as in his opinion they do) the failure of this new and elegant attempt of an ingenious Italian analyst, have thrown some additional light on the impossibility (though otherwise proved before) of resolving the general equation of the fifth degree by any finite combination of radicals and rational functions.

* "Dunque le H sono quantità costanti sotto la sostituzione di qualunque radice dell' equazione." To show that the constancy, thus asserted, does not exist, has been the chief object proposed in the present paper; to which the writer has had opportunities of making some additions, since it was first communicated to the Academy.

Fig: 1

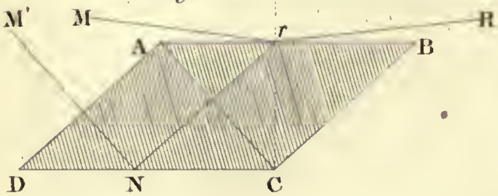


Fig: 4

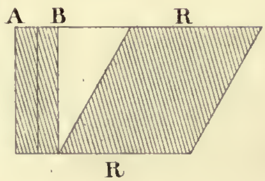


Fig: 3

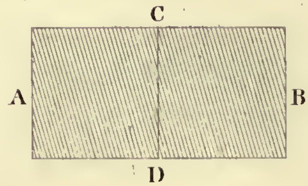


Fig: 2

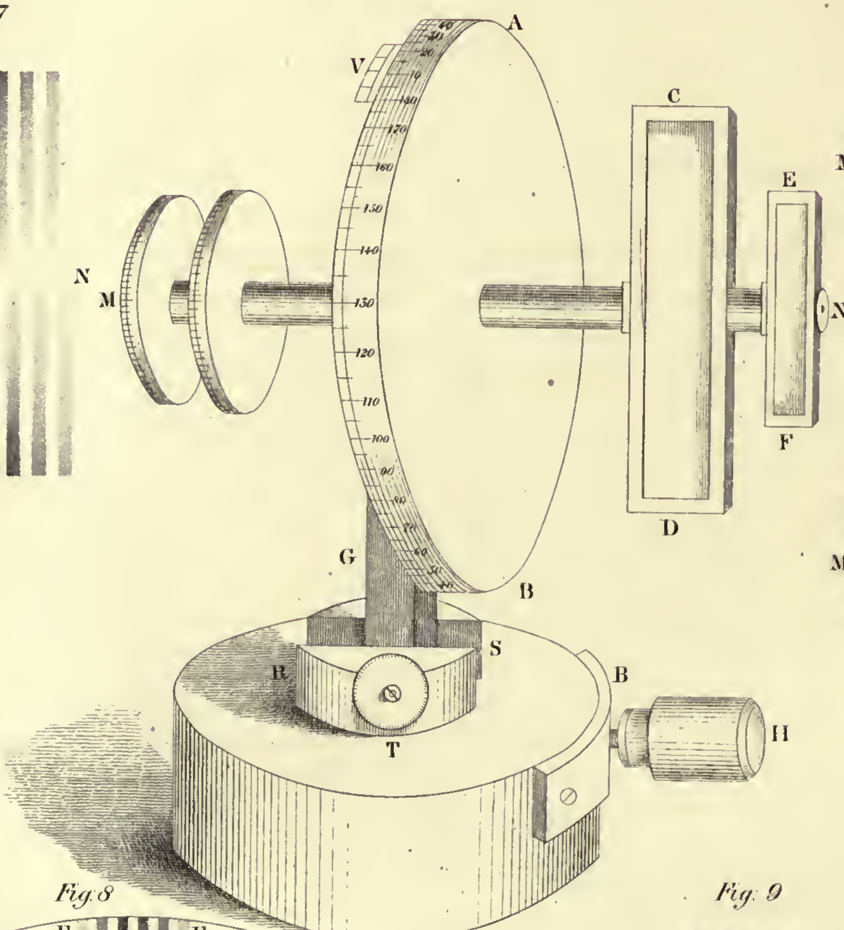


Fig: 7

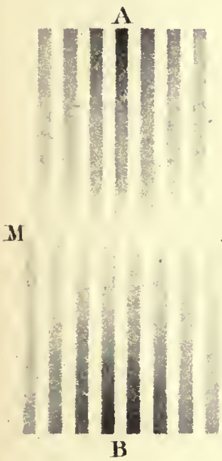


Fig: 5

Fig: 6

Fig: 8

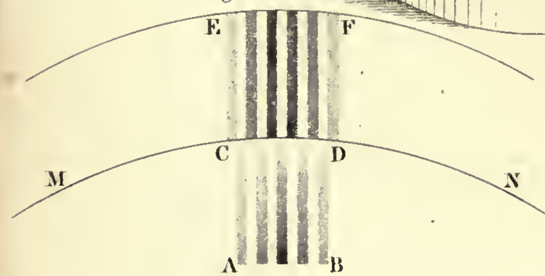
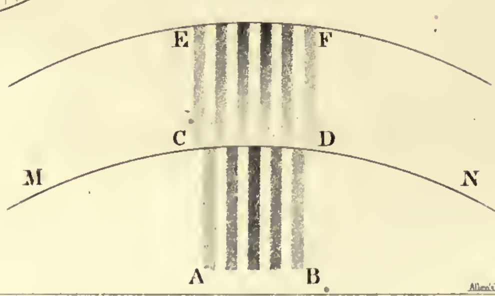


Fig: 9



XV.—*On the Compensations of Polarized Light, with the Description of a Polarimeter, for measuring Degrees of Polarization.* *By* Sir David Brewster, K. H., D. C. L., F. R. S., M. R. I. A., and V. P. R. S. Ed.

Read November 14, 1842.

In four papers, printed in the Philosophical Transactions for 1830, I have endeavoured to determine the general laws of the polarization of light, when reflected from or refracted by the first and second surfaces of bodies, or when suffering total or metallic reflexion. In opposition to the opinions of the most distinguished philosophers, I was led to the conclusion—that when light was reflected at any angle of incidence between 0° and 90° (excepting at the angle of complete polarization), or was refracted at these angles, it did not consist, as they maintained, of two portions, one of which was completely polarized, and the other completely unpolarized or common light; but that every portion of it had the same physical property, namely, that of having approximated more or less to the state of complete polarization. This general result, which enables us to compute all the phenomena of polarization by reflection and refraction, has, in so far as I know, never been called in question; but as the investigation was conducted on the supposition, that a pencil, composed of two pencils, polarized $+45°$ and $-45°$ to the plane of reflexion, was equivalent to a pencil of common light, it became important to have the general result confirmed by experiments made with common light itself; and though the inquiries, the results of which I am now about to explain, had not this object in view, yet it will be satisfactory to find in them a complete demonstration of my former views.*

In considering the condition of partially polarized light, it has always appeared to me probable that some method would be found of distinguishing it from a

* Philosophical Transactions, 1830, pp. 69, 133, 145, 287.

mixture of polarized and common light; and I have accordingly endeavoured at different times, though without success, to obtain such a test. While studying, however, the polarizing structure of the atmosphere, where it became desirable to ascertain the degree and kind of polarization which light reflected from different parts of it experienced, I was led to a series of experiments, which furnished me with the test of which I had been in search.

The comparative brightness of the two images in Iceland spar, directed to different parts of the sky, afforded a very imperfect indication of its state of polarization; and I had, therefore, been in the practice of employing the uniaxal or biaxal system of rings for this purpose.* Upon placing such a system between light partially polarized in one plane, and light partially polarized in an opposite plane, I found that the rings disappeared, the direct system being seen on one side of the plane of disappearance, and the complementary system on the other side.

In this experiment, the polarization of the light in one plane was compensated by the polarization of the same light in the opposite plane, and consequently both of the pencils that had undergone the two successive polarizing actions, had received the same degree of polarization in opposite planes. In virtue of these two equal and opposite polarizations, the light at the point of compensation, where the system of rings disappeared, had been restored from partially polarized to common light, and the light on each side of this point of compensation was in opposite states of partial polarization.

In order to have a more distinct idea of the nature of this experiment, let us suppose that light reflected once, at 24° of incidence, from glass, whose index of refraction is 1.525, is afterwards made to suffer one refraction at 80° by another surface of the same glass.† In this case, the partial polarization produced by reflexion is exactly compensated by the equal and opposite partial polarization produced by refraction. In like manner, a second reflexion at $83\frac{1}{2}$°, in an opposite plane, will compensate the first reflexion at 24°, or the refraction in the same plane at 80°.

Now, in these three cases of compensation, the quantity of polarized light in the three pencils is very different, as appears from the following table:

* See my Treatise on New Philosophical Instruments, 1813, p. 349.

† The action of one refraction is obtained by using a prism of well annealed glass, as shown in the Philosophical Transactions, 1830, p. 135, fig. 2.

Angles of Incidence.	No. of Rays, out of 1000 polarized by Reflection and Refraction.
24°	10.5
80°	158
83½°	139.3

Hence, it is obvious that the compensation is not produced by equal quantities of light polarized in opposite planes; and it would be absurd to suppose that the portions of common light existing in each of the partially polarized pencils performed any part in the compensation. But even if it did, it could act only by its quantity—that is, by the relation which it bore to the polarized portion of the beam. Now, in the three cases which we have noticed, the ratio of the common to the polarized portion of the pencil is not the same, although the compensation is perfect, as the following numbers show:

	Ratio of common and polarized Light.	
Angles of Incidence.	Reflected Pencil.	Refracted Pencil.
24°	4.15 to 1	. . . . .
80°	. . . . .	2.8 to 1
83½°	2.8 to 1	. . . . .

Hence, we are forced to the conclusion, that the compensation is produced neither by an equality of oppositely polarized rays, nor by a proportional admixture of common light, but by equal and opposite physical states of the whole pencil, whether reflected or refracted.

Let us now consider what takes place at the polarizing angle, or 56° 45′, in glass. The whole of the reflected light, or 79½ rays, is here wholly polarized, and the same quantity of oppositely polarized light, viz. 79½ rays, exists in the refracted beam. Now, this refracted beam is not capable of compensating the reflected one, notwithstanding their equality in point of polarized light, and though the reflected beam is not mixed with common light; so that, upon the old hypothesis, the refracted beam can owe its deficient power of compensation only to the large quantity of common light which it contains.

But though in the compensations already mentioned the proportions of common to polarized light are different; yet, in other cases of compensation, such as the following, the proportion is pretty nearly equal; but this equality is accidental, and is not the cause by which the compensation is produced.

Angles of Incidence.	Reflected Pencil.	Light polarized.	Refracted Pencil.	Ratio of common to polarized Light.
15° 40′	43.4	4.5	...	1/9.6
56° 45′	79.5	79.5	920	1/11.6
87° 51′	80.9	70	...	1/11.5

Hence, a pencil reflected at an incidence of 15° 40′, compensates another reflected at 87° 51′, and each of them compensates a pencil refracted at the polarizing angle 56° 45′, and the ratio of the common to the polarized light is nearly the same.

In support of the same views we shall examine what takes place at other three remarkable angles of incidence.

1. At 78° 7′ where the quantity of polarized light is a maximum, or 158 rays, the power of compensation by reflexion is less than at every angle of incidence between 78° 7′, and 30° where the quantity of polarized light varies from 158 to 17 rays.

At 78° 7′ the quantity of refracted light is double that of the reflected light, and is equal to two-thirds of the incident light, and the quantity of polarized light is nearly one-fourth of the reflected, and one-half of the refracted, light. Now, at this angle the power of compensation by reflexion and refraction is nearly in the inverse ratio of the quantity of light in the reflected and refracted beams, and not as the quantities of common light, which they are supposed to contain. For the powers of compensation are as 6° 50′ to 14° 7′; the ratio of light in each beam as 666 + 333, and the proportion of common light as 508 to 175.

2. At 85° 50′ 40″, when $i - i' = 45°$, when the refracted is one-half of the reflected light, and the quantity of polarized light one-third of the refracted light, one-sixth of the reflected light, and one-ninth of the incident light, the power of compensation by refraction is nearly double of that by reflexion,* being nearly in the inverse ratio of the quantities of light in the reflected and refracted beams, and not of the quantities of common light which they contain.

At other angles of incidence beside these two, the powers of compensation have no such relations.

3. At 82° 44′, a very remarkable angle, where $\cos(i + i') = \cos.^2 (i - i')$, and where the reflected is equal to the refracted light, the compensation by reflection is equal to the compensation by refraction, and the ratio of the polarized

* The one is 9° 44′, and the other 4° 48′.

to the common light, or to the total quantity in each beam, is the same;* but this equality is accidental, as appears from the fact already mentioned.

The remarkable phenomena produced at this angle in glass, and at the corresponding angle in all transparent bodies, where $\cos. (i + i') = \cos.^2 (i - i')$ require to be more minutely stated, and lead us to the construction of what may be called the *compensating rhomb*, which is shown in Plate, Fig. 1. It consists of a well annealed rhomb of glass, or of any other uncrystallized body ABCD, having, in the case of glass, the angles BAD, BCD $= 139° 25'$, and ABC $= 40° 35'$, when the index of refraction is 1.525. If a ray of light Rr, is incident upon AB, at an angle of 82° 44', exactly one-half of it will be reflected in the direction rM, and the other half refracted in the direction rN, having each the same quantity of polarized light, as already stated. But the ray rN is again reflected at N at an angle of 40° 35', and it will emerge from the face AD nearly perpendicularly, without suffering any perceptible refraction, in the direction NM'. If we now examine this ray M'N, we shall find it to be in the state of common light, although the incident ray rN contained 145 polarized rays, or nearly one-half of the pencil rN. In order to be satisfied of this, the compensating rhomb should be made of two equal and similar rectangular prisms, ABC, ADC, cemented to or nearly touching one another. By removing ADC, the ray rN emerging nearly perpendicularly from the face AC, will exhibit the state of its polarization, when it falls upon the face DC at the point N.

We have now obtained by this experiment a very singular result. If the pencil rN consists of 145 rays of polarized light, and $333 - 145 = 188$, of common light, the effect of a single reflexion at N has been to unpolarize polarized light! and to produce no change at all upon common light! a property of a reflecting surface hitherto unheard of, and incompatible with all our present knowledge of the polarization of light. After such a conclusion, it would be an unprofitable task to adduce any further arguments; and I shall therefore only state that all the phenomena of polarization, by successive reflexions and refractions, stand in direct contradiction of the views which I have been combating.

The restoration of the pencil rN to common light by reflexion at N, furnishes us at once with the principle of compensation, in conformity with the laws of polarization deduced in my papers of 1830. The whole of the ray rN has suffered a physical change by refraction at r, consisting of a rotation of its planes of polari-

* This is the only angle where this equality obtains.

zation towards a plane perpendicular to that of refraction, and the subsequent reflexion at N has exactly counteracted that rotation, by turning back the planes as many degrees towards the plane of reflexion. The reflexion at N has, therefore, brought back the ray *r*N into the same state as the original ray R*r*, that is, the ray NM′ is common light.

In order to ascertain if this principle is general, and to determine the laws of the compensation of partially polarized light, I shall now describe the instrument by which I have ascertained the physical condition on which compensation depends, and the leading facts on which the doctrine rests. From its property of measuring degrees of polarization, I have called this instrument a *Polarimeter*. It is represented in Fig. 2, and consists of two parts, one of which is intended to produce a ray of compensation, having a physical character susceptible of numerical expression, and the other to produce polarized bands, or rectilinear isochromatic lines, the extinction of which indicates that the compensation is effected. The first part of the instrument consists of a goniometer AB, carrying on its axis MN, a frame CD containing six or seven plates of glass, about the 70th of an inch thick, such as are now used for holding microscopic objects. This frame can be taken off and replaced by a black glass reflector highly polished, and free of all oxidation on its surface, or it may be fixed permanently at EF, alongside of the frame CD.*

The second part of the *Polarimeter* is a combination of two plates of rock crystal, or any other transparent doubly refracting mineral, such as I described in 1819, in my paper *on the Properties of Amethyst.*† The object which I had in view by this combination was to exhibit the colours of polarized light in *rectilinear bands*, and this is effected in the following manner. A plate of rock crystal, AB, Fig. 3, from the fiftieth to the tenth of an inch thick, is cut so that its faces are inclined 45° to the axis of the prism, which is the axis of double refraction. When the plate has been divided into two equal parts at the line CD, the one is placed transversely above the other, and cemented to it by Canada balsam, so that the two plates act in opposition to each other upon polarized light. When this plate is fixed at the end of a Nicol's prism (or a rhomb of calcareous spar, with a circular aperture just sufficient to separate the two images), as shewn in Fig. 4, the depolarizing axis of the plate being parallel to the prin-

* When much light is desired, a plate of a highly refracting substance, whose index of refraction is known, may be substituted for the glass.

† See Edinburgh Transactions, vol. ix. p. 148.

cipal section of the rhomb, we shall observe in polarized light a beautiful system of rectilineal bands, as exhibited in Fig. 5, where MN is a deep black neutral line, with the usual coloured bands on each side of it. With light polarized oppositely, the central band MN is white, as shewn in Fig. 6, in which the tints are complementary to those in Fig. 5.

Let us now suppose it required to determine the state or degree of polarization of any luminous surface from which light is reflected, or through which it is transmitted, or of any illuminated medium from which both reflected and refracted light are transmitted to the eye of the observer.

If the light is polarized in the plane of the meridian or a vertical plane, it may be more convenient to use the glass plate at CD, and in doing this the analyser with the rock crystal is fixed between the frame CD and the eye of the observer upon a pillar, or it may be held in the hand, so that the principal section of the rhomb is in a vertical plane. The rectilineal bands will then be seen distinctly crossing the luminous surface, when CD is perpendicular to the axis of vision. But if we incline CD from 0° of incidence towards 90, by turning round the goniometer, a position will be found when the rectilinear bands are interrupted by a neutral line, as in Fig. 7, and the bands at A on one side of the neutral line will be complementary to those at B on the other side. After marking the indication of the goniometer, when this takes place, suppose 160°, turn back the goniometer till the light from the luminous surface is nearly as much inclined to the plates on the other side of 0° of incidence, and vary the angle till the bands are interrupted as before, when the observer looks at the same point of the luminous surface. Having again observed the indication of the goniometer, suppose 10°, then $160° - 10° = 150°$ will be the inclination of two rays equally inclined to the plate, and the half of this, or 75°, will be the angle of incidence upon the plates, at which the polarization of the light from the luminous surface is compensated.

If the light from the luminous surface had been polarized horizontally, it would have been most convenient to have used the rock glass, or other reflector not metallic. In doing this, the luminous surface is reflected at the same angle between the polarizing angle and 90°, and also between the same angle and 0°, the analyser and rhomb being in each case interposed between the reflector and the eye, as before, and the angle of incidence varied till the neutral line MN is opposite to or seen upon the same part of the luminous surface. If the compensation takes place about 70°, it will also take place about 40°, and these angles will afford measures of the degree of polarization necessary to produce the compensation required.

In order to make these observations at different inclinations to the horizon, the pillar which carries the graduated circle of the goniometer, and also the pillar GH, must move upon a joint, as shewn in Fig. 2. By observations such as those above described, the following angles of compensation will be obtained:

Compensations between two Reflexions, one above and one below the polarizing Angle.

Below polarizing Angle.	Above polarizing Angle.
0°	90°
5	89½
10	89
16	88
20	86
24	83
30°	81°
34	77
39	74
44	70
48	65
52	62
56¾	56¾

Compensations between one Reflexion below the polarizing Angle, and one Refraction.

Reflexion.	Refraction.
2°	10°
5	22
10	42
15	59
20	74
24	80
25	81
28	87
30	89½

Compensations between one Reflexion above the polarizing Angle, and one Refraction.

Reflexion.	Refraction.
$89\frac{1}{2}$	22°
89	42
88	59
86	74
83	80
82	87
81	$89\frac{1}{2}$

If we now compare these results with the experimental and calculated ones given in my papers of 1830,* we shall find that one reflexion will compensate another reflexion, or one refraction, when the inclinations of the planes of polarization produced by the two reflexions are equal and opposite, or when the inclination produced by one reflexion is the complement of the inclination produced by one refraction; or more generally, in both cases, when the rotations produced in the plane of polarization are equal and opposite. Hence, it follows that the compensations of polarized light are produced by equal and opposite rotations of the planes of polarization.

Now, the inclination ϕ of the plane of polarization by reflexion at any angle of incidence i, is

$$\tan\phi = \tan.\,x\,\frac{\cos(i+i')}{\cos(i-i')},$$

and the inclination ϕ' for refracted light, is $\cot\phi' = \cot x \,.\, \cos(i-i')$. In the case of reflected light, the angles of incidence which compensate each other are those where ϕ has equal values; and in the case of reflected and refracted light, the one compensates the other, when $\phi+\phi'=90°$, or $\tan\phi + \cot\phi' = 1$, or when

$$\tan x\,\frac{\cos(i+i')}{\cos(i-i')} + \cot x \,.\, \cos(i-i') = 1.$$

Now, though we shall find that at the angles of compensation in the preceding

* Philosophical Transactions, 1830, pp. 74, 75, 78; 136, 138, 139, and 143.

table, the values of $+\phi$ and $-\phi$ in the case of reflexion, and of $\pm\phi$ and $90^\circ - \phi'$ in the case of a reflexion, and a refraction, are nearly equal; yet it requires to be proved, that when the planes of polarization are inclined at an angle, $\pm x$, to the plane of incidence, greater or less than 45°, another reflexion at another angle, which would give $\pm\phi$, or $90^\circ - \phi'$, of the same value, will restore the planes to their original inclination.

When $x = 45^\circ$, and when one reflexion has turned the planes of a ray polarized 45° into $37^\circ\ 21'$, or given the planes a rotation of $45^\circ - 37^\circ\ 21' = 7^\circ\ 39'$, the action of a refracting surface which produces the same rotation, or $52^\circ\ 39' - 45^\circ = 7^\circ\ 39'$ will bring the planes back to 45°, or restore the partially polarized light to common light. Call $x = 37^\circ\ 21'$, then in order that the refraction may restore the ray to 45° we must have $\phi' = 45^\circ$ or $\cot\phi' = \cot x \cos(i - i') = 1$. Now, $\cot\phi' = \cot x \cos(i - i')$, and when $x = 45^\circ$ and $\phi' = 52^\circ\ 39'$, $\cot\phi' = \cos(i - i')$. But $x = 37^\circ\ 21' = 90^\circ - \phi$, hence $\frac{1}{\cot x} = \cot\phi'$, and $\frac{1}{\cot x} = \cos(i - i')$, consequently $\cot x \cot(i - i') = 1$. In like manner ϕ' will be restored to 45° by a reflexion which gives ϕ such, that $\phi + \phi' = 90^\circ$, or $\tan\phi = \cot\phi'$. That is when $x = 45^\circ$, and $\phi = 37^\circ\ 21'$,

$$\tan\phi = \tan x \frac{\cos(i + i')}{\cos(i - i')} = 1. \quad \text{The general formula}$$

$$\tan\phi = \tan x \frac{\cos(i + i')}{\cos(i - i')} \quad \text{becomes, when } x = 45^\circ,$$

$$\tan\phi = \frac{\cos(i + i')}{\cos(i - i')},$$

But when $x = 52^\circ\ 39' = 90^\circ - \phi$, we have

$$\frac{1}{\tan x} = \tan\phi, \text{ and}$$

$$\frac{1}{\tan x} = \frac{\cos(i + i')}{\cos(i - i')}. \quad \text{Consequently,}$$

$$\tan x \frac{\cos(i + i)}{\cos(i - i)} = 1.$$

Having thus determined that light polarized in a plane whose inclination to the plane of reflexion is $+\phi$, will be compensated by oppositely polarized light, whose inclination is $-\phi$, if both the lights are reflected, or by refracted light whose inclination is $90^\circ-\phi$ or ϕ', we must next endeavour to discover at what angle of incidence the polarized light submitted to the polarimeter, has suffered reflexion or refraction, when we have the angle of incidence and the inclination of the plane of polarization, by which we have effected the compensation.

Let us first take the case when light partially polarized by reflexion is compensated by the polarization produced by refraction through one surface, at an incidence i of 80°. The index of refraction being 1.525, we shall have when

$$x=45^\circ,\ \cot\phi'=\cos(i-i'),\ \text{and}\ \phi'=52^\circ\,33'.$$

Now, the plane of the light polarized by reflexion must be inclined $90^\circ-\phi'$, or $37^\circ\,27'$; we must, therefore, find the angles of incidence above and below the polarizing angle, or the two values of i corresponding to this value of ϕ, namely, $37^\circ\,27'$, at one or other of which the original light must have been reflected. These values will be obtained from the expressions

$$\tan\phi=\frac{\cos(i+i')}{\cos(i-i')},\ \text{and}\ \sin i'=\frac{\sin i}{m}.$$

When $i+i'$ is less than 90°, or when the angle of incidence is less than the polarizing angle, $\tan\phi$ is *positive,* and we have

$$\sin i=\sqrt{\frac{(m^2+1)(1-\tan\phi)^2}{8\tan\phi}\left\{-1\pm\sqrt{1+\left(\frac{2m}{m^2+1}\right)^2\times\frac{4\tan\phi}{(1-\tan\phi)^2}}\right\}}.$$

When $i+i'$ is greater than 90°, and $\tan\phi$ *negative,* the formula becomes

$$\sin i=\sqrt{\frac{(m^2+1)(1+\tan\phi)^2}{-8\tan\phi}\left\{-1\pm\sqrt{1+\left(\frac{2m}{m^2+1}\right)^2\times\frac{-4\tan\phi}{(1+\tan\phi)^2}}\right\}}.$$

From these formulæ, whem $m=1.525$ and $\phi=37^\circ\,27'$, we obtain $i=24^\circ\,50'$, and $83^\circ\,30'$.

When the compensation of *refracted* light is effected by one *reflexion,* either above or below the polarizing angle, for example, at 15° 40', and 87° 51', we shall have

$$\tan\phi = \frac{\cos(i+i')}{\cos(i-i')} = 42^\circ\ 31'.$$

But in the refracted light thus compensated, we must have $\phi' = 90^\circ - 42^\circ\ 31 = 47^\circ\ 29'$, and, therefore, we must determine the angle of incidence i, at which the original light suffered refraction. The expressions from which we obtain i are $\cot\phi' = \cos(i-i')$, and $\sin i' = \frac{\sin i}{m}$, which give

$$\sin i = \frac{m}{\tan\phi'}\sqrt{\frac{\tan^2\phi' - 1}{m^2 + 1 - \frac{2m}{\tan\phi'}}},$$

from which we obtain, when $\phi' = 47^\circ\ 29'$, $i = 56^\circ\ 45'$, the maximum polarizing angle.

Hitherto we have supposed the compensation to be produced by one refraction, or by one reflexion; but it may be effected by several. In the case of reflected light this is not necessary, because we have all degrees of polarization by reflexion, from 0° of incidence to the polarizing angle, and from this again to 90° of incidence.

When the compensation, however, is made by *successive reflexions* at the same angle of incidence, or when light which is compensated has been so reflected, we may find the angle of incidence i, when n is the number of reflexions, by means of the formulæ

$$\tan\phi = \frac{\cos^n(i+i')}{\cos^n(i-i')},\quad \sin i' = \frac{\sin i}{m},\quad \text{and}\quad \sqrt[n]{\tan\phi} = \frac{\cos(i+i')}{\cos(i-i')},\text{*}$$

which give

$$\sin i = \sqrt{\frac{(m^2+1)(1-\sqrt[n]{\tan\phi})^2}{8\sqrt[n]{\tan\phi}}\left\{-1 \pm \sqrt{1+\left(\frac{2m}{m^2+1}\right)^2 \times \frac{4\sqrt[n]{\tan\phi}}{(1+\sqrt[n]{\tan\phi})^2}}\right\}},$$

when $i + i'$ is less than the polarizing angle, and $\tan\phi$ *positive*. But when $i + i$ is greater than 90°, and $\tan\phi$ *negative*, we have

$$\sin i = \sqrt{\frac{(m^2+1)(1+\sqrt[n]{\tan\phi})^2}{-8\sqrt[n]{\tan\phi}}\left\{-1 \pm \sqrt{1+\left(\frac{2m}{m^2+1}\right)^2 \times \frac{-4\sqrt[n]{\tan\phi}}{(1+\tan\phi)^2}}\right\}}.$$

* See Phil. Trans. 1830, p. 80, 81.

In the case of light polarized by *refraction,* the action of several surfaces may and must often be necessary to produce compensation, and in this case, or when the light compensated is polarized by *successive refractions,* we may find the angle of incidence by means of the formulæ

$$\cot \phi = \cos^n (i - i'), \text{ and } \sin i' = \frac{\sin i}{m}.$$

And since $\sqrt[n]{\cot \phi} = \cos (i - i')$, we have for n refractions,†

$$\sin i = \frac{m}{\sqrt[n]{\tan \phi}} \sqrt{\frac{\sqrt[n-1]{\tan \phi} - 1}{m^2 + 1 - \frac{2m}{\sqrt[n]{\tan \phi}}}}.$$

When the light has passed through a prism whose angle is ψ, then if the angle of the prism is equal to the angle of refraction, or $\psi = i'$, or $\sin \psi = \frac{\sin i}{m}$, the incidence i will be found from the formula for *one* refraction, because the ray will emerge perpendicularly from the second surface of the prism, and suffer no change in its planes of polarization.

If the angle of the prism is double the angle of refraction, or $\psi = 2i'$, and the deviation $i - i'$ a *minimum,* the incidence i will be found from the formula when n, the number of refractions, is *two;* the refraction, and consequently, the polarization at each surface being equal, and, therefore, the same, as for a plate when $\psi = 0$.

Having thus determined the laws of the compensation of polarized light, I shall conclude this paper by pointing out a few of their numerous applications.

1. The first and most important result of this inquiry is, that it affords a new and independent demonstration of the laws of the polarization of light by reflexion and refraction, given in my papers of 1830. As this result has been already referred to, I shall merely mention the following general proposition.

When a ray of common light is incident at any angle upon the polished surface of a transparent body, the whole of the reflected pencil suffers a physical change, bringing it more or less into a state of complete polarization; in virtue of which change, its planes of polarization are more or less turned into the plane

* See Phil. Trans. 1830, p. 137.

of reflexion, while the whole of the refracted pencil has suffered a similar, but opposite change, in virtue of which, its planes of polarization are turned more or less into a plane perpendicular to the plane of reflexion.

2. As the light of the sky and the clouds is more or less polarized, the employment of the light which they reflect may, in delicate experiments, be a serious source of error, if we are not aware of its properties. By the principle of compensation, however, we may convert this partially polarized light into common light, and thus make experiments with as great accuracy in the daytime, as we can do with the direct light of a flame. If the light from a particular part of the sky is admitted into a dark room, or otherwise employed, we have only to compensate its polarization either by reflexion or refraction, and employ, as *unpolarized* or *common light*, that part of the light which corresponds with the neutral line.

3. The laws of the compensation of polarized light enable us to investigate the polarizing structure of the atmosphere, and to ascertain the nature and extent of the two opposite polarizing influences, which I have found to exist in it, and by the compensation of which the neutral points are produced. But, as I shall soon submit to the Society the results of my observations on this subject, I shall not add any thing further at present.

4. In every case where reflected or refracted light reaches the eye of the observer, whether it comes from bodies near us, or from the primary or secondary planets of our system, the doctrine of compensation enables us to obtain important information respecting the phenomena presented by light thus polarized. The nature of the reflecting or refracting surface, the angles of reflexion or refraction, and the nature of the source of illumination, may, in certain cases, be approximately ascertained.

5. When the light of the sun, or any self-luminous body, is reflected from the surface of standing water, such as the sea or a lake, it is polarized according to laws which are well known ; but when the partially polarized light of the sky (light polarizes in every possible plane, passing through the sun and the observer) is reflected, a variety of curious compensations take place, which, when the position of the observer is fixed, vary with the season of the year, and the hour of the day. In some cases, there is a perfect compensation, the partially polarized light of the sky being restored to common light by the reflection of the

water. In other cases the light of the sky has its polarization increased by reflexion from the water in the same plane in which it was itself polarized; and in other cases, the compensation is effected only in particular planes. At sunset, for example, the light reflected from the sea at a great obliquity in two vertical planes inclined 45° to a vertical plane passing through the sun and the observer, is compensated in these two planes, or the plane of its polarization is inclined about 45° to the reflecting surface. The same observations apply to the light of the two rainbows when reflected from the surface of water.

6. When the light of the sky, or of the rainbow, is reflected from surfaces not horizontal, such as the roofs of houses, sheets of falling water, or surfaces of smoke and vapour, the compensations are more varied, and a perfect neutralization of the light by the second reflexion is more frequently obtained.

7. When the compensating rhomb, whose properties I have already described, is made of glass not highly polished, light that has suffered total reflexion is seen through the face AD, Fig. 1. As the faces AB, CD, are parallel, none of the light regularly refracted by the face AB can suffer total reflexion from CD. Upon examining this curious and unexpected phenomenon, I found that it was owing to light radiated, or scattered from the surface AB, which falling upon CD at angles greater than that of total reflexion, whose sine is $\frac{1}{m}$, necessarily suffered total reflexion. That this was the cause of the phenomenon, I proved by covering the surface AB with a film of dried milk, which radiated light from every part of its surface, and produced a beautiful zone of totally reflected light, increasing in brightness as the incidence upon AB became more oblique. In examining this totally reflected light, I was greatly surprised to find, that it was partially polarized, and exhibited an interesting example of compensation.

Let MN, Fig. 8, be the luminous zone of totally reflected light with its blue border. At the polarizing angle of the second surface of the rhomb, the polarization is incomplete; but at angles between that angle and 83°, the polarization gradually diminishes, and at 83° it becomes common light, the rectilineal fringes AB produced by the rock crystal passing into neutral light at CD, close to the boundary MN of total reflexion. From 83° to 90°, which corresponds to a very narrow space at CD, the light still appears compensated, though it is slightly polarized, in a plane perpendicular to that of reflexion.

At 83°, when this takes place, the totally reflected light MN is polarized, as shown at EF, Fig. 9, in a plane at right angles to that of reflexion. But as the angle of incidence diminishes, the polarization diminishes, till at an angle of 68° it becomes common light, the polarization produced by total reflexion at the second surface exactly compensating, as at CD, that produced by refraction at the first.

At angles less than 68°, the totally reflected light is partially polarized in the plane of reflexion, the polarization increasing till the scattered light disappears.

The polarization of the light that afterwards suffers total reflexion, is produced by its refraction at the first surface AB, Fig. 1 of the rhomb, and the phenomena above described arise from the opposite action of the reflecting surface CD; at one angle producing an inferior degree of polarization, at another compensating it, and at another overbalancing it.

ST. LEONARD'S COLLEGE, ST. ANDREW'S,
April 24th, 1841.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

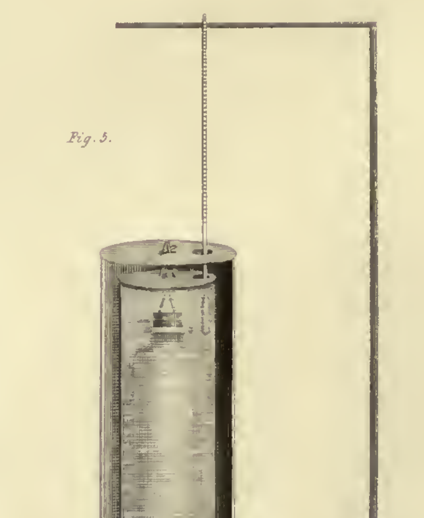
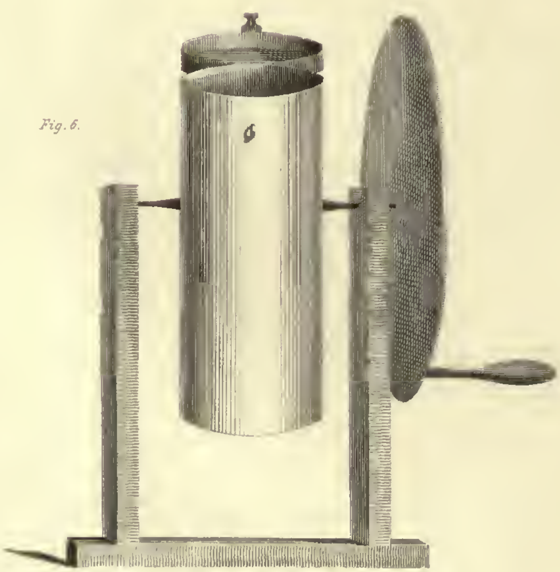


Fig. 6.



XVI. *On the Heat developed during the Formation of the Metallic Compounds of Chlorine, Bromine, and Iodine. By* THOMAS ANDREWS, M. D., M. R. I. A., *Professor of Chemistry in the Royal Belfast Institution.*

Read December 12, 1842.

1. IN pursuance of the train of investigation commenced in a preceding Memoir, I propose, in the present communication, to advance to the consideration of the more complicated thermal phenomena, which are accompanied by alterations in the state of aggregation of the combining bodies. To deduce general conclusions from such inquiries is extremely difficult, as the variation of temperature measured by the thermometer is in every instance the resultant of more than a single cause, each of which must be separately eliminated, before the heat arising from the chemical union can be determined. It has been my endeavour to furnish as many data as possible, in the cases I have examined, for the solution of these interesting problems.

2. That we may be enabled to measure with precision the heat developed during a chemical combination, it is necessary that the reaction should be very quickly completed; and the experiment is also greatly facilitated, when the action commences, by simple contact, without the application of external heat. These conditions are completely fulfilled, when chlorine, bromine, or iodine are brought into contact with zinc or iron, water being also present. To the success of the experiment the latter condition is indispensable, as these elementary bodies, at ordinary temperatures, and in the dry state, have no action upon one another.*

* The description generally given in chemical works of the rapid manner in which zinc, copper, antimony, &c. enter into combination with chlorine gas at common temperatures, is only true when the gas is in a moist state. Chlorine gas, when carefully dried, has no action whatever, at the ordinary temperature of the atmosphere, upon fine filings of zinc or iron, or upon copper reduced from

The relative proportion of water is also a matter of importance. The quantity present must be sufficient to dissolve, with facility, the resulting compound, and it ought not greatly to exceed that amount. In the following experiments I usually employed about 2.4 gr. of water, for every 0.42 gr. chlorine, 0.9 gr. bromine, and 1.5 gr. iodine, which entered into combination. If this precaution be attended to, and the mixture briskly agitated, the whole reaction will be completed in the course of a few seconds.

3. As our object is to ascertain the heat due to the combination of the reacting bodies in an anhydrous state, and as we actually obtain the result of the combination in a state of solution in water, it is obviously necessary, in the first instance, to apply a correction for the heat arising from the solution. The amount of this correction is easily discovered, by determining the heat evolved during the solution of a corresponding weight of the dry compound in the normal proportion of water. If the combining bodies do not unite in more than one proportion, there only now remains to be determined the heat evolved or absorbed during the changes of aggregation which occur in the course of the combination. Unfortunately we cannot attempt, by direct experiments, to discover the amount of this important correction.

4. If we now make

A = heat evolved during the reaction of chlorine, zinc (in excess), and water,

B = heat evolved during the solution of Zn Cl in a like proportion of water,

X = heat evolved or absorbed during the change of the constituents

its oxide by means of hydrogen gas, although the action, as is well known, is most energetic if moisture be present. On the contrary, the dry gas instantly combines with arsenic, antimony, and phosphorus. This striking difference appears to depend upon the circumstance that the compounds formed by chlorine with the former substances are solid at common temperatures and very fixed, while those formed with antimony and arsenic are fluid and volatile. The chloride of phosphorus is also very volatile. If, however, the chemical affinity be very intense, combination will take place although the resulting compound be quite fixed and solid. Thus potassium inflames in dry chlorine gas, but the chloride which is formed terminates the action before the whole of the metal has entered into combination. The fluidity of the metal also exercises an important influence in determining the combination,—as in the case of mercury, which slowly combines with dry chlorine. The preceding remarks may be also applied to the behaviour of dry bromine when brought into contact with the metals.

of Zn Cl, from the state of aggregation in which they exist, as gaseous chlorine and metallic zinc, to that state in which they exist in the dry chloride of zinc,

$x =$ heat due to the union of zinc and chlorine,

we shall have the following general equation:

$$x = \text{A} - \text{B} \pm \text{X}.$$

And, designating the corresponding values for bromine by A', B', X', x', and for iodine by A'', B'', X'', x'', we shall have

$$x' = \text{A}' - \text{B}' \pm \text{X}',$$
$$x'' = \text{A}'' - \text{B}'' \pm \text{X}''.$$

5. The class of metals forming more than one compound with chlorine, bromine, and iodine is very numerous; but none of them present the same facilities for this investigation as iron, to which accordingly I propose to confine my attention in the present paper. It is usually stated in chemical works that when chlorine, bromine, or iodine act upon an excess of iron filings, suspended in water, a solution of protochloride, protobromide, or protoiodide of iron is formed. But such a description gives a very imperfect idea of the successive series of phenomena which actually take place. We have only, indeed, to watch carefully the progress of the experiment, in order to discover that a sesquicompound ($Fe_2\,Cl_3$, $Fe_2\,Br_3$, $Fe_2\,I_3$) is formed in the first instance, which afterwards, by combining with an additional atom of iron, becomes converted into the protocompound ($Fe_2\,Cl_3 + Fe$, &c.) To prove this, we only require to filter the liquid before the reaction has terminated, when a red solution will be obtained, having all the properties of a solution of a sesquisalt of iron, and yielding by evaporation a red deliquescent mass. Whether the experiment be made with chlorine, bromine, or iodine, the same results will be obtained. An elegant illustration of a similar train of changes is afforded by the action of chlorine gas on metallic tin. If we agitate an excess of tin filings with a little water in a glass vessel of chlorine gas, till the colour of the gas has scarcely disappeared, and instantly filter, the liquid which passes through will produce only a faint opalescence, when dropped into a solution of the bichloride of mercury; but if the agitation be continued for only a few seconds after the disappearance of the chlorine, the filtered liquid will give a dense curdy precipitate when added to the same solution.

6. From these observations it follows, that the primary form of combination, into which the molecules of chlorine, bromine, and iodine enter with iron, is that represented by the formulas $Fe_2\,Cl_3$, $Fe_2\,Br_3$, $Fe_2\,I_3$, and that the so-called proto-compounds are, in reality, secondary combinations, formed by the union of the sesquicompounds with an additional atom of iron ($Fe_2\,Cl_3 + Fe$, &c.). This conclusion is farther confirmed by the well-known fact, that when these substances unite at elevated temperatures, the red or sesquicompounds are always formed.*

7. Let us now make

C = heat evolved during the reaction of chlorine, iron (in excess), and water.

D = heat evolved during the solution of $Fe_2\,Cl_3$ in a similar proportion of water.

E = heat evolved during the combination of $Fe_2\,Cl_3$ in solution with Fe.

Y = heat evolved or absorbed during the change of aggregation of the constituents of $Fe_2\,Cl_3$.

y = heat duc to the union of Fe_2 with Cl_3.

Let us also, as before, represent the corresponding values for bromine by C′, D′, E′, Y′, y', and for iodine by C″, D″, E″, Y″, y''. The following equations will then give the values of y, y', and y''.

$$y = \mathrm{C} - \mathrm{D} - \mathrm{E} \pm \mathrm{Y},$$
$$y' = \mathrm{C}' - \mathrm{D}' - \mathrm{E}' \pm \mathrm{Y}',$$
$$y'' = \mathrm{C}'' - \mathrm{D}'' - \mathrm{E}'' \pm \mathrm{Y}''.$$

8. Having thus endeavoured to lay down general formulas for the heat of combination, I proceed to describe the experiments by which the values of A, B, C, &c. have been determined.

9. The apparatus employed in these experiments consisted of several distinct parts. The combination was effected in a thin glass vessel of the form represcnted in fig. 1. When chlorine was the subject of experiment, this vessel was

* If the view, which regards Fe Cl as the primary form of combination, be preferred, it will be necessary to suppose that three successive changes occur,—first, the formation of the compound $Fe_2 + Cl_2$; secondly, its conversion into $Fe_2\,Cl_3$ by combining with Cl; and thirdly, the reconversion of the latter into $Fe_3\,Cl_3$ by its union with Fe.

filled with the gas in a moist state, and two very flimsy glass balls, such as those shown in fig. 4, were afterwards cautiously introduced. One of these balls contained a large excess of the metal in the state of fine filings; the other, a quantity of water, whose weight had been adjusted nearly in the proportions before described. On the other hand, when bromine and iodine were under examination, the metal and water were introduced into the vessel itself, while the bromine, or iodine, carefully weighed, was contained in one of the little balls. The vessel was in all cases closed by a good cork, which was rendered air-tight by cement. A small stud of iron wire was inserted into the cork to maintain the glass vessel in its proper position in the interior of the apparatus. This vessel, thus prepared, was agitated for some time in water adjusted to the proper temperature, and then placed in the light copper vessel, fig. 2, which was immediately filled with water, and its lid screwed on. In the top and bottom of the copper vessel, loops of copper wire were inserted, by means of which it could be suspended, without contact of the hand, in the centre of a cylindrical vessel of tin plate, fig. 3, having a detached cover above and below. The complete arrangement will be readily understood from an inspection of fig. 5. In the lids of the tin cylinder and copper vessel corresponding apertures existed, through which the bulb of a delicate thermometer could be introduced into the water in the interior of the latter. On withdrawing the thermometer the aperture in the copper vessel could be closed, in the course of two or three seconds, without touching the vessel itself. By this arrangement the copper vessel with its contents was suspended in a fixed position in the centre of, but not in contact with, an outer cylinder of tin plate, while at the same time the temperature of the water could be noted at any time without removing it from its situation. A larger cylindrical vessel, capable of being rapidly rotated round its shorter axis, completed the whole apparatus. It is shown in fig. 6.

10. When an observation was made the copper vessel was suspended in the cylinder, the opening in its lid closed, and the apparatus placed in a horizontal position, and then cautiously agitated (lest the glass balls should break), till a perfectly uniform temperature was established through the whole of the copper vessel and its contents. This being accomplished, the cylinder was again placed in the position represented in fig. 5, the temperature of the water carefully noted, and the cork replaced. It was then suddenly shaken, so as to rupture the

glass balls within, and immediately afterwards secured in the interior of the larger cylinder, fig. 6, where the whole was rapidly rotated, for the space of five and a half minutes, from the time of observing the temperature. It was then removed, and the temperature of the water again observed. In the case of bromine and iodine, all that now remained to complete the experiment was to weigh the water in the copper vessel, but, in the case of chlorine, the original volume of the gas had to be determined. For this purpose, the glass vessel was placed in a water-trough, and the cork withdrawn. From the quantity of water which rushed in, the bulk of the chlorine was easily estimated. It is almost unnecessary to add, that, in every instance, the whole of the chlorine had entered into combination; the small residue being atmospheric air, unavoidably introduced when the bulbs were inserted.

11. The accuracy of experiments of this kind greatly depends upon the heat which is gained or lost by the apparatus during the course of the experiment. In a vessel placed apart from other sources of heat, the losses and gains of heat will evidently be equal to one another for equal differences of temperature above and below that of the surrounding air. But in the apparatus I have just described, from the proximity of the person of the observer, and the necessity of grasping the tin cylinder while placing it in, and removing it from, the rotating machine, this middle point is no longer the temperature of the air, but $1^{\circ}.4$ above that point. Direct experiments also showed that the water had nearly attained its maximum point in $45''$, from the time when the glass balls were ruptured, and $15''$ usually elapsed from the observation of the first temperature to the latter moment. We may, therefore, assume that the water is at the maximum temperature during $4\frac{1}{2}'$, and at the minimum during $15''$. If we put e for the excess of the final temperature above the air, e' for the difference between the initial temperature and the same, and R and R$'$ for the corrections to be applied for the cooling and heating of the apparatus, during periods of $4\frac{1}{2}'$ and $15''$ respectively, we shall have

$$\text{R} = +(e - 1^{\circ}.4) \times 0.049,$$
$$\text{R}' = -(e + 1^{\circ}.4) \times 0.003 + 0^{\circ}.03.$$

12. The constant quantity $0^{\circ}.03$ is added to the correction for simple heating, as an allowance for the heat, transmitted by the hand through the apparatus, while rupturing the balls. The temperature of the water being generally so ad-

justed, that the mean point between the initial and final temperatures was from half a degree to one degree above that of the air, the entire correction required was in all cases very small.

13. The value in water of the different parts of the apparatus was estimated with as much precision as possible. The specific heat of the copper and brass of the copper vessel was assumed to be 0.095, that of the glass of the glass vessel and balls was determined by a careful experiment to be 0.140. The leather, cork, and cement were found to be nearly equivalent to 1.1 gr. of water, and the specific heat of the solution formed in each experiment was also determined.

14. In the description of the experiments I have used the following abbreviations:

Bar.—The height of the barometer.

Th. air.—The temperature of the air.

T^i.—The initial temperature of the water in the copper vessel.

T^f.—The final temperature of the same.

Inc. c.—The increment of temperature corrected for heating and cooling, according to the formulas given before.

Aq.—The weight of the water in the copper vessel.

Sn.—The weight of water equivalent to the solution of the compound formed. This is found by multiplying the absolute weight of the solution by its specific heat, which is also given.

Vss.—The weight of water equivalent to the vessels and other solid substances used in each experiment.

15. The temperatures are given in the degrees of Fahrenheit's scale; the height of the barometer in English inches; the volume of the chlorine in cubic centimetres; and the weight of the water, &c. in grammes. The volume of the chlorine gas requires to be corrected for moisture, as well as for temperature and pressure, and I have assumed the weight of 100 cubic centimetres of the dry gas at 32°, and under a pressure of 29.92 in. to be 0.317 grammes.

COMPOUNDS OF ZINC.

16. Zinc and chlorine, $Zn + Cl + Aq$.

Bar.	29.47 in. . . .	29.07 in. . . .	29.97 in.
Th. air.	50°.70 . . .	48°.50 . . .	50°.80

T^i.	47°.97	45°.22	49°.08
T^f.	55°.20	52°.18	54°.14
Inc. c.	7°.34	7°.03	5°.12
Aq.	136.6 gm.	143.0 gm.	143.6 gm.
Sn. (sp. heat 0.76)	2.4	2.4	1.7
Vss.	21.3	21.3	21.3
Cl.	141.4 c. c.	141.0 c. c.	100.4 c. c.
Heat of comb.	2802°	2820°	2811°

Mean heat referred to chlorine as unit, 2811°.

Mean heat referred to zinc as unit, 3086°.

The first number indicates the number of degrees through which a portion of water, equal in weight to the chlorine, would be raised by the heat extricated during the combination; the second, the corresponding number of degrees for a portion of water equal in weight to the zinc.

17. Zinc and bromine, Zn + Br + Aq.

Th. air.	63°.40	64°.10	68°.3
T^i.	61°.30	62°.07	66°.12
T^f.	66°.94	66°.91	71°.12
Inc. c.	5°.70	4°.87	5°.03
Aq.	152.8 gm.	155.0 gm.	158.4 gm.
Sn. (sp. heat 0.62)	2.3	2.0	2.1
Vss.	19.4	19.4	19.4
Br.	0.936	0.806	0.847
Heat of comb.	1063°	1066°	1068°

Mean heat referred to bromine as unit, 1066°.

Mean heat referred to zinc as unit, 2586°.

18. Zinc and iodine, Zn + I + Aq.

Th. air.	64°.0	63°.80	38°.4
T^i.	61°.08	60°.50	36°.74
T^f.	66°.72	67°.67	42°.42
Inc. c.	5°.66	7°.24	5°.77
Aq.	159.5 gm.	161.1 gm.	129.1 gm.
Sn. (sp. heat 0.56)	3.8	4.9	3.2

Vss.	19.7	19.8	21.6
I.	2.372	3.084	2.000
Heat of comb.	436°.7	436°.2	444°.0

Mean heat referred to iodine as unit, 439°.

Mean heat referred to zinc as unit, 1720°.

19. To ascertain in the preceding cases the heat due to the solution of the compound, portions of each, carefully dried, were introduced into the thin glass balls, and the weight accurately ascertained, while the normal proportion of water for their solution was placed in the glass vessel.

20. Chloride of zinc and water, Zn Cl + Aq.

Th. air.	36°.90	37°.20
T^i.	35°.71	36°.05
T^f.	39°.00	38°.72
Inc. c.	3°.29	2°.63
Aq.	131.4 gm.	129.9 gm.
Sn. (sp. heat 0.76)	10.6	8.4
Vss.	21.7	21.7
Zn Cl	3.516	2.750
Heat of comb.	292°	292°

Mean heat referred to chlorine as unit, 292°.

Mean heat referred to zinc as unit, 320°.

21. Bromide of zinc and water, Zn Br + Aq.

Th. air.	54°.00	55°.50
T^i.	53°.86	55°.35
T^f.	56°.36	57°.41
Inc. c.	2°.51	2°.06
Aq.	153.9 gm.	154.9 gm.
Sn. (sp. heat 0.62)	9.1	7.7
Vss.	19.4	19.4
Zn Br	5.077	4.310
Heat of comb.	127°	122°

Mean heat referred to bromine as unit, 124°.5.

Mean heat referred to zinc as unit, 302°.

22. Iodide of zinc and water, Zn I + Aq.

Th. air	58°.60	59°.10	38°.4
T^i	58°.02	59°.12	37°.58
T^f	59°.07	60°.21	40°.12
Inc. c.	1°.02	1°.06	2°.52
Aq.	159.1 gm.	159.6 gm.	125.6 gm.
Sn. (sp. heat 0.56)	4.8	5.0	10.7
Vss.	19.1	19.6	21.6
Zn I	3.52	3.92	8.42
Heat of comb.	66°.5	62°.6	59°.3

Mean heat referred to iodine as unit, 62°.8.

Mean heat referred to zinc as unit, 246°.

COMPOUNDS OF IRON.

23. Iron and chlorine, $Fe_2 + Cl_3 + Aq + Fe$.

Bar.	30.07 in.	29.97 in.	29.08
Th. air	50°.50	50°.50	48°.00
T^i	47°.47	47°.67	45°.78
T^f	53°.78	54°.08	51°.93
Inc. c.	6°.36	6°.47	6°.23
Aq.	133.8 gm.	143.9 gm.	143.9 gm.
Sn. (sp. heat 0.74)	2.2	2.4	2.4
Vss.	21.1	21.3	21.4
Cl.	131.7 c. c.	141.5 c. c.	141.5 c. c.
Heat of comb.	2503°	2534°	2505°

Mean heat referred to chlorine as unit, 2514°.

Mean heat referred to iron in Fe_2 as unit, 4921°.

24. It must be carefully observed that the unit here taken is not the whole of the iron dissolved, as in the case of zinc, but only two-thirds of it; because the remaining third does not enter directly into combination with the chlorine, as has been already explained.

25. Iron and brominc, $Fe_2 + Br_3 + Aq + Fe$.

Th. air	64°.10	49°.00
T^i	61°.81	47°.52
T^f	66°.89	53°.55
Inc. e.	5°.10	6°.14
Aq.	155.3 gm.	147.4 gm.
Sn. (sp. heat 0.60)	2.4	2.7
Vss.	19.4	19.4
Br.	0.994	1.145
Heat of comb.	909°	909°

Mean heat referred to bromine as unit, 909°.

Mean heat referred to iron in Fc_2 as unit, 3933°.

26. Iron and iodine, $Fe_2 + I_3 + Aq + Fe$.

Th. air	63°.40	63°.20	38°.10
T^i	61°.04	60°.30	36°.32
T^f	65°.99	65°.83	41.°44
Inc. c.	4°.97	5°.55	5°.17
Aq.	157.7 gm.	162.1 gm.	126.1 gm.
Sn. (sp. heat 0.54)	4.2	4.8	3.6
Vss.	19.6	19.5	21.6
I.	2.752	3.151	2.360
Heat of comb.	327°.8	328°.3	331°.5

Mean heat referred to iodine as unit, 329°.2.

Mean heat referred to iron in Fe_2 as unit, 2299°.

27. The object of the experiments detailed in the three following tables was to determine the heat evolved, when solutions of the sesquichloride, sesquibromide, and sesquiiodide of iron are eonverted into solutions of the protocompounds by agitation with an exeess of iron. The sesquiehloride of iron, obtained by the action of dry chlorine gas upon heated iron, was dissolved in water (the quantity being adjusted as usual) in the glass vessel, and an excess of iron filings was plaeed in one of the small balls. But I was obliged to have recourse to a differcnt method in order to proeure determinate quantities of the sesquibromide and sesquiiodide of iron in solution, from finding it impossible to

obtain these compounds in the dry state. At first I attempted to add an excess of bromine or iodine to solutions of known strength of the protocompounds; but, on endeavouring to expel the excess by heat, I found it difficult, even in the case of the sesquibromide of iron, to avoid the decomposition of the sesquicompound itself, when the solution was concentrated. The object in view was finally effected in a very complete and easy manner, by adding weighed quantities of bromine or iodine to solutions of the protobromide, or protoiodide of iron, containing more than twice as much bromine or iodine, as the quantity added. The object of employing a larger proportion of the proto-solutions than the bromine or iodine added would be capable of converting into the state of sesqui-compounds, was to prevent the possibility of any free bromine or iodine being present; and, as the results were the same, whether the excess of the proto-solution was greater or less, it evidently in no way interfered with the success of the experiment. In reducing the results we have, therefore, to remember that the sesqui-compound formed, contains three times the quantity of bromine or iodine added, designated in the tables by Br $\times$ 3 and I $\times$ 3.

28. Sesquichloride of iron and iron, $Fe_2\,Cl_3\,Aq + Fe$.

Th. air	61°.80	62°.50	43°.00
T^i	61°.85	61°.35	41°.21
T^f	63°.34	64°.29	45°.45
Inc. c.	1°.46	2°.92	4°.25
Aq.	132.8 gm.	144.3 gm.	151.4 gm.
Sn. (sp. heat 0.73)	3.0	6.8	10.4
Vss.	21.8	21.4	19.9
$Fe_2\,Cl_3$	0.856	1.895	2.900
Heat of comb.	406	402°	402°

Mean heat referred to chlorine in Cl_3 as unit, 402°.5.

Mean heat referred to iron in Fe_2 as unit, 788°.

29. Sesquibromide of iron and iron, $Fe_2\,Br_3\,Aq + Fe$.

Th. air	44°.40	46°.70	47°.20
T^i	44°.46	46°.23	45°.77
T^f	46°.68	49°.02	50°.84
Inc. c.	2°.23	2°.81	5°.14

Aq.	152.6 gm.	152.4 gm.	152.1 gm.
Sn. (sp. heat 0.60)	6.3	7.3	12.9
Vss.	19.6	19.6	19.6
Br × 3	2.163	2.739	5.199
Heat of comb.	184°.0	183°.9	182°.5

Mean heat referred to bromine in Br_3 as unit, 183°.5.

Mean heat referred to iron in Fe_2 as unit, 794°.

30. Sesquiiodide of iron and iron, $Fe_2\,I_3\,Aq + Fe$.

Th. air.	47°.40	47°.00	51°.10
T^i.	46°.41	46°.87	50°.15
T^f.	49°.22	49°.24	54°.66
Inc. c.	2°.80	2°.38	4°.58
Aq.	151.2 gm.	150.5 gm.	146.8 gm.
Sn. (sp. heat 0.54)	9.1	6.8	17.7
Vss.	20.0	19.9	19.8
I × 3	4.497	3.741	7.596
Heat of comb.	112°.3	112°.8	111°.1

Mean heat referred to iodine in I_3 as unit, 112°.1.

Mean heat referred to iron in Fe_2 as unit, 783°.

31. To complete this part of the inquiry, it only remains to determine the heat evolved during the solution of the sesquichloride, sesquibromide, and sesquiiodide of iron in water. This I have been able to accomplish only in the case of the sesquichloride of iron, from having failed, as has been already remarked, in all my attempts to obtain the other two compounds in a dry state. Even a concentrated solution of the sesquibromide of iron allows bromine to escape during the process of evaporation. If the evaporation be carried to dryness, and the dry mass heated just to the point of fusion, a red substance remains, which is composed of one atom of the protobromide and one atom of the sesquibromide of iron ($Fe_4\,Br_5$). An approximation, however, may be made to the heat which would be developed during the solution of these compounds, by assuming that it will bear the same relation to the heat developed during the solution of the sesquichloride of iron, which has been already ascertained to exist in the case of the analogous compounds of zinc (20, 21, 22).

32. Sesquichloride of iron and water, $Fe_2\,Cl_3 + Aq.$

Th. air.	60°.5 . . .	41°.4
T^i.	60°.2 . . .	41°.02
T^f.	61°.93 . . .	42°.10
Inc. c.	1°.68 . . .	1°.04
Aq.	132.8 gm. . .	120.4 gm.
Sn.	2.7 . . .	1.6
Vss.	21.7 . . .	19.3
$Fe_2\,Cl_3$	0.856 . .	0.504
Heat of comb.	466°. . .	441°

Mean heat referred to chlorine in Cl_3 as unit, 453°.

Mean heat referred to iron in Fe_2 as unit, 887°.

33. On the principle just stated, we may infer, as a rude approximation, that the heat disengaged during the solution of the sesquibromide of iron would be (referred to the iron as unit) 837°; and that disengaged during the solution of the sesquiiodide, 682°.

34. If we now substitute the numerical values, obtained by the preceding experiments, for the known quantities in the equations given before, we shall obtain

$$x = 3086° - 320° \pm \mathrm{x} \qquad (16, 20)$$
$$x' = 2586° - 302° \pm \mathrm{x}' \qquad (17, 21)$$
$$x'' = 1720° - 246° \pm \mathrm{x}'' \qquad (18, 22)$$

$$y = 4921° - 887° - 788° \pm \mathrm{Y} \qquad (23, 32, 28)$$
$$y' = 3933° - 837° - 794° \pm \mathrm{Y}' \qquad (25, 33, 29)$$
$$y'' = 2299° - 682° - 783° \pm \mathrm{Y}'' \qquad (26, 33, 30)$$

From these equations we deduce

$$x \text{ or } Zn + Cl = 2766° \pm \mathrm{x}$$
$$x' \text{ or } Zn + Br = 2284° \pm \mathrm{x}'$$
$$x'' \text{ or } Zn + I = 1474° \pm \mathrm{x}''$$

$$y \text{ or } Fe_2 + Cl_3 = 3246° \pm \mathrm{Y}$$
$$y' \text{ or } Fe_2 + Br_3 = 2302° \pm \mathrm{Y}'$$
$$y'' \text{ or } Fe_2 + I_3 = 834° \pm \mathrm{Y}''$$

35. It must be remembered that each of the letters x, x′, &c. represents two unknown quantities; first, the change of temperature due to the alteration of aggregation of the particles of the metallic elements, in passing from their ordinary form to that form in which they exist in the dry salt; and, secondly, the change of temperature arising from the like alteration of aggregation of the particles of the electro-negative element. The actual value of these quantities cannot be determined by direct experiments, but it is probable that for the combinations of the same metal, the differences between x, x′, and x″, and between Y, Y′, and Y″ will arise chiefly from the alterations of aggregation of the electro-negative, and not of the metallic element. Now, as the heat arising from the condensation of chlorine from the gaseous to what may perhaps be termed the saline solid state, must be far greater than that arising from the change of fluid bromine, or solid iodine, to the same state, it would be an object of great interest to determine the heat evolved or abstracted during the changes of these bodies from one physical condition to another, which would enable us to compare the heat of combination of each body in the same physical state. This I have only attempted yet to effect for the case of the solidification of bromine; and, as the result of a very imperfect experiment, it may be stated, that the heat evolved during the passage of that substance from the fluid to the solid state, would be sufficient to raise an equal weight of water through 24°. This amount of heat is evidently far too small to account for the differences observed in the values of x' and x'', and of y' and y''; from which it follows, that bromine and iodine, in the same physical state, evolve very different quantities of heat when combining with the metals.

36. On comparing the numbers deduced from the foregoing experiments (28, 29, 30) for the heat developed during the conversion of the sesqui-compounds of iron into the corresponding proto-compounds, by combining with half as much iron as they already contain, the very interesting general principle results, that, referred to the combining iron as unit, the heat evolved in all these cases of combination is the same. In fact, we have

$$Fe_2\,Cl_3\,Aq + Fe = 788°.$$
$$Fe_2\,Br_3\,Aq + Fe = 794°.$$
$$Fe_2\,I_3\,Aq + Fe = 783°.$$

The slight differences between these numbers are fully within the limits of the unavoidable errors of experiment, and leave no doubt of the truth of the principle just enunciated.

37. On a future occasion I hope to have an opportunity of describing a more extended series of experiments now in progress, on the heat developed during the combination of other elements with chlorine, bromine, and iodine; and, till that opportunity occurs, I shall reserve any further observations of a general character upon the preceding results. Meanwhile they may be thus recapitulated:

1. The heat developed during the combination of a given quantity of zinc with chlorine gas is sufficient to raise an equal weight of water through 2766°, while that evolved during the combination of the same metal with bromine, in the fluid state, is 2284°; and with iodine, in the solid state, 1474°.

2. The heat developed during the combination of iron with chlorine, bromine, and iodine (which always takes place under the form $Fe_2 Cl_3$, $Fe_2 Br_3$, $Fe_2 I_3$) is sufficient to raise an equal weight of water through 3246°, 2302°, and 834° respectively.

3. When solutions of the sesquichloride, sesquibromide, and sesquiiodide of iron become converted into proto-compounds by combining with iron, the heat evolved in all is the same for the same quantity of iron dissolved.

POLITE LITERATURE.

POLITE LITERATURE.

I. *A Memoir of the Medals and Medallists connected with Ireland. By the* Very Rev. Henry Richard Dawson, A.M., *Dean of St. Patrick's.*

Read 16th March, 1838.

O, when shall *Ireland*, conscious of her claim,
Stand emulous of Greek and Roman fame?

Pope.

THE increasing interest which has been of late years manifested respecting collections of medals, affords a strong proof of the value justly attached to them, both as commemorative corroborations of certain historical events, and also as specimens of skill, ingenuity, and taste amongst artificers in that line. In almost every country of Europe, excepting our own, its medallic history has at successive periods occupied not only the attention, but the pens of learned individuals, and their lucubrations have greatly contributed as well to stimulate the ingenuity of the artist, as to elucidate the facts connected with its exercise; so that many a political event, and many an heroic achievement, which had escaped the notice of contemporary historians, has, through their instrumentality, been rescued from oblivion, and brought under the notice of posterity in the almost imperishable materials of the precious metals.

The northern States of Europe can boast of Beskrivelse, Mechel, and Brenner illustrating and explaining their medals. Holland and the Netherlands have Van Mieris, Van Loon, and Bizot, in ponderous folios, with plates and text, describing each minute particular. In France, Le Blanc, Fleurimont, and Bouteroue have engraved both coins and medals; while in the later period of the glorious era of Andrieu, Laskey and Millingen have elaborately pointed out their beauties, and detailed their intentions. Italy can point to Anthony Count Caïetani explaining the various works of the middle ages contained in the cabinets of Mazzuchelli; and to Venuti, Nobili, and Molinet those of the Popes of Rome are indebted for a great addition to these attractions. England can refer to the works of Evelyn, Vertue, and Edwards noticing and illustrating the varied specimens of skill which have been produced by those artists whom the country encouraged, and whose works have served to perpetuate the actions, good or evil, of her devoted servants. I could refer to many other countries of Europe, where the proud records of their fame have found studious chroniclers both with pen and hand; but no attempt has yet been made to record historically the medals of Ireland; and while some pains have enabled me to rescue the works of her artificers from, I should say, undeserved oblivion, I venture to call the attention of the members of the Academy to some of the productions of the Irish Coining Press, as well as to some medals connected with our country, and executed by foreign artists, in the expectation that their countenance may be the means of eliciting some of the latent, and stimulating the neglected talents of our countrymen.

For some few years past I have been endeavouring to collect and arrange in historical order the medals connected with this part of the United Kingdom; and though with considerable diffidence I present these brief notices of my researches to you, (brief, because I find these records of our national deeds very few,) yet I am not without hopes that they may excite some interest even amongst those who have not hitherto turned their attention to this pursuit. I purpose, therefore, to offer you some notices of such medallists, and such designs, emanating from their studia, as have fallen under my observation. I regret to say my materials are scanty, owing, I believe, mainly to this, that the country has not hitherto fostered nor encouraged that beautiful branch of art.

The earliest medal that I have met with, as connected with Ireland, is of the time of Charles II.; a small silver piece, of very beautiful execution, and I consider it to be the work of some English or foreign artist, as both sides are obviously taken from two medals which were struck to commemorate the marriage of that Prince with Catherine of Portugal. It bears on the obverse a figure of St. Catherine with her wheel, and the legend PIETATE INSIGNIS. The reverse has Fame blowing a trumpet, and in her left hand she carries an olive branch. On the banner appended to the trumpet there is a small harp, the arms of Ireland, and were not that sufficient to appropriate this medal as belonging to our series, the inscription PROVINCIA CONNAGH, decides the matter. Now it is well known that Charles was married to Catherine of Braganza by Sheldon, Bishop of London, May 21st, 1662; but many think the ceremony was previously performed by a Roman Catholic priest to satisfy the scruples of the concealed as well as the avowed Romanist. This priest may have come from Connaught, and it is not improbable that this piece was struck, that at least some obscure evidence might remain of the event.

The Roettiers, the celebrated Dutch medallists, worked for Ireland; but their skill was, I believe, less exercised to commemorate the heroic achievements of her sons, than to promote the purposes of their unfortunate master; and those pieces generally known as the gun money of James II. are supposed to have been struck from dies executed by John Roettier. However base the materials of these coins, their neatness and execution afford reasonable grounds for attributing them to such a devoted follower as he was known to be of the exiled king. I should here observe that James Simon, the author of an essay on Irish coins, has engraved, Plate VII. No. 154, and described a silver medal, which he conceives alludes to the landing of James in Ireland, and his reception by his Irish subjects at Kinsale, March 12th, 1689. The obverse represents the king crowned, and in his royal robes, holding a baton in his hand. Behind him a ship, and before him a crowd hailing his approach, the legend JACOBUS · II. DEI · GRATIA. The reverse, two sceptres in saltire behind a crown, with the motto INTEMERATA, and the legend MAG. BR. FRA. ET · HIB. REX. 1689. Simon saw only a drawing of this medal, which was sent to him by Mr. Charles Smith of Dungarven; I have not been so fortunate as to meet with it myself, nor can I find any further record concerning it; but Simon is too accurate to allow me to doubt its existence in his day.

When William III. came to fight the battles for our liberty and his own sovereignty in this kingdom, his various victories were commemorated in Holland by his own countrymen, and so many medals were struck with the intent of perpetuating his renown, that it would be tedious here to enumerate them. The engravings and descriptions published by Van Loon inform us, that neither the Boyne nor Aughrim, nor Galway, nor Limerick, were considered undeserving of commemoration by those who were most conversant with the events which produced such an effusion of Irish blood. Nor were these memorials confined to the illustrious hero himself, for similar records are also found of his victorious generals, Schomberg and De Ginkle.

But in connexion with the history of this period, one medal only has been discovered, struck in Ireland, and this bears reference to Van Homrigh, a follower of William's, who settled in Ireland about this period. And as this medal has not been hitherto published, it may be interesting here to describe it, and to show upon what occasion it was struck. It appears from the records of the Corporation of Dublin, that in the year 1688 Sir Michael Creagh was Lord Mayor of the city, and as such was in possession of the parapharnalia connected with his office; in the following year two persons, Terence Dermot and Walter Motley, held the office, the one for nine, and the other for three months. They, it is supposed, never received the usual ensigns of dignity, but it is certain that in those troublesome times they were either lost or purloined, and to this day it is usual, at the triennial perambulations of the city boundaries by the Lord Mayor and his staff, for an officer to make proclamation that Sir Michael Creagh should appear and restore the collar and its appurtenances connected with the office, which he is alleged to have conveyed away. In the year 1698 William III. presented to the city a new collar of SS., to which is appended the noble medallion I am now about to describe, executed by James Roettier. Obverse, GULIELMUS . TERTIUS · D. G. MAG. BRIT. FRAN. ET · HIB. REX. Bust looking to the right, with flowing hair, in armour, with a scarf over it. Reverse, GULIELMUS III – ANTIQUAM ET FIDELEM – HIBERNIÆ METROPOLIN – HOC INDULGENTIÆ SUÆ MUNERE – ORNAVIT · BARTH VAN HOMRIGH ARM. URB. PRÆTORE · MDCXCVIII. This medallion is an important addition to our series, as few impressions can possibly come under public observation.

During the reign of Anne, though Croker exerted his talents in England to commemorate the distinguished events of her time, I have been unable to discover any medals immediately connected with Ireland; and this appears strange, since it is well known that Swift, then possessing great weight and authority, exerted his influence to procure that change in the coinage which called forth those pattern farthings, exhibiting records of remarkable circumstances, and which also have encouraged the preposterous notions so widely diffused respecting their extreme rarity and enormous value. He was a patriot, and it would appear from some memoirs connected with him, to a certain degree, a collector of medals; but his taste lay in a different line from that of encouraging artists or scientific pursuits.

Connected with the times of George I., I am able to produce, I think, one medallet, and that without any reverse. It is very small, and exhibits a three-quarter bust of my celebrated predecessor in the Deanery of St. Patrick, in his full wig and gown, with falling bands. It bears a strong resemblance to a portrait in my possession, which Swift is said to have given to Vanessa at the time he quarrelled with her. The legend is, J · S DD D · S · P · D · (Jonathan Swift, D. D., Dean of St. Patrick's, Dublin.) The execution is tolerably good, but I have not been able to ascertain either the artist or the occasion upon which it was struck.

In the succeeding reign, patronage or party feeling appears to have given some stimulus to the art, for I find no fewer than five medals connected with the period. As one only has been published, and that in a very incorrect and slovenly form, and none hitherto described, I shall here attempt to give some elucidation of them.

The first again refers to Dr. Swift, and exhibits his portrait three-quarter face to the left, with wig and gown, in a small oval frame, supported by a winged child upon clouds. To the right of Swift is Minerva seated, in armour, with spear and ægis, pointing with her right hand to a shield resting against her knee, and bearing the arms of Ireland. To the left a female also seated, leaning on a pile of books, and with her right hand holding a laurel crown over the Doctor's bust. Above there is a winged figure of Fame, and below a scroll inscribed REV. J. SWIFT, D. S. P. D. The reverse displays Hibernia seated, in her right hand an olive branch, and her left is supported by a harp. In the back ground a shepherd tending his flocks, and a view of the sea covered with ships. On the exergue is the date MDCCXXXVIII., J. R. FECIT. This medal, I conjecture, was intended to commemorate Swift's exertions for the advancement of commerce, manufactures, and agriculture. He was at that period in the zenith of his glory; and it cannot surprise us that the zealous friends, of whom he had many, should thus endeavour to perpetuate his fame. Of the artist I know nothing, and the execution is so rude, that I am indisposed to conjecture it to be the work of any artificer of eminence. The next in the suite gives better hopes for the progress of improved taste in the medallic art as connected with Ireland, and the subject is very interesting. The artist, T. Pingo, has not hesitated to put his name upon the work, and it fully supports the character he has obtained. The obverse presents three figures, on the right a female thrown upon the ground, emblematic of Ireland; at her feet a cap of liberty and a spear. A male figure in the centre is represented seizing her by the hair with his left hand, and with the right holds a dagger over her. On the left stands Justice, with her emblems, averting his uplifted arm, and the inscription reads, MAY GEORGE PROTECT WHAT JUSTICE TRYS TO SAVE. On the reverse, at the top, is the harp of Ireland, with some of the strings broken, and at the bottom a shield, bearing the arms of the city of Dublin, the sword, mace, cap, and collar of the city lying near it on the ground. Across the field is the legend, THE GLORIOUS – ATTEMPT – OF LXIV – TO PRESERVE THE – CONSTITUTION – MDCCXLIX. There is every reason to presume that this medal was struck to commemorate the defeat of the efforts put forth by the celebrated Charles Lucas in favour of the liberties of the Corporation of Dublin, as it was in this year he addressed his memorable letter to George II. on the charter of the liberties of the city of Dublin, complaining that the freemen and common council were defrauded of their rights and privileges by the Board of Aldermen, who, he alleged, were mere usurpers, and arrogated to themselves too much power in the election of the Lord Mayor.

I am now about to call your attention to a medal in the possession of many families in this country, which, in design and execution, will not be easily surpassed. As it has not, I believe, been published, and as it relates to an event considered very remarkable in the history of the Irish Parliament, I shall be excused for recording some details respecting it while they are yet attainable. By the Commons' Journals it appears that from the year 1692 the practice of the house was to call for and examine the public accounts. If there appeared a deficiency in the treasury, they provided for it; if a surplus remained after the purposes were served for which it had been granted, they proceeded to dispose of it for the public advantage, without asking permission from the Crown, or receiving any intimation that the king's prerogative was thereby invaded. It happened that in 1749 a considerable sum remained in the treasury, and upon the circumstance being reported, the Commons of Ireland framed the heads of a bill, according to the powers heretofore exercised by them, for applying a portion of it towards the discharge of the national debt. They were sent to England, returned without alteration or objection, and the same course would have continued, had not some mischievous intermeddling courtier discovered what he considered an invasion of the rights of the Crown, which it was determined by those in authority to repel. In the year 1751, the Lord Lieutenant, acting upon this suggestion, in his speech from the throne at the opening of the session, informed the House of Commons, "That he was commanded by the king to acquaint them, that his Majesty, ever attentive to the ease and happiness of his subjects, would graciously *consent* and recommend it to them, that such part of the money then remaining in the treasury, as should be thought consistent with the public service, should be applied to the further reduction of the national debt." This was assuming that the king had an exclusive property in it, and might, as an act of favour, permit the Parliament to dispose of it. The Commons in their Address paid no regard to this unprecedented claim. The heads of the bill were framed as usual, it passed the Commons and Privy Council, was sent to England, but returned with the word "consent" inserted in it. Then, though many members were dissatisfied with this infringement of their rights, it passed unanimously, and thus a precedent was made which was attempted to be used on the event which produced the present medal. In the year 1753 even a larger surplus was reported in the treasury. The Right Hon. Thomas Carter, Master of the Rolls, presented, on the 13th of December, a bill, entitled "An Act for the payment of £77,500, or so much thereof as shall remain due on the 25th of December, 1753, in discharge of the National Debt." This was read a first time on the following day, and a committee was appointed to inquire if any, and what alterations had been made in the preamble and enactments of the bill. On the 15th, Mr. Upton reported that an alteration, or rather an addition, had been made, by inserting in the preamble the following words: "And your Majesty, ever attentive to the ease and happiness of your faithful subjects, has been graciously pleased to signify that you would CONSENT, and to recommend it to us, that so much of the money remaining in *your Majesty's treasury* as should be necessary, be applied to the discharge of the *national* debt, or of such part thereof as should be thought expedient by *Parliament*." The house was again aroused to jealousy respecting an invasion of its privileges, and on the 17th it resolved itself into committee, when the Master of the Rolls reported from it, that they had agreed to the enacting paragraphs of the bill, but disagreed to the preamble; a division took place, and the bill was rejected by a majority of five voices.* Although the numbers on each side are not given in the Commons' Journals, I conceive, from the record of this and another medal, that the dissentients amounted to 124, a strong testimony to the feeling of parliamentary privilege that pervaded the house. I should add, that this bold assertion of right by her representatives produced no immediate advantage to Ireland, whatever may have been its future consequence, for his Majesty, by his letter, took that money out of the treasury which had been the subject of dispute. On the obverse of the medal the legend reads, UTCUNQUE FERENT EA FACTA MINORES VINCIT AMOR PATRIÆ. In the centre stands Hibernia, with a harp in her left hand, and behind her another figure holding a distaff, emblematic of the staple trade of the country. On her right stands another female grasping her hand, and holding in her right hand a roll inscribed LEGES. To her left is the Speaker of the House of Commons in his robes, placing a cap of liberty on her head, and holding in his left hand a heavy bag inscribed VINDICATA, and behind him three senators stepping out from a portico. Over the figures is Fame flying, and blowing a trumpet, with a banner appended, and inscribed CXXIV; she holds in her left hand a ribbon or band bearing the inscription, ERGO TUA JURA MANEBUNT. On the exergue are two human figures naked, the one with the head of a bird of prey, clutching at a quantity of money scattered on the ground, which the other with the head of a wolf, and loosed from a chain fastened to a rock, guards; behind them some open rolls. The legend on the reverse reads, QUIQUE SUI MEMORES ALIOS FECERE MERENDO. Across the field, SACRUM – SENATORIBUS CXXIV – QUI TENACES PROPOSITI – FORTITER AC PRUDENTER – JURA PATRIÆ RITE – VINDICARUNT XVII – DIE DECEMBRIS ÆRÆ – CHRISTIANÆ MDCCLIII – QUOCIRCA VIVITE – FORTES. I conjecture a medal in gold was given to each of the members who voted on the popular side, as I have seen several, and the one before me is engraved on the edge THO^{S}. MONTGOMERY, ESQ^{R}. 8 B^{R}. 1755. He was Member for the Borough of Lifford in that Parliament.

Another medal and medallet, both of similar type, were also struck upon the occasion of this triumph. Obverse, THE SPEAKER · AND LIBERTY. Bust three-

* In the "Universal Advertiser," Dublin, 1754, there is a list of the members who voted for and against the Altered Money Bill.

quarter face to the left, in wig and robe of office. The portrait is that of Henry Boyle, afterwards created Earl of Shannon, under whose banner the patriots opposed the corruption and tyranny practised by Primate Stone and the Court party. Reverse, THE 124 PATRIOTS OF IRELAND; in the field a harp with the royal crown over it. Exergue, DECEMBER 17 – 1753. The execution of both is indifferent, and the metal brass; they were probably struck immediately upon the occurrence of the event.

The next piece, and that too upon the same subject, refers to the Kildare family; on the obverse is seen a table covered with money, to the left a hand and arm stretched out from above grasping at it; to the right a man in full dress, in an attitude of defiance, with a drawn sword over the table, as if guarding the money, with the inscription, TOUCH NOT SAYS · KILDARE. Exergue, MDCCLV. Reverse, a harp with a crown over it; legend, PROSPERITY TO OLD IRELAND, 1754. This commemorates the celebrated memorial presented to the king by James Fitzgerald, Earl of Kildare, remonstrating against the withdrawal of money from Ireland, and the removal from public employment of those who favoured the popular cause.

The last medal but one connected with this reign had reference to a contested election for a member for the county, which took place in Louth in the year 1755. At that time a number of persons formed themselves into what they called an Independent Club, for the purpose of giving opposition to the gentlemen of the county of the high influential interests, and resolved to try and obtain the return of the members. In one instance they were successful in ousting Mr. Bellingham, and succeeded in returning Thomas Tipping, Esq., in conjunction with the Hon. W. H. Fortescue, to serve in Parliament. This medal commemorates their triumph. Obverse, FIRM TO OUR COUNTRY AS THE ROCK IN THE SEA. A large rock standing boldly in the sea, the four winds blowing against it, and on the top a figure of Hibernia, with her left hand resting upon a harp, and her right pointing upwards. Reverse, MAY THE LOVERS OF LIBERTY NEVER LOSE IT. Two hands united, with a heart over them; and underneath, in the field, BY OUR STRICT – UNION IN LOUTH – WE DISAPPOINTED THE – HOPES OF OUR ENEMIES – ON THE 1 OF NOVEM – 1755 IN THE 29 YEAR – OF THE REIGN OF – K · GEO · THE II – WHOM GOD LONG – PRESERVE. The artist has not given his name, but from the execution of the work he could not have been one of any note; and I may observe, that the design of the obverse seems to have been very closely copied from a medal by Dassier, to the memory of Dr. Samuel Clarke.

About the year 1756, there existed an Association of Painters and Sculptors in Dublin, who exhibited their works at a house in William-street, which they built as an Exhibition Room, with the assistance of a parliamentary grant; but not being incorporated, they were unable to hold the premises, and were eventually ejected from them by some persons who had advanced them money towards the completion of the building. They had a medal struck as an admission ticket, bearing on the obverse a boy sculpturing a bust, behind him another with pallet and colours, and in the back ground a column and a capital. The reverse is merely inscribed EXHIBITION TICKET, with a space left for the proprietor's name. This, I am aware, cannot legitimately be classed as a medal, but as it occasionally appears in collections, I have thought it desirable to record it here.

That I may not interrupt the course of this memoir, I shall here insert an account of a very remarkable medal which has been sent to me, (though I have been unable to procure an inspection of the piece itself,) and extracted from Faulkner's Dublin Journal of August 6th, 1768, which precludes the necessity of any further remark for its elucidation. "On Saturday last ended the poll for the election of Knights to represent the County of Westmeath in Parliament, when the numbers stood thus: for Lord Bellfield, 475; for the Hon. Colonel Rochfort Mervyn, 387; and for the Right Hon. A. Malone, 469, of whom 377 were single votes; when Lord Bellfield and Mr. Malone were declared duly elected, the latter by a majority of 82 over Colonel Rochfert Mervyn. After the return the free and independent electors, consisting of a most respectable majority of the gentlemen of the county, met together, and they (in testimony of the singularly constitutional conduct of their candidate, who stood forth at their call and nomination, with an exertion of his usual dignity and spirit,) formed a subscription for a gold medal with the following device: Liberty embracing with her right arm a pillar, and supporting herself by it, her left arm resting on her shield, her spear, casque, and other ensigns lying at her feet; the motto VINCIT AMOR PATRIÆ, ANNO 1768. On the reverse, a hand presenting a civic crown, and underneath, PRESENTED TO THE RIGHT HONOURABLE A. MALONE BY THE FREE AND INDEPENDENT ELECTORS OF THE COUNTY OF WESTMEATH, IN ACKNOWLEDGMENT OF HIS STRENUOUS AND SUCCESSFUL SUPPORT OF THEIR INTERESTS

ON THE 25TH OF JULY, 1768." I am pleased to have the opportunity of preserving this record of any testimonial to the merits of so celebrated a man as Malone, and the more so as I had vainly sought from the gentlemen of Westmeath any account of the occasion on which the medal was struck, as in fact it appeared totally unknown to those of whom I made the inquiry.

The reigns of the two last Georges constitute an æra in the medallic art, of which Ireland may be justly proud, as it produced two artists, who, notwithstanding the difficulties under which they laboured, were the authors of some specimens in the art, that will not lose by comparison with those of the most skilful in that line in any country. They were both natives of Dublin, and when I mention the names of William Mossop, father and son, every admirer of medals will justify me in endeavouring to rescue from oblivion such memorials of them as I have been able to obtain. Through the kindness of Edward Hawkins, Esq. of the British Museum, I have been put in possession of, and allowed to use, several letters and pieces of autobiography from William Stephen Mossop, jun., which give the Academy a security for their authenticity, but I shall state them very briefly, as they might otherwise extend this memoir to an unreasonable length. The series published by these two artists amounts to more than seventy pieces.

William Stephen Mossop, the elder, was born in Dublin A. D. 1751, and about 1765 was placed with Mr. Stone, at that time regarded in Dublin as a man possessed of considerable ingenuity as a die sinker, but whose talents never carried him higher than making a steel letter, or some other mechanical work. Here Mossop's time was thrown away, and his term of apprenticeship passed in the mere drudgery of a trade. Stone was employed in making seals for the Linen Board, and upon this work Mossop was chiefly engaged, and by his exertions mainly contributed to the support of his master's family. Stone soon fell a victim to intemperate habits, and was succeeded by his son, who following his lamentable example, died in the same wretched way. Mossop was then engaged to work for the Linen Board on his own account, and continued to execute their orders until 1781, when a change in the system of the Board threw him out of employment, burthened with a wife and growing family. At this period he was induced, from an accidental circumstance, to undertake some higher works of art. A person intending to purchase some medals, submitted them to the judgment of Mossop, who then, for the first time, had an opportunity of contemplating those beautiful results of human ingenuity. He gave an opinion in accordance with the impression produced on his own mind, recommended the purchase of them, but for some reason it was never completed, and eventually he bought them on his own account. From this hour his destiny was fixed; the flame had been kindled, and every moment he could spare from his other avocations was employed in the study of what was now become an absorbing pursuit. From admiring, he desired to imitate, and persuaded himself that though he might not succeed in the first or second attempt, he would ultimately accomplish something similar. In the year 1782 he produced his medal of Ryder the comedian, his first work, which as a debut in the arts will always be esteemed. When publicly announced, it attracted crowds to inspect and admire it: and yet, after a lapse of several months, but *one* was sold, and empty praise was for some time his sole reward.

At this period he executed a medallion, of which, I believe, only very few impressions remain. It represents the busts of the Right Hon. John Beresford and his wife, Miss Montgomery, side by side, and was engraved for a person who passed himself as a Turk, and kept baths in Dublin: he was called Solyman Achmet, but his real name was, I believe, Kerns. Having received some favour from Mr. Beresford, he caused this medal to be engraved, and set in the side of a silver cup, which he presented to him. The work is extremely delicate, and gives a faithful resemblance of his patron and lady.

Amongst those who were distinguished as encouragers of genius, Mossop found a friend and protector in the late Dr. Henry Quin. The first work he executed after his acquaintance with that gentleman was a head of his patron, and in it the artist had given an expression so true to nature, and had finished the whole with an air so closely resembling the antique, that it met the unqualified approbation of the excellent judge whose portrait it gives. The immediate occasion of this medal was as follows. Robert Watson Wade, Esq., first clerk of the treasury under Wm. Burton Conyngham, Esq., was affected with a violent imposthume in his side, which had baffled the skill of the faculty in Dublin, but having fortunately called in Dr. Quin, he obtained almost immediate relief, and as a token of gratitude presented him with this medal in gold, and inscribed on the reverse, OB SANITATEM RESTITUTAM EXCUDIT R W WADE. This was followed by orders for medals of Mr. La Touche, Mr. Alexander, Mr. Deane, and Viscount Pery. Of this nobleman it may not be unsuitable to record an anecdote, which affords an example worthy of imitation amongst those who may have an opportunity of patronizing arts and artists. When Mossop had finished the head of Lord Pery, he waited upon him with the work. His Lordship expressed himself highly pleased with the performance, and inquired what remuneration he expected; on Mossop's replying twenty guineas, the nobleman's surprise gave every reason to imagine that he conceived it an exorbitant demand; coldly remarking, that he thought the artist had not put a fair price upon his work, he observed, he hoped he would be satisfied to accept what he thought proper to give. With these words he presented Mr. Mossop with a paper, which he put into his pocket without examination, and in some confusion bowed and withdrew. If the artist was mortified under the impression that his price was to be reduced, we may imagine his gratification at finding he had been presented with an order for double the sum he had demanded.

Shortly after, in 1786, Mossop was employed to execute the Prize Medal of our Academy. The side with Hibernia and the emblems of art was the original device, to the other side was added the head of the Earl of Charlemont when he became our president. As this work may be justly considered the *chef d'œuvre* of the artist, and is, I regret to say, in the hands of so few of our members, it will be proper here more particularly to describe it. Obverse, JACOBUS · COMES · DE CHARLEMONT · PRÆS. The Earl is represented in the uniform of the Irish volunteers; the resemblance is most correct, and the execution of the head beautifully soft and fleshy; the modern costume, so ill adapted to classical art, is rendered agreeable by delicate and judicious management. Reverse, VETERES REVOCAVIT ARTES. Hibernia seated on a pile of books, surrounded by emblems of astronomy, chemistry, poetry, and antiquities. Exergue, ACAD · REG · HIB · INST · JAN . 28 – MDCCLXXXVI. The figure is bold and masterly, the drapery broad, and the drawing correct; while the disposition of the emblems is so tasteful, that in the variety of subjects embraced, nothing appears crowded or confused. The noble Earl was so pleased with this specimen of his skill, that he allowed the artist the use of his library, and free access to all his valuable collections.

Soon after the execution of this work Mossop received orders for the medal of Lord Rokeby the Primate; for that given at the Commencement in Trinity College; for the badges worn at various societies; and for tickets of admission to sundry institutions: in fact, he had arrived at the top of his profession, and in every thing connected with it in this country he was employed. His fame had reached England, so that Mr. Boulton, the intelligent proprietor of the Soho Factory at Birmingham, was induced to give him an invitation to go over to his employment in 1791, expressed in the most flattering terms, which, however, he thought proper to decline.

During the administration of the Marquis of Buckingham he produced a pattern piece, which he denominated the Union Penny, engraved after a design by Sir Joshua Reynolds. Only six impressions were struck before the die was destroyed, but so admirable was the execution, that two were thought worthy of a place in the cabinet of the reigning monarch. Afterwards he was employed to superintend the coinage of the copper money issued by Messrs. Camac, Kyan, and Camac, until the failure of the concern, by which he sustained considerable loss; and then he resumed his former pursuits. These led him in 1797 to commemorate the destruction of the French fleet at Bantry Bay by a beautiful medal, which is still worn by the members of a club established on the occasion in the neighbourhood; and he was further employed by the Orange Association and by the Farming Society, to design and make their badges and premium medals.

The Rebellion, and subsequently the Legislative Union in 1801, diverted the public mind from any consideration of the fine arts, and the medallic art, the object of our inquiry, shared the common neglect. With the exception of a medal for the Dublin Society, and a Premium Medal for the Navan Farming Society, no other work of importance was executed by Mossop; and when the former was undertaken, it was proposed that it should have an appropriate reverse for each of the objects which that Society was embodied to encourage. From the eminent skill exhibited in the part of the work which was completed, it is much to be regretted that the original plan was not persevered in. This medal, when at present used, is struck with a blank reverse, upon which is engraved the name of the person obtaining it, and the object for which it is adjudged.

In 1804 a paralytic affection, followed almost immediately by apoplexy, terminated in a few hours the life of this ingenious artist. Though his works are not numerous, they are interesting, and as the first of the kind produced in Ireland, are a lasting evidence of his natural ability in this department of art. Had he received the advantage of early preparatory study, there can be no doubt that he would have equalled any modern medallist, and rivalled those in former times of whom other countries are so justly proud. Besides his medals, he engraved several large official seals for corporate bodies in Dublin and elsewhere. He also executed a head in carnelion, and a small copy in ivory, from the celebrated gem of the marriage of Cupid and Psyche. In the domestic relations of son, husband, and father, he was most exemplary, and obtained respect wherever he was known.

William Stephen Mossop, jun., also a native of Dublin, was born in 1788, and after receiving a liberal education at the celebrated school of Samuel Whyte, he commenced in 1802 his studies in the fine arts at the academy of the Royal Dublin Society, under the care of Mr. Francis West, then master of the Figure School. The progress he made not proving satisfactory, he was placed amongst the private pupils of Mr. West, with whom he continued until his father's death left him, at the age of sixteen, very inadequately prepared to commence the practice of his profession; and the first work he produced was the medal for the Society incorporated for promoting Charter Schools. It was commenced in the life-time of his father, and finished shortly after his death, when the artist was not seventeen years of age. In 1806 he was employed by the Farming Society to execute a badge to be worn by such persons as were life members; and in 1809 he commenced a medal of considerable merit, for the purpose of commemorating the fiftieth year of the reign of George III. By his own account I find that in the following year he visited London for the first time; but, as he expresses it, "his stay was so short, and he was so much bewildered by the variety that surrounded him, that he did not derive all the advantages from it he might have done." However, his spirit was greatly aroused, for though after his return to Ireland he was much occupied in working at medals for various branches of the Farming Society, then in active operation, he found time to execute a medal, the die of which was afterwards purchased by the Feinaglian Institution as a Premium medal, and for which he obtained a premium himself from the Society of Arts at the Adelphi. In 1814 he obtained another premium from the same body for a head of Vulcan, which he engraved in compliance with an advertisement from that Society, who promised to purchase the die, but left it, through neglect, on his hands. Thus it appears his merit was acknowledged, but his works were very inadequately remunerated.

In 1820, I find from his letters, that he projected a series of medals of distinguished Irish characters, but I cannot discover that he put his design fully into execution, though medals of Ussher, Swift, Charlemont, Sheridan, and Grattan afford some evidence of a commencement. Their execution, and the fidelity of the likenesses they exhibit, are such as to make us regret the design was allowed to fall to the ground. The last die that I can discover of his workmanship is one of a noble medallion of the illustrious Wellington; it appears as if the subject, as well as the country of the hero, had sharpened his graver, and directed his hand, for it is in truth a spirited performance, having on the obverse a bust of the Duke to the left, and on the reverse the appropriate emblem of Victory crowning a warrior, who is seated, leaning upon his shield. There is also, by the same hand, a small medallet of the hero, a perfect gem; the die came into the hands of the late Mr. West of Skinner-row, and impressions from it are very rare. On one side it exhibits a bust inscribed DUKE OF WELLINGTON, and on the other the simple but expressive word WATERLOO, inclosed in a wreath; this reverse however was executed by another artist. Mossop died in 1827, having for some time previous been afflicted by mental aberration, brought on probably by intense application, and increased by those disappointments concomitant with unrequited genius and professional assiduity.

Unwilling to break in upon the account of the two Mossops, I must here insert a reference to some medals struck in the years 1797–8. Kirk, an artist well known in England, thought it no disparagement of his own talents to copy from Mossop's medal the head of Primate Robinson, and place it on a smaller one with his name, and bearing on the reverse an elevation of the library at Armagh, as a memorial of the liberality of that munificent prelate. The two next are miserable in point of design and workmanship. They were executed under the direction of a person named Brush, who was a silversmith, and as appears from them totally devoid of skill and judgment in that line. One I imagine to be the original badge of the Orange Society, and bears a figure of William III. on horseback within a border of orange lilies. On a scroll above, THE GLORIOUS MEMORY, and below, KING AND CONSTITUTION. Reverse, a sword and sceptre in saltire through a crown, in a wreath of orange lilies, and below on a scroll, GOD SAVE THE KING. The second bears the legend, CORPORATION AND CITIZENS OF LIMERICK,—a castle, with the armorial bearings of the city in a wreath of laurel and palm. Reverse, a crown within a laurel wreath inscribed TO THE HEROES OF COLOONY, 5TH · SEP^{R}. 1798. It was designed to commemorate the successful battle fought by the Limerick militia under Colonel Vereker, against General Humbert and the French, at Coloony, near Sligo. Another medal of this year, of beautiful workmanship, and executed by Hancock in England, commemorates the decisive victory obtained by Sir I. Borlase Warren over the French fleet off the coast of Donegal, on the 12th of October, 1798.

The visit of George IV. to his Irish dominions naturally called forth the emulative talents of various artists, both in this and the sister kingdom. On this occasion a medal was published by Mossop. Obverse, GEORGIVS IV D . G . BRIT . ET HIBERNIÆ REX F · D. The king's head laureated to the left. Reverse, ADVENIT REX CONCORDAT CIVITAS. Hibernia standing with a cornucopia in her right hand, and an Irish harp in her left; at her feet, on the right, a child with a lighted torch, setting fire to a pile of armour and military weapons; on her right a square altar, with a small flame arising from its top; in the exergue the arms of the city of Dublin, with the city mace, sword, and cap, MDCCCXXI. The die of the reverse of this medal was broken after a few impressions were struck off, and the artist speedily executed another, which differs a little from the one just described, having in the exergue, XII , AUG : MDCCCXXI.

Connected with his Majesty's visit, another medal was executed by Isaac Parkes, an artist still living, to commemorate the Installation held at St. Patrick's Cathedral. Obverse, GEORGIUS IIII · D : G : BRITANNIARUM REX · F : D : king's head laureated to the left, encircled by the collar of the order of St. Patrick. Reverse, south-east view of St. Patrick's cathedral; in the exergue, ROYAL INSTALLATION – AT ST PATRICKS DUBLIN – AUGUST XXVIII – MDCCCXXI. The view of the cathedral is very correct, and executed with ability.

There is a medal connected with this period, which, though executed in England, as it purports to be struck on Irish metal, it may be fitting to allude to. Obverse, GEORGIUS IIII D : G : BRITANNIARUM REX F : D : Bust to the left, with a laurel crown. Reverse, Hibernia with a harp, and a wolf dog at her feet, receiving the king, who is just landed from a boat bearing the royal standard. Howth, and some of the most conspicuous buildings of the city in the back ground. In the exergue, IN COMMEMORATION OF HIS MAJESTYS – MOST GRACIOUS VISIT TO IRELAND – 1821. W · HAMY DIREX. There is engraved on the edge, IRISH COPPER FROM THE MINES IN THE COUNTY OF WICKLOW. This is a work got up by Hamy and Mann, silversmiths in Dublin. The bust was executed by Benjamin Wyon, and the reverse by Mills, both artists of eminence, and are creditable to them.

I have but few medallists more to notice; as they are still living, and working in their profession, I should prefer finding that the Academy was about to take them under its fostering care, to occupying your time in criticising their performances. John Jones was employed in the establishment of the younger Mossop until the death of the latter, and has since produced some works from his own graver connected with the political events of these busy times. They speak for themselves, and I only regret that he has not been more employed, as his Premium Medal for the North East Agricultural Society, is, in taste and execution, a very beautiful performance. His tools and presses are now rusting in his workshop; and a talented professional native, educated in an excellent school, has the mortification of finding himself neglected, and English artists employed to record Irish events.

William Woodhouse, who is a native of England, and received his education at Birmingham, has also struck some few medals. I have no doubt, from the specimens I have seen, that were he to receive due encouragement, his talents would be well employed in the service of our country.

The last with whom I am acquainted is Isaac Parkes, a native of Birmingham also, but who came to this country in 1807, and served his apprenticeship to his brother, an eminent button manufacturer in this city. We are justified in considering Parkes as our own; for, here he served his time; here he received instructions in modelling from Sherwin, the pupil of Smyth, whose chisellings and figures adorn so many of our public buildings; and, here whatever proficiency he has attained to in the art has been elicited and nourished. If diligent attention to business, access to a well-chosen collection of models, and a considerable share of ingenuity and taste, can secure public patronage, Parkes well deserves it; and his large medallion of the late Duke of York is an evidence of his boldness and power in the art of die sinking,—for amongst all those of the middle ages, I have scarcely seen one that exceeds it in relief, and it has this superiority over them, that while they were invariably cast, this was raised out of the solid metal by the power of the screw.

The comparatively small number of medals I have been able to record from the time of Charles II. to the present day, affords a lamentable and humiliating proof of the small encouragement both arts and artists have hitherto received in Ireland. Our medallists, while labouring under great discouragements, have shown themselves capable of performances worthy a place in any cabinet; what might we not then expect if the liberal, the enlightened, the classical were once aroused to patronize an art which formed the boast of Ancient Greece and Rome in the days of their greatest power and highest civilization.

P. S.—It was my intention to have accompanied the preceding Memoir with an Appendix, giving a particular description of many other medals connected with Ireland, as well as those which have been noticed already, together with engravings of the most rare and interesting. But since I have more particularly directed my attention to the subject, my researches have led to the discovery of so many medals, of the existence of which I was before ignorant, already amounting in all to more than two hundred, that I shall for the present defer the publication of the appendix and engravings till I am enabled to present it to the Academy in a form as complete as I would wish, and as the subject deserves.

II. *On the Antiquity of the Kiliee or Boomerang.* *By* SAMUEL FERGUSON, ESQ., M. R. I. A.

"Forte tamen aliquis erit qui de *Aclide* certius aliquid in medium ferat."—*Pierii in Æneid.* l. vi. v. 730, *Comment.*

Read January 22, and February 12, 1838.

I.—OF THE CATEIA.

THE Kiliee or Boomerang, at present the peculiar weapon of certain Australian islanders, several varieties of which are represented in Plate I., appears to have been known to European and other Continental nations from a very remote period.

The name by which the Boomerang is most readily recognized in the works of Roman writers is *Cateia.* Of this, the earliest notice is found in the Æneid of Virgil, where, among various tribes who joined themselves with Turnus, mention is made of a people accustomed to whirl the Cateia after the Teutonic manner.

> "Et quos maliferæ despectant mænia Abellæ
> Teutonico ritu soliti torquere Cateias."
>
> *Virg. Æneid.* l. vii. v. 740.

The next mention of the Cateia occurs in the Punics of Silius Italicus, where the poet describes an individual of one of the Lybian tribes, who accompanied Hannibal to Italy, as being armed with the bent or crooked Cateia :

> "Tunc primum castris Phœnicûm tendere ritu
> Cinyphii didicêre Macæ : squalentia barbâ
> Ora viris : humerosque tegunt velamina capri
> Setigeri : pandâ manus est armata Cateiâ."
>
> *Sil. Ital. Punic.* l. iii. v. 274.

A third notice of the Cateia is found in the Argonautics of Valerius Flaccus, where, in an enumeration of the Maeotic nations which rose in arms against Jason, a people are described whose tents of raw hides were carried on waggons from the extremities of the poles of which their young men whirled Cateias.

> "Quin et ab Hyrcanis Titanius expulit antris
> Cyris in arma viros: plaustrisque ad prælia cunctas
> Coraletæ traxêre manus: ibi sutilis illis
> Et domus, et crudâ residens sub vellere conjunx,
> Et puer è primo torquens temone cateias."
>
> *Val. Flac. Argonaut.* l. vi. v. 83.

From these notices it may be collected,

1st. That the Cateia was an instrument of a curved shape, for this is the constant meaning of the adjective *pandus*. "Carinæ pandæ," (*Virg. Georg.* l. ii. v. 89.)—"Delphines pandi," (*Ovid. Trist.* l. iii. v. 9.)—"Fauces pandæ," (*Stat. Sylv.* l. iii. v. 15.)—"Rostrum pandum," (*Ovid. Metamor.* l. iv. v. 57.) —"Rami pandi," (*Ovid. Metamor.* l. xiv. v. 37.)—"Juga panda boûm," (*Ovid. Amor.* l. i. and *Eleg.* l. xiii. v. 4.)

2nd. That it was a projectile—"*e* temone torquens."

3rd. That it was dismissed with a rotatory motion—"torquens,"—"soliti torquere." For, although the verb *torqueo* is frequently applied to the projection of the straight missile, it is always with reference to the rotatory motion either of the *amentum*, by which several sorts of straight missile were thrown, or of the weapon itself round its own axis.

These marked characteristics of the Boomerang would, perhaps, furnish sufficient grounds for inferring an identity between it and the weapon under consideration; for, from recent experience, it might safely be asserted that no instrument having the peculiar shape ascribed to the Cateia could be projected with a rotatory motion, without also exhibiting the great distinguishing property of the Boomerang by a reciprocating flight. But the description of the Cateia, given by Isidore, Bishop of Seville, a writer of the end of the sixth and beginning of the seventh century, renders this line of argument unnecessary. He describes the Cateia as a species of bat, of half a cubit in length, which, on being thrown, flies not far, on account of its weight, but where it strikes, it breaks through with excessive impetus. *And if it be thrown by one skilful in its use, it returns* *back again to him who dismissed it.* The passage occurs in the " Origines," under the head CLAVA, viz.:

" CLAVA est qualis fuit Herculis, dicta quod sit clavis ferreis invicem religata, et est cubito semis facta in longitudine. Hæc et Cateia, quam Horatius *Caiam* dicit. Est genus Gallici teli ex materia quam maxime lenta; quæ, jactu quidem, non longe, propter gravitatem, evolat, sed ubi pervenit vi nimiâ perfringit. *Quod si ab artifice mittatur, rursum redit ad eum qui misit.* Hujus meminit Virgilius dicens

'Teutonico ritu soliti torquere Cateias.'

Unde et eas Hispani *Teutones* vocant."—*Isidor. Origin.* l. xviii. c. vii.

Thus, all the characteristics of the Boomerang, its use, its shape, its mode of projection, its extraordinary impetus, and its peculiar reciprocating flight, belong to the Cateia, from which it cannot but be concluded that these were the same weapon.

II.—OF THE ACLYS.

Another name by which a weapon of the same character would appear to have been known to Roman writers is *Aclis—aclidis*, and *Aclys—aclydis.* It is first mentioned by Virgil, speaking of the aborigines of Campania.

—— " Oscorumque manus: teretes sunt aclides illis
Tela; sed hæc lento mos est aptare flagello."

Virg. Æneid. l. vii. v. 730.

From which it appears that the Aclys was originally a hand weapon, as its discharge by means of a thong is mentioned as something unusual.

Silius also mentions the Aclys, after enumerating those tribes of Campania who allied themselves with Rome before the battle of Cannæ.

———— " Formabat Scipio bello.
Ille viris pila, et ferro circumdare pectus
Addiderat: leviora domo de more parentum
Gestabant tela; ambustas sine cuspide cornos;
Aclydis usus erat, factæque ad rura bippennis."

Sil. Ital. Punic. l. viii. v. 553.

And again, among the forces of Hannibal :

> "Jamque Ebusus Phœnissa movet, movet Artabrus arma
> Aclide vel tereti pugnax instare veruto."
>
> *Sil. Ital. Punic.* l. iii. v. 362.

Mention of the same weapon is found in the rescript directed to Zozimio, Procurator of Syria, empowering him to pay a certain annual stipend to Claudius, at that time tribune of one of the Roman legions, and afterwards Emperor, which document is embodied in the life of Claudius, by Trebellius Pollio. Here, among various articles of value, such as mantles, belts, and various sorts of weapons, are specified "Lanceæ Herculeanæ duæ—Aclides duæ—falces duæ, &c. (*Hist. Aug. Scrip. Minor.* v. ii. p. 149.)

These passages, although they may appear to distinguish the Aclys from straight missiles in general, yet do not afford more than a negative inference. A more satisfactory evidence of the shape of the weapon, may, however, be obtained from a passage of Valerius Flaccus in the above-mentioned enumeration of the Mæotic nations.

> "Nec procul albentes geminâ ferte aclyde parmas
> Hiberni qui terga Noæ, gelidumque securi
> Eruit, et totâ non audit Alizona ripâ.
>
> *Val. Flac. Argonaut.* l. vi. v. 99.

For "fert," Burmann reads "ferit," and considers the double Aclys as the instrument in eliciting a warlike sound from the struck shield. He also takes "albentes" to mean white, as having no device, in the same sense as "albus" in Virgil, "parmâ inglorius albâ." But "fert" is the reading of all the MSS., and, as "fert" cannot take an ablative to complete its meaning, "geminâ aclyde" must be referred to "albentes." Again, had Valerius intended to convey the same meaning with Virgil, he would have used "albas," or perhaps "albatas," but never "albentes," which means growing white from some other colour, and implies a proximate cause.—"Campique ingentes ossibus albent," (*Virg. Æneid.* l. xii. v. 36.)—"Caput quod videam canis albere capillis," *Ovid. Heroid.* Ep. xiii.) The meaning of the passage would, therefore, appear certainly to be, "close to him, the hewer of the crust of wintry Danube, who draws his water with his axe" (a quaint phrase parallel to that of Sidonius Apollinaris, "Ligerimque securi excisum, per frusta bibit.—*Carm.* v. v. 209.) advances shields charged with the white blazonry of the double aclys." Now, the general family to which this tribe belonged, appears as well from their being brought from the Alazonian or Amazonian river (it is also from the banks of the Danube that Seneca brings the Amazons in his Hyppolitus) as from some markedly Amazonian characteristics attributed to them. Of these the most striking is the adoration of pillar-stones, an Amazonian trait not to be mistaken. For, however fabulous that story was which appears to have originated in a vulgar etymology of the word Amazon, it is certain that there were nations of such a family, among whom the women took an active part in war, and that the worship of pillar-stones has been very generally ascribed to them by ancient writers. Plato mentions an amazonian pillar-stone at Athens. *Πλησιον ωκει των πυλων προς τη* Αμαζονιδι *στυλη* (*Plato in Axiocho.* v. iii. p. 365.) And the Argonauts of Apollonius are represented as finding a similar one in Pontus, near the Amazonian Temple of Mars.

> " Εισο δε μελας λιθος ηρηρειστο
> Ἱερος ῳ ποτε πασαι Αμαζονες ευχεταονται."
>
> *Apollon. Argonaut.* l. ii. v. 1177.

" Wherein was set up a black holy stone to which all the Amazonians offered their prayers." A stone of the same sort was shown in Colchis in the time of Arrian, and was said to have been the anchor of the Argo, (*Arrian. Peripl.* p. 9;) and even down to the thirteenth century, pillar-stones were of frequent occurrence throughout the plains bordering on the north of the Euxine, (*Rubruquis apud Hackluyt.* vol. i.) So that, in reference to the bearers of the shields blazoned with the double aclys, the following passage from Bryant's Analysis of Ancient Mythology may safely be submitted.

" The Amazonians were Arkites; hence it is, that they have ever been represented with lunar shields; many have thought that they were of a lunar shape, but this is a mistake, for most of the Asiatic coins represent them otherwise. The lunette was a device taken from their worship. It was their national ensign which was painted on their shields; whence it is said of them, 'Pictis billantur Amazones armis,' and in another place 'ducit Amazonidum lunatis agmina peltis, Penthesilea furens.' The Amazonian shields approached nearly to the form of a leaf, as did those of the Gothic nations. Pliny says of the Indian fig, 'Foliorum latidudo peltæ effigiem Amazoniæ habet.' Upon these shields they had more lunettes than one; and from them the custom was derived to the Turks and other Tartar nations."—(*Anal. Anc. Myth.* v. iii. p. 472.)

Whether or not the lunette, which is still the ensign of a very numerous nation, was an Arkite emblem, as this learned, but somewhat fanciful writer supposes, it is extremely probable, that if his interpretation of "lunatæ peltæ" be correct, this is the same blazonry described by Valerius Flaccus, whose omitting so marked a characteristic would otherwise be singularly inconsistent with the propriety observed throughout the remainder of his poem. "Albentes geminâ fert aclyde parmas" may then be rendered—"Advances shields charged with the white blazonry of the double *lunette*" and thus the curved form of the aclys, if this argument of Bryant be correct, will become as apparent as that of the "panda Cateia."

This view is strongly confirmed by the description given of this weapon by Servius. "The aclys," he says, "is a weapon of so great an antiquity, that the use of it in war has not been recorded (meaning probably, not otherwise than by poetical writers.) We read, however, that these were bats, of half a cubit in length, with horns projecting at either side, (*eminentibus hinc et hinc acuminibus,*) which were so cast against the enemy attached to a line, as to be capable of being retracted after having inflicted the wound;" (*Servius in Æneid.* l. vii. v. 730.) Here, while Servius clearly describes the shape, and refers to the peculiar flight of the Cateia, he seems to consider the latter as produced by the retraction of thongs to which the weapon was attached; and in this view he has been followed by all the commentators down to our time. He admits, however, immediately after, that this was but a guess, and refers to the tradition which appears to have preserved the true account; "putatur tamen esse teli genus quod per flagellum in immensum jaci potest," which will safely bear this translation, "some, however, are of opinion, that the thong was only used in its projection, and that by its means it could be cast to an immense distance."

Such was the Aclys, according to the uncertain report of Servius, and, whatever it may have appeared to him to be, *he identifies it with the Cateia,* making only this distinction, that the latter was a weapon of double the dimensions; "Cateiam quidam asserunt teli genus esse, *tale quale Aclides sunt,* ex materia quam maxime lenta, cubitûs longitudine, tota fere clavis ferreis illigata, quam in hostem jaculantes, lineis quibus eam adnexuerant, *reciprocam* faciebant;" (*Servius in Æneid.* l. vii. v. 741;) where it will be still observed, that he leaves it uncertain whether the reciprocating flight arose from the retraction of the lines, or was a consequence of the mode in which the weapon was thrown by their instrumentality.

To these we may add a testimony of considerable force, if the translation suggested should be deemed the true one, from Sidonius Apollinaris, Bishop of the Arverni, a writer of the fifth century. The passage occurs in that panegyric which Sidonius recited before the Emperor Majorian on his arrival at Lyons in the year 457. In this piece the Acquitanian prelate gives an interesting, though inflated account of a victory obtained a short time previously by Majorian over a predatory band of Vandals and Moorish slaves from Africa, who had attempted to carry off a prey from the coast of Campania. He depicts the fat Vandal starting from the benches of his galley, and arming himself for the support of his emissaries on shore, with certain poisoned missiles, which, according to what appears the most obvious translation,* *strike twice when once discharged;* and, in

* It may be argued that the words, "quæ feriant bis missa semel" have reference to the poison of the *arrows* alluded to in the preceding line, and mean, "which injure *doubly* by a single discharge." The other translation has, however, been preferred on the following grounds.

Both interpretations go on the assumption, that in the words "quæ feriant bis missa semel," the poet intended an antithesis between *bis* and *semel;* and the difference between the two interpretations consists in this, that in the one the antithesis is held to lie between the one discharge and the two *successive* effects; while in the other, it is held to lie between the one discharge and the two *simultaneous* effects.

It is true, *bis,* under certain circumstances, will mean double in simultaneous operation, as "bis periit amator," &c.; but never, it is submitted, when in opposition to *semel,* for semel has but one meaning, "once, *in point of time,*" and to be in opposition to it, *bis* must necessarily mean "twice, *in point of time.*" The interpretation which refers *bis* to a succession of blows, would, therefore, so far appear preferable to that in which *bis* is made to have reference to the double simultaneous operation of cutting and poisoning by one and the same blow.

Again, where the actions of two or more agents unite in one verb, the verb employed ought to be such as is proper to both or all. Thus, in expressing in English the idea supposed by the

the subsequent account of the engagement, represents some as slain by pikes, some by arrows, and others by the *Aclys.*

> " Tum concitus agmine totâ
> In pugnam pirata coït; pars lintre cavatâ
> Jam dociles exponit equos, pars ferrea texta
> Concolor induitur, teretes pars explicat arcus,
> Spiculaque infusum ferro latura venenum
> *Quæ feriant bis missa semel;* jam textilis anguis
> Discurrit per utramque aciem, &c. &c."

And again, after the battle joined:

> "Hunc conti rotat ictus equo, ruit *aclide* fossus
> Ille, veruque alius, jacet hic simil alite telo
> Absentem passus dexteram."
>
> *Sidon. Apollinar. Carm.* V. v. 328-415.

Thus, then, the notices which can be collected concerning the Aclys furnish evidence nearly as strong as that adduced in the case of the Cateia, shewing that

suggested interpretation to be conveyed by these words, we do not say, "which *poison* twice when once discharged," on the one hand, nor "which *cut* twice when once discharged," on the other; but select some equivalent for *feriant*, which is equally applicable to the infliction of a hurt by the incision of a cutting instrument, and by the operation of a poison, such as "wound," "hurt," "injure," &c.

But it is conceived that *ferio* is not capable of such an equivalent. It means essentially to "hit," to "strike," to "illide against," and is quite inapplicable, without a very strong metaphor, to the operation of a poison. But if there be two agents, as in this case, the common verb cannot be employed metaphorically, unless the metaphor be equally applicable to both agents. The meaning of the common verb cannot be split, so as to suggest two ideas, one metaphorical, and one simple, having reference severally to the respective agents. Had the poet intended the meaning suggested, he might properly enough have made use of either "noceo" or "lædo," both of which are applicable, as well in point of rythm as of meaning. Thus, "namque ut refecta est coluber, *nocuit* hominem protinus," (*Phædri*, l. i. fol. 18;) "*Lædere* aliquam vulnere," (*Ovid. in Ibin*, v. 484,) &c.

Further, *missa* seems to imply progressive motion, such as is more proper to successive than to simultaneous effects; and, therefore, had Sidonius intended the meaning suggested, he would probably have employed, not *missa*, but some such word as *acta, impacta,* or the like, which would carry the agents to their *locus in quo*, and leave them there.

To express the meaning suggested, the fittest words would be "quæ noceant dupliciter simplici ictu," which are all different from the words employed; but, to express the meaning adopted, it would be impossible to find apter words than those employed themselves.

all the chief characteristics of the Boomerang belong to this weapon also; whence it is concluded, that the Aclys was a weapon which differed from the Cateia only in dimensions.

III.—OF THE ANCYLE.

The etymology of the word *Aclys* points, in the third place, to another name by which a similar weapon seems to have been known to the Greeks. "Ego jacula crediderim, (says Turnebus, in his commentary on the 'duas Aclydes' of Trebellius, *Adversar.* lib. xxx. c. xi.), an sata, an amenta. Αγκυλαι autem Græciæ jacula quædam sunt; et per diminutionem inde Αγκυλιδες—inde *Aclydes.*" And this etymology is generally adopted by subsequent commentators. There exists, indeed, a remarkable connexion between the sounds *ak* and *ank*, which strongly supports the conjecture of Turnebus. Thus, as Vossius observes, from *κικιννος, cincinnus;* from *λειχω, lingo;* from *εχις, anguis.* In like manner *ank*, in the present of some verbs, assumes the form *ak* in the præterite, as stringo, *strixi;* fingo, *fixi;* frango, *fregi;* vinco, *vici;* pango, *pæxi, pegi, pepigi; pactum,* &c., (old præterite.) Thus, also, the *αγκυρα* of the Greeks, and *anchora* of the Latins, is found in the form *akkeri* in the Islandic, and *akkjeri* in the dialect of the Feroë islands.—(*Antiq. Americ. ante-Columb.*, p. 328.) So also in topographical nomenclature, the Sangar river, called by the barbarians Sagaris; the Ogygian gates, stated by Hesychius to be called the *Oncaian* gates by the Athenians, &c. Numerous similar instances may be had in the modern languages of Europe, as *against*, in the Anglo-Saxon onȝean, (*Skinner, Etymol. Mag. Ling. Ang.*); *aguillon*, the French needle, in the Teutonic, *angel*, (do.); *ache*, a pain, from the Anglo-Saxon anȝe, *vexatus*, (do.), &c.

Now the Αγκυλη of the Greeks, though commonly used synonymously with the Latin *Amentum*, meaning the thong or attached sling by which various sorts of missiles were discharged, has an independent signification as a distinct species of missile, as in that passage of the Orestes of Euripides, where certain Phrygians, speaking of their weapons, are made to say:

Ὁ μεν πετρους ὁ δε αγκυλας,
Ὁ δε ξιφος προκωπον εν χεροιν εχων.

Eurip. Orest. v. 1438.

On which the scholiast observes, *αγκυλας—τα ακοντια απο του επηγκυλιασθαι; ἢ διοτι απο της κατα μεσον αγκυλης λαμβανομενοι ριπτουσιν.*—"Ancylés, certain missiles, *so called from being of a curved shape,* or because weapons of that sort are thrown by means of an *ancyle* fastened to their middle." The *ancyle* is also given in Hesychius and Suidas, as *ειδος ακοντιου,* "a species of missile," along with its other significations.

If, then, the Aclys be truly a derivative of this name of a weapon known to Greeks as a missile of a curved form, there appear good grounds for considering the Ancyle also as belonging to the family of the Boomerang. These conclusions will receive further corroboration from an investigation of the meanings of the names so far sought to be identified.

IV.—OF THE RADICAL MEANINGS OF THE NAMES CATEIA, ACLYS, ANCYLE, AND TEUTON.

That Cateia means literally something curved, might be inferred from the application of the word in the Basque language to signify a reaping-hook,—Iguiteia, *falx,* (*Lhuid Archæol. Brit.*); and this inference is very amply borne out by an inspection of those words involving the idea of curvature, into which the element *kat* enters radically. Thus the Latin *catena,* a chain made up of twisted links, appears rather a derivative from, than the parent of the Belgic *catte,* a chain. That both signify something crooked or twisted, appears clearly from the application of the synonymous Welsh *kaduen* to mean both a chain and a *hook.* *Catte* also is the old Belgic anchor, whence our *cat-head,* that piece of timber, namely, from which the *cat* or anchor is suspended. *Guet,* again, in the Cornish, means a turning. In like manner, the Welsh *kad*-lys is found synonymous with the Irish *uir*-lis, or circular enclosure; an instance which may be considered conclusive in settling the meaning of the element *kad,* or *kat,* in the Celtic. Hence it appears, that the idea of rotundity or circularity, which is shewn in the Ordnance Memoir of Londonderry to enter into the signification of *gort, gard, villa, bally, urbs,* as applied to early cities, is also radically involved in the synonymous *cat*hair, whether spelled as in the Punic *gad*era, or as in the Gallic *cattur* of Ptolemy, or as in the Welsh and Cornish *cad*er of the present day; and hence a curious illustration of the *Γαδερα, τα περιφραγματα* of Hesychius, as well as of the other testimonies adduced by Bochart to shew that this word literally means a fenced enclosure.—(*Bochart. in Georg. Sac.*) It is worthy of observation, that the *kraals* of savage nations still retain this primitive form, which we see thus indicated in almost all the names used by European nations to signify a collection of habitations. These will be sufficient for the present to establish the necessary meaning of *Cateia.*

If the conjecture as to the etymological relation between the words Aclys and Ancyle be correct, it will only be necessary to investigate the radical meaning of the latter; and here we are introduced among a numerous family of words in which the idea of curvature is uniformly inherent: *αγκων, αγκυλη, αγκυρα*; *unguis, anguis, ancus, uncus, angulus, ango, angor, anxius, angle, ankle, hang, hank, hanker,* (synonymous with the *hake* of Lincolnshire, *Skinner,*) *hunkers, haunch,* (the Italian and Spanish *anca,* synonymous with *hough* or *hock,*) *hunch, hunch-backed,* (the Belgic *huckschoulderen,* from the Belgic and Teutonic *hucken,* to bend down,) in which last the connexion above contended for is strikingly manifested. These examples might be swelled to a great extent, but it is conceived that enough has been done to determine the essential meaning of *Ancyle,* and to shew a high degree of probability that a like signification is also involved in *Aclys;* so that as *Ancyle* appears to be nearly identical with the Sicilian *zancle,* a reaping-hook, *Aclys* may, in like manner, be the representative of our own *sickle.*

With regard to the passage from Isidore, which states that the Gauls and Spaniards of his time called the Cateiæ *Teutones,* as indicating the Teutonic origin of the weapon, it is to be observed, that the proper name of the Teutonic people is Tuitschen, or Duytschen, and that the word Teutones of the Latins was only a softened representation of that sound. Now Grial, commenting on this passage of Isidore, states that the Spaniards of his time continued to use certain instruments, which he conjectures to be the same. These he does not farther describe than by observing, that the name they then went by was *Chochones;* but *Chochono* in the Basque language is equivalent to the Castilian *Concavo,* (*Dictionar. Triling. ad verb.*); and hence it appears very probable, that the name Teutones was imposed on these weapons, not as indicative of their origin, but as descriptive of their shape.

It may, then, be concluded, with a strong degree of confidence, as well from the testimonies of ancient writers, as from the necessary signification of the names by which these weapons were known, that the Catciæ, Aclides, Ancylæ, and Teutones, of the classic authors, were true varieties of the Boomerang. The consideration of the name *Caia*, which also occurs in Isidore, but with marks of a corrupt reading, is reserved for another place.

V.—OF THE JAVELIN OF CEPHALUS AND AQUIFOLIA OF PLINY.

So far of the name or names by which weapons of this species were, or may have been, known to the ancients. That their peculiar flight was known, and has been markedly alluded to without the specification of any name, appears also from the classic writers. Ovid, in the fable of Cephalus and Aurora, has attributed the distinguishing property of the Boomerang to the weapon of Cephalus. "It pursues whatever it is aimed at: chance does not govern its flight; but it flies back of its own accord bloody from the wound it has inflicted."

> "Consequitur quodcunque petit; fortunaque missum
> Non regit, et revolat, nullo referente, cruentum."
>
> *Ovid. Metamor.* l. vii. v. 684.

From the context, however, it appears that Ovid does not ascribe any of the other characteristics of the Australian weapon to the one in question; on the contrary, he represents it as a straight and pointed dart.

> —— "jaculum cujus fuit aurea *cuspis*." (v. 675.)
> "Quâ tamen è sylvâ teneas *hastile* recisum
> Jamdudum dubito." (v. 677.)

Which would argue rather a hearsay acquaintance with the properties of the weapon, than any accurate knowledge of its shape or structure.

A passage, also, in the works of the elder Pliny, gives evidence of some acquaintance with the distinguishing properties of such missiles; though his attributing the peculiarity in question to an innate virtue of the wood will probably excite a smile. Speaking of the Aquifolia or Agrifolia, a species of holly, he says, a bludgeon of this wood, if thrown at any beast, and falling short of it, will glide nearer (query, to the beast or to the thrower?) in its rebound or descent. "Item baculum ex eâ factum, in quodvis animal emissum, etiam si citrà ceciderit defectu mittentis, *ipsum per se recubitu proprius adlabi*, tam præcipuam naturam inesse arbori."—(*Plin. Nat. Hist.* l. xxiv. c. 73.) On which the naturalist Bauhin observes: "At nos præcipuam in iis inesse superstitionem censemus qui istas nugas credant."—(*Hist. Plant.* l. viii. c. 3.) And indeed it is not surprising that properties so extraordinary should excite the ridicule of commentators not practically acquainted with the peculiarities of the weapon. Thus, Cerda, commenting on the words of Isidore, "Quod si ab artifice mittatur rursum redit ad eum qui misit," considering the alleged result as a consequence of some mystical sympathy between the weapon and a particular person, falls into the error of taking *artifex* to mean the maker of the instrument, and exclaims, "Nam cur non etiam redibit si mittatur ab alio quam ab artifice?"—(*Virg. Not. Var.*)

So far, then, it may be concluded that the Latin writers of the Augustan age were acquainted with weapons possessing all the characteristics of the Boomerang, but with that degree of uncertainty which would imply that their knowledge of them had been derived from a source very remote, either in point of distance or of time. This partial ignorance on the subject will account for any apparent discrepancy that might be charged against those evidences in which notices of the Cateia and Aclys, argued to be the same, are drawn from different passages of the same authors, who would thus appear *prima facie* to put a difference between them. That it was the extreme antiquity of the weapon which caused this uncertainty will appear the more probable from further considerations.

VI.—OF THE CLAVA OF HERCULES AND HAMMER OF THOR.

Isidore identifies the *Cateia* with the *Clava* of Hercules: "Clava est qualis fuit Herculis—hæc et Cateia;" an identity which, most probably, would not have been argued by one so well acquainted with the peculiarities of the Cateia without good grounds. That the Herculean weapon was a missile, appears from

Sextus Pompeius, "Clava, *teli* genus quâ Hercules utebatur;" for although, by a poetic license, Virgil has applied *telum* to a sword, yet the exactness necessary to a lexicographer like Festus precludes any uncertainty that might arise from his supposed adoption of this precedent. That his opinions, and those of Isidore, were recognized down to the tenth century, appears from the Anglo-Saxon Glossary of Ælfric, "Clava, vel Cateia, vel Teutona, anneꞅ cinneꞅ ꝣeꞅceoꞇ," i. e. "the Clava, Cateia, or Teutona, are missiles of one sort;—(*Ælfric. Glossar. ad calcem Dictionar. Somneri;*)—there are, therefore, sufficient grounds to justify some further inquiry into the truth of this assertion, although at first sight it may, perhaps, have appeared too startling for serious consideration.

That the idea of curvature is involved in the word *Clava,* as well as in those hitherto investigated, may be inferred from the application of numerous words of the same family. Thus *Clava* itself is used synonymously with *unguis,* to signify the twisted tendril of a vine; *claw,* our English for a hooked talon, is equivalent to *unguis* in another sense; and *clavus,* a crooked holdfast, or *clamp,* is another equivalent of *unguis,* as is indicated by our use of the synonymous *nail.* Thus *cluif,* in the Lowland Scottish dialect, is synonymous with *ungula;* and the word *clams* is still used in the same idiom for crooked forceps. Thus, also, *glomus,* our *clew,* or round ball of thread; *glom*ero, to gather in a circle; *clav*icula, the crooked key-bone of the shoulder, &c. Another confirmation may be drawn from the application of the Latin *clavis,* to signify a key; for, that the key was originally a crooked instrument appears clearly from all that can be collected from the works of the ancients concerning it; (*Salmasius in Exercitat. Plinian.*); and the very word *key,* by which this instrument is now known to us, is still the identical word used to express a club by the Sclavonic nations, (*Cluverius in Germ. Antiq.* p. 304,) and is very probably the same *caia* to which Isidore alludes in that description identifying the Clava and Cateia. Hence this conclusion seems quite legitimate, that the original form of the *Clava,* or artificial club, was like that of the *clavus,* or original holdfast; or like that of the *clavis,* or original key.

Hence the report of Servius concerning the Aclys, "Quod sit *clava,* cubito semis facta;" and the statement of Johannes de Janua, "Cateia—hasta quâ utebatur Hercules," appear by no means inconsistent with probability.

On these grounds, it may be expected that the club of Hercules will be found represented in ancient sculptures, drawings, or impressions on coins, of a curved shape. It appears, however, from an extended examination of glyptical and numismatical antiquities, as well as of the drawings which remain in the chief collections of Etruscan vases, or on sepulchral monuments, that the poetical Hercules is almost invariably represented with the straight, knotted weapon. The only marked exception which has been observed is in the contest of Hercules with Achelous, (Pl. I. fig. 9,) in the "Museum Etruscum," where the club in the hand of Hercules is represented of a form somewhat approaching to that of the common "hurl" of this country. It is apparently of an untrimmed stem of palm-tree, which, growing naturally straight, must have been reduced by artificial means to the curved shape; suiting well with the description given by Statius of the first attempt at forming an artificial weapon among a rude people.

> "Arcades hi: gens una viris, sed dissona cultu
> Scinditur; hi Paphias myrtos a stirpe *recurvant*
> Et pastorali meditantur prælia trunco."
>
> *Stat. Thebaid.* l. iv.

Where it is observable that the writer does not seem to consider the mere *fustis*, or stake, to be a legitimate *weapon* till bent into the curved form of the *Clava*.

But although the weapon with which Hercules almost universally appears armed in these poetical representations be undoubtedly a mere *fustis*, or knotted staff, there is one instance of a very differently shaped weapon, which appears certainly intended for the club of Hercules, being represented in ancient sculpture. The original is in the French King's collection, and has been described, and a drawing of it given, by Millin, (Pl. II. fig. 3.) The subject is a throne, on one side of which two young genii appear playing with a large, flat, curved instrument, which they seem with difficulty to support. Millin, following Viscenti, considers this instrument to be the *harpe*, or falciform weapon peculiar to Saturn and Perseus. This assumption is, however, quite gratuitous on the part of both. The sculptured instrument is blunt on the inner edge, and square at the broader extremity; whereas the *harpe* of Saturn is invariably represented as being sharp on the inner edge, and terminating in a point, (Pl. II. fig. 5;) while the *harpe* of Perseus (Pl. II. fig. 4) was a poetical combination of the sword and the Saturnian weapon, having a falciform projection at one side of a straight blade, and bears not the least resemblance to the sculpture. The sculptured instrument is, on the contrary, identical in shape with weapons represented in the hands of certain figures in the collection of Egyptian monuments published by Signor Rosellini; and these weapons are manifestly clubs, (Pl. I. figs. 10, 11.) It is clear, then, that the weapon in the sculpture is a *Clava*. That it is also intended to represent the particular Clava of Hercules may be inferred with a pretty strong degree of confidence from the accompaniments. It was a favourite practice with ancient artists to represent the influence of love over the sterner deities; as in the case of Mars, by young genii playing with his sword and helmet; in that of Jupiter, by their sporting with his thunderbolt; but particularly, and as a favourite study, they shewed the triumph of the softer passion over Hercules, by Cupids represented masquerading in his lion's skin, or toiling under the weight of his club. An inspection of any collection of ancient gems will give abundant evidence of the favourite character of the subject among classic artists. That the weapon in the sculpture is, therefore, the *Clava* referred to by Isidore, appears, on these considerations, highly probable.

There remains, besides, the practical test. If this weapon truly represent the club of Hercules alluded to by Isidore, an instrument formed on the model of it will exhibit the peculiar flight of the Cateia. The experiment has been tried, and the practical result confirms every induction drawn from the written testimony. Such an instrument exhibits the reciprocating flight almost, if not fully, as perfectly as the regularly shaped Boomerang. Indeed it is identical in shape with one variety of the crooked implement at present used by the inhabitants about Swan River, (Pl. I. figs. 5, 6, 8.)

It may, therefore, be concluded of this famous weapon, that the knotted *fustis* of ancient monuments is only its poetical representative; but that the true shape of the Herculean club, as understood by Festus, Isidore, Ælfric, and Johannes de Janua, is found in one variety of the Boomerang.

This conclusion is further corroborated by the fact, that a reciprocating flight has been ascribed to the weapon of Thor, who, it is well known, represents Hercules in northern mythology. "Lock gave to Thor a hammer, (says the Edda,) which he told him would be serviceable in combating giants; that it would never miss its mark; and that, though it should fly never so far off, it would return forthwith into his hand as often as he threw it." "Hammaren gaf hanu

Thor og tuad ai mundi bila throl ad lista og if hann yrpi hunum til tha mundi hann aldri missa og aldri fliuga so longt ad ei mundi hann soetia hond heim."—(*Edda Mythologica* lix. *Apud. Stephan. in not. uberior. in Sax. Gram.*) The name of this weapon was "*Miolner*," which means "the crusher," and with it Thor accomplished labours quite as wonderful as those of his southern prototype.

Now this weapon, although called *Clava* by Saxo, appears to have been regarded as of a hammer shape from a very early period; for it is related by Snorro, that when Haco, one of the first Christian kings of Scandinavia, was presented by his pagan subjects with the horn of Odin, and made upon it the sign of the cross, Sigurd, one of his counsellors, excused the apparent profanation, by telling the people that this was the sign of Thor's hammer, which the king had drawn upon the sacred vessel.—(*Snorro Sturl.* l. iii. c. 18.) Accordingly it is found that a T, or hammer-shaped instrument, exhibits the peculiar flight of the Cateia in a very perfect manner. The cross on many Scandinavian monuments, of an age apparently anterior to the introduction of Christianity, has been long since conjectured by Keysler and others to be a representation of this instrument. Hence it appears very probable that those double crosses which appear on the British coins of Cunobeline, and the single crosses in the hands of some of the Anglo-Saxon kings, (see *Ruding*,) are intended for weapons of the same description, (Pl. II. figs. 7, 8;) especially as it is found that instruments formed on the same model exhibit the reverse flight equally well with the common Boomerang; and as the tradition of cruciform missiles, called cριοραċ, having been used in war, is still preserved in some of the older Irish remains relating to Cuchillin and the Finns. It is true there is no peculiar flight ascribed to these weapons in the romances, at least so far as has been ascertained; but it is a remarkable fact, that the throwing of wooden crosses, having all the properties of the Boomerang, became a general amusement among the children of the lower orders here, immediately after the first introduction of the Australian instrument; and that this practice cannot be traced to any inventor among them, but appears to have sprung up spontaneously, as the revival of something that had been long disused, but was not altogether forgotten.

The ascertaining of these varieties in shape may, perhaps, prove useful in furnishing data for an investigation of the law which governs the flight of such missiles. For although, generally, any flat lamina, dismissed with a rotatory motion, will descend nearly in that plain at which its projectile force leaves it, and will, therefore, if projected upward, exhibit a reverse flight, yet the peculiar ascending flight of the Boomerang is found only to belong to varieties of the curved or angular instrument. To the property first alluded to, Plutarch appears to refer, in a remarkable passage in his inquiry, "Why the Pythian ceased to deliver her cracles in verse," a passage which would lead to the supposition that he had himself witnessed the flight of some such missile. "For," he says, "as the whirling of bodies that fall circularly downwards is nothing violent, but when upwards, forced by a preternatural circumgyration and whirlwind violence, two curling impetuosities become incumbered in one irregular circumrotation; so that divine rapture which is called enthusiasm," &c.—(*Phillips's Translation.*)

Here, again, as in the classic tradition, the evidences are accompanied by such marks of uncertainty, as imply a source in the most remote antiquity. Thus, while the form alone of the crosses of the Irish romances has been noticed, the peculiar flight, which ought to have been attributed to them at the same time, is transferred to a fabulous javelin like that of Cephalus, called the ꞅlaꞇ luᵹaıꝺ, or spear of Lewy, which is the subject of other and separate legends.

Thus, also, in Scandinavian history, the property of the hammer of Thor must have been considered fabulous in the time of Saxo, who regards the similar flight of a javelin as something preternatural. In describing a battle between Hacquin and King Harold Blaatand, he gives the following account. "A wondrous prodigy suddenly befel in the fleet of Hacquin; a javelin was observed to fly overhead, with so irregular and wandering a course, as to fill the minds of the beholders with no less awe than astonishment: for, carried hither and thither, with uncertain doublings, (in diversas partes dubiis reflexibus agitatum,) it appeared to be exploring a place for inflicting its wound. Which miraculous sight, while all were gazing at in horror and suspense, uncertain what a circumstance so extraordinary might portend, descending suddenly, it transferred the common danger to the sole head of Hacquin. Some think it was Gunnilda, the mother of Harold, who had procured the javelin by witchcraft, and thus took vengeance on the conquerer of her son."—(*Sax. Gram.* l. x.)

VII.—OF THE REMAINING NAMES OF THE CATEIA,—CAIA AND KAILE, AND OF ITS ORIGIN.

Among the different names by which weapons of this species have been so far sought to be identified, viz. Cateia, Teuton, Aclys, Ancyle, and Clava, there is none which approaches either of the appellations by which the Australian instrument is at present known. Now, however, that the close connexion of the crooked implement and club has been established, the following very remarkable testimony of Cluverius, regarding the latter, may be adduced. "The club," he says, "is still the only weapon known to many nations of the new world. Where Horace has called it *Caia*, as Isidore states, I cannot tell. This, however, I know, that at the present day, the Lusatians, a Sclavonic nation of Germany, call the club *Kai*; and that the Poles, also a people of Sclavonic stock, call it *Kiy*; but the Germans call it Kaile, and Keile, and Kiele, according to their different dialects: and whether these be all of one and the same original, I know not."—(*Cluver. Germ. Antiq.* p. 304.)

With regard to the apparently corrupt passage from Isidore, Lipsius well suggests, that for "Horatius," we should read "Dorcatio," a lost writer quoted elsewhere by Isidore. That *Caia*, the Latinized *Kai* of Cluverius, is the true reading, appears beyond question, whoever the writer may be that Isidore refers to. As to the meaning of *Kai*, it seems to be the radix of the entire family of words hitherto investigated, and to signify essentially something crooked.

Kay and *kayol* are the Welsh *cavus*; *kae* is the German *ballium*, or circular enclosure; *key*, *jetty*, and *wharf* have all their origin from verbs, of which *torqueo* is the common equivalent: hence it might, perhaps, be inferred that καὶ in the Greek has the same force as *vau* in the Hebrew, the *link*, namely, by which one part of the subject is connected with the other.

As to the *kiele* of Cluverius, it also is clearly of the same stock; we have it in our *keel* of a ship; the *ceola*, or curved vessel itself, of the Anglo-Saxons; the *galleon* of the Spaniards; and the English *yawl* and *galley*. We have it in the Latin *qualus*, and Welsh *kailh*, synonymous with the Irish *kliav*, the Belgic *kit*, and the Latin *cat*ina and *lan*cea, in all which, the same signification is conspicuous. Without a needless accumulation of examples, *kiele* may be taken as likewise descriptive of a crooked weapon; and when it is considered that this appropriate name is almost identical with the word *kiliee*, at present used by the natives about Swan River to indicate the same weapon, it cannot but excite speculations of great interest. And, wide as the difference is between the cultivated Germans of the present day, and the savages of Australia, it may not, perhaps, be too much to hope that this very striking point of coincidence may yet lead to the development of a perfect link between this widely and long separated race, and their kindred of the human family elsewhere.

We are now in a condition to form a conjecture as to the origin of the weapon. We have seen the strait *fustis* bent into the crooked *clava ;* then flattened, and used as a reciprocating missile : the elongation of the shorter limb of this clava would give the perfect *Cateia*, "Eminentibus hinc et hinc acuminibus," and thus the Boomerang would appear to be immediately sprung from the first offensive weapon used by man. Its place in the order of the invention of other weapons may be now investigated.

VIII.—OF THE COMPARATIVE ANTIQUITY OF THE BOOMERANG AND SPEAR.

It is a remarkable fact, that the names of the straight spear, in various languages, are either identical or radically connected with the names under which the weapons of this family have been hitherto found.

Thus, identical with *Cateia* are the straight Tudesque *Cateiæ* of Servius, "*Cateiæ* autem lingua Teuthisca hastæ dicuntur," (*Serv. in Æneid.* l. vii. v. 741 ;) the straight Persian *Cateia* of Johannas de Janua, "Cateia telum dicitur lingua Persarum et ut dicunt, lancea vel hasta," (*Catholicon ;*) and the *kadúayu* of Lhuid, which is the word still in use among the Welsh for a straight spear. To these may be added, as evidently looking to a like origin, the *chæts* of the Hebrews, the *kad*mos of the Cretans, (*Megisser*,) and the *got* of the Irish, all having a like signification.*

* An instance of the use of the word *Cateia* in the sense, as there would appear reason to believe, of a straight projectile, is furnished by the poem of Abbo, "De Obsessâ a Nortmannis Lutetiâ Parisiorum," printed with the works of Aimoinus, "De Gestis Francorum." The siege described in the poem took place in the year 885, and Abbo was an eye-witness. The word *Cateia*

The only names of the straight spear which appear to be connected with the word *Aclys*, are the Latin *jaculum*, and the Sclavonic *pikkel.* To the radix of *Ancyle* are clearly referrible *λαγχος, εγχος, εγχεια*, among the Greeks, and *lancea* among the Gauls. It is worthy of remark, that *εγχος* has been applied to the sword, and that the Latin equivalent, *ensis*, properly means the curved cimeter.

Clava, also, is a name common to the two classes of weapons, *glavea* in the old Latin signifying a straight spear : to this also, by a return to the original element *cam*, (from which *clam* and all its derivations are formed,) may be referred the Irish *gavla*, and the English *javelin.*

Identical with *Caia* are the Irish *gai*, the Welsh *guayu*, and the Cornish and Armorican *guayu ;* and perhaps to the same root may be traced the Gaulish *gæsa*, the Irish *keis*, and the German *speiss.*

The Irish *cuaille*, signifying a straight javelin, is in like manner identical with one of the present names of the crooked weapon ; and it is not impossible that, as the Oscans and Æolic Greeks said *pedor* for *quatuor*, *pilum* itself may be a form of *kilum*, especially as we find several words of this family, *pile*, *pole*, *pill*, *pale*, *pail*, for example, applied indifferently to signify a straight instrument, and a spherical body or vessel.

occurs in three different passages : first, where Abbo, personifying one of the towers of the city, represents it looking abroad over the hostile array brought against it.

> "Prospiciens turrisque nihil sub se nisi picta
> Scuta videt, tellus ab eis obtecta latebat :
> Inde super cernens lapides conspexit acerbos
> Et diras, ut apes, densè tranare Cateias."

l. i. p. 409.

Again, in the same book :

> "Pila dabat, rupesque simul, celeresque Cateias
> Plebs inimica deo."

l. i. p. 416, G.

And again, in the second book, speaking of Count Otho, one of the defenders :

> "Fossata volatu
> Transiliit propero, clypeum gestensque Cateiam."

l. ii. p. 419, C.

Hence the inference seems unavoidable, that, as the same names, and names radically connected, are found applied to those two distinct classes of weapons, and as these names are all radically and properly descriptive of the one class, but not at all descriptive of the other, that family of *crooked* missiles, the characteristic names of which are thus applied to the family of straight projectiles, must necessarily have been the older of the two, and the other must have originated from it. In other words, we must conclude that, as the club appears to have been the parent of the Boomerang, so does the Boomerang appear to have been the parent of the spear.

This conclusion, startling as it is, receives further confirmation from the fact, that the invention of the spear has been attributed to the Etruscans, who, although a very ancient people, were never looked on as the *aborigines* of their country; and it is very remarkable, that the name *coris*, which they are stated by Festus to have given to the weapon, (the *quiris* of the Sabines,) is so evidently associated with the idea of curvature, that the *Quirites*, *Curetes*, and *Corybantes*, have been argued to be the same, on the common affinity of these titles with the *χορος*, or circular dance of the priests of Mars.—(*Pezron*, *Antiq. of Nations*, c. iii.) Again, this *coris* of the Etruscans is one of the few words of their dialect which correspond with any part of a known language, being clearly identical with the Irish *corr*, still signifying a straight spear, and hitherto offering an unaccountable anomaly, as being the only one of a very numerous family which is not palpably applied to something curved or circular. For example, the Ordnance Survey of Derry contains a list, from O'Brien's Dictionary and Cormac's Glossary, of upwards of thirty words having *cor* for their radix, every one of which involves this peculiar meaning. A few of the most striking are subjoined.

> " *Cor*, a twist, a round or circular motion, a round hill; Latin, *curvus*.
> *Cór*, a choir; Latin, *chorus; chorea*, the circular dance.
> *Cor*, a round pit of water.
> *Coire*, a cauldron, a whirlpool.
> *Caor*, a berry.
> *Cuar*, crooked.
> *Corran*, a reaping hook," &c. &c.
>
> *Ordnance Survey of Londonderry*, p. 212-13.

IX.—OF THE TRANSIT OF THE NAMES OF THE CURVED MISSILE TO THE STRAIGHT WEAPON.

It has been seen that the *Aclys* and *Ancyle,* two varieties of the curved weapon, were thrown by means of an *amentum* or attached thong; and that the *Clava,* also, was thrown in this manner, appears from various representations both of the straight and crooked club having such an appendage, (Pl. II. fig. 11.) Now this, also, was the mode in which several varieties of the spear were thrown among the ancients, and in which a species of it is still thrown among the Australian savages.—(*Cooke's Voyages towards the South Pole.*) The word *lancea* itself has been derived by Isidore from this peculiarity, and εγχος *quasi* λαγχος is a received etymology for the Greek weapon. The *tragula* appears to be so thrown in Cæsar, (*De Bell. Gall.* l. v. cxlvi.); and the frequent allusions of other classic writers shew that the *amentum* was an usual appendage to the spear in general. Hence there would appear a probability, that the common name may have passed from one weapon to the other, through the medium of the common apparatus by which both were thrown; a probability which is considerably increased by the fact, that the amentum itself among the Greeks was also called αγκυλη, whence their μεσαγκυλον, or spear thrown by the *ancyle* attached to its middle. Whether this have anything to do with the *vinculum* of the Latins; and whether *aclys* may, in like manner, have given name to the Belgic *schacckel,* our *shackle; cateia* to the Belgic *catte,* a chain; and *caia* to our *guy,* or attached rope, I leave to the consideration of the curious.

X.—OF THE MODES OF THROWING THE CATEIA, Etc., AMONG THE ANCIENTS.

Whatever uncertainty may attend this portion of the inquiry, it is certain that the curved weapons under consideration were thrown by divers apparatus; and a consideration of what can be collected respecting these may, perhaps, furnish some practical hints towards devising similar appendages to the weapon as we have it at present.

The passages always quoted shew that the *Aclys* was thrown by means of a thong, and the expression of Servius, "teli genus quod per flagellum in immensum jaci potest," proves that this was not used in the retraction of the weapon, but must either have left the hand of the thrower along with the Aclys, or have been used as a sling, from which it may have been let slip, when it had acquired sufficient velocity. A horseman is represented on one of the British coins given by Ruding, (Pl. II. fig. 9,) who appears to be whirling an instrument of this sort round his head by a similar appendage. The same collection, also, affords a curious illustration of the use of the cruciform missiles already alluded to, (Pl. II. figs. 7, 8.) Here the ancient Briton is represented throwing his *criosach* from a sling, such as we may suppose Cuchullin, and the other heroes of Irish romance, to have done. The sling appears to be attached; but from the application of the epithet "eyed," or perforated, to the weapons of the Irish poems, there is reason to suppose that the artist intended to represent the missile here as on the point of slipping from the extremity of the thong.

Another apparatus used in hurling the *Clava*, if we are to credit the testimonies of northern mythology, was a haft or *manubrium*. It was by means of a haft of this sort Thor threw the *miolner;* and the efficacy of the apparatus is attested by various mythi, one of which, preserved by Saxo Grammaticus, gives the following characteristic account of a battle between Balder and Hother, in which a band of the Scandinavian deities took part with the former. "Then might be seen a battle waged by human against divine belligerants; for Hother, grit in his impenetrable mail, fearlessly assailed the thickest battalia of the gods, doing all that mortal man might against immortals. But Thor, upon the other hand, with such whirls of his club as had not been experienced till then, (*inusitato clavæ libratu,*) swept through every obstacle presented against him. There was no armour which did not yield before his strokes; no warrior who could sustain them, and live. Down went all he touched, the hurled oak bursting through helmet and shield. Bulk of body, and stoutness of heart, alike availed not. Then, indeed, the victory had fallen to the gods, had not Hother, perceiving the day to go against him, run, and rendered the club useless by cutting off its haft, (*clavam præciso manubrio inutilem reddidisset,*) deprived of which weapon, the gods betook themselves to sudden flight," &c.—(*Sax. Gram. Hist. Dan.* l. xvi.)

Now, it is stated in the Edda, that among the most precious things possessed by Thor was his *gauntlet,* which he always put on when he would throw the *miolner.* And there appears some probability that this and the *monubrium* of Saxo are one and the same, for the *haft* is not mentioned in the Edda, nor the *gauntlet* in the works of Saxo, while both describe the *miolner.* If so, it might, perhaps, be inferred that this was a sheath not for the protection of the hand, but for the reception of one limb of the weapon; and hence it is suggested, that an elastic haft, having a sheath attached, might also be found serviceable in throwing the Boomerang.

Many of the foregoing inferences will, doubtless, appear in a high degree speculative; and the writer is conscious, that, in pushing the inquiry in some directions to the length it has gone, the bounds of strict induction have been very closely approached; still it is submitted, that if the first step of the argument, namely, the identification of the *Cateia* with the Australian weapon, have been taken on sure ground, it will not be possible to stay the subsequent progress of the inquiry. And, that this step has been taken with great, indeed with extraordinary, certainty, appears as well from the minuteness with which all the peculiarities of the weapon in question are described in the passages already quoted, as from the fact that *unquestionable representations of the Boomerang* are found on ancient monuments. The representations in Pl. II. figs. 1 and 2, taken from Sig. Rosellini's "Egyptian Monuments," cannot be mistaken; and the reader who will take the trouble of referring to Mr. Wilkinson's work on the same subject, will there find still further confirmation of the acquaintance of this most ancient people with the very implement in question. In the latter instance, parties are represented throwing missiles of a form which, from experiment it is now certain, must have produced a reciprocating flight, at birds, reminding us strongly of that passage of Strabo, (l. iv. pp. 196, 7, *Ed. Causab.*,) where he describes the Belgæ of his time as using "a wooden weapon of the shape of a *grosphus*, which they throw out of the hand, and not by means of an *ancyle*, and which flies faster than an arrow, and is chiefly used in the pursuit of game." So, also, it is difficult to assign any other use to the instrument appearing in the hand of the Belgic Briton represented in Pl. II. fig. 6.

If any certainty could be had that the notices so far collected were all that antiquity could furnish on the subject, a new and very wide field of speculation, of perhaps a still more interesting character, would be opened, in the endeavour to trace the international resemblances between those people known to have used such weapons in the old world, and the tribes who still retain the use of them in the new. Even on the scanty materials here brought together, there is, however, sufficient to excite serious attention, in the fact, that amongst the ancient nations using the *Cateia* and its cognate weapons, certain peculiar characteristics are distinctly traceable, such as the prevalence among them, from the earliest periods, of Amazonian habits, and their being in almost every instance of the *white* variety of mankind, and of the *Xanthous* family of that variety, characteristics which point, in a very marked manner, to an Indo-European origin.

Now, there are in Australia two distinct races of men, one of which is clearly of the white variety, as appears from the coloured drawings which accompany M. Peron's Voyage to Van Dieman's Land and New South Wales, in 1824. What, then, shall we say? Has the European or Indo-European weapon, with its characteristic name, been introduced into Australia by these lighter-complexioned islanders; and are these far-separated savages members of the same great Japhetic stock of whom we have this testimony from the oldest and most authentic of human records, "By them were the isles of the Gentiles divided."—(*Gen.* c. x. v. 5.)

III. *On the Egyptian Stele, or Tablet.* By the Rev. Edward Hincks, D. D. (*Communicated by the President.*)

Read June 28, 1841.

Of the Egyptian monuments that are collected in European museums, there are none which ought to attract more attention than the *steles*, or funeral tablets; and yet I suspect that there are none which are more generally overlooked. They are certainly not so well calculated to arrest the attention of the uninitiated observer as many other objects; but they are much more likely to afford information. They in general record facts; and it not unfrequently happens that the facts recorded throw light on the history of the country, or on the state of society in it. Sarcophagi, on the other hand, mummy cases, sepulchral figures and cones very seldom determine any thing but the name and parentage of the deceased person whom they commemorate. The copious inscriptions, with which the former are often covered, contain merely extracts from the Ritual, or other general formulas, in which the names and offices of the deceased and of his parents are alone peculiar. There are some scarabæi, on which historical facts are recorded, and which are somewhat of the nature of medals. There is one, for example, in the museum at Liverpool, of which there is a duplicate at the Louvre, which records the name and parentage of the Queen of Amenothph III., and the northern and southern limits of his kingdom. These were probably sculptured in considerable numbers on the occasion of the marriage of that prince, which must have taken place when he was a mere child;* and which was in all probability an important political event, as transferring the actual government

* At the death of his father, this Amenothph and a twin brother, who shared with him the nominal sovereignty, were infants in arms; yet the scarabæi recording his marriage, are dated in the eleventh year of his reign.

from his mother to his wife or her father. There are other scarabæi of a similar nature; but the great majority of them are funereal, containing the name of a deceased person (or sometimes a blank for a name, the scarabæus having never been appropriated), followed by a speech from the Ritual respecting the heart of the speaker. The tablets, on the contrary, though essentially funereal, and containing much that is of a general nature, have, for the most part, a great deal which is peculiar to the deceased person. In this, they resemble our tombstones; and it is curious that they are of the same shape as those which we set up at the head of graves, and that they were set up in similar positions. Some tablets mention the King of Egypt whom the deceased person served, and the capacity in which he served him; some record the more important events in his life; some are dated either in the body of the inscription or at the top of the tablet, with the year of the king's reign, and often with the month and day of the month; and in some rare instances (would that they were more frequent!) the dates of the birth and death of the deceased person and the length of his life are all stated. I am aware of but two such tablets; but among the many which are in existence, that have not yet been examined, it is likely that there are others; and the immense importance of such tablets, which are probably the only means*

* Another means of equal value would exist, if we had records of the years of kings' reigns, in which the cyclical panegyries were held. These panegyries occurred at intervals of three years; ten of them forming a series, the τριακονταετηρις of the Rosetta stone. A tablet has been found at Silsilis, stating that a certain person presided over the first or grand panegyry in the thirty-first year of Rameses the Great, the second in his thirty-fourth year, the third in his thirty-seventh year, and the fourth in his fortieth year. Another tablet records that another individual presided over the sixth panegyry, in the forty-sixth year of the same king. Any of these records would prove that the first year of Rameses the Great was the first year of a τριακονταετηρις; and, of course, if the principles which I have endeavoured to establish elsewhere be correct, in a year B. C. of the form 1767—30 k. If now a record should be found of any given panegyry of the series occuring in any given year of any other king, the exact interval between the commencement of the two reigns could be determined from an approximate interval. Suppose, for example, that a record should be found of a grand panegyry occurring in the twenty-sixth year of Amenothph III. Knowing that the commencement of his reign was above 100 years before that of Rameses the Great, we should infer, that the interval between his twenty-sixth year and the first of Rameses, was ninety years; and, of course, that the interval between the beginnings of the two reigns was 115 years. Unfortunately, with the exception of the two tablets at Silsilis, I believe no record of this kind has been discovered.

by which the chronology of the Egyptian kings can be settled with accuracy, renders it highly desirable that they should be sought after.

In order to show the utility of tablets of this description, I will enter into some details respecting the two that are known; and I am the more disposed to do this, because a false inference has been drawn from one of them, and I believe the other has not been noticed by any one conversant with hieroglyphics.

One of these tablets, which is in the museum at Florence, records, that a person named Psammitich, was born in the third year of Necho, the tenth month, and first day; that he died in the thirty-fifth year of Amasis, the second month and sixth day; and that he lived seventy-one years four months and six days. When this tablet was first noticed, it was carelessly stated, that it counted seventy-one years from the third of Necho, to the thirty-fifth of Amasis; and from this it was inferred that there were thirty-nine years between the first of Necho and the first of Amasis. If, however, we take into account the months and days, we shall see that the true interval was forty years. This interval comprehends the reigns of three kings, the joint length of whose reigns is stated by Herodotus to be forty-seven years; by Africanus, from Manetho, to be thirty-one; and by Eusebius, who professes also to follow Manetho, to be forty-eight. We may judge of the degree of credit due to the Greek authorities by the gross blunders which they have, all of them, been detected in making, in this instance, where the truth is known from a cotemporary monument. We may likewise test their accuracy by the length of reign which they assign to Cambyses in Egypt. Herodotus, Diodorus Siculus, and Eusebius, are all agreed that he conquered that country in the fifth year of his reign; and of course that he reigned over it only three or four years. Africanus alone gives him a reign of six years;* but in this he is corroborated by the express testimony of a cotemporary monument,

* Καμβύσης ἔτη ε̉ τῆς ἑαυτοῦ βασιλείας Περσῶν ἐβασίλευσεν, Αἰγύπτου ἔτη ϛ'. So the text of Africanus exists in all MSS. and editions; but for ε̉ I would read θ'; correcting a mistake, into which a transcriber might easily fall, and rendering the statement perfectly consistent with truth. I would also correct the text of Africanus, by substituting ιϛ' for ϛ', as the length of reign of Necho II. This makes him agree as to the length of that reign with Herodotus; and as to the sum of the three reigns with the Florence tablet; for, where reigns are reckoned by complete years, months and days being neglected, the sum of sixteen, six, and nineteen years may be very well reduced to forty.

published by Mr. Burton;* and also by an obvious inference from the narrative in 2 Kings, xxiii, taken in connexion with the tablet above mentioned. Necho was king of Egypt before the death of Josiah, in 610, B. C.; but this could not have been the case, if Cambyses had only conquered Egypt in 525, B. C., as Amasis only reigned forty-four years, and Necho and the intermediate kings only forty. The true date of the death of Amasis, and of the conquest of Egypt by Cambyses, must therefore be 527, B. C.

The other tablet to which I have alluded is of the Ptolemaic age; and its dates are useful, not in determining the chronology of the reigns, which is already known from other sources,† but in ascertaining the power of a numeral character, which occurs for the first time in inscriptions of this age; and in determining to which of the Ptolemies a cartouche with certain titles belonged. This tablet belongs to Mr. Harris, of Alexandria, and it has been published by Mr. Sharpe, in the seventy-second and seventy-third Plates of his Egyptian inscriptions.

The person commemorated by this tablet was a priest at Memphis, named Psherin-phthah, son of a priest, who held a very high sacerdotal office, the name or precise nature of which I have not yet been able to ascertain. He is said to have been born in the x + 5 year of a Ptolemy, whose cartouche is

I have used the letter x to represent the unknown numeral, a bird's head, which is here accompanied by five vertical lines. He was born in the second month of this year, on the twenty-first day. When he was thirteen years old, his father died. He was promoted by Ptolemy "the new Osiris" (the Neo-Dionysus of the Greeks), in the tenth year of his reign, to the sacerdotal office which his father had held. After he had completed his forty-third year, he had his first

* An Egyptian functionary is said to have served under the Persians for *six* years of Cambyses, thirty-six of Darius, and twelve of Xerxes.—Burton Exc. Hier. 8.

† It appears from Ptolemy's canon, that the first year of Lathyrus was the 632nd of Nabonassar; the first of Neo-Dionysus, the 668th of Nabonassar; and the first of Cleopatra, the 697th of Nabonassar. Alexander's first year was 635, when his brother Lathyrus was driven to Cyprus; and the latter was restored to the throne of Egypt about 660.

son, who was named Imothph. He died, aged forty-nine years, in the eleventh year of Cleopatra and her son Cæsar, the eleventh month and twentieth day; and he was buried in the twelfth year on the thirtieth day of the first month. The usual interval between the death and burial was seventy days, and we see here that the Epagomenæ were not counted, being strictly *dies non*. His death took place, as appears from Ptolemy's canon, at the close of the 707th year of Nabonassar; and as he lived about forty-nine years, and was born at the beginning of a year, the year of his birth must have been the 659th of Nabonassar. This was the 25th year of Alexander, and certainly before the restoration of Lathyrus; as there is a papyrus at Berlin (Kosegarten, Plate XII.) dated in the twenty-sixth year of Alexander, the fourth month and nineteenth day; it is therefore certain that the cartouche above given, belongs to Ptolemy Alexander, though it does not contain his surname;* and that the bird's head, when used as a numeral, signified twenty.

It is a curious circumstance, that the tablet of the wife of this person, who was also his half-sister, is in the British Museum. It has been published by Mr. Sharpe in his 4th Plate; and by combining the information which the two tablets afford, we obtain much insight into the history of this family, which is perhaps not a bad illustration of Egyptian family history in general. It appears, that, after the death of the father of Psherin-phthah, his mother, Ho-onkh, married another priest named Hapi, by whom she had a daughter, Te-imothph, and a son, Imothph, who survived his half-brother and sister, and erected both their tablets. The first husband died when his son was thirteen years old, and therefore in the fifth year of Neo-Dionysus. Five years after, in his tenth year, and in the fourth month, Te-imothph was born; and in his twenty-third year, and in the eleventh month, she married her half-brother. The birth of their son, Imothph, is recorded as having taken place in the sixth year of Cleopatra, and in the eleventh month, just twelve years after her marriage; and she died in the tenth year of Cleopatra, the eighteenth day of the fifth month. Her age at her death is not stated on the tablet; but it must have been twenty-nine

* Unless indeed the sculptor committed the mistake of using the cartouche of the exiled, but afterwards restored king, *de jure*, instead of that of the intrusive king *de facto*. He might easily have done this after an interval of about fifty years.

years and a few weeks. By comparing the dates of the births of her son and of his father, the interval between them is found to be forty-three years and eight or nine months. This accords with the statement on the father's tablet: "I lived forty-three years before a son was born to me." Whether he had or had not daughters previously, is not stated. As they could not fill his sacerdotal office, the existence of such would be considered unimportant. That office was not strictly hereditary; for it appears from this tablet, that it was conferred by the sovereign. It is probable, however, that if it was not conferred, as a matter of course, on the heir of the former possessor, as soon as he attained a suitable age, it was limited to the members of a few particular families; and a desire to preserve the purity of the priestly stock, as well as to prevent it from becoming too numerous, may have led to such unnatural marriages as that of Psherin-phthah and his sister. Similar marriages were, however, common among all ranks of the Egyptians. It appears that the sacerdotal office, whatever it was, was conferred on this person, when he attained the age of eighteen. This may have been the age at which he was considered capable of filling it, and it may have been kept vacant for him; but it is also possible that it may have been held in the interim by some other person, on whose death it reverted to the son of the former incumbent.

In the remainder of this paper, it is my intention to resolve the inscription, which usually occurs on these tablets, into its several parts. I will treat of all these parts in succession; pointing out, as I go along, the *criteria* derived from each, by which the age of undated tablets may be ascertained; and likewise directing the Egyptian student to the parts in which he is to look for information respecting the person commemorated.

The following is the skeleton of an inscription in the most usual form:

D  C  B Pronoun, 3rd person.  A 

which I translate: "An act of homage to A; he has [*or, as the case may be*] given B unto C, who says D." The blank at A is filled up with the names and titles of deities; that at B with an enumeration of gifts; that at C with the name and description of the deceased person; and at D is the speech attributed to him. Sometimes the tablet is without a speech, the inscription closing at the end of C; and sometimes it begins with C, containing only the name and description of the deceased person and his speech. In a few tablets, the prefatory matter before C is somewhat different from the above; but the form given above is much the most usual.

I now remark, in the first place, that no record of facts, and, in short, nothing which would not answer equally well for any tablet, is to be expected till we come to C. The part before this is only valuable, as it may aid us in the study of the language, and as it may lead us to know the age of the tablet, supposing it to be without a date. To assist in this, I propose the following criteria, the result of a careful examination of a great number of tablets of known age.

1. If the lowermost of the two central introductory characters be omitted, the semicircle being placed over the triangle, the tablet may be presumed to be of the *most remote* antiquity. This is the case in the tablets, which have been found in the neighbourhood of the pyramids, and which bear the names of their builders, Cheoph (חוּף) and Kephren (כעפּרע). But if the introductory characters, being all present, be grouped in a different manner from what I have represented above, the tablet is not of *very great* antiquity. I speak, of course, comparatively. I mean, that I have met with no tablets, in which the initial group was differently arranged, which there was any reason to suppose anterior to the so called eighteenth dynasty.

2. If the initial group be followed by the preposition N *to*, the tablet can have no pretensions to antiquity: it is probably Ptolemaic or Roman.

3. If the names of more than one deity are combined in the space A, the tablet is not of the *most remote* antiquity. The earliest dated tablet, in which I have met this combination of divine names, is of the thirteenth year of Amenemhe II., the king whose cartouche was the first on the second line of the tablet of Abydos, at the time when that tablet was first copied. It has since, I believe, been broken off. If more than one deity be mentioned in tablets more ancient than this, the initial group is repeated for each; being, however, sometimes mutilated at its commencement for all after the first.

4. The mention of Osiris-Apis, or Apis-Osiris, the Serapis of the Romans, among the deities enumerated in A, is a proof that the tablet is Ptolemaic or

Roman. I do not think that any other inference can be safely drawn from the names of deities introduced.

5. The mode of writing Pente-pamente, a common title of Osiris, which occurs very frequently in A, furnishes more than one criterion. The use of a nose (the old Egyptian name of which was Phente) for the former part of this title was not introduced till the latter part of the eighteenth dynasty; and it is, of course, a proof that the tablet on which it occurs is not of *very* great antiquity. In the most ancient tablets, but not in them exclusively, this is written

 PNT ;

which is often reduced by abbreviation to the first character, a combination of water jars; either alone or with the small semicircle, which so commonly accompanies a single character when it stands for an entire word. The use of the square for P in this word is, comparatively speaking, modern.

6. The absence of a bird from the usual group representing Amente, whether in this title or elsewhere, is a proof that the tablet is not more ancient than the middle of the eighteenth dynasty. Anciently, the group without the bird, or the single character to which it was frequently reduced, signified "the west;" and the bird restricted the signification to "the divine west," or "the west of souls," that is, the Amente or Hades. About the middle of the eighteenth dynasty the bird was omitted. I have observed that, during a short interval of time previous to its omission, it had the usual sign of the plural number annexed to it. Should the word Amente occur on any tablet in that particular form, I should scarcely hesitate as to its being of the reign of Amenothph II., or one of his immediate predecessors or successors.

7. The omission of the connecting verb between A and B is, I think, a positive proof that the tablet is very ancient. We must not, however, conclude, that the insertion of the verb is a proof of the contrary; as it is found in tablets of the earliest age. The fact seems to be, that so long as the initial characters were grouped in the *primitive* manner (see 1), they might be translated in two ways; either "an act of homage for bounty to A," in which case the verb and pronoun were required before B; or "an act of homage; A has given," in which

case B should follow at once. It is well known, that the subject of an Egyptian verb, whether noun or pronoun, was always placed after it.

The connecting verb is followed by the pronoun of the third person, required by the contents of the space A. If a single male deity be there mentioned, the horned serpent, corresponding to the Hebrew ו, is invariably used; if a single female deity be mentioned, one of the usual characters for S is used; and if two or more deities be mentioned, the plural pronoun SN, with three small lines as a determinative sign, is employed. For convenience of grouping, a hand holding a small triangle is frequently substituted for the triangle itself. Thus, we have

 "he has given."

The contents of the space B were supposed by Dr. Young to be offerings to the gods, instead of gifts of theirs to the deceased person; and I believe the nationality of some English antiquarians leads them still to persist in this mistake. That it is such must be evident to any one who admits the first principles of hieroglyphic interpretation, from the use of the preceding verb and pronoun, as just explained. It is also evident from an examination of the contents of B; for, though many things there enumerated may be supposed to be given *to* the gods, as well as *by* them, this is by no means the case with all. We frequently meet among the gifts "a good burial;"—"that he may go in and go out in Noutehir, without being turned back at the gate of the abode of glory;"—"that he may adore the Sun in Heaven; that he may give aid in battle to Sebh upon the earth; that he may speak the truth (i. e. be justified or pronounced righteous) before Osiris in Amente." These are not the kind of gifts that a man would offer to a deity.

It may be asked, why I have translated the verb between A and B in the past tense, rather than in the optative mood. The latter appears more natural; and, as the letter N, the usual sign of the past tense, is not affixed to the verb, I should certainly have preferred "may he give;" did I not feel myself constrained by the authority of the Rosetta stone to adopt the other translation. In the fifth line of the hieroglyphic text of that inscription, we have an expression precisely similar to that in the tablets, in which the N of the past tense is equally wanting; and in the thirty-fifth line of the Greek version the verb is translated in the past tense. This appears to me decisive on the subject. The objection, which may occur to some, that the gifts enumerated were, in part at least, to be enjoyed hereafter, appears to me to have no force; and in truth the same objection might be made against the passage on the Rosetta stone; for among the gifts of the gods and goddesses there mentioned is "a kingdom established to him and to his children for ever." The answer is easy. The gift was past, though the enjoyment of it was future.

8. Very little dependence can be placed on the contents of B as determining the age of a tablet. It may, however, be stated that the abbreviated group,

which I believe means "the appointed nourishment of meat and drink," and which begins B in almost all tablets of the reign of Osortasen I., and of his successors to the very latest period, has not been met with, so far as I am aware, in any tablet of an earlier reign. Before his time the characters for meat and drink were placed after the words Hre taoue, "the appointed provision," or their abbreviation as above given; and accompanied either by a circle, representing a cake of bread, or by a long figure, resembling the prismatic spectrum, representing a number of such cakes. This character, however, is not to be translated in the present instance "bread" or "cakes," but "of all sorts." The Egyptian word having that meaning, being homophonous, or nearly so, with the word signifying bread, is often represented by the symbol for the latter; and it is so, I conceive, in this connexion.

The group which occurs between B and C was naturally translated "for the sake of" by those who imagined that B were offerings to the gods. As the deceased person could not make these offerings himself, they conceived that the survivors made them *for his sake*. It appears to me unaccountable that any should have retained the old translation of this group, who perceived the mistake in which it originated. I take the literal meaning of the group to be "to the receiving of," a compound proposition, more definite in its signification than the single N, which admitted a variety of meanings; and probably also more solemn, as being confined to the forms of religion. The middle character is a pair of arms held up, as if to receive a gift,* which ideagraphically denoted the

* This may derive confirmation from the speech of the ancestors of Rameses II. to that king, at the conclusion of the tablet of Abydos,—"We hold up our arms to receive offerings." It is true,

verb "to receive," and its derived noun; and which also denoted the same verb phonetically, according to the well-ascertained usage of the Egyptians, being the letter K, the first letter of the old Egyptian verb KI "to receive;" whence we have in Coptic ϫⲓ and ϭⲓ. After this character a small vertical line is frequently placed, signifying that it represents a word, and not a mere letter. Compound prepositions of this sort are of common occurrence in the Coptic language; and there are some well-known instances of them in Hebrew.

9. Now, I observe that, though this compound preposition en-ki-en, was substituted for the single preposition en, at a very remote period, it is not so remote a one as that instances to the contrary do not occur. The earliest dated tablet that I have seen, containing the compound preposition, is of the twenty-ninth year of Amenemhe II. In all tablets sculptured in the early part of the reign of this king, as well as in all those sculptured under his predecessor Osortasen I., or any of the preceding monarchs, the simple waved line, en, "to," is invariably used; if, indeed, the preposition be not omitted altogether.

The part of the inscription, which follows this simple or compound preposition, contains the name of the deceased person, preceded by an enumeration of the offices, sacerdotal, civil, or military, which he held, and followed in most instances by the names and offices of his father and mother (or at least one of them), and sometimes of his grandfather or other relatives. It is but seldom that the exact nature of all the offices held by the deceased person can be satisfactorily discovered. We can perceive, however, that the Egyptians in general, and especially the priests, were great pluralists. Occasionally, but very rarely, we meet in this part of the inscription with the name of a king, whom the deceased person served, and even with a fact respecting him of historical interest. Thus, in a tablet of the reign of Thothmos IV.,* belonging to Mr. Harris (Eg.

that the verb here used for "receive" is not ki; but is the equivalent verb chop, חף, preserved in the Coptic ϣⲉⲡ or ϣⲱⲡ, and corresponding to the Latin cap-ere.

* I mean the king, who is called Thothmos V. by Rosellini. The Italian antiquarian has imagined a king of this name, whom he calls Thothmos III., but who had no real existence. Having taken it into his head that Queen Amouneth ente heou, who erected the Karnac obelisks, was the *mother* of Thothmos Mephre, and finding that the name of the father of this king was Thothmos, he assumed the existence of a husband of the queen, whom he called Thothmos III.; and he styled Mephre, Thothmos IV. The fact is, however (as I conjectured in a note to my paper on the years and cycles of the ancient Egyptians, and as has since been completely established), that this queen

Ins. 93), the deceased person is called "the attendant upon the king in his journeys to the southern and northern countries, who went from Naharina (Mesopotamia) to Karai in the suite of his majesty." It is worthy of observation, that these are the identical limits of the Egyptian empire, which are recorded on the Liverpool and Paris scarabæi (as already noticed), in the eleventh year of Amenothph III., the son and successor of this king. This deceased person, whose name was Amenothph, was also "first prophet of Empe" and "superintendent of his Majesty's cattle stall;" and he held another office under the crown, the nature of which I do not understand.

After the name of the person commemorated by the tablet, there occurs very commonly, in inscriptions of all ages, an addition on which I will make a few remarks. It commences with the word Me (מע) "truth," expressed either symbolically, by an ostrich feather or a measure; phonetically, by the sickle and arm, which represent the two component letters of the word; or in both ways combined, the measure or feather, the sickle and arm being all used. This is followed by a club, T, representing the word Taoue, "speaking," the subsequent or complementary letters of which are but seldom expressed. And after this we occasionally meet characters which I consider to belong to the sentence; namely, Chal, (חל) a preposition, answering to the Hebrew אל or ל, "to," and either the name of Osiris, or the two N's, the hatchet and the pike, with which the words Nter, "god," and Naa, "great," are written, and which are commonly used as abbreviations of those words. I would then translate the entire addition, not as Champollion has done "the truth-speaking, *le veridique*,"* but "who has spoken the truth to Osiris," or "to the great god."† This expression I understand in a forensic sense, as meaning "who has been justified, or pronounced innocent, by Osiris." It has been expressly stated by Diodorus, that the president of every Egyptian court of justice wore a badge, which was called Truth,

was *sister* to Thothmos Mephre, and that they were children of King Thothmos II. It is therefore Mephre that we should call Thothmos III.; and his grandson, under whom this tablet was sculptured, must be Thothmos IV.

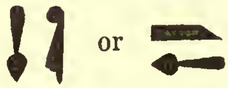
* I do not deny that the two former words would have this meaning, *if they stood alone;* as they do in the prænomen of the successor of Amenemhe III., whose phonetic name has not yet been ascertained, "The sun who speaks truth." But I conceive that in the addition of which I am speaking, the subsequent words, if not expressed, are always to be understood.

† Or as I have observed in one place, "To the lords of the abode of glory."

and which the monuments show us to have been an image of Thme, the goddess of Truth or Justice, who is represented sitting, with an ostrich feather on her head, and a bandage over her eyes. With this figure he touched the successful party in the suit; thus announcing to him that the decision of himself and his assessors was in his favour. This was as much as to say to him that "he had spoken the truth;" that his plea was true. In accordance with this, the unsuccessful accuser, the adversary of the deceased, is called in the ritual "the liar."

Here I cannot refrain from noticing the extraordinary mistake, into which Sir J. G. Wilkinson has fallen with respect to this badge, which he supposes to have been the same as that worn by the Jewish high priest; arguing from the similarity of the words Thme and Thummim. The resemblance between these words is merely apparent, and disappears when we reduce them to the radical forms. The initial Th of the Egyptian word is the feminine article, while the ת of the Hebrew word is radical; and, on the other hand, the Egyptian word has at the end of it a letter having the force of the Hebrew ע, to which there is nothing equivalent in the Hebrew word that has been supposed to correspond with it. The resemblance, then, between the names (מע and תם) it not real; nor were the purposes for which the two badges were worn at all similar.

The addition, of which I have been speaking, which is commonly abbreviated to two characters, such as



appears to belong to deceased persons exclusively; so that it might be translated "deceased," or "the late." It is contrasted with the characters,

which, when they follow the name of a man, imply that he is alive. Thus, on a broken tablet, in the British Museum (Eg. Ins. 27) the person commemorated is called Imothph, deceased, son of Hapi, still alive; and of a deceased mother, daughter of a deceased person, and sister to a living person. It was, however, in most cases, considered sufficient to express that a person was alive, if the characters for deceased were omitted after his name. Now, as these characters are wanting after the names of many persons commemorated on tablets, a question arises, whether these tablets were always funereal; whether they may not, in many instances, have been erected by individuals out of gratitude to the gods, for gifts conferred on them during their lives. That this was the case, in *some* instances, is highly probable; but I would by no means affirm that it was the case whenever the characters expressing death were wanting. It is, however, a question, which I do not feel myself called on to decide. One thing appears to me clear; namely, that the presence or absence of this addition is no criterion of the antiquity of the tablet.

10. It is otherwise with certain prefixes, which are found on very early and on very recent tablets, immediately after the preposition en, or enkien. Tablets of the Ptolemaic and Roman ages, and, perhaps I should add, tablets sculptured under the latest dynasties, have after this preposition the title "Osiris," which is never found on the more ancient tablets. I do not, by any means, intend to deny that it was customary, in ancient as well as in modern times, for the Egyptians to identify deceased persons with Osiris. I am aware that on that most ancient record, the coffin found in the third pyramid, this identification is distinctly made. What I mean to assert is simply this—that the title is not given to deceased persons *on ancient tablets.*

11. On the other hand, a title, which I interpret "the blessed," or "favoured," sometimes followed by a preposition, and the name of a deity, is almost peculiar to very ancient tablets. Instances, may, perhaps, occur, in which this title may be found on recent ones, or in which it may be wanting on ancient ones; but we may infer with tolerable certainty, that if this title be found on the stone, it is more ancient than the reign of Amenemhe III., and if it be not found on it, it is of that or some subsequent reign. I would be understood as speaking with the same qualification as I did with respect to the title Osiris. Deceased persons of all ages are spoken of as "blessed," or "possessed of blessing;" but it is only on ancient tablets that gifts are said to be given "to the blessed superintendent," &c., or the like.

The essential part of the title, to which I allude, is the character,

representing an object unknown to me. How this character came to signify "blessed," I cannot say; but Mr. Sharpe assigned this meaning to it by deciphering; and though I do not often assent to that gentleman's conclusions, I cannot avoid doing so in this instance. It may possibly represent the *idea* expressed by the word "blessed;" but it is possible also, and I think much more probable, that it represents some object, the name of which was pronounced in the same manner, or nearly so, as the Egyptian word for "blessed," or as the first syllable in this word. Along with this unknown character, there occur in this title, when written in full, the leaf, answering to the Hebrew Aleph, and which may be read by any vowel; the sickle M, the sieve CH, and either the pair of leaves EI, or the quail OU. The two latter characters are equivalent to our termination ed; and have the same effect as the corresponding Hebrew vowels י and ו, when placed before the last radical, in the participle Pahul or the verbal noun of the form Pahil. Rejecting then these servile letters, the Egyptian verb consists of three letters אמח, in addition to the unknown character; which I regard as merely *determinative*, unless it be used as a *substitute* for the whole word, or for its first syllable, or for the consonant M. To show the manner in which this peculiar character is introduced, I will set down a number of varieties which I have met with; putting for the common phonetic characters their Hebrew equivalents, and for the peculiar character an asterisk; and, for the sake of comparison, I will do the same thing with the word me, "truth," already mentioned; the asterisk in *it* representing *its* peculiar character, the ostrich feather or the measure.

Amach, to bless, is written, *אמח ; ח*אמ ; ח*א ; ח* ; *

Me, truth, is written, *מע ; מע* ; ע* ; *

The peculiar characters belonging to the word me, "truth," are known to be ideagraphic; but that which distinguishes the word amach, is unknown; and, as I have already observed, it may be significative of sound. If I must hazard a conjecture, it would be that it represented a vessel holding mud, with the mud flowing out of it; omi, or ome, is the Coptic for "mud;" and the old Egyptian word for it probably only differed from this in its vowels.*

* On communicating my views respecting this word to Mr. Birch, he proposed an objection to them, which I think it right to notice, as I trust I shall be able satisfactorily to remove it. He observed that the preposition used between this participle and the name of a deity was "to," not "by," as according to my views it should be. The proposition is חל, answering to the Hebrew

I now come to the most important part, as I think I may safely call it, of the inscription on a tablet, namely, the speech put into the mouth of the deceased person. It may be known by the group of hieroglyphics which precedes it, as in the skeleton inscription given above. These characters are כתו, "he says," that is, "who says;" for the Egyptians had no relative pronouns. If the person commemorated be a female, the broken line ס, "she," is used for the horned serpent, ו, "he." It must not be supposed that these spceches are always of importance, or even that they always convey information respecting the deceased person. Sometimes, the speech is a prayer addressed to Osiris, or some other deity; sometimes it is a statement of the happiness enjoyed by the deceased in Amente; sometimes it is an invitation to mankind in general, or to the priests, or to those who may approach the burial place, to pray for blessings to the deceased; but it is, in many instances, a brief narrative of the most important events in the life of the deceased person; and it is here, if any where in the body of the inscription, that wc may expect to find the time when he lived, or his age, stated.

It would be impossible, in such a papcr as this, to describe at any length the varied contents of this portion of the inscription. Nor is it necessary for my purpose, which is mercly to direct attention to this class of Egyptian antiquities, and to guide the purchaser or student to those which are of most value, either from their age or from their contents. It is a rule, which admits few exceptions, that very little information is to be derived from any tablet which does not contain a speech; but the converse of this is by no means true; many speeches contain no information whatever.

I havc mcntioned, as I went along, several critcria of the antiquity of tablets. It remains for me to notice one, the most striking of all, which lies not in the

אל or ל; and, no doubt, it signifies most commonly "to." It, however, has other meanings, *just as the corresponding* ל *has.* It is used before the name of a king, when the year of his reign is to be expressed. So is the Hebrew ל. And why may it not be also used for "of" in such expressions as "the blessed of Osiris," "the favoured of his master?" In that very ancient Hebrew passage, Genesis, xiv. 19, a document, which is probably of the same age with the tablets which contain this formula, the proposition ל is used for "of" in the similar expression, "Blessed be Abram of the most High God," לאל עליון. The Hebrew and the ancient Egyptian languages throw great light on each other; and it is not unreasonable to expect that the study of the Egyptian monuments will elucidate many passages of the sacred text that are now obscure.

inscription itself, but in the sculptures which accompany it. In the more ancient tablets, the figures which occur are exclusively those of the deceased person and his relatives; figures of deities are never introduced. On the contrary, a tablet of the eighteenth dynasty, or of any subsequent period, is seldom without the representation of some deity or deities. I must, however, remark, by way of caution, lest any one should infer from this that the Egyptians of the earlier ages did not represent their deities in a visible form, that in the inscriptions on these ancient tablets small images of the deities are used, either to represent their names, or as determinative signs after them. The difference between the two classes of tablets is not to be attributed to any change in the religious notions of the people; it seems to have been merely a difference of taste or fashion; the more ancient Egyptians representing the deceased person as entertaining his relatives at a feast, while those of after ages represented him as doing homage to the deities.

The dates of some tablets are conspicuously placed at the tops; the royal name and titles being inclosed in a cartouche, and the year of the king's reign, and sometimes the month and day, being prefixed. It is from a comparison of these dated tablets, the relative ages of which can admit of no question, that I have derived the criteria of antiquity which I have mentioned.

I say the *relative* ages, because there are gaps in Egyptian chronology, which render it impossible for us to assign as yet the years, or even the centuries, before our era, at which the earlier kings lived. We know that the eleven kings, who appear as the predecessors of Rameses II. in the tablet of Abydos, with the intervening kings and queens whose names are omitted, reigned together for about 300 years. These are included in the eighteenth dynasty of Manetho. We know also that from the commencement of the reign of Sheshonk I., who commenced the twenty-second dynasty of Manetho, to the Persian conquest, is within a trifle, in excess or in defect, of 450 years. But as to the interval between the accession of Rameses II. and that of Sheshonk I., we have as yet, so far as I am aware, no satisfactory evidence. We know both from Manetho, and from the royal tombs at Thebes and other monuments, that a great number of kings intervened; but we have no certainty, that they did not belong to two or more contemporaneous dynasties; or that in the same dynasty two or more brothers did not occupy the throne together. This interval, then, which is by some extended to 550 years, is reduced by others to less than the half of that period;* and thus an uncertainty to the extent of about 300 years exists as to the reign of each monarch of the so called eighteenth dynasty, when the date of its commencement is compared with any given era; although the order of most of the reigns is perfectly well ascertained, and the length of many of them is known also.

I have spoken of kings and queens belonging to this dynasty, whose names are omitted in the tablet of Abydos. That this should be the case should excite no surprise, because that tablet was only intended to include the royal ancestors of Rameses II. The non-appearance of a king's name in it is no evidence that he did not live during the interval of time which it comprehends. In point of fact, the monuments in existence exhibit to us no less than four royal personages, who lived between Thothmos IV. and Rameses I., the twelfth and fifteenth kings on the tablet, in addition to the two who appear as the thirteenth and fourteenth, viz., Amenothph III., and Horus (Har-em-hebee). The names of three of these kings are Amuntuonkh, Amunmes, and Amenothph IV.; that of the fourth, whose tomb is in the western valley at Thebes, is yet undetermined. There can be little doubt that Amuntuonkh was the brother of Amenothph III., who shared the sovereignty with him for a time. This was pointed out by Sir J. G. Wilkinson, who has, however, confounded this king, who probably died in his childhood, with Amenothph IV. This last king has deservedly excited much interest; and strange mistakes have been made respecting the age when he lived. M. Letronne, and other French writers, have supposed him to belong to a dynasty anterior to the shepherds, the immediate successors of the gods! Colonel Vyse, on the other hand, imagines him to be one of the Persian kings of the twenty-seventh or thirty-first dynasty! The monumental evidence is, however, conclusive as to his belonging to the Thothmos family. It appears, that having become a proselyte to sun worship, he changed his original name of Amenothph, which implies devotion to Amoun, for Vach-en-aten (בחנאתן),

* The most probable supposition appears to me to be that, which makes the date of the ceiling of the Memnonium about 1322 years B. C.; and which, to accord with this, assumes that the twentieth and twenty-first of Manetho's dynasties reigned contemporaneously after the nineteenth. If this be so, according to the principles laid down in a former note, Rameses the Great must have ascended the throne in 1347 B. C., about 400 years before Sheshonk.

"the adorer of the sun's disk."* The latter name is found at Karnac, cut over the former, the prænomen attached to it remaining unchanged. Not content with this, in the fervour of his religious zeal, he made war against the name of Amenothph, wherever he found it. It has been defaced in innumerable instances in the second cartouche of his grandfather (or perhaps his great grandfather), Amenothph III. In general, the name has been merely chiselled away; but in several places, a repetition of the prænomen has been cut over it; a plain proof that his hostility was not directed against his ancestor, but against the name which he bore. There is also a tablet of Mr. Harris's of the age of Thothmos IV. (already referred to in this paper), relating to a deceased Amenothph, the former part of whose name has been rudely defaced in every one of the four places where it occurs. A like hostility appears to have been directed against the goddess Mouth, the wife of Amoun. In a curious statue of the reign of queen Amuneth, in the collection of Sign. Athanasi, representing (as I conceive) this queen, when an infant, in the arms of her nurse, and commemorating the father of the nurse, whose name was Sen-Mouth; the latter part of this name, which occurs very frequently in the inscriptions, has been, in the majority of instances, more or less defaced. This statue is curious, not only on account of its subject, but on account of its exhibiting traces of two defacers; a political one, who obliterated the name of the queen on the accession of her brother; and a religious one, at a later period, who made war on the name of the goddess. I mention these facts, because they are not unconnected with the subject of the present paper; they furnish a criterion of the age of a tablet which may sometimes be applicable. If the name of Amoun, or Mouth, appears on a tablet with marks of a hostile tool, it may be considered as certain that it was anterior to the reign of Rameses I., perhaps to that of Horus; and as highly probable that it was *not very long anterior to it.* Very ancient tablets, which are now in existence, were in all probability buried in the days of the sun-worshipper.

* In an article in the *Foreign Quarterly Review*, which has appeared while these sheets were passing through the press, this king is called Oubasheniten, which is interpreted "the splendour of the disk." The Coptic word oubash, splendour, is in Egyption עבש, and can have no connexion with בח; the Coptic corruption of the latter might be bash or ouash, but it certainly could not be oubash. It has been demonstrated by Salvolini that this root signifies "to adore." Ouasht has this signification in Coptic, in which language a T is often paragogic.

Before the commencement of the eighteenth dynasty, the tablet of Abydos furnishes us with five royal names, to which we may add a sixth, ascertained from other monuments, who appear to have constituted the twelfth dynasty of Manetho, and to have reigned for about 160 years. These sovereigns have been commonly classed under the sixteenth and seventeenth dynasties of Manetho; but that writer's catalogue of the twelfth appears to me to be intended for them, though we must suppose it to be grossly corrupted. The five dynasties intervening between the twelfth and eighteenth, I conceive to have been either contemporaneous with the twelfth, or altogether imaginary.

The first two monarchs of this twelfth dynasty were Osortasen I.* and Amenemhe II.; the former of whom appears to have reigned forty-two years, and the latter thirty-two, before they took their respective successors into partnership with them. A great number of dated tablets are in existence, belonging to these two reigns. The first year of Amenemhe II. corresponded with the forty-third year of Osortasen I.; and the first of Osortasen II. with the thirty-third of Amenemhe II.; after whose death he appears to have reigned a very short time. We cannot, then, expect to have many monuments of his. After him comes Osortasen III., and then Amenemhe III. The first Amenemhe preceded Osortasen I., and belonged, according to Manetho, to the eleventh dynasty.

I have made the preceding statements advisedly, and on what I consider perfectly sure grounds, though they are at variance with the received opinions. Major Felix produced a supposed succession from Benihassan, from which he inferred that Amenemhe the First intervened between Osortasen I. and Amenemhe II. This error, for such it demonstrably is, has been adopted by Sir J. G. Wilkinson, and by Rosellini; and Mr. Cullimore has grounded upon it a restoration of the obliterated portion of the tablet of Abydos, which has been published, under the title of "Chronologia Hieroglyphica," by the Royal Society of Literature. I have the highest respect for the learning and ingenuity of Mr. Cullimore, but truth obliges me to pronounce this restoration to have been made on erroneous grounds, and to be of no authority whatever. The sole ground for supposing that the royal names at Karnac formed a connected series,

* Or Gesortasen, if the initial letter corresponding to ע be sounded in Greek as a G, as it is in Gaza, Gomorrah, &c. Hence, probably, the grossly corrupted reading of Manetho, Gesongosis.

like that of Abydos, was that the names of the three kings in question occurred among the names at Karnac ; and that they might be read with a little management in the order, in which the Benihassan inscription was *supposed* to indicate that the kings reigned. It is quite impossible, however, that the names at Karnac can be read with any management in the true order of succession, as indicated above ;* and therefore I conclude that the names at Karnac must have been set down without order, the inscription there having never been designed to be historical. Nor do I think that it at all follows, that these were names of Egyptian sovereigns *exclusively*. If Thothmos reigned over the country about Meroe, as I believe he did, his predecessors in that region might very well be represented as receiving homage from him, as well as his predecessors in Egypt.

I will now state the grounds on which I pronounce the received order of succession of these three kings to be erroneous. In one of Mr. Harris's tablets figured by Mr. Sharpe (Eg. Insc. 73), which is dated in the third year of Amenemhe II., the deceased person is made to say, that he was born in the reign of Amenemhe I., and was appointed to certain offices by Osortasen I. When first I saw this, I was lost in astonishment, having never doubted, after the confident statements of Mr. Cullimore, Sir J. G. Wilkinson, and Rosellini, that there was a clear indication at Benihassan of an order of succession inconsistent with this. To settle the question, however, I referred to the Benihassan inscription itself, which I found copied by Mr. Burton (Exc. Hier. 33). I certainly found the three royal names occurring there in an order, which might not unnatu-

* This remark has led to a friendly correspondence with Mr. Cullimore, the result of which I have been requested to communicate in a note. Mr. Cullimore and I are agreed, that there is a way of reconciling the facts above stated, which he does not dispute, with the authority of the Karnac tablet, namely, by supposing that Amenemhe I. usurped the government in the life-time of Osortasen I., but that he died before him, and the latter then resumed his authority ; so that he was, in fact, the predecessor both of Amenemhe II., as is testified by contemporary monuments, and of Amenemhe I., in accordance with the Karnac tablet. But Mr. Cullimore and I differ as to the claims of this tablet to be received as an historic document. He considers it to carry with it its own evidence that it is such, and to be sufficiently corroborated by other monuments. I, on the contrary, conceive it to be totally destitute of internal claims to be received as an authentic catalogue of kings ; I consider the evidence on which Mr. Cullimore relies, as corroborating it, to be inconclusive ; and I think that other parts of it, as well as the Osortasen succession, are inconsistent with contemporary monuments. Mr. Cullimore's services to the cause of literature have been great ; and while I am compelled to differ from him on this point, I readily acknowledge them.

rally be supposed to be the reverse order of their reigns. Amenemhe II. occurred first; it was followed by Amenemhe I., and that by Osortasen I. I observed, however, that there was a great deal of matter intervening between these royal names; and I found, on examination, that this intervening matter was of such a nature as completely to *disprove* the order of succession, which it had been supposed to *prove*. The inscription stated that Nebhothph had been appointed by Amenemhe II., in the nineteenth year of his reign, a "Repha-He," with the military government of a certain district; the same rank and government having been conferred on his father by Amenemhe I., and on his *elder brother* by Osortasen I. Of course, Osortasen I. intervened between the two Amenemhes. After this I became acquainted with a tablet in the Leyden Museum, the date of which made "assurance doubly sure;" being "the forty-fourth year of Osortasen I., which is the second year of Amenemhe II."

The importance of this inference, as setting aside the supposed series of kings at Karnac, will, I hope, be accepted as an excuse for this digression. I will only add, that of the kings preceding Amenemhe I., we know very little as to the order, and nothing as to the length of their reigns.

I have now completed the task which I had marked out for myself; and it is my earnest wish that what I have said on this branch of Egyptian antiquities may induce others of my countrymen to engage in the study of this interesting and important branch of literature. I trust that no preconceived opinion of the impossibility that hieroglyphic characters in ancient inscriptions should express phonetically the words of a language will cause them to shut their eyes against the fact that they do so. And I trust also that no unworthy national prejudice will lead them to undervalue this field of discovery, because, though it may be said to have been opened in England, its most successful cultivators have been *hitherto* foreigners. I well remember the time, when the current of national prejudice ran strong against what were contemptuously called "French Mathematics;" but the good sense of our countrymen at length prevailed, and those branches which were once regarded as exclusively French, have been pursued with as much success in England, and, I will add, in Ireland, as ever they were in France. Let us adopt the same course in respect to hieroglyphical literature; and, in place of decrying the labours of Champollion, and undervaluing his wonderful discoveries, let us apply ourselves to follow them up; correcting, as we go along, his errors where we find that he has committed them; but candidly acknowledging that he himself corrected most of his early errors in his grammar, and that those which remain are few and unimportant, when we take into account the number, the magnitude, and the importance of his discoveries.

IV. *On the true Date of the Rosetta Stone, and on the Inferences deducible from it.* By the Rev. Edward Hincks, D. D.

Read May 9, 1842.

In investigating the affairs of ancient nations by the help of the contemporary monuments that are yet in existence, there is no knowing beforehand how prolific a single truth may be; what a train of interesting and even important facts may be brought to our knowledge by combining that one truth with those that are already known. This should lead us to prize every new fact that can be ascertained, however unimportant it may appear in itself. And, on the other hand, a similar consideration should lead us to endeavour to correct every falsely assumed fact, no matter how trivial the error may appear; for falsehood is unfortunately as prolific as truth; and one falsehood, assumed as a fact, may give birth to errors without number.

A striking illustration of these general principles has lately occurred in M. Letronne's Edition of the Greek Inscription on the Rosetta Stone; in which, with the most perverse ingenuity, he draws inference after inference from the false date, which Dr. Young assigned to that monument; which inferences are all erroneous, and are in most cases the very reverse of those which should have been drawn.

The date, which Dr. Young erroneously assigned to that monument, was the 27th March, 196 B. C., according to the proleptic Julian reckoning; the true date was, according to the same reckoning, the 27th March, 197 B. C. I will first contrast the inferences which M. Letronne has drawn from Dr. Young's date, with the inferences that he would have drawn had he adopted the earlier date; placing, for greater clearness, the corresponding inferences, which are generally contradictory, in parallel columns. Having done this, I will bring forward reasons, on which I confidently pronounce it to be *impossible* that Dr. Young's date was the real date of the monument.

M. Letronne's inferences relate to the history of Epiphanes and to the mode of computing the years of his reign, and that of other Egyptian kings; and to the various priesthoods of royal personages that are mentioned on the Ptolemaic monuments. He begins with the latter of these; but it will be more convenient to take the former first. I will only premise that the ninth year of Epiphanes, according to Ptolemy's canon, and the Egyptian mode of dating, is admitted to have been that, the first day of which coincided with the 11th October, 197 B. C.

Assuming the Rosetta Stone to be dated in March, 196 B. C., M. Letronne infers:	If, however, it were dated in March, 197 B. C., the inferences would be:
1. That Philopator died in March, 204 B. C.	1. That Philopator died in March, 205 B. C. The decree bears date the day following the anniversary of his death; and, as it is said to be in his ninth year, while, according to the Egyptian computation, it was in his eighth, it must have been made on the day after the eighth anniversary of his death, when he had reigned eight complete years. It should be observed that the mention of the ninth year is in the Greek part of the inscription; the Egyptian date was on a part of the stone which is broken off.
2. That Epiphanes was born in October, 209 B. C.	2. That Epiphanes was born in October, 210 B. C.
3. That the interval between Philopator's death in March, 204, and the 1st Thoth in the following October, was counted as the first year of Epiphanes.	3. That the interval between Philopator's death and the 1st Thoth following, was counted as a continuation of the 17th of Philopator, which began on the preceding 1st Thoth; and that the first year of Epiphanes did not commence until the 1st Thoth after his father's death.
4. That, as a general rule, the portion of a year which elapsed between a king's death and the 1st Thoth following, no matter how small it might be, was counted as the first year of his successor.	4. That, in the case of a king succeeding peaceably to the throne in the latter part, or even in the middle of a year, the remainder of that year was called after his predecessor; and that his first year was not reckoned to begin till the 1st Thoth after his accession.

Previous to considering M. Letronne's inferences respecting the various royal priesthoods that are mentioned in Ptolemaic inscriptions, it will be right to mention the *data* which he uses in conjunction with the Rosetta Stone. There are three papyri in the Egyptian Museum at Paris, bearing date in Epiphi of the seventh year of Philopator, i. e. in August, 216 B. C.; in Pharmuthi of the 8th of Epiphanes, i. e. in May, 197 B. C.; and in Paophi of the 21st of Epiphanes, i. e. in November, 185 B. C. The important point, in which M. Letronne has erred, is that he supposes the second of these papyri to be dated ten months before the Rosetta Stone, when it is really dated two months after it.

On the first of these papyri and on the Rosetta Stone, Aetes or Aetos is mentioned as priest of Alexander and of the other deified kings; while on the second of the papyri Demetrius is mentioned as filling that office. On the second and third papyri, as well as on the Rosetta Stone, Hirene is mentioned as priestess of Arsinoe Philopator; but the Athlophora of Berenice Evergetis and the Canephora of Arsinoe Philadelphe are different in all the documents; Aria, however, the Canephora of the Rosetta Stone, being the Athlophora of the second papyrus. The inferences then are as follows:

5. Demetrius being priest of the kings before the decree recorded on the Rosetta Stone, while Aetos was priest at the time of that decree, and also at a period previous to it, the office of priest of the kings was not a permanent one, but was probably annual.

6. The offices of Athlophora, Canephora, and Priestess of Arsinoe, were all annual. It would be highly improbable, if this were not the case, that the persons holding them would in two out of the three cases, be changed during the short period of ten months.

7. The office of Athlophora was not placed first, as being a more important office than that of Canephora; for Aria held the former office in 197, and the latter in the following year. M. Letronne conjectures that the reason for the for-

5. Demetrius not being priest, so far as we know, till after Aetos had ceased to be so; there is no ground for supposing the office to be annual. Aetos probably held it from the commencement of the reign of Philopator till after the Rosetta decree. In the course of the next two months, he either died or was removed by the new sovereign, who, it will be recollected, assumed the reins of government at the date of that decree.

6. There is no reason as yet for supposing that any of the royal priesthoods was annual. The changes which took place between the dates of the Rosetta Stone, and of the second papyrus, were such as it was highly probable would take place, if the office were held during pleasure, in the two months next following the attainment of his majority by a minor sovereign.

7. The office of Athlophora, being always placed before that of Canephora, was a more important office. Aria, who held the latter in March, 197, was promoted to the former before May in that year, the former Athlophora dying,

mer being always named before the latter was, that Epiphanes, or those who acted for him in his minority, had a particular regard for the memory of his grandmother.

or being removed by the new king. The idea of these offices being annual ones appears to have first occurred to M. Champollion Figeac; but it is not necessary to suppose them to be so, in order to explain the observed facts; and the contrary supposition seems on every account preferable.

I come now to state my reasons for maintaining, that the Rosetta Stone records a decree which was made in March, 197 B. C. The date of the decree is given according to the Greek and Egyptian computations, so far as respects the month and day. It was the 4th of Xanthicus, being the 18th of Mechir. Now I am going to show that these dates could not possibly coincide in the year 196 B. C.; but that they could and did coincide in the preceding year.

It has been proved by Archbishop Ussher, that the Macedonian year was a solar one, similar to that which was introduced at Rome by Julius Cæsar. As, however, some may doubt whether this solar year was in use at so early a period as the date of the Rosetta Stone, and as it is generally believed that the Macedonians had also a lunar year; it will be necessary to show in the first place, that the 18th Mechir, that is, the 27th March, in the year 196 B. C., could not be the 4th of a lunar month. To do this, I need only quote M. Letronne's own words: "This year the full moon fell on the 29th March, or the 6th Xanthicus. The first of this month was then about the ninth day of the moon's age; whence it would follow that the calendar to which it belonged was not lunar, *unless this month was this year an intercalary one* (a moins que ce mois ne fût embolimique cette année)." The learned Frenchman has not explained how this removes the difficulty; though it is evident that he supposed it to do so. It is not very obvious how in any lunar calendar, whether the month was intercalary or not, the full moon could occur on the sixth day. In the preceding year the full moon fell on the 9th April; so that if the 27th March had been the fourth of a lunar month, the full moon would be on the 17th day of it. This is so much less astray from the correct time than in the year 196, that if it were certain that the Macedonian year were lunar, I think there could be no hesitation in fixing on the year 197 B. C., as that in which the fourth of a lunar month would coincide with the 18th Mechir. I am, however, decidedly of opinion, that the Macedonian year was solar; and I find that, by supposing it to have been so, an exact coincidence between the two dates occurred in the four years 200, 199, 198, and 197 B. C., but not in 196, or in any other year.

That the Macedonian year was a solar one, subsequent to the Julian reformation of the Roman calendar, is unquestionable. What I contend for is, that it was so at the time of the Rosetta Stone, more than 150 years before that reformation; and the double date of that monument appears to me to establish this interesting fact in chronology. The mode of proceeding, in order to investigate this matter, is a simple and obvious one. I will take those dates of the Macedonian solar year, as it existed under the Romans, which are recorded as being coincident with dates of the Julian year, or of the fixed Alexandrian year, the correspondence of which with the Julian is known. From these dates, and the known lengths of the Macedonian and Julian months, it is easy to ascertain with what day of the Julian year any given day of the Macedonian year, say the 4th of Xanthicus, coincided in each of the four years of the Julian cycle; and it is obvious that this coincidence must remain unaltered, if we compare Macedonian years, actual or proleptic, *at any period*, with proleptic Julian years.

Now it has been shown by Archbishop Ussher, that the Macedonian year, as used in Asia generally, differed in certain respects from the Macedonian year, as used in Macedonia. The commencement of both years was at the autumnal equinox; but the first month of the Asiatics was Hyperberetæus, while that of the Macedonians proper was Dius. The same difference remained through the other months, Xanthicus being the sixth in Macedonia, but the seventh in Asia. It is natural to suppose that Egypt would follow the Asiatic system in preference to that of the Europeans; and this is confirmed by the Egyptian date, with which one of these Asiatic dates which I am going to produce is stated to correspond. These dates (which I take from the treatise of Archbishop Ussher, "de Macedonum et Asianorum anno solari;" a valuable work, with which neither Dr. Young nor M. Letronne could have been acquainted) are, first, that of the martyrdom of the Apostle St. Paul; which is stated by Euthalius to have occurred on the 29th June, A. D. 67, being the 5th Panemus. Xanthicus, Artemisius, and Dæsius had the same number of days as March, April, and May. Therefore the 29th March in that year coincided with the 5th Xanthicus, and, of course, the 28th March with the 4th Xanthicus.

The second date is that of the martyrdom of St. Polycarp, which is shown by the learned Archbishop to be assigned by the most correct copy of the Acts thereof to the 2nd Xanthicus, and 26th March, A. D. 169; being the day of the great Sabbath, or that Sabbath which occurred at the Passover. In that year, therefore, the 4th Xanthicus also coincided with the 28th March.

The third date is that of the burial of the younger Valentinian, which is stated by St. Epiphanius to have fallen on the 23rd Artemisius, being the 21st Pachon (of the fixed Alexandrian year) and the 16th May, A. D. 392; the latter days are known to correspond. This correspondence gives us for the 4th Xanthicus in that year the 27th March. It is, therefore, evident that in bissextile years, the 4th Xanthicus corresponded with the 27th March, and in the other three years of the Julian cycle with the 28th March. This is, in truth, nothing more than what has been expressly asserted by the Archbishop, who shows in his treatise (pp. 46, 47, Ed. 1648), that in bissextile years the month of Xanthicus, which he specially notices on account of its connexion with Easter, began on the 24th March, and in the other three years on the 25th.

Now, as the year 197 B. C. was proleptically bissextile, according to the Julian computation, the 4th Xanthicus must in that year have coincided with the 27th March, and therefore with the 18th Mechir. In the three preceding years it would also coincide with the 18th Mechir, both dates coinciding with the 28th March; but in the following year, 196 B. C., and those after it, the 18th Mechir would coincide with the 27th March, while the 4th Xanthicus would coincide with the 28th.

It appears to me that this amounts to a complete demonstration, that the true date of the Rosetta Stone was 197 B. C., and that the date assigned to it by M. Letronne after Dr. Young was erroneous. Consequently, the seven inferences drawn by M. Letronne must be rejected; and the seven others, in most cases contradictory, which I have placed in the parallel columns, must be substituted for them.

V.—*An Essay upon Mr. Stewart's Explanation of certain Processes of the Human Understanding.* *By the* Rev. James Wills, A.M., M.R.I.A.

Read February 14, 1842.

CHAPTER I.

ARGUMENT STATED, AND MR. STEWART'S EXAMPLES ANALYZED, WITH A FEW ADDITIONAL CASES WHICH PRESENT THE SUBJECT UNDER A DIFFERENT ASPECT.

It is some years since I was very much struck by an argument of Mr. Stewart's with which many here are likely to be familiar: he endeavours to prove from several cases, that the mind, from habit, acquires a rapidity in the succession of distinct thoughts, so great as to escape the consciousness, a proposition which he endeavours to prove by examples, and from which he draws some important conclusions. Considering that all his instances are such as seem essentially to involve the principle of consciousness, I found it hard to acquiesce in his theory. But it was impossible not to admit that if Mr. Stewart has correctly stated his facts, the inference is in no way to be avoided. And I failed at the time to observe, that all these facts (as I shall presently show) are themselves results of a very complex nature, and requiring a minute analysis, before they could become the fair grounds of such inferences as Mr. Stewart's: I, therefore, with some reluctance, dropped a subject which seemed to offer some curious approaches to a more intimate knowledge of our intellectual nature. The popularity which Mr. Stewart's theory has acquired (chiefly owing to his very curious and interesting exposition of the phenomena of dreaming) has led me to reconsider the subject with more deliberate attention: and I now venture to advance a statement of the inferences which I propose to substitute for Mr. Stewart's.

To express Mr. Stewart's theory in his own language, it is this, "The wonderful effect of practice, in the formation of habits, has been often and justly taken notice of, as one of the most curious circumstances in the human constitution. A mechanical operation, for example, which we at first performed with the utmost difficulty, comes in time to be so familiar to us, that we are able to perform it without the smallest danger of mistake, even while the attention appears to be completely engaged with other subjects. The truth seems to be, that in consequence of the association of ideas, *the different steps of the process present themselves successively to the thoughts, without any recollection on our part, and with a degree of rapidity proportioned to the length of our experience;* so as to save us entirely the trouble of hesitation and reflection, by giving us every moment a precise and steady notion of the effect to be produced." According to this statement, a succession of acts of *attention* and *volition* are supposed to pass through the mind with a rapidity too great to be perceived, and for which, *therefore*, there *can* be no argument but the *necessity* of the thing; because, according to Mr. Stewart, no other will explain the phenomena. These notions are so involved in the entire of Mr. Stewart's Theory of the Mind, that were I to attempt a full analysis of his reasoning it would necessarily lead me into a very prolonged discussion, which should commence by a systematic exposition of those elementary views of the mind and its functions, which I conceive to be entangled with many errors by Mr. Stewart. The difficulty attendant on such an undertaking would be enormous: for I must confess that I cannot so easily satisfy myself as Mr. Stewart and other writers on the same subject seem to have done, with any definition of those elementary processes of the mind, on which so much reasoning is built.

The elementary fallacy in which I conceive Mr. Stewart's error to have originated, is comprised in his very first step. It is difficult to speak satisfactorily of a function so purely elementary as *consciousness*. Like light, it is chiefly apprehended by reflection from surrounding things: but it is not hard to point out the mistake which is implied in Mr. Stewart's view. He fails to observe that the mind apprehends by *wholes* before it perceives by *parts*. Consciousness, as it may be described (I do not pretend to define), appears to be the *sum of sensations and apprehensions* of whatever nature, which constitute the whole state of mind at any moment. The fallacy contained in Mr. Stewart's first examples, consists in an implication that every part of this aggregate is separately perceived. Had he distinctly asserted this proposition, he would have quickly seen his error, but he takes it for granted, and goes on to applications in which it misleads him. There is, in those who are in a state of consciousness, at all times a certain aggregate of things presented to the perception. Of these, some may become more prominently the objects of attention, and the rest will invariably, in the same proportion, become vague and indistinct. The perception of individual parts of this vague *whole* will, *in general*, not be separately recollected, because they have not been *separately* observed; and not, as Mr. Stewart assumes, because the observation has been too rapid. There is a process, it is true, by which, in a certain class of cases, the mind can recal and analyze a large combination of things; but this is not what Mr. Stewart has in view.*

I shall presently be in a condition to examine more closely some of Mr. Stewart's reasonings on this point, but I shall now proceed by a more simple and far shorter method, which Mr. Stewart himself has the great and signal merit of having pointed out, and in some measure exemplified. Instead of adopting definitions, and launching out upon the vague ocean of pure reasoning, I shall essay the humbler adventure of a coasting voyage along the safe shore of known and familiar facts; the only method that I suspect will be ever found to lead to any satisfactory result, in a science of which the first elements are so little tangible to strict observation as those of the mind.

The nature then of the analysis to which I beg to call the attention of the Academy is strictly this; I shall state in order a numerous train of well known and most common facts, in all of which the same process can be easily observed, and which will exhibit this process in a variety of aspects, so that it may thus appear what method of explanation will best agree with all. Among these I shall include Mr. Stewart's cases, and endeavour to show that his explanation, which is specious enough on a confined view of examples selected for the purpose, is negatived entirely when referred to other cases which cannot be regarded as specifically different.

The first case which Mr. Stewart states, with an explicit reference to the subject of this essay, has the advantage of offering a passing view of another

* Some of the examples by which Mr. Stewart illustrates his views concerning consciousness, perception, and attention, cannot be here satisfactorily discussed, until I shall have first fully explained the principle to be asserted in this essay. I shall, therefore, revert to them further on.

philosopher, who, though far less reasonable than Mr. Stewart upon the subject, offers the advantage of a different observation of the same phenomena.

Mr. Stewart quotes from Hartley his first example, which is that of a person playing upon the harpsichord. The fingers of the player perform a variety of movements from key to key, each of which, as Hartley observes, is at first an act of distinct *volition.* By degrees, however, the motions (according to his language) cling to each other, and the acts of volition grow less and less, until at last they become evanescent. On this case Mr. Stewart says, "thus in the ease of performance on the harpsichord, I apprehend that there is an act of the will preceding every motion of the finger, although the player may not be able to recollect these volitions afterwards, and although he may, during the time of his performance, be employed in carrying on a separate train of thought."

In supporting this proposition, Mr. Stewart observes, that the "player may vary his rate of movement, and play so slowly as to be able to attend to every separate movement:" and on this very justly observes Hartley's unreasonableness in assuming *two different* rules of mental action for the quick and the slow playing.

It is remarkable that Hartley's reasoning actually terminates in the *vulgar notion* upon that class of acts commonly called mechanical, from which his instance is drawn; a circumstance which at least seems to show that he has carefully observed, and correctly described the *phenomena*, though in his attempt to explain them he was (as usual) misled by a theory. The fact that the distinct acts *are not separately* the object of any *conscious* volition or attention, he recognized by direct *observation:* it was perhaps rash to infer the absence of these elements: but if Hartley knew any thing about the art from which he exemplified his reasoning, he must also have observed, that these separate attentions and volitions were in certain movements of the player *necessarily impossible*, and that, therefore, *some other law must be sought for:* the *automatic* movement is *very like* the truth, and *though liable* to Mr. Stewart's objections, would be far easier to support than his own solution. I trust to convince the Academy that there is no proof of the separate volitions *assumed by* Mr. Stewart, in either quick or slow movements. *Volitions* there must be, but executed under the intervention of another process; a process, it is true, still to be referred to the effect of HABIT, but carried on in the progress of its operation to a much more complete result than that contemplated by Mr. Stewart.

Let me call your attention to the actual instance: two or more notes are marked for the right hand to strike *together*, and perhaps as many more for the left, *all at the very same time*, and by *one movement* in which several others, all distinct in their effect and intent, are absolutely and *indivisibly combined* into one act: a single impulse giving simultaneous movement and synchronous directions to several members, and constituting, therefore, one conception in the mind of the mover. The difference between such a process and the most rapid succession that the nature of the thing can admit of,—say the vibrations of sound,—is as great as the difference between the mere confusion of substances called *mixture*, and the substantial union caused by chemical affinity: as that substance is one, so is the *effect* in this case absolutely one, executed by one act, governed by one *conception*—a single *complex idea*, the result of association. I agree with Mr. Stewart, or rather with the common notion, in assigning this complex act to habit; but habit acting, not by mere acceleration, but by a maturer process to which it is always tending, and which forms its main department of the mind; the combination of ideas which have been frequently presented, into recognized groups, of which each, losing its features of aggregation, acquires an integral and distinct identity of its own. Though I am anxious to avoid the adoption of any system of metaphysical language, yet it will be convenient to keep in view, that the results here described are the same which are called *complex ideas* by Mr. Locke, which term I shall retain through this Essay.

Let us dwell for a moment longer on this first case, and take one glance at the general progress of the performer in the *acquisition* of the art by which those complex movements are effected.

At first those signs must be *separately observed* by the learner, and the answerable movements separately made; two notes cannot be at the same instant *observed*, still less their movements (altogether amounting to four distinct acts of thought for one simultaneous act of the hands), be performed; though all are fully recognized, no velocity of will and attention can impart the simultaneous execution required: the movements can only come separately, and, *therefore*, cannot operate together. Slowly, however, and by continual repetition of the same efforts of attention, the combinations begin to be *seen as combinations*, and becoming virtually *single conceptions* are executed by *single movements.* One act of volition can direct the most complicated movement when it is once thus conceived. And it is a very remarkable and highly confirmatory fact, that the slightest attempt to direct the attention to any of the separate components or signs, would instantly disconcert the most practised skill. This Mr. Stewart would have seen and profited by seeing, had he not selected examples of which the component acts are not *necessarily* simultaneous. A performer on some kind of instrument requiring a succession of *uncompounded* movements, may undoubtedly, by playing more slowly, attend to his separate touches, but *then* he is not a case in point: for that species of acceleration of the mental processes which can be *actually observed*, is not that for which Mr. Stewart would contend. The point here to be established, is not that the mind may not operate with any imaginable velocity, but that the assumption of an acceleration so great as to escape all consciousness, is unnecessary for certain purposes, and a departure from an observable and well known process. It is one thing to assert that the mind can by distinct steps follow and regulate certain rapid changes of motion, and another to assume that this process may become so rapid, as to be impossible for the apprehension to follow it distinctly. The real difficulty which I shall have to surmount is this, that there appears in this case, and some others, to be two distinct trains of thought going on. I mean, further on, to show that this is but apparent, and I shall at the same time show that Mr. Stewart's assumption vastly aggravates this difficulty.

A curious instance of the effect of separate attentions and volitions in cases of complex action is not very uncommon. When a person of a very anxious temper is called on for an exhibition of skill in some act which requires very complex acts of mind, it sometimes occurs, that extreme anxiety to succeed forces the attention from the common process, as here described, to an intimate notice of the separate acts of the combination: and the links of complex volition are thus broken, so that embarrassed movements follow. The best illustration of this will occur farther on.

This last circumstance is most frequently observable in that extensive class of acts, which, in popular phrase, we call *mechanical.* They are, indeed, nearly decisive against Mr. Stewart; for, while they consist, for the most part, of complex movements, the separate acts of which they are framed have *never been recognized* *in separation*, and cannot be taken asunder by any power of attention. Of these, every person has his own share—one instance will be enough; that, suppose of unlocking some well known lock, which has become, by habit, so familiar, that it can be effected in the dark. Now let any person who is conscious of any such habit try to substitute his *reason* for the habit; he will at once, and I would say inevitably, fail; his *volitions* and *attentions* will put his hand astray. In fact, the operation of habit was to frame the *conception* of a movement, out of an actual movement which, by the *help of the sight*, was first repeatedly performed. Of such movements of frame and thought, are composed the *entire* actions of the player's hand, the dancer's foot, or the reader's eye. And here it may be useful to observe and bear in mind, that in all these cases, of every description, there exists at the same time a distinct succession of acts of will and attention, sometimes continuous and sometimes changing, but always fully apprehended by the consciousness; and that the mind is in fact thus guided from change to change, and from one complex act to another; while these latter alone are the processes in question here. According to Mr. Stewart, both must be going on together without intermission, at different rates, and having different objects; taking, for instance, the player on the harpsichord, we have the movements of the hands, the interpretation of the notes, the relative intent of each to a certain whole harmony, the moral sentiment belonging to the melody. Now had Mr. Stewart been asked to explain this medley of concurrent processes, he must have been forcibly conducted to the very theory which is here proposed to be substituted for his.

But I turn to Mr. Stewart's next example, suggested by a passage in the Latin writings of Doctor Gregory, who applies a similar example to prove or illustrate the rapidity of muscular action, for which he refers to the vast number and variety of intonations produced by muscular movements in the pronunciation of words. With the Doctor's application I am not concerned. Mr. Stewart says, "when a person, for example, reads aloud, there must, according to this doctrine, be a separate volition preceding every letter." Now, I do not here state Mr. Stewart's very indirect reasoning, because it consists altogether in combating objections which have not, I believe, been advanced, viz., objections to the *possibility* of the extreme rapidity of mental action required by the process he assumes. I do not, for *my part*, deny the fact of such *possible* velocity of the thinking power, though I see no force in Mr. Stewart's *reasons* for it. I only affirm that it is not proved by any of the alleged examples, and is not necessary for their explanation; and into the *assumption* of such a necessity, the entire argument of Mr. Stewart may be resolved.

This example is very convenient for illustration; I will, therefore, examine it fully. Now let it be distinctly kept in view, that though the process of reading is in both systems inferred to be the result of a power attained by *habit*, the difference is as to the nature of that attainment. Mr. Stewart's solution requires that it should be by *accelerating that succession of acts*, by which every letter of the word is *separately* noticed. If this be true, then, it is evident that the facility can in no way depend upon perceiving the combination, as it is the principle that every separate part must be *antecedently* recognized, and the perception of the combination is but consequent. *Therefore, it is quite immaterial how strange the order in which letters are combined*, when they are separately so far *known* as to be instantaneously recognized. Now this can be tested. If any reader who is sufficiently interested in the matter for an experiment, will take the trouble to write out a few lines of new combinations of letters, forming words of the ordinary number of letters, or get it done by another, and then try his skill in reading those words with the usual rapidity; he will immediately discover that, however expert he may consider himself to be, he will be compelled to go back to the old nursery discipline of *spelling*. Those extremely rapid attentions and volitions will be found to fail when they should be efficient, if the assumption of Mr. Stewart (for, after all, it is no more) be correct. Here, again, I might pause to dwell on the consequences of Mr. Stewart's assumption. The same law which demands successive distinct notices of the letters, essentially requires an equally distinct and separate succession of perceptions of the several parts which form the shape of the letter. The letters taken separately have each a sound different from their syllabic effect, and this again is variously modified according to the combination. Then comes to be recognized the sense which a word acquires from context; and lastly, the train of reason in which the intellect seems to be wholly engaged. If *all* these several trains are to be separately noticed, according to Mr. Stewart's law, it is evident what a complication of wholly distinct trains of thought must be simultaneously proceeding; but if Mr. Stewart should stop at any point short of this, it is plain that his whole theory fails; the explanation he must substitute at that point may serve as well for the whole; the *neces-*

sity of the assumption no longer exists. Let me now call the attention of the Academy to the law of progress, by which the requisite facility is actually attained, both in this and all the other cases to which Mr. Stewart's theory of acceleration can be applied.

So long as a direct and separate conscious attention is required to each of the several letters forming a word, the process is that of *spelling* only; the components are separately and successively noticed, but the result (a wholly different object of thought) is not perceived.

How, then, does the mind proceed? It slowly, and by much discipline of thought and repeated efforts, acquires a stock of *syllabic* and *vocal* associations; that is, it acquires a set of complex ideas and *represented* sounds. In these, it no more separately notices the separate parts of the syllable than the separate parts which constitute *the form of the letter*. And let it be observed, that in difficult handwriting, it is *by the syllable that the letter is known*, rather than the converse process. Again, it is pretty well known, that in correcting the press, it is exceedingly difficult to acquire the habit of perceiving literal errors; while compositors in printing offices have been heard to remark an occasional difficulty in reading words and sentences, from their habit of *attending* to the letters.

Just in proportion to the expertness of the reader, and his intimate acquaintance with written language, *the combinations* become more extended; and, in consequence, the number and extent of the parts *which escape notice* also increase; as the letter became lost in the syllable, so the syllable becomes lost in the perception of the word. Words acquire their visible symbols, and are discerned in such ill-formed scrawls, that no letter could be separately recognized; here it is evident that the general form of the word is enough for the mind. Even *common conventional forms of sentences* are read with one single act of thought, forming but one idea, registered by use; and if any one wants an illustration, I will refer him to the familiar fact, that in reading easy and idiomatic language, the omission of words is often unperceived. The omission is supplied by the mental eye; it has become *a portion of a known whole*. To complete our view of this case, a written word becomes identified with the *meaning* of which it is the visible symbol. By a further extension, a sentence becomes similarly identified with a process of thought. Every one possesses a certain range of thought, all of which habit has thus symbolized. And this range is various in its scope and breadth in different minds. Present any one with a wholly novel combination, and he must pause to analyse.

The facts so far observed are no more than an analysis of the process of learning. The scholar slowly acquires a class of complex ideas, called syllables; from these he acquires another more compounded, as they coalesce into another class called words. To this I may add, that, as ordinarily takes place in our complex ideas, the combination is entirely (or, to a great degree) different in character from the parts of which it is primarily composed. But, of this there are better examples; the sounds of the letters are to *some* extent preserved in most words. Another reason why the example was calculated to mislead is worth notice,—in speech, the *sounds* of most words are necessarily *successive;* and this alone might tend to conceal the simultaneousness of the mental act. But it will be at once recollected that, in reading, the eye has commonly passed over many words, before the tongue has performed its office.

The general inference is this,—that by habit, groups of *signs*, of *movements*, of *facts*, *thoughts*, *sensations*, or *phenomena*, acquire certain relations to each other; and these being acquired, it is the *combination* alone that becomes the object of thought.

The parts come simultaneously to the apprehension or sense; they do not even necessarily require to be *complete ;* it is enough if the *character* is kept. Hence the deceptions in drawing—the faces in the fire, and the innumerable illusions of the eye and ear; and, perhaps, all the senses.

I think that some more simple illustration of these facts may be satisfactory. Mr. Stewart employs several, but for the most part they are not sufficiently familiar to convey much in the way of illustration. Before I proceed to their analysis I shall, therefore, endeavour to apply the same investigation to some very common and familiar acts, with which most persons must be acquainted. In first learning to ride, there are certain niceties of posture and action, but still of a very simple and easy nature, to be simultaneously attended to. These the finished equestrian (unless he be a riding master) performs unconsciously, and perhaps has forgotten in their separate forms. A simple volition executes for him a compound posture of movement. But, look at the tyro, he learns in a few minutes all the simple rules that are to be taught ; but he cannot govern the gallop, or ride skilfully and with a firm and graceful seat over the bar or wall.

He forgets the leg, while attending to the inclination of the body; and the hand neglects its office, while he thinks of his feet; the saddle, bridle, stirrups, whip, and spurs belong to different systems, and war with each other, and the idea of preserving a graceful balance obliterates them all. Now, as the idea of succession is here *excluded*, and as the equestrian must keep all together, or roll in the dust, the process becomes more clearly indicated; he must necessarily acquire a *position of will* or attention, of which all these minutiæ are the components.

In shooting, there are three acts to be executed simultaneously—the motion of the gun, of the eye, and of the finger; they separately present no difficulty; the young sportsman is, however, aware how hard it is to think of them together; the veteran executes them as a simple act conceived by the will, and performed by the members. But this example offers a *side-glance* at the process: for in shooting there is an obstacle very often found from the operation of extreme anxiety to hit: the immediate effect of this is to cause a *minute attention to the means*, so that the *ordinary* act is thus interfered with. The complex volition is resolved into its component parts, and while the anxious marksman is securing some part of accuracy, he neglects some other. The sure marksman does not think of any methods; but hits without knowing how it was done: his *gun* seems to have learned its part, and comes up to his mark: he may tell you, if you ask, that he never takes an aim. The fact is, that men do not recollect, and often cannot find out the component ideas involved in their commonest acts: they act with a single effort complex in its motions, but uniform and *one* in the impulse of the mind.

It would be tedious to apply, at detailed length, the same reasoning to all the examples given by Mr. Stewart: but it is fit and just to touch upon them; in order to *indicate* at least their connexion with the general process. They may all indeed suggest much, which I shall not notice until further on, when I shall reach the more general statements which I think to be the results of this view.

The case of an *expert accomptant* is easily apprehended. The constant habit of arranging numbers into groups, each group indicating a certain sum, is the same process as that by which letters combine into words having each word a certain sense. This is too simply obvious to dwell upon.

But I would here call your attention, by the way, to the obvious difficulty, which makes the conception of all *unhabitual* operations very nearly impossible to the human mind. In truth, it is only when the habit is *actually acquired* that any idea of the act can be realized to conception; and it then escapes the powers of distinct analysis. But on this point I shall only need to remind you that the same difficulty must exist, however the matter be explained. It belongs not to the solution, but to the fact.

There is perhaps more real difficulty affecting the case of the jugglers, which is noticed by Mr. Stewart. And the more, because, as in many acts of the mind, it is in some degree entangled with *other laws of action.* Yet, so far as the main point, it is not really difficult to explain. The eye and hand, with all their involved rapidity, are still kept under the unerring government of a single conception of a complex continuous movement, every part of which is together present to the mind. Were it not for this, indeed, it would not be difficult to prove that this, and all other similar feats, would be utterly impracticable. A distinct interference of volition would arrest the juggler's flying and circling balls; as it would precipitate the rope dancer, another of Mr. Stewart's cases, from his dangerous height. In this case the movement and the balance are preserved by not thinking of the emergency of the instant: but yielding to the constant action of a conception and habitual impulse, which have been called mechanical, with a just regard to analogy, because they exclude the uncertainty of the deliberate and voluntary processes of the mind.

There is withal a distinction which I have hinted, but with which I did not wish to complicate the subject, which demands notice. The cases which I have referred to, as well as the numerous ones which might be mentioned, all fall into two general classes: that of *instantaneous* acts which present no difficulty, and lead the investigation with the simplicity of self-evidence to the nature of the operation; and those which, being *continuous*, appear at first less reconcileable to the solution which explains them into a single idea. This difficulty (if such it should be called) is but specious: there is no reason against the supposition of one idea being held for any length of time, which the purposes in question require. I am no more bound to the asssumption of a single instantaneous process than Mr. Stewart. I am not bound to disprove, that habit facilitates, and therefore accelerates any *constant* succession of ideas: but the inference is as to the *result*, when this *succession has apparently ceased.* And this result, according to the view here explained, is simply this, that the *limit* of such acceleration is a *coincidence.*

A result which, if this very faulty method of statement were to be allowed, would amount to something different from the metaphysical asymptotes, involved in Mr. Stewart's indefinite acceleration.*

There is one example brought forward by Mr. Stewart among the statements by which he is first led to the conclusion which I have been examining in this essay. I could not have noticed it much sooner without anticipating the inferences at which I have now arrived. The following is Mr. Stewart's statement: "It has been proved by optical writers, that in perceiving the distances of visible objects from the eye, there is a judgment of the understanding antecedent to the perception. In some cases this judgment is founded on a variety of circumstances combined together,—the conformation of the organ necessary for distinct vision; the inclination of the optic axis; the distinctness or indistinctness of the minute parts of the object; the distances of the intervening objects from each other, and from the eye; and, perhaps, on other circumstances besides these: and yet, in consequence of our familiarity with such processes from our earliest infancy, the perception seems to be instantaneous; and it requires much reasoning to convince persons unaccustomed to philosophical speculations that the fact is otherwise." I shall not here dwell on the very equivocal language used by Mr. Stewart. The purpose for which he uses the example is, however, such as to imply the more objectionable of two senses in which I might take his assertion of a "judgment of the understanding antecedent to the perception;" that is, that *antecedent* to the perception some distinct exercise of reason, referring to the separate incidents of the actual perception, occurs. In this sense, the mere statement is a sufficient reply; the notion conveys an utter absurdity. If, however, Mr. Stewart simply means the process of the understanding, by which inferences respecting the distances of visible objects have been gradually obtained; so that a judgment, grounded on such reasonings as he has stated, goes before and modifies the perception, forming, in accordance with his views, an antecedent part of it; while the extreme rapidity of the mind prevents any consciousness of the distinctness in time between the two processes; his fallacy is certainly less glaring, but I must observe, that it only becomes so by simplifying the assumed process. Now,

* The method is faulty, because it confuses two very distinct classes of phenomena: the aggregate perceptions of mere *consciousness*, and the complex formations of *association*.

the fact is, that the species of reasoning to which Mr. Stewart refers the judgment *has no existence in any case.* The reasons not only never occur to the understanding, but are not to be found by it, unless in the case of opticians, who are themselves so little aided by their reasons, that they have long disputed as to the means according to which vision is accompanied by a judgment of distance. The theory here stated reduces this question to a very simple and obvious law—the same long ago stated by Mr. Locke in his chapter on the Association of Ideas. By habit we are enabled to understand our perceptions as the indications of external things; the import of a habitual perception demands no reasons of any kind; it is become a part of it.* As the eye approaches or recedes, the appearances of things uniformly alter; and as the mind grows accustomed to these alterations, it insensibly learns to translate them into the constant fact. Should any occasion of doubt arise, the reasoning then steps in; it is, however, seldom derived from the laws of vision. When the judgment is not involved in the perception, it *follows* it. The artist whose business it is to imitate the appearances of things, imposes on the perception, by producing the same indications in a different way; it is then that the judgment becomes *antecedent*, and that the law of the appearances must be ascertained. In the common exercise of vision, distance is recognized as every other object of sight which constant recurrence has made familiar. By habit, the eye, ear, and all the senses acquire their proper scales of adaptation—a law involved in every movement of the frame, in every living thing.

There is another class of common facts, very curiously illustrative of the conclusion here aimed at. I mean the numerous *errors* arising from our tendency to combine, or from the habitual combinations of every individual. These, from their nature, must be mostly peculiar, and even singular. Every one may recollect some case in his own experience, and it is but a chance if any instance which one person may offer will have come within the observation of another. An instance may, however, be good for illustration. I recollect that once, on looking at a picture which represented the interior of a cottage, with very unusual force and truth, to have observed that the flame of the fire seemed to have the same quivering motion always accompanying the kind of flame represented. Now this could

* The perception is itself a complex state of mind; it is composed of certain sensations, and certain judgments.

not be the result of any *real* perception, but is easily explicable by the process already described; the form, hue, and motion of the flame had been so associated, that the incident wanting in the representation was supplied, before the judgment could come into operation.* Of this nature are those cases also, already slightly adverted to, of faces framed by the imagination out of accidental lines. Let me dwell a moment on this, for it is one of a large and diffusive class of results, to all of which the same explanation will apply. I mean that class of expressions and effects which must in part be referred to the fancy of the observer. The expression of the human countenance offers an instance in which several varied qualities of human character seem combined with certain dispositions of form, in such a manner, that while the expression is instantly presented to the observer, he can in few instances, and then but partially, and by much nicety of observation, ascertain the precise arrangements of feature to which the characteristic expression is due. I shall not encumber the case by an analysis of the origin of such combinations; it will be enough for the present purpose to observe, that the acquired tendency to read such undistinguished elements into meaning must be very deeply fixed; to all purposes, it might be considered as instinctive. For, while all can at once see and designate an ordinary expression, which is the result of certain lines of feature, the artist alone can discriminate the characteristic curve, and reproduce the effect on his canvass. But now observe the consequence of the associating tendency,—the strong prepossession which conveys ideas of expression from lines indistinctly discerned, will actually select and attach similar expression to similar lines, when they appear in any mass of confused and indiscriminate lines. The instant the eye rests on a single characteristic curve, this will be the key to all the lines in the mass which (if I may so speak) belong to the same face. The fierce eyebrow will impose on the eye a mouth of the same character, which will be seen in its proper place. This case is the plainest of its class; but all the forms of familiar things are similarly traced by the vacant eye, out of formless elements; for these alone leave it free to the stream of association. From this, I might proceed to the phenomena of dreams; but the subject demands a separate treatment, and must be referred to the conclusion of this Essay.

The cases so far stated to the Academy have exhibited the *simple continua-*

* The picture alluded to is the "Arran Fisherman's Drowned Child," by Burton.

tion of a process which we *can* trace, *to further phenomena of the same apparent nature* in which it *cannot* be so easily traced: but from which there seems no reason to exclude it, unless one which should be noticed before I venture to extend my theory to the explanation of some of the more complex operations of the intellect. This objection consists in the difficulty of attributing so many varied and continuous acts to one single conception, or moment of time. My answer to this objection (here) shall be very brief indeed, being no more than this,—that the self-same objection applies to Mr. Stewart's explanation of every example he adduces. If twenty acts of will, or attention, or reason, or any other mental process, take place in the *time of one*, the difficulty is not much diminished by saying they are *successive, instead of simultaneous.* In truth, no power of intellectual comprehension or resolution can distinctly conceive either one or the other; they are creatures of reason only. I am aware of the infinite divisibility of time, which is easily proved by the same argument which demonstrates the same proposition of a line, on the parts of which it is only necessary to conceive the idea of motion. I am also willing to assent to any proposition asserting the infinite velocity of the thoughts; I do not pretend to deny any thing on the mere ground of not being able to explain it; but I say that, so far as I can venture to assert, the proof has entirely failed. The *necessitas rei* of Mr. Stewart has no existence; and if any solution is to be tolerated of those processes of the mind which are so subtle, or so compounded, as to escape all direct analysis, there is none more likely to apply, than that which, in simpler cases, is plainly and manifestly applied to the same offices. On this point, let me recal your attention to Mr. Stewart's own argument against Hartley's theory, as I think we may now be better enabled to perceive that it equally destroys his own, while it is not applicable to that here offered. Hartley supposes the same processes, which are *voluntary* up to a certain rate of velocity, then to become automatic. Stewart very justly remarks the disadvantage of assuming two wholly different laws of action for the same processes, in different *degrees* of action. Now Mr. Stewart only escapes the same objection, by giving the same name to different things; this I have already shown. But in my own solution alone the same law is manifestly carried through, without the least abatement of its identity. Not being a summary operation, but the result of numerous operations, it does not in any way involve the principle of consciousness, more than the growth of the body involves sensation. Unconscious from the very commencement, the combining process is no worse than unconscious at the height and depth of its remotest combinations. And if—in the indefinite progress of intellectual power, which no thinking person will venture to limit—the elemental process which generates all our registered and tangible combinations should give birth to combinations more broad, or subtle, or varied, there is no reason why we should think it necessary to say that these are beyond the limits of its office.

It is easy to perceive, as a direct consequence, that the operation which I have explained by so many examples, must react upon all our perceptions, and therefore modify the very consciousness. All that we see or hear, and every intimation of the senses, must become variously involved with suggestion,—or combined into these complex notions which I have stated as an ultimate result. This process not only supplies the *successive* trains of recollection, which will arise at the sound of a name or the sight of a place : but it will, under circumstances, identify them into that indissoluble connexion, that often gives to place its peculiar aspect, or to countenance its familiar expression. Thus it is, that to different persons, the poet, painter, geologist, or agriculturist, the same prospect of a country presents so different a scene. The whole frame of intellect and perception are altered, and all that meets the sense formed into different combinations.

In the same manner, the moral structure of the mind is affected by the same law. It would demand a separate essay to shew the precise operations by which principles recognized by the intellect, and tendencies implanted in the nature, become variously involved, so as to become inseparable in thought from circumstances, acts, and courses of conduct. For a dissertation admirably illustrative of this, I would refer to Bishop Butler's chapter on Moral Habits. I shall here content myself with pointing out an important bearing of the principle. In proportion as we act upon a determining motive, there takes place and grows a combination which identifies the motive and the action, so that the principle becomes incorporated with the moving impulse. On the other hand, the converse process takes place, when a separate attention is frequently directed to laws of conduct which are rarely carried into effect. The habit of distinctly regarding those principles and observances, in proportion as it is cultivated, tends more and more to give them separate identities in the mind; so that the exercise of the reason becomes less and less capable of moving the active tendencies of our nature.

Hitherto the examples discussed have been more viewed as means of ascertaining a result, than for any interest of their own. I should, however, not have pursued them into so detailed a discussion, were there not applications to be made of more general interest and importance.

Before entering upon the application of the theory thus arrived at, to the explanation of more complicated phenomena, it may be advisable to clear away a slight difficulty which may otherwise appear to embarrass the language which I am compelled to use in common with other writers who have taken different views. Had I adopted a purely theoretical method, this explanation must have commenced my statement, in the regular form of definitions: the method here adopted has necessarily transferred these definitions to the conclusion: they are, in fact, the questions under discussion.

In common with Mr. Stewart's, the theory here explained involves the assertion of one law of operation pursued through different stages, in each of which, its results, though in principle the same, are apparently different, and actually tend to different uses. In these different stages, this operation has acquired different names; a circumstance which, while in ordinary language it undoubtedly contributes to clearness, tends, at the same time, to baffle the metaphysical inquirer. The river which winds through a hundred realms, is distinctly referred to these varied localities, by the hundred names, which only help to confuse the general map.

The term, *association*, is here used to signify the *process* by which ideas are combined, through all the stages of this operation. It is assumed to be the tendency of the mind to recal together, and permanently combine, oft recurring ideas or phenomena. As by repetition the effect of this tendency is increased, a consequence is that it must be experienced in different stages of progress: of these are the several classes of *suggestion*, in which one idea leads to the successive recurrence of another, which has been in *some way* associated with it. The next distinguishable stage, is that which it has been the purpose of this Essay to illustrate, and which, for distinctness, I have called *combinations*, or complex ideas of that kind *which are formed by association.**

* There are two distinct classes of complex ideas; viz., those framed by association, and those acquired from the immediate constitution of things.

CHAPTER II.

THE SAME ARGUMENT ILLUSTRATED BY A MORE EXTENDED APPLICATION—THE ORATOR.

In passing from cases in which the mental process approaches nearly within the ordinary range of that class of ideas, of which no one doubts the unity, it may be necessary to proceed with new caution. Hitherto our instances have had the advantage of the important character of being free from any element, not commonly recognized in single ideas : no difficulty has arisen from their *duration*, or apparent *variation ;* all, as I have endeavoured to show, being comprehended together within the *limits of duration* which appertain to single acts of thought. This last fact is especially important to be borne in mind ; as it offers the essential characteristic by which I would ascertain the *unity* of the mental process. But when I distinguish the instances now to be explained from those already offered, the distinction is only apparent. The difference in *this* respect is only just such as to present a difficulty to the apprehension : the intellectual processes are the same, and the reasoning, were it to be distinctly followed out, would be the same. This will now, however, be the less required, as I have some trust that the elementary process has been satisfactorily ascertained ; and the far more complicated nature of the example now to be noticed would render the same method hitherto followed, both tedious and difficult, and occupy an unwarrantable length of the Academy's time.

I have already endeavoured to shew, that there can be no reason for fixing any limits to the operation of the function which is known to be so active, or which has so large an ascertained compass, as the associating faculty. From the simplest commencement of its operation, where it is *merely suggestive*, to the completion of its task, when oft-repeated association is lost in the simultaneous unity of combination : from the simple combination which invests three or four letters with a moral or physical existence, to the wide and varied array of remotely related, or even discordant notions, forms, reasons, and abstractions, which, from their compass, variety, number, and even inconstant and fleeting connexions, reject the identifying stamp of a name ; all are still subject to the operation of a subtle process which is for ever going on, the most constant as well as the most powerful of the mental functions. In this, also, essentially different from all other mental functions of which we have any distinct notion, that it is independent of all volition and consciousness; and if the illustration be allowed, that it bears to the recognizable and conscious operations of the mind a relation analogous to that which the digestive and assimilative processes bear to the voluntary powers of the frame.

There is no discoverable limit to the operation of the process here described, though it only becomes distinctly cognizable as it comes within the province of language. But before this condition is attained, and beyond the bounded compass of language, there is an endless range of unfixed, local, and transitory combinations of ideas; some belonging to real existence, and some in their nature arbitrary and unreal: all, still, in some way connected with the ordinary operations of the mind. Of this vast stock of ideal elements, the wrought and unwrought materials of thought, there is a continuous transition in the progress of association: some are connected no further than the first stage of mere suggestion—these are the ordinary masses of our casual associations, and are, by the nature of things, unlimited; some have local relations, and are peculiar to times, places, individuals, and professions—these may acquire the form of *combination* in individual minds; others, lastly, from their uniform juxta-position in reality, acquire a permanent unity, and the indissoluble stamp of a name. These last alone are universally recognized in their real character; while the unlimited multitude of casual and transitory associations, appearing in the various stages of the common process, from the remotest suggestion to the most constant identification of an inseparable unity, thus afford a *seemingly* wide scope for metaphysical discriminations and classifications—while the process throughout is uniform. In following out this varied succession of changes, there would be, however, the utmost complication, as at every point the process becomes variously subjected to the active operations of the understanding, which derives from it the entire stock of its ideas. I shall now, therefore, aim to be compendious, and for this purpose select an example which involves the utmost difficulties to which this inquiry is liable.

The intellectual habits of the public speaker have been explained by Mr. Stewart, according to the theory which I have been endeavouring to supersede. Lord Brougham has described them with the accuracy of a philosopher, and the eloquence of a consummate orator. I quote this description, which is the more to my purpose from the metaphysical propriety of the language, which seems to indicate that Lord Brougham, had his attention been specially directed to the topics here discussed, would have followed it out to the same conclusion.* "Whoever (his Lordship writes) has observed the extraordinary feats performed by calculators, orators, rhymers, musicians—nay, by artists of all descriptions, can want no further proof of the power that man derives from the contrivances by which habits are formed in all mental exertions. The performances of the Italian Improvisatori, or makers of poetry off-hand upon any presented subject, and in almost any kind of stanza, are generally cited as the most surprising efforts in this kind. But the power of *extempore speaking* is not less singular, though more frequently displayed, at least in this country. A practised orator will declaim in measured and in various periods—will weave his discourse into one texture—form parenthesis within parenthesis—excite the passions, or move to laughter—take a turn in his discourse from an accidental interruption, making it the topic of his rhetoric for five minutes to come, and pursuing in like manner the new illustrations to which it gives rise—mould his diction with a view to attain or shun an epigrammatic point, or an alliteration, or a discord ; and all this with so much assured reliance on his own powers, and with such perfect ease to himself, that he shall even plan the next sentence while he is pronouncing off-hand the one he is engaged with, adapting each to the other, and shall look forward to the topic which is to follow, and fit in the close of the one he is handling to be its introducer; nor shall any auditor be able to discover the least difference between this and the portion of his speech he has got off by heart, or tell the transition from the one to the other."

In noticing the theoretical justness of the language here used, I overlook the fact that, notwithstanding his theory, Mr. Stewart's language is equally accommodated to what I consider the truth of nature ; a fact which, indeed, leads to the reflection—how much on the surface this truth is, had it been let alone. Mr. Stewart's common sense and sagacity intrude upon his ingenuity, which I must, in fairness, observe is not the characteristic of his sound understanding, and seldom

* The slight discrepancy will be accounted for by observing, that the subject occurs but incidentally in his Lordship's discourse, and that probably the outline is suggested by the perusal of Stewart.

leads him far astray from the track of observation. And it is, indeed, almost apparent from his language, that a second and more deliberate consideration would have led him to an inference, which, though opposed to his propositions, is directly involved in all his language. He had only to ask himself the question, why—having assigned so much of the very same operations to habit and association as he manifestly does—he should stop at a certain point, and not observe the strict analogy that pervades the entire work of the mind from first to last.

As the accomptant has insensibly treasured all the usual combinations of figures; as the fluent reader similarly possesses all the usual groups of letters, syllables with their wonted sounds; as the musician has the same possession of the two classes of simultaneous and successive indications of sound; so, in the separate pursuits of life, there is, incidental to every one, a peculiar range and grouping of the materials of professional avocation, all so ready at command, and so independent of separate attention and voluntary effort, as to admit to some extent of other trains of thought being at the same time engaged in. The poetical landscape painter can, with one glance of his imagination, throw together into one single whole, all the vast and boundless varieties of observed nature; the modifications of form, colour, light, and distance are at his command: sky with its blue depths and fantastic pageantry of cloudwork, earth with its varieties of hill and dale, forest and lake, from the mountain receding into etherial distance, to the flowers and weeds which diversify and animate his foreground. These, without conscious effort, roll together like new creations, at the very caprice of a moment. Nor is this all; with equal facility the groups of life, armies, processions, and all the bustle and pageantry of civil life start up in the conception, or fill an imaginary canvas with the additional incidents of representation, the adaptations of life and proportion which deceive the eye. These combinations,—and let me say, that I would not here dwell upon such a fact, did I not believe it, in different degrees, common to all minds,—offer a wide range of the most complicated conceptions of that kind which the mind most rapidly and easily throws together with the fertility of a kaleidescope, because being mainly conversant with visible images, they demand less attention and study in their acquisition, and form a great portion of the common stock. Every one is master of a certain stock of intellectual maps of familiar places and accustomed roads, as well as pictures and portraits, which supply the office of terms. From the same compen-

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dious source arise the similarly combined groups of our more purely intellectual stores. The lawyer, together with the stock of precedents, maxims, and forensic conventions and technicalities, which are to him an habitual language and rule of reason, is also possessed of his treasury of phrase, adapted to the exigency of his profession ; as he increases in practice, they grow together by the process of association, as insensibly as the muscles of the Athlete, and acquire command by training. With these he similarly obtains the habitual command of trains of considerations, which being variously adapted to the questions that engross his understanding, offer various and new points of relation to each other. These, however varied, subtle, and remote, must, in proportion as they are liable to recur in practice, become gradually arranged by some certain index of the mind with more or less familiar combinations, and, therefore, demanding a greater or less degree of separate attention to bring them together; the less familiar demanding more distinct and separate efforts of thought, because they are either not at all, or less, involved in the common process. But still, only in proportion as the combining processes have taken place, will the operation, so lucidly described by Lord Brougham, be performed. To the more experienced mind, or the more powerful and richer intellect, vast and seemingly boundless galleries (if I may use the metaphor) of views, combined in order, and ranged in their due subordination and distance, will start at every suggestion; and trains of reasoning, which hours are insufficient to express, will be placed like a picture before the mind. Of this, too, every mind possesses its share, but it is not given to all, or even to many, to look with a length and breadth of intellectual range that might well pass for inspiration along the chain of consequence to the remote conclusion.

Every pursuit and every character of mind has its own range, in which it gathers intellectual combinations of its own, incomprehensible to most others. It is needless, and would occupy a long discussion, to dwell on these unconscious commonplaces, the ideal or verbal associations of politicians and poets, moralists and preachers. I should use one description for all; the science does not exist, nor perhaps the intellect to produce it, which could reduce so wide a scope of method, arrangement, and material, into a practical compendium. It would hold the place to thought which logic does to reasoning, or rhetoric to language.

But here it may be useful to guard against the suspicion that two distinct processes are confused. Let it be observed, that in the whole of the operations to which I have adverted, I do not exclude the operation of any other process that may be insisted upon. I simply have endeavoured to place *due bounds* to an usurpation in favour of some known faculties, and to restore to another its own due jurisdiction. I am not to be understood as excluding the separate workings of attention and volition from their very observable place in every one of the operations just noticed. But what I have contended for is reducible to the nearly self-evident fact, that in the course of all habitual thoughts, there is a point where the *separateness* of associated ideas ceases to be perceived, and I say, that at the same point these separate acts of attention and volition also cease; they are neither *necessary* nor *conceivable*, or indicated by any sign, and their assumption is, therefore, altogether gratuitous.

The orator, as he follows out the details, which appear in the perspective of his ideas, will direct the minutest attention to each as it passes in array: while he is following out this long chain, he is obviously exerting a voluntary and conscious attention to the verbal evolution of its parts. And the very same law of association which offered the first summary glance of his whole argument, operates as he proceeds, and presents similar combinations at the separate stages. With this, suggestions, which are no more than imperfect associations, are starting up in proportion to the range of the speaker's mind. But reflect what an absurd medley of processes there should be, if we admit that throughout this lengthened operation the whole chain is still retained before him by a continued succession of iterations of the same rapid series of separate attentions and volitions; the necessary consequence of Mr. Stewart's assumption, that this chain is put together by this inconceivable operation: whereas, by the explanation which has been here offered, the formed combination is already there, lying like a text-book before a lecturer, and needing no jarring dance of *imperceptible* volitions and attentions; volitions unwilled, and attentions unattended to: no inconceivable analysis to supersede and frustrate those fundamental operations to which, by Mr. Stewart's own repeated admissions, direct or implied, the very power of thinking at all is due.

The view here offered may be illustrated with some precision. Every one may be supposed to dwell within some circle of familiar localities which are variously combined in his memory. Within this compass a hundred roads and by-paths are within the instant command of his recollection, and as in conception he places himself in each successive point, a wide variety of scenic combinations spontaneously arise on his mental vision, each of them filled with different successions of locality. Strictly analogous is the intellectual horizon of the practised professional speaker, within the range and compass of his habitual associations. The analogy may be further pursued even in the failures to which either is liable, when his thoughts attempt to travel out of the accustomed range: though he may possess a general knowledge of his line of road, the traveller must lose the changing combinations, the side views, and the shifting backgrounds; while the orator, in like manner, must want the varied suggestions, and the rapid transitions, so excellently described by Lord Brougham.

His language, supplied as language is by habitual combination, will become less appropriate, flowing, and effective; and should he not have the good sense to perceive quickly the really narrow limit of his power, and take due care to keep within its scope, he will soon become embarrassed by an effort to maintain his usual superiority.

There is another not unfamiliar affection to which unaccustomed speakers are occasionally subject, which may be considered to illustrate the elementary process in a different way. When a young speaker, in his great and earnest anxiety, instead of yielding his mind to the spontaneous processes already described, begins to exert an enforced voluntary effort, and to look for that language in one way which should be obtained in another; a total embarrassment often seizes him, he begins to look for the path on which he should be moving, and he can see nothing more than the preconceived outline, which it had been his design to clothe variously in effective language, and with all the popular artifices of rhetoric.

In thus dwelling on the example offered in this section, I cannot but observe, that I could have selected others far more illustrative of the argument; but I have thought it fairest and most satisfactory to pursue the subject as it has been argued by Mr. Stewart and others who have fallen into his views.

CHAPTER III.

APPLICATION TO DREAMS.

In dreaming, the ideas which press themselves are either such as have been previously connected by association, or not. If they have not, Mr. Stewart's theory cannot be applied, nor will such cases be found illustrative of the mode of explanation adopted in this essay. Both, though in very different ways, involve the principle of association.

Cases of dreaming occur in which the succession of thought appears too capricious to be easily referred to any of the waking habits of most minds, and though even these may be, to a considerable extent, explained according to the law of suggestion, yet it will be apparent enough that they cannot be considered as cases of that succession of thoughts, which has become accelerated from the effect of frequent iteration. In these it must be observed, that the process is *directly contrary* to the process of waking reason. Awake—certain ideas are accompanied by a rapid combination (or acceleration), such as not only to facilitate the course of the thoughts in some established direction, but to prevent any other; whereas, in sleep, the occurrence of the same idea leads mostly to a *different* train, which could not well take place if the same associative (or accelerating) faculty, instead of being more alert, were not itself asleep, or nearly so; and it is very curious to observe, how the suggestions of the waking faculties change in the very process of falling asleep, so as, indeed, to indicate very clearly that the faculty which governs the connexion of our thoughts has partially at least resigned its office. The most familiar things take monstrous forms, and begin to play strange anties, which are to be noticed as tending to show that *particular* operation of habit, on which Mr. Stewart relies for his solution, to be diminished, and rendered comparatively inert in sleep, just as the other faculties are.

Now, let us see what Mr. Stewart's notion involves. The associating faculty acts in sleep with *increased energy*, and according to a *new law*.

First, it acts with increased energy, or in other words, is more awake in sleep. When awake it can only read, play the piano, or execute such operations as it has learned from *repetition;* but asleep it acquires the power of accelerating all those thoughts over which it has no such power when awake; it can compose new novels with a rapidity unknown to Scott, and dramatize them with a facility beyond the joint efforts of Shakspeare and Garrick. No matter with what lumbering incapacity, or what inert and floundering dulness its waking thoughts may be combined, all at once in sleep, it can take the wings of Ariel and "Put a girdle round about the earth in forty minutes," or rather in the twinkling of an eye.—So much for increased energy.

But it acts according to a *new law.* Mr. Stewart says not. He meets the objection by those solutions which I have already gone through. But if these were *even granted*, the matter is not mended. For a moment, assuming Mr. Stewart's explanations to be all correct, it will yet appear that the sleeping and waking processes have the essential difference of a new law.

According to Mr. Stewart, the process of the mind, *when awake*, becomes so rapid that separate attentions and volitions grow imperceptible; if so, how does it happen that in a case of the *same supposed process in sleep they all become distinctly perceptible and conscious?*

The romance comprising a long succession of events, occurs in an instant, but all the parts of which it is composed are (according to Mr. Stewart) so separately attended to that they could not be more observed assunder, if they actually took a long period of time. Here, then, is one difference; there is not only an increase of power, but a different mode of action.

But I have another question to ask—if the assumed rapidity of ideas *does not* escape the attention, when asleep, and *does* when awake, why is not this character at least uniform? why, in fact, is it *reversed?*

Why, in sleep, do not all the other operations of habit become similarly resolved, by separate acts of attention, into their constituent parts? If this law were to be followed out into its consequences, there could be no such thing as a dream at all; thoughts would be thus resolved into their elements, and the mind could not think even for the purpose of dreaming. The case amounts to this; when awake, the effect of habit enables the mind to pursue a succession of musical notes, so fast that it cannot have a conscious perception of their separate occurrence: when asleep, it seems to have acquired a faculty the converse of this; that is, it accelerates a succession of slow operations, which, when awake, no power of conception could so compress together in the mind; and then it actually does perceive their separate occurrence. Now I will not undertake to deny the possibility of this mode of operation, because I do not think that any thing should be denied or affirmed without proof; but I say the case is clearly different from the former examples with which Mr. Stewart has attempted to illustrate and explain it. The attention which follows and dilates into a history, the rapid phantasmagoria of the dream, should, by the same power, separate the letters of a word, and the components of all our perceptions. It is plain that any *acceleration* supposed in the former cases, must involve some process different from the latter, and that the result also is opposite.

But it is needless to grapple with a theory which rests on nothing at all; the difficulties inseparable from Mr. Stewart's solution, entirely disappear when the process of habit is rightly comprehended, and directly applied.

When a complex conception, formed, as I have already explained, by the ordinary law of habit, offers itself to the mind, it presents *one undivided and simultaneous* combination. I am now to apply this principle to that class of dreams which can be considered instantaneous: to such alone the argument of this Essay extends.

I shall here for the present assume, for the assumption does not affect the argument, that there are two classes of dreams; those which are instantaneous, and those which are not. It is of the first I am here to speak. The first and greatest difficulty affects me in common with Mr. Stewart, for whether the aggregate of ideas which passes during the explosion of a pistol shot is successive or simultaneous, it is equally hard to comprehend. They take place in *the time of a single act of thought*, and I say, that they constitute but a single act; the nature of this I have fully explained, and it only remains to point out its probable application to this case.

In looking at a familiar combination of words, the intellect receives both the ideas of their appearance and their sense, long before the eye could have noticed all the separate letters, syllables, and words. In fact, only a part is looked at; but the mind, which is slow to analyze its own operations, is impressed with the sense of having separately noted all. Now such is the case of the dreamer; to understand it, no more is necessary than to recollect the observed fact, of which every one who dreams is aware,—I mean the tendency of the mind to realize its ideas in sleep. Think of a person, and he stands before you, and with him all the most prominent associations connected with him ; these, too, appear as *objects of sense*, being *realized* to the imagination. This fact is, indeed, well worthy of attention from those ingenious writers who have investigated the subject of dreams; and if I do not greatly err, it will be found to offer the specific principle from which all its peculiar phenomena arise. The effects of imagination cease to be distinguished from the effects of sensation. The conception, or *intellectual sign*, is in the dark isolation of sleep confounded with that thing, the presence of which it *habitually* signifies ; for though the intellect is obscured, and its action partial, yet so far as it does act, it follows the same laws of action as when awake; but the direct and manifest result is an illusion easily understood. The shadows of things being thus invested with the conditions of seeming reality, and exempted from the interference both of sensation and will, lead to a natural illusion. The mind, deceived by the whole combination, judges as we judge in looking at a perspective deception ; the whole of the accessory ideas becoming similarly realized, modify the process. It is not the person *only* who appears, but the person doing some characteristic act; which act carries with it the supposition of other accessories, in which may be involved the ideas of *distance* and *succession*. Thus a few characteristic facts may compose the illusory perception of a story, just as a few characteristic touches convey the illusion of a picture to the eye. The sole difficulty, indeed, which may seem to affect the entire process, is the *apparent succession and duration ;* the duration we know to be an illusion, and the succession (without duration) is resolved precisely into the common analogy of all the other examples I have noticed. There is, indeed, no reason why the idea of duration should not follow the common law of all our ideas. When awake, there is a *real* perception which is contradictory to the *illusory* perception. Asleep, the idea is subject to the general effect already stated as a common condition of the mental operations in dreaming ; with the conception in which it happens to be involved, it becomes seemingly *realized*, and consequently becomes a distinct feature of the illusion ; the *moment* has expanded into an *age*, because it seemed to embrace the occurrences of an age. If the thought of eternity should present itself, or of infinity, the imagination becomes oppressed with some vast field of darkness, or the burthen of some endless endurance. The idea of duration is subject to the same conditions by which all other ideas are affected. There is, perhaps, no idea so apt to be held in due subordination to the reality of things; and yet every one can at once recal cases enough in which it is liable to be variously falsified in the perspective of thought. The case of dramatic fiction may, perhaps, be considered most apposite; a train of occurrences, which involves the idea of time, is presented; and though the waking man is quite cognizant of the actual state of the case, yet a *latent* but operative *impression* follows the law of *habit* more quickly than the judgment of the reason; and the conditions of a fictitious succession are sufficiently realized, to affect the imagination. To produce such illusions, in the highest perfection, is indeed the end of a subtle art, by which the poet can impose his waking dream upon the reader.

> " Qui pectus inaniter angit,
> Irritat, mulcet falsis terroribus implet,
> Ut magus: et modo me Thebis, modo ponit Athenis."

But when, in sleep, a complex conception or train of ideas (for I suppose either case), involving the idea of succession, is presented, the idea then not merely affects the imagination with a latent impression—the impression takes the form of reality, and the conception becomes affected by the elements of time and space. A picture when dreamed of is likely to assume the appearance of reality, because the artifice of perspective suggests the impression of distance; and every other combination may convey similarly some impression, which, once received as real, alters the condition of the case. And here let it be observed, there can be no controversy on the point; however it may be explained, the idea of *duration* is *unreal;* it must at once be admitted to be but *a component idea*—involved, to be sure, in a very curious manner well worthy of attention, but offering absolutely no obstacle to any theory in question. But having gained this point, it suggests a good deal.

First, were we to look no farther, it seems plain that the same explanation may be applied to any other ideas which may seem to form parts of a dream; that (to use the short cut of illustration) the dream was but as a face seen in a fire, in which a few leading lines take the shape of a familiar combination, and, though imperfect, carry with them the entire of that which they partially represent. The same process (whatever it may be) which gives visible appearance to a mere idea, may be well supposed to give visionary completeness of outline to a few random touches of thought. This, let it be observed, has a very distinct parallel in the known illusions of the pencil; a few imperfect, but characteristic, lines can be so placed, as to convey as much as the most complete representation. But sleep seems to carry the process of deception much farther. I have, for instance, frequently observed, what must have occurred to many to notice, that in sleep the mind is strangely imposed on as to resemblances. The absurdity of the most fantastic changes and representations is seldom, if ever, noticed; and if a dream of any supposed incidents be attentively called over after waking, it will be observed, that in many instances the impressions were not only *unreal* but false.

Little now remains to be said, so far as the topic of dreaming is involved in this inquiry. Our thoughts, as I have shown, present themselves in varied aggregations. In different minds the constituent ideas of the aggregation are diversified by the habits and intellectual constitution of the individual; but while these aggregations are liable to be presented in sleep as in waking, there is just one condition of difference, which, without altering any of the primary laws of thought, by direct consequence changes the entire character of the result. This condition is simply the realizing of the idea. Under this operation, the slightest and most latent impression which constituted any part of the waking association, in sleep starts into shape, and becomes an efficient and distinguishable feature of the dream. A dream may thus be considered as *a picture* presented to the sleeper's fancy, sometimes full of meaning and orderly subordination, sometimes strange, fantastic, and unaccountable; at times the object is some preconceived association, and occupies the ordinary duration of thought, but still undergoes the effect of being dramatized in all its parts, because, in fact, such a consequence is absolutely involved in its being realized; and it is thus also that those seemingly instantaneous successions arise. Again, the *actually present* scene, or circumstances, may be part of a dream: and the sleeper will then awake under the sense of reality.

I shall now end with a few remarks upon the manner in which the ordinary law of association, considered simply as *suggestive*, may be supposed to operate in a state of sleep. For this purpose it must be observed, that the action and reaction of associations are mutual, and that, therefore, in sleep, if any moral affection of the mind is, as may happen to be, induced by some fantastic cause, it will, according to the known law of habit, immediately suggest some such occurrence as would *ordinarily* have caused it; suppose, for example, the parts of the frame which would be affected by violent weeping to be acted on by some cause purely physical: now, even when awake, the moral frame of mind is in some small degree liable to the species of external action here supposed; and the fact is general; there is no train of correlative affections either between mind or body, or between the thoughts and affections of the mind, that is not liable to commence at either end of the chain. When we are awake, this liability is regulated by the action of other causes; the processes of the mind are subject to both the will and the senses, there can, therefore (generally speaking), be no illusion; the scenes and occupations of reality are before us, and all the control of the active faculties is in operation. Now, to recur to the examples just given, a person, if he is of a delicate frame, may, under the influence of some nervous affection, be, even while awake, disposed to gloomy views of affairs; but let him fall asleep—he is instantly head and ears plunged into a bottomless abyss of perils, distresses, and labours, defined or undefined, taking form in the shape of some gigantic calamity, or clouding the prospect with the obscurity of terror and inconceivable ruin. It becomes a dream, or that species of oppressive consciousness which is called a nightmare.

Now, if the images of a dream are supposed to be presented *in succession*, a very different order of phenomena from those hitherto contemplated takes place; all, however, the result of the two main principles now stated, viz., the apparent realization of the idea, and the governing law of suggestion. The general condition will be best conceived by an illustrative method of statement; but first let me impress the two points to be illustrated. The moment the thought occurs, the thing appears: and as every thing is likely to present some suggestion, no sooner does it appear than some new fancy starts to mind, so as to place the whole in a new relation to the dreamer. This may be exemplified: a person dreams of some friend who lives in a distant city; the individual at once becomes present: this individual exercises some particular calling, or has habits which characterize him; these at once are suggested and *realized;* they absolutely imply the notion of some locality, and the locality becomes present. This implies a change of place, and at once, as if his night-cap were the wishing-cap of the fairy tale, the dreamer is transported with a thought over the intervening billows or mile-stones, and without any interruptions from collisions, explosions, or upsets, is set down in the well remembered street. No sooner is he there, than his friend, who is, perhaps, a great traveller, begins the story of some adventure in returning from the continent; or not being very hospitably disposed, asks him by what road he means to go home. Instantly at the word, a rush of waters, and the wind roaring in the shrouds, salutes his ear; or he is hurled away on the Liverpool railroad; and if he had the ill luck to have looked into any of the public journals that evening, he is startled into a terrified consciousness by the explosion of a boiler, or the shock of trains rushing into collision. Such is the fantastic chainwork, in which the same laws which contribute to maintain the coherence of our waking thoughts, operate to disarrange and confuse them into the obscure phantasmagoria of dreams.

CONCLUSION.

The subject of dreams has led me somewhat beyond the strict argument of this Essay. There is, perhaps, no class of affections to which the mind is liable, so adapted for the purpose of investigation on the elementary laws of association. Mr. Stewart's chapter on the subject of dreams offers also a singularly pleasing and instructive example of that just method of philosophical induction, of which there is generally so lamentable a dearth in all inquiries respecting the intellectual faculties.

But Mr. Stewart set out with a notion, which was not merely adapted to lead him into some important errors, but altogether to shut from his view the actual law which regulates the succession of thoughts in dreaming.

I regret this the more, because, if I am not very much mistaken, I shall hereafter show, that the elementary facts illustrated in this Essay would have otherwise offered to this sound-minded inquirer, a simpler and better evidenced foundation for the whole structure and action of human reason, than has yet been fully noticed by any of those who have turned their thoughts to the subject: this I trust to be enabled to explain satisfactorily hereafter.

VI.—*Memoir of Researches amongst the inscribed Monuments of the Græco-Roman Era, in certain ancient Sites of Asia Minor.* *By* James Kennedy Bailie, D.D., *late* F.T.C.D., *and Lecturer of Greek in the University.*

Read May 9 and 23, 1842.

PART I.

THE APOCALYPTIC CITIES.

I. THERE are few departments in the extensive field of classical antiquities which have excited greater interest, or to which scholars have applied themselves with more zeal, than the philology of inscriptions; those memorials of past ages which, more intimately than perhaps any other monuments, bring us into contact with the laws, the institutions, the manners, and, it may in a certain sense be added, the languages of the civilized nations of antiquity. On this point I feel assured, that it is quite unnecessary for me to enlarge at any great length in the hearing of my present auditory, composed as it is of persons who are fully prepared by their respective studies and accomplishments, to acquiesce in the truth of what is here stated; but as it has fallen to my lot, recently, to be placed in circumstances peculiarly favourable to the giving me a somewhat clearer insight into the various details of this branch of literature than I had ever possessed before, to a juster appreciation of its value, and to the improvement of my knowledge of it, by enabling me to prosecute my studies and my researches at the very fountain-head, it will not, perhaps, be regarded in the light of a presumptuous attempt on the part of the writer of the present memoir, to endeavour, by submitting it to their consideration, to awaken a spirit of inquiry commensurate to the importance of the subject. This, in the present state of literature and literary research, it would be difficult to overrate.

The Continental philologists, particularly those of Germany, have long since devoted their attention, proverbially so unwearied, to the elucidation of these remains. Their profound and exact learning has contributed in a pre-eminent degree to its establishment, as a most valuable and interesting department of literature. They have travelled with the zeal, and deciphered with the acumen, of devoted students; or from the professor's chair have poured fresh streams of light on the sense and construction of the monumental language. I here refer especially to the Germans; and, for evidence of what I state, I deem it sufficient to mention the names of Thiersch and Creuzer, of Müller and Böckh. The "Corpus Inscriptionum Græcarum," of the last of these scholars, will long remain a monument of his industry, learning, and profound research; it affords, at the same time, a convincing demonstration of the utility of this branch of philological science; for by his exact acquaintance with it, he has been enabled to clear up many points of extreme interest in the social economy of the ancient inhabitants of Greece, which had been involved in much obscurity before. It has supplied him with an extensive and a solid basis for the construction of his most valuable work, "Über die Staatshaushaltung der Athener," an attentive perusal of which is of such essential importance in the investigation of the Attic monuments, and the study of the Attic literature.

To the third of the abovementioned names, the deceased and lamented Müller, I cannot refrain from paying the tribute of a well-merited eulogium. He also was distinguished amongst the foremost in this, as well as in other departments of Greek learning. With the genuine ardour of a Philellenist, he visited the shores of Greece, penetrated into her territory, mixed with her children, disinterred from the sepulchres in which they had lain so long entombed, the sculptured monuments of her pristine magnificence, and gave them once more meaning and life. I shall not soon forget the impressions which were made upon me when visiting one of his favourite scenes. It was at Castri, the representative of the ancient Delphi. I was conducted by his host to the site of the Apolloneum, and within an enclosed space to which he directed my attention, on the very ground of the Peribolus, I found ranged the huge masses of engraved and sculptured blocks, which by Müller's perseverance had been laid open to view. Here was labour for months; I might say more truly, for years; for the entire extent was one continued series of engraved characters; the records of the Pythian shrine for generations on generations; and yet the part which had been exposed, formed, in all probability, but a small proportion of the monuments which still remained under ground; and which the deceased scholar would doubtless, had his life been spared, have rescued, as he had done their fellows, from their present state of oblivion.

The efforts which Müller made cost him dear. A few months before my arrival at Delphi, he had been carried off by a malignant fever, which had been brought on by his incessant labours. It is said that he was engaged in preparing a history of Greece, and that this visit to her shrine had been paid in the hope of discovering amongst its vast mass of inscribed monuments, inedited materials for his projected work. Nor would his expectations have been disappointed: for the little which I was enabled to observe, and the less to glean, amongst those treasures, sufficed to convince me that a rich and abundant harvest awaits the student in that spot, whether his attention be devoted to the sacred annals of Greece, or to researches into her dialects.

The great work of Professor Böckh to which I have referred, leaves, it is true, all other publications of the same class at a vast distance behind it. It may most justly be styled a national performance, and has been executed with talent proportioned to the munificence of the government under whose auspices it has been published. It is impossible to read a page of that work without being impressed with the highest admiration of the learning and critical acumen of the author. It is a vast repertory of political and philological learning. Under the first of these heads, I comprehend all subjects which relate to civil economy, all hieratic details, all private or domestic contracts; under the second, the philology of archaic forms, as well as the more known usages of the refined dialects of Greece and its dependencies.

But justice to the merits of British scholars demands a meed of praise to be awarded to them, for having contributed in no ordinary degree to the advancement of this literature. We all are acquainted with the names of Pococke, Chandler, Chishull, Clarke, and Rose. I mention these amongst a great number of others, as the representatives of their class, but not by any means as entitled to a monopoly of the honour which is due to talent, labour, and research. The "Antiquitates Asiaticæ" of the third of these, Edmund Chishull, was a publication in all respects worthy of the character which he had already acquired by his work on theBustrophedon Inscription of Sigeum, and which had brought him into a certain degree of collision, not derogatory to his scholarship, with the illustrious Bentley. This, and the publication which succeeded it, I reckon to be, on the whole, the most important of any which had appeared on palæography before the volume of Rose, who, in redeeming the pledge which his abilities and learning had given, had the advantage of an improved state of antiquarian knowledge, and of literary correspondence of the highest order.

His learned volume, entitled "Inscriptiones Græcæ vetustissimæ," was published in 1825, at the expense of the University of Cambridge, and is enriched with prolegomena and notes, evincing considerable research, a great part of them, moreover, the fruits of his intimacy with Professor Böckh.

A kindred spirit has animated the scholars of other nations; for example, Italy and France; the first of which can recount such names as Maffei, Lanzi, Visconti, amongst her contributors to this department of learning; whilst France has had her Spon, a traveller,—and amongst her antiquarians, a Barthélemy. a Raoul de la Rochette, and a Boissonade. I refrain from naming another who certainly made considerable noise in his day, but whose archæographical exploits in the Peloponnese have handed down his name to posterity with a somewhat worse than an equivocal reputation attached to it: for it is, I believe, a matter of notoriety, that the researches of Fourmont have not benefited scholars so much as his vain and dishonest pretences have occasioned them trouble in disengaging the ore from the dross, what was truly classical and authentic from the unlearned and spurious admixture.

The character of this traveller may be sufficiently estimated from the fact, that Professor Böckh has devoted an article of much length, in his great work, to the exposure of his forgeries. Nay more, it is even reported of him, but with what truth I can only judge from hearsay, that, such was his narrow-mindedness and illiberality, he caused, in many instances, monuments to be defaced, lest succeeding travellers should profit by their inspection. This at least I can state with certainty, that some instances of this ungenerous temper have been pointed out to myself during my tour in Greece.

In concluding this part of my subject, it may be interesting to my audience for me to remark, that the educated classes of Modern Greece are directing their attention to this amongst other branches of Hellenic literature. It was my good fortune, during my stay at Athens, to become acquainted with the gentleman* who is at present employed by the Greek government as Curator of Antiquities in that metropolis, and to benefit by many interesting conversations with him on the present state of learning in Greece, and the progress of his researches. He is himself an author, having given to the public a topographical account of ancient Athens, which has been translated into several of the modern languages. He has collected, moreover, in the Acropolis and the Theseium (which were the principal scenes of my labours), a considerable number of statues, busts, reliefs, and inscribed tablets, most, if not all of which, have been published in Ephemerides, and in his own work. This consideration, however, did not deter me from prosecuting my researches in the same field, and holding a converse on Minerva's height, or within the sanctuary of the hero-god of Athens, with her jurists, her priests, her statesmen, and her warriors.

But I press forwards somewhat too rapidly. Greece, though the principal scene of my labours, was one of the last; and it is my present intention to lay before my fellow-academicians, with all the respect which is due to so learned and distinguished a body, a summary of my researches in the order in which they were conducted. I might have observed a different, and, for some purposes, perhaps a more convenient arrangement; I mean by this, a classification of the documents which I have collected, according as they related to public or to private concerns, to secular or religious, to the historical or the purely legal. Of all these I possess examples, viz., treaties, lists of magistrates, treasury accounts, temple inventories, epitaphs, with a great variety of others, which have unfortunately been so mutilated and defaced, as to afford a wide scope to the student in such matters for the exercise of his palæographical sagacity.

Now, an arrangement under these several heads presents many advantages, when the subject is made a study: and a more convincing proof of its expediency cannot be cited than from the great work of Professor Böckh, wherein the reader is at a loss which to admire most, the lucidity of the disposition or the accuracy of the details. But as the circumstances under which I appear before the Academy, and hope shortly to present myself before the public, are somewhat different from those of the mere editor, I have deemed it best to be guided in a

* Ὁ κύριος Πιττάκι.

great measure by them, that is, to follow the course of my recent travels; to conduct my hearers over the ground which I have traversed; and at my halting-places to share with them my palace, my hovel, or my tent, as the case may be; and then to unpack before them my treasures of by-gone ages, whether sought in the desert, or amidst the habitations of my fellow-men; whether surrounded by the ruins of ancient splendor, or the tombs of departed greatness; whether exposed to the chilling blasts of the alpine region, or fanned by the zephyr of the valley, or scorched with the rays of a tropical sun. Limited as I was to a certain period of absence, it was quite impossible for me to consult my ease, or the state of the weather, in making my visits to ancient sites. With but rare exceptions, I was in constant motion; I was in consequence subjected to innumerable hardships and inconveniences, from which travellers in those imperfectly civilized regions, who have time at their command, are enabled to exempt themselves. I was accordingly forced to traverse the burning plains of Asia Minor in the dog-days, and to make my visit to Greece during mid-winter, in which region I shall not soon forget the perils my health and person encountered, more especially in the interior of the Morea, where the country has been, until very lately, a perfect wilderness; and the more civilized districts of which are but slowly emerging into social life, after the terrible vengeance wreaked upon the Moreotes by the hordes of the Egyptian Pasha. Roofless dwellings, wasted fields, ruined villages, and an impoverished people bade mournful welcome to my retinue and myself, after many an hour's exposure to "the pelting of the pitiless storm" in the alpine solitudes of the Peloponnese. Nor has that scourge of Greece, under the Mûsûlman rule, the pestilence of the Klepts, been wholly banished from the country; although, thanks to an improved system of police, and some vigorous measures adopted lately by the government, the evil has been materially diminished.

The researches of which I propose to give the Academy some account at present, commenced in Asia Minor, and embraced the following sites; Ephesus, Gheyerah (the representative of Aphrodisias), Aïlah Shehir (the ancient Philadelphia); Sart, that is, Sardes; Kîrkagatch, a Turkish town on the road from Thyatira to Pergamus, and which the inscriptions found there seem to prove to have been in some way connected with Stratonicea: Ak-Hissár, which occupies part of the site of the ancient Thyatira; Pergamus; Eski-Stanpûl, the site of

Alexandria Troadis;* Becrám, the representative of Assos;† and one or two other places of minor importance, in the Troad, on the site of Roman military stations, where I collected a few Latin inscriptions.

This list, to which is to be added a small collection which I made at Smyrna, comprehends my labours in the department of inscriptions during two excursions which I made from that city; one around the churches of the Apocalypse; and another to the Dardanelles, returning by the coast to Smyrna.

Of these sites, Aphrodisias and Thyatira furnished me with by far the greatest number of inscriptions. Indeed, so numerous are the inscribed monuments in the first of these places, that the principal trouble devolving upon the traveller is a selection of the most important, or those which illustrate best the ancient records of the place. I find fifteen of these inscriptions in my notebook; but at least ten times that number solicit the attention of the antiquarian: and accordingly the curious in such matters will find, in the last published volume of Mr. Fellows' travels in those regions, a much larger collection of the inscriptions of Aphrodisias than I have made. It will be borne in mind, however, that that gentleman worked at a great *mechanical* advantage, for, avowedly unacquainted with Greek literature himself, he adopted the plan of what may be termed mechanical copying; in which way two or three sheets of the soft Turkish paper will perform in a few minutes as much work as would cost ordinary drudges, who have the misfortune to know something of the language, as many hours to get through. Any one, however, who has seen his first volume, will clearly appreciate the advantages of this method. Whenever an inscription is at all defaced, and the most valuable are generally not the least so, the thousand lines which the chisel of time has indented in it, are as faithfully represented in the mechanical counterpart, as those of the epigraph itself; a source of error most prolific, as well as vexatious, to the decipherer afterwards, when threading his way through the palæographical labyrinth.

The strangest readings have, in consequence, found their way into that part of Mr. Fellows' first volume which relates to inscriptions. His second, which has recently made its appearance, I have not had time to examine with the minuteness which it seems to deserve.

* Acts, xvi. 8, 11. † Ibid. xx. 13, 14.

Rejecting, therefore, all such contrivances for facilitating or expediting labour, my uniform method was, to make myself acquainted, in each instance which presented itself, with the import of the words, when it was at all possible for me to do so. This, after some practice, was of great utility in enabling me to abridge the trouble of a repeated inspection, as established formulæ were of constant recurrence, and the known succession of words thus at once suggested itself to the mind. In cases, where the characters were so defaced or mutilated as to afford no clue, or next to none, to the sense, my practice was to read the several tituli *orthographically*, that is, to resolve them according to the known laws of termination of their components; I mean, according as they were nouns, verbs, or particles, thus to establish what may be termed resting-places for the eye, while the hand was occupied with the task of committing the record to paper.

This method, or rather what was consequent upon it, dexterity of transcription, effected often somewhat more than a mere abridgment of labour: for it is clear, that the same law of sequence which enabled me without actual inspection to anticipate sentences, supplied me also with the means of restoring them when broken off or effaced. I have thus been frequently guided to the general import, at least, of a document, the first appearance of which was most unpromising to the copyist.

An example, or two, may not be uninteresting.

There are few formulæ of more constant recurrence, particularly in the ancient sites of Asia Minor, than epigraphs on the coffers (*σοροὶ*) in which families of distinction laid their dead. By far the finest of these I met with was one in the upper quarter of Akhissár, the ancient Thyatira, it wanting only the operculum, but the body of the sarkophagos being in perfect preservation. The name of the individual who had caused it to be constructed is recited, the spot where he had it placed, the purposes which he had in view; and then follows a prohibition to all others meddling with, or in any way making use of, the soros, under a heavy penalty, which might appear to have been twofold; but this I shall explain more fully in its proper place.

The titulus concludes with stating, that the customary formality was observed, of a copy (*ἀντίγραφον*) being deposited in the office of the registries, (*τὸ ἀρχεῖον,*) in this case, perhaps, the senate-house; with the name of the pro-consul for the time being, the date of entry, and the name of the scribe (*δημόσιος*,) or registrar, by whom the document was entered.

The study of this most valuable monument enabled me to restore, in considerable part, three inscriptions to the same effect, which I found also at Akhissár, but in a different quarter of the town, namely, the Armenian cemetery. The extent to which they had been mutilated would otherwise have made it a hopeless task, as it is the custom of that people to re-work the ancient soroi for their own sepulchral purposes, and to provide room for emblematical devices, and epigraphæ in their own dialect, without much respect to the Græco-Roman monuments. Of this I observed more than one example at Akhissár: but the most remarkable instance I met with, was in a tomb at Kûtaïeh, the representative of the Cotyaion of Pliny.* The soros from which the Armenian selected his materials had belonged to a Greek family of the highest distinction, as is evident from the style of embellishment which it still exhibits. It is now covered with Armenian devices and characters, the former of which are easily distinguished from the reliefs of the more classical era.

The possession of this epigraph (to remark in passing), has enabled me to correct one of the oversights in Mr. Fellows' first volume, which was doubtless the result of his expeditious mode of transferring inscriptions abovementioned. This it has done by furnishing me with an important name, which had unquestionably been recited in that gentleman's inscription, but has been left out by him in his appended explanation as unintelligible. But this is not all. The consideration that this name was connected with Cotyaion restored another, and an important, reading in the same inscription, a geographical one, which had been totally disfigured by his mechanical process.

One of the inscriptions which I have brought home from Smyrna, supplies an excellent example of the mode of dealing with such as have reached us in so mutilated a state, as to preclude all hope of our arriving at a knowledge of their exact import. Such tituli as these are best studied *in situ;* and the resolution to which I have adverted above, should precede the process of copying, otherwise the chances are, that the most embarrassing mistakes will ensue.

The epigraph to which I now refer, was copied by me from an irregularly

* Histor. Nat. v. 41, 1.

fractured block of marble, which has been built into the east wall of the Venetian fort of San Pietro, and consists of ten lines, each numbering from seven to ten letters. It is plain, therefore, that but a meagre fragment of the entire monument remains, and, unfortunately, without any word of so precise an import as to throw light on its subject-matter or date. This is the more to be regretted, as there is something in the *air* of the inscription, which informs us that it was of a good era; and that the monument had been destined to perpetuate some remarkable event in the history of the town, perhaps the earlier, or that previous to the Roman dynasty. There is an allusion, in the first line, to an embassy, either to or from Greece; one, in the second and third, to the free constitution of Smyrna: another reference of the same import perhaps, in the fourth and fifth; in the remaining lines, more especially the ninth and tenth, the allusions are to its allies and confederates, but whether states or personages we have no means of determining. It may be, that the concluding expressions comprise both.

The learned Society which I address, will apply these hints to specific events in the Ionian history, in which the city of Smyrna was prominently engaged. We know, in general, that intercourse with Greece Proper was constantly maintained by the Asiatic confederation; in particular, that the games formed a most important centre of union.* Again, there was the treaty with Seleucus, which is not obscurely hinted at by the abbreviator of Trogus;† lastly, there was the league formed by the citizens of Lampsacum, Alexandria Troadis, and Smyrna, in favour of the Romans against Antiochus.‡

To which of these, if to any, the fragment under consideration refers, we have but scanty materials for determining. The terms in which it concludes, ΤΟΥΣΕΥΝΟΙ . . . . . ΑΣΣΥΝΕΡ . . . . that is, *cooperators in offices of good-will, &c.*, should lead us to infer, that the states of the Ionian alliance, either in whole or in part, had been mentioned in the document: but unfortunately, not a trace of their names has been preserved. It occurred to me, when studying the inscription on the spot, that possibly it had formed part of a supplement to the provisions of the treaty with the citizens of Magnesia (*ad Sipylum*), in support of the interests of Callinicus, which has been brought over to England by the Earl of Arundel, and the student of such matters will find published at

* Pausan. v. 8, 2. † Hist. xxvii. 2. ‡ Liv. Hist. xxxiii. 38; xxxv. 42.

length in Prideaux's volume.* The characters are certainly sufficiently antique to countenance this, or even the supposition of an earlier date : but beyond conjecture we have no data for proceeding.

Its allusions however, general as they are, cannot fail of inspiring much interest. In the hope of eliciting something more definite, I searched, in company with a gentleman of Smyrna, who most kindly attended me through the city, in every accessible quarter of the building, for the remainder of the monument, but without success. The rude hands of the semi-barbarous constructors of the fortress had, in all probability, consigned it to perpetual obscurity in laying the under-courses of the masonry. The portion which they had placed within sight, had been so chipped and otherwise defaced in the progress of the work, that it is probable, had the expeditious process of copying it been resorted to, the result would have exhibited an unintelligible mass of confusion.

There was some degree of inconvenience attendant on the study *in situ*, as the marble was at least five-and-twenty feet above the street-level, and I was obliged to employ a ladder placed against one of the buttresses, in order to obtain a sufficiently close inspection of its contents. This was in a densely inhabited quarter of the town, next the market-place ; and in a very short time I had more company with me than I could have desired. The generally received idea amongst the Turkish population is, that we explorers of ancient monuments can have no other object in encountering so much trouble for the sake of such obsolete reminiscences, than a vague notion that they point to some hidden treasure. Their cupidity is accordingly, still more than their curiosity, aroused ; and this has proved a fruitful source of the injury done by the Mahommedans to the finest treasures of the classical period :

> "Hoc fonte derivata clades
> In veterum monumenta fluxit."

My collection of inscriptions commenced, as I have said, at Ephesus. When I first reached Smyrna, having been limited by my diocesan, the Lord Primate, to an absence of but six months, it was my intention to visit the Apocalyptic sites alone, and that being effected, to return straight home. A period of sojourn

* Marmor. Oxon. p. 4, § 94, 95.

so brief, would evidently not have admitted my forming any collection worth mentioning of such treasures. I was soon, however, relieved from my fetters, by the extreme kindness of his Grace, who, in consideration of the object which I had in view, relaxed his parting injunction: for a letter that awaited my return from Pergamus, announced the gratifying intelligence, that my term of absence had been doubled; a great boon to a traveller in those regions, in which twenty-five or thirty miles is the ordinary length of a day's journey, and no facilities exist for expediting his movements beyond that limit; and, I must add also, eminently characteristic of the personage who conferred it.

I now proceed to enter somewhat more precisely into my details. I believe I have already mentioned, that the order which I mean to observe is that of my visits to the respective sites; a choice more agreeable to my recollections, and as fit as any other, perhaps, for presenting my acquisitions to the Academy. I now speak with reference to the Apocalyptic cities, reserving to myself the liberty of deviating from this rule in the case of others of less moment. I mean, however, in all cases, to classify each separate series, so as to avoid the chaotic jumble which one meets almost invariably in travellers' collections, as also, where the state of the monument at all admits it, to give a general outline of its contents.

II. Ephesus, at which celebrated site I arrived on the eighth of September, 1840, and where I commenced my labours in this department of research, furnished me with three. I could have had more, but I made choice of those which I had some reason to suppose had been little known or noticed before. I copied them from a cubical block of marble which lay half concealed in the midst of some agnus castus on the left-hand side of the road that skirts the citadel (called by the Turks *Aïasalûk*), and conducts to the lower town, if it be not a misnomer to apply the term to that wretched vestibule to the splendid ruins which overspread the valley of Coressus.

Each of the three inscriptions to which I now refer is mutilated, the introductory matter, or, as they may be termed, the preambles, being in a great measure wanting. This defect has arisen from the block of marble, on three faces of which they had been engraved, having passed, most probably, through the hands of some mason; I shall not say Turkish; for I regret to be obliged to remark, that the degenerate representatives of the ancient possessors of the country, are, for the most part, quite as ready as their masters, to appropriate to less worthy purposes the records of the civilization and the taste of their forefathers.

All these Ephesian inscriptions illustrate in the strongest manner the expressions of the sacred historian, Τίς γάρ ἐστιν ἄνθρωπος ὃς οὐ γινώσκει τὴν ἐφεσίων πόλιν νεωκόρον οὖσαν τῆς μεγάλης θεᾶς ἀρτέμιδος;* the first two having been framed with the avowed intention of enforcing and perpetuating the worship of the tutelary goddess of that celebrated emporium. Sufficient of the preliminary matter of the longest of these remains to inform us as to the grounds on which the ruling powers of Ephesus founded this and similar decrees; the document forming part of a Psephism which had been enacted by the senate and people. Its purport was to command the strict observance of the entire month Artemision, by a succession of festivals and assemblies, which are termed ἑορταὶ, ἱερομηνίαι, πανηγύρεις; the second being introduced, as appears evident, with a special reference to the Artemisiac solemnities which were ordained for a particular month. Thus the sacred month of the Nemean games, or rather the collective series of solemn observances which were enjoined as appropriate to that period, are termed by Pindar ἱερομνία νεμεάς.†

There are curious and interesting allusions in the preamble of this decree to the circumstances which we know from other sources to have existed amongst the Macedonians, the Egyptians, and the people of Laconia, namely, of their having had sacred months; the first and third, their Artemisius, for holding assemblies and celebrating feasts, called in this section of the Psephism ἐπιμήνια. I regret to observe, that the passage which completed the argument from example, by citing that of the Egyptians, has been exceedingly defaced; but sufficient has remained to enable me to determine with tolerable certainty, that this had not been forgotten, as, fortunately, the first syllable of the sacred month has escaped the ravages of the destroyer. Now, the names of the Egyptian months are perfectly well known, as are those also of the Macedonian, of which the learned Ideler has given a catalogue comparatively with the Athenian and the Syro-Macedonian.‡ In this, the Artemisius of the second of these peoples corresponds to the Munychion of the third, at least on Plutarch's authority; and

* Acts, xix. 35. † Nem. iii. 4.

‡ Handbuch der mathematischen und technischen Chronologie, Th. i.; p. 39 in Passow's Lexicon.

this again to the third of the Aratorial months, as represented in the sculptures which I saw in the Memnonium, and of which Sir Gardiner Wilkinson has given us an account in the first volume of the second series of his invaluable work.* Its name, both in his book and elsewhere,† is written *Phamenoth*. The query suggests itself, could this have been a contraction for Phtha-Amon-Thoth, a triad of Egyptian deities, and expressive of the conjunction of the intellectual with the generative and demiurgic powers? Two of the months of the season of the water-plants have been named after single divinities, Athyr and Thoth; why should not the same custom be observed in the case of a greater number, particularly as we know that it was usual for the Egyptians to form such groups? Thus, we have the triads of Thebes, Syene, Philæ, &c., the especial objects of adoration in those districts.‡

However this may be, it is certain that in the Ephesian inscription, the initial syllable of the desiderated month, which is expressly stated to correspond to the Macedonian Artemisius, is ΠΤΑ, and that the letters which are now effaced therefrom occupied a space about equal to its last two, supposing them to have been ΜΟΥΝΩΘ.

Here, however, a slight difficulty arises from the representative of Artemis in the Egyptian Pantheon having been Pasht, or as the Greeks expressed it, Bubastis. This may be met in two ways; firstly, by supposing that the framer of the decree merely intended to express the coincidence between the Artemisius of the Greeks and the Egyptian Phamenoth; for his words are, *And the most convincing proof of this religious veneration is, that the month denominated Pta* . . . . . . (by all the Egyptians) *has been called by the Macedonians and the rest, the Laconians, and the cities in their territory, Artemisius.* In the second place we may add the fact, that Pasht was a member of the great triad of Memphis, and the usual companion of Phtha, or Hephæstus, by whom she is stated in the hieroglyphic formulæ to be "the beloved."§ This makes it highly probable, that the great festival which Herodotus‖ mentions as having been celebrated at Bubastis in honour of Pasht, took place in the month of which we have been treating; and if this supposition be correct, the author of the Psephism

* Vid. pp. 377, s.

† Rosin. Antiqq. Rom. p. 954.

‡ Vid. Sir G. Wilkinson, vol. iv. p. 231.

§ Ibid. vol. iv. p. 280.

‖ Ibid, ubi supr. p. 279. Herod. ii. 59, s.

shewed great judgment in thus enforcing its provisions by an appeal to the religious usages of those who were the undoubted founders of the Greek mythological system.

I am now conducted to the second of these tituli, which is, as I have already observed, decapitated. Part, however, of the preamble remains, which was conceived in the same spirit with that of the foregoing. The observance of the Artemisiac festival is enforced by an appeal to the piety and the devotion of their predecessors; and then the decree concludes with consecrating certain days, doubtless, of the month Artemision, perhaps indeed the entire thereof, to the solemnities of that festival, during which Armistices (*ἐκεχειρίαι*) in particular were to be observed. We are further informed, that this was a decree of a grand convention, (*πανήγυρις*), the same which Thucydides terms a synodos,* and the whole concludes with the names of the Prostates, or president of the convention, and of the Agonothetes, or director of the games.† These are, Titus Aelius Marcianus Priscus, and Titus Aelius Priscus.

The next inscription, which also has been mutilated, comprises the latter half of a resolution or decree of a Panegyris in favour of some distinguished citizen, ordaining a statue (termed in the conclusion *τιμὴ*) to be erected in his honour. This is prefaced with an enumeration of his public services in the following instances; in matters which related to the panegyrical assembly, and the solemnities of the sacred month; in the establishment of what is here termed the Artemisiac Judgment (*ἡ ἀρτεμισιακὴ κρίσις*), by which I understand either the games themselves, or the court for the regulation of their details, over which the Asiarch for the time being presided; in augmentation of the prizes of the Athletes; lastly, in the erection of statues in honour of the successful candidates.

The only name preserved in this titulus is that of the individual to whom the convention had confided the office of providing for the erection of the statue, viz., L. Fænius Faustus. It might indeed be supposed that this individual had undertaken the office, of himself, and at his private cost; but I choose rather to think that he was the agent of the Panegyris, notwithstanding the use of *ἀναστήσαντος*, not *ἐπιμεληθέντος τῆς ἀναστάσεως*, as in an inscription of a similar purport which I copied at Philadelphia.

* Hist. iii. 104, μεγάλη ξύνοδος..... τῶν Ἰώνων.

† Hist. i. 127; ii. 179; vi. 127.

These Ephesian monuments cannot but be regarded as possessing much to interest us, from the notices which they contain of a prominent idolatry of the Panionian Confederacy. But interest of another order attaches to them also in the eyes of the Christian antiquarian, who will not fail to perceive in these strenuous efforts of individuals and bodies of men, marked indications of a decaying worship, and melancholy forebodings. The address of the silversmith of Ephesus* is familiar to all here, which presents so remarkable an instance of the admixture of low and sordid motives with the more elevated feelings of national vanity and pride: and doubtless, Demetrius was not only a skilful artist, but a sharp-sighted spectator of passing events. He well knew the versatile character of his fellow-citizens, and trembled for his craft; with what justice, these documents of a somewhat later era sufficiently attest: for to what are we to attribute these efforts of the heathen priesthood to reconstruct, to invest with additional solemnity, to fortify with more stringent sanctions, the worship of their tutelary, but the astounding fact, that the temple of the great goddess was fast falling into contempt, and that the magnificence of her, *whom all Asia and the world had worshipped*, was about to be destroyed? How truly the illiterate artisan predicted coming events! What a contrast his misgivings present to the assumed tone of confidence with which one of the state documents described above concludes; *inasmuch as this will conduce to the promotion of the honour of the goddess, which will continue more glorious and in higher repute, on those days, for all succeeding time!* The vaunted magnificence, and with it the decrees, of the proud Asiarchs of Ephesus, have crumbled, and are still crumbling, into dust, whilst the anticipations of her humble mechanic are inscribed in indelible characters on the ruins of her palaces and her shrines!

Between Ephesus and Laodicea, which was the next site that I wished particularly to visit, I took the road which included the towns of Aïdîn, Nâzeleh, Yeni-shehir, Gheyerah, and Seraï-kuî, which represent in their order the ancient names of Tralles, Nysa,† Antiocheia, Aphrodisias, Karoura.‡ Of these, the

* Acts, xix. 24, ss.

† Viz. according to D'Anville. Vid. Ansart, Not. in Plin. v. 29, 6. This, however, is questioned.

‡ Vid. "Visit to the Seven Churches," &c., by the Rev. Fr. V. J. Arundell, p. 73, and accompanying Map. Strab. xii. c. 8, p. 75. Tauchn.

fourth, or Gheyerah, presents highly interesting remains of temples and other public buildings, whilst inscribed monuments lie scattered on all sides in such profusion, as to render a judicious selection of their contents the chief difficulty of the traveller. I remained there for three days, during which interval I copied a considerable number of inscriptions in different quarters of the ancient site. The labour and difficulty of this operation was much enhanced by the extreme heat of the season, and my disinclination to adopt any mechanical device for curtailing either.

It is not my intention, at least for the present, to submit to the Academy the result of my sojourn at Aphrodisias, but to connect it with another series, and make these the subject of a separate memoir. I mean now to treat of those inscriptions alone which I have brought from the Apocalyptic sites, and one or two other places which lay in my road. The Aphrodisian Tituli, I mean the whole number which I found existing, would be sufficient to form a large volume of themselves.

The site of Tralles supplied me with none. I made anxious inquiries respecting them of the person who accompanied me in my excursions through the Acropolis and other quarters of the ancient town, but received the discouraging answer that all such monuments had disappeared. This gentleman (who was the Pasha's physician) chose, for obvious reasons, to convey his sentiments on this subject to me in Latin. I have a vivid recollection of his concluding words, which were uttered with strong emotion : "Lege Strabonem : ille omnia conspectui dabit : sed monumenta delevit barbara manus."

I pass over Eski-Hissár, the representative of Laodicea, and Pambûk-Kalessî, that of Hierapolis, as barren in the immediate materials of my present research. Desolation more utter and more disheartening can scarcely be conceived than that of Laodicea ; and the extraordinary vision which met my eyes at the second of those places, wholly engrossed my attention during the brief period of my stay. The remains of its baths, its temples, its amphitheatre, and more than all, the singular phenomena of its stalactitic concretions, render it one of the most interesting sites in the whole extent of Anatolia. But the feeling of utter loneliness and desolation is the same there as in the neighbouring locality of Laodicea. Not a habitation is to be seen, after the adventurous traveller has crossed the narrow ledge of rocks by which the ruins are approached from the plain of the Lycus.

The solitary Tûrkoman tending his charge, the jackal, and the viper, are now the only tenants of this once celebrated resort of the masters of the world and their Asiatic tributaries; for the saline baths of Hierapolis made it one of the most frequented watering-places in the Roman dominions.*

We shall now recross the Mæander, and penetrate the defiles of the Mesogis, on our way to Aïláh-shehir, *the fair city*, as it is called by the present possessors of the country, the representative of Philadelphia. It is usually set down in maps as Allah-shehr, that is, *the city of God;* a coincidence with its former ecclesiastical status, which, were it well-founded, would be remarkable, and which has been noticed :† but this is a mistake: the Turkish name of Philadelphia is but a variation of another which has been given by the present possessors of Asia Minor to other celebrated sites, distinguished, as the town of Attalus is, by the natural beauty of their position. I refer to the name *Ghiûzel-Hissár*, or *beautiful castle.* Thus, they call Tralles, and with the greatest justice, *Aïdîn-Ghiûzel-Hissár;* and Temnos, in the fine coast-country between the chain of the Sipylus, and the river Hermus, *Menamên-Ghiûzel-Hissár.* Philadelphia, which lies in one of the most beautiful recesses of the Tmolus, over the rich plain of the Katakekaumene, amply merits its present name.

But I must not forget my more immediate concern at present, the inscriptions of the ancient town. In these Philadelphia is by no means rich. I could discover but four or five: one on a block of marble, which now serves the town porters as a support for their loads, but had once been part of the pedestal of a statue erected in honour of a personage of consular dignity; two entaphial, and a fourth, which I discovered on the outer angle of one of those massive supports

* Vid. Plin. v. 29, 3. Strab. xiii. 4, p. 157. Tauchn.

† See the Rev. Mr. Arundell's *Visit, &c.*, p. 169. There is a strange confusion here. The author has written the name *Allah Sher*, and seems to think it capable of the double meaning: this is not the case: there are, in effect, three Turkish names, which closely approximate to each other in sound, but in meaning are quite different, which may be applied to Philadelphia, viz., *Alláh Shehir*, the city of God; *Aïláh Shehir*, the fair city; *Álla Shehir*, the red city. The second of these is the true Turkish name.

Were my classical associations to get the better of my veracity, the aspect of the Bûz-dagh (Tmolus) and of the bed of the Pactolus, would incline me to adopt the last of these. The stream still remains, at least in one sense, the Chrysorrhoas of the ancient naturalist.

that attest the former magnificence of the edifice to which they belonged, the church of St. John.

In the first of these, the name of the consular has been preserved, Flavius Archelaus Claudianus, as also that of the person to whom the erection of the statue had been confided, Glyko (or Glykis) Papias, whose rank as Bularch* is also mentioned.

The last cost me infinite pains to acquire, from its very elevated position, and the inconvenient manner in which the builder had placed the stone on which it had been engraved: I mean to explorers such as I am; for his own exigencies had compelled him to place the lines in a vertical position at the outer edge of the building. To add to my dissatisfaction, it turned out, after all the trouble I had taken to obtain possession of its contents, but a fragment, and that a meagre one, of the original composition. Sufficient, however, remained to direct my subsequent researches to its probable import. A name has been most fortunately preserved unmutilated, which is familiar to every reader of Claudian; and from the pages of his vindictive satire on the discarded favourite of Arcadius, I have been enabled to fill up the imperfect outline which the quoin of St. John's has supplied.

The name here alluded to is Eutropius, one most convenient to the purpose of the author of this epigraph, which was to bequeath to posterity a marble-graven record *in verse*, of the *courage* and *generalship* of an officer whom that courtier had employed in an important military operation. It occurs twice in the course of the inscription, which was composed in lines alternately hexameter and pentameter. Of eleven of these but the initial fragments remain, presenting only the first, or (and this in two instances alone) the first and second feet.

The historical fact which I brought to bear upon this monument, with a view to its elucidation and, if possible, restoring it, was that which has been detailed so amusingly and with such power of ridicule by Claudian, in the second of his poems against Eutropius, namely, the ill-concerted expedition of his general, the woolcomber Leo, against Tribigild, or as he is called by Claudian, Targibilus, the Ostrogothic leader, who had invaded Asia Minor, and was then occupied in devastating Pamphylia, where he had taken up a disadvantageous

* I have fully explained the import of this term (Βούλαρχος) in the commentary subjoined to my series of inscriptions of the Apocalyptic sites.

position between the Melas and the Eurymedon. By this, however, Leo failed to profit, and the result of the conflict was as might have been anticipated: he was defeated by Tribigild, and his army slaughtered or dispersed.*

The feature of the struggle which has, in my opinion, been drawn by the author of the inscription, is that where Leo terminates his career in a morass into which he is pursued, and where the poet has represented him as expiring from the mere influence of terror. This closing scene of the drama is described as follows :†

" Ast alios vicina palus sine more ruentes
Excipit, et cumulis immanibus aggerat undas.
Ipse Leo dama cervoque fugacior ibat,
Sudanti tremebundus equo; qui pondere postquam
Decidit implicitus limo, cunctantia pronus
Per vada reptabat, cœno subnixa tenaci.
Mergitur, et pingui suspirat corpore moles,
More suis, dapibus quæ jam devota futuris
Turpe gemit, quoties Hosius mucrone corusco
Armatur, cingitque sinus; . . . .
* * * * * * * * *
Ecce levis frondes a tergo concutit aura;
Credit tela Leo: valuit pro vulnere terror,
Implevitque vicem jaculi, vitamque nocentem
Integer, et sola formidine saucius efflat."

" The rest, in rude disorder hurrying, wild,
A marsh receives, full soon with corses pil'd.
Leo himself, more fleet than timid deer,
Flies on his panting steed, half dead with fear:
Anon his weight o'erpowers his courser's strength,
Who, 'tangled in the mud, with tottering length
Falls prone, and struggling in the slimy shoals
Wriggles in reptile effort, snorts and rolls,
Whilst the unwieldy bulk he bore, the pride
Of chieftains! wallows in the slimy tide,
Panting, expiring, as a well-gorg'd swine
Its gutturall screams when Hosius means to dine.
* * * * * * * * *

* Vid. Suid. in λέων, ii. p. 428. Ed. Kust. Gibbon, *Hist.* c. xxxii. p. 181.

† Lib. in Eutrop. ii. 438, ss.

The light breeze stirs the foliage in the rear;
The clash of weapons bursts on Leo's ear!
Affright performs the dreaded javelin's part,
And deals the blow which rives his dastard heart:
To vain affright he yields his parting breath,
Unconscious of a wound, and sinks in death!"

The author of the inscription has, as I conceive, availed himself of the incident of the discomfited army's betaking itself to the marsh, to represent its leader as desirous of visiting the water-nymphs of the district, whom he appears to have addressed in a mock style of supplication on behalf of this Ajax of the East.* Nor should I omit to observe, that a very unusual epithet occurs in the last verse but one, the nearest approximation I have found to which is the epithet of the hare, in a poem † of Nicander, *δερκευνής*, so beautifully descriptive of the particular habit it expresses.

Δερκεοκρήδεμνοι is that to which I now allude, and which I beg permission to translate, *ogling through your veils;* for I regard it as applied to the nymphs, and as intended to express a not unusual attribute of the sex, in which the classical mythics have been pleased to rank these offsprings of their fancy. If this conjecture be well-founded, the restoration I have ventured to offer may, perhaps, not be regarded as very far from the sense of the original composition. But however this may be, there can be no question of the felicity of the epithet under consideration.

The inscribed monuments of Sardes, which was the next site I visited, are not more numerous than those of Philadelphia. I am confident, however, that excavations in the vicinity of that once splendid structure, usually called the temple of Cybele, but of which only two columns have been left standing, would bring to light much curious and interesting information; I may add also, near the Gerusia,‡ or Old Man's Asylum, in the ancient city. I must, however, here remark, that I apply this name to the ruin to which I at present allude, rather in accordance with the presumptions of most of those who have preceded me in this route, than with my own belief. Mr. Arundell very naturally puts

* Vid. Gibbon, ubi supr. Claudian. in Eutrop. ii. 386. *Tunc Ajax erat Eutropii, &c.*

† Alexipharm. v. 67.

‡ Vitruv. de Architect. ii. 8, p. 64.

the question, after stating the measurement of the walls, and one of the rooms, "Might not this have been the Gymnasium?" *

It was in the neighbourhood of this ruin that I discovered the inscription which is numbered the eighth in my collection. The cubical block of marble on which it was engraved lay, with the inscribed face undermost, in the open ground to the east of this edifice, and had originally, I am persuaded, been set up within the precincts of the treasury of the ancient city. It is now, as I have said, prostrate, and is used by the Tûrkoman herdsmen and the villagers of Sart as a seat, in consequence of which it has been worn down to such a degree as almost to have ceased to attract the notice of the traveller. Mine it certainly would have escaped, had it not been pointed out to me by the sûrujî (or groom), who had the care of my horses, and attended me over the ground. I lost no time in making myself acquainted with its contents, but the labour of transferring them to my note-book was very considerable, and occupied nearly the whole of the time I could spare from visiting the other objects of interest in and around the site of Sardes.

The inscription numbered the ninth was copied by me from a Turkish grave which I observed when approaching the town. It was well chosen by the Mahommedan who had pressed it into this service, as the marble fragment on which it is inscribed, had itself once formed part of a soros, or sarcophagus; but the process which it has thus undergone has deprived it of its chief interest, the names and dates having been cut away to adapt it to the dimensions of the grave.

Such, however, is not the case with first-mentioned titulus, that near the Gerusia. Sufficient of this as yet remains to acquaint us with its general import. It supplies us in its names and historical references with data of no common interest to the classical antiquarian. It appears to have been a decree, or public act of the senate and people, directing a monument (*μνημεῖον*) to be raised in honour of one of the imperial benefactors of Sardes, with whom there is some reason to suppose a lady of Lampsacus, Publia, or Papia Patricia, to have been connected in his offices of kindness and liberality towards the distressed citizens. There is, as appears to me, distinct mention made of the names of Tiberius and Trajan: and, perhaps, in the portions which have been defaced or broken off, that of

* Visit, &c. p. 180.

Hadrian had also been introduced; for it is matter of history, that this great emperor had emulated his predecessors in the succour which he had afforded to the Sardians in their emergency. This monument, therefore, refers chiefly to a period, in which this metropolis had emerged from a dreadful national calamity, or rather a succession of calamities, in consequence of the earthquakes which so frequently devastated the volcanic region of the Katakekaumene. Those which had taken place during the reign of Tiberius are expressly recorded by Tacitus,* and Dio Cassius† refers to those which had occurred in Trajan's time, but in a general way, as the attention of that historian was more especially directed to Antiocheia, where Trajan was sojourning during the season of the catastrophe. The generosity of his successor, on a similar occasion, procured him, by a decree of the Sardians, the title of Neocorus,‡ one of great honour, and much sought after during the dynasty of pagan Rome, as well by communities as by individuals. It may be translated, *Temple-warden.*

The conclusion which appears, from the indistinct notices at the close of this titulus, to be probable is, that the funds at the disposal of the priesthood had mainly contributed to the erection of this testimonial.

We are informed also in the fifth, and as appears to me, in the thirteenth line also, that Sardes enjoyed, like Pergamus and a few other cities of principal note, the title of *δὶς νεωκόρος.* This expressed a much higher grade of honour than the single Neocore,§ to which, even by itself, the generality of cities esteemed themselves fortunate in being admitted.

The characters are, it is true, considerably effaced in both the instances to which I refer, and I did not venture to supply the Lacunæ until after a most careful consideration of the text, which points at once to the readings which I have introduced.

The simple epithet, *νεὼκόρος,* appears to have occurred towards the close, namely, in the twenty-third line. This, however, might have been *δὶς νεωκόρος* also, as a very considerable hiatus precedes the first syllable, which, together with the last, is the only remaining portion of the word.

There is a fragment preserved in the eighteenth line, belonging to a word

* Annal. ii. 47.

† Hist. Rom. lxviii. 24.

‡ Vid. Rees' Cyclopæd. Art. *Sardis.*

§ Vid. Vaillant. de Numism. Græc. Rom. pp. 266, ss.

which I have not met elsewhere, that is, γαζεῖον. The question is, what does this mean? We know what γάζα or γάζη, adopted from the Persian, was,* and that from it was derived the well-known γαζοφυλάκιον.† We have, likewise, the analogy of ἀρχεῖον, *a registry office*, ταμεῖον or ταμιεῖον, *a treasury*, formed from ἀρχὴ and ταμίας, and the like. If then γαζεῖον be the legitimate restoration in this passage, the conclusion appears at least to be probable, that the public building in which this monument was directed to be set up, was none other than the celebrated treasury of Crœsus, and therefore (supposing it to have been found *in situ*), that the spot it occupies was within the precincts of that building. I mention this, because, as I have remarked already, it has been very generally supposed that the Gerusia is represented by a considerable pile, which arrests the traveller's attention somewhat further on towards the west, and in the direction of the Pactolus.

However this may be, the propriety of the use of the term γαζεῖον is quite a distinct question. Ταμεῖον is that which I have found elsewhere, as, for example, in the Thyatirene Tituli. But the Persian invasion, and subsequent dynasty, account so satisfactorily for the former, that we may well allow the Sardian scribe the use of the term, without supposing him to have affected singularity.

I hasten, however, to conclude my remarks on this document, reserving more detailed ones for a fitter opportunity. The last I shall now offer is on the use of ἀπορίαν, of which almost the entire has been preserved in the eighth line, to which I may add that of ἔνδειαν (but of this I am not equally certain), in the seventeenth. These expressions illustrate very forcibly the picture which the Roman historian of those times draws, in his own brief yet graphic style, of the depth of misery into which the Sardians had been plunged by the catastrophe that had laid waste their devoted region.

The words of Tacitus are: "*Eodem anno duodecim celebres Asiæ urbes collapsæ nocturno motu terræ: quo improvisior graviorque pestis fuit. Neque solitum in tali casu effugium subveniebat, in aperta prorumpendi, quia diductis terris hauriebantur: 'Sedisse immensos montes: visa in arduo quæ plana fuerint: effusisse inter ruinam ignes,' memorant. Asperrima in Sardianos lues plurimum in eosdem misericordiæ traxit.*"‡

* Vid. Reland. Dissert. Misc. ii. p. 184. † Comp. S. Mark. xii. 41: S. Luke, xxi. 1.

‡ Annal. ii. 47. Comp. Strab. xiii. 4, p. 154. Tauchn.

The historian then proceeds to an enumeration of the other cities which had shared in the general calamity, as also in the imperial bounty: all had their tributes remitted to them for the time, and deputies of senatorial rank appointed to visit them, and take such measures for their relief as the exigencies of their cases demanded. The Sardians, in particular, exclusively of a temporary remission of their taxes, had a large grant from the imperial treasury.

My present circumstances forbid more than brief allusions to authorities. I therefore conclude this part of my subject with referring my learned audience to Pliny,* Strabo,† the medals of Tiberius which were struck in commemoration of this event,‡ and the Marble of Pozzuolo,§ for illustration of the historical document here noticed.

My road to Ak-Hissár, the Turkish town which occupies the site of the ancient Thyatira, lay through the battle-field of Cyrus, the Lydian tumuli, the western side of the Gygæan lake, and the town of Mermera, or Marmora, which some travellers suppose to be the representative of Exusta.‖ Whilst amongst those monuments of the Alyattic dynasty, the sepulchral mounds, I did not fail to visit in particular the largest, the tomb of Alyattes, of which Herodotus has left us an account.** The view which presented itself from the summit, of the lovely region beneath, of the long range of the Tmolus, the acropolis of Sardes, the lake of Koloë, and the plain of the Hermus studded with the monuments, in an endless profusion, of the remote age of the Mermnadæ, was one which will not soon be effaced from my memory.

Whilst on the summit of the Alyattic tumulus, I recalled to mind, in particular, that part of Herodotus' description, in which mention is made of the five *οὖροι*, or *termini*, which he affirms to have been placed there, with epigraphs inscribed upon them,†† specifying the amount of labour which the classes who had been employed in the task of erecting it had severally contributed. My curiosity was accordingly much excited, when I beheld on a narrow platform on the top of the mausoleum, and imbedded in a cavity in the centre thereof, an

* Hist. Nat. ii. 86, 1.

† Vid. xiii. p. 154. Tauchn.

‡ Spanheim. de Usu et Pr. Num. Diss. ix.

§ Vid. Gronov. Dissert. viii. Ernesti, Not. in Tacit. ubi supra.

‖ Vid. Smith, referred to in Mr. Arundell's work, p. 187.

** Vid. i. 98.

†† Herod. u. s. καί σφι γράμματα ἐνεκεκόλαπτο.

irregularly formed, oblong stone, to the best of my recollection, of granite, and on which I thought that I could trace certain marks, or indentations. These, however, may have been the effects of atmospheric influences: I could form no certain conclusion respecting them: still less am I enabled to assert with any degree of confidence that the rude block which I then saw before me had been also beheld by the Father of history: I wished, however, to believe the fact, and having travelled so far to test the accuracy of Herodotus, I found it no difficult matter to enlist my convictions under the banner of my imagination.

Thyatira, to which I am now conducted, furnished me with *nine* inscriptions, most of which were copied by me in a cemetery of the Armenians, lying a little off the road to the right, as the modern town is entered from the south-east. But by far the most perfect of the number is one which I had from a sarcophagus in the upper part of Ak-Hissár, where it lies in a field belonging to the Agha, who kindly granted me an escort thither, and his permission to examine the monument. Scarcely a letter of this has sustained any injury; and as the soros itself exists in all probability *in situ*, we may infer with some degree of confidence, that certain names which it supplies, designated of old the quarter in which it is now seen by the traveller.

I have already adverted to this titulus,* but in so general a way as to afford room for a more particular specification of its contents.

The erector of the soros was a person of the name of Fabius Zosimus. The spot which he selected was an unoccupied one before the city, contiguous to the Sambatheion, within the Peribolus, or precinct of the Chaldaron (perhaps Caldarium), and alongside of the public road.

These are local designations which it would, of course, be impossible for us, possessing as we do no notices whatever of the astygraphy of Thyatira, to explain satisfactorily. We know that περίβολος means what I have stated above, a precinct of any kind, whether wall, hedge, or rampart. We also know from Seneca,† Vitruvius,‡ and the younger Pliny,§ what the Romans termed *Caldarium*, or *Caldaria Cella*. The conclusion, therefore, to which we are conducted, is, that this opulent citizen of Thyatira had chosen a place of public resort wherein to erect this family monument; perhaps, from circumstances of owner-

* Vid. p. 118.

† Epistol. lxxxvi. 9.

‡ De Architect. v. 10. p. 152.

§ Epistol. v. 6. 26.

ship, or because he was prompted by his vanity* to a public display of so beautiful a monument as even the relic which I saw proves the tomb to have been when as yet uninjured by time or barbarism.

The inscription proceeds to inform us, that this soros was destined to his own use and to that of his *sweetest spouse* (γλυκυτάτῃ αὐτοῦ γυναικὶ) Aurelia Pontiana, exclusively, no other individual being privileged to make use of it for the purpose of interment: that any infringement of this notice was to be attended with a forfeiture *to the most illustrious city of the Thyatirenes, of one thousand five hundred denaria, and to the most sacred treasury* (τὸ ἱερώτατον ταμεῖον) *of two thousand five hundred:*† in addition to which, the parties so offending were *to incur the penalties of the law against breaking into tombs* (τυμβωρυχία). It is then added: *two fair copies of this inscription have been made, one of which has been entered* (ἐτέθη) *in the registry office* (ἀρχεῖον). *Done in the most illustrious city of the Thyatirenes, in the proconsulship of Catillius Severus, on the thirteenth of the month of Audnæus, in presence of Menophilus Julianus, Registrar.*

The following observations are suggested by this extract: firstly, that there were two classes of penalties to which tomb-breakers (τυμβωρύχοι) were made liable, one affecting their property, the other their persons, or, it may be, their civil rights. We know that amongst the Romans there were express laws against the violation of the receptacles of the dead,‡ as also that this department of legislature was not neglected by the Greeks: for Cicero's words, when treating of Solon's enactments on this and other matters relative to the common weal, are, "*Pœnaque est, si quis bustum (nam id puto appellari τύμβον), aut monumentum, aut columnam violarit, dejecerit, fregerit.* §

Secondly: that the framer of the inscription defines with great exactness the legal formalities which were observed, giving also names and dates.

Of these the proconsulship of Catilius Severus is the first. This name is

* The expressions of Rosinus prove that Zosimus shared this feeling in common with his countrymen: "*Communis Romanorum sepultura in viis publicis erat ut ex epitaphiis apparet, &c.*" Antiq. Rom. v. 39. fin.

† The value of the denarius was different at different times: but fixing it at a medium of eightpence halfpenny, these sums correspond respectively to £53 2*s.* 6*d.*, and £88 10*s.* 10*d.* of our money.

‡ Vid. Rosin. Antiq. *ubi supr.*

§ De Legibus, ii. 26.

found in the Consular Fasti in conjunction with T. Aurelius Fulvus, during the reign of Hadrian, and in the year U. C. 873. He had been sent previously into Bithynia, and filled shortly after the important office of Proconsul in Syria.*

The next date is given according to the Macedonian reckoning, and corresponds, in our's, to the fifth of December, that is, supposing Ussher's computation to be correct, which agrees sufficiently well with Ideler's table referred to above,† if we take the list of congruous months in the Calendars of Macedon and Athens with which Plutarch supplies us: but here there exists some diversity of opinion, a discussion of which I postpone to a more suitable occasion; contenting myself at present with stating the Athenian Poseideon, that is, half December, half January, to be the month I have selected as answering to Audynæus.

I have been induced by the value and fine state of preservation of this sepulchral inscription, to diverge somewhat from my regular course, as it is the sixth in the order of those from Thyatira. But it has saved me the trouble of commenting at any great length on most of the others, as of the nine which I have brought away from thence, perhaps *five*, certainly *four*, are entaphial records.

The following is a list of these, and a concise account of their contents.

a. A fragment of a Latin inscription, which I am inclined to think was the titulus of a statue erected by the citizens of Thyatira in honour of the proconsul Severus, the same who is mentioned in the foregoing. The high terms of eulogy in which the historian Dio‡ has written concerning this functionary, makes it at least probable, that his administration should have been distinguished with this mark of honour. I have accordingly ventured to restore it, and in conformity with the known rules of the Roman Sigla, on this hypothesis.

The marble on which it was engraved has been built into one of the walls of the old Greek church of St. Basil in Ak-Hissár, which is now used as a mosque. The entire thereof, with the exception of the part containing my inscription, has been covered in the Turkish fashion with a coarse plaster. I attempted to dislodge as much of this as might have enabled me at least to test the accuracy of my conjecture, but the fanaticism of the Imâm was aroused, and I judged it my most prudent course to forbear.

* Dio. Hist. Rom. lxix. 14. † Vid. p. 123. ‡ Hist. Rom. *ubi supr.*

b. The next inscription was copied from a mortar, formerly part of an altar, lying in the court of the Agha's residence in a village* through which I passed on my road from Pergamus to Magnesia (*ad Sipylum*). I was informed that it had been brought by the servants of that magistrate, Kara-Osman-Oglû, from Ak-Hissár, and I have therefore given it a place in the present series.

It records an honour which had been conferred by the senate and people (of Thyatira) on a distinguished matron, named Glykinna, in consideration of the public services of her husband, Publius Aelius Aelianus.

c. The third in order is also an honorary Titulus, commemorating the deserts of a victorious prize-man in the public games. It records the erection of a statue to his honour in a conspicuous position in Thyatira. The document having been mutilated in this part, I am unable to determine the name of the place with any degree of certainty; but I am disposed to think it was the Asium,† (τὸ ἀσεῖον,) and very probably one of the gymnasia, of which there were several in the ancient town. Apollonius Justus (of the first of these I am certain, but not equally so of the last) was the name of this fortunate candidate for so envied a distinction. The inscription mentions him as having been a victor in the torch-race (λαμπαδαρχήσαντα), as having been crowned (στεφανωθέντα), and, in general, as having excelled all other competitors (πρωτεύσαντα.)

d. The fourth inscription commemorates a similar testimony in favour of a successful athlete, Menander the son of Paullus, and on the part of the youths of the first Heraclean‡ gymnasia. This I copied from a beautifully sculptured marble slab in the Armenian cemetery mentioned above. It had once, perhaps, formed part of the pedestal of a statue, out of which it had been cut to adapt it to its present, or some other position.

e. The fifth cost me much trouble to decipher, nor am I yet assured of its real import. At first I regarded it as sepulchral. This opinion I have since abandoned for another, namely, that the marble fragment on which it appears,

* Yaïâ-keuî.

† The reading ἀστεῖον, which I have conjectured in a note on this inscription, is not by any means so probable: nor is there any authority for the use of the word, as for προάστειον in Herodian. *Hist. Rom.* i. 12.

‡ Or, *dedicated to Hercules.* The words are, οἱ περὶ τὸν ἡρακλέα τῶν πρώτων γυμνασίων νεανίσκοι .... ἐτίμησαν.

formed originally part of an altar, which had been erected by a lady named Aurelia Matria, in commemoration of the issuc of a suit (probably for disregarding her rights of sepulture), between her and a person of the name of Julius Atticus. If this conjecture be well-founded, it may follow that the altar in question was one of that class which the Romans styled *Aræ amicitiæ*, for mention of which my audience is referred to Tacitus.*

f. The sixth is sepulchral, that of Fabius Zosimus, to which I have already adverted.† The soros in which it appears wants the *operculum*, or cover, but in all other respects is in complete preservation. It is of greyish coloured and very fine grained granite. The ornamental sculpture is of a very simple kind, and there are no figured devices; but the chiselling of the cornices is in the best style of art, and the characters of the inscription deep, sharp, and beautifully even.

g. The seventh inscription is also entaphial. This I found on a flat and highly-ornamented stone covering an Armenian grave, intermixed with the devices of that people, and epigraphs in their language. It formed three columns, each making a consecutive sense with that which went before, and separated from it by highly ornamented sculptures in low relief. The names of the erectors of the monuments have been abstracted by the process of adapting the slab to its present position, but in the second and concluding compartments, I have found means to restore the names, firstly of the proconsul,‡ during whose tenure of office the monument was erected; secondly, of the emperor§ who then reigned; and thirdly, of certain Romans of distinction who were, by the provisions of the inscription, either admitted to a right of sepulture in the soros, or who witnessed the execution of the instrument; or, lastly, who contributed to the decoration of the monument. These were of the family of the Annii, of which Tacitus and other writers‖ make frequent mention.

The hand of time, and the liberties taken by the Armenian owners of this grave, have rendered any elucidation of this inscription almost a hopeless task. On certain points I am not as yet satisfied: but I hope much from the coope-

* Annal. iv. 74.

† Vid. pp. 118, 136.

‡ Lollianus, or Julianus. I incline to the former, on the evidence of an inscription which I copied in the Troad.

§ Trajan.

‖ Ex. g. Josephus, *Antiqq. Jud.* xviii. 2. 2. Compare Rosin. *Antiq. Elect.* p. 904.

ration of those who are best qualified to decide on the criticism of inscriptions, when my first part shall have made its appearance.

h. The next in order is also entaphial. A lady named Aurelia Tycha erected the soros for her own use, for that of her husband Aurelius (Rufus?) for that of their sons and daughters-in-law, and lastly, of the Olnetizi, a family of distinction, most probably, at the time of its erection, in Thyatira. At the close we again meet evidence of the Macedonian origin of that town in the date which is given, namely, the eighth of Dæsius, answering to the sixth before the Nones of May in the Roman reckoning, and to the second of that month in our calendar.

i. The ninth, and last of my Thyatirene tituli, also a sepulchral document, wants the name of the founder of the monument, but compensates for this by its mentioning at the close the existence in Thyatira of a public building for registries, called the Panionian Archium (*τὸ ἀρχεῖον πανιώνιον*), thus hinting some connexion with, or it may have been, a memorial of, the celebrated confederacy which bore that name. We observe in this also the name of Trajan as designative of the month which was called after that emperor, but in a part of the stone which had sustained so much injury as to be almost illegible.

It is proper, however, to apprize my audience, that my proofs for what has been here advanced, are by no means so satisfactory as to supersede other attempts to restore the true readings. I have accordingly, in my commentary on this part, proposed another series of these, and have accompanied it with a transcript of my original copy, to enable such inquirers as may feel an interest in the present subject to judge for themselves.

Of other remains of antiquity I could discover none whatever in Ak-Hissár, with the exception of capitals of columns, friezes with architectural sculpture, and pediments, the former of which have been employed for the most part in the construction of wells, which the traveller meets in every part of Asia Minor. Altar-pieces and capitals—the latter when of sufficiently massive proportions to admit of their being used for such purposes, are the materials one chiefly finds appropriated to these works of public utility; in one respect a fortunate application of those treasures of ancient art, and infinitely preferable to using them as street-pavement, or for the substructions of dwelling-houses. The most valuable inscriptions have thus been often preserved: but woe to the luckless monument which has had the misfortune of being decorated with reliefs of the features of the illustrious dead, or of embodying an artist's ideas of a superhuman beauty! On such as these the Mûsûlman Iconoclast has invariably been sure to wreak his fanatical wrath, and often the very circumstance of their attracting the admiration of the *dogs*, the polite appellation generally bestowed on Ghiours, or Infidels, by all true disciples of Islám, has proved a powerful auxiliary of this principle. An anecdote which has been related by the accomplished Cockerell, places this in a strong light.

It is thus that the work of demolition is, I fear, in rapid progress amongst the beautiful ruins of the temple of Aphrodite, in the vicinity of which the mud huts of the villagers of Gheyerah have been clustered, with large contributions from the sculptured relics of the ancient Aphrodisias.

My road to Pergamus lay to the north-west, through Bakîr, Kîrkagatch, and Soma, leaving Bûlléneh (the representative, as I think, of the Apollonia mentioned by Strabo*) to the left, in a direction south by west. I crossed the Ghedîz (the ancient Hermus), at a point a little less than half way between Ak-Hissár and the first of these towns. The second, Kîrkagatch, was my resting place for two days; and here I found some memorials of the Carian city Stratonicecia, which have led me to believe that the Turkish town has been in some way or another connected with the Macedonian colony, most probably through immigration of Greek families.

The memorials to which I here refer, are two of three inscriptions which I copied at Kîrkagatch.

a. The first commemorates the deserts of a citizen named Diodorus Philometor, son of Nicander, who had entitled himself to the honour thus conferred upon him by his patriotism and private benevolence. It was a public act or decree of the senate and people of the Hadrianopolitan Stratoniceans on behalf of this eminent person, who is mentioned as having discharged every magisterial office (*πᾶσαν ἀρχὴν*), as well as public service (*λειτουργίαν*), on the distinction between which it is unnecessary for me to dwell, in the hearing of those whose classical remembrances will immediately suggest to them the offices of the Archon and the Trierarch amongst the ancient Athenians.

* Προϊόντι δ' ἀπὸ τοῦ πεδίου καὶ τῆς πόλεως (Pergamus) ἐπὶ μὲν τὰ πρὸς ἕω μέρη, πόλις ἐστὶν ἀπολλωνία. Strab. xiii. 4, p. 150. Tauchn.

I found this inscription in the court of a private dwelling, belonging to a Greek family, in the higher quarter of the town. It was engraved on the upper part of a small column of *verde antico*, which served, as I conceive, to support a statue of the distinguished Stratonicean whose memory has thus been preserved.

b. The next inscription was found by me in the garden of the schoolmaster (*διδάσκαλος*) of the Greek church, supporting a Maltese flower-stand. From its supplying no information with respect to the site from which it had been brought to its present position, I am not as confident of its being a relic of Stratonicea, as of the one just mentioned. Some may suppose it to have been from Athens; but then the difficulty of transport from a place beyond sea is to be taken into account; yet, on the other hand, it must be acknowledged, that immigration into Asia Minor from the part of Greece over which King Otho bears sway, has been going on to a considerable extent since the accession of that prince, whose policy has been, to say the least, very generally distasteful to the proud and versatile people over whose regeneration he has been called upon to preside. This I can vouch for from experience, having frequently, during my sojourn in his majesty's dominions, involved myself in rather unpleasant altercations with my travelling companions, whilst reading them for their good, lessons of loyalty and subordination. Changes have, however, taken place since that time; amongst these the accession of Prince Mavrocordato to the councils of the Greek government, which may check this spirit of discontent, and operate beneficially for the future.

But to leave political matters to take care of themselves, and to return to my subject. The inscription at present under consideration was in honour of the emperor Hadrian, whose titles are enumerated, namely, Cæsar, August, Pan-Hellenian, and, I believe (but here the marble has been broken), Archon. The last two are specially illustrative of this great emperor's history, to whom, for his munificence towards them, the Greeks dedicated their Pan-Hellenium, and the Athenians, in particular, paid the compliment of an investiture with their chief magistracy.* I find moreover, amongst the inscriptions which Mr. Fellows has brought from Azani, one styling Hadrian *the god* and *Pan-Hellenian.*†

* Vid. Casaub. ad Spartian. *Hadrian.* p. 7, 4. Salmas. in Spartian. p. 34, e.

† Travels in Asia Minor, vol. i. p. 144.

It was this occurrence of a part of the word ἄρχοντα in the monument now under consideration, which induced me to suppose it of Athenian origin. But as the title in question was one of which Hadrian was deservedly proud, as it was a purely honorary distinction, there may hardly seem to exist sufficient reason for considering it as designative of place, at least in any such sense as to fix that of the monument. It is quite as reasonable to suppose, that the gratitude of the people of Stratoniceia, whose city had received substantial benefits from Hadrian, and had been dignified with his name, would lead them to select whatever title they judged would be most agreeable to that emperor's vanity.*

The erector of this statue (for the marble I saw is a fragment of what had once been a pedestal), was Julia Menylleina; and her special motive has been duly recorded, namely, to express her gratitude to Hadrian for his private acts of liberality towards her father, Julius Paterculus. The inscription concludes thus: ΓΑΙΟΥΙΟΥΛΙΟΥΠΑΤΕΡΚΛΟΥΠΑΤΡΟΣΙΔΙΟΥΑ ...... ΙΔΙΟΝ† ΕΥΕΡΓΕΤΗΝ.

I should have remarked, in connexion with this subject, viz., the intercourse in kind offices which subsisted between Hadrian and the citizens of Stratoniceia, the designation of the latter in the first of these two inscriptions: they are styled *Hadrianopolitan Stratoniceans*. Their city was one of the considerable number which, as having experienced their master's bounty, he had decreed should perpetuate the memorial thereof in their names. Thus, to cite another instance, Athens, at least that section which included within its precincts his gigantic structure, the Olympium. But in these, as in other instances, first associations overruled emotions of a more recent date, and their inhabitants soon recalled the ancient designations. In the case before us we observe a sort of transition state; a species of compromise effected between the old and the new. The additional title may have been imposed also for the sake of distinction.‡

c. Having travelled so far out of my course—for these inscriptions interfere with the regular series of the other from the Apocalyptic sites—I may as well conclude my notice of them with one which I had from the mosque *Yenî-Oglû*, formerly a Greek church. It is evidently of the Byzantine era; and appears,

* Anc. Univ. History, ii. 6, p. 503. † Or ΙΔΙΩΝ.

‡ I have enumerated in my Commentary ten cities which bore the name of Hadrianopolis.

from all that I have been able to decipher of it, to commemorate the erection of a church by a pious Greek, named Evander, whose virtues, as well as the character of his spouse, Aurelia Echneëa, are eulogized in language made up of extracts from the Iliad and Odyssey.

Whilst at Soma, on my road from Kîrkagatch to Pergamus, I met with a few inscriptions, but of such little importance as by no means to repay the trouble of committing them to my note-book. Some of these may be found in the first of Mr. Fellows' volumes of his recent travels in Asia Minor. I may say, indeed, that for this time at least my search after these remains had been arrested, as during my stay at Kîrkagatch I had been incapacitated for carrying my first design into execution, which was to include the Troad in my tour, by one of those mishaps which are ever likely to betide a traveller amongst the Greek population whether of Asia Minor or Greece Proper. In short, I was deprived of the means of doing so by the dishonesty of the persons with whom I lodged; to make the matter worse, Zantiote Greeks, and, therefore, in some sort fellow-subjects, and residing within the district of the Mûtsellîm of Pergamus, under a protection from the British Consul at Smyrna. I was accordingly forced to retrace my steps to the latter place as speedily as I could, to replace the funds of which I had been deprived.

This little disagreeable remembrance I may be pardoned for noticing for the sake of my motive in doing so, which is, to beseech those of my auditory, if such there be, who may entertain a design of penetrating into these regions, to take warning by my example, to confide less than I did in the integrity of their hosts, and keep constantly before their eyes the *Græculus esuriens*, and the *Græcia mendax* of the satirist of Aquinum.

I was not, however, prevented from visiting Pergamus, and thus completing my tour of the Apocalyptic sites. I then returned to Smyrna by the coast-road, leaving Magnesia (*ad Sipylum*) to the left. But a second excursion which I made from thence, namely, to the Dardanelles, and round by Bûnár-Bashî (usually regarded as the site of Ilium), and the Idæan region, to Pergamus, enabled me to fill up this blank. During the interval of which I speak, I visited also Alexandria (of the Troad), Assos, some Roman military stations, Lectos (the extreme point, to the south, of the *Priameïa regna*), Aïvalî (a town of recent date, and a conspicuous scene of action in the Greek revolution), Temnos (at least what has generally been supposed to be its site*), Magnesia (the Sipyleian), and added very considerably to my stock of inscriptions. Those of Pergamus which I now have the honour of submitting to the notice of the Academy, were, in a great measure, the fruits of this excursion. I propose, with the permission of the Council, to reserve for some future meeting, an account of my researches during this period amongst the other sites I have mentioned. Of this number, Yaiklî, a village on the road from Bûnár-Bashî to Eski-Stanpûl, where evident indications of Roman colonization meet the traveller's view on all sides, Eski-Stanpûl itself, the representative of Troas, and Beëram, that of Assos, furnished the greater part.

a. The Pergamene inscriptions are seven in number; four of the antebyzantine age; two of that period; and one of comparatively very recent date, in the modern language and style of writing. I copied it from the upper part of the architrave of the church of St. Theodore solely as a matter of curiosity, and submit a fac-simile which, I may observe, it was exceedingly difficult to take, from the intricacy of the character and the abbreviations employed by the engraver. The date of this is 1653, A.C. Those of the inscriptions of the Byzantine period are, respectively, 1433 and 1461.

b. One of these, the latest, was a testimony of affection on the part of a lady named Aelia Noma, towards a person of the other sex, of the name of Aelius Isidotus, but whether her husband, or in what degree related, is not mentioned. The just tribute to the virtues of his private character is not forgotten: and here, we may remark in passing, the peculiar and vitiated taste of the age is manifest. From the commencement of one of his names, Isidotus, and the terminating syllables of his professional title, Geometres, a sort of medley is formed to express his moral accomplishments, as will be evident to any one who compares the fourth with the two preceding lines.

c. Indeed, something analogous to the same taste may be observed in the other nearly coeval inscription which accompanies it. The subject of the eulogium in this case was Nicodemus, an architect, who had at his private cost repaired and embellished a public thoroughfare in Pergamus, called the Aediles' walk, or mall (*ἀγορανόμιος περίπατος*). The hint afforded by the name of

* Compare Plin. v. 31, 8. Arundell's *Visit, &c.* p. 297.

this public-spirited individual was too obvious and too tempting a one not to be fine-drawn, and accordingly we find subjoined to it the words ἅμα δὴ ὁ καὶ νικώνεος, thus making the following sentence, *To the divine and ever-sacred artists, the architect (Julius?) Nicodemus* (that is, people-vanquisher), *and who has at the same time approved himself Niconeus* (that is, youth-vanquisher), *&c.*, an attempt at paronomasia whereby I conceive were intended to be expressed his admirable fortitude and strength of mind in contributing of his substance to promote the comfort and ensure the safety of his fellow-citizens of Pergamus.

It will be perceived, that the writer of this encomiastic sketch was also a poet, on a small scale, as he terminates it with a catalectic tetrameter of the trochaic metre: but in judging of its merits we must exercise a little charity, and suppose that the gross blunder in the sixth foot is due to the oversight of the sculptor.

One additional observation, and I shall dismiss these inscriptions. It will be noticed, that series of the same letters range in one, with the first, fourth, and last lines;* in the other, with all.† What these mean, is the question. In the grave-yard of the church of St. Theodore, already mentioned, I observed similar series in all the epitaphs. I conceive them to be numerals. In the inscription of Isidotus, I think it is clear that they point at once to the year of our era; but in that of Nicodemus, the case appears to be otherwise, as the letters, supposing them to be numeral marks, correspond to 2000, 100, 80, 6. I conclude, therefore, that the reckoning in this last is the old Roman one, *ab urbe condita,* as in the Consular Fasti: and this agrees extremely well with the internal evidence which is supplied by the similarity of their style, this showing that their dates cannot have been very far asunder. I have, therefore, referred them, in my Commentary, to the years 1433 and 1461 after our Lord.‡

d. I now proceed to the earlier tituli, the first two of which concern the Emperor Hadrian. I have placed the more perfect one, though later in its date, the first, on account of its state of preservation. It was copied by me from a large cubical block of the finest Parian marble, which I found in the possession of a Greek resident in the upper quarter of the town, and which originally supported

* Viz. ΑΥΞΑ. † Viz. ΒΡΠΣ.

‡ I have referred inscription *b* to the Byzantine period, notwithstanding its dating eight years subsequent to the fall of the empire, as so brief an interval was not sufficient to produce any perceptible change in the style of these documents.

a statue of *the lord of the earth and sea*, as Hadrian is styled in this fine monument. If the execution of the sculptor was at all in proportion to that of the engraver, the whole work must have been in the highest degree splendid. The inscription is in every respect perfect, unless a critical eye would object to the diminutive size of the O, both long and short, which was, perhaps, intentional on the part of the lapicide, and designed to produce a better effect in the ranging of the lines. Perhaps he was apprehensive of not having sufficient space in some of the lines, which certainly approached very closely to the edge of the stone, even with the precaution he used, were he to engrave the full letters. It may be, that a little negligence contributed its share to this curtailment of the fair proportions of the letters in question: it certainly somewhat offends the eye.

This titulus informs us, that the *honour*, that is, the erection of the statue, was confided by the senate and people of the twice Neocore (*δὶς νεωκόρων*) Pergamenians to the prætors (*στρατηγοῖς*) of the time being, whose names are recited; and this is preceded by a very full list of the titles of the imperial object of their gratitude, who is styled August, Chief Pontiff, seven times of Tribunicial Authority, four times of Consular, the Lord of the Earth and Sea. His adoption also by Trajan, on which Dio Cassius* has thrown so much doubt, is implied in his being intituled the grandson of Nerva.

It is well known, that the learned Dodwell has introduced into his historical Prelections† an elaborate refutation of Dio's statements on this point: as also, that more recently, the eloquent author of the Decline and Fall of the Roman Empire has attempted a solution of the problem, by supposing that Trajan had, in a season of imbecility and irresolution, yielded to the entreaties of Plotina, and by a formal act of sonship, nominated her favourite his heir.‡ This is, in effect, deciding the question against Dio, with whom it is abundantly evident, notwithstanding the sentence of encomium of her he had before penned,§ that Plotina was no especial favourite: for to her efforts on behalf of Hadrian he applies the highly equivocal expressions *ἐρωτικὴ φιλία*. Yet the Greek historian speaks in the most positive tone, stating, moreover, that he had his information from his father, a grave authority unquestionably, but yet not inaccessible, constituted as the imperial court was, to the influence of less worthy motives.

* Hist. Rom. lxix. 1.

† Prælect. xvi. pp. 506, ss.

‡ Vid. Gibbon, ch. iii. p. 89.

§ Dio. u.s. lxviii. 5.

Now, it is certain, that the document of which I have just now given an account, proves nothing: it informing us only of an act of the Pergamene authorities, at a period when there existed every possible inducement to pay court to Hadrian, without the slightest risk attending the flattery. But with what an argument would Dodwell have been furnished, as well as Gibbon, who inclines to his opinion, had he been in possession of a document of import almost precisely similar to the one I have described, a public act of the authorities of Pergamus, passed during the life-time of Trajan, and conferring an honour on Hadrian: an act wherein he is styled the son of that emperor, virtually, under the title of the grandson of Nerva?

e. Such an act is the inscription to which I now beg to direct the attention of my audience, or rather somewhat more; for I have abundant reason to believe that, independently of being styled the grandson of Nerva, Hadrian is described in the very commencement as Publius Aelius Trajanus Hadrianus.

I found the marble on which it was engraved in the court of an obscure dwelling belonging to a Greek of Pergamus, set into one of the side-walls, and half-buried in the pavement of the yard. I was obliged, therefore, in order to copy it more perfectly, to employ persons to displace the stones. It was considerably defaced, as may be observed by the frequency of the dotted lines in my copy, which mark the passages where time and accident have impaired the distinctness of the characters: but of the substantial accuracy of the translation which I now offer, I am of opinion that no reasonable doubt can be entertained. It is as follows:

Publius Aelius Trajanus Hadrianus, Pro-consul of Pergamus, and Proprætor to the Emperor Nerva Trajanus, Cæsar, Augustus, Germanicus, Dacicus, of Syrophœnicia, Commagene; Grandson of the August Nerva; Curio of Nerva; late Demarch of the Antiocheans in the territory of the Chrysorrhoatæ; the Senate and People of the Pergamenes (have honoured) through Apollonius Dionysius . . . . and Malchio, and Cephalo Artemidorus, and Dionysius Demetrius, son of Amyntas . . . .

Such is the document: the questions which involve critical inquiry, it would not be expedient under present circumstances to enter into, or discuss, with any degree of minuteness: this I have reserved for a more suitable occasion : I content myself at present with giving the result, and conclude with expressing it as my firm belief, that this titulus goes far to establish Dodwell's opinion, and Hadrian's succession to the imperial purple *jure hæreditario.* It implies the fact, that there had been some public and recognized expression, at the least, of Trajan's intention ; one of superior stringency to a mere *sponsio adoptionis* which Dodwell supposes, and sufficient to authorize both the citizens of Pergamus to bestow, and Hadrian to accept, the highest title which could be conferred on a subject of the empire.

f. The inscription which I have placed next in order, was copied by me from a cippus in the finest state of preservation in one of the by-streets of the town. This also I was obliged to get cleared of the rubbish which had accumulated around it, so as almost entirely to conceal it from view. The inscribed face lay undermost, and it was with much difficulty that I succeeded in my object of acquainting myself with its contents, in consequence of the uneasy position I was forced to assume.

This monument decorated at one time the tomb of a citizen of considerable rank, M. Julius Major Maximianus, Quæstor, Propraetor, and Aedile (*ἀγορανόμος*) of the Romans, and is a curiosity in its way, from its being accompanied with a brief description of the personal appearance of the deceased functionary, namely, that he was well-favoured and of a ruddy complexion (*εὐσχήμων καὶ πυρσός.*)

g. The last of the series at present under review was copied by me from a marble near the ruins of the church of St. John. This also had been sepulchral; but farther than its general import, it conveys no information whatever, from its having been so completely mutilated. I copied it, however, as a memorial of the Acropolis, from a most fatiguing excursion through the remains of which I had just then descended : it had been brought down to its present position by a Turkish mason, and built into the upper course of his garden wall. It was, moreover, the only monument which I found in the city of Attalus, in the language of his self-constituted heirs.

I regret to mention, that Magnesia (*ad Sipylum*), in which I remained for two days, furnished me with no documents of this kind. Not but that I am convinced it contains some, but because the general alarm which seemed to have pervaded at that time the Greek population, rendered all my inquiries fruitless. On one occasion, indeed, I was conducted by a Greek to a fountain, on the upper part of which the word κατασκευάσας gave some promise of a reward to my perseverance; but no sooner did I stop to copy it and examine the ground adjacent in the hope of making a fresh discovery, than my guide made so precipitate a retreat, as in a few moments to be out of sight.

Thus began, and thus ended my search after tituli in the city of Antiochus: but in other respects I was amply rewarded for my visit to it, for the Sipyline Magnesia is, beyond all comparison, the most beautiful city I beheld in Asia Minor.

As I am not now writing a detail of my travels, I shall conduct my audience, by a far speedier and less rugged path than I was forced to traverse, over the heights and through the defiles of the giant Sipylus to the lovely Smyrna, the place of my first sojourning and of my last, in those regions of the myrtle and the zephyr. In Smyrna it was that I enjoyed the solace of refined society and Christian fellowship after many an arduous wandering beyond the pale of European civilization.

Of its ancient splendor Smyrna possesses now but scanty remains: of the monuments, which I am at present discussing, still fewer. A fragment of a decree or treaty, for it is impossible to decide which; a custom-house regulation, a votive thanksgiving, an epitaph, the name of the dedicator or of the architect of a temple, with about a half dozen other tituli, and some of these of the age of the lower empire, are all that I have been hitherto enabled to procure.

a. I have already ventured a few observations on the first of these,* since I penned which I have come to the conclusion, that it related to certain negociations between the Romans and the cities of the Ionian Confederacy which are detailed by Polybius and Livy. Yet as I have mentioned before, the evidence for this is extremely vague and uncertain, from the meagreness of the document.

b. The next in order is a titulus which related to the department of the customs of ancient Smyrna, and by the position of the marble from which I copied it, I think myself justified in fixing the locality of the Telonium of the port. It is now in the garden of an Armenian merchant, about five hundred yards eastward from the sea shore.

* Vid. page 119.

Smyrna is styled, in the commencement of this inscription, *The Neocore city of the Smyrnæans*, ἡ νεωκόρος σμυρναίων πόλις. This serves to fix the limits of the date of the monument, namely, that it was subsequent to the reign of Tiberius, in whose time the city became a Neocore, and prior to that of Hadrian, when it was admitted a second time to the honour, and was accordingly intituled *twice Neocore* (δὶς νεωκόρος.)

Caracalla conferred subsequently a third Neocoria on this favoured town, as he did also on Ephesus.*

c. The third of my Smyrnæan tituli was copied from a column in the mosque at Bûrnabát, a country retreat of the Frank merchants to the north-east of the city, and is said to have been brought from the ancient temple of Æsculapius. It was the votive offering of a convalescent, whose recovery is attributed to the favour of the deity Meles. The word with which it commences, ὑμνῶ, implies evidently, that it was intended as a metrical composition; and in effect, by merely retrenching the last word (ποταμὸν) of the second line, which was, in all probability, the gratuitous addition of an ignorant engraver, it forms two trimeter iambic lines. Superadded to this blunder, if I may be allowed to call it such, a second has been committed by my predecessors in this department; amongst the number, by Mr. Arundell.† These gentlemen never seemed to have imagined that the inscription was metrical; much less was the true metre ascertained. The consequence has been, that the learned public have been favoured with an inscription, evidently in trimeters, with a spondee in the second seat of one of the lines.

The following is a translation of this titulus:

"I hymn the god,
(The river) Meles,
My preserver;
Now that from pestilence of all kinds,
and distemper,
I have been set free."

* See Vaillant. *Numism. Imper. Græc.-Rom.* pp, 266. ss.

† Travels, &c. in Asia Minor, vol. ii. p. 406.

It was clearly a thanksgiving, after the cessation of some epidemic sickness, from which the writer had been preserved, or if affected, had recovered.

I had contented myself at first with the transcription which I had made from Mr. Arundell's volume. But I could not resist the curiosity which I experienced, in consequence of the occurrence of the false quantity in the second line, to test that gentleman's accuracy by an appeal to the original monument. It turned out precisely as I had anticipated: the inaccuracy rests with the traveller. He is, however, perfectly correct in his disposition of the lines, which to the unpractised eye of the mere metrist appears quite extraordinary, the following incongruous assemblage having been formed: a monometer iambic, a hypercatalectic of the same, a species of hypercatalectic trochaic, but with a spondee in the first seat, another iambic redundant by one syllable; next follows a cretic, and, last of all, a pure iambic monometer.

Horace says very truly, that in poetical compositions of a certain class, however you may break up their metrical arrangement,

"Invenias etiam disjecti membra poetæ."

With regard to the poetical merits of the verses under consideration, I venture not to offer an opinion, but unquestionably the resolution has been very complete, although not very happy in its sequence of metres.

The question naturally suggests itself, to whom are we to ascribe it? To which I return for answer, doubtless to the lapicide, who had been employed by this grateful votary of the health-restoring stream. I have been often quite astonished at the unconcern which the ancient Greeks seemed to have felt about the style in which their epigraphs were engraved. They seem to have left almost every thing to their workmen; and hence the capricious assemblages of characters which occur in some, and the violations of the rules of the language which we observe in others. Yet, on the whole, the persons of this class appear to have been of a very superior order (I express myself, of course, comparatively), and by no means unfit to be entrusted with the records which were, from time to time, entrusted to their care.

One word more, suggested by the votive inscription which I have just now noticed, and I shall dismiss it. The question has frequently been asked me, are the inscriptions which you have collected original? Have they never been seen, or copied, by any one else? And the answer which I have uniformly returned has been, that the circumstance of their having been so is perfectly unimportant to me: this, for two reasons, which will, I trust, be deemed as satisfactory by my learned auditory, as they are by myself. The first is, that I have reported no documents of this kind which have not been copied either by myself, or under my immediate superintendence, from the original monuments; and the second, that I have as yet seen but few, extremely few indeed, into the copies of which errors have not found their way, whether from haste, or inattention, or the absence of requisite accomplishments on the part of travellers. These oversights are, as is manifest, best and most satisfactorily eliminated by a careful collation with the monuments themselves, just in the same way as the mistakes of editors would be remedied by authors' manuscripts, and many an ingenious reading, many a conjectural emendation, over which vanity stands elated, prove but an impotent conclusion. This I state, at the same time that I believe I can with perfect confidence assure the Academy, that many of the inscriptions which I hope to have the honour to submit to its notice, have never before been seen, or at least considered by others, so as to have become the property of the public.

The greater number of the foregoing tituli is entitled to this distinction, as also the remaining ones of the Smyrnæan series, which will be found arranged from *f* to *k* in the copy, now before the President.

All these, with the exception of two of the Byzantine age, are mere fragments, from the existing contents of which it is impossible to pronounce any thing with certainty.

f. The first was copied from a piece of marble which has been built into the wall of the Turkish barracks, adjacent to the Jewish cemetery, at the foot of Mount Pagus. It contains the first and the last three letters of the Emperor Trajan's name, and vestiges of the words *ἀγῶνες* and *ἀγωνοθέτων*. We may conclude, therefore, that the subject of it bore some reference to games instituted in honour of that benefactor of his Asiatic provinces.

g. The next was taken from a piece of mosaic pavement which had been discovered at Chalka-bûnár, the name given by the Turks to that extent of low and swampy ground where the temple of Æsculapius formerly stood. It is also known by the name of Diana's Baths. The copy which I have given is a transcript of one I had from a gentleman resident in Smyrna, who accompanied it, at the same time, with a facsimile of part of the mosaic which had come into his possession. This I have subjoined.

The first part of this inscription was in so worn and illegible a state as to preclude the possibility of extracting from it any consistent sense. The latter half is, however, easily deciphered, with a few slight alterations. We read thus: ΓΑΝΥΜΗΔΟΥΣΔΙΟΙΚΗΤΟΥΠΑΚΙΑΛΗΣΛΑΜΠΡΟΤΑΤΗΣ: from which the inference is obvious, that the titulus was either commemorative of the virtues of that officer, or that it had been inlaid at his expense for some other purpose; very probably to hand down to posterity a memorial *of the most illustrious Pakiale*, his mistress.

h. The third of this series, which was copied from a marble in the wall of a khan, or Turkish inn, opposite to the Armenian church, was evidently sepulchral; but the fragment which remains of it contains no name to assist our researches.

i. The next is, as I have stated, an inscription of the Byzantine age, and was found engraved on a marble slab in one of those Greek churches which the Turks have converted into mosques, at some distance from Smyrna. It was a monumental tribute to the memory of an archbishop named Ætherichus, and commences accordingly with the *stavrós*.

k. The last of this series was copied from a cistern which has been imbedded in the wall of the same khan where the last but one was found. I present it as a curiosity, from its strange admixture of characters, without indulging in any vague conjectures as to their precise import.

The entaphial inscriptions from Kûtaïeh, which have been subjoined to the present fasciculus, may, I believe, with some degree of certainty, be reckoned amongst the inedited ones which I have collected. They were copied from two Armenian graves in the neighbourhood of the town, closed in, as usual, with marbles abstracted from ancient soroi, and worked up so as to suit the tastes and purposes of their more recent owners.

I have drawn sketches in outline of these interesting relics, the workmanship of which sufficiently attests the rank and consideration of the family whose property they were.

The summits of both are surmounted with a circular arch, which in one is repeated at an interval of about half a foot. The curves are marked by sculpture in low relief.

The bodies of each are divided into compartments, which are, in the one I have particularly referred to, rather more numerous, and more elaborately worked. To three oblong rectangular spaces, of unequal breadths, which cross the stone, succeeds a fourth of much ampler dimensions, divided into four square compartments, with intermediate areas, on which the Armenians have sculptured some characteristic devices, relating most probably to the occupations of the deceased, but without altogether effacing the Greek ornaments. They have also introduced here, as in most of their grave-stones which I saw at Ak-Hissár, inscriptions in their language, but have used some precaution, which I should conclude arose rather from the exigency of the case, than taste, in selecting such parts of the monuments for that purpose as had not been pre-occupied by the Hellenic.

These last are, in consequence, almost perfect, and inform us of the following particulars.

Firstly; that a lady named Nanas, erected this monument for the use of her husband Apollonius, and her own, which intention was subsequently carried into effect by their sons, Apollonius and Asalius.

Secondly; that a person of the name of Andromachus Latypus, I conclude of the same family as the abovementioned, had been interred in the same soros. This name occurs in the depressed space which intervenes between two of the reliefs that run along the breadth of the stone, and immediately above the square compartments, into which its body is divided.

Thirdly; that a person called Zelas Latypus, whose name was engraved as a heading to the second stone, lay in the soros of which it formed a part; thus proving what I have stated above as to the ownership of these monuments. It is then recorded, in an intermediate space, that Domna, the daughter of Proteas and Tatias (individuals doubtless of the family of the Latypi), had done honour to the memory of her parents, that is, had fulfilled their intentions in the erection of the soros, by depositing their remains therein.

I have deemed the observation with respect to the names of the Latypi worthy of being inserted here, as it leads at once to the restoration of an inscription which Mr. Fellows has copied from a grave in the same cemetery, but in a form which, I must be pardoned for observing, it would be difficult for the original engraver to recognize.*

I mention this also in illustration of the remarks on the subject of mechanical copying with which I commenced this memoir. The particular comment I reserve for a more suitable place than the pages of an abstract like the present.

I have thus conducted the audience which I have the honour of addressing, through those celebrated localities, the bare mention of the names of which awakens emotions of the deepest kind in the Christian's heart. However interesting their records—those I mean of their heathen state—may be in themselves, as conducing to the illustration of their history, their social institutions, or their local characteristics, I must for one confess, that such are not the sole causes which invest them in my eyes with their gorgeous and attractive drapery. I may say, with truth, that I never passed an hour within their mouldering palaces, their ruined halls, their prostrate shrines, their now silent and forsaken agoræ, their theatres, or their gymnasia, without the one absorbing reflection being present to my mind, that over these the beloved apostle of the blessed Jesus had exercised a spiritual rule, that here the apocalyptic angels had preached, and that within these precincts they had received those portentous warnings which but too truly, too faithfully, preluded the fate of their communities. There is an air and a sense of indescribable grandeur in those distant solitudes (for three of their number can be called by no other name), a grandeur incomparably superior to all that civilization, art, wealth, prosperity, could have bestowed on them. How is this? We know how difficult it is in the generality of cases to subject emotions to exact measures, or to reason with a geometrical precision on their causes; but here there is no occasion for any refined disquisitions. The very causes which are every day

* Travels, &c., vol. i. pp. 127, 323.

rendering them more valueless as schools of taste and design, which are every hour depriving them of their attractiveness in the eyes of the mere architect, or the mere virtuoso, are, in those of the reader of and believer in the Bible, enhancing their interest. The gorgeous ruins of the city of Diana, the desolated courts and shrines of Laodicea, the dethroned "Sardian Queen," address his heart with eloquence immeasurably more touching and more sublime than they could have done in the fulness of their beauty and magnificence. It is their position on the threshold of those prophecies which announce the events and develop the destinies of a better and higher than a mere political world: it is the Spirit quenched, the Candlestick removed, the Hour of retribution, the utter Rejection, which come home to his heart, imperishable monuments as they are of the righteous dealing, the truth, the providence of God.

ADDENDA ET CORRIGENDA.

Page 123, line 18, *for* *ἱερομνία* *read* *ἱερομηνία.*
—— 137, — 21, *for* legislature *read* legislation.
—— 138, — 23, *for* makes *read* make.

Note on Page 147, *Line* 19.

I have expressed the last of the Greek numerals, in this inscription, by Σ, which is the letter approaching nearest to the form in the original. But, accurately speaking, not Σ, but ΣΤ (*στῖ*), is the representative of 6 in the Greek notation.

The engraver of the titulus had, I feel persuaded, the last of these in view: and the reader will please to supply it in the note on the fifteenth line at mark †, or read the numerals thus, *͵βρʹπʹϛʹ*.

1 2 3 4 5 6 7

8

Varieties of the Boomerang, from Excursions in N. S. Wales by Lt. Breton. VOL. 1. page 235

"Sabre a ricochet" From Voyage de Decouvertes aux Terres Australes par M. Peron." Atlas. tab. XXX.

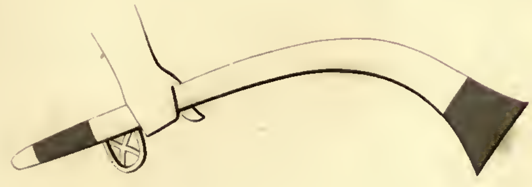
9

Combat of Hercules and Achelous Museum Etruscum V.I. tab. CXXII

10

Figure from Rosellini Mon. Civ. II Tab CXVI No. 8

11



From Rosellini Mon. Storici I Tab. XXVIII

J. Kirkwood Sc.

1

Figure from the Egyptian Monuments of S. Rosellini.
Mon. civ. II. tab. CXVII. 3.

2

Do. Do. Mon. civ. II. Tab. CXIX. 1.

4

Harpe of Perseus from a bronze in
Mus. Etrusc. V. I. tab. XXI.

3

Bas relief placé dans la salle de l'Apollon au dessus de Mercure dit l'Antinous du Belvidere
Monumens Antiques par A. L. Millin. T. I. p. 218. tab. XXIII.

5

Harpé of Saturn from a Marble in
Mus. Etrusc. V. I. tab. CXXIII.

6

Unknown British Coin Ruding pl. IV. N. 71.

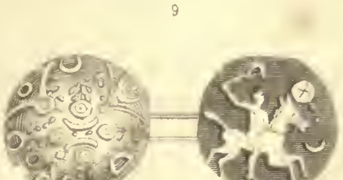
7

Reverse of Supposed Coin
of Cunobeline Stukely pl. X. N. 10

8

Ruding pl. 2. 30.

9



Unknown British Coin Stukeley pl. XIX. N. 5.

ANTIQUITIES.

ANTIQUITIES.

I. *On the Irish Coins of Edward the Fourth.* *By* Aquilla Smith, M.D., M.R.I.A.

Read 30th November, 1839.

THE study of the various coinages, which took place in Ireland, during the reign of Edward the Fourth, is peculiarly attractive, from the number and variety of his coins, which have reached our times; and the difficulties which have hitherto existed, in appropriating many of them to the exact period at which they were struck, give additional interest to the investigation.

My object in tracing the history of the coins of this reign, is, to endeavour to clear up some of the difficulties which have embarrassed our most skilful numismatists; and although I cannot pretend to remove all the obstacles which have been experienced, I trust I shall be able to bring forward some illustrations, particularly of one of the most interesting coinages of this reign, which will enable me to attain a greater degree of precision, in fixing the dates of some coinages, than has been the case heretofore.

I propose to notice, as briefly as possible, the several mintages which are described in the Acts of Parliament passed during this reign, of which we possess more records than of any of the preceding or subsequent reigns, for a long period. For this purpose, it will be convenient to divide the history of the coins into four sections, each distinguished by its peculiar type; and as there are a few coins known, of which we possess no records, except such as we derive from the pieces themselves, these will be described in connexion with the types, to which they bear the closest resemblance.

THE FIRST SECTION

Includes those coins, the type of which was peculiar to Ireland.

1461.—In the first year of this reign, at a parliament held at Dublin, it was enacted that a maille or halfpenny, and a quadrant or farthing of silver, be made in the Castle of Dublin, according to the rate of the new penny made in the last year of the reign of Henry the Sixth.* As none of these halfpence or farthings have been discovered, it is unnecessary to take any further notice of them.

1462.—In the next year, a farthing of copper, mixed with silver, was ordered to be made in the Castle of Dublin, having a crown on one side, with suns and roses in the circumferance of the crown; and on the other side, a cross, with the name of the place of mintage.† I am not aware of any of these farthings being in existence.

It appears that letters patent were granted to Germyn Lynch, of London, on the sixth of August, in the first year of Edward's reign, by which he was authorized to make coins within the Castles of Dublin and Trim, and in the town of Galway, to the tenor and effect of the statute or statutes, made by authority of a parliament held at Drogheda, in the last year of Henry the Sixth. The coins specified in the letters patent are, a groat of silver, whereof ten shall go to the ounce;‡ half groats and pennies were also authorized to be made, and a privy sign to be on every piece of silver money.§

Before I proceed to describe the coins made under the authority of the letters patent, it is necessary to refer to the statute of Henry the Sixth, according to the tenor and effect of which, Lynch was empowered to make coins.

In the year 1460, at a parliament held at Drogheda, it was enacted, that a groat should be made of the weight of three pence sterling, (forty-five grains Troy,) and to pass for four pence sterling, having on one side a crown, and on the other a cross, with the name of the place of mintage. And at an adjourned session of the same parliament, a penny of silver was ordered to be made, and to have the same impression as the groat.‖

* Simon, Appendix, No. VI.

† Simon, Appendix, No. VII.

‡ The Tower ounce = 450 grains troy.

§ Simon, Appendix, No. VIII.

‖ Simon, Appendix, No. V.

Simon has published a groat of this type, (Pl. III. fig. 61,) its weight forty-two grains, the crown is very shallow, and within a double tressure of twelve arches, in both of which particulars, it differs from the undoubted coins of Edward the Fourth, of a similar type; the ornaments at the points of the tressure are also different from those on Edward's coins. For these reasons, I appropriate this groat to Henry the Sixth.

Snelling, in his Supplement to Simon's Essay, has published a penny, (Pl. I. fig. 16,) its weight nine grains and a half, the crown is shallow, within a double tressure of twelve arches, and without ornaments at the points of the tressure. This penny I also consider as belonging to Henry the Sixth.

All the other coins, of a similar type, I appropriate to Edward the Fourth, for the following reasons. The crown on all of them is similar in form and workmanship, and very different from that on the coins just described; the double tressure round the crown consists of eight or nine arches, instead of twelve; and at each point of the tressure there are three pellets, instead of a trefoil with pointed leaves.

Of the groats there are four kinds. In the first, the crown, which is deep and broad, is within a double tressure of nine arches, with three pellets at each point of the tressure. On the reverse, a cross, with three pellets in each of its quarters; those in the first and third are connected by an annulet, (some pieces have the annulets in the second and fourth quarters of the cross;) legend, CIVITAS DVBLINIE, (Pl. I. fig. 1.) This groat weighs forty-four grains and a half, which is half a grain less than the standard: the deficiency may be accounted for by the remedy which was allowed to the mint-master, of six pence in the pound, or half a grain in each groat.*

* Simon, Appendix, No. VIII.

In every instance in which the *habitat* of the coin, if I may use the expression, is not mentioned, the reader will please to bear in mind, that the descriptions have been drawn up from coins which have been submitted to my inspection. And I avail myself of this opportunity of acknowledging my obligations, and expressing my grateful thanks, to the Very Reverend the Dean of Saint Patrick's, for the most unrestricted access to his extensive and very valuable collection, to which I am chiefly indebted for the illustrations of this paper. I am also under many obligations to the Reverend Mr. Butler, of Trim; Mr. Lindsay, and Mr. Sainthill, of Cork; and Lieutenant-Colonel Weld Hartstonge, of Dublin; for the loan of some of the rarest and most interesting coins of the Irish mints, and their permission to publish them.

The groat (Pl. I. fig. 9) has a double tressure of *ten* arches round the crown; the legend on the reverse is blundered, the s in Civitas is reversed, I is substituted for B, and an inverted L for N in Dublinie: it weighs only twenty-eight grains. The deficiency of its weight, although it is nearly as broad as the other groats, the blundered legend, the inferior workmanship, and the apparent impurity of the metal, lead me to believe that this coin is an ancient forgery.

The second kind of groat, (Pl. I. fig. 3,) differs only from the first in having three small crosses above the crown, in the angles outside the tressure; these crosses were, perhaps, privy marks, which by the letters patent were ordered to be placed on the silver coins; it weighs forty-four grains. Some minor distinctions on their reverses prove that there are, at least, three varieties of this kind.

The third kind of groat has the crown within a double tressure of *eight* arches, and a small sun in each angle outside the tressure. Reverse similar to the first kind. Weight, forty-four grains and a half.—(Pl. I. fig. 5.)*

The fourth kind differs only from the preceding one in having roses instead of suns outside the tressure. Weight, forty-two grains and a half.—(Pl. I. fig. 7.)

The suns and roses on these groats are sufficient evidence, as Mr. Lindsay remarks, that they belong to Edward the Fourth; they are the only coins of the type under consideration which he appropriates to this reign, and supposes they were coined in the first year.

Reluctant as I am to differ from so high an authority, I cannot help thinking they were coined in 1462, or early in 1463; for I have already shewn, that in 1462, a farthing of copper, mixed with silver, was ordered to be made in the Castle of Dublin, having suns and roses within (without?) the circumference of the crown; which enactment probably led to the alteration in the type of the groat. And the difference in the number of arches in the tressure may, I think, be accounted for, by supposing that the artist reduced them from nine to eight, to leave more room for the suns and roses in the angles outside the tressure. The groats of the first and second kind were probably coined in the first year of this reign.

* In Mr. Lindsay's "View of the Coinage of Ireland," a groat is described (page 39,) and engraved (Pl. V. fig. 106) as having *small roses* in the angles outside the tressure.

No half groats of the type under consideration have been discovered. Simon has published a coin, (Pl. IV. fig. 71,) which the Rev. Mr. Butler has referred to as a half groat of Henry the Sixth.* This coin is similar to the second kind of the groat which I have described; it is somewhat smaller, which has probably led to the supposition of its being a half groat; but its weight is thirty-seven grains, whereas the half groat should weigh only twenty-two grains and a half. I may also observe, that the diameter of the circle on the reverse corresponds exactly with that of the groats, which I have occasionally found very deficient in weight.

I think it is very probable that half groats of this type were never struck, notwithstanding they are mentioned in the letters patent, for the half groat was not ordered to be made by the statute of Henry the Sixth, according "to the tenor and effect" of which statute, Lynch was authorized to make coins. This opinion is supported by the fact of the half groat not appearing in either of the subsequent coinages, or previous to the year 1467.

There are pennies corresponding with the groats of the first and second kinds, (Pl. I. figs. 2, 4.) There is another which has only eight arches in the tressure; this may, possibly, be a penny of 1462; the form of the crown differs a little from the others, but it has not either the suns or roses outside the tressure.—(Pl. I. fig. 6.) The remarkable penny without the tressure, (Pl. I. fig. 8,) is, I believe, unique; I do not know of any groat similar to it. The same remark is applicable to the penny having a circle of small pellets instead of the tressure round the crown.† These pennies weigh from nine to twelve grains.

Mr. Lindsay remarks, that "the pennies of this coinage, do not appear to present any mode of distinguishing them from those of Henry the Sixth."‡ But if I am correct in my appropriation of the groats, the pennies I have noticed, all certainly belong to Edward the Fourth.

There are no coins of this type from any mint, except Dublin; and I am inclined to think that none were struck at Trim or Galway, for in the enactments of the first and second years of this reign, halfpence and farthings were ordered to be made in the Castle of Dublin *only*. The earliest coin known from the

* Numismatic Journal, vol. ii. p. 73.

† Editor's additional plate to Simon, fig. 15.

‡ View of the Coinage of Ireland, page 40.

mint of Trim, as I shall hereafter shew, was struck in the year 1467, and it does not appear that silver coins were made at any time in Galway.

It is evident that the coins I have described were minted before the year 1463, under the authority of the letters patent granted to Germyn Lynch, for in this year they were confirmed at a parliament held at Wexford, which confirmation was rather an indemnity for the coins made under the authority of the letters patent, than a renewal of the privilege for continuing a coinage of the same type; for by the same parliament, and in the same year, coins of a new type were ordered to be made.*

I proceed now to describe the brass and copper coins made under the same authority as the groats and pennies; and here again it is necessary to refer to the Act passed in the last year of Henry the Sixth.

At a parliament held at Drogheda, in 1460, it was enacted, that "a proper coyne, separate from the coyne of England, was with more convenience agreed to be had in Ireland, under two forms; the one of the weight of half-quarter of an ounce troy (Tower?) weight, on which shall be imprinted, on one side a lyon, and on the other side a crown, called an *Irelandes d'argent*, to pass for the value of one penny sterling; the other of vii. ob. (grains) of troy weight, having imprinted on one part of it a crown, and on the other part a cross, called a *Patrick*, of which eight shall pass for one denier." At an adjourned sitting of the same parliament, the former coin was declared to "be utterly void."†

The letters patent which authorized Germyn Lynch to make groats, half groats, and pennies of silver, gave him power to make "also eight pieces of brass, running at, and of the value of one penny of our said silver," and to "be imprinted, and bear scripture, and be of the weight, allaie and fyness, as is specified in the said statute or statutes" of Henry the Sixth. He was also empowered to make "four pieces of brass or copper, running at one penny of our said silver, to be imprinted with the figure of a bishop's head, and a scripture of this word PATRICIVS about the same head, on the one side, and with a cross with this word SALVATOR then (there‡) about, on the other side," and "that the weight and

* Simon, Appendix, No. VIII. † Simon, Appendix, No. V.

‡ So it is in Harris's edition of Sir James Ware's works, p. 212.

quantity of the said moneys of brass or copper be devised and made continually by the discretion of the master."*

These farthings and half-farthings were first published by the Rev. Mr. Butler, of Trim.†

The farthing has on one side a bishop's head, full face, with mitre richly ornamented; at the top, on the right side of the mitre, a sun of eight rays; on the left, a rose of six leaves; legend, PATRICIVS, divided below by the robed bust, which extends to the margin of the coin. On the other side, a cross, a sun in two of its quarters, and a rose in the alternate quarters; legend, SALVATOR, divided into four parts by the arms of the cross; suns and roses alternately between the two letters in each division of the legend: it weighs nine grains.—(Pl. I. fig. 10.)‡

Another has, at the right side of the mitre, a small cross instead of a sun; and at the left, a sun in place of a rose.—(Pl. I. fig. 11.)§

One variety of the half-farthing has, on one side, an open crown, within a circle of pellets, outside which is the word PATRIK; PA is separated from TRIK by a branch, and a similar branch is interposed between the termination and beginning of the word, and after the letter K there is a small annulet. On the other side, a cross, within a circle of pellets: it weighs eleven grains.—(Pl. I. fig. 12.)

In another, the crown is close; legend same as that just described; it has the letter P in one of the quarters of the cross on the reverse: it is corroded, and weighs nine grains.—(Pl. I. fig. 13.)

A third variety has the crown open, but of a very different form from that on the first variety; the legend, which is defaced, is evidently somewhat different from either of those described: it has not the letter P on the reverse, and weighs only seven grains.—(Pl. I. fig. 14.)

* Simon, Appendix, No. VIII.

† Numismatic Journal, vol. ii. p. 70.

‡ The coin published by Mr. Butler is represented as having a three-quarter face, owing to the imperfection of the coin from which the drawing was made.—*Numismatic Journal*, vol. ii. p. 75.

§ Fynes Moryson says, "there were lately found brass coins, by ploughing up the earth, whose stamp shewed that the bishops of Ireland had of old the privilege of coining."—*Itinerary*, Part i. Book iii. Chap. vi. vii. London, 1617.

A fourth variety has been recently discovered; it bears on its reverse a cross of a peculiar form; its weight is only six grains.—(Pl. I. fig. 15.)*

I have been particular in noting their weights, as on this ground I conclude that some of them, at least, belong to Edward the Fourth; and that all of them are not to be assigned to Henry the Sixth, for by the letters patent granted to Lynch, he was authorized to regulate their weights, at his discretion; whereas, by the Act of Henry the Sixth, the Patricks were ordered to be made of the weight of seven grains troy.

I have now described the coins comprised in the first section, the type of which was peculiar to Ireland; and proceed to notice the coins next in succession as to date, and which, from their type, may be denominated Hiberno-English.

THE SECOND SECTION,

Or Hiberno-English type, comprises those coins which bear devices peculiar to the Irish mint on the obverse, and the motto of the English mint, "Posui Deum Adjutorem Meum," on the reverse. They are of two kinds; one with the king's name and titles; the other with the king's head, name, and titles.

1463.—By the Act of the third year of Edward, which confirmed the letters patent to Germyn Lynch, a new coinage was ordered to be made, and the said Lynch was empowered to act according to the said letters, within the cities of Waterford and Limerick, during his life, in the same manner as is ordained to be done within the castles of Dublin and Trim; and that he shall make such scripture on the said coin of silver as ensues, viz., on the side of the crown, "Edwardus Dei Gratia, Dominus Hibernie;" and on the side of the cross, "Posui Deum Adjutorem Meum," together with the name of the place of mintage.†

The Dublin groat of this coinage has on the obverse a crown, within a double tressure of nine arches, trefoils at the points of the tressure, and outside it, a small annulet in each angle, all within a dotted circle; mint mark, a cross; legend, EDWARDVS DEI GRA DNS HYBERNIE, with small crosses interposed between the words. On the reverse, a cross, with three pellets in each quarter,

* From the small weight of this coin, and the remarkable form of the cross, it may possibly belong to Henry the Sixth.

† Simon, Appendix, No. VIII.

the pellets in the second and fourth quarters connected by an annulet. In the outer circle, POSVI DEVM ADIVTOREM MEV; in the inner circle, CIVITAS DUBLINIE. Weight, thirty-eight grains.—(Pl. I. fig. 16.)

There is a variety which has not the annulets in the alternate quarters of the cross, and the words on the obverse are separated by small annulets : it also weighs thirty-eight grains.—(Pl. I. fig. 18.)

The Waterford groat has on the obverse, small pellets, instead of annulets, in the angles outside the tressure; mint mark, a rose. On the reverse, it has not annulets connecting the pellets in the quarters of the cross; legend, POSVI, &c.; in the inner circle, CIVITAS WATERFORD : it weighs forty grains.—(Pl. I. fig. 20.)

These groats should weigh forty-five grains.

No half groat of this type has been discovered, nor is it to be expected.

A very fine and unique penny, resembling this type, has on one side a crown within a dotted circle; legend, EDWARD DI G DNS HYB; mint mark, a kind of lozenge, pierced in the centre. On the other side, a cross, with three pellets in each quarter; legend, CIVITAS DVBLIN : weight, nine grains and a quarter.—(Pl. I. fig. 17.)*

A fragment of a Waterford penny, the only specimen known, has the crown within a double tressure, with trefoils at its points; on the reverse, CIVITAS W . . . . . .—(Pl. I. fig. 19.)

Although this coin does not bear the king's name, like the Dublin penny, it certainly belongs to the coinage under consideration, for coins were not authorized to be made at Waterford previous to the year 1463; and besides, the trefoils, instead of pellets, at the points of the tressure, distinguish it from the coins of 1461 and 1462.

Halfpence and farthings were also ordered to be made at Waterford, but none of them have been discovered.

There are not any coins of this type known from the mints of Trim or Limerick.

1465.—A few specimens of a coinage are known, of which no record exists,

* This coin is remarkable for the absence of the tressure round the crown, yet, from its type, and bearing the king's name, it cannot be referred to any other period of this reign.

except such as the coins themselves afford, and according to the arrangement I have adopted, they must be placed in this division of the second section.

The groat has, on the obverse, a large rose of five leaves, with a small cross in its centre; there is a pellet in each angle, outside the double tressure of five arches, which surrounds the rose, all within a circle of pellets; mint mark, a cross; legend, EDWARDVS DEI GRA DNS HYBER.* Reverse, a sun of sixteen rays, having a large annulet in its centre; mint mark, a rose; in the outer circle, POSVI, &c.; in the inner circle, CIVITAS DUBLINIE. A piece of the coin is broken off, and it weighs only twenty-seven grains.—(Pl. I. fig. 22.)

The penny resembles the groat, and has not the tressure round the rose: the legend, as collected from the only two specimens which have come under my observation, is EDWAR DNS HYBER. Reverse, a sun of sixteen rays, like the groat; legend, CIVITAS DV . . . . . Weight, eight grains and a half.—(Pl. I. fig. 23.)

1465.—In the fifth year of this reign, at a parliament held at Trim, an Act was passed, the roll of which is lost; but a part of it, relating to the rise of the value of the gold noble, from eight shillings and four pence to ten shillings, is recited in the Act of the seventh year of this reign.†

Mr. Lindsay supposes that these coins were made in pursuance of the Act of 1465, an opinion which, in my mind, is strongly corroborated by the evidence furnished by the coins themselves.

The legend on the groat corresponds with that of 1463; and it is evident these coins must have been minted subsequent to that date, at which time the king's name was introduced on his Irish coins; and the absence of the king's head proves that they were minted previous to the year 1467, for in that year a new type, bearing the king's head, was ordered to be made. The rose on the obverse, and the sun on the reverse, also indicate for these coins a place in the series, between the years 1463 and 1467. In the latter year the king's head was substituted for the rose, and the sun was retained, having in its centre a small rose, instead of an annulet, as in the coins under consideration.

* The inscription on this coin is somewhat defaced; I have made up the deficiency by reference to Snelling's engraving, which has a small *rose* instead of an annulet in the centre of the sun.—*Snelling's Supplement to Simon*, Pl. I. fig. 19.

† Simon, Appendix, No. IX.

The weight of these pieces may also be adduced as evidence in favour of the date to which they are referred. It may be presumed that in 1465, when the value of the gold noble was raised one-fifth, that silver was raised in the same proportion in Ireland. And in the same year, the weight of the groat in England was reduced from sixty to forty-eight grains.*

The groat of 1463 weighed forty-five grains, and was afterwards probably reduced to thirty-six grains. The penny which I have described is well preserved, and weighs eight grains and a half, which nearly corresponds in proportion with the supposed weight of the groat; and I have already shewn that in the last year of Henry the Sixth the Irish groat was one-fourth less in weight than the English, and that the same relative weights were continued during the first three years of this reign. Hence the weight of the Irish groat of this year, which I suppose to have been thirty-six grains, still bears the same proportion to the English groat, and is exactly one-fourth less.†

It will presently appear that the value of silver was enormously raised in

* Ruding's Annals of the Coinage, vol. ii. p. 358, 2nd edit. 8vo.

† The rose was the badge of the House of York, and the sun was first introduced by Edward upon the coins. This impress he adopted in commemoration of an extraordinary appearance in the heavens, immediately before the battle of Mortimer's Cross in Herefordshire, (1461,) where three suns were seen, which shone for a time, and then were suddenly conjoined in one. As Edward was then victorious, he took a sun for his impress, which afterwards stood him in good stead at the battle of Barnet.—*Ruding's Annals of the Coinage*, vol. ii. p. 359, 2nd edit. 8vo.

" And on Ester day in the mornynge, the xiiij day of Apryl, [1471,] ryght erly, eche of them came uppone othere; and ther was suche a grete myste, that nether of them myght see othere perfitely; ther thei faughte, from iiij. of clokke in the mornynge unto x of clokke the fore-none. And dyverse tymes the Erle of Warwyke party hade the victory, and supposede that thei hade wonne the felde. But it hapenede so, that the Erle of Oxenfordes men hade uppon them ther lordes lyvery, bothe before and behynde, which was a starre withe stremys, wiche (was) myche lyke Kynge Edwardes lyvery, the sunne with stremys; and the myste was so thycke, that a man myghte not profytely juge one thynge from anothere; so the Erle of Warwikes menne schott and faughte ayens the Erle of Oxenfordes menne, wetynge and supposynge that thei hade been Kynge Edwardes menne; and anone the Erle of Oxenforde and his menne cryed 'treasoune! treasoune!' and fledde awaye from the felde withe viij. c. menne.—And so Kynge Edwarde gate the felde."—*Warkworth's Chronicle*, p. 16; edited by J. O'Halliwell, Esq.; printed for the Camden Society: London, 1839.

Ireland in 1467; and it is probable that so great a change was not suddenly adopted, but was rather preceded by the reduction I have supposed.

There is a small copper coin, of which only two or three specimens are known, and it presents some difficulties in assigning it to its proper place in this series. Obverse, a shield, bearing three crowns, two above, and one below; mint mark, a rose; legend, EDWARDVS D..... Reverse, a cross, having a small rose in its centre; and in each quarter of the cross three rays, which, with the four arms of the cross, present the appearance of a sun of sixteen rays, as on the coins of 1465; legend, CIVITAS DVBLINIE: it weighs nine grains.—(Pl. I. fig. 21.)

A coin of this type, in the cabinet of the Dean of St. Patrick's, has on the reverse CIVITAS DVBLIN; it evidently is not from the same die as the coin just described.

The value of this piece, concerning which no record has been discovered, may be supposed to have been a farthing, for its weight corresponds with that of the copper farthings minted in 1463.

Mr. Lindsay conjectures that this coin was struck about the latter end of this reign,* but the analogies of its type induce me to fix its date about the year 1467, the only period at which the sun, with a small rose in its centre, appears on the reverse of the coins of this reign. The three crowns on the shield will be explained in the fourth section.

1467. The next coinage of which any record exists, took place in the seventh year of this reign. Of this coinage, which comes within the second division of the Hiberno-English type, only one specimen was known to Simon.—(Pl. IV. fig. 72.) Snelling, in his Supplement to Simon's Essay, published four more, (Pl. I. figs. 20, 21, 22, 25,) and remarked that we had no record of them, except from the pieces themselves. Two pieces from the mint of Trim, and one of Drogheda, have been recently discovered, and have added considerably to the interest attached to this very remarkable coinage.

Mr. Lindsay is of opinion, that the coins published by Snelling were struck in 1467, as their reverses correspond with the description in the Act; and adds,

* View of the Coinage, p. 47.

that "*the obverse may have been changed by a subsequent proclamation.*"* This conjecture is not consonant with the evidence which I shall presently offer.

In the year 1467, at a parliament held in Dublin, it was enacted, as Ireland was destitute of silver, that a piece of silver called a *Double* should be coined, having on one side the print of a *crown*, with this inscription, "Edwardus Dei Gratia, Dominus Hyberniæ;" and on the other side a sun, with a rose, and the name of the place of mintage, which coin shall pass in Ireland for eight pence, and that ten such pieces shall make an ounce, according to the rightful standard of the tower of London. Groats, half groats, pence, half pence, and farthings, were also ordered; and the said coins to be made in the castles of Dublin and Trim, the cities of Waterford and Limerick, and the towns of Drogheda, Galway, and Carlingford.†

Hence it appears, that silver was at this time raised to double its former value in Ireland, for the Double was of the same weight as the groat of the last year of Henry the Sixth, according to which standard, the coinages of the three first years of this reign were regulated.

Some months ago, a coin, belonging to the Rev. Mr. Butler, of Trim, was submitted to my inspection. It has on one side the *king's head crowned, within a double tressure of nine arches;* on the other side, a sun of twenty-four rays, having a small rose in its centre: it weighs only ten grains.

The weight of this piece would lead one to suppose it was a penny, but it occurred to me that I had never seen either an English or Irish penny with the head *within a tressure;* hence I concluded that it must be a half groat of the year 1467; and as its type differed from every other coin described in the Acts of this reign, I conjectured that Simon had committed an error in transcribing the description of the *Double* in the Act of 1467.

Shortly after, I called on Sir William Betham, and mentioned to him my conjecture; he very kindly permitted me to inspect his manuscript notes from the Irish records, and immediately produced the volume containing the extract from the Act of the seventh year of Edward the Fourth. I was highly pleased to find my conjecture confirmed, for the coin called a "double" was described

* View of the Coinage, p. 41.

† Simon, Appendix, No. IX.

in Sir William's extract as "having an impression of a *face and crown* on one side," and on the other side, the device and inscription as given by Simon.*

I am also indebted to Sir William Betham for permission to publish a clause which he has transcribed, relating to the penny, half penny, and farthing, of this coinage. It states, that "in consequence of the smallness of the penny, it shall be lawful to insert the weight of ten pennies of alloy above the silver, at the king's cost, so that the eighty pennies shall weigh *an ounce and a half*, and contain the impression of the groat; and that the half pennies or farthings may be alloyed at the discretion of the Lord Lieutenant or Lord Deputy."

By this clause, it appears that the penny should weigh nearly eight grains and a half. The Act, as published by Simon, says, "Also that a piece be made, called a denier, (penny,) containing the half of the piece of two deniers, eighty of which shall go to the *ounce*, besides the alloy."†

Before I enter on the description of the coins, it is necessary to say a few words respecting the standard weight, as the writers on Irish coins have occasionally confounded the troy pound with that of the tower.

It should be recollected that the coinage of England and Ireland was regulated by the standard of the tower pound, which continued in use until the eighteenth year of the reign of Henry the Eighth, at which time it was abolished by proclamation, and the troy pound established in its stead.‡

The tower pound differed from the troy pound in weight only, being lighter by three quarters of an ounce; the denominations of their parts were the same. The troy ounce consisted of 480 grains; the tower, of only 450. It appears from the Act, that the coins of 1467 were ordered to be made "according to the rightful standard of the *tower* of London;" and consequently, the double, ten of which went to the ounce, should weigh forty-five grains.

A double groat was discovered in June, 1839, at Trim. Obverse, the king's head crowned, within a double tressure of nine arches; a trefoil, with

* This confirmation of my conjecture induced me to inquire into some other obscure points respecting Edward's coins, and ultimately led to the investigation, the fruits of which I now present.

† Simon, Appendix, No. IX.

‡ Ruding's Annals of the Coinage, vol. i. p. 18, 2nd edit. 8vo.

pointed leaves at six points of the tressure; mint mark defaced; legend, ... DVS DEI GRA DNS HYBER. Reverse, a large sun of twenty-four rays, having a small rose in its centre; legend, . ILLA DE DROG . . . . divided into four parts by suns and roses alternately: a portion of it has been broken off, and it weighs only thirty-eight grains. This unique and interesting coin is the earliest piece known from the mint of Drogheda.—(Pl. I. fig. 24.)

The double groat of the Dublin mint has a rose mint mark; legend, EDWARDVS DEI GRA DNS HYBERN. Reverse, CIVITAS DVBLINIE. This piece is in fine preservation, and weighs forty-four grains.—(Pl. II. fig. 25.)

A groat of the Dublin mint was the only coin of this type known to Simon, as was before observed; mint mark, a rose; legend, EDWARD DI GRA DNS HYBER; weight, twenty-two grains and a half.* The weight of this piece corresponds exactly with the standard fixed by the Act, and Simon referred it to its proper date; yet it is evident he did not clearly understand this coinage, for he describes a penny of a different type as belonging to it.†

A half groat of the Dublin mint was discovered at Trim, in 1834; type same as the groat; mint mark, a *sun;* legend, EDWA . . . . HYBERNIE. Reverse, CIVITAS DUBLINIE: weight, ten grains.—(Pl. II. fig. 26.)

The half groat now appears, for the first time, in the Irish series.

The Trim groat is unique; type similar to the others; it has not trefoils at the points of the tressure, as in the double groat. Reverse, . . . LA DE TRIM; it weighs twenty-three grains and a half, and is the earliest coin on which the name of this town appears.—(Pl. II. fig. 27.)

An interesting addition to the very few pieces of this type which are known, was discovered in August, 1839, near Castlecomer, county Kilkenny; it is the half groat of Trim, and is unique; mint mark, a rose; two small pellets over the crown; legend, EDWARDVS DI GRA DNS HYBE. Reverse, a sun of twenty-four rays; legend, VILLA DE TRIM; after the word Trim, there is a trefoil with pointed leaves, and pellets between them; its weight is eleven grains and a quarter, which accords exactly with the standard.—(Pl. II. fig. 28.)

I must now make a few remarks on the three small coins engraved in Snelling's Supplement to Simon.‡ They are described as "having a large sun of

* Simon, Pl. IV. fig. 72. † Simon, p. 26, and Pl. V. fig. 114. ‡ Pl. I. figs. 20, 21, 25.

fifteen rays" on their reverses; yet in the engravings, figs. 20 and 21 have suns of sixteen rays; and fig. 25, a sun of only ten rays, although it is full as large as fig. 21. Fig. 20, from its small size, and the *absence of the tressure* round the head, I believe to be the penny of this coinage; but its weight is said to be *eleven grains and a half,* which must be a mistake, as I have already shewn that the weight of the half groat ought to be eleven grains and a quarter; besides, according to the clause which I have given, on the authority of Sir William Betham, the penny should weigh about eight grains and a half; and by the Act, as published by Simon, it should weigh only about five grains and a half.—(See p. 16.) Fig. 21 corresponds in size with the Dublin half groat which I have published, but differs from it in having a rose for its mint mark; and the legends on the obverse and reverse are also different; besides, the sun has only sixteen rays, instead of twenty-four, the number on the five pieces in my plates. Its weight is stated to be twenty-two grains, being only half a grain less than the groat published by Simon.—(Pl. IV. fig. 72.)

Fig. 25 is very remarkable; its obverse is similar to an English penny of Edward the First or Third; yet from the sun on its reverse, it cannot be appropriated to any king but Edward the Fourth; it has no rose in its centre, and the legend, CIVITAS DVBLINI, is not divided into four parts by suns and roses, as in all the coins which I have published: its weight is said to be fourteen grains and a half.

Mr. Lindsay conjectures that this piece may have been a pattern for a penny; it presents several anomalies in its type, concerning which I cannot offer any explanation, as I have not seen the coin.

The Act of the seventh year of Edward authorized coins to be made in the castles of Dublin and Trim, the cities of Limerick and Waterford, and the towns of Drogheda, Galway, and Carlingford.

The coins from the Dublin mint are the most numerous, viz.: the double groat, groat, half groat, and penny. Of Trim, there are the groat and half groat, both unique. And of Drogheda, the double groat, which is also unique. None of Limerick or Waterford have been discovered; and it does not appear that silver coins were ever minted in Galway or Carlingford.

It is a remarkable circumstance, that during the first seven years of this reign, seven distinct coinages were issued from the Irish mints; some of them present several varieties of their types; and I may add, that the coins of this period are generally found to correspond in weight, very nearly, with that specified in the several Acts. But the history of the period on which I am about to enter is much embarrassed by the gross frauds which were practised in the authorized, as well as the illegal Irish mints.

Before I proceed to the consideration of the coins of the English type, it is necessary to notice a few from the mints of Drogheda and Dublin, which are not described in any of the Acts of this reign which have been published.

They are distinguished from the coins of the English type by having a rose in the centre of the reverse, instead of three pellets in each quarter of the cross, and for this reason I place them in this section.

The groat has the king's head crowned, within a double tressure of nine arches, a small sun at the right side of the crown, and left of the neck, and a rose at the left of the crown, and right of the neck; mint mark, a rose;* legend, EDWARDVS . . . GRA DNS HYBER. Reverse, a cross, with a rose in its centre; mint mark, a sun; legend, POSVI, &c., and in the inner circle, VILLA DROGHEDA. —(Pl. II. fig. 29.) In another, the suns and roses at the sides of the crown and neck are transposed; legend, EDWARDVS DI GRA DNS HYBER.; mint mark on the reverse, a rose.—(Pl. II. fig. 30.) They weigh from twenty-seven to twenty-nine grains.

No other coins of this type from the Drogheda mint have been discovered.

The groats of the Dublin mint present two varieties in the disposition of the suns and roses, like those of Drogheda; legend, EDWARDVS DII GRA DMS IBER. Reverse, POSVI, &c., and CIVITAS DUBLINIE; weight, thirty-two grains.—(Pl. II. fig. 31.)

The penny corresponding with the type of this groat weighs only six grains. —(Pl. II. fig. 32.)

The groat, Pl. II. fig. 33, has a different legend, EDWARDVS . . . . . FRAE D; weight, twenty-six grains.†

The penny of this variety weighs only six grains.—(Pl. II. fig. 34.)

* Simon, Pl. IV. fig. 82, has published one with a sun mint mark.

† The groats published by Simon, Pl. IV. figs. 80, 81, are both different from those I have described; the mint marks are a crown, and a sun.

Snelling, in his Supplement to Simon,* has published two halfpennies of this coinage, but has omitted to state their weight.

The Act of the first year of Richard the Third, which Simon speaks of as defaced by time and vermin, and which, as Mr. Lindsay remarks, "is evidently composed of parts of two Acts, and relate to coins of a very different description,"† enables me to fix the date of these coins in the year 1470.

In the first year of Richard, the master of the mint was authorized to make coins "in such manner and in such places, as is ordained by a Statute" of the tenth year of Edward the Fourth.‡ Now there are groats of Richard which correspond in every particular, except the king's name, with those of Edward; and my opinion as to their date, is supported by the fact of their deficiency in weight, for in 1472, Germyn Lynch, master of the mints in Ireland, was indicted, "for that when the Statute said, that every pound of bullion coined, should be forty-four shillings in money, he coined out of every pound forty-eight shillings, and that he coined at Drogheda one thousand groats, which being tried, it was found that *eleven* weighed but three quarters of an ounce,"§ instead of an ounce; so that the average weight of the groats was a little more than thirty grains, which agrees nearly with the weight of those now in existence.

There are several Dublin pennies which were probably coined about this time; they rarely exhibit the legends entire, but may be readily recognized by their reverses, which bear a cross, having a small rose in its centre, and the legend CIVITAS DUBLIN. In the quarters of the cross, there are alternately two roses and a sun, and two suns and a rose, instead of pellets, as in the coins of the next section.—(Pl. II. figs. 35, 36.) The former weighs nine grains, the latter only six.

The penny, fig. 37, is remarkable for the legend on its obverse, ED . . . DI GRA REX NGI F : it weighs nine grains and a half.

THE THIRD SECTION.

The coins included in this section are similar in type to the English coins of Edward.

* Plate I. figs. 23, 24.

† Lindsay, p. 47.

‡ Simon, Appendix, No. XVIII.

§ Simon, p. 27.

The value of silver in Ireland was raised enormously in 1467, the consequence of which was, that the price of every thing increased in proportion ; to remedy which evil, the next parliament held in Dublin, in 1470, enacted "that the master or masters of the coinage shall have power to make and strike within the castles of Dublin and Trym, and the town of Drogheda, five sorts of silver coynes, according to the fyness of the coynes struck in the Tower of London," viz. the groat, half groat, penny, halfpenny, and farthing. The groat to have on one side the print of a head crowned, with the writing, "Edwardus Dei Gratia, Rex Anglie Dominus Hibernie ;" and on the other side the print of a cross, with the pellets according to the groat made at Calais, and the motto, "Posui Deum Adjutorem Meum," with the name of the place of mintage ; of which groats, eleven shall make the ounce, troy (tower?) weight ; and that the fifth part of every pound be struck in small pieces. It was also enacted that the master might allay the halfpence and farthings according to the Statute made in the fifth year of this reign, which Statute cannot be found. By this Act, the coinage of 1467 was reduced to half its original value, and forbidden to be taken for a coin after the feast of the Purification next.*

1471.†—By an Act of this year, it appears that a great part of the coinage of 1470 was neither of full weight nor fine allay.‡

1472.—The Act of this year states, that false coins were made in Cork, Youghal, Kinsale, and Kilmallock.§

1473.—At a parliament held in Dublin, it was enacted, that the coins should be struck, for the time to come, within the castle of Dublin only, and in no other place in Ireland ; and that fourteen groats should make an ounce, according to the just *standard of the Tower of London ;* and to be made according to the fineness and alloy of the said tower ; and that Germyn Lynch be master of the said mint during good behaviour.‖

1475.—The groat made in England at this time was ordered to pass, if not clipped, for five pence ; and all the moneys to be struck in Ireland, to be of the

* Simon, Appendix, No. X.

† In Simon's Appendix, this Act is dated 1472 ; and at page 27, he calls it the Act "of the eleventh of this prince."

‡ Simon, Appendix, No. XI.

§ Simon, Appendix, No. XII.

‖ Simon, Appendix, No. XIII.

same value as they now are; and that all the mints in Ireland shall cease, except those of Dublin, Drogheda, and Waterford.*

1476.—The coin lately made in Cork, Youghall, Limerick, and other places in Munster, except Waterford, being neither lawful in itself, nor of lawful weight and allay, was declared void, and forbidden to be taken in payment.†

I have now given the substance of the several Acts which were passed from the year 1470 to 1476; and, from the number and variety of coins struck during this period, which are in existence; the obscurity and imperfections of the Acts of parliament; and the general deficiency of the coins in weight, the most convenient arrangement which can be adopted, is, to describe, first, the coins of the several mints; and afterwards endeavour to assign them to their proper dates.

CORK MINT.

Two varieties of the groat are known; one has the king's head, within a double tressure of nine arches; trefoils at six of its points; and at each side of the neck, a quatrefoil; legend, edwardvs dei gra dns hibernie. Reverse, a cross, with three pellets in each quarter; motto, POSI DEVM AIVTORE MEVM; in the inner circle, CIVITAS CORCAGIE; mint mark, a rose in three places; weight, thirty-eight grains.—(Pl. II. fig. 38.)

The other has a pellet at each side of the king's neck, and only a single pointed leaf at the points of the tressure; legend, EDWARDVS DEI GRA DNS IBERIЭ. Reverse, POSV . DEV . ADIVTOR MEV; in the inner circle, CIVITAS CORCAGIE; no mint mark on either side. This piece is well preserved, and weighs only thirty grains.—(Pl. II. fig. 39.)

DROGHEDA MINT.

The groat bears the king's head, within a double tressure of nine arches; legend, EDWARDVS DEI GRA DNS HYBER, or HYBERNI; mint marks, a crown, and a cross pierced in the centre. Reverse, POSVI, &c.; and in the inner circle, VILLA DE DROGHEDA. They weigh from thirty-three to thirty-four grains.—(Pl. II. figs. 40, 41.)

* Simon, Appendix, No. XIV.

† Simon, Appendix, No. XV.

The groats with the letter G on the king's bust are more numerous; mint mark, a cross pierced in the centre; legend, EDWARDVS DEI GRA DNS HYBERN. Some have an annulet at each side of the king's neck. The average weight of eight well preserved pieces is thirty-two grains.—(Pl. II. figs. 42, 43.)

A half groat has been recently discovered, and is unique; legend, EDWARD DI GRA DNS HYBER; mint mark, a sun; it has not trefoils at the points of the tressure. Reverse, POSVI, &c., and VILLA DE DROGHE: weight, fifteen grains. —(Pl. II. fig. 44.)

Of the pennies, there are four varieties.

The first has a pellet at each side of the king's neck. Reverse, VILLA DE DROGHE: weight, eight grains.—(Pl. II. fig. 45.)

The second has a small rose in the centre of the reverse, and weighs only six grains.—(Pl. II. fig. 46.)*

The third has an ornament, consisting of four loops united, so as to form a kind of quatrefoil, in the centre of the reverse;† legends, EDWARD DNS HYBER, and VILLA DE DROGHEDA: weight, seven grains.—(Pl. II. fig. 47.)

The fourth variety has a small sun at each side of the king's neck; and the legend on the obverse is different from all the others, viz. EDWARD REX ANG . FR; mint mark, a cross.‡ I do not know of any Drogheda groat with a similar legend.

DUBLIN MINT.

The legend on the groat is EDWARDVS DI GRA DNS HYBERNIE; mint marks, a rose, and a cross pierced in the centre. Reverse, POSVI &; and in the inner circle, CIVITAS DVBLINIE. They weigh from thirty-five and a half to forty-five and a half grains.—(Pl. III. fig. 48.) This is the heaviest piece of the English

* I should have placed this coin at the end of the second section, on account of the rose on its reverse, were it not that the pellets in the quarters of the cross identify it more closely with the coins described in this section. This piece, taken together with No. 36, exhibits the transition of the type from the coins of the Hiberno-English series to that of the English type described in this section.

† A similar ornament occurs on the York and Durham pennies of Edward the Fourth.—*Ruding*, Suppl. Pl. III. figs. 21, 28, 2nd edit.

‡ Simon, Pl. IV. fig. 92.

type which I have met with; it is more than four grains above the standard weight fixed by the Act under the authority of which it was coined.

The groats with the letter G on the king's bust are more numerous; the legends are, EDWARDVS DEI GRA DNS HYBER, HYBERN, and HYBERNI; mint marks, a sun, a cross, and a cinquefoil. They present many varieties, which it is unnecessary to particularize, and usually weigh about thirty-two grains each. —(Pl. III. figs. 49, 50.)

The legend on the half groat is, EDWARD DI GRA DNS HYBER; some have small pellets between the words, others small crosses; the latter is the most common on the coins of this type; mint marks, a sun and a cross. Reverse, POSVI, &c., and CIVITAS DUBLIN. They weigh seventeen grains.—(Pl. III. figs. 51, 52.)

The penny weighs seven grains and a half, and has a small cross at each side of the king's neck; legends, EDWARD DI GRA DNS HYBER, and CIVITAS DUBLINIE.—(Pl. III. fig. 53.)

Another has small pellets, instead of crosses, at each side of the king's neck.

A third variety has a kind of quatrefoil in the centre of the reverse, and the legend, CIVITAS DUBLIN; it weighs only six grains.—(Pl. III. fig. 54.)

LIMERICK MINT.

The groats present three varieties in the legends, EDWARD DI GRA REX ANGL ET FR or FRANC, and EDWARD DI GRA DNS HVBERNI. They all have the letter L on the king's bust, and have either a rose, a cross, or a cinquefoil, at each side of the neck; mint marks, on the obverse, a cross pierced in the centre, and a cinque foil at the beginning of the legend on the reverse; in the inner circle, CIVITAS LIMIRICI, and one of the pellets in the alternate quarters of the cross is replaced by a cinquefoil. They weigh in general about thirty-one grains.—(Pl. III. figs. 55, 56, 57.)

The only half groat which I have seen has the legends much defaced, yet it weighs seventeen grains; there is a quatrefoil at each side of the neck, and on the reverse, CIVITAS LIMIRICI, (Pl. III. fig. 58;) it has not the letter L on the king's bust, nor the cinquefoil instead of the pellet in the alternate quarters of the cross, like the groats, and the half groat published in the Editor's additional plate to Simon, (fig. 16.)

The only penny of this type which has been discovered is represented in the same plate, fig. 17.

Another penny has a kind of quatrefoil in the centre of the reverse, and weighs nine grains and a half.—(Pl. III. fig. 59.)

TRIM MINT.

The legend on the groat is EDWARDUS DEI GRA DNS HYBER, or HYBERN; mint marks, a rose, and a cross pierced in the centre. Reverse, POSVI, &c.; and in the inner circle, VILLA DE TRIM. One has a rose before the word POSVI, and another has a small cross in one of the quarters of the reverse. They weigh from twenty-eight to thirty-four grains.—(Pl. III. figs. 60, 61.)

The half groat of this type is unique; it was found at Trim, and weighs twenty-three grains.—(Pl. III. fig. 62.)

A penny, of any coinage, from this mint would be an interesting discovery; there can be no doubt that such pieces were minted.

WATERFORD MINT.

Several varieties of the coins from this mint are known. One groat has a ≼ on the king's bust, and a small plain cross at each side of the neck; mint mark, a rose; weight, forty-three grains.—(Pl. III. fig. 63.)

Another has a V on the king's bust, and weighs only twenty-eight grains.—(Pl. III. fig. 64.)*

Others have the letter G on the bust; mint marks, a rose, cinquefoil, and a cross pierced in the centre. They weigh from thirty-two to thirty-three grains.—(Pl. III. figs. 68, 69.)

There is a fourth variety, without any letter on the bust; mint marks, a rose, trefoil, and a cross pierced in the centre. Some have a quatrefoil at each side of the neck, others a plain cross, and some are without any mark in this place. They weigh, in general, about thirty-one grains each.—(Pl. III. figs. 65, 66, 67.)

* A trefoil is the mint mark of this variety, as appears from the coin published by Simon, Pl. IV. fig. 84.

Mr. Lindsay mentions a sun, as a mint mark on the Waterford groats, but does not say on which variety it occurs.

The legend on the obverse presents little variety ; and they all have on the reverse, CIVITAS WATERFORD, many of them having a small cross in the alternate quarters, with thc pellets.

No half groat of any type, from this mint, has been discovered.

There are several varieties of the pennies; one has a pellet at each side of the king's crown, and two small crosses at each side of the neck ; legend, EDWARD DI GR DNS IBERNIƎ ; mint mark, a cross. Reverse, CIVITAS WATERFORD ; weight, ten grains.—(Pl. IV. fig. 70.) A variety of this type has on the reverse, CIVITAS WATFORD.

Another has an annulet at each side of the king's neck ; it weighs nine grains and a half.—(Pl. IV. fig. 71.)

A third variety has a pellet at each side of the neck ; mint mark, an annulet. Reverse, CIVITAS WATFORD : weight, eight grains.—(Pl. IV. fig. 72.)

The legend of the fourth variety is, EDWARD DNS HYBER, and it has a small cross at each side of the neck. Reverse, CIVITAS WATFORD ; it also has a kind of quatrefoil in the centre, and weighs eight grains.—(Pl. IV. fig. 73.)

WEXFORD MINT.

The only kind of coin known from this mint is the groat, which was published by Simon, Pl. V. fig. 93, and represented as if in as good preservation, and as equal in workmanship to any of the coins of this reign. I am inclined to think the engraver has not given a correct delineation of the coin, as I have recently had an opportunity of seeing one, belonging to the Rev. Mr. Butler, of Trim, and it is remarkable for the rudeness of its execution ; it has the king's head crowned, within a double tressure of *ten* arches. The legends are very defective, and appear to have been greatly blundered. Reverse, VILLA WEISFOR ; the S is reversed, and on the coin it looks very like an X, for which it may have been intended ; the metal is apparently impure, and the coin weighs only twenty-six grains.—(Pl. IV. fig. 74.)

One small brass piece is known, which corresponds in type with the coins described in this section. It exhibits on one side the king's head crowned, and on the other, the cross and pellets; small strokes, or lines, appear to have been substituted for the legends: it weighs three grains and a half.—(Pl. IV. fig. 86.)

This may possibly be a farthing, as at one period of this reign, the Lord Lieutenant, or his Deputy, was empowered to allay the halfpence and farthings according to his discretion,* a privilege very likely to be exercised to its utmost extent.

Of the seven cities and towns in which the coins described in this section were minted, only four, viz. Drogheda, Dublin, Trim, and Waterford, are recognized as legal mints in the Acts which have been preserved.

I shall first dispose of the mints which were not legally qualified. The Cork groats appear to have been made between the years 1470 and 1473, for the Act of the year 1472 informs us of "there being divers coiners in the city of Cork, and the towns of Youghal, Kinsale, and Kilmallock, who make false coins without authority;"† and in 1473, it was enacted that the coins should "be struck for the time to come within the Castle of Dublin only, and *in no other place* in Ireland,"‡ and by this Act the weight of the groat was reduced to about thirty-two grains; hence it is clear, that one at least of the Cork groats which weighs thirty-eight grains was minted before 1473; and their blundered inscriptions, together with the apparent impurity of the metal, plainly indicate that they were the work of some fraudulent artist.

Wexford, as a place of mintage, is not mentioned in any of the Acts of this reign; and the only coin which I have seen from this mint is very deficient in weight, and bears evident proof of the fraudulent design of the person by whom it was executed. I am unable to assign any particular date to this piece.

The weight of the Limerick groats, which in no instance have I found to exceed thirty-two grains, makes it probable that they were not minted previous to the year 1473, at which period the standard weight of the groat was reduced from forty-one to nearly thirty-two grains; and as the privilege of making coins was restricted to Dublin *only* from 1473 to 1475, it is likely that the coins of this mint were issued during the latter year, for the Act of 1476 states, that "the silver coin *lately* made in Cork, Youghal, *Limerick*, and other places in

* Page 16.

† Simon, Appendix, No. XII.

‡ Simon, Appendix, No. XIII.

Munster, except Waterford, being neither lawful in itself, nor of lawful weight and allay," was declared void, and forbidden to be taken in payment.*

Although Limerick does not appear in the Acts as a legal mint, after the year 1467, I am disposed to think that city enjoyed authority to coin money at a subsequent period. The Limerick coins described in this section are as well executed as any pieces from the authorized mints; and besides the varieties of the groats which are known, there are also two varieties of the half groat and penny.—(Pl. III. figs. 55, 56, 57, 58, 59; see also Editor's additional plate to Simon, figs. 16, 17.) The number of coins issued from this mint distinguish it from those of Cork and Wexford, of which only groats of rude execution are known.

Of the coins from the authorized mints, those of Trim appear to have been made between the years 1470 and 1473, for in the latter year the privilege of striking money was withdrawn from this mint, and it does not appear to have been restored at any subsequent period.

The groats of Drogheda, Dublin, and Waterford, without the letter G on the king's bust, were all minted previous to the year 1473, as was also the Waterford groat with the letter ⋞ on the bust; the latter weighs forty-three grains, and is the heaviest piece of the English type which I have met with, except fig. 48, which weighs forty-five grains and a half.

The pieces with the letter G on the bust were all struck subsequent to the year 1473; some of those of Dublin may have been minted in that year, but the Drogheda and Waterford groats were probably issued in 1475, when the authority for making money was restored to those places.

I do not know of any half groats or pennies with the letter G on the bust.

Mr. Lindsay has stated, that the letter G is "probably the initial of Germyn Lynch,"† an opinion which I shall endeavour to corroborate.

Simon, on the authority of a manuscript in the Library of Trinity College, Dublin, states that in 1472, Germyn Lynch was indicted for making light groats at Drogheda.‡ But, independent of this authority, there is evidence in the Act of 1471, that Lynch had been deprived of his office of Master of the

* Simon, Appendix, No. XV.

† View of the Coinage, p. 43.

‡ Page 27.

Mint, for on the eighteenth of October, in the tenth year of this reign, (1470,) William Crumpe and Thomas Barby, merchants, were by letters patent constituted masters of the coinage;* and in 1473, it was ordered, that Germyn Lynch be Master of the Mint during good behaviour.†

It is reasonable to suppose, that Lynch, being restored to his office, would be anxious to adhere more strictly to the provisions of the Statutes; and as so many frauds had been committed in the coinage, he probably adopted the letter G as his privy mark; and I find that the groats with this mark on them are remarkable for the uniformity of their weight, and correspond pretty closely with the standard fixed in 1473. Lynch's coins are more numerous than the other varieties, which, with few exceptions, do not appear to be regulated by any standard.

There are four pennies described in this section, which I am unable to refer to any particular date, viz. Nos. 47, 54, 59, 73. No groats corresponding in type with them are known, and it is only from the larger pieces that the types described in the Acts can be satisfactorily determined.

There is one particular respecting the inscription on the coins of this period, which requires some notice. The Act of 1470 orders that the groat shall have the words REX ANGLIE in the inscription on the obverse. Now I have observed this title on only three coins, (figs. 37, 55, 56,) and on a Drogheda penny engraved in Simon's Essay.‡

Before I conclude my remarks on this section, I must say a few words respecting the weight of these coins. In 1470, it was enacted that eleven groats should make an ounce *troy*; each groat should, therefore, weigh very nearly forty-four grains, or $43\frac{7}{11}$. I presume the troy ounce has been erroneously substituted for that of the Tower, and consequently that the groat of this year should weigh very nearly forty-one grains, or $40\frac{10}{11}$. I only know of two coins which exceed the standard as fixed in 1470.§

* Simon, Appendix, No. XI. † Simon, Appendix, No. XIII.

‡ Plate IV. fig. 92.

§ Figs. 48, 63. The occasional extra weight is explained by the Act of 1470, which states: "And as the said money cannot always be made to agree according to the just standard, being, in default of the Master, sometimes made *too great*, and sometimes too small in weight or allay, by four deniers in every pound, which four deniers shall be a remedy for the said Master."—*Simon*, Appendix, No. X.

That the Tower ounce was the standard used in Ireland, is evident from the Act of 1467, which directs the coins to be made "according to the rightful *standard of the Tower of London;*" and from that of 1473, which enacts, that fourteen groats should make an ounce, "according to the just *standard of the Tower of London;*" and again, in 1479, "according to the fineness and *standard of the Tower of London;*" therefore, the groat of the year 1473 should weigh a little more than thirty-two grains, and not "about thirty-four grains to the groat," as stated by Mr. Lindsay.*

THE FOURTH SECTION

Comprises a class of coins of a very remarkable type, which were the last issued during this reign, and may be denominated the Anglo-Irish type. They have on the obverse a shield, bearing the arms of England and France quartered; and on the reverse, three crowns in pale, a device peculiar to the Irish coinage.

1478.—In the eighteenth year of this reign, at a parliament held at Trim, before Henry Lord Grey,† Deputy to George Duke of Clarence, it was enacted, that for the time to come, the liberty of Meath be restored and exercised, with all manner of liberties, in as ample a manner as was exercised and occupied in the time of Richard, late Duke of York, or his noble progenitors, lords of Meath; and that Henry Lord Grey, Lord Deputy, shall enjoy and exercise, by himself or his Deputy, the said liberty by the name of Seneschal and Treasurer of the said liberty of Meath, in as ample a manner and form as any Seneschal or Treasurer heretofore occupied and enjoyed the same. And further, this Act confirms a grant made by the king of the office of Seneschal and Treasurer of Meath to the said Henry, dated at Westminster the third day of March, in the seventeenth year of his reign. And by this Act, the said Henry, by himself or his officers, may for the future strike and coin all manner of coins of silver within the Castle of Trim, according to such fineness and allay as in the Statute for that purpose is provided.‡

* View of the Coinage, p. 42.

† Sir James Ware, in his Table of the Chief Governors of Ireland, does not mention Henry Lord Grey, Lord Deputy to George Duke of Clarence.

‡ Simon, Appendix, No. XVI.

The Statute here referred to is not to be found, but we learn from Sir James Ware, "that in the eighteenth year of Edward the Fourth, an Act passed a parliament held under Gerald Earl of Kildare, Lord Justice of Ireland, granting liberty to the Mint Master of coining pieces of *three* pence, two pence, and a penny;" and he adds, that "it is, however, worth observing that the impress on the coins of this time, on the reverse, was three crowns, denoting the three kingdoms of England, France, and Ireland."*

1479.—At a parliament held at Dublin, before Gerald Earl of Kildare, Deputy to Richard Duke of York, it was "enacted that Germyn Lynch, Master of the Minters, have power to strike coin at four shillings and ten pence per ounce, rendering to the merchant four shillings and four pence, and to the king and workmen six deniers, according to the fineness and standard of the Tower of London."†

1483.—"An indenture for Ireland was made with Thomas Galmole, Gent., Master and Worker of the Money of Silver, and Keeper of the Exchanges in the cities of Devylyn (Dublin) and Waterford. He was to make two sorts of monies; one called a Peny, with the king's arms on one side, upon a cross trefoyled on every end; and with this inscription, REX ANGLIE ET FRANCIE : and on the other side, the arms of Ireland, upon a cross, with this scripture, DNS HIBERNIE; of such Penyes in the pound weight of the Towere, iiii. c. l. pees, which is in nombre xxxvij s. vjd. The other money to be called the Halfpenny, with the like impression and inscription, and in weight one-half of the first, all of the old sterling."‡

These are the only records which remain of the last five years of this reign.

There are two varieties of the type of the coin issued during this period. One has on the obverse a shield, bearing the arms of England and France, quartered by a cross, the extremities of which are terminated each by three pellets; the shield is within a circle of pellets. Reverse, three crowns in pale, on a similar cross; mint marks, a trefoil, rose, and fleur de lis.

The other variety has a shield, quartered by a cross, whose arms are terminated each by three *annulets;* at each side of the shield is a smaller one, bearing

* Harris's Ware, vol. ii. p. 215. † Simon, Appendix, No. XVII.

‡ Ruding's Annals, 2nd edit. vol. ii. p. 376.

a saltire, The Arms of Fitzgerald Earl of Kildare and Lord Justice of Ireland in 1479;* all within a plain circle. The crowns on the reverse are closer, and of a more regular form, than those of the first variety, and are within a double tressure of eight, or more generally, nine arches; they invariably have a fleur de lis, on one or both sides, in some part of the legend, which is rarely found on the pieces of the first variety.

The following Table exhibits the most remarkable varieties of the legends which occur on the coins of the Anglo-Irish type.

WITHOUT THE FITZGERALD ARMS.

	GROATS.	
EDWAR REX ANGLIE FRANCI.	DOMINVS HYBERNIE.†	
EDWARDVS . . . ANGL	DEMINVS HYBERNIE.	Pl. IV. fig. 76.
EDWARDVS . . . . RANC.	ET : REX HYBERNIE.	,, 75.
REX ANGLIE FRANCIE.	ET REX HYBERNIE.	,, 77.
REX ANGLIE FRANCIE.	DOMINVS HYBERNIE.	,, 78.
DOMINVS HYBERNIE.	DOMINVS HYBERNIE.	,, 80.
	HALF GROATS.	
EDWARD DOM HYBE.	CIVITAS DVBLINIE.‡	
REX ANGL FRANCIE.	CIVITAS . . . LIN.	,, 87.
REX ANGL FRANCIE.	DOM HYBERNIE.§	
REX ANGL FRANCIE.	DOMINVS HIBERNIE.	,, 88.
REX ANGL FRANCIE.	DOMINS VBE.	,, 89.
REX ANE FRANCIE.	DOMINOS VRER.	,, 90.
DOMIN . . . RERIE.	DOMINOS V . .	,, 91.
	PENNIES.	
REX ANGL FRANCIE.	DOMNVS HYBENIE.‖	
REX ANG FRANC.	DOMINVS HIBERN . .	,, 93.

* The small shield which Simon represented as a figure of 8, (Pl. III. fig. 65,) and described as a mint mark, (p. 22,) was first recognized by the Rev. Mr. Butler as the arms of the Fitzgeralds.—*Numismatic Journal*, vol. ii. p. 73.

† Simon, Pl. IV. fig. 87.

‡ Ib. Pl. V. fig. 94.

§ Ib. Pl. V. fig. 95.

‖ Ib. Pl. IV. fig. 90.

WITH THE FITZGERALD ARMS.

GROATS.

REX ANLIE FRA.	DOMINOS VRERNI.	Pl. IV. fig. 82.
REX ANLIE FRA.	DOMINOS VRERNIE.	„ 83.
REX ANCIE CIE.	DOMINS VRER.	„ 84.
REX ANIE FRANCI.	DOMINOS VRENIE.	„ 85.

HALF GROAT.

DOMINOS.	DOMINO – .	„ 92.

Mr. Lindsay has published a small coin of this type, which he supposes to be a farthing,* and that "it may possibly belong to Henry the Seventh." This little piece is in the cabinet of the Dean of St. Patrick's, is greatly corroded and defaced, and weighs only two grains, which probably led to the supposition of its being a farthing. It is, however, the remains of a penny, for the diameter of the circle, and the size of the shield, correspond exactly with those of a well-preserved penny; and besides, Sir James Ware makes no mention of farthings of this type.

Some have thought, that as the arms of England and France are impressed on these coins, that they should be ascribed to Henry the Seventh, who was the first monarch who had these arms stamped on the English silver coins. To refute this opinion it is only necessary to refer to the coins of this type which bear the name of Edward.—(Pl. IV. figs. 75, 76.)

According to Simon, Henry the Eighth "having, in his thirty-third year, assumed the title of King of Ireland, was so proclaimed the thirteenth of June, 1541, in St. Patrick's Church, near Dublin;"† and Ruding informs us, that in the same year the title ET HYBERNIE REX was first used on the Great Seal of England.‡ Now the coins of Pl. IV. figs. 75, 77, not only prove that the arms of England and France appeared first on the Irish coins, but that the title of REX HYBERNIE was impressed on the coins of this country many years earlier than the date usually assigned to the introduction of this title. These pieces are therefore

* View of the Coinage, Pl. VI. fig. 128. † Simon, p. 33.

‡ Ruding's Annals, vol. ii. p. 443, 2nd edit.

indubitable evidences of a fact, the account of which has been imperfectly recorded by historians. Figs. 79 and 81 are peculiar in having the border of the shield formed of small pellets, instead of plain lines, and the former has a *fleur de lis* before the word REX; the only instance in which I have found this mint-mark on the groats without the Fitzgerald arms.

Some of these pieces are what are termed *mules* in numismatic language, e. g. the obverses of 78 and 80 are different, while their reverses are from the same die, as is evident from the blundered B in Hybernie.*

The many varieties, both in type and legends, which occur on the half-groats, require some notice. Of the six I have published, only one agrees in type with the groats of the first variety, and it is remarkable for having I instead of Y in Hibernie (fig. 88). The same peculiarity occurs on the penny, (fig. 93,) and I have seen a groat which corresponds in this particular with these two pieces.

Only one half-groat, bearing the Fitzgerald arms, is known, and it has the word DOMINOS on each side (fig. 92.)

The obverse of 87 and 89 corresponds with the groats of the first variety, while the reverse of each of them bears the cross with the annulets, and the plain circle, which, with the legend DOMINS VBE on the latter, identify them with the Fitzgerald type.

The former was struck at Dublin, and I do not know of any groat of this type from the same mint.

Figs. 90 and 91, although they have not the Fitzgerald arms on them, do, I presume, properly belong to the second variety of this coinage. The former bears a very close resemblance, in some particulars, to the groat, fig. 84.

Mr. Lindsay remarks, that "the half-groat has sometimes the initial of

* A curious fact may be learned from these two pieces, respecting the manner in which the letters were made on the die. They were formed with punches, or steel types, as is practised at the present time, for the artist manifestly put in the letter E by mistake, and to cover his blunder, he afterwards punched over it the letter B. Other instances in support of this opinion may be adduced, when, for instance, the artist substituted the reversed B for E, (Pl. IV. fig. 70,) and occasionally the letter L is represented in an ingenious manner by a double I, as in figs. 82 and 83. Such blunders, especially the latter, could scarcely happen had the artist used a graver, or cutting tool of any kind, in forming the letters.

the king's name before the word Rex."* I have not met with any such variety.

Sir James Ware says, that liberty to coin "pieces of *three*-pence, two-pence, and a penny," with three crowns on the reverse, was granted to the Mint Master in the eighteenth year (1478) of this reign. I conceive he has committed some error on this subject, for Moryson, who wrote many years before him, speaks of "cross-keale *groats*, with the Pope's triple crown."

Simon, relying on the correctness of Sir James Ware's account, endeavours to reconcile it with the standard fixed by the Act of 1479. He observes, "the standard of the Tower of London must be understood here only as to the allay, and not as to the weight of the Tower," and concludes that "the groat must have weighed forty grains, and ten (twelve?) of them to have been cut out of the ounce Troy, in which case silver was again *reduced* to near its former value;"† and in the next page informs us that "the pieces with three crowns" weigh from twenty-eight and a half to thirty grains, "the half piece fourteen to fifteen grains," and the penny "with the crowns seven grains."

It is difficult, if not impossible, to reconcile his opinions with the following facts:

In 1473 the weight of the Irish groat was reduced to nearly thirty-two grains, and in 1479 Germyn Lynch was empowered "to strike coyne at four shillings and ten pence per ounce, according to the fineness and standard of the Tower of London,"‡ which reduced the weight of the groat to thirty one grains.§

Sir James Ware represents these pieces in the proportion of three, two, and one, while Simon speaks of them as "pieces," and "half-pieces." I have weighed many of them, and in general they correspond with the weights, as stated by Simon; they also agree with the standard fixed in 1479,‖ and are in the pro-

* View of the Coinage, p. 46.

† Simon, p. 29.

‡ Simon, Appendix, No. XVII.

§ Simon evidently did not take a correct view of this coinage, for he understood the standard as applying to the allay, and not to the weight, whereas the Act expressly provides for both, in the words, "according to the fineness (allay) and standard (weight) of the Tower of London." He was in error in calculating the weight of the pieces according to the *Troy* ounce.

‖ Those of the Fitzgerald type are usually somewhat lighter than the others.

portions of four, two, and one, or in other words, groats, half-groats, and pennies.

The groat of this type rarely exceeds thirty, and never, I believe, thirty-two grains, a circumstance which cannot be reconciled with the Act of 1483, by which the penny was ordered to be made of the weight of twelve grains, or in the proportion of 450 to the pound Tower. Groats are not mentioned in this Act.

The coins without the Fitzgerald arms, were probably minted in the Castle of Trim, during the administration of Henry Lord Grey, in 1478; and those with the Fitzgerald arms were coined at the same place in the following years, under the authority of Gerald Earl of Kildare. The half-groat of Dublin, fig. 87, was probably minted by Germyn Lynch, in 1479.

It now only remains to offer some explanation of the meaning of the device of the three crowns, which has given rise to various conjectures.

Fynes Moryson, when enumerating the old coins which circulated in Ireland, says, "Also they had silver groats, called Cross-Keale groats,* stamped with

* As the meaning of this word, in its application to the groats, has not, I believe, been hitherto accounted for, I venture to offer an explanation of it. Reflecting on the subject, it occurred to me that the term was applied by the native Irish to the coin in reference to some peculiarity in the device, as several instances are well known in which coins obtained popular names, having a relation to their type, e. g. Rial or Royal, Angel, Harpers, &c.

As soon as I had made this conjecture, I expected to find its explanation in the Irish language; and on asking an Irish scholar the meaning of Cross-Keale, (cpoċ caol,) he without hesitation informed me it was "slender cross." The fitness of this name will be evident, on contrasting the cross on one of the three-crown groats with any of the coins of the English type, or those described in the first and second sections.

About this time, my attention was directed to a paper published in volume xv. of the Transactions of the Royal Irish Academy, by Mr. Hardiman, in which I found that the term "Cross-Keale money" was used in Ireland so early as 1419, in the reign of Henry the Fifth: "18 marks Cross-Keale money, with a penny addition in every groat," being mentioned as part of the payment of a mortage.—*Hardiman*, p. 50.

This circumstance at first appeared to set aside the reasonableness of my conjecture, but when I compared several groats belonging to the Henrys, I found those of Calais, with the "cross-crosslet" mint mark, were remarkable for the slender cross on the reverse, which served well to distinguish them from others as well as those of Edward the Third, which have a much broader cross, and they are all found in abundance in Ireland. The accompanying outlines of the reverses of two the Pope's triple crown; and these groats were either sent hither of old by the Popes, or for the honour of them, had their stamp set upon them."*

Sir James Ware considered the three crowns "as denoting the three kingdoms of England, France, and Ireland," an opinion in which Simon concurred.

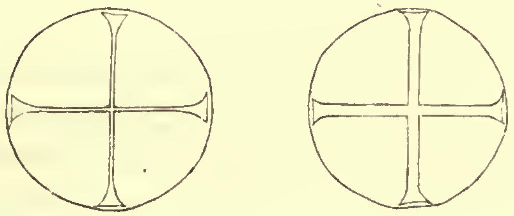
Neither of these opinions is correct; and it is a very remarkable circumstance, that this device, the meaning of which the learned research of Sir James Ware failed to discover, has, after the lapse of nearly four centuries since its introduction on the coins, been proved to be the arms of Ireland.

This highly interesting discovery was made by the Rev. Mr. Butler, of Trim; and I am much indebted to that learned gentleman for the following summary of the evidence which he has collected.

"Mr. Butler is of opinion, that the three crowns were the arms of Ireland, from the time of Richard the Second to the time of Henry the Seventh, for the following reasons.

"1. Richard the Second granted to Robert de Vere, permission to bear as his arms, *so long as he should be Lord of Ireland*, three crowns within a bordure.†

groats of Henry, in my possession, present a good illustration of the difference between the crosses, and tend to support my conjecture.



CALAIS. LONDON.

* Moryson's Itinerary, Part i. Book iii. p. 284, folio: London, 1617.

† Among the minor correspondence in the Gentleman's Magazine for June, 1840, the following note occurs:

"I take this opportunity of appropriating the arms on a pavement tile, engraved in the Gentleman's Magazine for October, 1818, which appears to have been found in Essex. The arms are described as three crowns quartering mullets. They are the arms of Robert de Vere, Earl of Oxford, who was the favourite of Richard II., and by him created Marquis of Dublin, and Duke of Ireland, on which occasion the king gave him for his arms, '*Azure, three crowns or, within a border*

"2. At Henry the Fifth's funeral, on the first car were emblazoned the ancient arms of England; on the second, those of France and England, quarterly; on the third, those of France; and on the fourth, three crowns on a field azure, which, although erroneously ascribed by Monstrelet, who gives this description, to King Arthur, were more probably the arms of Henry's great Lordship of Ireland.

"3. The crown first appears, on the first distinct and separate coinage for Ireland, issued according to an Act of parliament in 1460, declaring the independence of Ireland, and enacting that it should have a proper coin, separate from the coin of England.*

"4. The three crowns appear on the Irish coins of Edward the Fourth, Richard the Third, and Henry the Seventh; they are unknown to the English coinage; and when Henry the Eighth assumed the harp as the arms of Ireland, they appear no more.

"5. On the only silver coins on which the three crowns occur, they appear, as the harp does afterwards, on the reverse; the obverse bearing the arms of England; and when the legend, DOMINVS HYBERNIE is on the coin, it is on the same side with the three crowns, as it is afterwards on the same side with the harp.

"6. That these crowns are borne, not in a shield, but 'upon a cross,' is no objection to their being armorial bearings, as the harp was never borne on a shield, except on some coins of Queen Elizabeth, who instead of one harp, bore three in her coinage of 1561; as Edward the Fourth bore sometimes one, and sometimes three crowns. But that the three crowns were sometimes enclosed within a shield, is a fact which is incontestibly proved by a small copper coin,† two specimens of which were found at Trim, and another had been previously discovered at Claremont, near Dublin; the latter is in the cabinet of the Dean of St. Patrick's.

argent,' quartered with his own coat of De Vere, '*Quarterly gules and or, in the first quarter a mullet argent.*' He died without issue 16th Richard II., and was the *only* member of his family who bore this quartering of the three crowns. His arms are so remaining now, on the porch of the church at Lavenham, in Suffolk."

* Simon, Appendix, No. V.

† Plate I. fig. 21.

"7. In 1483, Thomas Galmole, Gentleman, Master and Worker of the Money of Silver, and Keeper of the Exchanges in the cities of Devylyn (Dublin) and Waterford, was bound by indenture to make two sorts of monies; one called a penny, with the king's arms on one side, upon a cross trefoyled on every end, and with this inscription, REX ANGLIE ET FRANCIE; and on the other side, *the arms of Ireland*, upon a cross, with this scripture, DNS HIBERNIE.*

"Some device must, therefore, have been as fully established as the arms of Ireland, as the fleur de lis and the lions were established as the king's arms. What were these arms, if they were not the three crowns?

"If we admit that the three crowns were the arms of Ireland, we have no difficulty about this indenture, and this coinage. If we deny it, the frequent appearance of the crowns on the Irish coins is still to be accounted for; we have to seek for the arms of Ireland, and to wonder at the total loss of all coins, in a rich and singularly varied coinage, which bear the stamp of the national heraldic bearings.

"The three crowns were relinquished as the arms of Ireland by Henry the Eighth, probably because they were mistaken for the Papal arms; and supported the vulgar notion, that the Pope was the sovereign of Ireland, and the king of England merely the lord under him. That such an opinion prevailed, appears from a letter of the Lord Deputy and Council of Ireland to Henry the Eighth, in 1540: 'And we thinke that they that be of the Irisherie wolde more gladder obey your Highnes by the name of King of this your lande, than by the name of Lorde thereof; having had heretofore a folisshe opinyon amonges them, *that the Bisshope of Rome should be King of the same*, for extirpating whereof we think it write under your Highness pardon, that by authority of Parliament, it shulde be ordeyned that your Majisty, your heirs, and successors, shulde be named Kings of this lande.'"†

* Ruding's Annals, vol. ii. p. 376, 2nd edit.

† State Papers, Ireland, No. cccxxxi. vol. iii. part iii. page 278.

Mr. Butler's original remarks on this interesting subject were first published in 1837, in the Numismatic Journal, vol. ii. p. 70, and additional evidence was given by him in Mr. Lindsay's "View of the Coinage, p. 46. His opinions appear to derive some support from Sir James Ware's account of the three crowns, as denoting the three kingdoms of England, France, and Ireland; for if we take into consideration the devices on both sides of the coins, we find the arms of England and

Simon was of opinion, "that the first pieces with the three crowns were struck in the reign of Henry the Sixth," during his brief restoration, in 1470. But it is very questionable whether Henry caused any money to be made in Ireland during that brief interval; and when we consider the weight of the pieces appropriated to him, and compare them with those of Edward, ordered to be made in 1470, in which year the standard of the Irish groat was fixed at nearly forty-one grains, it cannot be admitted that any money of the three-crown type, the groats of which rarely exceed thirty, and never, I believe, thirty-two grains, was coined previous to the year 1478; and from the Act of the latter year, it may be inferred, that the liberties of Meath had been in abeyance during the first eighteen years of Edward's reign, and that when they were restored, the new type was introduced, and that the privilege of striking money, granted to Lord Grey, the Lord Deputy, was indicated by placing on the coins the arms of the Lord of Ireland.

I have now concluded my remarks, which have extended to a far greater length than I anticipated, when I entered on this investigation; and I trust that when the opinions I have advanced, and the evidence I have adduced, shall be duly considered, it will be admitted that I have in some degree succeeded in clearing up several of the obscurities in which the history of the coins of this reign have been so long involved.

France quartered on the obverse; and on the reverse, the arms of Ireland. Now it is probable Sir James Ware knew Ireland had been represented by arms of some kind, but that he committed the mistake of supposing that the device on the reverse alone represented *three* kingdoms instead of *one*.

APPENDIX.

While these sheets were passing through the press, I received a communication from the Rev. Mr. Butler, expressing his desire to make known a conjecture which he had made respecting some of the three-crown groats, and offering at the same time to permit me to publish it as an Appendix to my paper. I gladly availed myself of the kind offer, and I trust that the originality of the conjecture, and the ability with which my learned friend has supported his views, will render it acceptable to my readers.

"Trim, 1840.

"My dear Sir,

"In Mr. Lindsay's very valuable 'View of the Coinage of Ireland,' he notices some newly discovered varieties of the money, commonly called the three-crown money, from its bearing on the reverse the ancient arms of Ireland.

"One of these varieties, he observes, bears the 'remarkable legend, Rex Anglie Francie et Rex Hibernie, the latter title being hitherto supposed to have been first adopted by Henry the Eighth.'

"Mr. Lindsay is of opinion that these coins, of which he engraves two specimens, (Nos. 126, 127,)* belong to Edward IV., and I believe that this appropriation of these coins has met with your concurrence. It is hazardous to oppose the judgment of two such numismatists, nor should I attempt to do so in a case which had been fully examined and decided; but it is probable that it did not occur, either to Mr. Lindsay or to you, to investigate the obscure claim which I shall now endeavour to urge upon you.

"The case we have to consider is this: We have coins bearing the title of Rex Hybernie. To what king are these coins to be assigned? From their pattern, their execution, and their weight, it is plain that they are of the time from Edward the Fourth to Henry the Seventh, inclusive; but the public title of all the recognized kings in that period, was Dominus Hybernie, which title appears upon the coins of Edward the Fourth, Richard the Third, and Henry

* See also Pl. IV. fig. 77, of this Essay.

the Seventh; and it is not to be supposed that Edward the Fifth coined money in Ireland with a new die, and a new title, who, if he coined any money, used in England his father's dies.

"If, therefore, we attribute these coins to any of these kings, we must suppose, either that one of them, at some uncertain time, for some reason, which we cannot conjecture, assumed this regal title, and afterwards as capriciously relinquished it; or that some Mint Master chose to give his sovereign a title which did not belong to him, and to impress it on his coins; a most improbable act in a Royal Mint Master, and one which a counterfeiter would carefully avoid.

"But there was another king to whom none of these reasonings apply, who, we have reason to think, coined money in Ireland, and who had a motive for assuming the title of King of Ireland; and (in the absence of direct evidence) to suppose that he did take that title, and coined money bearing it, is a less violent supposition than either of those which I have considered.

"In 1486, Lambert Simnel was received in Dublin with open arms by the Geraldines and the other Irish lords, as the representative of the House of York, which was always popular in Ireland, and 'as the son and lawful inheritor of the good Duke of Clarence, *their countryman* and protector during his life,'* and was proclaimed king, by the title of Edward the Sixth. Early in May, 1487, he was crowned in Christ Church, and 'the Parliament, Courts of Justice, Processes, Statutes, and Acts of the Council, came all out in his name.'†

"At that time there was a mint in Dublin,‡ and from the various patterns of

* Campion's History of Ireland, p. 103, Dub. 1633.

† Ware's Annals of Ireland, pp. 4—6, folio, 1705.

‡ If Thomas Galmole, alias Thomas Archibold, was Master and Worker of the Money of Silver, in Dublin, in the reigns of Richard the Third and Henry the Seventh, (and it is probable that he was so, for we find him so styled in 1483, (Ruding, vol. ii. p. 376,) and again, in 1506, (Rot. Can. Hib.) it is likely that some of the coins usually given to Henry the Seventh do not belong to the Royal Mint. The artist who could design and execute the Dominus Groat of Richard the Third, could not have perpetrated such barbarisms of spelling as Sivitas and Duxlin, or the barbarities of execution which disgraced these coins. If they belong to this reign they are probably some of the counterfeit money against which Henry the Seventh issued a proclamation in 1492, (Ware.) I may observe, that although more hastily executed, the Rex Groats, in the letters and whole style, appear to my not much-practised eye strongly to resemble the Dominus Groats of Richard the Third. Were they both the workmanship of Thomas Galmole?

Henry the Seventh's money, which are still extant, and from the fact that in 1483, 'the profits of the mint' were 'granted to the Earl of Kildare, in consideration of the charges he is at in the government, during the time he continues in it.'* It is to be inferred that there was, at that time, almost a constant coinage in Dublin, and if any money was coined in Dublin in the latter part of 1486, or in the beginning of 1487, it was Lambert Simnel's money, and bore his titles.

"It is extremely probable that he did coin money, for from his arrival in Ireland, he had at his command all the usual resources of the Irish Mint, and after the landing of the Earl of Lincoln, if from the first he was not supplied with money from Flanders, it was an obvious and easy method of multiplying his Flemish Groats, to melt them down and debase them to the Irish standard; a method not strange to the Irish Mint Master; and although Martin Swartz and his Almaines, would probably require to be paid in the pure grosses of Charles the Bold, some of which are still picked up in this country, and in the north of England, his Irish followers would be satisfied with money of the alloy, to which they were accustomed.

"Now, as it appears from the joy manifested by the Irish, at the passing of the Act proclaiming Henry the Eighth King of Ireland,† from the jibe of Henry the Seventh to the Irish lords at Greenwich, 'that if he did not come over soon they would crown apes,' and from other notices, that the Irish of that day were animated by an instinctive love of royalty, is it not probable that, too wise not to know the power of names and titles, the crafty counsellors of this mock king, the only English king ever crowned in Ireland, would not neglect to flatter the vanity of the Irish, on whose enthusiasm in his behalf they chiefly depended, by the cheap expedient of giving on Simnel's money, which was to circulate amongst them, in addition to his other imperial titles, the title of King of Ireland, thereby gratifying the national pride by nominally restoring Ireland to its ancient dignity as a kingdom, and obliterating a mark of vassalage, and of foreign domination.

"It is then probable that Lambert Simnel coined money in Dublin, and that on it he bore the title of King of Ireland, and it is not probable that that title

* Simon, Appendix, No. XVIII.

† State Papers, Ireland, vol. iii. part. iii. No. CCCXL. p. 304.

was borne by any other king to whom we can assign these groats; we shall therefore be justified in attributing them to Lambert Simnel, until some reason is shown to the contrary.

"It is true that the claim here put forward rests entirely upon conjecture, and that you and Mr. Lindsay, and other fully informed and experienced numismatists, may be aware of facts, which render it untenable; but the only evidence* which I know of at all inconsistent with it, is the legend of a half groat in the cabinet of the Rev. Mr. Martin, given by Mr. Lindsay in his Coins of Henry the Seventh, which reads, HENRIC DI GRAR HIBERNIE; but what inference can be drawn from so obscure a legend on a coin so blundered, that on the reverse it has CIVITAS DUXBLIN. Your beautiful engraving, which you were kind enough to send me, of one of these groats, from the *private* collection of the Dean of St. Patrick's, so truly called by Mr. Lindsay a *public* benefit, which has a legend hitherto unknown, and reads, EDWARDVS on the obverse, and on the reverse, ET REX HYBERNIE, (Pl. IV. fig. 75,) strengthens my position, that these coins were struck by the mock Edward the Sixth.

"Apologizing to you for the length of this letter, which has much exceeded my expectations,

"I am, my dear Sir,

"Yours most sincerely,

"R. Butler.

"*Dr. A. Smith.*"

TABLE OF THE WEIGHT OF THE GROAT AT DIFFERENT PERIODS DURING THIS REIGN.

1461 to 1465,	the groat weighed	45	grains.
1465 „ 1467,	„	36 ?	„
1467 „ 1470,	„	$22\frac{1}{2}$	„
1470 „ 1473,	„	$40\frac{10}{11}$	„
1473 „ 1479,	„	$32\frac{1}{7}$	„
1479 „ 1483,	„	31	„

* It is probable that decisive evidence on this subject is to be found in the unpublished Acts of Poynyng's Parliament.

NAMES OF CITIES AND TOWNS WHICH APPEAR ON THE IRISH COINS OF EDWARD THE FOURTH.

CORK.	CIVITAS CORCAGIE.	Pl. II. fig. 38.
DROGHEDA.	VILLA DE DROGHE.	,, 44.
	. . . . . DROGHEDA.	,, 29.
DVBLIN.	CIVITAS DVBLIN.	Pl. I. fig. 17.
	. . . . DVBLINI.	,, 8.
	. . . . DVBLINIE.	,, 1.
LIMERICK.	. . . . LIMIRICI.	Pl. III. fig. 55.
TRIM.	VILLA DE TRIM.	,, 60.
WATERFORD.	CIVITAS WATFORD.	Pl. IV. fig. 72.
	. . . . WATERFORD.	Pl. III. fig. 63.
WEXFORD.	VILLA WEISFOR.	Pl. IV. fig. 74.

TABLE SHEWING THE NUMBER AND DENOMINATIONS OF THE COINS ENGRAVED.

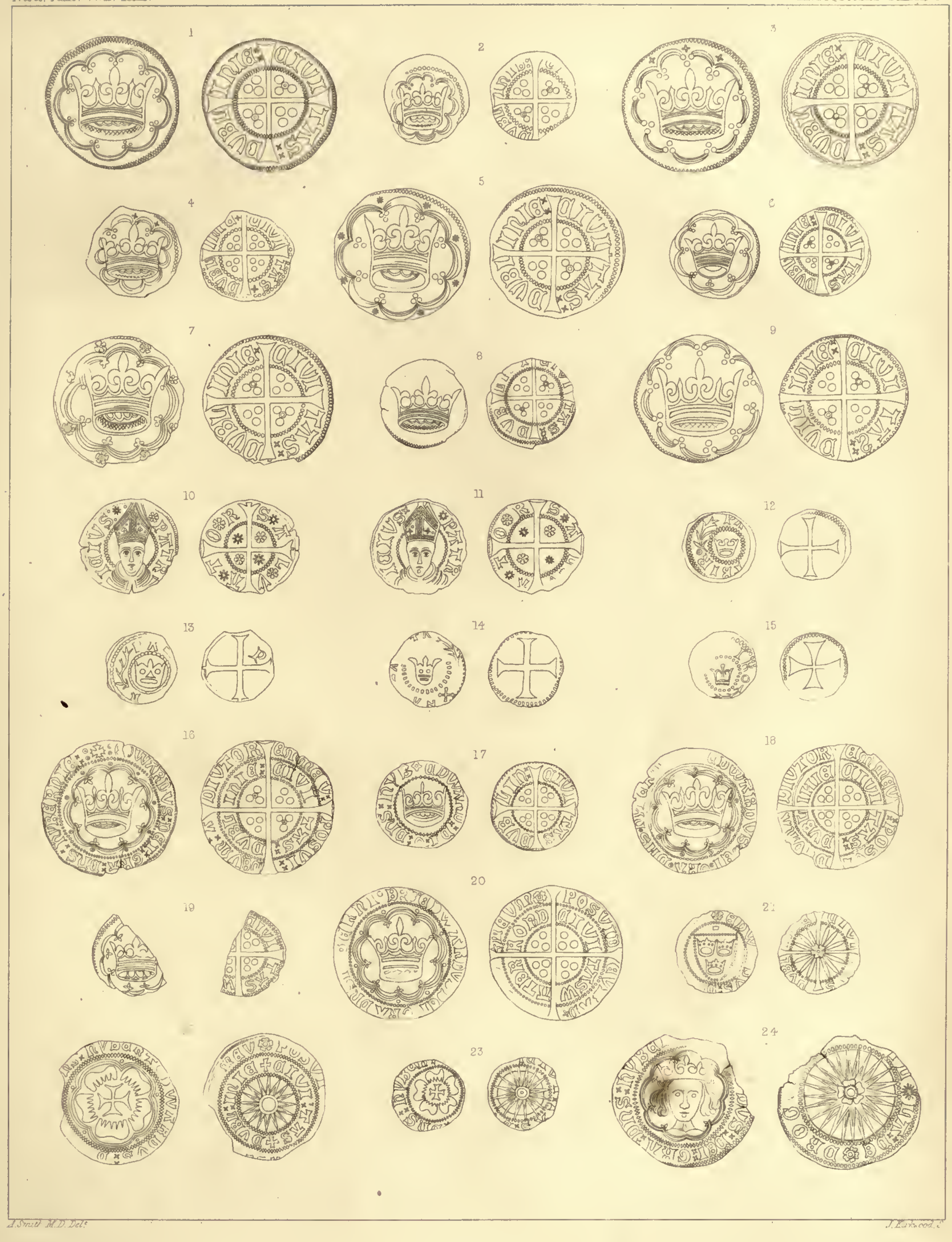
PLATE.	DOUBLE GROATS.	GROATS.	HALF-GROATS.	PENNIES.	COPPER AND BRASS.	TOTAL.
1	1	9	..	7	7	24
2	1	11	3	8	..	23
3	..	15	4	3	..	22
4	..	12	6	5	1	24
	2	47	13	23	8	93

EXPLANATION OF THE PLATES.

The numbers marked with an asterisk (*) have not been engraved before; several of them are only varieties of coins which have been published in other works.

Plate I.

NO.	DENOMINATION.	MINT.	DATE.	WEIGHT.	PAGE.	REFERENCE.
1	Groat.	Dublin.	1461	44½ grs.	5	Dⁿ. of St. Patrick's.
2	Penny.	,,	,,	9	7	,,
*3	Groat.	,,	,,	44	6	,,
*4	Penny.	,,	,,	12	7	,,
*5	Groat.	,,	1462	44½	6	,,
*6	Penny.	,,	,,	11	7	,,
7	Groat.	,,	,,	42½	6	,,
*8	Penny.	,,	?	10	7	,,
*9	Groat.	,,	?	28	6	,,
10	Farthing, copper.	?	,,	9	9	Lieut.-Col. Weld Hartstonge.
*11	,, ,,	?	,,	9	,,	Dⁿ. of St. Patrick's.
*12	Half-farthing, ,,	?	,,	11	,,	,,
13	,, ,,	?	,,	9	,,	Rev. Mr. Butler.
*14	,, ,,	?	,,	7	,,	,,
*15	,, ,,	?	,,	6	10	Dⁿ. of St. Patrick's.
16	Groat.	Dublin.	1463	38	11	,,
17	Penny.	,,	,,	9¼	,,	,,
*18	Groat.	,,	,,	38	,,	,,
19	Penny.	Waterford.	,,	(Broken.)	,,	Mr. Lindsay.
*20	Groat.	,,	,,	40	,,	Dⁿ. of St. Patrick's.
21	Farthing?	Dublin.	?	9	14	Rev. Mr. Butler.
22	Groat.	,,	1465	27	12	Dⁿ. of St. Patrick's.
23	Penny.	,,	,,	8½	,,	,,
*24	Double Groat.	Drogheda.	1467	38	17	Rev. Mr. Butler.



A. Smith M.D. Delᵗ. J. Kirkwood Sc.


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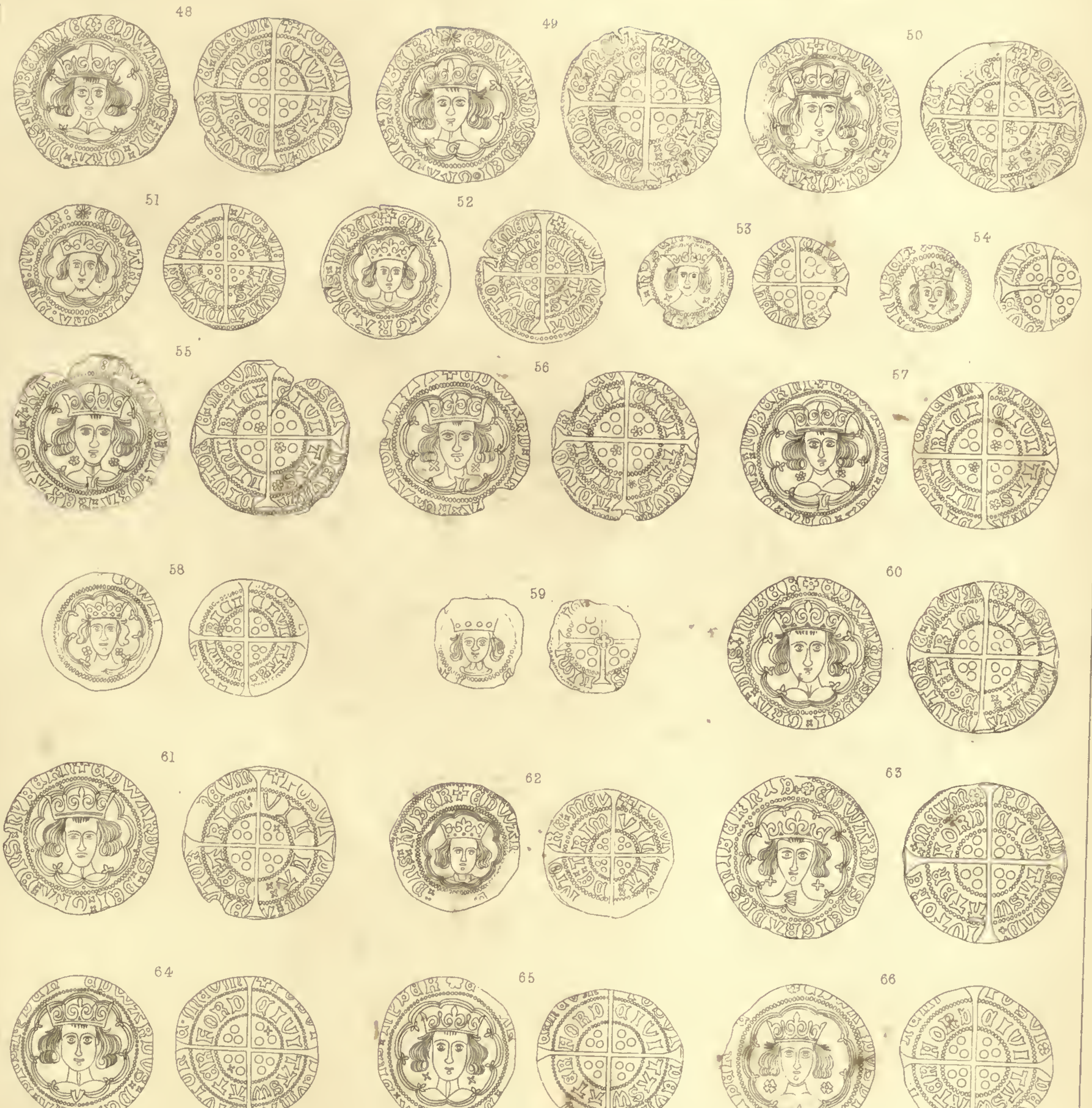
Plate II.

NO.	DENOMINATION.	MINT.	DATE.	WEIGHT.	PAGE.	REFERENCE.
25	Double Groat.	Dublin.	1467	44 grs.	17	Dn. of St. Patrick's.
*26	Half-groat.	"	"	10	"	Rev. Mr. Butler.
27	Groat.	Trim.	"	23½	"	Lieut.-Col. Weld Hartstonge.
*28	Half-groat.	"	"	11¼	"	Dn. of St. Patrick's.
*29	Groat.	Drogheda.	1470	29	19	"
*30	"	"	"	27	"	"
31	"	Dublin.	"	32	"	"
*32	Penny.	"	"	6	"	"
*33	Groat.	"	"	26	"	"
*34	Penny.	"	"	6	"	"
*35	"	"	?	9	20	"
*36	"	"	?	6	"	"
*37	"	"	?	9½	"	"
*38	Groat.	Cork.	1470–2	38	22	"
*39	"	"	"	30	"	"
*40	"	Drogheda.	"	33	"	"
*41	"	"	"	34	"	"
*42	"	"	1473–8	32½	23	"
*43	"	"	"	33	"	"
*44	Half-groat.	"	"	15	"	"
*45	Penny.	"	"	8	"	Mr. Lindsay.
*46	"	"	"	6	"	Dn. of St. Patrick's.
47	"	"	"	7	"	"

Plate III.

NO.	DENOMINATION.	MINT.	DATE.	WEIGHT.	PAGE.	REFERENCE.
*48	Groat.	Dublin.	1470–2	45½ grs.	23	Mr. Lindsay.
*49	,,	,,	1473–8	32	24	Dn. of St. Patrick's.
*50	,,	,,	,,	32	,,	,,
51	Half-groat.	,,	,,	17	,,	,,
*52	,,	,,	,,	17	,,	Mr. Sainthill.
*53	Penny.	,,	,,	7½	,,	Dn. of St. Patrick's.
54	,,	,,	,,	6	,,	Mr. Lindsay.
*55	Groat.	Limerick.	1473–6	31½	,,	Dn. of St. Patrick's.
*56	,,	,,	,,	31	,,	,,
*57	,,	,,	,,	31	,,	,,
*58	Half-groat.	,,	,,	17	,,	,,
59	Penny.	,,	,,	9½	25	Mr. Lindsay.
*60	Groat.	Trim.	1470–2	28	,,	Dn. of St. Patrick's.
*61	,,	,,	,,	34	,,	,,
*62	Half-groat.	,,	,,	23	,,	Rev. Mr. Butler.
63	Groat.	Waterford.	,,	43	,,	Mr. Sainthill.
64	,,	,,	,,	28	,,	Dn. of St. Patrick's.
*65	,,	,,	,,	31	,,	,,
66	,,	,,	,,	31	,,	,,
*67	,,	,,	,,	30½	,,	,,
*68	,,	,,	1473–8	33	,,	,,
*69	,,	,,	,,	32	,,	,,



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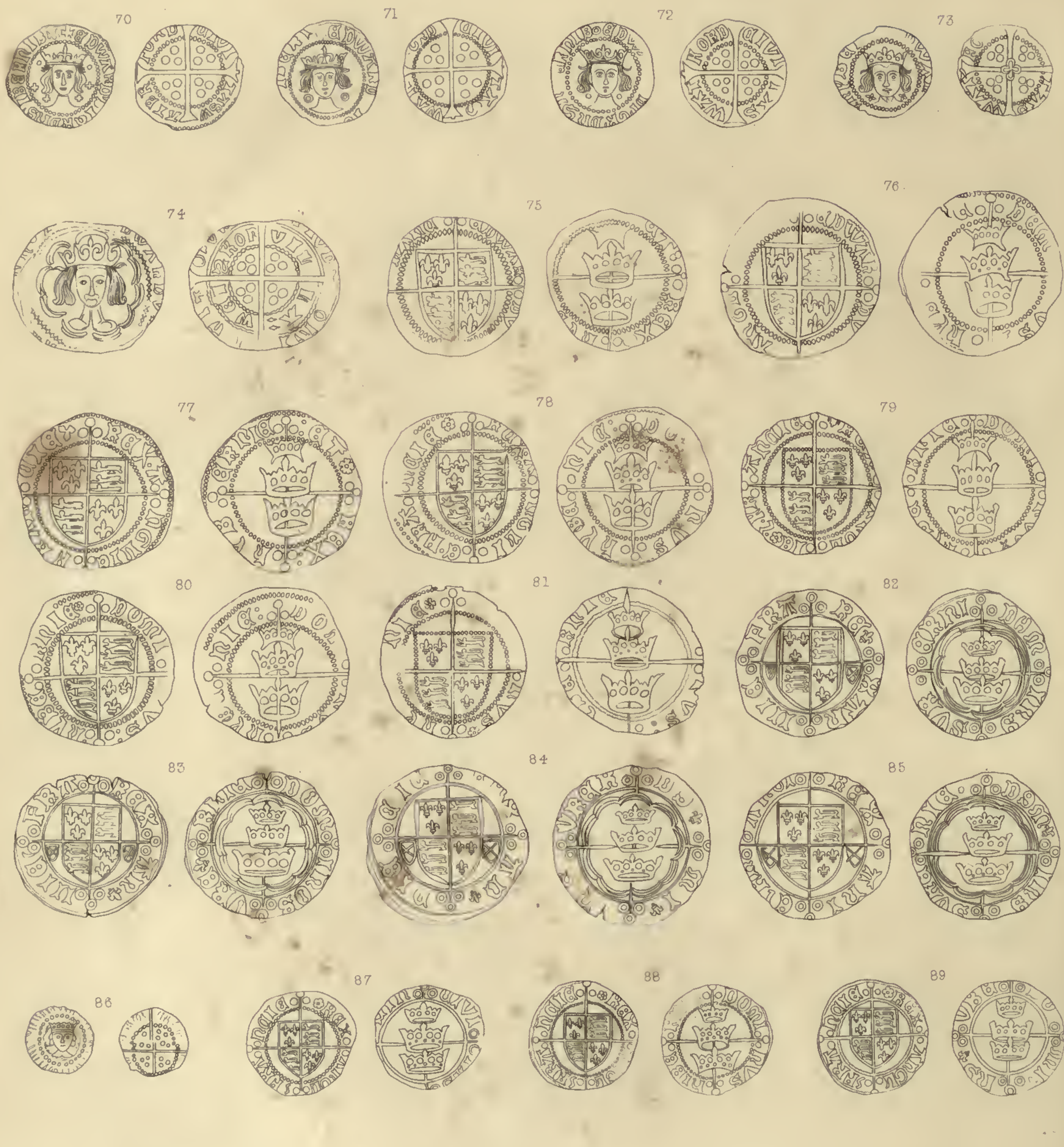

A Smith M D Del^t

J Kirkwood Sc

Plate IV.

NO.	DENOMINATION.	MINT.	DATE.	WEIGHT.	PAGE.	REFERENCE.
*70	Penny.	Waterford.	1473–8	10 grs.	26	Dn. of St. Patrick's.
*71	,,	,,	,,	$9\frac{1}{2}$	,,	,,
72	,,	,,	,,	8	,,	Mr. Sainthill.
73	,,	,,	,,	8	,,	,,
*74	Groat.	Wexford.	,,	26	,,	Rev. Mr. Butler.
*75	,,	Trim. ?	1478	$23\frac{1}{2}$	32	Dn. of St. Patrick's.
76	,,	,,	,,	29	,,	Mr. Lindsay.
77	,,	,,	,,	27	,,	Dn. of St. Patrick's.
*78	,,	,,	,,	30	,,	,,
79	,,	,,	,,	28	34	Mr. Lindsay.
*80	,,	,,	,,	30	32	Dn. of St. Patrick's.
*81	,,	,,	,,	30	34	Mr. Lindsay.
*82	,,	,,	1479	26	33	Dn. of St. Patrick's.
*83	,,	,,	,,	26	,,	,,
*84	,,	,,	,,	28	,,	,,
*85	,,	,,	,,	29	,,	,,
86	Farthing. ? Brass.	?	?	$3\frac{1}{2}$	27	,,
*87	Half-groat.	Dublin.	1478–9	11	32	,,
*88	,,	Trim. ?	,,	13	,,	,,
*89	,,	,,	,,	12	,,	,,
*90	,,	,,	,,	$13\frac{1}{2}$	,,	,,
*91	,,	,,	,,	$14\frac{1}{2}$	,,	Rev. Mr. Butler.
*92	,,	,,	,,	11	34	Dn. of St. Patrick's.
*93	Penny.	,,	,,	5	32	,,

II.—*On the Irish Coins of Henry the Seventh.* By Aquilla Smith, M.D., M.R.I.A.

Read 14th June, 1841.

INTRODUCTION.

As the coins which I am about to describe, belong to some of the Henrys, it appears to me that the best course which can be adopted, is, in the first place to inquire, whether any of them can be assigned to the predecessors of Henry the Seventh, who bore the same name; for by proceeding in this manner, the period, to which the coins can be appropriated, will be reduced to the smallest possible limit, and the inquiries which follow in the subsequent pages will be greatly facilitated.

Simon has pointed out the mistake committed by Bishop Nicholson, who says that "Henry the Fourth, in the year 1404, ordered the noble of his five immediate predecessors to pass in Ireland for ten shillings; and, from that time, all sorts of coin went at a higher value here than in England."*

The words referred to by the learned Prelate, who quoted from Sir John Davis's Reports, are these, "Mes le primer difference et inequalitie enter les standards del English moneys et Irish moneys est trove en 5 Edw. 4. Car donques fuit declare en parliament icy, que le noble fait en temps Edw. 3. R. 2. Hen. 4. Hen. 5. et Hen. 6. serroit de cest temps en avant currant en cest realm pur 10*s*. et issint le demy noble, et touts auters coines solonque mesme le rate. Vide Rot. Parliament, 5 Edw. 4. cap. 40. et 11 Edw. 4. cap. 6. et 15 Edw. 4. cap. 5. in le office del Rolles in Castro Dublin."†

The error of Bishop Nicholson in writing Henry IV., instead of Edward IV., is so palpable from his reference to Davis, that it would not require any notice

* Irish Historical Library, 8vo. 1724, p. 162.

† Davis's Reports, fol. 1674, p. 22.

here, had not Simon remarked, that "this last Act (15 Edw. IV.) seems to hint, that some kind of money was coined here in this reign, (Henry IV.,) as well as in that of Henry V."* He also conjectures that the great scarcity of money in England seems to have been a reason for coining the more money in Ireland, and therefore believes that the groats, figs. 56, 57, 58, 59, 60 in his 3rd Plate, belong to Henry the Fifth.

The Act of 1475, from which Simon drew his inference, ordains "that the coin called the gross, made in the reigns of Edward the Third, &c., not clipped, shall be of the value of six deniers. The gross made in England in the time of the present king, not clipped, shall pass for five deniers, *and all the moneys struck in Ireland to be of the same value as they now are.*"†

The latter part of this extract is the only passage in the Act which could give any support to his opinion; but it appears to me to have reference only to the numerous coins of various types, "struck in Ireland" in the first fifteen years of Edward's reign, during which period his Irish money was considerably less in value than his English.‡

In 1421, the ninth year of Henry the Fifth, in a parliament held at Dublin, before James Earl of Ormond, the Lords and Commons agreed to send a petition to the king, praying for the redress of several grievances. The petition contains nineteen articles, the third of which prays, "that certain money be struck in Dublin as in England, and that the necessary officers, moneyers, &c., be appointed."§

From this evidence it is probable, that no legal money was coined in Ireland for some time previous to the date of the petition, and it leaves no grounds whatever for Simon's appropriation of any Irish coins to Henry the Fifth, who died

* Essay on Irish Coins, p. 19.

† Simon, Appendix, No. XIV.

‡ I am indebted to my learned friend, the Rev. Richard Butler, of Trim, for directing my attention to several important records of the reigns of Henry the Fifth and Sixth, which have hitherto been unknown to writers on Irish coins, and which may be found in the "Rotulorum Patentium et Clausorum Cancellariæ Hiberniæ Calendarium," vol. i. pars 1.

§ "Art. 3. Petunt quod certe monete cudantur in Dublinia sicut in Anglia, cum omnibus officiariis, monetariis, &c., necessariis."—*Rot. Pat.* 9, *Hen. V. cap.* 111.

In the extracts from the Calendar, the words in full have been substituted for the contractions, which it would be useless and inconvenient to retain.

in 1422; but this subject may be more conveniently discussed hereafter, when I shall endeavour to support Mr. Lindsay's appropriation of the coins in question, to Henry the Seventh.

A writ, directed to the Sheriff of Dublin, in the first year of the reign of Henry the Sixth, recites, "that the king had learned that many merchants brought into Ireland large sums of counterfeit, washed, and clipped gold, and that they carried away the king's silver money."* And a roll of the same year, after reciting, "that Henry the Fifth had been informed, that there were counterfeiters of gold and silver, and washers, clippers, and weighers of the same in Ireland, and that he had caused proclamation to be made against such practices, under the penalty of loss of life and limbs, and that no person should presume to weigh or refuse gold (except such as was counterfeit or washed); appoints Janico Dartas, Nicholas Daly, and Richard Talloun, jointly and separately, to inquire after those who presumed to weigh the king's gold, and also of those who dared to carry clipped, washed, or counterfeit gold from England into Ireland, for the purpose of accumulating the king's silver money, and further gives the aforesaid officers power to arrest such offenders, together with their money, and commit them to prison."†

* "Breve vicecomiti Dublinie directum, in quo recitatur regem, ex gravi querela ligeorum Hibernie, accepisse quod quamplures mercatores ad Hiberniam venientes huc portant secum, causa vendendi et emendi, maximas summas auri Regis controfecti, loti, et tonsi, ad dictum populum decipiendum, et pecunias Regis argenteas hinc, ad opus suum, subdole extorquendum de die in die non desistunt."—*Rot. Claus.* 1 *Hen. VI. cap.* 40.

† "Rex (recitatur qualiter H. V., cum, ex gravi et clamosa insinuacione dominorum spiritualium et temporalium ac communium Hibernie in parliamento existentium, accepisset quod nonnulle persone extiterint controfectores cune monete auri et argenti, ac lotores, tonsores, et ponderatores ejusdem monete infra Hiberniam, per brevia sua fecerit proclamari quod ne quis, sub pena vite et membrorum, foret controfector, lotor, tonsor, vel ponderator dicte monete, et quod ne quis aurum in recepcionibus, &c., (auro loto et controfecto excepto) ponderare seu denegare presumeret,) assignavit Janico Dartas armigerum, Nicholaum Daly, et Ricardum Talloun, conjunctim et divisim, ad inquirendum de eis qui cum belanciis aurum Regis in vendicionibus &c., ponderare presumpserint, ac de illis qui aurum Regis tonsum, [aut lotum,] seu controfectum, extra Angliam in Hiberniam cariare presumpserint, ad monetam Regis argenti pro hujusmodi auro, vel alio modo accumulandum; et culpabiles, una cum mone [ta] Regis argenti sic accumulata, in quorumcumque manibus existat, capiendum, et ipsos prisone committendum. Dub, 10 Julii."—*Rot. Pat.* 1 *Hen. VI. Dorso, cap.* 109, *b.*

In the second year of this king, in a great council, held on the morrow of All Souls, before Edward Bishop of Meath, deputy of Edmund Earl of March, it was ordained, that the noble, half, and quarter noble (except counterfeit gold) should be universally received by weight, and that a standard weight should be deposited in the Irish Exchequer, and that all the sheriffs, mayors, &c., throughout the land, should have weights agreeing with the said standard, and that every liege subject should have access to the standard weight as often as he pleased, and that no person should refuse gold contrary to the aforesaid ordinance, under a penalty of ten shillings, to be paid to the king, and that any offender might be committed to gaol, and kept there until he made redemption and fine."*

It does not appear that the petition for the establishment of a mint in Dublin, in the ninth year of Henry the Fifth, was granted before the third year of Henry the Sixth, for on the 6th of February in that year, a grant of the office of master of the coinage in the Castle of Dublin, was made to John Cobbham, during the king's pleasure, provided that the money be made of the *same weight, allay, and assay, as the silver money which is made in London*, and that the said John may receive for the making of one lb. of money, to be made in the aforesaid Castle, only as much, and that he shall pay to the king as much, as the master of the coinage in London receives and pays for one lb. of the same sort, and he shall

* "In magno consilio, coram Edwardo episcopo Midie, deputato Edmundi comitis Marchie locum tenentis, in Crastino Animarum tento, ordinatum est, ad supplicacionem communium ad dictum consilium per brevia Regis electorum, quod nobilis, obolus, et quadrans auri (auro controfecto excepto) secundum pondus et valorem per ligeos ac alias gentes ad Hiberniam confluentes recipiantur per pondus universaliter: et quod unum standardum ponderis dicti auri standardo Anglie concordans sit, et in thesauro in custodia thesaurarii et camerariorum saccarii Hibernie, de cetero remaneret: et quod quilibet vicecomes, major, ballivus, senescallus, superior, et prepositus, per totam terram, ad eorum prosecucionem habeant pondera dicto standardo recte concordancia: et insuper, quod quilibet ligeus terre predicte habeat cursum ad dicta standarda in quolibet loco ubi assistunt, ad pondera standardi quociens sibi placuerit faciendum: et eciam, quod ligei, et indigene, et alienigene ad Hiberniam confluentes hujusmodi aurum, licet tonsum seu lotum, per pondus, secundum valorem et pondus ejusdem percipiant in futuro: et quod nullus hujusmodi aurum contra ordinacionem predictam refutet sub pena 10^s ad opus Regis solvendum: et quod corpus ejusdem delinquentis gaole committatur in ea moraturum quousque redempcionem et finem inde faciat &c."—*Rot. Claus.* 2 *Hen. VI. prima pars. cap.* 27.

be bound by indenture to perform the premises, in the same manner and form as the master in London is bound.*

In the third and fourth years of Henry, a grant of one hundred shillings a year, during the king's pleasure, was made to William Goldesmyth, the striker of the money in the Castle of Dublin.†

At a parliament held at Trim, in 1447, an Act was passed against clipping and counterfeiting the king's coin, and it was ordained "that no money so clipped be received in any place of said land, from the first day of May next to come, nor the money called the O'Reyly's money, or any other unlawful money, so that one coyner be ready at the said day to make the coyn."‡

In 1456, a parliament was held at Naas, and it was enacted, at the request of the Commons, that "whereas no mean could be found to keep the king's coin within the land of Ireland," all foreign merchants "shall pay for every pound of silver that they shall carry out of Ireland, forty-pence of custom to the king's customer, to the use of the king; and if any man shall do the contrary in concealing of the said custom, he shall pay for every penny, twenty shillings to the said customers, to the king's use,"§ and from the twelfth chapter of the same Act it appears that Ireland was greatly impoverished by the daily exportation of silver, and the great clipping of the coin, and that "the Irish money, called the O'Reyley's," daily increased; it was therefore enacted, that any person carrying silver out of Ireland shall pay for custom to the king twelve-pence for every ounce; "except lords and messengers going to England upon business of the public, who may carry plate with them, according to their degrees."||

* "Rex concessit Johanni Cobbham officium magistri cunagii in castro Dublinie faciendi, durante beneplacito, proviso quod moneta operata sit ejusdem ponderis, allaie, et assaie, sicut moneta argenti que in Londonio operata est, et quod dictus Johannes tantum pro factura 1 libre monete in castro predicto operate percipiat, et Regi tantum reddat, quantum magister monete in terra predicta pro hujusmodi libra percipit et reddit, et quod idem Johannes ad premissa facienda per indenturam obligetur, eisdem modo et forma quibus magister cunagii in terra predicta pro tempore obligatus existit. Trym, 6 Feb."—*Rot. Pat.* 3 *Hen. VI. cap.* 21.

† "Rex eisdem mandat quod Willelmo Goldesmy[th ?], percussori monete in castro Dublinie, 100s per annum ei per Regem concessos durante beneplacito annuatim solvant. [ ] Julii, anni predicti."—*Rot. Claus.* 3 & 4 *Hen. VI. cap.* 35.

‡ Simon, Appendix, No. III.

§ Ruding, 2nd edit. vol. ii. p. 341.

|| Simon, Appendix, No. IV.

The next and last Act of this reign relating to the coinage contains much that is important.

At a parliament held at Drogheda, in the year 1460, it was enacted, that the value of English gold coins should be raised one-fourth in Ireland, and that the gross of London, York, and Calais, not clipped within the extreme circle, should pass for five-pence in Ireland, and the smaller pieces in the same proportion. "And as not only the Dutchy of Normandy, but also the Dutchy of Guienne, when they were under the obedience of the realm of England, yet were no less separate from the laws and Statutes of England, and had also coynes for themselves different from the coyne of England; so Ireland, though it be under the obedience of the same realm, is nevertheless separate from it, and from all the laws and Statutes of it, only such as are there by the lords spiritual and temporal freely admitted and accepted of in parliament or great council, by which a proper coyne separate from the coyne of England, was with more convenience agreed to be had in Ireland under two forms; the one of the weight of half a quarter of an ounce troy weight, on which shall be imprinted on one side a lyon, and on the other side a crown, called an *Irelandes d'argent*, to pass for the value of one penny sterling; the other of VII. ob. of troy weight, having imprinted on one part of it a crown, and on the other part a cross, called a *Patrick*, of which eight shall pass for one denier. That a gross be made of the weight of three deniers sterling, and to pass for four deniers sterling, which shall have imprinted on it on one side a crown, and on the other side a cross like the coyne of Calais, bearing about the cross in writing, the name of the place where the coin is made; and that every person, who brings bullion to the mint, ought to receive and have for every ounce of silver, troy weight, nine of the said grosses of the value of three deniers. That the coyne called the *Jack*,* be hereafter of no value and void, and that the above coynes be made in the Castles of Dublin and Trymme;" and at an adjourned sitting of the same parliament it was enacted, "that the denier with the cross called Irelandes be utterly void, and that in lieu of it a penny be

* Having lately seen some copper pieces of *Jac*obus the Second of Scotland, which were found in Ireland, it occurred to me that the "Jacks" mentioned in the Act, might be these coins of James, who was contemporary with Henry the Sixth.

Since this note was written I find that the same term was applied to the brass shillings of James the Second. See "The Jacks put to their trumps," p. 123, in the Historical Songs of Ireland, printed for the Percy Society, 1841.

struck in silver, having the weight of the fourth part of the new gross of Ireland, to be imprinted and inscribed as the new gross."*

From the grant to Cobbham, in the year 1425, which provides that the money to be made in Dublin shall be of the *same weight, allay, and assay, as the silver money made in London,* and the appointment in the following year of a moneyer, with an annual salary of one hundred shillings, it is more than probable that some money was coined in Dublin about that time.

I know of only one coin which I can venture to assign to Henry the Sixth, during the early part of his reign. It has on the obverse, the king's head with an open crown fleury, within a circle of pellets, a star of six rays at the left side of the neck, mint mark a cross, legend HENRICVS DNS HIBNIE, an annulet at the end of the legend; reverse, a plain cross with three pellets in each quarter, legend CIVITAS DVBLINIE; there is an annulet after CIVI. It weighs twelve grains and a quarter.

This interesting coin, which is of the highest rarity, and in fine preservation, is in the cabinet of the Rev. J. W. Martin, of Keston, to whom I am indebted for the loan of it and several other Irish coins of great rarity.

That this coin, which on account of the absence of the tressure on the obverse, I believe to be a penny, was struck in the early part of the reign of Henry the Sixth, is very probable; evidence is now, for the first time, adduced, which proves that in 1425 Irish money was ordered to be made of the same standard as the English money, and the weight of this piece, which is equal to many of the English pennies of Henry the Sixth, and considerably more than the fourth part of any of the Irish groats of Henry the Seventh, which I believe never exceed thirty-two grains, and rarely weigh so much, shows clearly that it must have been coined during the reign of Henry the Sixth. The mint mark is similar to that which occurs on some of the English coins usually assigned to Henry, the annulets also, and the star, are marks which connect it with the same reign. The

* Simon, Appendix, No. V.

occurrence of the Roman N in three places in the legends of this coin, is very remarkable, I have not seen any other Irish coin from the time of Edward the Third, to that of Henry the Eighth, which has the Roman N in its legend, except a Dublin groat of the third year of Edward the Fourth.*

It is very doubtful, whether any money was coined under the authority of the Act of 1447, in which the provision for a new coinage depended on the coiner being ready against a certain day; and the great scarcity of silver, together with the daily increase of "the Irish money, called the O'Reyley's," mentioned in the Act of 1457, could scarcely have happened, had any legal money been coined in the meantime.

The Act of 1460 appears to warrant the inference, that if any money was coined in Ireland previous to that time, it must have been similar in *type* and *standard* to the penny already described; for by the same Act, "a proper coyne *separate from the coin of England*, was with more convenience agreed to be had in Ireland."

The type and weight of the coins ordered to be made in 1460, are so fully described in the Act, that it would appear there could be little difficulty, in determining which coins should be assigned to this date.

The penny called the "Irelandes d'argent," has not hitherto been discovered. The Act which ordered it to be made, came into operation on the 17th of March, and on the Monday after Trinity Sunday (8th June), the penny called "Irelandes" was declared to "be utterly void."

A few copper coins, of the type ascribed in the Act to the half-farthings called "Patricks," have been found, but most of them exceed, by several grains, the weight fixed by the Act. There is one† which I am inclined to appropriate to Henry the Sixth, because it weighs only six grains, and the form of the cross on the reverse is different from that on the heavier coins, which I believe were minted early in the reign of Edward the Fourth.

The type of the groat as described in this Act, agrees so far with some of the coins of Edward the Fourth, that it is still doubtful which of them are to be considered as belonging to Henry.

Taking for granted that the groat published by Simon (Pl. III. fig. 61) is

* See Irish Coins of Edw. IV. Pl. I. fig. 18, Trans. R. I. Academy, vol. xix.

† Ibid. Pl. I. fig. 15.

accurately represented, as having a tressure of *twelve* arches round the crown, which is very shallow, and a trefoil at each point of the tressure, I assign it to Henry the Sixth. It is much to be regretted that this coin cannot now be found in the numerous and extensive collections to which I have had access; but that such a piece was in Simon's possession can hardly be doubted, as the penny subsequently published by Snelling in his supplement (Pl. I. fig. 16) agrees with it in the number of arches in the tressure, and in the form of the crown, and such a coincidence can hardly be attributed to a mistake of the artist; this penny I also appropriate to Henry the Sixth.

I am aware that a distinguished collector in England does not believe that a groat with twelve arches in the tressure ever was in existence, on the grounds that no such piece is at present known; but a short time since, the same argument might have been applied to a coin of James the Second,* as no specimen of it was then known; two however have been lately discovered; one in pewter, which was found in a sewer in Dublin, is in the cabinet of the late Dean of St. Patrick's, and another in brass, in a good state of preservation, is in the possession of the author.

I shall now proceed to the investigation of the coins, which I conceive belong to Henry the Seventh, a task which I enter on with much diffidence, as it presents difficulties at almost every step of the inquiry.

THERE are many coins which may, without any doubt, be appropriated to Henry the Seventh, although very few documents relating to his Irish coins have been discovered, nor is it likely that any others have been preserved, from which direct evidence can be obtained.

The almost total absence of records connected with the coinage of this reign, is the more remarkable, as the greater part of the numerous Acts, relating to money coined during the reigns of Henry's immediate predecessors, Edward the Fourth, and Richard the Third, are still preserved among the State Papers in Ireland.

Ruding, on the authority of Snelling, states, that in the first year of Henry the Seventh "Robert Bowley" was "Maister of the Cunage and Mynt within the Cities of Dyvelin (Dublin) and Waterford."†

* Simon, Pl. VIII. fig. 177.

† Annals, vol. i. p. 90.

On the 9th of March, 1491, Nicholas Flint* was by the king's appointment "made overseer of the mints of Dublin and Waterford;" and on the 15th of April following, a proclamation was issued by the king at Greenwich, authorizing Gerald Earl of Kildare "to cause and prescribe certain laws for the prevention of false or mixt silver in coin within that his Lordship of Ireland."†

The English Act of his nineteenth year, 1504, states that "The coins, especially of silver, were so impaired as well by clipping as counterfeiting the same, and by bringing into the realm the *coin of Ireland*, that great rumour and variance daily increased among his subjects, for taking and refusing the same;" and in the same year it was enacted, that no person should bring into England "of the *coin of Ireland*, above the sum of three shillings and four pence, on pain of forfeiture and imprisonment, and fine and ransom, at the king's pleasure."‡

In 1506, the king granted to Thomas Galmole, alias Archibold, of Dublin, Goldsmith, the office of Master of the Coinage and Monies, made within the Castle of Dublin, and to hold the said office himself, or by his deputy, during the king's pleasure. §

These scanty records and the coins themselves, are the only sources from which evidence can be derived respecting the numerous coins of Henry which have been preserved; and before I enter on the description of them it will be convenient to inquire, whether it be possible to determine the standard by which the coinage was regulated. The want of any direct evidence on this subject compels me to revert to such facts as may be collected from the history of the preceding reigns.

* This person held several offices connected with the English mint, in the early part of this reign 1485—1487, he was, "Cont^r. Monete et Cunagii infra Turrim Lond." "Assaiator Monete et Cunagii"—"Sculptor de et pro ferris," "Campsor Monete et Cunagii infra Tur. London"—and on the 17th of May, 1486, he was appointed Keeper of the King's Exchange.—*Ruding*, vol. i. pp. 98, 106, 119, 161, and vol. iv. p. 194.

† Ware's Annals of Ireland, A.D. 1491.

‡ Ruding, vol. ii. pp. 397 and 399.

§ "18. Rex concessit Thomae Galmole de Dublinia, goldsmyth, alias Thomae Archibold, magisterium cunagii et numismatum infra castrum Dublinie fiendorum, habendum officium predictum per se vel deputatum, durante beneplacito. 6 Julii." *Rot. Pat.* 21 *Hen. VII. cap.* 18.

This Thomas Galmole was probably the same person who was "master and worker of the money of silver, and keeper of the exchanges in the cities of Devylyn and Waterford," in 1483. *Ruding*, vol. ii. p. 376.

I have already shewn, that in the third year of Henry the Sixth (1425), the master of the coinage in Dublin was bound, by indenture, to make the coins of the *same weight, allay, and assay, as the silver money, which was made in London*, from which time until the thirty-eighth year of the same reign (1460), it does not appear, nor is it probable, that any change in the standard took place; but in the latter year the Irish groat was ordered to be made "of the weight of three deniers sterling." The penny, or "denier sterling" of that time, weighed fifteen grains, consequently the Irish groat of 1460 should weigh only forty-five grains, and was a fourth less in weight and value than the English groat. And from this time "the first difference and inequality betwixt the standard of the English and Irish monies"* is to be dated, and not, as Sir John Davis supposed, from the fifth year of Edward the Fourth, at which time, however, the standard in Ireland was again changed, while its proportion to the English groat was preserved, which had been reduced in 1464 from sixty to forty-eight grains. During the subsequent years of Edward's reign, the standard of his Irish money was frequently altered, according to the exigencies of the times, and in the first year of Richard the Third, 1483, his Irish money was ordered to be made according to the standard of the twelfth year of Edward the Fourth, at which time the weight of the Irish groat was about thirty-two grains, or a third less than the English.

It has been just stated, that Edward reduced the English groat to forty-eight grains, which standard was adhered to in England, until the eighteenth year of Henry the Eighth. The Irish groat, during the latter part of Edward's reign and that of Richard, was about a third less than the English, and that the same proportion was observed in the early part of the reign of Henry the Seventh, is evident, from a passage in a letter, written by Octavian, Archbishop of Armagh, to the king in 1487, "recommending Arthur Magennis to that prince, for the bishopric of Dromore, wherein he says, that the revenue of that diocese is not worth above forty pounds, of the coin of Ireland, *which is less by the third part than the coin sterling*."† From this evidence and also from the fact, that some of Henry's groats, when in good preservation, weigh thirty-two grains, which I believe they never exceed, I conclude that the standard in Ireland was not altered during the reign of Henry, and that his Irish groat was always a third less than the English of the same period.

* See p. 50.

† Simon, p. 31.

Some arrangement is necessary, for the purpose of attempting to determine the order in which the several coins were issued from the mints. In the absence of documents by which the dates might be fixed, the only safe guide which remains are the coins themselves, and from deliberate consideration of the types and numerous varieties which have come under my observation, I have selected the cross on the reverse, as the character which best distinguishes the three sections into which I propose to divide them.

THE FIRST SECTION.

The coins included in this section have on the obverse a shield, bearing the arms of England and France, quartered by a cross, the extremities of which are generally terminated by three annulets; and on the reverse, three crowns in pale (the arms of Ireland),* with a similar cross: all the groats which I have seen have (with one exception) the letter H under the crowns; they usually weigh about twenty-eight grains, and never I believe exceed thirty.

The description of the numerous varieties of this type will be facilitated by dividing them into three classes: 1st, coins minted at Dublin; 2nd, those which bear the name of Waterford; 3rd, coins without the name of the place of mintage.

Of the Dublin mint there are groats, half-groats, and pennies.

The groat (Pl. V. Fig. 1) has the legends HENRIC DI GRACIA, and CIVITAS DVBLINIE. The lions on the shield have their tails doubled back in a manner which distinguishes this coin from the three crown money of Edward the Fourth and Richard the Third. The upper crown on the reverse has a double arch, surmounted by a ball and cross. It is evident that the artist at first inserted the letter E in the name of the city, and afterwards attempted to conceal his blunder by punching over it the letter V.

A groat has been lately found at Trim, the obverse of which is from the same die as fig. 1, the reverse has the cross and arches over the upper crown, but the legend is divided as follows: CIVIT-ASDV-BLIN-IEE, with a fleur-de-lis after the last letter. The coin is in the cabinet of the Rev. R. Butler, a small portion is broken off, and it weighs twenty-seven grains.

* See Irish Coins of Edward IV., p. 37.

It is not unlikely that the coin which Simon published (Pl. III. fig. 63) was partly defaced, and that in the attempt to restore the legend, REX was substituted for ACIA; the fleurs-de-lis in the legends are also omitted, and at the ends of the cross there are pellets instead of annulets.

All the half-groats have annulets at the ends of the cross on each side, but have not the letter H under the crowns; they weigh from twelve to thirteen grains.

Fig. 2 has the legends HENRICVS DI ORAI, and CIVITAS DVBBL-. The letter O has been substituted for G, as is also very evident on the obverse of fig. 3, which is undoubtedly from the same die; the legend on the reverse of the latter coin is CIV-ITA DEB-lin. On fig. 4, the legends are HENRICVS D, and CIVITAS DEBLIN, and fig. 5 reads HENRICVS DIO, and CIVITAS DEBLI.

The half-groat published by Simon (Pl. III. fig. 67), with the remarkable legend HENRIC DOM OBAR, if correctly represented, should perhaps be DOM VBER, an abbreviation of DOMINOS VBERNIE, the legend on several of the groats presently to be described.

The penny (Fig. 6) has a circle of pellets on each side, and *pellets* at the ends of the cross, the legends are he-NRICVS REX AN, and CIVITAS DVBLIN-, it weighs seven grains.

Groats are the only coins which are known from the mint at Waterford. The shield on the obverse is within a tressure of four single arches, outside which is a circle, sometimes formed of pellets, but more generally a plain line. The legend, in its most complete form, is, HENRICVS DI GRACIA REX, and on the reverse, CIVITAS WATERFORDE, one or more letters are generally omitted. The crowns on the reverse are within a tressure of double arches, the number of which is generally nine; the marks which occur in the legends are, a trefoil, a star of five rays, and a small cross.

Fig. 7 has the legends HENRICVS GRAIA REX, and CIVITAS WA-terfor-DE, the arms of the cross are terminated by *pellets*, as on some of the three crown groats of Edward the Fourth, the tressure on the reverse has only eight arches, there are small trefoils at its points, and in the angles outside it, and a *fleur-de-lis* at each side of the middle crown; another of similar type has the legend HENRICVS DI GRACIA RX.

Fig. 8 has the circle on each side formed of pellets, the legends are HENRICVS D GRACIA REX, and CIVITAS WATERFORD.

Fig. 9 has a circle of pellets on the obverse, and a plain circle on the reverse; the legends are HENRICVS DI GRACI REX, and CIVTAS WATERFOR.

The circle on each side of all the other varieties is formed by a plain line; the legends on fig. 10 are HENRICVS DI GRACIA R, and CIVITAS WATERFORDE, in two of the angles outside the tressure on the obverse there is a star of five rays.

On fig. 11 the legends are HENRICVS DI GRAE, and CIV-ITAS WATERFORD.

Fig. 12 has a star of five rays at each side of the lower crown, and the legends are HENRICVS DI GRACIA, and CIVITAS WATERFOR.

Figs. 13 and 14 are of ruder workmanship, and have a cross in the lower angles outside the tressure on the obverse; on the reverse of one, the legend begins below, and on the other, at the left of the crowns, while on a third specimen the legend commences in the usual place; these rude coins weigh from twenty-five to twenty-six grains. Fig. 14 is the only groat which I have seen without the letter H under the crowns.

There are other varieties which differ only from those described, in the arrangement of the letters in the quarters of the cross.

Of the coins without the name of the place of mintage, there are groats, half-groats, and pennies.

There are several varieties of the groats. Fig. 15 is a remarkably fine coin, it weighs thirty grains; a fleur-de-lis occurs in three places in the legends—HENRICVS DI GRACIA, and DOMINOS VBERNIE.* Fig. 16 reads REX ANLIE FRANC, and DOMINOS VBERNIE. Fig. 17 is remarkable for having DOMINOS VBERNIE on both sides, and the mint mark on the obverse is a cross formed by five small pellets. The next variety, fig. 18, has the borders of the shield, and the circles formed of pellets; the legends are REX ANGLIE F-rancie, and DOMINOS VBERNIE; and fig. 19, which is of a similar type, has on the reverse DOMINVS HIBERN.; it weighs only twenty-two grains.

These four last groats have the tails of the lions doubled back in the same manner as on the Dublin groat.

Fig. 20 (Pl. VI.) has the Fitzgerald arms at each side of the shield, the legends are REX ANLIE FRA, and DOMINOS VRERNIE. The letter H under the crowns distinguishes it from similar coins minted in the reign of Edward the Fourth.

* The king's name is invariably found on the groats of Dublin and Waterford, while on those without the place of mintage it occurs only on this groat.

Every groat of this type which I have seen, either of Edward the Fourth or Henry the Seventh, has VRERNIE on the reverse, but Simon gives one, Pl. III. fig. 65, which has HYBERNIE, and fig. 66 of the same plate has a tressure on each side like the Waterford groats, and the legends the same as his fig. 64.

Very few half-groats are known; fig. 21 has on the obverse a cross terminated by *pellets*, and a rose before the legend REX ANGL FRANCIE; reverse DOMNOS ----NIE, the letter H under the crowns, and over them a cross patee, instead of three annulets as at the other ends of the cross; it weighs thirteen grains. The cross patee on the reverse seems to identify this coin with the Dublin groat, fig. 1, while the obverse corresponds exactly with some of the half-groats of Edward the Fourth.*

Simon's half-groat, fig. 68, appears to have the same obverse as the coin just described, but the legend on the reverse is DOM HIBERNIE.

Pennies are also very rare; fig. 22 has a circle of pellets on each side, the cross on the obverse is without either pellets or annulets at its extremities, the legend probably was Rex angl-IE. On the reverse, which is *not* quartered by a cross, is the word VRERNI, divided equally by a small cross; it weighs six grains, and were it not for the H under the crowns, it would be difficult to assign this coin to its proper place in the Irish series.

Mr. Lindsay has published a penny, with H under the crowns, the legends are REX ANGLIE and DOM------.†

Simon did not hesitate to appropriate all the preceding coins to Henry the Sixth, for his words are, "Whether these coins were struck before the year 1460, or after the year 1470, during the short time this prince had reassumed the crown, is hard to ascertain; but by the letter H, which is on all the pieces with the three crowns, one might be tempted to believe, that they were coined during that short period, as it seems to be a distinguishing mark from those of Edward IV. struck before that time."‡

* See Irish Coins of Edward the Fourth, figs. 88, 89. The practice of using the dies of deceased monarchs was not unusual; it is well known that Henry the Eighth, in his first coinage, used his father's dies; and I have lately seen a coin in the cabinet of Mr. Cuff, which affords a more interesting illustration of the fact of old dies being altered. Mr. Cuff's coin is a Drogheda groat of Richard the Third, struck from a die used by Edward the Fourth, which was altered by punching the letters RIC, over EDW, the remains of which are very evident.

† Pl. VI. fig. 135.

‡ Page 22.

That the letter H was placed under the crowns as a distinctive mark, is very probable, but there is not any evidence whatever to support the appropriation of these coins to Henry the Sixth, who died eight years previous to the introduction of the three crown type into the Irish coinage.*

Simon's conjecture that these coins "were probably intended for three penny and three-halfpenny pieces,"† appears to have been grounded on Sir James Ware's statement, that, in 1478, liberty was granted to the master of the mint to coin "pieces of *three* pence, two pence, and a penny,"‡ that is, in the proportion of 3, 2, and 1, while the weights of the coins are as 4, 2, and 1, or groats, half-groats, and pennies, as they are denominated in 1 Ric. III. cap. 8, in which the type is particularly described.§

The appropriation of these coins to Henry the Sixth, was not questioned until Mr. Lindsay, in his "View of the Coinage of Ireland," transferred them to Henry the Seventh, and that they were struck early in his reign is probable, —from the style of workmanship and correspondence in weight between them and the coins of Edward the Fourth and Richard the Third of the same type,—from the fact of one of Edward's dies having been used for the obverse of the half-groat, fig. 21,—and the appointment in the first year of Henry the Seventh of a master of the coinage in the cities of Dublin and Waterford.

This is the most convenient place to notice a small coin, whose type is very different from any other known coin of any of the Henrys. The mint mark is a cross pierced in the centre, and the legend HE-nri-CVS DNS HIB, the words separated by small crosses; reverse, a plain cross with a rose on its centre, CIVIT is all that remains of the legend, it weighs five grains.—(Fig. 23.)

This coin is much defaced, but from the size of the circle and its weight, it appears to have been intended for a penny; it is difficult to assign it to any particular date, the rose proves that it was not struck previous to the time of Edward the Fourth, and as Richard the Third coined pennies with a rose on the reverse,|| and three crown groats, it is not unlikely that his successor coined money of different types. The rose pennies of Edward and Richard have suns and roses

* Ware's Antiq. by Harris, p. 215.

† Page 22.

‡ Ibid. p. 215.

§ Simon, Appendix, No. XVIII.

|| Snelling's Suppl. to Simon, Pl. I. fig. 27.

alternately on the field of the obverse, while on this coin of Henry neither of these badges appear.

On the other hand, it is now believed that Henry the Sixth coined money at London, Bristol, and York, during his brief restoration in 1470,* and although no documentary evidence exists to prove that Henry exercised his prerogatives in Ireland in 1470, it is not impossible that this penny may have been minted in that year. Without presuming to decide this difficult question, I may remark that the Dublin pennies coined by Edward, in 1470, have a rose on the centre of the reverse.

THE SECOND SECTION.

The cross patee extending to the edge of the reverse, with three pellets in each quarter, is the character common to all the coins in this section, which comprises two types; one having the king's head with an open crown—the other a crown with a double arch.

The Dublin groats with the open crown present several varieties, they weigh from twenty-six to thirty-one grains. Fig. 24 has the legend HENBICVS DI GRA DNS HYBEBNIE, one or two pellets between the words, no trefoils at the points of the tressure; reverse, two pellets before the motto POSVI DEVM ADIVTORE' MEVM, in the inner circle, CIVITAS DVBLINIE. Fig. 25 has a mint mark of four pellets, and DEI in the legend; reverse, a pellet after CIVITAS, in which E has been substituted for C. The legend of fig. 26 is, HENRICVS DEI GRA DNS HYBEB, there are trefoils at some of the points of the tressure; mint mark on the reverse, a cross pierced in the centre, and in the inner circle CIVITAS DVBLNNIE.

The four following groats have a small cross at the beginning of the legend, which is HENBICVS or HENRICVS DEI GRA DNS HIBER, small crosses between the words, and trefoils at the points of the tressure. The mint mark on the reverse of fig. 27, is a small cross patee; on fig. 28, a trefoil; fig. 29, has two small crosses, before the motto POSVI DEV ADIVTORE MEV. Fig. 30 has CIVITAS DVBLIN, and is without a mint mark on the reverse.

The name of the city on fig. 30 has been read DVBLYM, but it appears to me to have been blundered by punching the letters IN twice on the die; the letter taken for Y, is only the I doubled; and that taken for M, is a double N, as is evident from the projection at the top of the letter on the left, whereas the M

* Hawkins' Silver Coins of England, p. 108.

is always rounded at the top ; the coin is evidently blundered, and does not warrant the adoption of a reading for which there is no other authority.

Simon assigns to Henry the Sixth a groat of the same type as those now described, and conjectures that it was struck "*before* this unfortunate prince was *dethroned* by Edward the Fourth."* Mr. Lindsay assents to the appropriation, but thinks the coin was struck "after his restoration in 1470," as well as another groat which he has published.†

Although it cannot be proved that the coins already described in this section, belong to Henry the Seventh, there are many objections against assigning them to Henry the Sixth.

There is no evidence that any coins were minted in Ireland during Henry's brief restoration, nor even that his temporary authority was recognized in this country, and if coins had been struck at that time, it is not likely that he would have ventured to reduce the weight of the groat which in 1470 was nearly forty-one grains, to thirty-one, the greatest weight of any of these coins I have met with.

Until very lately it was universally believed, that Henry the Seventh did not coin any money with an open crown, but this opinion is now known to be erroneous, and to quote the words of Mr. Hawkins, it may be considered "as established beyond controversy, that Henry the Seventh did strike coins with an open crown."‡

The coin which led Mr. Cuff to this important discovery, is a York penny of Thomas Rotherham, who was archbishop of that see from 1480 to 1504. Mr. Hawkins, in his able and valuable work, describes a penny with the king's name on the obverse, and as having the archbishop's initial, "a T at one side of the neck and a (fleur-de-) lis at the other, with an H in the centre of the reverse;"§ but as the representation of the coin (fig. 367) is defective, inasmuch as it has not the "T at one side of the neck," I subjoin the figure of one which has lately come into my possession.

* P. 22, and Pl. III. fig. 70.

† P. 37, and Pl. V. fig. 104.

‡ Silver Coins of England, p. 120.

§ P. 120.

This little coin differs from the three varieties described by Mr. Hawkins, in having a small cross at one side of the neck; and it possesses additional interest in relation to some other Irish coins of Henry, as will appear hereafter.

It may not be amiss to notice a few particulars of the coins themselves. The small crosses on figs. 27, 28, 29, 30, as mint marks, are similar to those on coins to be described hereafter. The letter B is frequently substituted for R, a blunder which I have not observed on any of the coins of Edward the Fourth, struck in or about 1470, from which these coins are also distinguished by the absence of the hair on the king's forehead, a peculiarity common to the undoubted English and Irish coins of Henry the Seventh. The word HIBER in the legend is also remarkable, and I may add, it is not probable that such a variety of mint marks would have been adopted, during the very short period, within which these coins could have been struck by Henry the Sixth.

One groat of the Waterford mint is known; the letters which are preserved on the obverse are just sufficient to identify it as belonging to one of the Henrys. The legend appears to have been HENRIC DEI GRA REX ANGLI FRANC; reverse, POSVI, &c., and CIVITAS WATERFORD; it weighs thirty-two grains.—(Fig. 31.)

The last coin in this division has a large cross, mint mark, and the legend HENRIC DEI GRA REX ANGL FR, with small cinque-foils between the words; reverse, POSVI, &c., and CIVITAS DVBLINIE. The C is represented by E, and the D by an E reversed; it weighs twenty-nine grains.—(Fig. 32.)

This groat appears to be the link, as to type, between the preceding coins, and those with the double-arched crown in the next division.

The coins in the second division of this section, are distinguished by the double-arched crown, surmounted by a ball and cross. The number of arches in the tressure varies, and some have a pellet at each point of the tressure.

The legend on the groats is HENRIC DEI GRA REX ANGL FR. The C in the king's name is in most instances reversed, and the words are divided either by a small cross or two cinque-foils; reverse, POSVI DEVM AIVTORE MEVM, and in the inner circle, CIVITAS DVBIINIE; when in good preservation they weigh from thirty to thirty-two grains.—(Figs. 33, 34.)

Simon says he had some groats "with a *single*, and others with a double-arched crown."* I do not know of any such variety, and I have little doubt but his fig. 97 is incorrectly represented. The legend is HENRICVS DI GRA REX AGL

* P. 32.

& fr, and at each point of the tressure there is a small cross; now in all the arched crown groats of Henry which I have seen, they have only henric, nor have any of them crosses at the points of the tressure; in the next place, his coin has the motto posvi devm aivtorivm, which I have observed only on coins with a cross *fourchee* on the reverse.

All these differences can, perhaps, be accounted for, by supposing that Simon had before him a groat similar to my fig. 40, and it is very remarkable that the relative position of the letters on the reverses of his coin and mine are the same; thus posvi and inie are in the same quarter of the cross, instead of posvi and civi as on most other coins. It is probable, that the legend on the obverse was imperfect, and that the deficiency was supplied by copying from a groat with the arched crown, and the arches of the tressure may have been mistaken for those of the crown.

The half-groat has the crown apparently with a single arch, surmounted with a ball and cross, the hair in long flowing curls, trefoils at the points of the tressure, and on the breast the letter v inverted. The legend is henric di gra rex anlie; reverse, posvi devm adivtor, and civitas dvlin, with a cross after dv. It weighs twenty-one grains and a half.—(Fig. 35.)

The arches of the crown, which are plain, the arrangement of the hair, the v on the breast,* the meaning of which I cannot explain, the legends, and the trefoils at the points of the tressure, distinguish this coin from the groats. The small cross in the inner circle has been taken for an x, but a similar cross occurs at the end of the motto, and also on the reverse of the penny, fig. 22, on which it certainly does not represent a letter. The weight of this piece is considerably more than half of the groat; another specimen which I have seen weighs only fourteen grains and a half.

Henry the Seventh, in his fifth year, introduced the type of the arched crown on the English coins,† and shortly after (1491) Nicholas Flint, who held several offices in connexion with the English mint, in the early part of Henry's reign, was appointed master of the mint in Dublin and Waterford.

From these data I infer that the arched-crown groats were minted by Flint,

* Mr. Hawkins mentions a Durham penny of Edward the Fourth, with a v on the breast.—Silver Coins of England, p. 115.

† Hawkins, p. 107.

and this conjecture is supported by the very close resemblance between the English and Irish coins, in type and workmanship.

The half-groat, notwithstanding all its peculiarities, appears to be contemporary with the groats.

Mr. Lindsay supposes the Waterford groat (fig. 31) to have been struck by Henry the Sixth "after his restoration in 1470." The legend of the coin, however, is not in favour of this appropriation, and the form of the letters have some resemblance to those on the coins which I conceive were struck while Flint was master of the mints of Dublin and Waterford. This coin is remarkable for having the hair on the king's forehead.

I also consider fig. 32 to be the work of an English artist, it resembles some of the arched-crown groats in almost every particular except the crown, and even in this there is some resemblance, for if the cross was resting on a ball, the arches of the tressure might readily be taken for those of the crown.

THE THIRD SECTION.

All the coins in this section (with one exception) have the king's head on the obverse, and a cross fourchee with three pellets in each quarter, on the reverse. They may be divided into two classes; first, those having a double-arched crown; second, those with an open crown.

The coins in the first class have the arched crown, surmounted by a ball and cross; the arches are usually formed of pellets, but in some specimens they are plain lines; the number of arches in the tressure round the head varies, and there are generally three pellets at each point of the tressure, some have annulets within the tressure, and also between the words of the legend; the hair is always in long hanging curls, resembling in this respect the English groats of Henry. All the specimens which I have seen have the letter H* in the centre of the reverse, they are rudely executed and the legends are more or less defective; they appear to have been clipped, and weigh from twenty-six to twenty-eight grains.

* A boar's head is very neatly represented as occupying the centre of the reverse of a groat, published by Simon, Pl. V. fig. 99. In this instance, I suspect that he mistook the H for a boar's head, and the engraving seems to represent the coin in greater perfection than the original; my suspicion is supported, if not confirmed, by his own description; he says, "the last of these (arched-crown groats) has on the reverse, in the centre of the cross, a boar's head, mint mark; and though much *clipped* and *worn*, they weigh from *twenty-seven* to thirty-one grains."—p. 32.

Fig. 36 has the legend HENRIC DEI GRA REX ANLIE FR, and on the reverse CIVITAS DVBLINE; the motto appears to have been intended for POSVI DEVM ADIVTORIVM.

The legend on fig. 37 is HENRIES DEI GRA RIES ANLI, and on the reverse CIVITAS DVBILINI.

On fig. 38 the legends are HENRIES DI GR - - - REX A - - - - E, and CIVITAS DVBLINIE.

I do not know of any half-groats of this type.

The penny, fig. 39, has on the obverse a double-arched crown, and the letter H under it, the legend is HENR - - - - -; reverse, a cross pierced at each extremity, and the legend CIVITVS - - - - -, it weighs five grains and a half.

The pierced cross on this curious little piece, connects it with the coins in this section, but it is more particularly identified with them, by the form of the H in the king's name, which seems to be identical with the first letter in the legend on the obverse of fig. 38.

It is difficult to account for the peculiarities of this penny. The artist perhaps did not possess sufficient skill to execute a head on so small a scale, and as a substitute for it, transferred the initial of the king's name from the reverse to the obverse, the crown on which, resembles that on the coins in the first section, while the arches are the same as on the groat, fig. 34.

The arched crown, the long hanging curls, and the cross *fourchee* on the reverse, all concur in establishing the appropriation of these groats to Henry. It is now admitted, that the plain cross was not abandoned on the English coins until some time after the accession of Henry the Seventh; and in the Scotch series it does not appear, that the cross fourchee was adopted prior to the reign of James the Fourth, who was contemporary with Henry; nor does any instance of it occur on the numerous coins struck in Ireland during the reigns of Edward the Fourth and Richard the Third, while it invariably occurs, more or less modified, on all the Irish coins of Henry the Eighth; hence I conclude that these coins were struck subsequent to the arched-crown groats described in the second section, and the idea of placing the initial of the king's name on the reverse may have been derived from Rotherham's penny.* The rude manner in which they are executed makes it probable that they were not the work of an English artist,

* See p. 67.

while the occurrence of the words HENRIES and RIES, imply that they were executed by a Frenchman.

The coins in the second class have an open crown, and may be divided into those having a tressure round the head, and those without a tressure. The varieties of the first kind are numerous.

Fig. 40 (Pl. VII.) has the legend HENRICVS DEI GRACIA REX ALIE; reverse, POSVI DEVM AIVTORIVM, and in the inner circle CIVITAS DVBLINIE. Fig. 41, reads henri-cvs DEI GRATIA REX ANLIE; the motto is blundered, and in the inner circle it has SIVITAS DVBLINE, the D being represented by an inverted G. Fig. 42 is engraved to show the degree to which it is blundered on the reverse.

The number of arches in the tressure on these coins varies from eight to eleven, and at each point there is a small cross, the hair is in long hanging curls, just as it appears on the English groats of Henry with the arched crown; they weigh from twenty-seven to twenty-eight grains and a half.

The groat which Simon published (Pl. III. fig. 69) as belonging to Henry the Sixth, is evidently of the same type as my fig. 40.

Fig. 43 has a cross mint mark, the legend is HENRIC D-ei gra-CIA REX AGL, with small crosses between the words, there are three crosses within the tressure, and the hair is in long hanging curls; the motto is POSVI DVM ADIVTORIV MEVM, and in the inner circle CIVITAS DVBLINIE. The C is represented by E, and an inverted E is substituted for D; it weighs thirty grains.

Fig. 44 has the hair in short close curls; the legend is HENRI-C dc-I GRACIA REX ANGLE, with annulets between the words; the letter L is represented by a double I, as on some of three-crown groats;† reverse, POSVI, &c., and CIVITAS DVBLINIE; it has the letter H in the centre of the reverse, and weighs twenty-nine grains.

The mint mark on fig. 45 is a small cross, the tressure has only six arches, the crown is very flat, and there is a cross at each side of the neck. The legend is HENRIC dei gr-ACIA REX ALIE FR; reverse, POSVI DEVM ADIVTORIVM, and CIVITAS DVBLINI; it weighs only twenty-three grains.

Of the groats without the tressure round the head the varieties are very numerous.

Fig. 46 has a cross at each side of the crown, and the hair in long hanging

* See figs. 16, 18, 19, Pl.V.

curls; the legend is HENRICVS DI GRACIA REX ANI; reverse, SIVITAS DVBLINIE; the motto is blundered; it weighs twenty-nine grains. Fig. 47 is of the same type, but the legends on both sides are unintelligible; it weighs twenty-seven grains.

Simon's coin (Pl. III. fig. 59) is identified with this type, by wanting the tressure, and having the cross at each side of the crown; but if the details of his engraving are correct, the coin is very different from any I have seen.

Fig. 48 is a very remarkable coin, it has a rose or cinquefoil at each side of the crown, and also as a mint mark, the hair is in long full curls, and the bust is concealed by drapery, resembling a cloak, HENBIC is all that remains of the legend; the reverse is altogether unintelligible, and it weighs only twenty-four grains.

The coin in Simon's third Plate (fig. 60) is of this type, and is represented as being perfect in every respect; it is much to be regretted that many of the most curious coins which he possessed cannot now be discovered.

The remaining coins in this division are chiefly distinguished by the absence of the tressure round the head. The crown is open and very shallow—the hair is in short, close curls, which stand out from the face—the shoulders are more displayed than on any of the preceding coins, and are without drapery—and the mint mark is a cross. The legend on the obverse, in its most perfect form, is, HENRICVS DI GRACIA REX AGLIE FR; reverse, POSVI DEVM ADIVTORIVM, and in the inner circle CIVITAS DVBLINIE; a few have SIVITAS; the name of the city is generally abridged, and several are blundered to an extreme degree; they weigh from twenty-four to twenty-nine grains and a-half.

No half-groats or pennies of this type are known, and Dublin is the only place of mintage.

The following list exhibits the legends of the most remarkable varieties:

Fig. 49,	HENRCVS DI GRACIA REX AGLIE FR.	POSVI-DEVM -ADIVT-ORIVM.	CIVI-TAS -DVBL-INIE.
50,	HENRCVS DI GRACIA REX AGLIE FR.	POSVI-DEVMA -DIVTO-RIVM.	CIVI-TAS -DVB -LINI.
51,	HENRCVS DI GRACIA REX AGNIE.	POSVI-DEVM -ADIVT-ORIVM.	CIVI-TAS -DVB -LIN.
52,	HENRCVS DI GRACIA REX AGNIE.	POSV -IDEV -MADI -VTOR.	CIV -ITA -SDV -BL.
53,	HENRICVS DI GRACIA REX AGNI.	POSV -IDEV -MADI -VTOR.	CIV -ITA -SDV -BLI.
54,	HENRICVS DI GRACIA REX AGNI.	POSV -IDEV -MDEV -TORIV.	CIV -ITA -SD -VB.
55,	HENRICVS DI GRACIA REX AGN.	POSVI-DEVMI -ADIVT-ORIVM.	SIVI-TASD-DVB -LINE.
56,	HENRICVS DI GRACIA REX AGN.	IEMA -MIVI -TASD -VBLA.	CIVI-TAS -DVB -LIE.
57,	Blundered.	Blundered.	CIV -ITAS -DVB -IAII.

I have had occasion, in more than one instance, to doubt the accuracy of Simon's engravings; and it is plain that he sometimes erred in attempting to restore the legend of a defaced coin. His fig. 56, has GRA, but my fig. 55 has GRACIA, and is identified with Simon's, by having the letters NE in the name of the city united exactly as he has represented them; and my friend, the Rev. J. W. Martin, has a groat which certainly has been struck from the same die as mine, but defective in the legend exactly in the place where Simon's differs from fig. 55. Mr. Martin's coin has been traced to Simon's possession.

Of the many coins without the tressure which I have seen, I have not met with any so perfect as those engraved in Simon's Essay. The errors, for such I must consider them, which appear in the legends, &c., of figs. 56, 57, 58, may be accounted for by his attempting to restore partially defaced coins, while the letters in the inner circle correspond with pieces known at present.

In making these observations, I by no means intend to insinuate that Simon intentionally misrepresented the legends on any of his coins, on the contrary, I am satisfied that his errors are to be attributed to the want of opportunities enjoyed by his successors, and his work, which he "modestly styled an Essay only," has received a well merited eulogium from the able and impartial author of the "Annals of the Coinage of Britain."

Mr. Lindsay was the first writer who questioned the correctness of Simon's appropriation of the groats without the tressure to Henry the Fifth; and as several distinguished numismatists are still of opinion, that these groats are the earliest in the Irish series, it is necessary to enter at some length into the discussion of this question.

I shall first lay before my readers, an abstract of Mr. Lindsay's opinions, and then proceed to investigate the objections which have been urged against them.

"It must in the first place be observed," says Mr. Lindsay, "that no records have hitherto been discovered, which direct, or even refer to, an Irish coinage from the reign of Edward III., until the 38th Henry VI., 1459–1460."*

In the Introduction to this essay, I have quoted a roll of the 9 Henry V., and another of the 3 Henry VI., which, although unknown to Mr. Lindsay

* View of the Coinage of Ireland, p. 31.

when he wrote, tend to support his opinion that Henry the Fifth did *not* coin money in Ireland.

He next observes, "this Act (38 Henry VI.) would seem to imply that a separate coinage for Ireland, of a type and standard different from that of England, was then for the first time adopted; if so, the coins assigned to Henry V., viz., Nos. 56, 7, 8, 9, 60, of Simon, could not have been struck before that period, as they differ in type, and still more in weight from any English coins *hitherto* struck."

I have already shown, that if any money was coined in Ireland during the early part of the reign of Henry the Sixth, it ought to be of the same weight, allay, and assay, as the silver money made in London.* The difference in type will be noticed hereafter.

At an adjourned sitting of the parliament of the 38 Henry VI., it was ordered that the groat "shall pass for five-pence," and on these words, Mr. Lindsay remarks, "it is nearly certain that these coins must have been of the English standard, then sixty grains to the groat, otherwise they would not have been ordered to pass at the rate of a penny more than the new (Irish) groat of forty-five grains, and could not possibly have meant or included the groats given by Simon to Henry V.," and adds, "let us now consider the coins themselves, and compare them with the English coins of the Henrys. The first peculiarity which presents itself, is the want of the double tressure round the king's head"—the next, "is the cross fourchy on the reverse," then, "the king's title," and lastly, "their weight."

Mr. Lindsay, with the candour of an enlightened and impartial writer, concludes by saying, "having thus given to the coins an appropriation very different from that of Simon, or indeed I will admit of any other writer who has noticed them, I think it fair to lay before my readers, the opinion of a learned friend on whose judgment in matters relating to the English and Irish coinage, I have the greatest reliance."

With the arguments of Mr. Lindsay, in support of his appropriation, I fully concur, and therefore I feel imperatively called upon to institute a rigid inquiry into the objections of his learned friend, whose opinions are deservedly entitled to the highest respect.

* See p. 53.

The first objection is to the workmanship, of which he says, "comparing those groats assigned by Simon to Henry V., with the undoubted coinages of Edward IV. and Henry VII., I should say that the design and workmanship of the former is so very poor, imperfect, and barbarous, that coming from the *same* mint of Dublin, I cannot conceive them subsequent to Edward IV., and still less suppose them contemporaneous with those of the arched crown of Henry VII. To me they are evidently the *first* groats in the Irish series, the workmanship of very rude, ignorant artists, who had very imperfect command of the graver, could design little, and execute less."*

The appearance of the bust—the form of the letters—the blundered legends—the flat crown—the circle round the head, are all noticed; and he adds, "I cannot but repeat, that their appearance and fabric appear to me to exclude them altogether from the coinage of Henry VII."

The appearance of the bust and the workmanship on these coins is certainly very rude; yet the difference between the coins, "coming from the *same* mint of Dublin," may, in some measure, be accounted for, by the fact, that Nicholas Flint, who was "sculptor de et pro ferris," in the mint of London, in 1486, was made "overseer of the mints of Dublin and Waterford" in 1491, and was succeeded in his office in Dublin, in 1506, by Thomas Galmole alias Archibold, a *goldsmith in Dublin.*

"The letters are thin and uncertain" yet when they are compared with those on the rude coins of Henry the Seventh, with the arched crown (see figs. 36, 37, 38), it will be admitted, that if they are not identical, they bear a very close resemblance to them.

"The erroneous legends," are not more remarkable than the blunders which occur on some of the Irish groats of Henry the Eighth,† and are very similar to the legends on figs. 42 and 47, which, in my opinion, are identified with the time of Henry the Seventh, by having the hair in long hanging curls.

"The crown is quite level," but it is identical with that on the tressured groat (fig. 45), and bears a close resemblance to the crowns on some of the groats described in the first division of the second section.

"The head is encircled by a mere line, ana not a dotted circle," such, no doubt, appears to be the case on a few of these coins, but on most of them which

* Lindsay, p. 34.

† Ibid. Pl. VII. figs. 147, 148.

I have met with, the circle is more or less indented; on fig. 53 it is even roped, and several others have a circle of pellets very distinctly marked.

Mr. Lindsay's correspondent, relying on the objections which I have endeavoured to refute, says, "this is what may be termed the internal evidence furnished by the coin itself, and to me completely decides the question."

"The array of Acts of Parliament, weight of coins," &c., are not allowed to be of much importance; but I cannot consent to give up the evidence derived from such authorities, for the Irish coins of Edward the Fourth are generally found to be in strict accordance with the standard fixed by the Acts; and while it is admitted, that "the groat of Henry V. should weigh sixty grains," it appears to me incredible that any groats should be issued by him at so low a weight as "twenty-eight" grains.

It is also asserted, that no coinage took place in Ireland "from the death of Edward II. to the accession of Henry V.," and that "after such a lapse of time (nearly a hundred years), the attempt at a coinage may be expected to be very wretched, and so it is. Supposing, as is natural, that the Irish engraver would make the current English groat his copy, as near as his want of ability would allow him, the copy, such as we see it, is more Edward the Third's and Richard the Second's, than Edward the Fourth's,—in the former, a larger space was left unoccupied by the bust than on the latter; and where the artist could scarcely attempt the plain circle surrounding the head, it is no wonder that he abandoned the tressure."

Here again, the authority of authentic records is disregarded, for in 1336 (10 Edward III.) "a proclamation was then issued by the king and council, for the coining of pennies, halfpennies, and farthings in *Ireland*;"* and in 1339, a writ, entitled, "De cuneis in Hiberniam mittendis," was issued;† and if it be admitted that the English coins which have the name "Edwardus" belong to Edward the Third, this question is settled respecting the Irish coins; for in February, 1841, a farthing was found at Trim, on the obverse of which is a head within a triangle, and the legend EDW-ARDV-SREX; reverse, cross and pellets, with CIVITAS DVBLINIE. This coin is in the cabinet of the Rev. Richard Butler, of Trim. And if "nearly a hundred years" elapsed without any coinage taking place in Ireland, it does not follow that the first attempt should necessarily be

* Simon, p. 16. † Ibid. Appendix, No. II.

"very wretched," for the earliest groats minted in Ireland, of which we have any authentic records, were as well executed as the English coins of the same period; nor can I perceive that the coins in question are more like "the current English groat" than the Irish coins of Edward the Fourth; for on all the London groats of Richard the Second, and Edward the Third, which I have seen, the Roman N is used in the name of the city, while on these Irish coins of Henry it never occurs. The form of the letter I is also different; on Henry's coins it is always more or less forked, and never square at the ends, as is invariably the case on the supposed models. The objection of the plain circle round the head, has been already answered, and the striking resemblance in almost every respect (except the tressure and crosses at each side of neck), between fig. 45, and the untressured groats, induces me to believe that the artist "abandoned the tressure," rather from choice than inability to execute such a trifling ornament.

It also strikes me as very extraordinary, that an artist so ignorant as has been supposed, should invent a cross fourchee for the reverse of his rude coin; and how did the illiterate artist (who it is conjectured "could not spell") learn that the GRA on the supposed models, was only an abbreviation for GRACIA, which is found without exception on the untressured groats, as well as on some others of which little, if any doubt can exist, that they belong to Henry the Seventh, as the half-groat, fig. 35, and the tressured groats figs. 40 and 45; and why did not the copyist adopt the usual motto, but instead of it engrave on his die, POSVI DEVM ADIVTORIVM.?

Several authorities are cited to show "that REX AGL *may have been* also used in Ireland before the reign of Henry VII.;" but the Act of 10 Edward IV., which ordered that REX ANGLIE should form part of the legend on the coins, has not been noticed, and there is not any Irish coin known with this title, which can be referred to an earlier date. The penny of Henry the Sixth has the legend HENRICVS DNS HIBNIE.*

In bringing these observations to a conclusion, I feel bound to acknowledge, that, if I have been at all successful in establishing opinions different from those of preceding writers, it has been chiefly owing to the advantage I enjoyed of having so large a number of coins of the different types before me at one view. It now only remains for me to assign such reasons as appear to warrant the appropriation of the coins in the last plate to Henry the Seventh.

* See p. 56.

Assuming that it will be admitted that the groat with the arched crown, and the H in the centre of the reverse (fig. 36) belongs to Henry the Seventh, it can scarcely be doubted that figs. 40, 41, 42, are nearly contemporary with it—GRACIA in the legend—the arrangement of the hair—and the cross fourchee on the reverse are common to both. The cross on fig. 43 over the crown, which seems to have single arches, and the words REX AGL in the legend, connect this coin with the double-arched groats figs. 33, 34, while the crosses within the tressure, the word GRACIA, and the long curls, show how closely allied it is to figs. 40 and 44, the latter of which is remarkable for the H in the centre of the reverse. The cross at each side of the neck and the tressure on fig. 45, connect it with fig. 43, and in every other particular it is almost identical with fig. 50.

Notwithstanding all the objections which Mr. Lindsay's correspondent has made against the appropriation to Henry the Seventh, of the "groats assigned by Simon to Henry V.," he admits, "the curious groat in (Mr. Lindsay's) collection, *without a tressure*,* to be an early groat of Henry VII." To me this admission is important, yet I must in some measure dissent from it, in expressing my belief, that the coin was struck in the *latter* part of Henry's reign; the hair, and the cross at each side of the crown connect it with fig. 41, the absence of the tressure with fig. 55, and the word SIVITAS occurs on the three coins; fig. 47 is only a blundered variety of fig. 46, and fig. 48 is a very remarkable coin.

Of the remaining coins little need be said; the blundered legends on fig. 57 are not more remarkable than those on figs. 42, 47, and 48, and the want of the tressure is the chief distinction between them and fig. 45; the word GRACIA on the obverse—SIVITAS on three varieties, and the cross fourchee on the reverse—and the form of the letters, concur in making it probable, that all the coins in the last Plate were minted about the same time; and from the many varieties of type, and the bad style of workmanship of these coins, it is evident that the mint of Dublin was in a very unsettled state; under these circumstances it is not surprising to find the arched crown abandoned, and the open crown resumed in place of it.

I feel little hesitation now in appropriating these coins to the latter part of the reign of Henry the Seventh. It is not improbable that many of them were

* See fig. 46.

struck by Galmole, who was appointed master of the mint of Dublin on the 6th of July, 1506, and that he abandoned the tressure in imitation of Henry's latest English coinage.

I cannot conclude without acknowledging my obligations, and expressing my gratitude to those who have so kindly favoured me with the means of illustrating this very obscure period of the History of the Irish coinage.


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EXPLANATION OF THE PLATES.

Plate I.

NO.	DENOMINATION.	MINT.	WEIGHT.	PAGE.	REFERENCE.
1	Groat.	Dublin.	27 grs.	14	Mr. Cuff.
2	Half-groat.	,,	12½	15	,,
3	,,	,,	13	,,	Dr. A. Smith.
4	,,	,,	12	,,	,,
5	,,	,,	12	,,	Dean of St. Patrick's.
6	Penny.	,,	7	,,	,,
7	Groat.	Waterford.	26	,,	Mr. Sainthill.
8	,,	,,	28	,,	Dean of St. Patrick's.
9	,,	,,	28	16	,,
10	,,	,,	30	,,	,,
11	,,	,,	28	,,	Dr. A. Smith.
12	,,	,,	28	,,	Dean of St. Patrick's.
13	,,	,,	26	,,	Mr. Sainthill.
14	,,	,,	25	,,	Mr. Lindsay.
15	,,	,,	30	,,	Dean of St. Patrick's.
16	,,	?	30	,,	Dr. A. Smith.
17	,,	?	27	,,	,,
18	,,	?	28	,,	Rev. R. Butler.
19	,,	?	22	,,	Dean of St. Patrick's.

Plate II.

NO.	DENOMINATION.	MINT.	WEIGHT.	PAGE.	REFERENCE.
20	Groat.	?	26 grs.	16	Dean of St. Patrick's.
21	Half-groat.	?	13	17	Rev. J. W. Martin.
22	Penny.	?	6	,,	Rev. R. Butler.
23	,,	Dublin ?	5	18	,,
24	Groat.	Dublin.	26	19	Dean of St. Patrick's.
25	,,	,,	30	,,	Dr. A. Smith.
26	,,	,,	28	,,	Dean of St. Patrick's.
27	,,	,,	30	,,	,,
28	,,	,,	30	,,	,,
29	,,	,,	31	,,	Mr. Sainthill.
30	,,	,,	31	,,	,,
31	,,	Waterford.	32	21	,,
32	,,	Dublin.	29	,,	Dean of St. Patrick's.
33	,,	,,	32	,,	Mr. Sainthill.
34	,,	,,	30	,,	,,
35	Half-groat.	,,	21½	22	Rev. J. W. Martin.
36	Groat.	,,	27	24	Dean of St. Patrick's.
37	,,	,,	26	,,	,,
38	,,	,,	28	,,	Mr. Sainthill.
39	Penny.	,,	5½	,,	Rev. R. Butler.


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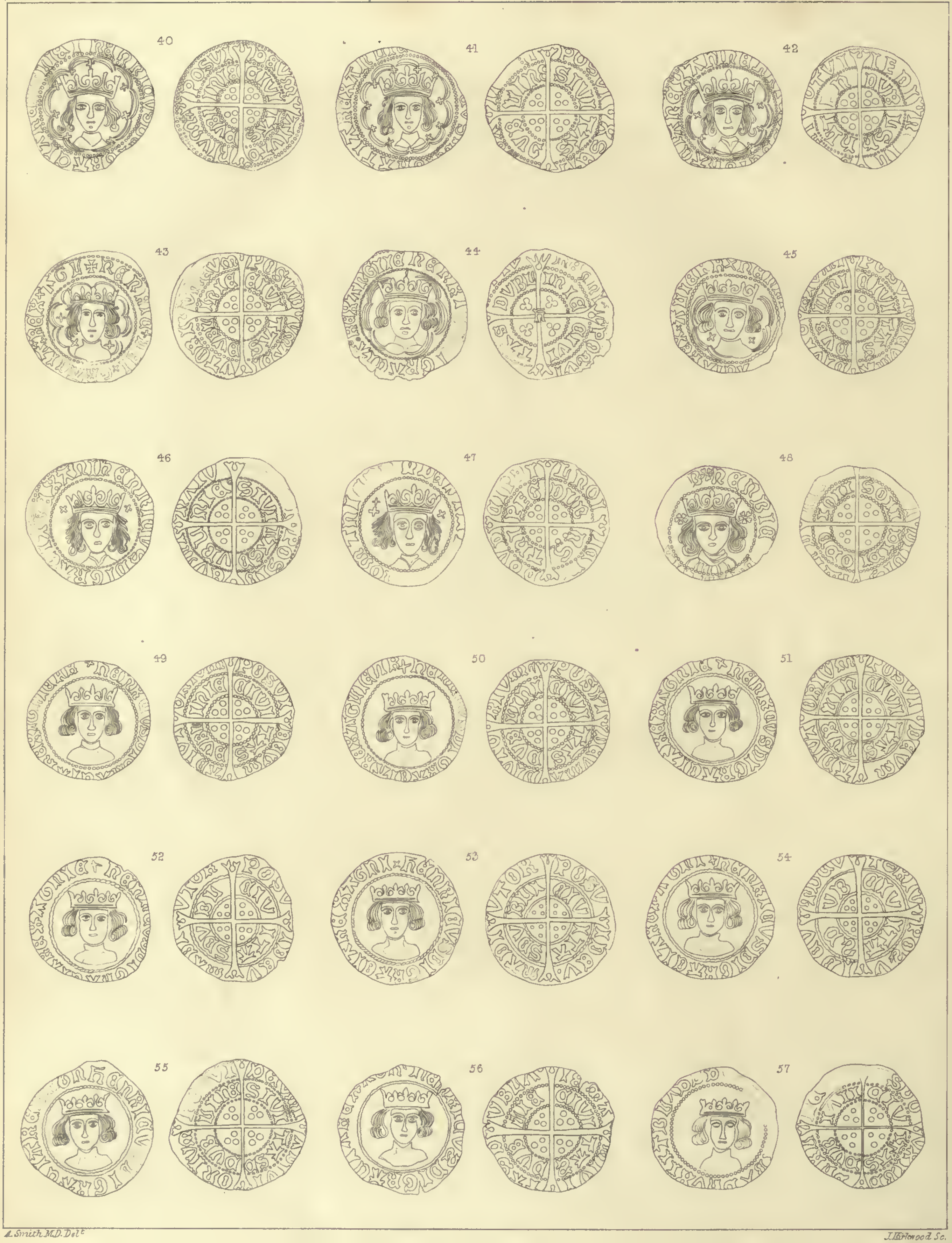

A. Smith M.D. Delt

J. Kirkwood Sc.

PLATE III.

NO.	DENOMINATION.	MINT.	WEIGHT.	PAGE.	REFERENCE.
40	Groat.	Dublin.	27 grs.	25	Dean of St. Patrick's.
41	,,	,,	28	,,	,,
42	,,	,,	27½	,,	,,
43	,,	,,	30	,,	Rev. R. Butler.
44	,,	,,	29	,,	Dean of St. Patrick's.
45	,,	,,	23	,,	,,
46	,,	,,	29	,,	Mr. Lindsay.
47	,,	,,	27	26	Dean of St. Patrick's.
48	,,	,,	24	,,	,,
49	,,	,,	26	,,	,,
50	,,	,,	24	,,	,,
51	,,	,,	28	,,	Dr. A. Smith.
52	,,	,,	25	,,	Dean of St. Patrick's.
53	,,	,,	28	,,	Mr. Sainthill.
54	,,	,,	25½	,,	Mr. Lindsay.
55	,,	,,	29½	,,	Dr. A. Smith.
56	,,	,,	28	,,	Dean of St. Patrick's.
57	,,	,,	29	,,	Dr. A. Smith.

III.—*On the Norse Geography of Ancient Ireland.** By George Downes, M.A.; M.R.I.A.; M.R.S.N.A., Copenhagen; F.H.M.M.S., Jena.

Read April 26th, 1841.

In the First Series of the Annals and Memoirs of the Royal Society of Northern Antiquaries, published in Copenhagen in 1837, there is a small Map of this country, annexed to an Essay on the Earliest Expeditions from the North to Ireland. This Essay is nearly identical with an English one, already published in the same city in 1836, and incorporated in the Address of the Society to its British and American Members. The Map in the latter publication exhibits some improvements on that in the former. A new locality is introduced, and an old error corrected, namely, the location of Clontarf to the north-west of Tara. The correction of this error is due to a distinguished member of the Academy, the late Dr. William West, by whose premature decease the progress of northern literature in this country has been greatly retarded.

The Norse Map of Ireland, though but a modern compilation, is so far interesting as it exhibits the scanty amount of the Irish localities, noticed in such of the Icelandic Sagas as were published previously to 1837. On these localities, which are mostly given both in Norse and English, I shall submit to the Academy a few observations, after which I shall undertake a slight extension of what may be termed the Norse Geography of Ancient Ireland. By Norse I mean Old Danish, which was originally denominated the Danish Tongue, afterwards Norræne, or Norse, but which has been long better known as Icelandic—the remote island, though but a colony, having imposed its name on the language of its un-

* A considerable time having elapsed since the reading of this paper, I have profited by the circumstance to introduce into it several corrections and improvements, in which I have received much assistance from a gentleman, acknowledged to be the best living authority on the subject of ancient Irish topography.

lettered founders, by virtue of its literary celebrity. The term *Runic*, so frequently applied to this language, even by such scholars as Parkhurst, is a misnomer, being applicable only to a peculiar form of its characters, like the term *Ogham* in Irish. In tracing to a foreign origin a few of our local names, I shall unavoidably startle vernacular prejudices, researches such as the present being but too frequently marked by a national bias. Local investigations recall local associations, and there is a charm about ancient things, by which the judgment becomes warped: a chastened imagination will indeed rather aid than obstruct inquiry into the topography of an imaginative people, but patriotism is a bad etymologist.

Of the four provinces of Ireland, which are all given in English on the Map, but two are given in Norse—*Úlaztir* and *Kunnáktir;* Leinster and Munster are, however, mentioned in the Essay, and two portions of the former are laid down on the Map—*Dýflinar-skíri*, or Dublinshire, and *Kunnjáttaborg*, which occupies much of the present county of Meath. The Danish writer asserts, after Chalmers, that *ster*, the termination of the names of three provinces, is a corruption of the Norse *staðr*, "place," not adverting to its occurrence without an *s* in Kunnáktir, where, however, it may have been omitted for euphony. It certainly has no connexion with the Irish τíɲ, which was invariably the leading word in local designations wherein it occurred, as in *Tir-Anlave*, or *Tirawley*—a name apparently Norse, but which is found, as *Tir-Amhalgaidh*, in the Book of Armagh, written about 680, a period anterior to the earliest northern invasion of Ireland on record, and which is misinterpreted in the Essay as *Olafs Höj*, or "Olave's Height." To the apparently idle tradition that Ulster owes its name to one Ullagh, a Norwegian, the Essay makes no allusion.

Though Leinster is not included among the Norse localities on the Map, Johnstone, in his edition of the *Lodbrokar-Quida*, or Death-Song of Lodbroc (otherwise called the *Krákumál*), printed in 1782, gives "Leinster" as the translation of "*Lindis-Eyri*," in a description of a sea-fight between the Northmen and the Irish: in the notes, however, he suggests that Lindisfarne may be intended, that is, Holy Island, off the coast of Northumberland (or now of Durham), and adds, that some suppose the Lindesnes, commonly called the Naze, in Norway, to be the locality in question. In Rafn's edition of the same poem, published in 1826, various opinions are cited. If *eyri*, "strand" (the

Danish *öre*, as in Elsin*ore*), be the correct reading, *Lindis* might be found in *Lindsay*, the northern part of Lincolnshire, did not the context almost directly point to Ireland. Olaus Wormius assigns as the scene of conflict an island on the Irish coast, and the presumption of the insular nature of the district intended is favoured by a different reading, *eyju*, suggested by Arni Magnússon, the founder of the Arna-Magnæan Commission, and perhaps the most consummate Icelandic scholar that has ever existed. If the opinion of these distinguished authorities be well-grounded, the locality in question may be the island of *Lambay*, laid down on Ptolemy's map as *Limnos* and *Limpnos*, forms not unlike the Norse *Lindis*, to which another form, *Linos*, bears a still stronger resemblance. This etymological conjecture seems also to admit of geographical support. In this part of the poem there appears to be a local progression. The naval battle-fields, mentioned in immediate connexion with Lindis-Eyri, are off the Scottish islands of Sky and Isla, and the Welsh island of Anglesey: it is, therefore, more natural to seek for Lindis-Eyri on the east coast of Ireland than on the east coast of England. Indeed, the achievements of Lodbroc on the coasts of Northumberland and Norway are alluded to in an earlier part of the poem; and the distinguished editor, Professor Rafn, himself is in favour of the Irish hypothesis.

Of our estuaries, but three are named on the Map. On the north-west coast appears *Jölduhlaup* [*Jöllduhlaup*], which is variously stated to be three, four, five, or eight days' sail from Iceland. "The name," says the English Essay, "signifies *the run* or *breaking of waves*, a designation applicable to no other place within the limits specified than Lough Swilly." I have elsewhere met with the assertion, that Jöllduhlaup is a translation of the Irish name of the lough, which, however, is not adduced. It may be reasonably doubted that the locality here assigned to Jöllduhlaup is the real one; and it is certain that Lough Swilly possesses no Irish name, which would admit of the above interpretation. In Olave Tryggvason's Saga this locality is expressly stated to be in Ireland, and distant five days' sail from Reykjanes, in the south of Iceland.

The site of *Úlfreksfjörðr* or *Úlfkelsfjörðr*, Ulfrek's or Ulfkel's Firth, as the Danish writer admits, cannot be ascertained, nor even with certainty referred to Ireland. The Sagas mention a battle fought, in 1018, between an Irish king, named Konofögr, supposed by Suhm to be Conochar O'Melachlin, king of

Meath, and the Orkneyan earl Einar, in this firth, which Schöning locates in the north of Ireland. However, as the eastern coast, in the neighbourhood of Dundalk, was equally the resort of the Scandinavian rovers, the matter has been compromised on the Map, where Lough Foyle figures as Úlfreksfjörðr, and Carlingford Bay as Úlfkelsfjörðr, with a note of interrogation added to each word, though Lough Foyle appears to have the stronger claim, the name *Carlingford* being itself evidently Norse.

Were the name alone of this firth taken into consideration, its locality might be reasonably sought in England. Ulfkell, surnamed *Snilling*, or Excellent, was a son-in-law of Ethelred II., from whom a great part, if not the whole, of East Anglia was named *Ulfkell Snilling's Land.* The estuary called the Wash, or Boston Deep, is adjacent to this territory; but the countries of the belligerents, Ireland and Orkney, render it unlikely that their place of encounter would be there. However, as Ulfkell appears to have at one period exercised a kind of vice-regal authority over the north of England, the firth in question may be one of those on its north-western shore. The Danish writer finds a similarity between the name Ulfkel and the Irish *O'Kelly*, in which *Kelly* is the Norse *Kjallak:* however, O'Kelly does not occur in Ireland as a topographical name so early as the time of Ethelred II. The name Ulfkel is of rare occurrence: one Thollak Ulfgelsön, or Thorlak Ulfgestson, is, however, mentioned in Inge Bardson's Saga. The other reading, Úlfreksfjörðr, seems to point to that branch of Morecambe Bay, in Lancashire, which runs up to Ulverstone.

The principal towns specified on the Map are *Dýflin*, *Hlimrek*, and *Veðrafjörðr*. Dýflin is a slightly modified adaptation of Ꝺuıḃ-lınn, the Irish name of Dublin. The opinion that the metropolis of Ireland was founded by the Danes can be easily confuted from its want of an original Norse name, and more satisfactorily from the consideration that it was a bishop's see before the arrival of the Northmen, and contained within its precincts a round tower, and a place of worship sacred to St. Michan (which is still perpetuated in the church of that name), as mentioned in the Calendar of Aengus, which dates so early as the eighth century. Hlimrek, in like manner, appears to be an adaptation of Luımneaċ, the Irish name of Limerick, for which various derivations have been proposed, and which was certainly an ancient appellation of the Lower Shannon. Veðrafjörðr, on the contrary, or Waterford, is pure Norse; and its etymology is given in the notes to the Death-Song of Lodbroc, already mentioned, from *vedr*, "*tempestas*," and *fiördr*, "*sinus*:" instead of *vedr*, *fadr*, or "father," has been suggested, meaning Odin; and the reading *Vatsfiord*, equivalent to *Vatnsfjörðr*—the name of two localities in Iceland—is given in the *Antiquitates Celto-Scandicæ*: of this reading *Waterford* is an exact translation; however, it would appear that Johnstone's derivation is to be preferred. A townland, designated *Ballyvedra alias Weatherstown*, exists in the neighbourhood of Waterford; but it seems not unlikely that it owes its name to the family of Madray, long settled in that part of the country. However this be, there is, perhaps, no district in Ireland more essentially Danish than the vicinity of Waterford. Hence it is the opinion of a high authority, that even the Irish name of that city, *Portlargy*, is derivable from the name of some northern warrior, perhaps the *Larac*, mentioned in the Annals of the Four Masters at the year 951, as having wasted Tigh Moling, on the Barrow, now St. Mullin's. There appears, however, to be a connexion between the name of the adjacent locality *Portlaw*, derived from laṁ, "hand," and *Portlargy*, derived from laiɲge, "thigh," to the shape of which member of the body the harbour is supposed to bear some resemblance.

Kunnjáttaborg, though laid down as an extensive district, would, from its termination, seem rather to have been a town, or castle. The nuptials of Brian Boru with Gormliath, whose Norse name is Kormlöd, are recorded to have been solemnized at Kunnjáttaborg; but in the Niála—a Saga of great authority, called after the distinguished Nial, by whom, about the year 1000, a kind of law-school was established in Iceland—the name is given as *Kantaraborg*, which, as Brian was king of Munster, Schöning identifies with Carbury, in the county of Cork. The Danish writer, however, infers from the context, that, notwithstanding its final syllable, the word is rather applicable to a tract of country; and this tract he, rightly and much to his credit, finds in *Kiennachtabregh*, or Bregia, in the county of Meath, which was within the range of Brian's conquests. In Johnstone's *Antiquitates Celto-Scandicæ* the reading *Kunnaktirborg* is given, and rendered "*urbi Connaciæ*." It seems strange that this reading is not noticed by the Danish writer: it must, however, be remembered, that both the text and version, in the work wherein it occurs, should be always consulted with suspicion. I say this by no means in disparagement of an industrious pioneer, who published sixty years ago, when the Arna-Magnæan Commission had but lately begun their severe labour of deciphering and collating the Icelandic manuscripts. *Kantaraborgar* is also given by Johnstone, and rendered similarly "*urbem Connaciæ.*"

Iniskillen is laid down, and described by the Danish writer, after the Royal Mirror, as a small island in *Logherne*, called in some manuscripts *Misdredan*—an ocular misconception of *Inisdredan*—in which a certain holy man, named Diermicius, possessed a church. The variations of orthography in the name concluded to be Iniskillen, as given in the *Antiquitates Celto-Scandicæ*, are so extraordinary as to render identification almost hopeless. Among the readings is *Inhiskladran*, perhaps *Inisclothran* in Lough Ree—cited as *Inis-Cloghran* by the Danish writer—where an abbot, named Dermit, resided. The site of the island may have been assigned to a wrong lake, or to the right one with some distortion of the name: *Ree* is convertible into *Erne* by a much less violent alteration than the name of the island has itself undergone.

Tara [*Teamuir*] and *Glendaloch* are likewise laid down after the Royal Mirror, in their Norse form, as *Themar* and *Glendelaga*, but the latter place is in the Essay located in Ulster.

There remains but one more Norse locality on the earlier Map, namely, *Smjörvik*, now Smerwick, on the coast of Kerry. The name is to all appearance Norse, but respecting its origin the Danish writer offers no opinion. The termination *wick* or *wich* (the Norse *vik*), so frequent in these countries, both in Scandinavian and Saxon localities, whether maritime or inland, is supposed to derive its applicability to either a bay or town, from the idea of *protection* implied in both. Although, as I shall hereafter show, there is room for doubting that the first syllable was originally *Smjör*, there are plausible grounds for this supposition. The word *smjör*, "butter," was in the North a frequent and sometimes absurd element both in local and personal names, as in those of Butterwaterheath in Iceland, Bjarn Caskbutter, Einar Butterback, Archbishop John Butterbelt, and Thorolf, who earned a nickname for life, by asserting that butter dripped from every blade of grass in Iceland. But the name Smerwick may have originated in a more important circumstance. That the Northmen carried on some kind of traffic with the south-west of Ireland would appear even from the surname of *Hlymreksfari*, or "Limerick trader," which was given to one Hrafn, who is supposed to have fought under the banner of Sigurd, earl of Orkney, at the battle of Clontarf. One article of this traffic may have been butter; and it is possible that Smerwick Harbour may have been in some way connected with a trade in this commodity.* The following curious tradition, to the sequel of which I shall have occasion to advert hereafter, shows at least, that on one of their homeward voyages from Ireland the Northmen had butter on board, either as an article of traffic, or diet. The sea-rover Leif, son of Hrodmar (who must not be confounded with the more celebrated Leif, son to Erick the Red), while ravaging the shores of Ireland, came to a large subterraneous house, lighted only by the gleaming of a sword, held by a man who had taken refuge within, but was slain by the Northman, who was thenceforward called *Hjörleif*, or "Sword-Leif," from the weapon, which was of great value. After continuing his devastations along a great extent of coast, Leif at length sailed for Norway, conveying, with other booty, ten or twelve Irish slaves, among whom one, named Duvthak, had the pre-eminence. In the following spring Leif sailed for Iceland with his slaves, accompanied by his foster-brother Ingolf, each in his own ship. The latter, on approaching the shore, flung overboard, according to usage, the columnar posts of the chief seat in his paternal mansion (which usually ended atop in the sculptured head of some deity, generally that of Thor); and at the spot where they were

* In an interesting paper on the Antiquities of the Church of Kilmelchedor, read before the Academy on the 11th of April, 1842, my derivation of Smerwick, from a word signifying *butter*, was treated as an absurdity, and the commission of it imputed to the Danish antiquaries, who, as I have stated in the text, are quite silent on the subject. The charge was grounded on the state of Smerwick Harbour, which was asserted to be so dangerous that no vessel could safely ride in it for many hours, even in the calmest weather. That this is a correct representation of its *present* state I entertain no doubt; but what says Dr. Smith, who wrote many centuries after the district was visited by the Northmen? "Beyond these is the haven of *Smerewick*, which lies up from N. to S., and is exposed to N. and W. winds. The whole is deep and good holding ground, the bottom being actually a *turf bog*, which vessels have pulled up with their anchors, which shews that it was once dry land: THERE IS NO DANGER IN SAILING INTO THIS PLACE."—*The Antient and Present State of the County of Kerry*, p. 360.

In the same paper another derivation of the name Smerwick was proposed, from the Irish ꞅméuꞃ (which is cognate both with the Icelandic *smjör* and the English *smear*), the inlet in question having a tendency to *spread* its waters over the adjacent shores. But, conceding for the sake of argument that the first syllable of the name is the Irish ꞅméuꞃ, I would ask, whether the poverty of the ancient language of Ireland was such, as to render it necessary to send to Iceland for the second syllable, expressive of so familiar an idea as *harbour*, or *bay*?

drifted ashore he founded the colony of *Ingolfshöfði*, or "Cape Ingolf." Leif, meanwhile, was driven so far westward, that the fresh water on board became at length exhausted, upon which one of the Irish slaves kneaded meal and butter together, asserting that this mixture would allay thirst. Rain falling soon after, what remained of the *mynnþak*, as the mixture was called by the slaves—and the first syllable of which word appears to be the Irish min, "meal"—was thrown overboard; and the place on the southern coast of Iceland, where it was drifted ashore, was thence named *Mynnþakseyri*, "Cape," or rather "Strand—Mynnþak."

But the word *Smerwick* admits of a more dignified etymology. By Fynes Moryson this locality is designated "St. *Mary Wic*, vulgarly called *Smerwick*," and on Mercator's map as "*Smerwik als St Mary wyk*." Of these names, the one would appear to be a contraction of the other: nor will this contraction seem forced when it is recollected, that *Marie-la-Bonne* has been degraded into *Marrowbone*, as the name of a lane in this city,—and seems also to have become, in a translated form, the parent of another word, very different both in sound and associations, namely, *gossamer*, *good St. Mary*—in French, *fil de la bonne vierge*—or, perhaps, *gauze o' Mary* (which is substantially a translation of the French expression), though the last syllable has been otherwise derived, from the French *mère* (*mère de Dieu*). Had the Danish writer been aware of the above explanation of *Smerwick*, he would doubtless have adverted to it in connexion with the Map, especially as a passage in Olave Tryggvason's Saga appears to throw a little twilight on the obscure subject. It is recorded of this celebrated wanderer, that in the year 993, when about twenty years of age, he was baptized in the largest of the Scilly Islands, at a monastery, situated in a place called in Norse, *Mariuhöfn*, and still *St. Mary's Haven*, and that he proceeded thence to England and Ireland, from which latter country he returned to Norway, two years after his baptism. Now, as Saxon localities are hardly found in Kerry, the termination *wick* seems to ascertain the Norse origin of the word; and no Northman was more likely to confer the honour of local perpetuation on the name of Mary than the individual, who, in addition to receiving the solemn rite of baptism at a seaport under her special protection, had been on the same occasion elated by a prediction, confirmatory of several preceding ones, that he would one day become king of Norway, which was uttered by the abbot who baptized him. Nay, the very preference of *wick* to *haven*, which has nearly the same meaning, would imply the wish to prevent confusion between two places, separated by only a short navigation.

In addition to the localities already noticed, *Kaupmannaey* appears on the more recent Map, at the entrance of Belfast Lough: the English name is not added, nor is the place mentioned in the Essay. This local name occurs, under an incorrect plural form, in the Anecdotes of Olave the Black, published by Johnstone, who translates it "Merchant Isles," but adds, "I know not what isles were so called." Yet it requires but a slight acquaintance with the northern languages to recognize *Kaupmannaey* as Copeland Island,—especially as it may be inferred from the narrative, that the place was in the vicinity of Cantire and the Isle of Man: besides, Johnstone was a resident of Copenhagen, and must have been aware that its name meant "Merchants' Haven." In English, *kaup* becomes *chap* in "chapman," and *Chip*, as the first syllable of "*Chipping*" (in such local names as *Chipping* Barnet, *Chipping* Norton, &c.), which is pronounced almost exactly as the Swedish *Köping*, however different in orthography, and, like it, signifies "market." The plural form in Johnstone's publication may have arisen from grouping the adjacent Light-House Island, and Mew Island, with Copeland: indeed the group is called on the spot the *Copeland Islands*.

To the preceding observations, suggested by the inspection of the Norse Map of Ireland, I would subjoin a brief consideration of some other localities, which, though not mentioned in any of the Sagas published antecedently to the Map, seem equally Norse in their origin with any of its meagre details.

There are three countries, in particular, where the Northmen have left topographical traces of their invasions, namely, Normandy, Eastland, and the British Islands. In Normandy, where they achieved a permanent conquest of the entire land, several classes of local names exist, originally Norse, and unknown in the rest of France: such are those ending in *fleur*, *beuf*, *tot*, and others, indicative of peaceful possession—the final settling-down of the invader, "*ut fons, ut campus, ut nemus placuit.*" In Eastland—called also Eastway, in contradistinction from Norway—which extended from Mecklenburgh to the White Sea, and included Vindland, or Northern Sclavonia, they founded a few settlements, which were exclusively maritime, such as Rostock, and Dantzick (Danes' Wick); for *Stargard*, or "Old Town," the name of two inland localities, is Sclavonian, notwithstanding its Norse aspect—*star* being cognate with the word *starost*, meaning "magistrate," or, literally, "elder" (which has been adopted into English by British travellers in Russia), and *gard* being equivalent to the Russian *gorod*, or "town," as in the name of the celebrated city of Novogorod, the *Holmgard* of the Northmen. In Ireland (to omit the other British Islands,) the Northmen never obtained a footing in the interior ; but as, in addition to planting a few commercial establishments on its shores, they also, during a long period, carried the trade of war to the very centre of the country, it seems likely that they would leave some topographical traces of their presence, and that such would be in some way commemorative of military enterprise, such, for example, as the fording of a river in the face of the enemy : and here it may be well to observe, that the meaning of the term *ford*—a frequent termination of Irish local names—is ambiguous, being equivalent to the Norse word *fjörðr*, "firth," when applied to a maritime locality, and to the Norse word *furða*, or "ford," when applied to an inland one. Examples of the former application of the term are found in *Carlingford* and *Strangford*, names of undoubted Northern origin,—of the latter, in *Odin's Ford*, the name of a locality on the Barrow, near Carlow, which (like *Odin's Fields*, in the county of Dublin) appears to owe its name to the great deity of the North, and, perhaps, in *Urlingford*, a town in the county of Kilkenny.

While the generality of our local names, terminating in *ford*, are either translations from the Irish, or originally English, the vernacular name of Urlingford —Aċ Uplann, or "Urlann's Ford"—seems to be an exception. Respecting the existence of any Irish individual of this name both history and tradition are silent; but, on turning to the records of the North, the name is found to bear a strong affinity to one of very frequent occurrence in the annals of Scandinavian warfare.

To what *Erling* the town in question may be indebted for its name there are no means of ascertaining, but it may be allowable to offer a conjecture. The name *Urlingford* may date from the celebrated expedition of the Norwegian king, Magnus Barefoot, to Ireland, who, confederated with the Irish king Myrjartak, or Murkertach, subjugated in 1103 the greater part of Ulster, and also Dublin, and Dublinshire already mentioned, from which they may have extended their conquests into the northern part of the present county of Kilkenny. Among the chieftains in Magnus's army was a son of Erlend, earl of Orkney, named Erling, who was slain with the Norwegian king on his second visit to

Ulster, and must therefore have been living when the allied monarchs ravaged Leinster; and, even if the conjecture that he gave name to Urlingford be groundless, it may have been called after some other Erling, a participator in one of the numerous expeditions, undertaken by the Danes from their settlements on the coast, during which they penetrated even to Clonmacnoise, in the very heart of the island: as *Urling* this name appears to be still extant in these countries, in connexion with a branch of manufacture. It is true that *Urlingford* is aspirated by the peasantry; but, as no tradition appears to exist, which would connect the name with a popular pastime, I would rather suppose the aspirated pronunciation to have originated in the circumstance, that the word *hurling* expresses an idea familiar to the mind, which *Urling* does not, in the same way as *Reginald's Tower*, on the quay of Waterford, has been converted into *Ring Tower*, to which corrupt denomination its round form gave a shade of plausibility.

Wexford, otherwise written *Weisford*, has a Saxon aspect: it may, however, mean West *fjörðr*, or "firth," as the Irish were denominated *Westmen* by the Northmen, in contradistinction from the name *Eastmen*, which they assumed themselves. Thus *Vestmannseyiar*, off the south of Iceland, means "Irishman's Islands;" and they owe their name to the following circumstance, which forms the sequel of the tradition respecting Leif, the sea-rover. Having at length effected a landing in Iceland, at a place called after him *Hjörleifshöfði*, or "Cape Hjörleif," where he built two houses, he in the following spring set about preparing the ground for sowing; and, although possessed of an ox, commanded his Irish slaves to yoke themselves to the plough. Duvthak, thereupon, concerted with his countrymen to destroy the ox, and say that a bear had killed it; and, when Leif and some of his followers went in quest of the bear, the Irish surprised and slew him, after which they fled in boats to the islands just mentioned, taking with them Leif's wives, and some of his effects. Meanwhile, two slaves, belonging to his foster-brother Ingolf, while in quest of the columnar seat-posts which had been flung into the sea, and on which the site of his future habitation was to depend, discovered the body of Leif, and informed their master of the circumstance. Ingolf, thereupon, having ascended a promontory to view the country, and ascertain, if possible, whither the homicides might have fled, descried the islands, and, rightly conjecturing that they had taken refuge there, pursued them, and slew them in a place thence called the *Slave's Isthmus*. As to the presumed change of *st* into the *x* in Wexford, it is borne out by that of *Ostmentown* into *Oxmantown*, a local name in this city.

Wicklow appears to have been at least partially a northern settlement, its Ostmen inhabitants being mentioned in history. Its present name is, however, Saxon, and a modification of *Winchiligillo*, or *Gwykingelo*, as Cambrensis writes it: as an actual Norse locality, the name would terminate in *wick* (*vik*).

I shall briefly advert to another class of names, likewise of Norse origin, which are scattered about all the coasts of the British Islands—I mean those terminating in *ey*, "island" (or one of its orthographical variations), which is found in the Irish aoi, and i, and even in the Hebrew אִי, but perhaps in its most extensive sense of a *maritime district*. Two examples of this class have been already noticed, namely, the Copeland Islands, and Lambay, or "Lamb Island"—a probable modification of its earlier Norse name, with *ey* annexed, and which occurs in a plural form among the islands of Greenland (*Lambeyjar*): to these may be added the *Saltees*. The names *Dalkey* and *Dursey* are doubtful, being likewise found far inland. That of a maritime parish, in the northern part of the county of Dublin, is derived from another Norse word for "island"—I mean *Holmpatrick*, a translation of the name of the neighbouring island of Inispatrick. The word *holm* implies *covering*, or *concealment*, and is usually applied to small uninhabited islands, as being best suited to such purposes. It is considered cognate with *hialmr*, "helmet," and is derived from the verb *hylia*, "conceal." The consistent first-fruits of the introduction of Christianity into Iceland, in the year 1000, was the legislative abolition of duelling; and some desert island was thenceforward chosen as the scene of conflict by individuals, who were too feebly imbued with the spirit of the mild religion to eschew sanguinary encounters: hence *holmgangr*, literally "island-going," became tantamount to "single combat." In the parish of Holmpatrick is a town, to which a neighbouring cluster of islets has given the name of *Skerries*, which in Norse means *rocks in the sea*, especially *covered* ones, and is probably found in the first syllable of the Norman locality *Cherbourg*, but which is equally derivable from the Irish sceir, "sharp sea rock." *Kalfr*, "calf," in modern Danish *kalv*, is a third Norse word for "island." It is applied to a small object in juxtaposition with a comparatively large one—for instance, to a hill beside a mountain, or an islet beside an island. Off the coast of Kerry are three islets—the *Bull*, the *Cow*, and the *Calf*. The last of these is close to Dursey Island, which, though small, is of much greater extent than the others, and the name Calf is perhaps of Norse origin : those of Bull and Cow may have been subsequently added, to make out the group, by persons unacquainted with the local meaning of *calf*. However this be, the *Calf of Man* is an undoubted example. In Normandy this word is supposed to be represented by *cauf*. The investigation of certain ruins, adjacent to one of the Greenland firths, was impeded by what are in Danish called *kalvisen*, by a number of which the firth was blocked up; this word, doubtless, means "ice-calves," or small masses of ice in the neighbourhood of large ones. The word *sound*, applied to some of our narrow straits, may be likewise of Norse origin.

In conclusion, I would with deference recommend to the attention of the Irish antiquary, and especially of the topographical and historical investigator, the hitherto neglected literature of the North. Although the most important works of the Scandinavian antiquaries are accessible through Latin versions, their minor publications teem with interesting and rapidly accumulating matter, locked up in languages which are in this country almost utterly unknown. Yet the comparative anatomy of antiquities cannot be too extensively cultivated. A fragment of an ancient object, found in one country, may be elucidated by comparing it with a corresponding fragment found in another ; and, what is of still greater importance, long-established errors may be thus removed. "The short sword or dagger," with which King, in his account of Richborough, has equipped a Roman bagpiper, would still maintain its belligerent masquerade, had not the discovery of a more perfect specimen in Scandinavia proved it to be the more appropriate appendage of a pipe ; and certain objects, deified in Sweden, the figures of which have been published by Pennant, might have long maintained their sanctity, had not the subsequent discovery of more perfect specimens in Denmark desecrated them into—knife-handles.

END OF VOLUME XIX.