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THE

TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

VOLUME XXII.

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SCIENCE.

TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

I.—*On the Relation between the Temperature of metallic Conductors, and their Resistance to Electric Currents. By the* Rev. Thomas Romney Robinson, *D.D., M.R.I.A., &c. &c.*

Read November 30, 1848.

In the year 1821, Sir H. Davy discovered that metallic wires resist a voltaic current more as their temperature is raised; and that this is the case whether they be heated by the current itself or some other means. His memoir contains many remarkable facts; but the imperfect state of rheometric knowledge at that time, and the unsteady action of the batteries which were then in use, prevented him from determining the law of the change. More recently the subject has been examined by E. Becquerel and Lenz, who found the increase of resistance proportional to the temperature. Their researches (to which however, I have not an opportunity of referring) were, I believe, made at temperatures little above that of boiling water; and it seemed desirable to extend them through a wider range, as facts of this nature have an unequivocal relation to the molecular forces and atomic structure of matter.

The transference of electricity through a wire has nothing in common with the movement of material fluids in a tube, except the analogy of the effects produced by enlarging the section of the conductor. A much more probable view of its nature is that which refers it to a successive change of tension in each molecule, acting by induction through the interval between them in a way corresponding to the charge and disruptive discharge of a coated plate. This in some degree accounts both for the heat produced by the current, and for its increasing with the resistance. It also seems to explain the increase of resistance; for the distance increases with the expansion, and the intensity required for discharge increases with the distance. That intensity must, by the ordinary theory, be as the square of the distance between the particles; and, therefore, the resistance is in the same ratio. Calling it A and the distance z; if this latter become by expansion $z+\zeta$, the proportional change of resistance

$$\frac{A'-A}{A}=\frac{2\zeta}{z}+\frac{\zeta^2}{z^2};$$

or, if $l+eP$ be the length of the expanded wire l,

$$A'-A=A\left(\frac{2eP}{l}+\frac{e^2P^2}{l^2}\right).$$

It will, therefore, increase faster than the expansion.

Or the transfer may be regarded in another view, as merely a momentary change of condition propagated in rapid succession like a wave, and modifying as it passes the normal action of the forces which keep the molecules in their state of equilibrium. A statement of this kind is of necessity vague, in our present imperfect knowledge (or rather conjectures); yet we are not without grounds for believing, that when these forces exert any specific action, they become less efficient as to others, which, under different circumstances, they would have maintained with energy. Thus, light interferes with affinity, and this with cohesion; thus also, pressure developes heat and electricity. It is, therefore, possible, that while producing thermic effects they may be unable to contribute to electric conduction. If so, we may expect that the change will be proportionate to its cause, to the heat developed in the conductor.

However this may be, whatever tends to throw light on a question so important cannot be without its use; and I hope the experiments which I have the honour to lay before the Academy may in some degree answer this purpose.

The rheometer and rheostat which I employed have been described by me in a previous communication*. To these is added a pyrometer, which measures the temperature by the expansion of the platinum wire which is the subject of experiment. It is shown at figs. 1 and 2. The wire *w*, 5.4

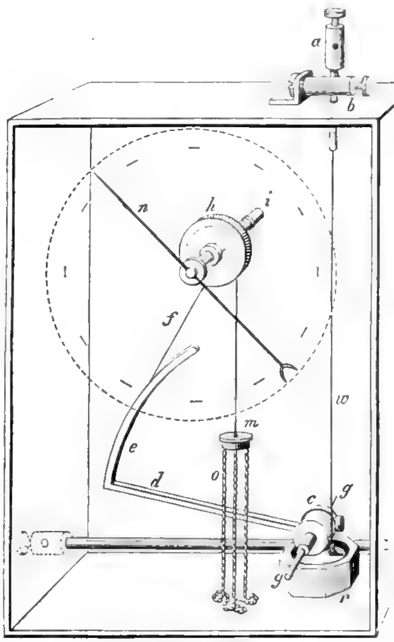


Fig. 1.

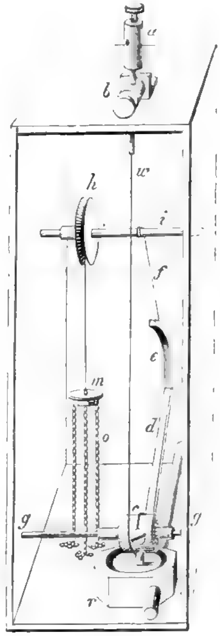


Fig. 2.

inches long, is held above by a clip attached to the binding-screw *a*, which slides in the piece *b*, and can be pinched in it at any position. Below, the wire is passed round the cylinder *c*, and secured in front by a screw. The arbor *g*, which carries the cylinder, has also an arm *d*, provided with a sectoral arc *e*, grooved on the exterior to lodge the silk thread *f*, attached to its lower extremity, and coiled above on the arbor *i*. This is furnished with a pulley *h*, on which is wound another thread bearing a counterpoise: it also carries the index *n*, which plays on a dial divided into sixty parts. It will easily be understood from this, that the counterpoise tends to pull up the arm *d*, and roll the wire on the cylinder *c*, but is resisted by its tensile force. If now the wire

* Transactions R. I. A. vol. xxi., Science, p. 291.

be elongated by expansion, the counterpoise will descend, and the index n will describe an arc of the dial proportional to that expansion. A strong platinum wire screwed into the lower part of c dips in the mercury-cup r, and by connecting this and the binding-screw a with a battery, any required current is passed through the wire w. Since the mass of the cylinder c, and its arbor, is very great in comparison of that part of the wire which is in contact with it, besides being connected with the metallic frame of the pyrometer, it is scarcely heated; and, therefore, its expansion may be neglected. The effective length of the wire may, therefore, be taken as the distance between the centre of the arbor and the bottom of the clip. Calling this l, the number of the pyrometer divisions P, and the value of one of them (in units of l) $= e$, the machine gives eP equal to the expansion—not of l,—but of a portion of it l', whose length, *when expanded*, is equal to l. We have, therefore,

$$l' = l - eP,$$

and obtain the expansion of l itself by the equation

$$d = \frac{eP \times l}{l - eP} = eP\left(1 + \frac{eP}{l} + \&c.\right).$$

In the instrument which I constructed, the length of $d = 3^{i}.00$; the radius of $c = 0^{i}.208$; and the diameters of the arbor i and the dial are respectively $0^{i}.1783$ and $3^{i}.24$. As each of the divisions $= 0^{i}.17$ nearly, the value of e is $0^{i}.17 \times \frac{0.208}{3.00} \times \frac{0.18}{3.24}$, or $0^{i}.00065$; and tenths of this are easily estimated. It was more exactly determined by lowering the clip an amount measured by a micrometer microscope, which gives as a mean of eighteen trials corresponding to 147^{P}, 0.09516, or,

$$e = 0.000643.$$

The diameter of the counterpoise-pulley $= 0^{i}.437$, and, therefore, any weight applied there causes a tension of the wire 35.3 times as great. The counterpoise consists of a weight $m = 31.3$ grains, which equilibrates the arm d and its sector; and of a piece of chain o which gives the tension. The use of this arrangement is, that when the wire is heated, and unable to bear much strain, the chain descends and rests on the bottom of the pyrometer, so that as it approaches the fusing point of platinum the tension has almost ceased. The front and back of the instrument are brass; the sides are glazed, but, except in photometric experiments, the glass is covered inside with slips of bright tin, to lessen the effect of radiation. The top is mahogany, to insulate the screw a; from its low conducting power this becomes very hot, and therefore exerts a cooling power on the wire much less than what acts at its other end.

To deduce the temperature from the corrected expansion, I have used the expansibility of platinum given by Dulong and Petit.* They assign

$$\text{Mean absolute dilatation from } 0^\circ \text{ to } 100^\circ \text{ Cent.} = \frac{1}{37700},$$

$$\text{. . . . . from } 0^\circ \text{ to } 300^\circ \text{ Cent.} = \frac{1}{36300}.$$

The corresponding temperatures, being measured by an air thermometer, might require a slight correction for the coefficient of gaseous expansion, which was Gay Lussac's; but such refinement is needless in the present research. The expansion-rate of the metal evidently increases with the temperature; its law is unknown, but we shall probably not err far by assuming

$$\epsilon = a.t + \beta.t^2,$$

and the above values give

$$0.0000088418 = a \times 180^\circ + \beta \times (180^\circ)^2,$$
$$0.0000091828 = a \times 180^\circ + 3\beta \times (180^\circ)^2;$$

the degrees being Fahrenheit's, but their origin at the freezing point of water. Hence

$$a = \log^{-1}(4.68282);\ \beta = \log^{-1}(0.72118).$$

But since ϵ is the absolute increase, divided by the length, we have

$$\frac{eP}{a.l} = t(1 + ft) = \eta P,$$

f being $\frac{\beta}{a} = \log^{-1}(6.03836)$, and $\eta = \frac{e}{al}$.

This quadratic may be solved in each experiment, or its positive root tabu-

* Annals of Philosophy, 1819.

lated for a decimal progression of P, and the intermediate values got by interpolation.

As these temperatures are reckoned from 32°, but the reading sets out from the actual temperature of the atmosphere, a correction is applied by adding to it the required amount.

When the heat was expected to be powerful, an additional resistance, sometimes as much as 1500^R, was included in the circuit, and gradually lessened till the full current passed. If this precaution be not attended to, the momentum which the counterpoise, &c., acquires in its rapid descent, is sufficient to produce a permanent elongation of the wire in its softened state, so that the index will not return to zero.

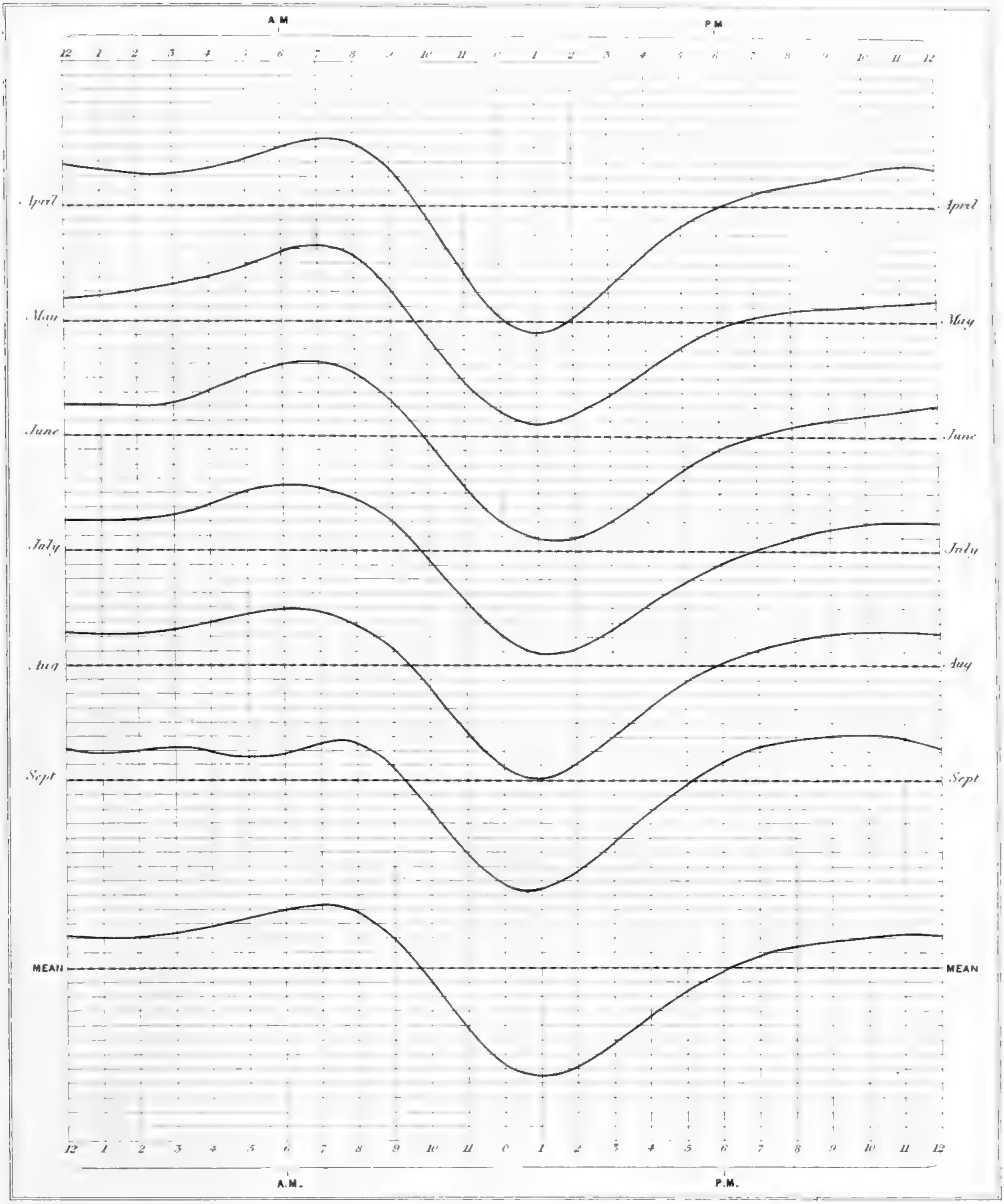
The battery used was at first on Daniell's principle, the acting surface of each metal being fifty-four square inches. With small wires this acts very well; but I found that, when the resistance of the circuit is little, the sulphate of copper is expended more rapidly than it can be supplied. In this respect, as also in giving a more powerful current, the chloride of copper is a better charge. Afterwards I used Grove's cells, each having 19.3 inches of platinum, and found them much more convenient. When the negative charge is 2 nitric acid, 2 water, and 1 sulphuric acid, they exceed in power twice as many of the others, and for a long time there is no extrication of nitric oxide. All inconvenience from this may be avoided by arranging them outside the window, and bringing the conductors through its wood-work.

When the circuit is completed through the pyrometer, its index moves very rapidly at first; several seconds, however, elapse before the wire becomes luminous, and $30^{s.}$ or $35^{s.}$ are necessary for its attaining its full heat: when it becomes stationary, a few light taps are given to the stand, to loosen any friction of the pivots. The ignition never extends to the extremities of the wire, especially the lower one; and the upper part of the wire is evidently the hottest, both for the cause already noticed, and the ascending current of heated air. It is a curious circumstance, that, when the temperature is above zero, the wire, which is then dazzling white, seems enlarged to three or four times its real diameter, an effect of irradiation which disappears when it is viewed through a darkly coloured glass. By reducing the current, and with it the intensity of the ignition, the dark portion at the bottom of the wire extends,

DIURNAL VARIATION OF THE MAGNETIC DECLINATION.

Summer Half Year. Scale 0.1 Inch = 1 Minute

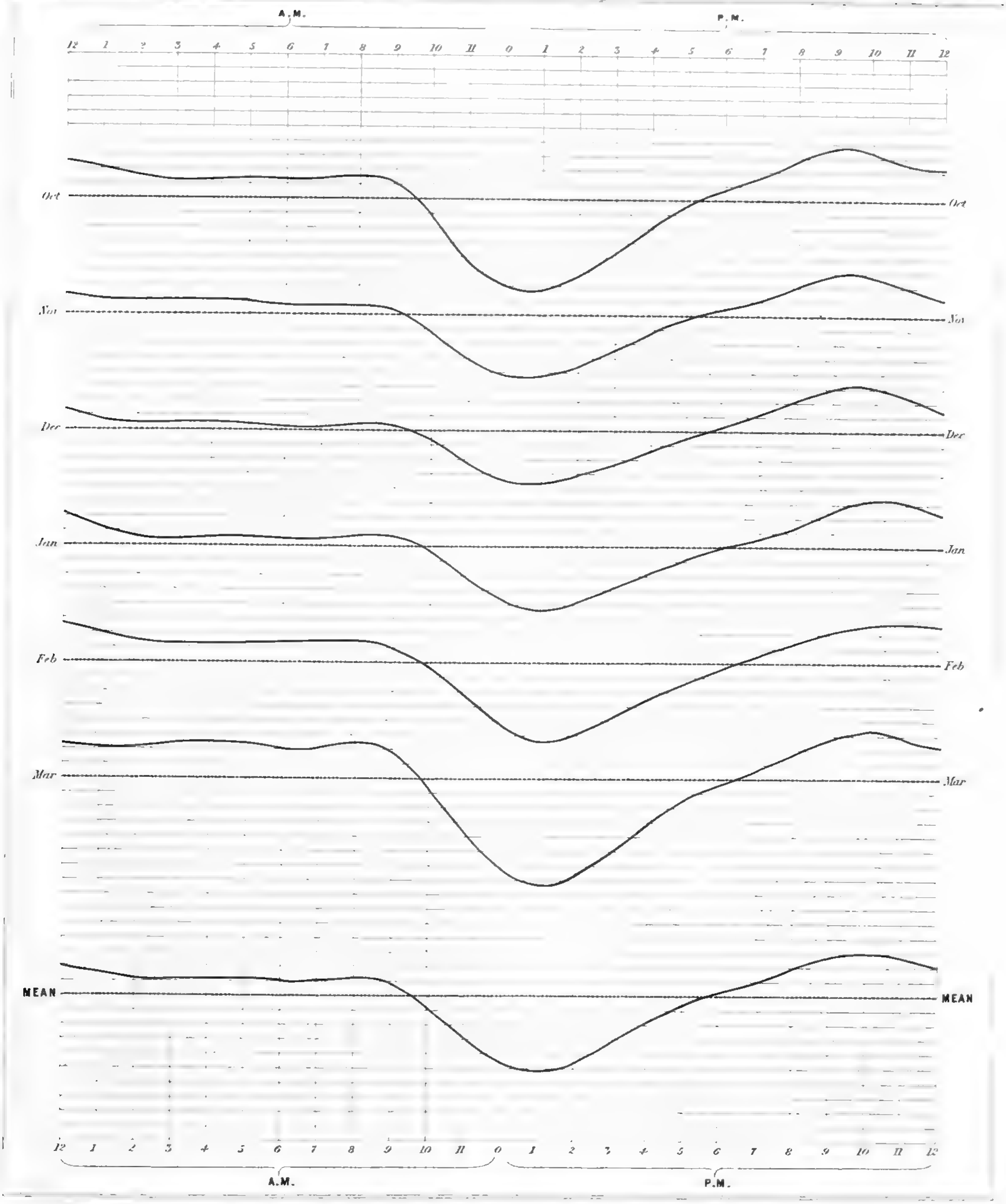
Positive Ordinates Correspond to Easterly Deflections of North end of Magnet



DIURNAL VARIATION OF THE MAGNETIC DECLINATION.

Winter Half Year. Scale 0.1 Inch 1 Minute.

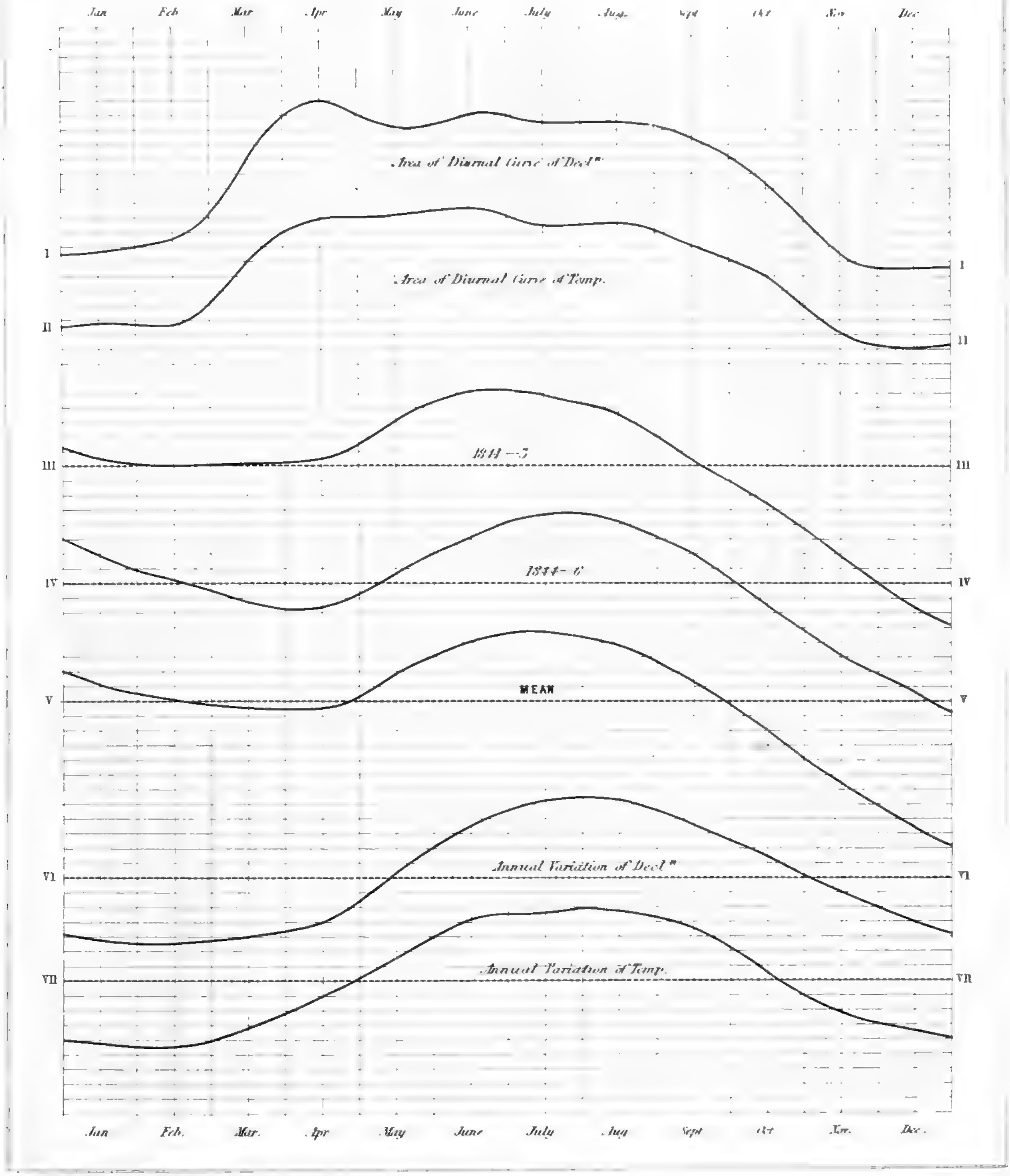
Positive Ordinates Correspond to Easterly Deflections North end of Magnet



ANNUAL VARIATION OF THE MAGNETIC DECLINATION.

III. IV. V. VI. Scale 0·2 Inch = 1 Minute

Positive Ordinates Correspond to Westerly Deflections of North end of Magnet



the upper one remaining nearly as before. If the room be completely darkened, this proceeds till only the upper inch of it is visible, and the *least additional* decrease of ignition makes all disappear, when the pyrometer shows about 550°.

The first used was originally $\frac{1}{104}$ of an inch diameter; but during the numerous preliminary experiments it was stretched till its thickness was only $\frac{1}{138}$: the results obtained are arranged in the following table, each being a mean of ten trials:

No.	Battery.	ϕ.	Current.	*P.*	*T.*	*A.*	REMARKS.
1	One D.	40°.8	0.809	12.1	289.7	305.5	
2	One G weak.	42°.0	0.858	13.4	319.9	320.8	
3	Two D.	44°.1	0.957	24.1	559.3	385.2	
4	Three D.	48°.2	1.168	41.6	930.9	497.9	
5	Four D.	49°.0	1.214	45.9	1020.6	523.8	Ignition visible in full day.
6	Five D.	51°.3	1.357	55.7	1215.0	591.6	Ignition strong red.
7	Seven D. + ρ.	52°.0	1.404	62.1	1338.7	643.8	48 resistance added.
8	Six D.	53°.7	1.515	67.8	1447.0	666.4	
9	Eight D. + ρ.	54°.0	1.538	73.1	1545.6	706.2	78.7 resistance added.
10	Seven D.	57°.0	1.794	85.0	1761.9	737.4	Yellow heat.
11	Eight D.	59°.0	2.089	96.6	1966.4	793.9	Almost white.
12	Six G.	60°.0	2.200	105.4	2116.0	840.5	Observed at night and doubtful.
13	Nine D.	60°.2	2.222	108.4	2166.2	827.8	
14	Twelve D.	61°.7	2.318	126.4	2461.6	898.2	Pure white and dazzling.

The column headed ϕ contains the deflection of the rheometer; the next gives the intensity of the current, its unit being that which deflects the rheometer to 45°, and disengages in a voltameter 6.57 inches of gases, at their normal temperature and pressure, in five minutes. The two next columns give P, the reading of the pyrometer corrected for the temperature of the air, and T, the temperature of the wire computed from it. The last gives A, the resistance of the wire expressed in revolutions of the rheostat.

The great increase of the resistance to more than four times its original value, and its steady progress at such high temperatures, are very remarkable.

The pyrometer was then placed in vacuo for a purpose that shall be subsequently noticed. As I feared that the sudden change of temperature might fracture a glass receiver, a box of strong copper was used, the lid of which was easily made air-tight by Whitworth's scraping process. Two apertures glazed with strong plate glass enabled me to read the index and inspect the wire; and an insulated wire passing through the top, and dipping in a mercury cup formed in the binding-screw *a*, connected it with the battery. The box was connected by a screw with the air-pump, which, however, was at the time not in good order. The following results are also each a mean of ten:

No.	Battery.	Pressure.	ϕ.	Current.	*P.*	*T.*	*A.*	Remarks.
15	One G. weak.	0ⁱ.64	41°.2	0.825	22.7	531.4	390.1	Charge had been used.
16	One G.	0ⁱ.42	43°.3	0.916	32.3	738.7	438.8	
17	Two G. weak.	0ⁱ.60	45°.5	1.025	45.0	1002.3	515.4	
18	Two G.	0ⁱ.40	50°.9	1.334	62.5	1346.4	606.2	
19	Three G. weak.	0ⁱ.51	. . .	. . .	69.6	1480.7	674.5	N. charge, 1 nit. acid + 1 water.
20	Three G.	0ⁱ.40	56°.0	1.716	89.4	1839.9	750.8	
21	Four G.	0ⁱ.46	57°.2	1.898	105.3	2116.0	829.5	
22	Five G.	0ⁱ.47	60°.8	2.211	123.2	2410.4	892.5	

The currents required to produce a given temperature are less, but the resistance is the same; its increase is therefore not due to any intrinsic quality of the current.

On attempting to pass the current of six Groves the wire gave way.

Another piece was substituted, to try whether feeble currents were similarly resisted, its diameter being $\frac{1}{104}$. On passing a current 0.279, the lowest which the actual position of the rheometer permitted me to measure, the wire was heated 24°, and its resistance $=189.4$. Increasing the current to 0.315, the wire was heated 9° more, and the resistance became 195.4. In fact, I believe it is impossible to pass any current whatever without changing the resistance in some degree; and think it highly probable, that this has given rise to the opinion entertained by some philosophers, that the resistance is a function of the current.

This wire failed also in an attempt to obtain higher temperatures,* and was

* In the first trial the zero of pyrometer was, before contact, -7.2; $P'=148.8$; zero after, $+25.2$. If the whole of the lengthening took place during the cooling, $P=159.9$ and $T=2981.3$. The resistance $=967.1$. In a second trial with an additional resistance of 50 in circuit, zero

replaced by one $\frac{1}{19}$ diameter. The counterpoise was changed till, after several trials, it was found that a tension of four ounces at zero of the pyrometer, was sufficient. I was surprised to find that a given battery produced nearly the same ignition in this as in the smaller wire, the increased current compensating the greater mass; but, from the greater quantity of heat evolved, the upper part of the pyrometer became very hot, so that the clip which holds the top of the wire was blued. This apparently made the resistance greater than the truth; but is not likely to have affected its changes, as the experiments were made in the inverse order of the table, so that the highest temperature was obtained first. After five results the wire was broken by an accident close to the cylinder; and as I had no more of that diameter, I rejoined it by a loop, which, being beyond the part that is ignited, might be expected not to interfere with the temperature produced.

Each result is a mean of five.

No.	Battery.	ϕ.	Current.	*P.*	*T.*	*A.*	Remarks.
23	One G. + 83.	60°.0	2.200	15.1	359.0	107.3	
24	. . . . +50.	62°.7	2.437	20.3	476.8	107.5	
25	One G.	67°.9	3.261	31.9	730.2	116.4	
26	Two G. + 50.	70°.1	3.768	46.4	1030.9	135.0	Not visible in strong day-light.
27	Two G.	72°.5	4.824	60.4	1306.5	151.0	Full red.
28	Four G.+105.	73°.9	4.827	70.3	1493.8	173.2	
29	. . . . +48.	75°.9	5.654	86.9	1795.5	189.1	Yellow heat.
30	Four G.	77°.1	6.272	99.3	2012.2	188.1	White heat.
31	Six G. + 20.	77°.7	6.588	106.5	2134.6	196.0	
32	Six G.	79°.0	7.286	121.9	2389.4	201.9	Splendid *in bright sunshine.*

The increase is evident here also, although the discordances are greater than in the last series.

Lastly, a wire of $\frac{1}{69}$ was used. At No. 40 the pyrometer was immersed in diluted alcohol, to ascertain whether the resistance increased with the current when the wire was kept cool. The fluid boiled round it with a sharp

$= -4.6$; $P' = 140.0$; zero after, $+24.0$. On the same supposition, $P = 148.1$; $T = 2803.0$; and the resistance $= 953.0$. In the third trial it gave way; the ends, however, were not fused, though a very little more would certainly have melted it. The temperature of the centre of the wire in the first of these experiments must have exceeded 3200°.

snapping, and its refraction may make the reading doubtful one or two tenths of a division. All but the last are means of five; that only of two, which, however, agree tolerably.

No.	Battery.	ϕ.	Current.	*P.*	*T.*	*A.*	Remarks.
33	One double G. +155.	56°.1	1.522	15.4	365.9	127.8	
34	. . . . . . +88.	58°.4	2.020	20.5	481.3	144.9	
35	. . . . . . +18.	61°.9	2.411	31.4	719.6	169.0	
36	Two G. + 22.6.	65°.1	2.758	46.0	1020.7	212.0	
37	Three G. + 80.	68°.1	3.226	60.6	1310.4	238.7	
38	. . . . +37.6.	69°.8	3.699	71.6	1518.0	270.1	
39	Four G.	72°.9	4.352	91.9	1883.9	318.6	
40	Five G.	81°.3	9.548	8.6	207.8	93.8	
41	Ditto.	73°.9	4.821	110.8	2206.7	346.0	Battery nearly exhausted and zero unsteady.
42	Six G.	74°.9	5.230	119.8	2355.1	351.4	

In examining these tables it is evident,

1. That the increased resistance is not occasioned by any condensation of the current. In No. 40 it passes more easily than in No. 33, though of six-fold power, and able to fuse a far thicker wire in air. It is also worth notice, that in these two cases the *productions* of heat are as 29 : 1; and, therefore, it is not by the mere employment of molecular forces in the production of that agent, but by its accumulating and becoming sensible, that conduction is impeded.

2. The magnitude of the change prevents me from attributing it to a mere change of molecular distance. On this hypothesis we have seen that it will be

$$\frac{A'-A}{A} = \frac{2eP}{l} + \frac{e^2P^2}{l^2}.$$

Now in No. 14, the highest of the set, $\frac{eP}{l} = 0.1$, which will give $\frac{A'-A}{A} = 0.30$, whereas it really $= 4.68$.

3. Nor is it proportional to the expansion: up to a certain point it may be expressed by the formula $A = a + bP$, but less accurately than by making it depend on the temperature; and

4. It appears to be correctly determined by the equation

$$A = a + bT; \tag{1}$$

where a is the resistance at 32° of Fahrenheit, and b its change for one degree.

Before endeavouring to deduce the values of these constants for each wire, it is to be remarked, that the resistances given in the preceding tables are too small, and require to be corrected for the heating effect of the current on the rheostat and resistance coils by which they were measured. The thickness of the wire in the former, and the immersion of the others in alcohol, might seem to guard against this danger; but, with powerful currents, both become warm to the touch. If we assume the truth of (1), the resistance measured is not A, but $A\,(1+b't')$, supposing all reduced to the freezing point. Now the heat generated is as $A \times$ square of current; and I have found by experiment, that the temperature of a wire follows the same law under 100°. Hence, for $1+b't'$, we may write $1+c.A.C^2$, and (1) becomes

$$A = a + bT - c.A^2.C^2.$$

Each result furnishes an equation of condition, which may be grouped together, and either by minimum squares or ordinary elimination the values of a, b, and c determined.

If the twenty-two that belong to the wire $\frac{1}{138}$ be thus combined, we have the equations

$$442.19 = a + b \times \ \ 674.1 - c \times \ \ 195855.0,$$
$$660.53 = a + b \times 1447.9 - c \times 1002800.0,$$
$$833.32 = a + b \times 2153.8 - c \times 3141477.0.$$

Hence,

$$a = 198.4\ ;\ \ b = 0.3412\ ;\ \ c = 0.00003182.$$

Computing with these the apparent values of A, and subtracting them from the observed, I have arranged the results according to T.

No.	*T.*	Obs. – Cal.	No.	*T.*	Obs. – Cal.	No.	*T.*	Obs. – Cal.
1	289.7	+10,2	6	1215.0	+ 4.0	11	1966.4	+10.5
2	320.0	+15.7	7	1338.7	+15.3	12	2116.0	+28.9
15	531.4	+13.6	18	1346.4	–30.8	21	2116.0	–12.0
3	559.3	+ 8.0	8	1447.0	+ 4.1	13	2166.2	– 2.0
16	738.7	+ 7.0	19	1480.7	– 3.3	22	2410.4	– 4.4
4	930.9	+ 6.8	9	1545.6	+18.0	14	2461.6	– 2.1
17	1002.3	+ 2.3	10	1761.9	– 1.0			
5	1020.6	+ 5.5	20	1839.9	–11.3			

With the exception of 18 and 12, the agreement between the formula and observation is sufficiently close; and even in them the error is not remarkable, if we consider that a degree of the pyrometer represents 20° of Fahrenheit, and that a little oxidation in the connexions may affect the resistance of an entire set. It even seems to me that this principle affords a very effectual method of measuring high temperatures in the arts.

With the wire $\frac{1}{19}$ the three equations are

$$114.55 = a + b \times 424.25 - c \times 130725.0,$$
$$171.10 = a + b \times 1531.93 - c \times 753848.0,$$
$$195.33 = a + b \times 2178.70 - c \times 1741069.0.$$

Hence,

$$a = 91.4\ ;\ b = 0.05898\ ;\ c = 0.0000141.$$

The value of a is certainly too large. I have mentioned the probable effect of the oxidation of the clip; possibly while looping the broken wire, the contact may have been improved, for I found afterwards, that by trying its resistance under water while attached to the clip, with a current = 2.743, the resistance was 76.1. On making the contact perfect it was only 46.

The values of observed − calculated resistances are

No.	T.	Obs. − Cal.	No.	T.	Obs. − Cal.
23	359	− 4.5	28	1494	+ 3.6
24	477	−11.1	29	1796	+ 7.9
25	730	−16.1	30	2012	− 2.4
26	1031	−13.5	31	2135	+ 2.2
27	1306	−11.6	32	2389	+ 0.1

Part of the errors of the first five are, I think, accounted for; and the rest is unimportant.

Lastly, for the wire $\frac{1}{69}$ the equations are

$$147.22 = a + b \times 522.27 - c \times 96511,$$
$$240.27 = a + b \times 1283.03 - c \times 644327,$$
$$338.67 = a + b \times 2148.57 - c \times 2694208.$$

Hence,

$$a = 81.9\ ;\ b = 0.1261;\ c = 0.000007188.$$

In these equations No. 40 is not included from the uncertainty of P. The comparison with observation gives,

No.	$T.$	Obs. – Cal.	No.	$T.$	Obs. – Cal.
33	366	0.0	38	1518	+ 2.0
34	481	+ 3.2	39	1884	+ 9.3
35	720	– 2.7	40	209	–10.1
36	1021	+ 3.2	41	2207	+ 0.5
37	1310	– 5.3	42	2355	– 9.7

Here also the conformity to observation is very satisfactory.

If we divide the values of b by a (assuming $a'' = 46$), we obtain 0.0017, 0.0013, and 0.0015, taking the mean of which we have for a platinum wire,

$$A = a\,(1 + 0.0015 \times T), \qquad (2)$$

a being, as before, the resistance at 32°.

It would be desirable to extend these experiments to other metals; but I could not find any determination of their expansions similar to that which Dulong and Petit have given for platinum, except what those philosophers have given for iron and copper. The first of these metals, however, I found to oxidate so rapidly in the pyrometer, that I could get no consistent results. It should for this purpose be surrounded by dry nitrogen. Copper also oxidates; but the film of oxide acts as a coating, and protects the interior, so that its diameter does not change up to 900°. The wire used was of the same diameter as the last platinum one, $\frac{1}{69}$. Each result is a mean of three closely agreeing.

No.	Battery.	ϕ.	Current.	$P.$	$T.$	$A.$	Obs. – Cal.
43	1 G. + 196	56°.9	1.808	11.1	102.3	30.1	+ 0.3
44	. . . + 105	64°.3	2.640	21.6	195.0	33.4	+ 0.3
45	. . . + 50	70°.6	3.902	40.0	348.9	37.8	– 0.6
46	. . . + 30	72°.9	4.352	53.0	452.4	41.8	– 0.2
47	1 G.	75°.8	5.608	81.0	664.1	49.6	+ 0.2

When the current was increased to produce ignition, $T = 919.8$, $A = 68.8$: but the zero changed, and the wire stretched till its diameter became $\frac{1}{92}$.

For this metal the coefficients of expansion are $\alpha = \log^{-1}$ (5.03826); $\beta = \log^{-1}$ (1.40598). As the resistance was measured by the rheostat alone, the correction cA^2C^2 need not be applied, and the equations are

$$33.77 = a + b \times 215.40,$$
$$45.90 = a + b \times 558.25.$$

Hence,

$$a = 26.37\,;\ b = 0.0348\,;\ \frac{b}{a} = 0.00133.$$

The value of $\frac{b}{a}$ is so near that of platinum as to make it an object of interest to ascertain whether the same equality prevails in other metals. The difference of conducting power in copper and platinum appears very strikingly here. At 32° the resistances are as 1 : 3.1; but in Nos. 46 and 39, when the current is the same, as 1 : 7.6. It may also be remarked, that these constants give, as the probable values of the correction cA^2C^2, quantities closely agreeing with those computed by the values of c given above.

The facility of bringing the wire of the pyrometer to a given temperature, and maintaining it, makes it a convenient source of light in photometers; but as the heat is not uniform along it, it seems worth inquiry according to what law it varies. Each section of the wire is traversed by the same current, and, therefore, under similar circumstances, would be equally heated: but the heat thus excited is dissipated by three cooling agencies. The first of these is radiation, which, though lessened by the bright metallic surface of the pyrometer, below what it would be in free space, is still very powerful. The second is the presence of air; and the third the conducting power of the wire itself, by which a portion of the heat escapes to the metallic supports which attach it to the instrument, in this case chiefly to the lower one. As long as their combined effects are inferior to the heating power of the current, the temperature must increase: while doing so, however, the resistance also increases, and with it (as shall be immediately shown) the heating power, the current being the same. On the other hand, the cooling causes also augment in energy, and in a still higher ratio. An equilibrium of these powers is therefore attained, to which belongs the thermic state shown by the pyrometer: it is, therefore, expressed by equating to cypher the differential equation, which expresses the rate of cooling for a differential section of the wire in function of the time. Calling x the distance of a point from the extremity of the wire, y the excess of its temperature above the surrounding medium, and $f(y)$, $f'(y)$, the heating power of the current, and that cooling one which depends on its surface, and on the surrounding media, the differential equation becomes

$$0 = H.f(y) - I.f'(y) - \frac{K.d^2y}{dx^2}.$$

The integral of which will determine y the temperature at x.

The heating power of a given voltaic current is known to be proportional to the resistance which it overcomes; but in all the experiments which have established these laws, the change of resistance which I have been considering was overlooked, because the conductors were kept at a comparatively low temperature. It might, therefore, be a question, whether the resistance to be used is the intrinsic (that at a given temperature) or that increased by heat. It is easily proved to be the latter by means of the apparatus (fig. 3). A is a thin jar, in whose neck are cemented copper wires terminating in the binding screws C, C. Their other ends are connected by platinum wire, W, of $\frac{1}{69}$ inch diameter, and 5.4 inches long. Over the wire is inverted another jar, B, formed of thin tube. If water be now poured in, B acts as a diving-bell, and the wire W is in contact with air. Passing a current through it, it may be intensely heated, and its resistance of course increased; but the heat which it gives off is employed in heating the glass and water by which it is surrounded, and can be measured by a thermometer immersed in the water. At low temperatures Newton's law of cooling is exact, and, therefore, the rise of the thermometer is proportional to the thermic power of the current. A single result will be sufficient. The jar

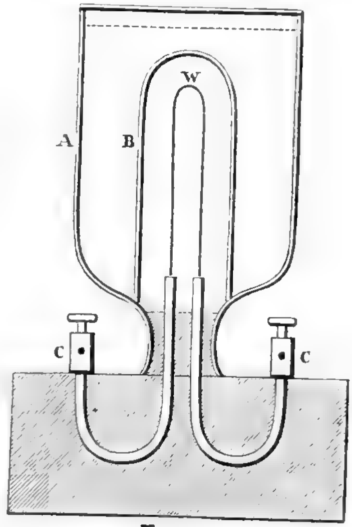


FIG. 3.

containing 5.5 cubic inches of water, a current = 3.527 was passed for twelve minutes, the air and water being both nearly 72°. The thermometer rose 77°.5, and the resistance A = 257.6. The wire was almost white, and its temperature must have been near 1500°. The instrument was then cooled, filled with water (which, by inclining it, was made to fill B), a resistance = 165.6 added in the circuit, and a current = 3.558 passed for twelve minutes. Now the thermometer rose only 29°.7, and the resistance = 89°.0. Here all was the same, except the increase of A by the heat; and that determined the greater heating power in the first experiment. The second was repeated, but without any interposed resistance in the circuit: bubbles of steam formed round the wire, which were condensed with sharp snapping; the current = 6.045, which would have melted it in air; the resistance = 94.5; and the rise of the thermometer = 83°.2.

From this it follows that $f(y) = a(1 + ry)$.

The function $f'(y)$ is far from being so easily determined. Fourier and Poisson, in their celebrated investigations, have assumed it $= ky$, following Newton. This, however, is quite at variance with observation. Dulong and Petit, in their memoir, have assigned expressions for the effect of the air and radiation, which represent their observations very exactly. According to them the effect of radiation is as $1.0077^{T} - 1$, and that of the air as $T^{1.23}$. The heated body which they employed was the bulb of a thermometer enclosed in a globe of copper a foot in diameter, blackened on its interior surface, and kept at an invariable temperature. The highest range was under 500°, and of course far below the point at which light is given off,—an agent which, doubtless, interferes with heat. Accordingly, their law of radiation fails altogether in my experiments, but the other is nearly exact. We are enabled to infer this from the law already mentioned, that the heating power of the current is as its square multiplied by the resistance.* Now in the experiments Nos. 1–14, the wire is in contact with air, while in Nos. 15–22, which were made *in vacuo*, the effect of that medium

* This law has been often verified, but the experiments just described gave a good illustration of it. In the first and third, the ratio of the heats excited is 1 : 1.074; while that of AC^2 is 1 : 1.077; the difference arising from this, that, the temperature being higher in the last, more was lost. It may also be mentioned as evidence of the heating power of that current, that it generated in the wire as much heat as would have ignited eight ounces of platinum to strong redness.

is nearly insensible. If from the first we deduce by interpolation the currents and resistances which correspond to the temperature of the vacuum series, we obtain one in which radiation and conduction must be the same as in it; and the difference between the AC^2 for the same temperature is evidently the measure of the cooling due to air. Comparing them with the corresponding values of T, I find that they are as its first powers. The deficiency from the experiment of Dulong and Petit proceeds, probably, from the air being heated.

In the vacuum series AC^2 must be as the combined effects of radiation and conduction. Omitting No. 15 as too low, if we divide AC^2 by T^2, we obtain the numbers

No. 22, ...	0.75
21, ...	0.67
20, ...	0.66
19, ...	0.64
18, ...	0.73
17, ...	0.54
16, ...	0.68

And if we allow for the residual air and the conduction, we may assume the radiation in this pyrometer to be as the square of the temperature. Hence, $f'(y) = Gy + I.y^2$, and as G is less than $H.ar$, the equation becomes,

$$\frac{d^2y}{dx^2} = my^2 - ny - p,$$

in which it must be remembered that m, the quotient of the coefficient of radiation by that of conduction, is constant for a given wire, but n and p vary with the current. The integration of this is facilitated by considering, that the effect of conduction must cease at a certain distance from the origin, beyond which y is constant. Let this value of y be θ, then we must have,

$$m\theta^2 - n\theta = p, \qquad (3)$$

substituting which, and writing μ^2 for $2m\theta - n$, and u for $\theta - y$, we have,

$$\frac{d^2u}{dx^2} = u\,(\mu^2 - mu).$$

Integrating

$$\left(\frac{du}{dx}\right)^2 = u^2\,(\mu^2 - \tfrac{2}{3}mu),$$

which requires no arbitrary constant, because du vanishes with u.

Integrating again, e being the base of the Neperian logarithms,

$$u = \frac{6\mu^2}{m} \times \frac{1}{\left(e^{\frac{\mu(x+\kappa)}{2}} + e^{-\mu\frac{(x+\kappa)}{2}}\right)^2}.$$

As y vanishes when $x = 0$, we have then

$$\theta = \frac{6\mu^2}{m} \times \frac{1}{\left(e^{\frac{\mu\kappa}{2}} + e^{-\frac{\mu\kappa}{2}}\right)^2}, \tag{4}$$

and hence deduce

$$\frac{y}{\theta} = 1 - \frac{(1 + e^{-\mu\kappa})^2}{e^{\mu x}\,(1 + e^{-\mu(\kappa+x)})^2}, \tag{5}$$

which determines the thermic condition of any point x.

As the pyrometer gives only T, the mean temperature, we must find its expression for any length x:

$$T = \frac{\int y\,dx}{x} = \theta + \frac{6\mu}{mx} \times \left(\frac{1}{e^{\mu(x+\kappa)} + 1}\right) + \kappa'.$$

In strictness this should be taken from 0 to the centre of that portion of the wire which is of uniform heat, supposing both supports to cool it equally; but as this part extends close to the upper support, I prefer taking it from 0 to λ, the place where the heat declines; in this instrument λ is five inches. Thence

$$\frac{T}{\theta} = 1 - \frac{1}{\mu\lambda}\left(1 + e^{-\mu\kappa} - \frac{(1+e^{-\mu\kappa})^2}{e^{\mu\lambda} + e^{-\mu\kappa}}\right). \tag{6}$$

It then remains only to determine θ, μ, and $e^{-\mu\kappa}$.

In (3) the quantity n is the difference of two quantities; one the heating power due to the increase of resistance by heat, which, being proportional to p, may be called rp, r being, as we have seen, 0.0015; the other representing the air's cooling power, which, like m, is constant, and therefore $= ms$. Equation (3) then becomes

$$m\theta^2 - \theta\,(rp - ms) = p,$$

which, combined with $\mu^2 = 2m\theta - n$, gives

$$\mu^2 = m\left(\frac{s + 2\theta + r\theta^2}{1 + r\theta}\right).$$

From the series in air and *in vacuo* already referred to, it appears that $s =$ 0.2822 ; it may, therefore, be neglected, and then

$$\mu^2 = 2m\theta\left(\frac{1 + \frac{1}{2}r\theta}{1 + r\theta}\right). \tag{7}$$

Substituting this in (4), we obtain

$$e^{\pm\mu\kappa} = \frac{5 + 2r\theta}{1 + r\theta} \pm \sqrt{\left\{\left(\frac{5 + 2r\theta}{1 + r\theta}\right)^2 - 1\right\}}. \tag{8}$$

Whence it appears, that though μ and θ increase, $e^{-\mu\kappa}$ is confined in narrow limits, ranging from 0.105 to 0.268, while θ passes from 0 to infinity.

Since the loss of heat must equal its production, we have $m\theta^2 + sm\theta$ as AC^2, or,

$$\theta^2 + s\theta = qAC^2,$$

and as s is small, $\theta = \sqrt{(qAC^2)}$. Hence the highest temperature attained is as the current, not as its square; and also as the square root of the resistance.

If we tabulate an equidistant series of θ, and compute for each the values of $e^{-\mu\kappa}$; $\dfrac{\mu^2}{2m} = f$, and $\dfrac{\theta \times (1 + e^{-\mu\kappa}) \times \sqrt{(2m)}}{\mu} = g$; and assume $\nu = \lambda \times \sqrt{(2m)}$, the equation (6) gives

$$\theta - T = \frac{g}{\nu}\left(1 - \frac{1 + e^{-\mu\kappa}}{e^{\sqrt{(f)}\times\nu} + e^{-\mu\kappa}}\right).$$

Now taking any two observed values of T, where A and C are known, we have the ratio of the θs; assuming one the other is known, and the other quantities can be taken from the table. A few trials of this kind show that

ν must be a number less than unity, and that $e^{\sqrt{(f)} \times \nu}$ must be so large that the last term of the equation may be neglected, whence

$$\theta - T = \frac{g}{\nu},$$
$$\mu = \frac{\nu}{\lambda} \times \sqrt{f}. \tag{9}$$

When ν is known θ is easily found; for, entering the table with T as argument, we get a first approximation, with which as argument the true value is obtained.

A still more accurate mode of obtaining this quantity is by observing the value of x at which the wire assumes a given temperature. That which I selected is the point at which platinum begins to be visible in total darkness. It is unknown; but by varying the current, so as to have different lengths of x, we may equate the values of y in (5). I measured x by a screen moved by a rack, which was lowered till it cut off all light. Its pinion was moved by a graduated head, and I found the measures very consistent.

With the wire $\frac{1}{69}$ I obtained

$T = 539.3$; $C = 2.213$; $A = 150.9$; no light visible.
$T' = 772.9$; $x = 1^{i}.29$; $C = 2.375$; $A = 180.9$;
$T'' = 850.5$; $x = 1^{i}.03$; $C = 2.516$; $A = 190.9$.

From these I find $\nu = 0.3$ nearly, and with this value

$$\theta = 710.9;\ \frac{\theta}{R} = 26.2.$$

$$\theta' = 911.5;\ \frac{\theta'}{R'} = 28.5;\ y' = 739.7.$$

$$\theta'' = 995.5;\ \frac{\theta''}{R''} = 28.6;\ y'' = 733.2.$$

R being $= \sqrt{A} \times C$.

The light of the wire is like Herschel's lavender ray, and is, perhaps, rather a phosphorescence than a true ignition.

For the wire $\frac{1}{138}$ the same process gives $\nu = 0.4$ nearly, as might be expected from its being proportional to $\sqrt{m}$; the temperature is also something higher, probably from the smaller quantity of light requiring a greater intensity to be visible.

I will conclude with a summary of the principal facts which I have endeavoured to establish in this memoir:—

1. When a wire of platinum is heated by a voltaic current, its resistance to the passage of that current increases without limit to the verge of its fusion.

2. That increase of resistance is not caused by the mere increase of the current.

3. It is not caused by the increased distance of the molecules.

4. It is not caused by the employment of the molecular forces in *generating* heat.

5. It is exactly proportional to the increase of temperature of the wire.

6. The same is the case with copper till the oxidation of the metal interrupts the experiment.

7. In both those metals a given elevation of temperature produces the same proportionate change of resistanee.

8. This change of resistance must always be attended to in rheometry; and the neglect of this precaution may explain some objections that have recently been made to the theory of Ohm.

9. The heat evolved by a current passing through a wire is as the square of the current, and as the actual resistance of the wire (that increased by the heat).

10. The highest temperature attained by any part of it is, however, as the current × square root of resistance.

11. The loss of heat by the air is as the difference of the temperatures of the air and wire.

12. That by radiation is in this pyrometer as the square of that difference nearly.

13. Thermic equation of the wire; from which it follows, that the temperature rises very rapidly in receding from the lower end of the wire, till at a small distance it becomes constant.

14. This constant temperature exceeds that given by the pyrometer by an amount varying from a seventh to a tenth.

15. The following is the table of the quantities involved in this equation:

θ.	$e^{-\mu\kappa}$.	f.	g.
400	0.1313	325.0	25.10
600	0.1422	457.9	32.03
800	0.1521	581.8	38.21
1000	0.1603	700.0	43.85
1200	0.1678	814.3	49.11
1400	0.1736	925.8	54.00
1600	0.1788	1035.3	58.69
1800	0.1839	1143.2	63.03
2000	0.1882	1250.0	67.21
2200	0.1922	1355.8	71.23
2400	0.1957	1460.9	75.08
2600	0.1990	1565.3	78.79
2800	0.2020	1669.2	82.38
3000	0.2047	1772.6	85.87

Observatory, Armagh,
Nov. 8, 1848.

II.—*On the Theory of Planetary Disturbance.* *By the* Rev. BRICE BRONWIN.

Read November 30, 1848.

1. IN this paper I shall consider, with M. Hansen, the disturbance as affecting the radius vector (or rather the mean distance) and the mean longitude; and I shall first, after his manner, employ two times. Various formulæ not noticed by him (one of them fundamental) are given, in the hope that they may some time be made useful. The principal equations are investigated in a way that leads to some very elegant formulæ, and the elimination of the quantities containing both the times is effected in a very simple manner. In finding the latitude, I propose to introduce the latitude itself and the reduction into the disturbance function; by which means the part of that function depending on the inclination of the orbit to the fixed plane is greatly simplified, and the determination of the latitude and reduction rendered easier. But it is not my intention in the present paper to develope the functions in series of sines or cosines.

The well-known differential equations of a planet's motion, referred to the plane of the orbit, are

$$\left.\begin{aligned} &\frac{d^2r}{dt^2} - \frac{h^2}{r^3} + \frac{\mu}{r^2} + \frac{dR}{dr} = 0, \\ &r^2\frac{dv}{dt} = h = h_0 - \int \frac{dR}{dv}\,dt, \end{aligned}\right\} \qquad (1)$$

$$R = \frac{m'r}{r'^2}\cos\chi - \frac{m'}{(r'^2 - 2r'r\cos\chi + r^2)^{\frac{1}{2}}}; \quad \cos\chi = \frac{xx' + yy' + zz'}{r'r};$$

where x, y, and z are the rectangular co-ordinates of the disturbed ($z = 0$), and x', y', and z' those of the disturbing body. The meaning of the other symbols is obvious.

When $m' = 0$, let the integrals of (1) be

$$r_0 = f(t, a_0, e_0, \pi_0, \epsilon_0), \quad v_0 = \phi(t, a_0, e_0, \pi_0, \epsilon_0).$$

And when the disturbing force is restored, let

$$r = f(t, a, e, \pi, \epsilon), \quad v = \phi(t, a, e, \pi, \epsilon).$$

Thus r and v are the same functions of t as when $m' = 0$, the elements a, e, &c., being variable, and determined in the usual manner. And, τ being a new time, we make

$$\rho = f(\tau, a, e, \pi, \epsilon), \quad \lambda = \phi(\tau, a, e, \pi, \epsilon).$$

We further suppose,

$$r_{,} = f(z, a_{,}, e_{,}, \pi_{,}, \epsilon_{,}), \quad v_{,} = \phi(z, a_{,}, e_{,}, \pi_{,}, \epsilon_{,});$$
$$\rho_{,} = f(\zeta, a_{,}, e_{,}, \pi_{,}, \epsilon_{,}), \quad \lambda_{,} = \phi(\zeta, a_{,}, e_{,}, \pi_{,}, \epsilon_{,});$$

the elements $a_{,}$, $e_{,}$, &c., and $h_{,}$, to be presently introduced, being constants, z a function of t, and ζ a function of τ and t. To these we must add the assumed relations,

$$r = r_{,}\beta, \quad \rho = \rho_{,}\xi, \quad v = v_{,} + \mathfrak{E}nt, \quad \lambda = \lambda_{,} + \mathfrak{E}nt.$$

By changing τ into t, we change ρ into r, λ into v, $\rho_{,}$ into $r_{,}$, $\lambda_{,}$ into $v_{,}$, ξ into β, and ζ into z.

From what has been given above, we have necessarily $r_{,}^2 \dfrac{dv_{,}}{dz} = h_{,}$, $\rho_{,}^2 \dfrac{d\lambda}{d\zeta} = h_{,}$. And from the way in which a, e, &c., are found, $\delta r = 0$, $\delta v = 0$, the characteristic δ denoting the variation relative to a, e, &c.; therefore, $\rho^2 \dfrac{d\lambda}{d\tau} = h$.

But $\rho^2 \dfrac{d\lambda}{d\tau} = \rho_{,}^2\xi^2 \dfrac{d\lambda_{,}}{d\tau} = \rho_{,}^2\xi^2 \dfrac{d\lambda_{,}}{d\zeta}\dfrac{d\zeta}{d\tau} = h_{,}\xi^2 \dfrac{d\zeta}{d\tau}$. Consequently $h_{,}\xi^2 \dfrac{d\zeta}{d\tau} = h$; or,

$$\frac{d\zeta}{d\tau} = \frac{h}{h_{,}\xi^2}. \tag{2}$$

Also $r^2 \dfrac{dv}{dt} = r^2 \dfrac{dv_{,}}{dt} + \mathfrak{E}nr^2 = r_{,}^2\beta^2 \dfrac{dv_{,}}{dz}\dfrac{dz}{dt} + \mathfrak{E}nr_{,}^2\beta^2 = h_{,}\beta^2 \dfrac{dz}{dt} + \mathfrak{E}nr_{,}^2\beta^2 = h$; or,

$$\frac{dz}{dt} = \frac{h}{h_{,}\beta^2} - \frac{\mathfrak{E}nr_{,}^2}{h_{,}}. \tag{3}$$

The two theorems now found are fundamental. The latter is not noticed by M. Hansen; the former, or one equivalent to it, is employed by him. But there is another fundamental theorem which he has not found; for we have necessarily

$$\frac{d^2\rho}{d\tau^2} - \frac{h^2}{\rho^3} + \frac{\mu}{\rho^2} = 0,$$

as in an undisturbed orbit; because, since d operates on τ only, the elements and h may vary in any manner whatever, and this equation will still hold.

Now $\frac{d\rho}{d\tau} = \rho_{,} \frac{d\xi}{d\tau} + \xi \frac{d\rho_{,}}{d\tau} = \rho_{,} \frac{d\xi}{d\tau} + \xi \frac{d\rho_{,}}{d\zeta}\frac{d\zeta}{d\tau} = \rho_{,} \frac{d\xi}{d\tau} + \frac{h}{h_{,}\xi}\frac{d\rho_{,}}{d\zeta}$, by (2); and $\frac{d^2\rho}{d\tau^2} = \rho_{,} \frac{d^2\xi}{d\tau^2} + \frac{d\rho_{,}}{d\zeta}\frac{d\zeta}{d\tau}\frac{d\xi}{d\tau} - \frac{h}{h_{,}\xi^2}\frac{d\rho_{,}}{d\zeta}\frac{d\xi}{d\tau} + \frac{h}{h_{,}\xi}\frac{d^2\rho_{,}}{d\zeta^2}\frac{d\zeta}{d\tau} = \rho_{,} \frac{d^2\xi}{d\tau^2} + \frac{h}{h_{,}\xi}\frac{d^2\rho_{,}}{d\zeta^2}\frac{d\zeta}{d\tau}$, by (2); $= \rho_{,} \frac{d^2\xi}{d\tau^2} + \frac{h^2}{h_{,}^2\xi^3}\frac{d^2\rho_{,}}{d\zeta^2}$, by the same.

But $\frac{d^2\rho_{,}}{d\zeta^2} - \frac{h_{,}^2}{\rho_{,}^3} + \frac{\mu}{\rho_{,}^2} = 0$. Therefore, substituting for $\frac{d^2\rho}{d\tau^2}$ and $\frac{d^2\rho_{,}}{d\zeta^2}$ their values, $\frac{h^2}{\rho^3} - \frac{\mu}{\rho^2}$ and $\frac{h_{,}^2}{\rho_{,}^3} - \frac{\mu}{\rho_{,}^2}$, we have

$$\frac{h^2}{\rho^3} - \frac{\mu}{\rho^2} = \rho_{,} \frac{d^2\xi}{d\tau^2} + \frac{h^2}{h_{,}^2\xi^3}\left(\frac{h_{,}^2}{\rho_{,}^3} - \frac{\mu}{\rho_{,}^2}\right);$$

or, since $\epsilon^3\rho_{,}^3 = \rho^3$,

$$-\frac{\mu}{\rho^2} = \rho_{,} \frac{d^2\xi}{d\tau^2} - \frac{h^2}{h_{,}^2\xi^3}\frac{\mu}{\rho_{,}^2};$$

which easily reduces to

$$\frac{d^2\xi}{d\tau^2} = \frac{\mu}{\rho^3}\left(\frac{h^2}{h_{,}^2} - \xi\right). \tag{4}$$

There are still two other theorems to be investigated, which are given by M. Hansen; but found by him in a manner very different from that which I shall employ.

$$\frac{1}{r} = \frac{\mu}{h^2} + \frac{\mu e}{h^2}\cos(v - \pi) \text{ gives } \frac{1}{r} - \frac{\mu}{h^2} = \frac{\mu e}{h^2}\cos(v - \pi) =$$

$$\frac{\mu e}{h^2}(\cos \pi \cos v + \sin \pi \sin v).$$

And differentiating relative to t,

$$\frac{1}{r^2}\frac{dr}{dt}=\frac{\mu e}{h^2}\sin(v-\pi)\frac{dv}{dt}\text{; or, since } r^2\frac{dv}{dt}=h,$$

$$\frac{1}{h}\frac{dr}{dt}=\frac{\mu e}{h^2}\sin(v-\pi)=\frac{\mu e}{h^2}(\cos\pi\sin v-\sin\pi\cos v).$$

Whence we easily deduce

$$\left(\frac{1}{r}-\frac{\mu}{h^2}\right)\cos v+\frac{1}{h}\frac{dr}{dt}\sin v=\frac{\mu e}{h^2}\cos\pi.$$

$$\left(\frac{1}{r}-\frac{\mu}{h^2}\right)\sin v-\frac{1}{h}\frac{dr}{dt}\cos v=\frac{\mu e}{h^2}\sin\pi.$$

In like manner,

$$\left(\frac{1}{\rho}-\frac{\mu}{h^2}\right)\cos\lambda+\frac{1}{h}\frac{d\rho}{d\tau}\sin\lambda=\frac{\mu e}{h^2}\cos\pi.$$

$$\left(\frac{1}{\rho}-\frac{\mu}{h^2}\right)\sin\lambda-\frac{1}{h}\frac{d\rho}{d\tau}\cos\lambda=\frac{\mu e}{h^2}\sin\pi.$$

Equating the two sets of values of $\frac{\mu e}{h^2}\cos\pi$ and $\frac{\mu e}{h^2}\sin\pi$,

$$\left.\begin{aligned}\left(\frac{1}{r}-\frac{\mu}{h^2}\right)\cos v+\frac{1}{h}\frac{dr}{dt}\sin v&=\left(\frac{1}{\rho}-\frac{\mu}{h^2}\right)\cos\lambda+\frac{1}{h}\frac{d\rho}{d\tau}\sin\lambda,\\ \left(\frac{1}{r}-\frac{\mu}{h^2}\right)\sin v-\frac{1}{h}\frac{dr}{dt}\cos v&=\left(\frac{1}{\rho}-\frac{\mu}{h^2}\right)\sin\lambda-\frac{1}{h}\frac{d\rho}{d\tau}\cos\lambda.\end{aligned}\right\}\quad(5)$$

Multiplying these by $\cos v$, $\sin v$, $\cos\lambda$, and $\sin\lambda$, and adding and subtracting results,

$$\left.\begin{aligned}\frac{1}{r}-\frac{\mu}{h^2}&=\left(\frac{1}{\rho}-\frac{\mu}{h^2}\right)\cos(\lambda-v)+\frac{1}{h}\frac{d\rho}{d\tau}\sin(\lambda-v),\\ \frac{1}{h}\frac{dr}{dt}&=-\left(\frac{1}{\rho}-\frac{\mu}{h^2}\right)\sin(\lambda-v)+\frac{1}{h}\frac{d\rho}{d\tau}\cos(\lambda-v),\\ \frac{1}{\rho}-\frac{\mu}{h^2}&=\left(\frac{1}{r}-\frac{\mu}{h^2}\right)\cos(\lambda-v)-\frac{1}{h}\frac{dr}{dt}\sin(\lambda-v),\\ \frac{1}{h}\frac{d\rho}{d\tau}&=\left(\frac{1}{r}-\frac{\mu}{h^2}\right)\sin(\lambda-v)+\frac{1}{h}\frac{dr}{dt}\cos(\lambda-v).\end{aligned}\right\}\quad(6)$$

The equations (5) may be put under the following form :

$$\frac{d}{dt}(r\sin v)-\frac{d}{d\tau}(\rho\sin\lambda)=\frac{\mu}{h}(\cos v-\cos\lambda).$$

$$\frac{d}{dt}(r\cos v)-\frac{d}{d\tau}(\rho\cos\lambda)=\frac{\mu}{h}(\sin\lambda-\sin v).$$

The first and second of (6) may be written thus:

$$\frac{1}{r}=\frac{\mu}{h^2}\{1-\cos(\lambda-v)\}+\frac{1}{h}\frac{d}{d\tau}\{\rho\sin(\lambda-v)\};$$

$$\frac{dr}{dt}=\frac{\mu}{h}\sin(\lambda-v)+\frac{d}{d\tau}\{\rho\cos(\lambda-v)\}.$$

These, and the third and fourth of (6), are very elegant equations, and are deserving of notice on the ground that, possibly, some use may be made of them. I shall only employ the third of (6). Differentiating it relative to t, there results,

$$\frac{1}{\rho^2}\frac{d\rho}{dt}=\left\{\frac{2\mu}{h^3}-\frac{2\mu}{h^3}\cos(\lambda-v)\right\}\frac{dh}{dt}+\left\{\frac{1}{h}\frac{d\lambda}{dt}\cos(\lambda-v)-\frac{1}{h^2}\frac{dh}{dt}\sin(\lambda-v)\right\}\frac{dr}{dt}$$
$$+\left(\frac{1}{r}-\frac{\mu}{h^2}\right)\frac{d\lambda}{dt}\sin(\lambda-v)+\left(\frac{1}{h}\frac{d^2r}{dt^2}-\frac{h}{r^3}+\frac{\mu}{hr^2}\right)\sin(\lambda-v).$$

Eliminating $\frac{dr}{dt}$ by the third and fourth of (6), and substituting for $\frac{d^2r}{dt^2}-\frac{h^2}{r^3}+\frac{\mu}{r^2}$ its value from (1), we find

$$\frac{1}{\rho^2}\frac{d\rho}{dt}=\frac{1}{h}\frac{d\rho}{d\tau}\frac{d\lambda}{dt}+\left\{\frac{1}{\rho}+\frac{\mu}{h^2}-\left(\frac{1}{r}+\frac{\mu}{h^2}\right)\cos(\lambda-v)\right\}\frac{dh}{hdt}-\frac{1}{h}\sin(\lambda-v)\frac{dR}{dr}.$$

Multiply this by h, put for $\frac{dh}{dt}$ its value from (1), and for $\frac{h}{\rho^2}$ in the first member its value $\frac{d\lambda}{d\tau}$; then making

$$T=\left\{\left(\frac{1}{r}+\frac{\mu}{h^2}\right)\cos(\lambda-v)-\left(\frac{1}{\rho}+\frac{\mu}{h^2}\right)\right\}\frac{dR}{dv}-\sin(\lambda-v)\frac{dR}{dr},$$

we have

$$\frac{d\rho}{dt}\frac{d\lambda}{d\tau}-\frac{d\rho}{d\tau}\frac{d\lambda}{dt}=T, \tag{7}$$

which is a very remarkable equation.

From this we deduce successively,

$$\frac{d\rho}{dt}\frac{d\lambda_{,}}{d\tau}-\frac{d\rho}{d\tau}\frac{d\lambda}{dt}=T+\epsilon n\frac{d\rho}{d\tau},$$

$$\frac{h_{,}}{\rho_{,}^{2}}\left(\frac{d\rho}{dt}\frac{d\zeta}{d\tau}-\frac{d\rho}{d\tau}\frac{d\zeta}{dt}\right)=T+\epsilon n\frac{d\rho}{d\tau};$$

but

$$\frac{d\rho}{dt}=\rho_{,}\frac{d\xi}{dt}+\xi\frac{d\rho_{,}}{d\zeta}\frac{d\zeta}{dt},\ \frac{d\rho}{d\tau}=\rho_{,}\frac{d\xi}{d\tau}+\xi\frac{d\rho_{,}}{d\zeta}\frac{d\zeta}{d\tau}.$$

Therefore, by substitution in the above,

$$\frac{h_{,}}{\rho_{,}}\left(\frac{d\xi}{dt}\frac{d\zeta}{d\tau}-\frac{d\xi}{d\tau}\frac{d\zeta}{dt}\right)=T+\epsilon n\frac{d\rho}{d\tau}. \tag{8}$$

This again is a very remarkable equation, and has a remarkable correspondence with (7). Dividing it by $\frac{h_{,}}{\rho_{,}}\frac{d\zeta}{d\tau}=\frac{h}{\rho_{,}\xi^{2}}=\frac{h}{\rho\xi}$, we have

$$\frac{d\xi}{dt}-\frac{d\xi}{d\tau}\left(\frac{\frac{d\zeta}{dt}}{\frac{d\zeta}{d\tau}}\right)=\frac{1}{h}\rho\xi T+\frac{\epsilon n\xi}{h}\rho\frac{d\rho}{d\tau};$$

or,

$$\frac{d\xi}{dt}=\frac{1}{h}\rho\xi T+\frac{\epsilon n\xi}{2h}\frac{d(\rho^{2})}{d\tau}+\frac{d\xi}{d\tau}\left(\frac{\frac{d\zeta}{dt}}{\frac{d\zeta}{d\tau}}\right). \tag{9}$$

Equation (7) multiplied by 2ρ gives

$$2\rho\frac{d\rho}{dt}\frac{d\lambda}{d\tau}-2\rho\frac{d\rho}{d\tau}\frac{d\lambda}{dt}=2\rho T.$$

But differentiating $\rho^{2}\frac{d\lambda}{d\tau}=h$, relative to t, we have

$$\rho^{2}\frac{d^{2}\lambda}{d\tau dt}+2\rho\frac{d\rho}{dt}\frac{d\lambda}{d\tau}=\frac{dh}{dt}.$$

The above, subtracted from this, gives

$$\rho^2 \frac{d^2\lambda}{d\tau\, dt} + 2\rho \frac{d\rho}{d\tau} \frac{d\lambda}{dt} = \frac{dh}{dt} - 2\rho T,$$

or,

$$\frac{d}{d\tau}\left(\rho^2 \frac{d\lambda}{dt}\right) = \frac{dh}{dt} - 2\rho T. \tag{10}$$

This is singular, and very worthy of notice. This theory, as we have treated it, gives rise to many very interesting formulæ.

$$\text{But } \rho^2 \frac{d\lambda}{dt} = \rho^2 \frac{d\lambda_{\prime}}{dt} + \epsilon n \rho^2 = \xi^2 \rho^2 \frac{d\lambda_{\prime}}{d\zeta} \frac{d\zeta}{dt} + \epsilon n \rho^2 = h_{\prime} \xi^2 \frac{d\zeta}{dt} + \epsilon n \rho^2.$$

Therefore (10) becomes

$$\frac{d}{d\tau}\left(h_{\prime}\xi^2 \frac{d\zeta}{dt}\right) + \epsilon n \frac{d(\rho^2)}{d\tau} = \frac{dh}{dt} - 2\rho T.$$

Or, since $h_{\prime}\xi^2 = \dfrac{h}{\frac{d\zeta}{d\tau}}$,

$$\frac{d}{d\tau}\left(\frac{\frac{d\zeta}{dt}}{\frac{d\zeta}{d\tau}}\right) = \frac{1}{h}\frac{dh}{dt} - \frac{1}{h} 2\rho T - \frac{\epsilon n}{h} \frac{d(\rho^2)}{d\tau}. \tag{11}$$

2. In the preceding section we have found all the fundamental formulæ, and indeed more than are absolutely required. We now proceed to discuss some points preparatory to integration.

We might immediately integrate (11) relative to t; but the integral of the second member would contain a very great number of terms with τ in their coefficients, or of the form $\tau f(t)$. We must, therefore, proceed otherwise.

Make $z = t + \omega$, $\zeta = \tau + \phi$. Then $\dfrac{d\zeta}{dt} = \dfrac{d\phi}{dt}$, $\dfrac{d\zeta}{d\tau} = 1 + \dfrac{d\phi}{d\tau}$. Substituting these values in (9) and (11), neglecting terms involving the fourth power of the disturbing force, and putting

$$\frac{1}{h}\rho\xi T + \frac{\epsilon n \xi}{2h}\frac{d(\rho^2)}{d\tau} = P, \quad \frac{1}{h}\frac{dh}{dt} - \frac{1}{h}2\rho T - \frac{\epsilon n}{h}\frac{d(\rho^2)}{d\tau} = S,$$

they become

$$\left.\begin{aligned}\frac{d\xi}{dt} &= P + \frac{d\xi}{d\tau}\frac{d\phi}{dt} - \frac{d\xi}{d\tau}\frac{d\phi}{dt}\frac{d\phi}{d\tau},\\ \frac{d^2\phi}{d\tau dt} &= S + \frac{d}{d\tau}\left(\frac{d\phi}{dt}\frac{d\phi}{d\tau} - \frac{d\phi}{dt}\frac{d\phi^2}{d\tau^2}\right).\end{aligned}\right\} \quad (1)$$

Let the integrals of these relative to t be

$$\xi = f(\tau) + \psi(\tau, t),\ \frac{d\phi}{d\tau} = f_{\prime}(\tau) + \phi(\tau, t):$$

$f(\tau)$ and $f_{\prime}(\tau)$ being the arbitraries of the integration. If we make $m' = 0$; then $\epsilon = 0$, and $\lambda = \lambda_{\prime}$. But in this case λ is a function of τ without t. Consequently, $\lambda_{\prime}$, ζ, and ϕ, are functions of τ without t; and, therefore, $\frac{d\phi}{dt} = 0$. Hence, all the terms in the second members of (1) vanish, and we have

$$\xi_0 = f(\tau),\ \frac{d\phi_0}{d\tau} = f_{\prime}(\tau);$$

ξ_0 and $\frac{d\phi_0}{d\tau}$ being the values of ξ and $\frac{d\phi}{d\tau}$ when $m' = 0$.

Now if we make the disturbing force to vanish in (2) and (4) of the first section, they become

$$\left.\begin{aligned}\frac{d\zeta_0}{d\tau} &= \frac{h_0}{h_{\prime}\xi_0^2},\\ \frac{d^2\xi_0}{d\tau^2} &= \frac{\mu}{\rho_0^3}\left(\frac{h_0^2}{h_{\prime}^2} - \xi_0\right),\\ 1 + \frac{d\phi_0}{d\tau} &= \frac{h_0}{h_{\prime}\xi_0^2};\end{aligned}\right\} \quad (2)$$

the last of these being derived from the first by putting for ζ_0 its value. These equations, being integrated, give us ζ_0 and ξ_0, or ξ_0 and ϕ_0.

To integrate the second of these, make $\xi_0 = \frac{h_0^2}{h_{\prime}^2}(1+\gamma)$, when it becomes

$$\frac{d^2\gamma}{d\tau^2} + \frac{\mu}{\rho_0^3}\gamma = 0,\ \text{or}\ \frac{\rho_0^3}{a_0^3}\frac{d^2\gamma}{d\tau^2} + n_0^2\gamma = 0.$$

To abridge, make $\frac{\rho_0^3}{a_0^3} = 1 + eE_1 + e^2E_2 + \&c.$, where E_1, E_2, &c., are known functions of the cosine of $n_0\tau + \epsilon_0 - \pi_0$ and its multiples ; and assume

$$\gamma = B + eB_1 + e^2B_2 + \dots.$$

By substitution, and equalling separately to nothing the terms multiplied by the different powers of e, we have

$$\frac{d^2B}{d\tau^2} + n_0^2B = 0, \quad \frac{d^2B_1}{d\tau^2} + n_0^2B_1 + E_1\frac{d^2B}{d\tau^2} = 0, \text{ \&c.}$$

The first of these gives $B = c \cos (n_0\tau + k)$. This value substituted in the second, it will give P_1 ; and so on. But in reducing the periodicals to one argument, we should have a series of sines and another of cosines, unless we make $k = \epsilon_0 - \pi_0$. In this case we must have $\epsilon_0 - \pi_0 = \epsilon_{,} - \pi_{,}$, and making $n_0 = n_{,} (1 + b)$, we must develope thus:

$$\cos (n_0\tau + \epsilon_0 - \pi_0) = \cos (n_{,}\tau + \epsilon_0 - \pi_0) - bn_{,}\tau \sin (n_{,}\tau + \epsilon_0 - \pi_0) - \text{\&c.}$$

The use of this is, by suitably determining b, to take away an improper term. In some cases, perhaps, both sines and cosines may be necessary.

But perhaps this will be more conveniently done thus:

$$\frac{1}{\rho_0} = \frac{\mu}{h_0^2}\{1 + e_0 \cos (\lambda_0 - \pi_0)\}, \quad \frac{1}{\rho_{,,0}} = \frac{\mu}{h_{,}^2}\{1 + e_{,} \cos (\lambda_{,,0} - \pi_{,})\}.$$

But $\lambda_{,,0} = \lambda_0$. Therefore we must have $\pi_0 = \pi_{,}$, or we should have both sines and cosines. And then we shall have

$$\left(\frac{h_0^2}{\mu\rho_0} - 1\right)\frac{1}{e_0} = \cos (\lambda_0 - \pi_0) = \cos (\lambda_{,,0} - \pi_{,}) = \left(\frac{h_{,}^2}{\mu\rho_{,,0}} - 1\right)\frac{1}{e_{,}};$$

or, after a little reduction,

$$\frac{h_0^2}{\mu\rho_0} = 1 - \frac{e_0}{e_{,}} - \frac{h_{,}^2}{\mu\rho_{,,0}}\,\frac{e_0}{e_{,}}.$$

But $\rho_0 = \rho_{,,0}\xi_0$. Substituting this value, we easily find

$$\frac{1}{\xi_0} = \left(1 - \frac{e_0}{e_{,}}\right)\frac{\mu}{h_0^2}\,\rho_{,,0} + \frac{h_{,}^2}{h_0^2}\,\frac{e_0}{e_{,}};$$

or,

$$\frac{1}{\xi_0} = \frac{e_{,} - e_0}{1 - e_0^2}\,\frac{\rho_{,,0}}{a_0e_{,}} + \frac{a_{,}e_0\,(1 - e_{,}^2)}{a_0e_{,}\,(1 - e_0^2)}.$$

Make $e_0 = e_{,}(1-\alpha)$, α being a small quantity of the order of the disturbing force; and we may write

$$\frac{1}{\xi_0} = \frac{a_{,}}{a_0} A + \alpha B \frac{\rho_{,.0}}{a_0}.$$

To which we may add from the third of (2);

$$\frac{d\phi_0}{d\tau} = \frac{a_0^{\frac{1}{2}}}{a_{,}^{\frac{1}{2}}} \frac{C}{\xi_0^2} - 1.$$

A, B, and C being functions of $e_{,}^2$ and α easily found. But $\rho_{,.0}$, a function of λ_0, and, therefore, a function of $n_0\tau$, must be converted into a function of $n_{,}\tau$ by Taylor's theorem. The results obtained are much more simple than those obtained by M. Hansen in the same case, and I think also more convenient.

If we wish to make $\pi_0 = \pi_{,} + \eta$, we have

$$\cos(\lambda_0 - \pi_0) = \cos\eta \cos(\lambda_0 - \pi_0) + \sin\eta \sin(\lambda_0 - \pi_0),$$

$$\frac{h_0}{\mu e_0} \frac{d\rho_0}{d\tau} = \sin(\lambda_0 - \pi_0) = \cos\eta \sin(\lambda_0 - \pi_{,}) - \sin\eta \cos(\lambda_0 - \pi_{,}),$$

$$\cos(\lambda_{,.0} - \pi_{,}) = \cos(\lambda_0 - \pi_{,}) = \left(\frac{h_{,}^2}{\mu\rho_{,.0}} - 1\right)\frac{1}{e_{,}}.$$

Between these we may eliminate $\sin(\lambda_0 - \pi_{,})$ and $\cos(\lambda_0 - \pi_{,})$ and obtain a result involving $\frac{d\rho_0}{d\tau}$; but I shall not pursue the subject further.

We may satisfy (2), and all the requisite conditions, by simply making $e_0 = e_{,}$, $\pi_0 = \pi_{,}$, $\xi_0 = \frac{a_0}{a_{,}}$, and $n_{,}\zeta_0 = n_0\tau$; but this would not leave us any arbitrary constant except a_0, and we might have an unsuitable term which we could not get rid of.

We must now proceed to another class of formulæ, some of which will be wanted. But I shall not confine myself to these.

Let $\overline{\left(\frac{d\xi}{d\tau}\right)}$, $\overline{\left(\frac{d\xi}{dt}\right)}$ denote that τ is to be changed into t in these quantities, after the operation of differentiation is performed. And thus this change will be denoted in other cases. Thus from (2) of the first section we have

$$\left.\begin{aligned}\overline{\left(\frac{d\zeta}{d\tau}\right)} &= \frac{h}{h_{,}\beta^2},\\ \overline{\left(\frac{d\phi}{d\tau}\right)} &= \frac{h}{h_{,}\beta^2} - 1.\end{aligned}\right\} \quad (3)$$

Eliminating $\frac{h}{h_{,}\beta^2}$ between the first of these and (3) of the first section, there results

$$\overline{\left(\frac{d\zeta}{d\tau}\right)} = \frac{dz}{dt} + \frac{\epsilon n r_{,}^2}{h_{,}};$$

or,

$$\overline{\left(\frac{d\phi}{d\tau}\right)} = \frac{dw}{dt} + \frac{\epsilon n r_{,}^2}{h_{,}}. \quad (4)$$

But it is obvious that $\overline{\left(\frac{d\phi}{d\tau}\right)} + \overline{\left(\frac{d\phi}{dt}\right)} = \frac{dw}{dt}$. Therefore,

$$\overline{\left(\frac{d\phi}{dt}\right)} = -\frac{\epsilon n r_{,}^2}{h_{,}}. \quad (5)$$

Also, $\frac{d\rho}{dt} = \rho\frac{d\xi}{dt} + \xi\frac{d\rho}{d\zeta}\frac{d\zeta}{dt}$. Whence

$$\overline{\left(\frac{d\rho}{dt}\right)} = r_{,}\overline{\left(\frac{d\xi}{dt}\right)} + \beta\frac{dr_{,}}{dz}\overline{\left(\frac{d\zeta}{dt}\right)}.$$

But $\overline{\left(\frac{d\rho}{dt}\right)} = \frac{\delta r}{dt} = 0$, from the manner in which the variable elements are found; and $\frac{d\zeta}{dt} = \frac{d\phi}{dt}$. Therefore,

$$r_{,}\overline{\left(\frac{d\xi}{dt}\right)} + \beta\frac{dr_{,}}{dz}\overline{\left(\frac{d\phi}{dt}\right)} = 0,$$

which by (5) gives

$$\overline{\left(\frac{d\xi}{dt}\right)} = \frac{\epsilon n r}{h_{,}}\frac{dr_{,}}{dz}; \quad (6)$$

and since also we necessarily have

$$\overline{\left(\frac{d\xi}{d\tau}\right)} + \overline{\left(\frac{d\xi}{dt}\right)} = \frac{d\beta}{dt};$$

therefore,

$$\overline{\left(\frac{d\xi}{d\tau}\right)} = \frac{d\beta}{dt} - \frac{\epsilon nr}{h_{\prime}} \frac{dr_{\prime}}{dz}. \tag{7}$$

The preceding are all the formulæ of this kind that are really wanted; but I shall give some others, on the ground that they may possibly be some time found useful. If we could change τ into t in higher differentials than the first, we might, perhaps, by differentiating the fundamental theorems, or any of the others, relative to τ or t, and then changing τ into t, find some new theorems which might be made available for simplification, or some way useful.

From (2), section (1), or $\frac{d\phi}{d\tau} = \frac{h}{h_{\prime}\xi^2} - 1$, putting $\tau + \phi$ for ζ, we have $\frac{d^2\phi}{d\tau^2} = -\frac{2h}{h_{\prime}\xi^3} \frac{d\xi}{d\tau}$. Therefore,

$$\overline{\left(\frac{d^2\phi}{d\tau^2}\right)} = -\frac{2h}{h_{\prime}\beta^3} \overline{\left(\frac{d\xi}{d\tau}\right)};$$

or by (7),

$$\overline{\left(\frac{d^2\phi}{d\tau^2}\right)} = -\frac{2h}{h_{\prime}\beta^3} \frac{d\beta}{dt} + \frac{\epsilon nh}{h_{\prime}^2\beta^2} \frac{d(r_{\prime}^2)}{dz}. \tag{8}$$

Equation (4), section (1), gives immediately

$$\overline{\left(\frac{d^2\xi}{d\tau^2}\right)} = \frac{\mu}{r^3}\left(\frac{h^2}{h_{\prime}^2} - \beta\right). \tag{9}$$

And from (2), referred to above, after putting $\tau + \phi$ for ζ, and differentiating relative to t, we have

$$\frac{d^2\phi}{d\tau dt} = \frac{1}{h_{\prime}\xi^2} \frac{dh}{dt} - \frac{2h}{h_{\prime}\xi^3} \frac{d\xi}{dt},$$

$$\overline{\left(\frac{d^2\phi}{d\tau dt}\right)} = \frac{1}{h_{\prime}\beta^2} \frac{dh}{dt} - \frac{2h}{h_{\prime}\beta^3} \overline{\left(\frac{d\xi}{dt}\right)};$$

or, by substitution from (6),

$$\overline{\left(\frac{d^2\phi}{d\tau dt}\right)} = \frac{1}{h_{\prime}\beta^2} \frac{dh}{dt} - \frac{\epsilon nh}{h_{\prime}^2\beta^2} \frac{d(r_{\prime}^2)}{dz}. \tag{10}$$

I might extend this list, but not without some difficulty; and as I am not sure of its utility, I shall here leave it.

We now turn to the equations (1) of this section. The first contains both ξ and ϕ; the second only ϕ. This last, therefore, is preferable; and as we do not want them both, we shall take this, which reduces to

$$\frac{d^2\phi}{d\tau dt}=S+\frac{d^2\phi}{d\tau dt}\left(\frac{d\phi}{d\tau}-\frac{d\phi^2}{d\tau^2}\right)+\frac{d\phi}{dt}\frac{d^2\phi}{d\tau^2}\left(1-2\frac{d\phi}{d\tau}\right).$$

For the first and second power of the disturbing force only,

$$\frac{d^2\phi}{d\tau dt}=S+\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau dt}+\frac{d\phi}{dt}\frac{d^2\phi}{d\tau^2}=S+S\frac{d\phi}{d\tau}+\frac{d\phi}{dt}\frac{d^2\phi}{d\tau^2}.$$

Putting this value in the second member of the above, we have

$$\left.\begin{aligned}&\frac{d^2\phi}{d\tau dt}=S+S\frac{d\phi}{d\tau}+\frac{d\phi}{dt}\frac{d^2\phi}{d\tau^2}-\frac{d\phi}{dt}\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau^2}.\\ &S=\left\{1+\frac{2\mu\rho}{h^2}-\left(\frac{2}{r}+\frac{2\mu}{h^2}\right)\rho\cos(\lambda-v)\right\}\frac{1}{h}\frac{dR}{dv}+\sin(\lambda-v)\frac{2\rho}{h}\frac{dR}{dr}-\frac{\epsilon n}{h}\frac{d(\rho^2)}{d\tau}\end{aligned}\right\}(11)$$

3. The values of ϕ and ξ would be very troublesome to find; for they would contain a great number of terms having $\tau-t$ in their coefficients, and which in w and β would vanish; and many which, when τ is changed into t, would unite with others. If, therefore, we could get rid of these quantities, we should greatly diminish the labour of integration. This, happily, we are able to effect. To accomplish it we will develope S relative to these quantities.

Equation (2) of section (1) gives $\xi=\frac{h^{\frac12}}{h_{,}^{\frac12}}\left(1+\frac{d\phi}{d\tau}\right)^{-\frac12}=\frac{h^{\frac12}}{h_{,}^{\frac12}}\left(1-\frac12\frac{d\phi}{d\tau}+\frac38\frac{d\phi^2}{d\tau^2}\right)$.
Also, $\frac{d(\rho^2)}{d\tau}=\frac{d(\rho_{,}^2\xi^2)}{d\tau}=\xi^2\frac{d(\rho_{,}^2)}{d\tau}+\rho_{,}^2\frac{d(\xi^2)}{d\tau}=\xi^2\frac{d(\rho_{,}^2)}{d\zeta}\frac{d\zeta}{d\tau}+\rho_{,}^2\frac{d(\xi^2)}{d\tau}=\frac{h}{h_{,}}\frac{d(\rho_{,}^2)}{d\zeta}+\rho_{,}^2\frac{d(\xi^2)}{d\tau}$. These values are to be substituted in that of S, after we have changed ρ into $\rho_{,}\xi$, v into $v_{,}+\epsilon nt$, and λ into $\lambda_{,}+\epsilon nt$. Make, therefore,

$$(S)=\left\{\frac{h_{,}^{\frac12}}{h^{\frac12}}+\frac{2\mu\rho_{,}}{h^2}-\left(\frac{2}{r}+\frac{2\mu}{h^2}\right)\rho_{,}\cos(\lambda_{,}-v_{,})\right\}\frac{1}{h_{,}^{\frac12}h^{\frac12}}\frac{dR}{dv}+\sin(\lambda_{,}-v_{,})\frac{2\rho_{,}}{h_{,}^{\frac12}h^{\frac12}}\frac{dR}{dr}-\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^2)}{d\tau},\text{ and}$$

$$(T)=\left\{\frac{2\mu\rho_{,}}{h^2}-\left(\frac{2}{r}+\frac{2\mu}{h^2}\right)\rho_{,}\cos(\lambda_{,}-v_{,})\right\}\frac{1}{h_{,}^{\frac{1}{2}}h^{\frac{1}{2}}}\frac{dR}{dv}+\sin(\lambda_{,}-v_{,})\frac{2\rho_{,}}{h_{,}^{\frac{1}{2}}h^{\frac{1}{2}}}\frac{dR}{dr}=$$

$$(S)-\frac{1}{h}\frac{dR}{dv}+\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^2)}{d\zeta}.$$

Putting the above values of ξ and $\frac{d(\rho^2)}{d\tau}$ in that of S, after developing relative to ξ; we have, making $\rho_{,}^0$ the value of $\rho_{,}$ (a function of $\tau+\phi$) when ϕ is made nothing.

$$S=(S)-(T)\left(\tfrac{1}{2}\frac{d\phi}{d\tau}-\tfrac{3}{8}\frac{d\phi^2}{d\tau^2}\right)-\frac{\epsilon n}{h}\rho_{,}^2\frac{d(\xi^2)}{d\tau}=$$

$$(S)-(T)\left(\tfrac{1}{2}\frac{d\phi}{d\tau}-\tfrac{3}{8}\frac{d\phi^2}{d\tau^2}\right)+\frac{\epsilon n\rho_{,}^2}{h_{,}}\frac{d^2\phi}{d\tau^2}\left(1-2\frac{d\phi}{d\tau}\right)=$$

$$(S)-(T)\left(\tfrac{1}{2}\frac{d\phi}{d\tau}-\tfrac{3}{8}\frac{d\phi^2}{d\tau^2}\right)+\frac{\epsilon n}{h_{,}}\rho_{,}^2\frac{d^2\phi}{d\tau^2}\left(1-2\frac{d\phi}{d\tau}\right)+\frac{\epsilon n\phi}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d^2\phi}{d\tau^2}. \quad (1)$$

We have now got rid of ξ. Let (S^0) and (T^0) be the values of (S) and (T) when ϕ is made nothing, or $\zeta=\tau$; then, by Taylor's theorem,

$$(S)=(S^0)+\phi\frac{d(S^0)}{d\tau}+\frac{\phi^2}{2}\frac{d^2(S^0)}{d\tau^2}, \quad (2)$$

$$(T)=(T^0)+\phi\frac{d(T^0)}{d\tau},$$

neglecting higher powers of the disturbing force; since we exclude those of the fourth order. Now ϕ enters nowhere but where it appears.

And putting $\frac{dh}{dt}$ for $-\frac{dR}{dv}$ for convenience,

$$(T^0)=(S^0)+\frac{1}{h}\frac{dh}{dt}+\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau},$$

$$\frac{d(T^0)}{d\tau}=\frac{d(S^0)}{d\tau}+\frac{\epsilon n}{h_{,}}\frac{d^2(\rho_{,}^{02})}{d\tau^2}$$

Therefore,

$$(T)=(S^0)+\frac{1}{h}\frac{dh}{dt}+\phi\frac{d(S^0)}{d\tau}+\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}+\frac{\epsilon n\phi}{h_{,}}\frac{d^2(\rho_{,}^{02})}{d\tau^2}. \quad (3)$$

Put in (1) the value of (S) given by (2), and that of (T) given by (3); and we have

$$S = (S^0)\left(1 - \tfrac{1}{2}\frac{d\phi}{d\tau} + \tfrac{3}{8}\frac{d\phi^2}{d\tau^2}\right) + \phi\frac{d(S^0)}{d\tau}\left(1 - \tfrac{1}{2}\frac{d\phi}{d\tau}\right) - \frac{1}{2h}\frac{dh}{dt}\frac{d\phi}{d\tau}\left(1 - \tfrac{3}{4}\frac{d\phi}{d\tau}\right) -$$
$$\frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d\phi}{d\tau}\left(1 - \tfrac{3}{4}\frac{d\phi}{d\tau}\right) + \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\frac{d^2\phi}{d\tau^2}\left(1 - 2\frac{d\phi}{d\tau}\right) + \frac{\epsilon n\phi}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d^2\phi}{d\tau^2} +$$
$$\frac{\phi^2}{2}\frac{d^2(S^0)}{d\tau^2} - \frac{\epsilon n\phi}{2h_{,}}\frac{d^2(\rho_{,}^{02})}{d\tau^2}\frac{d\phi}{d\tau};$$

and

$$S\frac{d\phi}{d\tau} = (S^0)\left(\frac{d\phi}{d\tau} - \tfrac{1}{2}\frac{d\phi^2}{d\tau^2}\right) + \phi\frac{d(S^0)}{d\tau}\frac{d\phi}{d\tau} - \frac{1}{2h}\frac{dh}{dt}\frac{d\phi^2}{d\tau^2} -$$
$$\frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d\phi^2}{d\tau^2} + \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau^2}.$$

We must now put these values of S and $S\frac{d\phi}{d\tau}$ in (11) of the last section, and we shall find as the result,

$$\frac{d^2\phi}{d\tau dt} = (S^0)\left(1 + \tfrac{1}{2}\frac{d\phi}{d\tau} - \tfrac{1}{8}\frac{d\phi^2}{d\tau^2}\right) + \phi\frac{d(S^0)}{d\tau}\left(1 + \tfrac{1}{2}\frac{d\phi}{d\tau}\right) - \frac{1}{2h}\frac{dh}{dt}\frac{d\phi}{d\tau}\left(1 + \tfrac{1}{4}\frac{d\phi}{d\tau}\right) +$$
$$\frac{d\phi}{dt}\frac{d^2\phi}{d\tau^2}\left(1 - \frac{d\phi}{d\tau}\right) - \frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d\phi}{d\tau}\left(1 + \tfrac{1}{4}\frac{d\phi}{d\tau}\right) + \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\frac{d^2\phi}{d\tau^2}\left(1 - \frac{d\phi}{d\tau}\right) + \frac{\phi^2}{2}\frac{d^2(S^0)}{d\tau^2} -$$
$$\frac{\epsilon n}{h_{,}}\frac{d^2(\rho_{,}^{02})}{d\tau^2}\frac{\phi}{2}\frac{d\phi}{d\tau} + \frac{\epsilon n\phi}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d^2\phi}{d\tau^2}. \qquad (4)$$

We can do nothing with the last in its present form, on account of the terms which contain ϕ and $\frac{d\phi}{dt}$. But happily these may be driven out by a little transformation. Neglecting terms containing the third power of the disturbing force, (4) gives

$$(S^0) = \frac{d^2\phi}{d\tau dt} - \tfrac{1}{2}(S^0)\frac{d\phi}{d\tau} - \phi\frac{d(S^0)}{d\tau} - \frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau^2} + \frac{1}{2h}\frac{dh}{dt}\frac{d\phi}{dt} - \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\frac{d^2\phi}{d\tau^2}$$
$$+ \frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d\phi}{d\tau} =$$
$$\frac{d^2\phi}{d\tau dt} - \tfrac{1}{2}\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau dt} - \phi\frac{d^3\phi}{d\tau^2 dt} - \frac{d\phi}{dt}\frac{d^2\phi}{d\tau^2} + \frac{1}{2h}\frac{dh}{dt}\frac{d\phi}{d\tau} - \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\frac{d^2\phi}{d\tau^2} + \frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d\phi}{d\tau}.$$

With this value we eliminate (S^0) from (4), in those terms where the disturbing force rises above the first power, and we thus find

$$\frac{d^2\phi}{d\tau dt} = (S^0) + \frac{d}{dt}\left\{\tfrac{1}{4}\frac{d\phi^2}{d\tau^2} + \phi\frac{d^2\phi}{d\tau^2} - \tfrac{1}{8}\frac{d\phi^3}{d\tau^3} - \tfrac{3}{2}\phi\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau^2} - \tfrac{1}{2}\phi^2\frac{d^3\phi}{d\tau^3}\right\} -$$
$$\frac{1}{2h}\frac{dh}{dt}\left\{\frac{d\phi}{d\tau}\left(1 - \tfrac{1}{4}\frac{d\phi}{d\tau}\right) - \phi\frac{d^2\phi}{d\tau^2}\right\} + \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\left\{\frac{d^2\phi}{d\tau^2}\left(1 - \tfrac{3}{2}\frac{d\phi}{d\tau}\right) - \phi\frac{d^3\phi}{d\tau^3}\right\} -$$
$$\frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\left\{\frac{d\phi}{d\tau}\left(1 - \tfrac{1}{4}\frac{d\phi}{d\tau}\right) - \phi\frac{d^2\phi}{d\tau^2}\right\}. \qquad (5)$$

This has been put under the simplest form. It still contains three terms which we cannot manage; but these may be easily eliminated. Neglecting the powers of the disturbing force above the first, and integrating relative to t, adding the correction $\frac{d\phi_0}{d\tau}$, or the value of $\frac{d\phi}{d\tau}$ when the disturbing force is made to vanish, we have

$$\frac{d\phi}{d\tau} = \frac{d\phi_0}{d\tau} + \int (S^0)\, dt = W \text{ suppose.} \qquad (6)$$

Now if we include both the first and second powers of the disturbing force, we have

$$\frac{d\phi}{d\tau} = W + \tfrac{1}{4}\frac{d\phi^2}{d\tau^2} + \phi\frac{d^2\phi}{d\tau^2} - \tfrac{1}{2}\int\frac{d\phi}{d\tau}\frac{dh}{h} + \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\int\frac{d^2\phi}{d\tau^2}\,dt - \frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\int\frac{d\phi}{d\tau}\,dt,$$

which gives us immediately,

$$\frac{d\phi}{d\tau} - \tfrac{1}{4}\frac{d\phi^2}{d\tau^2} - \phi\frac{d^2\phi}{d\tau^2} = W - \tfrac{1}{2}\int\frac{d\phi}{d\tau}\frac{dh}{h} + \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\int\frac{d^2\phi}{d\tau^2}\,dt - \frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\int\frac{d\phi}{d\tau}\,dt.$$

And

$$\frac{d^2\phi}{d\tau^2} - \tfrac{3}{2}\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau^2} - \phi\frac{d^3\phi}{d\tau^3} = \frac{d}{d\tau}\left\{\frac{d\phi}{d\tau} - \tfrac{1}{4}\frac{d\phi^2}{d\tau^2} - \phi\frac{d^2\phi}{d\tau^2}\right\}.$$

Make

$$\frac{d\phi}{d\tau} - \tfrac{1}{4}\frac{d\phi^2}{d\tau^2} - \phi\frac{d^2\phi}{d\tau^2} = X.$$

Now as all the terms in the value of X after W are of the second order, and will be multiplied in (5) by quantities of the first order, we may put W for $\frac{d\phi}{d\tau}$ in the value of this quantity. Then

$$X = W - \tfrac{1}{2}\int W\frac{dh}{h} + \frac{\epsilon n\rho_{,}^{02}}{h_{,}}\int\frac{dW}{d\tau}\,dt - \frac{\epsilon n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\int W dt, \qquad (7)$$

and (5) will become

$$\frac{d^2\phi}{d\tau dt} = (S^0) + \frac{d}{dt}\left(\frac{1}{4}\frac{d\phi^2}{d\tau^2} + \phi\frac{d^2\phi}{d\tau^2} - \frac{1}{8}\frac{d\phi^3}{d\tau^3} - \frac{3}{2}\phi\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau^2} - \frac{1}{2}\phi^2\frac{d^3\phi}{d\tau^3}\right)$$
$$- \frac{1}{2h}X\frac{dh}{dt} - \frac{\text{€}n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}X + \frac{\text{€}n\rho_{,}^{02}}{h_{,}}\frac{dX}{d\tau}. \qquad (8)$$

We must now eliminate $\frac{d\phi^2}{d\tau^2}$, $\frac{d^2\phi}{d\tau^2}$, &c. For this purpose we take

$$\frac{d\phi}{d\tau} = X + \frac{1}{4}\frac{d\phi^2}{d\tau^2} + \phi\frac{d^2\phi}{d\tau^2},$$

which squared, neglecting terms above the third order, gives

$$\frac{d\phi^2}{d\tau^2} = X^2 + \frac{1}{2}X\frac{d\phi^2}{d\tau^2} + 2\phi X\frac{d^2\phi}{d\tau^2} =$$

$$X^2 + \frac{1}{2}W\frac{d\phi^2}{d\tau^2} + 2\phi W\frac{d^2\phi}{d\tau^2} =$$

$$X^2 + \frac{1}{2}W^3 + 2\phi W\frac{dW}{d\tau} =$$

$$-W^2 + 2WX + \frac{1}{2}W^3 + 2\phi W\frac{dW}{d\tau}.$$

$$\frac{d^2\phi}{d\tau^2} = \frac{dX}{d\tau} + \frac{3}{2}\frac{d\phi}{d\tau}\frac{d^2\phi}{d\tau^2} + \phi\frac{d^3\phi}{d\tau^3} =$$

$$\frac{dX}{d\tau} + \frac{3}{2}W\frac{dW}{d\tau} + \phi\frac{d^2W}{d\tau^2}.$$

Substituting these values in (8), it becomes

$$\frac{d^2\phi}{d\tau dt} = (S^0) + \frac{d}{dt}\left(-\frac{1}{4}W^2 + \frac{1}{2}WX + \phi\frac{dX}{d\tau} + \frac{1}{2}\phi W\frac{dW}{d\tau} + \frac{1}{2}\phi^2\frac{d^2W}{d\tau^2}\right) -$$
$$\frac{1}{2h}X\frac{dh}{dt} - \frac{\text{€}n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}X + \frac{\text{€}n\rho_{,}^{02}}{h_{,}}\frac{dX}{d\tau}.$$

Let

$$Y = -\frac{1}{4}W^2 + \frac{1}{2}WX - \frac{1}{2}\int X\frac{dh}{h} - \frac{\text{€}n}{2h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\int Xdt + \frac{\text{€}n\rho_{,}^{02}}{h_{,}}\int\frac{dX}{d\tau}dt. \qquad (9)$$

By means of this the integral of the preceding is

$$\frac{d\phi}{d\tau} = W + \phi\left(\frac{dX}{d\tau} + \frac{1}{2}W\frac{dW}{d\tau}\right) + \frac{1}{2}\phi^2\frac{d^2W}{d\tau^2} + Y. \qquad (10)$$

Since $\overline{\left(\frac{d\phi}{d\tau}\right)} = \frac{dw}{dt} + \frac{\epsilon n r_{\prime}^2}{h_{\prime}}$; changing τ into t, we have

$$\frac{dw}{dt} = \overline{(W)} - \frac{\epsilon n r_{\prime}^2}{h_{\prime}} + w \overline{\left(\frac{dX}{d\tau} + \tfrac{1}{2} W \frac{dW}{d\tau}\right)} + \tfrac{1}{2} w^2 \overline{\left(\frac{d^2 W}{d\tau^2}\right)} + \overline{(Y)}. \quad (11)$$

We have now got entirely quit of ξ and ϕ; and τ is contained only in $\rho_{\prime}^{0}$ and $\lambda_{\prime}^{0}$, which are the same functions of τ and $a_{\prime}$, $e_{\prime}$, &c., as in an undisturbed orbit; and we may everywhere put them without the sign of integration, and then change τ into t. Thus we have virtually got rid of τ, and the result may be put under the same form as if it were obtained with only the ordinary time t.

If we change z into $t + w$, (3) of section (1) gives

$$\frac{h}{h_{\prime}\beta^2} = 1 + \frac{dw}{dt} + \frac{\epsilon n r_{\prime}^2}{h_{\prime}},$$

and

$$\beta = \frac{h^{\frac{1}{2}}}{h_{\prime}^{\frac{1}{2}}} \left\{ 1 - \tfrac{1}{2} \left(\frac{dw}{dt} + \frac{\epsilon n r_{\prime}^2}{h_{\prime}} \right) + \text{\&c.} \right\};$$

which will give β when $\frac{dw}{dt}$ is known. And we may change β into ϵ^{β}, and take the logarithm of both members, if we wish to have a result in the experimental form, which is that of M. Hansen. Or we may change β into $1 + \frac{\beta}{r_{\prime}}$; then we have

$$r = r_{\prime} + \beta.$$

But, changing ζ into $\tau + \phi$, we may employ (2) of section (1), which gives

$$\frac{h}{h_{\prime}\xi^2} = 1 + \frac{d\phi}{d\tau},$$

and

$$\xi = \frac{h^{\frac{1}{2}}}{h_{\prime}^{\frac{1}{2}}} \left(1 + \frac{d\phi}{d\tau}\right)^{-\frac{1}{2}}.$$

If we wish to have a result in the experimental form, this is, perhaps, the most convenient. And thus changing ξ into ϵ^{ξ}, and taking the logarithm of both members, we have

$$\xi = \tfrac{1}{2} \log \left(\frac{h}{h_{\prime}}\right) - \tfrac{1}{2} \log \left(1 + \frac{d\phi}{d\tau}\right).$$

M. Hansen employs this method, and differentiates relative to τ, by which means he gets rid of $\frac{1}{2}\log\left(\frac{h}{h_{\prime}}\right)$. Thus we have

$$\frac{d\xi}{d\tau} = -\frac{1}{2}\,\frac{\frac{d^2\phi}{d\tau^2}}{1+\frac{d\phi}{d\tau}},$$

and changing τ into t,

$$\overline{\left(\frac{d\xi}{d\tau}\right)} = -\frac{1}{2}\,\overline{\left(\frac{\frac{d^2\phi}{d\tau^2}}{1+\frac{d\phi}{d\tau}}\right)}.$$

Or, by (7) of the section referred to,

$$\frac{d\beta}{dt} = \frac{\mathfrak{C}nr}{h_{\prime}}\,\frac{dr_{\prime}}{dz} - \frac{1}{2}\,\overline{\left(\frac{\frac{d^2\phi}{d\tau^2}}{1+\frac{d\phi}{d\tau}}\right)}.$$

In this form M. Hansen leaves it. But we might employ (8) of the same section to reduce it, or to give a distinct form; but that would introduce $\frac{h}{h_{\prime}}$ again.

4. We now proceed to the determination of the latitude, and the reduction to a fixed plane. Make σ the sine of the latitude, i the inclination, and ϑ and θ the longitude of the node on the plane of the orbit and on the fixed plane respectively, ϑ having an origin fixed on the former plane. By making the plane of the orbit to turn round the radius vector an infinitesimal space, it is easily seen that,

$$d\vartheta = \cos i d\theta; \tag{1}$$

and we have the known formulæ,

$$\left.\begin{aligned} \frac{di}{dt} &= \frac{1}{h\sin i}\,\frac{dR}{d\theta} = \frac{\cos i}{h\sin i}\,\frac{dR}{d\vartheta}, \\ \frac{d\vartheta}{dt} &= -\frac{\cos i}{h\sin i}\,\frac{dR}{di}. \end{aligned}\right\} \tag{2}$$

I shall give some theorems here similar to (6) of the first section, without, however, making any use of them.

We have $\sigma = \sin i \sin (v - \vartheta) = \sin i (\sin v \cos \vartheta - \cos v \sin \vartheta)$, $\frac{d\sigma}{dt} = \sin i \cos (v - \vartheta) \frac{dv}{dt} = \frac{h}{r^2} \sin i \cos (v - \vartheta)$. Therefore, $\frac{r^2}{h} \frac{d\sigma}{dt} = \sin i \cos (v - \vartheta) = \sin i (\cos v \cos \vartheta + \sin v \sin \vartheta)$. From these we easily find

$$\sigma \sin v + \frac{r^2}{h} \frac{d\sigma}{dt} \cos v = \sin i \cos \vartheta.$$

$$-\sigma \cos v + \frac{r^2}{h} \frac{d\sigma}{dt} \sin v = \sin i \sin \vartheta.$$

Again, let κ be the same function of τ and the variable elements that σ is of t and the same elements ; so that

$$\kappa = \sin i \sin (\lambda - \vartheta), \frac{d\kappa}{d\tau} = \frac{h}{\rho^2} \sin i \cos (\lambda - \vartheta);$$

and, as before, we shall have

$$\kappa \sin \lambda + \frac{\rho^2}{h} \frac{d\kappa}{d\tau} \cos \lambda = \sin i \cos \vartheta,$$

$$-\kappa \cos \lambda + \frac{\rho^2}{h} \frac{d\kappa}{d\tau} \sin \lambda = \sin i \sin \vartheta.$$

Equating the two sets of values of $\sin i \cos \vartheta$ and $\sin i \sin \vartheta$,

$$\sigma \sin v + \frac{r^2}{h} \frac{d\sigma}{dt} \cos v = \kappa \sin \lambda + \frac{\rho^2}{h} \frac{d\kappa}{d\tau} \cos \lambda,$$

$$\sigma \cos v - \frac{r^2}{h} \frac{d\sigma}{dt} \sin v = \kappa \cos \lambda - \frac{\rho^2}{h} \frac{d\kappa}{d\tau} \sin \lambda.$$

By multiplying these by $\sin v$, $\cos v$, $\sin \lambda$, $\cos \lambda$, and adding and subtracting the products, we shall have no difficulty in deducing

$$\sigma = \kappa \cos (\lambda - v) \frac{\rho^2}{h} - \frac{d\kappa}{d\tau} \sin (\lambda - v).$$

$$\frac{r^2}{h} \frac{d\sigma}{dt} = \kappa \sin (\lambda - v) + \frac{\rho^2}{h} \frac{d\kappa}{d\tau} \cos (\lambda - v).$$

$$\kappa = \sigma \cos (\lambda - v) + \frac{r^2}{h}\frac{d\sigma}{dt} \sin (\lambda - v),$$

$$\frac{\rho^2}{h}\frac{d\kappa}{d\tau} = - \sigma \sin (\lambda - v) + \frac{r^2}{h}\frac{d\sigma}{dt} \cos (\lambda - v).$$

These two sets of equations correspond to (5) and (6) of the first section.

From $\frac{r^2}{h}\frac{d\sigma}{dt} = \sin i \cos (v - \vartheta)$, $\frac{d\sigma}{d\vartheta} = - \sin i \cos (v - \vartheta)$, and $\frac{d\sigma}{di} = \cos i$ $\sin (v - \vartheta)$; we easily find

$$\left.\begin{aligned} \frac{d\sigma}{d\vartheta} &= -\frac{r^2}{h}\frac{d\sigma}{dt}, \quad \frac{d\sigma}{di} = \sigma \frac{\cos i}{\sin i}, \\ \frac{d\kappa}{d\vartheta} &= -\frac{\rho^2}{h}\frac{d\kappa}{d\tau}, \quad \frac{d\kappa}{di} = \kappa \frac{\cos i}{\sin i}, \end{aligned}\right\} \qquad (3)$$

which may be found useful.

The part of the disturbance function depending on the inclination of the orbit to the fixed plane is very troublesome to express by means of i, ϑ, and θ, and the corresponding quantities i', ϑ', and θ', relative to the disturbing body; and when either ϑ is expressed by means of θ, ϑ' by means of θ', or the latter by means of the former, would contain a great number of terms. But it may be expressed very simply without these quantities, by means of the latitude ϕ and the reduction Δ, and the corresponding quantities ϕ' and Δ' relative to the disturbing body. Thus we should have $v - \Delta$, and $v' - \Delta'$, and $v' - \Delta'$ for the longitudes on the fixed plane, and

$$\frac{x}{r} = \cos \phi \cos (v - \Delta), \quad \frac{y}{r} = \cos \phi \sin (v - \Delta), \quad \frac{z}{r} = \sin \phi,$$

$$\frac{x'}{r'} = \cos \phi' \cos (v' - \Delta'), \quad \frac{y'}{r'} = \cos \phi' \sin (v' - \Delta'), \quad \frac{z'}{r'} = \sin \phi'.$$

But $\cos \chi = \frac{xx' + yy' + zz'}{rr'}$; therefore,

$$\cos \chi = \cos \phi \cos \phi' \{\cos (v - \Delta) \cos (v' - \Delta') + \sin (v - \Delta) \sin (v' - \Delta')\} + \sin \phi \sin \phi' = \cos \phi \cos \phi' \cos (v - v' - \Delta + \Delta') + \sin \phi \sin \phi' = (1 - \sigma^2)^{\frac{1}{2}}$$

$$(1 - \sigma'^2)^{\frac{1}{2}} \left\{ \cos (v - v') + \sin (v - v')(\Delta - \Delta') - \cos (v - v') \frac{(\Delta - \Delta')^2}{2} - \&c. \right\} + \sigma\sigma'. \quad (4)$$

We suppose here that σ and σ', and therefore also Δ and Δ', are small, as in the case of the moon and the old planets.

Making for a moment $r_{,}$ the projection of the radius vector, and $v_{,}$ the longitude on the fixed plane, and comparing the differentials of the areas described on this plane and that of the orbit, we have

$$\cos i\, r^2 dv = r_{,}^2\, dv_{,} = r^2 \cos^2\phi\, dv_{,},$$

and $\cos i\, dv = \cos^2\phi\, dv_{,} = \cos^2\phi\,(dv - d\Delta)$; or

$$(\cos^2\phi - \cos i)\frac{dv}{dt} = \cos^2\phi \frac{d\Delta}{dt}; \text{ or } \left(2\sin^2\frac{i}{2} - \sin^2\phi\right)\frac{h}{r^2} = \cos^2\phi\frac{d\Delta}{dt}.$$

Whence we derive

$$2\sin^2\frac{i}{2} = \sin^2\phi + \frac{r^2}{h}\frac{d\Delta}{dt}\cos^2\phi,$$

$$\frac{d\Delta}{dt} = \frac{h}{r^2\cos^2\phi}\left(2\sin^2\frac{i}{2} - \sin^2\phi\right).$$

And putting σ^2 for $\sin^2\phi$, $1-\sigma^2$ for $\cos^2\phi$,

$$\left.\begin{aligned} 2\sin^2\frac{i}{2} &= \sigma^2 + \frac{r^2}{h}\frac{d\Delta}{dt}(1-\sigma^2), \\ \frac{d\Delta}{dt} &= \frac{h}{r^2(1-\sigma^2)}\left(2\sin^2\frac{i}{2} - \sigma^2\right). \end{aligned}\right\} \qquad (5)$$

The equation $d\vartheta = \cos i\, d\theta$ gives $d\theta = \frac{1}{\cos i}d\vartheta$, and $d(\theta - \vartheta) = \left(\frac{1}{\cos i} - 1\right)d\vartheta$; or,

$d(\theta - \vartheta) = \dfrac{2\sin^2\frac{t}{2}}{\cos i}d\vartheta$. And by integration

$$\theta - \vartheta = 2\int\frac{\sin^2\frac{i}{2}}{\cos i}d\vartheta = a \text{ suppose.} \qquad (6)$$

Let $\Delta_{,} = (v - \vartheta) - (v_{,} - \theta)$, where $v_{,} = v - \Delta$. Then

$$\sin\Delta_{,} = \sin(v - \vartheta)\cos(v_{,} - \theta) - \cos(v - \vartheta)\sin(v_{,} - \theta).$$

But by the well-known theorems of spherical trigonometry, $\sin(v - \vartheta) =$

$\frac{\sin \phi}{\sin i}$, $\sin (v_{,} - \theta) = \frac{\tan \phi}{\tan i}$, and $\cos (v_{,} - \theta) = \frac{\cos (x - \vartheta)}{\cos \phi}$. Therefore, by substitution,

$$\sin \Delta_{,} = \tan \phi \cos (v - \vartheta) \left(\frac{1}{\sin i} - \frac{1}{\tan i}\right) = \tan \frac{i}{2} \tan \phi \cos (v - \vartheta) =$$

$$\frac{\sin i}{2 \cos^2 \frac{i}{2}} \tan \phi \cos (v - \vartheta).$$

But $\sin \Delta_{,} = \Delta_{,} - \frac{1}{6}\Delta_{,}^3$, and $\sin i \cos (v - \vartheta) = \frac{r^2}{h} \frac{d\sigma}{dt}$. Consequently, $_{,}\Delta - \frac{1}{6}\Delta_{,}^3 = \frac{\tan \phi}{2 \cos^2 \frac{i}{2}} \frac{r^2}{h} \frac{d\sigma}{dt} = \frac{r^2}{2h \cos^2 \frac{i}{2}} \frac{\sigma}{\sqrt{(1 - \sigma^2)}} \frac{d\sigma}{dt} = - \frac{r^2}{2h \cos^2 \frac{i}{2}} \frac{d \sqrt{(1 - \sigma^2)}}{dt}$.

and $\Delta_{,} = - \frac{r^2}{2h \cos^2 \frac{i}{2}} \frac{d\sqrt{(1 - \sigma^2)}}{dt} + \frac{1}{6}\Delta_{,}^3$.

Now $\Delta_{,} = v - v_{,} - \vartheta + \theta = \Delta - \vartheta + \theta = \Delta + \alpha$; and, therefore,

$$\Delta = - \frac{r^2}{2h \cos^2 \frac{i}{2}} \frac{d\sqrt{(1 - \sigma^2)}}{dt} - \alpha + \tfrac{1}{6}\Delta^3; \tag{7}$$

neglecting smaller quantities, observing that α is of the order of the disturbing force multiplied by $\sin^2 \frac{i}{2}$.

We want now to find $\frac{d\Delta}{di}$ and $\frac{d\Delta}{d\vartheta}$. To do this, we shall transpose $\frac{1}{6}\Delta^3$, and, after taking the partial differential, divide by $1 - \frac{1}{2}\Delta^2$, or multiply by $1 + \frac{1}{2}\Delta^2$; and by means of $\frac{d^2\sigma}{di dt} = \frac{h}{r^2} \cos i \cos (v - \vartheta) = \frac{\cos i}{\sin i} \frac{d\sigma}{dt}$, and $\frac{d^2\sigma}{d\vartheta dt} = \frac{h}{r^2} \sigma$, we shall find

$$\frac{d\Delta}{di} = \frac{r^2}{2h \cos^2 \frac{i}{2}} \frac{d\sigma}{dt} \frac{\sigma}{\sqrt{(1 - \sigma^2)}} \left\{ \cot i \left(\frac{2 - \sigma^2}{1 - \sigma^2}\right) + \tan \frac{i}{2} \right\} (1 + \tfrac{1}{2}\Delta^2). \tag{8}$$

$$\frac{d\Delta}{d\vartheta} = \frac{1}{2\cos^2\frac{i}{2}} \cdot \frac{1}{\sqrt{(1-\sigma^2)}} \left(\sigma^2 - \frac{r^4}{h^2}\frac{d\sigma^2}{dt^2}\frac{1}{1-\sigma^2}\right)(1+\tfrac{1}{2}\Delta^2) - \frac{2\sin^2\frac{i}{2}}{\cos i}(1+\tfrac{1}{2}\Delta^2).$$

But $\frac{r^4}{h^2}\frac{d\sigma^2}{dt^2} = \sin^2 i \cos^2 (v-\vartheta) = \sin^2 i - \sin^2 i \sin^2 (v-\vartheta) = \sin^2 i - \sigma^2$.

Therefore, putting this value in the preceding, we have

$$\frac{d\Delta}{d\vartheta} = \frac{1}{2\cos^2\frac{i}{2}} \cdot \frac{1}{\sqrt{(1-\sigma^2)}} \left(\frac{\sigma^2 - \sin^2 i}{1-\sigma^2} + \sigma^2\right)(1+\tfrac{1}{2}\Delta^2) - \frac{2\sin^2\frac{i}{2}}{\cos i}(1+\tfrac{1}{2}\Delta^2). \quad (9)$$

But the formulæ we have obtained are not convenient for actual application; it may be well, therefore, to give them in series. For this purpose we make $\sin i = s$, $\sin (v-\vartheta) = \eta$; then $\sigma = s\eta$. But as, in the first differentials, the elements do not vary, we must make $\frac{d\sigma}{dt} = s\frac{d\eta}{dt}$, and in $\frac{d\eta}{dt}$ we must not differentiate ϑ.

Now, expanding $\sqrt{(1-\sigma^2)}$ in (7), we find

$$\Delta = \frac{r^2}{2h\cos^2\frac{i}{2}}\frac{d\sigma}{dt}(\sigma + \tfrac{1}{2}\sigma^3 + \tfrac{3}{8}\sigma^5) - \alpha + \tfrac{1}{6}\Delta^3,$$

neglecting smaller terms. Or,

$$\Delta = \frac{r^2 s^2}{2h\cos^2\frac{i}{2}}\frac{d\eta}{dt}(\eta + \tfrac{1}{2}s^2\eta^3 + \tfrac{3}{8}s^4\eta^5) - \alpha + \tfrac{1}{6}\Delta^3.$$

But

$$\frac{1}{2\cos^2\frac{i}{2}} = \frac{2\sin^2\frac{i}{2}}{4\sin^2\frac{i}{2}\cos^2\frac{i}{2}} = \frac{2\sin^2\frac{i}{2}}{s^2} = \frac{1-\cos i}{s^2} = \frac{1-\sqrt{(1-s^2)}}{s^2} = \tfrac{1}{2} + \tfrac{1}{8}s^2 + \tfrac{1}{16}s^4.$$

Substituting this value, we have at length,

$$\Delta = \frac{r^2 s^2}{h}\frac{d\eta}{dt}\left\{\tfrac{1}{2}\eta + s^2(\tfrac{1}{8}\eta + \tfrac{1}{4}\eta^3) + s^4(\tfrac{1}{16}\eta + \tfrac{1}{16}\eta^3 + \tfrac{3}{16}\eta^5)\right\} - \alpha + \tfrac{1}{6}\Delta^3; \quad (10)$$

where some small quantities are retained, which, perhaps, are not wanted. In taking the partial differential coefficients of Δ, it will be better to find $\frac{d\Delta}{ds}$, and not $\frac{d\Delta}{di}$.

$$\frac{d\Delta}{ds} = \frac{2r^2s}{h}\frac{d\eta}{dt}\{\tfrac{1}{2}\eta + s^2(\eta + \tfrac{1}{2}\eta^3) + s^4(\tfrac{3}{16}\eta + \tfrac{3}{16}\eta^3 + \tfrac{9}{16}\eta^5)\} + \tfrac{1}{2}\Delta^2\frac{d\Delta}{ds}.$$

Write this for a moment, $\frac{d\Delta}{ds} = M + \frac{1}{2}\Delta^2\frac{d\Delta}{ds}$. Transpose the last term of the second member. Then $(1 - \frac{1}{2}\Delta^2)\frac{d\Delta}{ds} = M$; whence $\frac{d\Delta}{ds} = M(1 - \frac{1}{2}\Delta^2)^{-1} = M(1 + \frac{1}{2}\Delta^2)$; and

$$\frac{d\Delta}{ds} = \frac{2rs}{h}\frac{d\eta}{dt}\{\tfrac{1}{2}\eta + s^2(\eta + \tfrac{1}{2}\eta^3) + s^4(\tfrac{3}{16}\eta + \tfrac{3}{16}\eta^3 + \tfrac{9}{16}\eta^5)\}(1 + \tfrac{1}{2}\Delta^2). \quad (11)$$

Preparatory to finding $\frac{d\Delta}{d\vartheta}$, we may observe that $\frac{d\eta}{dt} = \cos(v - \vartheta)\frac{dv}{dt} = \frac{h}{r^2}\cos(v - \vartheta)$; and, therefore, $\frac{d^2\eta}{d\vartheta dt} = \frac{h}{r^2}\sin(v - \vartheta) = \frac{h}{r^2}\eta$. Also, $\frac{d\eta}{d\vartheta} = -\cos(v - \vartheta) = -\frac{r^2}{h}\frac{d\eta}{dt}$; and $\frac{r^4}{h^2}\frac{d\eta^2}{dt^2} = \cos^2(v - \vartheta) = 1 - \sin^2(v - \vartheta) = 1 - \eta^2$. We now find

$$\frac{d\Delta}{d\vartheta} = s^2\eta\{\tfrac{1}{2}\eta + s^2(\tfrac{1}{2}\eta + \tfrac{1}{4}\eta^3) + s^4(\tfrac{1}{16}\eta + \tfrac{1}{16}\eta^3 + \tfrac{3}{16}\eta^5)\} - \frac{2\sin^2\frac{i}{2}}{\cos i} + \tfrac{1}{2}\Delta^2\frac{d\Delta}{d\vartheta} - s^2(1 - \eta^2)\{\tfrac{1}{2} + s^2(\tfrac{1}{2} + \tfrac{3}{4}\eta^2) + s^4(\tfrac{1}{16} + \tfrac{3}{16}\eta^2 + \tfrac{15}{16}\eta^4)\}.$$

By further reduction, and treating the equation with regard to Δ as in the last case,

$$\frac{d\Delta}{d\vartheta} = s^2\{-\tfrac{1}{2} + \eta^2 + s^2(-\tfrac{1}{2} + \tfrac{1}{4}\eta^2 + \eta^4) + s^4(-\tfrac{1}{16} - \tfrac{1}{16}\eta^2 - \tfrac{11}{16}\eta^4 + \tfrac{9}{8}\eta^6)\}$$
$$(1 + \tfrac{1}{2}\Delta^2) - \frac{2\sin^2\frac{i}{2}}{\cos i}(1 + \tfrac{1}{2}\Delta^2).$$

But $\dfrac{2\sin^2\frac{i}{2}}{\cos i} = \dfrac{1-\cos i}{\cos i} = \dfrac{1}{\cos i} - 1 = \dfrac{1}{\sqrt{(1-s^2)}} - 1 = \frac{1}{2}s^2 + \frac{3}{8}s^4 - \frac{5}{16}s^6.$

Therefore,

$$\frac{d\Delta}{d\vartheta} = s^2\{-1+\eta^2+s^2\left(-\tfrac{7}{8}+\tfrac{1}{4}\eta^2+\eta^4\right)+s^4\left(-\tfrac{3}{8}-\tfrac{1}{16}\eta^2-\tfrac{11}{16}\eta^4+\tfrac{9}{8}\eta^6\right)\}(1+\tfrac{1}{2}\Delta^2). \quad (12)$$

Since $2\sin^2\frac{i}{2} = 1-\cos i = 1-\sqrt{(1-s^2)} = \frac{1}{2}s^2+\frac{1}{8}s^4+\frac{1}{16}s^6$. Putting this value in (5), and developing the terms, it becomes

$$\frac{d\Delta}{dt} = \frac{h}{r^2}\{s^2\left(\tfrac{1}{2}-\eta^2\right)+s^4\left(\tfrac{1}{8}+\tfrac{1}{2}\eta^2-\eta^4\right)+s^6\left(\tfrac{1}{16}+\tfrac{1}{8}\eta^2+\tfrac{1}{2}\eta^4-\eta^6\right)\}. \quad (13)$$

We will now transform (2), so as to introduce σ and Δ, or rather η and Δ.

$$\frac{ds}{dt} = \cos i\,\frac{di}{dt} = \frac{\cos^2 i}{h\sin i}\,\frac{dR}{d\vartheta} = \frac{1-s^2}{hs}\,\frac{dR}{d\vartheta},$$

$$\frac{d\vartheta}{dt} = -\frac{\cos^2 i}{h\sin i}\,\frac{dR}{ds} = -\frac{1-s^2}{hs}\,\frac{dR}{ds}.$$

Or, putting $s\eta$ for σ in R, and introducing the partial differentials of the new quantities,

$$\left.\begin{aligned}\frac{ds}{dt} &= \frac{1-s^2}{hs}\left(\frac{dR}{d\eta}\,\frac{d\eta}{d\vartheta}+\frac{dR}{d\Delta}\,\frac{d\Delta}{d\vartheta}\right),\\ \frac{d\vartheta}{dt} &= -\frac{1-s^2}{hs}\left(\frac{dR}{ds}+\frac{dR}{d\Delta}\,\frac{d\Delta}{ds}\right).\end{aligned}\right\} \quad (14)$$

It may not be amiss to find $2\sin^2\frac{i}{2}$ separately. Thus, i_0 being the mean value of i, or its value when the disturbing force is nothing, and, therefore, a constant quantity; we have

$$2\sin^2\frac{i}{2} = 2\sin^2\frac{i_0}{2} + 2\int d\sin^2\frac{i}{2} = 2\sin^2\frac{i_0}{2} + \int \sin i\,di =$$

$$2\sin^2\frac{i_0}{2} + \int\frac{dt}{h}\cos i\,\frac{dR}{d\vartheta} = 2\sin^2\frac{i_0}{2} + \int\frac{dt}{h}\cos i\left(\frac{dR}{d\eta}\,\frac{d\eta}{d\vartheta}+\frac{dR}{d\Delta}\,\frac{d\Delta}{d\vartheta}\right).$$

Make $\frac{1}{h}\frac{dR}{d\eta}\frac{d\eta}{d\vartheta}+\frac{1}{h}\frac{dR}{d\Delta}\frac{d\Delta}{d\vartheta}=P$, to abridge. Then

$$2\sin^2\frac{i}{2}=2\sin^2\frac{i_0}{2}+\int\cos iPdt=2\sin^2\frac{i_0}{2}+\int Pdt-2\int\sin^2\frac{i}{2}Pdt.$$

But substituting this value of $2\sin^2\frac{i}{2}$ under the integral sign of the second member,

$$2\sin^2\frac{i}{2}=2\sin^2\frac{i_0}{2}+\cos i_0\int Pdt-\tfrac{1}{2}(\int Pdt)^2+2\int Pdt\int\sin^2\frac{i}{2}Pdt.$$

By continuing these operations, we find

$$2\sin^2\frac{i}{2}=2\sin^2\frac{i_0}{2}+\cos i_0\int Pdt-\tfrac{1}{2}\cos i_0(\int Pdt)^2+\tfrac{1}{2\cdot 3}\cos i_0(\int Pdt)^3-\&c. \quad (15)$$

This may serve to eliminate $\sin^2\frac{i}{2}$. But since $2\sin^2\frac{i}{2}=1-\cos i=1-\sqrt{(1-\sin^2 i)}=\tfrac{1}{2}\sin^2 i+\tfrac{1}{8}\sin^4 i+\tfrac{1}{16}\sin^6 i+\&c.$, it may also serve to eliminate $\sin^2 i$ and $\sin i$, if we should find it convenient to do so.

It is much easier to find r and v on the plane of the orbit, than to find the values of the corresponding quantities on the fixed plane; but it is more difficult to find the latitude in the former case. It is very troublesome to find it directly from the variable values of the elements i and ϑ; and yet we must have the values of these quantities separately to a considerable degree of exactness in order to find α. M. Hansen has found the latitude by means of $\sin i \sin\vartheta$, $\sin i\cos\vartheta$; which is a much better method; and he has found α by means of these latter quantities, or rather functions derived from them. But this is attended with a great deal of trouble. I propose to pursue a different course, and to find $\sin i$, or s, separately, by the equation given above for the purpose, and then to find η. To do which I shall find

$$y=\sin(x-\vartheta-\gamma nt),$$

x being a constant quantity. This is finding a function of $\vartheta+\gamma nt$.

$$\frac{dy}{dt}=\frac{dy}{d\vartheta}\left(\gamma n+\frac{d\vartheta}{dt}\right)=$$

$$\frac{dy}{d\vartheta}\left\{\gamma n-\frac{1-s^2}{hs}\left(\frac{dR}{ds}+\frac{dR}{d\Delta}\frac{d\Delta}{ds}\right)\right\}=\frac{dy}{d\vartheta}Q \text{ suppose.}$$

It must be observed, that γnt is the uniform regression of the node, and γ is to be so determined as to take away the constant terms from Q.

Now $y_0 = \sin(x - \vartheta_0)$ the value when the disturbing force is made nothing; therefore, integrating,

$$y = \sin(x - \vartheta_0) + \int \frac{dy}{d\vartheta} Q dt.$$

For a first approximation, we make $\frac{dy}{d\vartheta} = -\cos(x - \vartheta_0)$; and after the integration is performed, we may make x anything we please. We shall make $x = v + \gamma nt = v_, + \epsilon nt + \gamma nt$. Then $y = \sigma$, and we shall have

$$\sigma = \sin(v + \gamma nt - \vartheta_0) + \int \frac{dy}{d\vartheta} Q dt. \qquad (16)$$

After this substitution has been made for x, and v replaced by $v_, + \epsilon nt$, $v_,$, being a given function of z, may be allowed to remain, or may be developed in terms of z, and z may be developed in terms of t.

We may make $y = \sin i \sin(x - \vartheta - \gamma nt) = \sin i \cos(\vartheta + \gamma nt) \sin x - \sin i \sin(\vartheta + \gamma nt) \cos x = p \sin x - q \cos x$.

$$p = \sin i \cos(\vartheta + \gamma nt),\ q = \sin i \sin(\vartheta + \gamma nt).$$

$$\frac{dp}{dt} = \cos i \cos(\vartheta + \gamma nt) \frac{di}{dt} - \sin i \sin(\vartheta + \gamma nt) \left(\frac{d\vartheta}{dt} + \gamma n\right).$$

$$\frac{dq}{dt} = \cos i \sin(\vartheta + \gamma nt) \frac{di}{dt} + \sin i \cos(\vartheta + \gamma nt) \left(\frac{d\vartheta}{dt} + \gamma n\right).$$

Or,

$$\frac{dp}{dt} = p \frac{\cos i}{\sin i} \frac{di}{dt} - q \left(\frac{d\vartheta}{dt} + \gamma n\right) = \frac{p}{s} \frac{ds}{dt} - q \left(\frac{d\vartheta}{dt} + \gamma n\right).$$

$$\frac{dq}{dt} = q \frac{\cos i}{\sin i} \frac{di}{dt} + p \left(\frac{d\vartheta}{dt} + \gamma n\right) = \frac{q}{s} \frac{ds}{dt} + p \left(\frac{d\vartheta}{dt} + \gamma n\right).$$

We may substitute in this, for $\frac{ds}{dt}$ and $\frac{d\vartheta}{dt}$, their values from (14); but as I prefer the former method, I shall not pursue this further; nor is it necessary, since any one who is desirous of doing it may easily carry it through. We may observe, however, that making δp, δq, δs, $\delta\vartheta$, the alterations produced in p, q, &c., by the disturbing force, we have for the first power of that force,

$$\delta p = \frac{p}{s}\delta s - q\delta\vartheta, \quad \delta q = \frac{q}{s}\delta s + p\delta\vartheta;$$

whence, observing that $s^2 = p^2 + q^2$, we have

$$\delta s = \frac{p\delta p + q\delta q}{s}, \quad \delta\vartheta = \frac{p\delta q - q\delta p}{s^2}.$$

Thus we find the alterations produced in s and ϑ from those produced in p and q. And in like manner we may find the alterations depending on the second power of the disturbing force.

In this section we have taken no notice of the development of r in the formulæ where it has appeared. Making $r = r_{,}\beta$, we may either let it stand thus, or put from (3) section (1) its value in terms of $\frac{dw}{dt}$, which we easily find to be

$$\beta = \frac{h^{\frac{1}{2}}}{h_{,}^{\frac{1}{2}}}\left(1 + \frac{dw}{dt} + \frac{\epsilon n r_{,}}{h_{,}}\right)^{-\frac{1}{2}}.$$

Then we shall have only $r_{,}$ and $v_{,}$, besides the terms depending on the latitude, to develope. These are given functions of $z = t + w$, and may, therefore, be developed together by Taylor's theorem.

We might have developed the value of S in (11), section (2), relative to ξ and ϕ, differently.

Since the difference only of λ and v enters into the composition of S, we may change these quantities into $\lambda_{,}$ and $v_{,}$, or into the anomalies $\lambda_{,} - \pi_{,}$ and $v_{,} - \pi_{,}$. Using for simplicity $\lambda_{,}$ and $v_{,}$, and making

$$M = \left(\frac{1}{r} + \frac{\mu}{h^2}\right)\frac{2\cos v_{,}}{h}\frac{dR}{dv} + \frac{2\sin v_{,}}{h}\frac{dR}{dr},$$

$$N = \left(\frac{1}{r} + \frac{\mu}{h^2}\right)\frac{2\sin v_{,}}{h}\frac{dR}{dv} - \frac{2\cos v_{,}}{h}\frac{dR}{dr},$$

we have

$$S = \left(1 + \frac{2\mu\rho}{h^2}\right)\frac{1}{h}\frac{dR}{dv} - \rho\cos\lambda_, M - \rho\sin\lambda_, N - \frac{\epsilon n}{h}\frac{d(\rho^2)}{d\tau}.$$

We eliminate ξ from this by putting $\rho_, \frac{h^{\frac{1}{2}}}{h_,^{\frac{1}{2}}}\left(1 - \frac{1}{2}\frac{d\phi}{d\tau} + \frac{3}{8}\frac{d\phi^2}{d\tau^2}\right)$ for $\rho = \rho_, \xi$.

Let (S) be the value of S when ρ is changed into $\rho_, \frac{h^{\frac{1}{2}}}{h_,^{\frac{1}{2}}}$, and we have

$$S = (S) + \left\{(\rho_, \cos\lambda_, M + \rho_, \sin\lambda_, N)\frac{h^{\frac{1}{2}}}{2h_,^{\frac{1}{2}}} - \frac{\rho_,}{h_,^{\frac{1}{2}}h^{\frac{1}{2}}}\frac{\mu}{h^2}\frac{dR}{dv}\right\}\left(\frac{d\phi}{d\tau} - \frac{3}{4}\frac{d\phi^2}{d\tau^2}\right) + \frac{\epsilon n}{h_,}\frac{d}{d\tau}\left\{\rho_,^2\left(\frac{d\phi}{d\tau} - \frac{d\phi^2}{d\tau^2}\right)\right\}.$$

Eliminate $(\rho_, \cos\lambda_, M + \rho_, \sin\lambda_, N)\frac{h^{\frac{1}{2}}}{2h_,^{\frac{1}{2}}}\left(\frac{d\phi}{d\tau} - \frac{3}{4}\frac{d\phi^2}{d\tau^2}\right)$ by

$$(S) = \left(\frac{1}{h} + \frac{2\mu\rho_,}{h_,^{\frac{1}{2}}h^{\frac{5}{2}}}\right)\frac{dR}{dv} - \frac{h^{\frac{1}{2}}}{h_,^{\frac{1}{2}}}(\rho_, \cos\lambda_, M + \rho_, \sin\lambda_, N) - \frac{\epsilon n}{h_,}\frac{d(\rho_,^2)}{d\tau}.$$

and we find as the result,

$$S = (S) - \left\{(S) - \frac{1}{h}\frac{dR}{dv} + \frac{\epsilon n}{h_,}\frac{d(\rho_,^2)}{d\tau}\right\}\left(\frac{1}{2}\frac{d\phi}{d\tau} - \frac{3}{8}\frac{d\phi^2}{d\tau^2}\right) + \frac{\epsilon n}{h_,}\frac{d}{d\tau}\left\{\rho_,^2\left(\frac{d\phi}{d\tau} - \frac{d\phi^2}{d\tau^2}\right)\right\}.$$

Make $(P) = (S) - \frac{1}{h}\frac{dR}{dv} + \frac{\epsilon n}{h_,}\frac{d(\rho_,^2)}{d\tau}$. Then,

$$S = (S) - P\left(\frac{1}{2}\frac{d\phi}{d\tau} - \frac{3}{8}\frac{d\phi^2}{d\tau^2}\right) + \frac{\epsilon n}{h_,}\frac{d}{d\tau}\left\{\rho_,^2\left(\frac{d\phi}{d\tau} - \frac{d\phi^2}{d\tau^2}\right)\right\}.$$

We now develope $\rho_,$ and $\lambda_,$ by the powers of ϕ; so that (S^0) and (P^0) will have only $\rho_,^0$ and $\lambda_,^0$, which are elliptic functions of τ only, with the constant elements $a_,$, $e_,$, &c.

$$(S) = (S^0) + \phi\frac{d(S^0)}{d\tau} + \frac{\phi^2}{2}\frac{d^2(S^0)}{d\tau^2};$$

$$(P)=(P^0)+\phi\frac{d(P^0)}{d\tau};\ \rho_{,}^2=\rho_{,}^{02}+\phi\frac{d(\rho_{,}^{02})}{d\tau}.$$

And by substitution,

$$S=(S^0)+\phi\frac{d(S^0)}{d\tau}-\tfrac{1}{2}(P^0)\frac{d\phi}{d\tau}+\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\frac{d\phi}{d\tau}+\frac{\epsilon n\rho_{,}^{02}}{h_{,}}\frac{d^2\phi}{d\tau^2}\left(1-2\frac{d\phi}{d\tau}\right)$$

$$+\frac{\phi^2}{2}\frac{d^2(S^0)}{d\tau^2}+\tfrac{3}{8}(P^0)\frac{d\phi^2}{d\tau^2}-\frac{\phi}{2}(P^0)\frac{d(P^0)}{d\tau}+\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}\phi\frac{d^2\phi}{d\tau^2}+\frac{\epsilon n}{h_{,}}\frac{d^2(\rho_{,}^{02})}{d\tau^2}\phi\frac{d\phi}{d\tau}.$$

To this we must add,

$$(P^0)=(S^0)-\frac{1}{h}\frac{dR}{dv}+\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}-\frac{\epsilon n}{h_{,}}\frac{d^2(\rho_{,}^{02})}{d\tau^2}\phi,$$

$$(S^0)=(P^0)+\frac{1}{h}\frac{dR}{dv}-\frac{\epsilon n}{h_{,}}\frac{d(\rho_{,}^{02})}{d\tau}+\frac{\epsilon n}{h_{,}}\frac{d^2(\rho_{,}^{02})}{d\tau^2}\phi,$$

5. In concluding this paper, I will give a brief sketch of a transformation of the differential equations which determine the place of a disturbed planet, which I have never seen noticed, but which I think is deserving of some consideration.

Make $Q=-\frac{\mu}{r}+R$; the equations with reference to a fixed plane are,

$$\frac{d^2x}{dt^2}+\frac{dQ}{dx}=0,\ \frac{d^2y}{dt^2}+\frac{dQ}{dy}=0,\ \frac{d^2z}{dt^2}+\frac{dQ}{dz}=0. \qquad (1)$$

Suppose now a plane always to make the constant angle i with the fixed plane, i being the mean inclination of the orbit to this plane. And let this plane slide on the fixed plane in such a manner that their intersection may have a uniform motion equal to that of the node, or so that the intersection of the two planes may be always the mean place of the node. And let $\theta-ant$ be the longitude of this intersection, θ being constant.

Let now $v_{,}$ and $y_{,}$ be co-ordinates making the angle $\theta-ant$ with x and y, $x_{,}$ lying in the intersection of the sliding and fixed planes. We have

$$x = x_{\prime} \cos (\theta - ant) - y_{\prime} \sin (\theta - ant),$$
$$y = x_{\prime} \sin (\theta - ant) + y_{\prime} \cos (\theta - ant).$$

We form the two equations,

$$\cos (\theta - ant) \left(\frac{d^2x}{dt^2} + \frac{dQ}{dx}\right) + \sin (\theta - ant) \left(\frac{d^2y}{dt^2} + \frac{dQ}{dy}\right) = 0.$$

$$- \sin (\theta - ant) \left(\frac{d^2x}{dt^2} + \frac{dQ}{dx}\right) + \cos (\theta - ant) \left(\frac{d^2y}{dt^2} + \frac{dQ}{dy}\right) = 0.$$

But

$$\cos (\theta - ant) \frac{dQ}{dx} + \sin (\theta - ant) \frac{dQ}{dy} = \frac{dQ}{dx_{\prime}},$$

$$- \sin (\theta - ant) \frac{dQ}{dx} + \cos (\theta - ant) \frac{dQ}{dy} = \frac{dQ}{dy_{\prime}}.$$

Therefore,

$$\cos (\theta - ant) \frac{d^2x}{dt^2} + \sin (\theta - ant) \frac{d^2y}{dt^2} + \frac{dQ}{dx_{\prime}} = 0.$$

$$- \sin (\theta - ant) \frac{d^2x}{dt^2} + \cos (\theta - ant) \frac{d^2y}{dt^2} + \frac{dQ}{dy_{\prime}} = 0.$$

Substituting for $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$, their values in $x_{\prime}$ and $y_{\prime}$, we have

$$\frac{d^2x_{\prime}}{dt^2} + \frac{dQ}{dx_{\prime}} + 2an \frac{dy_{\prime}}{dt} - a^2n^2x_{\prime} = 0,$$

$$\frac{d^2y_{\prime}}{dt^2} + \frac{dQ}{dy_{\prime}} - 2an \frac{dx_{\prime}}{dt} - a^2n^2y_{\prime} = 0.$$

Or dropping the distinctive marks of $x_{\prime}$ and $y_{\prime}$, as tending to produce confusion, and as no longer necessary:

$$\left.\begin{aligned} &\frac{d^2x}{dt^2} + \frac{dQ}{dx} + 2an \frac{dy}{dt} - a^2n^2x = 0, \\ &\frac{d^2y}{dt^2} + \frac{dQ}{dy} - 2an \frac{dx}{dt} - a^2n^2y = 0. \end{aligned}\right\} \quad (2)$$

We now take the new co-ordinates $y_{\prime}$ and $z_{\prime}$, $y_{\prime}$ lying in the sliding plane, and $z_{\prime}$ being perpendicular to it; and, consequently, we have

$$y = y_{\prime} \cos i - z_{\prime} \sin i, \quad z = y_{\prime} \sin i + z_{\prime} \cos i.$$

Proceeding by exactly the same steps as before, we find

$$\frac{d^2y_{\prime}}{dt^2} + \frac{dQ}{dy_{\prime}} - 2an \cos i \frac{dx}{dt} - a^2n^2 \cos i\,(y_{\prime} \cos i - z_{\prime} \sin i) = 0,$$

$$\frac{d^2z_{\prime}}{dt^2} + \frac{dQ}{dz_{\prime}} + 2an \sin i \frac{dx}{dt} + a^2n^2 \sin i\,(y_{\prime} \cos i - z_{\prime} \sin i) = 0.$$

Or again, dropping the distinctive marks of $y_{\prime}$ and $z_{\prime}$

$$\left.\begin{aligned}\frac{d^2y}{dt^2} + \frac{dQ}{dy} - 2an \cos i \frac{dx}{dt} - a^2n^2 \cos i\,(y \cos i - z \sin i) = 0.\\ \frac{d^2z}{dt^2} + \frac{dQ}{dz} + 2an \sin i \frac{dx}{dt} + a^2n^2 \sin i\,(y \cos i - z \sin i) = 0.\end{aligned}\right\} \quad (3)$$

Again, $x_{\prime}$ and $y_{\prime}$ lying in the sliding plane, let $x_{\prime}$ be the angle $\pi + \epsilon nt$ in advance of the line of intersection of the two planes. Then ϵ may be so determined, that $x_{\prime}$ shall lie in the mean place of the apse when projected on the sliding plane, or in any other position we please. And as before,

$$x = x_{\prime} \cos(\pi + \epsilon nt) - y_{\prime} \sin(\pi + \epsilon nt),$$
$$y = x_{\prime} \sin(\pi + \epsilon nt) + y_{\prime} \cos(\pi + \epsilon nt).$$

Making for a moment, in order to abridge,

$$A = 2an \frac{dy}{dt} - a^2n^2x, \quad B = -2an \cos i \frac{dx}{dt} - a^2n^2 \cos i\,(y \cos i - z \sin i);$$

we shall have from the first of (2) and the first of (3), by exactly the same process as in the two former transformations,

$$\cos(\pi + \epsilon nt)\frac{d^2x}{dt^2} + \sin(\pi + \epsilon nt)\frac{d^2y}{dt^2} + \frac{dQ}{dx_{\prime}} + A \cos(\pi + \epsilon nt) + B \sin(\pi + \epsilon nt) = 0.$$

$$-\sin(\pi + \epsilon nt)\frac{d^2x}{dt^2} + \cos(\pi + \epsilon nt)\frac{d^2y}{dt^2} + \frac{dQ}{dy_{\prime}} - A \sin(\pi + \epsilon nt) + B \cos(\pi + \epsilon nt) = 0.$$

Or, putting for $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ their values,

$$\frac{d^2x_{\prime}}{dt^2}+\frac{dQ}{dx_{\prime}}+L_{\prime}=0,$$

$$\frac{d^2y_{\prime}}{dt^2}+\frac{dQ}{dy_{\prime}}+M_{\prime}=0.$$

And again dropping the distinctive marks,

$$\left.\begin{aligned}&\frac{d^2x}{dt^2}+\frac{dQ}{dx}+L=0,\\&\frac{d^2y}{dt^2}+\frac{dQ}{dy}+M=0.\end{aligned}\right\}\qquad(4)$$

$$L=\frac{dy}{dt}\left\{2an\cos^2\frac{i}{2}-2\epsilon n+2an\sin^2\frac{i}{2}\cos 2(\pi+\epsilon nt)\right\}+2an\frac{dx}{dt}\sin^2\frac{i}{2}\sin 2(\pi+\epsilon nt)-$$

$$x\left\{a^2n^2+\epsilon^2n^2-2a\epsilon n^2\cos^2\frac{i}{2}-\tfrac{1}{2}a^2n^2\sin^2 i+\left(\tfrac{1}{2}a^2n^2\sin^2 i-2a\epsilon n^2\sin^2\frac{i}{2}\right)\cos 2(\pi+\epsilon nt)\right\}+$$

$$y\left(\tfrac{1}{2}a^2n^2\sin^2 i-2a\epsilon n^2\sin^2\frac{i}{2}\right)\sin 2(\pi+\epsilon nt)+a^2n^2\sin i\cos iz\sin(\pi+\epsilon nt).$$

$$N=\frac{dx}{dt}\left\{2\epsilon n-2an\cos^2\frac{i}{2}+2an\sin^2\frac{i}{2}\cos 2(\pi+\epsilon nt)\right\}-2an\frac{dy}{dt}\sin^2\frac{i}{2}\sin 2(\pi+\epsilon nt)+$$

$$y\left\{2a\epsilon n^2\cos^2\frac{i}{2}-a^2n^2-\epsilon^2n^2+\tfrac{1}{2}a^2n^2\sin^2 i+\left(\tfrac{1}{2}a^2n^2\sin^2 i-2a\epsilon n^2\sin^2\frac{i}{2}\right)\cos 2(\pi+\epsilon nt)\right\}+$$

$$x\left(\tfrac{1}{2}a^2n^2\sin^2 i-2a\epsilon n^2\sin^2\frac{i}{2}\right)\sin 2(\pi+\epsilon nt)+a^2n^2\sin i\cos iz\cos(\pi+\epsilon nt).$$

The terms in L and M containing z are of the order of the third power of the disturbing force multiplied by $\sin i$, and will not be wanted in these cases in which i is large, and, I think, cannot be wanted when it is small. Most of the other terms are of the second order; and as i is constant, they are all of a very simple character.

We must add from the last of (3)

$$\frac{d^2z}{dt^2}+\frac{dQ}{dz}+N=0.\qquad(5)$$

$$N=2an\sin i\frac{dx}{dt}+a^2n^2\sin i(y\cos i-z\sin i).$$

The term in N containing z is of the order of the third power of the disturbing force multiplied by $\sin^2 i$, and cannot, I conceive, in any case be wanted.

We may transform the co-ordinates x', y', and z', of the disturbing body in like manner to a sliding plane, and to a similar situation on that plane. Thus, marking all the corresponding quantities relative to this body with an accent, we have

$$x' = x'_1 \cos(\theta' - a'n't) - y'_1 \sin(\theta' - a'n't),$$
$$y' = x'_1 \sin(\theta' - a'n't) + y'_1 \cos(\theta' - a'n't), \quad z' = z'_1.$$
$$x'_1 = x'_2, \quad y'_1 = y'_2 \cos i' - z'_2 \sin i', \quad z'_1 = y'_2 \sin i' - z'_2 \cos i'.$$
$$x'_2 = x'_3 \cos(\pi' + \epsilon' n't) - y'_3 \sin(\pi' + \epsilon' n't),$$
$$y'_2 = x'_3 \sin(\pi' + \epsilon' n't) + y'_3 \cos(\pi' + \epsilon' n't), \quad z'_2 = z'_3,$$

But we may transform x', y', and z' to the plane of the orbit of the disturbing body, if we please.

To find the latitude, marking the letters in the second member with the number of the transformation to which they belong, we have

$$z = y_2 \sin i + z_2 \cos i = x_3 \sin i \sin(\pi + \epsilon nt) + y_3 \sin i \cos(\pi + \epsilon nt) + z_3 \cos i.$$

But σ being the sine of the true latitude, s the sine of the latitude relative to the sliding plane, and f the anomaly, or the longitude measured from the axis of x after the last transformation ;

$$z = r\sigma, \quad x_3 = r\cos f, \quad y_3 = r\sin f, \text{ and } z_3 = rs.$$

Therefore, by substitution in the last, and dividing by r,

$$\sigma = \sin i \{\sin(\pi + \epsilon nt)\cos f + \cos(\pi + \epsilon nt)\sin f\} + s\cos i;$$

or,

$$\sigma = \sin i \sin(f + \pi + \epsilon nt) + s\cos i. \tag{6}$$

Thus the latitude is very easily found.

Still marking the letters according to the transformation to which they belong,

$$y_1 = y_2 \cos i - z_2 \sin i = x_3 \cos i \sin(\pi + \epsilon nt) + y_3 \cos i \cos(\pi + \epsilon nt) - z_3 \sin i.$$

Let v be the longitude on the fixed plane, measured from its intersection with the sliding plane,

$$y_1 = r \sin v,$$

the rest as before ; then, by substitution,

$$\sin v = \cos i \{\cos f \sin (\pi + \epsilon nt) + \sin f \cos (\pi + \epsilon nt)\} - s \sin i.$$

Or,

$$\sin v = \cos i \sin (f + \pi + \epsilon nt) - s \sin i. \tag{7}$$

This will give the longitude when i is large ; and when it is small, we can without difficulty find v itself without finding its sine.

I must, in conclusion, repeat what I have before said, that I think this transformation worthy of consideration.

GUNTHWAITE HALL,
NEAR BARNSLEY, YORKSHIRE,
Nov. 13, 1848.

III.—*On the Mean Results of Observations. By the* Rev. Humphrey Lloyd, *D. D.*, President; *F. R. S.; Hon. F. R. S. E.; Corresponding Member of the Royal Society of Sciences at Gottingen; Honorary Member of the American Philosophical Society, of the Batavian Society of Sciences, and of the Society of Sciences of the Canton de Vaud, &c.*

Read June 12, 1848.

1. THE problem in which it is sought to determine the daily mean values of the atmospheric temperature or pressure, from a limited number of observed values, is one of fundamental importance in meteorology; and, accordingly, many solutions of it have been proposed by meteorologists. These solutions are derived, for the most part, from the known laws of the diurnal variation of these elements. Many of them are accordingly applicable only to the particular cases considered; while for others, which are really general in their nature, that generality is not established. It is the object of the following investigation to supply this deficiency, and to show in what manner the daily and yearly means may be obtained in all the periodical functions with which we are concerned in magnetism and meteorology.

2. It is known that the mean value of any magnetical or meteorological element, for any day, may be obtained, approximately, by taking the *arithmetical mean* of any number of *equidistant* observed values; the degree of approximation, of course, increasing with the number. A somewhat more exact mean may be deduced, as has been shown by Cotes and Kramp, by combining the equidistant observed values in a different manner; and Gauss has given a method, whereby the values of the integral, $\int_{-a}^{+a} U dx$, may be obtained with still greater accuracy from the observed values of the ordinate, U, corresponding to

certain definite abscissæ.* But in the case of periodical functions, it will appear from what follows that the refinement of COTES is unnecessary; and, in the case under consideration, there are practical reasons of another kind for adhering to the method of equidistant observations, and which, therefore, deprive us of the advantages of GAUSS's method.

3. Any periodical function U, of the variable x, may be represented by the series

$$U = A_0 + A_1 \sin(x + \alpha_1) + A_2 \sin(2x + \alpha_2) + A_3 \sin(3x + \alpha_3) + \&c.,$$

in which the first term, A_0, is the mean value of the ordinate U, and is expressed by the equation

$$A_0 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} U dx.$$

This is the quantity whose value is sought in the present investigation.

It is obvious that the values of U return again in the same order and magnitude when x becomes $x + 2\pi$; so that if $x = at$, the period is represented by $\frac{2\pi}{a}$. If then 2π be divided into n equal parts, so that the abscissæ of the points of division are x, $x + \frac{2\pi}{n}$, $x + \frac{4\pi}{n}$, &c., $x + \frac{2(n-1)\pi}{n}$, the sum of the corresponding ordinates will be

$$\Sigma(U) = nA_0 + A_1\Sigma \sin\left(x + \frac{2i\pi}{n} + \alpha_1\right) + A_2\Sigma \sin\left\{2\left(x + \frac{2i\pi}{n}\right) + \alpha_2\right\}$$
$$+ A_3\Sigma \sin\left\{3\left(x + \frac{2i\pi}{n}\right) + \alpha_3\right\} + \&c.$$

in which i denotes any one of the series of integer numbers, from 0 to $n - 1$ inclusive. The multiplier of A_m, in the general term of this series, is

$$\Sigma \sin\left\{m\left(x + \frac{2i\pi}{n}\right) + \alpha_m\right\}$$
$$= \sin(mx + \alpha_m)\,\Sigma \cos\frac{2im\pi}{n} + \cos(mx + \alpha_m)\,\Sigma \sin\frac{2im\pi}{n}.$$

* *Commentationes Societatis Regiæ Scientiarum Gottingensis*, tom. iii.

But, when m is not a multiple of n,

$$\Sigma \cos \frac{2im\pi}{n} = 0, \quad \Sigma \sin \frac{2im\pi}{n} = 0;$$

and, therefore, the preceding term vanishes. When m is a multiple of n,

$$\Sigma \cos \frac{2im\pi}{n} = n, \quad \Sigma \sin \frac{2im\pi}{n} = 0;\text{*}$$

and accordingly the term is reduced to

$$n \sin (mx + a_m).$$

Hence, all the terms of the series vanish, excepting those in which $m = kn$, k being any number of the natural series, and there is

$$\frac{1}{n} \Sigma(U) = A_0 + A_n \sin (nx + a_n) + A_{2n} \sin (2nx + a_{2n}) + \&c.$$

That is, the arithmetical mean of the n equidistant ordinates is equal to the sum of the terms of the original series of the order kn, whatever be the value of x.

The original series for U being always convergent, the derived series, which expresses the value of $\frac{1}{n}\Sigma(U)$, will be much more so; and, when the

* These results are easily established. The roots of the equation $y^n - 1 = 0$, being comprised in the formula $\cos \frac{2i\pi}{n} + \sqrt{(-1)} \sin \frac{2i\pi}{n}$, the m^{th} power of any one of these roots is $\cos \frac{2im\pi}{n} + \sqrt{(-1)} \sin \frac{2im\pi}{n}$; and the sum of the m^{th} powers of the roots is

$$\Sigma \cos \frac{2im\pi}{n} + \sqrt{(-1)}\ \Sigma \sin \frac{2im\pi}{n}.$$

Now, when m is not a multiple of n, this sum $= 0$, and therefore

$$\Sigma \cos \frac{2im\pi}{n} = 0, \quad \Sigma \sin \frac{2im\pi}{n} = 0;$$

as above. When m is a multiple of n, the sum of the m^{th} powers of the roots $= n$, and therefore

$$\Sigma \cos \frac{2im\pi}{n} = n, \quad \Sigma \sin \frac{2im\pi}{n} = 0.$$

This demonstration seems preferable to that derived from the general formulæ for the sum of the sines and cosines of arcs in arithmetical progression, which, in the latter of the two cases above mentioned, lead to illusory results.

number n is sufficiently great, we may neglect all the terms after the first. Hence, approximately, $A_0 = \frac{1}{n}\Sigma(U)$.

The error of this result will be expressed by the second term of the series, $A_n \sin(nx + \alpha_n)$, the succeeding terms being, for the same reason, disregarded in comparison; and accordingly the limit of error will be A_n. Thus, when the period in question is a day, we learn that the *daily mean value* of the observed element will be given by the mean of *two* equidistant observed values, nearly, when A_2 and the higher coefficients are negligible; by the mean of *three*, when A_3 and the higher coefficients are negligible; and so on.

4. The coefficient A_2 is small in the series which expresses the diurnal variation of temperature; and, consequently, the curve which represents the course of this variation is, nearly, the curve of sines. In this case, then, the mean of the temperatures at any two equidistant or *homonymous* hours is, nearly, the mean temperature of the day. The same thing holds with respect to the annual variation of temperature; and the mean of the temperatures of any two equidistant months is, nearly, the mean temperature of the year. These facts have been long known to meteorologists.

5. The coefficient A_3 is small in *all* the periodical functions with which we are concerned in magnetism and meteorology; and, therefore, the daily and yearly mean values of these functions will be given, approximately, by the mean of any *three* equidistant observed values.

In order to establish this, as regards the daily means, I have calculated the coefficients of the equations which express the laws of the mean diurnal variation of the temperature, the atmospheric pressure, and the magnetic declination, as deduced from the observations made at the Magnetical Observatory of Dublin during the year 1843. The observations were taken every alternate hour during both day and night; and the numbers employed in the calculation are the yearly mean results corresponding to the several hours. The origin of the abscissæ is taken at midnight.

6. The following is the equation of the diurnal variation of temperature:

$$\begin{aligned} U - A_0 = &+ 3^\circ{\cdot}60 \sin(x + 239^\circ{\cdot}0) + 0^\circ{\cdot}70 \sin(2x + 67^\circ{\cdot}2) \\ &+ 0^\circ{\cdot}26 \sin(3x + 73^\circ{\cdot}5) + 0^\circ{\cdot}03 \sin(4x + 102^\circ{\cdot}7) \\ &+ 0^\circ{\cdot}14 \sin(5x + 258^\circ{\cdot}6) + 0^\circ{\cdot}09 \sin(6x + 180^\circ). \end{aligned}$$

Hence the error committed, in taking the mean of the temperatures at any two equidistant hours as the mean temperature of the day, is expressed nearly by the term

$$0^\circ\cdot70 \sin (2x + 67^\circ\cdot2);$$

and consequently cannot exceed $0^\circ\cdot70$. To obtain the pairs of homonymous hours, whose mean temperature corresponds most nearly with that of the day, we have only to make $\sin (2x + 67^\circ\cdot2) = 0$; which gives for x the values

$$x = 56^\circ\cdot4, \quad x = 146^\circ\cdot4,$$

corresponding to the times

$$t = 3^h\ 46^m, \quad t = 9^h\ 46^m.$$

Accordingly, the best pairs of homonymous hours, so far as this problem is concerned, are $3^h\ 46^m$ A. M. and $3^h\ 46^m$ P. M., or $9^h\ 46^m$ A. M. and $9^h\ 46^m$ P. M.

The error committed, in taking the mean of the temperatures at any *three* equidistant hours as the mean temperature of the day, is, very nearly,

$$+\ 0^\circ\cdot26 \sin (3x + 73^\circ\cdot5);$$

and cannot therefore exceed $0^\circ\cdot26$. The best hours are those in which the angle, in the preceding expression, is equal to 180° or 360°. The corresponding values of x are

$$x = 35^\circ\cdot5, \quad x = 95^\circ\cdot5;$$

whence

$$t = 2^h\ 22^m, \quad t = 6^h\ 22^m.$$

Accordingly, the best hours of observation are

$$2^h\ 22^m \text{ A. M.}, \quad 10^h\ 22^m \text{ A. M.}, \quad 6^h\ 22^m \text{ P. M.};$$

and

$$6^h\ 22^m \text{ A. M.}, \quad 2^h\ 22^m \text{ P. M.}, \quad 10^h\ 22^m \text{ P. M.}$$

By taking the mean of any *four* equidistant observed values, the limit of error will, of course, be less. Its amount, which is the coefficient of the fourth term of the preceding formula, is only $0^\circ\cdot03$; and, accordingly, the mean temperature of the day is inferred from the temperatures observed at *any* four equidistant hours with as much precision as can be desired.

7. The law of the diurnal variation of the atmospheric pressure is contained in the following equation:

$$U - A_0 = + \cdot 0024 \sin (x + 244^\circ \cdot 3) + \cdot 0089 \sin (2x + 144^\circ \cdot 4)$$
$$+ \cdot 0008 \sin (3x + 27^\circ \cdot 9) + \cdot 0006 \sin (4x + 78^\circ \cdot 5)$$
$$+ \cdot 0001 \sin (5x + 228^\circ \cdot 7) + \cdot 0002 \sin (6x + 180^\circ).$$

The second term in this formula being the principal one, the mean of the pressures observed at any *two* equidistant hours, so far from approaching the mean daily pressure, may recede from it by the greatest possible amount within the limits of the diurnal variation. The error committed, in taking the mean of the pressures observed at *three* equidistant hours as the mean daily pressure, is, very nearly,

$$+ \cdot 0008 \sin (3x + 27^\circ \cdot 9) ;$$

and cannot therefore exceed ·0008. It is needless to inquire into the least value of this quantity, which is in all cases less than the probable error.

8. The law of the diurnal variation of the magnetic declination is expressed by the equation

$$U - A_0 = + 3' \cdot 29 \sin (x + 65^\circ \cdot 7) + 2' \cdot 08 \sin (2x + 224^\circ \cdot 5)$$
$$+ 0' \cdot 63 \sin (3x + 71^\circ \cdot 7) + 0' \cdot 30 \sin (4x + 237^\circ \cdot 5)$$
$$+ 0' \cdot 13 \sin (5x + 114^\circ \cdot 7) ;$$

the coefficient of the last term being evanescent. Hence the error to which we are liable, in taking the mean of the declinations observed at any three equidistant hours as the mean of the day, is, very nearly,

$$+ 0' \cdot 63 \sin (3x + 71^\circ \cdot 7) ;$$

and cannot exceed 0′·63. This term vanishes, and the mean of the three observed values will deviate from the true daily mean, by an amount less than the errors of observation, when

$$x = 36^\circ \cdot 1, \text{ or, } x = 96^\circ \cdot 1 ;$$

that is, when

$$t = 2^h\ 25^m, \text{ or, } t = 6^h\ 25^m.$$

Accordingly, the best hours of observation, for the elimination of the diurnal variation of the declination, are

$$2^h\ 25^m \text{ A. M.}, \quad 10^h\ 25^m \text{ A. M.}, \quad 6^h\ 25^m \text{ P. M.} ;$$

and

$$6^h\ 25^m \text{ A. M.}, \quad 2^h\ 25^m \text{ P.M.}, \quad 10^h\ 25^m \text{ P. M.} ;$$

which coincide, almost exactly, with the best hours for the determination of the mean temperature.

By taking the mean of the declinations observed at any *four* equidistant hours, as the mean of the day, the limit of error is reduced to $0'{\cdot}30$.

9. It appears from the preceding, that *any* three equidistant observations are sufficient to give the daily mean values (and, therefore, also the monthly and yearly mean values) for each of these elements, with nearly the requisite precision; and that, by a suitable choice of the hours, the degree of accuracy may be augmented as much as we please. But, in determining the *particular* hours for a continuous system of observations, this should not be made the primary ground of selection. The error of the daily means being in all cases reduced within narrow limits by the method already explained, we should choose the particular hours which correspond nearly to the maxima and minima of the observed elements, so as to obtain also the *daily ranges*. This condition will be fulfilled in the case of the *magnetic declination*, very nearly, by the hours

6 A. M., 2 P. M., 10 P. M.;

which will, moreover, give nearly the maximum and minimum of *temperature*, and of the *tension of vapour*, together with the maximum pressure of the *gaseous atmosphere*.* And, if we add the intermediate hours, 10 A. M. and 6 P. M., we shall have, nearly, the principal maxima and minima of the two other magnetic elements. Accordingly, for a limited system of magnetical and meteorological observations, at places for which the epochs of maxima and minima do not differ much from those at Dublin, the best hours of observation appear to be

6 A. M., 10, 2 P. M., 6, 10.

The conditions of the problem are altered, if at any place the laws of the diurnal variation have been already obtained from a more extended system of

* The ternary combination above proposed possesses the further advantage of coinciding, nearly, with one of those deduced above, as the *most* favourable for the determination of the *mean* temperature and *mean* declination. The errors of the resulting means are found by making $x = 90^\circ$ in the third terms of the general formulæ; and we thus find the error of temperature $= -0^\circ{\cdot}07$, while that of the declination $= -0'{\cdot}20$.

observations. In this case the mean of the day may be inferred from observations taken at *any hours* whatever, by the addition of a known correction; and the hours of observation should therefore be chosen chiefly, if not exclusively, with reference to the diurnal range of the observed elements.

10. The next question which presents itself for consideration, with respect to the daily means, is one which affects more nearly the reduction of the observations hitherto made at Dublin. In the extended system prescribed by the Council of the Royal Society in 1839, and followed at the Magnetical Observatory of Dublin during the four years commencing with 1840, observations were directed to be taken twelve times, at equal intervals, throughout the day,—namely, at the even hours of Gottingen mean time. In a system of observations so frequent, and extending over so considerable a time, blanks must unavoidably occur; and the question which presents itself here is,—in what way are the daily means to be deduced in such a case?

It has been shown that the effect of the *regular* diurnal variation may be nearly eliminated, and the mean of the day obtained, by taking the mean of three equidistant observed values. For the elimination of the *irregular* changes, however, the number of observations combined should be as great as possible; and in the case of the magnetic elements, in which these changes are often very considerable, this condition is an important one.

Now it is obvious that the twelve results of any day may be resolved into *two* groups of *six* equidistant results, or into *three* groups of *four*, or into *four* groups of *three*. Hence, when *one* result is wanting in the day, the mean may be inferred either from one group of six results, from two groups of four results, or from three of three. The last of these combinations, containing nine separate results, is, of course, to be preferred. When *two* results are wanting, the mean may be inferred from one group of four results, or from two groups of three; of which the latter combination, containing six results, is to be preferred. When *three* results are wanting, the mean of the day can only be inferred (in general) from one group of three; and when more than three are wanting, that mean cannot be generally obtained.

11. What has been said above applies to the irregular changes of short period, such, especially, as those to which the magnetic elements are subject. But there are also irregular changes of longer duration (as, for ex-

ample, those produced in the atmospheric pressure by the passage of the greater aerial waves), which complicate the problem, inasmuch as a different process is required for their elimination.

In the reduction of the magnetical and meteorological observations made at the Observatory of Dublin, the *civil* day is adopted; and the observations being made at the *odd* hours of Dublin mean time, very nearly, the *epoch* of the mean of all the twelve results is *mean noon.* But in the case of deficient observations, the epoch of the mean, inferred from the remaining observations, may deviate one or more hours from noon; and its amount, therefore (as compared with the mean *reduced to noon*), is affected by an error equal to the change which the observed element undergoes in that time. In the case of the atmospheric pressure, this error is often very considerable, and much exceeds that due to the changes of whose elimination we have hitherto spoken.

The law of the changes here referred to being unknown, we can only deal with them on the assumption that their course is uniform throughout the space of a day; and this assumption will, probably, seldom err much from the truth. Upon this principle, the effect of the irregular change will be eliminated by taking the mean of two or more results *equidistant from noon* (that is, the mean of a forenoon and afternoon result corresponding to the hours x and $12 - x$, or any combination of such means); and we have only to consider in what manner this process can be combined with the elimination of the *regular* diurnal change.

Let the mean of the *four* equidistant observed values commencing with the n^{th} hour be denoted, for brevity, by IV_n; then the *epochs* of the means IV_1, IV_3, IV_5, are 10 A.M., noon, and 2 P.M., respectively; so that the two conditions are satisfied by the combinations

$$\tfrac{1}{2}(IV_1 + IV_5), \text{ and } IV_3.$$

In like manner, the means of any *three* equidistant observed values being denoted by III_n, the epochs of the means III_1, III_3, III_5, III_7, are 9 A.M., 11, 1 P.M., and 3 respectively; so that both conditions are satisfied by the combinations

$$\tfrac{1}{2}(III_1 + III_7), \quad \text{and} \quad \tfrac{1}{2}(III_3 + III_5).$$

12. When, from the number and disposition of the blanks, none of these combinations can be had, and therefore both changes (regular and irregular)

cannot be eliminated, we must attend chiefly to that which is greater in amount. For the purpose of comparing their magnitude, I have taken the differences of the successive daily means, for the declination, the atmospheric pressure, and the temperature, as deduced from the observations of the year 1843 ; and have calculated the square root of the mean of the squares of these differences. The results, which may be taken as the measures of the irregular changes from day to day, are the following:

Mean Fluctuation from Day to Day.

Magnetic declination, . . . Fluctuation = 1′·04.
Atmospheric pressure, ... 0·214.
Atmospheric temperature, ... 3°·07.

Similarly, if we take the differences of the yearly means corresponding to the successive hours of observation, and combine them in the same way, we obtain the mean two-hourly fluctuations, arising from the regular diurnal change. These numbers are the following :

Mean Fluctuation in two Hours.

Magnetic declination, . . . Fluctuation = 2′·04.
Atmospheric pressure, ... 0·0065.
Atmospheric temperature, ... 1°·46.

These numbers, compared with the twelfth part of the former, serve to measure the relative magnitude of the regular and irregular changes to which the elements are subject in the same time. We thus find that, in the case of the magnetic declination, the irregular change (which is less than $\frac{1}{20}$th part of the regular) may be safely neglected; and we have only to attend to the diurnal changes, and to the irregular changes of *short* period. The daily means are, therefore, to be deduced from one of the combinations of Art. 10, giving the preference to that which contains the greatest number of individual results.

In the case of the atmospheric temperature, the irregular change (which is less than one-fifth part of the regular) is small; aud we must attend chiefly to the latter. The mean of the day is, therefore, to be inferred from one of the combinations of Art. 10, giving the preference to those of Art. 11, whose epoch is noon.

In the case of the atmospheric pressure, on the contrary, the irregular change (which is triple the regular) is the more important. The mean of the day is, therefore, to be deduced from any combination whose epoch is noon, giving, however, the preference to one of those of Art. 11, in which the diurnal change is also eliminated.

13. I now proceed to consider the reduction of the *monthly* means, in the case of deficient observations.

For the purpose of determining the regular diurnal variation of any magnetic or meteorological element, it is necessary to take the mean of an adequate number of separate results corresponding to each hour of observation, so as to eliminate the irregular and accidental changes. The results usually so combined are those of each month. Their number is, in general, sufficient for the purpose above-mentioned; while, on the other hand, the course of the diurnal change is sufficiently different from one month to the next, to demand a separate determination.

But in the case of deficient observations, the monthly means of the results corresponding to each hour will not exhibit, in general, the true course of the diurnal change without a correction. If a result be wanting at one hour of a day, in which all the results are much *above the mean*, it is obvious that the monthly mean corresponding to that hour will be *too small*, as compared with the means of the other hours; while, on the other hand, it will be *too great*, when all the results of the day in question are *below the mean*. The error will be greater, the greater the variation of the element observed from day to day. In the case of the atmospheric pressure it is so considerable, that the uncorrected monthly means afford no approximation to the law of the diurnal change, in the case of deficient observations.

The remedy which first suggests itself, in such a case, is to omit all the results of a day in which one or more are wanting. This process is inartificial and unsatisfactory. The *weight* of the mean is diminished in the proportion of the number of observations combined; and it is therefore important to employ all the observed results in its deduction, provided we can obtain a correction. Such a correction is easily found.

14. Let x denote the observed value of any element, at any hour on any day; and let a denote its *mean* value for that day; then

$$x = a + \xi,$$

in which ξ is the magnitude of the diurnal variation corresponding to the hour in question. Let there be n days of observation to be combined; then, summing the n results, dividing by n, and denoting the mean values by $\underline{x}$, $\underline{a}$, and $\underline{\xi}$,

$$\underline{x} = \underline{a} + \underline{\xi}.$$

Now, at any particular hour of any day, let *one* of the results be wanting; and let a' denote the mean for that day; summing the $n-1$ results,

$$S_{n-1}\, x = S_n a - a' + S_{n-1}\xi.$$

And dividing by $n-1$,

$$\underline{x} = \frac{S_n a - a'}{n-1} + \underline{\xi} = \underline{a} + \frac{\underline{a} - a'}{n-1} + \underline{\xi};$$

whence

$$\underline{x} + \frac{a' - \underline{a}}{n-1} = \underline{a} + \underline{\xi}.$$

The correction, therefore, is $+\dfrac{a' - \underline{a}}{n-1}$.

Similarly, if p results be wanting, we find

$$\underline{x} + \frac{Sa' - p\underline{a}}{n-p} = \underline{a} + \underline{\xi},$$

in which Sa' denotes the sum of the means of the days on which the deficiencies occur. Hence, the correction to be applied to the observed mean, $\underline{x}$, deduced from the $n-p$ values, is $+\dfrac{Sa' - p\underline{a}}{n-p}$.

15. The preceding correction depends, as might have been anticipated, on the difference of the daily means, for the days of deficient observations, and the *mean daily mean.* With the view of determining its probable amount, I have taken the differences between the mean of each day and the mean of the month, for the declination, the atmospheric pressure and temperature, as deduced from the

observations of the year 1843; and have calculated the square root of the mean of the squares of these differences, or the values of the expression $\sqrt{\frac{\{\Sigma(a-a)^2\}}{n}}$. The values of this quantity, which may be denominated the *mean daily error,* are the following:

Mean daily Error.

Magnetic declination, ...	Daily error = 0′·95.
Atmospheric pressure, ...	0·301.
Atmospheric temperature, ...	4°·25.

Now the mean value of n in each month (the Sundays being omitted) is 26. Hence the *mean correction,* in the case of a single deficient observation, is, for the magnetic declination, 0′·04; for the atmospheric pressure, 0·012; and for the temperature, 0°·17. In the case of the two meteorological elements, and especially in that of the atmospheric pressure, the correction is too considerable to be overlooked; in the case of the magnetic declination, and, probably, also in that of the other magnetic elements, it may be disregarded.

IV.—*Results of Observations made at the Magnetical Observatory of Dublin, during the Years* 1840–43. *By the* Rev. Humphrey Lloyd, *D. D.*, President; *F. R. S.; Hon. F. R. S. E.; Corresponding Member of the Royal Society of Sciences at Gottingen; Honorary Member of the American Philosophical Society, of the Batavian Society of Sciences, and of the Society of Sciences of the Canton de Vaud, &c.*

Read May 11 and 25, 1846.

FIRST SERIES.—MAGNETIC DECLINATION.

1. THE observations at stated hours, in the Magnetical Observatory of Dublin, commenced in November, 1838, and were at first taken twelve times during the day. Throughout the greater part of the following year, they were made at least eight times daily, with some variations as to the precise hours; and, at the beginning of the year 1840, the number of assistant observers was increased to three, and the observations were made *every alternate hour*, night and day, according to the comprehensive scheme recommended by the Council of the Royal Society, and followed at more than thirty observing stations in various parts of the globe. This plan has been in operation at the Dublin Magnetical Observatory until the end of the year 1843, when it was discontinued; four years' observations having been found sufficient for the determination of all the phenomena connected with the diurnal changes. The observations have been since continued upon a different and reduced scale, and with a view to other classes of phenomena.

I shall not, in this place, enter into any account of the instruments, or methods of observation, as these will be fully explained in the publication in which the observations themselves are presented in detail. I desire merely to lay before the Academy the principal conclusions already arrived at. In the present paper, accordingly, I shall give the results of the observations of the magnetic declination during the four years referred to;* and in those which I hope hereafter to communicate, I shall discuss in like manner the observations of the other magnetic and meteorological elements made during the same period.

Diurnal Variation.

2. A very limited series of observations is sufficient to exhibit the general features of the diurnal variation; but an extended one is necessary, if it be desired to ascertain with accuracy the mean amount of the changes. To determine these with precision, observations should be taken daily, at equal intervals not exceeding three hours, and be continued for one or more years. The course usually adopted in the reduction of such a series is, to combine separately the observations of each month, taking the arithmetical mean of all the results corresponding to the same hour. In this manner the course of the variation (which alters considerably throughout the year) is deduced for each month separately; and when the observations extend over several years, the monthly means of the separate years are to be again combined, each into a single mean.

Even this, however, is insufficient. The mean results thus obtained are deformed by the irregular fluctuations, which are often far greater than the regular changes; and it is necessary to omit the observations taken on days of disturbance, before we can deduce a correct mean from the results of any practicable series. This is proved in a striking manner by the observations of July, 1842. Owing to the great disturbance which took place on the 2nd and 4th of that month, the difference of the monthly means corresponding to 5 A.M., when these observations are retained and when they are omitted, amounts to $5'{\cdot}76$; so that the observations should be continued for fifty-seven years, in order to reduce the error to $0'{\cdot}1$.

In the final reduction of the Dublin observations, accordingly, all the results

* I have taken advantage of the delay which has occurred in the printing of this paper, to introduce the monthly mean results of the three following years, in the deduction of the annual and secular changes, and to make some other minor alterations of detail. The general conclusions originally arrived at are, however, not affected.

obtained on days of disturbance have been omitted,—those being defined to be days of disturbance, in which the sum of the differences between the separate results, and the monthly means corresponding to the same hours, exceeds a certain limit, which is about the double of its mean value. The number of separate observations actually combined, in deducing the monthly means for each hour, is, on the average, 86. The total number of observations employed exceeds 12,000.

3. The hours of observation, in accordance with the instructions of the Council of the Royal Society, were the even hours of Gottingen mean time. This being $1^h\ 4^m\ 50^s$ in advance of Dublin time, the observation hours are, nearly, the *odd hours of Dublin mean time.* The following are the differences of the monthly mean results corresponding to each hour, and the mean of the twelve, expressed in minutes. The *positive* numbers correspond to *easterly* deviations of the north pole of the magnet, and the *negative* to *westerly*.

TABLE I. DIURNAL VARIATION OF THE MAGNETIC DECLINATION.

	1 A.M.	3	5	7	9	11	1 P.M.	3	5	7	9	11
January, .	+1′·09	+0′·51	+0′·69	+0′·61	+0′·69	−2′·27	−4′·27	−2′·69	−0′·81	+0′·65	+2′·67	+3′·14
February,	+1·99	+1·15	+1·28	+1·46	+0·92	−2·46	−5·30	−3·44	−1·15	+0·59	+2·14	+2·80
March, . .	+2·08	+2·49	+2·37	+2·14	+1·85	−3·86	−7·21	−5·07	−1·17	+0·58	+2·94	+2·81
April, . . .	+2·36	+2·14	+3·15	+4·55	+2·52	−4·47	−8·75	−5·66	−1·51	+0·91	+1·94	+2·85
May, ...	+1·79	+2·54	+3·94	+5·16	+2·14	−4·12	−7·06	−5·13	−1·83	+0·29	+0·95	+1·31
June, ...	+2·09	+2·29	+4·18	+5·10	+2·55	−3·51	−7·16	−6·00	−2·42	+0·04	+1·02	+1·81
July, ...	+2·06	+2·54	+4·13	+4·31	+1·99	−3·56	−7·11	−5·64	−2·27	+0·09	+1·49	+1·98
August, .	+2·15	+2·52	+3·63	+3·70	+1·14	−4·68	−7·86	−4·98	−1·12	+1·05	+2·06	+2·37
September,	+1·77	+2·29	+1·52	+2·56	+0·85	−5·08	−7·58	−4·25	−0·32	+2·32	+3·02	+2·91
October, .	+2·06	+1·20	+1·37	+1·30	+0·99	−4·36	−6·16	−3·82	−0·28	+1·54	+3·56	+2·61
November,	+1·04	+1·08	+0·87	+0·58	+0·22	−3·02	−4·12	−2·30	−0·14	+1·02	+2·85	+1·91
December,	+0·59	+0·50	+0·44	+0·21	+0·21	−2·28	−3·45	−2·27	−0·45	+1·07	+3·03	+2·37
Summer, .	+2·04	+2·39	+3·43	+4·23	+1·87	−4·23	−7·59	−5·27	−1·58	+0·79	+1·75	+2·21
Winter, . .	+1·47	+1·15	+1·17	+1·05	+0·81	−3·05	−5·09	−3·27	−0·67	+0·91	+2.86	+2·60
Year, ...	+1.76	+1·77	+2·30	+2·64	+1·34	−3·64	−6·34	−4·27	−1·12	+0·85	+2·30	+2·40

4. The general features of the phenomenon, as deduced from these numbers, are the following:

I. Between 6 A. M. and 8 A. M. (the time varying with the season) the north pole of the magnet begins to move *westward*, and, therefore, the westerly declination *increases.* This movement continues until about 1 P. M., when the declination attains its *maximum.*

II. After 1 P. M. the north pole of the magnet moves *eastward*, and the declination *diminishes*, but at a slower rate than it had previously increased. This easterly movement continues until between 9 P. M. and 11 P. M., when the declination is a *minimum.*

III. There is a second, but much smaller, oscillation of the magnet during the night and morning; the north pole moving slowly to the *west* for a few hours before and after midnight, and afterwards returning to the *east* until between 6 A. M. and 8 A. M., when the declination is again a *minimum.*

IV. In summer the westerly movement during the night becomes nearly insensible. In winter, on the contrary, the easterly movement during the morning nearly vanishes; and the magnet is almost in a state of repose from 2 A. M. to 8 A. M.

V. In summer the morning easterly elongation is greater than the evening one; and, consequently, the greatest range is between 7 A. M. and 1 P. M. In winter, on the contrary, the evening easterly elongation is greater than the morning; and the greatest range is between 1 P. M. and 10 P. M. The total range is greater in summer than in winter.

These general characteristics of the diurnal variation may be most readily understood by a reference to Plates I. and II.

5. In order to determine the laws of the phenomenon with more precision, it will be desirable to express the difference between the declination at any hour, and the mean of the entire day, as a function of the time.

If Δ be taken to denote this difference corresponding to any time,

$$\Delta = \Sigma \left(A_i \cos ix + B_i \sin ix\right);$$

in which $x = n \times 15^\circ$, n being the number of hours, and parts of an hour, in the time reckoned from the epoch of the first observation, and i any number of the natural series. Then, since observation gives the values of Δ corresponding to $n = 0, 2, 4, \&c. \ldots 22$, we have twelve equations of condition, from which twelve coefficients of the periodical function may be deduced by elimination. The first of these, $A_0, = 0$; the following are the values of the remaining eleven.

Table II. Coefficients of the Equation of the Diurnal Curve of Declination.

	A_1	A_2	A_3	A_4	A_5	A_6	B_1	B_2	B_3	B_4	B_5
January, .	+ 2·426	− 1·119	+ 0·313	− 0·481	− 0·059	+ 0·009	− 0·260	− 0·942	− 0·362	+ 0·062	− 0·121
February,	+ 2·941	− 1·322	+ 0·607	− 0·313	+ 0·097	− 0·018	+ 0·264	− 0·802	− 0·257	+ 0·043	− 0·085
March, . .	+ 3·988	− 2·110	+ 0·777	− 0·598	− 0·119	+ 0·147	+ 0·688	− 0·739	− 0·112	+ 0·297	− 0·019
April, . . .	+ 4·374	− 2·912	+ 1.172	− 0·235	+ 0·009	− 0·051	+ 1·403	− 0·681	− 0·527	+ 0·133	− 0·109
May, ...	+ 3·747	− 2·670	+ 0·712	+ 0·047	− 0·034	− 0·010	+ 2·003	− 0·109	− 0·438	+ 0·173	− 0·006
June, ...	+ 3·928	− 2·597	+ 0·697	+ 0·018	+ 0·000	+ 0·044	+ 2·264	− 0·551	− 0·348	− 0·029	− 0·082
July, ...	+ 4·000	− 2·410	+ 0·545	− 0·163	+ 0·040	+ 0·047	+ 1·919	− 0·453	− 0·248	+ 0·014	− 0·072
August, . .	+ 4·241	− 2·617	+ 0·723	− 0·238	+ 0·041	+ 0·002	+ 1·032	− 0·121	− 0·367	+ 0·078	− 0·073
September,	+ 3·989	− 2·548	+ 0·890	− 0·233	− 0·205	− 0·124	− 0·128	− 0·355	− 0·282	+ 0·416	− 0·033
October, .	+ 3·452	− 1·991	+ 0·667	− 0·316	− 0·009	+ 0·256	− 0·335	− 0·625	− 0·285	+ 0·374	− 0·069
November,	+ 2·363	− 1·291	+ 0·253	− 0·369	− 0·036	+ 0·121	− 0·436	− 0·354	− 0·185	+ 0·322	+ 0·031
December,	+ 2·053	− 1·099	+ 0.055	− 0·392	− 0·088	+ 0·064	− 0·578	− 0·737	− 0·170	+ 0·200	− 0·021
Summer, .	+ 4·047	− 2·626	+ 0·790	− 0·136	− 0·022	− 0·017	+ 1·415	− 0·379	− 0·370	+ 0·131	− 0·065
Winter, .	+ 2·871	− 1·492	+ 0·445	− 0·410	− 0·036	+ 0·097	− 0·109	− 0·698	− 0·228	+ 0·216	− 0·049
Year, . .	+ 3·458	− 2·058	+ 0·620	− 0·272	− 0·029	+ 0·041	+ 0·653	− 0·537	− 0·298	+ 0·173	− 0·057

6. From the inspection of the numbers of this Table, we draw two important conclusions:

I. The values of the four latter coefficients, B_4, A_5, B_5, A_6, being small, all the terms of the series beyond the eighth may be neglected as inconsiderable. From this it follows, that *eight observations, made at equal intervals, are sufficient to determine the course of the diurnal variation.*

II. On comparison of the values of A and B for the separate months, it appears that there is a general resemblance in the course of the diurnal variation in the six months from April to September inclusive, as well as in the six months from October to March inclusive; and that thus the curves for the separate months distribute themselves naturally into two groups, in one of which the sun is to the north, and in the other to the south of the Equator.*

Hence, if we confine our attention to the three latter rows of the preceding Table, which give the values of the coefficients for the summer half-

* This fact appears likewise upon an examination of the immediate results of observation, as given in Table I.; and still more readily by the inspection of the curves in which these changes are graphically represented.—(See Plates I. and II.)

year, the winter half-year, and the whole year, respectively, the mean value of Δ at any hour will be expressed by the following equations, in which $x = n \times 15°$, n denoting the number of hours, and parts of an hour, *reckoned from midnight:*

Summer Half-year.

$$\Delta_n = 4'\cdot288 \sin(x + 55° 44') + 2'\cdot653 \sin(2x + 231° 47')$$
$$+ 0'\cdot872 \sin(3x + 70° 6') + 0'\cdot189 \sin(4x + 253° 56').$$

Winter Half-year.

$$\Delta_n = 2'\cdot873 \sin(x + 77° 10') + 1'\cdot647 \sin(2x + 214° 56')$$
$$+ 0'\cdot500 \sin(3x + 72° 7') + 0'\cdot463 \sin(4x + 237° 47').$$

Whole Year.

$$\Delta_n = 3'\cdot519 \sin(x + 64° 18') + 2'\cdot127 \sin(2x + 225° 22')$$
$$+ 0'\cdot688 \sin(3x + 70° 40') + 0'\cdot322 \sin(4x + 242° 27').$$

7. It is manifest that the coefficients of the equation of the diurnal curve may be generally expressed as periodical functions of the time, reckoned from a given epoch of the year. For this purpose we have only to apply to the values of A and B, belonging to the several months of the year (Table II.), the same process which has been already applied to the values of Δ, corresponding to the several hours of the day. We thus obtain the following formulæ, in which $x = n \times 30°$, n denoting the number of months, and parts of a month, reckoned from the 1st of January. The terms of the series which follow those here given are neglected as inconsiderable.

$$A_1 = +\,3'\cdot458 + 0'\cdot927 \sin(x + 280° 37') + 0'\cdot541 \sin(2x + 294° 56');$$
$$B_1 = +\,0'\cdot653 + 1'\cdot382 \sin(x + 297° 29') + 0'\cdot265 \sin(2x + 110° 21');$$
$$A_2 = -\,2'\cdot058 + 0'\cdot830 \sin(x + 96° 39') + 0'\cdot339 \sin(2x + 83° 41');$$
$$B_2 = -\,0'\cdot537 + 0'\cdot269 \sin(x + 239° 56') + 0'\cdot078 \sin(2x + 210° 48');$$
$$A_3 = +\,0'\cdot620 + 0'\cdot264 \sin(x + 297° 5') + 0'\cdot280 \sin(2x + 281° 40');$$
$$B_3 = -\,0'\cdot298 + 0'\cdot082 \sin(x + 119° 1') + 0'\cdot030 \sin(2x + 20° 26');$$
$$A_4 = -\,0'\cdot272 + 0'\cdot194 \sin(x + 269° 36') + 0'\cdot100 \sin(2x + 147° 17');$$
$$B_4 = +\,0'\cdot173 + 0'\cdot108 \sin(x + 144° 23') + 0'\cdot150 \sin(2x + 248° 50').$$

8. If we calculate the values of Δ, corresponding to the *even* hours of Dublin mean time, by means of the formula of Art. 5 and the numbers of Table II., we obtain the following values, in which the *positive* numbers correspond to *easterly* deviations of the north pole of the magnet, as before.

Table III. Diurnal Variation of the Declination (calculated Values).

	2 A.M.	4	6	8	10	Noon.	2 P.M.	4	6	8	10	12
January, .	+0′·48	+0′·73	+0′·54	+0′·88	−0′·47	−3′·77	−3′·74	−1′·65	−0′·12	+1′·64	+3′·31	+2′·19
February,	+1·42	+1·15	+1·39	+1·39	−0·40	−4·44	−4·76	−2·13	−0·29	+1·45	+2·62	+2·56
March, . . .	+2·22	+2·64	+2·01	+2·53	−0·59	−6·31	−6·70	−2·92	−0·21	+1·79	+3·27	+2·27
April, . . .	+2·06	+2·51	+3·98	+4·24	−0·64	−7·57	−7·80	−3·39	−0·08	+1·46	+2·48	+2·75
May, ...	+2·12	+3·12	+4·82	+4·35	−1·00	−6·29	−6·50	−3·44	−0·56	+0·73	+1·12	+1·53
June, ...	+2·02	+3·13	+4·95	+4·38	−0·35	−5·96	−7·10	−4·27	−0·91	+0·62	+1·41	+2·08
July, ...	+2·13	+3·36	+4·48	+3·55	−0·56	−5·95	−6·91	−3·94	−0·94	+0·91	+1·80	+2·05
August, . .	+2·17	+3·10	+3·89	+2·89	−1·58	−7·07	−6·93	−2·86	+0·15	+1·67	+2·30	+2·27
September,	+2·03	+1·95	+1·82	+2·49	−2·01	−7·19	−6·31	−2·19	+1·26	+2·75	+3·19	+2·19
October, .	+1·57	+1·27	+1·22	+1·64	−1·42	−6·04	−5·31	−1·89	+0·66	+2·76	+3·30	+2·28
November,	+1·04	+1·08	+0·60	+0·71	−1·25	−4·07	−3·44	−1·04	+0·36	+2·07	+2·69	+1·25
December,	+0·37	+0·61	+0·18	+0·46	−0·86	−3·24	−3·05	−1·31	+0·26	+2·16	+3·12	+1·34
Summer, .	+2·09	+2·87	+3·97	+3·67	−1·02	−6·69	−6·93	−3·35	−0·17	+1·35	+2·04	+2·15
Winter, . .	+1·19	+1·25	+0·98	+1·26	−0·83	−4·65	−4·49	−1·83	+0·12	+1·98	+3·05	+1·97
Year, ...	+1·64	+2·05	+2·49	+2·46	−0·93	−5·67	−5·72	−2·59	−0·01	+1·66	+2·55	+2·07

These numbers, together with those of Table I., are projected in curves in Plates I. and II. already referred to. Plate I. contains the curves of the six months of the summer half-year, together with the mean of the six; Plate II. those of the six months of the winter half-year, and the mean. The scale is one-tenth of an inch to a minute of arc.

9. The hours of *greatest* and *least* declination are deduced from the general equation, by making $\frac{d\Delta}{dx} = 0$; they are consequently given by the formula

$$\Sigma i \left(B_i \cos ix - A_i \sin ix \right) = 0.$$

Substituting for A_i and B_i their numerical values (Table II.), and solving the resulting equations by approximation, we obtain the following results:

Epoch of greatest Westerly Elongation.

Summer half-year, ...	0^h	58^m P. M.
Winter half-year, ...	0	47
Whole year, ...	0	54

Epochs of greatest Easterly Elongation.

Summer half-year, . . .	$6^h\ 50^m$ A. M. .	$11^h\ 8^m$ P. M.
Winter half-year, . . .	7 59	9 38
Whole year, ...	7 12	10 0

The hours of *mean* declination (or those at which the curve crosses the axis of abscissæ) are in like manner deduced from the equation

$$\Sigma(A_i \cos ix + B_i \sin ix) = 0.$$

The following are the results:

Epochs of Mean Declination.

Summer half-year, . . .	$9^h\ 36^m$ A. M. .	$6^h\ 3^m$ P. M.
Winter half-year, . . .	9 30	5 54
Whole year, ...	9 34	5 59

10. The critical hours for the separate months are given in the following Table:

Table IV. Hours of Greatest, Least, and Mean Declination.

Month.	Westerly Elongation.	Easterly Elongation.		Mean Declination.	
	P. M.	A. M.	P. M.	A. M.	P. M.
January, . .	$0^h\ 52^m$	$8^h\ 9^m$	$10^h\ 21^m$	$9^h\ 35^m$	$6^h\ 5^m$
February, . .	1 1	7 25	10 51	9 41	6 13
March, ...	1 0	8 7	9 55	9 41	6 10
April, ...	0 55	7 11	11 12	9 45	6 3
May, ...	0 58	6 55	———	9 31	6 30
June, ...	1 21	6 40	———	9 47	6 45
July, ...	1 11	6 9	———	9 43	6 43
August, . . .	0 52	6 11	10 55	9 20	5 47
September, .	0 35	7 31	9 25	9 13	5 1
October, . . .	0 30	8 7	9 3	9 25	5 9
November, .	0 25	7 55	9 23	9 7	5 12
December, .	0 30	8 9	9 35	9 11	5 31

The numbers of the preceding Table, notwithstanding some irregularities, exhibit very distinctly the influence of season upon the critical hours. The epoch of *greatest westerly elongation* occurs *latest* about the time of the summer solstice; and *earliest* in the last quarter of the year, or between the autumnal equinox and the winter solstice. The same thing holds with respect to the epochs of *mean declination*, which (as might have been expected) appear to be governed in great measure by the time of westerly elongation.

The epochs of *greatest easterly elongation* appear to be governed by the times of sunrise and sunset, and are, consequently, much more variable. The forenoon easterly elongation is *earliest* about the time of the summer solstice, and *latest* at that of the winter solstice; while the case of the afternoon easterly elongation is nearly the reverse. In the months of May, June, and July, in fact, there is no change in the direction of the movement during the night, but the needle is quiescent for a few hours after midnight, and then the north pole resumes its easterly movement until after 6 A. M.

The critical hours of greatest constancy throughout the year are those of the greatest westerly elongation, and those of the forenoon mean; the extreme difference between any of these hours, and the mean for the entire year, being twenty-eight minutes. The differences are much lessened, if *apparent* be substituted for *mean* time.

11. I proceed, in the next place, to state the results connected with the diurnal range.

The morning easterly elongation being greater than the evening one in summer, and less in winter, it follows that a complete view of the phenomena connected with the magnitude of the oscillation cannot be had, without taking into account the double range. This is accordingly done in the following Table, the first column of which gives the range of the *westerly* movement, between 7 A. M. and 1 P. M., nearly; and the second that of the succeeding *easterly* movement, from 1 P. M. to 10 P. M. nearly.

TABLE V. RANGES OF THE DECLINATION IN EACH MONTH.

Summer Half-year.			Winter Half-year.		
Month.	Westerly Movement.	Easterly Movement.	Month.	Westerly Movement.	Easterly Movement.
April, ...	13′·3	11′·6	October, ...	7′·8	9′·7
May, ...	12 ·2	8 ·4	November, .	4 ·8	7 ·0
June, ...	12 ·3	9 ·2	December, .	3 ·9	6 ·6
July, ...	11 ·6	9 ·2	January, ..	5 ·2	7 ·6
August, ...	11 ·8	10 ·2	February, ..	6 ·	8 ·1
September, .	10 ·1	10 ·8	March, ...	9 ·7	10 ·5
Mean, ...	11 ·9	9 ·9	Mean, ...	6 ·4	8 ·3

It appears from the foregoing Table, that (as above stated) the greatest range in summer is that of the westerly movement, its mean value being 11′·9; while, in winter, the greatest range is that of the easterly movement, and its mean value is 8′·3. It is remarkable, however, that the mean ranges of the easterly and westerly movements, for the entire year, are precisely equal, the mean value of each being 9′·1.

The *greatest* value of the maximum range is that of April, and its amount is 13′·3 ; the range then decreases until about the middle of July, and afterwards increases, attaining a second, but smaller maximum in August. The *least* value of the maximum range is that of December, and its amount is 6′·6, being one-half of the greatest value. The *mean* value of the maximum range, for the entire year, is 10′·1.

The unexpected fact, of the occurrence of the greatest ranges in April and August, was first noticed by BEAUFOY. He seemed to think, however, that the result was only an apparent one, and arose from the circumstance, that the times of observation approached more nearly the epoch of greatest elongation in April than in June. The fact has been since noted also by GAUSS, in his account of the Gottingen observations. " The differences" (of the declination at 8 A. M. and 1 P. M.), he observes, " are not greatest at the time of the summer solstice, but appear smaller in June and July than in April, May, and August ;" but he concludes, with BEAUFOY, that this was due to the accidental circumstance, that the *whole* range was not observed near the solstice, the time of the greatest easterly elongation being then earlier than 8 A. M. It is manifest, however, that such an explanation will not apply to the result deduced, as in the present instance, from the diurnal curve ; and there can be no longer any doubt of the reality of the phenomenon.

12. The physical dependence of the changes of declination upon the sun is evident from the fact that they observe a *diurnal* and an *annual* period. The conclusion deducible from this fact has been confirmed by the leading features of the diurnal movement. Thus it has been long ago observed, that the time of greatest westerly elongation follows the sun's meridian passage at a nearly constant interval ; and that the times of greatest easterly elongation, in the morning and evening, are in like manner connected, although not so closely, with the hours of sunrise and sunset. The greater magnitude of the range, in summer than in winter, is another obvious confirmation of the same view.

We may, I believe, disregard, as wholly untenable, the hypothesis originally proposed by Coulomb, in which the influence of the sun is assumed to be *direct*, and the effect of magnetic polarity in that body. It is easy to show that, if such an action exist at all, it cannot certainly account for the principal part of the observed effect. But, without dwelling on the negative side of the question, I hope to show that the sun acts indirectly, by means of his heating power exerted upon the earth's surface. This has been assumed by Canton, and since by Professor Christie, in the hypotheses which they have severally devised to account for the diurnal variation of the declination ; but the evidence upon which it rested did not extend beyond the facts which have just been stated. It will appear from the following examination, that the connexion between the changes of declination and those of temperature is more intimate than has been hitherto supposed.

13. The force which produces the deviation of the magnet from its mean position, at any moment of the day, is measured by the sine of the deviation,— or, since the deviation is small, by the angle of deviation itself, or by the ordinate of the diurnal curve ; and the sum of all these forces throughout the day, or the integral of the diurnal action, is measured by the area of the diurnal curve. If, then, the diurnal variation of the declination be the result of the diurnal variation of temperature, we should expect to find a marked correspondence between the areas of the diurnal curves of the two elements, throughout the year.

The following Table contains the computed values of these two functions, for the several months of the year, the units being one minute of declination, one degree of temperature, and one hour of time.

Table VI. Areas of the diurnal curves of Declination and Temperature.

	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Declination, ...	39·3	42·2	64·4	79·5	72·5	76·7	74·6	74·6	69·8	58·9	39·0	34·5
Temperature, . .	38.9	37·6	72·0	93·9	96·9	101.8	91·1	94·3	81·2	64·8	34·6	23·8

For the purpose of comparison, the preceding numbers are graphically projected in Plate III. figs. 1 and 2, one division of the scale corresponding to four minutes of declination, and to eight degrees of temperature. These curves exhibit in the clearest manner the correspondence between the two classes of phenomena, and leave no doubt whatever that they stand in the relation of effect and cause. The slight dissimilarities which exist between them are abundantly accounted for by the circumstance, that it is to the heating power of the sun, exerted upon the *earth's surface* (and not upon its atmosphere), that we must ascribe the changes of declination; and I venture to predict, that as soon as we are in possession of data respecting the diurnal changes of temperature of the earth's surface, sufficient for the purposes of a comparison such as that now made, the agreement of the laws will be found to be still more complete.

The same agreement appears, also, upon a comparison of the mean yearly values of the same functions, calculated for the four years; the following are the results.

Table VII. Areas of the diurnal curves of Declination and Temperature, for the separate years.

	1840.	1841.	1842.	1843.
Declination, ...	68·3	62·3	59·2	56·0
Temperature, . . .	84·0	66·0	62·7	55·6

Annual and Secular Variations.

14. The mean yearly values of the declination, for the seven years from 1840 to 1846, inclusive, are given in the following Table; the deviation of the north pole of the magnet from the astronomical meridian being measured from the north *eastward*. The third column of the Table contains the differences of the declination in the successive years, or the yearly amounts of the secular change.

Table VIII. Mean Yearly Values of the Declination, for the years 1840–46.

Year.	Declination.	Differences.
1840	332° 30′·69	
1841	34 ·65	+ 3′·96
1842	43 ·55	+ 8 ·90
1843	50 ·10	+ 6 ·55
1844	53 ·43	+ 3 ·33
1845	59 ·75	+ 6 ·32
1846	333° 7 ·22	+ 7 ·47

If n denote the number of years reckoned from any epoch, D_0 the declination at that epoch, and ϵ the change from year to year, the actual declination, D_n, will be given by the formula

$$D_n = D_0 + n\epsilon,$$

on the supposition that the change of declination is proportional to the time. But the middle of the year 1840 being taken as the epoch, the values of D_n, corresponding to $n = 0, 1, 2 \ldots 6$, are given in the preceding Table; so that we have seven equations for the determination of D_0 and ϵ. Combining these equations by the method of least squares, we obtain

$$D_0 = 332^\circ\ 30'{\cdot}30\,; \quad \epsilon = +\ 6'{\cdot}060.$$

Consequently the north end of the magnet moves to the *east* from year to year, and the westerly declination therefore diminishes, by 6′·06 annually, in its mean quantity. The amount (as will be seen from the Table) varies considerably in different years.

Subducting 3′·03 from the value of D_0 given above, the mean declination for January 1, 1840, is 332° 27′·27. Wherefore, the mean declination at any time is

$$D_n = 332^\circ\ 27'{\cdot}27 + 6'{\cdot}06 \times n,$$

n denoting the number of years, and parts of a year, reckoned from Jan. 1, 1840.

15. Subducting the mean values for each year from those of the separate months, we obtain a series of numbers which represent the course of the *annual variation*. During the year 1840 the declinometer was twice readjusted; and from this, and other causes, the monthly values of the absolute declination for that year cannot be relied upon with certainty. The following Table contains the values of the differences for the three following years, together with their means for the whole period:

Table IX. Annual Variation of the Declination for the years 1841–43.

Year.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1841, ...	+0′·75	+0′·92	−0′·70	−0′·12	−2′·04	−4′·11	−2′·48	−1′·80	−0′·41	+1′·24	+3′·37	+5′.40
1842, ...	−1·72	−1·52	−0·56	−1·01	−2·66	−3·61	−2·80	−0·76	+1·64	+2·76	+4·36	+5·94
1843, ...	+0·29	+0·51	+0·85	+0·32	+0·05	+0·13	−1·71	−2·76	−1·93	−0·18	+1·50	+2·99
Means,...	−0·23	−0·03	−0·14	−0·27	−1·55	−2·53	−2·33	−1·77	−0·23	+1·27	+3·08	+4·78

The following Table contains the corresponding numbers for the succeeding triennial period, with their means, and the means of all.

Table X. Annual Variation of the Declination for the years 1844–46, together with the mean of all.

Year.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1844, ...	−0′·01	+0′·24	+1′·13	+0′·08	−1′·72	−2′·56	−1′·79	−2′·18	−0′·74	+0′·46	+3′·44	+3′·66
1845, ...	−1·14	+0·29	+0·38	+0·35	−0·53	−1·16	−2·06	−2·15	−1·31	+1·19	+1·73	+4·44
1846, ...	−1·34	−0·73	+0·32	+1·99	+1·06	−1·09	−3·14	−1·84	−0·96	+0·78	+2·17	+2·76
Means,. . .	−0·83	−0·07	+0·61	+0·81	−0·40	−1·60	−2·33	−2·06	−1·00	+0·81	+2·45	+3·62
Means of six years,	−0·53	−0·05	+0·24	+0·27	−0·98	−2·06	−2·33	−1·92	−0·62	+1·04	+2·76	+4·20

The mean results for the two triennial periods, together with the means of all, are graphically represented in Plate III. figs. 3, 4, 5; the scale being 0·2 inch to one minute of arc. To facilitate comparisons hereafter instituted, the *positive* ordinates correspond to *westerly* deviations. The correspondence of the course of the variation during the two periods is as close as could be expected, the difference consisting chiefly in the epoch of the westerly maximum.

16. The following are the laws of the changes, as deduced from the mean results:

I. From the beginning of April to the beginning of July, the north end of the magnet moves to the *west;* the maximum of westerly declination takes place about the 8th of July, but the epoch varies considerably in different years.

II. During the remainder of the year (i. e. from the beginning of July to the beginning of April), the north end of the magnet moves to the *east;* the movement, however, is very slow during the first three months of the year.

III. The range of the westerly movement is 2·7 minutes; and that of the easterly 8·7 minutes. Thus, at the end of twelve months, the north end of the magnet has advanced to the east, by about 6·0 minutes, as has been already shown.

17. The annual variation of the declination was discovered by CASSINI in 1786.* It appeared from the observations of CASSINI, that the north pole of the magnet moved to the *east* from the vernal equinox to the summer solstice; and that, during the remaining nine months of the year, it moved to the *west.* The westerly movement, during the nine months, preponderated over the easterly, which took place during three; and thus the westerly declination was

* The determination of the annual variation is much more difficult than that of the diurnal, both on account of the much smaller frequency of the period, and the difficulty of preserving the instrument in the same unchanged condition during the much longer time, or of determining and allowing for its changes when they do occur. Accordingly, although the annual period may be traced in the observations of GILPIN, and is decidedly displayed in those of BOWDITCH, it has evaded the researches of recent observers. There is but a faint indication of its existence in the Gottingen observations, which were made at the hours of 8 A. M. and 1 P. M.; and Professor GAUSS and Dr. GOLDSCHMIDT find, in their analysis of these observations, no "important fluctuation dependent on season." A similar negative result is deduced by Dr. LAMONT from the Munich observations, which were made twelve times in the day.

greater at the close of the year than at the commencement. The difference was the yearly amount of the *secular* change.

The observations of CASSINI were made during five years, viz. from 1784 to 1788, inclusive. Although the annual variation at Paris was then greater than it is now at Dublin, the final means are less accordant; and M. KÆMTZ deduces from them the existence of a *double* oscillation. This conclusion, however, has arisen from what appears to be an erroneous mean value in the month of October, and is therefore not a legitimate interpretation of the results.

18. When we compare the course of the changes observed by CASSINI with those observed at Dublin, we find that the movements are precisely opposite. But, it is to be observed, the directions of the secular changes at the two periods are likewise opposed; and, putting together these facts, we are led to generalize the law as follows:

From a little after the vernal equinox until a little after the summer solstice, the movement of the north pole of the magnet is RETROGRADE, *or opposite in direction to the secular change; and during the remaining nine months of the year it is* DIRECT.

The remarkable relation between the annual and secular changes, here stated, may be observed on comparing the observations of BOWDITCH, in 1810, with those of CASSINI. At this time the westerly declination was *diminishing* at Salem, in Massachusetts, by about two minutes annually; and, in accordance with the preceding law, the direction of the annual movements is the *inverse* of that observed by CASSINI at Paris, in 1786, and agrees with that observed at Dublin at the present time. M. ARAGO, who notices these observations (*Annales de Chimie*, tom. xvi.), draws from them a different conclusion, and infers (although with an expression of doubt) that when the westerly declination diminishes from year to year, the period of CASSINI is transported from Spring to Autumn.

It further appears probable, that, at a given place, *the amount of the annual variation is related to that of the secular change, and vanishes when the latter vanishes.* This conclusion has been drawn by ARAGO, from the observations of GILPIN at London, in 1787–1793, and those of BEAUFOY in 1818–1820, as compared with those of CASSINI. At the former period, in fact, the secular change was only $+1'\cdot 0$ annually at London, and the annual variation was proportionally small; while, at the latter, both changes appeared to be evanescent.

19. The phenomena just described are, it is manifest, the resultants of two distinct changes,—namely, the *annual variation* properly so called, and the *secular change.* The amount of the latter is $+ 0'\cdot5 \times n$, n being the number of months elapsed. If this be subtracted from the numbers in the last row of Table X., reducing to the epoch July 1 (the middle of the year), we obtain the numbers of the following Table, which represent the course of the true annual variation. The *positive* numbers correspond to *easterly* deviations, as before.

Table XI. Periodical part of the Mean Annual Variation.

Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
$+2'\cdot22$	$+2'\cdot20$	$+1'\cdot99$	$+1'\cdot52$	$-0'\cdot23$	$-1'\cdot81$	$-2'\cdot58$	$-2'\cdot67$	$-1'\cdot87$	$-0'\cdot71$	$+0'\cdot51$	$+1'\cdot45$

The following is the equation of the curve of the annual variation, in which $x = n \times 30°$, n being the number of months, and parts of a month, reckoned from January 1. We may probably neglect, as inconsiderable, all the terms after the second.

$$\begin{aligned}\Delta D = 2'543 \sin (x + 52° 57') + 0'\cdot301 \sin (2x + 232° 33')\\ + 0'\cdot108 \sin (3x + 117° 46') + 0'\cdot112 \sin (4x + 26° 25')\\ + 0'\cdot057 \sin (5x + 295° 7') + 0'\cdot005 \sin 6x.\end{aligned}$$

The curve itself is represented in Plate III. fig. 6, the scale being 0.2 inch to one minute of arc. For the sake of the comparison with the annual curve of temperature, presently to be referred to, the *signs* are all changed, and the *positive* ordinates correspond to *westerly* deviations.

It appears from the inspection of this curve, that the course of the annual variation (unlike that of the diurnal in this respect) is represented by a *single* oscillation. The *minimum* occurs in the beginning of February, and the *maximum* in the beginning of August; and the whole *range* of the change is 5·0 minutes. The curve crosses the axis of the abscissæ in the middle of May and in the beginning of November.

To obtain the mean value of the declination corresponding to any month in any year, the value of ΔD, obtained above, must be added to that of D, given in Art. 14. The formula, therefore, is

$$D_n = 332^\circ\ 27'{\cdot}27 + 6'{\cdot}06 \times n + \Delta D;$$

n denoting, as before, the number of years reckoned from January 1, 1840.

20. We have already seen that the diurnal changes of declination and temperature are related in a very remarkable manner; and we should, therefore, naturally be led to expect a corresponding relation in the annual changes of the same elements. For the purpose of exhibiting it, I subjoin the differences between the mean temperatures of each month, and that of the entire year, as deduced from the observations made at the Magnetical Observatory during the years 1841–46.

TABLE XII. ANNUAL VARIATION OF TEMPERATURE.

Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
$-8^\circ{\cdot}38$	$-9^\circ{\cdot}04$	$-6^\circ{\cdot}24$	$-2^\circ{\cdot}03$	$+2^\circ{\cdot}95$	$+8^\circ{\cdot}93$	$+9^\circ{\cdot}18$	$+9^\circ{\cdot}46$	$+7^\circ{\cdot}22$	$-0^\circ{\cdot}90$	$-4^\circ.16$	$-7^\circ{\cdot}02$

The numbers of this Table are projected in Plate III. fig. 7, immediately below the corresponding curve of declination, the scale being ·05 of an inch to one degree of temperature. It will be seen, on a comparison of the two curves, that the annual variations of the declination and temperature present the most complete accordance, not only as to the hours of *maxima* and *minima*, but also in their entire course.* There is a slight, but systematic difference in the epochs of the *mean* values, those of the declination taking place about a *fortnight later* than those of the temperature. This is what we should be led *à priori* to expect, on the assumption that the magnetic changes are due to the

* Since this paper was written, I have learned that the correspondence between the annual changes of declination and temperature had been indicated by HORNER (*Gehler's Wörterbuch*) as a result of the Stockholm observations. The correspondence thus traced, however, does not extend beyond the fact, that the epochs of greatest and least declination coincide, nearly, with those of greatest and least temperature; and, in fact, the results themselves, although the means of observations extending over the space of thirty years (1786–1815), are manifestly encumbered with too large an amount of observation error, to render any more detailed comparison possible. The same remark applies, yet more strongly, to the results of the Manheim observations, as quoted by KÆMTZ. In both of these cases, the most *easterly* position of the needle occurs at the time of *greatest temperature* of the year, and *vice versâ.*

changes of temperature of the earth's crust; for it is known that the epochs of mean temperature, as well as those of the maximum and minimum temperature of the *soil*, are *retarded*, and follow the corresponding epochs for the temperature of the *air* by an interval which is proportional to the depth.

These retardations, when observation shall have determined them with greater precision, will probably be found (in accordance with the results of Professor FORBES'S experiments) to be different in different localities, depending upon the conductibility of the soil.

21. It remains to notice the bearing of the remarkable relations between the annual and the secular changes, stated in Art. 18. It would seem to follow from these relations, that the two classes of changes are physically connected; and therefore that the *secular*, as well as the *annual* variation, is due to the heating power of the sun exerted upon the earth's crust,—although not only the magnitude, but even the direction of the change is different at different times. It is not easy to frame even a conjecture as to the nature of such an agency, in the case of the secular change.

Disturbances.

22. Having examined the *periodical* and the *secular* variations of the declination, it remains to consider those which, from our ignorance of their laws, we have been accustomed to call "*irregular*."

Professor KREIL seems to have been the first to notify the remarkable fact, that magnetic disturbances occur more frequently at certain hours than at others; and, that the *direction*, as well as the *frequency* of these movements, has a dependence upon the time of the day. Colonel SABINE has since made a more complete and elaborate examination of this question, in the discussion of the Toronto observations, and has arrived at conclusions for the most part confirmatory of those obtained by Professor KREIL.

In these investigations, however, those disturbances only are taken into account which exceed a certain arbitrary limit; and of these the *frequency* is considered without any reference to their *magnitude*. In examining the question of the periodicity of disturbances, I have thought it advisable to pursue a different course. I have taken the differences between each result, and the monthly mean corresponding to the same hour, and combined these differences in the same manner as the errors of observation (to which they are analogous) are combined in the calculus of probabilities. The square root of the mean of the squares of these differences is, in fact, a quantity analogous to the *mean error*, and which we may therefore call the *mean disturbance*; and it is evident that its values, at the several hours of the day, and at the several seasons of the year, will serve to measure the probable disturbance to be expected at the corresponding times.

The values of this function have been deduced for the several hours of observation, in each month of the year 1843;* and those for the entire year are obtained from them by a repetition of the same process. They are given in the following Table:—

Table XIII. Values of the Mean Disturbance.

	1 A.M.	3	5	7	9	11	1 P.M.	3	5	7	9	11
January, ..	1′·88	2′·25	1′·65	1′·27	1′·21	1′·47	1′·40	1′·63	1′·14	3′·07	0′·83	3′·94
February, .	2·00	2·20	1·63	1·09	0·98	1·43	1·83	2·89	2·49	2·72	7·64	2·42
March, ...	2·91	2·69	1·12	2·07	1·43	1·34	1·72	2·99	1·69	1·79	2·37	2·73
April, ...	2·75	3·43	2·44	1·18	1·65	2·11	2·53	2·06	4·02	3·29	3·34	3·83
May, ...	2·90	3·01	1·97	1·61	1·61	1·56	1·72	1·93	1·87	0·92	2·85	10·96
June, ...	2·58	2·24	1·99	1·78	2·63	2·01	3·14	2·08	1·67	1·14	1·28	1·95
July, ...	3·67	3·33	2·56	2·61	3·25	2·99	3·30	1·63	1·78	1·36	4·89	3·47
August, ...	4·41	1·96	4·02	1·48	1·52	2·00	2·08	2·25	2·20	1·31	2·14	1·42
September,.	3·08	2·09	2·65	3·73	2·70	2·39	2·16	1·79	2·23	2·54	3·79	2·63
October, ...	2·76	3·39	1·16	2·46	1·96	1·76	2·04	1·89	1·33	3·95	3·22	2·58
November, .	2·14	1·21	1·79	1·09	1·17	1·75	1·68	2·17	1·52	1·70	1·12	1·64
December, .	1·18	1·03	1·16	0·80	0·99	1·74	1·56	1·41	1·93	3·16	2·01	1·67
Means, ...	2·81	2·52	2·16	1·93	1·89	1·93	2·17	2·11	2·12	2·44	3·47	4·07

These numbers show that the mean disturbance follows a law of remarkable regularity in dependence upon the hour. During the day,—i. e. from 6 A.M. to 6 P.M.,—it is nearly constant. At 6 P.M. it begins to increase, and arrives at a maximum a little after 10 P.M.; and it then decreases with the same regularity, and arrives at its constant day-value about 6 A.M.

* I have chosen this year, because in it the irregular changes were comparatively small; and, the number which expresses their frequency, in consequence, bearing a larger proportion to that which denotes their magnitude, any regular law to which they are subject will be more readily apparent.

23. The preceding results are independent of the *direction* of the disturbance. If, however, we take the sum of the squares of the easterly and westerly deviations separately, we find that the *easterly* disturbances preponderate during the night, and the *westerly* during the day; the former are, however, much more considerable than the latter, and the difference reaches a maximum about 10 P.M.

Let $\Sigma\Delta_+^2$ denote the sum of the squares of the positive, or easterly disturbances, and $\Sigma\Delta_-^2$ that of the negative, or westerly; then the mean values of the function $\sqrt{\left(\frac{\Sigma\Delta_+^2 - \Sigma\Delta_-^2}{n}\right)}$ are the following. The values in which the *easterly* disturbances preponderate are distinguished by the *positive sign*, and *vice versâ*.

TABLE XIV. MEAN VALUES OF $\sqrt{\left(\frac{\Sigma\Delta_+^2 - \Sigma\Delta_-^2}{n}\right)}$.

1 A.M.	3	5	7	9	11	1 P.M.	3	5	7	9	11
+1′·43	+1′·05	−1′·18	−1′·15	−1′·07	−0′·83	−0′·80	−1′·12	−0′·54	+1′·67	+2′·82	+3′·37

It thus appears that the mean disturbance observes a regular daily period, both in *magnitude* and *direction;* and this period, it is worthy of remark, is precisely the reverse of that of the regular diurnal movement,—the mean *position* of the magnet being nearly constant during the *night*, the mean *disturbance* during the *day*;—the principal oscillation of the magnet, in the *regular* movement, being to the *west* during the *day*, while that of the *irregular* movement is to the *east* during the *night*.

24. From these remarkable relations it seems evident that the two classes of phenomena are physically connected; and I am inclined to regard the disturbance which prevails about 10 P.M., as an *irregular reaction* from the regular day movement, and dependent upon it both for its periodical character and for its amount.

If this hypothesis be a just one, it will, of course, follow that the magnitude of the mean disturbance will vary in some direct proportion to the daily range, and should, therefore, be greater in summer than in winter. This appears to be established by the following Table, which contains the values of $\sqrt{\left(\frac{\Sigma\Delta^2}{n}\right)}$, corresponding to the several months of the year:

TABLE XV. ANNUAL VARIATION OF THE MEAN DISTURBANCE.

Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
2′·01	2′·96	2′·16	2′·85	3′·73	2′·11	3′·06	2′·42	2′·72	2′·51	1′·63	1′·67

These numbers are somewhat irregular; but if they be combined in periods of three months, taking the square root of the mean of the squares, we obtain the following numbers, in which the existence of an *annual period* is evident:

February, March, April, ...	2′·68
May, June, July, ...	3′·04
August, September, October, . . .	2′·55
November, December, January, . .	1′·77

The mean disturbance, for the entire year, is 2′·56.

25. It by no means necessarily follows, from the results now stated, that *all* disturbances have a periodical character. There probably are *two classes* of disturbances, the results of distinct physical causes, of which one observes a period, while the other is wholly irregular; and it is manifest that, in such a case, the period of the former will necessarily be impressed upon the *resultant* mean disturbance. We have, I think, also grounds for concluding that these two kinds of disturbances are further distinguished by the important characteristics,—that those of the former class are *local* (depending, as they do, upon the time at the place of observation), while those of the latter are *universal*, and belong to the phenomena which have hitherto so much engaged the attention of observers.

Of the periodical disturbances the principal (if not the only one) is that which occurs about 10 P. M., and which causes the north pole of the magnet to deviate to the east. The epoch of the maximum of easterly deflection varies, however, between very wide limits, being sometimes before 8 P. M., and sometimes later than 1 A. M.; and hence it is evident, that its effect on the monthly mean curve is to produce a *general increase* of the ordinate between these limits of time, as well as the *maximum* at 10 P.M.

We learn from the consideration of these facts, that the ordinary mode of grouping the observations, by taking the mean of all the results at the same hour,—although it truly gives the *mean* diurnal curve for the period embraced by the observations,—does not represent the average actual course of the movement during one day. In order to obtain the *representative*, or *type* curve (as it may be called), it seems necessary to treat the ordinates and abscissæ as independent variables, and to take,—not the means of the ordinates corresponding to certain definite abscissæ,—but the means, both of the abscissæ and ordinates, *corresponding to the time of the phenomenon.* The former of these will give the *mean epoch* of the disturbance, and the latter its *mean amount.*

We find, in this manner, that the mean epoch of the periodical disturbance is a few minutes before 10 P.M. Its mean magnitude is 10′·0; and its mean duration is an hour and a half.

V.—*On a Classification of Elastic Media, and the Laws of Plane Waves propagated through them.* *By the* Rev. Samuel Haughton, *Fellow and Tutor of Trinity College, Dublin.*

Read January 8, 1849.

In a Memoir on the equilibrium and motion of solid and fluid bodies, presented to this Academy in May, 1846, I deduced the laws of such bodies from the hypothesis of attracting and repelling molecules; since that time I have been led to consider the general laws of continuous bodies, without making any such restriction as to the nature of the molecular action. The present paper contains the results I have arrived at in this investigation, and may, perhaps, be considered interesting on account of the classification suggested as applicable to all elastic media. It consists of five sections; the first contains the general equations applicable to all media, and the properties of plane waves transmitted through them, which are readily deduced from an extension which I have given to a theorem originally stated by M. Cauchy, for a particular case; the second, third, and fourth sections contain respectively the laws of the three groups into which elastic media may be divided; these three groups consisting of,—first, bodies whose molecular action consists of exclusively normal pressures; secondly, bodies whose molecular action produces exclusively tangential forces; thirdly, bodies composed of attracting and repelling molecules. The fifth section contains a comparison of the mechanical theories of light proposed by Mr. Green and Professor Mac Cullagh, with some observations on the present state of the science of physical optics. Whatever theoretic objections may be made to the application of the theory of elastic media to optics, none such exist as to its application to solid and fluid bodies The mathematical investigations which, in the case of light, must be hypothetical, are, in the case of solids and fluids, essentially positive, and may be made the object of direct experiment. A general inquiry into the laws of elastic media, is an interesting application of rational mechanics, and although it must necessarily include cases purely hypothetical, it is not, therefore, to be considered as unimportant. In this respect it is analogous to an inquiry into the general theory of central forces, the importance of which is not confined to the investigation of those laws, of which examples occur in nature ; these are undoubtedly the most important, but the theory of central forces, considered as a branch of mechanics, would be incomplete, unless extended to all possible laws of central force.

SECTION I.—GENERAL EQUATIONS.

The formula of virtual velocities, which contains, as shown by LAGRANGE, the conditions necessary to be fulfilled in the interior and at the boundaries of a continuous body, is the proper starting point for a deductive theory of the mechanical structure of bodies. Every hypothesis which may be made, and every fact which experience has discovered, respecting the molecular constitution of bodies, may be expressed in its most simple form by the aid of this formula ; which, by its flexibility, and the facility it affords for deducing theoretic results, becomes of more importance in questions of this nature than in other mechanical problems. In order to express by means of it the conditions of equilibrium of a continuous body, it is necessary to distinguish the forces acting at each point into two classes, molecular and external forces ; including among the external forces the resultants of the attractions of the other points of the body, since these attractions arise from gravitation, and must not be confounded with the molecular forces. The formula of virtual velocities must also be stated in such a manner as not to involve any hypothesis as to the nature of molecular forces, so as to possess the requisite degree of generality. If the problem be dynamical, we must then add accelerating forces equal and opposite to those actually employed, so as to destroy the motion at each point, and consider the problem as one of equilibrium of forces. These negative accelerating forces must be considered as external forces.

The forces being thus divided into two classes, the formula will consist of two parts,

$$\Sigma\,(P\delta p + P'\delta p' + \&c.) + \Sigma\,(Q\delta q + Q'\delta q' + \&c.) = 0.$$

P, P', &c., denoting the external forces, and Q, Q', &c., denoting the molecular forces. If (x, y, z) denote the co-ordinates of a point at any instant of the motion, and a virtual displacement $(\delta x, \delta y, \delta z)$ be conceived, these quantities will be functions of four independent variables, which will be the initial values of (x, y, z), and the time. Let (u', v', w') denote the accelerating forces; hence

$$\Sigma\,(P\delta p + P'\delta p' + \&c.) = \iiint\{(X - u')\,\delta x + (Y - v')\,\delta y + (Z - w')\,\delta z\}\,dm.$$

I shall assume that the virtual moments of the molecular forces depend upon the differential coefficients of $(\delta x, \delta y, \delta z)$, by means of the following linear equation,*

$$\begin{aligned} Q\delta q + Q'\delta q' + \&c. = {} & P_1\frac{d\delta x}{dx} + P_2\frac{d\delta x}{dy} + P_3\frac{d\delta x}{dz} \\ & + Q_1\frac{d\delta y}{dx} + Q_2\frac{d\delta y}{dy} + Q_3\frac{d\delta y}{dz} \\ & + R_1\frac{d\delta z}{dx} + R_2\frac{d\delta z}{dy} + R_3\frac{d\delta z}{dz}. \end{aligned}$$

Inserting these values into the equation of virtual velocities, and integrating by parts, we find

$$\begin{aligned} 0 = {} & \iiint\{\rho\,(X - u')\,\delta x + \rho\,(Y - v')\,\delta y + \rho\,(Z - w')\,\delta z\}\,dxdydz \\ & - \iiint\left\{\left(\frac{dP_1}{dx} + \frac{dP_2}{dy} + \frac{dP_3}{dz}\right)\delta x + \left(\frac{dQ_1}{dx} + \frac{dQ_2}{dy} + \frac{dQ_3}{dz}\right)\delta y + \left(\frac{dR_1}{dx} + \frac{dR_2}{dy} + \frac{dR_3}{dz}\right)\delta z\right\}dxdydz \\ & + \iint(P_1\delta x + Q_1\delta y + R_1\delta z)\,dydz, \qquad (1) \\ & + \iint(P_2\delta x + Q_2\delta y + R_2\delta z)\,dxdz, \\ & + \iint(P_3\delta x + Q_3\delta y + R_3\delta z)\,dxdy. \end{aligned}$$

The equations of motion being determined by the triple integrals, and the conditions at the limits by the double integrals. The equations of motion formed from this equation will be

* The ground of this assumption is the fact, that molecular forces depend upon the *relative* displacements of the particles, and not on their absolute displacements.

$$\begin{aligned}
\rho(X-u') &= \frac{dP_1}{dx}+\frac{dP_2}{dy}+\frac{dP_3}{dz},\\
\rho(Y-v') &= \frac{dQ_1}{dx}+\frac{dQ_2}{dy}+\frac{dQ_3}{dz},\\
\rho(Z-w') &= \frac{dR_1}{dx}+\frac{dR_2}{dy}+\frac{dR_3}{dz}.
\end{aligned} \tag{2}$$

These equations are the same as those deduced by a totally different method (which will be given presently), and involve no restriction as to the extent of the deviation of (x, y, z) from their original values (a, b, c).

If we suppose $x = a + \xi$, $y = b + \eta$, $z = c + \zeta$, the expression for the moments of the molecular forces will become

$$\begin{aligned}
Q\delta q + Q'\delta q' + \&c. = P_1\delta\alpha_1 + P_2\delta\alpha_2 + P_3\delta\alpha_3\\
+ Q_1\delta\beta_1 + Q_2\delta\beta_2 + Q_3\delta\beta_3\\
+ R_1\delta\gamma_1 + R_2\delta\gamma_2 + R_3\delta\gamma_3;
\end{aligned}$$

where

$$\alpha_1 = \frac{d\xi}{dx},\quad \alpha_2 = \frac{d\xi}{dy},\quad \alpha_3 = \frac{d\xi}{dz};$$

$$\beta_1 = \frac{d\eta}{dx},\quad \beta_2 = \frac{d\eta}{dy},\quad \beta_3 = \frac{d\eta}{dz};$$

$$\gamma_1 = \frac{d\zeta}{dx},\quad \gamma_2 = \frac{d\zeta}{dy},\quad \gamma_3 = \frac{d\zeta}{dz}.$$

If we restrict the molecular forces by the condition

$$Q\delta q + Q'\delta q' + \&c. = \delta V,$$

we shall have the relations

$$P_1 = \frac{dV}{d\alpha_1},\quad P_2 = \frac{dV}{d\alpha_2},\quad P_3 = \frac{dV}{d\alpha_3},$$

$$Q_1 = \frac{dV}{d\beta_1},\quad Q_2 = \frac{dV}{d\beta_2},\quad Q_3 = \frac{dV}{d\beta_3},$$

$$R_1 = \frac{dV}{d\gamma_1},\quad R_2 = \frac{dV}{d\gamma_2},\quad R_3 = \frac{dV}{d\gamma_3},$$

$$V = F(\alpha_1, \alpha_2, \alpha_3,\quad \beta_1, \beta_2, \beta_3,\quad \gamma_1, \gamma_2, \gamma_3).$$

The kind of motion which it is the object of this paper to investigate is of the kind commonly called small oscillations; and for this kind of motion it is not necessary to use the most general equations, or to consider the unknown quantities of the problem as functions of (x, y, z, t). We may use, instead of (x, y, z), the co-ordinates (a, b, c) of the position of rest of the molecules. In fact, any differential coefficient of a function ϕ, taken with respect to (a, b, c), will be expressed by the equation,

$$\frac{d\phi}{da} = \frac{d\phi}{dx}\frac{dx}{da} + \frac{d\phi}{dy}\frac{dy}{da} + \frac{d\phi}{dz}\frac{dz}{da};$$

but, since $x = a + \xi$, $y = b + \eta$, $z = c + \zeta$, we obtain

$$\frac{dx}{da} = 1 + \frac{d\xi}{da}, \quad \frac{dy}{da} = \frac{d\eta}{da}, \quad \frac{dz}{da} = \frac{d\zeta}{da}.$$

Hence, neglecting quantities of the second order, we find

$$\frac{d\phi}{da} = \frac{d\phi}{dx},$$

and similarly for the other differential coefficients.

In the remaining part of this memoir (unless the contrary be expressed), I shall, therefore, consider (x, y, z) as the co-ordinates of the position of equilibrium of the molecules, and (ξ, η, ζ) as the small displacements of the molecule; the element of the mass will be expressed by the equation $dm = \epsilon dxdydz$, where ϵ denotes the density, not considered as a function of the time, since $(dxdydz)$ denotes the original element of the volume.

Two kinds of waves can pass through such a body as water; one, a surface wave, depending on the action of gravity for its propagation; the other, such a wave as propagates sound, and does not directly depend on external forces. This latter is the kind of wave described in this paper. The equations peculiar to it will be found by omitting (X, Y, Z) from the general equations; but though the external forces are not explicit in the formulæ, yet as they affect the density and structure of the body differently at different points, though they do not directly affect the wave, we must introduce them implicitly by rendering ϵ and the coefficients of V functions of (X, Y, Z), and therefore of (x, y, z)

The equation of virtual velocities thus modified will become, considering (x, y, z) as the positions of rest of the molecules,

$$\iiint \epsilon \left(\frac{d^2\xi}{dt^2} \delta\xi + \frac{d^2\eta}{dt^2} \delta\eta + \frac{d^2\zeta}{dt^2} \delta\zeta \right) dxdydz = \iiint \delta V dxdydz. \tag{3}$$

As V is a function of the quantities $(\alpha_1, \alpha_2, \alpha_3, \&c.)$ of a given form, we shall have

$$\begin{aligned} \delta V = & \frac{dV}{d\alpha_1} \delta\alpha_1 + \frac{dV}{d\alpha_2} \delta\alpha_2 + \frac{dV}{d\alpha_3} \delta\alpha_3 \\ & + \frac{dV}{d\beta_1} \delta\beta_1 + \frac{dV}{d\beta_2} \delta\beta_2 + \frac{dV}{d\beta_3} \delta\beta_3 \\ & + \frac{dV}{d\gamma_1} \delta\gamma_1 + \frac{dV}{d\gamma_2} \delta\gamma_2 + \frac{dV}{d\gamma_3} \delta\gamma_3. \end{aligned}$$

Substituting this value in equation (3), and integrating by parts, we obtain

$$\begin{aligned} \iiint \epsilon & \left(\frac{d^2\xi}{dt^2} \delta\xi + \frac{d^2\eta}{dt^2} \delta\eta + \frac{d^2\zeta}{dt^2} \delta\zeta \right) dxdydz = \iiint \delta V dxdydz \\ & = \iint \left(\frac{dV}{d\alpha_1} \delta\xi + \frac{dV}{d\beta_1} \delta\eta + \frac{dV}{d\gamma_1} \delta\zeta \right) dydz \\ & + \iint \left(\frac{dV}{d\alpha_2} \delta\xi + \frac{dV}{d\beta_2} \delta\eta + \frac{dV}{d\gamma_2} \delta\zeta \right) dxdz \\ & + \iint \left(\frac{dV}{d\alpha_3} \delta\xi + \frac{dV}{d\beta_3} \delta\eta + \frac{dV}{d\gamma_3} \delta\zeta \right) dxdy \\ & - \iiint \left(\frac{d}{dx} \cdot \frac{dV}{d\alpha_1} + \frac{d}{dy} \cdot \frac{dV}{d\alpha_2} + \frac{d}{dz} \cdot \frac{dV}{d\alpha_3} \right) \delta\xi dxdydz \\ & - \iiint \left(\frac{d}{dx} \cdot \frac{dV}{d\beta_1} + \frac{d}{dy} \cdot \frac{dV}{d\beta_2} + \frac{d}{dz} \cdot \frac{dV}{d\beta_3} \right) \delta\eta dxdydz \\ & - \iiint \left(\frac{d}{dx} \cdot \frac{dV}{d\gamma_1} + \frac{d}{dy} \cdot \frac{dV}{d\gamma_2} + \frac{d}{dz} \cdot \frac{dV}{d\gamma_3} \right) \delta\zeta dxdydz. \end{aligned} \tag{4}$$

The double integrals, as usual, denote the conditions at the bounding surface, and the triple integrals give the general equations of motion in the interior. The laws of propagation depend upon the triple integrals, and the laws of reflexion and refraction depend upon the double integrals.

The equations of motion derived from (4) will be,

$$\left.\begin{aligned}
-\epsilon\frac{d^2\xi}{dt^2} &= \frac{d}{dx}\cdot\frac{dV}{da_1}+\frac{d}{dy}\cdot\frac{dV}{da_2}+\frac{d}{dz}\cdot\frac{dV}{da_3}\,;\\
-\epsilon\frac{d^2\eta}{dt^2} &= \frac{d}{dx}\cdot\frac{dV}{d\beta_1}+\frac{d}{dy}\cdot\frac{dV}{d\beta_2}+\frac{d}{dz}\cdot\frac{dV}{d\beta_3}\,;\\
-\epsilon\frac{d^2\zeta}{dt^2} &= \frac{d}{dx}\cdot\frac{dV}{d\gamma_1}+\frac{d}{dy}\cdot\frac{dV}{d\gamma_2}+\frac{d}{dz}\cdot\frac{dV}{d\gamma_3}\,.
\end{aligned}\right\} \qquad (5)$$

These equations contain the laws of propagation of every variety of wave not depending on external forces; and as the function has not been restricted by any hypothesis as to the law of molecular action, they will be the dynamical equations of propagation of sound in air, water, solids, and of light, if we adopt the undulatory hypothesis. In all these cases, the difficulty is to ascertain the correct form of V peculiar to the particular case; the form being found, the coefficients must be determined by experiment for each body: so far as theory is concerned, the mechanical classification of bodies would be complete, if the form of V were known for all.

The conditions to be satisfied at the limits will be different, according as the surface is fixed, free, acted on by special forces, or in contact with other bodies. As there is no difficulty in forming the equations for any of these cases, I shall here give only the conditions at the limiting surface in contact with other bodies.

Let $F(x, y, z) = 0$ be the equation of the surface passing through the positions of rest of the molecules which at any instant form the actual limiting surface $f(x, y, z, t) = 0$, hence

$$\frac{dF}{dx}dx+\frac{dF}{dy}dy+\frac{dF}{dz}dz=0\,;$$

and if λ, μ, ν, be the angles which determine the position of the normal, we shall have

$$\cos\lambda = \kappa\frac{dF}{dx},$$

$$\cos\mu = \kappa\frac{dF}{dy},$$

$$\cos\nu = \kappa\frac{dF}{dz}.$$

If ω be an element of the surface

$$dydz = \omega \cos \lambda = \kappa\omega \frac{dF}{dx},$$

$$dxdz = \omega \cos \mu = \kappa\omega \frac{dF}{dy},$$

$$dxdy = \omega \cos \nu = \kappa\omega \frac{dF}{dz};$$

these equations will reduce the double integrals (4) to the form

$$\Delta = \iint \left(\frac{dV}{d\alpha_1} \cdot \frac{dF}{dx} + \frac{dV}{d\alpha_2} \cdot \frac{dF}{dy} + \frac{dV}{d\alpha_3} \cdot \frac{dF}{dz}\right) \kappa\omega\delta\xi$$

$$+ \iint \left(\frac{dV}{d\beta_1} \cdot \frac{dF}{dx} + \frac{dV}{d\beta_2} \cdot \frac{dF}{dy} + \frac{dV}{d\beta_3} \cdot \frac{dF}{dz}\right) \kappa\omega\delta\eta$$

$$+ \iint \left(\frac{dV}{d\gamma_1} \cdot \frac{dF}{dx} + \frac{dV}{d\gamma_2} \cdot \frac{dF}{dy} + \frac{dV}{d\gamma_3} \cdot \frac{dF}{dz}\right) \kappa\omega\delta\zeta.$$

These will be the double integrals resulting from one body, and from them must be subtracted similar terms derived from the body which bounds the one under consideration. $F(x, y, z) = 0$ is common to both bodies when at rest, also ξ, η, ζ, will be the same for all the bounding surface, $f(x, y, z, t) = 0$, during the motion; also $\delta\xi, \delta\eta, \delta\zeta$, are independent; hence the condition at the limits will be

$$\Delta' - \Delta'' = 0,$$

which is equivalent to three equations,

$$\left(\frac{dV_0'}{d\alpha_1} - \frac{dV_0''}{d\alpha_1}\right)\frac{dF}{dx} + \left(\frac{dV_0'}{d\alpha_2} - \frac{dV_0''}{d\alpha_2}\right)\frac{dF}{dy} + \left(\frac{dV_0'}{d\alpha_3} - \frac{dV_0''}{d\alpha_3}\right)\frac{dF}{dz} = 0,$$

$$\left(\frac{dV_0'}{d\beta_1} - \frac{dV_0''}{d\beta_1}\right)\frac{dF}{dx} + \left(\frac{dV_0'}{d\beta_2} - \frac{dV_0''}{d\beta_2}\right)\frac{dF}{dy} + \left(\frac{dV_0'}{d\beta_3} - \frac{dV_0''}{d\beta_3}\right)\frac{dF}{dz} = 0, \qquad (6)$$

$$\left(\frac{dV_0'}{d\gamma_1} - \frac{dV_0''}{d\gamma_1}\right)\frac{dF}{dx} + \left(\frac{dV_0'}{d\gamma_2} - \frac{dV_0''}{d\gamma_2}\right)\frac{dF}{dy} + \left(\frac{dV_0'}{d\gamma_3} - \frac{dV_0''}{d\gamma_3}\right)\frac{dF}{dz} = 0,$$

V', V'' denoting the functions proper to the first and second body, and V_0 denoting that the values of (x, y, z), deduced from $F(x, y, z) = 0$, have been substituted in V.

To the three equations (6) must be added the self-evident geometrical equations, which denote that the vibrating molecules at the bounding surface may be considered as belonging to either body; they are three in number,

$$\xi_0' = \xi_0'', \quad \eta_0' = \eta_0'', \quad \zeta_0' = \zeta_0''. \tag{7}$$

Equations (6) and (7) contain the laws of reflexion and refraction of vibrations for all bodies, and are completely determinate when the form of V is given for each of the bodies in contact.

Equations (5), (6), (7), are *necessary and sufficient* to determine the propagation and reflexion of waves, so far as they are connected with each other; and no mechanical theory of vibrations is correct which does not exhibit this connexion, or which assumes such laws of reflexion and refraction as contradict the laws of propagation; the connexion between these laws is no proof of the truth of any theory, but the want of a connexion would be a proof of the inconsistency of a theory.

The reduction of the general equations by the omission of the external forces may require some explanation. There are two kinds of waves, as I have stated in making the reduction, one only of which is the subject of our present inquiry. Fluid bodies, such as the atmosphere and the ocean, can propagate tidal waves depending on external forces; they are also capable of propagating waves of sound which depend directly on the molecular forces; solid bodies can only propagate the second species of waves. In considering this kind of vibration, we neglect the external forces, as they are of so much less intensity than the molecular forces, that they produce no effect on the motion; but if we suppose the motion to cease, and inquire into the state of equilibrium of the body, we should then use the general formula, which includes external forces; and, even in the case of motion, all the coefficients must be considered as variable, in consequence of the position of constrained equilibrium to which the body would return, if the motion ceased.

It is evident from an inspection of equations (5) and (6) that the differential coefficients of the function V, with respect to (a_1, a_2, &c.), occupy an important position in the theory of elastic media. Hitherto, I have only given to them a mathematical definition, I now proceed to explain their physical meaning, by the consideration of an elementary parallelepiped of the body. If we conceive a plane drawn in any direction in the interior of the body, and consider the parts of the body situated at opposite sides as acting upon any element of the plane; in general, the effect of the particles at one side will be to produce a normal force, and tangential forces acting in the element; these tangential forces may be resolved into two directions at right angles to each other. Let us now conceive an elementary parallelepiped, with one corner situated at the point (x,y,z) (these co-ordinates here denote the actual position of the molecule at any instant); let the forces acting on the side $(dydz)$ be denoted by (P_1, Q_1, R_1), P_1 being normal and parallel to the axis of (x), (Q_1, R_1) tangential and parallel to the axes of Y and Z; let the forces acting on the side $(dxdz)$ be (P_2, Q_2, R_2), Q_2 being the normal force; and let the forces at the side $(dxdy)$ be (P_3, Q_3, R_3), R_3 being the normal force. The forces acting on the opposite sides of the parallelepiped will be :—on the side opposite to $(dydz)$,

$$\left(P_1+\frac{dP_1}{dx}dx\right)dydz, \quad \left(Q_1+\frac{dQ_1}{dx}dx\right)dydz, \quad \left(R_1+\frac{dR_1}{dx}dx\right)dydz\,;$$

on the side opposite to $(dxdz)$,

$$\left(P_2+\frac{dP_2}{dy}dy\right)dxdz, \quad \left(Q_2+\frac{dQ_2}{dy}dy\right)dxdz, \quad \left(R_2+\frac{dR_2}{dy}dy\right)dxdz\,;$$

on the side opposite to $(dxdy)$,

$$\left(P_3+\frac{dP_3}{dz}dz\right)dxdy, \quad \left(Q_3+\frac{dQ_3}{dz}dz\right)dxdy, \quad \left(R_3+\frac{dR_3}{dz}dz\right)dxdy.$$

These forces, acting on the six sides of the parallelepiped, must equilibrate the forces (Xdm, Ydm, Zdm), applied at the centre of the parallelepiped, and arising from external causes; hence, the equations of equilibrium will be

$$Xdm+\left(\frac{dP_1}{dx}+\frac{dP_2}{dy}+\frac{dP_3}{dz}\right)dxdydz=0\,;$$

$$Ydm+\left(\frac{dQ_1}{dx}+\frac{dQ_2}{dy}+\frac{dQ_3}{dz}\right)dxdydz=0\,;$$

$$Zdm+\left(\frac{dR_1}{dx}+\frac{dR_2}{dy}+\frac{dR_3}{dz}\right)dxdydz=0\,;$$

or, since $\rho dxdydz=dm$, and the molecular forces must act in the direction opposite to the applied forces, including negative accelerating forces,

$$\rho(X - u') = \frac{dP_1}{dx} + \frac{dP_2}{dy} + \frac{dP_3}{dz};$$
$$\rho(Y - v') = \frac{dQ_1}{dx} + \frac{dQ_2}{dy} + \frac{dQ_3}{dz}; \qquad (8)$$
$$\rho(Z - w') = \frac{dR_1}{dx} + \frac{dR_2}{dy} + \frac{dR_3}{dz}.$$

These are identical with equations (2), and are true for all kinds of molecular action. In the particular case of a rigid parallelepiped, we should introduce another set of conditions arising from the equilibrium of couples. Let (Ldm, Mdm, Ndm) be external couples applied to the parallelepiped; these must equilibrate the couples arising from the molecular action of the surrounding parts of the body; it is easy to see that, neglecting the small forces arising from the differential coefficients of P_1, P_2, &c., the couples round the axes of x, y, z, will be

$$(R_2 + Q_3)\, dydz, \quad (P_3 + R_1)\, dxdz, \quad (Q_1 + P_2)\, dxdy.$$

Hence the required conditions will be

$$\epsilon L = R_2 + Q_3\ ;$$
$$\epsilon M = P_3 + R_1\ ;$$
$$\epsilon N = Q_1 + P_2\ ;$$

and, if no external couples be applied, the conditions will be,

$$R_2 = Q_3\ ;$$
$$P_3 = R_1\ ; \qquad (9)$$
$$Q_1 = P_2\ ;$$

since the couples (R_2, Q_3), &c., must act in opposite directions.

These conditions (9) were given by M. Cauchy,* and afterwards adopted by M. Poisson.† These writers seem to have considered them as necessary for all systems; but this is not true, as equations (8) exhibit all the relations which exist between the forces and the motions produced. Equations (9) are necessary

* Exercices de Mathematiques, tom. ii. p. 47.

† Journal de l'Ecole Polytechnique, cahier xx. p. 84.

for the equilibrium of a rigid parallelepiped, and will be shown in this memoir to be satisfied by the equations of equilibrium of bodies whose molecules attract and repel each other in the direction of the line joining them; but if no supposition be made as to the nature of the molecular action, there will be no condition resulting from the equilibrium of couples; for we have no right to assume, in the equilibrium of a parallelepiped, whose elements may alter their relative position, and thus develope new forces, that the same equations hold as in the equilibrium of an isolated rigid parallelepiped, for which case six equations of condition are necessary, arising from the equilibrium of forces and of couples.

Restoring the usual signification of (x, y, z) in equations (8), and omitting the external forces, we obtain

$$
\left.\begin{aligned}
-\epsilon\frac{d^2\xi}{dt^2} &= \frac{dP_1}{dx} + \frac{dP_2}{dy} + \frac{dP_3}{dz};\\
-\epsilon\frac{d^2\eta}{dt^2} &= \frac{dQ_1}{dx} + \frac{dQ_2}{dy} + \frac{dQ_3}{dz};\\
-\epsilon\frac{d^2\zeta}{dt^2} &= \frac{dR_1}{dx} + \frac{dR_2}{dy} + \frac{dR_3}{dz}.
\end{aligned}\right\} \qquad (10)
$$

These equations correspond with (5), and by comparing them we may deduce the following relations:

$$
\begin{aligned}
\frac{dV}{da_1} &= P_1, & \frac{dV}{da_2} &= P_2, & \frac{dV}{da_3} &= P_3;\\
\frac{dV}{d\beta_1} &= Q_1, & \frac{dV}{d\beta_2} &= Q_2, & \frac{dV}{d\beta_3} &= Q_3;\\
\frac{dV}{d\gamma_1} &= R_1, & \frac{dV}{d\gamma_2} &= R_2, & \frac{dV}{d\gamma_3} &= R_3;
\end{aligned}
$$

from which we obtain the following theorem:

"If through any point (x, y, z), three elements of planes be drawn parallel to the co-ordinate planes, the total action of the part of the body lying at one side of these planes will consist of three normal and six tangential forces; and these forces may be expressed by the differential coefficients of the function V, with respect to $(a_1, a_2, a_3, \&c.)$." This theorem is of importance in classifying bodies, as it enables us to pass directly from the resultant forces of the molecules to the form of the function which determines the laws of propagation and reflexion. Introducing these forces into the conditions at the limits (6), we obtain

$$P'_{01}\frac{dF}{dx}+P'_{02}\frac{dF}{dy}+P'_{03}\frac{dF}{dz}=P''_{01}\frac{dF}{dx}+P''_{02}\frac{dF}{dy}+P''_{03}\frac{dF}{dz};$$

$$Q'_{01}\frac{dF}{dx}+Q'_{02}\frac{dF}{dy}+Q'_{03}\frac{dF}{dz}=Q''_{01}\frac{dF}{dx}+Q''_{02}\frac{dF}{dy}+Q''_{03}\frac{dF}{dz};$$

$$R'_{01}\frac{dF}{dx}+R'_{02}\frac{dF}{dy}+R'_{03}\frac{dF}{dz}=R''_{01}\frac{dF}{dx}+R''_{02}\frac{dF}{dy}+R''_{03}\frac{dF}{dz}.$$

If the axis of z be made to coincide with the normal, we shall have

$$\frac{dF}{dx}=0,\quad \frac{dF}{dy}=0,\quad \frac{dF}{dz}=1.$$

Hence,

$$\begin{aligned} P'_{03}&=P''_{03}, & \xi'_0&=\xi''_0, \\ Q'_{03}&=Q''_{03}, & \eta'_0&=\eta''_0, \\ R'_{03}&=R''_{03}, & \zeta'_0&=\zeta''_0. \end{aligned} \tag{11}$$

These equations at the limits are the mathematical statement of two facts, of which one is mechanical and the other geometrical.

1. That the forces, normal and tangential (arising from molecular action), acting upon an element of the bounding surface, must be equal and opposite for the two bodies in contact.

2. That the motion of the particles in the bounding surface may be considered as common to both bodies.

I shall now return to the general equation (4); let the function V be divided into homogeneous parts, so that

$$V=\phi_0+\phi_1+\phi_2+\phi_3+\&c.,$$

where ϕ_0 is constant, ϕ_1 of the first degree with respect to (a_1, a_2), and so of the others; then, neglecting ϕ_0 and terms higher than the second, we obtain

$$V = \phi_1 + \phi_2,$$

$$\begin{aligned}\phi_1 = A_1\alpha_1 + A_2\alpha_2 + A_3\alpha_3 \\ + B_1\beta_1 + B_2\beta_2 + B_3\beta_3 \\ + C_1\gamma_1 + C_2\gamma_2 + C_3\gamma_3 .\end{aligned}$$

$$\phi_2 = (a_1^2)\ \alpha_1^2 + (a_2^2)\ \alpha_2^2 + \&c.$$
$$+ (a_1 b_2) \cdot \alpha_1\beta_2 + \&c., \quad + (c_3 a_2) \cdot \gamma_3\alpha_2 + \&c. ;$$

the expressions within the brackets denoting the corresponding coefficients.

ϕ_1 will contain nine coefficients, which will be constant, or functions of (x, y, z), according as there are no external forces, or *vice versâ;* and ϕ_2 will contain forty-five distinct coefficients. I shall consider these functions separately. Introducing ϕ_1 for V in equation (4), we obtain,

$$\begin{aligned}\iiint \epsilon \left(\frac{d^2\xi}{dt^2}\,\delta\xi + \frac{d^2\eta}{dt^2}\,\delta\eta + \frac{d^2\zeta}{dt^2}\,\delta\zeta\right) dxdydz = \iint (A_1\delta\xi + B_1\delta\eta + C_1\delta\zeta)\, dydz, \\ + \iint (A_2\delta\xi + B_2\delta\eta + C_2\delta\zeta)\, dxdz, \qquad (12) \\ + \iint (A_3\delta\xi + B_3\delta\eta + C_3\delta\zeta)\, dxdy\end{aligned}$$

$$-\iiint\left\{\left(\frac{dA_1}{dx}+\frac{dA_2}{dy}+\frac{dA_3}{dz}\right)\delta\xi+\left(\frac{dB_1}{dx}+\frac{dB_2}{dy}+\frac{dB_3}{dz}\right)\delta\eta+\left(\frac{dC_1}{dx}+\frac{dC_2}{dy}+\frac{dC_3}{dz}\right)\delta\zeta\right\}dxdydz,$$

whence we obtain

$$\begin{aligned}-\epsilon\frac{d^2\xi}{dt^2} = \frac{dA_1}{dx} + \frac{dA_2}{dy} + \frac{dA_3}{dz}, \\ -\epsilon\frac{d^2\eta}{dt^2} = \frac{dB_1}{dx} + \frac{dB_2}{dy} + \frac{dB_3}{dz}, \qquad (13) \\ -\epsilon\frac{d^2\zeta}{dt^2} = \frac{dC_1}{dx} + \frac{dC_2}{dy} + \frac{dC_3}{dz}.\end{aligned}$$

The terms on the right hand side of equation (13) must be added to the dynamical equations arising from ϕ_2, even in cases where (X, Y, Z) may be neglected on account of the intensity of the molecular forces. If we wish to take account of the external forces, we should use, instead of (13), the following:

$$-\epsilon\frac{d^2\xi}{dt^2}=\frac{dA_1}{dx}+\frac{dA_2}{dy}+\frac{dA_3}{dz}-\epsilon X;$$

$$-\epsilon\frac{d^2\eta}{dt^2}=\frac{dB_1}{dx}+\frac{dB_2}{dy}+\frac{dB_3}{dz}-\epsilon Y;$$

$$-\epsilon\frac{d^2\zeta}{dt^2}=\frac{dC_1}{dx}+\frac{dC_2}{dy}+\frac{dC_3}{dz}-\epsilon Z.$$

If no external forces act, equations (13) disappear, in consequence of A_1, A_2, &c., becoming constants; but the conditions at the limits (12) will still remain, unless the coefficients be not only constant, but zero.

The part of the differential equations of motion depending on ϕ_2 will be found by introducing $\delta\phi_2$ in place of δV in equation (3), and integrating by parts, according to the methods of the calculus of variations; but it may be found more readily by introducing ϕ_2 for V, in equation (4). Neglecting the equations of condition at the limits, we obtain for the equations of motion,

$$\left.\begin{aligned}
-\epsilon\frac{d^2\xi}{dt^2} &= (a_1^2)\frac{d^2\xi}{dx^2}+(a_2^2)\frac{d^2\xi}{dy^2}+(a_3^2)\frac{d^2\xi}{dz^2}\\
&+2(a_2a_3)\frac{d^2\xi}{dydz}+2(a_1a_3)\frac{d^2\xi}{dxdz}+2(a_1a_2)\frac{d^2\xi}{dxdy}\\
&+(a_1b_1)\frac{d^2\eta}{dx^2}+(a_2b_2)\frac{d^2\eta}{dy^2}+(a_3b_3)\frac{d^2\eta}{dz^2}\\
&+(a_2b_3+a_3b_2)\frac{d^2\eta}{dydz}+(a_1b_3+a_3b_1)\frac{d^2\eta}{dxdz}+(a_1b_2+a_2b_1)\frac{d^2\eta}{dxdy}\\
&+(a_1c_1)\frac{d^2\zeta}{dx^2}+(a_2c_2)\frac{d^2\zeta}{dy^2}+(a_3c_3)\frac{d^2\zeta}{dz^2}\\
&+(a_2c_3+a_3c_2)\frac{d^2\zeta}{dydz}+(a_1c_3+a_3c_1)\frac{d^2\zeta}{dxdz}+(a_1c_2+a_2c_1)\frac{d^2\zeta}{dxdy}.
\end{aligned}\right\}\quad(14)$$

$$\begin{aligned}
-\epsilon\frac{d^2\eta}{dt^2} &= (b_1^2)\frac{d^2\eta}{dx^2}+(b_2^2)\frac{d^2\eta}{dy^2}+(b_3^2)\frac{d^2\eta}{dz^2}\\
&+2(b_2b_3)\frac{d^2\eta}{dydz}+2(b_1b_3)\frac{d^2\eta}{dxdz}+2(b_1b_2)\frac{d^2\eta}{dxdy}
\end{aligned}$$

$$+ (b_1c_1)\frac{d^2\zeta}{dx^2} + (b_2c_2)\frac{d^2\zeta}{dy^2} + (b_3c_3)\frac{d^2\zeta}{dz^2}$$

$$+ (b_2c_3 + b_3c_2)\frac{d^2\zeta}{dydz} + (b_1c_3 + b_3c_1)\frac{d^2\zeta}{dxdz} + (b_1c_2 + b_2c_1)\frac{d^2\zeta}{dxdy}$$

$$+ (a_1b_1)\frac{d^2\xi}{dx^2} + (a_2b_2)\frac{d^2\xi}{dy^2} + (a_3b_3)\frac{d^2\xi}{dz^2}$$

$$+ (a_2b_3 + a_3b_2)\frac{d^2\xi}{dydz} + (a_1b_3 + a_3b_1)\frac{d^2\xi}{dxdz} + (a_1b_2 + a_2b_1)\frac{d^2\xi}{dxdy}.$$

$$-\epsilon\frac{d^2\zeta}{dt^2} = (c_1^2)\frac{d^2\zeta}{dx^2} + (c_2^2)\frac{d^2\zeta}{dy^2} + (c_3^2)\frac{d^2\zeta}{dz^2}$$

$$+ 2(c_2c_3)\frac{d^2\zeta}{dydz} + 2(c_1c_3)\frac{d^2\zeta}{dxdz} + 2(c_1c_2)\frac{d^2\zeta}{dxdy}$$

$$+ (a_1c_1)\frac{d^2\xi}{dx^2} + (a_2c_2)\frac{d^2\xi}{dy^2} + (a_3c_3)\frac{d^2\xi}{dz^2}$$

$$+ (a_2c_3 + a_3c_2)\frac{d^2\xi}{dydz} + (a_1c_3 + a_3c_1)\frac{d^2\xi}{dxdz} + (a_1c_2 + a_2c_1)\frac{d^2\xi}{dxdy}$$

$$+ (b_1c_1)\frac{d^2\eta}{dx^2} + (b_2c_2)\frac{d^2\eta}{dy^2} + (b_3c_3)\frac{d^2\eta}{dz^2}$$

$$+ (b_2c_3 + b_3c_2)\frac{d^2\eta}{dydz} + (b_1c_3 + b_3c_1)\frac{d^2\eta}{dxdz} + (b_1c_2 + b_2c_1)\frac{d^2\eta}{dxdy}.$$

By combining these equations with (12), and comparing (4), we obtain from the function $V = \phi_1 + \phi_2$ the following equations of motion,

$$\begin{aligned}
-\epsilon\frac{d^2\xi}{dt^2} &= \frac{d}{dx}\left(\frac{d\phi_2}{da_1} + A_1\right) + \frac{d}{dy}\left(\frac{d\phi_2}{da_2} + A_2\right) + \frac{d}{dz}\left(\frac{d\phi_2}{da_3} + A_3\right) \\
-\epsilon\frac{d^2\eta}{dt^2} &+ \frac{d}{dx}\left(\frac{d\phi_2}{d\beta_1} + B_1\right) + \frac{d}{dy}\left(\frac{d\phi_2}{d\beta_2} + B_2\right) + \frac{d}{dz}\left(\frac{d\phi_2}{d\beta_3} + B_3\right) \qquad (15) \\
-\epsilon\frac{d^2\zeta}{dt^2} &= \frac{d}{dx}\left(\frac{d\phi_2}{d\gamma_1} + C_1\right) + \frac{d}{dy}\left(\frac{d\phi_2}{d\gamma_2} + C_2\right) + \frac{d}{dz}\left(\frac{d\phi_2}{d\gamma_3} + C_3\right).
\end{aligned}$$

These are the equations of small oscillations of all media, for which the direct influence of the external forces may be neglected.

In equations (14) it will be observed that nine of the coefficients are composed of two terms, coefficients of the original function ϕ_2; these compound coefficients will, in the equations of motion, be equivalent to simple coefficients, so that the total number of distinct constants in the equations of wave-motion will be reduced to thirty-six; this reduction cannot, however, be introduced into the conditions at the limits, because each coefficient will at the limits be multiplied by a different function of (x, y, z). Hence may be deduced an important theorem, which I shall prove in a general manner.

"Two bodies, different in their molecular structure, may have the same laws of wave-propagation, but cannot have the same laws of reflexion and refraction."

The laws of wave-propagation (5) depend upon the differential coefficients of $\left(\frac{dV}{da_1}, \&c.\right)$, taken with respect to (x, y, z); but the laws of reflexion and refraction (6) depend on the quantities $\left(\frac{dV}{da_1}, \&c.\right)$ themselves. Each of these quantities is of the form

$$A\frac{d\xi}{dx} + B\frac{d\xi}{dy} + C\frac{d\xi}{dz} + D\frac{d\eta}{dx} + E\frac{d\eta}{dy} + F\frac{d\eta}{dz} + G\frac{d\zeta}{dx} + H\frac{d\zeta}{dy} + I\frac{d\zeta}{dz}.$$

If, therefore, for example, in the first of the equations (5), there be in $\left(\frac{dV}{da_1}\right)$ a term of the form $\left(B\frac{d\xi}{dy}\right)$, there will be in $\frac{dV}{da_2}$ a term of the form $\left(A'\frac{d\xi}{dx}\right)$; then, differentiating the first with respect to x, and the second with respect to y, we shall obtain, as part of the equations of motion, $(B + A')\frac{d^2\xi}{dxdy}$, while the corresponding part of the conditions at the limits will be $\iint B\frac{d\xi}{dy}\delta\xi dydz + \iint A'\frac{d\xi}{dx}\delta\xi dxdz$. If, now, we suppose another body such that $\left(A'\frac{d\xi}{dy}\right)$ and $\left(B\frac{d\xi}{dx}\right)$ are terms in $\left(\frac{dV}{da_1}\right)$ and $\left(\frac{dV}{da_2}\right)$, the part resulting from these quantities in the equations of motion will be $(A' + B)\frac{d^2\xi}{dxdy}$, which is identical with the former value, while the corresponding portion of the conditions at the limits will be $\iint A'\frac{d\xi}{dy}\delta\xi dydz+\iint B\frac{d\xi}{dx}\delta\xi dxdz$; which cannot be reduced to the former value, unless $A'=B$.

It is sufficient here to establish the general theorem. I shall return to it again, in comparing Professor MAC CULLAGH's theory of light and that advocated by M. CAUCHY and Mr. GREEN, which give the same laws of wave-propagation, but differ in the laws of reflexion and refraction.

I shall now integrate the equations of motion (14), for the particular case of plane waves and rectilinear vibrations. Let (l, m, n) be the direction cosines of the wave-normal, (α, β, γ) the direction of molecular vibration, and (v) the velocity of the wave; then the integral will be

$$\xi=\cos\alpha\cdot f\left\{\frac{2\pi}{\lambda}(lx+my+nz-vt)\right\},$$

$$\eta=\cos\beta\cdot f\left\{\frac{2\pi}{\lambda}(lx+my+nz-vt)\right\},$$

$$\zeta=\cos\gamma\cdot f\left\{\frac{2\pi}{\lambda}(lx+my+nz-vt)\right\}.$$

Introducing these values into the equations (14), we obtain

$$\begin{aligned}\epsilon v^2\cos\alpha &= P'\cos\alpha+H'\cos\beta+G'\cos\gamma,\\ \epsilon v^2\cos\beta &= Q'\cos\beta+F'\cos\gamma+H'\cos\alpha,\\ \epsilon v^2\cos\gamma &= R'\cos\gamma+G'\cos\alpha+F'\cos\beta;\end{aligned}\qquad(16)$$

where

$$\begin{aligned}P'&=(a_1^2)\,l^2+(a_2^2)\,m^2+(a_3^2)\,n^2+2(a_2a_3)\,mn+2(a_1a_3)\,ln+2(a_1a_2)\,lm;\\ Q'&=(b_1^2)\,l^2+(b_2^2)\,m^2+(b_3^2)\,n^2+2(b_2b_3)\,mn+2(b_1b_3)\,ln+2(b_1b_2)\,lm;\\ R'&=(c_1^2)\,l^2+(c_2^2)\,m^2+(c_3^2)\,n^2+2(c_2c_3)\,mn+2(c_1c_3)\,ln+2(c_1c_2)\,lm;\\ F'&=(b_1c_1)l^2+(b_2c_2)m^2+(b_3c_3)n^2+(b_2c_3+b_3c_2)mn+(b_1c_3+b_3c_1)ln+(b_1c_2+b_2c_1)lm;\\ G'&=(a_1c_1)l^2+(a_2c_2)m^2+(a_3c_3)n^2+(a_2c_3+a_3c_2)mn+(a_1c_3+a_3c_1)ln+(a_1c_2+a_2c_1)lm;\\ H'&=(a_1b_1)l^2+(a_2b_2)m^2+(a_3b_3)n^2+(a_2b_3+a_3b_2)mn+(a_1b_3+a_3b_1)ln+(a_1b_2+a_2b_1)lm.\end{aligned}$$

Equations (16) are the well-known equations for determining the axes of the ellipsoid whose equation is

$$P'x^2+Q'y^2+R'z^2+2F'yz+2G'xz+2H'xy=1.\qquad(17)$$

There are, therefore, three possible directions of molecular vibration for a given direction of wave plane; and there will be three parallel waves moving with velocities determined by the magnitude of the axes of the ellipsoid, the direction of vibration in each wave being parallel to one of the axes. The theorem involved in equation (16) was first proved by M. CAUCHY,* for bodies whose molecules act by attractions and repulsions in the line joining them. It is here extended to every kind of molecular action, and shown to be a fundamental property in molecular dynamics. It may be worth while to examine the reason of its truth. It arises from the homogeneity of equations (14) resulting from the absence of external forces. The right hand member of each of the three equations of motion consists of eighteen terms; there will, therefore, be in all fifty-four terms; if each of these were supposed to have different and independent coefficients, equations (16) would cease to be true, and the coefficients of ($\cos\alpha$, $\cos\beta$, $\cos\gamma$) would be nine distinct quantities, so that the theorem which represents the direction and magnitude of the molecular vibration by means of an ellipsoid whose coefficients are functions of the direction of the wave, would no longer be applicable. Equations (14) contain only thirty-six distinct constants, and this reduction in the number of constants arises from the assumption that the virtual moments of the system may be represented by $\iiint \delta V dxdydz$; which is equivalent to assuming that the virtual moments of the molecular forces applied at any point may be represented by the variation of a single function:

$$Q\delta q + Q'\delta q' + \&c. = \delta V.$$

The cubic equation whose roots are the squares of the reciprocals of the axes of the ellipsoid (17) is

$$(P'-s)(Q'-s)(R'-s)-F'^2(P'-s)-G'^2(Q'-s)-H'^2(R'-s)+2F'G'H'=0;$$

where $s=\epsilon v^2$. Hence, if (P, Q, R, F, G, H) denote the same functions of (x, y, z) that (P', Q', R', F', G', H') are of (l, m, n), it may be shown, in a manner similar

* Exercices de Mathematiques, tom. v. p. 32.

to the method for bodies whose molecules act in the line joining them, that the surface of *wave-slowness** will be

$$(P-1)(Q-1)(R-1)-F^2(P-1)-G^2(Q-1)-H^2(R-1)+2FGH=0. \quad (18)$$

$(P-1, Q-1,$ &c.) equated to zero, denoting the six fixed ellipsoids used in the memoir referred to ; these ellipsoids thus appear in the general theory of molecular dynamics, and will have the same use in interpreting the conditions at the limits in the general problem, as in the particular case of attracting and repelling molecules. As their use has been fully explained in my former memoir, I shall only refer to it, and pass on to other subjects.

I shall here state the general conditions necessary, in order that the molecular equilibrium of a body removed from the influence of external forces should be stable; this investigation will be found of use in the subsequent part of this memoir. The equation of virtual velocities, in the case supposed, will become

$$\iiint \delta V dx dy dz = 0 ;$$

or, as it may be more accurately stated,

$$\iiint \delta V dx dy dz = 0, \text{ or } < 0 ;$$

this sum never becoming positive for possible displacements. The equation of virtual velocities, as stated by M. Poisson and other writers, supposes no virtual displacements but those for which equal and opposite displacements are possible; the correct statement of the principle is, that the sum of the virtual moments of the system can never become positive for possible displacements. For the full development of this important correction of the equation of virtual velocities, as given by Lagrange, I shall refer to a memoir of M. Ostrogradsky, contained in the *Memoires de l'Academie de St. Petersbourg.*†

The application of the principle to the present case will be evident upon the statement of the question. As the form of V is given in terms of $(a_1, a_2,$ &c.), we shall have

$$\delta V = \frac{dV}{da_1}\delta a_1 + \frac{dV}{da_2}\delta a_2 + \text{\&c.};$$

* *Vide* Transactions of the Royal Irish Academy, vol. xxi. part ii. p. 172.

† Tom. iii. p. 130.

$\left(\frac{dV}{da_1}, \frac{dV}{da_2}, \&c.\right)$ denoting forces tending to alter the quantities $(a_1, a_2, a_3, \&c.)$, which are functions of (x, y, z). The whole body may, therefore, be divided into *couches* by a series of surfaces whose equation will be

$$a_1 - C = 0;$$

C denoting the parameter of the system. In a similar manner, the body may be conceived as divided into *couches* by other sets of surfaces corresponding to $(a_2, a_3, \&c.)$. In any one of these *couches* the corresponding function $(a_1, a_2, \&c.)$ will have a constant value, which will vary from one *couche* to another.

If $L = 0$ be the equation of a surface, we may conceive all space as divided into two portions, which will be distinguished by the property, that the portion lying at one side of the surface will have the function of (x, y, z) denoted by (L), positive; while for the rest of space, lying at the other side of the surface, the function (L) will be negative. Similarly, δL will be positive for all displacements made at one side of the surface; negative for all displacements on the opposite side; and zero for displacements in the surface itself.

Let us now resume the equation, $a_1 - C = 0$, which denotes a surface drawn in the interior of the body, along which the value of $\frac{d\xi}{dx}$ remains constant. It is necessary for the stable equilibrium of the body, that if the particles composing this surface be displaced from it, the molecular forces developed by the displacement should tend to restore them to the surface; this condition will require that $\left(\frac{dV}{da_1}, \frac{dV}{da_2}, \&c.\right)$, which are the forces developed by the displacements $(\delta a_1, \delta a_2, \&c.)$, should have signs opposite to those of the displacements. If no forces are developed tending to restore the molecules, we shall have

$$\frac{dV}{da_1} = 0, \quad \frac{dV}{da_2} = 0, \&c.;$$

and these equations will determine the *limits* of stable and unstable equilibrium in the body.

SECTION II.—LAWS OF NORMAL VIBRATIONS.

It has been shown that the differential coefficients of the function V, taken with respect to $(\alpha_1, \alpha_2, \&c.)$, denote the normal and tangential actions of the surrounding body on an elementary parallelepiped. I shall now suppose that the body is so constituted that the tangential forces vanish, and so deduce the laws of a body capable of transmitting pressure in a normal direction only. The condition that the tangential forces vanish will give the equations,

$$\frac{dV}{d\alpha_2} = 0, \quad \frac{dV}{d\beta_1} = 0, \quad \frac{dV}{d\gamma_1} = 0,$$
$$\frac{dV}{d\alpha_3} = 0, \quad \frac{dV}{d\beta_3} = 0, \quad \frac{dV}{d\gamma_2} = 0,$$

which will reduce the function to the form

$$2V = A\alpha_1^2 + B\beta_2^2 + C\gamma_3^2 + 2P\beta_2\gamma_3 + 2Q\alpha_1\gamma_3 + 2R\alpha_1\beta_2.$$

At first sight, it might appear that we might assume this or any other form of function to define a body, and proceed to deduce the laws of motion; but, as it is possible that an assumed form of the function may be only true for particular axes of co-ordinates, it is always necessary to examine whether a transformation of co-ordinates will introduce any new terms; if this be the case, then the assumed function will not represent completely the molecular structure of the body, as it will be merely a simplification of a more complex function, produced by assuming particular axes of co-ordinates, which are connected with the crystalline structure of the body. I shall examine the function just given for normal pressures by this method, and determine it so as to be independent of the axes of co-ordinates. The relations between the two systems of variables are

$$x = ax' + by' + cz',$$
$$y = a'x' + b'y' + c'z',$$
$$z = a''x' + b''y' + c''z';$$

also (ξ, η, ζ) will satisfy these equations. Let us now assume

$$u = \beta_3 + \gamma_2, \quad v = \gamma_1 + \alpha_3, \quad w = \alpha_2 + \beta_1,$$

we may immediately deduce, by the formulæ for transformation of the independent variables, the following values for the nine quantities (α_1, α_2, &c.),

$$\begin{aligned}
\alpha_1 &= a^2\alpha_1' + b^2\beta_2' + c^2\gamma_3' + bcu' + acv' + abw', \\
\beta_2 &= a'^2\alpha_1' + b'^2\beta_2' + c'^2\gamma_3' + b'c'u' + a'c'v' + a'b'w', \\
\gamma_3 &= a''^2\alpha_1' + b''^2\beta_2' + c''^2\gamma_3' + b''c''u' + a''c''v' + a''b''w'.
\end{aligned}$$

$$\begin{aligned}
\beta_3 &= \left\{\begin{array}{l} \quad a'\,(a''\alpha_1' + b''\alpha_2' + c''\alpha_3') \\ +\, b'\,(a''\beta_1' + b''\beta_2' + c''\beta_3') \\ +\, c'\,(a''\gamma_1' + b''\gamma_2' + c''\gamma_3') \end{array}\right\} &
\gamma_2 &= \left\{\begin{array}{l} \quad a''\,(a'\alpha_1' + b'\alpha_2' + c'\alpha_3') \\ +\, b''\,(a'\beta_1' + b'\beta_2' + c'\beta_3') \\ +\, c''\,(a'\gamma_1' + b'\gamma_2' + c'\gamma_3') \end{array}\right\} \\
\gamma_1 &= \left\{\begin{array}{l} \quad a''\,(a\alpha_1' + b\alpha_2' + c\alpha_3') \\ +\, b''\,(a\beta_1' + b\beta_2' + c\beta_3') \\ +\, c''\,(a\gamma_1' + b\gamma_2' + c\gamma_3') \end{array}\right\} &
\alpha_3 &= \left\{\begin{array}{l} \quad a\,(a''\alpha_1' + b''\alpha_2' + c''\alpha_3') \\ +\, b\,(a''\beta_1' + b''\beta_2' + c''\beta_3') \\ +\, c\,(a''\gamma_1' + b''\gamma_2' + c''\gamma_3') \end{array}\right\} \qquad (19) \\
\alpha_2 &= \left\{\begin{array}{l} \quad a\,(a'\alpha_1' + b'\alpha_2' + c'\alpha_3') \\ +\, b\,(a'\beta_1' + b'\alpha_2' + c'\beta_3') \\ +\, c\,(a'\gamma_1' + b'\gamma_2' + c'\gamma_3') \end{array}\right\} &
\beta_1 &= \left\{\begin{array}{l} \quad a'\,(a\alpha_1' + b\alpha_2' + c\alpha_3') \\ +\, b'\,(a\beta_1' + b\beta_2' + c\beta_3') \\ +\, c'\,(a\gamma_1' + b\gamma_2' + c\gamma_3') \end{array}\right\}
\end{aligned}$$

It is plain from the first three equations that the change of the co-ordinate axes will introduce into the function V three new variables (u, v, w). Hence, the function which I have assumed is only admissible for particular axes, and the true form of the function from which it is derived will be that of a function of the six variables used in my former memoir.

It is, however, possible to obtain such a function of $\left(\frac{d\xi}{dx}, \frac{d\eta}{dy}, \frac{d\zeta}{dz}\right)$ as shall not change its form with the transformation of co-ordinates. For, adding together the values of these quantities, we shall banish (u', v', w'), on account of the relations,

$$\begin{aligned}
bc + b'c' + b''c'' &= 0, \\
ac + a'c' + a''c'' &= 0, \\
ab + a'b' + a''b'' &= 0.
\end{aligned}$$

The result of the addition will be

$$\frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz} = \frac{d\xi'}{dx'} + \frac{d\eta'}{dy'} + \frac{d\zeta'}{dz'} = \omega.$$

This quantity (ω) will retain its value, independent of the directions of the co-ordinates, and consequently the proper form of the function V, for bodies which transmit only normal pressure in every direction, will be

$$V = F(\omega). \tag{20}$$

There are two other forms of the function V (considered as containing the nine differential coefficients) which possess this property of reproducing themselves by transformation of co-ordinates. Let (X, Y, Z) be determined by the following equations,

$$X = \beta_3 - \gamma_2, \quad Y = \gamma_1 - \alpha_3, \quad Z = \alpha_2 - \beta_1.$$

It appears immediately from equations (19) that (X, Y, Z) are expressed by the following relations in terms of (X', Y', Z'),

$$\begin{aligned} X &= aX' + bY' + cZ', \\ Y &= a'X' + b'Y' + c'Z', \\ Z &= a''X' + b''Y' + c''Z'. \end{aligned}$$

These formulæ are well known, and prove that another self-producing form of the function V will be

$$V = F(X, Y, Z). \tag{21}$$

I have proved in my former memoir that if $(a, \beta, \gamma, u, v, w)$ denote the coefficients $\left\{\frac{d\xi}{dx}, \frac{d\eta}{dy}, \frac{d\zeta}{dz}, \left(\frac{d\eta}{dz} + \frac{d\zeta}{dy}\right), \left(\frac{d\zeta}{dx} + \frac{d\xi}{dz}\right), \left(\frac{d\xi}{dy} + \frac{d\eta}{dx}\right)\right\}$, they will become, by transformation of co-ordinates, linear functions of $(a', \beta', \gamma', u', v', w')$; and that the formulæ of tranformation are the same as for (x^2), (y^2), (z^2), $(2yz)$, $(2xz)$, $(2xy)$. Hence, a function of these six quantities will reproduce itself, by transformation of axes of co-ordinates. The third form of V which possesses this property is, therefore,

$$V = F(a, \beta, \gamma, u, v, w). \tag{22}$$

The forms of the function V, which have just been determined, are perfectly general, and do not merely express properties of the body with reference to particular axes, but general properties of its molecular structure, independent of the directions of the co-ordinates.

I shall consider in the present section the function (20), which is peculiar to bodies which transmit normal pressure in every direction. Before deducing the equations of motion, it may be useful to show, in another manner, the necessity for reducing the function of (α, β, γ) to a function of $(\alpha + \beta + \gamma)$. By the theorem proved in my former memoir, (α, β, γ) are transformed by the same equations as (x^2, y^2, z^2); hence, $V = F(\alpha, \beta, \gamma)$ may be represented by a surface whose equation contains only the squares of the co-ordinates. It is evident that such a surface will be symmetrical with respect to the co-ordinate planes; and if this condition be satisfied for every system of co-ordinates, the surface must be a sphere; hence,

$$V = F(\alpha, \beta, \gamma) = F(\alpha + \beta + \gamma).$$

The equations of motion deduced from (20) will be the equations commonly used in hydrodynamics, and for this reason it may be useful to state them in their most general form. Let (x, y, z) denote, as in equations (1) and (2), the actual positions of the molecules; then, since

$$\delta V = F\left(\frac{d\delta x}{dx} + \frac{d\delta y}{dy} + \frac{d\delta z}{dz}\right),$$

we shall obtain from equations (2)

$$\begin{aligned} \frac{1}{\rho}\frac{dp}{dx} &= X - \frac{du}{dt} - u\frac{du}{dx} - v\frac{du}{dy} - w\frac{du}{dz}; \\ \frac{1}{\rho}\frac{dp}{dy} &= Y - \frac{dv}{dt} - u\frac{dv}{dx} - v\frac{dv}{dy} - w\frac{dv}{dz}; \\ \frac{1}{\rho}\frac{dp}{dz} &= Z - \frac{dw}{dt} - u\frac{dw}{dx} - v\frac{dw}{dy} - w\frac{dw}{dz}; \end{aligned} \qquad (23, a)$$

assuming $p = \frac{dV}{d\omega}$, and recollecting that (u', v', w') are the *total* differential coefficients of (u, v, w), taken with respect to (t). These are the mechanical equations of the problem, and, combined with the equation of continuity,

$$\frac{d\rho}{dt} + \frac{d(\rho u)}{dx} + \frac{d(\rho v)}{dy} + \frac{d(\rho w)}{dz} = 0, \qquad (24, a)$$

contain all that is requisite to determine the motion of points situated in the interior of the mass. The mechanical conditions to be satisfied at the limits are contained in the double integrals, which become, by the substitution of the element of the bounding surface,

$$\iint p dS\,(\delta x \cos \lambda + \delta y \cos \mu + \delta z \cos \nu),$$

(λ, μ, ν) denoting the direction of the normal. It may be easily shown from this expression, that if P denote a function of (x, y, z, t), expressing the normal forces applied at each point of the surface, the mechanical condition at the limits will be expressed by the equation,

$$p - P = 0. \tag{23, b}$$

To which must be added the equation of the bounding surface, deduced from geometrical considerations,

$$\frac{dF}{dt} + u\frac{dF}{dx} + v\frac{dF}{dy} + w\frac{dF}{dz} = 0. \tag{24, b}$$

It is unnecessary to enter further into this subject, as it is fully treated by LAGRANGE, in the second volume of the *Mecanique Analytique,* and is indeed the only example given by him of the application of his formulæ to the problem of bodies composed of continuous points.

Resuming the former signification of $(x, y, z, \xi, \eta, \zeta)$, we find from equations (5) or (15), since $2V = 2p\omega + A\omega^2$, and $\frac{dV}{d\omega} = p + A\omega$,

$$-\epsilon\frac{d^2\xi}{dt^2} = \frac{dp}{dx} + A\frac{d\omega}{dx},$$

$$-\epsilon\frac{d^2\eta}{dt^2} = \frac{dp}{dy} + A\frac{d\omega}{dy},$$

$$-\epsilon\frac{d^2\zeta}{dt^2} = \frac{dp}{dz} + A\frac{d\omega}{dz}.$$

If no external forces act, p and A will be constants; and, in this case, these equations become, assuming a negative sign for A, so that the equilibrium may be stable,

$$\epsilon \frac{d^2\xi}{dt^2} = A \frac{d\omega}{dx},$$

$$\epsilon \frac{d^2\eta}{dt^2} = A \frac{d\omega}{dy}, \qquad (25)$$

$$\epsilon \frac{d^2\zeta}{dt^2} = A \frac{d\omega}{dz}.$$

and, by differentiating with respect to (x, y, z), we obtain

$$\frac{d^2\omega}{dt^2} = \frac{A}{\epsilon}\left(\frac{d^2\omega}{dx^2} + \frac{d^2\omega}{dy^2} + \frac{d^2\omega}{dz^2}\right), \qquad (26)$$

which determines the cubical compression as a function of (x, y, z, t).

The equations (25) and (26) are applicable to the propagation of sound in gases, and to the longitudinal vibrations of elastic solids; but there will be an essential difference between the two cases. If the gas were freed from the action of external forces, and unconfined at its limits, its molecules would separate, and a displacement would develope no force tending to restore them to their original positions; hence, $\left(\frac{dV}{d\omega}\delta\omega\right)$ would become positive, which is inconsistent with stable equilibrium, and the velocity of wave propagation $\left(\sqrt{\frac{A}{\epsilon}}\right)$ would become imaginary. In order, therefore, that the equilibrium of the gas be stable, we must suppose a constant pressure exerted at the limits, sufficient to keep the molecules together, and restore them to their original positions, if displaced. No such condition is requisite in the elastic solid, for $\left(\frac{dV}{d\omega}\delta\omega\right)$ will be negative, without the assistance of forces applied at the bounding surface.

SECTION III. LAWS OF TRANSVERSE VIBRATIONS.

In order to obtain the form of the function V peculiar to transverse vibrations, we must suppose that, if a plane be drawn through the body in any direction, the molecular forces exerted on any element of the plane will be altogether tangential. Hence we obtain

$$\frac{dV}{da_1} = 0, \quad \frac{dV}{d\beta_2} = 0, \quad \frac{dV}{d\gamma_3} = 0\,;$$

and the function V will become

$$V = F(a_2,\ a_3,\ \beta_1,\ \beta_3,\ \gamma_1,\ \gamma_2).$$

But, as I have already shown, such a form must be assumed, as will reproduce itself by transformation of co-ordinates. The only function which satisfies this condition is the function (21),

$$V = F(X,\ Y,\ Z).$$

This function, deduced in a different manner, has been used by Professor MAC CULLAGH in his mechanical theory of light; and for the discussion of its properties and the laws of propagation, reflection, and refraction, deduced from it, I shall refer to his memoir in the Transactions of the Royal Irish Academy.* As, however, I shall have occasion to use them hereafter, I shall here state the differential equations of motion, and the conditions at the limits. On account of the form of the function, the following relations exist:

$$\frac{dV}{dX} = \frac{dV}{d\beta_3} = -\frac{dV}{d\gamma_2}\,;$$

$$\frac{dV}{dY} = \frac{dV}{d\gamma_1} = -\frac{dV}{da_3}\,;$$

$$\frac{dV}{dZ} = \frac{dV}{da_2} = -\frac{dV}{d\beta_1}.$$

Hence equations (5) will become

$$\left.\begin{aligned}
-\epsilon\frac{d^2\xi}{dt^2} &= \frac{d}{dy}\cdot\frac{dV}{dZ} - \frac{d}{dz}\cdot\frac{dV}{dY}\,;\\
-\epsilon\frac{d^2\eta}{dt^2} &= \frac{d}{dz}\cdot\frac{dV}{dX} - \frac{d}{dx}\cdot\frac{dV}{dZ}\,;\\
-\epsilon\frac{d^2\zeta}{dt^2} &= \frac{d}{dx}\cdot\frac{dV}{dY} - \frac{d}{dy}\cdot\frac{dV}{dX}.
\end{aligned}\right\} \qquad (27)$$

* Vol. xxi. p. 17.

These are the equations of motion; also equations (6) will become

$$\left(\frac{dV'_0}{dZ} - \frac{dV''_0}{dZ}\right)\frac{dF}{dy} - \left(\frac{dV'_0}{dY} - \frac{dV''_0}{dY}\right)\frac{dF}{dz} = 0;$$
$$\left(\frac{dV'_0}{dX} - \frac{dV''_0}{dX}\right)\frac{dF}{dz} - \left(\frac{dV'_0}{dZ} - \frac{dV''_0}{dZ}\right)\frac{dF}{dx} = 0; \qquad (28)$$
$$\left(\frac{dV'_0}{dY} - \frac{dV''_0}{dY}\right)\frac{dF}{dx} - \left(\frac{dV'_0}{dX} - \frac{dV''_0}{dX}\right)\frac{dF}{dy} = 0.$$

These are the conditions to be fulfilled at the limiting surface $F(x, y, z) = 0$. They are equivalent to two conditions only, as the third equation may be deduced from the first two. The reason of this is evident *à priori;* for the conditions at the limits express in general, that the normal and two tangential pressures arising from the molecular forces in each body equilibrate each other for every point of the separating surface; but in the present case there are no normal pressures; hence there can be only two mechanical conditions at the limits. To these conditions at the limits, arising from the mechanical equations, should be added the three geometrical conditions resulting from the equivalence of vibrations; these conditions are

$$\xi'_0 = \xi''_0, \quad \eta'_0 = \eta''_0, \quad \zeta'_0 = \zeta''_0.$$

These, together with the mechanical conditions, will be equivalent to five equations, which Professor Mac Cullagh reduces to four by the hypothesis that the density of the luminiferous medium is the same in all transparent bodies; this hypothesis is necessary in order to reduce the number of equations to the number of unknown quantities in the problem.

If no external forces act upon the system, the function V will be reduced to a homogeneous function of the second order,

$$2V = PX^2 + QY^2 + RZ^2 + 2FYZ + 2GXZ + 2HXY;$$

and since (X, Y, Z) are transformed by a change of co-ordinates, in the same manner as (x, y, z), it is evident that the coefficients of the rectangles will vanish for axes of co-ordinates which coincide with the axes of the ellipsoid,

$$Px^2 + Qy^2 + Rz^2 + 2Fyz + 2Gxz + 2Hxy = 1.$$

Hence the simplest form of the function will be

$$2V = PX^2 + QY^2 + RZ^2.$$

The axes of co-ordinates are in this formula the axes of elasticity.

SECTION IV. EQUATIONS OF A SYSTEM WHOSE MOLECULES ATTRACT AND REPEL EACH OTHER.

The third form of the function V, which possesses the property of reproducing itself by a transformation of co-ordinates, is given by equation (22),

$$V = F(\alpha, \beta, \gamma, u, v, w).$$

The six variables contained in this function are the quantities upon which a change in the distance between the molecules depends; hence the variation of this function will express the virtual moments of a system of forces tending to alter this distance. In my former communication* I have made use of this function, using definite integrals to represent the coefficients; the equations thus found contain a smaller number of constants, than if the coefficients had been assumed to be arbitrary, without any relation expressed by definite integrals. I shall here use the function in its most general form, as the use of definite integrals may, perhaps, involve an hypothesis which would be too restricted to represent all the bodies whose molecular forces act in the line joining the molecules.

The following relations will exist, in consequence of the form of the function,

$$\begin{aligned} \frac{dV}{du} &= \frac{dV}{d\beta_3} = \frac{dV}{d\gamma_2}; \\ \frac{dV}{dv} &= \frac{dV}{d\gamma_1} = \frac{dV}{d\alpha_3}; \\ \frac{dV}{dw} &= \frac{dV}{d\alpha_2} = \frac{dV}{d\beta_1}. \end{aligned} \tag{29}$$

These equations will reduce the equations of motion (5) to the following:

* Transactions of the Royal Irish Academy, vol. xxi. p. 151.

$$-\epsilon\frac{d^2\xi}{dt^2}=\frac{d}{dx}\cdot\frac{dV}{da}+\frac{d}{dy}\cdot\frac{dV}{dw}+\frac{d}{dz}\cdot\frac{dV}{dv};$$
$$-\epsilon\frac{d^2\eta}{dt^2}=\frac{d}{dy}\cdot\frac{dV}{d\beta}+\frac{d}{dz}\cdot\frac{dV}{du}+\frac{d}{dx}\cdot\frac{dV}{dw};\qquad(30)$$
$$-\epsilon\frac{d^2\zeta}{dt^2}=\frac{d}{dz}\cdot\frac{dV}{d\gamma}+\frac{d}{dx}\cdot\frac{dV}{dv}+\frac{d}{dy}\cdot\frac{dV}{du}.$$

These are equivalent to equations (24) of my former paper, and are an immediate consequence of (29), which express the relations among the resultants of the molecular forces consequent upon the restricted form of the function. Equations (29), or the corresponding equations (9),

$$Q_3 - R_2 = 0,\quad R_1 - P_3 = 0,\quad P_2 - Q_1 = 0,$$

are analogous to a statical theorem given by Fresnel in his Memoir on double Refraction. If no external forces act upon the system, or if their influence may be assumed to be constant at all points of the body, then the coefficients of V will become constants, and the linear part of V will introduce no terms into the differential equations, which will in this case depend upon a homogeneous function of the second order. There will be, therefore, in general, twenty-one constants, viz., the coefficients of the function

$$\begin{aligned}2V = {} & (a^2)a^2 + (\beta^2)\beta^2 + (\gamma^2)\gamma^2 + (u^2)u^2 + (v^2)v^2 + (w^2)w^2 \\ & + 2\{(\beta\gamma)\beta\gamma + (a\gamma)a\gamma + (a\beta)a\beta\} + 2\{(vw)vw + (uw)uw + (uv)uv\} \\ & + 2u\{(au)a + (\beta u)\beta + (\gamma u)\gamma\} + 2v\{(av)a + (\beta v)\beta + (\gamma v)\gamma\} \\ & + 2w\{(aw)a + (\beta w)\beta + (\gamma w)\gamma\}.\end{aligned}$$

The equations of motion deduced from this form of the function V are identical with the equations used by M. Cauchy in his theory of light.* They are deduced by M. Cauchy directly from the consideration of attractive and repulsive forces between the molecules. These equations have been used also by Mr. Green, who seems to have considered them as more general than M. Cauchy's equations.†

The function (22) is not altered in form by an alteration of the co-ordinate

* Exercices de Mathematiques, tom. v., p. 19.

† Cambridge Philosophical Society's Transactions, vol. vii. p. 121.

axes, and is the most general which can be used for the case considered in the present section. The direction of the molecular vibration will not be either normal or transversal, but will be determined by the axes of an ellipsoid first noticed by M. CAUCHY in treating of bodies whose molecules attract and repel each other. It may be worth while to examine whether there are any subordinate functions which possess the property of not being changed in form by the change of co-ordinates; as such functions, if they exist, will contain the laws of classes of bodies contained under the general function (22). I have already discussed one such function (20), which is evidently a particular case of (22). I shall now determine another remarkable subordinate function, which, while it is much less general than (22), yet contains most of the bodies whose properties are at all interesting. The following formulæ of transformation may be easily demonstrated.*

$$\begin{aligned}
\alpha &= a^2\alpha' + b^2\beta' + c^2\gamma' + bcu' + acv' + abw';\\
\beta &= a'^2\alpha' + b'^2\beta' + c'^2\gamma' + b'c'u' + a'c'v' + a'b'w';\\
\gamma &= a''^2\alpha' + b''^2\beta' + c''^2\gamma' + b''c''u' + a''c''v' + a''b''w';\\
u &= 2a'a''\alpha' + 2b'b''\beta' + 2c'c''\gamma' + (b'c'' + b''c')u' + (a'c'' + a''c')v' + (a'b'' + a''b')w';\\
v &= 2aa''\alpha' + 2bb''\beta' + 2cc''\gamma' + (bc'' + b''c)u' + (ac'' + a''c)v' + (ab'' + a''b)w';\\
w &= 2aa'\alpha' + 2bb'\beta' + 2cc'\gamma' + (bc' + b'c)u' + (ac' + a'c)v' + (ab' + a'b)w'.
\end{aligned}$$

If we now assume six functions of $(\alpha, \beta, \gamma, u, v, w)$, defined by the following equations:

$$\lambda = u^2 - 4\beta\gamma, \quad \mu = v^2 - 4\alpha\gamma, \quad \nu = w^2 - 4\alpha\beta;$$
$$\phi = 2\alpha u - vw, \quad \chi = 2\beta v - uw, \quad \psi = 2\gamma w - uv;$$

it may be shown without much difficulty, that these new functions are transformed by the following equations:

$$\begin{aligned}
\lambda &= a^2\lambda' + b^2\mu' + c^2\nu' + 2bc\phi' + 2ac\chi' + 2ab\psi';\\
\mu &= a'^2\lambda' + b'^2\mu' + c'^2\nu' + 2b'c'\phi' + 2a'c'\chi' + 2a'b'\psi';\\
\nu &= a''^2\lambda' + b''^2\mu' + c''^2\nu' + 2b''c''\phi' + 2a''c''\chi' + 2a''b''\psi'; \qquad (31)\\
\phi &= a'a''\lambda' + b'b''\mu' + c'c''\nu' + (b'c'' + b''c')\phi' + (a'c'' + a''c')\chi' + (a'b'' + a''b')\psi';\\
\chi &= aa''\lambda' + bb''\mu' + cc''\nu' + (bc'' + b''c)\phi' + (ac'' + a''c)\chi' + (ab'' + a''b)\psi';\\
\psi &= aa'\lambda' + bb'\mu' + cc'\nu' + (bc' + b'c)\phi' + (ac' + a'c)\chi' + (ab' + a'b)\psi'.
\end{aligned}$$

* Transactions of the Royal Irish Academy, vol. xxi. p. 160.

Hence a function of $(\lambda, \mu, \nu, \phi, \chi, \psi)$ will reproduce itself by transformation of co-ordinates; let the function be

$$2V = P\lambda + Q\mu + R\nu + 2F\phi + 2G\chi + 2H\psi.$$

I shall first prove the existence of three rectangular axes, for which this function reduces to its first three terms. If the axes of co-ordinates be transformed, and the coefficients of (ϕ', χ', ψ') equated to zero, we obtain

$$Pbc + Qb'c' + Rb''c'' + F(b'c'' + b''c') + G(bc'' + b''c) + H(bc' + b'c) = 0;$$
$$Pac + Qa'c' + Ra''c'' + F(a'c'' + a''c') + G(ac'' + a''c) + H(ac' + a'c) = 0;$$
$$Pab + Qa'b' + Ra''b'' + F(a'b'' + a''b') + G(ab'' + a''b) + H(ab' + a'b) = 0.$$

These equations will be satisfied by assuming for axes of co-ordinates the axes of the ellipsoid

$$Px^2 + Qy^2 + Rz^2 + 2Fyz + 2Gxz + 2Hxy = 1.$$

Hence, for these particular axes,

$$2V = P(u^2 - 4\beta\gamma) + Q(v^2 - 4\alpha\gamma) + R(w^2 - 4\alpha\beta). \qquad (32)$$

Using this value of V in equations (30), we obtain for the equations of motion,

$$\begin{aligned} -\epsilon\frac{d^2\xi}{dt^2} &= R\frac{dZ}{dy} - Q\frac{dY}{dz}; \\ -\epsilon\frac{d^2\eta}{dt^2} &= P\frac{dX}{dz} - R\frac{dZ}{dx}; \\ -\epsilon\frac{d^2\zeta}{dt^2} &= Q\frac{dY}{dx} - P\frac{dX}{dy}: \end{aligned} \qquad (33)$$

(X, Y, Z) denoting the same functions as in (27). If the function were

$$2V = PX^2 + QY^2 + RZ^2, \qquad (34)$$

equations (27) would be the same as (33).

These equations are those used by Professor MAC CULLAGH and by Mr. GREEN. They denote transverse vibrations, and will give FRESNEL'S wave-surface for plane waves, and also the vibrations parallel to the plane of polarization. Mr. GREEN has deduced his equations from a homogeneous function of the second order of $(\alpha, \beta, \gamma, u, v, w)$, by restricting it so as to be capable of propagating only normal and transverse vibrations; this restriction will reduce the constants from twenty-one to seven; six of which belong to the transverse vibrations, and the seventh to the normal vibration. It is to be remarked, however, that a function of $(\lambda, \mu, \nu, \phi, \chi, \psi)$ will represent only a part of the properties of bodies whose molecules attract and repel each other. Such a body is always capable of transmitting normal vibrations, and though it may transmit transverse vibrations following the laws of FRESNEL's wave-surface, yet the normal vibration cannot be supposed to vanish. If we include normal vibrations, the most general form of the subordinate function will be

$$V = F(\omega, \lambda, \mu, \nu, \phi, \chi, \psi). \tag{35}$$

The manner in which I have obtained equation (35) is not so direct as Mr. GREEN's method, and I have only used it for the sake of the intermediate equations (31), which exhibit a remarkable property of the quantities $(\lambda, \mu, \nu, \phi, \chi, \psi)$. The direct method of deducing it is the following. Let M. CAUCHY's ellipsoid (17) be constructed for the function V, which is homogeneous and of the second order with respect to $(\alpha, \beta, \gamma, u, v, w)$.

Equations (16) determine the directions of molecular vibration; in these equations, if (l, m, n) be substituted for $(\cos \alpha, \cos \beta, \cos \gamma)$, we shall obtain the following:

$$\begin{aligned}
(Q' - R')\, mn + F'\,(n^2 - m^2) + H'ln - G'lm &= 0,\\
(R' - P')\, nl + G'\,(l^2 - n^2) + F'ml - H'mn &= 0,\\
(P' - Q')\, lm + H'\,(m^2 - l^2) + G'nm - F'nl &= 0.
\end{aligned}$$

These equations express that one of the axes of the ellipsoid is normal to the wave-plane, and consequently that the other two axes are contained in the wave-plane: by stating analytically that these conditions are true, independent of the position of the wave-plane, we obtain the following relations among the coefficients of V,

$$\begin{gathered}
(\beta u) = 0, \quad (\alpha v) = 0, \quad (\alpha w) = 0,\\
(\gamma u) = 0, \quad (\gamma v) = 0, \quad (\beta w) = 0,\\
(\alpha u) + 2(vw) = 0, \quad (\beta v) + 2(uw) = 0, \quad (\gamma w) + 2(uv) = 0,\\
(\alpha^2) = (\beta^2) = (\gamma^2) = 2(u^2) + (\beta\gamma) = 2(v^2) + (\alpha\gamma) = 2(w^2) + (\alpha\beta).
\end{gathered}$$

These fourteen equations will reduce the function V to the form

$$2V = A\omega^2 + P\lambda + Q\mu + R\nu + 2F\phi + 2G\chi + 2H\psi. \qquad (35, a)$$

This is the function used by Mr. GREEN, and is a particular case of equation (35). The first term of this function will determine the normal vibration, and the last six will represent, as we have seen, transverse vibrations propagated according to the laws of FRESNEL's wave-surface.

If the body be homogeneous and uncrystalline, we shall have the relations,

$$P = Q = R,$$
$$F = 0, \quad G = 0, \quad H = 0,$$

which will reduce the function V to the form

$$2V = A\omega^2 + P(\lambda + \mu + \nu). \qquad (36)$$

This function will represent homogeneous solids, liquids, and gases, and will be the complete function for these bodies, provided there be neither external forces nor pressures at the limits. If there be such forces, however, it will be necessary to add to the function (36), which is of the second order, other terms of the first order, as in equations (15). If the function (36) be assumed to represent all the forces engaged, the equations derived from it will represent the motion of a body abandoned to its own molecular actions, and freed from all external influence, such as gravitation, pressure of an atmosphere, &c. The known properties of solids, liquids, and gases, enable us to determine the form of the function (36), and thus lead to the terms to be added in the general case for each species of body.

It is generally admitted that a solid body, if abandoned to itself, would be capable of vibratory motion, and that its molecules, if displaced, would tend to return to their former position. A gas, if abandoned to itself, would be dissipated by the repulsive force of its molecules, so that in this case the function (36) should lead to an impossible result, as vibratory motion is impossible without the addition of pressures at the limits, or some equivalent forces. A liquid occupies a position intermediate between a solid and a gas; and if we assume that a liquid abandoned to itself will be in a state of unstable equilibrium (i. e. its molecules, if displaced, will not return to their original position, while, if undisturbed, they will not be dissipated), we shall obtain from (36) the equations of liquid motion, including friction, which have been deduced by various writers from different considerations.

A liquid need not be supposed to be exactly in this state at all times; a slight cohesive or a slight repulsive force may be supposed to exist among its molecules, according to the quantity of caloric contained in it, or other physical circumstances, which may modify the intensity of the molecular actions. If such cohesive or repulsive forces be considered very small, as compared with the cohesive forces in a perfect solid, or the repulsive forces in a perfect gas, the equations deduced from the hypothesis, that these forces are zero, may still be used.

We obtain from equation (36) the following:

$$\frac{dV}{da} = A\omega - 2P(\omega - a), \qquad \frac{dV}{du} = Pu,$$

$$\frac{dV}{d\beta} = A\omega - 2P(\omega - \beta), \qquad \frac{dV}{dv} = Pv,$$

$$\frac{dV}{d\gamma} = A\omega - 2P(\omega - \gamma), \qquad \frac{dV}{dw} = Pw.$$

It is necessary and sufficient for stable equilibrium that these six forces should have signs contrary to the signs of (δa, $\delta\beta$, $\delta\gamma$, δu, δv, δw). If these be made positive, then the forces must have negative signs, and *vice versâ.*

Hence, for stable equilibrium, it is necessary that A and P be both negative, which will reduce the function (36) to the following,

$$-2V = A\omega^2 + P(\lambda + \mu + \nu). \tag{36, a}$$

Also the first three equations (changing the signs of A and P), added together, must be negative; hence the condition,

$$(-3A + 4P)\,\omega < 0. \tag{36, b}$$

If the equilibrium be stable, A cannot be less than $\frac{4P}{3}$; and if it be exactly equal to this value, we shall obtain the equations peculiar to liquids, because a displacement will produce no molecular force; and if $A < \frac{4P}{3}$, a molecular force will be developed which will tend to increase the displacement. The function (36, a) will, therefore, represent solids, liquids, or gases, according as $A >, =, < \frac{4P}{3}$.

I shall consider, first, the equations of homogeneous solids. The function (36, a), substituted in equations (30), leads to the following equations of motion.

$$
\begin{aligned}
\epsilon \frac{d^2\xi}{dt^2} &= (A - P)\frac{d\omega}{dx} + P\left(\frac{d^2\xi}{dx^2} + \frac{d^2\xi}{dy^2} + \frac{d^2\xi}{dz^2}\right), \\
\epsilon \frac{d^2\eta}{dt^2} &= (A - P)\frac{d\omega}{dy} + P\left(\frac{d^2\eta}{dx^2} + \frac{d^2\eta}{dy^2} + \frac{d^2\eta}{dz^2}\right), \qquad (37) \\
\epsilon \frac{d^2\zeta}{dt^2} &= (A - P)\frac{d\omega}{dz} + P\left(\frac{d^2\zeta}{dx^2} + \frac{d^2\zeta}{dy^2} + \frac{d^2\zeta}{dz^2}\right).
\end{aligned}
$$

These equations of motion of solid bodies were first given by M. CAUCHY;* equations identical in form, but with the relation $A = 3P$ between the coefficients, had been previously obtained by M. NAVIER.† M. POISSON deduced equations identical with those of M. NAVIER. Mr. GREEN has used the equations (37), with two independent constants, in his theory of light;‡ and Mr. STOKES has recently called attention to the importance of retaining the two coefficients (leaving their ratio to be determined by experiment for each solid), in a memoir published in the Cambridge Philosophical Society's Transactions.§

The relation $A = 3P$ is a consequence of the use of definite integrals for the coefficients of the function V, and only represents a particular elastic solid; its introduction does not alter the form of equations (37), nor does it render them more simple than they are in their present state.

The additions necessary to be made to the equations of hydrodynamics, in order to take into account the friction of the fluid particles, have been given by many writers. M. NAVIER first stated the equations in their corrected form for incompressible fluids.‖ M. POISSON has treated of the subject in a me-

* Exercices des Mathematiques, tom. iii. p. 180.

† Memoires de l'Institut., tom. vii. p. 389.

‡ Transactions of Cambridge Philosophical Society, tom. vii. p. 11.

§ Vol. viii. part 3.

‖ Memoires de l'Institut., tom. vi. p. 414.

moir published in the *Journal de l'Ecole Polytechnique;** and, more recently, M. BARRÉ DE SAINT VENANT† and Mr. STOKES‡ have written on the friction of fluids in motion.

The quantities to be added to the ordinary equations of hydrodynamics are the right hand members of equations (37), introducing the relation $\left(A = \frac{4P}{3}\right)$, which expresses that a displacement produces no molecular force.

The equations of the motion of gases have been already given, on the supposition that there is no tangential action, which is equivalent to assuming that the vibrations are normal, and therefore $(P = 0)$. If we wish to take account of the friction of gases, we should use the equations which have just been indicated for liquid motion.

SECTION V. COMPARISON OF MECHANICAL THEORIES OF LIGHT.

It is well known that different mechanical theories have been proposed to account for the phenomena of the movement of light in crystalline bodies, and that, although these theories differ in their fundamental hypotheses, yet, to some extent, they agree in representing the most obvious phenomena of double refraction. The laws of wave-propagation in crystals, are geometrical consequences of the properties of FRESNEL'S wave-surface; and no mechanical theory of light can be considered as even an approximation to the truth, unless it contains, as a deduction from its hypotheses, the wave-surface of FRESNEL. But it would be an error to conclude that any theory is correct, which satisfies this condition. There are, in fact, three different theories which satisfy this fundamental condition, and it is evident that they cannot all be true. The first of these theories was propounded by FRESNEL himself, in his memoir on Double Refraction.§ It is based on the hypothesis, that the luminiferous ether is composed of attracting and repelling molecules. The form of wave-surface known as FRESNEL'S is deduced by its author from this hypothesis, with the peculiarity that the vibrations of the molecules are perpendicular to the plane of polarization.

* Cahier xx. p. 139. † Comptes Rendus, tom. xvii. p. 1240.

‡ Transactions of Cambridge Philosophical Society, vol. viii. part 3.

§ Memoires de l'Institut, tom. vii., p. 45.

M. Cauchy afterwards gave the general equations peculiar to such a system, and deduced from them Fresnel's wave-surface, as a first approximation to what he considered as the more accurate laws of wave-propagation.*

In the year 1839, Mr. Green presented to the Cambridge Philosophical Society a memoir, in which, by a modification of M. Cauchy's equations, he obtained Fresnel's wave-surface as an exact deduction from the theory.† This modification consists, as I have already stated, in restricting the system to propagate normal and transverse vibrations. In M. Cauchy's or Mr. Green's theory, the vibration of the molecules is parallel to the plane of polarization. In the same year Professor Mac Cullagh presented to this Academy a mechanical theory of light, not founded on the hypothesis of attracting and repelling molecules. The vibrations in this theory also are parallel to the plane of polarization, and the form of the wave-surface is that given by Fresnel. These three theories of light, therefore, agree, so far as the laws of wave-propagation are concerned; and, excluding Fresnel's theory from the comparison (as the vibrations perpendicular to the plane of polarization make it distinct from the other two theories), there remain the mechanical theories of Mr. Green and Professor Mac Cullagh, which are identical so far as the laws of wave-propagation are concerned. The two theories are, however, really different in their fundamental assumptions; and this remarkable agreement in the laws of wave-propagation deduced from them admits of a simple explanation. I propose to account for the agreement, and to suggest the direction in which we should look for a true *experimentum crucis* between them.

The function V used by Mr. Green, when reduced to its simplest form, will be

$$-2V = A\omega^2 + P\lambda + Q\mu + R\nu; \tag{38}$$

and the simplest form of Professor Mac Cullagh's equations will be derived from the function

$$-2V = PX^2 + QY^2 + RZ^2. \tag{39}$$

It is evident from what I have stated in the first section, that (λ, μ, ν) will

* Memoires de l'Institut. tom. x., 1830.

† Transactions of Cambridge Philosophical Society, vol. vii. p. 121.

produce the same terms in the differential equations of motion, as X^2, Y^2, Z^2; for the squares will be the same in each, and the rectangles will be

$$(2\beta_3\gamma_2-4\beta_2\gamma_3,\ 2\gamma_1\alpha_3-4\alpha_1\gamma_3,\ 2\alpha_2\beta_1-4\alpha_1\beta_2),\ \text{and}\ (-2\beta_3\gamma_2,\ -2\gamma_1\alpha_3,\ -2\alpha_2\beta_1).$$

These two sets of rectangles will produce the same terms in the equations of motion, since we may transpose the differentiations without affecting the result, so far as the laws of propagation are concerned. The surfaces of wave-slowness deduced from (38) and (39) may be obtained immediately from equation (18). Equation (38) will give the following:

$$\begin{aligned} P' &= Al^2 + Rm^2 + Qn^2; & F' &= (A-P)\,mn; \\ Q' &= Am^2 + Pn^2 + Rl^2; & G' &= (A-Q)\,ln; \\ R' &= An^2 + Ql^2 + Pm^2; & H' &= (A-R)\,lm. \end{aligned} \tag{40}$$

And similarly from equations (39) will be found

$$\begin{aligned} P' &= Rm^2 + Qn^2; & F' &= -Pmn; \\ Q' &= Pn^2 + Rl^2; & G' &= -Qln; \\ R' &= Ql^2 + Pm^2; & H' &= -Rlm. \end{aligned} \tag{41}$$

These equations differ from the former only by not containing A.

The equation of wave-slowness (18) derived from (40) is

$$\begin{aligned} &\{A(x^2+y^2+z^2)-1\} \\ \times&\{(x^2+y^2+z^2)(QRx^2+PRy^2+PQz^2)-(Q+R)x^2-(P+R)y^2-(P+Q)z^2+1\}=0. \end{aligned} \tag{42}$$

The first factor of this equation represents a sphere whose radius is $\frac{1}{\sqrt{A}}$, and belongs to the normal vibration; the second factor is the equation of FRESNEL'S wave-surface, and in it the vibrations are transversal. The equation of wave-slowness deduced from Professor MAC CULLAGH'S function (39) will be the last factor of (42). So far, therefore, as the laws of wave-propagation are concerned, the functions (38) and (39) are equivalent, with this difference, that the function (38) introduces a normal wave, which does not enter into the equations derived from (39). It might be thought at first sight that we are at liberty to make $A = 0$, and thus reduce the function (38) to a function representing nothing but transverse vibrations; this, however, cannot be admitted, for as

(38) represents a body whose molecules act in the line joining them, a wave of normal compression is always possible. This will be rendered more evident by considering the conditions at the limits.

Let the limiting surface separating two bodies be the plane (x, y), then the equations of condition (11) will become, for the function (38),

$$\begin{aligned} Q'v'_0 &= Q''v''_0; & \xi'_0 &= \xi''_0; \\ P'u'_0 &= P''u''_0; & \eta'_0 &= \eta''_0; \\ A'\omega'_0 + Q'\alpha'_0 + P'\beta'_0 &= A''\omega''_0 + Q''\alpha''_0 + P''\beta''_0; & \zeta'_0 &= \zeta''_0. \end{aligned} \tag{43}$$

These equations are equal in number to the unknown quantities, provided normal waves be included, because the unknown quantities of the problem are the intensities of the reflected and refracted waves; it is impossible, therefore, for exclusively transverse waves to be produced by reflexion or refraction in such a body as (38) defines: in order to obtain unknown quantities whose number shall be equal that of the necessary conditions of the mechanical problem, we must introduce normal vibrations. The conditions at the limits deduced from (39) are

$$\begin{aligned} Q'Y'_0 &= Q''Y''_0; & \xi'_0 &= \xi''_0; \\ P'X'_0 &= P''X''_0; & \eta'_0 &= \eta''_0; \\ && \zeta'_0 &= \zeta''_0. \end{aligned} \tag{44}$$

The additional hypothesis made by Professor MAC CULLAGH, that the density of the medium is the same in the two bodies, reduces these equations to four. Accordingly, with this hypothesis, there is no necessity to have recourse to normal waves, as there will be four intensities to be determined in the transverse waves.

From these considerations it appears, that the *experimenta crucis* between the rival theories of light must be sought for among the laws of reflexion and refraction; but unfortunately these laws are not known with sufficient accuracy to enable us to decide the question. Mr. GREEN's theory contains the common laws of reflexion at the surfaces of ordinary media as first approximations, while Professor MAC CULLAGH's system has the advantage of giving these laws as exact results; nothing, however, but more accurate experiments can decide whether the approximation or the exact result be most in accordance with the truth; and as these experiments involve considerations of the intensity of light, it would be difficult to make them with sufficient accuracy. The present state of the wave theory of light certainly suggests grave doubts as to the nature of the foundation on which the whole system is based. We first assume the existence of an unknown medium, whose existence must remain unproved and unprovable by us; then, from supposed properties of this unknown medium, we deduce the laws of propagation, &c. Here a new difficulty arises; for we find several different theories capable of explaining the laws of propagation, and explaining with more or less exactness the most obvious of the laws of reflexion and refraction. How are we to decide among these conflicting theories? Are we to assume, with M. CAUCHY, that the observed laws of polarized light occupy, with respect to the mathematical laws deduced from his theory, the same position that the laws of KEPLER stand in with respect to the more accurate laws of planetary motion? or are we to assume that theory to be correct which agrees accurately with the common formulæ for reflected light, when it is well known that these formulæ themselves are doubtful for highly refracting substances? It appears certain, that we do not yet possess experimental knowledge sufficient to enable us to determine which of the theories of light is correct, or whether any of them be so. In a general point of view Professor MAC CULLAGH's theory possesses an important advantage, as compared with other theories. It contains no inexplicable normal wave, and does not render this difficult subject still more intricate, by the introduction of a useless vibration. It is greatly to be desired, that the attention of experimentalists were directed to the necessity which exists for more accurate and general researches into the laws of crystalline reflexion and refraction, and that the surface of FRESNEL were placed upon a purely experimental basis. From such researches, carefully conducted, might be deduced the geometrical laws of double refraction, and a foundation be laid for a complete and positive theory of the laws of polarized light.

VI.—*On the Rotation of a Solid Body round a fixed Point; being an Account of the late* Professor MAC CULLAGH's *Lectures on that Subject. Compiled by the* Rev. SAMUEL HAUGHTON, *Fellow and Tutor of Trinity College, Dublin.*

Read April 23, 1849.

THE following Essay, on the Rotation of a Solid Body round a fixed Point, has been compiled from my notes of Professor MAC CULLAGH's lectures, delivered in the Hilary Term of the year 1844, in Trinity College. A short account of some of the results contained in it was published by Professor MAC CULLAGH himself, in the Proceedings of the Royal Irish Academy.* As it has appeared to many of Mr. MAC CULLAGH's friends desirable that a somewhat more detailed account of his researches in this subject should be published, I have, in accordance with this desire, drawn up and presented to the Academy the following account of his lectures on Rotation. I have endeavoured to arrange the subject in a systematic order, and to give the results proved by him during the course of the lectures, carefully excluding all theorems and proofs of theorems, which were not originally given by him, as here stated.

SAMUEL HAUGHTON.

* Vol. ii. pp. 520, 542.

I.—*Composition of Rotations.*

Let O be the intersection of two axes of rotation, OR, OR′; and let the magnitudes of the rotations be represented by ω, ω'; then the motion impressed upon the body by these two rotations will be the same as the motion produced by a single rotation round an axis, which is represented in magnitude and position by the diagonal of the parallelogram formed by ω, ω'. For, draw through any point I of the body a plane perpendicular to the line OI, and project upon this plane the parallelogram formed by ω, ω';—the sides of this parallelogram will be ω sin ROI and ω' sin R′OI. Now the velocities impressed upon the point I by the rotations ω and ω', are OI.ω sin ROI, and OI.ω' sin R′OI; and the directions of these velocities are perpendicular to the sides of the projected parallelogram. Hence, if this parallelogram be turned in its plane through 90°, its sides will represent in magnitude and direction the actual velocities; the resultant of these velocities is perpendicular to the projection of the diagonal of the parallelogram (ω, ω'); this projection, turned round through 90°, will represent the actual velocity, which is therefore the same in magnitude and direction as would be produced by a single rotation represented by the diagonal of (ω, ω'). Hence rotations may be resolved along three rectangular axes by the same laws as couples, and they must be counted positive when the motion produced is from z to x, x to y, y to z, and *vice versâ.*

II.—*Linear Velocities produced by a given Rotation.*

Let the origin of co-ordinates be assumed on the axis of rotation, and let the magnitude of the rotation and of its components be represented by (ω, p, q, r): the velocity of any point (x, y, z) is in a direction perpendicular to the plane containing the axis of rotation and the point (x, y, z); and its magnitude is represented by the area of the triangle whose angles are situated at the origin, the point (x, y, z), and the point (p, q, r). Hence, the components of the linear velocity are represented by the projections of this triangle on the co-ordinate planes. These projections are

$$
\begin{aligned}
u &= qz - ry; \\
v &= rx - pz; \\
w &= py - qx.
\end{aligned} \tag{1}
$$

III.—*To represent geometrically the Moments of Inertia of a Body with respect to Axes drawn through a fixed Point.*

The moment of inertia of a body with respect to any axis (α, β, γ) is

$$
M = A' \cos^2 \alpha + B' \cos^2 \beta + C' \cos^2 \gamma - 2L' \cos \beta \cos \gamma - 2M' \cos \alpha \cos \gamma - 2N' \cos \alpha \cos \beta;
$$

where

$$
\begin{aligned}
A' &= \int (y^2 + z^2)\, dm, & L' &= \int yz dm; \\
B' &= \int (x^2 + z^2)\, dm, & M' &= \int xz dm; \\
C' &= \int (x^2 + y^2)\, dm, & N' &= \int xy dm.
\end{aligned}
$$

Assume $M = \frac{\mu}{r^2}$; μ being the mass of the body, and r a distance measured on the line (α, β, γ) and construct the ellipsoid whose equation is

$$
A'x^2 + B'y^2 + C'z^2 - 2L'yz - 2M'xz - 2N'xy = \mu; \tag{2}
$$

then it is evident that the moments of inertia of the body with respect to axes passing through the fixed point are represented by the squares of the reciprocals of the radii vectores of this ellipsoid. Assume $A = \mu a^2$, $B = \mu b^2$, $C = \mu c^2$, and let the axes of co-ordinates be the axes of the ellipsoid; its equation will thus become

$$
a^2x^2 + b^2y^2 + c^2z^2 = 1; \tag{3}
$$

and the equation of the reciprocal ellipsoid will be

$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \tag{4}
$$

This latter ellipsoid may be called the ellipsoid of gyration, as the perpendiculars on its tangent planes represent the radii of gyration; this is evident from the consideration, that these perpendiculars are reciprocal to the radii vectores of the ellipsoid (3). In fact, the moment of inertia with respect to any axis will be represented by the formula

$$M = (a^2 \cos^2 a + b^2 \cos^2 \beta + c^2 \cos^2 \gamma)\,\mu = \mu P^2 = \frac{\mu}{R'^2}; \qquad (5)$$

(R,P) denoting the radius vector and perpendicular on tangent plane of the ellipsoid (4); and $(R'P')$, the corresponding lines in the ellipsoid (3).

IV.—*To find the Magnitude, Position, and Direction of the Statical Couple produced by the Centrifugal Forces.*

If from any point (x, y, z) of the body, a perpendicular be let fall on the axis of rotation (a, β, γ), the centrifugal force will be represented by the product of the square of the angular velocity and this perpendicular; the corresponding elementary statical couple will be found by multiplying the centrifugal force by the distance from the foot of the perpendicular to the origin, which is represented by the quantity $(x \cos a + y \cos \beta + z \cos \gamma)$. The components of the elementary couple will be proportional to the projections of the triangle formed by the lines before mentioned. The components of the elementary couples must be integrated for the entire extent of the body, and the integrals thus found will be the components of the couple produced by centrifugal force: the expressions are as follows:

$$\omega^2 (x \cos a + y \cos \beta + z \cos \gamma)(z \cos \beta - y \cos \gamma)\, dm;$$
$$\omega^2 (x \cos a + y \cos \beta + z \cos \gamma)(x \cos \gamma - z \cos a)\, dm;$$
$$\omega^2 (x \cos a + y \cos \beta + z \cos \gamma)(y \cos a - x \cos \beta)\, dm.$$

If the axes of co-ordinates be principal axes, these expressions, when integrated, will become

$$\begin{aligned} &\omega^2 \cos \beta \cos \gamma\, (B - C) = qr\, (B - C); \\ &\omega^2 \cos a \cos \gamma\, (C - A) = pr\, (C - A); \qquad (6) \\ &\omega^2 \cos a \cos \beta\, (A - B) = pq\, (A - B): \end{aligned}$$

p, q, r being the components of the angular velocity ω. The position of the resultant couple may be expressed by means of the ellipsoid (4). If a tangent plane be drawn to this ellipsoid at the point (x, y, z), and perpendicular to the line (a, β, γ), it may be easily shown that the projections of the triangle formed by the radius vector and perpendicular are represented by the quantities

$$\cos\beta\cos\gamma\,(b^2-c^2),\quad \cos\alpha\cos\gamma\,(c^2-a^2),\quad \cos\alpha\cos\beta\,(a^2-b^2);$$

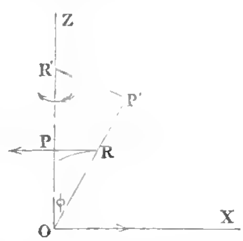
these three expressions multiplied by $\mu\omega^2$ will produce the quantities used in (6). Hence it appears, that the couple produced by the centrifugal forces lies in the plane of the radius vector and perpendicular to a tangent plane of the ellipsoid (4); the tangent plane being perpendicular to the axis of rotation. Also, the magnitude of the resultant couple is proportional to the triangle formed by the radius vector and perpendicular.

The differential equations of motion commonly used in the solution of this problem may be deduced immediately from equations (6). In fact, as the axes of co-ordinates are axes of permanent rotation, the increment of angular velocity round each axis will be equal to the statical couple of the applied forces (including centrifugal forces), divided by the moment of inertia round that axis; the statement of this fact, in analytical language, will give the equations of motion:

$$\begin{aligned} A\frac{dp}{dt} &= (B-C)\,qr + L;\\ B\frac{dq}{dt} &= (C-A)\,pr + M;\\ C\frac{dr}{dt} &= (A-B)\,pq + N: \end{aligned} \qquad (7)$$

(L, M, N) being the components of the applied statical couple.

The *position* and *magnitude* of the couple produced by the centrifugal forces are easily found by the method which has been just given; but the *direction* will be found more readily by taking more particular axes of co-ordinates. Let the axis of rotation be the axis OZ, and the plane of radius vector and perpendicular be the co-ordinate plane XOZ. In the accompanying figure OR′ and OP′ are the radius vector and perpendicular of the ellipsoid (2), and OR, OP the radius vector and perpendicular of the ellipsoid (4), which is reciprocal to the former; the rotation is positive, in the direction indicated by the arrow. As the rotation is round the



axis of z, it is easy to see that the statical couple produced by centrifugal force will have for components, round the axes of x and y respectively, the quantities $\omega^2\int yzdm$, $\omega^2\int xzdm$ taken with their proper sign; i. e. the components are $\pm\,\omega^2 L'$, $\pm\,\omega^2 M'$; $L'M'$, being coefficients in the equation of the ellipsoid

$$A'x^2 + B'y^2 + C'z^2 - 2L'yz - 2M'xz - 2N'xy = \mu.$$

The tangent plane to this ellipsoid, applied at the point (x, y, z) will be

$$(A'x - M'z - N'y)\,x' + (B'y - N'x - L'z)\,y' + (C'z - L'y - M'x)\,z' = \mu.$$

At the point R′ the tangent plane will be perpendicular to the plane XOZ, and will be found by making $x = 0$, $y = 0$, and destroying the coefficient of y' in the preceding equation. These conditions give us $L' = 0$, which proves that the statical couple produced by centrifugal force lies altogether in the plane XOZ. The equation of the tangent plane is the same as the equation of the line R′P′, and is

$$C'z' - M'x' = \frac{\mu}{z}.$$

Hence we obtain

$$\tan\phi = -\frac{M'}{C'}.$$

The value of the centrifugal couple is $\omega^2 M'$, which is found from the preceding equation by replacing C' and $\tan\phi$ by their values μP^2, and $\frac{Q}{P}$; Q being the line RP.

We thus obtain finally the centrifugal couple lying in the plane XOZ, and expressed by the equation

$$\omega^2\int xzdm = -\,\mu\omega^2 PQ. \tag{8}$$

It thus appears that the centrifugal couple lies in the plane of radius vector and perpendicular, is proportional to the area of the triangle ROP, and has a direction opposite to the direction of rotation.

V.—*To find the Relation between the Plane of principal Moments and the Axis of Rotation at any Instant.*

The motion of the body at any instant consists of a rotation of a certain magnitude round a certain axis; this rotation might be produced by an impulsive couple of a determinate magnitude and direction. The statical impulsive couple thus conceived is the couple of principal moments. Let this couple be represented by G, and act round the axis OR (fig. 1, p. 148); then the corresponding axis of rotation will be the perpendicular OP, and the relation between G and ω may be thus found. Let the axes of co-ordinates be the axes of the ellipsoid (4), the radius vector being determined by the angles (λ, μ, ν), and the axis of rotation by the angles (α, β, γ). From mechanical considerations we obtain the equations

$$G \cos \lambda = Ap = \mu\omega\, a^2 \cos \alpha \,;$$
$$G \cos \mu = Bq = \mu\omega\, b^2 \cos \beta \,;$$
$$G \cos \nu = Cr = \mu\omega\, c^2 \cos \gamma .$$

Hence we obtain

$$\left.\begin{aligned} \frac{\cos \lambda}{\cos \nu} = \frac{a^2 \cos \alpha}{c^2 \cos \gamma}, \quad \frac{\cos \mu}{\cos \nu} = \frac{b^2 \cos \beta}{c^2 \cos \gamma}, \\ \omega = \frac{G}{\mu PR} = \frac{G \cos \phi}{\mu P^2}. \end{aligned}\right\} \qquad (9)$$

The first two of these equations prove that the axis of rotation is the perpendicular on tangent plane of the ellipsoid, and the last equation gives the magnitude of the rotation in terms of the impressed couple and quantities determined by the nature of the body itself. Equations (9) are true, whatever be the forces acting on the body; if no forces act, G will be fixed in magnitude and position in space, by the principle of conservation of areas, but will change its position in the body, the axis of rotation accompanying it, and changing its position both in the body and in space.

VI.—*Rotation produced by Centrifugal Force; particular Properties of the Motion when no Forces act.*

The axis of rotation produced by the centrifugal couple always lies in the plane of principal moments. This theorem may be thus proved: Let the radius vector and perpendicular be drawn, which coincide with the axis of principal moment and axis of rotation at any instant; a line perpendicular to the plane of radius vector and perpendicular is the axis of centrifugal couple;

this line and the original radius vector are axes of the section of the ellipsoid made by their plane; at the point where the axis of the centrifugal couple pierces the ellipsoid let a tangent plane be applied; the perpendicular let fall on this tangent plane is the axis of rotation produced by centrifugal forces. From the construction it is evident that the plane of the second radius vector and perpendicular is perpendicular to the axis of G; hence the axis of the centrifugal couple and the axis of rotation produced by it, always lie in the plane of principal moment. Two important corollaries follow from the theorem just demonstrated, in the case where no forces act:—First, the component of angular velocity round the axis of primitive impulse is constant during the motion. Secondly, the radius vector which coincides with the axis of G is of constant length during the motion. The first theorem is obvious; for as the axis of rotation produced by centrifugal force is always perpendicular to the axis of G, it cannot alter the rotation round that axis. The second theorem follows from equation (9), from which we deduce

$$\omega \cos \phi = \frac{G}{\mu R^2}. \tag{10}$$

The left hand member of this equation is constant by the preceding theorem; and G is constant, since there is no external force; therefore R is constant.

As the axis of G is fixed in space, and the line R is constant, it is evident that the axis of G will describe in the body the cone of the second degree, determined by the intersection of the ellipsoid (4) with the sphere whose radius is R. The equation of this cone is

$$\frac{R^2 - a^2}{a^2} x^2 + \frac{R^2 - b^2}{b^2} y^2 + \frac{R^2 - c^2}{c^2} z^2 = 0. \tag{11}$$

As the axis of principal moments describes this cone in the body, it is accompanied by the axis of rotation, which is always the corresponding perpendicular on tangent plane of the ellipsoid. The cone described by the axis of rotation might be found thus. Let tangent planes be applied to the ellipsoid along the spherical conic in which the cone (11) cuts the ellipsoid. From the centre let fall perpendiculars on these tangent planes; the locus of these perpendiculars is the required cone.

VII.—*The Axis of principal Moments is fixed in Space.*

This is evident from D'ALEMBERT'S principle, but may be shown by geometrical considerations in the particular case under consideration. The axis varies in position in the body, in consequence of the centrifugal couple, which must be compounded with the impressed couple at each instant. Referring to equation (8), the value of the centrifugal couple is $-\mu\omega^2 PQdt$, the principal moment being $G = \mu\omega PR$ (vid. 9). Hence the angle through which the axis of principal moment shifts in an element of time is $-\frac{\omega Qdt}{R}$; this angle, multiplied by the constant radius vector, will give the elementary motion on the spherical conic traced by the axis of principal moment on the surface of the ellipsoid; this motion is therefore $-\omega Qdt$; but in the same time the point of the body which coincides with the point where the axis of moments pierces the spherical conic will describe the angle $+\omega Qdt$ in consequence of the angular rotation. Hence the axis of moments will remain fixed in space, and will move in the body with a velocity proportional to the tangent of the angle between the radius vector and perpendicular, the motion being in a direction opposite to the direction of the rotation; this is evident from the consideration that $Q\omega = P\omega \tan\phi$, $P\omega$ being constant and equal to $\frac{G}{\mu R}$ (vid. 9).

VIII.—*To find the Motion of the principal Axis in the Body.*

First Method.

The point of the principal axis of moments, which is situated at the distance R from the centre, moves on the spherical conic which has been determined. Let this point be projected on the three co-ordinate planes; then, since the spherical conic is projected into a conic section, the movement of the axis of moments is reduced to the movement of a point on a conic section, according to a law which must be determined. The radius vector describes an elementary triangle in the surface of the cone (11); let the projections of this triangle on the co-ordinate planes be (dA_1, dA_2, dA_3); we obtain easily

$$\frac{dA_1}{dt} = y\frac{dz}{dt} - z\frac{dy}{dt}, \quad \frac{dA_2}{dt} = z\frac{dx}{dt} - x\frac{dz}{dt}, \quad \frac{dA_3}{dt} = x\frac{dy}{dt} - y\frac{dx}{dt}.$$

Substituting in these equations the values of the velocities given by (1), we obtain

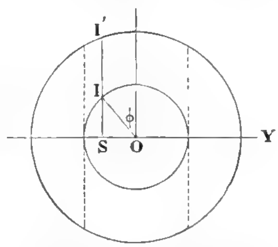
$$\begin{aligned} \frac{dA_1}{dt} &= P\omega\,\frac{R^2-a^2}{a^2}\,x = (R^2-a^2)\,p\,; \\ \frac{dA_2}{dt} &= P\omega\,\frac{R^2-b^2}{b^2}\,y = (R^2-b^2)\,q\,; \\ \frac{dA_3}{dt} &= P\omega\,\frac{R^2-c^2}{c^2}\,z = (R^2-c^2)\,r. \end{aligned} \tag{12}$$

These equations prove, that the *areolar velocity* of the projection on a co-ordinate plane varies at the ordinate to that plane. By means of the method of quadratures, we may determine from equations (12) the position of the projections of the principal axis at any instant, and hence deduce the position of the axis itself.

Second Method.

If the spherical conic be projected on a cyclic plane of the ellipsoid of gyration, by lines parallel to x and z, the projections will be two concentric circles, and the corresponding projections will lie on the same ordinate SII′ (fig. 2). The inner circle will belong to the projection parallel to x, if R be greater than b, and will belong to the projection parallel to z if R be less than b; and if R be equal to b, the two circles will coincide with each other and with the spherical conic, which in this case becomes the circular section of the ellipsoid. The projected point will revolve round the circumference of the inner circle, and will vibrate on the circumference of the outer circle, between the dotted lines. It is evident that the mean axis of the ellipsoid OY lies in the plane of the figure. Let SI and SI′ be equal to ρ, ρ', and let C, C'' denote the radii of the two circles: the velocities of the projections in the circles will evidently be

FIG. 2.



$$V = \frac{C}{\rho}\frac{dy}{dt}, \quad V' = \frac{C'}{\rho'}\frac{dy}{dt};$$

C and C' having the values

$$C = b\sqrt{\left(\frac{a^2 - R^2}{a^2 - b^2}\right)}, \quad C' = b\sqrt{\left(\frac{R^2 - c^2}{b^2 - c^2}\right)}.$$

The value of $\frac{dy}{dt}$ deduced from (1) is,

$$\frac{dy}{dt} = P\omega\left(\frac{1}{c^2} - \frac{1}{a^2}\right) xz = P\omega\left(\frac{1}{c^2} - \frac{1}{a^2}\right)\rho\rho' \sin\theta\cos\theta;$$

θ being the angle made by the plane of the circular section with the plane (x, y).

$$\sin\theta = \frac{c}{b}\sqrt{\left(\frac{a^2 - b^2}{a^2 - c^2}\right)}, \quad \cos\theta = \frac{a}{b}\sqrt{\left(\frac{b^2 - c^2}{a^2 - c^2}\right)}.$$

Introducing these values of $\frac{dy}{dt}$, $\sin\theta$ and $\cos\theta$, and for $P\omega$ its value $\frac{G}{\mu R}$, we obtain finally for the velocities

$$\left.\begin{aligned} V &= \frac{G}{\mu}\sqrt{\left\{\left(\frac{1}{R^2} - \frac{1}{a^2}\right)\left(\frac{1}{c^2} - \frac{1}{b^2}\right)\right\}}\,\rho' = K\rho'; \\ V' &= \frac{G}{\mu}\sqrt{\left\{\left(\frac{1}{R^2} - \frac{1}{c^2}\right)\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\right\}}\,\rho = K'\rho. \end{aligned}\right\} \quad (13)$$

The velocity of each projection, therefore, varies as the ordinate of the other. This theorem enables us to find a simple expression for the time. Using the angle (ϕ) marked in fig. 2, we obtain

$$\frac{Cd\phi}{dt} = K\sqrt{(C'^2 - C^2\sin^2\phi)};$$

(ϕ, C, K) belonging to the projection parallel to axis of x. If (ψ, C', K') be the corresponding quantities for the other projection, we obtain also

$$\frac{C'd\psi}{dt} = K'\sqrt{(C^2 - C'^2\sin^2\psi)};$$

or, since it is easily seen that $\frac{K}{K'} = \frac{C}{C'}$, we obtain finally

$$
\left.\begin{aligned}
K'dt &= \frac{d\phi}{\sqrt{\left(1 - \frac{C^2}{C''^2}\sin^2\phi\right)}}\,; \\
Kdt &= \frac{d\psi}{\sqrt{\left(1 - \frac{C''^2}{C^2}\sin^2\psi\right)}}.
\end{aligned}\right\} \qquad (14)
$$

The motion of the principal axis of moments is, therefore, expressed by an elliptic function of the first kind.

The motion of the axis of moments is determined by the magnitude of the radius vector of the ellipsoid, which is the axis of the original couple impressed upon the body; if this radius vector be greater than the mean axis of the ellipsoid, the corresponding spherical conic will have the axis of x for its internal axis; and if the radius be less than the mean axis, the axis of z will be the internal axis of the conic; in no case will the mean axis be the internal axis of the spherical conic. If the radius R be nearly equal to either the greatest or least semi-axis, the expression (14) for the time may be integrated. Let R be nearly equal to the greatest semi-axis. The first of the equations (14) belongs to the interior circle, which is of small dimensions in the case supposed; the second equation expresses the vibratory motion of the projection, through a small arc of the outer circle, which will have a radius much greater than the inner circle; we may, therefore, suppose the angle ψ to be equal to its sine. Multiplying both sides of the equation by $\frac{C'}{C}$ we obtain

$$
\frac{C'K}{C}\,dt = K'dt = \frac{\frac{C'}{C}\,d\psi}{\sqrt{\left(1 - \frac{C''^2}{C^2}\psi^2\right)}}.
$$

Hence

$$
\frac{C'\psi}{C} = \sin\,(K't + A). \qquad (15)
$$

If T_0 denote the time of a complete oscillation or revolution of axis of moments about the axis of x, and T_r the time of a revolution of the body round the axis of x, the following relation between these two periods may be readily deduced from (15):

$$T_0 = T_r \frac{bc}{\sqrt{\{(a^2 - b^2)(a^2 - c^2)\}}}. \tag{16}$$

If the axis of moments, and consequently the axis of revolution, be situated near the axis of greatest or least inertia, it will always continue near this axis; if, however, it be situated near the mean axis, the movement of the body will be determined by the following construction. Let the two cyclic planes of the ellipsoid be drawn through the mean axis; they will divide the ellipsoid into two regions, in one of which is situated the axis of maximum inertia, and in the other the axis of minimum inertia. The spherical conic described by the axis of principal moments will have the first or second of these axes for its internal axis, according as R is greater or less than the mean axis. If the axis of principal moments lie in one of the cyclic planes, the spherical conic becomes a circle, and its two projections become identical with itself (fig. 2, p. 148); the expressions (14) are reduced to the form

$$Kdt = \frac{d\phi}{\cos\phi};$$

which when integrated gives

$$Kt + A = \log\cot\left(\frac{\pi}{4} - \frac{\phi}{2}\right);$$

or,

$$\cot\left(\frac{\pi}{4} - \frac{\phi}{2}\right) = \cot\left(\frac{\pi}{4} - \frac{\phi_0}{2}\right)\epsilon^{Kt}; \tag{17}$$

ϕ_0 being the value of ϕ corresponding to $t = 0$, and K being expressed by the following quantity:

$$K = \frac{G}{\mu}\frac{\sqrt{\{(a^2 - b^2)(b^2 - c^2)\}}}{b^2ac}.$$

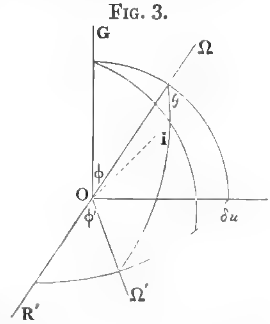
It is evident from the equation (17), that the axis of moments will coincide with the mean axis of inertia at the end of an infinite time.

IX.—*To find the Position of the Body in Space at the End of any given Time.*

First Method.

The radius vector of the ellipsoid, which is perpendicular to the plane containing the axes of principal moment and of rotation, always lies in the plane of principal moment, and describes in that plane areas proportional to the time.

FIG. 3.



Let OG, OΩ be the axes of principal moment and of rotation; OR′, OΩ′, the axes of centrifugal couple and of corresponding rotation; the plane ΩOΩ′ will contain the two successive positions of the axis of rotation. Let OI be the position of the axis of rotation at the end of the time δt; then δu will be equal to the angle described in the fixed plane by the line OR′. Let R' and P' be the radius vector and perpendicular corresponding to the centrifugal couple and its axis of rotation. The following relations are evident from the figure

$$\frac{\omega}{\omega'} = \frac{\sin \Omega' OI}{\sin \Omega OI} = \frac{\cos \phi'}{\sin \phi \delta u}; \text{ because } \sin \Omega' OI = \frac{\cos \phi'}{\sin \theta}, \sin \Omega OI = \frac{\sin \phi \delta u}{\sin \theta};$$

but from mechanical considerations,

$$\frac{\omega}{\omega'} = -\frac{P'R'}{PR\omega \sin \phi \delta t}; \text{ because, } \omega = \frac{G}{\mu PR}, \quad \omega' = -\frac{G\omega \sin \phi \delta t}{\mu P' R'}.$$

Hence, by equating the geometrical and mechanical expressions, we obtain

$$-R'^2 \delta u = \omega PR \delta t = \frac{G}{\mu} \delta t. \tag{18}$$

The position of the body in space is thus reduced to quadratures; but the problem may be solved more readily in the following manner.

Second Method.

The axis of principal moments, appearing to move in a direction opposite to the rotation, describes in the body the cone whose equation has been given (11).

If the cone reciprocal to this cone be described, one of its sides will lie in the fixed plane, and the whole motion of the body in space will be the same as the motion of this cone, which partly slides and partly rolls on the fixed plane, the sliding motion being uniform. This theorem is evident by resolving the angular velocity ω into two components, one round the axis of principal moments, and the other in a direction perpendicular to this, round the side of the reciprocal cone, which is in contact with the fixed plane. These components are $\omega \cos \phi$ and $\omega \sin \phi$; $\omega \cos \phi$ being constant and producing the sliding motion, while $\omega \sin \phi$ represents the angular velocity round the side of the cone in contact with the fixed plane. The angle described by the side of the reciprocal cone in the fixed plane at the end of a given time, is, therefore, the algebraic sum of two angles, one of which is proportional to the time, and the other is the angle described in the cone in consequence of the rotation $\omega \sin \phi$, and is, therefore, measured by the arc of a spherical conic. The position of the body at the end of the time t is thus found:—determine by equation (14) the position of the axis of principal moments in the cone (11); the corresponding position of the component axis of rotation in the reciprocal cone is therefore known. Hence the angle described in the time t in the fixed plane is

$$\Theta = \int \omega \cos \phi dt \pm \int \frac{ds}{R} = \omega \cos \phi . t \pm \frac{s}{R}. \tag{19}$$

The equation of the reciprocal cone is

$$\frac{a^2x^2}{R^2 - a^2} + \frac{b^2y^2}{R^2 - b^2} + \frac{c^2z^2}{R^2 - c^2} = 0. \tag{20}$$

In (19) the positive or negative sign must be used according as R is less or greater than the mean axis of the ellipsoid; this is evident from the composition of rotations, and from the consideration that in the former case the axis of rotation falls inside the cone (11), while in the latter case it falls outside.

X.—*To find a Point in a given Axis of Rotation, which being fixed, the Axis will be permanent.*

Let R′R″ (fig. 4) be the given axis, round which the body revolves with a rotation expressed by ω; describe the ellipsoid of gyration round the centre of gravity O, and draw OP′ parallel to R′R″. The centrifugal force $\omega^2 r dm$ at any point (x, y, z), may be resolved into two components, $\omega^2 \rho dm$ and $\omega^2 . R'P' . dm$; r and ρ denoting the distances of the point from the axes R′R″ and OP′ respectively; the effect of the rotation round R′R″ is therefore the same as an equal rotation round OP′, together with a number of parallel constant forces applied to each point of the body. The rotation round OP′ produces a centrifugal couple represented by $-\mu\omega^2 . OP.PR$ (vid. 8); or, determining the point R′ by thecondition $OP.PR = OP'.P'R'$, the centrifugal couple is $-\mu\omega^2 . OP'.P'R'$. The resultant of the parallel forces is a force applied at the centre of gravity, acting in the direction parallel to R′P′, and equal to $\mu\omega^2 . R'P'$. Comparing this with the centrifugal couple, it is evident that the forces at O destroy each other, and, therefore, the total result of the rotation round R′R″ is to produce a force acting at the point R′, which has been just determined. If this point be fixed, the axis R′R″ will be a permanent axis of rotation. The condition by which the point R′ is found is, that the triangle OR′P′ is equal to the triangle ORP; hence, if an ellipsoid confocal to the ellipsoid of gyration be described through the point R′, it will be perpendicular to the line R′R″. The general construction for permanent axes is, therefore, the following. Let the ellipsoid of gyration be described, and confocal ellipsoids; any line which pierces one of these ellipsoids at right angles is a permanent axis of rotation for the point of intersection.

FIG. 4.

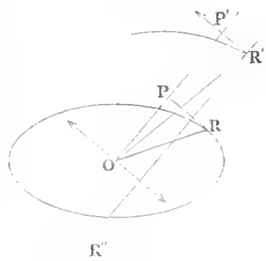


Plate IV.

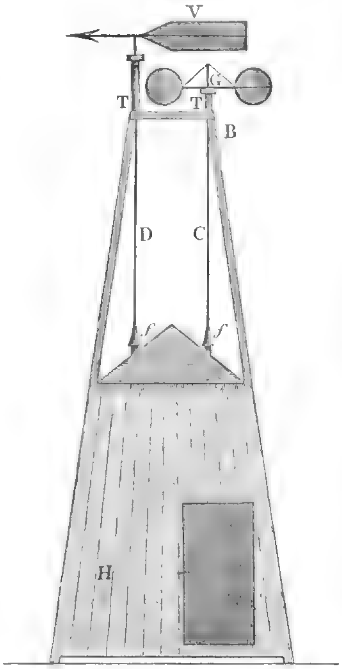


Fig. 1.

Plate V.

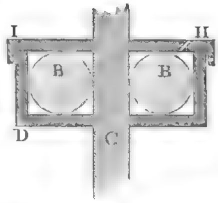


Fig. 2.

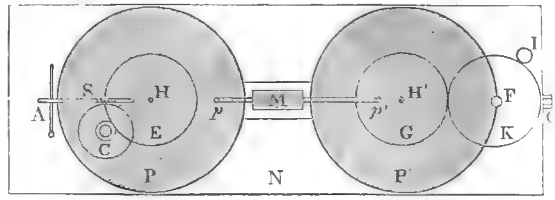


Fig. 6.

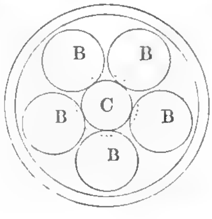


Fig. 3.

Plate VI.

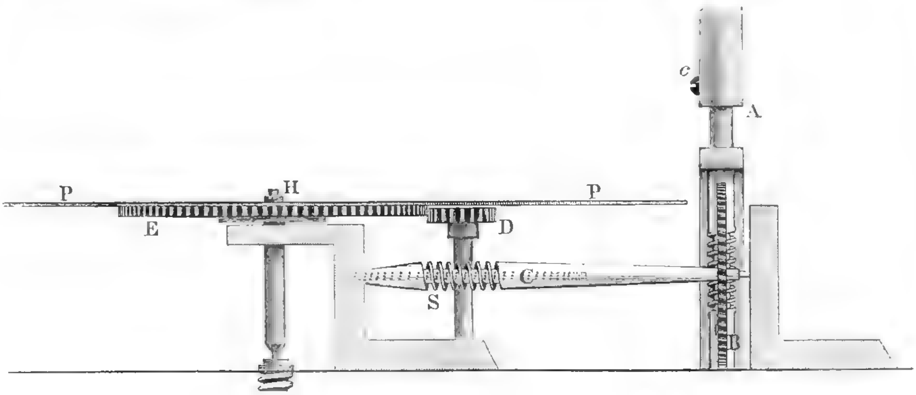


Fig. 4.

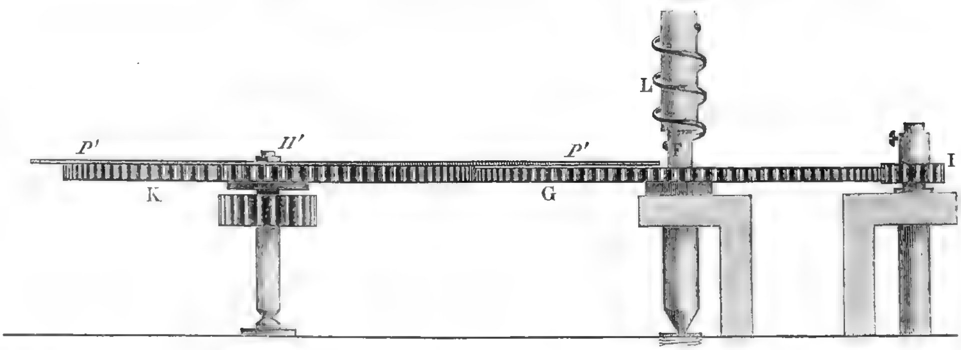


Fig. 5.

VII.—*Description of an improved Anemometer for registering the Direction of the Wind, and the space which it traverses in given intervals of Time.* By the Rev. T. R. Robinson, D.D., Member of the Royal Irish Academy, and of other Scientific Societies.

Read June 10, 1850.

Among the various branches of meteorology, none has been less successfully cultivated than anemometry. As a necessary consequence, we are almost totally ignorant of the causes which originate and the laws which govern the currents of the atmosphere, notwithstanding their interesting character as objects of physical research, and their importance as cosmical agents. This, however, is not to be attributed to neglect; we find Hooke and Derham pursuing the inquiry almost at the first dawn of physical science; and a variety of subsequent inventions connected with it shew that its importance was never forgotten. But a wrong path of observation was followed: the data which anemology requires are the direction and velocity of the wind at a given time; those which (with few exceptions) were sought, are its direction and pressure. Of the many ingenious machines which have been contrived for this purpose, those which are not mere anemoscopes may be reduced to three classes. In the first, originally devised by Hooke, the wind acted on a set of vertical wind-mill-vanes, which are kept facing it by a vane, or some equivalent contrivance, giving them motion round a vertical axis. They turn till the pressure on them equilibrates a graduated resistance of some kind, whose amount measures it. In the second, a square plane receives the impulse of the wind perpendicularly, and thus compresses a spiral spring which is connected with it. This, which was invented about a century ago by the celebrated Bouguer, has been lately brought into general use by Mr. Ossler, who has much improved it, and made it self-registering.* In the third class, of which LIND'S is the type, the pressure of the wind is measured by the column of water, or some other fluid, which it is able to support in an inverted siphon.

All these are liable to the following objections. First, wind fluctuates, both in velocity and direction, to an extent of which I had no conception till I entered on these researches. Instead of being a uniform flow of air, it may be likened to an assemblage of filaments moving with very unequal speed, and contorted in every direction; being in fact analogous to a river in flood, but with its eddies and counter-currents considerably exaggerated. Now, assuming the common equation $V^2 = mP$, we find $\frac{2dV}{V} = \frac{dP}{P}$, or the *relative* variations of pressure are twice as great as those of velocity; a record of the latter will therefore, be far less irregular. But the evil goes further; for in the fluctuations both of pressure and direction, the inertia of the moving parts of the anemometer carries them far beyond the point of balance, and makes the measure of pressure inaccurate, partly by exaggerating the amount of its changes, partly by the surface which receives the wind's impulse being at times wrongly placed with respect to its direction. The magnitude of this cause of error may be appreciated from these two facts, that I have seen LIND (the only pressure-gauge which I possess) range in a few seconds from 0 to 2·6 inches; and that in some winds a free vane will oscillate through arcs even of 120°.

2. The velocity can only be deduced from the pressure by experiment: if the relation between them be constant, this necessity is of little importance; but the fact is the reverse. In the case of the windmill-vanes, we have no information; and it is evident that the law which connects these variables must be very complex, in consequence of the wind which glances off the anterior surface modifying the *minus* pressure. In BOUGUER'S instrument it is commonly assumed that the pressure is the weight of the column whose height is that due to the velocity: there is, however, experimental ground for believing that it is nearly twice as great.† The excess is caused by the *minus* pressure,

* An anemometer of this kind, acting against a series of weights instead of a spring, was long used by the late Mr. KIRWAN, and is described in vol. xi. of the Academy's Transactions.

† See D'AUBUISSON, Hydraulique, p. 295. From DE BUAT'S investigations it is not unlikely that the velocities deduced from the records of OSSLER'S gauge are about one-third too great.

which, I may add, seems to follow a different law of velocity from the *plus* one. In LIND a similar uncertainty is produced by the negative action of the wind on the remote aperture of the gauge.

3. It has been well observed by FORBES, in his Second Report of the British Association on Meteorology, that little progress can be made in anemometry, except by the employment of self-registering instruments. If these record pressure, we cannot thence readily deduce the *mean* velocity, even admitting the law $V^2=mP$. Let V' and P' be their mean values; $V'+v$, $P'+\pi$, any others; n their number, then

$$nV'^2 + 2V' \times S(v) + S(v^2) = n\,mP' + mS(\pi),$$

or as $S(v)$ and $S(\pi) = 0$,

$$V'^2 = mP' - \frac{S(v^2)}{n},$$

in which the last term is often of very great magnitude; or if we take

$$V' + v = \sqrt{m} \times \sqrt{P},$$

we have

$$V' = \sqrt{m} \times \frac{S(\sqrt{P})}{n};$$

but I have found the trouble of computing the sums of the square roots, even for a few minutes, an insuperable objection.

These seem to me sufficient reasons for absolutely rejecting the pressure-gauge, and adopting instead of it one which gives directly the velocity, or rather its equivalent, the space traversed in a given time. Instruments fulfilling this object are by no means of recent date. One was described in 1749 by the Russian LOMONOSOFF; it consisted of a vertical wheel with float-boards like an undershot, half of which was screened, and which was kept in the plane of the wind's direction by a vane. This, by a train of wheel-work, indicated on a dial the revolutions of the wheel; there was no provision for recording these in connexion with time, but a very ingenious one for noting the quantities of wind which blow from each point of the compass. A much neater one was constructed in 1783, by the late Mr. EDGEWORTH, and used by him to measure the velocity of air currents, though designed for a different purpose.* It con-

* For measuring the ascent of a balloon; two years later it was used by our countryman CROSBIE in his perilous ascent, and was preserved by him when the rest of his apparatus was lost

sisted of four light windmill-vanes, delicately mounted, the arbor of which had an endless screw, that recorded its revolutions by means of the elegant arrangement now called the cotton-counter. This, the invention of which is attributed by WILLIS to the late Dr. WOLLASTON (who, I believe, learned it from Mr. EDGEWORTH), consists of two wheels of n and $n + 1$ teeth, driven by the same screw; a tooth of the first passes a fixed index for each revolution of the vanes, and an index borne by it passes a tooth of the second for every n revolutions.* In 1790, the hydrometric fly of WOLTMAN was proposed by its inventor as an anemometer; but Dr. WHEWELL is the first who appreciated in its full extent the importance of the space-measure, especially in its giving an integral instead of a differential result. His memoir, published in the sixth volume of the Cambridge Transactions, marks an era in the science, and, in my opinion, indicates the only path of its progress. The instrument described by him has been used by several observers, but most extensively by one whose energy and talents were well adapted to establish its character, Sir WILLIAM SNOW HARRIS. The results which he exhibited to the British Association in 1841, while they fully proved the value of the principle, shewed, at the same time, that the mechanical details were not sufficiently perfect to carry out the views of the inventor: in particular, the space traversed by the recording pencil is (at least in moderate winds) not as the velocity, but rather as its square. HARRIS proposed to investigate corrections for this, which, however, would be different in each anemometer, and probably variable even in the same one. This error arises from the small size of the vanes, which have, therefore, too little power compared to the friction; while that is greatly increased by the same cause, as, from their great angular velocity, a complicated train of wheel-work is required to bring down the speed of the recording point to a manageable amount. This report induced me to consider the subject carefully; and as it seemed possible to correct the defect in question, and some others which I had observed in a similar instrument used by Captain LARCOM, R. E., at

in the sea. The wheels have seventy-two and seventy-three teeth, and the revolution of the second wheel measures a mile.

* A very convenient portable anemometer is made by furnishing a set of my hemispherical vanes with such a counter. In that exhibited to the Academy, the radius of the circle described by their centres = 5·6 inches, and the diameter of the hemispheres = 3·1 inches. If the number of revolutions which it makes in a minute be divided by 10, the quotient is the velocity in miles per hour.

Mountjoy Barracks, I obtained permission from the governors of the Armagh Observatory (who had already directed me to erect an anemometer) to carry into effect my views. After some preliminary experiments, I constructed in 1843 the essential parts of the machine, a description of which I now submit to the Academy, and I added in subsequent years such improvements as were indicated by experience. It was complete in 1846, when I described it to the British Association at Southampton; so that I have had sufficient opportunity to ascertain its efficacy.

In contriving it, I was guided by the following principles:

1. The moving power should be so great in comparison of the friction, that the correction due to the latter may be inconsiderable. It should also be easily applied.

2. One means of effecting this is to have surfaces which receive the wind's impulse as far from their axis of motion as is consistent with strength. This satisfies the second condition, namely, that they shall be acted on by a large section of the current, and thus give an *average* result. When the vanes are as small as those used by WHEWELL, they may give measures far different from the general velocity, if met by those partial streams to which I have referred.

3. The movement of those surfaces should be as slow, relatively to that of the wind, as may be consistent with a sufficiency of moving power: this lessens the train required to bring down the speed of the recording point, and also diminishes the wear and tear of the whole machine.

4. It seems desirable that it should act without requiring any special provision for turning it in the direction of the wind.

5. The structure should be such, that all made after the same type will give identical results.

The third, fourth, and probably the fifth of these conditions, are against the vertical windmill as a measure. Its vanes never move slower than the wind, often three or four times as fast at their outer extremities.* With the best guiding ap-

* This must generate a considerable centrifugal motion in the air dragged round with the vanes, which will complicate the direct impulse of the wind. Its effect in large windmills is illustrated by a remarkable fact, observed in Holland by the late Mr. NIMMO, that in some of the best of them, the weathering at the extremity of the sail is *negative.* This can only act by preventing the escape of the air. An effect of this kind must be difficult to calculate.

paratus, its motions round the vertical axis will not exactly correspond with the oscillations of the wind; and very trifling variations in the angle of the vanes will make a great variation in their speed. The fourth condition excludes those horizontal windmills which act by a moveable screen. Of the remainder, in one class the vanes are made to turn during the revolution, so as to present a diminished surface to the wind while returning against it; these are objectionable, because the necessary machinery is liable to derangement, and involves much friction, which will vary during a long period of working, and change the space unit. There remain then those only in which the vanes are curved, so as to be unequally resisted on their opposite surfaces. Of these, the most elegant in principle and definite in action that I know, was suggested to me many years ago by Mr. Edgeworth. Its vanes are hollow hemispheres, whose diameters coincide with the arms that support them; the action on their concave surfaces exceeds that on the convex so much, that the machine is capable of being used as a motive power with considerable advantage; its simplicity of form is such that, without very great exactness of workmanship, similarity of action can be attained; and it combines great lightness with strength sufficient to resist very severe gales.*

The relation between the velocity of its vanes and that of the wind can be determined satisfactorily, in the actual state of hydrodynamics, only by experiment. In this instance, however, the problem is so modified by the antagonism of the returning vanes, that theory gives not merely the law which connects them, but a close approximation to their ratio, and the correction due to friction.

Let AH be an arm of the machine, bearing the hemispheres AIB, DKH, and revolving in the direction of the arrows, so that the velocity of their centres $= v$.†

* In the gale of December 15, 1848 (the anemometer diagrams of which are among the specimens exhibited to the Academy), the space recorded during the hour $2^h \cdot 3^h = 61{\cdot}5$ miles, but during $2\frac{1}{2}$ minutes it is $= 4{\cdot}27$, which gives 102·5 miles per hour for the velocity of that squall. Short as it was, it did much damage in the neighbourhood, but the instrument was unhurt. A still heavier gust is recorded in the diagram of the cyclone of March 29, 1850, where the velocity is nearly 130 miles per hour for 3 minutes.

† At least this point is assigned as the centre of effect by the common theory; it may, perhaps, be a little further out in the concave.

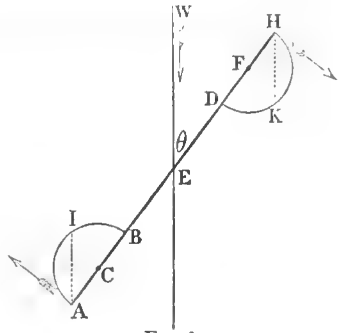


Fig. 1.

This rotation *in quiescent air*, will cause a resistance to the convex surface of each hemisphere $= a'v^2$; a' being a coefficient depending on its diameter. To this the wind, supposed to act in the direction WE, adds another resistance on the convex of AIB; but it also acts on the concave of DKH, with a force which tends to increase v; and as its coefficient a is considerably greater than a', v will increase. In consequence of this, the concave surface recedes from the wind, and the convex meets it more rapidly; the impelling force, therefore, diminishes, and the retarding forces increase. To the latter must also be added the centrifugal force expended in producing an outward current in the air that is dragged with the convex surfaces, and the effect of friction. Evidently, therefore, a speed will soon be attained, at which these forces balance each other. If $\theta =$ the angle WEH, V the wind's velocity, we have, by the theory of Borda for the undershot wheel,

$$\text{Force on DKH} = aV^2 \sin^2\theta - aVv \sin\theta.$$

$$\text{Force on AIB} = a'V^2 \sin^2\theta + a'Vv \sin\theta.$$

The force due to the rotation alone $= 2a'v^2$, and the centrifugal force being as v^2 may be assumed $= 2b'v^2$. Let f also $=$ the moment of friction at C, then the actual impelling force

$$F = (a - a')V^2 \sin^2\theta - (a + a')Vv \sin\theta - 2v^2(a' + b') - f.$$

We must, however, take the *mean* value of this through the semicircle. It is

$$\int_0^\pi \frac{Fd\theta}{\pi} = \frac{a - a'}{2} V^2 - \frac{a + a'}{\pi} \times 2Vv - 2v^2(a' + b') + f.\text{*} \qquad (1)$$

* This reasoning supposes a and a' to retain the same value through the semicircle. Experiment shows that they vary; but as the change is greatest when their influence on the velocity is least, the error of this assumption cannot have much influence. The centrifugal force cannot act on the concave, as there is no tendency in the air which it holds to escape in the direction of the arm.

As their mean force vanishes when the condition of permanent rotation is attained, if we equate it to cypher, we deduce

$$\frac{V}{v}=\frac{2}{\pi}\left(\frac{a+a'}{a-a'}\right)\left[1+\sqrt{\left\{1+\frac{\pi^2\,(a-a')}{(a+a')^2}\left(a'+b'+\frac{f}{2v^2}\right)\right\}}\right]. \qquad (2)$$

This shows that if we neglect the term introduced by friction, the ratio of the velocities V and v depends on the ratio of a and a' alone, being independent of their absolute magnitudes and also of v. It is, therefore, independent of the speed of the wind and the size of the machine.

Calling this ratio m, and making the instrument register $mv=V$, the true velocity of the wind $=V+u$, u being the correction due to friction, we have from (1)

$$\frac{a-a'}{2}(V+u)^2-\frac{2v}{\pi}(a+a')(V+u)-2v^2(a'+b')-f=0,$$

$$\frac{a-a'}{2}V^2-\frac{2v}{\pi}(a+a')V-2v^2(a'+b')=0\,;$$

whence

$$u^2+2u\times V\left\{1-\frac{2}{m\pi}\left(\frac{a+a'}{a-a'}\right)\right\}=\frac{2f}{a-a'}\,; \qquad (3)$$

the positive root of which may be tabulated for a series of values of V.

The constants of these equations must be given by experiment, and it is not easy to obtain them satisfactorily, especially the most important of them, a and a'. But for the unsteadiness of the wind,* both in force and direction, we might attach hemispheres to some weighing apparatus, with the concave and convex surfaces turned to the wind, and thus obtain absolute measures of them. This, however, could only be done by connecting the two with a pair of registers like those of OSSLER'S instrument, which would give the mean pressure for a considerable period; and such an apparatus is not at my command.

As, however, m depends on their ratio only, I found a method, which, though disturbed by the same cause, is tolerably successful. Two hemispheres, similar

* In illustration of this I may mention, that having placed two hemispheres on the arm, so that both concaves faced the wind (when, of course, they might be expected to remain in equilibrium), they oscillated with considerable force through arcs of 90°; the distance between their centres was 48·5 inches.

to those of the actual anemometer, are fixed on an arm in which, by means of a long slit, the axis of rotation can be shifted to any position; this axis causes a graduated circle to measure θ, the zero of which is determined by a vane above. The axis is shifted till the two pressures are equal, when, of course, a and a' are inversely as its distances from the two centres. In reducing this to practice, however, I found a difficulty which I had not anticipated. Since the forces on the hemispheres are as $V^2 \sin^2 \theta$, I concluded they would be at the maximum at 90°, and vanish at 0° or 180°; and began by observing them in the first of these positions. To my great surprise, I found that the equilibrium there is unstable, so that if the angle be changed the least either way, the concave predominates. This makes it hard to ascertain the true point of balance, as the direction of the wind is ever changing; but nevertheless I think I am warranted in concluding, with some confidence, from sixteen experiments made in four days with winds from a moderate breeze to a hard gale,

$$\frac{a}{a'} = 4 \cdot 011;$$

or, in round numbers, the action on the concave is four times that on the convex.

I was the more surprised at this predominance of the concave when the arm is inclined to the wind, because then the part HK of its convex acts against it.

From some other angles I obtained, though by fewer observations,

$$\theta = 80°, \quad . \quad . \quad \frac{a}{a'} = 4 \cdot 128.$$

75°, ...	4·378.
60°, ...	4·710.
45°, ...	4.783.
30°, ...	5·192.

Beyond this it is impossible to go, *as there the convex surface apparently ceases to act.* In fact, on removing the hemisphere DK, the other remains as in the wood-cut, making $\theta = 210°$ nearly, and oscillating as the direction of the wind changes. Whether this arises from the minus-pressure at the segment IA, or from the wind which passes at B eddying into the concave, I cannot decide;

but it is a striking illustration of the imperfect state of this branch of hydrodynamics.*

As to the coefficient b', its limits at least may be obtained by a process which I at first thought might give a and a',—the same which EDGEWORTH, HUTTON, BORDA, and VINCE, employed in their experiments on resistance. The resisting surface is placed at the extremity of a horizontal arm made to revolve round a vertical axis by a weight attached to a cord wound on the latter, and passing over a pulley. When the rotation becomes uniform,† the resistance must equal the accelerating force, and if this be constant, all the resistances must be equal, and, therefore, can be compared with the velocities. With this hope the apparatus described in the preceding section was placed in a tower of the Observatory, and alternately driven with its concave and convex surfaces foremost. Twenty revolutions were made before the time was noted; and then thirty were taken, giving $S = 381$ feet. The mean velocities varied

* From this it follows, that, if the friction do not prevent it, an anemometer of this kind should revolve even when its axis is in the direction of the wind. The small one already described does so, but this may be owing to the oscillation of direction.

† I fear the physicists just mentioned took the fulfilment of this condition for granted. This does not necessarily happen. Let F = the impelling power, K the moment of inertia of the apparatus, S the space described, T the time, V the velocity of the centre of the resistance.

$$\text{Accelerating force} = \frac{F - av^2}{K}.$$

$$v = \sqrt{\frac{F}{a}} \times \sqrt{\left(1 - e^{\frac{-2aS}{K}}\right)}.$$

$$T = \frac{K}{2\sqrt{(Fa)}} \times \log\left(\frac{\sqrt{F} + v\sqrt{a}}{\sqrt{F} - v\sqrt{a}}\right).$$

$$\text{Square of mean velocity or } \frac{S}{T} = \frac{F}{a} - \frac{K}{a} \times \frac{v}{T};$$

$$\text{mean square of velocity} = \frac{F}{a} - \frac{K}{a} \times \frac{v^2}{2S}.$$

From these expressions it is manifest, that v cannot be uniform till S is infinite. In my trials it continued to increase even till S was the largest I could command, 1143 feet. Hence also, the *mean square of velocity* (which ought to be used in computing the resistance) must differ from the square of the mean velocity; the latter, however, has always been used. As, moreover, no estimation has been made of the air's centrifugal force in the results which have hitherto been obtained in this way (and in fact it cannot be separated from the resistance), I am compelled to think they require revision, though they are at present received as standard facts.

from 2·23 to 8·50 feet. The values of F were determined by attaching to the centre of a hemisphere a fine thread perpendicular to its diameter, and passing over a delicate pulley (whose friction is known), to which weights were suspended, such that the driving weight just moved the apparatus when slightly jarred. These weights, divided by the mean squares of the velocities, give $a+b$ and $a'+b'$.* The result is

$$\frac{a+b}{a'+b'} = 2{\cdot}019,$$

and assuming $a=a'\times 4{\cdot}011$, we have

$$b' = a' \times 0{\cdot}9866 + b \times 0{\cdot}4953.$$

No means of determining the ratio of b to b' occurs to me; I could only satisfy myself that it is considerably less, by suspending a light body two feet outside the circle, and estimating the resultant of its deflection from the vertical in the direction of the radius. This made it evident that comparatively little air is thrown outwards by the concaves, the hollow, I suppose, carrying it round, and preventing its escape. We may, therefore, safely assume, that it is between the limits $b=b'$, $b=0$, and much nearer the latter. These suppositions giving the limits $b'=a'\times 1{\cdot}9866$; $b'=a'\times 0{\cdot}9866$.

If now we substitute these values and that of $\frac{a}{a'}$ in (2), we obtain

$$\frac{V}{v} = 3{\cdot}306, \text{ if } b'=b\,; \quad \frac{V}{v} = 2{\cdot}999, \text{ if } b=0.$$

It must, from what precedes, be much nearer the second; and if we also consider that the *mean* value of $\frac{a}{a'}$ through the semicircle is a little greater than that at 90°, we shall be justified in assuming the *theoretic* value of $m=3{\cdot}000$. It is in very unexpected (by me) agreement with that given by experiment.

The most obvious mode of determining this constant—placing the instrument on a carriage, and comparing its record with the space actually traversed—

* It is assumed in this, that a and a' are the same as in a current of air, which, however, may not be the fact. Especially it is possible, that the air of the apartment may be dragged round with the cups, and thus offer less resistance.

I found to fail, partly from the difficulty of eliminating the action of the wind, but still more from the fact, that a carriage *drags with it a quantity of air*, so that for many feet from it the anemometer does not feel the full effect of the motion. At low speeds, and on days of calm, I have got results which agree with that given by other methods, but more frequently the discordance destroys all confidence in it. The aerial log proposed by Sir W. S. Harris in the report at Plymouth, could not be applied, on account of the lofty position of my instrument; but I tried one far more delicate, by exploding small charges of powder at it, while my assistant noted the time required by the little globes of smoke (which in dry weather are not dissipated for many seconds) to traverse 150 feet. But the irregularity of the wind's motion makes all such trials unsatisfactory, and I got the most discordant results, the reason of which was evident by watching the track of the smoke; it rose, descended, twisted in eddies, and even occasionally came back many feet against a strong breeze. But in addition it can only give the movement of that *one* part of the current which it occupies, while the anemometer shows those of all that pass it in the same time, which are essentially distinct. I may add, that the impossibility of obtaining accurate measures of velocity by such means, was long since pointed out by Mr. Brice.*

The plan which succeeded consists in applying the whirling apparatus to carry the anemometer, as in the annexed figure. The anemometer has four hemispheres; it is similar to the actual one, and about a fourth of its dimensions: the distance AB is 45·6 inches, and as the diameter of the hemispheres is only 3 inches, we may assume the velocity of their centres to represent that of the wind. C is a counterpoise. I found that in this case the rotation produced no important outward current. The machine

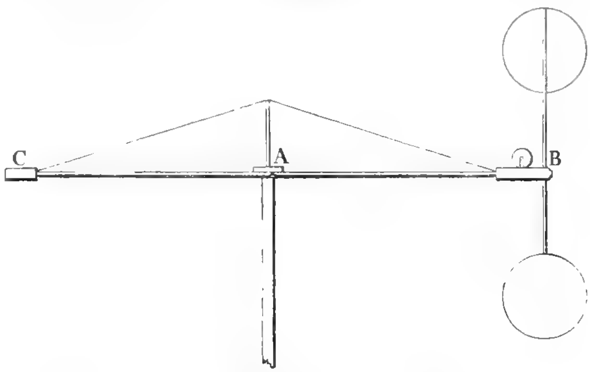


Fig. 2.

* Phil. Tra ns., 1766.

was permitted to make a few revolutions to come to its speed; and then the counter was put in action for a certain number of revolutions of the arm AC, generally 96. The time was also taken to give the mean velocity. I found

Driving Weight.	V in Feet.	$\frac{V}{v} = m.$	No. of Observations.
6 lbs.	7·09	3·562	6
15 „	11·53	3·133	13
21 „	13·75	3·004	9
27 „	15·66	3·004	7

I did not venture higher velocities, as the apparatus was not strong enough; but the above are sufficient to show that, after allowing for friction, the value of $m = 3{\cdot}000$.*

* Some facts observed during these experiments may be deserving a record.

1. The mouths of the hemispheres being covered with paper, so that planes were substituted for concaves, I found, with $V = 12{\cdot}80$ feet, $m = 5{\cdot}041$.

2. Cutting away the central paper, so as to leave merely a ring $0^{i}{\cdot}2$ broad, which (from what has been observed with PITOT'S tube) I thought might increase the effect, proved very disadvantageous.

3. Making the cups segments of 220° was also hurtful, for, with

$$V = 13{\cdot}87, \quad m = 5{\cdot}220.$$

4. A single hemisphere with a flat counterpoise presenting its edge to the air, gives

$$V = 7{\cdot}30, \quad m = 3{\cdot}700.$$

5. With three arms, two carrying *entire spheres*, and one a hemisphere,

$$V = 10{\cdot}30, \quad m = 7{\cdot}900.$$

6. Five vertical windmill-vanes (the best number), of the same outer diameter, but heavier, and set at 45°, give, in 96 revolutions of the arm,

$$V = 7{\cdot}63. \quad \text{No. rev.} = 497{\cdot}5.$$
$$V = 12{\cdot}29. \quad \text{No. rev.} = 516{\cdot}0.$$

The four hemispheres at the same time,

$$V = 13{\cdot}61. \quad \text{No. rev.} = 141{\cdot}2.$$

The tips of the vanes here move about 3·4 times as fast as the wind.

That this ratio holds for the large anemometer, as well as the small, is experimentally shown by placing the latter beside the other, and counting the revolutions made by it during eighty-eight of that. Fourteen such trials give, with a mean velocity of wind = 15·6 feet, the ratio of the revolutions = 4·12, the inverse of their dimensions being 4·29. The difference is due to the large one being above the other, and therefore getting the wind more freely.

As this relation does not depend on the elasticity of the impelling fluid, it should hold when the instrument is acted on by a stream of water, with the advantage of being much less affected by friction. I tried this in a large mill-course near Armagh, placing the small instrument in the central part of the current, where the velocity was found by floats = 1·613 feet. I obtained,

With four hemispheres, . . . m = 2·972.
With two, ... = 3·208.
With three, equidistant, ... = 3·041.

The trial with two was made rather from curiosity than from any dependance on the result which might be obtained, as, when passing the line of centres, the impelling force is so slight, that any eddy will produce a disturbance of the motion. It would not change the mean much, but I think should be rejected. That of the other two is 3·006, still probably a trifle too large, as the four are preferable on the same grounds to the three.

From all this I think we are warranted in laying down this law, that in a horizontal windmill of this description, the centres of the hemispheres move with one-third of the wind's velocity, except so far as they are retarded by friction.

This principle once established, its application is easy. Plate IV., fig. 1, shows the external appearance of my anemometer, as it stands on the flat roof of the dwelling-house. Its frame consists of four uprights, 3^i by 2^i, and $15^f\ 4^i$ long; $6^f\ 5^i$ asunder below, $2^f\ 4^i$ above. They support the strong frame B, in which a diagonal carries the bearings of the axles C and D. The part H is sheathed and roofed with plank (the roof covered with painted canvass); and it forms a very convenient room for the self-registering apparatus. The copper funnel F is attached to each axle, to prevent the entrance of wet. The great height of this frame is necessary to clear the dome of the west equatorial, which rises

S.E. of it; but it has the double disadvantage of causing additional friction by the weight of the long axles, and making the whole less stable. To obviate this last defect, about 3 cwt. of pig ballast is disposed round the floor of H; notwithstanding which, the machine was blown down in March, 1845. After this it was further secured by three iron shrouds attached to the walls in the directions S.E., S.W., and N.W.; and it has since withstood still heavier gales. The axle C bears the mill G for space; the axle D the vane V for direction.

The dimensions which I chose for the first of these are, 12 inches for the diameter of the hemispheres, and 23 inches for the distance of their centres from that of the axle. The latter might, perhaps, have been increased with advantage; but I was afraid of weakening the arms too much. The hemispheres are made of sheet zinc, strengthened by a wire rim; each weighs 1·31 lbs., but might have been lighter if made of thin copper. The arms which carry them are iron, 1·5 inches broad, and 0·1 thick, but feathered off to a sharp edge at each side, and kept from bending downwards by stays of wire. The hemispheres are four; for I found, by trials with the small anemometer, that this number is better than either five or three. Six is inferior to any lower number, not excepting two; probably because some eddy from the concaves reaches the convex surfaces. The iron tubes T, 8 and 18·5 inches long, are secured to the diagonal of the top frame, and carry boxes of bronze, in which are bronze balls, on and between which the axles C and D turn. This arrangement is the result of many experiments. At first they turned above in common brass journals, and their hardened points rested below on surfaces of hard steel. As, however, C with its appendages weighs 20·69 lbs., and makes, on an average, 1500 revolutions per hour,* the bearing surfaces were soon abraded; the friction also was far too great, being equivalent to 104 grains acting at the centre of a hemisphere. I then refashioned the pivot very carefully, and set it in an agate cup; but, though this was kept full of oil, after a year's work, I found that a hole of some depth had been drilled in it. I substituted for it one of sapphire, but even this failed after two years; and the friction was not so much lessened as I expected,

* Of this 6·69 is due to a piece of iron tube composing C, which I have recently replaced by a shaft of deal; this has reduced the weight to 16·23 lbs. The average velocity of the wind is about 10 miles per hour.

being 73 grains. This was finally replaced by the present mounting in May, 1849. It is shown in Plate V., figs. 2 and 3, where C is the axle, 0·82 inches diameter, D the box of bronze (8 copper to 1 tin); B, five balls of the same, 1·12 diameter; I a disc of iron truly turned on the axle; H an aperture for introducing occasionally a few drops of oil, which I find necessary for the lateral action of the balls. They bear both the lateral pressure and the weight; and, therefore, require only a slight lateral support below, which is given by the arbor of the endless screw. This arrangement shews no trace of wear after more than a year's work, and the total friction is but 53 grains:* the coefficient of that part of it which belongs to the balls, I find to be $\frac{1}{325\cdot6}$ of the load. As in high winds there is added to this a lateral pressure, in respect to which the balls do not act quite so advantageously, we may take it $\frac{1}{300}$. From this value of the friction, the correction of V is easily computed;† but it is in some

* Of this I find that 20·36 belong to the mill, and 32·64 to the registering apparatus: with the new shaft the total friction will be 48·61.

† For this it is also necessary to know the constant $a - a'$. I approximated to this as follows: a spring-balance is attached to a cord wound on the axle C, which, as $v = o$, measures with *four* cups the force $V^2 (a - a')$. Its slide moves a pencil parallel to the axis of a cylinder covered with paper, and made to revolve by clock-work, on which it traces the curve of time and force. The small anemometer already described gives V by comparison with the time. This V is reduced to that of the large instrument by comparative trials at the time of experiment. It must, however, be remembered, that it is a *mean velocity*, and that, therefore, the value of $a - a'$ thus obtained, is too small if the fluctuations be considerable. As V is affected by friction, the first values of $a - a'$ are used to correct it, and thus a more exact result is given by a second computation. By six diagrams I find,

Time	$V^2 (a - a')$	V	$a - a'$
$= 126^s \cdot 2$;	$= 828 \cdot 6$ grs.;	$= 4^m \cdot 99$	$= 33 \cdot 28$ grs.
98·8;	1760·6;	10·40	16·22
96·6;	1168·1;	6·27	29·71
184·2;	685·1;	2·85	51·57
101·2;	555·8;	5·04	21·04
98·0;	1698·7;	6·81	36·03

The second and fourth were marked as doubtful from excessive fluctuations; but as the mean of them and the fifth differs little from that of the other three which were considered satisfactory at the time of experiment, I retain them, and take $a - a' = 31 \cdot 997$, or 32 in round numbers. The equation (3) becomes, with the values previously given for $\frac{a}{a'}$ and m, respects preferable to correct mechanically by applying to C an auxiliary force equivalent to the friction. Besides the saving of labour, it extends the action of the instrument; as, from the data in the preceding note, it cannot move with less than 1·29 mile per hour. The dynamic effect of such a force, while the wind traverses 100 miles, $= \dfrac{53 \text{ grs.} \times 5280 \times 100}{3 \times 7000} = 1332 \text{lbs} \times 1$ foot; it would, therefore, require a weight of 37 lbs. falling 36 feet. The locality does not permit this; and I, therefore, purpose to use a remontoir, wound up by a small mill similar to the anemometer itself. Perhaps an electro-magnetic machine might be simpler; the expenditure of zinc and acid would be trifling, and their consumption proportionate to the work done. The chief difficulty would be the inconstancy of the current.

The vane V is three feet long by one and a half extreme breadth; it also is made of sheet zinc. From a wish to give it as little momentum as possible, it was at first a light wooden frame covered with varnished calico, which the wind soon destroyed. This axle turns also on balls.

It remains to describe the self-registering apparatus; and first, that for the space.

My first intention was to adopt a form resembling the charts of wind-paths given by Dr. WHEWELL in his memoir, but in which the curves should be drawn by the wind itself. The arrangement I proposed was to make the

$$u = V \times 0{\cdot}64637 \left\{ \sqrt{\left(1 + \frac{f}{a-a'} \times \frac{4{\cdot}78705}{V^2}\right)} - 1 \right\}; \qquad (4)$$

which, with the above values of f and $(a - a')$ is

$$u = V \times 0{\cdot}64637 \left\{ \sqrt{\left(1 + \frac{7{\cdot}92855}{V^2}\right)} - 1 \right\}. \qquad (5)$$

From this the following table is computed:

V	u	V	u
$V = 1^m$	$u = 1^m{\cdot}285$	$V = 6^m$	$u = 0^m{\cdot}068$
2	0·469	7	0·050
3	0·240	8	0·039
4	0·144	9	0·031
5	0·095	10	0·025

It is evident that above 5^m per hour the correction is insensible.

plane which holds the paper move by means of two slides at right angles to each other. Its motion should be given by a rack, travelling proportionally to the space, but revolving so as to be always in the direction of the wind. A pencil placed over the centre of its revolution would describe on the paper a track perfectly similar to that of the wind. At each hour and quarter hour, a clock was to print by punches a series of marks, which would represent the time.* The mechanical arrangements were planned, and certainly this method would have the great advantage of showing at one view the three variables, and speaking most distinctly to the eye; but I gave it up, from a conviction that it is much less adapted to give periodical means than the method of co-ordinates.

Of these I prefer the polar to the rectangular, for the following reasons. In the first place, the direction, being an angle, is at once recorded; secondly, a movement of rotation can be given to the paper-holder with much less friction than a rectilinear one; thirdly, this movement may be continued through many circumferences without inconvenience, while the other is limited by the length of the rack, or other contrivance for producing it. In the rectilinear direction-register there is also the inconvenience that, if the wind veer several times in the same direction round the horizon, a new series of graduations must commence. I may add, that there is, perhaps, a want of graphic propriety in representing angular veering by a right line, but none in measuring miles by a graduated arc. Fourthly, one form of printed paper serves for both. The only objection to the polar form, of which I am aware, is, that the scale is less near the centre than at the circumference; this, however, may be obviated in any case, when it is desired, by winding up the apparatus at shorter intervals, so as to keep the pencil near the latter.

First, then, as to the space: the dimensions which I have adopted for the windmill are such that, in 440 revolutions, the hemispheres travel one mile,

* August 16th. This, I find, has been applied by Mr. Ossler, who showed me at the late meeting of the British Association, some beautiful wind-curves, where the time is thus expressed. He checks the excursions in direction by using a windmill, and with great success. The x of his curves is the space, the y the direction. The time is shown by dots, single and multiple, struck in pairs at each side of the paper, and its record is very complete.

and the wind three. If degrees on the paper be miles of wind, the number of the former must be 440×120 for one of the paper-holder. A train which effects this very simply was arranged for me, by one whose recent loss I lament, not merely from personal regard, but from regret that science is deprived of aid so powerful as that of his high mechanical talents,—the late Mr. RICHARD SHARP. It is shown in Plate VI., fig. 4, where A is an arbor held loosely in the lower extremity of the axle, and carried round with it by the screw *c*. An endless screw on this drives the wheel B, of 88 teeth; a second endless screw S drives C, of 100; its pinion D, of 16, drives E of 96. On this the brass plate P, 14 inches in diameter, is fastened by a steady-pin and the nut H, which also assists in holding down the paper. The speed of the train is therefore $= 88 \times 100 \times 6 = 440 \times 120$.

The arrangement for direction is shown in Plate VI., fig. 5. The arbor F (which is also loose in the hollow of the vane axle) bears the wheel G of 96, which drives K of 96. On this the paper-holder P′ is secured by H′; its angular movement is therefore equal to that of the vane, while the paper can be more easily removed than if it were immediately carried by the vane-axle.

That axle is connected with the arbor F, not by any rigid attachment, but by the spiral spring L. This is necessary, not merely to prevent the destruction of the machinery in violent oscillations of the vanes, but still more to lessen their extent on the register-paper. Though Dr. WHEWELL had pointed out the magnitude of these oscillations, and the impossibility of preventing them, I was not at all prepared for what I found. It may be that these waverings of the wind are of greater amount at Armagh than elsewhere, owing to the exposed situation, and the undulating surface of the country; but, without some contrivance to check them, the direction-papers would be very unsightly objects. It must, however, be remembered, that they cannot be avoided entirely, nor is it desirable that they should be too much diminished; for I find that this is a distinctive character of some winds, independent of their velocity, and, therefore, implying some peculiarity in the origin or progress of the current. In particular I have remarked, that when excessive, it is connected with a roaring sound, that gives an exaggerated impression of their force. This was strikingly exemplified in the destructive tempest of February last, whose highest velocity did not exceed 40 miles per hour. On another occasion, when the velocity was of nearly the same amount, the sudden diminution of roar led me to suppose the gale was abating; but, on going to the instrument, I found the velocity had increased to 52, while the range of direction was only half its previous extent.

The contrivances which I have applied as checks on the direction-fluctuations seem to work well. In the first place, such as are completed in a second or two are chiefly expended in bending the spring L, being past before its tension can overcome the inertia of the paper-holder and its machinery. Secondly, the wheel G drives a regulator attached to the arbor of the pinion I, but not shown in the drawing. This consists of four vanes, shown in plan, Fig. 3, made of light deal frames covered with paper. Each is 37 inches high and 15 broad. As the whole is very light and turns on an agate, it yields to the slightest impulse of the vane, *if time be given*, but presents a very great resistance to rapid motion. Its speed is $\frac{96}{10}$ times that of the vane,* and this, combined with the action of the spring, will often reduce the oscillations to one-third of their absolute magnitude. As at first applied, the regulator was much smaller and immersed in water; but I was obliged to abandon that plan, in consequence of its action being interrupted during frost.

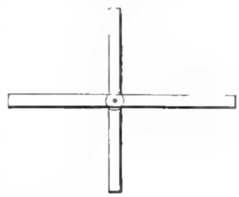


Fig. 3.

Lastly, Plate V., fig. 6, shows the method of connecting these two registers with that of time. N is a cast iron plate which bears the whole machinery, 40 inches by 14. P and P′ are the paper-holders; each has three spring-clips at its circumference, to hold the paper, which is further secured by the screw H passing through a hole punched in its centre; this screw serves also to centre it, being of the same size as one of its circles. One of these clips bears a fiducial line, with which the zero of graduation is made to coincide when a new paper is applied. M is a common clock movement, the weight and pendulum of which pass through openings in N. Its barrel carries a second wheel, which moves, by a rack, the bar *pp′* through six inches in twelve hours. This

* I have since added an intermediate wheel and pinion, which makes the speed $\frac{96}{10} \times \frac{60}{20} = 28{\cdot}8$, which is a considerable improvement. 24 might be immediately obtained, and would be, perhaps, the best.

bar slides in a dovetail on the front plate, and carries adjustable tubes at its extremities, in which pencil-holders are placed, and made to act by weights placed in their cups. Since, however, in the direction-register, the pencil, as at first arranged, and shown in the figures, travels from the circumference, I have found that in damp weather it occasionally has pulled the paper from the clips and torn it. I have, therefore, lately carried it by an additional piece, one end running in a guide at O, the other provided with a stud which fits in p': this complicates it a little, but remedies the inconvenience, and makes the time-reading the same in both registers.

The paper used is printed in red, from a plate engraved with a graduation of degrees and half degrees. Within this are a series of concentric circles, which represent portions of time. Those which correspond to hours are stronger than the rest, and half an inch apart; the intermediates show decimals of the hour. The mode of using it is this: the pencil p' being removed, the date is written on P near its pencil; the clock is then wound up, and p draws a line from the circumference to the centre. The paper on P′ is then removed or shifted, and if another be placed, it is similarly dated, with the addition of the degree, which is set at the fiducial line; and the pencil p' is replaced. Then, during the ensuing twelve hours, the action of the clock carries the pencils from the centre to the circumference. If there were no wind they would merely draw radial lines; but in general p traces a spiral, and p' shades an irregular sector. The clock should be adjusted so that the twelve hour circles should be exactly traversed. In general, a space-paper may contain four or six spirals, dating each winding line; and a direction one, two, or three sectors, shifting the zero point for each. This zero in my practice represents a wind from the south, and the graduation goes round from west to north. The papers are finally fixed with a weak solution of mastic in common whiskey, and preserved for reference.

In reducing these diagrams to a form available for computation, I have found no system preferable to the method pointed out by Dr. Whewell in his memoir. In the first instance, the centres of the papers are restored; in the space-papers, drawing radii through the intersections of the spirals with the hour-circles, the graduation gives the hourly spaces, which, if necessary, are corrected for friction: these are tabulated. In a second column is entered the direction at each hour. This is found by bisecting the arc of the hour-circle, which is shaded by the pencil.* The mean direction during each hour will, in general, not differ from the mean of those at its beginning and end; but if the eye perceives that this is not the case, those for the decimals of the hour may be taken. From this are computed two rectangular co-ordinates, which are given in the third and fourth columns; w the motion of the wind from the west, s that from the south. These are obtained by multiplying the hourly spaces into the sine and cosine of the mean direction. I have found it easiest to do this by a large sliding rule, having arranged a table of sines and cosines for each decimal of the degree. They need only be to three places of decimals, but should have a quadruple argument; its first column from 0° to 90°, its second from 180° to 270° (these on the left): its third from 360° to 270°; its fourth from 180° to 90° (these on the right): and over each column the appropriate signs. Details of this kind may seem trifling, but the waste of labour which they avoid is of great consequence when so great a mass of work has to be performed, as even one year of such a registry involves.

From these co-ordinates any final results may be obtained, as hourly, daily, monthly, or yearly means. Let W be such a mean of w, S of s, attending to the signs; then D, the mean direction for that time, and Σ, the mean space, are given by the equations

$$\tan D = \frac{W}{S}; \quad \Sigma = \frac{S}{\cos D} = \frac{W}{\sin D};$$

remembering that $\sin D$ has the same sign as W, and $\cos D$ as S, from which the quadrant of D is known.

As an example I annex the reductions of the twelve hours during which the centre of the cyclone already referred to passed the Observatory, as one which will illustrate the process in an extreme case.

* This is most rapidly performed by a plan explained in the figure. Let BC be the arc of the hour circle H; lay an edge of the ruler RT through C, and the centre I, so that its extreme point is on the hour circle. Then lay the parallel-ruler PL through that point and B; remove TR, and move the half of PL till it passes through I; the point G is in the line bisecting BC.

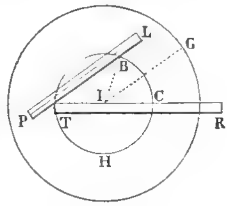


Fig. 4.

Date.	Space.	Direction.	W.	S.
March 29, 10 P. M.		303°·8		
11	33m·5	313·8	−26m·1	+21m·0
12	32·0	320·5	−21·8	+23·4
1 A. M.	31·1	307·3	−22·4	+21·5
2	29·4	314	−22·3	+19·1
3	30·3	294·7	−25·0	+17·1
4	31·5	$\overline{77}$·2*	−19·9	−24·4
5	30·5	78·8	+29·9	+ 6·3
6	31·1	66·7	+29·7	+ 9·3
7	31·1	69·1	+28·8	+11·7
8	32·9	$\overline{88}$·2	+24·5	−22·0
9	33·6	88·2	+33·4	+ 1·0
10	37·5	99·9	+37·5	− 0·3
Sum, ...	385·5		+46·3	+83·7

The means for the two irregular hours are taken from the reading for each tenth. We have $\tan D = \frac{46{\cdot}3}{83{\cdot}7}$, which, as both are positive, must be in first quadrant, therefore,

$$D = 28^{\circ}{\cdot}95, \text{ and } \Sigma = \frac{46{\cdot}3}{\sin 28^{\circ}{\cdot}57'} = 95{\cdot}65.$$

It appears, therefore, that during these twelve hours, the real movement of the air was only 95·6 miles, from a point 29° west of south.

For all purposes of physical investigation, this method of exhibiting the results is fully efficient; at the same time it is much to be desired, that some graphic method could be devised which would exhibit to the eye the relation

* At $3^h{\cdot}30^m$ the wind veered suddenly 217°·5, against the order of graduation, which is shown by the sign —. The mean direction for the hour = 219°·2. There also was at 7^h *exactly* another veer, in the same direction, of 210°·5. The mean direction for the hour = 132°·0.

of the space, direction, and time at one view. This might in some degree be performed by a delineation of the actual trajectory of the wind, either drawn by itself, or laid down from the co-ordinates w and s, on which the corresponding times are marked; but the analogy of curves described on a plane, and expressing the relation between two variables, naturally leads to the notion of a solid whose three dimensions would afford a triple representation. It would involve the construction of a model, or at least a contoured plan. For, in fact, if we conceive perpendiculars to be raised on one of my direction-papers, at each point of the shading, proportioned to the velocity at the corresponding instant, their totality would be limited by a relieved surface which would show by its undulations the state of the aerial movements, and might be contoured. Unfortunately, the changes of direction are so abrupt and large, that it is absolutely impossible to exhibit in this way the conditions of any short period; but it is probable that it may be different with the hourly or even annual mean of a considerable number of years; and I venture to recommend it, or some equivalent, as an object worth the attention of meteorological inquirers.

T. R. Robinson.

Armagh Observatory,
June 8, 1850.

VIII.—*On the Equilibrium and Motion of an Elastic Solid.* By the Rev. JOHN H. JELLETT, Fellow of Trinity College, and Professor of Natural Philosophy in the University of Dublin.

Read January 28, 1850.

1. THE problem which forms the subject of the present Memoir has already, at various times, occupied the attention of mathematicians. Although much of the interest which it has excited is due to its connexion with the undulatory theory of light, the importance of the problem itself, considered as a branch of rational mechanics, is fully admitted; and more than one writer has treated of it without regard to the real or supposed existence of a luminous ether. Nor can it, I think, be doubted, that such a distinction between the rational and the physical science, is in accordance with the dictates of just philosophy. The rational science would still be real, even though the existence of the ether were (if that were possible) disproved; and the admitted reality of the several solid and fluid bodies which are found in nature gives us, in such cases, the means of testing by experiment the accuracy of the laws arrived at. "Whatever theoretic objections," says Mr. HAUGHTON, "may be made to the application of the theory of elastic media to optics, none such exist as to its application to solid and fluid bodies. The mathematical investigations which, in the case of light, must be hypothetical, are, in the case of solid and fluid bodies, essentially positive, and may be made the subject of direct experiment. A general inquiry into the laws of elastic media is an interesting application of rational mechanics; and although it must necessarily include cases purely hypothetical, it is not, therefore, to be considered unimportant."*

* Transactions of the Royal Irish Academy, Vol. xxii. Part i. p. 97.

2. Two general methods have been adopted by the various authors who have treated of this problem. Of these, the one consists in forming expressions for the forces which act upon each particle in the medium under consideration, and then determining the laws of its equilibrium, or motion, by the general statical or dynamical equations. This method is followed by POISSON and CAUCHY. It is also adopted by NAVIER in the commencement of his Memoir, but soon abandoned, as being less complete than the second method. This latter, which is the method of LAGRANGE, and is followed by Mr. GREEN, Professor MAC CULLAGH, and Mr. HAUGHTON,* takes as its basis the equation derived from the combination of D'ALEMBERT's principle with that of virtual velocities, and is distinguished by the greater completeness of the solution which it affords; the same analysis giving both the general equations of equilibrium, or motion, and the particular conditions which must be satisfied at the bounding surface of the body or medium under consideration.† This is the method which I propose to adopt in the present Memoir. The discussion of a problem like the present must, of course, rest upon principles more or less hypothetical, inasmuch as the nature of molecular action cannot (at least in the present state of physical knowledge) be ascertained by direct experiment. The classification, however, with which the present investigation commences, cannot be considered as other than positive, inasmuch as the two kinds of force, upon the distinction between which it is founded, are known to exist in nature, and cannot, without a hypothesis, be reduced to one. The principle of this classification I shall now proceed to state.

* All these writers commence with the assumption that the sum of the internal moments of a medium may be represented by the variation of a single function. To this method it may, perhaps, be objected that it takes, as the foundation of a physical theory, a principle which is almost purely mathematical, and to which it appears difficult to give a definite physical meaning. This hypothesis, moreover, does not give to the equations of motion all the generality of which they are susceptible. I have, therefore, preferred taking, as the basis of the present Memoir, a principle essentially physical; more especially as the equations of motion derived from this principle are, in the case of homogeneous bodies, possessed of the full number of constants, and have, therefore, the greatest amount of generality which their form admits.

† The investigation of these conditions, according to the method ordinarily adopted, is, however, open to serious objections. These the reader will find noticed in a subsequent part of the present Memoir.

General Classification of Bodies.

I.—*Hypothesis of Independent Action.*

II.—*Hypothesis of Modified Action.*

3. The classification which I propose here to adopt, and which forms the basis of the present Memoir, is founded upon the following very obvious principle. The force, or influence, which one particle or molecule exerts on another, may show its effect either by causing a change in its *state*, or by causing a change in its *position*. Either or both of these changes may affect the influence which this particle in its turn exerts upon any of those around it. Thus, for example, if m, m', m'', be three particles acting upon each other by the ordinary attraction of gravitation, the action of m' upon m'' will be modified by the action of m only so far as their distance from each other is changed by it. The attraction of m has no power to change the attraction of m' upon any other particle, except by altering its distance from that particle. But the case would be altogether different if we supposed m, m', m'', to be *electrified* particles. In this case the action of m upon m' would modify the action of that particle upon m'', not only by changing the distance between them, but also by changing their electrical state, and, therefore, the force which each exerts upon the other. In the former case, if m' and m'' maintain the same relative position, the force which they mutually exert remains unchanged. In the second, even though the relative position of the two particles remains unaltered, their mutual action will be modified by the presence of a third particle.* From this distinction an obvious classification follows. In the first class we place all bodies whose particles exert upon each other a force which is *independent* of the surrounding particles; a force, therefore, which can be changed only by a displacement of one or both of the particles under consideration. In the second class, which includes all other bodies, the mutual action of two particles is supposed to be affected by that of the surrounding particles.

* I do not, of course, mean to say, that in a case like that of electrified particles, change of state in the particle itself may not be caused by change of position in the particles of some fluid which pervades it. It is sufficient for my purpose, that in such a case the force which two particles exert upon each other may be changed without a displacement of the particles themselves.

We shall now proceed to investigate the equations of equilibrium and motion, for bodies of the first class.

I.—Hypothesis of Independent Action.

4. Let the several particles of a body, which satisfies the hypothesis of independent action, be displaced from their original position of free equilibrium, this displacement being supposed to follow some regular law. Let it be required to determine the conditions of equilibrium of these particles in their new position, or, in other words, to assign the forces which should be applied to each of them in order to keep them at rest. Again, if the particles be left to themselves after the displacement, let it be required to determine the law of their motion.

In applying the method of Lagrange to any problem of equilibrium or motion, it is plainly necessary to commence with two assumptions, namely:—1. An assumed expression for the *intensity* of each of the acting forces. 2. An assumed expression for the *effect* which this force tends to produce; the effect of a force being defined by the quantity which it tends to change.

Let m, m' be two particles of the body under consideration, and let F be the force which, in their displaced position, they exert upon each other.

Let x, y, z be the co-ordinates of m in its original position, and ξ, η, ζ its resolved displacements. Let also $x', y', z', \xi', \eta', \zeta'$ be the co-ordinates and displacements of m'. Then, since, by the hypothesis of independent action, F does not depend upon the displacement of any of the other particles, and since, if the body have a regular constitution, the state of each particle must be a function of its position,

$$F = f(x, y, z, \quad x', y', z', \quad \xi, \eta, \zeta, \quad \xi', \eta', \zeta');$$

or, as it may be otherwise written,

$$F = f(x, y, z, \quad x', y', z', \quad \xi, \eta, \zeta, \quad \xi' - \xi, \quad \eta' - \eta, \quad \zeta' - \zeta).$$

But in all media with which we are acquainted, no internal force appears to be generated by a mere transference of the entire system from one position in space to another, the *relative* positions of the several particles remaining unchanged.

This being supposed universally true, we shall have, as is easily seen,

$$F=f\,(x,\ y,\ z,\quad x',\ y',\ z',\quad \xi'-\xi,\quad \eta'-\eta,\quad \zeta'-\zeta).$$

Let ρ, θ, ϕ be the polar co-ordinates of m' with regard to m; then since

$$x'=x+\rho\sin\theta\cos\phi,\quad y'=y+\rho\sin\theta\sin\phi,\quad z'=z+\rho\cos\theta\,;$$

it is plain that the foregoing expression for x may be written

$$F=f\,(x,\ y,\ z,\quad \rho,\ \theta,\ \phi,\quad \xi'-\xi,\quad \eta'-\eta,\quad \zeta'-\zeta).$$

Hitherto no assumption has been made either with respect to the magnitude of the distance between the particles m, m', or with respect to that of the displacements ξ, η, ζ, ξ', η', ζ'. But previously to proceeding further, it is necessary to make the following suppositions:

(1.) That the greatest distance between two particles which are capable of acting upon one another, or, as it is ordinarily termed, the radius of molecular activity, is indefinitely small compared with the intensity of the force generated.

(2.) That the sphere of molecular activity contains, nevertheless, an indefinitely great number of particles.

From the first of these assumptions, combined with the supposition that the displacements follow some regular law, we have

$$\begin{aligned}
\xi' &= \xi+\frac{d\xi}{dx}\,dx+\frac{d\xi}{dy}\,dy+\frac{d\xi}{dz}\,dz,\\
\eta' &= \eta+\frac{d\eta}{dx}\,dx+\frac{d\eta}{dy}\,dy+\frac{d\eta}{dz}\,dz,\\
\zeta' &= \zeta+\frac{d\zeta}{dx}\,dx+\frac{d\zeta}{dy}\,dy+\frac{d\zeta}{dz}\,dz.
\end{aligned}\qquad\text{(A)}$$

quantities of higher orders being neglected.

For the same reason,

$$F=F_0+A\,(\xi'-\xi)+B\,(\eta'-\eta)+C\,(\zeta'-\zeta).\qquad\text{(B)}$$

This expression consists, as will be seen, of two distinct parts, namely F_0, which represents the force which m' exerts upon m in the original position of these particles, and

$$A\ (\xi' - \xi) + B\ (\eta' - \eta) + C\ (\zeta' - \zeta),$$

the force generated by the displacement.

The supposition that the original state of the body was one of free equilibrium permits us to disregard the former of these parts. For it follows from that supposition, that if the several particles of the body receive equal displacements, the new position is also a position of equilibrium. Hence the suppositions,

$$\xi' = \xi, \quad \eta' = \eta, \quad \zeta' = \zeta,$$

must satisfy the general equation of equilibrium. But these suppositions give

$$F = F_0.$$

Hence the terms depending upon F_0 will disappear of themselves. We have, therefore, for the *effective* part of the force,

$$F = A\ (\xi' - \xi) + B\ (\eta' - \eta) + C\ (\zeta' - \zeta),$$

where A, B, C are in general of the form

$$f\ (x, y, z, \quad \rho, \theta, \phi).$$

Let α, β, γ be the angles which the direction of ρ makes with the axes, so that

$$\cos \alpha = \sin \theta \cos \phi, \quad \cos \beta = \sin \theta \sin \phi, \quad \cos \gamma = \cos \theta.$$

Then since

$$dx = \rho \cos \alpha, \quad dy = \rho \cos \beta, \quad dz = \rho \cos \gamma,$$

we shall have from equations (A),

$$\left.\begin{aligned}
\xi' - \xi &= \rho \left(\cos \alpha \frac{d\xi}{dx} + \cos \beta \frac{d\xi}{dy} + \cos \gamma \frac{d\xi}{dz}\right), \\
\eta' - \eta &= \rho \left(\cos \alpha \frac{d\eta}{dx} + \cos \beta \frac{d\eta}{dy} + \cos \gamma \frac{d\eta}{dz}\right), \\
\zeta' - \zeta &= \rho \left(\cos \alpha \frac{d\zeta}{dx} + \cos \beta \frac{d\zeta}{dy} + \cos \gamma \frac{d\zeta}{dz}\right),
\end{aligned}\right\} \quad (C)$$

and therefore,

$$F = A\rho\left(\cos\alpha\,\frac{d\xi}{dx} + \cos\beta\,\frac{d\xi}{dy} + \cos\gamma\,\frac{d\xi}{dz}\right)$$
$$+ B\rho\left(\cos\alpha\,\frac{d\eta}{dx} + \cos\beta\,\frac{d\eta}{dy} + \cos\gamma\,\frac{d\eta}{dz}\right) \qquad \text{(D)}$$
$$+ C\rho\left(\cos\alpha\,\frac{d\zeta}{dx} + \cos\beta\,\frac{d\zeta}{dy} + \cos\gamma\,\frac{d\zeta}{dz}\right).$$

Let α', β', γ' be the angles which the direction of this force makes with the axes, and X, Y, Z its components. Then

$$X = F\cos\alpha' = \rho\cos\alpha'\left\{A\left(\cos\alpha\,\frac{d\xi}{dx} + \cos\beta\,\frac{d\xi}{dy} + \cos\gamma\,\frac{d\xi}{dz}\right)\right.$$
$$+ B\left(\cos\alpha\,\frac{d\eta}{dx} + \cos\beta\,\frac{d\eta}{dy} + \cos\gamma\,\frac{d\eta}{dz}\right)$$
$$\left. + C\left(\cos\alpha\,\frac{d\zeta}{dx} + \cos\beta\,\frac{d\zeta}{dy} + \cos\gamma\,\frac{d\zeta}{dz}\right)\right\}; \qquad \text{(E)}$$

$$Y = F\cos\beta' = \rho\cos\beta'\left\{A\left(\cos\alpha\,\frac{d\xi}{dx} + \&c.\right) + \&c.\right\};$$

$$Z = F\cos\gamma' = \rho\cos\gamma'\left\{A\left(\cos\alpha\,\frac{d\xi}{dx} + \&c.\right) + \&c.\right\}.$$

We have next to consider the effect which this force tends to produce; and on this point the assumption here made is, *that the forces developed by the displacements of the several particles tend to change their relative positions only.* Hence it is evident, that the moments of the forces X, Y, Z will be

$$X\delta(\xi' - \xi), \quad Y\delta(\eta' - \eta), \quad Z\delta(\zeta' - \zeta),$$

respectively, or

$$\rho X\left(\cos\alpha\,\frac{d\delta\xi}{dx} + \cos\beta\,\frac{d\delta\xi}{dy} + \cos\gamma\,\frac{d\delta\xi}{dz}\right),$$
$$\rho Y\left(\cos\alpha\,\frac{d\delta\eta}{dx} + \cos\beta\,\frac{d\delta\eta}{dy} + \cos\gamma\,\frac{d\delta\eta}{dz}\right), \qquad \text{(F)}$$
$$\rho Z\left(\cos\alpha\,\frac{d\delta\zeta}{dx} + \cos\beta\,\frac{d\delta\zeta}{dy} + \cos\gamma\,\frac{d\delta\zeta}{dz}\right).$$

Substituting for X its value from (E), we find the following expression for the moment of that force:

$$\begin{aligned}\rho^2 \cos \alpha' & \left(\cos \alpha \frac{d\delta\xi}{dx} + \cos \beta \frac{d\delta\xi}{dy} + \cos \gamma \frac{d\delta\xi}{dz}\right) \\ \times & \left\{ A \left(\cos \alpha \frac{d\xi}{dx} + \cos \beta \frac{d\xi}{dy} + \cos \gamma \frac{d\xi}{dz}\right) \right. \\ & + B \left(\cos \alpha \frac{d\eta}{dx} + \cos \beta \frac{d\eta}{dy} + \cos \gamma \frac{d\eta}{dz}\right) \\ & \left. + C \left(\cos \alpha \frac{d\zeta}{dx} + \cos \beta \frac{d\zeta}{dy} + \cos \gamma \frac{d\zeta}{dz}\right) \right\}.\end{aligned} \tag{G}$$

This expression denoting the moment of that part of the force acting on m, which results from the relative displacement of m', it is evident that the complete moment of the forces X, which act upon m, will be found by multiplying (G) by the element of the mass, and integrating through the entire sphere of molecular action. Let ϵ be the density at the point x', y', z', and a the radius of the sphere of molecular activity. Then the element of the mass will be

$$d\mu = \epsilon\rho^2 \sin \theta \, d\rho \, d\theta \, d\phi;$$

and the limits of integration with respect to ρ, θ, ϕ, will be 0 and a, 0 and π, 0 and 2π, respectively. If then we assume,

$$\begin{aligned} A_{\alpha^2\alpha'} &= \iiint A\rho^2 \cos^2 \alpha \cos \alpha' \, d\mu = \int_0^\pi \int_0^{2\pi} \int_0^a A \epsilon \rho^4 \cos \alpha' \sin^3 \theta \cos^2 \phi \, d\rho \, d\theta \, d\phi, \\ A_{\alpha\beta\alpha'} &= \iiint A\rho^2 \cos \alpha \cos \beta \cos \alpha' \, d\mu, \\ A_{\alpha\gamma\alpha'} &= \iiint A\rho^2 \cos \alpha \cos \gamma \cos \alpha' \, d\mu, \\ &\quad \&c.\ ; \\ B_{\alpha^2\alpha'} &= \iiint B\rho^2 \cos^2 \alpha \cos \alpha' \, d\mu, \\ &\quad \&c.\ ; \\ C_{\alpha^2\alpha'} &= \iiint C\rho^2 \cos^2 \alpha \cos \alpha' \, d\mu, \\ &\quad \&c.\ ; \end{aligned}$$

we shall find for the complete moment of the forces X acting upon the particle m, the expression

$$L = A_{\alpha^2\alpha'}\frac{d\xi}{dx}\frac{d\delta\xi}{dx} + A_{\beta^2\alpha'}\frac{d\xi}{dy}\frac{d\delta\xi}{dy} + A_{\gamma^2\alpha'}\frac{d\xi}{dz}\frac{d\delta\xi}{dz}$$

$$+ A_{\beta\gamma\alpha'}\left(\frac{d\xi}{dy}\frac{d\delta\xi}{dz} + \frac{d\xi}{dz}\frac{d\delta\xi}{dy}\right) + A_{\alpha\gamma\alpha'}\left(\frac{d\xi}{dz}\frac{d\delta\xi}{dx} + \frac{d\xi}{dx}\frac{d\delta\xi}{dz}\right) + A_{\alpha\beta\alpha'}\left(\frac{d\xi}{dx}\frac{d\delta\xi}{dy} + \frac{d\xi}{dy}\frac{d\delta\xi}{dx}\right)$$

$$+ B_{\alpha^2\alpha'}\frac{d\eta}{dx}\frac{d\delta\xi}{dx} + B_{\beta^2\alpha'}\frac{d\eta}{dy}\frac{d\delta\xi}{dy} + B_{\gamma^2\alpha'}\frac{d\eta}{dz}\frac{d\delta\xi}{dz} \qquad \text{(H)}$$

$$+ B_{\beta\gamma\alpha'}\left(\frac{d\eta}{dy}\frac{d\delta\xi}{dz} + \frac{d\eta}{dz}\frac{d\delta\xi}{dy}\right) + B_{\alpha\ \alpha'}\left(\frac{d\eta}{dz}\frac{d\delta\xi}{dx} + \frac{d\eta}{dx}\frac{d\delta\xi}{dz}\right) + B_{\alpha\beta\alpha'}\left(\frac{d\eta}{dx}\frac{d\delta\xi}{dy} + \frac{d\eta}{dy}\frac{d\delta\xi}{dx}\right)$$

$$+ C_{\alpha^2\alpha'}\frac{d\zeta}{dx}\frac{d\delta\xi}{dx} + C_{\beta^2\alpha'}\frac{d\zeta}{dy}\frac{d\delta\xi}{dy} + C_{\gamma^2\alpha'}\frac{d\zeta}{dz}\frac{d\delta\xi}{dz}$$

$$+ C_{\beta\gamma\alpha'}\left(\frac{d\zeta}{dy}\frac{d\delta\xi}{dz} + \frac{d\zeta}{dz}\frac{d\delta\xi}{dy}\right) + C_{\alpha\gamma\alpha'}\left(\frac{d\zeta}{dz}\frac{d\delta\xi}{dx} + \frac{d\zeta}{dx}\frac{d\delta\xi}{dz}\right) + C_{\alpha\beta\alpha'}\left(\frac{d\zeta}{dx}\frac{d\delta\xi}{dy} + \frac{d\zeta}{dy}\frac{d\delta\xi}{dx}\right).$$

Similar expressions are found for the moments of the forces Y and Z.

5. Let X', Y', Z' be the external forces necessary to keep the particle at rest. Then, the equation of virtual velocities being in general

$$\iiint (X'\delta\xi + Y'\delta\eta + Z'\delta\zeta)\, dm + \iiint (L + M + N)\, dx\, dy\, dz = 0;$$

if we substitute for L, M, N their values found as above, we shall have the equation

$$\begin{aligned} 0 = &\iiint (X'\delta\xi + Y'\delta\eta + Z'\delta\zeta)\,\epsilon\, dx\, dy\, dz \\ &+ \iiint \left(P_1 \frac{d\delta\xi}{dx} + P_2 \frac{d\delta\xi}{dy} + P_3 \frac{d\delta\xi}{dz}\right. \\ &\quad + Q_1 \frac{d\delta\eta}{dx} + Q_2 \frac{d\delta\eta}{dy} + Q_3 \frac{d\delta\eta}{dz} \\ &\quad \left. + R_1 \frac{d\delta\zeta}{dx} + R_2 \frac{d\delta\zeta}{dy} + R_3 \frac{d\delta\zeta}{dz}\right) dx\, dy\, dz. \end{aligned} \qquad \text{(I)}$$

Where ϵ is the density, and

$$\begin{aligned} P_1 = &A_{\alpha^2\alpha'}\frac{d\xi}{dx} + A_{\alpha\beta\alpha'}\frac{d\xi}{dy} + A_{\alpha\gamma\alpha'}\frac{d\xi}{dz} \\ &+ B_{\alpha^2\alpha'}\frac{d\eta}{dx} + B_{\alpha\beta\alpha'}\frac{d\eta}{dy} + B_{\alpha\gamma\alpha'}\frac{d\eta}{dz} \\ &+ C_{\alpha^2\alpha'}\frac{d\zeta}{dx} + C_{\alpha\beta\alpha'}\frac{d\zeta}{dy} + C_{\alpha\gamma\alpha'}\frac{d\zeta}{dz}; \end{aligned} \qquad \text{(K)}$$

$$
\begin{aligned}
P_2 = A_{\alpha\beta\alpha'}\frac{d\xi}{dx} + A_{\beta^2\alpha'}\frac{d\xi}{dy} + A_{\beta\gamma\alpha'}\frac{d\xi}{dz} \\
+ B_{\alpha\beta\alpha'}\frac{d\eta}{dx} + B_{\beta^2\alpha'}\frac{d\eta}{dy} + B_{\beta\gamma\alpha'}\frac{d\eta}{dz} \\
+ C_{\alpha\beta\alpha'}\frac{d\zeta}{dx} + C_{\beta^2\alpha'}\frac{d\zeta}{dy} + C_{\beta\gamma\alpha'}\frac{d\zeta}{dz};
\end{aligned}
$$

$$
\begin{aligned}
P_3 = A_{\alpha\gamma\alpha'}\frac{d\xi}{dx} + A_{\beta\gamma\alpha'}\frac{d\xi}{dy} + A_{\gamma^2\alpha'}\frac{d\xi}{dz} \\
+ B_{\alpha\gamma\alpha'}\frac{d\eta}{dx} + B_{\beta\gamma\alpha'}\frac{d\eta}{dy} + B_{\gamma^2\alpha'}\frac{d\eta}{dz} \\
+ C_{\alpha\gamma\alpha'}\frac{d\zeta}{dx} + C_{\beta\gamma\alpha'}\frac{d\zeta}{dy} + C_{\gamma^2\alpha'}\frac{d\zeta}{dz};
\end{aligned}
$$

the values of Q_1, Q_2, Q_3 being deduced from these expressions by changing, in the suffixed letters, α' into β'; and those of R_1, R_2, R_3, by changing α' into γ'.

Integrating by parts, and equating to zero the coefficients of $\delta\xi$, $\delta\eta$, $\delta\zeta$, under the triple sign of integration, we find the equations of equilibrium to be

$$
\begin{aligned}
\epsilon X' &= \frac{dP_1}{dx} + \frac{dP_2}{dy} + \frac{dP_3}{dz}, \\
\epsilon Y' &= \frac{dQ_1}{dx} + \frac{dQ_2}{dy} + \frac{dQ_3}{dz}, \qquad \text{(L)} \\
\epsilon Z' &= \frac{dR_1}{dx} + \frac{dR_2}{dy} + \frac{dR_3}{dz}.
\end{aligned}
$$

The corresponding dynamical equations will be

$$
\begin{aligned}
\epsilon\left(X' - \frac{d^2\xi}{dt^2}\right) &= \frac{dP_1}{dx} + \frac{dP_2}{dy} + \frac{dP_3}{dz}, \\
\epsilon\left(Y' - \frac{d^2\eta}{dt^2}\right) &= \frac{dQ_1}{dx} + \frac{dQ_2}{dy} + \frac{dQ_3}{dz}, \qquad \text{(M)} \\
\epsilon\left(Z' - \frac{d^2\zeta}{dt^2}\right) &= \frac{dR_1}{dx} + \frac{dR_2}{dy} + \frac{dR_3}{dz}.
\end{aligned}
$$

If now we suppose that no external forces act, and replace P_1, P_2, &c., by their values (K), we shall have the three general equations of small oscillations in a body whose particles have been displaced from their original position of free equilibrium.

$$
\begin{aligned}
-\epsilon\frac{d^2\xi}{dt^2} = {} & A_{\alpha^2\alpha'}\frac{d^2\xi}{dx^2} + A_{\beta^2\alpha'}\frac{d^2\xi}{dy^2} + A_{\gamma^2\alpha'}\frac{d^2\xi}{dz^2} \\
& + B_{\alpha^2\alpha'}\frac{d^2\eta}{dx^2} + B_{\beta^2\alpha'}\frac{d^2\eta}{dy^2} + B_{\gamma^2\alpha'}\frac{d^2\eta}{dz^2} \\
& + C_{\alpha^2\alpha'}\frac{d^2\zeta}{dx^2} + C_{\beta^2\alpha'}\frac{d^2\zeta}{dy^2} + C_{\gamma^2\alpha'}\frac{d^2\zeta}{dz^2} \\
& + 2A_{\beta\gamma\alpha'}\frac{d^2\xi}{dydz} + 2A_{\alpha\gamma\alpha'}\frac{d^2\xi}{dxdz} + 2A_{\alpha\beta\alpha'}\frac{d^2\xi}{dxdy} \\
& + 2B_{\beta\gamma\alpha'}\frac{d^2\eta}{dydz} + 2B_{\alpha\gamma\alpha'}\frac{d^2\eta}{dxdz} + 2B_{\alpha\beta\alpha'}\frac{d^2\eta}{dxdy} \\
& + 2C_{\beta\gamma\alpha'}\frac{d^2\zeta}{dydz} + 2C_{\alpha\gamma\alpha'}\frac{d^2\zeta}{dxdz} + 2C_{\alpha\beta\alpha'}\frac{d^2\zeta}{dxdy} \\
& + \left(\frac{dA_{\alpha^2\alpha'}}{dx} + \frac{dA_{\alpha\beta\alpha'}}{dy} + \frac{dA_{\alpha\gamma\alpha'}}{dz}\right)\frac{d\xi}{dx} \\
& + \left(\frac{dA_{\alpha\beta\alpha'}}{dx} + \frac{dA_{\beta^2\alpha'}}{dy} + \frac{dA_{\beta\gamma\alpha'}}{dz}\right)\frac{d\xi}{dy} \\
& + \left(\frac{dA_{\alpha\gamma\alpha'}}{dx} + \frac{dA_{\beta\gamma\alpha'}}{dy} + \frac{dA_{\gamma^2\alpha'}}{dz}\right)\frac{d\xi}{dz} \\
& + \left(\frac{dB_{\alpha^2\alpha'}}{dx} + \frac{dB_{\alpha\beta\alpha'}}{dy} + \frac{dB_{\alpha\gamma\alpha'}}{dz}\right)\frac{d\eta}{dx} \\
& + \left(\frac{dB_{\alpha\beta\alpha'}}{dx} + \frac{dB_{\beta^2\alpha'}}{dy} + \frac{dB_{\beta\gamma\alpha'}}{dz}\right)\frac{d\eta}{dy} \\
& + \left(\frac{dB_{\alpha\gamma\alpha'}}{dx} + \frac{dB_{\beta\gamma\alpha'}}{dy} + \frac{dB_{\gamma^2\alpha'}}{dz}\right)\frac{d\eta}{dz} \\
& + \left(\frac{dC_{\alpha^2\alpha'}}{dx} + \frac{dC_{\alpha\beta\alpha'}}{dy} + \frac{dC_{\alpha\gamma\alpha'}}{dz}\right)\frac{d\zeta}{dx} \\
& + \left(\frac{dC_{\alpha\beta\alpha'}}{dx} + \frac{dC_{\beta^2\alpha'}}{dy} + \frac{dC_{\beta\gamma\alpha'}}{dz}\right)\frac{d\zeta}{dy} \\
& + \left(\frac{dC_{\alpha\gamma\alpha'}}{dx} + \frac{dC_{\beta\gamma\alpha'}}{dy} + \frac{dC_{\gamma^2\alpha'}}{dz}\right)\frac{d\zeta}{dz};
\end{aligned}
\tag{N}
$$

$$-\epsilon\frac{d^2\eta}{dt^2}=A_{\alpha^2\beta'}\frac{d^2\xi}{dx^2}+\&c.\,;$$

$$-\epsilon\frac{d^2\zeta}{dt^2}=A_{\alpha^2\gamma'}\frac{d^2\xi}{dx^2}+\&c.$$

Of these equations the second and third are deduced from the first by simply changing, in the suffixed letters, α' into β' and γ' respectively.

If the body be *homogeneous*, i. e. if all its points be absolutely similar, the quantities

$$A_{\alpha^2\alpha}\,,\ \&c.,\quad B_{\alpha^2\alpha}\,,\ \&c.,\quad C_{\alpha^2\alpha'}\,,\ \&c.,$$

will be evidently constant. The terms involving

$$\frac{d\xi}{dx},\ \&c.,\quad \frac{d\eta}{dx},\ \&c.,\quad \frac{d\zeta}{dx},\ \&c.,$$

will, therefore, disappear, and the equations (N) will become homogeneous partial differential equations of the second order with constant coefficients.* The number of constants which these equations contain is the same as the number of terms in the right hand members, namely, eighteen for each equation. Hence it is evident, that the equations which represent the small oscillations of a homogeneous medium satisfying the hypothesis of independent action, contain, in general, fifty-four constants. This is the greatest number of constants which these equations could be made to have without a change of form.

6. The conditions to be fulfilled at the limits of integration are, of course, obtained from the terms which appear under a double sign of integration in the equation derived from (I). These terms will evidently give

$$\begin{aligned}&\iint(P_1\delta\xi+Q_1\delta\eta+R_1\delta\zeta)\,dydz\\+&\iint(P_2\delta\xi+Q_2\delta\eta+R_2\delta\zeta)\,dzdx\\+&\iint(P_3\delta\xi+Q_3\delta\eta+R_3\delta\zeta)\,dxdy=0.\end{aligned}\qquad (N')$$

Let p, q, r be the angles which the normal to the surface bounding the given medium makes with the axes, and let dS' be the element of this surface. Then, if equation (N') be transformed in the usual way by making

* This conclusion does not hold for molecules situated at the surface of the body.—Vid. Art. 15.

$$dy\,dz = \cos p dS', \quad dz\,dx = \cos q dS', \quad dx\,dy = \cos r dS';$$

we shall have

$$\begin{aligned}&\iint (P_1 \cos p + P_2 \cos q + P_3 \cos r)\, \delta\xi\, dS'\\
+&\iint (Q_1 \cos p + Q_2 \cos q + Q_3 \cos r)\, \delta\eta\, dS'\\
+&\iint (R_1 \cos p + R_2 \cos q + R_3 \cos r)\, \delta\zeta\, dS'\\
+&\iint (X_1\, \delta\xi + Y_1\, \delta\eta + Z_1\, \delta\zeta)\, \epsilon_1\, dS' = 0;\end{aligned}$$

where X_1, Y_1, Z_1 are forces acting solely at the surface of the medium. The mode of treating this equation in the several cases which may occur having been fully given by Mr. HAUGHTON, I do not think it necessary to pursue this part of the subject further. On the most important of these cases, namely, the transmission of motion from one medium to another, vid. Art. 15.

7. We shall now proceed to integrate the equations (N), for the particular case of plane waves and rectilinear vibrations in a homogeneous body.

Assume,

$$\begin{aligned}\xi &= \cos l . f\,(ax + by + cz - vt),\\
\eta &= \cos m . f\,(ax + by + cz - vt),\\
\zeta &= \cos n . f\,(ax + by + cz - vt);\end{aligned}$$

where *a*, *b*, *c* are the cosines of the angles which the wave normal makes with the axes, and *l*, *m*, *n* are the angles made by the direction of vibration.

Substituting these values in (N), we find

$$\left.\begin{aligned}-\epsilon v^2 \cos l &= \Pi_1 \cos l + \Phi_1 \cos m + \Psi_1 \cos n,\\
-\epsilon v^2 \cos m &= \Pi_2 \cos l + \Phi_2 \cos m + \Psi_2 \cos n,\\
-\epsilon v^2 \cos n &= \Pi_3 \cos l + \Phi_3 \cos m + \Psi_3 \cos n;\end{aligned}\right\} \quad (\mathrm{O})$$

where

$$\left.\begin{aligned}\Pi_1 &= A_{\alpha^2\alpha'} a^2 + A_{\beta^2\alpha'} b^2 + A_{\gamma^2\alpha'} c^2 + 2A_{\beta\gamma\alpha'} bc + 2A_{\alpha\gamma\alpha'} ac + 2A_{\alpha\beta\alpha'} ab,\\
\Phi_1 &= B_{\alpha^2\alpha'} a^2 + B_{\beta^2\alpha'} b^2 + B_{\gamma^2\alpha'} c^2 + 2B_{\beta\gamma\alpha'} bc + 2B_{\alpha\gamma\alpha'} ac + 2B_{\alpha\beta\alpha'} ab,\\
\Psi_1 &= C_{\alpha^2\alpha'} a^2 + C_{\beta^2\alpha'} b^2 + C_{\gamma^2\alpha'} c^2 + 2C_{\beta\gamma\alpha'} bc + 2C_{\alpha\gamma\alpha'} ac + 2C_{\alpha\beta\alpha'} ab;\end{aligned}\right\} \quad (\mathrm{O}')$$

the values of Π_2, Φ_2, Ψ_2, Π_3, Φ_3, Ψ_3 being deduced from those of Π_1, Φ_1, Ψ_1, by changing, as before, α' into β' and γ'. Assuming

$$s = -\,\epsilon v^2,$$

and eliminating l, m, n between the equations (O), we find

$$(s-\Pi_1)(s-\Phi_2)(s-\Psi_3)-\Phi_3\Psi_2(s-\Pi_1)-\Psi_1\Pi_3(s-\Phi_2)-\Pi_2\Phi_1(s-\Psi_3)$$
$$-\Pi_2\Phi_3\Psi_1-\Pi_3\Phi_1\Psi_2=0. \qquad \text{(P)}$$

The value of s being determined by this equation, those of $\cos l$, $\cos m$, $\cos n$ are found at once from (O), combined with the condition

$$\cos^2 l+\cos^2 m+\cos^2 n=1.$$

Hence, the equation (P) being of the third degree, it appears that for each direction of wave plane there are in general three directions of molecular displacement; of these directions one is necessarily real, while the remaining two may be either both real or both imaginary. The vibration will not, however, be necessarily real, because its direction is so, as it is further necessary that the corresponding velocity of wave propagation should be real. Hence, as

$$s=-\epsilon v^2,$$

it is plain that at least one value of s must be negative. We infer, therefore, generally, that no body will be capable of transmitting a plane wave propagated by parallel rectilinear vibrations, unless equation (P) have at least one real negative root.

The surface of wave slowness, being the locus of a point upon the wave normal whose distance from the origin is inversely as the velocity of wave propagation, will be found by putting

$$s=-\frac{1}{r^2}$$

in the general equation (P). It is evidently a surface of the sixth order. In fact, if we put

$$P=A_{\alpha^2\alpha'}x^2+A_{\beta^2\alpha'}y^2+A_{\gamma^2\alpha'}z^2+2A_{\beta\gamma\alpha'}yz+2A_{\alpha\gamma\alpha'}xz+2A_{\alpha\beta\alpha'}xy,$$
$$Q=B_{\alpha^2\alpha'}x^2+\&c.,$$
$$R=C_{\alpha^2\alpha'}x^2+\&c.,$$

and denote by P', Q', R', P'', Q'', R'' the expressions derived from these by replacing α' by β' and γ' successively, we shall have, as the equation of the surface,

$$(P-1)(Q'-1)(R''-1)-R'Q''(P-1)-P''R(Q'-1)-QP'(R''-1) \\ +P'Q''R+P''QR'=0. \qquad (Q)$$

We shall next proceed to consider the two hypotheses which have been most frequently made by writers upon this subject, namely:—1. That the sum of the internal moments may be represented by the variation of a single function. 2. That the force which one molecule exerts upon another is a force of attraction or repulsion.

Hypothesis of the Existence of a single Function V, by whose Variation the Sum of the internal Moments of the Body may be represented.

8. This condition gives the equation

$$L+M+N=\delta V.$$

The three expressions (H), p. 187, must, therefore, when added together, give a complete variation. Now if we examine the value of L there given, we shall see that the first six terms, those, namely, which are multiplied by

$$A_{\alpha^2\alpha'},\ A_{\beta^2\alpha'},\ A_{\gamma^2\alpha'},\ A_{\beta\gamma\alpha'},\ A_{\alpha\gamma\alpha'},\ A_{\alpha\beta\alpha'};$$

form in themselves a complete variation, namely,

$$\delta\left\{\tfrac{1}{2}A_{\alpha^2\alpha'}\frac{d\xi^2}{dx^2}+\tfrac{1}{2}A_{\beta^2\alpha'}\frac{d\xi^2}{dy^2}+\tfrac{1}{2}A_{\gamma^2\alpha'}\frac{d\xi^2}{dz^2}+A_{\beta\gamma\alpha'}\frac{d\xi}{dy}\frac{d\xi}{dz}\right. \\ \left.+A_{\alpha\gamma\alpha'}\frac{d\xi}{dz}\frac{d\xi}{dx}+A_{\alpha\beta\alpha'}\frac{d\xi}{dx}\frac{d\xi}{dy}\right\}.$$

Similarly in the values of M and N the terms multiplied by

$$B_{\alpha^2\beta'},\ B_{\beta^2\beta'},\ \&\text{c.},\qquad C_{\alpha^2\gamma'},\ C_{\beta^2\gamma'},\ \&\text{c.},$$

respectively, form complete variations in themselves.

Let us now consider the term in the value of L

$$B_{\alpha^2\alpha'}\frac{d\eta}{dx}\frac{d\delta\xi}{dx}.$$

Corresponding to this term, we have in the value of M

$$A_{\alpha^2\beta'}\frac{d\xi}{dx}\frac{d\delta\eta}{dx};$$

and, therefore, since no such term can occur in N, we shall have in

$$L + M + N,$$

the group

$$B_{\alpha^2\alpha'} \frac{d\eta}{dx} \frac{d\delta\xi}{dx} + A_{\alpha^2\beta'} \frac{d\xi}{dx} \frac{d\delta\eta}{dx}. \tag{Q$'$}$$

Now if $L + M + N$ be a complete variation, it is plain that this expression must have been derived from a single term of the form

$$K \frac{d\eta}{dx} \frac{d\xi}{dx}.$$

Taking the variation of this term by the ordinary rule, and comparing it with (Q$'$), we have the condition

$$A_{\alpha^2\beta'} = B_{\alpha^2\alpha'}.$$

Proceeding in a similar way with the remaining terms, we find altogether eighteen equations of condition, namely,

$$\left.\begin{array}{lll} A_{\alpha^2\beta'} = B_{\alpha^2\alpha'}, & A_{\beta^2\beta'} = B_{\beta^2\alpha'}, & A_{\gamma^2\beta'} = B_{\gamma^2\alpha'}, \\ B_{\alpha^2\gamma'} = C_{\alpha^2\beta'}, & B_{\beta^2\gamma'} = C_{\beta^2\beta'}, & B_{\gamma^2\gamma'} = C_{\gamma^2\beta'}, \\ C_{\alpha^2\alpha'} = A_{\alpha^2\gamma'}, & C_{\beta^2\alpha'} = A_{\beta^2\gamma'}, & C_{\gamma^2\alpha'} = A_{\gamma^2\gamma'}, \\ A_{\alpha\beta\beta'} = B_{\alpha\beta\alpha'}, & A_{\alpha\gamma\beta'} = B_{\alpha\gamma\alpha'}, & A_{\beta\gamma\beta'} = B_{\beta\gamma\alpha'}, \\ B_{\alpha\beta\gamma'} = C_{\alpha\beta\beta'}, & B_{\alpha\gamma\gamma'} = C_{\alpha\gamma\beta'}, & B_{\beta\gamma\gamma'} = C_{\beta\gamma\beta'}, \\ C_{\alpha\beta\alpha'} = A_{\alpha\beta\gamma'}, & C_{\alpha\gamma\alpha'} = A_{\alpha\gamma\gamma'}, & C_{\beta\gamma\alpha'} = A_{\beta\gamma\gamma'}. \end{array}\right\} \tag{R}$$

These equations may be more briefly written as follows:

$$\left.\begin{array}{l} \iiint (A \cos \beta' - B \cos \alpha') (\xi' - \xi) (\eta' - \eta)\, dm = 0, \\ \iiint (C \cos \alpha' - A \cos \gamma') (\zeta' - \zeta) (\xi' - \xi)\, dm = 0, \\ \iiint (B \cos \gamma' - C \cos \beta') (\eta' - \eta) (\zeta' - \zeta)\, dm = 0. \end{array}\right\} \tag{S}$$

For if we substitute for

$$\xi' - \xi, \quad \eta' - \eta, \quad \zeta' - \zeta,$$

their values (C), and perform the integrations with regard to dm, the first of the foregoing equations may be written

$$(A_{\alpha^2\beta'} - B_{\alpha^2\alpha'})\frac{d\xi}{dx}\frac{d\eta}{dx} + (A_{\beta^2\beta'} - B_{\beta^2\alpha'})\frac{d\xi}{dy}\frac{d\eta}{dy} + (A_{\gamma^2\beta'} - B_{\gamma^2\alpha'})\frac{d\xi}{dz}\frac{d\eta}{dz}$$
$$+ (A_{\alpha\beta\beta'} - B_{\alpha\beta\alpha'})\left(\frac{d\xi}{dx}\frac{d\eta}{dy} + \frac{d\xi}{dy}\frac{d\eta}{dx}\right)$$
$$+ (A_{\alpha\gamma\beta'} - B_{\alpha\gamma\alpha'})\left(\frac{d\xi}{dx}\frac{d\eta}{dz} + \frac{d\xi}{dz}\frac{d\eta}{dx}\right)$$
$$+ (A_{\beta\gamma\beta'} - B_{\beta\gamma\alpha'})\left(\frac{d\xi}{dy}\frac{d\eta}{dz} + \frac{d\xi}{dz}\frac{d\eta}{dy}\right) = 0.$$

Since then these equations are supposed to hold for all possible displacements, we must have the six equations

$$A_{\alpha^2\beta'} = B_{\alpha^2\alpha'}, \quad A_{\beta^2\beta'} = B_{\beta^2\alpha'}, \quad A_{\gamma^2\beta'} = B_{\gamma^2\alpha'},$$
$$A_{\alpha\beta\beta'} = B_{\alpha\beta\alpha'}, \quad A_{\alpha\gamma\beta'} = B_{\alpha\gamma\gamma'}, \quad A_{\beta\gamma\beta'} = B_{\beta\gamma\alpha'}.$$

Six equations being furnished by each of the remaining equations (S), we shall have in all eighteen equations which are obviously identical with (R).

9. If the sum of the internal moments admit of being represented by the variation of a single function, the three directions of molecular displacement corresponding to a given wave plane will be at right angles to each other. This has been shown by Mr. HAUGHTON. We shall now proceed to prove the converse of this theorem, namely,

If the three directions of molecular displacement corresponding to the same wave plane be at right angles, and if this be true for every wave plane, the sum of the internal moments of the body may be represented by the variation of a single function.

We have seen that the directions of molecular displacement corresponding to a given wave plane are determined by the equations

$$-\epsilon v^2 \cos l = \Pi_1 \cos l + \Phi_1 \cos m + \Psi_1 \cos n,$$
$$-\epsilon v^2 \cos m = \Pi_2 \cos l + \Phi_2 \cos m + \Psi_2 \cos n,$$
$$-\epsilon v^2 \cos n = \Pi_3 \cos l + \Phi_3 \cos m + \Psi_3 \cos n.$$

Eliminating ϵv^2 between the first two of these equations, we have

$$\Pi_1 \cos l \cos m + \Phi_1 \cos^2 m + \Psi_1 \cos m \cos n$$
$$= \Pi_2 \cos^2 l + \Phi_2 \cos l \cos m + \Psi_2 \cos l \cos n.$$

Hence if $l_1, m_1, n_1,\ l_2, m_2, n_2,\ l_3, m_3, n_3$ be the three systems of values of l, m, n, we shall have

$$\Pi_1 \cos l_1 \cos m_1 + \Phi_1 \cos^2 m_1 + \Psi_1 \cos m_1 \cos n_1$$
$$= \Pi_2 \cos^2 l_1 + \Phi_2 \cos l_1 \cos m_1 + \Psi_2 \cos l_1 \cos n_1,$$

$$\Pi_1 \cos l_2 \cos m_2 + \Phi_1 \cos^2 m_2 + \Psi_1 \cos m_2 \cos n_2$$
$$= \Pi_2 \cos^2 l_2 + \Phi_2 \cos l_2 \cos m_2 + \Psi_2 \cos l_2 \cos n_2,$$

$$\Pi_1 \cos l_3 \cos m_3 + \Phi_1 \cos^2 m_3 + \Psi_1 \cos m_3 \cos n_3$$
$$= \Pi_2 \cos^2 l_3 + \Phi_2 \cos l_3 \cos m_3 + \Psi_2 \cos l_3 \cos n_3.$$

Adding these equations, and recollecting that, as the three directions of vibration are rectangular,

$$\cos^2 l_1 + \cos^2 l_2 + \cos^2 l_3 = 1,$$

$$\cos^2 m_1 + \cos^2 m_2 + \cos^2 m_3 = 1,$$

$$\cos l_1 \cos m_1 + \cos l_2 \cos m_2 + \cos l_3 \cos m_3 = 0,$$

$$\cos l_1 \cos n_1 + \cos l_2 \cos n_2 + \cos l_3 \cos n_3 = 0,$$

$$\cos m_1 \cos n_1 + \cos m_2 \cos n_2 + \cos m_3 \cos n_3 = 0\,;$$

we have

$$\Pi_2 = \Phi_1,$$

and similarly

$$\Phi_3 = \Psi_2, \quad \Psi_1 = \Pi_3,$$

or, substituting for Π_2, &c., their values from (O'),

$$(A_{\alpha^2\beta'} - B_{\alpha^2\alpha'})\,a^2 + (A_{\beta^2\beta'} - B_{\beta^2\alpha'})\,b^2 + (A_{\gamma^2\beta'} - B_{\gamma^2\alpha'})\,c^2 + 2(A_{\beta\gamma\beta'} - B_{\beta\gamma\alpha'})\,bc$$
$$+ 2(A_{\alpha\gamma\beta'} - B_{\alpha\gamma\alpha'})\,ac + 2(A_{\alpha\beta\beta'} - B_{\alpha\beta\alpha'})\,ab = 0,$$

$$(B_{\alpha^2\gamma'} - C_{\alpha^2\beta'})\,a^2 + (B_{\beta^2\gamma'} - C_{\beta^2\beta'})\,b^2 + (B_{\gamma^2\gamma'} - C_{\gamma^2\beta'})\,c^2$$
$$+ 2(B_{\beta\gamma\gamma'} - C_{\beta\gamma\beta'})\,bc + 2(B_{\alpha\gamma\gamma'} - C_{\alpha\gamma\beta'})\,ac + 2(B_{\alpha\beta\gamma'} - C_{\alpha\beta\beta'}) = 0,$$

$$(C_{\alpha^2\alpha'} - A_{\alpha^2\gamma'})\,a^2 + (C_{\beta^2\alpha'} - A_{\beta^2\gamma'})\,b^2 + (C_{\gamma^2\alpha'} - A_{\gamma^2\gamma'})\,c^2$$
$$+ 2(C_{\beta\gamma\alpha'} - A_{\beta\gamma\gamma'})\,bc + 2(C_{\alpha\gamma\alpha'} - A_{\alpha\gamma\gamma'})\,ac + 2(C_{\alpha\beta\alpha'} - A_{\alpha\beta\gamma'})\,ab = 0.$$

If these equations hold for all directions of wave plane, it is easily seen that the coefficients of

$$a^2,\ b^2,\ c^2, \quad ab,\ ac,\ bc,$$

must vanish of themselves. This condition will give eighteen equations which are evidently identical with the system of equations (R). The theorem, as stated above, is, therefore, true.

10. The total number of constants in $L+M+N$ being fifty-four, it is evident that the number of *distinct* constants contained in δV, and, therefore, in V, will be

$$54 - 18 = 36.$$

Now V, which is, as we have seen, a homogeneous quadratic function of the nine quantities

$$\frac{d\xi}{dx}, \frac{d\xi}{dy}, \frac{d\xi}{dz}, \frac{d\eta}{dx}, \frac{d\eta}{dy}, \frac{d\eta}{dz}, \frac{d\zeta}{dx}, \frac{d\zeta}{dy}, \frac{d\zeta}{dz},$$

will contain in general forty-five terms, and therefore, if it be subjected to no restriction, forty-five distinct constants. The function at which we have arrived is not, therefore, in its most general form. In fact, if we examine the composition of the terms in the value of L (H), we see that the quantities

$$\frac{d\eta}{dy}\frac{d\delta\xi}{dz}, \quad \frac{d\eta}{dz}\frac{d\delta\xi}{dy},$$

have the same coefficient, namely $B_{\beta\gamma\alpha'}$. Similarly, in the value of M we should have two terms

$$\frac{d\xi}{dz}\frac{d\delta\eta}{dy}, \quad \frac{d\xi}{dy}\frac{d\delta\eta}{dz},$$

with the common coefficient $A_{\beta\gamma\beta'}$. These coefficients being, by the equations (R), identical, the four terms enumerated above may be written

$$B_{\beta\gamma\alpha'}\left(\frac{d\eta}{dy}\frac{d\delta\xi}{dz}+\frac{d\xi}{dz}\frac{d\delta\eta}{dy}+\frac{d\eta}{dz}\frac{d\delta\xi}{dy}+\frac{d\xi}{dy}\frac{d\delta\eta}{dz}\right),$$

or,

$$B_{\beta\gamma\alpha'}\,\delta\left(\frac{d\eta}{dy}\frac{d\xi}{dz}+\frac{d\eta}{dz}\frac{d\xi}{dy}\right).$$

Hence it is evident, that the terms in V containing

$$\frac{d\eta}{dy}\frac{d\xi}{dz}, \quad \frac{d\eta}{dz}\frac{d\xi}{dy},$$

will be of the form

$$I\left(\frac{d\eta}{dy}\,\frac{d\xi}{dz}+\frac{d\eta}{dz}\,\frac{d\xi}{dy}\right).$$

Similar conclusions will be obtained for all terms of this form. These terms are distinguished by the technical rule, that, in the products which they severally contain, the same letter does not occur twice. Thus the conclusion at which we have arrived does not apply to terms of the form

$$G\frac{d\xi}{dx}\,\frac{d\xi}{dy},$$

where the same letter ξ occurs twice; nor to terms of the form

$$H\frac{d\xi}{dx}\,\frac{d\eta}{dx},$$

in which the letter x occurs twice. The preceding discussion gives us, therefore, the following general theorem:

If the constitution of a body, whose particles act independently, and whose original position is one of free equilibrium, be such, that the sum of the internal moments may be represented by the variation of a single function, this function must be of the form

$$V=\Sigma\left(F\frac{d\xi^2}{dx^2}\right)+\Sigma\left(G\frac{d\xi}{dx}\,\frac{d\xi}{dy}\right)+\Sigma\left(H\frac{d\xi}{dx}\,\frac{d\eta}{dx}\right)$$
$$+\Sigma\left\{I\left(\frac{d\eta}{dy}\,\frac{d\xi}{dz}+\frac{d\eta}{dz}\,\frac{d\xi}{dy}\right)\right\}. \qquad \text{(T)}$$

Each of the sums denoted by Σ will contain nine terms, thus giving thirty-six for the total number of distinct constants in V.

11. Previously to proceeding further, it may be well to compare this result with the investigations of Professor MAC CULLAGH and Mr. GREEN, in the undulatory theory of light. Both these writers assume the original state of the supposed luminous ether to be one of free equilibrium.* Both suppose also, that the sum of the internal moments may be represented by the variation of a

* Mr. GREEN has also investigated the problem under the supposition that the original position is not one of free equilibrium. The remarks in the text are, of course, only meant to apply to the first supposition.

single function. This function, in the system of Professor MAC CULLAGH, is in its simplest form given by the equation

$$-2V = a^2\left(\frac{d\zeta}{dy} - \frac{d\eta}{dz}\right)^2 + b^2\left(\frac{d\xi}{dz} - \frac{d\zeta}{dx}\right)^2 + c^2\left(\frac{d\eta}{dx} - \frac{d\xi}{dy}\right)^2.$$

The function used by Mr. GREEN is given by the equation

$$\begin{aligned}-2V = {}& \mu\left(\frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz}\right)^2 \\ & + L\left\{\left(\frac{d\eta}{dz} + \frac{d\zeta}{dy}\right)^2 - 4\,\frac{d\eta}{dy}\,\frac{d\zeta}{dz}\right\} \\ & + M\left\{\left(\frac{d\xi}{dz} + \frac{d\zeta}{dx}\right)^2 - 4\,\frac{d\xi}{dx}\,\frac{d\zeta}{dz}\right\} \\ & + N\left\{\left(\frac{d\xi}{dy} + \frac{d\eta}{dx}\right)^2 - 4\,\frac{d\xi}{dx}\,\frac{d\eta}{dy}\right\}.\end{aligned}$$

Comparing these successively with the general form (T), p. 198, we see that the function used by Professor MAC CULLAGH cannot by any supposition be identified with (T), inasmuch as it contains the products

$$\frac{d\zeta}{dy}\,\frac{d\eta}{dz},\quad \frac{d\xi}{dz}\,\frac{d\zeta}{dx},\quad \frac{d\eta}{dx}\,\frac{d\xi}{dy},$$

without the corresponding products

$$\frac{d\zeta}{dz}\,\frac{d\eta}{dy},\quad \frac{d\xi}{dx}\,\frac{d\zeta}{dz},\quad \frac{d\eta}{dy}\,\frac{d\xi}{dx}.$$

To identify the function used by Mr. GREEN with (T), we should have, as is easily seen,

$$L = M = N = \tfrac{1}{3}\mu.$$

These conditions would render the body uncrystalline, and therefore incapable of being generally identical with the luminous ether.

Hence we infer, that if the supposed luminous ether be a medium such as either of these writers assume it to be, the mutual action of its particles cannot be independent. In other words, we must suppose that in such a medium the capacity which each particle possesses of exerting force on any other particle, is *modified* by the action of the surrounding particles.

Bodies composed of attracting and repelling Molecules.

12. Two conditions may be supposed to be included in the supposition, that the molecular force is a force of attraction or repulsion, namely:

1. That the *direction* of the force is in the line joining the molecules. 2. That the *intensity* of this force, for each pair of molecules, is represented by a function of the distance. Retaining the former of these conditions, we may replace the second by the hypothesis made in the foregoing section, namely, that the sum of the internal moments may be represented by the variation of a single function. For as the effect of the force is in this case to change the distance between two molecules, if this force be represented by F, and the distance between the particles by ρ, the moment of the force F will be

$$F\delta\rho,$$

or dividing the force F as before into F_0 and F_1, and putting $\rho+\rho'$ for the distance as changed by displacement,

$$\text{effective moment} = F_1\delta\rho',$$

and therefore the complete moment is expressed by

$$\iiint F_1\delta\rho' dm,$$

which cannot be a complete variation, unless

$$F_1 = f(\rho').$$

Hence the proposition is evident. Instead, therefore, of defining the body to be one composed of attracting or repelling molecules, we shall define it to be "a body in which the molecular force acts in the direction of the line joining the molecules, and in which the sum of the internal moments is represented by the variation of a single function." We shall consider successively the simplifications which these two suppositions introduce into the general equations.

The first hypothesis, that, namely, which regards the *direction* of the molecular force, is mathematically represented by making

$$\alpha' = \alpha, \quad \beta' = \beta, \quad \gamma' = \gamma.$$

If these conditions be introduced into the general equations (N), it is easily seen that the number of distinct constants will be reduced to thirty, *sc.*,

$$A_{\alpha^3},\ A_{\beta^3},\ A_{\gamma^3},\ A_{\beta^2\gamma},\ A_{\gamma^2\beta},\ A_{\gamma^2\alpha},\ A_{\alpha^2\gamma},\ A_{\alpha^2\beta},\ A_{\beta^2\alpha},\ A_{\alpha\beta\gamma},$$
$$B_{\alpha^3},\ \&c.$$
$$C_{\alpha^3},\ \&c.$$

The equations so reduced will refer to a system of attracting or repelling molecules, in the more enlarged sense of the term attraction or repulsion, the force being defined solely by its direction, without any hypothesis as to its intensity. Using the words in this sense, we may state the conclusion at which we have arrived as follows:

The equations of equilibrium or motion in a system of attracting or repelling molecules, will in general contain thirty distinct constants.

13. We shall next proceed to consider what further simplification is introduced into these equations by the supposition that the sum of the internal moments may be represented by the variation of a single function. Making

$$\alpha' = \alpha, \quad \beta' = \beta, \quad \gamma' = \gamma,$$

in the equations of condition (R), we shall find that their number will be reduced to fifteen, the nine equations

$$\begin{array}{lll} A_{\gamma^2\beta'} = B_{\gamma^2\alpha'}, & C_{\beta\gamma\alpha'} = A_{\beta\gamma\gamma'}, & B_{\alpha\gamma\gamma'} = C_{\alpha\gamma\beta'}, \\ A_{\beta\gamma\beta'} = B_{\beta\gamma\alpha'}, & C_{\beta^2\alpha'} = A_{\beta^2\gamma'}, & B_{\alpha\beta\gamma'} = C_{\alpha\beta\beta'}, \\ A_{\alpha\gamma\beta'} = B_{\alpha\gamma\alpha'}, & C_{\alpha\beta\alpha'} = A_{\alpha\beta\gamma'}, & B_{\alpha^2\gamma'} = C_{\alpha^2\beta'}, \end{array}$$

being obviously equivalent to but six. Hence we infer that

The equations of equilibrium or motion of a body in which the molecular force acts in the line joining the molecules, and is represented by a function of the distance, will contain fifteen distinct constants.

This agrees with the result obtained by Mr. HAUGHTON, to whose Memoir the reader is referred for the further discussion of this case.*

* Vid. note at the conclusion of this Memoir.

II.—HYPOTHESIS OF MODIFIED ACTION.

14. Let m, m' be two molecules of the medium under consideration, m being that whose equilibrium or motion is required. Then if, as before, we suppose the force which m' exerts upon m to be composed of two parts, one depending upon the relative displacement of these two particles, and the other existing previously to the displacement of either, we shall still have, as in p. 183,

$$F = F_0 + A(\xi' - \xi) + B(\eta' - \eta) + C(\zeta' - \zeta).$$

Now it is easily seen that the difference between this case and the preceding will show itself in the nature of the quantity F_0. In the former case, in which the action of m' is independent of the other particles of the medium, F_0 must be of the form

$$f(x, y, z, \ \rho, \theta, \phi),$$

and may, as we have seen, be neglected in the case of a body whose original position is one of free equilibrium. But in the present case, in which it is supposed that the displacement of the other particles has itself the power of developing a force between m' and m, the form of F_0 is completely changed. Our first object, then, must be to determine the new form to be assigned to this quantity.

Let m'' be a third molecule of the given medium; ξ'', η'', ζ'', its displacements; and ρ_1, θ_1, ϕ_1, or ρ_1, α_1, β_1, γ_1, its polar co-ordinates with regard to m. Then it will appear, by reasoning similar to that of p. 183, that the mathematical expression for its effect in developing a force between m and m' will be

$$f(x, y, z, \ \rho, \theta, \phi, \ \rho_1, \theta_1, \phi_1, \ \xi'' - \xi, \ \eta'' - \eta, \ \zeta'' - \zeta, \ \xi'' - \xi', \ \eta' - \eta', \ \zeta'' - \zeta');$$

or, as it may be otherwise written,

$$f(x, y, z, \ \rho, \theta, \phi, \ \rho_1, \theta_1, \phi_1, \ \xi'' - \xi, \ \eta'' - \eta, \ \zeta'' - \zeta, \ \xi' - \xi, \ \eta' - \eta, \ \zeta' - \zeta).$$

Treating this expression as in p. 184, it becomes

$$f_0 + a\rho\left(\cos\alpha\frac{d\xi}{dx} + \cos\beta\frac{d\xi}{dy} + \cos\gamma\frac{d\xi}{dz}\right)$$
$$+ b\rho\left(\cos\alpha\frac{d\eta}{dx} + \cos\beta\frac{d\eta}{dy} + \cos\gamma\frac{d\eta}{dz}\right)$$

$$+ c\rho\left(\cos\alpha\frac{d\zeta}{dx} + \cos\beta\frac{d\zeta}{dy} + \cos\gamma\frac{d\zeta}{dz}\right)$$

$$+ a_1\rho_1\left(\cos\alpha_1\frac{d\xi}{dx} + \cos\beta_1\frac{d\xi}{dy} + \cos\gamma_1\frac{d\xi}{dz}\right)$$

$$+ b_1\rho_1\left(\cos\alpha_1\frac{d\eta}{dx} + \cos\beta_1\frac{d\eta}{dy} + \cos\gamma_1\frac{d\eta}{dz}\right)$$

$$+ c_1\rho_1\left(\cos\alpha_1\frac{d\zeta}{dx} + \cos\beta_1\frac{d\zeta}{dy} + \cos\gamma_1\frac{d\zeta}{dz}\right),$$

where f_0 is the force which is independent of the displacements of any one of the three particles. First let it be supposed that f_0 is independent of the displacements of any other particle.* Then, as the foregoing expression represents that part of the modifying force which results from the relative displacement of m'', it seems that the most general supposition which we can make as to the aggregate effect of all the particles is, that it is estimated by multiplying this expression by some function of the polar co-ordinates of m'', as also by the element of the mass, and integrating through the whole sphere of molecular activity. It is easily seen, that the result of this process will be an expression of the form

$$E_0 + A_1\frac{d\xi}{dx} + A_2\frac{d\xi}{dy} + A_3\frac{d\xi}{dz}$$

$$+ B_1\frac{d\eta}{dx} + B_2\frac{d\eta}{dy} + B_3\frac{d\eta}{dz}$$

$$+ C_1\frac{d\zeta}{dx} + C_2\frac{d\zeta}{dy} + C_3\frac{d\zeta}{dz};$$

where E_0, A_1, B_1, &c., are definite integrals depending upon the constitution of the medium, being in general of the form

$$f(x, y, z, \quad \rho, \theta, \phi).$$

Hence the general value of F ((D), p. 185) will become

* It is easily seen, that this supposition does not limit the generality of the result.

$$\begin{aligned}
E_0 &+ (A\rho \cos \alpha + A_1)\frac{d\xi}{dx} + (A\rho \cos \beta + A_2)\frac{d\xi}{dy} + (A\rho \cos \gamma + A_3)\frac{d\xi}{dz} \\
&+ (B\rho \cos \alpha + B_1)\frac{d\eta}{dx} + (B\rho \cos \beta + B_2)\frac{d\eta}{dy} + (B\rho \cos \gamma + B_3)\frac{d\eta}{dz} \\
&+ (C\rho \cos \alpha + C_1)\frac{d\zeta}{dx} + (C\rho \cos \beta + C_2)\frac{d\zeta}{dy} + (C\rho \cos \gamma + C_3)\frac{d\zeta}{dz}.
\end{aligned}$$

Resolving this force as before along the three axes, and proceeding as in p. 186, we find an expression for L similar to (H). There is, however, one important difference. In the value of L, which is derived from the principle of independent action, the quantities

$$\frac{d\xi}{dz}\frac{d\delta\xi}{dy}, \qquad \frac{d\xi}{dy}\frac{d\delta\xi}{dz},$$

have the same coefficient; and the same is true of the quantities

$$\begin{array}{ll}
\dfrac{d\xi}{dx}\dfrac{d\delta\xi}{dz}, & \dfrac{d\xi}{dz}\dfrac{d\delta\xi}{dx}, \\[2ex]
\dfrac{d\xi}{dy}\dfrac{d\delta\xi}{dx}, & \dfrac{d\xi}{dx}\dfrac{d\delta\xi}{dy}, \\[2ex]
\dfrac{d\eta}{dz}\dfrac{d\delta\xi}{dy}, & \dfrac{d\eta}{dy}\dfrac{d\delta\xi}{dz}, \\[2ex]
\dfrac{d\eta}{dx}\dfrac{d\delta\xi}{dz}, & \dfrac{d\eta}{dz}\dfrac{d\delta\xi}{dx}, \\[2ex]
\dfrac{d\eta}{dy}\dfrac{d\delta\xi}{dx}, & \dfrac{d\eta}{dx}\dfrac{d\delta\xi}{dy}, \\[2ex]
\dfrac{d\zeta}{dz}\dfrac{d\delta\xi}{dy}, & \dfrac{d\zeta}{dy}\dfrac{d\delta\xi}{dz}, \\[2ex]
\dfrac{d\zeta}{dx}\dfrac{d\delta\xi}{dz}, & \dfrac{d\zeta}{dz}\dfrac{d\delta\xi}{dx}, \\[2ex]
\dfrac{d\zeta}{dy}\dfrac{d\delta\xi}{dx}, & \dfrac{d\zeta}{dx}\dfrac{d\delta\xi}{dy}.
\end{array}$$

Now it is easy to see that in general this restriction has no effect in limiting the generality of the equations of motion of a homogeneous body. For whether the coefficients be equal or not, each of the foregoing pairs of quantities will furnish but *one* term to the equations of motion. Thus, if the quantities

$$\frac{d\xi}{dz}\frac{d\delta\xi}{dy}, \qquad \frac{d\xi}{dy}\frac{d\delta\xi}{dz},$$

enter into L in the form

$$A\frac{d\xi}{dz}\frac{d\delta\xi}{dy} + B\frac{d\xi}{dy}\frac{d\delta\xi}{dz},$$

the equations of motion will derive from them the single term

$$(A+B)\frac{d^2\xi}{dydz}.$$

It is evident, therefore, that the supposition

$$A = B$$

will not restrict in any way the generality of these equations. We have seen accordingly, that the principle of independent action gives to these equations the greatest number of independent constants which they can have, without a change of form. But the restriction may show itself in other ways. Thus, when we assume that the sum of the internal moments may be represented by the variation of a single function V, we find that in order to reconcile this supposition with the principle of independent action, it is necessary to assume further, that the coefficients of V are connected by nine equations of condition, and that, therefore, that principle does not admit of the existence of a function V in its most general form. This restriction has evidently been removed by supposing the state of each molecule to be modified by the action of the surrounding molecules. For, as we have just seen, this supposition enables us to obtain values for L, M, N in which the coefficients are completely independent of each other; and, with regard to the particular case of physical optics, we infer, as before, that if a luminous ether exist, whose constitution agrees with either of the hypotheses advanced by Professor MAC CULLAGH and Mr. GREEN respectively, each of the particles of that medium must be supposed to be capable of *modifying* the force exerted by any other particle within its sphere of action.

It is unnecessary to pursue the consequences of this principle further; for, as we have already seen, all the varieties of the general equations of motion, to the consideration of which the present Memoir is specially devoted, may be obtained from the more limited principle of independent action.

15. It is usual with writers upon the subject which has been here discussed, to consider the problem of the transmission of undulations from one body to another with which it is in mathematical contact. This problem, which, by an extension of the phraseology of optics, has been denominated the problem of refraction, has been investigated with special reference to a luminous ether, by Professor MAC CULLAGH, CAUCHY, GREEN, and others; and has been discussed by Mr. HAUGHTON for the case of solid bodies in general. But all these investigations appear to me to be liable to an objection to which I am unable to conceive any satisfactory answer. The nature of this objection, which has deterred me from following in this particular the steps of the writers in question, I shall now proceed to state.

On referring to p. 189 it will be seen, that the *form* of the general equations of motion, upon which the whole theory of undulation is based, depends upon the fact, that the coefficients are *constant* quantities, a fact which is, as we have seen, a result of the *homogeneity* of the medium; and the conclusions of p. 192 are evidently true, so long as the molecule under consideration is situated at a finite distance from the bounding surface of the medium. The functions to be integrated retaining the same form, and the limits of integration being the same, it is evident that the definite integrals will have the same value for every point.

Let us now consider the case of two media in contact. For the sake of simplicity, let the common surface of contact be an indefinite plane, which we shall take for the plane of xy. Let a, a' be the radii of molecular activity for the two media, and suppose that the molecule under consideration is situated at a distance from the plane of xy less than the greater of these. If now two*

* Instead of two spheres we may (as is easily seen) substitute a single sphere described with a radius not less than the greater radius of molecular activity. This substitution does not, however, in any way affect the reasoning in the text. The single definite integral

$$\iiint A \cos^2 a \cos a' dm$$

will still be replaced by

$$\iiint A \cos^2 a \cos a' dm + \iiint A_1 \cos^2 a_1 \cos a_1' dm;$$

the first being extended through the upper segment of the sphere, and the second through the lower segment. The value of the sum of these two quantities will evidently depend upon the distance of the point from the surface of separation.

spheres be described, with this molecule as their common centre, and with the radii a, a' respectively, each of the definite integrals of p. 186 will consist of two parts, the first being extended through all that portion of the first sphere which lies within the first medium; and the second through all that portion of the second sphere which lies in the second medium. Thus, instead of the definite integral

$$\iiint A \cos^2 \alpha \cos \alpha' \, dm,$$

taken through the entire of a sphere whose radius is a, we should have

$$\iiint A \cos^2 \alpha \cos \alpha' \, dm + \iiint A_1 \cos^2 \alpha_1 \cos \alpha_1' \, dm,$$

the limits of integration in each of these being determined as above stated. The limits of integration, and therefore the value of each of these integrals, depending upon the distance of the molecule from the plane of separation, it is evident that the coefficients in the general equations (N) will be functions of z, whose form will depend upon the constitutions of the two media, and will be, therefore, in general, unknown. The form of the equations of motion will therefore be completely altered, not only by the change of constant into variable coefficients. but by the introduction of terms of the first order,

$$\frac{d\xi}{dx}, \ \frac{d\xi}{dy}, \ \&c. \ \frac{d\eta}{dx}, \ \&c. \ \frac{d\zeta}{dx}, \ \&c.$$

The integral which represents wave motion will, therefore, be no longer applicable, nor will it be possible to give any integral of these equations without forming a number of additional hypotheses as to the constitution of the medium.

From these mathematical considerations, the following physical conclusions appear to be legitimately inferred:

(1.) That in the case of a single medium of limited extent, the molecules which are situated at a distance from the bounding surface less than the radius of molecular activity, move according to a law altogether different from that which regulates the motion of the particles in the interior.

(2.) That it is impossible to assign this law without forming one or more hypotheses as to the nature of the medium.

(3.) That if a plane wave pass through a homogeneous medium, it will not in general reach the surface; that is to say, the motion of the particles in and immediately adjoining the surface will not be a wave motion composed of rectilinear vibrations.

(4.) That if two media be in contact, there will be a stratum of particles extending on each side of the surface of separation to a distance equal to the greatest radius of molecular activity; and that the motion of the particles in this stratum is altogether different from that of the particles in the interior of either medium.

(5.) That, therefore, two media which are thus in contact, may be each perfectly capable of transmitting plane waves through them in all directions, and yet incapable of transmitting such a motion from one to the other; and that even in the case of reflexion, in which the motion is transmitted back again through the same medium, the vibrations may cease to be rectilinear. The phenomenon of total reflexion affords an instance of this.

Now in the investigation of the problem of refraction, it is supposed that the integral which represents plane waves is applicable to the motion of the molecules which are actually situated in the surface of separation, a supposition which the foregoing considerations prove to be generally untrue. Nor does the truth of this conclusion depend upon the method employed in the previous discussion. On whatever principle we investigate the motion of the particles of a medium, it is easily seen, that for all points situated in the stratum described above, the medium cannot be considered homogeneous, inasmuch as the force to which each molecule is subject varies with its distance from the surface of separation. Within this stratum, therefore, the molecules must be considered as forming a *heterogeneous* medium, whose constitution varies rapidly according to some unknown law. It is difficult to see what modification is thus introduced into the discussion of the problem of refraction, in which the two media are supposed to be homogeneous. But it appears to me, that the supposition of plane wave motion extending to the mathematical limits of a medium is in general untenable. Nor shall we remove the difficulty in question by the supposition, that the molecules of one medium are incapable of influencing those of another. The only effect of such a supposition, which is, besides, wholly gratuitous, would be the substitution of one integral such as

$$\iiint A \cos^2 a \cos a' dm,$$

for the sum of two,

$$\iiint A \cos^2 \alpha \cos \alpha' dm + \iiint A_1 \cos^2 \alpha_1 \cos \alpha_1' dm_1.$$

But as the limits of integration are still variable, the form of the general equations of motion will still be that described in p. 207. These equations do not, as we have seen, admit of an integral representing plane wave motion. It is easily shown that the difficulty here alluded to does not affect that part of the theory of light or sound in which the *direction* of the reflected or refracted ray is derived from the consideration of wave motion.

16. Before concluding the present Memoir, I think it necessary to say a few words on the applicability of the integral calculus to problems like the present, or more generally to any problems in which bodies are considered, not as continuous masses, but as assemblages of distinct molecules.

I may remark, in the first place, that the method and results of the present Paper would be in no wise affected by the rejection of the molecular hypothesis; all that is essential to the validity of the method here given being attained by defining a molecule to be *a particle so small, that the motion of the system may be fully represented by the motions of all these particles considered as units;* and without such a supposition no equations of motion of a continuous body appear to have a perfectly definite meaning.

But as the constitution of the bodies which we find in nature appears to favour the supposition of separate molecules, rather than that of perfect continuity, it becomes an important question to determine how far the methods of the integral calculus are applicable to such cases. This is the more necessary, as M. POISSON denies the applicability of these methods to any problems connected with molecular force; and, more generally, to any problems in which the force varies with extreme rapidity within the limits of integration:—" Au reste la formule d'EULER qui sert à transformer les sommes en intégrales, contient une série ordonnée suivant les puissances de la différence finie de la variable, qui n'est pas toujours convergente, quoique cette différence soit supposée très petite. L'exception a lieu surtout dans le cas des fonctions comme $f(r)$ qui varient très rapidement."*

It is quite true, that the methods of the integral calculus are in strictness applicable only to continuous masses, and that it is in such cases only that the

* Mem. de l'Inst. tom. viii. p. 399.

results which it furnishes are mathematically accurate. When the mass ceases to be continuous, these results become approximate, and would of course be valueless, unless we had some means of testing the degree of approximation attained. This we shall now proceed to consider.

Let m be the mass of any one of the separate molecules of which the body is composed, and let x, y, z be its co-ordinates. Let $mf(x, y, z)$ be the mathematical expression of some quality or power belonging to this molecule, of such a nature, that the corresponding quality of the entire body is mathematically expressed by the *sum* of the expressions which refer to the several molecules. Let also m', x', y', z', m'', x'', y'', z'', &c., be the masses and co-ordinates of the other molecules. Then, if we assume

$$u = f(x, y, z), \quad u' = f(x', y', z'), \text{ \&c.,}$$

the accurate expression sought for will be

$$mu + m'u' + m''u'' + \text{\&c.} = \Sigma mu.$$

Now let dv be the element of the volume geometrically considered, and ϵ the mean density of the matter which occupies it, so that its weight may be represented by

$$g\epsilon dv.$$

Then the approximate equation furnished by the integral calculus will be

$$\Sigma mu = \int u\epsilon dv.$$

In order to estimate the amount of the error which is involved in the use of the integral sign instead of the symbol of finite summation, we shall consider successively the several suppositions which are made in the interchange of these symbols, and the amount of the error introduced by each.

The object of this investigation being to determine, not the actual magnitude of the error, but merely its order, it is in the first place necessary to establish a notation to represent the respective orders of the several small quantities with which we are concerned.

Let e be an indefinitely small quantity which we take as the standard. Let the distance ω, between two consecutive molecules, be of the order i, or in other words let

$$\omega = ke^i;$$

where k is a finite magnitude. Let i' denote the degree of rapidity with which the function u varies, i. e. let it be supposed that u receives a finite increment in passing from one to another of two molecules, whose mutual distance ω' is given by the equation

$$\omega' = k'e^{i'}.$$

Suppose now the entire geometrical space which is occupied by the system of molecules, including also the small intervals or *pores* which separate them, to be divided into an indefinite number of equal portions, v, the linear dimension of each of which is a quantity of the order i''. We shall then have

$$v = k''^3 e^{3i''}.$$

Let $\Sigma_1 mu$ denote a finite summation extended to all the molecules contained in the first of these elements, $\Sigma_2 mu$ a similar summation for the molecules of the second element, $\Sigma_3 mu$ for the third, &c. Then

$$\Sigma mu = \Sigma_1 mu + \Sigma_2 mu + \Sigma_3 mu + \&c.$$

This equation is evidently exact.

Now let the following suppositions be made:

(1.) That u retains the same value for every molecule within the element v.

(2.) That the coefficient ϵ, which represents the mean density, is independent of the magnitude of the element.

These suppositions will give the following equations:

$$\Sigma_1 mu = u_1 \Sigma_1 m = u_1 \epsilon_1 v_1,$$

$$\Sigma_2 mu = u_2 \Sigma_2 m = u_2 \epsilon_2 v_2,$$

$$\&c. \quad \&c.;$$

and, therefore,

$$\Sigma mu = u_1 \epsilon_1 v_1 + u_2 \epsilon_2 v_2 + \&c. = \Sigma u \epsilon v.$$

Finally, instead of the symbol of finite summation Σ, let us substitute the symbol of integration $\int$, and we shall have

$$\Sigma mu = \int u\epsilon dv = \int u d\mu.$$

Let us now consider the order of the error introduced by each of these suppositions.

(1.) The supposition that u remains the same for all molecules situated within the element v, will introduce an error whose order is the same with that of the actual variation of u within that space. We assume here, that the function u varies continuously within the space ω'; in other words, that if ω' be divided into any number of equal parts, the variations which u receives in each of these parts are quantities of the same order of magnitude. Hence it is easily seen, that the variation of u within the space $k''e^{i''}$ will be represented by an expression of the form

$$k'''e^{i''-i'}.$$

For if ω' be divided into a number of parts, each equal to $k''e^{i''}$, the variations in these segments may be represented by

$$a_1 e^m, \ a_2 e^m, \ \&c.,$$

the exponent m being, in conformity with the foregoing assumption, the same for all. Hence we shall have for the complete variation of u,

$$(a_1 + a_2 + \&c.)\, e^m.$$

Now since a_1, a_2, &c., are finite quantities,

$$a_1 + a_2 + \&c.,$$

will be a quantity of the same order as their number. Denoting this number by n, we shall have

$$n = \frac{\omega'}{k''e^{i''}} = \frac{k'}{k''}\, e^{i'-i''}.$$

Hence it is evident, that the complete variation of u is of the order

$$m + i' - i''.$$

Since, therefore, this variation is by hypothesis finite, we must have

$$m + i' - i'' = 0,$$

or

$$m = i'' - i'.$$

Hence the expression for the partial variation of u is

$$k'''e^{i''-i'}.$$

Let u_1 be the least value of u within the element v, and $u_1 + k'''e^{i''-i'}$ the greatest, and let it be supposed, as the most unfavourable case, that u has throughout the value

$$u_1 + k'''e^{i''-i'}.$$

Substituting this expression in $\Sigma_1 mu$ we have

$$\Sigma_1 mu = u_1 \Sigma_1 m + k'''e^{i''-i'} \Sigma_1 m.$$

Hence the error in the equation

$$\Sigma_1 mu = u_1 \Sigma_1 m$$

is at most

$$k'''e^{i''-i'} \Sigma_1 m;$$

and, therefore, the error in

$$\Sigma mu = u_1 \Sigma_1 m + u_2 \Sigma_2 m + \&c.,$$

is, at most, a quantity of the form

$$Ke^{i''-i'}.$$

This equation will, therefore, be free from sensible error if

$$i'' > i'.$$

(2.) In estimating the error produced by the second supposition, we shall assume that the densities and magnitudes of the molecules vary with a *finite* degree of rapidity; and that, therefore, at any one point in the body, the sum of the masses of the molecules contained in an element is proportional to their number. Hence the equation

$$\Sigma_1 m = \epsilon_1 v,$$

is equivalent to an assumption, that the number of molecules contained in the element v is proportional to its volume.

To estimate the error involved in this assumption, let us compare, for the sake of greater generality, two elements whose bounding surfaces are wholly different in form. Suppose these elements to be similarly divided into rectangular prisms with the same transverse section, whose linear dimension is of the same order with the molecular distance. The error involved in such a supposition will be, for each of the prisms, represented by the expression

$$Ae^{3i};$$

and therefore, for the whole element, by

$$Ae^{3i} \times \text{number of prisms}.$$

But the number of these prisms, being directly as the volume of the element and inversely as the volume of each prism, will be represented by the expression

$$Be^{2(i''-i)}.$$

Hence the error in the foregoing division will be for each element

$$Ce^{2i''+i}.$$

Now since the bounding planes of these prisms, taken two and two, are symmetrically situated with regard to the molecules which they contain, if the extremities of the prisms were symmetrically situated with regard to the extreme molecules, the number of such molecules contained in these two prisms would evidently be as their lengths. But these extremities can always be made symmetrical by adding to one of the prisms a portion whose length is of the same order as the molecular distance. Hence the error involved in the assumption, that the number of molecules in each prism is proportional to its length, is represented by an expression similar to

$$Be^{2(i''-i)}.$$

The total error for each element is, therefore, expressed by a quantity similar to $Ce^{2i''+i}$. Let l, l', l'', &c., be the lengths of the several prisms into which the element v is divided, and let $\lambda, \lambda', \lambda''$, &c., denote the corresponding quantities for v'. Let also ω be the common transverse section. Then it follows from the assumptions which we have made, that

$$\Sigma_1 m = E\,(l + l' + l'' + \&c.),$$
$$\Sigma_2 m = E'(\lambda + \lambda' + \lambda'' + \&c.).$$

We have also

$$v = \omega\,(l + l' + l'' + \&c.),$$
$$v' = \omega\,(\lambda + \lambda' + \lambda'' + \&c.),$$

and therefore,

$$\frac{\Sigma_1 m}{\Sigma_2 m} = \frac{E}{E'} \cdot \frac{v}{v'}.$$

Hence, in general,

$$\Sigma_1 m = \epsilon_1 v.$$

Now we have seen that the error involved in the supposition from which this equation is derived, is for each element represented by an expression of the form

$$Ce^{2i''+i}.$$

The order of the total error will be found by multiplying this expression by the number of the elements. Now

$$\text{Number of elements} = \frac{\text{total mass}}{v} = \frac{M}{k''} e^{-3i''}.$$

Hence the order of the total error will be

$$i - i''.$$

The equation

$$\Sigma_1 m = \epsilon_1 v$$

will, therefore, be free from sensible error if

$$i > i''.$$

(3.) Lastly, it is easily shown, by reasoning similar to that of (1) and (2), that the error in the equation

$$\Sigma u\epsilon v = \iiint u\epsilon dv,$$

is at most of the order i''. The method here employed will, therefore, be free from sensible error if the three following equations hold:

$$i'' - i' > 0, \quad i - i'' > 0, \quad i'' > 0.$$

Hence we infer that

The methods of the integral calculus are applicable to questions of molecular mechanics, provided that the molecular force varies continuously within its sphere of action; and provided also that the sphere of molecular action is of such a magnitude as to admit of being subdivided into an indefinite number of elements, each element containing an indefinite number of molecules.

NOTE ON ARTICLE 12.

ON a reperusal of Article 12, it appears to me that I have not stated with sufficient accuracy the distinguishing characteristic of a system of attracting and repelling molecules, and I think it, therefore, necessary to add a few words in explanation of what I have there said.

The supposition that the molecular force is a function of the distance may have one of two meanings, namely:—1. That all molecules situated at the same distance from m exert upon it a force of the same intensity. 2. That the force which any one molecule m' exerts upon m cannot be changed, except by altering the distance between these two molecules. If we recollect that the symbol d denotes a passage from one molecule to another, and δ the displacement of the *same* molecule; and if we use the latter in its most general sense, as applied to any displacement, virtual or real, we may represent the first hypothesis by the equations

$$\frac{dF}{d\theta} = 0, \quad \frac{dF}{d\phi} = 0;$$

and the second by

$$\frac{\delta F}{\delta\theta} = 0, \quad \frac{\delta F}{\delta\phi} = 0.$$

It is in the latter of these significations that the second hypothesis made in Article 12 is to be understood.

Now it has been shown in the text, that if the sum of the internal moments be capable of being represented by the variation of a single function, we must have

$$F_1 = f(\rho') = a\rho',$$

since ρ' is indefinitely small. If then we use the symbol δ to denote an increment produced by a *real* displacement, we shall have

$$F_1 = \delta F, \quad \rho' = \delta\rho,$$

and, therefore,

$$\delta F = a\delta\rho.$$

Hence

$$\frac{\delta F}{\delta\theta} = 0, \quad \frac{\delta F}{\delta\phi} = 0,$$

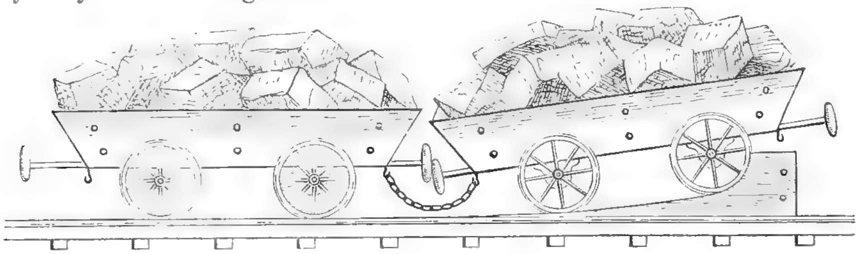
denoting, as we have seen, that the molecular force is, in the sense above defined, a force of attraction or repulsion. If, however, we confine ourselves to the strict sense of the terms attraction and repulsion, we ought to define the force solely by its *direction*, without any regard to its intensity. Properly speaking, therefore, the equations of motion of a system of attracting or repelling molecules will, in their most general form, contain thirty distinct constants.

IX.—*Account of Experiments made on a New Friction Sledge for stopping Railway Trains.* By the Rev. SAMUEL HAUGHTON, Fellow of Trinity College, Dublin.

Read June 4, 1850.

THE instrument which is here described, and with which the following experiments were made, is the invention of Mr. S. WILFRED HAUGHTON, Engineer, and Superintendent of the Dublin and Kingstown Railway Engine Factory at Ringsend Docks. It is intended to be used at the termini of railways, instead of the spring buffer usually employed to stop the train, in case it should happen, through the carelessness of the engine-driver or guards, to enter the terminus with too great speed. The objection to the use of the spring-buffer is twofold: the space through which the recoil can take place is too short, and the second recoil of the spring produces effects which are only less dangerous than those of the first shock; this latter inconvenience is sometimes remedied by the use of ratchets, which prevent the recoil of the spring after it has been compressed by the shock. But the first objection is founded on the use of the spring itself, and cannot be removed. A careful consideration of these objections led Mr. S. W. HAUGHTON to the invention of his Friction Sledge, as a substitute for the spring buffer.

The Friction Sledge consists of two strong wooden frames, shod with iron, and shaped as in the annexed diagram, which represents a side view of the sledge in action; these two frames are provided with iron flanges on the inside (so as to prevent them from slipping off the rails), and being placed parallel to each other at a distance equal to the interval between the rails, are strongly tied together by iron braces. The sledge, being placed upon the rails, is ready to perform its office of stopping a train moving with any moderate degree of velocity. The engine or foremost carriage of the train runs forward upon the sledge, and, striking against its curved front, receives a shock, which, if the sledge were immoveable, would be as fatal as the shock caused by impinging upon a stone wall; but the sledge, having sustained a portion of the shock, slides forward, and the remainder of the momentum is gradually destroyed by its friction against the rails.



The first time I had an opportunity of seeing the Friction Sledge in action was upon the 20th of April, 1849, on which occasion no accurate record was kept of its performance; it appeared, however, so completely successful in a practical point of view, that I was induced to attempt a few experiments, with a view of obtaining a correct knowledge, theoretical and practical, of its mode of action. I take this opportunity of acknowledging the kindness of the Board of Directors of the Dublin and Kingstown Railway, in allowing the use of waggons and a portion of the rails suited to the experiments; and of stating the obligations I am under to Mr. WILFRED HAUGHTON and the Rev. JOSEPH A. GALBRAITH, without whose assistance in conducting the experiments, and afterwards in calculating their results, I should have been unable to complete this account.

Our first experiments were performed on the 27th of April, 1849, with an engine and train of five empty passenger carriages,* in the presence of the Rev. T. ROMNEY ROBINSON, D.D., of Armagh Observatory, and Mr. BERGIN, Secretary to the Dublin and Kingstown Railway. On this occasion our mode of measuring

* The velocity of the train on this occasion was determined by counting with a chronograph the time occupied in performing the last four or five revolutions of the driving wheels of the engine; this determination, however, was rendered uncertain by the occasional slipping of the wheels on the damp rails.

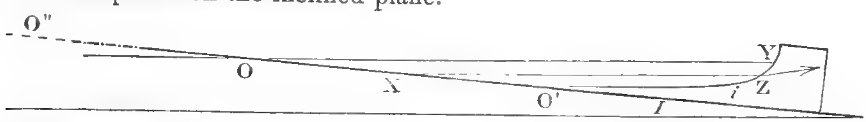
the velocity of the train was so imperfect, that I shall merely state the general results, without attempting to deduce any accurate conclusions from them.

Number of Experiment.	Velocity of Impact.	Length of Slide.
1	8·6 miles per hour.	13·66 feet.
2	9·4 ,, ,,	14·75 ,,
3	8·2 ,, ,,	12·58 ,,
4	9·4 ,, ,,	16·33 ,,
5	9·0 ,, ,,	15·50 ,,
6	10·5 ,, ,,	18·16 ,,

The weight of the engine and train was about 32 tons, and the sledge 7 cwt. The results of this trial are sufficient to prove the great efficiency of the Friction Sledge as a means of stopping a train in motion.

In order to obtain more accurate results, I determined to experiment with a single loaded waggon; and to measure the velocity with precision, I used a portion of the rails inclined to the horizon, and measured, by means of a pair of chronographs reading to the fifth of a second, the time of describing a given space, when the waggon was made to enter upon the inclined plane with an unknown velocity; or measured the space traversed by the waggon, when its whole motion took place on the inclined plane.

The subsequent experiments on the Friction Sledge were conducted as follows:—An inclined plane of about 160 feet, on the rails at the Engine Factory, Ringsend Docks, was selected, at the lower extremity of which the friction sledge was placed, and a loaded waggon was either pushed or allowed to run down the inclined plane, so as to impinge upon the sledge at the bottom. The velocity of impact and the length of the slide were the quantities to be measured, from which all others could be inferred by calculation; the velocity of impact was measured by noting the time taken by the loaded waggon to pass over a measured space on the inclined plane, or by allowing the waggon to start from fixed points on the inclined plane.



Let O″OO′ be a portion of the inclined plane, *I* the inclination of the plane to the horizon, i the angle which the axis of the waggon makes with the inclined plane when the front wheel is at the point Z. The line OY is horizontal; X is the point of bisection of OO′; and XZ is also horizontal. The velocity of the waggon at O′ is its velocity at the commencement of the shock; the velocity at O is equal (neglecting the friction of the wheels) to its velocity at Y; i. e., its velocity at the end of the shock.

I shall assume that the shock consists of a single blow given at the point Z with the velocity which animates the waggon at the point X.

If v' represent this velocity, then the velocity of impact actually imparted to the sledge will be

$$v = \frac{mv'}{m + m'},$$

m denoting the weight of the waggon,
m' . . . . sledge.

If V represent the velocity with which the sledge and waggon begin to move, and μ, k, represent the coefficients of friction of rest and motion respectively, we easily obtain (*neglecting the loss of momentum caused by imperfect elasticity*) the following equations:

$$V = v \cos i - \mu (g \cos I - v \sin i); \tag{1}$$

$$V^2 = 2gs (k \cos I - \sin I); \tag{2}$$

the first of which expresses the fact, that the momentum with which the sledge begins to move is equal to the difference between the original momentum and that destroyed by the friction of rest; the second equation is true on the hypothesis, that the friction of motion is a constant retarding force; eliminating V between these equations we obtain

$$v = A\sqrt{s} + \mu u \sec i; \tag{3}$$

A and u being defined by the following equations:

$$A \cos i = \sqrt{(2g [k \cos I - \sin I])},$$

$$u = g \cos I - v \sin i.$$

Each experiment tried with the Friction Sledge, should give particular values for s and v; which, substituted in (3), would afford a relation between

A and μ. A few trials soon convinced me that μ was not a constant, and that equation (3) did not represent the experiments; this result might have been foreseen, as the loss of motion arising from the shock of imperfectly elastic bodies was neglected in finding equation (3). I could not discover any direct method of introducing the loss of motion due to the shock, and therefore sought to modify equation (3) by experiment, so as to make it represent what actually occurred in each trial. After many failures I was induced to assume

$$\mu = Kv,$$

K being a constant. Introducing this value of μ into (3), and dividing by $\sqrt{s}$, we obtain

$$\frac{v}{\sqrt{s}} = A + K \sec i \, \frac{uv}{\sqrt{s}}. \tag{4}$$

I hope to be able to show that this equation represents faithfully the whole series of experiments, and that, too, with a degree of accuracy which seems to prove that it is the true expression of the facts which were observed.

The numerical values of the constants in the above equations were determined with care, and are as follows:

$$\begin{aligned} OO' &= 22{\cdot}5 \text{ feet}, \quad \sin I = {\cdot}0146, \quad \sin i = {\cdot}050\,; \\ O'X' &= 11{\cdot}25 \text{ feet}, \quad \cos I = {\cdot}999, \quad \cos i = {\cdot}998. \end{aligned}$$

The force down the inclined plane, allowing 10 lbs. per ton for friction, resistance of air, &c., is consequently,

$$f = {\cdot}4651 \text{ ft.}$$

The original measurements and calculated results are all given in the following tables, in which I have not suppressed a single experiment, although one of them is undoubtedly erroneous. The evidence on which I was induced to adopt the form (4) is completely given, and an opportunity thus offered for another interpretation of the experiments, although the accuracy with which equation (4) represents the results seems to preclude the possibility of an interpretation differing much from that which is here given.

FIRST SERIES.

January 25, 1850.—*Day damp and wet.*

	cwt.	qrs.	lbs.
Weight of waggon =	99	0	0.
Weight of sledge, =	4	3	21.

TABLE (1).—ORIGINAL MEASUREMENTS.

No.	Time.	Slide.		Observations.
	Secs.	Ft.	In.	
1	9·2	7	6	
2	17·0	4	$1\frac{1}{2}$	First explosion off hind wheel. Interval between wheels, 7ft. 6in.
3	10·4	6	$6\frac{1}{4}$	
4	9·2	7	0	
5	11·2	5	4	
6	11·0	5	$7\frac{1}{4}$	Explosion off hind wheel.
7	10·2	5	8	
8	9·4	6	9	

The first and second of these experiments were performed by placing two fog signals on the rails at O and O″, distant from each other 100 feet, and measuring with a chronograph the interval of time between the explosions, as registered in the first column; the remaining experiments were performed with one fog signal placed at O, the time of the front wheel passing O″ being observed by sight. The waggon was impelled by a number of men, who ceased pushing it before its arrival at O″.

TABLE (2).—CALCULATED RESULTS.

No.	v'.	$\sqrt{s}$.	v.	u.	$\frac{v}{\sqrt{s}}$	$\frac{uv}{\sqrt{s}}$
1	13·413	2·738	12·775	31·496	4·6658	146·954
2	9·947	2·031	9·474	31·661	4·6647	147·689
3	12·470	2·553	11·877	31·541	4·6521	146·734
4	13·413	2·645	12·775	31·496	4·8298	152·121
5	11·987	2·309	11·417	31·564	4·9445	156·070
6	12·479	2·367	11·886	31·540	5·0215	158·379
7	12·607	2·380	12·008	31·534	5·0453	159·101
8	13·234	2·598	12·605	31·504	4·8518	152·851

In this Table v' denotes the velocity of the waggon at the point X, situated 11 feet above the sledge; i. e. it is the mean of the initial and final velocities of the shock; it is computed from the velocity at O, by adding to its square the square of the velocity due to the space OX, and extracting the square root of the sum. The velocity at O is found from the formula

$$v'' = \frac{200 + {\cdot}4651\, t^2}{2t},$$

in which t denotes the time of describing OO″, in Table (1). In Exp. (2) we must substitute 185 for 200 in the formula; and in Exp. (6), 215 instead of 200, adding, in the latter case, in order to compute v', the square of the velocity due to 4 feet instead of 11·25 feet

In the second column, $\sqrt{s}$ is found from the second column of Table (1).

In the third column, $v = {\cdot}9525\, v'$; the numerical coefficient being deduced from the weights of the waggon and sledge.

In the fourth column, $u = 32{\cdot}134 - {\cdot}05\, v$; the coefficients being deduced from the equation which defines u (p. 222).

Second Series.

January 26, 1850.—*Wet day.*

Weights of waggon and sledge same as before.

Table (1).—Original Measurements.

No.	Time.	Slide.		Observations.
	Secs.	Ft.	In.	
1	11·0	6	$3\frac{1}{2}$	
2	11·8	6	$3\frac{3}{4}$	
3	13·6	5	$6\frac{1}{8}$	
4	17·2	4	2	
5	19·0	3	$11\frac{1}{2}$	Explosion off hind wheel.
6	17·4	3	$11\frac{1}{4}$	
7	17·8	3	$4\frac{1}{8}$	
8	17·4	3	$2\frac{5}{8}$	
9	18·4	3	$1\frac{1}{8}$	
10	8·8	9	$0\frac{7}{8}$	
11	8·4	11	$6\frac{3}{4}$	
12	8·4	10	9	

These experiments were tried in a manner similar to the last six experiments of the first series.

TABLE (2).—CALCULATED RESULTS.

No.	v'.	$\sqrt{s}$.	v.	u.	$\frac{v}{\sqrt{s}}$	$\frac{uv}{\sqrt{s}}$
1	12·112	2·508	11·536	31·558	4·5996	145·157
2	11·685	2·512	11·130	31·578	4·4307	139·914
3	11·012	2·347	10·489	31·610	4·4691	141·268
4	10·344	2·041	9·852	31·641	4·8270	152·733
5	10·256	1·989	9·769	31·646	4·9115	155·430
6	10·325	1·984	9·834	31·643	4·9566	156·843
7	10·290	1·828	9·801	31·644	5·3616	169·662
8	10·325	1·794	9.834	31·643	5·4816	173·454
9	10·249	1·758	9.762	31·646	5·5529	175·727
10	13·803	3·012	13·147	31·477	4·3648	137·393
11	14·238	3·400	13·561	31·456	3·9885	125·463
12	14·238	3·278	13·561	31·456	4·1370	130·132

The quantities here tabulated are the same as in the first series of experiments; and in calculating v' for Exp. 5, the same method was followed as in Exp. 6 of first series.

THIRD SERIES.

February 23, 1850.—*Fine day.*

Weights of waggon and sledge same as before.

The mode of estimating the velocity on this day was to allow the waggon to run down the inclined plane a measured distance, and thence compute the velocity of the impact.

TABLE (1).—ORIGINAL MEASUREMENTS.

No.		OO″.	Slide.	No.		OO″.	Slide.
		Ft.	In.			Ft.	In.
I.	1	25	8·80	III.	1	75	32·45
	2	,,	9·40		2	,,	33·80
	3	,,	10·60		3	,,	33·15
	4	,,	11·25	IV.	1	100	44·70
	5	,,	11·25		2	,,	46·00
II.	1	50	22·50		3	,,	46·75
	2	,,	22·85		4	,,	47·60
	3	,,	23·10		5	,,	47·50
	4	,,	22·60				

TABLE (2).—CALCULATED RESULTS.

No.	v'.	$\sqrt{s}$.	v.	u.	$\frac{v}{\sqrt{s}}$	$\frac{uv}{\sqrt{s}}$
I.	5·827	·924	5·550	31·857	6·0065	191·349
II.	7·563	1·377	7·203	31·774	5·2309	166·208
III.	8·970	1·661	8·544	31·707	5·1439	163·097
IV.	10·184	1·968	9·700	31·649	4·9288	155·994

The velocity v' at the point X is found from the formula

$$v'^2 = 2fs = \cdot 9302\ s,$$

s denoting OO″; and from v' and s the remaining quantities are calculated as before.

FOURTH SERIES.

March 5, 1850.—*Fine day.*

	tons.	cwts.	qrs.	lbs.
Weight of waggon =	2	4	3	8.
Weight of sledge =		4	3	21.

The velocity of impact was determined as in third series.

TABLE (1).—ORIGINAL MEASUREMENTS.

No.		OO″.	Slide.	No.		OO″.	Slide.
		Ft.	In.			Ft.	In.
I.	1	25	5·6	III.	1	75	23·1
	2	,,	6·025		2	,,	23·75
	3	,,	5·75		3	,,	24·0
	4	,,	5·55				
II.	1	50	14·6	IV.	1	100	32·5
	2	,,	15·2		2	,,	27·2
	3	,,	15·35		3	,,	30·1
	4	,,	15·7		4	,,	28·75

Note.—The point O in these experiments is 20 feet above the sledge.

TABLE (2).—CALCULATED RESULTS.

No.	v'.	$\sqrt{s}$.	v.	u.	$\frac{v}{\sqrt{s}}$	$\frac{uv}{\sqrt{s}}$
I.	5·623	·692	5·061	31·881	7·3136	233·164
II.	7·408	1·125	6·668	31·801	5·9244	188·488
III.	8·839	1·407	7·955	31·737	5·6538	179·437
IV.	10·069	1·571	9·062	31·681	5·7683	182·746

In this Table v is calculated by the formula

$$v = \cdot 9v';$$

the other quantities are deduced in the same manner as in the other experiments.

Having obtained the results calculated from the foregoing series of experiments, I had recourse to equation (4),

$$\frac{v}{\sqrt{s}} = A + K \sec i \, \frac{uv}{\sqrt{s}},$$

which contains two unknown quantities, A and $K \sec i$. Having found approximate values of these quantities in each of the four series of experiments, I then, by trial of successive numbers extending to the fourth decimal place, obtained finally the following results, which give the values of $K \sec i$ and A which best satisfy all the experiments.

FIRST SERIES.

$K \sec i$ = ·0313.	A =	1°	·0662
		2°	·0420
		3°	·0594
		4°	·0685
		5°	·0596
		6°	·0643
		7°	·0655
		8°	·0675
		Mean =	·0644

Rejecting the second experiment, which is manifestly erroneous.

SECOND SERIES.

$K \sec i = \cdot 0311.$	$A =$	1°	·0843	7°	·0851
		2°	·0794	8°	·0873
		3°	·0757	9°	·0879
		4°	·0771	10°	·0920
		5°	·0777	11°	·0867
		6°	·0788	12°	0900
				Mean =	·0835

THIRD SERIES.

$K \sec i = \cdot 0305.$	$A =$	1°	·1704
		2°	·1616
		3°	·1695
		4°	·1710
		Mean =	·1681

FOURTH SERIES.

$K \sec i = \cdot 0307.$	$A =$	1°	·1555
		2°	·1379
		3°	·1451
		4°	·1580
		Mean =	·1491

Referring to p. 222, we find

$$A \cos i = \sqrt{\{2g(k \cos I - \sin I)\}}.$$

Hence,

$$\frac{(A \cos i)^2}{2g} + \sin I = k \cos I;$$

or,

$$k = \cdot 0154\ A^2 + \cdot 0146.$$

Computing the four values of k from this formula, we obtain finally, for the two coefficients,

$$\begin{array}{ll} (1)\ K = \cdot 0313 & (1)\ k = \cdot 0147 \\ (2)\ K = \cdot 0311 & (2)\ k = \cdot 0147 \\ (3)\ K = \cdot 0305 & (3)\ k = \cdot 0150 \\ (4)\ K = \cdot 0307 & (4)\ k = \cdot 0149 \end{array} \qquad (5)$$

In order to appreciate the degree of accuracy obtained in these experiments, let us transform equation (4) into the following:

$$\frac{v}{\sqrt{s}} - A - K \sec i \frac{uv}{\sqrt{s}} = 0;$$

and substitute in its left hand side the values of $K \sec i$ and mean values of A found above. The results of this substitution for the four series of experiments are as follows, in which Φ denotes the left hand side of the equation just given.

FIRST SERIES.

$\Phi =$	1°	+ ·0018
	3°	− ·0050
	4°	+ ·0041
	5°	− ·0048
	6°	− ·0001
	7°	+ ·0011
	8°	+ ·0031

SECOND SERIES.

$\Phi =$	1°	+ ·0008	7°	+ ·0016
	2°	− ·0041	8°	+ ·0038
	3°	− ·0078	9°	+ ·0044
	4°	− ·0064	10°	+ ·0085
	5°	− ·0058	11°	+ ·0032
	6°	− ·0047	12°	+ ·0065

THIRD SERIES.

$\Phi =$	1°	+ ·0023
	2°	− ·0065
	3°	+ ·0014
	4°	+ ·0029

FOURTH SERIES.

$\Phi =$	1°	+ ·0064
	2°	− ·0112
	3°	− ·0040
	4°	+ ·0089

It is evident, on inspection of the foregoing results, that $\Phi = 0$ is true to *two* places of decimals; and it is not difficult to prove, that it may be inferred from this, that the values of K and k (5) are true to *three* places of decimals.

From the remarkable agreement of all these results, I think we are entitled to assume, that equation (4), or its equivalent $\Phi = 0$, is fully established.

In order to determine the lowest velocities for which equation (4) might be considered proved, I added to the third and fourth series of experiments a set of experiments, the object of which was to ascertain the point from which the waggon should be allowed to descend, so as just to move the Friction Sledge. The result of these trials was as follows:

	v'.	$\sqrt{s}$.	v.	u.	$\frac{v}{\sqrt{s}}$	$\frac{uv}{\sqrt{s}}$
Third day,	3·240	·142	3·078	31·88	21·6760	691·0308
Fourth day,	3·205	·110	2·885	31·99	26·2273	839·0017

Substituting these expressions in equation (4), and replacing A by ·1681 and ·1491 on the third and fourth days respectively, we obtain

$$K = \cdot 0311, \qquad K = \cdot 0310.$$

The agreement of these results with (5) is sufficient to prove, that the equation from which they are deduced is true for velocities as low as 3 feet per second.

The experimental truth of equation (4) being thus established, the results obtained from the foregoing series of experiments may be briefly summed up as follows:

1st. The momentum possessed by the waggon at the moment of impact on the Sledge is destroyed by two causes, which may be considered separately: first. the loss of momentum occasioned by the shock itself, and the adhesion of the Sledge to the rails; this I have called the *Friction of Rest:* secondly, the loss of momentum which takes place after the Sledge is set in motion, caused by friction against the rails; this is the *Friction of Motion.*

2nd. The friction of rest is directly proportional to the pressure against the rails, and to the velocity of impact, jointly.

3rd. The friction of motion is directly proportional to the pressure against the rails, and is independent of the velocity.

X.—*On certain Improvements in the Construction of Galvanometers: on Galvanometers in general: and on a new Instrument for measuring the relative Force of Magnetism in compound Needles intended to be nearly Astatic.* By MICHAEL DONOVAN, Esq.

Read May 22, 1848.

THE galvanometer, in the present day, has become a most important instrument of research, whether it be considered as a measure of electricity or of heat. In the latter capacity, it exceeds all others in sensibility and the promptness of its indications; and when it is recollected that by its aid facts have been ascertained which had been erroneously represented by the thermometer, that degrees of heat have been estimated to which the thermometer was in some cases almost insensible, and in others inapplicable, its value and capabilities need no encomium. But it is necessary that, for delicate purposes, we should have the instrument in its utmost attainable state of perfection. Under such impressions, I have made some efforts to improve it; and believe I have succeeded in rendering it more certain and accurate in its results, as well as more sensible in its indications.

The sensibility of a galvanometer is increased when its construction is such, that the two layers of the wire coil, between which the compound or double needle lies, are as near as possible to each other, the consequence being that the lower bar of the compound needle will be very close to both. The upper bar should be equally near the upper layer of the coil. The proximity of the bars to the coil is one of the great sources of sensibility; but, to permit this, many things must be attended to.

In the first place, however carefully the wire of the coil may have been covered with silk or cotton, there will always be a number of fibres projecting from the material. They are short it is true; but when the bars of the needle are very close to the three acting faces of the coil, these fibres are sometimes sufficiently long, resisting, and numerous, to obstruct the motions of the compound needle when in a state of great sensibility. The following plan succeeded in removing this source of uncertainty.

The wire, well covered with silk, is wound on a brass frame, each round so tight and close to the adjoining one, that the first layer may lie perfectly flat, without springing or bellying in any part. When the first layer has been wound from the beginning to the end of the frame, it is to be wound back again; and if the rounds of the first layer have been put on very compactly against each other, they will support the second layer without allowing the wire to force itself between any two which lie underneath. The flatness of the first layer will insure the same perfection for the second; and equal attention to tightness and flatness in winding the third layer, or fourth, if there be one, will produce a coil of great regularity and closeness, and of equal thickness,—qualities of the greatest utility.

Under these circumstances, it is obvious that the frame must be strong enough to sustain the collective tension of such a number of coils; and therefore brass, well covered in the touching parts with hard varnish, is to be preferred to ivory; although the latter is often used on account of its being a non-conductor. The four sides, which constitute the parallelogram on which the coil is wound, would not, without being inconveniently clumsy, afford sufficient resistance to this tension, but that the two end pieces of the frame, instead of being mere bars like the side pieces, constitute portions of a circle at the inner side, and are flat at the outer.

From what has been described, it is evident that the vacancy between the layers of the coil being but one-eighth of an inch, the numerous filaments projecting from the silk or cotton must be even more likely than in ordinary cases to obstruct the needle: and the very means adopted for securing sensibility would also react against that result, but for the following construction. Two plates of very thin, well-hammered brass are soldered to the two arches in such a manner as to connect them, and constitute what may be called a floor and a ceiling to the narrow chamber in which the lower bar of the compound needle rotates, thus effectually removing all possibility of obstruction by filaments, and greatly adding to the strength of the frame. The coil is wound round the outside of the frame and its brass plates, leaving the internal circular chamber of brass for the uninterrupted oscillations of the enclosed needle. Its vertical width is reduced by the two brass plates to one-eighth of an inch.

But the upper bar of the needle would still be liable to obstructions from filaments of the upper layer of coil, were it not that a circular plate of well-hammered silvered brass lies loosely on it, and exactly fills up the interior of the circle on which the graduation is engraved.

In order to permit the compound needle to be placed within, and removed from its berth in the chamber, as occasion may require, there is a slit along the middle of what I have (for want of a better name) called its ceiling, so narrow as barely to admit the lower bar of the needle. To keep the wire of the coil and its filaments in situation, without encroaching on this slit, its edges are guarded all round by an elevated margin* of very thin sheet brass, which stands exactly as high as the thickness of the coil. The circular silvered brass plate above mentioned, although described as one piece, really consists of two semicircles, the diametrical junction of which is so accurately fitted, that it appears as a straight engraved line upon one circular plate: it represents the magnetic meridian of the galvanometer. When the needle is to be removed, the semicircles are easily pushed up from below. Between the semicircles, and in their common centre, there is a small hole, which permits the spindle or connecting axis of the two bars of the compound needle to play freely. In the top of this spindle is a very small hole, barely large enough to receive the silk fibre which sustains the compound needle. Thus the two bars, which constitute the compound needle, are brought as near as possible to the three faces of the coil, without risk of obstruction. The chamber in which one bar of the needle rotates may be as vertically narrow as will allow free motion; one-eighth of an inch will be sufficient. In so narrow a space, it is obvious that any deviation from the horizontal position of the coil and frame would cause obstruction to the movements of the needle: the same would happen also if the needle were not accurately balanced on its spindle, and therefore on the suspending fibre of silk. The

* The coil must be wound as closely as possible against this margin on both sides; for here the energy of a voltaic current is most required to overcome the inertia of the needle.

horizontal portion is attainable by means of a detached spirit level and levelling screws; and the balance of the needle is regulated by a very small slider made of brass foil. It will be often found necessary to have a slider even on each member of the compound needle; for as the intensity of its magnetism is sometimes stronger, sometimes weaker, in consequence of spontaneous or induced changes, the dip will affect it more or less; and hence the necessity of the sliding equipoise to maintain the horizontal position in so narrow a chamber. The second slider becomes necessary when the poles are reversed in the manner hereafter described.

The level and levelling screws afford the means not only of rendering the plane of the chamber perfectly horizontal, but of giving true verticity to the pillar from the cross-bar of which the needle hangs. Without such a precaution the spindle of the needle would not freely rotate in the small hole made for it in the meridian line of the circular plate, or rather between the two semicircular plates.

To give greater precision to the centrality of this spindle, the hole in its top is made exceedingly small, and through it is looped the suspending silk fibre, the other end of which is looped through an equally small hole in a pin which may be secured by a thumb-screw, in the cleft end of a horizontal cross-bar, moveable in every direction at the top of the pillar. The use of having the holes so small is to secure the silk from shifting, and thus altering the balance or position of the needle. The length of silk fibre which I conceive to be sufficient is $7\frac{1}{2}$ inches, exclusive of what is looped above and below.

When occasion requires the compound needle to be removed from the galvanometer, the thumb-screw which screws against the pin in the cleft of the cross-bar at the top of the pillar is to be loosed; the pin with the silk fibre is to be drawn out; the two semicircular plates which lie within the graduated circle are to be pushed up from below, and the needle withdrawn. This facility of removing and replacing the needle is a great advantage when its magnetism requires regulation; and beside this the silk fibre often becomes twisted by the rapid spinning of the needle which certain experiments occasion and even require: it is prevented from untwisting by the polarity of the needle; but if the needle be held between the fingers while the pin hangs down by its fibre of silk, the latter will untwist itself; and when the pin ceases to spin, the whole may be replaced. Torsion, even of so slender a fibre, has more effect in impairing sensibility than might be supposed.

The frame of the coil is screwed on a strong circular plate of brass, surrounded by a hoop sufficient to secure the French shade which covers the whole. The solid vertical axis of this plate turns smoothly and slowly in a ground socket, by means of a crown wheel fixed to the plate, acted on by a pinion and thumb-screw attached to the tripod on which the whole instrument stands. The levelling screws pass through the tripod, and rest in so many holes in a circle of brass screwed to a mahogany stand, in the bottom of which should be a drawer to contain various necessaries. On the stand fits a mahogany cover or box, with fastenings. The ends of the coil pass down through two holes in the circular brass plate, bushed with ivory, and are soldered underneath the plate, each to an insulated binding screw, situated one on each side of the meridian line. Between the binding screws, and affixed to the circular brass plate, is a horizontal, hollow, square trunk, three inches in length: each side of the interior measures about three-sixteenths of an inch: it is fixed in the direction of the magnetic meridian line of the galvanometer, and has a continuation of this line engraved on its upper surface. At the end of the trunk, and on the meridian line, is erected a short, sharp brass point, on which may be occasionally placed a common compass needle. When the galvanometer is to be used, the compass needle is to be placed on the brass point, and the thumb-screw underneath turned until the point of the compass needle and the meridian line coincide precisely. The instrument is now set; the compass needle is to be removed, lest it interfere with the astatic needle: the latter should also have been previously removed, as it would equally interfere with the compass needle. The necessity of thus setting the instrument by a detached compass needle, and of not depending for this service on the astatic needle, although the latter is usually relied on, will abundantly appear hereafter.

The trunk is made hollow, because it has other duties to perform. It is intended to contain one prong of a magnet made exactly in the shape of what musicians call a "tuning fork," except that the prongs are nearer together, being just the distance that the two needles are from each other on their spindle. This magnet slides in and out of the trunk, which it exactly fits; it is so placed, that one prong is always vertical over the other when it is in use.

Its duty is occasionally to assist the directive power of the terrestrial magnetic meridian, and thus to lessen the influence of deflective forces on the needle. With needles of very great sensibility, very weak deflective forces will often produce deflection = 90°, and then the galvanometer is incapable of indicating any greater effect. The magnet is graduated in inches and tenths, on both prongs, in order to regulate its proximity to the astatic needle, and therefore its influence in experiments which require repetition or correspondence. The prongs are square, as well as the trunk in which one or the other slides; and one side of the trunk has a spring which presses on the magnet and enables it to slide evenly. Were the prongs cylindrical as well as the trunk, the magnet might turn to one side or the other, and then the other prong would exercise an undue influence on the astatic needle. When the magnet is not required in an experiment, it should be drawn out and put aside at a considerable distance. One prong is marked N, the other S; sometimes one must be uppermost, sometimes the other. It is easy to see how it should be placed in order to moderate the effect of a deflective force on the needle.

It is usual to graduate the circle from 0 to 90°, on each side of the magnetic meridian line. There is an inconvenience in this mode: in some experiments, the voltaic action, which is the cause of the deflection, exists but for an instant; there is no permanent effect to measure, nor any effect beyond one sudden start of the needle, and its immediate return. Yet sometimes the momentum is such as to carry the needle far beyond 90°, and even to whirl it round several times. I think it more convenient that the graduation should be carried to 180°.

The next thing to be considered in the construction of a galvanometer is the compound or so-called astatic needle, a subject of great importance and curious interest. A needle may be defective in two chief points: it may have too little sensibility, by having a strong directive tendency; or it may have too much, by having been brought too near the state of perfect astaticism: in the latter case, a weak deflecting force will produce a maximum deflection, and the needle will be insensible to all higher degrees of energy.

To avoid these imperfections, I employ the following construction, having discarded the common compound needle, which consists of two bars *permanently fixed* to one spindle. The lower bar of the needle is secured, in the usual manner, to the lower end of the spindle; but the upper one is moveable on the upper end of the spindle, and turns round on it as on an axis: it may be secured in any position by means of a minute nut which screws it against a shoulder on the spindle. Yet the total weight is trifling. One of my compound needles, thus constructed, weighs but $2\frac{1}{4}$ grains, including the nut; its bars are distant from each other $\frac{1}{5}$ of an inch; each is $2\frac{5}{6}$ inches in length, and $\frac{1}{24}$ inch in width; their sides are parallel. The best material for the bars is the mainspring of a small Geneva watch; it is very thin, and retains magnetism a long time. When saturated with magnetism, it no doubt dissipates a portion in a few days; but it retains about one-third for any length of time, and this is sufficient for ordinary purposes. It is obvious that the bars must be filed perfectly straight, and placed parallel to each other.

The following is the process which I employ for magnetizing the compound needle when great sensibility is not required. I caused a horse-shoe magnet of considerable power to be made, the limbs of which, from one extremity of the two polar faces to the other, are of such extent and thickness that the compound needle intended to be magnetized can rest its whole length on the faces. The limbs of the horse-shoe should approach each other within half an inch. In this way the compound needle will rest one edge of each bar on the terminal faces of the magnet, being placed exactly between the two limbs, and in the same situation that its keeper generally occupies. The compound needle will become saturated with magnetism in four hours. Before it is removed, the keeper of the horse-shoe should be put on laterally, close to the extremities, to facilitate the removal.

The needle is now capable of two different applications. If it be intended to measure powerful deflective forces, it is ready for that purpose, so far as its state of magnetism is concerned. But as the two north poles are together, the needle has acquired a dip, and it is necessary to balance this exactly by the counterpoise sliders already mentioned. The needle, when properly placed in the galvanometer, being not astatic, it will give short, quick oscillations, will require no small voltaic energy to deflect it to 90°, and will return to zero with great precision and promptitude. It is now in its least sensible form.

Should greater sensibility be required, we have only to turn the upper bar round on its axis until it be precisely reversed, and to remove the counterpoise towards the centre. To reverse the bars with precision is no easy matter, as will be seen hereafter.

It will be found very convenient thus to have a needle susceptible of a strong directive tendency by causing the poles to act in concert, or of greater sensibility by their acting in opposition; for in this way, and with the occasional aid of the forked magnet, a great range of voltaic forces may be measured. With ordinary galvanometers, if the sensibility be great, a very feeble deflecting force carries the needle to its maximum, and no greater force can be estimated.

When the bars have been reversed, as described above, the instrument will be found to possess the usual sensibility of astatic needles; but not a sufficiency for all purposes, especially when it is intended as a measure of heat. In this case, other proceedings must be resorted to, which, after a few preliminary observations, I will describe.

During some experiments on different modes of communicating magnetism to needles, results were obtained which I am not aware have been observed by others, or made the subject of inquiry. I sometimes produced a compound needle which found its position directly across the magnetic meridian, instead of coinciding with it. As often as it was moved into the right position, it would return to the wrong one; and the more care I took to insure an equally distributed magnetism, the more certainly would this perplexing anomaly recur. It would be useless to trouble the Academy with all the particulars; it will suffice to say that, in order to investigate the cause of these failures, and provide a remedy, I was obliged to contrive a new instrument. As it would be difficult to refer to its employment without giving it a name, I will here call it a volta-magnetometer, the prefix being sufficient to distinguish it from the magnetometer used for a different purpose.

The volta-magnetometer consists of a horizontal brass graduated circle, or ring, fixed directly over and parallel to a circular brass plate of the same total diameter. Their distance from each other, maintained by three stout brass studs, is a quarter of an inch, sufficient to allow free space for the oscillation of the compound needle of the galvanometer, which is to be transferred to it as occasion may require, one bar of the needle lying above the graduated ring, and the other between the latter and the circular brass plate. On the circular brass plate is engraved a circle, corresponding with that of the upper graduated ring: both circles are graduated and numbered with such precision, that each degree on the upper circle is exactly vertical to the corresponding one underneath. The degrees on the lower circle are carried a little farther in towards the centre than those on the upper circle, to enable the observer to take the two corresponding degrees in his view, along with the interposed points of the needle, when his eye is directed vertically over the four objects, so that they shall all coincide. As a further facility, the numbers are engraved *inside* the circle. The instrument stands on levelling screws, which are received in holes in a brass ring screwed to a wooden stand; and the whole is covered by a French shade. A stop which acts underneath the stand checks tedious oscillations.

From one of the three studs which support the upper circle, proceed horizontally two long brass blades hinged to it in such a manner that they can be made to approach towards or recede from each other. When brought together, they fit closely by their straight edges, and constitute a kind of forceps capable, by its long handles, of firmly grasping the spindle of the compound needle when it is suspended in the centre of the graduated circles. By means of a sliding clip, they can be retained in this position. The silk fibre, which sustains the compound needle, is suspended from a pin in a cross bar at the top of a pillar, in the same manner as in the galvanometer, and is similarly circumstanced in all respects with regard to the adjustments which have been already mentioned. A compound needle, with its silk fibre and pin, may thus be transferred from one instrument to the other, and will fit exactly in either.

Experiments made with the volta-magnetometer soon convinced me that the apparently anomalous position assumed by the needle, which I had taken so much pains to render astatic, and which nevertheless stood perpendicularly to the magnetic meridian, was in strict accordance with the circumstances under which it was placed. To explain the matter, it is necessary to advert to what the properties of a compound needle would be, if, as its name expresses, it were really astatic. If the four poles of the needle, supposed to be perfectly similar in magnetic power, be placed in the same vertical plane, with the synonymous poles contiguous to each other, the directive power will be at its maximum. If in this state of things the bars be opened out, no matter what the angles subtended by them may be, the law of the parallelogram of forces comes into operation: the compound needle will take such a direction, that the resultants of the parallelograms, two sides of which are the intersecting magnetic axes, will bisect the vertical angles which include the magnetic meridian, and will, therefore, coincide with it. The resultants will represent the intensity of the directive forces of the compound needle, and in reasoning on the subject may be substituted for them. In proportion as the bars of the needle are opened, and the vertical angles which include the magnetic meridian are enlarged in consequence, the resultants or directive forces diminish in energy, until at length the bars are entirely opened and lie with their dissimilar poles contiguous, exactly in the same vertical plane: the resultants then vanish; there is no longer any directive force; and consequently the needle will remain in any position.

This state of indifference depends on the condition laid down, that the four poles possess exactly the same intensity and distribution of magnetism, and that the bars lie precisely in the same vertical plane with the synonymous poles contiguous to each other. If this position of the bars be ever so little disturbed, by turning one of them on the common axis away from the other, even to the amount of one or two degrees, the resultants begin to exert their influence, weakly, it is true, but sufficiently to cause the compound needle, previously indifferent, to take up a position at right angles with the magnetic meridian, because the resultants coincide with it. In that position the needle will permanently remain, and if disturbed, will, after a few oscillations, return to it. Thus the apparently anomalous phenomenon, which surprised me because it had never been previously observed, is very easily understood.

If, instead of equal distribution of magnetism, the power of one bar of the compound needle exceed that of the other, the needle, instead of becoming indifferent when both bars are brought into the same vertical plane, will obey the predominant power of the stronger bar, and pass at once into the magnetic meridian. Indeed the tendency to do so may be exhibited before the two bars are brought into the same vertical plane; the resultants, still feebly in operation, may more or less antagonize the predominant power of the stronger bar; a balance of the two forces will take place; the compound needle will take up a position nearer to or farther from the magnetic meridian, according to the degree of resistance which the resultants offer: but if the bars be brought into the same vertical plane, the resultants will be eventually overpowered.

In order to bring the compound needle to a right angle with the magnetic meridian, the deviation of its bars from the vertical plane must be more or less according to the greater or less predominance of magnetism in one of them: and the more equal the distribution of magnetism is amongst the four poles, the nearer to the same vertical plane may the bars be brought without causing them to swerve from the right angle at which they stand with the magnetic meridian; although, when they are precisely in the same vertical plane, the needle loses all tendency to that or any other position.

The condition of greatest sensibility is that in which the resultant is barely so far overpowered by the predominant magnetism of one bar, that the needle turns very slowly into the magnetic meridian from being at right angles with it, and in passing through 180° occupies from 30 to 36 seconds.

There is sometimes great difficulty in obtaining the results described: the magnetic axes may not coincide with the metrical axes, so that when the bars appear to be in the same vertical plane, the magnetic axes are not so: the true poles may not be equidistant from the centres of the bars: the magnetism may be irregularly distributed, owing to peculiarities in the steel; and all these circumstances may combine. Empirical trials are, in such cases, the only resources; and patience will insure success.

The volta-magnetometer is capable of imparting sensibility to galvanometer needles in two ways: first, by affording means of distributing the total quantity of magnetism between the four poles in a degree so nearly equal, that the feeblest directive power only will remain, and thus the least resistance will be offered to weak deflecting forces. Secondly, by causing the needle to assume a true position with regard to the magnetic meridian, and thus enabling it to give a *just* estimate of a deflecting force acting on it, which, as we have seen, it does not always present. By means of this instrument, we can prepare needles of any degree of directive power, and can describe that degree in giving an account of experiments. Results obtained by persons at a distance from each other may be compared, when the sensibility of the compound needle made use of is expressed numerically: it may be so expressed by stating the number of degrees by which the magnetism of the two bars of the compound needle differ, when tested on the graduated circles of the volta-magnetometer: this will very nearly give the sensibility, but absolute precision cannot be attained.

I have already described the process of magnetization by which the compound needle acquires sufficient sensibility for ordinary purposes; but if required to be in its condition of greatest sensibility, we must proceed as follows. It is to be transferred to the volta-magnetometer, and its spindle or common axis is to be confined between the blades of the forceps. The upper bar is to be turned on the common axis until it form any angle with the lower one, suppose 80°: the forceps being opened, the needle will oscillate, and finally settle in such a position, as will show the relative intensity of the poles by the degrees pointed to on the graduated circles. If the magnetism be equal, the four poles will stand at 40°. But it has been already shown, that this equality may be but apparent, owing to the want of coincidence between the magnetic and metrical axes. In order to test this, the bars are to be closed until they fall into the same vertical plane, the poles being reversed. If the compound needle, when liberated, after oscillating a while, turn very slowly into the magnetic meridian, no more need be done; for although some little irregularity in the distribution of the magnetism is thus manifested, the desired effect is obtained. If the needle do not turn into the magnetic meridian, the bars are to be again opened to an angle of 80°, one of the poles is to be touched with the opposite pole of a strongly magnetized steel wire, or sewing needle, until there be a difference of 1° on the indications of the bars. This, or less, will be sufficient difference, when the bars are closed, to carry the compound needle into the magnetic meridian. Should the difference be more than 1°, that bar nearer the magnetic meridian line should be touched with the similar pole of the magnetized wire, and by lessening or increasing its power, the difference of 1° may be attained. The spindle being then caught in the forceps, the upper bar of the needle is to be turned round on the common axis, until the reversed poles appear in the same vertical plane. But a want of precise coincidence in this respect, between the reversed poles of the bars, to an amount often undiscoverable by the eye, will cause the compound needle to lie at right angles with the magnetic meridian. Hence, to produce perfect coincidence, the eye must be assisted by a magnifier, and the compound needle must be executed with great precision so that the bars shall be straight, and identical in all their dimensions. The mode of viewing the needle is of great consequence. The eye must be placed in such a situation that the four objects concerned shall be seen in the same vertical line; namely, the degree on the lower circle, the point of the lower bar, the corresponding degree on the upper circle, and the point of the upper bar. The needle being adjusted, the spindle is to be disengaged from the forceps. If the reversed poles precisely coincide in the same vertical plane, the needle will oscillate, and finally settle in the magnetic meridian, provided that it retains sufficient directive tendency. If the needle do not lie directly north and south, other trials must be made, and generally the process of perfect adjustment is tedious and troublesome.

It sometimes happens, as already observed, that the magnetism of the bars is equal, and the needle has no directive power. In such a case, the slightest touch of a magnetized wire to any one of the poles of the compound needle will increase or lessen its power, and thus alter the balance: the slighter the alteration the better.

The predominant magnetism should be so feeble, that the needle will very slowly fall into the magnetic meridian; but the predominance should be adequate to produce that effect. If the bars be not precisely in the same vertical plane, they will have a weak tendency to cross the magnetic meridian: but having also a directive power, they will be acted upon by the two forces, and will point at the degree on the graduated circles which expresses the balance of forces. In this state the needle is unfit for service; for although the deflection from the magnetic meridian thus produced may amount to a few degrees only, it will resist a weak tendency to deflection in the opposite direction, when the galvanometer is employed to measure voltaic action, and may modify even a stronger one. Thus the condition of the needle most conducive to sensibility is that of being retained *precisely* in the magnetic meridian, with the feeblest predominance of magnetism in one bar, which is adequate to this effect.

A needle is frequently found to lie in the magnetic meridian, not because its bars coincide with precision in the same vertical plane, for perhaps they do not at the time, but because the magnetism of one or the other bar is so strong as to overpower the great error which would have arisen in consequence. The volta-magnetometer will detect the offending bar, by the inequality of the angles to which the poles will point on the graduated circles when the bars are opened out, and it may be weakened to the necessary degree by the similar pole of the magnetized wire. Needles which point truly north and south, on account of this inordinate predominance of power in one bar, are deficient in sensibility; and when a needle has extreme sensibility, it may be that it points erroneously for want of sufficient predominance.

In this state of extreme sensibility the needle is subject to making unaccountable excursions, amounting to 5°, 10°, or 12° east or west. On one occasion, when I had suspended a needle by a new silk fibre, it hung for that day exactly north and south. Next morning, at five o'clock, it was found 8°; at nine o'clock it was 0, and so remained all day. These changes recurred every day, about the same hours, during the month of April. On some days, the excursion and return took place twice. About the fifth week, the needle stood constantly at 8° for two days; it then shifted to 10° in the opposite direction, but returned to 0 in the middle of the day. I endeavoured to trace these changes to torsion by an hygrometric quality of the silk, or to alterations of temperature, but could not come to any certain conclusion, although I still attribute them to either or both of these causes, knowing no other that could operate.

Be this as it may, these variations in the direction of the needle would be a source of false estimation of any deflective force, were the galvanometer planted according to the indications of a needle thus in error: and variations will more certainly occur, the greater the sensibility of the needle. The remedy, however, is easy: the pin which sustains the silk fibre may be turned round ever until the needle point accurately; but this mode of rectification is only admissible on the condition that the position of the galvanometer has been rectified by its independent compass needle, and its magnetic meridian line: and here is another use of this needle and line; without them, what errors might in such cases be committed.

I have frequently found, that when the compound needle was adjusted exactly north and south by the volta-magnetometer, it pointed 10° or 12° differently when carried to a different room where the galvanometer was stationed to receive it: hence the rectification of the needle by the volta-magnetometer should be effected beside the galvanometer.

From what has been said, it is evident that the volta-magnetometer is only necessary when needles of exceeding sensibility are required. In ordinary cases, the method of magnetizing already described, or that commonly practised, is sufficient.

Having now brought under notice the erroneous bearing which the compound needle is apt to assume, and the possibility of its being in error, even to the amount of 90°, when the deviation of one bar from the vertical plane of the other is so small as to be undiscoverable by the eye, it is obvious how bad a guide such a needle is for determining the position of the galvanometer in order to render it fit for use. Yet, as hitherto made, we have no other means of setting the instrument in the magnetic meridian than to turn it round on its axis until the compound needle point north and south on the graduated circle. It is true that so great an error will only be possible when the needle is brought unusually near the astatic state; but it will be considerable in proportion to the sensibility attained. The magnetic meridian line and compass needle, which I have added to the galvanometer, afford a protection against this source of fallacy.

One immediate ill consequence of the error of the needle, should it exist undiscovered and uncorrected, is, that it affects those degrees of the circle which are of most value in delicate galvanometry. The first twenty degrees are in the direct ratio of the deflecting force, which no other degrees on the circle are. The effect of terrestrial magnetism is as the sine of the angle comprised between the magnetic meridian and the magnetic axis of the needle on which it acts. The sine of 20° is as nearly as possible double the sine of 10°; but beyond 20° the virtual ratio cannot be determined without submitting each particular galvanometer to an experimental investigation. A nd more than this, if the experimenter shall have taken the trouble of ascertaining the value of the degrees above the twentieth, by MELLONI's, or any other method, it will prove unavailing; for the whole scale of ratios of deflecting forces to angular deflections becomes deceptive if the magnetic meridian line of the galvanometer do not correspond with the terrestrial magnetic meridian. When the instrument is used as a galvanoscope, a small error is perhaps of little consequence, except that it causes the needle to represent deflecting forces weaker or stronger than they really are; but when it is used as a measure of heat, and for some other nice purposes, a want of coincidence between the magnetic meridian of the instrument and the terrestrial magnetic meridian would be productive of serious error. It is therefore evident how useful is the addition of the magnetic meridian line, with its point and compass needle, for setting the galvanometer due north and south: it at once detects the error of the astatic needle, if such exist.

On the subject of the coil, it may be proper to mention that, although it is often made of copper wire covered with cotton, such a covering is altogether unfit. I have observed that one of my galvanometers, which is furnished with a cotton-covered coil, although sometimes as sensible as most others, often becomes singularly otherwise. On such occasions, it will not be affected by a thermo-electric current generated by the heat of the fingers on two wires, bismuth and antimony, soldered together, which at another time would move the same needle 60° or 80°. I have not been able to connect these failures with any particular states of the weather, although such states may be the cause, acting perhaps on the hygrometric properties of the cotton. Coils covered with silk are not subject to this uncertainty in their action.

As silver is said to be the best of all conductors of electricity, it might be supposed that the wire of the coil should be of that metal. I made comparative trials of silver and copper coils, each resembling the other in every respect except the metal. I could perceive no decided difference in their effects, but imagined the copper to have some little advantage, and therefore adopted it. The difficulty of covering *pure* silver wire with silk is, as I am informed, great; and this, along with the cost of the silver, made a difference in the total cost of the galvanometer of more than £3. The advantage, if there be any, is certainly not commensurate.

I conceive that, in the generality of galvanometers, the coil and its frame are too narrow: there is certainly an advantage in having them so broad that they extend on each side nearly to the whole diameter of the graduated circle which covers them.

To conclude:—the improvements in the construction of galvanometers, here suggested, may be summed up as follow:—1. The addition of means, independent of the astatic needle (which may greatly err), for setting the instrument in the magnetic meridian. 2. The close approximation of the needles to the coil. 3. The removal of obstructions to the rotation of the needle. 4. The means of inducing in the bars of the needle the least difference of polarity that is consistent with their function. 5. A method of detecting and preventing derangement of the needle arising from forces which cause in it a tendency to stand transversely to its true position. 6. A construction of the needle which renders available the operation of a strong or a weak directive force, as may be required. 7. The introduction of a controlling graduated magnetic power for increasing or diminishing the deflecting influence of voltaic forces on the needles.

I venture to hope that these improvements, along with the several adjustments and facilities added, will render the instrument more convenient, will increase its sensibility, and contribute to the accuracy of its indications.

Some persons may conceive that the sensibility, to attain which I have taken so much trouble, is practically redundant. I can only say that I have experienced the absolute necessity of the arrangement described, during my late investigation of the laws of tribothermo-electricity, some of which would have remained unknown but for the excellence of the galvanometer made use of. I need not again refer to the delicacy required for thermometrical experiments.

NOTE.

I EXTRACT from the Proceedings of the Academy the following observations on this communication, made by the Rev. Dr. LLOYD, President:

"The President observed, that all the facts respecting the position of equilibrium of the astatic needle, to which Mr. DONOVAN had directed the attention of the Academy, and which (as far as he was aware) he has been the first to notice, were immediate consequences of theoretical laws.

"When two magnetic needles are united by a fixed vertical axis passing through their centres, and perpendicular to both, the moment of the force exerted by the earth upon them is the sum of the moments which it exerts upon each needle separately, and is, therefore,

$$X\,(M \sin u + M' \sin u');$$

in which M and M' denote the *magnetic moments* of the two needles, u and u' the angles which their *magnetic axes* make with the magnetic meridian, and X the horizontal component of the earth's magnetic force. In the state of equilibrium this moment is nothing; so that if u_0 and u_0' denote the corresponding values of u and u', there is

$$M \sin u_0 + M' \sin u_0' = 0. \qquad (1)$$

Consequently, if two lines be taken from any point of the vertical axis, parallel to the magnetic axes of the two needles, and proportional to their magnetic moments, M and M', the diagonal of the parallelogram constructed upon them must lie in the magnetic meridian, when the compound needle is at rest.

"Again, if we substitute $u = u_0 + v$, $u' = u_0' + v$, in the general expression of the statical moment, it becomes, in virtue of (1),

$$X\,(M \cos u_0 + M' \cos u_0') \sin v$$

Hence the compound needle is acted upon as a single needle, whose *magnetic axis* lies in the direction of the diagonal of the parallelogram above mentioned, and whose *magnetic moment* is

$$\mu = M \cos u_0 + M' \cos u_0'. \tag{2}$$

Accordingly, the diagonal of the parallelogram already referred to will represent in magnitude the magnetic moment of the compound needle. For, if the equations (1) and (2) be squared, and added together, and the angle contained by the magnetic axes of the two needles, $u_0' - u_0$, be denoted by α, we have

$$\mu^2 = M^2 + 2MM' \cos \alpha + M'^2. \tag{3}$$

"In the case of the *astatic* needle, $\alpha = 180 - \delta$, δ being a very small angle, and $\cos \alpha = -\cos \delta = -1 + \frac{1}{2}\delta^2$, *q.p.* whence

$$\mu^2 = (M - M')^2 + MM'\delta^2. \tag{4}$$

Accordingly, whem $M - M'$ is not a very small quantity, the second term may be neglected in comparison with the first, and $\mu = M - M'$, nearly. On the other hand, when $M - M' = 0$, we have $\mu = M\delta$.

"Returning to (1), and substituting for u'_0 its value, $u_0 + \alpha$, we have

$$\tan u_0 = \frac{-\sin \alpha}{\frac{M}{M'} + \cos \alpha}; \tag{5}$$

by which the position of the needle with respect to the magnetic meridian, when at rest, is determined. In the case of the astatic needle the preceding equation becomes

$$\tan u_0 = \frac{-M'}{M - M'} \cdot \delta \sin 1'. \tag{6}$$

From this we learn,

"1. That the tangent of the angle of deviation of the astatic needle from the magnetic meridian varies, *cæteris paribus*, as the angle δ, contained by the magnetic axes of the two component needles.

"2. That, however small that angle be, provided it be of finite magnitude, the tangent of the deviation may be rendered as great as we please, and therefore the deviation be made to approach to 90° as nearly as we please, by diminishing the difference of the moments of the two needles."

XI.—*On the Original and Actual Fluidity of the Earth and Planets.* By the Rev. SAMUEL HAUGHTON, M. A., Fellow of Trinity College, and Professor of Geology in the University of Dublin.

Read May 12, 1851.

THE communication which is here offered to the Academy contains a brief examination of the three following questions:

1st. Whether the nebular hypothesis of LAPLACE affords an explanation of the equality of the mean movements of rotation and revolution of the moon and other satellites.

2nd. Whether the evidence of the original fluidity of the earth and planets, afforded by their observed figures, is satisfactory with respect to all the planets.

3rd. Whether we possess, from the data afforded by astronomy, sufficient knowledge of the structure of the interior of the earth to enable us to draw conclusions respecting it, which are of geological value.

The answer which I have given to each of these questions is in the negative, and the object I have had in view in offering this communication will be accomplished, if it should in any way assist inquirers in estimating at their just value speculations relating to the original condition of the earth. The importance of such speculations has been, I believe, greatly overrated, and they have been too readily applied to the explanation of some geological facts, for which other and more probable causes can be assigned; such as the changes of climate which have taken place on the surface of the earth, and the increase of temperature as we descend below its surface. I have, therefore, examined these questions with the view of proving that, if we confine ourselves to the facts which we certainly know respecting the earth and planets, neither the nebular hypothesis, nor the hypothesis of the internal fluidity of the earth, is entitled to take a place in the list of positive facts.

I.—*On the Physical Cause of the Equality of the Mean Angular Movement of Revolution and Rotation of the Moon and other Satellites.*

The exact equality which exists between the mean angular motion of revolution and rotation of the moon has given rise to many investigations and speculations as to its physical cause. The French Academy of Sciences proposed as the subject for a prize essay, in 1764, the theory of the Libration of the Moon. This prize was obtained by LAGRANGE, who showed, that if there were in the beginning a very small difference between the movements of revolution and rotation of the moon, the attraction of the earth would be sufficient to establish a rigorous equality between these motions. LAPLACE, in his *Systeme du Monde*, p. 472,* has made some remarks on the physical cause of this remarkable phenomenon, which is not peculiar to the moon, but has been proved to exist in the case of the four satellites of Jupiter, and the eighth satellite of Saturn; according to him, there must have been some physical cause which first brought the difference between the angular motions of revolution and rotation of the satellites within the narrow limits, in which the attraction of the planet could establish their perfect equality; and subsequently the libration caused by the establishment of this exact equality must have been destroyed by the operation of the same cause, at least in the case of the moon, since the observations of MAYER, BOUVARD, and NICOLLET have proved that no such libration now exists in that satellite. A physical cause capable of producing both these effects, LAPLACE believed might be found in the nebular hypothesis proposed by himself to account for other remarkable phenomena of the planetary system, such as the movement of the planets in the same direction, and nearly in the same plane; the movement of the satellites in the same direction as that of the planets; the movement of rotation of these different bodies, and of the sun, in the same direction as their movement of revolution, and in planes nearly the same; and the small eccentricity of the orbits of the planets and satellites. According to this hypothesis, the moon, when existing in a gaseous condition,

* Vid. note.

would, by the powerful attraction of the earth, be forced to assume the form of an ellipsoid of unequal axes, having its longest diameter directed towards the earth; the terrestrial attraction continuing to operate in the same direction, must, according to the theory, have at length, by approximating the movements of revolution and rotation, brought their difference within the limits in which an exact equality would begin to be established; the libration produced in the greatest axis of the moon in this manner must have been subsequently destroyed by the internal friction of its particles, and thus the singular appearance produced of an exact equality between the angular motions of revolution and rotation. The illustrious author of the nebular hypothesis having thus explained by means of his theory this remarkable fact, proceeds to apply it to the explanation of the relation which exists between the mean motions of revolution of the first three satellites of Jupiter. Into this further application of the theory it is not my intention to inquire, as the facts may be accounted for by friction *ab extra*, acting on the satellites, the existence of which may be readily admitted without adopting the nebular hypothesis. But with reference to the explanation offered by LAPLACE, of the equality of the movements of revolution and rotation of the six satellites whose time of rotation has been observed, it appears to be natural to inquire, whether the explanation does not prove too much, and whether it would not apply equally well to establish the equality of the motions of revolution and rotation of the planets. Unless, in fact, it can be shown, that the physical cause assigned for the explanation of the fact relating to the satellites operated upon those bodies more powerfully than upon the planets, the explanation cannot be admitted; for it will be granted, that the attraction of the sun must, on the nebular hypothesis, have operated in the same manner on the planets, as the attraction of the latter on their satellites. The question is therefore one of degree, and not of kind, for the same cause operated in both cases. With the view of ascertaining whether the cause assigned by LAPLACE acted more powerfully on the satellites than on the planets, I have made the following calculations.

I shall commence by investigating the figure of a planet, supposed homogeneous and gaseous, revolving on its axis and round the central body in the same time and in the same plane; it is easy to prove that the conditions of equilibrium are satisfied by its surface assuming the figure of an ellipsoid with three unequal diameters, the least being the axis of revolution, and the greatest being directed towards the central body.

The components of attraction of the fluid mass upon a particle at its surface* are

$$Ax, \quad By, \quad Cz;$$

where

$$A = \frac{3Mf}{a^3} L, \quad B = \frac{3Mf}{a^3} \frac{d.\lambda L}{d\lambda}, \quad C = \frac{3Mf}{a^3} \frac{d.\lambda' L}{d\lambda'},$$

M denoting the mass of the fluid, a the least semi-axis of the ellipsoid, f the dynamical measure of attraction of two units of mass at the unit-distance,

$$L = \int_0^1 \frac{u^2 du}{\sqrt{1+\lambda^2 u^2}\sqrt{1+\lambda'^2 u^2}},$$

$$\lambda^2 = \frac{b^2 - a^2}{a^2}, \quad \lambda'^2 = \frac{c^2 - a^2}{a^2};$$

the equation of the ellipsoid being

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Let the axis of rotation be the axis of x, and the axis of y be directed towards the central body; if u denote the distance from the centre of the sun to any particle of the planet, δ the distance between the centres of the sun and planet, E the mass of the central body, and ω the angular velocity; it is easy to see that the equation of the surface, deduced from hydrostatical principles, will be

$$\left(A + \frac{fE}{u^3}\right) xdx + \left(By - \frac{fE}{\delta^2} + \frac{fE(\delta - y)}{u^3} - \omega^2 y\right) dy + \left(C - \omega^2 + \frac{fE}{u^3}\right) zdz = 0;$$

but, neglecting small quantities,

$$\frac{E}{u^3} = \frac{E}{\delta^3}\left(1 + \frac{3y}{\delta}\right), \text{ and } \frac{E}{\delta^2} - \frac{E(\delta - y)}{u^3} = \frac{2Ey}{\delta^3},$$

therefore, the equation of the surface becomes

* Vid. DUHAMEL, "Cours de Mecanique," Tom. I. p. 198.

$$\left(A+\frac{fE}{\delta^3}\right)xdx+\left(B-\frac{2fE}{\delta^3}-\omega^2\right)ydy+\left(C+\frac{fE}{\delta^3}-\omega^2\right)zdz=0.$$

Combining this with the assumed equation,

$$\frac{xdx}{a^2}+\frac{ydy}{b^2}+\frac{zdz}{c^2}=0,$$

we find the following equations of condition,

$$A+\frac{fE}{\delta^3}=(1+\lambda'^2)\left(C+\frac{fE}{\delta^3}-\omega^2\right);$$

$$A+\frac{fE}{\delta^3}=(1+\lambda^2)\left(B-\frac{2fE}{\delta^3}-\omega^2\right).$$

Substituting in these the values of A, B, C, and making

$$3\phi=\frac{E}{M}\frac{a^3}{\delta^3},\qquad 3\varepsilon=\frac{\omega^2 a^3}{fM},$$

we obtain

$$\left.\begin{aligned}L+\phi&=(1+\lambda'^2)\left(\frac{d.\lambda' L}{d\lambda'}+\phi-\varepsilon\right);\\ L+\phi&=(1+\lambda^2)\left(\frac{d.\lambda L}{d\lambda}-2\phi-\varepsilon\right).\end{aligned}\right\}\quad(1)$$

But since $\phi=\varepsilon$, by the third law of KEPLER, equations (1) become simply

$$\left.\begin{aligned}L+\phi&=(1+\lambda'^2)\frac{d.\lambda' L}{d\lambda'},\\ L+\phi&=(1+\lambda^2)\left(\frac{d.\lambda L}{d\lambda}-3\phi\right).\end{aligned}\right\}\quad(2)$$

If the definite integral L be expanded, it becomes

$$L=\frac{1}{3}-\left(\frac{1}{2}\cdot\frac{\lambda^2+\lambda'^2}{5}\right)+\left(\frac{1.3}{2.4}\frac{\lambda^4+\lambda'^4}{7}+\frac{1.1}{2.2}\frac{\lambda^2\lambda'^2}{7}\right)-\&c.,$$

substituting this value in (2), and neglecting quantities higher than λ^2, λ'^2, we find,

$$\lambda^2=30\phi,\qquad \lambda'^2=\tfrac{15}{2}\phi;$$

or if ϵ, ϵ', denote the ellipticities of the principal sections, passing through the greatest and least diameter, and mean and least, respectively; since $\lambda^2 = 2\epsilon$, $\lambda'^2 = 2\epsilon'$, we obtain finally for the ellipticities of the principal sections

$$\epsilon = 5\,\frac{E}{M}\frac{a^3}{\delta^3}, \qquad \epsilon' = \tfrac{5}{4}\cdot\frac{E}{M}\cdot\frac{a^3}{\delta^3}; \tag{3}$$

from which it appears that the ellipticity of the section passing through the greatest and least diameters is four times greater than the ellipticity of the section passing through the mean and least diameters.

If the planet be supposed to revolve on its axis with an angular rotation different from that of its revolution round the central body, the equality $\phi = \gamma$ will no longer subsist, and we should therefore use equations (1) to determine the ellipticities of the principal sections. The result is

$$\epsilon = \tfrac{15}{4}\,(3\phi + \gamma), \qquad \epsilon' = \tfrac{15}{4}\,\gamma; \tag{4}$$

ϕ and γ being the quantities already defined, and depending on the central body and rotation of the planet respectively. If the central body be supposed so remote as to produce no effect on the figure of the planet, then $\phi = 0$, which renders the ellipticities equal, and corresponds to the figure of revolution assumed by the planet, if acted on only by its own attraction, and the centrifugal force caused by its rotation.* If, therefore, we suppose the spheroid of revolution, whose ellipticity is $\epsilon = \frac{15}{4}\gamma$, described, having the axis of rotation for its least diameter, the effect produced by the attraction of the central body will be measured by the shape and magnitude of the couche included between this spheroid of rotation and the ellipsoid which forms the actual surface of the planet. The friction between this couche and the interior spheroid, which would constitute the surface of the planet, if the central body ceased to exist, will tend to render the motions of rotation and revolution of the planet equal to each other, and when the difference of these motions has fallen within the narrow limits indicated by analysis, will destroy the libration produced by the action of the central body in rendering those motions exactly equal. It may be proved by simple geometrical considerations, that if the planet separates from the central body, as a nodular or annular mass, without much friction, that

* Vid. POISSON, "Traité de Mecanique," Tom. II. p. 544.

its times of rotation and revolution at the period of separation will be nearly equal; and since we have no reason to assume any difference in the mode in which the planets and satellites were thrown off from the central mass, we may suppose, in order to render our calculations possible, that at the period of separation, the movements of rotation and revolution were so nearly equal as to justify us in using equations (3) instead of (4). Equations (4) might be used as well as (3), but require an additional hypothesis as to the time of rotation of the planet; but as this hypothesis should be the same for the planets and satellites, the generality of the reasoning is not affected by the use of equations (3). In these equations, the only quantity which is unknown is a, the radius of the planet or satellite at the time of its separation. We may obtain a value for a, in terms of the actual radius of the planet and its past and present moments of inertia, by the ordinary principles of mechanics; and if we assume as the measure of contraction of each planet the ratio which its original time of rotation bears to its actual time of rotation, we can calculate the value of ϵ and ϵ' for each planet and satellite. It will be shown afterwards, that the amount of contraction thus assumed is much too small for the planets which are attended with satellites, and probably for all the planets; but it will be useful to make the calculation upon this supposition in the first instance.

Let i, I denote the former and present moment of inertia of the planet, supposed homogeneous; a, $\mathbf{a}$, its former and present radius, and n the number of rotations contained in one revolution; then

$$a^5 : \mathbf{a}^5 :: i : I :: n : 1,$$

therefore,

$$a : \mathbf{a} :: \sqrt[5]{n} : 1,$$

or,

$$a^3 = \mathbf{a}^3 \sqrt[5]{n^3};$$

and substituting this value in (3), we find

$$\epsilon = 5 \frac{E}{M} \frac{\mathbf{a}^3}{\delta^3} \sqrt[5]{n^3}. \qquad (5)$$

The data from which I have calculated the values of ϵ corresponding to each planet and satellite are contained in the following Tables.

TABLE I.*

SATELLITES.		n	a : δ	$E : M$	ϵ
Moon, ...		1·00	$\frac{2153}{7926 \times 59{\cdot}964}$	87·73	$\frac{1}{24524}$
Satellites of Jupiter,	1st, .	1·00	$\frac{2508}{87 \times 6048}$	$\frac{1000000000}{17328}$	$\frac{1}{32{\cdot}00}$
	2nd, .	1·00	$\frac{2068}{87 \times 9623}$	$\frac{1000000000}{23235}$	$\frac{1}{308\ 32}$
	3rd, .	1·00	$\frac{3377}{87 \times 15350}$	$\frac{1000000000}{88497}$	$\frac{1}{1094{\cdot}5}$
	4th, .	1·00	$\frac{2890}{87 \times 26998}$	$\frac{1000000000}{42659}$	$\frac{1}{4580{\cdot}3}$

TABLE II.*

PLANETS.	n	a : δ	$E : M$	ϵ
Mercury, ...	87·6	$\frac{3140}{190 \times 387098}$	4865751	$\frac{1}{36082}$
Venus, ...	230·9	$\frac{7800}{190 \times 723331}$	401839	$\frac{1}{103970}$
Earth, ...	365·25	$\frac{7926}{190000000}$	389551	$\frac{1}{205121}$
Mars, ...	669·7	$\frac{4100}{190 \times 1523692}$	2680337	$\frac{1}{529573}$
Jupiter, ...	10468	$\frac{87000}{190 \times 5202776}$	1047·871	$\frac{1}{1084480}$
Saturn, ...	24631	$\frac{79160}{190 \times 9538786}$	3501·600	$\frac{1}{1589015}$
Uranus, ...	77524	$\frac{34500}{190 \times 19182390}$	24905	$\frac{1}{11029750}$

* The figures contained in the first three columns of Table I. are taken from the third edition of Sir JOHN F. W. HERSCHEL'S Astronomy, pp. 331, 649, 650. The corresponding figures of Table II. are calculated from the Tables of the same book, pp. 647, 648.

From the foregoing Tables, it would appear that the effect of the planets in elongating the figures of their satellites was greater than the effect of the Sun upon the planets; and so far the conclusion to be drawn from the calculation accords with the idea of LAPLACE. But a slight consideration will show that the amount of contraction assigned to the planets is much too small. In fact, we are entitled by the nebular hypothesis to assume that each planet, at the time of its separation, extended at least as far as the orbit of its most distant satellite; this consideration supplies us with another and safer measure of the contraction of those planets which have satellites.

The following Table contains the values of $\sqrt[5]{n}$, which express the amount of contraction used in Tables I. and II., and also the value of the ellipticity of each planet, supposed homogeneous and extending to the orbit of its outermost satellite.

TABLE III.

PLANETS.	$\sqrt[5]{n}$	$a : c$	ϵ
Mercury, ...	2·4462	—	—
Venus, ...	2·9695	—	—
Earth, ...	3·2547	$\frac{7926 \times 59{\cdot}964}{190000000}$	$\frac{1}{32{\cdot}801}$
Mars, ...	3·6743	—	—
Jupiter, ...	6·3675	$\frac{87000 \times 26{\cdot}998}{190 \times 5202776}$	$\frac{1}{14228}$
Saturn, ...	7·5560	$\frac{79160 \times 64{\cdot}359}{190 \times 9538786}$	$\frac{1}{2571{\cdot}3}$
Uranus, ...	9·5035	$\frac{34500 \times 22{\cdot}8^{*}}{190 \times 19182390}$	$\frac{1}{798591}$

From the first column of this Table, it appears that the original radius of the planets used in Tables I. and II. in no case exceeded ten times the present radius, which is too small for the planets with satellites, especially the Earth and Saturn, and probably too small for all the other planets. From a comparison

* These figures refer to the fourth satellite.

of the ellipticities in Tables I., II., III., we are led to infer that the action of the Sun in elongating Jupiter, and so by internal friction causing his movements of rotation and revolution to become equal, was much less powerful than the corresponding action of Jupiter upon his satellites; hence the physical cause assigned by LAPLACE for this equality may be admitted in the case of Jupiter's satellites. But this conclusion will not apply to the Earth. From Table I. it appears, that the elongating action of the Earth upon the Moon is represented by the fraction $\frac{1}{24 \cdot 524}$; while Table III. shows that the similar action of the Sun upon the Earth is represented by the fraction $\frac{1}{32 \cdot 801}$.

Before quitting this subject it may be useful to consider the various explanations which might be offered to explain the difficulty which undoubtedly exists in the case of the Earth and Moon.

We are not at liberty to assume that the planets separated from the central mass as annuli, and the satellites as nodules, which would give to the planets a quicker rotation than to the satellites. In this case $\varpi > \phi$, and therefore $\epsilon < 4\epsilon'$; hence the couche, on the friction of which the effect in question depends, would be less for the planets, *ceteris paribus*, than for the satellites. But this assumption is not admissible, since the only annuli with which we are acquainted in the solar system occur among the satellites. Neither are we at liberty to assume greater friction among the particles of the satellites than of the planets, for, according to the nebular hypothesis, they are probably composed of the same materials. It is possible to explain the difficulty by assuming a sufficient amount of contraction in the Moon. It is, in fact, easy to prove that the effect of the Earth upon the Moon would be equal to that of the Sun upon the Earth and Moon, supposed to extend as far as the orbit of the Moon, provided the Moon extended to a distance represented by the equation

$$\frac{\delta}{a} = 24 \cdot 322, \quad \text{or,} \quad \frac{a}{\mathrm{a}} = 9 \cdot 076\,;$$

and this amount of contraction is physically possible, since it is less than the distance from the Moon at which a particle would be equally attracted by the Moon and Earth. But how are we to reconcile this amount of contraction with the observed facts, without tacitly assuming that the internal friction of the Moon, supposed fluid, was greater than that of the Earth; an assumption which is purely arbitrary, and made to explain the difficulty.

There remains one real difference between the case of the planets and satellites, which, so far as it operates, is a *vera causa*, and acts in the direction required. The effect of the internal friction in destroying the increment of angular velocity must be greater in proportion as the mass of the planet or satellite is less; as we observe small rivers more retarded by the friction of their bed than large rivers. But it may be doubted whether this cause is sufficient to account for the remarkable difference which exists between the planets and satellites.

The conclusion which the foregoing calculations appear to warrant us in drawing is the following: that the nebular hypothesis does not explain the equality of the mean movements of revolution and rotation of the satellites, although it cannot be said to be absolutely inconsistent with it.

II.—*Figure of the Earth and Planets.*

It is well known that on the hypothesis of the original fluidity of the planets, it is necessary that the ellipticity of each planet should lie between two limits, which are, respectively, five-fourths and one-half of the fraction which expresses the ratio of centrifugal force to gravity at the surface of each planet;* the first or major limit corresponding to the case of homogeneity, and the second or minor limit corresponding to the case of infinite density at the centre. It is possible to compare this theory with observation in the case of five planets and the Moon. In the following Table, m denotes the ratio of centrifugal force to gravity at the surface of each planet, gravity being expressed in feet, and calculated from the formula

$$G = g\frac{P}{E}\frac{R^2}{r^2}, \tag{6}$$

in which G, g, denote gravity on the surface of the planet and Earth respectively; P, E, the masses of the planet and Earth; R, r, the radii of the Earth and planet. The centrifugal force at the equator of each planet is calculated from the ordinary formula

$$f = 4\pi^2\frac{r}{T^2},$$

in which r is expressed in feet, and T, the time of rotation, in seconds.

* CLAIRAUT, Figure de la Terre, p. 294.

TABLE IV.

PLANETS.	Gravity.*	Centrifugal Force.	Minor Limit, $\frac{1}{2} m$	Major Limit, $\frac{5}{4} m$	Observed Ellipticity. ϵ	Observer.
Earth, . . .	32·088	0·111	$\frac{1}{578}$	$\frac{1}{231}$	$\frac{1}{299 \cdot 152}$	Bessel.
Mars, ...	17·428	0·054	$\frac{1}{644}$	$\frac{1}{258}$	$\frac{1}{16 \cdot 3}$	W. Herschel.*
					$\frac{1}{38 \cdot 8}$	Arago.†
Jupiter, . .	99·007	7·090	$\frac{1}{27 \cdot 92}$	$\frac{1}{11 \cdot 17}$	$\frac{1}{17 \cdot 7}$	Arago.‡
Saturn, . . .	35·787	5·792	$\frac{1}{12 \cdot 35}$	$\frac{1}{4 \cdot 94}$	$\frac{1}{10 \cdot 37}$	W. Herschel. §
Uranus, . .	26·490	3·074	$\frac{1}{17 \cdot 22}$	$\frac{1}{6 \cdot 88}$	$\frac{1}{9 \cdot 92}$	Mädler.

On comparing the observed ellipticities with the limits calculated in the preceding Table, it appears that the ellipticity of Mars exceeds the major limit admissible on the fluid hypothesis; the inference from which fact is, either that gravity is not perpendicular to the surface of Mars, or that his interior structure is not that which would be assumed by a fluid body. The first of these suppositions appears inadmissible from the fact, that there is reason to believe, that there are degrading and disintegrating forces at work on the surface of that planet, similar to those now in operation on the Earth, and which would render the surface perpendicular to gravity, if not so originally. The second supposition would appear to be inconsistent with the idea that Mars derived his present figure from having been originally fluid; at least, we are scarcely justified

* Transactions of Royal Society of London, for the year 1784. The ratio of the axes of the planet Mars, deduced from observation, is 1355 : 1272.

† Exposition du Systeme du Monde, p. 37. The ratio of axes deduced from observation by ARAGO is 194 : 189.

‡ Exposition du Systeme du Monde, p. 39. The ratio of axes is 177 : 167.

§ Transactions of Royal Society of London, for the year 1790. The ratio of axes is 2281 : 2061.

in assuming the original fluidity of all the planets, when there exists so remarkable an exception in the case of the planet Mars.*

III.—*On the Structure of the Earth, supposed partly Fluid and partly Solid.*

In the following investigation I shall suppose the Earth composed of elliptical couches of small ellipticity, the density of each couche being constant and a function of its distance from the centre. The surfaces bounding the couches must be perpendicular to the resultant of the forces acting upon the particles composing them, in the parts of the Earth which are supposed fluid, and also at the boundary between the solid and fluid parts, since the friction of the fluid would render the bounding surface perpendicular to the resultant, if not so originally. The only external forces supposed to act upon the particles are the centrifugal forces arising from the earth's rotation.

The condition that any surface bounding one of the couches of equal density should be perpendicular to gravity is contained in the following equation:

$$\text{const} = V + N, \tag{7}$$

in which V is the potential of the earth, and

$$N = \tfrac{1}{3}\omega^2 r^2 - \tfrac{1}{2}\omega^2 r^2 s\,; \tag{8}$$

r denoting the radius of the surface, ω the angular velocity, and $s = \cos^2\theta - \frac{1}{3}$, θ being the angle contained between the radius vector and the axis of rotation.† The potential contained in (7) is composed of two parts, one relating to the couches inside the surface considered, and the other to the couches outside the same surface. The value of the potential of a body constituted as we have supposed the earth, on an external point, is,

* It has been remarked by LAPLACE (Mec. Cel. Tom. II. p. 370, and Tom. V. p. 287), that the ellipticities of the principal sections of the Moon, deduced from the moments of inertia obtained by the observations of TOBIAS MAYER and NICOLLET, are nearly $\frac{1}{1675}$ and $\frac{1}{1773}$, and that both these ellipticities are greater than those of the figure of the Moon, if supposed fluid and homogeneous, which would give the maximum ellipticity. We have, therefore, in the Moon a case similar to Mars, viz., the actual ellipticity is greater than the major limit of the fluid hypothesis; but it is easier to admit that gravity is not perpendicular to the surface in the case of the Moon.

† Mec. Celeste, Tom. II. p. 66.

$$V = \frac{4\pi \int \rho a^2}{r} - \frac{4\pi \varepsilon}{5r^3} \int \rho \frac{d \, . a^5 e}{da}; \tag{9}$$

in which ρ is the density of any couche, a the radius of its equi-capacious sphere, and e its ellipticity.

The potential of a shell composed of couches arranged in the manner supposed, on an internal point, is,

$$V = 4\pi \int \rho a - \frac{4\pi r^2 \varepsilon}{5} \int \rho \frac{de}{da}. \tag{10}$$

The radius vector of the surface of each couche is given by the following equation,

$$r = a\,(1 - e\varepsilon); \tag{11}$$

from which may be deduced the values of the equatorial and polar axes, viz., $a\,(1 + \frac{1}{3}e)$, and $a\,(1 - \frac{2}{3}e)$. Substituting from the foregoing equations in (7), we find

$$\begin{aligned} \text{const} = {} & \frac{4\pi}{a}(1 + e\varepsilon) \int_0^a \rho a^2 - \frac{4\pi}{5a^3} \varepsilon \int_0^a \rho \frac{d \, . a^5 e}{da} \\ & + 4\pi \int_a^{\mathbf{a}} \rho a - \frac{4\pi a^2}{5} \varepsilon \int_a^{\mathbf{a}_1} \rho \frac{de}{da} - \frac{4\pi a^2}{5} \varepsilon \int_{\mathbf{a}_1}^{\mathbf{a}} \rho \frac{de}{da} \\ & + \frac{4\pi a^2 m}{3\mathbf{a}^3} \int_0^{\mathbf{a}} \rho a^2 - \frac{4\pi a^2 m}{2\mathbf{a}^3} \varepsilon \int_0^{\mathbf{a}} \rho a^2; \end{aligned}$$

$\mathbf{a}$ denoting the mean radius of the external surface, $\mathbf{a}_1$ the mean radius of the internal surface of the shell supposed solid, and m the ratio of centrifugal force to gravity at the equator. This equation consists of two parts, one independent of ε, which is satisfied by means of the constant; the second, which is the coefficient of ε, gives the condition,

$$\frac{e}{a} \int_0^a \rho a^2 - \frac{1}{5a^3} \int_0^a \rho \frac{d \, . a^5 e}{da} - \frac{a^2}{5} \int_a^{\mathbf{a}} \rho \frac{de}{da} - \frac{ma^2}{2\mathbf{a}^3} \int_0^{\mathbf{a}} \rho a^2 = 0. \tag{12}$$

This equation expresses the fact, that each fluid surface is perpendicular to the resultant of all the forces acting upon the particles composing it.

Differentiating this equation, so as to banish the integrals, we obtain,

$$\frac{d^2 e}{da^2} + \frac{2\rho a^2}{\int_0^a \rho a^2} \frac{de}{da} - \frac{6e}{a^2} \left(1 - \frac{\rho a^3}{3 \int_0^a \rho a^2}\right) = 0. \tag{13}$$

This equation is identical with that derived from the supposition that the Earth is completely fluid, and is therefore independent of the law of density and ellipticity of the solid parts of the Earth; it determines the relation which necessarily exists between the law of density and ellipticity of the fluid portions of the Earth. If the law of density of the fluid parts be given, the integral of this differential equation will give the law of ellipticity, involving two constants, one of which is determined by the condition that the density does not become infinite at the centre, and the other constant may be expressed in terms of the ellipticity of the surface which bounds the fluid. If we suppose that there is a fluid nucleus inside the Earth, whose radius is $\mathbf{a}_1$, and ellipticity ϵ_1, equation (12) will give for the bounding surface of the nucleus the following,

$$\frac{\epsilon_1}{\mathbf{a}_1}\int_0^{\mathbf{a}_1}\rho a^2 - \frac{1}{5\mathbf{a}_1^3}\int_0^{\mathbf{a}_1}\rho\frac{d.a^5e}{da} - \frac{\mathbf{a}_1^2}{5}\int_{\mathbf{a}_1}^{\mathbf{a}}\rho\frac{de}{da} = \frac{m\mathbf{a}_1^2}{2\mathbf{a}^3}\int_0^{\mathbf{a}}\rho a^2. \tag{14}$$

If, also, we assume, as we may in the case of the Earth, that the external surface is perpendicular to gravity, equation (12) may be applied to this surface, although not fluid. Hence we obtain,

$$\frac{2}{5}\int_0^{\mathbf{a}}\rho\frac{d.a^5e}{da} = (2\epsilon - m)\,\mathbf{a}^2\int_0^{\mathbf{a}}\rho a^2. \tag{15}$$

Equations (14) and (15) assert, respectively, that the inner and outer surfaces of the solid shell are perpendicular to gravity.

In the case of the Earth, the integral at the right-hand side of these equations is known, because the mean density of the Earth is known. The integral at the left-hand side of equation (15) is also known; since it may be expressed in terms of the difference of the moments of inertia with respect to the polar and equatorial axes, which is given by the inequalities of the Moon's motion produced by the structure of the Earth, or by the phenomena of precession and nutation, which are produced by the same cause. In fact, if C, A denote the moments of inertia with respect to the polar and equatorial axis respectively.

$$C - A = \frac{8\pi}{15}\int_0^{\mathbf{a}}\rho\frac{d.a^5e}{da}. \tag{16}$$

Also the first and second integrals, on the left-hand side of equation (14) are known from the differential equation (13), if we assume the law of density of the fluid parts to be known.

There remains, however, the third integral on the left-hand side of (14), which cannot be known without assuming a law of density and also of ellipticity for the solid portion of the Earth.

We are thus led to the conclusion, that it is necessary to assume *three* hypotheses with respect to the internal structure of the Earth, before we can be in a position to assert how far it is solid and how far fluid. The three necessary hypotheses are:—1st. The law of density of the fluid parts. 2nd. The law of density of the solid parts. 3rd. The law of ellipticity of the solid parts.

If we suppose that these are given, then equations (14), (15) will become,

$$\left.\begin{aligned} F(\mathbf{a}, \mathbf{a}_1, \epsilon, \epsilon_1, m) &= 0\,; \\ f(\mathbf{a}, \mathbf{a}_1, \epsilon, \epsilon_1, m) &= 0\,; \end{aligned}\right\} \qquad (17)$$

in which F, f denote known functions. In these equations $\mathbf{a}$, ϵ, m are known, and $\mathbf{a}_1$, ϵ_1, are determined by the equations themselves.

If we suppose that the fluid parts of the earth are bounded on both surfaces by solids, we should then have three equations, analogous to (14) and (15), belonging to the two surfaces of the fluid, and to the external surface respectively. From these, assuming the law of density of the fluid, and of density and ellipticity of the solid parts, we should obtain

$$\left.\begin{aligned} \Phi\,(\mathbf{a}, \mathbf{a}_1, \mathbf{a}_2, \epsilon, \epsilon_1, \epsilon_2, m) &= 0\,; \\ \mathrm{X}\,(\mathbf{a}, \mathbf{a}_1, \mathbf{a}_2, \epsilon, \epsilon_1, \epsilon_2, m) &= 0\,; \\ \Psi\,(\mathbf{a}, \mathbf{a}_1, \mathbf{a}_2, \epsilon, \epsilon_1, \epsilon_2, m) &= 0\,; \end{aligned}\right\} \qquad (18)$$

$\mathbf{a}_2$, ϵ_2 being the radius and ellipticity of the second surface of the fluid. In equations (18), as before, $\mathbf{a}$, ϵ, m are known; but the number of unknown quantities is greater than the number of equations, the unknown quantities being four, viz., $\mathbf{a}_1$, $\mathbf{a}_2$, ϵ_1, ϵ_2, while there are only three equations. The problem is therefore not so definite as the last, and requires an additional hypothesis.

Confining our attention to the simplest case (17), we see that before a single step can be made towards using equations (14) and (15), we must assume three laws, respecting facts of which we have no certain knowledge, and probably never shall. The subject would thus appear to be excluded from the domain of positive science, and to possess an interest for the mathematician alone.

I shall conclude this investigation by examining the structure of the Earth on the simple but improbable hypothesis of homogeneity, and by determining how far the density belonging to the rocks of the surface may extend to the materials composing the interior of the Earth.

If the Earth be supposed to be composed of a solid shell, having the density of the rocks at its surface, and of a fluid homogeneous nucleus, equations (14) and (15) will become

$$\frac{2}{5}\rho\epsilon_1 - \frac{3}{5}\rho_0(\epsilon - \epsilon_1) = \Delta\frac{m}{2}, \tag{19}$$

and

$$\frac{6}{5}\{\rho_0\epsilon\mathbf{a}^5 + (\rho - \rho_0)\epsilon_1\mathbf{a}_1^5\} = (2\epsilon - m)\Delta\mathbf{a}^5; \tag{20}$$

in which ρ_0 signifies the density of the rocks of the shell, ρ the density of the nucleus, and Δ the mean density of the whole Earth. To equations (19) and (20) must be added the following, which expresses that the mass of the Earth is equal to the sum of the masses of its shell and nucleus.

$$\rho - \rho_0 = (\Delta - \rho_0)\frac{\mathbf{a}^3}{\mathbf{a}_1^3}. \tag{21}$$

Eliminating ρ from (19) and (20) by means of (21), they become respectively

$$\frac{2}{5}\left\{\rho_0 + (\Delta - \rho_0)\frac{\mathbf{a}^3}{\mathbf{a}_1^3}\right\}\epsilon_1 - \frac{3}{5}\rho_0(\epsilon - \epsilon_1) = \Delta\frac{m}{2}, \tag{22}$$

and

$$\frac{6}{5}\left\{\rho_0\epsilon\frac{\mathbf{a}^2}{\mathbf{a}_1^2} + (\Delta - \rho_0)\epsilon_1\right\} = (2\epsilon - m)\Delta\frac{\mathbf{a}^2}{\mathbf{a}_1^2} \tag{23}$$

In the case of the Earth $\Delta = 2\rho_0$; substituting this value of the mean density, and solving equations (22) and (23) with respect to ϵ_1, we find

$$\epsilon_1 = \frac{5m + 3\epsilon}{5 + 2\phi^3}; \tag{24}$$

$$\epsilon_1 = \frac{7\epsilon - 5m}{3}\phi^2; \tag{25}$$

ϕ being used to denote the fraction $\frac{\mathbf{a}}{\mathbf{a}_1}$.

These are the equations which correspond, on the supposition of homogeneity, to the equations (17). Equating the values of ϵ_1, we obtain the following equation to determine ϕ:

$$2\phi^5 + 5\phi^2 = 3\,\frac{5m + 3\epsilon}{7\epsilon - 5m}\,. \tag{26}$$

Substituting in this equation for m and ϵ their values in the case of the Earth, viz., $\frac{1}{289}$ and $\frac{1}{300}$, we find,

$$2\phi^5 + 5\phi^2 = 13{\cdot}57743. \tag{27}$$

Applying STURM's theorem to this equation, it is easy to prove that it has only one real root, which lies between $\phi = 1$ and $\phi = 2$. The numerical value of this root is

$$\frac{\mathbf{a}}{\mathbf{a}_1} = \phi = 1{\cdot}2407.$$

Hence, since $\mathbf{a} = 3958$ miles; $\mathbf{a}_1 = 3190$ miles, and

$$\mathbf{a} - \mathbf{a}_1 = 768 \text{ miles}. \tag{28}$$

This is the thickness of the earth's crust, on the hypothesis that both the crust and nucleus are homogeneous, and the surfaces of both perpendicular to gravity.

I shall now prove that this thickness of crust is a major limit to the depth to which the density of the rocks at the surface can extend into the interior; the density being supposed heterogeneous.

The difference of the moments of inertia of the nucleus with respect to its polar and equatorial axis may be expressed as follows:

$$C - A = \frac{8\pi}{15}\int_0^{\mathbf{a}_1} \rho\,\frac{d.a^5 e}{da} = \frac{8\pi}{15}\,\rho\,\frac{\epsilon_1}{\sigma}\,\mathbf{a}_1^5, \tag{29}$$

σ denoting an unknown number, depending on the structure of the nucleus, and which, if the nucleus be supposed fluid, is greater than unity.

Substituting from (29) in equations (14) and (15) we find

$$\epsilon_1\left(\frac{1}{3} - \frac{1}{5\sigma}\right)\{\rho_0 + (\Delta - \rho_0)\,\phi^3\} - \frac{1}{5}\,\rho_0\,(\epsilon - \epsilon_1) = \frac{1}{6}\,\Delta m\,; \tag{30}$$

and,

$$\frac{2}{5}\,[\rho_0\,(\epsilon\phi^5 - \epsilon_1) + \frac{\epsilon_1}{\sigma}\,\{\rho_0 + (\Delta - \rho_0)\,\phi^3\}] = \frac{1}{3}\,\Delta\,(2\epsilon - m)\,\phi^5. \tag{31}$$

Solving these equations with respect to ϵ_1, and making $\Delta = 2\rho_0$ we find

$$\epsilon_1 = \frac{m + \frac{3}{5}\,\epsilon}{\frac{3}{5} + \left(1 - \frac{3}{5\sigma}\right)(\phi^3 + 1)}\,; \tag{32}$$

$$\epsilon_1 = \frac{(7\epsilon - 5m)\,\phi^5}{\frac{3\,(\phi^3 + 1)}{\sigma} - 3}\,. \tag{33}$$

Eliminating ϵ_1 and solving for σ, we find,

$$\frac{1}{\sigma} = \frac{(3\phi^5 + A) + 5\phi^5\,(\phi^3 + 1)}{(3\phi^5 + A)\,(\phi^3 + 1)}\,, \tag{34}$$

in which $A = 3\,\dfrac{5m + 3\epsilon}{7\epsilon - 5m}$.

But the nucleus being supposed fluid, the denominator of the right-hand member of (34) is greater than its numerator; consequently we have the inequality

$$2\phi^5 + 5\phi^2 < 3\,\frac{5m + 3\epsilon}{7\epsilon - 5m}\,. \tag{35}$$

The value $\phi = 1{\cdot}2407$ renders the left-hand member of (35) equal to the right, and therefore ϕ must be less than $1{\cdot}2407$, and, consequently, the depth to which the density of the surface extends is less than 768 miles.

The results which have just been obtained are to be regarded merely as examples of the manner in which equations (14) and (15) should be used, if we were acquainted with the laws of density and ellipticity of the fluid and solid parts of the Earth. So long as we are ignorant of these laws, we cannot calculate numerical values, and indeed the chief use of the investigation I have just given appears to be, to enable us to estimate at their just value speculations relating to the interior of the Earth, of whose real structure we are, and must remain, hopelessly ignorant.

NOTES.

No. I., *referred to in page* 252.—" Un des phénomènes les plus singuliers du système solaire, est l'égalité rigoureuse que l'on observe entre les mouvemens angulaires de rotation et de révolution de chaque satellite. Il y a l'infini contre un à parier qu'il n'est point l'effet du hasard. La théorie de la pesanteur universelle fait disparaître l'infini, de cette invraisemblance, en nous montrant qu'il suffit pour l'existence du phénomène, qu'à l'origine, ces mouvemens aient été très peu différens. Alors l'attraction de la planète a établi entre eux, une parfaite égalité; mais en même temps, elle a donné naissance à une oscillation périodique dans l'axe du satellite, dirigé vers la planète, oscillation dont l'étendue dépend de la différence primitive des deux mouvemens. Les observations de MAYER sur la libration de la lune, et celles que MM. BOUVARD et NICOLLET viennent de faire sur le même objet, à ma prière, n'ayant point fait reconnaître cette oscillation, la différence dont elle dépend, doit être très petite; ce qui indique avec une extrême vraisemblance, une cause spéciale qui d'abord a renfermé cette différence dans les limites fort resserrées où l'attraction de la planète a pu établir entre les mouvemens moyens de rotation et de révolution, une égalité rigoureuse, et qui ensuite a fini par détruire l'oscillation que cette égalité a fait naître. L'un et l'autre de ces effets résultent de notre hypothèse; car on conçoit que la lune à l'état de vapeurs, formait par l'attraction puissante de la terre, un sphéroïde allongé dont le grand axe devait être dirigé sans cesse vers cette planète, par la facilité avec laquelle les vapeurs cèdent aux plus petites forces qui les animent. L'attraction terrestre continuant d'agir de la même manière, tant que la lune a été dans un état fluide, a dû à la longue, en rapprochant sans cesse les deux mouvemens de ce satellite, faire tomber leur différence, dans les limites où commence à s'établir leur égalité rigoureuse. Ensuite, cette attraction a dû anéantir peu à peu l'oscillation que cette égalité a produite dans le grand axe du sphéroïde, dirigé vers la terre. C'est ainsi que les fluides qui recouvrent cette planète, ont détruit par leur frottement et par leur résistance, les oscillations primitives de son axe de rotation, qui maintenant n'est plus assujetti qu'à la nutation résultante des actions du soleil et de la lune. Il est facile de se convaincre que l'égalité des mouvemens de rotation et de révolution des satellites a dû mettre obstacle à la formation d'anneaux et de satellites secondaires, par les atmosphères de ces corps. Aussi l'observation n'a-t-elle jusqu'à présent, rien indiqué de semblable."—LAPLACE, *Exposition du Systeme du Monde*, pp. 472, 473.

No. II., *added March* 25, 1852.—Since the foregoing communication was offered to the Academy, I have become acquainted with Mr. HENNESSEY'S Researches in Terrestrial Physics, published by the Royal Society of London in the Philosophical Transactions, Part II., for 1851. In these Researches, pp. 544, 545, Mr. HENNESSEY obtains numerical values for the major and minor limit of the thickness of the Earth's crust, the interior being supposed fluid. These limits are 600 miles and 18 miles respectively. The first limit is obtained by assuming LAPLACE'S law of density for the fluid nucleus of the Earth, and the same law for the solid shell, with an alteration of the constants to correspond with the supposed alteration of density of the shell in passing from the fluid to the solid condition. As the hypotheses used to obtain this limit are arbitrary, the limit itself must be considered only as of the same value as the limit in equation (28), deduced from the improbable hypothesis of homogeneity in the shell and nucleus. The other limit is more interesting, being assumed to be a minor limit to the thickness of the Earth's crust, and independent of the law of density of the interior.

On a careful examination of the hypotheses on which the determination of this limit depends, I believe that it will be found, that one of them is inadmissible, and others arbitrary. If I understand Mr. HENNESSEY aright, the following are the statements from which he deduces his minor limit of the thickness of the Earth's crust:

1st. The shell is homogeneous and of the density of the rocks at the surface.

2nd. The shell is bounded by similar surfaces, whose ellipticity is $\frac{1}{300}$.

3rd. The internal surface of the shell is perpendicular to gravity.

4th. The external surface of the shell is not perpendicular to gravity, and its ellipticity, if it were so, would be $\frac{1}{294}$.

The fourth of these statements appears to me to be inadmissible for the following reasons: the ellipticity of the surface perpendicular to gravity is assumed by Mr. HENNESSEY to be $\frac{1}{294}$, which is a mean between the ellipticities $\frac{1}{288}$ and $\frac{1}{300}$, deduced from the pendulum, and lunar inequalities,* but the ellipticity deduced from the lunar observations, $\frac{1}{300}$, is identical with that deduced from the measurement of meridian arcs, and although there may be some chance in this agreement, yet it is sufficient to suggest the idea, that the surface of the Earth is rigorously perpendicular to gravity, and that the pendulum experiments are influenced by variations of local attraction, arising from variable density in the rocks, or from the position of land and water. Such are the usual explanations of the difference between the ellipticity obtained from the pendulum and that deduced from lunar observations; and unless some explanation be offered of the agreement between the ellipticity of the actual surface obtained from meridian arcs, and the ellipticity of the surface perpendicular to gravity deduced from the lunar inequalities, it is not allowable to assume, that the mean of the results of the pendulum and lunar observations gives the surface perpendicular to gravity.

In fact, the observations of the pendulum and of the Moon should give exactly the same ellipticity, and would do so, were it not that the pendulum is liable to local variations, from which the other method is exempt; the result of the latter is, therefore, more trustworthy, and this result is almost identical with the ellipticity of the actual surface. It is certainly unphilosophic to take the mean of observations which differ more from each other than they differ from the quantity with

* The figures here given are those adopted by Mr. HENNESSEY, and are probably as near the truth as any others which have been deduced. The ellipticity deducible from SABINE's pendulum experiments is $\frac{1}{288 \cdot 7}$; and from BOUVARD, BURCKHARDT, and BURG's lunar observations, is $\frac{1}{304 \cdot 1}$. (Mec. Cel., Tom. v. p. 45.)

which we wish to compare them, and then to assume that the difference between the mean so found and that quantity is a real difference.

Adopting the four hypotheses above mentioned, Mr. HENNESSEY has deduced from his formulæ the following value for the ratio of the radius of the nucleus to the radius of the exterior surface, p. 545;

$$a_1^5 = \frac{2}{3} + \frac{1}{3}\,\frac{\frac{5}{4}\,m - (e)}{\frac{5}{4}\,m - e}. \tag{1}$$

In this equation a_1 denotes the ratio of the radius of the nucleus to the radius of the external surface, which is assumed equal to unity; $m = \frac{1}{289}$ is the ratio of centrifugal force to gravity at equator; $e = \frac{1}{300}$ is the ellipticity of the actual surface of the Earth; and $(e) = \frac{1}{294}$ (the mean of the fractions $\frac{1}{288}$ and $\frac{1}{300}$, obtained from the pendulum and lunar observations), is the ellipticity of the surface, if perpendicular to gravity. Substituting these values in equation (1), Mr. HENNESSEY obtains $a_1^5 = 0{\cdot}97714$, and $a_1 = 0{\cdot}99539$, $1 - a_1 = 0{\cdot}00461$, from which he infers, that "consistently with observation, the least thickness of the Earth's crust cannot be less than 18 miles." It is very easy to prove, that if the shell be bounded by similar surfaces, both of which are perpendicular to gravity, that its thickness is zero; this I believe to be the true minor limit of the thickness of the crust.

But even admitting Mr. HENNESSEY's assumption, that the outer surface of the Earth is not perpendicular to gravity, I am unable to agree with him as to the formula from which its thickness should be calculated. In equation (1), which is deduced from the previous equations, a_1 is the reciprocal of the quantity I have called ϕ. This equation contains only the fifth power of a_1 or ϕ, whereas, the equation deducible from the investigation which I have given contains both the fifth and third powers of ϕ, and gives a numerical result which differs materially from Mr. HENNESSEY's. The investigation is as follows. Assuming $\epsilon_1 = \epsilon = e$ in equation (30), which asserts that gravity is perpendicular to the inner surface of the crust and is deduced from (14), and solving for σ, we find, making $\Delta = 2\rho_0$,

$$\frac{3}{5\sigma} = \frac{(\phi^3 + 1)\,e - m}{(\phi^3 + 1)\,e}. \tag{2}$$

In equation (15), the external surface is supposed perpendicular to gravity, and, therefore, the ellipticity ϵ of its right-hand member must be replaced by (e); the integral at the left-hand side of this equation is proportional to the difference of the moments of inertia of the Earth with respect to its polar and equatorial axes (16), and does not require the surface to be perpendicular to gravity; in fact, the left-hand side of this equation may be supposed to belong to any body having the same difference of moments of inertia as that belonging to the Earth. Separating the integral into two parts, belonging respectively to the shell and nucleus of the Earth, the external surface being supposed similar to the inner, and not perpendicular to gravity, we find,

$$3\,(\phi^5 - 1)\,e + 3\,(\phi^3 + 1)\,\frac{e}{\sigma} = 5\,\{2(e) - m\}\,\phi^5;$$

which might have been deduced directly from (31), by making $\epsilon = \epsilon_1 = e$ on the left-hand side, $\epsilon = (e)$ on the right, and $\Delta = 2\rho_0'$. Solving this equation for σ, we find,

$$\frac{3}{5\sigma} = \frac{\{10(e) - 3e - 5m\}\,\phi^5 + 3e}{5e(\phi^3 + 1)}. \tag{3}$$

Eliminating σ from equations (2) and (3), we obtain finally,

$$\phi^5 + \frac{5m - 2e}{10(e) - 3e - 5m} = \frac{5e}{10(e) - 3e - 5m}\,\phi^3. \tag{4}$$

In this equation ϕ is the reciprocal of a_1, and the other letters are the same as the corresponding letters used in equation (1). Equation (4) differs widely from the equation (1) obtained by Mr. HENNESSEY; the hypotheses used in obtaining it are the four hypotheses used by him; and yet I am unable to perceive any error in the process by which (4) is found.

Substituting for $m, e, (e)$; their values $\frac{1}{289}, \frac{1}{300}$, and $\frac{1}{294}$, we find,

$$\phi^5 + 1\cdot58425 = 2\cdot48290\,\phi^3. \tag{5}$$

Applying STURM's theorem to this equation, I find that it has three real roots, one negative and two positive; the latter lying between $\phi = 1$ and $\phi = 2$. These roots are

$$\phi = 1\cdot0436; \qquad \phi = 1\cdot3626.$$

Rejecting the negative root, as being not applicable to the question in hand, it would appear at first sight as if there were two solutions, corresponding to the two real positive roots just found; but it is evident, by referring to equation (28), that the second value of ϕ, being greater than 1·2407, is to be rejected as well as the negative root; in fact, the second value of ϕ would give a thickness to the crust of the Earth greater than the depth to which the density of the rocks at the surface can extend; and such a thickness, as has been already shown, is inconsistent with the supposition of a fluid nucleus. Calculating the thickness of crust corresponding to the least positive root of equation (5), we find,

$$\mathbf{a} - \mathbf{a}_1 = 166 \text{ miles.} \tag{6}$$

This result differs materially from that obtained from the same data by Mr. HENNESSEY, but as the hypothesis on which it is founded is untenable, the result itself is of little value, except so far as it illustrates the use of the equations already given. As I have before stated, the thickness of the crust would be zero, if we were to admit the first three statements, and combine with them an assertion that the surface of the Earth is perpendicular to gravity. This I believe to be the true minor limit of the thickness of the Earth's crust; and the major limit appears to me to require for its numerical calculation a knowledge of facts, respecting which we must be content to remain in ignorance.

XII.—*On the Homology of the Organs of the Tunicata and the Polyzoa.** By GEORGE JAMES ALLMAN, M. D., F. R. C. S. I., M. R. I. A., Professor of Botany in the University of Dublin.

Read January 26, 1852.

THOUGH the close affinity between the *Tunicata* and the *Polyzoa* has been generally acknowledged, yet the full extent to which the organization of the one is represented by that of the other does not appear to have been hitherto recognised by the zoologist. I propose in the present communication to point out some apparently unnoticed instances of homological identity, while I shall endeavour to show that almost every modification of form in the organization of the one is, by the easiest transition, convertible into a corresponding form in the other; that they are both, therefore, constructed on precisely the same type, and must constitute one and the same great natural group.

In order to render this subject intelligible, it will be necessary in the first place to fix the terms indicative of the various aspects of the *Tunicata* and the *Polyzoa*, terms which are so vaguely used by different authors as to give rise to great confusion in description. In the determination of the *anterior* and *posterior* aspects, there would seem to be no difficulty, as the former must manifestly be assumed as that to which the mouth is directed, while the posterior will then of course be the aspect directly opposed to this. The determination of the *dorsal*, or *superior*, and of the *ventral*, or *inferior* aspects, is not so easy. I believe, however, that the *cephalic ganglion*, or its homologue, must be here our true guide, and that its position will always correspond with the dorsal, or superior aspect of the animal, to which the ventral will then consequently be diametrically opposed. Mr. HUXLEY, in his admirable memoir on *Salpa* and

* *Polyzoa* THOMPSON, synonymous with *Bryozoa* EHRENBERG. THOMPSON'S name has priority of date over that of EHRENBERG, and should, therefore, in justice to its founder, and in obedience to the laws of Natural History nomenclature, be adopted.

Pyrosoma, assumes the *heart* as indicating the dorsal aspect of the *Tunicata*;* the cephalic ganglion, however, in those inferior members of the animal kingdom in which the dorsal and ventral aspects are already indicated by other characters, is invariably placed on the dorsal side of the alimentary canal, and though it be admitted that the almost universal position of the heart among invertebrate animals is also dorsal; yet where, as in the *Tunicata*, we find the ganglion and heart placed on opposite sides, the superior importance of the ganglion will, I think, justify us in assuming its position as the *constant* one, and concluding that it is the heart therefore and not the ganglion that has changed place. The only apparent difficulty in assuming the ganglion as the index of the back results from its not being always obvious that the nervous mass before us is homologous with a true cephalic, or supra-œsophageal ganglion; there will, however, I think, always be found marks sufficient to decide this point; we shall subsequently see that the ganglion, both in the *Tunicata* and the *Polyzoa*, undoubtedly contains a supra-œsophageal element, which from its pre-eminent importance will determine the

FIG. 1.

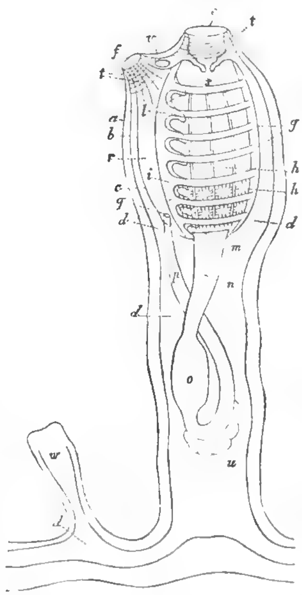


FIG. 2.

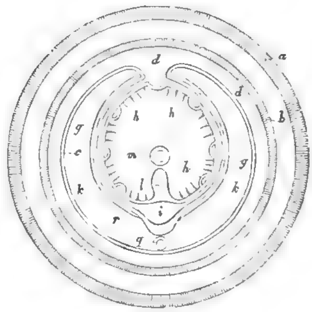


FIG. 1. Diagramatic view of *Clavelina*.
FIG. 2. Diagramatic transverse section of *Clavelina*.†

a, external tunic; *b*, middle tunic; *c*, internal tunic; *d*, *d*, *d*, sinus system; *e*, respiratory orifice; *f*, cloacal orifice; *g*, *g*, transverse respiratory bars; *h*, *h*, longitudinal respiratory bars; *i*, branchial sinus; *k*, *k*, proper membrane of respiratory sac; *l*, languets; *m*, mouth; *n*, œsophagus; *o*, stomach; *p*, intestine; *q*, anus; *r*, cloaca; *s*, tentacula; *t*, muscular fibres in middle tunic; *u*, heart; *v*, nervous ganglion; *w*, gemma.

* "Observations upon the Anatomy and Physiology of Salpa and Pyrosoma, together with Remarks on Doliolum and Appendicularia." By Thomas Henry Huxley. Phil. Trans., 1851.

† In all the figures accompanying this paper, the same letters are used with the homologous organs.

dorsal position of the ganglionic mass, even though the latter should also perform functions usually devolving on ganglia situated below the alimentary canal.

1. *Respiratory System.*—As it is in the respiratory organs of the two groups that the leading peculiarities of their structure are to be found, our attention must be first directed to this portion of the organization, with the view of determining how far the respiratory apparatus in the one has its homologue in the other. Now two distinct notions have prevailed on this point, some zoologists* maintaining that the respiratory sac of the Ascidian has its representative in the pharynx of the Polyzoon, and that the rudimental tentacula at the orifice of this sac are homologous with the tentacula of the Polyzoon; while others† assert, that the branchial sac of the *Ascidiæ* is homologous with the tentacular crown of the *Polyzoa*, the longitudinal bars of the sac corresponding to the tentacula of the *Polyzoa*, and the transverse bars becoming extinct. Now, neither of these views appears to me to represent the exact truth, for, while I conceive that the tentacular crown of the *Polyzoa* has undoubtedly its true homologue in the respi-

Fig. 3.

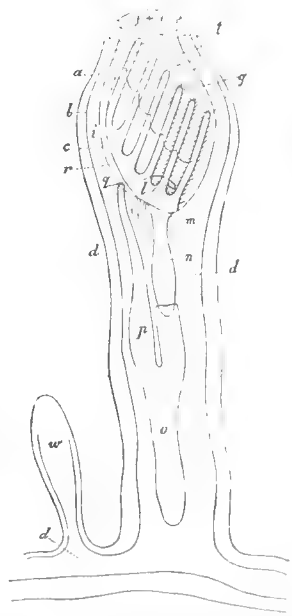


Fig. 4.

Fig. 3. Diagramatic view of a *hippocrepian Polyzoon* (retracted). Fig. 4. Diagramatic transverse section of a *hippocrepian Polyzoon.*

a, ectocyst; *b*, endocyst; *c*, tentacular sheath; *d, d, d*, perigastric space; *f* + *e*, external orifice of cell; *g, g*, tentacula; *i*, lophophore; *k, k*, caliciform membrane; *l*, oral valve-like organ; *m*, mouth; *n*, œsophagus; *o*, stomach; *p*, intestine; *q*, anus; *r*, cavity of tentacular sheath; *t*, muscular fibres in endocyst; *v*, nervous ganglion; *w*, gemma.

* See Dr. A. Farre, "Observations on the Minute Structure of some of the higher Forms of Polypi." Phil. Trans. 1837.

† See Van Beneden, "Sur les Ascidies Simples." Mem. de l'Acad. Roy de Belgique, Tome xx., 1847.

ratory sac of the *Ascidiæ*, I believe that it is to the *transverse*, and not to the longitudinal bars of this sac that the tentacula of the Polyzoa are homologous; and this is a very important distinction, the non-recognition of which has rendered all previous attempts at comparison between the tentacular crown of the *Polyzoa* and the respiratory sac of the *Ascidiæ* untenable.

On this subject much light is thrown by the *hippocrepian Polyzoa*, or those fresh-water genera which, like *Plumatella*, have their tentacula arranged on a crescentic "lophophore;"* and we shall best perceive the relations in question by comparing an ascidian Tunicate with one of these *Polyzoa*, a *Clavelina* (Figs. 1 and 2), for example, with a *Plumatella* (Figs. 3 and 4). In *Clavelina*, the great "branchial sinus" of Milne-Edwards,† from each side of which the transverse bars or vessels of the respiratory sac are given off, will correspond to the elongated lophophore in *Plumatella*, and the richly ciliated transverse bars to the ciliated tentacula, while the delicate membranous sac, to the interior of which the respiratory bars are adherent, and which Milne-Edwards has shown to be perforated in the intervals of these bars by the "respiratory stigmata," will have its homologue in the calyx-like membrane adherent to the base of the tentacular plume in *Fredericella* and the hippocrepian *Polyzoa*. This correspondence will be rendered more obvious by imagining the branchial sinus to be rotated round its oral extremity in a vertical plane through an angle of 90°, towards the superior or anal side of the Tunicate; its position from longitudinal will thus be changed to transverse, while the transverse bars will become longitudinal, and the branchial sinus and its bars will then have the same direction as the exserted lophophore and tentacula of *Plumatella;* while it is interesting to observe that, during the retracted state of the Polyzoon, the lophophore assumes the normal direction of the branchial sinus in the Tunicate.

That the tentacula of the *Polyzoa* are not homologous with the unciliated

* In a Report on the Fresh-Water *Polyzoa*, read before the Edinburgh Meeting of the British Association for 1850, I found that our increased knowledge of the structure of the *Polyzoa* rendered it necessary to make some change in the terminology hitherto employed in their description; and the terms used in that Report are also adopted in the present memoir. The *Polypide* is the retractile portion of the Polyzoon as distinguished from its cell; the *Ectocyst* is the external tunic of the cell; the *Endocyst* is the internal tunic; the *Lophophore* is the kind of disc or stage which surrounds the mouth and bears the tentacula.

† See M. Milne-Edwards's beautiful memoir, "Sur les Ascidies Composeés."

rudimentary tentacula at the entrance of the respiratory sac in the *Ascidiæ* is also apparent, not only from the difference of structure, but from the fact, that while the tentacula of the *Polyzoa* are in immediate relation with the digestive tube, those of the *Ascidiæ* are evidently mere appendages of the internal tunic. It is true, that in accordance with this view, we can find no homologue in the *Polyzoa* for the tentacula of the *Ascidiæ;* we must therefore conclude, that these organs have absolutely died out in the *Polyzoa*, a circumstance for which we have been already prepared by their disappearance in *Salpa* and other Tunicates.

In connexion with the tentacular crown, there is another part of the organization of the *Polyzoa* for which we have still to find an equivalent, and which, without comparison with the *Tunicata*, would remain inexplicable, namely, the curious valve-like organ which overhangs the mouth in *Fredericella* and the hippocrepian *Polyzoa*. Now this is plainly homologous with the tongue-like bodies, the "languets" of MILNE-EDWARDS, which are attached along the branchial sinus in *Clavelina* and certain other Tunicates, and thence project into the interior of the branchial sac, and which in *Salpa* are represented by a single one. The languet in *Salpa* is connected with a peculiar ciliated cavity lying immediately at its base, and which seems also to have its representative in the excavation of the lophophore at the base of the oral appendage in *Plumatella* and the allied forms; and through which the cavity of this appendage appears to communicate with the perigastric space. Further observation will, in all probability, prove that the interior of the languets in the *Tunicata* communicate in these with the great "sinus system,"* which is equivalent with the perigastric space of the *Polyzoa*. MILNE-EDWARDS believes the languets in *Clavelina* to exhibit a kind of erection, a phenomenon which would suggest as its explanation such a communication as that here supposed, and which, at all events, renders still more striking the resemblance between the languets of the *Tunicata* and the oral appendage of the *Polyzoa*, an organ which seems to present an analogous phenomenon. In both groups the bodies in question would seem to be organs of special sense, probably of taste.

* This name has been given by HUXLEY to the whole of the space included between the internal and middle tunics in the *Tunicata*, and through which the blood, uninclosed in proper vessels, vaguely circulates. See HUXLEY, loc. cit.

We now need only a few unimportant modifications in order to complete the resemblance between the branchial sac of *Clavelina* and the tentacular crown of *Plumatella;* we have only to imagine the oral extremity of the branchial sinus to be prolonged with its bars for a short distance downwards, so as to surround the mouth, the transverse bars to become free at their extremities, where, opposite to the branchial sinus, they communicate with the "thoracic sinus," the longitudinal bars to be suppressed, and the "languets" to be reduced to *one* situated in the immediate vicinity of the mouth; a series of changes involving no essential modification of structure; and we shall then have an organ only wanting in a deep crescentic depression of the distal extremity of the branchial sinus, to resemble, even in minute details, the tentacular crown of *Plumatella.*

Now nearly all the changes which we have thus hypothetically supposed to take place in *Clavelina*, in order to convert its branchial sac into the tentacular crown of *Plumatella*, do actually occur in other genera of *Tunicata*, some in one, and some in another. The predominant importance of the transverse over the longitudinal bars of the branchial apparatus in the *Tunicata* is sufficiently manifest; in most cases they are larger and more evident than the longitudinal; in *Pyrosoma*, as appears from Mr. HUXLEY's account of this genus,* they are not only the better developed, but they alone carry cilia; the transverse bars, moreover, are constant in all the genera, while the longitudinal actually disappear in *Salpa* and in *Doliolum*, unless, indeed, we adopt the ingenious view of Mr. HUXLEY, who supposes that the lower division ("Epipharyngeal Band" of HUXLEY) of the gill in *Doliolum* is homologous with the longitudinal bars in *Pyrosoma* and other tunicates; an opinion, however, which is surely opposed by the fact, that in two species of *Salpa* examined by SAVIGNY, this naturalist has pointed out the existence of a small inferior gill, maintained by Mr. HUXLEY to be the homologue of the inferior division of the gill in *Doliolum;* and yet the superior or constant gill in one of these *Salpæ* shows at the same time traces of longitudinal bars as in *Pyrosoma*. In *Doliolum*, as it would appear from Mr. HUXLEY's short but interesting account, the superior and inferior divisions of the gill are directly continuous with one another behind; indeed they are evidently one and the same organ carried across the thoracic chamber

* Loc. cit.

(Fig. 5); the gill in *Doliolum* then plainly consists of a great branchial sinus, carrying its respiratory bars on each side as in *Clavelina*, but differing from the disposition of parts in the latter genus by having its posterior extremity prolonged downwards till it reaches the inferior wall of the thoracic chamber, along which it then runs forwards parallel to the superior portion. The mouth perforates this inferior prolongation of the sinus, and thus becomes related to the sinus and its bars exactly as the mouth in the *Polyzoa* is to the lophophore and tentacula in these. Savigny informs us, that the mouth opens between the inferior and superior gill in the *Salpæ* examined by him; but it is not easy to determine from his description whether these portions are directly continuous, as in *Doliolum*. In *Doliolum*, moreover, the remote extremities of the branchial bars of one side are quite separate from those of the other, and thus present the open condition which characterizes the tentacular crown in the *Polyzoa*, so that the gill of *Doliolum* constitutes the exact link by which the branchial sac of the *Ascidiæ* passes immediately into the tentacular crown of the *Polyzoa*. In *Pyrosoma* we have also an approach to the open condition of the tentacular crown, for the inferior extremities of the transverse bars of one side are separated from those of the other by a considerable space, and, according to Lesieur, even become free for some distance from their extremities in the species which he describes.

Fig. 5.

Fig. 5. Ideal longitudinal section of *Doliolum*.

a + *b*, external and middle tunic united: *c*, internal tunic; *d*, *d*, sinus system: *e*, respiratory orifice; *f*, cloacal orifice; *g*, respiratory bars; *i*, *i*, branchial sinus; *m*, mouth; *n*, œsophagus; *o*, stomach; *p*, intestine; *q*, anus; *r*, cloaca; *v*, nervous ganglion.

The structure and connexions, then, of the ascidian tentacula, together with the modifications actually experienced by the longitudinal and transverse bars in the different forms of *Tunicata*, and the fact that the tentacular crown in the hippocrepian *Polyzoa* will admit of a satisfactory explanation in accordance alone with the views here taken, afford evidence that the homologues of the tentacula in the *Polyzoa* are neither the rudimentary tentacula at the entrance of the branchial sac of the *Ascidiæ*, nor the longitudinal bars of this sac, as maintained by those naturalists who have yet recognised in the branchial sac of the *Ascidiæ* an organ homologous with the tentacular crown of the *Polyzoa;* but that their true equivalents must be sought for in the transverse bars, and this is further borne out by the observation of the ascidian embryo in which the longitudinal bars would seem to make their appearance subsequently to the transverse ones; the respiratory sac thus passing in the course of its development through a stage more nearly corresponding to the simpler condition which we meet with in the respiratory crown of the *Polyzoa*.

In *Salpa* the languets are reduced to a single one, that, however, which remains in this genus is not, as we might be led to expect from the comparison we have made between these organs and the oral appendage of *Plumatella*, the languet nearest the mouth, but on the contrary (if we may judge from its position), the most remote from this part of the animal. It is, however, particularly worthy of attention, that both the existing languet of *Salpa* and the oral appendage of *Fredericella*, and the hippocrepian *Polyzoa*, are quite similarly related to the great nervous ganglion. This ganglion we shall presently see to be homologous in the *Tunicata* and *Polyzoa*, and it is manifestly it, and not the mouth, that determines the place of the persistent languet.

However interesting the hippocrepian *Polyzoa* may be in directly indicating the relations here dwelt on, the infundibulate genera present no difficulty, for they exhibit, after all, but an unimportant modification of the former, and are connected to them by a series of intermediate forms. The arms of the lophophore in *Plumatella* have only to become obsolete in order to transform this genus into a *Fredericella*, in which, however, the lophophore still retains a bilateral figure, which is rendered yet more decided by the presence of the oral valve-like organ. In *Laguncula* Van Ben., the oral appendage has disappeared, but the lophophore still presents a slight bilaterality. Finally, in the fresh-water genus, *Paludicella*, and most of the marine genera, not only has the oral appendage disappeared, but all trace of bilaterality has now vanished from the lophophore.

2. *Dermal System.*—M. Milne-Edwards has proved by the anatomy of *Clavelina*, that there exist in this genus, and probably in all *Tunicata*, three distinct

envelopes, which, however, may be variously united with one another in the different genera.* Now all these have their homologues in the *Polyzoa;* the external sac or test of the *Tunicata* corresponds to the external investment, or ectocyst, of the *Polyzoa;* the middle sac, or mantle, of the *Tunicata*, to the internal investment, or endocyst, of the *Polyzoa;* and the internal tunic of the *Tunicata*, which surrounds the branchial sac, and forms the "thoracic chamber" of MILNE-EDWARDS (and which is divided into two portions, one inferior, containing the proper branchial sac, and the other superior, constituting the cloacal chamber), will be equivalent with the tentacular sheath of the *Polyzoa.* The homology of the two outer tunics of the *Tunicata* with the ectocyst and endocyst of the *Polyzoa* is obvious, and need not here be further dwelt upon; but the homology of the third or innermost tunic of the *Tunicata* with the tentacular sheath of the *Polyzoa* is very important, and will require to be considered more in detail. If we examine this tunic in *Clavelina*, we shall find that it is continuous with the mantle at the respiratory and cloacal orifices, and becomes attached to the alimentary canal, just behind the mouth and anus. It thus holds to the surrounding parts in the *Tunicata* exactly the same relation that the tentacular sheath or inverted tunic in the *Polyzoa* does to the corresponding parts of these during the retracted state of the animal. In the *Polyzoa* there is, properly speaking, but one external orifice, namely, that through which the tentacular crown is projected and retracted; but this is equivalent to the respiratory and cloacal orifices of the *Tunicata* united, and the point where the rectum opens externally in the *Polyzoa* is not, therefore, as supposed by VAN BENEDEN and others, the homologue of the cloacal orifice in the *Tunicata*, with the cloacal chamber itself become extinct,—a view which evidently originated in the too exclusive contemplation of the Polyzoon in its exserted state,—but rather corresponds to the point where the rectum penetrates the internal tunic in the *Tunicata*, and the cloaca in the latter will then be represented by the superior or dorsal portion of the space between the tentacular crown and sheath in the *Polyzoa*, this space becoming obliterated in the exserted state of the polypide.†

* See HUXLEY, loc. cit.

† To the normal structure both of the *Tunicata* and the *Polyzoa*, *Appendicularia* presents a remarkable exception. In this singular little Tunicate, as described by HUXLEY, the branchiæ are reduced to a mere rudiment, and while the thoracic chamber formed by the internal tunic is largely

3. *Digestive System.*—The form, structure, and peculiar course of the alimentary canal in the *Tunicata*, closely resembles what we find in the *Polyzoa*. This canal in the *Polyzoa* consists of three distinct portions: œsophagus, stomach, and intestine; the œsophagus communicates with the stomach by a well-defined cardiac orifice, and the cardiac extremity of the stomach frequently presents a cylindrical elongation, with the œsophagus opening into its anterior end; the stomach is separated from the intestine by a well-marked pylorus. The alimentary canal in the *Tunicata* is also divided into œsophagus, stomach, and intestine; in some instances these divisions are obscurely marked, but in others they are as well defined as in the *Polyzoa*. Now if, in accordance with the views attempted to be established in the present memoir, we consider the branchial sac of the Ascidian as the homologue of the tentacular crown of the Polyzoon, we shall have the three regions of the alimentary canal of the one exactly homologous respectively with the three regions in the other. If, on the contrary, the branchial sac of the Ascidian be homologous with the first region —the pharynx or œsophagus—of the alimentary canal of the Polyzoon, then, in order to find a homologue in the Polyzoon for that portion of the canal which intervenes between the branchial sac and the stomach in the Ascidian, and which is without doubt a true œsophagus, differing altogether in structure from the stomach, wherever in the *Tunicata* the alimentary canal acquires its proper development, we must take the cardiac prolongation of the stomach in the *Polyzoa* for an œsophagus, a view not borne out either by its structure or its functions; for independently of the fact that it is not always present, this prolongation obviously belongs to the proper stomach, having, it is true, special muscles sometimes developed in it, so as to give it the structure and office of a gizzard;* but more frequently being a simple prolongation of the gastric cavity, in no respect differing from the remainder of this cavity either in structure or func-

developed, the intestine does not open into it, but passes forwards and downwards to perforate the middle and external tunics, and thus open directly outwards. There is consequently here no cloaca. *Appendicularia* at a first glance appears to afford the connecting link between the *Tunicata* and the *Polyzoa;* but a little consideration will show that the most important point by which it differs from the normal *Tunicata*, namely, the absence of a cloaca, is that which also separates it at the greatest distance from the *Polyzoa*.

* See "Report on Fresh-water Polyzoa." Rep. Brit. Assoc., 1850, p. 310.

tion. In both the *Tunicata* and the *Polyzoa* the intestine is invariably bent on the first portion of the alimentary tube as it passes forward to the anal outlet; but there is a curious difference between the two groups in this respect, namely, that while in the *Tunicata* the first bend of the intestine, as noticed by Mr. Huxley, is always towards the lower side, or that opposite to the ganglion, its whole course in the *Polyzoa* is as invariably towards the upper, or ganglionic side, a difference, however, in no degree invalidating the homological identity of the parts. The structure of the walls of the alimentary canal in the *Tunicata* reminds us strongly of that in the *Polyzoa*. In some *Tunicata* there is a well-developed liver; in others, however, this organ is entirely absent, or only represented by a peculiar coloured layer on the interior of the walls of the alimentary canal, exactly as in the *Polyzoa*.

4. *Circulatory System.*—The circulatory system of the *Tunicata* admits of a very interesting comparison with that of the *Polyzoa*. The degraded condition of the vascular system in the former, where the heart scarcely advanced beyond the embryonic condition, is alternately branchial and systemic; and the undefined or extra-vascular circulation in the whole of the abdominal region conduct us at once to the complete absence of the heart in the *Polyzoa*, where the circulation—altogether extra-vascular, except so far as the tubular tentacula and lophophore represent a vascular system—is effected by the propulsive action of vibratile cilia. The condition of the circulatory system in the *Polyzoa* has already been quite anticipated in the curious Tunicate genus *Pelonaia*,* where the heart itself has disappeared. The great "sinus system" of the *Tunicata*, filled with the vaguely circulating blood, has its exact homologue in the perigastric space of the *Polyzoa*, occupying, like the latter, the interval between the middle and internal tunics.

5. *Muscular System.*—The muscles on which devolves the office of the retraction of the polypide in the *Polyzoa* are of course absent in the *Tunicata*, but notwithstanding this, we have some interesting points of correspondence between the muscles of the two groups. In the middle tunic or mantle of the *Ascidiæ* there is, as is well-known, a large development of muscular tissue in the form of circular and longitudinal fibres, which give to this tunic its cha-

* See Forbes and Goodsir in Edinburgh New Phil. Jour., vol. xxxii. p. 29.

racteristic contractility. Now these muscles are exactly represented by equivalent fibres which are developed in the homologous tunic or endocyst of the *Polyzoa*, and constitute the "parietal muscles" of these animals. The circular bands of *Salpa* and *Doliolum* appear to be developed in the *internal* tunic, and have their representatives in the sphincters occurring in the inverted tunic of the *Polyzoa*. Striated muscular fibre exists in many, if not in all the *Polyzoa*, and a similar condition of this tissue has been detected by ESCHRICHT and HUXLEY in *Salpa*.

6. *Nervous System.*—Between the great nervous ganglion in the *Tunicata* and the *Polyzoa* there is apparently a marked difference in position, this ganglion in the *Tunicata* being placed between the respiratory and cloacal orifice, while in the *Polyzoa* it is situated on the œsophagus near its oral extremity, and this difference might at first lead to the belief, that the homological identity which we have witnessed between the other organs of the two groups fails to show itself in the nervous system; still, however, it can be rendered evident, that no exception is here offered to the unity of plan already demonstrated, and that the two ganglia are strictly homologous. The ganglion is manifestly identical in function in the two groups, for in each we have nerves passing off from it both to the respiratory apparatus and to the œsophagus and region of the mouth, a distribution in which it corresponds with that of both the branchial and cephalic ganglia of the higher *Mollusca*, whose offices it thus seems to combine.

In several of the *Tunicata*, a well-defined otolithic capsule has been discovered in connexion with the ganglion; and Mr. HUXLEY has suggested to me that this ganglion ought therefore to be considered as homologous with the pedal ganglion of the Lamellibranchiate *Mollusca*, since in these the otolithic capsule is always found in connexion with the pedal ganglion. To this view, however, several objections appear to me to present themselves; the ganglion of the *Tunicata* and of the *Polyzoa* has functions devolving on it which we never see performed by the pedal ganglion of the *Lamellibranchiata;* the development of the pedal ganglion, moreover, bears a constant relation to that of the foot, and though the obliteration of the foot does not necessarily bring with it the absence of the ganglion—as in *Teredo*, for example, where the researches of QUATREFAGES have shown the existence of a pair of minute ganglia, manifestly representing the pedal ganglia of those *Lamellibranchiata*, in which the foot is not suppressed,—yet the pedal ganglion presents us under such circumstances with its lowest condition of development, and analogy will not permit us to suppose that in the absolutely footless Tunicate or Polyzoon this ganglion acquires its maximum, and even becomes here the only nervous centre present. It would, indeed, seem as if the solitary nervous centre of the *Tunicata* and *Polyzoa* combined the functions of the several separate centres of the *Lamellibranchiata*, while the superior importance of the cephalic element determines its supra-œsophageal position.

If we now carefully consider the difference of position between the two ganglia, we shall find that this is, after all, unimportant; in the *Tunicata*, while the ganglion is always placed between the two external orifices, it is at the same time situated in the interval between the internal and middle tunic, and is consequently in the midst of the sinus; in the *Polyzoa*, the two orifices coalescing, the ganglion can no longer occupy the position it held in the *Tunicata*; it is, therefore, carried backwards, and, still bathed in the fluid of the sinus, now becomes situated on the œsophagus, a difference of position which, it will easily be seen, involves no important change of relations, and which is necessarily connected with the difference in the arrangement of the other organs in the respective groups. In the *Polyzoa*, from their constant motions of retraction and exsertion, the ganglion could not occupy the fixed position which it does in the *Tunicata*, and, therefore, comes to be situated upon the polypide itself, all whose motions it then necessarily follows.

7. *Generative System.*—The construction of the generative system in the *Tunicata* and *Polyzoa* is also in conformity with the views of the present memoir. Both are hermaphrodite; in both we have, besides true sexual generation, generation by gemmation, the gemma in the *Polyzoa* being formed exactly as in the *Tunicata* from a diverticulum of the sinus system.

Though our knowledge of the developmental phenomena is in many respects so deficient as to afford much less assistance in the present inquiry than could be desired, yet if we compare the embryological development of an Ascidian as given by MILNE-EDWARDS or VAN BENEDEN, with that of a Polyzoon, we shall still find the results in accordance with the views of the present paper. In the embryo-Ascidian, after the internal organs have begun to assume the definite form which is subsequently to characterize them, we find that the interior of the body presents from behind forward four cavities, more or less distinguishable from each other, and which there is no difficulty in recognising as the future intestine, stomach, œsophagus, and respiratory sac. As yet, however, there is no trace of longitudinal or transverse bars in the respiratory sac, and it is only at a subsequent period that these bars come to line its walls. Observations are here deficient, but so far as they go it would seem that the transverse bars first make their appearance, that the longitudinal then show themselves; and lastly, that the sac becomes pierced by the respiratory stigmata. The circumstances under which the minute tentacula within the orifice of the respiratory sac become developed have not yet been satisfactorily observed. So many difficulties oppose themselves to our observation of the development of the *ovum* in the *Polyzoa*, that no facts of importance in the determination of the present question can thence be derived; but if we examine the corresponding development of the *bud* of *Paludicella*, we shall find after a time, that the nascent Polyzoon presents three distinct cavities, which are to become intestine, stomach, and œsophagus, and which are manifestly homologous with the cavities to which we give the same names in the embryo-Ascidian. Instead, however, of the closed cavity which in the Ascidian lies anterior to the œsophagus, and is to constitute the respiratory sac, we have here the anterior extremity of the œsophagus surrounded by a ring—the future lophophore—round whose outer margin a number of minute tubercles soon show themselves, and these then, becoming elongated, constitute the tentacula of the Polyzoon. Now between the formation of these tentacula and that of the respiratory bars of the *Ascidiæ*, the resemblance appears quite complete; in *Paludicella* and most other *Polyzoa*, there is, it is true, nothing homologous with the proper membrane of the respiratory sac of the *Ascidiæ* (the caliciform membrane of *Fredericella* and the hippocrepian *Polyzoa* being here absent), and consequently the closed præbuccal chamber of the *Ascidiæ* does not exist in them; but the essential part of the respiratory apparatus—the transverse bars of the Ascidian and the tentacula of the Polyzoon—entirely correspond in their order and mode of development, and so far the evidence derived from the phenomena of development coincides with that afforded by anatomy. In *Fredericella* and the hippocrepian *Polyzoa*, the proper membrane of the sac shows itself in the form of a delicate calyx, which surrounds the base of the tentacular plume; the difficulty of observing the development of the bud through the more opaque tissues of these *Polyzoa* has rendered us here deficient in the class of facts now under discussion, and we are not, therefore, yet prepared to institute an actual comparison between the development of the branchial membrane in the *Ascidiæ* and the caliciform membrane in the hippocrepian *Polyzoa;* so far, however, as our imperfect observations go, the facts are still in accordance with the views of the present paper; and though we have but little positive evidence to assist us in our conclusions, yet there is not a single observation tending to disprove the position that the branchial membrane of the one, and the caliciform membrane of the other, present in the circumstances of their development the conditions of homologous organs.

Among the other points of resemblance between the two groups, it is interesting to observe the frequent occurrence among the *Tunicata* of definite compound phytoidal forms resulting from gemmation, exactly as in the *Polyzoa.*

From what has now been stated it must be manifest, that the *Tunicata* and the *Polyzoa* are more nearly related to one another than either to any other branch of the animal kingdom; that they really belong to one and the same great structural type; and that the differences between them are non-essential modifications of this type, rendered for the most part necessary by the new power superadded upon the *Polyzoa* of alternately projecting and retracting the respiratory crown and anterior portion of the digestive organs through the external orifice of the cell.

The homology of the organs in the *Tunicata* and the *Polyzoa*, which it has been the object of the present paper to demonstrate, will be rendered more apparent by bringing together the equivalent organs of the two groups in the following two parallel series:

TUNICATA.		POLYZOA.
External tunic, ...	=	Ectocyst.
Middle tunic, ...	=	Endocyst.
Internal tunic, ...	=	Tentacular sheath.
Sinus system, ...	=	Perigastric space,
Respiratory orifice, . . . } Cloacal orifice, ... }	=	External orifice of cell.
Transverse respiratory bars,	=	Tentacula.

TUNICATA.		POLYZOA.
Branchial sinus, ...	=	Lophophore.
Membrane of respiratory sac,	=	Caliciform membrane.
Languet, ...	=	Oral valve.
Cloaca, ...	=	Space between tentacular crown and sheath.
Œsophagus, ...	=	Œsophagus.
Stomach, ...	=	Stomach.
Intestine, ...	=	Intestine.
Muscles of middle tunic, .	=	Parietal muscles.
Muscles of internal tunic (*Salpa, Doliolum*), . . .	=	Sphincters of internal tunic.
Ganglion, ...	=	Ganglion.
Tentacula, ...	=	0
Longitudinal respiratory bars,	=	0
Heart, ...	=	0

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XIII.—*Experimental Researches on the Lifting Power of the Electro-Magnet.* Part I. By the Rev. T. R. Robinson, D. D., Member of the Royal Irish Academy, and of other Scientific Societies.

Read June 14, 1852.

As soon as Oersted's great discovery had led to the construction of electro-magnets, high expectations were formed that they might afford a motive power as energetic and more economical than the steam-engine. The prodigious force which they manifest when excited by even a feeble current, and the power of annulling or reversing it in an instant, might seem to justify the hope; and an immense amount of inventive talent has been expended in attempts to realize it. These attempts, however, have shown that electro-magnetic engines can scarcely ever be either a cheap or a very efficient source of power. Electricity is now known to have a definite mechanical equivalent; the zinc and acids required to produce it are more costly than the coal, which will evolve isodynamic heat; and the hitherto contrived methods of converting electro-magnetism into moving force involve much more loss than the mechanism of the steam-engine does in respect of heat. I may add, that the great magnetic force which I have referred to exists only in contact; on the least separation of the keeper it decreases rapidly, not merely because magnetic force follows the law of the inverse square of the distance, but because that separation destroys in a very great degree the actual magnetism of the magnet. It must, however, be kept in mind that there are many cases where economy and intensity are of less consequence than facility of application and convenience; in which, therefore, the electro-magnetic engine deserves a preference even for industrial purposes, and much more for the work of the experimental physicist, although its action may be more costly. In particular, the absence of all danger, and perfect quiescence when not put in action, and the capability of being moved to any locality where a couple of wires can be led from its battery, deserve special consideration. Such views several years ago induced my friend, Mr. T. F. Bergin, to experiment on the construction of a machine suitable to the workshop of the amateur, or the laboratory of the philosopher; and I hope he will at no distant period lay his invention before the Academy. In its progress he occasionally consulted me as to the form and mass of the magnets to be employed; the distribution and kind of wire in their helices; and the intensity of the currents transmitted through them which might be expected to give the highest dynamic effect from a given consumption of materials. On all these points I was surprised to find that there was little or no exact information extant; I therefore determined to look for it myself; and since the beginning of 1848 have given to this object such attention as was permitted by my other avocations. In carrying it out I have derived much valuable aid from Mr. Bergin, not merely in the contriving and constructing the necessary apparatus, but also in making many experiments which I had not the means of performing. During this period several German physicists have been engaged in similar investigations;* but if I do not deceive myself, neither their results, nor those of Mr. Joule,† go so far as to make the present communication unnecessary; and I trust it will be found not merely useful to the practical magnetician, but also valuable, as affording data which have been carefully determined, to those, who like Dr. William Thomson, are investigating the theory of magnetic induction.

Before describing my methods of experimenting, a brief account of what occurs in the action of the electro-magnet may make their object more intelligible. If we conceive the cylindric core divided into thin sections perpendicular to its axis, and confine ourselves to the uppermost of them; on passing a current through the helix, its two surfaces will possess opposite polarities, derived mainly from the inducing power of those spires which are in its plane, but also in a decreasing amount from those which are below it. The intensity of these polarities depends on that of the inducing forces and of those which oppose them; the former is known to be proportional to the intensity of the

* Translated by Dr. Tyndall in the Philosophical Magazine, March, 1851.

† Philosophical Magazine, October, 1851.

electric current and a function of the number and diameters of the spires, but the other is almost totally unknown. It is admitted that these polarities comport themselves as if they were two fluids, each repelling itself or attracting the other inversely as the square of the distance, and becoming latent when permitted to unite; we might, therefore, suppose that under the influence of the excited helix, they will separate until the increasing repulsion of themselves, and attraction of each other, balance its influence. But there is yet another force which prevents this separation from proceeding quite so far; it is called the coercive force, and may be described as a resistance which the molecules of iron present to any alteration of their polar condition, whether the change be union or separation of polarities. The first of these will be as the polarity, the last some function of it, of which, I believe, nothing is known. Let now a second section be placed below the first, and in contact with it; it will be excited to an equal intensity; but the heteronymous polarities partially neutralize each other at the contact surface, and the remaining two being at twice the former distance have less power to oppose the induction of the helix. Therefore it will produce a still greater separation of the polarities, and so on, till the helix is filled with these sections. For this we may evidently substitute a solid bar; the intervals between its molecules being analogous to the surfaces of contact, and as evidently it can be showń that the extremities of the bar will exhibit opposite polarities, whose intensity gradually decreases towards the centre till it vanishes at that point. If now a keeper of the same section be placed on one extremity of the magnet, suppose the Boreal one, it will also become a magnet, and its Austral polarity will neutralize much of the Boreal of the other; the action of the helix will therefore evolve a still higher degree of magnetism in the latter, till a new equilibrium of forces is attained. In this instance, however, we can measure the new polarity, for it is proportional to the force with which the keeper is attracted by the polar extremity of the magnet. On the same principles the development of the magnetism will be carried still higher if the remote extremity of the keeper be connected with the Austral extremity of another magnet; and it will reach its maximum if the remaining poles of the two magnets be united as in the ordinary horse-shoe, and thus a magnetic circuit be completed. In this case there would be scarcely any free magnetism evident, and the forces which oppose H, the action of the helix on each molecule, are the coercive force C; and the differences of polar attractions across the molecule itself M, and across the intervals between it and those which adjoin it D. At the contact of the keeper this interval must be far greater than in the continuous iron; and the constant of attraction may also be different, but I think the attractions will be in a constant, perhaps an assignable, ratio.* If in this state of things the current in the helices be stopped, the polarities of the molecules tend to re-unite by the forces $M-D$, and are prevented by the force C, which now maintains the magnetic state as it opposed its production. As M must be always greater than D, and, as I have said, proportional to it, the magnetism must sink until $M-D=C$, and then remain permanent. It has long been known that the keeper of an electro-magnet adheres to it with considerable force when the current ceases; but I am not aware that the meaning of this fact has been interpreted, or measures of it taken. Now lifting the keeper, D is destroyed at the polar surfaces, and the forces are $M-C$, so that the magnetism will decrease till $M=C$, but will not necessarily vanish even in this case.

It is not my intention to go further into the theory of electro-magnetism, which I hope will be fully developed by the able geometrician to whom I have already referred; and I merely call attention to these elementary principles of it for the purpose of indicating the sort of information which I have endeavoured to obtain, and the way in which it seems to bear on these molecular forces.

The power of an electro-magnet may be examined either by measuring the force required to detach a keeper from its poles; secondly, by observing its attraction of a mass of iron at a small distance; or thirdly, by its deflection of a magnetic needle. The second of these methods appears to me objectionable, from the complication introduced by the rapid curvature of the lines of magnetic action near the poles, and from the great diminution of the forces by a very small interval; this is even more felt in the third, as the needle must be placed at a very considerable distance from the magnet. In both the varying distribution of the magnetism must be taken into account, and neither of them seems

* The contact will be closer when the attraction is powerful, and therefore the adhesion of the keeper something greater than what is due to the mere intensity of the magnetism, but I do not know whether this effect is appreciable.

to offer any mode of distinguishing between the forces M, D, and C. The first may be perhaps less accurate as to individual measures, or at least requires greater care and more numerous repetitions than the method of deflection; but these are probably more than compensated by the magnitude of the quantity to be measured. In applying it I have examined—

1. The relation between a magnet's power and the intensity of the current passing through its helices.

2. The effect of varying the number of spires in the helices and their distribution on the magnet.

3. The change produced by varying the unexcited portion of the magnetic circuit.

4. The difference between electro-magnets of iron and those of steel; and

5. The influence of the length and diameter of the magnet.

The first of these is the subject of the present communication, reserving the others for another opportunity.

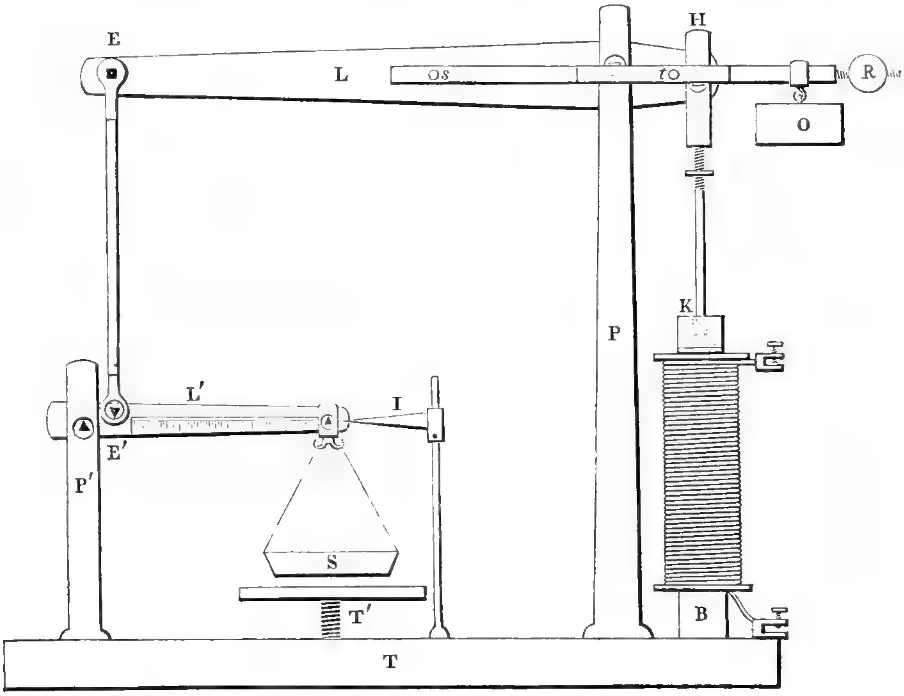
The apparatus which I used in making these experiments consists of an electro-magnet, a weighing apparatus, and the instruments for measuring and regulating the exciting current, each of which requires some notice.

1. The magnet consists of two cylinders of iron (the softest and most homogeneous that I have ever seen), each twelve inches long and two in diameter. They were made hollow, as from BARLOW's experiments I had imagined that the central portion added little to the effect; and I purposed to experiment at temperatures above boiling water, by introducing heaters in these cavities. I find that in this I was mistaken,* but the results are merely reduced in proportion to the transverse section, or as 3 : 4, the cavity being one inch diameter. The cylinders are screwed, with their axes 6 inches apart, into a base of the same iron, 2 inches deep, and $2\frac{1}{4}$ broad; together they weigh 26 lbs. The keeper is a rectangular prism, the same size as the base, weighing 7 lbs. It was planed and fitted so carefully, by scraping, to the polar surfaces of the cylinders, that it all but adheres to them by atmospheric pressure; and was then fitted with guides, so as always to insure uniformity of contact.

* Mr. BERGIN, with my helices on a solid magnet of the same dimensions, obtained with a current = 1·0117, a lift of 670·8 lbs. This magnet, with the same current, gives 509·2; the numbers are as 4 : 3·03.

The helices are made of lapped copper wire, No. 12, or $\frac{1}{9}$ inch diameter, coiled in four layers on mahogany bobbins, $2\frac{3}{8}$ diameter, and 10·9 long. The two have 638 spires, and 483 feet of wire; each layer being well soaked with lac varnish. I used wood for these bobbins, to prevent the magnet from being much heated when powerful currents are employed, but in all subsequent helices used copper, as the wires were sometimes so hot that I feared for their covering.* The external diameter of the helices is 33 inches.

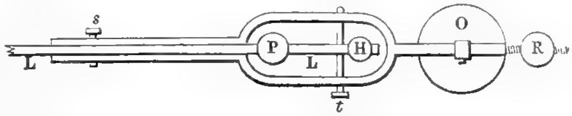
2. The weighing apparatus is shown in the wood-cut. It consists of a strong



oak table, T, 32 by 16 inches, and 2 inches thick, in which are, inlaid and secured by strong wood screws, two pieces of $\frac{3}{8}$ boiler-plate. On one of these is fixed the magnet by a strong bolt tapped into the centre of its base B, and set

* On one occasion, with metal bobbins, the magnet and its keeper were heated 35° in 70 minutes.

vertical by adjusting screws not shown in the figure.* The same iron plate bears the pillar P, also iron, 27·5 inches high, 2 and $1\frac{1}{4}$ diameter at its extremities, firmly screwed below, and steadied by oblique braces of $\frac{3}{4}$-inch round iron (not shown), bolted to the iron at the other end of the table. This bears in rings of hard steel the fulcrum knife-edge of the lever L, which is of spring-steel, $\frac{5}{8}$ thick, 3 deep, tapering to 2 and $1\frac{3}{4}$. Its arms are 21 and 3·5. Its short arm carries by knife-edges the cylinder H, in which is tapped a strong steel screw passing through a hole in the centre of the keeper K, and bearing it by a hemispheric head fitted in a corresponding cavity. The other arm is similarly linked by EE′ to a second lever L′, whose fulcrum is in the pillar P′ 12 inches high. Its arms are 10 and 1 inches; and at its outer extremity it carries the scale dish S. A slit in the direction of its length enables it to act as a steel-yard, by shifting along it small weights suspended by a loop of fine iron wire; and for this object it has a division from $1^{i}.9$ to $9^{i}.5$. The whole apparatus (except the scale) is counterpoised by attaching to L a piece shown in plan, fig. 2, by the screw *s* and the steady pin *t*. The box O contains shot, and the ball R, which is tapped on a fine screw, makes the adjustment exact.



The mode of using this instrument is easily understood. When the magnet is excited, and weights nearly equivalent to its lift are placed in the scale, the screw of the keeper must be turned till a mark on L′ stands at the index I. This index, which is hinged to P, so that it can be turned out of the way, shows when the lower edge of the slit in L′ is horizontal. Then a check-nut on the screw must be turned into firm contact with H, to preserve this adjustment

* This arrangement of the magnet did not admit of its being removed, and replaced with the requisite precision; and latterly it was changed for one which Mr. BERGIN contrived to meet this difficulty. A very strong rectangular frame of brass is secured on the table, 2 inches deep, and able to receive within it the base of the magnet, with an inch play all round. The magnet is slightly excited, so that it may hang freely from its keeper in this space. Then steel screws tapped in the brass, one in front, two behind, and two at the ends of the frame, are brought up so as to pinch the base equally, and thus I am certain that the pull which separates the keeper from the magnet will always be direct. This I find acts most satisfactorily.

during a series of measures. The least of the sliding weights is now hung to its loop and cautiously moved, till either it lifts the keeper, or arrives at the end of the division. In the latter case it is changed for a heavier. If none of them overcome the magnet, a scale weight, equivalent to the greatest moment of the last of the steelyard weights, is placed in the dish, and so on. Those which I use are 0·1 lb., 0·2, and 0·6 for the steelyard; the others are 0·5, 1, 2, 4, 7, and 14; the dish also = 0·5. They were carefully verified by a set of grain-weights belonging to me, and another of Professor STEVELLY. The leverage of the machine was determined with equal care. By means of the above weight and a balance, for the use of which I am indebted to my friend Mr. MALLET,* two of 28 lbs. and two of 56 were verified. Suspending them to the keeper, I found the weights required to counterpoise 56, 112, 168, and 199 lbs., and obtained their ratio = 59·730. In these trials additions to the load of 0·031, 0·046, and 0·094 lb. were easily detected; an error of about 1 lb. in the ton.

A machine of this kind is of course not expected to equal the accuracy of an ordinary balance; but for the work which it has to do it is far preferable on two accounts. To lift the keeper by weights equal to its attraction would be very dangerous, for the sudden descent of 8 or 9 cwt. would cause a fearful concussion; while the fall of its equivalent, 15 on the pad T′, is scarcely felt. Besides, when the separation is nearly attained, the most delicate manipulation is necessary; and it is far easier to avoid jar in sliding a light weight, than in placing in a scale one sixty times as heavy. But in fact the force to be measured is itself fluctuating to an extent which far passes any errors of the weighing.

3. I have measured the intensity of the voltaic current by a tangent rheometer; and this mention of it might suffice, were it not that even in an instrument so well known the details of its use are not without value, and that its results cannot be duly compared to those of another without a distinct knowledge of its individuality. I prefer it to the one described in a former commu-

* It is the smallest of those mentioned in his Report to the British Association on the Corrosion of Railway Bars; when loaded with 56 lbs. in each scale it turns decidedly with three grains. All these comparisons were made by the method of double weighing.

nication,* as including a wider range, and being independent of the intensity of its needle's magnetism. Its circular conductor consists of five copper rings, each 0·5 broad and 0·05 thick, the innermost of which has 16 inches internal diameter. This is commonly used alone, but the others can be combined with it. The connectors descend from the nadir of the rings within the wooden stem which supports them, pass through its base (which is provided with levelling screws), and then, proceeding about 18 inches in the magnetic meridian, turn at right angles, and proceed parallel, and almost in contact, for three feet, to a commutator which connects them with the general circuit. By thus reversing the current, not merely in the rheometer, but also in so great a length of the connectors, I designed to eliminate their influence ; and experience shows that such a precaution is quite necessary. Concentric with the rings, and perpendicular to their plane, is fixed a brass circle 9 inches diameter, divided to half degrees, at whose centre stands a point of hard steel, very carefully finished to an angle of 60°.† On this turns a needle $1^{i}.77$ long, $0^{i}.25$ deep, and 0·05 thick ; it has a ruby cap, and pointers of palladium long enough to reach the divisions ; and it weighs altogether 75 grains. It has been shown by WEBER (Poggendorf, vol. lv.) that if the ratio of the ring's diameter to the length of the needle be greater than 4 or 5, the tangent of deflection is proportional to the force. This ratio, however, is too low for high deflections. When it is 4 I find the law fails at 33°, and when 4·8 at 50. In this rheometer it is 9. As, however, it was necessary to ascertain whether the influence of the connectors was injurious, I at the same time examined its sufficiency by the voltameter, and found for 28 angles from 20° to 75°,‡ that the tangents are *exactly* as the quantities of mixed gases evolved in a given time, supposed dry, and at the normal temperature and pressure. The factor by which the tangent gives the current force F depends on the unit assumed for that quantity. WEBER, in the memoir referred to, uses one derived from the intensity of terrestrial magnetism at the

* Transactions of the Royal Irish Academy, vol. xxi. p. 303.

† It was formed by traversing it while rapidly revolving in the drill apparatus of a slide-rest, inclined at 30° along the surface of a cylindric lap also rapidly revolving, and charged first with very fine emery, and then with crocus. It bears examining with a power of 120 diameters, and is far more perfect than any point which I have seen in a theodolite or compass.

‡ The greatest which 18 Groves' could produce with the voltameter.

place of observation, a quantity variable in itself, and by no means easy to ascertain. Dr. W. Thomson has more recently proposed* one expressed in terms of the mechanical effect to which the current is equivalent; which, however, must be regarded as a scientific conception, rather than of practical use. A standard, to be available, must be of easy access and application, and in these respects I see no reason for preferring any to one which is in frequent use, the electrolysis of water. The most obvious current unit is that which can decompose a grain of water in a unit of time. It seems to me, however, that if a second, or even a minute, be taken as time unit, the values of current will be inconveniently fractional, if an hour, as much too large, and therefore I take five minutes. Adopting this, all that is required to make these rheometers speak a given language is, to note the seconds in which a known volume of the gases is evolved, and reduce it to that due to 300 seconds; to compute its normal volume G by means of the formulæ in treatises of Pneumatics, and measure carefully the deflection ϕ', then we have

$$m = \frac{G \times \text{cotan}\, \phi}{\log^{-1}(0{\cdot}89310)}\,; \quad F = m \times \tan \phi\,;$$

the experiments for which can be completed in a single day.

Both ends of the needle are read with direct, and again with reversed current, to eliminate excentricity and zero errors; the readings are made with a prismatic microscope, and can be depended on to 2′.

The rheostat is used in these experiments merely to equalize the current, and therefore has no necessary connexion with their results; but as in a former communication† I mentioned its peculiar construction, and promised further details, I take this opportunity of stating my conclusion as to its working. As exhibited to the Academy on that occasion, it consisted of a wire of platinum, whose length was varied by raising it out of mercury, while it was cooled by being surrounded with distilled water; and I expected that by measuring the temperature of this latter fluid I might apply the necessary correction for the change of resistance due to the heat evolved by the passage of the current. Unless this be attended to, I am satisfied that no measures can be made

* Philosophical Magazine, 1851; p. 551.

† Transactions of the Royal Irish Academy, vol. xxi. p. 303.

deserving full confidence, and that it is necessary *even with the feeblest transference of electricity.* This especially applies to those methods in which a current is divided between two conductors, and its respective quantities in them are estimated from their relative resistances, previously determined. That relation involves the temperature of each, and varies with the current. In many respects this rheostat was a great improvement on that which I previously used, these probable errors being $0^{i}.16$ and $0^{i}.28$; but I soon found it could not invariably be trusted. Occasionally a film of water would adhere so obstinately to the platinum, that its contact with the mercury did not occur till two inches below the surface of the latter ; and this state would continue for several days. A little solution of potassa lessened this tendency, but made the water too good a conductor ; I therefore abandoned the mercury in that part of the instrument, and made the contact by a spring clip of platinum. This change enables me to use a wire of palladium instead of platinum ; the former resisting twice as much with the same section, and, what is more important, varying its resistance ten times less by a given change of temperature ; being, in this respect, the lowest of all the metals which I have examined. These alterations have improved the accuracy of the rheostat, its probable error being now only $0^{i}.06$. The wire is $\frac{1}{43}$ diameter, and its range 15 inches, read to $0^{i}.01$, by a vernier.* If greater resistance be required, 19 equivalents of the same wire, also immersed in water, can be added to the circuit. I wish I could give some more definite statement of this wire's resistance than is contained in the mention of its diameter, for that alone is not sufficient. Platinum wire I find, even when drawn in a gemmed hole, and heated white hot after its passages, resists unequally in different parts of the same piece,—much more may different specimens be expected to differ. A tolerable approximation to it, however, is given by the fact, that if we use the current unit just described, the intensity (or the electro-motive force of the contact theory) of a Groves' cell, determined by the tangent rheometer, = 47·282 inches of this wire.†

Another measure (which I hope may ultimately prove an accurate one) is afforded by the electrolytic intensity of water (the imaginary polarization of

* Equal to 970 inches of $\frac{1}{12}$ copper wire.

† Mean of the last 20 I observed, the greatest being 48·675, the least 45·345.

electrodes of the contact theory). As I have formerly shown, it varies by heat. I assigned 0·04986 as the change for 1° Fahr., but this value was obtained by dividing the current, and without means of correcting the rheostat for temperature. I have since obtained by better methods—

$$e \text{ at } 60° = 62{\cdot}229, \quad \text{change for } 1° = 0{\cdot}06735.$$

It is not affected by the quantity of sulphuric acid mixed with the water to increase its conducting power, being almost identical whether this be $\frac{1}{9}$ or $\frac{1}{90}$ of the electrolyte. Nor is it (within very wide limits) by the size of the electrodes; being the same when they oppose surfaces of 19 square inches (the size of the platinum in the battery), of 3, or of 0·75, the intensity of the battery being given.

But there is a change, real or apparent, depending on that intensity. The value above given was obtained with two Groves'; with three it is 69·137 at 60°, and with four 75·052. It is my present belief that this seeming increase is caused by two things: by the internal resistance of the cells decreasing in consequence of being heated by the current, and by the rheostat wire being hotter within than at its surface. The thermometer immersed in the water gives merely the latter temperature, and therefore the resistance correction is too small.* This, however, I hope soon to be able to determine.

After this long preface (which I hope will not be useless to any one who may engage in these or similar researches), I proceed to state in the following

* Taking the equation

$$m \tan \phi = F = \frac{E}{R+r},$$

and introducing a resistance ρ, which produces the deflection ϕ',

$$E = \frac{m(\rho + dR)}{\cot \phi' - \cot \phi},$$

dR being any change of the cells' resistance. Introduce now the voltameter, and a similar equation gives $E-e$. Now if the wire be hotter than we reckon, we use a value of ρ less than the truth, $E-e$ is therefore too little, as we compute it; but E, as separately determined, is also too little, nay, even more so, because the current is stronger when the voltameter is not in circuit. Therefore e will be too great. To obtain access to the truth, it will be necessary, first, to determine the law of the cell's resistance as connected with its temperature; and secondly, to measure the wire's temperature not by an immersed thermometer, *but by its own expansion.*

Table my results, subjoining an explanation of each of its columns, and any miscellaneous facts which could not be easily tabulated.

TABLE.

No.	Obs.	F	T	L at 60°.	$\frac{dL}{dF}$	λ	Λ
1	10	6·8528	78°·1	775·24	. .	4·66	131·81
2	10	5·2015	85·4	722·70	20	4·26	136·34
3	10	4·6566	94·2	713·79	31	4·77	131·31
4	10	3·9366	65·1	677·00	50	5·46	125·51
5	15	3·5843	66·0	659·28	42	4·79	131·74
6	10	3·1303	72·4	645·14	32	5·52	131·07
7	15	2·5496	67·9	632·72	45	5·60	128·31
8	10	2·1769	65·5	610·02	78	4·30	130·80
9	10	1·8876	63·9	588·07	75	4·65	126·10
10	19	1·5384	62·7	568·51	99	4·74	. . . .
11	15	1·4107	61·5	553·81	102	4·56	134·16
12	20	1·2565	63·7	540·61	101	4·60	128·50
13	25	1·1071	62·9	521·12	128	4·87	131·41
14	25	0·9589	62·3	499·80	180	4·41	130·39
15	10	0·7909	62·3	462·53	264	5·77	131·14
16	15	0·6272	61·8	412·52	305	4·37	125·71
17	10	0·5482	62·0	388·42	340	4·59	124·88
18	10	0·4693	61·7	358·81	492	4·96	127·29
19	10	0·3921	62·0	312·04	638	4·38	116·12
20	10	0·3145	61·1	259·95	735	4·53	110·10
21	10	0·2340	61·1	195·49	799	5·06	90·78
22	10	0·1565	62·2	133·67	928	4·87	77·49
23	10	0·1164	60·2	93·79	1028	4·36	62·37
24	10	0·0794	59·2	54·61	1029	4·04	37·50
25	10	0·0389	59·3	14·18	621	4·12	10·08
26	50	0·0000	. . .	4·44	41	. . .	. . . .
27	10	− 0·0389	58·8	+ 2·19	49	+ 2·75	+ 3·28
28	5	− 0·0798	59·5	− 2·81	134	+ 3·00	− 3·67
29	5	− 0·1162	62·8	− 29·30	1148	− 1·81	− 11·19
30	5	− 0·1551	60·0	− 91·26	1503	− 3·79	− 47·77
31	5	− 0·2343	62·1	− 166·72	926	− 4·20	− 77·58
32	4	− 0·3140	62·1	− 238·46	776	− 4·69	− 92·76
33	5	− 0·3925	64·7	− 289·81	735	− 4·49	. . . . .
34	3	− 0·4707	61·2	− 353·64	. . .	− 6·49	− 114·39
35	5	− 0·6243	60·2	− 404·64	. . .	− 3·64	. . . . . .

The second column of this Table contains the number of experiments on which the value of L given in the fifth is based. In general, two sets of five each were taken on separate days, and if they were in close agreement this was thought sufficient. But it sometimes happens that though each set is perfectly consistent, the two differ as much as 20 lbs., which occurs especially when the magnet's lift is about half its maximum, at which point the coercive force of the iron seems to make some abrupt change. In these cases other sets were taken, till from the uniform spread of the differences I felt satisfied that I had obtained a fair average.

The third column, headed F, contains the values of the currents expressed in the unit which has been described above. It must be remembered, however, that they act on 638 spires. I consider them true to 0·001 at least of their assigned amount. The negative sign indicates that in these instances the direction of the current is reversed in the helices.

The fourth column gives T, the temperature of the magnet, as shown by a thermometer dipped in mercury, which filled the upper inch of the cavity in the northern cylinder of the magnet; at first both cylinders were tried, but this was found useless. It is necessary to know the temperature, for the force of electro-magnets, as of common ones, varies with it. To investigate the correction, 40 feet of leaden pipe, $\frac{3}{8}$-inch external, and $\frac{1}{4}$ internal diameter, were coiled on helices, containing 316 spires of the same wire used in the others, but coiled on tin tubes. These worms had each 25 turns; they were covered with thick cloth, and connected by a tube of vulcanized caoutchouc with a small boiler, so that a current of steam could be passed through them, and the condensed water escaped from their open extremity.* As the keeper and base (which were also covered with cloth) presented much cooling surface, the temperature could not be maintained above 180°, but could be kept very steady. The lift of the magnet being then determined, the magnet was left to cool, and the observation was repeated at the ordinary temperature, and with the same current as nearly as could be managed. As I was not aware of any reason for supposing that the effect of temperature changes its law under that of boiling water, I assumed the change to be as the temperature, or, L being the lift at 60°,

$$l = L \times \{1 + \tau(T - 60)\}.$$

* To the last it was turbid with sulphuret of lead, so that this material cannot be depended on as a conductor of steam.

Hence

$$\tau = \frac{l' - l}{l(T' - T)} + \left\{\frac{l' - l}{l(T' - T)}\right\}^2 \times (T - 60^\circ) + \&c.$$

Then I obtained

$$1. \begin{cases} l' = 461{\cdot}55 \;\ldots\; T = 167^\circ{\cdot}0 \;\ldots\; F = 1{\cdot}5830 \\ l = 480{\cdot}94 \;\ldots\ldots\; 69{\cdot}1 \;\ldots\ldots\; 1{\cdot}5868 \end{cases}$$

$$2. \begin{cases} l' = 583{\cdot}04 \;\ldots\ldots\; 174{\cdot}3 \;\ldots\ldots\; 2{\cdot}6198 \\ l = 601{\cdot}83 \;\ldots\ldots\; 70{\cdot}5 \;\ldots\ldots\; 2{\cdot}6198 \end{cases}$$

$$3. \begin{cases} l' = 646{\cdot}80 \;\ldots\ldots\; 175.4 \;\ldots\ldots\; 3{\cdot}5634 \\ l = 661{\cdot}62 \;\ldots\ldots\; 73{\cdot}8 \;\ldots\ldots\; 3{\cdot}5761 \end{cases}$$

$$4. \begin{cases} l' = 329{\cdot}39 \;\ldots\ldots\; 172{\cdot}5 \;\ldots\ldots\; 0{\cdot}8613 \\ l = 343{\cdot}08 \;\ldots\ldots\; 69{\cdot}3 \;\ldots\ldots\; 0{\cdot}8608 \end{cases}$$

Interpolating for the difference of F in each pair,* I obtain from these—

1. $\tau = -\ 0{\cdot}000385$
2. . . . $-\ 0{\cdot}000300$
3. . . . $-\ 0{\cdot}000220$
4. . . . $-\ 0{\cdot}000385$

Mean, . $-\ 0{\cdot}000322$

The three first might induce a suspicion that τ diminishes as F increases; but the fourth disproves this; and as the third set was less consistent than the rest, I regard the difference as mere error. I use the value 0·00033.

Subsequent to these experiments Dr. Lloyd has discovered that the inductive power of terrestrial magnetism is *increased* by a small elevation of temperature. Before this came to my knowledge I had applied the wire of these helices to other purposes, or I would have examined the coefficient τ at intermediate temperatures; I have, however, made a similar observation with respect to steel electro-magnets, and suspect it depends on the coercive force bearing a high ratio to the inducing force. In the present instances I do not

* The interpolation was deduced from a special series; these values of L not being comparable to those of the Table, as the helices have only half the number of spires, are of less diameter, and their tin bobbins add something to the mass of the magnet.

think any change of sign occurs: were it otherwise I must have noticed its effect; as in many of these experiments the magnet has been heated by the current above 100°, and an increase of L must have been produced contrary to all my experience. I may, however, have occasion to re-examine the question, and will not neglect it.

The fifth column gives L, the number of pounds required to lift the keeper, obtained by reducing the observed number to 60° by the coefficient τ. It may seem an easy matter to obtain this, but no one who has not tried it will be prepared for the many precautions that are necessary.

1. The utmost stability in the apparatus, absence of tremors, and delicacy of touch, are required. With a heavy lift, when approaching the limit of adhesion, the agitation caused by a step, the shutting of a distant door, or the action of a gust of wind on the building, will determine a break of contact, with a deficiency of 10 or even 20 lbs.

2. These magnets (and it is the case also with permanent magnets) will bear a much greater load if the strain be gradually increased, than if it be applied abruptly, the difference being sometimes 40 lbs. Therefore the weight of the steelyard must be slided along very gradually (and I need scarcely say with cautious handling), and allowed to rest at each step a few seconds, as it were, to let the acting forces adjust themselves. I do not see why this should be, unless, perhaps, the state of tension which is produced favours the development of magnetism, but the fact is very striking; when the keeper is detached and immediately replaced, it will not nearly resist the load, even if that be upheld, and then lowered to its bearing.

3. Time is an important element: I do not think any current which the wire of this magnet can conduct is capable of developing its full power in a few seconds. With the highest power which I have applied it must act for five minutes *at least*, and from $F=0{\cdot}3$ downwards for full fifteen. This has been noticed, though in a far less degree, by Faraday, who observed the circular polarization caused by the action of electro-magnets on dense glass to increase for a minute and a half after making the contact. That, however, is not a very delicate test; and as the poles of his magnet were not connected by a keeper, the molecular excitement must have been far less intense than in this case. As a specimen of this sluggishness of inductivity (which, by the way, is a

serious impediment to electro-magnetic engines), I give the set which first decidedly convinced me of its influence.

Time = 10^m . . .	L = 205·26 . . .	F = 0·2751
6	191·10	0·2779
13	213 52	0·2655
8	194·05	0·2603
12	207·62	0·2568

The increase of time more than compensates for a considerable diminution of current. I have regulated the duration of each set according to what I conceived to be a sufficient allowance of time.

4. These causes are uniform in their action, and can be avoided or corrected, but there exists another, which is the chief source of error in these experiments, namely, the molecular change which iron suffers when exposed to powerful magnetization. In consequence of this, however pure and soft it may be, it becomes capable of retaining permanent magnetism, *and in the same proportion less susceptible of excitation by its helices.* This magnetism (which I call λ) is variable ; it may, perhaps, be intense while the magnet is excited, but on lifting the keeper it declines rapidly till it attains a certain amount, which is, however, not invariable ; and it always increases during a set, though after a few hours it returns to its ordinary quantity. It, however, occasionally happens that when the magnet has been powerfully excited for many days, its iron becomes disturbed in this respect, and then the values of L fall far short of their legitimate magnitude. In such cases it is best to leave it at rest for a few weeks ; but I have found that if the current be reversed the L becomes higher, and have therefore in many instances performed this for the alternate measures. The results thus obtained are tolerably uniform, but are always less than those given by a magnet that has never been excited, or has been long in repose. With excitation less than what is given to this magnet by a current = 1, this cannot be done, because then it will be seen from the Table that there is a real difference between the L's produced by the direct and reverse currents. One consequence of this change deserves notice, which may be observed in almost every series,—the gradual decrease of the successive measures of a set. Thus in one taken with peculiar care,

Time $= 7^{m}$ . . . $L = 661{\cdot}65$ . . . F kept at 3·5938
7 . . . . 658·34
7 . . . . 648·10
9 . . . . 647·44
8 . . . . 644·58

From all this it follows that the values given in this Table can be offered only as a first approximation, but I hope a close one. The negative values imply that the polarity is reversed.

The sixth column contains the factor which gives the change of L due to a small variation of F. It is the coefficient of the first power of the variable in the formula for interpolation when the distances of the values from which it is derived are unequal; and besides its use in correction of results when there are small differences of power, it is given here, because it must be, nearly, what I call it at the head of the column, the first differential coefficient of L in respect of F, and as such may be useful in testing the hypotheses which we form respecting the functional relation of these quantities. Into this inquiry, I have stated that I do not intend to enter, and I will at present merely direct attention to the entire want of proportionality between L and F.

Ascending from No. 26 we find that a current = 0·04 produces a power of 10 lbs.; the addition of a second 0·04 adds 40; of a third, the same; after which the rate of increase goes on decreasing. No. 13 shows that a unit current will excite an L of 500 lbs., and No. 1 that one nearly sevenfold will add to this only its half. The result is even more striking if we consider column 6. Were L as F, the numbers there should be constant; whereas they decrease from 1028 to 20; and were the series continued upwards, must vanish at a certain value of L not very much greater than 800. This leads us to the remarkable conclusion that L cannot exceed a certain magnitude A, however intense the exciting power may be,* and as a necessary inference, that the separation of magnetic polarities has a limit. For if we revert to the conditions of an excited electro-magnet, which I have noticed at the commencement of this memoir, it is clear that at the surface of contact of the keeper we must have

$$0 = aH + bD - cM - eC.$$

* This has been announced by Mr. Joule (Phil. Mag.); it was, however, recognised by me long before I knew of his paper.

Now there is good reason to believe that C has a limit, or, in other words, that the molecules of iron can oppose only a limited resistance to induction ; D and M are as L, and therefore if H be infinite, so must the latter also be, unless M too have a physical limit. What that limit is,—whether the expansion of the hypothetic fluid, or the impossibility of exciting vibratory movement beyond a certain extent,—I do not pretend to determine. For this magnet I believe the A to be under 1000 lbs. Secondly, I would call attention to some other facts that seem important. At No. 26 is found 4·44 for L when there is no exciting force, it is the permanent magnetism λ. If we apply a direct current 0·04, it adds to this 9·74 ; if a reverse, it only subtracts 2·25 ; L therefore is a function of λ as well as of F. At Nos. 24 and 28 the differences are $+ 50{\cdot}17$, $- 7{\cdot}25$; at 23 and 29 still wider asunder, after which they begin to approach, but are not the same exactly till F passes $\pm 0{\cdot}7$. At $F = 1{\cdot}24$, the direct and reverse results are identical. It follows from this that the coercive force consists of at least two terms, one changing sign with F, the other depending on the habitual direction of the excitation ; the latter is not overpowered completely until $L = \frac{1}{2} A$, and does not vanish even when $L = 0$, as is manifest from Nos. 27 and 28. As a practical deduction, we may infer that in all machines involving the reversion of an electro-magnet's polarity, its excitation should not fall short of this. On the other hand, it should not much exceed it ; for the increase of power gained by a given increase of current is constantly lessening, and the consumption of materials augments even faster than the current.

The seventh column gives the value of λ the residual magnetism, which, for reasons already stated, should be known, in order to compare L with any formula. These numbers are given by from six to ten observations ; and it will be observed that they do not vary much. When the reverse happens (as in Nos. 4–7) it is evident from the irregularity of the quantity $\frac{dL}{dF}$, that the values of L are discordant from the rest of the series. It will also be noticed, in confirmation of what was stated as to the coercive force, that in No. 28 λ remains positive, although diminished, although L is negative. The highest value of it which I ever got is 8·88, which (the keeper being removed in the mean time) gradually decreased till, after 36 hours, it was permanent at 3·17.

The last column gives Λ, the residual excitation, or the force which the magnet retains when, after being excited, the current is withdrawn, and it is left with the keeper down, and which I consider to be the phenomenon that promises the most direct information as to the law of the coercive force. This state seems to continue for an indefinite time; at least I have never found any diminution of its intensity after many weeks. Accordingly, in observing it, I have either left it 10^m or during the night. It corresponds to the state $M-D=C$. During the previous excitation these forces were of much greater amount, and the force C had aided $M-D$ against H: when the latter is withdrawn, M re-unites a portion of the opposite polarities, and decreases in consequence, so of course does D. As to C, there is some reason for believing that it may in the first instance aid this re-union; and that a certain decrease of M is required to develop the molecular action on which it depends in the opposite direction. If so, it will depend not on the M which co-exists with it, but a previous one. However, we know that it must also soon begin to oppose the force $M-D$. Now it is obvious that if C were constant, the final value of $M-D$, and therefore of Λ, must also be so; if it were proportional to M, Λ would vanish; and if it were in any inverse ratio of it, the differences of L and Λ would lessen as they increased. On inspecting the Table it does appear that all above No. 15 may be considered of the same value, in its mean 130·68: which amounts to this, that the force C cannot arrest the decrease of the magnetism *as long as it exceeds half the maximum A.* Is it constant above this, where, as has been shown, it yields equally to excitation in either direction, or does it merely suffer there some abrupt change of magnitude? For lower values of L it decreases, bearing always an increasing ratio to it; thus in No. 21 it is nearly half, in No. 23 two-thirds; and in the negative values this continues to hold, although, as in the case of L, they are long less than the positive. If, while the magnet be in this condition, we pass through its helices a current that would in the ordinary mode give it a force equal to its Λ, its entire effect is not superadded to the other. Thus I found that, having passed $F=0·9864$, which on this occasion gave $\Lambda=125·18$, if I passed then $F=0·1395$, which would have produced $L=124·46$, I had $L'+\Lambda=169·81$; so that it only added 44·63. This was to be expected from the principles already explained; but I cannot so well explain an experiment which shows that a current which can give $L=\frac{1}{2}A$ produces the same results even if the magnet have residual excitation. If a negative current be passed, it destroys this condition, unless very feeble, but even then it lessens it; thus 0·0127 reduces Λ from 129·41 to 117·51. I may add, that even the fifteenth of this will excite this magnet, and change its residual magnetism.

While the magnet is thus circumstanced, it shows faint traces of free magnetism; each cylinder having its accustomed polarity at its acting surface, the opposite at its other extremity, and a neutral point in the middle. The ends of the keeper and base have the same polarities as those of the cylinders with which they are in contact. If one cylinder only be excited, the value of Λ is the same as for the two, but the distribution of the magnetism is modified, as might be expected.

XIV.—*Report on the Chemical Examination of Antiquities from the Museum of the Royal Irish Academy.* By J. W. Mallet, A. B., Ph. D.

Presented April 11, 1853.

THE examination of the antiquities to which the present paper refers was undertaken in the hope that more extensive and accurate chemical information, as to the nature of some of the materials employed by the craftsmen who so many centuries ago formed the numerous implements used for purposes of war and peace, which now are to be found in the Museum of the Academy, might be found of value in elucidating the history of the ancient arts by which these implements were produced.

This Museum has afforded peculiar facilities for a research of the present nature, as from the great extent and variety of the objects which it contains, and its general completeness as regards *Irish* antiquities, it was easy to procure a sufficient number of really typical examples in each of the departments examined, without injuring the collection of specimens, as such.

The greater number of the articles submitted to investigation were metallic; the universal applicability of the metals for the purposes of peace and war, of use and ornament, rendering everything calculated to throw light on the materials and processes employed in ancient metallurgy, most important and interesting.

The specimens of this class were most carefully selected, and may, I think, be fairly taken as types of this department of the Museum.

Commencing, then, with the ancient metals and alloys, the first to be described are the

GOLD ORNAMENTS,

of which class of Celtic antiquities I have seen no record of any previous analyses.

Of this metal I analyzed eight specimens, viz.:

No. 1. Fragments of one of the twisted "torques," supposed, I believe, to have been worn round the neck (Museum mark, 513 D). It consisted of a strip of thin plate gold, twisted so as to form a spiral, this being then bent into a circle, and the ends turned into two small hooks, by which the torque was clasped. The ornament had been broken up by the finder into pieces of about two inches long, but when entire its circle must have been ten inches in diameter. The part examined consisted of the two end hooks. The colour of the gold was a pale, rather sickly, yellow, and its specific gravity was 15·377.

No. 2. Fragment of a torque similar to No. 1, and most probably found along with it, in the county of Sligo; but the locality of neither is certain. Museum mark, 516 D. This specimen, which was of a rather deeper yellow colour than the last, was from the middle of the torque. Its specific gravity, 15·444.

No. 3. Part of a twist of wires of about a tenth of an inch in diameter each, the whole length of the twist, which is straight, being about six inches. Locality unknown. This may have formed part of a bracelet, but there is no second specimen in the Academy Museum, and from its workmanship it does not seem likely to be by any means of so ancient a date as the majority of these gold ornaments. The colour was a very deep rich gold yellow, and the specific gravity, 18·593.

No. 4. Two fragments of a lunette-shaped ornament, made of very thin gold plate, and having a little pattern round each edge. The whole must have measured ten or twelve inches across, and the greatest breadth of the flat plate itself was about two inches. It was probably a neck gorget, or ornament for the head, similar to many others preserved in the Museum of the Academy. The locality of the specimen is unknown. It is of about the same colour as standard gold, and of specific gravity, 17·528.

No. 5 was a small plate or spatula of gold, about an inch and a half long, and a quarter of an inch wide. It was probably *unmanufactured* gold, not intended for any special use in its present form. It is not known where it was found. The colour was a little lighter than that of No. 4, and specific gravity, 17·332.

No. 6. Fragment of very thin plate gold, which formed part of a boss or convex ornament, about four inches in diameter, very like those which cover the ends of the ornaments supposed to be diadems, in the Academy Museum. Locality unknown. It was of nearly the same colour with No. 4, and its specific gravity, 15·306.

No. 7. Specimen of supposed Celtic ring-money. It consisted of a bit of gold wire, of about three-fourths of an inch long, and nearly an eighth of an inch in diameter, bent into a circle, the ends being quite close, but not fastened to each other. It has been stated by Sir WILLIAM BETHAM* that the weights of these rings used for money were graduated with reference to the unit of twelve grains, or half a pennyweight, Troy. This specimen weighed 62·13 grains, or 2 dwt. 12 grs., five of Sir W. BETHAM's units, and 2·13 grs. over. Colour about the same as No. 5. Specific gravity, 17·258.

No. 8. Another specimen of ring-money. It was rather larger than No. 7, but composed of thinner wire. The colour was very much the same with the last, and specific gravity, 16·896. Its weight was 30·04 grains, which is exceedingly close to 1 dwt. 6 grs., or two and a half of Sir W. BETHAM's units. Hence it was about half the weight of No. 7. The localities where these specimens were found are not known.

The results of the analyses† of the gold ornaments were as follow:—

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
Gold, . . .	71·54	79·48	96·90	88·64	88·72	81·10	86·72	85·62
Silver, . .	23·67	18·01	2·49	11·05	10·02	12·18	12·14	12·79
Copper, . .	4·62	2·48	Trace.	·12	1·11	5·94	1·16	1·47
Lead, . . .	Trace.	. . . .	. . . .	. . . .	. . . .	·28	Trace.	. . . .
Iron, . . .	. . . . .	. . . .	. . . .	. . . .	·02	. . . .	Trace.	. . . .
	99·83	99·97	99·39	99·81	99·87	99·50	100·02	99·88

We observe here considerable diversity of composition, and on the whole

* Transactions of the Royal Irish Academy, vol. xvii. Antiquities, p. 7.

† The process of analysis calls for no particular remark, except that the gold was precipitated from the solution made nearly neutral by evaporation, by adding (hot) a slight excess of sulphate of ammonia, which re-agent throws down the metal in the form of a compact sponge, and does not produce the effervescence occasioned by oxalic acid.

the existence of a greater amount of alloy than one would expect from reading the accounts of gold ornaments to be found in various books on antiquities, in which they are often described as of "pure gold," "fine gold," &c., the colour being apparently very often the only guide to such a belief. Although the analyses here given differ much from each other, yet we find some traces of connexion between the composition of the alloys and the forms into which they were manufactured.

Thus, Nos. 1 and 2 are greatly below the standard of the others, and these are both specimens of the same kind of ornament, the torque, and the only specimens examined. They do not differ much from the composition of the electrum of the ancients, as given by Pliny and others.*

No. 6 is about on a par with these as to the quantity of gold, but contains a larger proportion of copper, and less silver.

Nos. 7 and 8 accord very closely with each other, a circumstance particularly interesting from the probability of their having been used as money. For if, as is urged in the memoir on this subject before referred to, these rings really represent a metallic currency of graduated weight, based upon a fixed standard, it surely would be a strong confirmation of this opinion, as well as a fact highly illustrative of the advanced state both of commerce and of metallurgical skill on the part of the fabricators of these rings, if they were shown to be also of constant composition, and that therefore their relative values were actually represented by their proportionate weights. To decide this question, however, more numerous analytical results would be indispensable.

Nos. 4 and 5 in the Table also agree very closely, from which we might surmise that the latter, which probably was not intended for use in the condition in which it was found, was perhaps in process of manufacture into one of the thin lunette-shaped ornaments, like No. 4, which have often been found in Ireland. Its small size, however, renders this more doubtful.

No. 3 is of a much higher standard than any of the others, and approaches fine gold. From its being wire-drawn in the ordinary way through a draw-plate, it is probably not nearly so ancient as the other specimens examined.

* "Ubicunque quinta argenti portio est, electrum vocatur."—*Plinii Hist. Nat.* lib. xxxiii. c. 4. "Alia (species electri) ex partibus auri tribus et una argenti conflatur."—*Margerit. Philos.* 1523.

In the earliest ages wire appears to have been made by cutting thin plates of metal into strips* and rounding these upon the anvil; and Beckmann, in his History of Inventions,† seems to think that the modern method dates no earlier than about the middle of the fourteenth century.

If these ornaments presented no appearance of determined composition, and on the whole contained less silver, it might be supposed that they were made of native gold, merely fused, and worked into the required shapes; but from the results actually obtained, although they are by no means conclusive on this point, I think it appears more likely, on the contrary, that these articles were made from alloys artificially produced, and perhaps from determinate quantities of the constituent metals. If this supposition be correct, no information can be derived from these analyses as to the *geographical source* of the surprising quantity of gold found in the manufactured state in Ireland. In Cornwall along with the stream tin, in Scotland, and in much larger quantity in Ireland itself, in the county of Wicklow, native gold has been obtained, and this metal (as well as silver, iron, tin, and lead) is mentioned by Strabo‡ among the products of Britain. It is therefore conceivable that much of the precious metal used in this country may have been found at home, though its quantity would seem to indicate foreign commerce as the more likely channel by which it was procured, unless native gold was anciently much more abundant in Ireland than it has been in more modern times.

SILVER ORNAMENTS.

These are much rarer in Ireland, and throughout the north of Europe, than those of gold, as indeed might be expected in collecting the relics of so distant a period, when we consider that the latter metal occurs, it may be said, invariably in the native state, while the former is found so but rarely, and, in Europe at least, not in any very great quantity; and that the silver ores from which it is most abundantly obtained require the application of much metallurgical skill for the extraction of the metal. Apart, too, from the initial difficulties attendant upon the smelting of its ores, silver, when obtained, is by no means so malleable or easily worked as gold, a circumstance which in some

* Exodus, xxxix. 3. Homer, Odyss. lib. viii. 273–278. † Vol. i. Art. Wiredrawing.
‡ Lib. iv. 30.

degree accounts for the rude workmanship of very many of the Celtic antiques of this material.

Of the specimens in the Royal Irish Academy collection I selected and analyzed the following eight:—

No. 1. A small ingot of silver, cast in an open mould. Museum mark, $\frac{703}{c}$ $\boxed{157}$. It was of a long oval shape, about two inches in length, and half an inch wide. It had two small nicks in one side, close together, as if to mark its weight or value. Its weight was 377·23 grs. = 15 dwts. 12 grs. (+ 5·23 grs.). Hence if used for money it would have been equivalent to 31 or perhaps to 32 of the half-pennyweight units. It was a little tarnished by superficial sulphuret of silver. Its specific gravity = 10·225.

No. 2. A piece of *hexagonal* wire, very neatly made, probably by hammering, about an inch and a half long, and an eighth of an inch in diameter. Museum mark, 809 c. It was bent into the shape of a horse-shoe, and the ends were cut sharply off, so as to induce the belief that it too may have been used for money. It weighed 103·86 grs. = 4 dwts. 6 grs. (+ 1·86 grs.) or about eight and a half units. Its specific gravity = 10·253.

No. 3. End of a taper bangle or penannular bracelet of very rude workmanship. Also, perhaps, occasionally used as money. It was very hard, and rather brittle, breaking with a fine earthy fracture of a yellowish white colour. Specific gravity = 8·770.

No. 4 was a specimen which appeared at first sight to be part of a flat silver bracelet or armlet, stamped with the triangular indentations so common on the silver ornaments of Celtica and Scandinavia, and not broken, but cut across at the ends. Museum mark, $\frac{702}{c}$. On attempting to cut it again, however, it turned out to be a counterfeit, consisting in fact of a core of iron covered with an exceedingly thin plate of silver, which was so skilfully joined as to deceive the eye even on careful observation. This imitation of articles in the precious metals has been observed before in gold rings, which are sometimes found on a thin shell of the valuable material covering a large core of copper or occasionally of lead; but I can find no recorded instance of silver counterfeits of this kind being found among presumed early Celtic antiquities. The iron core of this specimen was much corroded, and the silver was tarnished by sulphuret. Specific gravity of the silver = 10·379.

No. 5. Fragment of a flat armlet, broken across at the ends, and stamped with small square indentations. Museum mark, $\frac{490}{c}$. There were traces of chloride of silver upon the surface, which was much worn. Specific gravity = 10·335.

No. 6. Two fragments of round wire, forming part of a torque large enough for the neck. They are stamped with a small pattern of alternate squares and little pellets in relief. Specific gravity = 10·519.

No. 7. Two fragments of square wire, part of a number of wires twisted into a spiral bundle so as to form an almost solid cylinder. The twist formerly united two silver boxes covered with filigree work. Specific gravity = 10·468.

No. 8. Part of the hinge of a chased hollow bangle, said to resemble common modern Egyptian workmanship; found, it is believed, along with No. 7, and numerous other articles, in a railway cutting near Navan. Museum mark, $\frac{490}{c}$. The silver seems superior in malleability to that of any of the other ornaments examined. Specific gravity = 10·198.

The analysis of these specimens gave the following results:—

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
Silver, . .	93·93	79·84	37·05	94·69	94·01	92·82	95·87	92·38
Copper, . .	5·44	17·73	60·26	3·11	4·34	5·85	3·59	7·21
Gold, . . .	·42	1·34	1·11	1·80	1·31	·89	·17	·30
Lead, . . .	. . . .	Trace.	·10	. . . .	·06	. . . .	. . . .	. . . .
Tin, ...	. . . .	. . . .	·61	. . . .	. . . .	. . . .	. . . .	. . .
Iron, . . .	. . . .	. . . .	. . . .	·04	. . . .	. . . .	. . . .	. . . .
Sulphur, .	. . . .	. . . .	. . . .	Trace.	. . . .	. . . .	. . . .	. . . .
	99·79	98·91	99·13	99·64	99·72	99·56	99·63	99·89

The composition of these silver articles does not seem so varied, nor is there the same agreement between the constitution of the particular specimens destined for the same use, as in the case of the gold ornaments. With the exception of Nos. 2 and 3, the whole set contain from 92 to 96 per cent. of silver with 7 to 3 per cent. of copper, and a little gold. The copper might certainly have been derived from the silver ore smelted, and exist in purely accidental quantity; but this seems improbable from the very small quantity of lead detected, as this metal is a much more frequent mineralogical concomitant of silver than copper is, and from the fact that at least some of the ancients treated their silver ores, just as at the present day, with lead, either in the metallic state, or as sulphuret* (Galena), and subjected the alloy thus obtained to cupellation, which latter process would of course remove the copper. The silver was therefore, in all probability, intentionally alloyed. If *all* these ornaments were, though used as such, occasionally employed also for money, as WORSAAE† and others seem to think, one would be led to suspect that with the silver, as with the gold ring-money, something like a recognised standard metal existed when these articles were in use. This may perhaps be too hasty a conclusion.

Nos. 2 and 3 of these specimens are the only ones which differ remarkably in composition from the others, especially No. 3, which actually contains one-half more copper than silver, though preserving the colour and general appearance of the latter metal. Some, at least, of this large quantity of copper was probably added in the state of bronze, as shown by the presence of a little tin in the silver alloy. On dissolving the silver in nitric acid, the tin remained behind, with the gold forming a "purple of Cassius," of a very good purple colour verging on red.‡ The uniform presence of a little gold in all the silver articles examined is not surprising, since the ancients were, it is almost certain, unacquainted with the process of parting.

I may mention here, in connexion with the silver antiquities, a bluish semi-metallic substance, something like dull or tarnished steel, but very much softer, and brittle, used in the inlaying of small shrines, relic-cases, croziers, &c., of the middle ages. It has not much lustre, but from its colour contrasts very well with either silver or brass, into works in which latter metal it was frequently introduced, but always sparingly. I had very little material to operate on, only about a grain and a half, and consequently was unable to do more

* "Excoqui non potest nisi cum plumbo nigro, aut cum vena plumbi. Galænam vocant, quae juxta argenti venas plerumque reperitur."—*Plinii Hist. Nat.* lib. xxxiii. c. 6.

† Primeval Antiquities, pp. 59, 60.

‡ "Purple of Cassius" was obtained in a similar way by M. H. Feneulle (*Ann. de Chim. et de Phys.*, xxxii. 320), on dissolving a number of ancient Roman silver coins in nitric acid. The results of his rather numerous analyses certainly seem to prove that the *Romans* fixed no standard for their silver coinage.

than analyze it qualitatively. It was of a dark bluish-gray colour, approaching black, very brittle, and exhibited a small lamellar fracture when broken.

It consisted, as had been previously known, or at least generally supposed, for the most part of silver; but contained besides, antimony, sulphur, and traces of lead and copper. It may very probably have been made by the partial reduction of some of the antimonial ores of silver, its sole essential constituents being, I believe, silver, antimony, and sulphur.

Having discussed the results of the examination of the antiquities of gold and silver, which principally belong to the class of ornaments, the metallic remains next to be considered are those composed of the important alloy bronze, in primitive ages the universal material for all instruments or utensils in which tenacity and hardness were required, and that are formed of iron in more modern times.

WEAPONS AND IMPLEMENTS OF BRONZE.

These, the forms of many of which are very peculiar, and sometimes very beautiful, and their workmanship frequently such as would not disgrace the artificers of the present day, have early directed the attention of archæologists to the processes used in their formation by the smiths and metallurgists of the epoch to which they belonged. Hence we find several inquiries, more or less extended, on record, aiming at an elucidation of some of these processes by the assistance of chemical analysis. Thus, of specimens found in the British Isles, Mr. ALCHORN,* His Majesty's Assay-master in 1774, examined two bronze swords found in a bog at Cullen, Co. Tipperary, and announced as the result, that the metal was "chiefly copper, interspersed with particles of iron, and perhaps some zinc, but without containing either gold or silver;" adding, "But I confess myself unable to determine anything with certainty."

In 1796, DR. PEARSON† communicated to the Royal Society an account of his analysis of seven specimens of bronze, found in the bed of the river Witham in Lincolnshire, in which he found, copper, 85·7 to 91 per cent; tin, 14 to 9; and in one instance, 0·3 of silver. In 1816, Professor CLARKE,‡ of

* Archæologia, vol. iii. p. 355. † Philos. Trans. 1796. ‡ Archæologia, vol. xviii. p. 343.

Cambridge, analyzed portions of bronze vessels found near Sawstone, Cambridgeshire, and found them to consist of 88 per cent. of copper, and 12 per cent. of tin. This same composition has recently been found for Irish specimens by Dr. ROBINSON of Armagh.* Professor E. DAVY,† Mr. O'SULLIVAN,* Mr. DONOVAN,* and Mr. J. A. PHILLIPS,‡ have published more complete analyses of antiquities from the latter country, in which foreign metals (as lead, silver, and iron) have been carefully sought for, and their quantity determined; and a similar accurate examination of Scottish relics of bronze has been made by Mr. WILSON, Hon. Sec. of the Society of Antiquaries of Scotland.§ Of these investigations that of MR. PHILLIPS is the most important as regards Ireland.

The specimens from the Museum of the Academy which I have examined were all found in Ireland; and are, as a group, completely illustrative of the principal classes of antiquities belonging to that country. They are sixteen in number.

No. 1. A flat celt or kind of hatchet, the most common weapon of bronze found in Ireland, Museum mark, $\frac{639}{B}$. This specimen was discovered, it is believed, in the county of Cavan; it is a fine hard bronze, of a deep brass-yellow colour, the "Celtic brass" of antiquaries. It was in excellent preservation, being scarcely even tarnished on the surface. Specific gravity, 8·631.

No. 2. Another flat celt, with rounded edges; locality unknown. Museum mark $\frac{433}{B}$ $\boxed{395}$. It was slightly and uniformly corroded on the exterior, and on being filed proved to be a much softer bronze than No. 1; of a copper-red colour, a little lighter than that of pure copper. Specific gravity, 8·303.

No. 3. A long hollow celt, resembling in shape specimens which have been found in Denmark, discovered in the county of Wicklow. Marked, M'Enty. It was a hard and rather brittle bronze, of about the same colour as No. 1; slightly and uniformly corroded. Specific gravity, 7·960.

No. 4. A short hollow celt, of very good workmanship, and exhibiting scarcely a trace of corrosion; supposed to be from the county of Cavan. Museum mark,

* Proceedings of the Royal Irish Academy, vol. iv. pp. 430–469.

† Wilson's Archæology of Scotland, p. 247.

‡ Quarterly Journal of the Chemical Society, October, 1851.

§ Wilson's Archæology of Scotland, p. 245.

Farnham, 38. The metal was very soft, and resembled No. 2 in colour, but was not quite so red. Specific gravity, 8·428.

No. 5. A long spear-head, ribbed upon each side; of excellent workmanship, and not at all corroded. Mark, $\frac{93}{B}$. The bronze was hard and uniform, and had received and retained a very good edge; colour about the same as No. 1. Specific gravity, 8·581.

No. 6. Portion of a spear-head, marked $\frac{114}{B}$; a flat, thin blade, with a beautiful edge; the surface perfectly smooth and polished, but tarnished of a deep brown colour, resembling, I believe, the appearance of the bronzes called "Cinque cento." This skin of brown upon the outside was eaten through in some places by superficial corrosion. When filed, the metal was found to be exceedingly hard, and of a yellow colour, something deeper than No. 5. Specific gravity, 7·728.

No. 7. A flat scythe, found in the county of Roscommon. Museum mark, $\frac{976}{A}$. Several similar articles were found with this; they were slightly curved blades, of about twelve or fourteen inches long, and tapered in breadth from about three inches at one end to a rounded point at the other. They had been attached to a handle at the broad end by three rivets. The specimen examined was a copper-coloured bronze of no great hardness, and but slightly corroded on the surface. Specific gravity, 8·404.

No. 8. Portion of a sword-handle; locality unknown. Museum mark, $\frac{891}{A}$ $\boxed{182}$. It was part of the characteristic Celtic weapon, in which the heft of the handle and blade were cast in a single piece, the former part being generally remarkable for its shortness as compared with those of modern times. This specimen was made of a beautiful compact metal, very hard, and of a yellow colour like that of No. 1, but a little deeper. No corrosion upon the surface. Specific gravity, 8·819.

No. 9. Part of the blade of a sword of the same character as the last, but made from a metal by no means so hard or good. Mark, $\frac{908}{A}$ $\boxed{188}$. It was similar in colour internally to No. 8, but was much more corroded on the outside. Specific gravity, 8·487.

No. 10. Portion of a dagger or Irish knife (found near Newry?) Marked $\frac{4}{B}$. A good hard bronze, very like No. 8 in colour and external appearance, and rather more malleable. It was scarcely tarnished. Specific gravity, 8·675.

No. 11. Fragment of a chisel, marked $\frac{198}{B}$, made of very inferior bronze, copper-coloured, soft, and not uniform in texture. It contained cavities produced by air-bubbles in the casting, and was very much corroded; oxide of tin, carbonate of copper, and the *red dinoxide of copper* were observable on the surface. Specific gravity, 7·896.

No. 12. Specimen of the bronze ring-money which is found in such great quantity in Ireland. It was a small ring (about an inch in diameter), in form a simple circle, cast in a single piece, and having no opening like those in the specimens of gold and silver, by which the rings might be strung together into a chain. Its weight was 100·53 grs. = 4 dwts. 4·53 grs., or about eight of the units spoken of before. The bronze was moderately hard, of a deep brass-yellow colour, very little corroded, but having a slight film of green "ærugo" on the surface. Specific gravity, 8·072.

No. 13. Specimen of ring-money of a larger size than the last, being about two inches in diameter: locality unknown. It differed from it also in being *hollow*, cast upon a core of fine siliceous sand, which had not been extracted, but remained firmly imbedded in the bronze. There were two small projections on opposite edges of the ring outside, and the metal and core were pierced at these bosses by a hole apparently intended to allow a string to pass, by which the rings might be strung together. If used as money, this method of attaching the separate pieces would be certainly less convenient than merely stringing the rings themselves through the centre. Might not these articles have been made as parts of necklaces, or other ornaments for the person, though perhaps also used occasionally as a circulating medium? The weight of the specimen, *including the sand core*, = 388·43 grs. = 16 dwts. 4·43 grs. = about 32 of the half-pennyweight units. The bronze was very like that of No. 12, but much more brittle. Its surface had been smooth and polished, but was slightly pitted in some places by corrosion. Specific gravity, 8·231.

No. 14. Fragment of a large cauldron or tall vessel of thin sheet bronze. From its size (about 2 ft. 6 in. high), and the thinness of the plates of which it was made, it displays a degree of skill and neatness in the treatment of bronze most remarkable as existing at so early a period as this vessel probably belongs to. The metal is not very hard, but extremely tough, and is of a beautiful rich bronze-yellow colour ("gold bronze"), scarcely altered by time. Specific gravity, 8·145.

No. 15. Portion of a small oval-shaped bell, made of a deep yellow bronze or bell-metal, hard and brittle. The surface was rough, but not much corroded. Specific gravity, 8·094.

No. 16. Fragment of a small square bell; the metal about as hard as No. 15, nearly of the same colour also, but not so brittle. It was more corroded, and did not seem so good a material for the purpose. Specific gravity, 7·708.

These specimens, being carefully analyzed, gave the results contained in the following Tables.

(In each case a minute qualitative analysis was first made, and the absence of other metals than those afterwards estimated in quantity ascertained).

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
Copper, ...	86·98	98·74	88·30	95·64	86·28	84·64	95·85	87·07
Tin, ...	12·57	1·09	10·92	4·56	12·74	14·01	2·78	8·52
Lead, ...	. . . .	. . . .	·10	·25	·07	. . . .	·12	3·37
Zinc, ...	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
Iron, ...	. . . .	·08	Trace.	. . . .	·31	Trace.	1·32	. . . .
Cobalt, ...	. . . .	. . . .	. . . .	. . . .	·09	. . . .	. . . .	. . . .
Gold, ...	Trace.	Trace.	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
Silver, ...	·37	·06	. . . .	·02	. . . .	. . . .	. . . .	. . . .
Arsenic, . . .	. . . .	. . . .	Trace.	. . . .	. . . .	. . . .	. . . .	. . . .
Antimony, . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
Sulphur, . . .	. . . .	. . . .	. . . .	. . . .	. . . .	·03	. . . .	Trace.
	99·92	99·97	99·32	100·47	99·49	98·68	100·07	98·96

	No. 9.	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.
Copper, ...	87·94	90·72	91·03	87·24	85·89	88·71	84·78	85·92
Tin, ...	11·35	8·25	8·39	9·58	13·83	9·46	12·90	11·73
Lead, ...	·28	·87	. . . .	2·79	Trace.	1·66	2·04	1·72
Zinc, ...	Trace.	. . . .	. . . .	Trace.	. . . .	. . . .	·06	. . . .
Iron, ...	. . . .	. . . .	. . . .	. . . .	. . . .	·03	. . . .	Trace.
Cobalt, ...	. . . .	. . . .	Trace.	. . . .	. . . .	. . . .	. . . .	. . . .
Gold, ...	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
Silver, ...	. . . .	. . . .	. . . .	Trace.	. . . .	. . . .	Trace.	. . . .
Arsenic, ...	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	·03	Trace.
Antimony, . .	. . . .	. . . .	·04	. . . .	. . . .	. . . .	. . . .	. . . .
Sulphur, . . .	Trace.	. . . .	. . . .	. . . .	Trace.	. . . .	. . . .	. . . .
	99·57	99·84	99·46	99.61	99·72	99·86	99.81	99·37

The composition of these specimens agrees *in general* with that of the articles which have been examined by the authors referred to before, and also accords pretty closely with the quantities synthetically employed by the ancient metallurgists of whose labours we have any account. Thus, Pliny tells us* of the method adopted in his day for making bronze, which, however, he obviously treats of principally as a material for statues and public monuments. "Massa proflatur in primis, mox in proflatum additur tertia portio æris collectanei, hoc est, ex usu coempti. Peculiare in eo condimentum atritu domiti, et consuetudine nitoris veluti mansuefacti. Miscentur et plumbi argentarii pondo duodena ac selibræ, centenis proflati. Appellatur etiamnum et formalis temperatura æris tenerrimi, quoniam nigri plumbi decima portio additur et argentarii vigesima: maximéque ita colorem bibit, quem Græcanicum vocant. Novissima est quae vocatur ollaria, vase nomen hoc dante, ternis aut quaternis libris plumbi argentarii in centenas æris additis. Cyprio si addatur plumbum, color purpuræ fit in statuarum prætextis." The analyses also of antiquities, not of Celtic origin, as those by KLAPROTH,† DIZE,‡ MONGEZ,§ GÖBEL,|| and pupils of ERDMANN,** all approach each other within rather narrow limits, and differ little from those at present under consideration.

The cause of this general accordance is obvious, namely, that the physical properties required in these alloys of copper and tin are only to be found within a small range of variation in chemical composition. Dr. ROBINSON†† has given it as his opinion, that, when used for weapons, the atomic constitution of Celtic bronze was constantly 14 Cu + Sn; but though this formula may and probably does represent the best alloy for the manufacture of implements for warlike purposes, or others in which similar requirements exist, yet it cannot be said that it invariably accords with the actual composition of the antiquities in question, as No. 6 of the analyses in the Table is near 11 Cu + Sn, while No. 4 approaches 39 Cu + Sn, and Nos. 2 and 7 contain still more copper, though perhaps the former of these should hardly be considered as bronze.

As on the one hand we must not conclude that a simple and invariable

* Hist. Nat. lib. xxxiv. c. 9.

† Gehlen's Journal, No. 15, and Journal des Mines, Mars, 1808, p. 161.

‡ Journ de Phys, 1790.

§ Mém. de l'Instit.

|| Schweig. Journn. 60, 407.

** Journ. für pr. Chem. xl. 374.

†† Proc. Royal Irish Acad., vol. iv.

proportion existed in the quantities of the component metals of these bronzes, so, on the other, it seems erroneous to infer, as Mr. Wilson, in his Archæology of Scotland* has done, that the absence of such invariable composition necessarily proves that these antiquities were the work of native artists, who were unable to combine the metals they used with the accuracy and certainty of foreign metallurgists of the same epoch. For, passing over all the difficulties which the primitive modes of reduction of the respective ores, and the impurities consequently retained by the metals, must have presented to the early manufacturers of any nation, and supposing the copper and tin used to be each perfectly pure, the task of producing from these materials an alloy of definite and uniform composition is, even at the present day, one requiring great skill, both in the actual process of melting, and in the previous construction of furnaces, &c., so that for the purpose and at the period of the manufacture of the articles in question it must almost be deemed impossible of accomplishment.†

Hence the observed variations of composition between the several bronze antiquities found in Ireland by no means negative the possibility, to say the least, of their having been produced by a single people, and that one, far advanced in the art of metallurgy for the age to which these articles are referred.

Yet, although these differences of composition are probably to a very great extent owing to the want of sufficient skill and appliances to produce from the same materials uniform results,—difficulties enhanced to the last degree where many *small* articles are to be cast, at separate operations, and from alloys formed in small quantities,—and are therefore to be looked upon as unintentional,—yet some marks of design may perhaps be traced in the differences between the alloys of articles intended for different purposes. Thus we find two of the celts (Nos. 2 and 4) and the war scythe (No. 7) consist almost entirely of copper, the quantity of tin amounting in No. 2 to only 1·09 per cent., a proportion so small that it might be supposed to be derived merely from the addition of fragments of old bronze to the copper, or from imperfect reduction of the ore.

* Page 249.

† The principal difficulty arises from the "burning out" of the tin, which takes place with great rapidity on access of air to the melted bronze. The column in the Place Vendome, Paris, was a remarkable instance of mismanagement in this respect, almost the whole of the tin having disappeared from the metal employed to cast some of the blocks of bronze.

The only analysis of Celtic bronze containing so much copper, that I have seen, is that of a broken spear-head found in Ireland, examined by Mr. PHILLIPS,* from which he obtained 99·71 per cent. of copper, and ·28 of sulphur. The composition of the metal used in casting the celebrated Quadriga of Chios (better known in this country as the Horses of St. Mark's, Venice), as determined by KLAPROTH,† was also very near that of the present specimens, being 99·13 per cent. copper, and ·87 tin. Some ancient nails analyzed by the same chemist,‡ and Greek and Roman coins examined by Mr. PHILLIPS,§ and pupils of ERDMANN,‖ also appear to have been made from nearly pure copper.**

The other celts (Nos. 1 and 3), one of the spear-heads (No. 5), and one of the swords (No. 9), agree pretty closely in composition; containing about 87 or 88 per cent. copper, and 13 or 12 of tin, if we disregard all traces of foreign metals. This is the composition assigned by Dr. ROBINSON as the best for the purpose, and his opinion seems in fact borne out by the results before us, as the specimens numbered 1, 3, and 5, were certainly very far superior to most of the others in hardness, toughness, and uniformity; and the alloy having this constitution may be considered as the normal one, *at least where other metals than copper and tin are present only in insignificant quantity;* for 2 or 3 per cent. of a foreign metal, as lead, seems to exercise a very great influence in changing the character of the whole. Thus the sword (No. 8) in which the tin amounted to only 8·52 per cent., but which contained besides 3·37 per cent. of lead, fully equalled the weapons just mentioned in hardness, and perhaps even exceeded them in malleability and facility in working.

The second of the two spear-heads (No. 6) was exceedingly hard, and had received a good edge, but it had not the same toughness as the others, and had broken across without bending; hence so large a proportion of tin as it contains, 14·01 per cent., does not seem to yield a metal so well adapted for weapons.

The two daggers or knives (Nos. 10 and 11) agree very closely in composition, yet the difference in physical properties is most marked.

* Quart. Journ. Chem. Soc., *loc. cit.* † Beiträge, vi. 89. Gehlen's Journal, No. 15.

‡ Gehlen's Journal, *loc. cit.* § *Loc. cit.* ‖ Journ. pr. Chem. xl. 374.

** The presence of sulphur is highly interesting as giving the strongest presumption that the copper of these ancient alloys was obtained from the imperfect reduction of sulphuretted ores of that metal.

No. 10 was a bronze of excellent quality, a little softer than No. 8, but still sufficiently hard, tough, and uniform, and not at all corroded; while No. 11 was soft, full of cavities, not uniform in texture, and was covered with the results of corrosion. (Of course in this, as in every other instance in which the corrosion of metals is examined, regard ought to be had to the situation in which they have been discovered; but unfortunately, in the present case, information on this head is entirely wanting.) Additional analyses of very inferior bronzes and those which have suffered most from corrosion, taking care to examine fragments taken from different parts of the same article, might yield results of interest, and possibly of practical importance.

The cauldron or vase of thin sheet bronze (No. 14) contained about the same per-centage of copper as the bright yellow-coloured alloy for weapons (rather more than 88 per cent.), but not quite so much tin, its place being partly supplied by about 2 per cent of lead, which tends to make the alloy more malleable.*

The two specimens of ring-money (Nos. 12 and 13) contain quantities of copper differing by nearly 1·5 per cent., while the proportion of tin varies still more, being 9·58 per cent. in the former, and 13·83 in the latter. In the one we find 2·79 per cent. of lead, but in the other a mere trace of that metal is perceptible. Hence, it is obvious that, as far as these specimens are to be considered as representing the ancient Celtic currency of bronze, no very accurate standard of alloy was observed in its production. This is not surprising; the formation of such a definite and constant alloy being, as above mentioned, attended with so much difficulty, and the inferior value of the material rendering it by no means as important as in the case of gold or silver. The rings being merely cast, and not struck like ordinary coins, the physical properties of the metal did not need so much attention as in the case of arms or implements, where they were of the first importance.

The samples of bell-metal examined, numbered 15 and 16, differ but little from each other. The quantity of copper is considerably greater than that generally employed at the present day. This may have been owing to the desire of the early artists to avoid brittleness in the metal, but more probably to the

* Modern workmen are well aware that the addition of about 2 per cent. of lead greatly improves the working qualities of brass and bronze, causing both to cut smooth and sharp in the lathe, or, as French brass-workers express it, to cut "*seché*."

"burning out" of the tin by some rude and slow process of melting. The composition of these specimens is very simple, copper and tin being, as in all the other ancient bronzes, almost the sole constituents. Modern bell-metal is occasionally much more complex, containing, according to THOMPSON,* 80 per cent. of copper, 5·6 of zinc, 10.1 of tin, and 4·3 of lead.† Those Irish bells are, it is needless to say, of much more recent date than the bronze weapons, and belong, it is believed, to about the eighth or ninth century.

With respect to the foreign metals found in minute quantities in these alloys, although, with the exception perhaps of lead, they may all be fairly considered as accidental, and merely introduced as impurities of the constituent metals, yet they are not to be neglected, as in the consideration of antiquities they may occasionally yield some valuable collateral information. The lead which is found in many of the specimens, and has been previously detected in much larger quantity in some bronzes, might have existed as an impurity of the copper, but was more probably added either intentionally in the separate state, or existing in older bronze remelted, which often contained this metal, particularly when used for statues. *Zinc* was only observable in minute traces in three of the bronzes, but its presence in these *was distinctly ascertained.* In all probability it was introduced along with the copper, and was derived from blende occurring along with the ore of that metal, and imperfectly separated from it. Though found in large quantity in some early Roman coins, and by GÖBEL‡ in wire from a Livonian tomb, I believe it has not before been detected in Celtic bronze. Iron might have come in with either of the constituent metals, and has been observed in previous analyses. Indeed, from its universal diffusion in nature, its absence in these alloys would be more surprising than its presence. Our finding the rarer metal, cobalt, though only in two instances, is more remarkable; these, however, are not the only antique bronzes in which it has been observed, as Mr. PHILLIPS, in his valuable paper

* Ann. Phil. 2. 209.

† M. GIRARDIN found the composition of the "cloche d'argent," an ancient bell at Rouen (previously thought to have contained a large proportion of silver, from oblations made at the time of its founding), to be, copper, 71; tin, 26; zinc, 1·80; and iron, 1·20 = 100. (Ann. de Chim. et de Phys. 50. 205.) The proportion of copper here is very small.

‡ Schweig. Journal, 60. 407.

alluded to above, notices it and nickel as occurring in several early coins, and in one Irish specimen, a celt, to the amount of 0.34 per cent. (with a trace of nickel). The minute quantities of the Precious Metals were perhaps derived from fragments adhering to old ornaments of bronze, which were afterwards re-melted. It has been supposed that traces of silver found in one or two previous analyses were owing to the lead not having been freed from this metal, and this was probably often the case, but in two instances here (Nos. 1 and 2) it could not have been so, as no lead was present. Arsenic and antimony have not, I believe, except in one instance, been hitherto noticed in similar alloys,* and existing in such very small quantity, are not easily detected; but by employing a separate portion of bronze for the purpose, I determined rigidly the question of their presence or absence. The source of these traces found in the alloy is not difficult of explanation. Indications of sulphur (and of carbon, by Mr. DONOVAN) have been observed in several specimens previously analyzed; these, and the traces of arsenic and antimony, are interesting, as rendering it at least probable that some of the copper used by the ancients was smelted from sulphuret of copper or copper pyrites (probably the chalcitis or misy of Pliny), or other ores of the same class, and that native copper, red oxide of copper, and malachite, did not, as some authors seem to suppose, constitute their only sources of the metal.

The question, from what countries were the copper and tin, employed to such an immense extent by the nations of antiquity, derived, is one of great interest, and has been already treated of, especially with reference to the source of the tin, by several authors of celebrity. They seem generally agreed that the former of these metals was discovered, and extracted at a very early period in several places in the south and east of Europe, and adjoining portion of Asia. At the period of the Trojan War, and at the time of the building of Solomon's Temple, the supply of copper must have been most abundant, and the name frequently occurs in the Pentateuch. The art of casting statues of bronze is ascribed by Pausanias to Rhœcus and Theodorus of Samos (about 700 or 800 B. C.), at which time it must of course have become common, though we have

* Jahn (Ann. Pharm. 27·338) found 8·22 per cent. of antimony in an ancient weapon from the ruined castle of Henneberg.

but little knowledge of the localities from which it was derived. When Pliny wrote, its principal sources were Cyprus, Campania, Gaul, and Spain, especially the last-named country. Although England now supplies a very large proportion of all the copper made use of in the world, there are no traces in history of any having been smelted here so early as the Celtic period, and the contrary seems to be proved by a passage of Cæsar, De Bell. Gall.,* where he says of Britain: "Nascitur ibi plumbum album in Mediterraneis regionibus, in maritimis ferrum, sed ejus exigua est copia, *ære utuntur importato.*" Strabo† also enumerates gold, silver, iron, tin, and lead, among the products of Britain, but does not mention copper, the sixth of the then well-known metals, which it is improbable he would have omitted if its being found there were familiarly known.

It seems on the whole most probable that by far the largest portion of the tin used in the manufacture of the bronze of antiquity was brought from Cornwall by Phœnician or other merchants, and by them distributed over the south of Europe, Syria, and Asia Minor. Strabo‡ and Pliny§ indeed state that it came from Spain, and tinstone is known to exist in that country in the province of Gallicia, but the quantity there found is not likely to have supplied the whole demand for this metal; and their account is easily explained by the consideration that the great commercial depot of the eastern merchants was probably situated somewhere near Gades (now Cadiz) in the south of Spain, and that to this place the tin was brought from the west, and from it was again distributed to the consumers in the Mediterranean. Aristotle distinctly mentions, "*τὸν κασσιτερον τὸν Κελτικὸν*" in his treatise, De Mirab. Auscult.

Though Pliny‖ tells us of tin, "Nulli rei sine mixturâ utile," yet it was obviously well known in the separate state, as in Homer we read of the breastplate of Agamemnon:

> "*Τοῦ δ' ἤτοι δέκα οἶμοι ἔσαν μέλανος κυάνοιο,*
> *Δώδεκα δὲ χρυσοῖο, καὶ εἴκοσι κασσιτέροιο.*"
>
> —Il. xi. 24.

And the same poet mentions metallic tin in several other places. Hesiod does

* Lib. iv. c 34. † Lib. iv. 305. ‡ Lib. iii. p. 219, ed. Almel. § Lib. xxxiv. c. 16. ‖ *Loc. cit.*

so too, and Aristotle and Pliny speak of it as a known and common substance. Yet antiquities of this metal unalloyed are very rare; indeed there have been, I believe, but three instances recorded in which such have been found in Great Britain,—and these all in England.* Hence a single additional specimen found in the collection of the Academy acquires considerable interest. It filled the interior of a hollow bronze ring of about four inches and a half in external diameter, the thickness of the ring being about half an inch. It was easily recognised as tin by its colour and the resistance it offered to the knife, and on chemical examination it proved to be nearly pure, containing mere traces of iron and lead. Where partially exposed to the atmosphere it had acquired a coating of peroxide. From the way in which it is attached to the bronze, and the character of the latter, it would seem with little doubt to belong to the same period as the other early bronze antiquities in the Museum.

In the collection there is an earthen vessel, apparently intended to be placed in the fire, in which were found several small fragments of bronze very much corroded, a brown earthy powder in which particles of the "ærugo" of bronze were observable, and a bit of a white metal of considerable lustre, and exhibiting a somewhat lamellar structure. This latter was hard and very brittle, so as to be easily reduced to powder in a mortar. There were no traces of corrosion on the surface. Specific gravity, 8·107. On analysis it gave in 100 parts,

Copper,	66·12
Tin,	30·62
Silver,	·13
Antimony,	1·91
Sulphur,	·11
	98·89

Thus, though an alloy of copper and tin, it differs totally from bronze in the proportion of its ingredients. The only analysis I have seen which comes near this is that of an antique Roman mirror by Klaproth,† in which he found, copper, 62; tin, 32; and lead, 6; = 100. Whether the Irish alloy was intentionally

* Phil. Trans. vol. xxiii. p. 1129, and vol. li. p. 13. Archæologia, vol. xvi. p. 137.

† Scherer's Allgem. Journ. d. Chemie, No. 33.

made to be used for a similar purpose,—a supposition in some degree countenanced by the presence of a little antimony,—or that the proportion of tin was accidentally large, and the specimen was about to be remelted, and perhaps copper added, is not easy to decide. The pulverulent substance found in the same vessel was neither an ore of any kind, nor a furnace product, but appeared on examination to have been merely dust of an earthy character mixed with the results of bronze corrosion (probably the sweepings of a workshop or some such place). The vessel and its contents constitute an interesting relic of early metallurgy.

Although *Zinc* in the metallic state was unknown, at least until the twelfth century, it seems certain that some of its ores were worked and used for making Aurichalcum or brass at an extremely early period, the valuable properties of that alloy being well known and appreciated in Aristotle's time, although he speaks of it under the title χαλκὸς Μοσσύνοικος as a rare variety of bronze. This Pliny also considered it, and it seems never to have come into common use amongst either the Greeks or Romans. Hence it is not surprising that no example of a brazen article of decidedly Celtic manufacture has yet been discovered. I analyzed a fragment of a shallow basin from the district of Castlebernard, County of Cork, recently presented to the Academy Museum, and found it to contain :—

Copper, ...	72·58
Zinc, ...	25·29
Iron, ...	1·16
Lead, ...	·02
Tin, ...	Trace.
	99·05

The specimen was soft, of a bright brass-yellow colour, and specific gravity, 7·717. From its form, however, and composition, which is quite that of ordinary modern brass, it is probably of very little antiquity, and does not at all belong to the same class with the really Celtic articles in the Museum; an alloy therefore of copper and zinc of equal age with the latter is still to be sought for. Amongst German antiquities GÖBEL (Schweig. 60. 407.) found some of this composition, but I do not know to what date or people his specimens have been assigned by archæologists.

Lead is another of the metals which was extracted from its ores and applied to many of the purposes for which it is used at the present day, at probably a a period nearly as ancient as that of the introduction of copper. With the exception of native metals, no ore would be more likely to have come under the notice and attracted the attention of primitive metallurgists than galena, the commonest form in which lead occurs, and from which its extraction would present scarcely any difficulty even with the rudest means for smelting. Mention is accordingly made of this metal among the Egyptians, Hebrews, Greeks, Romans, and other nations of antiquity, who not only used it in its separate state for water-pipes and other mechanical purposes, but were also acquainted with the method of employing it in the refinement of gold* and silver.† In Britain, particularly in Derbyshire, numerous remains of lead-workings have been discovered, probably belonging to the time of the Roman occupation, but I have seen no account of any relics of this metal of similar antiquity having been found in Ireland; and in the Museum of the Academy, the only traces of lead that I could find were the cores or filling of one or two reliquaries of thin gold plate or foil, which are, I believe, considered as specimens of mediæval art. The interior surface of one of these cores, which was itself hollow, was considerably corroded, being covered with a grayish crust of carbonate, the production of which was probably accelerated by the contact of the gold. Some ancient lead from an abbot's coffin in the Cathedral of Christ Church, Dublin, which was deeply pitted by corrosion, yielded traces of sulphate along with the carbonate upon its surface, and on cupellation of the metal itself left a minute bead of silver.

Of the numerous

WEAPONS AND IMPLEMENTS OF IRON

I examined four, namely:—

No. 1. A sword found at Kilmainham, near Dublin. It is long and straight, adapted for both cutting and thrusting, and is one of those examined by M. Worsaae on visiting the Museum a few years ago, and declared by him to

* Theognis—*Γνωμαι*. l. 1101. † Jeremiah, vi. 28. Pliny. lib. xxxiii. c. 6.

agree in appearance with Norse swords preserved at Copenhagen and Stockholm. The blade was covered with a thick coat of rust, on removing which a portion of metal beneath was reforged as the simplest way of determining its character. It turned out not to be steel, but moderately good soft iron, incapable of being hardened by quenching when hot in water. On solution in dilute sulphuric acid it left a very slight black sediment, consisting of carbon with traces of phosphorus.

No. 2. A knife, also found at Kilmainham, marked $\frac{365}{D}$; still more corroded than the last, there being in fact very little metal left. The fracture was very close-grained, and of a bluish-white colour; and on reforging it proved to be steel of an inferior quality, leaving, on solution in a dilute acid, carbon, containing phosphorus and silica, the quantity of which was, however, not determinable. The specimen is probably a very modern one, and ought not to be classed with the others here described.

No. 3. A nail from Dunshaughlin in the county of Meath. It was not nearly so much corroded as the last two specimens, and the rust was hard and closely adherent, whereas that on the sword and knife was loose and easily detached. On the crust of oxide some traces of the blue phosphate of iron were observable, and more was to be found on breaking it off from the metal. The latter, on reforging, proved, as might have been expected, to be soft iron, containing, as was found on chemical examination, a large proportion of phosphorus as compared with the other specimens.

No. 4 was another knife, of a small size, narrow-bladed, and thick on the back, discovered at Strokestown in the county of Roscommon. It was covered by very little rust as I got it, but had apparently had some corrosion previously removed. It consisted, not of steel, which it somewhat resembled in appearance, but merely of malleable iron, of excellent quality, which dissolved almost perfectly in dilute sulphuric acid, leaving a barely visible trace of carbon, and affording no indications of phosphorus.

From these experiments we see, so far as their limited number renders it allowable to judge, that the really ancient weapons of this class found in Ireland do not consist of steel, but of soft iron. This would be in itself an interesting fact if confirmed by further investigation, as showing that the early Scandinavians and Celts, or those who supplied them, though able to make good malleable iron in the first instance, but were ignorant of the methods of giving it the properties of steel by any after-process, though it is plain from passages of Greek and Roman authors* that the latter substance was known to them. The subject, however, demands further investigation.

Having concluded the account of the metallic antiquities, I proceed now to the examination of some other objects, principally used for ornament. And first of—

PRECIOUS STONES.

Of these I found seven varieties, at least according to the classification of the jeweller, though not all distinct mineralogical species, viz. sapphire, beryl, turquoise, garnet, amethyst, clear rock-crystal, and chalcedony. The determination of the periods when these were used as ornaments is of course altogether an antiquarian question, and I do not know to what dates these individual specimens are referred; but with the exception, perhaps, of some of the articles of crystal, none of them seem to belong to the period of the Celtic makers and users of the gold and bronze objects above examined. Some of the amethysts found in crosses and other ecclesiastical relics are merely the uncut terminations of quartz crystals of the common form, no part of the prism, but only the hexagonal pyramid, being visible. Some of the sapphires, which are small and uncut, have been most probably found in the county of Wicklow, with rolled pebbles of the mineral occurring in which county they agree perfectly in external characters.†

Four beads of *amber* which I examined, of different degrees of transparency and colour, were quite unchanged in chemical properties, and, except one, presented nothing remarkable in appearance; this, on being split, was found to be white and nearly opaque (wax amber) in the interior, while the exterior, including the surface of the hole, piercing the bead, was orange-yellow and transparent, like the more ordinary variety of this substance, to a depth of about the twentieth of an inch. This change of molecular condition must, I imagine, have

* See Aristotle, De Mirab. Pliny, lib. xxxiv. c. 14. Dr. PEARSON (Phil. Trans. 1796) found all the supposed iron weapons, from Lincolnshire, which he examined, to consist of steel.

† See the author's Examination of Gold Sand of Wicklow. Proceedings of Geological Society of Dublin for 1851–52.

been the result of age ; unless indeed the bead had at some time been immersed in oil or any similar fluid, which had penetrated it to the extent described.

Two or three fragments of *jet* examined proved, as might be expected, unaltered specimens of this variety of coal.

COLOURED GLASS BEADS.

These occur of several distinct colours ; the most common are, two shades of blue, a black (in reality intensely deep green), a very pale sea green, and white ; of each of which I received a specimen for qualitative analysis.

No. 1 was a very fine dark-blue bead, from Kilmainham, quite resembling good modern cobalt glass in colour, but full of minute air bubbles. By fluxing with an alkali, solution in muriatic acid, and the application of the usual re-agents, the colouring matter was found to be oxide of cobalt, but the glass also contained a trace of copper. Whether the latter was accidental, or, being known to tinge glass blue or green, was added with the intention of improving the colour, it would be impossible to say. It was contended by GMELIN* that the blue glass of the ancients was not stained by cobalt but iron (that it was analogous to ultramarine) ; and an ancient specimen of a sapphire blue colour, analyzed by KLAPROTH,† gave no indications of the former metal ; but Sir H. DAVY‡ found cobalt in all the glass vessels of this colour from the tombs of Magna Græcia, and the same colouring material has been detected in the beads found upon Egyptian mummies. The present is therefore but an additional instance of the use of a compound of this metal for a special purpose being known long before the metal itself or any of its preparations had been obtained in a state approaching purity.

No. 2. A bead of so dark a bottle-green colour as to appear by reflected light quite black and opaque. In very thin splinters it was translucent, and of the above tinge. The colouring material was oxide of iron, in very large quantity, and traces of manganese were also distinctly perceptible. The specimen was from Templepatrick in the county of Antrim, nearly spherical, well shaped, and had a finely polished surface.

No. 3 was a blebby, light blue bead, verging on green, from Kilmainham.

* Götting. gel. anz. 1776.

† Beiträge. s. 144.

‡ Phil. Trans. 1815, p. 108.

But for the contained air bubbles it would have been nearly transparent. The colour was due to oxide of copper; and both in the staining of the glass, and in forming the bead, the specimen was a very rude result of early art.

No. 4 was a flattened bead, also from Kilmainham. It was more nearly transparent than any of the others, and had only a very faint tinge of sea-green, so pale that it probably was not intentional; on the contrary, this would seem more likely to have been an attempt at colourless glass, which we know was more highly valued by the ancients, at least in the south of Europe, than any other. I could detect no colouring metallic oxide in the present specimen, except the merest trace of oxide of iron.

No. 5. An opaque, white bead, of a flattened form, from the same locality with the last. On examination it proved not to be glass at all, but pure crystalline white marble (carbonate of lime), which had been very neatly cut to the required shape, and the surface well polished. This material has not, I believe, been hitherto noticed among those employed for these primitive ornaments.

The results of the examination of these glasses agree very well with those of some of the specimens of Klaproth and Sir H. Davy; but further investigation of the Celtic articles (and indeed of those from the south of Europe) would be important in order to elucidate the history of this ancient manufacture, as it is only from the analysis of numerous examples, varying in date and locality, that we can hope to derive any valuable general information on the subject.

Another highly interesting branch of an inquiry as to the means of decoration possessed by the ancients is that concerning their

PIGMENTS,

and hence I have been most anxious to examine such remains of this kind as might be in existence in Ireland; but have only succeeded in obtaining specimens (used in fresco painting) from a single locality, namely, Slane Abbey, in the county of Meath; and these probably do not belong to an earlier date than 1512, as the Abbey, originally established in the seventh century, was re-founded in that year. These specimens were not contained in the Academy Museum, but were detached by Mr. F. W. Burton, a member of the Academy, and presented for the purposes of this examination. There were six varieties of colour examined, all of which had been laid on upon a uniform white ground of about the twentieth of an inch in thickness, or perhaps a little thicker, as part of the ground had no doubt been lost in removing the stucco from the walls. The coats of colour were a little thinner, but were not uniform, being thicker in some places than in others; they were all mixed with an oily substance used in very small quantity, which was soluble in alchol and ether, reprecipitable from the former on the addition of water; want of sufficient material made it impossible to determine its nature more accurately. The ground or basis upon which the colours were laid consisted of carbonate of lime mixed with a little silica, or rather white siliceous clay, which, as well as the colours themselves, had been carefully and finely ground. The examination of the individual pigments gave the following results:—

No. 1 was a dull red, almost a brick colour, but somewhat brighter. Heated before the blowpipe on charcoal, it fused into a black shining bead, and in the reducing flame gave globules of a soft, white metal, which on examination proved to be lead. Digested in diluted nitric acid it partially dissolved with effervescence. The solution gave with hydro-sulphuric acid a black precipitate of sulphuret of lead, and with ammonia, after filtration and heating, a slight reddish-brown one of peroxide of iron, containing a trace of alumina. On fluxing the residue, insoluble in nitric acid with carbonate of soda, it was found to consist of highly ferruginous silica. Hence this colour appears to be an impure oxide of iron, probably iron ochre, mixed with carbonate of lead; or possibly may have been red lead, mixed with ground hæmatite, the former having altered in chemical composition in the lapse of time, by the action of air and moisture, &c.

No. 2. A pale yellow, verging on Naples yellow or yellowish white. Before the blowpipe it behaved nearly in the same manner as the red, but became much darker by the first application of the heat, before fusion. Treated in the same way as the last, it proved to be a light yellow ochre, mixed with a large proportion of ceruse, and containing a good deal of the oily matter with which the colours appear to have been mixed. Originally it may have been of an orange colour, and the lead, as in the last case, in the state of red lead.

No. 3. A light blue; the only one of the colours which had any pretensions to brilliancy. It invariably occurred over a coat of the red, No. 1; which probably was picked out or cut through in some places, so as to produce a pattern of blue in relief on the red ground. It dissolved to a great extent with effervescence when heated in nitric acid, and the solution, on adding excess of ammonia, gave a pale blue solution of oxide of copper and a copious precipitate of oxide of lead. The insoluble residue was fluxed, and consisted of silica with alumina and oxide of copper, and probably an alkali. This colour, therefore, appears to have been partly a copper frit of the same kind with that found by Sir H. Davy, in the blue pigments examined by him in Italy, but differing from these latter in that some of the copper existed in a state soluble in acids, I believe as carbonate. Before the blowpipe, this, like the other colours, yielded metallic lead when heated on charcoal, and empyreumatic products in a closed glass tube.

No. 4. White. This turned out to be slightly impure carbonate of lime, the same, in fact, as the basis of all the colours, though laid on in a separate layer. The fact of this being the white employed, and not white lead, which yet (or minium) was mixed with the other pigments, would seem to indicate either that the ceruse was prepared of so impure a character as not to be a good white, or that it was known to darken by long exposure, where traces of sulphureted hydrogen were present in the atmosphere, and therefore was rejected as not a permanent colour.

No. 5. A grayish black. It became white by the gentle application of the blowpipe flame, and dissolved in nitric acid with copious effervescence, leaving a slight carbonaceous residue of a black colour, perfectly dissipated, with the exception of a trace of silica, by heating to redness for an instant on platina foil. The nitric acid solution contained nothing but lime. This therefore was a mixture of carbon in some form, probably lamp-black, with carbonate of lime.

No. 6 was a dull brown, which proved to be an ochre, containing silica, alumina, lime, and a large quantity of oxide of iron analogous to Nos. 1 and 2, but of a different shade. It was mixed, like them, with carbonate of lead.

These results with respect to ancient Irish pigments agree, as far as they go, to a remarkable extent with those of Sir H. Davy's interesting investigation* of the ancient Roman pigments above referred to. The materials used

* Philosophical Transactions, 1815.

are in each instance the same, or nearly so. This is not wonderful, however, as we find that most of the materials are of the commonest and most easily obtained substances for the purpose, requiring but little preparation, and are of a durable and stable character; hence naturally selected by the early artist, and, when once in use, not likely to cease to be so from the knowledge of them dying out, as that of difficult and rare preparations might easily have done. Two points of difference are, however, to be found between the Roman pigments and these Irish ones, namely, the *use of an oily medium* for mixing the colours from Slane Abbey, whereas such was not apparently employed by the Romans in their fresco paintings; and the occurrence in the former of large quantities of ceruse or white lead, which, although known and described by Pliny and Vitruvius as a common colour, was not found by Sir H. DAVY in any of the specimens examined by him. Both circumstances tend to prove the more modern character of the Irish pigments.

Such are the results of the examination of these antiquities so far obtained. It will be seen that some classes of objects in the Academy Museum have not been spoken of at all, and that this investigation is very far from exhausting the subject with respect even to the departments discussed. Conscious of my unfitness for the task, I have left the bearings of these results upon archæology almost untouched; yet the analyses and experiments above described are not, perhaps, absolutely barren in results of interest, and at least put on record a number of facts concerning the materials employed at early periods in Ireland for various purposes of the arts, which may possibly in some degree assist the researches of archæologists. In the hope that such may be the case I have ventured to lay them before the Royal Irish Academy, to the liberality of whose Council I am indebted for the specimens principally experimented upon.

XV.—*On the Properties of Inextensible Surfaces.* By the Rev. JOHN H. JELLETT, A. M., Fellow of Trinity College, and Professor of Natural Philosophy in the University of Dublin.

Read May 23, 1853.

1. ALTHOUGH the celebrated theorems of GAUSS have received from mathematicians much and deserved attention, inducing them to bestow considerable labour upon obtaining for these theorems simple and elegant demonstrations I do not find that any attempt has been made to extend his discoveries upon this subject. Yet the highly interesting character of the theorems alluded to might naturally induce the expectation of other important results connected with the theory of inextensible surfaces, sufficient to repay the labour of a more general consideration of the question than has been (so far as I am aware) as yet attempted. I propose, therefore, in the present Memoir to consider generally what are the conditions to which the displacements of a continuous inextensible membrane are subject. These conditions are expressed (as will be seen) by a system of three partial differential equations of a very simple form, which contain the solution of all questions connected with this theory. From these equations I shall deduce general expressions for the variations which the differential coefficients,

$$\frac{dz}{dx}, \quad \frac{dz}{dy}, \quad \frac{d^2z}{dx^2}, \quad \frac{d^2z}{dxdy}, \quad \frac{d^2z}{dy^2},$$

undergo in consequence of the displacement of the membrane. These expressions give immediately the two theorems of GAUSS. I shall then proceed to consider how far the flexibility of the membrane is destroyed by rendering rigid any curve traced upon its surface. I shall in the next place investigate the laws which govern the displacement of a surface which is partially extensible (as hereafter explained), and how far the preceding theorems are applicable to such surfaces; and, finally, I shall consider how far these conclusions are applicable to the laminæ which we find in nature, which are neither wholly inextensible nor wholly devoid of thickness. The results arrived at will be found, I think, sufficiently remarkable to attract the attention of mathematicians to this subject.

2. *Definition of an Inextensible Surface.*—Two definitions of inextensibility have been given by LAGRANGE and GAUSS respectively. According to the former, who defines the force which resists extension to be the force which resists the increase of superficial area, a surface is inextensible if it be impossible to change the superficial area of any portion of it. But this definition seems to be hardly consistent with the meaning ordinarily attached to the word "inextensible." For if we conceive a membrane admitting of being indefinitely extended in any direction, but of such a nature, that an extension in any one direction is always accompanied by a corresponding contraction in another, so as to preserve the area unchanged, such a membrane would be, according to LAGRANGE'S definition, *inextensible.* But it appears more consistent with ordinary ideas to consider an inextensible surface to be one which does not admit of *any* extension, rather than one whose capacities of extension and contraction counterbalance one another in the manner above described. I shall, therefore, in the present Memoir adopt the definition of GAUSS, as more exactly embodying the ordinary ideas on the subject, adding to it the definition of partially extensible surfaces, a class not noticed by GAUSS, but presenting some remarkable properties. These definitions are as follows:

I. *A surface is said to be inextensible, when the length of a curve traced arbitrarily upon it is unchangeable by any force which can be applied to it.*

II. *A surface is partially extensible, if there be at each of its points one or more inextensible directions; in other words, if it be possible to trace at each point one or more inextensible curves.*

We shall now proceed to consider how these definitions may be mathematically expressed, commencing with the case of inextensible surfaces.

3. *Deduction of the Equations which connect the Displacements of an Inextensible Surface.* Let ds be the element of a curve traced in any direction upon the surface, and let δ be the symbol of displacement, i. e. a symbol denoting the

passage of a molecule, or physical point, from one geometrical point of space to another. Then, since the curve of which ds is an element, is by the assumed definition inextensible, we must have

$$\delta ds = 0\,;$$

or, putting for ds its value,

$$\sqrt{(dx^2 + dy^2 + dz^2)}\,;$$

and performing the operations indicated by δ,

$$dx d\delta x + dy d\delta y + dz d\delta z = 0\,; \qquad \text{(A)}$$

recollecting that δ is a commutative symbol. But since the displacements δx, δy, δz, refer to a point on the surface, we must have

$$d\delta x = \frac{d\delta x}{dx}\,dx + \frac{d\delta x}{dy}\,dy,$$

$$d\delta y = \frac{d\delta y}{dx}\,dx + \frac{d\delta y}{dy}\,dy,$$

$$d\delta z = \frac{d\delta z}{dx}\,dx + \frac{d\delta z}{dy}\,dy\,;$$

x, y, being the independent variables.

Let

$$dz = pdx + qdy,$$

be the equation of the surface.

Substituting for $d\delta x$, $d\delta y$, $d\delta z$, dz, in equation (A), we have

$$\left(\frac{d\delta x}{dx} + p\frac{d\delta z}{dx}\right)dx^2 + \left(\frac{d\delta y}{dx} + \frac{d\delta x}{dy} + q\frac{d\delta z}{dx} + p\frac{d\delta z}{dy}\right)dxdy + \left(\frac{d\delta y}{dy} + q\frac{d\delta z}{dy}\right)dy^2 = 0.$$

But since the condition expressed in this equation is supposed to hold for *every* curve traced upon the surface, it must be true for all values of

$$\frac{dy}{dx}.$$

We have, therefore,

$$\left.\begin{aligned} &\frac{d\delta x}{dx} + p\frac{d\delta z}{dx} = 0,\\ \frac{d\delta y}{dx} + \frac{d\delta x}{dy} &+ q\frac{d\delta z}{dx} + p\frac{d\delta z}{dy} = 0,\\ &\frac{d\delta y}{dy} + q\frac{d\delta z}{dy} = 0. \end{aligned}\right\} \qquad \text{(B)}$$

These equations may be put under a somewhat simpler form, by assuming

$$u = \delta x + p\delta z,$$
$$v = \delta y + q\delta z,$$
$$w = \delta z.$$

Making these substitutions, we find

$$\left.\begin{aligned} &\frac{du}{dx} - wr = 0, \\ &\frac{du}{dy} + \frac{dv}{dx} - 2ws = 0, \\ &\frac{dv}{dy} - wt = 0; \end{aligned}\right\} \qquad \text{(C)}$$

where r, s, t, are used in their ordinary sense to denote the differential coefficients

$$\frac{d^2z}{dx^2}, \quad \frac{d^2z}{dxdy}, \quad \frac{d^2z}{dy^2};$$

derived from the equation of the surface. Any one of the quantities u, v, w, may be determined by means of a differential equation of the second order. Thus, for example, eliminating w between the equations (C), we find,

$$\frac{1}{r}\frac{du}{dx} = \frac{1}{2s}\left(\frac{du}{dy} + \frac{dv}{dx}\right) = \frac{1}{t}\frac{dv}{dy}.$$

Hence,

$$\frac{dv}{dx} = \frac{2s}{r}\frac{du}{dx} - \frac{du}{dy},$$

$$\frac{dv}{dy} = \frac{t}{r}\frac{du}{dx}.$$

Differentiating the first of these equations with respect to y, and the second with respect to x, and subtracting, we find easily,

$$r\frac{d^2u}{dy^2} - 2s\frac{d^2u}{dxdy} + t\frac{d^2u}{dx^2} = \frac{1}{r}\frac{d(rt - s^2)}{dx}\frac{du}{dx}; \qquad \text{(D)}$$

and similarly for v,

$$r\frac{d^2v}{dy^2} - 2s\frac{d^2v}{dxdy} + t\frac{d^2v}{dx^2} = \frac{1}{t}\frac{d(rt - s^2)}{dy}\frac{dv}{dy}. \qquad \text{(E)}$$

The equation for w may readily be deduced from (C). Differentiating the first of these equations with respect to y, and the second with respect to x, and subtracting, we have

$$\frac{d^2v}{dx^2} = 2s\frac{dw}{dx} - r\frac{dw}{dy} + w\frac{ds}{dx}.$$

Differentiating this equation with respect to y,

$$\frac{d^3v}{dx^2dy} = 2s\frac{d^2w}{dxdy} - r\frac{d^2w}{dy^2} + 2\frac{ds}{dy}\frac{dw}{dx} + w\frac{d^2s}{dxdy}.$$

Again, differentiating the third of equations (C) twice with regard to x,

$$\frac{d^3v}{dx^2dy} = t\frac{d^2w}{dx^2} + 2\frac{dt}{dx}\frac{dw}{dx} + w\frac{d^2t}{dx^2}.$$

Subtracting these equations one from the other, we find,

$$r\frac{d^2w}{dy^2} - 2s\frac{d^2w}{dxdy} + t\frac{d^2w}{dx^2} = 0. \qquad \text{(F)}$$

Some interesting results followed at once from the fundamental equations. Thus, for example, if the displacements of the surface be all parallel to the same plane, we shall have, taking this plane for the plane of xy,

$$w = 0.$$

The equations (C) are thus reduced to

$$\frac{du}{dx} = 0, \quad \frac{du}{dy} + \frac{dv}{dx} = 0, \quad \frac{dv}{dy} = 0.$$

Integrating this system of equations, we find, without difficulty,

$$u = A + By, \quad v = C - Bx;$$

or since $\delta z = 0$

$$\delta x = A + By, \quad \delta y = C - Bx;$$

A, B, C, being constants. These equations express the following theorem:

If the displacements of an inextensible surface be all parallel to the same plane, the surface moves as a rigid body.

More generally, if we make

$$w = \delta z = ax - by + e,$$

in the equations (C) we shall find without difficulty the solution:

$$\delta x = cy - az + e',$$
$$\delta y = bz - cx + e''.$$

Hence we infer that—

If the movement of an inextensible surface, parallel to any one line, be that of a rigid body, the entire movement is that of a rigid body.

4. *Variations of the Differential Coefficients.*—If we denote by δ' the variation, properly so called, i. e., the change which the function receives in consequence of a change of form, it is evident that

$$\left.\begin{aligned} \delta z &= p\delta x + q\delta y + \delta z', \\ \delta p &= \frac{dp}{dx}\delta x + \frac{dp}{dy}\delta y + \frac{d\delta z'}{dx} = \frac{dp}{dx}\delta x + \frac{dp}{dy}\delta y + \frac{d\,(\delta z - p\delta x - q\delta y)}{dx} \\ &= \frac{d\delta z}{dx} - p\frac{d\delta x}{dx} - q\frac{d\delta y}{dx}, \\ \delta q &= \frac{d\delta z}{dy} - p\frac{d\delta x}{dy} - q\frac{d\delta y}{dy}. \end{aligned}\right\} \quad (G)$$

Eliminating

$$\frac{d\delta x}{dx}, \quad \frac{d\delta y}{dy},$$

from these equations, by means of the first and third of equations (B), we have

$$\delta p = (1 + p^2 + q^2)\frac{d\delta z}{dx} - q\left(\frac{d\delta y}{dx} + q\frac{d\delta z}{dx}\right),$$

$$\delta q = (1 + p^2 + q^2)\frac{d\delta z}{dy} - p\left(\frac{d\delta x}{dy} + p\frac{d\delta z}{dy}\right).$$

In the same way we find,

$$\delta r = \frac{d\delta p}{dx} - r\frac{d\delta x}{dx} - s\frac{d\delta y}{dx},$$

$$\delta s = \frac{d\delta p}{dy} - r\frac{d\delta x}{dy} - s\frac{d\delta y}{dy} = \frac{d\delta q}{dx} - s\frac{d\delta x}{dx} - t\frac{d\delta y}{dx},$$

$$\delta t = \frac{d\delta q}{dy} - s\frac{d\delta x}{dy} - t\frac{d\delta y}{dy}.$$

Substituting for δp, δq, and reducing the resulting expressions by means of equations (D), (E), (F), we find ultimately,

$$\delta r = (1 + p^2 + q^2)\frac{d^2\delta z}{dx^2} - r\left(\frac{d\delta x}{dx} + \frac{d\delta y}{dy}\right) - 2\left(r\frac{d\delta x}{dx} + s\frac{d\delta y}{dx}\right),$$

$$\delta s = (1 + p^2 + q^2)\frac{d^2\delta z}{dxdy} - 2s\left(\frac{d\delta x}{dx} + \frac{d\delta y}{dy}\right) - \left(r\frac{d\delta x}{dy} + t\frac{d\delta y}{dx}\right), \qquad \text{(H)}$$

$$\delta t = (1 + p^2 + q^2)\frac{d^2\delta z}{dy^2} - t\left(\frac{d\delta x}{dx} + \frac{d\delta y}{dy}\right) - 2\left(s\frac{d\delta x}{dy} + t\frac{d\delta y}{dy}\right).$$

From the two equations (G) it is easy to verify that the element of the superficial area remains constant; for if we multiply the first of these equations by p, and the second by q, and add them, we find, recollecting the second of equations (B),

$$p\delta p + q\delta q = (1 + p^2 + q^2)\left(p\frac{d\delta z}{dx} + q\frac{d\delta z}{dy}\right);$$

or from the first and third of equations (B),

$$p\delta p + q\delta q + (1 + p^2 + q^2)\left(\frac{d\delta x}{dx} + \frac{d\delta y}{dy}\right) = 0, \qquad \text{(I)}$$

which is obviously equivalent to

$$\delta\sqrt{(1 + p^2 + q^2)}\, dxdy = 0.$$

Again, multiplying the first of equations (H) by t, the second by $2s$, and the third by r, and subtracting the second product from the sum of the other two, we have

$$t\delta r + r\delta t - 2s\delta s = \delta(rt - s^2)$$

$$= (1 + p^2 + q^2)\left(t\frac{d^2\delta z}{dx^2} - 2s\frac{d^2\delta z}{dxdy} + r\frac{d^2\delta z}{dy^2}\right) - 4(rt - s^2)\left(\frac{d\delta x}{dx} + \frac{d\delta y}{dy}\right)$$

$$= -4(rt - s^2)\left(\frac{d\delta x}{dx} + \frac{d\delta y}{dy}\right).$$

Hence, and from equation (I), it is easy to see that

$$\frac{\delta(rt - s^2)}{rt - s^2} - 4\frac{(p\delta p + q\delta q)}{1 + p^2 + q^2} = 0,$$

which is plainly equivalent to

$$\delta \frac{rt - s^2}{(1 + p^2 + q^2)^2} = \delta \frac{1}{RR'} = 0. \tag{K}$$

This equation is the analytical statement of GAUSS's celebrated theorem, namely, that

In all the possible movements of an inextensible surface, the product of the principal radii of curvature at every point of the surface is constant.

Let S be a portion of the surface bounded by any closed curve. Conceive this curve to be referred to the surface of a sphere, by radii drawn parallel to the normals, and let S' be the included portion of the spherical surface. Then, if the radius of the sphere be supposed to be unity,

$$S' = \iint \frac{dS}{RR'},$$

and therefore,

$$\delta S' = \iint \delta \frac{dS}{RR'} = 0.$$

Hence, *In all possible motions of an inextensible surface, the area of the spherical curve corresponding to any closed curve described upon the surface* (denominated by GAUSS the "curvatura integra") *remains constant.*

This is the second theorem of GAUSS.

5. We shall next proceed to consider the effect of *fixing* any curve upon the surface. The determination of the displacement of the surface in this case will obviously depend upon the following analytical problem :—" To find three functions u, v, w, which shall satisfy the partial differential equations,

$$\frac{du}{dx} - wr = 0,$$

$$\frac{du}{dy} + \frac{dv}{dx} - 2ws = 0,$$

$$\frac{dv}{dy} - wt = 0,$$

and shall, moreover, have the values

$$u = 0, \quad v = 0, \quad w = 0,$$

for all points of a given curve or portion of a curve."

Let

$$dy = mdx,$$

be the equation of the projection of the given curve upon the plane of xy. Then since u, v, w, vanish for a continuous portion of this curve, we must have

$$\left.\begin{aligned}\frac{du}{dx} + m\frac{du}{dy} &= 0,\\ \frac{dv}{dx} + m\frac{dv}{dy} &= 0,\\ \frac{dw}{dx} + m\frac{dw}{dy} &= 0.\end{aligned}\right\} \qquad (L)$$

But if we make

$$w = 0$$

in the first and third of equations (C), we shall have

$$\frac{du}{dx} = 0, \qquad \frac{dv}{dy} = 0.$$

Hence and from equations (L) we have

$$\frac{du}{dy} = 0, \qquad \frac{dv}{dx} = 0.$$

Differentiating these equations upon the same principle, we have

$$\left.\begin{aligned}\frac{d^2u}{dx^2} + m\frac{d^2u}{dxdy} = 0, &\qquad \frac{d^2u}{dxdy} + m\frac{d^2u}{dy^2} = 0,\\ \frac{d^2v}{dx^2} + m\frac{d^2v}{dxdy} = 0, &\qquad \frac{d^2v}{dxdy} + m\frac{d^2v}{dy^2} = 0.\end{aligned}\right\} \qquad (M)$$

Hence it is easily seen that the equations (D) and (E), p. 346, become for this curve

$$\left.\begin{aligned}(r + 2sm + tm^2)\frac{d^2u}{dxdy} &= 0,\\ (r + 2sm + tm^2)\frac{d^2v}{dxdy} &= 0.\end{aligned}\right\} \qquad (N)$$

Hitherto the reasoning employed has been perfectly general, embracing surfaces of every class. But in our subsequent investigations we must discuss severally the three great classes into which surfaces are divided with respect to their curvature, namely :

1. Surfaces whose principal curvatures are *similar*, or those in which

$$rt - s^2 > 0.$$

2. Developable surfaces, in which

$$rt - s^2 = 0.$$

3. Surfaces whose principal curvatures are *dissimilar*, or those in which

$$rt - s^2 < 0.$$

I. *Surfaces whose principal curvatures are similar.*—In this case it is plain that the equation

$$r + 2sm + tm^2 = 0$$

is impossible, whatever be the value of m. Therefore the equations (N) can only be satisfied by making

$$\frac{d^2u}{dxdy} = 0, \qquad \frac{d^2v}{dxdy} = 0.$$

Hence, and from equations (M), we find

$$\left.\begin{array}{lll} \dfrac{d^2u}{dx^2} = 0, & \dfrac{d^2u}{dxdy} = 0, & \dfrac{d^2u}{dy^2} = 0, \\ \dfrac{d^2v}{dx^2} = 0, & \dfrac{d^2v}{dxdy} = 0, & \dfrac{d^2v}{dy^2} = 0, \end{array}\right\} \qquad \text{(O)}$$

which must hold throughout the fixed curve. Again, differentiating equations (D) and (E) (which are true generally) with regard to x, and rejecting differential coefficients of the first and second order, which vanish for the fixed curve, we have

$$\left.\begin{array}{l} r\dfrac{d^3u}{dxdy^2} - 2s\dfrac{d^3u}{dx^2dy} + t\dfrac{d^3u}{dx^3} = 0, \\ r\dfrac{d^3v}{dxdy^2} - 2s\dfrac{d^3v}{dx^2dy} + t\dfrac{d^3u}{dx^3} = 0, \end{array}\right\} \qquad \text{(P)}$$

which must hold throughout the fixed curve.

Differentiating equations (M) as before, and eliminating

$$\frac{d^3u}{dx^2dy}, \qquad \frac{d^3u}{dx^3},$$

from equations (P), we have

$$\left.\begin{aligned}(r+2sm+tm^2)\frac{d^3u}{dxdy^2}=0,\\(r+2sm+tm^2)\frac{d^3v}{dxdy^2}=0.\end{aligned}\right\} \qquad (Q)$$

Hence it is easy to infer, as before, that all the differential coefficients of the third order vanish for points of the surface situated on the fixed curve; and a very slight examination will show that by proceeding in the same manner we shall find that *all* the differential coefficients of u, of all orders, vanish for the limiting curve. Now if u be a function of the same form throughout the surface, it is plain that these conditions can only be satisfied by the supposition that u vanishes at every point. The same conclusion will hold if u change its form. For if u be supposed to have the same form for all points between the limiting curve and any other curve drawn arbitrarily, it is plain, from what has been said, that its value can be no other than zero. Hence for all points of the second curve

$$u=0.$$

Now it is evident that the same reasoning which was before applied to the limiting curve is equally applicable to this second curve, and so on for any number of curves bounding those parts of the surface for which the form of u is the same. It appears, therefore, from the foregoing reasoning, that we must have throughout the entire surface

$$u=0.$$

By precisely similar reasoning it may be shown that we have throughout the entire surface

$$v=0;$$

and on referring to equations (C), it will be seen that it follows at once from these equations that

$$w=0.$$

Replacing u, v, w, by their values in terms of δx, δy, δz, we have for every point of the surface

$$\delta x = 0, \quad \delta y = 0, \quad \delta z = 0.$$

Hence we infer the following theorem:

If any curve be traced upon an inextensible surface, whose principal curvatures are finite and of the same sign, and if this curve be rendered immovable, the entire surface will become immovable also.

More generally, let it be required to determine a system of values of u, v, w, which shall satisfy the equations (C), and which shall have at all points of a given curve, or part of a curve, the given values

$$u = u_1, \quad v = v_1, \quad w = w_1.$$

Then it is easy to show from the foregoing discussion, that there is but one such system.

For, if possible, let there be two systems of values,

$$u = U, \quad v = V, \quad w = W,$$
$$u = U', \quad v = V', \quad w = W',$$

which satisfy the given conditions. Then since the equations (C), which these two systems of values are supposed to satisfy, are linear, it is plain that if we form a third system,

$$u = U - U', \quad v = V - V', \quad w = W - W',$$

this system will also satisfy equations (C). But as the values of u, v, w are given for the limiting curve, the two assumed systems must be coincident throughout this curve, and therefore we must have for all its points,

$$U - U' = 0, \quad V - V' = 0, \quad W - W' = 0.$$

Now we have seen in the foregoing discussion that if u, v, w be a system of values satisfying these two conditions, we must have generally

$$u = 0, \quad v = 0, \quad w = 0.$$

Hence it is plain that at every point of the surface

$$U - U' = 0, \quad V - V' = 0, \quad W - W' = 0.$$

The two systems of values are therefore identical. Hence we infer the theorem—

If a curve be traced upon an inextensible surface, whose principal curvatures are finite, and of the same sign, and if any given determinate motion be assigned to this curve, the motion of the entire surface is determinate and unique.

Thus, for example, it is easily shown that if the limiting curve be made *rigid*, the entire surface will become rigid also.

II. We shall next consider the case of *developable* surfaces, or those in which

$$rt - s^2 = 0.$$

This case may be subdivided into two, which require to be considered separately. These cases are—

1. When the fixed curve is either a rectilinear section of the surface or the *arête de rebroussement.*

2. When the fixed curve is not either of these.

1. Let the fixed curve be a rectilinear section. Then it is plain that this curve must satisfy the equation

$$r + 2sm + tm^2 = 0,$$

which expresses the fact that the radius of curvature of the normal section passing through this line, i. e. in the case of a developable surface, of the line itself, is infinite.

Hence the equations (N) become identically true, without supposing that

$$\frac{d^2u}{dxdy} = 0, \qquad \frac{d^2v}{dxdy} = 0.$$

It is plain, therefore, that the reasoning by which it was shown that the several differential coefficients of u, v vanish for the fixed curve, is no longer applicable, and that the several conditions of the problem may be satisfied *without* supposing u, v to vanish at every point of the surface. We infer, therefore, that,

In a developable surface composed of an inextensible membrane, any one of its rectilinear sections may be fixed without destroying the flexibility of the membrane.

And it is easily seen that the same conclusion will hold if the fixed curve be the *arête de rebroussement* of the developable surface.

2. Let the fixed curve be neither a right line nor the *arête de rebroussement.* Then since this curve does not satisfy the equation

$$r + 2sm + tm^2 = 0,$$

we must have, as in the first case,

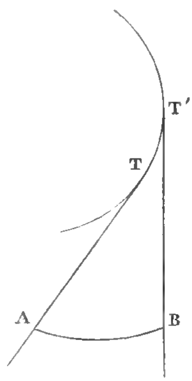
$$\frac{d^2u}{dxdy} = 0, \qquad \frac{d^2v}{dxdy} = 0.$$

All the reasoning of that case is therefore strictly applicable, and it will appear, as before, that *all* the differential coefficients of u must vanish for the limiting curve. Hence, if u preserve the same form, it can have no value but zero. Now let it be supposed that u may change its form; then it is easily seen that the zero value of u can only change in passing across a curve whose equation is

$$r + 2sm + tm^2 = 0.$$

Every part of the surface, therefore, which can be reached from the fixed curve *without* crossing either the *arête de rebroussement* or a rectilinear section, is necessarily fixed. The remainder of the surface is capable of motion. Hence we have the following construction:

Let AB be a fixed curve drawn on the given membrane. Draw through the extreme points A, B, the rectilinear sections of the developable surface, and produce them to touch the *arête de rebroussement.* Then it is evident, from the foregoing analysis, that all that part of the surface which lies between the two lines, and on the same side of the *arête de rebroussement* with the fixed curve, will itself be fixed. Beyond these lines the surface is flexible.



To determine more accurately the nature of the motion of which the surface is capable, we shall now proceed to integrate the equation (D), which, for a developable surface, is in general possible.

Since $rt - s^2 = 0$, equation (D) becomes in the present case

$$r\frac{d^2u}{dy^2} - 2s\frac{d^2u}{dxdy} + t\frac{d^2u}{dx^2} = 0. \qquad \text{(R)}$$

The equations of the characteristic are therefore

$$r + 2s\frac{dy}{dx} + t\frac{dy^2}{dx^2} = 0, \tag{S}$$

$$rd\,.\frac{du}{dy} + t\frac{dy}{dx}\,.\,d\,.\,\frac{du}{dx} = 0. \tag{T}$$

Let

$$p = Q$$

be the equation of the given surface; then

$$r = Q's, \qquad s = Q't,$$

where

$$Q' = \frac{dQ}{dq}.$$

Substituting these values in equation (S), we find easily

$$\frac{dy}{dx} = -\,Q'.$$

Now since equation (S) represents a rectilinear section of the surface, it is evident that in this equation Q' must be constant. Hence it becomes

$$y + Q'x = \text{const.}$$

Again, substituting for r, t, and $\frac{dy}{dx}$ in equation (T), and integrating, we find

$$Q'\frac{du}{dy} - \frac{du}{dx} = \text{const.}$$

Then the integral of equation (R), which may readily be obtained in the ordinary way, will be

$$u = xf(q) + F(q). \tag{V}$$

The following general property of the motion may be deduced from this equation:

*The rectilinear sections of the surface are rigid.**

For, since in a developable surface q is constant for the same rectilinear section, the value of u for such a section will be

$$u = Ax + B,$$

A, B being constants.

* It is easily seen, however, that these sections may all bend at the *arête de rebroussement.*

Similarly we shall have

$$v = A'x + B'.$$
$$w = A''x + B''.$$

Hence it is easy to see that if x', y', z' be the co-ordinates of the new position of the point x, y, z, we shall have

$$x' = ax + b,$$
$$y' = a'x + b',$$
$$z' = a''x + b'',$$

where a, a', a'', b, b', b'', are constant for the same rectilinear section. From these equations it is plain that the locus of the points x', y', z' is still a right line.

III. *Concavo-convex surfaces, or those in which*

$$rt - s^2 < 0.$$

It is a well-known property of surfaces of this class that at each point of the surface there are two real directions satisfying the condition

$$r\cos^2\alpha + 2s\cos\alpha\cos\beta + t\cos^2\beta = 0\,; \qquad \text{(W)}$$

an equation which expresses the geometrical fact, that the normal section which passes through either of these directions will have at that point an infinite radius of curvature. We may therefore conceive the entire surface to be crossed by two series of curves, such that a tangent drawn to either of them at any point shall possess this geometrical property. These curves we shall denominate (for a reason which will appear subsequently) *curves of flexure*. We shall consider separately (as before for developable surfaces) the two different cases which arise, according as the fixed curve is or is not a curve of flexure.

1. When the fixed curve is a curve of flexure it is evident, as in the case of developable surfaces, that the equation

$$(r + 2sm + tm^2)\,\frac{d^2u}{dxdy} = 0$$

becomes identically true *without* supposing

$$\frac{d^2u}{dxdy} = 0.$$

We conclude, therefore, as before, that any one of these curves may be fixed without destroying the flexibility of the surface. The reason for the name "curve of flexure" is thus explained. In fact we see that these curves, when fixed, allow the surface to *bend* round them, the flexure commencing at the curve itself. We shall presently show that this property is peculiar to the curves of flexure as above defined.

2. When the fixed curve is not a curve of flexure, the reasoning before given in the case of developable surfaces will show that δx, δy, δz, and all their differential coefficients, vanish for the fixed curve. If, therefore, these functions retained throughout the same form it is plain that the value of each could be no other than zero. Before proceeding to consider how far this conclusion is modified by a change in the forms of the functions, we shall prove the following theorems, which are essential to our purpose.

(I.) If the functions which represent the displacements of an inextensible surface have different forms at different points of the surface, the parts of the surface for which these functions retain the same forms are bounded by curves of flexure.

This theorem is proved by reasoning nearly identical with that of p. 354. For, if possible, let the forms of these functions change in passing across a curve which is not a curve of flexure. Let

$$u = U, \qquad v = V, \qquad w = W,$$

be the values which hold at one side of the curve, and

$$u = U', \qquad v = V', \qquad w = W',$$

those which hold at the other. Then it will appear precisely as in p. 354 that if we form a third system of values,

$$u = U - U', \qquad v = V - V', \qquad w = W - W',$$

this system will satisfy the equations (C), and will, moreover, be such that for every point of the curve in question we shall have

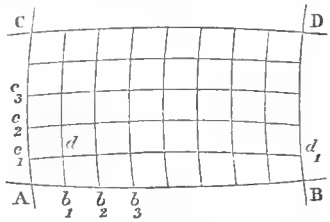
$$u = 0, \qquad v = 0, \qquad w = 0.$$

Since, then, the bounding curve is not a curve of flexure, and since U, V, W, U', V', W', are functions of determinate form, it is plain that we must have generally

$$U - U' = 0, \qquad V - V' = 0, \qquad W - W' = 0.$$

No one, therefore, of the functions u, v, w, can change its form, except in passing across a curve of flexure. Hence the proposition is evident.

(II.) Let AB, AC be two arcs of curves of flexure commencing at the same point A. Through B, C draw the curves of flexure BD, CD, meeting in D. Then if AB, AC be fixed, the entire quadrilateral ABDC is fixed also.



The truth of this theorem is nearly evident from the theory of partial differential equations, combined with the principle laid down in (I.), but it may be strictly proved as follows:

Since u, v, w, can only change their forms in passing a curve of flexure, we may suppose them to retain the same form throughout the entire of the quadrilateral $Ab_1d_1c_1$, formed by drawing the curves of flexure b_1d, c_1d.

Let $$\theta = c, \qquad \theta' = c',$$

be the equations of the two series of curves of flexure. Then, since the functions θ, θ' satisfy the differential equations

$$r\frac{d\theta^2}{dy^2} - 2s\frac{d\theta}{dx}\frac{d\theta}{dy} + t\frac{d\theta^2}{dx^2} = 0,$$

$$r\frac{d\theta'^2}{dy^2} - 2s\frac{d\theta'}{dx}\frac{d\theta'}{dy} + t\frac{d\theta'^2}{dx^2} = 0,$$

if the independent variables x, y, be changed into θ, θ', the equation

$$r\frac{d^2w}{dy^2} - 2s\frac{d^2w}{dxdy} + t\frac{d^2w}{dx^2} = 0$$

will (as is well known) assume the form

$$\frac{d^2w}{d\theta d\theta'} + P\frac{dw}{d\theta} + Q\frac{dw}{d\theta'} = 0, \tag{X}$$

P, Q, being functions of θ, θ'. Now since w vanishes for the curve

$$\theta = c,$$

it is plain that we must have throughout this curve,

$$\frac{dw}{d\theta'} = 0, \quad \frac{d^2w}{d\theta'^2} = 0, \quad \frac{d^3w}{d\theta'^3} = 0, \text{ \&c.},$$

and for the curve

$$\theta' = 0,$$

$$\frac{dw}{d\theta} = 0, \quad \frac{d^2w}{d\theta^2} = 0, \quad \frac{d^3w}{d\theta^3} = 0, \text{ \&c.}$$

For the point A, therefore, which is the intersection of these curves, both these systems of equations must be satisfied. Putting, then, in equation (X),

$$\frac{dw}{d\theta} = 0, \quad \frac{dw}{d\theta'} = 0,$$

we have

$$\frac{d^2w}{d\theta d\theta'} = 0,$$

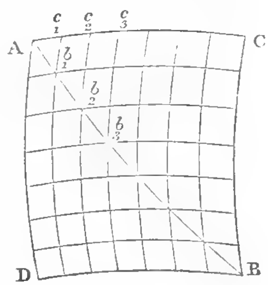
and it is easily seen that neither P nor Q will become infinite; and by following the same reasoning with that of p. 352, we shall find that for the point A all the differential coefficients of w must vanish. Hence as the form of w remains the same throughout the quadrilateral Ab_1dc_1, we must have for the whole of that quadrilateral

$$w = 0.$$

Now it is evident that the reasoning which we have applied to the point A is in every respect applicable to b_1, and thus in succession to b_2, b_3, &c. The value of w will therefore be zero for all points of a second curve of flexure c_1d_1. And by pursuing the same method we see evidently that w must vanish throughout the whole of the quadrilateral ABDC. Hence, the direction of the axis of z being indeterminate, we shall have in general,

$$\delta z = 0, \quad \delta y = 0, \quad \delta x = 0,$$

throughout ABDC. The whole of this quadrilateral is therefore fixed. We shall now proceed to consider the general case.



Let AB be any arc of a curve (not a curve of flexure) traced upon the surface. Through A, B, draw the curves of flexure, AC, AD, BC, BD. Then if AB be fixed, the quadrilateral ACBD is fixed also.

For whatever law or laws we suppose the displacements to follow, it is plain that we may assume a number of points, b_1, b_2, b_3, &c. so close that one of these displacements, w, for example, shall retain the same form throughout each one of the quadrilaterals

Ab_1, b_1b_2, b_2b_3, &c., formed by drawing curves of flexure through b_1, b_2, &c. Hence, and from p. 359, it is evident that w must vanish throughout the entire of each of these quadrilaterals. But if

$$w = 0$$

for the quadrilaterals Ab_1, b_1b_2, it follows from Theorem II. p. 360, that w must vanish for the quadrilateral b_1c_2; and by pursuing the same method we shall easily see that we must have

$$w = 0$$

for each of the quadrilaterals into which ACBD is divided. Hence the truth of the proposition is evident. This proposition may be expressed by saying that

If an arc of a curve traced upon an inextensible surface be rendered fixed or rigid, the entire of the quadrilateral, formed by drawing the two curves of flexure through each extremity of the curve, becomes fixed or rigid also.

6. We shall now proceed to consider the case of surfaces which, without being wholly inextensible, have at each point one or more inextensible directions.

Reverting to the discussion of p. 345, and making

$$\frac{dx}{ds} = \cos\alpha, \qquad \frac{dy}{ds} = \cos\beta,$$

we find easily

$$\frac{\delta ds}{ds} = \left(\frac{du}{dx} - wr\right)\cos^2\alpha + \left(\frac{du}{dy} + \frac{dv}{dx} - 2ws\right)\cos\alpha\cos\beta + \left(\frac{dv}{dy} - wt\right)\cos^2\beta. \quad \text{(Y)}$$

From this equation it is plain that, unless the coefficients of

$$\cos^2\alpha, \qquad \cos\alpha\cos\beta, \qquad \cos^2\beta,$$

vanish separately, there can be, for each law of displacement, but two values of

$$\frac{\cos\alpha}{\cos\beta},$$

which will satisfy the equation

$$\delta ds = 0.$$

If these coefficients vanish separately, δds will vanish for every direction round the point. Hence it is easy to infer the following theorems :—

If a surface have at each point three or more inextensible directions, it is wholly inextensible.

A surface may have at each point one or two inextensible directions, without being wholly inextensible.

Suppose that the given surface has at each point two inextensible curves included in the equation

$$R dx^2 + 2 S dx dy + T dy^2 = 0,$$

or

$$R \cos^2 \alpha + 2 S \cos \alpha \cos \beta + T \cos^2 \beta = 0.$$

Then, as this equation must be identical with

$$\left(\frac{du}{dx} - wr\right) \cos^2 \alpha + \left(\frac{du}{dy} + \frac{dv}{dx} - 2ws\right) \cos \alpha \cos \beta + \left(\frac{dv}{dy} - wt\right) \cos^2 \beta = 0,$$

we shall have

$$\frac{1}{R}\left(\frac{du}{dx} - wr\right) = \frac{1}{2S}\left(\frac{du}{dy} + \frac{dv}{dx} - 2ws\right) = \frac{1}{T}\left(\frac{dv}{dy} - wt\right). \qquad \text{(Z)}$$

These two equations contain the entire theory of the surfaces under consideration.

Suppose, for example, that the surface is one of dissimilar curvatures, and that its curves of flexure are inextensible. We have then

$$R = r, \qquad S = s, \qquad T = t,$$

and the equations (Z) become

$$\frac{1}{r}\frac{du}{dx} = \frac{1}{2s}\left(\frac{du}{dy} + \frac{dv}{dx}\right) = \frac{1}{t}\frac{dv}{dy}, \qquad \text{(A')}$$

being identical with the equations which are found by eliminating w between the general equations (C), p. 346. The displacement w remains indeterminate. From these considerations it is easy to deduce the following theorem :—

If the curves of flexure traced upon a surface with dissimilar curvatures be inextensible, the most general displacement of which the surface is capable may be found by supposing it first to move as an inextensible surface, and then to receive at each point a normal displacement of arbitrary magnitude.

Let W be an arbitrary function of x and y. Then the equations (A′) being put under the form

$$\frac{du}{dx} = Wr, \qquad \frac{du}{dy} + \frac{dv}{dx} = 2Ws, \qquad \frac{dv}{dy} = Wt,$$

the expression for the extension of any small arc ds (Y) will become

$$\delta ds = (W - w)\, ds\, (r \cos^2 \alpha + 2s \cos \alpha \cos \beta + t \cos^2 \beta).$$

Hence for the class of surfaces under consideration we infer that—

The extension of any small arc of a curve commencing at a given point, divided by the arc itself, varies inversely as the radius of curvature of the normal section which passes through it.

7. Having thus investigated the case of inextensible and partially inextensible surfaces, we should, in the next place, proceed to consider how far the results arrived at are applicable to the various membranes which we find in nature, and which are neither perfectly inextensible nor altogether devoid of thickness. But before entering upon this question we shall briefly examine the case of inextensible *bodies*.

Conceive a curve to be traced in the interior of a body, passing through the *successive* physical points or molecules a, b, c, d, &c. Suppose now that the several points of the body receive small displacements, and take the curve which is the locus of the points a, b, c, d, &c. in their new position. If the length of the second curve be equal to that of the first, and if this be true of all curves which can be so drawn, the body may be said to be inextensible. Adopting this definition, we shall have the following theorems :—

I. *Every body which is perfectly inextensible is also perfectly rigid.*

II. *Any body may, without being wholly inextensible, have at each of its points an infinite number of inextensible directions, and these directions will be situated upon a cone of the second order.*

Let δx, δy, δz, be the displacements of any point in the body, and let ds be an element of a curve, making with the axes of co-ordinates the angles α, β, γ. Then it is easily seen, that the variation of this element will be given by the equation

$$\frac{\delta ds}{ds} = \frac{d\delta x}{dx}\cos^2\alpha + \frac{d\delta y}{dy}\cos^2\beta + \frac{d\delta z}{dz}\cos^2\gamma \qquad (\text{B}')$$

$$+\left(\frac{d\delta z}{dy}+\frac{d\delta y}{dz}\right)\cos\beta\cos\gamma+\left(\frac{d\delta x}{dz}+\frac{d\delta z}{dx}\right)\cos\gamma\cos\alpha+\left(\frac{d\delta y}{dx}+\frac{d\delta x}{dy}\right)\cos\alpha\cos\beta.$$

Now if the body be inextensible, we must have for all values of α, β, γ,

$$\delta ds = 0.$$

Hence we have the six equations,

$$\frac{d\delta x}{dx} = 0, \quad \frac{d\delta y}{dy} = 0, \quad \frac{d\delta z}{dz} = 0\,; \qquad (\text{C}')$$

$$\frac{d\delta z}{dy} + \frac{d\delta y}{dz} = 0, \quad \frac{d\delta x}{dz} + \frac{d\delta z}{dx} = 0, \quad \frac{d\delta y}{dx} + \frac{d\delta x}{dy} = 0.$$

Integrating this system of equations, which may be effected without difficulty, we find,

$$\delta x = a + Bz - Cy,$$
$$\delta y = b + Cx - Az,$$
$$\delta z = c + Ay - Bx\,;$$

the well-known expression for the displacements of a rigid body. These being the most general values which δx, δy, δz admit of, the truth of the first theorem is evident.

With regard to the second theorem, if the body is so constituted that the displacements δx, δy, δz must satisfy the equations

$$\frac{d\delta x}{dx} = A, \quad \frac{d\delta y}{dy} = B, \quad \frac{d\delta z}{dz} = C, \qquad (\text{D}')$$

$$\frac{d\delta z}{dy} + \frac{d\delta y}{dz} = 2A', \quad \frac{d\delta x}{dz} + \frac{d\delta z}{dx} = 2B', \quad \frac{d\delta y}{dx} + \frac{d\delta x}{dy} = 2C'.$$

A, B, C, a, b, c, being functions of x, y, z, the body will have at each point an infinite number of inextensible directions situated on the cone (real or imaginary),

$$A\cos^2\alpha + B\cos^2\beta + C\cos^2\gamma + 2A'\cos\beta\cos\gamma + 2B'\cos\gamma\cos\alpha + 2C'\cos\alpha\cos\beta = 0.$$

If the constitution of the body be given, A, B, C, &c., will be given functions. In this case the equations (D′) furnish the means of determining

δx, δy, δz. Thus, for example, if the body be homogeneous, A, B, C, &c., will be constants, and it is not difficult to prove that δx, δy, δz will be of the form

$$\begin{aligned} \delta x &= ax + by + cz + d, \\ \delta y &= a'x + b'y + c'z + d', \\ \delta z &= a''x + b''y + c''z + d''. \end{aligned} \qquad \text{(E')}$$

Let x', y', z', be the co-ordinates of the molecule in its new position. Then since

$$\begin{aligned} \delta x &= x' - x, \\ \delta y &= y' - y, \\ \delta z &= z' - z; \end{aligned}$$

we have

$$\begin{aligned} x' &= (a + 1)\,x + by + cz + d, \\ y' &= a'x + (b' + 1)\,y + c'z + d', \\ z' &= a''x + b''y + (c'' + 1)\,z + d''. \end{aligned} \qquad \text{(F')}$$

Hence it is easy to infer the following theorem:

If a homogeneous body have at each point a cone of inextensible directions, and if in the interior of the body there be described an algebraic surface of any order, all the molecules situated upon that surface will after displacement be situated upon a surface of the same order.

In general, whatever be the nature of the body, if ds be an element making with the axes the angles α, β, γ, which satisfy the equation

$$\frac{d\delta x}{dx}\cos^2\alpha + \frac{d\delta y}{dy}\cos^2\beta + \frac{d\delta z}{dz}\cos^2\gamma$$
$$+\left(\frac{d\delta y}{dz} + \frac{d\delta z}{dy}\right)\cos\beta\cos\gamma + \left(\frac{d\delta z}{dx} + \frac{d\delta x}{dz}\right)\cos\alpha\cos\gamma + \left(\frac{d\delta x}{dy} + \frac{d\delta y}{dx}\right)\cos\alpha\cos\beta = 0;$$

it is plain that we shall have

$$\delta ds = 0.$$

Hence,

Whatever be the law of the displacement, there will be at each point of the body an infinite number of directions (forming a cone of the second order), for which the length of the element will be unchanged.

We shall now return to the case of surfaces.

8. The preceding discussion of the properties of inextensible surfaces is of course a mathematical abstraction, not strictly applicable to any substance which we find in nature. Every membrane with which we are acquainted is possessed of some extensibility; and all substances have of course a certain thickness. Our definition, therefore, of an inextensible surface is not strictly true for any really existing substance. But as there are in nature many substances for which this definition is very approximately true, it becomes a question of some interest to determine how far the results of the preceding investigation are applicable to such substances. We shall, therefore, proceed to consider the case of a membrane whose thickness is indefinitely small as compared with its other dimensions, and whose extensibility is such that in any displacement of the membrane, the variation in the *length* of any arc of a curve traced upon its surface is indefinitely small compared with the displacement of any of its parts. Thus, if x, y, z be the co-ordinates of any point on the surface, and s an arc of a curve traced upon it, the assumption which we shall make as to the inextensibility of the membrane may be mathematically expressed by saying that δs is indefinitely small compared with δx. If it be necessary to take *thickness* into account, we must suppose s to be a curve traced arbitrarily in the substance of the membrane. Supposing, for the sake of greater generality, that this is the case, we may state the problem under discussion as follows:

To determine the possible displacement of a membrane very slightly extensible, and whose thickness is very small compared with its other dimensions.

Let x', y', z' be the co-ordinates of a point in the substance of the membrane; x, y, z, the co-ordinates of a point on the surface, indefinitely near to the first; and i, a quantity of the same order of magnitude as the thickness of the membrane.

Through the point $x'y'z'$ let a normal be drawn to the surface of the membrane, and let n represent the part of the normal between $x'y'z'$ and its intersection with the surface, which we shall denote by x, y, z. Then if α, β, γ be the cosines of the angles which the normal makes with the axes, we shall have

$$\begin{aligned} x' &= x + \alpha n, \\ y' &= y + \beta n, \\ z' &= z + \gamma n. \end{aligned} \qquad (\text{G}')$$

Differentiating these equations, and rejecting $nd\alpha$, $nd\beta$, $nd\gamma$, on account of the small quantity n, which is of the same order as the thickness of the membrane, we have

$$\left.\begin{aligned} dx' &= dx + \alpha dn, \\ dy' &= dy + \beta dn, \\ dz' &= dz + \gamma dn. \end{aligned}\right\} \qquad (\text{H}')$$

If now we represent by ds' an arc of a curve traced in the substance of the membrane, we shall have, as before,

$$ds'\delta ds' = dx'd\delta x' + dy'd\delta y' + dz'd\delta z'. \qquad (\text{I}')$$

But if we regard $\delta x'$, $\delta y'$, $\delta z'$ as functions of the three variables x, y, n, we shall have

$$\begin{aligned} d\delta x' &= \frac{d\delta x'}{dx}dx + \frac{d\delta x'}{dy}dy + \frac{d\delta x'}{dn}dn, \\ d\delta y' &= \frac{d\delta y'}{dx}dx + \frac{d\delta y'}{dy}dy + \frac{d\delta y'}{dn}dn, \\ d\delta z' &= \frac{d\delta z'}{dx}dx + \frac{d\delta z'}{dy}dy + \frac{d\delta z'}{dn}dn. \end{aligned} \qquad (\text{K}')$$

Substituting in (I′) the values (H′) and (K′), and putting for dz its value $pdx + qdy$, we have

$$\begin{aligned} \frac{d\delta s'}{ds} = {} & \left(\frac{d\delta x'}{dx} + p\frac{d\delta z'}{dx}\right)\frac{dx^2}{ds'^2} + \left(\frac{d\delta y'}{dy} + q\frac{d\delta z'}{dy}\right)\frac{dy^2}{ds'^2} \\ & + \left(\frac{d\delta x'}{dy} + \frac{d\delta y'}{dx} + p\frac{d\delta z'}{dy} + q\frac{d\delta z'}{dx}\right)\frac{dx}{ds'}\frac{dy}{ds'} \\ & + \left(\frac{d\delta x'}{dn} + p\frac{d\delta z'}{dn} + \alpha\frac{d\delta x'}{dx} + \beta\frac{d\delta y'}{dx} + \gamma\frac{d\delta z'}{dx}\right)\frac{dx}{ds'}\frac{dn}{ds'} \\ & + \left(\frac{d\delta y'}{dn} + q\frac{d\delta z'}{dn} + \alpha\frac{d\delta x'}{dy} + \beta\frac{d\delta y'}{dy} + \gamma\frac{d\delta z'}{dy}\right)\frac{dy}{ds'}\frac{dn}{ds'} \\ & + \left(\alpha\frac{d\delta x'}{dn} + \beta\frac{d\delta y'}{dn} + \gamma\frac{d\delta z'}{dn}\right)\frac{dn^2}{ds'^2}. \end{aligned} \qquad (\text{L}')$$

Now since $d\delta s'$ is by hypothesis indefinitely small, as compared with any one of the quantities $d\delta x'$, $d\delta y'$, $d\delta z'$, and since this is true for all directions of the arc ds', it is plain that the coefficients of each of the quantities

$$\frac{dx^2}{ds'^2},\quad \frac{dy^2}{ds'^2},\quad \frac{dx}{ds'}\frac{dy}{ds'},\quad \frac{dx}{ds'}\frac{dn}{ds'},\quad \frac{dy}{ds'}\frac{dn}{ds'},\quad \frac{dn^2}{ds'^2},$$

must be indefinitely small as compared with δx, δy, δz. We shall, in the first place, consider the coefficients of the first three of these quantities.

If we neglect, as before, quantities of the second order, we may evidently substitute in these coefficients δx, δy, δz, for $\delta x'$, $\delta y'$, $\delta z'$. We shall have then

$$\frac{d\delta x}{dx} + p\frac{d\delta z}{dx} = ia,$$

$$\frac{d\delta x}{dy} + \frac{d\delta y}{dx} + p\frac{d\delta z}{dy} + q\frac{d\delta z}{dx} = 2ib,$$

$$\frac{d\delta y}{dy} + q\frac{d\delta z}{dy} = ic,$$

be satisfied, where a, b, c are functions of x and y of the same order of magnitude as δx, δy, δz.

Transforming these equations as in p. 346, we find

$$\frac{du}{dx} - wr = ia,$$

$$\frac{du}{dy} + \frac{dv}{dx} - 2ws = 2ib, \qquad (\text{M}')$$

$$\frac{dv}{dy} - wt = ic.$$

Now it is well known, that such a system of equations may always be satisfied by the values

$$u = u' + iu_1,$$
$$v = v' + iv_1,$$
$$w = w' + iw_1;$$

where u', v', w', satisfy the equations

$$\frac{du'}{dx} - w'r = 0,$$

$$\frac{du'}{dy} + \frac{dv'}{dx} - 2w's = 0,$$

$$\frac{dv'}{dy} - w't = 0.$$

Hence it is plain, that the displacements of a surface which is but slightly extensible will differ from those of an inextensible surface, by quantities which are of the same order of magnitude as the extensibility of the surface. From this it is easy to infer, that all the theorems which are *rigorously* true for an inextensible surface are *approximately* true for a surface possessed of an indefinitely small amount of extensibility.

Let us now consider the coefficients of the quantities

$$\frac{dx}{ds'}\frac{dn}{ds'}, \quad \frac{dy}{ds'}\frac{dn}{ds'}, \quad \frac{dn^2}{ds'^2}.$$

These coefficients give the equations

$$\left.\begin{aligned} \frac{d\delta x'}{dn} + p\frac{d\delta z'}{dn} + \alpha\frac{d\delta x'}{dx} + \beta\frac{d\delta y'}{dx} + \gamma\frac{d\delta z'}{dx} &= iA, \\ \frac{d\delta y'}{dn} + q\frac{d\delta z'}{dn} + \alpha\frac{d\delta x'}{dy} + \beta\frac{d\delta y'}{dy} + \gamma\frac{d\delta z'}{dy} &= iB, \\ \alpha\frac{d\delta x'}{dn} + \beta\frac{d\delta y'}{dn} + \gamma\frac{d\delta z'}{dn} &= iC, \end{aligned}\right\} \quad (\mathrm{N}')$$

A, B, C being of the same order as $\delta x'$, $\delta y'$, $\delta z'$. Since α, β, γ are independent of n, the third of these equations may be integrated at once. Performing the integration, and supposing the integrals to begin when

$$x' = x, \quad y' = y, \quad z' = z,$$

we have

$$\alpha\delta x' + \beta\delta y' + \gamma\delta z' = \alpha\delta x + \beta\delta y + \gamma\delta z + i\int_0^n C dn. \quad (\mathrm{O}')$$

Now it is evident that

$$\alpha\delta x' + \beta\delta y' + \gamma\delta z' = \delta n,$$
$$\alpha\delta x + \beta\delta y + \gamma\delta z = (\delta n)_0,$$

denoting by $(\delta n)_0$ the normal displacement of the point on the surface. Equation (O′) becomes, therefore,

$$\delta n = (\delta n)_0 + i\int_0^n C dn.$$

Now the definite integral

$$\int_0^n C dn$$

is evidently a small quantity of the second order; if therefore we neglect quantities of the third order, we shall have

$$\delta n = (\delta n)_0.$$

Hence we infer that—

In all possible displacements of a thin membrane or lamina which is very slightly extensible, the normal displacements of points situated on the same normal to the surface are equal.

This would also follow from the next theorem.

Substituting in the first two equations (N′) for α, β, γ their values in terms of p and q, we have

$$\left.\begin{aligned}\frac{d\delta x'}{dn} + p\frac{d\delta z'}{dn} + \frac{1}{\sqrt{(1+p^2+q^2)}}\left(\frac{d\delta z'}{dx} - p\frac{d\delta x'}{dx} - q\frac{d\delta y'}{dx}\right) = iA,\\ \frac{d\delta y'}{dn} + p\frac{d\delta z'}{dn} + \frac{1}{\sqrt{(1+p^2+q^2)}}\left(\frac{d\delta z'}{dy} - p\frac{d\delta x'}{dy} - q\frac{d\delta y'}{dy}\right) = iB.\end{aligned}\right\} \quad (\mathrm{P}')$$

Now it is plain that without altering the form of these equations we may substitute, in the last three terms of each, δx, δy, δz for $\delta x'$, $\delta y'$, $\delta z'$. For this substitution merely amounts to the addition of quantities of the same order as iA, iB, to the right-hand members of these equations. Again, referring to p. 348, we have

$$\frac{d\delta z}{dx} - p\frac{d\delta x}{dx} - q\frac{d\delta y}{dx} = \delta p,$$

$$\frac{d\delta z}{dy} - p\frac{d\delta x}{dy} - q\frac{d\delta y}{dy} = \delta q.$$

Making these substitutions in equations (P′), we have

$$\frac{d\delta x'}{dn} + p\frac{d\delta z'}{dn} + \frac{\delta p}{\sqrt{(1+p^2+q^2)}} = iA,$$
$$\frac{d\delta y'}{dn} + q\frac{d\delta z'}{dn} + \frac{\delta q}{\sqrt{(1+p^2+q^2)}} = iB.$$

Integrating these equations between the limits 0 and n, and neglecting as before quantities of the third order, we have

$$\left.\begin{aligned}\delta(x'-x) + p\delta(z'-z) + \frac{n\delta p}{\sqrt{(1+p^2+q^2)}} &= 0,\\ \delta(y'-y) + q\delta(z'-z) + \frac{n\delta q}{\sqrt{(1+p^2+q^2)}} &= 0.\end{aligned}\right\} \quad (Q')$$

But since

$$\frac{n}{\sqrt{(1+p^2+q^2)}} = z' - z,$$

these equations may evidently be written

$$\delta\{x' - x + p(z'-z)\} = 0,$$
$$\delta\{y' - y + q(z'-z)\} = 0.$$

Hence recollecting that the points xyz, $x'y'z'$, were originally on the same normal, we have still, *after* the displacement,

$$x' - x + p(z'-z) = 0,$$
$$y' - y + q(z'-z) = 0.$$

We infer, therefore, that—

In every possible displacement of a thin membrane or lamina whose extensibility is very small, all points which were originally situated on the same normal to the surface will remain so after the displacement.

This important theorem, which is assumed as an hypothesis by most writers on the equilibrium of elastic laminæ, is thus established, independently of any theory of molecular force, as a mathematical consequence of the small amount of extensibility which is possessed by the lamina.

It may be well, before concluding, to say a few words in explanation of the rule which we have followed in the rejection of small quantities.

Small quantities of the first order, as δx, &c., have been retained throughout.

Small quantities of the second order are rejected in the expressions for dx', dy', dz', (H′), because the retention of these quantities would leave the form of the equations (M′) and (N′) altogether unchanged.

Small quantities of the third order are rejected in equations (O′) and (Q′), because the differences

$$\delta n - (\delta n)_0, \quad \delta x' - \delta x, \&c.$$

ought properly to be of the second order. If we retain these terms we may enunciate the foregoing theorem rigorously as follows :

If a membrane which is but slightly extensible receive a finite displacement, the separation of any point from the normal drawn through the corresponding point on the surface, is indefinitely small, as compared with the distance of these points from each other.

With respect to the comparative magnitude of the two small quantities i and n, depending respectively upon the extensibility and the thickness of the lamina, it may have been observed that throughout the preceding discussion they have been treated as quantities of the same order. Let us consider what would be the effect of a violation of this rule.

As the thickness of the membrane is not supposed to be insensible, we cannot suppose n to be indefinitely small as compared with i, without assigning to the membrane an amount of extensibility *not* indefinitely small. This would remove it from the class of substances which we have been considering.

If we had supposed i to be indefinitely small as compared with n, we should not have been justified in rejecting $nd\alpha$, $nd\beta$, $nd\gamma$ in forming the expressions (H′), p. 368. Our investigation would not, therefore, have differed in any material respect from that of the displacements of a *body* of finite dimensions and of an indefinitely small amount of extensibility ; and in such a case it would readily appear from the discussion of p. 365 that the body would be, q. p., rigid. We see then that—

No membrane can be flexible which does not possess an amount of extensibility finite, as compared with its thickness.

It is, perhaps, superfluous to add, that it is not necessary to the truth of the preceding theorems that the membrane should be absolutely or approximately inextensible by any imaginable force. It is sufficient for our purpose if the forces which are supposed to be applied to the membrane are incapable of extending it. And in such a case all the foregoing theorems will hold, if we substitute for "all possible displacements," "all displacements which can be effected by any amount of force which is supposed to be present."

Some interesting practical conclusions follow from this discussion. Thus, if we desire to take advantage of the very slight extensibility of many species of laminæ, to enable them to resist flexure, it appears, from p. 359, that we must be careful to form the lamina originally, while in a soft, semi-fluid, or otherwise extensible state, into a surface whose curvatures are similar, otherwise it will always be liable to bend along a curve of flexure. If sufficient force be used to make the lamina bend along any other curve or in any way violate the conditions which have been established, it will be found that there is always produced a *crease*, in other words a curve, along which the separation between one molecule and the next is not indefinitely small. In such a case there will in general be a permanent alteration in the substance of the lamina. Thus, for example, it is easy to fold a sheet of paper into the form of a cone, without breaking or in any way injuring it. Let the base of this cone be rendered rigid by being attached to a ring, and it will be found that any further attempt to bend the paper will produce a *crease*, or curve of permanent alteration in its substance.

Again, from the discussion of p. 361, we may deduce the practical conclusion, that the strength by which a surface of dissimilar curvatures resists flexure may be greatly increased, if it be traversed by a rigid rod attached to its substance, along any curve not a curve of flexure.

NOTE.

SINCE the foregoing sheets were printed, I have arrived at the following theorem, which is of some interest, as connected with the class of surfaces which we have been examining:

If a closed oval surface be perfectly inextensible, it is also perfectly rigid.

To prove this, let us denote, as before, by δx, δy, δz, the resolved displacements of any point on the surface. Let $\delta' x, \delta' y, \delta' z$ be its most general displacements considered as a rigid body; then it is known that

$$\delta' x = a + Cy - Bz,$$
$$\delta' y = b + Az - Cx,$$
$$\delta' z = c + Bx - Ay,$$

a, b, c, A, B, C, being constants. Now if we form a third system—

$$\Delta x = \delta x + \delta' x,$$
$$\Delta y = \delta y + \delta' y,$$
$$\Delta z = \delta z + \delta' z,$$

it is plain that Δx, Δy, Δz will satisfy the conditions of the problem contained in equations (B) or (C). Moreover, if $x_1 y_1 z_1$, $x_2 y_2 z_2$ be two given points on the surface, the constants a, b, c, A, B, C, can always be so determined as to satisfy the equations

$$\Delta x_1 = 0, \quad \Delta z_1 = 0,$$
$$\Delta x_2 = 0, \quad \Delta z_2 = 0,$$

without in any way limiting the generality of the displacements δx, δy, δz. Suppose now that we assume, as in p. 346,

$$u = \Delta x + p\Delta z, \quad w = \Delta z,$$

it is plain that u, w will satisfy the first of equations (C), and will vanish at the two points $x_1 y_1 z_1$, $x_2 y_2 z_2$. Let these points be P, Q, and suppose, to fix our ideas, that the axis of z passes through them. The plane of xz will then intersect the surface in a closed curve, $PRQS$, passing through these points. Now since u vanishes at the points P, Q, if we trace its values in passing along the curve $PRQS$, we shall find a *maximum* value (disregarding its sign) somewhere between P and Q as at R, and again somewhere between Q and P as at S. We have, therefore, for each of the points R, S,

$$\frac{du}{dx} = 0,$$

since the equation of the curve $PRQS$ is

$$dy = 0.$$

The first of equations (C) gives us then at each of these points

$$w = 0.$$

But since the position of the axis of x is indeterminate, it follows from what has been said, that, on *every* section of the surface made by a plane passing through the axis of z, there will be at least two points, for which

$$w = 0.$$

Hence it is plain that there will be on the surface one or more closed curves for which this condition will hold. It will be sufficient to consider one of these curves, which, for the sake of distinctness, we may call an equator.

We have seen, p. 347, that w must satisfy the equation

$$r\frac{d^2w}{dy^2} - 2s\frac{d^2w}{dxdy} + t\frac{d^2w}{dx^2} = 0,$$

or, as it may be otherwise written,

$$\frac{d}{dy}\left(r\frac{dw}{dy} - s\frac{dw}{dx}\right) + \frac{d}{dx}\left(t\frac{dw}{dx} - s\frac{dw}{dy}\right) = 0.$$

Multiply this equation by $dxdy$, and integrate it through the whole of either of the segments into which the surface is divided by the equator. We have then

$$\int\left(r\frac{dw}{dy} - s\frac{dw}{dx}\right)dx + \int\left(t\frac{dw}{dx} - s\frac{dw}{dy}\right)dy = 0,$$

the single integrations being extended through the whole of the bounding curve. But since, for every point in this curve we have

$$w = 0,$$

if this equation be transformed according to the usual rule (*Calculus of Variations*, p. 218) it will become

$$\int\left(r\frac{dw^2}{dy^2} - 2s\frac{dw}{dy}\frac{dw}{dx} + t\frac{dw^2}{dx^2}\right)\Omega ds = 0,$$

where ds is the element of the bounding curve, and

$$\Omega = \left(\frac{dw^2}{dx^2} + \frac{dw^2}{dy^2}\right)^{-\frac{1}{2}}$$

Now since in the class of surfaces which we are considering,

$$rt - s^2 > 0,$$

it is easily seen that all the elements of the foregoing definite integral must have the same sign. The total integral cannot therefore vanish unless each of its elements vanishes. Hence it is plain that we must have at each point of the equator

$$\frac{dw}{dx} = 0, \quad \frac{dw}{dy} = 0.$$

If we now follow the same reasoning as in p. 352 we shall readily see that *all* the differential coefficients of w will vanish at the equator, and therefore that we must have *generally*

$$w = 0.$$

Hence, and from p. 347, it is evident that the displacements represented by Δx, Δy, Δz, are those of a rigid body. Since then $\delta' x$, $\delta' y$, $\delta' z$ are by hypothesis the displacements of a rigid body, it is evident that the differences between these quantities, $\Delta x - \delta' x$, $\Delta y - \delta' y$, $\Delta z - \delta' z$, or δx, δy, δz, are so likewise. We infer, therefore, that—

The most general displacement which a closed, oval, inextensible surface admits of, is that of a rigid body.

Such a surface is therefore inflexible.

XVI.—*On the Attraction of Ellipsoids, with a new Demonstration of* CLAIRAUT'S *Theorem, being an Account of the late* Professor MAC CULLAGH'S *Lectures on those Subjects. Compiled by* GEORGE JOHNSTON ALLMAN, LL. D., *of Trinity College, Dublin.*

Read June 13, 1853.

[THE following Memoir contains the substance of a Series of Lectures delivered by the late Professor MAC CULLAGH to the Candidates for Fellowship in Trinity College, Dublin, in Hilary and Michaelmas Terms, 1846.

It is now published by the Academy, with the view of securing to Professor MAC CULLAGH the merit of whatever is original in the investigation or its results.

The present Paper may be regarded as a Sequel to the Account of Professor MAC CULLAGH'S Lectures on Rotation, given by the Rev. SAMUEL HAUGHTON in a former part of the present volume of the Transactions of the Academy.]

PROPOSITION I.

If P *be any point on the surface of an ellipsoid, and* PC_1 *be drawn perpendicular to an axis* OC, *and an ellipsoid be described through* C_1 *concentric, similar and similarly placed to the given ellipsoid; then the component of the attraction of the given ellipsoid on* P *in a direction parallel to* OC *is equal to the attraction of the inner ellipsoid on the point* C_1.

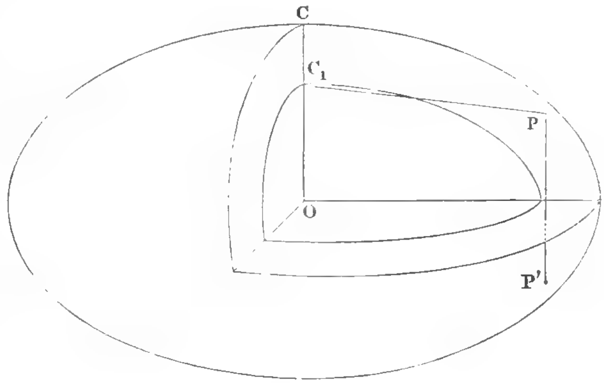


FIG. 1.

This theorem is an extension of that given by MAC LAURIN* relating to the attraction of a spheroid on a point placed on its surface. It may, moreover, be established by means of the same geometrical proposition from which MAC LAURIN deduced his theorem.

Through the point P let a chord PP′ of the given ellipsoid be drawn parallel to the axis OC; now suppose both ellipsoids to be divided into wedges by planes parallel to each other, and passing respectively through this chord and the parallel axis of the inner; and suppose the wedges to be divided into pyramids, the common vertex of one set being at P, and of the other at C_1. Observing that any two of these parallel planes cut the two surfaces in similar ellipses, such that the semi-axis of one is equal to the parallel ordinate of the other, it is easy to see that the reasoning employed by MAC LAURIN may be used to establish the truth of the theorem stated above.

* De Caus. Phys. Flux. et Refl. Maris, sect. 3. Or see Airy's Tract on the Figure of the Earth, Prop. 8.

PROPOSITION II.

*To calculate the attraction of an ellipsoid on a point placed at the extremity of an axis.**

Let the semi-axes of the ellipsoid be a, b, c, where $a > b > c$, and let the point on which it is required to find the attraction be C, the extremity of the least axis.

Suppose the ellipsoid to be divided by a series of cones of revolution which have a common vertex C and a common axis CC′, C′ being the vertex of the ellipsoid opposite to C; it will be sufficient to find an expression for the attraction of the part of the ellipsoid contained between two consecutive conical surfaces, whose semi-angles are θ and $\theta + d\theta$ respectively. Suppose now the part of the ellipsoid between two consecutive cones to be divided into elementary pyramids with a common vertex C. Let CP be one of these elementary pyramids, whose solid angle is ω; let PQ be drawn perpendicular to CC′; from the centre O draw a radius vector OR parallel to CP, and from the extremity R let fall a perpendicular RS on the axis CC′.

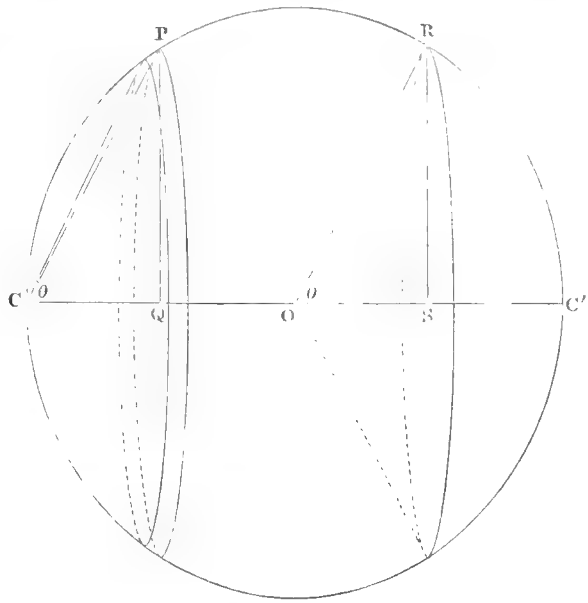


FIG. 2.

Now the attraction of the elementary pyramid CP on the material point μ, placed at its vertex $= \mu f \rho \omega \, . \, \mathrm{CP}$; and the component of this attraction in the direction of the axis is

* Proceedings of the Royal Irish Academy, vol. iii. p. 367.

$$\mu f \rho \omega \,.\, \mathrm{CQ} = 2\,\mu f \rho \omega \,.\, \frac{\overline{\mathrm{OR}}^2 \cos^2\theta}{c}\,.$$

Now suppose the radius vector OR to revolve around the axis OC′, then the attraction on the point C of the portion of the ellipsoid bounded by the two cones of revolution, whose semi-angles are θ and $\theta + d\theta$ respectively, since it is made up of the components in the direction CC′ of the attractions of all the elementary pyramids CP, is

$$\frac{2\,\mu f \rho}{c} \cos^2\theta\, \Sigma\, (\overline{\mathrm{OR}}^2\, \omega) = \frac{2\,\mu f \rho}{c} \cos^2\theta d\theta \Sigma\, (\overline{\mathrm{OR}}^2\, d\phi),$$

$d\phi$ being the angle between two consecutive sides of the cone generated by the revolution of OR.

But $\Sigma\ (\overline{OR}^2 d\phi)$ is equal to twice the superficial area of the part of this cone which is enclosed within the ellipsoid; moreover, the projection on the plane ab of this portion of the surface of the cone is an ellipse whose semi-axes are $r_1 \sin\theta$, $r_2 \sin\theta$, and whose area is $\pi r_1 r_2 \sin^2\theta$, r_1 and r_2 being the maximum and minimum values of OR: the superficial area of the portion of the cone within the ellipsoid is therefore $\pi r_1\, r_2 \sin\theta$.

Hence it follows that

$$\Sigma\, (\overline{\mathrm{OR}}^2 d\phi) = 2\pi r_1 r_2 \sin\theta.$$

The attraction on the point C of the part of the ellipsoid contained between the two cones of revolution, whose common vertex is at C, and whose semi-angles are θ and $\theta + d\theta$ respectively, is therefore

$$\frac{4\,\pi\mu f\rho}{c} \cos^2\theta\; d\theta\, r_1\, r_2 \sin\theta,$$

where

$$\frac{1}{r_1} = \sqrt{\left(\frac{\cos^2\theta}{c^2} + \frac{\sin^2\theta}{a^2}\right)}, \text{ and } \frac{1}{r_2} = \sqrt{\left(\frac{\cos^2\theta}{c^2} + \frac{\sin^2\theta}{b^2}\right)}.$$

On substituting these values, the expression given above becomes

$$4\pi\mu f\rho\, \frac{abc \cos^2\theta \sin\theta d\theta}{\sqrt{(a^2 \cos^2\theta + c^2 \sin^2\theta)}\ \sqrt{(b^2 \cos^2\theta + c^2 \sin^2\theta)}}.$$

Hence the attraction of the solid ellipsoid on the point C at the extremity of the least axis is

$$4\pi\mu f\rho \int_0^{\frac{\pi}{2}} \frac{abc \cos^2\theta \sin\theta d\theta}{\sqrt{(a^2\cos^2\theta + c^2\sin^2\theta)}\ \sqrt{(b^2\cos^2\theta + c^2\sin^2\theta)}}.$$

Let $\cos\theta = u$, and this expression becomes

$$4\pi\mu f\rho \int_0^1 \frac{abc\, u^2\, du}{\sqrt{\{c^2 + u^2(a^2 - c^2)\}}\ \sqrt{\{c^2 + u^2(b^2 - c^2)\}}}. \qquad (1)$$

In the same way it may be shown that the attraction of the ellipsoid on a point μ placed at the extremity of the mean axis, is

$$4\pi\mu f\rho \int_0^1 \frac{abc\, u^2\, du}{\sqrt{\{b^2 + u^2(c^2 - b^2)\}}\ \sqrt{\{b^2 + u^2(a^2 - b^2)\}}};$$

and on a point at the extremity of the greater axis,

$$4\pi\mu f\rho \int_0^1 \frac{abc\, u^2\, du}{\sqrt{\{a^2 + u^2(b^2 - a^2)\}}\ \sqrt{\{a^2 + u^2(c^2 - a^2)\}}}.$$

It will be seen in a subsequent proposition, that these three expressions are not independent of each other, the values of the three attractions in question being connected by an equation.

PROPOSITION III.

*To give geometrical representations of the attraction of an ellipsoid on points placed at the extremities of its least and mean axes.**

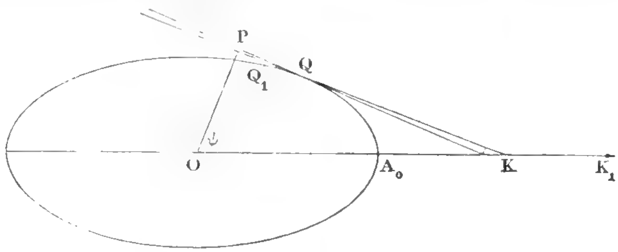


FIG. 3.

On the greater axis OA_0 of the focal ellipse assume a point K_1 such that

$$OK_1 = \frac{b}{c} OA_0;$$

from the point K_1 draw a tangent K_1Q_1 to the focal ellipse, and let $T =$ tangent K_1Q_1 − arc A_0Q_1, then the attraction of the ellipsoid on the particle μ placed at the extremity C of the least axis is

$$\frac{4\pi\mu f\rho abc^2}{(a^2 - c^2)(b^2 - c^2)} T. \qquad (2)$$

* Proceedings of the Royal Irish Academy, vol. iii. p. 367.

For let a point K be assumed on the greater axis OA_0 of the focal ellipse, such that

$$OK = \frac{OA_0}{c}\sqrt{\{c^2 + u^2(b^2 - c^2)\}};$$

from K let a tangent KQ be drawn to the focal ellipse, and let OP be the perpendicular let fall from O on KQ, then ψ denoting the angle A_0OP,

$$\overline{OK}^2 . \cos^2\psi = \frac{a^2 - c^2}{c^2}\{c^2 + u^2(b^2 - c^2)\} . \cos^2\psi.$$

Moreover,

$$\overline{OK}^2 . \cos^2\psi = \overline{OP}^2 = (a^2 - c^2)\cos^2\psi + (b^2 - c^2)\sin^2\psi.$$

Equating these values, and solving for $\sin^2\psi$, we get

$$\sin^2\psi = \frac{(a^2 - c^2)u^2}{c^2 + u^2(a^2 - c^2)}.$$

Now

$$d . (\tan KQ - \text{arc } A_0Q) = \sin\psi d . OK^*$$

$$= \frac{(a^2 - c^2)(b^2 - c^2)}{c} \frac{u^2 du}{\sqrt{\{c^2 + u^2(a^2 - c^2)\}}\sqrt{\{c^2 + u^2(b^2 - c^2)\}}}.$$

By comparing this expression with (1) given in the last proposition, it appears that the attraction on the point C of the portion of the ellipsoid contained between the two conical surfaces whose semi-angles are θ and $\theta + d\theta$ respectively, is

$$\frac{4\pi\mu f\rho abc^2}{(a^2 - c^2)(b^2 - c^2)} d . (\tan KQ - \text{arc } A_0Q).$$

Now in order to obtain the attraction of the whole ellipsoid on the point C, we have to integrate the expression given above between the limits $u = 0$ and $u = 1$, or $OK = OA_0$ and $OK = OK_1$; from which it appears that its value is

$$\frac{4\pi\mu f\rho abc^2}{(a^2 - c^2)(b^2 - c^2)} T.$$

It is easy to see that the attraction of the part of the ellipsoid contained within the conical surface, whose semi-angle θ is equal to $\cos^{-1} u$, is

* Transactions of the Royal Irish Academy, vol. xvi. p. 79. Proceedings of the Royal Irish Academy, vol. ii. p. 507.

$$4\pi\mu f\rho \frac{abc^2}{(a^2-c^2)(b^2-c^2)}(T-t), \tag{3}$$

where $t = \tan \mathrm{KQ} - \mathrm{arc}\ \mathrm{A_0 Q}$.

To represent the attraction on a point μ placed at the extremity of the mean axis, assume on the transverse axis $\mathrm{OA_0}$ of the focal hyperbola a point $\mathrm{K_1}$ such that $\mathrm{OK_1} = \mathrm{OA_0}\frac{c}{b}$, and from $\mathrm{K_1}$ draw a tangent $\mathrm{K_1 Q_1}$ to the hyperbola, and let $T = \tan \mathrm{K_1 Q_1} - \mathrm{arc}\ \mathrm{A_0 Q_1}$, then the attraction of the ellipsoid on the point μ is

$$-4\pi\mu f\rho \frac{ab^2c}{(a^2-b^2)(c^2-b^2)}T. \tag{4}$$

To prove this, assume a point K such that $\mathrm{OK} = \frac{\mathrm{OA_0}}{b}\sqrt{\{b^2 + u^2(c^2-b^2)\}}$; from K draw a tangent KQ to the hyperbola, and from O let fall a perpendicular OP on this tangent, then if $\psi = \text{angle } \mathrm{A_0OP}$,

$$\sin^2\psi = \frac{(a^2-b^2)u^2}{b^2+u^2(a^2-b^2)}.$$

Hence by following a method similar to that used in finding the representation of the attraction on a point at the extremity of the least axis, the expression given above may be easily obtained.

The attractions C, B of the ellipsoid on points placed at the extremity of the least and mean axes are thus represented by means of arcs of the focal ellipse and hyperbola respectively. In consequence of the third focal conic of the ellipsoid being imaginary, no direct geometrical representation can be given for the attraction A on a point placed at the extremity of its greater axis. It will, however, be found, as was intimated above, that a simple relation exists between the three attractions, which enables us to represent this last by means of arcs of both focal conics.

The relation alluded to is

$$\frac{A}{a} + \frac{B}{b} + \frac{C}{c} = 4\pi\mu f\rho.* \tag{5}$$

* Proceedings of the Royal Irish Academy, vol. ii. p. 525.

This can be easily proved by the help of the following geometrical theorem :

If from the extremities A, B, C of the three axes of an ellipsoid, three parallel chords Ap, Bq, Cr, be drawn, and if these chords be projected each on the axis from whose extremity it is drawn, then the sum of these three projections, Aα, Bβ, Cγ, divided respectively by the lengths of the axes AA′, BB′, CC′, on which they are measured, will be equal to unity.

Now conceive three chords Ap, Ap', Ap'', to be drawn from A, making each with the other two very small angles, and so forming a pyramid with a very small vertical solid angle ω ; and from B and C let two systems of chords Bq, Bq', Bq'', and Cr, Cr', Cr'', be drawn, each system forming a very small pyramid whose three edges are parallel to the three edges Ap, Ap', Ap'', of the pyramid which has its vertex at A.

The attractions of the three pyramids, reduced each to the direction of the axis passing through its vertex, will be equal to $\mu f\rho\omega.\mathrm{A}\alpha$, $\mu\rho f\omega.\mathrm{B}\beta$, $\mu f\rho\omega.\mathrm{C}\gamma$ respectively, and, therefore, the sum of those attractions divided respectively by the lengths of the axes will be

$$\mu f\rho\omega\left(\frac{\mathrm{A}\alpha}{\mathrm{AA'}}+\frac{\mathrm{B}\beta}{\mathrm{BB'}}+\frac{\mathrm{C}\gamma}{\mathrm{CC'}}\right)=\mu f\rho\omega.$$

Let pyramids thus related be indefinitely multiplied, and the ellipsoid will be simultaneously exhausted from the three points A, B, C.

Hence the sum of the whole attractions at A, B, C, divided respectively by the lengths of the corresponding axes, will be $2\pi\mu f\rho$, or,

$$\frac{A}{a}+\frac{B}{b}+\frac{C}{c}=4\,\pi\mu f\rho.$$

PROPOSITION IV.

To find an expression for the potential V of a system of particles at a point M *whose distance from the centre of gravity of the system is very great compared with the mutual distances of the particles.*

It is proved by POISSON,* that if the origin of co-ordinates be at the centre of gravity,

* Mecanique, tome i. p. 178.

$$V = \frac{M}{r'} + \frac{3}{2r'^5} \Sigma (xx' + yy' + zz')^2 \, dm - \frac{1}{2r'^3} \Sigma (x^2 + y^2 + z^2) \, dm,$$

$x'\ y'\ z'$ being the co-ordinates of the distant point, and r' its distance from the origin. Let now the principal axes at that centre be taken as axes of co-ordinates; then, since

$$\Sigma xydm = 0, \qquad \Sigma xzdm = 0, \qquad \Sigma yzdm = 0;$$

$$V = \frac{M}{r'} + \frac{3}{2r'^5} \Sigma (x^2x'^2 + y^2y'^2 + z^2z'^2) dm - \frac{1}{2r'^3} \Sigma (x^2 + y^2 + z^2) \, dm.$$

Hence, if A, B, C be the three principal moments of inertia, and I the moment of inertia round OM,

$$V = \frac{M}{r'} + \frac{1}{2r'^3} (A + B + C - 3I). \tag{6}$$

PROPOSITION V.

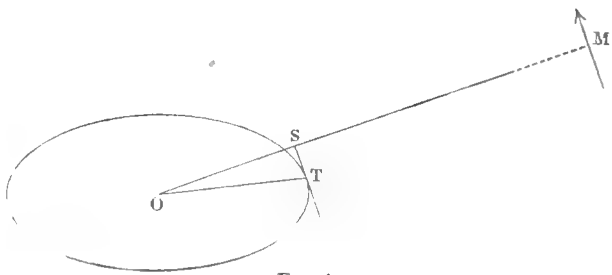


FIG. 4.

A system of material particles attract a point M, *whose distance from the centre of gravity* O *of the attracting mass is very great compared with the mutual distances of the particles; then if a tangent plane be drawn to the "ellipsoid of gyration,"* perpendicular to* OM, *the whole attraction lies in the plane* OST, *where* S *is the point in which this tangent plane intersects* OM, *and* T *the point of contact.*

* The centre of this ellipsoid is at the centre of gravity; its axes are in the directions of the principal axes, and their lengths are determined by the equations

$$Ma^2 = A, \quad Mb^2 = B, \quad Mc^2 = C.$$

This ellipsoid is used by Professor MAC CULLAGH in his Theory of Rotation; see Rev. S. HAUGHTON'S Account of Professor MAC CULLAGH'S Lectures on that subject, Transactions R. I. A. vol. xxii. p. 139.

Let α, β, γ be the direction angles of OT; α', β', γ', of OM; $\alpha_1, \beta_1, \gamma_1$, of TS; and $\alpha_0, \beta_0, \gamma_0$, of the normal to the plane OST; and let OS, OT, and the angle SOT, be denoted by p, r, and ϕ respectively. It will be sufficient to prove, that the component Q of the attraction in the direction of the normal to the plane OST is cypher.

We shall first find the components X, Y, Z, of the attraction in the directions of the axes, and thence deduce the value of Q.

Now,

$$X = -\frac{dV}{dx'} = \frac{M}{r'^2}\cos\alpha' + \frac{3}{2r'^4}(A+B+C-3I)\cos\alpha' + \frac{3}{2r'^3}\frac{dI}{dx'},$$

$$Y = -\frac{dV}{dy'} = \frac{M}{r'^2}\cos\beta' + \frac{3}{2r'^4}(A+B+C-3I)\cos\beta' + \frac{3}{2r'^3}\frac{dI}{dy'},$$

$$Z = -\frac{dV}{dz'} = \frac{M}{r'^2}\cos\gamma' + \frac{3}{2r'^4}(A+B+C-3I)\cos\gamma' + \frac{3}{2r'^3}\frac{dI}{dz'};$$

but,

$$\frac{dI}{dx'} = \frac{2(A-I)\cos\alpha'}{r'}; \quad \frac{dI}{dy'} = \frac{2(B-I)\cos\beta'}{r'}; \quad \frac{dI}{dz'} = \frac{2(C-I)\cos\gamma'}{r'}.$$

Hence we have

$$\begin{aligned} X &= \frac{M}{r'^2}\cos\alpha' + \frac{3}{2r'^4}(A+B+C-5I)\cos\alpha' + \frac{3A\cos\alpha'}{r'^4}, \\ Y &= \frac{M}{r'^2}\cos\beta' + \frac{3}{2r'^4}(A+B+C-5I)\cos\beta' + \frac{3B\cos\beta'}{r'^4}, \\ Z &= \frac{M}{r'^2}\cos\gamma' + \frac{3}{2r'^4}(A+B+C-5I)\cos\gamma' + \frac{3C\cos\gamma'}{r'^4}. \end{aligned} \qquad (7)$$

Now,

$$Q = X\cos\alpha_0 + Y\cos\beta_0 + Z\cos\gamma_0;$$

but,

$$\begin{aligned} \sin\phi\cos\alpha_0 &= \cos\beta\cos\gamma' - \cos\gamma\cos\beta', \\ \sin\phi\cos\beta_0 &= \cos\gamma\cos\alpha' - \cos\alpha\cos\gamma', \\ \sin\phi\cos\gamma_0 &= \cos\alpha\cos\beta' - \cos\beta\cos\alpha'; \end{aligned}$$

the following relations moreover exist,

$$a^2\cos\alpha' = rp\cos\alpha, \quad b^2\cos\beta' = rp\cos\beta, \quad c^2\cos\gamma' = rp\cos\gamma; \qquad (8)$$

hence, by substitution, we have

$$\cos a_0 = \frac{b^2 - c^2}{pr \sin\phi} \cos\beta' \cos\gamma', \qquad \cos\beta_0 = \frac{c^2 - a^2}{pr \sin\phi} \cos\gamma' \cos a',$$

$$\cos\gamma_0 = \frac{a^2 - b^2}{pr \sin\phi} \cos a' \cos\beta'.$$

Substituting these values for $\cos a_0$, $\cos\beta_0$, $\cos\gamma_0$, in the expression for Q, and observing that

$$\cos a' \cos a_0 + \cos\beta' \cos\beta_0 + \cos\gamma' \cos\gamma_0 = 0,$$

we get

$$Q = \frac{3M}{r'^4} \frac{a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)}{pr \sin\phi} \cos a' \cos\beta' \cos\gamma' = 0. \quad (9)$$

PROPOSITION VI.

The same things being supposed, to find the components of the attraction, namely, R in the direction of the centre of gravity MO, *and P in the transverse direction* TS.

To find R;

$$R = X \cos a' + Y \cos\beta' + Z \cos\gamma',$$

$$\therefore \quad R = \frac{M}{r'^2} + \frac{3}{2r'^4}(A + B + C - 5I) + \frac{3I}{r'^4},$$

$$R = \frac{M}{r'^2} + \frac{3}{2r'^4}(A + B + C - 3I). \quad (10)$$

To find P;

$$P = X \cos a_1 + Y \cos\beta_1 + Z \cos\gamma_1;$$

but,

$$\sin\phi \cos a_1 = \cos a' \cos\phi - \cos a,$$

$$\sin\phi \cos\beta_1 = \cos\beta' \cos\phi - \cos\beta,$$

$$\sin\phi \cos\gamma_1 = \cos\gamma' \cos\phi - \cos\gamma.$$

Substituting for $\cos a$, $\cos\beta$, $\cos\gamma$, their values from (8), we get

$$\cos a_1 = -\frac{a^2 - p^2}{pr \sin\phi} \cos a', \qquad \cos\beta_1 = -\frac{b^2 - p^2}{pr \sin\phi} \cos\beta',$$

$$\cos\gamma_1 = -\frac{c^2 - p^2}{pr \sin\phi} \cos\gamma'.$$

Substituting these values of $\cos\alpha_1$, $\cos\beta_1$, $\cos\gamma_1$, and observing that

$$\cos\alpha'\ \cos\alpha_1 + \cos\beta'\ \cos\beta_1 + \cos\gamma'\ \cos\gamma_1 = 0,$$

we have

$$P = -\frac{3M}{r'^4}\,\frac{p^2(r^2-p^2)}{pr\sin\phi} = -\frac{3M}{r'^4}(pr\sin\phi)\,;$$

or,

$$P = -\frac{3M}{r'^4}(\mathrm{OS}\times\mathrm{ST}). \qquad (11)$$

The negative sign indicates* that the force P acts in the direction TS, i.e. from the radius vector towards the perpendicular of the ellipsoid of gyration. If the force P be resolved into three others in the directions of the axes, it is evident from the values given in Proposition V. for X, Y, Z, that these components are

$$\frac{3(A-I)}{r'^4}\cos\alpha',\quad \frac{3(B-I)}{r'^4}\cos\beta',\quad \frac{3(C-I)}{r'^4}\cos\gamma'.\dagger \qquad (12)$$

PROPOSITION VII.

An ellipsoid is composed of ellipsoidal strata of different densities and of variable but small ellipticities; find the components, central and transverse, of its attraction on an external point.

The values found in the last Proposition for the components of the attraction of any mass on a very distant point, will be found to hold in the present

* The direction of the force P, which Professor MAC CULLAGH determines by the interpretation of the negative sign, may be very clearly seen from the following considerations. This force exists in every case where the three principal moments of inertia are not all equal, that is, when the ellipsoid of gyration is not a sphere. The greatest axis of that ellipsoid is manifestly towards that part of the body in which there is a deficiency of attracting matter. If we consider now the position of a perpendicular on a tangent plane of an ellipsoid with relation to the corresponding radius vector, we shall find that it always lies away from the greatest axis. But the transverse force has been shown to be in the plane of radius vector and perpendicular. Therefore, the direction of the transverse force, being towards the preponderating matter, must be parallel to TS.

† The results given by Professor MAC CULLAGH in Propositions V. and VI. may be otherwise

case, whatever be the position of the attracted point. In order to show this, we shall first prove it for a homogeneous ellipsoid of small ellipticities. Such an ellipsoid being given, another, confocal with it, can be constructed so small, that the distance to the attracted point may be regarded as very great, compared with the axes of this ellipsoid: the components of the attraction of this small ellipsoid on the distant point are given by the expressions (10) and

obtained, and, perhaps, with greater facility, by introducing the consideration of the *statical moment* of the attracting force.*

If the three principal moments of inertia were equal to each other, then the whole attraction would be in the direction of the centre of gravity, and its magnitude would be

$$\frac{M}{r'^2}.$$

In general, however, the attracting mass will be of an irregular shape; there will exist then, in addition to the principal part of the attraction which will be central, a transverse force which will cause a motion of rotation about the centre of gravity.

The components of the moment of this transverse force in the three principal planes are

$$x'Y - y'X, \quad y'Z - z'Y, \quad z'X - x'Z;$$

but from (7),

$$x'Y - y'X = -\frac{3(A-B)}{r'^3}\cos\alpha'\cos\beta' = -\frac{3M}{r'^3}(a^2 - b^2)\cos\alpha'\cos\beta',$$

$$y'Z - z'Y = -\frac{3(B-C)}{r'^3}\cos\beta'\cos\gamma' = -\frac{3M}{r'^3}(b^2 - c^2)\cos\beta'\cos\gamma',$$

$$z'X - x'Z = -\frac{3(C-A)}{r'^3}\cos\gamma'\cos\alpha' = -\frac{3M}{r'^3}(c^2 - a^2)\cos\gamma'\cos\alpha'.$$

Now it is well known, that $\frac{1}{2}(a^2-b^2)\cos\alpha'\cos\beta'$, $\frac{1}{2}(b^2-c^2)\cos\beta'\cos\gamma'$, $\frac{1}{2}(c^2-a^2)\cos\gamma'\cos\alpha'$, are the areas of the projections of the triangle OST on the principal planes. Hence it follows, that the resultant moment lies in the plane of the radius vector OT, and the perpendicular OS to a tangent plane of the ellipsoid of gyration; the tangent plane being perpendicular to OM. It appears also, that the magnitude of the resultant moment is

$$-\frac{3M}{r'^3}(\text{OS} \times \text{ST}),$$

and therefore that the transverse component of the attraction

$$P = -\frac{3M}{r'^4}(\text{OS} \times \text{ST}).$$

Or the values of the central force and the moment of the transverse force may be obtained directly from the expression (6) for the potential V. This function is of such a nature, that its differential coefficient with relation to any line (the sign being changed) is equal to the re-

* See Rev. R. Townsend, in the University Examination Papers, 1849, p. 51.

(11); now the attractions of two confocal ellipsoids on an external point are in the same direction, and proportional to their masses; the components of the attraction of the proposed ellipsoid will, therefore, be

$$R = \frac{M}{r'^4} + \frac{3}{2r'^4}\frac{M}{M_1}(A_1 + B_1 + C_1 - 3I_1),$$

$$P = -\frac{3M}{r'^4}(\mathrm{OS}_1 \times \mathrm{S}_1\mathrm{T}_1),$$

the letters with suffixes referring to the small ellipsoid.

The attracting ellipsoids being confocal, their ellipsoids of gyration are confocal also; hence it follows, that

$$\frac{M}{M_1}(A_1 + B_1 + C_1 - 3I_1) = A + B + C - 3I,$$

and

$$\mathrm{OS}_1 \times \mathrm{S}_1\mathrm{T}_1 = \mathrm{OS} \times \mathrm{ST}.$$

It appears from this, that the central and transverse components of the attraction of a soid ellipsoid of uniform density, and whose ellipticities are small,

solved part of the attraction in that direction; and the differential coefficient with relation to any angle (the sign being changed as before) gives the component in the plane of that angle of the moment of the attractive force.

Hence,

$$R = -\frac{dV}{dr'} = \frac{M}{r'^2} + \frac{3}{2r'^4}(A + B + C - 3I),$$

since

$$\frac{dI}{dr'} = 0.$$

Again, if N be the component of the moment of the attractive force round OZ,

$$N = -\left(x'\frac{d}{dy'} - y'\frac{d}{dx'}\right)V;$$

but

$$\left(x'\frac{d}{dy'} - y'\frac{d}{dx'}\right)F(x'^2 + y'^2) = 0, \text{ where } F \text{ is any function.}$$

$$\therefore \quad N = \frac{3}{2r'^3}\left(x'\frac{d}{dy'} - y'\frac{d}{dx'}\right) = \frac{3}{2r'^3}\left(x'\frac{d}{dy'} - y'\frac{d}{dx'}\right)\left(\frac{Ax'^2 + By'^2 + Cz'^2}{r'^2}\right).$$

$$\therefore \quad N = -\frac{3(A-B)}{r'^3}\cos\alpha'\cos\beta'.$$

The two other components of the moment may be similarly obtained. The remainder of the proof is the same as in the former part of this note.

on any external point whatever, are given by the same formulæ as the corresponding components of the action of any mass on a distant point.

Now it is a property of moments of inertia, that they are *subtractive*, that is, the difference of the moments of inertia of two masses with relation to any axis is equal to the moment of inertia of the difference of those masses with relation to the same axis. And the values at which we have arrived for the central force, and for the three components of the transverse force, contain in each term either the mass or a moment of inertia in the first power, and, therefore, these values also are subtractive. Hence the two components of the attraction of a homogeneous mass contained between two concentric and coaxal ellipsoids of small ellipticities, are given by formulæ (10) and (11). Now suppose an ellipsoidal mass to be composed of strata bounded by ellipsoids of different but small ellipticities, each stratum being homogeneous throughout its extent, while the density varies from one stratum to another according to any law; then, since those formulæ hold for the action of each stratum separately, and since the terms of which they are made up are in their nature *additive*, they hold for the entire mass.*

PROPOSITION VIII.

An oblate spheroid is composed of spheroidal strata of different densities and of variable but small ellipticities; find the components of its attraction on any external point.

The expressions given in the last Proposition for R and P become simplified in this case. Let OZ be the axis of revolution, and let λ denote the angle which OM makes with the plane XY; then since A and B are equal, we have

$$I = A \cos^2\lambda + C \sin^2\lambda,$$

and therefore,

$$A + B + C - 3I = (C - A)(1 - 3\sin^2\lambda),$$

also,

$$M(\text{OS} \times \text{ST}) = (A - C) \sin\lambda \cos\lambda.$$

Substituting these values in the expressions for R and P, we have

* See Professor MAC CULLAGH, in the University Examination Papers, 1833, p. 268.

$$R = \frac{M}{r'^2} + \frac{3}{2}\frac{C-A}{r'^4}(1 - 3\sin^2\lambda), \tag{13}$$

$$P = 3\frac{C-A}{r'^4}\cos\lambda\sin\lambda. \tag{14}$$

The direction of the force P is towards the plane of the equator; this appears from the shape of the "ellipsoid of gyration," which in this case is a prolate surface of revolution.

Prop. IX. Clairaut's Theorem.

Whatever be the law of variation of the earth's density at different distances from the centre, if the ellipticity of the surface be added to the ratio which the excess of the polar above the equatorial gravity bears to the equatorial gravity, their sum will be $\frac{5}{2}q$, *where* q *is the ratio of the centrifugal force at equator to the equatorial gravity.*

For suppose the attracted point M to be on the surface of the earth, which is known to be an oblate spheroid of small ellipticity. Then, from the principles of Hydrostatics, since the tangential force is cypher, we have

$$R\cos\theta - P\sin\theta - \omega^2 r\cos\lambda\cos(\theta - \lambda) = 0, \tag{15}$$

where ω denotes the angular velocity, and θ the angle which the tangent to the meridian through the attracted point makes with the radius vector; developing $\cos(\theta - \lambda)$ and arranging, we obtain

$$(R - \omega^2 r\cos^2\lambda)\cos\theta = (P + \omega^2 r\cos\lambda\sin\lambda)\sin\theta. \tag{16}$$

But from the property of the elliptic section made by the plane of the meridian, we have

$$\cot\theta = \frac{e^2\sin\lambda\cos\lambda}{1 - e^2\cos^2\lambda} = 2\epsilon\sin\lambda\cos\lambda, \quad q.\,p.,$$

where e is the excentricity and ϵ the ellipticity of this ellipse.

Substituting in (16) this value of $\cot\theta$, and the values of R and P from (13) and (14), the equation of equilibrium becomes

$$\left\{\frac{M}{r^2}+\frac{3}{2}\frac{C-A}{r^4}(1-3\sin^2\lambda)-\omega^2 r\cos^2\lambda\right\}2\epsilon\sin\lambda\cos\lambda=(3\frac{C-A}{r^4}+\omega^2 r)\sin\lambda\cos\lambda,$$

or, approximately,

$$\left\{\frac{M}{a^2}+\frac{3}{2}\frac{C-A}{a^4}(1-3\sin^2\lambda)-\omega^2 a\cos^2\lambda\right\}2\epsilon=3\frac{C-A}{a^4}+\omega^2 a.$$

If we neglect quantities of the second order, this equation becomes

$$\frac{2\epsilon M}{a^2}=3\frac{C-A}{a^4}+\omega^2 a. \tag{17}$$

We have thus arrived at a relation which enables us to express the unknown quantity $C-A$, in terms of quantities which are all known, and, therefore, to eliminate the former from any other equation in which it may occur.

Now let R_e and R_p denote the equatorial and polar attractions respectively; we have from the general value of R (13),

$$R_e=\frac{M}{a^2}+\frac{3}{2}\frac{C-A}{a^4},$$

$$R_p=\frac{M}{c^2}-3\frac{C-A}{c^4};$$

but

$$c=a(1-\epsilon),\ \therefore\ \frac{1}{c^2}=\frac{1}{a^2}(1+2\epsilon)\text{ and }\frac{1}{c^4}=\frac{1}{a^4}(1+4\epsilon),$$

$$\therefore\ R_p=\frac{M}{a^2}+\frac{2M\epsilon}{a^2}-3\frac{C-A}{a^4}.$$

But,

$$G_p=R_p\text{ and }G_e=R_e-\omega^2 a;$$

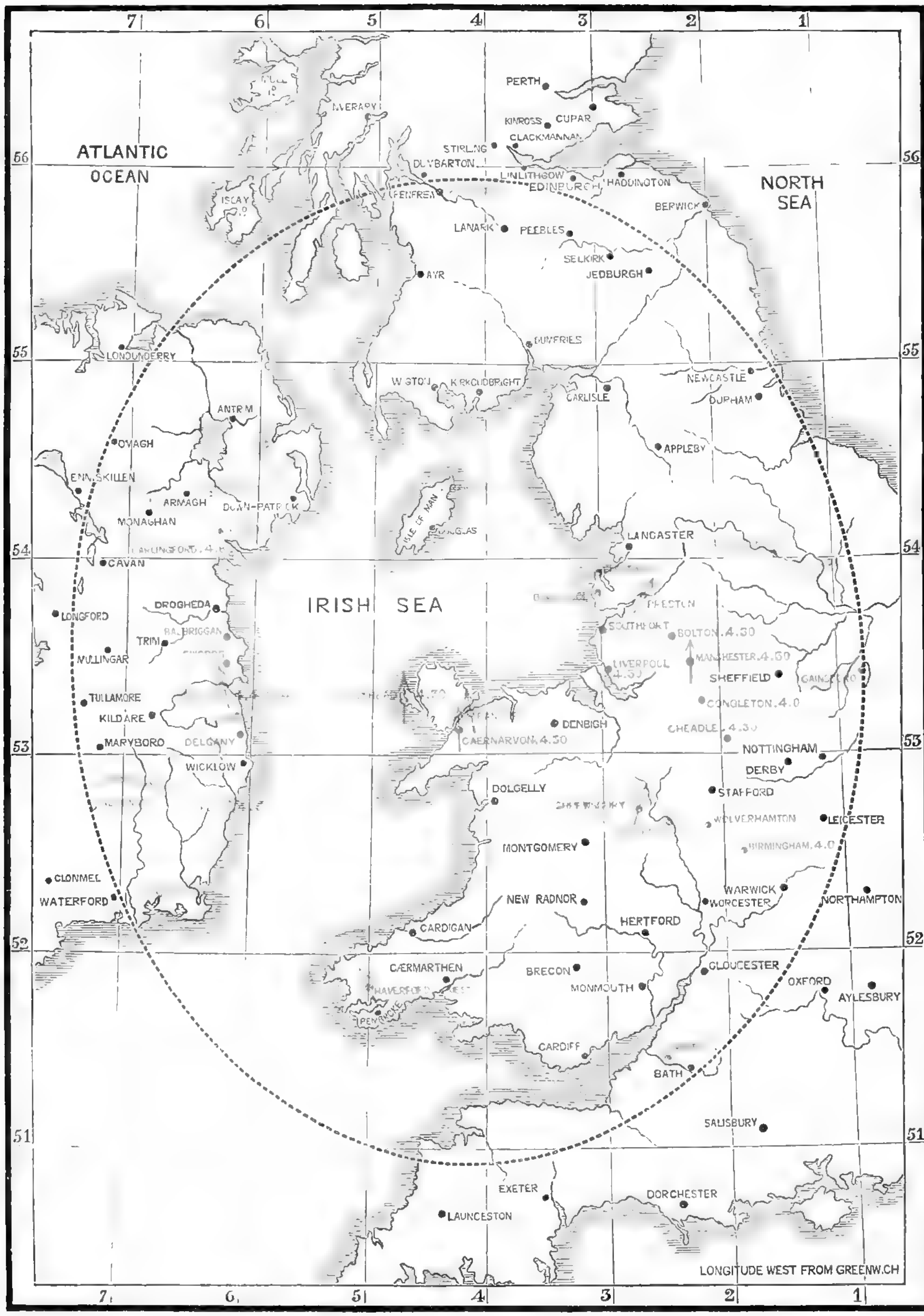
$$\therefore\ G_p-G_e=\frac{2\epsilon M}{a^2}-\frac{9}{2}\frac{C-A}{a^4}+\omega^2 a.$$

Eliminating $\dfrac{C-A}{a^4}$ by means of equation (17), we get

$$\frac{G_p-G_e}{G_e}=-\epsilon+\frac{5}{2}\frac{\omega^2 a}{G_e};$$

or,

$$\frac{G_p-G_e}{G_e}+\epsilon=\frac{5}{2}q. \tag{18}$$



ATLANTIC OCEAN
NORTH SEA
IRISH SEA
PERTH
KINROSS
CUPAR
CLACKMANNAN
STIRLING
DUMBARTON
LINLITHGOW
EDINBURGH
HADDINGTON
BERWICK
RENFREW
LANARK
PEEBLES
SELKIRK
JEDBURGH
AYR
DUMFRIES
LONDONDERRY
ANTRIM
WIGTON
KIRKCUDBRIGHT
CARLISLE
NEWCASTLE
DURHAM
APPLEBY
OMAGH
ENNISKILLEN
ARMAGH
DOWNPATRICK
MONAGHAN
ISLE OF MAN
DOUGLAS
LANCASTER
CAVAN
DROGHEDA
LONGFORD
PRESTON
SOUTHPORT
BOLTON.4.30
MANCHESTER.4.30
LIVERPOOL 4.30
SHEFFIELD
TRIM
MULLINGAR
TULLAMORE
KILDARE
MARYBORO
DELGANY
WICKLOW
DENBIGH
CAERNARVON.4.30
CONGLETON.4.0
CHEADLE.4.30
NOTTINGHAM
DERBY
STAFFORD
DOLGELLY
WOLVERHAMTON
LEICESTER
MONTGOMERY
BIRMINGHAM.4.0
CLONMEL
WATERFORD
WARWICK
WORCESTER
NORTHAMPTON
NEW RADNOR
HERTFORD
CARDIGAN
GLOUCESTER
CAERMARTHEN
BRECON
MONMOUTH
OXFORD
AYLESBURY
CARDIFF
BATH
SALISBURY
EXETER
DORCHESTER
LAUNCESTON
LONGITUDE WEST FROM GREENWICH

XVII.—*Notice of the British Earthquake of November* 9, 1852. By Robert Mallet, *C. E., M. R. I. A.*

Read February 13, 1854.

Although earthquakes are recorded as occurring in very considerable numbers in Great Britain, yet their effects have usually been so slight and transient, that a new one is always an object of popular interest, though, fortunately, from these circumstances, of no more abiding importance generally. For objects beyond merely learned curiosity, as aiding in the compilation of that base of induction that is yet destined to make the earthquake part of exact science, it seemed desirable to collect and arrange, in as authentic and clear a form as the author found possible, the facts of the earthquake of 1852,—one of the most widely diffused and simultaneously felt shocks of any recorded as affecting our islands.

For this purpose, shortly after the occurrence, the author published in several newspapers an invitation to all observers of the earthquake to forward to our fellow-labourer, Mr. Edward Clibborn, on his behalf, communications as to such facts as they might be in possession of respecting the event, and accompanied his invitation by the statement of the four most important points of fact to which attention was principally desirable.

He also applied officially to the heads of the Dublin Metropolitan Police, requesting a systematic examination of the men on duty on the night of November 9, 1852, and that their answers to certain questions given, should be transmitted back to him.

The author, with regret, deems it due to science to mention, that the leading London journal to which he transmitted his request for English communications, with an apathy or ignorance scarcely credible, declined publishing it.

The police authorities promptly answered the author's desire, and the testimony of the police on duty having been taken by one of the inspectors, they forwarded to him documents of which the following embraces the sum.

1. A great number of the police observed a shock.

2. Several of the men state that the motion was sideways, others that it was up and down. They are also divided in opinion as to whether it was from east to west, or from north to south.

3. All agree in stating that the time was two or three minutes before or after 4 o'clock, A.M.

4. All heard a rumbling noise, somewhat like distant thunder; they also heard a rattling of windows as if shaken by a concussion.

The result proves how little reliance as to accuracy or amount of information is to be expected from persons untrained in habits of exact and faithful observation.

Through the zealous co-operation of Mr. Clibborn, a very large number of private letters and other communications were received, and many others, as well as newspaper notices, were transmitted directly to the author. The great mass of these, however, were liable to the remarks just before made. A very few, selected for their graphic character, were sufficient to read to the Academy, though not to publish *in extenso.* All that appeared worthy of credit for accuracy, &c., were arranged and discussed in the form following, very much upon the model of the Great Earthquake Catalogue of the Transactions of the British Association; and from the combination of information from all sources the Seismic map accompanying this paper was prepared. Upon it all places at which a record exists of the shock having been felt are marked in red letters. Wherever the time of the occurrence of the shock was noted it is marked after the name of the place of observation. The time in Ireland is assumed as that for the meridian of Dublin; that in Great Britain is, with one exception (Congleton), Greenwich mean time. Wherever the horizontal direction was noted, it is marked by a red arrow passing through the place in that direction. The broad shaded line generally circumscribes the space within which the shock is recorded to have been actually felt, or may be inferred that it might have been felt. But it will be understood that such a boundary is wholly imaginary, and serves merely to convey a general notion of the form of the territory shaken, as the motion due to the earth-wave (like all other elastic waves in media of indefinite dimensions) passes away from the point of greatest disturbance or summit of the wave, and is gradually lost in all directions to observation, whether unaided or instrumental. Were our earth perfectly elastic, in fact every earthquake, however slight, would shake the whole globe.

The author's own experience at Glasnevin, near Dublin, was that of being suddenly aroused from sound sleep, with some slight sense of alarm by, as it seemed to him, a tremulous shock, with a dead, heavy, thump-like sound, such as a very heavy bag of wet sand might make if dropped from some feet above, upon a large planked floor; his immediate thought was, that some heavy man had jumped upon the floor of the room beneath his bedroom; and conceiving the possibility of house-breakers, he looked out of the window and listened attentively for a few seconds. Not a sound disturbed the singular stillness of the dull, dark-gray leaden haze that hung over the winter morning; he fancied his wife must have started in sleep, and returned directly to bed again. The notion of an earthquake never occurred to him, and it was not until its occurence was remarked to him by others at noon, that he connected his disturbance with such an event. Several families residing in the author's neighbourhood, however, were so much alarmed by the disturbance (more particularly in a few instances where some of the members had been familiar with earthquakes abroad, and at once recognised this as one), that they remained up all the rest of the night, or rather early morning.

On arriving in town he found a large framed drawing of 4 feet 9 inches long, by 2 feet 7 inches deep, and weighing about 7 lbs., which had hung by two brass rings attached by leather straps to the frame against a wall of his office, ranging S. by W. and E. by N. fallen down and wedged diagonally in its own plane between two walls which started at right angles from the wall against which it hung, and at an interval not much wider than the breadth of the framed drawing. The leather straps of the rings were torn asunder, and on examination proved to have been a little decayed by age and drought; but the leather, on trial, was still found to possess such toughness that a weight much beyond that of the drawing and frame would have been incapable of rending either of them.

The author made these observations before the occurrence of an earthquake had been noticed to him; he afterwards returned to the matter and carefully observed the conditions in which the drawing hung, and under which it was found fallen. Of this, more hereafter.

BRITISH EARTHQUAKE OF NOVEMBER 9, 1852.

SCOTLAND, ENGLAND, AND WALES.

LOCALITY.	APPARENT DIRECTION.	SHOCKS, NUMBER.	DURATION AND TIME.	OBSERVED PHENOMENA.	AUTHORITY.
Glasgow,	. . . .	One or more.	. . . .	A lady awaked by shaking of her bed.	Correspondent of North British Mail, November 16, 1852.
Ramsay,	. . . .	One.	A few seconds; 4 15 A.M.	Awakened by noise as of house-breakers; shook every house in the place; weather for some days before wet and boisterous.	Correspondent of the Times, November 13, 1852.
Harrogate,	. . . .	One.	4 A.M.	Numbers roused from sleep; windows, doors, and movables violently shaken; dogs barked.	Correspondent of Times, November 13, 1852.
Southport,	. . . .	One.	Some seconds; 4 30 A.M.	Beds shaken, doors rattled, &c.	Saunders' News-letter, November 13, 1852.
Lytham,	. . . .	One.		A lady awakened at 4½ A.M. Then recollected having felt something of the sort before earlier in the same night.	
Fleetwood,	. . . .	One or more.			
Liverpool,	. . . .	One.	4 30 A.M.	Furniture shaken; a strong oscillation of the ground and buildings; a deep rumbling noise distinctly heard by many persons; children screamed when awakened. One observer felt five or six vibrations; day dark and misty, with drizzling rain.	Liverpool papers quoted by Times, November 10, 1852.
Manchester,	North and South, or N. E. & S. W.	One.	From 3 or 4 to 30 seconds. 4 30 A.M.	A tremulous vibratory motion, like that of the rolling of many waggons; furniture seen to move; several persons experienced nausea; dogs trembled and were much frightened; no noise was heard but that produced by movables shaken. The shock considered by those who felt it both more violent than that of March, 1843; temperature at the time about 50° Fahr. The morning very dark, but calm and fair.	Correspondent of Saunders' News-letter, November 11, 1852.
Holyhead,	. . . .	One.	4 30 A.M.	A shock felt, accompanied by a very loud noise; wind S. E.; cloudy.	Correspondent of Times, November 10, 1852.
Beaumaris,	S. to N., or 10° to 15° W. of N.	One.	While counting rapidly 100; 20 to 30 seconds.	Large things, that required an effort to shake by hand, shook audibly in bed-room; a noise before, during, and after, heard by several.	Andrew Ramsay, Esq., Geological Survey, Beaumaris; private letter, November 9, 1852.
Bangor,	. . . .	One.	4 30 A.M.	A shock accompanied by a very loud noise; wind S. E.; fog.	Electric Telegraph to Times, November 10, 1852.
Caernarvon,	S. and N.	A continued vibration.	20 or 30 seconds; time not given.	A fearful rolling noise, like a whole brigade of fire-engines running over a paved street, suddenly broke an oppressively calm morning, it died gradually away; everything shook.	Evening Post, November 13, 1852, copied probably from a Manchester paper.
Llanberis,	. . . .	One	4 30.	A tremendous blow; slates clattered on the roof, and everything shook; a rumbling sound, which seemed to travel away into distance.	Rev. R. Eyre; letter in Western Star, November 12, 1852.
Gifach, in Wales,	W. to E. (?); oscillatory.	Continued vibration about 60 seconds.	4 30.	A hollow noise like distant thunder, coming and going from N. W. to S. E.; a trembling increasing to a rapid rocking.	A mining captain's letter in Saunders, November 13, 1852.
Congleton,	. . . .	One.	4 A.M.	A smart shock.	Electric Telegraph to the Times, November 10, 1852.
Birmingham,	. . . .	. . . .	4 30 A.M.	In Birmingham felt but slightly; noise and shock both perceived.	
Cheadle and various places in Staffordshire,	. . . .	. . . .	. . . .	At Cheadle, seemed to most persons like the shock and noise of a heavy person falling out of bed. At Barnage the sound was like that of a rushing wind; some evidence of a previous shock on the night of the 8th of November, at 10½ P. M.	

SCOTLAND, ENGLAND, AND WALES—*continued.*

LOCALITY.	APPARENT DIRECTION.	SHOCKS, NUMBER.	DURATION AND TIME.	OBSERVED PHENOMENA.	AUTHORITY.
Shropshire, Shrewsbury, Iron Bridge, Bromley, &c.	W. S. W. and N. N. E.	Three or four.	4 30 A.M.	A severe shock felt all over the county, getting fainter to the westward; pheasants crowed at Acton Reynal preserves; sleepers awakened; doors rattled; houses shook bodily.	
Wolverhampton,	. . . .	At Fitz the shock lasted 30 seconds, according to Rev. D. Nihil.	. . . .	The weather had been wild and stormy, with lightning and hail, some days before, but was quite calm at the time; no sound was observed; bells rung at Ellesmere church; at Newton a wood bridge over the Severn thrown down; 30 yards of strong wall thrown down at Shrewsbury.	
Haverford-west,	. . . .	One.	. . . .	A severe shock; blowing and hailing before; many people up and awake.	Mr. Clibborn, M.R.I.A. private communication
Gloucester and Bristol,	. . . .	. . . .	. . . .	Stated to have been felt plainly in both.	Belfast Mercury, November 12, 1852.

IRELAND.

LOCALITY.	APPARENT DIRECTION.	SHOCKS, NUMBER.	DURATION AND TIME.	OBSERVED PHENOMENA.	AUTHORITY.
Co. Antrim,	. . . .	One.	4 30 A.M.	The shock distinctly felt in several parts of the country.	Coleraine Chronicle, November 14, in Saunders' Newsletter, November 15, 1852.
Armagh City,	. . . .	. . . .	. . . .	Noticed by two persons.	Private comunication from Dr. Robinson, November 10, 1852.
Belfast,	Probably from South to North.	One.	Shortly after 4 o'c.	Undoubtedly felt, accompanied by a dull, heavy sound; houses shaken; the motion vibratory; thirty printers at work in the Belfast newspaper offices were not concious of the shock.	Belfast Mercury, November 11, 1852.
Newtownards,	. . . .	One.	. . . .	A person awakened by oscillation of his bed, and noise of the window bars of his bed-room.	Belfast Mercury.
Tanderagee,	. . . .	Two within a few seconds.	4 A.M.	The whole room shook twice; furniture creaked and was displaced; the observer was awake and reading.	Saunders' Newsletter, November 10, 1852.
Glaslough,	. . . .	Two.	4 10, or 4 12 A.M.	The bed of the observer was twice heaved up as if by a large dog turning nimbly underneath, the second heave shorter than the first; all in the house felt it and got up from bed and assembled at once.	Correspondent of Saunders' Newsletter, November 12, 1852.
Carlingford,	. . . .	One, lasting 15 seconds.	4 8 A.M.	Every movable article set in motion from top to bottom of the house; many of the family did not awaken; great terror produced.	P. Darcy, correspondent of the Evening Post, November 13, 1852.
Newry,	. . . .	One.	. . . .	Experienced with great alarm by many individuals while in bed.	Newry Telegraph and Saunders of November 12, 1852.
Balbriggan, the Naul, Swords, Ardgillan Castle,	. . . .	One.	. . . .	Houses shook at Swords; felt very intensely at the Naul; the family awakened at Ardgillan Castle, and the steward fired a gun out of the window, fancying he heard housebreakers.	Saunders' Newsletter, November 11, 1852.

IRELAND—*continued.*

LOCALITY.	APPARENT DIRECTION.	SHOCKS, NUMBER.	DURATION AND TIME.	OBSERVED PHENOMENA.	AUTHORITY.
Dublin City,	S. to N. or S. E. by S. and N. W. by N. A telescope standing on end fell towards the north in Nassau-st.—*G. Yeates.* A picture was shaken down from its fastenings on a wall runing S. by W., and N. by E., and so circumstanced as to prove that the direction of emergence of shock was upwards at a considerable angle from S. to N.—*R. Mallet.*	One, and probably one at a previous part of the night. It seems very probable that there was a previous shock on the night of the 9th of Nov. at about 12 o'clock, and some likelihood of another or several minor ones having been felt on the night of the 11th November, 1852. (*Letters from Mr. Clibborn and Mr. Malone.*) Some observers were conscious of three distinct heaves during the continuance of the tremor. (*Letter, I. Farrell, Gt. Brunswick-street.*)	4 to 4 15 A.M. Most probally at 4 5 Dublin time. Whole time of tremor about 8 or 10 seconds.	The shock perceived by multitudes both awake and suddenly aroused by it from sleep; those who were awake and standing, or in motion, felt little; those who leaned against walls or other objects were fully alive to the reality and extent of the motion. Observers who were awake differ as to accompanying noise, but evidence for its occurrence preponderates. The sound is variously described as of "a rushing wind," a "rumbling sound like a fire-engine on pavement," &c. Almost all sleepers suddenly aroused by the shock were conscious of a heavy, hollow sound, like the fall of a heavy soft body on a large hollow floor. The motion is generally described as vibratory, ending in one or two sudden heaves. It is uncertain whether the sound accompanied or closely succeeded the shock; most probably the latter. Houses were heavily shaken; a shattered chimney thrown down at Phibsborough; water thrown out of full vessels. A few minutes after the shock, the street gas-lights were observed to be agitated as in a storm, arising obviously from the agitation of the water in the gasometer tanks at the works. (Letter from Mr. Wilson, Christ-Church-place.) The balance-weights of window-sashes swung against the sash-casings, north side of Dublin. Sparrows were thrown from their roosting-places, Great Southern and Western Railway goods shed, and Mountjoy-square, and many picked up dead on the ground in the morning. Watchmen at the Dublin and Drogheda Railway terminus saw the drag-chains hanging from trucks set to oscillate. Caged birds in some instances began to sing; dogs barked; the printers at work in Saunders' News Office were unconscious of anything unusual. Chairs standing on an oilcloth floor in Lincoln-place slid along the floor out from the wall. The night of the 9th was oppressive and sultry for the time of year; a leaden sky, more than usually light for the time and season, obscuring all stars, and a death-like calm. A few drops of rain fell in some places.	Public papers of Dublin, and sundry private communications.
				Passengers in the Liverpool and Holyhead steam-boats felt no shock.	Evening Post correspondent.
Suburbs of Dublin, Kingstown, Rathmines, Mountpleasant.	. . . .	. . . .	. . . .	Phenomena similar to those felt in the city and parts immediately adjacent.	Sundry private communications. Private communications and Saunders' Newsletter November 11, 1852.
Bray, Delgany, Ballyboden, Rathfarnham,	. . . .	Two heavy thumps with a continuous vibratory jar.	Delgany, 4 15. Bray, 4.	At Delgany a bunch of keys was shaken and rattled in a chamber candlestick, where they had been left. Sound generally described as continuing after the principal shocks.	
Wicklow,	. . . .	. . . .	. . . .	Phenomena generally as at Dublin.	
Arklow, Newtown-mountkennedy,	. . . .	Shaking for several seconds.	Arklow, a few minutes after 4. Arklow, 4 5. Castle Howard, 4 5.	Sleepers awakened; houses shook; a chimney-piece clock stopped at a place near Wicklow town, at 700 feet elevation over sea. The shock very severe at Castle Howard, described to have shaken to its foundation; it lies on a hill-side.	Saunders' Newsletter, November 10.
Kilkenny,	. . . .	. . . .	. . . .	Houses vibrated in an alarming manner; the oscillations were not thought to be accompanied by any noise.	Kilkenny Moderator, November 10, 1852.
Wexford,	. . . .	. . . .	. . . .	A slight shock which shook the observer's house and bed, and those of his neighbours.	Correspondent of Saunders' Newsletter, November 12, 1852.

In the preceding columns the directions, where given, are those of true meridian. The times of shock (as in map) are Greenwich mean time in Great Britain (except as to Congleton); and in Ireland, assumed Dublin time. Most of the newspaper correspondence adopted was authenticated. The names of private correspondents are not in all cases given ; and their information must be taken on the author's authority.

The more important points to arrive at in every earthquake, of course, are:—

1. The direction of emergence of the earth-wave of shock.
2. The moment of its emergence in time.
3. Its velocity of emergence.
4. The dimensions of the wave, or rather its altitude at each point of observed emergence.

In the present instance the observations collected afford but a very meagre basis even for approximate answers to any one of these inquiries, and such must ever be the case until self-registering seismometers are to be found in all our observatories, &c.

The following conclusions, however, are justifiable:—

1. The general direction of emergence of the earth-wave was from south to north, making a considerable vertical angle with the horizon, i. e. emerging upwards from the ground.

The following is some of the evidence that it had both a horizontal and a vertical component of motion. As regards horizontal direction (that which is commonly best observable, and popularly assumed to be the only element of direction), there is abundant testimony that it was from south to north, varying more or less to the eastward or westward. There is also the decisive evidence of the fall of a pocket telescope to the northward, which stood on end in a glass-case in Mr. Yeates' optician shop, in Grafton-street. Its fall *towards* the north does not invalidate the other evidence that the primary motion was from south to north; as there exist numerous observed cases of objects *disturbed* by the primary or forward movement of the earth-wave, and *thrown down* by its return movement. (See First Report on Earthquakes, Trans. Brit. Ass.)

As respects the vertical component, in addition to the testimony of many who *felt* an up and down movement, the conditions of fall of the picture in the author's premises, already noticed, afford conclusive evidence. The wall against which it hung ranged nearly east and west ; a shock coming horizontally, or nearly so, from the north towards the south could produce no effect but that of pressing it closer to the wall; or, if from south to north, of causing it to swing outwards from the wall, like a pendulum, suspended from the rings. No motion (within the limits here in question) from east to west, horizontally, that is, in the plane of the wall, could affect it at all. The vertical element of motion is indispensable to account for its having fallen.

If this vertical motion had been one emerging upwards from north to south, unless the angle were almost perfectly vertical, its effect would be merely to increase a little the strain upon the points of support for the moment, and to cause the lower part of the picture to swing out from the wall. But if the direction of motion were diagonally upwards from south to north, the whole force due to the vertical component, less the friction of the picture against the wall, would be expended in straining the points of support, and that due to the horizontal component in throwing the picture bodily off from the face of the wall. The picture was actually found thrown forward about eighteen inches from the wall at the point where it was arrested in its descent, after a fall of about four feet. I conclude, therefore, that the actual direction of emergence of the wave in the city of Dublin was from south to north (with probably a few degrees westerly bearing), and upwards at an angle of from 25 to 30 degrees with the horizon. The following diagram may make this more intelligible.

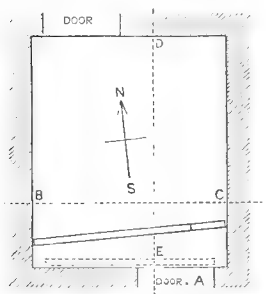


Fig. 1.

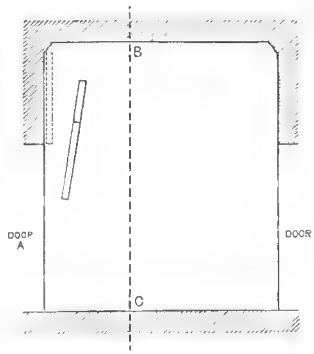


Fig. 2.

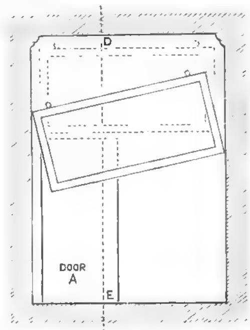


Fig. 3.

Fig. 1. Plan of passage.

Fig. 2. Vertical section through BC, looking southwards.

Fig. 3. Vertical section through DE, looking westwards.

The dotted lines show the original position of the picture; the hard lines its position when founddislodged and wedged between the side-walls.

The general form of the area of sea and land shaken assumes on the map the form of a large irregular ellipse of small eccentricity. Were observations to the south-west of this region available, it is likely that it might enlarge the curve in that direction.* Within this space the point of maximum disturbance seems to have been at and about Shrewsbury, where more serious results of the shock were experienced than in any other quarter. A strong wall of thirty yards in length was there overthrown; and at Newton, not far distant, a wooden bridge over the Severn is said to have fallen; while the bells rang in Ellesmere church.

The circumstances both of direction and of centre of maximum surface disturbance, therefore, seem to point to the great volcanic focus of which the Azores, Portugal, and the Canary Islands, form the well-known centres of convulsion—as the region of the origin of the shock in the present case—the point from which the blow was delivered, which transmitted the elastic wave, would, on following the general direction to the south, pass through the group of the Canaries. And assuming that we have arrived with any tolerable approximation at the angle of emergence, the vertical depth of the origin, if taken below these islands, would be very great indeed. It is quite likely, however, that an intermediate point of convulsive energy exists in or about the latitude of Lisbon, and thus at a less profound depth.

It is worthy of remark that the circumstances of this shock, viz., the previous increasing tremor, accompanied by the "Bramidos," the horrible subterranean thunder, and ending with the violent single or double shock, are precisely those of the terrible Lisbon earthquake of 1755, and of all others in that region.

The Portuguese focus was in energetic action about the precise time of our earthquake, as the following notice proves:—

"A shock of an earthquake had been felt at Malaga, which spread general consternation among the inhabitants of that city. At half-past 1 o'clock A.M.

* I have obtained no observations made at sea. Professor Haughton stated when the author's paper was read, that the shock had been felt at Clogheen, county of Tipperary; and the Rev. S. Smith, that it had been felt at Enniskillen. As these gentlemen, however, did not communicate their information at the proper time, the author has been unable to adopt it, or to know to what extent the evidence may be received.

strong oscillations shook all the edifices. The people immediately sallied out of their houses, and sought refuge at La Alameda and in the public squares. Fortunately, the shock was not renewed. The temperature was suffocating; the cloudy aspect of the sky induced a belief that another earthquake would take place the following night. Many families accordingly retired on board the vessels in the harbour. The shock was preceded by a loud noise."—"Times" Newspaper, 10th November, 1852.

The date actually referred to will be the 7th or 8th of the month.

It may be remarked, that the basin of the Greek Archipelago appears to have been in activity a little before the same period, an earthquake having overthrown the magnificent columnar remains of the Temple of Jupiter Olympias at Athens. ("Times," 24th November, 1852.)

As respects the time of the shock, it appears to have been felt almost, if not altogether, simultaneously over the whole area shaken in Great Britain and in Ireland,—a circumstance in itself corroborating the evidence for the considerable angle at which it emerged, for shocks nearly horizontal are always observed to have a progressive translation over the shaken country.

There is no evidence of a sufficiently precise character to warrant any conclusions, either as to the velocity of emergence of the wave, or as to its altitude, i. e. the actual range of shaking produced at any point by the shock. Comparing the evidences of disturbance in Dublin and other places, with the effects of many other earthquakes, the author is disposed to attribute the safety from serious calamity fortunately experienced by us, in great measure, to the vertical element in the direction of the shock, which, with only the same velocity and range, had it been been much more nearly horizontal, might probably have produced great disaster.

On examining the lines of horizontal direction for different localities as recorded on the map, they will be found to differ considerably. In this there is nothing unusual or irreconcilable with faithful observation or with science. Local changes in the true direction, and still more in the apparent or horizontal direction, of translation of the earth-wave are due to many causes, principally to changes in the geological formations at different points, or to the structure of the earth's crust, and to abrupt changes in the physical features of the surface of the country. Instances are not wanting of the same shock being felt in directions almost opposite, at the same moment, in places not far apart, and this circumstance constitutes one of the difficulties of disentangling the true elements of almost every shock, and of the construction of seismometrical instruments.

Some very local points of greater disturbance were observed. Thus, at Castle Howard, in the Vale of Avoca, county of Wicklow, Ireland, the shock was experienced with great severity, due to the circumstances of its position. It stands upon a spur of mountain jutting out towards the westward from a north and south range. It rests on slate rocks, having a generally north and south strike, and its elevation on the hill is considerable. A shock from south to north would therefore affect it with exaggerated power. The shock was not felt at all by any one on board any of the steam-vessels passing either way between Liverpool or Holyhead and Dublin, on the night of the 9th November, nor was it felt by the printers up and at work in the several newspaper offices in Dublin, Liverpool, or Manchester. In all these cases the local vibrations going on by the machinery at work obviously prevented the earthquake jar being observed, or confounded it with those taking place from the local causes.

A few of the more remarkable secondary effects observed may be noticed. The flickering of the gas-lights in the streets of Dublin, observed by the writer from Christ-Church-place (see *ante*), occurring some minutes after the shock, was doubtless due to the depression of the gasometers at the gas-works into the tanks by the vertical direction of the shock, and by the surging of the water in the tanks themselves, the time that elapsed being that necessary to transmit such disturbance from the gasometers through the elastic fluids in the street mains, tubes, &c., to the lights.

Very many small birds, chiefly sparrows, were found dead upon the ground on the morning after the shock, as at the goods sheds of the King's-bridge Terminus, Great Southern and Western Railway, and in Mountjoy-square. This, which has been often observed in earthquake countries, is due to the creatures being shaken while asleep off their roosting-places, the involuntary muscles of the claws, which hold them on, not being prepared to resist so sudden and unexpected a shock.

Clocks were stopped in some places, unfortunately without the time of stopping being noted.

As the relations of earthquakes with meteorology are as yet uncertain, it appeared desirable to obtain returns on this subject from several stations within the region of shock for the day of the earthquake, and for the one preceding and following. The results are given in the subjoined Table.

Meteorological Table, *referring to the British Earthquake of November* 9, 1852, *for five principal Stations.*

Instrument.	Date.	Observing Stations.				
		Dublin.	Armagh, reduced to 50° Fahr. 9 A.M.	Sligo, Markree Castle, 10 o'clock A.M.	York, 9 o'clock A.M.	Greenwich means for the day.
	Nov.					
Barometer, .	8,	29·820	29·504	29·712	29·848	29·976
	9,	30·116	29·994	30·105	30·208	30·025
	10,	30·044	29·933	30·002	30·104	29·915
Thermometer,	8,	58°·5	57°·0	52°·9	55°·0	52°·8
	9,	59 ·0	48 ·2	48 ·7	48 ·0	51 ·5
	10,	52 ·0	43 ·2	44 ·9	40 ·0	40 ·3
	Inches.	Inches.	Inches.	Inches.	Inches.	
Rain-gauge,	8,	0·	0·136	0·313	0·05	0·
	9,	0·280	0·036	0·063	0·05	0·
	10,	0·030	1·043	0·001	0·00	0·
			Miles, last 12 hours travelled.			
Wind, . . .	8,	S. W.	251 S.	W. N. W.	S.	W.S.W.
	9,	S.	46 S. S. W.	S. E.	S. W.	W.-S.W.-W.S.W.
	10,	S. E.	25 E. N. E.	E. N. E.	S. W.	calm, N.E.-E.N.E.
						On the 11th Aurora, and magnets greatly disturbed.

Note.—The whole winter of 1852 was one of unusual wetness, followed in early spring by hard and protracted frosts, and was probably the most severe winter, on the whole, recorded for twenty or thirty years in the British Islands.

These results confirm the view (so far provisionally) deducible from all earthquakes, namely, that meteorological phenomena have probably no true causative connexion with these disturbances, although they are often acted on and modified by the secondary effects due and subsequent to earthquakes.

There are good grounds for supposing that during the night of the 9th November, 1852, several minor shocks of earthquake took place, and were felt more or less in and around Dublin by different individuals, who, before the severe shock of 4 o'clock A.M., did not attribute them to any natural cause: and a letter to our fellow-member, Mr. Clibborn, from a trustworthy observer, renders it probable that on the succeeding night there was a continuance of subterranean commotion of a diminished character, and such, very probably, might have continued during the intervening day also, but without attracting the attention, which the *silence and increase of sound* at night induced.

The last shock of earthquake of any note occurring in Great Britain was experienced with various degrees of intensity throughout the greater part of Lancashire, and the adjacent districts of Westmoreland, Cumberland, Cheshire. Flintshire, and the Isle of Man, and took place a few minutes before 1 o'clock on the morning of Friday, March 17, 1843. On that occasion, as on the present, somewhat varying statements were made as to the duration and severity of the visitation, but there was a material difference in the state of the weather and of the atmosphere. After the last shock in 1843, accounts were received from the West Indies announcing that a severe earthquake had taken place there about the same time, and that a great number of lives had been sacrificed.

Of preceding British earthquakes, out of 116 recorded by Milne, 31 (according to him) had their centres in Wales, 31 along the south coast of England, 14 on the borders of Yorkshire and Derbyshire, and 5 or 6 in Cumberland.

In the south of England he is of opinion that most shocks have an E. and W. direction, while those of Anglesea, North Wales, and Cheshire are N. W. and S. E., in both cases coinciding with the general lines of great faults.

He considers, from the discussion of 130 Scottish and of 116 English earthquakes, that there is a maximum of occurrence for the former in November, and for the latter in September; taken all together, there occurred 74 in the three winter months, 44 in spring, 58 in summer, and 79 in autumn; or 50 in the summer half year against 89 in the winter half.

M. Perrey, in his Memoir on British earthquakes, from a discussion of 234 recorded, extending over a period of ten centuries, viz., from the ninth to the nineteenth, distributes them thus :—

Winter, ...	56
Spring, ...	42
Summer, ...	52
Autumn, ...	67

On the whole, whether it may ultimately appear that earthquakes have some distinct relations to season all over the globe, in Great Britain the base of induction is too small, and the numbers approach too near to equality, to warrant such conclusions at present.

M. Perrey has also classified the directions of shocks (horizontal or apparent directions only) of the preceding British earthquakes, as follows :—

S. to N.,.	0·48
N. E. to S. W., ...	0·48
E. to W., ...	1·70
S. E. to N. W., ...	0·73
S. to N., ...	3·73
S. W. to N. E., ...	1·46
W. to E., ...	1·46
N. W. to S. E., ...	0·97

If we unite those having the same direction, but merely opposite primary motions, we have,

North and South, ...	1·21
East and West, ...	3·16
Intermediate points to the Eastward of North, . . .	1·94
Intermediate points to the Westward of North, . . .	1·70

If this result be relied upon as on a sufficient basis, it would indicate that British earthquakes most frequently come from other and more distant centres of disturbance than that assignable to the shock here treated of.

XVIII.—*Notes on the Meteorology of Ireland, deduced from the Observations made in the Year* 1851, *under the Direction of the Royal Irish Academy. By the* Rev. HUMPHREY LLOYD, *D.D., F.R.S.; Hon. F.R.S.E.; V.P.R.I.A.; Corresponding Member of the Royal Society of Sciences at Gottingen; Honorary Member of the American Philosophical Society, of the Batavian Society of Sciences, and of the Societè de Physique et d'Histoire Naturelle of Geneva, &c. &c.*

Read June 27 and December 12, 1853.

THE science of meteorology is, perhaps more than any other, dependent upon *co-operation* and upon *method*. Individual observers may investigate successfully certain detached meteorological problems, such as the laws of the diurnal and annual changes of temperature, pressure, and humidity, at a given place; but little progress can be made in Climatology, or in the knowledge of the greater movements of the atmosphere, and their relation to the non-periodic variations of temperature and pressure, without the co-operation of many observers distributed over a large area, and acting upon a common plan.

For this task the voluntary association of individuals is insufficient. However zealous such persons may be, it is not possible to bind them to that uniformity of system without which little can be effectively done. Observations taken at different hours, or by different methods, can never be compared satisfactorily; and *any* comparison will involve an amount of labour in the processes of reduction which may render them impracticable. In addition to this, certain rules of observation are imposed by the conditions of some of the great problems of meteorology; and no co-operation in which these rules are deviated from can contribute to their solution.

For these and other reasons it is desirable that, in every country, such observations should be provided for by the Government, and placed under the direction of one of its official departments. And there can be no doubt of the services which meteorology, properly studied, may be made to contribute to those interests which it is the duty of every Government to promote. The health of man, the operations of agriculture by which he procures his food, and many other of his material interests, are dependent upon climatological relations, which must be known and studied before they can be applied. Every one acknowledges the fact, that the salubrity of a district, and its adaptation (or the reverse) to particular human constitutions, is intimately connected with its meteorological conditions. And the same thing is true of all organized beings, and especially of those which are subservient to the uses of man. Thus, the question of the naturalization of exotic plants is, mainly, a meteorological problem, dependent upon the climatological relations of the region to which the plant is indigenous, and of that to which it is to be transferred; and the importance of obtaining accurate data for its solution will be recognised, when it is borne in mind that, in Europe, most of the plants useful to man belong to this class, and that those hitherto acclimatized probably bear a very small proportion to the whole. Lastly, the processes of cultivation, to which these vegetables are to be subjected, are also connected in an intimate manner with meteorological knowledge. We may instance this connexion in the operations of irrigation, and of drainage, both of which are dependent upon the knowledge of the amount of rain-fall in the district to be operated on.

It is true that meteorological science has been hitherto comparatively barren in such applications; and the fact itself, with many persons, would be accepted as evidence that abstract and practical knowledge are wholly separate and unconnected. But, when properly understood, it leads to a different conclusion. *Superficial* knowledge in this science can indeed yield but few practical results; and those by whom such results have been hitherto sought have expected to find them at the surface. There are indeed cases—such, for example, as the one last referred to—in which the connexion between meteorological science and its applications is obvious and simple, and in which, accordingly, that connexion has been traced and made use of. But in general it is otherwise. In a subject so complex as the laws which govern the aerial envelope of the earth, and where so many causes are in operation, practical applications can be obtained only from *mature* theoretical knowledge. Thus, it may be shown that the knowledge of the phenomena of temperature, requisite for the determination of the possible geographical limits of a single species of plants, is by no means inconsiderable;* and when to this we add the consideration of the other agencies which are at work in the atmosphere, all influencing vegetable life, it is plain that we are not in a condition to deduce any useful result connected with the distribution of species, until we have mastered a much larger amount of theoretical knowledge than is usually brought to bear in such deductions.

It would seem, therefore, to be the duty of the Government of every civilized state to provide the statistical data which have so many important bearings upon the material welfare of the people, and in the form best fitted for their discussion and examination. And to the lover of truth itself, for its own sake, the fulfilment of this duty would, fortunately, supply the wants of science in the most complete and satisfactory manner.

In many countries, accordingly, provision has been made by their respective Governments for the collection and discussion of meteorological data upon a uniform and well-digested plan. The Government of Prussia appears to have taken the lead in this important labour. Its example has been followed by those of Russia, Austria, Bavaria, and Belgium; and the names of Dove, Kupffer, Kreil, Lamont, and Quetelet, to whom the superintendence of these observations has been intrusted, afford the surest warrant of their successful prosecution.† But perhaps the most important undertaking of this nature is

* For each plant there is a lower limit of temperature, below which it will cease to vegetate; while, in order that it may blossom and bear fruit, it must receive, between the two seasons of this minimum temperature, a certain amount of heat beyond this limit which is constant for each species. It is upon this *integral of effective heat*, as has been shown by De Candolle, that the existence of the species depends. For information on this and other subjects connected with the applications of meteorology, see the interesting introduction, by M. Martins, to the *Annuaire Meteorologique de la France.*

† The results of many of these series have been already published. Professor Dove has published the results of the observations made in Prussia in the years 1848 and 1849. The observations made at the Russian observatories have been published from time to time by M. Kupffer, in the *Recueil des Observations faites dans l'Empire de Russie.* The results of the Bavarian obser-

the recent organization of a system of meteorological observations *at sea* by the Government of the United States. There are, at the present time, nearly 1000 masters of ships, belonging to the navy and merchant services of the United States, engaged in such observations; and the discussion of the results, by Lieutenant Maury, has led to many consequences of great value to the sciences of meteorology and hydrography, and rich in practical applications to navigation. The Government of the United States has earnestly sought the co-operation of the Governments of the several maritime nations of Europe in this enterprise, and the demand has led to a Conference at Brussels, for devising a uniform system of meteorological observations at sea. This Conference, held in August and September last, was attended by individuals representing the respective Governments of Belgium, Denmark, France, Great Britain, Netherlands, Norway, Portugal, Russia, Sweden, and the United States.

Impressed with the conviction that it was the duty of each country to take its part in these labours, and especially in the investigation of its own climatology, the Council of the Royal Irish Academy directed their attention, early in the year 1850, to the object of organizing a uniform system of meteorological observations in Ireland. And the peculiarity of the climate of this island perhaps more than balances the smallness of its extent, in giving an interest to the investigation. Situated as it is at the north-western extremity of Europe, and exposed to the full influence of the northern branch of the gulf stream which sweeps its western shores, its winter temperature is as high as that of the southern shores of the Euxine; while, on the other hand, the great precipitation of vapour, due to the same cause, gives it a summer heat as low as parts of Finland.

The questions, whose solution was aimed at by this measure, are thus stated by the Council in their second Report:—

1. The distribution of temperature, humidity, and rain, as affected by geographical position and by local circumstances; and the other phenomena of climate.

2. The effect of season (combined with the influences already referred to)

vations have been given by Dr. Lamont, in the *Annalen der Meteorologie;* and those of the Belgian system, in the admirable series of papers drawn up by M. Quetelet, *Sur le Climat de Belgique.*

upon the distribution of temperature, and the varying position of the isothermal lines *from month to month.*

3. The *non-periodic* variations of pressure, temperature, and humidity, and their connexion with the course and direction of the aerial currents.

4. The phenomena and laws of storms, whether revolving or otherwise.

5. The periodical winds prevailing during certain seasons, and their modifications from geographical position or local causes.

6. The course and rate of progress of atmospheric waves.

Concurrently with the meteorological observations, it was determined to institute an extended series of observations on the phenomena and laws of the tides around the coasts of Ireland, the results of which will shortly be laid before the Academy by Mr. HAUGHTON. The observations of the former class having been intrusted by the Council to my care, for reduction and discussion, I now proceed to lay before the Academy their principal results. It will be necessary, however, in the first instance to describe the plan of observation itself.

Stations.—The meteorological stations are:—

1. The Coast-guard stations at Portrush, Buncrana, Donaghadee, Courtown, Dunmore East, Castletownsend, Cahirciveen, and Kilrush; and, for observations of sea temperature only, those of Cushendall and Bunown. At all of these the observations were taken, with the permission of the Lords of the Treasury and of the Comptroller-General, by the boatmen belonging to the Coast-guard Service, the individuals having been specially selected for the duty by the inspecting officers, and having been instructed in the mode of observing by members of the Council of the Academy.

2. The Lighthouses at Killough, Inishgort, and Killybegs, where, with permission of the Ballast Board, the observations were made by the lightkeepers, instructed as before.

3. The Astronomical Observatories of Armagh and Markree, where the observations were taken by the Observatory assistants, with the permission of Dr. ROBINSON and Mr. COOPER; the Magnetical Observatory of Dublin, where they were made with the permission of the Board of Trinity College; and the stations at Portarlington and Athy, where they were undertaken by Dr. HANLON and ALFRED HAUGHTON, Esq.

In addition to these the Academy has received observations, made upon the prescribed plan, from the Royal Observatory of Dublin, and from the Queen's Colleges at Belfast and Galway, which could not conveniently be included in the following discussions, not having extended over the whole of the period discussed. The observations at the Royal Observatory, and at the Queen's College, Belfast, commenced in April, 1851, and have been continued to the present time; the necessity for their omission is the more to be regretted, as they appear to have been made with every possible care.

The positions of the several stations, together with the heights (in feet) of the cisterns of the barometers above the mean sea level,* are given in the annexed Table. They are shown in Plate VII.

Table I. Names and Positions of the Meteorological Stations.

No.	Station.	County.	Latitude.	Longitude.	Height above Sea.	Locality.
I.	Portrush, ...	Antrim, ...	55° 13′	6° 41′	29	Coast-guard station.
II.	Buncrana, . . .	Donegal, ...	55 8	7 27	48	Do.
III.	Donaghadee, . .	Down, ...	54 38	5 33	16	Do.
IV.	Killybegs, . . .	Donegal, ...	54 34	8 27	20	Lighthouse.
V.	Armagh, ...	Armagh, ...	54 21	6 39	211	Observatory.
VI.	Killough, ...	Down, ...	54 13	5 40	23	Lighthouse.
VII.	Markree, ...	Sligo, ...	54 14	8 28	132	Observatory.
VIII.	Westport, ...	Mayo, ...	53 50	9 37	17	Lighthouse.
IX.	Dublin, ...	Dublin, ...	53 21	6 15	19	Magnetical Observatory.
X.	Portarlington, .	King's County,	53 9	7 12	230	Dr. Hanlon's residence.
XI.	Athy, ...	Kildare, ...	53 0	6 58	200	Mr. Haughton's residence.
XII.	Courtown, . . .	Wexford, . . .	52 39	6 13	34	Coast-guard station.
XIII.	Kilrush, ...	Clare, ...	52 38	9 30	19	Do.
XIV.	Dunmore, . . .	Waterford, . .	52 8	6 59	66	Do.
XV.	Cahirciveen, . .	Kerry, ...	51 56	10 13	52	Do.
XVI.	Castletownsend,	Cork, ...	51 33	9 9	18	Do.

The instruments were furnished by the Academy to the coast-guard and lighthouse stations; and were constructed under the direction of the Council, and upon a common plan. They consist of a barometer, a pair of ordinary thermometers (dry and wet bulb), a pair of self-registering thermometers, a

* At Portarlington and Athy these heights have been taken from the Contour Maps of the Ordnance Survey, and must, therefore, be considered as only approximate: at all the other places they have been obtained by actual levelling from the nearest Ordnance bench-marks.

wind-vane, Lind's anemometer, a rain-gauge, and (at the coast-guard stations) a thermometer adapted to the observation of sea temperature. The thermometers were previously compared in Dublin with the standards belonging to the Magnetical Observatory, and their errors exactly determined. The barometers were compared with the Dublin standard after they were placed at the several stations, by means of good portable barometers; and the heights of the cisterns above the sea were ascertained by levelling. All this was done by Members of the Council, under whose superintendence the instruments were erected.

The following were the positions of the instruments:—

PORTRUSH.—The barometer was put up in the guard-house, which is situated on an eminence facing the harbour; and the thermometers and the rain-gauge in a small attached garden. The four thermometers at this, and at every other station, were inclosed in a shallow box with a sloping roof, and wire-gauze front. A vertical gnomon was fixed at most of the stations in the window-sill of the guard-house, for the purpose of determining the time of noon; and the observers were furnished with a Table of the equation of time computed for the year 1851, and for the mean longitude of Ireland.

DONAGHADEE.—The meteorological instruments were favourably placed: the barometer in the guard-house, and the thermometers and rain-gauge in an inclosed yard connected with it. The meridian line was traced on the sill of a window in the guard-house, the shadow being given by a vertical iron bar.

KILLOUGH.—Lighthouse, St. John's Point.—The barometer was put up in the hall of the light-keeper's dwelling; the other meteorological instruments were well placed in a garden attached to it. The meridian line was traced on the flagging, at the south side of the house, the shadow being given by a vertical iron rail.

COURTOWN HARBOUR.—The barometer was erected in the guard-house of the station; the thermometers in an inclosed yard at the rear, attached to a wall facing northward; and the rain-gauge on an eminence behind it.

DUNMORE EAST.—The barometer was put up in the guard-house of the station; the thermometers were attached to the northern external wall, and were not completely guarded from radiation. The rain-gauge was fixed to a wall in front.

Buncrana.—The meteorological instruments were put up at the guard-house,—the barometer within, and the thermometers on one of the external walls facing to the north; the site was not favourable.

Killybegs.—This lighthouse is admirably circumstanced for meteorological observations. The Academy's barometer was not put up, the barometer belonging to the lighthouse being found sufficiently good; it was favourably placed in the sitting-room of the light-keeper's dwelling. The thermometers were fixed in an angle of the yard at the rear of the house; the rain-gauge was attached to an iron railing in the front yard. There is a sun-dial in the front yard, the position of which was examined, and found correct.

Westport—Inishgort Lighthouse.—The meteorological instruments were erected at the lighthouse of Inishgort, in charge of the light-keeper. The barometer belonging to the lighthouse was found sufficiently good for the observations; it is placed in the sitting-room of the light-keeper. The thermometers were fixed to one of the external walls facing northward, and the rain-gauge in the small garden attached to the lighthouse.

Kilrush.—The meteorological instruments were erected at the guard-house, close to the quay; the barometer within the guard-house, and the thermometers attached to an external wall. The rain-gauge was fixed at the foot of the flag-staff.

Cahirciveen.—The barometer was erected in the house occupied by the boatman in charge, in the town of Cahirciveen, and the thermometers and rain-gauge in the yard and garden attached to it. Their site was not favourable.

Castletownsend.—The barometer was placed in the guard-house, and the thermometers on one of the external walls facing northward. The rain-gauge was fixed at the foot of the flag-staff. The time of noon was found by means of a dipleidoscope belonging to the officer in command of the station.

Plan of Observation.—It is probable that over a tract of country so limited as this island, the distribution of temperature, humidity, and rain, does not vary materially from one year to another; and that, consequently, a tolerable approximation to the laws of this distribution may be obtained from the results of a single year, if every precaution be adopted to insure the perfect comparability of the results. It was arranged, accordingly, that the observations should be continued at the coast-guard stations until the end of the year 1851, so as to embrace a period of at least one year reckoned from the time when the observers had acquired the power of observing with accuracy. The monthly means for this year may be reduced to their *absolute* mean values, by the help of the more extended series of observations made in Dublin, by which the deviations of any monthly result from its absolute mean value is sufficiently known.

The Committee, upon whom the duty of superintending these arrangements devolved, were desirous that the plan of observation should be the least onerous that could lead satisfactorily to the results aimed at. One of the principal of these—the determination of the movements of masses of air, whether in storms, or in the displacement of atmospheric waves,—demands, as has been said, that the observations should be taken at *equal intervals* of time; and the only condition imposed by the other meteorological problems is, that these times should be so chosen as to furnish the *daily means* of the elements sought. Now *any three* observations, taken *at equal intervals* throughout the day, are sufficient to eliminate the diurnal variation, and therefore to give the *daily means* of *all* the meteorological elements; and undoubtedly, where such a system is practicable, the observations should be taken at 6 A. M., 2 P. M., and 10 P. M., which has been shown to be preferable to any other eight-hourly group for meteorological purposes.*

At the coast-guard stations, however, such a plan of observation would have been incompatible with the regular duties of the men; and it was advisable to adopt a less complete system, which might be followed at all the stations, and in which interruptions were not likely to occur. Fortunately, two observations in the day, taken at equal intervals, are sufficient to give the daily means of all the meteorological elements, excepting the atmospheric pressure; and, as the diurnal variation of the pressure is very small,—much smaller than its irregular fluctuations in these latitudes,—it may be disregarded, and the objects for which the present system was instituted may be attained by taking *two* observations in the day, at *homonymous hours.*

* Transactions of the Royal Irish Academy, vol. xxii. p. 65.

The best pair of homonymous hours, for the determination of the *mean temperature*, and nearly also for that of the *mean humidity*, are $9^h\ 46^m$ A. M., and $9^h\ 46^m$ P. M.* Limiting themselves to the exact hours, the Committee might accordingly have chosen either 9 A. M. and 9 P. M., or 10 A. M. and 10 P. M.; the former pair was adopted, its superior convenience seeming to outweigh the advantage of the latter in accuracy.

For the fuller elucidation of some of the questions proposed, it was further arranged that *hourly observations* should be taken at all the stations for twenty-four hours, at the equinoxes and solstices, according to the plan laid down by Sir John Herschel. It was likewise provided, that hourly observations should be taken occasionally, under special circumstances, such as storms, unusual disturbances of barometric equilibrium, &c.

For further details of the plan of observation, the reader is referred to the "Instructions" prepared by the Council of the Academy. I now proceed to the results of the observations.

Temperature of the Air.

Corrections.—It has been already stated, that the thermometers employed in measuring the temperature and humidity of the air were carefully compared with a standard thermometer, and their errors noted. When the errors differed by more than $0^\circ{\cdot}2$ in different parts of the scale, the instrument was rejected; when they did not, the mean of the observed errors was adopted as a constant error for the whole scale of the instrument. Table II. gives the numbers thus obtained for the several instruments; these numbers are applied, with the contrary signs, as corrections to the observed results.

It has been stated that the mean of the temperatures observed at 9 A. M and 9 P. M. is, very nearly, the mean of the entire day. The small corrections required, in order to reduce the former to the latter, are obtained from the bi-hourly observations made at Dublin in the years 1840–1843. Table III. contains the results of that series, giving the mean differences between the temperature at each hour of observation, and that of the entire day.

* See the paper already referred to. The hours $9^h\ 30^m$ A. M., and $9^h\ 30^m$ P. M., are better for humidity.

Table II. Errors of the Thermometers.

Station.	Dry Therm.		Wet Therm.	
	No. of Inst.	Error.	No. of Inst.	Error.
Portrush, ...	10	+ 0·5	7	+ 0·3
Buncrana, ...	19	+ 0·4	11	+ 0·4
Donaghadee, . . .	5	+ 0·6	8	+ 0·6
Killybegs, ...	12	+ 0·5	22	+ 0·4
Armagh, ...	—	0·0	—	0·0
Killough, ...	37	– 0·3	33	– 0·1
Markree, ...	1	+ 0·4	2	+ 0·4
Westport,*	—	—	—	—
Dublin, ...	—	+ 0·1	—	+ 0·2
Portarlington,* .	—	—	—	—
Athy, ...	26	0·0	35	– 0·2
Courtown, ...	30	0·0	34	+ 0·2
Kilrush, ...	14	+ 0·2	21	+ 0·3
Dunmore, ...	28	– 0·3	32	– 0·3
Cahirciveen, . . .	16	+ 0·2	18	+ 0·2
Castletownsend, .	—	+ 0·5	27	– 0·1

Table III. Mean Differences between the Temperature at each Hour of Observation, and that of the entire Day, at Dublin.

	1 A.M.	3.	5.	7.	9.	11.	1 P.M.	3.	5.	7.	9.	11.
January, . .	– 1°·3	– 1°·8	– 2°·1	– 1°·9	– 0°·8	+ 2°·2	+ 3°·6	+ 3°·0	+ 1°·0	– 0°·2	– 0°·5	– 0°·8
February, .	– 1 ·2	– 1 ·6	– 2 ·2	– 2 ·1	– 0 ·4	+ 2 ·4	+ 4 ·0	+ 3 ·3	+ 1 ·0	– 0 ·4	– 0 ·7	– 0 ·9
March, . . .	– 3 ·5	– 3 ·7	– 3 ·7	– 3 ·1	+ 0 ·7	+ 4 ·1	+ 6 ·0	+ 5 ·5	+ 2 ·9	– 0 ·4	– 1 ·7	– 2 ·4
April, ...	– 4 ·9	– 5 ·4	– 5 ·6	– 1 ·3	+ 2 ·5	+ 5 ·4	+ 6 ·6	+ 6 ·4	+ 4 ·1	– 0 ·1	– 2 ·7	– 3 ·9
May, ...	– 5 ·7	– 6 ·5	– 5 ·2	+ 0 ·5	+ 2 ·8	+ 4 ·9	+ 5 ·8	+ 6 ·1	+ 4 ·1	+ 0 ·6	– 2 ·9	– 4 ·4
June, ...	– 6 ·2	– 7 ·0	– 4 ·9	+ 2 ·0	+ 3 ·1	+ 4 ·7	+ 5 ·4	+ 5 ·2	+ 4 ·2	+ 1 ·0	– 2 ·8	– 4 ·6
July, ...	– 4 ·9	– 5 ·7	– 4 ·9	+ 0 ·8	+ 2 ·5	+ 4 ·6	+ 5 ·1	+ 5 ·1	+ 3 ·7	+ 0 ·6	– 2 ·7	– 4 ·2
August, . . .	– 5 ·0	– 5 ·4	– 5 ·7	– 0 ·9	+ 2 ·6	+ 5 ·1	+ 5 ·9	+ 5 ·6	+ 3 ·9	+ 0 ·2	– 2 ·6	– 3 ·7
September, .	– 3 ·7	– 4 ·2	– 4 ·2	– 2 ·4	+ 1 ·7	+ 4 ·8	+ 6 ·0	+ 5 ·0	+ 2 ·9	– 0 ·6	– 2 ·1	– 3 ·2
October, . .	– 2 ·6	– 3 ·2	– 3 ·9	– 3 ·5	+ 0 ·2	+ 4 ·3	+ 5 ·5	+ 4 ·6	+ 1 ·9	– 0 ·1	– 1 ·3	– 1 ·9
November, .	– 1 ·2	– 1 ·5	– 1 ·7	– 1 ·8	– 0 ·6	+ 2 ·2	+ 3 ·5	+ 2 ·5	+ 0 ·6	– 0 ·1	– 0 ·8	– 1 ·0
December, .	– 0 ·8	– 1 ·1	– 1 ·3	– 1 ·2	– 0 ·8	+ 1 ·6	+ 2 ·6	+ 1 ·5	+ 0 ·4	– 0 ·2	– 0 ·4	– 0 ·5
Year, ...	– 3°·4	– 3°·9	– 3°·8	– 1°·2	+ 1°·1	– 3°·9	+ 5°·0	+ 4°·5	– 2°·6	+ 0°·0	– 1°·8	– 2°·6
Summer, . .	– 5 ·1	– 5 ·7	– 5 ·1	– 0 ·2	+ 2 ·5	+ 4 ·9	+ 5 ·8	+ 5 ·6	+ 3 ·8	+ 0 ·3	– 2 ·6	– 4 ·0
Winter, . . .	– 1 ·8	– 2 ·1	– 2 ·5	– 2 ·3	– 0 ·3	+ 2 ·8	+ 4 ·2	+ 3 ·4	+ 1 ·3	– 0 ·2	– 0 ·9	– 1 ·2

* At Westport and Portarlington the errors of the thermometers were not determined.

From the preceding Table we obtain the following corrections, which are to be applied to the means of the observed temperatures at 9 A. M. and 9 P. M., in order to reduce them to the mean of the day:—

April, . . .	corr. =	$+0^{\circ}\cdot1$	October, . .	corr. =	$+0^{\circ}\cdot5$
May, . . .	,,	$+0\cdot1$	November,	,,	$+0\cdot7$
June, . . .	,,	$-0\cdot1$	December,	,,	$+0\cdot6$
July, . . .	,,	$+0\cdot1$	January, .	,,	$+0\cdot7$
August, . .	,,	$+0\cdot0$	February, .	,,	$+0\cdot6$
September,	,,	$+0\cdot2$	March, . .	,,	$+0\cdot5$

It hence appears that the correction is nearly constant throughout the summer, and throughout the winter months, respectively. The mean summer correction is $+0^{\circ}\cdot1$; the mean winter correction $+0^{\circ}\cdot6$.

Mean Monthly Temperatures.—The mean temperatures have been obtained, at all but three of the stations, from the observations at 9 A. M. and 9 P. M., by the application of the preceding corrections. At Markree the observations were taken at 10 A. M. and 10 P. M.; and the reducing numbers are therefore somewhat different, and smaller in amount. At Portarlington and Athy the observations were taken but *once* in the day, namely, at 9 A. M.; and at these stations, accordingly, the mean temperatures are inferred from the maximum and minimum temperatures as given by the self-registering thermometers. The formula employed is that of Kæmtz, viz.:—

$$mean\ temp. = min. + a\,(max. - min.)$$

The mean value of the coefficient,* as deduced from the observations at the observatories of Armagh, Markree, and Dublin, is $a = 0\cdot41$.

The following Table contains the resulting values of the mean temperature for the several months of the year 1851:—

* The coefficient in Kæmtz's formula appears to vary considerably at different places, both in its mean amount, and in the law of its variation from month to month. At Armagh and Markree its greatest value is in December, and its least in July; at Dublin, it is the reverse. I have taken above the mean of the yearly values for the three stations.

Table IV. Mean Temperatures for each Month of the Year 1851, at the several Stations.

Station.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Portrush, ...	42°·0	42°·3	42°·9	45°·7	50°·2	55°·3	56°·5	58°·8	55°·5	51°·3	44°·4	44°·7	49°·1
Buncrana, ...	41 ·6	41 ·8	43 ·3	45 ·4	50 ·5	55 ·8	56 ·9	59 ·3	55 ·8	51 ·2	42 ·6	44 ·3	49 ·0
Donaghadee, . . .	42 ·8	43 ·1	43 ·3	46 ·6	50 ·8	55 ·8	57 ·0	58 ·8	56 ·3	51 ·8	43 ·9	45 ·4	49 ·6
Killybegs, ...	43 ·8	43 ·9	45 ·1	47 ·1	51 ·4	56 ·6	58 ·1	60 ·7	58 ·0	52 ·7	45 ·6	46 ·3	50 ·8
Armagh, ...	41 ·4	41 ·7	42 ·5	45 ·4	50 ·3	56 ·2	57 ·1	58 ·6	55 ·3	50 ·4	41 ·0	43 ·2	48 ·6
Killough, ...	44 ·6	44 ·0	43 ·9	46 ·7	51 ·0	55 ·2	58 ·1	59 ·1	57 ·2	53 ·2	43 ·0	46 ·6	50 ·2
Markree, ...	39 ·4	41 ·2	42 ·3	45 ·3	50 ·1	55 ·7	56 ·6	58 ·7	55 ·1	49 ·8	41 ·8	42 ·2	48 ·2
Westport, ...	46 ·3	45 ·2	46 ·2	47 ·3	52 ·1	56 ·5	58 ·3	60 ·9	58 ·4	53 ·7	47 ·2	47 ·8	51 ·7
Dublin, ...	43 ·6	43 ·6	44 ·0	46 ·8	52 ·5	58 ·8	60 ·2	62 ·0	55 ·9	51 ·9	41 ·2	43 ·3	50 ·3
Portarlington, . .	40 ·6	40 ·6	40 ·5	43 ·1	49 ·1	54 ·8	57 ·3	57 ·7	53 ·7	50 ·1	39 ·4	40 ·3	47 ·3
Athy, ...	40 ·7	40 ·9	41 ·8	45 ·4	50 ·6	56 ·9	58 ·2	60 ·8	53 ·6	51 ·0	39 ·7	41 ·3	48 ·4
Courtown, ...	43 ·3	43 ·8	44 ·3	46 ·6	52 ·5	57 ·3	59 ·5	60 ·7	56 ·5	51 ·8	41 ·6	45 ·1	50 ·3
Kilrush, ...	44 ·2	45 ·5	45 ·2	47 ·0	51 ·7	56 ·3	59 ·1	60 ·8	58 ·1	52 ·5	44 ·7	45 .1	50 ·9
Dunmore, ...	45 ·4	44 ·8	44 ·8	47 ·4	53 .3	58 ·2	61 ·6	62 ·3	59 ·1	53 ·8	42 ·6	46 ·2	51 ·6
Cahirciveen, . . .	45 ·6	46 ·2	46 ·4	48 ·5	53 ·9	58 ·2	60 ·2	62 ·0	58 ·7	54 ·1	46 ·5	47 ·0	52 ·3
Castletownsend, .	45 ·4	46 ·0	46 ·1	47 ·9	53 ·6	57 ·0	60 ·7	61 ·8	59 ·6	54 ·1	45 ·7	46 ·9	52 ·1

Before we proceed to discuss the mean temperatures in the several months of the year 1851, it is important that we should know the *absolute mean temperatures* at some one station, and thereby the *deviations from the means* in the several months of the year in question. Over a tract of country so limited as Ireland, these deviations will not differ much in different localities; and therefore, knowing them for one station, we are enabled to reduce the results of the single year, with probably sufficient exactness, to their absolute mean values at all the rest.

The absolute mean temperatures of the several months are known, at Dublin, by means of the series of observations made during twelve years at the Magnetical Observatory. The monthly mean temperatures, deduced from that series, are given in the following Table. From the year 1840 to 1843, inclusive, the daily means are those of twelve equidistant hours; from 1844 to 1850, inclusive, they are inferred from the temperatures observed at 10 A. M. and 10 P. M.; and in 1851, from those of 9 A. M. and 9 P. M. In the last line of the Table are given the deviations of the monthly means in 1851, from the *mean* monthly means, as deduced from the twelve years.

TABLE V. MEAN MONTHLY TEMPERATURES AT DUBLIN.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1840	41°·9	39°·7	42°·0	45°·4	54°·4	59°·4	59° 2	62°·1	53°·1	47°·2	44°·0	39°·8	49°·0
1841	36 ·7	39 ·7	46 ·1	47 ·9	55 ·2	56 ·5	57 ·3	59 ·4	57 ·1	48 ·3	42 ·3	41 ·8	49 ·0
1842	38 ·6	41 ·8	45 ·3	46 ·7	53 ·0	60 ·5	58 ·9	61 ·6	57 ·3	46 ·5	45 ·4	48 ·8	50 ·4
1843	42 ·8	37 ·9	44 ·3	47 ·1	51 ·3	56 ·4	60 ·3	60 ·4	58 ·2	47 ·2	44 ·9	49 ·2	50 ·0
1844	41 ·8	38 ·4	42 ·9	50 ·6	52 ·9	58 ·7	59 ·8	57 ·0	57 ·1	49 ·7	47 ·4	38 ·7	49 ·6
1845	41 ·7	40 ·3	38 ·7	48 ·2	51 ·3	58 ·9	58 ·0	57 ·8	54 ·6	51 ·1	46 ·3	41 ·3	49 ·0
1846	46 ·3	45 ·2	43 ·8	46 ·4	54 ·4	63 ·3	61 ·7	60 ·7	59 ·1	51 ·0	48 ·5	36 ·7	51 ·4
1847	42 ·3	38 ·7	43 ·3	46 ·2	53 ·6	57 ·7	64 ·0	59 ·9	54 ·6	51 ·9	48 ·8	45 ·5	50 ·5
1848	37 ·0	44 ·3	43 ·6	45 ·8	56 ·9	56 ·5	60 ·4	56 ·6	56 ·6	49 ·7	43 ·2	43 ·7	49 ·5
1849	42 ·6	44 ·4	44 ·7	43 ·5	53 ·6	57 ·4	60 ·3	60 ·0	56 ·7	50 ·5	47 ·0	41 ·6	50 ·2
1850	39 ·1	46 ·6	43 ·4	49 ·0	51 ·8	60 ·0	61 ·2	58 ·9	55 ·0	47 ·8	47 ·5	45 ·6	50 ·5
1851	43 ·6	43 ·6	44 ·0	46 ·8	52 ·5	58 ·8	60 ·2	62 ·0	55 ·9	51 ·9	41 ·2	43 ·3	50 ·3
Means	41°·2	41°·7	43°·5	47°·0	53°·4	58°·7	60°.1	59° 7	56°·3	49°·4	45°·5	43°·0	50°·0
Diffs.	− 2 ·4	+ 1 ·9	+ 0 ·5	− 0 ·2	− 0 ·9	+ 0 ·1	+ 0 ·1	+ 2 ·3	− 0 ·4	+ 2 ·5	− 4 ·3	+ 0 ·3	+ 0 ·3

It will be seen from this Table, that the temperature in the months of January, February, and October, 1851, was higher than the average temperature, while, in November, it was considerably lower. The mean temperature of the entire year was only 0°·3 above the average.

The depression of temperature in the month of November is a remarkable case of those non-periodic fluctuations to which the attention of meteorologists has been drawn by Professor Dove. This fluctuation appears to have proceeded from north-east to south-west, and to have been nearly obliterated when it reached the western coast of the island. At the northern and eastern stations the unusual cold began on the 24th day of the month; at the southern and western it commenced on the 26th and 27th. It reached its maximum about the 30th, and ceased about the 3rd of December. When we compare the mean temperatures of November and December at Killough, Dublin, Courtown, and Dunmore, on the eastern coast, with those at Killybegs, Westport, Kilrush, and Cahirciveen, on the western, we observe that the temperature of November is less than that of December by 3°·3 at the former stations, while the defect is only 0°·6 at the latter.

Upon a comparison of the mean yearly temperatures of the several stations, we observe that those of the *inland* stations are *in defect*, as compared with the corresponding coast stations. Thus the mean temperature of Armagh (48°·6) is less than that of Donaghadee by 1°, and less than that of Killough by 1°·6. The mean temperature of Markree (48°·2) is less than that of Killybegs by 2°·6, and than that of Westport by 3°·5. The mean temperatures of Portarlington and Athy (47°·3 and 48°·4) are in like manner in defect, when compared with those of Dublin and Courtown, and by an intermediate amount. I shall return to this subject hereafter, and merely notice it at present for the purpose of observing that no satisfactory conclusion can be drawn as to the dependence of temperature upon geographical position, unless the inland and coast stations be compared separately.

Confining ourselves for the present to the coast stations, which are the most numerous and the most widely distributed, we observe that there is an increase of mean annual temperature in proceeding *from north to south* of the island, the mean temperature of Portrush and Buncrana being 49°·0, and that of Dunmore, which is nearly on the intermediate meridian, 51°·6. Similarly there is an increase of temperature in proceeding *from east to west*, the mean temperature of Killough and Dublin being 50°·2, and that of Westport, which is nearly on the intermediate parallel, 51°·7.

But for an accurate determination of the rate of increase of temperature in the two directions, it is necessary to combine the results by the method of least squares. For this purpose let t denote the observed mean temperature of any month, at any given station; T the *probable* temperature of the same month at an *assumed* central station; and let the distances (in geographical miles) of the former from the latter, measured on the meridian and perpendicular to the meridian to the north and west, respectively, be denoted by y and x; then, if V and U be the increase of temperature corresponding to a single mile in each direction,

$$t = T + Ux + Vy.$$

There will be a similar equation for each station; and combining them by the method of least squares, we shall obtain the most probable values of the unknown quantities T, U, and V.

The simplest mode of employing this method in the present instance is to take, as the arbitrary central station, that whose latitude and longitude are the arithmetical means of the latitudes and longitudes of the stations of observation. The resulting equations are thus reduced to the following:—

$$nT = \Sigma\,(t),$$
$$U\Sigma\,(x^2) + V\Sigma\,(xy) = \Sigma\,(xt),$$
$$U\Sigma\,(xy) + V\Sigma\,(y^2) = \Sigma\,(yt).$$

For the reason already stated, I shall employ in this calculation only the results obtained at the coast stations. These are, in the order of latitude, Portrush, Buncrana, Donaghadee, Killybegs, Killough, Westport, Dublin, Courtown, Kilrush, Dunmore, Cahirciveen, Castletownsend. The mean latitude and longitude of these stations are 53° 29′, and 7° 39′ respectively. And we find

$$\Sigma\,(x^2) = 39094, \qquad \Sigma\,(xy) = -\,22569, \qquad \Sigma\,(y^2) = 65811.$$

Substituting and eliminating between the second and third equations, we obtain—

$$U = \cdot 0000319\ \Sigma\,(xt) + \cdot 0000109\ \Sigma\,(yt);$$
$$V = \cdot 0000109\ \Sigma\,(xt) + \cdot 0000189\ \Sigma\,(yt).$$

By these formulæ the values of T, U, and V, for each month are calculated. They are given in the following Table:—

TABLE VI. ELEMENTS OF MONTHLY ISOTHERMAL LINES.

	T		U	V	W	u
	1851.	Mean.				
January, ...	44°·1	41°·7	+ ·0080	− ·0102	·0130	52°
February, ...	44 ·2	42 ·3	+ ·0093	− ·0119	·0151	52
March, ...	44 ·6	44 ·1	+ ·0131	− ·0064	·0146	26
April, ...	46 ·9	47 ·1	+ ·0043	− ·0070	·0082	59
May, ...	52 ·0	52 ·9	+ ·0012	− ·0139	·0140	85
June, ...	56 ·8	56 ·7	− ·0031	− ·0109	·0114	106
July, ...	58 ·9	58 ·8	− ·0049	− ·0202	·0208	104
August, ...	60 ·6	58 ·3	+ ·0029	− ·0121	·0124	77
September, ...	57 ·4	57 ·8	+ ·0101	− ·0090	·0135	42
October, ...	52 ·7	50 ·2	+ ·0059	− ·0070	·0092	50
November, ...	44 ·1	48 ·4	+ ·0304	+ ·0077	·0313	− 14
December, ...	45 ·7	45 ·4	+ ·0103	− ·0017	·0104	9
Year, ...	50 ·7	50 ·3	+ ·0073	− ·0085	·0112	49°

The values of U and V being known, the positions of the isothermal lines are determined. The inclination of the isothermal lines to the meridian, measured from north to west, u, and the rate of increase of temperature in the direction perpendicular to them, W, are known by the formulæ

$$\tan u = \frac{V}{U}, \qquad W = \surd(U^2 + V)^2.$$

Their values for the several months are given in the foregoing Table.

We see then that, on the mean of the whole year, the isothermal lines are inclined to the meridian by the angle N. 49° W.; and that the temperature increases in a direction perpendicular to these lines, by ·0112 of a degree for each geographical mile, or at the rate of 1 degree for 89 miles. The increase of temperature, in proceeding from north to south, is $V =$ ·0085, or 1° in 118 geographical miles; the corresponding increase, in proceeding from east to west, is $U =$ ·0073, or 1° in 137 geographical miles.

We learn further, that the *mean annual* isothermal lines furnish a very inadequate representation of the progression of temperature; and that when we follow the course of these lines *from month to month*, we find them to vary within very wide limits. The extreme positions of these lines, as given in the preceding Table, are those for the months of June and November. But the result obtained for the latter month must, I think, be regarded as anomalous, on account of the irregularity in the distribution of temperature already noticed; and, rejecting it, the extreme positions correspond to the two solstitial months. They are the following:—

June, . . . $u =$ N. 106° W., $W =$ ·0114,
December, $u =$ N. 9° W., $W =$ ·0104;

so that the direction of the isothermal lines varies through an angle of 97° in the course of the year, being nearly parallel to the meridian in December, and nearly perpendicular to it in June. (See Plate VII.)

We may now employ the formula

$$t = T + Ux + Vy,$$

to deduce the *probable* temperature at any place, and compare it with that actually *observed;* we shall thus find the effect due to local causes. Making this calculation for the four *inland* stations, we obtain the results given in the following Table:—

Table VII. Calculated Temperatures at Inland Stations.

Station.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Armagh, ...	43°·2	43°·2	43°·8	46°·4	51°·2	56°·3	58°·0	59°·9	56°·6	52°·1	43°·4	45°·3	50°·0
Markree, ...	43 ·8	43 ·9	44 ·7	46 ·7	51 ·4	56 ·2	57 ·8	60 ·1	57 ·3	52 ·5	45 ·3	45 ·9	50 ·5
Portarlington, .	44 ·2	44 ·3	44 ·6	47 ·0	52 ·2	57 ·0	59 ·3	60 ·8	57 ·4	52 ·7	43 ·5	45 ·6	50 ·7
Athy, ...	44 ·2	44 ·3	44 ·5	47 ·0	52 ·4	57 ·2	59 ·6	60 ·9	57 ·4	52 ·8	43 ·1	45 ·5	50 ·7

Defect of observed Temperatures.

Station.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Armagh, ...	1°·8	1°·5	1°·3	1°·0	0°·9	0°·1	0°·9	1°·3	1°·3	1°·7	2°·4	2°·1	1°·4
Markree, ...	4 ·4	2 ·7	2 ·4	1 ·4	1 ·3	0 ·5	1 ·2	1 ·4	2 ·2	2 ·7	3 ·5	3 ·7	2 ·3
Portarlington, .	3 ·6	3 ·7	4 ·1	3 ·9	3 ·1	2 ·2	2 ·0	3 ·1	3 ·7	2 ·6	4 ·1	5 ·3	3 ·4
Athy, ...	3 ·5	3 ·4	2 ·7	1 ·6	1 ·8	0 ·3	1 ·4	0 ·1	3 ·8	1 ·8	3 ·4	4 ·2	2 ·3

We learn that the defect of temperature due to inland position is, as might have been expected, least in summer and greatest in winter. A small part of this defect is due to elevation: but it is easily eliminated. The mean height of the instruments at the coast stations above the level of the sea is 30 feet. We have, therefore, only to subduct this from the known heights at the inland stations, and to correct for the difference of level at the rate of 1° Fahr. for 276 feet, which is the mean of the determinations made by Mr. Welsh in his balloon ascents, for the lower portion of the atmosphere lying beneath the great vapour plane. The mean yearly results at the four inland stations, thus corrected, are as follow:—

	Observed Defect.	Height above Sea.	Correction.	Reduced Defect.
Armagh, ...	1°·4	211	− 0°·7	0°·7
Markree, ...	2 ·3	132	− 0 ·4	1 ·9
Portarlington, . . .	3 ·4	230	− 0 ·7	2 ·7
Athy, ...	2 ·3	200	− 0 ·6	1 ·7
			Mean =	1°·8

Diurnal Ranges of Temperature.

Climatology depends upon the *ranges* of temperature (whether diurnal, monthly, or annual), no less than upon *mean values;* and their investigation is accordingly a necessary part of the present inquiry. In the present series of observations, the *diurnal ranges* of temperature are given by means of the results obtained with self-registering thermometers. These results are the least satisfactory portion of the whole series. It is well known that the ordinary self-registering thermometers are extremely apt to get out of order, the maximum by the index becoming entangled in the mercury, and the minimum by the distillation of the spirit into the upper part of the tube; and although the observers were carefully instructed in the mode of remedying these derangements, no one (I believe) who has handled such instruments will wonder that men previously unaccustomed to them should have sometimes failed in what is in all cases a somewhat delicate operation. The blanks in the Table of *maximum* temperature at Buncrana and Killybegs, and those in the Table of *minimum* temperature at Killybegs and Dunmore, are due to this cause.

But there is another source of error affecting the *maximum* thermometer, which it is still more difficult to avoid. If the instrument be exposed to the influence of radiation for *any* portion of the day, however short, it will, from its construction, retain the impression made upon it; and consequently, if the abnormal temperature to which it has been thus subjected exceed the greatest temperature of the air in the day, an erroneous result will be recorded. The difficulty of guarding thermometers completely from such influences is well known; and although some trouble was taken to insure this protection, the observations themselves show that it was not effective at all the stations. I have, accordingly, been compelled to reject a portion of the results obtained with the maximum thermometer at Killough, Courtown, Kilrush, and Dunmore, as defective from this cause.

The results are given in the following Tables. Table VIII. contains the monthly means of the *maximum* temperature in each day; Table IX. those of the *minimum* temperature; and Table X. the differences of the two preceding, or the monthly means of the *diurnal ranges*.

Table VIII. Maximum Temperatures (Monthly Means).

Station.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Portrush, . . .	47°·5	47°.7	47°·7	52° 2	56°·5	61°·9	63°·9	65°·7	63°·3	56° 6	49°·0	48°·2
Buncrana, . .	45 ·9	47 ·1	48 ·8	52 ·6	57 ·1	63 ·9	—	—	—	—	—	—
Donaghadee, .	47 ·8	48 ·0	48 ·5	52 ·3	57 ·6	63 ·5	64 ·2	66 ·3	63 ·2	57 ·0	47 ·8	48 ·8
Killybegs, . . .	47 ·8	48 ·4	50 ·1	55 ·1	59 ·2	62 ·4	—	—	—	—	—	—
Armagh, . . .	47 ·5	47 ·6	48 ·4	52 ·8	57 ·4	64 ·5	64 ·8	67 ·0	62 ·9	56 ·4	45 ·7	47 ·0
Killough, . . .	48 ·8	47 ·6	48 ·8	—	—	—	—	—	—	59 ·1	48 ·5	48 ·6
Markree, . . .	46 ·6	48 ·0	49 ·5	53 ·3	58 ·4	67 ·4	65 ·9	67 ·2	64 ·1	56 ·3	48 ·0	46 ·9
Dublin, ...	50 ·1	49 ·5	49 ·8	53 ·3	58 ·3	65 ·5	65 ·7	68 ·4	62 ·9	58 ·4	46 ·7	48 ·6
Portarlington,	48 ·0	48 ·3	48 ·8	52 ·8	58 ·8	66 ·0	66 ·0	67 ·6	64 ·9	57 ·9	46 ·6	46 ·6
Athy, ...	47 ·5	47 ·7	49 ·2	54 ·2	59 ·0	66 ·6	66 ·7	69 ·6	62 ·6	57 .7	45 ·8	46 ·4
Courtown, . .	—	—	—	—	—	—	67 ·2	69 ·2	64 ·6	58 ·7	47 ·4	48 ·9
Kilrush, ...	49 ·0	49 ·3	50 ·4	—	—	—	—	—	—	57 3	49 ·0	48 ·1
Dunmore, . . .	48 ·6	48 ·4	50 ·5	—	—	—	—	—	—	54 ·9	45 ·0	48 ·1
Cahirciveen, .	50 ·2	49 ·6	49 ·7	—	—	—	—	—	63 ·5	57 ·1	50 ·2	49 ·7
Castletownsend,	50 ·1	50 ·5	52 ·0	54 ·1	61 ·3	64 ·3	66 ·7	68 ·3	65 ·3	57 ·9	47 ·6	47 ·0

Table IX. Minimum Temperatures (Monthly Means).

Station.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Portrush, . . .	36°·2	36°·8	37°·2	40°·0	45°·4	49°·8	52°·5	53°·0	49°·6	46°·1	39°·6	38°·8
Buncrana, . .	37 ·1	36 ·8	38 ·1	39 ·4	45 ·4	49 ·9	52 ·4	53 ·5	49 ·1	45 ·8	37 ·8	38 ·9
Donaghadee, .	39 ·3	40 ·0	39 ·6	43 ·4	46 ·9	50 ·9	53 ·0	54 ·3	53 ·0	47 ·6	39 ·9	41 ·9
Killybegs, . .	39 ·9	39 ·7	41 ·1	41 ·5	46 ·5	49 ·1	—	—	—	—	—	—
Armagh, . . .	35 ·7	36 ·9	36 ·7	39 ·3	44 ·3	50 ·2	52 ·3	53 ·3	50 ·3	45 ·9	36 ·6	38 ·6
Killough, . . .	40 ·2	39 ·0	39 ·2	41 ·6	44 ·6	49 ·8	52 ·3	53 ·6	51 ·6	47 ·0	36 ·5	40 ·0
Markree, . . .	34 ·4	36 ·0	36 ·1	38 ·9	44 ·8	50 ·0	52 ·9	54 ·4	49 ·7	46 ·8	38 ·1	38 ·0
Dublin, ...	41 ·0	41 ·2	40 ·6	43 ·0	47 ·0	52 ·0	54 ·8	57 ·2	51 ·0	48 ·4	38 ·7	40 ·6
Portarlington,	35 ·5	35 ·1	34 ·8	36 ·4	42 ·4	47 ·1	51 ·2	50 ·7	45 ·9	44 ·7	34 ·4	35 ·9
Athy, ...	36 ·0	36 ·1	36 ·7	39 ·2	44 ·8	50 ·1	52 ·4	54 ·7	47 ·3	46 ·3	35 ·4	37 ·7
Courtown, . .	36 ·2	38 ·0	38 ·0	41 ·2	46 ·0	50 ·9	54 ·1	55 ·6	50 ·6	46 ·3	36 ·1	40 ·4
Kilrush, . . .	39 ·5	40 ·2	40 ·3	42 ·1	47 ·4	49 ·0	52 ·2	54 ·0	50 ·0	45 ·2	38 ·3	41 ·4
Dunmore, . . .	39 ·2	39 ·9	40 ·0	42 ·7	47 ·5	53 ·0	56 ·6	58 8	55 ·5	—	—	—
Cahirciveen, .	41 ·2	42 ·3	42 ·2	43 ·9	49 ·1	53 ·2	56 ·0	58 ·6	54 ·4	50 ·2	43 ·0	43 ·1
Castletownsend,	40 ·7	41 ·6	40 4	42 ·9	47 ·7	51 ·3	55 ·1	57 ·4	55 ·0	49 ·6	40 ·7	42 ·7

Table X. Diurnal Ranges of Temperature (Monthly Means).

Station.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Portrush, . . .	11°·3	10°·9	10°·5	12°·2	11°·1	12°·1	11°·4	12°·7	13°·7	10°·5	9°·4	9°·4
Buncrana, . .	8 ·8	10 ·3	10 ·7	13 .2	11 ·7	14 ·0	—	—	—	—	—	—
Donaghadee, .	8 ·5	8 ·0	8 ·9	8 ·9	10 ·7	12 ·6	11 ·2	12 ·0	10 ·2	9 ·4	7 ·9	6 ·9
Killybegs, . .	7 ·9	8 ·7	9 ·0	13 ·6	12 ·7	13 ·3	—	—	—	—	—	—
Armagh, . . .	11 ·8	10 ·7	11 ·7	13 ·5	13 ·1	14 ·3	12 ·5	13 ·7	12 ·6	10 ·5	9 ·1	8 ·4
Killough, . . .	8 ·6	8 ·6	9 ·6	—	—	—	—	—	—	12 ·1	12 ·0	8 ·6
Markree, . . .	12 ·2	12 ·0	13 ·4	14 ·4	13 ·6	17 ·4	13 ·0	12 ·8	14 ·4	9 ·5	9 ·9	8 ·9
Dublin, ...	9 ·1	8 ·3	9 ·2	10 ·3	11 ·3	13 ·5	10 ·9	11 ·2	11 ·9	10 ·0	8 ·0	8 ·0
Portarlington,	12 ·5	13 ·2	14 ·0	16 ·4	16 ·4	18 ·9	14 ·8	16 ·9	19 ·0	13 ·2	12 ·2	10 ·7
Athy, ...	11 ·5	11 ·6	12 ·5	15 ·0	14 ·2	16 ·5	14 ·3	14 ·9	15 ·3	11 ·4	10 ·4	8 ·7
Courtown, . .	—	—	—	—	—	—	13 ·1	13 ·6	14 ·0	12 ·4	11 ·3	8 ·5
Kilrush, . . .	9 ·5	9 ·1	10 ·1	—	—	—	—	—	—	12 ·1	10 ·7	6 ·7
Dunmore, . . .	9 ·4	8 ·5	10 ·5	—	—	—	—	—	—	—	—	—
Cahirciveen, .	9 ·0	7 ·3	7 ·5	—	—	—	—	—	9 ·1	6 ·9	7 ·2	6 ·6
Castletownsend,	9 ·4	8 ·9	11 ·6	11 ·2	13 ·6	13 ·0	11 ·6	10 ·9	10 ·3	8 ·3	6 ·9	4 ·3

In the following Table are given the results of the last of the preceding, combined in yearly and half-yearly periods, retaining only those stations at which one or other of the two half-years is complete:—

Table XI. Diurnal Ranges (Half-yearly and Yearly Means).

Station.	Summer.	Winter.	Year.
Portrush, ...	12°·2	10°·3	11°·3
Donaghadee, . . .	10 ·9	8 ·3	9 ·6
Armagh, ...	13 ·3	10 ·4	11 ·8
Killough, ...	—	9 ·9	—
Markree, ...	14 ·3	11 ·0	12 ·6
Dublin, ...	11 ·5	8 ·8	10 ·1
Portarlington, . .	17 ·1	12 ·6	14 ·9
Athy, ...	15 ·0	11 ·0	13 ·0
Kilrush, ...	—	9 ·7	—
Cahirciveen, . . .	—	7 ·4	—
Castletownsend, .	11 ·8	8 ·2	10 ·0
Coast Stations, .	11°·6	8°·9	10°·3
Inland do., . . .	14 ·9	11 ·3	13 ·1
Differences, . .	3 ·3	2 ·4	2 ·8

From the mean results of the preceding Table, we learn that the diurnal range is greater at the *inland* than at the *coast* stations, the mean excess being 2·8 degrees. The excess is greater in summer than in winter, being 3°·3 in the former, and 2°·4 in the latter season.

We are now in a position to refer to one, at least, of the practical inferences which may be deduced from the preceding results.

The climatological conditions connected with temperature, which favour the prevention or cure of diseases of the lungs, are, firstly, a high winter temperature; and secondly, a small amount of diurnal range. It has been already stated that Ireland is well circumstanced as to these conditions; let us now inquire which is its most favourable region as respects them.

The months of lowest temperature in Ireland, and which are on that account the most trying to the patients above alluded to, are those of December, January, February, and March. During these months the mean temperature varies very little, the mean range at Dublin being from 41°·7, in January, to 45°·4, in March, or only 3·7 degrees. Now the mean direction of the isothermal lines for these four months is N. 37° W.; so that the highest mean temperature for these months is to be found on the south-western coast, not far from Valentia.

The second condition above mentioned, although not frequently taken into account, is, perhaps, still more important. In proof of this it may be mentioned that in Norway, which is remarkable for the small amount of the diurnal range of temperature, consumption is uncommon, even in the highest latitudes; while in parts of Sweden, where this condition does not hold, it is very prevalent. Now, we learn from Table xi., that among the stations at which observations were made in 1851, the winter diurnal range of temperature is least at Cahirciveen. Both conditions, therefore, point to the south-western coast of Kerry as the region in Ireland most favourable to patients affected with these formidable maladies.

I am not in possession of any statistical data bearing upon this question, and am therefore unable to say how far the conclusion thus drawn is borne out by facts.

Temperature of the Sea.

Provision was made that the temperature of the sea should be observed at all the places at which tidal observations were made. For this purpose each station was furnished with a thermometer, having its bulb inclosed within a small reservoir of copper, for the double purpose of guarding it from accident, and of protecting it (by means of the contained water) from rapid changes of temperature when it was lifted into the air for observation. The observer was instructed to note its indications twice in the day, at intervals of about twelve hours, the thermometer being attached to a pole, and plunged to the depth of about one foot in deep water. The diurnal change of the temperature of the sea being very small, it is completely eliminated by two such observations. At many of the stations the instrument was lost, or broken, in the attempt to use it during boisterous weather. We are, therefore, only in possession of results from six stations, which are contained in the following Table:—

Table XII. Temperature of the Sea (Monthly Means).

Station.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Portrush, ...	46° 9	45°·6	45°·7	47 ·9	50°·6	54°·3	57°·5	58°·7	58°·7	54°·9	49°·6	47°·8	51°·5
Cushendall, . . .	46 ·6	45 ·7	45 ·6	47 ·2	49 ·1	52 ·1	55 ·4	57 ·0	58 ·3	55 ·6	51 ·6	49 ·4	51 ·1
Donaghadee, . .	46 ·5	45 ·6	45 ·6	48 ·1	50 ·3	52 ·5	55 ·8	57 ·4	57 ·5	54 ·7	50 ·1	49 ·3	51 ·1
Bunown, ...	48 ·7*	49 ·4	50 ·9	51 ·4	54 ·4	57 ·8	60 ·2	62 ·9	61 ·5	55 ·1	49 ·3	48 ·1	54 ·1
Courtown, ...	46 ·0*	45 ·1	45 ·8	48 ·3	53 ·2	58 ·4	61 ·8	63 .5	60 ·9	56 ·1	47 ·9	46 ·8	52 ·8
Castletownsend,	47 ·1	46 ·6	46 ·8	49 ·7	53 ·5	56 ·3	60 ·3	61 ·3	60 ·3	54 ·8	49 ·2	48 ·6	52 ·9
Means, ...	47 ·0	46°·3	46°·7	48°·8	51°·9	55° 2	58°·5	60°·1	59°·5	55°·2	49°·6	48°·3	52°·3
Differences, . . .	−5 ·3	−5 ·9	−5 ·6	−3 ·5	−0 ·4	+3 ·0	+6 ·2	+7 ·9	+7 ·2	+3 ·0	−2 ·6	−3 ·9	

In the last two lines of the Table are given the mean results of the six stations, and the differences between them and the mean of the entire year. These

* At Bunown and Courtown no observations of sea temperature were made in January. The results in the Table for that month are the means of the temperatures of December and February.

numbers accordingly exhibit the law of the *annual variation* of sea temperature, around the coasts of Ireland; and the remarkable regularity in their progression shows that, even from the results of a single year, we obtain a close approximation to the actual law.

We learn from these numbers that the annual change of the sea temperature, at the surface, differs considerably from that of the air above it, the difference consisting chiefly in a *retardation* of the epochs of maximum and minimum. Thus the minimum temperature occurs in the middle of February, and the maximum in the middle of August,—or about a month after the corresponding epochs of the temperature of the air. The annual range is also, as might have been expected, considerably less than that of the air. These results accord sufficiently well with the conclusions drawn by Kæmtz, from a comparison of the results of many voyagers.

But the most interesting result is that concerning the relation between the temperature of the sea at the surface, and that of the superincumbent air. Upon this subject the greatest discordance exists in the statements of different observers. According to Humboldt, the mean temperature of the Atlantic Ocean, at the surface, is in all cases higher than that of the atmosphere above it. This conclusion is confirmed by the observations of Peron and Fitzroy, and is contradicted by those of Irving, Forster, and Kotzebue. From an elaborate discussion of the observations of many voyagers, Kæmtz infers that the temperature of the sea is *less* than that of the air over the land in the lower latitudes, while in the higher latitudes it is *greater;* the difference seldom, however, exceeding 1° Fahr. The original conclusion of Humboldt, however, seems to be placed beyond all doubt by the recent observations of Captain Duperrey, which appear to be more numerous, and taken with more precautions to insure accuracy, than any preceding. It seems now to be generally admitted that, in the *temperate* and *polar* regions, the temperature of the sea is *higher* than that of the air; and the only question that remained was as to the tropics. Now the observations of Duperrey were made all round the globe, between 10° N. and 10° S. latitude; and they were taken at intervals of four hours, so as completely to eliminate the effects of the diurnal change. From these observations it appears that the temperature of the sea is higher than that of the air within the zone already mentioned, the mean excess in the Atlantic being 0°·83 Fahr., and in the Great Ocean about half that amount.

The present observations possess much interest in connexion with these questions. In order to perceive their bearing, I have, in the Table which follows, given the half-yearly and yearly means of the sea-temperature at the several stations, together with the differences between them and the corresponding means of the temperature of the air. At Cushendall and Bunown no observations of the temperature of the air were actually made; and for these stations, consequently, the latter means are calculated from the isothermal lines.

Table XIII. Temperature of the Sea (Yearly and Half-yearly Means).

Station.	Summer.		Winter.		Year.	
	Temp.	Excess.	Temp.	Excess.	Temp.	Excess.
Portrush, ...	54°·6	+1°·0	48°·4	+3°·8	51°·5	+2°·4
Cushendall, . . .	53 ·2	−1 ·0	49 ·1	+4 ·4	51 ·1	+1 ·7
Donaghadee, . . .	53 ·6	−0 ·6	48 ·6	+3 ·6	51 ·1	+1 ·5
Bunown, ...	58 ·0	+2 ·4	50 ·2	+3 ·2	54 ·1	+2 ·8
Courtown, ...	57 ·7	+2 ·2	47 ·9	+3 ·0	52 ·8	+2 ·6
Castletownsend, .	56 ·9	+0 ·1	48 ·9	+1 ·5	52 ·9	+0 ·8
Means, ...	. . . .	+0°·7	. . . .	+3°·3	. . . .	+2°·0

It appears from the last line of this Table, that the temperature of the sea is, upon the mean of the entire year, 2°·0 higher than that of the air above the coast. The excess is 3°·3 in winter, and 0°·7 in summer. There appears also to be considerable diversity in the amount of the excess at the different stations; it is greatest, on the mean of the entire year, at Bunown, and least at Castletownsend.

This excess of the temperature of the sea above that of the air furnishes the explanation of the fact already noticed,—namely, the diminution of the temperature of the air in proceeding from the coasts inland; for it is obvious that the air in the vicinity of the sea must have its temperature raised by contact with the water.

It follows also, that the *absolute* excess of sea temperature considerably exceeds that above stated. Thus, we have seen, the temperature of the sea, on the average of the entire year, exceeds that of the air *over the coasts* by 2°·0; while the latter temperature exceeds that of the *air inland* (for the same latitude and longitude) by 1°·8. The total excess of the sea temperature above that of the air amounts, therefore, to 3°·8 Fahrenheit.

This excess, which appears to be much greater than has been observed elsewhere, is to be ascribed, mainly, to the influence of the gulf-stream upon the temperature of that part of the ocean which bathes our shores. But there is likewise another cause which undoubtedly contributes also to the effect. It has been shown by Mayer and Joule, that heat is generated by the friction of fluids in motion, and the latter experimentalist has established the important physical law, that there is a definite relation between the heat so produced, and the mechanical power expended by the moving mass. Mr. Rankine has already applied this principle to explain the fact, observed by M. Renou, namely, that the temperature of the river Loire at Vendôme is higher than that of the air above it; and it is obvious that a similar explanation is applicable to the phenomenon under consideration. There is no doubt as to the reality of the cause; the only question can be as to the magnitude of the effect to be ascribed to it. That such effect is, at all events, *sensible*, I infer from two circumstances. The first of these is, that the phenomenon of the excess of sea temperature appears to be *general*, and must, therefore, be the effect of some general cause; the second is, that on the coasts of Ireland there is no sensible difference between the amount of the excess on the *eastern* and on the *western* shores.*

Should the effect of this cause be found to be sensible, and its amount be determined, our views of the cycle of meteorological phenomena would be much enlarged. The elevation of temperature rarefies the air; the denser air flows in to supply the partial vacuum, and wind is produced; and finally, this wind,

* There is another corroborating circumstance which perhaps deserves also to be mentioned. Most bathers have, I believe, noticed the fact that the sea appears warmer, *cæteris paribus*, when agitated than when at rest. I am not aware that any direct thermometrical measures have ever been made to establish the fact thus evidenced by the senses; and I need not say that, if established, it would bear the whole weight of the hypothesis above proposed.

both by its own motion, and by that of the ocean which is so subject to its power, restores again the heat which had been converted. Thus the normal condition of temperature is preserved, not only throughout the changes which render it latent and sensible, in the generation and condensation of vapour, but also in its conversion into mechanical power and its reproduction, in the phenomena of the tempest and of the billowy sea.

Barometric Pressure.

The following Table contains the monthly means of the observed pressures, diminished by 28 inches, and reduced to 32° Fahr.:—

Table XIV. Monthly Means of Barometric Pressure.*

Station.	Jan.	Feb.	Mar.	April	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Portrush, . . .	1·444	1·901	1·628	1·840	2·030	1·888	1·790	1·939	2·150	1·703	1·976	2·123
Buncrana, . . .	1·377	1·827	1·580	1·777	1·968	1·849	1·760	1·886	2·118	1·681	1·972	2·102
Donaghadee, .	1·496	1·930	1·652	1·852	2·036	1·915	1·798	1·948	2·152	1·728	1·950	2·148
Killybegs, . . .	1·415	1·868	1·622	1·822	2·032	1·918	1·828	1·938	2·126	1·718	2·008	2·112
Armagh, . . .	1·284	1·722	1·448	1·640	1·836	1·716	1·603	1·757	1·958	1·540	1·796	1·942
Killough, . . .	1·514	1·932	1·666	1·852	2·030	1·922	1·806	1·963	2·138	1·751	1·943	2·083
Markree, . . .	1·356	1·833	1·537	1·741	1·954	1·828	1·730	1·859	2·051	1·648	1·933	2·040
Westport, . . .	1·323	1·877	1·655	1·831	2·042	1·900	1·802	1·909	2·092	1·705	2·013	2·074
Dublin, ...	1·569	1·989	1·718	1·887	2·088	1·986	1·860	2·046	2·206	1·815	2·040	2·221
Portarlington,	1·286	1·687	1·419	1·595	1·834	1·646	1·592	1·696	1·897	1·520	1·764	1·922
Athy, ...	1·259	1·658	1·416	1·566	1·763	1·665	1·556	1·668	1·889	1·533	1·753	1·914
Courtown, . .	1·570	1·974	1·710	1·861	2·066	1·983	1·852	2·008	2·192	1·836	2·020	2·205
Kilrush, ...	1·428	1·852	1·646	1·783	1·998	1·896	1·798	1·925	2·117	1·781	2·047	2·120
Dunmore, . . .	1·511	1·910	1·666	1·804	2·010	1·930	1·804	1·942	2·130	1·786	1·980	2·144
Cahirciveen, .	1·484	1·884	1·678	1·787	2·026	1·926	1·826	1·949	2·121	1·797	2·078	2·158
Castletownsend,	1·528	1·948	1·741	1·838	2·072	1·986	1·854	1·980	2·152	1·831	2·084	2·192

The next Table contains the constant corrections to be applied to the preceding results. The first column gives the diameters of the tubes; the second, the instrumental corrections obtained by comparison of the several instruments with the Dublin standard by means of portable barometers; the third, the reductions to the sea-level, calculated at the rate of ·0011 of an inch for each

* The pressure for January, at Westport, is the mean of 17 days only, viz., Jan. 15–31; and that for August, at Dublin, is the mean of 15 days, viz., Aug. 1–14.

foot of altitude; and the fourth, the sums of the two preceding, or the total corrections.

These results are incomplete, no comparisons having been made of the barometers at the four inland stations. For this reason, and also because of the uncertainty attending the comparison of barometers by means of portable instruments, I have thought it necessary to seek the corrections also by comparison of the observed results themselves. In comparisons of the latter kind, where the stations are widely separated, it seems necessary to employ the means of somewhat extended series of observed results, during which the fluctuations of barometric pressure are small. I have accordingly selected for the purpose the monthly means of May, July, and September, in which months there was but little variation of barometric equilibrium. The defects of the means at each station compared with those at Dublin, for these months, are given in the fifth, sixth, and seventh columns of the Table; and the last column contains the inferred corrections, which are equal to the mean differences + ·021 (the reduction of the Dublin results to the sea-level).

Table XV. Barometric Corrections.

Station.	Diameter of Tube.	Instrumental Correction.	Reduction to Sea-level.	Total Correction.	Difference from Dublin.			Correction by Comparison.
					May.	July.	Sept.	
Portrush, . . .	·28	+ ·032	+ ·032	+ ·064	·058	·070	·056	+ ·082
Buncrana, . . .	·34	+ ·017	+ ·053	+ ·070	·120	·100	·088	+ ·124
Donaghadee, .	·30	+ ·049	+ ·018	+ ·067	·052	·062	·054	+ ·077
Killybegs, . . .	—	− ·052	+ ·022	− ·030	·056	·032	·080	+ ·077
Armagh, . . .	·57	—	+ ·232	—	·252	·257	·248	+ ·273
Killough, . . .	·28	− ·030	+ ·025	− ·005	·058	·054	·068	+ ·081
Markree, . . .	—	—	+ ·145	—	·134	·130	·155	+ ·161
Westport, . . .	—	− ·007	+ ·019	+ ·012	·046	·058	·114	+ ·094
Dublin, ...	·55	+ ·003	+ ·021	+ ·021	·000	·000	·000	+ ·021
Portarlington,	—	—	+ ·253	—	·254	·268	·309	+ ·298
Athy, ...	—	—	+ ·220	—	·325	·304	·317	+ ·336
Courtown, . .	·28	+ ·028	+ ·037	+ ·040	·022	·008	·014	+ ·036
Kilrush, ...	·32	+ ·095	+ ·021	+ ·116	·090	·062	·089	+ ·101
Dunmore, . . .	·32	+ ·019	+ ·073	+ ·092	·078	·056	·076	+ ·091
Cahirciveen, .	·38	+ ·034	+ ·057	+ ·091	·062	·034	·085	+ ·081
Castletownsend,	·26	+ ·012	+ ·020	+ ·072	·016	·006	·054	+ ·046

It will be seen by comparing the resulting corrections, in the fourth and last columns of the preceding Table, that the agreement is satisfactory, except at the four stations, Buncrana, Killybegs, Killough, and Westport; the corrections inferred from a comparison of the *results* being, at these stations, considerably greater than those deduced by a comparison of the *instruments*. This may possibly be due, in part, to the entrance of air into the barometer tubes; but there seems reason to apprehend that the discrepancy may be also partly due to errors of observation, and that of a systematic kind,—the observers at the foregoing stations, having shown less aptitude than others for this kind of duty. Under the circumstances, the corrections deduced by the latter method seem the more reliable. They have accordingly been applied to all the selected observations hereafter discussed.

Applying the foregoing corrections to the results of Table XIV., we obtain the numbers of the following Table:—

Table XVI. Monthly Means of Barometric Pressure, corrected and reduced to Mean Sea-level.

Station.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Portrush, . . .	1·526	1·983	1·710	1·922	2·112	1·970	1·872	2·021	2·232	1·785	2·058	2·205	1·950
Buncrana, . . .	1·501	1·951	1·704	1·901	2·092	1·973	1·884	2·010	2·242	1·805	2·096	2·226	1·949
Donaghadee, .	1·573	2·007	1·729	1·929	2·113	1·992	1·875	2·025	2·229	1·805	2·027	2·225	1·961
Killybegs, . . .	1·492	1·945	1·700	1·900	2·110	1·995	1·905	2·015	2·203	1·795	2·085	2·190	1·944
Armagh, . . .	1·557	1·995	1·721	1·913	2·110	1·989	1·876	2·030	2·231	1·813	2·069	2·215	1·960
Killough, . . .	1·595	2·013	1·747	1·933	2·111	2·003	1·887	2·044	2·219	1·832	2·024	2·164	1·964
Markree, . . .	1·517	1·994	1·698	1·902	2·115	1·989	1·891	2·020	2·212	1·809	2·094	2·201	1·954
Westport, . . .	1·417	1·971	1·749	1·925	2·136	1·994	1·896	2·003	2·186	1·800	2·107	2·168	1·946
Dublin, ...	1·590	2·010	1·739	1·908	2·109	2·007	1·881	2·067	2·227	1·836	2·061	2·242	1·973
Portarlington,	1.584	1·985	1·717	1·893	2·132	1·944	1·890	1·994	2·195	1·818	2·062	2·220	1·953
Athy, ...	1·595	1·994	1·752	1·902	2·100	2·001	1·892	2·004	2·225	1·870	2·090	2·250	1·973
Courtown, . .	1·606	2·010	1·746	1·897	2·102	2·019	1·888	2·044	2·228	1·872	2·056	2·241	1·976
Kilrush, ...	1·529	1·953	1·747	1·884	2·100	1·997	1·900	2·026	2·218	1·882	2·148	2·221	1·967
Dunmore, . . .	1·602	2·001	1·757	1·895	2·101	2·021	1·895	2 033	2·221	1·877	2·071	2·235	1·976
Cahirciveen, .	1·565	1·965	1·759	1·868	2·107	2·007	1·907	2·030	2·202	1·878	2·159	2·239	1·974
Castletownsend,	1·574	1·994	1·787	1·884	2·118	2·032	1·900	2·026	2·198	1·877	2·130	2·238	1·980

In order to perceive more clearly the simultaneous variations in the distribution of pressure, I have, in the following Table, combined the stations, and their results as given above, into four groups, as hereafter described in treating of the observations of wind-force. These are the following:—

Table XVII. Monthly Means of Barometric Pressure for the Four Groups of Stations.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
North-east, . .	1·575	2·005	1·732	1·925	2·111	1·995	1·879	2·035	2·226	1·817	2·040	2·201	1·962
North-west, . .	1·475	1·970	1·715	1·908	2·120	1·992	1·897	2·012	2·200	1·801	2·095	2·186	1·948
South-east, . .	1·600	2·007	1·748	1·900	2·104	2·016	1·888	2·048	2·226	1·862	2·063	2·240	1·975
South-west, . .	1 556	1·971	1·764	1·879	2·108	2·012	1·902	2·028	2·206	1·879	2·146	2·233	1·974

The phenomena of the distribution of pressure are very clearly shown in the foregoing Table. It will be seen from it that, on the average of the entire year, there is an *excess of pressure in the south* of the island, and a defect in the north, the *minimum* being at the *north-western* extremity. This excess of pressure in the south is likewise shown in the means for the seasons of summer, autumn, and winter, respectively; and the cause of it will, I think, hereafter appear upon the discussion of the phenomena of storms. In the separate months, the points of greatest and least pressure vary somewhat irregularly; but they are, in almost every month, at opposite extremities of the island. Thus, in January, the *maximum* pressure is in the *south-east*, and the *minimum* in the *north-west;* and so for the others. This circumstance is what should have been expected *à priori;* and it affords satisfactory evidence of the general accuracy of the results themselves.

Direction and Force of the Wind.

Direction of Wind.—The direction of the wind was observed, at most of the stations, by means of the ordinary wind-vane. Much care was, however, taken, not only in placing these instruments truly in azimuth, but also in selecting positions for them which seemed least exposed to eddies or other local irregularities. At Armagh and Dublin the direction of the wind was recorded continuously, by means of self-registering anemometers.

The following Tables give the number of times, out of 100, in which the wind blew from each of the *eight* points at the several stations; Tables xviii. and xix. containing the results for the summer and winter half-years, respectively, and Table XX. those for the entire year. The winds from the *intermediate* points, when observed, were divided equally between the two adjacent *principal* points:—

TABLE XVIII. FREQUENCY OF THE SEVERAL WINDS (SUMMER).

Station.	N.	N. E.	E.	S. E.	S.	S. W.	W.	N. W.
Portrush, . . .	20	5	6	6	24	15	11	13
Buncrana, . . .	13	8	6	12	11	19	12	19
Donaghadee, . .	23	6	5	13	11	14	16	11
Killybegs, . . .	15	9	14	7	8	13	21	14
Armagh, ...	12	9	7	6	16	19	18	15
Killough, . . .	11	7	14	12	16	18	8	14
Markree, ...	14	5	4	17	14	15	10	21
Westport, . . .	10	10	14	10	3	3	32	19
Dublin, ...	2	10	12	13	8	23	11	19
Portarlington, .	5	28	2	11	7	14	13	21
Athy, ...	13	1	2	12	16	12	25	19
Courtown, . . .	13	17	5	7	8	23	15	13
Kilrush, ...	12	10	14	8	8	13	19	17
Dunmore, . . .	15	8	14	5	7	18	8	16
Cahirciveen. . .	11	9	12	9	12	18	16	14
Castletownsend,	8	9	11	12	2	37	15	6

TABLE XIX. FREQUENCY OF THE SEVERAL WINDS (WINTER).

Station.	N.	N. E.	E.	S. E.	S.	S. W.	W.	N. W.
Portrush, . . .	15	3	4	7	35	23	9	4
Buncrana, . . .	10	5	5	10	15	27	15	13
Donaghadee, . .	9	7	4	7	14	25	26	9
Killybegs, . . .	11	6	8	9	13	19	18	15
Armagh, ...	6	5	2	5	26	35	12	10
Killough, . . .	9	4	4	5	18	25	12	24
Markree, ...	12	4	4	19	19	23	9	10
Westport, . . .	13	4	12	10	6	8	26	21
Dublin, ...	2	1	2	14	14	38	14	13
Portarlington, .	5	11	1	4	10	19	20	30
Athy, ...	7	2	1	15	28	11	23	13
Courtown, . . .	7	5	4	4	18	23	24	16
Kilrush, ...	10	9	10	6	16	23	13	14
Dunmore, . . .	16	4	3	5	14	20	22	16
Cahirciveen, . .	8	8	13	12	12	20	17	10
Castletownsend,	13	5	3	6	5	35	21	13

Table XX. Frequency of the several Winds (Year).

Station.	N.	N. E	E.	S. E.	S.	S. W.	W.	N. W.
Portrush, . . .	17	4	5	6	30	19	10	8
Buncrana, . . .	12	7	6	11	13	23	14	16
Donaghadee, . .	16	7	4	10	13	19	21	10
Killybegs, . . .	13	8	11	8	10	16	20	15
Armagh, ...	9	7	4	5	21	27	15	12
Killough, . . .	10	5	9	8	17	21	10	19
Markree, ...	13	4	4	18	17	19	10	16
Westport, . . .	11	7	13	10	4	5	29	20
Dublin, ...	2	6	7	14	11	31	13	16
Portarlington, .	5	19	1	7	9	16	16	25
Athy, ...	10	2	1	13	22	12	24	16
Courtown, . . .	10	11	4	6	13	23	19	14
Kilrush, ...	11	9	12	7	12	18	16	16
Dunmore, . . .	15	6	9	5	11	19	20	16
Cahirciveen, . .	9	8	13	10	12	19	17	12
Castletownsend,	10	7	7	9	3	36	18	9

The following are the *mean* results for the *whole island*:—

	N.	N. E.	E.	S. E.	S.	S. W.	W.	N. W.
Summer,. . .	12·3,	9·4,	8·9,	10·0,	10·7,	17·1,	16·3,	15·7.
Winter, . . .	9·6,	5·2,	5·0,	8·6,	16·4,	23·4,	17·6,	14·4.
Year, ...	10·8,	7·3,	6·9,	9·2,	13·6,	20·2,	17·0,	15·0.

We learn from them that, in the year 1851, the wind blew, on the average of the entire year, most frequently from between S.W. and W., and least frequently from between N. E. and E. The same thing holds also for the summer half-year, the point of maximum frequency being, very nearly, W. S.W., and that of minimum frequency E. N. E. In the winter half-year the point of maximum frequency is more nearly S.W., that of the minimum being as before. The ratio of the numbers representing the greatest and least frequency is greater in winter than in summer.

It is not necessary to enter more minutely into the discussion of the numbers of the preceding Tables, as it is probable that the results of a single year, as to the frequency of the several winds, will deviate considerably from the means of several. I may observe, however, that they afford some indications of a law of distribution, depending upon the aspect of the coast. Thus, on comparing the numbers denoting the frequency of any particular wind at the several stations, with their mean for the whole island, it would seem that easterly winds are slightly in excess on the western coast, and westerly winds on the eastern. In other words, there appears to be a preponderating tendency of the wind to blow *from the land*, at each place, as compared with the mean of all. It will remain for future inquiry to ascertain whether this holds good in other years, and is, therefore, to be referred to a general law. If so, it is probably the effect of the land and sea breezes, the former preponderating in the average of the winds at 9 A.M. and 9 P.M.

Pressure of the Wind.—For the measurement of the pressure of the wind, a Lind's anemometer was furnished to each station. The difficulty of obtaining accurate results with this little instrument arise, partly, from the smallness of its indications, and, partly, from the oscillations of the fluid in the tube; the latter are so considerable as to render the instrument of little value, except in the hands of a patient and somewhat practised oberver. After some trial, accordingly, it was deemed advisable that the force of the wind should be in all cases *estimated*, and that the use of Lind's anemometer should be limited to that of furnishing a check upon this estimation in the case of the stronger winds.

The first thing to be determined, then, was the choice of a scale of force. The scales in use are various: in one of them there are *four* degrees of wind-force; in another, *six;* and in a third (the Admiralty scale) there are *twelve*. The last of these appears to be too minute for the ordinary powers of unaided estimation, and the first not sufficiently so. The intermediate scale (from 0 to 6), was accordingly adopted; and it appears to be further recommended by the circumstances,—1, that it is the subdivision most generally used on the Continent; and 2, that, as its numbers represent the same degrees of wind-force with the alternate numbers of the Admiralty scale, the latter are convertible into the former by simply dividing by two. The six degrees of wind-force were designated as follow:—1. *Light breeze;* 2. *Moderate breeze;* 3. *Strong breeze;* 4. *Moderate gale;* 5. *Strong gale;* 6. *Storm.*

In order to know the amount of confidence which may be placed in such observations, it is necessary to determine how far, in respect of accuracy, six degrees of wind-force can be estimated, the observations being supposed to be made by practised observers. And to be able to apply the observations, we must further know, what are the pressures and velocities of the wind corresponding to the several terms of the scale. For these purposes I made a somewhat extended series of observations, estimating the force of the wind according to the prescribed scale, and, at the same time, measuring its velocity by means of Robinson's anemometer. The following Table gives the mean results of these observations. The numbers in the first column are the terms of the scale; those in the second are the corresponding times of 100 revolutions of the instrument, expressed in seconds;* the third column contains the corresponding velocities of the wind, in feet per second; and the fourth the calculated velocities, deduced as hereafter described.

Table XXI. Velocities of the Wind corresponding to the Terms of the Scale (0 – 6).

n	T	V (observed).	V (calculated).
I.	71	12	12
II.	35	25	23
III.	25	35	35
IV.	20	43	46
V.	16·8	51	58
VI.	11·6	75	70

* Dr. Robinson has shown (Transactions, vol. xxii. p. 167), that the velocity of the wind is to that of the centres of the hemispherical cups, as 3 to 1. But r being the length of the horizontal arms of the instrument, measured to these centres, the circumference of the circle described by them is $2\pi r$; and if r be expressed in feet, and n be the number of revolutions performed in a second, their velocity is $2\pi r \times n$. The corresponding velocity of the wind therefore is $V = 6\pi r \times n$. In the instrument in my possession, the radius is 5·5 inches. Hence $2r = \frac{11}{12}$, and substituting for π its numerical value, $V = 8{\cdot}64 \times n$.

Instead of noting the number of revolutions, and parts of a revolution, performed in a given time, I have found it convenient to observe the *time* of performing 100 revolutions. I have had

We see that the terms of the estimated scale correspond, nearly, to an arithmetical progression of *velocities*, and not of *pressures*. This fact has been already noticed by Dr. ROBINSON.

The common difference in this series, which is equal to its first term, is obtained from the numbers of the third column by means of the formula $V = nV_1$. The following are the deduced values:—

I.	$V_1 = 12{\cdot}0$	IV.	$V_1 = 10{\cdot}8$
II.	$12{\cdot}5$	V.	$10{\cdot}2$
III.	$11{\cdot}7$	VI.	$12{\cdot}5$

The mean of these values is $V_1 = 11{\cdot}6$. The calculated values of V, contained in the last column of the foregoing Table, are, accordingly, obtained from the formula

$$V = 11{\cdot}6 \times n;$$

their agreement with the *observed* values is sufficient to establish the assumed law.

As a verification of the preceding result, I took also a tolerably extended series of measurements of the *pressures* of the wind, corresponding to the highest term of the scale, with Lind's anemometer. Their mean gave 2·06 inches for the reading of the instrument corresponding to that term; and the corresponding pressure on one square foot of surface, computed in the proportion of 5·20 pounds to the inch, is 10·7 pounds. Hence, the pressure belonging to the unit of the scale is $P_1 = 0{\cdot}30$. The corresponding velocity is inferred from the formula $V^2 = 437\,P$. Its value is $V_1 = 11{\cdot}5$; a result which agrees very closely with that already deduced from ROBINSON's anemometer.

The results hitherto given rest only on my own estimations; it remains

the instrument accordingly provided with a little hammer, which is pressed against the registering wheel by a spring, and which, being raised by a projecting pin at one point of its circumference, falls again with a sharp noise when this has passed. The interval between two such strokes of the hammer, therefore, is the time of one whole revolution of the registering wheel, or of 100 revolutions of the arms. Accordingly, a chronometer being held close to the ear, the whole observation is effected by the help of that organ. The velocity of the wind in this case is given by the formula $V = \frac{864}{T}$, T being the observed time of 100 revolutions.

to see how far they accord with those of other observers. I have selected for this purpose the results of the observations with Lind's anemometer, made at Portrush and Donaghadee by two of the best of the coast-guard observers, and have placed my own beside them, for comparison. The results, converted into pressures (expressed in pounds on the square foot), are contained in the following Table. The numbers in the last column are the *calculated* pressures, deduced from the formula

$$P = P_1 n^2,$$

n being the number of the term of the scale, and P_1 (= 0·30) the pressure corresponding to the first term.

Table XXII. Observations of the Pressures of the Wind corresponding to the Terms of the Scale (0 – 6).

Term.	Dublin.	Portrush.	Donaghadee.	Calculated.
I.	0·5	0·4	0·5	0·3
II.	1·3	1·3	1·1	1·2
III.	3·0	2·9	2·9	2·7
IV.	4·2	5·3	5·3	4·8
V.	7·0	7·3	7·9	7·5
VI.	10·7	—	—	10·8

It will be seen, that the differences of the corresponding numbers at the three stations are small, and that their means agree very well with the calculated pressures. It seems therefore to be fully proved, that the velocity of the wind may be estimated to six degrees, by practised observers, with sufficient accuracy.

The following Table contains the monthly means of the observed wind-force, in the terms of the prescribed scale. At Buncrana and Westport the force appears to have been over-estimated; at the other stations it seems to have been correctly observed:—

TABLE XXIII. MONTHLY MEANS OF THE FORCE OF THE WIND.

Station.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Portrush, . . .	2·0	1·7	2·0	1·7	1·7	2·0	1·7	1·7	1·3	1·9	1·8	1·7
Buncrana, . . .	2·5	2·0	2·2	2·3	2·2	2·4	2·0	2·1	1·9	2·3	1·8	2·1
Donaghadee, . .	2·2	1·7	1·8	1·7	1·6	1·5	1·6	1·3	1·3	1·8	2·0	1·4
Killybegs, . . .	1·9	1·2	1·8	1·1	1·5	1·7	1·3	1·4	1·4	2·2	1·5	1·9
Armagh, ...	2·6	1·9	1·7	1·5	1·3	1·4	1·1	1·1	1·0	1·6	1·1	1·8
Killough, ...	2·2	1·4	1·6	1·4	1·2	1·9	1·7	1·9	1·5	2·2	1·8	2·3
Markree, ...	2·0	1·6	2·1	1·5	1·7	1·9	1·9	1·5	1·6	1·9	1·6	1·7
Westport, . . .	3·9	2·4	2·7	2·1	2·2	2·5	2·4	2·2	1·8	3·0	2·2	2·5
Dublin, ...	1·9	1·3	1·3	1·3	1·4	1·7	1·6	1·0	1·0	1·4	1·1	0·9
Portarlington, .	1·3	1·3	1·2	1·3	1·2	2·0	1·4	1·4	1·2	1·7	1·1	1·0
Athy, ...	1·3	1·1	1·5	1·4	1·7	1·5	1·7	1·5	0·7	1·3	1·0	1·0
Courtown, . . .	1·9	1·7	1·5	1·5	1·4	1·4	1·2	1·5	1·4	1·5	1·8	1·8
Kilrush, ...	2·7	1·9	2·0	1·6	1·6	2·0	1·9	1·7	1·5	2·2	1·6	1·7
Dunmore, . . .	2·5	2·0	1·9	1·9	1·8	2·0	1·9	1·9	1·8	2·0	1·7	2·1
Cahirciveen, . .	2·8	1·9	2·1	1·8	1·8	2·1	1·8	1·9	1·4	2·2	1·8	2·2
Castletownsend,	2·9	2·2	2·4	2·3	1·7	2·1	1·7	2·1	1·7	2·5	1·7	2·6
Means, . .	2·29	1·71	1·86	1·65	1·63	1·88	1·68	1·64	1·41	1·98	1·60	1·79

The lowest line of the preceding Table contains the mean results of all the stations, or the average forces of the wind in the several months for the whole island. The mean force for the entire year is 1·76, corresponding to a velocity of 20·4 feet per second. The force is, of course, greater in winter than in summer, the mean force for the winter half-year being 1·87, and that for the summer half-year 1·65. The law of the annual progression is, however, greatly masked by irregularities; thus, the minimum force for the year 1851 occurs in the month of September, while the force in June is above the average of the entire year.

In the following Table are given the results of the preceding for the entire year, and for its two principal divisions. The excess of the force in winter appears at all the stations, excepting Dublin, Portarlington, and Athy.* At these three stations, also, the force of the wind is much *below* the average.

* Buncrana is likewise an exceptional case; but the exception is there probably due to inaccuracy of observation.

Table XXIV. Mean Force of the Wind for the Summer and Winter Half-Years, and for the whole Year.

Station.	Summer.	Winter.	Year.
Portrush, ...	1·68	1·85	1·77
Buncrana, ...	2·15	2·15	2·15
Donaghadee, . . .	1·50	1·82	1·66
Killybegs, ...	1·40	1·75	1·58
Armagh, ...	1·23	1·78	1·51
Killough, ...	1·60	1·92	1·76
Markree, ...	1·68	1·82	1·75
Westport, ...	2·20	2·78	2·49
Dublin, ...	1·33	1·32	1·33
Portarlington, . .	1·42	1·27	1·34
Athy, ...	1·42	1·20	1·31
Courtown, ...	1·40	1·70	1·55
Kilrush, ...	1·72	2·02	1·87
Dunmore, ...	1·88	2·03	1·96
Cahirciveen, . . .	1·80	2·17	1·98
Castletownsend, .	1·93	2·38	2·16

If, to eliminate local irregularities, we combine the preceding results in groups, according to the arrangement hereafter described, we find the following values for the mean forces of the entire year:

North-east, . . .	1·64	North-west, . . .	1·94
South-east, . . .	1·61	South-west, . . .	2·00

From this it appears that the mean force of the wind is considerably greater in the *west* of the island than in the *east*, the ratio being somewhat greater than that of 1·2 to 1. There is but little difference between the forces in the northern and southern portions of the island.

Cyclonic Movements.

In analyzing the phenomena of rotation, the first step was to note those cases in which the mean directions of the wind, in any two districts, differed by 90°, or upwards. It was soon perceived, that no conclusion could be drawn as to a general movement of the atmosphere, when the wind was very moderate, the direction being then greatly influenced by local causes. Accordingly, excluding those cases in which the wind did not exceed a light breeze at most of the stations, the remainder were examined in detail, by laying down the simultaneous directions of the wind upon a series of skeleton charts prepared for the purpose; and there was no difficulty in ascertaining, by the inspection of these charts, the existence or non-existence of rotatory movement. The same means sufficed to determine, very nearly, the position of the centre of the vortex at each epoch; and the places of the centre being thus found, for epochs distant by intervals of twelve hours, the direction and velocity of its progressive movement are ascertained.

The position of the centre of the vortex at any instant may be determined, more accurately, by calculation. Thus, if y and x denote the distances (in geographical miles) of the place of observation from any assumed central point, measured on the meridian, and on the perpendicular to the meridian, respectively; y_0 and x_0 the corresponding co-ordinates of the centre of the vortex; and θ the angle which the direction of the wind at the point (y, x) makes with the meridian, measuring from north to east;

$$y - y_0 + (x - x_0) \tan \theta = 0,$$

the direction of the wind being perpendicular to the line connecting the points (y, x) and (y_0, x_0). Now, all the quantities in this equation are given, excepting y_0 and x_0; so that, if the direction of the wind be accurately known at two stations, the co-ordinates of the centre of the vortex may be completely determined. The irregularities due to local causes, and the errors of observation themselves, forbid this; and, in order to lessen their influence, it is necessary to know the direction of the wind at several stations. There will then be as many equations of the preceding form, as there are places of observation; and the unknown quantities, y_0 and x_0, are to be determined by combining these equations by the method of least squares.

It is found, that the centre of the vortex is also the point of *least barometric pressure*, and that the pressure increases regularly with the distance from it. Hence the position of the centre may be inferred from the barometric observations alone. The positions thus determined have been found to coincide in all cases, very nearly, with those deduced from the observed directions of the wind.

The following are the well-marked instances of aerial rotation which have occurred in Ireland in the course of these observations. No case has been included in the enumeration, in which the simultaneous directions of the wind did not differ, at two points, by at least 90°; and thus, probably, many cases of cyclonic movement are passed over, in which the centre of the vortex was remote. The observations themselves are given in detail in Table XXXIII., at the end of this Paper.* The following are their principal results :—

1850. Oct. 6, 7.—Cyclone and storm, moving from S.W. to N.E., with a velocity of about 290 geographical miles *per diem*. (Plate VIII. Figs. 1, 2, 3.)

Oct. 6, 9 A. M.—Centre of the vortex on the south-western coast of Ireland, west of Kilrush. Least pressure at Cahirciveen. Mean velocity of the wind = 25 feet per second; greatest do. (on the west coast) = 45 feet. The atmosphere at the northern stations unaffected by the vortex at this epoch.

Oct. 6, 9 P. M.—Centre of the vortex over the north of Ireland, a few miles north of Killybegs. Absolute barometric minimum (= 28·836) at Killybegs; increase of pressure in 100 miles = 0·30 inch. Mean velocity of wind = 35 feet per second; greatest do. (Markree) = 70. Southern stations unaffected by the vortex.

Oct. 7, 9 A. M.—Centre on south-western coast of Scotland. Least pressure at Donaghadee. Mean velocity of wind = 45 feet per second; greatest do. (north coast) = 60 feet. Hail fell at Markree; wind amounting to a gale in the north, in the evening of the same day.

The diameter of the vortex may be estimated with tolerable precision in this case, by measuring from the centre to the limits of the region affected by the movement; it was about 280 geographical miles.

Oct. 22, 23.—An interesting and instructive case of conflicting currents generating a rotatory movement. The velocity of the wind was uniform throughout the island, and was from 30 to 35 feet per second. (Plate VIII. Figs. 4, 5, 6.)

Oct. 22, 9 P. M.—Wind from N. W. in the north of Ireland, and from S. W. in the south-east. The central point of junction of these currents was over the channel, to the north-east of Dublin. Least pressure at Donaghadee.

* It seems certain that a careful study of the *simultaneous* atmospheric phenomena, even in a limited district, will throw more light upon the "law of storms," than any other mode of inquiry; and for this reason, as well as for the authentication of my own inferences, I have thought it right to give these observations *in extenso*. It is probable that an attentive examination of them may elicit many conclusions which have escaped my notice.

Oct. 23, 9 A. M.—A distinct rotatory movement, whose centre was a little to the north-east of the point of junction above referred to, not far from Donaghadee. Least pressure at Donaghadee, as before.

Oct. 23, 9 P. M.—Rotatory movement continued. Centre of vortex had moved from S. W. to N. E., at the rate of about 100 miles *per diem*. Absolute minimum of pressure (= 29·360) at Donaghadee; increase of pressure in 100 miles = 0·10 inch.

Nov. 18, 19.—A cyclone, with violent storm, crossing the island from W. S. W. to E. N. E. (Plate IX. Figs. 1, 2, 3.) The movement of the centre of the vortex appears to have been curvilinear, and to have varied considerably in velocity. Between 9 P. M. of the 18th, and 9 A. M. of the following day, its path was from S. W. to N. E., and its velocity about 320 miles *per diem;* in the succeeding twelve hours its course was nearly from W. to E., with a greatly diminished velocity. The mean velocity of the wind, throughout the storm, was from 45 to 50 feet per second.

Nov. 18, 9 P. M.—Centre of the vortex on the south-western coast, about 30 miles to the north of Cahirciveen. Least pressure at Kilrush. Maximum velocity of wind (in south of island) = 60 feet per second.

Nov. 19, 9 A. M.—At this epoch the wind was blowing from N. at Killybegs, and from S. at Donaghadee; it was blowing from S. E. at Portrush, and from N. W. at Castletownsend; from S. S. E. at Armagh, and from N. N. W. at Markree. The centre of the vortex was therefore over Ireland at that time, and between the stations above mentioned, its exact position being about 15 miles to the west of Armagh. Absolute minimum of pressure (= 28·248) at Armagh; increase of pressure = 0·31 inch. Maximum velocity of wind (in south) = 65 feet per second.

Nov. 19, 9 P. M.—Centre over the Channel, to the south-east of Donaghadee. Absolute minimum of pressure (= 28·410) at Donaghadee; increase of pressure = 0·28 inch. Maximum velocity of wind (in south) = 55 feet per second.

We have seen that the centre of the vortex was between Armagh and Markree at 9 A. M. of the 19th; and, as the direction of its progressive movement was not far from the line connecting these places, it must have passed nearly centrally over both. Hence we should expect there the peculiar phenomena—the *lull* of the wind, and the *sudden reversal* of its direction—which are observed to occur at places in the path of the centre of a cyclone. I shall therefore briefly describe the series of changes at these two stations. The observations at Armagh are taken from the records of the self-registering anemometer, which were, of course, *continuous;* those at Markree were made at short intervals.

At Armagh the wind began to blow at 7 P. M. of the 18th, with a velocity of 32 feet per second. The maximum velocity, with the exception of a short squall* at 5 A. M., occurred at 7 A. M. of the 19th, and amounted to 43 feet per second. From this time the wind abated rapidly *almost to a calm*, its velocity at noon amounting only to 6 feet per second; but at 3 P.M. it rose again, with a velocity of 22 feet. The initial direction of the gale was from the E. S. E. From 9 P. M. on the 18th, to 1 A.M. on the 19th, it veered to S., at which point it continued for several hours, including the period of greatest force of the gale. At 11 A. M. its direction had returned to S. E., and it then *suddenly shifted* to W. N. W., altering through 160° in 24 minutes. The minimum of pressure took place at $11^{h.}\ 30^{m.}$, at the close of this movement; its amount was 27·930 inches.†

* During the squall, which lasted only three minutes, the velocity reached 90 feet per second.

† The following are the anemometric observations above referred to. The direction is measured from S. through W. to N.; the velocity is expressed in miles per hour. On the 19th, from 4 A. M. to 8 A. M., the direction-registering pencil was thrown out of gear, but there appears to have been no change of any magnitude in the interval:

	Nov. 18 A. M.		Nov. 18 P. M.		Nov. 19 A. M.		Nov. 19 P. M.	
Hour.	Vel.	Dir.	Vel.	Dir.	Vel.	Dir.	Vel.	Dir.
0	12·2	28°·8	4·8	212°·3	23·4	335°·2	4·1	124°·0
1	8·3	28 ·2	5·3	243 ·6	24·4	354 ·3	10·5	104 ·4
2	5·9	47 ·8	8·7	276 ·2	22·2	353 ·4	12·0	93 ·9
3	7·2	49 ·8	7·8	274 ·5	23·6	345 ·6	15·4	90 ·2
4	8·1	48 ·1	14·7	278 ·8	23·2	—	16·9	108 ·8
5	3·8	41 ·3	16·1	281 ·7	25·3	—	13·8	136 ·7
6	2·9	75 ·0	16·6	281 ·9	27·4	—	15·1	138 ·3
7	3·7	162 ·3	22·0	283 ·5	29·4	—	16·2	151 ·1
8	1·5	162 ·0	19·2	285 ·5	29·0	—	16·6	153 ·6
9	2·3	212 ·4	19·3	282 ·1	17·6	339 ·8	15·8	151 ·2
10	4·9	205 ·4	17·6	321 ·3	14·6	320 ·6	9·2	147 ·5
11	5·9	209 ·2	20·1	330 ·6	7·6	324 ·5	11·8	159 ·5

At Markree the gale commenced at $4^{h.}\ 30^{m.}$ P. M. of the 18th, with a rapidly falling barometer. At 7 P. M. the wind abated to a breeze, the barometer still falling. It recommenced at 10 P. M. from the S. E.; and at 3 A. M. on the 19th it appears to have attained its maximum. At 6 A. M. the wind again abated; and at 7 A. M. *there was a calm.* The minimum pressure took place at this time, and amounted to 28·170 inches. At 9 A. M. the wind rose again from the N. N. W., but not with such force as before; and in the afternoon there was a strong gale again.*

From these facts it is evident, that the centre of the vortex passed nearly over Markree at 7 A. M., and over Armagh at $11^{h.}\ 30^{m.}$ A. M. At Donaghadee, which is nearly in the prolongation of the line connecting the two former places, the wind ceased at 1 P. M., and recommenced at 5 P. M.; so that the vortex passed nearly centrally over this station at about 3 P. M. From these data we learn that the cyclone moved from W. S. W. to E. N. E.; and that the velocity of the progressive movement was then about 12 miles per hour.

* The following are the extra observations at Markree above referred to. Reduction of barometer to sea-level = + 0·161 inch:—

Date.	Hour.	Bar.	Therm.	Remarks.
Nov. 18,	$4^{h.}\ 30^{m.}$	1·124	46°·8	Blowing a gale; storm and rain began about noon.
	6	1·002	48 ·8	Ditto; rain.
	7	0·926	49 ·5	Strong breeze; heavy rain.
	8	0·835	50 ·5	Ditto; ditto.
	10	0·652	53 ·7	Gale.
	11	0·577	54 ·0	Wind rising to gale; mizzling rain.
	12	0·538	52 ·5	Strong breeze; ditto.
Nov. 19,	3 A. M.	0·315	50 ·5	Wind higher than at any previous time.
	5	0·120	46 ·5	Gale; mizzling rain.
	6	0·027	45 ·5	Strong breeze.
	7	0·009	47 ·3	Calm.
	8	0·023	48 ·2	Light breeze; mizzling rain.
	9	0·067	45 ·8	Gale.
	10	0·124	46 ·0	Strong breeze; wind N. N. W.
	3 P. M.	0·303	48 ·8	Strong breeze.
	7	0·483	49 ·5	Strong gale.
	10	0·533	48 ·0	Gale from N. W.; light showers.
	11 30	0·578	49 ·0	Strong gale.
Nov. 20,	10 A. M.	1 084	48 ·4	Ditto; showers.
	11	1·128	48 ·3	Ditto.
	1 30	1·227	47 ·5	Ditto.
	3	1·282	46 ·5	Ditto; heavy rain.
	5	1·376	44 ·8	Moderate gale.

The dimensions of the vortex may likewise be collected from the same data. The interval between the commencement of the storm, and the passage of the centre, at Armagh, was $16\frac{1}{2}$ hours; and, the velocity being 12 miles an hour, the radius of the vortex was about 200 miles. The magnitude of the nearly quiescent portion of air in the centre of the vortex is better defined. At Armagh the lull lasted from three to four hours; at Markree three hours; and at Donaghadee four hours. The diameter of the quiescent central portion was, therefore, about 40 miles.

We may now refer to some particulars connected with this gale, which appear to merit attention—although probably, in the present state of knowledge on this subject, we should not be justified in offering any suggestions in explanation.

Among the first of these are the abnormal variations in the rotatory movement, especially along the track of the centre. The most curious of these irregularities is that of the direction. At Armagh this began to change rapidly at 9 P. M. of the 18th. At 9 P. M. it was E. S. E.; at 10 P. M., S. E.; at midnight, S. S. E.; and at 1 A. M. on the 19th, S. At this latter point it remained for several hours; and the direction then *retrograded* through an arc of about 45°. At 9 A. M. on the 19th, it was S. S. E.; and at 11 A. M. it came back to S. E., after which the *sudden* shift to W. N. W., already noticed, took place.

The next point which seems to merit notice is the fact, that the force of the gale was considerably greater to the south of the line of passage of its centre, than on that line itself, or to the north of it. Thus, at Killiney, where I made frequent observations during the gale, I found the maximum velocity to be 80 feet per second; at Armagh it was little more than half that amount.

It has been already mentioned that the greatest force of the storm occurred at Armagh and Markree, *before* the epoch of minimum pressure, the interval at both places being about four hours and a half. A similar interval took place at Killiney, but in the *opposite* direction, the epoch of greatest intensity *following* that of least pressure by four hours and a half.

The last point which appears to demand notice is the fact, that there was a considerable interval between the epochs of the greatest intensity of the storm at Dublin and at Killiney, places only ten miles apart. The greatest force of the gale, at Dublin, took place between 1 P. M. and 2 P. M.; at Killiney it occurred between 5 P. M. and 6 P. M. There is a similar interval between the times of minimum pressure at the two places, the least height of the barometer occurring later at Killiney than at Dublin by two or three hours. These differences are probably connected with the difference of altitude of the places of observation.

1851. Jan. 15, 16.—A remarkable case of a double cyclone with storm, and a double minimum of pressure. (Plate IX. Figs. 4, 5, 6.) The first of the two vortices crossed the island from S. to N. on the 15th, and the second traversed the north-western portion of it, from S. W. to N. E., on the following day. The velocity of the former is not well determined; that of the latter is about 270 miles *per diem.* The mean velocity of the wind was from 30 to 35 feet per second on the former day, and from 55 to 60 on the latter.*

Jan. 15, A. M.—Centre of vortex about 20 or 30 miles south of Dunmore.

* The following extra observations were taken at Markree, January 15, 16. Reduction of barometer to sea-level = 0·161 inch:—

Date.	Hour.	Bar.	Therm.	Wind.		Cloud.		Remarks.
Jan. 15,	$1^{h.}$ $30^{m.}$	0·786	42°·1	N. W.	4	10	N	Mizzling rain.
	2 30	0·832	41 ·5	N. W.	4	10	N	Mizzling rain.
	4	0·921	40 ·8	W. N. W.	4	10	N	Rain.
	5	0·986	41 ·5	W. N. W.	4	10	N	Rain.
	6	1·040	42 ·5	W. N. W.	4	10	N, C K	Clouds breaking all round.
	10	1·204	37 ·9	S. W.	2	0	0	Began to clear at $6^{h.}$ $30^{m.}$
	12 5	1·242	34 ·4	S. W.	1	3	C	Clouds rapidly rising from south-west.
	12 17	1·240	34 ·1	S. W.	1			
Jan. 16,	1 A. M.	1·225	34 ·3	S. W.	1	9	C	A large halo round moon.
	8 30	0·776	42 ·0	S. E.	5	10	N	Mizzling rain.
	9	0·751	43 ·7	S. E.	5	10	N	
	9 30	0·726	45 ·5	S. S. E.	5	10	N	
	10	0·718	47 ·2	S. S. E.	5	10	N	Lind's anemometer = 0·85 inch.
	11	0·668	49 ·7	S.	5	10	N	Wind not quite as strong as at 10 A. M.
	12	0·614	50 ·0	S.	4	10	N	Rain; rough, with heavy rain a little
	1 P. M.	0·566	51 ·5	S.	4	10	N	[before this time.
	2	0·534	50 ·5	S.	4	9	N, K S	Clouds began to break at 1 P. M.
	9	0·620	46 ·8	S. W.	5	9	N	
	9 30	0·661	46 ·7	S. W.	5	7	N, K	Gusts very high occasionally.
	10	,0·715	46 ·1	S· W.	5	9	N, C K	Lind's anemometer = 0·80 inch.
	11	0·794	45 ·2	S. W.	4	8	N, K S	
	11 30	0·824	44 ·8	S. W.	5	10	N	Rain.
	12	0·835	43 ·0	S. W.	4	2	·K S, C K	
Jan. 17,	1 A. M.	0·890	43 ·5	S. W.	4	1	K	
	2	0·920	42 ·7	S. W.		2	K S, C	

Absolute minimum of pressure (= 28·718) at Dunmore; increase of pressure = 0·15 inch. Maximum velocity of wind (west coast) = 60 feet per second.

Jan. 15, 9 P. M.—Centre of vortex appears to have been at this time a few miles north of Buncrana; the cyclonic movement was, however, not distinctly marked, probably owing to the influence of the second cyclone. Least pressure at Buncrana. Velocity of wind uniform throughout the island.

Jan. 16, 9 A. M.—Centre of second vortex to the south-west of Westport. Least pressure at Westport.

Jan. 16, 9 P. M.—Centre about 20 miles west of Buncrana. Absolute minimum of pressure (= 28·671) at Buncrana; increase of pressure = 0·20 inch.

Jan. 30, 31.—A very interesting cyclone traversing the western portion of the island, in direction from N. to S. nearly, at the rate of about 150 miles *per diem*. The wind light, the mean velocity being about 20 feet per second.

Jan. 30, 9 P. M.—Centre of vortex over north-western portion of island, a little to the north of Killybegs. Least pressure at Killybegs. Maximum velocity of wind (in south-west) = 40 feet per second.

Jan. 31, 9 A. M.—Centre a little to the eastward of Westport. Absolute minimum of pressure (= 29·032) at Westport; increase of pressure = 0·10 inch. Maximum velocity of wind (in south-west) = 25 feet per second. Lightning observed in north in evening of this day and day preceding.

March 18.—A cyclone, with storm, traversing the island from S. to N., at the rate of about 200 miles *per diem*.*

* The following extra observations were taken at Markree, March 18, 1851:—

Date.	Hour.	Bar.	Therm.	Wind.		Cloud.		Remarks.
March 18,	7 A. M.	1·150	41°·9	S. E.	4	10	N	100 rev. in 29ˢ·; pouring rain.
	8	1·128	43 ·5	S. S. W.	4	10	N	100 rev. in 42ˢ·; light rain.
	9	1·148	46 ·5	S. W.	3	9	N	
	10	1·170	46 ·0	S. W.	3	9	N, C S	100 rev. in 60ˢ·; a shower.
	11	1·176	47 ·5	S. W.	4	6	N, K	100 rev. in 36ˢ·; intermittent sunshine.
	12	1·193	47 ·3	S. W.	4	10	N	Rain.
	4 30	1·238	45 ·5	S. W.	5	10	N	100 rev. in 22ˢ·; rain; blows a strong gale when raining, but a moderate gale between showers.
	6	1·266	42 ·0	S. W.	4	9	N	

March 18, 9 A. M.—Centre of vortex near Markree. Absolute minimum of pressure (= 29·328) at Armagh; increase of pressure = 0·10. Mean velocity of the wind = 45 feet per second; greatest do. (west coast) = 50 feet.

March 18, 9 P. M.—Centre of vortex north of the island. Absolute minimum of pressure (= 29·371) at Portrush; increase of pressure = 0·13 inch. Mean velocity of the wind = 35 feet per second; greatest do. (north-west) = 50 feet.

March 19, 9 A. M.—Rotatory movement broken up, and wind lessened. Barometer fell, and wind rose again to a gale in the evening; greatest velocity (north-west) = 65 feet per second.

March 25.—A distinct rotatory movement at 9 A. M. of this day, the centre of which was a little to the north of Westport. Absolute minimum of pressure (= 29·408) at Westport; increase of pressure = 0·13 inch. The velocity of wind uniform, and about 30 feet per second. The wind was very light at the preceding and subsequent observations, so that the progressive movement of the vortex cannot be traced.

June 11, 12.—Cyclone crossing the island from S. W. to N. E., with a velocity of about 260 miles *per diem.*

June 11, 9 P. M.—Centre of the vortex a little to the west of Cahirciveen. Least pressure at Cahirciveen. Mean velocity of wind = 40 feet per second.

June 12, 9 A. M.—Centre over the island, between Kilrush and Westport. Absolute minimum of pressure (= 29·347) at Kilrush; increase of pressure = 0·04 inch. Mean velocity of wind = 25 feet per second.

June 12, 9 P. M.—Centre over the channel, to the east of Killough. Least pressure at Dublin. Mean velocity of wind = 20 feet per second.

July 27, 28.—Cyclone traversing the western coast, in direction from S. S. W. to N. N. E. Velocity of wind = 30 feet per second.

July 27, 9 A. M.—Centre of vortex west of Cahirciveen. Least pressure at Cahirciveen. Greatest velocity of wind in south-west. The wind at the north-eastern stations uninfluenced by the vortex.

July 27, 9 P. M.—Centre south of Westport. Absolute minimum of pressure (= 29·559) at Markree; increase of pressure = 0·10 inch. Velocity of wind uniform.

July 28, 9 A. M.—General current from S.W.; mean velocity = 30 feet per second.

August 23, 24.—Well-defined cyclone advancing in a curvilinear path, the movement of the centre being at first from N. W. to S. E., and afterwards from S.W. to N. E.*

Aug. 23, 9 P. M.—Centre of the vortex north-west of the island. Least pressure at Buncrana. Mean velocity of wind = 25 feet per second. Lightning along the whole of the eastern coast during the day.

Aug. 24, 9 A. M.—Centre near Armagh. Absolute minimum of pressure = 29·439) at Armagh; increase of pressure = 0·13 inch. Mean velocity of wind = 35 per second; greatest do. (south) = 55 feet. The centre of the vortex appears to have passed over Donaghadee about noon. At 9 A. M. the direction of the wind there was E. S. E.; at 12 (noon) W. S. W.; and at $1^{h.}$ $30^{m.}$ P. M. W. N. W., the shift being accompanied by strong gales and heavy rain.

Aug. 24, 9 P. M.—Centre north-east of the island. Least pressure at Donaghadee. Mean velocity of wind = 25 feet per second.

September 29, 30.—Interesting cyclone and storm, crossing the island from S. S. W. to N. N. E., with a velocity of about 270 miles *per diem.*

* The following extra observations were taken at Donaghadee on this day (August 24). Reduction of barometer to sea-level = + 0·077. The force of the wind is expressed in inches of Lind:—

	Barometer.	Direction.	Force.	Remarks.
Noon,	1·274	W. S. W.	0·1	Light showers.
1·30 P. M.	. . .	W. N. W.	1·0	Squall, with heavy rain.
3·0	1·404	N. N. W.	2·5	Rain.
4·0	1·504	W. N. W.	1·5	

(Plate x. Figs. 1, 2, 3.) Mean velocity of wind on the 29th = 45 feet per second.*

Sept. 29, 9 A. M.—Centre of vortex off the south-western coast, to the west of Cahirciveen. Force of wind greatest at the same station at 3 A.M.; but the barometer continued to fall until noon, when the pressure was 28·970. Increase of pressure = 0·22 inch. Greatest velocity of wind (north-west) = 60 feet per second.

Sept. 29, 9 P. M.—Centre over the island, about midway between Kilrush and Dublin. Absolute minimum of pressure (= 29·030) at Markree; increase of pressure = 0·12 inch. Least pressure in south-east at 6 P. M. Greatest velocity of wind (north-east) = 55 feet per second.

Sept. 30, 9 A. M.—Centre near Malin Head, at northern extremity of the island. Absolute minimum of pressure (= 29·020) at Portrush; increase of pressure = 0·15 inch. Mean velocity of wind = 35 feet per second; greatest do. (north-west) = 55 feet.

Sept. 30, Oct. 1.—Cyclone moving apparently in curvilinear path, its course being at first from W. to E., until it reached the centre of the island, and afterwards from S. S. W. to N. N. E. Mean velocity of wind between 25 and 30 feet per second.

* The following extra observations were taken at Markree, Sept. 29, 30, and Oct. 1:—

Date.	Hour.	Bar.	Therm.	Wind.		Cloud.		Remarks.
Sept. 29,	7 A. M.	1·197	50°·3	S. E.	5	10	N	Rain; 100 rev. anem. in 29ᵐ·
	8	1·169	51 ·1	S. E.	5	10	N	Ditto.
	9	1·145	51 ·1	S. E.	5	10	N	Ditto.
	10	1·125	51 ·4	S. E.	5	10	N	Ditto; 100 rev. anem. in 29ᵐ·
	11	1·111	52 ·7	S. E.	5	10	N	Ditto.
	12	1·070	54 ·2	S. E.	5	10	N	Ditto.
	1 P. M.	1·059	55 ·0	S. E.	5	10	N	Ditto.
	3	0·998	56 ·2	S. E.	4	10	N, S	Ditto.
	5	0·955	55 ·3	E.	4	10	N	Ditto.
	7	0·932	55 ·0	E.	5	10	N	Ditto.
	10	0·869	55 ·0	N. E.	1	10	N	100 rev. anem. in 164ᵐ·; aurora.
Sept. 30,	7 A. M.	0·952	55 ·0	W.	4	10	N	A little rain.
	8	0·979	54 ·4	W. N. W.	4	10	N	Mizzling rain.
	10	0 948	54 ·0	W.	4	10	S, N	
	11	1·062	54 ·2	W. S. W.	4	10	S, N	
	11 P. M.	0·833	50 ·2	S. E.	5	4	S, KS	
	12	0·801	50 ·2	S. E.	5	10	N, S	
Oct. 1,	1 A. M.	0·752	51 ·0	S. E.	5	10	N	
	2	0·700	51 ·0	S. E.	4	10	N	

Sept. 30, 9 P. M.—General southerly current. Centre of vortex to the west of the island; least pressure on west coast. Greatest velocity of wind (on west coast) = 45 feet per second.

Oct. 1, 9 A. M.—Centre of vortex over the island, between Kilrush and Courtown. Absolute minimum of pressure (= 28·838) equally distant from Dublin, Courtown, and Dunmore. Northern stations beginning to be affected by vortex. Greatest velocity of wind (north-east) = 50 feet per second.

Oct. 1, 9 P. M.—Centre north of Portrush. Absolute minimum of pressure (= 28·853) at Portrush; increase of pressure = 0·09 inch. At Donaghadee a sudden shift of the wind from S. S. E. to W. took place at $4^{h.}$ $30^{m.}$ P. M.

Oct. 4, 5.—Distinct cyclone moving from W. S. W. to E. N. E., and passing over (or near) the northern extremity of the island. (Plate x. Figs. 4, 5, 6.) Mean velocity of wind = 35 feet per second. General electrical disturbance.

Oct. 4, 9 A. M.—General current from S. W.; centre of vortex north-west of island. Greatest velocity of wind (on west coast) = 45 feet per second.

Oct. 4, 9 P. M.—Centre close to northern extremity of the island. Absolute minimum of pressure (=29·182) at Portrush; increase of pressure = 0·11 inch. Greatest velocity of wind (north-west) = 55 feet per second.

Oct. 5, 9 A. M.—Centre north of the island; least pressure at Portrush. Greatest velocity of wind (north) = 60 feet per second.

From the facts above stated, we may draw the following general conclusions:—

1. The occurrence of cyclonic movements in the atmosphere is not infrequent in Ireland, and may be traced even in the case of moderate winds.

2. The rotatory movement is invariably in the same direction, namely, that opposite to the diurnal movement of the sun in azimuth.

3. This rotation is always accompanied by a considerable disturbance of barometric equilibrium, which is greater in proportion to the velocity of the rotatory movement, the pressure being a minimum at the centre of the vortex, and increasing regularly with the distance from that point.

4. The place of greatest velocity appears to have no very definite relation to that of the centre of the vortex, sometimes nearly coinciding with it, and

at others being situated in front, or in the rear, on the right hand or on the the left, of the centre. In the remarkable cyclone of Nov. 18, 19, 1850, the wind raged with greatest violence on the *right hand* of the centre (looking in the direction of the progressive movement); and this appears to be the case of most frequent occurrence.

5. The vortex itself has a *progressive* movement, at the rate of from 100 to 300 miles *per diem*, the average velocity of those observed being 220 miles *per diem*. The direction of this movement is generally from S.W. to N.E.

6. If a line be drawn through the centre of Ireland, in the direction from S.W. to N.E., the track of the centres of by far the greater number of the cyclones, passing over or near Ireland, lies to the north of that line.

7. There is reason to conclude, that these rotatory movements are caused by the conflict of two rectilinear currents moving in different directions.

Storms.

For the purpose of eliminating local irregularities, and (to a certain extent also) inequalities of estimation, I have, in examining the distribution of the higher winds, combined the stations into four groups, omitting Portrush and Buncrana, which lie somewhat apart. These groups are as follow:—

I. North-eastern.—Donaghadee, Killough, Armagh. Mean latitude = $54° 24'$, mean longitude = $5° 57'$.

II. North-western.—Killybegs, Markree, Westport. Mean latitude = $54° 13'$, mean longitude = $8° 51'$.

III. South-eastern.—Dublin, Courtown, Dunmore. Mean latitude = $52° 43'$, mean longitude = $6° 29'$.

IV. South-western.—Kilrush, Cahirciveen, Castletownsend. Mean latitude = $52° 2'$, mean longitude = $9° 37'$.

The line joining groups i. and iv. lies, almost exactly, N.E. and S.W.; and that joining groups ii. and iii., N.W. and S.E.

The following are the numbers of times in which the average force of the wind, in each of these groups, amounted to a *strong breeze;* or the average velocity to 35 feet per second, and upwards.

Table XXV. Number of Times in which the Velocity of the Wind was 35 Feet per Second and upwards.

Month.	North-East.	North-West.	South-East.	South-West.
January, ...	18	19	13	29
February, ...	7	15	10	13
March, ...	5	15	4	14
April, ...	3	11	3	12
May, ...	2	10	1	4
June, ...	2	15	5	10
July, ...	3	9	4	9
August, ...	0	9	3	7
September, ...	2	11	3	6
October, ...	3	21	4	17
November, ...	0	14	1	6
December, ...	3	9	9	14
Spring, ...	10	36	8	30
Summer, ...	5	33	12	26
Autumn, ...	5	46	8	29
Winter, ...	28	43	32	56
Year, ...	48	158	60	141

From the foregoing numbers it appears, that high winds are much more frequent on the western than on the eastern coast, the numbers denoting the relative frequency, on the average of the entire year, being nearly as 3 to 1. This preponderance of high winds on the western coast holds at all seasons of the year, the maximum occurring at the *north-western* extremity in autumn, and at the *south-western* in winter. The greatest frequency is in the north-west, on the average of the entire year.

The following are the cases in which the mean force of the wind, over the whole island, amounted to *a gale;* or in which the mean velocity was 45 feet per second and upwards:—

Nov. 23, 24, 1850.—Storm along the western coast, blowing at first from S. S. W., and veering through S. W. to W. Least pressure in north-west throughout.

Nov. 23, 9 p. m.—Storm began at south-western extremity of the island; velocity = 45 feet per second.

Nov. 24, 9 A. M.—Wind continued to blow in same district; velocity increased to 60 feet per second. Absolute barometric minimum (north-west) = 28·644.

Nov. 24, 9 P. M.—Storm extended over whole of western coast; velocity of wind = 55 feet per second.

Dec. 14.—Storm affecting the whole island, but chiefly the western coast. Wind at first from S. S. W., but veering to W. S. W. at 9 P. M. Least pressure in north-west throughout. Electrical disturbance over the whole island.

Dec. 14, 9 A. M.—Velocity on western coast = 65 feet per second. Absolute barometric minimum (north-west) = 28·952.

Dec. 14, 9 P. M.—Velocity on western coast = 50 feet per second.

Dec. 31, Jan. 1, 1851.—Storm from S. W. and S., beginning on western coast, and extending over whole island.*

Dec. 31, 9 A. M.—Velocity on western coast = 50 feet per second. Direction S. S. W. and S. W.

Dec. 31, 9 P. M.—Gale affecting whole island, except north-eastern extremity. Greatest in south-west; velocity = 60 feet per second. Direction as before. Absolute barometric minimum (north) = 29·177.

Jan. 1, 9 A. M.—Wind abated.

Jan. 1, 9 P. M.—Gale from S. W. and S. over whole island, except north-western extremity. Velocity (south-east) = 55 feet per second. Absolute barometric minimum (north) = 28·975.

In this case, therefore, there were two storms succeeding each other on consecutive days, with a double fall of the barometer. The direction of the wind Jan. 1 P. M. was remarkable. The prevailing current was from S. W., and

* The following extra observations were taken at Markree on December 31, 1850:—

Date.	Hour.	Bar.	Therm.	Wind.	Cloud.	Remarks.
Dec. 31,	11 A. M.	1·122	52°·0	S. 4	10 N	Light rain.
	12	1·090	53 ·7	S. 4	10 N	Ditto.
	1 P. M.	1·070	53 ·75	S. 4	10 N	Mizzling rain.
	3	1·040	53 ·9	S. 4	10 N	Rain.
	5	1·036	52 ·0	S. 3	10 S, N	Wind falling since 4 P. M.

extended over the central parts of the island; while there appears to have been an *indraught* towards it, from the north-western and south-eastern quarters.

Jan. 12, 13.—Storm from S. and S.W., beginning in the north-west, and advancing in the direction from N.W. to S.E. Velocity of wind = 60 feet per second. Least pressure in north-west throughout.*

Jan. 12, 9 P.M.—Gale in north-west.

Jan. 13, 9 A.M.—Storm advanced to line joining north-east and south-west centres. Absolute barometric minimum (north-west) = 29·174; pressure least at Markree at noon.

Jan. 27.—Storm from S. and S.W. in the afternoon of this day, chiefly along the western coast. Velocity of wind = 55 feet per second. Absolute barometric minimum (north-west) = 29·309.†

* The following extra observations were taken at Markree, January 12, 13, 1851:—

Date.	Hour.	Bar.	Therm.	Wind.	Cloud.	Remarks.
Jan. 12,	11 P.M.	1·365	44·0	S.S.E. 4	10 N	Rain.
13,	7 A.M.	1·058	48·5	S. 5	10 N	Moderate rain
	8	1·054	48·8	S. 5	10 N	Light rain.
	11	1·000	50·3	S.S.E. 5	10 N	
	12	0·981	51·1	S.S.E. 4	10 N	
	2 P.M.	0·994	50·1	S. 5	3 K, N	Bright sunshine.
	3	1·031	48,8	S.S.W. 4	9 N, CK	Faint sunshine; a little rain.
	4	1·070	47·8	S.S.W. 3	9 N	A little rain.

† The following extra observations were taken at Courtown on this day:—

Date.	Hour.	Bar.	Dir.	Force.	Remarks.
Jan. 27,	Noon.	1·755	S.S.W.	1	Cumulus clouds, with cirrus above; partially overcast.
1851.	1 P.M.	1·737	S.S.W.	1	Sky becoming overcast.
	2	1,697	S.	2	Wind veered from S.S.W. to S.
	3	1·665	S.	3	Cloudy and sultry. Lind = 0·4 inch.
	4	1·637	S.	3	Wind increasing. Lind = 0·6 inch.
	5	1·597	S.	4	Ditto; dark in S.S.W.
	6	1·563	S.	4	Rain; force of wind greatest. Lind = 0·8 inch.
	7	1·517	S.	4	Ditto; darkly overcast. Lind = 0·8 inch.
	8	1·507	S.	4	Ditto; ditto. Lind = 0·8 inch.
	9	1·500	S.	3	Sky brightening in W. Lind = 0·6 inch.
	10	1·500	S.	3	Flashes of lightning in S.S.W. and W. Lind = 0·4 inch.
	11	1·500	S.	2	Sky brightening.
	12	1·501	S.W.	2	At 11·45 wind veered from S. to S.W.

June 15, 16.—Gale from S.W. and W., on the western coast. Velocity about 50 feet per second.

June 15, 9 A.M.—Wind from S.W. Velocity on western coast = 50 feet per second. Least pressure in north-west.

June 15, 9 P.M.—Velocity = 45 feet per second. Absolute barometric minimum (north) = 29·575.

June 16, 9 A.M.—Wind from W. Velocity = 50 feet per second.

July 13, 14.—Storm chiefly in north-west, blowing at first from S.S.W., and veering through S.W. to W. This appears to have been a cyclonic gale, the centre of the cyclone passing to the north of the island; it is not included in the former series on account of this circumstance. The velocity of the wind was greatest in the north-west throughout; the barometric pressure was least in the north-west on the 13th, and in the north-east on following day.

July 13, 9 A.M.—Storm from S.S.W., in the north-west of the island. Velocity = 60 feet per second.

July 13, 9 P.M.—Gale veered to S.W., and affected a large portion of the island. Velocity of wind = 60 feet per second, as before. Absolute barometric minimum = 29·052.

July 14, 9 A.M.—Wind veered to W. Velocity in north-west increased to 65 feet per second.*

Dec. 7.—Storm began in south-western extremity of the island, and extended thence over the whole. Direction of wind between S. and S.W.

* The following extra observations were taken at Markree, July 14, 1851. The wind column contains the time of 100 revolutions of Robinson's anenometer:—

Date.	Hour.	Bar.	Therm.	Wind.		Cloud.		Remarks.
July 14,	6 A.M.	1·040	55°·2	W.	34s.	10	N	Strong gale; mizzling rain.
	7	1·065	54 ·2	W.	28	10	N	Strong gale; rain.
	8	1·086	55 ·4	W.	28	10	N	Ditto.
	9	1·124	58 ·4	W.	33	10	N	Ditto.
	10	1·173	55 ·3	W.	19	9	N	Ditto; a little rain.
	11	1·200	59 ·4	W.		9	N, K	Ditto.
	12	1·225	58 ·8	W.		9	N, K	Ditto.
	1 P.M.	1·260	57 ·8	W.	18	9	N	Ditto.
	3	1·317	53 ·9	W.N.W.	34	5	N, K	Moderate gale.

Dec. 7, 9 A. M.—Gale from S. S. W. in the south-west.

Dec. 7, 9 P. M.—Storm over the whole island. Greatest velocity and least pressure in north-west. Velocity = 70 feet per second. Absolute barometric minimum = 29·267. At Cahirciveen the barometer fell until 7 P. M.; and the wind shifted from S. to W. at the same time.*

Dec. 9.—Storm from S.W. along the western coast. Least pressure in north-west throughout.

Dec. 9, 9 A. M.—Velocity of wind in west = 50 feet per second.

Dec. 9, 9 P. M.—Velocity = 60 feet per second. Absolute barometric minimum = 29·632.

Dec. 20.—Gale blowing from S. S. W., beginning on western coast, and advancing to eastern. Least pressure in north and north-west.

Dec. 20 A. M.—Gale on west coast. Velocity = 55 feet per second.

Dec. 20 P. M.—Gale transferred to east coast. Velocity = 50 feet per second. Absolute barometric minimum (north) = 29·457. At Markree there was a sudden shift of the wind from S. S. W. to N. W. at $7^{h.}$ $35^{m.}$ P. M.

From the foregoing facts we may draw the following conclusions:—

1. The greater gales are much more frequent on the western, than on the eastern coast, the numbers denoting the relative frequency being nearly as 5 to 1. The frequency of storms is nearly the same in the northern and southern portions of the island.

2. The direction of the wind, in all the cases enumerated, was between S. and W. In about half of these cases the wind blew, throughout, from the

* The following extra observations were taken at Cahirciveen on this day:—

Date.	Hour.	Bar.	Wind.		Remarks.
Dec. 7, 1851.	4 P. M.	1·550	S.	4	Heavy rain, and squally.
	5	1·517	S.	5	Ditto, ditto.
	6	1·446	S.	5	Ditto, ditto.
	7	1·426	S. W.	5	Squally; light rain.
	8	1·444	W. S. W.	4	Ditto, ditto.
	9	1·522	W.	4	Drizzling rain.
	10	1·564	W.	3	Cloudy.

same point; in half it veered from 4 to 6 points of the compass, the veering being in the direction produced by a cyclone moving from S.W. to N.E., and having the path of its centre to the north of the island.

3. The axis of the gale is in some cases transferred parallel to itself, *to the eastward.* Remarkable instances of this movement occur in the gales of January 12, 13, and December 20.

4. The least barometric pressure occurs, in almost every instance, in the *north-western quarter* of the island.*

5. The locality of the *highest wind* sometimes coincides with that of *least pressure*, and sometimes does not. In the latter case, the axis of least pressure is generally to the *westward* of the axis of the storm.

6. On either side of the axis of a storm, the wind appears to blow *towards that line.* A remarkable instance of this phenomenon occurred in the storm of January 1.†

We are now in a position to consider the question, whether *all* storms are *cyclonic?* And if not, what *proportion* do rotatory storms bear to the whole? Of the greater storms which have occurred since the commencement of these observations, the rotatory character of five (those of October 6, 1850, November 18, January 15, 1851, March 18, and September 29) has been completely established. We have seen in this section, that the same character may be predicated, with great probability, of five more; while there remain five in which the wind has blown, throughout, in the same direction. In fifteen months, accordingly, there have occurred fifteen storms, of which *two-thirds* were *cyclonic.* As respects the remaining one-third, the phenomena are characterized, not only by the absence of any *veering* of the wind, but also by the fact, that the pressures appear to increase with the distance from a *line or axis* of minimum pressure, rather than from a *point;* or, in other words, that the *isobaric* lines are parallel right lines, instead of concentric circles. It is true

* In one case only, the locality of least pressure shifted from the north-western to the north-eastern extremity of the island. This is consistent with the supposition, that the storm in question was a cyclone, whose centre had a progressive motion eastward.

† The conclusions numbered 3, 5, 6, have already been drawn by Mr. ESPY, from an examination of the storms in the United States in the early months of the year 1843.—*First Report on Meteorology.*

that these facts are by no means decisive in disproving rotatory movement; for they are consistent with a rotation of the wind in a plane perpendicular, or highly inclined, to the horizon. Still we are perhaps not justified in assuming the existence of a rotation of this kind, without further evidence; and it seems more reasonable, in the present state of our knowledge, to admit two different kinds of winds, than to endeavour to reduce all to one by the help of a gratuitous hypothesis.

Hourly Observations.—It has been already stated, that hourly observations were appointed to be made during twenty-four consecutive hours, at the equinoxes and solstices, in the hope that their results might throw light upon the simultaneous atmospheric changes occurring over the island, and especially upon the direction and rate of progress of atmospheric waves. The observations on the first two of these term-days (March 21 and June 21) at six of the stations, are given in detail at the end of this Paper (Table xxxiv.). Those of the two latter (September 22 and December 22) have been omitted, no atmospheric change of a marked kind having occurred during them.

March 21.—A gale occurred on this day, accompanied by a marked barometric depression. The minimum of pressure took place during the observations, the time of its occurrence varying considerably at the different stations. At Cahirciveen, there was a sudden fall of the barometer between 9 A.M. and 10 A.M. followed by a sudden rise between 12 and 1 P.M., the mercury being nearly stationary from 10 A.M. to 12. A similar change took place at Dunmore East, and at the same hours. For these two stations, accordingly, the epoch of minimum pressure may be taken to be 11 A.M.; the subsequent changes were small and irregular. At Courtown, the barometer descended very slowly and gradually until 5 P.M.; it then ascended until 10 P.M., after which it descended again. All the changes were, however, very small.

At the northern stations the fall of the barometer was more considerable, and more regular. At Markree, where it was most rapid, it amounted to 0·210 in 6 hours. The minimum at Markree occurred between 3 P.M. and 4 P.M.; at Armagh, the minimum took place at 6 P.M.; and at Portrush, at 8 P.M.

From these results it would appear that the trough of the wave travelled from south to north, nearly, with a velocity of about 22 miles per hour. The barometric depression was greatest at Markree, where the barometer stood at

28·689, when lowest. The lowest pressure increased from that point in the south-easterly direction, being 28·972 at Dunmore.

At Markree the wind shifted from S. S. E. to S. S. W. at the time of greatest depression. The same phenomenon took place at Armagh and Portrush, although not with such precision; the change of direction at the former station being from S. S. E. to S., and at the latter from S. E. to S. No similar change occurred at the southern stations.

It should be observed, that the foregoing phenomena are not necessarily to be ascribed to the transit of a rectilinear wave. They are all consistent with the effects of a cyclone, coming from the S. or S. W., the track of its centre lying to the west of the island.

June 21, 22.—The changes of the direction and of the pressure of the wind, on this day, are manifestly the effects of a cyclonic movement, the centre of the vortex sweeping round the north coast of Ireland, in a somewhat curvilinear path, from west to east. It has not been included in the former series, the force of the wind having been below the limit there adopted. At 9 A. M. of the 21st, the centre of the vortex was off the north-west coast, to the west of Killybegs. At 9 P. M. of the same day, it had arrived to the north of Portrush; and at 9 A. M. of the 22nd, it was to the north-east of Donaghadee.

The veering of the wind was, on the average, about 90°; its duration was very different at the different stations, being shortest for those near the path of the centre of the vortex, and longest for those remote. The wind, which was very light throughout, fell about the time of veering at most of the stations.

The descent and subsequent rise of the barometer were regular, and the minimum well-defined. The time of least pressure coincided at all the stations, very nearly, with the middle of the time of veering of the wind; it was earliest on the western coast, and latest on the eastern, the epoch of its occurrence being between 12 and 1 P. M. at Markree and Cahirciveen, and between 5 P. M. and 6 P. M. at Dublin and Courtown. The barometric depression was small, the mean pressure at the epoch of minimum being 29·74.

Humidity of the Air.

The following Tables give the results of the psychrometrical observations. Table XXVI. contains the monthly means of the *temperatures of evaporation* at the several stations; and Table XXVII. those of the *tension of vapour*, calculated by Regnault's Table:—

Table XXVI. Temperature of Evaporation.

Station.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Portrush, . . .	40 ·3	40 ·5	40 ·8	43 ·3	48 ·1	52 ·8	54 ·8	56 ·8	53 ·8	49 ·3	42 ·0	43 ·2	47 ·1
Buncrana, . . .	39 ·4	39 ·6	40 ·6	42 ·5	47 ·5	52 ·3	53 ·9	56 ·2	53 ·1	48 ·4	40 ·2	42 ·1	46 ·3
Donaghadee, . .	41 ·1	40 ·9	41 ·0	43 ·5	47 ·6	52 ·7	54 ·3	56 ·5	54 ·0	49 ·4	41 ·1	43 ·4	47 ·1
Killybegs, . . .	41 ·6	41 ·7	42 ·6	44 ·6	49 ·3	53 ·6	55 ·5	58 ·1	55 ·7	50 ·1	43 ·4	44 ·4	48 ·4
Armagh, ...	39 ·4	39 ·5	40 ·2	42 ·9	47 ·2	52 ·3	54 ·0	55 ·5	52 ·7	18 ·0	38 ·9	41 ·1	46 ·0
Killough, . . .	42 ·3	41 ·8	41 ·6	44 ·8	48 ·9	53 ·2	56 ·1	57 ·2	55 ·0	50 ·7	40 ·6	44 .2	48 ·0
Markree, . . .	39 ·2	40 ·7	41 ·4	43 ·7	48 ·0	52 ·9	54 ·8	56 ·9	53 ·9	49 ·3	41 ·3	41 ·9	47 ·0
Westport, . . .	44 ·2	44 ·0	44 ·7	46 ·3	51 ·1	55 ·7	57 ·8	60 ·3	58 ·1	52 ·7	46 ·3	46 ·8	50 ·7
Dublin, . . .	41 ·2	41 ·2	41 ·5	43 ·8	48 ·6	54 ·2	55 ·8	58 ·8	53 ·1	49 ·3	39 ·1	41 ·3	47 ·3
Portarlington,	41 ·9	42 ·0	42 ·8	44 ·4	51 ·6	55 ·8	56 ·2	59 ·1	55 ·2	51 ·4	41 ·3	42 ·0	48 ·6
Athy, ...	40 ·4	40 ·5	41 ·4	44 ·6	49 ·5	55 ·6	56 ·5	58 ·7	53 ·3	49 ·8	38 ·5	41 ·3	47 ·5
Courtown, . . .	41 ·7	41 ·8	42 ·0	44 ·2	48 ·9	53 ·9	56 ·4	57 ·9	53 ·9	49 ·9	39 ·4	43 ·2	47 ·8
Kilrush, ...	42 ·8	43 ·7	43 ·4	45 ·3	49 ·7	54 ·4	56 ·7	59 ·2	56 ·3	50 ·8	43 ·1	43 ·4	49 ·1
Dunmore, . . .	43 ·5	42 ·9	42 ·8	45 ·5	50 ·8	54 ·4	57 ·4	59 ·3	55 ·8	50 ·7	40 ·0	43 ·9	18 ·9
Cahirciveen, . .	43 ·5	44 ·1	44 ·2	45 ·6	50 ·4	54 ·8	57 ·4	59 ·6	55 9	51 ·7	44 ·3	45 ·0	49 ·7
Castletownsend,	43 ·8	43 ·9	43 ·5	45 ·2	50 ·7	54 ·4	58 ·1	59 ·9	57 ·8	51 ·7	43 ·4	45 ·1	49 ·8

Table XXVII. Tension of Vapour.

Station.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Portrush, . . .	·243	·242	·239	·256	·317	·382	·414	·442	·399	·337	·252	·274	·316
Buncrana, . . .	·227	·230	·229	·243	·301	·365	·387	·421	·379	·316	·233	·256	·299
Donaghadee, .	·250	·241	·239	·250	·299	·370	·395	·434	·397	·338	·236	·268	·310
Killybegs, . . .	·248	·248	·253	·269	·334	·389	·415	·461	·423	·342	·268	·282	·328
Armagh, ...	·231	·228	·231	·250	·295	·357	·388	·412	·376	·320	·224	·246	·297
Killough, . . .	·256	·249	·245	·279	·329	·389	·434	·451	·414	·352	·237	·271	·326
Markree, ...	·244	·252	·252	·267	·315	·378	·411	·443	·407	·349	·263	·271	·321
Westport, . . .	·277	·285	·288	·306	·370	·443	·476	·523	·486	·398	·315	·321	·374
Dublin, ...	·248	·243	·242	·258	·306	·377	·402	·466	·382	·336	·229	·252	·312
Portarlington,	·250	·245	·247	·249	·332	·390	·401	·457	·384	·359	·250	·259	·319
Athy, ...	·254	·249	·252	·275	·324	·417	·432	·475	·403	·356	·232	·261	·328
Courtown, . . .	·258	·251	·250	·266	·312	·386	·424	·455	·395	·351	·227	·269	·320
Kilrush, ...	·269	·275	·270	·285	·339	·412	·436	·488	·437	·364	·271	·275	·343
Dunmore, . . .	·271	·264	·262	·288	·349	·380	·427	·477	·415	·349	·230	·273	·332
Cahirciveen, . .	·269	·277	·274	·277	·333	·400	·444	·488	·421	·371	·277	·290	·343
Castletownsend,	·278	·274	·264	·275	·343	·400	·460	·500	·465	·370	267	·291	·349

Very few results of a general nature can be drawn from these observations, the distribution of vapour being governed by the proximity of the station to the sea, or by other local circumstances. It will be seen, from the last column of Table XXVII., that the yearly mean *tension of vapour* increases, although not in any regular progression, in proceeding from the north to the south of the island. Its mean value for the entire island is 0·326 of an inch; its greatest value (at Westport) is 0·374.

The following Table contains the values of the relative *humidity*, the state of complete saturation being represented by 100:—

Table XXVIII. Humidity of the Air.

Station.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Portrush, . . .	92	90	87	82	86	86	90	88	90	90	87	93	88
Buncrana, . . .	87	87	82	79	81	81	83	83	84	85	86	88	84
Donaghadee, . .	92	87	86	78	80	82	84	87	87	89	83	89	85
Killybegs, . . .	88	87	85	82	87	84	85	86	87	87	88	91	86
Armagh, ...	89	87	86	81	80	78	82	83	85	88	88	89	85
Killough, . . .	88	87	86	86	87	88	89	89	88	88	86	86	87
Markree, ...	99	96	92	87	86	84	89	89	93	96	98	99	92
Westport, . . .	89	95	93	93	94	96	97	97	99	97	98	98	96
Dublin, ...	88	86	85	80	77	75	76	83	85	88	89	90	84
Portarlington,	87	84	80	72	72	72	74	77	74	86	90	91	80
Athy, ...	98	95	91	85	81	84	85	88	95	95	95	98	91
Courtown, . .	93	89	86	83	78	81	83	85	86	92	87	90	86
Kilrush, ...	94	91	90	87	87	90	86	91	90	93	92	92	90
Dunmore, . . .	90	90	89	87	85	78	77	84	82	85	85	88	85
Cahirciveen, . .	89	89	88	80	79	82	84	87	85	90	88	91	86
Castletownsend,	92	89	86	81	82	85	86	90	90	89	88	92	88

The distribution of *humidity* is still more under the influence of local circumstances, and therefore still less regular. Thus, Portrush and Castletownsend—the one at the northern, the other at the southern extremity of the island—have nearly the same mean humidity; while Portarlington and Athy—places near each other, and both inland—are almost at the opposite extremities of the scale. The *driest* station is Portarlington; the *most humid*, Westport. At the latter place, in fact, the air is nearly saturated with moisture, the place of observation being entirely surrounded by water, and but a few feet above the sea. The *mean humidity* for the entire island, for the year 1851, is 87.

Rain.

Before proceeding to the observations of rain-fall throughout Ireland in the year 1851, it is important that we should know its *normal* amount at one or more stations, as deduced from the mean of several years. We have, for this purpose, two series of observations, one at Dublin, and the other at Armagh, extending uninterruptedly over eleven and twelve years respectively. The results of these two series are contained in the following Tables.

Table XXIX. Monthly Fall of Rain at Dublin, in Inches (1841–1851).

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1841	2·41	1·00	2.12	1·12	1·94	1·84	2·52	2·22	1·88	4·89	3·58	2·98	28·50
1842	1·41	3·11	2·47	0·81	3·18	2·00	2·57	1·24	4·60	1·54	4·37	0·78	28·08
1843	2·15	1·84	1·65	3·34	4·51	2·60	1·93	2·12	0·67	3·44	3·10	0·35	27·70
1844	1·54	2·37	2·34	0·95	0·21	1·69	1·83	3·77	2·55	3·32	5·24	2·60	28·41
1845	4·48	0·81	1·72	0·88	1·54	4·00	3·33	2·52	0·95	3·92	3·56	3·77	31·48
1846	3·43	1·42	2·85	5·97	2·14	1·50	3·14	4·33	2·98	4·72	2·80	0·83	36·11
1847	3·57	2·63	1·22	3·42	2·21	1·91	0·64	1·43	1·35	2·13	2·09	3·20	25·80
1848	1·88	3.23	2·40	3·15	0·93	3·92	2·37	5·10	2·42	4·38	1·50	2·83	34·11
1849	3·30	0·72	1·07	2·57	2·07	0·47	2·69	3·33	3·83	3·93	1·80	4·03	29·81
1850	2·25	1·47	1·14	3·63	2·42	1·69	2·26	1·38	1·99	1·24	2·32	2·39	24·18
1851	5·28	0·49	2·38	1·77	1·31	2·71	3·48	2·01	1·81	3·27	1·01	0·88	26·40
Means	2·88	1·74	1·94	2·51	2·04	2·21	2·43	2·68	2·28	3·34	2·85	2·24	29·14

Table XXX. Monthly Fall of Rain at Armagh, in Inches (1840–1851).

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1840	4·94	2·75	0·46	0·76	3·03	2·50	3 15	2·66	2·42	1·26	3·45	2·82	30·20
1841	2·00	2·27	3·65	1·70	1·55	2·60	2·35	2·90	2·31	4·00	3·02	3·50	31·85
1842	2·79	2·73	4·23	0·03	4·08	2·42	3·01	2·98	2·85	1·98	4·71	3·01	34·82
1843	2·25	1·27	1·93	2·93	3·94	3·34	4·16	3·84	1·22	4·00	3·19	2·24	34·31
1844	2·64	3·24	2·89	1·67	0·04	4·47	2·36	3·07	2·23	4·35	3·00	0·53	30·49
1845	4·99	1·33	1·62	3·16	0·39	5·57	3·63	1·88	2·83	4·84	4·76	5·26	40·26
1846	4·58	1·86	3·79	2·85	1·68	2·10	3·85	3·55	3·35	4·93	3·30	1·63	37·47
1847	3·03	1·97	1·46	3·15	2·48	1·91	1·08	1·10	2·67	3·78	3·78	5·86	32·27
1848	1·87	6·75	3·77	3·32	1·24	2·73	3·92	3·48	2·38	3·15	3·70	3·01	39·32
1849	6·30	2·51	1·48	2·09	3·00	0·87	3·98	2·89	3·55	4·39	2·73	3·26	37·05
1850	4·08	5·04	1·24	3·51	2·41	2·37	3·14	2·72	2·71	2·24	3·21	2·46	35·13
1851	5·53	2·83	2·55	1·54	1·92	3·45	3·66	2·81	2·44	2·90	1·41	2·11	33·15
Means	3·75	2·88	2·42	2·23	2·15	2·86	3·19	2·82	2·58	3·48	3·35	2·97	34·68

The lowest line in each gives the *mean monthly* fall of rain. It will be seen, from an inspection of the numbers, that there is no regular progression in the amount of rain-fall throughout the year, such as is observed in the phenomena of temperature or humidity. In Dublin the greatest rain-fall, in the mean of the eleven years, occurs in October, and the least in February; their amounts are 3·34 and 1·74 inches respectively. At Armagh the maximum is in January, and the minimum in May; and they amount to 3·75 and 2·15 inches. The *mean yearly* rain-fall at Dublin is 29·14 inches; that at Armagh is 34·68 inches.

The following Table gives the monthly fall of rain in the year 1851, at all the meteorological stations:—

Table XXXI. Monthly Fall of Rain in the Year 1851, at all the Meteorological Stations.

Station.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Portrush, ...	5 69	2.71	3·45	1·20	1·91	3·12	2·86	3·52	1·36	5·10	4·34	1·98
Buncrana, . . .	4·91	2.89	3·36	1·78	1·69	2·83	3·41	3·30	2·48	5·20	5·04	2·39
Donaghadee, . .	5·45	1·51	3·38	1·30	1·40	2·93	2·16	3·18	0·96	1·82	2·38	1·46
Killybegs, . . .	4·26	2·84	2·20	1·58	1·82	3·08	2·84	2·93	2·84	3·46	3·25	2·10
Armagh, ...	5.81	2·83	2·65	1·54	1·92	3·13	3·81	2·56	1·84	3·48	1·47	2·01
Killough, ...	4·00	1·18	2·47	1·03	1·29	2·63	2·73	2·06	1·00	2·69	1·19	0·92
Markree, ...	5·01	2·89	3·07	2·23	1·53	2·67	5·20	5·15	2·40	4·42	3·68	2·06
Westport,* . . .	5·06	4·37	5·61	4·85	1·24	4·55	3·78	3·43	2·02	5·55	2·60	2·80
Dublin, ...	5·28	0·49	2·38	1·77	1·31	2·71	3·48	2·01	1·81	3·27	1·01	0·88
Portarlington, .	4·11	0·52	2·07	0·94	0·82	2·51	2·59	1·61	0·84	2·59	1·15	1·48
Athy, ...	5·10	0·93	2·37	1·29	1·35	2·68	2·73	2·91	1·24	3·24	1·41	1·49
Courtown, . . .	7·98	0·95	3·44	2·14	1·17	3·08	2·67	1·65	0·63	3·41	0·55	1·97
Kilrush, ...	6·42	1·69	3·99	1·75	0·96	2·50	2·83	2·85	0·79	4·44	2·10	2·26
Dunmore, . . .	8·33	0·97	3·87	1·69	1·26	3·82	2·67	2·80	0·66	4·29	0·74	2·41
Cahirciveen, . .	11·22	4·90	6·41	2·17	2·31	4·71	5·51	5·31	2·63	7·74	2·13	4·33
Castletownsend,	9·76	3·67	4·03	1·53	1·87	4·87	4·79	3·80	0·68	4·59	0·82	2·12

It will be seen from the foregoing Table, that the greatest diversity exists in the amount of rain-fall in different localities. To render this more apparent. and to facilitate the examination of the causes which influence the distribution, I have, in the following Table, given the yearly rain-fall at the several stations arranged in the order of magnitude, beginning with the smallest:—

* The amount for the month of January at Westport is incomplete, the observations having commenced in the middle of the month.

Table XXXII. Total Rain-Fall in the Year 1851, at the several Meteorological Stations.

20—25 inches,	Portarlington,	21·23 inches.
	Killough,	23·19 „
25—30 „	Dublin,	26·40 „
	Athy,	26·74 „
	Donaghadee,	27·93 „
	Courtown,	29·64 „
30—35 „	Kilrush,	32·58 „
	Armagh,	33·05 „
	Killybegs,	33·20 „
	Dunmore,	33·54 „
35—40 „	Portrush,	37·24 „
	Buncrana,	39·28 „
40—45 „	Markree,	40·31 „
	Castletownsend,	42·53 „
45—50 „	Westport,	45·86 „
50—60 „	Cahirciveen,	59·37 „

Thus, it will be seen, the greatest rain (at Cahirciveen) is *nearly treble* of the least (at Portarlington). The *mean rain-fall* throughout Ireland, in the year 1851, is 34·50 inches.

If we assume the *proportion* of rain at the different stations to be constant, or nearly so, the numbers of the preceding Table may all be reduced to their *mean values*, by multiplying by the factor which expresses the relation of the rain of 1851 to the mean at any one station. We already possess two such mean values: viz., at Armagh and Dublin. They are 29·14 and 34·68 inches respectively; and the factors thence deduced are 1·10 and 1·05.

When we examine the results of the preceding Table, taken in connexion with the geographical position and physical circumstances of the stations, we arrive at the following conclusions:—

1. The places of *least rain* are either inland, or on the eastern coast; while those of *greatest rain* are at, or near, the western coast. Thus the stations at which the yearly fall of rain exceeds 40 inches are all on the western and south-western coasts; while those at which it is below 30 inches are either inland or on the eastern.

2. The amount of rain is greatly dependent on the proximity of a mountain chain or group, being always considerable in such neighbourhood, unless the station be to the east or north-east of the same. Thus, of the places of *least rain*, Portarlington lies to the north-east of Slieve-bloom; Killough, to the north-east of the Mourne range; Dublin, to the north-east of the Dublin and Wicklow range; while, on the other hand, the places of *greatest rain*,—Cahirciveen, Westport, and Castletownsend—are in the vicinity of high mountains, but on a different side.

These facts are easily explained. The prevailing wind blows from the S. W., and reaches this island loaded with the vapour of the gulf-stream. This vapour is condensed and precipitated in rain, when it first meets the colder air over the land, namely, on the western and south-western shores. But the principal condensing centres are the mountains, in the neighbourhood of which, consequently, the precipitation is more abundant, and especially on their western and south-western sides. And the same circumstance which causes the greater precipitation at these points must also protect the region over which the wind next passes (the north-east), the air being thus deprived of a large portion of its vapour before arriving there.

TABLES.

The following Tables contain the portions of the individual observations, the results of which are referred to in pages 450–469.

Table XXXIII. contains the selected observations on days of storm, or of marked cyclonic movement. It comprises the direction and force of the wind, the pressure, temperature, and amount of cloud, at the time of observation; as also the greatest and least temperatures, and the quantity of rain fallen, in the preceding twenty-four hours. The pressures are, for comparison, reduced to the mean sea-level; the numbers in the Table are the excesses above 28 inches. The force of the wind is expressed in terms of the scale $(0-6)$.

Table XXXIV. contains the hourly observations on the term-days, March 21, 22, and June 21, 22, at Portrush, Armagh, Markree, Courtown, Dunmore, and Cahirciveen. The *velocity* of the wind at Portrush, Armagh, Markree, and Courtown, was observed by means of ROBINSON'S anemometer; it is expressed in feet per second.

Table XXXIII. Selected Observations.

Station.	1850. October 6, 9 A. M.						October 6, 9 P. M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	2	1·573	46°·7	8	·04	S.	2	0·952	50°·5	56°·0	41°·0
Buncrana, . .	S.	1	1·527	49 ·6	10	·36	S.	0	1·022	50 ·6	55 ·0	43 ·0
Donaghadee, .	S. W.	0	1·590	49 ·7	10	·01	S. W.	5	0·998	51 ·1	54 ·7	43 ·5
Killybegs, . . .	S.	—	1·387	48 ·7	—	·47	N. W	5	0·836	53 ·0	55 ·0	48 .0
Armagh, . . .	S.	1·9	1·558	47 ·3	10	·03	S.	3·0	0·906	49 ·2	55 ·0	44 ·5
Markree, . . .	S. E.	4	1·383	47 ·1	10	·27	W.N.W.	6	0·908	52 ·0	55 ·5	40 ·2
Dublin, ...	S. E.	1	1·562	49 ·1	10	·09	S.	4	1·063	52 ·5	60 ·5	45 ·3
Courtown, . .	S.	2	1·550	50 ·5	10	·10	S. W.	2	1·180	52 ·2	59 ·2	42 ·0
Kilrush, . . .	S. W.	3	1·392	54 ·3	—	·04	S. S. W.	5	1·242	49 ·3	59 ·0	49 ·5
Dunmore, . . .	S.	3	1·490	53 ·8	—	·27	W.	3	1·242	53 ·3	53 ·5	47 ·5
Cahirciveen, .	W.S.W.	4	1·309	—	10	·00	N. W.	4	—	—	—	—
Castletownsend,	W.S.W.	3	1·423	55 ·5	10	—	S. S. W.	5	1·382	51 ·5	61 ·0	50 ·5

Station.	October 7, 9 A.M.						October 22, 9 P. M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	N. W.	5	1·245	48°·5	7	·42	N. W.	2	1·645	50°·7	54°·0	41° 0
Buncrana, . .	N.N.W.	4	1·458	49 ·6	5	·58	N. W.	3	1·721	48 ·6	—	36 ·0
Donaghadee, .	N. W.	5	1·190	52 ·2	10	·25	N. W.	3	1·610	49 ·4	—	41 ·0
Killybegs, . . .	N. W.	—	1·460	52 ·8	—	·15	N. W.	—	1·708	51 ·0	53 ·5	42 ·3
Armagh, . . .	N. W.	2·6	1·340	51 ·0	8	·28	N. W.	2·3	1·640	47 ·5	49 ·9	41 ·0
Markree, . . .	W. N. W.	5	1·493	51 ·0	7	·23	N. N. W.	3	1·682	48 ·8	52 ·6	31 ·7
Dublin, ...	S. W.	4	1·425	50 ·9	1	·07	S. W.	3	1·649	48 ·9	51 ·2	35 ·5
Courtown, . .	W.	4	1·488	50 ·5	3	·12	S. W.	—	1·736	45 ·2	48 ·0	34 ·0
Kilrush, ...	N. W.	4	1·590	49 ·8	—	·15	N. W.	4	1·851	52 ·3	54 ·0	39 ·0
Dunmore, . . .	W. N. W.	3	1·588	52 ·3	—	·10	S. W.	4	2·233?	43 ·3	49 ·5	36 ·0
Cahirciveen, .	N. W.	3	1·226?	—	6	·00	W. N. W.	2	1·831	53 ·6	55 ·0	39 ·0
Castletownsend,	W. S. W.	1	1·714	54 5	3	—	W. S. W.	3	1·837	51 ·5	55 ·0	39 ·0

Remarks.

Oct. 6, 9 A. M.—Rain throughout the island; heavy rain at Cahirciveen.

Oct. 6, 9 P. M.—Storm in north-west from 10 30 P. M. to 3 A. M. of following day. Showers in south; hail at Markree.

Oct. 7, 9 A. M.—Showers at Portrush, Donaghadee, and Cahirciveen; hail at Markree.

Oct. 22, 9 P. M.—Heavy clouds at Buncrana, rising from the horizon in north-east, at this and at succeeding observation. Rain in south.

TABLE XXXIII. (*continued*). SELECTED OBSERVATIONS.

Station.	October 23, 9 A. M.						October 23, 9 P. M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	N. E.	2	1·489	47°·0	4	·24	N.	2	1·439	47°·3	47°·0	43°·0
Buncrana, . .	N. N. E.	2	1·543	40 ·6	6	·68	N. N. E.	4	1·512	44 ·6	—	38 ·0
Donaghadee, .	N. E.	3	1·422	47 ·6	5	·13	N. N. W.	2	1·360	44 ·4	50 ·0	42 ·6
Killybegs, . .	N.	—	1·540	43 ·3	—	·27	N.	—	1·464	46 ·8	49 ·2	42 ·0
Armagh, . . .	N. N. W.	1·1	1·478	40 ·5	2	·23	W. N. W.	1·1	1·414	41 ·8	47 ·8	38 ·5
Markree, . . .	N.	3	1·552	44 ·9	9	·33	N.	3	1·469	44 ·3	48 ·2	41 ·0
Dublin, ...	N. W.	2	1·454	44 ·4	1	·05	—	3	1·407	43 ·1	50 ·5	39 ·3
Courtown, . .	N. W.	—	1·445	44 ·0	2	·08	N. W.	—	1·413	43 ·0	51 ·0	42 ·2
Kilrush, . . .	N. N. W.	5	1·628	47 ·3	—	·27	N.	4	1·593	45 ·4	49 ·0	44 ·2
Dunmore, . .	N. W.	3	1·470	45 ·8	—	·18	N. W.	3	1·425	43 ·3	48 ·0	41 ·5
Cahirciveen,	N.	3	1·619	49 ·8	10	·57	N. E.	3	1·621	46 ·8	48 ·0	45 ·6
Castletownsend,	N.	1	1·610	48 ·5	9	·00	N.	1	1·544	44 ·5	52 ·0	46 ·0

Station.	November 18, 9 P. M.						November 19, 9 A. M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. E.	2	1·169	47°·8	55°·0	43°·0	S. E.	2	0·408	47°·5	9	·67
Buncrana, . . .	S. S. E.	5	1·205	46 ·8	—	42 ·0	S.	3	0·411	45 ·7	10	·88
Donaghadee, .	S. E.	4	1·195	48 ·5	—	41 ·5	S.	5	0·438	48 ·9	10	·59
Killybegs, . . .	S. E.	5	0·991	49 ·5	54 ·0	43 ·8	N.	—	0·307	44 ·9	—	·50
Armagh, . . .	S. E.	2 ·3	1·055	50 ·0	54 ·5	39 ·0	S. S. E.	3·4	0·248	44 ·5	10	·70
Markree, . . .	S. E.	5	0·813	53 ·3	54 ·4	35 ·9	N. N. W.	3	0·285	45 ·6	10	·61
Dublin, ...	S. S. E.	2	1·126	55 ·4	58 ·2	44 ·0	S.	5	0·383	45 ·2	10	·51
Courtown, . .	S. S. W.	3	1·214	55 ·2	57 ·0	42 ·0	S. S. W.	5	0·589	48 ·0	10	·70
Kilrush, ...	S. E.	5	0·916	48 ·6	55 ·6	46 ·0	W. N W.	6	0·542	56 ·2?	10	2·40
Dunmore, . . .	S. W.	5	1·146	49 ·8	53 ·5	46 ·0	W.	6	0·671	48 ·8	—	·72
Cahirciveen, .	S. W.	5	0·923	55 ·8	58 ·5	45 ·8?	W. N. W.	5	0·825	48 ·2	10	·97
Castletownsend,	S. W.	5	1·148	54 ·5	56 ·0	42 ·0	N. W.	5	0·864	47 ·0	10	1·00

REMARKS.

Oct. 23, 9 A. M.—Squally day. Showers on west coast; hail at Markree.
Oct. 23, 9 P. M.—Showers throughout, except south-eastern quarter; hail at Buncrana and Markree.
Nov. 18, 9 P. M.—Rapid fall of barometer. Continued rain throughout the island.
Nov. 19, 9 A. M.—Surface of mercury *concave* at Dublin. Rain throughout; heavy at Kilrush and Castletownsend. Spray from the sea supposed to have reached the gauge at Kilrush.

Table XXXIII. (*continued*). Selected Observations.

Station.	November 19, 9 P. M.						November 20, 9 A. M.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Max.	Min.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	N. E.	3	0·587	49°·5	52°·0	44°·0	N. E.	3	1·113	49°·6	6	·31
Buncrana, . . .	N.	3	0·610	47 ·5	—	44 ·0	N. N. E.	4	1·172	44 ·4	7	·16
Donaghadee, .	N. E.	4	0·410	46 ·4	50 ·0	43 ·0	N. E.	4	1·001	49 ·0	10	·23
Killybegs, . . .	N.	—	0·714	49 ·3	51 ·8	44 ·0	N.	—	1·214	47 ·5	—	·04
Armagh, . . .	N. W.	2·6	0·571	46 ·0	54 ·0	44 ·0	N. N. W.	2·0	1·096	48 ·5	9	·21
Markree, . . .	N. W.	4	0·694	47 ·6	50 ·8	40 ·2	N. W.	5	1·245	48 ·0	10	·14
Dublin, ...	W.	4	0·549	47 ·9	50 ·5	45 ·7	N. N. W.	3	1·003	48 ·5	9	·17
Courtown, . .	W. S. W.	4	0·585	47 ·5	51 ·0	49 ·0?	N. N. W.	2	0·947	47 ·0	6	·40
Kilrush, ...	W. N. W.	5	0·794	45 ·3	—	44 ·5	N. W.	6	1·354	47 ·3	10	·90
Dunmore, . . .	W. N. W.	5	0·711	46 ·8	49 ·5	46 ·5	N. W.	4	1·071	48 ·8	—	·25
Cahirciveen, . .	W. N. W.	5	1·147	48 ·8	50 ·0	47 ·0	N.	5	1·435	46 ·2	10	·10
Castletownsend,	W.	3	0·949	45 ·5	56 ·0	47 ·0	N.	3	1·240	48 ·5	8	·33

Station.	November 23, 9 P. M.						November 24, 9 A. M.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Max.	Min.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	S. W.	3	1·235	43°·4	52°·0	44°·0	S. W.	2	0·741	48°·2	9	·30
Buncrana, . . .	W.	2	1·197	42 ·1	—	44 ·2	S. W.	2	0·770	47 ·8	9	·20
Donaghadee, .	S. S. W.	1	1·362	44 ·0	50 ·0	46 ·0	S. W.	1	0·775	48 ·9	9	·26
Killybegs, . . .	S. W.	1	1·243	46 ·5	52 ·5	47 ·3	S. W.	3	0·684	50 ·5	—	·25
Armagh, . . .	S. S. W.	2·6	1·294	42 2	52 ·0	41 ·1	S. W.	2·9	0·723	50 ·0	10	·26
Markree, . . .	S.	3	1·210	42 ·9	51 ·4	42 ·5	S. W.	3	0·605	49 ·6	8	·37
Dublin, ...	S.	3	1·389	43 ·4	57 ·3	45 ·5	S. W.	2	0·797	51 ·5	6	·25
Courtown, . .	S. S. W.	1	1·446?	43 ·0	56 ·5	42 ·0	S. W.	3	0·870	54 ·0	10	·46
Kilrush, ...	S. S. E.	3	1·247	48 ·3	56 ·0	47 ·5	W. S. W.	5	0·718	52 ·3	10	·25
Dunmore, . . .	S. W.	4	1·380	53 ·3	53 ·5	44 ·5	S. W.	5	0·895	52 ·8	—	·40
Cahirciveen, .	S. W.	4	1·215	52 ·8	57 ·5	47 ·4	W. S. W.	5	0·834	51 ·6	8	·61
Castletownsend,	S. W.	5	1·376	49 ·5	55 ·0	47 ·0	W. S. W.	5	0·922	53 ·5	10	·52

Remarks.

Nov. 19, 9 P. M.—Showers throughout, except south-eastern quarter.
Nov. 23, 9 P. M.—Rain on west coast.
Nov. 24, 9 A. M.—Showers in various places; thunder and lightning at Cahirciveen.
Nov. 24, 9 P. M.—Showers throughout the island.

Table XXXIII. (*continued*). Selected Observations.

Station.	November 24, 9 p. m.						December 14, 9 a. m.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. W.	4	0·568	47°·9	52°·0	41°·0	S.	4	1·164	40°·2	9	·03
Buncrana, . .	N. W.	5	0·498	46 ·6	—	41 ·1	S. S. W.	5	1·079	42 ·6	10	·06
Donaghadee, .	W.	3	0·586	47 ·4	51 ·4	43 ·1	S. S. W.	4	1·353	42 ·4	10	·16
Killybegs, . . .	W. N. W.	5	0·715	48 ·5	52 ·3	45 ·0	S.	6	0·948	47 ·5	—	·06
Armagh, . . .	W. S. W.	2·6	0·690	45 ·2	51 ·8	42 ·0	S. S. E.	4·1	1·185	42 ·2	10	.03
Markree, . . .	W. S. W.	5	0·777	42 ·6	50 ·8	41 ·2	S. S. W.	6	0·955	44 ·6	10	·23
Dublin, ...	S. W.	5	0·776	45 ·4	56 ·5	44 ·2	S. S. E.	2	1·347	47 ·4	10	·04
Courtown, . .	W. S. W.	3	0·850	45 ·0	55 ·5	40 ·0	S. W.	3	1·429	39 ·0	10	·00
Kilrush, ...	W. N. W.	5	0·914	44 ·8	53 ·5	48 ·5	S. W.	5	1·174	43 ·3	10	·55
Dunmore, . . .	W.	5	0·947	52 ·3?	50 ·0	49 ·5	S. W.	4	1·386	49 ·3	—	·23
Cahirciveen, .	W.	5	—	47 ·8	56 ·0	47 ·0	W.	5	1·242	45 ·2	8	·50
Castletownsend,	W.	5	1·013	48 ·5	55 ·0	46 ·0	S. W.	5	1·297	50 ·5	10	·26

Station.	December 14, 9 p. m.						December 15, 9 a. m.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, · · ·	S. W.	3	1·055	38°·6	47°·0	37°·0	S. W.	4	1·055	40°·0	3	·32
Buncrana, . . .	W. S. W.	3	0·982	38 ·6	—	38 ·0	W.	4	1·026	39 ·6	4	·41
Donaghadee, .	W. S. W.	3	1·118	39 ·4	48 ·0	40 ·0	W.	3	1·137	37 ·4	8	·41
Killybegs, . . .	W.	5	1·104	42 ·3	45 ·0	38 ·0	W.	5	1·146	42 ·3	—	·43
Armagh, . . .	S. W.	3·4	1·163	36 ·7	46 ·8	34 ·0	S. W.	3·0	1·163	37 ·5	9	·48
Markree, . . .	S. W.	3	1·156	36 ·2	45 ·7	37 ·8	S. W.	1	1·061	39 ·6	9	·30
Dublin, ...	S. S. W.	4	1·276	38 ·4	52 ·8	41 ·0	S. S. W.	2	1·255	42 ·4	10	·58
Courtown, . .	S. W.	3	1·314	37 ·5	41 ·5	36 ·0	S. W.	1	1·316	40 ·0	10	·61
Kilrush, ...	W. S. W.	5	1·326	39 ·8	45 ·0	41 ·0	W. S. W.	2	1·294	46 ·3	10	·40
Dunmore, . . .	N. W.	5	1·396	40 ·8	51 ·0	41 ·5	S. W.	2	1·310	46 ·3	—	·25
Cahirciveen, .	W.	5	1·404	42 ·8	43 ·0	41 ·0	S. W.	2	1·213	47 ·4	10	·23
Castletownsend,	W. S. W.	5	1·409	44 ·5	51 ·0	42 ·0	S. W.	3	1·300	46 ·5	10	·69

Remarks.

Nov. 24, 9 p. m.—Gale from 6 p. m. to 9 p. m. at Markree.
Dec. 14, 9 a. m.—Thunder and lightning at Cahirciveen and Kilrush.
Dec. 14, 9 p. m.—Thunder and lightning throughout the island. Hail fell in several places during the day.

Table XXXIII. (*continued*). Selected Observations.

Station.	December 31, 9 A. M.						December 31, 9 P. M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	3	1·430	44°·4	9	·12	S.	3	1·173	50°·0	56°·0	40°·0
Buncrana, . . .	S. S. W.	4	1·400	47 ·6	10	·00	S. S. W.	4	1·180	48 ·6	—	42 ·0
Donaghadee, .	S. S. W.	3	1·525	44 ·9	10	·04	S. W.	2	1·291	49 ·4	52 ·0	41 ·5
Killybegs, . .	S. S. W.	4	1·317	50 ·2	—	·06	S. S. W.	5	1·171	49 ·4	52 ·0	44 ·0
Armagh, . . .	S.	3·8	1·463	47 ·0	10	·05	S.	3 0	1·264	51· 1	53 ·8	46 ·0
Markree, . . .	S.	4	1·311	50 ·2	10	·11	S. S. W.	3	1·204	50 ·2	54 ·2	38 ·6
Dublin, ...	S.	2	1·543	51 ·9	10	·03	S.	3	1·306	54 ·9	57 ·8	48 ·5
Courtown, . .	S.	3	1·631	50 ·5	10	·00	S. S. W.	5	1·399	53 ·2	55 ·0	48 ·0
Kilrush, ...	S. W.	3	1·455	52 ·3	10	·15	S. S. W.	5	1·270	51 ·3	—	42 ·5
Dunmore, . . .	S. W.	3	1·606	51 ·8	—	·02	S. S. W.	5	1·405	52 ·8	52 ·5	47 ·0
Cahirciveen, .	W.	5	1·405	55 ·2	10	·00	S. W.	5	1·314	51 ·6	56 ·0	51 ·0
Castletownsend,	S. S. W.	5	1·580	52 ·0	10	·10	S. W.	6	1·379	53 ·5	54 ·0	48 ·5

Station.	1851. January 1, 9 A. M.						January 1, 9 P. M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. W.	3	1·282	46°·4	4	·12	S.	1	0·949	47°·0	51°·0	43° 0
Buncrana, . . .	S. S. W.	2	1·298	44 ·6	8	·08	N. N. W.	2	1·000	43 ·6	—	43 ·0
Donaghadee, .	W. S. W.	2	1·370	47 ·4	9	·08	S. W.	4	1·055	49 ·9	53 ·0	45 ·2
Killybegs, . . .	S. W.	2	1·285	47 ·5	—	·15	W. N. W.	1	1·066	43 ·5	47 ·2	45 ·0
Armagh, . . .	S. W.	3·8	1·383	48 ·0	10	·29	S.	3·8	0·998	53 ·1	54 ·5	41 ·5
Killough, . . .	S. W.	3	1·444	48 ·3	7	·00	S.	6	1·108	47 ·5	52 ·0	48 ·0
Markree, . . .	S.	2	1·319	44 ·6	10	·38	N. W.	3	1·078	42 ·7	48 ·3	44 ·0
Dublin, ...	S. S. W.	3	1·421	51 ·5	10	·20	S.	4	1·109	56 ·5	58 ·0	52 ·2
Courtown, . .	S. S. W.	3	1·455	51 ·5	10	·15	S.	5	1·230	53 ·5	55 ·5	50 ·0
Kilrush, ...	S. W.	2	1·503	50 ·3	10	·08	S. S. W.	5	1·029	53 ·3	—	49 ·0
Dunmore, . . .	S. W.	3	1·417	51 ·8	—	·15	S.	5	1·210	53 ·8	54 ·0	49 ·5
Cahirciveen, .	S. W.	3	1·427	50 ·8	10	·60	W.	2	—	46 ·8	56 .0	49 ·4
Castletownsend,	S. W.	5	1·463	51 ·5	10	·55	S. S. W.	6	1·144	52 ·5	55 ·0	48 ·0

Remarks.

Dec 31, 9 A. M.—Rain on west coast.

Dec. 31, 9 P. M.—Rain in south; light rain on the east coast. Lightning observed at Killybegs. At Markree gale abated at 4 P.M.

Jan. 1, 9 A. M.—Lightning observed at Markree. Rain in south-west.

Jan. 1, 9 P. M.—Rain throughout; continued rain in north.

Table XXXIII. (*continued*). Selected Observations.

Station.	January 12, 9 p.m. Wind. Direction.	Force	Barom.	Therm.	Max.	Min.	January 13, 9 a.m. Wind. Direction.	Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	S.	3	1·697	42°·5	49°·0	42°·0	S.	4	1·283	48°·4	10	·27
Buncrana, . . .	S.	4	1·669	43 ·4	—	42 ·0	S. S. W.	5	1·254	49 ·4	10	·35
Donaghadee, .	S.	3	1·800	44 ·4	49 ·5	45 ·5	S. S. W.	4	1·407	47 ·6	10	·01
Killybegs, . . .	S.	6	1·535	45 ·3	51 ·2	42 ·2	S.	3	1·157	49 ·6	—	·05
Armagh, . . .	S.	2·9	1·720	43 ·2	50 ·0	40 ·8	S.	4·5	1·327	49 ·0	10	·22
Killough, . . .	S.	3	1·776	46 ·8	53 ·0	43 ·0	S. W.	5	1·377	49 ·3	8	·00
Markree, . . .	S. S. E.	4	1·560	43 ·2	40 ·8	36 ·6	S.	5	1·190	49 ·5	10	·05
Dublin, ...	S. S. E.	3	1·797	44 ·7	53 ·0	42 ·6	S. S. E.	3	1·455	51 ·2	9	·00
Courtown, . .	S.	2	1·830	45 ·0	51 ·5	42 ·0	S.	3	1·507	50 ·5	10	·00
Kilrush, ...	S. W.	3	1·651	47 ·3	51 ·0	42 ·5	S. W.	5	1·543?	49 ·8	10	·30
Dunmore, . . .	S. S. W.	3	1·801	46 ·8	50 ·5	41 ·5	S. W.	5	1·463	50 ·3	—	·00
Cahirciveen, .	S. W.	4	1·628	48 ·6	52 ·0	44 ·0	S. W.	5	1·244	52 ·2	10	1·03
Castletownsend,	S. W.	5	1·719	47 ·5	51 ·0	42 ·5	S. S. W.	6	1·347	50 ·0	10	·10

Station.	January 15, 9 a.m. Wind. Direction.	Force	Barom.	Therm.	Cloud.	Rain.	January 15, 9 p.m. Wind. Direction.	Force	Barom.	Therm.	Max.	Min.
Portrush, . . .	S. E.	2	0·967	43°·2	10	·76	W.	3	1·212	44°·7	47°·0	41°·0
Buncrana, . . .	S. E.	2	0·970	44 ·1	10	·55	N. N. W.	4	1·186	42 ·6	—	43 ·0
Donaghadee, .	S. E.	4	0·996	44 ·9	10	·82	N. W.	3	1·218	43 ·9	47 ·5	44 ·6
Killybegs, . . .	E. S. E.	0	0·949	43 ·8	—	·68	W. N. W.	3	1·321	44 ·0	46 ·0	42 ·5
Armagh, . . .	E. S. E	1·5	0·946	44 ·9	10	·76	W.	1·5	1·312	41 ·2	47 0	36 ·0
Killough, . . .	S. E.	3	0·980	45 ·9	7	·61	N. W.	3	1·349	42 ·5	48 ·0	45 ·0
Markree, . . .	N. W.	2	0·906	42 ·4	10	·78	S. W.	2	1·365	37 ·5	47 ·6	40 8
Westport, . . .	N. E.	5	0·972	42 ·0	—	·56	—	—	—	—	—	—
Dublin, ...	E.	2	0·868	47 ·4	10	·81	S. W.	3	1·361	40 ·5	49 ·5	44 ·8
Courtown, . . .	E.	3	0·807	48 ·2	10	·90	W. S. W.	2	1·424	40 ·0	50 ·0	42 ·5
Kilrush, ...	N.	5	0·862	39 ·8	10	1·06	W.	3	1·428	39 ·8	51 ·5	39 ·5
Dunmore, . . .	E. S. E.	4	0·718	47 ·8	—	·98	W. S. W.	1	1·485	39 ·8	49 ·5	45 ·5
Castletownsend,	N.	5	0·827	40 ·5	10	1·15	N. W.	2	1·499	40 ·5	52 ·0	41 ·0

Remarks.

Jan. 12, 9 p.m.—Showers on west coast. Lunar halo observed at Buncrana, Donaghadee, and Courtown.

Jan. 13, 9 a.m.—Rain in several places, but not universal.

Jan. 15, 9 a.m.—Rain throughout the island.

Jan. 15, 9 p.m.—At Markree, gale lasted from 1 p.m. to 6 p.m. Lunar halo observed at Markree and Donaghadee.

Table XXXIII. (*continued*). Selected Observations.

Station.	January 16, 9 A.M.						January 16, 9 P.M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	3	1·138	40°·3	10	·28	S.	3	0·723	49°·0	52°·0	35°·0
Buncrana, . . .	S.	5	1·087	41 ·6	10	·35	S. S. W.	5	0·671	48 ·1	—	37 ·0
Donaghadee, .	S.	6	1·254	43 ·4	10	·49	S. W.	4	0·872	48 ·4	50 ·1	39 ·0
Killybegs, . .	S. E.	5	0·925	45 ·3	—	·28	W. N. W.	6	0·772	48 ·0	54 ·2	38 ·5
Armagh, . . .	S. S. E.	5·0	1·072	44 ·1	10	·61	S. W.	4·5	0·858	47 ·8	52 ·2	38 ·0
Killough, . . .	S.	6	1·153	45 ·4	10	·42	S. W.	6	0·948	48 ·3	51 ·0	40 ·0
Markree, . . .	S. E.	5	0·879	46 ·8	10	·31	S. W.	5	0·876	45 ·7	52 ·0	33 ·7
Westport, . . .	S.	6	0·812	54 ·0	—	·69	—	—	—	—	—	—
Dublin, ...	S. E.	4	1·182	47 ·1	10	·73	S.	4	0·966	49 ·3	56 ·0	39 ·7
Courtown, . .	S.	5	1·209	47 ·5	10	·54	S. S. W.	3	1·059	48 ·0	53 ·5	33 ·0
Kilrush, . . .	S. W.	5	0·922	50 ·8	10	·50	S. W.	5	0·980	48 ·8?	52 ·0	39 ·5
Dunmore, . . .	S.	5	1·144	48 ·8	—	·96	W. S. W.	5	1·112	47 ·3	51 ·5	38 ·0
Cahirciveen, .	S. S. W.	5	0·863	54 ·0	10	·00	W.	5	1·041	46 ·0	55 ·0	—
Castletownsend,	S. W.	5	0·857	51 ·5	10	·73	W. S. W.	6	1·093	48 ·5	54 ·0	40 ·0

Station.	January 27, 9 P.M.						January 28, 9 A.M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. E.	3	1·456	45°·7	47°·0	33°·0	S.	1	1·547	38°·8	2	·12
Buncrana, . . .	S.	5	1·378	45 ·6	46 ·0	35 ·0	S. S. W.	3	1·506	39 ·6	4	·03
Donaghadee, .	S. S. E.	5	1·575	45 ·6	47 ·0	36 ·5	S. W.	1	1·634	39 ·9	2	·22
Killybegs, . . .	S.	5	1·285	45 ·7	47 ·5	38 ·0	W. S. W.	3	1·487	42 ·7	—	·03
Armagh, . . .	S.	3·8	1·439	45 ·9	46 ·0	35 ·0	S.	3·0	1·588	39 ·4	0	·08
Killough, . . .	S.	3	1·542	44 ·5	45 ·0	36 ·0	S. W.	3	1·665	41 ·6	2	·18
Markree, . . .	S.	4	1·333	41 ·4	45 ·8	29 ·8	S. E.	2	1·507	38 ·6	10	·23
Westport, . . .	—	—	—	—	—	—	S. W.	3	1·461	46 ·0	—	·48
Dublin, ...	S. E.	2	1·516	45 ·5	49 ·0	35 ·0	S.	1	1·645	40 ·9	6	·15
Courtown, . .	S.	3	1·542	43 ·8	49 ·0	31 ·2	S. W.	1	1·690	37 ·0	3	·34
Kilrush, ...	S. W.	5	1·329	41 ·8	48 ·0	37 ·5	S.	3	1·514	42 ·3	10	·23
Dunmore, . . .	S. S. W.	5	1·516	46 ·3	47 ·0	35 ·0	S. W.	2	1·658	47 ·3	—	·25
Cahirciveen, . .	S. W.	5	1·349	43 ·8	49 ·6	36 ·6	S. W.	4	1·470	44 ·4	10	·67
Castletownsend,	W. S. W.	5	1·452	44 ·5	50 ·0	37 ·0	S W.	5	1·620	44 ·5	10	·33

Remarks.

Jan, 16, 9 A. M.—Lightning at Westport. Rain throughout the island.
Jan. 16, 9 P. M.—Lunar halo at Dublin and Courtown. Showers at various places.
Jan. 27, 9 P. M.—Rain at both extremities of the island; hail in south. Gale lasted from noon to 3 P. M. at Castletownsend, from 4 P. M. to 7 P. M. at Markree.

Table XXXIII. (*continued*). Selected Observations.

Station.	January 30, 9 p. m.						January 31, 9 a. m.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Max.	Min.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	S. W.	1	1·162	35°·1	39°·0	34°·0	E.	1	1·168	33°·4	6	·00
Buncrana, . . .	S. S. W.	2	1·217?	33 ·6	39 ·0	35 ·0	S. E.	3	1·090	36 ·6	8	·00
Donaghadee, .	W. S. W.	1	1·189	32 ·6	40 ·0	35 ·0	S. S. E.	1	1·175	35 .4	9	·01
Killybegs, . . .	S. W.	3	1·128	37 ·5	41 ·5	36 ·0	S. E.	2	1·082	38 ·7	—	·05
Armagh, . .	S. W.	1·4	1·185	31 ·9	38 ·8	33 ·0	S. S. E.	0·5	1·137	35 ·9	10	·01
Killough, . . .	W.	1	1·191	41 ·8?	47 ·0	37 ·0	S. E.	1	1·168	42 ·3	8	·00
Markree, . . .	S. W.	1	1·149	32 ·3	38 ·7	32 ·7	S. E.	1	1·058	34 ·9	10	·26
Westport, . . .	—	—	—	—	—	—	N.	2	1·032	44 ·0	—	·50
Dublin, ...	S. W.	2	1·220	35 ·0	42 ·5	35 ·5	S. E.	0	1·127	35 ·9	6	·08
Courtown, . . .	W. S. W.	2	1·221	34 ·0	41 ·0	32 ·7	S.	1	1·124	35 ·0	4	·07
Kilrush, ...	W. N. W.	3	1·224	38 ·3	44 ·0	35 ·5	W.	2	1·100	42 ·8	10	·24
Dunmore, . . .	W.	2	1·215	36 ·8	41 ·5	35 ·5	W.	2	1·157	38 ·8	—	·07
Cahirciveen, . .	W. N. W.	3	1·255	41 ·6	45 ·0	37 ·8	W. N. W.	3	1·070	43 ·6	9	·25
Castletownsend,	W. S. W.	4	1·338	38 ·5	42 ·5	37 ·0	W. S. W.	2	1·184	41 ·5	10	·33

Station.	January 31, 9 p. m.						March 18, 9 a. m.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Max.	Min.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	N.	2	1·440	40°·7	44°·0	30°·0	S. E.	3	1·395	42°·9	10	·09
Buncrana, . . .	S. E.	1	1·484	34 ·8	41 ·0	33 ·0	S.	4	1·373	41 ·6	10	·27
Donaghadee, .	N. E.	3	1·441	40 ·9	44 ·0	31 ·5	S. S. E.	4	1·475	44 ·6	10	·13
Killybegs, . . .	N. E.	0	1·525	36 ·5	43 ·3	34 ·8	W. N. W.	4	1·325	41 ·3	—	·22
Armagh, . . .	N.	1·2	1·485	32 ·6	40 ·8	31 ·0	S. E.	3·5	1·328	45 ·1	10	·23
Killough, . . .	N. E.	0	1·460	34 ·4	44 ·0	32 0	S.	3	1·460	45 ·3	10	·26
Markree, . . .	N.	2	1·531	35 ·3	42 ·1	29 ·8	S. W.	3	1·331	45 ·6	9	·25
Westport, . . .	—	—	—	—	—	—	W.	6	1·366	47 ·0	—	·56
Dublin, ...	N.	3	1·382	40 ·0	44 ·8	34 ·3	S. W.	3	1·381	50 ·9	10	·39
Courtown, . .	N. E.	4	1·288	40 ·7	44 ·5	32 ·5	S W.	3	1·460	50 ·7	10	·33
Kilrush, ...	N. N. E.	2	1·380	38 ·8	44 ·0	34 ·5	W. S. W.	5	1·465	44 ·8	8	·20
Dunmore, . . .	E.	4	1·258	41 ·8	44 ·0	37 ·0	W. S. W.	4	1·504	48 ·8	—	·26
Cahirciveen, .	N. E.	4	1·268	42 ·4	46 ·2	37 ·0	W. S. W.	4	1·458	48 ·2	6	·40
Castletownsend,	N.	2	1·243	43 ·5	48 ·0	36 ·0	W. N. W.	5	1·523	50 ·5	8	·24

Remarks.

Jan. 30, 9 p. m.—Lightning at Markree and Buncrana. Rain, snow, and hail in west.
Jan. 31, 9 a. m.—Lightning observed in north in the evening. Light rain on west coast. Hail and sleet in some places.
March 18, 9 a. m.—Rain throughout island, south-eastern quarter excepted.

TABLE XXXIII. (*continued*). SELECTED OBSERVATIONS.

Station.	March 18, 9 P. M.						March 19, 9 A. M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. W.	3	1·371	42°·4	49°·0	39°·0	S. W.	3	1·449	44°·3	4	·46
Buncrana, . . .	W. S. W.	3	1·376	41 ·1	50 ·0	40 ·0	S. W.	2	1·395	46 ·6	5	·31
Donaghadee, .	W.	3	1·454	41 ·4	50 ·5	41 ·5	W. S. W.	2	1·520	44 ·4	9	·46
Killybegs, . . .	W.	5	1·408	45 ·0	49 ·5	41 ·0	W. S. W.	0	1·450	47 ·1	—	·00
Armagh, . . .	S. S. W.	2·5	1·470	40 ·8	49 ·5	40 ·0	S.	2	1·487	44 ·5	10	·25
Killough, . . .	S. W.	3	1·500	43 ·3	50 ·0	41 ·0	W.	3	1·510?	44 ·5	2	·18
Markree, . . .	S. E.	3	1·465	42 ·7	48 ·7	39 ·5	S. E.	2	1·386	46 ·3	10	·14
Westport, . . .	W.	5	1·495	47 ·0	—	—	S.	2	1·428	47 ·0	—	·14
Dublin, ...	S. S. W.	3	1·557	43 ·7	54 ·3	41 ·5	S. E.	0	1·518	46 ·5	10	·03
Courtown, . .	S. W.	1	1·608	41 ·2	55 ·0	41 ·5	S.	1	1·534	46 ·5	10	·00
Kilrush, ...	W.	4	1·608	46 ·8	48 ·5	40 ·5	S. E.	2	1·411	46 ·8	10	06
Dunmore, . . .	W.	3	1·634	45 ·3	57 ·5	43 ·0	S. S. W.	2	1·504	47 ·3	—	·16
Cahirciveen, .	W. S. W.	3	1·671	45 ·6	51 ·0	46 ·4	E. S. E.	1	1·345	46 ·4	10	·32
Castletownsend,	W.	3	1·662	47 ·5	53 ·0	46 ·5	S. W.	5	1·393	47 ·5	10	·10

Station.	March 19, 9 P. M.						March 25, 9 A. M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. E.	3	1·121	44°·1	51°·0	40°·0	S. E.	3	1·611	44°·2	10	·01
Buncrana, . . .	S.	3	1·112	44 ·4	52 ·0	41 ·0	S.	3	1·579	45 ·4	9	·00
Donaghadee, .	S.	3	1·170	45 ·4	50 ·3	40 ·5	S S. E	3	1·676	45 ·4	10	·00
Killybegs, . . .	S. S. E.	6	0·953	45 ·3	50 ·3	42 ·3	S. E.	3	1·437	46 ·0	—	·10
Armagh, . . .	S.	2·8	1·108	43 ·0	49 ·5	39 ·0	S. E.	2·5	1·590	45 ·5	10	·01
Killough, . . .	S.	3	1·240	45 ·4	51 ·0	41 ·0	S.	2	1·677	46 ·4	10	·00
Markree, . . .	S.	5	0·900	41 ·3	47 ·6	38 ·0	W. S. W.	4	1·444	46 ·2	10	·20
Westport, . . .	S. W.	6	0·773	45 ·0	—	—	W.	3	1·408	50 ·0	—	·35
Dublin, ...	S.	3	1·143	44 ·5	49 ·5	42 ·0	S. E.	2	1·592	45 ·9	10	·25
Courtown, . .	S. S. W.	3	1·180	45 ·8	51 ·5	39 ·8	S. E.	3	1·599	47 ·0	10	·34
Kilrush, . . .	S. W.	5	1·014	43 ·8	52 ·0	44 ·5	W. S. W.	3	1·513	50 ·3	10	·34
Dunmore, . . .	S. S. W.	4	1·207	46 ·8	50 ·5	47 ·5?	S. S. W.	1	1·487	49 ·8	—	·56
Cahirciveen, .	W. S. W.	4	1·069	45 ·6	53 ·4	45 ·2	W.	3	1·460	51 ·8	9	·68
Castletownsend,	W.	5	1·129	48 ·5	52 ·5	46 ·0	S. W.	3	1·520	52 ·5	10	·30

REMARKS.

March 18, 9 P. M.—Lightning observed at Markree, Buncrana, and Donaghadee. Rain in the north-west. Lunar halo observed at Armagh.

March 19, 9 A. M.—At 2 P. M. the wind shifted to S. S. E. at Donaghadee, and to E. S. E. at Killybegs; the shift being followed by a gale. Rain in the south. Solar halo observed at Armagh.

March 25, 9 A. M.—Light rain throughout, south-western quarter excepted.

Table XXXIII. (*continued*). Selected Observations.

Station.	June 11, 9 P. M.						June 12, 9 A. M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	2	1·748	51°·3	56°·0	46°·0	N. E.	3	1·455	49°·1	10	·58
Buncrana, . . .	N.	3	1·689	51 ·6	63 ·0	41 ·0	S. E.	5	1·466	52 ·6	8	·34
Donaghadee, .	S.	2	1·806	50 ·6	59 ·5	44 ·6	S. W.	1	1·425	57 ·6	10	·54
Killybegs, . . .	S. E.	5	1·683	49 ·2	67 ·0	47 ·2	E.	0	1·443	56 ·4	—	·64
Armagh, . . .	S. E.	3	1·710	48 ·5	60 ·0	42 ·3	N. W.	0·5	1·407	55 ·3	10	·76
Killough, . . .	S.	3	1·760	52 ·7	64 ·0	45 ·0	S.	3	1·531	54 ·4	10	·96
Markree, . . .	E.	4	1·523	48 ·0	61 ·4	43 ·1	N. E.	2	1·381	57 ·7	10	·49
Westport, . . .	S. E.	3	1·614	54 ·0	—	—	N. E.	2	1·396	54 ·0	—	1·04
Dublin, ...	S. E.	4	1·675	49 ·1	65 ·7	42 ·5	S.	1	1·401	66 ·4?	9	·28
Courtown, . . .	S. E.	4	1·695	49 ·5	61 ·0	39 ·2	S. W.	3	1·445	59 ·0	10	·85
Kilrush, ...	S.	2	1·463	54 ·8	59 ·0	44 ·5	S. W.	2	1·347	57 ·8	10	·35
Dunmore, . . .	S. S. E.	5	1·556	53 ·8	57 ·5	45 ·0	S. W.	2	1·440	56 ·3	—	1·00
Cahirciveen, . .	S.	3	1·412	56 ·4	—	41 ·4	S.	2	1·348	57 ·6	10	1·12
Castletownsend,	S. W.	5	1·493	55 ·0	57 ·0	41 ·0	S. W.	3	1·378	56 ·0	10	·85

Station.	June 12, 9 P. M.						June 15, 9 A. M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	N.	1	1·558	52°·0	57°·0	46°·0	S.	3	1·687	54°·1	10	·31
Buncrana, . . .	E. S. E.	2	1·593	48 ·1	59 ·0	46 ·0	S. W.	4	1·663	53 ·6	10	·19
Donaghadee, .	N. N. E.	3	1·510	50 ·2	59 ·6	48 ·0	S. S. W.	2	1·796	54 ·4	10	·47
Killybegs, . . .	N.	2	1·624	51 ·5	58 ·8	47 ·0	S. W.	3	1·661	55 ·2	—	·09
Armagh, ...	N. N. W.	0·5	1·557	50 ·4	58 ·0	47 ·0	S.	3	1·737	54 ·8	10	·10
Killough, . . .	N.	3	1·474	52 ·7	65 ·0	46 ·0	S.	3	1·817	52 ·7	9	·08
Markree, . . .	N. N. W.	1	1·622	48 ·5	59 ·0	48 ·6	S. S. W.	4	1·694	57 ·1	10	·10
Westport, . . .	N.	2	1·676	51 ·0	—	—	W.	5	1·681	57 ·0	—	·06
Dublin, ...	N. W.	2	1·479	59 ·5	66 ·7	48 ·5	S. S. W.	1	1·842	58 ·4	10	·00
Courtown, . . .	S. W.	1	1·487	57 ·0	61 ·5	49 ·3	S. S. W.	2	1·900	56 ·3	10	·02
Kilrush, ...	N.	1	1·563	53 ·8	65 ·0	45 ·5	W. S. W.	5	1·764	55 ·8	10	·18
Dunmore, . . .	W.	2	1·521	54 ·8	57 ·5	50 ·5	S. W.	4	1·888	56 ·3	—	·03
Cahirciveen, . .	N.	3	1·625	53 ·0	—	49 ·2	S. W.	3	1·811	56 ·4	10	·67
Castletownsend,	W.	2	1·555	56 ·5	61 ·5	49 ·0	S. W.	5	1·874	55 ·5	10	·40

Remarks.

June 11, 9 P. M.—Heavy rain throughout the island.
June 12, 9 A. M.—Light rain at most of the stations.
June 12, 9 P. M.—Rain at Armagh, Killough, and Castletownsend; light rain at Donaghadee and Killybegs.
June 15, 9 A. M.—Rain throughout the island; heavy rain in south-west.

Table XXXIII. (*continued*). Selected Observations.

Station.	June 15, 9 P.M.						June 16, 9 A.M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. W.	3	1·586	53°·9	61°·0	51°·0	W.	5	1·727	53°·7	9	·14
Buncrana, . . .	W.	4	1·565	53 ·9	62 ·0	52 ·0	N. W.	5	1·785	55 ·4	8	·04
Donaghadee, .	W. S. W.	3	1·635	54 ·6	61 ·4	52 ·5	N. W.	5	1·742	57 ·4	9	·11
Killybegs, . . .	S. W.	5	1·658	54 ·5	61 ·2	52 ·4	W.	5	1·870	56 ·7	—	·10
Armagh, . . .	S. W.	2·8	1·655	54 ·9	63 ·4	52 ·5	W.	3·5	1·824	56 ·1	7	·08
Killough, . . .	S.	3	1·930?	52 ·8	64 ·0	51 ·0	S. W.	3	2·167?	55 ·7	4	·10
Markree, . . .	W.	3	1·679	52 ·8	62 ·4	52 ·4	W.	4	1·911	56 ·7	9	·01
Westport, . . .	W.	5	1·681	56 ·0	—	—	W.	5	1·909	55 ·0	—	·02
Dublin, ...	S. W.	3	1·730	55 ·0	64 ·2	53 ·0	S. W.	3	1·881	59 ·1	10	·16
Courtown, . .	W. S. W.	2	1·779	55 ·5	64 ·0	54 ·5	W.	4	1·890	57 ·5	4	·22
Kilrush, ...	W.	4	1·814	53 ·8	58 ·0	50 ·5	W. N. W.	5	1·813	55 ·8	4	·00
Dunmore, . . .	W. S. W.	2	1·826	54 ·3	60 ·0	55 ·0	W.	2	1·978	59 ·3	—	·26
Cahirciveen, .	W.	3	1·821	56 ·0	—	53 ·4	W.	3	2·048	57 ·3	2	·08
Castletownsend,	S. W.	4	1·872	56 ·0	60 ·0	53 ·0	W. S. W.	4	2·023	59 ·0	7	·42

Station.	July 13, 9 A.M.						July 13, 9 P.M.					
	Wind.		Barom.	Therm.	Cloud	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	3	1·503	60°·3	10	·08	S. W.	3	1·038	59°·9	66°·0	54°·0
Buncrana, . . .	S. W.	5	1·474	61 ·1	10	·15	W. S. W.	5	1·075	58 ·6	—	56 ·5
Donaghadee, .	S. W.	2	1·613	64 ·4	10	·05	S. W.	3	1·143	59 ·9	67 ·0	58 ·6
Killybegs, . . .	S. S. W.	5	1·501	60 ·0	—	·14	S. W.	5	1·042	58 ·1	—	—
Armagh, . . .	S. W.	3	1·568	62 ·7	10	·15	S. S. W.	4	1·091	59 ·0	65 ·3	58 ·6
Killough, . . .	S. W.	3	1·476	57 ·0	7	·10	S. W.	6	—	54 ·4	68 ·0	51 ·0
Markree, . . .	S. S.W.	4	1·479	61 ·1	10	·03	S.	4	1·032	57 ·0	65 ·8	58 ·4
Westport, . . .	S. W.	6	1·460	63 ·0	—	·25	W.	6	1·082	58 ·0	—	—
Dublin, ...	S.	4	1·656	70 ·6	7	·00	S. W.	3	1·209	62 ·5	73 ·2	60 ·5
Courtown, . .	S. W.	1	1·721	66 ·5	7	·00	S. W.	4	1·311	61 ·3	69 ·5	59 ·5
Kilrush, ...	S. W.	5	1·590	62 ·8	10	·00	S. W.	5	1·308	59 ·8	64 ·0	56 ·5
Dunmore, . . .	S. W.	2	1·756	65 ·8	—	·00	S. W.	3	1·345	60 ·8	62 ·0	59 ·5
Cahirciveen, .	S. W.	3	1·664	62 ·2	10	·00	W.	5	1·349	59 ·2	—	60 ·4
Castletownsend,	S. W.	3	1·779	63 ·0	10	·00	W.	5	1·423	58 ·5	65 ·0	59 ·0

Remarks.

June 15, 9 P.M.—Light rain, chiefly in north-west.

July 13, 9 A.M.—Rain in north and north-west.

July 13, 9 P.M.—Gale highest at 1 P.M. at Cahirciveen, and at 2 P.M. at Donaghadee. Rain at Portrush, Buncrana, Killough, and Westport; light rain at Dublin and Dunmore.

Table XXXIII. (*continued*). Selected Observations.

Station.	July 14, 9 A.M.						July 27, 9 A.M.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	W.	4	1·058	54°·3	10	·56	—	0	1·827	58°·0	9	·04
Buncrana, . . .	N. W.	6	1·182	53 ·6	10	·41	S. W.	1	1·884	61 ·1	7	·24
Donaghadee, .	W.	4	1·112	56 ·4	10	·36	W. S. W.	1	1·864	60 ·4	9	·02
Killybegs, . . .	W.	6	1·256	56 ·7	—	·36	S.	2	1·823	60 ·9	—	·09
Armagh, ...	W.	3	1·204	54 ·3	10	·47	S.	0·5	1·841	58 ·9	7	·00
Killough, . . .	W.	6	1·251	52 ·6	10	·15	W.	0	1·856	57 ·3	2	·01
Markree, . . .	W.	5	1·334	54 ·9	9	·45	S. E.	2	1·795	57 ·8	10	·02
Westport, . .	N. W.	6	1·376	58 ·0	—	·32	S. E.	3	1·745	54 ·0	—	·10
Dublin, ...	S. W.	4	1·312	60 ·9	10	·14	S. E.	1	1·854	60 ·9	4	·00
Courtown, . . .	W. S. W.	4	1·418	61 ·0	8	·21	E. S. E.	1	1·866	61 ·0	8	·00
Kilrush, ...	W. N. W.	5	1·513	57 ·8	5	·20	S.	2	1·757	55 ·8	10	·05
Dunmore, . . .	W.	3	1·465	61 ·3	—	·22	S.	2	1·828	63 ·8	—	·00
Cahirciveen, . .	W.	4	1·577	60 ·0	6	·48	S. S. W.	3	1·684	58 ·4	10	·14
Castletownsend,	W.	5	1·589	63 ·0	8	·50	E. S. E.	3	1·740	57 ·5	10	·23

Station.	July 27, 9 P.M.						July 28, 9 A.M.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Max.	Min.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	S. E.	2	1·626	56°·3	66°·0	53°·0	S. W.	3	1·553	60°·9	8	·36
Buncrana, . . .	S. S. E.	2	1·578	56 ·6	—	53 ·0	S. S. W.	2	1·551	62 ·6	8	·35
Donaghadee, .	S. E.	3	1·671	55 ·0	65 ·5	53 ·0	S. S. W.	2	1·603	63 ·4	10	·33
Killybegs, . . .	S. W.	3	1·594	59 ·3	—	—	W.	2	1·613	61 ·4	—	·06
Armagh, . . .	S. S. E.	1	1·607	57 ·9	63 6	48 ·4	S. S. W.	3·5	1·585	61 ·4	8	·50
Killough, . . .	S. W.	3	1·747	55 ·4	63 ·0	50 ·0	S. W.	3	1·625	60 ·4	6	·14
Markree, . . .	S.	2	1·559	57 ·7	65 ·4	45 ·6	S. S. W.	2	1·579	65 ·2	9	·59
Westport, . . .	W.	4	1·583	61 ·0	—	—	W.	3	1·580	59 ·0	—	·41
Dublin, ...	S. S. E.	3	1·664	63 ·2	71 ·4	50 ·0	S.	2	1·634	70 ·2	4	·16
Courtown, . . .	S. S. W.	2	1·713	61 ·2	66 ·3	49 ·5	S. S. W.	3	1·662	64 ·2	5	·12
Kilrush, ...	W. S. W.	3	1·630	61 ·8	64 ·0	48 ·5	W. S. W.	2	1·597	61 ·8	8	·32
Dunmore, . . .	S. W.	4	1·707	61 ·8	63 ·5	58 ·0	W. S. W.	2	1·671	62 ·8	—	·20
Cahirciveen, . .	S. W.	3	1·595	61 ·6	—	52 ·8	S· W.	3	1·597	62 ·4	10	·42
Castletownsend,	S. W.	5	1·446	61 ·0	65 ·0	52 ·5	S· W.	5	1·631	62 ·5	9	·19

Remarks.

July 14, 9 A. M.—Rain throughout the north.
July 27, 9 A. M.—Rain along west coast. Arch in N. W. observed at Markree.
July 27, 9 P. M.—Lightning seen in north-west at Donaghadee. Rain throughout island.
July 28, 9 A. M.—Solar eclipse in the afternoon of this day; clouds of slate colour, as observed at Markree, at time of greatest obscuration. Showers on west coast.

Table XXXIII. (*continued*). Selected Observations.

Station.	August 23, 9 p. m.						August 24, 9 a. m.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S. W.	2	1·684	54° 9	64°·0	55°·0	S. E.	1	1·593	52°·4	10	·14
Buncrana, . .	W. S. W.	3	1·663	55 ·6	—	56 ·0	S.	1	1·587	53 ·1	10	·18
Donaghadee, .	W. S. W.	2	1·717	54 ·9	67 ·0	55 ·0	E. S. E.	3	1·545	52 ·9	10	·59
Killybegs, . .	W.	4	1·728	57 ·3	—	—	N.	3	1·584	47 ·7	—	·41
Armagh, . . .	S. W.	1·6	1·746	53 ·1	66 ·4	55 ·1	N. E.	2·2	1·439	50 ·7	10	·30
Killough, . . .	S. W.	3	1·772	57 ·4	63 ·0	56 ·0	S. E.	3	1·737?	57 ·0	8	·62
Markree, . . .	S. S. E.	1	1·736	51 ·2	65 ·5	53 ·2	N. N. W.	2	1·539	46 ·6	10	·88
Westport, . . .	W.	3	1·747	58 ·0	—	—	N. W.	2	1·603	53 ·0	—	·77
Courtown, . .	S. W.	1	1·815	56 ·5	73 ·3	54 ·3	S. W.	4	1·523	59 ·2	5	·30
Kilrush, ...	W.	3	1·850	54 ·8	63 ·0	57 ·5	N. W.	5	1·630	51 ·8	10	·57
Dunmore, . . .	W.	1	1·845	56 ·8	60 ·5	57 ·0	W.	5	1·588	59 ·8	—	·37
Cahirciveen, .	W.	2	1·827	57 ·2	—	58 ·5	W. N. W.	4	1·700	57 ·4	4	·88
Castletownsend,	S. W	5	1·860	57 ·0	68 ·0	59 ·5	W.	5	1·677	57 ·5	5	·35

Station.	August 24, 9 p. m.						September 29, 9 a. m.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	N.	2	1·823	55°·1	57°·0	49°·0	S. E.	2	1·531	52°·2	10	·02
Buncrana, . .	N.	2	1·772	53 ·6	—	51 ·0	S.	4	1·482	51 ·6	10	·14
Donaghadee, .	W. N. W.	2	1·773	50 ·2	61 ·3	50 ·0	S. S. E.	3	1·577	53 ·9	10	·03
Killybegs, . . .	N. W.	2	1·890	55 ·0	—	—	S. E.	5	1·373	52 ·0	—	·06
Armagh, . . .	W.	1	1·838	50 ·8	57 ·2	49 ·7	S. W.	3	1·485	51 ·3	10	·09
Killough, . . .	N.	3	1·817	52 ·7	64 ·0	49 ·0	S. E.	6	1·244?	48 ·4	10	·08
Markree, . . .	N. W.	2	1·922	50 ·5	60 ·0	50 ·2	S. E.	5	1·286	51 ·0	10	·16
Westport, . . .	N.	3	1·896	57 ·0	—	—	S. E.	5	1·215	54 ·0	—	·70
Dublin, ...	—	—	—	—	—	—	S. E.	2	1·478	52 ·7	10	·05
Courtown, . .	W. S. W.	2	1·851	54 ·5	63 ·0	50 ·0	S. E.	5	1·513	53 ·5	10	·11
Kilrush, . . .	N. W.	3	1·946	54 ·8	62 ·0	46 ·5	S.	5	1·109	58 ·8	6	·04
Dunmore, . . .	W. N. W.	2	1·914	56 ·3	72 ·5?	55 ·0	S.	5	1·349	56 ·3	—	·25
Cahirciveen, .	N. W.	2	1·988	57 ·6	—	53 ·6	S.	5	1·032	59 ·5	10	·93
Castletownsend,	W.	2	1·921	57 ·5	66 ·0	55 ·0	S. W.	5	1·120	60 ·5	10	·40

Remarks.

Aug. 23, 9 p. m.—Lightning observed throughout the eastern coast during the day. Rain at Killough and Castletownsend; showers at Killybegs, Westport, and Courtown.

Aug. 24, 9 a. m.—At Courtown gale commenced at 5 a. m., and ended at 2 p. m. Rain throughout island, but chiefly in north.

Aug. 24, 9 p. m.—Aurora observed at several places. Showers in north.

Sept. 29, 9 a. m.—Gale highest in the south-west at 3 a. m. Rain throughout the island.

Table XXXIII. (*continued*). Selected Observations.

Station.	September 29, 9 P. M.						September 30, 9 A. M.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Max.	Min.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	S. E.	2	1·271	54°·0	58°·0	48°·0	S.	2	1·020	56°·4	10	·16
Buncrana, . . .	S.	5	1·287	53 ·6	—	49 ·0	W. S. W.	3	1·056	56 ·1	10	·59
Donaghadee, .	S. E.	5	1·315	51 ·0	60 ·0	50 ·0	S. S. W.	3	1·106	57 ·8	10	·25
Killybegs, . . .	N. N. E.	5	1·162	54 ·1	—	—	W. N. W.	5	1·148	55 ·9	—	1·02
Armagh, ...	S. E.	3·5	1·178	53 ·8	54 ·3	49 ·5	S. W.	2·5	1·107	55 ·0	10	·50
Killough, . . .	S.	6	1·149	48 ·1	57 ·0	45 ·0	S.	2	1·157	55 ·0	8	·37
Markree, ...	N. E.	1	1·030	54 ·5	56 ·3	46 ·8	W.	4	1·200	52 ·9	10	·79
Westport, . . .	N. E.	3	1·132	56 ·0	57 ·0	50 ·0	N. W.	5	1·265	55 ·0	—	·30
Dublin, ...	S. E.	4	1·134	55 ·9	58 ·0	51 ·5	S. W.	3	1·209	55 ·6	10	·78
Courtown, . . .	S. S. E.	5	1·147	56 ·5	58 ·5	46 ·0	W. S. W.	2	1·270	56 ·5	8	·48
Kilrush, ...	N. N. W.	4	1·138	57 ·8	63 ·0	50 ·5	N. W.	4	1·402	53 ·8	10	·03
Dunmore, . . .	S.	3	1·089	58 ·3	—	50 ·0	W.	2	1·325	57 ·3	—	·20
Cahirciveen, .	N. W.	3	1·225	56 ·6	61 ·4	52 ·2	N. W.	2	1·400	54 ·6	6	·35
Castletownsend,	W. S. W.	3	1·194	57 ·5	63 ·0	51 ·0	S. W.	3	1·374	55 ·5	6	·00

Station.	September 30, 9 P. M.						October 1, 9 A. M.					
	Wind. Direction.	Wind. Force	Barom.	Therm.	Max.	Min.	Wind. Direction.	Wind. Force	Barom.	Therm.	Cloud.	Rain.
Portrush, . . .	S.	1	1·178	50°·2	59°·0	50°·0	S. E.	3	0·855	54°·4	10	·28
Buncrana, . . .	S. W.	3	1·185	51 ·1	—	50 ·0	S. S. E.	1	0·889	50 ·6	10	·45
Donaghadee, .	S. S. W.	1	1·240	51 ·8	59 ·5	53 ·0	S. S. E.	4	0.936	54 ·9	10	·07
Killybegs, . . .	E. S. E.	5	0·973	53 ·3	—	—	W. N. W.	3	0·873	52 ·7	—	.03
Armagh, . . .	S.	2	1·189	49 ·8	57 ·3	50 ·8	S. S. W.	2·5	0·846	52 ·0	10	·72
Killough, . .	S.	3	1·172	55 ·1	59 ·0	47 ·0	S.	6	—	54 ·4	10	·47
Markree, ...	S. E.	4	1·048	50 ·0	57 ·6	53 ·2	W.	0	0·834	53 ·0	10	·11
Westport, . . .	S.	3	1·062	56 ·0	57 ·0	53 ·0	N.	2	0·882	52 ·5	—	·30
Dublin, ...	S.	2	1·231	51 ·4	58 ·7	52 ·2	S.	0	0·838	54 ·0	10	·33
Courtown, . .	S.	1	1·275	52 ·2	59 ·7	53 ·3	S.	1	0·844	52 ·0	10	1·03
Kilrush, ...	S. S. E.	3	1·142	53 ·8	58 ·0	47 ·5	N. N. W.	2	0·912	49 ·3	5	·53
Dunmore, . . .	—	—	—	—	60 ·0	56 ·0	S. S. W.	1	0·831	53 ·8	—	1·97
Cahirciveen, . .	—	3	1·094	56 ·3	58 ·6	52 ·2	N. N. W.	2	0·923	51 ·4	10	·79
Castletownsend,	S. W.	5	1·173	57 ·0	62 ·5	52 ·5	W.	2	0·892	54 ·5	5	·82

Remarks.

Sept. 29, 9 P. M.—Wind fell at 7 P. M. at Markree. Aurora. Rain throughout, except south-western quarter.

Sept. 30, 9 A. M.—Rain, for the most part light, at Buncrana, Killough, Armagh, Markree, and Kilrush.

Sept. 30, 9 P. M.—Gale commenced at 8 P. M. at Markree. Rain in south-west.

Oct. 1, 9 A. M.—Rain, chiefly in north and east.

Table XXXIII. (*continued*). Selected Observations.

Station.	October 1, 9 P. M.						October 4, 9 A. M.					
	Wind		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	W.	2	0·853	49°·9	56°·0	47°·0	S. W.	2	1·309	50°·0	1	·10
Buncrana, . . .	W. S. W.	2	0·898	49 ·6	—	44 ·0	W. S. W.	3	1·314	50 ·6	3	·15
Donaghadee, .	S. W.	1	0·901	48 ·9	57 ·3	50 ·5	W. S. W.	1	1·472?	51 ·2	1	·13
Killybegs, . . .	W. N. W.	2	0·916	49 ·7	—	—	S. S. W.	3	1·347	53 ·0	—	·30
Armagh, . . .	S. W.	1	0·927	45 ·0	52 ·7	49 ·7	S. W.	2	1·270	50 ·2	0	·11
Killough, . . .	W.	2	—	53 ·7	58 ·0	48 ·0	S.	2	1·386	51 ·7	7	·04
Markree, . . .	S.	3	0·900	50 ·3?	55 ·0	49 ·3	S. S. W.	4	1·304	51 ·8	3	·13
Westport, . . .	W.	5	0·912	52 ·5	56 ·0	50 ·0	S. W.	5	1·304	52 ·5	—	·50
Dublin, ...	S. W.	2	0·980	44 ·1	57 ·0	53 ·2	S. W.	2	1·406	51 ·9	0	·25
Courtown, . .	W.	1	1·012	43 ·0	55 ·0	51 ·7	S. W.	2	1·480	50 ·3	2	·14
Kilrush, ...	W. N. W.	4	1·045	49 ·8	56 ·0	45 ·5	S. W.	5	1·468	49 ·8	8	·48
Dunmore, . . .	W. N. W.	1	1·031	48 ·3	—	54 ·0	W. S. W.	2	1·477?	53 ·3	—	·35
Cahirciveen, . .	W.	2	1·056	49 ·6	55 ·4	50 ·0	W. S. W.	4	1·457	52 ·2	5	·27
Castletownsend,	S. W.	2	1·092	48 ·5	61 ·0	49 ·5	W.	5	1·600	53 ·5	5	·25

Station.	October 4, 9 P. M.						October 5, 9 A. M.					
	Wind.		Barom.	Therm.	Max.	Min.	Wind.		Barom.	Therm.	Cloud.	Rain.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	1	1·182	46°·2	58°·0	47°·0	W.	4	1·346	51°·0	10	·40
Buncrana, . . .	S. W.	2	1·201	47 ·6	—	47 ·0	N. W.	5	1·391	51 ·1	8	·57
Donaghadee, .	S. W.	1	1·236	48 ·9	57 ·4	49 ·5	W. N. W.	2	1·371	49 ·4	10	·08
Killybegs, . . .	W. N. W.	4	1·208	51 ·0	—	—	W. N. W.	6	1·458	53 ·1	—	·43
Armagh, . . .	S. S. W.	0·5	1·225	47 ·2	56 ·3	46 ·8	W.	1·5	1·439	50 ·8	8	·22
Killough, . . .	S.	3	1·418	49 ·0	57 ·0	47 ·0	W.	2	1·573	51 ·7	4	·20
Markree, . . .	W.	4	1·245	46 ·6	54 ·2	44 ·6	W.	3	1·528	48 ·6	3	·86
Westport, . . .	N.	6	1·276	53 ·0	54 ·0	51 ·0	N. W.	6	1·520	53 ·5	—	·56
Dublin, ...	S. W.	3	1·260	48 ·0	59 ·2	49 ·0	W.	2	1·502	49 ·4	4	·14
Courtown, . . .	W. S. W.	1	1·345	48 ·0	57 ·0	46 ·5	S. W.	2	1·574	49 ·5	7	·11
Kilrush, ...	W. N. W.	5	1·423	46 ·8	54 ·0	43 ·5	N. W.	5	1·720	47 ·8	8	·32
Dunmore, . . .	W.	2	1·380	51 ·3	58 ·0	—	W. S. W.	2	1·633	51 ·8	—	·25
Cahirciveen, . .	W.	3	1·461	51 ·6	54 .8	50 ·2	W.	3	1·728	52 ·0	9	·43
Castletownsend,	W.	5	1·426	53 ·5	57 ·0	49 ·0	W.	3	1·691	57 ·5	10	·40

Remarks.

Oct. 1, 9 P. M.—Aurora observed in several places. Rain, chiefly in south-west.
Oct. 4, 9 A. M.—Rain, chiefly in south-west; hail and rain at Castletownsend.
Oct. 4, 9 P. M.—Lightning observed throughout Ireland during this day. Rain at Kilrush and Dunmore; hail and rain at Westport and Castletownsend.
Oct. 5, 9 A. M.—Lightning observed at Cahirciveen. Showers, chiefly on west coast.

TABLE XXXIII. (*continued*). SELECTED OBSERVATIONS.

Station.	December 7, 9 A. M.						December 7, 9 P. M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	2	1·940	51°·3	10	·02	S.	4	1·364	49°·5	53°·0	46°·0
Buncrana, . .	S. W.	3	1·906	51 ·1	9	·00	S. S. W.	5	1·494	49 ·6	—	46 ·0
Donaghadee, .	S. S. W.	2	1·970	50 ·4	10	·00	S. S. W.	5	1·541	50 ·4	52 ·0	47 ·0
Killybegs, . .	S. W.	2	1·902	51 ·3	—	·10	S. S. W.	6	1·225	52 ·9	—	—
Armagh, . . .	S. W.	3·5	1·941	51 ·0	10	·01	S.	6	1·423	50 ·0	52 ·5	46 ·4
Killough, . . .	S. W.	2	2·031	50 ·4	6	·05	S. W.	6	1·570	49 ·2	52 ·0	46 ·0
Markree, . . .	S. S. W.	3	1·852	50 ·6	10	·03	S. S. W.	6	1·274	51 ·5	53 ·3	47 ·4
Westport, . . .	S. W.	5	1·815	55 ·0	—	·07	S. W.	6	1·301?	57 ·0	58 ·0	51 ·0
Dublin, ...	S. S. W.	2	1·997	51 ·6	6	·00	—	3	1·560	52 ·8	57 ·0	48 ·8
Courtown, . .	S. S. W.	3	2·058	51 ·0	10	·01	S.	5	1·666	50 ·5	54 ·0	48 ·0
Kilrush, ...	S. S. W.	3	1·966	51 ·8	10	·03	S. S. W.	5	1·448	53 ·8	55 ·0	51 ·5
Dunmore, . . .	S. S. W.	3	2·016	50 ·8	—	·03	S.	5	1·638	50 ·8	52 ·5	—
Cahirciveen, .	S.	4	1·962	52 ·3	10	·20	W.	4	1·603	53 ·0	55 ·4	50 ·8
Castletownsend,	S. W.	5	2·033	51 ·5	10	·11	S. S. W.	5	1·579	53 ·5	57 ·5	50 ·0

Station.	December 9, 9 A. M.						December 9, 9 P. M.					
	Wind.		Barom.	Therm.	Cloud.	Rain.	Wind.		Barom.	Therm.	Max.	Min.
	Direction.	Force					Direction.	Force				
Portrush, . . .	S.	3	1·781	52°·0	10	·10	S.	3	1·667	55°·5	58°·0	41°·0
Buncrana, . . .	S. W.	3	1·761	53 ·6	9	·06	S. W.	4	1·677	55 ·1	—	43 ·0
Donaghadee, .	S. S. W.	3	1·895	49 ·9	10	·05	S. S. W.	3	1·760	52 ·9	—	42 ·4
Killybegs, . .	S. W.	5	1·762	54 ·0	—	·18	S. W.	5	1·667	54 ·5	—	—
Armagh, . . .	S. S. W.	4·5	1·767	53 ·1	10	·11	S.	4	1·726	54 ·5	57 ·4	41 ·8
Killough, . . .	W.	2	1·747	48 ·7	3	·04	S. W.	2	1·787	47 ·4	50 ·0	45 ·0
Markree, . . .	S. W.	4	1·770	54 ·6	10	·05	S. S. W.	4	1·625	55 ·0	55 ·7	41 ·7
Westport, . . .	S. W.	5	1·696	58 ·0	—	·04	W.	6	1·604	58 ·0	59 ·0	49 ·0
Dublin, ...	S. S. W.	2	1·885	57 ·5	10	·01	S. S. W.	2	1·834	55 ·8	59 ·6	45 ·0
Courtown, . .	S. S. W.	3	1·970	53 ·3	10	·02	S. S. W.	4	1·867	53 ·5	56 ·0	37 ·5
Kilrush, . . .	W. S. W.	4	1·893	55 ·8	10	·00	S. W.	4	1·750	54 ·8	57 ·0	48 ·5
Dunmore, . . .	S. S. W.	2	1·997	52 ·8	—	·03	S. S. W.	5	1·883	52 ·8	53 ·5	—
Cahirciveen, .	S. W.	4	1·913	56 ·2	10	·25	S. S. W.	6	1·780	55 ·4	57 ·4	49 ·0
Castletownsend,	S. W.	5	1·980	54 ·0	10	·04	S. W.	5	1·865	54 ·0	53 ·0	45 ·0

REMARKS.

Dec. 7, 9 A. M.—Rain at Killough; light rain at Markree and Kilrush.

Dec. 7, 9 P. M.—Storm lasted from 6 P. M. to 9 P. M. in the south; least pressure at 7 P. M. at Cahirciveen. Lightning observed at Buncrana in south-west. Rain at most of the stations.

Dec. 9, 9 A. M.—Light rain, chiefly on west coast.

Dec. 9, 9 P. M.—Lunar halo observed at Donaghadee. Light rain, chiefly on west coast.

Table XXXIII. (*continued*). Selected Observations.

Station.	December 20, 9 A. M.						December 20, 9 P. M.					
	Wind.						Wind.					
	Direction.	Force	Barom.	Therm.	Cloud.	Rain.	Direction.	Force	Barom.	Therm.	Max.	Min.
Portrush, . . .	S.	3	1·676	49°·8	9	·00	S.	3	1·442	51°·4	54°·0	38°·0
Buncrana, . . .	S. S. W.	4	1·717	51 ·1	10	·00	S. S. W.	5	1·472	53 ·6	—	42 ·0
Donaghadee, .	S. S. W.	3	1·982	49 ·6	10	·00	S. W.	4	1·580	51 ·0	—	42 ·0
Killybegs, . . .	S. S. W.	5	1·641	53 ·4	—	·00	N.	4	1·470	46 ·3	—	—
Armagh, . . .	S.	4	1·738	50 ·2	9	·02	S.	4·5	1·480	52 ·0	53 ·6	38 ·0
Killough, . . .	S.	6	1·667?	48 ·7	5	·07	S.	6	1·642	48 ·7	51 ·0	42 ·0
Markree, . . .	S. S. W.	4	1·644	51 ·1	10	·01	N. W.	2	1·459	44 ·6	54 ·0	40 ·2
Westport, . . .	S. W.	6	1·590	58 ·0	—	·05	S.	0	1·491	48 ·0	59 ·0	47 ·0
Dublin, ...	S.	1	1·812	52 ·1	10	·00	S.	3	1·578	52 ·8	57 ·0	35 ·5
Courtown, . . .	S.	3	1·882	51 ·5	10	·02	S.	4	1·617	52 ·5	53 ·7	45 ·0
Kilrush, ...	S. W.	3	1·712	53 ·8	10	·03	N. W.	2	1·525	46 ·8	49 ·0	46 ·5
Dunmore, . . .	S. S. W.	5	1·847	51 ·3	—	·02	S. S. W.	5	1·550	52 ·3	52 ·5	—
Cahirciveen, . .	S.	6	1·762	54 ·5	10	·25	S. W.	2	1·580	48 ·6	51 ·8	49 ·8
Castletownsend,	S. W.	5	1·837	52 ·0	10	·10	S. W.	5	1·559	51 ·5	55 ·0	48 ·0

Remarks.

Dec. 20, 9 A. M.—Rain throughout the island, but chiefly on west coast.
Dec. 20, 9 P. M.—Wind veered from S. S. W. to N. W. at $7^{h.}\,30^{m.}$ P. M. at Markree. Rain throughout the island.

Note.—At Armagh the velocity of the wind is recorded, in miles per hour, by means of Robinson's anemometer. The numbers so given are, in the preceding Table, reduced to the scale (0 – 6) employed at the other stations. The velocity of the wind was also occasionally observed at Portrush, Markree, Dublin, and Courtown, by means of small anemometers constructed on the same principle.

Table XXXIV. Hourly Observations.

Cahirciveen.							Armagh.						
Lat. = 51° 56′. Long. = 10° 13′. Height = 52 feet. Barometric Correction = + 0·081. Force of Wind expressed in terms of scale (0 – 6).							Lat. = 54° 21′. Long. = 6° 39′. Height = 211 feet. Barometric Correction = + 0·273. Velocity of Wind expressed in feet per second						
Day.	Hour.	Barom.	Therms.		Wind.		Barom.	Therms.		Wind.		Cloud.	
			Dry.	Wet.	Direction.	Force		Dry.	Wet.	Direction.	Vel.	Amt.	Form.
Mar. 21,	6 A. M.	0·879	50°·6	....	S. W.	3	0·801	39°·9	38°·5	S. E.	18	6	C S, K S
	7	0·878	46 ·0				0·786	41 ·3	40 ·2	S. E. *b* E.	18	9	C S, K S
	8	0·870	47 ·0				0·769	44 ·0	42 ·7	S. E.	18	6	C S, K S
	9	0·848	47 ·2	....	W. S. W.	4	0·755	46 ·3	43 ·7	S. E.	29	9	K S
	10	0·780	48 ·4				0·732	46 ·7	44 ·0	S. E. *b* S.	22	10	K S, N
	11	0·777	48 ·8				0·705	47 ·1	45 ·0	S. E.	26	8	K S, N
	12	0·780	48 ·8	....	W.		0·686	47 ·5	44 ·8	S. E.	35	9	K S
	1 P. M.	0·847	49 ·0				0·659	47 ·1	45 ·2	S. E.	35	10	N
	2	0·866	49 ·2				0·627	48 ·2	45 ·7	S. E.	29	8	K S, K
	3	0·862	49 ·2				0·600	48 ·3	45 ·9	S. E. *b* S.	29	10	N
	4	0·873	49 ·2				0·584	46 ·3	44 ·5	S. *b* E.	28	10	N
	5	0·864	49 ·0				0·571	45 ·5	43 ·7	S. S. E.	37	10	N
	6	0·863	48 ·6				0·566	45 ·1	43 ·2	S. S. E.	32	10	N
	7	0·868	47 ·8				0·570	44 ·2	42 ·8	S. *b* E.	32	10	N
	8	0·856	47 ·0	....	W. S. W.	3	0·577	44 ·2	42 ·8	S.	34	9	N
	9	0·865	46 ·8	....	W. S. W.	3	0·585	43 ·9	42 ·0	S.	29	8	—
	10	0·861	46 ·0				0·595	43 ·0	41 ·4	S. *b* W.	29	10	N
	11	0·860	45 ·0				0·606	42 ·9	41 ·1	S. *b* W.	27	10	N
Mar. 22,	0	0·850	45 ·0	....	S. W.	2	0·607	42 ·5	40 ·3	S.	30	10	N
	1 A. M.	0·786	44 ·5				0·611	41 ·5	39 ·6	S.	24	10	N
	2	0·779	44 ·2	....	S. S. W.	1	0·608	40 ·2	38 ·5	S.	29	9	N
	3	0·753	43 ·8	....	S. W.	1	0·605	40 ·8	38 ·6	S.	32	9	K S, N
	4	0·793	44 ·0				0·614	40 ·8	38 ·4	S. *b* E.	27	8	K S
	5	0·753	43 ·6	....	N. E.		0·617	40 ·3	38 ·0	S. *b* E.	33	8	K S, C
	6	0·755	42 ·8	....	N. E.	1	0·625	40 ·3	38 ·1	S.	28	9	K S

Remarks. (Cahirciveen)

The observations of temperature were taken, by mistake, with one of the registering thermometers.
Cloudy for the most part throughout the day.
Heavy showers from March 21, 6 A. M. to 3 P. M.

Remarks. (Armagh)

Squally throughout the day.
March 22, 0 A. M.—Wind unsteady.
Raining, with little interruption, from March 21, 10 A. M. to March 22, 2 A. M.; amount = 0·235 inch.

Table XXXIV. (*continued*). Hourly Observations.

Courtown. Lat. = 52° 39′. Long. = 6° 13′. Height = 34 feet. Barometric Correction = + ·036. Velocity of Wind expressed in feet per second.

Markree. Lat. = 54° 14′. Long. = 8° 28′. Height = 132 feet. Barometric Correction = + 0·161. Velocity of Wind expressed in feet per second.

		Courtown					Markree						
Day.	Hour.	Barom.	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Force	Barom	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Force.	Cloud. Amt.	Cloud. Form.
Mar. 21,	6 A. M.	1·042	41°·2	41°·2	S. S. W.	13	0·782	41°·0	40°·3	S. S. E.	25	9	N.
	7	1·030	43 ·8	43 ·5	S.	14	0·776	41 ·7	41 ·0	S. S. E.	27	10	N.
	8	1·023	44 ·8	43 ·0	S.	17	0·758	43 ·6	42 ·6	S. S. E.	29	10	N.
	9	1·013	45 ·2	44 ·2	S.	22	0·738	45 ·2	43 ·7	S. E.	22	9	N, K
	10	0·994	47 ·0	46 ·0	S.	35	0·711	46 ·4	44 ·6	S. E.	32	10	N, K, S
	11	0·980	48 ·8	46 ·5	S. S. W.	35	0·691	47 ·5	45 ·3	S. S. E.	27	10	K, N, S
	12	0·970	49 ·8	46 ·5	S. S. W.	43	0·657	46 ·7	44 ·7	S. S. E.	26	10	N, S, K S
	1 P. M.	0·959	50 ·0	46 ·8	S. S. W.	43	0·615	47 ·3	46 ·2	S. S. E.	25	10	N
	2	0·955	50 ·8	47 ·0	S. W.	43	0·560	45 ·1	44 ·9	S. S. E.	35	10	N
	3	0·942	50 ·2	46 ·8	S. W.	58	0·528	47 ·5	46 ·9	S.	29	10	N
	4	0·934	50 ·0	46 ·0	S. W.	43	0·528	48 ·7	47 ·1	S. S. W.	28	10	N
	5	0·930	47 ·8	45 ·0	S. W.	38	0·555	47 ·0	45 ·7	S. W.	19	10	N
	6	0·933	47 ·0	44 ·0	S. W.	35	0·563	46 ·0	45 ·1	S. S. W.	26	10	N
	7	0·942?	45 ·8	43 ·0	S. W.	31	0·583	45 ·1	44 ·5	S. S. W.	15	10	N
	8	0·952	45 ·5	43 ·0	S. W.	29	0·597	44 ·3	43 ·4	S. S. W.	22	9	N, S
	9	0·956	43 ·2	42 ·0	S. W.	29	0·600	43 ·4	42 ·9	S.	20	3	N
	10	0·957	43 ·2	41 ·2	S. W.	17	0·613	43 ·8	43 ·3	S.	23	7	N
	11	0·956	43 ·0	41 ·0	S. W.	17	0·619	43 ·3	43 ·0	S.	16	9	N, S
Mar. 22,	0	0·956	42 ·0	40 ·0	S. W.	15	0·617	42 ·9	42 ·3	S.	20	9	N, K S
	1 A. M.	0·943	41 ·2	39 ·0	S. W.	14	0·635	43 ·4	43 ·0	S. S. E.	16	10	N, K S
	2	0·936	41 ·0	40 ·0	S. S. W.	13	0·637	42 ·9	42 ·4	S.	18	10	N, K, S
	3	0·925	41 ·0	40 ·0	S. W.	13	0·640	42 ·4	42 ·1	S.	16	7	N, S
	4	0·913	40 ·0	39 ·0	S. W.	12	0·642	42 ·0	41 ·7	S.	15	9	N, S
	5	0·902	38 ·2	38 ·0	S. W.	7	0·645	41 ·5	41 ·4	S.	14	9	S, K S, N
	6	0·894	38 ·0	37 ·0	S. W.	7	0·654	41 ·7	41 ·6	S.	14	8	N

Remarks (Courtown).

Mar. 21.—Intermittent sunshine; partially overcast, with cumulus.
3 P. M.—Squall, with drops of rain.
5 P. M.—Halo round sun.
Mar. 22, 2 A. M. & 3 A. M.—Halo round moon.

Remarks (Markree).

Mar. 21, 6 A. M. to 12 P. M.—Moderate gale; at 2 P. M. strong gale.
Rain began at 0 30 P. M., and continued until 7 P. M.; amount = 0·102. Light rain at 10 and 11 P. M.
4 P. M.—Clouds very dense in W. and N. W.

Table XXXIV. (*continued*). Hourly Observations.

Dunmore East.

Lat. = 52° 8′. Long. = 6° 59′. Height = 66 feet.
Barometric Correction = + 0·091.
Force of Wind expressed in terms of scale (0 – 6).

Portrush.

Lat. = 55° 13′. Long. = 6° 41′. Height = 29 feet.
Barometric Correction = + 0·082.
Velocity of Wind expressed in feet per second.

Day.	Hour.	Dunmore East Barom.	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Force	Portrush Barom.	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Force.	Cloud. Amt.	Cloud. Form.
Mar. 21,	6 A. M.	0·964	45°·0	43°·5	S.	3	1·011	40°·8	39°·9	S. E.	18	5	N
	7	0·954	45 ·0	43 ·5	S. S. W.	4	0·998	40 ·6	39 ·7	S. E.	18	7	N
	8	0·951	46 ·0	44 ·5	S. S. W.	3	0·988	42 ·8	41 ·8	S. E.	25	6	N
	9	0·945	47 ·5	46 ·0	S. S. W.	3	0·971	45 ·4	43 ·9	S. E.	30	7	N
	10	0·885	48 ·0	47 ·0	S. S. W.	4	0·945	47 ·4	45 ·4	S. E.	35	8	N
	11	0·881	48 ·5	47 ·0	S. S. W.	4	0·923	49 ·5	46 ·9	S. E.	35	6	N
	12	0·881	48 ·5	47 ·0	S. S. W.	4	0·901	49 ·1	46 ·5	S. E.	35	6	N
	1 P. M.	0·907	49 ·0	47 ·5	S. S. W.	4	0·878	48 ·6	46 ·1	S. E.	43	6	N
	2	0·913	49 ·5	47 ·0	S. S. W.	4	0·844	49 ·9	47 ·5	S. E.	41	6	N
	3	0·901	49 ·0	47 ·5	S. S. W.	5	0·819	48 ·5	45 ·7	S. E.	35	7	N
	4	0·905	47 ·5	45 ·0	W. S. W.	5	0·793	48 ·4	46 ·0	S. E.	39	8	N
	5	0·905	48 ·5	46 ·0	S. W.	5	0·765	47 ·9	45 ·6	S. E.	35	9	N
	6	0·909	47 ·5	45 ·0	W. S. W.	5	0·765	46 ·6	44 ·6	S. E.	35	9	N
	7	0·914	46 ·0	44 ·0	W. S. W.	5	0·744	45 ·7	43 ·8	S.	37	9	N
	8	0·919	45 ·5	43 ·5	W. S. W.	3	0·727	45 ·5	43 ·7	S.	39	9	N
	9	0·905	45 ·0	43 ·0	W. S. W.	4	0·737	45 ·3	43 ·7	S.	36	9	N
	10	0·905	44 ·5	42 ·5	S. W.	3	0·744	45 ·1	43 6	S.	48	8	N
	11	0·903	44 ·0	42 ·5	S. W.	3	0·752	44 ·9	43 ·3	S.	37	7	N
Mar. 22,	0	0·882	45 ·0	43 ·0	S. W.	4	0·760	44 ·4	42 ·8	S.	39	9	N
	1 A. M.	0·883	45 ·0	43 ·0	S. S. W.	4	0·764	44 ·0	42 ·4	S.	37	9	N
	2	0·881	45 ·0	43 ·0	S. W.	3	0·770	43 ·6	42 ·0	S.	36	10	—
	3	0·855	44 ·5	42 ·5	S. W.	3	0·728	43 ·0	41 ·0	S.	29	10	—
	4	0·830	43 ·5	41 ·5	S. W.	2	0·764	42 ·1	40 ·6	S.	39	9	N
	5	0·811	43 ·0	41 ·5	S.	2	0·776	41 ·9	40 ·4	S.	35	9	N
	6	0·811	43 ·0	41 ·5	S.	—	0·780	41 ·9	40 ·5	S.	35	9	N

Remarks. (Dunmore East)

Mar. 21.—Squally throughout day, and for the most part clouded.
2 P. M.—Heavy shower.
Greatest force of wind from 3 P. M. to 5 P. M.; amounted to 8 lbs. on the square foot.

Remarks. (Portrush)

Faint sunshine until 4 P. M.; afterwards overcast and misty.
Noon.—Light showers.
6 P. M.—Continued rain.
9 P. M.—Showers.

Table XXXIV. (*continued*). Hourly Observations.

Cahirciveen. Lat. = 51° 56′. Long. = 10° 13′. Height = 52 feet. Barometric Correction = + 0·081. Force of Wind expressed in terms of scale (0 – 6).

Armagh. Lat. = 54° 21′. Long. = 6° 39′. Height = 211 feet. Barometric Correction = + 0·273. Velocity of Wind expressed in feet per second.

		Cahirciveen.					Armagh.						
Day.	Hour.	Barom.	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Force	Barom	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Vel.	Cloud. Amt.	Cloud. Form.
June 21,	6 A. M.	1·757	55°·6	....	S. W.	1	1·584	57°·2	54°·6	S. S. W.	7	10	S
	7	1·743	58 ·2				1·580	58 ·7	55 ·3	S.W. *b* S.	13	5	S, C
	8	1·734	59 ·0				1·577	60 ·4	56 ·8	S.W. *b* S.	18	5	S, C, K
	9	1·734	59 ·6	....	S. W.	1	1·549	61 ·8	57 ·3	S.W. *b* S.	17	10	K S
	10	1·724	61 ·0				1·530	63 ·3	58 ·3	S.W. *b* S.	12	10	S
	11	1·721	60 ·6				1·517	65 ·1	59 ·3	S.W. *b* S.	14	10	K S
	12	1·713	61 ·0	....	W.	2	1·505	65 ·7	59 ·0	S.W. *b* S.	21	10	K S
	1 P. M.	1·713	62 ·4				1·498	64 ·9	60 ·0	S.W. *b* S.	17	10	K
	2	1·719	58 ·6				1·480	62 ·9	61 ·1	S.W. *b* S.	18	10	K S, N
	3	1·724	57 ·4	....	N. W.	3	1·467	62 ·2	58 ·6	S.W. *b* W.	16	10	K S, S
	4	1·726	56 ·4				1·455	61 ·3	58 ·3	S. W.	14	10	K S, S
	5	1·730	56 ·8				1·469	55 ·9	53 ·9	W. N.W.	10	10	K S, S
	6	1·751	56 ·8	....	N. N. W.	2	1·476	53 ·7	51 ·1	W *b* N.	8	10	N, K S
	7	1·762	56 ·2				1·474	53 ·2	51 ·7	W.	7	10	C K
	8	1·773	55 ·6				1·484	53 ·9	51 ·6	W.	8	3	K
	9	1·792	54 ·8	....	N. W.	2	1·495	50 ·6	48 ·6	W. *b* N.	6	1	K S
	10	1·805	54 ·4				1·498	50 ·1	48 ·1	W.	9	1	K S
	11	1·808	54 ·2				1·509	50 ·1	48 ·2	W.	6	7	K
June 22,	0	1·815	54 ·0				1·524	50 ·8	48 ·9	W. *b* S.	10	7	K
	1 A. M.	1·835	53 ·8	....	W. N. W.	2	1·522	49 ·0	47 ·6	W. *b* N.	5	4	C K
	2	1·851	53 ·4	....	N. W.	2	1·529	49 ·0	47 ·3	W	6	7	S, C K
	3	1·866	53 ·4	....	N. N. W.		1·533	49 ·4	47 ·3	W.	7	7	C K
	4	1·884	53 ·6				1·548	51 ·2	48 ·8	W.	9	9	K, N
	5	1·911	53 ·4				1·583	50 ·5	48 ·9	W. *b* N.	8	10	K
	6	1·937	53 ·4	....	N. N. W.	2	1·604	51 ·9	49 ·0	N. W.	17	9	K

Remarks. (Cahirciveen.)

The observations of temperature were taken, by mistake, with one of the registering thermometers.

June 21, 22.—Clouded throughout day.

June 21, 2 P. M.—Drizzling rain.

June 22, 0 A. M.—Showers; 2 A. M. ditto.

Remarks. (Armagh.)

June 21.—Rain from 2 P. M. to 6 P. M.; amount = 0·076.

June 22, 4 A. M., 6 A. M.—Drizzling rain.

Table XXXIV. (*continued*). Hourly Observations

Courtown. Lat. = 52° 39′. Long. = 6° 13′. Height = 34 feet. Barometric Correction = + 0·036. Velocity of Wind expressed in feet per second.

Markree. Lat. = 54° 14′. Long. = 8° 28′. Height = 132 feet. Barometric Correction = + 0·161. Velocity of Wind expressed in feet per second.

		Courtown					Markree						
Day.	Hour.	Barom.	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Vel.	Barom.	Therms. Dry.	Therms. Wet.	Wind. Direction.	Wind. Vel.	Cloud. Amt.	Cloud. Form.
June 21,	6 A. M.	1·864	59°·0	58°·0	—	0	1·668	56°·1	54°·9	S. S. E.	9	10	S, K S
	7	1·851	60 ·0	58 ·0	S. E.	5	1·652	57 ·0	55 ·4	S. S. E.	9	10	N, S
	8	1·839	61 ·2	59 ·2	S. E.	8	1·650	58 ·8	56 ·1	S. S. W.	8	10	N, S
	9	1·804	63 ·8	61 ·2	S. E.	12	1·623	59 ·5	56 ·9	S. S. W.	13	10	N, S
	10	1·794	64 ·2	61 ·2	S. E.	11	1·607	61 ·0	57 ·2	S. S. W.	9	10	S, K S
	11	1·778	64 ·8	61 ·0	S. E.	12	1·607	62 ·6	58 ·2	S. S. W.	16	10	N, K
	12	1·767	67 ·0	63 ·0	S. E.	9	1·599	63 ·4	58 ·6	S. S. W.	12	10	N
	1 P. M.	1·738	67 ·2	63 ·0	S. E.	11	1·593	57 ·5	55 ·0	N. W.	12	10	N, S
	2	1·718	63 ·8	60 ·0	—	0	1·611	52 ·8	51 ·9	N. N. W.	12	10	N
	3	1·698	63 ·0	60 ·5	—	0	1·618	53 ·2	53 ·6	W. N. W.	2	10	N
	4	1·680	65 ·5	63 ·5	W. S. W.	14	1·600	54 ·1	53 ·4	N. W.	3	10	N, K, S
	5	1·668	64 ·0	60 ·5	W. N. W.	17	1·586	57 ·0	53 ·1	N. W.	14	4	N, K
	6	1·668	63 ·2	60 ·0	W. N. W.	17	1·596	55 ·0	52 ·1	W. N. W.	11	9	N, K
	7	1·676	61 ·2	57 ·0	N. W.	14	1·612	55 ·2	51 ·9	W. N. W	13	3	N, K
	8	1·695	60 ·0	57 ·0	N. W.	29	1·624	52 ·9	50 ·0	W.	10	10	K, N
	9	1·726	56 ·5	53 ·2	N. W.	19	1·631	51 ·1	49 ·7	W.	14	4	K, K S
	10	1·740	56 ·0	53 ·0	N. W.	—	1·645	50 ·6	50 ·0	W.	9	10	N, K
	11	1·754	54 ·8	50 ·2	N. W.	—	1·645	49 ·4	49 ·0	W. N. W.	13	7	N, K S
June 22,	0	1·773	52 ·2	49 ·2	N. W.	—	1·649	50 ·2	49 ·8	W. N. W.	14	8	N, K
	1 A. M.	1·786	51 ·8	49 ·0	—	0	1·663	49 ·9	49 ·9	W. N. W.	11	10	N
	2	1·791	51 ·2	49 ·0	—	0	1·680	50 ·4	50 ·3	W.	14	10	N
	3	1·805	50 ·2	48 ·2	W.	5	1·693	49 ·1	49 ·4	N. N. W.	10	10	N
	4	1·825	50 ·0	48 ·0	W. S. W.	5	1·717	48 ·7	47 ·7	N. N. W.	14	4	C S, C K
	5	1·846	52 ·8	51 ·2	S.	6	1·641	51 ·9	50 ·1	N. N. W.	12	9	N, C S
	6	1·857	53 ·0	51 ·0	W.	7	1·779	51 ·7	48 ·8	N. N. W.	19	3	K, N

Remarks. (Courtown)

June 21, 1 P.M.—Overcast. Thunder, followed by heavy drops of rain; air sultry.

Remarks. (Markree)

June 21, 6 A. M.—9 A. M.—Light rain.
3 P. M.—Heavy rain; amount = ·060.
10 P. M.—Heavy shower.
June 22, 10 A. M.—Moderate rain.

Table XXXIV. (*continued*). Hourly Observations.

		Dunmore East.					Portrush.						
		Lat. = 52° 8'. Long. = 6° 59'. Height = 66 feet. Barometric Correction = + 0·091. Force of Wind expressed in terms of scale (0 – 6).					Lat. = 55° 13'. Long. = 6° 41'. Height = 29 feet. Barometric Correction = + 0·082. Velocity of Wind expressed in feet per second.						
Date.	Hour.	Barom.	Therms.		Wind.		Barom.	Therms.		Wind.		Cloud.	
			Dry.	Wet.	Direction.	Force		Dry.	Wet.	Direction.	Vel.	Amt.	Form.
June 21,	6 A.M.	1·784	60°·5	57°·0	S. S. W.	1	1·761	57°·0	56°·3	S.	15	10	—
	7	1·774	62 ·0	58 ·5	S. S. W.	2	1·744	57 ·5	56 ·6	S.	17	10	—
	8	1·761	63 ·0	60 ·0	S. S. W.	2	1·732	58 ·4	57 ·3	S.	17	10	—
	9	1·737	65 ·0	60 ·5	S. S. W.	2	1·718	58 ·5	57 ·4	S·	25	10	—
	10	1·727	66 ·0	62 ·5	S.	2	1·705	58 ·6	56 ·9	S.	26	10	—
	11	1·716	68 ·0	64 ·5	S.	2	1·689	59 ·6	58 ·2	S	28	10	—
	12	1·688	69 ·0	64 ·5	S.	2	1·673	60 ·4	57 ·7	S.	31	10	—
	1 P.M.	1·676	69 ·0	63 ·0	S.	2	1·667	61 ·0	58 ·9	S.	35	10	—
	2	1·661	67 ·0	62 ·0	S. S. W.	2	1.655	60 ·4	57 ·6	S. W.	35	10	—
	3	1·647	66 ·0	60 ·5	S. S. W.	2	1·642	60 ·4	57 ·1	S. W.	35	10	—
	4	1·632	65 ·5	60 ·5	W. S. W.	1	1·651	54 ·9	53 ·2	N. W.	36	10	—
	5	1·632	63 ·0	58 ·0	W. N. W.	1	1·652	52 ·9	51 ·5	W.	36	10	—
	6	1·644	62 ·0	58 ·0	N. W.	2	1·654	53 ·3	51 ·0	W.	23	9	N
	7	1·649	61 ·0	57 ·0	N. W.	2	1·655	54 ·4	52 ·3	W.	17	8	N
	8	1·658	58 ·5	54 ·0	N. W.	2	1·659	54 ·2	51 ·8	W.	34	4	N
	9	1·667	56 ·5	52 ·0	N. W.	2	1·654	53 ·1	51 ·0	W.	39	4	N
	10	1·714	54 ·5	50 ·5	N. W.	2	1·665	52 ·5	50 ·8	W.	27	3	N
	11	1·739	53 ·5	49 ·5	N. W.	1	1·673	52 ·3	51 ·3	W.	36	10	—
June 22,	0	1·749	52 ·0	49 ·0	N. W.	1	1·682	52 ·2	50 ·7	W.	35	4	N
	1 A.M.	1·813	50 ·5	47 ·5	N. W.	1	1·674	52 ·3	50 ·8	W.	37	9	N
	2	1·808	48 ·0	46 ·0	W. N.W.	1	1·685	52 ·5	50 ·8	W.	39	5	N
	3	1·811	48 ·5	46 ·0	W. N. W.	1	1·693	52 ·0	50 ·9	N. W.	41	10	—
	4	1·813	48 ·0	46 ·0	W. N. W.	1	1·715	51 ·9	50 ·3	N. W.	48	8	N
	5	1·813	51 ·0	49 ·0	W. N. W.	2	1·748	51 ·6	48 ·9	N. W.	48	9	N
	6	1·835	54 ·5	52 ·0	W. N.W.	2	1·760	51 ·6	48 ·4	N. W.	48	9	N

Remarks (Dunmore East).

June 21.—Cloudy until 10 P.M. Raining from 4 P.M. to 6 P.M.

Clear from June 21, 11 P.M. to June 22, 5 A.M.

Remarks (Portrush).

June 21.—Overcast until 6 P.M. Light rain until 1 P.M.

June 22, 3 A.M. to 6 A.M.—Showers with intermittent sunshine.

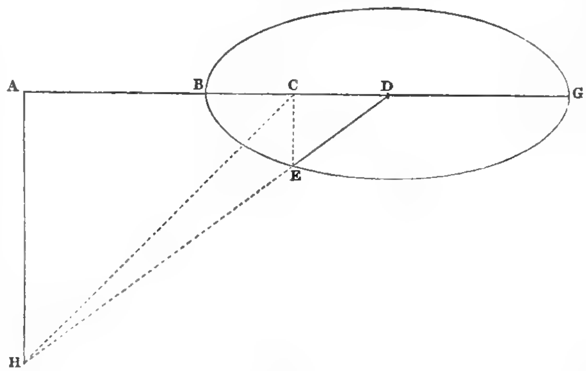
XIX.—*Experimental Researches on the Lifting Power of the Electro-Magnet. Part II. Temperature Correction; Effects of Spirals and Helices.* By the Rev. T. R. Robinson, D. D., *President of the Royal Irish Academy, and Member of other Scientific Societies.*

Read June 26, 1854.

In my former communication on the subject, I examined the relation between the lifting power of the electro-magnet and the force of the current which excites it; and shewed that the first increases much more slowly than the second, so that it cannot pass a limit which depends on the size of the magnet by any assignable amount of current force. But besides the magnitude of that force, the magnet's power depends even more on the number and distribution of the spires of its helices; we can dispose of a very restricted amount of current. The most advantageous mode of employing a given battery is when its internal and external resistance are equal, its action therefore $= \frac{E}{2R}$. This for the Grove's which I use, exposing 19 inches of platinum, is 6 of my units; and for my Callan's of 90 square inches is 14.5, the last of which would only excite my magnet with a single spire to one-sixtieth of its maximum. But however we increase the number of spires, they have all an exciting power; and if this acted equally for each on the magnet, the effect of the current might be increased without limit. It is true that the increased resistance would require a larger battery, but this can always be commanded. But this equality of action does not exist; the exterior spires act more feebly on account of their greater distance, and those at a distance from the polar surfaces exert little influence on them, both from distance and obliquity of force; and secondly, though they do excite fully the parts near their plane, yet the magnetism developed there is greatly weakened by the induction which transmits it to the poles. As far as I know, these disturbing causes have not been studied; and I hope that the following results will not be without their value to the construction as well as theory of the electro-magnet.

The electro-dynamic laws, discovered by Ampere, Biot, and others who have followed in their steps, may seem sufficient to establish its theory; and it is desirable to ascertain whether they succeed in doing so, for the simple case of a circular or helical current.

Let BEG be a differential slice of the magnet, E one of its elements, H a current element below its plane, whose direction is perpendicular to the plane HAG, the power of H to attract and, as we may infer, to magnetise E is, as its magnitude dc, as its energy, μF (F being the force of the current), as the inverse square of the distance HE, and as the cosine of the angle CHE; or putting $HA = z$, $AC = x$, $CE = y$, $DE = r$, $AD = b$, and $BDE = \theta$, the magnetism of E,



$$dM' = \frac{\mu F . dc . E \sqrt{(x^2 + z^2)}}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}.$$

A difficulty occurs here however. The polarity given to the element E is such, that its axis is perpendicular to HE; but, as in the case before us, the current surrounds the magnet, it is evident that unless $z = 0$, each portion of it must produce a different axis. Is this physically possible in the molecules of iron? If so, their polarities must be very irregularly distributed. Or do the axes coalesce into one resultant, whose intensity may be estimated by the composition of forces? This seems most probable; and, therefore, assuming it, and resolving dM' in the direction of z,

$$dM = \frac{\mu F E dc \times x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}.$$

As the magnet has cylindric arms, BEG is a circle; E, therefore, $= rdrd\theta dl$, transporting the origin to the centre, we have

$$dM = \frac{\mu F dcdl\,(b - r\cos\theta)\,.\,rdrd\theta}{(b^2 + r^2 + z^2 - 2br\cos\theta)^{\frac{3}{2}}}. \qquad (1)$$

Integrating this for θ from 0 to 2π, and for r from 0 to r', we obtain the magnetic force of a slice of the magnet whose thickness $= dl$, due to the action of H.

This, however, assumes that each molecule is susceptible of magnetism up to the full influence of the current on it, which can scarcely be the fact. Those nearest the helix being most excited, must tend to induce polarities opposite to their own on those next within, on which also the direct action is less energetic; and we may, therefore, expect to find a zone of intense magnetism succeeded by one weaker, null, or even reversed, followed by a series of similar alternations. This does occur in compound magnets to a great extent; and is manifest in those experiments of Plücker, which prove that a mass of iron is less attracted than filings of the same metal, and these less than powder of iron, more sparsely distributed by being diffused through lard. Of course the same inductive interference occurs in the case before us; but we know too little of its laws to be able to introduce it into the calculation.

The first integral belongs to a class which presents considerable difficulty when its modulus is so near unity, as must be the case with the innermost spires; and among the methods of approximation which have been devised by Euler and Legendre, none, on the whole, are as convenient for my purpose as the common development by the Binomial theorem. Let $b^2 + r^2 + z^2 = u^2$, $rdr = udu$; and expanding $\left(1 - \frac{2br\cos\theta}{u^2}\right)^{-\frac{3}{2}}$, and omitting odd powers of $\cos\theta$, because the terms introduced by their integration vanish between the limits, we obtain

$$dM = \frac{\mu F dcdldud\theta}{u^2}\left\{\begin{array}{l} b + A \times \dfrac{2br^2}{u^2}\left(\dfrac{5}{4} \times \dfrac{2b^2}{u^2} - 1\right)\cos^2\theta \\ \quad + C \times \dfrac{(2b^3)r^4}{u^6}\left(\dfrac{9}{8} \times \dfrac{2b^2}{u^2} - 1\right)\cos^4\theta \\ \quad + E \times \dfrac{(2b)^5 r^6}{u^{10}}\left(\dfrac{13}{12} + \dfrac{2b^2}{u^2} - 1\right)\cos^6\theta \\ \quad + \qquad \&c. \qquad \&c. \end{array}\right\}$$

The general term being

$$\frac{3\,.\,5\,\ldots\,.\,2n-3\,.\,2n-1}{2\,.\,4\,\ldots\,.\,2n-4\,.\,2n-2}\times\frac{(2b)^{n-1}\times r^n\,.\cos^n\theta}{u^{2n-2}}\times\left(\frac{2n+1}{2n}\times\frac{2b^2}{u^2}-1\right).$$

When n is even,

$$\int_0^{2\pi}\cos^n\theta d\theta=2\pi\left\{\frac{1\,.\,3\,.\,5\,\ldots\,.\,n-1\,.\,.}{1\,.\,2\,.\,3\,\ldots\,.\,\frac{1}{2}n\,\ldots\,.}\times\frac{1}{2^{\frac{n}{2}}}\right\},$$

and, therefore, the integral of the corresponding term is

$$\pi\times\frac{3\,.\,5\,\ldots\,.\,2n-3\,.\,2n-1}{2\,.\,4\,\ldots\,.\,2n-4\,.\,2n-2}\times\frac{1\,.\,3\,.\,5\,\ldots\,n-1}{1\,.\,2\,.\,3\ldots\frac{1}{2}n}\times\frac{2^{\frac{n}{2}}\,.\,b^{n-1}\,.\,r^n}{u^{2n-2}}\left(\frac{2n+1\,.\,b^2}{n\,.\,u^2}-1\right),$$

and it is derived from that which precedes it by multiplying the latter by the factor

$$\frac{2n-3\,.\,2n-1}{n-2\,.\,n}\times\frac{b^2r^2}{u^4}\,; \tag{2}$$

we thus obtain

$$dM=\mu F\,.\,dc\,.\,dl\,.\,du\,.\,\pi\left\{\begin{array}{l}\dfrac{b}{u^2}+\dfrac{3br^2}{2u^4}\left(\dfrac{5b^2}{2u^2}-1\right)+\dfrac{3\,.\,5\,.\,7\,.\,b^3r^4}{2\,.\,2\,.\,4\,.\,u^8}\left(\dfrac{9b^2}{4u^2}-1\right)\\ +\dfrac{3\,.\,5\,.\,7\,.\,9\,.\,11\,.\,b^5\,.\,r^6}{2\,.\,2\,.\,4\,.\,4\,.\,6\,.\,u^{12}}\left(\dfrac{13b^2}{6u^2}-1\right)+\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ +\dfrac{3\,.\,5\,.\,7\ldots 2n-3\,.\,2n-1\,.\,b^{n-1}\,.\,r^n}{2\,.\,2\,.\,4\,\ldots\,.\,n-2\,.\,n\,.\,u^{2n}}\left(\dfrac{2n+1\,.\,b^2}{nu^2}-1\right).\end{array}\right.$$

This must now be integrated for r. As $rdr=udu$, integrating by parts,

$$\int\frac{r^n du}{u^m}=\frac{-r^2}{m-1\,.\,u^{m-1}}-\frac{nr^{n-2}}{m-1\,.\,m-3\,.\,u^{m-3}}-\frac{n\,.\,n-2\,.\,r^{n-4}}{m-1\,.\,m-3\,.\,m-5\,.\,u^{m-5}},\ \&c.,$$

which must be continued till the exponent of r vanishes. Each value of n gives two sets of terms on account of the factor $\frac{(2n+1)b^2}{n\,.\,u^2}-1$; if then we combine the first set of the n^{th} with the second of the $(n+2)^{th}$, we have the sum

$$Pb^{n+1}\times\frac{2n+1}{n}\times$$

$$\left\{\begin{array}{l}-\dfrac{r^n}{2n+1\,.\,u^{2n+1}}-\dfrac{nr^{n-2}}{2n+1\,.\,2n-1\,.\,u^{2n-1}}-\dfrac{n\,.\,n-2\,.\,r^{n-4}}{2n+1\,.\,2n-1\,.\,2n-3\,.\,u^{2n-3}}-\&c.\\ +\dfrac{r^{n+2}}{n+2\,.\,u^{2n+3}}+\dfrac{r^n}{2n+1\,.\,u^{2n+1}}+\dfrac{nr^{n-2}}{2n+1\,.\,2n-1\,.\,u^{2n-1}}+\&c.\end{array}\right\}$$

which destroy each other, except the term

$$\frac{Pb^{n+1} \cdot \overline{2n+1} \cdot r^{n+2}}{n \cdot \overline{n+2} \cdot u^{2n+3}},$$

which belongs to the exponent $n+2$. If the term of $n = T_n$,

$$T_n = \frac{Pb^{n-1} \cdot r^n}{\overline{2n-1} \cdot u^{2n-1}},$$

and the next,

$$T_{n+2} = T_n \times \frac{\overline{2n+1} \cdot \overline{2n-1} \cdot b^2 \cdot r^2}{n \cdot \overline{n+2} \cdot u^4}, \tag{3}$$

from which the successive terms of the integral (which all vanish when $r=0$) are easily formed. We thus find, calling the section of the magnet $\pi r^2 dl = A$,

$$dM = \mu FA \left\{ \begin{array}{l} \dfrac{b}{2u^3} + \dfrac{1 \cdot 3 \cdot 5 \cdot b^3 r^2}{2 \cdot 2 \cdot 4 \cdot u^7} + \dfrac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot b^5 \cdot r^4}{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot u^{11}} \\ + \dfrac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot b^7 \cdot r^6}{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot u^{15}} + \&c. \end{array} \right\} \times dc. \tag{4}$$

This converges sufficiently, unless b is nearly $= r$; then, notwithstanding the simplicity of the law of continuation, the computation is tedious. But as soon as n is so large that $\frac{1}{n^2+2n}$ may be neglected, it can be much simplified; for. calling $\frac{4b^2r^2}{u^4} = \rho$, (3) becomes

$$T_{n+2} = T_n \rho \times \frac{n}{n+2}; \quad T_{n+4} = T_n \rho^2 + \frac{n}{n+4};$$

and x being the number of steps,

$$T_{n+2x} = T_r \rho^x + \frac{n}{n+2x}. \tag{5}$$

Even this is too slow; but it enables us to compute x terms *per saltum*. The sum of them is $(m = \frac{1}{2}n)$,

$$T_n \left\{ \frac{\rho m}{m+1} + \frac{\rho^2 m}{m+2} + \frac{\rho^3 m}{m+3} \cdots + \frac{\rho^x m}{m+x} \right\},$$

or developing and arranging according to the powers of $\frac{1}{m}$, and putting

$$\rho + \rho^2 \dots + \rho^x = \frac{\rho}{1-\rho}(1-\rho^x) = A,$$

$$\rho + 2\rho^2 + \dots + x\rho^x = \frac{\rho}{(1-\rho)^2}\{1-(x+1)\rho^x + x\rho^{x+1}\} = B,$$

$$\rho+4\rho^2+9\rho^3\ldots+x^2\rho^x=\frac{\rho}{(1-\rho)^3}\left\{1+\rho-(x+1)\rho^x+(2x^2+2x-1)\rho^{x+1}-x^2\rho^{x+2}\right\}=C;$$

we have

$$S(T_{n+1}\ldots T_{n+2x})=T_n\left\{A-\frac{B}{m}+\frac{C}{m^2}\right\}, \tag{6}$$

which is sufficient for practice. I take $x=10$ or 20, and thus obtain the value of the integral very rapidly. This must now be integrated for c, which admits of two cases. In the first the current is a circle whose radius $=b$, therefore $\int dc=2\pi b$; and if $S=$ the sum of the series in (4), the action of a single ring or convolution of a spiral, whose plane is perpendicular to the axis.

$$M=\mu F\,.\,A\,.\,S\times 2\pi b. \tag{7}$$

No sensible error can arise from considering it a circle ; and I have computed the following table of the coefficient of $\mu\,.\,F\,.\,A$ for the magnet and spirals which I use, in which $r=1$, and the least value of $b=1.13$.

Table I.

b	$z=0$.	$z=2^{\prime}$.	$z=4^{\prime}\cdot0$.	$z=6^{\prime}\cdot0$.	$z=8^{\prime}\cdot0$.	$z=10^{\prime}\cdot0$.
1·13	4·64861	0·27331	0·05165	0·01904	0·00738	0·00390
	·47328					
1·33	3·17533	0·34107	0·06874	0·02306	0·01019	0·00541
	·65685					
1·53	2·51848	0·40382	0·08722	0·02971	0·01325	0·00703
	·41086					
1·73	2·10762	0·45736	0·10757	0·03720	0·01671	0·00887
	·28624					
1·93	1·82138	0·50204	0·12805	0·04526	0·02048	0·01093
	·20683					
2·13	1·61455	0·53619	0·14628	0·05351	0·02457	0·01313
	·16073					
2·33	1·45382	0·56135	0·16393	0·06206	0·02890	0·01552
	·13127					
2·53	1·32255	0·57794	0·18134	0·07068	0·03349	0·01810
	·10852					
2·73	1·21403	0·58788	0·19831	0·07956	0·03806	0·02076
	·9134					
2·93	1·12269	0·59570	0·21376	0·08832	0·04286	0·02349
	·7785					
3·13	1·04484	0·59226	0·22864	0·09676	0·04762	0·02632
	·6758					
3·33	0·97726	0·58978	0·24042	0·10541	0·05268	0·02941
	·5932					
3·53	0·91794	0·58268	0·25166	0·11340	0·05764	0·03253
	·5194					
3·73	0·86600	0·57465	0·26148	0·12155	0·06261	0·03555
	·4457					
3·93	0·82143	0·56522	0·27009	0·12913	0·06756	0·03870

It appears from the second column of this table that the power of a ring decreases with an increase of its diameter, very rapidly at first, but more slowly afterwards, so that its action continues sensible to a considerable distance. But out of its plane the case is different, the total effect is much less; but if z have any considerable magnitude, it increases with the diameter of the ring.

The case of a spiral is, however, not that of most ordinary occurrence, the wire being generally disposed in a helix. To obtain its effect on the magnet's element A, we substitute for dc in (4) the differential of the helix. In this curve if $e =$ the slope of the wire,

$$z = b\theta \times \tan e, \quad dc = \frac{dz}{\sin e};$$

as, however, the curve is inclined and its induction is in a plane perpendicular to it, dc must be resolved in the direction of its base, and we have

$$dM = \frac{\mu F . A \times dz}{\tan e} \times \left\{ \frac{br^2}{2u^3} + \frac{1.3.5.b^3r^4}{2.2.4.u^7} + \&c. \right\}$$

Putting $b^2 + r^2 = t^2$, the integral consists of a series of terms,

$$\frac{\int b^{n-1} dz \times r^n}{(t^2+z^2)^{\frac{2n-1}{2}}} = b^{n-1} \times r^n z \left\{ \begin{array}{l} \dfrac{1}{2n-3 . t^2 . u^{2n-3}} + \dfrac{2n-4}{2n-3 . 2n-5 . t^4 . u^{2n-5}} \\ \dots\dots\dots\dots\dots\dots \\ + \dfrac{P . 2n-2i}{2n-(2i+1)\, t^2 . u^{2n-(2i+i)}} \end{array} \right\}$$

which vanish with z, and therefore require no constant.

The series terminates when $i = n$, and its last term is

$$Q_n = \frac{r^n . b^{n-1}}{t^{2n-2} . \times u} \left\{ \frac{2.4.6 \dots 2n-4.1}{1.3.5 \dots 2n-5.2n-3} \right\}.$$

The term preceding this is obtained by multiplying it by $\frac{1.t^2}{2.u^2}$; the next by the additional factor $\frac{3.t^2}{4.u^2}$, and so on till the last factor is $\frac{2n-5.t^2}{2n-4.u^2}$.

Having obtained Q_n, the next term,

$$Q_{n+2} = Q_n \times \frac{2n-2.2n}{2n+1.2n-1} \times \frac{r^2 b^2}{t^4};$$

and as each term of (4) has a coefficient, whose law of derivation is $\frac{2n+1 \,.\, 2n-1}{n \,.\, n+2}$, we have

$$Q_{n+2} = Q_n \times \frac{n-1}{n+2} \times \frac{4r^2b^2}{t^4},$$

so that the successive integrals can be computed with facility. This expression, however, is not so well adapted as that for a spiral, to the process of *summary* computation which becomes desirable when b is near r. The $(n+2x)^{th}$ term

$$Q_{n+2x} = Q_n \times \frac{4br^r}{t^4} \times \frac{n-1 \,.\, n+1 \,.\,.\,.\, n+2x-3}{n+2 \,.\, n+4 \,.\,.\,.\, n+2x};$$

whence, putting as before $\rho = \frac{4b^2r^2}{t^4}$,

$$\log Q_{n+2x} = Q_n + \log \rho \times x + \log\left(1 - \frac{3}{n+2}\right) + \log\left(1 - \frac{3}{n+4}\right) \,.\,.\,.\,.\, + \log\left(1 - \frac{3}{n+2x}\right).$$

Developing the logarithms, and stopping at $\frac{1}{n^3}$, this becomes

$$\log.\, Q_{n+2x} = \log.\, (Q_n \times \rho^x) - \text{modulus} \left\{ \frac{3x}{n} - \frac{3x}{2n^2}(2x-1) + \frac{x}{n^3}(4x^2 - 3x + 2) \right\},$$

which for $x = 10$ or 20 is sufficiently rapid.

The intermediate terms in this instance are more easily obtained by the method of quadratures, their sum being $\int_n^{n+2x} Q_n dx$. This process gives

$$S\{ Q_{n+2} + Q_{n+4} \,.\,.\,.\, + Q_{n+2x} \} = Q_n \left\{ A' - \frac{B'}{n} + \frac{C'}{n^2} \right\}, \tag{9}$$

in which

$$A' = x \pm \frac{x^2.\log \rho}{2} + \frac{x^3 \log^2 \rho}{6} \pm \&c.$$

$$B' = \frac{x^2}{2} \{ \pm 3 + 2x \log \rho \}$$

$$C' = \frac{x^2}{4} \{ \mp 3 - 2x \log \rho + 10x \},$$

sufficiently exact, and easier for computation than their true values, $\dfrac{\rho^x - 1}{\log \rho}$, &c. In practice I found it best to take $x = -4$ and $+5$.

Having obtained the sum of any number of the terms Q, the sums of the preceding terms are successively obtained by the factors already given, and the multiplication must be continued till the products are certainly of an order that may be neglected.

If the sum of all these integrals $= S'$,

$$M = \frac{\mu FA}{\tan e} \times S' . z = \mu FA \times b\theta S',$$

and as $\theta = 2\pi \times$ number of spires in helix $(= s)$,

$$M = \mu FA \times 2\pi b S'. \tag{10}$$

The computation of S' is much facilitated by the terms Q containing only the inverse first power of u as a factor, so that when their sum is once got for any values of z, it is known for any other with a given b. The terms derived from Q are similarly computed in sum.

I have tabulated a few values of it, which will suffice to make an approximate comparison of this theory with observation.

TABLE II.

b.	$z = 1$.	$z = 2$.	$z = 3$.	$z = 4$.	$z = 5$.	$z = 6$.	$z = 7$.	$z = 8$.	$z = 9$.	$z = 10$.
1·130	2·1902	1·3456	0·9580	0·7367	0·5976	0·5017	0·4320	0·3791	0·3377	0·3044
1·564	1·9663	1·3382	0·9798	0·7670	0·6280	0·5294	0·4574	0·4023	0·3589	0·3242
1·998	1·4987	1·1642	0·8800	0·7052	0·5845	0·4973	0·4318	0·3811	0·3409	0·3081
2·432	1·3014	1·0418	0·8142	0·6516	0·5598	0·4817	0·4207	0·3727	0·3343	0·3042
2·866	1·1040	0·9206	0·7676	0·6443	0·5485	0·4749	0·4172	0·3712	0·3339	0·3031

For any point within or without the helix,

$$S = \frac{S'z' \pm S''z''}{z' + z''}.$$

It is useless to pursue the analytic part of the inquiry further at present, because the distribution of the magnetism excited by the spires in a closed circuit (which is quite a different problem from that of a magnetic bar) depends on the law of induction from molecule to molecule, which is altogether unknown. Sir W. S. HARRIS has inferred, in the case of a permanent magnet inducting at a distance, that this law is the inverse of the distance. Without, however, inquiring how far his observations justify this conclusion, it manifestly cannot apply in the present instance, as the facts rather indicate an exponential function. If the coercive force of the metal did not interfere, the negative logarithm of the induction through iron should be proportional to the distance ; but the law of this force also is unknown.*

In examining the action of spires, and, still more, facts of induction, it is necessary tohave a magnet of variable length, as no satisfactory conclusion can be drawn if several be employed, owing to the various qualities of iron. It is almost impossible to get two pieces of equal power, since the slightest difference in forging, turning, or planing, influences this property. That which was used in these experiments is of the same dimensions as the one described in my former paper, differing in being solid, and having its base of brass. The cylinders are connected by a slide, composed of two pieces of the same iron, each one and a quarter by two inches in section, in which semicircles are cut out to receive them ; and by steady-pins and screws it can be firmly attached at any height. From the excellence of the fitting the contact is very close ; and experience shows that it makes no interruption of the magnetic circuit. Setting it to leave four inches of each cylinder, I found that with 0·85 current force, and helices (F) containing 641 spires, the lift of the magnet is 817 lbs., when the screws are tight, and 800 when they are slackened, and the contact maintained by the attraction alone. If we allow for the decrease of magnetism mentioned in my first paper, these two may be considered identical. The sufficiency of the contact may also be inferred, from the parts of the cylinders below the slide showing no free magnetism. I found this to be the case even when this magnet was excited to the highest power which I have yet obtained with it.† I may add, that no part of the lifting power is due to the action of the spires on the keeper or slide : when the helices, even excited to the great power mentioned

* Bringing into contact with my magnet's N pole an iron tube, three-quarters of an inch in diameter, and nine feet long, seven feet of it are N, and the remaining two S.

† The cylinders were 2·1 long, the helices those already mentioned, and $\psi = 3005{\cdot}04$, $T = 81^{\circ}{\cdot}8$, L' was 1374·17.

in the note, were laid on the keeper in the same position as they had on the magnet, its extremities would neither attract a small key nor hold a horse-shoe keeper.

I have already pointed out the gradual decrease of the magnet's power during a series of experiments, and the fact that this decrease is prevented by continually reversing the current. On this plan most of the experiments whose results follow were conducted. It is made more effectual by exciting the magnet, and, without disturbing the keeper, suddenly reversing. It seems that the abrupt change of magnetic tension keeps the molecules of the iron in a state of neutrality, which prevents them from assuming permanent polarity.* To perform this easily, a commutator is attached to the magnet, and each set consists of six, half direct and half reverse. By this method the results have become far more consistent. The rest of the apparatus is unchanged except the rheometer, which is now on the construction discovered by M. GAUGAIN, and demonstrated by M. BRAVAIS.† Six rings, the largest 19^{i} in diameter, are placed parallel on a frustum of a cone, whose base is four times its axis: the centre of the needle is in the vertex of the cone, the needle is three inches long. This arrangement has the advantage of giving the proportionality of the force to the tangent of deflection—a far wider range than in the ordinary construction, and enabling to measure much higher currents. For such the largest ring alone is used; for small currents the whole six. A set of fifteen observations with the voltameter gives for its constants in the first case,

$$F = \tan\phi \times \log^{-1}(0{\cdot}58298)\{1 + \log^{-1}(6{\cdot}7607) \times (\sin^2\phi - \tfrac{3}{2}\sin^4\phi)\},$$

which will serve for any instrument of the same dimension.

In comparing the efficiency of different arrangements of spires or magnets, the most obvious method is to excite them till the lifts of the magnet are the same, when the *mean* efficiency of each spire, $= \mu$, must be inversely as ψ, the product of the force and number of spires. It would, however, involve an immense waste of time to ascertain this equality, and therefore it is better to refer them to a common standard. That which I have chosen is the action of

* This has been so effectual that the residual magnetism is insensible, though the number of times that it has been excited exceeds 1200.

† Comptes rendus, Jan., 1853.

a pair of spirals (A), possessing a definite character, and acting on the magnet under the most favourable circumstances, namely, when its cylinders are reduced so as merely to lodge the wire, in which case the action with a given ψ is the greatest possible. With these I obtained a series of values of L and the corresponding ψ', from which can be found, by interpolation, the ψ' corresponding to any L. If that L is obtained by any other spirals, helices, or altered length of the magnet, we have, assuming the mean efficiency of $(A)=1$,

$$\mu=\frac{\psi'}{\psi}.$$

As, however, in using different currents, the magnet is unequally heated, it is necessary, in the first instance, to determine

THE TEMPERATURE CORRECTION.

In my former paper I investigated the correction by heating the magnets to about 70° and 170°; and, assuming that the decrease was uniform throughout this interval, I deduced the coefficient of decrease = 0·00033. The experiments of Dr. Lloyd on the temperature change of the magnetism induced by the earth on soft iron have led me to doubt the correctness of this assumption, and institute further experiments, which show that the law is much more complicated in the case of the electro-magnet, and that the coefficients which express it vary with the nature of the magnet. The magnet used in the first instance is one belonging to Mr. Bergin's collection (to whom I am indebted not only for the use of much valuable apparatus, but for still more valuable assistance in these researches), extremely convenient for the work. The cylinders are five and a half inches diameter, their centres six asunder; they are hollow, their thickness being half an inch; they are eight inches long, and the base and keeper have a section equal to theirs. The helices are those designated (I). The balance used with it is well worth notice. It has only one lever (whose ratio is 11·625), the longer arm of which carries a platform, on which weights, multiples of 30 lbs., can be placed: below the platform is suspended, by a spring balance, a tin vessel, into which shot is poured, whose weight is seen by the index up to 30 lbs., the limit of the spring. If this be not sufficient, the shot is permitted to escape by a valve at the bottom of the vessel, another weight is set on the platform, and the process is repeated. The manipulation is easier than in mine, and the accuracy superior, but the concussion is greater. The hollow cylinders receive copper vessels, which may be filled with hot water or ice, and their magnitude is sufficient to preserve a nearly uniform temperature for some time. The experiments were, unfortunately, much disturbed by the perpetual passage of carriages through the street, which caused the loss of many results; indeed we could only work during the night, and even then had much disturbance. The temperature was measured in the middle of the keeper, the middle of the base, and the top of the cylinders, and the mean taken. F was kept at 0·4734, giving $\psi = 143·91$; T is the mean.

TABLE III.

	T.	*L.*	*O – C.*	No Obs.
1	41°·98	871°·83	– 22·68	6
2	53 ·56	876 ·48	+ 19·49	8
3	58 ·07	859 ·25	+ 12·85	6
4	64 ·78	842 ·23	+ 7·67	5
5	73 ·41	812 ·48	– 13·11	8
6	76 ·83	822 ·36	+ 1·42	9
7	81 ·85	823 ·61	+ 0·78	8
8	87 ·62	818 ·05	– 4·73	8
9	92 ·38	822 ·74	– 3·44	9
10	96 ·56	826 ·72	– 4·20	6
11	102 ·68	831 ·99	– 3·23	6
12	107 ·40	855 ·17	+ 13·98	6
13	111 ·89	854 ·83	+ 7·48	9
14	116 ·14	845 ·53	– 8·13	5
15	121 ·85	856 ·82	– 5·91	5
16	127 ·44	876 ·68	+ 4·74	8

The third column shows that the value of L diminishes as the temperature increases; becomes a minimum at about 75°; it then increases to the highest temperature in the Table; after that it would certainly again decrease; but we could not easily, with this arrangement, pass 127°. If there be no subsequent maxima and minima, these facts imply a formula such as

$$L = L' \{1 - at + bt^2 - ct^3\},$$

which, in fact, is the form given when the coefficients are determined by CAUCHY's method of interpretation.

By this, reckoning t from 60°, is derived

$$L = 842.54\,\{1 - t \times \log^{-1}(7\cdot35439) + t^2 \times \log^{-1}(5\cdot78313) \\ - t^3 \times \log^{-1}(3\cdot46064)\}. \qquad (11)$$

The fourth column contains the difference between the observed L and that calculated by this formula; the discordance is considerable, especially in the three first, but not greater than might be expected under the circumstances, and the errors being often effected with contrary signs shows that they are casual. I may also remark, that the current was not reversed in these observations. The difference between the correction given by this formula, and the change of L which I had obtained with the hollow two-inch magnet, showed the necessity of instituting similar experiments for that which I was using in the present series; and I found that by surrounding it and its slide with a covering of thick cloth, and the keeper with a similar one, I could raise the temperature to 220°, the limit of the thermometer which I used, by placing a gas flame on the base. The slide is the hottest; but its temperature, that of the cylinders at their top, and that of the keeper, were taken, and the mean deduced by giving each weight as the length of the piece.

The first was with the cylinders $= 0^{i}\cdot15$, and the spirals (A) with $\psi = 170\cdot79$. I obtained

Table IV.

No.	*T.*	*L.*	*O − C.*	No. Obs.
17	69°·6	615·90	+ 2·89	18
18	81 ·0	603·44	− 3·84	6
19	100 ·5	601·40	+ 0·94	12
20	130 ·0	598·89	+ 3·14	12
21	161 ·7	591·53	− 4·08	12
22	207 ·4	598·90	− 1·95	12

whence I similarly deduce

$$L = 618\cdot96\,\{1 - t\,.\log^{-1}(7\cdot03890) + t^2\,.\log^{-1}(4\cdot99503) \\ - t^3\,.\log^{-1}(2\cdot43656)\}. \qquad (12)$$

Secondly. With the cylinders $= 2^{i}\cdot1$, the helices (B) and $\psi = 553\cdot75$.

TABLE V.

No.	$T.$	$L.$	$O - C.$	No. Obs.
23	69°·1	902·36	0·00	16
24	96 ·1	888·98	+ 5·54	12
25	132 ·4	867·98	− 5·57	12
26	173 ·2	871·28	− 0·01	12
27	220 ·3	860·95	− 0·02	12

giving

$$L = 911{\cdot}85 \{1 - t \,.\, \log^{-1}(7{\cdot}09736) + t^2 . \log^{1}(5{\cdot}08868) \\ - t^3 . \log^{-1}(2{\cdot}61666)\}. \qquad (13)$$

Thirdly. With the cylinders = 10·1, with the helices (B), (J), (K), (L) and (M), containing 1002 spires, and with $\psi = 553{\cdot}75$, I obtained,*

TABLE VI.

No.	$T.$	$L.$	$O - C.$	No. Obs.
28	64°·4	729·45	− 2·25	12
29	97 ·5	707·34	+ 4·91	12
30	125 ·5	692·12	− 5·39	12
31	164 ·6	701·31	+ 0·50	12
32	208 ·6	690·43	− 0·01	12

giving

$$L = 735{\cdot}69 \{1 - t \,.\, \log^{-1}(7{\cdot}30919) + t^2 . \log^{-1}(5{\cdot}40470) \\ - t^3 . \log^{-1}(2{\cdot}98826)\}. \qquad (14)$$

It must be remembered, that these equations are mere formulæ of interpolation, and not the actual functions expressing the change due to temperature : yet there is an evident correspondence between them and the forces which are

* As the slide was at the bottom of the magnet, the plan of heating it, used for the others, was not available ; but by placing a brass curved funnel over a double argand gas-burner, a stream of heated air was thrown within the covering of the magnet ; and by placing its orifice so that part impinged on the slide, while the rest circulated within the confined space, I insured the same temperature in the slide and cylinders.

concerned. They all comprise three terms affected with t, and, of course, show a decrease at first to a minimum, then an increase to a maximum, and a subsequent decrease. Now, as I formerly noticed, the L represents the polarity at the contact surfaces of the magnet and keeper: this depends on, first, the polarization of those molecules which are excited by the helices; secondly, on that of the remainder of the magnetic circuit, and therefore its amount depends on the intensity of these polarizations, and on the facilities with which their influence is transmitted by induction. The intensity, again, depends on the susceptibility of the molecules directly, and inversely as the coercing force. Now each of these may be expected to change with the temperature. The correlation of heat with other molecular forces is such that, *à priori*, we would anticipate its lessening such forces as we are considering; and we find that at a red heat even iron is scarcely attracted by the most powerful magnets, which must depend on the molecules ceasing to be excitable. A diminution of this power, and of that which transmits the magnetism from one particle to another, must lessen L, while a contrary effect will arise from the diminution of the coercing force. All these influences will be functions of the lengths of the magnetic current, and of the excited cylinders; and accordingly we find that the coefficients in (12), (13), and (14), increase with the latter. Calling them α, β, and γ, and the length of the cylinder z, the three values of γ are exactly represented by the formula $a + bz$, and those of α nearly by $a' + b'^z$; from which one might infer that γ corresponds to that part of the change which belongs to the excitabiles, and α to the conduction. I have not found any simple expression for β, but since it gives the intermediate increase of magnetic attraction, which (as I hope to show in a future communication) depends on the coercive force, we may refer it to that.

I did not think it necessary to investigate these corrections for the other lengths of cylinder, as these three give sufficient data for interpolation, and from them,*

* Although it is not safe to interpolate beyond the limits of the observations, yet, computing from these the constants for twelve-inch cylinders, and reducing by them the temperature experiments with my first magnet, I find the higher gives in each pair an L' less than that of the lower by 7·56, 6·92, 4·44, 6·24, or in the mean 6·29 for a difference of temperature = 101° 6. This, therefore, shows both that the same law holds in this magnet, though hollow, and that these constants will serve to reduce the observations made with it.

$$\alpha = 0{\cdot}001094 + \frac{z}{2} \times \log^{-1}(6{\cdot}17319) + \frac{z^2}{4} \times \log^{-1}(4{\cdot}90309).$$

$$\beta = 0{\cdot}00000989 + \frac{z}{2} \times \log^{-1}(4{\cdot}34242) + \frac{z^2}{4} \times \log^{-1}(3{\cdot}25527).$$

$$\gamma = 0{\cdot}00000002733 + \frac{z}{2} \times \log^{-1}(2{\cdot}14768).$$

The case is, however, very different if those parts of the circuit, which are not directly excited, be lengthened. With the cylinders $= 10{\cdot}1$, and the helices (B), two inches high, placed in contact with the keeper, I got with $\psi = 544{\cdot}83$:

Table VII.

No.	$T.$	$L.$	$O-C.$	No. Obs.
33	64°·4	806·70	− 0·71	12
34	119 ·2	772·97	− 0·15	12
35	161 ·1	752·33	+ 0·25	12
36	197 ·9	732·81	− 0·08	12

These give

$$L = 810{\cdot}84\,\{1 - t\,.\log^{-1}(6{\cdot}99120) + t^2\,.\log^{-1}(4{\cdot}62797) - t^3\,.\log^{-1}(2{\cdot}21015)\}, \qquad (15)$$

when the coefficients are less than even in (12), but the law is the same as in the rest. The places of the minimum and maximum are much higher, and the change of L is greater than in any of the others.

The terms within the brackets in these expressions are, I think, independent of ψ; for in my former paper it is shown that, with the magnet there used, the decrease of L for a given difference of t is the same, with very considerable variations of the current.

ACTION OF SPIRALS.

I. In the first instance, I give the Table already referred to of the spirals (A), which I assume as my standard. Their constants are

$$b = 1{\cdot}14\ ;\ b' = 2{\cdot}80\ ;\ s = 40.$$

Their external diameter was intended to be as large as the distance of the cylinders would admit, and the cylinders themselves are shortened to 0·15, giving the greatest number possible of spires, and the shortest magnet. The observed values of L are reduced to 60° by the coefficients of (12).

Table VIII.

No.	$T.$	Spiral.	$L'.$	$\Delta L'$	$\psi'.$	$\frac{\Delta\psi'}{\Delta L'}.$	No. Obs.
37	68°·0	*A*.	696·96		201·49		12
				43.36		0·34503	
38	68 ·0	„	653·60		186·55		12
				23·41		0·67322	
39	72 ·6	„	630·19		170·79		12
				54·10		0·42218	
40	72 ·3	„	576·09		147·95		12
				76·40		0·30301	
41	65 ·6	„	499·69		124·80		18
				67·18		0·38538	
42	65 ·3	„	432·51		98·97		12
				77·84		0·23523	
43	67 ·0	„	354·67		80·66		12
				86·15		0·19900	
44	63 ·9	„	272·51		64·31		12
				89·71		0·16252	
45	64 ·3	„	182·80		49·73		12

As higher values could not easily be obtained without some risk of destroying the spirals by the evolved heat, I use, when necessary, the numbers of Table XII. obtained with the helices B, whose ratio to A is known.

II. We can now compare spirals of different diameters, and thus ascertain how far the preceding theory agrees with experiments. The cylinders were set to $0^{i}{\cdot}6$, which permits the use of spirals $7^{i}{\cdot}5$ diameter by overlapping them. The spirals are made of flatted copper wire 0·2 by 0·05, to enable the employment of powerful currents with such batteries as I possess;* but experience makes me regret this arrangement, for it requires a greater length of cylinder, and the action of the current is probably not quite uniform

* The current employed with (C) would evolve in a voltameter 18 cubic inches of mixed gases per minute.

through the section of the wire. The error, however, must be trifling. Their constants are

$$\begin{array}{llll} (C) & b = 1{\cdot}135. & b' = 1{\cdot}905. & s = 18. \\ (D) & \;\;,, \;1{\cdot}130. & \;\;,, \;2{\cdot}770. & \;\;,, \;40. \\ (E) & \;\;,, \;1{\cdot}150. & \;\;,, \;3{\cdot}873. & \;\;,, \;60. \end{array}$$

I obtained with them

TABLE IX.

No.	Spiral.	T.	L'.	ψ.	ψ'.	μ.	Theoretic μ.	No. Obs.
46	(C)	69°·6	641·86	187·32	178·63	0·9684	1·3772	12
47	(D)	66 ·0	598·85	188·02	156·12	0·8303	1·0000	12
48	(E)	66 ·2	606·07	193·78	158·27	0·8167	0·8229	12

L is reduced by (12), as from the shortness of the cylinders it must nearly be exact for them also. These results are in a ratio so different from what I anticipated, that I suspected some error was produced by the overlapping of the spirals E. I therefore repeated the experiments with one of Mr. Bergin's magnets, in which the distance of the axes is 7·5, and the length of the cylinder $0^{i}{\cdot}5$; but it gave similarly

TABLE X.

No.	Spiral.	T.	L.	ψ.	ψ'.	μ.	Theoretic μ.	No. Obs.
49	(C)	63°·6	502·32	193·38	125·69	0·6497	1·3772	12
50	(D)	64 ·3	457·56	189·14	107·53	0·5685	1·0000	12
51	(E)	63 ·2	458·38	191·74	107·84	0·5624	0·8229	12

Both sets agree in showing that the power of a spire does not decrease nearly so fast by an increase of diameter as the equation (7) assigns. In both the μ is diminished relatively to that of A by the greater length of the cylinder, and (in the second) of the entire magnet and its keeper, as will be more evidently shown hereafter. As, however, (D) is almost identical in its diameter with A, by taking it as unit, we get a more distinct comparison of the relative values of μ.

Table XI.

	Tab. IX.	Tab. X.	Theoretical.
C	1·1663	1·1428	1·3772
D	1·0000	1·0000	1·0000
E	0·9836	0·9893	0·8229

The formula gives more power to the spires of C, less to those of the zones $D-C$ and $E-D$ than is observed: if, in fact, we determine what parts of the effect of E belong to them, we find for

$$\begin{array}{lll} C \text{ effect} = & 10{\cdot}4967 = & 9 \times 1{\cdot}1663. \\ D-C \quad \text{,,} & 9{\cdot}5033 = & 10 \times 0{\cdot}9503. \\ E-D \quad \text{,,} & 9{\cdot}5080 = & 10 \times 0{\cdot}9508. \end{array}$$

It appears *probable*, from Table I., that the two or three innermost spires act much more powerfully than the rest; and therefore it seems that, from them to a considerable distance, *the exciting force of the others is constant.* The discrepancy between the computed and observed μ is far too great to be caused by error of observation; for again taking D as the standard, the theoretic μ will give L' 48 greater for C, and 61 less for E, than the true values, while the probable errors of the latter are only 2·04 and 1·99.

ACTION OF HELICES.

Here also the discrepancy between theory and observation is considerable, independent of the interference of induction. This is shown, even without measures, by some striking facts; for instance, the helices (F) being placed above the polar surfaces of the cylinders set to 4·1, but separated by plates of zinc $\frac{1}{32}$ thick, and excited to have $\psi = 552$, the magnetism produced was scarcely sensible when a keeper was applied across the cylinder, immediately below them; I had no means of measuring it, but its attraction was not more than a pound or two. Had the helices been on the cylinders, L would be 850. If one of these helices be placed in the same way above one polar surface, there is scarcely any attractive power developed at the other: in this case, however, as the magnetic circuit is incomplete, the force is much less; but I expected to find 30 or 40 pounds at least. Similar results were obtained with a magnet, the upper two inches of whose cylinders are iron, the rest brass. When the helices were on the brass, just below the iron, the lift was but 0·18 of its amount when they were on the iron. At the extremity of the helix, the exciting force might be supposed to be little less than in its interior. The constants of the helices which I used are

(B)	$b = 1·130.$	$b' = 1·965.$	$z = 1·8.$	No. layers $= 8.$	$s = 214.$
(F)	„ 1·130.	„ 2·866.	„ 1·8.	„ 20.	„ 641.
(G)	„ 1·110.	„ 1·210.	„ 7·6.	„ 2.	„ 324.
(H)	„ 1·650.	„ 1·750.	„ 7·6.	„ 2.	„ 320.
(I)	„ 2·890.	„ 2·990.	„ 7·6.	„ 2.	„ 304.
(J)	The same dimensions as (B).			„ 8.	„ 192.
(K)				„ 8.	„ 188.
(L)				„ 8.	„ 183.
(M)				„ 8.	„ 225.

(G), (H), and (I) are from Mr. Bergin's collection; the two last were kept concentric with the magnet by wooden framing, though a considerable error in this respect seems to have little effect.

The results obtained with (B) are given in a separate Table, as they were intended to be used for interpolation beyond the range of Table VIII., and, therefore, the first differences are included in it. L' is reduced by (13), the cylinders being 2·1.

TABLE XII.

No.	$T.$	$L'.$	$\Delta L'.$	ψ	$\frac{\Delta\psi}{\Delta L'}.$	No. Obs.
52	68°·1	1098·90		1059·35		12
			63·47		4·02143	
53	69 ·1	1035·43		804·11		12
			124·55		2·01991	
23	69 ·2	910·88		552·53		16
			96·25		1·36820	
54	64 ·4	814·63		420·84		12
			111·22		1·25265	
55	64 ·2	703·41		281·52		12
			114·74		0·70865	
56	65 ·4	588·67		200·21		12
			110·80		0·44052	
57	62 ·6	477·87		151·40		12
			145·03		0·34793	
58	63 ·8	332·84		100·94		12

The four last are comparable to Table VIII., and give for μ,

$$\begin{array}{r} 0{\cdot}7571 \\ 0{\cdot}7606 \\ 0{\cdot}7577 \\ 0{\cdot}7498 \\ \hline \text{mean} = \log^{-1}(9{\cdot}87912) = 0{\cdot}7570 \end{array}$$

The others are given in

TABLE XIII.

No.	Helices.	T.	L'.	ψ.	ψ'.	μ.	Calc. μ.	Cylinder.	No. Obs.
55 58	} B)	. . .	. . . .	. . . .	. . . .	0·7570	0·8288	2·1	48
59	(F)	71·4	874·96	550·72	380·42	0·6927	0·7038	2·1	10
60	($B+J$)	67·0	848·91	544·20	353·21	0·6490	0·5452	4·1	11
61	60 + (K)	66·9	810·70	547·78	318·61	0·5816	0·4073	6·1	17
62	(G)	63·5	767·41	547·19	272·53	0·4981	0·3280	8·1	11
63	(H)	63·2	747·34	541·89	249·41	0·4603	0·3211	8·1	11
64	(I)	61·6	737·58	543·49	240·74	0·4430	0·2695	8·1	11
65	61 + (L)	67·4	765·12	547·65	270·35	0·4937	0·3228	8·1	11
28	65 + (M)	64·4	733·44	553·75	237·48	0·4289	0·2552	10·1	12
33	B	64·4	810·13	544·83	314·12	0·5765	. . . .	10·1	12
66	F	71·7	797·39	548·04	301·56	0·5502	. . . .	10·1	10

Here also the difference between the theory and observation is considerable; more so than appears at first sight. The computed values of μ given in the eighth column assume that the intensity in every part of the magnetic circle is equal; or, in fact, that magnetism is transmitted by induction without any diminution. That this is not the case is evident from comparing Nos. 55, 58, and 33, in which the difference of sixteen inches in the two cylinders of the magnet reduces the actual efficiency of the spires of B from 0·757 to 0·5765. Therefore, all these calculated values are much too great, and yet all of them, except the first, are less than those given by observation, while in the spirals they are greater.

The decrease of efficiency depending on an increase of the spire's diameter is less than that assigned by theory, still more than in the spiral. In (F) and (B), which are nearly of the same radii as (D) and (C), the ratio of μ is 0·915 with the cylinders 2·1, and 0·954 with 10·1 (which case does not properly belong to this part of my subject, but is given for the comparison), instead of 0·849.

This discrepancy is still more evident with the cylinder 8·1, where (G), (H), and (I) are mere cylindric annuli, but where the ratio of (I) to (G) is found to be 0·8894 instead of 0·8217.

The same is the case with respect to the influence of the cylinder's length; as is shown in the following Table of the ratio of the observed to the theoretic μ, for helices equiradial to (B); in which I have also given it for the spirals and three helices.

TABLE XIV.

Cylinder.	$\frac{\text{Obs. }\mu}{\text{Calc. }\mu}$ Helices.	Mean Diameter of Spiral.	$\frac{\text{Obs. }\mu}{\text{Calc. }\mu}$ Spiral.
0·6	0·7031	C 1·520	0·7031
2·1	0·9123	D 1·700	0·8303
4·1	1·1904	E 2·511	0·9925
6·1	1·4279	G 1·16	1·5221
8·1	1·5294	H 1·70	1·4335
10·1	1·6807	I 2·94	1·6438

This divergence was not expected by me; for the principles on which the equation (1) is founded have been found to give correctly the action of helical currents on each other, and their deflection of a magnetic needle. There is, however, one marked difference between these and the case of the electro-magnet. The polarities of two currents cannot be in any way altered by their mutual action; those of the molecules of the needle are kept nearly permanent, both in intensity and direction, by the coercive force of the hard steel; so that the ordinary methods of statics apply with certainty to them. In the case of soft iron, both these vary with the condition of the current, and according to laws which I do not think are fully known. From the excess of activity of the outer and lower spires, I am inclined to suspect that the resolution of the exciting power in the direction of x and z is the main cause of error, though some of it, as I have already indicated, must also belong to the mutual induction of the molecules.

Some practical inferences may be drawn from these experiments, for the construction of electro-magnets intended to act with a closed magnetic circuit.

1. The nearer the spires can be kept to the polar surfaces, the better, for their activity is much diminished as they recede from it: the great superiority of spirals over helices shows this. Thus, the efficiency of (A) is 1·4436 times that of the equiradial (F); their mean distances from the poles being 0·07 and 1·07.

2. The very small decrease caused by increasing the diameter (at least, as far as 7·5, and probably beyond it to an extent not likely to occur in practice) leads to the conclusion that the helices should be as wide as the distance of the cylinders permits, or that b' *shall be half that distance;* b will, of course, be as nearly as possible the radius of the cylinders.

3. The height of the helices and cylinders should be as little as is consistent with lodging a sufficiency of wire to employ to the best advantage the power of the battery which is used.

4. This height $= z$ may be determined thus :—

Let E and R be the constants of the batteries; $d =$ the diameter of the wire; $d + c$ that of it when covered with thread (c being in my wire $= 0{\cdot}03$); $2s =$ the number of spires in the helices; and ρ' their resistance. We have

$$2s = \frac{2z\,(b'-b)}{(d+c)^2};\ \rho' = \frac{8\rho\,.\,(b'+b)\,(b'-b)\times z}{d^2(d+c)^2},$$

ρ being a constant, such that the resistance of a unit of the wire $= \dfrac{4\rho}{\pi d^2}$: for copper I find $\rho = \log^{-1}(5{\cdot}71018)$. Then we have for the exciting power with a unit current,

$$\mu\psi = \frac{2s\mu E}{R+\rho'} = \frac{2E\,(b'-b)}{R} \times \frac{\mu z}{(d+c)^2 + \dfrac{8\rho z\,(b'+b)\,(b'-b)}{Rd^2}};$$

or, putting $a = \dfrac{8\rho\,(b'+b)\,(b'-b)}{R}$,

$$\mu\psi = \frac{2E\,(b'-b)}{R} \times \frac{\mu z}{(d+c)^2 + \dfrac{az}{d^2}}.$$

If in this we consider d and z as variables, μ a function of z, and differentiate for the maximum, we obtain the equations,

$$o = d^3 (d + c) - az,$$
$$o = \frac{zd\mu}{dz} \left\{ 1 + \frac{az}{d^2 (d + c)^2} \right\} + \mu. \qquad (16)$$

Substituting, in the second, for az its value in (16),

$$o = \frac{zd\mu}{dz} \left\{ 1 + \frac{d}{d + c} \right\} + \mu. \qquad (17)$$

When z is determined by any particular condition, (16) gives the most advantageous diameter of wire, and *vice versâ.*

If it be not, and if the relation between it and μ be known, the two equations give the d and z for the absolute maximum. In the case of my magnet (when $b' = 3$, $b = 1{\cdot}13$) Nos. 55, 58, 60, 61, 65, and 28, give the means of expressing that relation by an interpolation formula, $A - Bz + Cz^2 - Dz^3$. Supposing the battery to consist of ten Groves' such as I use, $R = 47$, with these I obtain

$z = 8{\cdot}39$,
$d = 0{\cdot}14725$, or nearly No. 9 of the wire gauge,
$s = 999$,
$\mu\psi = 2780{\cdot}29$.

A much higher power, however, would be obtained if the ten cells were grouped as five double cells, and the helices made to suit this condition. In this case $R = 11{\cdot}75$; and we find for the best arrangement,

$z = 8{\cdot}33$,
$d = 0{\cdot}2106$, a little more than No. 6,
$s = 538{\cdot}18$,
$\mu\psi = 3549{\cdot}60$.

5. I suppose the current to traverse the helices consecutively; but they are frequently used collaterally with the notion of obtaining a more powerful current. This is not to be recommended in ordinary cases. If the numbers of the spires be s' and s'', and their resistances ρ' and ρ'',

$$\text{consecutive } \psi : \text{collateral } \psi_{,} :: s' + s'' : \frac{s'\rho'' + s''\rho}{\rho' + \rho''} \times \frac{R + \rho' + \rho''}{R + \dfrac{\rho'\rho''}{\rho' + \rho''}}.$$

If, as is usual, $s' = s''$, $\rho' = \rho''$, the ratio becomes

$$\psi : \psi_{,} :: 2R + \rho' : R + 2\rho'.$$

When ρ' is greater than R the collateral arrangement is best, but then it must be observed, the expenditure of zinc and acid is twice as great as in the ordinary arrangement.

In these experiments the cylinders were entirely covered with spires; when this is not the case, and particularly when the helices are at a distance from the polar surfaces, the action is diminished, because it is imperfectly transmitted by induction. Such observations seem calculated to advance our knowledge respecting that power, and I hope soon to submit to the Academy those which I have made with this view; and some respecting electro-magnets of hard steel and cast iron.

XX.—*Some Account of the Marine Botany of the Colony of Western Australia.* *By* W. H. HARVEY, M. D., M. R. I. A., *Keeper of the Herbarium of the University of Dublin, and Professor of Botany to the Royal Dublin Society, &c.*

Read December 11, 1854.

THE land vegetation of Western Australia is now tolerably well known, chiefly through the labours of Mr. JAMES DRUMMOND and of Dr. L. PREISS, who have separately explored almost all the settled districts; and the former has also pushed his researches far to the northward and eastward, beyond the range of any colonist's settlement. Lesser collections of land plants have been made by Baron HÜGEL, Captain MANGLES, the late Mrs. MOLLOY, Mr. J. S. ROE, and other amateurs.

The vegetation of the seaboard of the colony is much less known. Our earliest acquaintance with West Australian *Algæ* is derived from small but interesting collections, made by some of the early French exploring expeditions; and by Dr. ROBERT BROWN, who accompanied FLINDERS. Many of the less common species of these collections are only known to botanists by description or figures. By far the largest series of Algæ brought from this coast is that procured during four years' exploration of the colony by Mr. L. PREISS, to whom great credit is due for having collected 141 species, as, from the nature of his engagements, but little time could be devoted to this branch of botany. We owe to Dr. SONDER, of Hamburgh, a very able analysis and description of PREISS'S Algæ; and the Dublin University Herbarium is indebted to the liberality of Senator BINDER, of the same free city, for a tolerably perfect set of these Algæ. I have thus had the great advantage of examining authentic specimens of most of the new genera and species discovered by PREISS, and described by SONDER. A parcel containing between sixty and seventy species of Western Australian

Algæ, collected by Mr. Mylne, was presented to me by the late Dr. Charles Lemann, of London, and is now incorporated with the Dublin University Herbarium. This series, though small, contains several not ascertained by Preiss, and the specimens are generally more copiously collected, and in better order. I have received a few others from my friend J. Backhouse, of York, who procured them at Fremantle, during his visit to the colony. Collections of Algæ, I am informed, have been repeatedly made in this colony by amateurs, chiefly ladies; but respecting their contents the botanical world is no wiser, as they have been dispersed hither and thither among friends at home.

This is all the information I possess respecting previous algological researches in Western Australia. My own observations were made between January and August, 1854, at a few widely separated points on this extensive coast; not, perhaps, at the best possible collecting stations, but at those which were most accessible. These were King George's Sound and Cape Riche, on the southern coast; and Fremantle, Garden Island, and Rottnest Island, all in the immediate vicinity of Swan River, on the western coast. I shall briefly describe the features of the coast of these places.

I landed at King George's Sound in January, and remained till the end of February; and I revisited this shore in August. My head-quarters were at the little town of Albany, situated on the shores of Princess Royal Harbour, an oval, land-locked, lake-like basin, with a very narrow entrance; and I made frequent excursions on foot to the coasts in the vicinity, chiefly to Middleton Bay, distant about three miles; and also dredged repeatedly in various parts of the Sound between Bald Head and the opposite shores. The vegetation of the enclosed harbour is, as might be expected, very different from that of the more exposed Sound. Its shores are generally sandy, shoaling to a considerable distance from the margin, leaving a very broad marginal belt of less than two fathoms in depth at high water, and in many places of less than one fathom. The tides rise and fall very irregularly, being much influenced by the wind. The rise varies from two to four feet; and there is generally but one tide in the twenty-four hours. Now and then, however, I have observed two tides. The depth of the central basin varies from five to seven fathoms. About the entrance the shores are rocky and rather steep, the rocks being coarse granites perhaps the least adapted of any to the growth of Algæ. In all the shallow water round the northern and north-eastern beaches grows abundance of *Polyphysa peniculus*, a very remarkable little Alga, known only in this locality, where it was detected by Dr. R. Brown. It is invariably found attached to dead shells, chiefly to the separated valves of a common *Venus* (like *V. aurea?*), and is very frequently infested by a peculiar *Polysiphonia* (*P. infestans H.*), which I have found nowhere else. *Hormosira Labillardieri*, a fucoid plant, resembling strings of beads, and the only representative of the littoral fuci which I have met with, occurs on rocks near high-water mark, and extends to half-tide level. All the other fucoid plants of this coast commence at low-water mark, and are rarely left dry, even at the greatest recess of the tide. The deeper parts of the harbour appear to be occupied by immense strata of *Dictyota furcellata*, a slender, excessively branched species; and of *Stilophora Lyngbyæi*, with a liberal sprinkling of *Hypneæ*, and of a very luxuriant variety of *Spyridia filamentosa*. On the leaves of *Zostera*, and on the stems of *Caulinia antarctica*, both which form vast meadows in water from two to six feet deep, grows a profusion of small parasites, and on scattered stones, in the same zone of depth, *Laurencia Tasmanica*, and *Cystophyllum muricatum*, flourish abundantly.

At Middleton Bay is an extensive strand, some miles in length, reaching to the entrance of Oyster Harbour, and a narrow belt of rocky shore at the southern end, where, at the low-water of spring tides, many interesting species of the *Laminarian zone* may be gathered. *Ecklonia radiata*, the only laminarioid plant of this coast, fringes the whole of these rocks, and extends some distance within the heads of Princess Royal Harbour. Outside the heads, in the more open bay, the leaves are generally rough with prickles, and the whole plant grows stronger, being the state described by authors as *E. biruncinata* or *E. exasperata;* while in the tranquil water of the harbour the surface of the fronds is generally *smooth*, being the *E. radiata* of Agardh. From personal observations I conclude that these supposed species are not distinct, as originally stated by Turner. In summer time the rocks at Middleton Bay, between high and low water, are either completely bare, or produce a scanty vegetation of obscure *Calothrices;* or of a very minute *Polysiphonia*, with starved varieties of *Gelidium corneum;* the power of the sun being probably too great to admit of the growth of a fucoid vegetation, such as clothes rocks similarly exposed in colder climates. But in winter these same rocks are all densely covered with *Chorda lomentaria* and *Ectocarpus siliculosus*, two plants of rapid growth, and both belonging to forms which are rare in the warmer, and abundant in the colder waters of the sea. Just above the laminarian belt, and extending into it, several social *Laurenciæ*, both here and on other parts of the coast, cover the rocks, often in very wide patches.

Nothing of any interest was collected in Oyster Harbour; nor was dredging in the Sound attended with any very remarkable result. Very little of the amount dredged had been detached by the dredge; the greater portion consisted of drifting plants, collected by currents and eddies on various parts of the sandy bottom. The deepest fucoid plant, observed *in situ*, was *Scaberia Agardhii*, which abounds on every part of the coast explored by me in 2–5 fathom water. Wherever *Caulinia antarctica* can find a footing, its wiry stems, but rarely its leaves, are generally found covered with parasites, many of which (such as *Thuretia*, *Halophlegma*, and various *Dasyæ*) are very curious and beautiful. The parasites on *Zostera*, on the contrary, usually grow on the leaves, not on the stem; and here are found *Chondriæ*, *Griffithsiæ*, *Callithamnia*, *Wrangeliæ*, *Crouaniæ*, &c.

I spent the month of March at Cape Riche, a bold promontory, about 60 miles by compass, and 70 or 80 by land, to the east of King George's Sound; and famous for the beauty and variety of flowering plants found on the hills in its neighbourhood. Here I was the guest of George Cheyne, Esq., who has a farm and sheep-run at the Cape. The dry season had advanced too far to permit my seeing this beautiful district to the best advantage, or to allow of my making an extensive gathering of land plants; and the sea-shore proved to be singularly barren in Algæ. The ordinary *Fucoideæ* (*Sargassa* and *Cystophoræ*), with *Ecklonia radiata*, chiefly occupy the laminarian zone; and the smaller *Rhodospermeæ*, scattered among them, are few, and of little interest. Here, nevertheless, I collected a new Genus (*Lasiothalia*), and a remarkably fine *Liagora* (*L. Cheyniana*).

Early in April I started, overland, for Swan River, and on the 21st reached Fremantle, where I remained till the 21st of May; and returned again for the first fortnight in July. At this place the algologist must depend, either on the dredge, or on the western gales, which frequently throw drifted plants ashore. The coast at both sides of the town, which is built on a little calcareous promontory, consists of long, sandy beaches that extend for many miles. On these, in stormy weather, many beautiful plants are cast up; but, owing to the fineness of the weather during nearly the whole of my stay, my success must have fallen far short of that of a collector in average seasons. I am convinced of this from the reports I heard from many persons at Fremantle; and also from the fact that thirty of the species found by Preiss were not ascertained by me. Nevertheless, I more than doubled my previous list, finding very many species not in Preiss's collection. Some of these were dredged in the bay, in 5 or 6 fathoms water, but the greater number were picked up on the beach. Amongst the most remarkable of the Fremantle plants are *Claudea elegans* (found by George Clifton, Esq.), and *Kallymenia cribrosa*. *Halophlegma Preissii* is very common; so also is *Dasya tenera*, which, in a very few minutes after it has been removed from the water, melts into a rose-coloured, gelatinous mass. *Halosaccion firmum* and *H. Hydrophora*, apparently identical with the Kamtchatkan plants, are also very frequent; and *Eucheuma speciosum*, the jelly or blanc-mange weed of this colony, floats on shore in great abundance after winter gales.

Whilst residing at Fremantle, I made three excursions to Garden Island, distant about nine miles in a S.W. direction, landing each time on the northern and north-eastern beaches. On all these excursions I made very considerable collections of drifted plants, finding several species not seen or very rarely met with elsewhere. Among these the most remarkable were *Sarcomenia delesserioides* and *S. hypnœoides;* and *Lenormandia spectabilis*, which is here extremely abundant, varying greatly in size, and in the breadth of the frond. I noticed that several species found at this island were much more luxuriant than individuals of the same kind collected at Rottnest Island, a few miles to the north. This is especially the case with *Griffithsia Binderiana*,—the specimens from Garden Island being four times the size of those from Rottnest. This I attribute to the fact, that at Rottnest this species always grows on *Zostera;* whilst at Garden Island it attaches itself to various Algæ; and the observation (coupled with other similar ones elsewhere made), seems to render it probable that Algæ really derive nourishment from the soil on which they grow.

From Fremantle I moved to Rottnest Island, about the end of May, and remained till the end of June, a period of six weeks. This little island is situated about twelve miles W. by N. from Fremantle; and its land Flora is remarkable for the total absence of *Proteaceæ* and of grass trees (*Xanthorrhœa*), and for the paucity of *Myrtaceæ*, *Epacrideæ*, and *Leguminosæ* (with the exception of *Templetonia*, and two or three *Acacias*). It is seven miles long, and about three wide; it contains several large lakes of salt water, and is indented with many small bays, some of them with sandy beaches, and others rocky. Almost the whole island is surrounded by limestone reefs, at greater or less distances from the shore. The limestone seems of very recent formation, and is of similar character to that at Arthur's Head, and in other localities near Fremantle, already described by several geologists. It is remarkable for very fantastic and diversified forms. The reefs are generally flat-topped, but the surface is very rough, either thickly bristling with sharp points, a few inches high; or broken into miniature mountains and valleys,—strongly recalling to mind the *raised map* of Switzerland. Other reefs are ridged; the ridges parallel to each other, but variously directed towards the shore. The outer face of the bordering reef is generally very steep, often perpendicular or overhanging; and frequently it goes down, like a quay wall, into two or three fathoms water. At the N.E. angle of the island, a very remarkable quay-like reef, called the "Natural Jetty," runs out many hundred yards into the sea. Its surface is laid bare, at low-water, of spring tides, which rise and fall from 2 to 3½ feet. Many of the detached reefs are shaped like round tables, or mushrooms, being fixed on a slender central stalk, often only a few feet in diameter; the horizontal ledge, or table, spreading out to many yards on all sides. Sometimes two or three of these tables are joined together by narrow stone bridges; and sometimes large holes, through which you can look down two or three fathoms into the clearest water, are found in the table; and the swells rise through them, and flow over. I often wondered how these *filigree* reefs could so long withstand the beating of the waves in winter storms. Almost all of them offer good harvests to the algologist; and beautiful pictures to any one who can appreciate the loveliness of living vegetable forms. The surfaces of most are well clothed with the smaller *Rhodospermeæ* (*Laurenciæ*, *Hypneæ*, *Acanthophora*, &c.); and thickly studded with a *Caulerpa* (*C. lætevirens*, *Mont?*) with short stems, clothed with brilliant club-shaped leaves, resembling miniature clusters of grapes. At every few yards, deep basin-like hollows, of greater or lesser size, break the surface of the reef, and afford well-sheltered nooks for a variety of beautiful Algæ. The water in these basins is always intensely transparent; the bottom frequently of white sand; and the steep and craggy sides clothed with Algæ vegetation, in which the brightest tints of green, purple, carmine, and olive, and the most graceful waving forms, are mingled in rich variety. Here is the favourite locality of some eight or ten species of *Caulerpa*, of several very distinct forms, and every one a beautiful object. All these are green; but the tints vary from the darkest bottle-green to the pale, fresh green of an opening *beech* leaf. Some resemble soft ostrich feathers; others, branches of the Norfolk Island pine; others, strings of beads; others, squirrels' or cats' tails; and *C. scalpelliformis* is like a double saw. Under the shelter of the *Caulerpæ* the smaller *Rhodosperms* (such as *Dasyæ* and *Callithamnia*) are often found. But these are most numerous on the perpendicular sides of the border reefs, where also rich meadows of *Caulerpæ* are seen waving in the clear water, from a foot beneath the surface to a considerable depth. Various *Fucoideæ* and *Ecklonia radiata* are scattered here and there through the deeper pools, and on the sides of the reef. None of these are ever left dry at low water.. In many places a profusion of a *Bryopsis* (*B. Australis*) enlivens the rocks with its silky tufts of green, each tuft separate from its neighbour. Some of the shallower reefs, near high-water mark, are partially covered with sand: and this is the habitat of *Penicillus arbuscula*, a little green Alga, which may be compared either to a miniature tree, or to a shaving-brush. *Struvea plumosa* abounds on all the reefs, at about half tide level, generally growing on the very edges of the rock-pools and border-reefs. I obtained from Mr. SANFORD, Colonial Secretary, a specimen of a new *Struvea*, sent by Mrs. DRUMMOND from Champion Bay, differing from *S. plumosa* in its vastly larger size, and more compound network. The specimen has been bleached white, and in this state strongly resembles a beautiful pattern of *old point-lace*, and might be made into ladies' collars, as it is of a tough substance.

I shall conclude this summary with a few remarks on the geographical distribution of the species collected.

The annexed descriptive catalogue contains 352 species: of which 277 are (so far as we yet know) peculiar to the Australasian Flora, and 75 belong either to *pelagic* species, or to more or less distant botanical regions. They are grouped as follows:—

	Whole number collected.	Australian.
Ser. 1. *Melanospermeæ*,	42	26
„ 2. *Rhodospermeæ*,	270	216
„ 3. *Chlorospermeæ*,	40	35
	352	277

These numbers do not show the whole of the *Melanospermeæ* observed; some 15 or 20 species of *Sargassum* and *Cystophora* not having been examined, and having therefore been omitted from the list.

Still, the great preponderance of *Rhodospermeæ* is a remarkable feature. But the most singular fact is the proportion between the *Australian* and *pelagic* species of *Chlorospermeæ*, a group whose species are, generally speaking, much less local than those of either of the other divisions. The comparatively great number of *Siphoneæ* in Australia is one reason of this anomaly; another may be, that I have not yet minutely examined the species of *Cladophora* and *Calothrix*. Nevertheless, there is a marked deficiency in W. Australia of the common littoral *Chlorosperms*.

The *Pelagic* species, or those which inhabit many very distant places and dissimilar climates, are :—

Chorda lomentaria.
Dictyota dichotoma.
Asperococcus echinatus.
Ectocarpus siliculosus.
Gelidium corneum.
Plocamium coccineum.
Spyridia filamentosa.
Centroceras clavulatum.
Ceramium rubrum.
——— *fastigiatum.*
Gracilaria confervoides.
Codium tomentosum.
Ulva latissima.
Enteromorpha compressa.

Species showing affinity with the vegetation of the Red Sea and Indian Ocean, are :—

Turbinaria vulgaris.
Cystoseira prolifera.
Dictymenia fraxinifolia.
Leveillia jungermannioides.
Dasya Lallemandi.
Callithamnion thyrsigerum.

Connecting the W. Australian with the Flora of the South Pacific, are :—

Dictyota Kunthii ; Rhodymenia corallina ; Ceramium miniatum.

The Cape of Good Hope is represented by,—

Martensia elegans ; Dasya pellucida ; and *Halophlegma.*

Of Antarctic species are *Callithamnion simile* and *Delisia pulchra*, both found by Dr. HOOKER at Kerguelin's Land.

Representing the North Pacific, from S. Francisco to Kamtchatka, are *Halosaccion firmum* and *H. hydrophora*, identical, so far as my judgment goes, with the specimens from high northern latitudes.

The characteristic vegetation of the Mediterranean Seas (of Europe and Mexico) is more largely developed, as shown by the following list :—

Dictyota ciliata.	*Polysiphonia Havanensis.*	*Liagora distenta.*
Hydroclathrus cancellatus.	——— *obscura.*	——— *viscida.*
Asperococcus sinuosus.	——— *pennata.*	*Halymenia Floresia.*
Chondria sedifolia?	*Dasya mollis.*	*Dudresnaia coccinea.*
Polysiphonia breviarticulata.	*Wrangelia penicillata.*	*Crouania attenuata.*
	Peyssonelia rubra.	*Halimeda macroloba.*
	Helminthora divaricata.	

The following 27 are natives of the coasts of the British Islands, as well as of those of W. Australia :—

Chorda lomentaria.	*Champia parvula.*	*Ceramium fastigiatum.*
Dictyota dichotoma.	*Laurencia obtusa.*	——— *gracillimum.*
Stilophora Lyngbyæi.	*Gracilaria confervoides.*	*Dudresnaia coccinea.*
Asperococcus Turneri.	*Gelidium corneum.*	*Crouania attenuata.*
——— *echinatus.*	*Helminthora divaricata.*	*Callithamnion sparsum?*
Sphacelaria cirrhosa.	*Plocamium coccineum.*	*Codium tomentosum.*
Ectocarpus siliculosus.	*Rhodophyllis bifida.*	*Ulva latissima.*
Chondria dasyphylla.	*Spyridia filamentosa.*	*Enteromorpha compressa.*
Polysiphonia obscura.	*Ceramium rubrum.*	*Calothrix cæspitula?*

I hope this outline may prove not uninteresting to botanists, and trust to be permitted, after my return to Europe, to lay before the Academy a more full memoir on this subject, accompanied by copious descriptions of the new species, and plates illustrative of the new genera, and some of the more remarkable species.

W. H. HARVEY.

MELBOURNE, *September* 11, 1854.

Catalogue *of Marine Algæ, collected by* Dr. W. H. Harvey *in Western Australia, from January to August*, 1854; *with short Descriptions of the New Genera and Species.*

Note.—The *Numbers* between parentheses ( ) in this List are those under which the Species stand in a running Catalogue, kept by Dr. H., as the Collection proceeds.

Series I.—MELANOSPERMEÆ.

Order I.—FUCACEÆ.

Sargassum, Cystophora, } Several species of these genera have been collected and packed away without examination, and are not now accessible.

1. Turbinaria *vulgaris*, J. Ag. Fragments on the beach, Fremantle (288).
2. Scytothalia *dorycarpa*, Grev. Rocks below low-water mark, common. King George's Sound, and Rottnest (300).
3. Scytothalia *dorycarpa β. xiphocarpa; S. xiphocarpa*, J. Ag. Thrown up from deep water at King George's Sound and Cape Riche. I consider that the characters which distinguish this plant from the preceding depend on depth of water, and exposure to currents (301).
4. Scaberia *Agardhii*, Grev. Very common everywhere, in 2–3 fathoms (83).
5. Cystophyllum *muricatum*, J. Ag. Common in Princess Royal Harbour, King George's Sound; and in the Swan River, from Perth to Fremantle (73).
6. Cystoseira *prolifera*, J. Ag. A single specimen on the beach, Fremantle, after a gale (287).
7. Hormosira *Labillardieri*, Bory. Common near high-water mark, and at half-tide, in Princess Royal Harbour, King George's Sound. Rare at Cape Riche. Not seen elsewhere (76).
8. Carpoglossum *quercifolium*, Kütz. Rottnest, on the reefs ( ).
9. Carpoglossum *angustifolium*, Sond. Cast ashore at Cape Riche and Fremantle ( ).
10. Myriodesma *serrulatum*, Dne. A few specimens picked up at Cape Riche and Fremantle, after storms (159).
11. Myriodesma *latifolium*, n. sp.; on the beach at Fremantle (278). My specimens not being at hand, I cannot at present further characterize this new species than by saying that it has the ramification of *M. serrulatum*, but the segments are an inch broad, densely dotted with innumerable scaphidia. It is quite different from *M. quercifolium*, Bory.
12. Notheia *anomala*, Bail. and Harv. Harv. in Hook. Fl. Nov. Zel. cum icone. Parasitical on *Hormosira* in Princess Royal Harbour.

Order II.—SPOROCHNACEÆ.

13. Sporochnus *comosus*, Ag. (?) King George's Sound and Fremantle, two or three feet long, and much stouter and more rigid than *S. pedunculatus* (13).

14. SPOROCHNUS *sp.* Fremantle (157). Being uncertain whether this or the preceding be Agardh's plant, I defer the description of either.
15. SPOROCHNUS *radiciformis*, Ag. Fremantle and Cape Riche (156).
16. SPOROCHNUS *scoparius*, n. sp.; fronde tereti rigidâ crassâ dendroideâ (2–3 pedali); caule strato velutino vestito; ramis creberrimis undique egredientibus decomposito-pinnatis angulatis glabris, minoribus erectis strictis sparsè spinosis subalternis; receptaculis ovalibus v. oblongis pedicellum ipsis multiplo longius coronantibus. At Cape Riche, and Garden and Rottnest Islands (248). I collected this at first as *Fucus inermis*, R. Br., or *F. caudatus*, Lab.; but my plant is a true *Sporochnus*, and not always unarmed.

ORDER III.—LAMINARIACEÆ.

17. ECKLONIA *radiata*, Turn. *E. radiata* and *E. exasperata*, J. Ag. Lining most of the rocky shores at extreme low-water mark. Examination on the sea shore disposes me to unite these two supposed species. They vary extremely in roughness and smoothness, and in the comparative length of the rachis, all the forms imperceptibly running together (75).
18. CHORDA *lomentaria*, Lyngb. Clothing tidal rocks, in winter, at King George's Sound. My specimens are not fully grown, being in the state called *Asp. castaneus*, Carm. (323).

ORDER IV.—DICTYOTACEÆ.

19. HALISERIS *Mulleri*, Sond.; stipite elongato ramoso; fronde dichotomâ v. suppressione ramorum alternè ramosâ, sinubus obtusiusculis, segmentis erectis latis linearibus integerrimis sæpè alternè divisis; laminâ crassiusculâ enervi; antheridiis sparsis. King George's Sound, Cape Riche, and Fremantle (102). Much larger and thicker in substance than *H. polypodioides* with rounded sinuses.
20. HALISERIS *pardalis*, n. sp.; stipite brevi; fronde dichotomâ, sinubus rotundatis, segmentis patentibus linearibus integerrimis repetitè furcatis subundulatis obtusis; laminâ tenui-membranaceâ enervi; soris dispositis in lineas recurvas è costâ ad marginem proficiscentibus. Fremantle, rare (155). A beautiful and distinct species, elegantly marked in dotted lines like a leopard's skin.
21. PADINA *Frazeri*, Grev. Fremantle and Rottnest, common (158).
22. ZONARIA *nigrescens*, Sond. Rocky shores, common (49). Very near *Z. variegata*, if distinct.
23. ZONARIA *interrupta*, Ag. var. *spiralis;* segmentis spiraliter tortis. Cape Riche and Rottnest (295).

METACHROMA, nov. gen. *Caulis* basi radicans, cartilagineus, tereti-compressus, alternè ramosus. *Rami* infernè costati, lineares, pinnatifidi, lacinulis bicuspidatis. *Sporæ* (?) per superficiem laciniarum sparsæ, prominentes, intra perisporum hyalinum singulæ nidulantes. *Alga* Australasica radice ramoso-fibrillosâ, caule ramosissimo, ramis spiraliter tortis, laciniis tortione spuriè trifariis.

24. METACHROMA *thuyoides*, n. sp.; Middleton Bay, King George's Sound; and Cape Riche at low-water mark (21). *Frond* 12–18 inches long, much branched. The generic name alludes to a remarkable change of colour, from olive to verdigris green, when thrown into fresh water.

25. Dictyota *Kunthii*, Ag. Key West and Rottnest (81 and 225).
26. Dictyota *fastigiata*, Sond. Abundant at Cape Riche and Fremantle (167). A true *Dictyota.*
27. Dictyota *radicans*, n. sp.; fronde estuposâ stipitatâ basi fibris crassis sparsis è stipite et laminâ emissis radicante dichotomo-pinnatifidâ, segmentis cuneatis, lateralibus erectis, sinubus angustis, apicibus obtusissimis; soris effusis in medio parte frondis collectis. Rottnest and Garden Island (184). This species is readily marked by its rooting by a few rope-like filaments.
28. Dictyota *paniculata*, J. Ag. Common (14). If I rightly understand this plant it varies much in breadth and degree of ramification.
29. Dictyota *furcellata*, Ag.? *D. minor*, Sond. Excessively common in Princess Royal Harbour, King George's Sound, and elsewhere. In summer it comes ashore in vast banks, and is often the only plant raised from the bottom, by the dredge or hooks, in shallow water (24).
30. Dictyota *dichotoma*. King George's Sound and Rottnest (15).
31. Dictyota *ciliata*, J. Ag.? Carnac and Rottnest Islands, on shallow reefs, growing with *D. dichotoma*, from which its greener colour and ciliate margins best distinguish it (154).
32. Stilophora *Lyngbyæi*, J. Ag. Princess Royal Harbour in summer, very common (25).
33. Hydroclathrus *cancellatus*, Bory. Cape Riche, Fremantle, and Rottnest (183).
34. Asperococcus *sinuosus*, Ag. King George's Sound and Rottnest, &c. (27).
35. Asperococcus *Turneri*, Hook. *A. bullosus*, Auct. King George's Sound and Fremantle (26).
36. Asperococcus *echinatus*, Lx. King George's Sound ( ).

Order V.—CHORDARIACEÆ.

37. Cladosiphon? sp. . . . King George's Sound (17). This has the habit of *Mesogloia virescens*, and I should so name it, but that the frond is certainly hollow, which character would put it in *Cladosiphon.* I am by no means, however, satisfied that this is a character of any generic importance in these plants.
38. Mesogloia *filum*, n. sp.; fronde simplici v. ramo uno v. altero donatâ, basi et apice attenuatâ. King George's Sound (82).

Order VI.—ECTOCARPACEÆ.

39. Sphacelaria *paniculata*, Suhr. Cape Riche (297).
40. Sphacelaria *Novæ Hollandiæ*, Sond. Cape Riche, on rocks and shells in shallow water, common. Dredged at Fremantle (296).
41. Sphacelaria *cirrhosa*, Ag. On *Zostera* leaves, Fremantle, common (153).
42. Ectocarpus *siliculosus*, Lyngb. Very abundant at King George's Sound, in winter. Just commencing at Rottnest in June; and at Cape Riche in March (322).

Series II.—RHODOSPERMEÆ.

Order I.—RHODOMELACEÆ.

43. Claudea *elegans*, Ag. Fremantle, very rare, June, *Geo. Clifton, Esq.* (276).

44. Martensia *elegans*, Her. *M. Brunonis*, Harv. MS. Garden Island and Rottnest, rare, May and June. My specimens seem identical with the South African ones (170).

45. Martensia *denticulata*, n. sp.; frondibus sessilibus cæspitosis tenui-membranaceis repetitè dichotomis, laciniis cuneatis ultimis non raro flabelliformibus; margine crispato denticulato; fenestro apice ciliato v. lobato, lobulis demum elongatis fenestratisque. Species valdè variabilis. Garden Island and Rottnest, on reefs near low-water mark, June (171).

46. Martensia *Australis*, n. sp.; stipite cartilagineo brevi in frondem multilobatam membranaceam basi incrassatam desinente, margine hic illic minutissimè denticulato; fenestro apice angustissimè marginato denticulato. King George's Sound, rare, February (88).

47. Thuretia *quercifolia*, Dne. King George's Sound and Garden Island (65).

48. Sarcomenia *delesserioides*, Sond. Garden Island and Fremantle (130). Three varieties occur together, viz.: *a. latifolia*, phyllodiis lato-lanceolatis; *β. lancifolia*, phyllodiis lineari-lanceolatis; *γ. cirrhosa*, phyllodiis angustissimis, supremis sæpiùs cirrhiferis. The plant described by me in Ner. Austr. as *S. delesserioides* is a *Delesseria*, namely, *D. corifolia*, H. I have now gathered Sonder's plant in abundance.

49. Sarcomenia *hypneoides*, n. sp.; fronde lineari angustissimâ compressâ distichè ramosissimâ, ramis ramulisque oppositis attenuatis acutis basi nec angustatis; stichidiis lanceolatis sparsis v. fasciculatis. Garden Island and Fremantle. Certainly a congener with the preceding, to which it bears precisely the same relation that *Desmarestia viridis* does to *D. ligulata*. Both this and the preceding species are gray and iridescent when living, but turn a brilliant rosy red after a few minutes' exposure to the air, and this colour is preserved in drying (276).

50. Lenormandia *spectabilis*, Sond. Garden Island, abundant; rare at Rottnest (113). *L. latifolia*, Harv. Ner. Austr. is only a broad-leaved variety. This plant varies extremely in size.

51. Jeannerettia *frondosa*, n. sp.; caule dichotomo cartilagineo alato v. denudato; phyllodiis cuneatis dichotomis crispatis, costâ infra medium laminæ evanescente; fasciculis stichidiorum sparsis. Garden Island, rare (112). This plant is intermediate in character between *Jeannerettia* and *Pollexfenia*.

52. Pollexfenia *pedicellata*, Harv. Ner. Austr., t. 5. King George's Sound, Garden Island, and Rottnest, common (33). *β. multipartita;* fronde angustiore, regulariter dichotomâ (100). *P. multipartita*, Harv. in Herb. T. C. D. Having collected both these forms in abundance, I am forced to unite them under one specific name.

53. Polyphacum *proliferum*, Ag. King George's Sound and Fremantle (89).

54. Thamnoclonium *proliferum*, Sond. King George's Sound, cast ashore (318).

55. Thamnoclonium *flabelliforme*, Sond. Fremantle, in fragments only (319).

56. Thamnoclonium *Lemannianum*, n. sp.; caule corneo crasso (pedali et ultrà) echinulato infernè tereti supernè alato ramoso; ramis quoquoversum directis alatis phyllodia proliferè ferentibus; phyllodiis furcatis v. dichotomis costatis basi cuneatis apice obtusis, segmentis lateralibus erectis plus minus incisis. Fremantle, cast ashore in July (320). I first received this truly noble species in a collection of Western Australian Algæ, made by *Mr. Mylore*, and presented to *Herb. T. C. D.* by my late lamented friend Dr. Charles Lemann, of London, to whose memory this plant is now consecrated.

57. Dictymenia *fraxinifolia*. *Fucus fraxinifolius*, Turn. Rottnest, rare (241). I abandon the genera *Epineuron* and *Spyrymenia* as not being distinguishable from *Dictymenia*.

58. Dictymenia *fimbriata*, Grev. Garden Island, rare (110).

59. Dictymenia *tridens*, Grev. Garden Island, Rottnest, and King George's Sound (111).

60. Dictymenia *spiralis*, Sond. Common everywhere (20).

61. Dictymenia *pectinella*, n. sp.; fronde infernè valdè costatâ supernè sub-costatâ lineari distichè ramosâ planâ; ramis erecto-patentibus oppositis v. abortu alternis linearibus obtusis tenuissimè costatis ciliato-fimbriatis; ciliis oppositis argutè pectinato-pinnatifidis involutis; antheridiis magnis ovalibus ad apices ciliarum fasciculatis. Garden Island, very rare (290). A very distinct and beautiful species.

62. Kützingia *canaliculata*, Sond. Abundant everywhere. Often 2 or 3 feet in length (61).

63. Kützingia *angusta*, n. sp.; fronde infernè costâ cartilagineâ percursâ decompositè pinnatâ; ramis angustè-linearibus planis, superioribus tenuissimè costatis v. ecostatis; ramulis oppositis erecto-patentibus obtusis apice involutis. Rottnest, rare (242). A very much smaller, narrower, and thinner plant than *K. canaliculata*, of which it has the structure.

64. Kützingia *serrata*, n. sp.; fronde basi cartilagineâ denudatâ v. alato-marginatâ bi-tripinnatifidâ et è costâ primariâ proliferâ; laciniis membranaceis planis tenuissimè costatis, junioribus, lacinulisque argutè serratis. Rottnest, very rare (291).

65. Rytiphlæa *Australasica*, Mont. King George's Sound, common. Rare at Garden Island (31).

66. Rytiphlæa *elata*. (*Rhodomela elata*, Sond.!) dendroidea (1–2 pedalis); caule tereti crassissimo (2–3 lineas diametro) opaco ramoso; ramis decomposito-ramosissimis di-tri-chotomis v. vagè divisis, minoribus ramulisque patentibus transversim striatis; striis approximatis; axillis latissimis; ceramidiis ovatis pedicellatis; stichidiis ad latera ramulorum fasciculatis; siphonibus primariis 5–6 magnis, strato crasso cellularum minutarum corticatis. Cast ashore at Fremantle (304). A gigantic species, quite unlike any known to me.

67. Trigenia *Australis*, Sond. Cast ashore in July, Fremantle (292).

68. Acanthophora *dendroides*, n. sp.; caule incrassato indiviso infernè nudo supernè ramis alternis spiraliter evolutis vestito; ramis decompositis circumscriptione lanceolatis; ramulis spinosis, spinulis solitariis sparsis. Rottnest on the reefs, near low-water mark (224). Much the largest and most robust of the genus.

69. Alsidium? *spinulosum*, n. sp.; fronde tereti crassâ dendroideâ decompositè ramosissimâ; ramis ramulisque erectis quoquoversum sistentibus; ramulis spinæformibus sparsis; ceramidiis ramulos terminantibus. Garden Island, Rottnest, and Cape Riche (180). Primary tubes in the stem, 5, very large, and full of granular endochrome.

70. CHONDRIA *dasyphylla*, Ag. King George's Sound, August (293).

71. CHONDRIA *sedifolia*, Harv. Ner. Bor. Amer. *C. zostericola*, and *C. Curdicana*, Harv., MS. Common on *Zostera* leaves, King George's Sound, and Rottnest (29).

72. CHONDRIA *corynephora*, n. sp.; fronde tereti succosâ siccitate roseâ robustâ quoquoversum ramosissimâ; ramis indivisis patentibus è latere bis terve ramosis; ramulis oppositis, fasciculatis, v. sparsis, sæpiùs incurvis cylindraceis basi constrictis obtusissimis. Cape Riche and Garden Island (114). Much more robust than *C. dasyphylla*. It soon breaks to pieces in fresh water, by which character and others it is readily known from the following.

73. CHONDRIA *verticillata*, n. sp.; fronde tereti succosâ siccitate badiâ bis-terve umbellatim divisâ; ramulis fasciculato-verticillatis saccatis oblongis obtusissimis basi constrictis; tetrasporis in ramulis nidulantibus. Garden Island, rare (273).

74. CHONDRIA *Umbellula*, n. sp.; fronde pusillâ ($\frac{1}{2}$–1 unciali) simplici saccato-clavatâ apice ramulis 5–10 conformibus umbellatim coronatâ; ramulis nunc apice umbellulatis; ceramidiis ovatis sessilibus; tetrasporis sparsis (190). Rottnest, on *Zostera* leaves. A very curious and pretty little species.

75. CHONDRIA *lanceolata*, n. sp.; fronde pusillâ (1–2 unciali) compressâ cartilagineâ alternè ramosâ sub-distichâ; ramis ramulisque alternis basi et apice attenuatis acutis; ceramidiis ovatis pedicellatis; tetrasporis sub apicibus ramulorum congestis. Rottnest, on *Zostera* leaves (191).

76. LEVEILLIA *jungermannioides*. *L. Schimperi*, and *L. gracilis*, Dne. Abundant on a variety of *Algæ* at Fremantle, Garden Island, and Rottnest (123).

77. POLYZONIA *Sonderi*, Harv. Ner. Austr. Garden Island, on *Fucoids* (284).

78. POLYZONIA *flaccida*, n. sp.; caule primario repente; ramis erectis simplicibus ramosisve tenuissimis flaccidis oligosiphoniis; foliis (v. ramulis) alternis pectiniformibus, pectinis lacinulis 5–6 filiformibus articulatis *monosiphoniis* acutis; stichidiis arcuatis rostratis. On *Fucoids*, King George's Sound, Garden Island, and Rottnest. Much more slender, and of softer texture than *P. Sonderi*, and readily known by its one-tubed lacinulæ (34).

79. POLYSIPHONIA *Hystrix*, Harv. Ner. Austr., t. 14. Cast ashore, Garden Island (121).

80. POLYSIPHONIA *Mallardiæ*, Harv. Ner. Austr., t. 13. With the preceding (117).

81. POLYSIPHONIA *breviarticulata*, Ag. Abundant on the reefs, near low water, Rottnest (188).

82. POLYSIPHONIA *Havanensis*, Mont. (?) With the preceding, profusely common. More robust than the American plant, but otherwise very similar (118).

83. POLYSIPHONIA *infestans*, n. sp.; pallida, siccitate fuscescens; frondibus (2–3 uncialibus) cartilagineis chartæ arctè adhærentibus setaceis sursum attenuatis pellucidè articulatis ramosissimis; ramis patentibus pluries alternè v. vagè divisis ramulisque conspersis; ramulis capillaribus simplicibus patentibus; axillis latis; articulis 4-siphoniis subtorulosis, inferioribus diametro brevioribus, superioribus æqualibus v. sublongioribus. Common on *Polyphysa peniculus*, at Princess Royal Harbour, King George's Sound. It has the habit of *P. fibrillosa*, but is more nearly allied to *P. Harveyi* and *P. Binneyi* than to any other that I remember (22).

84. POLYSIPHONIA *mollis*, Harv. Ner. Austr. On *Zostera*, at Fremantle (120).

85. Polysiphonia *mutabilis*, n. sp.; mollis, aëre cito deliquescens, versicolor, siccitate rosea, frondibus aggregatis (2–3 uncialibus) tenuissimè corticatis articulatis supernè ecorticatis dichotomis ramosissimis; ramis minoribus subalternè divisis erecto-patentibus; ramulis sparsis basi et apice attenuatis acutis; articulis 6-siphoniis, ramorum diametro æqualibus, ramulorum brevioribus. On *Zostera*, at Fremantle (116). Pale brown when fresh, but almost instantly changing to rose red, and soon decomposing. I have neglected to make a section of the living stem, and it is impossible to cross-cut the dried frond, and very difficult to remove from the paper the smallest scrap for examination. Three primary tubes are seen in the front view of each articulation; and in most of the branches a series of external, shorter, secondary cells appear, being the commencement of a cortical layer, which is more evident in the lower parts of the frond.

86. Polysiphonia *Roeana*, n. sp.; punicea; frondibus (3–6 uncialibus) cæspitosis capillaribus mollibus chartæ arctè adhærentibus decompositè ramosissimis; ramis alternè compositis sæpè subsecundis pluries divisis; ramulis ultimis filiformibus elongatis sparsis omnibus eximiè patentibus; axillis latissimis; articulis pellucidè 4-siphoniis, inferioribus diametro 4–6-plo, superioribus duplo, ramulorum sesqui-longioribus. Dredged at Fremantle in 4–5 fathoms (119). A beautiful species, allied to *P. formosa*, but quite distinct. I name it in honour of J. S. Roe, Esq., Surveyor-General of the colony, from whom I received much kind attention during my stay at Perth, and who, though not a botanist, never neglects an opportunity of promoting the science.

87. Polysiphonia *rufolanosa*, n. sp.; siccitate rosea; frondibus pusillis (vix uncialibus) densissimè intertextis arachnoideis dichotomis ramosissimis suffastigiatis; ramis ramulisque patentissimis divaricato-squarrosis crispisque; axillis distantibus; articulis 4-siphoniis diametro sesquilongioribus. On the stems of *Caulinia antarctica*, Princess Royal Harbour, King George's Sound (39). To the naked eye this little plant looks like a small *Callithamnion*, or like delicate flocks of fine crimson silk. The stems are about $\frac{1}{600}$ of an inch in diameter.

88. Polysiphonia *scopulorum*, n. sp.; badia; frondibus pusillis (vix uncialibus) cæspitosis basi radicantibus rigidulis capillaribus tetragonis erectis parcè ramosis infrà simplicibus suprà ramis lateralibus plus minùs onustis; ramis sæpè secundis erectis simpliciusculis vel ramuliferis; ramulis paucis consimilibus; axillis angustissimis; articulis diametro subduplo-longioribus, superioribus æqualibus; ceramidiis ovatis sessilibus. On littoral rocks, Rottnest, common (187). Allied to *P. rudis*, but smaller. It slightly adheres to paper in drying.

89. Polysiphonia *implexa*, Hook. and Harv. Nov. Zel. Parasitic on *Corallines* and on *Caulinia* at King George's Sound (79).

90. Polysiphonia *prostrata*, n. sp.; parasitica, omnino prostrata, discis rameis prorepens, rubra, siccitate fuscescens; frondibus pusillis (1–2 uncialibus) è centro radiantibus subparallelis secundè ramosis; ramis filiformibus simplicibus repentibus apice involutis; ramulis liberis paucissimis brevissimis; articulis 4-siphoniis diametro subduplo-brevioribus; ceramidiis ovatis longiusculè pedunculatis (ramos v. ramulos terminantibus). Parasitical on the fronds of *Zonaria nigrescens*, which it sometimes completely covers over with cobweb-like threads, Fremantle, rare (305).

91. Polysiphonia *neglecta*, MS. Sand-covered rocks, at Middleton Bay, King George's Sound, mixed with *P. pennata* and *Callith. cymosum.* I have not fully determined this species, which requires a careful comparison with some others of similar habit (11).

92. Polysiphonia *forcipata*, n. sp.; pallida, siccitate purpureo-nigrescens; frondibus subsolitariis (2–3 uncialibus) crassis cartilagineis pellucidè articulatis repetitè dichotomis v. abortu scorpioideo-secundis; ramulis ultimis bis terve furcatis apice forcipatis! articulis 6–siphoniis diametro brevioribus; ceramidiis ovatis sessilibus. On *Zostera* at Rottnest and King George's Sound (186). A distinct species, looking like a *Ceramium* to the naked eye.

93. Polysiphonia *cancellata*, Harv. Ner. Austr., t. 15. King George's Sound, common (35).

94. Polysiphonia *nigrita*, Sond. Garden Island and Rottnest (122).

95. Polysiphonia *aurata*, n. sp.; fusco-rubra, madefactâ aurea; frondibus cæspitosis (2–3 uncialibus) capillaribus cartilagineo-membranaceis articulatis decompositè ramosis; ramis dichotomis alternisve erecto-patentibus; ramulis alternis v. secundis apice furcellatis; articulis 10–siphoniis inferioribus diametro 2–3-plo-longioribus, superioribus æqualibus; septis angustissimis; ceramidiis ovatis sessilibus; tetrasporis magnis subsolitariis. King George's Sound, rare (307). Allied to *P. furcellata* in ramification, and to *P. versicolor* in substance and colour.

96. Polysiphonia *versicolor*, Harv. Ner. Austr., t. 16. King George's Sound (36), var. *β. tenuior.* With the preceding (37).

97. Polysiphonia *rostrata*, Sond. On *Caulinia*, &c. Rottnest and Fremantle (115).

98. Polysiphonia *pennata*, Ag. Sand-covered rocks, Middleton Bay, King George's Sound (12).

99. Polysiphonia *pectinella*, n. sp.; siccitate roseo-purpurea; frondibus pusillis (uncialibus) basi radicantibus ramosis arachnoideis; ramis paucis alternis v. sparsis filiformibus simplicibus per totam longitudinem pectinatis; ramulis secundis patentissimis simplicibus brevibus obtusis; articulis 8–siphoniis diametro æqualibus v. duplo-longioribus. On mud, near high-water mark, Princess Royal Harbour, King George's Sound. A larger variety at Rottnest (38). Certainly allied to *P. Pecten Veneris*, but a far more delicate and more brightly coloured species.

100. Polysiphonia *obscura*, Ag. Sand-covered rocks at Middleton Bay, King George's Sound; mixed with *P. pennata* and *P. neglecta* (47).

101. Polysiphonia *Calothrix*, n. sp.; minuta, densè cæspitosa, rupestris, badia; surculo prostrato radicibus numerosissimis elongatis apice mamilloso-squamosis radicante; ramis erectis secundis simplicissimis brevissimis approximatis subacutis; articulis 10–12-siphoniis, surculi diametro duplo-brevioribus, ramorum adultorum sesquiduplo-longioribus; tetrasporis paucis in ramis nidulantibus. On rocks at half-tide level, King George's Sound (337). This spreads in wide patches, like those of *Calothrix scopulorum*, which it so closely resembles in aspect, that I had actually dried and set it aside for that plant, nor did I discover my error till after I had applied the microscope. It is a larger plant than *P. prorepens*, and very much smaller than *P. obscura*, to which it is allied.

102. Polysiphonia *prorepens*, Harv. Ner. Austr. Parasitical on *Dicranema Grevillii*, at King George's Sound (306).

103. Polysiphonia *cladostephus*, Mont. Garden Island and King George's Sound (271).

104. Dasya *Gunniana*, Harv. Ner. Austr., t. 17. On the reef called "The Natural Jetty," Rottnest (211).

105. Dasya *elongata*, Sond. Abundant at Fremantle, and Rottnest, and King George's Sound (59).

106. Dasya *Cliftoni*, n. sp.; caule elongato (pedali et ultrà) tenui flexuoso v. scandente glabro omnino corticato subdistichè ramoso bi-tripinnato, pinnis patentibus glabris; pinnulis alternis remotiusculis ramellosis; ramellis multoties divaricato-dichotomis vix attenuatis obtusis monosiphoniis, articulis cylindraceis, diametro 3–4-plo-longioribus. Dredged in Fremantle Harbour, by G. Clifton, Esq., after whom this beautiful plant is deservedly named. I also collected it at Garden Island and Rottnest, and afterwards at King George's Sound (164).

107. Dasya *frutescens*, n. sp.; caule (2–4 unciali) vagè ramosissimo glabro corticato; ramis quoquoversum directis patentibus bis-terve divisis attenuatis, minoribus ramellis vestitis; ramellis pluries dichotomis vix attenuatis obtusis, segmentis falcato-recurvis v. incurvis, articulis diametro 2–3-plo-longioribus; ceramidiis sessilibus urceolatis ore porrecto; stichidiis minutis sessilibus oblongis acutis. Rottnest, on *Zostera*. Something like a small form of *D. elongata*, but with much more slender and longer jointed ramelli. It is perhaps nearer to *D. arbuscula*, with which, however, it does not agree (303).

108. Dasya *proxima*, n. sp.; fronde crassâ corticatâ vagè ramosâ; ramis elongatis virgatis simplicibus vel ramos 2–3 consimiles lateraliter ferentibus, ramis omnibus ramulos breves quoquoversum emittentibus; ramulis corticatis simplicibus v. iterum ramosis, junioribus ramellis vestitis; ramellis subverticillatis dichotomis è basi latâ conspicuè attenuatis, axillis patentibus, apicibus filiformibus obtusis, articulis diametro 3–4-plo-longioribus; ceramidiis ramulos primarios terminantibus urceolatis ore brevi prominulo. Cast ashore at Middleton Bay, King George's Sound, August. Nearly allied to *D. elongata*, but the ramelli are very different, quickly melting in fresh water. It is a much larger plant than *D. naccarioides*, with larger ramelli and longer joints (336).

109. Dasya *collabens*, Harv. Ner. Austr., t. 21. King George's Sound, rare (58).

110. Dasya *Wrangelioides*, n. sp.; caule gracili (2–3 unciali) pellucidè articulato 10–12-siphonio distichè ramoso omnibus partibus ramellis vestito; ramis patentibus sursum curvatis simplicibus v. iterum alternè ramosis; ramellis densissimis multoties divaricato-dichotomis acutis, articulis diametro sesquilongioribus; ceramidiis . . . . . . ; stichidiis minutissimis ovato-acuminatis. Parasitical on *Caulinia antarctica*. Fremantle, King George's Sound, and Cape Riche. A very distinct species, named from its external resemblance to *Wrangelia velutina* (272).

111. Dasya *multiceps*, n. sp.; caule subnullo (ferè bulboso!) mox in ramos numerosissimos erectos diviso; ramis (2–3 uncialibus) simplicibus pellucidè articulatis, articulis diametro subbrevioribus polysiphoniis, pinnatis v. apice bipinnatis, ambitu linearibus v. lineari-spathulatis; pinnis oligosiphoniis alternis approximatis brevissimis superioribus sensim longioribus ramellosis; ramellis alternis pluries dichotomis parum attenuatis obtusis. On sand-covered rocks, half buried in sand, on the Natural Jetty reef, Rottnest, June. The specimens are not in fruit, and probably but half grown. There is an evident tendency in the upper pinnæ to lengthen and become compound (251).

112. Dasya *plumigera*, n. sp.; caule elato (pedali et ultrà) crasso villis stipato sub-dichotomo, segmentis ramiferis; ramis secundariis longissimis (1–2 pedalibus) caule multò tenuioribus glabris corticatis simplicibus infernè sæpè denudatis supernè pulcherrimè plumoso-pinnatis; pinnis alternis crebris horizontalibus plus minùs ecorticatis polysiphoniis iterum pinnulatis; pinnulis oligosiphoniis brevissimis ramelliferis; ramellis dichotomis attenuatis obtusis, articulis diametro 2–4-plo-longioribus; ceramidiis magnis pedicellatis inflato-ovatis ore prominulo; stichidiis minutis oblongis acutis. King George's Sound, and Cape Riche, and Garden Island; cast ashore and dredged. Also sent by *Dr. Curdie* from Cape Northumberland. A superb species, with branches like ostrich feathers (32).

113. Dasya *villosa*, Herv. Ner. Austr., t. 20. Garden Island, Rottnest, and King George's Sound (109). Very variable in size and ramification, putting on as many phases as *D. elegans*, its representative species.

114. Dasya *mollis.* Harv. Ner. Bor. Amer. King George's Sound, rare (64).

115. Dasya *Callithamnion. Polysiphonia Callithamnion*, Sond.! in Pl. Preiss. Abundant on the stems of *Caulinia antarctica*, &c. Rottnest and Fremantle (106).

116. Dasya *tenera*, n. sp.; cartilaginea, mox aëre diliquescens, siccitate rosea; fronde tetrasiphoniâ corticatâ decompositè ramosissimâ subdichotomâ flexuosâ; ramis irregulariter divisis, minoribus sæpè secundis, ultimis attenuatis acutis, omnibus denudatis v. ramellis tenuissimis laxè vestitis; ramellis verticillatis basi ramosis subsimplicibus strictis cylindraceis obtusis; ceramidiis ovatis pedicellatis; stichidiis sparsis v. fasciculatis lanceolatis è ramulis enatis. Very common in May at Fremantle. Dredged in January and February at King George's Sound; and in March at Cape Riche. When growing it is a very pale brown, and is then crisp and brittle; but almost immediately it grows flaccid in the air, assumes a brilliant rosy red, and soon melts into a gelatinous mass (78).

117. Dasya *Lallemandi*, Mont.! *D. gracilis*, Harv. MS. Perpendicular sides of the Jetty reef, at Rottnest, and rarely on *Zostera* leaves, June. I have compared my specimens with one from the Red Sea, given me by *Dr. Montagne*, and find them to agree in all essential characters. The colour, when growing, is brownish red, becoming purple in drying. *Dr. Montagne's* specimen is faded (212).

118. Dasya (*Stichocarpus*) *crassipes*, n. sp.; caule incrassato hispido (3–4 unciali) vagè diviso corticato ramis articulatis onusto; ramis (2–3 uncialibus) simplicibus glabris plus minùs distinctè articulatis polysiphoniis densissimè pinnatis ambitu linearibus; pinnis brevissimis (2–3 lineas longis) oligosiphoniis dichotomo-multifidis, segmentis ultimis solùm monosiphoniis acutis, articulis diametro æqualibus vel subbrevioribus; ceramidiis magnis inflato-globosis pedicellatis. Rottnest, on the perpendicular sides of the Jetty reef, and cast ashore (189). It sometimes forms large tufts 6–8 inches in diameter, is very rigid, resists the action of fresh water; is carmine when fresh, but becomes brown in drying, and scarcely adheres to paper.

119. Dasya *pellucida*, Harv. Ner. Austr., t. 27. King George's Sound, very rare (308). More squarrose than the Cape of Good Hope plant, but otherwise the same.

ORDER II.—LAURENCIACEÆ.

120. DELISIA *pulchra*, Grev. Rottnest Island, rare (239).

121. ASPARAGOPSIS *Sanfordiana*, n. sp.; surculo valido ramosissimo repente caules plures emittente; caulibus erectis simplicibus è basi-longè nudis suprà ramellis thyrsoideo-penicillatis; penicillis ramellorum quoquoversum egredientibus eximiè obtusis; pinnellis oppositis filiformibus crispato-incurvis; ceramidiis globosis infernè in pedunculo clavato attenuatis. Garden Island and Rottnest. A very distinct and noble species, much larger and more robust than *A. Delilei*, with which, however, I cannot at present further compare it. The much-branched surculi are as thick as crowquills; the stems, equally thick, are 3–8 inches long, or more, ending in a very dense, deep purple coma. The fasciculi of ramelli are remarkably *obtuse* in outline. I name it in honour of W. A. SANFORD, Esq., Colonial Secretary of Western Australia, with whom I had some pleasant sea-side walks, and to whom, during my stay in the colony, I am indebted for much kind attention and assistance (124).

122. ASPARAGOPSIS *armata*, n. sp.; surculo ultra-setaceo parum ramoso repente caules plures emittente; caulibus erectis ramosis usque ad basin ramellis obsitis v. brevissimè nudis; ramis secundariis consimilibus ad basin armatis ramulis subternis nudis retrorsùm aculeatis; penicillis ramellorum subdistichis ambitu ovatis acutis; pinnellis oppositis; ceramidiis globosis; pedunculo cylindraceo. Garden Island and King George's Sound (193). Also from Tasmania, *R. Gunn, Esq.* Whether this be what I have figured for *A. Delilei*, in Ner. Austr., t. 35, I cannot at present say, not having the book at hand. If not, I at least confounded it with that species. It differs from the European plant in having branched stems, feathered with ramelli nearly to the base; and in having two or three naked branchlets armed with reflexed prickles issuing from the lower side of every main branch, near the base. The frond is from 6–10 inches long, twice as thick as hog's bristle, and of a pale red colour.

123. LAURENCIA *Forsteri*, Grev. On *Caulinia* stems, &c., very common (103 and 126). No. 126 is var. *β. elata*, Sond. A much larger and stronger form than the common one.

124. LAURENCIA *obtusa*, Lx. King George's Sound and Rottnest, on *Algæ* (67).

125. LAURENCIA sp. . . . On rocks, King George's Sound and Rottnest, near low-water (6). Either a larger form of *L. obtusa*, or a new species.

126. LAURENCIA *affinis*, Sond. Cape Riche (310).

127. LAURENCIA *arbuscula*, Sond. Cape Riche (309).

128. LAURENCIA *cruciata*, n. sp.; livido-purpurea, cæspitosa; fronde tereti rigidâ quoquoversum ramosâ; ramis ramulisque patentissimis oppositis verticillatisve rarò alternis, ramulis junioribus cylindricis truncatis, fructiferis verrucoso-glandulosis. This requires to be compared with *L. paniculata*, J. Ag., of which I have no specimen. My plant is extremely hard and rigid, scarcely adhering to paper after two days' maceration in fresh water. Agardh compares his plant with *L. obtusa*, with which mine cannot be confounded. On *Caulinia* stems, Rottnest (209).

129. LAURENCIA *heteroclada*, n. sp.; densissimè cæspitosa, è surculis repentibus orta; fronde livido-purpureâ tereti rigidâ tenaci; juniori pluries secundè ramosâ, ramis ramulisque erecto-

appressis, axillis angustissimis; adultâ apice paniculatâ, ramis quoquoversum egredientibus elongatis patentibus, ramulis alternis spiraliter insertis corymboso-multifidis; ceramidiis ovatis sessilibus. Clothing the borders of reefs laid bare at low water, and covering wide spaces, Rottnest (210). Nothing can be more dissimilar in ramification than the young and the full-grown plant.

130. LAURENCIA sp. . . . On rocks near low-water mark, King George's Sound (7). I have not determined this species.

131. LAURENCIA *Tasmanica*, Hook. and Harv. Abundant on stones in shallow water in Princess Royal Harbour, King George's Sound (5).

132. LAURENCIA *elata*, Harv. Ner. Austr., t. 33. Garden Island, Rottnest, and King George's Sound (125).

133. LAURENCIA *Grevilleana*, n. sp.; purpureo-coccinea; fronde complanatâ eximiè distichâ decomposito-pinnatâ; pinnis in rachide strictâ alternis erecto-patentibus; pinnulis oblongis inciso-crenatis v. pinnatifidis, inferioribus minutis glandula-formibus, fructiferis . . . Abundant on the under surface of flat-topped reefs, near low-water mark, Rottnest (196). Allied to *L. pinnatifida*, but of softer substance, and very different colour. When fresh it is a beautiful rosy carmine, partially preserved in drying. I name it in honour of Dr. GREVILLE, the first reformer of this genus.

134. LAURENCIA sp. . . . Rottnest (197). Near *L. distichophylla*, J. Ag.? It requires further examination. Besides these species of Laurencia here enumerated, I have collected two or three others in small quantity, which for the present I suppress.

135. LOMENTARIA *zostericola*, n. sp.; fronde pusillâ (1–2 unciali) paniculatim ramosâ ambitu ovatâ; caule basi inconspicuè articulato suprà toruloso; ramis ramulisque patentibus suboppositis v. verticillatis (nunc sparsis) obtusis articulato-constrictis, articulis diametro brevioribus v. subæqualibus; ceramidiis globosis sparsis v. aggregatis. On *Zostera* at Rottnest (195). The spores are affixed to a very large placenta, nearly filling the cavity of the ceramidium.

136. CHAMPIA *parvula*. *Lomentaria parvula*, Ag. King George's Sound and Rottnest (57).

137. CHAMPIA *affinis*. *Lomentaria affinis*, Ag. King George's Sound, Rottnest, and Garden Island (194).

138. CHAMPIA *compressa*, Harv. Rottnest, rare (245).

ORDER III.—WRANGELIACEÆ.

139. WRANGELIA *penicillata*, Ag.! *W. plumosa*, Harv.! Alg. Tasm. On *Zostera* leaves at Rottnest, abundant (198). Much more robust than a Mediterranean specimen with which I have compared it, but very similar to one from Florida. My *W. plumosa* from Tasmania seems to differ solely in being more luxuriant, so far as I can judge from a very poor specimen now before me.

140. WRANGELIA? *Agardhiana*, n. sp.; fronde cartilagineâ (6–8 unciali) corticatâ decompositè ramosissimâ; ramis ramulisque dichotomo-alternis pluries divisis patentissimis ad genicula verticillatim ramellosis; ramellis minutissimis dichotomo-multifidis obtusis; articulis ramel-

lorum diametro sesquilongioribus. Dredged in 6–7 fathoms in King George's Sound (40). I have seen no fruit, but have little hesitation in referring this fine species to *Wrangelia*. It seems nearly allied to a plant from Cape Northumberland, distributed by me under the MS. name of *Crouania insignis*, but which is perhaps also a *Wrangelia*.

141. Wrangelia *velutina*, H. *Dasya velutina*, Sond.! Common at Rottnest and Garden Island, rare at King George's Sound (108). I have found both the cystocarpic and tetrasporic fruits, which are exactly as in other species of *Wrangelia*.

142. Wrangelia *myriophylloides*, n. sp.; fronde rigidiusculâ è basi articulatâ ecorticatâ infernè stuposâ pinnatim ramosâ; ramis patentibus simplicibus v. iterum pinnatis ad genicula verticillatim ramellosis; ramellis pluries trichotomis segmentis patentibus apice trifurcis acutissimis; fructu . . . Parasitical on the larger *Fucoids*, Rottnest (246). A very distinct species.

143. Wrangelia *Nitella*, n. sp.; fronde membranaceâ flaccidâ è basi articulatâ (articulis diametro 4–6-plo-longioribus) ecorticatâ decompositè pinnatâ; ramis ramulisque sæpiùs oppositis distichis ad genicula verticillatim ramellosis; ramellis di-tri-chotomo-multifidis segmentis patentibus acutissimis; tetrasporis globosis ad ramellos sessilibus; cystocarpiis . . . Cast ashore at King George's Sound and Rottnest, rare (213). Very similar in external habit to *W. multifida*, but much more nearly allied to *W. squarrulosa* and *W. myriophylloides*. It is a much smaller and more flaccid plant than the latter, and closely adheres to paper in drying. Many of the branches, on my specimens, end in nearly naked cirrhose prolongations, indicating that they come from deep water.

144. Wrangelia *Halurus*, n. sp.; rosea, gelatinoso-membranacea (aquâ dulci cito deliquescens); fronde è filo repente ortâ articulatâ ecorticatâ vagè ramosâ; ramis elongatis simplicibus basi et apice attenuatis ad genicula verticillatim ramellosis; ramellis dichotomo-multifidis patentibus obtusis; articulis ramorum diametro 2–3-plo, ramellorum multiplo-longioribus; cystocarpiis ramulos abbreviatos coronantibus. On *Caulinia* stems at Fremantle and King George's Sound (127). Very similar in aspect to *Halurus equisetifolius*, but much softer, of paler colour, and soon decomposing. The *cystocarps* are those of a *Wrangelia*.

145. Wrangelia? *abietina*, n. sp.; fronde cartilagineâ crassâ elongatâ (6–10 uncias longâ) corticatâ decompositè pinnatâ; pinnis pinnulisque alternis distichis subhorizontalibus, ultimis subarticulatis tenuiter corticatis, ad genicula verticillatim ramellosis; ramellis dichotomis incurvis obtusis; articulis diametro 3–4-plo-longioribus. Garden Island, rare (270). Possibly a species of *Halurus*.

146. Wrangelia? *tenella*, n. sp.; pusilla ($1\frac{1}{2}$ uncialis), cæspitosa; fronde tenuissimâ membranaceâ è basi articulatâ ecorticatâ vagè ramosâ; ramis subsimplicibus nunc iterum ramosis elongatis virgatis per totam longitudinem bipinnatis; pinnis brevissimis (vix semilineam longis) oppositis v. verticillatis, pinnulis 2–3-cellularibus obtusis; articulis ramorum diametro 4-plo, pinnarum 2–3-plo, pinnularum sesquilongioribus. On the Jetty reef, Rottnest, rare (285). I am doubtful whether to place this species in *Wrangelia* or *Callithamnion;* but place it provisionally in the former, on account of the tendency to verticillation in the pinnæ and ramelli.

Order IV.—CORALLINACEÆ.

147. Amphiroa *charoides*, Lx. King George's Sound, Cape Riche, and Rottnest, on rocks (9).

148. Amphiroa *intermedia*, n. sp.; fronde gracili (biunciali) fastigiatâ sub-tetrachotomâ, ramulis stellatim patentibus verticillatis; articulis cylindraceis basi et apice nodoso-incrassatis, superioribus diametro 8-plo-longioribus; geniculis angustissimis; ceramidiis ad ramulos secundis. On *Caulinia* stems, Rottnest (282). A much smaller plant than *A. charoides;* and differing from *A. stelligera* in the shorter nodes, &c.

149. Amphiroa *stelligera*, Dne. On *Caulinia*, King George's Sound, and Rottnest, common (4).

150. Amphiroa *gracilis*, n. sp.; fronde lapidescente di-tri-chotomâ fastigiatâ; articulis cylindraceis basi et apice truncatis diametro multoties (10–14-plo) longioribus; geniculis diametro æqualibus; ceramidiis numerosissimis quoquoversis. King George's Sound and Rottnest, common (218).

151. Amphiroa *granifera*, n. sp.; fronde lapidescente di-tri-chotomâ fastigiatâ; articulis cylindraceis, inferioribus basi et apice nodoso-incrassatis, superioribus simplicibus diametro 6–8-plo-longioribus; geniculis diametro æqualibus, inferioribus calcareo-granulosis, superioribus cartilagineis nudis; ceramidiis ad ramulos secundis. On *Caulinia* at King George's Sound and Rottnest, common (283).

152. Amphiroa *Ephedra*, Lx. Fremantle, *G. Clifton*, Esq. (289).

153. Amphiroa *anceps*, Lx. Rottnest, not common (281).

154. Amphiroa *australis*, Sond. In dark hollows of the reefs, Rottnest (217).

155. Amphiroa sp. . . . Rottnest, growing with *A. australis*, to which it is allied (219). The specimen retained for description has become broken in travelling, and I therefore leave this plant undescribed for the present.

156. Cheilosporum *pulchellum*, n. sp.; fronde pusillâ brevi stipitatâ dichotomâ flabelliformi fastigiatâ; articulis sagittatis medio costatis sæpè transversim rugulosis diametro sesquilongioribus, lobis brevibus acutis erectis; ceramidiis . . . . At Rottnest, parasitical on *Algæ* (250). A much smaller and more delicate plant than *C. sagittatum*, and differing from that and *C. cultratum*, to which it is more nearly allied, in the erect, not patent, and shorter lobes of the articulations.

157. Jania *micrarthrodia*, Lx. Common on *Caulinia* and *Algæ*, &c. (53).

158. Jania *affinis*, n. sp.; fronde pusillâ dichotomâ, ramis ramulisque erectis strictiusculis; axillis acutis; articulis omnibus cylindraceis diametro triplo-longioribus; ceramidiis parvis urnæformibus. Rottnest ( ). The size of *J. micrarthrodia*, but with much longer joints and more erect growth. It may be *J. pacifica*, Aresch.

159. Jania *Cuvieri*, Lx. Many varieties of this species abundant (3).

169. Mastophora *plana*, Sond. Extremely common on rocks, Rottnest (50).

170. Mastophora *Lamourouxii*, Dn. King George's Sound and Cape Riche ( ).

Order V.—SPHÆROCOCCOIDEÆ.

171. Delesseria *denticulata*, n. sp.; fronde costatâ dichotomâ rigidiusculâ; segmentis lato-linearibus crispato-undulatis margine denticulatis; costâ opacâ cartilagineâ apicem versus evanescente; membranæ cellulis parvis rotundato-hexagonis; venis nullis; soris in sporophyllis muricatis è costâ prorumpentibus. Parasitic on *Algæ*, Rottnest (235). Of a rigid substance, scarcely adhering to paper. 3–4 inches high, the branches $\frac{1}{4}$ inch broad.

172. Delesseria *crispatula*, n. sp.; pusilla (1–2 uncialis); fronde costatâ dichotomâ; segmentis linearibus integerrimis undulato-crispatis, costâ articulatâ 3-siphoniâ; venis nullis; soris in sporophyllis propriis è costâ enatis v. raró in segmentis terminalibus. Fremantle, on *Caulinia*, rare (129). Analogous to *D. alata*, but differing in the articulated midrib and absence of lateral veins.

173. Delesseria *spathulata*, Sond.? On *Zostera*, *Caulinia*, and various *Algæ*. Rottnest and King George's Sound. I am not quite sure that my plant and Sonder's are the same. Mine is analogous to *D. ruscifolia*, as the following is to *D. Hypoglossum* (203).

174. Delesseria *hypoglossoides*, n. sp.; pusilla, decumbens; fronde costatâ foliolis è costâ tenui articulatâ trisiphoniâ prorumpentibus ramosâ; foliolis lineari-lanceolatis planis utrinque acutis, venis nullis. In crevices of rocks at Garden Island and Rottnest (172). So like *D. Hypoglossum* as not to be known without microscopic examination. Then indeed the articulated midrib at once characterizes it.

175. Delesseria *dendroides*, n. sp.; caule elongato nudo carnoso-cartilagineo crassissimo (2–3 lineas diametro) apice in frondem ramosissimam desinente; fronde costatâ foliolis è costâ validâ prorumpentibus ramosâ; foliolis geminis exactè oppositis lineari-lanceolatis utrinque acutis, adultis costâ cartilagineâ opacâ, junioribus costâ articulatâ percursis; venis nullis; membranæ cellulis strato unico dispositis magnis oblongis. Fremantle, rare, *G. Clifton, Esq.* (269). A superb species of the *Hypoglossum* section, resembling a beautiful tree, a foot or 18 inches high, with a trunk-like stem 6–8 inches long, supporting a large head of branches. The ramification is similar to that of *D. oppositifolia*, but the substance of the leaf is of a very different structure. It closely adheres to paper.

176. Delesseria *revoluta*, n. sp.; fronde costatâ foliolis a costâ validâ infra apicem revolutum prorumpentibus ramosâ; foliolis ovalibus latitudine sesqui vel subduplo-longioribus tenuimembranaceis undulatis denticulatis apice obtusè acuminatis revolutis; soris? ——. On other *Algæ*, King George's Sound, and Rottnest, rare. 2–3 inches high. Very unlike any previously described species (311).

177. Delesseria *corifolia*, n. sp.; fronde costatâ foliolis a costâ crassâ prorumpentibus ramosâ; foliolis cartilagineo-carnosis crassis opacis lanceolatis basi ovatis obtusis; membranæ cellulis pluriserialibus, interioribus magnis, superficialibus minutissimis; cystocarpiis sorisque in sporophyllis propriis è costâ enatis. Garden Island and Rottnest, rare (279). My specimens are few and far from complete, but sufficient to establish a very distinct species, with remarkably thick and densely cellular leaves. It most resembles *D. nervifolia*, but has a very different structure. It was small scraps of this plant which I described in Ner. Austr. under *Sarcomenia delesserioides*.

178. Hemineura *crispata*, n. sp.; fronde pinnatifido-decompositâ, lobis oblongis basi et apice angustatis obtusis oppositis margine subintegerrimis undulato-crispatis demum crispatissimis; costâ immersâ supernè evanescente, costulis obsoletis; coccidiis in costâ loborum sessilibus ore producto rostratis; soris rotundato-hemisphæricis convexis secus marginem seriatis. Rottnest and King George's Sound. Sent also by *Dr. Curdie* from Cape Northumberland (312). A smaller plant than *H. frondosa.*

179. Nitophyllum *cartilagineum*, n. sp.; fronde sessili aveniâ cartilagineo-membranaceâ rigidâ crassâ dichotomâ; laciniis linearibus pluries divisis crispato-undulatis obtusis patentibus; axillis rotundatis; soris minutis impressis per totam frondem sparsis. Garden Island, not uncommon (131). Remarkably thick in substance, shrinking in drying, and imperfectly adhering to paper. Colour, brownish red.

180. Nitophyllum *fimbriatum*, n. sp.; fronde pusillâ (1–2 unciali) bifidâ v. pluries furcatâ basi cuneatâ stipitatâ; stipite brevi in costâ mox evanescente prolongato; laciniis rotundatis; margine processibus minutis ramosis densè fimbriato; soris per totam laminam sparsis. Parasitical on *Ptilota coralloidea*, at Garden Island, rare (268). I suspect my specimens are not fully grown, though one of them is in fruit. The elegantly fringed margin at once marks the species.

181. Nitophyllum *pulchellum*, n. sp.; pusillum (sub-biunciale), tenuissimè membranaceum, roseum, cæspitosum; fronde sessili aveniâ dichotomâ fastigiatâ; laciniis lato-linearibus v. cuneatis undulato-crispatis patentibus obtusis; axillis rotundatis; soris rotundatis majusculis per totam frondem sparsis. King George's Sound and Rottnest, on various *Algæ.* Like a miniature *N. punctatum*, to which species it is perhaps too nearly allied (60).

182. Nitophyllum *minus*, Sond. Garden Island and Rottnest (181).

183. Nitophyllum *ciliolatum*, n. sp.; fronde cæspitosâ sessili angustè-lineari dichotomâ ramosissimâ ciliolis marginalibus et superficialibus passim echinulatâ. On *Caulinia*, &c., King George's Sound (30). Very similar to *N. minus*, except in the presence of the ciliæ, which I find constant in very numerous specimens examined.

184. *Nov. Gen.?* My number (141) from Garden Island appears to belong to a new genus, allied to *Nitophyllum;* but without the cystocarpic fruit it is impossible to determine it.

185. Phacelocarpus *Labillardieri*, Endl. Common at Rottnest and Garden Island (134).

186. Phacelocarpus *alatus*, n. sp.; fronde costatâ; costâ elevatâ benè definitâ utroque latere laminâ angustâ alatâ; ciliis subulatis distichis. Rottnest (261). Half the breadth of *P. Labillardieri*, with a more strongly defined midrib and less deeply pinnatifid lamina. I suspect that several species are confounded under the name *Labillardieri.*

187. Heringia? *filiformis*, n. sp.; fronde cæspitosâ, è surculis repentibus ortâ setaceâ filiformi v. apice compressâ vagè ramosâ subdichotomâ rigidiusculâ. Garden Island, rare (182). Similar to *H. mirabilis* in structure, but the fruit is unknown.

188. Dicranema *filiforme*, Sond. Garden Island (133).

189. Dicranema *Grevillii*, Sond. King George's Sound, Cape Riche, Garden Island, and Rottnest (97).

190. Dicranema *revolutum*, Ag. On *Caulinia*, in shallow water. Cape Riche (128).

191. Dicranema *pusillum*, n. sp.; fronde unciali subdichotomâ v. vagè ramosâ, apicibus fructiferis strictis; tetrasporis in ramulis immutatis sparsis. Dredged near Emu Point, King George's Sound, on *Caulinia* stems. About the size of *D. revolutum*, but readily known by its straight apices, those bearing tetraspores not swollen. The cystocarps are near the tips of the branchlets (80).

192. Calliblepharis? *Preissii*, Ag. Garden Island and Fremantle (138). I have not satisfactorily ascertained the genus of this plant.

193. Calliblepharis *conspersa*, n. sp.; fronde stipitatâ cartilagineâ simplici vel parcè dichotomâ a margine pinnatâ; pinnis variè lobatis et fimbriatis nunc multifidis margine dentato-aculeatis ciliatisve; disco aculeis v. lobulis ramosis consperso; coccidiis per totam laminam sparsis. Garden Island (132). Like *C. ciliata* in habit, and very variable in form, and readily known by its scattered cystocarps.

194. Calliblepharis? *pannosa*, n. sp.; fronde stipitatâ rubro-sanguineâ v. purpurascente cartilagineo-corneâ rigidâ dichotomâ; laciniis linearibus è margine densissimè pinnato-fimbriatis; pinnis angustissimis patentibus simplicibus v. pinnatim compositis vagè dentatis v. ciliatis; coccidiis . . . . . . Abundant on rocks near low-water mark, Middleton Bay, King George's Sound, and at Rottnest, cast ashore (98). I have seen no fruit, but the habit and structure agree with those of *Calliblepharis.*

Sarcocladia, nov. gen. *Frons* plana, cartilagineo-carnosa, crassa, multifida, duplici strato constituta; stratum interius cribroso-spongiosum è cellulis brevibus anastomosantibus et lacunis intercellularibus; exterius è cellulis minutis verticaliter seriatis constitutum. *Cystocarpia* marginalia, elevata, hemisphærica, umbilicata; pericarpium cellulosum, crassum; sporæ minutæ in filis è placentâ centrali radiantibus seriatæ. *Tetrasporæ* . . . . *Alga* livido-rubra, siccitate nigrescens, ramosissima, subdichotoma; margine revoluto.

195. Sarcocladia *obesa*, n. sp.; abundant at King George's Sound and Rottnest (280).

196. Thysanocladia *oppositifolia*, Ag. *T. pectinata*, Harv.! Ner. Austr. Common at Garden Island and Rottnest. Sometimes two feet long (165).

197. Thysanocladia *laxa*, Sond.; fronde livido-purpureâ siccitate fuscescente planâ, infernè medio-incrassatâ v. subcostatâ, supernè ecostatâ, distichè decomposito-pinnatâ; pinnis lato-linearibus approximatis patentibus suboppositis; pinnulis erectiusculis lato-linearibus planis basi angustatis simplicibus vel trifurcis; axillis pinnularum eximiè rotundatis; soris tetrasporarum in apicibus dilatatis immersis. Rottnest, rather rare (237). Livid purple, with a slight bloom when fresh. Very distinct from *T. oppositifolia.*

198. Thysanocladia *costata*, n. sp.; fronde planâ costâ validâ percursâ distichè decomposito-pinnatâ ambitu ovatâ; pinnis patentibus approximatis suboppositis costatis; pinnulis argutè serratis subcostatis; coccidiis . . . Rottnest (260). A very handsome plant, 12–14 inches high, readily known by its strong midrib.

199. Thysanocladia *coriacea*, Harv. Ner. Austr., t. 36. Rottnest and Garden Island, common (105). The cystocarps are crowded near the ends of the ramuli exactly as in *T. dorsifera.*

200. Gracilaria *confervoides*, Grev. Abundant at Fremantle (166).

201. Gracilaria *dactyloides*, Sond. Garden Island and Rottnest, not uncommon (178). My plant

is a true *Gracilaria*, but requires to be compared with Sonder's, which is said to be *terete*, while mine is strongly compressed.

202. Gracilaria *fruticosa*, n. sp.; fronde rubro-coccineâ siccitate fuscescente compressâ quoquoversum ramosâ; ramis crebris patentissimis bis terve divisis; ramulis alternis v. secundis vagè spinoso-armatis acutis; coccidiis . . . Fremantle, rare (179). Nearly allied to *G. armata*, but of softer substance, and compressed. The peripheric cells are in a single row.

203. Gracilaria sp. . . . King George's Sound (95). Not in fruit. I have not been able to determine this species satisfactorily.

Order VI.—SQUAMARIEÆ.

204. Peyssonelia *rubra*, Grev.? Rottnest, a solitary specimen (316). If not the same as the Mediterranean plant, it is very nearly allied to it.

205. Cruoria? *australis*, n. sp.; fronde pusillâ ovali roseâ, filis verticallibus simplicibus, articulis diametro subduplo-longioribus, cystocarpiis è basi frondis erectis magnis oblongis. Parasitical on *Amphiroa australis*, at Rottnest (317). I am doubtful of the genus, not having found tetraspores on many specimens examined. The filaments most resemble those of a *Cruoria* or *Petrocelis;* but the habit is that of an *Actinococcus*. The cystocarps in my plant are oblong, consisting of dichotomous strings of spores, either whorled round a vertical axis, or proceeding from a central point.

Order VII.—GELIDIACEÆ.

206. Gelidium *corneum*, Lx. King George's Sound, not common (43). Some of the very dwarf varieties are frequent, near high-water mark, on all the rocky shores. Near Arthur's Head, Fremantle, grows abundance of what I suppose to be *Acrocarpus ramellosus* of Pl. Preiss. One or two specimens of a dichotomous *Gelidium*, resembling *G. variabile*, were gathered at Rottnest.

207. Gelidium *proliferum*, n. sp.; fronde infernè semiterete crassissimâ, supernè compresso-planâ v. applanatâ decompositè pinnatâ et proliferâ, setis minutis demum foliaceis densissimè muricatâ; pinnis pinnulisque lato-linearibus planis, pinnulis erecto-patentibus; cystocarpiis bilocularibus in processis filiformibus simplicibus v. pinnatis è pinnulis emissis immersis. Fremantle, thrown up after storms (244). A very distinct species, much the largest of the genus. I have long possessed imperfect specimens collected by *Messrs. Mylne* and *Backhouse*.

208. Pterocladia *lucida*, J. Ag. King George's Sound and Rottnest (44). The King George's Sound specimens agree closely with those from New Zealand. The Rottnest plant may possibly belong to a new species, but requires very careful examination.

209. Eucheuma *speciosum*, J. Ag. Fremantle and Rottnest (232). The *Jelly* or Blanc-mange weed of the colonists.

210. Solieria *australis*, n. sp.; fronde dendroideâ (1–2 pedali) robustâ decomposito-ramosissimâ; ramis alternis sparsisve approximatis pluries alternè compositis; ramulis ultimis (1–2 uncialibus) linearibus acutis basi setaceo-attenuatis; cystocarpiis in ramulis semi-immersis. Fremantle and King George's Sound (150). A noble species, much more robust and branching than *S. chordalis*, and readily known, even in fragments, by the *acute*, but not *acuminate* apices.

211. Hypnea *musciformis*, Ag. King George's Sound and Rottnest, common (16).

212. Hypnea *episcopalis*, Hook. and Harv. Rottnest, rare (252). My specimens have fruit of both kinds, further establishing this species, whose crosier-like tendrils and scarlet colour are truly episcopal.

213. Hypnea *setaculosa*, J. Ag. Rottnest and King George's Sound (70).

214. Hypnea *divaricata*, J. Ag. King George's Sound (69).

215. Hypnea sp. . . . Rottnest, on the reefs (253).
216. Hypnea sp. . . . Rottnest, on the reefs (222).
} Not ascertained.

Order VIII.—CHÆTANGIEÆ.

Hennedya, nov. gen. *Caulis* teres, ramosus; ramis apice in frondem planam dichotomam stratis tribus contextam dilatatis; stratum *medullare* è filis tenuissimis anastomosantibus densissimè intertextis; *intermedium* cellulis magnis vacuis uniseriatis; *periphericum* cellulis minimis verticaliter ordinatis compositum. *Cystocarpia* hemisphærica, elevata, umbilicata, demum poro pertusa, ad apices laciniarum sessilia, fasciculos sporarum secus parietes loculi dispositos foventia. *Tetrasporæ* . . . . *Alga* australis, fusco-rubra, rigidè membranacea, multoties dichotoma; laciniis crispatis lato-linearibus apice emarginatis.

217. Hennedya *crispa*, n. sp.; Garden Island and Rottnest, abundant (168). Readily known from *Chætangium* by the single row of large cells forming the intermediate stratum of the frond, and by the completely external fruit. It grows in large tufts, often a foot in diameter. The frond is deep red when growing, and remarkably crisped and curled. The cystocarps are formed in a little notch at the extreme end of the laciniæ. The generic name is given in honour of Mr. Roger Hennedy, of Glasgow, a most able and indefatigable investigator of the Algæ of the West of Scotland.

Order IX.—HELMINTHOCLADIEÆ.

218. Helminthora *divaricata*, J. Ag. Rottnest and King George's Sound in winter, common (234).

219. Liagora *viscida*, Ag. King George's Sound and Cape Riche, common (8).

220. Liagora *distenta*, Ag. Cape Riche, rare (313).

221. Liagora *Cheyniana*, n. sp.; fronde gelatinosâ compressâ siccitate subcanaliculatâ dichotomâ ramosissimâ; ramis erecto-patentibus argenteis villo purpureo tomentosis, apicibus divaricatis; filis periphericis liberis cylindraceis furcatis. At Cape Riche (294). *Frond* 6–8

inches high, nearly a line in diameter, much branched, dichotomous, rarely with lateral branches. The peripheric threads extend beyond the calcareous portion, and form a purple tomentum to the branches, as in *Microthoe.* This fine plant is named in compliment to George Cheyne, Esq., of Cape Riche, at whose hospitable house I resided during my residence on that part of the coast.

222. Microthoe *lapidescens*, Dne.? *Galaxaura lapidescens*, Lx.? Reefs at Rottnest (221). This is certainly a *Rhodosperm*, and nearly related to *Liagora.* When living it is clothed with dense, dark purple villosity, composed of *Callithamnoid* filaments.

223. Microthoe *marginata*, Dne.? On the reefs, at Rottnest, and cast ashore at King George's Sound (96). I have no authentic specimen at hand to compare with. Mine spring from short, dichotomous, cylindrical, woolly stems, which, had they been found disconnected, would pass for a separate species. The upper frond is flat, slightly inflexed at the margin when dry, repeatedly dichotomous, and deep purple red.

Order X.—RHODYMENIACEÆ.

224. Hymenocladia? *divaricata*, n. sp.; fronde planâ roseâ gelatinoso-membranaceâ decompositè pinnatâ, rachide flexuosâ basi et apice attenuatâ, pinnis pinnulisque lineari-lanceolatis attenuatis patentibus, pinnulis ultimis setaceis minutis horizontali-divaricatis; cystocarpiis ad discum vel marginem laminæ insidentibus sparsis; tetrasporis magnis triangulè divisis per ramos majores distributis. King George's Sound (68). I venture to refer this plant to *Hymenocladia*, J. Ag., a genus founded on *Fucus Usnea*, R. Br., whose cystocarps are unknown, and which is temporarily placed by J. Agardh in *Laurenciaceæ.* My plant has a similar habit and internal structure, and similar tetraspores; but the nucleus of its cystocarp is formed of strings of cells radiating from a basal placenta; if I mistake not, on the plan of those of a *Rhodymeniacea*, though the spores are of unusually large size in this order, and more resemble those of a *Sphærococcoid* plant. The external habit is not unlike that of *Gigartina Teedii.*

225. Hymenocladia? *Ramalina*, n. sp.; fronde planâ roseâ membranaceâ ramosissimâ, ramis subpinnatim 2–3-divisis alternis oppositisque patentibus basi et apice attenuatis, ramulis ultimis subulatis v. filiformibus elongatis horizontaliter patentibus; fructu. . . . King George's Sound, rare (87). A less gelatinous plant than the last, imperfectly adhering to paper, more irregularly branched, less compounded, and with much longer ramuli.

226. Plocamium *procerum*, Ag. Very common everywhere (94).

227. Plocamium *Mertensii*, Grev. Rottnest (140 and 259).

228. Plocamium *Preissianum*, Sond. King George's Sound and Rottnest (86).

229. Plocamium *coccineum*, Lyngb. King George's Sound and Rottnest (72).

230. Rhodophyllis *bifida*, Kütz. Garden Island, rare (145).

231. Rhodophyllis *volans*, n. sp.; cæspitosa, è filis intertextis orta; fronde membranaceâ roseâ subdichotomâ vel vagè partitâ, segmentis linearibus patentibus margine simplicibus vel sæpissimè pinnatis; pinnis ovalibus oblongisve obtusis basi attenuatis subpetiolatis; cysto-

carpiis per discum frondis sparsis; tetrasporis in pinnis nidulantibus zonatim divisis. King George's Sound (93) and Rottnest (142). A pretty little species, with the habit of *Hemineura frondosa* in miniature; and readily known by its scattered, not marginal, cystocarps.

232. Rhodymenia *corallina*, Grev. King George's Sound and Rottnest (85).

233. Rhodymenia (*Acropeltis*) *australis*, Sond. Abundant at Rottnest (144). I have gathered both kinds of fruit. The *cystocarps* are in every respect similar to those of *Rhodymenia.*

234. Rhodymenia (*Acropeltis*) *phyllophora*, n. sp.; caulescens; stipite alato ramoso, ramis in frondes pergamenas crassas infernè costâ validâ evanescente donatas dichotomo-multifidas abeuntibus; segmentis linearibus cuneatisve, margine incrassato plano; soris maculam depressam infra apicem frondis formantibus. *Hab.* Rottnest (238). Frond 1–2 feet high, much branched; segments $\frac{1}{4}$–$\frac{1}{2}$ inch broad. This is probably the same as *Acropeltis phyllophora*, H. and H., but I have not had the opportunity of comparing it with that plant.

235. Rhodymenia *elata*, n. sp.; caulescens; stipite plano-compresso subcanaliculato ramoso, ramis in frondes pergamenas infernè subcostatas pinnato-dichotomas abeuntibus; rachide flexuosâ, segmentis alternis linearibus angustis dichotomis erecto-patentibus obtusis, axillis rotundatis. Rottnest, rare (233). A noble species, two feet high, and much branched, very distinct from *R. flabellifolia*, with which alone it can be confounded.

236. Rhodymenia? *obtusata*, Sond. Rottnest and Garden Island, common (143). I have not examined the cystocarps minutely, and my specimens are not now accessible. I think it scarcely of this genus.

237. Rhodymenia? *rosea*, n. sp.; stipite brevi compresso mox ampliato, fronde basi cuneatâ tenuimembranaceâ flaccidâ roseâ subpalmatifidâ, segmentis lato-cuneatis variè lobatis, lobis acutis. Fremantle, *G. Clifton*, Esq. I have seen only a single immature specimen, sufficient to establish a distinct species, but not to fix the genus. It may possibly be a *Rhodophyllis.* A transverse section shows a double row of large empty cells in the medullary layer, and a thin cortical layer of minute cellules.

Areschougia, nov. gen. (*Harv. MS. Herb. T. C. D.*) *Frons* linearis, compressa, immersè costata, distichè ramosissima, è filo centrali articulato et stratis tribus cellularum constituta; stratum *medullare* è filis articulatis longitudinalibus anastomosantibus laxè intertextis, *intermedium* è cellulis rotundis majusculis pluriseriatis, *corticale* è cellulis minimis verticalibus formatum. *Cystocarpia* fronde immersa, inter fila strati intermedii suspensa, reticulo filorum velata, carpostomio demum aperta, fila sporifera a placentâ centrali emissa continentia; sporæ subrotundæ, seriatæ. Genus *Rhabdoniæ* proximum; differt filo centrali articulato, et habitu. Dixi in honorem Prof. J. E. Areschoug, Upsaliensis, Algologi eximii.

238. Areschougia *australis. Halymenia australis, Sond. Pl. Preiss. Phacelocarpus australis, Sond. Bot. Zeit.* 1845, p. 55. *Areschougia ligulata, Harv. MS. olim in Herb. T. C. D.* Common at Rottnest (173). The structure of the frond is very similar to that of *Phacelocarpus;* that of the cystocarp to *Rhabdonia.*

239. Areschougia *Laurencia, Harv. in Herb. T. C. D. Thamnocarpus ? Laurencia, H. and H. olim.* Rottnest, rare (236). I have seen no fruit; but the structure of the frond nearly agrees with that of *A. australis*, and the habit is not dissimilar.

240. Rhabdonia ? *Sonderi*, J. Ag. Cast ashore at Fremantle (139). I have not seen fruit.

ORDER XI.—CRYPTONEMIACEÆ.

241. MYCHODEA *carnosa*, Hook. and Harv. Cape Riche and King George's Sound (99). The cystocarps in this and in the following species are external, hemispherical, sessile on the sides of the ramuli, by which character, and the very large size of the intermedial cells of the frond, this genus differs from *Cystoclonium;* to which, however, it is closely allied.

242. MYCHODEA *membranacea*, H. and H. King George's Sound (42).

243. CALLOPHYLLIS *coccinea*, H. Garden Island (137). My (263) is probably only a very narrow variety of this variable plant.

244. CALLOPHYLLIS sp. . . . . King George's Sound (151). Delicately membranous, with marginal fruit.

245. KALLYMENIA *cribrosa*, n. sp.; stipite brevi in frondem maximam simplicem v. bipartitam rotundato-reniformem ampliato, laminâ basi cordatâ gelatinoso-membranaceâ foraminibus circularibus crebris pertusâ; cystocarpiis sparsis. Fremantle and King George's Sound, rare. June (274). A very remarkable species, elegantly perforated, like an *Agarum.*

246. GIGARTINA *disticha*, Sond. Fremantle (262). A solitary specimen only.

GATTYA, nov. gen. *Frons* membranacea, compressa, disticha, pinnatifida, è filo centrali verticillatim ramelloso composita. *Filum* centrale articulatum, callithamnoideum, ad genicula fila verticillata dichotoma emittens, ramellorum apicibus in stratum periphericum membranaceum arctè cohærentibus. *Cystocarpia* et *Tetrasporæ* ignotæ. *Alga* tenella, parasitica; structurâ ferè *Endocladiæ;* habitu diversissimo; affinitate magis ad *Catenellam* accedens. The generic name is given in honour of MRS. MARGARET GATTY, of Ecclesfield, Yorkshire, a diligent explorer of British Algæ and Marine animals.

247. GATTYA *pinnella*, n. sp.; parasite on *Sarcocladia*, and on *Corallines*, Rottnest (223). A beautiful little plant, fit to bear a lady's name, and of a very curious structure. Though the fruit is unknown, I have no hesitation in proposing the genus.

HOREA, nov. gen. *Frons* carnoso-membranacea, plano-compressa, è stratis tribus cellularum composita; stratum *medullare* è cellulis maximis inanibus demum sæpè ruptis; *intermedium* cellulis pluriseriatis minoribus coloratis; *corticale* filis moniliformibus verticalibus dichotomis muco cohibitis formatum. *Favellæ* intra pericarpium proprium apice spinis coronatum, poro pertusum, ad placentam basalem affixæ; filis arachnoideis laxè circumdatæ, sporas conglobatas angulares foventes. *Tetrasporæ* sparsæ, cruciatim divisæ. *Algæ* Australasicæ, roseæ, distichè decomposito-pinnatæ v. dichotomæ, chartæ arctè adhærentes. The name is given in honour of REV. W. S. HORE, of St Clement's, Oxford, an excellent algologist, and ardent and successful explorer of the Algæ of Plymouth Sound, &c., to whom I am indebted for large numbers of beautifully preserved specimens of rare British Algæ.

248. HOREA *halymenioides*, n. sp.; fronde subdichotomâ, segmentis decomposito-pinnatis ambitu ovatis, pinnis pinnulisque divaricato-patentibus nunc spuriè anastomosantibus attenuatis acutis, pinnulis setaceis. Fremantle, common (152).

249. HOREA *flabelliformis*, n. sp.; fronde flabelliformi subfastigiatâ dichotomâ, laciniis dichotomo-

multifidis margine integris v. parcè lobatis, lobulis deltoideo-subulatis acutis. King George's Sound, rare (341). Frond broader and more dichotomous than in the preceding, spreading from a central point like a fan.

250. Chrysymenia *obovata*, Sond.! King George's Sound and Rottnest (104). I have seen no fruit, and can throw no light upon the genus. But J. Agardh must have got hold of something very different, or he would not refer this plant to *Rhabdonia*, to which it bears neither internal nor external resemblance.

251. Chylocladia *secunda*, Hook. and Harv.! King George's Sound (340). I have not compared with New Zealand specimens; but refer this plant from memory and description.

252. Chylocladia *opuntioides*, n. sp.; fronde (6–10 uncias altâ) infernè cartilagineâ solidescente obsoletè constrictâ dichotomâ, supernè di-tri-chotomâ articulato-constrictâ membranaceâ succo aquoso repletâ, ramulis ad genicula verticillatis articulatis; articulis ramorum puncto affixis (citò in aquâ dulci sejunctis) ovali-oblongis basi et apice obtusissimis; cystocarpiis . . . Rottnest, Fremantle, and King George's Sound (192). Either this or the following appears to be the "*Ch. articulata*" of Australian botanists, but both differ essentially from each other, and from the European species so called. The present is remarkable for the rapidity with which its branches and ramuli fall to pieces, without dissolving, when thrown into fresh water. An hour or two is sufficient to denude a large specimen, leaving nothing behind but the cartilaginous main stem. The colour is a beautiful rosy purple.

253. Chylocladia *Cliftoni*, n. sp.; fronde (6–8 uncias longâ) tenui membranaceâ succo gelatinoso repletâ roseâ è basi articulato-constrictâ trichotomâ v. umbellatim ramosâ; ramis ternis ferè ad singula genicula egredientibus; ramulis sæpè numerosis; articulis inferioribus clavatis diametro 4-5-plo-longioribus, superioribus obovatis, ultimis ellipsoideis utrinque obtusis. Fremantle, *G. Clifton, Esq.* (265). A much more delicately membranous plant than *Ch. articulata*, of larger size, closely adhering to paper in drying, and soon dissolving in fresh water. It is nearly allied to *Ch. Mülleri*, Sond.! but quite distinct.

254. Halosaccion *firmum*, Post. and Rup.? Fremantle, common (135, *a*).

255. Halosaccion *hydrophora*, Post. and Rup.? With the preceding; also at King George's Sound (135, *β*). These are very similar in form to the Kamtchatkan plants to which I refer them; but they closely adhere to paper, and are filled, when recent, with very slimy mucus. Both produce cystocarps. I am doubtful, whether as species they are sufficiently distinct one from another.

256. Halymenia *Floresia*, Ag. Fremantle (314); also found by *G. Clifton, Esq.*

257. Halymenia *Kallymenioides*, n. sp.; fronde planâ gelatinoso-membranaceâ foliaceâ informi variè lobatâ et sinuatâ, margine glanduloso, laciniis acutis, cystocarpiis sparsis. Cast ashore at Fremantle, rare (174). This has the habit of *Kallymenia*, but exactly the structure of *Halymenia*.

258. Gelinaria *ulvoidea*, Sond. Fremantle and King George's Sound (136). The structure, as already stated by Kützing, is very similar to that of *Halymenia*. The only difference is, that in *Gelinaria* the peripheric membrane is very thick and fleshy, composed of two or three rows of small polygonal cells, protected externally by a thick stratum of vertical, moniliform

filaments, formed of very minute oblong, cells. The colour, when fresh, is a bright, but very fugacious, rosy pink. I have seen no fruit.

259. Nemastoma? *gelinarioides*, n. sp.; fronde gelatinoso-carnosâ roseâ planâ decomposito-pinnatâ, pinnis approximatis erecto-patentibus pinnatis v. bipinnatis, segmentis basi parum attenuatis sublanceolatis acutis, ultimis lato-subulatis acutiusculis. King George's Sound, rare (84). Very like some of the more branching forms of *Gelinaria ulvoidea*, but of much denser and different structure. The structure is as dense as in *Schizymenia*.

260. Nemastoma *damæcornis*, n. sp.; fronde gelatinoso-carnosâ extereti compresso-planâ dichotomo-multifidâ subfastigiatâ; segmentis patentibus cuneatis, terminalibus filiformibus obtusis; axillis omnibus eximiè rotundatis; tetrasporis sparsis cruciatim divisis. At Fremantle and Rottnest, rare (315). It requires to be compared with the Mediterranean *N. dichotoma*, which it closely resembles, and from which it may not be sufficiently distinct.

Order XII.—SPYRIDIACEÆ.

261. Spyridia *filamentosa*, H. Abundant all along the coast (18).

Order XIII.—CERAMIACEÆ.

262. Centroceras *clavulatum*, Ag. Common on littoral rocks and on *Zostera*, &c. (2).

263. Ceramium *rubrum*, Ag. Rottnest and King George's Sound, in winter (258).

264. Ceramium *puberulum*, Sond.! *C. monile, H. and H.!* On *Zostera*, Rottnest, and King George's Sound (66).

265. Ceramium *isogonum*, n. sp.; fronde pusillâ (1–2 unciali) subsetaceâ dichotomâ fastigiatâ, segmentis erecto-patentibus terminalibus forcipatis; articulis corticatis omnibus diametro æqualibus lineâ hyalinâ centrali notatis medio parumque constrictis; favellis subterminalibus bilobis ramellis 1–2-fulcratis; tetrasporis . . . . . On *Algæ*, Garden Island (286). Quite distinct from any of the *rubrum* section.

266. Ceramium *miniatum*, Suhr.? *C. Filicula, Harv. M. S.;* filo primario repente frondes minutas (semiunciales) sparsas erectas emittente; fronde compressâ distichè subpinnatâ, pinnis dichotomo-fastigiatis, segmentis terminalibus brevissimis dentiformibus, articulis diametro brevioribus sacculo roseo coloratis, omnibus nisi supremis interstitiis nudis, tetrasporis secus marginem segmentorum utrinque longitudinaliter seriatis. Parasitical on *Dictyota Kunthii* at Rottnest (220). I have little hesitation in referring this to *C. miniatum*, Suhr. (first found on the Peruvian Coast), although Agardh makes no mention of the primary creeping filament, and there are some other slight differences in the description.

267. Ceramium *australe*, Sond.! Garden Island, rare (285). Near *C. Deslongchampsii.*

268. Ceramium *fastigiatum*, Harv. Parasitical on *Zostera*, Rottnest, rare (257).

269. Ceramium *gracillimum*, Kütz. Parasite on *Algæ*, on mud-banks, King George's Sound, January (23).

270. Ptilocladia *pulchra*, Sond.! Garden Island, rare (147 and 148).

271. Halophlegma *Preissii*, Sond.! Very abundant on the reefs at Rottnest; also on *Caulinia*, &c. (63).

272. Hanowia *australis*, Sond.! Fremantle, rare (56).

273. Hanowia *robusta*, n. sp.; fronde (vix evolutâ) compressâ latâ; filis setaceis, articulis primariis ovoideo-cylindraceis ad genicula contractis diametro 2-3-plo-longioribus, endochromate amplâ. Fremantle, very rare ( ). My specimens are immature. The filaments are much more robust and more laxly woven than in *H. australis.*

274. Hanowia *arachnoidea*, n. sp.; fronde compressâ latâ furcatâ v. dichotomâ, filis arachnoideis, articulis primariis cylindraceis diametro 6–8-plo-longioribus. King George's Sound, very rare (52). *Frond* 1–2 inches high, the segments ¼ to ½ inch broad, compressed. Filaments much more slender than in *H. australis*, with much longer joints.

Lasiothalia, nov. gen. *Frons* filiformis, membranacea, ramosa, hirsuta, è filis longitudinalibus intertextis anastomosantibus, filoque centrali majori contexta; filis periphericis externè fila callithamnoidea subsimplicia horizontalia libera emittentibus. *Fructus?*

275. Lasiothalia *hirsuta*, n. sp.; Cape Riche, very rare (321). I found only two or three specimens. The largest is about 6 inches long, irregularly divided, with lateral branches and slender filiform ramuli. Every part of the plant is clothed with short, simple, or slightly branched, horizontal, jointed hairs. There is no trace of gelatine, and the plant but slightly adheres to paper.

276. Dudresnaia *coccinea*, Bonn.! King George's Sound, very rare (325).

277. Crouania *attenuata*, *β. australis.* On *Zostera*, &c., King George's Sound (62). Much larger and less gelatinous than the British plant usually is, but scarcely otherwise different.

278. Crouania *vestita*, n. sp.; fronde ultra-setaceâ decompositè ramosissimâ membranaceâ (vix gelatinosâ), ramis ramulisque patentibus, omnibus ramellis densissimè velatis, ramellis divaricato-multifidis; favellis solitariis reniformibus in ramulis minoribus inter ramellos immersis; tetrasporis sphæricis triangulè divisis. Rottnest and King George's Sound, on *Zostera*, &c. (338). Much more robust than *C. attenuata*, much less gelatinous, and not moniliform in any part of the frond.

279. Dasyphila *Preissii*, Sond.! On the stems of *Fucoideæ*, Garden Island, common (149).

280. Ptilota *coralloidea*, J. Ag. Garden Island, Rottnest, and King George's Sound, common (91).

281. Ptilota sp. King George's Sound (92). Possibly only a variety of the last, with articulated ramelli.

282. Ptilota *striata*, n. sp.; fronde ancipiti siccitate transversim ruguloso-striatâ decompositè ramosissimâ, ramis majoribus sparsis alternè divisis vix pinnatis, minoribus linearibus pectinato-pinnatis, pinnulis subulatis alternis simplicissimis; favellis minimis ad latus superius pinnularum pedicellatis involucratis, involucro è filis callithamnoideis multiseriatis composito; tetrasporis ad processos proprios ramosos è lateribus pinnularum emissis. Rottnest, not uncommon (240). A most distinct and beautiful species with the habit of *Phacelocarpus Labillardieri.* It most resembles *P. Rhodocallis*, H. (*Rhodocallis elegans*, *Kütz.*), but differs essentially from that species in the position and nature of the involucres, &c.

283. Ptilota *siliculosa*, n. sp.; fronde complanatâ costatâ decompositè ramosissimâ, ramis majoribus alternis sparsisve, minoribus linearibus pectinato-pinnatifidis, pinnulis è basi lato subulatis alternis simplicissimis; tetrasporis in glomerula siliculiformia è pinnularum latere superiore enata congestis, ad fila callithamnoidea brevissima circum axim verticillata affixis. Rottnest, rare (243). Very like the preceding in habit; but evidently ribbed, and rather inciso-pinnatifid than pinnate, and not obviously transversely striate; and abundantly characterized by the strangely metamorphosed fructification.

284. Thamnocarpus *Gunnianus*, Harv. Common at Garden Island and Rottnest; but not in fruit (169).

285. Griffithsia *ovalis*, n. sp.; fronde erectâ (sub-bi-unciali) di-tri-chotomâ subfastigiatâ crassissimâ, segmentis erecto-patentibus, articulis diametro 3–4-plo-longioribus, inferioribus clavatis, mediis superioribusque obovatis inflatis ad genicula maximè constrictis; fertilibus conformibus; involucris tetrasporarum circa genicula involucratis è ramellis minimis conflatis. Parasitical on *Zostera*, King George's Sound (41). Also sent by *Dr. Curdie*, from Cape Northumberland. Very much more robust than *G. corallina*, with nodes contracted like those of an *Opuntia*. It is as robust as *Chylocladia articulata*.

286. Griffithsia *monilis*, n. sp.; fronde basi radicante cæspitosâ (1–2 unciali) dichotomâ fastigiatâ crassissimâ, segmentis erecto-patentibus; articulis diametro sesquilongioribus globoso-inflatis siccitate sub-collapsis et ovalibus ad genicula maximè constrictis; fertilibus conformibus, involucris tetrasporarum circa genicula verticillatis. Parasitical on *Algæ* at Garden Island, and on *Zostera* at Rottnest (326). When fresh it resembles beautiful strings of ruby-coloured beads, but fades much in drying.

287. Griffithsia *Binderiana*, Sond.! Garden Island on *Algæ*, Rottnest on *Zostera* (199).

288. Griffithsia *Teges*, MS. Cast ashore at Fremantle (146). I do not describe this species, as the fruit is unknown. It forms enormous, coarse, mat-like strata, one or two feet in breadth, composed of filaments resembling those of *G. secundiflora*, but very irregularly branched.

289. Corynospora *australis*, n. sp.; fronde (biunciali) setaceâ gelatinoso-membranaceâ dichotomo-decompositâ et alternè ramosâ, ramulis pluries dichotomis, articulis longissimis ad genicula nec contractis, ramellis superioribus tenuissimis dichotomis, apicibus longè filiformibus arachnoideis; tetrasporis ad genicula ramorum majorum subsessilibus oblongis nucleo indiviso. Rottnest, in June, very rare (344). Fremantle, July, *G. Clifton, Esq.* A very distinct species, readily known by its attenuated apices.

290. Corynospora *gracilis*, n. sp.; fronde pusillâ (unciali) tenui alternè ramosâ v. subdichotomâ, ramulis quoquoversum egredientibus inferioribus furcatis superioribus bis-terve dichotomis, apicibus subattenuatis obtusiusculis; tetrasporis . . . ? Garden Island, rare, July (266). The habit and substance of the plant are those of *Corynospora*.

291. Callithamnion *thyrsigerum*, Thw. MS.; filo primario repente, secundariis erectis cæspitosis capillaribus (1–1½ uncialibus) vagè ramosis, ramis minoribus sæpissimè secundis filiformibus simplicissimis acuminatis; articulis diametro 3–5-plo-longioribus cylindraceis; tetrasporis circa genicula suprema ramorum verticillatis pedicellatis, pedicellis ramosulis thyrsoideo-paniculatis; favellis in ramulo terminalibus involucratis. On *Algæ* and *Zostera*

King George's Sound and Rottnest (51). A beautiful and very distinctly characterized species of the *C. Turneri* section, which I first gathered at Belligam Bay, Ceylon, in company with my friend *G. H. K. Thwaites, Esq.*, of Peradenia Botanical Gardens.

292. Callithamnion *cymosum*, n. sp.; densissimè cæspitosum; filis primariis repentibus intricatis, secundariis erectis arachnoideis (uncialibus) vagè ramosis, ramis subdichotomis v. alternis minoribus filiformibus erectis longè simplicibus obtusis, articulis diametro multoties (8–12-plo) longioribus cylindraceis; tetrasporis in cymis veris æqualibus v. scirpoideis secus ramos evolutis dispositis; favellis . . . . . ? On sand-covered rocks, Middleton Bay, King George's Sound and Rottnest; often half buried in sand (10). The cymoid inflorescence is very peculiar, and beautifully accurate to the typical cyme.

293. Callithamnion *delicatulum*, n. sp.; pusillum, arachnoideum, filo primario repente; secundariis erectis (vix uncialibus) parum ramosis è quoque geniculo plumulatis, plumulis oppositis per paria decussatis infra apicem articuli egredientibus tenuibus laxè pinnatis, pinnulis inferioribus sæpiùs oppositis reliquis alternis è rachide flexuosâ emissis omnibus attenuatis simplicibus v. ramulo uno alterove auctis; fructu . . . . . Parasite on *Solieria australis*, at King George's Sound. A very delicate and beautiful little plant (339).

294. Callithamnion *gracilentum*, n. sp.; minutum (1–2 lineas altum); filo primario repente crasso ramos suboppositos liberos emittente; ramis filo primario quadruplo-angustioribus pinnatis, pinnis oppositis patentibus simplicibus v. latere inferiori subramellosis subattenuatis obtusiusculis; articulis fili primarii diametro sesqui v. subduplo, ramorum 4–5-plo, ramulorum sesquilongioribus. Parasite on *Fucoids*, Rottnest, rare (327). Apparently nearly allied to *C. leptocladum*, Mont.; but scarcely the same?

295. Callithamnion *aculeatum*, n. sp.; filo primario repente; secundariis erectis (sub-uncialibus) capillaribus subdichotomis v. alternè ramosis corymboso-fastigiatis; ramis omnium serierum quoquoversum egredientibus, minoribus caule duplo-angustioribus, ramulis ad genicula ferè omnia verticillatis spinæformibus patentissimis brevissimis simplicibus subacutis; tetrasporis solitariis ad ramulos lateralibus; articulis ramorum diametro 2–3-plo-longioribus. On *Zostera*, at King George's Sound, rare (343).

296. Callithamnion *spinescens*, Kütz.? *Cal. tomentellum, Harv. MS.* Very common, everywhere on *Algæ*, &c. This species is so common that it can hardly have escaped Preiss, and therefore I suppose it the *C. spinescens* of Sonder's list. But the ramuli are not *whorled;* but *opposite* and *decussated;* one pair spreading one way, the next at right angles to them. In all my specimens the articulations of the stem are very short. In habit, it has much resemblance to *Jungermannia tomentella* (48).

297. Callithamnion *horizontale*, n. sp.; filis erectis (uncialibus) capillaribus solitariis parum ramosis, ramis 3–4-lateralibus simplicibus patentibus cum filo primario è quoque articulo oppositè plumulatis; plumulis è medio articuli egredientibus subdistichis *horizontaliter* patentibus (latus planum sursum vertentibus) ambitu ovatis pinnatis; pinnâ infimâ simplici, cæteris furcatis; articulis omnibus diametro æqualibus v. sesquilongioribus; apicibus acutis; tetrasporis solitariis ramulum pusillum pinnarum terminantibus. Parasitic on *Griff. Binderiana* at Rottnest; and on *Pol. nigrita* at Garden Island (254).

298. CALLITHAMNION *verticale*, n. sp.; filis erectis (uncialibus) capillaribus subsolitariis parum ramosis, ramis 1–2-lateralibus brevibus cum filo primario è quoque articulo oppositè plumulatis; plumulis è medio articuli egredientibus distichis *verticaliter* patentibus (latus planum ad latera vertentibus) ambitu ovatis pinnatis; pinnis omnibus plus minùs furcatis; articulis diametro æqualibus v. sesquilongioribus; apicibus acutis; tetrasporis solitariis ramulum pusillum pinnarum terminantibus. Parasite on *Algæ* at Garden Island (267). Very nearly allied to the preceding; but having a different aspect, from the different direction of the flat surface of the plumules.

299. CALLITHAMNION *pulchellum*, n. sp.; pusillum (semi-unciale); filo primario ramisque primariis prostratis repentibus demum ramos secundarios erectos simplices v. parum ramosos emittentibus; ramis omnibus è quoque articulo oppositè v. cruciatim plumulatis; plumulis 2–4 infra apicem articuli egredientibus patentibus ambitu ovatis pinnatis; pinnis simplicissimis approximatis obtusis; articulis ramorum diametro 2–4-plo-longioribus, pinnarum et pinnellarum diametro brevioribus; favellis simplicibus rachidem plumuli terminantibus; tetrasporis à pinnellis abbreviatis formatis. Parasitic on various *Algæ;* especially on *Areschougia australis.* Rottnest and Cape Riche (230). At first I supposed this beautiful little plant to be *C. australe*, J. Ag., but on comparison with his description, my plant must be different. The plumules on the younger part of the frond are always *opposite* and *vertical;* those on the older erect branches are frequently in fours, *cruciate* and *horizontal.* Can this be *C. Preissii*, Sond.? The specimens with cruciate plumules would be near Sonder's description.

300. CALLITHAMNION *simile*, Hook. and Harv. On *Fucoideæ* at King George's Sound and Rottnest (90). I have compared the specimens with one from Kerguelin's Land, and find them to agree.

301. CALLITHAMNION *Wollastonianum*, n. sp.; fronde ultra-setaceâ elatâ (4 unciali) basi tenuiter corticatâ sursum longè pilis squarrosis stuposo-hirsutâ subdistichè ramosissimâ; ramis alternis decomposito-pinnatis, penultimis distichis pellucidè articulatis alternè plumulatis; plumulis patentibus longissimis ambitu linearibus; pinnis tenuibus erectiusculis brevibus, inferioribus simplicibus, superioribus sæpiùs furcatis v. pinnulatis; tetrasporis solitariis ad ramulos brevissimè pedicellatis; articulis diametro 2–4-plo-longioribus. Middleton Bay, King George's Sound, rare in August (329). A very beautiful species, which I name in affectionate regard to the family of ARCHDEACON WOLLASTON, from whom I received unvarying kindness during the whole of my stay at King George's Sound. It is nearly allied to *C. latissimum*, but differs in several respects.

302. CALLITHAMNION *Brownianum*, n. sp.; fronde ultra-setaceâ elatâ (4 unciali) subecorticatâ sursum longè pilis squarrosis stuposo-hirsutâ quoquoversum ramosissimâ; ramis pluries alternè decompositis, penultimis quoquoversis pellucidè articulatis nodosis (parietibus cellularum crassis gelatinosis), alternè plumulatis; plumulis quoquoversis brevibus crispis pinnatis, pinnis capillaribus longissimis maximè curvatis inflexis; articulis pinnularum diametro 4-plo-longioribus; tetrasporis brevissimè pedicellatis solitariis v. geminis ad latera pinnularum enatis. On *Zostera* at Rottnest, Fremantle, and King George's Sound

(264). Much resembling the last in aspect, but not distichous in any part; and with remarkably curled pinnules. I name it in compliment to Mrs. Richard Brown of Fremantle, an amateur collector of Algæ, from whom and her estimable husband I received much kind attention during my stay in their neighbourhood.

303. Callithamnion *laricinum*, n. sp.; fronde cartilagineâ setaceâ (1–3 unciali) ferè ad apices ramorum corticatâ glabrâ quoquoversum ramosâ ambitu pyramidali; ramis alternis patentibus supernè sensim brevioribus ramulis dichotomo-multifidis undique obsessis; ramulis pluries dichotomis, segmentis patentibus, ultimis brevissimis spinæformibus; favellis geminis oblongis! simplicibus v. furcatis; tetrasporis globosis ad latera ramulorum sparsis. On *Zostera* at Rottnest, common (200). This has the aspect and substance of *C. tetragonum;* but is more nearly related to *C. granulatum* or *C. grande.*

304. Callithamnion *flabelligerum*, n. sp.; fronde erectâ crassiusculâ alternè decomposito-ramosâ omninò ecorticatâ; ramis ramulisque quoquoversum egredientibus, terminalibus corymboso-flabellatis, ramulis dichotomo-multifidis fastigiatis; apicibus obtusis patentibus; favellis geminis rotundatis ramulis stipatis (quasi involucratis). On *Zostera* at Rottnest, and at Garden Island on *Algæ* (201). Nearly allied to *C. corymbosum*, but a more robust, though smaller plant; with cells more like those of a *Griffithsia* than of a *Callithamnion.*

305. Callithamnion *multifidum*, n. sp.; fronde pusillâ (unciali) arachnoideâ ecorticatâ densè cæspitosâ alternè ramosâ; ramis simplicibus ramosisve, ramulis alternis quoquoversis dichotomo-multifidis; segmentis patentibus obtusis; articulis ramorum basi incrassatis diametro 4-plo, ramulorum cylindraceis diametro 2–3-plo-longioribus. On sand-covered rocks, half-tide level, generally buried in the sand, the grains of which adhere closely to the filaments. Reefs at Rottnest, May and June (229).

306. Callithamnion *crispulum*, n. sp.; fronde pusillâ ($\frac{1}{2}$–$\frac{3}{4}$ unciali) capillari ecorticatâ cæspitosâ infernè quoquoversum, supernè distichè ramosâ; ramis superioribus è rachide flexuosâ alternè plumulatis; plumulis brevissimis alternè pinnatis, pinnis 3–4 simplicissimis filiformibus elongatis obtusis eximiè arcuato-inflexis; articulis omnibus diametro sesquilongioribus; favellis geminis; tetrasporis . . . . . In shady crevices of rocks, at half-tide level, Rottnest. Near *C. Borreri*, but a much smaller plant, and sufficiently characterized as above (228 α).

307. Callithamion *pusillum*, n. sp.; fronde pusillâ (vix unciali) capillari ecorticatâ cæspitosâ infernè simpliciusculâ supernè quoquoversum ramosâ; ramis infernè plumulatis, supernè alternè ramosis, ramis minoribus è rachide strictiusculâ quoquoversum plumulatis; plumulis brevissimis vix pinnatis; pinnis 2–3 alternis v. secundis elongatis obtusis arcuatis inflexis; articulis omnibus nisi basilaribus diametro 3-plo-longioribus; favellis geminis; tetrasporis globosis ad latera pinnarum solitariis. Crevices of rocks, at half-tide, Rottnest (228 β). At first I had this for a variety of *C. crispulum*, but it differs in not being in any part distichous, and in the longer articulations.

308. Callithamnion *Scopula*, n. sp.; fronde pusillâ (unciali) capillari ecorticatâ quoquoversum ramosâ, ramis paucis cum ramulis ambitu clavatis quoquoversum plumulatis; plumulis inferioribus brevibus, superioribus elongatis pinnatis; pinnis simplicibus filiformibus longis-

simis arcuato-incurvis obtusis; articulis omnibus diametro 2-3-plo-longioribus; tetrasporis ellipsoideis numerosis secus pinnas sessilibus. Crevices of rocks, at half-tide, Rottnest (328). This is certainly near *C. roseum* in miniature. To the naked eye it looks very like *Dasya ocellata*, or like a bunch of little bottle brushes.

309. CALLITHAMNION *debile*, n. sp.; fronde pusillâ (vix unciali) tenuissimâ ecorticatâ cæspitosâ infernè quoquoversum supernè distichè ramosâ; ramis paucis alternè divisis, ramis minoribus distichè ramulosis, ramulis patentissimis inferioribus simplicibus spinæformibus superioribus furcatis v. subpinnulatis; articulis inferioribus diametro 5-8-plo, ramulorum 3-4-plo-longioribus; tetrasporis solitariis ad ramulos sessilibus. Rottnest, rare (330). Unlike any Australian species; and most like some starved form of *C. polyspermum*, but of a very fragile substance and pale colour.

310. CALLITHAMNION *radicans*, n. sp.; nanum, parasiticum, velutino-cæspitosum; fronde minutâ (2 lineas altâ) basi fibrillis crispatis radicante, è basi ramosissimâ; ramis primariis alternis secundisve 2-3-ties decompositis, minoribus ramulisque secundis strictis; articulis cylindraceis diametro 4-5-plo-longioribus; ramulis fructiferis prope basin ramorum sparsis simplicibus v. parum ramosis; tetrasporis ellipsoideis terminalibus. On *Zostera* leaves, Fremantle (331). This resembles *C. luxurians*, J. Ag., externally, but seems sufficiently marked by its *rooting* filaments and longer articulations.

311. CALLITHAMNION *botryocarpum*, n. sp.; nanum, penicillato-cæspitosum; fronde minutâ (1-1½ lineas altâ) è basi ramosissimâ, ramis alternis v. secundis patentibus flexuosis nunc subsquarrosis; articulis diametro 4-plo-longioribus; tetrasporis magnis triangulè divisis in glomerula ad axiles ramorum densissimè aggregatis; antheridiis, botryoideis è quoque ferè articulo ramorum sæpè evolutis. Abundant on *Chorda lomentaria*, at King George's Sound, in August (324). Externally very like *C. Daviesii*, but I suppose distinctly characterized by its fruit. The tetraspores are very large for this section of the genus. The antheridia resemble little clusters of grapes, ranged along the upper branches of fertile specimens.

312. CALLITHAMNION *sparsum*, Harv.(?) Parasite on *Sporochnus*, at Garden Island. This requires to be compared with British specimens; and also with Kützing's *C. humile* from the Cape of Good Hope. It is quite different from either of the preceding, very sparingly branched, of a deep purple colour, and rather rigid texture, with very short articulations.

SERIES III.—CHLOROSPERMEÆ.

ORDER I.—SIPHONACEÆ.

313. CAULERPA *simpliciuscula*, Ag.? On the reefs, Rottnest. A much dwarfer, and more branching form than that figured by Turner, if the same. Possibly my plant may be rather akin to *C. lentifera*, J. Ag. (207).

314. CAULERPA *lætevirens*, Mont.? Extremely abundant on the surface of shallow reefs, exposed at low water, Rottnest. I have not compared with Montagne's plant (208).

315. CAULERPA *cylindracea*, Sond. King George's Sound, rare (54).

316. Caulerpa *tenella*, n. sp.; surculo setaceo glabro; frondibus filiformibus simplicibus v. parcè ramosis, ramis vagis, foliis spiraliter laxè insertis subtristichis erecto-patentibus subulatis brevibus mucronatis læteviridibus. On the Natural Jetty at Rottnest, very rare (215). A slender species, 1–2 inches high.

317. Caulerpa *hypnoides*, R. Br. Abundant in tide-pools and borders of reefs, at Rottnest and Garden Island (185).

318. Caulerpa *Mulleri*, Sond.! surculo crasso squamulis cylindraceis dichotomis densè muricato; fronde erectâ stipitatâ oblongâ obtusâ pinnatâ; stipite pinnisque foliolis undique densissimè obtectis, foliolis geminis basi unitis cylindraceis obtusis apice bi-mucronulatis erectis imbricatis intensè viridibus. On border reefs and sides of deep tide-pools at Rottnest (205). Nearly related to *C. hypnoides*, but a much stronger and coarser plant, readily known at a glance, when the two are seen together, though difficult to characterize. In *C. hypnoides* the surculus and base of stem are clothed with far more densely set and muricated squamæ, and the folioli are much smaller, softer, more patent, more laxly set, and more acute.

319. Caulerpa *obscura*, Sond.! Abundant at King George's Sound; and in tide-pools, &c., Rottnest (77). The fronds are often 12–18 inches long.

320. Caulerpa *furcifolia*, Hook. and Harv. A few fragments cast ashore at King George's Sound, February ( ).

321. Caulerpa *geminata*, n. sp.; surculo glabro; frondibus erectis simplicibus (brevibus) articulato-constrictis glabris, foliis parvis oppositis ovoideis distichis v. tortione caulis quoquoversum directis. On very shady rocks, usually on the under surface of table-reefs, Rottnest. The distichous form is readily distinguishable; but that with leaves turned to all sides resembles *C. sedoides* in miniature; but is readily known by its articulate stem and opposite leaves (214). I suspect that it is *S. sedoides*, of Sonder in Pl. Preiss.

322. Caulerpa *corynephora*, Mont. King George's Sound, and in deep tide-pools, Rottnest (101).

323. Caulerpa *scalpelliformis*, R. Br. King George's Sound, and on border reefs, Rottnest (206).

324. Struvea *plumosa*, Sond. Abundant on all the shallow reefs at Rottnest, but scarcely in season in June, when I visited the island (216).

325. Struvea *macrophylla*, n. sp.; fronde oblongo-ovali maximâ (4–5 uncias longâ, 3 uncias latâ) crenatâ, tubulis anastomosantibus pluries pinnatis. Champion Bay, *Mrs. Drummond*, *Jun.* A single specimen, bleached white, was sent by *Mrs. Drummond* to *Mr. Sanford*, who kindly presented it to me. The frond closely resembles a beautiful structure of "old point-lace," and as it is very tough and strong, it might be manufactured into ladies' *natural-lace collars*, by merely tacking on a border of net.

326. Polyphysa *Peniculus*, Ag. *Fucus Peniculus*, R. Br. Extremely abundant, at all seasons, in Princess Royal Harbour, King George's Sound, growing on old shells. Not seen elsewhere (1).

327. Penicillus *Arbuscula*, Mont.? Abundant, on shallow, sand-covered reefs at Rottnest (204). It varies much in size. The stem is sometimes scarcely twice as thick as a hog's bristle; sometimes as thick as a goose-quill. I have not compared with Montagne's plant.

328. Halimeda *macroloba*, Dne. Cape Riche and Rottnest, on the reefs (226).

329. Codium *tomentosum*, Ag. Abundant everywhere (45).

330. CODIUM *laminarioides*, n. sp.; stipite brevi cuneato mox in frondem amplissimam (2–3 pedalem) planam subsimplicem v. parcè lobatam expanso. At Rottnest and King George's Sound, on the under surface of table-shaped rocks. If this be only a form of *C. elongatum* it is indeed an extraordinary one. The undivided frond is often three feet wide by two feet long, resembling a piece of green cloth (227).

331. CODIUM *spongiosum*, n. sp.; fronde sessili molli polymorphâ variè lobatâ et spongioideâ; filis interioribus laxiusculis in gelatinâ immersis, periphericis cylindraceis v. pyriformibus obtusis; spermatiis fusiformibus basi et apice acutis. On shells and stones, &c., about low-water mark, common ( ). I do not wonder that this has not been brought to Europe, as it is almost impossible to prevent the spongy mass decomposing (with a very unsavoury smell) during the process of drying.

332. CODIUM *mamillosum*, n. sp.; fronde globosâ vel reniformi puncto affixâ solidâ; filis interioribus densissimè intertextis arachnoideis gelatinâ subsolidâ obvallatis, periphericis maximis inflato-cylindraceis, eorum apicibus ad superficiem frondis quasi mamillis directis, siccitate sericeo-nitentibus. Fremantle and King George's Sound, cast ashore (162). It forms a very solid, green, mammillated ball, composed internally of very slender, densely packed threads, throwing off to all sides externally, radiating branches, whose apices, closely set together, give the mammillated appearance to the surface.

333. BRYOPSIS *australis*, Sond.? Very common on rocks, at Rottnest and Carnac (161).

334. BRYOPSIS sp. On *Zostera*, Rottnest (175).

335. BRYOPSIS sp. Perhaps *B. foliosa*, Sond. On sand-covered rocks, Rottnest (249).

336. DICTYOSPHÆRIA *sericea*, n. sp.; fronde umbilicatâ medifixâ variè lacerâ (nunquam vesicatâ) sericeâ; vesiculis minimis globoso-polyhedris. On rocks near low-water mark, King George's Sound, Cape Riche, and Rottnest (160). Very distinct from *D. favulosa* at all ages.

ORDER II.—CONFERVACEÆ.

337. CLADOPHORA *valonioides*, Sond. Common on rocks and in shallow water (55).

338. CLADOPHORA sp. Sand-covered rocks, King George's Sound (46).

339. CLADOPHORA sp. *C. anastomosans*, MS. Cast ashore at Fremantle (163).

340. CLADOPHORA sp. Fremantle (176).

341. CLADOPHORA sp. Fremantle (177).

342. CLADOPHORA sp. Near *C. pellucida*. Rottnest, on reefs (275).

343. CLADOPHORA sp. Allied to *C. glaucescens* (333). I have neither books nor specimens at hand sufficient to determine whether these species have been previously described.

ORDER III.—ULVACEÆ.

344. PHYCOSERIS *Ulva*, Sond. Garden Island.

345. PHYCOSERIS *latissima*. *Ulva latissima*, Auct. I cannot say to which of Kützing's species

these specimens should be referred, but I fear that author has needlessly multiplied the names in this genus.

346. Enteromorpha *compressa.* Submerged rocks and woodwork, everywhere.

Order IV.—OSCILLATORIACEÆ.

347. Rivularia *australis*, n. sp.; fronde maximâ (fronde 1–1½ uncias diametro) solitariâ hemisphæricâ solidâ lubricâ olivaceo-viridi. On rocks near low-water mark, Cape Riche (298). I suppose this belongs to Kützing's genus *Euactis*, but I have not minutely examined it. It is the largest of the genus known to me.

348. Rivularia sp. Near *R. plicata*, Carm. Upper end of Princess Royal Harbour, on stones and wood in shallow water (19).

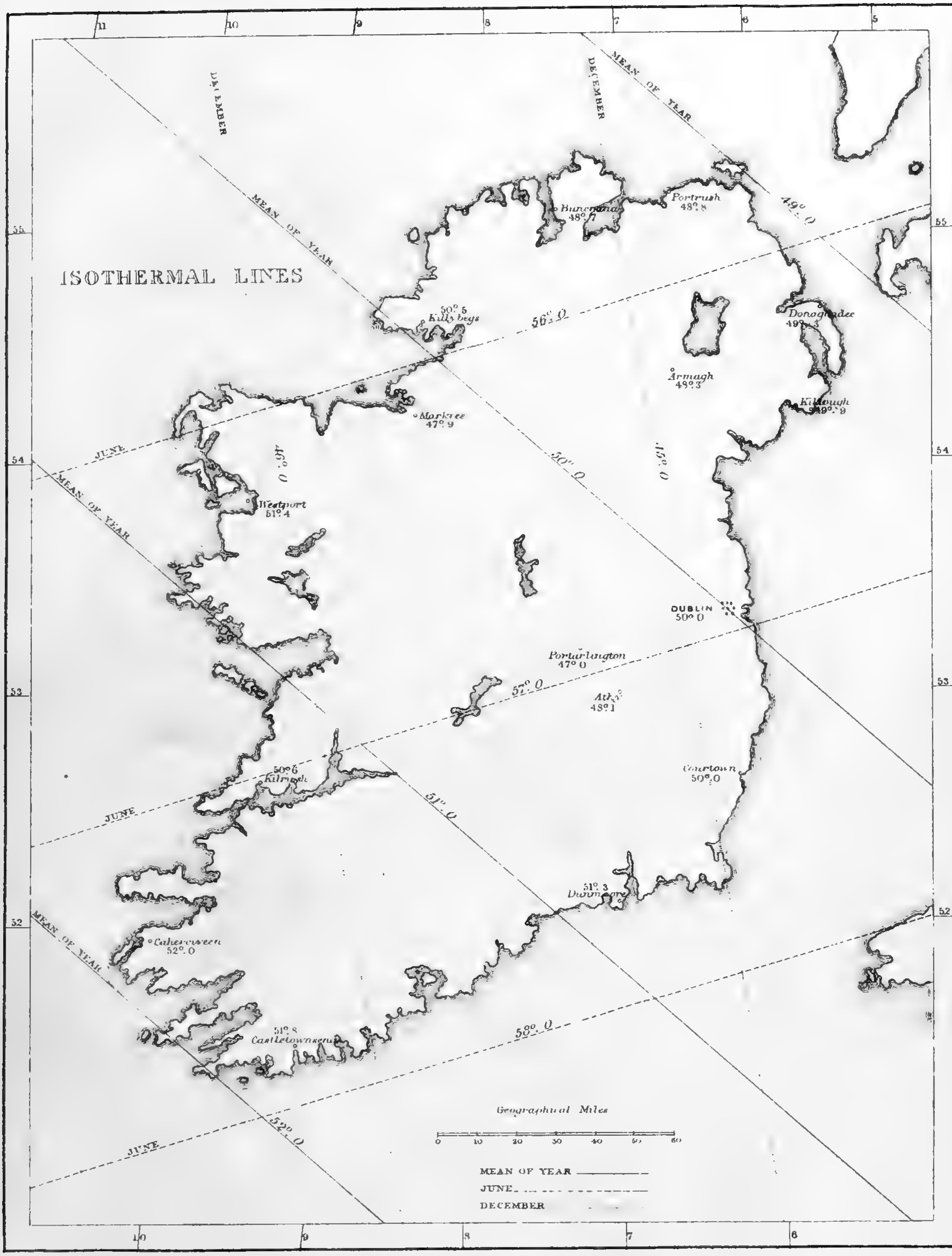
349. Calothrix *cæspitula*, Harv.? Parasitical on *Algæ*, in tide-pools at Cape Riche (299). This requires to be compared with the European plant, to which, if not the same, it is closely related.

350. Calothrix *limbata*, MS. Littoral rocks, Rottnest (277).

351. Calothrix sp. Cape Riche (334).

352. Calothrix sp. Cape Riche (335). I cannot at present identify these species; and have besides two others, collected in smaller quantity.

At Sea, *September* 4, 1854.


ISOTHERMAL LINES
DECEMBER
DECEMBER
MEAN OF YEAR
MEAN OF YEAR
MEAN OF YEAR
MEAN OF YEAR
JUNE
JUNE
JUNE
Portrush 48°.8
Donaghadee
Armagh 49°.3
Killough
Markree 47°.9
Westport 51°.4
DUBLIN 50°.0
Portarlington 47°.0
Athy 48°.1
Courtown 50°.0
Kilrush 50°.6
Cahersiveen 52°.0
Dunmore 51°.3
Castletownsend 51°.8
Killybegs 50°.5
56°.0
57°.0
58°.0
49°.0
50°.0
51°.0
52°.0
Geographical Miles
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MEAN OF YEAR
JUNE
DECEMBER

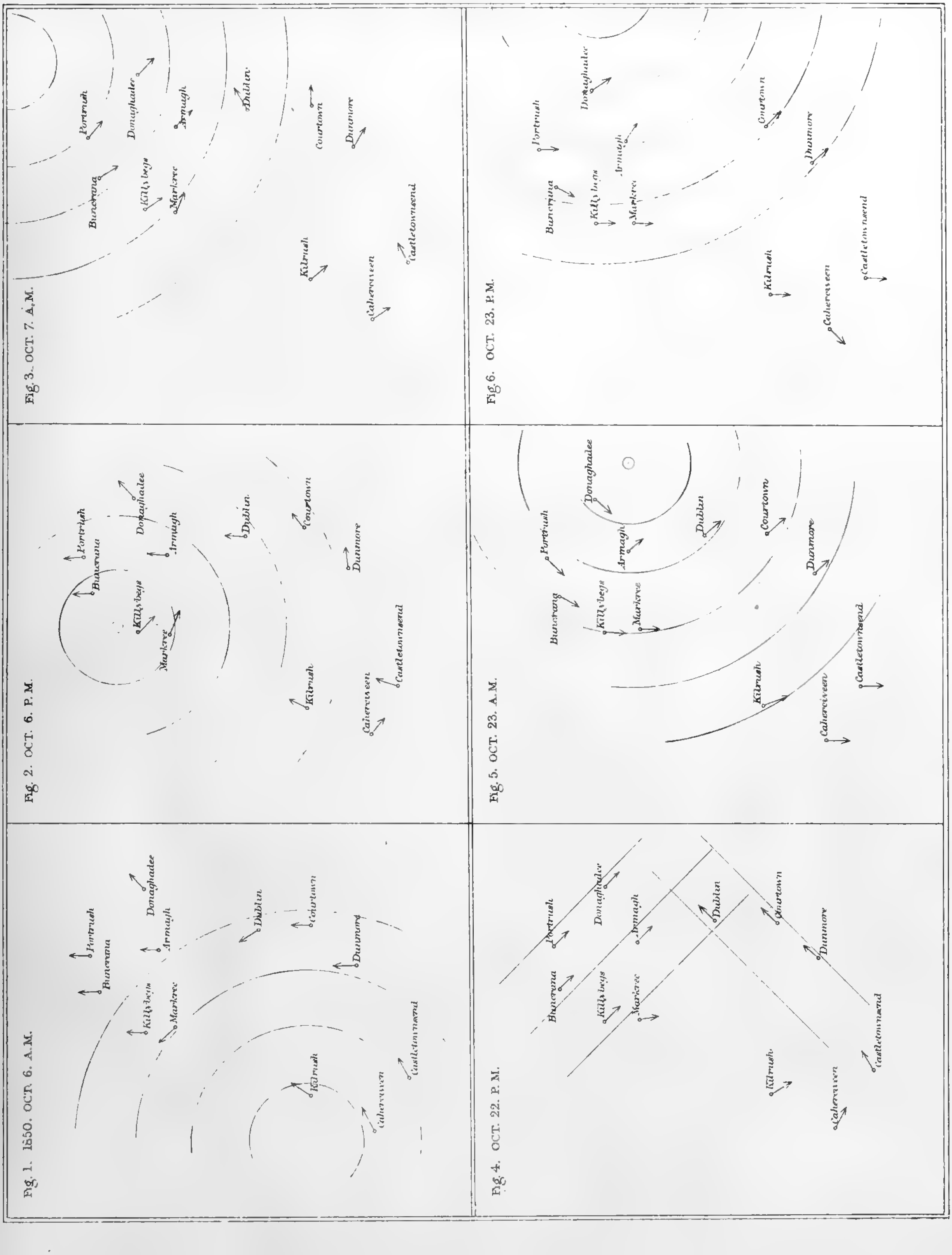


Fig. 1. 1850. OCT. 6. A.M.

Fig. 2. OCT. 6. P.M.

Fig. 3. OCT. 7. A.M.

Fig. 4. OCT. 22. P.M.

Fig. 5. OCT. 23. A.M.

Fig. 6. OCT. 23. P.M.

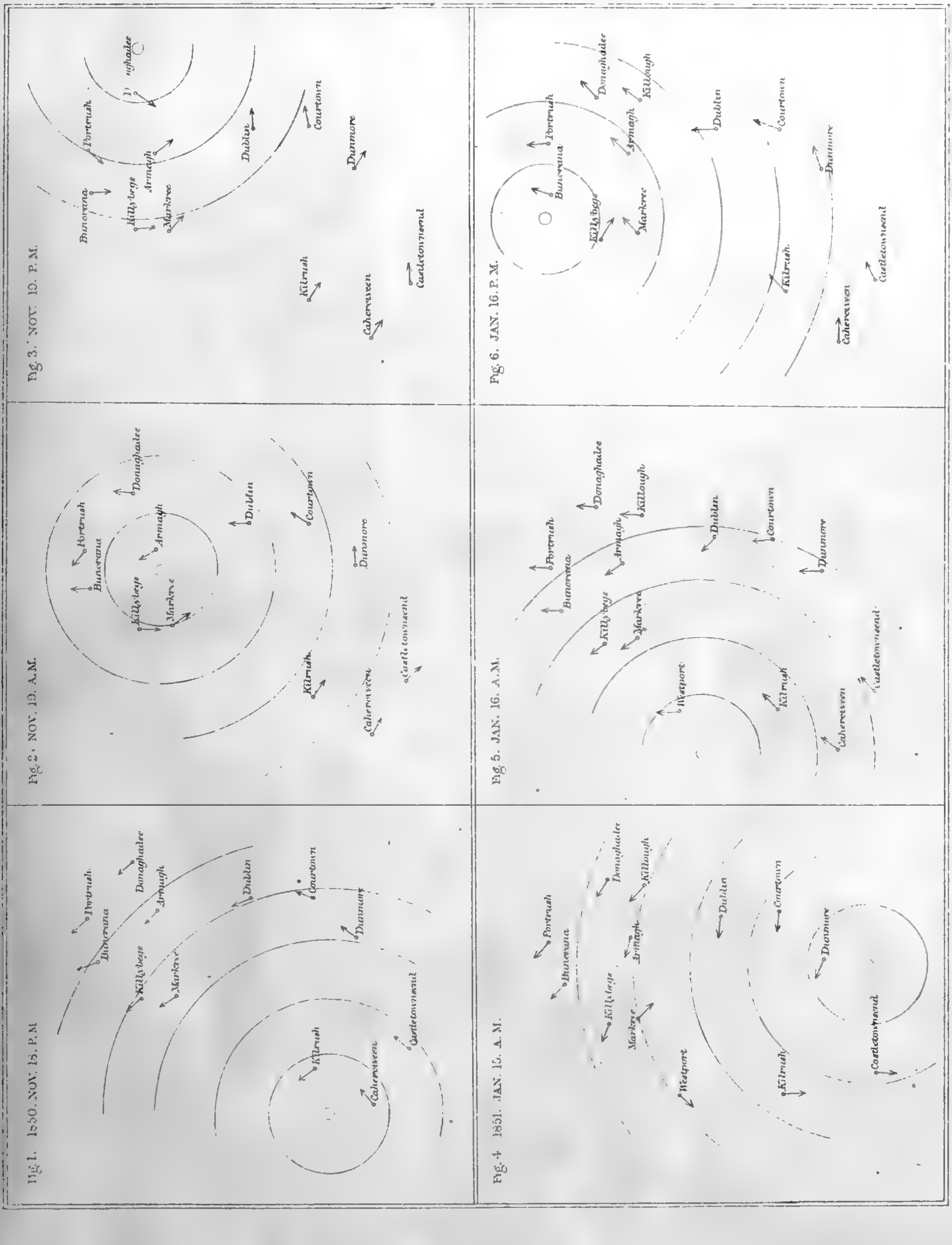


Fig. 1. 1850. NOV. 15. P.M.
Portrush
Bunerana
Donaghadee
Armagh
Killybegs
Markree
Dublin
Courtown
Dunmore
Kilrush
Castletownsend
Caherciveen
Fig. 2. NOV. 19. A.M.
Portrush
Bunerana
Donaghadee
Armagh
Killybegs
Markree
Dublin
Courtown
Dunmore
Kilrush
Castletownsend
Caherciveen
Fig. 3. NOV. 19. P.M.
Portrush
Bunerana
Armagh
Killybegs
Markree
Dublin
Courtown
Dunmore
Kilrush
Caherciveen
Castletownsend
Fig. 4. 1851. JAN. 15. A.M.
Portrush
Bunerana
Donaghadee
Killough
Armagh
Killybegs
Markree
Dublin
Courtown
Dunmore
Westport
Kilrush
Castletownsend
Fig. 5. JAN. 16. A.M.
Portrush
Bunerana
Donaghadee
Killough
Armagh
Killybegs
Markree
Dublin
Courtown
Dunmore
Westport
Kilrush
Caherciveen
Castletownsend
Fig. 6. JAN. 16. P.M.
Portrush
Bunerana
Donaghadee
Killough
Armagh
Killybegs
Markree
Dublin
Courtown
Dunmore
Kilrush
Caherciveen
Castletownsend

Fig. 1. 1851. SEP. 29. A.M.

Fig. 2. SEP. 29. P.M.

Fig. 3. SEP. 30. A.M.

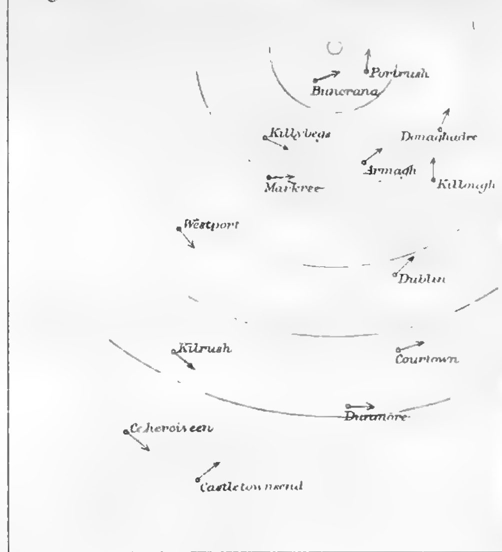
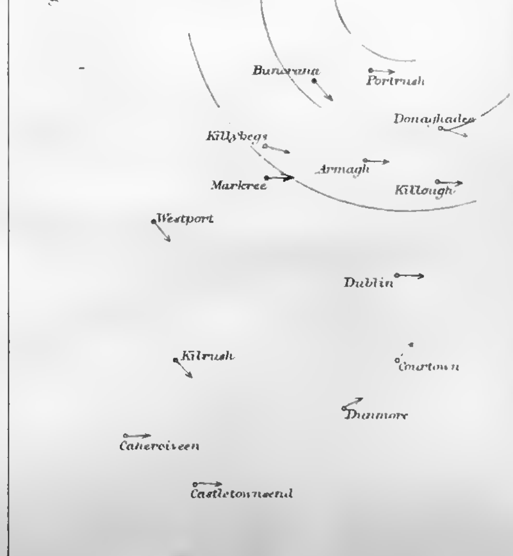


Fig. 4. OCT. 4. A.M.

Fig. 5. OCT. 4. P.M.

Fig. 6. OCT. 5. A.M.



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